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HYDRAULICS AND ITS APPLICATIONS

HYDRAULICS AND ITS APPLICATIONS

BY

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PREFACE TO FIRST EDITION

WERE water a perfectly non-viscous, inelastic fluid, whose particles, when in motion, always followed sensibly parallel paths, Hydraulics would be one of the most exact of the sciences.

But water satisfies none of these conditions, and the result is that in the majority of cases brought before the engineer, motions and forces of such complexity are introduced as baffle all attempts at a rigorous solution.

This being so, the best that can be done is to discuss each phenomenon on the assumption that the fluid in motion is perfect, and to modify the results so obtained until they fit the results of experiment, by the introduction of some empirical constant which shall involve the effect of every disregarded factor.

It is worth while here impressing on the student of the science that, apart from these experimentally-deduced constants, his theoretical results are, at the best, only approximations to the truth, and may, if care be not taken in their interpretation, be actually misleading.

On the other hand, it may be well to answer the criticism of those who would cavil at such theoretical treatment, by pointing out that the results so obtained provide the only rational framework on which to erect the more complete structure of hydraulics.

In the following pages an attempt has been made to consider the science, and its application to the design of Hydraulic Machinery, in a manner suitable for a student who has some initial knowledge of mechanics.

While written primarily with the needs of a student in view, it is, however, hoped that the book may prove of value to such as are actively engaged in the practice and profession of Hydraulic Engineering. Although it has not been attempted to elude the largely imaginary difficulties of a mathematical treatment involving some knowledge of the Differential and Integral Calculus, the knowledge of this subject which is necessary for a thorough grasp of the greater part of the book is very slight.

Where, as in some few paragraphs, a somewhat more extended

mathematical knowledge is required, the work is such as may safely be left by all but the more advanced student of the subject.

In the section devoted to Hydraulic Machinery, it has not been attempted to deal in any way exhaustively with the subject, and only such machines have been illustrated or described as are typical of their class, represent good modern design, and illustrate some definite principle of construction. For many of these illustrations the Author is indebted to the manufacturers, and while reference has been made to these in the text, he would take this opportunity of thanking them collectively for the help which they have so courteously tendered.

As was essential, reference has been freely made to the minutes of the proceedings of the various English and American societies, and to the English and German technical press, as well as to standard works on the subject. Of these the Author is particularly indebted to the Councils of the Institution of Civil Engineers, the Institution of Mechanical Engineers, the American Society of Civil Engineers, the *Zeitschrift des Vereins deutscher Ingenieure*, *The Engineer*, and *Engineering*.

The greatest debt of all is, however, owing to the teachings and published papers of his old professor and some time chief, Osborne Reynolds. Old students of the Professor will readily recognise to what extent any slight merit which the book may possess is due, directly or indirectly, to the influence of one to whom the science of Hydraulics owes so much.

In conclusion the Author would tender his thanks to Mr. S. Chapman, to Mr. E. Magson, and to Mr. C. H. Lander and Mr. F. Pickford, of the Manchester University, each of whom has revised a portion of the proofs and to whose kindly criticism and suggestion the book owes much.

A. H. GIBSON.

MANCHESTER,
February, 1908.

PREFACE TO FIFTH EDITION

RELEASE from other professional duties has at last enabled the author to revise "Hydraulics and its Applications." Some obsolete matter has been cut out, and new material added, and it is hoped that it now gives a fairly up-to-date picture of its subject.

Of recent years the subject of Fluid Mechanics has been much to the fore and many excellent text-books are available dealing with this aspect of fluid motion. Some hundreds of papers have been published on the subject of turbulence alone, and while the author is full of admiration for the assiduity with which the problem has been attacked and for the high degree of mathematical ability shown by its exponents, from the point of view of the professional engineer who is concerned with the behaviour of water in pipes and open channels, the results, so far, seem small in comparison with the expenditure of energy.

Reference has been made in the present edition to the work of Prandtl, von Karman, Blasius and their followers, but the book is more definitely meant for the student of "Hydraulics" as opposed to "Fluid Mechanics."

It is hoped that it may help him to obtain a working solution to many of the problems which may afterwards face him in his professional career.

During the last twenty years the author in collaboration with his colleague J. Allen—now Professor in the University of Aberdeen—has been actively engaged in the construction and operation of many scale models of rivers, tidal estuaries, and of hydraulic structures such as weirs and siphon spillways. A chapter dealing with the basic principles of such work has been added to the present edition. The engineer actively concerned in this type of investigation would be well advised to consult Professor Allen's recent book "Scale Models in Hydraulic Engineering" (Longmans, Green & Co.), which treats the whole subject in much detail.

A. H. GIBSON.

MANCHESTER,
January, 1951.

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HYDRAULICS AND ITS APPLICATIONS

SECTION I.

CHAPTER I.

Introductory—Historical *Résumé*—Physical Properties of Water—Density—Compressibility—Cohesion—Adhesion—Capillarity—Surface Tension—Viscosity.

Introductory—Physical Properties of Water.

Art. 1.—Hydromechanics.

The science which deals with liquids at rest or in motion may be divided into two branches: **Hydrostatics**, which deals with the equilibrium of liquids at rest; and **Hydrodynamics**, which deals with the problems connected with their motion. The term **Hydraulics** is usually broadly applied to that portion of the latter branch which deals with the motion of water in so far as this is of importance in the problems brought directly under the notice of the engineer.

A knowledge of the fundamentals of Hydrostatics is, however, so essential to a thorough grasp of the principles of Hydrodynamics, and is of such direct importance to the hydraulic engineer, that a treatise on Hydraulics would not be complete without some preliminary treatment of this branch of the subject.

The origin of the science is of great antiquity, and no attempt will be made to give a detailed historical *résumé* of its growth. Some few of the principles of Hydrostatics were enunciated by Archimedes (B.C. 250), and it is a remarkable fact that for 1,800 years from this date—until the time of Stevinus, Galileo, and Torricelli—practically no further progress was made.

The construction of the elaborate series of aqueducts and of service pipes for supplying Rome with water indeed shows that the Romans possessed some knowledge of the properties of water when at rest and

when in motion in pipes and open channels, but we have no record that this knowledge was based on any quantitative laws.

A treatise by Stevinus, written about 1585, would appear to follow historically that of Archimedes. In this the method of obtaining the pressure of a liquid on the sides and base of a containing vessel was first demonstrated.

Galileo, in a treatise published in 1612, discussed the Hydrostatic Paradox and also the flotation of bodies in water.

Shortly afterwards Torricelli made an important investigation into the behaviour of a jet when issuing vertically from an orifice, while, since the middle of the seventeenth century, numerous investigators have been at work deducing by experimental observation and theoretical reasoning the laws governing the various manners of motion of liquids, and applying these laws to the development of the science of Hydraulics. Of these experimentalists perhaps Mariotte, Bernoulli, and D'Alembert, with Poiseuille, Darcy, and Bazin in France; Rankine, Froude, Osborne Reynolds and James Thomson in England; Eytelwein, Weisbach, and Hagen in Germany; Venturi in Italy, with Francis and Hamilton Smith in America, are most worthy of note.

In spite, however, of all the work which has been so ably accomplished by these and other observers, Hydraulics cannot yet be classed as an exact science. The laws governing many of its phenomena are still imperfectly understood, and the difficulties—chiefly analytical—to be overcome before all the disturbing factors can be taken fully into account are very great. In such cases, experience, based on the results of experiment, forms the only safe guide. In other cases, however, the deductions of theory are found to be perfectly in accord with observed phenomena, and an attempt will be made in the course of this work to indicate to what extent our knowledge of the forces controlling any phenomenon is sufficiently accurate and comprehensive to enable theory to be an exact guide, and where, on the other hand, theory, based on insufficient data, is only useful as indicating in what direction a true solution is to be found.

Art. 2.—Physical Properties of Water.

In its pure state, water is an almost colourless, transparent, odourless liquid, and one of the best solvents to be found in Nature. Its maximum density occurs at 4° C., or 39·1° F., and under atmospheric pressure it freezes at 32° F. and boils at 212° F. The freezing point is lowered, and the boiling point raised by an increase in pressure. The specific gravity at

maximum density is unity, and the specific heat varies slightly with temperature, increasing from 1·000 at 32° F. to 1·013 at 212° F.

Weight of Water.- Authorities differ as to the precise value of the weight at maximum density, the lowest value given being about 62·379 lbs. per cubic foot and the highest 62·425 lbs. The latter value - that of Rankine—is commonly adopted as being most nearly correct. The following table, calculated from Rankine's formula, gives the weight per

Temp. Fahr.	Weight per cubic foot.	Temp. Fahr.	Weight per cubic foot.	Temp. Fahr.	Weight per cubic foot.	Temp. Fahr.	Weight per cubic foot.
32°	62·42	80°	62·23	130°	61·56	180°	60·55
40°	62·42	90°	62·13	140°	61·37	190°	60·32
50°	62·41	100°	62·02	150°	61·18	200°	60·07
60°	62·37	110°	61·89	160°	60·98	210°	59·82
70°	62·31	120°	61·74	170°	60·77		

cubic foot at different temperatures. For the purpose of all calculations relating to Hydraulics it is sufficiently accurate to take the weight per cubic foot at 62·4 lbs., more especially as the water with which the engineer has to deal is never perfectly pure, but contains more or less of the soluble salts. Unless otherwise stated, the above value will be adopted throughout this treatise.

The density of sea water varies slightly with the locality, but is about 1·026 times that of fresh water. Its weight, at the temperatures commonly met with in practice, may be taken as 64·0 lbs. per cubic foot.

Compressibility.—Water is very slightly compressible, the compressibility varying with the temperature, the pressure, and with the amount of air in solution. If δV be the change in volume corresponding to an increment in pressure δP, the values of the bulk modulus K, which equals $-\frac{\delta P}{\frac{\delta V}{V}}$, as obtained by different observers are as follows. These values are in lbs. per square inch.

The mean value may be taken in round numbers as 300,000 lbs. per square inch or 43,200,000 lbs. per square foot.

The compressibility is so slight that in all practical calculations con-

Authority.	Temp. °F.							
	32°	40°	50°	60°	70°	80°	90°	
Landholt & Bornstein.[1]	284,000	293,000	303,000	312,000	319,000	328,000	335,000	At low pressures.
Tait[2]	283,000	291,000	301,000	311,000	320,000	328,000	336,000	At low pressures.
	292,000	300,000	311,000	321,000	329,000	336,000	342,000	At 1 ton sq. in.
	300,000	310,000	320,000	328,000	336,000	342,000	346,000	At 2 tons sq. in.

cerning water at rest or in a state of steady motion it may be assumed to be an incompressible fluid. In certain important phenomena, however, notably those involving a sudden initiation or stoppage of motion, this compressibility becomes an important, and often the predominating factor, and in the treatment of such cases the above mean value of K will be adopted.

In such cases it should be noted that it is the ratio of the bulk modulus to the density of a fluid which forms the true criterion of its compressibility or elasticity, as it is this ratio which governs the wave propagation on which such phenomena depend. In this respect air is only about eighteen times as compressible as water.

At ordinary temperatures and pressures, water is capable of dissolving about 2 per cent. of its own volume of air. As the temperature is raised part of this is liberated. The volume in a saturated solution under a pressure of one atmosphere is approximately as follows[3] :—

Temp. F.°	32°	40°	60°	90°	120°	150°
Percentage volume of air in solution	2·88	2·56	2·10	1·78	1·62	1·49

At a given temperature the volume of gas absorbed is sensibly independent of the pressure, so that the weight absorbed is proportional to the pressure. Any reduction in pressure tends to liberate part of the air, and the hissing which is so often noticeable in jet pumps and injectors, or

[1] From Landholt and Bornstein, Grassi, "Annales de Chimie et Physique," 1851, vol. 31, p. 437, gives results which are in close agreement with these.

[2] "Math. and Phys. Papers," Sir Wm. Thompson, vol. iii., 1890, p. 517.

[3] A. Winkler, "Zeits. f. Phys. Chemie," 1892, vol. 9, p. 171, and Bohn and Bock, "Wied Ann.," 1891, vol. 48, p. 319

where water is escaping at high velocities past the restricted area of a valve seat, is due to the reduction of pressure and subsequent liberation of bubbles of air, which occurs under these circumstances. The trouble which is experienced in keeping the air vessels on the delivery side of a pump charged is due to the facility with which water dissolves gases under high pressures.

Ice Formation.—In still water, under normal conditions, when ice is formed this exists as a surface layer. In very cold climates, in addition to existing in the ordinary surface form, ice is found in the form of anchor ice and of frazil. **Anchor ice** consists of an agglomerated mass of coarse crystals clinging to the bed of the channel and formed there by loss of heat by radiation, the cooling of the bottom thus produced causing ice to form. This can only occur with a clear sky and where the surface ice—if any—is transparent. Surface ice will in general effectually prevent its formation. It is usually found in rapid streams, where surface ice cannot form.

Frazil consists of fine spicular ice crystals floating in the water and formed by slight supercooling below 32° (probably about ·001° being sufficient). This adheres to the surface ice and to the anchor ice, and may finally choke up the stream. In engineering work frazil ice is particularly objectionable, as it adheres to the racks and strainers and to the gates of a turbine, and if, due to exposure to air, these are slightly supercooled, freezes into one solid mass.

In common with all other liquids, water also possesses the properties of **Cohesion, Adhesion,** *and* **Viscosity.**

Art. 3.—Cohesion

is that property of a liquid, or solid, which enables neighbouring molecules to resist any stress of the nature of a tension. **Adhesion** is that property which enables it to adhere to a solid body with which it may be in contact. Thus a drop of water exhibits cohesion in its hanging together, and adhesion in virtue of its clinging to a solid body, the force of gravity being overcome both by cohesion and by adhesion. Both phenomena are due to molecular attraction, cohesion between neighbouring molecules of the liquid, and adhesion between those of the solid and liquid.

The phenomenon known as Capillarity, which appears in the tendency of a liquid to rise or fall from its normal level in a tube of fine bore

according as the liquid does or does not wet the tube, would appear to be due to cohesion and adhesion; while it is probable that cohesion between the surface molecules of a liquid gives rise to **Surface Tension**, the property of a liquid which gives it the appearance of having an elastic skin at its surface of separation from a gas, or from any other liquid. Experiment shows that the tension in any such surface film is everywhere the same, and that the tension across any imaginary line in the surface is normal to that line. Also the tension is independent of the curvature of the surface; decreases with an increase in temperature; is constant for a given temperature, for the surface of separation of any particular liquid and gas, or of any two given liquids; and for any given liquid and solid in the presence of air, the angle of inclination, α, of the surfaces is constant.

Thus, for water and glass at a temp. of 68° F. . . $\alpha = 25° 32'$
for mercury and glass . . . . . $\alpha = 128° 52'$

while the surface tension, expressed in pounds per foot run of the line of contact, and at the above temperature, has the following values (from Quincke's experiments):—

For water and air . . . $T = \cdot 005548$ lbs. per foot
For mercury and air . . . $T = \cdot 03698$ „

No satisfactory explanation of the precise nature of surface tension has been formulated. Beyond doubt, however, it is a molecular phenomenon, and is inseparable from that of cohesion, the intensity of the one probably determining that of the other.

Many theories have been educed to explain the phenomena of capillarity. Of these, the two which have been most generally accepted are due respectively to Poisson and Young. Briefly outlined, the former assumes that when a liquid wets a plate the attraction between the layer of molecules in intimate contact with the plate and the plate itself is greater than the intermolecular attraction of the liquid. The molecular motion of the molecules forming the surface film is thus reduced, while in consequence of their greater freedom, that of the molecules immediately distant from this film is increased. The density of this secondary layer of fluid is thus reduced below the normal, and the resultant upward pressure of the surrounding fluid causes it to rise up the plate until a state of statical equilibrium is attained between the cohesive forces and the action of gravity on the supported fluid.

Where a liquid does not wet the plate, the density of this secondary film is increased by the molecular action between the plate and fluid, with a consequent fall in the surface level in the immediate neighbour-

hood of the plate. That this variation of density actually occurs has been indirectly proved in other ways, while results obtained by the application of this theory are amply confirmed by experiment.

The second theory attributes the phenomena to the action of a series of surface tensions, which are assumed to exist at every surface of contact of any liquid or gas with any solid, and also at the surface of contact of any liquid with any other liquid, or with a gas. Thus at the common line of intersection of a solid, liquid, and gas, *i.e.*, at the line passing through P (Fig. 1) and perpendicular to the plane of the paper, three surface tensions, of intensity T_{as}, T_{al}, and T_{ls} per unit length of this line, are in existence. For equilibrium then, their directions and magnitudes will be

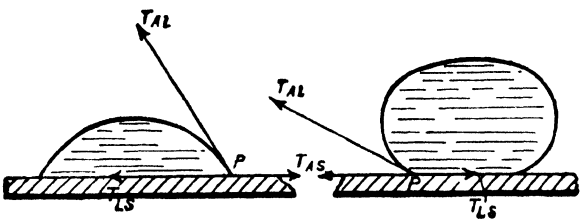


FIG. 1.

related according to the ordinary laws of statical equilibrium. Thus for contact with a plane surface we have—

$$T'_{as} = T'_{ls} + T'_{al} \cos a,$$

and the angle a will be acute or obtuse according as $T'_{as} - T_{ls}$ is positive or negative.

If $T_{as} = T_{ls} + T_{al}$, $a = 0$ and for this, and all greater relative values of T_{as}, the fluid will immediately spread to cover the surface, the effect being as though the liquid were pulled outwards in every direction by the tension T_{as}, the resultant of the tensions T_{ls} and T_{al} being insufficient to resist this motion. On the other hand if $T_{as} - T_{ls}$ is negative, cos a is negative and the angle a is obtuse. If $T_{ls} - T_{as} = T_{al}$, $a = 180°$, and the liquid, if in sufficiently small masses, assumes the spherical state.

On these assumptions, justified in so far as the results obtained by their application go, the various phenomena of capillary action easily lend themselves to mathematical treatment.

Rise of Liquid in a Capillary Tube.—Let h be the height of the liquid in the tube, of diameter d feet, and let w be the weight of unit volume of the liquid. Here h is measured to a horizontal plane tangential to the

curved surface at the top of the column (Fig. 2). Let V = the (small) volume of liquid above this plane.

The height of the column when in a state of equilibrium may be deduced from the principle of virtual work,[1] *i.e.*, by equating the work done against gravity in any small vertical displacement of the column, to that done by the surface tensions.

If δx is this displacement we have, for equilibrium

$$(T_{ls} - T_{as})\,\pi d\,\delta x + w\,\delta x\left\{\frac{\pi d^2 h}{4} + V\right\} = 0$$

$$\therefore h = \frac{4}{w d}(T_{as} - T_{ls}) - \frac{4V}{\pi d^2}$$

But $T_{as} - T_{ls} = T_{al}\cos a$

$$\therefore h = \frac{4\,T_{al}\cos a}{w d} - \frac{4V}{\pi d^2} \qquad (1)$$

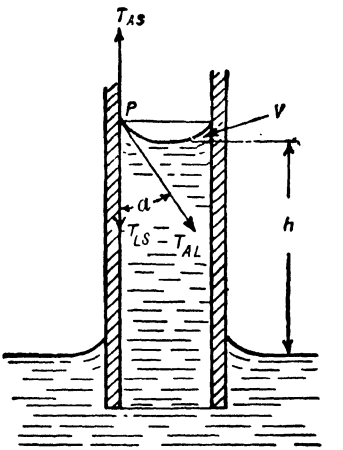


FIG. 2.

In general V is very small, so that without sensible error (1) may be written

$$h = \frac{4\,T\cos a}{w d} \qquad (2)$$

Similarly it may be shown that the vertical rise or fall between two parallel plates at a distance d apart $= \dfrac{2\,T\cos a}{w d}$ (3)

It is with the former case of capillary action that we are chiefly concerned in hydraulics, as affecting the accuracy of measurements of pressure in a liquid, when these depend upon the height of a supported column of the liquid. Thus with a piezometer, in which pressure is measured by means of a water column, the artificial elevation of the pressure column by capillary action at about 68° F. is given by

$$h = \frac{4 \times \cdot 005548 \cos 25°\,32'}{62{\cdot}4\,d} \text{ feet.}$$

[1] More simply, it may be considered that the whole weight of the supported column is carried by the surface films, and that this weight is equal to the vertical component of the surface tension. This leads, as before, to the equation

$$w d \,.\, T\cos a = w\frac{\pi d^2}{4} h,$$

$$\therefore h = \frac{4\,T\cos a}{w d}$$

With liquid in its spherical state we have $2\pi r\,T = \pi r^2 p$, or $p = \dfrac{2T}{r}$, giving p, the excess of internal over external pressure. Thus with small values of r a comparatively small value of T may be accompanied by a large value of p.

If d be measured in inches, this artificial elevation in inches is given by

$$h = \frac{\cdot 04624}{d} \text{ inches.}$$

In the case of a mercury column, taking the specific gravity of mercury to be 13·596, we have $\cos a = -\cdot 6276$

$$\therefore h = -\frac{4 \times \cdot 03698 \times \cdot 6276 \times 144}{62\cdot 4 \times 13\cdot 596\, d} \text{ inches}$$

$$= -\frac{\cdot 01576}{d} \text{ inches}$$

where d is measured in inches.

The negative sign here indicates that the surface of the mercury column is depressed below the level corresponding to the statical pressure alone. The following table gives values of h at 68° F. corresponding to various tube diameters.

DIAMETER OF TUBE IN INCHES.

		·02	·04	·06	·08	·10	·125	·15	·20	·25	·30	·50
h in inches	Water in glass tube . .	2·312	1·156	·771	·578	·462	·370	·308	·231	·185	·154	·092
	Mercury in glass tube .	·788	·394	·263	·197	·158	·126	·105	·079	·063	·053	·032

Evidently then, tubes of small bore are not advisable for use in such pressure-measuring apparatus, especially where small pressures are to be measured, except where the difference of height in two similar tubes is used to indicate the pressure. Here, so long as both tubes are clean, the effects of capillarity, being the same for each tube, neutralise each other. Again, since the specific gravity of mercury is 13·596, the proportional error introduced by neglecting capillarity in the case of a mercury column will be $\frac{13\cdot 596 \times \cdot 01576}{\cdot 04624} = 4\cdot 63$ times that introduced with a water column, and will be in the opposite direction.

ART. 4.—VISCOSITY.

Every known fluid offers a resistance, analogous to friction, to the relative sliding motion of any two adjacent layers, and the physical

property of the fluid to which this is due is termed **Viscosity.** This property—only noticeable when the fluid is in motion—is the cause of all so-called fluid friction and gives the fluid the appearance of being able to withstand a shear stress between adjacent layers.

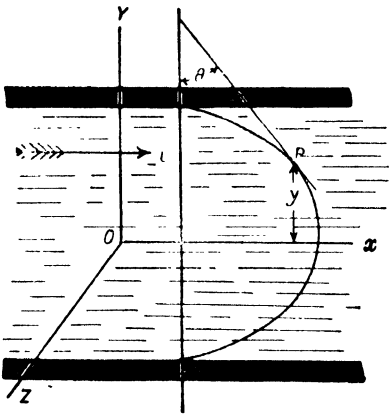


FIG. 3.

The magnitude of this shear or distortive stress over any plane is proportional to the rate of distortion, and hence to the rate of change of velocity with space perpendicular to the plane. Thus if co-ordinate axes OX, OY, OZ (Fig. 3) be taken at some point O of a stream, and if f_s be the distortive stress accompanying relative motion of adjacent layers in a direction parallel to the axis of x, the velocity in this direction at any point P, distant y from that axis, being denoted by v, and if the velocity in the direction of OZ is zero, we have, at the point P

$$f_s \propto \frac{dv}{dy} \tag{1}$$

If a curve, having ordinates representing values of v, be plotted on a base parallel to OY, and if θ be the angle which the tangent to the curve at P makes with this base line, the values which f_s adopts as y varies will be represented to scale by the corresponding values of tan θ.

Putting $f_s = \mu \frac{dv}{dy}$, the coefficient μ is termed the **Coefficient of Viscosity.**[1]

The value of this coefficient varies with different fluids, and varies with temperature for any particular fluid.

For water, experiments by Poiseuille indicate that it follows the law

$$\mu = \frac{\cdot 00003716}{\cdot 4712 + \cdot 01435\, T + \cdot 0000682\, T^2}$$

[1] Expressed dimensionally we have $\mu = f_s \div \frac{dv}{dy}$ = stress ÷ $\frac{\text{vel.}}{\text{space}}$ = stress × time

But stress = $\frac{\text{force}}{\text{area}} = \frac{\text{mass} \times \text{acceleration}}{\text{area}} = \frac{M \times V}{L^2 \times T} = \frac{M}{LT^2}$.

$\therefore$ μ is of dimension $\frac{M}{LT^2} \div \frac{1}{T} = \frac{M}{LT}$.

where T is in degrees F., and the corresponding value of the distortive stress f_s is given in pounds per square foot.

With the pound, foot, and degree Centigrade as units,

$$\mu = \frac{\cdot 00003716}{1 + \cdot 03368\, T + \cdot 000221\, T^2}$$

In C. G. S. units, the unit of force being taken as the dyne, and the temperature in degrees Centigrade, we have

$$\mu = \frac{\cdot 0179}{1 + \cdot 03368\, T + \cdot 000221\, T^2}.$$

The following table indicates the values of μ for water corresponding to different temperatures, f_s being expressed in pounds per square foot, and in C. G. S. units.

Temperature.		μ.	μ.
Fahrenheit.	Centigrade.	ft. lb. units.	C. G. S. units.
32°	0°	·00003716	·01793
50°	10°	·00002735	·01310
68°	20°	·00002109	·01016
86°	30°	·00001685	·00810
104°	40°	·00001376	·00663
122°	50°	·00001150	·00554
140°	60°	·00000971	·00469

Experiments indicate that above 32° F. the viscosity of water, in addition to varying with temperature, also increases very slightly with an increase in pressure. It may be noted that whereas with liquids the viscosity decreases with an increase in temperature, with gases the reverse is the case. Researches by Dr. Grindley and the author show that in the case of air the viscosity is independent of the pressure, and, if temperatures are measured in degrees Centigrade, increases with the temperature according to the law $\mu = \{355{\cdot}3 + 1{\cdot}168\, T' - \cdot 00248\, T^2\} \times 10^{-9}$ ft. lb. sec. units, or approximately

$$\mu = \{358 + \cdot 932\, T\} \times 10^{-9} \text{ ft. lb. sec. units}$$

between the limits of temperature 0° C. and 100° C.[1]

This is as might be inferred from the kinetic theory of fluids, for layers of fluid moving with different velocities are continually interchanging molecules by diffusion. Thus the more rapidly moving layers are continually losing momentum by interchange with the slower layers, and a

[1] "Proc. Roy. Soc.," vol. 80 A., 1908, p. 114.

continually applied force is necessary to maintain this state of motion. Since the diffusivity increases with an increase of temperature both in liquids and gases, it would be expected that in both cases the viscosity would increase with temperature.

It is, however, extremely probable that cohesion plays a great part in producing viscosity in liquids, and while in gases the molecules exert forces on each other by collision only, so that here diffusivity is the important factor in producing viscosity, in liquids the greater cohesion at lower temperatures more than counterbalances the diminished diffusivity and hence increases the viscosity.

Since the interchange of molecules will be proportional to the area over which such interchange takes place, the resistance to distortion will be proportional to this area. Evidently then if eddies are formed in the course of a stream of fluid, since the area over which interchange of momentum may take place is greatly increased, the viscous resistance to motion will also be increased.

Viscosity, being a physical property of a fluid, is independent of the velocity of translation of its particles, and where motion takes place in parallel straight lines, the resistance to motion can be directly inferred from a knowledge of the viscosity. If eddies are formed, however, the motions and forces involved are of such complexity as to prevent any attempt at a solution from *à priori* reasoning based solely on a knowledge of the physical properties of the fluid.

Although from the definition of viscosity, the stresses in different fluids under similar circumstances of motion will be proportional to their values of μ, yet in comparing the effects of viscosity in modifying such motions it is the ratio of these stresses to the inertia of the fluid which must be taken into account. Thus the determining factor is $\mu \div \rho$. (Cf. p. 71.) This ratio is known as the " kinematic viscosity " and is usually denoted by the symbol ν.

The following are values of ν for water in C. G. S. units.[1] and in foot-pound second units.

Temp. °C.	0°	5°	10°	15°	20°	30°	40°
ν C. G. S.	·0179	·01522	·01311	·0114	01008	·0080	·00662
ν ft.-lb.-sec.	$1{\cdot}9 \times 10^{-5}$	$1{\cdot}64 \times 10^{-5}$	$1{\cdot}41 \times 10^{-5}$	$1{\cdot}23 \times 10^{-5}$	$1{\cdot}08 \times 10^{-5}$	$0{\cdot}865 \times 10^{-5}$	$0{\cdot}713 \times 10^{-5}$

From this point of view air is a much more viscous fluid than water, as will be seen from the following figures.

[1] Hosking, R., " Phil. Mag.," 1909.

		32° F.	60° F.	100° F.
μ ft. lb. sec. units	air	$35{\cdot}5 \times 10^{-6}$	$37{\cdot}2 \times 10^{-6}$	$39{\cdot}6 \times 10^{-6}$
	water	$37{\cdot}2 \times 10^{-6}$	$23{\cdot}0 \times 10^{-6}$	$14{\cdot}0 \times 10^{-6}$
ρ *i.e.*, weight per cub. ft. $\div g$.	air	$25{\cdot}1 \times 10^{-4}$	$23{\cdot}8 \times 10^{-4}$	$22{\cdot}1 \times 10^{-6}$
	water	1·938	1·936	1·925
$\frac{\mu}{\rho}$	air	$14{\cdot}15 \times 10^{-5}$	$15{\cdot}61 \times 10^{-5}$	$17{\cdot}91 \times 10^{-5}$
	water	$1{\cdot}92 \times 10^{-5}$	$1{\cdot}19 \times 10^{-5}$	$\cdot 727 \times 10^{-5}$
Ratio of values of $\frac{\mu}{\rho}$ . .	$\frac{\text{air}}{\text{water}}$	7·37	13·1	24·6

From these it appears that at 60° F. the relative effect of viscosity in modifying the motion is practically thirteen times as great in the case of air as in the case of water.

The energy absorbed in overcoming viscous resistance in any fluid motion finally appears in the form of heat which is dissipated by conduction and radiation.

CHAPTER II.

Hydrostatics—Principles—Pressure Intensity—Transmissibility of Pressure—Atmospheric Pressure—Pressure Gauges—Resultant Pressure—Centre of Pressure—Equilibrium of Floating Bodies—Metacentric Height—Oscillations of Ships—Strength of Pipes and Cylinders.

ART. 5.—HYDROSTATICS.

COMMONLY, any substance which, at ordinary temperatures, possesses in a marked degree the property of accommodating itself to the shape of any vessel into which it may be placed, is termed a *fluid*.

Fluids may be divided into two classes; gases and liquids; according as they are easily, or with difficulty, compressible.

Definition of a Perfect Fluid.—By a perfect fluid we mean a substance such that the pressure exerted by it on any surface with which it may be in contact is everywhere normal to that surface.

From the above definition it follows that with a perfect fluid any action of the nature of friction between solid and fluid is impossible, since this would necessitate the action of some force tangential to the surface. It follows that there can be no frictional resistance to the motion of a solid body through the fluid, or to the steady motion of the fluid through any pipe or channel having solid boundaries, and that in consequence any portion of the fluid may be separated from any other portion by a force however small, if applied for a sufficient length of time. Further, the perfect fluid is incapable of existing in a state of tension.

Such a conception is useful, although no such substance as a perfect fluid is known in Nature All known fluids, in virtue of their properties of cohesion and viscosity, offer some resistance of the nature of friction to the motion of any solid surface with which they may be in contact.

Evidently then the laws governing the behaviour of a perfect fluid when in motion, are not applicable, without some modification, to water; nor are the laws governing its statical equilibrium applicable to that of water in cases where, as when under the influence of capillary action, the effect of cohesion is great. When dealing, however, with water in bulk, impregnated as it usually is with air in solution, the effect of cohesion may be neglected, and when at rest water may be considered as satisfying the

essential condition of a perfect fluid, viz., that it exerts a normal pressure on all surfaces with which it may be in contact.

Art. 6.—Pressure at a Point.

The average pressure intensity over any area A equals the total pressure P on the area divided by the area, or equals $\frac{P}{A}$. If the pressure varies from point to point of the area, and if this be divided into a large number n of small areas δA, the pressure on any one of these containing a given point being δP, then the limiting value to which the ratio $\frac{\delta P}{\delta A}$ tends, as n is made infinitely large, is taken as the pressure intensity—or more shortly the pressure—at the point under consideration.

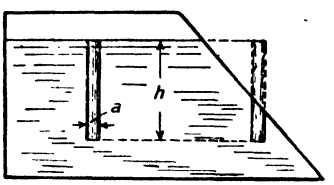


Fig. 4.

The units in which pressure intensity is measured depend on those of force and space.

In English practice the unit is usually the pound per square inch, or per square foot. In the metric system of units the usual unit is the kilogramme per square centimetre.

These units are connected by the relationship that 1 kilogramme per square cmm. = 14·223 lbs. per square inch.

Art. 7.—In a Liquid at Rest, the Pressure Intensity is everywhere the same at the same Depth, and is the same in all Directions.

The truth of the first of these propositions may be seen by considering the equilibrium of a small vertical column of the liquid of cross sectional area a and having its base at the depth h below the surface (Fig. 4). Wherever this column is taken in the liquid its weight will be the same and must be balanced by the vertical upward pressure on the base, which will therefore be the same. Since the sectional area of the column is

unaltered, it follows that the pressure intensity on the base must everywhere be the same.

If W is the weight of unit volume, or the intrinsic weight of the liquid, the weight of the column $= W\,a\,h$.

If p is the pressure intensity at a depth h, the upward pressure on the base $= p\,a$.

$$\therefore\ p\,a = W\,a\,h.$$

$$\therefore\ p = W\,h.$$

The pressure of still water against the sides or bottom of any vessel is then simply due to the "head," or height of the level of the free surface of the water above the point considered. Each square foot of the surface at a depth h may be considered as supporting a column of water of 1 square foot cross sectional area, and of height h, and therefore of weight 62·4 h lbs.

$\therefore$ Pressure per square foot, per foot of head = 62·4 lbs.

Pressure per square inch, per foot of head = ·433 lb.

$\therefore$ Head equivalent to a pressure of 1 lb. per square inch = 2·308 feet.

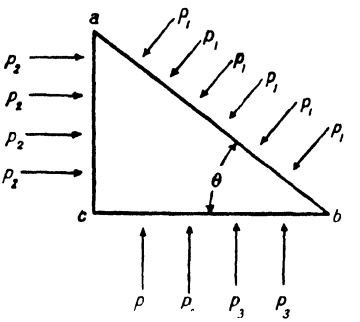


FIG. 5.

The second of the above propositions may be deduced by considering the equilibrium of a triangular prism of the liquid of unit length having its edges horizontal, and its ends perpendicular to the sides. If $a\ b\ c$ (Fig. 5) be a cross section of the prism, $a\ c\ b$ being a right angle and $b\ c$ being horizontal, and p_1, p_2, p_3 be the mean intensities of pressure on the sides $a\ b$, $a\ c$, $c\ b$, we have for equilibrium, resolving parallel to the sides $b\ c$ and $c\ a$,

$$p_2\,ac = p_1\,ab \sin\theta$$

$$p_3\,cb = p_1\,ab \cos\theta + W\frac{ac\ cb}{2}.$$

Putting $ac = ab \sin\theta$; $bc = ab\cos\theta$, we get

$$p_2 = p_1$$

$$p_3 = p_1 + \frac{W}{2}\,ac.$$

If now the sides of the prism be indefinitely diminished p_1, p_2, and p_3 become, in the limit, the pressure intensities at the same point, but in different directions. Also $a\ c$ ultimately becomes infinitely small, so that in the limit

$$p_1 = p_2 = p_3.$$

It follows from these theorems that the free surface of still water is a surface of equal pressure, and that all equipotential surfaces in the fluid are parallel to this. This surface is not plane, but is everywhere normal to the direction of the force of gravitation, and therefore to the direction of a plumb line.

Art. 8.—Transmissibility of Pressure in a Fluid.

If a pressure be applied to the surface of a fluid, this pressure is transmitted equally to all parts of the fluid. This may be seen by considering a closed vessel filled with water, and fitted with a piston of area A, to which a force P is applied, producing a pressure intensity at this point of $P \div A$. Considering any other area A of the surface, the two may be supposed connected by a cylinder of the fluid, having imaginary boundaries (Fig. 6).

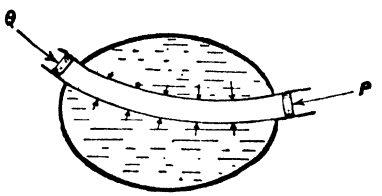


Fig. 6.

Suppose piston P to be displaced through a small distance x. Then in virtue of the incompressibility of the fluid, Q will be displaced through the same distance. Also, since the reaction of the cylinder walls is everywhere perpendicular to the direction of motion of the contained fluid, no work is done against this reaction, and in consequence the work done by P = work done on Q. Since the areas and displacements of P and Q are the same, the pressure intensities over their surfaces, introduced by the action of the external forces, must also be equal. The total pressure intensity at either P or Q will then be obtained by adding to the pressure produced by the force P, that due to the weight of the liquid. Altering the plane of the pistons at P and Q will evidently not affect the pressure intensity on either surface.

This property is taken advantage of in many hydraulic machines, notably in Bramah's Hydraulic Press, and in machines of a like type.

In the Hydraulic Press, illustrated diagrammatically in Fig. 7, water is forced by means of a small pump F, whose plunger has an area a, into the cylinder C of the press whose area is A. Neglecting friction, a force P, applied to the plunger of the force pump, will then produce a pressure intensity of $\frac{P}{a}$ in the pump and press cylinder, and hence a force of $Q = P\frac{A}{a}$ on the press plunger. A particular case of the transmissibility of

pressure is found in the transmission of the pressure of the atmosphere on the surface of water to every part of its depth. Thus the pressure intensity at any depth h is strictly that due to the weight of the superposed water, together with the atmospheric pressure.

Atmospheric Pressure.—The pressure of the atmosphere varies from day to day at the same place, and from place to place at the same time. Its mean value at sea level is about equivalent to that at the base of a mercury column 30 inches in height, or 14·7 lbs. per square inch. This is equivalent to the pressure at the base of a water column of approximately 34 feet in height, and this is usually taken as the height of the water barometer. Thus the true pressure intensity at a depth h below the free surface of water is given by 62·4 $(h + 34)$ lbs. per square foot.

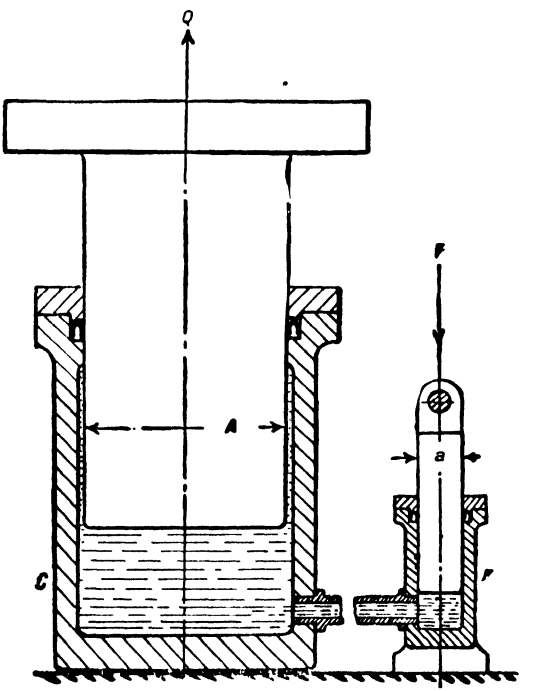


FIG. 7.—Bramah's Press.

In most hydrostatic problems, however, it is the pressure in excess of that due to the atmosphere which is required, so that the pressure at a submerged point is commonly taken as being that due to the head of water alone.

ART. 9.—PRESSURE GAUGES.

The most accurate method of measuring the difference between the pressure at any point in the length of a pipe and that of the atmosphere, is by means of a piezometer or manometer in which the difference is measured by the height of a column of liquid supported by the excess pressure. Where the difference of pressure is not more than one or two pounds per square inch, this liquid may conveniently be water, when the pressure difference is given by $h \div 2{\cdot}31$ or $\cdot 433\,h$ lbs. per square inch,

h being measured in feet. Fig. 8 A and B, shows such a gauge fitted to a pipe conveying respectively air—in A—and water—in B. 0 in each case marks the zero of the scale, and, in B, h measures the pressure at

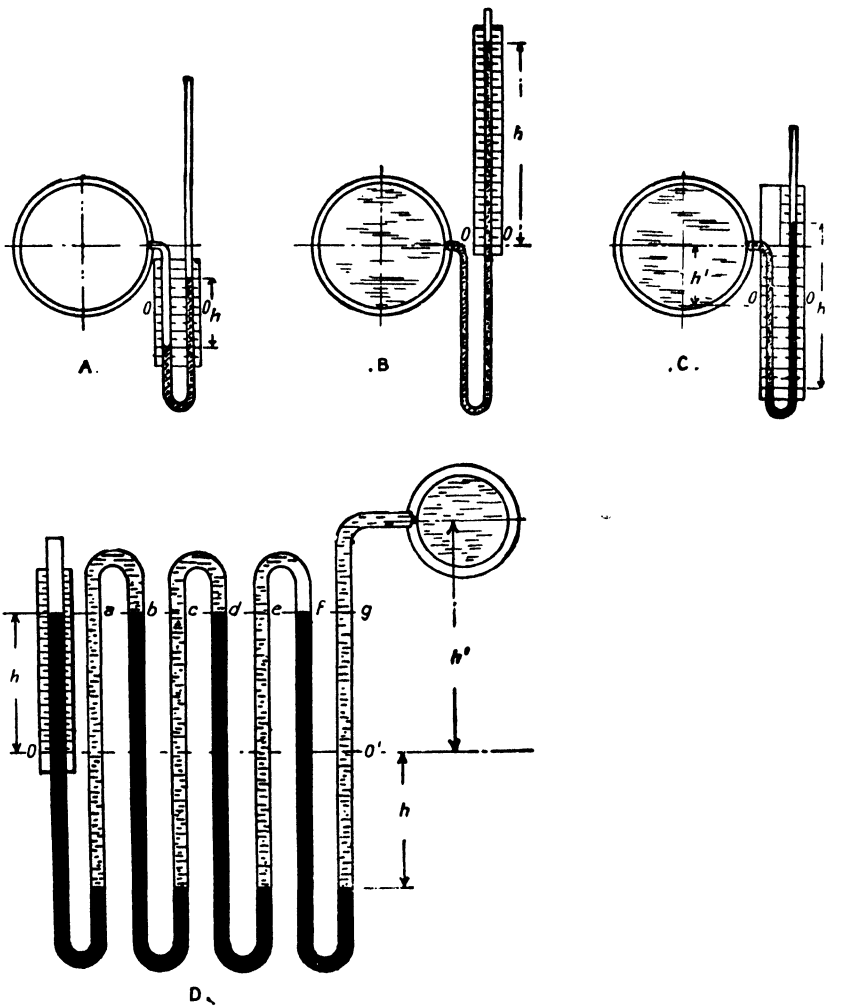


FIG. 8.

the centre of the pipe, from which the pressure at any other point may be calculated from purely statical considerations. If the pressure is so great as to give an inconveniently long column of water, mercury (specific gravity 13·596) may be used as shown in Fig. 8 C. If $O\ O$ is the common

level of the mercury columns with no water in the pipe, the pressure at the centre of the pipe under the conditions shown is that due to a column of mercury h feet high, minus that due to a column of water of height $h' + \frac{1}{2} h$ feet, and is therefore equal to $13{\cdot}596\ h - {\cdot}5h - h' = 13{\cdot}096\ h - h'$ feet of water. Where the pressure is in excess of 100 feet of water, such a gauge gives an inconveniently long mercury column and a type of gauge in which a large range of pressures may be recorded on a comparatively short scale is shown in Fig. 8 D. This consists of a series of n U tubes connected as shown, the lower half of each being filled with mercury and the upper half with water. If this gauge be coupled up to a pipe, and if $O\ O'$ be the common level of the surfaces of separation of mercury and water when the connecting tube is full of water, but with atmospheric pressure in the main, and if 0 be taken as the zero of the scale, the pressure in the main corresponding to a recorded height of h feet is $2\ n \times 12{\cdot}6\ h + h - h' = (25{\cdot}2\ n + 1)\ h - h'$ feet of water. This follows since the pressure at b = pressure at a = atmospheric pressure + pressure due to a column of mercury $2\ h$ feet high — pressure due to a column of water $2\ h$ feet high, so that the pressure at $b = 2 \times 12{\cdot}6\ h$ feet of water above the atmosphere. Again, pressure at d = pressure at c = pressure at $b + 2 \times 12{\cdot}6\ h$ feet of water $= 4 \times 12{\cdot}6\ h$ feet of water above the atmosphere. Similarly pressure at $f = 6 \times 12{\cdot}6\ h$, and at $g = 8 \times 12{\cdot}6\ h = 2n \times 12{\cdot}6\ h$ feet of water.

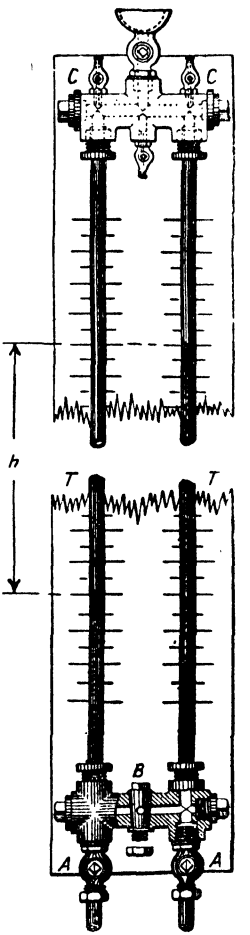


FIG. 9.

Differential Gauges.—Where it is required to determine the difference of pressure at two points in the length of a pipe conveying water, some form of differential gauge is commonly used. Such a gauge is shown in Fig. 9. Here the tubes $A\ A$ are coupled up to the required points in the pipe, the water from which partially fills the branches TT of the inverted U tube. If the difference of pressure is moderately large, the upper portion of this tube contains air, and the difference of level of the free surfaces then gives the difference in pressure

in feet of water.[1] An air cock is provided at C, by which air may be pumped into or withdrawn from the tubes; so that whatever the mean pressure in the pipe, the free surfaces of the columns may be brought to a convenient height on the gauge scales. For very small pressure differences the upper part of the U is filled with some liquid whose specific gravity is slightly less than that of water. Gasoline (specific gravity approx. ·71); kerosene (specific gravity approx. ·8); toluene (methyl-

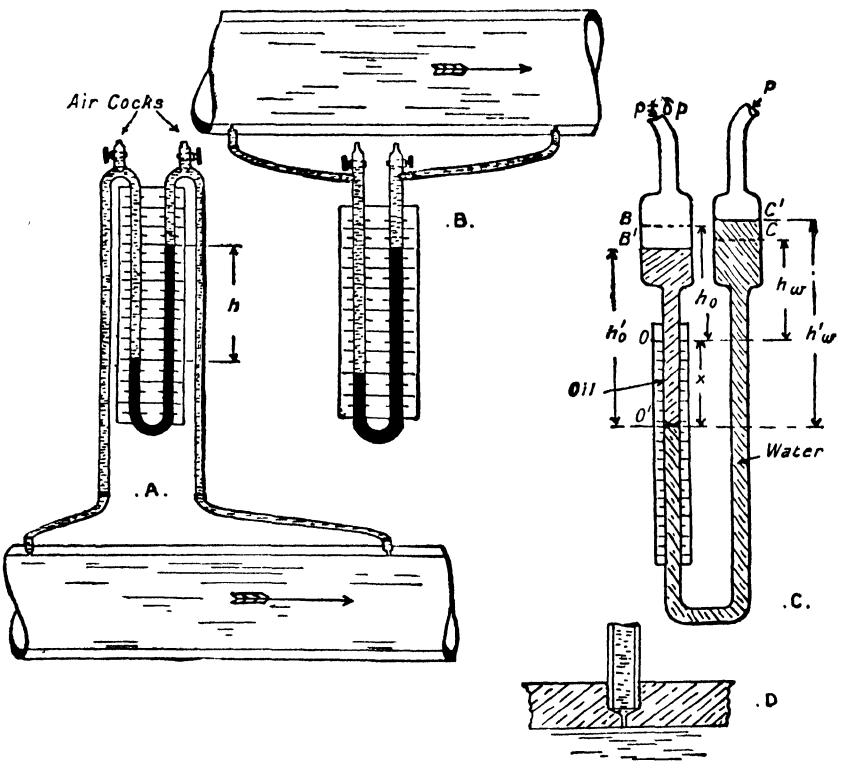


FIG. 10.

benzene, specific gravity approx. ·87); or sperm oil, are suitable liquids for this purpose. If S is the specific gravity of this liquid, and h is the difference of level of the surfaces of separation in the two tubes, the difference of pressure is that due to the difference in the weights of two columns, one of water and the other of this secondary liquid, and is

[1] Actually, the difference between the pressures produced by a column of water and one of air, each of the same height h. Owing to the small specific gravity of air the weight of the latter may, however, be safely neglected, except where the pressure in the pipe is very great.

therefore such as would be produced at the base of a column of liquid of specific gravity $1 - S$. This gives a difference of pressure of $(1 - S)\,h$ feet of water, and the gauge reading is thus multiplied in the ratio $1 \div 1 - S$, as compared with that of a simple water gauge recording the same difference of pressures. This multiplying factor will be termed the "gauge coefficient" in future, and will be denoted by C. The three-way cock at B allows of any air in the connecting pipes being readily expelled, and allows the quantity of air or of oil in the upper part of the tubes to be regulated as required.

A second type of gauge, in which the gauge fluid is heavier than water, is shown in Fig. 10 A or B. If the pressure difference is large, mercury may be used in the gauge, in which case if h is the difference of level, the difference of pressure equals $(13{\cdot}596 - 1)\,h = 12{\cdot}596\,h$ feet of water, and the gauge coefficient $= {\cdot}0794$. If the pressure difference is small, a mixture of carbon tetrachloride and of gasoline with a specific gravity of 1·25 gives a gauge whose coefficient is $1 \div (1{\cdot}25 - 1) = 4$.

A type of differential gauge which is useful for measurements of air flow is shown in Fig. 10 C, and consists of a U-tube whose branches are enlarged at their upper ends. Water is poured into one and some lighter fluid (specific gravity S) into the other leg of the tube. Let a and A be the sectional area of the small and large portions of the tubes. If 0 is the zero of the scale reading, let O, B and C be the levels of the surface of separation of oil and water and of the free surfaces when the latter are exposed to atmospheric pressure, and let O', B', C' be these levels when the pressure difference at the free surfaces is δp, these pressures being respectively p and $p + \delta p$.

Then
$$p + h'_w = p + \delta p + Sh'_o$$
$$\therefore \delta p = h'_w - Sh'_o$$
$$\text{But } h'_o = h_o + x - x\frac{a}{A} = h_o + x\left(1 - \frac{a}{A}\right).$$
$$\text{And } h'_w = h_w + x + x\frac{a}{A} = h_w + x\left(1 + \frac{a}{A}\right).$$
$$\therefore \delta p = h_w + x\left(1 + \frac{a}{A}\right) - S\left\{h_o + x\left(1 - \frac{a}{A}\right)\right\}$$
$$= x\left\{\frac{a}{A}(1 + S) + 1 - S\right\}.$$

As compared with a simple water gauge, the coefficient in such a gauge is $1 \div \left\{\frac{a}{A}(1 + S) + 1 - S\right\}$.

By using two fluids of specific gravity S' and S, such as a mixture of

alcohol and water of specific gravity, say ·85, and kerosene of specific gravity, say ·82, the coefficient becomes $1 \div \left\{\frac{a}{A}(S'+S)+(S'-S)\right\}$.

Thus in the latter case taking $\frac{a}{A} = \frac{1}{16}$, the coefficient becomes 7·43, while with water and kerosene in the same tube the coefficient is 6·1.

Errors of Fluid Differential Gauges.—Any error made in estimating the specific gravity of one of the fluids in a differential gauge will lead to an error in the gauge coefficient, and this error will be proportionately greater the larger the coefficient. Where the heavier fluid is water, an error of one-tenth of 1 per cent. in the estimation of S will lead to the following percentage errors in the value of C.

Assumed value of C.	2	5	10	20
Per cent. error in the estimation of C . .	·10	·40	·90	1·9

In a gauge which may be required to record under widely differing pressures, an appreciable error may be introduced, unless allowance is made for the different compressibilities of the two fluids. In such a gauge as is shown in Fig. 9, where air and water are the fluids, the gauge reading is usually taken as giving the true pressure difference in feet of water, although, strictly speaking, this is the difference between the pressures due to this same height of columns of air and of water. At atmospheric pressure the error thus introduced is negligible, but at higher pressures it may become important, as indicated in the following table.

Mean pressure in pipe.		Specific gravity at 50° F.		C.	Per cent. error.
Lbs. per square inch above atmosphere.	Feet of water.	Water.	Air.		
0	0	·99969	·00125	1·00125	·125
14·7	34	·99974	·00250	1·0025	·25
44·1	68	·99983	·00375	1·00375	·375
88·2	136	·99998	·00750	1·0075	·75
176·4	272	1·00027	·01500	1·0150	1·5
352·8	544	1·00085	·03000	1·030	3·0

The percentage error is smaller where two liquids are used, owing to the smaller difference in their compressibilities, although with toluene and

water (specific gravity of toluene ·866 at 50° F.—compressibility ·000079 per unit volume per atmosphere) the error amounts to 1·02 per cent. at 20 atmospheres pressure and to 0·5 per cent. at 10 atmospheres pressure. With kerosene—compressibility ·0000696 at 60° F.—as the gauge fluid, this error would be correspondingly less. A variation of temperature, by altering the relative specific gravities of the two fluids, may alter the gauge coefficient appreciably, as indicated below. The values of C are the gauge coefficients for a gauge using water in connection with the particular fluid.

Temp. °F.	Specific gravity (mean).				C.		
	Water.	Sperm oil.	Toluene.	Kerosene.	Sperm oil.	Toluene.	Kerosene.
40°	1·000	·8872	·8709	·8115	8·85	7·73	5·30
50°	·9997	·8836	·8660	·8079	8·58	7·45	5·20
60°	·9990	·8800	·8611	·8026	8·32	7·19	5·06
70°	·9980	·8764	·8562	·7940	8·08	6·95	4·84

Over such a range of temperatures, the percentage error, corresponding to a temperature range of 10° F., has a mean value of 2·4 per cent. with sperm oil, 2·8 per cent. with toluene, and 2·4 per cent. with kerosene, its magnitude increasing with the temperature. With water and mercury, the coefficients of expansion are so nearly identical that the error caused by this effect is practically negligible.

Slight differences in the bores of the tubes and in their degree of cleanliness also affect the readings by altering the form of the meniscus, while where oil and water are used, the attraction between oil and glass causes it to act in the gauge as though its specific gravity were higher than is actually the case. In view of these facts, it is not sufficient to calculate the gauge coefficient from a knowledge of the specific gravities alone if the gauge is required for accurate work, and the only satisfactory way is to calibrate it against a head of water under the same conditions as to pressure and temperature as are likely to be experienced in use.

In order that a piezometer may give an accurate record of the pressure in a pipe containing water in motion, it is important that the surface of the latter should be smooth in the neighbourhood of the piezometer opening, and especially important that any burr or roughness produced on the pipe wall by the drilling of the latter should be removed. Where a pipe is of small bore, and where the pressure is to be measured at a point at some distance from one end, this is a matter of some difficulty, and in such a

case a hole should be drilled at one operation through both walls of the pipe. The entering hole may then be plugged up and the second hole, which, with care, will be found to be free from any burr, may be used as the pressure opening. Such an opening should be small—one-thirty-second of an inch will usually be found to be ample, the connection appearing as shown in Fig. 10 D.

The point of attachment of the pressure tubes to the main is immaterial, the free surfaces or surfaces of separation in the gauge rising to the same height whether the point of attachment is at the highest or lowest point of the main.

Where a continuously recording gauge is to be used, the possibilities of a single pressure opening becoming choked render it advisable to adopt the arrangement shown in connection with a Venturi meter in Art. 196.

The *Chattock* gauge or manometer (Fig. 11) affords one of the most accurate means of measuring small pressure differences. As shown it is suitable for measuring the difference of air pressures between the piezometer connections

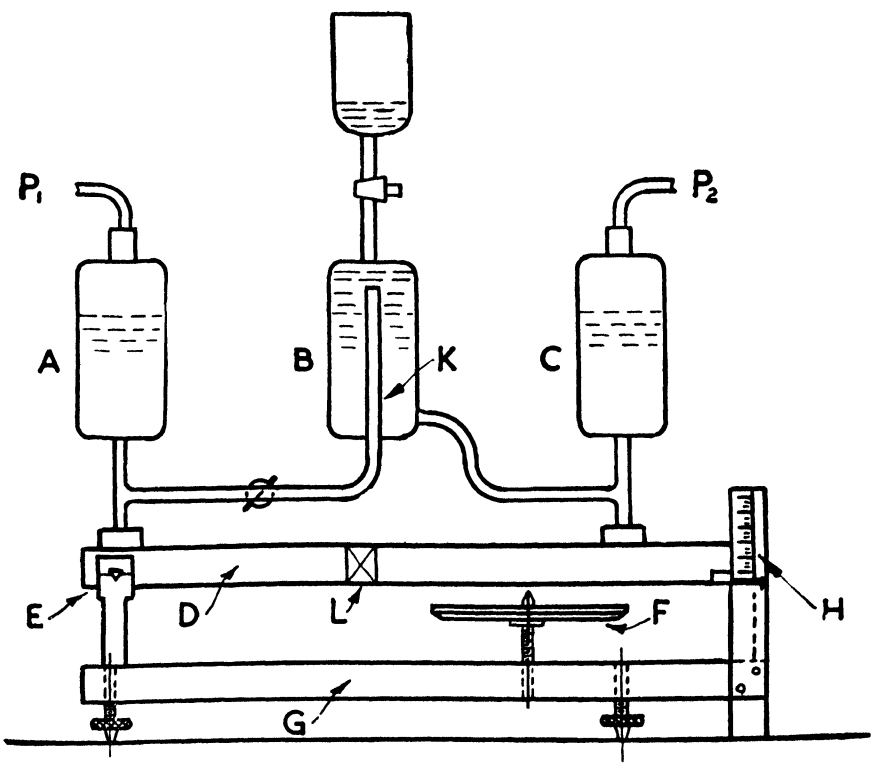


FIG. 11. Chattock Manometer.

P_1 and P_2. A and C are water vessels, B is filled with some lighter transparent fluid such as castor oil. The vessels are carried on a tilting frame D mounted on knife edges at E. This can be raised or lowered by a screw carrying a large graduated disc F, which is itself mounted on a table G levelled by adjusting screws. The table G carries a vertical scale H with vernier graduations, read against an index on the tilting frame. When the frame is levelled and the pressures P_1 and P_2 are equalised, there will be a meniscus in the vertical tube K, between the oil and water. This is observed by a microscope mounted at L, whose fiducial line is adjusted vertically so as to bring the meniscus into coincidence. When exposed to differing pressures P_1 and P_2 the position of the meniscus will change and it is again brought into the fiducial line by adjusting the tilting frame. If the difference between E and F is l inches and that between the vessels A and C is L inches ; if the screw at F has 1 mm. pitch and if there are 100 graduations on the edge of the disc, one unit $= \frac{L}{l} \times \cdot 01$ inches of water.

In order to maintain a clean meniscus it is advisable to use pure castor oil and salt water of density about 1·07. The readings of the gauge must then be corrected for the density of the salt solution.

Where pressure differences in liquids are to be measured, some other liquid such as carbon tetrachloride (sp. gr. 1·6) may be used in vessels A and C.

Art. 10.—Resultant Pressure and Centre of Pressure of a submerged Area.

If p denote the mean pressure intensity over a small element of area δA, the total pressure on this element will be given by $p\,\delta A$ and the total pressure on the whole submerged area will be the sum of all these small normal pressures, and will be represented by $\Sigma\, p\, \delta\, A$.

The resultant of all these elementary forces is termed the Resultant Pressure on the area, while the point in which the line of action of this resultant meets the area is termed the Centre of Pressure.

If the Centre of Gravity—or centroid—of an element δA, be at a depth x below the free surface of the liquid, the total pressure on the whole submerged area will thus be given by

$$\Sigma\, W\, x\, \delta\, A.$$

If $\bar{x}$ be the depth of the centroid of this area it can be easily shown that $\Sigma\, x\, \delta\, A = \bar{x}\, \Sigma\, \delta\, A = \bar{x}\, A$.

$$\therefore \quad \Sigma W x \delta A = W A \bar{x},$$

i.e., the total pressure on a single face of any submerged area is equal to the area multiplied by the depth of its centroid below the free surface and by the intrinsic weight of the fluid.

Example 1.—Calculate the total pressure on the internal curved surface of an hemispherical bowl of radius r, placed with its diametrical plane horizontal, and just filled with water.

Here $\bar{x} = \frac{r}{2}$; $A = 2\pi r^2$.

$$\therefore \quad \text{Total pressure} = 62{\cdot}4 \times 2\pi r^2 \times \frac{r}{2} = 62{\cdot}4\,\pi r^3 \text{ lbs.}$$

In this case the resultant pressure will, for equilibrium, be vertical and equal to the weight of the water;

$$\therefore \quad \text{Resultant pressure} = 62{\cdot}4 \times \frac{2}{3}\pi r^3 \text{ lbs}$$

In the case of a plane surface, such as a dock gate, the total pressure on a face will be the same as the resultant pressure, both being normal to the face.

Example 2.—In the dock gate shown in Fig. 12, the width of gate being 20 feet, the depth of the centroid of the submerged portion on the right-hand side is 5 feet, and on the left-hand side is 2 feet. The areas of these submerged surfaces are 200 feet and 80 feet respectively, so that the total pressures, and also the resultant pressures on the two faces, are $5 \times 200 \times 62{\cdot}4 =$ 62,400 lbs. and $2 \times 80 \times 62{\cdot}4 =$ 10,000 lbs. The resultant of the two pressures will then be a single force of 62,400 − 10,000 = 52,400 lbs. acting from right to left. The magnitude of the resultant pressure intensity, and of its distribution over the gate are indicated in Fig. 11. Here the pressure intensity at any depth is indicated by the horizontal distance between the surface of the gate and the straight lines $A\,C$ and $F\,G$. The resultant pressure at any depth is then to the left, and is represented by the horizontal width of the shaded area. Evidently at all points below the lower surface level, the resultant pressure intensity will be constant, since the pressure intensity increases at equal rates on both sides of the gate. The resultant force to the left per foot run of the gate is represented by the shaded area.

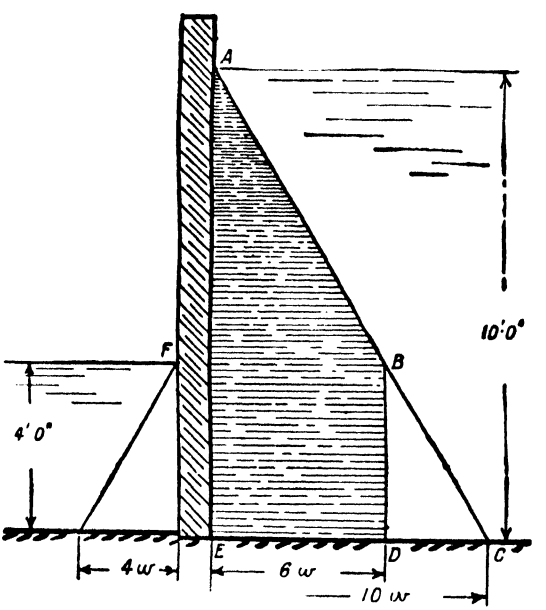


FIG. 12.

The determination of the position of the Centre of Pressure of an area is the same as that of the

line of action of the resultant of a series of statical forces each normal to the surface under consideration. Referring this to rectangular co-ordinates $O\,X$ and $O\,Y$ (Fig. 13) of which $O\,X$ is vertical and $O\,Y$ in the surface, we have, taking moments about these axes and representing the co-ordinates of the Centre of Pressure by $X\,Y$,

$$\left.\begin{aligned}\Sigma\,(p\,\delta\,A\,\cos\theta)\,X &= \Sigma\,(p\,\delta\,A\,\cos\theta\,x)\\ \Sigma\,(p\,\delta\,A\,\cos\theta)\,Y &= \Sigma\,(p\,\delta\,A\,\cos\theta\,y)\end{aligned}\right\} \quad (1),\ (2)$$

writing $p = W\,x$ these become

$$\left.\begin{aligned}\Sigma\,(W\,\delta\,A\,\cos\theta\,x)\,X &= \Sigma\,(W\,\delta\,A\,\cos\theta\,x^2)\\ \Sigma\,(W\,\delta\,A\,\cos\theta\,x)\,Y &= \Sigma\,(W\,\delta\,A\,\cos\theta\,x\,y)\end{aligned}\right\} \quad (3),\ (4)$$

Here θ is the angle between the tangent plane to the surface at the point $(x\,y)$ and the plane $X\,O\,Y$. It follows that $\Sigma\,(\delta\,A\,\cos\theta)$ is the projection of the surface on this plane. Call this A', and let $\bar{x}'$, $\bar{y}'$, be the co-ordinates of the centroid of this projection on the reference plane.

Then (3) becomes

$$W\,A'\,\bar{x}'\,X = W\,\Sigma\,(\delta\,A\,\cos\theta\,x^2)$$

$$\therefore\ X = \frac{\Sigma\,(\delta\,A\,\cos\theta\,x^2)}{A'\,\bar{x}'} \quad (5)$$

Similarly

$$Y = \frac{\Sigma\,(\delta\,A\,\cos\theta\,xy)}{A'\,\bar{x}'} \quad (6)$$

It follows that the centre of pressure of any curved surface has the same co-ordinates as that of its projection on the plane of reference.

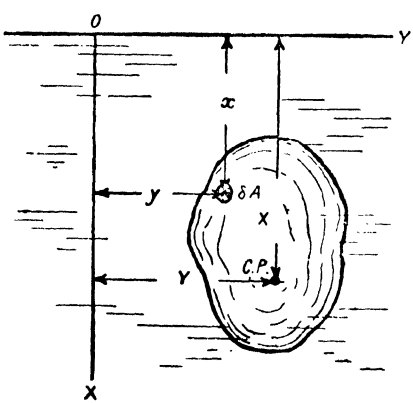


FIG. 13.

Example.—The centre of pressure of the curved surface of a hemisphere having its diametrical plane vertical, and immersed with its upper edge in the surface of the water, is at the same depth as that of the vertical diametrical plane.

Since $\Sigma\,(\delta\,A\,\cos\theta\,x^2)$ is the moment of inertia of the projection of the surface, about the axis $O\,Y$, expression (5) may be written

$$X = \frac{A'\,k'^2}{A'\,\bar{x}'} = \frac{k'^2}{\bar{x}'}$$

where k' is the radius of gyration about the axis $O\,Y$ of the projection of the surface on the plane $X\,O\,Y$.

In the case of the hemispherical surface just considered—radius r—

$$\bar{x}' = r \qquad A' = \pi\,r^2$$

$$A' k'^2 = \frac{\pi r^4}{4} + \pi r^2 . r^2 = \pi r^4 \left\{ \frac{5}{4} r^2 \right\}$$

$$\therefore X = \frac{5}{4} r.$$

Where the surface is plane, the axes $O X$, $O Y$ may be taken in the plane itself, when $\theta = 0°$, and the above expressions reduce to

$$X = \frac{A k^2}{A \bar{x}} = \frac{k^2}{\bar{x}}$$

$$Y = \frac{\Sigma (x y \delta A)}{A \bar{x}},$$

where A is the area and k its radius of gyration about the axis $O Y$ in the surface, $\bar{x}$ now being the distance of the centroid of the area from the surface, measured in the plane of the area.[1]

In the case of a rectangle, having its side b in the surface, and its side d inclined at an angle θ to the vertical,

$$A k^2 = \frac{1}{12} b d^3 + \frac{b d^3}{4} = b d \left(\frac{d^2}{3} \right)$$

$$\bar{x} = \frac{d}{2}$$

$$\therefore X = \frac{2}{3} d,$$

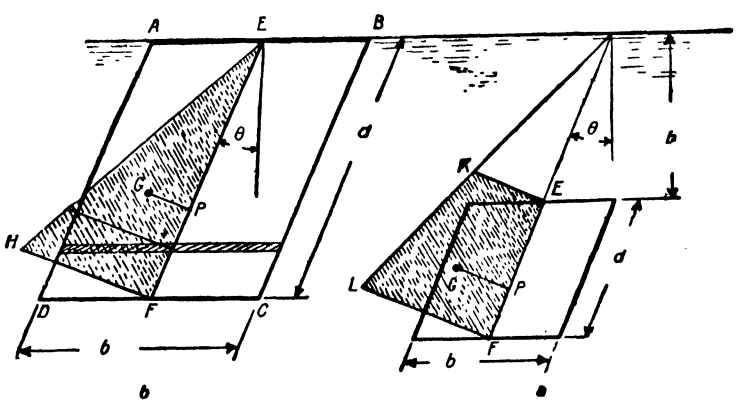


FIG. 14.

i.e., whatever the inclination, the C. P. is on the median line at a point distant $\frac{2}{3}$ the length of the rectangle from the surface.

[1] If k_1 is the radius of gyration of the figure about an axis in its plane parallel to the surface and passing through its C. G. we have $k^2 = k_1^2 + \bar{x}^2$, so that the distance of the C. P. below the C. G. is $X - \bar{x} = \frac{k^2}{\bar{x}} - \bar{x} = \frac{k_1^2}{\bar{x}}$.

Where the upper edge of the rectangle is horizontal and at a depth h below the surface (Fig. 14 *a*)

$$k^2 = \frac{d^2}{12} + \left(h \sec\theta + \frac{d}{2}\right)^2$$

$$\bar{x} = h \sec\theta + \frac{d}{2}$$

$$\therefore \quad X = \frac{\frac{d^2}{12} + \left(h \sec\theta + \frac{d}{2}\right)^2}{h \sec\theta + \frac{d}{2}}$$

Where the plane of the rectangle is vertical $\theta = 0°$ and the above becomes

$$X = \frac{\frac{d^2}{12} + \left(h + \frac{d}{2}\right)^2}{h + \frac{d}{2}}$$

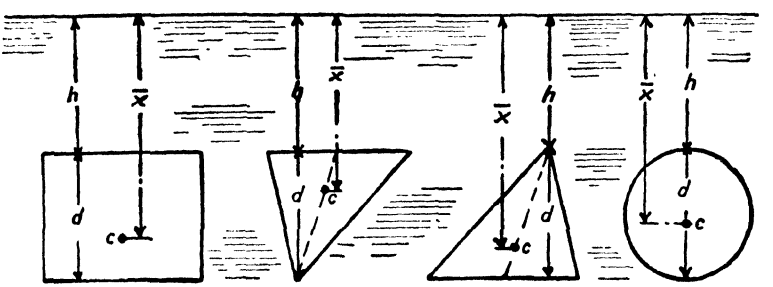


FIG. 15.

The following table gives the position of the centre of pressure in some cases of frequent occurrence in practice (Fig. 15 A, B, C, D). The distances d, h and $\bar{x}$, are measured in the plane of the figure in each case.

Figure.	A	B	C	D
$\bar{x}$	$\frac{3}{2}\left\{\frac{3h^2 + 3hd + d^2}{2h + d}\right\}$	$\frac{6h^2 + 4hd + d^2}{6h + 2d}$	$\frac{6h^2 + 8hd + 3d^2}{6h + 4d}$	$h + \frac{d}{2} + \frac{d}{16\left(h + \frac{d}{2}\right)}$

Art. 11.—Equilibrium of Floating Bodies.

When a body is freely floating in a liquid, the conditions of equilibrium may be inferred by imagining the body removed and the space occupied by it to be filled with the liquid. The reactions of the surrounding liquid will be unaltered by the change, the whole being still in equilibrium, and it is clear that the resultant pressure of the surrounding liquid is vertically upwards, and is equal to the weight of the displaced liquid, and also that the line of action of this resultant pressure passes through the Centre of Gravity of the displaced liquid. It follows that for equilibrium the weight of the floating body is equal to the weight of the liquid which it displaces, and that the Centres of Gravity of the body and of the displaced liquid are in the same vertical line.

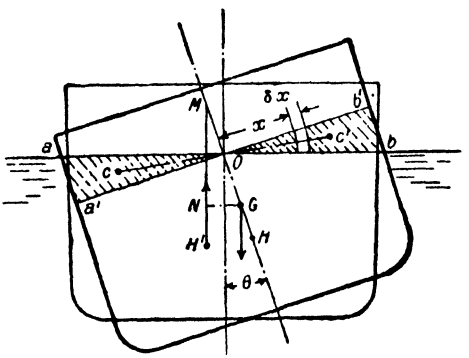


Fig. 16.

The Centre of Gravity of the displaced liquid is called the **Centre of Buoyancy.**

Stability of Equilibrium.—If a floating body be slightly displaced from its equilibrium position so that the line joining the C. G. and the Centre of Buoyancy is no longer vertical, the forces now acting may tend to restore the body to its original position; to move it still further from that position; or to maintain it in equilibrium. In the first case the equilibrium is said to be stable; in the second, unstable; and in the third, neutral.

Let G (Fig. 16) be the C. G. of the floating body; H the Centre of Buoyancy when in the equilibrium position; and H' the C. B. in the displaced position shown. Through H' draw a vertical $H'\ M$ to meet $H\ G$ in M.

The weight of the body now acts vertically downwards through G, and the equal force of buoyancy vertically upwards through H', each of these forces being equal to W, the weight of the body, and together forming a couple, whose arm is $G\ N$, the perpendicular from G on to $H'\ M$.

Obviously if M is above G this couple tends to restore the body to its equilibrium position, so that the equilibrium is stable, unstable, or neutral, according as M is above, below, or coincides with G.

If the angle of displacement $= \theta$, the magnitude of the righting moment $= W \,.\, G\,N = W \,.\, G\,M \sin\theta$.

As θ is increased the position of the intersection M of the verticals through H and H' will in general move and will approach or recede from G. The point M, for an infinitely small angle of displacement, is called the **Metacentre** of the body, and the distance $G\,M$ is called the **Metacentric Height.**

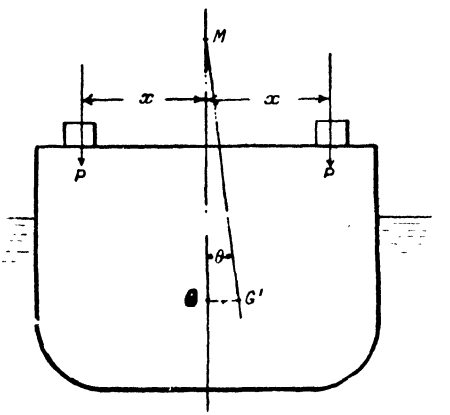


FIG. 17.

Evidently in ship design it is of the highest importance that the metacentre should be above G, under all conditions of loading and under any circumstances of rolling.

The height $G\,M$ may be determined experimentally by placing two equal weights P at equal distances x from the centre line of the vessel, when floating on an even keel and in its equilibrium position.

Let W be the weight of the vessel, including the weights P.

Then if both weights be moved through a distance $\delta\,x$ to the right, the C. G. of the vessel will move through a distance $G\,G'$ (Fig. 17) where

$$G'\,G = \frac{2\,P}{W} \cdot \delta\,x.$$

Also $G\,G'$ will be parallel to the direction in which P is moved, *i.e.*, will be perpendicular to $G\,M$, since if any portion of a body be moved in a given direction, the C. G. of the whole moves in the same direction.

If θ be the angle of heel produced by this shift of the weights

$$G\,G' = G\,M \tan\theta.$$

$$\therefore\ G\,M = G\,G' \cot\theta = \frac{2\,P}{W} \cdot \delta\,x \cot\theta.$$

θ may be measured by noting the change in inclination of a long pendulum as the weights are moved. The experiment should be repeated for

different values of δx, measured both to the right and to the left, and a curve may then be drawn on an angle base, showing values of $G M$. By exterpolation the value of $G M$ in the limit when $\theta = 0$ can then be determined.

Since the righting couple $= W \cdot G M \sin \theta$, this equals $2 P \delta x \cos \theta$, so that the same experiment enables us to draw the Stability Curve, showing the value of this righting couple for different angles of heel.

If the small masses be moved about, the weight, and therefore the volume of water displaced remaining constant, the locus of the Centre of Buoyancy is termed the Surface of Buoyancy. Since, for equilibrium, the vertical through the C. B. must pass through the C. G., and since for small displacements, the line joining two successive positions of the C. B. is parallel to the surface, it follows that the tangent plane to the surface of buoyancy at the C. B. is parallel to the water surface, and therefore that the vertical through the C. G. of the body is normal to the surface of buoyancy. In other words, any curve of buoyancy H_1 H_2 H_3 is an involute of the corresponding curve of metacentres M_1 M_2 M_3 (Fig. 18).

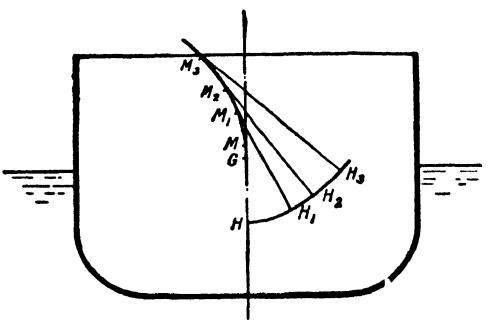


FIG. 18.

In general in the case of a ship, owing to the fact that the under water contours are not symmetrical about an amidships section, as they are about a longitudinal section, the vertical through the centre of buoyancy in the displaced position will not intersect the line H G, since the C. B. is now displaced in a different plane from that of the rotation of the boat. By projecting the verticals through the successive centres of buoyancy on to two vertical planes, one running fore and aft, and the other perpendicular to this, we get one series of intersections on each plane, and thus get two metacentric heights, the first for pitching displacements (Fig. 19), and the other, previously obtained, for rolling displacements.

The latter is in general, for the ordinary type of ship, by far the more

important, although the stability in a longitudinal direction may be seriously affected by the flooding of one or more watertight compartments. In the case of vessels of the submarine type, the longitudinal stability becomes of the greatest importance, lack of such stability causing a tendency to dive suddenly.

The metacentric height for fore and aft displacements may be experimentally determined just as for rolling displacements.

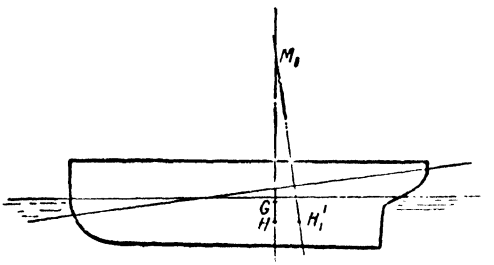


FIG. 19.

The position of the metacentre may be determined theoretically as follows, if the positions G and H of the C. G. of the vessel and of the centre of buoyancy are known.

Let A = area of section of vessel made by the plane of water line, or plane of flotation.

„ K = radius of gyration of this area about a longitudinal axis through its Centroid.

„ V = volume of water displaced by vessel.

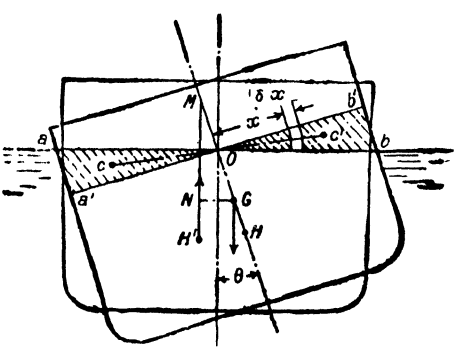


FIG. 20.

Let G, H, H', M, have the meanings previously attached to them.

„ O be the elevation of the line of intersection of the planes of flotation in the equilibrium and displaced positions (Fig. 20).

Then if a small angular displacement θ be given to the vessel, the volume displaced, being proportional to the weight of the vessel, does not change.

$\therefore$ Volume of wedge $a\ o\ a'$ = volume of $b\ o\ b'$,

Again if y be the depth, perpendicular to the paper, of any element δx of the wedge at a distance x from o, the volume of this element $= y\ x\ \theta\ \delta x$.

$$\therefore \text{ Volume of wedge } a\ o\ a' = \theta \int_0^a x\ y\ d\ x$$

$\therefore$ Since volume displaced is unaltered we have

$$\theta \int_o^a x\, y\, d\, x = \theta \int_o^b x\, y\, d\, x$$

and for this to be true the axis through O must pass through the centroid of the water line section. Also, due to the immersion of the wedge $a\ o\ a'$ and the emersion of $b\ o\ b'$, we get an upward force of buoyancy transferred from C. G. of wedge $b\ o\ b'$ to that of $a\ o\ a'$, or since if $w = wt$ of a cubic foot of water, the moment of buoyancy due to the immersion of any element of the wedge as shown is $w.\ x^2.\ \theta.\ y\ \delta x$.

$$\therefore \text{ Moment of buoyancy of wedges } a\ o\ a' \text{ and } b\ o\ b' \Big\} = w\,\theta \int_{-b}^{a} y\, x^2\, d\, x$$
$$= w\,\theta\, A\, K^2$$

But by the transference of the wedge of water, the centre of buoyancy is moved parallel to $c\ c'$; c and c', being the C. G^s. of the wedges; through a distance $H\ H'$, where $H\ H'\ .\ V\ w = w\ \theta\ A\ K^2$ (as before).

$$\therefore \quad H\ H' = \frac{A\ K^2}{V}\ \theta.$$

Also, if θ is small $\dfrac{H\ H'}{H\ M} = \theta$ very nearly.

$$\therefore \quad H\ M = \frac{A\ K^2}{V}.$$

Knowing $H\ M$, and determining $G\ M$ experimentally, $H\ G$ can be determined, from which, if the position of H is known, that of G may then be found. H may in general be determined with comparative ease since the contours of a vessel at various levels, and hence the volumes displaced between these levels, are usually accurately known. The problem then simply resolves itself into finding the C. G. of these volumes by taking moments about the water line.

ART. 12.—TIME OF OSCILLATION OF A ROLLING SHIP IN STILL WATER.

When a floating body is freely oscillating, the resistance of the water being neglected, its motion is similar to that of a pendulum except that the body does not now oscillate about a fixed axis.

Just, however, as in the case of a pendulum, the righting couple is proportional to the sine of the angle of displacement, so that the time of a complete double oscillation is given—as in the case of a pendulum—by

$$T = 2\,\pi \sqrt{\frac{\text{Moment of inertia of body about axis of oscillation}}{g \times \text{righting moment for unit angle of displacement.}}}$$

As the displacement increases, the curve, to which the plane of flotation is everywhere tangent, is known as the Curve of Flotation, and evidently in rolling the motion of the body is exactly the same as if an imaginary curve of flotation fixed in the vessel were to roll on a fixed horizontal surface. The position of the instantaneous axis of oscillation may then be determined by noting that since the weight of the vessel and the buoyancy, both vertical, are the only forces acting on the body, the C. G. of the vessel must move vertically, if at all, so that the instantaneous axis is in the horizontal line through G (Fig. 21). Again since the curve of flotation rolls on a horizontal surface, the instantaneous centre must also be in the vertical through the centre of flotation F, *i.e.*, the axis is at O, the point of intersection of $G\ O$ and $F\ O$. For small oscillations O will sensibly coincide with G.

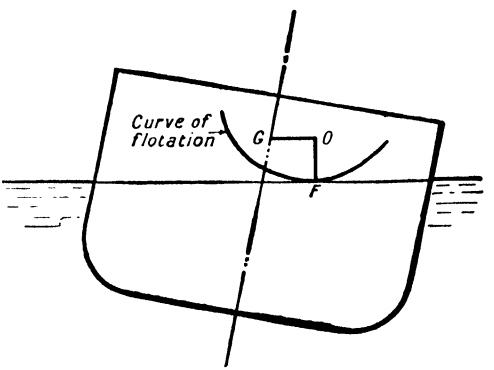


FIG. 21.

If k = radius of gyration of a body of weight W about an axis through its C. G., and if m = metacentric height for rolling displacements, the equation of motion may be written

$$\frac{d^2\theta}{dt^2} \cdot \frac{W k^2}{g} + W m \sin\theta = 0$$

or for small displacements

$$\frac{d^2\theta}{dt^2} \cdot \frac{W k^2}{g} + W m \theta = 0$$

from which we get $T = 2\pi\sqrt{\frac{k^2}{g m}}$, the relation given above.

Although a certain unknown mass of water will move along with the vessel, increasing the inertia of the moving mass without increasing the restoring couple and thus tending to increase the time of oscillation, yet in practice very close agreement is found between the calculated and experimental periods.

E.g. In the Devastation, the calculated time was 7 secs.[1]
,, experimental ,, 6·75 secs.

[1] These are the times of a single oscillation.

E.g. In the Monitor (U.S.N.) calculated time was 2·5 secs.
„ experimental „ 2·7 secs.

Froude, experimenting on ships fitted with and without bilge keels, found that the effect of the keel is to extinguish the oscillations more rapidly, but that the effect on the period of rolling is very slight, the average difference produced by the addition of these keels being about 6 per cent. Also the extinctive effect is greater when the ship is moving than when stationary.

The effect of an increase in the metacentric height is to stiffen the vessel and to diminish the period of the oscillation, while any increase in its radius of gyration tends to increase the period. Too great stiffness is inadvisable because of the tendency of the forces brought into play to strain the vessel, and it is often advisable in the case of a cargo vessel to arrange the cargo so that its heavier portions are as near to the skin of the vessel as possible. This increases the radius of gyration, and thus the period of rolling, without seriously affecting the stability.

The metacentric height in the case of merchant ships varies of course with the loading, but when fully loaded is usually between $1\frac{1}{2}$ and 4 feet. For small vessels, such as tugs and torpedo boats, it varies from about 1 to 2 feet.

Art. 13.—Strength of Pipes and Cylinders.

This will be briefly discussed as being of great importance to the hydraulic engineer.

In a thin pipe the stress over the whole thickness of metal may be taken as being sensibly uniform, and by considering a section made by a diametrical plane we see that the force per unit length of pipe tending to rupture it across this plane $= 2\,r\,p$, where

r = pipe radius in inches.

p = internal pressure in lbs. per square inch.

If t = thickness of pipe in inches and f = stress per square inch in metal, we have the force resisting rupture along this plane $= 2\,f\,t$.

$$\therefore \text{ for equilibrium } f = \frac{p\,r}{t} \text{ or } t = \frac{p\,r}{f}.$$

For steel or wrought-iron pipes this rule is sufficiently accurate, and here the working value of f varies from 7,500 to 8,500 lbs. per square inch for wrought iron, and 10,000 to 12,000 lbs. per square inch for steel,

increasing with the size of pipe. In a riveted pipe the efficiency of the riveted joint must be taken into account in determining this thickness. This may be taken at about 55 per cent. for single and 70 per cent. for double riveting. A minimum thickness of about $\frac{3}{16}$ inch should be adopted to allow for any reduction by corrosion.

Where the pipes are to be exposed to considerable pressure, and especially when made of cast iron, the distribution of pressure over the walls is not so simple, those fibres of the metal nearer the centre being more heavily stressed than those further removed. In this case if r_1 and r_0 are the inner and outer radii, the stress in the pipe, under an internal pressure p_1, will have its maximum value when $r = r_1$, *i.e.*, at the inner circumference, and at this point.

$$f_{(max.)} = p_1 \frac{r_1^2 + r_0^2}{r_0^2 - r_1^2}$$

This may be written

$$r_0 = r_1 \sqrt{\frac{f + p_1}{f - p_1}}.$$

The value adopted for the working stress f varies with the pipe diameter, and with cast-iron pipes increases gradually from about 2,000 lbs. per square inch in a 2-inch pipe to 3,000 lbs. per square inch in an 8-inch pipe, and 3,500 lbs. per square inch in a pipe or cylinder of 24 inches diameter.

The following table indicates the thickness of cast-iron pipes and cylinders for heavy pressures, as calculated on the above assumption.

Pipe diameter in inches.	Working pressure, lbs. per square inch.		
	750	1000	1120
2	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{7}{8}$
3	$\frac{11}{16}$	$\frac{15}{16}$	$1\frac{1}{8}$
4	$\frac{13}{16}$	$1\frac{1}{8}$	$1\frac{3}{8}$
5	$\frac{29}{32}$	$1\frac{5}{16}$	$1\frac{9}{16}$
6	$1\frac{1}{32}$	$1\frac{7}{16}$	$1\frac{23}{32}$
8	$1\frac{5}{32}$	$1\frac{11}{16}$	$1\frac{29}{32}$
10	$1\frac{13}{32}$	2·0	$2\frac{5}{16}$
12	$1\frac{23}{32}$	$2\frac{5}{16}$	$2\frac{21}{32}$
16	$2\frac{5}{32}$	$2\frac{15}{16}$	$3\frac{13}{32}$
20	$2\frac{5}{8}$	$3\frac{9}{16}$	$4\frac{1}{8}$
24	$3\frac{1}{32}$	$4\frac{1}{8}$	$4\frac{3}{4}$

Collapsing Pressures of Cylindrical Pipes.—The sudden discharge of water from a hydraulic pipe line, such as may follow a burst at its lowest point, may, unless special provision be made, lead to the production of a partial vacuum in the pipe and thus give rise to an effective external collapsing pressure which is increased by the pressure of the earth filling in the case of a buried pipe. Very few reliable experiments are available as to the effect of such pressures on pipes of large diameter. Such experiments as are available lead to the conclusion that the collapsing pressure of a long steel welded pipe is given very approximately by [1]

$$p = \frac{74000}{\frac{d}{t} + \frac{1}{810}\frac{d^3}{t^3}} \text{ lbs. per sq. in.}$$

where d and t are in the same units.

If stiffening rings are provided at suitable distances,

$$p = \frac{74000\,t}{d} \text{ lbs. per sq. in.}$$

The correct spacing of the rings is given by s, where

$$\frac{s}{d} = 1400\sqrt{\frac{t^3}{d^3}}.$$

Examples.

1. In a hydraulic press the ram is 10 inches and the pump plunger 1 inch diameter, the leverage for working the pump 16 to 1. What is the velocity ratio of the pump handle and ram? Actually a force of 30 lbs. exerts a pressure of 44,000 lbs. on the press table. What is the efficiency?

Ans.: 1,600; 91·7 per cent.

2. A dock gate is 12 feet broad and 14 feet deep, and the water rises 12 feet on one side and 3 feet on the other side above its lower edge. Find the resultant pressure and centre of pressure for each side of the gate and find the magnitude and position of the resultant of these.

Ans.: 53,914 lbs.; 3,370 lbs.; centres of pressure 4 feet; 1 foot from lower edge.

Single resultant 50,544 lbs. at 4·20 feet from lower edge.

[1] See "Trans. Am. Soc. Mech. E.," 1906, p. 730; 1907, p. 123; "Trans. Inst. C. E. Ireland," 1910, p. 138; "Phil. Mag.," vol. 29, 1915, p. 67.

3. A rectangular sluice gate, 5 feet broad and 6 feet deep, having its upper edge at a depth of 4 feet, is inclined at 45° to the vertical and admits water to an empty penstock. The sluice is lifted by a force applied parallel to its plane. Determine the necessary magnitude of this force if the coefficient of friction between gate and guides is ·12.

Ans.: 1,375 lbs.

4. A circular conduit, 6 feet diameter, which just runs full, is fitted with a sluice, and it is required to balance this about a horizontal axis. Show that this axis should be placed 2 feet 3 inches above the bottom of the conduit.

5. A vertical wall, 10 feet high and 3 feet thick, is exposed to water pressure on one side. What is the maximum depth of water in order that the lowest joint should not be anywhere under tension? Weight of masonry per cubic foot = 170 lbs.

Ans.: 6·26 feet.

6. A battleship weighs 13,000 tons. On filling the ship's boats on one side with water—this weighing 60 tons and its mean distance from the centre of the boat being 30 feet—the angle of displacement of a plumb line is 2° 16′ (tan $\theta = ·0396$). Determine the metacentric height for rolling displacements.

Ans.: 3 feet 6 inches.

7. Show that a solid cylinder of length l, radius r, and specific gravity s, floating with its axis vertical, is in stable equilibrium if

$$r > l\sqrt{2s(1-s)}.$$

8. A single-riveted steel pipe, 30 inches internal diameter, is exposed to a head of 450 feet of water. Taking $f = 12{,}000$ lbs. per square inch, and the efficiency of the joints = 60 per cent., what should be its thickness?

Ans.: $t = ·407$ inch $= \frac{13}{32}$ inch.

9. A cast-iron pressure pipe, 4 inches diameter, is exposed to a pressure of 1,100 lbs. per square inch. Taking $f = 2{,}800$ lbs. per square inch, what is the requisite thickness?

Ans.: $t = 1·032$ inch $= 1\frac{1}{32}$ inch.

SECTION II.

CHAPTER III.

Motion of Viscous Fluids—Stream Line and Eddy Motion—Conditions regulating the two modes of Motion—Reynold's Researches—Critical Velocity in Parallel Pipes—In Converging Pipes—Motion of a Fluid—Equations of Motion—Hele Shaw's experiments—Flow against Viscous Resistance—Between Parallel Plates—Through Circular Tube—Genesis of Eddy Formation.

ART. 14.—MOTION OF VISCOUS FLUIDS.

WHEN a rigid body moves, the motion at any point in its mass can be accurately determined once the motions of any three points, not in the same straight line, are fixed. In this case, when the boundary conditions are known, the determination of the internal motion is simple. But with a fluid such as water the motion of the mass of fluid as a whole does not necessarily give the internal motion. While giving the mean motion, it does not fix the absolute motion of each particle of the fluid relatively to any fixed point. This is quite apart from intermolecular motion, which gives rise to diffusion in a mass of water even when apparently motionless.

By the earlier experimentalists the motion of water in mass was imperfectly understood. On the assumption of a motion simple and analogous to that of a rigid body, it was impossible to reconcile the results experimentally obtained by many observers, while the very transparency, or uniform opacity of most fluids made it impossible to see the internal motion. Probably the first indication of there being more than one kind of fluid motion was obtained from the appearance of the free surface of flowing water. Where not otherwise disturbed this may have two appearances, corresponding to different modes of motion. In the one objects are reflected from the surface without distortion, while in the other an irregular motion of the surface may be noted, and reflection is accompanied by distortion. Where motion occurs in a passage having solid lateral boundaries, however, even this indication is absent, and the introduction of floating particles of solid matter does not help to any extent in showing the nature of the action which is taking place.

Matters remained in this state for many years, and it was left for

Osborne Reynolds, by his method of colour bands,[1] to prove conclusively that the motion in a mass of water may be of two kinds; to make clear the simplicity of the one, and the complexity of the other; and to demonstrate the reasons for, and the laws governing each kind of motion.

The conclusions to be drawn from Professor Reynolds's experiments are that the motion of a fluid may be of two kinds. In the one there is a continuous steady motion of the particles, in which, if $\frac{dv}{dt} = 0$, the motion at a fixed point remains constant, while, in the other, owing to eddy formation, the motion at a fixed point varies, but according to no definite law. This is termed **turbulent, sinuous,** or **unsteady** motion.

Introducing the idea of **stream lines**, *i.e.*, of imaginary lines in the fluid, such that at any point the direction of motion is tangential to the line, it

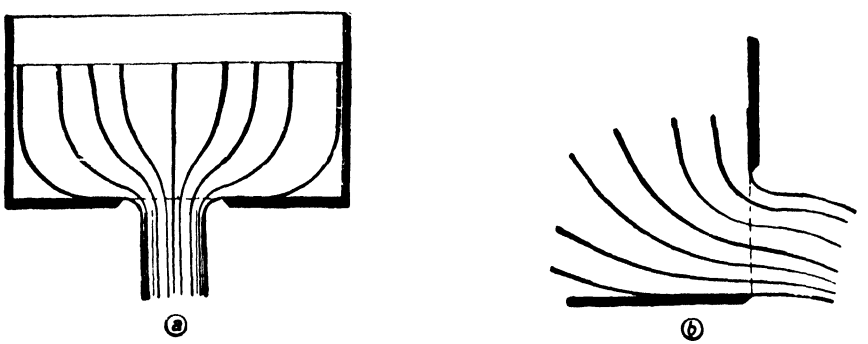


FIG. 22.

follows that in steady motion these stream lines become fixed, and this type of motion is therefore known as **stream line motion.**

If the stream line be imagined to form the axis of a tube of finite sectional area having imaginary boundaries, and such that its area, at different points in its length, is inversely proportional to the velocity at those points, this is known as a **stream tube.**

If the motion at a fixed point varies, and if it is still possible to find a definite motion for these points such that the motion of the fluid relative to the points may remain constant, the latter may be reduced to steady motion by considering the motion relative to these moving points. The fluid may then be said to have a continuous, steady, but unequal motion.

[1] For a full account of this method of investigating the two manners of motion of water see a paper by Osborne Reynolds, "Phil. Trans. Royal Society," 1883.

In almost all the cases of fluid motion which are of practical importance in hydraulics, the motion is found to be turbulent.

Art. 15.—Conditions regulating the Two Manners of Motion.

Several conditions combine to determine whether in any particular instance the motion of water shall be steady or unsteady. Osborne Reynolds[1] came to the conclusion that the conditions tending to stability and steadiness of motion are:—

(1) An increase in viscosity.

(2) Converging solid boundaries.

(3) Free (exposed to air) surfaces.

(4) Curvature of the path, with the greatest velocity at the outside of the curve.

(5) A reduced density of the fluid.

The reverse of these conditions tends to produce turbulence, as does a state of affairs in which a stream of fluid is projected into a body of fluid at rest.

The effect of solid boundaries in producing instability would appear to be due rather to their tangential than their lateral stiffness. One very remarkable instance of this effect of boundaries possessing tangential stiffness, however small this may be, occurs when a film of oil is allowed to form on the surface of water. Here the oil film exerts a small but appreciable tangential constraint, with the result that motion which was originally stable becomes unstable. This results in the formation of eddies below the surfaces of the oil and water, and the energy which was originally imparted by the action of the wind to form and maintain stable wave motion is now applied to the institution of this eddy motion, with the well-known result as to the stilling of the waves.

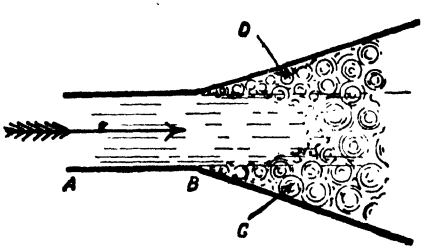


Fig. 23.

Where one stream of liquid is in contact with a second stream moving with a different velocity, the common surface of separation is found to be in a most unstable condition. Reynolds showed this by allowing the two liquids, Carbon Bisulphide and water, to form a horizontal surface of separation in

[1] "Phil. Trans. Royal Society," 1883.

a long horizontal tube. The tube was then slightly tilted so as to produce a relative axial motion of the fluids, when it was found that for extremely small values of the relative velocity the motion was unstable.

In this manner may be explained why diverging boundaries are a cause of instability. In such a case as that shown in Fig. 23, the motion from A to B in the parallel portion of the channel may be perfectly steady, depending on the dimensions of the channel and the velocity of flow. On leaving B, however, at any but the smallest velocities the water appears to be projected in the form of a core of the same dimensions as the channel, through the mass of dead water at C and D. Here all the conditions necessary for instability are present.

This instability attending the relative motion of fluids of different density affords an explanation of the interesting phenomenon known as "deadwater," which is sometimes noticed when a ship encounters a surface layer of fresh water, overlying the denser sea-water. Under such circumstances the resistance to motion is greatly increased. The phenomenon has been investigated by V. W. Ekman (Norwegian Polar Expedition Researches, 1893, No. 15), who found that the resistance is due

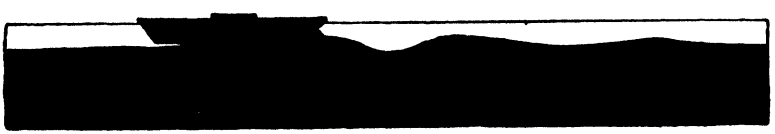


FIG. 24.

to the formation and maintenance of a train of waves at the unstable surface of separation of the fluids, as shown in Fig. 24.

More recent experiments[1] tend to show that the foregoing conclusions as to the effect of curvature of the path in affecting the manner of motion are only true where the outer boundary of the fluid is formed by a solid surface, and that, as shown at the impact of a steady jet on a plane surface, at the efflux of a jet from a sharp-edged orifice, and in motion in a free vortex, curved motion with the velocity greatest at the inside, not at the outside of the curve, tends to steady motion. Generally speaking, wherever the velocity of flow is increasing and the pressure diminishing, as in the case where lines of flow are converging, there is an overwhelming tendency to steady motion. In a tube with converging boundaries, it is this which leads to steadiness of flow, and it is because this effect is sufficiently pronounced

[1] By the Author, "Memoirs, Manchester Lit. and Phil. Soc.," Vol. 55, 1911, No. 18.

to overcome the tendency to sinuous motion which all solid boundaries, of whatever form, produce, that the motion in such tubes is steady for very high velocities of flow.

Art. 16.—Critical Velocity.

The experiments by which Professor Reynolds demonstrated the nature of the two modes of motion of water were carried out on glass tubes of

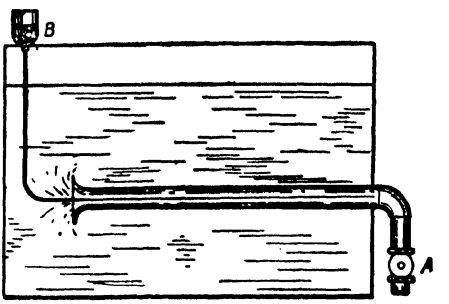


Fig. 25.

various diameters up to 2 inches, and about 4 feet 6 inches long. These were fitted with bell-mouth entrances, and were immersed horizontally in a tank of clear water having glass sides (Fig. 25). In carrying out the experiment the water in the tank is allowed to come absolutely to rest, and the valve A is then slightly opened, allowing water to flow slowly through the tube. A little water, coloured with aniline dye, is introduced into the mouthpiece through a fine tube supplied from the vessel B.

At first this coloured water is drawn out into a single stream tube,

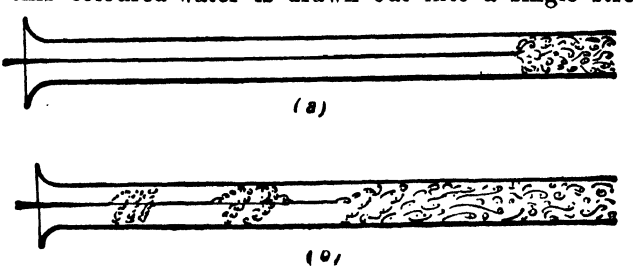


Fig. 26.

extending through the whole length of the tube, as shown in Fig. 25, the whole appearing to be motionless unless a slight motion of oscillation is given to the water in the supply tank, when the stream line sways gently from side to side, but without in the least losing its definition. As the valve A is further opened the velocity through the tube increases, and the

stream tube is drawn out more and more, still retaining its definition until at a certain velocity eddies begin to be formed intermittently near the outlet end of the tube (Fig. 26 *a*).

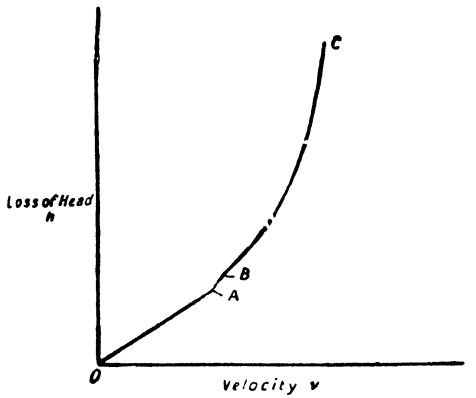


FIG. 27.

The formation of these eddies is accompanied by the almost instantaneous diffusion of the colour band. As the velocity is still further increased the point of initiation advances towards the mouthpiece. The apparent lesser tendency towards eddy function near the inlet end of the tube is due to the stabilising influence of the convergent mouthpiece. Finally the motion throughout becomes unsteady (Fig. 26 *b*).

Any initial disturbance of the water tends to reduce the velocity at which the motion changes from steady to turbulent, and which is termed the **Critical Velocity.**

The velocity at which eddy formation is first noted in a long tube in such experiments is termed the "higher critical velocity." There is also a "lower critical velocity" at which the eddies in flow originally turbulent die out, and this is, strictly speaking, the true critical velocity. It has a much more definite value than the higher critical velocity, which is extremely sensitive to any initial disturbance of the fluid. Over the range of velocities between the two critical values, the fluid, if moving in stream line flow, is in an essentially unstable state, and the slightest disturbance may cause it to break down into turbulent motion.

The determination of the lower critical velocity is not possible by the colour-band method, and Reynolds took advantage of the fact that the law of resistance changes at the critical velocity to determine the values by measuring the loss of head accompanying different velocities of flow in pipes of different diameters. On plotting a curve showing velocities and losses of head (Fig. 27) it was found that up to a certain velocity, *A*, for any given tube, these points lie on a straight line passing through the origin of co-ordinates. From *A* to *B* there is a range of velocities over which the plotted points are very irregular, indicating general instability, while for greater velocities the points lie more or less on a smooth curve, indicating that the loss of head is possibly proportional to v^n.

To test this, and if so to determine the value of n, the logarithms of the loss of head and of the velocity were plotted (Fig. 28), since if

$$h = k v^n$$

$$\log h = \log k + n \log v$$

which is the equation to a straight line, inclined at an angle θ to the axis of $\log v$ (where $\tan \theta = n$), and cutting off an intercept $= \log k$ on the axis of $\log h$.

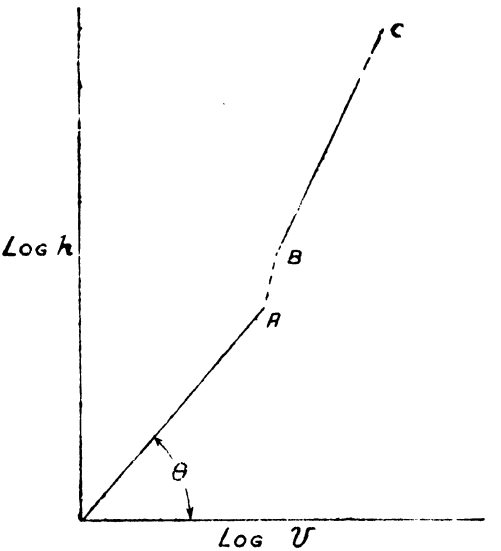


FIG. 28.

It was then found that with turbulent motion at the entrance to the pipe, the plotted points lie on a straight line up to a certain point A, the value of θ for this line being 45°. Up to this point n is unity and h is proportional to v. At A, which marks the *lower* critical velocity, the law suddenly changes and h increases very rapidly. The relation between h and v, however, follows no definite law, until the point B is reached. Reynolds found that the points from B to C and onwards lay on a straight line, indicating a law of resistance $h = k v^n$. We now know that this is not strictly true, and that for large changes of velocity, with smooth pipes the line diverges from the straight, and the index n changes somewhat.

The values of n determined in this way by Reynolds are :—

Material of Pipe.	n
Lead	1·79
Varnished Glass	1·82
New Cast Iron	1·79
Old Cast Iron	1·88
	2·00

Between A and B the value of n is greater than before A or after B, and the increased resistance accompanying an increase in velocity is greater than even when the motion is altogether sinuous. This is due to the fact that within this range of velocities eddies are being initiated in the tube, and the loss of head is due, not only to the maintenance of a more or less uniform eddy *régime*, but also to the absorption of energy in eddy formation.

As a result of his experiments, Reynolds concluded that the critical velocity is inversely proportional to the diameter d of the pipe, and is given by the formula

$$v_c = \frac{K\mu}{d\rho} = \frac{K\nu}{d} \quad \ldots\ldots\ldots\ldots \quad (1)$$

The value of K for the lower critical velocity is approximately 2300. This applies to a straight parallel pipe and is applicable to any fluid and any system of units.

The expression vd/ν is known as the Reynolds Number. From his own experiments Reynolds concluded that the higher C. V. was about six times as large as the lower C.V., but more recent experiments indicate much higher values than this. Much depends on the exact degree of disturbance at the entrance to the tube.

As an indication of the magnitude of the lower C. V. we have for example for water at 60° F., $\nu = 1{\cdot}19 \times 10^{-5}$ in foot-pound-second units so that

$$v_c = \frac{{\cdot}0274}{d} f.s. \text{ where } d \text{ is in feet,}$$

while for air at the same temperature $\nu = 15{\cdot}61 \times 10^{-5}$

$$\therefore\ v_c = \frac{{\cdot}359}{d} f.s.$$

It is evident that in engineering practice, except in the cases where fluids of very high viscosity such as fuel oils are involved, the velocities will be well outside the range of laminar flow.

In this general connection Fig. 29 [1] is of interest. It shows the relationship between $R/\rho v^2$ and vd/ν found by Stanton and Pannell in experiments on the flow of air and of water through a series of smooth brass pipes of diameters ranging from 0·144 inches to 4·96 inches. Here R is the resistance per unit area of wall surface and its value is $f/2$ where f is the frictional coefficient used in pipe flow formulae.

The curve consists of two parts AB and CD, connected by the portion BC over which the plotted points representing the observations are irregularly

[1] Stanton and Pannell, "Phil. Trans. Roy. Soc. A.," vol. 214.

disposed. AB represents the region of stream line or laminar flow, and is calculated from the formula

$$R = \frac{8\mu v}{d}.$$

At B the value of vd/ν is approximately 2300.

The region BC represents the transitional range over which turbulence is initiated. At C, vd/ν is approximately 3000.

Over the range CD the flow is partially but not wholly turbulent. If it were, the resistance would be proportional to v^2 and the curve would become horizontal.

In these smooth pipes the resistance is proportional to v^n where n is less than 2. Also the experiments indicate that the value of n is not strictly constant but depends on the value of vd/ν. However, over the range of values of vd/ν from 3000 to 150,000, Davies and White [1] show that a close approximation to the curve is given by the formula

$$R \propto \frac{lv^{1\cdot75}\nu^{0\cdot25}}{d^{1\cdot25}}.$$

It should be noted that all the experimental points, both for water and air lie in a very narrow band about the line CD.

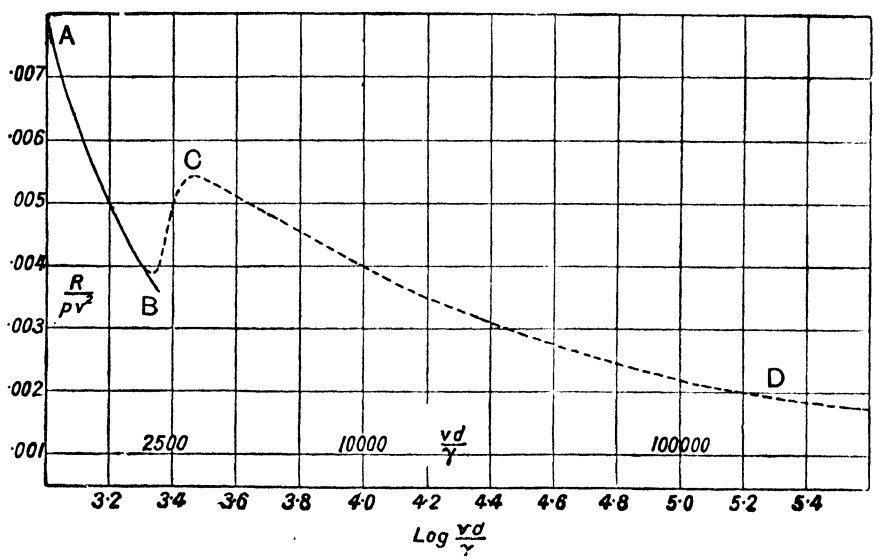


FIG. 29.

[1] Davies and White, "A Review of Flow in Pipes and Channels," "Engineering," July 19th, 1929, p. 71.

If the Reynolds Number exceeds 4000, Prandtl [1] has shown that the equation

$$\frac{1}{\sqrt{4f}} = 2 \log_{10} (R_e \sqrt{4f}) - 0{\cdot}8 \text{ where } R_e = \frac{vd}{\nu}$$

gives a very close approximation to the results.

In a rough pipe the form of the curve CD is very different and the curve becomes horizontal at values of the Reynolds Number which depend on the surface roughness. This is considered in detail in Chapter IX.

Art. 17.—Criterion for Critical Velocity.

As may be shown by an application of the principle of dynamical similarity,[2] formula (1) is a particular case of the general formula

$$v_c = \frac{K\nu}{l}$$

which is applicable to all cases of fluid motion. Here l is the length of some one definite dimension of the body. The value of the constant K depends only on the form of the surfaces over which flow is taking place. Thus in flow past similar plates immersed in water and in air, Eden [3] has shown by visual observation, that the type of flow, especially in the rear of the plate, is identical for identical values of vl/ν where l is the length of any particular side of the plate.

Art. 18.—Critical Velocity in Converging Tubes.

In a converging tube the angle of convergence of the sides has a large effect on the critical velocity. At all ordinary velocities the motion in tubes or nozzles having more than a few degrees of convergence may be considered as non-sinuous. Experiments on the flow of water through circular pipes having sides converging uniformly at an angle θ, gave the following approximate values for the lower critical velocity, at 14° C.[4]

[1] Prandtl, "Aerodynamic Theory," vol. 3, p. 143.
[2] See Chap. IV., Art. 33.
[3] "Advisory Committee for Aeronautics," 1911–12, p. 96.
[4] "Proceedings, Roy. Soc. B.," vol. 83, 1910, p. 376.

Values of θ.	5°.	7½°.	10°.	15°.
C. V. feet per second—				
At large section (3-inch diam.) . .	1·5	1·94	2·44	3·25
„ throat (1½-inch diam.) . . .	6·0	7·76	9·77	12·9
„ point where diam. is 2¼ inches . .	2·7	3·45	4·34	5·73

The lower critical velocity in a parallel pipe of the same mean diameter (2¼ inches) at this temperature is ·133 feet per second, while in a 3-inch and a 1½-inch pipe it is respectively 0·10 and 0·20 feet per second, the higher critical velocities being respectively 0·86, 0·65, and 1·29 feet per second. Should the ratio of higher to lower critical velocities have the same value in a conical pipe as in a parallel pipe, this would mean that in the case of a 1½-inch jet discharged from a converging nozzle with steady flow in the supply pipe, the critical velocity would have the following values—

Value of θ.	5°.	7½°.	10°.	15°.
C. V. (higher), feet per second . . .	39·0	50·4	63·5	83·8

and this would explain the glassy appearance of the high velocity jet, leaving the nozzle of a Pelton wheel. Further experiments are, however, needed to settle this point and also to determine how, with the same values of θ but with different inlet and outlet diameters, the critical velocity depends on the latter factor.

Art. 19.—Equations of Motion of a Fluid.

The motion of any particle of fluid acts upon by external forces and by its own weight may be considered from two points of view. In the first, by equating the work done on the mass to the increase of energy in the potential, pressure, and kinetic form, together with the loss by dissipation, as by internal friction, which converts mechanical energy into heat, we get an expression for the velocity in terms of the applied forces. With steady motion of a non-viscous fluid this method is always applicable.

Where a mass of fluid has unsteady—sinuous—motion, however, the impossibility of determining the kinetic energy possessed by its eddies in virtue of their rotation renders the application of the energy equation impossible, and it becomes necessary to consider the motion from the

point of view of the production of momentum, since the momentum of the fluid forming a vortex is unaltered by its motion of rotation.

With unsteady motion, moreover, of a viscous fluid, the magnitude and direction of the forces—including those due to viscosity—acting at any particular point become indeterminate, so that the molecular motion at the point is then indeterminate, and the general equations of motion become in general impossible of application. Even in the case of the steady motion of a viscous fluid, these, when stated in terms of the viscosity, become so unwieldy that, except in one or two particular cases, *e.g.*, those of steady flow between parallel plates or through a circular pipe, they are unfitted for application to the solution of any practical problem, although where so applicable the solution becomes perfectly accurate.

A simplification of these equations may be obtained by neglecting the effect of viscosity—*i.e.*, assuming the liquid to be a perfect fluid—and it is in this form that they are usually stated. Evidently the solution of any such equation can only be made to apply to the results of any given problem by the introduction of some constant obtained by experiment which itself has the effect of modifying the solution so as to take into account the effect of viscosity, and it is to this extent that hydraulics is to be considered an experimental science. If it were possible in every case to apply the equations of motion in full, the science would become exact.

The equations in full are deduced in the following pages, and the terms involving viscosity are afterwards eliminated, so as to give the form as usually stated for a non-viscous incompressible fluid.

Art. 20.—Equations of Motion for a Viscous Fluid.

Taking a fixed point O in the fluid as the origin of co-ordinates, let OX, OY, OZ be three co-ordinate axes, and let u, v, w be the components of the velocity of a particle parallel to these three axes. u, v, and w, will be supposed finite and continuous, and, since they vary with the position of the particle and the time, are functions of x, y, z, and t.

The velocity of a particle may be considered from two points of view. Considering any fixed point, the velocity of successive particles as they pass that point may vary, and since x, y, z are now constant, the rates of variation parallel to the three axes are represented by the partial differentials $\frac{\partial u}{\partial t}$, etc. For steady motion these are separately zero.

If, however, we consider the variation of velocity of any one particle as

it moves from point to point of its path, since this now also includes the variation of x, y, z with t, its components will now be represented by $\frac{du}{dt}$ etc., and are the components of the true acceleration of the particle. These are not necessarily zero for steady motion.

If then a particle moves from a point (x, y, z) to a second point $(x + \delta x; y + \delta y; z + \delta z)$ in time δt, its change of velocity δu is given by

$$\delta u = \frac{\partial u}{\partial x}\delta x + \frac{\partial u}{\partial y}\delta y + \frac{\partial u}{\partial z}.\delta z + \frac{\partial u}{\partial t}.\delta t$$

and in the limit

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial u}{\partial y}\cdot\frac{dy}{dt} + \frac{\partial u}{\partial z}\cdot\frac{dz}{dt} + \frac{\partial u}{\partial t} \qquad (1)$$

But from the definitions of u, v, w, we have

$$u = \frac{dx}{dt} \quad : \quad v = \frac{dy}{dt} \quad : \quad w = \frac{dz}{dt}$$

$$\therefore \quad \frac{du}{dt} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t} \qquad (2)$$

similarly
$$\frac{dv}{dt} = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t} \qquad (3)$$

and
$$\frac{dw}{dt} = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t} \qquad (4)$$

Next consider an elementary parallelopiped in the fluid bounded by edges $\delta x, \delta y, \delta z$ (Fig. 30). For continuity of motion the difference between the amounts of fluid which flow into and out of its faces during time δt must be equal to the increase in the mass which they enclose. Expressing this analytically, we get the equation of continuity.

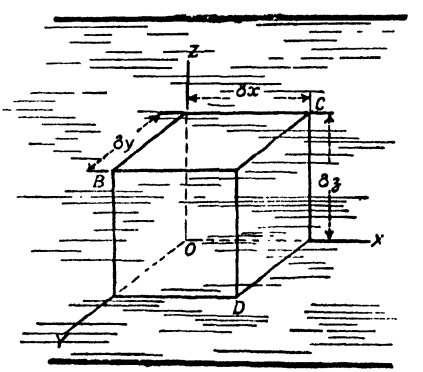


FIG. 30.

Now the mass of fluid entering across the face $O\,B$ in time $\delta t = \rho\, u\, \delta y\,.\,\delta z\,.\,\delta t$.

And the mass of fluid leaving across the face $C\,D$ in time $\delta t = \rho\, u\, \delta y\,.\,\delta z\,.\,\delta t + \frac{\partial}{\partial x}(\rho u)\,\delta x\,.\,\delta y\,.\,\delta z\,.\,\delta t$.

$$\therefore \text{ Gain across these faces } = -\frac{\partial}{\partial x}(\rho\, u)\,\delta x\,.\,\delta y\,.\,\delta z\,.\,\delta t.$$

$\therefore$ Total gain

$$= -\left\{ \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) \right\} \delta x \,.\, \delta y \,.\, \delta z \,.\, \delta t.$$

But the mass contained at time $t = \rho \, \delta x \,.\, \delta y \,.\, \delta z.$

and " " " " $t + \delta t = \left(\rho + \frac{\partial \rho}{\partial t} \,.\, \delta t \right)$

$$\delta x \,.\, \delta y \,.\, \delta z.$$

$$\therefore \text{ Gain in time } \delta t = \frac{\partial \rho}{\partial t} \,.\, \delta t \,.\, \delta x \,.\, \delta y \,.\, \delta z.$$

Equating these expressions for the gain, we have

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} + \frac{\partial (\rho w)}{\partial z} = 0 \tag{5}$$

from which, if ρ is constant, *i.e.*, if the fluid is incompressible, we have

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{6}$$

as the equation of continuity for an incompressible fluid.

In the case of a gas ρ is not constant, but we may have $\frac{\partial \rho}{\partial t} =$ constant, in which case we have regular motion in the gas.

Now if the stresses in a viscous fluid (which follows the same stress strain law as an elastic solid) be represented by the notation p_{xx}, p_{xy}, p_{yz}, etc., where each of these symbols denotes a stress on the plane perpendicular to the axis of co-ordinates represented by the first suffix, in the direction of the second suffix, so that, for example, the stress p_{xy} is a stress on the plane perpendicular to $O\,X$, in the direction $O\,Y$, and is therefore a tangential stress on this plane, the relations holding between the various stresses for equilibrium are given by the following equations.[1]

$$\left.\begin{aligned} p_{xx} &= -p - \frac{2}{3}\mu\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) + 2\mu \frac{\partial u}{\partial x} \\ p_{yy} &= -p - \frac{2}{3}\mu\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) + 2\mu \frac{\partial v}{\partial y} \\ p_{zz} &= -p - \frac{2}{3}\mu\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) + 2\mu \frac{\partial w}{\partial z} \end{aligned}\right\} \tag{7}$$

$$\left.\begin{aligned} p_{xy} &= p_{yx} = \mu\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) \\ p_{yz} &= p_{zy} = \mu\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right) \\ p_{zx} &= p_{xz} = \mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \end{aligned}\right\} \tag{8}$$

[1] Stokes on "Theories of the internal friction of fluids in motion and of the equilibrium and motion of elastic solids"; also Lamb's "Motion of Fluids," p. 219, or "Hydrodynamics."

where $p = -\frac{p_{xx} + p_{yy} + p_{zz}}{3}$, and μ = Coefficient of Viscosity.

But, considering the elementary volume $\delta x \,.\, \delta y \,.\, \delta z$, we see that the total force acting on this volume, due to any variation of stress on opposite faces of the volume in the direction $O X$, is

$$\left\{\frac{\partial p_{xx}}{\partial x} + \frac{\partial p_{yx}}{\partial y} + \frac{\partial p_{zx}}{\partial z}\right\} \delta x \,.\, \delta y \,.\, \delta z.^1$$

So that if an external force, having a component X per unit mass in this direction, also act on the element, we have

$$\left.\begin{array}{l}\text{Total force on element}\\ \text{in direction } O X\end{array}\right\} = \left[\left(\frac{\partial p_{xx}}{\partial x} + \frac{\partial p_{yx}}{\partial y} + \frac{\partial p_{zx}}{\partial z}\right) + \rho X\right] \delta x \,.\, \delta y \,.\, \delta z.$$

$$\left.\begin{array}{l}\text{This equals mass} \times \text{acceleration}\\ \text{of element in direction } O X\end{array}\right\} = \rho \,.\, \delta x \,.\, \delta y \,.\, \delta z \times \frac{d u}{d t}.$$

$$\therefore \quad \rho X + \left(\frac{\partial p_{xx}}{\partial x} + \frac{\partial p_{yx}}{\partial y} + \frac{\partial p_{zx}}{\partial z}\right) = \rho \frac{d u}{d t}.$$

Similarly, considering the accelerations in directions $O Y$ and $O Z$, we get

$$\left.\begin{array}{l}\rho Y + \left(\frac{\partial p_{xy}}{\partial x} + \frac{\partial p_{yy}}{\partial y} + \frac{\partial p_{zy}}{\partial z}\right) = \rho \frac{d v}{d t}\\ \rho Z + \left(\frac{p_{xz}}{\partial x} + \frac{\partial p_{yz}}{\partial y} + \frac{\partial p_{zz}}{\partial z}\right) = \rho \frac{d w}{d t}\end{array}\right\} \quad (9)$$

1895, p. 508. In this discussion, p_{xx}, etc., are reckoned positives when tensions, while p, as is common in hydrodynamical problems, is reckoned positive when a compression. This accounts for the negative sign before p in (7).

[1] This may be proved as follows. The difference of normal stress on the faces $O B$ and $C D$ (Fig. 30A) $= \frac{\partial p_{xx}}{\partial x} \delta x$.

$\therefore$ Difference of normal force on these faces $= \frac{\partial p_{xx}}{\partial x} \delta x \,.$ $\delta y \,.\, \delta z$.

Also the difference of tangential stress in the direction $O X$, on the faces $O C$ and $B D = \frac{\partial p_{yx}}{\partial y} \delta y$, while the resultant force due to these tangential stresses in this direction $= \frac{\partial p_{yx}}{\partial y} \,.\, \delta x \,.\, \delta y \,.\, \delta z$.

Similarly the tangential stresses on the faces $O D$ and $B C$ in the direction $O X$ give rise to a resultant force $\frac{\partial p_{zx}}{\partial z} \,.\, \delta x \,.\, \delta y \,.\, \delta z$.

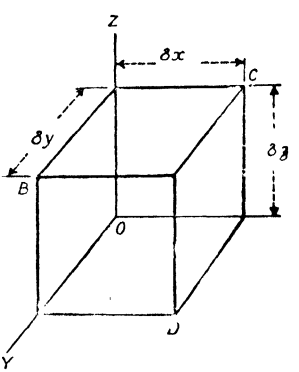


FIG. 30A.

These include all the forces due to variation of stress across the element, which have a component in the direction $O X$. Summing these, we get the resultant force in the direction of increasing x. Similarly for the forces in the directions $O Z$ and $O Y$.

Differentiating the equations (7) and (8), so as to obtain the values of $\frac{p_{xx}}{\partial x}$, etc., in the case of a viscous fluid, and inserting the values thus obtained in equations (9), the first of these becomes

$$\left.\begin{aligned}\rho X - \frac{\partial p}{\partial x} - \frac{2}{3}\mu \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) + 2\mu \frac{\partial^2 u}{\partial x^2} \\ + \mu\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 v}{\partial x \,.\, \partial y}\right) + \mu\left(\frac{\partial^2 w}{\partial x \,.\, \partial z} + \frac{\partial^2 u}{\partial z^2}\right)\end{aligned}\right] = \rho \frac{d u}{d t}.$$

From the equation of continuity we have

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Also $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x\, \partial y} + \frac{\partial^2 w}{\partial x\, \partial z} = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0.$

So that the equations become

$$\left.\begin{aligned}\rho \frac{d u}{d t} &= \rho X - \frac{\partial p}{\partial x} + \mu (\nabla^2 u) \\ \rho \frac{d v}{d t} &= \rho Y - \frac{\partial p}{\partial y} + \mu (\nabla^2 v) \\ \rho \frac{d w}{d t} &= \rho Z - \frac{\partial p}{\partial z} + \mu (\nabla^2 w)\end{aligned}\right\} \qquad (10)$$

where ∇^2 denotes the operator $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)$.

The terms involving μ in these equations are complex, and for purposes of practical application to hydraulic problems are usually neglected. If this is done, the equations of motion for a non-viscous, incompressible fluid become

$$\left.\begin{aligned}\rho \frac{d u}{d t} &= \rho X - \frac{\partial p}{\partial x} \\ \rho \frac{d v}{d t} &= \rho Y - \frac{\partial p}{\partial y} \\ \rho \frac{d w}{d t} &= \rho Z - \frac{\partial p}{\partial z}\end{aligned}\right\} \qquad (11)$$

or, writing $\frac{d u}{d t}$ in terms of $\frac{\partial u}{\partial x}$, etc., from equation (2), and dividing throughout by ρ

$$\left.\begin{aligned}X - \frac{1}{\rho}\frac{\partial p}{\partial x} &= u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t} \\ Y - \frac{1}{\rho}\frac{\partial p}{\partial y} &= u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t} \\ Z - \frac{1}{\rho}\frac{\partial p}{\partial z} &= u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}\end{aligned}\right\} \qquad (12)$$

the Eulerian equations of motion.

Here the axes of co-ordinates have been taken to coincide with the principal axes of stress. If, however, these are transferred, exactly similar equations are obtained.

In the case where gravity is the only external force acting on the fluid, $X = o$, $Y = o$, and $Z = -g$, g acting in the opposite direction to that of z increasing.

So far, the equations are applicable to motion of any kind, whether steady or unsteady, but in their present form are not obviously useful for the solution of any practical problem. So long as the motion is sinuous and irregular nothing further can be done with them.

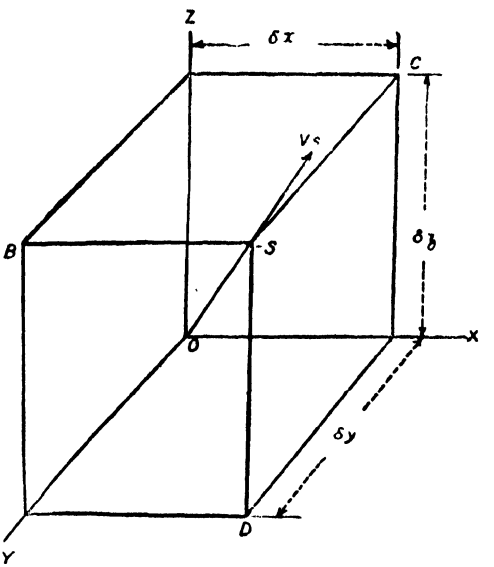


FIG. 30B.

ART. 21. — APPLICATION TO STREAM LINE MOTION.

If, however, the fluid be moving with definite stream line motion, these equations can be considerably simplified. Suppose a particle moving with stream line motion from O, in the direction OS (Fig. 30B), with velocity V_s, the space OS being δs. The direction cosines of this motion in the directions OX, OY, OZ, are $\frac{dx}{ds} \cdot \frac{dy}{ds} \cdot \frac{dz}{ds}$

Also

$$\begin{cases} \frac{dx}{ds} = \frac{u}{V_s} = l \\ \frac{dy}{ds} = \frac{v}{V_s} = m \\ \frac{dz}{ds} = \frac{w}{V_s} = n \end{cases}$$

Again, geometrically we get

$$V_s^2 = u^2 + v^2 + w^2 \qquad (13)$$

And

$$V_s = l\,u + m\,v + n\,w. \qquad (14)$$

For stream line motion, and with only gravity acting, the general equations (12) may then be written

$$\left.\begin{aligned} -\frac{1}{\rho}\frac{\partial p}{\partial x} &= \frac{\partial u}{\partial t} + V_s\left(l\frac{\partial u}{\partial x} + m\frac{\partial u}{\partial y} + n\frac{\partial u}{\partial z}\right) \\ -\frac{1}{\rho}\frac{\partial p}{\partial y} &= \frac{\partial v}{\partial t} + V_s\left(l\frac{\partial v}{\partial x} + m\frac{\partial v}{\partial y} + n\frac{\partial v}{\partial z}\right) \\ -g - \frac{1}{\rho}\frac{\partial p}{\partial z} &= \frac{\partial w}{\partial t} + V_s\left(l\frac{\partial w}{\partial x} + m\frac{\partial w}{\partial y} + n\frac{\partial w}{\partial z}\right) \end{aligned}\right\} \quad (15)$$

Now $$\left(l\frac{\partial u}{\partial x} + m\frac{\partial u}{\partial y} + n\frac{\partial u}{\partial z}\right) = \frac{dx}{ds}\cdot\frac{\partial u}{\partial x} + \frac{dy}{ds}\cdot\frac{\partial u}{\partial y} + \frac{dz}{ds}\cdot\frac{\partial u}{\partial z}$$

$$= \frac{du}{ds}$$

$$\left.\begin{aligned} \therefore\quad -\frac{1}{\rho}\frac{\partial p}{\partial x} &= \frac{\partial u}{\partial t} + V_s\frac{du}{ds} \\ -\frac{1}{\rho}\frac{\partial p}{\partial y} &= \frac{\partial v}{\partial t} + V_s\frac{dv}{ds} \\ -g - \frac{1}{\rho}\frac{\partial p}{\partial z} &= \frac{\partial w}{\partial t} + V_s\frac{dw}{ds} \end{aligned}\right\} \quad (16)$$

Multiplying these equations respectively by l, m, and n, and adding, gives

$$-ng - \frac{1}{\rho}\left(l\frac{\partial p}{\partial x} + m\frac{\partial p}{\partial y} + n\frac{\partial p}{\partial z}\right) = l\frac{\partial u}{\partial t} + m\frac{\partial v}{\partial t} + n\frac{\partial w}{\partial t} + u\frac{du}{ds} + v\frac{dv}{ds} + w\frac{dw}{ds}. \quad (17)$$

From (14) we get $\frac{\partial V_s}{\partial t} = l\frac{\partial u}{\partial t} + m\frac{\partial v}{\partial t} + n\frac{\partial w}{\partial t}$.

From (13) we get $\frac{dV_s^2}{ds} = 2\left(u\frac{du}{ds} + v\frac{dv}{ds} + w\frac{dw}{ds}\right)$,

Also $$l\frac{\partial p}{\partial x} + m\frac{\partial p}{\partial y} + n\frac{\partial p}{\partial z} = \frac{dp}{ds}.$$

$\therefore$ Substituting, equation (17) becomes

$$-ng - \frac{1}{\rho}\frac{dp}{ds} = \frac{\partial V_s}{\partial t} + \frac{1}{2}\frac{d}{ds}(V_s^2)$$

or, since $n = \frac{dz}{ds}$

$$-g\frac{dz}{ds} - \frac{1}{\rho}\frac{dp}{ds} = \frac{\partial V_s}{\partial t} + \frac{1}{2}\frac{d}{ds}(V_s^2),$$

or, $$\frac{d}{ds}\left(\frac{p}{\rho} + \frac{V^2}{2} + gz\right) = -\frac{\partial V_s}{\partial t}.$$

If W = weight of unit volume of the fluid, $W = \rho g$, so that the equation may be written

$$\frac{d}{ds}\left(\frac{p}{W} + \frac{V_s^2}{2g} + z\right) = -\frac{1}{g}\frac{\partial V_s}{\partial t}. \quad (18)$$

Integrating this we get $\frac{p}{W} + \frac{V_s^2}{2g} + z = -\frac{1}{g}\int \frac{\partial V_s}{\partial t} . ds$ (19)

In steady stream line motion, the velocity at any fixed point remains constant, so that $\frac{\partial V_s}{\partial t} = 0$, and the equation then becomes

$$\frac{p}{W} + \frac{V_s^2}{2g} + z = \text{constant.} \quad (20)$$

This equation holds throughout any particular stream line, but the constant varies from one stream line to another. When all stream lines start under the same conditions, as in the case of a flow from a small orifice in the base of a vessel, the constant is the same throughout.

Art. 22.—Change of Pressure across Stream Lines.

If water be moving with stream line motion, these stream lines being curved, the pressure varies from one stream tube to another. This change in pressure may be determined by considering the equilibrium of an elementary column of fluid of sectional area δa (Fig. 31), having its axis radial, and resting with its two ends in two stream tubes, whose pressures are p and $(p + \delta p)$ respectively. The centrifugal force on the column is balanced by the difference of pressure on the two ends and we have, for equilibrium,

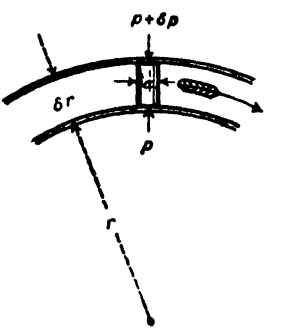


Fig. 31.

$$W . \delta a . \delta r . \frac{v^2}{g r} = \frac{d p}{d r} . \delta r . \delta a$$

$$\therefore \frac{W v^2}{g r} = \frac{d p}{d r}$$

Art. 23.—Hele-Shaw's Experiments.

A geometrical construction for the stream lines in a perfect fluid has been deduced by Professor Rankine, while Professor Hele-Shaw has verified the accuracy of this construction by experiments on a viscous fluid flowing past an obstacle, the motion taking place between parallel glass plates at a very small distance apart.[1] In this case the motion is governed almost entirely by viscosity, all eddy motion is prevented, and, as proved by Professor Sir G. G. Stokes,[1] the effect as regards stream-line formation is the same as in the case of a perfect fluid.

[1] British Association Report, 1898; also "Transactions Inst. Naval Architects," 1898, vol. 85.

This may be shown as follows : [1]

At all points in the same horizontal plane in a perfect fluid we have. if u be the velocity of flow, $\frac{p}{W} + \frac{u^2}{2g} = \text{const.}$

$$\therefore \frac{dp}{W} + u\frac{du}{g} = 0 \qquad (1)$$

If the stream lines are curved, and if δp be the change in pressure, due to centrifugal action, across a stream tube of radial width δr, and of radius of curvature r, we have (p. 59)

$$\frac{dp}{dr} = \frac{W u^2}{g r}$$

and on substituting this value in (1),

$$\frac{u}{r} + \frac{du}{dr} = 0 \qquad (2)$$

In the case of a viscous fluid flowing with stream line motion between two parallel plates, if s be the direction of flow at any instant, we have (p. 61) $\frac{dp}{dy} = 0$; $\frac{dp}{dz} = 0$; $\frac{dp}{ds} = \mu\frac{d^2u}{dy^2}$; while if $\bar{u}$ be the mean velocity of flow this latter equation reduces to

$$\frac{dp}{ds} = -\frac{3\mu}{h^2} \cdot \bar{u} \text{ . (p. 63).}$$

The first two of these equations show that lines of equal pressure cut the stream lines at right angles. If then in Fig. 32, A and C, as also B and D, are points of equal pressure on two adjacent stream lines $A\ B$ and $C\ D$, over which the mean velocities are $u + \delta u$ and u, the corresponding values of δs for equal values of δp are $A\ B$ and $C\ D$, so that

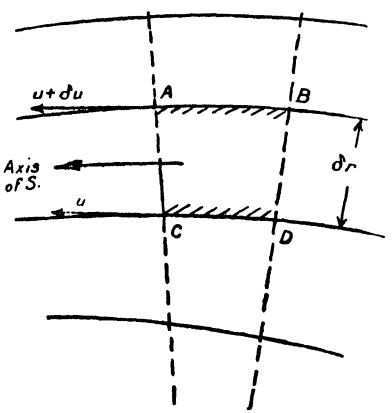


Fig. 32.

$$\frac{A\ B}{C\ D} = \frac{u}{u + \delta u}$$

But the centre of curvature of the stream lines is at the point of intersection of $A\ C$ and $B\ D$ produced, so that $\frac{A\ B}{C\ D} = \frac{r + \delta r}{r}$

$$\therefore \frac{u}{u + \delta u} = \frac{r + \delta r}{r}$$

[1] Dunkerley, "Transactions Inst. Naval Architects," 1900, vol. 42, p. 227.

and, neglecting small quantities of the second order, this reduces to

$$\frac{u}{r} + \frac{du}{dr} = 0$$

which is identical with equation (2) above.

Art. 24.—Flow against Viscous Resistance.

The stream-line motion of a fluid through a pipe or channel is not accompanied by any slip at the boundary of solid and fluid, and the resistance to flow is due to the viscous resistance to the shear of adjacent parallel planes. It follows from this that with steady motion the resistance is independent of the nature of the solid surface.

Steady Flow between Horizontal Parallel Plates.

Take OX, OY, OZ, co-ordinate axes in the fluid (Fig. 33). Let the direction of motion be parallel to the axis of x, and the plates perpendicular to that of y. Suppose the boundaries in the direction of OZ to be infinitely distant, so as not to affect the motion, and neglect the effect of gravity. Let $2h$ = distance between plates, and let OX bisect this distance.

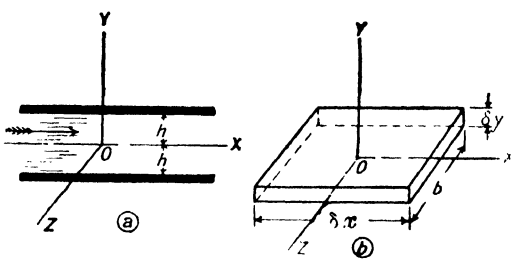


Fig. 33.

Let u, v, and w denote velocities of flow in directions OX, OY, OZ. Then $v = 0$, $\therefore$ if p is the intensity of normal pressure on any plane perpendicular to OX we have $\frac{dp}{dy} = 0$. The variations of velocity and pressure in the direction OZ may be neglected, since the boundaries in this direction and therefore the points at which the pressure may be zero, are at an infinite distance.

It follows that $\frac{dp}{dz} = 0$, so that on any plane perpendicular to OX the intensity of normal pressure is uniform.

The tractive—or shear—force on any plane perpendicular to OY and of area A is $\mu \frac{du}{dy} \cdot A$, where μ = coefficient of viscosity (Art. 4).

The difference of tractive force on the two faces of a stratum of thickness δy, and of area A (Fig. 33 b), is thus given by

$$\frac{d}{dy}\left\{\mu \frac{du}{dy} \cdot A\right\} \delta y = \mu \frac{d^2 u}{dy^2} A \, \delta y.$$

i.e., by the rate of change of this force with respect to y, multiplied by the change in y.

But if b is the width of the stratum, and δx its length, the area of each end $= b\,\delta y$, while its area $A = b\,\delta x$.

Also, since for equilibrium the difference of traction on the two faces of the stratum is equal to the difference of normal pressures on the two ends,

$$b \cdot \delta y \cdot \frac{dp}{dx} \cdot \delta x = \mu \frac{d^2 u}{dy^2} \cdot A\,\delta y = \mu \frac{d^2 u}{dy^2} \cdot b\,\delta x\,\delta y$$

$$\therefore \quad \frac{dp}{dx} = \mu \frac{d^2 u}{dy^2}.^1$$

Integrating this expression, we get

$$u = C + By + \frac{1}{2\mu} \cdot y^2 \cdot \frac{dp}{dx}, \text{ where } C \text{ and } B \text{ are constants.}$$

Since the motion is symmetrical about the axis of x, *i.e.*, is the same for equal positive and negative values of y, the term involving the first power of y must vanish, since this would change sign with y, so that $B = o$.

Again for uniformity of flow $\frac{dp}{dx}$ is constant, while, assuming no slip at the boundaries, we have $u = o$ for $y = \pm h$.

Determining the constant C, so that these boundary conditions are satisfied, we get

$$u = -\frac{1}{2\mu}\left\{h^2 - y^2\right\}\frac{dp}{dx}$$

i.e., the pressure diminishes as x increases. Evidently, too, from the form of the equation the curve of velocities is a parabola.

The flux over unit width of the plates is given by

$$\int_{-h}^{h} u\,dy = -\frac{2}{3}\frac{h^3}{\mu} \cdot \frac{dp}{dx}$$

If the measurements are taken in pounds, feet, and seconds, the volume per second in cubic feet, Q, is given by

$$Q = \frac{2}{3}\frac{h^3}{\mu} \cdot \frac{dp}{dx}.$$

[1] This may be deduced directly from the general equations of motion, p. 56, for since $v = o$; $w = o$; $\frac{du}{dt} = o$; equations (10) reduce to

$$\frac{\partial p}{\partial x} = \mu\,(\nabla^2 u).$$

Since $\frac{dp}{dy} = o$; $\frac{dp}{dz} = o$; $\frac{du}{dz} = o$; $\frac{du}{dx} = o$, this becomes $\frac{dp}{dx} = \mu \frac{d^2 u}{dy^2}$.

If $p_1 - p_2 =$ fall of pressure between two points at a distance l apart this becomes

$$Q = \frac{2}{3}\frac{h^3}{\mu} \cdot \frac{p_1 - p_2}{l}.$$

The maximum velocity occurs at the axis, where $y = 0$, and equals $\frac{1}{2\mu} \cdot h^2 \cdot \frac{dp}{dx}$.

$$\text{The mean velocity} = \frac{\text{flux over a section}}{\text{area of section}} = \frac{Q}{2h} = \frac{1}{3\mu} h^2 \frac{dp}{dx}$$

$$\therefore \frac{\text{Maximum velocity}}{\text{Mean velocity}} = \frac{3}{2}.$$

The shear stress on the boundaries is given by

$$\mu \frac{du}{dy} = y \frac{dp}{dx} = h \cdot \frac{p_1 - p_2}{l}.$$

Art. 25.—Steady Flow through a Circular Pipe.

Suppose the pipe to be horizontal, and, neglecting the effect of gravity, assume the flow to be produced by a uniform difference of pressure head along its length. Let the axis of x be the axis of the tube, and let a be its radius. Using the same notation as in the preceding example, and assuming the velocity everywhere parallel to OX, we have $v = o$, $w = o$.

$$\therefore \frac{dp}{dy} = 0\,; \frac{dp}{dz} = 0, \text{ as in the previous case.}$$

The tangential stress, or tractive force per unit area, on a plane perpendicular to a radius $= \mu \frac{du}{dr}$.

Hence for a cylindrical shell concentric with the pipe, of length δx and having inner and outer radii r and $r + \delta r$, the difference of the tangential force on the inner and outer surfaces will be $\frac{d}{dr}\left\{ 2\pi r \cdot \delta x \cdot \mu \frac{du}{dr} \right\} \delta r$, and this must be balanced by the difference of normal pressure on the two ends of the shell.

Since $\frac{dp}{dr} = o$, the pressure intensity p at any point of the section is uniform.

$$\therefore \quad 2\pi r \cdot \delta r \cdot \frac{dp}{dx} \cdot \delta x = \frac{d}{dr}\left\{ 2\pi r \cdot \delta x \cdot \mu \frac{du}{dr} \right\} \delta r$$

$$\therefore \quad \frac{d}{dr}\left\{ r \frac{du}{dr} \right\} = \frac{dp}{dx} \cdot \frac{r}{\mu}.$$

Integrating, we get

$$u = \left\{ \frac{1}{4\mu} \frac{dp}{dx} \right\} r^2 + A \log r + B. \tag{1}$$

Since the velocity at the axis, where $r = o$, cannot be infinite and since $\log o = \text{inf.}$, $A = o$.

Determining B, so that $u = o$ when $r = \pm a$, *i.e.*, for no slip at the boundaries, gives

$$u = -\frac{1}{4\mu} \cdot \frac{dp}{dx} \cdot \left\{ a^2 - r^2 \right\} \tag{2}$$

So that the flux through the pipe, which equals

$$\int_o^a u \cdot 2\pi r \, dr = \frac{\pi a^4}{8\mu} \cdot \frac{dp}{dx} \text{ cubic feet per second.}$$

Writing $\frac{dp}{dx}$ as $\frac{p_1 - p_2}{l}$ where p_1 and p_2 are the pressure intensities at a distance l apart, along the axis of the pipe, we have

$$Q = \frac{\pi a^4}{8\mu} \cdot \frac{p_1 - p_2}{l} \text{ cubic feet per second.}$$

The maximum velocity is obtained by putting $r = o$ in equation (2) and equals $\frac{p_1 - p_2}{4\mu l} \cdot a^2$ feet per second.

$$\text{The mean velocity} = \frac{Q}{\text{area of section}} = \frac{p_1 - p_2}{8\mu l} \cdot a^2 \text{ feet per second.} \tag{3}$$

$\therefore$ Maximum velocity = 2 (mean velocity).

On equating expressions (2) and (3) it is readily shown that the filament of mean velocity is found at a radius ·707 a.

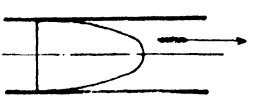


FIG. 34.

From equation (2) we see that the curve of velocities across a diameter is a parabola, and that the surface of velocities for the pipe will be a paraboloid of revolution (Fig. 34).

If $\bar{v}$ = mean velocity, we have $p_1 - p_2 = \frac{32\mu l \bar{v}}{d^2}$, Poiseuille's form of the equation.

The shear stress at the boundary is given by

$$\mu \frac{du}{dr} = \frac{r}{2} \cdot \frac{dp}{dx} = \frac{a}{2} \cdot \frac{p_1 - p_2}{l}. \tag{4}$$

Art. 26.—Steady Flow through a Circular Tube, assuming Slip at the Boundaries.

If we assume slip at the boundaries of a circular tube, some assumption must be made as to the magnitude of the slip before any determination of the pressure fall along the tube can be made. Assuming, as seems rational, that the velocity at the boundaries is proportional to the tangential stress $u\left(\frac{d\,u}{d\,r}\right)_{r=a,}$ we have if u' = velocity at boundaries, then

$u' \propto \mu\left(\frac{d\,u}{d\,r}\right)_{r=a}$ or $\quad u' = -k\left(\frac{d\,u}{d\,r}\right)_{r=a} = \frac{k\,a}{2\,\mu}\cdot\frac{d\,p}{d\,x}$ from (4), Art. 25.

Using this to determine B in equation (1), Art. 25, and proceeding as before, we finally get

$$Q = \frac{\pi\,a^4}{8\,\mu}\left\{1 + \frac{4\,k}{a}\right\}\frac{p_1 - p_2}{l} \text{ cubic feet per second.}$$

If this formula be used to calculate the value of μ, experiment shows that different values are obtained with different diameters, whereas if formula (3), p. 64, be used the values obtained for μ are independent of the diameter, showing that the assumption that there is no slip at the boundaries is justified.

Art. 27.—Eddies—Cause of Eddy Formation.

It may be shown that the conditions for dynamical similarity of flow in two similar pipes of different diameters, or past two submerged similar bodies of different size, are only satisfied when the value of $v/v\,l$ is the same for each. The ratio of the forces acting on two similar particles, due respectively to viscosity and to inertia, are then the same for each, and it will therefore be at some definite value of this criterion that the change over from stream line to eddy motion takes place.

Since the viscous forces are proportional to v, and inertia forces to v^2, the relative value of the former forces becomes less as the velocity increases. At sufficiently low velocities the stabilising effect of viscosity is paramount, and any tendency to deviation from stream line motion, such as may be produced by roughness of the surface, is damped out without producing eddies. As the speed increases, however, the inertia forces become so relatively large that, at some definite velocity, viscosity is no longer able to prevent deviations from stream line flow, and any surface irregularities cause rapidly rotating particles of fluid to be projected from the surface to form eddy nuclei.

The axis about which the rotation of an eddy takes place may be either

straight, curved, or may return on itself in which case a vortex ring is formed. With an eddy of one of the former types, the motion is essentially unstable unless the ends of the axis are in contact with some solid surface. Thus, such an eddy as is usually formed is rapidly damped out by viscous resistances and its energy is dissipated in the form of heat.

On the other hand, a stable vortex, whether with a rectilinear or a curvilinear axis, may be projected through the surrounding fluid with surprisingly small loss of energy. An example of this may be found in the case of the ordinary smoke ring in air, or of a similar vortex ring in water.

Such a stable vortex consists of a rapidly rotating nucleus, surrounded by a " field " in which the motion is cyclical and irrotational, and in which the velocity diminishes rapidly with the distance from the axis.

When a stream of fluid flows past an immersed solid, at all but the slowest velocities eddies are formed in the rear of the solid, which, however, are not of the type already described, in that the motion is not now cyclic, the mass comprising an eddy being composed partly of fluid flowing around the edges of the solid and partly of fluid drawn from its rear face. As a consequence of this there is a continual backward flow towards the central portion of this face, to make good the loss of fluid abstracted by the eddies at its sides, the state of affairs being represented in Fig. 35 A.

Where the body extends from the surface to the bottom of the stream, so that the motion is sensibly in two dimensions, the eddy formation is a discontinuous process. In such a case eddies may be formed either simultaneously at each edge, or alternately at the two edges of the body as shown in Fig. 35 B.

The eddies as first formed are small, and gradually grow until sufficiently large to impede the rearward flow in the wake which is necessary to feed them. They then break away and join the procession of eddies forming the boundary of the wake.

When formed in three dimensions, eddies may be produced either continuously or discontinuously. In the former case they are at different stages in their growth at different points of the perimeter of the body, and appear in the wake as a spiral (Fig. 35 C). This type of vortex formation is usually found in the rear of a plate, normal or slightly inclined to the current. If the plate be inclined to the current at an angle exceeding about 30° the formation becomes discontinuous, as indicated in Fig. 35 D,[1] which shows the eddies formed behind a square plate inclined at 40° to

[1] Tech. Report, Aviation Committee, 1911-12.

the stream. In this type of motion two spirals are formed from the sides of the plate and are linked together to form a chain of eddies by a sheet of fluid flowing from the trailing edge of the plate.

From the manner of eddy formation it is evident that the pressure on the rear surface of the body in a current of fluid will be less than normal, and subject to a periodic variation, the period depending on the time of formation of an eddy and so on the shape and dimensions of the body and on the velocity of flow.

If the body is supported so as to be capable of vibration, and the period

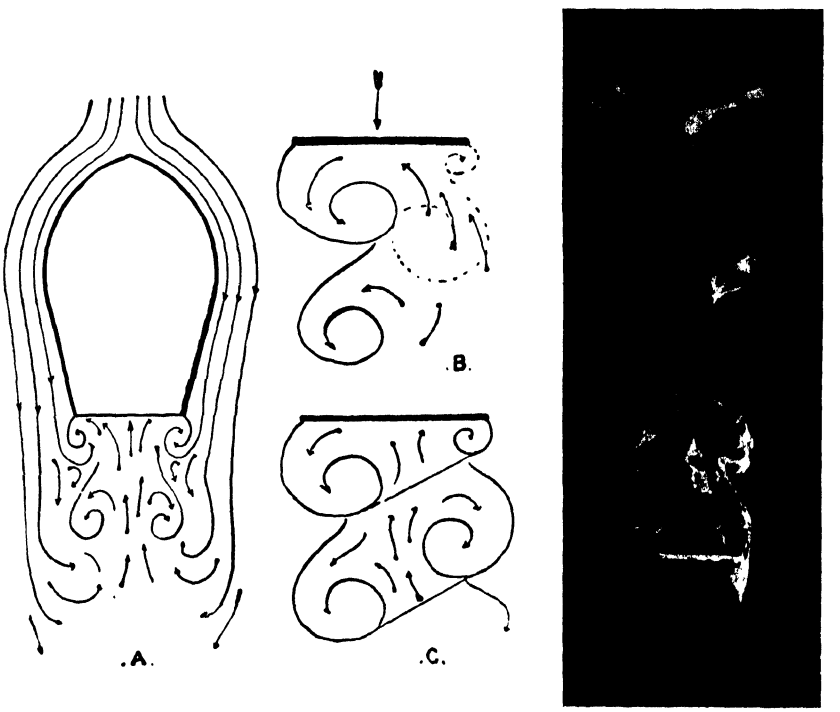


FIG. 35.

of eddy formation synchronises, or nearly so, with that of its natural vibration, oscillations are set up which may become very large. This effect may often be noted where a flexible bough dips into a steady stream. On account of this action, current meters, in which the pressure on a stationary flat plate is taken as a measure of the velocity, are not satisfactory.

CHAPTER IV.

Hydrodynamical Resistance.

General Considerations—Relative Velocity—Dynamical Similarity—Corresponding Speeds—Theoretical Formulae—Pipe Flow—Distribution of Velocity—Resistance of Submerged Bodies—Of Partially Submerged Bodies—Scale Effects.

Art. 28.

A body in steady motion through a fluid, or at rest in a moving current, experiences a resistance whose magnitude depends upon the relative velocity, the physical properties of the fluid, the size and form of the body, and, at velocities above the critical, also upon its surface roughness.

At velocities below the critical, where the flow is "stream line," the resistance is due essentially to the viscous shear of adjacent layers of the fluid. It is directly proportional to the velocity, to the viscosity, and, in bodies of similar form, to the length of corresponding dimensions. There is no slip at the boundary of solid and fluid, and the physical characteristics of the surface do not affect the resistance.

At velocities above the critical, where the motion as a whole is definitely turbulent, there is still a layer of fluid in contact with the surface in which the motion is non-turbulent.[1] This is the so-called "Boundary Layer." The thickness of this diminishes as the velocity increases, being approximately proportional to $1/R_e$. If the irregularities of the solid surface are so small as not to project beyond this layer the surface is in effect smooth. If they do project they promote eddy formation, whose magnitude depends on the degree of roughness of the surface. The resistance is then due in part to the viscous shear in the surface layer but mainly to eddy formation in the main body of the fluid.

Although the viscosity of a fluid provides the mechanism by which eddy formation becomes possible and by which the energy of the eddies when formed is dissipated, it has only a very small effect on the magnitude of the resistance in turbulent motion, and, as will be shown later, it would have no direct effect in a system in which the resistance was wholly due to eddy formation and, consequently, proportional to v^2.

[1] Stanton, "Proc. Roy. Soc. A.," vol 97, 1920.

Experiments carried out over a limited range of velocities have usually shown that with turbulent flow the resistance of any given body is proportional to v^n, where n is slightly less than 2, although experiments on flow in rough pipes, on the resistance of cylinders, of inclined plane surfaces, and of airship bodies, show that in such cases the variation from the index 2 may be within the limits of experimental error. With smooth pipes, however, n may be as low as 1·75, and with ship-shaped bodies of fair form in water is usually about 1·85.

More recent experiments indicate that no one constant value of n holds over a very wide range of velocities, but that n increases with the velocity, and that a formula of the type

$$R = A\,v + B\,v^2 + C\,v^3,$$

where A and B and C are constants depending upon the form and roughness of the body and on the physical properties of the medium, more nearly represents the actual results. Over a moderate range of velocities, however, a single value of n can usually be obtained which gives the resistance within the errors of observation, and in view of the convenience of such an exponential formula, it is commonly adopted in practice.

At velocities above the critical, the direct influence of viscosity increases with the departure of the index n from 2. When $n = 2$ the resistance is proportional to the density of the fluid, and, in similar bodies, to the square of corresponding dimensions.

Between the low velocities at which the motion is stream line, and the high velocities at which it is definitely turbulent, there is a range over which it is extremely unstable, and in which the resistance may be affected considerably by small modifications in the form, presentation, or surface condition of the body. Thus the resistance of a sphere at a certain velocity, whose magnitude depends on the diameter, is actually diminished instead of being increased by increasing its roughness.

In problems occurring in practice, however, velocities are, in general, well above the critical point. One noteworthy exception is to be found in the flow of oil through pipe lines in which, owing to the high viscosity of the fluid, the motion is usually non-turbulent.

ART. 29.

In hydrodynamical problems it is usual to assume that the resistance depends solely on the relative velocity of fluid and body, and that it is

immaterial whether the body is at rest in a current of fluid or is moving through fluid at rest. Although there is not much direct experimental evidence on this point, it is probable that while with stream line motion the resistance is identical in both cases, in turbulent motion it is not necessarily so, and that it may be sensibly greater when the fluid is in motion than when the body is in motion.

This is to be expected when it is realised that in a fluid in motion with a mean velocity v many of its particles have a higher velocity, so that the kinetic energy is greater than that given by the product of the mass and the square of the mean velocity. Any difference arising from this effect will, in general, only be small, but comparatively large differences may be expected over the range of velocities in which the motion is turbulent owing to the fact that, with a stationary body, the interaction occurs in a medium which has an initial tendency to instability owing to its motion.

Thus the system of eddy formation in the rear of any solid body advancing into still water may reasonably be expected to differ from that behind the same body in a current of the same mean velocity, owing to the unstability in the latter case of the medium in which, and from which in part, it is being maintained.

Art. 30.—Dimensional Homogeneity and Dynamical Similarity.

Except in the case of stream line flow, the laws of hydrodynamical resistance can only be deduced experimentally. Much information can, however, be obtained regarding these laws from an application of the principles of "dimensional homogeneity" and "dynamical similarity."

The first of these states that all the terms of any correct and complete physical equation must have the same dimensions. That is, if the numerical value of any one term in the equation depends on the size of one of the fundamental units, every other term must depend upon it in the same way, so that if the size of the unit is changed, every term will be changed in the same ratio, and the equation will still remain valid.

Two systems, involving the motion of fluid relative to geometrically similar bodies, are said to be dynamically similar when the paths traced out by corresponding particles of the fluid are also geometrically similar and when the velocities are such that all corresponding forces acting on two such particles are in the same ratio, which ratio must be that of the mass $\times$ acceleration, since this is fixed.

In two systems, denoted by (1) and (2), if w be the weight of unit volume, ρ the density, v the velocity, l any definite linear dimension, and r the radius of curvature of the path, these forces are in the ratio

$$\frac{F_1}{F_2} = \frac{w_1 v_1^2}{r_1} \cdot \frac{r_2}{w_2 v_2^2} = \frac{\rho_1 l_1^3 r_2}{\rho_1 l_2^3 r_1} \left(\frac{v_1}{v_2}\right)^2$$
$$= \frac{\rho_1}{\rho_2} \cdot \frac{l_1^2}{l_2^2} \left(\frac{v_1}{v_2}\right)^2.$$

It follows that for dynamical similarity corresponding velocities must be such as to make the corresponding forces due to each physical factor proportional to $\rho l^2 v^2$. The velocities so related are termed "Corresponding Speeds."

ART. 31.

Where the only physical factor involved is the weight of the fluid, since the force due to this is proportional to ρl^3, the required condition will evidently be satisfied if $\frac{v_1}{v_2} = \sqrt{\frac{l_1}{l_2}}$.

On the other hand, if viscous forces are all important, since the force due to viscosity equals $\mu \frac{dv}{dl}$ per unit area, where μ is the coefficient of viscosity,

$$\frac{F_1}{F_2} = \frac{\mu_1}{\mu_2} \cdot \frac{l_1^2}{l_2^2} \cdot \frac{v_1}{v_2} \cdot \frac{l_2}{l_1}$$
$$= \frac{\mu_1 l_1 v_1}{\mu_2 l_2 v_2},$$

and for this to equal
$$\frac{\rho_1 l_1^2 v_1^2}{\rho_2 l_2^2 v_2^2}$$

it is necessary that
$$\frac{\rho_1 l_1 v_1}{\mu_1} = \frac{\rho_2 l_2 v_2}{\mu_2},$$

i.e., that
$$\frac{v_1}{v_2} = \frac{\mu_1 \rho_2 l_2}{\mu_2 \rho_1 l_1} = \frac{\nu_1 l_2}{\nu_2 l_1},$$

where ν is the "kinematic viscosity," $\frac{\mu}{\rho}$.

Generally speaking, wherever the influence of gravity is involved in the interaction between a solid and a fluid, as is the case where surface waves or surface disturbances are produced, and where the direct influence of viscosity is negligibly small, corresponding speeds are proportional to the square root of corresponding linear dimensions; while where gravitational forces are not involved, and where the forces are due essentially to viscous resistances, corresponding speeds are inversely proportional to corresponding linear dimensions, and directly proportional to the corresponding kinematic viscosities.

The flow of water from a sharp-edged orifice under the action of gravity is

typical of the first type of interaction, while the resistance of an airship, or of a submarine submerged to such a depth that no surface waves are produced, is an example of the second type.

The resistance of a surface vessel is one of a series of cases, of importance in practice, in which both gravity and viscosity are involved, and in which, therefore, no two corresponding speeds will satisfy all requirements. In other words, the speeds which give geometrically similar wave formations around two similar ships will not give similar stream lines in those parts of the systems subject to viscous flow. If, however, the influence of one of these factors is much greater than that of the other, approximately similar results, which may be of great value in practice, can be obtained by using corresponding speeds chosen with reference to the important factor. Thus, in tank experiments on models of floating vessels, the corresponding speeds are chosen with reference to the wave and eddy effects, and are proportional to the square root of corresponding linear dimensions. This involves a scale error for which a correction is made, as explained later (Art. 134).

Art. 32.

In view of the value of the results which may be obtained by the use of the principle of dynamical similarity in problems involving fluid resistance, the method of its more general application will now be outlined.

The physical relationship involved in any interaction between solids and fluids, giving rise to motion, may comprise some or all of the following quantities :—

Quantity.	Symbol.	Dimensions.
Length . . .	l	l
Time . . .	t	t
Velocity . . .	v	lt^{-1}
Acceleration . .	g	lt^{-2}
Density . . .	ρ	Ml^{-3}
Force . . .	R = mass × accn.	Mlt^{-2}
Viscosity . . .	μ = stress × $\frac{l}{v}$	$Ml^{-1}t^{-1}$
Elasticity . . .	E = stress	$Ml^{-1}t^{-2}$

All these may be defined in terms of three fundamental units. The most convenient units are usually M, l, t, although any such combination as v, l, t, or R, v, l, containing the three units M, l, t, may be used if desired.

In those problems of Hydraulics in which only steady flow is involved, the elasticity of the fluid is unimportant, and the only factors involved are R, l, v, ρ, μ, g.

Suppose some physical relationship to involve the first four of these quantities in the form

$$R \propto l^x v^y \rho^z \mu^p.$$

Then for this relationship to be true whatever units of mass, length, or time, are used, the equation must be dimensionally homogeneous, and, since a mass can only be equal to a mass, a length to a length, and a time to a time, these quantities must each be identical for both sides of the equation.

Expressing the equation dimensionally gives

$$M l t^{-2} = l^x . l^y t^{-y} . M^z l^{-3z} . M^p l^{-p} . t^{-p},$$

and, equating the indices of like quantities, we have

$$\left.\begin{aligned} 1 &= z + p \\ 1 &= x + y - 3z - p \\ -2 &= -y - p \end{aligned}\right\} \text{ whence } \begin{aligned} z &= 1 - p \\ y &= 2 - p \\ x &= 2 - p \end{aligned}$$

so that

$$R \propto l^{2-p} . v^{2-p} . \rho^{1-p} . \mu^p$$

$$\propto l^2 v^2 \rho \left(\frac{\mu}{v l \rho}\right)^p$$

Since p is unknown, this may be written as

$$R = l^2 v^2 \rho \phi \left(\frac{\mu}{v l \rho}\right) \quad . . . . . (1)$$

where ϕ represents some function whose form can only be deduced from experiment.

If the effect of viscosity is negligible, but if the weight of the fluid is involved in the phenomena, the relationship becomes

$$R \propto l^x v^y \rho^z g^p$$

or

$$M l t^{-2} = l^x . l^y t^{-y} . M^z l^{-3z} . l^p t^{-2p},$$

from which

$$\left.\begin{aligned} 1 &= z \\ 1 &= x + y - 3z + p \\ -2 &= -y - 2p \end{aligned}\right\} \text{ whence } \begin{aligned} z &= 1 \\ y &= 2 - 2p \\ x &= 2 + p \end{aligned}$$

so that

$$R \propto l^{2+p} . v^{2-2p} . \rho . g^p$$

$$\propto l^2 v^2 \rho \left(\frac{lg}{v^2}\right)^p$$

or

$$R = l^2 v^2 \rho \phi \left(\frac{lg}{v^2}\right) \quad . . . . . (2)$$

If the force depends on both the weight and viscosity of the fluid, the relationship becomes

$$R \propto l^x \cdot v^y \cdot \rho^z \cdot \mu^p \cdot g^q$$

or $$M\,l\,t^{-2} = l^x \cdot l^y\,t^{-y} \cdot M^z\,l^{-3z} \cdot M^p\,l^{-p}\,t^{-p} \cdot l^q\,t^{-2q}$$

from which

$$\left.\begin{aligned} 1 &= z + p \\ 1 &= x + y - 3z - p + q \\ -2 &= -y - p - 2q \end{aligned}\right\} \text{ whence } \begin{aligned} z &= 1 - p \\ y &= 2 - p - 2q \\ x &= 2 - p + q \end{aligned}$$

so that $$R \propto l^{2-p+q} \cdot v^{2-p-2q} \cdot \rho^{1-p} \cdot \mu^p \cdot g^q$$

$$\propto l^2 v^2 \rho \left(\frac{\mu}{v\,l\,\rho}\right)^p \cdot \left(\frac{l\,g}{v^2}\right)^q$$

or $$R = l^2 v^2 \rho\, \psi\left(\frac{\mu}{v\,l\,\rho} \cdot \frac{lg}{v^2}\right) \quad \ldots\ldots\ldots\ldots \quad (3)$$

In this case, for true dynamical similarity of flow past two similar bodies of different size it would be necessary for the ratios $\mu/v\,l\,\rho$ and $l\,g/v^2$ to be simultaneously identical for both cases of flow. If the same fluid is used in each, it is evident that this is impossible, since the first ratio requires $v \propto 1/l$ while the second requires $v \propto \sqrt{l}$.

Art. 33.—Resistance to Uniform Flow through a Parallel Pipe.

An examination of the factors involved in the non-accelerated flow of a fluid through a pipe indicates that the resistance to flow per unit length of the pipe may depend in some way upon the diameter d, the velocity of flow, and on the density and viscosity of the fluid. It is difficult to imagine any other physical factor likely to affect the resistance except the roughness of the walls. If the roughness is proportional to the diameter, the only factors involved are R, l, d, v, ρ, μ, while in similar pipes l is proportional to d. Under these conditions we have (equation 1, p. 73),

$$R = l^2 v^2 \rho\, \phi\left(\frac{\mu}{v\,l\,\rho}\right) = l\,d\,v^2 \rho\, \phi\left(\frac{\mu}{v\,d\,\rho}\right)$$

Here R is the total resistance to flow, so that if p be the pressure drop over the total length of pipe, $R = \pi\,d^2\,p \div 4$,

and $$\frac{p}{l} = \frac{v^2 \rho}{d}\, \phi\left(\frac{\mu}{v\,d\,\rho}\right)$$

In stream line flow, experiment shows that p/l is proportional to v. It follows that $\phi\left(\frac{\mu}{v\,d\,\rho}\right)$ must equal $\frac{k\mu}{v\,d}$, and that $\frac{p}{l} = \frac{k\,\mu\,v}{d^2}$,

where k is a numerical constant. This is Poiseuille's expression for the resistance to viscous flow, k having the value 32.[1]

If the flow is turbulent, experiment shows that the pressure gradient is nearly proportional to v^n, where n is between 1·75 and 2·0, in which case $\phi\left(\frac{\mu}{v d \rho}\right)$ must be such as to make $\phi\left(\frac{1}{v}\right) = v^{n-2}$, so that

$$\frac{p}{l} \propto \frac{\rho v^2}{d}\left(\frac{\mu}{\rho v d}\right)^{2-n}$$

$$= \frac{k' \rho v^n \nu^{2-n}}{d^{3-n}} \qquad \text{where } \nu = \frac{\mu}{\rho}$$

or

$$h = \frac{k' v^n \nu^{2-n}}{d^{3-n}} \cdot l$$

where h is the difference of head at two points distant l apart, expressed as a length of a column of the fluid. This is the Reynold's formula for pipe flow.[2]

If the foregoing assumptions are correct, on plotting observed values of $\frac{p d}{l \rho v^2}$ against simultaneous values of $\frac{\nu}{v d}$ in any series of experiments in which different liquids or pipes of different diameters are used, the points should lie on a single curve.

That this is the case for fluids so widely different as air, water, and oil, has been shown by various observers, notably by Stanton and Pannell [3] (Fig. 36). The experimental points for both air and water lie evenly about the two curves shown, the upper referring to a rough pipe, and the lower to a smooth pipe. The agreement in the case of air, can, however, only be expected to be close where the drop in pressure is so small that the effect of the change of density is negligible.

Art. 34.

Equation 1 (p. 73) shows that with an incompressible fluid, if the resistance to flow varies as v^2, $\phi\left(\frac{\nu}{v d}\right)$ degenerates into a numerical constant. Viscosity ceases to have any direct influence on the resistance,[4] and true similarity of flow should be obtained in all pipes at all velocities. If n is less than 2, the equations show that for similarity of flow in two pipes of different

[1] Art. 25.
[2] Osborne Reynolds, "Scientific Papers," vol. ii., p. 97.
[3] "Phil. Trans. Roy. Soc.," vol. A, 214, 1914, p. 199.
[4] See footnote on p. 79.

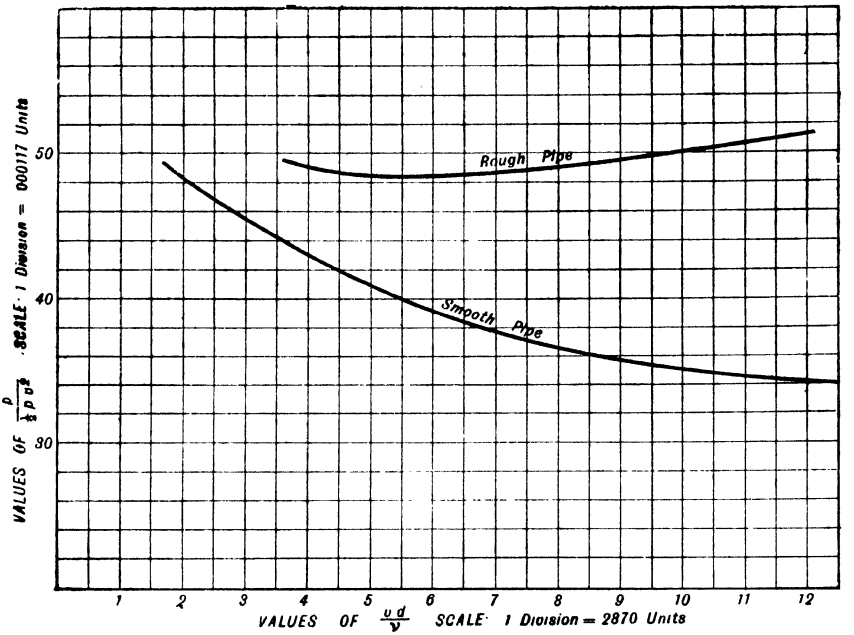

Rough Pipe
Smooth Pipe
50
40
30
1
2
3
4
5
6
7
8
9
10
11
12
VALUES OF $\frac{p}{\frac{1}{2}\rho v^2}$ SCALE· 1 Division = 000117 Units
VALUES OF $\frac{v d}{\nu}$ SCALE· 1 Division = 2870 Units

FIG. 36.

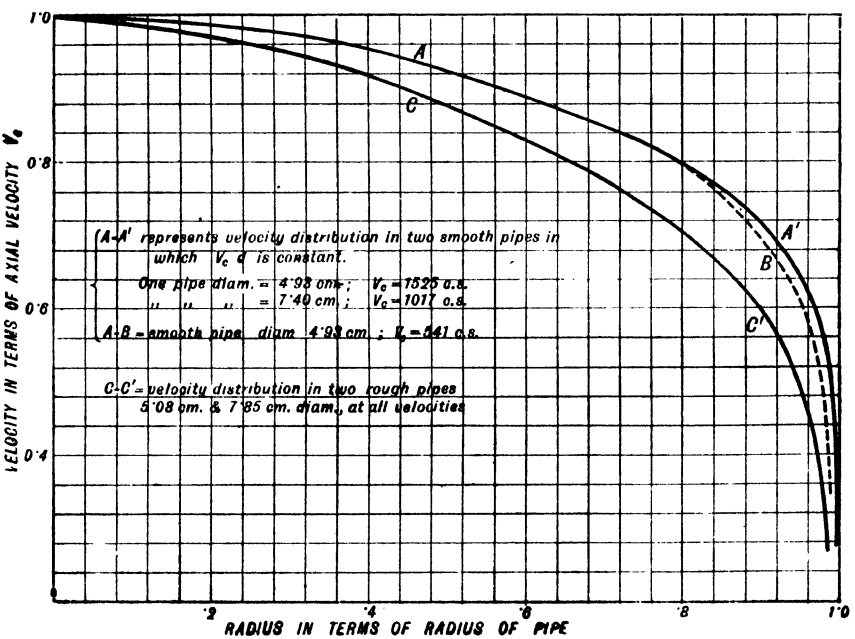

A
C
A'
B
C'
(A-A' represents velocity distribution in two smooth pipes in which V_c d is constant.
One pipe diam. = 4·93 cm. ; V_c = 1525 c.s.
" " " = 7·40 cm. ; V_c = 1017 c.s.
A-B = smooth pipe diam 4·93 cm. ; V_c = 541 c.s.
C-C' = velocity distribution in two rough pipes 5·08 cm. & 7·85 cm. diam., at all velocities
1·0
0·8
0·6
0·4
·2
·4
·6
·8
1·0
VELOCITY IN TERMS OF AXIAL VELOCITY V_c
RADIUS IN TERMS OF RADIUS OF PIPE

FIG. 37.

diameters, or conveying different fluids, it is necessary that $\frac{\nu}{vd}$ should be the same in both cases.

This has been shown to be true by Stanton,[1] who measured the distribution of velocity with turbulent flow across the diameters of two rough pipes of different diameters, and repeated the measurements for two smooth pipes. In the rough pipes ($n = 2$) the velocity distribution (CC', Fig. 37) was the same in both pipes and at all velocities. In the smooth pipes ($n < 2$) identical curves were obtained only when ν/vd was the same in each pipe. Under other conditions the curves were sensibly identical from the centre up to a radius of about 0·8 times the radius of the pipe, but differed appreciably at larger radii (AA' and AB, Fig. 37).

In any type of pipe the "critical velocity" at which the type of motion changes from stream line to turbulent is obtained with a constant value of ν/vd, and this is generally true for fluid motion under other circumstances.

As already indicated, experiment shows that the resistance to flow in a smooth pipe, where n is less than 2, is not strictly proportional to any one power of the velocity, and Dr. C. H. Lees [2] has shown that Stanton and Pannell's results for smooth-drawn brass pipes are very closely represented by the empirical relationship

$$\frac{p}{l} = \frac{\rho v^2}{d}\left\{ a\left(\frac{\nu}{vd}\right)^{\alpha} + b \right\}$$

where $\alpha = 0{\cdot}35$ and a and b are constants, so that

$$\phi\left(\frac{\nu}{vd}\right) = a\left(\frac{\nu}{vd}\right)^{\cdot 35} + b$$

In Stanton's rough pipe the ratio of friction to v increases with velocity, and the authors suggest, for both rough and smooth pipes, an expression of the form

$$F = \rho v^2 \left\{ A\frac{\nu}{vd} + K + B\frac{vd}{\nu} \right\}$$

in which K depends only on the roughness of the pipe.

Art. 35.—Resistance of Wholly Submerged Bodies.

Where a body is submerged to such a depth that no surface waves are formed, gravity has no effect on the resistance. This will be the case for a deeply submerged submarine, or for an airship. If there is no acceleration

[1] "Proc. Roy. Soc. A.," vol. 85, 1911.

[2] See also Art. 74.

the factors involved in the resistance are the same as in the case of flow through a pipe and the resistance is given by

$$R = \rho l^2 v^2 \phi \left(\frac{lv}{\nu}\right)$$

The value of the unknown function ϕ may be found by plotting observed values of $R/l^2 v^2 \rho$ against simultaneous values of lv/ν, and by finding an empirical equation to represent the plotted points. Since the values of both terms in the expression for R are dependent on v, the form of the function can be determined from experiments on a single body at different speeds in the same medium.

In model experiments, if the medium is the same for model and original, and if the suffix " m " denotes the model, then if the speeds be chosen so that $v_m l_m = v l$, we have

$$\frac{lv}{\nu} = \frac{l_m v_m}{\nu},$$

so that $\phi\left(\frac{lv}{\nu}\right)$ becomes a constant, and

$$\frac{R}{R_m} = \left(\frac{lv}{l_m v_m}\right)^2 = 1.$$

The speeds thus related are " corresponding speeds," and at these speeds the model and its original are " dynamically similar." In this case the corresponding speeds are inversely proportional to the linear dimensions, and at these speeds the resistance of the model and of the original are equal.

Unfortunately, this relationship would involve such high speeds in the case of the model as to be of no practical value. If, however, the model experiments can be carried out in a medium whose kinematic viscosity is less than that of the original, the corresponding speeds are reduced. For example, the tests may be carried out in a compressed air wind tunnel say under a pressure of 25 atmos. In this case $\frac{\rho_m}{\rho} = 25$ while $\frac{\nu_m}{\nu} = \frac{1}{25}$, since in a gas μ is independent of pressure.

Then $\frac{lv}{l_m v_m} = \frac{\nu}{\nu_m} = 25$ and $\frac{v}{v_m} = 25\,\frac{lm}{l}$.

At these corresponding speeds $\frac{R}{R_m} = 625\,\frac{\rho}{\rho_m} = 25$.

Screw Propellers or Air Screws.—Here the turbulence is so great that the direct effect of viscosity may be neglected. Assuming the thrust T to be given by $T \propto d^x v^y \rho^z n^a g^b$, where v is the velocity of translation and n the revs. per min., then since $n \propto \frac{1}{t}$ we have

$$\frac{Ml}{t^2} = l^x \left(\frac{l}{t}\right)^y \left(\frac{M}{l^3}\right)^z \left(\frac{1}{t}\right)^a \left(\frac{l}{t^2}\right)^b$$

from which in the usual way we get

$$T \propto \rho d^2 v^2 \left(\frac{dn}{v}\right)^a \left(\frac{dg}{v^2}\right)^b.$$

Adjusting the speeds of translation so that $dn \div v$ is the same for model and original, and making $v \propto \sqrt{d}$ (corresponding speeds) we have

$$\frac{T}{T_m} = \frac{\rho}{\rho_m} \left(\frac{d}{d_m}\right)^3.$$

If gravity does not affect the results, as in the case of an air screw or deeply submerged propeller we have $T \propto \rho d^2 v^2 = K \rho d^2 v^2$ and the model experiments can be carried out at any speed.

Art. 36.—Resistance of Partially Submerged Bodies.

When a body is partially submerged, or although submerged is so near the surface that surface waves are produced, part of the resistance to motion is due to this wave formation. The influence of gravity and of viscosity must now be taken into account, and, as shown on p. 74, the resistance is given by

$$R = l^2 v^2 \rho \psi \left(\frac{\nu}{vl} \cdot \frac{lg}{v^2}\right).$$

In the case of model experiments it is necessary for dynamical similarity that each of the terms of ψ shall have the same value for both model and original. But if, as is usual in practice, both are to operate in water, ν is sensibly constant, while g is also constant, so that for similarity both vl and v^2/l would require to be constant. Since this is impossible, it follows that the lines of flow and the wave formation around a ship and its model in the same fluid cannot simultaneously be made dynamically similar. It remains to be seen whether any of the arguments involved in the function ψ may reasonably be neglected so as to give an approximation which is likely to be of use in practice.

Experiment shows that the resistance of a ship-shaped body at the speeds usual in practice is proportional to v^n, where n is approximately 1·83, and the nearness of this index to 2·0 indicates that the direct effect of viscosity is small. If it be assumed that this influence of viscosity is negligible,[1] the term $\nu/l\, v$ may be omitted from the equation, which now reduces to

$$R = l^2 v^2 \rho \phi \left(\frac{g\, l}{v^2}\right).$$

[1] This does not involve the assumption that skin friction is unimportant, or that viscosity plays no part in the phenomenon. It is, in effect, assuming that skin friction, instead of being proportional to v^n where n is slightly less than 2, is proportional to v^2. In this case the resistance is due to the steady rate of formation of eddies at the surface of the body, and once these have been formed and have left the immediate vicinity of the body, the rate at which they are damped out by viscosity has no effect on the drag.

If, now, l/v^2 be made the same both for the model and the original,

$$\frac{R}{R_m} = \frac{\rho}{\rho_m} \cdot \frac{l^2}{l_m^2} \cdot \frac{l}{l_m} = \frac{\rho}{\rho_m} \left(\frac{l}{l_m}\right)^3.$$

In this case the "corresponding speeds" at which wave and eddy formation are the same for ship and model, are given by

$$\frac{v}{v_m} = \sqrt{\frac{l}{l_m}}.$$

In practice these corresponding speeds are used, but allowance is made for the different effects of viscous resistances in the two cases by the well-known "Froude" method (Art. 134).

Art. 37.—Scale Effects.

From what has already been said, it will be evident that in most model experiments, some one of the factors involved tends to prevent exact similarity of flow. Where, as in many cases, this effect is small, and when its general result is known, the results of model experiments can be used with confidence to predict the performance of a large scale prototype.

The resistance of square plates exposed normally to a current affords a typical example of scale effects. Expressing the resistance of such plates as

$R = C \rho v^2$ . . in absolute units (British or C. G. S.)

$= K \rho v^2$ . . where K is in pounds weight per square foot and v is in feet per second

experiments show the following values of C and of K :—

	C	K	Size of Plate.
Dines [1] . .	·56	·00135	1 foot square.
Canovetti [2] . .	·56	·00134	3 feet diameter (circular)
Eiffel [3] . .	·55	·00133	10 inches square.
,, . .	·56	·00136	14 ,, ,,
,, . .	·59	·00142	20 ,, ,,
,, . .	·61	·00147	27 ,, ,,
,, . .	·62	·00150	39 ,, ,,
Stanton [4] . .	·52	·00126	2 ,, ,,
,, . .	·62	·00148	5 feet square.
,, . .	·62	·00149	10 ,, ,,

[1] "Proc. Roy. Soc.," vol. 48, p. 252.
[2] Société d'Encouragement pour l'Industrie Nationale, "Bulletin," 1903, vol. i., p. 189.
[3] Eiffel, "Résistance de l'Air."
[4] "Collected Researches, N. P. L.," vol. i., p. 261.

Such experiments show that while $R/l^2 v^2 \rho$ is almost independent of v, it increases by about 18 per cent. as the size of the plate is increased from 2 inches to 5 feet square. The most probable explanation of this would appear to be that while experiments on any one plate have been taken as showing the resistance to be proportional to v^2, this is only approximately true.

An examination by Messrs. Bairstow and Booth [1] of all reliable experiments on square plates leads to the conclusion that an empirical formula of the type

$$R = a\,(v\,l)^2 + b\,(v\,l)^3$$

gives a closer approximation to the observed results. This indicates that $\phi\left(\frac{l\,v}{\nu}\right)$ of equation (1) equals $m + n\,v\,l$, where m and n are constants for any particular fluid.

It may be noted that experiments [2] show that the pressures on the upstream side of a plate are not subject to a dimensional affect, but that the whole variation can be traced to changes in the negative pressure behind the plate.

[1] "Tech. Report, Advisory Committee for Aeronautics," 1910–11, p. 21.
[2] "Proc. Inst. C. E.," vol. clxxi.

CHAPTER V.

Bernoulli's Theorem—Elementary Proof—Experimental Verification—Applications—Converging Flow—Loss due to sudden change in Section of a Stream—Special Cases—Gradual Enlargement of Section—Vortex Motion—Forced Vortex—Free Vortex—Compound Vortex.

ART. 38.

Bernoulli's Theorem.—A particle of water forming part of a larger body may be capable of doing work in virtue of its pressure, its velocity, or its elevation. The expression for its total energy is

$$\frac{p}{W} + \frac{v^2}{2g} + z.$$

Each of these terms is of the dimensions of a length, and, if the units of force and length are the pound and foot, each represents the corresponding energy in foot lbs. per lb. Thus, if the fluid is utilised in some form of prime mover in which its velocity and elevation remain unaltered, it is capable of doing work at the rate of p/w foot lbs. per lb., so long as its connection with the surrounding fluid enables the pressure p to be maintained.

Similarly, under conditions where its pressure and elevation remain unaltered, it can do work at the rate of $v^2/2g$ foot lbs. per lb., while if used in a gravity machine, where its pressure and velocity are unchanged, if discharged at datum level it may do z foot lbs. of work per lb.

The theorem expressed in the equation

$$\frac{p}{W} + \frac{v^2}{2g} + z = \text{constant},$$

as applied to the fluid throughout any given stream tube, is known, from its discoverer, as Bernoulli's theorem.

ART. 39.—ELEMENTARY PROOF OF BERNOULLI'S THEOREM.

Making the assumption that the fluid is non-viscous and therefore exerts only normal forces on any bounding surface, the proof of this theorem is easily deduced. Consider the element—length δs—of a stream tube in

such a fluid, the only forces tending to produce motion of this element being its own weight and the normal pressures of the surrounding fluid (Fig. 38).

Let its cross sectional area at the top be "$a + \delta a$," and at the bottom "a," the pressure intensities at the top and bottom being $p - \delta p$ and p respectively. Let the average normal pressure on the sides of the element be q. Actually q will lie between p and $p - \delta p$.

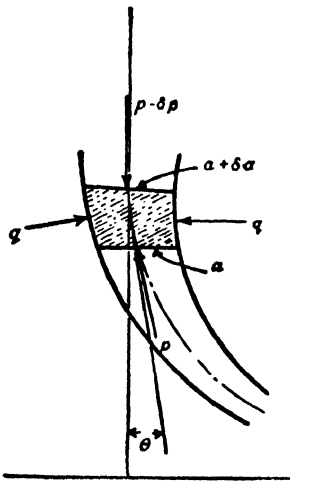


FIG. 38.

This normal pressure will have an unbalanced component $q \cdot \delta a$ along the axis, and, since the fluid is non-viscous, the only effect of this pressure in producing motion in the direction of the axis will be due to this component.

Let the direction of the axis make an angle θ with the vertical.

Then the magnitude of the resultant of forces on top and bottom faces of element, in the direction of its motion $\Big\} = \begin{cases} (p - \delta p)(a + \delta a) - p a \\ = p\,\delta a - a\,\delta p. \end{cases}$

Resultant of normal pressures on sides of element in same direction $\Big\} = -q\,\delta a.$

The only other force acting on the element is its weight, and the resolved part of this in the same direction

$$= \left\{ W \left(a + \frac{\delta a}{2}\right) \delta s \cos\theta \right\}$$

$\therefore$ Resultant force in direction of motion $\Big\} = p\,\delta a - a\,\delta p - q\,\delta a + W\left(a + \frac{\delta a}{2}\right)\delta s \cdot \cos\theta$

$$= \text{mass} \times \text{acceleration}$$

$$= \frac{W\left(a + \frac{\delta a}{2}\right) d s}{g} \times \frac{d v}{d t} \quad \text{where } v = \text{velocity of element.}$$

Putting $q = p - k\,\delta p$, and neglecting small quantities of the second order, we get

$$\frac{W a\,\delta s}{g} \cdot \frac{\delta v}{\delta t} = W a\,\delta s \cos\theta - a\,\delta p \qquad (1)$$

But $v = \frac{\delta s}{\delta t}$ and $\delta s \cos\theta = -\delta z$, z being measured vertically upwards and δs being the difference of level of the two ends of the element.

$$\therefore \text{ Equation (1) becomes } \frac{W}{g} . v\,\delta v = -W\,\delta z - \delta p$$

or
$$\frac{W}{g} v\,\delta v + W\,\delta z + \delta p = 0. \qquad (2)$$

When δs, δp, are indefinitely small we get, in the limit, on integrating

$$\frac{W}{g} . \frac{v^2}{2} + W z + p = \text{constant}$$

or
$$\frac{v^2}{2g} + \frac{p}{W} + z = \text{constant}. \qquad (3)$$

ART. 40.

Since v is the velocity along a stream line, any attempt to apply the equation to the motion of a mass of fluid by taking v as the mean velocity of the mass will obviously lead to error unless the square of this mean velocity, multiplied by the mass itself, is equal to the sum of the squares of the various stream tube velocities, each multiplied by the mass contained in its own stream tube; and this is only true when all the stream lines are parallel and have the same velocity, and when, in consequence, no internal work is being performed against viscosity.

While this state of affairs is never accurately realised in the case of the motion of water in pipes or open channels, yet the equation may be made to apply to such cases by the introduction of a term involving losses of energy due to viscous resistance and eddy formation.

E.g., if the suffixes (1) and (2) refer to two successive positions of a particle of viscous fluid, we may say that

$$\frac{p_1}{W} + \frac{{v_1}^2}{2g} + Z_1 = \frac{p_2}{W} + \frac{{v_2}^2}{2g} + Z_2 + {}_1H_2,$$

where ${}_1H_2$ represents this loss of energy between the positions (1) and (2).

Applying this to the case of flow through a pipe or channel of varying area, it is usual to assume that the equation still holds when v_1 and v_2 are the mean velocities of flow in the direction of the axis at the two sections (1) and (2).[1]

Where the flow changes from steady to unsteady between (1) and (2) the term ${}_1H_2$ will then include the kinetic energy due to motion in directions perpendicular to the axis of the pipe.

[1] In the case of flow in a straight pipe with the transverse curve of velocities in the form of a semi-ellipse, the actual kinetic energy of flow parallel to the axis is about 2 per cent. greater than the value computed from the mean velocity, when the latter has a value of 5 feet per second.

Art. 41.—Experimental Verification of Bernoulli's Theorem.

The apparatus shown (Fig. 39) consists of a horizontal passage having parallel vertical sides of plate glass, the first half forming a converging, and the second half a diverging channel. The passage connects two reservoirs, *A* and *B*, and pressure tubes are erected at frequent intervals.

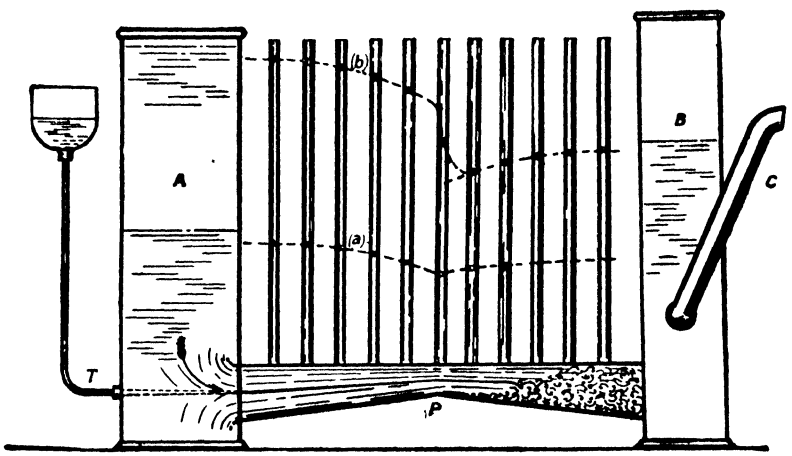


Fig. 39.

Water is fed into the tank *A*, which is of large area compared with the passage, and is led away by the overflow *C*. By suitable regulation of the flow into *A*, and of the height of the overflow *C*, the velocity of flow through the channel and the pressure at any point may be regulated as required. Aniline dye may be introduced into the incoming stream of water by means of the capillary tube *T*.

At very low speeds the motion throughout the passage is steady, and the curve joining the tops of the pressure columns is as shown in (*a*). At any point to the right of the throat the pressure head will be less than that at the corresponding point to the left, because of viscous resistance, so that, since the velocity head is the same at the two points, the total head is less on the right.

As the velocity is increased, the first indication of unsteady motion is provided by an occasional eddying of such colour bands as get below the horizontal through the lowest point of the throat.

The velocity at which this occurs depends on the angle of divergence of the sides of the passage. As the velocity is increased this unsteady

motion extends to the whole of the water in the diverging portion of the passage, the motion in the converging part being uniformly steady. The pressure curve now appears as shown in (*b*), and is discontinuous at the throat.

On plotting the energy curve, $\frac{p}{W} + \frac{v^2}{2g} + z = y$, the curve shown in

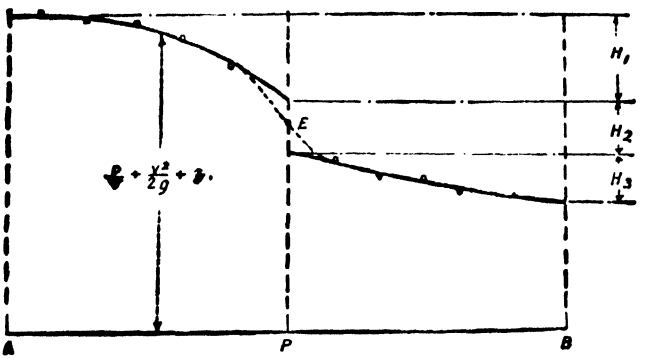


FIG. 39A.

Fig. 39A is obtained, and shows very clearly the magnitude of the various losses. Here H_1 represents loss of head, due to viscous resistance between A and P : H_2, the loss due to eddy formation at the throat ; and H_3, the loss due to viscous resistance and to eddy formation between P and B.

H_3 may be either less or greater than H_1, depending on the velocity, as, with fairly low velocities, the eddies formed at the throat die out to some extent before reaching B, their kinetic energy being transformed into available pressure energy and thus reducing H_3. At suitable velocities, indeed, H_3 may be negative, indicating that the energy rendered available by the dying out of eddy motion is more than equal to that absorbed in additional eddy formation and viscous resistance after the throat is passed.[1]

In every case, however, the head loss ($H_2 + H_3$) is greater than H_1, and experiments show very clearly that, while it is possible to change pressure head to velocity head without appreciable loss of energy, it is impossible to change velocity head to pressure head by the reverse operation without loss, except at velocities too low to be considered in practice.

[1] Since the velocity across the section is unequal, the K. E. ($\Sigma(mv^2)$) is greater than $M\bar{v}^2$ where $\bar{v}$ is the mean velocity. Thus the equalisation of the velocities tends to increase the apparent, though not the actual. K. E.

This is one important factor in the difference between the efficiencies of centrifugal pumps and turbines.

Art. 42.—Flow in Converging Channels. Radial Flow.

Where flow takes place in a converging channel, the motion is steady, and, neglecting viscosity,[1] the energy throughout any stream tube is constant, so that

$$\frac{p}{W}+\frac{v^2}{2g}+z=\text{constant.}$$

But for continuity of flow, if A = area of channel at some point where the velocity is v,

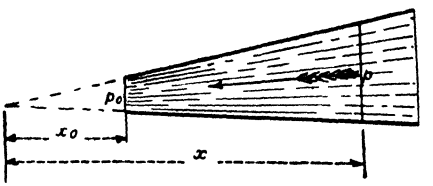


Fig. 40.

$$A\,v=\text{constant}=A_0\,v_0$$

where A_0 and v_0 are the area and velocity at some point distant x_0 from the point of convergence of the boundaries (Fig. 40).

Putting $A=k\,x$ we have $v=\frac{x_0\,v_0}{x}$, and if z is constant, *i.e.*, if the stream lines are horizontal,

$$\frac{p}{W}+\frac{x_0^2\,v_0^2}{2\,g\,x^2}=c \tag{1}$$

$$\therefore\ \frac{p}{W}=c-\frac{x_0^2\,v_0^2}{2\,g\,x^2}. \tag{2}$$

$$\text{But from (2) } \frac{p_0}{W}=c-\frac{v_0^2}{2g} \tag{3}$$

$$\therefore\ c=\frac{p_0}{W}+\frac{v_0^2}{2g}$$

$$\therefore\ \frac{p-p_0}{W}=\frac{v_0^2}{2\,g}\left\{1-\frac{x_0^2}{x^2}\right\}\text{ feet of water.} \tag{4}$$

This applies to flow through a converging channel having parallel upper and lower boundaries and to the case of (inward) radial flow towards a centre. The result gives the fall in pressure between two points radially distant x and x_0 from the origin, and may be applied to flow in an inward flow radial turbine. Since with outward flow between diverging boundaries the motion is unsteady, Bernoulli's equation ceases to hold, so that the formula is inapplicable to the case of a radial (outward flow) turbine.

[1] For the viscous resistance in a converging channel, see "Phil. Mag.," July, 1909, p. 25.

In the case of flow through a circular converging pipe or nozzle, $A = k\,x^2$.

$$\therefore v = \frac{x_0^2\, v_0}{x^2}, \text{ and assuming } s \text{ constant,}$$

we have $$\frac{p}{W} + \frac{x_0^4\, v_0^2}{2g\,x^4} = c.$$

As before $$c = \frac{p_0}{W} + \frac{v_0^2}{2g}$$

$$\therefore \frac{p - p_0}{W} = \frac{v_0^2}{2g}\left\{1 - \frac{x_0^4}{x^4}\right\} = \frac{v_0^2}{2g}\left\{1 - \frac{A_0^2}{A^2}\right\}.$$

It follows that if a pipe of area A suffer a gradual contraction of section to a, and if p_A and p_a are the corresponding pressures and V_A the velocity at entrance to the pipe

$$\frac{p_A - p_a}{W} = \frac{V_A^2}{2g}\left\{\left(\frac{A}{a}\right)^2 - 1\right\} \text{ feet.}$$

$$\therefore V_A = \sqrt{\frac{2g\,(p_A - p_a)}{W\left\{\left(\frac{A}{a}\right)^2 - 1\right\}}} \text{ f. s.}$$

This enables the velocity of flow and the discharge to be obtained from a measurement of $\frac{p_A - p_a}{W}$, the difference of pressures in feet of water at the entrance and throat of the pipe. The Venturi meter (Art. 225) depends directly on this principle for its action.

An interesting phenomenon accompanying radial outward flow is noticed if a plate on which a jet is impinging normally be brought gradually nearer to the orifice, until a point is reached where the escaping stream touches both plate and nozzle (Fig. 41A).

Let r_1, p_1, v_1, a_1, be the radius, pressure, velocity, and area of the issuing stream at the exit from the orifice and at the commencement of its flow between the plate and the nozzle.

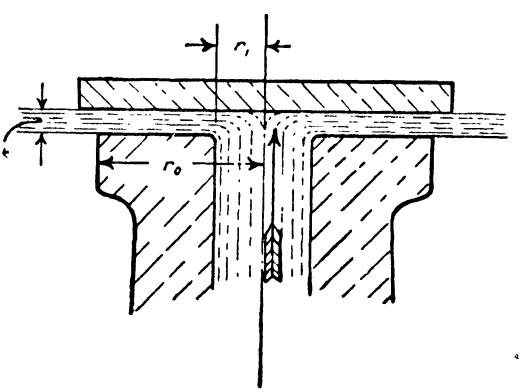


FIG. 41A.

Let r_0, p_0, v_0, and a_0 represent similar quantities at the point of escape of the stream into the atmosphere.

Then

$$\begin{cases} a_1 = 2\,\pi\,r_1\,t, \text{ where } t = \text{thickness of the escaping stream.} \\ a_0 = 2\,\pi\,r_0\,t. \\ v_0 = v_1 \dfrac{a_1}{a_0} = \dfrac{v_1\,r_1}{r_0}. \\ p_0 = \text{atmospheric pressure.} \end{cases}$$

Neglecting frictional and eddy losses between (1) and (0), we have

$$\frac{p_1}{W} + \frac{v_1^2}{2g} = \frac{p_0}{W} + \frac{v_0^2}{2g} = \frac{p_0}{W} + \frac{v_1^2}{2g} \cdot \frac{r_1^2}{r_0^2}$$

$$\therefore \frac{p_0 - p_1}{W} = \frac{v_1^2}{2g}\left\{1 - \left(\frac{r_1}{r_0}\right)^2\right\} \qquad (1)$$

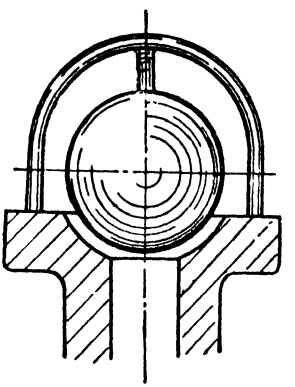


FIG. 41B.

Since r_1 is less than r_0, $p_0 - p_1$ is positive, and the pressure at (1) is less than atmospheric. It follows that since the pressure between (1) and (0) is everywhere less than atmospheric, while that on the corresponding portion of the outer face of the plate is atmospheric, there will be a resultant force tending to force the plate up to the orifice.

When the distance t becomes very small, the pressure over the area πr_1^2 approximates to the static head and the plate is forced away from the orifice, so that an intermittent action is set up, the plate vibrating to and from the orifice, but never being driven far away.

The ball-nozzle (Fig. 41B), designed to give a thin evenly distributed discharge from a fire hydrant, acts on exactly the same principle.

Here a cage is fitted to prevent the loss of the ball on the first impact of the jet.

ART. 43.—LOSS DUE TO THE SUDDEN ENLARGEMENT OF CROSS SECTION OF A STREAM.

Whenever the cross section of a pipe or channel increases abruptly, the water, on passing the enlargement, is thrown into a state of unsteady motion, with a consequent loss of available energy. If the mean velocities before and after passing the enlargement are known, the equations of momentum may be applied to determine the magnitude of this loss.

For, consider the case of flow through a horizontal pipe (Fig. 42), suffering a sudden change of area from a to A.

Let p and v be the pressure and mean velocity immediately before the change of section.

Let P and V be the pressure and mean velocity in the large pipe when the motion has steadied after impact.

Let p' be the mean pressure on the end of the pipe at EE.

Neglecting frictional effects, which will be small, the forces tending to

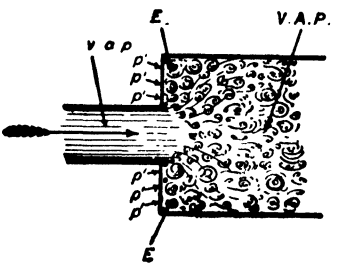


FIG. 42.

produce a change of momentum in the direction of flow between the sections at which p and P are measured are:—

(1) the pressure over the area at $p = pa$.[1]

(2) „ „ „ „ „ $EE = p'(A - a)$.

(3) „ „ „ „ „ at $P = PA$.

∴ The force producing change of momentum in the direction of motion $= \{p\,a + p'(A - a) - P\,A\}$ lbs. (1)

This must equal the change of momentum per second in the direction of this force

$$= \frac{W}{g} A\,V^2 - \frac{W}{g} a\,v^2 = \frac{W}{g}\{A\,V^2 - a\,v^2\}. \quad (2)$$

The assumption is commonly made that the pressure p' over the face EE is uniform, and is sensibly equal to p. Although the actual distribution and magnitude of this pressure probably vary with the ratio of the areas and relative position of the two branches, and with the velocity, we have not sufficient experimental data to frame any definite law connecting the two. The above assumption, however, appears to be fairly well justified by the results of such experiments as are to hand, and making use of this we have, on combining equations (1) and (2)

$$(p - P)\,A = \frac{W}{g}\{A\,V^2 - a\,v^2\} \quad (3)$$

Also for continuity of flow $A\,V = a\,v$ ∴ $v = \frac{A\,V}{a}$

∴ $$(p - P)\,A = \frac{W}{g}\left\{1 - \frac{A}{a}\right\} A\,V^2. \quad (4)$$

[1] The pressure p being measured at the mean depth of the section.

If now H' = loss of head due to shock at the enlargement, we have

$$\frac{p}{W} + \frac{v^2}{2g} = \frac{P}{W} + \frac{V^2}{2g} + H' \tag{5}$$

$$\therefore \quad \frac{p - P}{W} + \frac{V^2 A^2}{2a^2 g} - \frac{V^2}{2g} = H' \tag{6}$$

Substituting for $\frac{p - P}{W}$ from (4) we get

$$H' = \frac{V^2}{2g}\left(\frac{A}{a} - 1\right)^2 \tag{7}$$

$$= \frac{v^2}{2g}\left(1 - \frac{a}{A}\right)^2 \tag{8}$$

$$= \frac{(v - V)^2}{2g} \tag{9}$$

Writing m for $\frac{A}{a}$, relations (7) and (8) become:—

$$H' = \frac{V^2}{2g}(m - 1)^2 \tag{10}$$

$$= \frac{v^2}{2g}\left(1 - \frac{1}{m}\right)^2 \tag{11}$$

Experiments by the author [1] indicate that the percentage loss increases slightly with the ratio of enlargement, and in pipes with the same ratio of enlargement is greater the smaller the pipe.

Denoting the ratio of enlargement by m, and the smaller diameter by d, the loss at a sudden enlargement for values of m between 2 and 12 and for pipe diameters ranging from ·50 inch to 6 inches, can be expressed, within narrow limits, by the relationship—

$$\text{loss of head} = \frac{102{\cdot}5 + {\cdot}25m - 2{\cdot}0d}{100}\left\{\frac{(v_1 - v_2)^2}{2g}\right\} \text{ feet.}$$

The following table shows the results obtained by the use of the formula, against those experimentally obtained:—

Diameters of pipes.	Value of m.	Loss expressed as percentage of $\frac{(v_1 - v_2)^2}{2g}$.	
		Experimental.	By formula.
·65 to 2·15 inches	10·96	103·5	103·9
·50 „ 1·5 „	9·0	102·8	103·7
1·0 „ 3·0 „	9·0	102·1	102·8
1·5 „ 3·0 „	4·0	101·7	100·5
2·0 „ 3·0 „	2·25	99·2	99·1
3·0 „ 6·0 „	4·0	97·5	97·5
4·0 „ 6·0 „	2·25	92 (approx.)	95·0

[1] "Trans. Roy. Soc. Edinburgh," vol. 48, 1911, p. 97.

Art. 44.—Loss in Pipes and Passages with a gradually increasing Cross-section.

The loss of head accompanying a gradual enlargement of section would naturally be expected to be less than that experienced at a sudden enlargement of section between the same initial and final areas. To determine the extent to which this conclusion is justified a somewhat extended series of experiments on such pipes was carried out some time ago by the author.[1]

Circular Pipes with Uniformly Diverging Boundaries.—Calling θ the angle of conicity of a pipe, the following table, as also Fig. 43, shows the mean of the results obtained.

Loss of Energy expressed as a Percentage of $\frac{(v_1 - v_2)^2}{2g}$, the Theoretical Loss at a Sudden Change of Velocity from v_1 to v_2.

Value of θ		180°.	90°.	60°.	40°.	20°.	10°.
Pipe diameters in inches.	·65 to 2·15	103·5	...	...	...	...	...
	·50 „ 1·50	102·8	102·6	102·8	82·0	45·0	16·6
	1·0 „ 3·0	102·1	104·1	101·3	80·8	44·0	...
	1·5 „ 3·0	101·7	111·1	120·5	101·7	42·5	17·5
	2·0 „ 3·0	99·2	112·1	...	88·7	41·9	18·6

From these it appears that as θ is diminished from 180° the loss increases, attains a maximum value for some value of θ in the neighbourhood of 65°, and afterwards diminishes rapidly with θ until θ is about 5° 30′. This value gives a minimum loss of approximately 13·5 per cent. Any further diminution in θ is accompanied by an increased loss owing to the large value of the wall friction in pipes of the comparatively great length accompanying such small values of θ. The value of θ giving rise to the same loss as is experienced with a sudden change of section, varies from 41° to 60°, being slightly greater for a given area-ratio the larger the pipes, and, for a given mean diameter, increasing in an irregular manner with the area-ratio. Its mean value over the range of ratios considered is approximately 50°, and where, in the design of hydraulic machinery, it is necessary for this value to be exceeded, a sudden enlargement of section gives a more efficient transformation of energy than does a uniformly tapering pipe. For values of θ between 7·5° and 35° the loss may be expressed with a fair degree of accuracy by the relationships—

$$\text{loss} = \cdot 0110\ \theta^{1\cdot 22}\ \frac{(v_1 - v_2)^2}{2g} \text{ feet, where } \theta \text{ is in degrees,}$$

[1] "Proc. Roy. Soc. A.," vol. 83, 1910. "Trans. Roy. Soc. Edinburgh," vol. 48, 1911, p. 97; see also K. Andres, "Zeitschrift d. V. d. Ingenieure," Sept. 17, 1910, p. 1585.

or

$$\text{loss} = 3{\cdot}50 \left(\tan \frac{\theta}{2}\right)^{1{\cdot}32} \frac{(v_1 - v_2)^2}{2g} \text{ feet.}$$

This latter relationship becomes of importance in the design of trumpet-shaped pipes to give a minimum loss of energy.

Rectangular Pipes with Uniformly Diverging Boundaries.—Three sets of rectangular pipes were examined, these having one pair of sides parallel and 1·329 inch apart in every case. The areas of the small and of the large ends of these pipes were, in the case of the pipes having

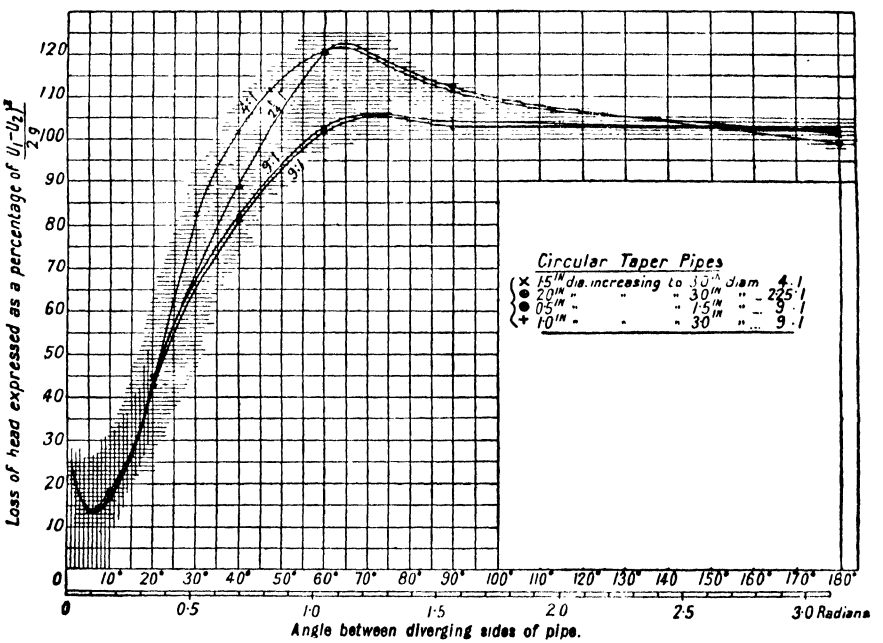


FIG. 43.—Loss of Head in Circular Taper Pipes

area-ratios of 4 : 1 and 9 : 1, identical with those of the circular taper pipes having the same ratios of enlargement.

The results of the means of the experiments on these pipes are plotted in Fig. 44. From these it appears that the percentage loss is very approximately the same for all ratios of enlargement between 2·25 : 1 and 9 : 1 for values of θ between 10° and 40°, and that it varies but little with the dimensions of the pipe. The minimum loss is obtained when θ is approximately 11°, the percentage loss under these circumstances being about 17·5 per cent. As θ is increased the loss increases

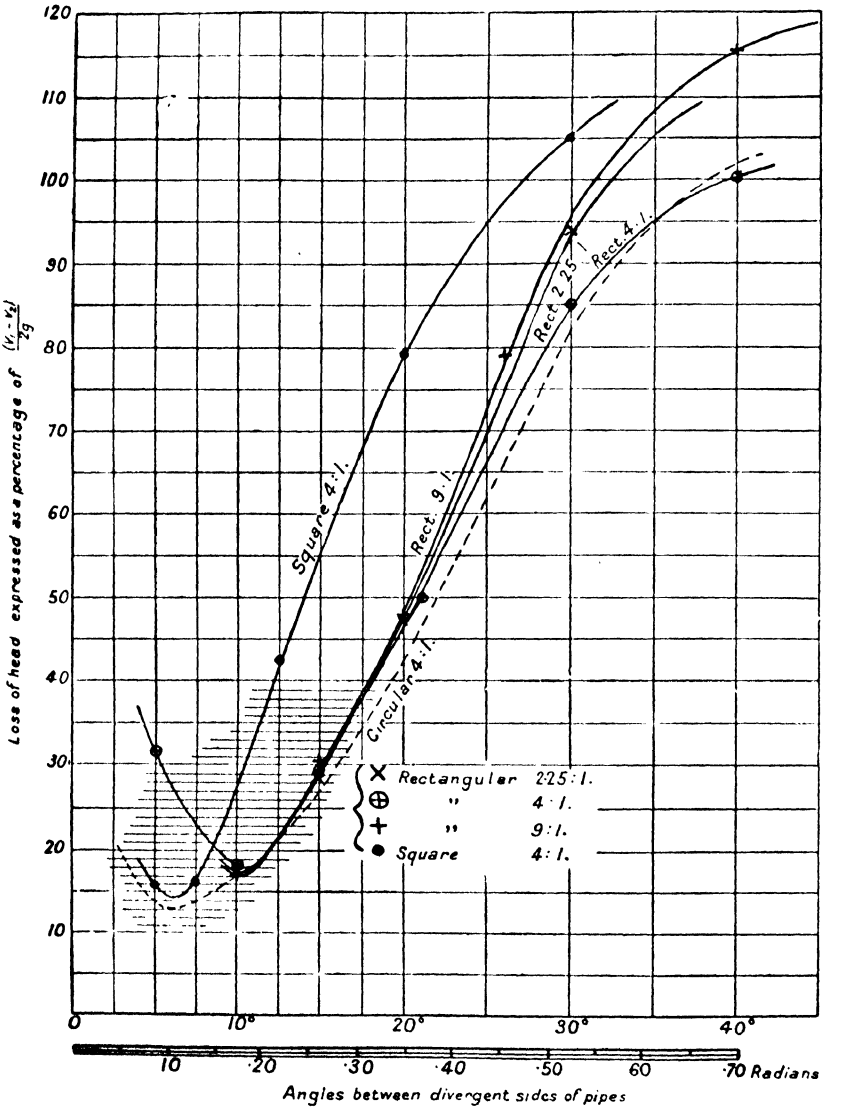


FIG. 44.—Percentage Loss of Head in Straight Taper Pipes of Square and Rectangular Section.

rapidly, and attains a value of 100 per cent. when θ is between 31° and 40°, the value of this critical angle being less with the smaller ratios of enlargement and with the pipes having the smaller mean sectional areas.

Experiments on rectangular pipes having an enlargement ratio of 9 : 1,

and having θ respectively 40°, 50°, and 90°, showed corresponding losses of 115, 122·3, and 119 per cent. This indicates a maximum loss when θ is approximately 70°, or when it has sensibly the same value as gives maximum loss in the corresponding circular pipe.

Pipes of Square Section with Uniformly Diverging Boundaries.—Five pipes, having a smaller section 1·329 inch square and a larger section 2·658 inches square and with θ respectively 5°, 7·5°, 12·5°, 20°, and 30°, were examined, the results being plotted in Fig. 44. The percentage loss is a minimum for a value θ in the neighbourhood of 6°, practically the same as for a circular pipe, and has a value of about 14·5 per cent.

Trumpet-Shaped Pipes.—From *à priori* reasoning it would appear that when the initial and final areas and the length of a pipe are fixed, the loss of head may be reduced by making the pipe trumpet shaped, the angle of divergence of its sides being least where the velocity is greatest and vice versâ. To test this point several pipes were examined, these being designed so as to make either the retardation $\left(\frac{d\,v}{d\,t}\right)$ constant, the change of velocity per unit length of pipe $\left(\frac{d\,v}{d\,x}\right)$ constant, or the loss of kinetic head per unit length of pipe $\frac{d(v^2)}{d\,x}$, constant. The latter pipes were found to be the more efficient, the saving, over a straight taper pipe of the same length ranging from 20 per cent. to 60 per cent. and being greater the shorter the pipe.

In a rectangular pipe of length l, satisfying this condition and having one pair of sides parallel, if the half-breadths at the small and the large ends are respectively y_1 and y_2, the half-breadth y, at a distance x from the small end is given by

$$\frac{1}{\sqrt[4]{y}} = \frac{1}{\sqrt[4]{y_1}} - \frac{x}{l}\left(\frac{1}{\sqrt[4]{y_1}} - \frac{1}{\sqrt[4]{y_2}}\right),$$

while in a circular pipe whose smaller and larger radii are r_1 and r_2, the radius at a distance x from the small end is given by

$$\frac{1}{\sqrt[4]{r^5}} = \frac{1}{\sqrt[4]{r_1^5}} - \frac{x}{l}\left(\frac{1}{\sqrt[4]{r_1^5}} - \frac{1}{\sqrt[4]{r_2^5}}\right).$$

Where the length of pipe is great, or the ratio of areas small, the curves thus formed may, at the smaller end of the pipe, diverge at an angle less than that (6° in a circular pipe and 11° in a rectangular pipe) giving minimum loss, and in such a case the pipe would be made to diverge uniformly at this best angle up to a point where its straight sides would intersect the calculated curved sides.

If, on the other hand, the length of pipe is small or the ratio of areas large, the calculated curves may, towards the larger end of the pipe, diverge at an angle greater than that giving a loss equal to that at a sudden enlargement.

In such a case a more efficient pipe is obtained by enlarging the pipe to its final section by a sudden enlargement at the point at which the angle of divergence becomes equal to this critical value.

A still more efficient pipe is obtained if, from the point at which the angle of divergence becomes equal to the critical angle, the section is enlarged gradually, the best angle of divergence being found to vary but slightly in such circumstances and in any cases likely to be found in practice, being approximately 20° for a rectangular pipe, and 10° for a circular pipe.

Compound Pipes and Passages.

The loss of head in a pipe whose section increases gradually from A_1 to A_3, and which then suffers a sudden enlargement of area to A_2 (Fig. 45), is approximately equal to the sum of the separate losses which would be experienced in the taper portion of the pipe and at the sudden enlargement, if these were independent of each other. By reducing the angle of divergence of the first portion of such a tube, the sudden enlargement of section and the accompanying loss is made greater, but the loss in the diverging portion is reduced in a double degree, since not only is the numerical coefficient expressing such loss as a percentage of $(v_1-v_3)^2/2g$ diminished, but A_3 is diminished at the same time, and thus the factor v_1-v_3 is also diminished. A diminution in the angle of divergence therefore causes a rapid diminution in this portion of the loss, which may, or may not, be counter-balanced by the loss at the sudden enlargement of section. Owing to the comparatively low velocities at the large end of the pipe, however, except in pipes whose length is comparatively very short, and whose ratio of enlargement is small, this latter loss is comparatively small and the total loss is a minimum with a pipe—straight or curved– whose angle of divergence—

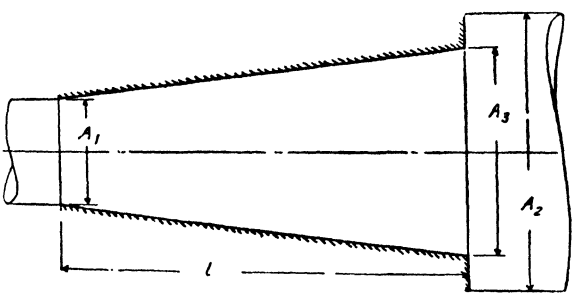


FIG. 45.

actual or effective—is little, if any, greater than that giving minimum loss in the diverging portion of the pipe alone.

It thus becomes possible to design a pipe—often with a considerable reduction in length—in which the boundaries are straight, and in which the loss is still appreciably less than in a straight taper pipe giving the full enlargement of section with the best possible value of θ.

The total loss of head in such a pipe (shown in Fig. 45) is theoretically equal to

$$K \frac{(v_1 - v_3)^2}{2g} + \frac{(v_3 - v_2)^2}{2g} \text{ feet}$$

where K is obtained from the curves of Figs. 43 and 44.

The following table shows values of θ giving minimum loss of head in such a pipe, along with the corresponding loss of head in the pipe, and in a straight taper pipe of the same length and with the same initial and final areas.

	Ratio of areas.	Value of θ giving minimum loss.	Length of pipe (inches).	Value of θ in straight taper pipe of same length.	Loss of head expressed as a percentage of $\frac{(v_1 - v_2)^2}{2g}$	Loss of head expressed as a percentage of the loss in straight taper pipe.
Rectangular Pipes.	9 : 1 (Sections ·59 inch × 1·329 inch and 5·315 inches × 1·329 inch)	10° 20′	17·95	15°	16·2	54·9
		10° 40′	8·81	30°	15·3	16·0
		11° 00′	6·49	40°	16·2	14·2
		11° 45′	4·08	60°	20·3	...
	4 : 1 (1·329 inch square to 1·329 inch × 5·315 inches)	10° 20′	15·15	15°	15·6	53·8
		11° 00′	11·30	20°	15·3	32·6
		12° 00′	7·45	30°	18·2	22·0
		12° 45′	5·48	40°	22·0	21·1
		15° 30′	3·45	60°	29·4	...
	2·25 : 1 (1·329 inch square to 1·329 inch × 2·99 inches)	10° 40′	6·30	15°	15·2	51·5
		13° 20′	3·10	30°	22·8	26·8
		15° 40′	2·29	40°	28·2	25·2
Circular Pipes.	9 : 1 ·50 inch diam. to 1·50 inch diam.	7° 10′	3·80	15°	13·6	52·3
		11° 00′	1·87	30°	18·2	27·2
	4 : 1 1·50 inch diam. to 3·0 inches diam.	7° 00′	8·57	10°	12·7	73·5
		13° 30′	2·80	30°	20·8	22·2
	2·25 : 1 2·0 inch diam. to 3·0 inches diam.	7° 00′	5·71	10°	13·4	72·5
		15° 00′	1·87	30°	24·3	35·2

By this method of construction the loss may be reduced in favourable circumstances to about 90 per cent. (in rectangular) pipes) and to about

96 per cent. (in circular pipes) of the minimum possible loss in a uniformly tapering pipe undergoing the full enlargement section. By designing the pipe from A_1 to A_8 with curved boundaries, the loss may be still further reduced. In the majority of cases occurring in practice, however, the additional trouble of calculation and cost of template will not be counterbalanced by the slight increase in efficiency which they render possible.

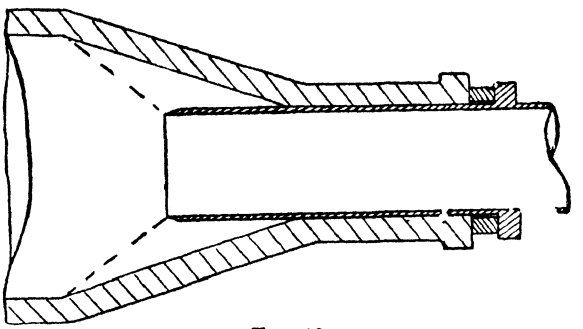


Fig. 46.

Effect of projecting Smaller Pipe into Space bounded by Diverging Walls.—For the purpose of examining this effect, a thin sleeve of internal diameter ·90 inch and external diameter 1·0 inch was prepared and used inside the various 1-inch pipes, as shown in Fig. 46.

The results of the experiments are as follow :—

Percentage Loss of Head.

Value of θ . . .	40°	60°	90°
Taper pipe without sleeve . . .	80·8%	101·3%	104·1%
Projecting pipe with diameters ·90 inch and 3 inches . .	Pipe projecting— ·50 inch . 98·0% 1·00 inch . 108·0%	Pipe projecting— ·25 inch . 104·5% ·50 inch . 107·2% 1·00 inch . 112·0%	Pipe projecting— ·25 inch . 107·7%

In every case the effect of the projection is to increase the loss, the effect being greater the greater the length of the projection and also the less the angle of divergence of the conical sides. The loss is, in fact, greater in every case than would be experienced if the pipe walls were tapered off from the extremity of the projecting pipe, as shown by the dotted lines in the sketch.

Art. 45.—Loss due to Sudden Contraction of Area of a Stream.

When water flowing along a channel or through a pipe meets with a sudden contraction of the area of the pipe, as shown in Fig. 47, we also get a loss of energy which is due, not directly to the contraction of the stream, but to the subsequent re-enlargement which always takes place.

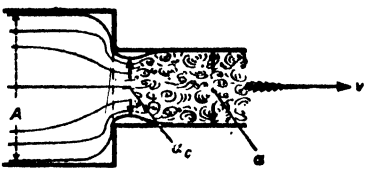


Fig. 47.

The stream, after passing the contraction, suffers a further diminution in area, and for a short distance fails to fill the pipe, afterwards re-expanding as shown.[1] Up to the contracted section the loss is solely that due to simple viscous resistance.

If A represents the larger stream area
„ a „ „ smaller „
„ a_c „ „ *vena contracta*

V, v and v_c representing the velocities at these sections, the loss of energy due to this enlargement is approximately

$$\frac{(v_c - v)^2}{2g} \text{ ft. lbs. per lb.}$$

Writing C_c for $\frac{a_c}{a}$, the above may be expressed in the form

$$\frac{(v_c - v)^2}{2g} = \left(\frac{1}{C_c} - 1\right)^2 \cdot \frac{v^2}{2g} = F\,\frac{v^2}{2g}$$

Here F is called the coefficient of hydraulic resistance, and can only be determined experimentally.

A series of experiments carried out by the author on the apparatus shown in Fig. 47, having a value of $m = 10{\cdot}96$, and having pipe diameters of 2·15 inches and ·65 inch, gave a mean value of ·600 for C_c, over a range of values of v from 1·35 to 8·69 feet per second. The value diminished from ·610 with the lowest to ·587 with the highest of these velocities. Experiments on a 6-inch pipe, contracting respectively to a 3-inch and to a 4-inch pipe, give in the former case a value of $C_c = {\cdot}595$, and in the latter case $C_c = {\cdot}692$.

If C_c has the value ·586, the loss $= {\cdot}50\,\frac{v^2}{2g}$. This is approximately the value of C_c, where the area A is so large as not to affect the stream

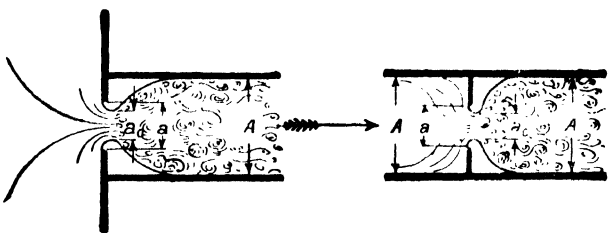
[1] This action may occur even though the pipe is full of water, the space shown empty in Fig. 47 being now occupied by dead water which takes no part in the flow through the pipe.

line production, and gives the loss usually found where a pipe is led out of a reservoir by means of a sharp-edged junction.

Art. 46.—Loss caused by a Diaphragm in a Parallel Pipe.

Much work in this direction has been done by Weisbach, to whom the following experimental results are due.

With a diaphragm having an aperture a leading from a reservoir of large area into a pipe of area A (Fig. 48A), calling the loss $\left\{\frac{A}{C_c\, a} - 1\right\}^2 \frac{v^2}{2g}$



Figs. 48A and 48B.

$= F\frac{v^2}{2g}$, the following values were obtained. Here $C_c\, a$ = area of *vena contracta.*

$\frac{a}{A}$	·1	·2	·3	·4	·5	·6	·7	·8	·9	1·0
C_c	·616	·614	·612	·610	·607	·605	·603	·601	·598	·596
F	231·7	50·99	19·78	9·612	5·256	3·077	1·876	1·169	·734	·480

With a diaphragm inserted in a pipe of area A (Fig. 48B), the following values were obtained.

$$\text{Loss} = \left(\frac{A}{C_c\, a} - 1\right)^2 \frac{v^2}{2g} = F\frac{v^2}{2g}.$$

$\frac{a}{A}$	·1	·2	·3	·4	·5	·6	·7	·8	·9	1·0
C_c	·624	·632	·643	·659	·681	·712	·755	·813	·892	1·00
F	225·9	47·77	17·48	7·80	3·753	1·796	·797	·29	·06	·00

[1] See also Gaskell, "Proc. Inst. C. E.," 1914; Judd, "Journal Am. Soc. M. E.," Sept. 1916: Davis and Jordan, "Bulletin, University of Illinois," No. 14, Dec. 2, 1918.

A formula connecting the area of the *vena contracta* with that of the aperture in the diaphragm, in terms of the relative area of pipe and of aperture, is given by Rankine, and is

$$C_c = \frac{a_c}{a} = \frac{\cdot 618}{\sqrt{1 - \cdot 618 \frac{a^2}{A^2}}}.$$

Thus when $a = A$, $C_c = 1$.

,, a is very small, C_c has the limiting value ·618.

The effect of the portion of the pipe before the orifice is to produce a closer approximation to parallelism of the stream lines, and in consequence a *vena contracta* of larger area is obtained, with a reduced after-enlargement, and hence less loss than where this constraint is absent.

ART. 47.—VORTEX MOTION. PRESSURE IN A ROTATING LIQUID.

Where a mass of liquid moves as a whole with vortex motion, this may occur in either of two ways.

The first is seen when a vessel containing water is rotated, or when the contained water is stirred so as to make it rotate. Such a vortex is termed a **Forced Vortex.**

The second is seen when water flows through a hole in the bottom of a vessel. Here a vortex is usually formed naturally, some initial disturbance of the water determining the direction of rotation, although in the northern hemisphere the earth's rotation would itself tend to cause a rotation in an anti-clockwise direction as viewed from above. This is termed a **Free Spiral Vortex.** The water moves spirally towards the centre with stream line motion, so that, neglecting viscosity, its energy per unit mass is everywhere the same. If, while the mass is rotating, the orifice be plugged, the motion becomes one of simple rotation in horizontal planes, and forms a **Free Cylindrical Vortex.**

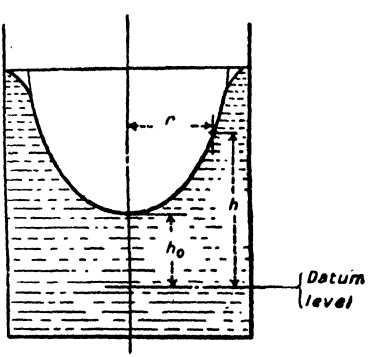


FIG. 49.—Forced Vortex.

Forced Vortex Motion with Uniform Angular Velocity.—Since the angular velocity w is constant, we have at any radius r, $v = w r$. (1)

The increase in pressure radially is given by

$$\frac{d\,p}{d\,r} = \frac{W}{g} \cdot \frac{w^2\,r^2}{r} = \frac{W}{g}\,w^2\,r.$$

Integrating between the limits r_1 and r_2 we have

$$\frac{p_1 - p_2}{W} = \frac{w^2}{2\,g}\,(r_1^2 - r_2^2). \qquad (2)$$

If $p = p_0$ where $r = o$,

$$\frac{p - p_0}{W} = \frac{w^2}{2\,g} \cdot r^2$$

or, putting $\frac{p}{W} = h$ (Fig. 49),

$$h - h_0 = \frac{w^2}{2\,g} \cdot r^2 \quad \therefore \quad h = h_0 + \frac{w^2\,r^2}{2\,g}$$

which is the equation to a parabola.

Since the pressure at any point is that equivalent to the column of water supported at the point, it follows that all surfaces of equal pressure, including the free surface of the vortex, form paraboloids of revolution having the axis of rotation as their common axis. Near the sides of the vessel the liquid lags owing to viscosity, and here the surface level will fall below that of the paraboloid.

Free Cylindrical Vortex Motion.—Here, since we have stream line motion, the equation $\frac{p}{W} + \frac{v^2}{2\,g} + z = \text{constant}$, holds.

$\therefore$ in any horizontal plane $\frac{p}{W} + \frac{v^2}{2\,g} = \text{constant}$.

Differentiating,

$$\frac{1}{W} \cdot \frac{d\,p}{d\,r} + \frac{v}{g} \cdot \frac{d\,v}{d\,r} = 0. \qquad (1)$$

Introducing the condition for rise in pressure across a stream tube, viz., $\frac{d\,p}{d\,r} = \frac{W\,v^2}{g\,r}$, this becomes

$$\frac{v^2}{g\,r} + \frac{v}{g}\frac{d\,v}{d\,r} = 0$$

$$\therefore \quad \frac{d\,v}{d\,r} + \frac{v}{r} = 0, \text{ or } -\frac{d\,r}{r} = \frac{d\,v}{v}.$$

Integrating we get $\log_e r + \log_e v = \text{constant} = B$.

$$\therefore \quad v\,r = \text{constant} = e^B = B_1.$$

$$\therefore \quad v = \frac{B_1}{r} \qquad (2)$$

i.e., in a free vortex the velocity varies inversely as the distance from the axis of rotation.

It follows that the increase of pressure with radius is identical with that in inward radial flow.

Thus if p_1, v_1, r_1, are the attributes of a point in the same horizontal plane as p, v, r,

$$\frac{p - p_1}{W} = \frac{v_1^2}{2g}\left\{1 - \frac{r_1^2}{r^2}\right\} \qquad (3)$$

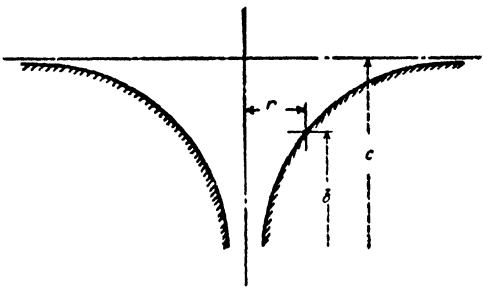


FIG. 50.—Free Vortex.

Putting $\frac{p}{W}$ constant in Bernoulli's equation we get the equation to the curve of equal pressure, that is $\frac{v^2}{2g} + z = \text{constant} = C$, and substituting for v in terms of r from (2) we have

$$\frac{B_1^2}{2g\,r^2} + z = C$$

$$\therefore\ C - z = \frac{B_1^2}{2g\,r^2}$$

the equation to a hyperbolic curve of the nature $y\,x^2 = A$, and which is asymptotic to the axis of rotation and to the horizontal through $z = C$ (Fig. 50).

A Free Spiral Vortex may be considered as a case of cylindrical vortex and radial motion combined in the required proportions, since in each case the velocity is inversely proportional to the radius. The angle between the stream line and the corresponding radius vector at any point will then be constant, and the stream lines will form a series of equiangular or logarithmic spirals. The difference of pressure between any two points may then be found by adding the pressure differences due to the two methods of flow taken separately.

Strictly this should only be applied to cases of inward flow, since outward flow causes instability.

In a free spiral vortex formed under normal conditions, the surface resistances tend to prevent the attainment of such high velocities near the centre of rotation as are indicated by the preceding analysis. In consequence the actual values of $C - z$ for small radii are less than those calculated. The following observations by the author on the surface profile of such a vortex, produced by the discharge from a circular

cylindrical vessel 2 feet in diameter under a constant head of 9 inches, bear out this point.

Radius inches.		12.	10.	8.	6.	4.	3.	2.	1.
$(C - z)$ inches. Orifice 1 inch diameter	observed	·0	·02	·05	·08	·13	·19	·40	1·14
	calculated $(C - z) = \frac{1{\cdot}71}{r^2}$		·02	·03	·05	·11	·19	·43	1·71
$(C - z)$ inches. Orifice 1·5 inch diameter	observed	·00	·03	·06	·13	·26	·43	1·06	2·96
	calculated $(C - z) = \frac{3{\cdot}82}{r^2}$		·04	·06	·11	·24	·45	·95	3·82

Since the velocity varies inversely as the radius, and since this velocity cannot be infinite at the axis where $r = o$, an air column is essential at the centre of a free vortex. When this air column cannot be maintained, we get a combination of a forced vortex—at and near the axis—and a free vortex at points further removed (Fig. 51).

If a be the radius at which the two surface curves intersect, the depth of the central depression below the general level of the surface may be shown to be given by $\frac{w^2 a^2}{g}$ feet.

This is termed a **Compound Vortex**, and the state of affairs there existing is of importance in its application to the theory of the flow of

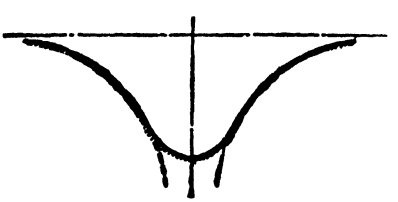


FIG. 51.—Compound Vortex.

water in a centrifugal pump fitted with a Vortex Chamber. Here the water in the impeller forms a forced vortex with outward radial flow, while when free of the vanes an approximation to a free spiral vortex is formed, and the pressure increases as the velocity diminishes outwards.

The formation of a vortex in a discharging vessel increases the time of discharge by diminishing the effective area of the orifice, and by reducing the inclination of the issuing particles to the plane of the orifice. Experi-

ments by the author[1] showed that when discharging water under heads varying from 8 to 12 inches in a free spiral vortex, the coefficient of discharge was sensibly independent of the head and had a value ·287 for a 1-inch orifice and ·178 for a 1½-inch orifice. When no vortex formed, the coefficient was ·62.

Examples.

(1) In a 12-inch Venturi meter the throat diameter is 4 inches. Taking C to be constant = ·982, determine the difference of head at the entrance and throat when discharging

(*a*) 100
(*b*) 500 } gallons per minute.
(*c*) 1,000

N.B.—1 gallon = 10 lbs.

Answer. (*a*) Vel. = ·340 $f.$ $s.$ — ·149 feet.
(*b*) „ = 1·701 $f.$ $s.$ — 3·73 feet.
(*c*) „ = 3·402 $f.$ $s.$ — 14·92 feet.

(2) Inward radial flow takes place between two parallel circular plates, 3 feet diameter and 6 inches apart. Discharge takes place through an orifice 18 inches diameter in the centre of the lower plate. Neglecting viscous resistances, determine the pressure at the entrance when 1,000 cubic feet per minute pass the plates, assuming the pressure at the edge of the discharge orifice to be atmospheric.

Answer. ·585 feet of water = ·254 lb. per square inch.

(3) If in the previous example the plates are fitted with vanes and rotate at 400 revolutions per minute, determine the difference of pressure at inlet and outlet due solely to the production of a forced vortex.

Answer. 46·04 feet of water = 20 lbs. per square inch.

(4) Taking the above (2) and (3), as an example of an inward radial flow turbine, determine the total pressure drop between inlet and outlet.

Answer. 46·62 feet of water.

(5) A centrifugal pump has an impeller 12 inches internal, 24 inches external diameter and runs at 800 revolutions, discharging into a vortex chamber where a free vortex is formed. If the outer diameter of the

[1] "Memoirs, Manchester Lit. and Phil. Soc.," 1911.

vortex chamber is 3 feet, determine the rise in pressure due to the vortex motion.

(*a*) in the wheel; (*b*) in the vortex chamber.

Neglect all losses and assume a perfectly efficient vortex chamber.

Answer. (*a*) 81·9 feet of water.

(*b*) 60·6 „ „

(6) A flat cylindrical disc, 18 inches diameter, keyed on to the lower end of a 3 inch vertical shaft serves as a hydraulic footstep bearing. Its lower face is plane and bears against radiating ribs cast in the pressure cylinder, while its upper face carries a series of radial ribs which bear against the plane upper lid of the pressure cylinder. Thus water above the disc rotates with the disc, while the water below is at rest. The upper and lower sides are in free communication around the periphery of the disc. Determine the resultant upward pressure on the shaft at a speed of 400 revolutions per minute.

Answer. 844 lbs.

(7) A pipe 4 inches diameter is suddenly enlarged to 6 inches diameter. Determine the consequent loss of energy per lb. of water with a velocity of flow in the small pipe of 6 feet per second.

Answer. ·173 foot lb. per lb.

CHAPTER VI.

Flow from a small Orifice—Coefficients of Contraction, Velocity and Discharge—Dynamical Similarity of Orifices—Suppressed Contraction—Borda's Mouthpiece—Sharp-edged Orifice—Converging Mouthpiece—Bell-mouthed Orifice—Diaphragm in Pipe Line—Velocity of Approach—Time of emptying Reservoir—Submerged Orifice—Form of Jets.

Art. 48.—Flow from a Small Orifice.[1]

If an opening be made in the side or base of a tank containing water, the introduction of a few drops of aniline dye shows that steady stream line motion is set up in the mass of fluid, these stream lines converging towards the orifice from every side. At the boundary of the issuing jet the stream lines are, as already indicated (Art. 14), tangential to the edges of the orifice, and with a sharp-edged orifice the general stream line motion is as shown in Fig. 22. It follows that, after passing the plane of the orifice, the section of the jet gradually diminishes, and its boundaries do not become parallel until some finite distance from the orifice. The section at which the jet becomes parallel is termed the *vena contracta*, or contracted vein, and with a small circular orifice is at a distance from the orifice equal to about ·498 times the diameter.

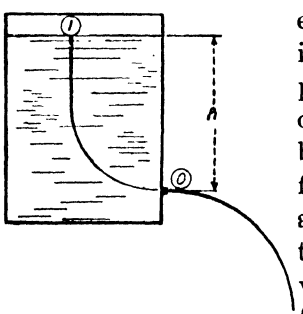


Fig. 52.

Now suppose a particle of weight w lbs. to travel along a stream line from the surface to the orifice and on to the *vena contracta* (Fig. 52). Since the motion is steady, the energy throughout is constant (neglecting viscosity); and if we suppose the surface area to be large, so that the surface velocity may be neglected, when in the surface its kinetic energy is zero. If the orifice is at a mean depth h below the free surface, and if its level be taken as datum, the potential energy of the particle is wh foot lbs., while its pressure energy is zero, its pressure being that of the atmosphere which is taken as datum pressure.

[1] By a small orifice we mean one whose dimensions are small in comparison with the head of water above its centre, so that at any point in its area the head may be taken as equal to that at its centre without sensible error. The subject-matter of Arts. 48—57 refers to such orifices.

Immediately after passing the orifice, its potential energy is zero, so that if we can determine its pressure energy, its kinetic energy and therefore its velocity may be inferred. Now on passing the orifice the jet is exposed to atmospheric pressure, and if the pressure is the same throughout its pressure energy is zero. If the jet is parallel this condition is satisfied,[1] but so long as its boundary is curved the centrifugal action of the outer layers necessitates an increase in pressure along the radius of curvature towards the centre of the jet. It follows that in the plane of the orifice, and up to the *vena contracta*, the pressure in the interior of the jet is greater than that of the atmosphere.

The distribution of pressure and of velocity across a horizontal diameter in the plane of the orifice is substantially as shown in Fig. 53. With free lateral contraction, the velocity at the centre of the jet varies from ·62 to ·64 $\sqrt{2\,g\,h}$, while with lateral contractions suppressed this velocity is about ·69 $\sqrt{2\,g\,h}$ (Bazin).

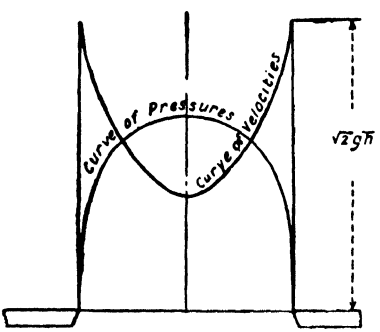


FIG. 53.

At the *vena contracta*, however, the pressure across the jet is uniform and atmospheric,[1] the pressure energy at the orifice having been converted into kinetic energy, with a consequent increase in velocity from orifice to *vena contracta*.

The pressure energy at the *vena contracta* then is zero, the potential energy is zero, while the kinetic energy is $\frac{v^2}{2\,g}$ ft. lbs. per lb.

Denoting the surface by the suffix (1), and the *vena contracta* by the suffix (0), we have

$$w\left[\frac{p_0}{W}+\frac{v_0^2}{2\,g}+z_0\right]=w\left[\frac{p_1}{W}+\frac{v_1^2}{2\,g}+z_1\right]$$

where $p_1=p_0=0$; $z_0=0$; $v_1=0$

$$\therefore\quad \frac{v_0^2}{2\,g}=z_1$$

$$\therefore\quad v_0=\sqrt{2\,g\,z_1}=\sqrt{2\,g\,h}\text{ ft. per sec.}$$

i.e., theoretically the velocity of efflux at the *vena contracta* is the same as

[1] Even in a parallel jet the pressure in the interior will be greater than atmospheric because of the surface tension. The magnitude of this excess pressure, in feet of water, is given by $h'=\frac{2\,T}{d\,W}$, where T is the surface tension in lbs. per foot and d is the diameter in feet

if the particles had fallen freely through a height h feet under the action of gravity.

This theorem was first demonstrated by Torricelli,[1] who showed that a vertical jet of fluid would rise very approximately to the free level in the vessel from which it was supplied.

Due, however, to the viscosity of the liquid, to surface tension, and to the resistance of the air, the velocity is always slightly less than this theoretical value, and is given by $C_v \sqrt{2gh}$, where C_v is called the **Coefficient of Velocity.** The value of C_v can only be determined experimentally, but with a sharp-lipped orifice increases with h from about ·96 to ·994. Weisbach gives the following values for an orifice of ·033 feet diameter:—

h feet . .	·066	1·64	11·5	56·0	338
C_v . . .	·959	·967	·975	·994	·994

At the *vena contracta* the stream lines for the first time become parallel and perpendicular to the cross section of the stream, so that if a_c be the sectional area of the *vena contracta* in square feet, the volume discharged per second $= C_v \sqrt{2gh} \times a_c$ cubic ft. If a is the area of the orifice, the ratio $\frac{a_c}{a}$ is termed the **Coefficient of Contraction,** and is denoted by C_c. The discharge is thus given by $C_c\, C_v \sqrt{2gh} \times a = C \sqrt{2gh} \times a$ cub. ft. sec.

Here C is termed the **coefficient of discharge.**

These coefficients vary with the head h; with the area of the orifice; and with the shape of the orifice and its position.

The following table, giving values of C, abridged from Hamilton

Head from centre of orifice in feet.	Square Orifices; lengths of side of square in feet.				
	·02	·04	·07	·12	·20
·4	—	·643	·628	·616	—
·6	·660	·636	·623	·613	·605
1·0	·648	·628	·618	·610	·605
6·0	·623	·612	·607	·605	·604
20·0	·606	·604	·602	·602	·602
100·0	·599	·598	·598	·598	·598

[1] De motu gravium naturaliter accelerato (1643).

Head above centre of orifice in feet.	Circular Orifices; diameters in feet.				
	·02	·04	·07	·12	·20
·4	—	—	·628	·612	—
·6	·655	·630	·618	·609	·601
1·0	·644	·623	·612	·605	·600
6·0	·618	·607	·602	·599	·598
20·0	·601	·599	·597	·596	·596
100·0	·593	·592	·592	·592	·592

Smith's "Hydraulics," indicates the nature of these variations. In each case the orifice had full contraction, had free discharge into air, and the inner face of the plate in which the orifice was formed had sharp corners, so that the escaping jet only touched these inner edges.

More recent experiments throw some doubt on the assumption, based chiefly on the work of Smith, that the coefficient decreases up to a head of 100 feet, under which head it has a constant value of ·592 for all diameters from ·5 inches to 12 inches. Among these may be quoted those of *Messrs. Judd and King* at the Ohio State University,[1] which were carried out on orifices of diameters ¾ inch, 1 inch, 1½ inches, 2 inches, 2½ inches, under heads varying from 4 to 93 feet. The results showed a very small change in the coefficient for an increase of head. The following are from tests on the 2-inch orifice :—

Head in feet . . .	5·00	9·08	17·80	23·24	36·12	47·02	57·70	69·99	92·01
Coefficient of discharge	·6084	·6083	·6080	·6083	·6082	·6088	·6081	·6080	·6080

Experiments by *H. J. Bilton*[2] indicate that if an orifice is less than about 2·5 inches diameter perfect contraction is impossible, the degree of imperfection becoming more marked as the diameter decreases. This is true however high the head. For orifices of greater diameters the contraction appears to be complete and constant for heads above about 17 inches, attaining a value ·598. Smaller orifices likewise have a "critical" head, greater than 17 inches, above which the contraction is constant but

[1] *Engineering News*, New York, September 27, 1906.
[2] *Engineer*, June 19, 1908.

greater than ·598. The coefficient is the same for a given orifice under a given head, whatever be the direction of discharge. The following tables give the essential results of these experiments :—

Diameter of orifice in inches	0·15	0·2	0·25	0·3	0·4	0·5	0·6	0·75	1·0 and over.
Normal coefficient of discharge	0·631	0·630	0·628	0·627	0·624	0·621	0·618	0·613	—
Critical head in inches	65	55	45	32	25	22½	20	18	17

COEFFICIENTS OF DISCHARGE FOR STANDARD CIRCULAR ORIFICES.

Head in inches.	Diameter of Orifice in Inches.						
	2½ and over.	2	1½	1	¾	½	¼
45 and over	0·598	0·599	0·603	0·608	0·613	0·621	**0·628**
22	—	—	—	—	—	**0·621**	0·638
18	—	—	—	—	**0·613**	0·623	0·643
17	**0·598**	**0·599**	**0·603**	**0·608**	0·614	0·625	0·645
12	0·600	0·601	0·606	0·612	0·618	0·630	0·658
9	0·604	0·606	0·612	0·619	0·623	0·637	0·660
6	0·610	0·612	0·618	0·626	0·632	0·643	0·669
3	—	—	—	0·640	0·646	0·657	0·680
2	—	—	—	—	—	0·663	0·683

The critical heads and the corresponding normal coefficients are in bold figures.

T. P. Strickland,[1] from experiments on sharp-edged orifices of 1 inch and 2 inches diameter, deduces the formula $C = ·5925 + \frac{·018}{\sqrt{h \cdot d^{\frac{3}{2}}}}$, where d is the diameter in inches and h the head in feet.

With a rectangular orifice, the coefficient is rather greater than with a square orifice of the same area, and generally increases with the ratio of the perimeter to the area of an orifice.

Messrs. Smith and Walker [2] give the following experimental results for sharp-edged circular orifices :—

[1] "Proc. Inst. C.E.," vol. 181, p. 531.
[2] "Proc. Inst. Mechanical Engineers," Jan., 1923, p. 23.

COEFFICIENT OF DISCHARGE C FOR CIRCULAR SHARP-EDGED ORIFICES.

Head over Centre.	Diameter of Orifice, in Inches.				
Feet, H.	0·75	1·0	1·5	2·0	2·5
0·2	0·684	0·645	0·617	0·611	0·609
0·4	0·675	0·640	0·615	0·609	0·607
0·8	0·666	0·636	0·613	0·607	0·606
1·2	0·659	0·634	0·612	0·607	0·606
1·6	0·654	0·632	0·612	0·607	0·606
2·0	0·651	0·630	0·611	0·607	0·606
4·0	0·641	0·627	0·611	0·607	0·606
8·0	0·635	0·626	0·611	0·607	0·606
10·0	0·635	0·625	0·611	0·607	0·606
20·0	0·635	0·625	0·611	0·607	0·606
40·0	0·635	0·625	0·611	0·607	0·606
60·0	0·634	0·624	0·611	0·607	0·606
80·0	0·634	0·624	0·611	0·607	0·606
100·0	0·634	0·624	0·611	0·607	0·606

TABLE 2.

COEFFICIENT OF CONTRACTION C_c FOR CIRCULAR SHARP-EDGED ORIFICES.

Head over Centre.	Diameter of Orifice, in Inches.				
Feet, H.	0·75	1·0	1·5	2·0	2·5
0·2	0·720	0·676	0·644	0·631	0·622
0·4	0·710	0·670	0·638	0·626	0·618
0·8	0·699	0·663	0·631	0·622	0·616
1·2	0·690	0·658	0·627	0·619	0·615
1·6	0·684	0·654	0·625	0·618	0·615
2·0	0·680	0·652	0·624	0·617	0·614
4·0	0·671	0·644	0·621	0·617	0·614
8·0	0·668	0·640	0·620	0·617	0·614
10·0	0·667	0·639	0·620	0·616	0·614
20·0	0·667	0·639	0·618	0·616	0·613
40·0	0·666	0·639	0·617	0·615	0·613
60·0	0·666	0·639	0·617	0·615	0·613
80·0	0·666	0·638	0·616	0·614	0·612
100·0	0·666	0·638	0·616	0·614	0·612

COEFFICIENT OF VELOCITY C FOR CIRCULAR SHARP-EDGED ORIFICES.

Head over Centre.	Diameter of Orifice, in Inches.				
Feet, H.	0·75	1·0	1·5	2·0	2·5
0·2	0·951	0·955	0·958	0·967	0·980
0·4	0·951	0·956	0·964	0·973	0·983
0·8	0·953	0·960	0·971	0·977	0·986
1·2	0·955	0·963	0·975	0·983	0·989
1·6	0·956	0·965	0·978	0·984	0·989
2·0	0·957	0·966	0·980	0·984	0·990
4·0	0·956	0·973	0·983	0·984	0·990
8·0	0·951	0·977	0·985	0·984	0·990
10·0	0·953	0·977	0·985	0·986	0·990
20·0	0·953	0·978	0·988	0·986	0·993
40·0	0·954	0·978	0·990	0·988	0·993
60·0	0·954	0·979	0·990	0·988	0·993
80·0	0·954	0·979	0·992	0·990	0·993
100·0	0·954	0·979	0·992	0·990	0·993

An examination of the foregoing data shows that there is an appreciable difference between the results of experiments by different observers, and that the difference is more pronounced for small orifices and low heads. This may be due partly to slight differences in the finish of the edges and upstream faces of the orifices, and partly to the effect of surface tension and to differences in the temperature and viscosity of the water. For orifices having a diameter exceeding 1·5 inches and under heads greater than 10 feet, the values in general agree within about 1 per cent. If smaller orifices are to be used for measuring purposes they should be calibrated as nearly as possible under the conditions likely to obtain in use.

Suppressed Contraction.—Any interference with the free production of stream lines, such as occurs, for example, where the depth of water above the orifice is too small, or where this is near the sides or bottom of a tank as indicated in Fig. 22 b, curtails the full flow to the orifice, and tends to prevent the full contraction of section at the *vena contracta*. In any such case the value of C_c and of C will be greater than indicated above, the true values being entirely dependent on the circumstances governing the flow to the orifice.

From experiments on a rectangular orifice, ·177 feet long and ·089 feet wide, in which the contraction could be suppressed on part or the whole of the perimeter by means of flat plates perpendicular to the plane of the orifice and ·22 feet long, *Bidone* deduced the formula

$$C = ·608 + ·0925\ x,$$

where x is the fraction of the perimeter over which the contraction is suppressed. For circular orifices

$$C = ·608 + ·079\ x.$$

With this device, however, contraction of section takes place at the inner edges of the baffle plates, and when the orifice is entirely surrounded this simply forms a Borda's mouthpiece running full (p. 122) with C about ·75.

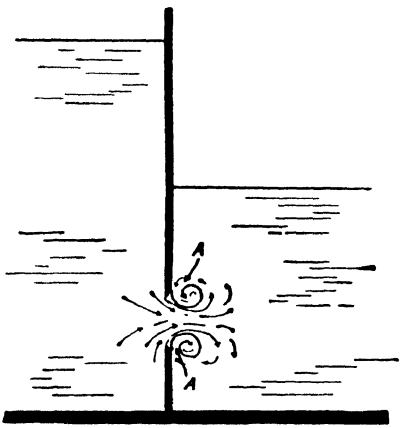


FIG. 54.

Experiments by *Lesbros* on a vertical orifice, ·656 feet square, showed mean values of the coefficient, for heads between 1·5 and 10 feet, in the ratios 1·00, 1·032, 1·043, 1·116, according as the contraction was unaffected, suppressed at the lower edge, at one side, and at the lower edge and sides, and thus agree fairly well with Bidone's results.

Where an orifice is made in the thick side of a vessel, or is fitted with an external pipe so that the jet, after springing clear of its inner edges, again touches the sides of the orifice before escaping, or where fitted with re-entrant mouthpiece, the physical conditions governing the flow are entirely changed, and the coefficients assume new values. Thus the coefficient of contraction may vary from ·5 to 1·0 under different conditions of working, with the same size of orifice and the same head. There is, however, nothing mysterious about this change — it is governed entirely by the conditions of flow, and, as will be seen, may be predetermined with some exactitude when those conditions are known.

Submerged Orifices.—If an orifice discharges below the surface of the water in a second vessel instead of into the air, this has the effect of increasing the pressure against which discharge takes place, and thus of reducing the effective head, and also of slightly reducing the value of the coefficient of discharge.

The reduction of the coefficient of discharge is probably a secondary

effect of the eddy formation which occurs in the dead water surrounding the jet at A and B, Fig. 54, the dead water comprising the eddy projecting into the stream and reducing its effective area.

In the sketch (Fig. 55), if the suffix $(_1)$ refer to the free surface in the upper vessel and suffix $(_0)$ to the *vena contracta*, we have, neglecting viscosity,

$$\frac{p_1}{W} + \frac{v_1^2}{2g} + z_1 = \frac{p_0}{W} + \frac{v_0^2}{2g} + z_0$$

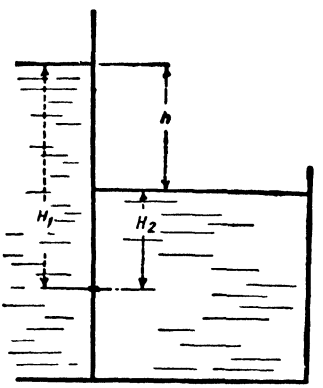


FIG. 55.

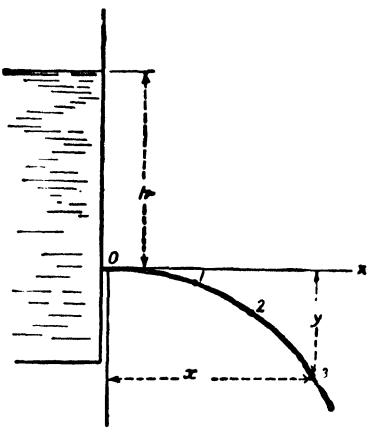


FIG. 56.

Taking the datum level at the centre of the orifice, and assuming the area at the surface in the upper vessel to be large, we have

$$p_1 = 0;\ v_1 = 0;\ z_0 = 0;\ z_1 = H_1;\ \frac{p_0}{W} = H_2$$

$$\therefore\quad H_1 - H_2 = \frac{v_0^2}{2g} \text{ feet}$$

$$\therefore\quad v_0 = \sqrt{2g\,(H_1 - H_2)} = \sqrt{2gh},$$

where h = difference in level of the free surfaces on the two sides of the orifice.

ART. 49.—EXPERIMENTAL DETERMINATION OF THE COEFFICIENTS.

(1) **Coefficient of Velocity, for an Orifice in a Vertical Plate.**—Keeping h constant, mark on a flat board parallel to the plane of the jet, points 1, 2, 3, 4, etc., by means of a set square, these points marking as nearly as possible the centre of the jet. Draw a smooth curve through the points, and draw the horizontal OX through O (Fig. 56). Taking any point on

the curve we get, if v is the velocity of efflux, measuring horizontal distances from the *vena contracta*,

$$x = v\,t;\ y = \frac{1}{2}g\,t^2$$

$$\therefore\ v = \frac{x}{t} = \frac{x}{\sqrt{\frac{2y}{g}}} = \sqrt{\frac{g\,x^2}{2\,y}}.$$

But $\frac{v}{\sqrt{2\,g\,h}} = C_v$. $\qquad \therefore\ C_v = \sqrt{\frac{x^2}{4\,y\,h}}$.

If the issuing jet make an angle α with the horizontal, then, the origin of co-ordinates being taken at the *vena contracta*,

$$x = v\cos\alpha\,.\,t;\ y = \frac{1}{2}g\,t^2 - v\sin\alpha\,t.$$

Since $v\sin\alpha\,t = x\tan\alpha$, on substituting for t^2 we get:

$$y + x\tan\alpha = \frac{g\,x^2}{2\,v^2\cos^2\alpha}$$

$$\therefore\ v = \sqrt{\frac{g\,x^2\sec^2\alpha}{2\,(y + x\tan\alpha)}}.$$

Then $C_v = v \div \sqrt{2\,g\,h}$. If α is not known, it a second pair of values x_1, y_1, are measured, we get

$$v = \sqrt{\frac{g\,x_1^2\sec^2\alpha}{2\,(y_1 + x_1\tan\alpha)}},$$

and from these two equations α may be calculated.

A second method of determining the coefficient of velocity is to allow a jet of water to escape from an orifice in the vertical side of a tank supported on knife edges (Fig. 57), the level of the water in the tank being maintained constant by the influx of a vertical stream. The position of a pointer fixed to the tank is noticed before the orifice is opened, and when the jet is allowed to escape, this pointer is brought back to the same mark by means of the weight W, placed at a leverage x.

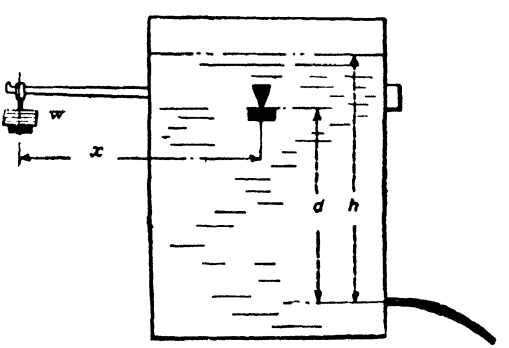


FIG. 57.

The weight of water w leaving the vessel per second is determined by

weighing. Then if v be its mean velocity in a horizontal direction, the momentum per second leaving the tank in this direction $= \frac{w\ v}{g}$. To produce this flux of momentum a constant horizontal force of magnitude $\frac{w\ v}{g}$ lbs. must be impressed on the jet, a force which can only be due to an otherwise unbalanced pressure on the side of the tank opposite to the orifice.

We have then, taking moments about the knife edge:—

$$W\ x = \frac{w\ v}{g}\ .\ d \text{ where } d = \text{depth of orifice below knife edge}$$

$$\therefore \quad v = \frac{W}{w}\ .\ g\ .\frac{x}{d} \text{ feet per second}$$

$$\therefore \quad C_v = \frac{W\ g\ x}{w\ d\ \sqrt{2\ g\ h}}.$$

This method is only suitable for experiments on a small scale. A third method, which has been applied very successfully, consists in measuring the actual velocity at several points in the cross section of the *vena contracta* by means of a Pitot Tube (Arts. 84, 118). If then v is the velocity at any radius x, the total discharge $= \int_0^r 2\ \pi\ v\ x\ dx$ cubic feet per second. If a curve be plotted having as abscissæ the values of x and as ordinates the corresponding values of $2\ \pi\ x\ v$, the area under this curve will represent the discharge per second, from which the mean velocity may be obtained by dividing by the area of the jet.

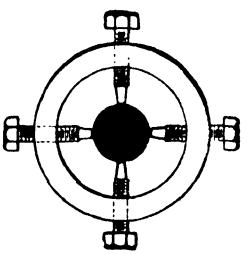


FIG. 58.

(2) The **coefficient of contraction**, C_c, may be measured by using a ring (Fig. 58), surrounding the jet at the *vena contracta*, and fitted with micrometer measuring screws, which may be adjusted until just touching its surface.[1]

(3) The **coefficient of discharge**, C, may be determined directly by measuring the quantity Q in cubic feet discharged per second from the orifice a, under a head h.

Then $$C = \frac{Q}{a\sqrt{2\ g\ h}}.$$

[1] For a sketch of a contraction gauge, and for experiments on the measurements of the coefficients in the case of a large jet forming part of a Pelton Wheel installation, see a paper by W. R. Eckhart, "Proc. Inst. Mech. Engineers," 1909—10. Also *Engineering*, January 14, 1910, p. 59.

ART. 50.—DYNAMICAL SIMILARITY IN ORIFICE FLOW—RATIONAL FORMULA FOR DISCHARGE.

Since the flow from an orifice discharging freely into the air is affected both by the weight and viscosity of the fluid, it is necessary for the similarity of the lines of flow from two similar orifices, that $\nu/v\,h$ and hg/v^2 shall be simultaneously the same for both (Art. 32).

If the orifices are similarly situated, so that the head on each is proportional to its diameter, the second of these relationships is satisfied if $v \propto \sqrt{h}$, as is very approximately true. If both orifices are discharging the same fluid, so that ν is the same for both, the first relationship will not be satisfied, so that viscosity prevents exact similarity of the streams. It follows that the coefficient of discharge will vary somewhat with the size of orifice and with the head.

It appears, however, that if viscosity has any effect on the velocity, the latter cannot be accurately proportional to $\sqrt{h}$, for if v depends upon h, μ, ρ and g, so that

$$v \propto h^x \mu^y \rho^z g^a$$

dimensionally,

$$l t^{-1} = l^x \cdot M^y l^{-y} t^{-y} \cdot M^z l^{-3z} \cdot l^a t^{-2a},$$

which is satisfied if

$$x = 3a - 1; \quad y = 1 - 2a; \quad z = 2a - 1.$$

$$\therefore \quad v \propto h^{3a-1} \cdot \left(\frac{\mu}{\rho}\right)^{1-2a} \cdot g^a$$

$$\propto \left(\frac{h^3 g}{\nu^2}\right)^a \cdot \frac{\nu}{h}.$$

If $v \propto \sqrt{h}$ it follows that $a = \cdot 5$, and, therefore, that v is independent of ν, while if v is not independent of ν it cannot be proportional to $\sqrt{h}$.

If d is the diameter, or the length of any side of an orifice, Q will depend upon d.

let $$Q \propto h^x \mu^y \rho^z g^a d^m.$$

Expressing this dimensionally, and equating indices of like quantities as before gives

$$x = 1 + 3a - m; \quad y = 1 - 2a; \quad z = 2a - 1;$$

so that $$Q \propto h^{1+3a-m}, \nu^{1-2a} \cdot g^a \cdot d^m.$$

$$\propto \left(\frac{h^3 g}{\nu^2}\right)^a \left(\nu h\right) \left(\frac{d}{h}\right)^m \quad \ldots\ldots\ldots \quad (1)$$

The values of a and m can only be determined from experiments in which the head and size of orifice are varied independently.

Thus Gourley and Crimp, from experiments on rectangular weirs with end contractions (p. 156), deduced the empirical formula

$$Q = 3{\cdot}10\, b^{1{\cdot}02}\, h^{1{\cdot}47},$$

where b is the length of the weir crest. In this case

$$1 + 3a - m = 1{\cdot}47\ ; \quad m = 1{\cdot}02,$$

making $a = {\cdot}4967$. In this case $1 - 2a$, the index of ν, equals ·0067, indicating that at different viscosities the discharge would be proportional to $\nu^{{\cdot}0067}$. A 100 per cent. increase in the viscosity, such as would be obtained by altering the temperature of the water from 90° F. to 40° F., would then increase the discharge by about ·6 per cent.

In similar orifices similarly situated, so that $d \propto h$, equation (1), becomes

$$\frac{Q}{\nu h} \propto \left(\frac{h^3 g}{\nu^2}\right)^a \propto \left(\frac{h^3}{\nu^2}\right)^a \quad \ldots\ldots\ldots \quad (2)$$

and, if the viscosity is the same in both,

$$\frac{Q}{h} \propto h^{3a}$$

It follows that if values of $\frac{Q}{h}$ are plotted against values of h, the curve obtained will apply equally well to a large weir or orifice or to a small scale model having a head proportionally smaller.

Since the effects of surface tension have been neglected in deducing the foregoing relationships, this conclusion only applies to models sufficiently large for this effect to be negligibly small.

The triangular weir is an example in which, by altering the head, a series of similar orifices are formed. For a right-angled notch, experiments by Barr [1] and by Gaskell [2] give

$$Q = 2{\cdot}48\, h^{2{\cdot}48} \text{ c.f.s.}$$

while Gourley and Crimp [3] give

$$Q = 2{\cdot}48\, h^{2{\cdot}47} \text{ c.f.s.}$$

Since $Q \propto h^{3a+1}$, these indices 2·48 and 2·47 correspond respectively to values of a equal to ·4933 and ·4900, while the corresponding indices of ν are ·0133 and ·020.

[1] *Engineering*, April 8 and 15, 1908.
[2] "Proc. Inst. C. E.," vol. cxcvii.
[3] "Proc. Inst. C. E.," vol. cc., 1914–15, pt. ii.

Art. 50a.—Dynamical Similarity in Submerged Orifices.

In a submerged orifice the discharge is not affected by gravity, and, consequently, the condition $vl/\nu = \text{constant}$ must be satisfied for dynamical similarity (Art. 31). Thus in flow past diaphragms in pipe lines, or through Venturi meters, identical coefficients of discharge should be obtained for identical values of this ratio. This has been confirmed experimentally by W. N. Bond [1] in the case of flow through the same submerged orifice using liquids of widely differing viscosities.

Art. 51.

Theoretically we may always calculate the flow from an orifice in the side or bottom of a vessel, by equating the flux of momentum across the plane of the orifice per second to the unbalanced statical pressure on the opposite vertical side in the one case, and to weight of the unbalanced column of water in the other, *i.e.*, by expressing the fact that the total unbalanced force in the direction of flow is taken up in producing this flow. With an ordinary opening in the side or bottom of a vessel, however, the calculation of this unbalanced force is impossible, since, due to the fact that there is stream line motion along the face containing the

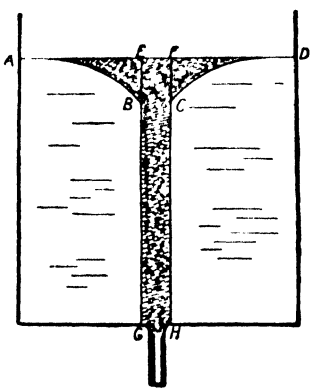


Fig. 59.

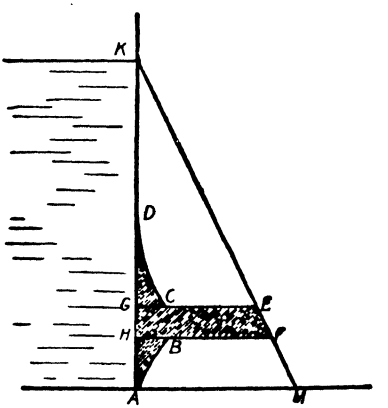


Fig. 60.

opening, giving the water kinetic head, the pressure head at any point in this face is less than that simply due to the statical head at that point by an amount which is indeterminate.[2] The resultant unbalanced pressure is therefore in this case indeterminate. In the case of an opening in the

[1] "Proc. Physical Society of London," vol. 33, pt. 4, June 15, 1921.

[2] Since part of the head produces acceleration towards the point.

flat, horizontal bottom of a vessel, the pressure over this face is not uniform, since the kinetic head is greatest near the orifice, the distribution of pressure being as represented in Fig. 59, where the ordinates from the base to the curve *A B C D* measure the pressure on the base.

When the orifice *G H* is closed, the pressure on *G H* is that represented by the shaded area *E F H G*, but when open the unbalanced pressure becomes that represented by the whole shaded area *A B G H C D*.

Similarly in the case of an opening in the vertical side *A K* of a vessel (Fig. 60). Here the area *G E F H* represents the pressure on *G H* when the orifice is closed, while the horizontal distances between the line *K M* and the curves *A B* and *C D* measure the pressures on the side of the vessel with an issuing jet. The shaded area then represents the unbalanced pressure, which is to be equated to the outflow of momentum per second from the orifice.

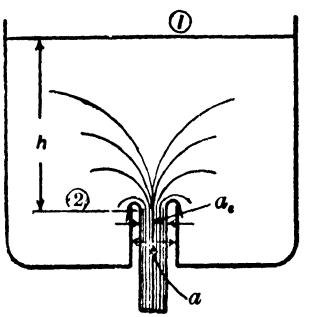


FIG. 61.

By returning the mouthpiece for some distance into the vessel, as in the Borda mouthpiece (Fig. 61), this flow over the face containing the orifice is prevented, and the pressure on this face now approximates to that due to the statical head alone. The equations of momentum can therefore be applied.

ART. 52.—BORDA'S MOUTHPIECE.

Let a be the area of the orifice and a_c the area of the stream as soon as it has assumed a parallel cylindrical form, *i.e.*, at the *vena contracta*. Let h be the depth of this point below the surface. If the suffixes (1) refer to the surface and (2) to the *vena contracta*, we have, neglecting viscosity,

$$\frac{p_1}{W} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{W} + \frac{v_2^2}{2g} + z_2.$$

Also $p_2 = p_1 = 0$; $z_1 - z_2 = h$; $v_1 = 0$ if surface area is large,

$$\therefore \quad v_2^2 = 2gh.$$

Mass carried away per second $= \frac{W}{g} \cdot a_c v_2$.

$\therefore$ Momentum carried away per second $= \frac{W}{g} \cdot a_c v_2^2$.

This momentum is produced by a force equal to the weight of a column of water of area a and of height h (approximately).

$$\therefore \quad \text{Force producing motion} = W\,a\,h \text{ lbs.}$$

$$\therefore \quad W\,a\,h = \frac{W}{g} \cdot a_c\, v_2^2$$

$$= \frac{W}{g} \cdot a_c \cdot 2\,g\,h$$

$$\therefore \quad a_c = \cdot 5 a,^1$$

i.e., the coefficient of contraction in the case of a Borda's mouthpiece, where the effluent stream does not touch the sides of the mouthpiece, is one half.

Borda's Mouthpiece running full.—With a vertical Borda, where the water in the vessel is initially quite steady, the issuing jet after becoming parallel does not again come into contact with the sides of the mouthpiece. If, however, the water in the vessel be agitated, the jet, after contracting to form a *vena contracta*, again expands to fill the mouthpiece (Fig. 62). This is termed running full as opposed to running freely as in the previous case. The issuing motion is now turbulent, not steady as is the case with a free Borda. It is worthy of note that under no circumstances with a free Borda, or indeed with any jet having non-sinuous motion, is any splash produced on meeting a solid surface. With sinuous motion splashing always occurs.

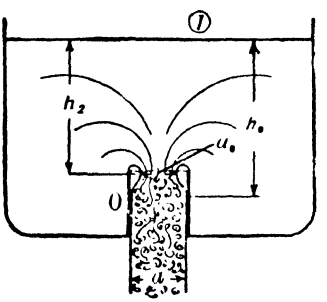


FIG. 62.

We now get a loss of head due to the expansion of the stream from a_c to a after contracting, so that

$$\frac{p_1}{W} + \frac{v_1^2}{2\,g} + z_1 = \frac{p_0}{W} + \frac{v_0^2}{2\,g} + z_0 + \text{loss due to expansion.}$$

The loss due to expansion (Art. 44) $= \frac{v_0^2}{2\,g}(m-1)^2$,

$$\text{where } m = \frac{a}{a_c}$$

$$\therefore \quad \frac{p_1}{W} + \frac{v_1^2}{2\,g} + z_1 = \frac{p_0}{W} + \frac{v_0^2}{2\,g}\left[(m-1)^2 + 1\right] + z_0$$

But $p_1 = 0$ and $v_1 = 0$. Also $z_1 - z_0 = h_0$.

$$\therefore \quad \frac{p_0}{W} + \frac{v_0^2}{2\,g}\left[(m-1)^2 + 1\right] = h_0. \qquad (1)$$

[1] Actually $v_2^2 = C_v^2 \times 2\,g\,h$, so that $\frac{a_c}{a} = \frac{1}{2\,C_v^2}$. The coefficient of contraction for the Borda running full is modified in a similar manner.

Now equating the momentum passing the section at O per second, to the force producing it, we get

$$W\,a\,h_0 - p_0\,a = \frac{W}{g}\,a\,v_0^2$$

$$\therefore \quad h_0 - \frac{p_0}{W} = \frac{v_0^2}{g}$$

But $h_0 - \frac{p_0}{W} = \frac{v_0^2}{2\,g}\,[(m-1)^2 + 1]$ from (1)

$\therefore$ Equating these values we get $\frac{(m-1)^2+1}{2} = 1$

$$\therefore \quad (m-1)^2 = 1 \qquad \text{or } m = 2,$$

i.e., the coefficient of contraction at the neck in a Borda's mouthpiece running full is one half.

If the jet discharges into the atmosphere so that $p_0 = 0$ we have

$$v_0^2 = g\,h_0, \text{ so that } v_0 = \frac{\sqrt{2\,g\,h}}{\sqrt{2}} \text{ (approximately),}$$

i.e., the velocity of discharge is $\frac{1}{\sqrt{2}}$ times the velocity when flowing free, and since the area of full jet = 2 (area of free jet).

$\therefore$ Quantity flowing per second from full Borda $= \sqrt{2}$ (quantity from free Borda).

The coefficient of discharge from such an orifice is thus equal to $\sqrt{2}$ times the coefficient for a free Borda mouthpiece and therefore to

$\frac{1}{C_v^2\sqrt{2}}$. Taking $C_v = \cdot 975$ this would make $C = \cdot 745$.

Experiments by H. J. Bilton[1] on such mouthpieces, $2\frac{1}{2}$ diameters in length, gave the following values for C, no variation of C with head being noted for heads as low as 6 inches :—

Diameter (ins.).	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{3}{4}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$
C	·91	·87	·85	·83	·81	·79	·77	·76	·75

The difference between the calculated and experimental values of C is due to the fact that the area at the *vena contracta* is actually greater than $\cdot 5\,a$ because of the effect of viscosity in modifying the contraction of area. Eddy loss at the re-expansion of the stream is consequently reduced with a corresponding increase in discharge.

[1] "Victorian Inst. of Engineers," April 1, 1908.

Denoting the contracted section by the suffix (2) we have

$$\frac{p_2}{W} + \frac{v_2^2}{2g} + z_2 = \frac{p_0}{W} + \frac{v_0^2}{2g} + z_0 + \text{loss due to expansion.}$$

But loss due to expansion $= \frac{v_0^2}{2g}(2-1)^2 = \frac{v_0^2}{2g}$ (Art. 44)

Also $v_2 = 2v_0$,

$$\therefore \quad \frac{p_0 - p_2}{W} = \frac{4v_0^2 - 2v_0^2}{2g} + (z_2 - z_0)$$

$$\therefore \quad \frac{p_0 - p_2}{W} = \frac{v_0^2}{g} + (z_2 - z_0).$$

Again $v_0^2 = g h_0$,

$$\therefore \quad \frac{p_0 - p_2}{W} = h_0 + z_2 - z_0.$$

In a horizontal mouthpiece $z_2 = z_0$,

$$\therefore \quad \frac{p_0 - p_2}{W} = h_0 \text{ feet,}$$

i.e., the gain of pressure from (2) to (0) is equal to a head of h_0 feet.

If the jet issues into the atmosphere $p_0 = 0$.

$$\therefore \quad \frac{p_2}{W} = -h_0 \text{ feet.}$$

Or the pressure at (2) is less than that of the atmosphere by an amount equivalent to a head h_0 of water.

This reduction of pressure at section (2) explains why the velocity at this point, and hence the discharge, is greater with a full Borda than with one running free. The conditions from the surface to the section at (2) are exactly the same whether the mouthpiece runs full or free, but in one case the jet is discharging at this point against atmospheric pressure, while in the other case this pressure is in part removed.

The effect is then substantially the same as if the free Borda were subjected to an additional head h_0.

The theoretical limit of this h_0 is 34 feet, but practically, the liberation of dissolved air as the pressure falls, prevents the formation of anything approaching an absolute vacuum.

Art. 53.—Sharp-edged Orifice in a flat Plate, with External Tube.

With an external tube of the same diameter as the orifice, the effluent stream, after forming a *vena contracta*, always re-expands to fill the tube (Fig. 63).

In this case, as already explained, the coefficient of contraction cannot be theoretically deduced by an application of the equations of momentum. Its value is probably about ·62, but almost certainly varies with the head and size of orifice. The final area of the jet $= a$, so that the coefficient of velocity is the same as the coefficient of discharge. This coefficient varies with the size of orifice, with the length of tube, and probably to some extent with the head. Experiments by Weisbach gave

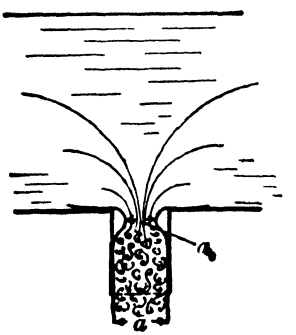


FIG. 63.

the following values of C_v, for tubes having a length equal to about three diameters:—

Diameter (feet).	·032	·066	·098	·131
C_v	·843	·832	·821	·810

The value of C_v was found to vary with the length of the tube approximately as follows:—

Length ÷ diameter .	1·0	3·0	10·0
C_v	·88	·82	·78

Sharp-edged Orifice with Converging Mouthpiece.—If a conical converging pipe be fitted external to the orifice, with its larger diameter equal to that of the orifice, the coefficients of velocity and of discharge at exit depend on the angle of convergence.

The following values are taken from experiments by Castel, and indicate that a maximum discharge is to be obtained when the angle of convergence is about 13°·20′.

Diameter of mouthpiece = ·61 inches.

Angle of Convergence.	0°·0′	1°·36′	3°·10′	5°·26′	7°·52′	10°·20′	12°·4′	13°·24′	14°·28′	16°·36′	21°·0′	29°·58′	40°·20′	49°·0′
C_v	·830	·866	·894	·924	·931	·950	·955	·962	·966	·971	·971	·975	·980	·984
C	·829	·866	·895	·920	·929	·938	·942	·946	·941	·938	·918	·896	·869	·847

Diameter = ·787 inches.

Angle of Convergence.	2°·50′	5°·26′	6°·54′	10°·30′	12°·10′	13°·40′	15°·2′	18°·10′	38°·52′
C_v	·906	·930	·938	·953	·957	·964	·967	·970	·979
C	·910	·928	·938	·945	·949	·956	·949	·939	·920

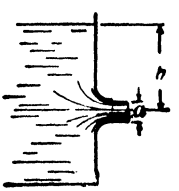


FIG. 64.

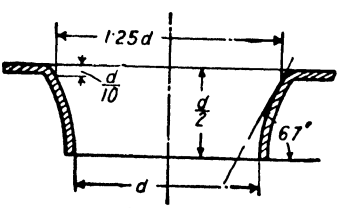


FIG. 65.

The mean of three experiments[1] on a large rectangular converging mouthpiece under a head of 9·6 feet gave a value of $C = ·981$. In this mouthpiece the length was 9·6 feet; the large end 2·4 × 3·2 feet; and the small end ·044 × ·623 feet, opposite faces enclosing angles respectively of 11°·38′ and 15°·18′.

ART. 54.—BELL-MOUTH ORIFICE.

With a bell-mouth orifice (Fig. 64) having a curvature approximately the same as that of the natural stream lines, the pressure over the walls of the mouthpiece will be zero; the coefficient of contraction, unity; and the only loss of head, that due to viscosity.

Such an orifice, if well designed, has a coefficient C or C_v varying from about ·95 for small diameters to ·995 for large diameters, and is well adapted for measuring purposes.

[1] "Practical Hydraulics," Downing, 1861. By Lespinasse et Languedoc.

Experiments by Weisbach indicate that the general proportions should be as shown in Fig. 65, the sides of the mouthpiece making an angle of 67° with the plane of the orifice at a point distant about $\frac{d}{10}$ from that plane.

$$\text{If } C_v = \cdot 975\,; \quad \frac{v^2}{2g} = \cdot 95\,h$$

$\therefore$ loss of energy due to this type of mouthpiece $= \cdot 05\,h$

$$= \cdot 05\,\frac{v^2}{2g} \text{ ft. lbs. per lb.}$$

Art. 55.—Diaphragm in Pipe Line.

Adopting the Venturi formula (p. 88)

$$Q = C\,A\,\sqrt{\frac{2\,g\,h}{n^2 - 1}} \quad \text{c.f.s.}$$

for the discharge through a pipe of sectional area A fitted with a diaphragm (Fig. 66), having a central orifice whose area in $1/n$ times that of the pipe under a difference of pressure h feet of water or the two sides of the diaphragm, the value of C depends on the type of orifice and diaphragm, and on the points at which the pressures are measured.

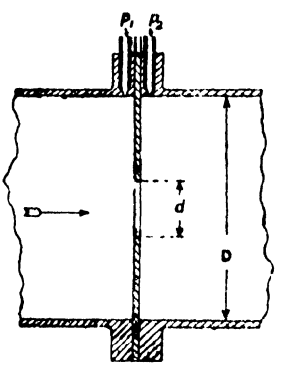
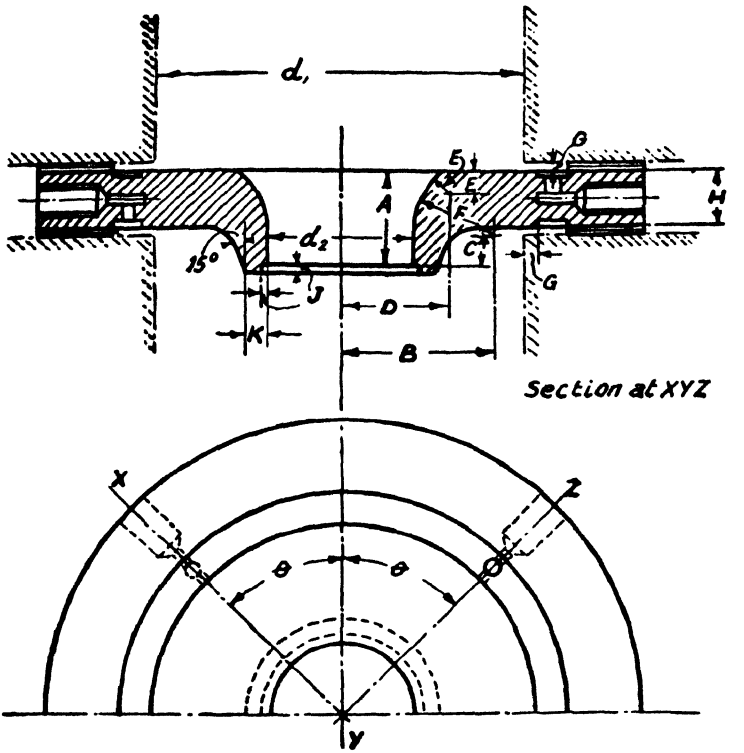


Fig. 66.

Adopting as standard a square-edged orifice in a thin diaphragm, whose thickness, at the orifice, does not exceed 1/30 the diameter and which is bevelled off at 45° on the down-stream side, the following values of C[1] may be taken if the up-stream pressure is measured at the wall of the pipe at a point one pipe diameter upstream from the orifice, and the down-stream pressure at a point ·5 diameter below the orifice.

Value of $1/n$.	·3	·4	·5	·6	·7
C	·614	·615	·618	·624	·634

[1] Hodgson. See also "Proc. Inst. C. E.," 1916–17, pt. ii., p. 108.



Dimensions.	d_2	A	B	C	D	E	F	G	H	J	K	d_1	⊙
Times d_2 .	1	0·605	0·83	0·3	0·75	0·2	0·33	Less than 1/20	*	1/30	1/12	1·4 to 3·15	10°–80°

* H not to exceed 0·35 d_2, but may be less if convenient.

FIG. 67.

The coefficient depends on the points at which the up-stream and down-stream pressures are measured. Convenient points on a specially designed diaphragm are immediately upstream and downstream of the plate as shown in Fig. 66. In this case the British Standards Institution,[1] after considering all available data, state that for standard test purposes the value of C may be taken as 0·605 with an accuracy of $\pm$ 2 per cent., so long as the area of the orifice is not greater than 0·5 that of the pipe.

With a rounded orifice of the proportions shown in Fig. 67, the Institution[1] has adopted the following values of C :—

[1] British Standards Institution, Specification for Pump Tests, No. 599, 1935.

n . .	10	5	3·3	2	
C . .	0·985	0·980	0·970	0·950	

For these coefficients to apply the orifice should be remote from any bends. It should be preceded by at least ten diameters of straight pipe and be followed by at least five diameters of straight pipe.

Owing to the convenience of its application to an existing pipe line the diaphragm-orifice method of water measurement has much to recommend it where it is not intended to be a permanent installation. The thin plate diaphragm can often be inserted in an existing pipe line by springing the flanges. In each case the accuracy is comparable with the much more expensive Venturi meter. The disadvantage for a permanent installation is the much greater loss of head as compared with that in the Venturi meter.

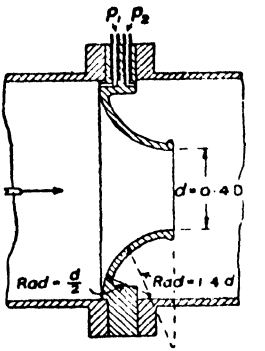


FIG. 68.

With a rounded orifice of the proportions shown in Fig. 68, experiments[1] show that the coefficient of discharge is sensibly ·99 if the pressures are measured at the points shown.

ART. 56.—VELOCITY OF APPROACH.

Where the surface area of a vessel is not so large as to allow of the velocity of approach of the water to an orifice being neglected, if the suffixes (1) and (2) refer to the surface and to the *vena contracta* respectively, we have

$$\frac{p_1}{W} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{W} + \frac{v_2^2}{2g} + z_2.$$

Putting $p_1 = 0$ and $p_2 = 0$, we have

$$\frac{v_2^2}{2g} = \frac{v_1^2}{2g} + z_1 - z_2$$

$$= \frac{v_1^2}{2g} + h \qquad (h = \text{head over orifice})$$

$$\therefore \quad v_2^2 = 2gh + v_1^2.$$

Here v_1 is the velocity of approach.

In words, the kinetic energy at efflux is equivalent to the potential energy at the surface, together with the kinetic energy due to the velocity of approach.

[1] *Engineering*, Dec. 1, 1922, p. 690.

The statical head which would give the same velocity of efflux is now

$$H = \left[h + \frac{v_1^2}{2g} \right] = h + h'.$$

So that the effect of the velocity of approach in increasing the outflow is theoretically the same as that of an additional head h' where $h' = \frac{v_1^2}{2g}$.

ART. 57.

Time of emptying a Vessel through a small, freely discharging Orifice.— (1) Suppose the surface area A of the vessel to be uniform and large compared with the area a of the orifice.

Let v = velocity of efflux at any instant, and h the height of free surface above the *vena contracta* at the same instant.

Now assuming that $v = C_v \sqrt{2gh}$, we have, since

$$\frac{\text{velocity of fall of surface}}{\text{velocity of efflux}} = \frac{a_c}{A}, \text{ that}$$

$$\text{velocity of surface} = -\frac{dh}{dt} = C_v \frac{a_c}{A} \sqrt{2gh} \qquad (1)$$

$$\therefore \quad C_v \frac{a_c}{A} \sqrt{2g} \cdot dt = -\frac{dh}{\sqrt{h}} \qquad (2)$$

Integrating between any two limits of height h_1 and h_2 we get the time $t_2 - t_1 = t$, necessary to lower the surface through the distance $h_1 - h_2$.

$$\therefore \quad C_v \frac{a_c}{A} \sqrt{2g} \cdot t = 2 \left\{ h_1^{\frac{1}{2}} - h_2^{\frac{1}{2}} \right\}$$

$$\therefore \quad t = \frac{2A}{C_v a_c \sqrt{2g}} \left\{ h_1^{\frac{1}{2}} - h_2^{\frac{1}{2}} \right\} \qquad (3)$$

If C be the coefficient of discharge for the orifice the time will then be given by

$$t = \frac{2A}{C a \sqrt{2g}} \left\{ h_1^{\frac{1}{2}} - h_2^{\frac{1}{2}} \right\} \qquad (4)$$

Tank with constant inflow Q cusecs and discharging through an orifice a.

Here

$$\frac{dh}{dt} = \frac{Q}{A} - \frac{Ca}{A} \sqrt{2gh}$$

$$= B - K\sqrt{h}$$

$$\therefore \quad \int_{T_1}^{T_2} dt = \int_{h_1}^{h_2} \frac{dh}{B - K\sqrt{h}} = \frac{1}{K} \int_{h_1}^{h_2} \frac{dh}{\frac{B}{K} - \sqrt{h}}.$$

The solution is

$$T_2 - T_1 = \frac{2}{K}\left[\sqrt{h_1} - \sqrt{h_2} + \frac{B}{K}\log_e \frac{\frac{B}{K} - \sqrt{h_1}}{\frac{B}{K} - \sqrt{h_2}}\right]$$

where $\frac{B}{K} = \frac{Q}{Ca\sqrt{2g}}$.

Time of emptying a Reservoir of varying Cross-section, by small freely discharging Orifice.—Here A is no longer constant but will be a function of h, so that equation (1)

$$-\frac{d\,h}{d\,t} = \frac{C\,a}{A}\sqrt{2\,g\,h}$$

may be integrated if A is an integrable function of h.

Example I.

Reservoir with uniformly varying cross-sectional Area.—Let $k\,D$ = area of reservoir at the orifice.

Then $A = k\,(D + h)$,

$$\text{and } -k\int_{h_2}^{h_1}\left(\frac{D+h}{\sqrt{h}}\right)d\,h = C\int_{t_2}^{t_1}(a\sqrt{2\,g})\,d\,t,$$

$$\therefore\quad t_2 - t_1 = t = \frac{k}{C}\left[\frac{2\,D\left(h_1^{\frac{1}{2}} - h_2^{\frac{1}{2}}\right) + \frac{2}{3}\left(h_1^{\frac{3}{2}} - h_2^{\frac{3}{2}}\right)}{a\sqrt{2\,g}}\right]$$

Where C = coefficient of discharge.

Example II.

Hemispherical Bowl—Radius r—emptied through Hole in the Bottom.

Here $A = \pi\{r^2 - (r-h)^2\} = \pi\{h^2 - 2\,r\,h\}$

$$\therefore \int_{h_2}^{h_1}\frac{A}{\sqrt{h}}\,d\,h = \pi\int_{h_2}^{h_1}\left(h^{\frac{3}{2}} - 2\,r\,h^{\frac{1}{2}}\right)d\,h$$

$$= \pi\left\{\frac{2}{5}\left(h_1^{\frac{5}{2}} - h_2^{\frac{5}{2}}\right) - \frac{4}{3}\,r\left(h_1^{\frac{3}{2}} - h_2^{\frac{3}{2}}\right)\right\}$$

$$\therefore \quad t_2 - t_1 = t = \frac{\pi}{C}\left[\frac{\frac{2}{5}\left(h_1^{\frac{5}{2}} - h_2^{\frac{5}{2}}\right) - \frac{4}{3}r\left(h_1^{\frac{3}{2}} - h_2^{\frac{3}{2}}\right)}{a\sqrt{2g}}\right]$$

Where the lower limit of height is less than about four times the diameter of the orifice, a vortex, with an air core, is usually formed, with the effect that the area of discharge is reduced (Art. 48).

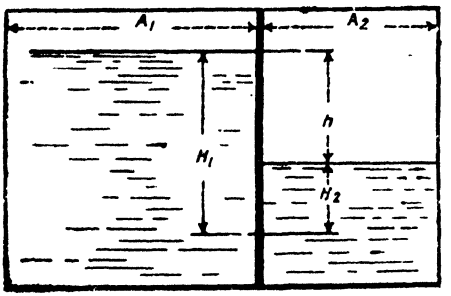


FIG. 69.

Time of Discharge from a Submerged Orifice. — If two vessels whose surface areas are A_1 and A_2 (Fig. 69) are connected by an orifice of area a, and if at any instant H_1 and H_2 are the heights of their free surfaces above the level of the orifice, we have, neglecting viscosity, and assuming

$$v = \sqrt{2g(H_1 - H_2)} = \sqrt{2gh}$$

Velocity of surface $A_1 = v \cdot \frac{a}{A_1}$; velocity of $A_2 = v\frac{a}{A_2}$

$$\therefore \quad -\frac{dh}{dt} = C\sqrt{2gh} \cdot a \cdot \left\{\frac{1}{A_1} + \frac{1}{A_2}\right\}$$

$$\therefore \quad t_2 - t_1 = t = \frac{2}{C\left\{\frac{1}{A_1} + \frac{1}{A_2}\right\} a \cdot \sqrt{2g}}\left\{h_1^{\frac{1}{2}} - h_2^{\frac{1}{2}}\right\}$$

where C = coefficient of discharge.

Where the upper surface remains at the same level we have the state of affairs which holds during the filling of a canal lock through a submerged orifice in the lock gate. The upper surface is in effect now of infinite area, so that putting $A_1 = \infty$ in equation (1), we get the time of filling the lock.

$$\therefore \quad t = \frac{2A_2}{Ca\sqrt{2g}}\sqrt{H_1 - H_2}.$$

With submerged orifices the value of C is about ·61, diminishing slightly with an increase in the effective head $H_1 - H_2$.

ART. 58.—FORM OF JET.

Where a jet issues from an orifice in the horizontal base of a vessel, its sectional area gradually diminishes as the velocity increases, until it finally breaks up into a series of detached drops. With a circular orifice

this is the most noticeable feature. With any other form of orifice, however, the jet after escaping suffers a continuous change in the form of its cross section. This is due to the effect of the tension of its surface film, which tends in the first place to bring the jet into the circular form. The inertia of the particles of water causes this effect to be slightly overdone, so that a continual alteration and realteration of section takes place, the jet finally—if sufficiently large in diameter and of sufficient height—settling down into a circular section. The action is periodic and approximately isochronous, and consequently with a steady jet the section at any fixed point is constant.

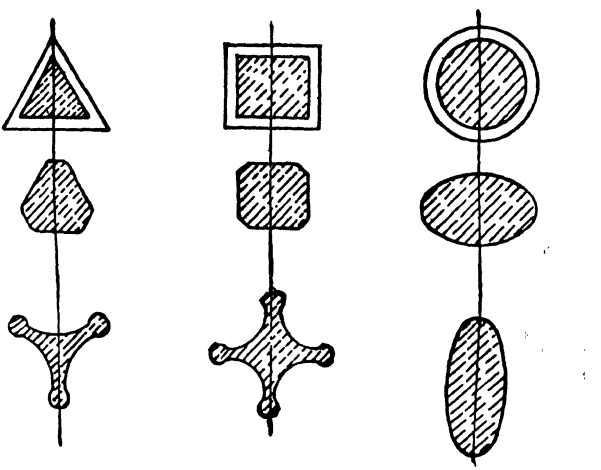


FIG. 70.

Where a jet escapes from an orifice in the vertical side of a vessel a second disturbing factor now affects its form, for since the velocities of efflux at points on a vertical diameter of the orifice increase with the depth, the trajectories of any two particles situated in the same vertical plane will intersect if produced.

This has the effect of narrowing the jet in the direction of its depth, and, by the consequent impact of particles, causing it to become wider in a horizontal direction. Surface tension prevents dispersion of the stream, and brings the diverging particles back towards the axis of the jet and in a horizontal plane. The consequent impact causes the jet to become narrower in a horizontal direction and increases its depth, and a continual succession of such reactions produces a continual state of change of section along its length. The action is periodic and approximately isochronous,

and consequently, with a steady jet, the points at which the cross section is of the same general shape as the orifice are relatively stationary. With a circular orifice, the section varies from an ellipse with major axis horizontal near the *vena contracta* to the same with major axis vertical. With a polygonal orifice of n sides the section ultimately becomes a star of n points, these points having thickened extremities. The angular

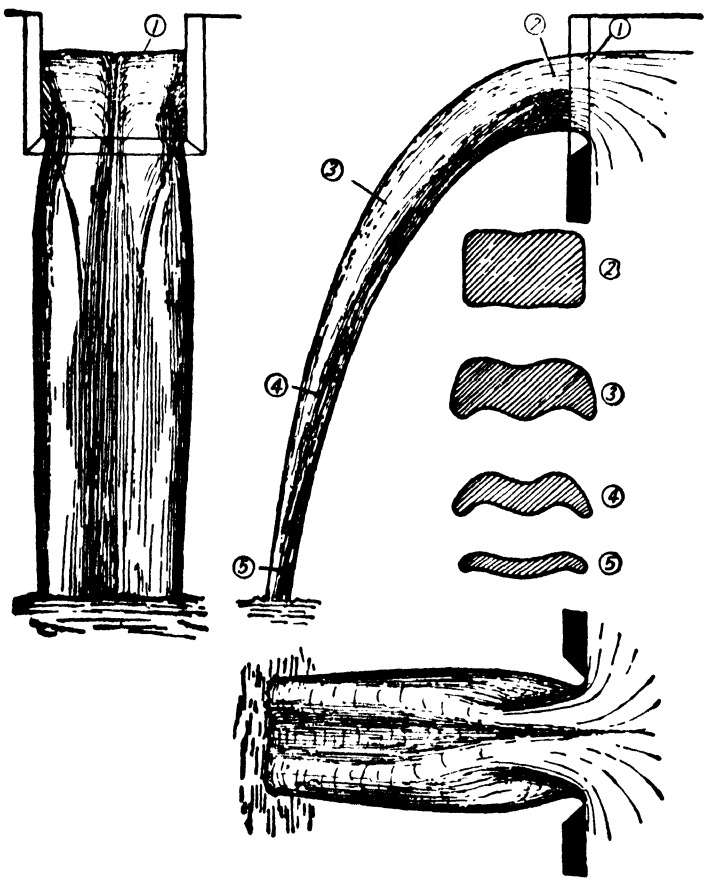


FIG. 70A.

points either coincide in direction with those of the orifice or intersect a corresponding face, always being symmetrical with respect to the orifice. Fig. 70 illustrates the general form taken by the jet from a triangular, square, or circular orifice, the upper section in each case being at, and the second

slightly past the *vena contracta*, and the lower giving the star form assumed by the jet.[1]

Fig. 70A shows the form taken by a stream issuing from a sharp-edged rectangular notch. The effect of the impact of converging particles is here strongly marked. The cross sections at various points on the jet are indicated in the figure.

Examples.

(1) A circular orifice 1 square inch in area is made in the vertical side of a large tank. If the jet fall vertically through $1\frac{3}{4}$ feet while moving horizontally through 5 feet, at the same time discharging 16 gallons per minute, determine the horizontal force on the tank.

Answer. 1·286 lbs.

(2) If the tank of the preceding example is suspended from knife edges 5 feet above the level of the orifice, and if, when the head of water in the tank is 4 feet, the discharge is 261 lbs. per minute, while a weight of 10·62 lbs., with a horizontal leverage of 1 foot, is required to keep the tank vertical, determine the coefficients of velocity, contraction, and discharge.

Answer. $C_v = \cdot 98$; $C_c = \cdot 638$; $C = \cdot 625$.

(3) A boat, having jet propulsion, moves at 10 miles per nour. The water leaves the nozzles with a relative velocity of 40 feet per second. If 1,500 lbs. per second are passed through the pumps, determine the propelling force on the vessel.

Answer. 1,180 lbs.

[1] For further information, a paper by Lord Rayleigh ("Proceedings, Royal Society," vol. 71) should be consulted.

CHAPTER VII.

Large Orifices—Rectangular Orifice—Circular Orifice—Submerged and partially Submerged Orifices—Sluice Gates—Notches and Weirs—Theoretical Formulae—Boussinesq's Theory—Experimental Results and Formulae—Francis—Bazin—Fteley and Stearns—Hamilton Smith—Inclination of Weir Face—Clinging Nappe—Triangular Notch—Trapezoidal Weir—Submerged Weirs—Broad-crested Weirs—Side Weirs—Time of Discharge over Weirs.

Art. 59.—Large Orifices.

Where an orifice is large, except where formed in the horizontal base of a vessel the assumption that the mean velocity over its whole area is sensibly equal to that at its centre of area is no longer true, and it becomes necessary to take account of the variation of velocity at different depths in its plane. As in the case of a small orifice a *vena contracta* is formed of approximately the same sectional shape as the orifice, and depending for its magnitude on the shape, dimensions, and head above the latter, and on the circumstances governing the formation of stream lines in the approaching vein.

In the usual theoretical discussion of the flow from such an orifice the two fundamental assumptions on which the theory is based are open to objection, and while the results obtained are not without value as forming the basis of useful empirical formulae, the treatment cannot be looked upon as scientific.

Briefly outlined, the method consists in assuming that at all points at the same depth in the plane of an orifice the velocity of efflux is the same, being that corresponding to the head of water above the point, and that the direction of flow at each point is perpendicular to the plane of the orifice.

Calculating the discharge over a small element δa of the area, *i.e.*, $\sqrt{2gh} \cdot \delta a$ and summing such discharges over the elements which go to make up the whole area, gives what is termed the theoretical discharge $\int \sqrt{2gh} \cdot \delta a$, and this, when multiplied by an empirical constant termed the coefficient of discharge, gives the true discharge.

Fig. 53, which indicates roughly how the velocity at the orifice varies at different points in the cross section, sufficiently shows the error involved in the first, while a consideration of the stream line formation

shows the error of the second assumption. Fig. 71 indicates how the velocity perpendicular to the orifice varies, and how it differs from the theoretical velocity $\sqrt{2gh}$, as given by the horizontal ordinates of the parabola $m\ n\ q$. Here the actual velocity is represented in elevation and plan by the corresponding ordinates of the curves $a\ c\ b$, and $a'\ c'\ a''$.

The solid represented in plan and elevation by the shaded areas will then represent to scale the volume discharged per second; while the solid bounded by the plane of the orifice; the perpendiculars an, bq, $a'\ p'$, $a''\ p''$; and the curved surface of the parabola, represented by $n\ p\ q$, will represent to the same scale the theoretical discharge. The ratio of these volumes thus gives the coefficient of discharge.

In spite of the recognised fallacies embodied in the method of treatment outlined above, the difficulties encountered in a rigorous treatment are so many and the results obtained so cumbrous, that we are still compelled to fall back on the more simple, though inaccurate, formulae, together with these experimentally deduced coefficients.

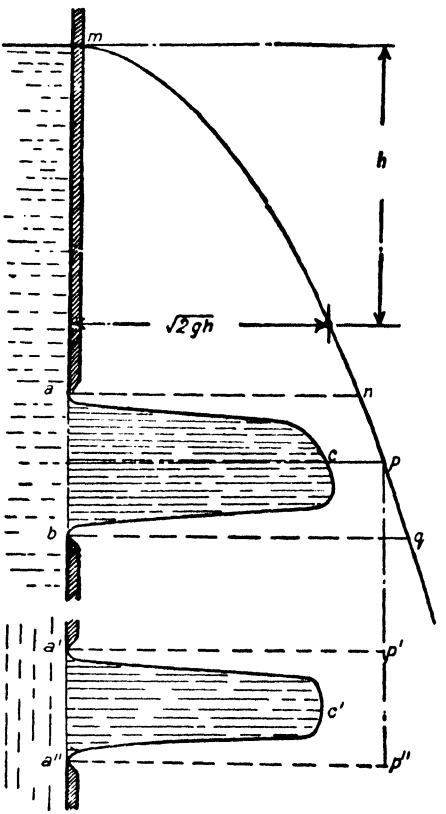
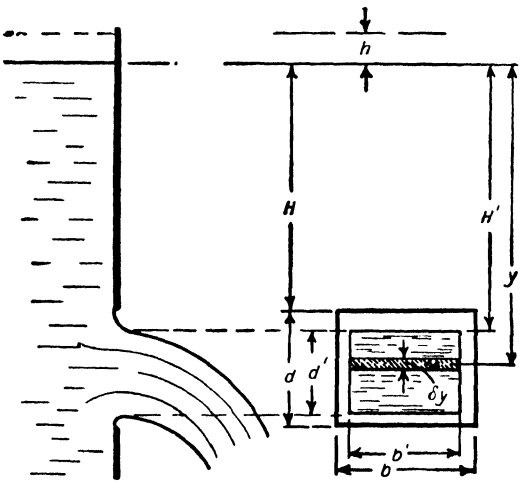


FIG. 71.

The assumptions, however, become more rational if the state of affairs at the *vena contracta* be considered instead of at the orifice. Here we may assume with some reason that the flow is everywhere perpendicular to the cross section of the stream, and that the pressure throughout is sensibly uniform and equal to that of the atmosphere, so that, except for the retarding effect of the atmosphere at the boundaries, the velocity at any point in this section, at a depth h, below the free surface in the vessel, is sensibly equal to $\sqrt{2gh}$. The more important cases of flow will now be considered on this assumption.

Art. 60.—Large Vertical Rectangular Orifice, neglecting Velocity of Approach.

Let H' = height of free surface above top of *vena contracta*.
„ b = breadth of orifice.
„ d = depth „
„ b' = breadth of *vena contracta* } assumed to be a rectangle.
„ d' = depth „ „ }



Consider the flow over any element of the *vena contracta* at a depth y below the free surface, and having an area $b' \delta y$.

The velocity at this depth $= C \sqrt{2 g y}$ feet per second.

$\therefore$ Flow over this area per second $= C_v b' \delta y \sqrt{2 g y}$, c.f.

Integrating this expression, and giving y the limits $H' + d'$ and H' we get the flow over the whole area.

$$\therefore Q = C_v b' \sqrt{2 g} \int_{H'}^{H' + d'} \sqrt{y}\, d y = \frac{2}{3} C_v b' \sqrt{2 g} \left[y^{\frac{3}{2}} \right]_{H'}^{H' + d'}$$

$$= \frac{2}{3} C_v b' \sqrt{2 g} \left\{ (H' + d')^{\frac{3}{2}} - H'^{\frac{3}{2}} \right\} \qquad (1)$$

The difficulty of obtaining accurate measurements of b', H' and d' now arises, since these vary with, but do not bear a fixed ratio to b, H, and d,

while experimental evidence as to the precise nature or the variation is wanting. The best way out of the difficulty appears to be to write

$$C = C_v \frac{b' \left\{ (H' + d')^{\frac{3}{2}} - H'^{\frac{3}{2}} \right\}}{b \left\{ (H + d)^{\frac{3}{2}} - H^{\frac{3}{2}} \right\}}$$

where H = depth of top of orifice below the free surface. This then gives us

$$Q = \frac{2}{3} C b \sqrt{2g} \left\{ (H + d)^{\frac{3}{2}} - H^{\frac{3}{2}} \right\} \qquad (2)$$

The reason for this substitution is that the value of C in this formula has been determined with some accuracy for various types of orifice and for different heads.

Formula (2) is that obtained by the inaccurate process of reasoning outlined at the beginning of this article, for, assuming that the flow through any element $b\ \delta\ y$ of the orifice is the same as that through the corresponding element of the *vena contracta* of area $b' \frac{d'}{d} . \delta\ y$, and that this flow is therefore given by $C\ b\ \delta\ y \sqrt{2\ g\ y}$, where C has the same value for each element, and therefore for the whole area, we have

$$Q = C\ b \sqrt{2g} \int_H^{H+d} \sqrt{y} . d\ y$$

$$= \frac{2}{3} C\ b \sqrt{2g} \left[(H + d)^{\frac{3}{2}} - H^{\frac{3}{2}} \right]$$

Comparing this with the simple formula

$$Q = C\ b\ d \sqrt{2\ g \left(H + \frac{d}{2} \right)},$$

obtained by considering the head over the section as sensibly equal to that at its centroid, we see that the two values of Q are in the ratio $\frac{2}{3} \frac{(H + d)^{\frac{3}{2}} - H^{\frac{3}{2}}}{d \sqrt{H + \frac{d}{2}}}$. For values of H greater than $\frac{d}{2}$ the difference amounts to less than 1·0 per cent., so that for all larger values of H, the simpler formula is to be preferred.

The velocity of approach may be taken into account by increasing the head by an amount h as explained on p. 129, thus making the effective head $= H + h$.

Then $Q = \frac{2}{3} C\ b \sqrt{2\ g} \left[(H + h + d)^{\frac{3}{2}} - (H + h)^{\frac{3}{2}} \right]$ (2)

As is clear from the expression assumed above for the value of C, this is not a constant, but varies both with the depth of the water and with the dimensions of the orifice, and can only be determined by experiment.

With sharp-edged orifices, and a free discharge into air, the values of C for a square opening are very approximately as given below (Hamilton Smith).

Head above centre of orifice in feet.	Length of side of square in feet.			
	·40	·60	·80	1·0
·6	·601	·598	—	—
1·0	·603	·601	·600	·599
3·0	·605	·604	·603	·603
6·0	·604	·603	·602	·602
10·0	·603	·602	·602	·601
20·0	·601	·601	·601	·600
100·0	·598	·598	·598	·598

For vertical rectangular orifices the value of C varies with the ratio of the width b to the depth d.

The following table gives the value of the coefficient for orifices 1 foot wide:—

Head in feet from centre of orifice.	Values of d in feet.						
	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1·0	1·5	2·0
1·0	·632	·632	·618	·612	·606	·626	—
3·0	·627	·627	·615	·610	·605	·614	·619
6·0	·615	·615	·609	·604	·602	·606	·610
10·0	·606	·603	·601	·601	·601	·601	·602
20·0	·602	·601	·601	·601	·601	·601	·602

Art. 61.—Flow through a Circular Orifice.

Let H = depth of centre of orifice.

„ R = radius of orifice.

„ r = „ „ *vena contracta*.

Then the area of an elementary strip taken in the *vena contracta* at a distance y from the centre, as shown, is $2\,x\,\delta\,y$.

The head over the element $= H - y$,

$\therefore$ Flow over this element $= \sqrt{2\,g\,(H-y)} \times 2\,x\,\delta\,y$

$\therefore \quad \delta\,Q = 2\sqrt{2\,g} \times \sqrt{(r^2 - y^2)\,(H-y)}\,\delta\,y.$

For convenience in integrating, expand $\sqrt{H-y}$ by the Binomial Theorem, *i.e.*, put

$$\sqrt{H-y} = \sqrt{H}\sqrt{1-\frac{y}{H}} = \sqrt{H}\left\{1 - \frac{y}{2\,H} - \frac{y^2}{8\,H^2} + - \text{etc.}\right\}$$

$$\left(\text{since } (1-x)^n = 1 - n\,x + \frac{n\,(n-1)}{2}\,x^2 - \text{etc.}\right)$$

Now put $y = r\cos\theta$.

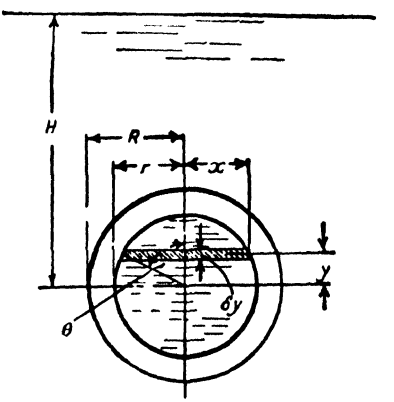
Then $\dfrac{d\,y}{d\,\theta} = -\,r\sin\theta.$

$\therefore \quad \delta\,y = -\,r\sin\theta\,\delta\,\theta,$

and $\sqrt{r^2 - y^2} = \sqrt{r^2\,(1-\cos^2\theta)}$

$= r\sin\theta.$

So that $\delta\,Q = 2\sqrt{2\,g\,H}\left\{-\,r^2\sin^2\theta\left(1 - \dfrac{r\cos\theta}{2\,H} - \dfrac{r^2\cos^2\theta}{8\,H^2} + - \text{etc.}\right)\right\}\delta\,\theta.$



Integrating this expression and giving θ the limits π and o, since these are the values of θ corresponding to the values $\mp\,r$ of y, we get, on introducing C_v, the total flow Q over the whole section.

$$\therefore \quad Q = 2\,C_v\sqrt{2\,g\,H}\int_o^{\pi} r^2\sin^2\theta\left(1 - \frac{r\cos\theta}{2\,H} - \frac{r^2\cos^2\theta}{8\,H^2} + - \text{etc.}\right)d\,\theta$$

$$= C_v\,\pi\,r^2\sqrt{2\,g\,H}\left\{1 - \frac{1}{32}\,\frac{r^2}{H^2} + - \text{etc.}\right\}$$

the succeeding terms being negligible.

If C be the coefficient of discharge for the orifice, then since $C_v = \dfrac{\pi\,r^2}{\pi\,R^2}$ we may write

$$Q = C\,\pi\,R^2\sqrt{2\,g\,H}\left\{1 - \frac{1}{32}\,\frac{r^2}{H^2}\right\}$$

As in the case of a rectangular orifice, the coefficient in general decreases as the orifice increases and also as the head increases.

The following table gives approximate values of C for circular orifices (Hamilton Smith) :—

Head above centre of orifice (feet).	Diameter of orifice in feet.			
	·40	·60	·80	1·0
1·0	·597	·595	·593	·591
2·0	·599	·597	·596	·595
4·0	·598	·597	·597	·596
6·0	·598	·597	·596	·596
10·0	·597	·596	·596	·595
20·0	·596	·596	·595	·594
100·0	·592	·592	·592	·592

Art. 62.—Submerged Orifices—Sluice Gates.

If the water level on the discharge side is above the highest point of the opening we get a submerged orifice.

Let H_1 = depth of water above upper edge of orifice on the incoming side, in feet.

Let H_2 = similar depth on discharge side.

Then the effective depth at all points in the orifice is the same, viz., $H_1 - H_2$.

$\therefore$ Theoretical velocity of efflux $= \sqrt{2\,g\,(H_1 - H_2)}$ ft. sec.

$\therefore$ Quantity per second through orifice $= C\,A\,\sqrt{2\,g}\,\sqrt{H_1 - H_2}$, where A is the area of the orifice.

Taking into account the velocity of approach, v_1, the effective head will now be $H_1 + h - H_2$ feet, where $h = \frac{v_1^2}{2\,g}$, and the discharge will be given by

$$Q = C\,A\,\sqrt{2\,g}\,\sqrt{H_1 + h - H_2} \text{ cub. ft. sec.}$$

The value of C for a submerged orifice may be taken as being about 1 per cent. less than its value for the same orifice under the same effective head, and when discharging freely into air.

Hanbury Brown[1] gives the following as being approximate values of C for sluices and regulator openings in masonry piers :—

[1] "Irrigation," Constable & Co., London, 1907.

Ordinary lock sluices	$C =$	·62
Small regulator openings with shallow water	„	·57
Regulator openings up to 6 feet wide with recesses in the piers	„	·62
Ditto between 6 feet and 13 feet wide	„	·72
Ditto above 13 feet wide	„	·82
Regulator openings up to 6 feet wide with straight and continuous piers	„	·72
Ditto between 6 feet and 13 feet wide	„	·82
Ditto above 13 feet wide	„	·92

A series of experiments[1] on a submerged orifice 4 feet square through a wooden bulkhead 3·75 inches thick, with various forms of entry, gave the following values of C:—

—	Form of entrance.	Values of C.		
		Max.	Min.	Mean.
	Square corners	·625	·605	·61
a	Contraction suppressed by means of guides curved to arcs of ellipses, so as to gradually divert the incoming stream — on bottom	·665	·625	·64
b	„ „ and one side	·735	·675	·69
c	„ „ and two sides	·825	·757	·78
d	on all four sides	·970	·919	·95

In every case the minimum value of C was obtained with a mean velocity of flow ranging from 1·9 to 2·3 feet per sec., the velocities of flow ranging from approximately 1·0 to 4·0 feet per sec.

The experiments were extended by the addition of tubes of lengths ·62, 1·25, 2·50, 5·0, 10·0, and 14·0 feet on the discharge side of the orifices, and the curves of Fig. 72 denote to what extent these affected the discharge.

For the tubes having square corners at the entrance, the coefficient of discharge increases at first almost directly as the length of tube increases, then at a gradually decreasing rate until the length "L" of the tube becomes between 3 and 3·5 times the side of the square. For lengths

[1] "Bulletin of the University of Wisconsin," No. 216. Engineering Series. vol. 4. No. 4.

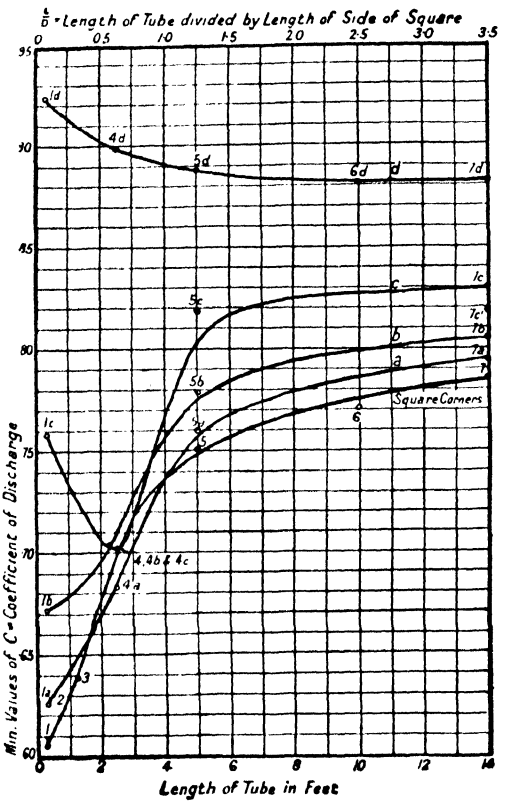


FIG. 72.

less than about 5 feet observations showed an upstream current located at the upper side of the tube, this reducing the effective discharge area. This effect was found to become less marked as the velocity was increased. For greater lengths of outlet tube this current was absent, and the conditions permitted the formation of a region of less than normal pressure at the *vena contracta*. If n represents the number of sides on which the contraction is suppressed, the following relationships give the minimum values of C within about 2 per cent. :—

For $L \div D \leqq 0{\cdot}1$;

$$C = {\cdot}605 + {\cdot}019\, n^2.$$

For $L \div D \geqq 3{\cdot}0$; $C = {\cdot}786 + {\cdot}0055\, n^2$.

Experiments by the author [1] on a sluice gate 3 feet wide, with bottom and side contractions entirely suppressed, gave values of C agreeing within 1 per cent. with the formula.

$$C = {\cdot}610 + {\cdot}21 \frac{\sqrt{d}}{H_2 + \frac{d}{2}}$$

where d is the depth of the opening in feet. The coefficient is sensibly independent of the effective head. The sluice gate consisted of an iron plate, $\frac{1}{4}$ inch thick, with its bottom edge planed square, and the depths of the opening ranged from ·1 to ·3 feet. The velocity of approach is taken into account in computing the head.

[1] "Proc. Inst. C. E.," vol. ccvii., 1918–19. pt. i.

Chatterton,[1] from experiments on sluice gates between 5·25 feet and 6 feet wide, with effective heads up to 1·2 feet, obtained the result

$$C = \cdot 615 + \cdot 007 \times 2^{(5 - h_2)}$$

Thus when

$h_2 =$	·4	·6	·8	1·0	1·5	2·0
$C =$	·77	·76	·74	·72	·69	·67
C (Author) means for all openings	·79	·75	·71	·69	·67	·66

In these experiments the velocity of approach was neglected, so that the coefficients will of necessity be relatively high.

Benton [2] from experiments on sluice gates having widths between 4 feet and 10 feet, with openings between ·19 feet and 3·20 feet, and with effective heads ranging from ·07 feet to 4·85 feet, gives the formula

$$C = \cdot 720 + \cdot 0074\, w,$$

where w is the width of the gate in feet.

In these experiments the side contractions were not suppressed, which accounts for the term involving w. The gates, moreover, had an angle bar riveted along the discharging edge, forming, in effect, a thick gate, while the velocity of approach was neglected. Both these factors would tend to increase the value of C.

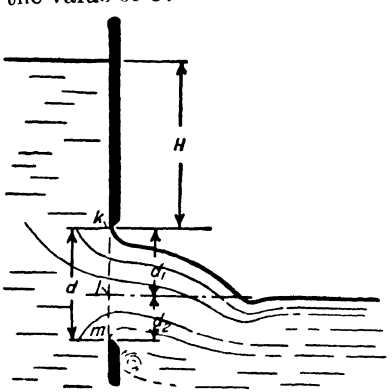


FIG. 73.

Partially Submerged Orifice (Fig. 73).—Here the portion of the orifice l m, below the normal water level on the discharge side, may be considered as a submerged orifice, while the portion k l above this level may be looked upon as discharging freely into air, the quantity passing each portion being calculated separately on these assumptions.

Thus in a rectangular orifice of breadth b and depth d, having a head of water on the supply side $= H$ above

[1] "Hydraulic Experiments in the Kistna Delta," No. 2 of the Professional Papers of the College of Engineering, Madras, 1901.

[2] Punjab Irrigation Branch Papers, No. 8.

the top of the orifice, with free discharge over a depth d_1, and submerged over the remainder d_2 of its depth we have

$$\text{Discharge over depth } d_1 = Q_1 = \frac{2}{3} C_1 b \sqrt{2g} \left\{ (H + d_1)^{\frac{3}{2}} - H^{\frac{3}{2}} \right\}$$

$$\text{,, } \quad \text{,, } \quad \text{,, } \quad d_2 = Q_2 = C_2 b d_2 \sqrt{2g} (H + d_1)^{\frac{1}{2}}.$$

$$\therefore \text{ Total discharge } Q = Q_1 + Q_2 = b \sqrt{2g} \left[\frac{2}{3} C_1 \left\{ (H + d_1)^{\frac{3}{2}} - H^{\frac{3}{2}} \right\} + d_2 C_2 (H + d_1)^{\frac{1}{2}} \right]$$

If $C_1 = C_2 = C$ we get

$$Q = C b \sqrt{2g} \left[(H + d_1)^{\frac{1}{2}} \left\{ \frac{2}{3} (H + d_1) + d_2 \right\} - \frac{2}{3} H^{\frac{3}{2}} \right]$$

If $h =$ the head equivalent to the velocity of approach, the formula becomes

$$Q = C b \sqrt{2g} \left[(H + d_1 + h)^{\frac{1}{2}} \left\{ \frac{2}{3} (H + d_1 + h) + d_2 \right\} - \frac{2}{3} (H + h)^{\frac{3}{2}} \right]$$

The difficulty in exactly estimating d_1 and d_2, and the undulatory motion of the surface which is usually found in a partially submerged orifice, renders this is a most unsatisfactory method of estimating the discharge.

From a scientific point of view the above treatment, due to Dubuat, is not satisfactory. A rational treatment would require to take into account the distribution of pressures and velocities in the issuing stream, and from the nature of the case would lead to results too cumbersome for practical application.

Adopting the simple formula $Q = C b d \sqrt{2g H}$, experiments by the Author [1] on a sluice gate 3 feet wide, with no side or bottom contractions, gave the following values of C, when discharging freely :—

d feet.	H feet.							
	·1	·2	·4	·6	·8	1·0	1·5	2·0
·106	·630	·635	·638	·640	·643	·645	·646	·647
·206	·580	·585	·598	·603	·607	·610	·614	·615
·308	·536	·555	·570	·576	·581	·585	·588	—

[1] "Proc. Inst. C. E.," vol. cevii., 1918–19, pt. i.

ART. 63.—FLOW OVER NOTCHES AND WEIRS.

Where the free surface on the supply side is below the level of the upper edge of an orifice, this is termed a notch, and, if of large dimensions, a weir.

The theoretical treatment for flow over such notches follows exactly similar lines to that for flow through large orifices, and is subject to the same erroneous assumptions. The errors involved in assuming that the velocity of efflux perpendicular to the plane of the notch, at the notch, at any depth h, is proportional to $\sqrt{2\,g\,h}$, is indicated in Fig. 74, where the horizontal ordinates of the parabola $m\ n\ q$ show the theoretical velocity at any depth below the free surface of the still water at S, while the ordinates of the curve $a\ n\ c\ b$ denote the actual velocities in this direction in the plane of the notch. The coefficient of discharge will then be equal to the ratio of the volumes represented in elevation by the shaded area $a\ n\ c\ b$ and the area $m\ n\ q\ b$.

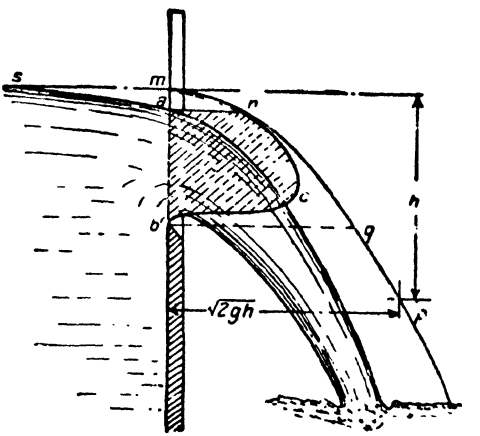


FIG. 74.

The fact that owing to the curvature of the stream lines the pressure over a cross section is nowhere uniform, renders a rigorous treatment impossible, and the next best method is to consider the state of affairs in the plane of the notch.

An examination of the contour of the escaping stream shows that in the plane of the notch its upper surface is lower than the free surface at a point a short distance up-stream where the motion is steady.

This fall from s to a (Fig. 74) is essential if the surface filaments are to have the required velocity of efflux at the notch, and since their motion is approximately perpendicular to the plane of the orifice and is unaffected save by the resistance of the air, their velocity may be taken as sensibly equal to $\sqrt{2\,g\,(a\,m)} = \sqrt{2\,g\,H'}$ (Fig. 75).

In general H' is approximately equal to ·14 H, where H = height of water measured to still water level, above the crest of the notch or weir, so that $(H - H')$ = depth of water at crest in feet.

ART. 64.—THEORETICAL FORMULAE FOR FLOW OVER WEIRS.

Rectangular Weirs (Fig. 75).—Let b = length of weir in feet.

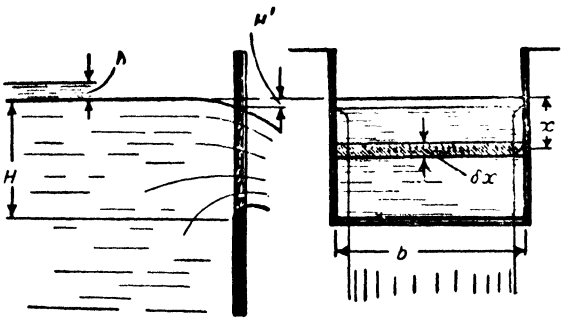


FIG. 75.

Consider the flow across any element of area $b\,\delta x$, in the plane of the weir, at a depth x below the still water surface of the supply.

The volume passing this element per second $= \delta Q = C'\,b\,\delta x\,\sqrt{2gx}$ cub. ft. per sec.

Integrating this expression and giving x the upper and lower limits H and H', and assuming that C' has the same value for each element, we have

$$Q = \frac{2}{3}\,C'\,b\,\sqrt{2g}\left\{H^{\frac{3}{2}} - H'^{\frac{3}{2}}\right\} \text{ cub. ft. per sec.} \qquad (1)$$

The value of this coefficient C' has not been experimentally determined, so that the formula in its present form cannot be applied to determine the discharge in any practical case. If, however, we write

$$C'\left\{H^{\frac{3}{2}} - H'^{\frac{3}{2}}\right\} = C\,H^{\frac{3}{2}}$$

the formula becomes

$$Q = \frac{2}{3}\,C\,b\,\sqrt{2g}\,H^{\frac{3}{2}} \text{ cub. ft. per sec.} \qquad (2)$$

and the value of this coefficient is known with fair accuracy for different values of H.

Formula (2) may be deduced with less reason by assuming the depth of water in the plane of the notch to be H, *i.e.*, neglecting the fall from still water level, and assuming that the velocity of the surface filaments is zero, and therefore by integrating the expression $\delta Q = C\,b\,\delta x\,\sqrt{2gx}$ between the limits H and 0. It might also be deduced from equation (2) of p. 139 by writing $H = 0$; $d = H$.

The value of C, in addition to varying with the breadth and depth of the weir, varies largely with its position with respect to the sides and bottom of the approach channel. Where the sides of this channel are so far removed as not to affect the contraction of section at either side of the weir, this is said to have two end contractions, while where the channel

is of the same breadth as the weir, the latter has no end contractions, and is termed a *Suppressed Weir*. The bottom contraction is also affected by the nearness of the sill of the weir to the bottom of the channel of approach and by the inclination of the weir face.

Where the "*nappe*," or falling sheet of water, comes into contact with the top or down-stream face of the weir, as shown in Figs. 80 and 88, a further and in general irregular variation takes place in *C*. In what immediately follows, only those weirs will be considered in which the *nappe* springs clear of the crest and discharges freely into the air, the crest being vertical, narrow in comparison with the head, and having a sharp upstream edge.

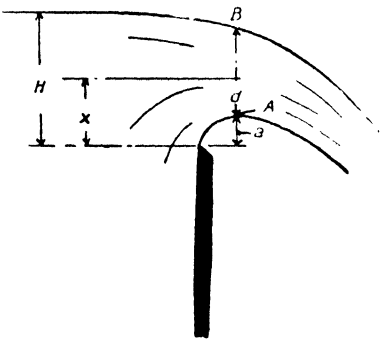


FIG. 76.

Boussinesq's Theory.—A theoretical treatment of the flow over a suppressed weir, which is of much interest, is due to *Boussinesq*.

Let H = depth above sill.

,, a be the amount by which the arched under side of the vein rises above the sill. (Assumed, and verified by Bazin, to be a definite fraction (·13) of H) (Fig. 76).

At the vertical section, at which a is a maximum, let d be the vertical thickness of the vein.

If we assume that, over this section, the stream lines are portions of concentric circles and are normal to the section, and also assume that there is no loss of energy up to this section we have the conditions obtaining in free vortex flow and as in that case, if r is the radius of curvature of any stream line, $v\,r$ is constant.

Consider any filament, x above the sill, of velocity v, and radius r.

Let v_a = vel. at A ; v_b = vel. at B.

Then $v_a = \sqrt{2\,g\,(H-a)}$; $v = \sqrt{2\,g\,(H-a)} \cdot \dfrac{r_a}{r_a + x - a}$

$$\therefore\ Q = b \int_a^{d+a} \sqrt{2\,g\,(H-a)} \cdot \frac{r_a}{r_a + x - a} \cdot dx$$

$$= b\,\sqrt{2\,g\,(H-a)} \cdot r_a \log \frac{r_a + d}{r_a}$$

Let $r_a = n\,d$

Then $Q = b\sqrt{2g(H-a)} \cdot n\,d \log\left(1+\frac{1}{n}\right)$

And since $\left(\frac{r_b}{r_a}\right)^2 = \left(\frac{v_a}{v_b}\right)^2 = \frac{H-a}{H-a-d}$

$$\therefore\quad d = (H-a)\left\{\frac{2n+1}{(n+1)^2}\right\}^{1}$$

$$\therefore\quad Q = b\sqrt{2g}\left\{H-a\right\}^{\frac{3}{2}}\left[\frac{n(2n+1)}{(n+1)^2}\log\left(1+\frac{1}{n}\right)\right]$$

But for permanence of *régime* the flow will adjust itself until the discharge is a maximum, and in this case $\frac{dQ}{dn} = 0$.

On differentiating [2] this gives $\frac{2n+1}{3n+1} = \log_e\left(1+\frac{1}{n}\right)$

$$\text{or } n = \cdot 8814 \qquad \left(\log\left(1+\frac{1}{n}\right) = \cdot 7581\right)$$

$$\therefore\quad Q = b\sqrt{2g}\,(H-a)^{\frac{3}{2}}\left\{\frac{\cdot 8814 \times 2\cdot 7628}{(1\cdot 8814)^2}\right\} \times 7851$$

$$= b\sqrt{2g}\,H^{\frac{3}{2}}\left\{1-\frac{a}{H}\right\}^{\frac{3}{2}} \times \cdot 522.$$

Taking $a \div H = \cdot 13$ this becomes

$$Q = \cdot 423\, b\sqrt{2g}\,H^{\frac{3}{2}}$$

$$= 3\cdot 39\, b\, H^{\frac{3}{2}} \text{ c. f. s.}$$

a value in very close agreement with experimental results.

[1] $\left(\frac{r_b}{r_a}\right)^2 = \left(\frac{n+1}{n}\right)^2 = \frac{H-a}{H-a-d}$

$$\therefore\quad d = (H-a)\left\{1-\left(\frac{n}{n+1}\right)^2\right\}$$

$$= (H-a)\left\{\frac{2n+1}{(n+1)^2}\right\}.$$

[2] $\frac{dQ}{dn} = K\left[\frac{n(2n+1)}{(n+1)^2}\cdot\frac{n}{n+1}\left(-\frac{1}{n^2}\right) + \log\left(1+\frac{1}{n}\right)\left\{\frac{(n+1)^2(4n+1)-n(2n+1)2(n+1)}{(n+1)^4}\right\}\right]$

$$\therefore\quad 2n+1 = \log\left(1+\frac{1}{n}\right)\left\{(n+1)(4n+1) - 2n(2n+1)\right\}.$$

$$= (3n+1)\log_e\left(1+\frac{1}{n}\right).$$

Art. 65.—Experimental Results and Formulae.

Adopting the base formula $Q = K\,b\,H^{\frac{3}{2}}$ cub. ft. per sec., where $K = \frac{2}{3}\sqrt{2g}\,C$, many experiments have been carried out to determine the value of K.

Francis,[1] in a preliminary series of experiments carried out in 1848 and 1851, assumed the formula $Q = K\, b\, H^x$, and found x to vary slightly with the head, having a mean value 1·47.

In his later experiments, however, carried out in 1852, he adopted, for convenience, the constant value 1·5, and determined the corresponding values of K.

From these experiments, carried out on weirs from 8 to 10 feet long, and with heads ranging from ·62 to 1·56 feet, he deduced the empirical formula

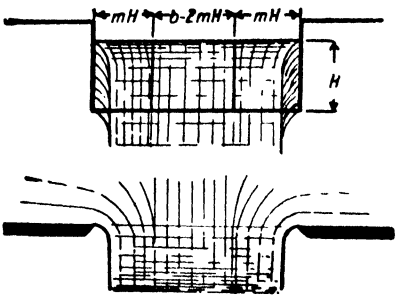


FIG. 77.

$$Q = 3{\cdot}33 \left(b - \frac{n}{10} H \right) H^{\frac{3}{2}} \text{ cub. ft. per sec.}$$

for the flow over a sharp-edged rectangular notch, where $n = 2$, 1, or 0, according as the stream has 2, 1, or 0 end contractions, the bottom contraction being free in every case.

Thus for two end contractions $Q = 3{\cdot}33\ (b - {\cdot}2\ H)\ H^{\frac{3}{2}}$ c.f.s.

„ one „ contraction $Q = 3{\cdot}33\ (b - {\cdot}1\ H)\ H^{\frac{3}{2}}$ c.f.s.

„ no „ „ $Q = 3{\cdot}33\ b\ H^{\frac{3}{2}}$ c.f.s.

That this formula, though empirical, has a rational basis may be seen if we consider the state of affairs in a long and shallow notch having two end contractions (Fig. 77).

If the notch is sufficiently broad, a certain section in the middle will have stream lines which, as seen from above, are parallel, while the two end sections have stream lines which suffer an amount of curvature depending on their distance from the ends. The distance over which this curvature extends is found to be approximately $m\ H$, where $m = 1{\cdot}5$. This end contraction evidently diminishes the discharge by an amount which is constant for a given depth, and it is reasonable to assume that it therefore diminishes the effective breadth of the notch by an amount which is also constant for a given depth, and which depends only on the depth. The law of its variation with the depth has not been accurately determined, but it was assumed, as appears to be approximately true, that the diminution in effective length is directly proportional to the depth

[1] Lowell Hydraulic Experiments (New York, 1883).

and that the effective length is therefore equal to $(b - n\ k\ H)$ where n is the number of contractions and k is a constant to be determined experimentally. Francis then assumed (though not on theoretical grounds) that the discharge from notches of equal effective length varies according to a constant power of the depth, so that the formula became [1]

$$Q = K\ \{b - n\ k\ H\}\ H^x.$$

Though neither of the assumptions made is quite correct, yet if x be given the value 1·5, K the value 3·33, and k the value $\frac{1}{10}$, they lead to results closely in accord with experiment.

In general the Francis formula is suitable for use for all heads above 6 inches where the bottom contraction and end contractions, if any, are free, and where the length of crest is greater than three times the head.

Even where the ratio of length to head is less than three, the formula gives fairly accurate results, the discharge as calculated being always slightly less than observed. Recent experiments at Cornell University [2] on a suppressed weir 6·56 feet wide and with heads ranging from 2·0 to 4·85 feet give mean results 98·4 per cent. of those calculated by its use, and indicate its reliability for heads up to 5 feet.

For perfect contractions Francis specifies a distance to side of approach channel $= 2\ H$, and a depth below the weir crest $= 3\ H$, and states that a reduction of the bottom clearance to $2\ H$ and of the side clearance to H increases the flow by about 1 per cent.

Effect of the Velocity of Approach.—As in the case of a submerged orifice, the fact that the stream has kinetic energy in virtue of its velocity of approach to the weir may be taken into account, and correction made for this by adding to the measured head H a supplementary head h, where $h = \frac{v^2}{2g}$. Here v is the velocity of approach, and is necessarily the computed mean velocity over the whole section of the approach channel. But the distribution of velocity over this section is not at all uniform, the filaments nearer the surface and nearer the centre having a velocity greater than the mean, depending on the depth, width, and surface condition of the channel. The particles which suffer the least change in their direction of motion on passing the weir are thus those which have a velocity of approach greater than the mean, and since the velocity of approach in

[1] A formula of this form was suggested by Boyden in 1846.

[2] "Trans. Am. Soc. C.E.," vol. 44, p. 397. The maximum variation from Francis's formula, viz., 4 per cent., occurred with 2·64 feet head.

these particles has a greater proportional effect in increasing the flow past the weir than in those which approach its plane in a more oblique direction, the effective velocity of approach will be greater than that corresponding to the calculated head h. Most experimenters, therefore, put the effective head as equal to $H + a\,h$, where a has some value greater than unity, and write

$$Q = K\,b\,(H + a\,h)^{\frac{3}{2}} = K\,b\,D^{\frac{3}{2}}.$$

Francis, however, allows for the velocity of appoach by putting $H + h$ and h as his limits of integration in formula (1) of this article, and thus gets

For a suppressed weir $Q = 3{\cdot}33\,b\left\{(H+h)^{\frac{3}{2}} - h^{\frac{3}{2}}\right\}$

With one end contraction $Q = 3{\cdot}33\,(b - {\cdot}1H)\left\{(H+h)^{\frac{3}{2}} - h^{\frac{3}{2}}\right\}$

With two end contractions $Q = 3{\cdot}33\,(b - {\cdot}2\,H)\left\{(H+h)^{\frac{3}{2}} - h^{\frac{3}{2}}\right\}$.

In using Francis's formula, care should be taken to apply the correction in this manner.

The value of $h = \dfrac{v^2}{2\,g}$ is determined by approximation. Thus Q is first determined from the measured H, and this value of Q is used to determine v from the known area of the channel.

Fteley and Stearns,[1] from experiments on suppressed weirs 5 and 19 feet long, and with heads ranging from ·07 to 1·60 feet, and also from a discussion of Francis's results, obtained the formula for a suppressed weir

$$Q = 3{\cdot}31\,b\,H^{\frac{3}{2}} + {\cdot}007\,b \text{ cub. ft. per sec.}$$

For a weir with end contractions, b is to be replaced by $(b - {\cdot}1\,n\,H)$. The velocity of approach is to be allowed for by putting the effective head $= H + a\,h$, where

$a = 1{\cdot}5$ for a suppressed weir.

$a = 2{\cdot}05$ for a weir having two end contractions.

These are mean values. Actually the experiments showed that a varies with H, as shown approximately in the following table:—

[1] "Trans. Am. Soc. C.E.," 1883, pp. 1—118.

Depth of channel of approach below crest.	Suppressed weir—channel 5 feet wide, depth on crest in feet.					
	·2	·4	·6	1·0	1·5	2·0
·5	1·70	1·53	1·52	1·48	—	—
1·0	1·87	1·79	1·71	1·61	1·52	—
1·7	1·66	1·63	1·60	1·54	1·46	1·38
2·6	1·51	1·49	1·47	1·43	1·38	1·33

Hamilton Smith,[1] from experiments by Fteley and Stearns, Francis, and self, with weir lengths from ·66 up to 19 feet, heads up to 2 feet, altered Francis's formula so as to take a correction $H + a\,h = D$, for the velocity of approach, giving a the values 1·4 for a contracted, and 1·33 for a suppressed weir.

Then $Q = 3{\cdot}29 \left\{ b + \frac{D}{7} \right\} D^{\frac{3}{2}}$ for a suppressed weir.

$Q = 3{\cdot}29\, b\, D^{\frac{3}{2}}$ for one end contraction.

$Q = 3{\cdot}29 \left\{ b - \frac{D}{10} \right\} D^{\frac{3}{2}}$ for two end contractions.

Smith also suggests determining v, and therefore h, by measuring the surface velocity in the approach channel, the ratio of surface velocity to mean velocity being usually approximately equal to a.

Bazin,[2] from his own experiments on suppressed weirs from 1·65 to 6·56 feet long, and with heads ranging from ·17 to 1·97 feet, and also from an examination of the results of Fteley and Stearns, deduced the formula

$$Q = \left\{ {\cdot}405 + \frac{{\cdot}00984}{H} \right\} b \sqrt{2g}\; H^{\frac{3}{2}} \text{ c.f.s.}$$

for a weir with no velocity of approach. For a weir with velocity of approach, the effective head $= H + 1{\cdot}69\, h$, and on substituting for h in terms of the depth of approach channel Bazin finally gets

$$Q = \left\{ 1 + {\cdot}55 \left(\frac{H}{P + H} \right)^2 \right\} \left\{ {\cdot}405 + \frac{{\cdot}00984}{H} \right\} b \sqrt{2g}\; H^{\frac{3}{2}} \text{ c.f.s.}$$

Approximately, for heads from 4 inches to 1 foot we have

$$Q = \left\{ {\cdot}425 + {\cdot}21 \left(\frac{H}{P + H} \right)^2 \right\} b \sqrt{2g}\; H^{\frac{3}{2}} \text{ cub. ft. per sec.}$$

[1] Hamilton Smith, "Hydraulics," pp. 99 and 137.

[2] "Annales des Ponts et Chaussées," 1898, 2me trimestre.

correct within about 2½ per cent. In these formulae P = height of the weir crest above the bed of the approach channel. All dimensions are in feet. Expressing the formula in the form $Q = K\,b\,H^{\frac{3}{2}}$, the following values of K are deduced from Bazin's experimental results :—

H feet.	Height of Crest of Weir above Bottom of Channel (feet).						
	1·5	2·0	3·0	4·0	6·0	8·0	10·0
·4	3·531	3·500	3·473	3·462	3·454	3·451	3·450
·6	3·534	3·481	3·435	3·414	3·397	3·391	3·388
·8	3·572	3·499	3·430	3·400	3·374	3·364	3·359
1·0	3·612	3·532	3·443	3·402	3·366	3·352	3·344
1·2	3·675	3·573	3·465	3·413	3·366	3·347	3·337
1·4	—	3·615	3·491	3·429	3·372	3·347	3·337
1·6	—	3·659	3·520	3·449	3·381	3·351	3·335
1·8	—	—	3·549	3·468	3·390	3·355	3·336
2·0	—	—	3·579	3·490	3·402	3·362	3·340

Rehbock, as a result of experiments on suppressed weirs, suggests the formula

$$Q = \left(3{\cdot}23 + 0{\cdot}434\,\frac{H}{P}\right) b\,(H + 0{\cdot}0034)^{1{\cdot}5} \text{ cusecs.}$$

for values of P/H greater than 1·5, and this formula has been adopted by the British Standards Institution[1] as being acceptable for standard tests on pumps or turbines.

For weirs with end contractions, the Institution has adopted the formula

$$Q = 3{\cdot}29(b - 0{\cdot}1H)H^{1{\cdot}5} \text{ cusecs.}$$

This may be used for heads from 3 inches to 24 inches, provided that b/H is greater than 2.

It is doubtful, however, whether the discharge based on the measurements of head or any large weir can be relied upon as being accurate within ± 2 per cent. Laboratory experiments show that the flow immediately behind the weir, under apparently identical conditions, may adopt either of two forms. In the one the flow takes place as would be expected. In the other a roller eddy with horizontal axis is established behind the weir. The discharge in the two cases may differ by as much as ± 2 per cent.

[1] British Standard Specification for Pump Tests, No. 599, 1935.

Although in the formulae so far quoted the discharge is assumed to be proportional to $H^{1.5}$, the true index is somewhat less than this. As indicated on p. 151, Francis obtained the value 1·47 and this has been confirmed by recent observations. Thus Gourley and Crimp, from measurements on rectangular weirs with end contractions having lengths ranging from 6 inches to 36 inches and with heads from ·15 feet to 1·0 feet, deduced the formula

$$Q = 3{\cdot}10\, b^{1{\cdot}02}\, H^{1{\cdot}47} \text{ c.f.s.}$$

Discharge over a Curved Weir—Bell Mouth Spillways.[1]—When a weir whose crest, in plan view, is curved, with a large radius of curvature and discharges radially inwards, the discharge per foot run is sensibly the same, but slightly less than that of a straight weir having the same cross section of crest and the same crest length.

In some modern reservoirs the overflow, instead of being discharged over a spillway on the crest of the dam, takes place over a circular spillway built at some little distance upstream of the dam. The diameter of this may be anything up to about 90 feet. The discharge enters a vertical shaft terminating in a horizontal discharge tunnel. In such a spillway there is always a tendency for whirling flow to be set up at the entrance, which would reduce the discharge, and to prevent this a series of radial baffles are erected on the crest. In a large spillway six of these, equally spaced, are adequate.

An extreme case of the curved weir with inward or outward flow is found in the case of flow over the upper edge of an open vertical stand pipe.

Experiments carried out at Cornell University [2] on a series of such pipes of diameters 2, 4, 6, 9, and 12 inches with outward flow, show that, if D is the diameter in feet, for heads less than ·028 $D^{1{\cdot}04}$ the flow is similar to that over a sharp-crested weir, while for heads greater than ·107 $D^{1{\cdot}03}$ the flow becomes similar to that of a jet.

The discharge for weir flow is given by the formula

$$Q = 8{\cdot}8\, D^{1{\cdot}29}\, H^{1{\cdot}29} \text{ cub. ft. per sec.}$$
$$= 2{\cdot}01 \left\{ l\, H \right\}^{1{\cdot}29} \quad \text{,,} \quad \text{,,}$$

Here l is the length of the crest, l and H both being in feet.

For jet flow the discharge is given by

$$Q = 5{\cdot}7\, D^2\, H^{0{\cdot}53} \text{ c.f.s.}$$
$$= C\, A \sqrt{2g}\, H^{{\cdot}53} \text{ c.f.s.}$$

where A is the area and $C = {\cdot}905$.

[1] See "Scale Models in Hydraulic Engineering," J. Allen, Longmans, Green & Co., 1947, p. 84. See also a paper by Binnie and Wright, *Journal Inst. C. E.*, Jan. 1941, p. 197.

[2] "Proc. Am. Soc. C.E., 1906, p. 479.

In similar experiments on pipes of 6·9, 10·1, 13·7, 19·4, and 25·9 inches outside diameter, with flow radially inwards,[1] the discharge was given by

$$Q = K\,l\,H^{1\cdot42} \text{ cub. ft. per sec.}$$

where K has the following values :—

Diameter of weir (inches).	6·9	10·1	13·7	19·4	25·9
Value of K . . .	2·93	2·94	2·97	2·99	3·03

These constants hold for heads up to one-fifth of the diameter of the weir.

Effect of a Change in Head on the Discharge from a Rectangular Weir.

Since $Q = K\,H^{\frac{3}{2}}$, $\therefore \dfrac{d\,Q}{d\,H} = 1{\cdot}5\,K\,H^{\frac{1}{2}}$,

$$\therefore \delta\,Q = 1{\cdot}5\,K\,H^{\frac{3}{2}}\,\delta\,H, \text{ and } \frac{\delta\,Q}{Q} = 1{\cdot}5\,\frac{\delta\,H}{H},$$

i.e., the proportional change in Q is 1·5 times that in H.

Effect of Inclination of Weir Face.

As might be expected from its effect on the crest contraction, an

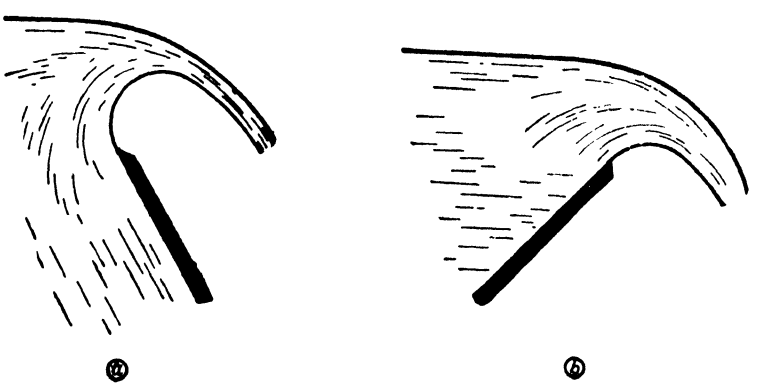


FIG. 78.

up-stream inclination of the weir face (Fig. 78 *a*) reduces, while a down-stream inclination (Fig. 78 *b*) increases, the discharge.

[1] H. J. F. Gourley, "Proc. Inst. C.E.," vol. 184, 1910—11, pt. 2, p. 297.

Bazin obtained the following comparative results with inclined weirs :—

Direction of inclination.	Up-stream.			Weir, vertical.	Down-stream.				
Slope : horizontal distance given first . . .	1 to 1	2 to 3	1 to 3	0	1 to 3	2 to 3	1 to 1	2 to 1	4 to 1
Relative discharge . .	·93	·94	·96	1·00	1·04	1·07	1·10	1·12	1·09

The maximum discharge was obtained with a down-stream inclination of 7 to 4.

Professor G. S. Williams, in experiments on weirs with a down-stream inclination of 1 to 1. obtained a mean comparative increase of 7·2 per cent. with a crest height of 6·65 feet, and of 8·8 per cent. with a crest height of 11·25 feet. The heads in these experiments ranged from ·5 to 4 feet.

Effect of a Sloping Apron on the Down-stream Side of a Sharp-crested Weir.

If such a weir, having a vertical up-stream face and a sharp crest is fitted with a sloping apron on the down-stream side, the discharge is increased to an extent which depends on the slope of the apron. *Bazin*, experimenting on such weirs having heights of 1·64 and 2·64 feet and with heads ranging from ·3 to 1·5 feet, deduced the approximate formula $K = \frac{3{\cdot}85}{S^{\frac{1}{12}}}$ for values of the slope S between 1 and 12.

Effect of Rounding the Up-stream Edge.

A slight rounding of the up-stream edge, by diminishing the contraction

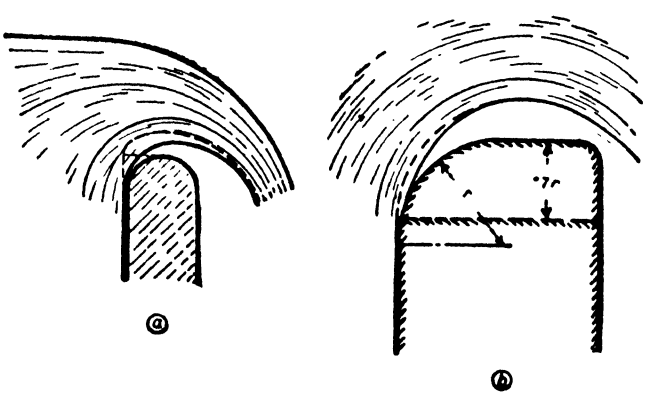


FIG. 79.

of section as indicated in Fig. 79 *a*, always causes an increased discharge. Messrs. *Fteley and Stearns*, from experiments with crest radii up to 1 inch, found that, so long as the *nappe* was fully aerated, the effect of the rounding was to increase the effective head by h', where $h' = \cdot 7\ r$ (Fig. 79 *b*).

Effect of Clinging Nappe.

The experiments of Bazin showed very clearly the importance of taking into consideration the particular form assumed by the *nappe*.

If discharging freely, with free admission of air underneath, such a *nappe* is said to be fully aerated. In a suppressed weir discharging between the walls of a discharge channel of its own width, the *nappe* will remain in contact with these walls, and tends to eject the air enclosed in the underlying space. For free aeration it is necessary for air holes to be provided in these wing walls.

If no provision is made for such an air supply, a partial vacuum is

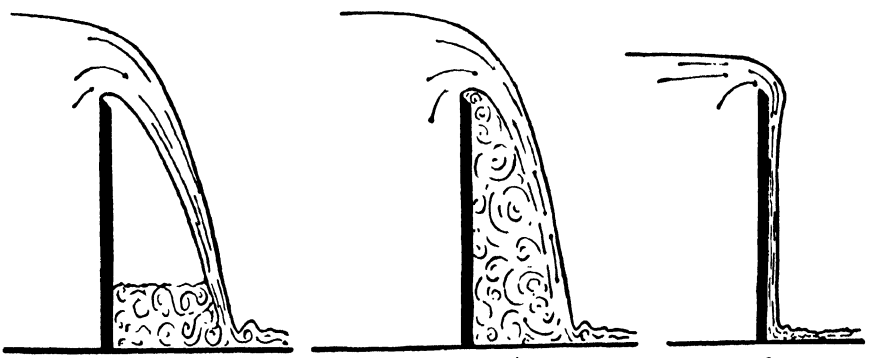


FIG. 80.—Depressed, Drowned, and Clinging Nappes.

produced beneath the *nappe*; the latter is said to be depressed, and the discharge is increased (Fig. 80 *a*).

In an extreme case of the depressed *nappe* the whole of the air behind the *nappe* may be ejected, when this space is occupied by a turbulent eddying mass of water which does not itself join in the general motion of the *nappe*. The latter is then said to be drowned or wetted underneath (Fig. 80 *b*).

With very low heads the *nappe* may adhere to both crest and downstream face of weir (Fig. 80 *c*).

Bazin gives the following as the relative discharges from a thin-edged weir 2·46 feet high and with a head of ·656 foot under different *nappe* conditions:—

Nappe.	Relative discharge.
Free discharge into air with full aeration	1·000
Nappe depressed with partial vacuum beneath . . .	1·060
Nappe wetted underneath; down-stream water level ·42 foot below crest	1·148
Nappe adhering to down-stream face	1·279

Except with a freely discharging *nappe* the state of affairs is very unstable. Any pulsation in the stream flow, any floating body piercing the *nappe* and allowing admission of air, or even a sudden gust of wind, blowing the *nappe* into contact with the weir face may totally alter the conditions of flow, and no attempt should be made to use a weir, except when discharging freely, as a measuring device.

For steep down-stream slopes where the *nappe* tends to break free from the apron, the discharge is increased by the partial vacuum formed beneath the *nappe*. As this slope becomes flatter the conditions approach those for a horizontal crest, while for slopes of 3 to 1 the discharge is approximately the same as for a thin-crested, freely discharging weir.

Art. 66.—Triangular Notch.

Let H be the head of water above the vertex P of the notch (Fig. 81), H being measured to the level of the still water surface behind the notch. Let B = breadth of notch at height H.

Then assuming the velocity of efflux at any depth x to be given by $\sqrt{2gx}$, a formula for the discharge can be deduced as for the rectangular notch.

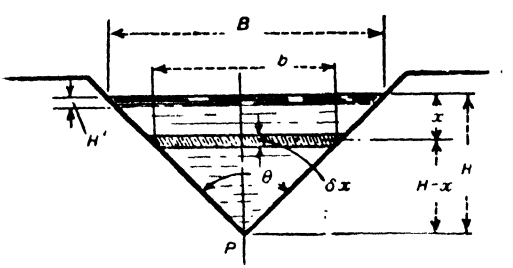


Fig. 81.

Thus the area of an element of the area at a depth x and of width δx is $b\,\delta x = B \cdot \frac{H-x}{H} \cdot \delta x$.

∴ Flow over this element

$$= \delta Q = B\sqrt{2gx} \cdot \frac{H-x}{H} \cdot \delta x \text{ c.f.s.} \qquad (1)$$

$$\therefore \quad Q = \frac{CB\sqrt{2g}}{H} \int_0^H (H\sqrt{h} - h^{\frac{3}{2}})\, dh$$

$$= \frac{4}{15}\, CB\sqrt{2g} \,.\, H^{\frac{3}{2}} \text{ cusecs.} \qquad (2)$$

Putting $\frac{B}{2} = H \tan\frac{\theta}{2}$ where θ is the angle included between the sides of the notch, this becomes

$$Q = \frac{8}{15} C\sqrt{2g} \,.\, \tan\frac{\theta}{2} \,.\, H^{\frac{5}{2}} \text{ cusecs}$$

$$= 4{\cdot}28\, C \tan\frac{\theta}{2} \,.\, H^{\frac{5}{2}} \text{ cusecs.} \qquad (3)$$

Since the ratio $\frac{B}{H}$ is constant for any one notch, it is to be expected that the value of C with different heads will be more nearly constant than in the case of a rectangular notch. Experiments by *Professor James Thomson*[1] indicate that this is so, the value increasing very slightly as the head diminishes. With a right-angled notch the variation was less than 1 per cent. under a range of heads from 2 inches to 7 inches, while with a notch having sides inclined at 2 horizontal to 1 vertical ($\tan\frac{\theta}{2} = 2$) the value of C increased by about 2 per cent. as the head was reduced.

As the result of these experiments Professor Thomson estimated the mean value of C for a right-angled notch as ·593, thus giving a discharge

$$Q = 2{\cdot}536\, H^{\frac{5}{2}} \text{ cubic feet per second.}$$

With a notch having side inclinations of 2 to 1 ($\tan\frac{\theta}{2} = 2$), the mean value of C was found to be ·618, making

$$Q = 5{\cdot}29\, H^{\frac{5}{2}} \text{ cubic feet per second,}$$

while as the angle is still further increased, C appears to approach a limiting value ·620.

[1] "British Association Report," 1861, p. 351.

With these wide-angled notches the value of C, however, fluctuates between much wider limits than in the case of the right-angled notch and this renders the former type not so suitable for measuring purposes.

Thomson's formula for a right-angled notch may be written in the form

$$Q = \cdot 305\ H^{\frac{5}{2}} \text{ cub. ft. per minute}$$

where H is the head in inches.

A series of very careful experiments has been carried out by *Mr. James Barr*,[1] on such notches, with heads up to 10 inches. The chief results of these are shown graphically in Fig. 82, where curves C, A, and

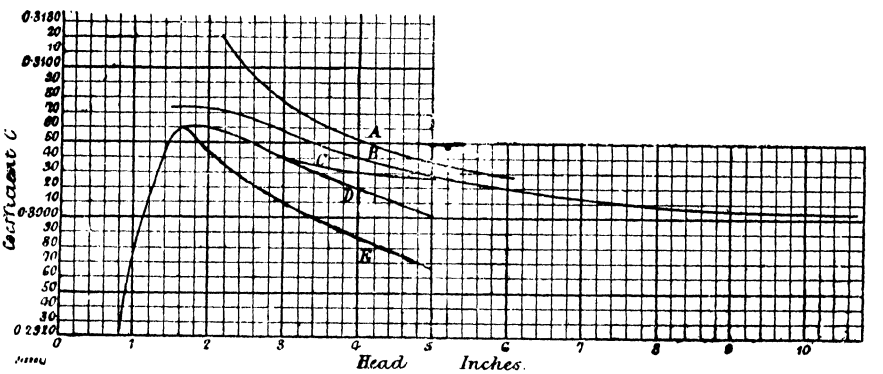


FIG. 82.—Value of Coefficient C in $Q = CH^{\frac{5}{2}}$ c.f.m. for a Right-angled Notch.

B respectively represent the results obtained on a notch with an extremely fine edge, on one with a crest $\frac{1}{12}$ inch broad, and on one with a crest $\frac{1}{16}$ inch broad. In these experiments the approach channel was 4 feet wide, and the depth of the floor of the channel below the vertex of the notch was 2 feet.

To find the effect of the width of the channel of approach upon the discharge, false sides made of wood were placed in the flume. They extended the full depth of the flume, and for a distance of 3 feet up-stream.

Two sets of experiments were made, in one of which a head of 3 inches was maintained and in the other a head of 4 inches. Fig. 83 gives the results. They show that in order that the flow may be independent of the channel width, the latter must be at least eight times the head.

[1] *Engineering*, April 8 and 15, 1910.

A sheet of zinc 36 inches broad and 42 inches long was used as a

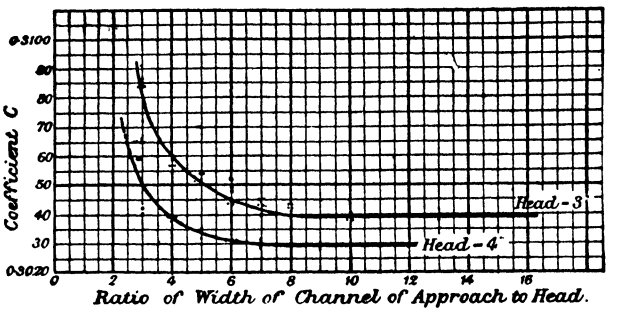


FIG. 83.

temporary floor to investigate the effect of the depth of the channel of approach upon the discharge. The results are given in Fig. 84, which shows that the presence of the floor produces a slight diminution in the discharge, and that this effect disappears when the channel depth is about three

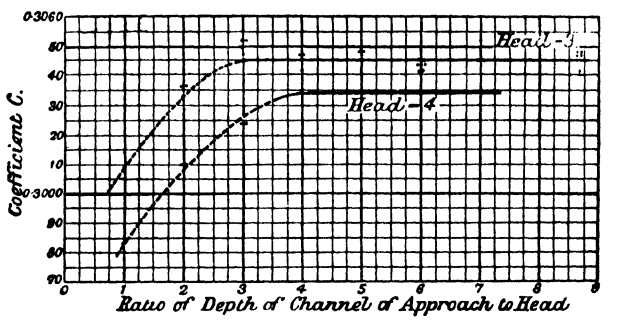


FIG. 84.

times the head with a 3 inch head and about four times the head with a 4 inch head.

Experiments were also made in which the head was varied. For the results given in *E* (Fig 82) the floor was maintained level with the vertex of the notch, and it was at a depth 6 inches below the vertex for the results given in *D* (Fig. 82).

By coating the up-steam face of the weir plate with coarse emery the discharge was increased by 2·4 per cent. with a 3 inch head and by 1·7 per cent. when the head was 4 inches.

The results of the whole series of experiments are in extremely close accord with those previously obtained by Professor Thomson.

An examination of the experimental data of Thompson and Barr, and of more

recent experiments by Gaskell [1] and by Gourley and Crimp,[2] indicates that the discharge over a triangular notch is given very closely by the formula

$$Q = 2{\cdot}48 \tan \frac{\theta}{2} H^{2{\cdot}48}$$

This formula has been adopted by the British Standards Institution.[3] The conditions specified are that the lip of the notch shall be $\frac{1}{16}$ inch thick; the head shall be measured at a point upstream equal to four times the head; the vertex shall be not less than 12 inches above the bottom of the channel for heads up to 9 inches and not less than 18 inches for higher heads; and the width of the channel shall be not less than four times the head.

While suffering from the disadvantage that only small quantities of water are passed with a comparatively high head, the triangular has the advantage over the rectangular notch where the flow is very variable, that with either large or small heads it is equally easy to take accurate measurements of the head, while with the rectangular notch with very low heads this is practically impossible.

On allowing for the velocity of approach, equation (3) becomes

$$Q = \frac{8}{15} C \tan \frac{\theta}{2} \sqrt{2g} \left\{(H + h)^{\frac{5}{2}} - h^{\frac{5}{2}}\right\} \text{ cub. ft. per sec.}$$

Here h may be taken as $1{\cdot}4 \frac{v^2}{2g}$, where v = velocity of approach.

In using the notch, care should be taken that the sides are equally inclined to the vertical.

Variation of Discharge with Head.

Since $$Q = K H^{\frac{5}{2}}$$

$$\therefore \frac{dQ}{dH} = 2{\cdot}5 K H^{\frac{3}{2}}$$

$$\therefore \frac{\delta Q}{Q} = 2{\cdot}5 \frac{\delta H}{H}$$

It follows that a small change in, or error in estimating H, produces 2·5 times the percentage change or error in Q.

Art. 67.—Trapezoidal Notch (Fig. 85).

Let b be the bottom breadth of the notch. Let θ be the inclination to the vertical, of the sides.

[1] "Proc. Inst. C. E.," vol. cxcvii.
[2] "Proc. Inst. C. E.," vol. cc., 1914–15. pt. ii.
[3] British Standards Specification for Pump Tests, March, 1935.

The notch then becomes equivalent to a rectangular notch of breadth b,

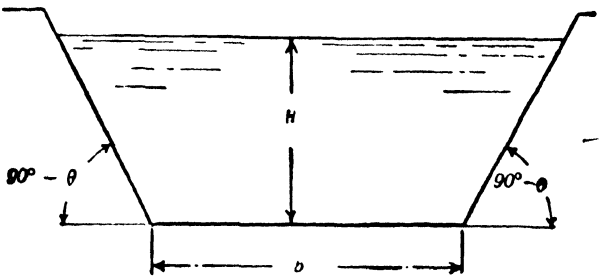


FIG. 85.

together with a triangular notch having an angle 2θ between its inclined sides, and the discharge Q due to a head H is obtained by adding the values of Q, as calculated for two such notches.

$$\therefore \quad Q = \sqrt{2g}\left\{\frac{2}{3} b\, C_1 H^{\frac{3}{2}} + \frac{8}{15} C_2 \tan\theta\, H^{\frac{5}{2}}\right\} \qquad (1)$$

Assuming $C_1 = C_2 = C$ this becomes

$$Q = C\sqrt{2g}\, H^{\frac{3}{2}}\left\{\frac{2}{3} b + \frac{8}{15} H \tan\theta\right\}. \qquad (2)$$

Francis's formula for a rectangular notch shows that the two end contractions reduce the effective breadth by $\cdot 2\, H$, and the discharge by an amount

$$\cdot 2\, H \times \frac{2}{3} C\sqrt{2g}\, .\, H^{\frac{3}{2}} = \frac{2}{15} C\sqrt{2g}\, .\, H^{\frac{5}{2}}.$$

If then the ends of the weir, instead of being vertical, are inclined outwards so that the added area counterbalances the increased contraction of section of the stream, the coefficient K in the formula $Q = K\, b\, H^{\frac{3}{2}}$ should be independent of the head.

For this to be so we have

$$\frac{2}{15} C\sqrt{2g}\, .\, H^{\frac{5}{2}} = \frac{8}{15} C\sqrt{2g} \tan\theta\, H^{\frac{5}{2}},$$

so that $\tan\theta = \frac{1}{4}$, or the sides are to be inclined outwards with a slope of 1 in 4. This is termed a **Cippoletti** [1] **Weir.**

From his own experiments and those of Francis, on heads from 3 to 24 inches, *Cippoletti* made $Q = 3{\cdot}367\ b\ H^{\frac{3}{2}}$ cub. ft. per sec., the Francis velocity correction being used.

[1] First described by C. Cippoletti. See "Giornale del Genies Civiles," 1886

Experiments by *Messrs. Flinn and Dyer*[1] on weirs having sill lengths of from 3 feet to 9 feet, and heads from ·3 foot to 1·25 feet, gave, as the mean of thirty-two experiments, $K = 3{\cdot}283$. In this formula, however, the velocity correction of Hamilton Smith (effective head $= H + 1{\cdot}4\ h$) was used. Had the Francis correction been applied this coefficient would have been in close accord with that of Cippoletti.

If $C_1 = {\cdot}623$ (Francis mean value) and $C_2 = {\cdot}593$ (Thomson's value for a right-angled notch), the inclination of the sides becomes 1 to 4·2. Experiments by *J. C. Stevens*[2] on weirs having lengths of 6 inches and of 1, 2, and 3 feet, with side inclinations of 1 to 4, and with heads ranging from ·08 foot to ·8 foot, indicate that for small lengths, the discharge is greater than is given by the formula, the error increasing as the length is diminished. For lengths less than 3 feet the side inclinations should apparently vary as follows:—

Length of weir (feet)	·5	1·0	2·0	3·0
Cotan θ . . .	4·25	4·2	4·1	4·0

Messrs. Gourley and Crimp give the formula

$$Q = 3{\cdot}10\ b^{1{\cdot}02}\ H^{1{\cdot}47} + 2{\cdot}48 \tan\theta\ H^{2{\cdot}47}$$

for trapezoidal weirs.

Art. 68.—Submerged Weirs.

Where the water on the down-stream side of a weir rises above the level of the sill we get a submerged weir.

If H_1 and H_2 (Fig. 86) be the heights of the free surfaces above the sill of a rectangular weir, the flow over the upper part of the section, of a depth $H_1 - H_2$, may be considered as a free discharge into air, and that over the lower part, of depth H_2, as a discharge through a submerged orifice.

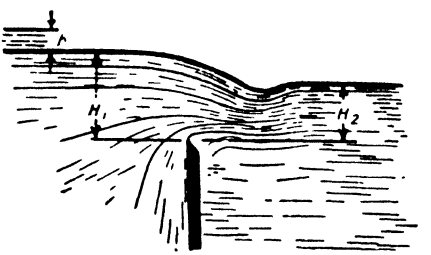


Fig. 86.

Thus for the upper portion $Q_1 = \frac{2}{3} C_1\ b \sqrt{2g}\ (H_1 - H_2)^{\frac{3}{2}}$

„ „ lower „ $Q_2 = C_2\ b \sqrt{2g}\ H_2\ (H_1 - H_2)^{\frac{1}{2}}$

[1] "Trans. Am. Soc. C.E.," vol. 32, 1894, pp. 9—33.

[2] *Engineering News*, New York, August 18, 1910, p. 171.

If we assume $C_1 = C_2 = C$, we have for the total flow

$$Q = C\,b\,\sqrt{2\,g\,(H_1 - H_2)}\left\{\frac{2}{3}(H_1 - H_2) + H_2\right\}$$

$$= C\,b\,\sqrt{2\,g\,(H_1 - H_2)}\left\{\frac{2}{3}\,H_1 + \frac{1}{3}\,H_2\right\}.$$

Taking into account the velocity of approach, we have

$$Q_1 = \frac{2}{3}\,C_1\,b\,\sqrt{2\,g}\left\{(H_1 - H_2 + h)^{\frac{3}{2}} - h^{\frac{3}{2}}\right\}$$

$$Q_2 = C_2\,b\,\sqrt{2\,g}\,H_2\,(H_1 - H_2 + h)^{\frac{1}{2}}$$

$$\therefore\ Q = C\,b\,\sqrt{2\,g\,(H_1 - H_2 + h)}\left\{\frac{2}{3}(H_1 - H_2 + h) + H_2 - \frac{\frac{2}{3}\,h^{\frac{3}{2}}}{(H_1 - H_2 + h)^{\frac{1}{2}}}\right\}$$

As a very close approximation we have

$$Q = C\,b\,\sqrt{2\,g\,(H_1 - H_2 + h)}\left\{\frac{2}{3}(H_1 + h) + \frac{1}{3}\,H_2\right\}.$$

The value of C varies with the ratio $\frac{H_2}{H_1 + h}$, and can only be deduced by experiment.

The following mean values are obtained from the results of experiments by Francis, and by Messrs. Fteley and Stearns :—

Francis.—Depth of water at sill ·85 to 2·3 feet.[1]

Ratio $\frac{H_2}{H_1 + h}$	·00	·1	·3	·5	·7	·9
C . . .	·623	·625	·606	·594	·594	·596

Messrs. Fteley and Stearns.[2]

$\frac{H_2}{H_1 + h}$	·1	·3	·5	·7	·9
C . .	·630	·605	·590	·585	·595

[1] Suppressed weir 22·2 feet long. "Trans. Am. Soc. C.E.," vol. 12.

[2] Suppressed weir 5 feet long. Values of H_1 from ·3 to ·8 foot. Crest 3·2 feet from bottom of channel. "Trans. Am. Soc. C.E.," vol. 12.

The results of experiments on submerged weirs 1 foot and 2 feet long, with two end contractions, are given in *Engineering News*, New York, August 18, 1910, p. 174.

Redtenbacher makes $C_1 = \cdot 57$ and $C_2 = \cdot 62$ in the above formula, while *Pestalozzi* also makes C_2 equal to ·62 and makes C_1 vary from ·534 to ·566, diminishing as the ratio of H_2 to H_1 increases. It is indeed to be expected that C_2 will be greater than C_1, since owing to eddy formation behind the weir the pressure on the discharge side of the crest will be less than that corresponding to the assumed statical head $H_1 - H_2$, thus giving rise to an increased discharge over the lower portion of the stream.

The above discussion of submerged weir flow is due to Dubuat, and is admittedly unsatisfactory from a scientific point of view. The difficulties in the way of a more rational investigation of the problem based on the

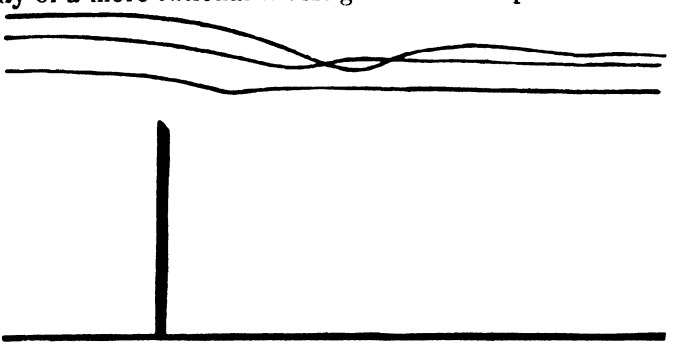


FIG. 87.—Surface Curves in Neighbourhood of Submerged Weirs.

variations of pressure and velocity in the stream are, however, extremely great, as will be realised from a study of Fig. 87.

These curves show the profile of the stream as measured by the author in the neighbourhood of such a weir, under different heads.

Recent experiments by the author on models of weirs *having rounded crests* show that, using the simple formula $Q = CbH^{\frac{3}{2}}$, where H is the up-stream head, if H_s is the down-stream depth of submergence the relative values of the coefficient C as compared with a freely discharging weir, are as follows :—

$\frac{H_s}{H}$	0	·1	·2	·3	·4	·5	·6	·7	·8	·9
$\frac{C_s}{C}$	1·0	·995	·984	·975	·960	·940	·910	·860	·780	·630
$\frac{C_s}{C}$	1·0	·991	·983	·972	·956	·937	·907	·856	·778	·621

The figures in the third line are from similar experiments by U.S. Deep Waterways (U.S. Water Supply Paper No. 200, p. 146). See also *Soucek*, *Proc. Am. Soc. C.E.*, Vol. 68, No. 8, Part I, October, 1942, p. 1352. It will be seen that even for considerable degrees of submergence the effect on the discharge is relatively small.

Art. 69.—Broad-crested Weirs.

Experiments by Bazin indicate that in general the *nappe* will clear the crest of a sharp-edged weir if this is of less width than ·5 H. For widths between ·5 H and ·66 H the condition is unstable, and any variation in the flow will cause the *nappe* to cling or to break free, with a corresponding variation in the discharge, while for widths greater than this the *nappe* clings to the crest in every case.

More recent experiments by G. S. Williams indicate that the ratio of crest width w to head, at which the *nappe* springs clear, depends on the head and is usually less than ·5, as shown in the following table :—

Crest width (feet)	·48	·93	1·65
Head at which *nappe* becomes clear (feet)	1·5	2·5	*Nappe* not clear when $H = 4$ feet.
Ratio of $w : H$ . .	·32	·38	$< ·41$

Where the crest is sufficiently wide to cause adherence of the *nappe*,

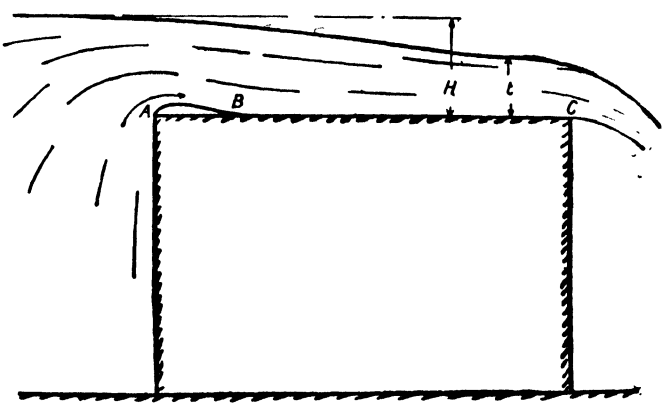


Fig. 88.

the weir is termed broad-crested. In such a weir the stream springs clear of the crest at its up-stream edge A and again makes contact at B, as in Fig. 88. A partial vacuum is produced from A to B, and if the width of the crest is little greater than is necessary to cause this contact, the discharge is

greater—up to about 15 per cent., depending upon the head—than with a thin-crested weir having free discharge. As the crest is lengthened the friction of the surface from B to C reduces the discharge, which now depends appreciably on the roughness of the surface.

Bazin experimenting on flat-crested weirs having crest widths ranging from ·164 foot to 6·56 feet, and with heads from ·25 foot to 1·5 feet, found

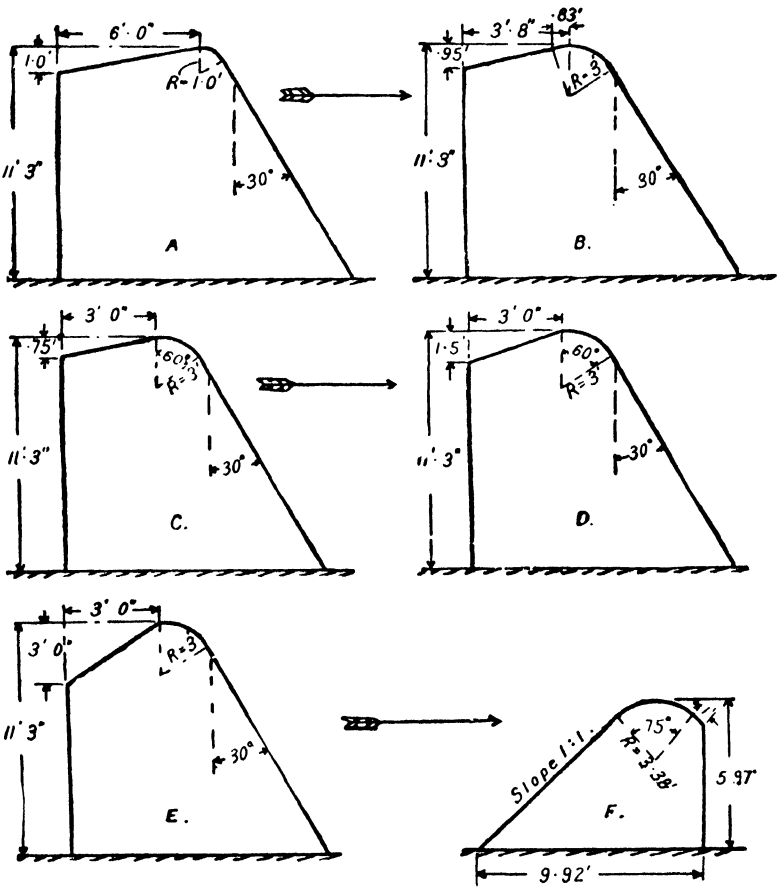


FIG. 89.

that where the *nappe* adheres to the top but not to the down-stream face of such a weir, the discharge may be expressed by

$$Q = K' \, b \, H^{\frac{3}{2}}, \text{ where } K' = K \left\{ \cdot 70 + \cdot 185 \, \frac{H}{w} \right\}.$$

Here K is the coefficient for the corresponding thin-crested weir, as calculated from Bazin's formula

For wider crests, and for heads greater than 1·5 feet, this formula gives rather low results, while if applied to the Francis formula, the results are much too low for crest widths greater than 3 feet and for heads greater than 1 foot.

An examination by the author, of experiments by G. S. Williams[1] on such weirs, with widths ranging from ·48 foot to 16·3 feet and with heads from ·5 foot to 4·0 feet, shows that, adopting the Francis formula, the discharge is given within about 3 per cent. for all widths and heads, so long as

$$w > 2\ H, \text{ by writing } K' = K\left\{\cdot 75 + \cdot 1\sqrt{\frac{H}{w}}\right\}.$$

Fteley and Stearns, using crest widths of from 2 to 10 inches, and heads

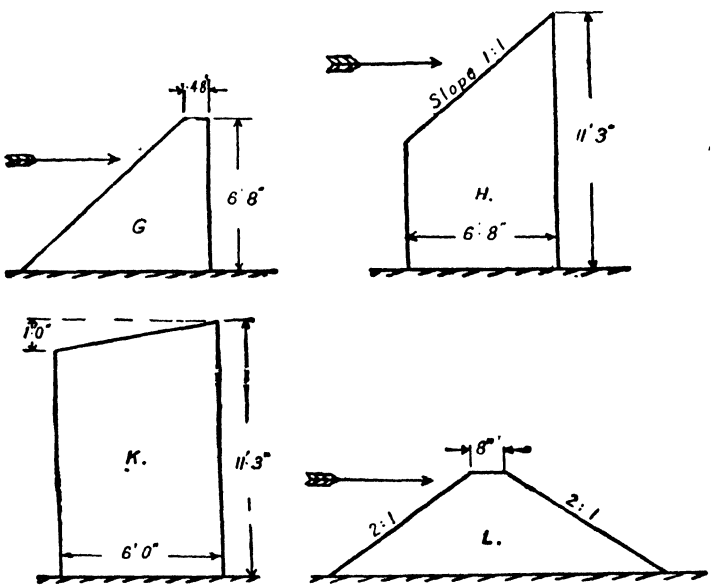


FIG. 90.

from ·116 to ·894 feet, deduced the formula $Q = K\,b\,(H + k)^{\frac{3}{2}}$, where $k = \cdot 2016\sqrt{(\cdot 807\,b - H)^2 + \cdot 2146\,b^2} - \cdot 1876\,b$, and K is the coefficient for a thin-crested weir.

The most reliable experiments on flow over broad-crested weirs and dams are those by Bazin[1] and those carried out at Cornell University between 1898 and 1904 under the supervision of Professor G. S. Williams.[1]

[1] For a full discussion of these results, as well as those of Bazin, see "Water Supply and Irrigation," Paper No. 200, U.S. Geological Survey; "Weir Experiments, Coefficients and Formulas," by R. E. Horton. Also see "Rafter," "Trans. Am. Soc. C.E.," vol. 44, 1900.

In the latter experiments which were made for the United States Geological Survey and for the United States Board of Engineers on Deep Waterways the weirs were 16 feet in length and heads up to 4 feet were used. The Cornell experiments were by far the more extensive, and the chief results of these are shown by the following tables and the diagrams of Fig. 89, 90, and 91 On all weirs having vertical down-stream faces the *nappe* was fully aerated. In the weirs with inclined down-stream faces no air was admitted under the *nappe*.

RECTANGULAR FLAT-CRESTED WEIRS.

Values of K' in formula, $Q = K' \, b \, H^{\frac{3}{2}}$.

H (feet).	Values of w.							
	0·48 ft.	0·93 ft.	1·65 ft.	3·17 ft.	5·89 ft.	8·98 ft.	12·24 ft.	16·30 f
0·5	3·01	2·76	2·73	2·66	2·61	2·61	2·61	2·61
1·0	3·24	3·01	2·93	2·70	2·67	2·66	2·65	2·64
1·5	3·33	3·19	3·03	2·73	2·69	2·67	2·67	2·65
2·0	3·33	3·29	3·08	2·73	2·68	2·67	2·66	2·65
3·0	3·33	3·33	3·12	2·71	2·65	2·64	2·62	2·61
4·0	3·33	3·33	3·15	2·69	2·63	2·61	2·60	2·59

COMPOUND WEIRS (A TO F). FIG. 89.

H (feet).	A.	B.	C.	D.	E.	F.
0·5	3·21	3·10	3·22	3·23	3·23	3·23
1·0	3·42	3·27	3·35	3·46	3·46	3·27
1·5	3·54	3·38	3·44	3·61	3·64	3·40
2·0	3·55	3·44	3·47	3·68	3·75	3·46
3·0	3·30	3·48	3·48	3·75	3·87	3·87
4·0	3·14	3·51	3·48	3·81	3·96	3·65

From experiments on weirs of ogee cross section (A to F), the formula

$$K' = \left\{ 3{\cdot}62 - {\cdot}16\,(s - 1) \right\} H^{\frac{1}{20}}$$

was found to give the coefficient with reasonable accuracy. Here s is the slope of the up-stream face.

Example: $s = 2:1$; $H = 4{\cdot}0$; $\therefore K = 3{\cdot}46 \times 4^{\frac{1}{10}}$; $\log 4^{\frac{1}{10}} = {\cdot}0301$; $\therefore K = 3{\cdot}46 \times 1{\cdot}0716 = 3{\cdot}70$.

This formula holds for weirs with an up-stream slope more than from 3 feet to 4·5 feet broad, and with a down-stream radius (above from 2 feet

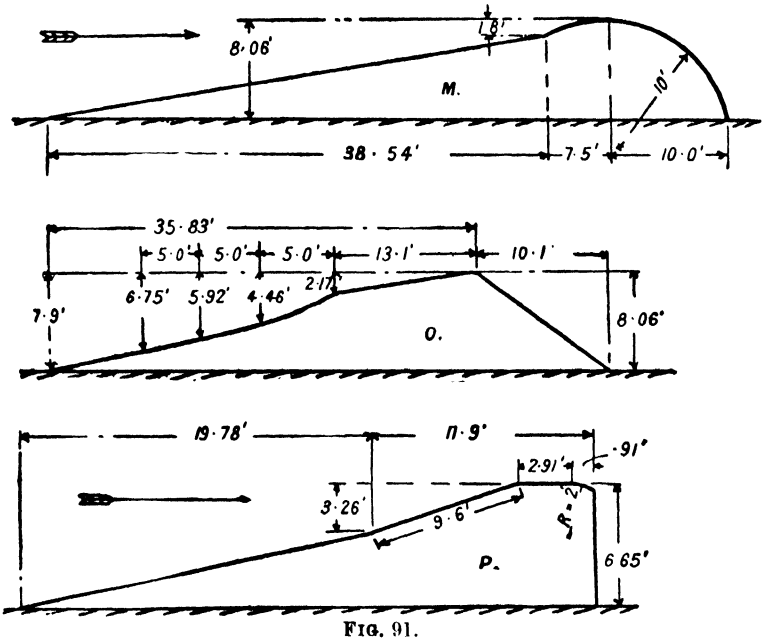


FIG. 91.

to 3 feet) great enough to retain the *nappe* in contact, and yet not so large as to simulate a broad, flat crest.

TRAPEZOIDAL AND TRIANGULAR WEIRS (G TO L). FIG. 90.

Type G.

Up-stream Slope.		1 to 1.	2 to 1.		3 to 1.	4 to 1.	5 to 1
Crest Width (feet).		·48	·33 [1]	·66 [1]	·66 [1]	·66 [1]	·66 [1]
Head (feet).	·5	3·22	3·35	3·22	3·64	—	3·31
	1·0	3·57	3·68	3·44	3·82	3·44	3·33
	1·5	3·59	3·83	3·59	3·83	3·46	3·34
	2·0	3·60	3·77	3·66	3·69	3·48	3·35
	3·0	3·58	3·68	3·68	3·55	3·48	3·38
	4·0	3·55	3·70	3·70	3·55	3·48	3·39

[1] "Rafter," crest height = 4·7 feet.

Types H to L.

Head (feet).	H.	K.	L.
·5	3·53	3·47	3·14
1·0	3·59	3·46	3·42
1·5	3·64	3·45	3·52
2·0	3·65	3·42	3·61
3·0	3·63	3·35	3·66
4·0	3·61	3·29	3·66

Compound Weirs (M to P). Fig. 91.

Head (feet).	M	N[1]	O	P
·5	3·21	2·91	3·65	3·06
1·0	3·21	3·16	3·63	3·05
1·5	3·20	3·33	3·61	3·04
2·0	3·16	3·42	3·56	3·11
3·0	3·06	3·51	3·45	3·20
4·0	3·01	3·58	3·38	3·27

From a consideration of these various tables it appears that with dams having an ogee cross section, or having an up-stream slope, the discharge usually increases according to some higher power of the head than the 1·5th. Recent gaugings of the flow over large dams of these types[2] indicate that this power may attain a value as high as 1·75, though it usually lies between 1·50 and 1·65.

Effect of Condition of Nappe, on Discharge over wide, flat-crested Weirs.

Just as with a thin weir the *nappe* from a wide-crested weir may either spring clear of the down-stream face, be depressed, or be drowned. A simple depression of the *nappe* has, however, little effect on the discharge, slightly increasing it for low heads and diminishing it for high heads. If the *nappe* is drowned the effect is very slight so long as the stream

[1] Type N is identical with M, but with the addition of a 12 inch × 12 inch timber baulk on its crest.

[2] *Engineering News*, New York, September 29, 1910, p. 321.

clings to the crest over its whole width. In some cases a mass of turbulent water separates the *nappe* from the weir crest. Under such circumstances Bazin found that the discharge is the same as from a thin weir with drowned *nappe*, and is given by

$$Q = K' b H^{\frac{3}{2}}, \text{ where } K' = K \left\{ \cdot 878 + \cdot 128 \frac{P}{H} \right\},$$

P being the depth of the approach channel below the weir crest, and K being the coefficient for a thin-crested weir freely discharging under the same head.

ART. 70.—RATIONAL FORMULA FOR FLOW OVER BROAD-CRESTED WEIRS.

As was first pointed out by *Dr. W. C. Unwin*, a rational formula for flow over a weir of this type may be deduced if the crest be assumed to be so wide that the filaments form a parallel stream of thickness t (Fig. 88) before leaving the crest, and that in this stream the pressure at any point is that statically corresponding to its depth. We then have the velocity at the surface and at every point in this stream $= \sqrt{2g(H-t)}$, while $Q = b\, t \sqrt{2g(H-t)}$.

The value of t will adjust itself to give a maximum discharge, and this theoretical value for maximum flow may be determined by equating $\frac{dQ}{dt}$ to zero. This gives $t = \frac{2}{3} H$, and substituting this value, we get $Q = \cdot 385\, b \sqrt{2g}\, H^{\frac{3}{2}}$ as the maximum possible discharge. Writing this in the usual form $Q = \frac{2}{3} C b \sqrt{2g}\, H^{\frac{3}{2}}$, we get $C = \cdot 578$, and K = 3·087.

This method of treatment becomes more rational if account be taken of the fact that in a parallel stream flowing in an open channel, the distribution of velocity over any vertical is not uniform, being a maximum at or near the surface and a minimum at the bottom.

Experiments show that the ratio of the mean velocity over the section of such a stream to the maximum surface velocity, while varying with the depth, width, and roughness of the bottom of the channel, lies between the limits ·82 and ·87 for such surfaces and depths as are common on the crest of such weirs, this ratio increasing with the depth of water.

Assuming, as is practically the case, that the maximum surface velocity in the case of the weir is equal to $\sqrt{2g(H-t)}$, the mean velocity will equal $k\sqrt{2g(H-t)}$, and the discharge will be given by

$$k \cdot K \cdot b H^{\frac{3}{2}} = 3{\cdot}087\ kbH^{\frac{3}{2}} = K'bH^{\frac{3}{2}} \text{ cub. ft. sec.}$$

Thus corresponding to the values ·82 and ·87 of k, the values of K become 2·53 and 2·69.

The validity of this formula receives remarkable confirmation from the results of the tests on such weirs at Cornell (p. 172), where for all heads between ·5 foot and 4 feet, and for all crest widths greater than 5 feet the values of K' lie between 2·59 and 2·69.

A sloping crest increases the discharge, as does any rounding of the up-stream corner. The effect of this rounding is not so pronounced as with a thin-crested weir, diminishing as the crest width increases and also as the head increases. Fteley and Stearns, experimenting on a crest 4 inches wide, with radii of one-fourth, one-half, and one inch respectively, found the effective head to be increased in the ratio $\left\{1 + \frac{\cdot 41\ R}{H}\right\}$, this correction being applicable for heads of not less than ·17 and ·26 feet on weirs with radii of one-fourth and one-half inch respectively.

Bazin's experiments on crests having widths of 2·62 and 6·56 feet, heads from ·25 foot to 1·50 feet, with an up-stream crest radius of 4 inches, showed a mean increased discharge of 13·5 per cent. with the narrower and 10 per cent. with the broader crest, while the United States Deep Waterways experiments on a weir 22 feet wide, with 6 : 1 slope on each face, showed an average increase of 2 per cent. with an up-stream edge rounded to a radius of 4 inches. Experiments on weir B, Fig. 89A, showed that a rounding of the up-stream edge to a radius of 4 inches increased the discharge by about 4 per cent. at the higher heads. The condition of the crest as regards roughness is found not to influence the discharge by more than about 2 per cent.

ART. 71.—SIDE WEIRS.

A side weir consists of a rectangular notch formed in the side of a culvert with a view of discharging all storm water rising above its sill to a separate storm relief channel alongside. Experiments by G. S. Coleman and Dempster Smith[1] show that the necessary length of the sill of such a weir is given by

$$l = 29{\cdot}1\ w^{1{\cdot}4}\ h^{\cdot 513} \text{ feet,}$$

and that the discharge is given by

$$Q = 1{\cdot}674\ w\ l^{\cdot 72}\ h^{1{\cdot}645}$$

where w is the width of the main channel.

[1] "Proc. Inst. C. E.," Selected Engineering Papers, No. 6, 1923.

ART 72.

Time of lowering the Level in a Reservoir through a Rectangular Notch.—With the usual notation, the volume discharged per second with a head H behind the notch is given by

$$Q = K\,b\,H^{\frac{3}{2}} \text{ cubic feet.}$$

∴ At this instant we have the velocity of the free surface in the reservoir given by

$$-\frac{d\,H}{d\,t} = \frac{K\,b\,H^{\frac{3}{2}}}{A}$$

Integrating this, the time $(t_2 - t_1$ seconds, to lower the level through a distance $H_1 - H_2$ feet, is given by

$$t_2 - t_1 = t = \frac{2\,A}{K\,b}\left\{\frac{1}{H_2^{\frac{1}{2}}} - \frac{1}{H_1^{\frac{1}{2}}}\right\} \text{ seconds.}$$

Similarly the time of lowering the level through a right-angled vee notch whose discharge is $Q = 2{\cdot}536\ H^{\frac{5}{2}}$ cusecs (p. 162) is given by

$$t_2 - t_1 = 0{\cdot}262\ \mathrm{A}\left\{\frac{1}{H_2^{\frac{3}{2}}} - \frac{1}{H_1^{\frac{3}{2}}}\right\}.$$

ART. 73.—TIME TO ALTER LEVEL IN RESERVOIR, WITH A FLOOD INFLOW AND DISCHARGE OVER A SPILLWAY WEIR.

In a problem of this kind it is important, having a knowledge of the probable rate and increase of flood flow over a period, sometimes of days, to be able to pre-determine the rate of rise in the reservoir, and the maximum height to be expected with a spillway of given length. If A be the surface area of the reservoir in square feet; Q the rate of inflow at any instant, and if h be the head at this time on the weir crest, the excess of inflow over outflow is $Q - k\,b\,h^{\frac{3}{2}}$, so that

$$\frac{d\,h}{d\,t} = \frac{Q - k\,b\,h^{\frac{3}{2}}}{A}.$$

This can be integrated if Q and A are constants. Writing $Q = k\,b\,H^{\frac{3}{2}}$ where H is the head on the crest which would give the same discharge as the inflow Q, and putting $h/H = r$, the solution is

$$T_2 - T_1 = \frac{2A}{3\,k\,b\sqrt{H}}\left[\log_e \frac{\sqrt{1-r-\sqrt{r}}}{1-\sqrt{r}} - \sqrt{3}\left\{\tan^{-1}\frac{2}{\sqrt{3}}(\sqrt{r}+\tfrac{1}{2}) - \frac{\pi}{6}\right\}_{r_1}^{r_2}\right]$$

$$= B\left[\phi(r)\right]_{r_1}^{r_2} \quad \text{where } B = \frac{2A}{3\,k\,b\sqrt{H}}.$$

$\phi(r)$ is known as Gould's function and values of this have been calculated for a wide range of values of h/H or r.

Once the maximum height in the reservoir has been reached and the level begins to fall, $Q - k\,b\,H^{\frac{3}{2}}$ is negative and the solution becomes

$$T_2 - T_1 = B[\phi(r_1) - \phi(r_2)]$$

where $\phi(r)$ has the value

$$\left[\log_e \frac{\sqrt{r}-1}{\sqrt{1+r+\sqrt{r}}} + \sqrt{3}\left(\tan^{-1}\frac{2\sqrt{r}+1}{\sqrt{3}}\right)\right].$$

Values of these functions, both for rising and falling levels, are tabulated below :—

VALUES OF GOULD'S FUNCTION $\phi(r)$ (LEVEL RISING).

r	·00	·01	·02	·03	·04	·05	·06	·07	·08	·09
·0	·0000	·0153	·0306	·0459	·0613	·0750	·0919	·1072	·1226	·1378
·1	·1520	·1685	·1838	·1992	·2155	·2300	·2483	·2646	·2810	·2973
·2	·3110	·3301	·3464	·3628	·3791	·3950	·4137	·4319	·4501	·4683
·3	·4830	·5047	·5229	·5411	·5593	·5750	·5957	·6139	·6321	·6534
·4	·6730	·6960	·7173	·7386	·7598	·7760	·8024	·8237	·8450	·8663
·5	·8880	·9137	·9399	·9660	·9921	1·009	1·044	1·070	1·097	1·113
·6	1·142	1·171	1·199	1·229	1·259	1·291	1·323	1·355	1·389	1·424
·7	1·459	1·496	1·534	1·573	1·614	1·656	1·700	1·745	1·792	1·841
·8	1·893	1·947	2·004	2·063	2·127	2·194	2·266	2·342	2·425	2·515
·9	2·613	2·721	2·841	2·977	3·134	3·319	3·544	3·834	4·243	4·938

LEVEL FALLING.

r	1·05	1·10	1·15	1·20	1·25	1·30	1·40	1·50	1·60	1·70
$\phi(r)$	− 2·438	− 1·750	− 1·362	− 1·086	− 0·874	− 0·704	− 0·438	− 0·236	− 0·074	+ 0·061
r	1·80	1·90	2·0	2·5	3·0	4·0	5·0	6·0	7·0	8·0
$\phi(r)$	0·175	0·274	0·360	0·682	0·895	1·170	1·474	1·502	1·571	1·648

Intermediate values may be obtained by drawing a graph or by interpolation. In any case falling values are not generally important.

In practice the rate of inflow is not constant, but by taking its mean value over an interval of time relatively short compared with the whole flood period the problem may be solved in stages. Thus at the end of the first interval we have

$$T_1 - T_o = B\left\{\phi\left(\frac{h_1}{H}\right) - \phi\,(o)\right\}$$

where $B = \dfrac{2A}{3\,k\,b\,\sqrt{H}}$, and H is the weir head corresponding to the mean inflow during this period. Similarly at the end of the second period

$$T_2 - T_1 = B^1\left\{\phi\left(\frac{h_2}{H^1}\right) - \phi\left(\frac{h_1}{H^1}\right)\right\}$$

where H^1 is the head corresponding to the mean inflow during the second period, and where B^1 includes the effect of an increase in the area of the reservoir, if this is significant.

As an example of this method take the hypothetical case where the flood inflow increases uniformly from zero to 21,000 cusecs in thirty-six hours and then falls more gradually to 5000 cusecs in another ninety-six hours. The value of A, which is sensibly constant, is 400×10^6 square feet. The weir length b is 660 feet, and the weir coefficient K is 3·0. Taking the time interval as twelve hours, we have :—

First Twelve Hours.—Mean $Q = 3500$ cusecs ;

$$H = \left(\frac{3500}{660 \times 3}\right)^{\frac{2}{3}} = (1{\cdot}77)^{\frac{2}{3}} = 1{\cdot}461, \text{ so that } \sqrt{H} = 1{\cdot}21.$$

Then $T_1 - T_o = 12 \times 3600 = \dfrac{2 \times 400 \times 10^6}{9 \times 660 \times 1{\cdot}21}\left\{\phi\left(\dfrac{h_1}{1{\cdot}461}\right) - \phi\,(o)\right\}$

$\therefore\ \phi\left(\dfrac{h_1}{1{\cdot}461}\right) = 0{\cdot}388$, so that by interpolation from the table $\dfrac{h_1}{1{\cdot}461} = 0{\cdot}245$

$\therefore\ h_1 = 0{\cdot}358$ feet.

Second Twelve Hours.—Mean $Q = 10{,}500$ cusecs.

$\therefore\ H = \left(\dfrac{10{,}500}{660 \times 3}\right)^{\frac{2}{3}} = 3{\cdot}04. \quad \therefore\ \sqrt{H} = 1{\cdot}742.$

Then $T_2 - T_1 = 12 \times 3600 = \dfrac{800 \times 10^6}{9 \times 660 \times 1{\cdot}742}\left\{\phi\left(\dfrac{h_2}{3{\cdot}04}\right) - \phi\left(\dfrac{0{\cdot}358}{3{\cdot}04}\right)\right\}$

$\therefore\ \phi\left(\dfrac{h_2}{3{\cdot}04}\right) - \phi\,(0{\cdot}118) = {\cdot}5590.$

Since $\phi\,(0{\cdot}118)$ by interpolation equals 0·1805 we have

$$\phi\left(\frac{h_2}{3{\cdot}04}\right) = 0{\cdot}7395 \quad \therefore\ \frac{h_2}{3{\cdot}04} = 0{\cdot}431 \text{ (tables) and } h_2 = 1{\cdot}310.$$

Proceeding in this way we get $h_3 = 2{\cdot}60$ feet ; $h_4 = 3{\cdot}58$ feet ; $h_5 = 3{\cdot}96$ feet ; $h_6 = 4{\cdot}00$ feet ; $h_7 = 3{\cdot}92$ feet. Here h_6 or the level after seventy-two hours marks the peak after which the level begins to fall.

In dealing with an actual problem it would be better to take shorter time intervals—say six hours rather than twelve, though in this particular example

the difference in the final level as calculated by taking six-hour intervals is only about 2 inches.

Change of level in reservoir with flood inflow and discharge over a Vee notch.—This is not so important in practice as when discharge takes place over a rectangular spillway. Taking the discharge as $Kh^{\frac{5}{2}}$, the solution becomes

$$t_2 - t_1 = \frac{A}{k\,h^{\frac{3}{2}}}\left\{\phi\,(r_2) - \phi\,(r_1)\right\}.$$

Values of this function have been calculated by Mathieson.[1] Typical values are given in the following tables, from which intermediate values may be obtained by plotting or by interpolation.

$\phi\,(r)$ Level Rising.

r	·00	·02	·04	·06	·08
0·1	·1001	·1202	·1403	·1605	·1807
0·2	·2010	·2215	·2420	·2626	·2834
0·3	·3043	·3256	·3468	·3684	·3902
0·4	·4123	·4347	·4575	·4809	·5047
0·5	·5282	·5528	·5779	·6037	·6303
0·6	·6574	·6856	·7148	·7451	·7767
0·7	·8100	·8448	·8815	·9205	·9622
0·8	1·006	1·055	1·109	1·169	1·237
0·9	1·314	1·410	1·532	1·701	1·984

$\phi\,(r)$ Level Falling.

r	·00	·02	·04	·06	·08
1·0	∞	1·594	1·316	1·161	1·052
1·1	·9692	·9010	·8450	·7976	·7559
1·2	·7192	·6848	·6557	·6292	·6051
1·3	·5831	·5625	·5433	·5254	·5092
1·4	·4930	·4786	·4638	·4516	·4390

[1] Lecturer in Hydraulics at Manchester University.

CHAPTER VIII.

Skin Friction—Froude's Experiments—Zahm's Experiments—Disc Friction.

ART. 74.—SKIN FRICTION.

WHENEVER a liquid flows over a solid surface, or when a submerged plane moves in the direction of its length through a liquid, a resistance to motion is experienced. This is commonly termed skin friction, and should not be confused with the wave-making resistance which is experienced owing to the formation of surface waves, when a partially submerged body is in motion.

Though initially due to viscosity, the laws governing fluid friction are usually very different from those of simple viscous resistance, because of the fact that except at extremely low speeds the motion of the fluid becomes unsteady, eddies are formed, and the energy absorbed in fluid friction now chiefly consists of the energy of formation of these eddies. This energy is finally absorbed in overcoming the viscous resistance of the fluid at points remote from the surface at which the eddies are generated.

The most important series of experiments to determine the resistance to the motion of submerged planes, are those carried out in 1872 by Mr. Froude at Torquay. Here a series of flat boards, having differently prepared surfaces, was held vertically and suspended from a carriage which was driven at a uniform speed, and was thus towed endwise through the still water in a large basin. The carriage was fitted with a dynamometer and automatically recorded the velocity and resistance of the board. These boards were $\frac{3}{16}$ inch wide, 19 inches deep, and varied in length from 1 foot to 50 feet. The top edge was submerged to a depth of $1\frac{1}{2}$ inches and the boards were fitted with a cut-water, the resistance to this being determined separately. In these experiments Mr. Froude determined that—

(1) The resistance varies greatly with the condition of the surface, the

Length of surface .	2 feet.				8 feet.				20 feet.				50 feet.			
Value of n . .	2·00	1·95	2·16	2·00	1·85	1·94	1·99	2·00	1·85	1·93	1·90	2·00	1·83		1·83	2·00
Mean resistance in lbs. per square foot . . .	·41	·38	·30	·90	·325	·314	·278	·625	·278	·271	·262	·534	·250		·246	·488
Resistance per square foot over last foot in lbs. .	·390	·370	·295	·730	·264	·260	·263	·488	·240	·237	·244	·465	·226		·233	·456
	A	B	C	D	A	B	C	D	A	B	C	D	A	B	C	D

Here *A* refers to varnished surfaces, or to the painted surface of an iron ship.

„ *B* „ surfaces coated with paraffin wax.

„ *C* „ „ „ „ tinfoil.

„ *D* „ „ covered with sand of medium coarseness.

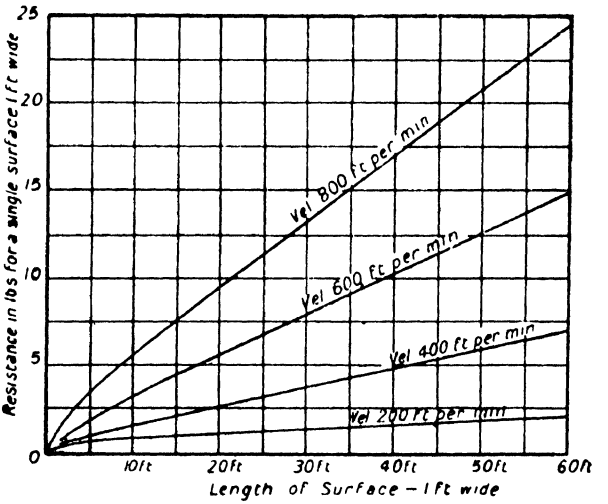

Resistance in lbs for a single surface 1 ft wide
25
20
15
10
5
0
Vel 800 ft per min
Vel 600 ft per min
Vel 400 ft per min
Vel 200 ft per min
10ft
20ft
30ft
40ft
50ft
60ft
Length of Surface – 1 ft wide

FIG. 92.

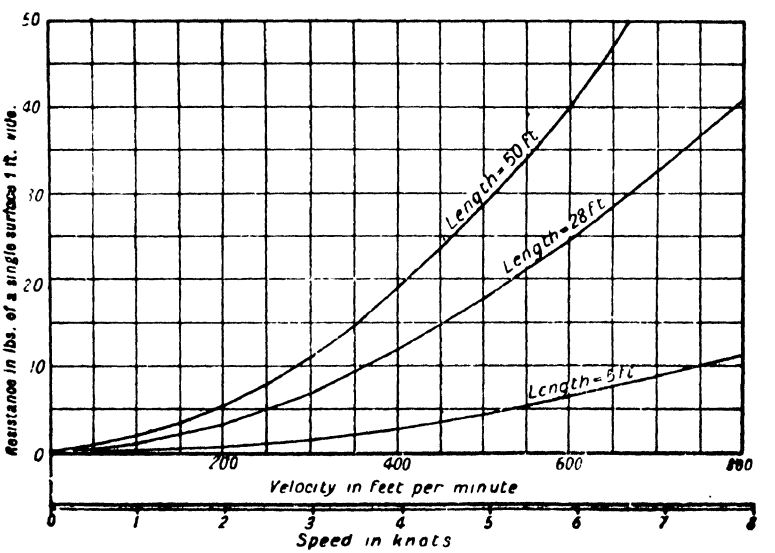

Resistance in lbs. of a single surface 1 ft. wide.
50
40
30
20
10
0
Length = 50 Ft.
Length = 28 Ft.
Length = 5 Ft.
200
400
600
800
Velocity in Feet per minute
0
1
2
3
4
5
6
7
8
Speed in knots

FIG. 93.

resistance for boards 50 feet long at a velocity of 10 feet per second being with a coating of

Varnish or smooth paint of such composition as is found on the bottom of iron ships . .	·250	lbs. per sq. ft.
Tinfoil	·246	„ „
Fine sand	·405	„ „
Calico	·470	„ „
Sand of medium coarseness	·488	„ „

(2) The resistance is proportional to v^n, where

n { (*a*) depends on the surface,
(*b*) decreases, up to a certain limit, with an increase in length,
(*c*) is sensibly independent of the velocity.

(3) The total resistance increases with the length, though the resistance per square foot decreases as the length increases.

Writing: Resistance $= f S v^n$

where S = area of surface, it was found that

(*a*) depends on the surface
(*b*) decreases with an increase in length, becoming approximately constant when the length is large.
(*c*) is independent of the pressure.
(*d*) is proportional to the density of the fluid and diminishes very slightly as the temperature increases.

Owing to viscous drag those parts of the surface near the prow communicate motion to the water, so that the relative motion is smaller over the rear part of the surface and the drag per unit area is consequently less. The velocity of the accompanying current increases until at some point in the surface a balance is obtained between the amount of energy given to the accompanying stream per second, and the energy dissipated by eddy formation in the surrounding fluid and in producing motion of a greater volume of this water against viscous resistances. After this point is reached, the velocity of the accompanying current and the resistance per square foot of surface remain approximately constant. It will be noted that the mean resistance per square foot of area diminishes very slightly for lengths above 50 feet.

A short *résumé* of Mr. Froude's results is given on p. 182, these particular experiments being carried out at a velocity of 10 feet per second.[1]

Curves (Figs. 92 and 93) have been prepared from the results of these experiments, and show respectively the resistance with length at constant

[1] From the British Association Report, 1874.

speed, and with speed for a given length, for a varnished or painted iron surface.

For ships the value of n is usually taken as 1·83, and the following values of f are appropriate for the surfaces usual in practice.[1]

Length in feet.	Iron bottom (painted).	Copper sheathing.	
		Smooth.	Rough.
200	·00944	·00943	·01170
300	·00923	·00930	·01152
400	·00910	·00926	·01140
500	·00904	·00926	·01186

Art. 75.—Skin Friction of Air.

For the case of air, the most reliable work appears to have been done by Zahm ;[2] who has made observations in an air tunnel 6 feet square, on smooth boards ranging from 2 to 16 feet in length, at velocities from 5 to 25 miles per hour. The results are similar to those obtained by Froude in water, in that the resistance per square foot diminishes with the length, and, for smooth surfaces, varies as $v^{1·85}$. The following results, corresponding to a velocity of 10 f.s., show that the resistances under similar conditions with short boards are approximately proportional to the densities of the two fluids. Thus for a smooth board (Froude's surface A) the resistance is 790 times as great in water as in air.

Length, feet.	2	4	8	12	16
Mean resistance, lbs. per square foot . . .	·000524	·000500	·000475	·000467	·000457

For strict comparison, experiments carried out with the same values of $\frac{vl}{\nu}$ should be considered. Thus, taking the ratio of ν for air and water as 13 : 1, a velocity of 10 f.s. with the 4-feet board in water would correspond to a velocity of 32·5 f.s. with the 16-feet board in air. Taking the resistance in

[1] "Inst. Naval Architects," March, 1898.
[2] "Phil. Mag.," vol. viii., 1904, pp. 58–66.

air as proportional to $v^{1\cdot85}$, the resistance per square foot of the 16-feet board at this speed is

$$\cdot000457 \times (3\cdot25)^{1\cdot85} = \cdot00402 \text{ lbs.}$$

and $R \div v^2 = \cdot00000380$.

The value of $R \div v^2$ for the 4-feet board in water is ·00325, the ratio of the two being 855, a value only about 4 per cent. greater than the relative density of water and air at 60° F.

In view of the fact that one set of experiments was carried out in still water and the other in an air current whose flow was not perfectly uniform, the agreement between the two sets of results is very close.

Art. 76.—Disc Friction.

The resistance to the rotation of a disc in water at high speeds is of importance in the design of centrifugal pumps and turbines, the energy expended in overcoming such resistances amounting in the case of some high-speed pumps to as much as 15 per cent. of the total energy given to the shaft.

Theory of Disc Resistance.

Assuming the resistance of an elementary ring of mean radius r and of radial width δr to be given by $f \cdot 2\pi r \delta r \cdot v^n$, where f is a coefficient of friction and v is the velocity of the element,[1] then if ω is the angular velocity of the disc in radians per second $v = \omega r$, and the moment of this resistance $= 2\pi f \omega^n r^{n+2} \delta r$. Then the moment of resistance of the two faces of a disc having a radius of R feet

$$= 4\pi f \omega^n \int_0^R r^{n+2} \, dr \text{ foot lbs.}$$

$$= \frac{4\pi f \omega^n}{n+3} \cdot R^{n+3} \text{ foot lbs.}$$

Assuming the resistance of the edge of the disc to follow the same law, and therefore to be given by $2\pi b \cdot f \omega^n R^{n+2}$, where b is the breadth in feet, the total moment of disc resistance, M, will be given by

$$M = 2\pi f \omega^n R^{n+2} \left\{ \frac{2R}{n+3} + b \right\} \text{ foot lbs.}$$

[1] The friction per square foot at a velocity v feet per second is, from this definition, $f v^n$ lbs.

Dr. W. C. Unwin has carried out a series of experiments on discs 10, 15 and 20 inches diameter rotating inside a casing whose side clearance could be varied from 1·5 inches to 6 inches.[1] The maximum speed attained in these experiments was about 470 revolutions per minute. More recent experiments by A. Ryan and the author have extended these speeds up to 2,200 revolutions per minute on discs of 9 and 12 inches diameter.[2]

In the apparatus used in the latter experiments a horizontal, motor-driven shaft carries the disc to be tested. The shaft passes through easily fitting bushes in the sides of a casing surrounding the disc, as shown in Fig. 94.

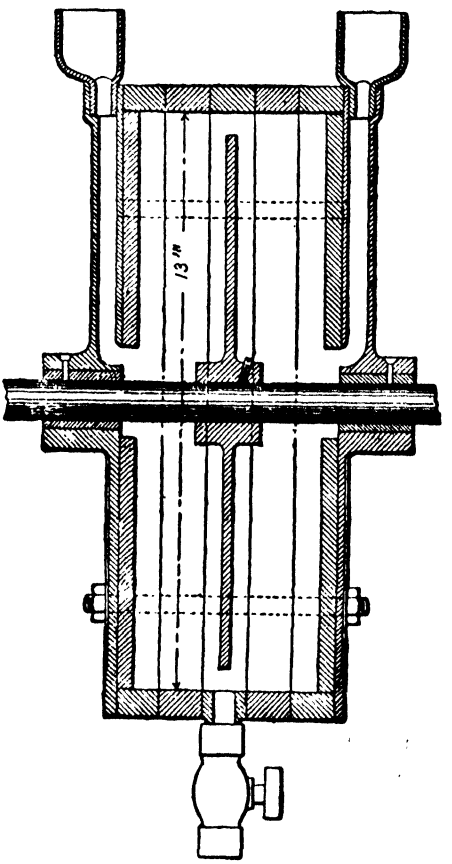


FIG. 94.

The two parallel sides of the casing are provided with adjustable plates with different surfaces, and are separated by a series of cast-iron rings 13 inches in internal diameter, these forming the body and enabling the side clearance to be varied at will from $\frac{1}{8}$ inch to $2\frac{1}{8}$ inches.

Six plain discs in all were examined, these being either 9 inches or 12 inches in external diameter and about ·2 inch thick. They were formed of polished brass, of rough cast-iron, and of rough cast-iron painted and varnished. The surface of the rough cast-iron discs was left as received from the foundry, except that all outstanding roughness was dressed off. These would be considered excellent castings and had a comparatively smooth skin. After being used they received two coats of quick-drying varnish-paint, and were then used for the third series of experiments. Each disc is carried on a central boss $1\frac{1}{4}$ inches in diameter and $\frac{3}{4}$ inch deep.

[1] "Proc. Inst. C.E." vol. 80, 1885, p. 221.
[2] "Proc. Inst. C.E.," vol. 179, 1909–10, pt. I.

The results throughout were very consistent, and there is no reason to suspect an error of more than about 2 per cent. in those of any series.

Variation of Disc Resistance with Temperature.—As it was recognised that it would be impossible to carry out a series of experiments without some variation in the mean temperature of the water, preliminary experiments were made to determine how such a variation affected the resistance.

A possible law of such variation may be deduced from purely theoretical considerations, the only assumptions made being that the resistance of each element of the rotating surface is proportional to the same power n of its velocity, and to some power of the viscosity μ, and of the density ρ of the fluid, both of which vary with temperature.[1]

On this assumption it may be shown that the resistance, other things being equal, is, as in the case of pipe flow, probably proportional to $\mu^{2-n}\rho^{n-1}$, where n is that power of the velocity to which the resistance at constant temperature is proportional.

For the sake of comparison the experimental results have been plotted in Fig. 95 against the curves representing the relationship, Resistance at $t°$ F. = Resistance at 65° F. $\times \left\{\frac{\mu}{\mu_{65}}\right\}^{2-n} \cdot \left\{\frac{\rho_t}{\rho_{65}}\right\}^{n-1}$, for corresponding values of n. From these it appears that the theoretical curves fit the experimental results remarkably closely, quite sufficiently closely indeed to justify the adoption of this formula.

To render the series more complete the theoretical curves for $n = 1{\cdot}9$ and $n = 2{\cdot}0$ have also been added.

The results show that the resistance diminishes with an increase in temperature, the amount of the variation with a given temperature-difference increasing as the temperature diminishes and also as n diminishes. Its value in the neighbourhood of 65° F. with a polished brass disc ($n = 1{\cdot}8$) is about one-third of 1 per cent. per degree Fahr. When $n = 1{\cdot}9$ this falls to one-seventh of 1 per cent. per degree Fahr., and when $n = 2{\cdot}0$ it becomes inappreciably small.

[1] By the theory of dimensions. The method is applied to pipe flow on p. 74.

Experimental Results.

The main results of the experiments at constant temperature are given in the table on p. 190. Speeds varying between 450 and 2,200 revolutions per minute were used for the determinations.

From these results it is apparent that in no case does n attain a value

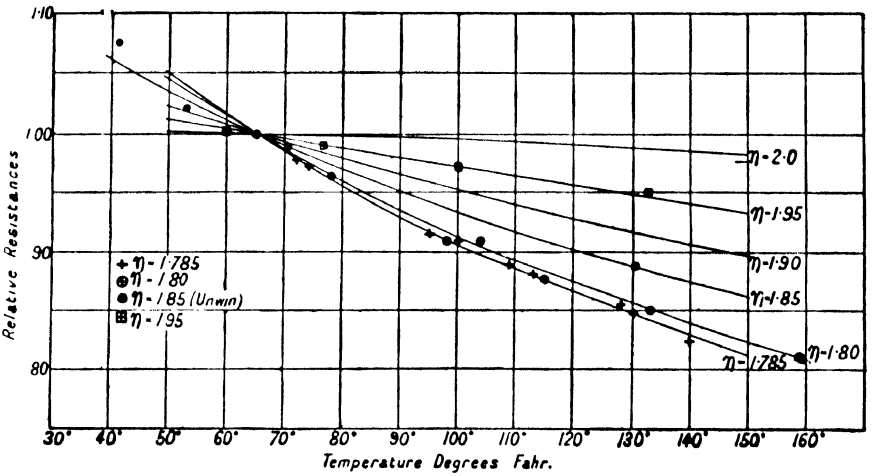


FIG. 95.—Variation of Disc Resistance with Temperature.

so high as 2, its maximum value, even with a rough cast-iron disc, being only 1·915. A comparison of these results with those of Dr. Unwin shows that in every case the values of n obtained by the latter experimenter, at speeds between 67 and 350 revolutions per minute, are higher than those obtained by the author at these greater speeds. The following table indicates the different values obtained with various discs :—

VALUES OF "n" WITH SMOOTH CASING.

—	Polished brass disc.	Painted and varnished cast-iron disc.	Rough cast-iron disc.
Unwin, 10 inch . .	1·85	1·94	2·00
Author, 12 inch . .	1·79	1·797	1·808
,, 9 inch . .	1·83	1·83	1·85

From this it would appear that n is not a constant for all speeds, but diminishes slightly as the speed increases.

RESULTS OF EXPERIMENTS.

		Side clearance.	Value of "n."	Value of "f."	Moment in foot lbs. at 1,500 revolutions per minute	Friction in lbs. per square foot at an *average* speed of	
						10 feet per sec.	50 feet per sec.
		Inches.					
Polished brass disc 12 inches in diameter	Rough cast-iron casing	$\frac{1}{8}$	1·800	0·00409	3·772	0·260	4·72
		$\frac{3}{8}$	1·800	0·00422	3·890	0·268	4·86
		$1\frac{1}{8}$	1·810	0·00414	3·950	0·269	4·98
		$1\frac{3}{8}$	1·800	0·00432	3·981	0·274	4·99
		$2\frac{1}{8}$	1·800	0·00474	4·367	0·301	5·46
	Painted cast-iron casing	$\frac{1}{8}$	1·792	0·00360	3·119	0·222	3·92
		$\frac{3}{8}$	1·797	0·00359	3·251	0·226	4·07
		$1\frac{1}{8}$	1·810	0·00356	3·396	0·231	4·27
		$1\frac{3}{8}$	1·800	0·00359	3·308	0·228	4·14
	Smooth metal casing	$\frac{1}{8}$	1·816	0·00346	3·311	0·227	4·22
		$\frac{3}{8}$	1·809	0·00373	3·459	0·239	4·40
		$1\frac{1}{8}$	1·785	0·00438	3·772	0·267	4·70
		$1\frac{3}{8}$	1·772	0·00421	3·428	0·251	4·37
		$2\frac{1}{8}$	1·766	0·00471	3·750	0·278	4·78
Ditto 9 inches diameter	Painted casing	$\frac{3}{8}$	1·830	0·00342	3·9053	0·232	4·42
Rough cast-iron disc 12 inches in diameter	Rough cast-iron casing	$\frac{3}{8}$	1·912	0·00300	4·385	0·247	5·31
		$1\frac{1}{8}$	1·915	0·00301	4·457	0·250	5·40
		$1\frac{3}{8}$	1·912	0·00297	4·345	0·244	5·25
		$2\frac{1}{8}$	1·915	0·00302	4·467	0·251	5·42
	Painted cast-iron casing	$\frac{3}{8}$	1·798	0·00428	3·886	0·270	4·85
		$1\frac{1}{8}$	1·807	0·00426	4·018	0·273	5·00
		$1\frac{3}{8}$	1·813	0·00424	4·098	0·278	5·11
		$2\frac{1}{8}$	1·813	0·00416	4·018	0·273	5·02
	Rough cast-iron casing with two concentric baffles ½ inch deep on each side	$\frac{3}{8}$	1·883	0·00337	4·365	0·261	5·38
		$1\frac{1}{8}$	1·883	0·00337	4·365	0·261	5·38
Ditto 9 inches diameter	Painted casing	$\frac{3}{8}$	1·850	0·00372	1·099	0·263	5·15
Painted and varnished cast-iron disc 12 inches diameter	Rough cast-iron casing	$\frac{3}{8}$	1·850	0·00353	3·959	0·249	4·90
	Painted cast-iron casing	$\frac{3}{8}$	1·795	0·00361	3·232	0·236	4·05
		$1\frac{1}{8}$	1·798	0·00374	3·354	0·234	4·21
Ditto 9 inches diameter	Painted casing	$\frac{3}{8}$	1·830	0·00340	0·8995	0·224	4·26
		Clearance over vanes					
12-inch brass disc with four radial vanes on each face, in painted casing	Vanes ½ inch deep	$\frac{1}{8}$	1·910	0·00903	13·00	0·730	15·77
		$\frac{3}{8}$	1·910	0·01067	15·35	0·861	18·60
	Vanes ¼ inch deep	$\frac{3}{16}$	1·950	0·00668	11·22	0·598	13·74
		$\frac{3}{8}$	1·950	0·00687	11 56	0·615	14·14

Power absorbed in Disc Resistance.—Since the resisting moment is given by $\frac{4\pi f\omega^n}{n+3} R_1^{n+3}$ foot lbs., the work absorbed in overcoming this resistance is given by $\frac{4\pi f\omega^{n+1}}{n+3} R_1^{n+3}$ foot lbs. per second

$$= \frac{4\pi f\left(\frac{\pi N}{30}\right)^{n+1}}{550\,(n+3)} R_1^{n+3} \text{ horse-power.}$$

The following table indicates the magnitude of the horse-power absorbed at 1,500 revolutions and at 2,000 revolutions per minute, in a few typical cases with a $\frac{5}{8}$ inch side clearance:—

HORSE-POWER ABSORBED.

—	12-inch polished brass disc.		12-inch rough cast-iron disc.		12-inch painted cast-iron disc.		9-inch polished brass disc, in painted casing.	12-inch brass disc with four radial vanes ½ inch deep on each face, in painted casing.
	In rough cast-iron casing.	In painted casing.	In rough cast-iron casing.	In painted casing.	In rough cast-iron casing.	In painted casing.		
HP. at 1,500 revolutions	1·11	0·928	1·25	1·11	1·13	0·923	0·258	4·38
HP. at 2,000 revolutions	2·49	2·07	2·9	2·49	2·57	2·06	0·584	10·12

The results are of interest as showing that the minimum resistance is found with the minimum side clearance. Also that the resistance of a smooth disc in a rough casing is sensibly the same as that of a rough disc in a smooth casing. The reason would appear to lie in the fact that as rotation occurs a strong circulatory current is set up, outwards over the surface of the disc and inwards over the face of the casing, where, its tangential velocity having been reduced by the roughness of the casing, it is again fed in to the disc. The formation of such currents is reduced by a reduction in the side clearance, and this accounts for the reduced frictional resistance accompanying such a reduction.

CHAPTER IX.

Pipe flow—Experimental Formulae—Darcy—Hagen—D'Aubuisson—Prony—Eytelwein—Weisbach—Kutter—Rational formula for pipe flow—Reynolds—Unwin—Lawton—Thrupp—Tutton—Values of f and C for various pipes—Fire hose—Resistance with oil—Sand—Mean velocity—Distribution of velocity—Measurement of discharge—Pitot tube—Relation of pipe diameter to volume discharged—Gradual and sudden stoppage of motion in an uniform pipe—Water hammer.

Art. 77.—Pipe Flow.

One very important effect of fluid friction is experienced in the resistance to the flow of water through a pipe. This resistance can only be overcome by a gradual fall of pressure in the liquid, in the direction of motion, and, reasoning from analogy to the resistance experienced by a plane surface moving through water, it might be inferred that, with sinuous motion, the total resistance R would equal $f S v^n$,

where $\begin{cases} f \text{ depends chiefly on the surface of the pipe and to a smaller extent on viscosity.} \\ S = \text{area of wetted surface.} \\ n \text{ depends on the pipe surface, and is approximately equal to 2.} \end{cases}$

Putting A = sectional area of pipe in square feet.

„ P = length of perimeter of pipe.

„ $p_1 - p_2$ = fall in pressure in lbs. per square foot over a length l feet of pipe.

This becomes

$$(p_1 - p_2)\, A = f\, P\, l\, v^n$$

$$\therefore \quad p_1 - p_2 = f \frac{P}{A} \cdot l\, v^n. \tag{1}$$

Here $\frac{A}{P} = \frac{\text{area}}{\text{perimeter}}$ is termed the **hydraulic mean depth**[1] and is commonly denoted by m, so that (1) may be written

$$p_1 - p_2 = \frac{f\, l\, v^n}{m}. \tag{2}$$

In the case of a circular pipe $m = \frac{\pi r^2}{2 \pi r} = \frac{r}{2} = \frac{d}{4}$, so that, if in

[1] If the mass of water in the pipe be imagined as distributed over a horizontal surface of the same area as the walls of the pipe, its depth will then be the same as the hydraulic mean depth for the pipe.

addition we express $p_1 - p_2$ as a difference h of head in feet of water, equation (2) becomes

$$h = \frac{f\, l\, v^n}{62{\cdot}4\ m} = \frac{f'\, l\, v^n}{d}. \qquad (3)$$

The analogy between the two cases is, however, not exact, in that, while with a solid moving through a large body of water any disturbance set up at the surface may be propagated over any unknown distance, becoming less marked as the distance from the solid increases and finally dying out altogether, any such disturbance in a pipe has a strictly limited range of extension, but is in general communicated to the whole mass of water in motion.

Where the motion through the pipe is everywhere steady, it is entirely governed by the law of simple viscous resistance and the conditions are accurately stated by the formulae of Art. 25. In turbulent flow $n = 2$. Since the dimensions of $\frac{f l v^2}{d}$ are $f \,.\, L \,.\, \left(\frac{L}{T}\right)^2 \,.\, \frac{1}{L} = f\left(\frac{L}{T}\right)^2$, in order to make f a constant applicable to any system of units, we must introduce a term of dimensions $T^2/L = T/v = acc^{-1}$ or $\frac{1}{g}$. The equation may now be written in the usual form

$$h = \frac{f l v^2}{2 g d} = \frac{4 f l v^2}{2 g m}$$

or in the form adopted by De Chezy $v = C\sqrt{\frac{m h}{l}}$. As explained on p. 49, there is in pipe flow a transitional stage after the critical point, over which the flow is only partially turbulent and in which the resistance is proportional to v^n where n is less than 2 and in which we now know that n varies with the Reynolds number R_e. In smooth pipes this stage may range over values of Re from 3000 to 20×10^6. In rough pipes the range is much less and depends on the degree of roughness. At higher velocities the v^2 law holds.

Many experimental researches have been carried out during the last 100 years on rough and smooth pipes and many empirical formulae have been proposed to fit the observed results. While these are valid over the range of their particular experiments, since many of them were carried out at velocities in the transitional range it is unsafe to extend their application much beyond their own range.

One of the earliest reliable series of experiments on pipe flow is that of

Darcy who, in 1857, from tests on cast-iron pipes having diameters from 0·5 inches to 20 inches deduced the relationship

$$h = \frac{f v^2 l}{2 g d}\left\{1 + \frac{1}{12 d}\right\}$$

where for clean surfaces $f = 0{\cdot}005$

,, ,, old incrusted surfaces $f = 0{\cdot}010$.

Hagen (1854) deduced from experiments by Bossut, Couplet and Dubuat the formula

$$h = \frac{f l v^{1{\cdot}75}}{d^{1{\cdot}25}}$$

but did not discover any law of variation of these indices or of f with the surface or diameter of the pipe. This appears to be the first of the exponential formulae.

As a matter of interest it may be noted that D'Aubuisson, Prony and Eytelwein assumed that the resistance to motion is composed of two parts, one due to simple frictional resistance at the boundaries and proportional to v, and the second due to eddy formation and proportional to v^2. Thus f in the standard formula became

$$f = a + \frac{b}{v}.$$

Their values are as follows :—

	a	b
D'Aubuisson .	·00670	·00121
Prony . .	·00684	·00112
Eytelwein .	·00549	·00114

Weisback, assuming that the first part of the resistance was proportional to $v^{\frac{3}{2}}$, made $f = {\cdot}0036 + \frac{{\cdot}00429}{\sqrt{v}}$.

Ganguillet and Kutter put

$$\frac{f}{2g} = \left\{\frac{1 + \left(41{\cdot}6 + \frac{{\cdot}00281}{h}\right)\frac{N}{\sqrt{m}}}{\frac{1{\cdot}811}{N} + 41{\cdot}6 + \frac{{\cdot}00281}{h}}\right\}^2$$

where N depends on the roughness of the pipe surface and varies from ·010 to ·019, its value for clean cast iron or asphalted pipes being ·013, for new riveted pipes ·014, and for old pipes ·019.

These formulae, excepting that of Kutter, neglecting as they do any change in resistance due to a change in the condition of the pipe surface, are obviously useless unless the experimental conditions can be accurately reproduced.

Kutter's formula, introducing as it does a roughness coefficient N, has a wider range of application. It does however assume the resistance to be proportional to v^2 and there is evidence to show that many of the experiments on which this formula is based were carried out on velocities within the transitional range.

Smooth Pipes.— For smooth pipes, after the transition point, the value of f in the formula $h = \frac{f l v^2}{2 g m}$ diminishes as R_e increases as indicated in the curve of Fig. 95A. For values of R_e between 3000 and 150,000 Davies and White [1] have shown that the empirical formula

$$f = 0{\cdot}08 \left(\frac{v d}{\gamma}\right)^{-{\cdot}25}$$

agrees well with the observed results. Beyond the range of R_e covered by Stanton and Pannell's experiments, in fact up to values of $R_e = 25 \times 10^6$, the curve is represented very closely by Prandtl's formula

$$\frac{1}{\sqrt{4f}} = 2 \log_{10} (R_e \sqrt{4f}) - 0{\cdot}8 \quad \text{where } R_e = \left(\frac{v d}{\gamma}\right)$$

Corresponding values of f and of R_e are shown in the following table :—

R_e	10^4	10^5	10^6	10^7	10^8
f	·0080	·0045	·0029	·0020	·00152

The results of experiments on a 60-inch pipe with a smooth sand cement lining [2] in which the value of R_e ranged up to 10^6 and on the 18-ft. Ontario power tunnel,[3] with its rubbed concrete surface, where R_e ranged up to

[1] Davies and White, *Engineering*, July 19, 1929, p. 71.
[2] Barnes, Inst. Water Engineers, 1933.
[3] Bulletin 852, Agricultural Dept. U.S.A. (F. C. Scobey).

$2{\cdot}6 \times 10^7$, show agreement within the errors of experimental measurement, with this formula.

Rough Pipes.—Our knowledge of the effect of surface roughness has been extended more recently, notably by the work of Nikuradse [1] and of Colebrook and White.[2] In the experiments of Nikuradse the surface of the pipe was artificially roughened by sand grains of selected sizes. Calling d/k the ratio of pipe diameter and of surface roughness, the results of these experiments

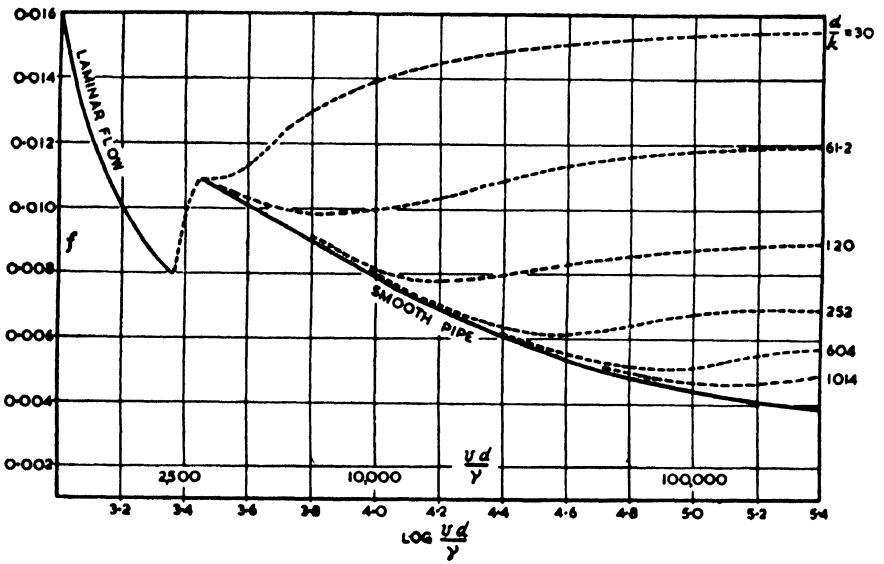


FIG. 95A.

are indicated in the curves of Fig. 95A. The values of f now depart from the smooth pipe law, the earliest departures being with the roughest surfaces. There is in each case a transition stage in which f increases with vd/γ, after which it tends to become constant, indicating that the v^2 law of resistance and full turbulence has been attained.

Nikuradse shows that the values of f for speeds at which the flow becomes truly turbulent are given by the formula

$$f = \frac{1}{4}\left(2 \log_{10} \frac{3{\cdot}7\, d}{K}\right)^{-2}.$$

[1] Nikuradse, J., *Forschungsh. Ver. dtsch. Ing.* No. 361, 1933. English translation in the *Petroleum Engineer*, May, 1940, p. 78.

[2] Colebrook and White, *Proc. Roy. Soc. A.*, clxi, 1937, p. 367.

These values, along with the corresponding values of the Reynolds number at which full turbulence occurs, are given in the following table. The last line shows the values of f for a smooth pipe.

$\frac{d}{k}$	30	60	120	250	500	1000
$\frac{v\,d}{\gamma}$	6×10^4	2×10^5	3×10^5	4×10^5	6×10^5	8×10^5
f	·0152	·0113	·0089	·0071	·0058	·0047
f (smooth)	·005	·004	·0036	·0034	·0032	·0030

Since the value of γ for water at 15° C. is $1{\cdot}23 \times 10^{-5}$ in ft. lb. sec. units, the values of $v\,d$ at which full turbulence is established with water at this temperature are as follow :—

$\frac{d}{k}$	30	60	120	250	500	1000
$v\,d$ for full turbulence . .	0·7	2·5	3·7	4·9	7·4	9·8

A difficulty in applying the foregoing results in practice arises from the fact that a rough pipe is not usually uniformly rough, especially in the case of a corroded surface. In experiments by Colebrook and White [1] variable roughnesses were imposed on different sections of a pipe. Starting with a pipe having uniform roughness in which $d/K = 153$, they found that by superposing grains of ten times this size on about 2 per cent. of the area, the resistance was increased by 12 per cent. at high velocities and by 20 per cent. when $R_e = 10{,}000$. This number of large grains alone increased the resistance as compared with the uniform roughening with the smaller grains, by 40 per cent. when $R_e = 10{,}000$, but reduced it by 7 per cent. when $R_e = 100{,}000$.

[1] Colebrook and White, " Experiments with Fluid Friction in Roughened Pipes," *Proc. Roy. Soc. A.*, clxi, 1937, p. 367. See also Colebrook, *Jour. Inst. C. E.*, April, 1939, p. 133, and Allen, *Jour. Inst. C. E.*, April, 1943, p. 91.

At the World Power Conference in 1933, Eric Lindquist presented a paper on Velocity Formulae for Open Channels and Pipes in which he dealt in detail with the work of previous investigators including the theoretical investigations of Prandtl, Blasius and von Karman. In 1936 the Institute of Civil Engineers set up a Committee on Velocity Formulae for Open Channels and Pipes. This committee has presented a report [1] having taken into account Lindquist's paper and all other available evidence.

As a matter of interest, their general conclusions are, briefly, as follows :—

(1) Prandtl and von Karman have, on theoretical grounds, with some experimental determination of the constants, established their "smooth pipe" formula

$$\frac{1}{\sqrt{4f}} = 2\log\frac{v\,d}{\gamma}\sqrt{4f} - 0{\cdot}8 = 2\log\frac{v\,d\sqrt{4f}}{2{\cdot}51\,\gamma} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where f is the coefficient in the formula $h = \dfrac{f\,l\,v^2}{2g\,m}$.

Writing $C^1 = \dfrac{1}{\sqrt{4f}} = \dfrac{C}{\sqrt{8g}}$ where C is the coefficient in the Chezy formula $v = C\sqrt{m\,i}$, formula (1) may be written

$$C^1 = 2\log\frac{d\sqrt{2g\,d\,i}}{2{\cdot}51\,\gamma} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

New commercial pipes with centrifugally spun linings obey the same law up to high values of the Reynold's number R_e. The effect of wall roughness first becomes noticeable when the size k of the larger asperities exceeds $2\log\dfrac{2{\cdot}25\,d}{k}$. For values of k up to twice this value, the divergence from the smooth pipe law is negligible.

Many commercial pipes, after a few years' service, obey the Prandtl-von Karman rough-pipe laws

$$C^1 = 2\log\frac{3{\cdot}7\,d}{k}.$$

Concrete pipes obey this law over the usual working range (velocity above 2 f.s. and diameters up to 6 feet).

[1] *Jour. Inst. C. E.*, No. 2, 1948–9, p. 173.

For the transition stage between the smooth pipe law and the fully rough-pipe law, the formula

$$C^1 = -2 \log \left[\frac{k}{3{\cdot}7\, d} + \frac{2{\cdot}51\, C^1}{R_e} \right]$$

give reasonable results for practical purposes.

Recent work shows that the effect of roughness depends on the shape and distribution of the asperities as well as on their size. While the theoretical investigation of Blasius, von Karman and Prandtl have thrown light on the general phenomena and mechanism of turbulence, it cannot be said that they have been of much direct help to the engineer faced with the problem of deciding what size of pipe to instal to give a required discharge under a given head.

Art. 78. Exponential Formulae.

Reynolds, using dimensional reasoning, showed (p. 75) that a rational formula for flow through similar pipes should be of the form

$$h = \frac{K\, v^n \gamma^{2-n}}{d^{3-n}} \cdot l \quad \ldots\ldots\ldots\ldots \quad (1)$$

where K depends on the roughness of the pipe wall. In this case the sum of the indices of v and of d is 3.

This, however, only holds for truly similar pipes in which the size of the asperities is proportional to the pipe diameter. If k be the size of the asperities dimensional reasoning leads to the relationship $h \propto \frac{v^n k^q}{d^{3-n+q}}$, at constant temperature, so that the sum of the indices, for pipes of different diameters but of the same wall roughness, unless very smooth, should be somewhat greater than 3.

Adopting the basic formula $h = \frac{f l v^n}{d^x}$, many observers have deduced values of f, x and n from their own and other experiments.

Professor Unwin,[1] from the results of experiments on pipes ranging from 2 inches to 48 inches diameter, obtained the following values :—

[1] "Industries," 1886, Vol. I, p. 51.

Surface.	Diam. of pipe (ins.).	f	x	n
Wrought iron . . .	2	·000351	1·210	1·75
Asphalted . . .	10 to 48	·000395	1·127	1·85
Riveted	10 to 26	·000405	1·390	1·87
New cast iron . . .	3 to 20	·000334	1·168	1·95
Cleaned cast iron . .	3 to 12	·000378	1·168	2·00
Incrusted cast iron . .	2 to 9	·000685	1·160	2·00

While Professor F. C. Lea (Hydraulics), including additional data from pipes from 3 to 72 inches diameter, and assuming a constant value of 1·25 for x, gives the following mean values for f and n :—

Surface.	f	n
New cast iron . . .	·00036	1·93
Old ,, ,, . . .	·00060	2·00
Riveted pipes . . .	·00050	2·00
Asphalted pipes . .	·00034	1·75
Wood	·00060	1·75

Other writers, such as Barnes,[1] Manning [2] and Williams and Hagen,[3] using much the same experimental data, have deduced their own values of f, n and x. As is to be expected, their formulae give within reasonable limits the same results, so long as the velocities do not greatly exceed those used in the experiments in question. Fortunately the velocities common in engineering practice are usually between 3 f.s. and 10 f.s., and within these limits the results may be adopted with some confidence.

As an example, the following comparative figures have been calculated for new cast-iron pipes, of diameter 12 inches and 36 inches and at different velocities.[4]

[1] "Hydraulic Flow Reviewed."
[2] "On the Flow of Water in Channels and Pipes."
[3] Hydraulic Tables by G. S. Williams and A. Hagen. John Wiley & Sons, New York, 1905.
[4] With modifications from. *Am. Soc. C.E.*, Oct. 1949, p. 1212.

				Diam. in inches.		
				12	36	36
				Vel f.s.		
	f	n	x	3·0	2·0	10·0
Barnes	·000343	1·953	1·172	2·9	0·37	8·5
Lea	·000360	1·93	1·25	3·0	0·35	7·8
Manning	·000344	2·00	1·33	3·1	0·32	8·0
Tutton	·000345	1·96	1·295	3·2	0·31	7·6
Unwin	·000334	1·95	1·168	2·8	0·36	8·2
Williams and Hagen	·000366	1·85	1·168	2·8	0·37	7·2

The last three columns give the loss of head per 1000 feet of pipe. For the 12-inch pipe at 3 f.s. the differences amount to ± 7 per cent. For the 36-inch pipe at 2 f.s., from + 7 per cent. to − 10 per cent., and at 10 f.s. from + 7 per cent. to − 9 per cent. Taking the averages, the Lea formula agrees almost exactly for both diameter and for all velocities.

Deterioration of Pipes with Age.—The internal fouling of a pipe due to corrosion or tuberculation, by increasing its roughness and reducing its internal area will generally, except with concrete pipes, reduce the value of f appreciably after a few years' use. Much depends on the alkalinity of the water and on the initial degree of protection of the walls.

Experiments by Herschell on a 4-feet riveted pipe consisting of alternate large and small rings with projecting rivet heads and coated with asphalt gave, when new, values of f ranging from 0·0065 at 1 f.s. to 0·0056 at 6 f.s. After four years the corresponding values of f were 0·0085 and 0·0065.

Experiments on a 38-inch riveted steel pipe [1] show that f increased from 0·0038 to 0·0079 after thirty years' service, while in a 7-inch cast-iron pipe f increased from 0·0053 to 0·0257 in the same time.

Tests [2] on a 72-inch bare riveted pipe with butt joints when new gave $f = 0·0052$ at all velocities from 1 to 4 f.s. After two years the values of f were 0·0098 at 1 f.s. and 0·0062 at 4 f.s.

Barnes [3] suggests that to allow for corrosion, etc., a pipe should be designed to give an initial discharge in excess of requirements. The following allowances are suggested for different types of pipe :—

[1] *Journal New England Waterworks Association*, 1935, p. 239.
[2] *Trans. Am. Soc. C.E.*, 1898 and 1900.
[3] "Hydraulic Flow Reviewed." Also *Proc. Inst. C. E.*, 1918–19, Part I, p. 1.

Type of pipe.	Uncoated C.I.	Asphalted C.I.	Asphalted riveted pipe.	Wood stave.	Cement or concrete.
Designed discharge	1·55 Q	1·45 Q	1·33 Q	1·08 Q	1·06 Q

In this connection a paper by Colebrook and White [1] on the reduction of the carrying capacity of pipes with age is of much interest.

Variation of Resistance with Temperature or with a change in the Kinematic Viscoscity

From the form of equation (1), p. 199, it appears that if n is less than 2 the resistance varies as ν^{2-n}. Experiments by the author on the resistance of rotating discs (p. 186) confirmed this and additional confirmation is obtained by experiments by Mr. Mair,[2] on a brass pipe in which $n = 1{\cdot}79$.

A somewhat striking confirmation of the validity of this equation was obtained by the author [3] by experiments on a copper pipe whose walls were corrugated circumferentially. The diameters at the top and bottom of the corrugation were 1·8 inches and 2·0 inches. Each corrugation was approximately in the form of a sine curve with a pitch of 0·4 inches. The tests showed the unexpected result that the resistance was proportional to $v^{2\cdot135}$. According to equation (1) the resistance should therefore decrease with an increase in viscosity. Experiments were made at different temperatures in which the viscosity was varied over a range of 3 : 1. These showed that the resistance was, in fact, reduced by an increase in viscosity and that, within the limits of experimental error, the reduction was in agreement with that deduced from theory.

For values of n in the neighbourhood of 2 the effect of temperature variation is very small. When $n = 1{\cdot}8$ the resistance alters about one-third of 1 per cent. per degree Fahrenheit, while when $n = 1{\cdot}9$ the change is one-seventh of 1 per cent. per degree.

For engineering calculations, however, it is convenient to be able to express resistance to flow in the forms

$$h = \frac{f l v^2}{2 g m} \text{ or } v = C\sqrt{m i}$$

and to enable this to be done with reasonable accuracy values of f and of C as deduced from the experimental results of many observers have been collected in the following tables. These give, within the range of velocities

[1] *Journal Inst. C. E.*, Nov. 1937, p. 99.
[2] *Proc. Inst. C. E.*, Vol. 84, 1886, p. 424.
[3] A. H. Gibson, *Phil. Mag.*, Vol. I, 1925, p. 199.

common in practice, their variation with v and with the pipe diameters, for pipes of different types.

MEAN VALUES OF f AND C, IN FORMULAE $h = \frac{f l v^2}{2 g m}$ AND $v = C \sqrt{m i}$.

LEAD.

Velocity, feet per second.		Diameter (inches).			
		1.	2.	3.	4.
2	f	·00793	·00747	·00721	·00705
	c	90	93	94·5	95·5
4	f	·00660	·00622	·00600	·00586
	c	99	101·5	103·5	105
6	f	·00603	·00568	·00549	·00535
	c	103·5	106·5	108·5	109·5
8	f	·00563	·00530	·00512	·00500
	c	107	110	112	113·5
10	f	·00530	·00500	·00482	·00471
	c	110	113·5	116	117

WOOD.

Velocity, feet per second.		Diameter (inches).					
		6.	12.	18.	24.	36.	48.
2	f	·00841	·00686	·00607	·00560	·00496	·00456
	c	87·5	97	103	107	113·5	119
4	f	·00817	·00667	·00590	·00543	·00483	·00442
	c	89	98·5	105	109	116	120·5
6	f	·00805	·00657	·00581	·00535	·00475	·00436
	c	89·5	99	105·5	109·5	116·5	121·5
8	f	·00797	·00650	·00575	·00530	·00470	·00431
	c	90	99·5	106	110	117	122·5

CEMENT OR ASPHALTED PIPE.

Velocity, feet per second.		Diameter (inches).						
		6.	9.	12.	18.	24.	36.	48.
2	f	·00657	·00623	·00601	·00570	·00550	·00521	·00503
	c	99	102	103·5	107	108	111	113·5
4	f	·00585	·00555	·00535	·00507	·00489	·00468	·00447
	c	105	108	109·5	112·5	115	117	120
6	f	·00550	·00520	·00502	·00476	·00459	·00435	·00420
	c	108	111·5	113·5	116·5	118·5	121·5	124
8	f	·00521	·00495	·00477	·00453	·00436	·00415	·00399
	c	111·5	114	116	119·5	121·5	124·5	127·5
10	f	·00506	·00480	·00463	·00439	·00423	·00401	·00387
	c	113	116	118	121	123·5	127	129

BARE WROUGHT IRON.

Velocity, feet per second.		Diameter (inches).						
		3	6	12	24	36	48	60
2	f	·00612	·00543	·00482	·00427	·00400	·00380	·00367
	c	102·5	109	116	123·5	127·5	130	132
4	f	·00530	·00471	·00417	·00370	·00347	·00329	·00319
	c	110	117	124	131·5	136·5	140·5	142
6	f	·00487	·00434	·00384	·00341	·00319	·00303	·00292
	c	115·5	122	129·5	137	142	146	148·5
8	f	·00459	·00408	·00361	·00321	·00300	·00285	·00275
	c	119	125·5	132	141·5	147	151	153·5
10	f	·00437	·00388	·00344	·00306	·00285	·00272	·00262
	c	121·5	129	137	145	151	154	157

RIVETED WROUGHT IRON OR STEEL PIPES.

Velocity, feet per second.		Diameter (inches).					
		12	24	36	48	60	72
2	f	·00615	·00495	·00437	·00400	·00372	·00351
	c	102	114	121·5	127·5	131·5	137
4	f	·00553	·00445	·00393	·00358	·00333	·00316
	c	108	120	128	132·5	138	143
6	f	·00520	·00419	·00369	·00338	·00315	·00297
	c	111·5	124	132·5	137	143	147·5
8	f	·00496	·00400	·00352	·00322	·00300	·00284
	c	114	127·5	135·5	141·5	147	151
10	f	·00481	·00388	·00342	·00312	·00291	·00275
	c	116	129	136·5	143·5	148·5	153·5

NEW CAST IRON.

Velocity, feet per second.		Diameter (inches).						
		3	6	9	12	18	24	36
2	f	·00691	·00588	·00535	·00500	·00455	·00425	·00387
	c	96·5	104·5	109·5	113·5	119·5	123·5	129
4	f	·00654	·00556	·00505	·00473	·00430	·00403	·00367
	c	99	108	113	117	122·5	127·5	133
6	f	·00635	·00539	·00490	·00459	·00417	·00390	·00356
	c	100·5	109	114·5	119	124·5	129	134·5
8	f	·00617	·00525	·00479	·00447	·00407	·00380	·00346
	c	102	111	117	119·5	125·5	130	136

CLEANED CAST IRON.

At all Velocities.	Diameter (inches).						
	3	6	9	12	18	24	36
f	·00770	·00685	·00640	·00609	·00569	·00541	·00505
c	91·5	97	100·5	103	106·5	109	113

OLD, LIGHTLY TUBERCULATED CAST IRON.

Velocity, feet per second.		Diameter (inches).						
		3	6	9	12	18	24	36
2	f	·0147	·0126	·0115	0108	·00983	·00921	·00835
	c	66	71·5	75·5	77·2	81	83·7	88
4	f	·0146	·0125	·0113	·0106	·00965	·00905	·00825
	c	66·3	72	76	77·7	81·5	84·1	88·5
6	f	·0145	·0124	·0112	·0105	·00958	·00897	·00817
	c	66·6	72·3	76·4	78·2	82	84·5	89
8	f	·0144	·0122	·0111	·0104	·00950	·00890	·00810
	c	67	72·6	76·8	78·7	82·4	85·0	89·5

A series of experiments by Freeman [1] on the flow through fire hose, give the following mean values of f, at velocities ranging from 10 to 30 feet per second.

Unlined linen hose . . . $f = ·0084$

Smooth rubber lined hose . $f = ·0044$

It is probable that at such velocities as are common in practice, *viz.*, about 100 feet per second, these values would become about ·0080 for the unlined, and ·0040 for the rubber-lined hose.

Mr. R. A. Barnes, in a recent discussion of experiments by himself and other observers, gives the following values as applying to new and cleaned pipes :— [2]

[1] "Transactions American Society Civil Engineers," 1889, p. 303.

[2] Barnes, "Hydraulic Flow Reviewed," Spon, 1916.

Material.	Formulae for		
	Mean Vel., f.s. $v =$	Friction Head, h ft. $h =$	Q, c.f.s. $Q =$
New uncoated cast-iron pipes . . .	$136{\cdot}6\ m^{\cdot 60}\ i^{\cdot 51}$	$\cdot 000343\ \frac{l\ v^{1\cdot 953}}{d^{1\cdot 172}}$	$46{\cdot}7\ d^{2\cdot 60} \times i^{\cdot 512}$
New asphalted cast-iron	$174{\cdot}1\ m^{\cdot 769}\ i^{\cdot 529}$	$\cdot 000136\ \frac{l\ v^{1\cdot 891}}{d^{1\cdot 454}}$	$47{\cdot}1\ d^{2\cdot 769} \times i^{\cdot 529}$
New asphalted single-riveted wrought-iron and steel pipes .	$171{\cdot}4\ m^{\cdot 723}\ i^{\cdot 527}$	$\cdot 000386\ \frac{l\ v^{1\cdot 898}}{d^{1\cdot 372}}$	$49{\cdot}4\ d^{2\cdot 723} \times i^{\cdot 527}$
Do., double-riveted with taper or cylinder joints	$129{\cdot}9\ m^{\cdot 44}\ i^{\cdot 52}$	$\cdot 000279\ \frac{l\ v^{1\cdot 923}}{d^{1\cdot 346}}$	$55{\cdot}4\ d^{2\cdot 44} \times i^{\cdot 52}$
New smooth wood-stave pipes . . .	$223{\cdot}3\ m^{\cdot 66}\ i^{\cdot 586}$	$\cdot 00047\ \frac{l\ v^{1\cdot 707}}{d^{1\cdot 126}}$	$71{\cdot}3\ d^{2\cdot 66} \times i^{\cdot 586}$
New unplaned wood-stave pipes . .	$182{\cdot}5\ m^{\cdot 666}\ i^{\cdot 569}$	$\cdot 000541\ \frac{l\ v^{1\cdot 757}}{d^{1\cdot 171}}$	$58{\cdot}5\ d^{2\cdot 666} \times i^{\cdot 569}$
Clean neat cement pipes	$136{\cdot}3\ m^{\cdot 635}\ i^{\cdot 484}$	$\cdot 00024\ \frac{l\ v^{2\cdot 066}}{d^{1\cdot 312}}$	$42{\cdot}0\ d^{2\cdot 635} \times i^{\cdot 484}$

ART. 79.

More recent investigations indicate that the function in formula 12, p. 196,

$$\frac{h\,d}{l\,v^2} = \phi\left(\frac{\nu}{v\,d}\right)$$

is very complex, being of the form $A\left(\frac{\nu}{v\,d}\right)^n + B$,[1] where A, B and n are constant for all similar pipes, *i.e.*, for pipes whose ratio of surface roughness

[1] C. H. Lees, "Proc. Roy. Soc.," A., vol. 91, 1914, p. 46.

to diameter is the same. A, B and n all increase with the relative roughness of the surface.

It is, however, not necessary to know the form of the function ϕ in order to make use of this relationship, for if experiments be carried out on any one pipe at different velocities of flow, and if a curve be plotted having values of $\frac{h\,d}{l\,v^2}$, as ordinates and $\nu/v\,d$ as abscissae, corresponding points obtained from any *similar* pipe of any diameter will lie on the same curve.

It is to be noted that pipes of the same surface but of different diameters

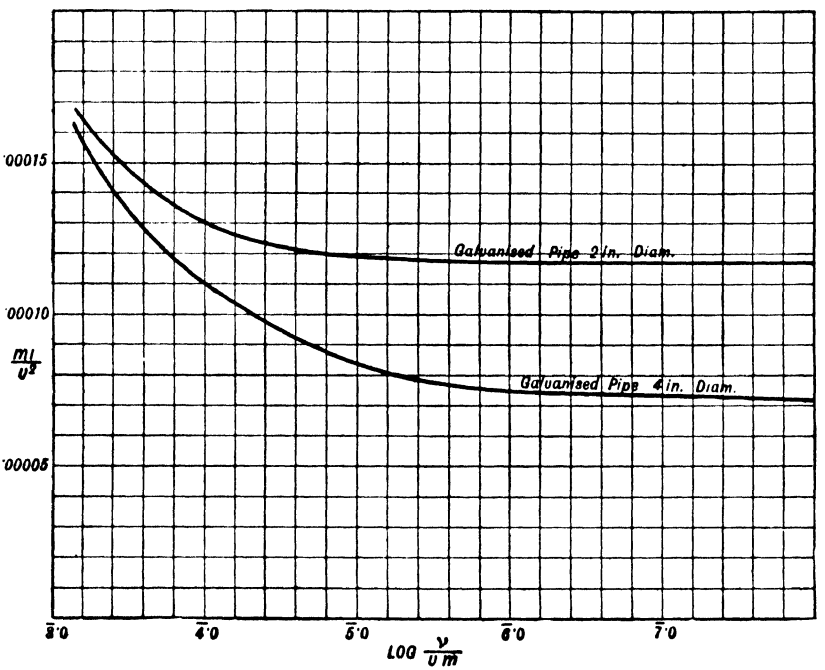


FIG. 95B.

are not similar, and the difficulty in applying this method of investigation lies in fixing any satisfactory scale of roughness. If, for example, it were determined that a 1-inch drawn brass pipe had the same relative roughness as a 6-inch drawn steel pipe, a 48-inch concrete pipe, and a 120-inch riveted steel pipe, the curve drawn for the 1-inch brass pipe would serve equally well for each of the others.

While attempts have been made to deduce such a scale of roughness, the

results are not as yet sufficiently definite to be of great practical value. An investigation now proceeding in the Author's laboratory indicates that if roughness of the surface of a galvanised iron pipe be taken as unity, that of other surfaces is approximately as follows :—[1]

New uncoated cast iron pipes	1·40
New asphalted cast iron pipes	3·55
New wood stave pipes	5·65
Concrete pipe very carefully hand-finished . .	1·50
Concrete pipes with ordinary good finish . .	6·00
Rough concrete pipes	18·50

Fig. 95B shows curves obtained from a 2-inch and a 4-inch galvanised pipe, the abscissae being log $v/v\,m$. The curve for the 4-inch pipe would therefore apply equally well to a cast-iron pipe, 5·6 inches diameter, a wood-stave pipe, 22·6 inches diameter, or a rough concrete pipe, 74 inches diameter.

Adopting the formula

$$\frac{m\,i}{v^2} = \phi\left(\frac{\nu}{v\,d}\right) = A\left(\frac{\nu}{v\,d}\right)^n + B$$

where m = the hydraulic mean radius and $i = h/l$, Lees obtained the values $a = \cdot 35$; $A = \cdot 001463$; $B = \cdot 00028$, for smooth drawn brass pipes.[2] Although m varied from ·0103 foot to ·1036 foot in the experiments A, B, and n were apparently constant over the whole range, indicating that the degree of roughness of the surface was so small that the pipes, although having the same surface, were sensibly similar.

For rough pipes, however, A, B, and n all vary with the diameters. Thus the following are the values for 2-inch and 4-inch galvanised iron pipes.

	A	B	n
2 inches . .	·1455	·000117	·757
4 inches . .	·0783	·000070	·455

[1] See also papers by Mr. Evan Parry, "Proc. New Zealand Society of Civil Engineers," vol. iii., 1917, and "English Electric Journal," Oct., 1920.

[2] In foot pound second units.

Since the physical condition of a pipe surface and any deviation from straightness of the pipe vitally affect the resistance, and since the determination of the relative physical condition of two surfaces, even though of the same material, is a matter of the greatest difficulty, no pipe-flow formula is to be relied upon as giving the discharge corresponding to a given loss of head, with any great degree of accuracy in a proposed pipe line. The difficulty in exactly reproducing the conditions of the experiments from which the formulae have been deduced prevents this, and the engineer who predicts to within 10 per cent. the discharge from a pipe under given pressure conditions has every reason to be satisfied with his choice of constants.

Art. 80.—Most Economical Diameter for a Power Pipe Line—Working Velocities.

Other things being equal, the best diameter will be that which leads to the minimum over-all cost of operation and maintenance.

The annual cost of the supply pipe of a power scheme is the sum of the interest charges on the capital cost, the maintenance and depreciation, together with the value of the power lost due to pipe friction.

Taking interest, maintenance, and depreciation as proportional to the capital cost £C, and equal to nC per annum, if Q be the normal rate of flow in c.f.s. :—

$$Q = \frac{\pi d^2}{4} \cdot v; \quad h_f = \frac{4 f l v^2}{2 g d}$$

$$\therefore h_f = \frac{4 f l}{2 g d} \times \frac{16 Q^2}{\pi^2 d^4} = \frac{f l Q^2}{10 d^5} \text{ (approx.)}$$

$$\therefore \text{h.p. lost} = \frac{Q \times 62{\cdot}4 \times h_f}{550} \cdot \eta = \frac{62{\cdot}4 f l Q^3}{5500 d^5} \eta = P,$$

where η is the efficiency of the turbine.

Then, if the value per h.p. is £m per annum, the effective cost of the pipe line per annum

$$n C + m P$$

and the diameter should be such as to make this a minimum. If the cost of the pipe line is $K l d^x$:—

$$n K l d^x + \frac{62{\cdot}4 f l Q^3}{5500 d^5} \cdot \eta m$$

is to be a minimum, and differentiating with respect to d and equating to zero :—

$$n\,K\,x\,d^{x-1} = \frac{62{\cdot}4\,f\,Q^3\,\eta\,m}{1100\,d^6}$$

$$\therefore \quad d^{5+x} = \frac{62{\cdot}4\,f\,Q^3\,\eta\,m}{1100\,n\,K\,x}$$

For steel pipe lines, the cost of the pipe, including erection, appears to be roughly proportional to d^3, in which case if $\eta = {\cdot}80$:—

$$d^{6{\cdot}5} = \frac{f\,Q^3\,m}{33\,.\,n\,K}.$$

As an example, let $f = {\cdot}0055$, $Q = 100$ c.f.s., $m = £4$, $n = {\cdot}065$, $K = £1{\cdot}0$,

Then $$d^{6{\cdot}5} = \frac{{\cdot}0055 \times 10^6 \times 4}{33 \times {\cdot}065} = 10250$$

$$\therefore \quad d = 4{\cdot}14 \text{ feet.}$$

The area of this pipe is 13·4 square feet, making the velocity 7·45 f.s. If $Q = 10$ c.f.s., $d = 1{\cdot}43$ feet, and the velocity becomes 6·35 f.s.

Since the cost of the pipe line, and therefore K, increases for a given diameter with the working head, the most economical velocity will generally increase with the head, and where, owing to special circumstances, the cost of the supply conduit is relatively high, as when it consists of a pressure tunnel excavated through rock and lined, the most economical velocity may be as high as 15 feet per second.

In practice it is often necessary to adopt lower working velocities because the difficulties of speed regulation with a long pipe line increase with the velocity of flow. This effect becomes less important as the length of the supply pipe is diminished, and with comparatively short pipes velocities up to 18 f.s. are not uncommon. In a normal power installation they range from about 5 f.s. to 9 f.s., while in systems for domestic supply velocities from 3 f.s. to 6 f.s. are usual.

Art. 81.—Friction with Fluids other than Water.

Since the formula

$$\frac{h\,d}{l\,v^2} = \phi\left(\frac{\nu}{v\,d}\right)$$

is true for all liquids, it follows that if the loss of head accompanying the flow of water at different velocities through a given pipe be known, so that a

curve can be plotted showing values of $\frac{h\,d}{l\,v^2}$ against values of $\frac{\nu}{v\,d}$, this curve will apply equally well to a fluid of any other known viscosity. Thus the loss of head due to the flow of oil of known viscosity through a pipe line can readily be obtained if the loss of head corresponding to a given velocity of flow of water is known. In the case of oil flow, however, owing to the high viscosity, the flow is usually stream line at the velocities and range of pipe diameters commonly used, in which case Poiseuille's formula is directly applicable. With crude oils the loss of head is very heavy, owing to the great viscosity. Attempts have been made to reduce this loss of head by the addition of some 10 per cent. of water. This reduces the loss considerably but the discharge takes the form of an emulsion from which the water is removed with difficulty. By using a pipe having rifled grooves along its walls, centrifugal action keeps the heavier water in contact with the walls, and reduces both emulsification and loss of head. Tests made by the Southern Pacific Railroad Co.,[1] give the following values of C in the formula $v = C\sqrt{mi}$.

8-in. plain pipe conveying	oil	... ...	$C = 5{\cdot}46$
8 „ „ „ „	90 % oil and 10 %	water	$C = 7{\cdot}11$
8 „ rifled „ „	90 % oil and 10 %	„	$C = 65$
8 „ pipe „	water only	... ...	$C = 113$
8 „ plain „ „	oil	... ...	$C = 3{\cdot}76$
8 „ rifled „ „	90 % oil and 10 %	water	$C = 79$
8 „ pipe „	water only	... ...	$C = 100$

Water Charged with Sand.—When a stream of water flows over a mass of loose material which may be carried in suspension, at moderate velocities of flow the material is rolled along the bottom of the pipe or channel, with comparatively great loss of head. As the velocity increases, at a certain point, depending on the size and weight of the particles, these are picked up and carried in suspension in the water. Experiments on sand, having grains varying from ·16 m.m. to ·75 m.m. in diameter show that for a 1-inch pipe this velocity is about 3·5 feet per second, while for a 3-inch pipe it is about 4 feet per second, and for a 32-inch pipe about 9 feet per second.[2] With grains of approximately uniform size the minimum resistance is attained with this velocity. With grains of widely varying size the minimum resistance is attained with a velocity somewhat less than is necessary to pick up the larger grains. With fine sand the additional loss of head is about 25 per cent. of that due to the water alone, for each 1 per cent. of added sand. For velocities higher

[1] *Engineering News*, New York, June 7, 1906. *Engineering Record*, May 23, 1908.

[2] "Transactions American Society Civil Engineers," Vol. 57, 1906.

than are necessary to cause suspension, the proportion of solid matter which may be carried is approximately independent of the velocity. Experiments show that the proportion of sand to water may exceed 50 per cent. Where a sand ejector is used this proportion depends largely on the form of ejector.[1] More recent experiments [2] on rifled pipes delivering a mixture of water with sand, gravel, silt, or clay, indicate that rifling reduces the resistance up to about 20 per cent. with coarse sand or gravel, but increases it with silt or clay.

Mixtures of Water and Air.—See Art. 215.

Art. 82.—Variation of Velocity over the Cross-Section of a Cylindrical Pipe.

Experiments show that with sinuous motion through a pipe the velocity of flow is much greater at the centre than near the walls. From experiments on pipes with diameters ranging from 8 inches to 19 inches Darcy deduced the formula

$$v = v_{max} - \frac{k}{a} x^{\frac{3}{2}} \sqrt{i} \quad . \quad (1)$$

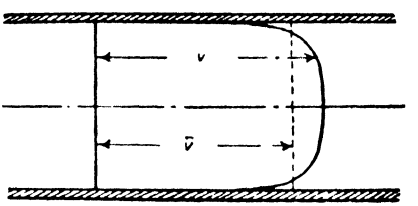


Fig. 96.

as giving the velocity v at any radius x of a pipe of radius a. When the unit of length is the metre, $k = 11{\cdot}3$, and when the foot, $k = 20{\cdot}4$. The volume of the solid of revolution (Fig. 96), whose height represents the velocity at any radius, is given by

$$\therefore \text{ mean height} = \frac{\int_0^a 2\pi xv dx}{\pi a^2} = \frac{2\pi \int_0^a xv dx}{\pi a^2}$$

Substituting for v from equation (1) we have

$$\text{mean velocity, } \bar{v} = v_{max} - \frac{4}{7} k \sqrt{ai}.$$

$$\therefore \left(\frac{x}{a}\right)^{\frac{3}{2}} = \frac{4}{7}$$

$$\therefore x = {\cdot}689\, a$$

[1] "Transactions American Society Civil Engineers," Vol. 57, 1906.

[2] *Expt. Station Bulletin*, Vol. 2, No. 1, Feb. 1939, pp. 1–5, "Rifling in Dredge Pipe Lines." Also G. W. Horner, *Proc. Am. Soc. C. E.*, 1940, p. 1324.

giving the radius at which the velocity is equal to the mean over a cross-section.

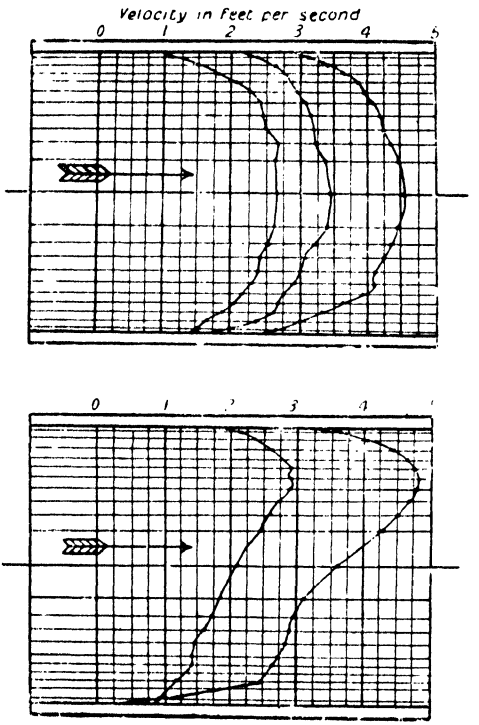


FIG. 97.

More recent experiments by Bazin[1] on a cement pipe 2·63 feet diameter, and by Messrs. Williams, Hubbell, and Fenkell[2] on cast iron asphalted pipes having diameters of 12″, 16″, 30″, and 42″, indicate that the curve of velocities is very nearly an ellipse to which the pipe walls are tangential. Calling u the minimum velocity at the walls, and V the maximum velocity at the centre, the equation to the curve would then be

$$\frac{x^2}{a^2} + \left(\frac{v-u}{V-u}\right)^2 = 1.$$

The volume of the solid of revolution bounded by this curve is equal to

$$\pi a^2 u \times \tfrac{2}{3} \pi a^2 (V-u) = \pi a^2 \bar{v}$$

$$\therefore \bar{v} = u + \tfrac{2}{3}(V-u).$$

The mean velocity occurs at a radius $x = \cdot 75\, a$.

Messrs. Williams, Hubbell, and Fenkell estimated the surface velocity to be practically one-half that at the centre,[3] in which case the mean velocity is ·83 times the maximum. The ratio was found to increase with the mean velocity. Figs. 97*a* and 97*b* show velocity curves obtained on the 30-inch pipe respectively on the straight and at a bend in these experiments.

The results of Bazin's experiments can be represented very approximately by the formula

$$v = V - k\left(\frac{x}{a}\right)^3 \sqrt{ai}$$

[1] "Mem. de l'Académie des Sciences," xxxii. 1897.
[2] "Proc. Am. Soc. C. E.," vol. xxvii., 1901, p. 314.
[3] See the discussion on surface velocity on p. 214.

where k has a mean value, with the foot as unit, of 38, though a more accurate formula is

$$v = V - 38 \left\{ \left(\frac{x}{a}\right)^3 + 1{\cdot}29 \left(\frac{x}{a}\right)^2 \left(1 - 1{\cdot}1 \frac{x}{a}\right)^2 \right\} \sqrt{a\,i}$$

The mean results of these experiments gave $u = {\cdot}51\ V$; $\bar{v} = {\cdot}855\ V$; and showed the mean velocity to be attained at a radius ${\cdot}74\,a$.

Assuming $\bar{v} = C\sqrt{mi} = {\cdot}707\ C\sqrt{ai}$, from Bazin's simpler formula assuming $x = {\cdot}74\ a$, we have

$$\bar{v} = V - 15{\cdot}4\sqrt{ai}$$
$$\therefore \frac{V}{\bar{v}} = 1 + \frac{21{\cdot}8}{C}$$

The value $\bar{v} = {\cdot}855\ V$, thus corresponds to a value of $C = 128$. It is indeed to be expected that as here indicated, the ratio of mean to maximum velocity will vary with the diameter and surface conditions of the pipe, both of which affect C. Experimentally the ratio is found to range from ·79 to ·86, increasing with the diameter and smoothness of the pipe, the following table indicating how, from Bazin's simple formula, this ratio varies with C or with the value of f.

C	80	90	100	110	120	130	140
f	·0105	·0079	·0064	·0053	·0045	·0038	·0033
$\bar{v} \div V$	·785	·804	·820	·834	·846	·857	·865

Other experiments have been carried out by Cole[1] and by Morrow,[2] from which, together with those already quoted, it may be concluded that in gauging the discharge of a pipe by means of a Pitot tube (p. 217), the mean of observations at a radius varying from ·69 a in small, to ·75 a in

[1] "Trans. Am. Soc. C. E.," 1902, vol. 47, pp. 63—276.
[2] "Proc. Roy. Soc.," 1905, vol. 76, p. 205.

large pipes, will give the mean velocity directly, while the mean of observations at the centre of the pipe when multiplied by a constant varying from ·79 to ·86, with an average value of ·84, will also give the mean velocity with a fair degree of accuracy.

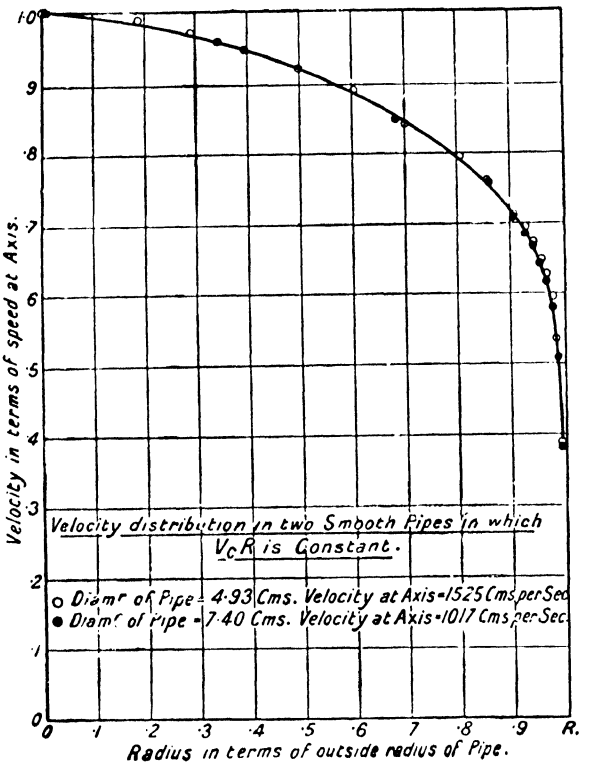


FIG. 98.

Experiments by R. Threlfall,[1] on the flow of air through gas pipes from 6 to 36 inches diameter, indicate that here the radius of the circle of mean velocity is about 0·775 of the pipe radius.

The ratio of mean to maximum velocity in these experiments was very constant over a wide range of velocities — 22 *f.s.* to 41 *f.s.*—and had the mean value ·873.

Experiments by Michael Longridge [2] on air flow through a 16-inch pipe at velocities ranging from 77 to 210 feet per second showed that the ratio of mean to maximum velocity varied by less than 1½ per cent. from its mean value 0·921. The radius of mean velocity in these experiments was ·80 of the pipe radius.

Surface Velocity.—Since the Pitot tube, with which velocities are invariably measured, cannot be used to determine velocities at points nearer to the wall than the radius of its orifice, the manner in which the velocity varies at points very near the wall is difficult to determine with any degree of accuracy. By producing the curve obtained by plotting

[1] "Proceedings Institute Mechanical Engineers," 1904.

[2] "Engineer's Report of Brit. Engine and Boiler Insurance Co., Ltd., for 1910," p. 92.

velocities across the pipe, the earlier experimenters concluded that the surface velocity was approximately one-half the maximum velocity. More recent experiments by Morrow on a pipe 2 inches in diameter, and by Stanton[1] on the flow of air through pipes of 5 and 7·4 cmm. diameter, however, indicate that the velocity of the film in actual contact with the wall is zero, even with sinuous flow, and that the velocity at first increases very rapidly as the centre of the pipe is approached. The film of fluid in the immediate neighbourhood of the wall appears to be moving with steady viscous flow, this state of viscous flow merging almost imperceptibly into that of sinuous flow, the thickness of the film undergoing rectilinear motion being extremely small.

In the latter experiments a Pitot tube having an orifice only ·25 millimetre deep was used. In rough pipes (resistance proportional to v^2) the velocity curve was found to be similar from the centre to within 3 millimetres of the walls, for all velocities, and to be a parabola, having the equation

$$v = V_c - A\,r^2.$$

For smooth pipes the curve was also parabolic up to a radius of $0{\cdot}8\,a$. For exact similarity of the curves over the whole range of radii, it is necessary that the central velocities should be inversely proportional to the pipe diameters. Fig. 98 shows velocity curves obtained in these experiments.[2]

Art. 83 —Mixing of Adjacent Layers During Flow of Liquid in Pipes.

From the fact that the velocity near the centre of a pipe is much greater than near the walls, it is evident that there must be a continual admixture of the faster moving liquid with those more slowly moving particles nearer the walls ahead of it. Thus the particles of a column of the fluid, originally of unit length, will, at any subsequent time, be found to be distributed over a length greater than that which they originally occupied. This matter is of some importance in connection with oil transmission pipe lines, through which from time to time different grades of oil are to be pumped, and in which it is desirable to know the probable volume of contaminated oil likely to follow a change over from grade to grade.

In experiments on a wooden pipe line conveying water and consisting of a length of 13,200 feet of 10-inch pipe in series with 47,500 feet

[1] "Proc. Roy. Society," A., vol. 85, 1911, p. 366; also vol. 97, p. 413.

[2] In the above formula V_c is the velocity at the centre of the pipe.

of 16-inch pipe,[1] aniline dye or bran was admitted for a measured time to the intake, and the discharge from a relief valve on the 10-inch pipe, situated 12,700 feet from the intake, and from the outlet from the 16-inch pipe, was carefully watched for the appearance and disappearance of the colouring matter or bran. With the dye the exact instant of appearance and disappearance, particularly the latter, could not be determined with any very great accuracy, but the time of passing the relief valve was apparently about one minute greater than the time of admission. As the mean velocity was 582 feet per minute, this means that the mixing had extended over a length equal to $58,200 \div 12,750 =$ 4·6 per cent. of the distance traversed. The bran apparently took eighteen minutes longer than the time of admission to pass the outlet from the 16-inch pipe, indicating an admixture over about 8·5 per cent. of the distance traversed. Experiments carried out with colouring matter on a 4-inch and a 6-inch pipe line at Cornell[2], conveying water at velocities ranging between about ·9 and 4·5 feet per second showed that the admixture extended over a mean distance equal to about 13·3 per cent. of the distance traversed in the 6-inch pipe, and 11 per cent. of this distance in the 4-inch pipe. Both these pipes were spiral riveted, but the 4-inch pipe had an asphalted surface. On the whole it would appear that the percentage distance for similar pipes diminishes with an increase in diameter, and for pipes of the same diameter increases with the roughness, apparently varying little with the mean velocity.

Experiments by the Standard Oil Co.[2] on the admixture of oil of different grades in a 6-inch pipe line, 24 miles long, showed that with light amber oil, specific gravity ·79, displacing heavy black oil, specific gravity ·85, with a mean velocity of flow of about 4 feet per second, the length of admixture was between 8·5 and 12·1 per cent. of the distance travelled, while, where the heavy displaced the light oil, the length was only ·44 per cent. Similar experiments on an oil-pipe line 210 miles long between Morgantown and Millway, with light oil displacing heavy oil, showed a length of admixture varying from 10 to 12 per cent.

Art. 84.—Measurement of Pipe Discharge.

The volume being discharged by a pipe may be determined in several ways.

(1) The most accurate method consists in collecting and weighing the

[1] J. L. Campbell, *Engineering News*, August 27, 1908, p. 227.

[2] *Cornell Civil Engineer*, Dec. 1911, p. 122, by Prof. E. W. Schoder.

quantity discharged in a given time. This is, in general, only possible with small volumes.

(2) The **Venturi meter** (Art. 225) affords a simple method of measuring the flow, and one which is capable of giving excellent results. When fitted to a pipe line of diameter exceeding about 2 inches its indications are, under normal conditions, thoroughly reliable so long as the velocity exceeds 1 foot per second, and the discharge may be predicted, even without calibration, to within about 1·5 per cent.

The orifice diaphragm (Art. 55) may be used in place of a Venturi meter.

(3) The **Pitot Tube.**—For measurements of the flow in pipes or in unconfined streams, where the velocity is fairly high, the Pitot tube is capable of giving good results. The instrument usually consists of a bent tube terminating in a small orifice pointing up-stream, which is surrounded by a second tube whose direction is parallel to that of flow. A series of small holes in the wall of the outer tube admits water, at the mean statical pressure in their vicinity, to its interior, which is connected to one leg of a manometer. The other leg is connected to the central tube carrying the impact orifice. If v is the velocity of flow immediately up-stream from this orifice, the pressure inside the orifice where the velocity is zero is equal to the sum of the statical pressure at the point, plus $C\,v^2/2g$ feet of water, where c is a constant whose value approximates closely to unity in a well-designed tube. It follows that the difference of level in the two legs of the manometer equals $C\,v^2/2g$.

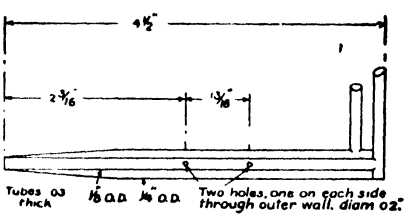


Fig. 99.

Figs. 99 to 102 show types of Pitot tube. Fig. 99 shows the type used for measuring the air speed of aeroplanes, and for wind tunnel investigations. A tube of this type, having the dimensions shown, gave $C = 1{\cdot}00$ within 1 per cent.

The tube illustrated in Fig. 100 gave a value of $C = {\cdot}926$ when rated by towing through still water and ·895 when calibrated in a 2-inch pipe. The low value of C in still water is probably due to the fact that the pressure orifices are too near the shoulder of the pressure tube If this were lengthened with the orifices further back, the coefficient would probably be higher. The difference between the calibration in still water and in the small pipe is to be expected, since the velocity at the section of the pipe containing the pressure orifices is of necessity increased by the presence of the tube, and the pressure as recorded by the static pressure column will consequently be less than in

the plane of the impact orifice. This effect will increase with the ratio of the diameter of tube to that of the pipe, and unless this ratio is small, the tube should be calibrated in a pipe of approximately the same dimensions as that in which it is to be used. It is preferable, for pipe work, to use a simple Pitot tube having only an impact orifice (Fig. 101), and to obtain the static pressure from a pair of small orifices in the pipe walls in the plane of the impact orifice and diametrically opposite each other. These should open

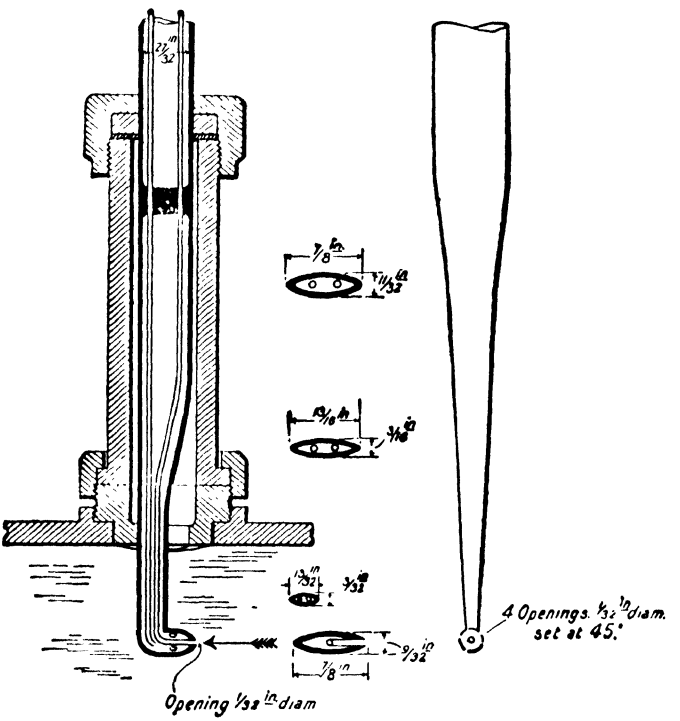


FIG. 100

flush with the wall of the pipe, any burr or roughness at the point of entry being carefully removed. Using the tube in this way, the coefficient C may usually be taken as ·99 within 1 per cent.[1]

In the type of Pitot tube shown in Fig. 102, and known as the "Pitometer," the pressure at the down-stream orifice is less than the statical pressure in the pipe and the coefficient is less than in the normal type. For the

[1] "The Theory and Development of the Pitot Tube," Gibson. *The Engineer*, July 17, 1914, p. 59. R. Winkel, "V.D.I.," June 9, 1923, p. 568, gives values of C for a large number of types of Pitot tube.

tube shown, calibrations in flowing water in pipes give a mean value of $C = \cdot 916$. Owing to eddy formation at the down-stream orifice the coefficient of such a tube fluctuates within fairly wide limits.

The Pitot tube may be calibrated either in still water or in a current. In the latter case the mean velocity is computed from readings taken at a large number of points in a cross section, and the coefficient of the instrument is adjusted so as to make this mean velocity agree with that obtained from weir flow, current meter, or gravimeter measurements on the same stream.

Without exception, observers have found that a still water rating gives a

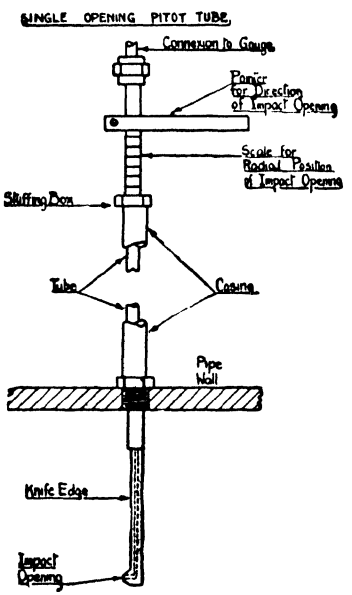


FIG. 101.

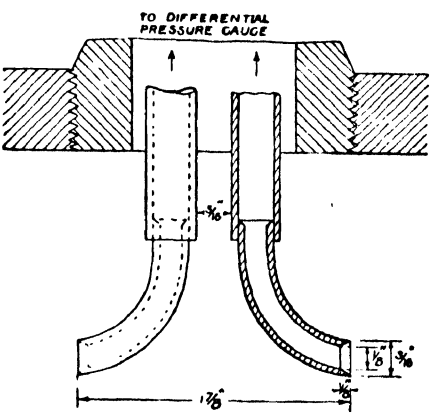


FIG. 102.

somewhat higher value of C. The explanation would appear to be two-fold. In the first place, the velocity of flow in a moving current is never quite steady, but suffices a series of periodic fluctuations, and since the Pitot tube is an instrument which essentially measures the mean momentum, or the mean (v^2) of the flow and not its mean velocity, any such fluctuation superposed on a given mean velocity will give an increased head reading. In the second place, when metering a flowing current, the tube cannot be used at points very near the boundary where the velocity is least, and for this reason also the mean recorded velocity tends to be too high.

It follows that, although a still-water or still-air rating represents the true coefficient of the instrument, this requires to be reduced somewhat for use in a current, the effect increasing with the unsteadiness of the current. Where a high degree of accuracy is required, the rating should be carried out under conditions nearly as possible resembling those under which measurements have afterwards to be made.

For measurements of the flow in pipes the instrument should be used at a section remote from any bend or source of disturbance. For approximate work the velocity of the central filament may be measured. This, when multiplied by a coefficient which varies from ·79 in small pipes to ·86 in large pipes, gives the mean velocity. Alternatively, the velocity may be measured at the radius of mean velocity, which varies from about ·7 a in small pipes to ·75 a in large pipes, where a is the radius of the pipe. These values, however, only apply to a straight stretch of the pipe, and if it is necessary to make

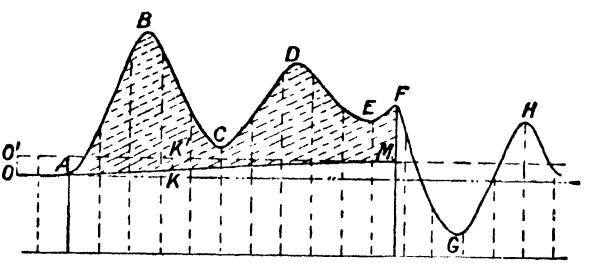


FIG. 102A.

measurements near a bend, and in any case for accurate results, the pipe should be traversed along two diameters at right angles and the velocities measured at a series of radii. If δr is the width of an elementary annulus containing one series of such measurements whose mean value is v, the discharge over this element is equal to $2\pi r \delta r . v$, and the total discharge Q $(= \Sigma_0^a 2\pi r v \delta r)$ is obtained by summing the discharges over the whole of the elements.

(4) **Chemical and Electrical Methods.**—Methods involving the introduction of salt or some other suitable chemical into the pipe, and the analysis of a sample abstracted at a point down-stream, or the measurement of the electrical resistance at a point down-stream are described in connection with the measurement of stream flow (Art. 120). They are equally applicable to pipe flow, especially if the pipe is long and the flow disturbed.

(5) **Pressure-rise Method.**—Mr. N. R. Gibson [1] has recently developed a method of determining the velocity in a turbine supply pipe, by taking a

[1] "Proc. Am. Soc. Civil Engineers," 1919, p. 173; also "Power," March 22, 1921, p. 452.

pressure-time diagram at the delivery end of the pipe line during slow closure of the turbine gates. Fig. 102A shows the form of diagram usually obtained, closure occupying the interval from A to F. If $O\,O'$ represents the sum of the friction and kinetic head at the beginning of motion, this term will be represented by some smaller value, $K\,K'$ at K, and will be zero when the valve is closed. The ordinates of the curve $A\,B\,C\,D\,E\,F$, measured above the line $B\,K\,M$, represent the head available at each instant, to check the velocity. If the height of the diagram represents pressures in feet of water, the area between A and F is then equal to $\frac{l}{g}(v_1 - v_2)$, so that if v_2 is zero, or if not zero is known, v_1 can be calculated.

Intermediate points, such as K, on the "recovery line" $A\,K\,M$, may be determined, since the area under the curve $A\,B\,D\,F$ up to any instant during closure is proportional to the reduction of velocity up to that instant, while the residual kinetic plus friction head is proportional to the square of the residual flow. As a first approximation, it may be assumed that the velocity is reduced uniformly.

Experiments show that under favourable circumstances the method is capable of giving good results. It is not suitable if the pipe line is less than about 150 feet in length.

Art. 85.—Relation of Diameter of Pipe to Quantity Discharged.

With rough pipes, since $Q = \frac{\pi d^2}{4} \cdot v$, and since the loss of head H in any length l is given by

$$H = \frac{v^2}{2g}\left\{\frac{f\,l}{m}\right\}$$

$$\therefore \quad v = \sqrt{\frac{2\,g\,H\,m}{f\,l}}$$

$$\therefore \quad Q = \frac{\pi d^2}{4}\sqrt{\frac{2\,g\,H\,d}{4\,f\,l}} = \frac{\pi}{4}\sqrt{\frac{2\,g\,H}{4\,f\,l}} \cdot d^{\frac{5}{2}}$$

$$\therefore \quad Q = C'\sqrt{\frac{H}{l}} \cdot d^{\frac{5}{2}}, \text{ where } C' = \sqrt{\frac{g}{2f}} \cdot \frac{\pi}{4}.$$

For smooth pipes, the loss of head is proportional to a lower power of the velocity than the second, and to a power of the diameter slightly different from the first, *i.e.*,

$$H = k\frac{v^n\,l}{d^x}$$

$$\therefore \quad v = \left(\frac{H}{k} \frac{d^x}{l} \right)^{\frac{1}{n}}$$

$$\therefore \quad Q = C' \left(\frac{H}{l} \right)^{\frac{1}{n}} . d^{2 + \frac{x}{n}}$$

Professor Unwin gives coefficients for formulae deduced by Hagen, which for clean cast iron pipes make

$$Q = C \left(\frac{H}{l} \right)^{\frac{1}{2}} d^{2 \cdot 63}$$

C' having the value 41·9.

In experiments carried out by M. Vallot, Q was found to be proportional to $d^{2.66}$

ART. 86.—INITIATION AND STOPPAGE OF MOTION IN A PARALLEL PIPE.

(*a*). **Gradual Stoppage.**—Neglecting the effect of the elasticity of the water, and thus assuming that during acceleration or retardation in a pipe line, the velocity throughout the pipe is uniform at any instant, if a retardation $-\frac{dv}{dt}$ be produced in any way, as by the gradual closing of a valve at the outlet to the pipe, or by the retardation of a plunger giving motion to the water column, the rise in pressure behind the valve or piston due to this retardation will be given by

$$p' a = - \frac{w}{g} . l . a . \frac{dv}{dt}$$

$$\therefore \quad p' = - \frac{wl}{g} . \frac{dv}{dt} \qquad \text{lbs. per square foot . (1)}$$

where "l" is the length, and "a" the sectional area of the column in square feet. The minus sign is used since $\frac{dv}{dt}$ being an acceleration, is itself negative if the motion is being retarded. This pressure difference at the two ends of the pipe is superposed on that due to steady flow with the velocity obtaining at the given instant. Thus if the loss of pressure from entrance to exit, due to steady flow with velocity v feet per second, is $\frac{wv^2}{2g} \left(1 + \frac{fl}{m} \right)$ lbs. per square foot, and if v and $\frac{dv}{dt}$ are respectively the velocity and the acceleration at a given instant, the pressure at the valve will be less than the statical pressure under conditions of no flow, by an amount

$$\frac{w}{g} \left\{ l \frac{dv}{dt} + \frac{v^2}{2} \left(1 + \frac{fl}{m} \right) \right\} \text{ lbs. per square foot.}$$

A general expression for the pressure changes, accompanying changes of velocity in a pipe line may be obtained from Bernoulli's equation. If

at any point in the pipe the acceleration is $\frac{dv}{dt}$, the force per pound necessary to produce this acceleration is $\frac{1}{g}\frac{dv}{dt}$, and the work done by this force while motion takes place through a small distance δx is $\frac{1}{g}\cdot\frac{dv}{dx}\cdot\delta x$.

Similarly, if the work done against frictional resistances is expressed as

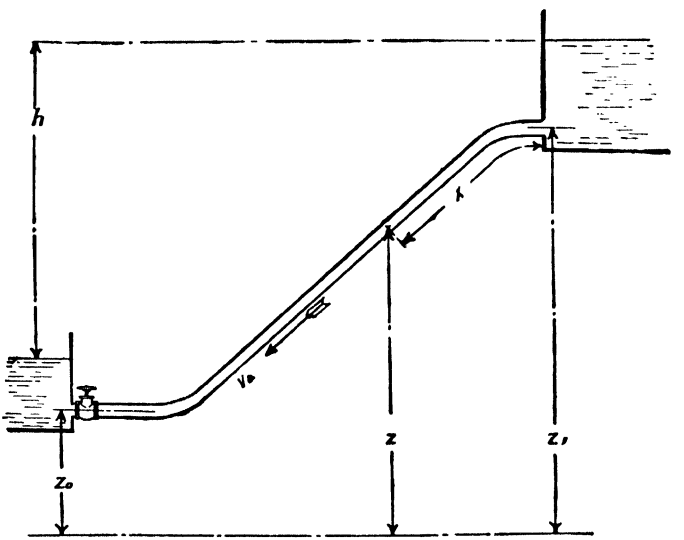


FIG. 103.

$\frac{fv^2}{2gm}$ per lb., per unit length of the pipe, as is usual, the equation as finally modified for friction and for acceleration becomes

$$\frac{d}{dx}\left\{\frac{p}{w}+\frac{v^2}{2g}+z\right\}=-\left\{\frac{1}{g}\frac{dv}{dt}+\frac{fv^2}{2gm}\right\} \qquad (2)$$

where z is the height of a given point in the fluid, above datum level, and x is its distance from some abitrary point, measured in the direction of flow. This equation is true even if the pipe line be not uniform in diameter.

Integrating both sides of (2) with respect to x we get

$$\frac{p}{w}+\frac{v^2}{2g}+z=-\frac{1}{g}\int_0^x\frac{dv}{dt}.\,dx-\frac{f}{2gm}\int_0^x v^2dx+c \qquad (3)$$

Whenever the acceleration is a known function of the time or of the distance travelled by a particle, equation (3) may be solved and the pressure at any point obtained.

As an example, consider the uniformly retarded flow through a pipe of uniform cross-sectional area of and length l. Let $\frac{dv}{dt} = -a$, and let the suffixes v and 1 refer to the pipe immediately behind the valve and to the inlet at the top respectively. Let x be measured from the inlet (Fig. 103), and let v_a be the velocity in the pipe line.

Then at the inlet, where $x = 0$, we have $p = p_1$, $z = z_1$,

$$\int_0^x v^2 \, . \, dx = 0, \text{ so that } c = \frac{p_1}{w} + \frac{v_a^2}{2g} + z_1 \text{ from (3)}$$

When $x = l$, *i.e.*, behind the valve, we have $p = p_v$, $z = z_v$,

$$\int_0^l v^2 \, dx = v_a^2 l, \int_0^l \frac{dv}{dt} \, dx = -a \int_0^l dx = -al;$$

$$\therefore \quad \frac{p_v}{w} + \frac{v_a^2}{2g} + z_v = \frac{a}{g} \, . \, l + \frac{p_1}{w} + \frac{v_a^2}{2g} + z_1 - \frac{fl}{2gm} v_a^2 \qquad (4)$$

But $\frac{p_1}{w} + z_1 + \frac{v_a^2}{2g} - z_v$ is the head equivalent of the statical pressure p_s at the valve with no flow through the pipe, so that we get

$$\frac{p_v}{w} = \frac{a}{g} l + \frac{p_s}{w} - \frac{v_a^2}{2g} \left\{ 1 + \frac{fl}{m} \right\} \text{ feet of water,} \qquad (5)$$

$$\text{or} \quad p_v = p_s + \frac{w}{g} \left\{ a l - \frac{v_a^2}{2} \left(1 + \frac{fl}{m} \right) \right\} \text{ lbs. per square foot.}$$

the result previously obtained from general considerations. Obviously this expression has its maximum value when $v_a = 0$, *i.e.*, at the instant the valve reaches its seat.

In order to get uniform retardation of a column by closing a valve at its lower end, the rate of closure of this valve would, however, need to be somewhat complicated. If a is the pipe area, and a_0 the effective valve area at any instant (the effective valve area is the actual area multiplied by the coefficient of discharge), and if v_0 is the corresponding velocity of efflux, v_a being the corresponding velocity of pipe flow, we have $v_a = \frac{a_0 v_0}{a}$, so that

$$\frac{dv_a}{dt} = \frac{1}{a} \left\{ a_0 \frac{dv_0}{dt} + v_0 \frac{da_0}{dt} \right\}.$$

If a_1 is the effective valve area when the valve begins to close, and if v_1 is the corresponding velocity of pipe flow, the value of a_0 after t seconds is given by

$$a_0 = a \left\{ \frac{v_1 + at}{\sqrt{2al + 2gh - \frac{fl}{m}(v_1 + at)^2}} \right\}^1$$

See "Water Hammer in Hydraulic Pipe Lines," p. 8. Gibson. Constable & Co., 1908.

In an experiment carried out by Prof. I. P. Church,[1] on a pipe line 1 inch in diameter and 2,395 feet long, fitted with a nozzle 2 inches diameter at its lower end and discharging into the atmosphere, the nozzle was closed in 25 seconds, so as, it is stated, to give uniform retardation in the pipe. The statical head at the valve was 302 feet (131 lbs. per square inch), and the pressure at the nozzle during steady flow was 108 lbs. per square inch. The maximum pressure attained was 143 lbs. per square inch, so that $p_v - p_s = 12$ lbs. per square inch.

Here, assuming a coefficient of velocity of ·985, the velocity of efflux would be $\cdot 985\ \sqrt{2\,g \times 108 \times 2{\cdot}31} = 125$ feet per second, so that the velocity in the pipe would be 7·81 feet per second.

This makes—a=·312 feet per sec., per sec., and makes

$$-\frac{w}{g}.a\,l = \frac{62{\cdot}4 \times {\cdot}312 \times 2395}{32{\cdot}2} = 1{,}455 \text{ lbs. per square foot,}$$

$$= 10{\cdot}1 \text{ lbs. per square inch,}$$

as compared with the observed value 12 lbs. per square inch.

Uniform Closure of Valve.—Where the outlet valve is closed uniformly, the acceleration varies from instant to instant according to a complicated law.

Let the pipe line, of uniform area a square feet, discharge at its lower end through a valve into a chamber where the pressure is uniformly p_0 lbs. per square foot. Let times be measured *backward* from the instant the valve reaches its seat, so that, if the valve be closed uniformly in T seconds, using the same notation as before we have $a_0 = a_1 \frac{t}{T}$ as giving the valve opening at an instant t seconds *before closure is complete.* Let v_a be the velocity in the pipe line. Equation (3) now becomes

$$\frac{p}{w} + \frac{v^2}{2g} + z = \frac{1}{g}\int_0^x \frac{d\,v}{d\,t}\,.\,d\,x - \frac{f}{2\,g\,m}\int_0^x v^2\,d\,x + c \qquad (6)$$

δt being negative.

Also since, when $x = 0$, $\left\{\begin{array}{l} p = p_1,\ z = z_1,\ \int_0^x \frac{d\,v}{d\,t}\,d\,x = 0, \\ v = v_a,\ \int_0^x v^2\,d\,x = 0, \end{array}\right\}$

$$\therefore \quad c = \frac{v_a^2}{2\,g} + z_1 + \frac{p_1}{w}.$$

Again, when $x=l$, *i.e.*, on the outlet side of the valve, neglecting losses in the valve, we have

$$\left\{\begin{array}{ll} p = p_0,\ z = z_0, & \int_0^l \frac{d\,v}{d\,t}\,.\,d\,x = l\,\frac{d\,v}{d\,t} \\ v = v_0,\ \int_0^l v^2\,d\,x = v_a^2\,l, & \end{array}\right\};$$

[1] Journal of Franklin Institute, April and May, 1890.

so that $\frac{v_0^2 - v_a^2}{2g} + \frac{p_0 - p_1}{w} + z_0 - z_1 = \frac{l}{g}\left\{\frac{d\,v_a}{d\,t} - \frac{f\,v_a^2}{2\,m}\right\}$ (7)

Writing $\frac{p_1}{w} + \frac{v_a^2}{2g} + z_1 - \frac{p_0}{w} - z_0 = h$ feet, where h is the difference of statical head on the two sides of the valve with no flow taking place, equation (7) becomes

$$\frac{v_0^2}{2g} - h = \frac{l}{g}\left\{\frac{d\,v_a}{d\,t} - \frac{f\,v_a^2}{2\,m}\right\} \quad (8)$$

On substituting for $\frac{d\,v_a}{d\,t}$, its value $\frac{1}{a}\left\{a_0\frac{d\,v_0}{d\,t} + v_0\frac{d\,a_0}{d\,t}\right\}$, this becomes

$$a_0\frac{d\,v_0}{d\,t} + v_0\frac{d\,a_0}{d\,t} = \frac{a}{l}\left\{\frac{v_0^2}{2}\left(1 + \frac{a_0^2}{a^2}\frac{fl}{m}\right) - g\,h\right\} \quad (9)$$

and on dividing throughout by the coefficient of v_0^2, we get

$$k\,a_0\frac{d\,v_0}{d\,t} = v_0^2 - b\,v_0 - c,$$

or $$\int\frac{d\,v_0}{v_0^2 - b\,v_0 - c} + D = \frac{1}{k}\int\frac{d\,t}{a_0} = \frac{T}{k\,a_1}\int\frac{d\,t}{t} \quad (10)$$

where $b = \frac{\frac{2\,l}{a}\cdot\frac{d\,a_0}{d\,t}}{1 + \frac{fl}{m}\left(\frac{a_0}{a}\right)^2}$; $c = \frac{2\,g\,h}{1 + \frac{fl}{m}\left(\frac{a_0}{a}\right)^2}$; $k = \frac{2\,l}{a\left\{1 + \frac{fl}{m}\left(\frac{a_0}{a}\right)^2\right\}}$.

If a_0/a is small, so that the term $\frac{fl}{m}\left(\frac{a_0}{a}\right)^2$ is small in comparison with unity, and may therefore be neglected, b, c, and k become constants having the values

$$b = \frac{2\,l}{a}\cdot\frac{d\,a_0}{d\,t};\quad c = 2\,g\,h;\quad k = \frac{2\,l}{a}.$$

This will always be the case as the valve gets close to its seat, and when in consequence the hammer effect is most noticeable. In such a case both sides of (10) become integrable. Writing this in the form

$$-\int\frac{d\,v_0}{\left(\sqrt{c + \frac{b^2}{4}}\right)^2 - \left(v_0 - \frac{b}{2}\right)^2} + D = \frac{T}{k\,a_1}\log_e t \quad (11)$$

and determining D from the consideration that when $t = T$, v_0 has a known value $\bar{v}_0$, we have

$$\frac{1}{2\sqrt{c + \frac{b^2}{4}}}\log_e\left\{\frac{\sqrt{c + \frac{b^2}{4}} - \frac{b}{2} + v_0}{\sqrt{c + \frac{b^2}{4}} + \frac{b}{2} - v_0}\cdot\frac{\sqrt{c + \frac{b^2}{4}} + \frac{b}{2} - \bar{v}_0}{\sqrt{c + \frac{b^2}{4}} - \frac{b}{2} + \bar{v}_0}\right\} = \frac{T}{k\,a_1}\log_e\frac{T}{t}$$

$$\text{or } \frac{1}{m}\log_e\left\{\frac{r+v_0}{q-v_0}\cdot\frac{q-v_0}{r+v_0}\right\}=\frac{T}{k\,a_1}\log_e\frac{T}{t} \tag{12}$$

$$\therefore\quad v_0=\frac{\dfrac{r+\bar{v}_0}{q-\bar{v}_0}\cdot q\cdot\left(\dfrac{T}{t}\right)^{\frac{mT}{ka_1}}-r}{1+\dfrac{r+\bar{v}_0}{q-\bar{v}_0}\left(\dfrac{T}{t}\right)^{\frac{mT}{ka_1}}}\text{ feet per second} \tag{13}$$

where $r=\sqrt{c+\frac{b^2}{4}}-\frac{b}{2}$; $q=\sqrt{c+\frac{b^2}{4}}+\frac{b}{2}$; $m=2\sqrt{c+\frac{b^2}{4}}$

This gives the velocity of efflux, from which the velocity v_a at any instant within the range of valve opening over which $\frac{fl}{m}\left(\frac{a_0}{a}\right)^2$ is negligibly small, may be readily obtained.[1]

Writing p_v for the difference of pressure on the two sides of the valve at any instant during closure, we get, as in equation (5),

$$p_v=p_s+\frac{w}{g}\left\{\frac{d\,v_a}{d\,t}\cdot l-\frac{v_a^2}{2}\left(1+\frac{fl}{m}\right)\right\}\text{ lbs. per square foot,}$$

and on substituting for $\frac{d\,v_a}{d\,t}$ from (8) this becomes

$$p_v=p_s+\frac{w}{g}\left\{\frac{v_0^2}{2}-\frac{v_a^2}{2}-g\,h\right\}\text{ lbs. per sq. ft.} \tag{14}$$

Evidently this has its maximum value when v_0 is a maximum and therefore, from (13), when t vanishes, *i.e.*, at the instant the valve reaches its seat. At this instant v_0 attains the limiting value q, while v_a becomes zero, so that we get

$$(p_v)_{max}=p_s+\frac{w}{g}\left\{\frac{q^2}{2}-g\,h\right\}\text{ lbs. per sq. ft.}$$

$$\text{But } \frac{q^2}{2}-g\,h=\frac{c}{2}+\frac{b^2}{4}+\frac{b}{2}\sqrt{c+\frac{b^2}{4}}-g\,h$$

$$=\left(\frac{l}{a}\cdot\frac{d\,a_0}{d\,t}\right)^2+\frac{l}{a}\cdot\frac{d\,a_0}{d\,t}\cdot\sqrt{2\,g\,h+\left(\frac{l}{a}\cdot\frac{d\,a_0}{d\,t}\right)^2}$$

It follows that the rise in pressure behind the valve at the instant when closure is complete, above that obtaining with no flow through the pipe, is given by

$$p'=\frac{w}{g}\left[\left(\frac{l}{a}\cdot\frac{a_1}{T}\right)^2+\frac{l}{a}\cdot\frac{a_1}{T}\sqrt{2\,g\,h+\left(\frac{l}{a}\cdot\frac{a_1}{T}\right)^2}\right]\text{ lbs. per sq. ft.} \tag{15}$$

[1] Where $\frac{fl}{m}\left(\frac{a_0}{a}\right)^2$ is not negligible the treatment follows the lines outlined on p. 27. "Water Hammer," *ante cit.*, p. 224. This case is, however, not of great practical importance, since the rise in pressure before the valve gets near to its seat, and hence before the state of affairs hypothecated obtains, is usually very small.

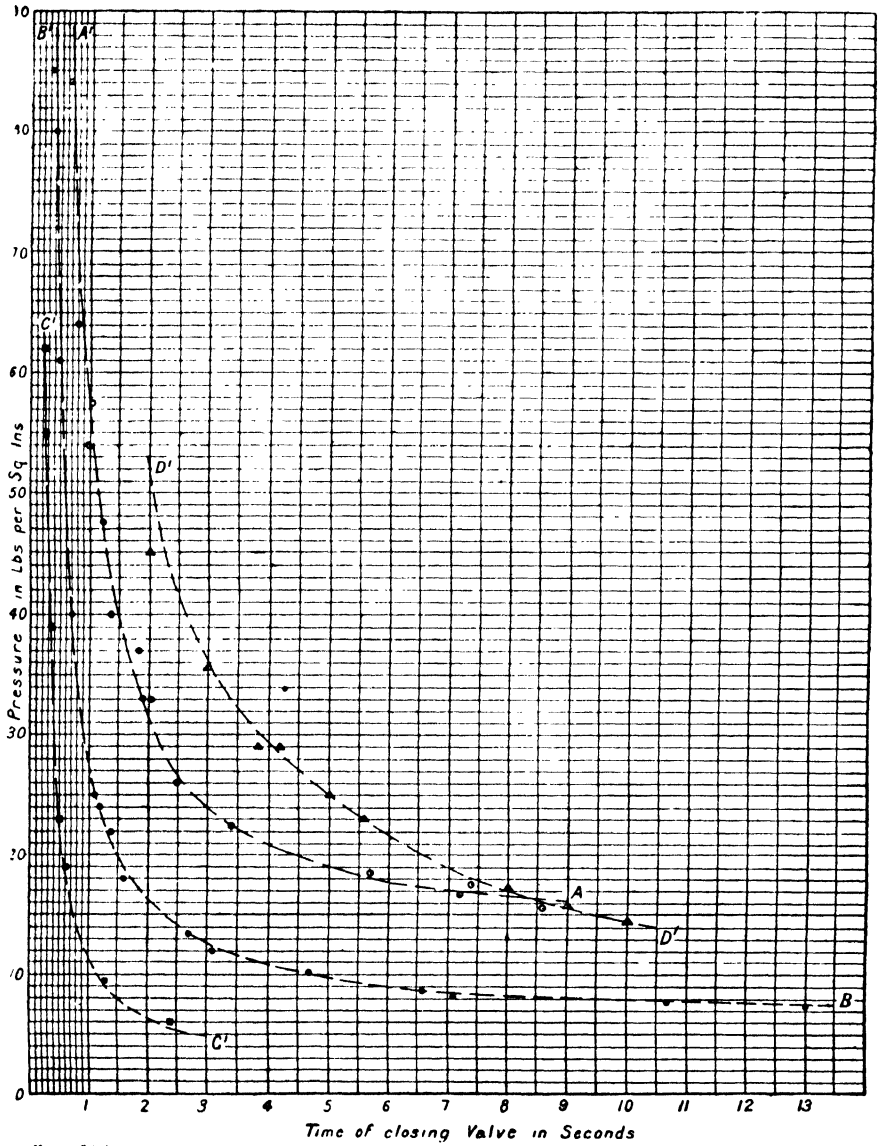


FIG. 104.—Pressures Attained Behind Valve in Excess of those Obtaining with Steady Flow with Valve Open.

Experiments by the author on a cast-iron pipe line 550 feet long and 3·75 inches diameter, with a range of values of the ratio $a : a_1$ from 9 to 79 and with values of T ranging from ·22 to 13 seconds, show results in close

accordance with those calculated from this formula. In Fig. 104 the

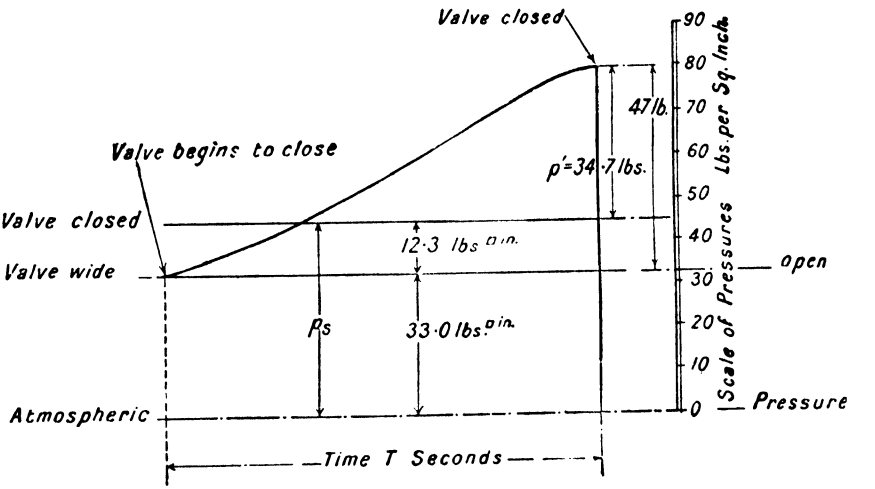


FIG. 105.

experimental results are plotted against the dotted curves which represent the calculated results.

Series A.	$a \div a_1 = 19{\cdot}5$	h	$= 104{\cdot}6$	ft.
„ B.	„ $= 34{\cdot}7$	„	$= 104{\cdot}6$	„
„ C.	„ $= 72{\cdot}5$	„	$= 105{\cdot}5$	„
„ D.	„ $= 8{\cdot}6$	„	$= 104{\cdot}0$	„

Pressures were measured by an indicator whose drum received its rotary motion directly from the spindle of the valve, and a typical diagram under these conditions is shown in Fig. 105.

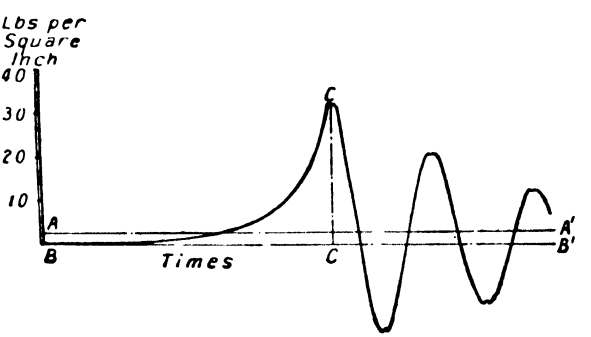


FIG. 106.

When the valve has reached its seat the pressure falls below that corresponding to the statical head, then rises again, a wave of pressure being reflected backwards and forwards along the pipe. This portion of the phenomenon, due solely to the elasticity of the water column, will

be explained later. If the indicator drum be rotated uniformly and independently of the valve spindle, a diagram similar to Fig. 106 is obtained. Here $A\ A'$ marks the pressure in the main before opening the valve, $B\ B'$ the pressure at the valve with steady flow, so that the distance from $A\ A'$ to $B\ B'$ represents $\frac{f\,l\,v^2}{2\,g\,m}$. C marks the instant of closing of the valve and $C\ C'$ indicates the excess pressure p' due to closure.

Both theory and practice emphasise the importance of a relatively slow motion of the valve as it approaches its seat, and particularly at the instant of closing.

Gradual Opening of Valve.—If the valve be gradually opened, so that both valve area and velocity of pipe flow increase with time, equation (6) becomes $$\frac{p}{w}+\frac{v^2}{2\,g}+z=-\frac{1}{g}\int_0^x\frac{d\,v}{d\,t}\,d\,x-\frac{f}{2\,g\,m}\int_0^x v^2\,d\,x+C \qquad (6')$$

If the valve is opening uniformly, on proceeding as before (9) becomes $$a_0\frac{d\,v_0}{d\,t}+v_0\frac{d\,a_0}{d\,t}=-\frac{a}{l}\left[\frac{v_0^2}{2}\left\{1+\left(\frac{a_0}{a}\right)^2\frac{fl}{m}\right\}-g\,h\right] \qquad (9')$$

and on making the assumption that $\left(\frac{a_0}{a}\right)^2\frac{fl}{m}$ is small, this becomes

$$\int\frac{d\,v_0}{v_0^2+b\,v_0-c}+D=-\frac{1}{k}\int\frac{d\,t}{a_0}.$$

When $t=0$, let $v_0=\bar{v}_0$, the flow being steady, and let $a_0=a_1$. Then $a_0=a_1+t\frac{d\,a_0}{d\,t}=a_1+Kt$, where $K=\frac{d\,a_0}{d\,t}$.

$$\therefore\quad\left\{\frac{v_0-r}{v_0+q}\cdot\frac{\bar{v}_0+q}{\bar{v}_0-r}\right\}=\left(\frac{a_1}{a_1+Kt}\right)^{\frac{m}{Kk}}$$

from which we get

$$v_0=\frac{\frac{\bar{v}_0-r}{\bar{v}_0+q}\cdot q\left(\frac{a_1}{a_1+Kt}\right)^{\frac{m}{Kk}}+r}{1-\frac{\bar{v}_0-r}{\bar{v}_0+q}\left(\frac{a_1}{a_1+Kt}\right)^{\frac{m}{Kk}}}\text{ ft. per second,}$$

where r, q, m and k have the meanings attached to them on p. 227, viz., $r=\sqrt{c+\frac{b^2}{4}}-\frac{b}{2}$; $q=\sqrt{c+\frac{b^2}{4}}+\frac{b}{2}$; $m=2\sqrt{c+\frac{b^2}{4}}$; $k=\frac{2\,l}{a}$:[1]

This gives the velocity of efflux after an interval of t seconds from the commencement of the motion, in terms of the valve opening and velocity

[1] Here $b=\frac{2\,l}{a}\cdot\frac{d\,a_0}{d\,t}$ and $c=2\,g\,h$.

of efflux at the latter instant. This only holds so long as the valve is opening. Suppose the valve to be stopped at an instant when its opening is a_0', the velocity of efflux at this instant being calculated to be v_0'. Let $a_0' \div a = n$.

From this time onwards $\frac{d\,v_a}{d\,t} = n \cdot \frac{d\,v_0}{d\,t}$, and equation (9′) becomes

$$l\,n\,\frac{d\,v_0}{d\,t} = -\left[\frac{v_0^2}{2}\left\{1 + n^2\frac{fl}{m}\right\} - g\,h\right]$$

So that

$$\int \frac{d\,v_0}{c' - v_0^2} + D = \frac{1}{k'}\int d\,t$$

where $c' = \dfrac{2\,g\,h}{1 + n^2\dfrac{fl}{m}}$; $k' = \dfrac{2\,l\,n}{1 + n^2\dfrac{fl}{m}}$:

Integrating we get

$$\frac{1}{2\sqrt{c'}}\log\frac{\sqrt{c'} + v_0}{\sqrt{c'} - v_0} + D = \frac{t}{k'}.$$

When $t = 0$, *i.e.*, immediately the valve comes to rest, $v_0 = v_0'$.

Using this to determine D, we finally get

$$v_0 = \frac{\dfrac{\sqrt{c'} + v_0'}{\sqrt{c'} - v_0'} \cdot e^{\frac{2\sqrt{c'}}{k'}t} - 1}{\dfrac{\sqrt{c'} + v_0'}{\sqrt{c'} - v_0'} \cdot e^{\frac{2\sqrt{c'}}{k'}t} + 1}\ \sqrt{c'} \text{ feet per second} \qquad (16)$$

as the velocity of efflux after t seconds from the stoppage of the valve.

It will be noted that as t increases, this tends to the limit

$$\sqrt{c'} = \sqrt{\frac{2\,g\,h}{1 + n^2\dfrac{fl}{m}}}.$$

The method of treatment so far outlined, while giving results which are rigorously true for an incompressible fluid in a rigid pipe line fails to account for many of the phenomena actually observed during the stoppage of motion in a long column of water, since these are largely due to the elasticity of the water column. For example, an examination of equations (1) or (2) indicates that an instantaneous stoppage of motion, involving an infinite retardation, will necessitate an infinite retarding force, and hence an infinite pressure at the closed end of the pipe, a conclusion which is not at all borne out by the result of experiment.

Actually, when the column is brought instantaneously to rest, compression takes place; a wave of compression is reflected from the closed end of the pipe; and the initial kinetic energy of the water is transformed into resilient energy or energy of strain.

When the retardation is gradual, part of the kinetic energy is absorbed in doing work against the retarding force, and part in compressing the column, the latter factor becoming increasingly important as the rate of retardation is increased.

(b) Sudden Stoppage of Motion—Ideal Case.

If a column of water, flowing with velocity v along a uniform pipe (supposed rigid), have its motion checked by the instantaneous closure of a rigid valve, the phenomena experienced are due entirely to the elasticity of the column, and are analogous to those obtaining in the case of the longitudinal impact of an elastic bar against a rigid wall.

At the instant of impact, the motion of the layer in contact with the valve is suddenly stopped, and its kinetic energy is changed into resilience, or energy of strain, with a consequent sudden rise in pressure. This stoppage and rise in pressure is almost instantaneously transmitted to the adjacent layer, with the result that a state of zero velocity and high pressure (this at any point being p above the pressure obtaining at that point with steady flow at velocity v) is propagated as a pressure wave along the pipe, with velocity V_p.[1]

This wave reaches the open end of the pipe after t seconds, where $t = l \div V_p$. At this instant the whole of the column is instantaneously at rest in a state of uniform compression.

This is not a state of equilibrium, since the pressure immediately inside the open end of the pipe is "p" greater than that in the surrounding medium. In consequence, the strain energy of the end layer is reconverted into kinetic energy, its pressure falls to that of the surrounding medium, and it rebounds with its original velocity v towards the open end of the pipe. This relieves the pressure on the adjacent layer, with the result that a state of normal pressure and of velocity $(-v)$ is propagated as a wave towards the valve, reaching it after a second interval $l \div V_p$ seconds. At this instant the whole of the column is at normal pressure and is moving towards the open end with velocity v. The end of the column tends to leave the valve, but cannot do so unless the pressure drops to zero, or so near to zero that any air in solution is liberated. Its motion is consequently checked and its kinetic energy goes to reduce the strain energy to a value below that corresponding to normal pressure. The pressure drops suddenly by an amount equal to that through which it originally rose, and a wave of zero velocity and of pressure p below

[1] V_p is the velocity of propagation of sound waves in the medium, and is equal to $\sqrt{\frac{Kg}{w}}$ where w is the weight per cubic foot, and K is in lbs. per square foot.

normal is transmitted along the pipe, to be reflected from the open end as a wave of normal pressure and velocity towards the valve. When this wave reaches the valve $4l \div V_p$ seconds after the instant of closure, the conditions are the same as at the beginning of the cycle and the whole is repeated.

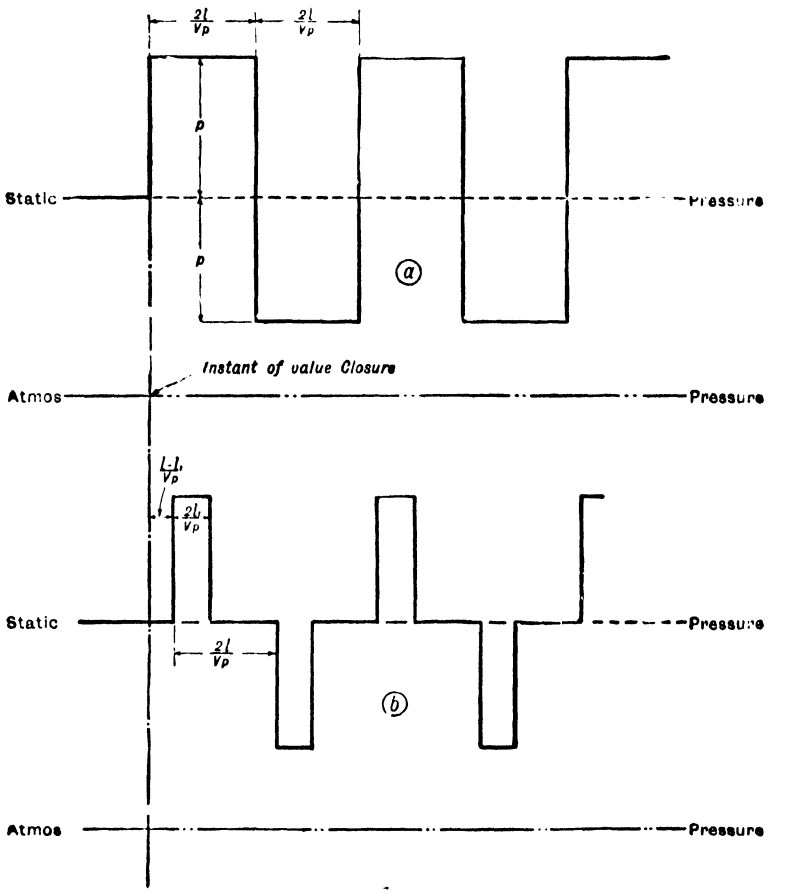


FIG 107.

Under such ideal conditions the state of affairs at the valve would be represented by Fig. 107 *a*. At any other point, at a distance l_1 from the open end, the pressure-time diagram would appear as in Fig. 107 *b*.

If the velocity were such as to make p greater than the normal absolute pressure in the pipe, the first reflux wave would tend to reduce the pressure below zero. Since this is impossible, the pressure could only fall to zero, and

the subsequent rise in pressure would be correspondingly reduced. Actually, at such low pressures, any dissolved air is liberated and the motion rapidly breaks down.

Effect of Friction in the Pipe Line.

The effect of friction in the pipe line modifies the phenomena in a complex manner. In the first place the pressure, with steady flow, falls uniformly from the open end towards the valve, and the pressure at the valve will be represented by such a line as *A B*, Fig. 108. On closure, the pressure here will rise by an amount p as before.

When the adjacent layer is checked its rise in pressure will also be p, but

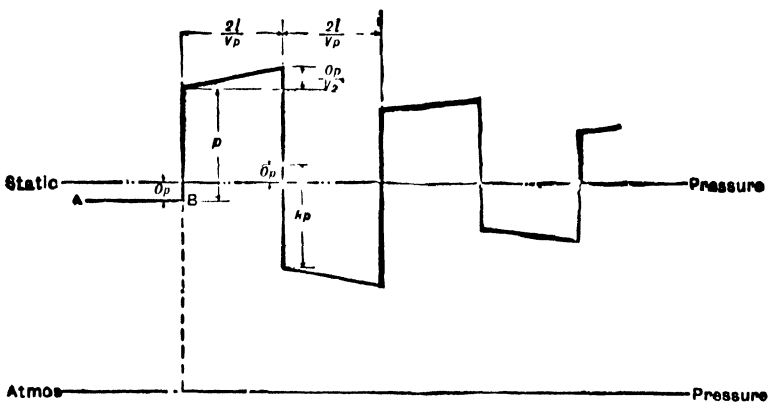


FIG. 108.

since its original pressure was higher than that at the valve, its new pressure will also be higher. It will therefore tend to compress that portion of the column ahead of it and will lose some of its strain energy in so doing. This will result in the pressure at the valve increasing as layer after layer is checked, but since this secondary effect travels back from each layer in turn with a velocity V_p, the full effect at the valve will not be felt until a time $2l \div V_p$ after closure. At this instant the pressure will have risen by an amount which to a first approximation may be taken as $\delta p \div \sqrt{2}$ [1] where δp is the pressure difference at the ends of the pipe under steady flow.

[1] With steady flow the pressure distant x from the valve will be greater than that at the valve by an amount $\delta p \cdot \frac{x}{l}$.

$\therefore$ Excess strain energy of a layer of length δx at this point, due to this pressure

$$\propto \delta x \left(\frac{\delta p \times x}{l}\right)^2$$

Assuming this excess energy to be distributed over the length x between this layer and the

When reflux takes place, the end layer, having transmitted part of its energy along the pipe, can no longer rebound with the original velocity v. Moreover, since at the instant when the disturbance again reaches the valve, and the column is moving towards the open end, frictional losses necessitate the pressure at the valve being δp greater than at the open end, the pressure drop will be less than in the ideal case.

If the velocity of the first reflux be $k v$, where k is somewhat less than unity, the pressure diagram will be modified sensibly as in Fig. 108.

Friction thus causes the pressure wave to die out rapidly without affecting the periodicity appreciably.

Magnitude of Rise in Pressure, following Sudden Closure.

Assuming a rigid pipe, on equating the loss of kinetic energy per lb. of fluid to the increase in its strain energy or resilience :—

$$\frac{v^2}{2g} = \frac{p^2}{2Kw}^{1}$$

$$\therefore \qquad p = v\sqrt{\frac{Kw}{g}} = v\,V_p\,\frac{w}{g}$$

where p is the rise in pressure, v the velocity of flow in feet per second, and V_p the velocity of propagation of sound waves in the medium.

Putting $K = 43{\cdot}2 \times 10^6$ lbs. square foot.
,, $w = 62{\cdot}4$ lbs. c.f.
,, $g = 32{\cdot}2$.
this becomes $p = 9160\,v$ lbs. per square foot.
$= 63{\cdot}7\,v$ lbs. per square inch.

Effect of Elasticity of Pipe Line.

Owing to the elasticity of the pipe walls, part of the kinetic energy of the moving column is expended in stretching these, with a resultant increase in the complexity of the phenomena ; a reduction in the maximum pressure

---

valve, it will cause a rise in pressure p', where $(p')^2\,x = \delta\,x\left(\frac{\delta\,p\,.\,x}{l}\right)^2$.

Integrating to obtain the effect of all such layers from 0 to x gives

$$(p')^2 = \left(\frac{\delta\,p}{l}\right)^2 \int_0^x x\,d\,x = \left(\frac{\delta\,p}{l}\right)^2 \frac{x^2}{2},$$

and when $x = l$, $p = \frac{\delta\,p}{\sqrt{2}}$.

This is only a first approximation, since the equalisation of pressures will be accompanied by surges, which will introduce additional frictional losses.

[1] If a cube of unit side be subject to a pressure increasing from 0 to p, the change in volume will be $p \div K$, and since the mean pressure during compression is $p \div 2$, the work done is $p^2 \div 2K$.

attained, and an increase in the rate at which the pressure waves die out. The state of affairs is then indicated in Figs. 109 *a* and *b*, which are reproduced from diagrams taken by the writer from a cast-iron pipe line 3·75 inches diameter and 450 feet long.[1]

Fig. 109 *a* was obtained from behind the valve and Fig. 109 *b* at a point 15 feet from the open end of the pipe.

The elasticity of the pipe line may modify the results in two ways. If free to stretch longitudinally, at the instant of closure the end of the pipe

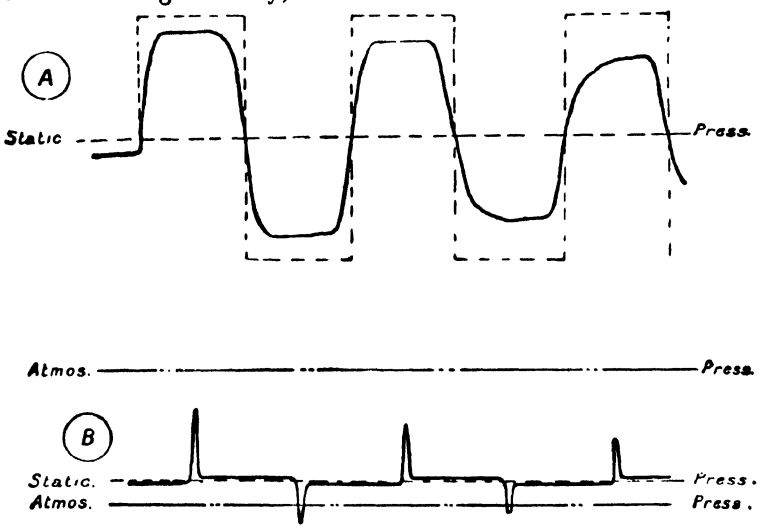


FIG. 109.

and the water column will move together with a common velocity u,[2] less than v, and a wave of extension will travel along the pipe wall. The instantaneous rise in pressure at the valve will now be equal to

$$(v - u)\sqrt{\frac{K\,w}{g}}.$$

Since the velocity of propagation is much greater in metal than in water, this wave will be reflected from the open end of the pipe, and will reach the closed end again before the reflected wave in the water column.

At this instant the closed end of the pipe will rebound towards the closed end with velocity u, and will produce an auxiliary wave of pressure equal t

[1] Gibson, "Water Hammer in Hydraulic Pipe Lines," Constable & Co., 1908.

[2] It may readily be shown that $u = v\left\{\frac{1}{1 + \frac{w_m\ a_m\ V_m}{w_a\ V_p}}\right\}$

where w_m, a_m and V_m refer to the weight per c.f., the cross-sectional area and the velocity of propagation in the metal, and w, c, V_p to the corresponding quantities for water.

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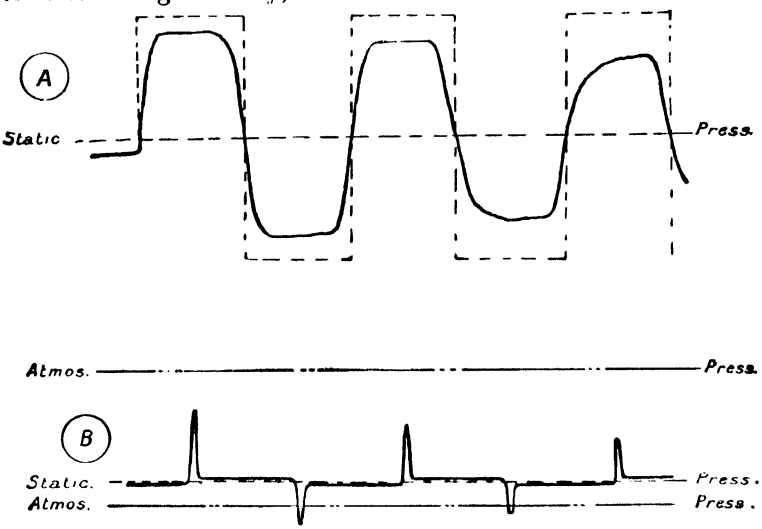

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where w_m, a_m and V_m refer to the weight per c.f., the cross-sectional area and the velocity of propagation in the metal, and w, c, V_p to the corresponding quantities for water.

$$u\sqrt{\frac{K\,w}{g}}$$

in the water column. This will increase the pressure to the value

$$v\sqrt{\frac{K\,w}{g}}$$

which would obtain if the pipe were anchored. So that the effect on the maximum pressure attained during this first period $2\,l \div V_p$ is zero. The effect on the subsequent history of the phenomenon is complex. The net effect, however, is to superpose on the normal pressure wave a subsidiary wave of high periodicity ($4\,l \div V'_p$, where V'_p is the velocity of propagation of waves in metal) and of magnitude $\pm\, u\sqrt{\frac{K\,w}{g}}$.

If, as usual in practice, the pipe is anchored so that no appreciable movement of the end is possible, this effect will be small.

The second effect of the elasticity of the pipe line is due to the fact that, since the walls extend both longitudinally and circumferentially under pressure, the apparent diminution of volume of the fluid under a given increment of pressure is greater than in a rigid pipe.

The effect of this is to reduce the virtual value of K to a value K', where

$$\frac{1}{K'} = \frac{1}{K} + \frac{r}{2\,t\,E}\left(5 - \frac{4}{\sigma}\right),^{1}$$

[1] If at any section of the pipe the increase in pressure due to retardation is p' lbs. per square foot, the increase in the circumferential stress in the pipe walls is $\frac{p'\,r}{t}$ and in the longitudinal stress is $\frac{p'\,r}{2\,t}$ lbs. per square foot.

If, then, $\delta\,x$ is the change in length of an element of the pipe at this point, whose original length was x, we have

$$\frac{\delta\,x}{x} = \frac{p'\,r}{2\,t\,E} - \frac{p'\,r}{\sigma\,t\,E};\quad \therefore\ \delta\,x = x\,\frac{p'\,r}{2\,t\,E}\left\{1 - \frac{2}{\sigma}\right\}$$

$$\text{Also } \frac{\delta\,r}{r} = \frac{p'\,r}{t\,E} - \frac{p'\,r}{2\,\sigma\,t\,E}$$

$$\therefore\ \delta\,r = r\,\frac{p'\,r}{t\,E}\left\{1 - \frac{1}{2\,\sigma}\right\}.$$

The change in the volume of this element is therefore given by

$$\pi\left\{(r + \delta\,r)^2\,(x + \delta\,x) - r^2 x\right\},$$

$$= \pi\left\{2\,r\,x\,\delta\,r + r^2\,\delta\,x\right\} \left\{\begin{array}{l}\text{neglecting small quantities}\\ \text{of the second order.}\end{array}\right.$$

$$= \pi\,r^2\,x\left\{\frac{2\,p'\,r}{t\,E}\left(1 - \frac{1}{2\,\sigma}\right) + \frac{p'\,r}{2\,t\,E}\left(1 - \frac{2}{\sigma}\right)\right\}$$

$$= \pi\,r^2\,x \times \frac{p'\,r}{2\,t\,E}\left\{5 - \frac{4}{\sigma}\right\}$$

$$\therefore\ \frac{\delta\,V}{V} = \frac{p'\,r}{2\,t\,E}\left\{5 - \frac{4}{\sigma}\right\}.$$

But the actual new volume of the liquid $= \pi\,r^2\,x\left(1 - \frac{p'}{K}\right)$ while its apparent new

Experiment.	Velocity before closing Valve.	Time of Closing.	Rise in pressure.	
			Calculated ($p = 54\ v$).	Experimental.
	feet per second.	seconds.	lbs. per square inch.	lbs. per square inch.
1	·3625	·070	19·5	19·5
2	,,	·090	,,	19·5
3	,,	·140	,,	18·7
4	,,	·270	,,	16·8
5	,,	·280	,,	14·7
6	·551	·065	29·7	29·8
7	,,	·090	,,	29·3
8	,,	·175	,,	29·3
9	,,	·255	,,	25·0
10	,,	·275	,,	24·4
11	,,	·275	,,	26·2
12	·720	·125	38·9	37·5
13	,,	·135	,,	38·1
14	,,	·150	,,	38·1
15	,,	·250	,,	37·0
16	,,	·270	,,	35·7
17	,,	·270	,,	35·7
18	1·094	·110	59·0	57·5
19	,,	·150	,,	58·7
20	,,	·245	,,	55·0
21	,,	·285	,,	46·3
22	1·444	·160	77·9	75·0
23	,,	·210	,,	73·7
24	,,	·215	,,	73·0
25	,,	·300	,,	62·5
26	,,	·370	,,	58·8

For values of $T > 4l \div V_p$, formula (15), p. 227, may be used without serious error.

Experimental Results with Sudden Closure of Valve. In the author's

experiments $l = 550$ feet; $K' = 251{,}000 \times 144$; $E = 10^7 \times 144$ lbs. per square foot; $a : a_m = 1{\cdot}275$; formula (1) p. 235, gives

$$p' = 7780\ v \text{ lbs. per square foot,}$$
$$= 54{\cdot}0\ v \text{ lbs. per square inch.}$$

For comparison the observed results are shown in the table on p. 239, against those obtained by using this formula.

From these results it appears that so long as T is less than ·13 seconds ($l \div V_p = {\cdot}13$) the calculated and observed pressures are in every case in close agreement. This agreement is substantially maintained until $T =$ about ·21 seconds, while when $T = {\cdot}26$ seconds, the mean error involved in using the uncorrected formula in this case is about 14 per cent.

In a series of experiments carried out by M. Joukowsky[1] on cast-iron pipes of 4 inches and 6 inches diameter, having lengths of 1,050 and 1,066 feet respectively, the time of valve closing being ·03 seconds in each case, the observed rise in pressure agrees closely with the formula, $p = 57\ v$. The following are some of the results obtained by interpolation from the plotted results of these experiments.

4-INCH PIPE.

Velocity, feet per second	·5	2·0	3·0	4·0	9·0
Observed pressure—lbs. per square inch .	31	119	172	228	511
$p = 57\ v$	29	114	171	228	513

6-INCH PIPE.

Velocity, feet per second	·6	2·0	3·0	7·5
Observed pressure—lbs. per square inch .	43	113	173	426
$p = 57\ v$	34	114	171	427

[1] *Stoss in Wasserleitungsröhren*, St. Petersburg, 1900. The author has been unable to ascertain the thickness of the walls of these pipes. Abstract by O. Simin in Trans. Am. Waterworks Ass., 1904.

The pressure in the last experiment, calculated on the assumption that the water is incompressible, is 3,585 lbs. per square inch, a result which sufficiently indicates the nature of the errors involved in extreme cases by neglecting the effect of elasticity.

These sudden increases of pressure, commonly known as water hammer, may evidently become most serious with large values of v, and in such cases the ill effects due to the too sudden closing of a valve should be guarded against by a relief valve or other similar device placed as near as convenient to the main valve. In the case of a pipe line for a power plant, where the head is not excessive it is usual to provide a stand pipe, or simple vertical pipe capable of taking the whole discharge of the main, and open at the top to the atmosphere. This should be coupled to the supply main just before its entrance to the power-house, and the height should be so arranged that when standing under the statical pressure of the supply reservoir, the water level is within a short distance of the top. Under these conditions, the maximum pressure which may occur in the pipe line is that due to the statical pressure in the stand pipe, together with that necessary to produce motion up this pipe.

It is usual in practice to make some allowance for possible water hammer by designing the pipes to withstand a pressure of 100 lbs. per square inch in excess of that due to the statical head, thus, in effect, allowing for an instantaneous stoppage at a velocity of 1·6 feet per second.

Sudden Opening of a Valve.

If the valve at the lower end of a pipe line be suddenly opened, the pressure behind the valve falls by an amount p lbs. per square inch, and a wave of velocity v towards the valve $\left\{v = p\sqrt{\frac{g}{K'w}} \text{ (approx.)}\right\}$, and of pressure p below statical, is propagated towards the pipe inlet.

The magnitude of p depends on the speed and amount of opening of the valve, and if the latter could be thrown wide open instantaneously the pressure would fall to that obtaining on the discharge side. In the author's experiments, with the valve thrown open through ·5 of a complete turn the maximum drop in pressure was 40 lbs. per square inch, the statical pressure being 45 lbs. per square inch.

With the valve opened through ·10 of a complete turn the maximum drop was 20 lbs. per square inch, and with $\frac{1}{20}$ of a complete turn the drop was 11 lbs. per square inch. In each case the time of opening was less than ·13 seconds ($l \div V_p$).

In the case of a horizontal pipe, or one which is so situated that the absolute statical pressure is everywhere greater than p, this pressure wave reaches the pipe inlet with approximately its original amplitude, and at this instant the whole column is moving towards the valve with velocity v and pressure p below normal.

The pressure at the inlet is, however, maintained normal, so that the wave returns from this end with normal pressure and with velocity $2v$. At the valve this wave is reflected with a velocity which is the difference between $2v$ and the velocity of efflux at this instant, and with a corresponding pressure. As the velocity of efflux will now be greater than v, the wave velocity will be less than v, and the rise in pressure less than p above normal. This wave is reflected from the inlet to the valve, and here the cycle is repeated, the amplitude of the pressure wave diminishing rapidly until steady flow ensues.

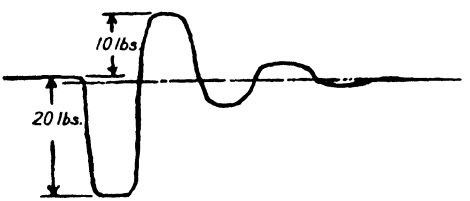


FIG. 110.

Fig. 110 shows the diagram obtained by the author from the experimental pipe line under these conditions.

Where the pipe slopes upwards towards its inlet, so that, beyond a certain point in its length the absolute statical pressure is less than the

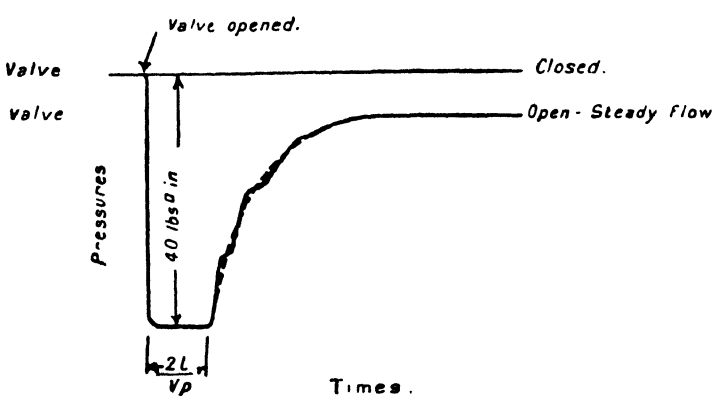


FIG. 110A.

drop in pressure caused at the valve by sudden opening, then on the passage of the first wave of negative pressure the wave motion becomes

partially discontinuous after this point is reached,[1] and the wave travels on to the inlet with a gradually diminishing amplitude. The amplitude with which it reaches the inlet, and which will be probably 2 or 3 lbs. per square inch less than the absolute statical pressure at inlet, decides the state of velocity of the reflected wave. This will evidently be less than in the preceding case, and under such circumstances the wave motion dies out very quickly. As the valve opening becomes greater, the efficiency of the valve as a reflecting surface becomes less, so that with a moderate opening the pressure may never even attain the pressure due to the statical head.

This is shown in Fig. 110A, which is a diagram obtained by the author from the experimental pipe line when the outlet valve was opened suddenly (time $<\cdot 13$ seconds) through half a complete turn.

Neglecting the effect of elasticity of the water column, the velocity of efflux accompanying a sudden valve opening may be obtained as follows:

Imagine the effective valve opening to assume instantaneously a value a_0 and to remain of this value, so that $\frac{dv_a}{dt} = \frac{a_0}{a} \cdot \frac{dv_0}{dt}$. Equation (6), (p. 225), now becomes

$$\frac{p}{w} + \frac{v^2}{2g} + z = -\frac{1}{g}\int_0^x \frac{dv}{dt}\,dx - \frac{f}{2gm}\int_0^x v^2 dx + c \qquad (6'')$$

while equation (9) becomes

$$-a_0 \frac{dv_0}{dt} = \frac{a}{l}\left\{\frac{v_0^2}{2}\left(1 + \frac{fl}{m}\left(\frac{a_0}{a}\right)^2\right) - gh\right\} \qquad (9'')$$

$$\text{or } k\frac{dv_0}{dt} = c^2 - v_0^2$$

$$\text{where } \begin{cases} k = \dfrac{2la_0}{a} \cdot \dfrac{1}{1 + \dfrac{fla_0^2}{ma^2}} \\ c^2 = \dfrac{2gh}{1 + \dfrac{fl}{m} \cdot \dfrac{a_0^2}{a^2}} \end{cases}$$

$$\text{so that } \int \frac{dv_0}{c^2 - v_0^2} + D = \frac{1}{k}\int dt.$$

From this we get

$$\frac{1}{2c} \log \frac{c + v_0}{c - v_0} + D = \frac{t}{k}, \qquad (11'')$$

[1] Actually before this, since water gives up its dissolved air rapidly when the pressure falls to within 2 or 3 lbs. of a complete vacuum.

while since $v_0 = 0$ when $t = 0$, we have $D = 0$.

$$\therefore \quad v_0 = c \left\{ \frac{1 - e^{\frac{2c}{k} \cdot t}}{1 + e^{\frac{2c}{k} \cdot t}} \right\} \text{ feet per second}^1 \qquad (12'')$$

giving the velocity of efflux at an instant t seconds after the valve opens.

As t increases, this approximates to the value c or

$$\sqrt{\frac{2gh}{1 + \frac{fl}{m} \cdot \frac{a_0^2}{a^2}}}$$

By substitution in (9'') the value of $\frac{dv_0}{dt}$ at any instant and therefore of $\frac{dc}{dt}$ may be obtained, and knowing this the pressure behind the valve may be readily obtained.

Examples.

(1) Experiments by Dr. Lampe on the 16-inch asphalted pipes of the Dantzic main gave results which for velocites between 1·6 and 3·0 feet per second agree with the value as deduced from the formula $\frac{h}{l} = ·000371 \frac{v^{1.85}}{d^{1.15}}$. Determine the corresponding values of f in the formulae $h = \frac{f l v^2}{2 g m}$ when $v = 1·6$ and 3·0 feet per second.

Answer. $\begin{cases} ·00531. \\ ·00485. \end{cases}$

(2) Darcy's experiments on a cast-iron pipe ·617 feet diameter give the result $\frac{h}{l} = ·000380 \frac{v^{1.95}}{d^{1.05}}$ with fair accuracy. Determine the velocity of flow and the discharge per minute from a similar pipe 1,000 feet long under a head of 50 feet.

Answer. $\begin{cases} \text{Velocity} = 2·893 \text{ feet per second.} \\ \text{Volume} = 51·9 \text{ cubic feet per minute.} \end{cases}$

(3) The jet of a Pelton wheel has an effective diameter of 2 inches. The supply pipe is 6,000 feet long and is an asphalted pipe 15 inches

[1] "Water Hammer" *ante cit.*, p. 51. This formula receives close experimental confirmation from the results of experiments on the velocity of flow down the drive pipe of a hydraulic ram, published in the Bulletin of the University of Wisconsin, No. 205, 1908, p. 143.

diameter, and the supply head is 800 feet. Determine the probable discharge in cubic feet per second, and the horse-power of the jet. Assume $c_v = \cdot 972$: $f = \cdot 0050$.

Answer. Velocity of jet = 218 f.s.; discharge = 4·75 cubic feet per second; 396 horse-power.

(4) With a given slope, a clean cast-iron pipe 9 inches diameter, is found to give a discharge of 25,000 gallons per hour. Determine the necessary diameter of pipe, having twice the slope, to give a discharge of 50,000 gallons per hour. (Take Unwin's values for the index of d and of v, p. 202.)

Answer. 10·3 inches.

(5) The outlet valve from a main 5,000 feet long is closed in 1 second. Determine the rise in pressure if the initial velocity of flow is 4 feet per second, and the pipe is rigid.

Answer. 254·8 lbs. square inch.

(6) A uniform pipe, 200 feet long, is fitted with a plunger which, originally moving at 6 feet per second, is brought to rest uniformly in 1·5 second. Assuming the water to be incompressible, determine the pressure on the piston caused by the retardation.

Answer. 10·76 lbs. per square inch.

(7) If, in the above example, the plunger is driven from a crank 1 foot long, and making 100 revolutions per minute, and with S. H. motion, determine the pressure produced by retardation at the end of the stroke.

Answer. 295 lbs. per square inch.

For a fairly complete treatment of the subject of water hammer, including its effect in complicated systems of pipe lines, a "Symposium on Water Hammer," published by the Am. Soc. Mech. E. (1933) might be consulted.

CHAPTER X.

Losses in a pipe line—Hydraulic gradient—Losses at Valves—Bends—Elbows—Exit—Flow in long pipes—Time of discharge through pipe line—Equivalent diameter of uniform main—Branch mains—Multiple supply—Bye-pass—Distribution of water—Pipe line accessories—Syphons—Inverted Syphons—Flow through nozzles—Form of nozzle—Maximum discharge of kinetic energy through a nozzle.

Art. 87.—Pipe Line Losses.

In constructing a pipe line to connect two reservoirs, or to distribute the discharge from any reservoir, the problem which usually presents itself to the engineer is that of determining the minimum size of pipe which, under given conditions of head loss, shall be capable of discharging a given quantity of water per minute. In the construction of a supply pipe line for a power station, the problem is much the same, and in every case it is first of all necessary to determine the conditions involving loss of head, and the magnitude of these losses.

Commencing at the supply end of the pipe, we have

(1) Loss due to friction and eddy formation at the entrance to the pipe.

(2) Frictional losses in the pipe itself.

(3) Losses at valves, sluices, etc.

(4) Losses at all elbows, bends, or deviations from the straight.

(5) Losses at pipe junctions.

(6) Losses at sudden enlargements, or contractions in the area of the pipe.

(7) Loss at exit, due to the rejection of kinetic energy.

In every case these losses are approximately proportional to the square of the velocity, so that if v be the velocity we may write total loss of head $= {}_1H_7 = F v^2$.

Where a pipe line connects two reservoirs, the difference of head between their free surfaces must then equal $F v^2$, for the pipe to run full with velocity v.

If in Fig. 111 a horizontal $A B$ be drawn through the upper free surface, and if a series of ordinates be drawn vertically downwards from $A B$ representing on the vertical scale of the drawing the total loss of pressure energy per lb. from the pipe entrance to the particular point

considered, the ends of all such ordinates being connected give a curve called the **hydraulic gradient** for the pipe line. If now any datum line *C D* be taken, such that the height *C A* represents the potential energy of the water at *A*, the height of any point on the gradient line, above *C D*, will evidently represent the head available for producing flow at that point, while the pressure energy at all points on the hydraulic gradient will be zero. It follows that if a series of open stand pipes are erected at various points on the pipe line the free surfaces in these pipes will lie on the gradient line, while the pressure in the pipe at any

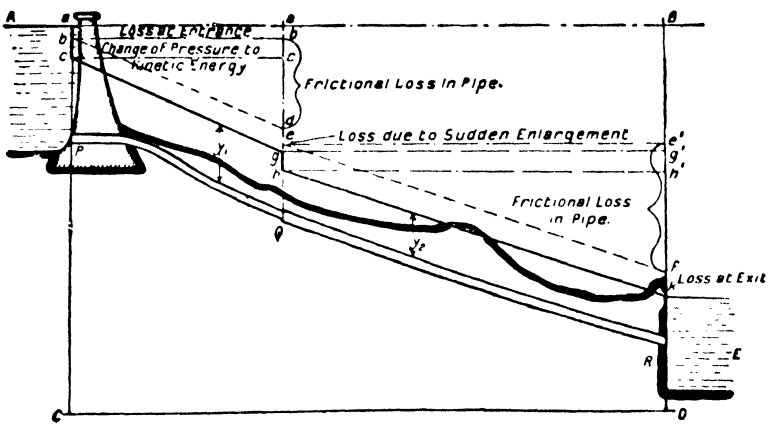


FIG. 111.—Hydraulic Gradient.

point will be measured by the vertical distance y_1, y_2, etc., of that point below the gradient.

The slope of the hydraulic gradient is termed the "**virtual slope**" of the pipe.

Fig. 111 represents the hydraulic gradient for the pipe line shown in elevation, which consists of a parallel pipe *P Q*, having a sudden enlargement of section at *Q*, remaining parallel from *Q* to *R*, and discharging at *R* into the service reservoir *E*.

Here *a b* represents frictional and eddy loss at entrance: *b d* represents frictional loss from *P* to *Q* $= \frac{f\, l_1\, v_1^2}{2\, g\, m_1}$ ft.-lbs. per lb.: *d e* represents loss due to sudden enlargement of section $= \frac{(v_1 - v_2)^2}{2\, g}$ ft.-lbs. per lb.: *e' f* represents loss due to friction between *Q* and *R* $= \frac{f\, l_2\, v_2^2}{2\, g\, m_2}$ ft.-lbs. per lb.: *f k* represents loss due to rejection of kinetic energy at exit.

The vertical distance from the datum to the broken line $a\ b\ d\ e\ f\ k$, now represents the sum of the (kinetic + pressure + potential) energies at any point, and if $c\ g$ be drawn parallel to $b\ d$ at a vertical distance $b\ c$ below $b\ d$, equal to $\frac{v_1^2}{2\ g}$, the distance from the datum to the line $c\ g$ now gives the sum of the (pressure + potential) energies at any point from P to Q, If similarly a line $h\ k$ be drawn parallel to $e\ f$, at a vertical distance below this equal to $\frac{v_2^2}{2\ g}$, the whole line $a\ b\ c\ g\ h\ k$ now represents the hydraulic gradient for the pipe.

The pressure in the pipe line will be everywhere greater than that of the atmosphere, so long as the pipes nowhere rise above the hydraulic gradient. If part of the pipe line be laid above the gradient line, the pressure in this portion of the pipe will be less than atmospheric, and any leakage at a joint allows air to be drawn in with a possible stoppage of flow. If the pipe rises above the gradient line by a distance equivalent to the barometric height, 34 feet, the flow will of necessity stop completely. Owing to the discharge of dissolved air from water at low pressures, the maximum height practically attainable is however much less than this. The syphon (Art. 82) is an instance of the pipe line being above the hydraulic gradient.

If then the suffixes A and E refer to the surfaces in the two reservoirs and if Z_A and Z_E are the heights of these free surfaces above some datum, we have, if the pipe discharges below the surface in the lower reservoir

$$Z_A - Z_E = {}_AH_E = \text{losses at entrance} + \text{losses in pipe} + \text{losses at exit.}$$

Art. 88.—Detailed Losses in Pipe Line.

(1) **Losses at Entrance.**—These depend on the form of entrance adopted. Thus with a bell mouthpiece the loss of head is about $\cdot 05\ \frac{v^2}{2\ g}$ feet of water (p. 127), while with a pipe projecting into the reservoir and forming a re-entrant mouthpiece, the loss, when running full, is $\frac{v^2}{2\ g}$ feet of water (p. 123).

Where the pipe opens flush with the side or bottom of the reservoir the loss of head becomes about $\cdot 47\ \frac{v^2}{2\ g}$ feet

From what has already been said, it will be seen that this loss of energy occurs simply in getting the water into the pipe, due to the

formation of a *vena contracta* and the subsequent re-enlargement of the stream to fill the pipe. This reduces the energy available for producing motion along the pipe and gives a reduced flow with the same total head.

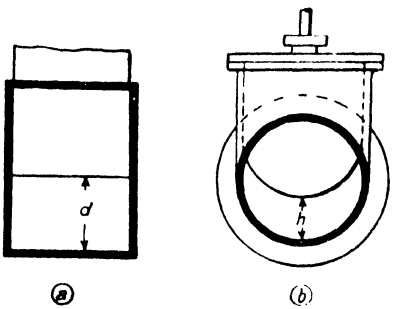


FIG. 112.

(2) **Frictional Losses in the Pipe.**—These have been considered in Chapter IX.

(3) **Losses at Valves, etc.**—The loss of head caused by a valve or sluice in a pipe line is partly due to the sinuous motion set up by the expansion of the stream to fill the pipe, after its contraction in passing the valve and partly to the irregularities in the shape of the water-way through the valve. The globe valve causes the greatest loss of head, and even when wide open gives a resistance equal to that of from 20 to 35 feet of clear straight pipe of its own diameter.

Loss caused by Sluice in Rectangular Pipe (Fig. 112 *a*).—Depth of pipe D. Depth of sluice opening d.[1]

$\frac{d}{D}$	·1	·2	·3	·4	·5	·6	·7	·8	·9	1·0
F	193	44·5	17·8	8·12	4·02	2·08	·95	·39	·09	·00

Here contraction is prevented in three directions, and for small openings the result of this is marked by the reduction in the value of F.

Gate Valve in Circular Pipe (Fig. 112 *b*).

$\frac{h}{D}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1·0
F	97·8	17·00	5·52	2·06	·81	·26	·07	·00

[1] Loss $= F\frac{v^2}{2g}$ feet. Results due to Weisbach.

Cock in Cylindrical Pipe (Fig. 112 *c*).—Here a = area of section through valve : A = area of pipe : θ = angle through which valve is turned.

θ	5°	10°	15°	20°	25°	30°	35°	40°	45°
$\frac{a}{A}$	·93	·85	·77	·69	·61	·535	·46	·385	·315
F	·05	·29	·75	1·56	3·1	5·47	9·68	17·3	31·2

θ	50°	55°	60°	65°	82°
$\frac{a}{A}$	·25	·19	·14	·09	Valve closed.
F	52·6	106	206	486	—

Throttle Valve in Cylindrical Pipe (Fig. 112 *d*).—These experiments

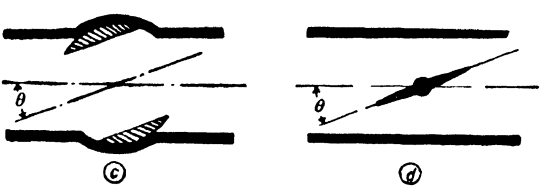


FIG. 112.

were carried out on pipes and valves of slightly under 2 inches diameter.

θ	5°	10°	20°	30°	40°	50°	60°	70°	90°
F	·24	·52	1·54	3·91	10·8	32·6	118	751	—

Experiments on a gate valve for a 24-inch pipe [1] gave values of F as indicated in the following table :—

[1] Kuichling, "Transactions American Society Civil Engineers," 1892, vol. 26.

$\frac{r}{D}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{7}{8}$
F	·8	1·6	3·3	22·7	41·2

This loss due to alteration in area and shape of section is serious in all hydraulic machinery when working at high velocities, and especially in machines of the piston type.

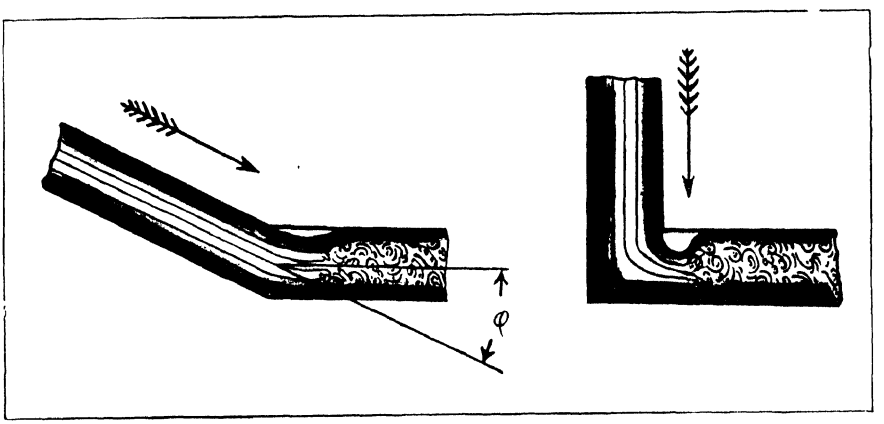


FIG. 113.

(4) **Losses at Bends and Elbows.**—Whenever the direction of motion of a stream is abruptly changed, as at a sharp bend or elbow, a loss of head is experienced which appears to be due to the formation of a *vena contracta* on passing the elbow and to the subsequent re-enlargment and shock which then takes place (Fig. 113). With an easy bend, experiments[1] by *Dr. Brightmore* show that the state of affairs is very different, the water flowing round the bend with a motion approximating to that in a free vortex, its velocity being greatest at the inside and least at the outside of the bend. At the same time a transverse circulatory current is set up. In these experiments it was shown that in such a case the loss due to the bend does not occur so much in the bend itself as in the portion of straight pipe immediately following, where the equalisation

[1] "Proceedings Institute Civil Engineers," 1906–7, vol. 169, p. 315.
[2] See also J. Eustice, "Proc. Roy. Soc.," A., vol. 85, 1911, p. 119

of velocities in the stream of water gives rise to eddy production. Using cast-iron pipes of 3″ and 4″ diameter, the loss in the bend itself was found to be almost identical with (sometimes less than) the loss in a corresponding length of straight pipe. Experiments tend to show that so long as the degree of curvature of the bend is sufficient to set up the same distribution of velocity, the overall loss is sensibly independent of the curvature. With reflex bends, unless there is an adequate length of pipe between adjacent bends, the loss is greater than in similar bends having continuous curvature in one direction. Otherwise the total loss is much the same.[1]

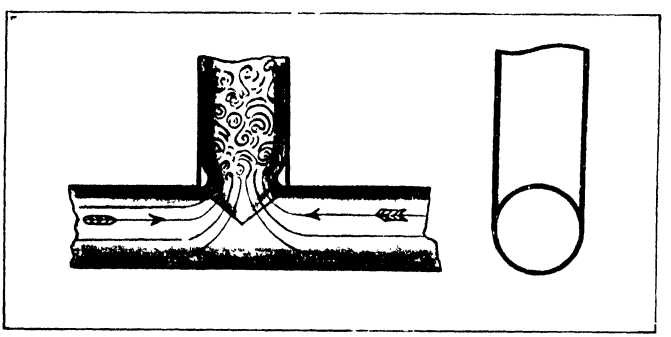


FIG. 114.

The first experiments of any note on the resistance of bends are due to *Weisbach*, who from experiments on pipes of $1\frac{1}{4}$ inches diameter, deduced the formula

$$h_B = \text{loss of head due to bend} = F \frac{v^2}{2g} \times \frac{\theta^\circ}{180^\circ} \text{ feet,}$$

where θ is the angle through which the bend is carried.

For circular pipes, F has the value $\left[\cdot 131 + 1\cdot 847 \left(\frac{r}{R}\right)^{\frac{7}{2}}\right]$ where r is the radius of the pipe, and R the radius of the bend.

For pipes of rectangular section, $F = \cdot 124 + 3\cdot 104 \left(\frac{S}{2R}\right)^{\frac{1}{2}}$ where S is the length of side of a section parallel to the radius of curvature of the bend.

These experiments were, however, not numerous and were only carried

[1] "Flow of Water through Rectangular Pipe Bends," Davies and Puranik, *Jour. Inst. C. E.*, No. 4, Feb. 1936, p. 83. See also Williamson, *Jour. Inst. C. E.*, No. 6, April, 1939, p. 451, and Addison, Paper 5084, *Jour. Inst. C. E.*, No. 8, Oct. 1937, p. 561.

out on bends of small radius, so that not much reliance is to be placed in the results.

For sharp bends or elbows (Fig. 113), Weisbach deduced the formula

$$\text{Loss} = F \frac{v^2}{2g} \text{ feet}$$

$$\text{where } F = \cdot 946 \sin^2 \frac{\theta}{2} + 2 \cdot 05 \sin^4 \frac{\theta}{2},$$

θ being the angle of deviation of the elbow.

With a tee branch pipe (Fig. 114) the loss is rather greater than in a right-angled elbow.

Recent experiments by the author on elbows of rectangular section, $\frac{1}{2}$ in. $\times$ 1 in., showed the loss to be proportional to v^2 for all velocities up to 22 feet per second, and gave the following values for F:

θ	90°	60°	45°	30°	15°
F. (Author)	1·20	·492	·263	·111	·0240
F. (Weisbach)	·99	·365	·183	·0728	·0222

These correspond to the relationship $F = \cdot 0000676\ \theta^{2 \cdot 17}$ where θ is in degrees. The values are considerably greater than those given by Weisbach's formula, and while the difference may be due to some extent to the difference in the shapes and sizes of the passages, recent experiments on elbows of circular cross section tend rather to confirm the author's value when $\theta = 90°$.

Experiments by *Alexander*,[1] by *Williams*,[2] and by *Brightmore*,[3] the former using varnished wooden pipes of $1\frac{1}{4}$ inches diameter, Williams using asphalted pipes of 12, 16, and 30 inches diameter, and Brightmore using cast-iron pipes of 3 and 4 inches diameter, indicate that the additional loss due to the curvature of a pipe does not, as might be expected, diminish uniformly as the radius of curvature increases, but, after attaining a minimum value for a value of $R = 5\,r$ (Williams and Alexander), $R = 7 \cdot 5\ r$ (Brightmore), increases slightly to a point where

[1] "Proceedings Institute Civil Engineers," vol. 159, p. 341.
[2] "Proceedings American Society Civil Engineers," 1901, p. 314.
[3] "Proceedings Institute Civil Engineers," 1906–7, vol. 169, p. 315.

R = about 13 r, and afterwards diminishes to zero with a curve of infinite radius. The experiments further show that the power of v, to which the loss is proportional in a gradual bend, is the same for the bend as for the straight pipe. Alexander, as the result of his own and of Williams' experiments, concludes that the additional loss due to a bend of radius $R = 5\ r$, is equivalent to that offered by a length of straight pipe equal to 3·38 l, where l is the length of the curved portion of the pipe. Thus if the angle of the bend is 90° this makes the equivalent length equal to 13·3 d, where d = pipe diameter in feet. This agrees with the results of a number of experiments made at the Yorkshire College,[1] from which it was concluded that the resistance of an easy right-angled bend is equivalent to that in a straight pipe 10 to 15 diameters in length, while for a sharp right-angled bend or elbow the equivalent length is from 30 to 36 diameters.

The following values of F are deduced from Brightmore's results:—

Value of $R \div r$.

—	Vel. f.s.	Elbow.	4	8	12	16	20
Pipe, 3″ diam.	5·0	1·14	·43	·32	·37	·32	·26
,,	7·6	1·26	·43	·32	·40	·33	·19
,,	10·0	1·17	·42	·31	·39	·35	·19

Value of $R \div r$.

—	Vel. f.s.	Elbow.	4	8	12	16	20
Pipe, 4″ diam.	5·0	1·14	·37	·26	·37	·26	·26
,,	7·6	1·19	·38	·30	·31	·27	·21
,,	10·0	1·17	·39	·30	·32	·30	·21

Experiments by *E. W. Schoder*[2] on right-angled bends having radii from 1·34 to 20 pipe diameters, on a 6-inch wrought-iron pipe, give the

[1] *Engineering*, September 25, 1896, p. 390.

[2] "Trans. American Society Civil Engineers," 1908, pp. 34, 416–445.

following approximate values for F. The resistance curves appear to attain a minimum where $R = 10\ r$; rise to a maximum for $R = 14\ r$; attain a second and lower minimum for $R = 30\ r$; and afterwards show increased values as R increases within the limits of the experiments.[1]

Vel. f.s.	$R \div r$.						
	2·68	5	10	15	20	30	40
5	·41	·36	·36	·39	·36	·34	·41
10	·45	·32	·27	·34	·27	·23	·37
16	·49	·36	·34	·38	·32	·28	·40

The general conclusion to be drawn from the foregoing results is that the radius of a bend should, if possible, be between 5 r and 10 r, when the loss of head will be very approximately $\cdot 3\ \frac{v^2}{2g}$.

Loss of Head in Commercial Screw Pipe Elbows and Tees.— Experiments

Loss of Head in Commercial Elbows and Tees.

(1) Experimenter.	(2) Fitting name and type.	(3) Velocity in pipe feet per second.	(4) Excess loss of head caused by fitting feet of water.	(5) Length of straight pipe required to give same excess loss of head (In feet)	(6) (In pipe diameters)	(7) Value of F in loss of head $= F\frac{v^2}{2g}$
Bain . . .	¾-inch black mall. elbow (old)	2	0·051	1·54	23	0·82
		5	0·29	,,	,,	0·76
		10	1·10	,,	,,	0·72
,, . . .	¾-inch galv. mall. elbow (new)	2	0·0355	1·08	16	0·57
		5	0·206	,,	,,	0·53
		10	0·78	,,	,,	0·50
,, . . .	1-inch black mall. elbow (old)	2	0·047	1·90	23	0·76
		5	0·27	,,	,,	0·70
		10	1·02	,,	,,	0·67

[1] See also L. R. Balch, "Bulletin, University of Wisconsin," No. 578, 1913, for experiments on bends in a 3-inch pipe. See also *Jour. Inst. C. E.*, No. 4, 1947–8, p. 350.

LOSS OF HEAD IN COMMERCIAL ELBOWS AND TEES (*continued*).

(1) Experimenter.	(2) Fitting name and type.	(3) Velocity in pipe feet per second.	(4) Excess loss of head caused by fitting feet of water.	(5) Length of straight pipe required to give same excess loss of head. (In feet)	(6) (In pipe diameters)	(7) Value of F in loss of head $= F\frac{v^2}{2g}$
Bain.	1-inch cast-iron elbow (old)	2	0·063	2·55	31	1·02
		5	0·365	,,	,,	0·95
		10	1·37	,,	,,	0·90
Davis	2-inch mall. iron elbow	2	0·046	4·81	28	0·74
		5	0·28	5·82	34	0·72
		10	1·07	6·46	38	0·69
,,	2-inch cast-iron elbow	2	0·080	8·37	49	1·29
		5	0·52	10·8	63	1·34
		10	2·04	12·3	72	1·32
Daley	3-inch cast-iron elbow	1	0·0088	5·58	20	0·57
		5	0·211	6·20	24	0·54
		10	0·84	6·67	26	0·51
		25	5·15	7·23	28	0·53
,,	4-inch cast-iron elbow	1	0·0107	9·10	27	0·69
		5	0·234	,,	,,	0·61
		10	0·89	,,	,,	0·58
		25	5·10	,,	,,	0·54
Schoder	6-inch cast-iron elbow	3	0·077	15·3	30	0·55
		5	0·195	14·4	28	0·50
		10	0·75	14·5	29	0·48
		16	1·90	14·7	29	0·48
Davis	2-inch cast-iron tee and plug (water leaving branch)	2	0·115	12·0	70	1·85
		5	0·74	15·4	90	1·91
		10	2·92	17·6	102	1·88
,,	Same tee (water entering branch)	2	0·089	9·3	54	1·43
		5	0·60	12·5	72	1·55
		10	2·52	15·2	88	1·63
Daley	3-inch cast-iron tee and plug (water entering branch)	1	0·0392	24·9	96	2·53
		5	0·950	27·9	104	2·45
		10	3·76	29·8	115	2·43
		25	22·65	31·8	123	2·33
,,	Same tee (water leaving branch)	1	0·0242	15·4	59	1·56
		5	0·563	16·5	64	1·45
		10	2·22	17·6	68	1·43
		25	13·40	18·8	73	1·37
,,	3-inch cast-iron tee and dead end (water entering branch)	1	0·0202	12·8	50	1·30
		5	0·352	10·6	41	0·91
		10	1·41	11·2	43	0·91
		25	8·65	12·2	47	0·89

LOSS OF HEAD IN COMMERCIAL ELBOWS AND TEES (*continued*).

(1) Experimenter.	(2) Fitting name and type.	(3) Velocity in pipe feet per second.	(4) Excess loss of head caused by fitting feet of water.	(5) Length of straight pipe required to give same excess loss of head. (In feet)	(6) (In pipe diameters)	(7) Value of F in loss of head $= F\frac{v^2}{2g}$
Daley . . .	Same tee (water leaving branch)	1	0·0161	10·2	40	1·04
		5	0·275	8·2	32	0·71
		10	1·16	9·2	36	0·75
		25	7·40	10·4	40	0·76
,, . .	4-inch cast-iron tee and plug (water entering branch)	1	0·0291	24·8	73	1·88
		5	0·600	22·7	67	1·55
		10	2·185	21·8	64	1·41
		25	11·80	20·1	59	1·22
,, . .	Same tee (water leaving branch)	1	0·0223	18·9	55	1·44
		5	0·483	18·3	54	1·21
		10	1·82	18·3	54	1·17
		25	10·70	19·2	54	1·10
,, . .	4-inch cast-iron tee and dead end (water entering branch)	1	0·0199	16·9	50	1·28
		5	0·455	17·4	51	1·25
		10	1·75	17·6	52	1·13
		25	10·40	17·7	52	1·07
,, . .	Same tee (water leaving branch)	1	0·0175	14·9	44	1·13
		5	0·378	14·3	42	0·97
		10	1·50	15·1	44	0·97
		25	9·20	15·7	46	0·95
,, . .	4-inch cast-iron tee filled in to make a square elbow	1	0·0173	14·7	43	1·12
		5	0·427	16·2	48	1·10
		10	1·66	16·7	49	1·07
		25	10·40	17·7	52	1·07
,, . .	3-inch coupling			0·25	1·0	
,, . .	4-inch coupling			0·65	1·9	

by Professor *E. W. Schoder*, *Messrs. D. H. Daley*, *P. Bain*, and *G. J. Davis*,[1] on common cast-iron and malleable-iron short turn, screw end elbows and tees in wrought-iron pipe lines give the results shown in the preceding table. The range of sizes is from $\frac{3}{4}$ inch to 6 inches diameter. In these fittings the inside "burr" at the ends of the pipes, caused by the wheel pipe cutter, had been reamed out. Two sets of experiments were carried out with each tee pipe. In the one, a short length of pipe with a

[1] The *Cornell Civil Engineer*, Vol. 20, No. 3, December, 1911.

cap was screwed on to the idle end forming a "dead end," while in the other the idle end of the tee was capped directly by a screw plug.

(5) **Losses at Sudden Changes in Section of the Pipe.**—These have been already considered in Art. 44.

(6) **Losses at the Exit from the Pipe.**—Where a pipe discharges below the surface, it is possible by using a diverging outlet, to convert a certain proportion, up to about ⅔, of the kinetic energy of pipe flow, into pressure energy. With this exception the whole of the kinetic energy is dissipated in eddy production in the surrounding mass of water.

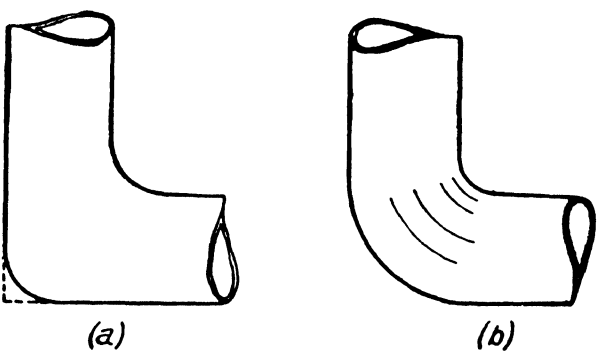


FIG. 114A.

Experiment [1] shows that by reducing the outer radius of a bend so as to increase the area at the mid section as in Fig. 114A (*a*) the losses are less than in the normal type. An extreme case is the "Kaplan" bend, which has been extensively used in turbine draft tubes, and in which the outer radius is zero.

In large bends, in pipes or ducts of rectangular cross section, the loss may be appreciably reduced by the introduction of guide vanes Fig. 114A (*b*) over the outer radius of the bend.

ART. 89.—FLOW IN LONG PIPES.

A consideration of the preceding section will show that in a long pipe the losses at bends, at entrance and exit, and at changes of section, are

[1] *Trans. Am. Soc. Mech. E.*, Vol. 43, 1921, p. 277. Also Gibson, Aspey and Tattersall, *Proc. Inst. C. E.*, Vol. 231, 1930-1, Part I, p. 211.

usually so small in comparison with the friction losses as to be negligible, so that for a long pipe connecting two reservoirs, the whole resistance may be taken to be given by

$$\Sigma \frac{f\,l\,v^2}{2\,g\,m} = \frac{f}{2\,g}\left\{\frac{l_1 v_1^2}{m_1} + \frac{l_2 v_2^2}{m_2} + \frac{l_3 v_3^2}{m_3} + - - -\right\} \text{ feet head}$$

v_1, v_2, etc., being the velocities, and m_1, m_2, etc., the hydraulic mean depths, in the lengths l_1, l_2, etc., of the pipe.

In short pipes the losses due to velocity changes become of greater importance as the length of pipe diminishes, and for pipes of lengths less than 100 diameters will, in general, be important.

Art. 90.—Time of Discharge Through an Uniform Pipe Line.

If two reservoirs of area A_1 and A_2 are connected by a single pipe of diameter d and length l, and if v be the velocity in the pipe when h is the difference in surface level in the two reservoirs, we have

$$h = \frac{v^2}{2\,g}\left\{K + \frac{4\,f\,l}{d}\right\} \text{ feet.}$$

where K = coefficient of loss at entrance and exit.

If the pipe is long this may be written

$$h = \frac{v^2}{2\,g}\left\{\frac{4\,f\,l}{d}\right\} \text{ without sensible error.}$$

In this case $v = \sqrt{\frac{2\,g\,h\,d}{4\,f\,l}}$ feet per second.

Also $-\frac{d\,h}{d\,t}$ = relative velocity of surfaces A_1 and A_2

$$= v \cdot \frac{\pi\,d^2}{4}\left\{\frac{1}{A_1} + \frac{1}{A_2}\right\}$$

$$\therefore \quad -\frac{d\,h}{d\,t} = \sqrt{\frac{2\,g\,d}{4\,f\,l}} \cdot \frac{\pi\,d^2}{4} \cdot \left\{\frac{1}{A_1} + \frac{1}{A_2}\right\}\sqrt{h}$$

The time ($t_2 - t_1 = t$) necessary to reduce the difference in level from H_1 to H_2 is then got by integrating this expression between the given limits

$$\therefore \quad t = \frac{8\sqrt{4\,f\,l}}{\pi\,d^2\left\{\frac{1}{A_1} + \frac{1}{A_2}\right\}\sqrt{2\,g\,d}}\left\{H_1^{\frac{1}{2}} - H_2^{\frac{1}{2}}\right\} \text{ secs.}$$

$$= \frac{2\sqrt{f\,l}}{\pi\left\{\frac{1}{A_1} + \frac{1}{A_2}\right\}d^{\frac{5}{2}}}\left\{H_1^{\frac{1}{2}} - H_2^{\frac{1}{2}}\right\} \text{ secs.} \qquad (1)$$

Thus the time of filling a reservoir A_2 from a second reservoir A_1, whose level remains constant, *i.e.*, $A_1 = \infty$ is given by

$$t = \frac{2\sqrt{f}\,l \cdot A_2}{\pi\, d^{\frac{5}{2}}} \left\{ H_1^{\frac{1}{2}} - H_2^{\frac{1}{2}} \right\} \text{ secs.}$$

If $h = \dfrac{f' l v^n}{d^e}$, equation (1) becomes

$$t = \frac{4\,(f\,l)^{\frac{1}{n}}}{\pi \left(\dfrac{1}{A_1} + \dfrac{1}{A_2} \right) d^{2+\frac{e}{n}}} \cdot \frac{n}{n-1} \cdot \left\{ H_1^{\frac{n-1}{n}} - H_2^{\frac{n-1}{n}} \right\}.$$

Art. 91.—Branch Mains.

Where a large main is divided into a number of branches, d_1, d_2, d_3, etc., of different lengths and discharging into reservoirs against different heads, the flow along each of these branches may be determined as follows. Let Fig. 115 represent the arrangement diagrammatically, the reservoir A supplying the pipe a, which in turn supplies the reservoirs B, C, and D through pipes b, c, and d. Let Z_A, Z_B, Z_C, Z_D represent the heights above some common datum, of the free surfaces in the respective reservoirs, and let Z_J represent the height of the junction J. Let v_a, v_b, v_c, etc., represent the velocities in the various pipes, and let A_a, A_b, A_c, etc., represent their areas.

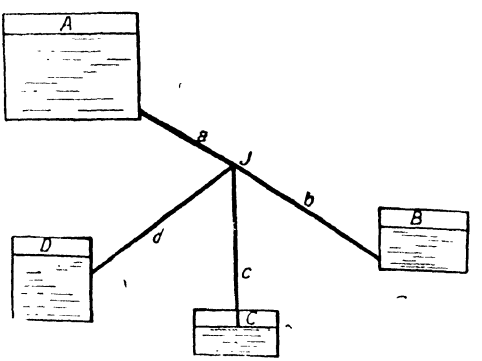


Fig. 115.

Then

$$Z_A = Z_J + \frac{p_J}{W} + \frac{v_a^2}{2g} + {}_AH_J \quad \left\{ \begin{array}{l} \text{where } {}_AH_J \text{ represents the} \\ \text{loss of head from } A \text{ to } J. \end{array} \right. \qquad (1)$$

Similarly

$$Z_J + \frac{p_J}{W} + \frac{v_b^2}{2g} = Z_b + {}_JH_B \qquad (2)$$

$$Z_J + \frac{p_J}{W} + \frac{v_c^2}{2g} = Z_c + {}_JH_C \qquad (3)$$

$$Z_J + \frac{p_J}{W} + \frac{v_d^2}{2g} = Z_D + {}_JH_D \qquad (4)$$

Also if all the pipes run full we have for continuity of flow

$$A_a v_a = A_b v_b + A_c v_c + A_d v_d. \qquad (5)$$

Since, in these five equations, the only unknowns are the four velocities and the pressure at J (the losses ${}_AH_J$, ${}_JH_B$, etc., being determinate in terms of the velocities), the equations are perfectly determinate and a solution will give the velocities in the various pipes in terms of known quantities. The quantities discharged through each pipe may then be determined.

EXAMPLE.

A reservoir A supplies three supplementary reservoirs B, C, D through a single 24-inch pipe divided at J into two 12-inch and one 18-inch pipe leading respectively to reservoirs B, C, and D. If the lengths of these pipes are $l_a = 500$ feet, $l_b = 1{,}000$ feet, $l_c = 1{,}500$ feet, $l_d = 3{,}000$ feet; and if $Z_A = 50$ feet, $Z_B = 30$ feet, $Z_c = 10$ feet, $Z_D = 20$ feet, $Z_J = 40$ feet, determine the velocities of flow in each pipe. Take $f = \cdot 005$, and neglect all except friction losses.

Here ${}_AH_J$ in equation (1) above becomes

$$\frac{\cdot 005 \times 500 \times 4}{2} \frac{v_a^2}{2g} = \frac{5 v_a^2}{2g}.$$

Determining all such values, and substituting, equations (1) to (5) above become

$$\frac{p_J}{W} + 6 \frac{v_a^2}{2g} = 10 \qquad (1)$$

$$\frac{p_J}{W} - 19 \frac{v_b^2}{2g} = -10 \qquad (2)$$

$$\frac{p_J}{W} - 29 \frac{v_c^2}{2g} = -30 \qquad (3)$$

$$\frac{p_J}{W} - 39 \frac{v_d^2}{2g} = -20 \qquad (4)$$

$$4 v_a = v_b + v_c + 2{\cdot}25 v_d. \qquad (5)$$

Subtracting (2) from (1), (3) from (4), and (3) from (2) we eliminate $\frac{p_J}{W}$, and get

$$6 v_a^2 + 19 v_b^2 = 40 g \qquad (6)$$

$$29 v_c^2 - 39 v_d^2 = 20 g \qquad (7)$$

$$29 v_c^2 - 19 v_b^2 = 40 g \qquad (8)$$

Determining v_b, v_c, and v_d in terms of v_a from equations (6), (7), and (8), and substituting these values in (5), this becomes, on writing $g = 32{\cdot}2$, and simplifying

$$4 v_a = \sqrt{67{\cdot}75 - {\cdot}316 v_a^2} + \sqrt{88{\cdot}8 - {\cdot}207 v_a^2} + 2{\cdot}25 \sqrt{49{\cdot}5 - {\cdot}154 v_a^2}. \qquad (9)$$

Writing this as 4 $v_a - \phi(v_a) = F(v_a)$ (where $\phi(v_a)$ is the right-hand side of this equation) and giving v_a the value

6·0 this makes $(F\ v_a) = -7{\cdot}32$.

Putting $v_a = 6{\cdot}5$ „ „ $F(v_a) = -5{\cdot}13$.

„ $v_a = 7{\cdot}5$ „ „ $F(v_a) = +\ {\cdot}26$.

Plotting these values of v_a and $F(v_a)$ we find that $v_a = 7{\cdot}48$ makes $F(v_a) = 0$, and therefore satisfies the above equation (9).

Substituting this value of v_a in equations (6), (7) and (8) gives $v_b = 7{\cdot}08$, $v_c = 8{\cdot}79$, $v_d = 6{\cdot}39$, while from equation (1) we have

$$\frac{p_J}{W} = 10 - 6\frac{v_a^2}{2g} = 4{\cdot}78 \text{ feet of water.}$$

Art. 92.—Multiple Supply.

Where more than one reservoir or source of supply feeds into one common pipe, the surfaces of the water in the supply reservoirs being at different levels, it becomes necessary to determine what share of the total flow each of these sources of supply contributes. *E.g.*, in the case illustrated in Fig. 115A the two reservoirs A and B, with surface levels Z_A and Z_B above datum, feed through pipes of areas A_a and A_b, into a common pipe of area A_c, at J, the joint flow passing into a reservoir at C, with its surface at a height Z_c above datum level.

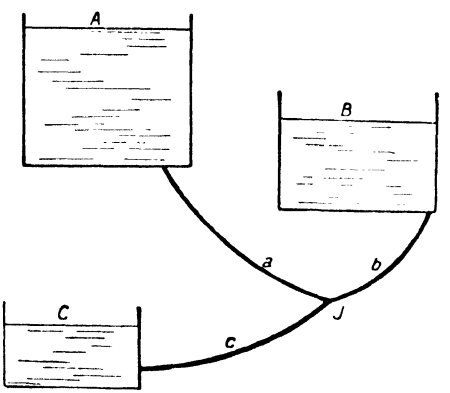


Fig. 115A.

Here we have

$$Z_A = Z_J + \frac{p_J}{W} + \frac{v_a^2}{2g} + {}_AH_J \quad \left\{ {}_AH_J \text{ representing the loss from } A \text{ to } J. \right. \tag{1}$$

$$Z_B = Z_J + \frac{p_J}{W} + \frac{v_b^2}{2g} + {}_BH_J \tag{2}$$

$$Z_C + {}_JH_C = Z_J + \frac{p_J}{W} + \frac{v_c^2}{2g} \tag{3}$$

Also if all the pipes run full we have for continuity of flow

$$A_a v_a + A_b v_b = A_c v_c. \tag{4}$$

Then since ${}_AH_J$, ${}_BH_J$, ${}_JH_C$ are respectively proportional to v_a^2, v_b^2, v_c^2, and may be determined in terms of these velocities when the construc-

tion and sizes of pipes are known, the only unknowns are the three velocities v_a, v_b, v_c, and the pressure p_J. These equations may then be solved and the unknown factors determined. A similar method of solution will apply to any case of multiple supply, with any number n of sources of supply. Here we shall have $n + 2$ equations formed, from which the $(n + 1)$ velocities and the pressure at the junction may be determined.

Art. 93.—Flow along a Bye-Pass.

If p_1 and p_2 lbs. per square foot are the pressures in the main pipe at the entrance to and exit from the bye-pass pipe or diversion (Fig. 116), we have, putting V and v for the velocities in the large and small pipes respectively,

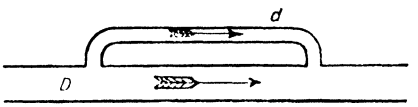


Fig. 116.

$$\frac{p_1 - p_2}{W} = \frac{4 f L V^2}{2 g D} = \frac{4 f l v^2}{2 g d} + K \frac{v^2}{2 g}$$

where $K \dfrac{v^2}{2 g}$ represents the loss in the bye-pass at entrance and exit and at any valves.

From this we have, assuming f to have the same value in the two pipes,

$$\frac{L V^2}{D} = \frac{l v^2}{d} + K' v^2 = \left\{ \frac{l}{d} + K' \right\} v^2$$

$$\therefore \quad \frac{V^2}{v^2} = \frac{D}{L} \left\{ \frac{l}{d} + K' \right\}.$$

If Q and q are the quantities flowing along the two pipes

$$Q = \frac{V . \pi D^2}{4} \qquad q = \frac{v \pi d^2}{4}$$

$$\therefore \quad \frac{Q}{q} = \frac{V D^2}{v d^2} = \sqrt{\frac{D}{L} \left\{ \frac{l}{d} + K' \right\}} \cdot \frac{D^2}{d^2}$$

$$= \sqrt{\left(\frac{D}{d} \right)^5 \cdot \frac{l + K' d}{L}}$$

The proportion of the whole flow which passes along the diversion or $\dfrac{q}{Q + q}$ is given by

$$q = \frac{1}{1 + \sqrt{\left(\dfrac{D}{d} \right)^5 \cdot \dfrac{l + K' d}{L}}} (Q + q).$$

Art. 94.—Flow through Pipes coupled up in Parallel (Fig. 117).

Here pipes d_1, d_2, etc., of length l_1, l_2, etc, couple up two vessels whose pressures at the points of exit and inlet are p_1 and p_2 lbs. per sq are foot. We now have

$$\frac{p_1 - p_2}{W} = 4f\frac{l_1 v_1^2}{2g d_1} = 4f\frac{l_2 v_2^2}{2g d_2} = 4f\frac{l_3 v_3^2}{2g d_3} = \text{etc.},$$

neglecting all losses except those due to friction.

If Q= total flow through pipes in cubic feet per second, we have

$$Q = \frac{\pi}{4}\{v_1 d_1^2 + v^2 d_2^2 + v_3 d_3^2 + \text{etc.}\}$$

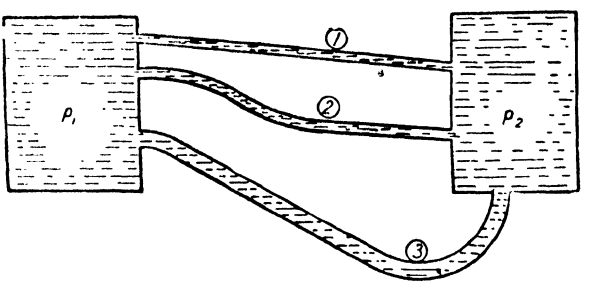


Fig. 117.

$\therefore$ Writing $v d^2 = \sqrt{\frac{l v^2}{d}} \cdot \frac{d^{\frac{5}{2}}}{l^{\frac{1}{2}}}$ we get

$$Q = \frac{\pi}{4}\sqrt{\frac{l_1 v_1^2}{d_1}}\left\{\frac{d_1^{\frac{5}{2}}}{l_1^{\frac{1}{2}}} + \frac{d_2^{\frac{5}{2}}}{l_2^{\frac{1}{2}}} + \frac{d_3^{\frac{5}{2}}}{l_3^{\frac{1}{2}}} + \text{etc.}\right\}.$$

And since $\frac{p_1 - p_2}{W} = \frac{4f}{2g} \cdot \frac{l_1 v_1^2}{d_1}$, this becomes

$$Q = \frac{\pi}{4}\sqrt{\frac{p_1 - p_2}{W} \cdot \frac{g}{2f}}\left\{\Sigma \frac{d^{\frac{5}{2}}}{l^{\frac{1}{2}}}\right\}.$$

Example.

Determine the diameter of a pipe of length l, to give the same discharge as two parallel pipes of the same length and of diameter d.

Discharge from two small pipes $= \frac{\pi}{4}\sqrt{\frac{p_1 - p_2}{W} \cdot \frac{g}{2f}}\left(\frac{2\,d^{\frac{5}{2}}}{l^{\frac{1}{2}}}\right)$

„ „ large pipe $= \frac{\pi}{4}\sqrt{\frac{p_1 - p_2}{W} \cdot \frac{g}{2f}}\left(\frac{D^{\frac{5}{2}}}{l^{\frac{1}{2}}}\right)$

$\therefore$ For same discharge $D^{\frac{5}{2}} = 2\,d^{\frac{5}{2}}$

$$\therefore D = 2^{\frac{2}{5}}\,d = 1{\cdot}32\,d.$$

Thus, assuming the resistance to vary as v^2, one pipe 15·84 inches diameter would give the same discharge as two 12-inch pipes.

Assuming more correctly the results of the experiments quoted on p. 222, and taking

$$Q = C\left(\frac{H}{l}\right)^{\frac{1}{2}} d^{2\cdot 58} = \frac{C}{l^{\frac{1}{2}}}\sqrt{\frac{p_1 - p_2}{W}}\,d^{2\cdot 5},$$

we should have in the above example, $D^{2\cdot 58} = 2\,d^{2\cdot 58}$

$$\therefore D = 2^{\frac{1}{2\cdot 58}} \cdot d = 1{\cdot}308\,d.$$

or one pipe 15·70 inches would now give the same discharge as the two 12-inch pipes.

This duplication in parallel of supply pipes is very common in large water supply systems, and is also of use where, in the case of a fire main, the pressure at the nozzle is insufficient to give the required velocity of flow. By coupling a second line of hose between the supply main and the hydrant box, the fall in pressure at the nozzle is much reduced and consequently a higher jet is obtained.

Art. 95.—Main of Uniform Diameter in which the Discharge diminishes as the Length increases.

In water mains for domestic supply, the main pipe is tapped at intervals by service pipes, so that the volume carried gradually diminishes. Let the rate of diminution be uniform and equal to q cubic feet per second per foot run of the pipe. Let l be the total length; H the total loss of head in the pipe; Q the initial supply; V the initial velocity of flow. Then the volume passing any point P distant x from the entrance, will be $Q - qx$ *c. f. s.* Let h_x be the loss of head up to this point.

Then since the loss of head in a length δx at P

$$= \frac{f v^2}{2\,g\,m}\,\delta x = \frac{f \cdot \delta x \cdot V^2}{2\,g\,m}\left(\frac{Q - q\,x}{Q}\right)^2 \text{ feet,}$$

$$\text{the loss } h_x, \text{ up to } P = \frac{f\,l^2}{2\,g\,m\,Q^2}\int_0^x (Q - q\,x)^2\,dx$$

$$= \frac{f\,l^2}{2\,g\,m\,Q^2}\left[Q^2\,x - Q\,q\,x^2 + \tfrac{1}{3}\,q^2\,x^3\right] \text{ feet,}$$

and since, if Q_x is the flow at this point, $Q = Q_x + qx$, this can be written

$$h_x = \frac{f\,l^2}{2\,g\,m\,Q^2}\left[Q_x^2\,x - Q_x\,q\,x^2 + \tfrac{1}{3}\,q^2\,x^3\right] \text{ feet,}$$

At the discharge end of the pipe line $Q_x = o$; $q\,x = Q$; $x = l$.

$$\therefore H = \frac{f\,V^2}{2\,g\,m\,Q^2}\cdot\left[\tfrac{1}{3}\,Q^2\right] l = \tfrac{1}{3}\,\frac{f\,l\,V^2}{2\,g\,m},$$

or the loss of head is one-third that which would occur in the same pipe line if the flow were uniform instead of gradually diminishing.

Art. 96.—Distribution of Water.

The consumption of water for domestic and trade purposes varies largely from town to town. In Great Britain the domestic supply ranges from about 18 to 25 gallons per head per day, while in the United States the consumption ranges from about 40 to 200 gallons per day. During the summer months the consumption per day may be as much as 40 per cent. greater than the average for the whole year. The hourly fluctuation in demand necessitates a maximum rate of supply about 33 per cent. greater than the average daily rate taken over 24 hours, so that the supply pipes should be sufficiently large to give a temporary supply at least 85 per cent. greater than the yearly average. It is usual to allow for a maximum rate of flow equal to at least twice the mean rate to be anticipated when reasonable increases of population have been taken into account.

In order to render the fluctuating run-off from the catchment area, available for the fluctuating demand, a storage reservoir is necessary to catch and store the run-off when in excess of the temporary demand, and when the average monthly run-off and probable demand is known with fair accuracy, the necessary volume of this may be calculated. Usually the reservoir is at some considerable distance from the town to be supplied, and to avoid laying down a long pipe line to supply water at the maximum hourly rate demanded, and to minimise to some extent the inconvenience caused by a possible break in this line, a smaller "service" reservoir, capable of storing about three day's supply is constructed as near to the town as is convenient, and, if feasible, at a

height to permit the service mains to be supplied with water under a pressure of from 100 to 150 feet. Where this is not possible it becomes necessary to adopt some pumping system and to pump water under pressure into the pipes as required.

Where the difference of level between the supply and service reservoirs is great, the whole of the supply pipe line may be put under considerable statical pressure if a valve is fitted at its outlet, and the cost of a pipe line designed to withstand such a pressure may be excessive. With a view of reducing the maximum pressures, the regulating valve may be fitted at the pipe entrance, or one or more small reservoirs—known as **break-pressure reservoirs**—may be provided at suitable intervals in the

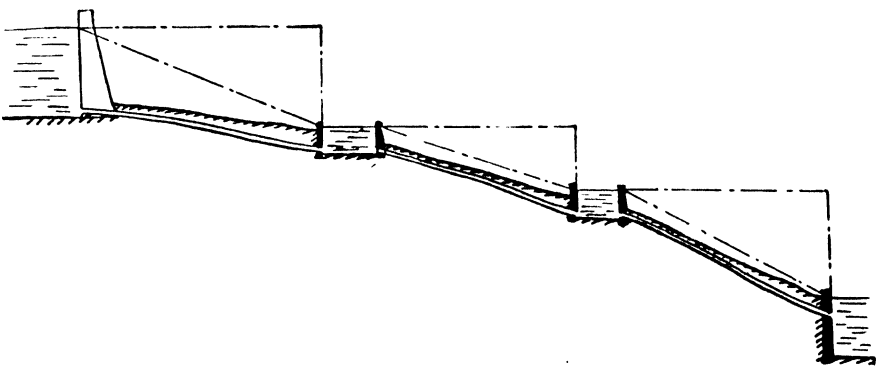


FIG. 118.—Break-Pressure Reservoirs.

line, as shown in Fig. 118. The service level in each of these reservoirs is in the gradient line for the system, and the hydrostatic pressure in any length of the pipe cannot now exceed that due to the next higher reservoir.

In order to reduce the risk of a total stoppage of the supply in case of accident to the supply main, the latter may be laid in duplicate, two pipes of smaller diameter taking the place of the single larger pipe. Where the lines are long, there should be cross connections with valves, so that in case of a break in either line a short section only need be cut out. This gives a greater discharge than would be possible with the line single throughout.

For pipes up to about 24 inches in diameter the cost of the pipe itself, the jointing, excavating, and laying, is roughly proportional to the pipe diameter, and the cost of a twin pipe with cross connections and valves is about 50 per cent. greater than that of a single pipe line. For larger

pipes, up to 72 inches diameter, the cost increases more rapidly than the diameter, being roughly proportional to its 1·5th power, and the cost of a twin line is about 30 per cent. greater than that of a single line.

Pipe Line Accessories.—Where the summit of a pipe line approaches the gradient line, and generally at the highest point of each vertical bend, an **air valve** should be provided to discharge any air which might tend to accumulate. This valve usually takes the form of a ball float which falls as air accumulates and allows it to escape. Fig. 119*a* shows a small valve of this type, while Fig. 119*b* shows the type of valve fitted

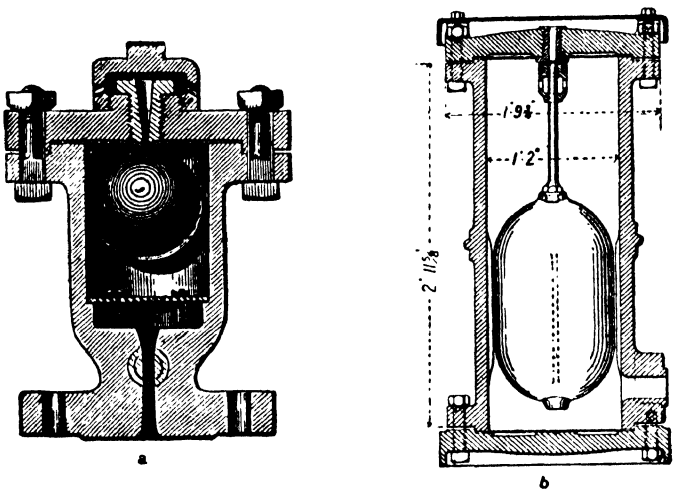


FIG. 119.—Air Valves.

to the 42-inch supply pipe lines of the Loch Leven power works.[1] In the latter case the float, working between guides, carries the air-valve spindle at its upper end, the upper portion of the valve casing serving as an air vessel to reduce shocks in the pipe.

Such valves also serve to admit air to the pipe line and to prevent the formation of a vacuum in case of a rapid eflux of water following a fracture at some lower point of the line. This is of importance in the case of large steel pipes which are usually too thin to withstand an external pressure of any magnitude without collapsing.

To guard against the discharge of water backwards from the main in the case of a fracture occurring in an inverted syphon below the level of the discharge reservoir, or in any similar position, a **reflux valve** is

[1] "Proc. Inst. C.E.," vol. 187., 1911-12, pt. 1, p. 28.

usually introduced at the inlet to a reservoir or near the outlet from such an inverted syphon. This consists of an enlargement in the pipe fitted with a diaphragm pierced with a series of valve openings. These are furnished with flap valves opening in the direction of normal flow, and automatically close if the direction of the current is reversed.

Where serious damage would follow fracture of a main it is usual to fit an **automatic stop valve.** Such a valve is placed, if possible, under comparatively light pressure, near a deep depression in the pipe line. In this type of valve a disc which is held by a lever, and projects into the water way, is thrown back when the velocity of flow exceeds a predetermined limit. This releases a trigger and an attached weight which closes the valve. The speed of closure may be regulated by means of a dashpot. Such an arrangement is shown diagrammatically in Fig. 120. In other valves of the same class the main valve is of the sluice type with parallel faces and is opened or closed by means of a loaded piston in a hydraulic cylinder surmounting the valve. Water under pressure is admitted to the under or upper side of this piston by a four way valve, actuated either by the release of a trigger as in the former type, or by the motion of a loaded piston in an auxiliary cylinder which is coupled to the main.

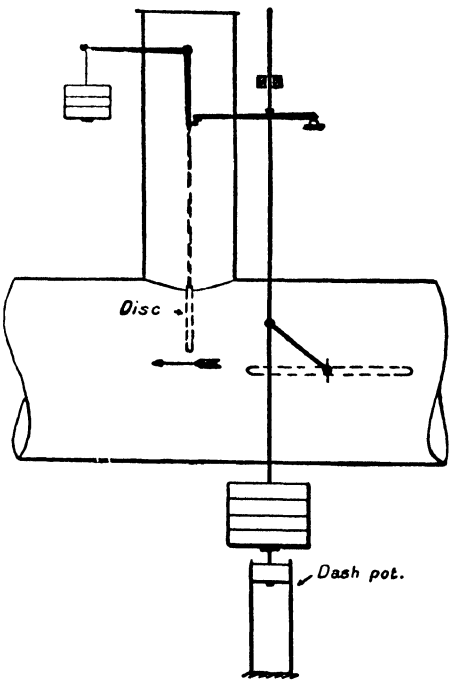


FIG. 120.—Automatic Throttle Valve.

Where different parts of a town are at widely different elevations, the head necessary to supply the more elevated sections may lead to an excessive pressure in the more low lying portions of the supply system, and in such a case it is usual to divide the area into zones at respectively lower levels, each zone carrying an independent distribution system fed from the main. The lower zones are then supplied through a series of **reducing valves,** each set to maintain the required maximum pressure in its respective zone. **In case of fire it is advisable that the full head be**

available, and for this purpose the type of reducing valve shown diagrammatically in Fig. 121 has been devised.[1] This consists essentially of a double-beat equilibrium valve whose opening is normally regulated by the pressure on its discharge side. In case of an abnormal draught on the main, such as might occur in case of fire, the pressure falls sufficiently low for the weight W to open the auxiliary valve V, and the under side of the plunger P is put into free communication with the atmosphere. This causes the weight, and the valve, to fall to its lowest limit, giving an uninterrupted flow past the main valve.

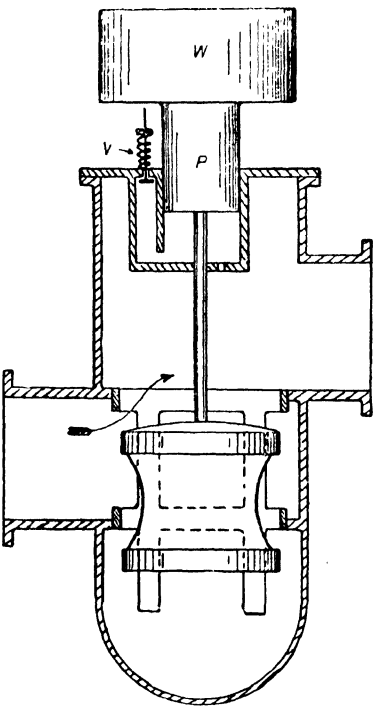


FIG. 121.—Reducing Valve.

Two methods of lay-out are common in the demand area of supply systems. In the first of these a large main is laid through the centre of the district with branch pipes extending from it to form a gridiron covering all parts of the district, the outer ends of these branches being connected by a small main so as to eliminate dead ends.

In the second system, commonly termed the "ring" system, a large main is laid to enclose the district, and from this main branch pipes are taken to form a grid covering the whole area.

If both systems are designed to give the same discharge at the same pressure at all points of the district, the former system usually costs considerably less than the latter, and is to be preferred.

ART. 97.—SYPHONS.

Where a pipe line is to be laid to connect two reservoirs at different levels, over ground which is higher than either water level, the cost of excavation is often so great as to preclude the use of a pipe line which

[1] By Mr. G F. Deacon. This device is used on the distribution system of the Liverpool Water Works.

shall fall below the hydraulic gradient, and in such cases a syphon (Fig. 122) is commonly used.

In its simplest form this consists of an inverted U-tube (Fig. 123), both legs being full of water, and the flow is generally calculated by equating the total head producing flow, *i.e.*, the head due to the unbalanced column of water $Z_A - Z_C$, or the difference of heads in the two reservoirs, to the sum of the frictional and other losses in the pipe and of the velocity head produced.

Thus $Z_A - Z_C =$ loss at entrance and exit $+ \frac{f l v^2}{2 g m}$, l being the total length of the syphon $A' B C'$.

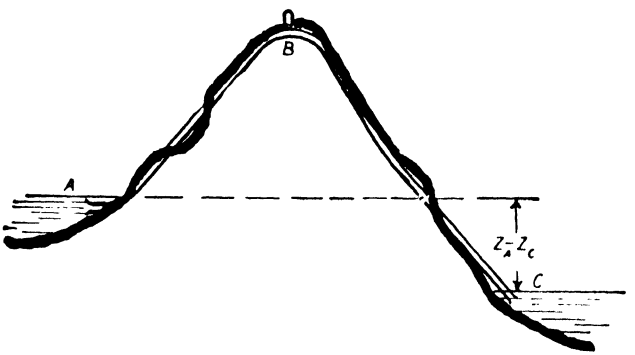


FIG. 122.

This may be seen by considering the flow along each leg of the syphon separately.

Along $A' B$, we have

$$\frac{p_{A'}}{W} + Z_{A'} = \frac{p_B}{W} + Z_B + \frac{v^2}{2 g} + \frac{f l_{A'B} v^2}{2 g m} + \text{loss at entrance,}$$

$$\frac{p_{A'}}{W} + Z_{A'} - Z_B - \frac{p_B}{W} = \frac{v^2}{2 g}\left\{1 + \frac{f l_{A'B}}{m} + K\right\}. \qquad (1)$$

Along $B C'$ we have

$$Z_B + \frac{v^2}{2 g} + \frac{p_B}{W} = Z_{C'} + \frac{v^2}{2 g} + \frac{p_{C'}}{W} + \frac{f l_{BC'} v^2}{2 g m}.$$

$$\therefore Z_B - Z_{C'} + \frac{p_B}{W} = \frac{p_{C'}}{W} + \frac{f l_{BC'} v^2}{2 g m}. \qquad (2)$$

$\therefore$ From (1) and (2) we get on adding

$$Z_{A'} - Z_{C'} = \frac{p_{C'} - p_{A'}}{W} + \frac{v^2}{2 g}\left\{1 + \frac{f l}{m} + K\right\}$$

But

$$\left.\begin{aligned} Z_{A'} + \frac{p_{A'}}{W} &= Z_A \\ Z_{C'} + \frac{p_{C'}}{W} &= Z_C \end{aligned}\right\}$$

$$\therefore Z_A - Z_C = \frac{v^2}{2g}\left\{1 + \frac{fl}{m} + K\right\}$$

= loss at exit + friction loss + loss at entrance.

The assumption that both legs of the syphon run full is one which is not always justified in practice. Evidently the height $B\,A$ must be less than the barometric height by an amount $-\frac{v^2}{2g}\left\{1 + \frac{f\,l_{A'B}}{m} + K\right\}$ or we should have an absolute vacuum formed at points below B, and the flow would cease. Theoretically, the limit of possible flow up the inlet leg is reached when the pressure at the summit B is absolute zero ($-$ 34 feet of water),[1] and is then given by the equation

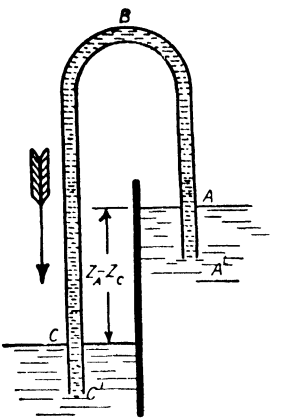


FIG. 123.

$$34 + \frac{p_{A'}}{W} - (Z_B - Z_{A'}) = \frac{v_1^2}{2g}\left\{1 + \frac{f\,l_1}{m} + K\right\}$$

Or writing $K = \cdot 5$ and Z_A for $Z_A + \frac{p_{A'}}{W}$ we get

$$34 - (Z_B - Z_A) = \frac{v_1^2}{2g}\left\{1 \cdot 5 + \frac{f\,l_1}{m}\right\}$$

the suffix (1) referring to the inlet leg.

If the actual flow is less than this, p_B is greater than zero, and we get an additional head aiding the flow down the outlet leg.

The velocity of flow down the outlet leg with maximum flow up the inlet is given by the equation

$$Z_B - Z_C - 34 = \frac{f\,l_2\,v_2^2}{2\,g\,m}$$

the suffix (2) referring to this leg.

If the syphon is to run full, we must have v_2 not greater than v_1

i.e., $$\frac{2\,g\,m}{f\,l_2}\left\{Z_B - Z_C - 34\right\} \not> \frac{2\,g\,m}{1 \cdot 5\,m + f\,l_1}\left\{34 - (Z_B - Z_A)\right\}$$

[1] Actually less than this, because of liberation of air at low pressures.

or
$$\frac{(Z_B - Z_C) - 34}{34 - (Z_B - Z_A)} \gtrsim \frac{f l_2}{1{\cdot}5\, m + f l_1}$$

If the syphon be long, so that $1{\cdot}5\, m$ may be neglected in comparison with $f l_1$ we have

$$\frac{(Z_B - Z_C) - 34}{34 - (Z_B - Z_A)} \gtrsim \frac{l_2}{l_1}$$

or
$$\frac{Z_A - Z_C}{34 - (Z_B - Z_A)} \gtrsim \frac{l_1 + l_2}{l_1}$$

$$\therefore \frac{l_1}{l} \gtrsim \frac{34 - (Z_B - Z_A)}{(Z_A - Z_C)}$$

EXAMPLE.

If $\begin{cases} Z_A - Z_C = & 50 \text{ ft.} \\ l = & 1{,}000 \text{ ft.} \\ Z_B - Z_A = & 10 \text{ ft.} \end{cases}$

$$l_1 \gtrsim \frac{1{,}000 \times 24}{50} = 480 \text{ ft.}$$

or the outlet leg will not run full if the inlet leg is more than 480 feet in length.

With a longer inlet and shorter outlet the flow up the inlet will not be able to keep pace with that down the outlet, and this will then run only partly full. Also the velocity up the inlet will not now be so great as with a shorter inlet, so that the discharge will be less. Evidently, then, the position of the apex of the syphon has a great influence on the discharge.

With a shorter inlet and a longer outlet, the total length being the same, the discharge will be unaltered, but the syphon will have the advantage of working under a greater absolute pressure at the apex, and is therefore less likely to be affected by air leakage at the joints.

In practice it is necessary to place an air chamber at the highest point of the syphon, into which air gradually accumulates during its working. This air is then removed at frequent intervals, either by some form of air pump, or by means of a steam ejector.

Where the syphon discharges into the atmosphere, any failure of the outlet leg to run full, by admitting air to the apex at once breaks the vacuum and stops the flow.

Figure 124 shows the hydraulic gradient for a syphon, the straight line $A\ C$ being the gradient line. In drawing this the only losses taken into account have been those due to friction. If a second line $A'\ C'$ be drawn parallel to and at a vertical distance from $A\ C$ equal to the barometric

height, the distance of the syphon below A' C' will give the absolute pressure at any point. In the sketch, syphons A B_1 C, A B_3 C, and A B_2 C are shown connecting A and C, all rising to the same height h, above the surface at A. Here, although B_2 is not nearly 34 feet above A, an absolute vacuum would be attained before reaching B_2 and the syphon will consequently not work. A comparison of 1 and 3 shows that there is a greater pressure in the air vessel at B_1 than at B_3, and the syphon 1 will thus run longer without removal of air from this

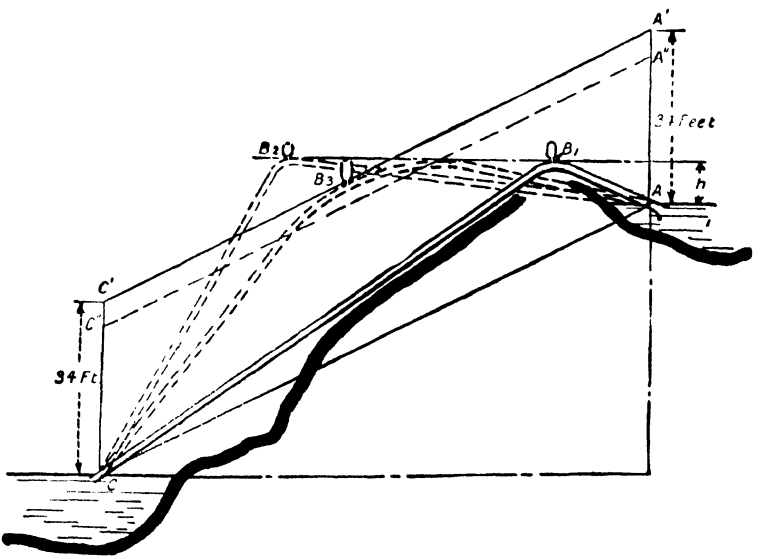


FIG. 124.

chamber than will 3. Leakage at joints is not likely to have so serious an effect as with 3. Where a regulating valve is to be used on a syphon, this should always be placed on the outlet leg.

ART. 98.—SIPHON SPILLWAYS.

The siphon spillway is a device for discharging surplus water over a dam. It utilises a much higher head than the spillway weir and thus gives a higher discharge for a given discharge area. Fig. 125 shows a simple form of siphon spillway. It is constructed in concrete and is rectangular in cross section. The level of the crest of the spillway is that of normal level in the reservoir. Air vents are provided slightly above crest level. If the water level rises flow takes place down the outlet leg. When the air vents are covered the falling water entrains and ejects the air in the throat and the siphon primes.

The outlet leg then runs full and discharges under an effective head which is the difference between the reservoir level and the centre line of the outlet if the syphon is discharging freely into air, or the level of the tail water if the outlet is submerged.

If the reservoir level falls the air vents in due course become uncovered,

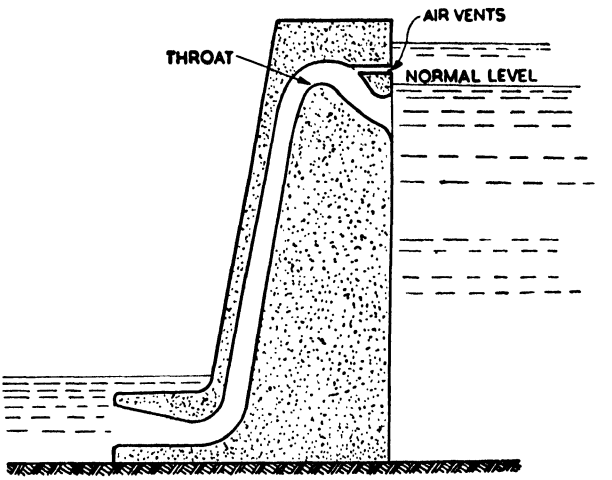


FIG. 125– Siphon Spillway.

air is admitted, the siphon action is destroyed and discharge automatically ceases. The siphon spillway thus provides an automatic means of maintaining the reservoir level within circumscribed limits.

If A is the throat area, the discharge is $Q = K A\sqrt{2gh}$ where K depends on the design, and may attain a value as high as 0·85.[1] The more usual value, with normal designs is about 0·65. If the outlet is submerged the value of K depends largely on the design of the outlet bend and outlet passage. With a divergent outlet as shown in Fig. 125 the discharge is greatly increased. Experiments by the author indicate that the best angle of flare is about 8 degrees.

ART. 99.—FLOW THROUGH NOZZLE AT END OF PIPE LINE.

Where a supply of water moving with a high velocity is required, as for fire extinguishing, or for the jets of an impulse turbine or Pelton wheel, a converging nozzle is fitted to the delivery end of the supply pipe.

[1] The effect of modifications in design as well as illustrations of many different types may be found in a paper " Experiments on Siphon Spillways," Gibson, Askey and Tattersall, *Proc. Inst. C. E.*, Vol. 231, 1930–1, Part I.

The energy of the issuing jet is now entirely in the kinetic form.

Theoretically, if pointed vertically upwards, and if no energy losses were experienced, the height of the jet would be the same as that of the free surface of the supply reservoir, or that corresponding to the pressure head inside the nozzle. The various frictional resistances, however, reduce the nozzle pressure and the issuing velocity, after which the resistance of the air and that due to the impact of falling particles of water tend to retard the upward motion of the rising particles, so that the height to which the jet rises is considerably less than that given by $h = \frac{v^2}{2g}$, v being the velocity of efflux.

The following results are deduced from experiments carried out by *J. T. Fanning*[1] with ordinary converging fire nozzles and with a stream slightly inclined to the vertical, so that the effect of the falling particles would not be so great as with a vertical stream.

Nozzle diameter.	Pressure at Nozzle.	Measured Velocity of Efflux.	Velocity theoretically corresponding to Trajectory.	Percentage, Actual, of theoretical height.
1 inch	46·5 lbs. per sq. inch	83·1 ft. per sec.	67·0 ft. per sec.	81·0 %
1 ,,	130·0 ,, ,, ,,	127·1 ,, ,,	80·0 ,, ,,	63·0 %
1⅛ ,,	44·5 ,, ,, ,,	79·9 ,, ,,	67·0 ,, ,,	84·0 %
1⅛ ,,	103·0 ,, ,, ,,	120·8 ,, ,,	80·0 ,, ,,	66·3 %
1¼ ,,	43·0 ,, ,, ,,	80·0 ,, ,,	67·0 ,, ,,	84·0 %
1¼ ,,	93·3 ,, ,, ,,	120·0 ,, ,,	80·0 ,, ,,	66·6 %
1⅜ ,,	41·5 ,, ,, ,,	79·6 ,, ,,	67·0 ,, ,,	83·0 %
1⅜ ,,	88·0 ,, ,, ,,	116·9 ,, ,,	80·0 ,, ,,	66·6 %

Forms of Nozzle.—These are usually of circular section and may either converge uniformly to a short parallel neck at the orifice (Fig. 126*a*), or

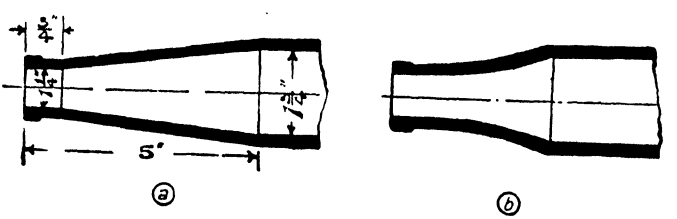


FIG. 126.

have a convergence which becomes more gradual as the outlet is approached (Fig. 126*b*).

[1] *Engineering News*, July 14, 1892.

The latter form is to be preferred. Its coefficient of contraction is unity, and the coefficient of discharge about ·98.

The following values of the latter coefficient are deduced from experiments by *Freeman*[1] on smooth conical nozzles.

Diameter of nozzle . .	¾ in.	⅞ in.	1 in.	1⅛ in.	1¼ in.
Value of C	·983	·982	·972	·976	·971

A form known as the ring nozzle is used to a less extent, and consists of a converging nozzle having a circular orifice, which is fitted with a short annular ring of square section (Fig. 126*c*). This, with a ring ⅛-inch square, gives a stream having a *vena contracta* of area about ·764 times that of the orifice. The value of C_v is about ·97, the coefficient of discharge being about ·74. Since the efficiency of a nozzle depends entirely on the value of its coefficient of velocity C_v the ring nozzle would appear to have no advantage over the ordinary smooth nozzle, and in practice, with the same pressure head and size of nozzle, throws a jet to a slightly less height. It has been proposed to use a small re-entrant mouthpiece, so as to form a Borda orifice in connection with this nozzle

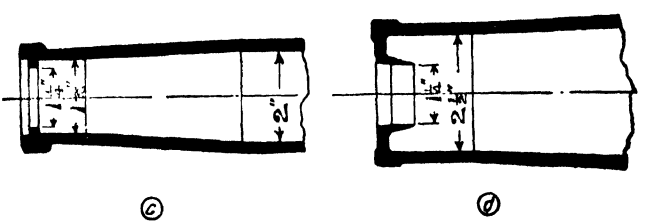


FIG. 126.

(Fig. 126 *d*), and to use this as a standard nozzle to determine experimentally the horse-power of a pumping engine by measuring the pressure inside a nozzle or series of nozzles supplied by the engine. The coefficient of discharge of such a nozzle as shown in Fig. 126 *d* is given by Freeman as ·582. Since, however, the ordinary type of nozzle shown in Fig. 126 *a* and *b* may be relied upon to give discharges which are identical within the limits of practical measurement under given pressure con-

ditions, certainly within 1 per cent., these nozzles are very satisfactory for such a purpose, this accuracy being quite as high as that attending the use of a standard weir for measuring purposes.

The form of nozzle used in connection with the Pelton wheel is discussed in Arts. 144 and 147.

Velocity of Flow through a Nozzle at the end of a Pipe Line.

Let A, L, and v_A refer to the supply pipe, and a and v_a to the nozzle.

Then $\frac{v_A}{v_a} = \frac{a}{A}$.

If H = supply head in feet we have

$$H = K\frac{v_A^2}{2g} + \frac{f\,l}{2\,g}\frac{v_A^2}{m} + \frac{v_a^2}{2g} + k\frac{v_a^2}{2g}$$

where K and k represent the coefficients of loss at the entrance to the pipe and at the nozzle.

Putting $v_A = v_a \cdot \frac{a}{A}$ we get

$$H = \left\{ K\frac{a}{A} + \frac{f\,l\,a^2}{m\,A^2} + 1 + k \right\} \frac{v_a^2}{2g}$$

$$\therefore\ v_a = \sqrt{\frac{2\,g\,H}{1 + k + K\frac{a}{A} + \frac{f\,l\,a^2}{m\,A^2}}} \qquad (1)$$

Since $k = \cdot 05$ (approximately), while $K\frac{a}{A}$ is in general small, neglecting these terms we get

$$v_a = \sqrt{\frac{2\,g\,H}{1 + \frac{f\,l\,a^2}{m\,A^2}}} = \sqrt{\frac{2\,g\,H}{1 + \frac{4\,f\,l\,d^4}{L^5}}} \qquad (2)$$

If p = pressure in pounds per square inch at the entrance to the nozzle and if $\frac{k\,v_a^2}{2\,g}$ = loss in nozzle

$$\frac{p \times 144}{W} + \frac{v_A^2}{2g} = \frac{v_a^2}{2g}(1 + k)$$

$$\therefore\ 2{\cdot}3\,p = \frac{v_a^2}{2g}\left\{ 1 + k - \frac{a^2}{A^2} \right\}$$

$$\therefore\ v_a = \sqrt{\frac{4{\cdot}6\,p\,g}{1 + k - \frac{a^2}{A^2}}}. \qquad (3)$$

This formula may be used to determine k, and hence the coefficient of velocity $\sqrt{(1 - k)}$, by measurement of p and of the quantity discharged.

With a ring nozzle the preceding formula becomes

$$v_a = \sqrt{\frac{4{\cdot}6\ p\ g}{1 + k - \frac{a_c^2}{A^2}}} \tag{4}$$

where a_c is now the area of the *vena contracta* of the issuing stream.

In any case, when k and the coefficient of contraction for any given nozzle are known, the value of v_a can be directly determined in terms of p. Then since the energy discharged at the nozzle per second

$$= \frac{W\ a_c\ v_a^3}{2\ g} \text{ ft. lbs.},$$

we have the horse-power delivered at the nozzle given by

$$\text{H.P.} = \frac{W\ a_c\ v_a^3}{2\ g \times 550} = \frac{62{\cdot}4\ a_c\ v_a^3}{1{,}100\ g} = {\cdot}00176\ a_c\ v_a^3$$

$$= 3{\cdot}17\ a_c \left\{\frac{p}{1 + k - \frac{a_c^2}{A^2}}\right\}^{\frac{3}{2}}. \tag{5}$$

EXAMPLE.

With a uniformly converging nozzle, 1½ inch diameter, taking $k = {\cdot}03$, and assuming a supply pipe of 3 inches diameter, we have $a_c = a =$ area of 1½ inch pipe $= {\cdot}01227$ square feet, while $\frac{a_c^2}{A^2} = \frac{1}{4}$. Thus if the pressure at the nozzle is 80 lbs. per square inch, we have

$$\text{H.P. at nozzle} = 3{\cdot}17 \times {\cdot}01227 \left\{\frac{80}{1{\cdot}03 - {\cdot}25}\right\}^{\frac{3}{2}}$$

$$= 40{\cdot}5 \text{ H. P.}$$

If water is supplied at the pipe entrance under a constant pressure P lbs. per square foot, neglecting the difference in head between pipe entrance and nozzle, we have $\frac{P}{W} = H$,

$$\therefore \quad v_a = \sqrt{\frac{2\ g\ H}{\left(1 + \frac{4\ f\ l}{D^5}\ d^4\right)}} \text{ ft. per sec.}$$

Without the nozzle we should have $a = A$, $v_a = v_A$,

and $$v_A = \sqrt{\frac{2\ g\ H}{1 + \frac{4\ f\ l}{D}}} \text{ ft. per sec.}$$

so that the velocity with, is greater than without the nozzle.

The discharge per second without the nozzle is given by

$$v_A \times A = \sqrt{\frac{2gH}{1+\frac{4fl}{D}}} \times A \text{ cub. ft.,}$$

and the kinetic energy by $\frac{v_A^3}{2g}$. $A \times 62{\cdot}4$ ft. lbs.

$$= \left\{\frac{2gH}{1+\frac{4fl}{D}}\right\}^{\frac{3}{2}} \cdot \frac{62{\cdot}4}{2g} A \text{ ft. lbs.}$$

With the nozzle, the quantity per second is

$$v_a \times a = \sqrt{\frac{2gH}{1+\frac{4fld^4}{D^5}}} \times a$$

$$\therefore \frac{Q \text{ with nozzle}}{Q \text{ without nozzle}} = \sqrt{\frac{\frac{D^5}{(D^5+4fld^4)}\left\{\frac{\pi^2 d^4}{16}\right\}}{\frac{D}{(D+4fl)}\left\{\frac{\pi^2 D^4}{16}\right\}}}$$

$$= \sqrt{\frac{d^4(D+4fl)}{(D^5+4fld^4)}} = \sqrt{\frac{Dd^4+4fld^4}{D^5+4fld^4}}$$

or the quantity passing the pipe is always less with, than without the nozzle.

Since the kinetic energy varies as the product of two factors (the quantity and the (velocity)^2), one of which increases, while the other diminishes as the nozzle area is increased, a maximum amount of kinetic energy will be delivered at the nozzle with some definite ratio of pipe and nozzle areas.

To obtain this ratio we have

K.E. with nozzle $= \frac{v_a^3}{2g} \times 62{\cdot}4\ a$ ft. lbs. per sec.

$$= \left\{\frac{2gH}{1+\frac{4fld^4}{D^5}}\right\}^{\frac{3}{2}} \times \frac{62{\cdot}4\ \pi d^2}{2g \times 4}$$

$$= 395\ d^2 \left\{\frac{H}{1+\frac{4fld^4}{D^5}}\right\}^{\frac{3}{2}} \text{ ft. lbs. per sec.} \qquad (6)$$

$$= 395 \frac{a}{A} D^2 \left\{\frac{H}{1+\frac{4fl}{D}\cdot\frac{a^2}{A^2}}\right\}^{\frac{3}{2}} \text{ ft. lbs. per sec.} \qquad (7)$$

Differentiating this with respect to $\frac{a}{A}$, and equating the result to zero we have the condition that the K. E. should be a maximum This gives[1]

$$\left\{\frac{d}{D}\right\}^4 = \frac{D}{8fl}$$

or

$$\frac{a}{A} = \sqrt{\frac{D}{8fl}}$$

the dimensions being taken in feet.

If for example the pipe supplying the nozzle of a Pelton wheel be 1 foot in diameter and 500 feet long, the maximum K. E. is delivered when

$$\frac{a}{A} = \sqrt{\frac{1}{8 \times 500 f}}$$

and if $f = \cdot 01$ this becomes

$$\frac{a}{A} = \sqrt{\frac{1}{40}} = \cdot 158.$$

By substituting this value of $\frac{a}{A}$ or of $\frac{d^2}{D^2}$ in equation (7) above, the value of this K. E. is given.

Thus $(\text{K. E.})_{\max.}$

$$= 395 \times \cdot 158 \left\{\frac{H}{1 + \frac{4 \times \cdot 01 \times 500 \times (\cdot 158)^2}{1}}\right\}^{\frac{3}{2}} \text{ ft. lbs. per sec.}$$

$$= 62 \cdot 4 \left\{\frac{H}{1 \cdot 50}\right\}^{\frac{3}{2}}$$

$$= 34 H^{\frac{3}{2}} \text{ ft. lbs. per second.}$$

If the Pelton wheel have an efficiency of 80 per cent. the work done by the wheel per second will be given by

$$\cdot 8 \times 34 H^{\frac{3}{2}} \text{ ft. lbs.}$$

$$= \frac{27 \cdot 2}{550} H^{\frac{3}{2}} \text{ horse-power}$$

$$= \cdot 0495 H^{\frac{3}{2}} \text{ horse power.}$$

[1] $\frac{d(K.E.)}{d\left(\frac{a}{A}\right)} = \left(1 + \frac{4fl}{D} \cdot \frac{a^2}{A^2}\right) - \frac{a}{A} \times \frac{3}{2} \sqrt{1 + \frac{4fl}{D} \cdot \frac{a^2}{A^2}} \times 8 \frac{fl}{D} \cdot \frac{a}{A}$.

Equating this to zero we have

$$\frac{12fl}{D} \cdot \frac{a^2}{A^2} = 1 + \frac{4fl}{D} \frac{a^2}{A^2}$$

$$\therefore \frac{a^2}{A^2} = \frac{D}{8fl}.$$

With a circular nozzle this makes $d = \sqrt{\frac{D^5}{8fl}}$.

Thus with a head of 100 feet the wheel would develop 49·5 horse-power

As the nozzle area is increased, starting from zero, the power developed thus increases to a maximum of 49·5 H.P. with a nozzle opening $a = \cdot 158\ A$, and then again diminishes.

If in the above problem f be taken $= \cdot 005$, we get the maximum K. E. when $\frac{a}{A} = \cdot 224$. The value of this K. E. $= 48 \cdot 1\ H^{\frac{3}{2}}$ foot lbs. per second, so that with the same head and wheel efficiency the maximum power developed would be 70·0 H.P.

In a fire nozzle, the problem is to obtain a stream moving with as high a velocity as possible, and this is evidently to be obtained by making the expression $\left[1 + k + K\frac{a}{A} + \frac{f\,l\,a^2}{m\,A^2}\right]$, the denominator of the fraction expressing the velocity, as small as possible. This is done by increasing, as far as practicable, the ratio $\frac{A}{a}$.

In the case of the supply to a Pelton wheel for successful governing, the maximum nozzle area should not exceed that corresponding to the critical value, so that everywhere within the range of opening $\frac{d\,(K\,E)}{d\,(a)}$ may be positive, and the supply of energy may increase with the nozzle opening. The curves (Fig. 127) derived from equation (6) of this article, show the kinetic energy (expressed as horse-power) delivered from pipes of diameters 1·0 and 2·0 feet, 1,000 feet long, under a head of 200 feet, with different values of the ratio $\frac{a}{A}$. The value of f has been taken at ·005. In each case the maximum nozzle opening for successful governing is that corresponding to the point E.[1]

While the above investigation is interesting, it is not usual, in a well-designed pipe line, to come across a case in which the nozzle area is greater than the maximum for successful governing since this would entail a velocity of flow in the main much greater in general than that (3 to 6 feet per second) adopted in practice.

Examples.

(1) Two reservoirs are connected by a pipe 500 yards long. For the first 250 yards its diameter is 4 inches, increasing for the second 250

[1] For a further consideration of this subject see an article by Professor John Goodman in *Engineering*, November 4, 1904.

yards to 8 inches. A re-entrant mouthpiece is fitted at the entrance, while discharge takes place directly into the side of the lower reservoir at a depth of 6 feet below the surface. If the difference in the surface level in the reservoirs is 10 feet, determine the discharge in gallons per minute. $f = \cdot 005$.

Answer. 121 gallons per minute.

(2) A gate valve in a 12-inch cylindrical C. I. pipe is lifted through $\frac{3}{8}$ of its full opening. Determine the equivalent length of a straight pipe offering the same resistance. $f = \cdot 005$.

Answer. $l = 276$ feet.

(3) Two reservoirs of 80,000 and 10,000 square feet area are connected by a 6-inch pipe, 500 feet long. Initially, the difference of surface level

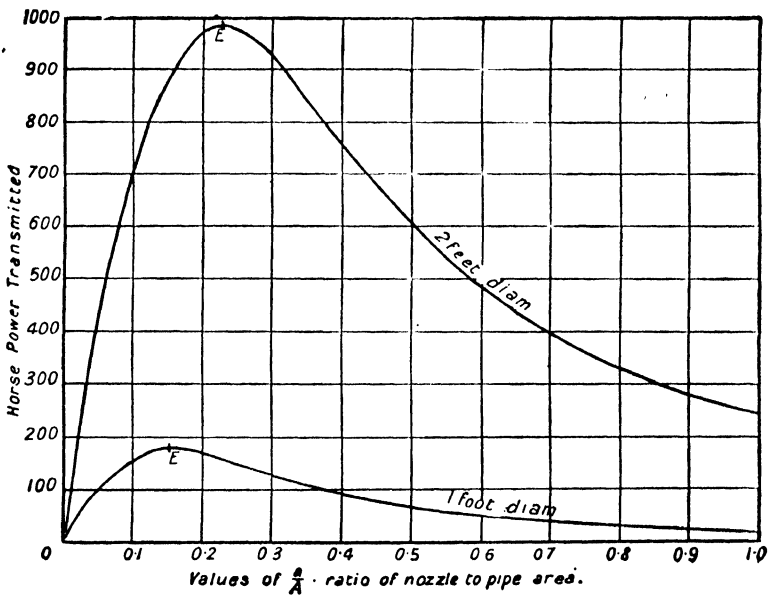


FIG. 127.

is 16 feet. Determine how soon the level in the two reservoirs will be the same. Take $f = \cdot 0075$.

Answer. About 69 hours.

(4) Two reservoirs, surfaces 15 and 25 feet above a common datum feed through a 12-inch and an 18-inch pipe respectively, into a common 24-inch pipe at a height 5 feet above datum level, while this in turn feeds a reservoir whose surface level is taken as datum. If the lengths of the 12-inch, 18-inch, and 24-inch pipes are respectively 1,000 feet, 1,500

feet, and 2,000 feet, determine the velocity of flow in and the discharge through each pipe, neglecting all losses of head except those due to friction. Also determine the pressure at the junction of the three pipes. Take $f = \cdot 005$.

Answer.
$$\begin{cases} 12 \text{ in. pipe.} & v = 4 \cdot 58 \text{ f.s.} & Q = 3 \cdot 60 \text{ c.f.s.} \\ 18 \text{ ,, ,,} & v = 7 \cdot 29 \text{ f.s.} & Q = 12 \cdot 89 \text{ c.f.s.} \\ 24 \text{ ,, ,,} & v = 5 \cdot 22 \text{ f.s.} & Q = 16 \cdot 40 \text{ c.f.s.} \\ \text{Pressure at junction} = 3 \cdot 48 \text{ ft. of water.} \end{cases}$$

(5) Assuming $Q \propto \sqrt{\frac{H}{l}}\, d^{2 \cdot 58}$ determine the discharge from a 12-inch pipe, if a similar 8-inch pipe of half the length gives a discharge of 10,000 gallons per hour under the same head.

Answer. 20,153 gallons per hour.

(6) A pumping engine supplies water at a pressure of 75 lbs. per square inch through a short pipe 20 inches diameter to a hydrant box, from which it is discharged by four parallel nozzles 2 inches in diameter. Taking the coefficient of loss in the nozzles to be ·03, determine the water horse-power of the engine.

Answer. 181 horse-power.

(7) The difference of surface level in two reservoirs which are connected by a syphon is 25 feet. The length of the syphon is 2,000 feet and its diameter is 12 inches. Assuming $f = \cdot 005$, and that the syphon runs full, determine the discharge. Also if the vertex of the pipe line is 16 feet above the surface level in the upper reservoir, determine the maximum length of inlet leg for the pipe to run full. Neglect all but friction losses.

Answer. $Q = 4 \cdot 99$ c.f.s.
$= 1{,}865$ gallons per minute.
Maximum length of inlet $= 1{,}440$ feet.

CHAPTER XI.

Flow in an open channel—Formulae—Darcy and Bazin—Prony—Eytelwein—Kutter—Manning—Fidler—Thrupp—Williams—Critical velocity—Form of channel—Channel of constant mean velocity—General equation of flow—Non-uniform flow—Standing waves—Backwater function—Effect of dams—Bridge piers—Passage of boat through a narrow canal—Flow round river bends—Distribution of velocity in an open channel—Erosion and Silting—Gauging of flow in streams and channels—Current meters—Floats—Pitot tube—Chemical methods—Rating Tables—Gauging of ice-covered streams.

ART. 100.—FLOW IN AN OPEN CHANNEL.

THE term "open channel" includes all rivers, artificial canals, aqueducts, and conduits, and, in addition, sewers and pipes of whatever section which run partially full, and which consequently do not present a solid boundary to every side of the contained liquid. The force producing flow cannot now be provided by any external head, but is solely due to the slope or gradient of the channel.

If a circular pipe be laid almost horizontally and if the surface level of water flowing through the pipe be allowed to rise, the change from the state in which the flow is governed by the laws appertaining to an open channel, to that in which the ordinary laws of pipe flow hold, is not abrupt, and it is to be inferred that a general formula is deducible which by satisfactory adjustment of constants shall fit either type of flow.

Still, the comparative simplicity of the conditions holding in the case of a circular pipe, the complications which must of necessity be introduced where water flows through a channel of uneven section, and the ease with which accurate observations are made in the one case and the difficulty with which even such a fundamental observation as the difference in level at points widely distant is accurately determined in the other, render it impossible that the laws governing the flow in open channels should be so definite and of such universal application as those already considered.

Assuming the resistance, R, to flow, to be proportional to the wetted perimeter P of the channel, this may be expressed as

$$R = f' S v^n,$$

where f' is a coefficient depending on the condition of the surface and probably, from analogy to pipe flow, also on the velocity v, while n is a

number probably varying from 1·79 to 2·00, depending on the surface and on the velocity v, and $S = P \times l$, where l = length of channel.

If A = sectional area of channel beneath water line, and if we assume the resistance to be equally divided over the area we have, if p = resistance per unit area of the stream

$$p = f' \cdot \frac{P}{A} \cdot l\, v^n.$$

Here $\frac{A}{P}$ or m is the hydraulic mean depth of the section.

If the channel be of uniform slope $\frac{h}{l}$, where $\frac{h}{l} = i = \sin\theta$ (θ being the angle of inclination), then the weight of water in this length l, per unit area of the channel, being $W\, l$ lbs., the resolved part of this weight in the direction of motion $= W\, l \frac{h}{l} = W\, h$ lbs.

∴ If the velocity is constant so that this force is entirely expended in overcoming friction and not in producing acceleration, we have

$$W\, h = f' \frac{P}{A}\, l\, v^n$$

$$h = \frac{f''\, l\, v^n}{m}$$

$$\text{or } h = \frac{f\, l\, v^n}{2\, g\, m}$$

In an open channel n may be taken as being approximately equal to 2, so that the formula becomes

$$h = \frac{f\, l\, v^2}{2\, g\, m} \tag{1}$$

This may be written in the form adopted by *Chezy, viz.*,

$$v = \sqrt{\frac{2\, g}{f} \cdot \frac{h}{l}\, m} = C \sqrt{m\, i} \tag{2}$$

where $$C^2 = \frac{2\, g}{f}$$

Many experiments have been devoted to determining the values of C or of f for channels having different physical characteristics, and the results of the more important of these are as follow, the numerical values of the coefficients obtained by the various observers being collected and tabulated on pp. 291—295.

Darcy and *Bazin* (1855—9), as the results of experiments carried out on the Bourgoyne Canal, gave C the value $\frac{1}{\sqrt{a + \frac{b}{m}}}$ where a and b

(p. 290) vary only with the material and condition of the bed and sides of the channel.

These channels were of many different forms and dimensions; were lined with different materials, and had slopes varying from ·001 to ·01.

Prony, from experiments by Chezy and Dubuat on earthen channels and on wooden channels of small section put

$$a\,v + b\,v^2 = m\,i$$

or
$$C = \sqrt{\frac{1}{b + \frac{a}{v}}}.$$

The corresponding value of $f = 2\,g\left(b + \frac{a}{v}\right) = A + \frac{B}{v}$

where $\begin{cases} \frac{1}{a} = 22{,}472. & A = ·00607. \\ \frac{1}{b} = 10{,}607. & B = ·00286. \end{cases}$

Eytelwein, from experiments on the Rhine channel, gave the same type of formula, his coefficients being

$$\frac{1}{a} = 41{,}211. \quad A = ·00719.$$

$$\frac{1}{b} = 8{,}975. \quad B = ·00156.$$

In both these cases the unit of length is 1 foot. With moderate velocities $a\,v$ is small compared with $b\,v^2$ and may be neglected, when the formulae reduce to

Prony $\quad v = \frac{1}{\sqrt{b}}\sqrt{m\,i} = 103\sqrt{m\,i}. \quad f = ·00607.$

Eytelwein $\quad v = - - - = 95\sqrt{m\,i}. \quad f = ·00714.$

These coefficients, being independent of the condition of the surface, are obviously only applicable to channels having the same physical characteristics as those experimented upon.

Bazin (1897), as the result of a very large number of experiments on canals and conduits of all sections and dimensions, deduced for C the value

$$\frac{157·6}{1 + \frac{N}{\sqrt{m}}} \text{ in foot units.}$$

N varying with the character of the surface.

Values of N for different types of surface are given on p. 290

This gives a value of $f = \cdot 000259 \left\{1 + \frac{2\,N}{\sqrt{m}} + \frac{N^2}{m}\right\}$.

This form of equation is in common use in France, and has given good results with velocities not exceeding 4 feet per second.

Ganguillet and *Kutter* deduced the coefficient

$$C = \frac{41{\cdot}6 + \frac{\cdot 00281}{i} + \frac{1{\cdot}8112}{N}}{1 + \left(41{\cdot}6 + \frac{\cdot 00281}{i}\right)\frac{N}{\sqrt{m}}} \text{ in foot units.}$$

the equation being identical with that used for pipe flow. N depends on the character of the surface, and has values given on p. 293.

The complication of this formula is largely due to an attempt to make it conform to the results of experiments made on the Mississippi. In certain of these, however, the results should have been corrected for the error introduced by the use of double floats, while in others the slope of the water surface was too slight to be measured with any degree of accuracy.

More recent investigations render it extremely doubtful whether the value of C does depend on i as this formula indicates, and the simpler formula of *Bazin* would appear to give results at least as accurate, except possibly in the case of very large channels. The formula is, however, of very general application in Great Britain, India, Germany and the United States, and the inconveniences due to its complication are removed by the use of hydraulic tables which have been prepared giving the values of C for all probable values of i and of N.

A graphical solution of problem involving the formula may be obtained by the use of the diagram of Fig. 128.[1]

Manning[2] gives the formula

or $$C = \frac{1{\cdot}486}{N} m^{\frac{1}{6}} \quad v = \frac{1{\cdot}486}{N} m^{\cdot 67} i^{\cdot 5}$$

for pipes or channels, N being the same as Kutter's N for the same class of surface.

From the nature of the case it would appear hopeless to obtain any strictly mathematical solution for flow in open channels and rivers of irregular section, and even to observe and record correctly the physical data required is almost impossible. An examination of the results given

[1] F. C. Scobey, U. S. Dept. of Agriculture, Bulletin 852, 1920.
[2] "On the Flow of Water in Channels and Pipes."

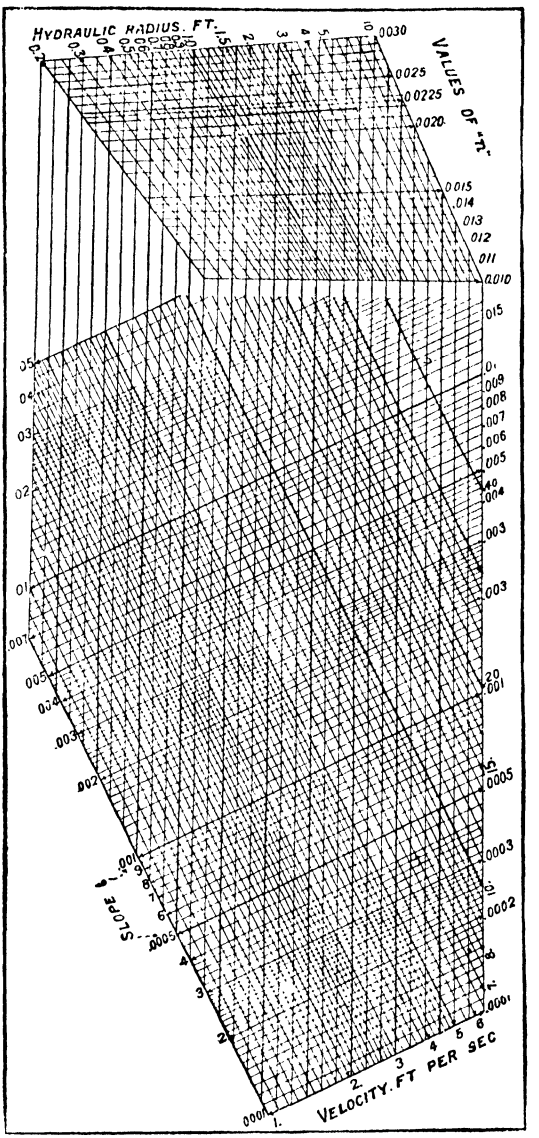


FIG. 128.—Diagram for Graphical Solution of Problems involving Kutter's Formula.

From the intersection of the hydraulic radius "m" and n follow the vertical line to the intersection of the slope "i" and v, or from the intersection of i and v follow the vertical to the intersection of m and n.

by an application of the various formulae shows that for abnormal sections or velocities a difference of 50 per cent. is not uncommon.

COEFFICIENTS FOR FLOW IN OPEN CHANNELS.

Character of Surface.	*Darcy.*		*Bazin.*	*Kutter.*
	a	*b*	*N.*	*N.*
Smooth cement or planed timber	·000046	·0000045	·109	·009 to ·010
Unplaned timber, flumes, slightly tuberculated iron, ashlar and well-laid brickwork . .	·000058	·0000133	·290	·012 to ·013
Rubble masonry and brickwork in an inferior condition. Fine gravel well rammed . .	·000073	·000060	·833	·017
Rubble in inferior condition. Canals with earth beds in very good condition . . .				·020
Canals with earth beds in good condition			1·540	·0225
Ditto in moderate condition .	·000085	·00035		·025
Canals and rivers in rather bad order			2·355	·030
Ditto in very bad order . .	·00012	·00070	3·170	·035

Here a channel is said to be in very good order when it is free from boulders, hollows in its bed and banks, sharp bends, snags and weeds. When badly choked with weeds the value of N in Kutter's formula may become much greater than ·035. The following values of N in the latter

formula are taken from Jackson's tables, and are probably slightly more accurate than those given above.

	N.
Planed timber accurately jointed—glazed or enamelled surfaces	·009
Smooth cement or plaster	·010
Unplaned timber well jointed—new brickwork well laid	·012
Unglazed stonework—iron—brick and ashlar	·013
Wooden troughs with battens inside, $\frac{1}{2}$ inch apart	·015
Rubble set in cement	·017

If any of these are in bad order the next higher value for N is to be taken.

For convenience in applying the results of these formulae the values of f in the formula $h = \frac{f\,l\,v^2}{2\,g\,m}$, and of C in the formula $v = C\sqrt{m\,i}$ have been calculated and are tabulated opposite.

The following table gives values of C in the formula $v = C\sqrt{m\,i}$, calculated from Bazin's formula in which $C = \frac{157{\cdot}6}{1 + \frac{N}{\sqrt{m}}}$.

Hydraulic mean Depth.	Values of *N*.					
m ft.	·109	·290	·833	1·54	2·35	3·17
·5	137	112	72	50	35	29
1·0	142	122	86	62	47	38
1·5	145	128	94	70	54	44
2·0	146	131	100	76	59	49
2·5	147	133	104	80	64	53
3·0	148	135	107	84	67	56
4·0	149	138	111	89	72	61
5·0	150	140	115	93	77	65
6·0	151	141	118	97	80	69
8·0	152	143	122	102	86	74
10·0	152	145	125	106	90	79
15·0	153	147	130	113	98	87
20·0	154	148	133	117	103	92
25·0	155	149	135	121	107	96
30·0	155	149	137	123	110	100
50·0	156	150	144	129	119	108

Character of Surface.		Darcy.			Bazin			Kutter { Slope 1 in 1,000.		
		$m = \cdot5$	$m = 1$	$m = 2$	$m = \cdot5$	$m = 1$	$m = 2$	$m = \cdot5$	$m = 1$	$m = 2$
Smooth cement or planed timber	$f =$	·00354	·00325	·00310	·00345	·00318	·00300	·00334	·00264	·00219
	$c =$	135	141	144	136·7	142	146	139	156	171
Unplaned timber, flumes, slightly tuberculated iron, ashlar and well laid brickwork	$f =$	·00544	·00459	·00416	·00515	·00431	·00376	·00517	·00396	·00320
	$c =$	109	118·5	124·5	112	122	131	120·3	127·6	143·2
Rubble masonry and brickwork in an inferior condition—fine gravel well rammed	$f =$	·01241	·00856	·00663	·01227	·00870	·00653	·01208	·00865	·00663
	$c =$	72·0	86·7	98·5	72·5	86·0	99·4	73·1	86·4	98·5
Rubble in inferior condition. Canals with earth beds in perfect condition	$f =$							·01671	·01260	·00936
	$c =$							62·0	71·5	83·0
Canals with earth beds in good condition	$f =$				·02620	·01670	·01130	·02400	·01648	·01200
	$c =$				49·6	62·0	75·5	51·8	62·5	73·3
Canals with earth beds in moderate condition	$f =$	·0550	·0280	·0167				·0312	0·210	·0151
	$c =$	34·2	48·0	62·0				45·5	55·4	65·5
Canals and rivers in rather bad order, weeds on bottom	$f =$				·0543	·02916	·0184	·0490	·0318	·0221
	$c =$				34·5	47·0	59·1	36·2	45·0	54·0
Canals and rivers in very bad order—choked with boulders	$f =$	·0979	·0528	·0302	·0779	·0450	·0266	·0710	·0452	·0305
	$c =$	25·6	35·0	45·1	28·8	37·8	49·2	30·1	37·6	46·0

Rational Formula.

From analogy with pipe flow it would appear probable that a formula of the type

$$h = \frac{f\,l\,v^n}{\left(\frac{A}{P}\right)^x} = \frac{f\,l\,v^n}{m^x}$$

where, as in the case of a pipe, n is in general less than 2, would most nearly represent the law of channel flow.

Claxton Fidler[1] has determined the values of f, n, and x from many experimental results of Darcy, Bazin, Smith, Stearns, and other observers, and the following table is abridged from values given by him :—

Form of Section.	Material of Surface.	n	x	f
Circular	Smooth neat cement	1·75	1·167	·0000676
Rectangular.	,, ,, ,,			
Circular	Cement and sand	,,	,,	·0000787
,,	Smooth brick.			
Rectangular.	Smooth ashlar	,,	,,	·0000904
Circular	Bare metal pipes with rivetted joints.	1·77	1·18	·0000871
,,	Rough brickwork	1·80	1·20	·0000977
Rectangular.	Unplaned timber	,,	,,	·0000944
,,	Rough Brickwork or ashlar	,,	,,	·0001122
Circular	Lined with fine gravel	1·90	1·33	·0001202
Rectangular.	,, ,, ,, ,,	1·96	1·40	·0001521
,,	,, ,, coarse gravel	2·10	1·50	·0001862
,,	Rubble masonry	,,	,,	·0002240

While further investigation may slightly alter these values, it is extremely probable that the final solution of the problem of flow in regular channels will be found in this type of formula.

The pipe flow formula of *Thrupp*

$$h = \frac{C^n\,v^n\,l}{m^x}$$

where m is the hydraulic mean depth is also applicable to channel flow, the following being the values of the quantities C^n, n, and x. When m is small, $x + a\sqrt{\frac{b}{m}} - 1$ should be substituted for x.

[1] Fidler, "Calculations in Hydraulic Engineering," Part II. (Longmans & Co.) 1902.

Surface.	n.	C^n.	x.	a.	b.
Neat cement { semi-circular section	1·74	·0000680	1·165		
Neat cement { rectangular section	1·95	·0000494	1·190		
Brickwork, well laid	2·00	·0000600	1·220		
Unplaned plank	2·00	·0000714	1·230		
Brickwork, rough	2·00	·0000780	1·250	·0670	·50
Lined with fine gravel	2·00	·0001399	1·320	·0245	·50
Chiselled masonry	2·00	·0001246	1·320	·0788	·60
Lined with coarse gravel	2·00	·0002005	1·410	·1565	1·00
Earth in fair condition	2·00	·0002360	1·440	·1518	1·00
Rough earth	2·00	·0002600	1·560		

The results obtained by this formula compare very well with those of Fidler, the latter probably on the whole giving the better results.

Prof. G. S. Williams, adopting this exponential formula, gives n and x constant values respectively equal to 1·9 and 1·25. The formula then becomes $h = \frac{K\, v^n\, l}{m^x}$, or $v = C'\, m^{\cdot 67} \times i^{\cdot 54}$, where K and C have the following values:—

	C'.	K.
Very smooth channels	180 to 190	·00030 to ·00035
Unplaned plank	145 ,, 165	·00045 ,, ·00050
Sewer crock	140 ,, 160	·00050 ,, ·00075
Brick sewers	130 ,, 160	·00050 ,, ·00080
Earth channels	60 ,, 80	·0010 ,, ·0020
Rough natural channels	40 ,, 50	·0020 ,, ·0050

Writing the formula in the form

$$v = K\, m^x\, i^y,$$

Barnes,[1] from an extended investigation of data by many observers, gives the following values and formulae for clean surfaces:—

Surface.	k	x	y	i
Planed wooden flumes	223·3	·660	·586	$\frac{\cdot 000098\, v^{1\cdot 707}}{m^{1\cdot 126}}$
Unplaned wooden flumes	182·5	·666	·569	$\frac{\cdot 000107\, v^{1\cdot 757}}{m^{1\cdot 171}}$
Neat cement channels	136·3	·635	·484	$\frac{\cdot 000039\, v^{2\cdot 066}}{m^{1\cdot 312}}$

[1] A. A. Barnes, "Hydraulic Flow Reviewed," Spon, 1916, p. 47.

Surface.	k	x	y	i
Smooth-faced concrete . .	95·1	·567	·471	$\frac{\cdot 000063\ v^{2\cdot 124}}{m^{1\cdot 204}}$
Hard well-pointed brick . .	92·1	·602	·466	$\frac{\cdot 000061\ v^{2\cdot 146}}{m^{1\cdot 292}}$
Dressed masonry channels, well set in cement. . . .	109·7	·713	·483	$\frac{\cdot 000060\ v^{2\cdot 07}}{m^{1\cdot 476}}$
Hammer-dressed dry masonry .	70·0	·820	·500	$\frac{\cdot 000204\ v^{2\cdot 00}}{m^{1\cdot 64}}$
Earth channels in average condition free from vegetation .	58·4	·694	·496	$\frac{\cdot 000275\ v^{2\cdot 016}}{m^{1\cdot 399}}$

Art. 101.—Critical Velocity in an Open Channel.

As in pipe flow there will be some velocity in an open channel below which the flow will become laminar, and the resistance will be proportional to v.

Experiments by Mr. E. C. Thrupp [1] on the Thames and the Kennet, the former having a mean depth of 7·6 feet and the latter of 2·4 feet, are stated to have shown that velocities of 0·66 f.s. in the former and 0·64 f.s. in the latter were below the critical. At such velocities the difficulties of measuring with accuracy the gradient and indeed the mean velocities in a natural stream are insuperable. Laboratory experiments by Allen [2] in which extreme accuracy was possible, show that in a rectangular channel laminar flow exists up to a value of $R_c = \frac{v\,m}{\gamma} = 300$. Then follows a range in which transverse currents [3] are imposed on the laminar flow and in which the resistance increases rapidly. The velocity at which turbulence is set up is given by $R_c = 1{,}400$. This value is independent of the surface roughness, so long as m/k, where k is the size of the excrescences exceeds 25. Thus in a wide channel 3 feet deep, if the water temperature is 15° C., the C.V. is only about 0·07 inches per second, and is about 0·20 inches per second if the depth is 1 foot.

Since in a natural stream the bends and irregularities of the channel tend to reduce the C.V., it is evident that in any such stream the motion will be essentially turbulent.

[1] *Proc. Inst. C.E.*, Vol. 171, p. 346.
[2] Allen, "Stream Line and Turbulent Flow in Open Channels," *Phil. Mag.*, Vol. XVII, June, 1934, p. 1081.
[3] Art. 112.

Allen's work indicates that in turbulent flow through smooth rectangular channels the expression

$$f = 0{\cdot}0234 \left(\frac{v\,m}{\gamma}\right)^{-0{\cdot}15}$$

applies with considerable accuracy.

ART. 102.—FORM OF CHANNEL.

Since for a channel of given sectional area A, the hydraulic mean depth $A \div P$ varies with the form of its section, while the resistance to flow increases as $A \div P$ diminishes, it becomes important to determine what form of channel will give the maximum value of $A \div P$ for a given value of A, since this will be the channel of maximum discharge for a given slope. Further, if this sectional area is a minimum, the cost of excavation is a minimum, and since in general the perimeter increases with the sectional area, the cost of pitching the faces of the channel is also a minimum. Theoretically, the best form of channel is one in which the bed is a circular arc, since this gives a minimum ratio of wetted perimeter to sectional area.

An investigation into the properties of different sections will be simplified if the coefficient C in the formula $v = C\sqrt{\frac{A}{P}\,.\,i}$, be assumed constant for a given surface. On this assumption we have:—

$$Q = A\,v = C\sqrt{\frac{A^3}{P}\,.\,i} \text{ cub ft. per sec.}$$

For v to be a maximum, $\frac{A}{P}$ must be a maximum, so that $d\left(\frac{A}{P}\right) = 0$

$$\therefore \quad P\,d\,A - A\,d\,P = 0. \tag{1}$$

Again, for Q to be a maximum, $\frac{A^3}{P}$ is to be a maximum, so that $d\left(\frac{A^3}{P}\right) = 0$.

$$\therefore\ 3\,P\,A^2\,d\,A - A^3\,d\,P = 0. \tag{2}$$

Where A is fixed, $d\,A = 0$ and both conditions are satisfied if $d\,P = 0$.

EXAMPLE.

(1) **Rectangular Channel.**—Breadth $2\,b$ — depth d.

Here $$A = 2\,b\,d \;:\; P = 2\,b + 2\,d = \frac{A}{d} + 2\,d$$

$$\therefore\ \frac{d\,P}{d\,(d)} = -\frac{A}{d^2} + 2.$$

Putting $A = 2\,b\,d$ and equating $\frac{d\,P}{d\,(d)}$ to zero, we have $b = d$.

Putting the full breadth $= B$, we have $d = \frac{B}{2}$, *i.e.*, for maximum flow the depth must equal one-half the breadth.

We then have

$$Q = C\sqrt{\frac{A^3}{P} \cdot i}$$

$$= C\sqrt{\frac{8\,b^3\,d^3}{2\,b + 2\,d} \cdot i}$$

$$= C\sqrt{2\,i} \cdot b^{\frac{5}{2}}$$

$$= \frac{C}{4}\sqrt{i}\,B^{\frac{5}{2}} \qquad (1)$$

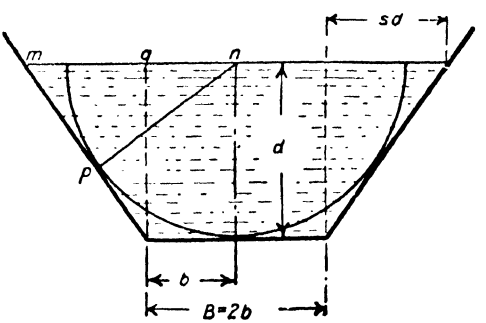


FIG. 129.

(2) Trapezoidal Channel.—Fig. 129.

Let b = half bottom breadth; d = depth; s = cotangent of angle of slope of sides.

Then $A = 2\,b\,d + s\,d^2$.

$P = 2\,(b + d\sqrt{1 + s^2})$.

For Q to be a maximum with a given area of channel or for the cost of construction to be a minimum, it is necessary that $\frac{d\,P}{d\,(d)} = 0$

But $$P = 2\left\{\frac{A}{2\,d} - \frac{s\,d}{2} + d\sqrt{1 + s^2}\right\}$$

$$\therefore \quad \frac{d\,P}{d\,(d)} = 2\left\{-\frac{A}{2\,d^2} - \frac{s}{2} + \sqrt{1 + s^2}\right\}$$

$\therefore$ For maximum value of Q

$$\sqrt{1+s^2} = \frac{2\,b\,d + s\,d^2}{2\,d^2} + \frac{s}{2}$$

$$= \frac{b}{d} + s$$

$$\therefore \quad (1+s^2)\,d^2 = (b + s\,d)^2 \tag{2}$$

But from the figure it will be seen that if a circle having its centre in the surface of the water can be drawn to touch the sides and bottom, we have

$$m\,n = b + s\,d\,;\ m\,p = m\,q = s\,d\,;\ n\,p = d$$

and since $(m\,n)^2 = (m\,p)^2 + (n\,p)^2$, the above equation is satisfied. These proportions then give the best results.

The value of s depends largely on the material in which the channel is excavated. The following may be taken as the minimum permissible values.

Earthen canal with faced sides $s = 1{\cdot}0$.

„ „ „ natural „ $s = 1{\cdot}5$.

„ „ in light soil $s = 2{\cdot}0$.

The latter value is usually adopted for all unfaced earthen sides.

Substituting this value of s in equation (2) we have

$$5\,d^2 = (b + 2d)^2$$

$$\therefore d = 4{\cdot}24\,b \tag{3}$$

$\therefore$ For maximum discharge $d = 2{\cdot}12\,B$, where B is the bottom breadth.

We then have

$$Q = C\sqrt{\frac{A^3}{P}} \cdot i$$

$$= C\sqrt{\frac{(2\,b\,d + s\,d^2)^3}{2\,(b + d\sqrt{1+s^2})}} \cdot i$$

Substituting for d from (2) we have

$$Q = C\,b^{\frac{5}{2}} \frac{(2\sqrt{1+s^2} - s)}{\sqrt{2}\,(\sqrt{1+s^2} - s)^{\frac{5}{2}}} \cdot \sqrt{i}$$

and giving s the value 2

$$Q = 20{\cdot}4\ C\,b^{\frac{5}{2}} \text{ cub. ft. per sec.}$$

if C and b are taken in foot units

or

$$Q = 36{\cdot}1\ C\,B^{\frac{5}{2}}.$$

Whatever the slope of the sides, the trapezium of best shape will be that in which the sides are made tangent to the circle of radius $= d$ having its centre in the surface, and as may be readily shown, all such channels have the same hydraulic mean depth $= \frac{d}{2}$. It follows then that the velocity of flow when the channel is full will be independent of the slope of the sides, and will depend solely on the gradient of the bed. The discharge of any two trapezoidal channels of the best form and of the same gradient and depths, will, when running full, be proportional to their respective mean widths.

The following table indicates how the top and bottom widths for a section of this type, vary with the slope of the sides:—

Slope.	Angle of inclination of sides.	Width.	
		Top.	Bottom.
0 to 1	90°	2·000 d	2·000 d
·25 to 1	75° . 58′	2·062 d	1·562 d
·5 to 1	63° . 26′	2·236 d	1·236 d
·75 to 1	53° . 8′	2·500 d	1·000 d
1·0 to 1	45° . 0′	2·828 d	·828 d
1·5 to 1	33° . 41′	3·606 d	·606 d
2·0 to 1	26° . 34′	4·472 d	·472 d
2·5 to 1	21° . 48′	5·385 d	·385 d
3·0 to 1	18° . 26′	6·325 d	·325 d

Circular Section.—Fig. 130.

Let d = diameter of circle.

„ θ = angle at centre subtended by wetted perimeter.

Then $$A = \frac{d^2}{8}(\theta - \sin\theta)\,;\; P = \frac{d}{2}\theta$$

$$\therefore \frac{A}{P} = \frac{d}{4}\left(1 - \frac{\sin\theta}{\theta}\right).$$ Suppose d fixed.

For maximum velocity $$\frac{d\left(\frac{A}{P}\right)}{d\theta} = 0$$

$$\therefore \theta = \tan\theta$$

$$\therefore \theta = 257\tfrac{1}{2}^\circ.$$

For maximum discharge $\dfrac{d\left(\dfrac{A^3}{P}\right)}{d\,\theta} = 0$

$$\therefore\ 3P\frac{dA}{d\theta} - A\frac{dP}{d\theta} = 0$$

$$\therefore\ 3\theta(1 - \cos\theta) = \theta - \sin\theta$$

$$\therefore\ 2\theta - 3\theta\cos\theta + \sin\theta = 0.$$

The value of θ which satisfies this equation is 308°, so that a circular conduit will give its maximum discharge when the depth of water is about ·95 of the diameter, the discharge then being about 5 per cent. greater than when completely full.

The discharge corresponding to any depth of water is given by

$$Q = C\sqrt{\frac{A^3}{P}\cdot i}$$

$$= C\sqrt{\frac{d^5}{256}\ \frac{(\theta - \sin\theta)^3}{\theta}\cdot i}$$

$$= \frac{C}{16}\,d^{\frac{5}{2}}\,i^{\frac{1}{2}}\sqrt{\frac{(\theta - \sin\theta)^3}{\theta}}$$

when $\theta = 180° = \pi$

$$Q = \frac{C}{5{\cdot}1}\,d^{\frac{5}{2}}\,\sqrt{i}\ \text{cub. ft. per sec.}$$

The semi-circular section when running full has a hydraulic mean depth of $\frac{d}{4}$, and since this is greater than that of any other form of channel of the same area, this section is well fitted for an open channel.

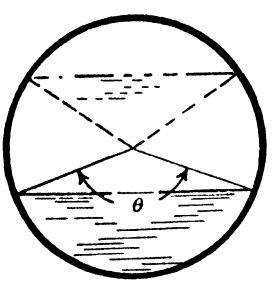


FIG. 130.

Where a polygonal channel is used, the hydraulic mean depth is greatest when the sides and bottom of the channel are designed so as to be tangent to a circle having its centre in the water line. The trapezoidal section and rectangular section of greatest flow, are particular cases of this. Where vertical sides are to be used the most suitable form of bottom consists of a circular arc, concave upwards.

Channel of Constant Mean Velocity.—Where the depth of water in a channel may vary within wide limits, it is in general desirable to design this so that the velocity of flow may be as nearly as possible independent of the depth. Otherwise, in an open canal, the velocity may become so great as to damage the sides and bottom by scouring (Art. 114), while in a sewer, with low heads, the velocity may become insufficient to produce

the necessary flushing. On the assumption that $v = C\sqrt{m\,i}$, where $C =$ constant, the only essential condition to be satisfied for v to be independent of the depth is that the hydraulic mean depth shall also be independent of the depth of water.

Thus the required channel must have sides formed by a continuous curve such that the area bounded by the sides, and any two horizontals varies as the length of the arcs intercepted between these horizontals. No curve can be found to satisfy these conditions, though close approximations may be obtained.

Obviously, a rectangular section of great depth compared with its width would satisfy the conditions approximately, and especially if its

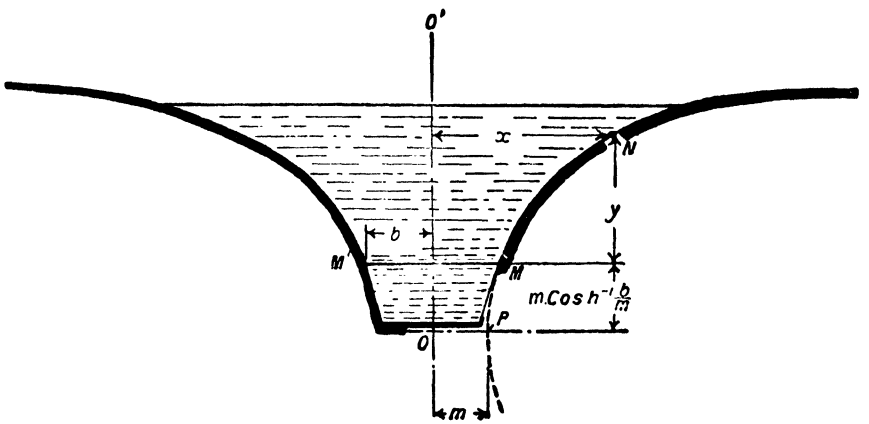


FIG. 131.

bottom were constructed so as to offer less resistance per unit area than its sides.

A construction which gives accurate results between certain limits may, however, be obtained as follows.

In Fig. 131 let x be the half breadth of the section at a height y above $M'\,M$ where the half breadth is b, and let s be the length of the arc $M\,N$

The position of the axis $M'\,M$ and the breadth b are usually fixed from a consideration of the minimum discharge to be expected through the channel, a trapezoidal channel having an upper breadth $M'\,M\,(= 2\,b)$ being designed to take this minimum discharge when running full. Let m be the hydraulic mean depth of this small channel, let p be its half perimeter, and a its half area. It is required to continue the sides of this channel so as to give a section for which the hydraulic mean depth $A \div P$ shall be equal to m for all depths of water.

Considering the section of the complete channel to one side of the axis $O\,O'$, we have

$$\frac{A}{P} = \frac{a + \int x\,dy}{p + s} = \text{constant} = m$$

$$\therefore \quad a + \int x\,dy = m\,(p + s).$$

Differentiating, this gives $x = m\,\frac{ds}{dy}$.

But
$$\frac{ds}{dy} = \sqrt{1 + \left(\frac{dx}{dy}\right)^2}$$

$$\therefore \quad x^2 = m^2\left\{1 + \left(\frac{dx}{dy}\right)^2\right\}$$

$$\therefore \quad \frac{dy}{m} = \frac{dx}{\sqrt{x^2 - m^2}}.$$

Integrating this we have

$$y = m\cosh^{-1}\frac{x}{m} + D$$

$$= m\log_e\{x + \sqrt{x^2 - m^2}\} + D'.$$

But $x = b$ when $y = 0$

$$\therefore \quad D = -\,m\cosh^{-1}\frac{b}{m}$$

$$D' = -\,m\log_e\{b + \sqrt{b^2 - m^2}\}$$

$$\therefore \quad y = m\left(\cosh^{-1}\frac{x}{m} - \cosh^{-1}\frac{b}{m}\right) \qquad (1)$$

$$= m\log_e\left\{\frac{x + \sqrt{x^2 - m^2}}{b + \sqrt{b^2 - m^2}}\right\}. \qquad (2)$$

From equation (2) the curve of the side may be plotted by calculating values of y, corresponding to a series of values of x.

Since $v = C\sqrt{m\,i}$ this velocity with any given gradient may be adjusted to any given value by designing the small channel so as to give the required value of m. The only restriction is that m cannot exceed $b \div 2$, this being its value when the lower channel is semicircular, or rectangular with a breadth equal to twice its depth.

EXAMPLE.

To design a channel to give an uniform velocity of flow of 4 feet per second, the half breadth b being 2·5 feet, and C having a value 90.

Here, assuming a rectangular section for the lower channel, of depth 2·5 feet, we have $m = 1{\cdot}25$ feet.

$$\therefore \quad 4 = 90\sqrt{1{\cdot}25\,i}$$

$$\therefore \quad i = {\cdot}00158.$$

Again when $x = 5$ we have

$$y = 1{\cdot}25 \log_e \left\{ \frac{5 + \sqrt{25 - 1{\cdot}5625}}{2{\cdot}5 + \sqrt{6{\cdot}25 - 1{\cdot}5625}} \right\}$$

$$= 1{\cdot}25 \log_e \left\{ \frac{5 + \sqrt{23{\cdot}4375}}{2{\cdot}5 + \sqrt{4{\cdot}6875}} \right\}$$

$$= 1{\cdot}25 \log_e 2{\cdot}108$$

$$= 1{\cdot}25 \times 2{\cdot}302 \log_{10} (2{\cdot}108)$$

$$= {\cdot}933 \text{ feet.}$$

Obtaining a series of such values of y, corresponding to definite values of x, the section may be constructed. For this particular example the following table shows how the half breadth of the section increases with the depth:—

x	2·5	5·0	10·0	15·0	20·0	30·0	50·0
y	0	·933	1·817	2·320	2·520	2·832	3·300

The curve is a portion of a catenary, and, writing its equation in the form $\frac{y - D}{m} = \cosh^{-1} \frac{x}{m}$,

or $$m \cosh \left(\frac{y - D}{m} \right) = x,$$

it will be seen that this catenary has its axis parallel to and at a distance $(-D) = m \cosh^{-1} \frac{b}{m}$ below the axis $M'M$, while its vertex P is at a horizontal distance m from the centre line $O\,O'$ (Fig. 131).

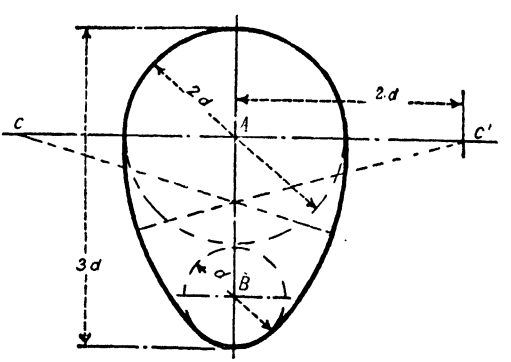


FIG. 132.

In a closed channel, or sewer, it is impossible to make the mean depth, and therefore the velocity, constant for all depths of water. To approximate to this as far as possible the egg shaped sewer (Fig. 132) is often used. In section this consists of two circular arcs centred at A and B, and connected by a second pair of circular arcs centred

at C and C'. The proportions often adopted in practice are indicated in Fig. 132.

Figure 133 *a* and *b* shows sections which are sometimes adopted for large sewers to the same end, the hydraulic mean depth being fairly high even with a small discharge.

Effect of varying *m* or *i*.

Assuming $$v = C\sqrt{m\,i}$$

we have $$\frac{dv}{dm} = \frac{C}{2}\sqrt{\frac{i}{m}}$$

or $$dv = \frac{C}{2}\sqrt{\frac{i}{m}} \cdot dm = \frac{v}{2m} \cdot dm$$

$$\therefore \quad \frac{dv}{v} = \frac{dm}{2m}.$$

Thus a small increase in m produces one-half its percentage increase in v.

Similarly it may be shown that $\frac{dv}{v} = \frac{di}{2i}$, so that in calculating v, and therefore the discharge, from this formula, any error in the value

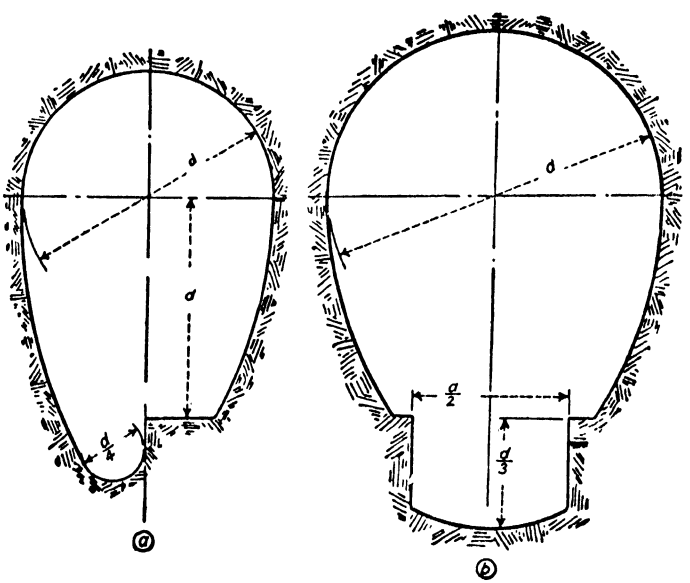


FIG. 133.

assumed for the slope will lead to one-half the proportional error in the estimated discharge.

Again, since $$Q = C\sqrt{\frac{A^3}{P}} . i$$

$$\frac{dQ}{dA} = \frac{3}{2} C \sqrt{\frac{i}{P}} . A^{\frac{1}{2}}$$

$$\therefore \quad \frac{dQ}{Q} = \frac{3}{2}\frac{dA}{A},$$

so that, neglecting the small increase in the wetted perimeter accompanying an increase in depth, any small increase in the cross sectional area produced by such a change in depth will be accompanied by 1·5 times this proportional increase in the discharge.

Again assuming A to be known and a small error to be made in the estimated value of P, we have

$$\frac{dQ}{dP} = C\sqrt{i}\left(-\frac{1}{2}\frac{A^{\frac{3}{2}}}{P^{\frac{3}{2}}}\right)$$

$$\therefore \quad \frac{dQ}{Q} = -\frac{1}{2}\frac{dP}{P},$$

so that this error leads to one-half the proportional error in the estimation of Q.

Since these errors should be severally small and may all occur in the same direction, the total possible error will be equal to their sum.

Thus if the probable error in the estimation of $i = p$ per cent.

„ „ „ „ „ „ „ $A = q$ per cent.

„ „ „ „ „ „ „ $P = r$ per cent.

The possible area in the estimation of the discharge, assuming C to have its correct value, will be given by

$$\left\{\frac{p + r}{2} + 1{\cdot}5\ q\right\} \text{ per cent.}$$

Art. 103.—General Equation of Flow in an Open Channel.

Consider a steady stream of cross-sectional area A, flowing over a bed having an inclination θ to the horizontal, where $\sin\theta$ = slope $= i$.

Let A B (Fig. 134) be any stream tube, the vertical depths of A and B below the surface being y_A and y_B.

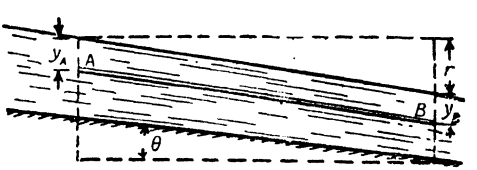


Fig. 134.

Let ${}_Ah_B$ be the loss of head in this stream tube from A to B, due to frictional resistances.

Then applying Bernoulli's equation of energy, we have

$$\frac{p_A}{W} + \frac{v_A^2}{2g} + z_A = \frac{p_B}{W} + \frac{v_B^2}{2g} + z_B + {}_Ah_B.$$

Let the difference in the level of the water surface over the points A and B be r.

Then $z_A + y_A = z_B + y_B + r$

$$\therefore z_A - z_B = r + y_B - y_A$$

$$\therefore \frac{p_A}{W} + \frac{v_A^2}{2g} + r + y_B - y_A = \frac{p_B}{W} + \frac{v_B^2}{2g} + {}_Ah_B.$$

If the stream is sensibly parallel over the length A B, as will be the case if θ is not large, we have

$$\frac{p_A}{W} = y_A \text{ and } \frac{p_B}{W} = y_B$$

$$\therefore \frac{v_A^2}{2g} + r = \frac{v_B^2}{2g} + {}_Ah_B$$

$$\therefore r = \frac{v_B^2 - v_A^2}{2g} + {}_Ah_B.$$

If now we imagine the area A divided into n elementary sections, each equal to $\frac{A}{n}$ we get, for each stream tube of area $\frac{A}{n}$:—

$$\frac{A}{n} \cdot r = \frac{A}{n} \cdot \frac{v_B^2 - v_A^2}{2g} + \frac{A}{n} \cdot {}_Ah_B$$

and, summing these over the whole section,

$$\Sigma_0^n\left(\frac{A}{n} \cdot r\right) = \Sigma_0^n\left(\frac{A}{n} \cdot \frac{v_B^2 - v_A^2}{2g}\right) + \Sigma_0^n\left(\frac{A}{n} \cdot {}_Ah_B\right).$$

If we assume that the velocity is constant over any cross section, the above equation reduces to

$$A\,r = A\,\frac{v_B^2 - v_A^2}{2g} + {}_AH_B$$

where ${}_AH_B$ is the total frictional loss between the cross sections at A and B.

This is still true of the whole mass of water in the stream if the velocity at a cross section is not uniform, provided that the distribution of velocity is such that the total kinetic energy at that section is equal to the mass of water multiplied by the square of the mean velocity at the section. In this case v_B and v_A become the mean velocities. Experiments by Messrs. Fteley and Stearns on the flow of water in the Sudbury conduit, 9 feet wide and 3 feet deep, in which the velocity was measured at 97 different points in a cross section, gave results showing that the error in assuming this to be true was less than 1 per cent. The error

will be greater in a shallow channel having a rough bed, but in general the results calculated on this assumption may be taken as substantially correct.

If P = wetted perimeter of section, and if $(\bar{v})^2$ = mean square of the velocity from A to B

$$_AH_B = \frac{f(\bar{v})^2}{2g} \cdot P \cdot A\ B,$$

$$\therefore\ r = \frac{v_B^2 - v_A^2}{2g} + \frac{f(\bar{v})^2}{2g} \cdot \frac{P}{A} \cdot A\ B.$$

If $A\ B = \delta\ l$ we have $r = \frac{dr}{dl} \cdot \delta\ l$, while if $v_A = v$, $v_B = v + \frac{dv}{dl} \cdot \delta\ l$ and $(\bar{v})^2 = \left(v + m\frac{dv}{dl}\delta\ l\right)^2$, where m is less than unity.

So that, neglecting small quantities of the second order

$$\frac{dr}{dl}\delta\ l = \frac{2v\frac{dv}{dl}\delta\ l}{2g} + \frac{fv^2}{2g} \cdot \frac{P}{A}\delta\ l$$

or

$$\frac{dr}{dl} = \frac{v}{g} \cdot \frac{dv}{dl} + \frac{fv^2}{2g} \cdot \frac{P}{A}. \qquad (1)$$

This is the general equation of flow in an open channel, v being the mean velocity at a cross section, and though the assumptions made in its conception are not altogether justified by the result of experiment, yet it forms a useful guide and is capable of a wide range of application in the general problems of channel flow.

If h is the depth of water at A (measured vertically from the surface) the depth at B is given by

$$h - \frac{dr}{dl} \cdot \delta\ l + i \cdot \delta\ l.$$

Again the depth at B is given by

$$h + \frac{dh}{dl} \cdot \delta\ l$$

$$\therefore\ i\ \delta\ l = \frac{dr}{dl} \cdot \delta\ l + \frac{dh}{dl} \cdot \delta\ l$$

$$\therefore\ \frac{dr}{dl} = i - \frac{dh}{dl}.$$

Substituting this value in (1) we get

$$i - \frac{dh}{dl} = \frac{v}{g}\frac{dv}{dl} + \frac{fv^2}{2g} \cdot \frac{P}{A}, \qquad (2)$$

giving the general equation in terms of the slope of the bed.

The physical interpretation of this equation is that the total loss of (potential and pressure) energy per unit length of the channel, due to the

fall in the level of the bed and in the depth of the water, is equal to the increase of kinetic energy together with the loss in friction per unit length of channel.

For uniform flow such as occurs in a culvert with slope, velocity, and depth of water constant, we have $\frac{d\,h}{d\,l} = 0, \frac{d\,v}{d\,l} = 0$

$$\therefore\ i = \frac{f\,v^2}{2\,g} \cdot \frac{P}{A}$$

or the total potential energy is absorbed in overcoming frictional resistances. This gives the relation between the slope and the velocity, or the discharge Q, for since $v = \sqrt{\frac{2\,g\,i\,A}{f\,P}}$

$$\therefore\ v\,A = Q = \sqrt{\frac{2\,g\,i\,A^3}{f\,P}},$$

so that for a given slope we can find the section for v to be a constant, and to give any required discharge.

With a rectangular section, breadth b, we get for either uniform or non-uniform flow, if b is constant and if Q is constant

$$Q = v\,b\,h = \text{const.}$$

$$\therefore\ v\,h = \text{const.}$$

$$\therefore\ h\frac{d\,v}{d\,l} + v\frac{d\,h}{d\,l} = 0$$

$$\therefore\ \frac{d\,v}{d\,l} = -\frac{v}{h} \cdot \frac{d\,h}{d\,l}.$$

Substituting this value, equation (2) becomes

$$i - \frac{d\,h}{d\,l} = -\frac{v^2}{g\,h}\frac{d\,h}{d\,l} + \frac{f\,v^2}{2\,g} \cdot \frac{P}{A}$$

$$\therefore\ \frac{d\,h}{d\,l} = \frac{i - \frac{f\,v^2}{2\,g} \cdot \frac{P}{A}}{1 - \frac{v^2}{g\,h}} \qquad (3)$$

Here $-\frac{v^2}{g\,h} \cdot \frac{d\,h}{d\,l}$ still represents the rate of increase of kinetic energy with length, and shows that the K. E. increases when the depth diminishes, *i.e.*, when $\frac{d\,h}{d\,l}$ is negative.

If b be great in comparison with h, we may write $\frac{P}{A} = \frac{b + 2\,h}{b\,h} = \frac{1}{h}$ (approximately), especially if, as is very usual in open channels, the bottom is rougher than the sides.

Substituting this value in equation (3) we have, for such a channel,

$$\frac{d\,h}{d\,l} = \frac{i - \frac{f\,v^2}{2\,g\,h}}{1 - \frac{v^2}{g\,h}} = i\left\{\frac{1 - \frac{f\,v^2}{2\,g\,h\,i}}{1 - \frac{v^2}{g\,h}}\right\} \qquad (4)$$

This gives the slope of the water surface, and from a solution of this equation, the profile of this surface may be determined.

Thus for steady uniform flow $\frac{d\,h}{d\,l} = 0$, *i.e.*, the depth is constant throughout.

$$\therefore\ 1 - \frac{f\,v^2}{2\,g\,h\,i} = 0$$

or
$$h = \frac{f\,v^2}{2\,g\,i} \qquad (5)$$

Art. 104.—Non-Uniform Flow.

If, as is usual, the motion while being constant at a point, varies from point to point in the length of the stream, so that $\frac{d\,v}{d\,l}$ and therefore $\frac{d\,h}{d\,l}$, is not zero, let H be the equivalent depth of an uniform stream of breadth b which would give the same discharge as the variable stream in unit time.

Then
$$H = \frac{f\,V^2}{2\,g\,i} \qquad \text{from (5)}$$

where
$$Q = V\,b\,H \text{ or } V = \frac{Q}{b\,H}$$

$$\therefore\ \frac{Q^2}{b^2\,H^2} = \frac{2\,g\,i\,H}{f} \text{ or } \frac{Q^2}{b^2\,h^2} = v^2 = \frac{2\,g\,i\,H^3}{h^2\,f}$$

Substituting $\frac{2\,g\,i\,H^3}{h^2\,f}$ for v^2 in (4), this becomes

$$\therefore\ \frac{d\,h}{d\,l} = i\left\{\frac{1 - \frac{H^3}{h^3}}{1 - \frac{H^3}{h^3}\cdot\frac{2\,i}{f}}\right\} \qquad (6)$$

This is the differential equation to the curve forming the longitudinal profile of the surface.

Since the value of $\frac{d\,h}{d\,l}$ depends both on the ratio $\frac{h}{H}$ which may be artificially adjusted in any stream, and on the ratio $\frac{2\,i}{f}$ which is fixed

once the slope and physical condition of the bed is fixed, an investigation of any particular case of flow must take into account both these factors.

The surface curves corresponding to a few particular cases of flow will now be investigated.

First suppose $\frac{2\,i}{f}$ less than unity, the state of affairs existing in a channel of slope less than $\frac{f}{2}$.

The following may be taken as approximate values of f at such velocities as are common in practice (Bazin), the foot being the unit of length.

Smooth cemented surface .	$f = \cdot0030$
Ashlar or brickwork . .	$= \cdot0037$
Rubble masonry . . .	$= \cdot0065$
Earth	$= \cdot0120$

so that for $\frac{2\,i}{f}$ to be less than unity, with a rubble masonry channel the slope would not exceed ·00325 feet per foot, or 1 in 310.

Case 1 (*a*). Let $\frac{2\,i}{f} < 1$ and also $h^3 < \frac{2\,i}{f} H^3$.

Both numerator and denominator of the right-hand side of equation (6) are now negative, so that $\frac{d\,h}{d\,l}$ is positive, *i.e.*, the depth of water increases down stream.

Also as h increases it finally reaches the critical value $\sqrt[3]{\frac{2\,i}{f}}$. *II.* Here the denominator becomes equal to zero, and in consequence the value of $\frac{d\,h}{d\,l}$ becomes ∞, or the surface curve at this point becomes vertical (Fig. 135), and the phenomenon known as the standing wave is produced.

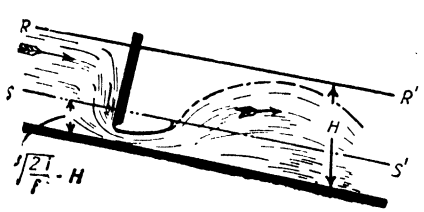


FIG. 135.

In the figure, suppose the dotted line $S\,S'$ to be drawn parallel to the bed, to represent a depth $\sqrt[3]{\frac{2\,i}{f}}$. *II.* If by means of a sluice we can

get the water surface below this level, then on passing the sluice the depth of water increases as shown. Finally, when $h = \sqrt[3]{\frac{2i}{f}} \cdot H$, the curve should be vertical where it intersects this line. Before this limit is reached, however, the hypothesis that the stream lines are sensibly parallel ceases to be even approximately true, and the curve becomes modified as shown in the dotted lines. This is exemplified in the case of a sluice fitted in a channel having a very small slope.

If h_1 and h_2 be the depths of the stream before and after its sudden change of level, the value of h_2 may be calculated.

Let v_1 and v_2 be the velocities at sections h_1 and h_2.

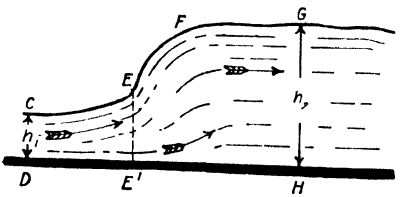


FIG. 136.

It is not now legitimate to assume that the loss due to shock at the sudden change of section is $\frac{(v_1 - v_2)^2}{g}$ as in the case of pipe flow, since the pressure over the area $E'E$ (Fig. 136) is no longer uniform and equal to that from E to F but varies with the depth, and hence one of the fundamental assumptions made in deducing this formula is unjustified.

On applying the equation of momentum, however, we have

Difference of forces acting in the direction of motion, on the faces $C\ D$ and $G\ H$ $\Big\} = p_1 A_1 - p_2 A_2$,

where p_1 and p_2 are the mean pressures over the areas A_1 and A_2.

Then $$p_1 = \frac{W h_1}{2} \qquad p_2 = \frac{W h_2}{2}.$$

The change of momentum per second, in passing the sections $C\ D$ and $G\ H$ $\Big\} = \frac{W}{g}\left\{A_2 v_2^2 - A_1 v_1^2\right\}$.

Also $A_1 v_1 = A_2 v_2$, and if the section is rectangular, $h_1 v_1 = h_2 v_2$, so that on equating the momentum per second to the force producing it, we have

$$\frac{h_2}{2}\frac{v_1}{v_2} - \frac{h_1}{2} = \frac{v_1^2 - v_1 v_2}{g}$$

$$\therefore \frac{h_2}{2} \cdot \frac{h_2}{h_1} - \frac{h_1}{2} = \frac{v_1^2}{g}\left\{1 - \frac{h_1}{h_2}\right\}$$

$$\therefore \frac{v_1^2}{g} = \frac{(h_2^2 - h_1^2)}{2 h_1} \cdot \frac{h_2}{h_2 - h_1} = \frac{(h_2 + h_1)}{2 h_1} h_2.$$

If $h_2 - h_1 = x$ this reduces to

$$\frac{v_1^2}{g} = \frac{(x + 2\,h_1)\,(x + h_1)}{2\,h_1}$$

$$\therefore\ x = \sqrt{\frac{h_1^2}{4} + \frac{2\,h_1\,v_1^2}{g}} - \frac{3}{2}h_1.$$

This gives the height of the standing wave.[1]

After rising to the level, h_2, where $h_2 \begin{matrix} > \\ < \end{matrix} \sqrt[3]{\frac{2\,i}{f}} \cdot H$, we have the state of affairs considered in Case 1 (*b*).

Case 1 (*b*). Here $\frac{2\,i}{f} < 1$, while h is greater than $\sqrt[3]{\frac{2\,i}{f}}$. H and is less than II.

In equation (6) the numerator is now negative, while the denominator is positive, so that $\frac{d\,h}{d\,l}$ is negative, or the depth h diminishes down-stream.

As the velocity increases and h diminishes, the denominator of the fraction $\frac{h^3 - H^3}{h^3 - \frac{2\,i}{f}\,II^3}$ vanishes before the numerator, so that $\frac{d\,h}{d\,l}$ tends to a limiting value $-\infty$ where $h = \sqrt[3]{\frac{2\,i}{f}} \cdot H$.

At this point the surface curve becomes vertical as shown in Fig. 137. If produced by a sudden drop in the bed of a stream, as shown in this figure, h increases up-stream, and approaches more nearly to H as this distance increases, the surface curve being asymptotic to the line $R\,R'$.

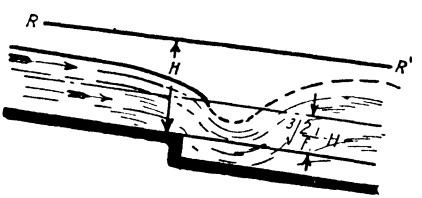


FIG. 137.

Such a drop in the bed may cause an appreciable increase in the velocity of flow for a considerable distance up-stream and may thus affect the foundations of structures (bridges, etc.) which may be placed up-stream, besides causing serious erosion of the bed.

In the case of the sluice (Fig. 135), the water after rising to the height h_2 is governed by this second set of conditions, so that the level again falls until $h = \sqrt[3]{\frac{2\,i}{f}} \cdot H$. Inertia then causes the level to fall below this, when we have the conditions of Case 1 (a) repeated. Thus a

[1] See "Proc. Inst. C. E.," vol. 197. 1913–14. pt. 3. p. 232.

series of stationary waves are produced, the level alternately rising and falling above and below that given by $h = \sqrt[3]{\frac{2\,i}{f}}\,.\,H$. At each successive jump a loss of energy occurs, and the velocity energy after the jump is therefore diminished. It follows that the value of h_2 must be greater after each successive jump, and ultimately will become equal to H, after which steady flow occurs.

The same reasoning applies to the stream after passing the drop in the bed (Fig. 137) the depth ultimately settling down to H.

The state of affairs outlined in this second case may be met with where a flume having a slight inclination delivers water to a penstock from which one or more turbines or water wheels are fed. It then becomes important that the proportions of the flume should be such as to prevent the water level from falling below a certain minimum. This problem will afterwards be considered in detail (p. 322).

Case 1 (*c*).

$$\text{Let } \begin{cases} \frac{2\,i}{f} < 1. \\ h > H. \end{cases}$$

Here the surface is everywhere above the line $R\,R'$ (Fig. 138). Both the numerator and denominator in the fraction.

$$i\left\{\frac{h^3 - H^3}{h^3 - \frac{2\,i}{f}H^3}\right\} \text{ (equation 6) are positive}$$

$$\therefore \quad \frac{d\,h}{d\,l} \text{ is positive.}$$

Down-stream then the depth increases, and $\frac{h^3 - H^3}{h^3 - \frac{2\,i}{f}H^3}$ ultimately tends to the limit unity, *i.e.*, $\frac{d\,h}{d\,l}$ tends to the limit i, the slope of the bed. It follows that the down-stream surface tends to become horizontal.

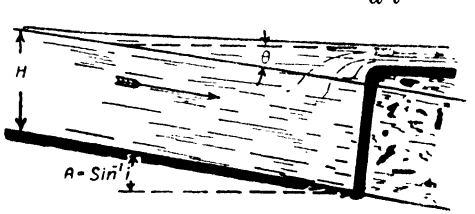


FIG. 138.

Up-stream h tends to the limit H, and $\frac{d\,h}{d\,l}$ to the limit zero, so that the curve tends to become asymptotic to the line $R\,R'$.

This is the form of surface curve produced by a weir or dam in a stream of small slope, and is of importance since the introduction of such

a dam causes what may be a serious raising of the backwater level for some considerable distance up-stream.

Next let $\frac{2\,i}{f}$ be greater than unity, the state of affairs usually existing in artificial water supply channels.

Case 2 (*a*).

$$\text{Let } \begin{cases} \frac{2\,i}{f} > 1. \\ h < H. \end{cases}$$

$\frac{d\,h}{d\,l}$ (equation 6) is now positive and the depth increases down-stream. Since, as h increases, the numerator vanishes before the denominator $\frac{d\,h}{d\,l} = 0$ in the limit, *i.e.*, the surface curve tends to become asymptotic to the line $R\,R'$ (Fig. 139).

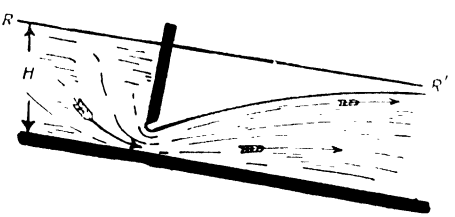


FIG. 139.

This state of affairs is attained at a sluice in a stream having a slope greater than $\frac{f}{2}$.

Case 2 (*b*).

Let $\frac{2\,i}{f} > 1$, and let h be greater than H and less than $\sqrt[3]{\frac{2\,i}{f}}\,.\,H$.

Here $\frac{d\,h}{d\,l}$ (equation 6) is negative and the depth diminishes down-stream. As h diminishes the numerator vanishes before the denominator and in the limit $\frac{d\,h}{d\,l} = 0$, or the curve becomes asymptotic to the line $R\,R'$, and the stream settles down to the uniform depth H. This state of affairs is realised where an obstacle in a river bed may have caused the level to rise to within the required limits. The surface curve is then as shown in Fig. 140. Up-stream, as h increases it finally reaches the value $\sqrt[3]{\frac{2\,i}{f}}\,.\,H$. Here $\frac{d\,h}{d\,l}$ is $-\infty$, and the curve becomes perpendicular to the bed of the stream. As h increases still further the state

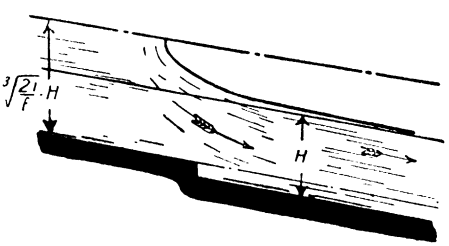


FIG. 140.

of affairs considered in Case 2 (*c*) is attained. This vertical front is seen when a sudden rush of water, such as may be produced by the bursting of an embankment, is caused in a channel of fairly rapid slope. It is also seen in the bores which occur at certain states of the tide in the Seine between Havre and Rouen, and in various other rivers and contracted channels.

Case 2 (*c*). Let $\begin{cases} \frac{2i}{f} > 1 \\ h > \sqrt[3]{\frac{2i}{f}}\, H. \end{cases}$

Here $\frac{dh}{dl}$ is positive $\therefore$ h increases down-stream. Down-stream $\frac{h^3 - H^3}{h^3 - \frac{2i}{f}H^3}$ tends to the limiting value unity, so that in this direction the limiting value of $\frac{dh}{dl}$ is i, or the surface tends to become horizontal (Fig. 141).

Up-stream, as the depth diminishes, we reach a point where $h = \sqrt[3]{\frac{2i}{f}}\, H$, and, for this value of h, $\frac{dh}{dl} = \infty$ or the surface curve here becomes perpendicular to the bed of the stream, a standing wave being produced.

This is the curve obtained where an under-water obstruction such as a dam or broad-crested weir is placed across a stream of rapid slope. Since the possibility of $\frac{dh}{dl}$ becoming infinite, depends on $\frac{2i}{f}$ being greater than unity, the production of a standing wave under these circumstances is only possible where this latter condition is satisfied.

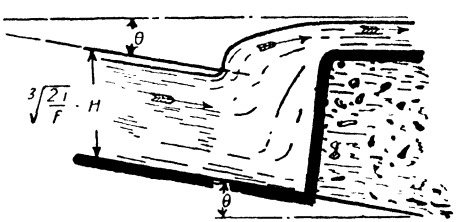


FIG. 141.

In practice the two most important cases are those represented in 1 (*b*) and 1 (*c*). In the first of these, the effect of a sudden drop in the bed of a stream may, as already explained, be serious, while the case of the reduction in level in a fore-bay feeding a power plant, caused by the sudden demand for energy by the turbines also comes under this heading. In the second, the effect of a dam in increasing the surface elevation at points further up-stream is important. The investigation of each of these cases resolves itself into determining, from a solution of

equation (6), the value of h corresponding to any point at a distance l from some datum, since when this is known, the rise or fall from normal, and consequently the change in velocity, can be determined.

To obtain a solution for the equation, we have

$$\frac{d\,h}{d\,l} = i\left\{\frac{\dfrac{h^3}{H^3} - 1}{\dfrac{h^3}{H^3} - \dfrac{2\,i}{f}}\right\}. \tag{6}$$

Writing $\frac{h}{H} = m$, so that $\frac{d\,h}{d\,m} = H$, this becomes

$$H\frac{d\,m}{d\,l} = i\left\{\frac{m^3 - 1}{m^3 - \dfrac{2\,i}{f}}\right\}$$

$$\therefore d\,m\left\{1 + \frac{1 - \dfrac{2\,i}{f}}{m^3 - 1}\right\} = \frac{i}{H}\,d\,l.$$

Integrating this expression,

$$\frac{i}{H}\,l = m + \left(1 - \frac{2\,i}{f}\right)\int\frac{d\,m}{m^3 - 1} + \text{const.} \tag{7}$$

while

$$\int\frac{d\,m}{m^3 - 1} = \frac{1}{2}\int\left\{\frac{m+1}{(m^2 + m + 1)\,(m-1)} - \frac{1}{m^2 + m + 1}\right\}d\,m$$

$$= -\frac{1}{2}\left\{\frac{1}{3}\log_e\frac{m^2 + m + 1}{(m-1)^2} + \frac{2}{\sqrt{3}}\tan^{-1}\frac{2\,m+1}{\sqrt{3}}\right\}$$

$$\therefore \frac{i}{H}\,l = \left[m - \left(1 - \frac{2\,i}{f}\right)\right.$$

$$\left.\left\{\frac{1}{6}\log_e\frac{m^2 + m + 1}{(m-1)^2} + \frac{1}{\sqrt{3}}\tan^{-1}\frac{2\,m+1}{\sqrt{3}}\right\}\right] + C. \tag{8}$$

The expression

$$\left\{\frac{1}{6}\log_e\frac{m^2 + m + 1}{(m-1)^2} + \frac{1}{\sqrt{3}}\tan^{-1}\frac{2\,m+1}{\sqrt{3}}\right\}$$

is often termed the "backwater function." Writing this as $\phi\,(m)$, the equation becomes

$$\frac{i}{H}\,l = m - \left(1 - \frac{2\,i}{f}\right)\phi\,(m) + C$$

$$\therefore\ i\,l = h - H\left(1 - \frac{2\,i}{f}\right)\phi\,(m) + C$$

$$\therefore\ h = i\,l + H\left(1 - \frac{2\,i}{f}\right)\phi\,(m) + C \tag{9}$$

Thus if h_1 and h_2 are the depths at points distant l_1 and l_2 from the datum, we have

$$h_1 - h_2 = i\,(l_1 - l_2) + H\left(1 - \frac{2\,i}{f}\right)\left\{\phi\,(m_1) - \phi\,(m_2)\right\} \quad (10)$$

The following table gives values of $\phi\,(m)$ for different values of $\frac{h}{H}$ in the case of a dam, where h is always greater than H.

From these values a curve may be constructed if required for use, and intermediate values obtained by interpolation.

$\frac{h}{H}$	$\phi\,(m)$	$\frac{h}{H}$	$\phi\,(m)$	$\frac{h}{H}$	$\phi\,(m)$	$\frac{h}{H}$	$\phi\,(m)$
1·000	∞	1·020	2·098	1·10	1·587	2·20	1·015
1·001	3·090	1·025	2·025	1·15	1·468	2·50	·989
1·002	2·860	1·030	1·966	1·20	1·387	3·0	·963
1·003	2·725	1·036	1·908	1·30	1·280	4·0	·939
1·004	2·629	1·044	1·843	1·40	1·211	5·0	·927
1·005	2·555	1·050	1·803	1·50	1·162	7·0	·915
1·007	2·445	1·056	1·763	1·60	1·125	10·0	·911
1·010	2·326	1·060	1·745	1·70	1·096	15·0	·909
1·012	2·266	1·070	1·697	1·80	1·073	20·0	·908
1·015	2·192	1·080	1·656	2·00	1·039	50·0	·907

In the case of a fall down-stream, $\frac{h}{H}$ is always less than unity. The following table gives values of $(\phi\ m)$ for this case.

$\frac{h}{H}$	$\phi\,(m)$	$\frac{h}{H}$	$\phi\,(m)$	$\frac{h}{H}$	$\phi\,(m)$	$\frac{h}{H}$	$\phi\,(m)$
1·000	∞	·985	2·183	·850	1·367	·400	·709
·999	3·090	·980	2·085	·800	1·253	·350	·656
·998	2·859	·975	2·009	·750	1·159	·300	·605
·997	2·723	·970	1·946	·700	1·078	·250	·553
·996	2·628	·960	1·847	·650	1·006	·200	·503
·995	2·552	·950	1·769	·600	·939	·150	·453
·994	2·491	·940	1·705	·550	·877	·100	·402
·992	2·395	·920	1·602	·500	·819	·050	·352
·990	2·319	·900	1·522	·450	·763	·000	·302

As an example of the use of these tables, calculate the rise in level at a point ½ mile up stream, produced by a dam arranged so as to raise the level at its crest through 8 feet. The original depth of the stream, supposed uniform, was 2 feet, the slope of the bed 1 in 500 (=·002), and the value of $f = ·006$.

The necessary height of the dam may be calculated as indicated on p. 177. Assuming this to be done, we have, in the preceding formulae

$$H = 2. \quad 1 - \frac{2}{3}\frac{i}{f} = 1 - \frac{2}{3} = \frac{1}{3}.$$

Let the suffix (1) refer to a point just above the dam, and the suffix (2) to the given point. Then since the positive direction of l is down-stream, we have in equation (10), $h_1 = 10$; $l_1 = 0$; $l_2 = -2640$, and from the tables we get $\phi\ (m_1) = \phi\left(\frac{10}{2}\right) = ·927$, so that this equation becomes

$$10 - h_2 = ·002\ (2640) + ·666\left\{·927 - \phi\left(\frac{h_2}{2}\right)\right\}$$

$$\therefore\ h_2 - \frac{2}{3}\phi\left(\frac{h_2}{2}\right) = 4·102.$$

This equation can only be solved by trial.

Let $$h_2 - \frac{2}{3}\phi\left(\frac{h_2}{2}\right) - 4·102 = y$$

Then if $h_2 = 5$, $y = 5 - ·6593 - 4·102 = + ·2387$

,, $h_2 = 4$, $y = 4 - ·6927 - 4·102 = - ·7947$

For a solution of the equation, y must equal 0, and since the value of a continuous function such as y cannot change from + to − without passing through the value zero, it follows that for some value of h_2 between 4 and 5, $y = 0$.

Evidently, too, the correct value of h_2 is nearer 5 than 4. Try $h_2 = 4·75$.

If $h_2 = 4\ 75$, $y = 4·75 - ·6656 - 4·102 = - ·0176$. The value of h_2 is then between 4·75 and 5·0.

A close approximation to the correct result can then be obtained by drawing a curve connecting those values of y and of h_2 already found. Where this curve intersects the axis of h_2, we shall have the value of h_2 which makes $y = 0$, and therefore which satisfies the equation. In the problem, $h_2 = 4·78$ provides a very close approximation to the correct value. At a distance up-stream equal tc 4,000 feet the value of h, determined in the same way, is 2·34 feet. Since the slope is ·002 the height of the bed at this latter point, above that at the dam, is ·002 × 4,000 = 8·0 feet.

The surface at this latter point is therefore ·34 feet higher than at the dam. With the dam removed and the flow per minute unchanged, the flow being uniform and the depth of channel equal to H, the difference of level instead of being ·34 feet would be 8·0 feet.

Figure 142 illustrates the form of backwater curve observed by D'Aubuisson on the Wesser. Here the mean slope of the bed was 2·33 feet per mile = ·000141, and the depth before introducing the dam was 2·46 feet. The effect of the dam in raising the surface level was apparent for 4·33 miles up-stream.

In a second series of observations on the Werra, the following results were obtained.

Mean depth $H = 1{\cdot}7$ feet; width = 80 feet; fall = 3·88 feet per mile

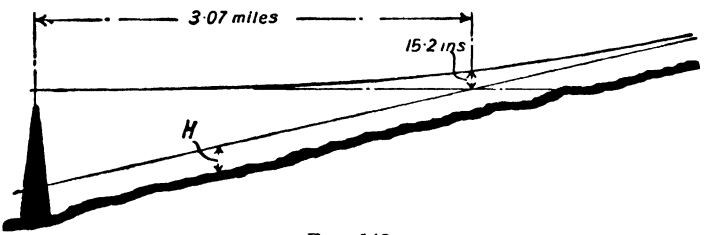


FIG. 142.

A dam 15·66 feet in height was placed across the stream, and the height of water over the sill was found to be 1·13 feet.

The following table indicates the observed and calculated depths, and rises in surface level at points above the dam.

Distance of observed point from Dam.		0.	·85 Miles.	1·5 Miles.	3·5 Miles.
Depth of water in feet .	observed	16·79	12·30	11·28	3·01
	calculated	—	13·50	11·35	3·20
Rise of level in feet .	observed	15·09	10·60	9·58	1·31
	calculated	—	11·80	9·65	1·50

In these calculations the value of f has been taken as ·02.

As an example of the effect of a drop in the bed of a stream in pro

ducing an increased velocity at points up stream, consider the same stream as before to have a fall at some point in its length (Case 1 (*b*)), and suppose that this causes a lowering of the surface just above the fall through a depth of 6 inches.

Then as before $H = 2$; $1 - \frac{2\,i}{f} = \frac{1}{3}$.

Also in equation (10), we have

$$h_1 = 1{\cdot}5\,;\ l_1 = 0\,;\ \phi\,(m_1) = \phi\left(\frac{1{\cdot}5}{2}\right) = 1{\cdot}159 \text{ (from table).}$$

To determine the depth at a point 50 feet up stream we have $l_2 = -50$ feet, so that the equation becomes

$$1{\cdot}50 - h_2 = {\cdot}002\,(50) + \frac{2}{3}\left\{1{\cdot}159 - \phi\left(\frac{h_2}{2}\right)\right\}$$

$$\therefore\ h_2 - \frac{2}{3}\phi\left(\frac{h_2}{2}\right) - 1{\cdot}50 - {\cdot}10 - {\cdot}773$$

$$= {\cdot}627.$$

Putting $h_2 - \frac{2}{3}\,\phi\left(\frac{h_2}{2}\right) = {\cdot}627 = y$ we get

if $h_2 = 1{\cdot}8$, $y = 1{\cdot}8 - 1{\cdot}0146 - {\cdot}627 = +{\cdot}1584$
„ $h_2 = 1{\cdot}9$, $y = 1{\cdot}9 - 1{\cdot}1793 - {\cdot}627 = +{\cdot}0937$
„ $h_2 = 1{\cdot}95$, $y = 1{\cdot}95 - 1{\cdot}3393 - {\cdot}627 = -{\cdot}0163$

On plotting values of h_2 and y, the curve shows that y_2 is zero when $h_2 = 1{\cdot}946$ (approximately), and this, therefore, gives the depth at the given point.

At a point 20 feet up stream we have

$$h_2 - \frac{2}{3}\,\phi\left(\frac{h_2}{2}\right) = {\cdot}667$$

On solving this equation in the same way, we find that the depth here is 1·932 feet (approximately). Since the breadth is constant, the mean velocity at any point is inversely proportional to the depth, so that at the two points 20 and 50 feet up stream the velocities are increased by the fall in the ratios $\frac{2}{1{\cdot}932}$ and $\frac{2}{1{\cdot}946}$ respectively.

This action becomes increasingly important as the slope is diminished. For example, in the previous case, if the slope were diminished to ·001, the velocities at the same two points would be increased by the fall in the ratios $\frac{2}{1{\cdot}56}$ and $\frac{2}{1{\cdot}72}$ respectively.

As a further example of the use of these formulae, consider the case of a flume of rectangular section, feeding a forebay from which a turbine is to be supplied.

The breadth of flume is 20 feet, the slope 1 in 1,000, the length 1,000 yards, and the value of $f = \cdot 003$. The discharge required is 500 cubic feet per second.

The value of H, the uniform depth necessary to give this supply is given by

$$H = \sqrt[3]{\frac{f Q^2}{b^2 2 g i}} = \sqrt[3]{\frac{\cdot 003 \times 25 \times 10{,}000}{61 \cdot 4 \times 400 \times \cdot 001}}$$
$$= 3 \cdot 08 \text{ feet.}$$

If at the upper end the depth of water is greater than this, say 4 feet, we have (Case 1 (*c*)) and everywhere $h > H$. The depth of water thus increases down stream.

Applying equation (10), we now have

$$\begin{cases} h \text{ (at entrance to flume)} = 4 \cdot 0 ; \; l_1 = 0 \\ H = 3 \cdot 08 ; \; l_2 = 3{,}000 \text{ feet} ; \; \frac{2i}{f} = \frac{\cdot 002}{\cdot 003} = \frac{2}{3} \\ \phi \left(\frac{h_1}{H} \right) = \phi \left(\frac{4}{3 \cdot 08} \right) = \phi \, (1 \cdot 3) = 1 \cdot 280 \text{ (tables)} \end{cases}$$

$$\therefore \quad 4 - h_2 = - \cdot 001 \, (3{,}000) + \frac{3 \cdot 08}{3} \left\{ 1 \cdot 280 - \phi \left(\frac{h_2}{H} \right) \right\}$$

$$\therefore \quad 7 - 1 \cdot 315 = h_2 - 1 \cdot 027 \, \phi \left(\frac{h_2}{H} \right)$$

$$\therefore \quad h_2 - 1 \cdot 027 \, \phi \left(\frac{h_2}{H} \right) - 5 \cdot 685 = y.$$

If $h_2 = 6 \cdot 70$ $\quad y = 1 \cdot 015 - 1 \cdot 027 \, (1 \cdot 017) = - \cdot 030$

„ $h_2 = 6 \cdot 75$ $\quad y = 1 \cdot 065 - 1 \cdot 027 \, (1 \cdot 015) = + \cdot 022.$

The correct value of h_2, the depth in the forebay, is approximately 6·73 feet. The depth of the water in the flume at different points in its length can be calculated in the same way, and the necessary height of side ascertained.

If, at the upper end, the depth of water is equal to H, this will remain constant throughout, while if less than H and greater than $\sqrt[3]{\frac{2i}{f}} H$ or ·871 H, *i.e.*, between 3·08 feet and 2·68 feet, we get Case 1 (*b*). The height will now decrease down stream until it reaches the value $\sqrt{\frac{2i}{f}}$, after which a series of waves will be produced, and the depth will, as explained in Case 1 (*b*), finally settle down to 3·08 feet.

The critical point is found by putting $h_2 = \sqrt[3]{\frac{2i}{f}} H = 2 \cdot 68$ in equation (10). Let $h_1 = 3 \cdot 0$ feet.

Then we have at the critical point

$$3 - 2{\cdot}68 = {\cdot}001\,(-l_2) + \frac{3{\cdot}08}{3}\left\{\phi\left(\frac{3{\cdot}0}{3{\cdot}08}\right) - \phi\left\{\frac{2{\cdot}68}{3{\cdot}08}\right\}\right\}$$

$$\therefore \quad {\cdot}32 = {\cdot}001\,(-l_2) + 1{\cdot}027\,\{\,1{\cdot}996 - 1{\cdot}429\,\}$$

$$\therefore \quad {\cdot}262 = {\cdot}001\,l_2$$

$$\therefore \quad l_2 = 262 \text{ feet.}$$

If, at the entrance, the depth is less than 2·68 feet, as might occur if this flume were fed from a sluice, we have Case 1 (*a*) repeated, and the depth would finally increase up to 3·08 feet. With an open channel leading directly out of a reservoir, the required discharge could only be obtained by having the depth at entrance equal to or greater than H.

Art. 105.—Channel with Horizontal Bed.

Here i is zero, so that equation (6) of the last article ceases to apply. Making $i = o$ in equation (3), p. 308, we get

$$\frac{d\,h}{d\,l} = \frac{\dfrac{f\,v^2}{2\,g}\cdot\dfrac{P}{A}}{1 - \dfrac{v^2}{g\,h}} \tag{1}$$

Writing $v^2 = \dfrac{Q^2}{b^2\,h^2}$ we have, if the channel is rectangular and broad, so that $\dfrac{P}{A} = \dfrac{1}{h}$ (sensibly),

$$\frac{d\,h}{d\,l} = \frac{\dfrac{f\,Q^2}{2\,g\,b^2\,h^3}}{1 - \dfrac{Q^2}{g\,b^2\,h^3}}$$

$$\therefore \quad d\,h\left\{\frac{2\,g\,b^2\,h^3}{f\,Q^2} - \frac{2}{f}\right\} = d\,l. \tag{2}$$

Integrating between the limits l_1 and l_2 we get

$$l_1 - l_2 = \frac{1}{f}\left\{\frac{g\,b^2}{2\,Q^2}\,(h_1^4 - h_2^4) - 2\,(h_1 - h_2)\right\} \tag{3}$$

from which the difference in level $(h_1 - h_2)$ at any two points distant $l_1 - l_2$ from each other, may be calculated when a given quantity Q cubic feet per second is flowing along the channel.

Expressing (2) as

$$\frac{d\,h}{d\,l} = \frac{1}{\dfrac{2}{f}\left\{\dfrac{g\,b^2\,h^3}{Q^2} - 1\right\}} \tag{4}$$

we have, for $\dfrac{d\,h}{d\,l}$ to be infinite, the condition

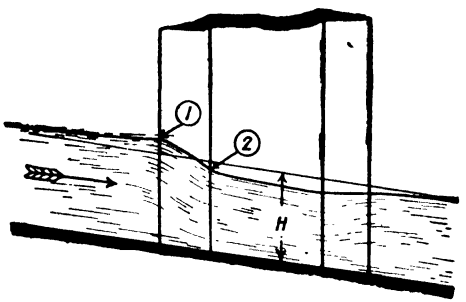
$$g\,b^2\,h^3 = Q^2 = v^2\,b^2\,h^2$$

$$\therefore \quad v^2 = g\,h,$$

so that if by any means such as drawing off a considerable amount of water suddenly by opening a lock gate, the velocity can be made equal to $\sqrt{g\,h}$, a wave with vertical crest will be produced.

ART. 106.—CHANGE OF LEVEL IN A STREAM PRODUCED BY BRIDGE, PIERS, ETC.

Where a series of piers are placed across the bed of a stream, the effect is to raise the up-stream level exactly as if a dam were placed in the stream, and the form of the surface curve will depend on whether the stream satisfies the conditions of Case 1 (*c*) or 2 (*c*) (Art. 105).



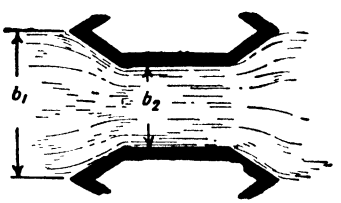


FIG. 143.

The height will be a maximum at the up-stream end of the pier. On arriving at the contracted section of the stream, the velocity will be increased, the increase in kinetic energy necessitating a corresponding loss of potential energy, and the depth is diminished. On again arriving at the open channel the velocity diminishes and the depth increases (Fig. 143).

Neglecting losses by friction between the sections (1) and (2), if b_1 and b_2 are the effective breadths and h_1, h_2 the depths at these points, we have

$$h_1 + \frac{v_1^2}{2g} = h_2 + \frac{v_2^2}{2g}$$

assuming the kinetic energy to be that given by $\frac{v_1^2}{2g}$ and $\frac{v_2^2}{2g}$ where v_1 and v_2 are the mean velocities at the two sections, an assumption which within narrow limits is justified by experiment.

$$\therefore \quad v_2^2 = v_1^2 + 2\,g\,x, \text{ where } x = h_1 - h_2.$$

If the discharge through the contracted area at (2) is given by $c\,b_2\,h_2\,v_2$, we have

$$v_2 = \frac{Q}{c\,b_2\,h_2}$$

Also $$v_1 = \frac{Q}{b_1 h_1}$$

$$\therefore \quad \frac{Q^2}{(c\, b_2\, h_2)^2} = \frac{Q^2}{(b_1 h_1)^2} + 2 g x$$

$$\therefore \quad x = \frac{Q^2}{2g}\left\{ \frac{1}{c^2 b_2^2 h_2^2} - \frac{1}{b_1^2 h_1^2} \right\}$$

or $$x = \frac{Q^2}{2g}\left\{ \frac{1}{c^2 b_2^2 (h_1 - x)^2} - \frac{1}{b_1^2 h_1^2} \right\}.$$

The value of c varies with the form of pier, but with pointed cutwaters is about ·95 (Eytelwein), diminishing to ·85 for a bridge having square or rectangular piers. By considering the problem as one of flow through a weir or notch having a submerged crest (this crest being level with the bed of the stream), and under an effective head x, we may obtain a second expression for Q in terms of x, and by equating these two expressions the value of h_1 may be obtained in terms of Q and of b_1 and b_2. From this, an application of equation (9) (p. 316) will give the depth at any point up stream, and the entire up-stream profile may then be plotted. On the down-stream side of the obstacle there is a gradual rise of the surface level as the depth increases to a uniform value H.

ART. 107.

With radial outward flow over a horizontal bed, such as occurs when a vertical stream impinges on such a surface, we have, if h is the depth at a radius r, Q the quantity per second, and v the velocity at radius r,

$$Q = v \times 2\pi r h = \text{const.}$$

$$\therefore \quad v r \frac{d h}{d r} + r h \frac{d v}{d r} + v h = 0$$

$$\therefore \quad \frac{d v}{d r} = -\left\{ \frac{v}{h}\frac{d h}{d r} + \frac{v}{r} \right\} \qquad (1)$$

Substituting this value in equation (2) (p. 307), we have

$$i - \frac{d h}{d r} = -\left\{ \frac{v^2}{g h}\frac{d h}{d l} + \frac{v^2}{g r} \right\} + \frac{f v^2}{2 g} \cdot \frac{P}{A}. \qquad (2)$$

But with a horizontal bed $i = o$.

Also $$\frac{P}{A} = \frac{2\pi r}{2\pi r h} = \frac{1}{h}, \text{ and } l = r$$

$$\therefore \quad (2) \text{ becomes } \frac{d h}{d r} = \frac{\frac{v^2}{g r} - \frac{f v^2}{2 g h}}{1 - \frac{v^2}{g h}} = \frac{\frac{v^2}{g}\left\{ \frac{1}{r} - \frac{f}{2 h} \right\}}{1 - \frac{v^2}{g h}}. \qquad (3)$$

This becomes infinite if $v^2 = g\,h$,

i.e., if $$\frac{Q^2}{4\,\pi^2\,r^2\,h^2} = g\,h$$

if $$h^3 = \frac{Q^2}{4\,\pi^2\,g} \cdot \frac{1}{r^2}. \qquad (4)$$

At the radius at which this relation holds, a standing wave will be formed.

The height of this wave may be estimated, as in the case of the wave produced in a rectangular channel.

If v_1 and v_2 are the velocities and h_1 and h_2 the depths immediately before and after the rise, so that $h_2 - h_1 = x$, we have, as on p. 312,

$$\frac{v_1^2}{g} = \frac{(h_2 + h_1)\,h_2}{2\,h_1}$$

from which

$$h_2 - h_1 = x = \sqrt{\frac{h_1^2}{4} + \frac{2\,h_1\,v_1^2}{g}} - \frac{3}{2}\,h_1.$$

ART. 108.—CHANGE OF LEVEL PRODUCED BY THE PASSAGE OF A BOAT THROUGH A NARROW CANAL WITH HORIZONTAL BED.

Let A = cross sectional area of canal.

,, a = sectional area of vessel amidships, beneath water line, by a plane perpendicular to its axis.

Let v = velocity of vessel.

Here the state of affairs may be simplified if we first consider the water to be a perfect fluid. As the vessel moves along through this fluid, the volume displaced by the forepart passes along backwards between the vessel and the sides and bottom of the canal to fill the space vacated in the rear. In this case we get a backward current extending from the prow to the stern of the boat, its velocity increasing as the effective area of the channel diminishes, and having a maximum value $v\left(\frac{a}{A-a}\right)$ at the amidships section.

To produce this current a surface gradient is necessary, the surface falling from its normal level at the prow to a minimum at the amidships section, and from this point rising to its normal level at the stern.

Due to the viscosity of the fluid, however, a mass of water is dragged along with the boat, forming a current confined mainly to the centre and surface of the canal.

Since, for permanence of the *régime*, the backward flow across any section of the canal must equal the corresponding forward flow, the back-

ward bottom current must now be sufficiently great to supply an additional mass of water equal to this, so that its velocity at the minimum section will be $\left(K + \frac{a'}{A - a'} \right) v$, its velocity at the bows being $K\ v$. Here $a' = a + a_1$ where a_1 is the area of the channel occupied by the forward current at the amidships section. $A - a'$ is then the effective area of the backward current.

But to produce a backward velocity of flow at the bows of the boat the surface level at the bows must be less than at some distance ahead, and will thus be below the normal. The result is that the water level in the canal falls as the boat approaches, has its minimum value near the amidships section, and then rises to attain its normal value. The effect of the bow waves in modifying the level at the bows is here neglected.

By applying equation (3) (p. 322), the difference of level at any two

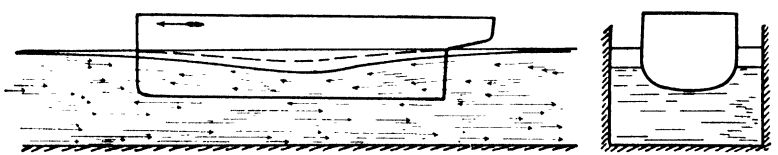


FIG. 144.

points in advance of the boat may be deduced in terms of $K\ v$, since $Q = K\ v\ b\ h_1$, where h_1 is the depth of water at the bows.

Fig. 144 shows the surface curves for a perfect fluid (dotted lines) and for water.[1]

If the boat is nearer to one side of the channel the velocities of flow are greater on this side of the boat, the pressures, particularly abaft its beam are consequently less, with the result that it tends to sheer off towards the further side.

ART. 109.—SUCTION EFFECT BETWEEN PASSING SHIPS.

Even in open water, where one boat is overtaking another in moderately close proximity on a parallel path their mutual action, due to interference of currents between their hulls may have serious effects. The mass of water displaced by the forepart of the leading boat, returning to fill the space vacated by its stern causes a continual influx of water towards the

[1] For the further investigation of the change of level round a moving vessel, a paper in *The Engineer*, vol. 63, p. 252, may be consulted.

stern of the latter, and this is increased by the influx necessary to provide the water thrown astern by the propeller. Further astern the impact of the streams converging from the two sides of the hull produces a region of outflowing currents.

The bows of the overtaking boat first come within the influence of this region which tends to produce a slight outward sheer. As it creeps further ahead the bows come within the influence of the inflowing currents while the stern is still being repelled, with a resultant tendency to inward sheer which has often led to serious collisions. This effect is a maximum where the bows of the follower are about one-third of its length aft the bows of the leading ship. As the follower draws further ahead the tendency to sheer diminishes and is replaced by a tendency to bodily inward drift, while when almost abreast, the bows of the follower become exposed to the outflowing currents from the leader's bows while its stern is still being attracted, with a consequent tendency to an outward sheer.[1]

The effect depends largely on the sizes, speeds, and relative speeds of the vessels, increasing with the size of the leader, and with the common speed, and diminishing as the relative speed increases.

Art. 110.—Flow round River Bends.

A river flowing through an alluvial plain always tends to gradually increase any winding which may occur in its course, until finally a new channel is cut through the narrow neck of land thus formed. The following explanation of this scouring of the outer bank of a bend and the deposition of detritus on the inner bank has been given by Professor James Thompson.[2]

In consequence of the centrifugal force, the pressure at any level in a transverse section of the stream increases outwards, so that the level of the free surface is highest near the outer bank. Near the bottom, however, the resistance of the bed reduces the velocity and consequently the centrifugal force of the water, which now becomes insufficient to overcome the tendency to inward flow produced by the higher level of the free surface at the outside of the curve. The water near the bottom, then has a tendency to flow inwards and to carry with it gravel and other detritus which is left at the inner bank. Experiments in a model river

[1] For a full discussion of the phenomena of interaction an article in "Bedrock" Vol. I, No. 1, pp. 66—87, may be consulted.

[2] "Proceedings Royal Society," 1877, p. 356.

bend, in which the direction of flow was indicated by coloured stream lines and by the behaviour of threads tied to pins fixed in the bed of the stream, as well as by floating particles of matter, indicated a state of affairs as represented in Fig. 145. Here the dotted line *A B* indicates the path of a particle floating in the surface, while the curves shown in full represent the motion near the bed of the stream.

As indicated, a counter-current flows from inner to outer bank over the upper portion of the stream, but since the same volume of water is moved by the two currents, and since the sectional area of the outer current is comparatively very large, its effect in carrying suspended matter to the outer bank is negligible.

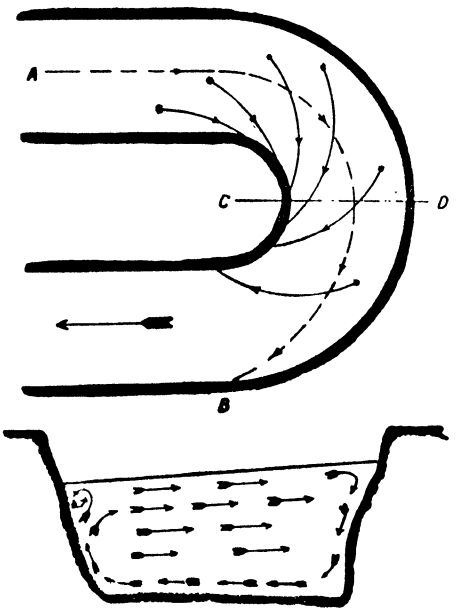


FIG. 145.

While this theory undoubtedly accounts for a portion of the erosion, and for the deposition of detritus at the inner bank, it is probable that the impact of the stream on the concave bank is a more potent factor in actually causing erosion, and more particularly is this the case when the stream is in flood and when in consequence the erosive effects are most serious. Under such circumstances observation shows that the surface velocity is a maximum near the outer, and not the inner, bank.

ART. 111.—LOSS OF HEAD PRODUCED BY BENDS IN AN OPEN CHANNEL.

Very little experimental evidence is available regarding the loss of head due to bends in an open channel. Experiments on a cement lined semicircular conduit, 9·8 feet in diameter, divided into four consecutive sections *A B C D*, showed the following results.[1]

Section *A* is a tangent 640 feet long.

„ *B* is 120 feet long and includes a curve of 100 feet radius.

[1] *Engineering Record*, Oct. 21st, 1911. By E. G. Hobson. See also Yarnell and Woodward, *Tech. Bull.* 526, U. S. Dept. of Agriculture, Oct. 1936.

Section *C* is 220 feet long and includes two **50 feet radius** reverse curves.

„ *D* is 1075 feet long and is practically straight.

The horizontal curves were approximated to in construction by 10 foot tangents. The mean depth throughout was approximately 4 feet and the hydraulic mean radius 2·14 feet.

Section.	*A*	*B*	*C*	*D*
Mean velocity . .	7·10	6·86	6·94	7·15
Co-efficient *C* . .	129	114	90	119
Kutter's *n* . . .	·0132	·0149	·0189	·0142

The low value of the coefficient *C* in *D* as compared with that in *A* is doubtless due to the loss produced in this section during the redistribution of velocities produced by bend *C* and ought strictly to be debited to that bend, while similarly, a certain portion of the loss really due to bend *B* would appear due to the bends in the next section *C*.

Probably the following values of *C* would be more approximately correct.

	Value of *C*.
A. and *D*. Straight channel . .	128
B. Curve of 100 ft. radius . . .	118
C. Reverse curves of 50 ft. radius .	88

Art. 112.— Distribution of Velocity in an Open Channel

Depression of the Filament of Maximum Velocity.

As in the case of a closed pipe, the resistance introduced by the solid boundaries causes the velocity to diminish in the neighbourhood of the

sides and bottom of an open channel, and from analogy with pipe flow it might be expected that the maximum velocity in any cross section would be found in the surface and at the centre of the stream.

While this is commonly the case in a broad, rapid, and shallow stream

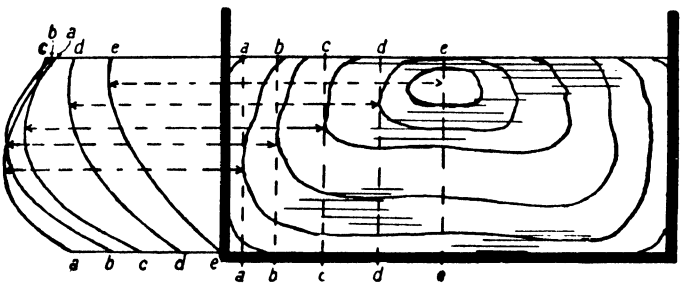


FIG. 146.

in any other case the filament of maximum velocity is usually found below the surface even with a down-stream wind. Its depth varies with the direction of the wind, with the depth and physical characteristics of the stream, and with the velocity of flow. On a calm day it usually lies at a depth between ·1 h and ·4 h (where h is the depth of the stream) and for depths above 5 feet, has a mean depth of about ·3 h.

Fig 146, taken from a gauging by Darcy of a rectangular channel ·25

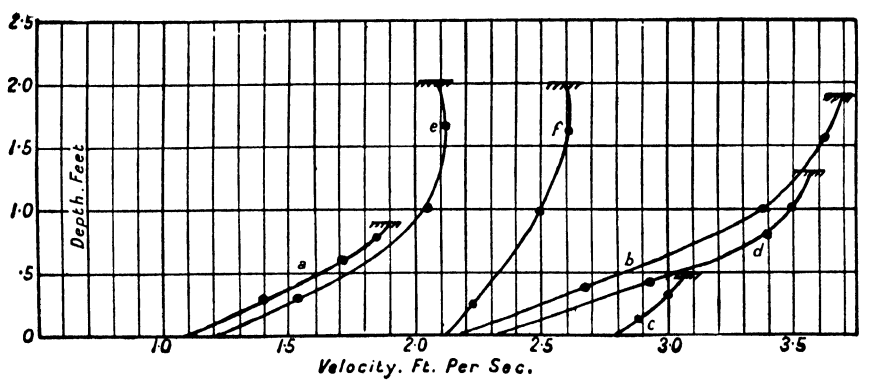


FIG. 147.

metre deep and ·8 metre wide, shows the general distribution of velocity over various vertical sections of a rectangular channel and also the equi-velocity contours in a cross section, while Figs. 147 and 148, show

the results of gaugings on the experimental channel of the Cornell University.[1]

This is of rectangular section, with concrete sides and bottom having a slope of 1 in 500, and has a width of 16 feet. Velocity measurements were made in eight verticals in a cross-section by means of current meters.

The curves in Figs. 147 and 148 show the variations of velocity in a vertical plane in typical of these experiments, each plotted point giving the mean of all eight observations at that depth in the cross-section.

The effect of a large ratio of width to depth in raising the filament of maximum velocity is evident from a comparison of the curves of Fig. 147

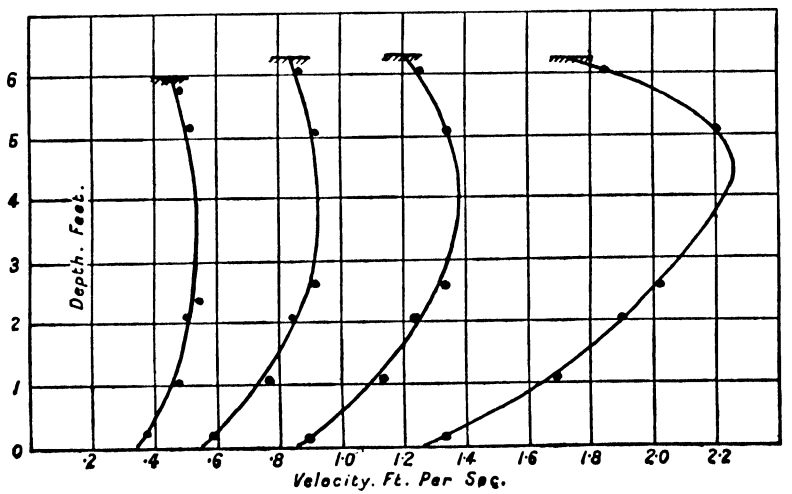


FIG. 148.

and of Fig. 148, while the effect of an increased velocity of flow in raising the filament is evident from a comparison of the several curves of Fig. 148.

The depression of this filament of maximum velocity is mainly due to the action of the sides of the channel. Frictional losses at the sides reduce the energy and thus the head of the water in their neighbourhood, with the result that the surface level at the sides is lower than near the centre of the stream, and the cross-sectional profile of the water surface is a curve concave to the bed. Owing to this super-elevation of the water near the centre and to its tendency to find its own level, transverse currents are set up which travel downwards near the centre of the stream; outwards along the bottom to either bank, upwards along the sides, and, for permanence of *régime*, inwards along and near the surface.

[1] U. S. Geological Survey. Water Supply and Irrigation Papers, No. 95.

Since the inward surface drifts consist of water which has travelled up the sides and has come from the region of minimum velocity, they will evidently have the effect of reducing the surface velocity and of depressing the filament of maximum velocity.

The sketches in Fig. 149 *a* and *b* show respectively the directions

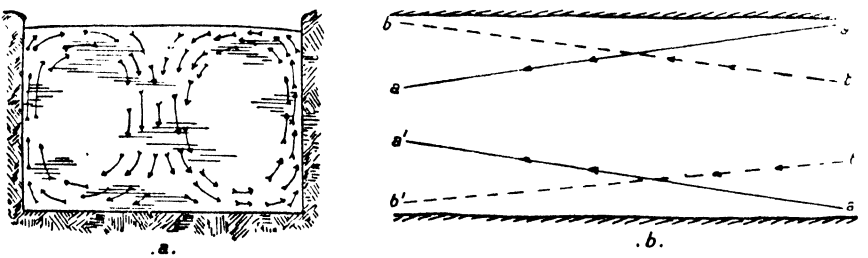


FIG. 149.

of the transverse currents, and of the resultant motion of the stream, the full lines $a'a'$, aa, in Fig. 149 *b* representing the direction of the surface currents, and the dotted lines $b'b'$, bb, those of the bottom currents. The presence of such currents in channels of various sizes has been experimentally demonstrated by the author.[1]

In addition to this action of the sides, any retardation of the flow

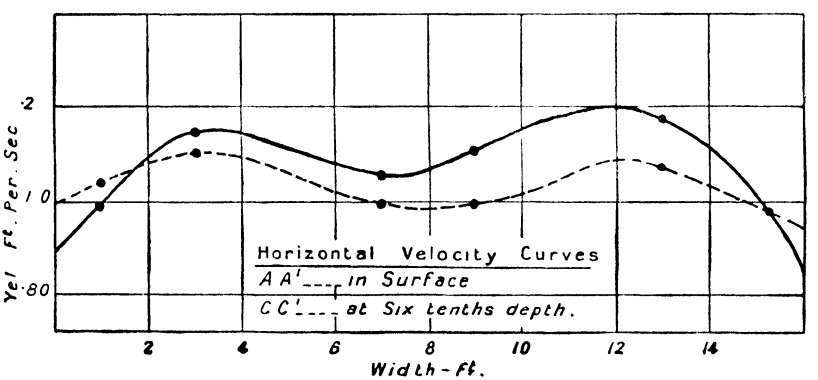


FIG. 150.

being more marked over the central and swifter portions of the stream tends to increase the super-elevation of the central surface and the formation of these transverse currents, while an acceleration of the flow

[1] Proc. Roy. Soc., A., Vol. LXXXII., 1909, pp. 149—159.

tends to reduce them. The author's experiments however indicate that for any such non-uniform flow as is likely to be experienced at a gauging station, the influence of the sides is the all-important factor.

This theory explains why, as is found in practice, the depth of the filament of maximum velocity, and, as will be seen later, also that of mean velocity in any vertical,

(*a*) is greater as the influence of the sides increases and hence as the ratio of depth to width of the stream increases.

(*b*) is less as the roughness of the bottom increases, since this roughness retards the transverse current without having any compensating effect.

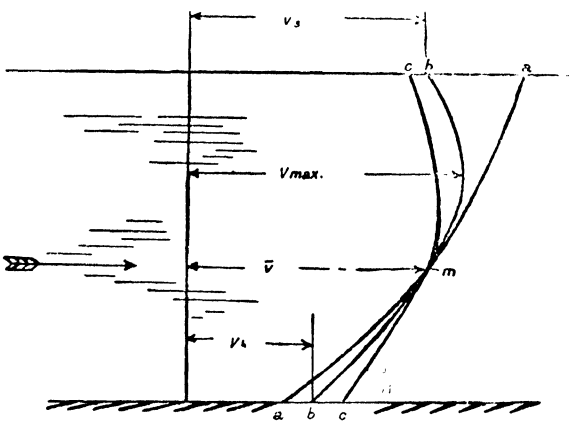


FIG. 151.

(*c*) in the case of a rectangular channel, is greater nearer the sides.

It also explains why, on measuring the velocities across a horizontal in a stream, two points of maximum velocity are often found, these being one on each side of the centre as shown in Fig. 150, which is taken from a gauging of the Cornell channel.[1]

The effect of the wind on the curve of velocities in a vertical is indicated in Fig. 151, which shows the curves

(*a*) with a strong down-stream wind.

(*b*) with no wind.

(*c*) with a strong up-stream wind.

It is found that although both the magnitude and position of the filament of maximum velocity is affected, that of the filament of mean velocity (*m* in Fig. 151) is sensibly independent of the state of the wind. The probable explanation of this is that an up-stream wind banks up the head waters and so increases the surface gradient of the stream, thereby increasing the velocity of flow over its lower portions to an extent which compensates for the reduced velocity of the surface layers.

[1] From U. S. Geol. Survey, Water Supply Papers. No. 95, pp. 73 and 74.

Mean Velocity in a Vertical.

Actually the position of the filament of maximum velocity in a vertical is not of great importance. That of the filament of mean velocity, which is at a greater depth, is however very important in stream gauging, since, if it be known, the operation of gauging reduces itself to the measurement of a single velocity at this depth in each of a series of verticals distributed across the stream.

The depth of the latter filament varies from about $\cdot 5h$ to $\cdot 7h$, having the former value in a wide and shallow stream of depth less than about 2·0 feet, and the latter value in a smooth wooden or cement channel whose depth is approximately one-half the width. In the great majority of cases in practice it lies between $\cdot 55h$ and $\cdot 65h$ increasing with the depth and diminishing with the roughness of the channel. An examination of a large number of river and canal gaugings by members of the U.S. Geological Survey [1] leads to the following as the most probable values of its depth.

Condition of bed.	Very rough with boulders.					Large gravel and small boulders.				
Depth of stream, feet . . .	0 to 2	2 to 4	4 to 6	6 to 10	above 10	0 to 2	2 to 4	4 to 6	6 to 10	above 10
Ratio $\frac{\text{depth of filament of mean vel.}}{\text{depth of stream.}}$	·50	·56	·59	·63	·67	·54	·58	·62	·66	·69

Condition of bed.	Small gravel and sand.					Very smooth wood or cement.				
Depth of stream, feet . . .	0 to 2	2 to 4	4 to 6	6 to 10	above 10	0 to 2	2 to 4	4 to 6	6 to 10	above 10
Ratio $\frac{\text{depth of filament of mean vel.}}{\text{depth of stream.}}$	·57	·60	·65	·69	·71	·61	·65	·68	·70	·72

Generally speaking, the velocity at six-tenths depth in any vertical will give the mean velocity in that vertical within 5 per cent. except in

[1] U. S. Geol. Survey, Water Supply and Irrigation Papers, No. 95.

abnormal cases, while the mean of the velocities at two-tenths and at eight-tenths of the depth may also be relied upon as giving the mean velocity within narrow limits.

The velocity in a vertical is least at the bottom, the ratio of bottom to mean velocity ranging from ·6 to ·9. It usually lies between ·75 and ·85, but varies widely in a short interval in the same stream.

The ratio of the mean to the surface velocity in a vertical also varies within somewhat wide limits and depends largely on the direction and force of the wind. On a calm day it lies between ·80 and ·90, diminishing with the velocity of flow, with the roughness of the channel, and with the ratio of breadth to depth. While usually inadvisable to use the surface velocity in computing the discharge of a stream, it is sometimes impossible, in time of flood to make any other measurements. Under such circumstances the surface velocity, multiplied by ·85 will give the mean velocity in a vertical with a fair degree of approximation.

The ratio of the mean velocity over the whole section to the maximum surface velocity varies considerably with the depth and state of the channel and with the direction of the wind. On a calm day it usually lies between ·60 and ·85, increasing with the depth of the stream. In a gauging of the Rhine—depth 6 to 19 feet—its value was ·73 while Harlacher obtained the same value in gauging the Elbe—depth 4—7 feet. Gaugings of the Eger at Falkenau (·62 to 1·1 feet deep) gave a value of ·58. The following values of this ratio are deduced from Bazin's formula (p. 342), by writing $v_{mean} = C\sqrt{m \sin. \theta}$ and by giving C its appropriate values.

Hydraulic Mean Depth of Section.	Material of Bed.			
	Planed Planks, Cement, etc.	Brickwork.	Rubble Masonry.	Earth.
1·0 feet	·85	·83	·77	·65
2·0 ,,	,,	,,	·79	·71
3·0 ,,	,,	,,	·80	·73
4·0 ,,	,,	,,	·81	·75
5·0 ,,	,,	,,	,,	·76
6·0 ,,	,,	·84	,,	·77
10·0 ,,	,,	,,	·82	·78
20·0 ,,	,,	,,	,,	·80

In a very shallow stream the vertical velocity curves approximate to straight lines, and in this case the means of surface and bottom velocities give a close approximation to the mean velocity.

The main results of a large number of river gaugings by members of the U.S. Geological Survey are given below.[1]

Here the symbol *S* means sandy; *G* gravelly; *R* rocky; *B* boulders.

River.	Approximate Width Feet.	Range of depth. Feet.	Coefficient for reducing to mean velocity in any vertical the vel. observed at following points.			Per cent. of depth at which thread of mean vel. is found.	Character of bottom.
			Six-tenths depth.	Mean of top and bottom.	Top.		
Appomattox, Va.	150	2·5— 4·9	1·00	1·11	·81	61·5	S
James, Va.	850	3·0— 4·4	1·01	1·08	·88	59·6	S
Roanoke, Va	110	1·8— 3·3	·99	1·01	·81	60·9	R and S
Staunton, N.C.	400	2·0— 6·0	·96	1·12	·95	65·7	G and S
Dan, N.C.	600	1·4— 3·7	·99	1·01	·86	61·6	S
Dan, Va.	350	3·5— 9·8	·98	1·09	·90	63·6	S
Reddie, N.C.	85	1·7— 2·5	1·02	·97	·83	57·7	S
Yadkin, N.C.	160	2·7— 6·3	·96	1·12	·91	66·5	S and B
Yadkin, N.C.	450	4·0—11·3	1·02	1·06	·81	59·2	S and R
Catauba, N.C.	200	1·8— 5·0	1·00	1·04	·82	60·6	S and G
Catauba, N.C.	200	6·3— 7·0	1·00	1·08	·88	58·5	M
Catauba, N.C.	110	3·5	1·01	1·05	·80	59·0	S
Wateree, S.C.	300	12·3—17·7	1·01	1·13	·90	58·4	M
Broad, S.C.	500	5·0— 8·9	·96	1·22	·89	65·4	M and S
Saluda, S.C.	800	3·0— 8·5	·97	1·03	·82	62·2	S
Little Tennessee, N.C.	660	3·6— 6·0	1·03	1·06	·83	58·7	B
Nolichucky, Tennessee	300	1·6— 5·1	1·02	1·02	·80	59·1	B
Fishkill, N.Y.	90	2— 5	1·036	—	·79	58·7	G
Wallkill, N.Y.	130	3—17	·98	—	·85	63·7	S
Farad Flume, Cal.	10·1	5·98	·95	1·25	1·09	76·0	Wood
Cornell Canal	16·0	7·2— 8·3	—	—	—	65·7	Concrete
" "	16·0	·55— 1·9	—	—	—	54·3	"

The ratio of the velocity at mid depth to the mean velocity in any vertical appears to vary very little, its usual value ranging from 1·02 to 1·06, increasing with the depth of the stream. A value of 1·04 may be relied upon as giving the mean velocity within 3 per cent. for all normal sections and velocities of flow.

Art. 113.—Distribution of Velocity over a Vertical through the Centre of the Stream.

In spite of very many experiments which have been carried out to determine the distribution of velocity, and of many attempts to formularise the results of such investigations, so many and so varied are the factors

[1] U. S. Geol. Survey, Water Supply and Irrigation Papers, No. 95, pp. 150—158.

which influence this distribution, that, as might be expected from the nature of the case, the formulae so far collected can only be considered as giving useful approximations to the required result, and this is more particularly the case where the flow in a natural channel of irregular section is under consideration.

By making one or two assumptions as to the circumstances governing the flow in an open channel, a theoretical formula may be deduced, which, while only applying so far as these assumptions are justified, may still serve as the rational basis of a more exact empirical formula, for giving the distribution of velocity. Such a formula will now be considered.

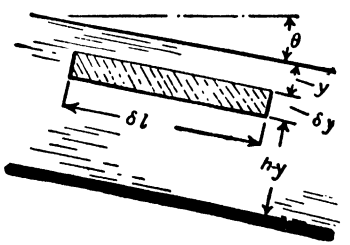


FIG. 152.

Suppose the stream to be sensibly parallel; of width which is great in comparison with its depth; flowing steadily; and that the resistance to flow is due entirely to simple viscous shear, a state of affairs never exactly realised in practice.

Let y be the vertical distance from the surface, of a stratum of the fluid, δy the thickness, δl the length, and b the breadth of the stratum (Fig. 152).

The weight of this element of fluid $= W b \delta y \delta l$.

The resolved part of this weight in the direction of motion $= W b \delta y \delta l . \sin \theta$.

The difference of tractive force on the upper and lower faces of the stratum $= \mu \frac{d^2 v}{d y^2} . b . \delta l . \delta y$, where μ is the coefficient of viscosity (p. 62) and where v is the velocity of flow in the direction of the stream. The pressures on the two ends of the stratum are equal since these are at the same depth and are of the same area. Also since the stream is wide, the variation of shear on the two vertical sides of the stratum may be neglected.

$$\therefore \quad W b \delta y \delta l \sin \theta = - \mu \frac{d^2 v}{d y^2} . b \delta l . \delta y$$

$$\therefore \quad \frac{W \sin \theta}{\mu} = - \frac{d^2 v}{d y^2},$$

the negative sign denoting that the resultant shear force acting upon the element is in opposite direction to the force $W \sin \theta$.

Integrating this expression twice, we get

$$v = C + B y - \frac{W \sin \theta}{2 \mu} y^2 \qquad (1)$$

If v_s = surface velocity, *i.e.*, where $y = 0$, $C = v_s$

$$\therefore \quad v = v_s + B\,y - \frac{W \sin\theta}{2\,\mu}\,y^2 \tag{2}$$

$$= v_s - \frac{W \sin\theta}{2\,\mu}\left(y - \frac{B\,\mu}{W \sin\theta}\right)^2 + \frac{B^2\,\mu}{2\,W \sin\theta}$$

$$\therefore \quad v = v_s + \frac{B^2\,\mu}{2\,W \sin\theta} - \frac{W \sin\theta}{2\,\mu}\left(y - \frac{B\,\mu}{W \sin\theta}\right)^2 \tag{3}$$

the equation to a parabola having a horizontal axis at a depth

$$\frac{B\,\mu}{W \sin\theta} = y_1.$$

Since $$\frac{d\,v}{d\,y} = \frac{W \sin\theta}{\mu}\left\{y - \frac{B\,\mu}{W \sin\theta}\right\}$$

we have $\frac{d\,v}{d\,y} = 0$, *i.e.*, the velocity is a maximum where

$$y = \frac{B\,\mu}{W \sin\theta}, \text{ i.e., at a depth } y_1$$

$$\therefore \quad v_{max} = v_s + \frac{B^2\,\mu}{2\,W \sin\theta}$$

$$= v_s + \frac{W \sin\theta}{2\,\mu}\cdot y_1^2 \tag{4}$$

$$\therefore \text{ from (2) and (4) we have } v_{max} - v = \frac{W \sin\theta}{2\,\mu}\,(y_1 - y)^2. \tag{5}$$

If v_b = bottom velocity, where $y = h$

$$v_b = v_s + B\,h - \frac{W \sin\theta}{2\,\mu}\,h^2 \text{ from (2)}$$

$$\therefore \quad B = \frac{v_b - v_s}{h} + \frac{W \sin\theta}{2\,\mu}\,h$$

$$\therefore \quad v = v_s + \frac{v_b - v_s}{h}\cdot y + \frac{W \sin\theta}{2\,\mu}\,(h\,y - y^2). \tag{6}$$

Also $$v - v_b = B\,(y - h) + \frac{W \sin\theta}{2\,\mu}\,(h^2 - y^2)$$

$$= \frac{W \sin\theta}{2\,\mu}\left\{2\,y_1\,(y - h) + (h^2 - y^2)\right\}$$

$$\therefore \quad v_{max} - v_b = \frac{W \sin\theta}{2\,\mu}\left\{h - y_1\right\}^2. \tag{7}$$

While different observers have deduced different forms for the vertical velocity curve, the parabola with its vertex either in or below the surface appears to fit the majority of cases fairly well.

The foregoing analysis is therefore interesting, as giving the correct form of curve.

Bazin, experimenting on a stream having the maximum velocity in the surface, obtained

$$v_{s\,(max)} - v_{mean} = 25{\cdot}4 \sqrt{m \sin \theta}$$
$$v_{mean} - v_b = 10{\cdot}87 \sqrt{m \sin \theta}$$
$$v_{s\,(max)} - v_b = 36{\cdot}27 \sqrt{m \sin \theta}$$

the general equation being

$$v = (v_s)_{max} - 36{\cdot}27 \sqrt{m \sin \theta} \left(\frac{y}{h}\right)^2.$$

Here v_{mean} is the mean velocity over the whole section, and m is the hydraulic mean depth, the dimensions being taken in feet and the velocities in feet per second. He also states that wherever the position of maximum velocity, the relation

$$v_{max} - v_b = 36{\cdot}27 \sqrt{m \sin \theta}$$

holds true. In the vertical plane containing the filament of maximum velocity, we have from equation (7)

$$36{\cdot}27 \sqrt{m \sin \theta} = \frac{W \sin \theta}{2 \mu} \{h - y_1\}^2$$

Substituting the value of $\frac{W \sin \theta}{2 \mu}$ thus found, in equation (5), when the maximum velocity is below the surface

$$v_{max} - v = 36{\cdot}27 \sqrt{m \sin \theta} \left\{\frac{y - y_1}{h - y_1}\right\}^2 \qquad (8)$$

a formula which gives fairly accurate results in practice.

Rankine states that the maximum, mean, and bottom velocities may be taken as being in the ratio 5 : 4 : 3 in ordinary cases, and in the ratio 4 : 3 : 2 in very slow currents, and these ratios may be taken as being approximately correct for streams and rivers of moderate size.

Velocity at Mid-depth.

From equation (5) of this article, we may obtain the mean velocity over any vertical by integrating the sum of such terms as $v \delta y$ over the vertical, and by dividing this sum by its length h. Thus

$$\bar{v} = \frac{\int_0^h \left\{v_{max} - \frac{W \sin \theta}{2 \mu} (y_1 - y)^2\right\} d y}{h}$$

$$= v_{max} - \frac{W \sin \theta}{2 \mu} \left\{\frac{h^2}{3} - h y_1 + y_1^2\right\} \qquad (9)$$

while from (5) we may obtain the velocity at mid-depth, *i.e.*, where

$$y = \frac{h}{2}.$$

Thus $$v_{\frac{h}{2}} = v_{max} - \frac{W \sin \theta}{2 \mu} \left\{ \frac{h^2}{4} - h\, y_1 + y_1^2 \right\}$$

$$\therefore\ v_{\frac{h}{2}} - \bar{v} = \frac{W \sin \theta\, h^2}{24 \mu}$$

from which, by determination of the mid-depth velocity, the mean velocity may be determined.

In general the mid-depth velocity is from 1·02 to 1·06 times the mean, and from ·94 to ·98 of the maximum.

If $\bar{y}$ be the depth at the point having a velocity equal to the mean, we have, from equations (5) and (9)

$$\frac{h^2}{8} - h\, y_1 + y_1^2 = \{ y_1 - \bar{y} \}^2$$

$$\therefore\ \bar{y} = y_1 \pm \sqrt{\frac{h^2}{8} - h\, y_1 + y_1^2}.$$

Putting $y_1 = \cdot 2\ h$ this gives $\bar{y} = \cdot 62\ h$.
„ $y_1 = \cdot 3\ h$ „ „ $\bar{y} = \cdot 65\ h$.

Art. 114.—Permissible Velocity in Open Channels; Erosion and Deposition of Silt.

Water in motion exerts an erosive or scouring action on the bed and sides of the containing channel, and the maximum permissible velocity thus depends on the nature of the bed.

Particles of matter once disturbed, may be transported either by being rolled along the bed of the stream or by being carried in suspension, and for each material a certain critical velocity must be attained, depending on its size and specific gravity, before this is set in motion. Once in motion, however, the velocity may be reduced somewhat below this critical value before the material is again deposited.

While the erosive power of water varies as the square of its velocity, its transporting power, or the power to move boulders, etc., which may lie in its path, varies approximately as v^6. This may be seen if we consider that the force exerted by the stream on any body is equal to the change of momentum produced in the stream passing the body, and since the area of that portion of the stream affected is proportional to the sectional area, a^2, of the body, this force will be equal to $K\, v^2\, a^2$ lbs. The force resisting motion is that of the friction of the body on the

bottom of the stream and is proportional to its weight, and therefore to its volume a^3.

Material.	Bottom Velocities, in ft. per sec., at which		
	Transportation Begins.	Material is in Equilibrium.	Deposition Begins.
Coarse sand	1·07	·71	·62
Gravel—			
Size of pea	·71	·62	
„ small bean . .	1·56	1·07	·71
Shingle—rounded, one inch or more in diameter . .	3·2	2·14	1·56
Flints—Size of hen's egg .	4·0	3·2	2·14

Equating these forces, we have $K v^2 a^2 = c a^3$

$$\therefore a \propto v^2$$
$$\therefore a^3 \propto v^6$$

i.e., the weight of the solid moved is proportional to the sixth power of the velocity. Obviously this only holds so long as the bodies are similar, the velocity necessary to move a sphere being much less than that to move a cubical block of the same weight.

The size of particle moved by a stream over a smooth sandy bed is given approximately by $d = \frac{45\ v^2}{w - 64}$ inches, where w is its density in lbs. per cubic foot, and v is the velocity in feet per second.[1]

A stream which carries a certain amount of fine material in suspension has a greater capacity for transporting larger material than one which carries only the larger material. Experiments show that a stream will carry more than four times the weight of sand of ·4 to ·5 mm. diameter in the presence of a certain weight of sand of ·3 mm. diameter than in its absence.[2]

While an excessive velocity of flow leads to erosion of the channel, a too

[1] Dr. G. S. Owens. *Engineer*, May, 15, 1908, p. 511.
[2] *Engineering News*, New York, vol. 63, 1910 (p. 580).

sluggish flow favours the growth of aquatic plants, while any change of velocity from high to low in a stream carrying material in suspension, causes a deposition of a portion of the material, and a consequent silting up of the channel. In order to prevent deposit in small sewers or drains, a mean velocity of not less than 3 feet per second is necessary. For sewers from 12 to 24 inches diameter the velocity should not be less than 2·5 feet per second, while with larger sizes than this the velocity may be reduced to 2 feet per second.

Mr. R. G. Kennedy,[1] from observations on a large number of Indian irrigation canals, concludes that there is a certain critical velocity at which a long canal will maintain its channel in silty equilibrium. This velocity is given by $v_o = c\ h^{\cdot 64}$ feet per second, where c has the following values :—

Light sandy soil	c = ·82.
Coarse sandy soil	·90.
Sandy loam	·99.
Coarse silt	1·07.

Where a main canal supplies or is supplied by feeders, the various depths and velocities of flow should be adjusted to suit this relationship in order that there may be no silting or erosion in the main or feeder canals.

If v be any other velocity, and if q_o and q be the amounts of silt carried respectively at v_o and v.

$$q = q_o \left(\frac{v}{v_o}\right)^{\frac{5}{2}} \text{ approximately.}$$

Taking c = ·84, the following table shows suitable mean velocities of maximum flow for equilibrium of such channels in sandy soil :—

Depth of channel (feet) . .	2	3	4	5	6	7	8
Mean velocity (feet per sec.) .	1·3	1·7	2·0	2·4	2·65	2·9	3·2

Ganguillet and Kutter give the following as the safe bottom and mean velocities, but state that these are probably too small rather than too large :—

[1] "Proc. Inst. C.E.," vol. 119, 1874–5, p. 281.

Material of Channel.	Safe Bottom Velocity. Feet per Second.	Safe Mean Velocity. Feet per Second.
Soft brown earth	25	·33
Soft loam	·50	·66
Sand	1·00	1·32
Gravel.	2·00	2·64
Pebbles	3·00	3·94
Broken stone or flint . . .	4·00	5·58
Conglomerate	5·00	6·56
Stratified rock	6·00	8·20
Hard rock	10·00	13·13

Actually, as indicated by Kennedy, the safe mean velocity depends also upon the depth. More recent work shows that at medium depths through firm loamy soil a mean velocity of 3·0 to 3·5 feet per second is safe, while with fine well-rammed gravel or loose rock this may be increased to from 5 to 7 feet per second. In a concrete-lined channel faced with cement, the maximum safe velocity with water which carries solid material in suspension is about 9 feet per second. A higher velocity wears and roughens the bottom until the roughness thus produced reduces the velocity sufficiently to prevent further erosion. With an ordinary brick or heavy dry-laid rubble channel, the velocity should not exceed 15 feet per second, any higher velocity necessitating a carefully-laid facing of heavy masonry with cemented joints.

Recent experiments by Mr. A. B. Buckley [1] tend to show that with a given surface slope the velocity is increased by the deposition of fine silt on the bed of a canal, possibly because the relatively high viscosity of the bottom layers of heavily silted water tends to reduce eddy formation. He suggests a formula of the type

$$v = \{ c + a\,(z - b)^{\cdot 383} \}\, m^{\cdot 85} \,.\, i^{\cdot 72},$$

where z is the average amount of silt in suspension expressed in grams per cubic metre.

b is the minimum silt content.

a is a numerical coefficient.

Art. 115.—Gauging of Flow in Streams and Open Channels.

Many methods are available for obtaining the discharge of a stream, these differing widely in the accuracy of their results and the cost and

[1] "Proc. Inst. C. E.," 1922–23.

difficulty of their application. The method to be adopted in any case depends largely on the degree of accuracy required and on the size of the stream.

The accuracy of a discharge measurement, whatever be the method adopted, depends greatly on the physical characteristics of the stream at the point of measurement. If possible, this should lie on a straight reach and away from the influence of a bend, the bed should be permanent and not strewn with boulders, and the slope and wetted perimeter such that at all stages of the stream the velocity at all parts of the section may be easily measurable. The banks should be sufficiently high to prevent overflow in times of flood, and the section should be outside the sphere of influence of bridge piers or mill dams.

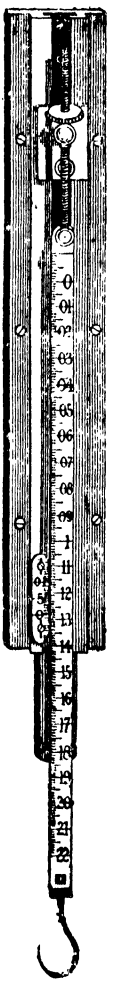


FIG. 153.—Hook Gauge.

In a small natural stream it may be possible to install a weir of standard design, always provided that the rise in the upstream level, which this involves, is permissible. In an artificial channel either a weir or a *Venturi Flume* (Art. 116A) may be used.

The latter device has the great advantage that it allows a free unobstructed flow, does not trap *débris*, while the loss of head which it produces is negligible compared with that produced by a weir.

In a large stream or river it may be possible to utilise an existing weir, but in this case the only safe method is to construct a scale model of the weir and of its upstream and downstream surroundings and to obtain the appropriate weir coefficients from experiments on this. Failing this the only methods of obtaining the discharge are to obtain as nearly as possible the mean velocity (v feet per second) of the stream, to multiply this by the cross sectional area (A square feet), and to get the discharge Q by the relation:—

$Q = v A$ cubic feet per second, or to adopt the chemical method of gauging.

The value of v may be approximated to in many ways.

(*a*) By using a current meter or Pitot tube to give the velocity at a point or series of points in a cross section, and by deducing the mean velocity from such observations;

(*b*) By using one or other type of float, and by measuring the time

necessary for a series of these to traverse a given length of the channel.

(*c*) By injecting colouring matter or some suitable chemical into the stream and by noting the time occupied by this in traversing a measured distance.

Before considering these methods in detail, a few general observations as to their relative advantages and disadvantages may be made. Experiment shows that the motion at any point in an open channel is never steady and uniform, but suffers a series of pulsations, the periodic time of which may vary from a few seconds to two or three minutes. These are due to a variety of causes. Eddies formed at the sides and bottom drift away to every portion of the stream; snags and hollows in the bed, bends, and falls, all produce some (irregular) disturbance of the flow, with the result that the velocity at a point in the surface may vary by 20 per cent., and at a point near the bed by as much as 50 per cent. (Harlacher) in a short interval of time.

In experiments on the St. Clair River (1899) the velocity-time curves showed two sets of waves, small ones of 15 to 60 seconds amplitude and larger ones of 3 to 6 minutes amplitude. The range of velocities as found from the larger waves was in some cases 35 per cent. of the mean velocity shown by the meter reading taken over ten minutes. These experiments indicate that the pulsations are very limited in extent in a direction at right angles to the current. The whole depth of the river is affected by them, although their effect decreases from the bottom towards the surface.

It follows that a float, measuring as it does the velocity due to a single pulsation, may give results which are greatly in error, and the only chance of obtaining a fair estimate of the mean velocity over a single section of the stream by floats is to take the mean of a large number—40 or 50—of observations. The complexity of the motion is very evident when floats are used. Of a series dropped into a stream at the same point, no two will trace out the same path, and as may be well understood when the number of observations necessary to give any pretensions to accuracy is remembered, this method, though at first sight so simple, may easily prove the most expensive method of determining the discharge. With current meters, on the other hand, the mean velocity at any point may be obtained with reasonable accuracy, provided the period of observation is sufficient to cover a series of the pulsations of velocity. Professor Unwin found that the mean time of successive 100 revolutions of such a meter in the Thames, when plotted, gave a very irregular curve, while the mean times of succes-

sive 500 revolutions gave an almost straight line. In general the time of a single observation should not be less than five minutes, a period of six to ten minutes being advisable.

Art. 116.—Weir Measurements.

For an artificial channel a sharp-edged weir is a very suitable means of measuring the flow, provided that the obstruction of the channel and the change in upstream level and loss of head due to its installation are not objectionable.

For comparatively large flows the rectangular weir, either suppressed or with end contractions may be used, while for small flows the triangular notch is more suitable. For the precautions to be adopted to obtain accurate results with such weirs see Chapter VII.

Art. 116a.—The Venturi Flume.

The **Venturi Flume** consists of a channel, usually in concrete, which in plan view contracts gradually to a parallel throat and then re-expands to its

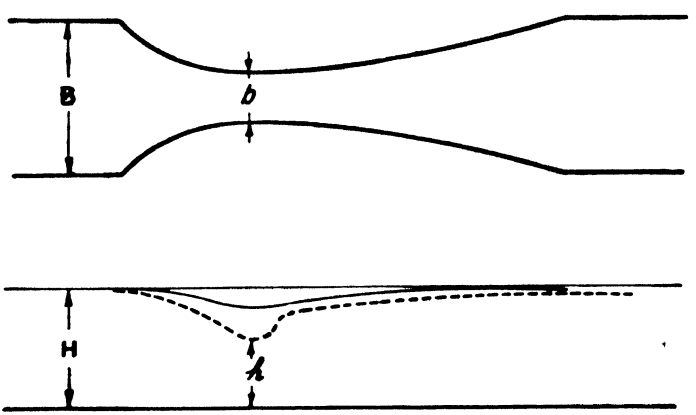


Fig. 153a.—Venturi Flume.

original width (Fig. 153a). Alternatively, the channel may be parallel and the contraction be produced by a " hump " in the bottom of the channel. In either case, if B and b ; $\overline{V}$ and v ; H and h, are the breadths, velocities and depths of water at entrance and throat, and if Q is the discharge in cusecs, $\overline{V} = \frac{Q}{BH}$; $v = \frac{Q}{bh}$, and, applying Bernoulli's theorem,

$$H - h = \frac{Q^2}{2g}\left\{\frac{1}{b^2 h^2} - \frac{1}{B^2 H^2}\right\}$$

$$\therefore\ Q = C\sqrt{\frac{2g(H-h)}{\frac{1}{b^2 h^2} - \frac{1}{B^2 H^2}}} = C\frac{b\,h}{\sqrt{1 - \left(\frac{b\,h}{B\,H}\right)^2}}\sqrt{2g(H-h)} \quad . \ . \quad (1)$$

where C is the coefficient of discharge which is usually between 0·96 and 0·99.

Two main types of flow are possible in the Venturi Flume. If the discharge is small the downstream rise from the throat is gradual as shown by the full line curve in Fig. 153A. If sufficiently great a standing wave is formed below the throat as shown by the dotted line. In this case, as in a broad crested weir (p. 175) the throat level adjusts itself so as to give a maximum discharge. The depth h then becomes $\frac{2}{3}$ H and, as in the weir, the discharge is given by $Q = 3{\cdot}09\,C\,b\,H^{\frac{3}{2}}$, where the effective value of H is H plus the head equivalent to the velocity of approach, and equals $H + \frac{V^2}{2g}$.

With this type of flow the only head reading required is that of H with the appropriate correction for velocity of approach which may be obtained by successive approximations. Equation (1), however, which takes into account the velocity of approach applies to either type of flow.

The total loss of head with a well-designed flume varies from about 0·06 H at small flows to about 0·20 H at full discharge. It depends in the latter case largely on the proportions of the flume.[1] Best results would appear to be obtained when the length of the throat is at least twice H, and when the mean angle of diversion between the two sides of the outlet is approximately 15 degrees.

Art. 117.—Current Meter Measurements.

Meters in use at the present day may be divided into two classes: (1) those in which the revolving part carries a series of helicoidal vanes mounted on a horizontal axis, and (2) those in which a series of conical or hemispherical cups is mounted on arms, as in an anemometer, on a vertical axis. The former type is illustrated in Fig. 154, which shows the Amsler meter, and in Fig. 155, which shows the Haskell meter, while the latter is shown in Fig. 156, which illustrates the Price meter. The latter type of meter has some advantages over the former in that friction is usually less, since it practically all comes on one point which is easily protected from any grit in the water, while in addition this type will start in a current of less

[1] For full discussions of types of flume and their characteristics, see "Notes on Standing Wave Flumes," C. C. Inglis, Govt. of Bombay P.W.D. Tech. Paper No. 15, 1928. Also F. V. A. Engel, *The Engineer*, Aug. 3rd and 10th, 1934, and A. H. Jameson, "Water and Water Engineering," March, 1930.

velocity than will move the other, and yet will not revolve as rapidly under the same conditions of high velocity flow.

The meter is fitted with a guide vane which keeps its axis perpendicular to the direction of the current. The wheel may be either geared

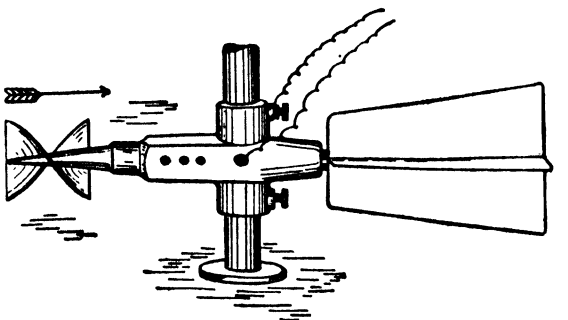


FIG. 154.—Amsler Current Meter.

to a counter which records the revolutions direct and is put into and out of gear by means of a cord from the point of suspension of the meter, or may make and break the contact in an electrical circuit at each revolution, thus enabling the number of revolutions to be indicated by means of a counter or buzzer placed on shore or in a boat. The advantages of the latter method in reducing the resistance to rotation and the tendency

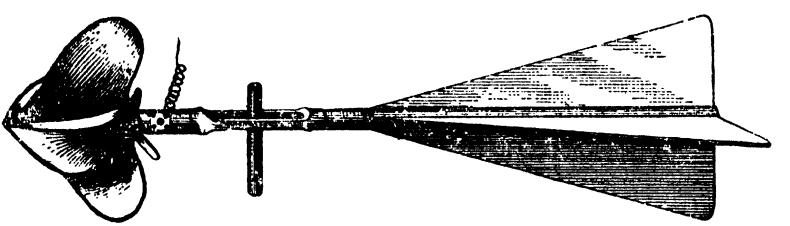


FIG. 155.—Haskell Current Meter.

to clog are obvious, and the mechanically operated meter is becoming obsolete.

The instrument is previously calibrated by towing at known velocities through still water, the number of revolutions corresponding to these velocities being recorded. It has the disadvantages that it cannot be used where floating grass or weed is prevalent, and that it requires rating at frequent intervals. Further, it cannot be used at very low velocities. The minimum permissible velocity depends on the type of meter, but in general varies from 3 to 6 inches per second.

There are two methods of using the meter. In the first, the "**point**" method, it is held successively at certain points in a cross section. In a shallow stream this may be done by clamping it to a staff which is carried by an observer in waders, and which is held vertically at the required points, with one end resting on the bed of the stream. In deeper streams it is attached to a heavy sinker and is suspended from a convenient bridge or cable placed across the stream where possible, or

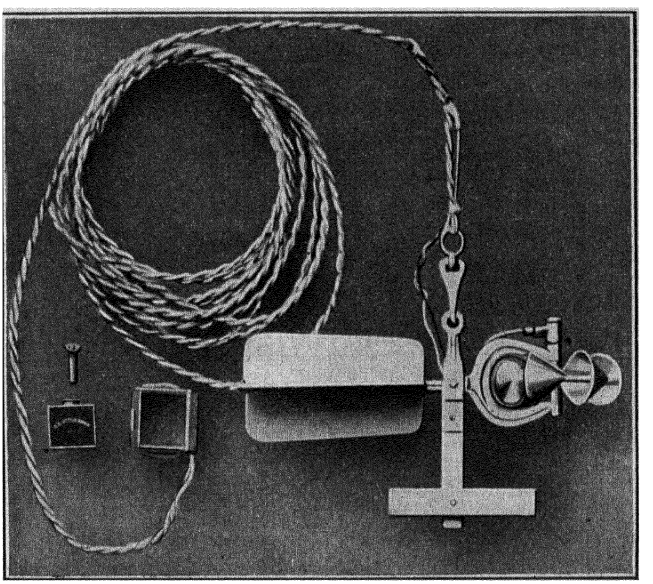


FIG. 156.—Price Current Meter.

from an outrigger fixed to an anchored boat where the width precludes this. When the "point" method is used, the meter may either be held, (1) at several equidistant points in certain equidistant verticals, the mean velocity being deduced from these readings as explained later; (2) at six-tenths, or at mid-depth in a series of equidistant verticals, the mean velocity in each of these verticals then being found by applying a factor; (3) at the surface, bottom, and mid-depth in a series of verticals; (4) at the surface and bottom only, or at two-tenths and eight-tenths of the depth in a series of verticals, in which case the mean of the two readings is taken as the mean velocity in the vertical. While the first method is likely to give the best results in a steady stream, yet, as previously

indicated, the length of time necessary to obtain the many observations is a serious drawback in a stream of any considerable size.

In a large stream where it is impossible to see the bottom, owing to the impossibility of fixing the meter very near to the sides and bottom where the velocities are least, the results tend to be too high. To obviate this the meter should not be placed nearer to the surface than one foot.

The mid-depth point is used because the factor, about ·96, which is used to obtain the mean velocity is more constant for it than for any other point on the vertical; while the six-tenths point gives very approximately the true mean velocity on the vertical. These factors are discussed at further length on pp. 334 and 336. Observations taken at either of these points are capable of giving excellent results.

Method (3) was adopted by Moore in his gauging of the Thames.[1] Assuming the vertical velocity curve to be a parabola, its area is given by the formula $A = \frac{h}{6}\{ v_s + 4\, v_{\frac{h}{2}} + v_b \}$, h being the depth, v_s and v_b the surface and bottom velocities respectively. The discharge per second flowing between the two end verticals is given by

$$\text{Vol.} = \frac{d}{3}\{ \Sigma(A_1 + A_n) + 4\,\Sigma(A_2 + A_4 + \ ) + 2\,\Sigma(A_3 + A_5 + \ ) \} \text{ c.f.s.}$$

where d is the breadth of the successive vertical strips; the first term is the sum of the areas of the first and last velocity curves; the second term is four times the sum of the even sections; and the last term is twice the sum of the odd sections excluding the first and last. The total discharge is then obtained by adding the small volume flowing between each end section and the shore. On account of the large variation in bottom velocity with a given mean flow this method is, however, not to be recommended.

Method (4), in which the surface and bottom velocities are measured is only advisable for very shallow steams. Experiments at Cornell University show that the results thus obtained agree closely with those given by a weir if the bed is smooth or gravelly, the depth from ·4 to 1·0 feet and the velocities from ·4 to 1·5 feet per second. For a gravelly bed the meter should be held with its centre from 3 to 4 inches above the bottom and about 2 inches below the surface, while with a smooth bed each distance should be about 2 inches. With depths between 5 and 10 feet, the mean of velocities ·5 feet above the bottom and ·5 feet below the surface gave results too low by as much as 30 per cent. In such cases the mean of readings at two-tenths and at eight-tenths the depth gives good results.

[1] "Proc. Inst. C E.," vol. 45, p. 220.

In the "**integration**" method, the meter is kept in motion during the whole period of its immersion. It may either be moved uniformly from the surface to the bottom of the stream in a series of vertical lines; diagonally across from one side to the other, at the same time being moved from the surface to the bottom several times; or across the stream at a given depth. The recorded velocity is then taken as the mean for the particular vertical or for the whole section as the case may be. Although an observation by this method can be carried out in considerably less time than by the point method, the results are not nearly so accurate. The velocity recorded being the resultant of the velocities of the meter and of the water is always higher than the true velocity, the error increasing with the speed of movement of the meter and also increasing as the velocity of flow diminishes. It is only to be recommended where a stream is rising or falling rapidly and where in consequence the speed with which the observations can be made is a great advantage.

Simultaneously with the velocity observations, soundings should be taken from which the cross section of the stream may be obtained. In a narrow stream these should be taken at intervals of from 2 to 5 feet, while where the breadth exceeds 100 feet, they should be taken at intervals of from 10 to 25 feet, depending on the roughness of the bed.

Field Notes.—The following shows the method adopted for entering up field observations and computing mean velocities in the case where velocities are measured at several points on a cross section.

[Gauging made January 23, 1904, by B.S.D. Meter No. 349, on Dan River, Madison, N.C. Gauge height; beginning 2·10 feet.; end 2·26 feet; river rising.]

Distance from initial point.	Depth of stream.	Depth of observations.	Time in seconds.	Total number of revolutions.	Revolutions per second.	Velocity per second.	Per cent. of depth.
40	0·3	0·5	500	600	1·20	2·88	17
		1·0	500	580	1·16	2·78	33
		1·5	500	540	1·08	2·59	50
		2·0	500	470	·94	2·27	67
		2·5	500	380	·76	1·83	83

These observations are recorded for a series of verticals in the cross section. They are then plotted on squared paper, depths as ordinates and velocities as abscissæ, and a smooth curve is drawn through the plotted points, care being taken to give them as nearly as possible equal weight if they do not all fall on a smooth curve. From this curve

velocities are read off at top and bottom and at equal intervals of, say, each ·5 foot, and are set down in order. Thus from the above curve we get—

0·0 ... $v_s = 2·90$	2·0 ... $v_4 = 2·25$
0·5 ... $v_1 = 2·88$	2·5 ... $v_5 = 1·88$
1·0 ... $v_2 = 2·77$	3·0 ... $v_b = 1·31$
1·5 ... $v_3 = 2·58$	

The mean velocity in this vertical is then computed from the prismoidal formula for seven abscissæ as follows:—

$$v_m = \tfrac{1}{18} \{ v_s + v_b + 4 (v_1 + v_3 + v_5) + 2 (v_2 + v_4) \}$$

In this case we have:—

$$\begin{aligned} v_s + v_b &= 2·90 + 1·31 = 4·21 \\ 4 (v_1 + v_3 + v_5) &= 4 \{ 2·88 + 2·58 + 1·88 \} \\ &= 29·36 \\ 2 (v_2 + v_4) &= 2 (2·77 + 2·25) \\ &= 10·04 \end{aligned}$$

$$\therefore v_m = \tfrac{1}{18} \{ 4·21 + 29·36 + 10·04 \}$$

$$= 2·42 \text{ f.s.}$$

The cross section having been plotted, the areas of the various compartments having such verticals as their centre lines may be obtained, either by direct measurement by planimeter or by calculation, and the discharge calculated as follows:—

Compartment.	Area of section square feet.	Mean velocity. feet per second.	Discharge c.f.s.
1	15·1	1·32	19·9
2	28·2	1·97	55·6
3	36·5	2·42	88·3
4	32·1	2·56	82·1
5	23·7	1·99	47·2
6	13·5	1·33	17·9
		Total ...	311·0 c.f.s.

When the vertical velocity curves have been obtained the discharge may be computed somewhat more accurately by considering the discharge between any two such verticals as being represented by the volume of the solid having these curves bounding opposite parallel sides as shown in Fig. 157. For example, the discharge between the verticals 2 and 3 in this figure is given by

$$\frac{{}_2d_3}{3}\left\{\bar{v}_2\,h_2+\bar{v}_3\,h_3+\sqrt{\bar{v}_2\,h_2\,\bar{v}_3\,h_3}\right\}\text{ c.f.s.}$$

where $\bar{v}_2$ and $\bar{v}_3$ are mean velocities in the verticals 2 and 3, and where h_2 and h_3 are the corresponding soundings, ${}_2d_3$ being the distance between the verticals. The discharge between the two end soundings is then given by the sum of such terms as the above between these soundings.

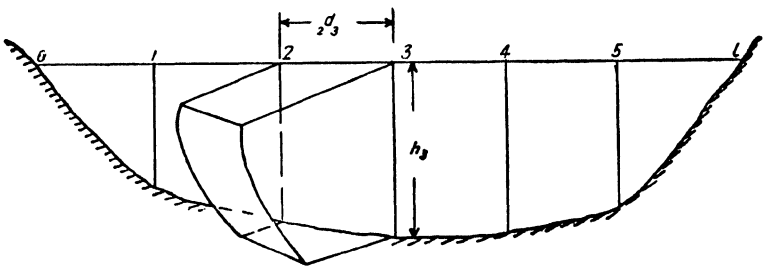


FIG. 157.

To this must be added the discharge over these sections outside the end soundings, which is given by

$$\tfrac{1}{3}\left\{v_1\,h_1\times{}_0d_1+\bar{v}_5\,h_5\times{}_5d_l\right\}\text{ c.f.s.}$$

Calibration of Current Meters.—In rating a meter it is usually suspended from a car or a boat, and is towed with a uniform velocity through still water at a depth of 2 or 3 feet. The length of a run varies from 100 to 300 feet, with sufficient of a starting run to attain a steady velocity before entering the measured length. It is moved in either direction from end to end of the run to eliminate the effect of a current in either direction, and the time of the run and the number of revolutions of the meter are recorded by means of a chronograph.

The range of velocities employed in rating should be those for which the meter is to be used, and no attempt should be made to extend the rating table beyond its experimental limits.

When in use the meter may either be suspended from a cable, in which case its axis is free to move about both a vertical and a horizontal axis, or it may be fixed to a vertical rod in such a way as to remove the second degree of freedom. Experiments indicate that the same rating table is not strictly accurate for the two cases, and that with a given velocity of flow the revolutions increase as the freedom of motion decreases.[1] The

[1] U.S. Water Supply and Irrigation Paper, No. 95, pp. 83–89; also p. 81. Also *Engineering News*, 1914, vol. 71, p. 1083.

difference depends on the type of meter, and for velocities of 1 foot per second is usually about 2 per cent.

A further source of inaccuracy, particularly with very low velocities of flow, is due to the fact that a rating carried out in still water is not quite accurate when applied to the same relative velocities in moving water.

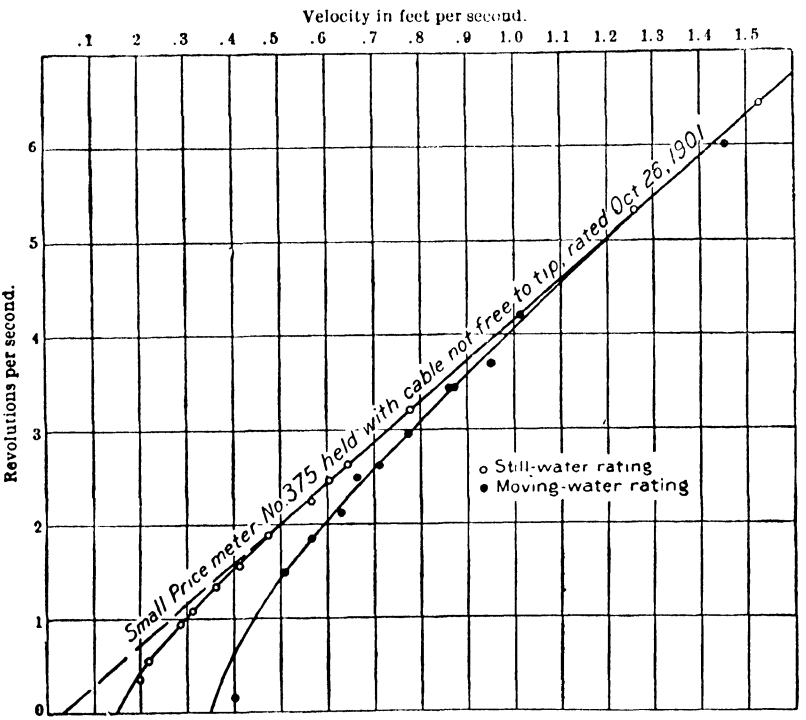


FIG. 158.—Rating Curves of Price Current Meter.

Experiments [1] indicate that the meter, particularly of the cup type, does not indicate so high a velocity when held in a current as when dragged through still water, the difference varying from 1 to 4 per cent. at a velocity of 1 foot per second with different types of meter. The difference is about 1 per cent. at a velocity of 2 feet per second.

In calibrating a current meter it is usual to plot the curve connecting velocity of current and number of revolutions of meter per second. In the majority of cases this is found to be of the form shown in Fig. 158. At very low speeds the friction of the instrument varies between fairly

[1] U.S. Water Supply and Irrigation Paper, No. 95, pp. 83–89; also p. 81. Also *Engineering News*, 1914, vol. 71, p. 1083.

wide limits, but diminishes as the speed increases. This causes the plotted points to lie more or less on a curved line, but renders this portion of the calibration unreliable. At a certain critical speed of the instrument the friction takes a fairly constant value, and the curve becomes very approximately a straight line of the form

$$v = a\,n + b.$$

It is not advisable to use the meter to register speeds below the critical. This depends on the instrument, but is usually from 3 to 6 inches per second, and in general it may be taken that on this account the meter is not a suitable instrument for the measurement of the discharge of a stream if the velocity over more than 15 per cent. of its area is less than 6 inches per second.

The current meter should not be used where the turbulence is abnormally great. Experiments on the Nile [1] indicate that where the turbulence is excessive the estimated discharge tends to be too high, but in any normal stream, with a well-chosen gauging station, any such error will only be small.

Art. 118.—Pitot Tube Measurements.

The velocity at any point in the cross section may also be estimated by means of the Pitot tube (Art. 84).

This method is not so well adapted for measuring such low velocities as usually occur in open channels as for higher velocities such as are more common in pipe flow.

For small, shallow, and rapid streams it is, however, capable of giving fairly good results. Fig. 159 shows a tube, with positive and negative pressure openings as used for the rating of such streams by the United States Geological Survey. This tube was rated in still water in a reservoir and in moving water, being placed for the latter rating at about 30 points in the cross section of a channel 1 foot wide and ·6 feet deep. The former rating

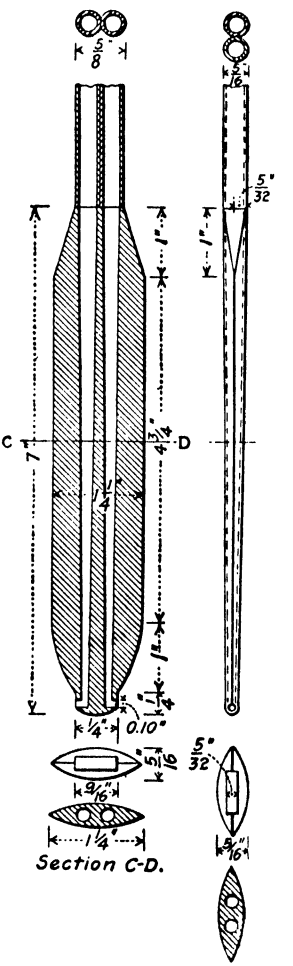


Fig. 159.—Pitot Tube for Steam Rating.

[1] E. H. B. Wade, "Phys. Dept. Paper," No. 4, Ministry of P. W., Egypt, 1921.

gave values of C in the formula $V = C\sqrt{2gh}$ ranging from ·855 to ·87, with a mean value of ·86. The velocities as found by this tube from the still-water rating were invariably greater than were given by the moving water rating, the average difference being 6·4 per cent.[1] This great difference is, however, probably due partly to the comparatively large disturbing effect of the tube in such a small channel, and partly to the impossibility of taking measurements near the walls where the velocity is least.

Art. 119.—Estimation of Velocity by Floats.

These are liberated at a series of points in a long straight reach (Captain Cunningham, from experiments on the Ganges Canal[2] recommends that this length should not be less than 200 feet) and the time occupied in covering a measured distance is noted.

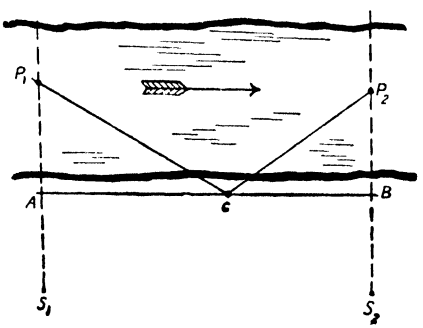


Fig. 160.

Floats may be divided into four classes:—

(1) Surface floats.
(2) Sub-surface floats.
(3) Twin floats.
(4) Velocity rods.

(1) **Surface Floats.**—These consist of any easily seen masses of light material, painted cork or discs of light wood for example, of small size so as to move along with, and register, the velocity of the surface filaments. A series of trials are necessary to get the maximum surface velocity v_s of the stream, from which the mean velocity of flow may be estimated from Bazin's formula (p. 339). It is preferable, however, to deduce the mean velocity in each of a number of sections of the stream from repeated observations of the surface velocity in each of these sections. The sections may be marked in a stream of moderate dimensions by ropes hanging from a bridge or temporary support.

In a large river, observations with the theodolite are necessary to determine the track of the float. This may be satisfactorily carried out as follows:—A base line $A\ B$ (Fig. 160) is chained out parallel to the river for a length of about 250 feet, depending on the width of the river. At the two ends stakes are erected, while second stakes are erected in

[1] *Engineering News*, vol. 62, No. 7, p. 174.
[2] "Proceedings Inst. Civil Engineers," 1882, vol. 71.

lines ranged perpendicularly to the base line, as at S_1, S_2. An observer with a theodolite is stationed near the centre of the base line at C, and an observer is stationed at each stake, S_1, S_2. The float being liberated up stream, the theodolite observer keeps the line of collination of his instrument on this. As it passes the line of sight $S_1 A$, the observer at S_1 gives an audible signal and the theodolite observer notes the angle $A C P_1$. On passing the line $S_2 B$ a second signal is given at the angle $B C P_2$ noted. The line $P_1 P_2$ can then be plotted. With a stream of moderate velocity the same observer may give the signals both at A and B. The effect of wind on the surface velocity, however, together with the tendency of the floats to follow every variable cross current and to be affected by every surface eddy, renders the results obtained by this method unreliable except as approximations to the truth.

(2) **Sub-surface Floats.**—These consist of bodies having surfaces of large area, as illustrated for example in Fig. 161, attached to small surface floats for ease of observation, the length of connection being adjusted so as to allow the true float to remain at any given depth. The velocity of the float will then be approximately that of the current at the required depth. The figure shows the float used in the Connecticut River survey in 1874. The sub-surface float was a hollow annulus of tin $8\frac{1}{2}$ inches high, $8\frac{1}{2}$ inches outside diameter, and $7\frac{1}{2}$ inches inside diameter. This was weighted with 28 oz. of lead. The surface float was an ellipsoid of tin 6 inches in diameter and 1·5 inches deep, the connecting cord being ·036 inches in diameter. A series of such floats liberated at different points in the cross section of a stream and at different depths may be used to give by their mean velocity the mean of that of the stream, or by arranging a single row, the depth of each being $\frac{3}{5}$ that of the stream at the point of introduction, these may be taken as giving the mean velocities in their respective sections. While this type is more reliable than the surface float it suffers from the disadvantage that it is impossible to determine the exact position or depth of the lower float, for while the position of the upper float may be known, that of the lower float varies

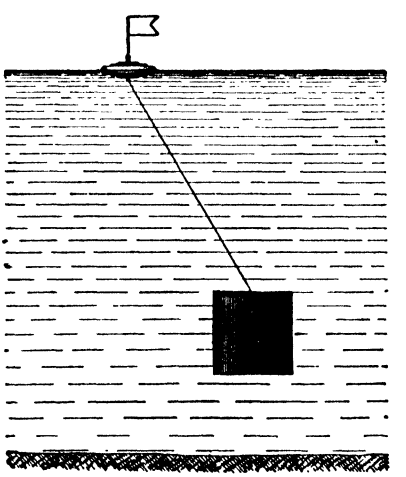


FIG. 161.

with the direction and velocity of the wind and with the length of cord connecting the two floats. Also the upper float may either drag or be dragged by the lower, and the upper is on this account likely to retard the lower where the latter is above the filament of maximum velocity, and to accelerate it when it is at a greater depth than this. As this latter effect extends over a greater proportion of the depth than the former, it would tend to make the velocities of flow recorded by the floats too high. Experiments made by T. G. Ellis, 1874, on the Connecticut River (mean velocity 2·1 ft. per sec.) with current meters and with double floats[1] showed that the mean velocity as found by floats was from 6 to 26 per cent. greater than by meter, the difference increasing with the velocity. Marr—October, 1879, on the Mississippi,[2] the width being about 2,000 feet and the depth 16·4 feet (mean velocity 2·6 ft. per sec.) found the mean float velocity to be about 3·5 per cent. greater than the meter velocity, while Henry—1869—on the St. Clair River[3]—45 feet deep—and with a mean velocity of 3·4 feet per second, found the mean float velocity about 10 per cent. greater than the meter velocity. In this case the float velocity was less than the meter velocity to a depth of about 7 feet. Below that depth the float velocity was the greater, the difference increasing with the depth.

(3) **Twin Floats.**—These consist of two masses of equal size, usually spheres coupled together by means of a wire, the lower of which is weighted so as to remain vertically below the upper, which floats at the surface. The velocity of the float then gives the mean of the velocities at the surface and at the depth of the lower mass. If this is adjusted so as to just clear the bottom, the velocity of the float will be approximately the mean velocity for the vertical in which the instrument floats.

(4) **Velocity Rods.**—The velocity rod, or rod float, consists of a light wooden rod or tin tube about 1 inch in diameter, and made in adjustable lengths. The lower end of the bottom length is weighted and the length adjusted until the rod floats vertically with its lower end clearing the bottom by a few inches. In a large river and where these are not likely to interfere with navigation, logs of wood about 12 inches in diameter, having their lower ends weighted with iron and their upper ends painted white, may be used.

The velocity of the rod is approximately the same as the mean over its depth, and gives the mean velocity over the vertical in which it floats. The

[1] Report Chief Eng., U.S.A., 1878, Appendix B.
[2] McKenzie, A. Report on Current Meter Observations. Burlington. 1884.
[3] "Journal Franklin Inst.," vol. 62, p. 322.

difficulty in using the rod lies in its tendency to drag over shoals and weeds, and to obviate this its lower end may be arranged to float at a height h^1 above the bed of the stream.

For such a case Francis gives the empirical formula

$$v_m = v_r \left(1{\cdot}012 - {\cdot}116 \sqrt{\frac{h^1}{h}}\right)$$

giving the mean velocity in the vertical containing the rod in terms of the velocity of the rod (v_r), h^1, and h the depth of the stream. Here h^1 should be less than ·25 h.

Of all floats the velocity rod gives the best results, and for channels of moderate and uniform depth, encumbered with floating weeds and grass, this is probably the best method of obtaining the velocity. In a series of experiments on the Loch Katrine Aqueduct; concreted surface; concave bottom; width 9′ 1″; radius of curvature 20′ 10½″; hydraulic mean depth 2·87 feet; velocity rods 2′ 2″ long, gave results which agreed within ¾ of 1 per cent. with results as obtained by weir measurement, while the velocity as obtained from the maximum surface velocity, and an application of the formula $v = v_s - 25{\cdot}4 \sqrt{m\,i}$, was 18 per cent. too low.

An elaborate series of experiments was carried out in 1856 by J. B. Francis on the Lowell Canal[1] to determine the relative accuracy of weir and rod float measurements, this canal being 27·75 feet wide where the first 63 experiments were made and 14 feet wide where the remaining 52 were made. The length of run was 70 feet, the floats being loaded tin tubes, 2 inches in diameter. From these it was found that the mean difference in the discharge as obtained by the two methods was less than 2 per cent. in all but three of the experiments, the mean difference being about 1 per cent. The mean velocity in these experiments varied from ·5 to 5 feet per second.

Experiments in 1900 at Cornell University[2] showed about the same degree of accuracy in the case of flow in a canal 16 feet wide and with depths of water ranging from 5 to 10 feet, and velocities of flow up to 2·07 feet per second. The immersed portions of the rods varied from 75 per cent. to 95 per cent. of the depth of the stream, and the length of measured run varied from 7 to 25 feet, depending on the velocity. In every case the float velocity was slightly less than that given by the weir.

[1] Lowell, Hydraulic Experiments, p. 170.
[2] "Trans. Am. Soc. C.E.," vol. 12, p. 301.

ART. 120.—COLOUR METHOD AND SALT VELOCITY METHODS OF GAUGING.

The mean velocity of a stream over a measured reach may be measured by injecting some suitable colouring solution and by noting the time required for this to traverse a known distance. Red or green aniline dye forms a suitable substance, and permanganate of potash may be used in waters free from organic matter. The solution should be introduced by a hand pump in one sudden injection. The coloured area is gradually extended in length as it passes down-stream, and the mean of the times at which the first and last traces of colour pass the measuring station should be noted.

Salt Velocity Method.[1]—The salt velocity method of water measurement

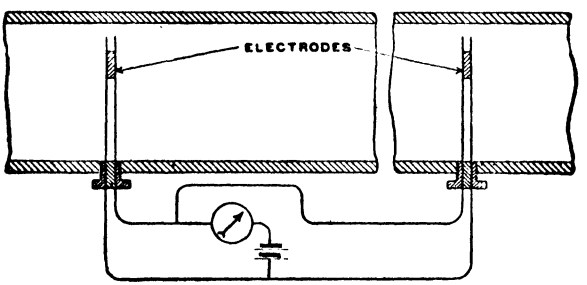


FIG. 161A.

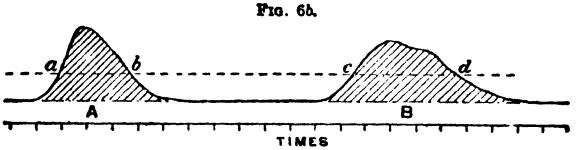


FIG. 161A.

is based on the fact that salt in solution increases the electrical conductivity of water.

Two pairs of insulated electrodes are mounted in the conduit at a measured distance apart. These are coupled to a battery, or to some convenient D.C. supply with a voltmeter or ammeter in the circuit (Fig. 161A). Salt in solution is injected at a point above the upper pair of electrodes, and the passage of this over the electrodes is indicated by the deflection of the needle of the recording instrument.

The salt solution should be injected in a single burst, and, in a large pipe, simultaneously at a number of points distributed evenly over the cross

[1] C. M. Allen and E. A. Taylor, "Trans. Am. Soc. Mech. E.," vol. xlv., 1923; also "Mechanical Engineering," January, 1924, p. 13. E. Crewdson, *Engineer*, December 14, 1923, p. 635.

section. The electrodes may consist of insulated copper or steel plates from ¼ inch to 1 inch apart, and should preferably extend across the conduit.

If an autographic recording electrical instrument is used the diagram is usually of the type indicated in the figure. The curves at *A* and *B* are not symmetrical, and experiments show that the time interval to be taken is that between the centres of area of the shaded areas at *A* and *B*. In a short pipe line it is necessary for accuracy to obtain such diagrams and to determine the positions of the centres of area. In a long pipe line, or one in which the time to traverse the distance between the electrodes is not less than about twenty seconds, it is possible to dispense with an autographic recorder without appreciable sacrifice of accuracy. The instants *a*, *b*, *c*, *d*, at which the needle of the voltmeter indicates a definite voltage about one-third of the maximum, are then to be taken by chronometer, and the correct time interval is that between the mid-points of *a b* and *c d*.

The salt velocity method is quicker, requires much less salt, is cheaper, and probably more accurate than the chemical method (Art. 121). It is applicable to any conduit whose length exceeds about 60 feet, although its accuracy increases with the length of the conduit.

Art. 121.—Salt Solution Method in Gauging.

The method consists in adding to the stream at a uniform and known rate in the form of a strong solution, some chemical salt, for which sensitive reagents are available, and by determining by analysis the percentage of the salt in samples taken at a point sufficiently far down-stream for thorough admixture with the water to have taken place.

The accuracy of the results depends essentially on the sample being truly representative of the water at the collecting station. For this reason the method is only permissible under conditions facilitating a thorough admixture between the points of introduction of the salt and of collection of the samples. Where the water passes through turbines between these points, or where the stream is very turbulent, these conditions are likely to be fulfilled, and experiment indicates that with care the accuracy of this method is then comparable with that of a weir. Where a stream is of regular section and of moderate velocity it is extremely difficult to ensure thorough admixture, and the method is not to be recommended unless great care is taken to introduce the salt simultaneously at a number of points evenly distributed over the cross section, and to collect the samples at a similar series of points down-stream.

Unless the stream is very brackish, common salt is a suitable chemical. This is introduced as a strong solution of about 16 lbs. per cubic foot of water. The rate of introduction should be such that the water of the stream after admixture contains not less than 1 part in 75,000 of salt. Sufficient of the solution should be used to give a uniform supply for between 20 and 30 minutes. It should be introduced at a number of points in a cross section, and a sample of it should be taken during its introduction.

The samples of the dosed stream water should be taken over a number of

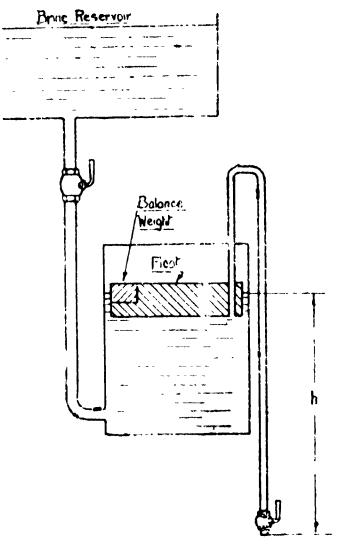


FIG. 162.

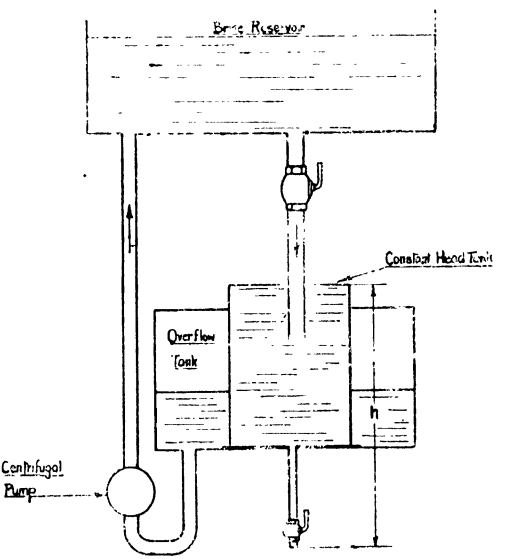


FIG. 163.

points in the cross section. They may be taken by a pump, through a flexible pipe whose open end is traversed across the stream during the process. A surface float will give the approximate time necessary for the first of the mixture to reach the sampling section. A few minutes should then be allowed before beginning to take the sample. Since many streams contain some salt in solution, a sample of the natural stream water should be taken during the tests, and should be tested for salinity.

The relative salt contents of the various samples should be determined by titration against a standard silver nitrate solution by an experienced chemist.

If Q = discharge of stream in c.f.s.
q = quantity of solution introduced in c.f.s.
c_1 = amount of salt in natural stream water in lbs. per c.f.
c_2 = amount of salt in the down-stream sample in lbs. per c.f.
c = amount of salt in the dosing solution in lbs. per c.f.

Then

$$Q = \frac{c - c_2}{c_2 - c_1} q,$$

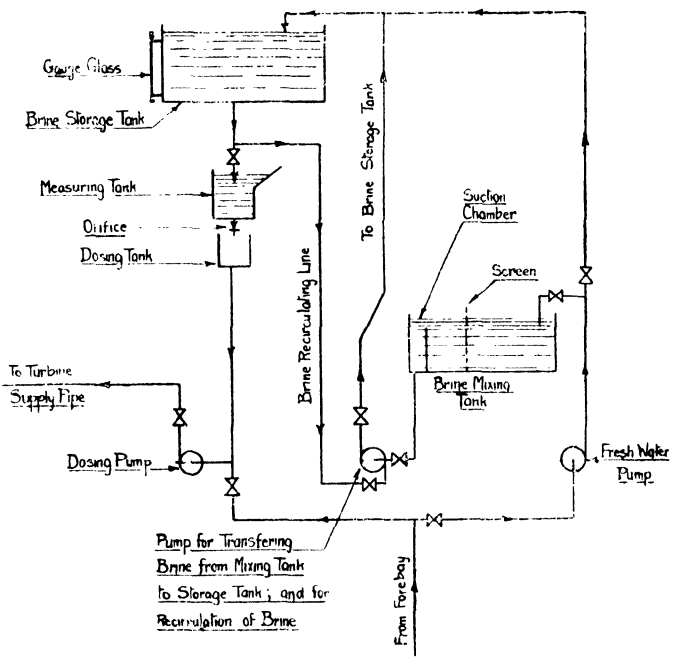


FIG. 164.

and if V_1, V_2 and V are the volumes of silver nitrate solution respectively necessary to titrate unit volumes of normal stream water of the down-stream sample, and of the dosing solution,

$$Q = \frac{V - V_2}{V_2 - V_1} q.$$

A uniform rate of flow of the solution may be ensured by supplying this through an orifice under a constant head. Two methods of doing this are indicated in the sketches of Figs. 162 and 163.

A lay-out recommended in the Turbine Test Code of the American Society

of Mechanical Engineers is shown in Fig. 164. Here the measured dosing solution runs into a dosing tank communicating with the suction side of a centrifugal pump which draws stream water from the forebay and discharges under pressure into the stream or pipe line. The level in the dosing tank is kept constant by means of a valve on the pipe connecting it to the pump suction. By introducing the solution under pressure in this way, diluted with a considerable volume of stream water, the difficulty of ensuring a satisfactory admixture is greatly reduced.

Art. 122.—Travelling Screen Method.

A method which may be used [1] for measuring the discharge of a parallel conduit consists in the provision of a light but rigid apron of canvas over a framework of angle iron, suspended vertically from a light carriage which runs on rails fixed on either side of the conduit. When lowered, the apron fills the conduit with very little clearance and is carried along with the same velocity as the stream. Its velocity is then measured by a chronometer and electrical contacts. This method necessitates a conduit of 60 to 80 feet in length, and has been applied to conduits up to 20 feet wide and 12 feet deep, for the measurement of the water supplied to turbines under test.

Art. 123.—Stream Rating Tables.

The usual object of velocity measurements in a river is the construction of a rating table, which shall show the relation at a given point between the height of water, referred to some permanent bench mark, and the discharge of the river. In order to prepare such a table it is necessary to obtain the discharge at various stages of a stream, covering the usual range of fluctuation. Usually the data are embodied in the form of a rating curve, showing graphically the relationship between discharge and height of surface level. The length of time during which such a curve can be safely applied depends on the class of channel. Where the channel is constantly shifting it cannot be used for many months unless the soundings are frequently checked with reference to the original datum level.

In such a channel an artificial control should, if possible, be constructed below the gauging station. Such a control may consist of a low dam, whose height is sufficient to prevent its being drowned in times of flood, by banking up of the down-stream level. With such a control the relationship between gauge height and discharge becomes stable and independent of any change of section at the gauging station itself.

[1] By Prof. E. Andersson of Stockholm. See *Zeitschrift des Vereins Deutscher Ingenieur*, April 20, 1907.

Art. 124. Gauging of Ice-Covered Streams.

When a river is ice bound its flow becomes somewhat similar to that in a closed flume, the water now flowing under pressure. A series of measurements of such streams by members of the U.S. Geol. Survey[1] leads to the following conclusions:

(1) The maximum velocity occurs at a point between 35 per cent. and 40 per cent. of the depth measured from the underside of the ice. The ratio of mean to maximum velocity ranges from about ·80 with a depth of 3 feet to ·92 with a depth of 16 feet, having a mean value of ·85.

(2) There are two points of mean velocity on a vertical, the first lying between ·08 and ·013 of the depth, and the second between ·68 and ·74 of the depth.

(3) The vertical velocity curve becomes more concave as the river rises, owing to the increased head.

(4) In making gaugings of such streams the vertical velocity curve method, or the integration method, should be adopted in preference to any of the single-point and co-efficient methods.

Examples.

(1) A canal whose depth is 4 feet, having slopes 2 to 1, has a bottom width of 10 feet. The bed is of earth (Kutter's N = ·025), and the gradient is 1 foot per mile. Determine the discharge in cubic feet per second.

Answer { Hydraulic mean depth = 2·58 feet.
$C = 68·8$.
Discharge = 109·5 cubic feet per second.

(2) A rectangular flume 4 feet wide and 2 feet deep is roughly constructed of unplaned timber, and is required to deliver 80 cubic feet of water per second. Determine the necessary gradient, and assuming it to supply water to a power station distant 5 miles from the supply reservoir, determine the percentage loss in transmission if the difference of level between the supply reservoir and tail race is 500 feet. $C = 127·6$.

Answer. Gradient 1 in 162·9.
Loss of energy = 32·4 per cent.

(3) A flat-bottomed channel is required to have a constant velocity of flow for all depths of water. The bottom breadth is 5 feet. Determine

[1] Water Supply and Irrigation Paper. No. 95, p. 158.

the depth at a section where the breadth is 20 feet, if the hydraulic mean depth is 1·25 feet. $C = 60$.

Answer. 4·315 feet.

(4) The original depth of a wide stream is 3 feet, and the slope of its bed is 1 in 1,000, the value of f being ·0131, $C = 70$. A dam 10 feet high is erected across the stream. Determine the rise in the water level immediately behind the dam and at points $\frac{1}{2}$ and 1 mile up stream. (Assume the coefficient of discharge for the dam to be ·560.)

Answer. Rise behind dam = 9·46 feet.
„ $\frac{1}{2}$ mile up stream = 6·87 feet.
„ 1 „ „ „ = 4·34.

(5) It is required to excavate a canal out of rock to be of rectangular section and to bring 500 cubic feet of water per second from a distance of 4 miles with a velocity of $7\frac{1}{2}$ feet per second. Determine the gradient and the most suitable section for the canal. Take $C = 150$.

Answer. Section 11·54 feet wide, 5·77 feet deep.
Gradient 1 in 1,154.

(6) In carrying out field operations to determine the discharge of a river, a straight reach 500 feet long is available. The slope is approximately $\frac{1}{10000}$, and the levelling is possibly accurate within $\frac{1}{500}$ foot. The possible error in determining the wetted perimeter is 8 per cent., and in determining the mean sectional area is 5 per cent. To what degree of accuracy are the final results likely to approximate.

Answer. Within about $13\frac{1}{2}$ per cent.

(7) The value of f for a stream having a slope of $\frac{1}{1000}$ is ·0050. Normally, the stream is of depth 4 feet and breadth 60 feet, but is passed through a sluice having an opening $2\frac{1}{2}$ feet deep. Determine whether the conditions are such as to lead to the formation of a standing wave, and if so determine the probable height of the crest of this wave above the upper edge of the sluice.

Answer. Yes. ·93 feet.

CHAPTER XII.

Impact of Jets on Stationary and Moving Vanes—Distribution of Pressure over Plate—Graphical Construction to determine Pressure on a Vane—Centre of Pressure on Vane—Compounding of Jets—Resistance of Submerged Plane Surfaces—Rudder Action—Resistance of Submerged Bodies—Resistance and Propulsion of Ships—Jet Propulsion.

ART. 125.—IMPACT OF JETS.

WHEN a steady jet of water impinges on any solid surface there is none of the rebound which follows the impact of two solid bodies. Instead of this, a thin stream is formed which glides along the surface until it reaches the boundaries, when it leaves approximately tangential to the surface.

When the initial and final directions and velocities of an impinging jet are known, the force which it exerts on the surface in any direction may be calculated by equating this force to the total change of momentum of the jet per second in this direction.

It is important at this stage to differentiate between the "absolute" and "relative" velocities of a jet. Thus, if the jet be projected from a fixed nozzle and strike a moving surface, its velocity may be considered from the point of view of a person standing by the nozzle and who notes its velocity relatively to the earth, or from that of a person moving with the surface struck, and who notes the motion relative to this moving surface. The first person then notes the absolute, and the second the relative velocity of the jet.

In the case of impact on a fixed surface the change of relative velocity at impact is identical with the change of absolute velocity, and in applying the equation of momentum either may be considered. In any other case, however, the change of momentum must be measured by the change of *absolute* and not of relative velocity.

If friction be neglected and also losses due to shock, the velocity relative to the surface will be unaffected by the impact. Also, the pressure exerted on the surface at any point will be in the direction of the normal at that point. On these assumptions we may consider the following cases :—

(1) **Normal Impact on a Stationary Plane Surface (Fig. 165).**

Let a = sectional area of jet in square feet.

v = velocity of jet in feet per second.

Then the weight of water impinging on the plane per second $= W\,a\,v$ lbs., where W is the weight of 1 cubic foot of water.

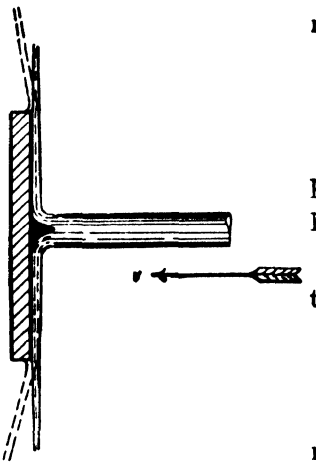


FIG. 165.

The initial momentum of this per second normal to the plane

$$= \frac{W\,a\,v^2}{g} \text{ ft. lb. units.}$$

If the final velocity is tangential to the plane, the final momentum normal to the plane $= 0$.

$\therefore$ Change of momentum per second normal to plane $= \dfrac{W\,a\,v^2}{g}$.

$\therefore$ Normal pressure on plane $= \dfrac{W\,a\,v^2}{g}$ lbs.

If θ be the angle which the sheet of water makes with the plane of the surface on leaving, the final velocity per second normal to the plane $= v \sin \theta$, and the momentum in this direction

$$= \frac{W\,a\,v^2}{g} \sin \theta.$$

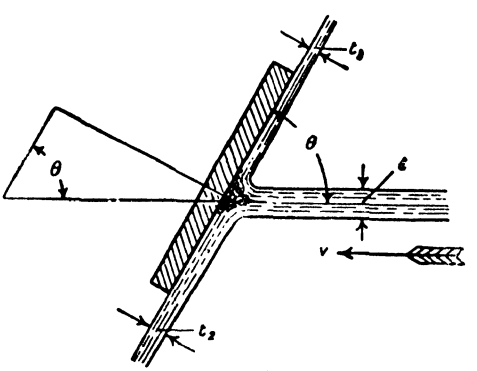


FIG. 166.

The change of momentum per second and therefore the pressure normal to the plane is now $\dfrac{W\,a\,v^2}{g}(1 - \sin \theta)$ lbs.

(2) Oblique Impact on a Stationary Plane Surface (Fig. 166).

Assuming the jet to be of the same width as the plane and to be pre-

vented from spreading sideways by flanges, part of the stream will escape from each end of the plane. The sectional area of each of these streams can be calculated by expressing the fact that the change of momentum parallel to the plate is unchanged by impact if friction be neglected.

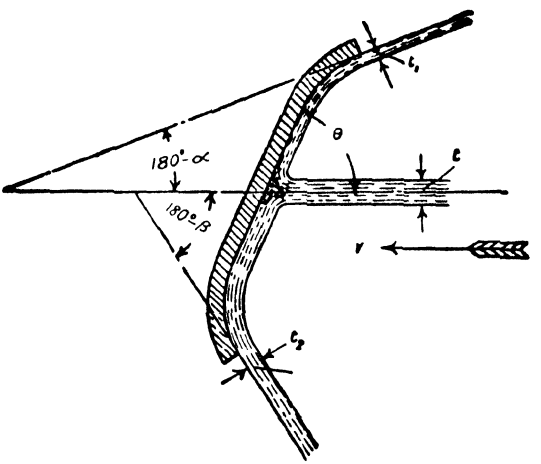


FIG. 167.

Suppose the jet of unit width and of area t, the thickness of the discharge streams being t_1 and t_2.

Let θ be the (acute) angle between the direction of the jet and the plane, v the velocity of the jet.

Then initial momentum parallel to plane $= \dfrac{W\,t\,v^2}{g} \cos\,\theta$

and final momentum parallel to plane $= \dfrac{W\,v^2}{g}(t_2 - t_1)$

$$\therefore\ t\cos\theta = t_2 - t_1$$

But $$t = t_2 + t_1$$

$$\therefore\ \frac{t_1}{t_2} = \frac{1-\cos\theta}{1+\cos\theta},\ i.e.,\ t_2 > t_1.$$

Also $$t_1 = t\left(\frac{1-\cos\theta}{2}\right)$$

$$t_2 = t\left(\frac{1+\cos\theta}{2}\right).$$

Again, the initial momentum, in a direction perpendicular to the plane $= \frac{W\,t\,v^2}{g}$. sin θ, and the final momentum in this direction = 0.

$\therefore$ Normal force on plane $= \frac{W\,t\,v^2}{g}$. sin θ lbs.

$\therefore$ Force in direction of jet $= \frac{W\,t\,v^2}{g}$. sin $^2\theta$ lbs.

(2) **Impact on Stationary Curved Vane** (Fig. 167).

If the inclined plane of the previous case be fitted with ends curved so as to deflect the escaping streams into directions making angles α and β with that of the jet, the pressure on the vane in the direction of the jet will be increased or diminished according as α and β are greater or less than 90°.

As before $$t_1 = t\left(\frac{1-\cos\theta}{2}\right).$$

$$t_2 = t\left(\frac{1+\cos\theta}{2}\right).$$

The final momentum per sec. in the direction of the jet $= \frac{W}{g}v\{t_1 v\cos\alpha + t_2\cos\beta\}$

$$= \frac{W\,v^2}{g}\{t_1\cos\alpha + t_2\cos\beta\}$$

$\therefore$ Change of momentum per sec. in this direction, by impact $= \frac{W\,v^2}{g}\{t - t_1\cos\alpha - t_2\cos\beta\}$

$\therefore$ Pressure on vane in direction of jet

$$= \frac{W}{g}v^2\,t\left\{1 - \frac{\cos\alpha + \cos\beta}{2} - \frac{\cos\theta\,(\cos\beta - \cos\alpha)}{2}\right\} \text{ lbs.}$$

This is a maximum when $\alpha = 180°$ and $\beta = 180°$, *i.e.*, when the discharge is returned parallel to the jet, and then has the value $\frac{2\,W}{g}v^2\,t$ lbs. In this particular case, as whenever $\alpha = \beta$, the pressure is independent of θ, the angle of impact.

(3) **Tangential Impact on a Fixed Vane.**

Let the vane deflect the jet through an angle α.

Then the initial momentum in the direction of the jet $= \frac{W\,A\,v^2}{g}$.

Then the final momentum in the direction of the jet $= \frac{W\,A\,v^2}{g}$. cos α.

$\therefore$ Change of momentum per second = force in this direction

$$= \frac{W A v^2}{g} (1 - \cos \alpha) \text{ lbs.}$$

Thus, if $\alpha = 180$ (semicircular vane) the force is $\frac{2 W A v^2}{g}$ lbs

ART. 126.—ACTUAL FORCE OF IMPACT.

In practice, skin friction reduces the relative velocity of a jet over its sur-

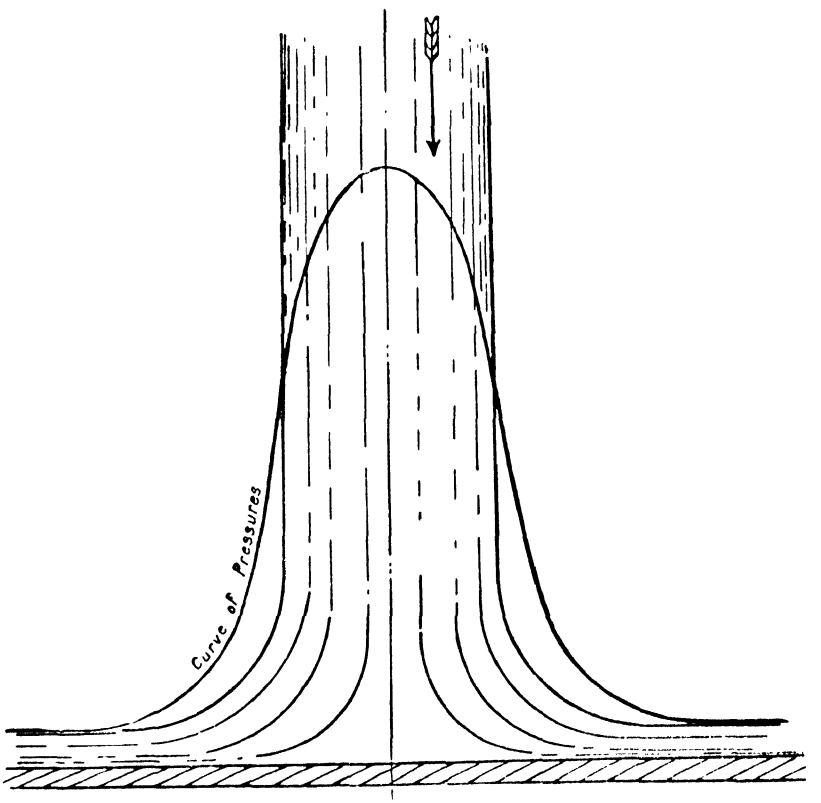


FIG. 168.

face, and so affects the force of impact. Calling K the ratio of final to initial relative velocity K depends on the roughness of the surface, on the length of the path of the water over the surface, and on the thickness of the stream. Unless the vane has easy curves, energy may also be wasted in eddy formation, and this still further reduces the value of K.

The final direction of a free stream is seldom tangential to the surface of

the vane. Surface tension, and, in a curved vane, centrifugal force, cause the deflection to be less than that corresponding to the vane angle. This latter effect depends on the form of the vane and on the velocity of impact. With impact on a hemispherical vane the angle between the jet and the tangent to the surface at the point of discharge may be as much as 10 degrees.

Art. 127.—Distribution of Pressure over Surface at Impact.

Owing to the curvature of the stream in the neighbourhood of the plate (Fig. 168), and to the centrifugal force caused thereby, the pressure increases along the radius of curvature towards the surface of the plate and the centre of the jet, the velocity suffering a corresponding diminution.

Assuming no loss of energy before impact, the energy per lb. of water will be $\frac{v^2}{2g} = h$ ft. lbs., where, in the case of a vertical jet, h is the height of the free surface of the supply reservoir above the plate. Thus, at the centre of the jet at the plate where the velocity is zero, the pressure intensity will have its maximum value $W h = \frac{W v^2}{2g}$ lbs. per square foot. From this point the pressure intensity falls off radially as indicated in the figure, the pressure curve being approximately as shown.

Experiments show that the pressure at the centre of the jet is practically the same as (slightly less than) that corresponding to $\frac{v^2}{2g}$ feet of water.

The following table,[1] shows the pressure exerted at different radii by a vertical jet $1\frac{9}{16}$" diameter, when impinging normally on a fixed plate, under a head of about 27 inches:—

Radial distance from axis of jet in inches	·000	·125	·225	·325	·425	·525	·625	·725
Pressure intensity expressed as a fraction of "h"	·991	·986	·971	·966	·944	·906	·860	·805

Radial distance from axis of jet in inches	·825	·925	1·125	1·325	1·525	2·025	2·350	—
Pressure intensity expressed as a fraction of "h"	·749	·666	·509	·302	·170	·151	·075	—

[1] "Proc. Inst. C. E.," vol. 60, p. 436.

In general, with a plate having a diameter from two to three times that of the jet, the total pressure exerted will exceed 90 per cent. of the theoretical.

Since the force theoretically exerted on a hemispherical vane by the impact of a jet with velocity v is $\frac{2\,W\,a\,v^2}{g}$ lbs., this is equivalent to a pressure intensity over its surface of $\frac{2\,a\,v^2}{g\,A}$ feet of water, where A is its projected area, and since the maximum pressure intensity which can be produced by a jet is $v^2/2\,g$ feet, it follows that full deviation through 180 degrees is impossible unless A equals, or is greater than, 4 a.

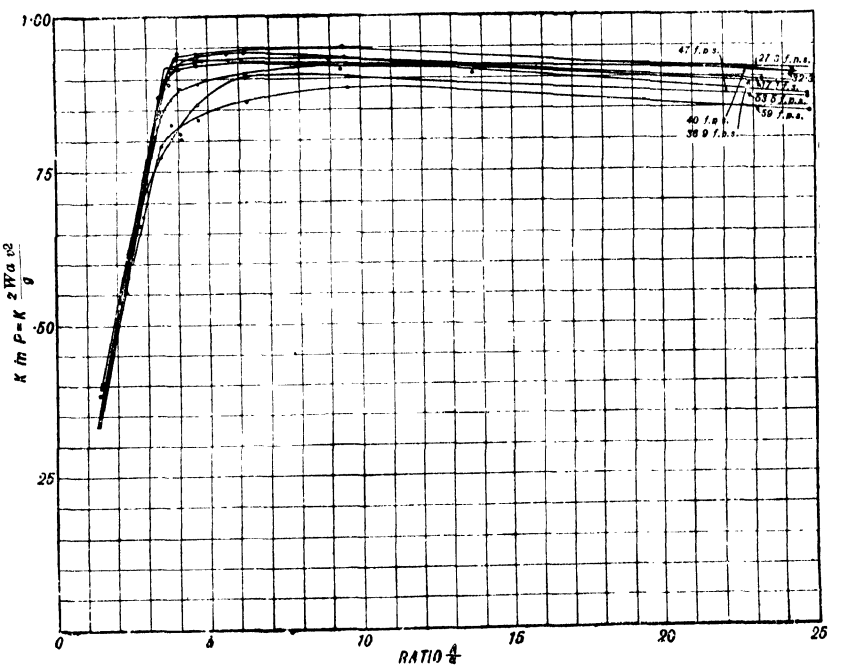


Fig. 169.

Fig. 169 shows the result of impact tests on such vanes of different diameters,[1] carried out in the Author's laboratory. From this it appears that the force on the vane is very nearly a maximum when $A : a$ is 4, and that for smaller values of A it falls off rapidly.

[1] "Phil. Mag.," vol. 45, January, 1923, p. 229.

Art. 128.—Impact on Moving Vanes.

Let v be the velocity of the jet in feet per second.
,, u be the velocity of the vane in the direction of motion of the jet.
Then the relative velocity of jet and vane $= v - u$.

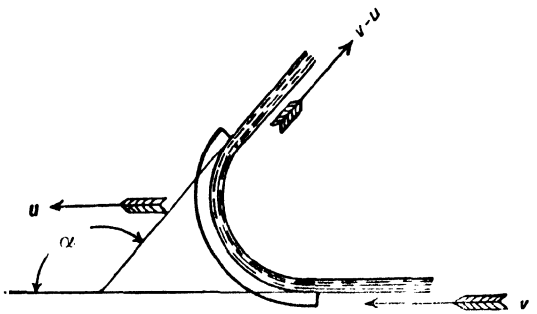


Fig. 170.

(1) **Impact on a single vane curved through an angle a, and moving in the direction of the jet** (Fig. 170).

Here the weight of water striking the vane per sec. $= W\ a\ (v - u)$ lbs.

Initial momentum in direction of motion $= \dfrac{W\ a\ (v - u)\ v}{g}$

Final velocity in direction of motion $= u + (v - u) \cos a$

$\therefore$ Final momentum in direction of motion $= \dfrac{W\ a}{g} \{(v - u)\ u + (v - u)^2 \cos a\}$

$\therefore$ Change of momentum in this direction $= \dfrac{W\ a}{g} \{(v - u) [v - u - (v - u) \cos a]\}$

$= \dfrac{W\ a}{g} (v - u)^2 (1 - \cos a)$

$\therefore$ Work done on vane $= \dfrac{W\ a}{g} (v - u)^2 (1 - \cos a)\ u$ ft. lbs. per sec.

Initial kinetic energy of jet $= \dfrac{W\ a\ v^3}{2\ g}$ ft. lbs.

$\therefore$ Efficiency $= \dfrac{2\ (v - u)^2\ (1 - \cos a)\ u}{v^3} = \eta.$

For maximum efficiency $\dfrac{d\ \eta}{d\ u} = 0$

$\therefore\ 2\ (v - u)^2 - 4\ (v - u)\ u = 0$

$\therefore$ either $v = u$ or $v - u = 2\ u$

so that $v = 3\ u.$

In the first case the velocity of the jet is unchanged and the work done on the vane is zero. In the second case we have :—

$$\text{Work done on vane} = \frac{W A}{g}\left(\frac{2}{3}v\right)^2 \cdot \frac{v}{3}(1 - \cos a) \text{ ft. lbs. per sec.}$$

$$= \frac{4}{27}\frac{W A}{g} \cdot v^3 (1 - \cos a) \text{ ft. lbs. per sec.}$$

$$\therefore \text{Efficiency (max.)} = \frac{8}{27}(1 - \cos a).$$

This has its greatest value when $a = 180°$, and then equals $\frac{16}{27}$.

When $a = 90°$, *i.e.*, in the case of normal impact on a single flat plate, we have, as before, the maximum efficiency when $u = \frac{v}{3}$, and then efficiency (max.) $= \frac{8}{27}$.

(2) **Impact on a Series of Moving Vanes.**

If, instead of a single vane, we have a series of vanes successively placed in the path of the jet at frequent intervals, the weight of water striking these vanes per second becomes $W a v$ lbs.

If the vanes form surfaces of revolution having their axes in the line of action of the jet, neglecting losses due to friction we have, if a be the angle through which the vane is curved :—

$$\left.\begin{array}{l}\text{Change of momentum per second in direction of jet}\end{array}\right\} = \frac{W a v}{g}(v - u)(1 - \cos a)$$

$$\therefore \text{Total force on vanes} = \frac{W a v}{g}(v - u)(1 - \cos a) \text{ lbs.}$$

$$\left.\begin{array}{l}\text{Work done on vanes per second}\end{array}\right\} = \frac{W a v u}{g}(v - u)(1 - \cos a) \text{ ft. lbs.}$$

$$\therefore \text{Efficiency} = \frac{2(v u)(v - u)(1 - \cos a)}{v^3} = \eta$$

$$\text{For maximum efficiency } \frac{d\eta}{du} = 0$$

$$\therefore v(v - u) - v u = 0$$

or

$$v = 2u$$

$$\therefore \text{Efficiency (max.)} = \frac{v^2 \times \frac{v}{2}(1 - \cos a)}{v^3}$$

$$= \frac{1 - \cos a}{2}$$

This is a maximum for $a = 180°$, *i.e.*, with a series of hemispherical cups and then equals unity. With a series of flat plates having normal incidence $a = 90°$ and the efficiency equals ·5.

If K is the ratio of the relative velocities at exit and entrance, the force on the vanes becomes

$$\frac{W}{g} a v (v - u)(1 - K \cos a)$$

and the maximum efficiency $\frac{1 - K \cos a}{2}$.

ART. 129.—IMPACT ON ONE OF A SERIES OF RECURVED VANES WHOSE DIRECTION OF MOTION MAKES AN ANGLE WITH THAT OF THE JET.

This problem is one of much importance in the design of impulse turbines. Let a be the angle between the directions of v and of u, and

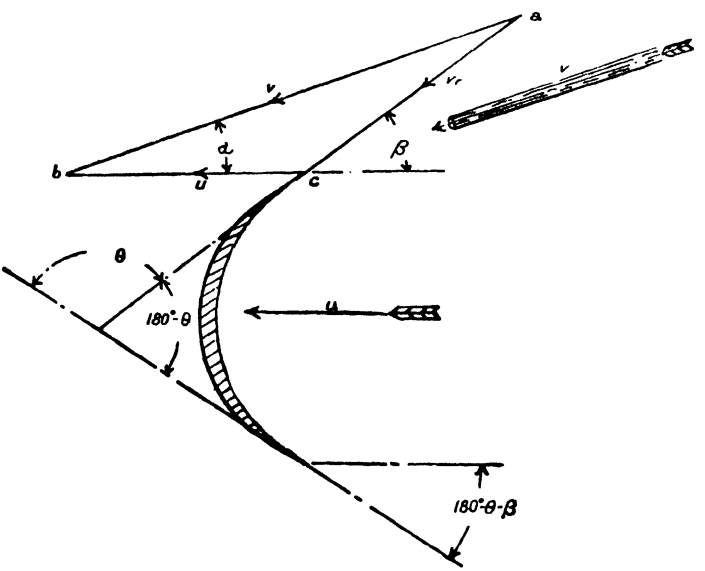


FIG. 171.

let θ be the total angle through which the vane is recurved (Fig. 171). Then if, as is usual, the incidence is tangential, the relative velocity v_r of jet and vane at impact is given by

$$v_r^2 = v^2 + u^2 - 2 v u \cos a \text{ (triangle } a\ b\ c.)$$

and, neglecting friction and eddy losses, the relative velocity at discharge will be the same as this.

Also, for tangential incidence, the direction of the vane at incidence must be parallel to the direction of the jet relative to the vane and must

therefore make an angle β with the direction of motion of the vane where β is obtained from the relationship (triangle $a\ b\ c$).

$$\text{Sin } \beta = \frac{v}{v_r} \text{ . sin } a.$$

The initial velocity of jet in the direction of motion of vane $= v \cos a$

„ final „ „ „ „ „ „

$$= u - v_r \cos (180° - \theta - \beta)$$
$$= u + v_r \cos (\theta + \beta)$$

$\therefore$ Change of momentum per sec. in direction of motion of vane $= \frac{W Q}{g} \left\{ v \cos a - u - v_r \cos (\theta + \beta) \right\}$

$\therefore$ Force on vane $= \frac{W Q}{g} \left\{ v \cos a - u - v_r \cos (\theta + \beta) \right\}$ lbs.

Taking friction into account, this becomes

$$\text{Force} = \frac{W Q}{g} \left\{ v \cos a - u - K v_r \cos (\theta + \beta) \right\} \text{ lbs.}$$

ART. 130.—GRAPHICAL CONSTRUCTION TO DETERMINE THE PRESSURE EXERTED ON A VANE (Fig. 172).

(1) Impact on a Fixed Vane.

Let the vector BA represent the initial velocity of the jet in feet per second.

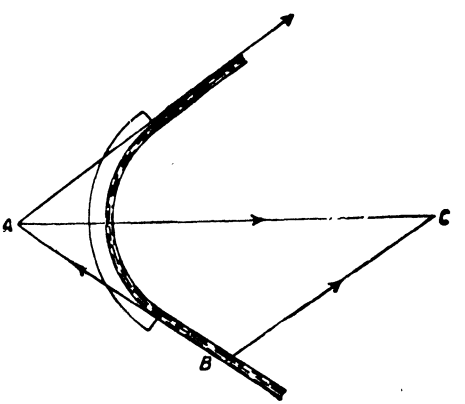


FIG. 172.

Let the vector $\overline{BC}$ represent its final velocity on leaving the vane.

Then „ „ $\overline{AC}$ represents the change of velocity, and therefore gives a measure of the resultant pressure on the vane, for if $W\ Q$ lbs. of water strike the vane per second, the change of momentum in the direction AC per second $= W\ Q . \overline{AC} \div g$.

$\therefore$ Resultant pressure on vane acts in the direction $C\,A$ and is of magnitude $\frac{W\,Q}{g}\,.\,\overline{C\,A}$ lbs.

(2) **Impact on One of a Series of Moving Vanes.**

Let $\overline{B\,A}$ (Fig. 173 *a* and *b*) represent the initial velocity of the jet in magnitude and direction.

Let $\overline{B\,C}$ represent the velocity of the vane.

Then $\overline{C\,A}$ represents the relative velocity of the water over the surface of the vane.

Draw $E\,F = C\,A$ tangential to the vane at exit and make $F\,G$ equal and parallel to $B\,C$. The vector $\overline{E\,G}$, which represents the velocity com-

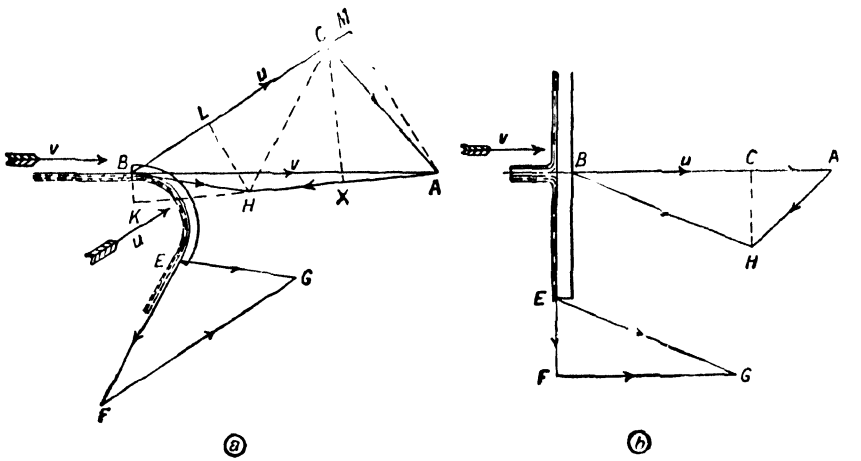


FIG. 173.

pounded of the velocity of the vane and of that of the water relative to the vane, now gives the absolute velocity of the discharge stream.

Drawing $B\,H$ equal and parallel to $E\,G$, the acceleration which is given to the water by the vane is represented by $\overline{A\,H}$, and the resultant pressure on the vane $P = \frac{W.\,Q}{g}\,.\,HA$ lbs. The line of action of this pressure is parallel to $H\,A$.

The effective pressure is the component of this in the direction of motion of the vane. Thus in Fig. 173 *a*, this is given by $\frac{W.\,Q}{g}\,.\,\overline{LM}$ lbs., and the work done per second by $\frac{W.\,Q}{g}\,.\,\overline{LM}\,.\,\overline{BC}$ ft. lbs. In Fig. 173 *b*,

the effective pressure $= \frac{W.\,Q}{g} \,.\, \overline{CA}$ lbs. and the work per second $= \frac{W.\,Q}{g} \,.\, \overline{CA} \,.\, \overline{BC}$ ft. lbs.

By producing $A\ H$ to K (Fig. 173 a) and dropping perpendiculars $B\ K$, $C\ X$, on to $A\ H$, we can prove that

$$\overline{BA}^2 - \overline{BH}^2 = \overline{HA}\ (\overline{HA} + 2\,\overline{KH}).$$

Also, since $\overline{CA}$ represents the initial relative velocity of jet and vane and $\overline{CH}$ their final relative velocity, we have—neglecting friction—$\overline{CH} = \overline{CA}$. $\therefore\ \overline{HA} = 2\ \overline{HX}$

$$\therefore\quad KA = KH + 2\,HX$$

$$\therefore\quad KA + KH = HA + 2\,KH = 2\,(KH + HX)$$

$$\therefore\quad HA + 2\,KH = 2\,KX$$

$$\text{Also } \frac{2\,KX}{2\,BC} = \frac{LM}{AH}$$

$$\therefore\quad HA\,(HA + 2\,KH) = 2\,(LM \,.\, BC)$$

$$\therefore \text{ Work done} = \frac{W.\,Q}{2\,g}\,(\overline{BA}^2 - \overline{BH}^2)$$

$$= \frac{W.\,Q}{2\,g}\,(v_1^2 - v_2^2)$$

where v_1 and v_2 are the initial and final absolute velocities of the water.

$$\therefore \text{ Efficiency} = \frac{\text{Work done on vane per second}}{\text{Total energy of jet}}$$

$$= \frac{\frac{W.\,Q}{2\,g}\,(v_1^2 - v_2^2)}{\frac{W.\,Q}{2\,g}\,v_1^2}$$

$$= 1 - \left(\frac{v_2}{v_1}\right)^2$$

Centre of Pressure on Vane.—The position of the centre of pressure on any vane receiving a jet tangentially may be determined as follows. Consider any small arc $P\ Q$ (Fig. 174 a) of the vane. If the velocity is supposed unaltered by friction, $P\ A$ and $A\ Q$, tangents at P and Q represent to some scale the (equal) velocities at P and Q, while $R\ Q$, perpendicular to the chord $P\ Q$, represents to the same scale the change of velocity between P and Q. Normals $P\ C$ and $Q\ C$ to the curve intersect in C, the centre of curvature of the arc, and $\frac{RQ}{AQ} = \frac{PQ}{PC}$, so that if $P\ C$ represent the velocity v at P, the chord $P\ Q$ represents the change of

velocity between P and Q and the resultant pressure on the arc $P\,Q$ is given by $\frac{W.\,Q}{g}\,.\,\overline{PQ}$ lbs.. if Q is the volume in cubic feet striking the vane per second.

Thus if a series of normals to the curve are drawn intersecting at C_1, C_2, C_3, etc. (Fig. 174 *b*), the pressure on the corresponding arcs will be given by $\frac{W.\,Q}{g}\,.\,\left\{\frac{\text{arc}}{\text{radius}}\right\}\,v$ lbs., and these pressures act outwards through the middle points of their respective arcs. Drawing in the lines

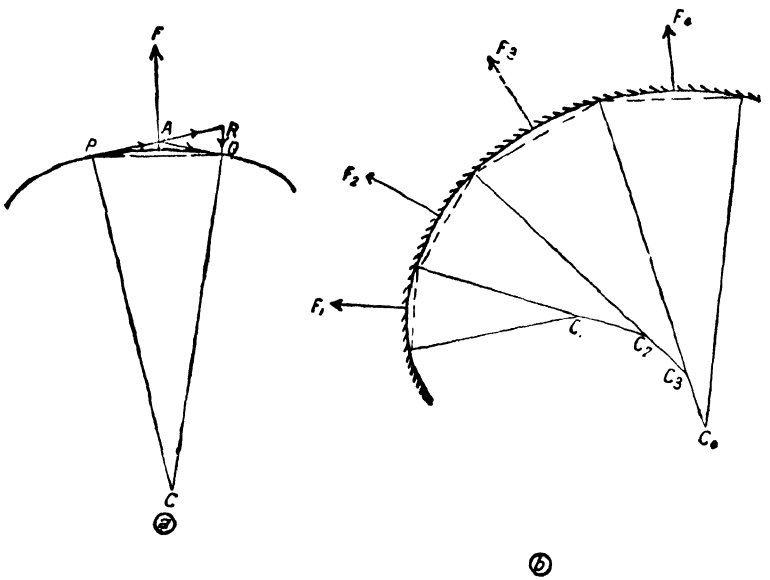


FIG. 174.

representing these pressures F_1, F_2, F_3, etc., the funicular polygon for these forces may be drawn, and the line of action of their resultant obtained as explained in any work on applied mechanics. The point of intersection of this line with the surface of the vane gives the centre of pressure on the vane.

If the velocity varies from point to point of the vane the same general principles apply except that now the velocity v in the expression—pressure $= \frac{W.\,Q}{g}\,\left\{\frac{\text{arc}}{\text{radius}}\right\}\,v$. lbs.—varies from point to point. Where the law of its variation is known, the problem becomes perfectly determinate.

Art. 131.—Compounding of Jets.

If two steady streams are allowed to impinge on each other under atmospheric pressure, they combine to form a single stream.

Let streams (1) and (2) combine to form stream (3) (Fig. 175). Then if a, v, and Q represent the sectional areas, velocities of flow, and quantity carried by each stream, we have, if θ be the angle made by an impinging stream with the direction of the resultant stream :—

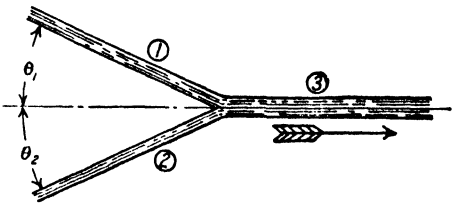


Fig. 175.

For continuity of flow, $v_1, a_1 + v_2, a_2, = v_3, a_3$ (1)

The equations of momentum give :—

$$Q_1 v_1 \sin \theta_1 - Q_2 v_2 \sin \theta_2 = 0 \quad (2)$$

$$Q_1 v_1 \cos \theta_1 + Q_2 v_2 \cos \theta_2 = Q_3 v_3 \quad (3)$$

while the equation of energy gives :—

$$\frac{W Q_1 v_1^2}{2g} + \frac{W Q_2 v_2^2}{2g} = \frac{W Q_3 v_3^2}{2g} + \text{loss at impact.}$$

$$= \frac{W (Q_1 + Q_2) v_3^2}{2g} + H'$$

$$\therefore \quad H' = \frac{W}{2g} \{ Q_1 v_1^2 + Q_2 v_2^2 - (Q_1 + Q_2) v_3^2 \}$$

$$= \frac{W}{2g} \{ Q_1 (v_1^2 - v_3^2) + Q_2 (v_2^2 - v_3^2) \} \quad (4)$$

By substitution from equations (2) and (3) above, this loss may be determined in terms of v_1, v_2, θ_1 and θ_2.

In two particular cases H' may be zero. In each of these it is necessary that $\left.\begin{matrix} v_1^2 - v_3^2 = 0 \\ v_2^2 - v_3^2 = 0 \end{matrix}\right\}$ simultaneously, and therefore that $\pm v_1 = \pm v_2 = \pm v_3$.

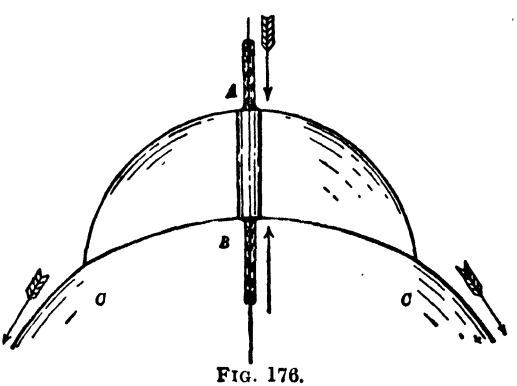


Fig. 176.

The first case is found where $v_1 = v_2 = v_3$, *i.e.*, where two parallel streams moving with the same velocity combine to form a single stream, with an area equal to the combined area of the two. The second case is found where $v_1 = -v_3$. each of these being

numerically equal to v_3, and corresponds to the direct impact of two streams of equal velocity. A film of water is then formed, the velocity of whose mass centre (v_3 in the equation of momentum) is zero, but which has a velocity of outward flow equal to v_1 or v_2. Professor Osborne Reynolds illustrated this by allowing two streams of equal velocity to meet by direct impact, and noting the clear and glassy appearance of the resultant film. If a cyliudrical prism having plane and parallel ends be placed in the path of the stream (Fig. 176), the films from A and B to the point of contact C, are still perfectly clear. After C, however, the frosted appearance of the film indicates eddy formation and the institution of sinuous motion.

Compounding of Confined Streams.—Loss at Impact.

When the impinging streams are confined, the pressure is no longer the same before and after impact and the available data is insufficient to allow the equations of momentum to be applied. Experiments by the author[1] on the loss following the impact of such streams, only one of which is deviated by the impact, show that this is given by

$$\text{loss} = a\frac{v_1^2}{2g} + b\frac{v_2^2}{2g} \text{ ft. lbs. per lb. of the impinging jet} \qquad (2)$$

By the impinging jet is meant that which suffers deviation. The velocity of this jet is v_2 while the velocity of the primary or undeviated stream is v_1. The values of the constants a and b depend on the angle of impact θ, and on the ratio m of areas of the primary and impinging streams. The values of θ in the experiments were varied from 5° to 90°, while m was varied from 1 to 5. The area of the impinging stream was $\frac{1}{2}'' \times 1''$ throughout, and the area of the primary stream was the same before and after impact. The velocities ranged up to 23 ft. per sec. Under these conditions the following are the values of a and of b.

θ	Values of m, *i.e.* ratio of area of primary and impinging streams.									
	1		2		3		4		5	
	a	b	a	b	a	b	a	b	a	b
90°	3·0	1·53	1·85	·97	1·57	·915	1·42	·921	1·33	·925
60°	2·0	·71	·49	·59	·38	·70	·33	·75	·31	·79
45°	1·5	·44	·32	·45	·22	·60	·19	·68	·17	·73
30°	1·0	·22	·20	·35	·12	·53	·10	·62	·083	·68
15°	·50	·060	·090	·28	·050	·48	·040	·58	·034	·652
5°	·17	·008	·030	·254	·013	·447	·011	·566	·009	·642
0°	·00	·00	·00	·250	·00	·444	·00	·563	·00	·640

[1] "Proc. Roy. Soc. Edinburgh," 1912–13.

It will be noted that when θ is large, and particularly where m is small, the v_1^2 term is all important, while for small values of θ and large values of m the v_2^2 term becomes the more important.

Art. 132.—Resistance of Submerged Plane Surfaces.

If an entirely submerged plane surface be moved normally or obliquely with uniform velocity through still water, the resistance to motion depends slightly on frictional resistances, but to a much larger extent on the change of momentum produced in the surrounding water during the passage of the plane, and on the eddy production in the rear of the plane.

Normal Motion of a Plane through Still Water.

Here, in front of the plane, stream line motion is set up, and if a be the area of the plate, and v its velocity, a column of water of sectional area a and of length v feet, is transferred from front to rear of the plane per second. The effect of this in producing resistance, may be seen by considering the flow of a steady stream past such a submerged plane. As before, up to the plane the motion is steady, and the stream line formation is as indicated in Fig. 177.[1] The plane, therefore, affects the momentum in a direction normal to its plane, of a mass of water of volume $a\,v$ cubic feet, per second. Since the velocity of the mass in this direction is initially v feet per second, if it were possible to destroy the whole of its momentum the total pressure on the front of the plane would be $\frac{W\,a\,v^2}{g}$ lbs. From a consideration of the stream line formation, it is, however, clear that it is only those stream lines at and near the centre of the plane which are actually diverted at right angles to their normal direction, the outer layers being diverted through a smaller angle depending on their distance from the centre, so that the change of momentum in the direction of motion, and hence the pressure on the front of the plane, is less than $\frac{W\,a\,v^2}{g}$. At only one point, *i.e.*, the centre of the plane, is the momentum actually destroyed, and here the pressure intensity amounts to $\frac{W\,v^2}{2\,g}$, the head corresponding to the velocity of flow. Immediately after passing the plane the motion becomes sinuous,

[1] From a paper by Professor Hele Shaw, "Trans. Inst. Naval Architects," 1898, vol. 85.

eddies are formed,[1] and since the energy of eddy production has to be supplied in the form of extra work done on the plate, this directly increases the resistance to motion. In other words, while the pressure on the front of the plane is unaffected, that on the rear face is reduced by this eddy production, and since the resistance to motion is equal to the difference of pressure on the two faces, this is increased. Plates of different shapes have different effects as regards eddy production, the circular shape giving least resistance for a given area, while generally the resistance increases slightly with the ratio of the length of periphery to the area of the plane. Also it would appear that

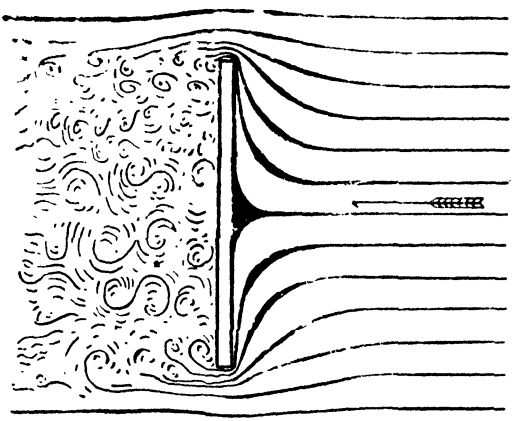


FIG. 177.

as the size of plate increases, the proportional effect of the eddy production increases slightly. Experiments show that for the normal motion of a submerged plane through still water, where the boundaries are so remote as not to affect the resistance, this is given by $k \frac{W a r^2}{g}$ lbs., where k is a coefficient depending on the size and shape of the plate, and diminishing slightly as the speed increases.

With a circular plate, k varies from about ·560 in a plate of 1 inch diameter to ·650 with a diameter of 3 inches and ·720 with a diameter of 6 inches, afterwards increasing slightly with the diameter. Dubuat and Duchemin obtained a mean value of ·717 for a plate 1 foot square moving through still water, k being ·50 for the front of the plate and ·217 for the

[1] See Art. 27, p. 63.

back. Experiment also tends to show that the resistance is somewhat greater when the plate is stationary in a moving current than when it is moved through still water. This is probably due to the fact that the water when in motion is in a much more unstable condition than when at rest, so that the sphere of influence of eddies projected from the boundaries of the plane is greater, and the consequent dissipation of energy is greater in the former than in the latter case.

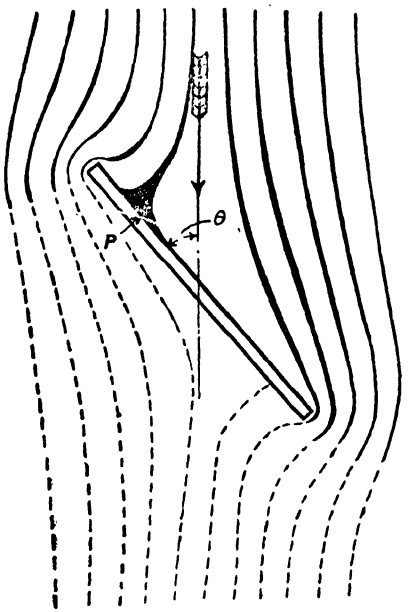


FIG. 178.

With a plate 1 inch square held in a moving current, Stanton[1] obtained a value of $k = \cdot 620$.

Submerged Plane in Pipe of Small Cross-Sectional Area.

The effect of placing the plane in a pipe of restricted area, is to increase the velocity with which the water escapes over the edge of the plane. This causes an increased eddy production, a consequent reduction of pressure on the rear of the plane, and hence an increased resistance.

Oblique Motion of Submerged Planes.

Where an oblique plane makes an angle θ, with its own direction of motion (Fig. 178), the velocity of the water normal to the plane $= v \sin \theta$.

The column of water whose momentum is affected, is now of sectional area $a \sin \theta$, so that the change of momentum produced by the passage of the plane is proportional to

$$\frac{W}{g} v \, a \sin \theta \, v \sin \theta, \text{ } i.e., \text{ to } \frac{W a}{g} v^2 \sin^2 \theta.$$

It might then be expected that the resistance to the oblique motion of a plane would equal

$$\text{Direct resistance} \times \sin^2 \theta.$$

Due, however, to the unequal eddy formation at the rear of the plate with the two kinds of motion, and to the fact that as θ is diminished,

[1] "Trans. Inst. N.A.," 1909.

frictional resistances form an increasingly large portion of the whole, this law does not hold. Lord Rayleigh,[1] indeed, showed that a rational expression for the normal pressure on the front face of the plane when moving through an infinite mass of fluid is given by

$$P = \frac{W}{g} \cdot \frac{\pi \sin \theta}{4 + \pi \sin \theta} \cdot v^2 \text{ lbs.}$$

this neglecting the effect of eddy formation.

From this we have, if P' = normal pressure on oblique plane, and if P = corresponding pressure on a normal plane,

$$P' = P \cdot \frac{(4 + \pi) \sin \theta}{4 + \pi \sin \theta} = P \frac{\sin \theta}{\cdot 56 + \cdot 44 \sin \theta}.$$

As the result of experiment M. Joessels, of the French Navy, deduced an empirical formula of this type, making

$$P' = P \frac{\sin \theta}{\cdot 39 + \cdot 61 \sin \theta},$$

while Duchemin deduced the relationship

$$P' = P \cdot \frac{2 \sin \theta}{1 + \sin^2 \theta}.$$

The following table shows results experimentally obtained by Colonel Beaufoy and by Stanton, and also some results of experiments by Vince quoted by Rayleigh, together with corresponding values as calculated by the $\sin^2\theta$ formula, and by those of Rayleigh, of Joessels, and of Duchemin.

Angle made by plane with direction of motion		90°	80°	70°	60°	50°	40°	30°	20°	10°
Ratio $\frac{P'}{P}$ experimentally obtained	Vince	1·00	—	·974	—	·873	—	·663	·458	·278
	Beaufoy	1·00	·915	·845	·828	·722	·579	—	·321	·272
	Stanton (square plane)	1·00	1·00	1·00	·97	·93	·915	·89	·59	·27
$\sin^2\theta$		1·00	·97	·88	·75	·587	·413	·250	·117	·030
$\frac{\sin \theta}{\cdot 56 + \cdot 44 \sin \theta}$ (Rayleigh)		1·00	·990	·965	·919	·854	·751	·641	·481	·273
$\frac{\sin \theta}{\cdot 39 + \cdot 61 \sin \theta}$ (Joessels)		1·00	·994	·975	·944	·894	·821	·715	·571	·351
$\frac{2 \sin \theta}{1 + \sin^2 \theta}$ (Duchemin)		1·00	1·00	1·00	·99	·965	·91	·80	·61	·34

[1] "Scientific Papers," I., p. 287, and III., p. 491; or see Lamb's "Hydrodynamics," p. 93.

The resistance in the direction of motion is given by $P' \sin \theta$.

Probably the experiments by Stanton are the more reliable, and for values of θ greater than 15°, the agreement of these results with those calculated by the formula of Duchemin, is very close.

As in the case of the normal plane, there is only one point P, this in the median plane of the plate, at which the velocity is zero and at which the maximum pressure $\frac{W v^2}{2 g}$ is therefore attained. This is in advance of the centre of the plane, and its distance from the centre in terms of the length l of the plane, as obtained theoretically, is given in the following table.[1] The distance $\bar{x}$ of the centre of pressure from the centre of the plane, is theoretically given by the formula $\bar{x} = \frac{3}{4} \cdot \frac{\cos \theta}{4 + \pi \sin \theta} \cdot l$,[1] and values of $\bar{x}$ are also tabulated below.

θ	90°	70°	50°	30°	20°	10°
$\bar{x}$	·000	·037 l	·075 l	·117 l	·139 l	·163 l
Distance of point where stream divides, from centre of plane . .	·000	·232 l	·402 l	·483 l	·496 l	·500 l

It is found, moreover, that with a rectangular oblique plane the total pressure for a given value of θ depends largely on whether the long or short edges of the plane are perpendicular to the stream, the resistance being greatest when the long edges are so placed. The reason for this is explained by Lord Rayleigh as follows. Although there is only one point of maximum pressure whatever the manner of presentation of the plane, yet with the long edges perpendicular to the stream the motion is approximately in two dimensions, and a region of almost maximum pressure extends over the greater part of the length. The case is very different, however, when the short dimension is perpendicular to the stream, for then, along the greater part of the length the flow is rapid and the pressure in consequence low.

This is of importance in the design of the wing surfaces of aeroplanes, since, area for area, a short (in the direction of motion) wide surface is con-

[1] From Lamb's "Hydrodynamics," p. 94.

siderably more efficient than a long narrow one, the total pressure approximating more nearly to $\frac{W a v^2}{2g}$ as the surface is made shorter and wider.[1]

The following table shows the ratios of P' to P experimentally obtained by Stanton on rectangular planes.

Angle of inclination . .	90°	80°	70°	60°	50°	40°	30°	20°	10°
Rectangle length = 2 breadth	1·00	1·01	1·02	1·00	1·11	1·12	·80	·45	·165
Rectangle length = ½ breadth	1·00	1·01	1·00	·973	·91	·77	·71	·71	·50

The maximum ratio, 1·15, for the former plane was attained when θ was 44°, while with the latter plane the pressure curve had a characteristic hump (ratio = ·74) when θ was 25°.

Art. 133.—Resistance to Motion of Submerged Bodies.

If the plane already considered forms the end of a solid prism whose axis is parallel to the direction of motion and whose length is more than about three times the diameter of the plane, the directions of the stream lines are modified as shown in Fig. 179. Eddies are still formed at C, after which the motion becomes parallel to the axis of the prism, until at B a second formation of eddies takes place.

The total eddy formation at B and C is now less than at B alone with a plane surface, and this results in a higher pressure on the rear of the prism than on the plane and a consequent smaller resistance to motion. Even including skin friction, with a moderate ratio of length to diameter the resistance of the prism is less than that of the plane. Putting the resistance equal to $k \frac{W a}{g} v^2$, we have $k = ·55$ with a prism having plane ends and a length of about three times its diameter. If fitted with a tapering cutwater the production of eddies at C is obviated and the value of k becomes about ·40 Fitted also with a tapering stern eddy formation

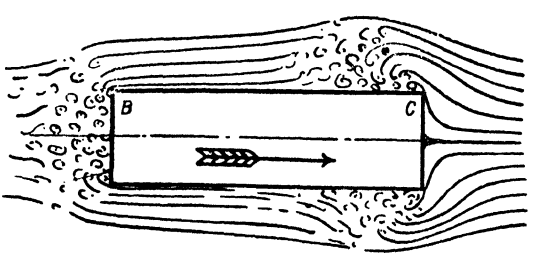


Fig. 179.

[1] For curves showing the effect of varying the manner of presentation of glider surfaces for aeroplanes, see Lord Rayleigh's paper, *ante laq.*

is largely prevented at B, and k is reduced to about ·125. The resistance is now largely due to skin friction and should be calculated on that assumption.

Experiments by Dubuat, on a square prism, section $a \times a$, length l, gave resistances in the following ratios:

$l \div a$	·03	1	2	3	6
k	·93	·78	·67	·66	·73

Art. 134.—Resistance of Ships.

The resistance to the motion of a ship is due mainly to skin friction, but also to the formation of surface waves and of eddies (chiefly at the stern). Mr. Froude found that although the velocities of gliding vary largely at different points of the hull of a ship, no sensible error is involved if the frictional resistance be calculated on the assumption that the wetted surface is equivalent to that of a plane of equal area and length in the direction of motion, and moving at the same speed. The frictional

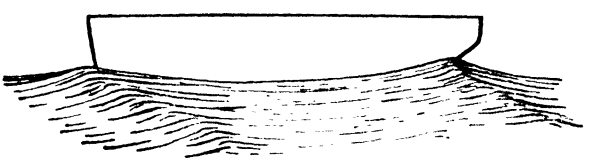


Fig. 180.

resistance of the ship may thus be calculated from Froude's experimental results on the resistance of plane surfaces (Art. 74).[1]

Wave production is a very complicated phenomenon, and depends in magnitude and in form largely on the form and speed of the boat. In general it may be taken that the motion is accompanied by the formation of bow and stern waves, and by a fall of the water surface amidships (Fig. 180). The fact that outward stream line flow must commence in front of the bows, and that the head necessary to produce this can only be obtained by a relative elevation of the surface level, accounts for the bow wave, while the reduction in velocity of the accompanying stream at the stern and the consequent increase in pressure accounts for the stern wave.[2] Also the

[1] See also Art. 36.

[2] For an article on the changes of level around a vessel, see the *Engineer*, vol. 63, p. 252.

increased velocity amidships is accompanied by a diminution in pressure, and accounts for the lowering of the surface level at this point.

Eddy formation, apart from that due to skin friction, is largely confined to the stern, and is due to flow in what is in effect a channel with diverging boundaries. Mr. Froude, in his experiments, found it necessary not only to use a cutwater, but also to taper off the stern of his planks in order to get a resistance — length curve, which should pass through the origin of co-ordinates, the effect being as indicated in Fig. 181.

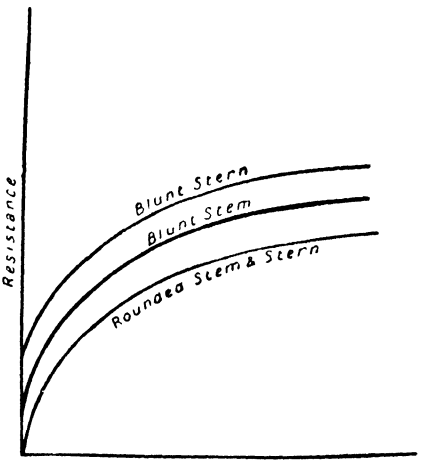


FIG. 181.

As a factor in reducing resistance in fact, a finely tapered stern or run is of much greater value in a ship-shaped body than is a fine stem. With a model as shown in Fig. 182, the directions of motion for least and greatest resistances are as indicated.

Experiments by Stanton[1] on the resistance of models having one end hemispherical and the other conical, give the following values of the relative resistance R.

Angle of convergence of conical end.	Method of application.	R.
12°	Hemispherical end first	·51
	Conical ,, ,,	·69
20°	Hemispherical end first	·59
	Conical ,, ,,	·76
40°	Hemispherical end first	·99
	Conical ,, ,,	1·00

[1] "Transactions Inst. Naval Architects," 1909.

Taking S to be the projected area of the stern, the reduction in pressure, due to eddy production is approximately proportional to v^2, and the resistance thus produced proportional to $S\,v^2$.

In general, however, skin friction accounts for the greater part of the total resistance, varying from 80-90 per cent. of the total for speeds of 6 to 8 knots, to 50 or 60 per cent. at very high speeds—this with clean bottoms. Eddy production, in a well-formed ship, accounts for 8-10 per cent. of the total, and wave-making resistance for the remainder.

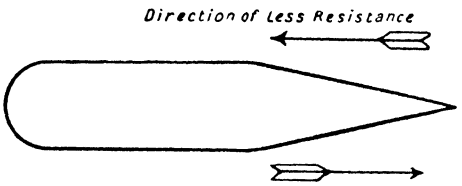


FIG. 182.

The total resistance, at speeds up to about 10 knots, is approximately proportional to v^2, while Sir William White[1] states that as the speed increases further, the power of the velocity to which the resistance is proportional increases to a maximum value of about 3, owing to the increasing magnitude of the wave-making resistance, and then falls again to a value slightly below 2. Thus in the case of an 80-foot boat exampled by Sir William White

The resistance up to 10 knots was approximately $\propto v^2$
" " at 13 " " " $\propto v^3$
" " " 17-18 " " " $\propto v^{1.9}$

while with the Iris—300 feet long,

the resistance up to 13 knots was approximately $\propto v^2$.
" " at 18 " " " $\propto v^{2.8}$.

Ship Model Experiments.—As indicated in Art. 36, the wave and eddy formation around a ship and its model are similar if their speeds are in the ratio $V_s : V_m = \sqrt{l_s} : \sqrt{l_m}$, and at these "corresponding speeds," their wave-making and eddy resistances are in the ratio $l^3_s : l^3_m . = D^3$, where D is the scale ratio.

Also, if S_s and S_m be the areas of their wetted surfaces, their skin frictional resistances are given by $f_s\ S_s\ V_s^n$ and $f_m\ S_m\ V_m^n$, where f_s, f_m and n can be deduced from Froude's experiments (Art. 74).

In determining the resistance of any proposed ship, a scale model is made usually of paraffin wax, and is towed through still water, the resistance corresponding to any speed being noted. Thus in Fig. 183,

[1] "Manual of Naval Architecture," by W. H. White, p. 466. London, John Murray.

$A\ A$ represents a resistance curve, the vertical ordinates from the line $O\ X$ representing to scale the total resistance of the model.

The area and length of its wetted surface being known, and the coefficient of friction determined, the frictional resistance can now be calculated for any speed, and the curve $B\ B$, of frictional resistances drawn.

If now the horizontal scale be increased in the ratio $\sqrt{D} : 1$, and the vertical scale in the ratio $D^3 : 1$, the curve $A\ A$ serves as the total resistance curve for the ship. Thus if the model is $\frac{1}{12}$ the size of the ship $D = 12$, the corresponding speeds of ship and model are in the

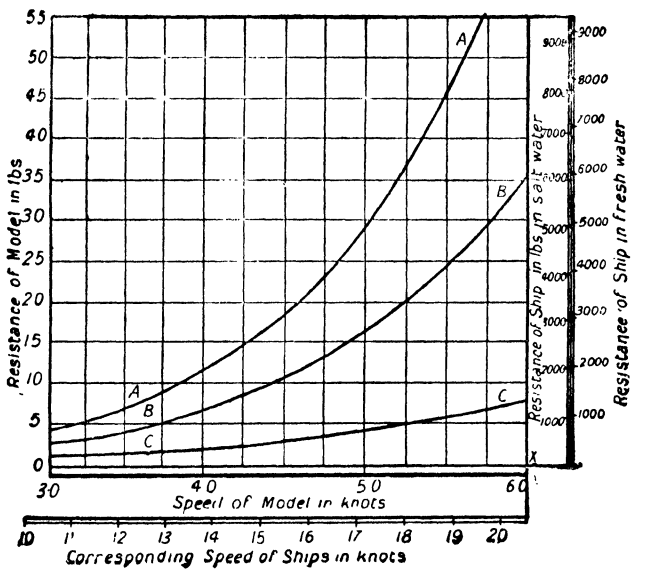


FIG. 183.

ratio $\sqrt{12} : 1 = 3{\cdot}461 : 1$, and at this corresponding speed the resistances are in the ratio $12^3 : 1 = 1728 : 1$.

If fresh water is used in the experimental tank the vertical scale is to be increased again in the ratio of the densities of salt and fresh water, while a further final correction is necessary because of the different values of f for the ship and model. This is got over by calculating the actual frictional resistance of the ship for her true length at various speeds, and by setting the values thus obtained as ordinates downwards from the curve $B\ B$, to form the curve $C\ C$. The true resistance of the ship at any speed is then given by the corresponding vertical intercept between $A\ A$ and $C\ C$.

The resistance at any given speed V_s, is calculated from model observations at the corresponding speed V_m, as follows:—

Total resistance of model (observed) $= R_m$ lbs.

Frictional ,, ,, ,, (calculated) $= f_m S_m V_m^n$. lbs.

$\therefore$ Wave-making and eddy resistance of model $= R_m - f_m S_m V_m^n . = E$ lbs.

$\therefore$ Wave-making and eddy resistance of ship in salt water $= D^3 E \times \frac{64}{62{\cdot}4}$. lbs.

$$\therefore \text{ Total resistance of ship} = \frac{64}{62{\cdot}4} D^3 E + f_s S_s V_s^n \text{ . lbs.}$$

A second method of carrying out tank experiments is due to Colonel English, and has certain advantages. Suppose the resistance of a ship in commission to be known at velocity V_1, that of another proposed ship to be determined at a velocity V_2. Make models of the two ships. Let suffix 1 refer to existing ship; suffix 2 to that to be built, and let prefixes s and m refer to ship and model respectively.

Let $\frac{{}_mV_1}{{}_sV_1} = \sqrt{\frac{{}_ml_1}{{}_sl_1}}$; $\frac{{}_mV_2}{{}_sV_2} = \sqrt{\frac{{}_ml_2}{{}_sl_2}}$; and let ${}_mV_1 = {}_mV_2$

i.e., make ${}_ml_1$ and ${}_ml_2$ satisfy the conditions that

$${}_sV_1 \sqrt{\frac{{}_ml_1}{{}_sl_1}} = {}_sV_2 \sqrt{\frac{{}_ml_2}{{}_sl_2}} \text{ or } \frac{{}_ml_1}{{}_ml_2} = \left(\frac{{}_sV_2}{{}_sV_1}\right)^2 . \frac{{}_sl_1}{{}_sl_2}.$$

If now the models be towed simultaneously, each attached to one of the ends of a horizontal lever with an adjustable fulcrum, the position of this fulcrum may be readily adjusted until the lever floats in its mid position, and the relative resistances of the two models are then in the ratio of the lengths of the arms of the lever. Let resistance of model (2) be n times that of model (1).

Let E represent wave-making and eddy resistances; F = skin friction.

Thus resistance of model (2) $= {}_mE_2 + {}_mF_2 = n \{{}_mE_1 + {}_mF_1\}$

But $${}_mE_2 = {}_sE_2 \left(\frac{{}_ml_2}{{}_sl_2}\right)^3$$

$$\therefore \quad {}_sE_2 \left(\frac{{}_ml_2}{{}_sl_2}\right)^3 = n \left\{ {}_sE_1 \left(\frac{{}_ml_1}{{}_sl_1}\right)^3 + {}_mF_1 \right\} - {}_mF_2$$

$\therefore$ Total resistance of new ship is given by

$${}_sR_2 = {}_sE_2 + {}_sF_2 = {}_sF_2 + \left(\frac{{}_sl_2}{{}_ml_2}\right)^3 \left[n \left\{ {}_sE_1 \left(\frac{{}_ml_1}{{}_sl_1}\right)^3 + {}_mF_1 \right\} - {}_mF_2 \right]$$

Of the terms in this expression ${}_sF_2$, ${}_mF_1$, and ${}_mF_2$ can be readily calculated, while ${}_sE_1$ being equal to ${}_sR_1 - {}_sF_1$, can also be obtained, since the total resistance ${}_sR_1$ of ship (1) is known, while its frictional resistance ${}_sF_1$ can be readily calculated.

Art. 135.—The Propulsion of Ships.

Of the many systems of mechanical propulsion which have from time to time been devised, only three have attained any measure of success. These respectively use the paddle wheel, the screw propeller, and the hydraulic jet as their propelling agents.

The principle underlying all these systems is the same, the propeller being devised so as to create a sternwards current of water, and the corresponding reaction on the propeller providing the propelling force on the ship. Thus, whatever system be adopted, if a mass of water of weight C lbs. per second have its sternward velocity increased by v feet per second by the action of the propeller, the change of momentum per second in this direction $= \frac{C\,v}{g}$ foot-pound units, and the thrust on the propeller is of the same magnitude, $\frac{C\,v}{g}$ lbs., and in the opposite direction.

Since the thrust varies as the product of C and v, any one of these factors may be changed without affecting the thrust if the other factor suffer a corresponding change, so that a given thrust may be obtained by the comparatively slow movement of a large mass of water, or by the quick movement of a smaller mass. Other things being equal, then, an increase in the area of the moving stream and a diminution in its velocity will tend to efficiency by reducing shock and kinetic losses.

While this conclusion is verified in practice, in the case of the paddle wheel and of the hydraulic jet (see Art. 136) practical considerations soon limit the maximum possible area (sooner in the case of the jet than of the paddle wheel). In the case of the screw propeller, moreover, frictional losses rapidly increase with the area of the screw, while an attempt to increase the area by increasing the radius, increases the centrifugal action of the water, and by reducing the pressure near the centre of the propeller, tends to reduce the effective thrust.

Paddle Wheels.—These give excellent results in smooth water, and where the draught and therefore the depth of immersion of the paddles is not likely to vary largely. They are well adapted for river navigation, particularly for shallow draught vessels, but are quite unfitted for use in

any ocean-going boat which may be exposed to rough weather, and whose draught may differ by some feet on the outward-bound and homeward-bound journeys.

If R = mean foot radius,

„ N = number of revolutions per second,

then $2\pi R N = V_P$ = velocity of paddles relative to the ship in feet per second.

If V_s be the absolute velocity of the ship, $(V_P - V_s)$ is termed the slip, or more correctly the apparent slip, and $100\,\frac{V_P - V_s}{V_P}$ is termed the percentage slip. This in general varies from 20 to 30 per cent. The chief losses are due to shock at entrance and exit from the water and are reduced as far as possible by the use of feathering floats arranged so as to enter the water without shock. Where, as is usual, the paddle wheels are placed amidships, the stream-line motion over the run of the ship is greatly affected, at least over those layers between the surface and the bottom of the wheel, and since the relative velocity of flow over this portion of the ship is largely increased, the effect is to augment the resistance as compared with the towing resistance with paddles removed.

The Screw Propeller is by far the most important of the three types. Its advantages consist in its possibilities of adoption to high speed prime movers; its depth of immersion, which makes its efficiency largely independent of a varying draught, and its reduced liability to accident, in virtue of its sheltered position.

If P = mean pitch of propeller blades, *i.e.* the distance the boat would travel per revolution if the screw were supposed to rotate in a fixed solid nut, and if V_p is the corresponding axial velocity so that $V_p = N\ P$, then $V_p - V_s$ is usually termed the slip, or more correctly the apparent slip of the screw, and, if this worked in still water, would give the sternward velocity in the propeller race relative to the surrounding water. The real slip is the change actually produced by the screw on the velocity of the water in the propeller race, and since the propeller works in an accompanying current of water which has an initial absolute velocity V_w in the direction of the ship, this is given by $(V_p - V_s + V_w)$.

Evidently the apparent is always less than the true slip, and may, if V_w is sufficiently great, become negative. In general, however, it is positive and, expressed as a percentage of V_p, has a value of about 20 per cent.

The real slip is always positive if the screw is to have any propelling effect, as will be evident if it be remembered that it measures the change in the sternward momentum of the water.

Owing to the centrifugal action of the water, the pressure over the front face of the propeller is negative, so that part of the total change of velocity occurs before the water actually reaches the propeller. There is thus a tendency to draw water from the stern of the vessel and to reduce the forward pressure over this portion of its surface, with the result that the resistance, as compared with the towing resistance, is considerably augmented.

The velocity with which water can be supplied to the screws is entirely dependent on the atmospheric pressure augmented by the head of water over the screw. Should this be insufficient to give the necessary velocity of inflow the column of water in front of the screw is broken and "cavitation" is caused, just as in a pump which is being driven at too high a speed. This may be prevented either by increasing the depth of immersion or by reducing the speed of the propeller.

The maximum efficiency of the propeller would appear to be about 75 per cent., the magnitude of the various losses then probably approximating to the following values:—

Friction losses	9 per cent.
Eddy losses	10 „
Losses by kinetic energy rejected in wake .	6 „

Assuming an efficiency for the engine and shafting of 80 per cent. this gives a combined efficiency of 60 per cent.

In spite of much experimental work and theoretical investigation, very little is actually known as to the best design of screw for any given conditions. Mr. Froude, after an extended series of experiments, concluded that the mean effective angle of the screw blade should be 45°, this making the pitch equal to about twice the extreme diameter, and that the true slip should be about $12\frac{1}{2}$ per cent. The blade area, projected on a plane perpendicular to the axis of the screw should then be equal to $\frac{8{\cdot}9\ R}{V_s^2}$ square feet, where R = resistance of ship in pounds at the maximum velocity V_s.[1]

[1] For a further investigation into the action of the propeller the reader is referred to a paper on the action of propellers by Professor Rankine, "Transactions Inst. Naval Architects," vol. 6; also papers by Mr. Froude in vols. 6, 8 and 19 of the same Transactions, and to White's "Naval Architecture," p. 543. Also to papers in the "Proc. Inst. C. E.," vol. 102, p. 74; vol. 122, p. 51; vol. 165, p. 293; and in recent "Transactions Inst. Naval Architects."

Power Necessary for Propulsion.

If R = resistance of ship in lbs., and V_s its velocity in feet per second we have

Work done in propelling vessel = $R\ V_s$ foot-pounds per second.

$\therefore$ Horse-power to overcome resistance = $\frac{R\ V_s}{550}$.

The I.H.P. of the ship's engines will be considerably more than this, because of mechanical friction losses in the engine and propeller shaft and, to a still greater extent, because of the inefficiency of the propeller itself. In general it may be taken that from 45 per cent. to 60 per cent. of the energy developed in the engine cylinders is utilised in doing useful work.

Since R is approximately proportional to V_s^2, the horse-power will vary as V_s^3, except at very high speeds, where the wave-making resistance becomes abnormal.

Art. 136.—Jet Propulsion.

When a jet of water escapes from an orifice in the side of a vessel, the force necessary to produce this outflow is equal in magnitude to the flux of momentum per second across the *vena contracta* of the jet, and acts in the direction of flow of the jet. This force necessitates an otherwise unbalanced pressure or reaction on the side of the vessel opposite to the orifice, which reaction tends to move the vessel in the opposite direction to that of the jet.

Advantage has been taken of this in a system which has been applied (though with only moderate success) to the propulsion of large vessels. Here a supply of water is drawn into the boat, usually through a vertical pipe opening amidships, by a centrifugal pump driven by the main engines, and is then discharged directly astern. The escaping stream, in virtue of its reaction on the vessel, does work in propulsion, while the theoretical efficiency of the system may be calculated as follows:—

Let A = sectional area of discharge orifices in square feet.

u = velocity of vessel in feet per second.

v = velocity of efflux of stream relative to boat.

Then $v - u$ = absolute velocity of efflux of stream, *i.e.*, its velocity relative to the surrounding water.

The initial velocity of the water before being drawn into the vessel $= 0$

Its final velocity in the direction of motion of the vessel $= u - v$ ft. per sec.

The weight of water discharged per second $= W A v$ lbs.

$\therefore$ The change of momentum of the water in the opposite direction to that of motion of the vessel $= \frac{W A v}{g}(v - u)$ per sec.

$\therefore$ Propelling force on boat $= \frac{W A v}{g}(v - u)$ lbs. (1)

Work done in propulsion per second $= \frac{W A v u}{g}(v - u)$ ft. lbs. (2)

Kinetic energy rejected per second in the discharge $= \frac{W A v}{2 g}(v - u)^2$ ft. lbs. (3)

$\therefore$ Total energy given to water per second neglecting eddy and frictional losses $= \frac{W A v}{2 g}\left\{(v - u)^2 + 2 u (v - u)\right\}$ ft. lbs.

$$= \frac{W A v}{2 g}(v^2 - u^2) \text{ ft. lbs.} \quad (4)$$

$\therefore$ Theoretical efficiency of jet $= \frac{\text{useful work done by jet}}{\text{energy given to jet}} = \frac{2 u (v - u)}{v^2 - u^2}$

$$= \frac{2 u}{v + u}. \quad (5)$$

This has its maximum value, unity, when $v = u$.

An examination of equation (1) will, however, show that under these circumstances the propelling force is zero, and that with a large propelling force it is impossible to work under conditions which conduce to high efficiency.

Water may be taken into the pumps either through one or more vertical or horizontal pipes at right angles to the axis of the vessel, through pipes facing forwards in the direction of the axis, or through vertical or horizontal pipes fitted with a scoop facing in the direction of motion of the vessel. In the latter case the effect is the same as with the inlet pipes facing forwards except that the scoop introduces a slight additional resistance to motion. If h be the depth of the inlet, the head producing flow along the inlet pipe is given by $h + \frac{u^2}{g}$ feet in the two latter cases, and by h feet

if the inlet opens flush with the sides or bottom of the boat, and since in every case the delivery pressure is that corresponding to the depth of immersion of the outlet, less energy is required of the pump per lb. of water to discharge at a given velocity with the inlet facing forwards than with a plain side or bottom inlet. In any case the total change of momentum per second is the same provided the discharge velocities are the same.

ART. 137.—VARIATION OF EFFICIENCY WITH SIZE OF ORIFICES.

Assuming the energy of propulsion to be given by $k\,u^3$ ft. lbs. per second (p. 397), where k is constant for a given vessel,

$$k\,u^3 = \frac{A\,W}{g}\,u\,v\,(v-u)$$

$$\therefore\quad k\,u^2 + u\,.\,\frac{A\,W}{g}\,v - \frac{A\,W}{g}\,v^2 = 0$$

or
$$u = -\frac{A\,W}{2\,g\,k}\,v\left\{1 - \sqrt{1 + \frac{4\,g\,k}{A\,W}}\right\}.$$

Substituting this value of u in equation (5), we have

$$\text{Efficiency} = \frac{2\,u}{u+v} = \frac{2\left\{1 - \sqrt{1 + \frac{4\,g\,k}{A\,W}}\right\}}{1 - \sqrt{1 + \frac{4\,g\,k}{A\,W}} - \frac{2\,g\,k}{A\,W}}$$

Multiplying numerator and denominator by $1 + \sqrt{1 + \frac{4\,g\,k}{A\,W}}$ this finally gives

$$\text{Efficiency} = \frac{4}{3 + \sqrt{1 + \frac{4\,g\,k}{A\,W}}},$$

so that the efficiency of propulsion increases as the ratio $\frac{A}{k}$ increases, *i.e.*, in the case of any given boat, as the area of the orifices increases. But as the quantity discharged per minute, and also the size of pump necessary to give this discharge increase with A, the limit of efficiency in this direction is soon reached.

EXAMPLE

If $k = 2{\cdot}4$ and if $A = 2$ square feet, we have efficiency of jet

$$= \frac{4}{3 + \sqrt{1 + \frac{4{\cdot}8}{2}}} = \frac{4}{4{\cdot}85} = 82{\cdot}5 \text{ per cent.}$$

Suppose the boat to travel at 15 miles per hour, $u = 22$ feet per second.

$$\text{Then efficiency} = \frac{2u}{u+v} = \cdot 825$$

$$\therefore \quad 1\cdot 175\, u = \cdot 825\, v$$

$$\therefore \quad v = \frac{1\cdot 175 \times 22}{\cdot 825} = 31\cdot 33 \text{ feet per second.}$$

$$Q = 31\cdot 33 \times 2 = 62\cdot 66 \text{ cubic feet per second.}$$

Assuming the combined efficiency of engine, pump, and jet to be $\cdot 80 \times \cdot 60 \times \cdot 825 = \cdot 396$, we have :—

$$\text{I.H.P. of engine} = \frac{k\, u^3}{550} \times \frac{1}{\cdot 396}$$

$$= \frac{2\cdot 4 \times 22^3}{550 \times \cdot 396} = 117 \text{ I.H.P.}$$

The most noteworthy experiments on hydraulic propulsion have been carried out by the British Government. In 1866 the Admiralty built two almost similar gunboats, the *Waterwitch* and the *Viper*, the former being fitted with jet propulsion and the latter with a screw propeller. Their displacements were, *Waterwitch* 1,161 tons, *Viper* 1,180 tons. The *Waterwitch* took water in through a vertical opening amidships, passed it through a 14-foot centrifugal pump, and discharged astern through two 24-inch nozzles. When discharging 5·2 tons of water per second with a relative velocity of 29 feet per second, the engines indicated 760 I.H.P., and gave a speed of 9·3 knots (15·71 feet per second) with a jet efficiency of 70 per cent. The *Viper*, with 696 I.H.P., gave 9·58 knots. This comparison is, however, rather unfair to the *Viper*, as its speed suffered from the provision of a double bilge keel, and from its slightly fuller run.

In 1878, the Swedish Government built two torpedo boats, 58 feet long —10′ 9″ beam—and with 20 and 21 tons displacement, the heavier machinery of the hydraulic boat necessitating the larger displacement. The latter boat took in water vertically, and with 78 I.H.P. gave a speed of 8·12 knots. The screw boat with 90 I.H.P. gave 10 knots.

In 1882, the Admiralty had one of a batch of torpedo boats fitted with hydraulic propulsion.

The details of the boats were as follows :—

	Length.	Beam.	Draught.	Displacement.	I.H.P.	Speed.
Screw .	63·0 ft.	7′ 6″	3′ 8½″	12·89 tons	170	17·3 knots
Hydraulic .	66·4 ft.	7′ 6″	2′ 6″	14·4 tons	167	12·6 knots

The hydraulic boat was fitted with a centrifugal pump 2′ 6″ diameter, running at 428 revolutions per minute, and discharged astern through two 9-inch outlets. The pump delivered approximately one ton of water per second at a velocity of 37·25 feet per second (as against 21·28 feet per second of the boat), and was fitted with a vertical inlet pipe, provided with a scoop facing forward.

In each case the efficiency of the pump was approximately 48 per cent. The last-mentioned torpedo boats gave the highest jet efficiency, viz., 72·8 per cent. The combined efficiency of jet and pump was then 34·9 per cent. as compared with an average efficiency of about 65 per cent. in the case of a screw propeller. Taking an efficiency of 80 per cent. for the engine and shafting in each case, though this would be slightly greater in the case of the pump, we have the combined efficiency of propelling mechanism

Hydraulic.	Screw.
27·9 %	52·0 %

The low efficiency of the centrifugal pump has, in the past, prevented the adoption of the system of jet propulsion on any large scale. As a propeller, the jet itself is more efficient than the screw, and there would appear to be no valid reason why, with the more efficient modern types of pump, the combination should not be made as efficient as the screw propeller.[1]

It should be noted that the system works equally well with the discharge either above or below water level, except that in the former case energy is wasted in lifting the water to the level of the discharge orifice. The advantages of the system are

(1) No racing of propellers in rough weather;
(2) No under water obstacles to become entangled in wreckage;
(3) Ease of control from some central station on the bridge;
(4) Great facilities for manœuvring.

Examples.

(1) A horizontal jet issuing from an orifice 1 inch in diameter discharges 2 gallons per second and impinges normally on a large fixed

[1] See *The Engineer*, Feb. 11, 1921, p. 140, for the results of some tests.

plane. Determine the force exerted on the plane and also, neglecting losses, the maximum pressure intensity likely to be attained.

Answer. { 36·5 lbs.
23·25 lbs. per square inch.

(2) If the above plate has edges curved so as to deflect the stream through a total angle of 150°, determine the pressure on the plate.

Answer. 68 lbs.

(3) A jet from a rectangular orifice 6″ wide × 1″ deep impinges on a curved vane 6″ wide, with a velocity of 60 feet per second. On incidence the jet makes an angle of 30° with the plane of the vane, and the portions into which it is divided are deflected through total angles of 120° (the smaller portion) and 90° (the larger portion). Determine the pressure on the plane in the direction of the jet.

Answer. 300 lbs.

(4) Assuming a series of vanes similar to that in question (3) to move in the direction of the jet with a velocity of 30 feet per second, determine the efficiency of the arrangement.

(5) Two free jets of sectional area 2 square inches and 1 square inch, and of velocities 20 f.s. and 30 f.s., are inclined to each other at an angle of 60°, and combine to form a single stream. Determine the direction and velocity of this stream and the loss of energy at impact.

Answer. { Resultant stream makes an angle of 31° 58′ with direction of first stream.
Velocity = 21·06 feet per second.
Loss of head = 80·3 feet lbs. per second.

(6) A jet propelled vessel has a wetted surface area of 800 square feet, and a coefficient of resistance of ·004. If the area of the nozzle openings is 3·0 square feet, determine the maximum theoretical efficiency of the jet considered as a propeller. Also, assuming the pump efficiency to be 50 per cent. and the mechanical efficiency of the engines to be 85 per cent., determine the efficiency of propulsion.

Answer. { Efficiency = ·836.
Total efficiency = ·355.

(7) If the boat of the preceding example is travelling at 12 miles per hour, determine the necessary jet velocity and the discharge per second, for this efficiency.

Answer. { Velocity = 24·56 feet per second.
Discharge = 2·05 tons per second.

(8) Water flows at 20 feet per second through a 6″ pipe forming a right-angled bend. Find the magnitude of the resultant force tending to move the pipe.

Answer. 215 lbs.

(9) A jet delivering 3 cubic feet of water per second at 100 feet per second impinges tangentially on one of a series of vanes moving at 50 feet per second in a direction inclined at 20° to that of the jet. If the vanes deflect the jet backwards so that its final direction of motion is the same as that of the vane, and if the final relative velocity of jet and vane is ·8 times the initial relative velocity, determine the pressure on the vane in its direction of motion.

Answer. 515 lbs.

(10) Experiments on a 20-feet model are carried out to determine the probable horse-power required in a ship 600 feet long when doing 24 knots. Determine the corresponding speed of the model, and, assuming the value of f for the ship to be ·00910 and that for the model to be ·00935. determine the probable horse-power of the engines if the resistance of the model at the corresponding speed is 9·54 lbs. Assume 60 per cent. of the I.H.P. to do useful work at the propeller, and assume wetted area of ship = 48,000 square feet; n for ship = 1·83; for model 1·85.

Answer. { Corresponding speed = 4·38 knots.
I.H.P. = 24,600.

CHAPTER XIII.

Waves—of Transmission—of Oscillation—Ripples

Art. 138.—Waves.

Waves may be divided into three main classes according as they are

(*a*) Waves of Transmission.

(*b*) Waves of Oscillation.

(*c*) Ripples.

In a **Wave of Transmission**, not only does the wave form advance, but each particle of water over which the wave passes is translated forward during its passage and is finally left at some distance ahead of its original

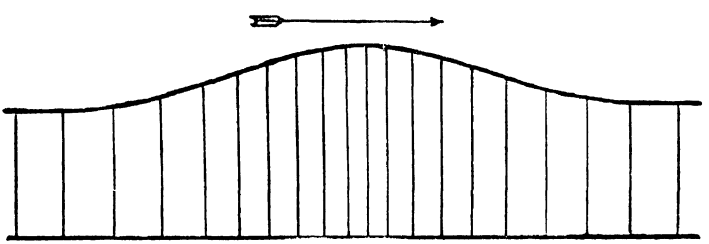


Fig. 184a.

position. Perfect waves of this type are solitary and the wave itself is either wholly raised above or wholly depressed below the general surface of the fluid. In the former case it is termed a positive; in the latter case a negative, wave. Such positive waves are formed by the sudden opening of a sluice gate admitting a body of water into a long, level canal, or may be formed in a long trough by the sudden immersion at one end, of a solid body of volume equal to that of the required wave. The negative wave of transmission is very unstable, and always gives rise to a train of waves of oscillation.

Method of Transmission.—If the water when at rest be supposed subdivided by a series of equidistant vertical planes, perpendicular to the direction of the wave's transmission, the columns which they enclose suffer the distortion shown in Fig. 184a, during the passage of the wave As the front of the wave approaches one of these columns the pressure on

the plane forming its rear face is increased, and under this pressure the plane moves forwards, heaping up the water ahead of it. When the column has been raised to its greatest height, at the crest of the wave, it presses equally on both front and rear planes, accelerating the velocity of the column ahead, retarding that behind, and so increasing its own width until it finally comes to rest at its original level. Each successive column is in turn displaced forward in the direction of transmission, and the result is that a volume of water equal to that of the wave is displaced in this direction. The motion of each particle is very approximately a semi-ellipse, with its major axis horizontal, horizontal displacements being uniform, and vertical displacements proportional to the distance above the bottom of the channel.

Velocity of Propagation.—Imagine the wave propagated in a stream

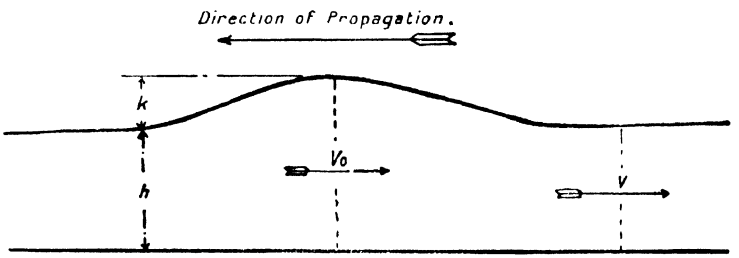


FIG. 184B.

running in the opposite direction with such uniform velocity V as to keep the wave form stationary relative to the banks.

Then if h be the original depth of water; k the height of wave; V_o the velocity at the crest of the wave (Fig. 184B)

we have $$V_o = V \frac{h}{h+k}$$

Applying Bernoulli's equation to the surface filaments where the pressure is constant

$$h + k + \frac{V_o^2}{2g} = h + \frac{V^2}{2g}$$

$$\therefore \frac{V^2}{2g}\left\{1 - \left(\frac{h}{h+k}\right)^2\right\} = k$$

$$V^2 = 2g\frac{(h+k)^2}{2h+k} = 2g \cdot \frac{h+k}{1+\frac{h}{h+k}}$$

Usually k is small compared with h, in which case

$$V = \sqrt{g\,(h + k)} \text{ (approx.)}$$

and if k is comparatively very small, this approximates to

$$V = \sqrt{g\,h}$$

Since the distribution of pressure or motion is not altered by the

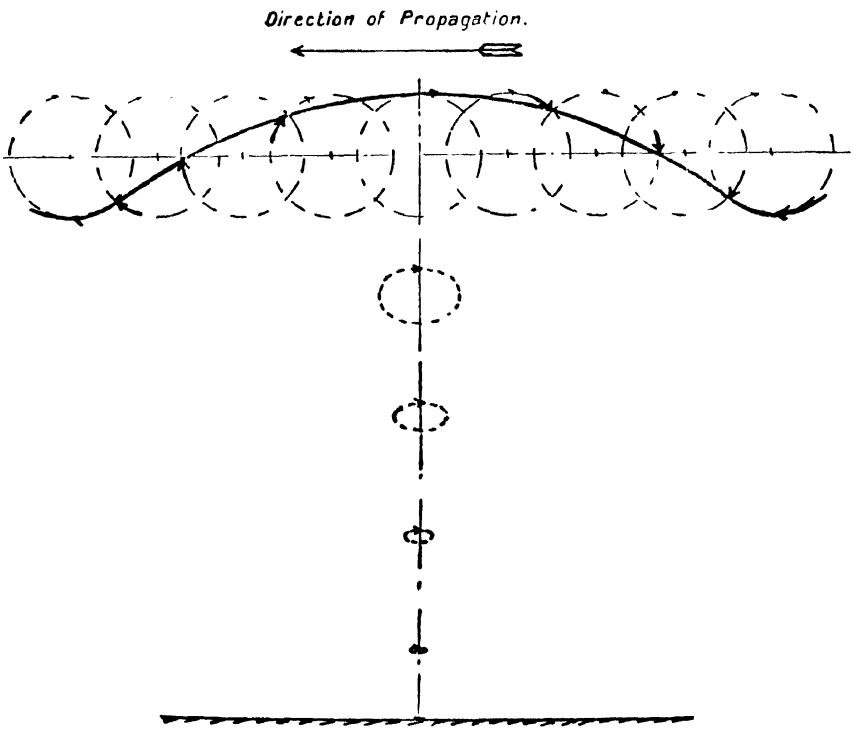


FIG. 184c.—Wave of Oscillation.

motion of the stream, V is the same as the velocity of propagation of the wave form in still water.

The maximum velocity of horizontal translation of the particles forming the wave is given by $V - V_0$, *i.e.*, by $V\,\frac{k}{h + k}$.

Experiment shows that as k approaches h in value, the wave breaks at the crest.

ART. 139.—WAVES OF OSCILLATION.

In waves of oscillation, as exemplified in the ordinary deep sea wave, the particles involved in the motion are not transported in the direction

of propagation, but oscillate with uniform angular velocity about a mean position, in a vertical plane parallel to that direction. In deep water the orbit of a particle near the surface is sensibly circular, and the wave form is trochoidal. In shallower water the orbit becomes approximately elliptical with the major axis horizontal. In either case the particles in the surface have the largest orbits, the extent of the motion diminishing as the depth below the surface increases. The vertical motion diminishes more rapidly than the horizontal motion, so that the deeper a particle the flatter is its orbit. In deep water the maximum diameter of the orbit diminishes in geometrical progression as the depth below the surface increases in arithmetical progression. The following table[1] gives the maximum horizontal and vertical displacements of the particles at different depths for waves whose lengths are respectively equal to, and ten times as great as the depth of the water they are traversing, the greatest horizontal displacement of the particles at the bottom being represented by unity.

Depth below surface.	Length of wave = depth of water.		Length of wave = 10 times depth.	
	Horizontal displacement.	Vertical displacement.	Horizontal displacement.	Vertical displacement.
At surface . .	267·7	267·7	1·204	·670
·10 depth . .	142·8	142·8	1·164	·596
·20 „ . .	76·2	76·2	1·129	·524
·30 „ . .	40·6	40·6	1·098	·454
·40 „ . .	21·7	21·2	1·072	·386
·60 „ . .	6·2	6·1	1·031	·254
·80 „ . .	1·9	1·6	1·008	·126
At bottom . .	1·0	0·0	1·000	·000

The motion of the particles in a wave of oscillation in fairly deep water is shown in Fig. 184c. Those particles in the crest of the wave move forward in the direction of propagation; those in the trough move backwards; while those at the mean level have simply vertical motion.

Velocity, Length, and Height of Waves of Oscillation in Deep Water.—Taking the surface particles to describe circular orbits of radius r with

[1] Airy, "Tides and Waves," Table IV.

angular velocity ω, the height of the wave is $2r$. Let l be its length, measured from crest to crest. Imagine the wave form brought to rest by imposing on it a velocity equal and opposite at every point to that of its normal motion, *i.e.*, imagine the orbital circle concentric with a circle of radius R, where $2\pi R = l$, rolling, as shown in Fig. 184D, on a horizontal plane with angular velocity ω, and with a linear velocity V equal to that of propagation. Then $V = \omega R = \dfrac{\omega l}{2\pi}$.

Under these conditions the velocity of a particle at Q is compounded of a velocity $\omega . OK$ perpendicular to OK, and $\omega . OQ$ perpendicular to OQ,

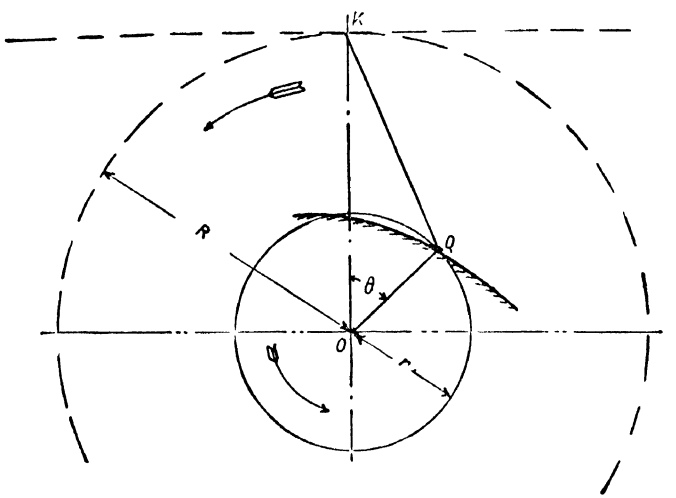


FIG. 184D.

and so equals $\omega . KQ$ perpendicular to KQ, so that KQ is normal to the surface at Q. If now v_c be the velocity of a particle in the crest, and v the velocity at any other point Q in the free surface, and therefore at the same pressure,

$$\therefore \quad \frac{v^2}{2g} + r\cos\theta = \frac{v_c^2}{2g} + r$$

$$\therefore \quad \frac{\omega^2}{2g}\{\overline{QK}^2 - (R-r)^2\} = r(1-\cos\theta).$$

But

$$\overline{QK}^2 = (R - r\cos\theta)^2 + (r\sin\theta)^2$$
$$= R^2 - 2Rr\cos\theta + r^2$$

and on substituting this value and simplifying we have—

$$\left\{ \frac{\omega^2 R}{g} - 1 \right\} r(1 - \cos\theta) = 0 \qquad (1$$

$$\therefore \quad \omega = \sqrt{\frac{g}{R}}$$

$$\therefore \quad V = \omega R = \sqrt{g R}$$

$$= \sqrt{\frac{g l}{2\pi}}$$

i.e., in deep water the velocity is independent of the height of wave and of the depth of water, and depends only on the length of wave. This is true, within small limits, so long as the depth of water is greater than one-half the length of the wave.

In shallow water the velocity of propagation becomes equal to $\sqrt{\frac{g l}{2\pi} \cdot \frac{b}{a}}$ where a and b are the horizontal and vertical axes of the elliptical orbit, and vary with the depth of water. In waves which are very long compared with the depth, the velocity approximates to $\sqrt{g h}$, where h is the depth, so that as the water becomes shallower and shallower the velocity approximates more nearly to that of the wave of translation. The wave of oscillation in shallow water is indeed intermediate between the true wave of oscillation and the wave of translation.

There is no definite relationship between the length and height of waves. Both depend largely on the velocity of the wind which has raised them, and on the distance travelled under its influence. Where this distance is great the velocity is practically identical with the mean velocity of the wind.[1] For short "fetches," such as are usual in reservoirs, Stevenson gives the formula.

$$k = 1{\cdot}5\sqrt{D} + 2{\cdot}5 - \sqrt[4]{D}$$

where k is the height of wave in feet, and D is the "fetch" in miles. As deep sea waves approach a shallowing beach, the orbital motion of their lower particles is checked, they partake to an increasing extent of the nature of waves of translation, and finally usually break when the depth of water is little more than the height of the wave.

Art. 140.—Ripples.

So far it has been assumed that the only forces involved in wave formation and propagation are those due to the inertia of the particles of

[1] Dr. Vaughan Cornish, "Cantor Lecture," 1912.

water. In large waves this is sensibly true, but in small waves, the curvature of whose surface is comparatively great, the modifying effect of the forces called into play by surface tension becomes appreciable, and in very small waves becomes the predominating factor. The effect of surface tension is to increase the sub-surface pressure under the crests and to reduce that under the troughs of the waves. If T be the surface tension in lbs. per linear foot of surface, and if ρ be the radius of curvature at any point, the resultant increase or diminution of pressure in feet of water is given by $\frac{T}{W\rho}$ feet.

Assuming the wave to be of trochoidal form, the horizontal and vertical displacements of a particle since leaving the crest are given by—

$$x = R\theta \text{ (approx.)}: \quad y = r(1 - \cos\theta).$$

$$\therefore \quad \frac{1}{\rho} = \frac{d^2y}{dx^2}\text{(approx.)} = \frac{r}{R^2}\cos\theta.$$

If the pressures at the crest and at any other point Q (Fig. 18 D) are not equal, but are respectively p_c and p lbs. per square foot, we have :—

$$\frac{v^2}{2g} + r\cos\theta - \frac{v_c^2}{2g} - r = \frac{p_c - p}{W}$$

$$= \frac{T}{W}\left(\frac{1}{\rho_c} - \frac{1}{\rho}\right)$$

$$= \frac{T}{W}\frac{r}{R^2}(1 - \cos\theta)$$

But $\frac{v^2}{2g} + r\cos\theta - \frac{v_c^2}{2g} - r = \left(\frac{\omega^2}{g}R - 1\right) r(1 - \cos\theta)$ (1) p. 409.

$$\therefore \quad \omega^2 = \frac{g}{R} + \frac{Tg}{WR^3}$$

$$\therefore \quad V^2 = gR + \frac{Tg}{WR}$$

$$= \frac{gl}{2\pi} + \frac{2\pi g}{Wl}\frac{T}{}.$$

For water at 60° F, T = ·005 lbs. per foot (approx.), and on substituting this value, the quadratic becomes

$$l = \cdot 0975\ V^2 \pm \sqrt{\cdot 0095\ V^4 - \cdot 00324}.$$

For this to have real roots V^4 must be not less than ·342, and V not less than ·765 f.s., or 9·2 inches per second. When V has this value, l = ·67 inches.

For higher values of V, l has two values, one less and one greater than ·67. The greater value represents the length of the ordinary wave

of oscillation in which surface tension plays a comparatively small part, while the smaller value represents the length of a wave in which surface tension is the predominating factor. Such waves are called ripples. No wave or ripple can be produced by a moving object whose velocity is less than this critical value of 9·2 inches per second, and a breeze of velocity less than this has no power to ruffle the surface of a still pool.

SECTION III.

CHAPTER XIV.

Hydraulic Prime Movers—Impulse Wheels and Pressure Turbines—The Pelton Wheel—Losses—Form and Number of Buckets—Speed Regulation—Jets from Needle-nozzles—The Girard Turbine—The Turgo Impulse Turbine.

ART. 141.—HYDRAULIC PRIME MOVERS.

ALTHOUGH the question of the utilization of natural water powers has always been one of great economic importance, the introduction and perfection of electrical manufacturing processes, and the possibility of transmitting electrical energy without great loss or expense to a great distance from the place of its generation, has of recent years made it practicable to take advantage of many water powers far remote from large centres of industry and has raised the whole question to an altogether higher plane of importance, while the consequent demand for hydraulic prime movers capable of developing large powers in single units, and of satisfying the exhaustive demands of such installations in the way of speed regulation and efficiency, has led to a great transformation in the design of such motors.

The first hydraulic prime mover consisted of a wooden paddle water-wheel dipping into the current of a stream, and as such a motor was only required to do the work previously performed by an animate agency, the power required was small and the efficiency of only secondary importance.

The construction was at first of the most primitive type, but was gradually improved; iron took the place of wood; improvements in design led to increased efficiency; the demand for greater powers led to the necessity for utilizing larger falls and the consequent development of the breast and overshot wheels, until a type of wheel was evolved, which within its limitations was as efficient as the most modern of turbines. Its chief disadvantages lay in its slow speed of rotation, the impossibility of close speed regulation, and in the large size of wheel required for even small powers; and while for the purposes for which the motor was first required these were not serious, the introduction of more modern machinery, more particularly for textile purposes, involved the necessity for a motor which, having a fairly high speed of rotation in order to avoid

excessive loss in gearing, should be capable of close speed regulation and of taking advantage of higher falls and of large quantities of water. For such purposes the water-wheel was almost entirely superseded by one or other type of turbine.

The introduction of electric driving with its large and almost instantaneous changes of load, while giving an additional fillip to the manufacture of high-speed turbines, had its greatest effect in modifying and perfecting the methods of speed regulation, and in increasing the size of the unit, while at the same time rendering it imperative to design a motor which should be highly efficient under a wide range of loads.

The success which has attended the attempt to satisfy these onerous conditions may be inferred when it is remembered that many manufacturers will now guarantee to construct a turbine which shall give an efficiency of over 80 per cent. over a range of loads of 50 per cent., and which shall respond to an increased demand for power of 33 per cent. with less than 3·5 per cent. variation in speed. With smaller load variations the speed variation is almost infinitesimal, and it becomes easy to run a series of alternating current machines in step with such motors.

The design of hydraulic motors has thus proceeded by well-defined stages, the size and efficiency steadily increasing until at the present time there are units in existence developing 70,000 H.P., and giving efficiencies approximating to 90 per cent.

Wherever a continuous supply of water at a sufficient elevation, or in motion as in a stream, is available, the potential or kinetic energy which this possesses may be turned into useful work.

Before embarking on any power scheme for utilizing such energy, it is however of the highest importance that the true possibilities of the scheme should be ascertained, for as the usefulness of the supply depends in most cases on its uniformity over long periods of time, the maximum available power is strictly regulated by the least power which is available after the longest probable period of drought.

This minimum supply can only be satisfactorily ascertained by investigation of past records extending over many years. Where such records are not available, every attempt should be made by a close investigation of the rainfall records for the particular districts over a long period of years, and of the character, condition, and area of the gathering ground, to estimate the minimum supply likely to be available under the worst probable combination of circumstances, and the scheme may then, and not until then, be developed.

The method of utilizing the supply depends largely on its magnitude,

form, and locality. Where, though comparatively small, it is continuous, the available horse-power may be largely increased by the formation of a storage reservoir capable of impounding at least a 24 hours' inflow. By this means energy may be utilized for the eight hours or so comprising a working day at a rate greatly in excess of the mean rate of inflow.

Where the natural configuration of the country necessitates the power plant being placed at some distance from the storage reservoir, the supply is usually led through an open canal or ditch having a slight gradient, into a smaller storage reservoir termed the forebay, which is placed as near to the power plant as possible. From the forebay the supply is then taken to the prime mover by means of a closed pipe termed the penstock.

In the case of a water-wheel installation the penstock may consist of an open channel.

The supply of water to the prime mover is regulated by means of sluices or gates, which usually form an integral part of the machine.

After doing work, the water is rejected into a discharge channel termed the tail-race.

The most suitable type of prime mover for any particular case depends on—

(1) The quantity of water available.
(2) The supply head.
(3) The purpose for which power is required.

Those types in general use consist of—

(1) Water-wheels.
(2) Turbines.
(3) Piston engines.

Each has its own sphere of usefulness, and in determining the type to be adopted each installation demands special consideration, guided by the circumstances peculiar to the case.

In general, the water-wheel is only suitable for small powers and for comparatively low heads and where close speed regulation is not essential. Its efficiency is greatly affected by a variation in the supply and in the head or tail race levels. It is of great size and weight in proportion to the power developed, and has a low rotative speed. It is now, however, almost obsolete, and for all industrial motive power installations, whether using high or low heads, one or other type of turbine is used.

In certain cases for comparatively small powers, and where a high head is available and a slow rate of rotation is required, the piston engine is most satisfactory.

In passing through a prime mover, water may do work either by changing potential energy or kinetic energy or pressure energy into work; or by a combination of these processes.

In the overshot water-wheel, for example, rotation is produced almost entirely by the weight of the water; in impulse wheels deriving their motive force from the impact of a high velocity jet of water, work is done solely in virtue of the kinetic energy of the jet; in turbines of the reaction type the pressure energy of the water is partly changed into kinetic energy in the wheel itself, this being absorbed in producing rotation of the wheel; while in a piston engine the water does work in virtue of its pressure, its velocity being so small as to be negligible.

In designing any type of hydraulic prime mover, certain general principles should be borne in mind.

(*a*) All shock, whether of water on moving or stationary surfaces, or on water moving with a low velocity, should be avoided as being productive of loss of energy in eddy formation.

This may be prevented by arranging that as far as possible any stream of water on meeting a solid surface is moving tangentially to the surface, and that passages conveying the working fluid are not subject to abrupt changes of sectional area or of form.

(*b*) Frictional losses should be reduced to a minimum by reducing the area of the wetted surfaces to a minimum compatible with easy curves of flow, and by reducing the relative velocity of flow over such surfaces to a minimum.

(*c*) As far as possible, the motive fluid should be rejected devoid of energy, and therefore moving with as low an absolute velocity as will suffice to carry it out of the motor.

The possibility of conforming to these general principles varies with the type of motor.

ART. 142.—HYDRAULIC TURBINES.

Modern turbines may be divided into two main classes, impulse and reaction turbines.

In an impulse turbine, the whole head of the supply water is converted into kinetic energy before the wheel is reached. The water leaves the supply nozzle or nozzles in one or more high-velocity jets at the pressure (usually atmospheric) obtaining in the turbine casing. It then impinges on a series of vanes or buckets carried by the wheel, and, in virtue of the change of

direction and, hence, of tangential momentum produced by these buckets, exerts a driving force and so does work on the shaft. Its direction is freely deviated by, and its pressure remains uniform during, its passage through the buckets.

The **Pelton wheel** (Fig. 185) is the only form of impulse turbine which has been used in important installations during recent years.

The **Pressure** or **Reaction turbine** consists essentially of a wheel or runner provided with vanes into which water is directed over the whole periphery by a series of guide vanes. The water on leaving these guide vanes is under pressure, and supplies energy partly in the kinetic and partly in the pressure

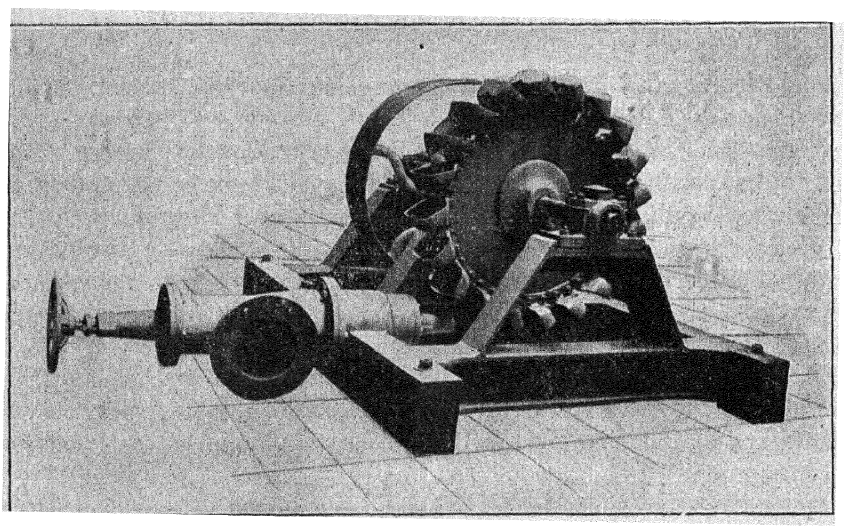


FIG. 185.—Pelton Wheel.

form. In its passage through the runner the pressure energy is utilised in increasing the relative velocity of flow between the vanes, and the water finally leaves the runner at the pressure obtaining in the discharge pipe or draft tube.

The change of momentum accompanying this change from pressure head to kinetic head necessitates an equivalent reaction on the moving vanes, and work is thus done on the turbine shaft. This turbine, as well as the impulse type, therefore owes its propelling force, not to statical pressure, but to changes of momentum, the pressure difference over the concave and convex faces of a bucket being produced, as in the impulse wheel, by the change of momentum in the stream passing the bucket. In the case of a wheel having vertical downward flow, the weight of the water also adds to this propelling force.

Since the turbine works under pressure, the buckets should always remain full of water, and to this end admission should take place continuously all around the circumference of the wheel. If not, those buckets which happen to be idle will either be empty, having discharged their contents into the discharge pipe, or will contain dead water. In the first case the buckets must be refilled before the pressure at the circumference can be utilised, while, in the second case, the necessity for imparting momentum to this still water causes loss of energy by impact. In the impulse wheel, on the other hand, the supply may be admitted to the whole, or to a part of, the circumference without loss of energy.

In the earliest of the reaction turbines, the Fourneyron, the guide vanes were inside the runner, forming an outward flow turbine. This was followed by the Jonval turbine, in which the guide vanes are above the runner and the water flows axially into and through the wheel, giving an axial flow turbine. Both these types have been, to all intents, obsolete for some years, and the Francis, or inward flow turbine, in which the guide vanes surround the outer periphery of the runner, has been in general use. In the earlier Francis turbines the discharge was also radially inwards, but in modern turbines, in order to obtain a larger discharge area with a given diameter of runner, the form of the buckets has been modified so as to give a discharge in a direction which is more or less parallel to the axis of the turbine. In the most recent turbines for low-head plants, the design is indeed tending to a turbine which, as regards the runner, is essentially of the axial flow type. Inward-flow guide vanes are used, but there are signs that even in this respect the design of low-head turbines is tending to revert to a modified Jonval type. At the present moment low-head high-speed turbine design is in a state of rapid development, and it is probable that the standard of a few years hence will differ very appreciably from the Francis type.

Art. 143.—General Comparison of Impulse and Reaction Turbines.

The peripheral velocity of a Pelton wheel for maximum efficiency is slightly less than one-half the spouting velocity of the jet (usually approximately $\cdot 46\sqrt{2gH}$, where H is the head), while that of an impulse turbine of the Girard type varies from about $\cdot 40\sqrt{2gH}$ to $\cdot 60\sqrt{2gH}$, depending upon the vane angles. On the other hand, that of the reaction turbine varies from about $\cdot 65\sqrt{2gH}$ to $1 \cdot 45\sqrt{2gH}$, depending on the head and design. Because of this, the Pelton wheel is well adapted for very high heads, which may then be utilised with moderate speeds of rotation, while the relatively

high speed of the reaction turbine enables reasonably high rotative speeds to be obtained with low heads.

The Pelton wheel cannot well be designed to utilise efficiently more than two jets on a single wheel, and as the maximum practicable jet diameter is about 12 inches, the volume of water which can be handled, and the output of the turbine, become small under low heads. The reaction turbine, with its full peripheral admission, is well adapted for large volumes. It is not suited for small powers under high heads, since the volume of water is then small, the waterways are of very small sectional area and easily become choked by floating *débris*, and the fluid frictional losses become relatively high.

The Pelton wheel is not well adapted for use with a suction or draft tube, and, where the tail-race level varies appreciably, must be installed above the highest tail-water level, with some sacrifice of head. The reaction turbine lends itself readily to the use of a draft tube, and has the further advantage that it may be drowned without loss of efficiency. The efficiency of the reaction turbine is not so sensitive to changes of head as that of the Pelton wheel, and, since the percentage variation in head is usually greater in low-head than in high-head plants, this is another reason why the Pelton wheel is not well adapted for low heads.

If operated under constant head and constant speed, the efficiency of the Pelton wheel does not fall off so rapidly at part loads as that of the reaction turbine. On the other hand, the modern reaction turbine has a slightly higher full-load efficiency, so that the average efficiency from half- to full-load is sensibly the same in a well-designed machine of either type. The following table shows typical values of the part-load efficiencies of modern turbines of both types of large size, installed under equally favourable conditions :—

Proportion of maximum discharge .	·2	·3	·4	·5	·6	·7	·8	·9	1·0
Efficiency of reaction turbine . .	·63	·70	·75	·79	·82	·87	·90	·89	·88
Efficiency of Pelton wheel . .	·73	·79	·82	·83	·84	·85	·86	·85	·83

While efficiencies exceeding 90 per cent. are on record for large reaction turbines, such values can only be attained by the most careful attention to the design not only of the turbine but also of its setting, and, in general, the full-load efficiency does not greatly exceed 87 per cent. in the case of a large reaction turbine, and 83 per cent. in the case of a large Pelton wheel. The possibilities of accurate speed regulation are about equal in the two types.

For large units the reaction turbine is generally preferable for heads up to 400 feet. For heads above 750 feet the Pelton wheel is more suitable, while between these limits the choice depends largely upon local circumstances and on the power required. The greater simplicity and accessibility of the parts requiring replacement, due to natural wear and tear, renders the Pelton wheel more suitable when the supply is taken from a stream carrying an appreciable amount of grit or silt in suspension.

Art. 144.—The Pelton Wheel.

The Pelton wheel consists essentially of a disc having buckets around its periphery, on which impinge one or more jets from a suitably designed

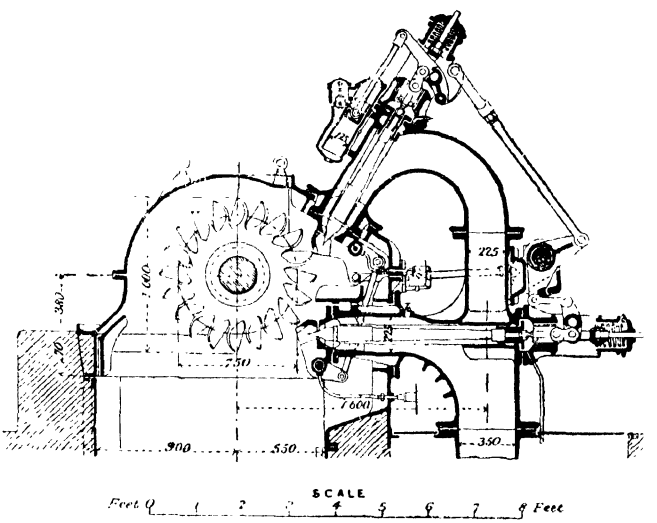


Fig. 186.—Double-jet Pelton Wheel, 6,200 H.P. at 1,500 revolutions per minute; head 2,700 ft.

nozzle or nozzles. The line of action of each jet is tangential to the pitch circle of the buckets; hence the machine is sometimes termed a tangential impulse turbine.

Except for very large units the wheel is usually constructed as a horizontal shaft machine, owing to the simplicity of construction and facility for inspection afforded by this type. Although two, or even four, jets have been used on one wheel, the single jet is almost universal. Where two jets are used they are usually placed approximately at some such angle as shown in Fig. 186. Owing to the splash from one jet affecting the other, and to the fact that one

of the nozzles must be at a greater elevation than the lowest point of the bucket-pitch circle, the efficiency is somewhat lower than that of a single-jet machine, and where the power required from a unit is greater than can be obtained from a single jet, it is usually preferable to mount two single jet wheels side by side on the same shaft (Fig. 187).

Nozzles.—The modern Pelton wheel is always fitted with a circular nozzle having a needle regulator. This is a cylindrical needle or spear of tapering section fitted inside the nozzle axially with the jet (Fig. 186). Discharge takes place through the annulus between needle and nozzle, giving a solid

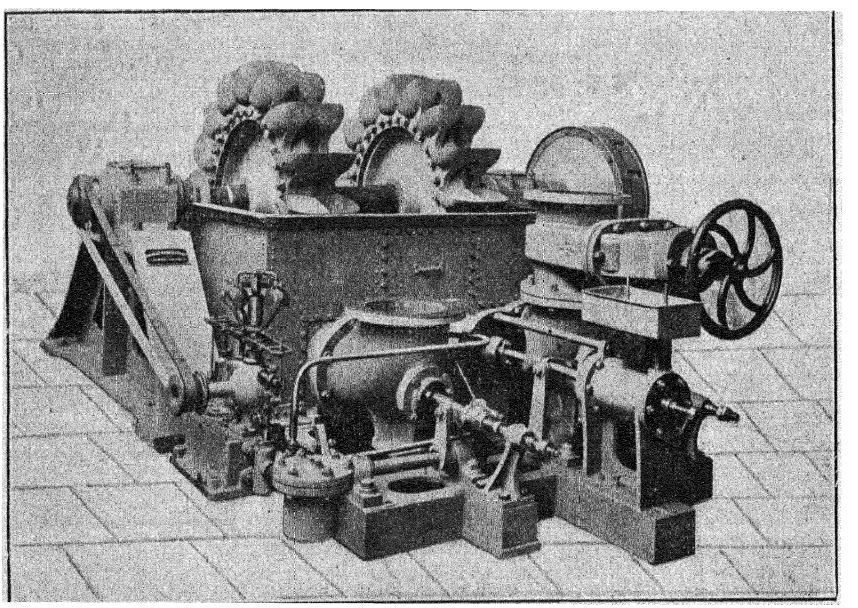


FIG. 187.—Twin Pelton Wheel with Hydraulic Relay Governor.

cylindrical jet on leaving the needle. The size of jet is determined by the axial position of the needle. This may be regulated by hand, or, as in all important installations, by the governing mechanism. It is important that the needle be central in the nozzle and be supported so as to prevent all vibration. Also it should be so formed that the least section of the annulus is in the plane of the nozzle for all openings.

Other shapes of jet have been tried, but all suffer a greater windage loss than the circular. Also all tend to become circular and to become unsteady in the process.

The maximum diameter of jet yet used in practice is about 12 inches.

Jets from Needle-Nozzle.

The presence of the central needle in a nozzle provided with needle regulation causes a reduction in the velocity of the central filament.

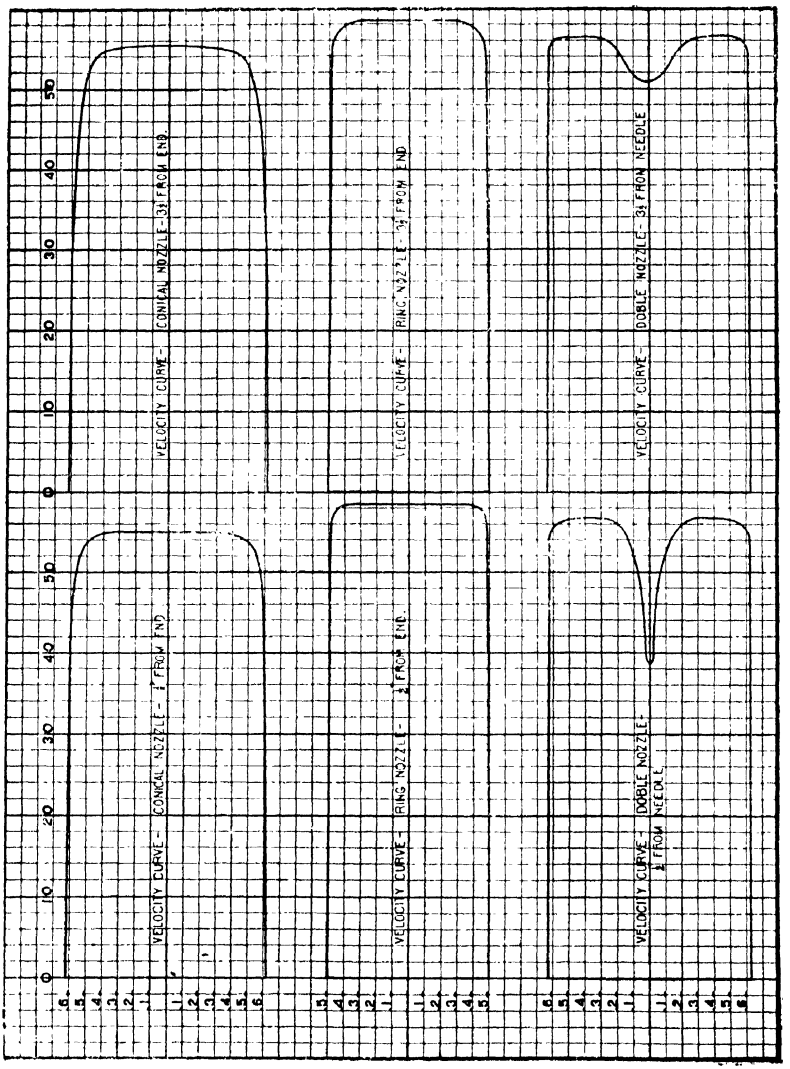


FIG. 188.

and to this extent tends to reduce the efficiency of the jet. Fig. 188 shows the velocity obtained at different points in the cross section of a jet obtained respectively from a plain conical nozzle, a ring nozzle

and a Pelton wheel nozzle with needle regulator.[1] From these it appears that the central velocity at a point distant ½ inch from the tip of the needle is only ·68 of the maximum velocity. At a section 3½ inches from the tip this ratio becomes ·90, while when the distance is 9½ inches it becomes ·96.

At mid opening (diameter 1·25 inches) the coefficient of velocity diminishes slightly as the head increases, from about ·992 with 23 feet head to ·978 with 120 feet head. With a given head the velocity was slightly the

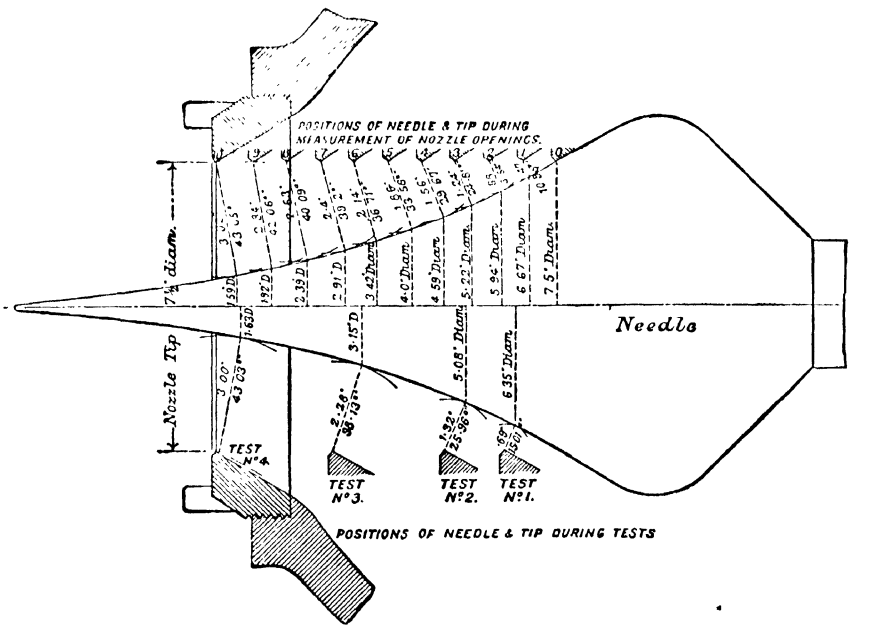


FIG. 189.

greatest with the nozzle half open. The efficiencies in these experiments varied from ·964 to ·993. The maximum jet diameter was 1·50 inches.

Experiments on a larger nozzle, giving a jet up to 7 inches diameter under heads up to 850 feet,[2] showed the following results :

Distance from centre of jet (inches).	0·0	·5	1·0	2·0	3·0
Velocity (feet per second)	212·7	228·7	229·3	229·9	22?·8

[1] From a Thesis by H. C. Crowell and G. C. Lenthe (Massachusetts Institute of Technology, 1903).

[2] W. R. Eckhart, Inst. Mech. Engineers, January, 1910.

The coefficient of velocity increased from ·971 to ·989 as the nozzle area was increased, the coefficient of discharge diminishing at the same time from ·965 to ·838, and the efficiency increasing from ·958 to ·986. Fig. 189 shows the shape of needle and tip used in these experiments.

It is advisable to fit a series of guides behind the needle, parallel to the axis, so as to eliminate whirl in the water approaching the jet. Any such whirl causes the jet to spread rapidly and greatly reduces its efficiency. Also, in order to reduce windage losses, the nozzle should be as near to the buckets as mechanical considerations permit.

Buckets.—The original Pelton buckets were of rectangular section (Fig. 190 *a*). These have been superseded by the elliptical bucket (Fig. 190 *b*), in which that part of the lip in the line of the jet is omitted. The lip and ridge of the original bucket deflect the jet in two planes at right angles, and as the paths of the streams thus formed cross, a certain amount of energy is dissipated by their impact. Also, the lip tends to deflect the jet radially inwards towards the rim of the wheel, in which case some fouling of the succeeding bucket is inevitable. The sharp curves and corners of this type of bucket cause an appreciable loss in eddy formation, and tests show that the efficiency obtained with the modern form of bucket is from 6 to 10 per cent. greater than with the older form.

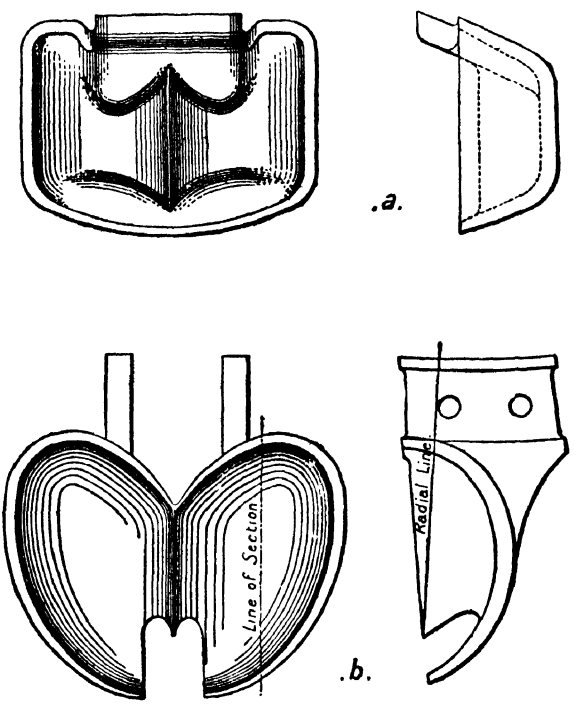


FIG. 190.

The angle through which the jet is deflected by the bucket should be as nearly 180° as possible. In order that the discharge from one shall clear the back of the following bucket, in practice this angle is limited to a maximum of about 165°.

The friction loss in the buckets increases with the wetted area, and to reduce this the number of buckets should be as small as is consistent with continuous impact, while they should be made no larger than is necessary to give the required change of direction with easy curves and without shock. The surface should be as smooth and well finished as possible. In a high-speed wheel the runner consists of a steel disc, to which the buckets are bolted. The latter usually carry two lugs which straddle the wheel rim and

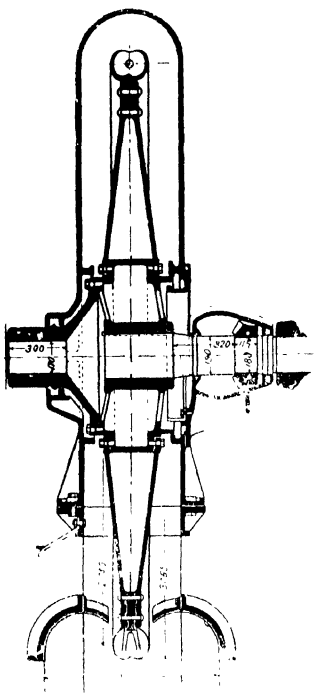


Fig. 191.

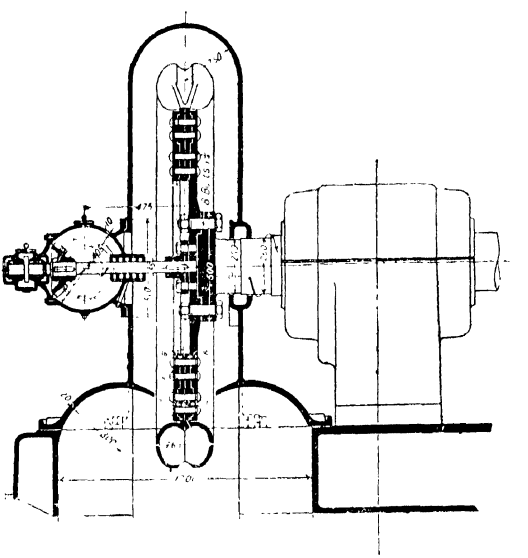


Fig. 192.

are attached to it by two bolts. In some cases a double-wheel disc has been used, with the buckets carrying three lugs. Two of these straddle the discs and the third fits between the discs. The lugs are so designed that a single bolt passes through the rear lugs of one bucket and the forward lugs of the next. This type of construction is advantageous where large buckets are to be fitted to a wheel of comparatively small diameter. For exceptionally high peripheral speeds, it is inadvisable to rely on bolts to resist the large centrifugal forces, and in one type of construction, which has been success-

fully used under such conditions, each bucket carries two lugs which closely straddle the disc, and are kept in position by two fitting bolts and by two steel rings, one on each side of the wheel.

Figs. 191 and 192 [1] show two types of construction adopted at the Arniberg Power Station. Here units, developing respectively 3,000 H.P. and

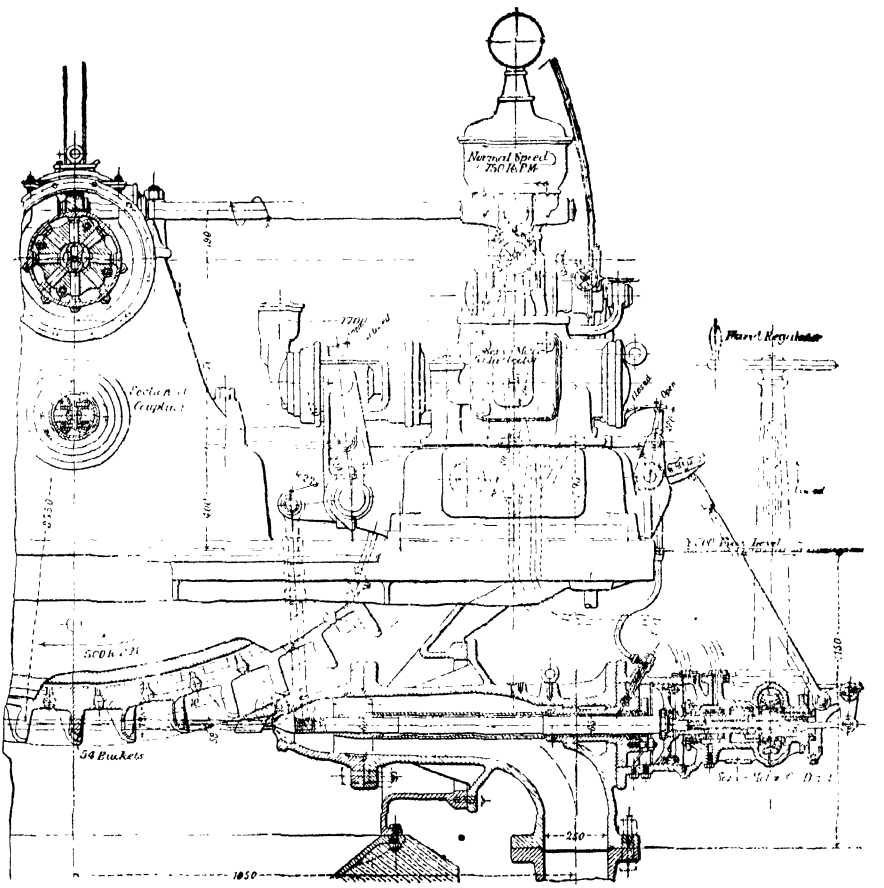


FIG. 193.

1,300 H.P. under 2,800-foot head, have wheels of about 5 feet 5 inches and 9 feet 6 inches diameter, making respectively 630 r.p.m. and 360 r.p.m. In these wheels rivets and not bolts are used, so that individual buckets cannot so readily be replaced.

[1] See *Engineering*, December 15th, 1922.

In an installation at the Fully Hydro-Electric Station in Switzerland,[1] operating under the extremely high head of 5,412 feet, the velocity of the buckets is 303 feet per second. In this case the buckets are of forged steel. Each is provided with two lugs, which fit into a wedge-shaped slot in the forged-steel disc (Fig. 193). A series of openings is made in the periphery of the wheel through which the lugs are inserted, and the required spacing is obtained by distance pieces of triangular section inserted between the buckets. The last distance piece is inserted while the rim is heated, and the contraction on cooling binds the whole firmly together.

Form of Buckets.—Where the dividing ridges of the buckets are straight in profile, these are not fixed radially but are inclined backwards from the direction of rotation at such an angle as to give normal incidence on the first impact of the jet. If placed radially the jet would be deflected into the rim of the wheel during the first half of the period of impact and would tend to produce serious inefficiency.

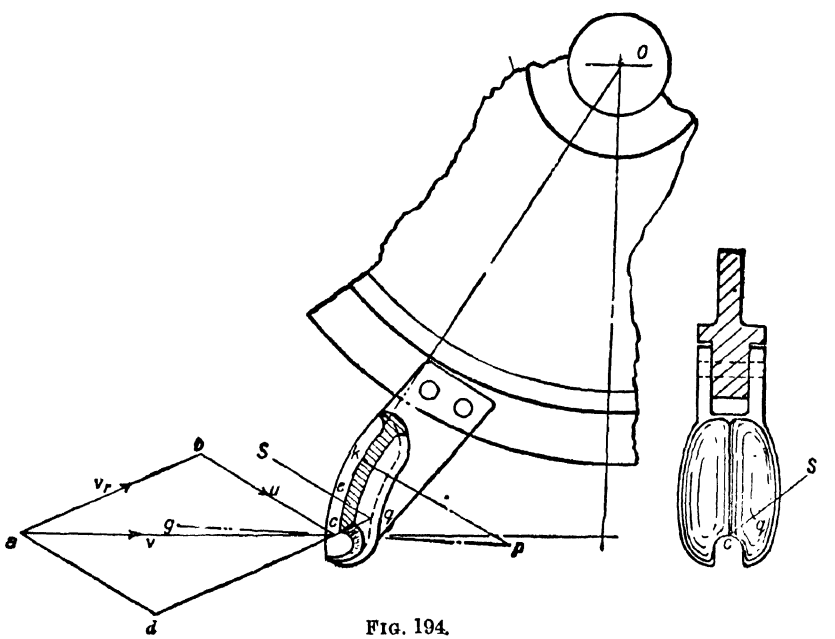


FIG. 194.

A type of construction which is more theoretically correct, and which is found to give better results in practice, is indicated in Fig. 194.

[1] *The Engineer*, February 28, 1913.

Let $\overline{ac} = v$ = velocity of jet.

Let $\overline{bc} = u$ = velocity of bucket.

Then $\overline{ab}$ = relative velocity of jet and bucket.

If ω = angular velocity of wheel, and if r = radius at the point of impact, we have $u = \omega r$ and $\overline{bc}$ is perpendicular to $\overline{oc}$.

Draw $d\,c$ parallel to $a\,b$. Then $\overline{d\,c}$ represents the direction of the jet relative to the moving bucket.

For the jet to leave the bucket with zero absolute tangential velocity, its final direction must be parallel to, and the component parallel to the plane of the wheel of its final relative velocity must be equal to $\overline{cb}$. If then $\overline{gc}$, the bisector of the angle $\overline{b\,c\,d}$, be a normal to the surface of impact at c, the required conditions will be fulfilled, the jet striking the vane in the direction $\overline{d\,c}$ and leaving (relatively to the wheel) in the direction of $\overline{c\,b}$. The direction and magnitude of $\overline{ac}$, and the direction of $\overline{b\,c}$ being fixed, if the final relative velocity be approximately known, the magnitude of $\overline{bc}$ and therefore the most efficient speed of rotation may be determined. In a well-designed bucket the final relative velocity may be taken as ·75 of the initial. If c and k be the first and last points at which the jet impinges on the ridge and if a third point l be taken midway between c and k, the directions of such normals as $c\,g$ may be determined for these three points, and a smooth curve drawn through these points and having the required normals will give the correct curve for a longitudinal section of the receiving edge of the bucket. In general, a circular arc with centre at p, the intersection of the normals through c and k will give a very close approximation to the curve.

Strictly, since the path of the mid particles of the jet relative to the bucket is given by $c\,q\,s$, the normal at q should bisect this angle. If, however, the curve through q be made parallel to $c\,l\,k$, the approximation to the correct curve will be sufficiently near.

A close approximation may also be obtained by determining graphically the points of intersection of the bucket with the axis of the jet for different positions of the bucket, and by drawing a smooth curve such that the axis of the jet is normal to the curve in every position of the bucket.

To prevent the jet striking the back of the bucket, this should be everywhere above the line $d\,c$, while to reduce splash on passing through the jet the edge at c should be as sharp as possible.

Experiments by Mr. J. Nakahara [1] on buckets having a straight profile

[1] "Memoirs of the College of Engineering," Kyushu Imperial University, Fuknoka, 2, 143–174.

show that the maximum efficiency was obtained when the ridge was at right angles to the jet at the mean position between the beginning and end of full jet.

More recent experiments in the author's laboratory [1] show that with the type of bucket shown in Fig. 190 *a*, best results were obtained when the bucket was at right angles to the jet, at the mean position between the beginning and middle of full jet, while with the type shown in Fig. 190 *b*, maximum efficiency occurred when the bucket was at right angles to the jet at the beginning of full jet. In the first case, the buckets were inclined 3° backwards, and, in the second case, 9° backwards, from the position indicated as being best by Nakahara. The effect of a small variation in this angle is, however, small, a 5° alteration on either side of the best position only reducing the efficiency by about 1 per cent.

In modern practice the width of the buckets is between three and five times the diameter of the jet, the ratio diminishing as the size of jet increases, while the wheel diameter should not be less than about ten times the jet diameter. If, on settling the number of revolutions and peripheral speed, and hence the diameter of a wheel, this is less than the required multiple of the diameter of a single jet to give the required power, duplicate jets should be used.

Number of Buckets.—For minimum loss these must be as few as is consistent with the jet being wholly intercepted for all bucket positions, so that the entering bucket may entirely intercept the jet before the leaving bucket begins to free itself. From this consideration, a simple geometrical construction shows that if n be the minimum possible number of buckets, R the extreme outer radius over the receiving edges of the buckets, r the pitch circle radius, and t the thickness or diameter of the jet, n is given very approximately by the relationship

$$n = \frac{\pi}{\sqrt{1 - \frac{\left(r + \frac{t}{2}\right)^2}{R^2}}}$$

If $R = r + s$, so that s is that portion of the bucket projecting beyond the pitch circle, we have

$$n = \frac{\pi}{\sqrt{1 - \left(\frac{r + \frac{t}{2}}{r + s}\right)^2}}$$

[1] W. W. Broadbent, M.Sc. Thesis, Manchester University, September, 1923.

Giving s a value ranging from ·60 t in the case of a wheel of less than 3 feet diameter to ·565 t where the wheel is upwards of 6 feet in diameter, values of n in close accord with modern practice are obtained. Generally, values of n given by the formula $n = k\sqrt{\frac{r}{t}}$, where k ranges from 7·0 to 8·0 as the wheel diameter decreases from 6 feet to 3 feet, will be found

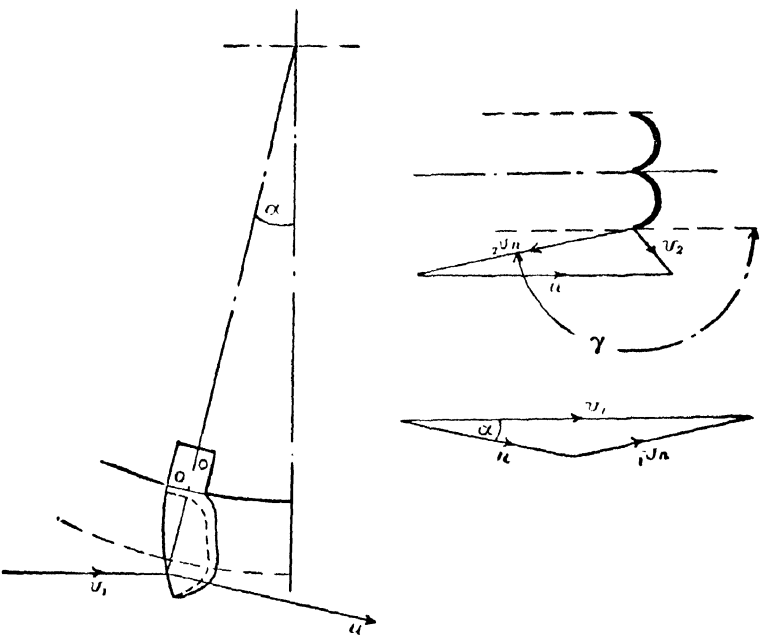


FIG. 195.

to give results which are sufficiently near for all practical purposes. The theoretical value of s thus being obtained, a little additional overlap is usually given to allow for any slight variation in the axial position of the jet.

ART. 145.—EFFICIENCY OF PELTON WHEEL.

Let u = peripheral speed of buckets at pitch circle.

$= \frac{2\pi r N}{60}$ where r = perpendicular distance from axis of jet to centre of wheel; N = revs. per minute.

Let v = initial velocity of jet.
Let v_2 = final absolute velocity.
Let ${}_1v_r$ = relative velocity of jet and bucket at entrance.
Let ${}_2v_r$ = relative velocity at discharge.
Let α = mean angle between jet and tangent at point of contact.
Let γ = total angle of deviation of buckets (Fig. 195).

Then initial velocity of jet in direction of tangent at point of impact $= v \cos \alpha$.

Component, parallel to tangent at discharge, of final velocity relative to bucket $= {}_2v_r \cos \gamma$.

$\therefore$ Absolute velocity in this direction at discharge $= u + {}_2v_r \cos \gamma$.

$\therefore$ Change of tangential momentum per second, per lb.

$$= \frac{1}{g} \{ v \cos \alpha - u - {}_2v_r \cos \gamma \}.$$

$\therefore$ Work done per lb. of water per second

$$= \frac{u}{g} \{ v \cos \alpha - u - {}_2v_r \cos \gamma \} \text{ ft. lbs.}$$

$$\therefore \text{ Efficiency} = \frac{u}{g\,h} \{ v \cos \alpha - u - {}_2v_r \cos \gamma \} \text{ ft. lbs.}$$

The loss due to friction and eddies in buckets $= \frac{{}_1v_r^2 - {}_2v_r^2}{2\,g}$ ft. lbs. per lb.

where ${}_1v_r = \sqrt{v^2 + u^2 - 2\,v\,u \cos \alpha}$ (Fig. 199).

The loss due to rejection of kinetic energy $= \frac{v_2^2}{2\,g}$ ft. lbs. per lb.

where $v_2^2 = u^2 + {}_2v_r^2 + 2\,u\,{}_2v_r \cos \gamma$.

Experiments indicate that ${}_2v_r$ may be as low as from ·6 to ·7 times ${}_1v_r$. It is certain, however, that in a well-designed bucket, having a ratio of bucket width to jet diameter not less than about 3·3, this ratio is greater, and approximates to ·80.

If the angle of deviation of the buckets were 180°, the theoretical value of the peripheral speed for maximum efficiency would be $v \cos \alpha \div 2$, or approximately $v \div 2$, since α is small. When account is taken of the disc windage loss which increases rapidly with the speed of rotation, and of the fact that γ is less than 180°, the best peripheral speed lies between ·44 v and ·48 v.

Taking $\alpha = 10°$; $\gamma = 165°$; $u = ·46\ v$; ${}_2v_r = ·80\ {}_1v_r$; this makes ${}_1v_r = ·553\ v$; ${}_2v_r = ·443\ v$; and the hydraulic efficiency = ·876, a value agreeing closely with the results of the best modern practice. In this case the loss in the buckets is 11 per cent., and at discharge 1·4 per cent.

Test Number.	1.	2.	3.	4.
Revolutions per minute	302	300	301	299
Output in kilowatts	962	1724	2296	2527
Diam. of jet, ins.	4·36	5·61	6·56	6·80
Mean velocity of jet, f.s. . . .	221·4	222·8	224·2	227·4
Ratio of bucket to jet velocity . .	·406	·401	·400	·399
Discharge, c.f.s.	22·9	38·2	52·9	57·6
Nozzle efficiency, per cent. . . .	95·8	96·8	98·2	98·6
Relative velocity of jet entering bucket .	132·1	134·0	135·1	138·9
„ „ „ leaving „ .	77·5	79·2	65·1	63·9
Distribution of Energy of Jet.				
Useful work, per cent.	74·4	74·2	70·4	68·1
Bucket friction and eddies, per cent. .	23·0	23·2	27·7	29·2
Kinetic energy in discharge, per cent. .	1·1	1·0	1·8	1·9
Other hydraulic losses, due to splash at entrance, changing angle of application of jet, etc.	1·5	1·6	1·1	0·8

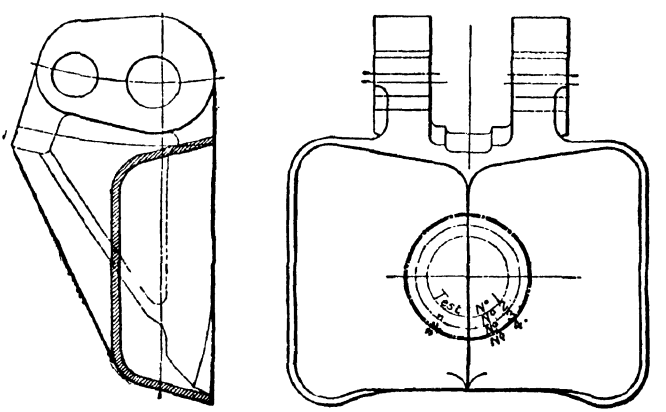


FIG. 196.

The above table shows the distribution of energy as obtained in a series of tests on a wheel of which the following are the details[1]:—

Number of buckets . . 15; angle $a = 4° 41·5'$.
Width „ „ . . 19·5 inches; angle $\gamma = 166°$.
Projected area „ . . 252 square inches; head, 810 feet.

[1] "Proceedings Inst. Mechanical Engineers," January, 1910.

Radius of pitch circle . . 33·5 inches.
,, outside tip of buckets . 40·5 ,,
Revolutions per minute . . 300.
Nozzle—needle nozzle, with tip 7·5 inches diameter.

The large value of the bucket losses, particularly in trials 3 and 4, is undoubtedly due partly to the comparatively small ratio of width of

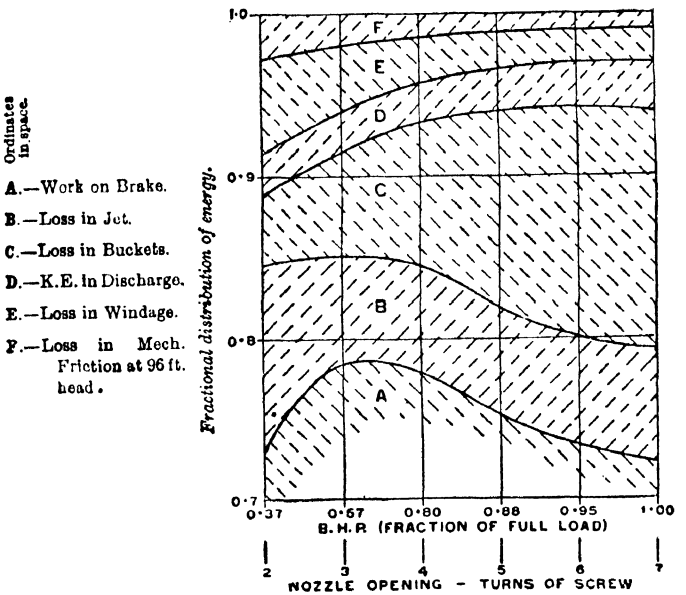


FIG. 197.—Distribution of Energy in Tests on Pelton Wheel.

bucket to diameter of jet, and partly to the design of bucket, which is shown in Fig. 196.

The following table, showing results obtained in recent tests of Pelton wheels,[1] well illustrates the extent to which the high efficiency is maintained at part loads in a well-designed wheel.

No.	Number of Nozzles.	Fall in Metres.	Revolutions per Minute.	Efficient Horse-power at Full Gate.	a = per cent. of Full Load. b = per cent. Efficiency of the Turbine.											
					a	b	a	b	a	b	a	b	a	b	a	b
X.	2	90	355	326	100·0	76·0	89·3	79·7	86·5	81·6	71·4	83·6	39·6	84·2	—	—
XI.	1	100	180	300	100·0	88·9	85·4	89·3	72·7	87·8	56·0	86·0	42·0	87·8	24·0	77·0
XII.	2	350	376	6,050	100·0	83·5	89·9	81·0	75·8	82·8	55·1	83·8	27·1	80·5	14·2	75·9
XIII.	1	850	630	3,710	100·0	86·2	89·4	86·9	59·2	85·2	71·2	85·1	46·7	82·9	22·3	78·5

[1] By Professor F. Prazil. "Proceedings Inst. Mechanical Engineers," 1910-11.

Fig. 197 shows graphically the distribution of energy obtained during tests by the author of a Pelton wheel under a head of 96 feet.[1] Measurements of the wheel show that $\alpha = 18°$, and that $\gamma = 160°$. Maximum efficiency was attained when $u \div v = \cdot 47$. The buckets were of the type shown in Fig. 198 and were obviously too small for the size of jet at full load.

Design of Pelton Wheel.

Example.

To design a Pelton wheel to work under an effective head of 500 feet and to develop 800 H.P. at 360 revolutions per minute.

Assuming a coefficient of velocity = ·985, the velocity of efflux of the jet $= \cdot 985 \times \sqrt{500 \times 64 \cdot 4}$

$= 177$ feet per second.

Taking the velocity of the pitch circle of the wheel as ·46 times that of the jet, we have

Peripheral velocity of wheel = 81·3 feet per second.

$\therefore$ Radius of pitch circle $= \dfrac{81 \cdot 3 \times 60}{2\pi \times 360} = 2 \cdot 158$ feet.

$\therefore$ Diameter of pitch circle = 4 feet $3\frac{3}{4}$ inches.

Next assuming an efficiency of 85 per cent., we have the energy passing the nozzle per second given by $\dfrac{800 \times 550}{\cdot 85}$ ft. lbs. = 518,000 ft. lbs., and since each cubic foot of water contains $\dfrac{62 \cdot 4 \times (177)^2}{2g}$ ft. lbs. = 30,380 ft. lbs. in the form of kinetic energy, this requires $\dfrac{518{,}000}{30{,}380} = 17 \cdot 06$ cubic feet per second.

The required area of the nozzle is thus $\dfrac{17 \cdot 06}{177} = \cdot 0964$ sq. ft.

$= 13 \cdot 89$ sq. ins.

giving a jet diameter of 4·20 inches.

Taking $n = 7 \cdot 5 \sqrt{\dfrac{r}{t}}$, this gives the number of buckets as equal to $7 \cdot 5 \sqrt{\dfrac{25 \cdot 9}{4 \cdot 22}} = 18 \cdot 6$, or say 20 for convenience in balancing.

Next applying the formula $n = \dfrac{\pi}{\sqrt{1 - \left(\dfrac{r + \frac{t}{2}}{r + s}\right)^2}}$ we get, on

[1] "Proc. Inst. Mech. E.," May, 1922, p. 643.

substituting for n, r and t, on reduction $s = 2{\cdot}5$ inches, giving the amount by which the buckets must project beyond the pitch circle for continuous impact. For safety it is usual to increase this slightly, say to 2·75 inches, giving an extreme wheel diameter of 4 feet 9¼ inches.

The buckets would in this case be about 21 inches wide.

Art. 146.—Specific Speed.

As for a reaction turbine, the specific speed (Art. 167) of a Pelton wheel is given by

$$N_s = \frac{N\sqrt{P}}{H^{\frac{5}{4}}} \text{ r.p.m.}$$

where N is the working speed; P is the horse power per jet; and H the head.

But $N = \dfrac{60\,u}{\pi D}$, where D is the diameter of the pitch circle

and $P = \dfrac{W\,v^3\,\pi\,d^2\,\eta}{2g \times 4 \times 550}$, where d is the diameter of the jet and η is the efficiency.

$$= \frac{\cdot 00176\,\pi\,d^2}{4}\,\eta\,(2\,g\,H)^{\frac{3}{2}}$$

$$= \cdot 610\,d^2\,H^{\frac{3}{2}} \text{ if } \eta = \cdot 85$$

$$\therefore \quad \sqrt{P} = \cdot 780\,d\,H^{\frac{3}{4}}.$$

Also, $u = \cdot 46\sqrt{2\,g\,H}$ (approx.).

$$\therefore \quad N = \frac{60 \times \cdot 46\sqrt{2\,g\,H} \times \cdot 780\,d\,H^{\frac{3}{4}}}{\pi D H^{\frac{5}{4}}}$$

$$= \frac{55\,d}{D}.$$

If $D \div d = 12$, this makes the specific speed approximately 4·6. This is the highest value of the specific speed for a single-jet Pelton wheel for high efficiency. Thus the maximum speed obtainable with a single jet for an output of, say, 2,500 B.H.P. under a head of 900 feet, would be given by

$$4{\cdot}6 = \frac{N\sqrt{2500}}{(900)^{\frac{5}{4}}},$$

from which

$$N = \frac{4{\cdot}6 \times (900)^{\frac{5}{4}}}{\sqrt{2500}} = 455 \text{ r.p.m.}$$

With two jets, or two wheels each with a single jet, the maximum value with the same ratio of $D \div d$ becomes approximately 6·5. By reducing the ratio $D : d$ to 10, the specific speed, with a single jet, becomes approxi-

mately 5·5. The full-load efficiency of such a wheel is about 5 per cent. less than that of a wheel with a specific speed of 4·5, and in order to obtain this efficiency the buckets require to be very carefully designed.

Art. 147.—Speed Regulation of Pelton Wheels.

Since the efficiency of a Pelton wheel, or other impulse wheel, depends on the maintenance of the correct ratio of peripheral velocity of bucket and velocity of jet, if high efficiency is to be expected at all loads the method of governing must be such as to keep the latter velocity as nearly as possible constant. Where this is the case, there is no reason, except for the greater proportional effect of windage and mechanical friction at part loads, why

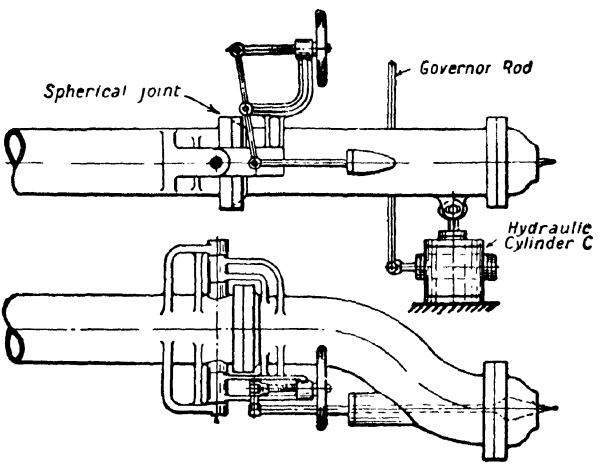


Fig. 198.—Combined Needle and Deflecting Nozzle.

the efficiency should not be independent of the load. There are three general methods of regulating the speed of Pelton wheels. These are :—

(1) By deflecting the jet from the wheel by a deflecting nozzle or hood.
(2) By a combined needle regulator and pressure regulator.
(3) By a combined needle regulator and deflector or deflecting nozzle.

In the first method the jet is deflected wholly or partly from the wheel by the action of a relay piston operated from the governor or, in small plants, by the governor directly. The flow in the pipe line is independent of the load, no pressure surges are produced, and the method gives excellent governing. It is, however, very wasteful under a variable load, and is only to be condoned where, as in some irrigation schemes, or in some installations on the

higher reaches of a stream, the user is under an obligation to discharge water at a minimum rate for users lower down the stream. The loss may be reduced by using the deflector in conjunction with a hand-regulated needle. This is set by hand at intervals to give the maximum discharge likely to be required during the next period, and any fluctuation of load up to this maximum is handled by the deflector.

In the older forms of deflecting nozzle (Fig. 198) the nozzle is connected to the pipe line through a swivel joint packed by means of a leather ring, and

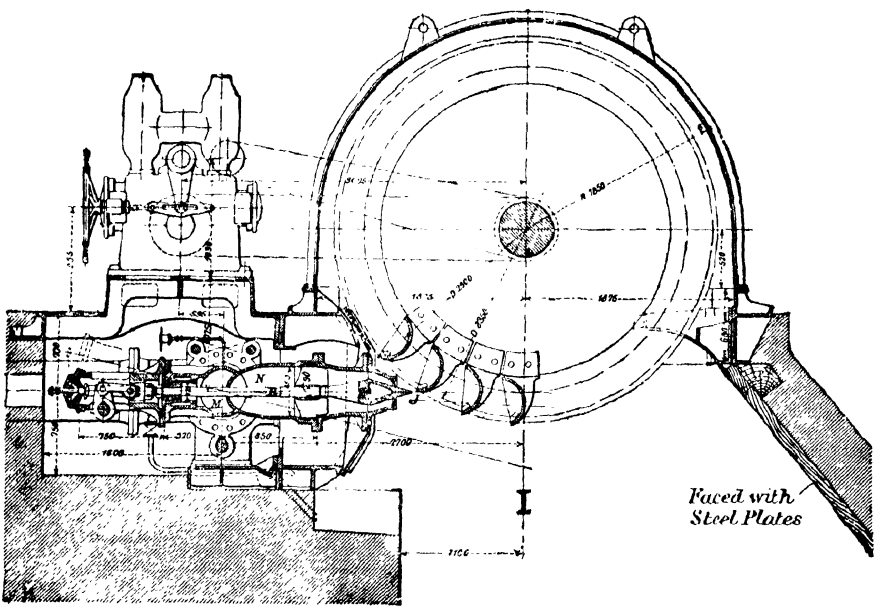


Fig. 199.—Double runner Pelton Wheel developing 13,500 H.P. at 250 revolutions per minute. Head 1,330 ft. Pitch circle diameter 9·52 ft.

is carried by the plunger rod of the relay cylinder. The pressure below this plunger and, hence, its position, are regulated by a pilot valve operated by the governor.

Fig. 199[1] shows a deflecting nozzle, in which the nozzle, with the needle-regulating mechanism, is mounted on a hollow trunnion, through which the pressure water is conveyed. This is carried through a stuffing box which is easily kept tight. Instead of a deflecting nozzle, the jet may be deflected by means of a movable hood or *deflector*, fitted between the nozzle and the wheel, and whose position is regulated by the governor (Figs. 191 and 202).

[1] *Engineering*, August 28, 1914.

This is lighter, cheaper, and gives less trouble in maintenance than the deflecting nozzle.

In the Seewer deflector[1](Fig. 199A), a series of guide vanes are mounted on trunnions in the body of the nozzle. Their angular position is regulated by the axial position of the controlling shaft *S*, which is controlled by the governor. When the guide vanes are inclined to the axis they produce whirl in the jet which spreads rapidly on leaving the nozzle. A fixed hood is fitted between the nozzle and the wheel, having an opening only slightly greater

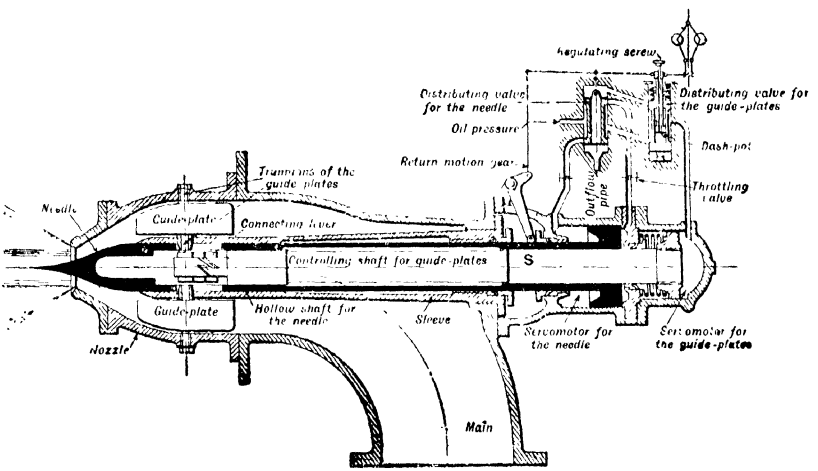


FIG. 199A.—Seewer System of Pelton Wheel Regulation.

than the maximum diameter of the parallel jet, so that when spreading occurs only a part of the water reaches the wheel. The design of the connections is such that when the wheel is running at the normal speed with any load the vanes are parallel to the axis.

In the second method of regulation, the position of the needle is regulated by the governor, while, to prevent pressure surges accompanying a sudden reduction of load, a *pressure regulator* is fitted.

Fig. 200 shows details of such a device applied in this case to a twin Pelton. Here the cross lever *L* is connected to the piston rods of the two relay cylinders, and carries the dashpot rod *A*. Its connected plunger works in the weighted dashpot *C*, which itself carries the needle valve V_1. Pressure water supplied through the small pipe *P* keeps the main escape valve *V* closed so long as the valve V_1 is closed. If this valve is opened, however,

[1] Seewer, "Recent Developments in Hydro-Electric Engineering," *Proc. Inst. Mech. E.*, Vol. 134, 1936, p. 283.

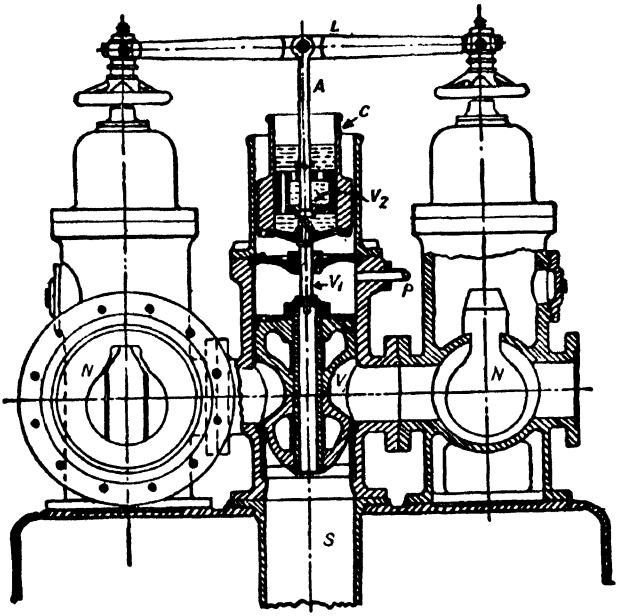


FIG. 200.—Pressure Regulator for Pelton Wheel.

by a sudden upward motion of L, the pressure above the main valve is relieved, and the valve opens, relieving the pressure at the nozzles N N.

The valve V_2 permits of a sudden depression of L, without unduly stressing the dashpot rod A.

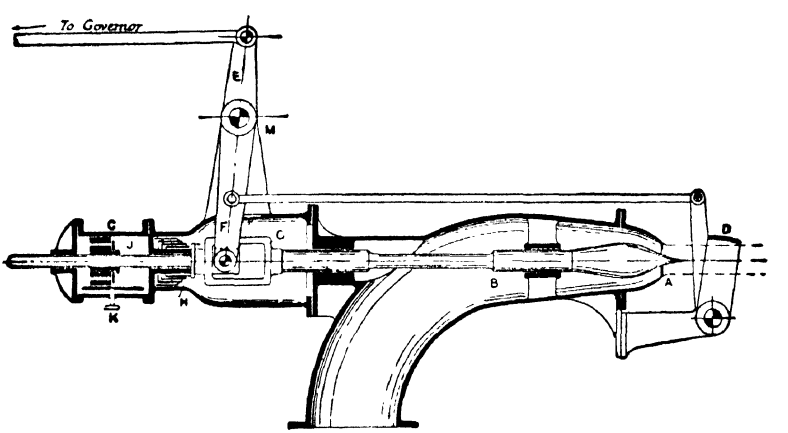


FIG. 201.

While, owing to the smaller volumes of water to be handled, the pressure regulator is more suitable for Pelton wheels than for reaction turbines, it suffers from the liability of the valve to stick and from the difficulty of ensuring its synchronous action, and in most recent plants the third method of regulation has been used.

Here both deflector and needle are regulated by the governor. The connection is so arranged as to allow of a rapid movement of the deflector on a

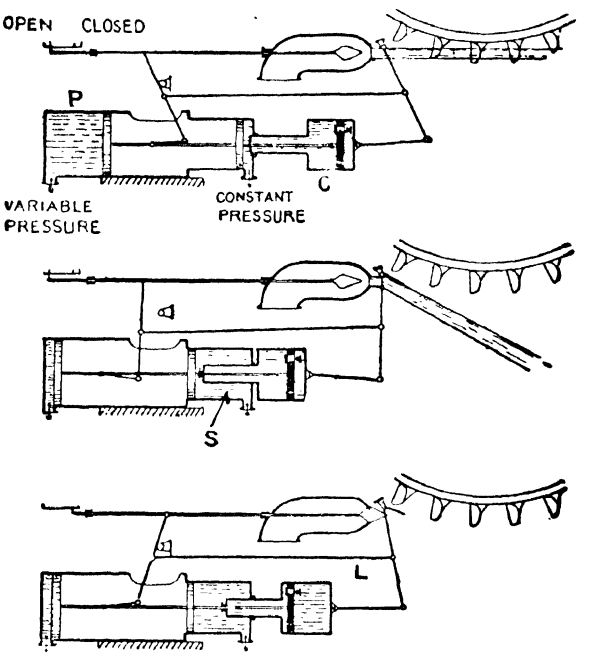


FIG. 202.—Governing arrangement for Lac Fully Pelton Wheels.

reduction of load, followed by a slow movement of the needle closing the nozzle. When in its final position the deflector is tangential to the reduced jet, ready for an immediate response to any fresh change of load. One arrangement of this kind is shown in Fig. 201. The needle carries a slotted extension *G*, and is forced to the right by a spring *H*, whose action is retarded by the action of the oil dashpot *C*. If the load is reduced the lever *F* is forced to the right by the servo-motor, deflecting the jet, while the needle follows slowly until the slot is again in contact with the end of the lever *F*, with the needle tangential to the jet. On an increase in load the lever is forced to the left, and carries with it both the needle and deflector. The valves *J* in the dashpot plunger allow free motion in this direction.

Fig. 202 shows another method of obtaining the same result as applied to the units of the Lac Fully installation. A pilot valve operated by the

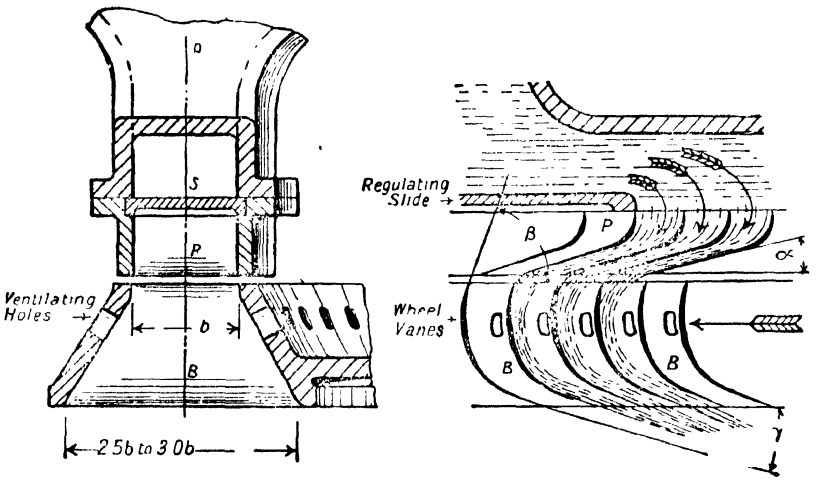


Fig. 203.—Axial Flow Girard Turbine.

governor regulates the admission of pressure water to the left-hand side of piston *P*. On throwing off the load this pressure is reduced, and the piston

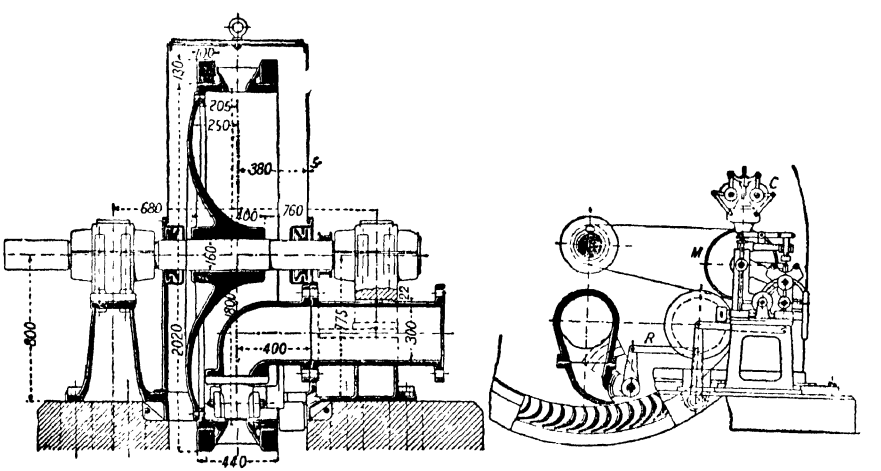


Fig. 204.—Partial Admission Girard Turbine. 1,000 H.P. at 500 revolutions per minute under 1,640 ft. head.

line, along with the cylinder *C*, is forced to the left, deflecting the jet. Under the action of the constant pressure in the space *S*, the dashpot cylinder *C* is

then forced slowly to the right, carrying with it the link L and the spear rod and closing the jet.

Art. 148.—The Girard Turbine.

This turbine, which, although now almost obsolete, was in extensive use up to the end of last century, may be constructed either as an axial flow machine (Fig. 203), in which case the axis is usually vertical, or with outward radial flow (Figs. 204 and 205). The axial flow type is more suitable for large powers under comparatively low heads, where full circumferential admission is required.

Pressure water supplied through the pipe Q is guided by the converging passages P P, its pressure diminishing as its velocity increases, and is discharged at atmospheric pressure into the moving buckets B B. These are ventilated to prevent the jet expanding to touch the rear vane, and since the width of bucket diminishes with the angle of inclination of the vanes, they are also splayed out from the inlet to the discharge side, the breadth at discharge usually varying from 2·5 to 3 times that at inlet. The outward deviation produced by this splaying of the buckets, while slightly diminishing the efficiency of working, cannot be avoided. The guide and vane angles at entry and exit are so designed that water enters the buckets without shock, and is discharged with an absolute velocity which is only sufficiently great to ensure its ready removal from the wheel.

When less power is required one or more of the inlet passages may be cut off by means of the slide S, the motion being regulated either by hand or by a governor. By the provision of a series of supply ports which may be completely cut out of action one by one, the loss of energy which is inevitable through contraction and re-enlargement of section whenever a stream is throttled, is reduced to a minimum. Where only one admission port is used, the supply should be regulated by means of a sluice or hood working between the port and the entrance to the buckets. By this means the velocity of efflux is unaltered, and the only loss introduced is that which may be caused by any alteration in the angle of impact of the jet.

Fig. 204 shows such a turbine, built by Messrs. Piccard, Pictet & Co., of Geneva, and developing 1,000 H.P. at 500 revolutions per minute, under a head of 1,640 feet.

While specially well fitted for heads of 100 feet and upwards, this turbine, with full circumferential injection, gives good results with heads from 10 to 50 feet, and has indeed been used with a head as low as 16½

inches. With low heads and full injection the wheel must be horizontal, since with a vertical wheel the jet velocity at the highest and lowest points would be very different, while at the same time the loss of head due to the difference in level of the guide vanes at exit and the tail-race would become serious. To avoid the latter loss as far as possible in the case of a vertical wheel with partial injection, the guide vanes are placed as near the bottom of the wheel as practicable. The horizontal wheel offers the further advantage for large volumes of water, in that it affords greater facilities for getting rid of a complete circumferential discharge.

In common with all impulse turbines, the part gate efficiency is high, while under suitable conditions the full load efficiency may amount to 80

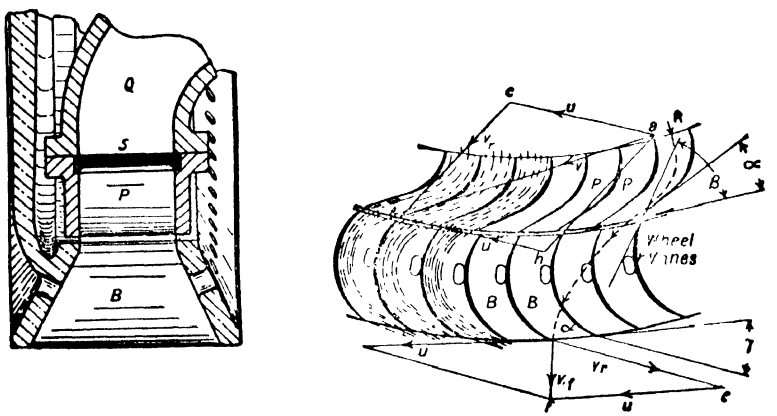


FIG. 205.

per cent. In the case of an outward flow Girard turbine described in the "Proceedings of the Institution of Civil Engineers," [1] and giving 400 H.P. under 594 feet head, the outer diameter of the wheel was 8′ 11″, inside diameter 7′ 10¼″, the vanes, 110 in number, were 4·7 inches wide at the entrance, and were splayed out to 15¾ inches. A single inlet passage was used 4·31 inches in width. Under this head the jet velocity was 181·6 feet per second, and the efficiencies were as follows :—

B.H.P.	Revolutions.	Efficiency
82·5	211	59·5 %
341	210	76·4 %
400	209	79·0 %

[1] 1881-2, Part 4.

In this type of impulse wheel if α is the angle which the incoming jet makes with the plane of the wheel in an axial flow turbine and with the tangent plane to the wheel at the point of impact in a radial flow machine, and if γ be the angle which the discharge tips of the vanes make with the same plane (Figs. 203 and 205), α is generally made about 24° and γ about 21°. These values, however, depend upon the head, increasing from about 12° and 13° respectively with large heads and small volumes of water, to about 30° and 28° with low heads and large volumes. β, the angle of inclination of the vanes at entrance is then made so that the entering stream slides along the vanes without shock.

The construction for determining the vane angles is shown in Fig. 205. Here $a\ b$ represents v, the velocity of the stream leaving the guide vanes, in magnitude and direction, while $h\ b = a\ c$ represents u, the velocity of the wheel buckets at the point of entry. $c\ b = a\ h$ then represents the relative velocity at entrance to the buckets, and marks the correct inclination of the bucket tips.

In order to take advantage of the full head any impulse turbine working under atmospheric pressure should be located at as small an elevation above the tail-race as is possible. While keeping this in view, however, the situation of the wheel must be such that it is not liable to become submerged by a raising of the tail-race level in time of flood. If this should occur the buckets run full, the wheel works as a pressure turbine, the conditions for which it is designed are entirely violated, and efficient working becomes impossible.

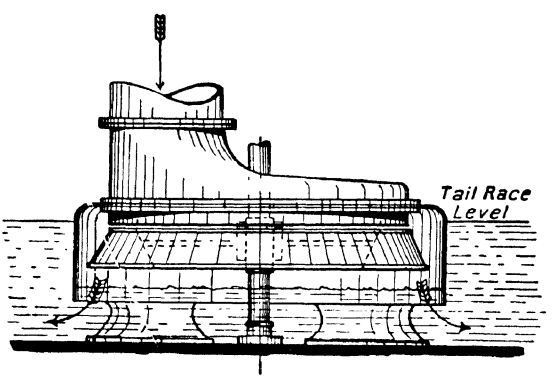


FIG. 206.—Axial Flow Girard Turbine with Full Circumferential Injection and with Girard's System of Hydro-pneumatization.

This often necessitates the sacrificing of a portion of the available head, and, with a low fall, renders the use of such a turbine inadvisable. With a high fall the loss is in general so small as to be negligible.

Attempts have been made to remove this drawback to the impulse turbine. Thus Girard, in his system of Hydro-pneumatization, placed the whole turbine in an air-tight casing, the lower end of which opened out beneath the surface in the tail-race (Fig. 206). By means of an air pump driven by the turbine the air in this casing was maintained at such a pressure as to keep the water level inside the casing below that of the

turbine wheel, whatever the rail-race level. The complication and expense thus introduced, however, together with the fact that power is required to work the air pump, prevented the general adoption of this idea.

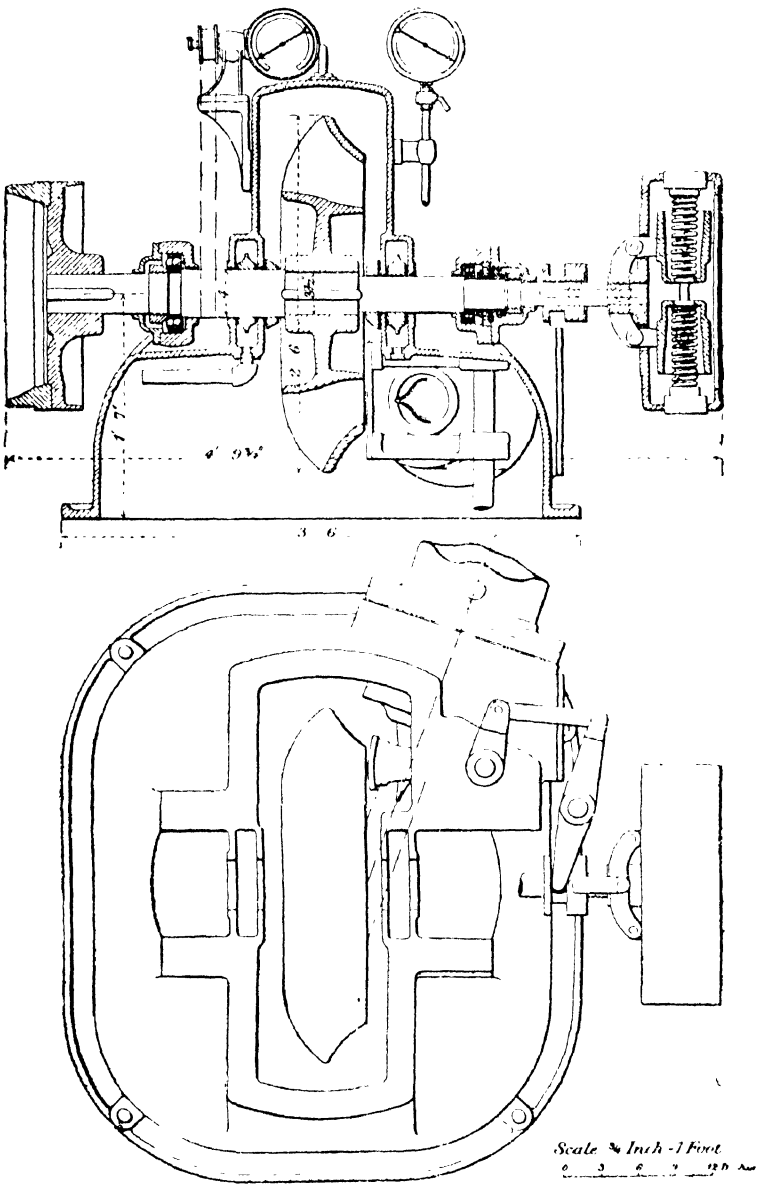


FIG. 207.—"Turgo" Impulse Turbine.

By mounting the wheel in an air-tight casing at some distance above the tail-race and using a discharge pipe or draught tube, delivering below the surface of the tail-race, the difficulty may be overcome. On starting up the turbine the escaping water ejects the air from the casing and creates a partial vacuum. An air valve, actuated by a float in a chamber connected with both casing and draught tube, then admits sufficient air to prevent the water level from rising as high as the wheel.

This method has not been adopted often in the past. In view of the somewhat serious loss due to windage at the speeds necessitated by the high heads now being utilised, there is reason to think that it may again come into use, since windage losses are directly proportional to the density of the air.

In the *Haenel* "limit" turbine, which is essentially the same as the Girard, the buckets were so designed as to run full when working as an impulse turbine, the areas of the wheel passages being approximately the same throughout. The flooding of the turbine does not then affect its efficiency, except in so far as it affects the available head, since the machine then acts as a pressure turbine. It proved fairly good for low falls with a head which does not vary greatly, giving efficiencies varying from about 60 per cent. at half gate to a maximum of about 72 per cent. It is now obsolete.

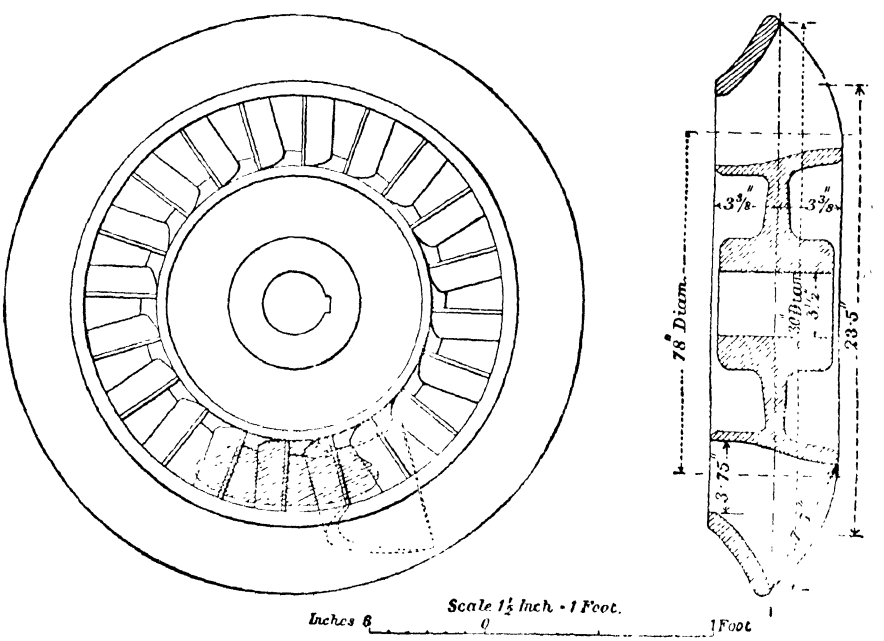


FIG. 208.—Runner for "Turgo" Turbine.

Art. 149.—The "Turgo" Impulse Turbine.

This turbine, which has been developed during the past few years, consists essentially of an axial flow Girard runner, used in conjunction with one or more nozzles of the Pelton wheel type. It thus combines the ease of governing of the Pelton wheel with the higher speed of rotation of the Girard turbine. The diameter of the runner is appreciably less than that of a Pelton wheel using the same jet; the peripheral speed is somewhat higher; and the specific speed about twice as great. It has the further advantage

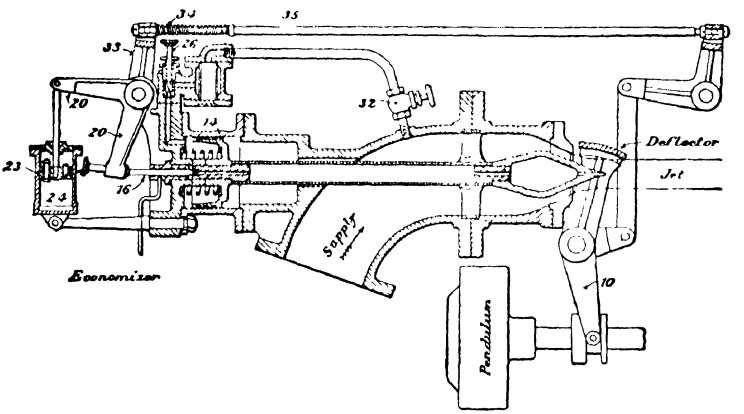


Fig. 209. Governing mechanism of "Turgo" Turbine.

that the windage resistance of the runner is much less than that of a Pelton wheel. On the other hand, a thrust bearing becomes necessary to deal with the unbalanced end force on the shaft.

Figs. 207 and 208 show such a turbine designed for 150 H.P. at 700 r.p.m. under a head of 250 feet,[1] and Fig. 209 shows the arrangement of the governing mechanism.

Here pressure water is admitted through valve 32 behind the spring load and plunger 14. The spear rod is hollow, and if valve 16 is off its seat this water is vented through the spear rod and a small orifice in the tip of the spear. If valve 16 is on its seat, however, the pressure on the two sides of plunger 14 is equalised, and the spear is forced towards its seat under the action of its spring, closing the nozzle.

The deflector is connected by a linkage to the bell crank lever 20, one end of which presses against valve 16. If the load is thrown off, the deflector is forced into the jet; the valve is pushed hard up against the end of the spear rod; and the nozzle is slowly closed. As the area of the jet is reduced the

[1] "Proc. Inst. C. E.," vol. 213, 1921–1922, part I., p. 396.

deflector is lifted out of the jet until, finally, the deflector and spear come to rest with the deflector just clearing the jet.

If the load be thrown on, the deflector is raised, the bell crank lever 20 is withdrawn from the valve and the pressure on the front of the plunger 14 forces the spear backwards, increasing the area of the nozzle.

A slow movement of the spear, combined with a rapid movement of the deflector, is ensured by connecting rod 35 to lever 33 through a spring 34. The rapid movement of the deflector compresses the spring which keeps valve 16 against its seat. As the spear rod moves to the right it is followed by valve 16, but the rate of travel of this valve is regulated by the resistance of the dashpot 24. A non-return valve in the piston of this dash-pot allows the oil to pass readily from its upper to its lower side, and then permits the lever 20, and with it the valve, to be withdrawn rapidly when the load is suddenly increased. This is followed by a comparatively rapid opening of the nozzle.

The direct connection of the governor to the deflector gives extremely rapid and close speed regulation on a sudden reduction of load.

Art. 150.—Theory of the Impulse Turbine.

Let u represent the peripheral velocity of the wheel.
v represent the absolute velocity of the water.
w represent the tangential component of velocity of the water.
f represent the radial component of velocity of the water.
v_r represent the relative velocity of water and vane.
suffix 2 represent the conditions at entrance to the wheel.
suffix 3 represent the conditions at discharge from wheel.

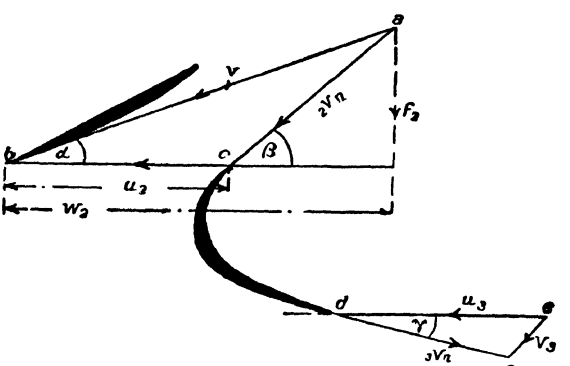


Fig. 210.—Velocity Diagram for Axial Flow Impulse Turbine.

Then for entry to take place without shock the velocities at entrance are represented by the triangle of velocities, *a b c*, (Fig. 210), and the state of affairs at exit is indicated by triangle *d e f*. From these it appears that :—

$$f_2 = w_2 \tan a = (w_2 - u_2) \tan \beta.$$

$${}_2v_r = \sqrt{v_1^2 + u_2^2 - 2\,v_1 \,.\, u_2 \cos a}$$

$$v_3 = \sqrt{{}_3v_r^2 + u_3^2 - 2_3\, v_r \,.\, u_3 \cos \gamma}.$$

But for the effect of skin friction and eddy formation in the vanes the relative velocities at entrance and exit would be the same if a given particle of water entered and left the wheel at the same radius. In an axial flow wheel this is approximately true, but not in a radial flow wheel. In the latter case the relative velocity of the points at which a particle leaves and enters is $u_3 - u_2$, and the component of this in the direction of the discharging stream is $(u_2 - u_3) \cos \gamma$.[1] It follows that, neglecting friction, the relative velocity at discharge is given by

$${}_3v_r = {}_2v_r + (u_3 - u_2) \cos \gamma.$$

If friction be taken into account

$${}_3v_r = k \{ {}_2v_r + (u_3 - u_2) \cos \gamma \},$$

where k apparently has a value of from ·90 to ·95 in a well-designed wheel.

The initial moment of momentum of the water about the axis of the wheel

$$\text{per lb.} = \frac{v_1 \cos a}{g} \,.\, r_2$$

[1] If different portions of the surface with which a particle comes in contact have different velocities, as in the case of a radial flow turbine, the relative velocity is not constant. It may be determined graphically, since it will be the resultant of the relative velocity at inlet, and of the component in the direction of the vane at the required point, of the relative velocity of the vane at that point and at inlet. Thus, if in Fig. 211,

$n\,a$ = relative velocity at inlet
$a\,b$ = velocity of vane at inlet
$c\,d$ = velocity of vane at outlet
$\angle\, m\,c\,p = \gamma$.

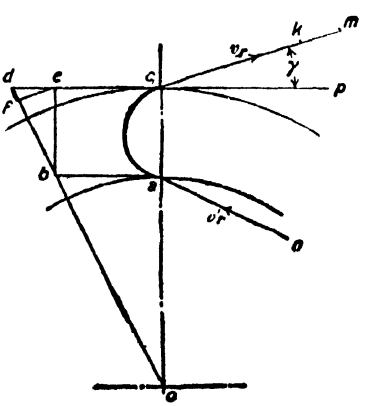


FIG. 211.

Then $d\,e$ = relative velocity of vane at outlet and at inlet, and $e\,f$, drawn parallel to $c\,m$, represents the component of this in the direction of the vane at outlet. If then $c\,k = n\,a$, and if $c\,k$ be produced to m where $k\,m = e\,f$, the relative velocity at outlet is represented by $c\,m$. Where, in the case of a Girard turbine, the ratio of outer and inner radii = 1·25, with a value of $\gamma = 21°$, the actual relative velocity at outlet is approximately 1·23 times that at inlet. Frictional resistance will, however, reduce this by some unknown amount, and will probably bring the ratio down to about 1·10. In any case, the effect on the value of γ for maximum efficiency will be slight, the effect being to reduce this value, and this should be taken into account in arranging the design.

The final moment of momentum of the water about the axis of the wheel

per lb. $= \dfrac{(u_3 - {}_3v_r \cos \gamma)\, r_3}{g}$.

If ω is the angular velocity of rotation, the work done, U, per lb. per second is given by

$$U = \frac{\omega}{g}\left\{ v_1 \cos \alpha \,.\, r_2 - (u_3 - {}_3v_r \cos \gamma)\, r_3 \right\} \text{ft. lbs.}$$

The hydraulic efficiency of the machine, neglecting mechanical losses and disc windage, is then equal to $U \div H$, while the hydraulic efficiency of the runner equals $2\,g\,U \div {v_1}^2$.

The loss due to skin friction and eddy formation in the buckets

$$= \frac{({}_2v_r + (u_3 - u_2) \cos \gamma)^2}{2g}\left\{1 - k^2\right\} \text{ft. lbs. per lb.}$$

The loss due to the kinetic energy of discharge

$$= \frac{{v_3}^2}{2\,g} \text{ ft. lbs. per lb.}$$

In an axial flow turbine, $u_2 = u_3$; $r_2 = r_3$, and these expressions become

$$U = \frac{\omega}{g}\left\{ v_1 r_2 \cos \alpha - (u_2 - {}_3v_r \cos \gamma)\, r_2 \right\}$$

$$= \frac{u_2}{g}\left\{ v_1 \cos \alpha - u_2 + {}_3v_r \cos \gamma \right\} \text{ ft. lbs. per lb.}$$

$$\text{Bucket Loss} = \frac{{}_2{v_r}^2 (1 - k^2)}{2\,g}$$

$$= \frac{{}_2{v_r}^2 - {}_3{v_r}^2}{2\,g} \text{ ft. lbs. per lb.}$$

For maximum hydraulic efficiency, assuming the vane angles designed to give entry without shock, the sum of the losses due to bucket friction and to rejection of kinetic energy must be a minimum. In any wheel of normal design, the exit angle γ should be as small as possible. In practice, this angle cannot be made much less than 15° without making the breadth of the wheel at exit abnormally large. Adopting this value, the angles α and β, and the speed u_2, are capable of relative adjustment. As indicated by the triangle of velocities, if the angle α and the velocity of the jet are fixed, the peripheral speed of the wheel for entry without shock is increased as β is increased. At the same time the relative velocities, and, therefore, the bucket losses, are

reduced, but the kinetic energy of discharge is increased. Similarly, if the bucket angle β be fixed, an increase in α reduces the peripheral speed for entry without shock. At the same time, it increases the relative velocities, and may increase or diminish the discharge velocity depending on the angles. Assuming the relative velocity at discharge to be ·9 times that at entrance, the following tables show how the peripheral speed, the losses, and the efficiency vary :—

$\beta = 30°$.

α	15°	20°	10°
$u_2 \div v_1$	·518	·350	·689
${}_2v_r \div v_1$	·518	·680	·347
$v_3 \div v_1$	·137	·290	395
Loss in buckets (as fraction of $v_1^2 \div 2g$)	·052	·088	·023
Loss at discharge (as fraction of $v_1^2 \div 2g$)	·019	·084	·156
η	·929	·828	·821

$\beta = 45°$.

α.	15°	20°	22°	25°	30°
$u_2 \div v_1$	·707	·598	·556	·495	·368
${}_2v_r \div v_1$	·367	·483	·525	·590	·707
$v_3 \div v_1$	·394	·205	·158	·148	·147
Loss in buckets (as fraction of $v_1^2 \div 2g$)	·025	·048	·052	·068	·095
Loss at discharge (as fraction of $v_1^2 \div 2g$)	·155	·042	·025	·022	·022
η	·820	·910	·923	·910	·883

$\beta = 60°$.

α	20°	25°	30°	35°
$u_2 \div v_1$ . . .	·742	·669	·580	·490
${}_2v_r \div v_1$ . . .	·396	·485	·580	·664
$v_3 \div v_1$ . . .	·409	·268	·148	·176
Loss in buckets . .	·029	·045	·064	·084
Loss at discharge . .	·167	·072	·022	·031
η	·827	·883	·914	·885

$\beta = 75°$.

α	20°	25°	30°	35°	40°	45°
$u_2 \div v_1$ . . .	·848	·792	·735	·660	·585	·520
${}_2u_r \div v_1$ . . .	·355	·435	·519	·592	·665	·735
$v_3 \div v_1$ . . .	·545	·430	·309	·253	·170	·206
Loss in buckets. .	·024	·035	·052	·066	·084	·103
Loss at discharge .	·296	·185	·095	·037	·029	·042
η	·680	·780	·853	·897	·887	·855

$\beta = 90°$.

α.	30°	35°	40°	45°	50°
$u_2 \div v_1$ . . .	·865	·815	·764	·707	·645
${}_2v_r \div v_1$ . . .	·503	·574	·645	·707	·769
$v_3 \div v_1$ . . .	·441	·336	·250	·193	·190
Loss in buckets . .	·049	·064	·080	·099	·110
Loss at discharge . .	·194	·112	·062	·037	·036
η	·757	·824	·858	·864	·854

From these figures, and the corresponding curves of Figs. 212 and 213, it appears that for any given value of β, the maximum efficiency is attained when α is almost exactly equal to $\beta \div 2$. The maximum attainable hydraulic efficiency falls off as β and α are increased, but for values of β between 30° and 60° the effect is not large. The values of the peripheral velocity for entry

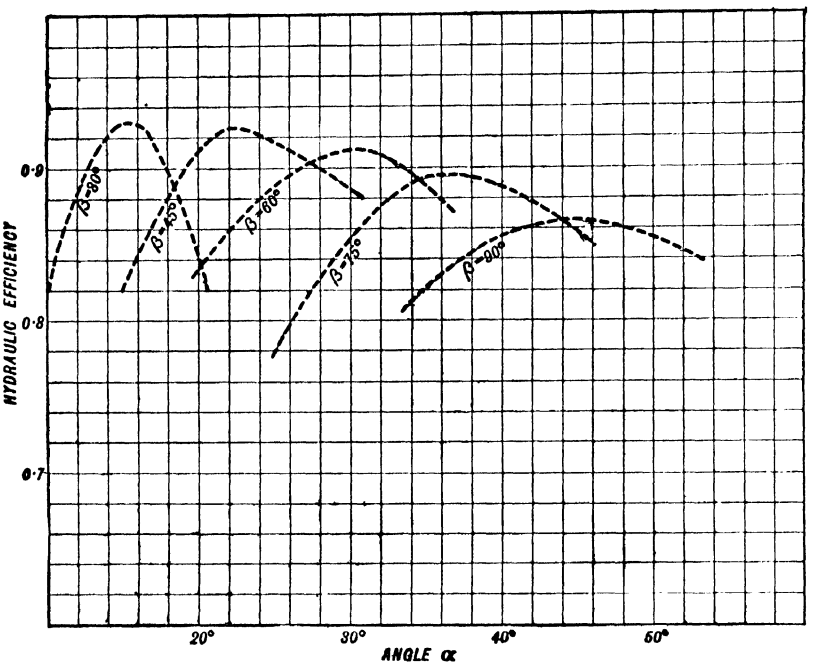


FIG. 212.

without shock for any value of β working in conjunction with the best value for α, are as follows :—

β. . .	30°	45°	60°	75°	90°
u_2/v_1 . .	·52	·55	·58	·63	·71

So that since windage losses increase with the speed of rotation, the overall efficiency with a given size of jet and wheel will tend to fall off somewhat more rapidly than the hydraulic efficiency, with an increase in β.

Moreover, since the effective sectional area of the wheel passages at inlet

$= 2\pi r_2 b_2 \sin\beta$, and at outlet $= 2\pi r_3 b_3 \sin\gamma$ (neglecting the thickness of the vanes), and since the area should be greater at outlet than at inlet in the ratio, $1 : k$, in order to ensure free deviation of the jet, we must have

$$\frac{r_3 b_3 \sin\gamma}{r_2 b_2 \sin\beta} = \frac{1}{k}.$$

In an axial flow machine this makes

$$\frac{b_3}{b_2} = \frac{\sin\beta}{k \sin\gamma}.$$

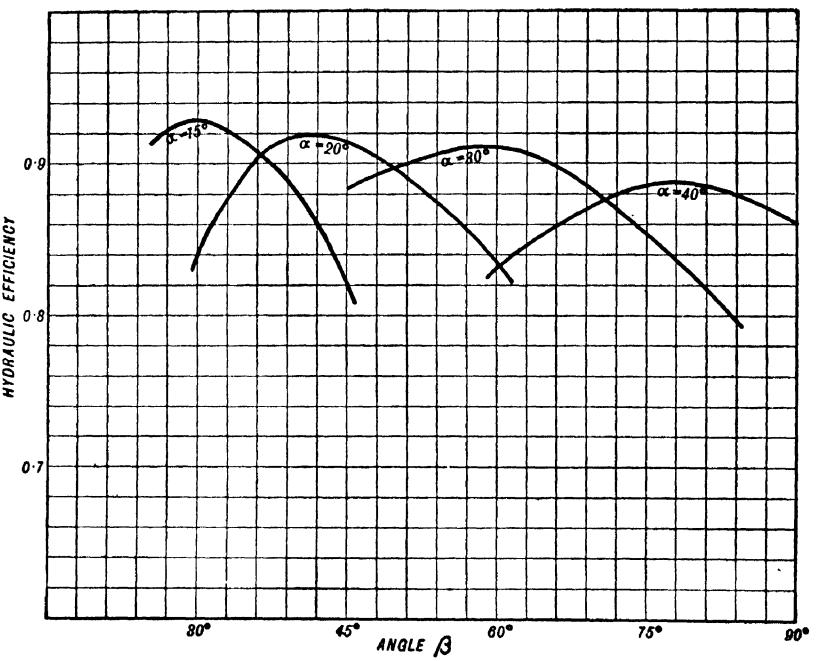


FIG. 213.

To satisfy this requirement the buckets must be splayed out towards the exit, as shown in Figs. 203 and 208. The width of the buckets at the discharge edge thus becomes very large with large values of β.

On the other hand, in a turbine using a Pelton nozzle, an increase in the values of β and of a enables the nozzle to be brought nearer to the wheel and reduces the loss due to windage and dispersion of the jet. In a machine of this type the best value of β apparently lies between 40° and 60°.

In the turbine shown in Fig. 208, $a = 20°$; β (measured to the back of the blades) is 42°; $\gamma = 12° 30'$. The mean diameter at entrance is 18

inches and at exit 23·5 inches, so that $u_3 \div u_2 = 1{\cdot}30$. Taking k as ·95, then, from the triangle of velocities, $u_2 = {\cdot}560\, v_1$; ${}_2v_r = {\cdot}51\, v_1$; ${}_3v_r = k \{ {}_2v_r + (u_3 - u_2) \cos \gamma \} = {\cdot}638\, v_1$; $v_3 = {\cdot}155\, v_1$. The hydraulic efficiency, based on the energy of the jet, is ·933 ; the loss in the buckets is ·043 ; and the loss at discharge ·024. Tests on the wheel,[1] under a head of 248 feet, showed a maximum overall efficiency of ·81, which, as the efficiency of the jet was ·90, corresponds to ·90 measured on the energy of the jet. As windage and mechanical friction losses amounted to approximately ·040, the agreement between the calculated and measured efficiency is very close. The maximum efficiency was attained with a value of $u_2 = {\cdot}46\, v_1$. The corresponding velocity at the outer periphery of the bucket is ·56 v_1. It is to be expected that the best speed at the mean radius of the vanes will be somewhat less than that giving entry without shock at this point, since at the latter speed there will be impact on the back of that portion of each vane which lies outside this radius. Also, as the speed is reduced, the windage and mechanical losses are reduced. It may be taken that the best results are likely to be attained in practice when the speed of the outer periphery of the runner at entrance satisfies the conditions of the velocity triangle for entry without shock.

Effect of Centrifugal Action.

In an axial flow turbine centrifugal action tends to heap up the water towards the outside of the buckets. Thus, in Fig. 214, a particle of water entering at P (in plan) tends to follow a path $P\,A$ instead of its actual path

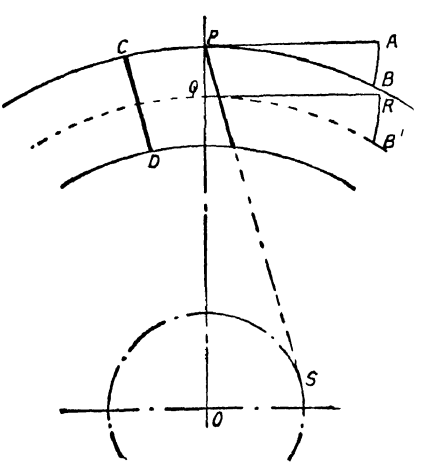


FIG. 214.

[1] "Proc. Inst. C. E.," vol. 213, 1921–22, Part I., p. 407.

$P\,B$. The outer particles are prevented from following the path $P\,A$ by the action of the outer walls of the bucket, but at points nearer the centre of rotation this constraint is absent and the particles tend to follow their natural paths. The relative motion which then takes place may be prevented by designing the buckets so that the actual path of each particle (in plan) is a straight line perpendicular to the radius. In this case, if Q is the middle point of the bucket at inlet, R will be its middle point at outlet, and the bucket will be splayed out symmetrically about R instead of B'.

The actual paths of a particle through the runner may be drawn as for a pressure turbine, except that now the relative velocity of the water over the vanes is known approximately, while the velocity of flow varies.

A second method which has been adopted consists in making the outlet edge at P parallel to $P\,S$, instead of being radial, where $\widehat{S\,P\,O} = \widehat{A\,P\,B}$. The inlet edge $C\,D$ is then made parallel to this, with the result that the length of path traversed by the various particles becomes more nearly equal, and the relative motion is largely prevented.

Art. 151.—The Banki Turbine.

A new type of radial flow impulse turbine, invented by Professor Banki, of Budapest, is shown diagrammatically in Fig. 215. The velocity is com-

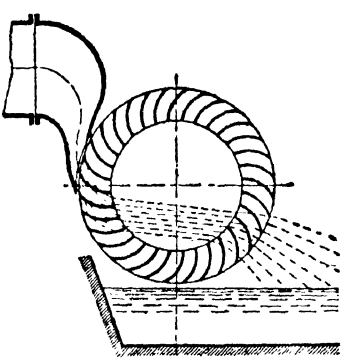


Fig. 215.—The Banki Hydraulic Turbine

pounded in two stages, the water passing twice through the wheel, as shown in the sketch. The entry angle of the wheel vanes is about 70°, and the peripheral speed approximately $\cdot 48\sqrt{2\,g\,H}$. It is claimed that efficiencies of between 80 and 90 per cent. have been obtained with specific speeds much higher than those possible with a Pelton wheel.

CHAPTER XV.

Pressure Turbines—Barker's Mill—Borda Turbine—Fourneyron—Jonval—Thomson—Francis—Kaplan Turbines—Draft Tubes—Mixed Flow Turbines—Head and Tail Races—Ice Troubles.

ART. 152.—PRESSURE OR REACTION TURBINES.

ALL pressure turbines of the radial outward flow type may be considered as modifications of the old Scotch turbine—Reaction wheel, Barker's mill

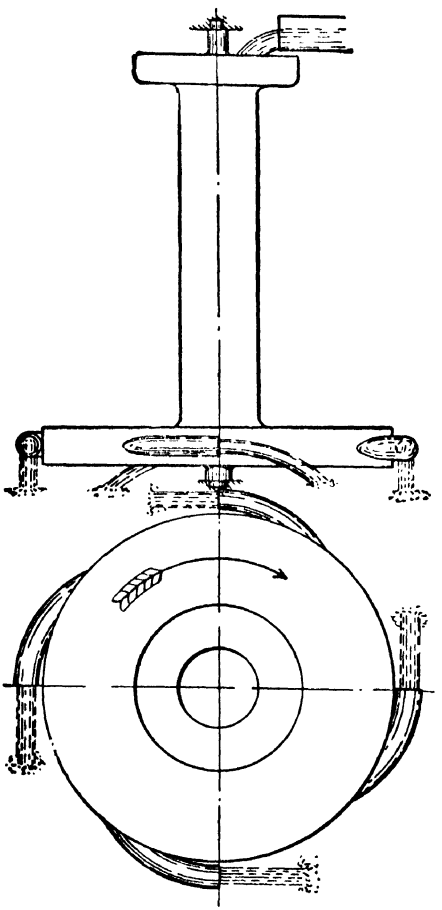


FIG. 216.—Barker's Mill.

or Segner's turbine, as it is variously called. In this turbine (Fig. 216), water is admitted through a vertical supply pipe, flows outwards through straight or curved horizontal arms, and escapes through orifices so placed in these arms as to give a series of horizontal jets perpendicular to the diameters containing the orifices. The reaction of the jets then produces rotation of the wheel. This has, however, only been made in small sizes and is of no practical importance.

ART. 153.

The **Borda** wheel was in all probability the first practical pressure turbine to be constructed. This consists of a couple of concentric cylindrical casings (Fig. 217) mounted on a vertical shaft, the space between these being provided with a series of inclined vanes. The casing is usually of some considerable depth and the mean radius of the vanes large. Water is led into the casing in a direction almost normal to the vanes and acts partly by impact. The pressure produced by its weight is, however, the chief factor in producing rotation. Strictly speaking, the wheel is a pressure or impulse turbine according as the buckets run wholly or partially full, this depending on the distance between consecutive vanes. In the Borda, in common with all other axial flow turbines, since the direction of the flow is parallel to the axis, the effect of centrifugal force on the flow may be neglected. In spite of its crudity, efficiencies of up to 70 per cent. have been obtained with this, which is the origin of all modern turbines of the parallel flow type.

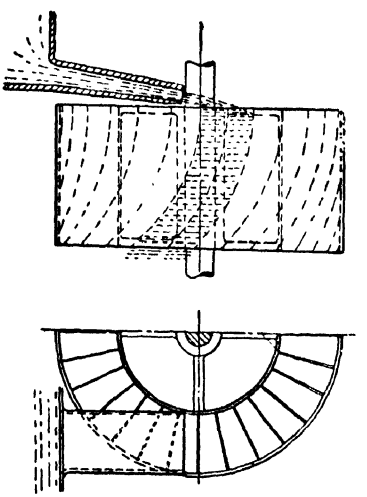


FIG. 217.—Borda Turbine.

ART. 154.—THE FOURNEYRON TURBINE.

The first highly efficient pressure turbine was developed by *Fourneyron* (1827). The Fourneyron turbine is an outward radial flow reaction wheel (Fig. 218). Water is supplied through the pipe *S*, which is closed at the bottom and terminates in an inverted cone, the outside of the pipe forming the entrance to a series of guide passages *P*. Guided by these passages, the water enters the wheel moving in the same direction as the wheel buckets,

impinges on the curved vanes B, has its direction of motion changed, and finally escapes around the periphery of the wheel.

This type of turbine has been used with heads from 1 foot to 350 feet, and with moderate heads is capable of an efficiency of about 75 per cent. at full power. Speed regulation is usually performed either by throttling the supply at its entrance to the wheel by means of a sliding cylindrical or ring gate, or sluice, fitting between the fixed and moving vanes (Fig. 219) and actuated by means of a governor or by throttling the discharge by means of a similar ring gate fitting outside the moving vanes (Fig. 220). This, by increasing the pressure in the wheel, reduces the effective head-producing rotation.

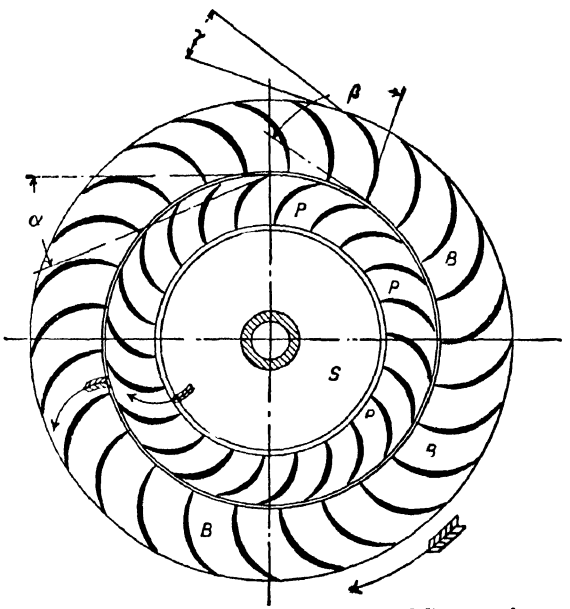


FIG. 218.—Section through Guides and Runner of Fourneyron Turbine.

The former method has the disadvantage that the entering streams of water, after their contraction in escaping past the edge of the ring gate, re-expand to fill the moving buckets, with consequent loss of energy. On account of this the efficiency at part gate is low. It may be improved if the turbine wheel be divided by parallel diaphragms, as indicated in Figs. 219 and 220, into what is in effect a series of wheels in parallel.

In such a wheel this enlargement of section after cut off can only affect one chamber, and the part gate efficiency is in consequence increased. The method has the drawback, however, that the areas of the apertures are reduced and also that frictional resistances to flow are increased by the diaphragms, so that the full gate efficiency suffers. In spite of this, it forms the most general and satisfactory method of governing turbines of this type.

The second method, by increasing the pressure at exit, diminishes the effective head and increases the loss of kinetic energy at exit, and is hence

very inefficient at part gate. In the first turbines erected at Niagara Falls, which were of the Fourneyron type, and of 5,500 H.P., this method of governing was adopted with satisfactory results as regards speed regulation.[1]

Outward flow turbines are in general difficult to govern, because an increase in speed caused by a reduction in load increases the centrifugal

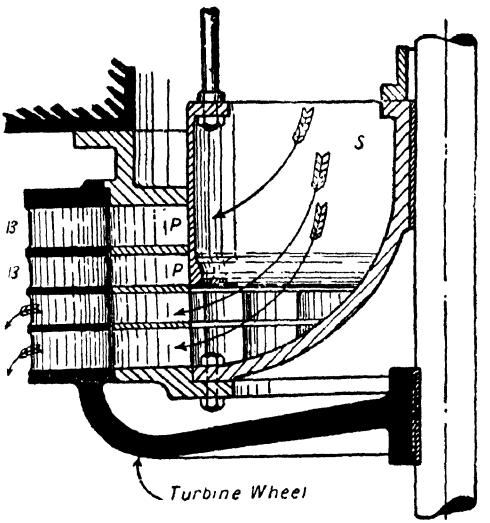


FIG. 219.—Section of Multiple Chamber Fourneyron Turbine Runner and Guides, with Inside Cylinder Gate.

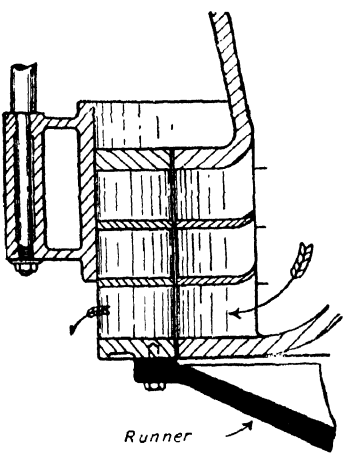


FIG. 220.—Outside Cylinder Gate.

pressure of the water in the turbine wheel, and thus causes an increased outward flow, which tends to increase the speed still further.

[1] These turbines were designed by Messrs. Faesch and Piccard (1895), and work under 132 feet head. A full load efficiency of 82·5 per cent. is said to have been obtained. In this first series of turbines the ring gates are arranged to open downwards, the idea being that it is safer to allow the gates to open suddenly in case of accident to the coupling rods than to close suddenly, because of the probable effect of water ram in the latter case. The fact that the lower section of both wheels is not opened for the escape of water except at full load, and the possibility of a dangerous accumulation of detritus taking place in these sections, along with the nuisance caused by the violent upward escape of water against the turbine deck, led to the gates in the second series of turbines being designed to shut downwards. The gates are regulated by a mechanical relay governor having the gearing, which moves the gates operated by clutches put in motion by changes of speed, the motion of the governor balls allowing one or another pawl to gear with a ratchet wheel, which gives motion through the clutches and gearing to a rack coupled to a lever on the lay shaft from which the gates are directly worked.

A counterbalance weight is used to balance the weight of the gates and coupling rods.

While these turbines have given satisfactory results, the whole of the later series of turbines are of the inward radial flow type.

The efficiency of the Fourneyron turbine may be increased by the addition of the diffuser invented by Boyden about 1844.

This consists of a fixed annular casing (Fig. 221), surrounding the wheel and fitting closely to the outer periphery of the moving vanes. The sides of this casing diverge gradually, its depth increasing from that of the buckets to about twice this, and its width being about four times that of the wheel. By this means, part of the kinetic energy of the discharge is converted into pressure energy, and since the pressure at the outside of the diffuser is that corresponding to the depth of immersion, the pressure at the inside and at the exit from the turbine wheel is less than this, so that the effective head

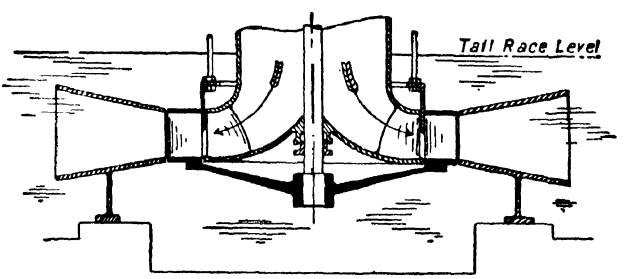


FIG. 221.—Outward Radial Flow Turbine with Diffuser.

is increased. In this manner the efficiency may be increased by about 6 per cent. in a well-designed turbine under moderate head, the proportional increase being less as the head increases.

The outward flow turbine suffers in efficiency from the fact that its passages are of necessity divergent and that flow through these diverging passages is always accompanied by loss of energy in eddy production. It is, moreover, an expensive machine, not easy to govern well, and has been generally replaced by turbines of the Francis or some other more modern type.

The following results of tests on a Fourneyron turbine fitted with an internal cylinder gate are given by Unwin :—

Gate opening. . .	1·0	·875	·625	·375
Efficiency . . .	·62	·60	·43	·30

ART. 155.—THE JONVAL TURBINE.

By shortening the casing of the Borda turbine, and adding suitable guide vanes to direct the water under pressure into the wheel, Jonval (1843)

devised a form of turbine which had many advantages over and largely displaced the Fourneyron turbines then in use. In this, which is an axial flow turbine, water is directed by means of radial guides into a series of radial vanes (Fig. 222). Since each particle of water in its motion through the wheel remains at approximately the same distance from the axis, the effect of centrifugal force on the flow becomes negligible, and the difficulty in governing which is so pronounced in the Fourneyron turbine is thus removed.

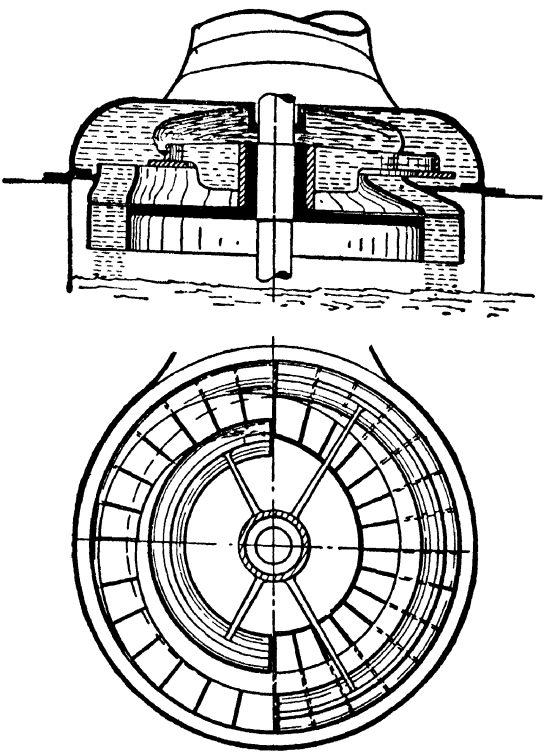


FIG. 222.—Jonval Turbine.

In all axial flow turbines, however, whether of the pressure or impulse type, the linear velocity of points at the entrance to the buckets varies with the radius, and as the velocity of efflux from the guide passages is approximately uniform, there is, with radial vanes, only one particular radius at which the ratio between the velocities of the wheel and of efflux is suited to any given blade angles. For efficient working, and to avoid undue shock at entrance, the radial width of the buckets must then be small, compared with the radius of the wheel, and in general should not exceed one-fifth of the latter.

Governing was usually performed by cutting off the supply to one or more guide passages by a circular slide (Fig. 223), or by a scroll gate, and this greatly reduces the efficiency at part gate.

The most important improvement in the design of the Jonval turbine consisted in the subdivision of the wheel into concentric compartments, each forming a complete turbine.

Then, by regulating the water supply to each compartment in turn, and

if necessary, completely cutting off the supply to one or more compartments, the wheel could be utilised with fairly high efficiencies, and, by supplying compartments at different distances from the axis, could be given a constant speed under varying heads. Thus, with a double turbine, the outer compartment, having a higher velocity, would be used with a minimum supply and a maximum head, while the outer and inner compartments together would be designed to give the same power with a reduced head and an increased supply. The vane angles should be different in the two compartments.

This type of turbine combined the advantages of fairly high efficiency

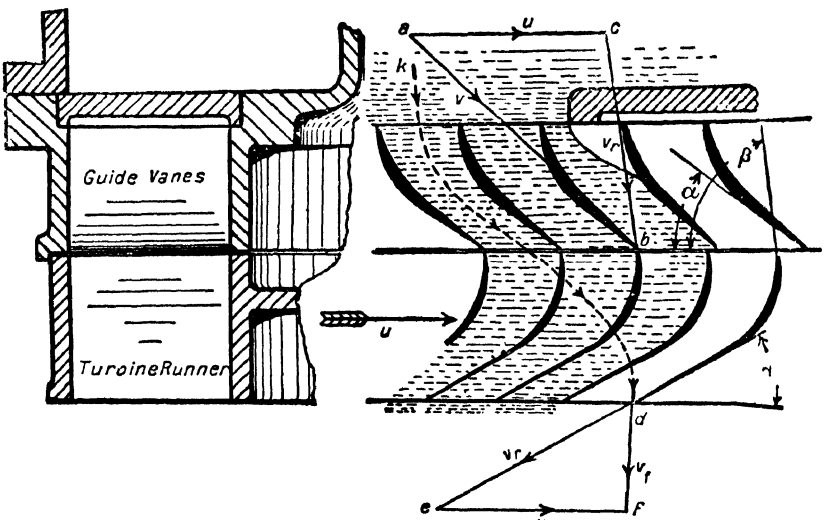


FIG. 223.—Section through Runner and Guides of Downward Flow Jonval Turbine.

at part gate and of constant speed under variable head in a manner which is unequalled by any but the radial inward flow or Francis turbine, and was therefore suitable for variable conditions of working. It could be applied to heads from 2 feet to 180 feet, and for speeds from 20 to 400 revolutions per minute, but is more particularly fitted for low and medium heads with large quantities of water, the large area of the water passages enabling it to pass large volumes of water. Since European users are in general compelled to take advantage of such sources of power, this largely accounts for the favour with which the Jonval turbine was received and for its large development in Europe before the introduction of the Francis turbine.

Art. 156.—The Suction or Draft Tube.

The invention of the draft tube about 1840 greatly increased the possibility of adapting the pressure turbine to suit local conditions. This consists in arranging the discharge pipe so that its lower end always discharges below the surface level in the tail race. By this means the turbine may be elevated above tail-race level without loss of head, for since (neglecting the kinetic head in the tube) the pressure in the draft tube at tail-race level is equal to that of the atmosphere, if the diameter is uniform that at the top of the tube will be less than atmospheric by an amount equivalent to the difference of level between turbine and tail race. The available head, measured from head water to the discharge side of the turbine, is thus the same as if the turbine were erected at tail-water level and discharged under atmospheric pressure.

In such a draft tube, the velocity of flow into the tail race is the same as that of discharge from the turbine, and the corresponding kinetic energy is lost.

The mean velocity of discharge from the runner varies from about $\cdot 2\sqrt{2gh}$ in a slow speed turbine under high head to as much as $\cdot 55\sqrt{2gh}$ in a modern high-speed, low-head turbine. In the latter case the discharge still contains, in the form of kinetic energy, some 30 per cent. of the energy available in the supply water, and it becomes vitally important to provide means of recovering a large proportion of this in the form of pressure energy before it enters the tail race.

This may be accomplished by using a draft tube of gradually increasing diameter, so as to reduce the velocity from its original value v_1 to v_2 before discharge. The head now lost is equal to $v_2^2 \div 2g$ feet plus the loss due to friction and eddy formation in the tube. Expressing this loss as $k(v_1 - v_2)^2 \div 2g$, the head regained is

$$\frac{v_1^2 - v_2^2 - k(v_1 - v_2)^2}{2g} \text{ feet.}$$

Since the pressure at the point of discharge from the draft tube is constant, being regulated solely by the depth of its submersion, any such reconversion of kinetic into pressure energy reduces the pressure on the discharge side of the turbine, and so increases the effective head.

In a straight tube the value of κ depends on the angle θ included between the opposite sides of the tube, as indicated in the following table [1] :—

[1] See Art. 45.

ϑ.		5°	10°	15°	20°	25°
κ		·13	·17	·27	·42	·62
Total loss of head expressed as a fraction of $v_1^2 \div 2g$ where	$v_2 = \frac{v_1}{4}$	·13	·16	·21	·30	·41
	$v_2 = \frac{v_1}{3}$	·17	·19	·23	·30	·39
	$v_2 = \frac{v_1}{2}$	·28	·29	·32	·35	·40

The last three lines of the table show the total loss of head, including that due to the final velocity v_2, when v_2 is respectively one-fourth, one-third, and one-half of v_1.

With a fixed length of draft tube, the choice often lies between a small angle of flare, with a relatively small ratio of $v_2 \div v_1$, or a larger angle and a larger velocity ratio. The latter gives a greater loss in the tube but a smaller loss at discharge, and the best proportions are such as give the minimum total loss. If, for example, the diameter of the tube at entrance is 4 feet, and the available length is 20 feet, an angle of flare of 5° will make the outlet diameter 5·75 feet; the ratio $v_1 \div v_2 = 2{\cdot}06$; and the total loss of head $\cdot 27\, v_1^2 \div 2g$. An angle of 10° will make $d_2 = 7{\cdot}5$ feet; $v_1 \div v_2 = 3{\cdot}52$; and the loss of head $\cdot 18\, v_1^2 \div 2g$, while with an angle of 15°, $d_2 = 9{\cdot}27$ feet; $v_1 \div v_2 = 5{\cdot}37$, and the loss of head $= \cdot 21\ v_1^2 \div 2g$. In this case the best angle would be slightly over 10°.

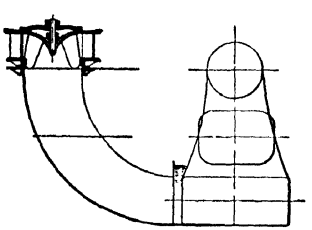


FIG. 224.

By bending the tube so as to discharge in the direction of flow in the tail race (Fig. 224) a longer tube may be used with a given turbine elevation, and the kinetic energy of discharge is partially utilised in assisting this flow and is not entirely wasted. The losses in a bent tube are, however, appreciably greater than in a straight tube.[1]

In the conventional theory of turbine design it has been usual so to arrange the vane angles as to discharge the water from the runner without any tangential velocity. Even with a slow-speed, high-head turbine, however, this is only the case at one gate opening, while with a high-speed, low-

[1] "The Efficiency of Regain in Straight and Bent Draft Tubes," Gibson and Latrow. Inst. C.E. Selected Engineering Papers No. 34, 1926.

head runner there is a considerable velocity of whirl at all gate openings. It is usual to assume that this whirl is analogous to that in a free vortex, in which case we have

$$\frac{p_2 - p_1}{w} = \frac{{w_1}^2 - {w_2}^2}{2g} = \frac{\omega^2}{2g}\left\{\frac{1}{{r_1}^2} - \frac{1}{{r_2}^2}\right\}$$

where ω is the angular velocity ; and r_1 and r_2 are radii of which r_2 is the greater.

Allowing for the eddy losses accompanying outward radial flow, the gain of pressure due to the conversion of the kinetic energy of whirl in a draft tube having radii of r_1 and r_2 at entrance and exit would be equal to

$$\frac{k^1}{2g}({\omega_1}^2 - {\omega_2}^2)$$

where k' is less than unity. There is little definite data as to the actual value of k', but it is probably about ·6 in the average draft tube.

The straight conical draft tube utilises a certain proportion of this energy of whirl. Thus, if $r_2 = 2\,r_1$ and if $k' = ·6$, some 45 per cent. will be con-

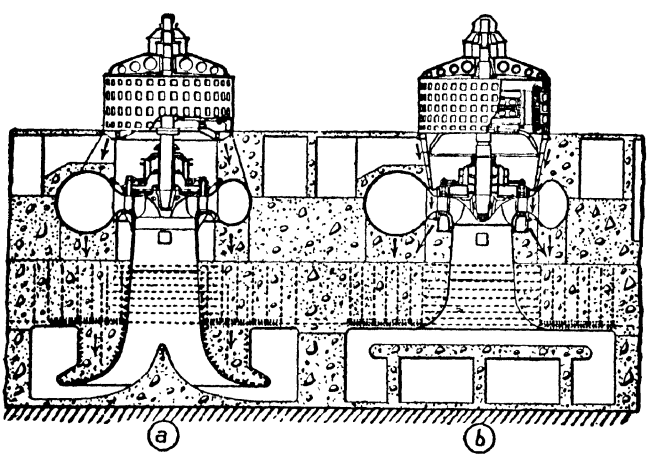


FIG. 225.

verted into pressure energy. With a view of still further reducing the loss, two types of tube have been introduced of recent years in which the water, at the point of discharge from the conical draft tube, is constrained to flow outwards between parallel surfaces of considerable radius. This gives a very low velocity to both the components of discharge.

It also enables the overall length of the draft tube to be diminished and, in a vertical turbine, enables the water to be discharged in the direction of

flow in the tail race, so that its residual kinetic energy is directly utilised in assisting this flow.

These types of draft tube are known respectively as the "spreading draft tube" and the "hydracone regainer." In the latter (Fig. 225 *b*) the water flowing down the draft tube is discharged against a flat plate, which changes its direction, while in the former a conical surface is provided for the same purpose (Fig. 225 *a*).[1] Since, in a free vortex, the pressure is a minimum at the centre, trouble may be caused if the velocity of whirl is large by the liberation of air at the centre of the draft tube immediately underneath the runner, and to prevent this in some recent low-head plants the central cone has been extended up the tube as far as the runner.

Incidentally, a conical draft tube tends to improve the speed regulation. With quick regulation on a falling load, the inertia of the suction column tends to break the column at the top of the tube and to produce a vacuum in the turbine casing. Any such separation is followed by a reflux up the draft tube which may cause severe water-hammer effects. Even a comparatively small change of load may, in a long draft tube, set up pulsations which are detrimental to steady running and which are reduced by the use of a conical tube. As an example of this effect, consider the case of a vertical parallel draft tube l feet long dipping h_d feet below the surface in the tail race. If the head equivalent to the kinetic energy of the discharge for the runner is h_k, if the retardation is α f.s.s., and if the barometric height is 33 feet, separation will occur if

$$33 - h_k - (l - h_d) = \frac{l}{g}\alpha$$

i.e., if

$$\alpha = \frac{32}{l}\left\{33 - h_k - l + h_d\right\}.$$

If, for example, $l = 25$; $h_d = 2$; $h_k = 4$, the necessary retardation is only 7·7 f.s.s.

The maximum elevation at which a turbine may be placed depends largely upon the velocity of discharge and especially on the velocity of whirl. It thus depends essentially upon the working head and the design of the runner. Moreover, owing to the greater tendency to form an air core in tubes of large diameter, the maximum elevation diminishes somewhat with the size of the turbine. If w and f be the maximum components of tangential and axial velocity for any gate opening, the working elevation should not exceed

$$30 - \frac{w^2 + f^2}{2g} - \frac{l}{g}\alpha.$$

[1] The diagram shows the two types of tube as fitted to two of the 40,000-H.P. units of the Niagara Falls Power Company.

Thus in a high-speed turbine, under a head of, say, 40 feet, in which $\frac{w^2+f^2}{2g} = \cdot 40\,h$, and in which a possible retardation of 8 f.s.s. may occur on a sudden reduction of load, the permissible elevation would not exceed 13 feet, whereas in a small slow-speed turbine it might be as much as 25 feet.

The draft tube is applicable to any type of pressure turbine, but more particularly, for mechanical reasons, to the axial discharge type. It is not commonly used with impulse turbines, though it may be adapted if desired.

The pressure turbine thus fitted possesses many advantages over the submerged type. The turbine runs dry when the head water is cut off, and is easily accessible : a horizontal shaft machine may frequently be adopted where otherwise this would be impossible, and the general accessibility for examination and repairs is greatly increased, while the risk of damage to machinery by floods is diminished.

With a single horizontal shaft turbine, or with two horizontal turbines on the same shaft, if these discharge outwards, it is necessary for the wheel shaft to pass through the draught tube as in Fig. 238, and a stuffing box becomes necessary to prevent leakage of air into the tube. Tightness is commonly assured by means of a water seal, consisting of a chamber C surrounding the shaft (Fig. 226), and supplied with pressure water from the penstock by means of a small pipe P. Any slackness at the gland then allows this pressure water to escape outwards, or into the draught tube, and does not lead to air leakage.

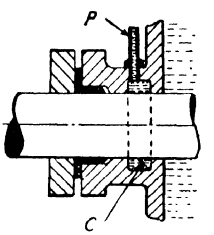


FIG. 226.—Water Seal for Draught Tube Stuffing Box.

ART. 157.—THE INWARD FLOW TURBINE.

The next important step in turbine development was due to J. B. Francis (1849) in America and Professor James Thomson in England, who, placing the guide vanes outside the wheel, obtained an inward radial flow turbine. Several important advantages accompanied this change in design.

(1) The guide and wheel passages, now being convergent, the tendency to eddy formation is reduced.

(2) The increased accessibility of the guides makes it possible to use improved methods of regulating the flow of water to the wheel.

(3) Any increase in the speed of rotation by increasing the centrifugal pressure at the outer periphery of the wheel tends to reduce the flow into the wheel, which becomes, to a certain extent, self-regulating.

(4) The centrifugal pressure of the water balances the pressure due to an

appreciable fraction of the supply head. This fraction depends upon the vane angles, but may be as high as ·66. Only a portion of the supply head is absorbed in producing velocity of efflux from the guide vanes, the remainder

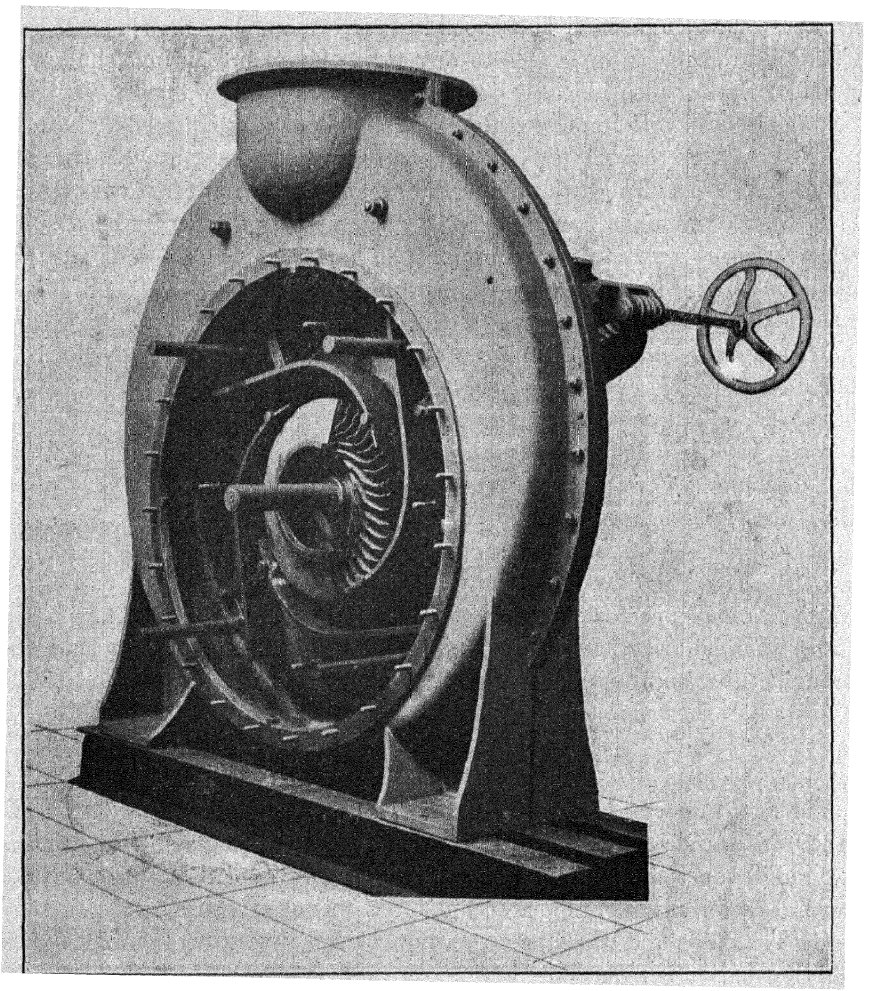


FIG. 227.—Thomson Vortex Turbine.

being gradually absorbed in its passage through the wheel. The velocity of efflux thus may be comparatively low, hydraulic friction losses are reduced, and the corresponding reduction in the peripheral speed of the wheel enables it to be applied successfully to very high heads up to about 800 feet.

(5) The supply water, when moving with its maximum velocity (at

entrance), is admitted to the most rapidly moving part of the wheel, and leaves it at points where its peripheral velocity is comparatively low. This

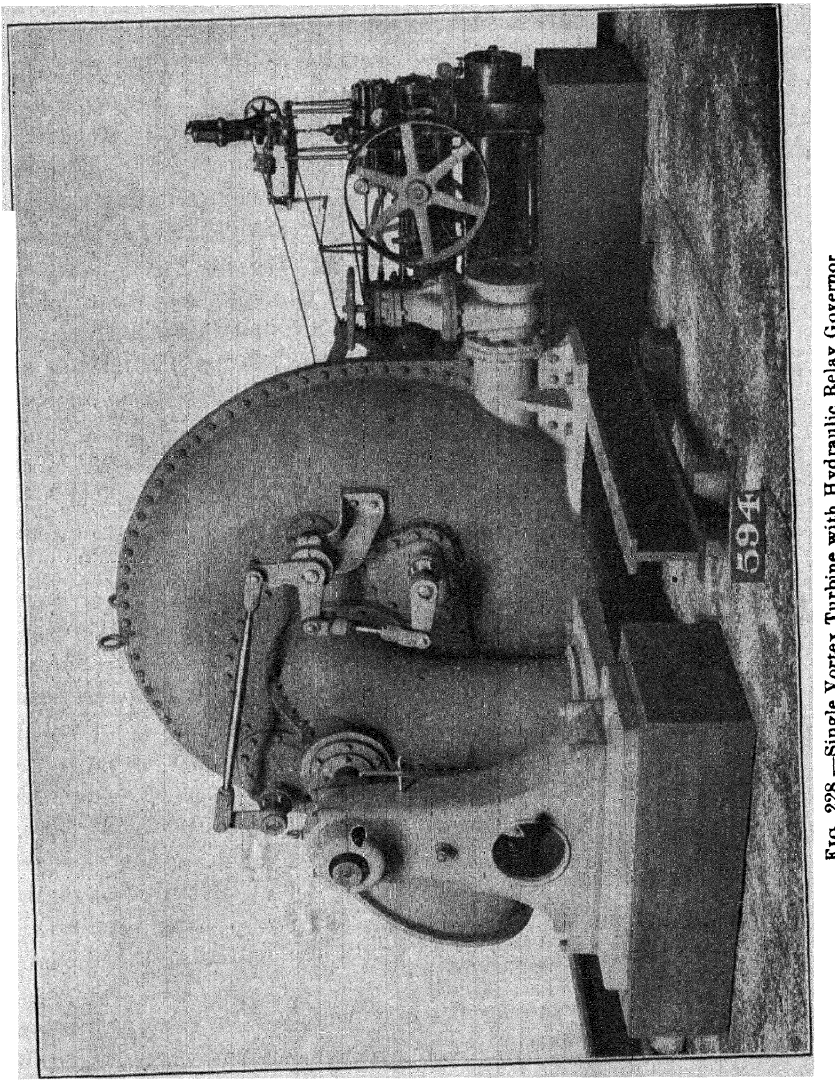


FIG. 228.—Single Vortex Turbine with Hydraulic Relay Governor.

facilitates the adoption of vane angles designed to give entry without shock and discharge without excessive loss of kinetic energy.

The type of turbine designed by Professor Thomson is illustrated in Fig. 227. It is a purely inward radial flow machine, with the runner mounted

inside a cast iron casing, termed the vortex chamber. In the original designs this casing was in the form of a spiral, the supply water entering tangentially at the largest part of the spiral and sweeping around the casing, whose area was so designed as to give a uniform velocity of whirl at all points of the circumference. Fig. 227 [1] shows a less correct type of construction, in which the water enters the casing radially. The entering water is directed into the wheel buckets by a series of guide vanes placed around the outer circumference, these being designed so as to follow the lines of flow in a spiral vortex. These guides vary in number from four to eight, and are movable, being pivoted near their inner ends, so as to be approximately in balance. They are coupled together by a series of bell-crank levers and links as indicated in Fig. 228,[2] so as to rotate together and shut off water equally from all parts of the wheel. Motion is given to these guide vanes either by hand or by an automatic governor.

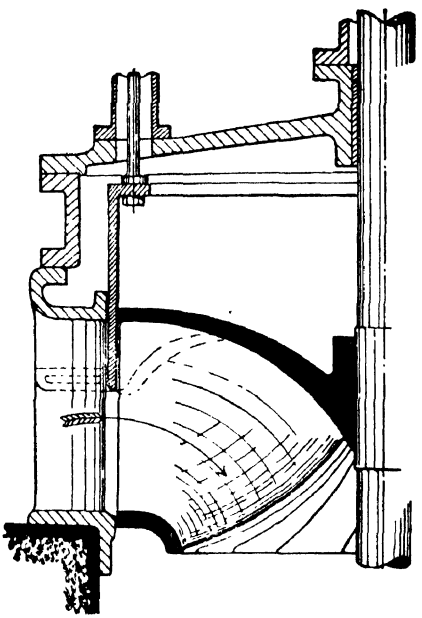


FIG. 229.—Francis Turbine with Inside Cylinder Gate Regulation.

This turbine may be constructed either with a double or single wheel. In the double wheel a series of vanes is fixed on either side of the runner disc. Discharge takes place radially at the centre, and the water is then diverted axially through two discharge pipes, or draught tubes, placed one on each side of the wheel. This gives a wheel which is in balance as regards end thrust. To facilitate the discharge, the wheel vanes are usually curved at their outlet edges so as to direct the water in the direction of the axis.

In the single wheel the vanes are fixed on one side of the rotating disc, and discharge takes place on one side only. The disc is therefore subject to end thrust, which must either be hydraulically balanced or taken up by means of a suitable thrust bearing. This disadvantage, which is common to all single discharge pressure turbines, may be overcome by mounting a pair of

[1] By courtesy of Messrs. G. Gilkes & Co., Ltd., Kendal.

[2] By courtesy of the Platt Iron Works, Dayton, Ohio.

similar wheels on the same shaft, the flow being in opposite directions through the two wheels. The two end thrusts thus balance each other.

In the earlier Francis turbines, regulation was usually performed by

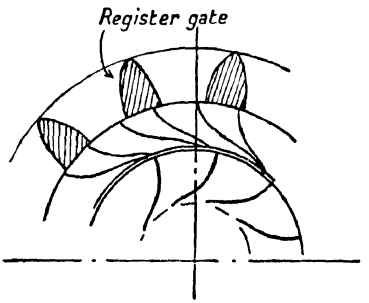


FIG. 230.

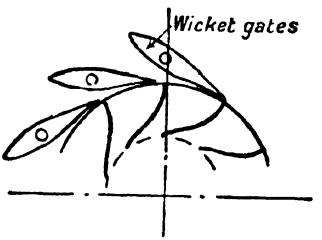


FIG. 231.

cylinder gates or register gates. The cylinder gate (Fig. 229) consists of a plain cylinder sliding axially between the guide vanes, which are fixed, and the runner. Its axial position is regulated by the governor. The register

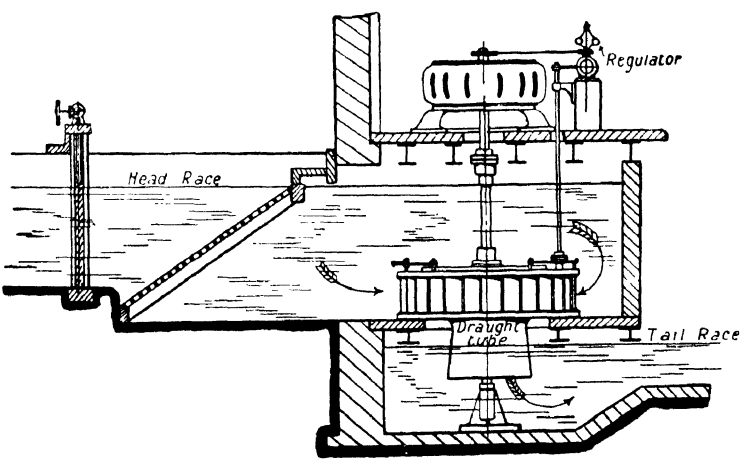


FIG. 232.—Arrangement of Vertical Shaft Single Vortex or Combined Flow Turbine working under Low Head in Open Forebay.

gate (Fig. 230) consists of a cylinder having appropriate waterways and capable of rotation about the axis of the turbine. It was usually fitted outside the guide-vane ring.

With either type excessive eddy formation is produced at part gate, giving low part-load efficiencies. The register gate is now obsolete, but

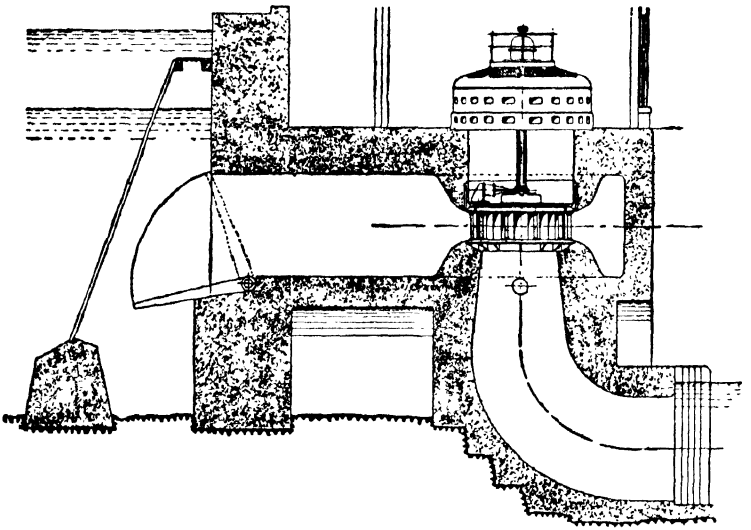


FIG. 233. Francis Turbine developing 3,600 H.P. at 100 revolutions per minute under a head of 35 feet. (Side elevation.)

the cylinder gate is still used in small plants having a fairly constant load and where low part gate efficiency is unimportant. It is cheap and not so easily deranged as the wicket gate, as the system of pivoted guide vanes (Fig. 231) is termed. The latter arrangement is, however, the only one now used in important installations. Such guide vanes give guide passages of gradually convergent form, offering easy curves to the supply water at all gates, and the part gate efficiency is high. They are comparatively light and easily moved, and greatly facilitate speed regulation under sudden load variations. The original form of guide vane, as fitted to the early Thomson turbine, has since been modified by Fink and others. It is now shorter and is pivoted about its centre of pressure, while the number of guides is made approximately equal to the number of wheel vanes. The present-day Thomson turbine and the modified Francis turbine are sensibly identical, and both are commonly known as Francis turbines.

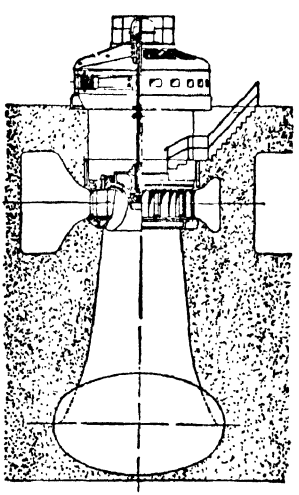


FIG. 234.—Francis Turbine. (End elevation.)

In a low-head installation the turbine may be erected in the open forebay, as shown in

Fig. 232. This method has the disadvantage that the guide-vane mechanism is submerged, and cannot be inspected or repaired without draining the wheel pit, and in most recent important medium- and low-head installations the guide-vane ring is surrounded by a spiral volute chamber, from which the pressure water is delivered with uniform velocity around the entire periphery of the guide ring.

The velocity in this volute ranges from $\cdot 15\sqrt{2gH}$ to $\cdot 25\sqrt{2gH}$, the

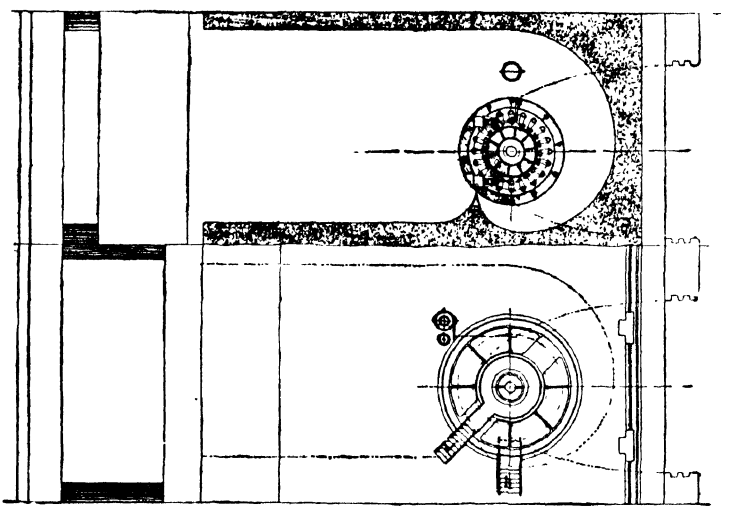


FIG. 235.—Francis Turbine. (Plan.)

higher value applying to low-head plants and the lower to heads exceeding 300 feet.

For heads not exceeding about 100 feet, modern practice has favoured the moulding of the volute chamber in the concrete of the substructure (Fig. 235). Such a construction has many advantages. As compared with an installation in an open forebay, it is more efficient, and only the guide vanes and the runner are submerged. The cross section of the volute may be made of any required shape, while the concrete lends itself to a smooth finish and to the formation of easy curves, both of which tend to efficiency in operation.

For higher heads, considerations of strength necessitate a metal casing, which may be of cylindrical section, but which, for single-runner machines, is usually of spiral volute form (Figs. 236, 237 and 240). This may be of steel plate, cast iron, or cast steel. Owing to the risk of flaws in the casting, and of its unsuitability for withstanding sudden shocks, cast iron is not very

suitable for large casings subject to high heads and liable to water-hammer shocks.

The two sides of the casing are tied together by a series of spacers surrounding the guide-vane ring (Fig. 236). These are usually shaped so as to guide the water into the guide vanes, and form what is termed the "speed ring." The speed ring is also fitted where the volute is moulded in the con-

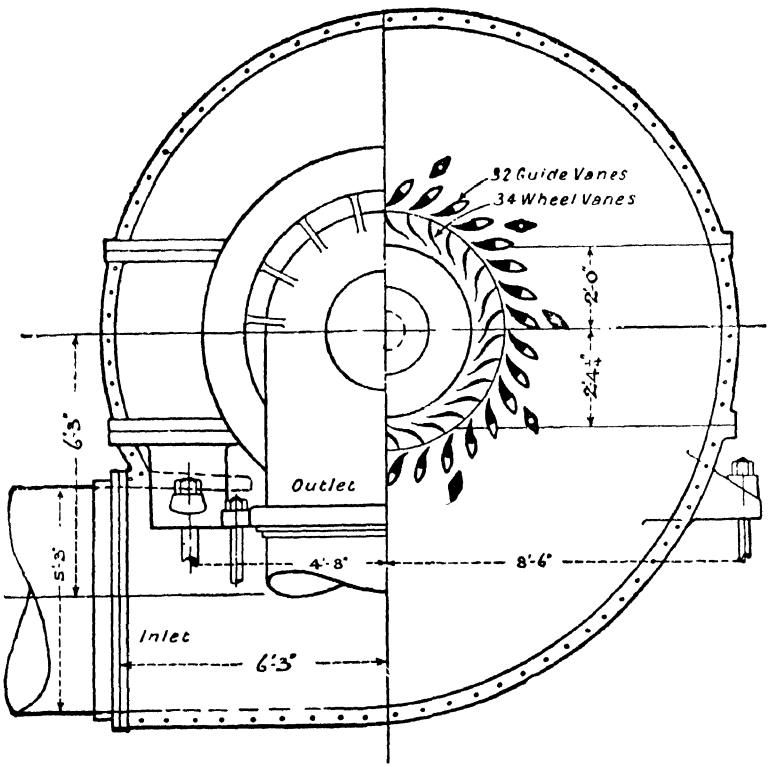


FIG. 236.—Cross Section and End Elevation of Turbine for Snoqualmie Falls.

crete substructure, and then consists of two speed-ring crowns connected by spacers. In modern vertical-shaft units of this type the load is transmitted vertically downwards through the concrete below the generator to the speed ring, and through its vanes or spacers, which act as columns, to the base of the substructure.

Guide Vanes.—The guide vanes or gates are commonly made of cast steel. The stems may be cast in one piece with the vanes, or, in large units, may be keyed to the vanes so as to facilitate the removal of worn parts. Under very

high heads, where the water carries an appreciable amount of grit, bronze guide vanes are advantageous for resisting erosion.

The stems project through stuffing boxes in the turbine casing. Each stem carries a lever, which is coupled to a common regulating ring concentric with the turbine shaft (Fig. 238), whose position is regulated by the govern-

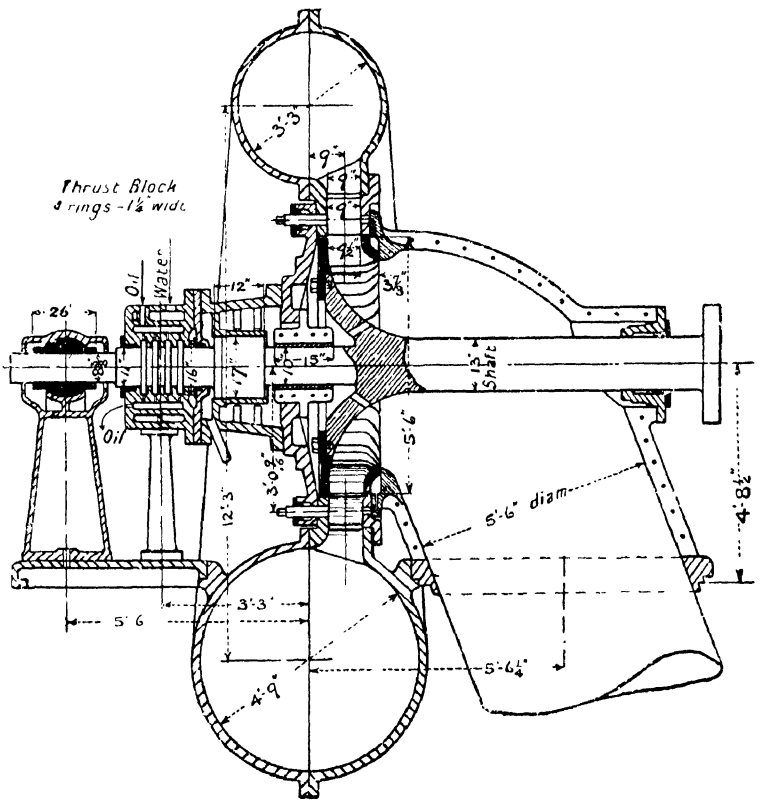


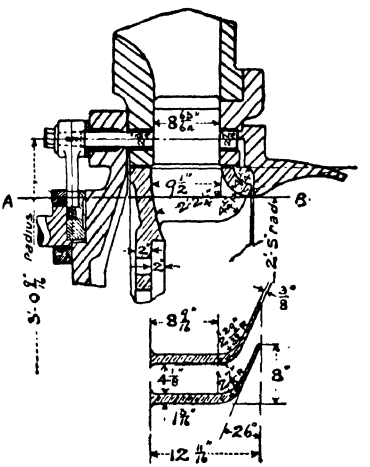
FIG. 237.—Longitudinal Section of Single Wheel Horizontal Shaft Francis Turbine developing 10,000 H.P. at 300 R.P.M. under 260 ft. effective head. Outer diameter of runner 66 ins.

ing mechanism, so that all the guide vanes are opened or closed simultaneously.

The gate stems should be strong enough to resist the stress which would be produced in the case of an obstruction between two vanes with the full effort of the governor concentrated upon them. The links between the levers and the regulating ring should be the weakest part of the system.

Turbine Runners.—For turbines of moderate size under low and moderate

heads, the iron runner, cast in a single piece, is general. In order to facilitate shipment and to provide greater assurance of sound castings in very large machines, the runner is usually cast in four quadrants, which are tied together by a heavy cast-steel crown at the top, and by a cast-steel ring which is a force fit around the lower band of the runner (Fig. 239).[1]



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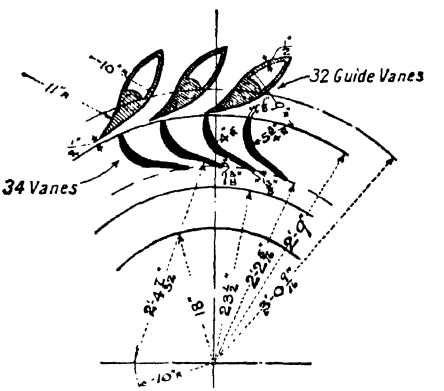
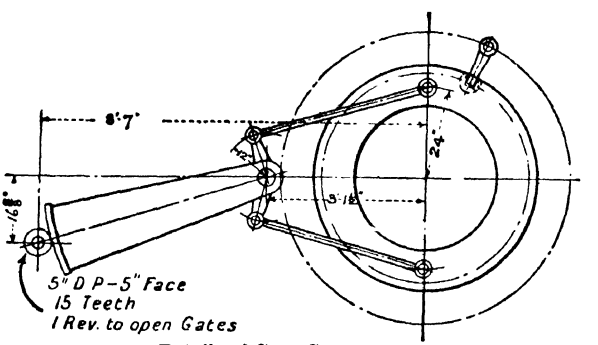


Details of Runner and Guide Vanes.



Details of Gate Connections.

FIG. 238.—Details of Turbine for Snoqualmie Falls.

For slow-speed runners steel-plate vanes may be used with advantage. These are pressed or hammered to shape against a former, and are cast into the turbine crowns. They have the advantage of being thinner and smoother than the cast vane.

In the early development of turbines for high heads, considerable diffi-

[1] By courtesy of the Cramp Shipbuilding Co., Philadelphia.

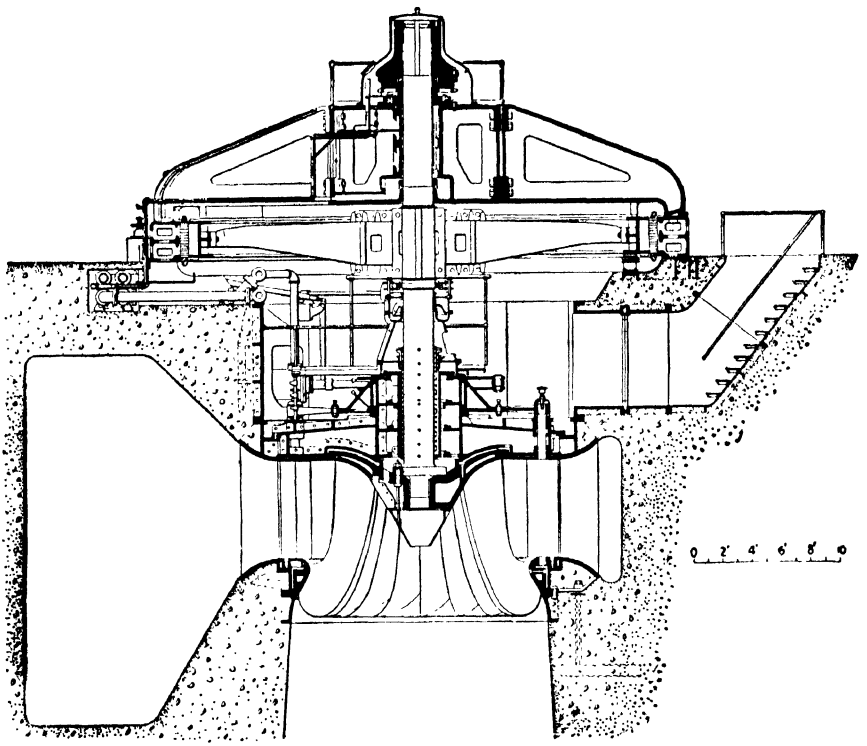


FIG. 239.[1]—18,000 H.P.; 17 ft. 8 in. diameter; 30 ft. head; 55·6 r.p.m.

culty was experienced, due to a corrosion and wastage of the runner especially near the discharge edges of the vanes where mean pressures are low and where, due to eddy formation, local pressures may be much less than the

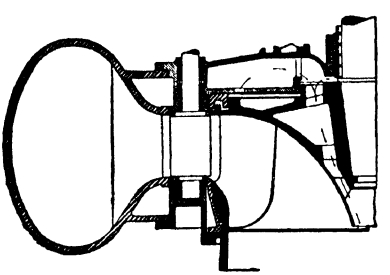


FIG. 240.

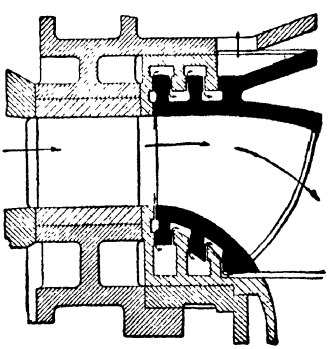


FIG. 241.

[1] By courtesy of the Cramp Shipbuilding Co., Philadelphia

mean. This leads to the phenomenon of *cavitation*. Vapour cavities are formed at regions of the lowest pressures, and these collapse when brought into a region of higher pressure. If in contact with a vane very high impact

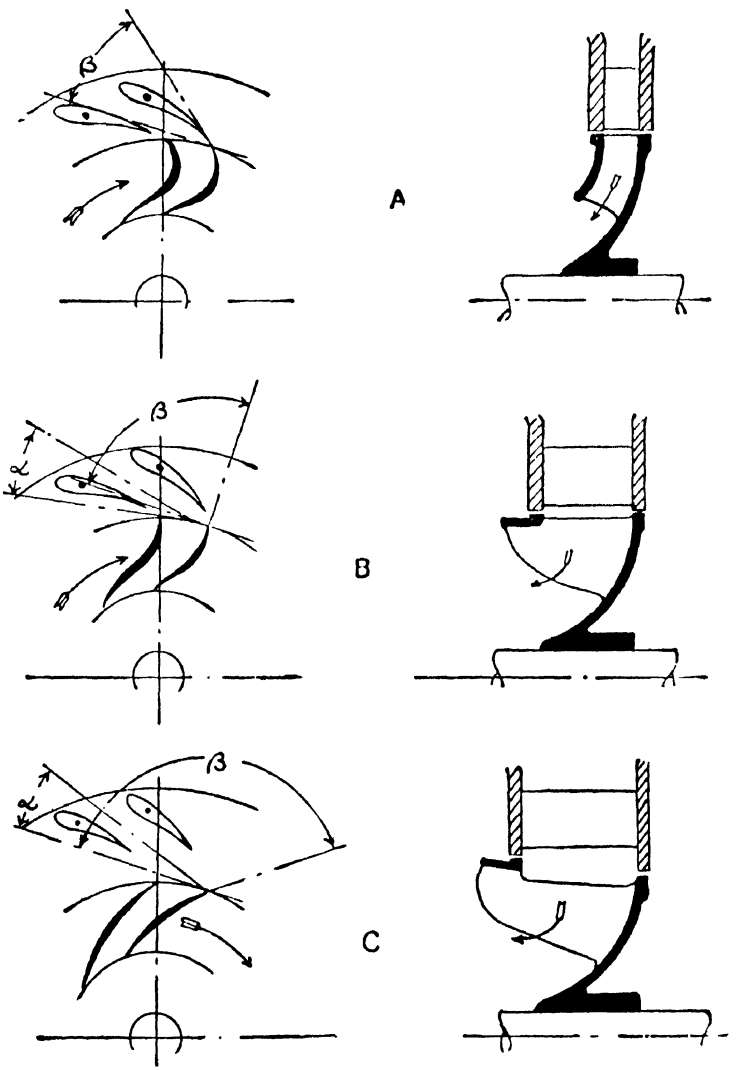


FIG. 242.

pressures of several tons per square inch may be set up at collapse, and these have a disintegrating effect on the metal of the vane. Such corrosion can largely be eliminated by correct design. Corrosion is pronounced at high

heads if the turbines are operated much at part gate, while the practice of running for long periods without load is especially bad for the runners. Where the supply of water is clean, cast-iron runners are to be preferred, as being smoother than cast steel, except where the speed is so high as to necessitate cast steel being used to withstand the centrifugal stresses. Where the water carries grit or sand in suspension the runner is usually made of cast steel if large, and of phosphor bronze if small. Leakage between the periphery of the runner and the turbine casing, which may become a serious source of loss in a high-head plant, may be reduced by the provision of a

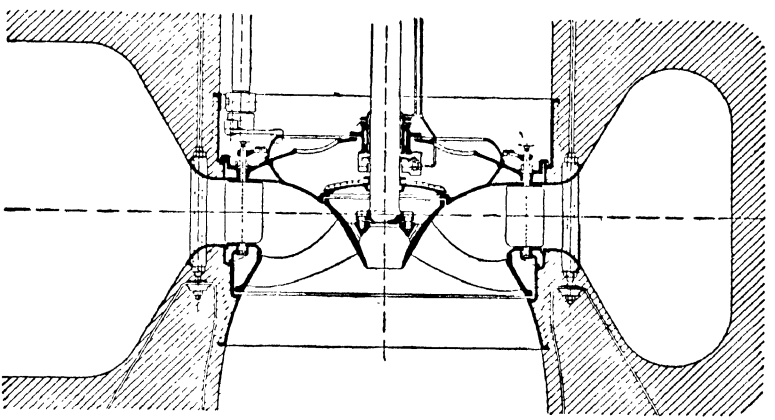


FIG. 243.—Specific speed 118·5 giving 10,000 H.P. under 32·8 ft. head at 93·7 revolutions per minute. Efficiency at full load, 85·0 per cent.

labyrinth seal. Fig. 240 shows the application of a single labyrinth, while Fig. 241 shows a multiple labyrinth adapted for very high heads.

The increasing demand for turbines of high capacity, and capable of operating at fairly high rotative speeds under low heads, has led to a gradual change in the form and proportions of the runner, as indicated in Fig. 242 and in Fig. 243,[1] which shows diagrammatically a modern type of low-head runner. The change has been in the direction of increasing the depth of the buckets, and at the same time of maintaining or increasing the ratio of the discharge area at exit to that at entrance. Also, whereas it was formerly considered essential that the space between the guide vanes and the runner should be reduced to a minimum, the most recent low-head turbines show a very large radial clearance.

During the last ten years turbines of this type have been developed by

[1] By courtesy of Messrs. Vickers, Ltd.

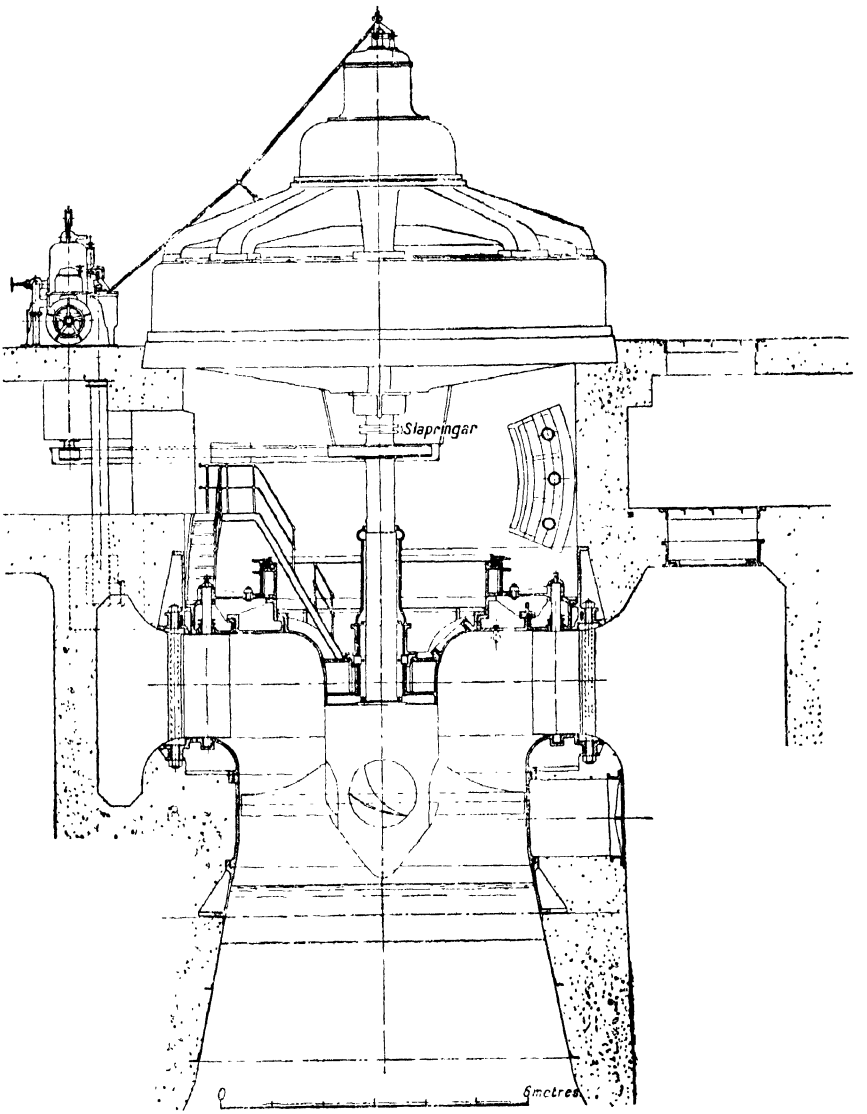


FIG. 244—Section of Kaplan Turbine Runner, diameter 19·0 ft. 11,200 H.P under 21·3 ft. head.

Kaplan [1] in Sweden and Nagler [2] in America. They are characterised by the small number of vanes, which varies from two to four, and by their shape,

[1] "Zeitschrift fur des Gesamte Turbinewesen," November 20th, 1919.

[2] American Soc. Mech. Engineers. December, 1919.

which is sensibly the same as that of the blades of a screw propeller. In the Kaplan turbine these vanes may be rotated so as to suit the direction of

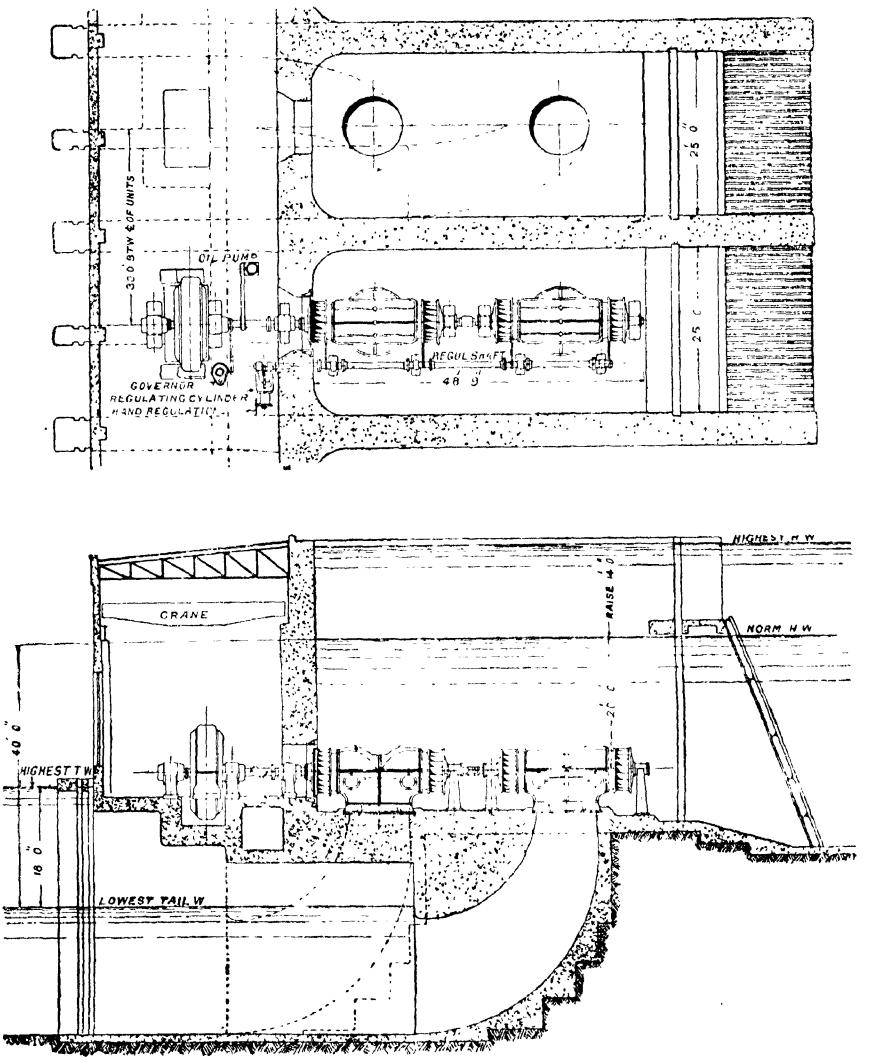


FIG. 245.—Quadruplex low-head Turbine in open flume setting.

inflow of the water at all gate openings. Fig. 244 [1] shows such a turbine arranged to give a maximum of 12,000 H.P. under a head of 6·5 metres

[1] From a paper by G. Molinder, Swedish Water Power Association, April, 1922.

(21·3 feet) at 62·5 r.p.m., corresponding to a specific speed (Art. 167) of 144, or some 50 per cent. greater than is efficiently attainable with the more conventional design of inward flow runner.

Where the maximum speed of rotation possible with a single runner is too low, higher speeds may be obtained by the use of a number of smaller runners, mounted on the same shaft. Fig. 245 shows an installation having four such runners. This arrangement, however, has a number of disadvantages as compared with the single runner. These are :—

(1) A separate series of guide vanes and gate mechanisms is required for each runner, each of which is submerged.

(2) Torsional deflection in the governor layshaft renders it difficult to ensure equal gate openings on all runners.

(3) Owing to shock and interference between the discharge streams from the runners of a double-runner turbine, a greater proportion of the kinetic energy of discharge is lost than in the case of a single-runner turbine. This effect is increased by the impossibility of avoiding sudden changes of direction of flow in the water leaving the casing of the double turbine. Since a high velocity of discharge is a feature of a high-specific-speed turbine, this factor then becomes of special importance.

For these reasons the multi-runner, low-head turbine is being, to an increasing extent, displayed by the single-runner, high-specific-speed machine.

End Thrust on Shaft.—With a single-runner turbine, owing mainly to the static pressure behind the runner due to leakage between the runner and the casing, there is, unless special means are adopted to prevent it, an unbalanced end thrust on the shaft.

Where the head is low, the pressure behind the runner may be relieved by a series of vent holes through the runner crown near the axis. In order to prevent the water behind the runner whirling and so increasing in pressure outwards, due to centrifugal action, a number of radial vanes, almost touching the wheel, are carried by the turbine casing. Any force still unbalanced is taken up by a small thrust block. Where the head is high this method is inadequate, and with a large turbine the end thrust may be so large as to make the provision and maintenance of a suitable mechanical thrust bearing a matter of some difficulty. For this reason the greater part of the end thrust is balanced by hydraulic rather than mechanical means. One method of doing this is shown in Fig. 237.

The greater part of the pressure thrust is eliminated by venting the space behind the wheel into the draft tube through six holes in the wheel disc. There is, however, a resultant thrust towards the draft tube, which increases with the gate opening, and a closer balance is obtained by means

of a balancing piston behind the back head of the wheel casing. This is a forged enlargement of the shaft, 17 inches in diameter, and works in a water-packed brass sleeve.

The chamber in front of the piston is supplied with pressure water from the supply pipe through a strainer, while the space behind the piston is drained into the draught tube. A constant thrust towards the left is thus

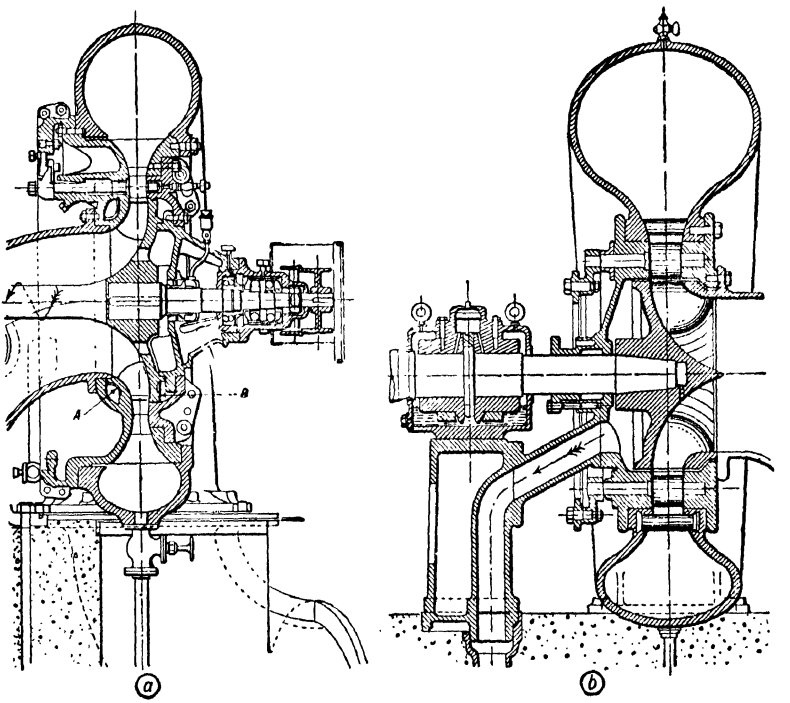


FIG. 246.

produced, and since there is a small leakage past the piston this thrust may be very accurately adjusted to suit the conditions of running by adjustment of the supply valve. Any remaining thrust due to change of load is taken up by a collar thrust bearing situated behind the balance piston. An adjustment of the balance piston supply valve to give balance at $\frac{5}{8}$ full load renders the thrust bearing liable to a possible thrust of about 25,000 lbs. at maximum, or very low loads.

In a second method the areas of the runner at *A* and *B* (Fig. 246 *a*) are made equal. Leakage from the spaces at *A* and *B* is reduced to a minimum by cutting down the radial clearance between runner and casing, and by the

use of labyrinth joints. Any unbalanced force is taken up by a small thrust bearing.

In a third method the space behind the runner is vented to the draft tube by a pipe of sufficiently ample dimensions to ensure the pressure in this space being sensibly the same as that in the draft tube (Fig. 246 *b*). Here, again, any appreciable wear of the runner may render the discharge pipe inadequate in area, giving rise to an unbalanced force for which a thrust block must be provided.

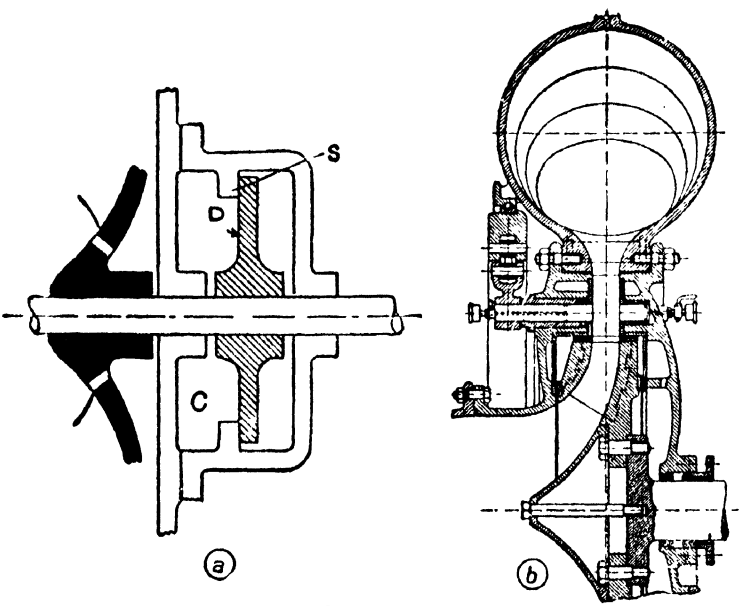


FIG. 247.

Fig. 247 *a* shows a balancing arrangement in which no thrust block is necessary. The space *C* to the left of the balancing disc *D* is supplied with pressure water from the penstock through a regulating valve, while the space to the right is freely vented. When the disc is in contact with the annular ring *S*, the force exerted by the pressure water is sufficient to overcome the force acting in the opposite direction on the runner, and the shaft moves to the right. A very small motion allows sufficient leakage between the disc and the seating ring to reduce the pressure in the chamber *C* to that necessary to produce equilibrium, and, in practice, the shaft oscillates laterally about this position within very narrow limits. In general, the lateral movement does not exceed about ·01 inch. Fig. 247 *b*,[1] shows one modern design in

[1] Bergstrom, "Proc. Inst. C. E.," February, 1920.

which the runner has been modified so as to act as its own balancing disc. Here the radial clearance between the runner and the casing is reduced to a

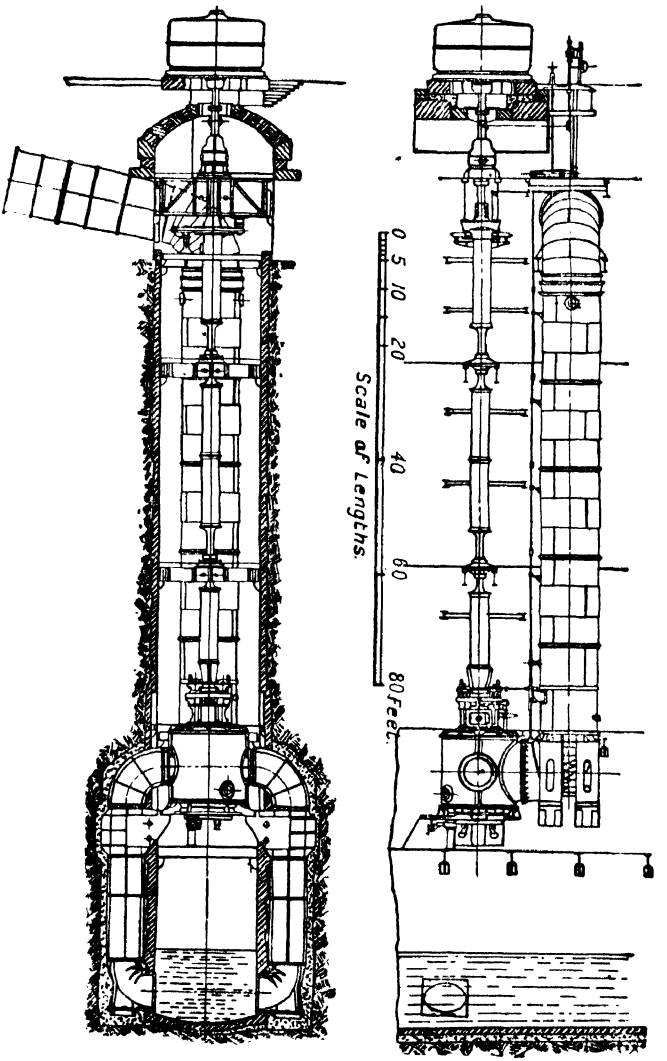


FIG. 248.—General arrangement of Double Francis Turbine developing 10,250 H.P. at 250 revs. per min. under 133 feet head. Canadian Niagara Power Co.

minimum. The two faced rings on the front and back faces of the runner work, with a normal clearance of about ·02 inches, over corresponding rings on the turbine casing. Any unbalanced force on the runner—say, to the left—

causes it to move to the right, increasing the clearance between the left-hand rings and reducing that between the right-hand rings. This reduces the pressure in the left-hand space and increases that in the right-hand space until a position of equilibrium is attained.

Balance Piston for Vertical Turbines.—In the case of a vertical shaft

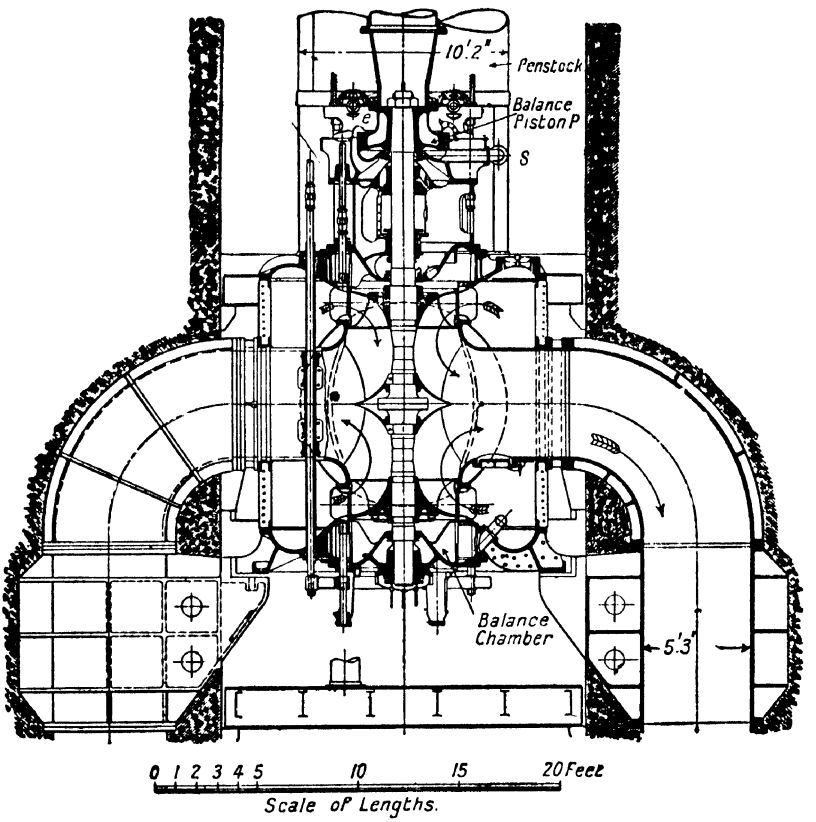


FIG. 249.—Sectional Elevation of Double Francis Turbine for 10,250 H.P. Canadian Niagara Power Co.

turbine, installed at the bottom of a wheel pit and transmitting its power through a long vertical shaft, Fig. 248, the vertical load to be carried is very large, and to obviate the necessity of carrying the whole of this load on a mechanical thrust bearing, balancing pistons have been used in a number of cases. Fig. 249 shows this method as applied in the case of early double Francis turbines of the Canadian Niagara Power Company. Here the weight of the rotating parts is 120 tons, and is balanced partly by the upward pres-

sure on the bottom face of the lower runner, water under the full pressure of the supply head, 133 feet, being admitted to a balance chamber beneath this runner, and partly by the upward pressure on the rotating balance piston, which is subject to the same pressure. Leakage past this piston is drained away to the tail race, and, by adjusting the valve on the supply pipe *S*, the upward pressure may be regulated with great nicety.

Any unbalanced load is supported by the suspension bearing (Fig. 250) which is placed on the upper deck. In this bearing oil, under a pressure of 375 lbs. per square inch, is supplied to the annular chamber *C* surrounding the bush *B*, and escapes outwards between the fixed and rotating discs at *D*. These discs have an outside diameter of 36 inches and a bearing area of 780 square inches. Any slight swing or lateral wear of the shaft is permitted by the spherical bearing of the lower disc.

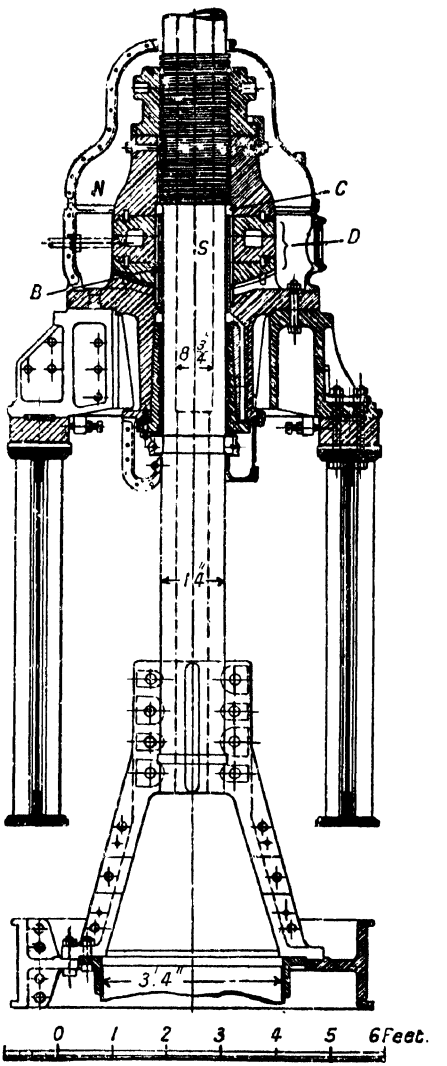


FIG. 250.—Main Footstep Bearing for 10,250 H.P. Turbine.

Thrust Bearings.—The thrust bearing for a horizontal shaft unit is usually of the simple collar type, incorporated as part of one of the main bearings. Where the end thrust is comparatively small, ring-oiled, self-lubricating bearings or ball thrust bearings are satisfactory. For large pressures forced lubrication is advisable, with some cooling device for dissipating the heat generated at the bearing. The bearing may be water-jacketed, or the oil may be cooled by a cooling coil containing circulating water in the oil tank. Bearing surfaces should be babbitted. A bearing forming part of, or bolted to, the turbine casing is preferable

to one separately supported, since the latter is more likely to get out of alignment.

Bearings for Vertical Shafts.—In a small unit, erected in an open forebay, the thrust bearing often consists of a plain footstep bearing supporting the lower end of the shaft (Fig. 258), and provided with a lignum vitæ bearing pad and bearing strips. In large units the bearing is never submerged. It

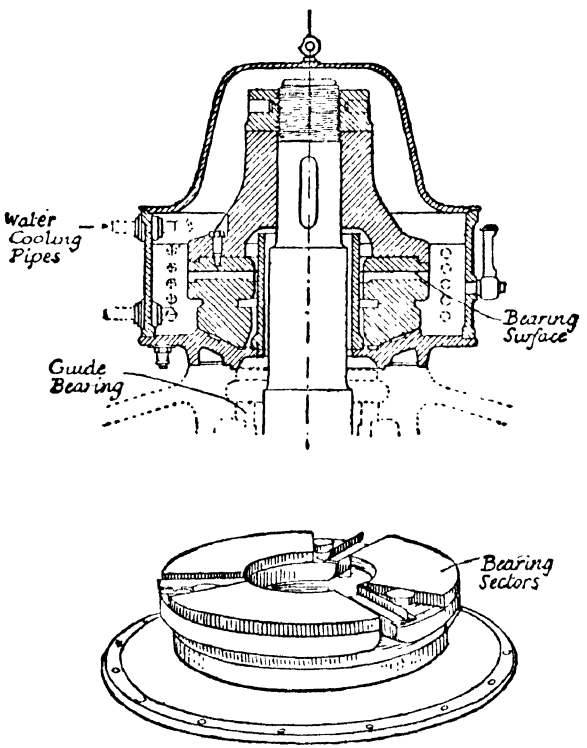


FIG. 251.

may be carried by a cast-iron truss, supported from the speed ring of the turbine, and situated below the generator as in Fig. 252, or may be situated above the generator, as shown in Fig. 239. The latter method has the advantage of being more accessible, and gives a more compact construction. The upper guide bearing of the generator is located immediately below the thrust bearing.

Owing to the fact that the whole of the weight of the rotating parts of both turbine and generator is carried by the thrust bearing, this forms a very

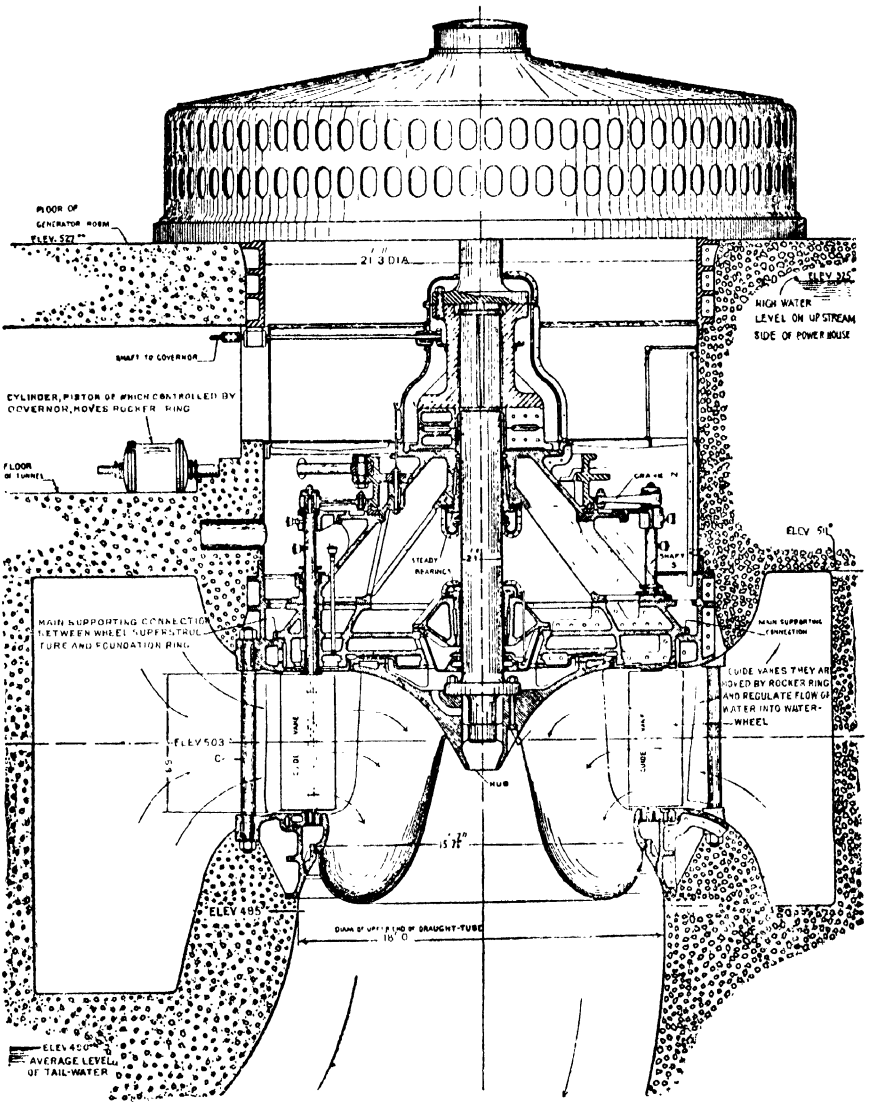


FIG. 252.—Keokuk Turbines of the Mississippi River Power Co. Overload capacity 13,000 H.P., under 34 feet head.[1]

important feature in the design of such a unit. Until comparatively recently, the type of oil-pressure bearing shown in Fig. 250 was used almost exclusively for large units. The danger accompanying any failure of the oil supply

[1] By courtesy of the Cramp Shipbuilding Co., Philadelphia.

has, however, led to the more general use, in modern units, of some form of bearing not requiring a pressure oil-supply. By replacing the lower disc by a series of blocks, each of which has an alternate inclined and flat sector, the oil, supplied under gravity, is fed into the segments by the rotation of the shaft, and a simple bearing (Fig. 251) is produced which is capable of good results under large loads. Bearings of this type have been used for loads up to 80 tons [1] at speeds up to 375 r.p.m. Tests show them capable of withstanding pressures of approximately 1,400 lbs. per square inch at peripheral speeds of 70 feet per second.

An elaboration of this idea has led to the so-called Michell or Kingsbury bearing, in which the stationary disc is made up of several babbitted segments, each of which is mounted on a pivot to enable it to adjust its angle of inclination to the rotating disc, so as to ensure an approximately even distribution of loading over its surface. This bearing also has the advantage that the oil supply does not need to be under pressure. In such bearings, applied to the Keokuk turbines (Fig. 252), the diameter is 56 inches, the total load 255 tons, and the mean bearing pressure 350 lbs. per square inch. Tests show that the friction loss in these bearings amounts to between 7·5 and 10 kw. at the normal speed of 100 r.p.m., or approximately one-tenth of 1 per cent. of the output of the turbine.

In some more recent installations a roller bearing of reduced dimensions is placed inside the main bearing. The roller bearing is made with a slight clearance, and only comes into operation if any wiping of the shoes of the main bearing causes the bearing plate to settle.

Roller bearings have given excellent results in many modern plants, under loads up to 250 tons and at speeds up to 300 r.p.m., while for light loads ball bearings are very satisfactory. Both types of bearing are sensibly independent of any oil supply.

In some recent large units spring thrust bearings have been used in which a flexible plate divided into sections by from six to ten grooves takes the place of the pivoted blocks of the Michell bearing. This is supported by a large number of short, stiff, spiral springs, with a view of obtaining an even distribution of loading over the bearing. There is, however, a danger of vibration being set up with such a bearing.

A number of typical test results of small- and medium-sized Francis turbines in different types of setting are given in the following table,[2] the general arrangement of each turbine being shown in Fig. 253.

[1] At the Hauterive Installation (Fribourg, Switzerland).

[2] By Professor F. Prazil, Proc. Inst. Mech. Engineers, 1910–11.

DIMENSIONS AND TEST RESULTS OF FRANCIS TURBINES.

No. of Turbine.	System.		DETAILS OF DESIGN.										TEST RESULTS.								
			Base Data.				Runner Dimensions.									Percentage of Full Gate Effective H.P.							
							Inlet.									100	90	80	70	60	50	40
			Fall in Metres.	Water Flow. Cubic Metres per Second.	Revolutions per Minute.	Full Load. H.P.	Dia. D. Metres.	Breadth B. Metres.	Number of Blades.	Peripheral Speed, Velocity Due to Revolutions $u = D\pi n/60$.	Velocity Due to Fall $C = \sqrt{2gH}$.	Velocity Co efficient $K_u = u/C$.	Fall in Metres.	Revolutions per Minute.	Full Load H.P.	Percentage Efficiency.						
I.	I. & II. vertical axis; III. & IV. horizontal axis	Single	4·6	3·000	100	142	1·25	0·300	17	6·30	9·5	0·67	4·6	100	158·3	88·1	88·5	87·5	85·5	82·2	78·0	...
II.		Open	4·4	4·940	100	220	1·30	0·435	17	6·82	9·3	0·72	4·4	100	260	82·5	87·0	87·2	87·0	84·0	80·5	...
III.		Single	6·6	5·400	127	372	1·30	0·495	16	8·70	11·35	0·78	7·0	128	434·5	76·9	83·5	85·0	83·5	79·7	74·5	...
IV.		Double	10·4	11·26	172	1250	1·30	2×0·325	15	11·7	14·2	0·81	10·4	160	1250	81·3	85·5	86·5	86·5	85·0	83·5	...
V	Horizontal axis, spiral wheel-case	Single	42·6	1·300	428	550	0·80	0·100	15	17·8	28·5	0·615	42·0	411	550	81·5	82·0	81·5	80·0	78·0	75·0	70·5
VI			62·0	5·000	400	3200	1·10	0·220	17	23·1	34·8	0·67	64·0	400	4000	...	...	...	87·5	86·5	84·5	81·5
VII.		Double	60·0	4·000	500	2500	0·90	2×0·0975	...	23·4	34·2	0·68	60·0	496	2696	78·6	87·5	87·5	84·5	78·0	...	...
VIII.			87·0	2·875	500	2500	0·95	2×0·110	19	25·0	41·2	0·60	87·0	490	2620	83·5	85·5	84·5	83·0	80·5	77·7	...
IX.			143·0	2·000	600	3000	1·10	2×0·0400	...	34·2	52·8	0·65	147·0	601	3458	77·2	79·5	78·5	77·0	74·5	71·8	...

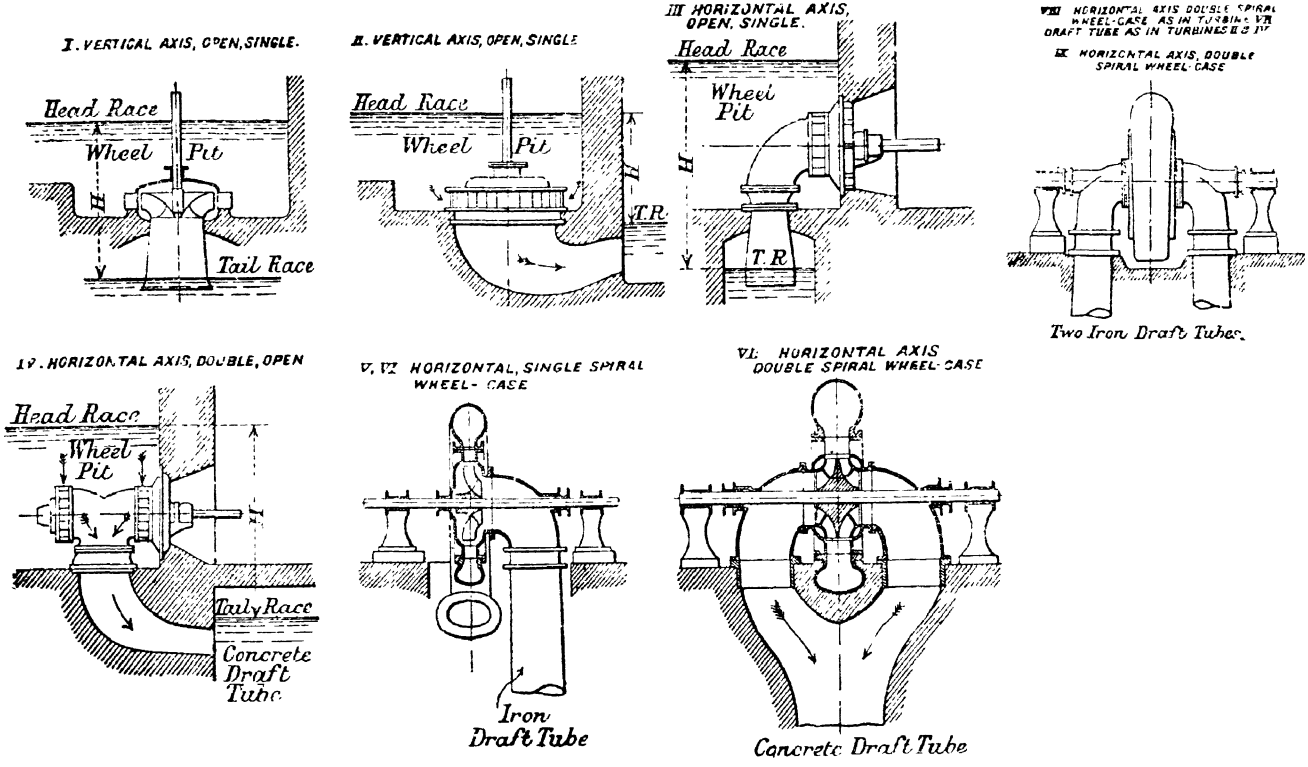


FIG. 253.—Sketch Classification of Francis Turbines I. to IX.

ART. 158.—THE "AMERICAN" TYPE OR "MIXED FLOW" TURBINE.

While the improved Francis or Thomson turbine is undoubtedly the most efficient and offers advantages in respect of ease of regulation and high part-gate efficiency which are unequalled by any other type, the high first cost prohibits its use in many instances. The demand for a turbine suitable for low and medium falls, which must above all things be cheap,

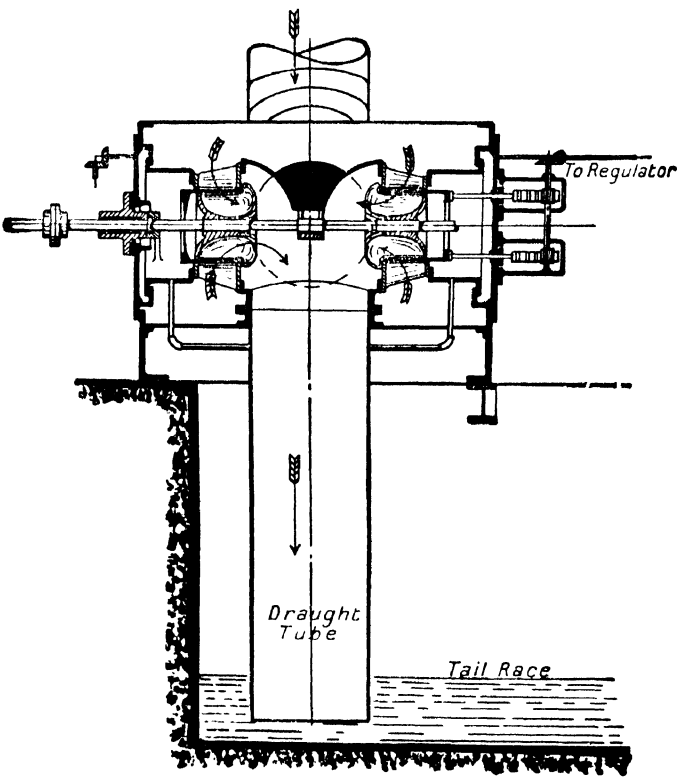


FIG. 254.—General Arrangement of Twin Mixed-Flow Turbine with Horizontal Shaft.

and in which the efficiency need not necessarily be very high, has led to the development of a type of machine which is mainly confined to the United States. The Hercules and Victor low-pressure turbines may be taken as representative of this class.

These machines, which are still often fitted with fixed guide vanes and regulated by means of the cylindrical gate or ring sluice, have inward radial

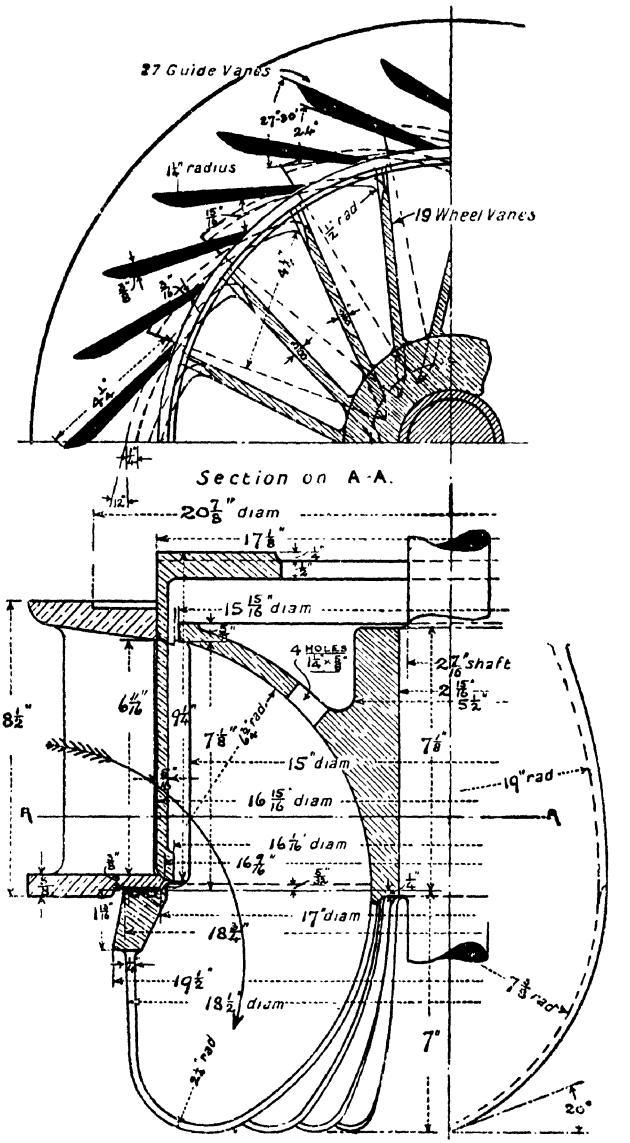


FIG. 255.—Section through Guides and Runners of Victor Mixed-Flow Turbine with Cylinder Gate; 179 H.P. at 664 revs under 50 ft. head.

flow as in the Francis turbine. After the inlet the wheel buckets are curved both laterally and vertically, the water in its passage through the wheel tracing out a path which is approximately a quadrant of a circle, and being

finally discharged partly in an axial and partly in an outward radial direction.

Fig. 254 shows the general arrangement of an enclosed double horizontal shaft turbine of this type, while Figs. 255 and 256 show details of the guide vanes and runner of a Victor turbine, as designed to develop 180 H.P. when running at 665 revolutions per minute under a head of 50 feet. The vanes at the outlet are spoon-shaped. By this type of construction an extremely large discharge area is ensured, while by making the wheel deep at inlet, the inlet area is correspondingly increased. The wheel thus becomes of very

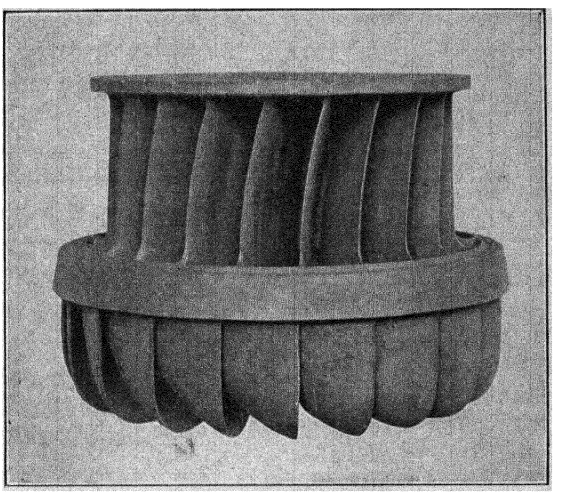


FIG. 256.—Runner for 15 in. Mixed-Flow Victor Turbine.

compact construction, and is capable of dealing with a large volume of water, but has the defect that in virtue of this great depth its efficiency at part gate is comparatively low.

In the turbine illustrated the wheel diameter is 15 inches and the depth $7\frac{1}{8}$ inches at entrance, while at exit the diameter is $18\frac{1}{2}$ inches, the over-all depth of the vanes being $14\frac{1}{8}$ inches. The guide vanes, twenty-seven in number, are fixed, and give a mean inlet angle of about 26°. The wheel vanes, nineteen in number, are flat at entrance, and are inclined forwards towards the direction of rotation, making an angle of about 110° with the tangent to the inlet circle.

The vane angle at exit varies from 12° to 20°, the minimum value being that affecting the radial outward discharge. As may be readily understood, the difficulty of designing these exit edges so as to give the correct inclination

at each radius is almost insuperable, and experiment proves the only safe guide as to the precise curvature to give to the vanes. Governing is performed by means of a cylinder gate set between guide vanes and wheel, sliding axially and receiving its motion from two draw-bars which carry racks gearing with pinions which are actuated either by an automatic governor or by hand (Fig. 254.).

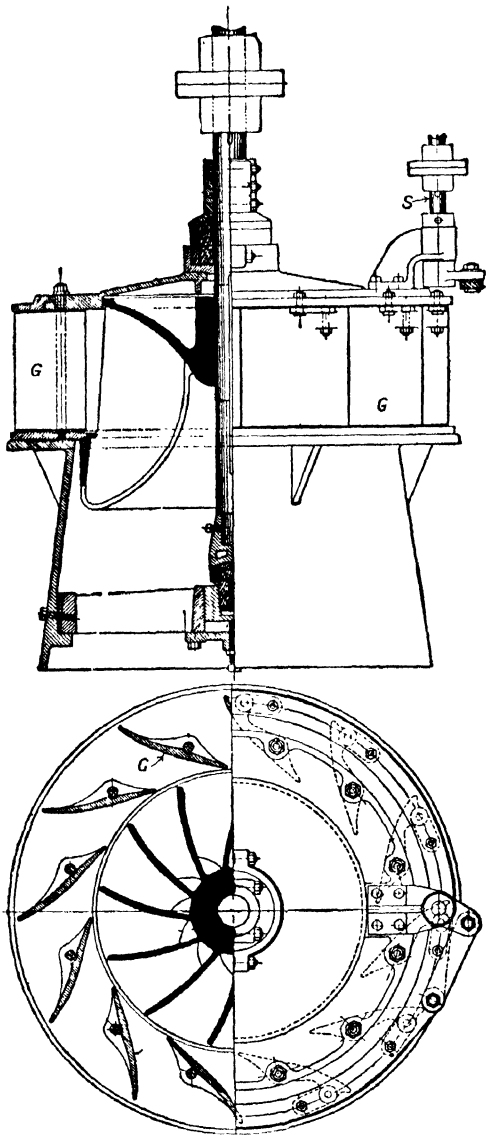


FIG. 257.—"New American" Turbine.

As thus constructed the turbine is capable of a full-load efficiency of about 82 per cent., falling to about 65 per cent. at half gate.

In the Hercules turbine the wheel vanes are provided with a series of horizontal wings or projections, which to some extent serve the purpose of separate compartments in confining the effect of throttling to one portion of the wheel and thus increasing the part-gate efficiency, at the expense, however, of that at full gate.

Fig. 257 shows the general arrangement of a vertical New American turbine, in which the admission of water is controlled by the wicket gates *G*. These are operated through a ring, which is moved by a link connected with the governor through the shaft *S*.

Fig. 258 illustrates the Samson-Leffel turbine, which combines a radial inflow runner of the Francis type with an inward radial admission and axial discharge runner of the

typical American type of turbine. Here again the gates are of the wicket type and are operated from the governor through the pinion *P* and geared sector *S*.

If specially designed for the particular head, flow, and speed of rotation, this class of wheel is well adapted for moderate powers and for medium heads up to about 35 feet where the head is fairly constant, and where part-gate efficiency is unimportant as compared with low first cost. The general practice of manufacturing it in stock sizes, and of supplying that size which most nearly meets the requirements of the purchaser, instead of designing the machine to suit its location, together with the fact that such American machinery has in the past been characterised by a flimsiness of construction unusual in English and Continental practice, has, however, had the effect of discrediting this class of turbine among European engineers to an extent greater than its inherent disadvantages deserve. Evolution in this type of machine would appear to be tending in the direction of fewer and deeper buckets with wider openings to avoid obstruction.

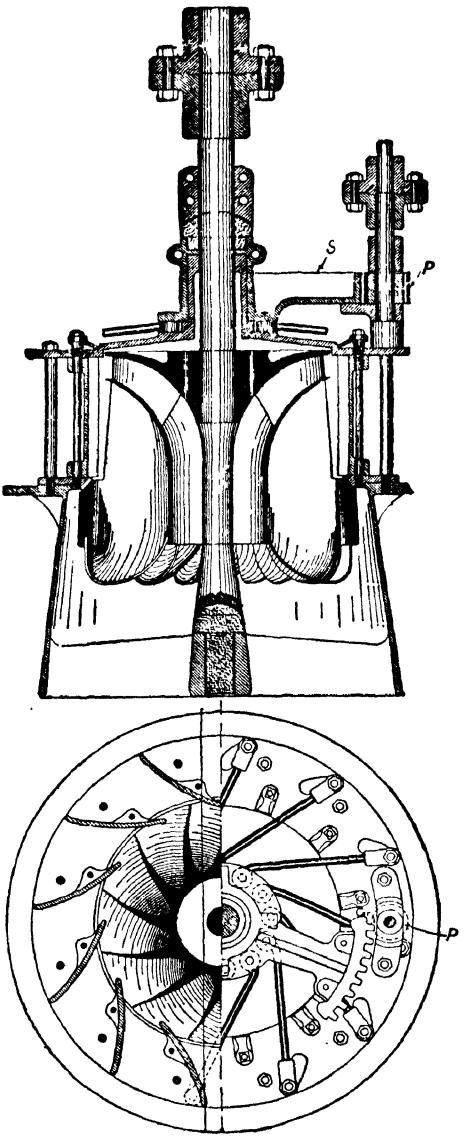


FIG. 258.—Samson-Leffel Turbine.

ART. 159.—DESIGN OF HEAD AND TAIL-RACES.

The construction of the head and tail-races for a turbine plant requires careful consideration. The forebay should be free from sharp turns and sudden changes of section, the

velocity of flow not exceeding from 3 to 4 feet per second. The tail-race should be of the same capacity and should be formed with a well under the turbine outlet, which should contain from 2 to 4 feet of dead water when the turbines are idle. If this be not attended to, a serious loss of head may be caused.

Where a long penstock is installed, this should be supplied at its highest point with an air pipe, whose open end is above the level in the head race to permit the escape of accumulated air, and also to prevent the collapse of the pipe due to the formation of a vacuum should the turbine gates be opened with the head gates closed.

A strainer is always fitted at the exit from the forebay, or supply canal, and consists of a series of wooden or metal grids. These are usually spaced about 1 inch apart, and, in order to reduce the loss due to the contraction and subsequent expansion of sections of the stream, should be as narrow as is consistent with strength. The angle of inclination of the strainer should be not less than 45°, in order to give a large area of waterway.

Effect of Ice Formation.—Although the hydraulic power plant is not greatly hampered by ice formation in Great Britain and the western portion of the continent of Europe, ice troubles may become serious in such climates as are found in Canada and the northern portion of the United States, and may necessitate some modification in the design of the head-race.

The trouble which is often experienced, particularly with frazil ice (Art. 2, p. 5), due to its tendency to adhere to the racks, strainers and gates of a turbine, can, however, be prevented by a slight heating of the racks or turbine gates. At the Ottawa Electric Company's Power House No. 1 a line of steam pipes laid above water level and against the face of the rack was found to answer perfectly. In the same installation, consisting of three 39-inch wheels, 30-feet head, using 100,000 cubic feet of water per minute, steam was supplied by a small pipe to each of the wheel housings when the unit began to lose capacity. To supply this, 20 tons of coal were used during four months of winter, with eleven days on which frazil was bad, only occasional injection of steam being found to be necessary. Electric heating of the same racks has also been tried successfully, 600 amperes at ·3 volts removing the ice at once from a single rack bar with the air temperature at 150° Fahr.[1] These bars were $\frac{5}{16}$ inches thick and 18 feet long.

In a power plant it is usually advisable that the water at the intake should be covered with surface ice, as being the most effective preventative of the formation of both frazil and anchor ice. If there are large stretches of

[1] These results have been taken from "Ice Formation," by Barnes, Wiley & Son, N.Y., 1907. Here the whole subject is considered in detail.

open water above the surface ice, however, frazil is formed and adheres to the lower surface, and may result in a stoppage of the channel. In such a case, or when located at the foot of rapids, it is better to construct a head-race of sufficient size to serve as a settling basin for the ice drawn in. Even then it may sometimes be necessary to blast a channel in the surface sheet. Where a long narrow canal is fed from a stretch of open water the ice difficulty becomes very great. A surface covering is then harmful, as encouraging the adherence of frazil.

CHAPTER XVI.

Theory of Turbine Design—Design of Stand Pipes.

Art. 160.—Theory of Turbine Design.

The turbine system, as a whole, may be divided into the supply pipe, the turbine itself, and the discharge pipe. Losses of head are experienced in each branch of the system, so that if H be the total difference of level between open head and tail-race, the head available for doing work on the turbine will be less than H by the amount necessary to overcome the frictional and other losses in the supply and discharge pipes.

The loss due to this cause is approximately proportional to the lengths of the pipes and to the square of the velocity of flow, and its relative importance diminishes as the gradient of the pipe line and its sectional area are increased. Theoretically, by making the pipes of sufficiently large area the loss may be made quite negligible. This increase in area is, however, accompanied by a corresponding increase in the first cost, and it appears to be fairly well agreed that in practice it does not pay to reduce the pipe line velocity below from 6 to 8 feet per second, the value to be adopted increasing with the gradient and with the head.

In a large power plant where the head is high and the penstock direct, this velocity may be increased without serious proportional loss of head up to a maximum of about 16 feet per second.

If, in any type of turbine, v be the velocity of the supply water at its exit from the guide vanes, and if α be the angle between the guide vanes and the direction of motion of the wheel at the point of entry, v may be resolved into two components; $v \sin \alpha$, perpendicular to the direction of motion of the vanes, which is usually termed the velocity of flow, and $v \cos \alpha$ parallel to this direction, which is termed the velocity of whirl. Throughout the following discussion the velocity of flow will be denoted by f, and that of whirl by w. Thus in a radial-flow turbine f is radial and w tangential to the wheel, while in an axial-flow machine f is parallel to the turbine axis and w is in the plane of the wheel.

In virtue of the velocity of whirl, the supply water possesses momentum in the plane of the wheel, and it is the change in the moment of this momentum about the axis of the wheel during the passage of the water

through the wheel buckets which provides the turning moment on the shaft. For this turning moment to be a maximum with a given value of v and a given quantity of water, it follows that the value of w should be a maximum, and therefore that a should be as nearly as possible zero.

On the other hand, the volume of water which a given wheel is capable of handling per second depends on f, since it is in virtue of this component of velocity that water is carried into the turbine buckets. As a is diminished, therefore, the size of turbine for a given power, and the first cost, increases, and the problem before the designer is to determine at what point the diminution in efficiency due to an increase in a becomes of more importance than the corresponding diminution in first cost. A similar state of affairs holds at exit. For maximum hydraulic efficiency the sum of energy rejected in the discharge and the losses in the wheel should be reduced to a minimum. This in general requires the absolute velocity of discharge from the wheel to be as small as possible. The minimum permissible velocity of discharge is, however, governed by the necessity for getting a given volume of water per second through the limited sectional area of the buckets at this point, and here also an increase in the velocity of flow at the expense of the velocity of whirl will enable a larger volume of water to be dealt with and a greater power to be obtained, but with a sacrifice of the hydraulic, though possibly not of the economic, efficiency.

Where an abundant supply of water is available at all times, the efficiency of the turbine may become relatively unimportant as compared with its prime cost, and a cheap but comparatively inefficient turbine may be preferable on all counts. In such a case high efficiency at part loads is a minor consideration and may be entirely subordinated to considerations of cheapness and of ease of governing. Here, however, it must be remembered that where a long supply pipe or channel is necessary, a decrease in the efficiency of the turbine, particularly at full load, necessitating as it does an increased water supply, may cause the initial cost of construction of such channels or pipes to more than counterbalance the decreased prime cost of the motor.

Again, where the supply is variable and where in times of drought barely sufficient water is available, it is highly important that high efficiency at all gates be the first consideration.

Thus the turbine designer must keep many conflicting possibilities in view, each of which has its own effect on the most suitable design to be adopted, and no hard and fast rules can be formulated for the design of any type of machine. Certain broad principles may, however, be laid

down, to which the design must conform if the turbine is to be efficient and these will now be briefly indicated.

In the following demonstrations :—

Let ω = angular velocity of the wheel in radians per second.

r = radius in feet at any point indicated by a suffix.

v = velocity of supply stream at its exit from the guide vanes, in feet per second.

$u = \omega r$ = velocity of wheel at point indicated by a suffix.

f = velocity of flow.

w = velocity of whirl.

α = angle between guide vanes and direction of motion of wheel at entrance.

β = angle between lip of moving vanes and direction of motion of wheel.

γ = angle between discharge edges of moving vanes and direction of motion of wheel.

Q = flow in cubic feet per second.

W = weight of 1 cubic foot of water = 62·4 lbs.

Art. 161.—General Case of Inward Radial Flow Turbine (Fig. 259).

Let the suffix (*a*) refer to the state of affairs in the supply pipe.

(*b*) refer to the state of affairs in the discharge pipe.

(1) refer to the state of affairs at the exit from guide vanes.

(2) refer to the state of affairs at the inlet to wheel vanes.

(3) refer to the state of affairs at the exit from wheel vanes.

Then a consideration of the diagram shows that if the vane angles are proportioned so as to avoid shock at entrance:

$$\begin{cases} f_2 = w_2 \tan \alpha = (w_2 - u_2) \tan \beta, \ \therefore \ u_2 = w_2 \left(1 - \dfrac{\tan \alpha}{\tan \beta}\right); \\ f_3 = (u_3 - w_3) \tan \gamma; \ {}_2v_r = f_2 \operatorname{cosec} \beta; \ {}_3v_r = f_3 \operatorname{cosec} \gamma. \end{cases}$$

$$\left.\begin{array}{l}\text{The moment of momentum of the}\\ \text{water leaving the guide vanes per}\\ \text{sec. about the axis of rotation}\end{array}\right\} = \frac{WQ}{g} w_1 r_1 \text{ ft. lb. units, and since,}$$

if losses at the exit from the guide vanes are neglected, $w_1 = w_2$, while, neglecting clearance, $r_1 = r_2$, this becomes $\dfrac{WQ}{g} w_2 r_2$ ft. lb. units

$$\text{The final moment of momentum} = \frac{W}{g} Q\, w_3\, r_3 \text{ ft. lb. units.}$$

$$\left.\begin{array}{l}\therefore \text{ Change of moment of momentum} \\ = \text{turning moment}\end{array}\right\} = \frac{W}{g} Q \{ w_2 r_2 - w_3 r_3 \} \text{ ft. lbs.}$$

$$\left.\begin{array}{l}\therefore \text{ Work done by this moment per second}\end{array}\right\} = \frac{W}{g} Q \{ w_2 r_2 - w_3 r_3 \} \omega \text{ ft. lbs.}$$

$$= \frac{W}{g} Q \{ w_2 u_2 - w_3 u_3 \} \text{ ft. lbs.} \qquad (1)$$

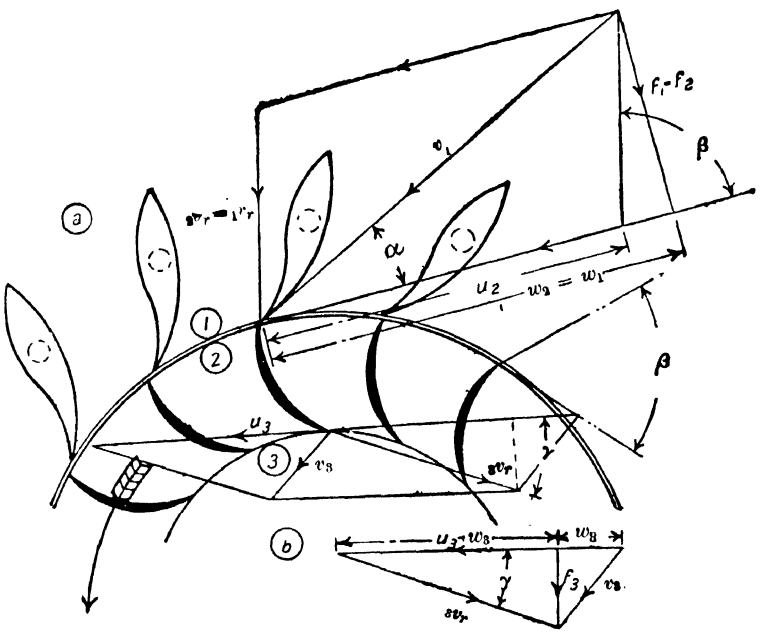


FIG. 259. –Guide and Vane Angles for Inward Radial Flow Turbine.

In an ideal turbine, having no friction or eddy losses, neglecting differences of level in the wheel,

$$\frac{p_2}{w} + \frac{{v_2}^2}{2g} = \frac{p_3}{w} + \frac{{v_3}^2}{2g} + \text{work done per lb. between (2) and (3).}$$

In this case the efficiency will be a maximum when the energy rejected in the discharge is a minimum, *i.e.*, when v_3 is a minimum. This occurs when $v_3 = f_3$ and consequently when $w_3 = 0$, and the work done per lb. then equals $w_2 u_2 \div g$.

Assuming the vane angles designed for entry without shock, and for this state of affairs at exit, if H' is the effective head in such a turbine :—

$$H' = \frac{w_2 u_2}{g} + \frac{f_3^2}{2g} \quad \ldots \ldots \ldots \quad (2)$$

from which, writing $f_3 = f_2 \frac{b_2 r_2}{b_3 r_3} = w_2 \tan \alpha \frac{b_2 r_2}{b_3 r_3}$,

we get

$$w_2 = \sqrt{\frac{2gH'}{2 + \left(\frac{b_2 r_2}{b_3 r_3} \tan \alpha\right)^2 - 2\frac{\tan \alpha}{\tan \beta}}} \quad \ldots \ldots \quad (3)$$

while since

$$u_2 = w_2 \left(1 - \frac{\tan \alpha}{\tan \beta}\right),$$

this gives the peripheral speed of the wheel for maximum efficiency.

The effect of a variation in α or β on the peripheral speed for maximum efficiency is shown in the following table, which gives the theoretical values of k (where $u_2 = k\sqrt{2gH'}$) in the case where $f_2 = f_3$

Values of β.	Values of α.							
	0°	5°	10°	12½°	15°	20°	25°	30°
60°			·658		·636	·604	·564	·516
75°			·685		·669	·648	·625	·596
90°	·707	·705	·702	·698	·695	·685	·672	·655
105°			·724		·729	·732	·733	·730
120°			·741		·748	·756	·764	·770

On the foregoing assumptions, the maximum theoretical hydraulic efficiency of the turbine is given by

$$\eta = \frac{\text{work done per second by water}}{\text{energy supplied per second}} = \frac{\frac{WQ}{g} \cdot w_2^2 \left(1 - \frac{\tan \alpha}{\tan \beta}\right)}{WQH'}$$

$$= \frac{2\left(1 - \frac{\tan a}{\tan \beta}\right)}{2\left(1 - \frac{\tan a}{\tan \beta}\right) + \frac{b_2^2 r_2^2}{b_3^2 r_3^2} \tan^2 a}$$

$$= \frac{1}{1 + \frac{1}{2}\left(\frac{b_2 r_2}{b_3 r_3}\right)^2 \frac{\tan^2 a}{1 - \frac{\tan a}{\tan \beta}}} \qquad (4)$$

This expression does not take into account any of the losses due to mechanical or fluid friction in the turbine. The actual efficiency of the machine is the ratio of the useful work delivered at the turbine shaft to the energy supplied to the turbine, and is given by the ratio

$\frac{2\pi N M}{W Q H'}$ where N = revolutions per second
M = turning moment on shaft in foot lbs.

Here M is less than the hydraulic turning moment $\frac{W Q}{g}(w_2 r_2 - w_3 r_3)$ foot lbs. by an amount depending on the frictional resistances of the turbine.

From (4) it appears that the efficiency should theoretically increase as the ratio $\frac{b_2 r_2}{b_3 r_3}$ diminishes, *i.e.*, as the breadth of the turbine is more rapidly increased towards the centre. Even theoretically, however, this possible increase is only small because of the smallness of $\tan^2 a$, while such a construction, by giving wheel passages which have a sectional area increasing in the direction of flow, tends to produce unsteady motion with a consequent loss of energy in eddy production.

Again from equation (4) it appears that the efficiency increases as β increases, and diminishes with an increase in a, the table on p. 506 giving values of η corresponding to different values of these angles, in the case where $\frac{b_2 r_2}{b_3 r_3} = 1$, and where in consequence $f_2 = f_3$.

From this it would appear that it is an advantage to make β as large as possible. But since we have $f_2 = w_2 \tan a = u_2 \left\{\frac{\tan a}{1 - \frac{\tan a}{\tan \beta}}\right\}$, the velocity of flow for a given peripheral wheel speed diminishes as β increases, so that an increase in β necessitates either a larger and more expensive turbine or a higher peripheral speed. In the latter case frictional losses are increased, and in view of these facts it has become very usual to make $\beta = 90°$ for medium heads. For very high heads β may range where 60° to 90°, and for low heads from 90° to 145°.

Values of β.	Values of α.				
	10°	15°	20°	25°	30°
60°	·981	·959	·922	·871	·800
75°	·983	·962	·931	·888	·835
90°	·984	·964	·936	·902	·857
105°	·985	·966	·941	·910	·874
120°	·986	·968	·946	·920	·889

In the case where $\beta = 90°$ the hydraulic efficiency is given by

$$\eta = \frac{2}{2 + \left(\frac{b_2 r_2}{b_3 r_3} \tan \alpha\right)^2}. \qquad (5)$$

Similarly, although the hydraulic efficiency decreases as α increases, yet the volume passing through the turbine, and consequently its horse power, also increases, the maximum power being obtained when the product of Q and η is a maximum. It follows that the most satisfactory value of α depends on the purpose for which the turbine is desired. In a high-class turbine, α will be as small as mechanical considerations permit—generally between 10° and 15°, and the turbine will gain in efficiency at the expense of a higher prime cost. Where a cheap turbine is required α may have any value up to about 35°.

The volume of water passing through the turbine per second is given by $Q = 2 \pi r_2 b_2 f_2 = 2 \pi r_2 b_2 w_2 \tan \alpha$ cubic feet,[1] and when the turbine is working under conditions of maximum efficiency this becomes

$$Q = 2 \pi r_2 b_2 \tan \alpha \sqrt{\frac{2 g H'}{2 + \left(\frac{b_2 r_2}{b_3 r_3} \tan \alpha\right)^2 - 2 \frac{\tan \alpha}{\tan \beta}}} \qquad (6)$$

[1] This assumes that the passages run full, and neglects the effect of the thickness of the vanes. The latter factor may readily be allowed for, and is considered later (Art. 164). The construction of the vanes may, however, cause some contraction of the stream, in which

so that both the velocity of the turbine for maximum efficiency and the volume of water passed through the wheel vary as $\sqrt{H}$, while since the output of the turbine is proportional to $Q\,H$, it is proportional to $H^{\frac{3}{2}}$.

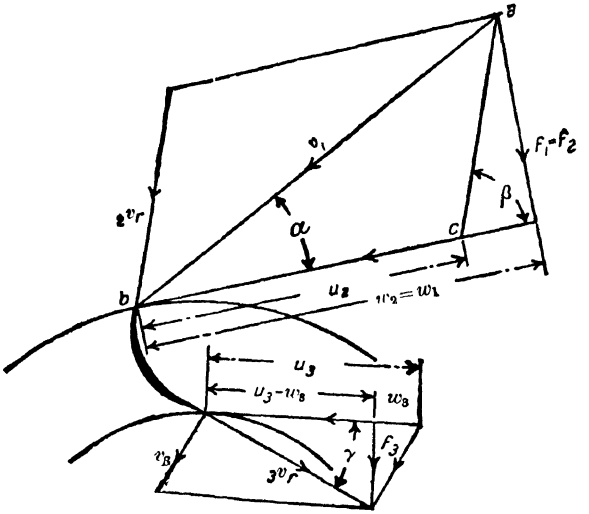


FIG. 260.—Vane Angles for Inward Radial Flow Turbine.

The discharge angle γ (Fig. 260) may be determined from the consideration that, if $w_3 = 0$,

$$\tan\gamma = \frac{f_3}{u_3} = \frac{f_2}{u_3}\frac{b_2}{b_3}\frac{r_2}{r_3}$$

$$= \frac{w_2}{u_3}\tan a\,\frac{b_2}{b_3}\frac{r_2}{r_3}$$

As will be seen later, however, this is not the best value for γ when friction in the wheel is taken into account.

Change of Pressure through Wheel.—Since the motion of a particle of water at any point in the wheel may be compounded of its motion in a forced vortex with angular velocity ω, and of its motion parallel to the wheel vanes with (variable) velocity v_2, the difference of pressure at any two points in the wheel will be the algebraic sum of the differences necessary to produce these motions.

---

case the actual area over which flow takes place is less than that of the passages. We then get

$$Q = k \times 2\pi r_2 b_2 f_2 = k \times 2\pi r_2 b_2 w_2 \tan a \text{ cubic feet,}$$

where k equals the coefficient of discharge of the passages. This, which includes both the coefficient of velocity and of contraction, is usually taken as about ·95 and must be introduced in the application of these formulae to any specific example.

Thus, due to the vortex motion,

$$\frac{p_2' - p_3'}{W} = \frac{u_2^2 - u_3^2}{2g} \text{ feet,}$$

while due to the flow between the vanes,

$$\frac{p_2'' - p_3''}{W} = \frac{{}_3v_r^2 - {}_2v_r^2}{2g} \text{ feet.}$$

Summing these we have :—

Total difference of pressure at inlet and outlet. $\Big\} = \dfrac{p_2 - p_3}{W} = \dfrac{u_2^2 - u_3^2}{2g} + \dfrac{{}_3v_r^2 - {}_2v_r^2}{2g}$

But ${}_3v_r = f_3 \operatorname{cosec} \gamma$, and ${}_2v_r = f_2 \operatorname{cosec} \beta$, while $u_3 = \dfrac{u_2}{n}$,

$$\therefore \quad \frac{p_2 - p_3}{W} = \frac{u_2^2}{2g}\left(1 - \frac{1}{n^2}\right) + \frac{f_3^2 \operatorname{cosec}^2 \gamma - f_2^2 \operatorname{cosec}^2 \beta}{2g}. \qquad (7)$$

The pressure p, at any radius r between inlet and outlet may be obtained if the angle θ made by the vanes at this radius with the tangent to the corresponding circle, and the velocity of flow, f, be known.

Here, if u be the velocity of the wheel at r,

$$u = \frac{u_2\, r}{r_2}, \text{ while } v_r = f \operatorname{cosec} \theta,$$

$$\therefore \quad \frac{p_2 - p}{W} = \frac{u_2^2}{2g}\left\{1 - \left(\frac{r}{r_2}\right)^2\right\} + \frac{f^2 \operatorname{cosec}^2\theta - f_2^2 \operatorname{cosec}^2\beta}{2g}$$

$$\text{or} \quad p = p_2 - W\left[\frac{u_2^2}{2g}\left\{1 - \left(\frac{r}{r_2}\right)^2\right\} + \frac{f^2 \operatorname{cosec}^2\theta - f^2 \operatorname{cosec}^2\beta}{2g}\right]$$

lbs. per square foot. (8)

The pressure at any point, and therefore the whole pressure on that portion of the runner which carries the vanes, may be determined in this way.

The pressure over the face of the runner inside the vanes $= p_3$ lbs. per square foot, so that the total pressure over this face may be determined. Owing to leakage past the outer periphery of the runner the pressure on the rear face may, however, amount to as much as p_2 lbs. per square foot, and owing to the large difference of pressure thus produced on the two faces the end thrust on the shaft may become excessive. Various methods of balancing this end-thrust have been adopted, and have been illustrated in the preceding chapter.

Summary.—Collecting the more important of the results so far obtained we have in the case of an inward radial flow turbine, working without shock at entrance and rejecting its discharge water without any tangential velocity, and therefore (neglecting the effect of other losses) working at its maximum efficiency,

$(1)_a$ $$u_2 = \left(1 - \frac{\tan a}{\tan \beta}\right) w_2$$

$$= \left(1 - \frac{\tan a}{\tan \beta}\right) \sqrt{\frac{2 g H'}{2 - 2\frac{\tan a}{\tan \beta} + \left(\frac{n b_2}{b_3} \tan a\right)^2}}$$

$(2)_a$ $$f_2 = w_2 \tan a \text{ feet per second.}$$

$(3)_a$ $$\tan \gamma = n \frac{w_2}{u_3} \frac{b_2}{b_3} \tan a$$

$(4)_a$ $$f_3 = n \frac{b_2}{b_3} f_2 \text{ feet per second.}$$

$(5)_a$ $$p = p_2 - \frac{W}{2g}\left[u_2^2 \left\{1 - \left(\frac{r}{r_2}\right)^2\right\} + f^2 \operatorname{cosec}^2\theta - f_2^2 \operatorname{cosec}^2\beta\right]$$

lb. per square foot.

$(6)_a$ Hydraulic efficiency $$\eta = \frac{2}{2 + \left(\frac{b_2}{b_3} n \tan a\right)^2} \times \frac{1}{1 - \frac{\tan a}{\tan \beta}}$$

$(7)_a$ $$\text{Revolutions} = \frac{u_2 \times 60}{2 \pi r_2} \text{ per minute.}$$

Since α is always small, we see from (1 *a*) that if $\beta = 90°$ the peripheral velocity of the wheel for maximum efficiency is very approximately given by

$$u_2 = w_2 = \sqrt{\frac{2 g H'}{2}} = \cdot 707 \sqrt{2 g H'},$$

so that with radial tips the peripheral velocity at entrance is nearly that due to a fall through a height $H' \div 2$, and the supply water at this point has changed only approximately one half of its total head into kinetic energy.

The velocity and, therefore, the pressure at discharge from the guide vanes, with a given available head, depends on the values α, β and u_2. Thus if α and u_2 are fixed, an increase in β necessitates a reduction in v_2

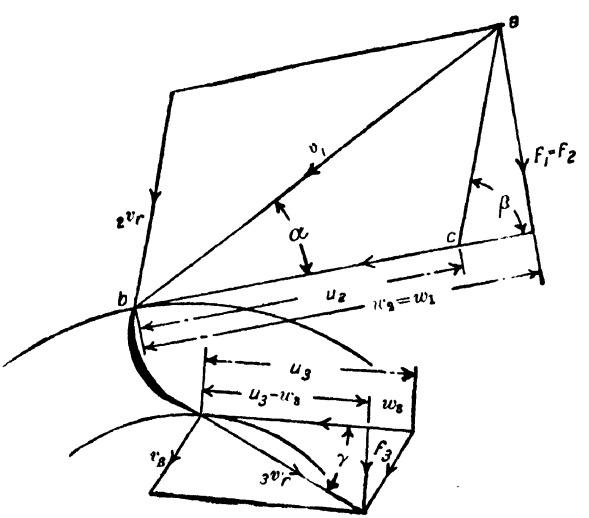


FIG. 261.—Vane Angles for Inward Radial Flow Turbine

(Fig. 261) and in f_2, which is accompanied by a corresponding increase in p_2. The ratio of p_2 to the pressure equivalent to the supply head is termed the "degree of reaction." Its value is approximately ·5 in a turbine having radial vane tips, but by suitable adjustment of the vane angles and peripheral speed it may be made anything from about ·75 to zero. In the latter case the turbine becomes an impulse turbine.

Since f_2, α, u_2 and β are interdependent, it is necessary to decide upon the values to be given to two of these quantities before the remainder can be determined. In practice it is very usual to fix values of α and of f_2, and the following table indicates the usual range of these :—

Type	f_2.	u_2.	α.	$n = \frac{r_2}{r_3}$
Inward radial flow turbine—				
Thomson type . . .	About ·125 $\sqrt{2gH}$	About ·66 $\sqrt{2gH}$	About 11°	2
Francis type	From ·125 $\sqrt{2gH}$ to ·25 $\sqrt{2gH}$	From ·66 $\sqrt{2gH}$ to ·80 $\sqrt{2gH}$	From 11° to 14°	From 1·25 to 1·50
Mixed flow turbine of the American type	From ·35 $\sqrt{2gH}$ to ·40 $\sqrt{2gH}$	From ·72 $\sqrt{2gH}$ to ·80 $\sqrt{2gH}$	From 22° to 28°	
Outward radial flow turbine .	About ·25 $\sqrt{2gH}$	From ·56 $\sqrt{2gH}$ to ·62 $\sqrt{2gH}$	From 20° to 25°	From ·80 to ·85
Axial flow turbine of high specific speed	From ·35 $\sqrt{2gH}$ to ·55 $\sqrt{2gH}$	From ·80 $\sqrt{2gH}$ to 1·4 $\sqrt{2gH}$	From 16° to 22°	1·0

Art. 162.—Axial Flow Pressure Turbine.

Here the general theory is the same as for the inward and outward radial flow types of turbine, and the same demonstrations hold good if it be remembered that $r_2 = r_3 = r$; $u_2 = u_3 = \omega r$; $n = 1$.

If, as assumed in the previous cases, the plane of the wheel is horizontal, the equation of energy now becomes :—

$$\frac{p_2}{W} + \frac{{v_2}^2}{2g} = \frac{p_3}{W} + \frac{{f_3}^2}{2g} + \{ \text{work done in wheel} + \text{losses from (2) to (3)} - h \} \text{ per lb.}$$

where h = depth of wheel.

This assumes no velocity of whirl at exit, so that $f_3 = v_3$.

We now have

$$\frac{p_2 - p_3}{W} = \frac{{f_3}^2 \operatorname{cosec}^2\gamma - {f_2}^2 \operatorname{cosec}^2\beta}{2g} - h$$

from equation (7), p. 508, the term $\frac{{u_2}^2}{2g}\left(1-\frac{1}{n^2}\right)$ expressing the centrifugal effect, now vanishing.

Here f_2 generally equals from [·175 to ·225] $\sqrt{2g(H'-h)}$, while u_2 varies from [·62 to ·68] $\sqrt{2g(H'-h)}$. α varies from 16° to 22°, while γ varies from 15° to 25°.

Evidently with a given value of α and of β, at only one point in the radius of the wheel for any given speed, will the conditions be suitable for entrance without shock. For entrance without shock at all radii, the value of β should change continuously, so as to suit the corresponding peripheral speed. This construction is seldom carried out in practice. In the modern high specific speed low head turbine which is essentially of this type, f_2 may have a value as high as $\cdot 55\sqrt{2gH'}$, and u_2 as high as $1\cdot 4\sqrt{2gH'}$.

Art. 163.—Mixed Flow Turbines.

Here, as in the purely inward flow type of wheel, the turbine is to a certain extent self-regulating. The design of the vanes at inlet is regulated by the laws governing the design of the inward flow wheel and at outlet by the laws governing that of the axial flow type. In general, a higher peripheral velocity is adopted for this type, this varying from ·7 to ·75 $\sqrt{2gh}$ for maximum efficiency at full gate. Where regulated by cylinder gates, the speed for maximum efficiency falls to from ·57 to ·61 $\sqrt{2gH}$ at half-gate, the efficiency under these conditions varying from about 80 per cent. at full gate to 65 per cent. at half gate.

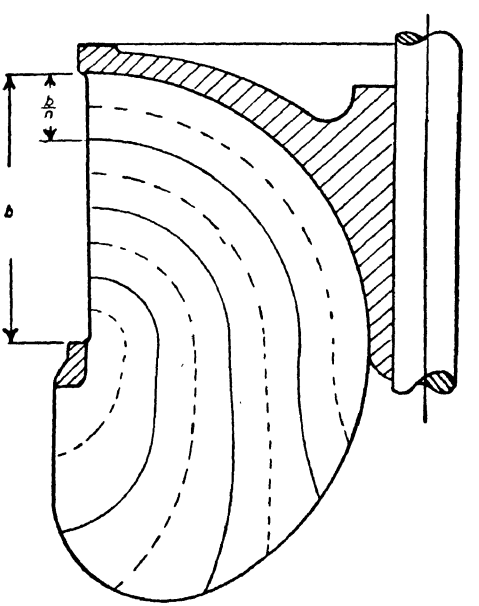


Fig. 262.

The correct inclination of the tangent plane to the vane at discharge, at any radius, may be determined on the assumption that when the wheel runs full there is no relative radial interchange of the particles of water. The turbine may then be imagined as subdivided into n elements each

having a depth at inlet $= b \div n$, and a width at outlet $= l \div n$ where b is the depth of the wheel at inlet and l is the length of the discharging periphery of each bucket (Fig. 262). It then becomes easy to calculate the respective angles of inclination to the horizontal and the vertical at the mid-point of each element at outlet in order that the discharge may be as nearly as possible axial, and so to obtain the contours of the vane.

This method cannot, however, be relied upon to give very accurate results, since the velocity of flow across the whole outlet area will not be equal. This follows because of the reduced resistance to flow through those elements which offer the shorter path to the water, and in which, at the same time, because of their shorter radial length, the resistance to inward flow caused by the centrifugal action of the water is least. Thus the velocity of flow will be greater in those elements of the discharge area which are at a greater distance from the centre.

It thus becomes necessary for accurate results to treat each section as a separate turbine with given inlet and outlet pressures, and so to calculate the relative flow per unit area across each section.

This involves very elaborate calculations,[1] and the more usual method in practice is to determine the correct mean angle for the outflow at the point of mean radius on the assumption that this outflow is uniform over the whole section.

Art. 164.—Thickness of Vanes.

This correction, in the case of an inward flow turbine, may be made as follows :—

Let t be the vane thickness, and n the number of vanes which cut the circumference under consideration.

At inlet, the area of guide vane circle occupied by these vanes $= b_1 n_1 t_1$ cosec α, while the area of wheel vane circle $= b_2 n_2 t_2$ cosec β.

At outlet the area of wheel vane circle $= b_3 n_2 t_3$ cosec γ, where α, β, and γ are the angles as calculated when neglecting the blade thickness.

Thus the outlet area is reduced in the ratio

$$\frac{2\pi r_3 b_3 - n_2 t_3 b_3 \text{ cosec } \gamma}{2\pi r_3 b_3}, \text{ so that—}$$

$$f_3 \text{ (true)} = f_3 \text{ (approx.)} \left\{ 1 - \frac{n_2 t_3 \text{ cosec } \gamma}{2\pi r_3} \right\}.$$

[1] For a further consideration of this matter the reader may consult an article by Professor Lorenz, *Zeitschrift des Vereines Deutscher Ingenieure*, October, 1905 (p. 1670); *Zur Theorie der Francis Turbinen*, Fritz Oesterien, Berlin, Julius Springer; "Vorlesungen über Wasserkraft Machinen," Dr. Camerer; "Wasser Turbinen," Professor R. Thomann; "Hydraulic Turbines," R. L. Daugherty.

For f_3 to be kept the same we must then either have $\frac{\tan\gamma\ (\text{true})}{\tan\gamma\ (\text{approx.})} = \frac{1}{1 - \frac{n_2\, t_3\, \text{cosec}\,\gamma}{2\pi\, r_3}}$, or the breadth b_3 must be increased in the same ratio.

Similar corrections may be applied at the inlet to the wheel and the outlet from the guides.

Allowance should be made for the difference in radii between the guide and wheel vane circles, and in order to keep the velocity of flow constant where the water leaves the guides and enters the wheel, the breadth b of the guides may be made slightly different to that of the vanes, so as to keep the area through which radial flow takes place, constant. We then have—

$$b_1 = \left\{ \frac{2\pi\, r_2 - n_2\, t_2\, \text{cosec}\,\beta}{2\pi\, r_1 - n_1\, t_1\, \text{cosec}\,\alpha} \right\} b_2.$$

This is only approximate since the passage of the wheel vanes before the guide passages tends to diminish the effective area of the latter, while the effective area of the wheel at inlet is similarly diminished by the presence of the guide vanes. Sufficient data are not available to fix the best number of wheel vanes in any particular case; the greater the number, the more perfect is the guidance given to the water, although at the same time frictional losses are increased. The longer the water passages the fewer the vanes necessary to give sufficient guidance. In the case of a number of modern inward radial flow turbines examined by the author, and of sizes ranging from 8 inches diameter to 66 inches diameter, the number of vanes was given with fair accuracy by the relation $n = k\sqrt[3]{d}$, where d = diameter in inches and where k is a coefficient varying from about 7·7 in the smaller to 8·4 in the larger wheels and having a mean value = 8.

In general, with swivelling vanes, the number of guide vanes is made slightly less than the number of wheel vanes, while with stationary guides the number is usually slightly greater. The thickness t varies from $\frac{1}{8}$ inch in a small turbine with steel plate vanes to about $\frac{1}{2}$ inch in a large turbine with cast vanes. In high-class turbines the vanes have rounded edges at entrance and exit, so that the effective value of t generally varies from about $\frac{1}{16}$ inch to $\frac{5}{16}$ inch.

Art. 165.—Curvature of Vanes.

So long as the inlet and outlet angles of the vanes are correctly proportioned, the shape of the vane between these points only affects the

efficiency in so far as it tends to give steady or unsteady motion in the stream. To this end the design should be such that the passages are nowhere divergent, and that any changes of curvature are as gradual as possible. The first of these requirements is more easily satisfied in the inward flow than in the outward flow turbine. With this requirement

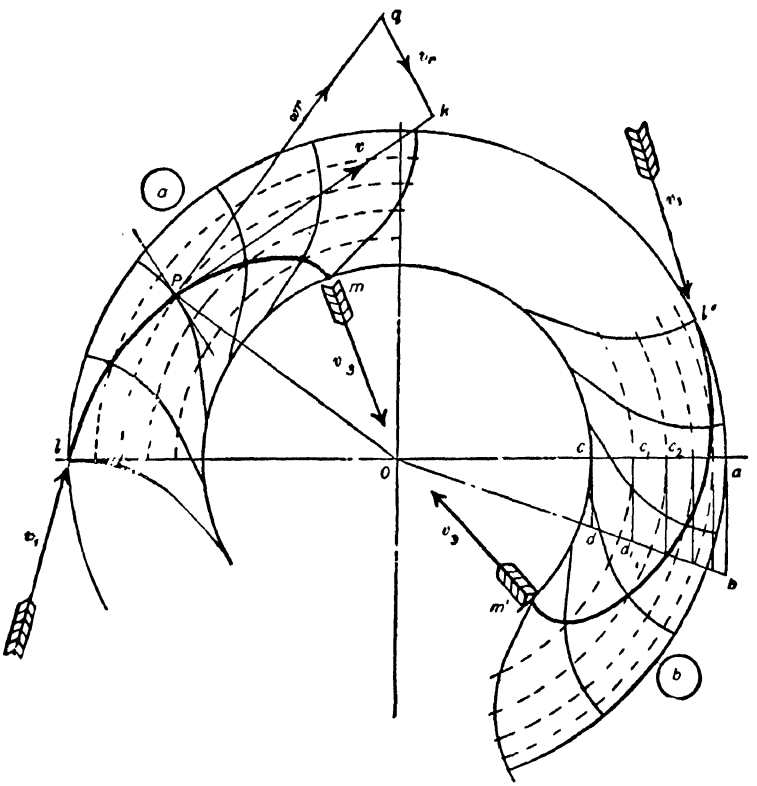


FIG. 263.—Sketch showing actual path of particles of water through an Inward Radial Flow Turbine.

fulfilled, the most efficient vane curve will be that with which the change in curvature of the path of the stream is most gradual, and to determine this it is advisable to set out a diagram showing for any proposed vane the true path of the particles of water in passing through the wheel. Two such diagrams are shown in Fig. 263, in which (*a*) shows the true path where the velocity of flow is uniform, and (*b*) where the velocity of flow varies inversely as the radius. To construct diagram (*a*), a series

of equidistant circles are set out between the inner and outer vane circles. If s be the radial distance between each pair of these, the time for a particle to pass from one to the other $= \frac{s}{f}$ seconds. Next set out the same number of equidistant vanes, the distance apart on the outer vane circle being $\omega\, r_2 \frac{s}{f}$ feet. A particle leaving the intersection of the first circle and first vane will then be at the intersection of the second circle and vane after an interval of time $\frac{s}{f}$ seconds and so on, so that the path of the particle can be sketched in, as at $l\ m$.

Where the velocity of flow is not uniform, the only difference in the construction is that the intermediate circles are not equidistant but are so spaced that the particle travels radially from one to the other in equal intervals of time. Thus in case (*b*) a radius $o\ a$ being drawn, a right-angled triangle $o\ a\ b$ is constructed on this and lines $c\ d$, $c_1\ d_1$, etc., drawn, cutting off equal areas $c_2\ d_1$, $c_1\ d$, etc., from the triangle. Circles drawn through c_1, c_2, etc., will now, by their intersections with the equidistant vanes, give points on the path $l'\ m'$ of a particle.

These curves may be utilized to obtain the absolute velocity of the water and the relative velocity of water and vane at any radius, for if at any point P of radius r, $P\ q$ be drawn perpendicular to $O\ P$ and equal to $\omega\ r$, and if $q\ k$ be drawn parallel to the tangent to the vane at P and $P\ k$ be drawn tangential to the curve at P, then since the actual velocity of the particle at P is compounded of the velocity of the vane at P, *i.e.*, of $\omega\ r$, and of its velocity relative to the vane, and since its actual velocity is tangential to the curve at P and its relative velocity is parallel to the vane, $P\ q\ k$ serves as the triangle of velocities at P, and $\overline{P\ k} =$ the absolute velocity v, while $\overline{q\ k} =$ the relative velocity v_r of water and vane.

By obtaining v_r in this way the pressure "p" at any radius r may be easily obtained by an application of the formula

$$p = p_2 - \frac{W}{2\,g}\left\{ u_2^2 - u^2 + v_r^2 - {}_2v_r^2 \right\} \text{ lbs. per square foot,}$$

where u and v_r refer to a point at radius r.

Example.

A turbine runner has an outer diameter of 5 feet 6 inches and an inner diameter 4 feet 8 inches. It makes 300 revolutions per minute. The space occupied by the blades being divided into five concentric strips,

each $1\frac{1}{2}$ inches wide, it is found graphically that the relative velocities at the centres of these strips, commencing at the outside are,

26 ; 24 ; 26 ; 40 ; 65 feet per second.

The relative velocity at entrance is 27 feet per second and at exit is 76 feet per second. The pressure head at inlet is 125·5 feet.

Denoting these strips by a, b, c, d, e, we have :—

$$p_a = p_2 - \frac{62{\cdot}4}{64{\cdot}4}\{(86{\cdot}5)^2 - (84{\cdot}5)^2 + (26)^2 - (27)^2\}$$
$$= p_2 - 280.$$
$$p_b = p_2 - \frac{62{\cdot}4}{64{\cdot}4}\{(86{\cdot}5)^2 - (80{\cdot}5)^2 + (24)^2 - (27)^2\}$$
$$= p_2 - 820.$$
$$p_c = p_2 - \frac{62{\cdot}4}{64{\cdot}4}\{(86{\cdot}5)^2 - (76{\cdot}5)^2 + (26)^2 - (27)^2\}$$
$$= p_2 - 1{,}510.$$
$$p_d = p_2 - \frac{62{\cdot}4}{64{\cdot}4}\{(86{\cdot}5)^2 - (72{\cdot}6)^2 + (40)^2 - (27^2)\}$$
$$= p_2 - 3{,}010.$$
$$p_e = p_2 - \frac{62{\cdot}4}{64{\cdot}4}\{(86{\cdot}5)^2 - (68{\cdot}8)^2 + (65)^2 - (27)^2\}$$
$$= p_2 - 6{,}040.$$

At exit $p_3 = p_2 - \frac{62{\cdot}4}{64{\cdot}4}\{(86{\cdot}5)^2 - (66{\cdot}8)^2 + (76)^2 - (27)^2\}$

$$= p_2 - 7{,}810.$$

$$\therefore \quad \frac{p_2 - p_3}{W} = \frac{7{,}810}{62{\cdot}4} = 125{\cdot}3 \text{ feet.}$$

This affords a check as to the accuracy of the construction. Assuming the pressure over the corresponding portion of the back face of the disc to be p_2 lbs. per square foot, the total unbalanced pressure over this portion, in the direction of the suction tube is equal to

$$\frac{2\pi \times 1{\cdot}5}{12}\left[\left(\frac{32{\cdot}25}{12} \times 280\right) + \left(\frac{30{\cdot}75}{12} \times 820\right) + \left(\frac{29{\cdot}25}{12} \times 1{,}510\right) + \left(\frac{27{\cdot}75}{12} \times 3{,}010\right) + \left(\frac{26{\cdot}25}{12} \times 6{,}040\right)\right] \text{ lbs.}$$
$$= {\cdot}7854\{752 + 2{,}105 + 3{,}680 + 6{,}960 + 13{,}210\}$$
$$= 21{,}000 \text{ lbs.}$$

Art. 166.—Sources and Magnitude of Losses in the Pressure Turbine.

So far the various losses in the turbine and its appendages have been neglected, and while it is impossible to take these fully into account in a

theoretical discussion, yet a more detailed examination will be of value as indicating their relative importance. In general, these losses are due to :—

(1) Leakage between guides and wheel.

(2) Shock at entrance if the vane angles are not adapted to the speed of the wheel.

(3) Friction during motion of the water over the vanes and crown of the wheel.

(4) Eddy formation caused by incorrect curvature of the vanes, by changes of section, or divergence of the passages.

(5) Shock caused by sudden enlargement of the stream section on entering the buckets at part gate.

(6) Rejection of kinetic energy in the discharge.

(7) Mechanical friction at bearings.

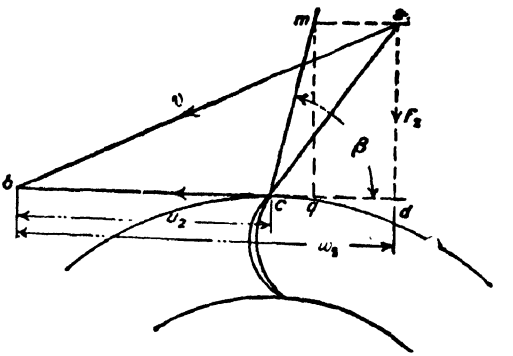


FIG. 264.

(1) **Loss by Leakage.**—This loss being proportional to the possible area through which leakage may take place and to the velocity of efflux over this area, is proportional to the product of the periphery of the wheel and the radial clearance at entrance, and to the square root of the head at entrance. Its magnitude is greatest in turbines of the inward flow type, and may amount to from 2 to 4 per cent. of the total energy supplied.

(2) **Loss due to Shock at Entrance.**—If angle β is not correctly proportioned.

Let $\overline{a\,b} = v$ (Fig. 264) represent the velocity at the exit from guides.

Let $c\,b = u_2$ represent the velocity of vane tips.

Then before entering the buckets the relative velocity of water and of wheel in the direction of the tangent at $c = \overline{c\,d}$.

Also if $\widehat{m\,c\,q} = \beta$, the relative velocity in the same direction on entering the buckets $= c\,q$.

$$\therefore \text{Loss of head due to shock at entrance} = \frac{(\overline{c\,d} - \overline{c\,q})^2}{2\,g} \text{ feet of water.}$$

But $\overline{c\,d} = w_2 - u_2$

And $\overline{c\,q} = \overline{m\,q} \cot \beta = f_2 \cot \beta$.

$$\therefore \text{ Loss of head } = \frac{\{w_2 - u_2 - f_2 \cot \beta\}^2}{2g} \text{ feet}$$

$$= \frac{{w_2}^2}{2g}\left\{1 - \frac{u_2}{w_2} - \frac{\tan a}{\tan \beta}\right\}^2 \text{feet.}$$

The actual loss due to shock is in general less than that calculated, since only a portion of the whole supply stream suffers the extreme change of velocity.

Experiments by Fliegner [1] indicate that the loss due to shock at entry is less when the angle β has a value slightly different from its theoretical value, and indicate that the best value for β is greater than the theoretical for values of β less than about 105°, and less than the theoretical for values of β greater than this.

The following table gives some of Fliegner's results :—

Value of discharge angle γ	15°	30°	15°	30°	15°	15°
Value of β for no shock at entrance. . .	120°	120°	90°	90°	60°	60°
Experimental value of β for maximum efficiency.	111°	109°	105°	102°	98½°	85°

It follows that with angles designed for entry without shock, the most efficient speed will be less than the theoretical for values β of less than 105° and will be greater for greater values of β, a conclusion which is verified by the results of experiment.

Fliegner's results led him to the conclusion that the best value for β is given by the formula $\beta' = 90° + \frac{\beta - \gamma}{5} - 30\frac{b_3 - b_2}{b_2}$, where β and γ are the values of inlet and discharge angles calculated in the usual way for maximum efficiency, and where b_2 and b_3 are the buckets widths at entrance and exit.

(3) **Frictional Losses in the Wheel**, H_F.—These may be divided into the losses caused by (*a*) Flow over the surfaces of the vanes and crowns ; (*b*) Disc friction due to rotation of the turbine crown or crowns through the surrounding water.

(*a*) If v_r = mean relative velocity of flow through turbine, m the hydraulic

[1] *Zeitschrift des Vereines Deutscher Ingenieure*, vol. 23, p. 459.

mean depth of turbine passages, l the effective length of passages, F the coefficient of friction, this loss is given by

$$H_F = \frac{F\,l\,v_r^2}{2\,g\,m} \text{ feet of water.}$$

where F will vary from about ·004 with well-finished vanes to ·010 with cast-iron vanes.

(*b*) The disc friction due to rotation of the turbine crowns has been considered in detail in Art. 76, p. 186.

Its magnitude depends considerably on the type of turbine. In a single wheel radial flow turbine the whole of the rear face of the runner will, in general, be subject to this resistance, as will that portion of the front face which lies between the shaft and the inner tips of the vanes, while in the case of a double discharge turbine of the Thomson vortex type, this rubbing area is almost doubled. A large increase in disc friction is also experienced in the case of a turbine balanced by the addition of a rotating balance piston.

In a wheel of the parallel flow type the design may be such as to cause a similar resistance at the outer circumference whose radius is r_2 and breadth b_2, the loss of energy in this case being given by $f\,\omega^3\,r_2^4\,b_2$ foot lbs. per second.

In any case this loss of energy per second is proportional to ω^3, and the loss per lb. to ω^2, since Q is proportional to ω. The magnitude of the loss may be from 2 to 5 per cent.

(4) and (5) **Eddy Losses.** — These losses, due to eddy formation at changes of curvature and to shock at entrance at part gate, do not admit of even approximate calculation. They may, however, be minimised by designing all passages to have as easy a curvature as possible, and by the adoption of gates of the swivel type. With this type of gate these losses may account for between 1 and 8 per cent. of the total head, while with cylinder regulation they may amount to as much as 20 per cent. at half gate.

(6) **Loss due to Rejection of Kinetic Energy in the Discharge.** — This equals $k\,v_3^2 \div 2\,g$ feet, where k depends on the turbine setting. Where the turbine discharges directly into the tail race or into a parallel draft tube, practically the whole of this energy is lost. With a well-designed divergent draft tube a portion, up to about ·85, may be converted into pressure energy, so that k may be as low as ·15.

At full load, this source of loss usually accounts for between 2 and 8 per cent. of the supply head, being greatest in low head turbines of high specific speed. It is relatively greater at part gate.

Effect of Frictional Losses in the Wheel.[1]—An examination of the triangle of velocities at discharge (Fig. 259) shows that if w_3 is increased, keeping u_3 and f_3 constant, v_3 is increased but ${}_3v_r$ is diminished. This will lead to an increased rejection of K.E. but to a reduced loss due to vane friction, and as maximum efficiency is attained when the sum of the losses is a minimum the correct discharge angle will not, as previously assumed, be that giving radial discharge, and, therefore, the minimum loss of K.E. but will be somewhat greater.

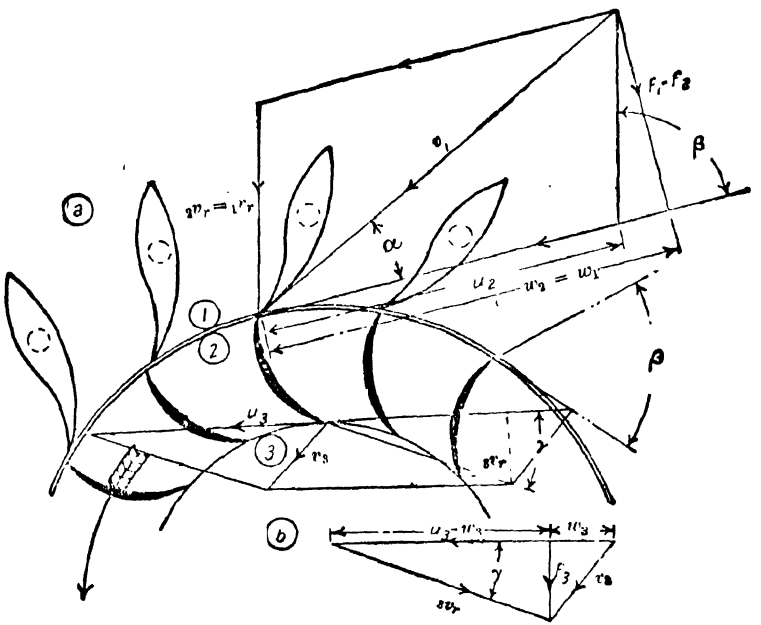


FIG. 259.—Guide and Vane Angles for Inward Radial Flow Turbine.

Expressing the vane friction loss as $k_f\,{}_3v_r{}^2 \div 2g$, the total loss

$$h_l = \frac{1}{2g}\left\{k_f\,{}_3v_r{}^2 + k\,{v_3}^2\right\}$$

$$= \frac{1}{2g}\left\{k_f[\,(u_3 - w_3)^2 + f_3{}^2] + k\,(w_3{}^2 + f_3{}^2)\right\} \quad . \quad . \quad . \quad . \quad (9)$$

For a given u_3 and f_3, differentiating with respect to w_3 and equating to zero, we have, for minimum total loss

$$2\,w_3\,(k_f + k) = 2\,k_f\,u_3$$

$$\therefore \qquad w_3 = \left(\frac{k_f}{k_f + k}\right)u_3 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

[1] See also Art. 168.

$$\therefore \quad \frac{f_3}{u_3 - w_3} = \tan\gamma = \frac{f_3}{u_3}\left(\frac{k + k_f}{k}\right) \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Substituting for w_3 in (9) from (10), the minimum total loss becomes

$$\frac{k_f k}{k_f + k} \cdot \frac{u_3^2}{2g} + (k_f + k)\frac{f_3^2}{2g} . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

Witn this angle the discharge will have a component of whirl in the direction of motion of the runner.

The value of k_f varies widely with the type and size of runner. With a large runner of the low head axial flow type, having not more than four blades, it may be as low as ·025, while it may be as high as ·075 in a small inward flow runner.

ART. 167.—SPECIFIC SPEED OF A TURBINE.

In a given turbine :—

(1) The speed N is proportional to $\sqrt{H}$,

(2) The discharge Q is proportional to $\sqrt{H}$,

(3) The horse-power P is proportional to $H\sqrt{H} = H^{\frac{3}{2}}$.

In order to afford a basis of comparison of turbines of different diameters and proportions operating under different heads, the term known as " specific speed " has been introduced. This may be defined as the speed at which a runner would operate if reduced geometrically to such a size that it would develop 1 H.P. under unit working head. The figures for specific speed given in the following pages refer to a unit head of 1 ft. If the metre be adopted as the unit, these figures require to be multiplied by 4·45.

To determine the value of the specific speed, imagine the head to be reduced from H to h, the dimensions remaining unaltered. Then since the peripheral speed for maximum efficiency is proportional to $\sqrt{H}$, while the output is proportional to $H^{\frac{3}{2}}$,

$$\frac{n}{N} = \sqrt{\frac{h}{H}}, \text{ while } \frac{p}{P} = \left(\frac{h}{H}\right)^{\frac{3}{2}}.$$

Now imagine all the proportions of the turbine to be reduced in the same ratio. Since for a given head the number of revolutions is proportional to the diameter, and since the quantity of water is proportional to the inlet area, and therefore to the square of the diameter :—

$$\frac{p}{P} = \left(\frac{h}{H}\right)^{\frac{3}{2}} \times \left(\frac{d}{D}\right)^2$$

and

$$\frac{n}{N} = \sqrt{\frac{h}{H}} \times \frac{D}{d}$$

$$= \sqrt{\frac{h}{H}} \sqrt{\frac{P}{p}} \times \left(\frac{h}{H}\right)^{\frac{1}{2}}$$

$$= \sqrt{\frac{P}{p}} \cdot \left(\frac{h}{H}\right)^{\frac{5}{4}}.$$

If now h and p be made equal to unity, n becomes the specific speed N_s, so that

$$N_s = \frac{N\sqrt{P}}{H^{\frac{5}{4}}}.$$

In the case of a turbine having more than one runner, P represents the output

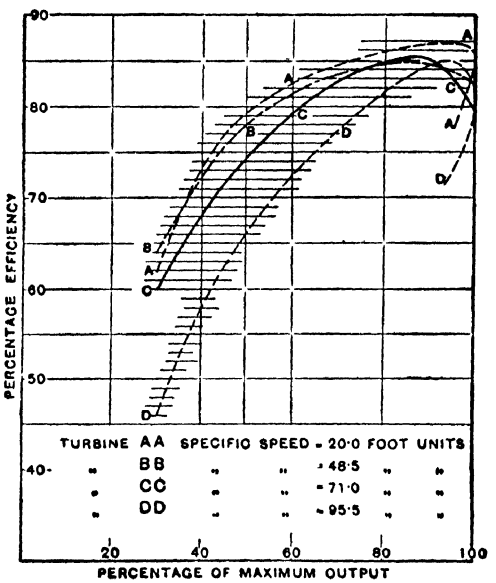


FIG. 265.

per runner. If P is taken as representing the total output, this value of N_s is to be divided by the square root of the number of runners.

The specific speed of a reaction turbine may be varied by varying the diameter of the runner, the angle of the guide vanes, and the angle of the wheel vanes. By modifying the design, as indicated in the sketches of Figs. 242 and 243, it is possible, while maintaining high efficiencies at full load, to increase the specific speed from about 15, its minimum value, with the type shown in Fig. 242 *a*, to about 120 with the type shown in Fig. 243. Specific

speeds as high as 150 are possible with the Kaplan type of turbine with some sacrifice in efficiency. The turbine shown in Fig. 243 has a specific speed of 118·5 at its designed speed of 93·7 r.p.m. under a head of 32·8 ft. Under these conditions efficiency at full load is 85 per cent., at three-quarters load 81 per cent., and at half load 71 per cent. Under a head of 19·7 ft. at the same speed of rotation its output is 4,200 H.P., its specific speed 146, and its full load efficiency 68 per cent. The type of runner shown in Fig. 242 *a* is well adapted for high heads and relatively small volumes ; *b* is suitable for medium speeds ; *c* for high speeds and medium heads ; and Fig. 243 for low heads and high speeds. These high specific speeds are extremely valuable for low-head installations since they enable the size and cost of the turbine, of its setting, and of the generator to be greatly reduced. In fact, many existing low-head installations would have been commercially impracticable but for the development during recent years of the high-speed turbine.

High specific speeds are, however, attended by some disadvantages, particularly for medium and high heads. The part-gate efficiency in general falls off as the specific speed increases, broadly as indicated by the curves of Fig. 265, which show the efficiencies of a series of model test turbines of approximately the same size operating under the same head of 4 m.[1] Also if the speed is unduly high it becomes very difficult to avoid corrosion troubles. At the present stage of design, the maximum specific speeds to be used under normal circumstances with various heads are approximately as follows :—

Head feet . .	40	60	80	100	150	200	300	400	500	600
Specific speed .	160	130	110	100	80	65	50	42	37	35

ART. 168.—VARIATION OF DISCHARGE ANGLE AND EFFICIENCY WITH SPECIFIC SPEED.

In a turbine of given specific speed, since $P \propto Q\,H \propto A_3 f_3 H$, and $N \propto u_3 d_3$,

$$\therefore \quad N_s \propto \frac{u_3 d_3 \sqrt{A_3 f_3 H}}{H^{\frac{5}{4}}} \propto \frac{u_3 \sqrt{f_3}}{H^{\frac{3}{4}}}$$

But

$$u_3 = \sqrt{v_3^2 - f_3^2} + \sqrt{{}_3v_r^2 - f_3^2} \quad \text{(Fig. 266 } a\text{)}.$$

$$\therefore \quad N_s \propto \sqrt{f_3}\left\{ \sqrt{v_3^2 - f_3^2} + \sqrt{{}_3v_r^2 - f_3^2} \right\} \text{ for a given head.}$$

[1] By the courtesy of Messrs. Boving & Co., Ltd. See also "High Specific Speed Water Turbines," *Proc. Inst. Mech. E.*, 1934, A. A. Fulton, Vol. 128, p. 361.

If now f_3 is varied by varying the discharge angle γ and u_3 (Fig. 266 *b*), keeping v_3 and ${}_3v_r$ constant, so that the sum of the losses due to K.E. and vane friction is constant, N_s will be a maximum when $\frac{d(N_s)}{df_3} = 0$, *i.e.*, when

$$f_3^2 = \frac{\sqrt{(v_3^2 - f_3^2)({}_3v_r^2 - f_3^2)}}{2}$$

i.e., when

$$f_3 = \sqrt{\frac{w_3\, w_r}{2}} \quad \ldots\ldots\ldots \quad (13)$$

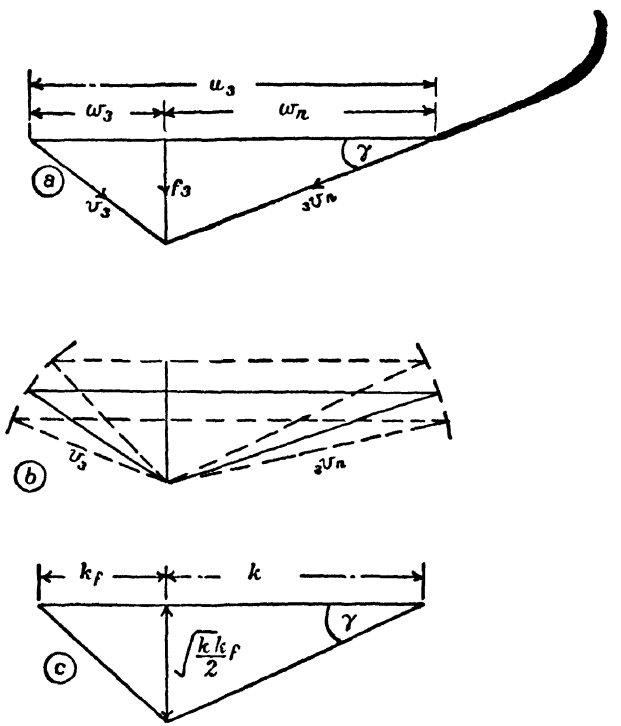


FIG. 266.

But from equation (12), p. 521 it appears that the maximum loss for a given u_3 and f_3, is given by

$$h_l = \frac{k_f\, k}{k_f + k} \cdot \frac{u_3^2}{2\,g} + (k_f + k)\frac{f_3^2}{2g}.$$

and since

$$N_s \propto \frac{u_3\sqrt{f_3}}{H^{\frac{1}{2}}} = \frac{m\, u_3\sqrt{f_3}}{H^{\frac{1}{2}}}$$

$$2\,g\,h_l = \frac{k_f\, k}{k_r + k} \cdot \frac{H^{\frac{1}{2}} \cdot N_s^2}{m^2 f_3} + (k_f + k) f_3^2$$

For minimum loss with a given specific speed, $\frac{d\,h_l}{d\,N_s} = 0$,

$$\therefore \quad -\frac{k_f\,k}{k_f+k}\cdot\frac{H^{\frac{1}{2}}\,.\,N_s^{\,2}}{m^2 f_3^{\,2}} + 2\,(k_f+k)\,f_3 = 0.$$

$$\therefore \quad 2\,(k_f+k)\,f_3 = \frac{k_f\,k}{k_f+k}\cdot\frac{u_3^{\,2}}{f_3}$$

$$\therefore \quad \frac{f_3}{u_3} = \frac{\sqrt{\frac{k_f\,k}{2}}}{k_f+k} \qquad \ldots\ldots\ldots (14)$$

This agrees with (13) if $\begin{cases} w_3 = \dfrac{k_f}{k_f+k}\cdot u_3 \\ w_r = \dfrac{k}{k_f+k}\cdot u_3 \end{cases}$.

The triangle of velocities satisfying these conditions is shown in Fig. 266 *c*, from which it follows that $\tan\gamma = \sqrt{\frac{k_f}{2\,k}}$.

Substituting in (2) the value of f_3/u_3 from (3) gives

$$2\,g\,h_l = 3\,(k_f+k)\,f_3^{\,2}.$$

and since $\frac{N_s\,H^{\frac{1}{2}}}{m} = u_3\sqrt{f_3} = \frac{k_f+k}{\sqrt{\frac{k\,k_f}{2}}}\,(f_3)^{\frac{1}{2}}$ . . . . from (14),

$$\therefore \quad 2\,g\,h_l = 3\left\{\frac{(k_f\,k)^2\,.\,N_s^{\,4}}{4\,(k_f+k)\,.\,m^4}\right\}^{\frac{1}{3}} H \qquad \ldots\ldots (15)$$

and the efficiency, neglecting leakage and mechanical losses, is given by

$$\eta = 1 - \frac{h_l}{H}.$$

The kinetic energy of discharge from the runner is $v_3^{\,2}/2\,g$, and since,

$$\begin{aligned} v_3^{\,2} &= f_3^{\,2} + w_3^{\,2} \\ &= f_3^{\,2} + \left(\frac{k_f}{k_f+k}\right)^2 u_3^{\,2} \\ &= f_3^{\,2}\left\{1 + \frac{2\,k_f}{k}\right\} \\ &= \left\{\frac{(k_f\,k)^2\,.\,N_s^{\,4}}{4\,(k_f+k)^4\,m^4}\right\}^{\frac{1}{3}}\left\{1+\frac{2\,k_f}{k}\right\} H \end{aligned}$$

$$\therefore \quad \frac{v_3^{\,2}}{2\,g\,H} = \frac{1}{2\,g}\left\{\frac{(k_f+k)^2\,N_s^{\,4}}{4\,(k_f+k)^4\,m^4}\right\}^{\frac{1}{3}}\left\{1+\frac{2\,k_f}{k}\right\} \qquad \ldots\ldots (16)$$

In this expression $m = \frac{60}{\pi\,d_3}\sqrt{\frac{62{\cdot}4}{550}\,A_3\,\eta}$. In a wheel having axial discharge, A_3 is the discharge area of the runner measured perpendicular to the

axis, and d_3 is the mean diameter of the runner at discharge. Taking this as $\frac{2}{3}$ the over-all diameter, and taking the efficiency η as ·85, m becomes 7·9.

The values of $v_3 \div 2gH$ and of γ for minimum over-all loss depend essentially on the relative values of k_f and k, while the value of $v_3^2 \div 2gH$ also depends on the specific speed. The following tables indicate the best values of $v_3^2 \div 2gH$, and the corresponding over-all loss for a few typical values of k_f and k.

N_s	20	50	100	200	300	γ
$k_f = ·03$ $k = ·15$	·013	·046	·117	·291	·50	17° 32′
$k_f = ·06$ $k = ·15$	·021	·071	·180	·451	·78	24° 6′
$k_f = ·03$ $k = ·20$	·010	·035	·087	·22	·38	15° 18′
$k_f = ·06$ $k = ·20$	·017	·058	·145	·37	·63	21° 40′

The corresponding values of the total loss expressed as a fraction of the total head H are :—

N_s	20	50	100	200	300
$k_f = ·03$ $k = ·15$	·005	·018	·045	·114	·193
$k_f = ·06$ $k = ·15$	·007	·025	·063	·157	·273
$k_f = ·03$ $k = ·20$	·005	·018	·046	·117	·200
$k_f = ·06$ $k = ·20$	·008	·028	·071	·180	·307

The foregoing results indicate that the efficiency of a turbine of high specific speed cannot be expected to be as high as one of low-specific speed. They also indicate the necessity for reducing the vane friction (k_f) to a minimum if reasonable efficiencies are to be obtained at high speeds. This is accomplished in the modern high-speed runner by reducing the number and length of the vanes.

Art. 168a.—Measurement of Head in Turbine Tests.

The efficiency of a unit consisting of a turbine and its setting is given by the ratio

$$\frac{\text{Brake Horse Power of Turbine}}{\text{Water Horse Power available at Turbine}}.$$

(*a*) **Reaction Turbine.**—The net head to be used in calculating the water horse power available in the case of a reaction turbine is the total head at entrance to the turbine (measured from the level of maximum elevation in the tail race, at a point at which the immediate disturbance due to discharge from the turbine has died away) less the kinetic head at this section of the tail race.

In an *encased reaction turbine* the head at entrance is to be taken at the level of the centre of the supply pipe as near the entrance to the turbine casing as is feasible. In a *reaction turbine in an opening setting*, the head is to be measured at a point in the surface of the water as near the turbine setting as possible. The mean velocity of flow at this section of the flume or forebay is to be used in computing the kinetic head, and this is to be taken into account in determining the available head, as in the case of an encased turbine.

Where the tail race near the point of discharge from the draft tube is inaccessible, its surface level at the nearest accessible point is to be taken, and allowance may be made for the difference of level between this point and the point of discharge by an application of the formula

$$V = C\sqrt{mi}$$

using an appropriate value for C.

Where the efficiency of that part of the turbine comprising the casing, guide vanes and runner is required, the net head is the difference between the total heads at the entrance to the casing and at the entrance to the draft tube.

(*b*) **Pelton Wheel or Impulse Turbine.**—In a Pelton wheel or impulse

turbine, the net head to be used in calculations of the efficiency is the sum of the potential pressure, and kinetic heads at a point in the supply pipe line immediately behind the nozzle if only one nozzle is used, and behind the point where the branches to the nozzles diverge if more than one nozzle is

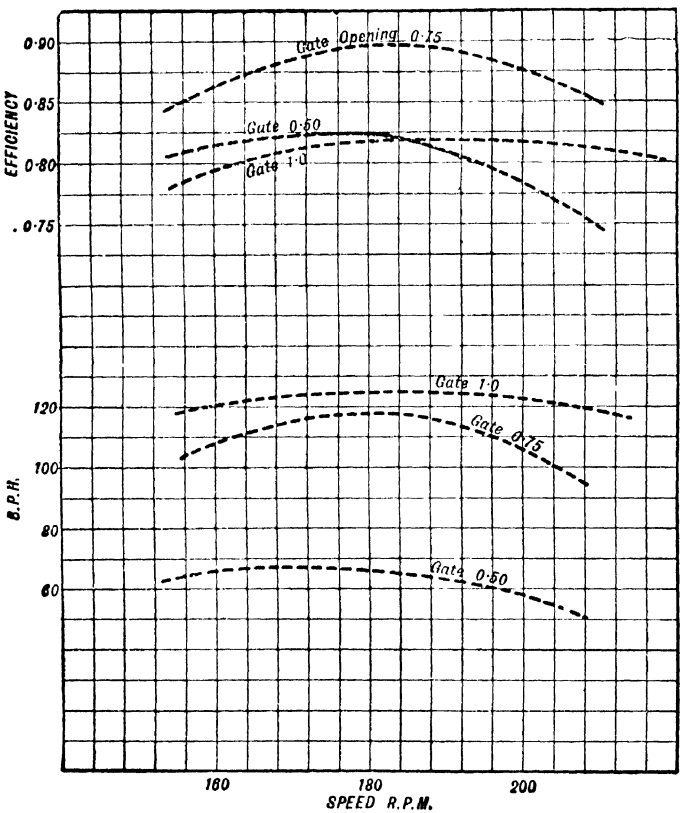


FIG. 267.

used. The datum level for measuring the potential head is to be taken as that of the lowest point of the pitch circle of the buckets.

This takes no account of the height of the wheel above the tail race level. It affords a correct basis for comparing one Pelton wheel with another, but not for direct comparison of a Pelton wheel and of a reaction turbine as applied to the same scheme, since the latter type of turbine, fitted with a draft tube, is able to utilise a larger proportion of the gross head.

Art. 169.—Characteristic Curves for Reaction Turbines.

During the tests of a turbine or model, the head is usually maintained as nearly as possible constant, and the output and efficiency are measured for a series of gate openings and speeds. From these results, speed-power and speed-efficiency curves, as shown in Fig. 267, may be drawn for a series of

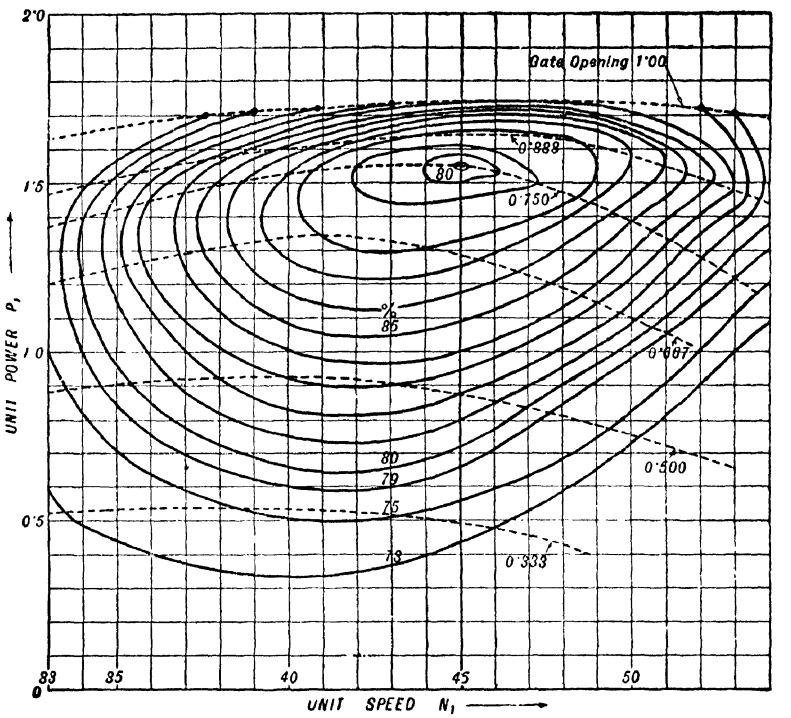


Fig. 268.

gate openings. In order, however, to show graphically the behaviour of the turbine under a wide range of operating conditions, a series of "characteristic curves" is usually prepared from the test data. Such a diagram, deduced from tests on a Francis turbine designed for 124 B.H.P. at a speed of 187 r.p.m. under a head of 17·3 feet, is shown in Fig. 268. To construct this diagram, the values obtained from the tests and shown in Fig. 267 are reduced to those corresponding to a unit head of 1 foot. The speed-power curves under this head at the various gate openings are then drawn as

shown in the dotted lines of Fig. 268 [1] on a base of unit speeds, the unit speed being $N \div \sqrt{H}$. In the above case the unit speed corresponding to 187 r.p.m. is $187 \div \sqrt{17{\cdot}3} = 45{\cdot}0$ r.p.m. From the speed-efficiency curves, the speeds at which the same efficiences are obtained under different gate openings are obtained. These speeds are marked on the corresponding speed-power curves, and through these points a series of continuous curves are drawn. These are the "characteristic curves."

This type of diagram is valuable in an investigation of the effect of any probable change in operating conditions, or of the suitability of the turbine for conditions other than those for which it is designed. If, for example, this particular turbine were required to operate under a 20-feet head, the power developed at full gate would be

$$1{\cdot}73 \times 20^{\frac{3}{2}} = 154{\cdot}8 \text{ B.H.P.},$$

and the normal speed would be

$$45 \times \sqrt{20} = 201 \text{ r.p.m.},$$

while the efficiencies under these conditions would be the same as shown in the diagram for the normal unit speed. If, however, the speed were required to be, say, 200 r.p.m., the corresponding unit speed would be

$$220 \div \sqrt{20} = 49{\cdot}2 \text{ r.p.m.}$$

The corresponding efficiencies may be read off the diagram. For this turbine the values are shown in the second line of the following table. The efficiencies at normal speed are given in the third line of the table.

Gate opening . . .	1·0	·883	·750	·667	·500
Efficiency, per cent. . .	82·8	87·6	86·4	83·0	75·5
Efficiency at normal speed.	83·0	88·2	90·0	87·3	82·0

If the speed of 220 r.p.m. were to be maintained, and the head to be increased to, say, 30·3 feet, the unit speed would become

$$220 \div \sqrt{30{\cdot}3} = 40{\cdot}0 \text{ r.p.m.}$$

[1] Bergstrom, "Proc. Inst. M. E.," February, 1920.

The unit power at full gate is now 1·71, and the output is

$$1{\cdot}71 \times 30{\cdot}3^{\frac{3}{2}} = 285 \text{ B.H.P.}$$

The efficiency under these conditions would be as follows :—

Gate opening . . .	1·0	·883	·750	·667	·550
Efficiency per cent. . .	80·6	85·4	86·0	86·5	83·0

This particular turbine has a specific speed of 59.

Art. 170.—Model Tests.

Although the preceding theoretical investigations lead to results of much value, and, in a turbine of simple form, enable the design to be worked out with a fair degree of accuracy, its limitations are important and must be kept in view. The primary assumption that the water is completely guided by the vanes is certainly only approximately true, while, unless the vanes are parallel for some short distance before the point of exit, the issuing stream may fail to occupy the total exit area. It is usual to allow approximately for this effect by making the calculations on the assumption that the coefficient of contraction at entry to and exit from each series of vanes is ·95. At part gate the conditions become even more uncertain, while in a mixed flow runner of complex form, and in a parallel flow turbine of high specific speed, the true value of the theory lies in its power of giving the main outlines of a preliminary design which may afterwards be modified by the results of experiment.

The more complex the form of the runner, the more necessary does it become to rely on experiment for the final design, and for an estimate of the performance and output under varying conditions of speed, gate opening, and head, and the very high efficiencies attained by some modern turbines have only been obtained by correct theoretical design aided by extended experimental investigation. Testing plants are now utilised by most high-class manufacturers, in which all standard runners are accurately analysed, and in which new designs are tested. In the case of a large turbine, a geometrically similar model is tested. For such tests it is not necessary that the head should be the same as in the large turbine. If the suffix m refer to the model, and if S be the scale ratio of runner and model, and N the revolutions

per minute, then if the turbine and its model are installed in similar settings, their efficiencies will be very approximately equal if

$$\frac{N_m}{N} = S\sqrt{\frac{H_m}{H}},$$

and if, at the same time, the volumes of water passing are in the ratio

$$\frac{Q_m}{Q} = \frac{1}{S^2}\sqrt{\frac{H_m}{H}},$$

the horse-powers will then be in the ratio

$$\frac{\text{H.P.}}{(\text{H.P.})_m} = S^2\left(\frac{H}{H_m}\right)^{\frac{3}{2}}.$$

The relative effect of friction is somewhat greater in the model than in the

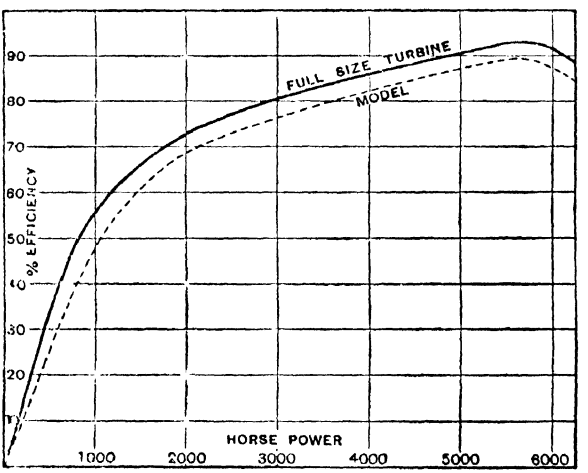


FIG. 269.

full-size machine, and as the losses in the setting are usually also greater in the model, the efficiencies obtained from tests on the complete installation are normally rather higher than from the model tests.

Fig. 269 shows such efficiency curves as obtained from a Francis turbine developing 6,000 H.P. at 116 r.p.m. under a head of 49 feet.[1] The runner diameter is 7 feet 6¼ inches, while that of the model is 2 feet 3⅜ inches. In this case the design of the setting in the large plant was better than in the model tests. Where the settings are similar, the coincidence between the two curves is usually closer than in this particular example.

[1] *Gen. El. Review*, June, 1914.

As a result of comparison between full-scale and model tests Moody[1] suggested the relationship

$$e = 1 - (1 - e_m)\left(\frac{d}{D}\right)^{\cdot 25}\left(\frac{h}{H}\right)^{\cdot 10}$$

where e is the full-scale efficiency and e_m is that of the model under corresponding conditions.

Later he decided to delete the term $\left(\frac{h}{H}\right)^{\cdot 10}$. Pardoe,[2] as a result of further experience, replaces the index 0·25 by 0·20, making

$$e = 1 - (1 - e_m)\left(\frac{d}{D}\right)^{\cdot 20}.$$

Camerer suggests the formula

$$e = 1 - (1 - e_m)\frac{1{\cdot}4 + \frac{1}{\sqrt{D}}}{1{\cdot}4 + \frac{1}{\sqrt{d}}}.$$

where the dimensions are in metres. If in feet this becomes

$$e = 1 - (1 - e_m)\frac{0{\cdot}81 + \frac{1}{\sqrt{D}}}{0{\cdot}81 + \frac{1}{\sqrt{d}}}.$$

Thus if $d = 1$ ft.; $D = 5$ ft.; $e_m = 0{\cdot}85$ the values of e as calculated respectively from the Moody, Pardoe and Camerer formulae become 0·900; 0·891; and 0·896.

Examples.

(1) Show that in a reaction turbine working under conditions of maximum efficiency, the efficiency is given by

$$\eta = 2\,(1 - e)\cos^2 a\left(1 - \frac{\tan a}{\tan \beta}\right)$$

where e measures the proportion of energy existing in the form of pressure energy, in the water at entrance to the wheel.

(2) The angle of the guide blades in an I.F. reaction turbine is 12°, the peripheral speed is 32 feet per second, and the velocity of the water at inlet is 36 feet per second. Determine theoretical vane angles at inlet

[1] L. F. Moody, *Proc. Am. Soc. C.E.*, Aug. 1925, p. 1009.
[2] W. S. Pardoe, *Proc. Am. Soc. C.E.*, 1940, p. 180.

and outlet, the inner being $\frac{2}{3}$ the outer diameter and the velocity of flow constant.

Answer. $\begin{cases} \text{Inlet angle} & = 67°. \\ \text{Outlet angle} & = 19° 20'. \end{cases}$

(3) If in the previous example the head is 36 feet, the B.H.P. 50, the gross efficiency 82 per cent., what must be the effective area of the wheel inlet, assuming a coefficient of contraction of ·95.

Answer. 2·10 square feet.

(4) The external and internal diameters of an I.F. reaction turbine are 4 feet and 2 feet 3 inches, the vanes are radial at inlet, the velocity of flow through the wheel is $= \frac{1}{8}\sqrt{2\,g\,H'}$, and the peripheral velocity is that due to a fall through $\frac{H'}{2}$. Determine the vane angle at outlet for the water to be discharged without any tangential velocity.

Answer. 17° 27'.

(5) If in the turbine of the preceding question the vanes at outlet make an angle of 20° with the circumference, determine the speed at which the wheel should run; the velocity of flow; and the pressure and kinetic heads in the supply chamber, in terms of the available head H', and also determine the angle which the guide blades make with the circumference.

Answer.

Revolutions per minute	$= 4{\cdot}72\sqrt{g\,H'}$	$a = 11° 34'$.
Velocity of flow, f	$= {\cdot}203\sqrt{g\,H'}$	
Pressure head	$= {\cdot}492\,H'$	
Kinetic head	$= {\cdot}508\,H'$.	

CHAPTER XVII.

Speed Regulation—Governors—Pressure Regulators—Inertia of Rotating Parts—Surge Tanks—Differential Surge Tanks—Closed Stand Pipes—Fluctuations of Speed in Practice.

ART. 171.—SPEED REGULATION.

FOR a hydraulic turbine to be capable of close speed regulation under a varying load, it is necessary that it be provided with a reliable governor, that the inertia of the rotating parts be sufficiently great, and that the power conduit be so designed or so equipped that the rise or fall in pressure at the turbine following any probable change of load is only a small fraction of the working head.

As regards the latter factor, the easiest plant to control is one in which the turbine is installed in an open forebay fed directly from the supply canal. Here a demand for power is instantly met by an increased flow into the turbine with the velocity corresponding to the working head, and in this case and also when the turbine gates are closed on a diminishing load inertia effects are negligible. Both head and tail race should be of ample size, so that any fluctuation in flow may not cause an appreciable change in either level, while the approach channel should have easy curves and well-finished surfaces, in order to reduce the possibility of any periodic wave formation.

Wherever possible the use of a long pipe line should be avoided, for it may be taken as generally true that the difficulties of governing increase as the slope from open head water to tail water is diminished.

Where it is necessary to supply the turbine through a pipe line, the force necessary to produce the required acceleration of the supply column on an increasing load, and its retardation on a diminishing load, can only be obtained by a fall or rise in pressure at the turbine. If l feet be the length of the conduit, if v_1 and v_2 feet per second are the velocities of flow required to give the initial and final steady loads, and if t seconds be the interval of time over which acceleration takes place, then, if the acceleration were uniform, the drop in pressure at the turbines below that obtaining with steady flow would be given by

$$p = \frac{v_2 - v_1}{t} \times \frac{l}{g} \text{ feet of water.}$$

Since, for a given percentage increase of load, $v_1 - v_2$ is proportional to the mean velocity of flow, the corresponding drop in pressure will be proportional to this mean velocity, so that, other things being equal, the difficulty of governing increases with the working velocity adopted for the pipe line.

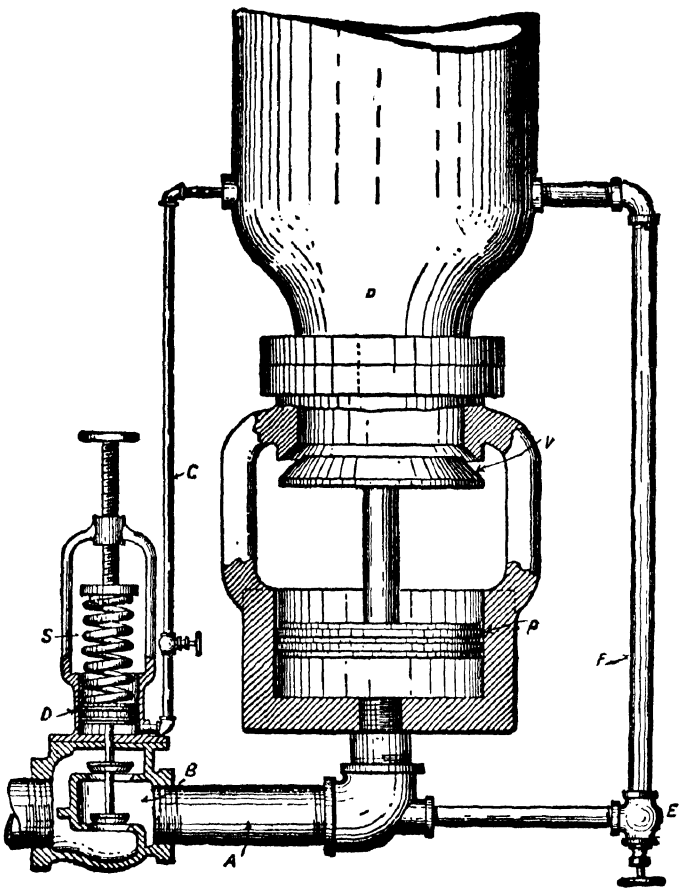


FIG. 270.—Lombard Relief Valve.

With a long pipe line the magnitude of these inertia effects would be too great to permit of successful governing, and to reduce them some form of pressure-regulating device is necessary. The form which this should take depends to some extent on the type of installation.

If the pipe line is steep, with a gradient exceeding about 45°, a demand for energy on an increasing load receives a sufficiently rapid response, and speed regulation in this direction, with moderate velocities of flow, is com-

paratively easy. The increase of pressure on a diminishing load must, however, be guarded against by the provision of a surge tank (Art. 175), automatic pressure regulator, or relief valve. The pressure regulator consists of a valve usually operated through the governing mechanism, and so arranged as to open immediately the speed of the turbine increases, and to close very gradually (Art. 174).

The relief valve consists of a valve opening outwards placed on the penstock near to the turbine casing. This is adjusted so as to open directly the pressure exceeds the normal by a few pounds per square inch, and its area should be such that, if the gates are closed suddenly due to a throwing-off of the entire load, it is capable of taking the whole discharge. If A be the minimum total cross-sectional area of the guide passages when these are wide open, in order to allow for exceptional circumstances the total relief-valve area should not be less than $\cdot 7\ A$.

The ordinary spring-loaded relief valve is not usually very successful on a large plant, one or other type of hydraulically operated valve being commonly fitted. Such a one is the Lombard valve, illustrated diagrammatically in Fig. 270. Here P is the penstock, with its relief valve V, which is held up to its seat by the water pressure on the piston P. The space behind this piston is connected through the pipe A to the waste valve B. This is a balanced valve held closed by means of the spring S against the pressure of the penstock water, which acts through the pipe C on the piston D. When the pressure in the penstock becomes greater than normal the waste valve B is forced open, allowing water to escape and relieving the pressure behind the piston P. The relief valve V then opens, relieving the pressure in the main. When this pressure falls below normal the valve B again closes, and the pressure behind the piston P gradually increases to that in the penstock and again closes the valve V. The rate of closing can be regulated to prevent surging by suitable adjustment of the throttle valve E on the connecting pipe F.

Special consideration of the circumstances of each installation is necessary to determine which of these regulating devices is likely to give the best results.

If the head is not too great to allow it to be constructed at a reasonable cost, a surge tank affords the most satisfactory solution, since it also reduces the inertia effects on an increasing load. Where the pipe line is long and the gradient comparatively small, a surge tank is essential for governing the turbine on an increasing load. If the head is too great to permit of an open surge tank, a closed tank may be used. Without such a device it becomes impossible to obtain close speed regulation of a reaction turbine,

except by the provision of an abnormally heavy fly-wheel, and in an extreme case the only installation capable of giving satisfactory operation consists of a Pelton wheel fitted with a deflector hood, or deflecting nozzle, discharging full bore at all loads except as regulated from time to time by hand.

In the case of a low-head plant fed through a long closed pipe line, care should always be taken to ensure that the drop in pressure accompanying the maximum acceleration possible in practice is not so great as to reduce the pressure appreciably below that of the atmosphere. This condition often renders the use of a surge tank at the power house essential.

The characteristics of the turbine also have an important bearing on its speed regulation. Where the turbine efficiency falls off appreciably between three-quarters and full load, as may be the case, the difficulty of close regulation over this range of loads is greatly increased, since an increase of load requires a greater additional supply of water than would be the case with a constant or an increasing efficiency.

Art. 172.—Governors.

The requirements of a successful governor are :

(1) **Sensitiveness**, *i.e.*, it should alter the position of the turbine gates with as small an increase of speed as possible. With a good hydraulic governor, a fluctuation of speed not exceeding ·5 per cent. is usually adequate to produce motion of the gates.

(2) **Rapidity of Action.**—In general the quicker the response on an increasing load the better. On a falling load, however, a too rapid response tends to set up water-hammer, except in the case of a Pelton wheel with jet deflector or deflecting nozzle. No definite rule can be laid down for the best speed of operation. A greater speed is possible with a short penstock and low velocities of flow than with a longer penstock or higher velocities. In general a governor which will give the full range of gate opening within two or three seconds will satisfy all requirements ; but, if necessary, the modern governor is capable of reducing this time to one second or even less.

Owing to the inertia and considerable frictional resistance of the turbine gates, the centrifugal governor is not sufficiently powerful in itself to give the required motion, and some form of relay becomes necessary. In the earlier turbines various types of mechanical relay were used. Fig. 271 [1] shows such a device, in which a double ratchet is actuated by a link from a lay shaft belt-driven from the turbine shaft. A ratchet wheel, mounted on a second lay shaft by which the turbine gate is directly operated, is wholly

[1] By courtesy of Messrs. Gilbert Gilkes & Co., Ltd., Kendal.

or partially masked by a plate whose position depends on the height of the governor. Thus, under normal conditions of working, the plate covers the wheel so as to put both pawls out of gear. Any increase in speed then raises the governor, rotates the guard plate, and allows one pawl to gear with the ratchet wheel, rotating the lay shaft and closing the turbine gates. A decrease in speed causes the second pawl to be put into gear, and thus produces an opposite rotation of the lay shaft and an opening of the gates.

With any such type of mechanical device, however, some considerable time is required to entirely open a sluice gate, and while for such a purpose

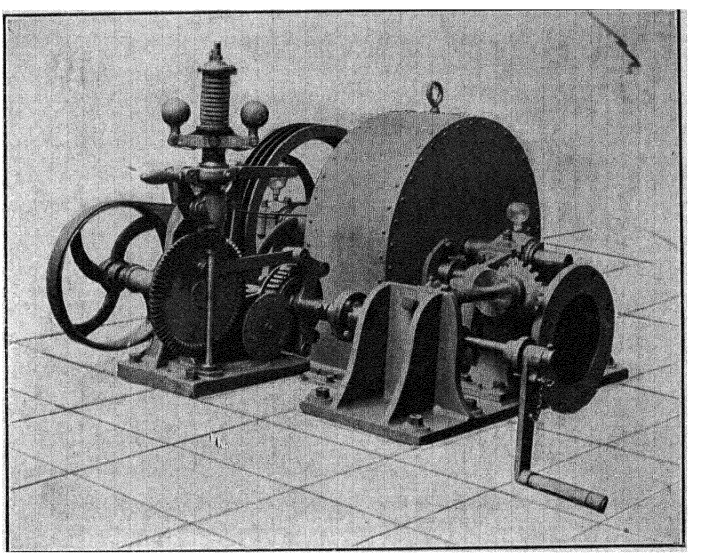


FIG. 271.—Governor with Mechanical Relay, as fitted to Girard Turbine.

as driving textile machinery, where the changes of load are relatively small, this type fulfils the requirements, it has proved inadequate to satisfy the more exacting demands of electric generation. For such a purpose the pressure-operated relay provides the only satisfactory solution. Here the centrifugal governor operates a regulating valve which admits fluid under pressure to one side or other of a piston in a relay cylinder, this piston being connected to and operating the gate mechanism. The operating fluid may consist either of water or of oil. If water, multi-stage centrifugal pumps may be used for producing the pressure ; if oil, plunger pumps or gear-wheel pumps are used. The pumps deliver into a pressure tank provided with a large air chamber, which enables the draft on the pressure supply during the

short periods of gate motion to be made at a greater rate than that of the pump supply. Owing to the possibility of sticking due to grit in the water and to corrosion, the water-pressure governor has been generally displaced, except in the largest systems, by the oil-pressure governor. In the case of a large system the use of water, with the addition of soluble oil, potassium bichromate, or glycerine, has some advantages in cheapness and in that it enables the more convenient form of high-speed centrifugal pump to be used.

(3) **Stability**, *i.e.*, the governor should bring the gates to their proper position with as little hunting or as few oscillations as possible about this

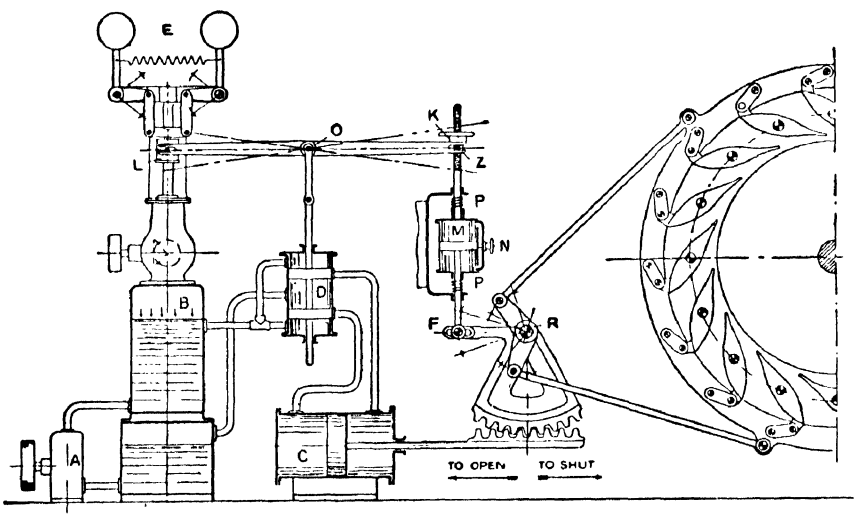


FIG. 272.

position. To ensure this, some form of relay return or compensating device is usually necessary. The reason for this will be evident if it be considered that, as the speed falls, the gates are opened and the supply column is accelerated, this action persisting until the supply of energy per unit time is equal to the demand. But the acceleration of the water column goes on for an appreciable time after the gate opening has ceased, and in consequence the supply becomes too great for the requirements of the wheel, the speed increases, and the governor commences to close the gates. This checks the motion of the supply column, and in virtue of its inertia produces an increased pressure at the valve, and a temporarily increased velocity of flow through the gates. The speed of the wheel thus increases still further, and the gates are closed until an instantaneous balance is again set up between supply and

demand As the inertia pressure falls, the supply now becomes less than the demand, the speed falls, the gates commence to reopen, and the state of hunting which is here outlined may not die out for some considerable time. To prevent hunting, it is necessary that the governor should cause the gates to overrun their final position slightly, and to bring them back slowly to their final position, corresponding to the altered load. This latter operation is performed by means of some form of compensating device.

Fig. 272 shows diagrammatically one such typical arrangement,[1] where *A* represents the oil pump, *B* the pressure-oil receiver, *C* the servo-motor, *D* the distributing valve, and *F* the compensating device. The governor is so adjusted that at normal speed the sleeve *L* and the distributing valve are in their central positions as shown. An increase in speed lifts the valve about *Z* as a fulcrum, forcing the servo-motor piston to the right and closing the gates. This motion lowers the points *F* and *Z*, bringing the distributing valve back to its central position and preventing any further motion of the piston of the servo-motor. The distance between *F* and *Z* can be altered by hand so as to enable the speed to be varied slightly while running if required. In this arrangement each position of the guide vanes corresponds to a definite speed of rotation. In order to obtain a sensibly constant speed at all loads, a spring-loaded oil dashpot *M* is introduced between the points *F* and *Z*. On any rapid motion of the gates following a sudden change of load, this dashpot acts as a rigid connection, and the controlling springs *PP* are extended or compressed. Under the action of these springs, the dashpot cylinder is gradually brought back to its central position in its guides, while its piston remains stationary. This brings the point *Z* back to its central position. The accompanying motion of the distributing valve causes the speed either to increase or diminish slightly until *L* attains its central position corresponding to constant turbine speed.

Similar devices, as applied to Pelton wheel installations, have already been illustrated in Figs. 201, 202 and 209.

Art. 178.—Pressure Regulators.

In high-head plants, where the pipe line is long and circumstances do not permit of the provision of a surge tank, the governor may be operated in conjunction with a pressure regulator or relief valve arranged so as to open and discharge water into the tail race as the turbine gates close. One such regulator, as applied to a Francis turbine, is shown diagrammatically in Fig. 273. A rapid closure of the gates lifts the weighted dashpot *A*, and

[1] Messrs. The English Electric Co., Ltd.

along with it the relief valve. The valve then gradually closes under the action of this weight, the time of closure being regulated by the amount of opening of the needle valve, which controls the rate of flow of the oil from one side to the other of the piston *B*.

In a large unit the mechanical operation of a large relief valve necessitates a very powerful servo-motor, and to obviate this the type of regulator shown in Fig. 200 may be used.

Fig. 274[1] shows another type of pressure regulator in which the operating force is provided by the pressure of the supply water. The lever *A* is coupled

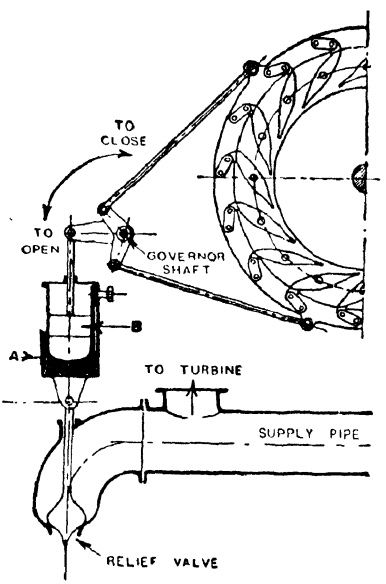


FIG. 273.

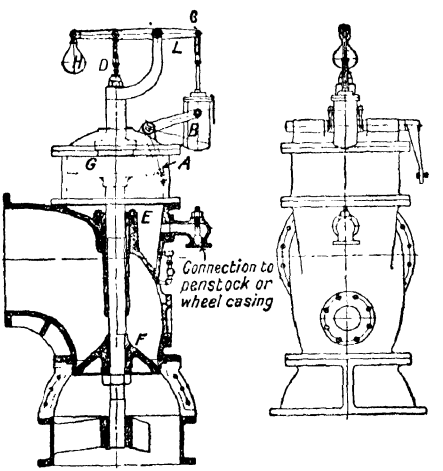


FIG. 274.—Pressure Regulator.

to the gate mechanism, and a closure of the gates moves this lever from right to left. This lowers the dashpot *B*, and, if the motion is sufficiently rapid, carries down the dashpot piston, lowering the end *C* of the lever *L*, and opening the pilot valve *D*. This allows pressure water to escape from the chamber *E*, the pressure on the relief valve *F* overcomes the upward pressure on the balance piston *G*, and the valve opens. The pilot valve is then slowly closed by the action of the weight *H* acting against the resistance of the dashpot, and the pressure in chamber *E* gradually increases and closes the relief valve.

The capacity of a pressure regulator should be between 50 and 75 per

[1] The Wellman Seaver Morgan Co.

cent. of the maximum turbine discharge, depending on the type of plant and the conditions of operation.

From the nature of the case the pressure regulator is much better adapted for use under high heads and with small volumes of water than where the volumes to be handled are large.

Art. 174.—The Surge Tank.

The surge tank or stand pipe consists essentially of a vertical open pipe whose lower end is connected to the penstock as near to the turbines as

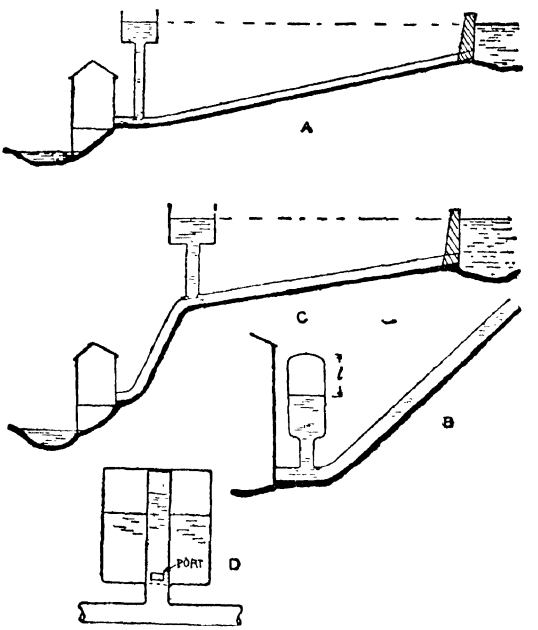


Fig. 275.

possible, and whose upper end is above the surface-level in the supply reservoir (Fig. 275 *a*). Any closure of the turbine gates is then accompanied by a flow up the stand pipe. The resultant retardation of the supply column and the rise in pressure, with a surge tank of adequate size, is greatly reduced.

The surge tank is, however, of greatest service on an increasing load, where, owing to the necessity for accelerating the supply column, the drop in pressure at the turbines would be excessive. In such a case any sudden demand at the turbines is met by the flow down the surge tank, with a consequent reduction in the necessary acceleration in the penstock.

Where the head is so great as to render an open stand pipe impracticable, a closed stand pipe may be installed, as shown in Fig. 275 *b*. Compressed air is now supplied to the upper part of the tank by a small compressor of sufficient capacity to make up any leakage or absorption of air by the water.

Where there is high ground at some distance from the power house, an open stand pipe may be installed there (Fig. 275 *c*), and if the length of closed penstock between this and the turbines is too great for good regulation, a small auxiliary closed stand pipe may be installed adjacent to the turbines.

In the following investigation of the simple surge tank, let

L = length of supply pipe line from forebay to surge tank;
R = ratio of sectional area of surge tank and main;
c = friction coefficient, so that friction loss in pipe line under velocity $V = cV^2$;
V_1 = velocity in pipe line under steady flow before change of load;
V_2 = velocity in pipe line under steady flow after change of load;
V = velocity in pipe line at any instant during unsteady period;
V' = velocity in pipe between stand pipe and turbines at any instant during unsteady period;
y = height of water in stand pipe at any instant, above the level corresponding to steady flow with velocity V_1 (Fig. 276).

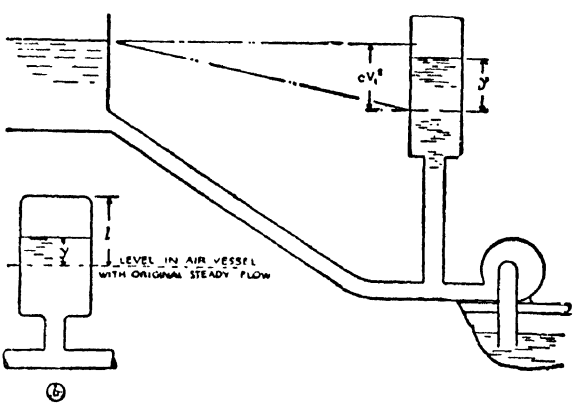


FIG. 276.

Then at an instant t seconds after a given change of load, the head available for producing retardation in the pipe line is

$$y - c(V_1^2 - V^2),$$

and the equation of motion in the pipe line is

$$y - c(V_1^2 - V^2) = -\frac{L}{g}\frac{dV}{dt} \quad \ldots\ldots\ldots \quad (1)$$

Also, since the quantity of water entering the turbines equals the volume flowing down the pipe line, less that entering the tank,

$$R\frac{dy}{dt} = V - V' \quad \ldots\ldots\ldots \quad (2)$$

Multiplying (1) and (2) gives

$$[y - c(V_1^2 - V^2)]\,dy = -\frac{L}{gR}(V - V')dV \quad \ldots \quad (3)$$

$$\int y\,dy = \frac{L}{gR}\int (V' - V)\,dV + c\int (V_1^2 - V^2)\,dy \quad \ldots \quad (4)$$

At the point of maximum surge $V' = V_2$, and (4) becomes

$$\int_o^{y_m} y\,dy = \frac{L}{gR}\int_{V_1}^{V_2} (V_2 - V)\,dV + c\int_o^{y_m} (V_1^2 - V^2)\,dy \quad . \quad (5)$$

or

$$y_m^2 = \frac{L}{gR}(V_2 - V_1)^2 + 2c\int_o^{y_m} (V_1^2 - V^2)\,dy \quad \ldots \quad (6)$$

The last term cannot be integrated mathematically unless V is known in terms of y. Since V is unknown, some approximate method is necessary to obtain a solution. Denoting the variable $c(V_1^2 - V^2)$ by z, it appears that when y is small, z increases slowly and dz is less than dy. As y approaches its maximum, z increases more rapidly than y, and dz becomes greater than dy. In view of this, R. J. Johnson [1] assumes that a sufficiently close approximation is attained by putting $dz = dy$ as an average over the whole summation, so that

$$c\int_o^{y_m} (V_1^2 - V^2)\,dy = \int_{z_1}^{z_2} z\,dz = \left[\frac{z^2}{2}\right]_{z_1}^{z_2}$$

$$= \frac{c^2}{2}\left[(V_1^2 - V^2)^2\right]_{V_1}^{V_2} = \frac{c^2}{2}(V_1^2 - V_2^2)^2,$$

leading to the final expression

$$y_m^2 = \frac{L}{gR}(V_1 - V_2)^2 + c^2(V_1^2 - V_2^2)^2 \quad \ldots \quad (7)$$

This expression holds for both acceleration and retardation.

[1] "Trans. Am. Soc. C. E.," Vol. lxxviii. (1915), p. 760. Also 1908, p. 443.

Neglecting friction and any governor action, the motion in the tank would be simple harmonic, with a time T of complete oscillation, given by

$$T = 2\pi\sqrt{\frac{RL}{g}} \text{ seconds} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

while $y_{max.}$ would be equal to $\pm \sqrt{\frac{L}{gR}}(V_1 - V_2)$ . . . (9)

Taking friction into account, the time of oscillation is increased. Johnson finds that the expression

$$T = \frac{\pi}{2}\sqrt{\frac{RL}{g} + c^2 R^2 (V_1 + V_2)^2} \text{ sec.} \quad . \quad . \quad . \quad . \quad . \quad (10)$$

gives a very close approximation to the actual time required to reach the first crest or trough of the wave.

After the first surge the oscillation is gradually damped out by the pipe friction, except where affected by the action of the governor. If the governor operates so as to close the gates during the upward surge, and to reopen them on the downward surge, keeping in step with the natural oscillation in the tank, the oscillations, instead of dying out, may, with a tank of comparatively small cross-sectional area, increase with time, in which case successful operation is impossible.

In deducing expression (7), it is assumed that the velocity of flow to the turbines at the instant of maximum surge is the same as V_2, the velocity of flow under steady flow when conditions have settled down after the change. Owing to the fact that the head at the turbines is not the same at the two instants, this will not, in general, be the case. Apart from this, expression (7), depending as it does on an approximation to the effect of friction, may be appreciably in error if the friction loss is relatively high. Thus, while the expression is very useful in making preliminary investigations, it should not be used for the determination of final designs. For such a purpose, the method of arithmetical integration should be used. This method, while somewhat tedious, enables the effect of any known and variable factor to be taken into account.

Arithmetical Integration and Solution of Stand Pipe Surge.—Suppose the initial velocity to be V_1, and suppose a sudden alteration in the demand to V_2, which is to be constant after the initial change. If V_2 be less than V_1 the water will rise in the surge tank.

After a small interval of time δt let the velocity up the standpipe be v_1. Then the mean velocity during this period will be $\frac{v_1}{2}$ and the height y_1 above that obtaining before the change will be $\frac{v_1 \delta t}{2}$ at the end of the period. At

this instant the velocity in the main pipe line will be $V_2 + v_1 R$, where R is the ratio of surge-tank area to pipe area, so that the mean acceleration in the main during this period will be

$$-\left\{\frac{V_1 - (V_2 + v_1 R)}{\delta t}\right\}.$$

The mean velocity in the main during the period is $V_2 + \frac{v_1 R}{2}$, while the mean height y is $\frac{v_1 \delta t}{4}$, so that equation (1), p. 545, becomes

$$\left\{\frac{V_1 - (V_2 + v_1 R)}{\delta t}\right\}\frac{L}{g} + c\left\{V_1^2 - \left(V_2 + \frac{v_1}{2} R\right)^2\right\} = \frac{v_1 \delta t}{4},$$

from which, by substituting the known values of V_1, V_2 and δt, v_1 may be obtained. Now taking a second period δt, and calling v_2 the velocity up the surge tank at the end of this period, the mean velocity during the period is $(v_1 + v_2) \div 2$, and the height y_2 at the end of the period is equal to

$$y_1 + \left(\frac{v_1 + v_2}{2}\right)\delta t.$$

The mean height during the period is $y_1 + \left(\frac{v_1 + v_2}{4}\right)\delta t$, and the mean velocity in the pipe line is $V_2 + \frac{v_1 + v_2}{2} . R$, while the acceleration during the period is $\left(\frac{v_2 - v_1}{\delta t}\right) R$, so that

$$\left\{\frac{v_1 - v_2}{\delta t}\right\}\frac{RL}{g} + c\left\{V_1^2 - \left(V_2 + \frac{v_1 + v_2}{2} R\right)^2\right\} = y_1 + \left(\frac{v_1 + v_2}{4}\right)\delta t$$

for which v_2 and hence y_2 may be obtained

Similarly, at the end of a third period, we have

$$\left\{\frac{v_2 - v_3}{\delta t}\right\}\frac{RL}{g} + c\left\{V_1^2 - \left(V_2 + \frac{v_2 + v_3}{2} . R\right)^2\right\} = y_2 + \left(\frac{v_2 + v_3}{4}\right)\delta t$$

giving v_3, and so on.

If, for example, $L = 500$ feet; $R = 8{\cdot}0$; $c = {\cdot}03$; $V_1 = 4{\cdot}77$ f.s.; $V_2 = 1{\cdot}94$ f.s., and if an interval δt of 5 seconds be adopted, we have :—

During period (1) :—

$$\frac{(4{\cdot}77 - 1{\cdot}94 - 8\,v_1)\,500}{5 \times 32{\cdot}2} + {\cdot}03\left\{22{\cdot}75 - (1{\cdot}94 + 4\,v_1)^2\right\} = 1{\cdot}25\,v_1$$

which reduces to $v_1^2 + 55{\cdot}4\,v_1 - 19{\cdot}5 = 0$

giving $v_1 = {\cdot}352$ f.s.

$$y_1 = \frac{{\cdot}352 \times 5}{2} = {\cdot}88 \text{ feet.}$$

and $V = 1{\cdot}94 + (8 \times {\cdot}352) = 4{\cdot}756$ f.s.

During period (2) :—

$$\frac{(\cdot 352 - v_2)\ 500 \times 8}{5 \times 32 \cdot 2} + \cdot 03 \left[22 \cdot 75 - \{ 1 \cdot 94 + 4 (\cdot 352 + v_2) \}^2 \right] =$$

$$\cdot 88 + \left(\frac{\cdot 352 + v_2}{4} \right) 5$$

reducing to $v_2^2 + 56 \cdot 1\, v_2 - 16 \cdot 2 = 0,$

giving $v_2 = \cdot 289$ f.s.

$$y_2 = \cdot 88 + \left(\frac{\cdot 352 + \cdot 289}{2} \right) 5 = 2 \cdot 48 \text{ feet.}$$

and $V = 1 \cdot 94 + (8 \times \cdot 289) = 4 \cdot 252$ f.s.

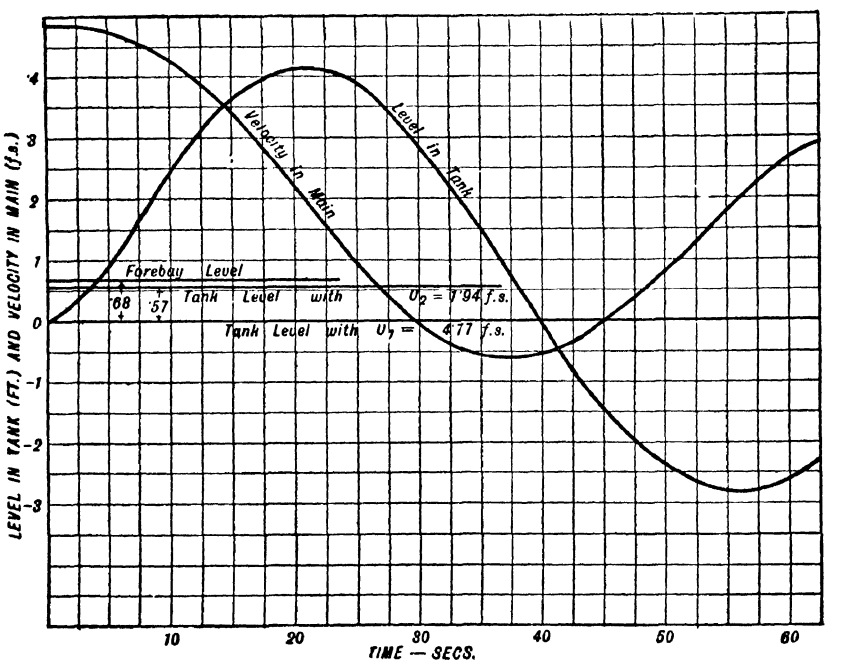


FIG. 277.

Proceeding from step to step in this way the curves of Fig. 277 are obtained. An upward surge of 4·21 feet is followed by a downward surge of 2·80 feet below the original level in the surge tank, which would be followed by a smaller upward surge and so on until the level settles down to that obtaining under the new steady conditions.

The time to reach the crest of the first surge is approximately 21·0 seconds.

It will be noticed that the water in the pipe line is brought to rest and then slightly reversed in motion during a portion of the cycle.

Calculating the height of the first surge and the time to reach the maximum height by formulae (7) and (10), gives a height of 3·98 feet and a time of 17·7 seconds.

In investigating a surge-tank problem to fix the final design, the intervals of time should be taken more closely than in this example. Usually eight or ten intervals are advisable up to time of the first surge.

Art. 175.—Closed Stand Pipe.

In the case of a closed stand pipe let l be the length of the tank above the original water-level (Fig. 276 *b*) and p the original absolute air pressure in feet of water. Then equation (1), p. 545, becomes

$$\frac{p\,l}{l-y} - p + y - c(V_1^2 - V^2) = -\frac{L}{g}\frac{dV}{dt},$$

while equation (7) becomes

$$y_m^2 = 2\,p\,l\log_e\left(\frac{l-y_m}{l}\right) + 2py_m + \frac{L}{g\,R}(V_1 - V_2)^2 + c^2(V_1^2 - V_2)^2.$$

This expression holds for both acceleration and retardation by giving the correct sign to y. From its form it is evident that a given volume of air chamber is most effective when the sectional area is as large and the length l as small as possible.

The closed stand pipe suffers from the disadvantage that an air compressor is necessary to keep the air vessel charged. Unless very carefully designed to suit the characteristics of the particular installation, sympathetic surges are apt to be set up, which render successful regulation impossible.

Art. 176.—The Differential Surge Tank.

With a view to preventing any augmented oscillations in the surge tank due to governor action or to synchronism of load changes, R. D. Johnson has suggested the use of the differential surge tank.

This consists of a simple stand pipe or riser, supplemented by a larger tank, which surrounds the riser and communicates with it through one or more openings of restricted size near the bottom of the tank (Fig. 276 *d*). The diameter of the riser is usually from ·75 to 1·0 times that of the conduit.

Fig. 278 shows a typical arrangement of such a tank, as applied to a 40,000 H.P. installation.[1] Freezing in winter is prevented by the circulation of hot air around the top of the tank from a heater at the foot of the riser.

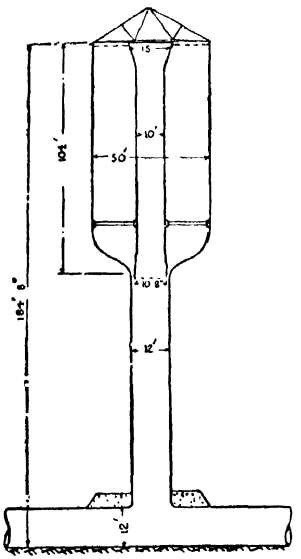


FIG. 278.

In the case of a sudden change of load the level in the riser alters quickly, allowing the acceleration or retardation in the conduit to take place more rapidly than with a simple surge tank. The difference of level between the water in the riser and the tank then causes a flow through the connecting ports, which gradually equalises the levels. In the correctly designed tank, the area of the tank and port are so arranged as to enable this to take place just as the velocity in the conduit attains its final steady value.[2]

It is evident that since the initial pressure change at the turbines is greater than with the simple surge tank, extra work will be thrown on the governors, and the inertia of the rotating parts will require to be greater in order to maintain the same temporary fluctuation in speed. Actually, however, the modern governor is capable of dealing satisfactorily with such changes as may occur in any normal plant.

ART. 177.—MODEL SURGE TANK INVESTIGATIONS.

The surge tank problem, especially when the layout is complicated, is most satisfactorily handled by the use of a model. Usually a comparatively simple set up is required, and experiments[3] show that the results obtained are in very close agreement with the behaviour of the prototype.

In the case of a surge tank and its model the ratio of corresponding vertical displacements y is fixed by the ratio of the friction heads at the corresponding initial steady velocities of flow, so that if the suffix m refers to the model,

$$y/y_m = C\bar{V}^n/C_m V_m^n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

[1] Salmon River Power Co., *Engineering News*, July 16, 1914, p. 150.

[2] For an approximate treatment of the theory of the differential surge tank, see Mr. Johnson's original paper, "Trans. Am. Soc. C. E.," vol 78, 1915, p. 760, or "Hydro-Electric Engineering," vol. 1, p. 198: "Gibson." Blackie & Son.

[3] "Investigation of the Surge Tank Problem by Model Experiments," Gibson, *Proc. Inst C.E.*, Vol. 219, 1924–5, Part I, p. 161 and Vol. 235, 1932–3, Part I, p. 327.

For equation (1) to be dimensionally homogeneous it follows that

$$y/y_m = \frac{L \cdot \bar{V} \cdot t_m}{L_m \cdot V_m \cdot t} \text{ so that } \frac{t}{t_m} = \frac{L\bar{V}y_m}{L_m V_m y} = \frac{C_m \cdot L\,\bar{V}_m^{n-1}}{C \cdot L_m\,\bar{V}^{n-1}} \quad . \quad . \quad . \quad . \quad . \quad (9)$$

and for equation (2) to be dimensionally homogeneous :—

$$R/R_m = \frac{\bar{V}\,t\,y_m}{V_m\,t_m\,y} = \frac{C_m^{\,2} \cdot L\,\bar{V}_m^{\,2n-2}}{C^2 \cdot L_m\,\bar{V}^{2m-2}} \quad . \quad . \quad . \quad . \quad . \quad (10)$$

A model of an average installation, if to natural scale, would usually need to have a very large-scale ratio, if the range of movements in the surge tank were to be such as to make accurate measurements possible. Moreover, the diameter of the pipe line would often be so small that the velocity would be below the critical.

Fortunately experience shows that very different scale-ratios may be adopted for vertical and horizontal dimensions and in practice a much smaller ratio is used for the vertical dimensions.

A pipe line of any convenient diameter may be used, so long as the ratio R_m of the areas of the model surge tank and the model pipe line at any given level is related to the ratio R of the areas of the large surge tank and pipe line at the corresponding level as indicated by formula (10).

In practice a convenient horizontal scale of length, of pipe diameter, and of working velocity $\bar{V}_1$ are closer for the model. The values of n and of the frictional loss $C_m\bar{V}_m^{\,n}$ in the model at this velocity are determined by experiment ($= y_m$). Knowing the diameter and length of the large pipe line the value of $C\,\bar{V}_1^{\,n}$ or of y is computed. The ratio of these values of y gives the vertical scale ratio of the model.

The correct area of the model surge tank is then determined from (10).

If the time of opening or closing the turbine gates is known the corresponding time in the model is then given by the relationship (9), and the elevations of the water surfaces in the surge tanks at corresponding times will be in the ratio

$$\frac{y_m}{y} = \frac{C_m\,\bar{V}_m^{\,n}}{C\,\bar{V}_n}.$$

As an example take the case of a 4-foot pipe line 500 feet long. It has a surge tank 11·33 feet diameter, so that $R = 8{\cdot}0$. The initial steady velocity of flow is 4·77 f.s. and this is suddenly reduced to 1·94 f.s. The observed friction loss in the pipe is 0·61 feet at 4·77 f.s. The model has a 3-inch pipe 33·3 feet long. The friction loss is $C\,\bar{V}^n$ where, by experiment, $C = {\cdot}086$ and $n = 1{\cdot}82$. Assuming the same index for the large pipe, the corresponding value of C is 0·0357.

Choosing an initial velocity of 1 f.s. for the model as being convenient, the final velocity will be 0·406 f.s.

The ratio of the values of y is then $\frac{0{\cdot}086}{0{\cdot}0357} \times \left(\frac{1{\cdot}0}{4{\cdot}77}\right)^{1{\cdot}82} = \frac{1}{7{\cdot}12} = \frac{y_m}{y}$

while $$\frac{R_m}{R} = \frac{33{\cdot}3}{500} \times \left(\frac{0{\cdot}0357}{0{\cdot}086}\right)^2 \times \left(\frac{4{\cdot}77}{1}\right)^{1{\cdot}64} = \frac{1}{6{\cdot}70}$$

and $$\frac{t_m}{t} = \frac{33{\cdot}3 \times 0{\cdot}0357}{500 \times 0{\cdot}086} \times \left(\frac{4{\cdot}77}{1}\right)^{0{\cdot}82} = \frac{1}{10{\cdot}01}.$$

The value of R for the model is $8 \div 6{\cdot}7 = 1{\cdot}196$, so that its surge tank should have an area of 8·45 square inches and a diameter of 3·28 inches. The time to attain the crest of the first surge and the period of successive surges will be 10·01 times as great as in the model and the amplitude of the surges will be 7·12 times as great.

In such models the constant level reservoir is represented by an open tank with levelled upper edges. In operation the supply to this tank is such that a slight overflow takes place under initial conditions. When shut down takes place this overflow increases, but with a tank say 3 feet by 3 feet the rise in level is insignificant.

Art. 178.—Effect of Inertia of Rotating Parts.

During any sudden change of load, there is a certain period, whose duration depends on the design of the installation and on the sensitiveness and rapidity of action of the governor, during which the supply of energy to the turbine is greater or less than the demand, and, in order to prevent the temporary fluctuation in speed during this period becoming excessive, some appreciable fly-wheel effect is necessary. This should be sufficient for the difference in the kinetic energy of the rotating parts, at the highest and lowest permissible speeds, to be equal to the maximum excess or defect of energy supply and demand during the period of changing load.

If the governor takes t seconds to complete the gate movement corresponding to a given change of load, and if the variation of head at the turbine during this period and the law of gate closure be known, the difference between the energy developed by the turbine and that absorbed at the turbine shaft during this period can be calculated. If this energy, measured in foot lbs., be represented by E, and if w_1 and w_2 are the initial and final angular velocities of the shaft, the kinetic energy liberated or stored up during the change of speed

$$= \frac{1}{2}\frac{I}{g}(w_1^2 - w_2^2) = \frac{1}{2}\frac{I}{g}(w_1 + w_2)(w_1 - w_2) = \frac{I}{g}\, w \,.\, \delta w,$$

where I is the moment of inertia of the rotating parts and equals $w k^2$, k being the radius of gyration and w the weight in lbs.; and where w is the mean angular velocity.

If N be the mean speed in revolutions per minute ($w = \pi N/30$), we have

$$\frac{I}{g} w \,.\, \delta w = E$$

$$\therefore \quad \frac{\delta w}{w} = \frac{E g}{I w^2}$$

$$\therefore \quad \frac{\delta N}{N} = \frac{900 E g}{\pi^2 N^2 I} = \frac{2940 E}{N^2 I}.$$

For example, in the case of a turbine developing 5,000 H.P. at 250 r.p.m. if 50 per cent. load be thrown off instantaneously, and if the governor is capable of giving the required gate motion in 2 seconds, the average horse-power generated by the turbine during this period, if the rate of change of input follows a straight-line law, will be $(5{,}000 + 2{,}500) \div 2 = 3{,}750$ H.P. The output is 2,500 H.P., so that E is the energy equivalent to 1,250 H.P. or 2 seconds, or $1{,}250 \times 550 \times 2 = 1{,}370{,}000$ foot lbs. If under these conditions the increase in speed is not to exceed 5 per cent., so that $\delta N \div N = \cdot 05$ the necessary moment of inertia, I, of the rotating parts is equal to

$$\frac{2{,}940 \times 1{,}370{,}000}{62{,}500 \times \cdot 05} = 1{,}290{,}000 \text{ foot lb. units.}$$

Where no surge tank is fitted, or where any long length of pipe line is fitted between the surge tank and the turbines, it becomes necessary, in order to obtain an accurate estimate of the size of fly-wheel necessary to give governing within definite limits, to determine the pressure at each instant during the change of load, at the turbine gates or turbine nozzle. While this requires a somewhat tedious mathematical investigation, it may be done when the law of closure or of opening of the gates or nozzle is known (pp. 225, 241).

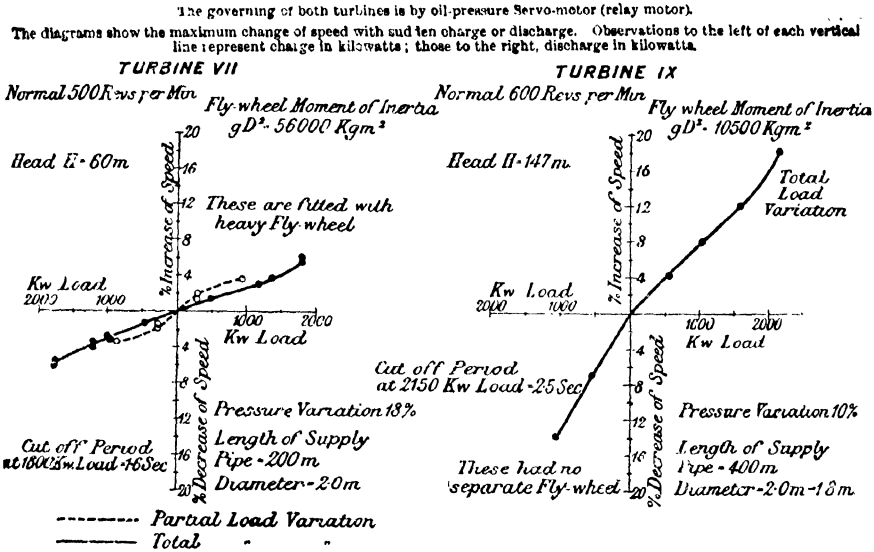
Fluctuations of Speed in Practice.—As already indicated, the most perfect governor is incapable of giving close speed regulation if the hydraulic conditions of the installation are essentially bad. With reasonably good hydraulic conditions, and without any special flywheel, a good modern governor will enable the speed of a reaction turbine to be maintained within about

2·5	per cent.	for a	25	per cent.	load charge,
5–6	,,	,,	50	,,	,,
12–15	,,	,,	100	,,	,,

These are the instantaneous fluctuations of speed. The final fluctuation in speed may in general be kept within

1	per cent. for a		25	per cent. load change	
2	,,	,,	50	,,	,,
4	,,	,,	100	,,	,,

In a Pelton wheel installation, with a jet deflector and governor-operated needle, the fluctuations of speed are greater on an increasing than on a



FIGS. 279, 280.—Governing Tests of Pressure Turbines.

diminishing load, owing to the necessity for the acceleration of the supply column on an increasing load. The following shows typical values of the instantaneous fluctuation in speed in such an installation :—

12–15	per cent. for a		100	per cent. load change		(load on).
4– 5	,,	,,	100	,,	,,	(load off).
5– 7	,,	,,	50	,,	,,	(load on).
3– 4	,,	,,	50	,,	,,	(load off).
3– 4	,,	,,	25	,,	,,	(load on).
2– 3	,,	,,	25	,,	,,	(load off).

The final fluctuations are sensibly the same as in a reaction turbine.

In a Pelton wheel the turning moment suffers a slight periodic variation, depending on the number of buckets and the speed of the wheel. On account of this and of the comparatively small weight of the wheel for a given power, a flywheel is often essential for such close governing as is necessary for the driving of alternators in parallel.

Figs. 279 and 280 show the results of governing tests on turbines VII. and IX. (pressure turbines) of Fig. 254.

CHAPTER XVIII.

The Hydraulic Engine—Brotherhood—Rigg—Swash Plate Engine—Theory of Action—Losses—Port Areas.

ART. 179.—THE HYDRAULIC ENGINE.

WHERE a supply of high pressure water is available and where rotary motion at a moderate speed is desired, the reciprocating piston engine has certain advantages for small powers, particularly where it is able to work at or near full load and where the speed variation may be excessive, as occurs, for example, in the working of a capstan. For such work the rotary motor is out of the question, both on account of the necessity for gearing to reduce its necessarily high speed, and of the great reduction in its efficiency under variable speed conditions. The reciprocating engine, however, having an efficiency which is approximately independent of its speed, and being compact, is particularly well adapted for such work.

One of the most widely used types of engine is the Brotherhood (Fig. 281), which is designed to work with pressures from 60 lbs. to 1,050 lbs. per square inch.

Here three single acting cylinders fitted with trunk pistons are fixed radially at 120° to an external cylindrical casing, the three connecting rods working on a single crank pin. Each cylinder is fitted with a single inlet and outlet port, the opening of this to supply and exhaust being regulated by the rotary valve shown at D in Fig. 281.

This valve is rotated by the crank shaft, and carries passages connecting with the pressure supply and the exhaust which are alternately presented before the port of each cylinder in turn. The method of keeping the rotating joints tight against such high pressures is indicated in the sketch. Here A is a leather joint washer, on which the back nut B rotates, the play due to wear on the joint being taken up by the expansion of the rubber ring C, which rotates with the valve and back nut. The valve itself bears against a lignum-vitæ seat. All pistons are packed with "L" leathers, and the cylinders and all working parts are lined with brass.

The engine as thus constructed forms a compact and serviceable motor, has no dead centre, and is built in sizes up to about 30 B.H.P. The

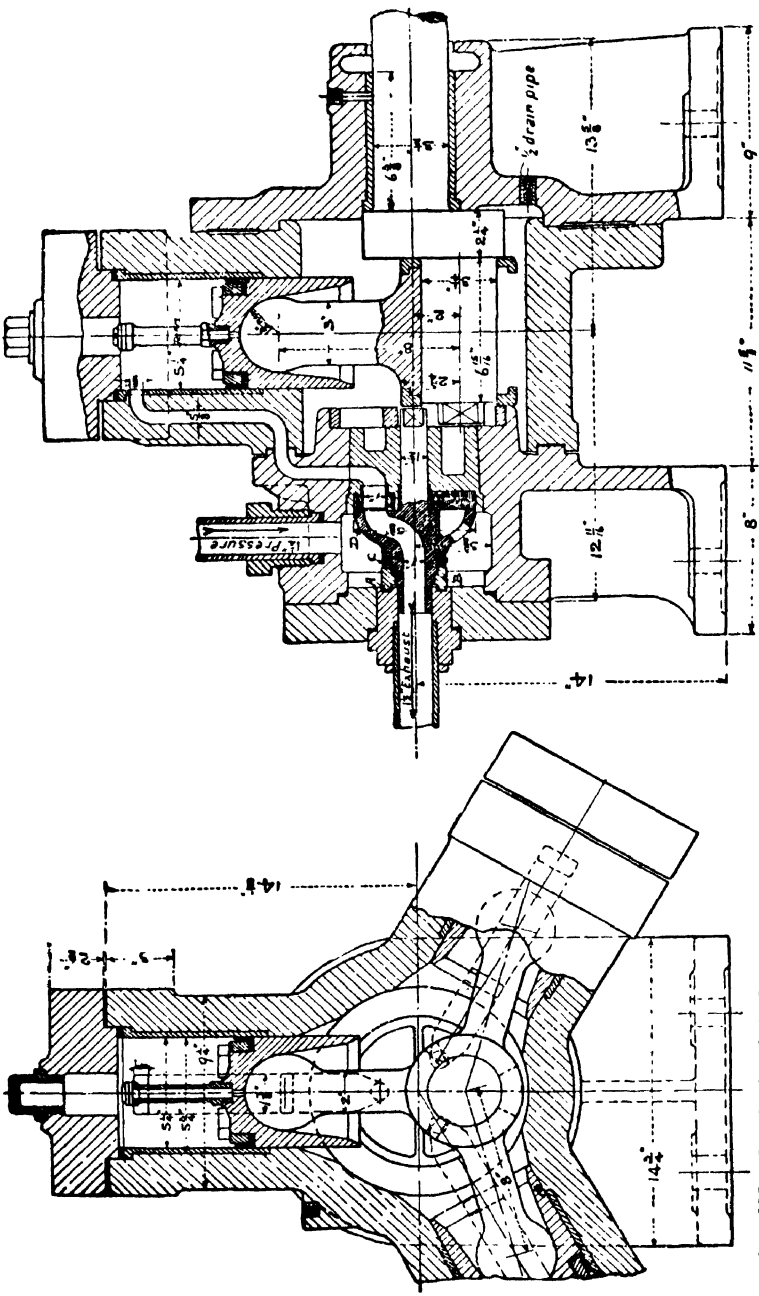


FIG. 281.—Brotherhood Hydraulic Engine : Cylinders 5¼ inches diameter × 4 inches stroke, developing 13 B.H.P. at 47 revs. per min., with a supply pressure of 700 lbs. per square inch. (By Hydraulic Eng. Co., Ltd., Chester.)

piston speed is about 30 feet per minute for all sizes, the engine shown in Fig. 265 having cylinders 5¼ inches diameter × 4-inch stroke and developing 13 B.H.P. at forty-seven revolutions per minute when supplied with water at 700 lbs. pressure.

The full power efficiency, as measured on the brake horse power, varies with the working pressure from about 52 per cent. with 60 lbs. pressure, to 65 per cent. at 1,050 lbs. This is equivalent to an hydraulic efficiency of from 61 per cent. to 76 per cent. These values are probably too low rather than too high.

All constant stroke engines, however, suffer from the fact that water cannot be used expansively, and since, to avoid shock, it is necessary to keep the cylinders full of water, the same energy is used whatever the load on the engine, causing the efficiency to be low at light or variable loads. It has been attempted to overcome this difficulty, but without great success, by cutting off the pressure supply before the end of the stroke and admitting low pressure water from an auxiliary supply for the rest of the stroke.

A device due to Meyer consists in providing an air chamber at each end of the (double acting) cylinder. Cut-off may then take place before the end of the stroke, the air afterwards expanding and driving the piston. No gain in efficiency is to be anticipated from this method of working.

The more usual device is to reduce the stroke and piston displacement to suit the varying demands for power. In the Brotherhood engine this has been accomplished by the Hastie regulating device.

As thus constructed, power is transmitted through a hollow crank shaft to the driving pulley by means of a volute spring, an increase in the load producing a further coiling of this spring. The inner end of the spring is fixed to the hollow crank shaft, while the driving pulley carries a disc keyed to a cam shaft working inside the crank shaft. Any variation in the load then produces a relative rotation of the crank and cam shaft. A cam keyed to the latter shaft utilizes this relative rotation to increase or diminish the crank radius, and thus to adapt the volume of piston displacement to the demand for power. This device is, however, seldom now fitted to the engine.

Of all variable power engines, that of Mr. R. Rigg [1] has achieved the greatest success in practice.

This engine is provided with three cylinders, fitted with plungers which are pivoted at their outer ends to an external cylindrical ring, which is

[1] See *Engineering*, vol. 45, p. 61.

itself keyed to the driving shaft. The cylinders face radially outwards at 120°, are single acting, and are rigidly fixed at their inner ends to a ring which rotates on a fixed hollow shaft forming the supply pipe, eccentric with the external casing, and carrying the inlet and outlet ports. The rotation of the outer ring is thus accompanied by a reciprocating motion of the pistons in their cylinders, the stroke of each being twice the eccentricity of the two rings. There are three ports, one to each cylinder, and as these rotate each in turn is presented before an inlet and an outlet port in the central hollow shaft. Regulation of power at constant speed is obtained by altering the eccentricity, and therefore the stroke of the engine, by means of an hydraulic relay governed by a centrifugal governor. The reduction in water consumption is thus proportional to the reduction in the stroke. When regulating the eccentricity by hand, at constant power, a decrease in stroke will be accompanied by a corresponding increase in speed, the work done per second then being approximately the same. This method of working may be adopted for capstans and the like, where when hauling in slack, etc., a rapid rotation with very short stroke is required, while when the full pull is to be exerted by the rope, a long stroke and slow rate of rotation is essential. By reversing the relative position of the centre the direction of rotation is reversed.

The speed of the engine may be anything up to about 500 revolutions per minute, though probably 250 revolutions marks the limit of its really efficient performance. At the latter speed the inventor claims to obtain efficiencies up to 80 per cent., though this value would appear to be improbably high.

In the Armstrong type of engine, oscillating cylinders mounted side by side and driving on to a common crank shaft are adopted. Three cylinders driving cranks at 120° are usual, and are fitted, either with externally packed rams forming a single acting engine, or with differential rams having areas in the ratio 2 : 1. The smaller of these is exposed to a constant pressure head, and the engine becomes double acting with equal efforts on each stroke. The single acting engine is more commonly used. Admission and discharge take place through a circular slide valve working within the hollow trunnions which carry the cylinders. This engine is made to develop up to about 70 B.H.P.

Swash Plate Engine.—Fig. 282 shows a type of hydraulic engine which has been extensively used for the training of large guns in battleships. Seven cylinders are used on either side of the swash plate, and transmit their pressure to the plate through the connecting rods *D*. The swash plate *C* is carried on trunnion bearings in a gimbal ring *E*, which, in turn, is mounted on

trunnion bearings *F* found in the casing. The revolving swash plate *G*

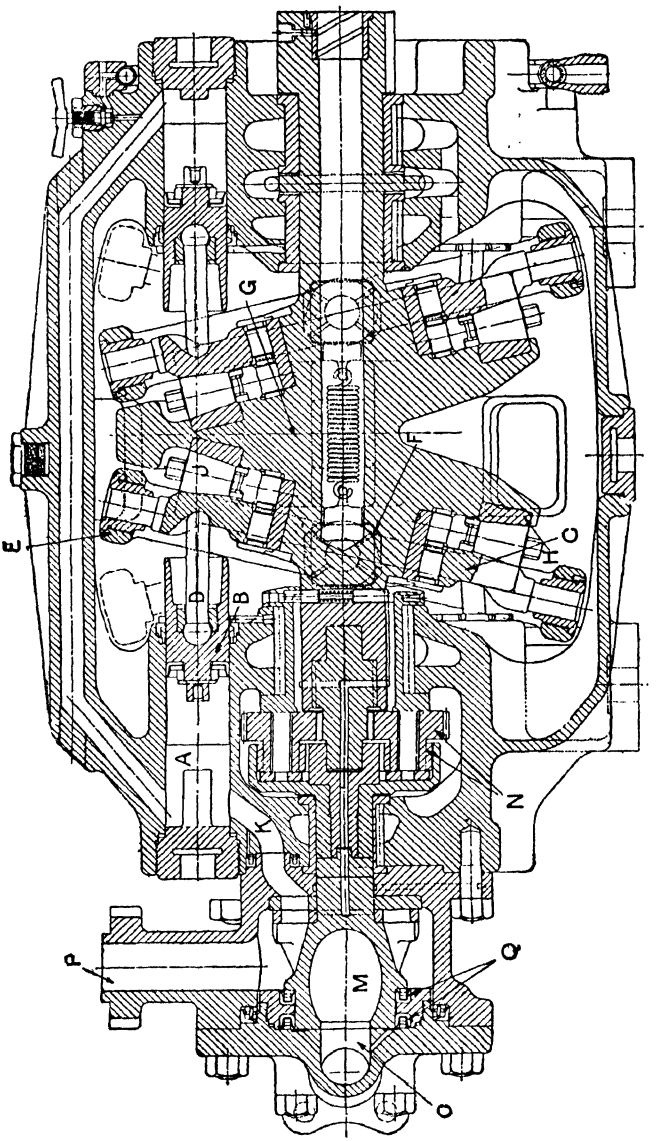


FIG. 282.—Sectional drawing of a Hydraulic Swash Plate Engine.

consists of a circular inclined plate forming part of the main driving shaft. Each face carries a hardened roller path and roller or ball bearings, and are

fitted between the rocking plate C and the revolving plate G. Pressure water is admitted to the cylinders by the parts K and the valve M, which is rotated by the gear wheels N, which are driven in an opposite direction to that of the shaft. The valve is rotated at one-sixth engine speed and has twelve equidistant ports, and during each revolution of the engine each cylinder connects with a pressure port and an exhaust port consecutively. Six of the twelve ports communicate with pipe O through the interior of the valve and six with pipe P through the outside of the valve, the interior and exterior passages being separated by cup leathers Q, so that by using pipe P for pressure and O for exhaust, or conversely, the direction of rotation may be changed.

Art. 180.—Theory of the Hydraulic Engine.—Horse Power.

If K = number of cylinders—supposed single acting.
p = mean effective pressure in lbs. per square inch in each cylinder.
N = number of revolutions per minute.
l = length of stroke in feet.
A = ram area in square inches.

Then I.H.P. $= K \cdot \frac{p\,l\,A\,N}{33{,}000}$.

The mechanical efficiency varies with the size and type of engine, and with its condition, but will be generally about 85 per cent.

If Q = volume of water used per minute in cubic feet.
H = supply head in feet.

The hydraulic efficiency

$$= \frac{\text{indicated work on piston}}{\text{energy in supply water}} = \frac{K \cdot p\,l\,A\,N}{62{\cdot}4\,Q\,H},$$

or writing $$Q = K\,l\,A\,N \div 144,$$

we have the hydraulic efficiency $= \frac{144\,p}{62{\cdot}4\,H} = \frac{2{\cdot}31\,p}{H}$.

The total efficiency of the system $= \frac{33{,}000\ (\text{B.H.P.})}{62{\cdot}4\,Q\,H}$

= hydraulic efficiency × mechanical efficiency.

Pressure on Piston.—As in the reciprocating pump, the pressure on the piston at any point of the stroke is affected by the inertia of the supply column. The engine may in fact be considered as a reversed pump with mechanically operated valves, and the investigation into pressure conditions in the cylinder proceeds on exactly similar lines to that in the case

of the pump. For a further investigation of these pressure effects the reader is referred to Art. 189. It may be noted, however, that at any point of the stroke, where a is the piston acceleration and v its velocity, if a_s is the area and l_s the length of the supply pipe, m being its hydraulic mean depth, we have:—

$$\left.\begin{matrix}\text{Head on piston}\\ \text{in feet}\end{matrix}\right\} = \left[\begin{matrix}\text{supply head}\\ \text{in feet}\end{matrix} - \frac{A}{a_s} \cdot \frac{l_s}{g} \cdot \left\{a + \frac{f v^2 A}{2 m a_s}\right\} \text{ ft.}\right].$$

Here a is positive during the first, and negative during the last part of the stroke.

A weighted accumulator feeding the supply pipe line has the effect of increasing acceleration pressures as explained in Art. 218. The provision of an air vessel on the supply side of the engine materially improves the smoothness of running, and by maintaining an approximately uniform flow in the supply main, reduces frictional and shock losses and so increases the efficiency of the motor. The necessary dimensions of this air vessel depend on the number and arrangement of the cylinders; on the speed and dimensions of the engine; and on the length of supply pipe. If more than one cylinder is used, the cranks being placed at equal angles round the shaft, the velocity of flow through the supply pipe is considerably steadied, and indeed with the usual type of three-cylinder engine having cranks at 120°, becomes so nearly uniform that it becomes possible to dispense with the air vessel.

For a further consideration of the questions deciding the size of the air vessel in any particular case reference should be made to Art. 194. With the high pressures usually adopted in these engines it becomes imperative either to provide some mechanical device for maintaining the charge of air in the vessel, or to make the vessel of ample area to maintain its mean working level approximately constant over long periods of working. Thus the necessary size of air vessel for the engine will in general be slightly greater than that for the corresponding pump.

CHAPTER XIX.

Pumping Machinery.—The Scoop Wheel—The Archimedean Screw—The Reciprocating Pump—Types—Valves—Theory of Action—Air Vessels—Positive Rotary Pump.

Art. 181.—Pumps.

Most hydraulic prime movers are, with slight modifications, capable of being reversed, and when driven by an external agency provide a means of raising water from a lower to a higher level.

Thus the overshot water wheel corresponds to the chain and bucket pump; the breast or Sagebien wheel to the scoop wheel; the reversed pressure turbine gives the centrifugal pump, and the reversed reciprocating engine the reciprocating pump. The capacity of each of these machines as a prime mover also to a large extent indicates its capacity, when reversed, as a pump, and just as the reciprocating engine and turbine are the most important types of prime mover, so the reciprocating piston pump and the centrifugal pump are the most important types of pump.

In addition to those already mentioned, we have a series of special types of pump, each of which is specially adapted to some particular combination of circumstances. Of these, the Archimedean screw, the jet pump, the positive rotary pump, the hydraulic ram, and the air lift pump, are interesting examples and will be considered in due course.

Art. 182.—The Scoop Wheel.

This has in the past been largely applied to the drainage of fen districts, and simply consists of a Sagebien wheel reversed and driven usually by steam power, the water then being lifted in the buckets from the lower to the higher level. It is fitted for lifts up to about 6 feet and may be constructed to deliver up to about 400 tons per minute, giving an efficiency under favourable conditions of upwards of 75 per cent. Its chief advantages appear in cases where a large quantity of *débris* is encountered. As in the Sagebien wheel, the inclination of the floats should be such as to prevent shock at entrance and losses at exit as far as possible. It is found that the best results are obtained when the angles which the vanes make with the water surface at entrance and

exit are equal. If any variation is made from this design, it is better to increase the angle of egress, since this reduces the loss due to lifting the water above head-race level. While increasing the loss at entrance due to shock, and to driving the water back from the floats, this is in general less serious than the former source of loss. In some cases the angle of egress is made equal to twice the angle of ingress. The floats are found

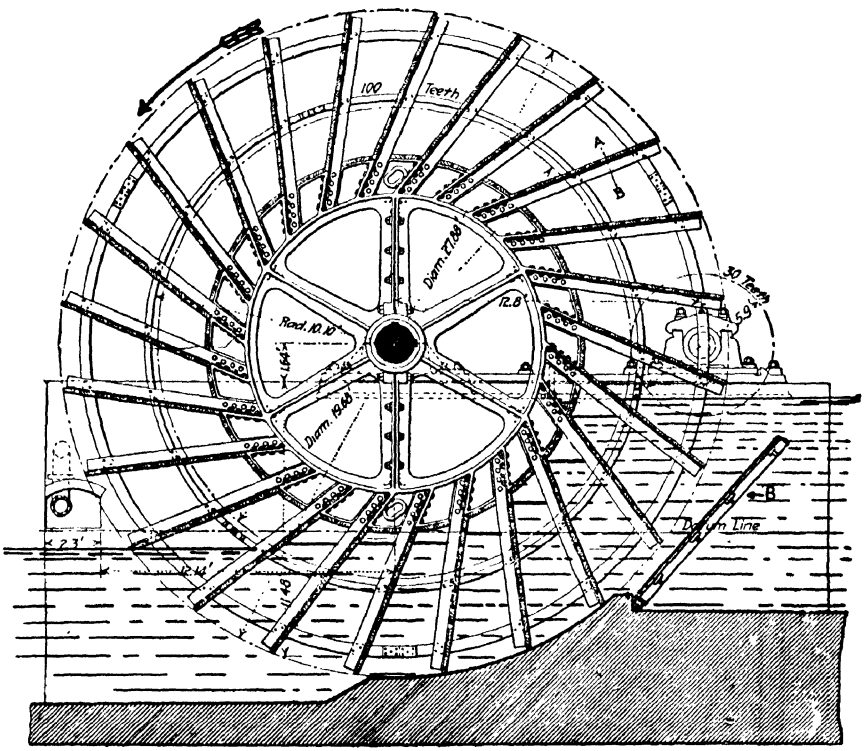


FIG. 283.—Scoop Wheel with movable breast.

to give better results when flat than when curved, and are arranged to make an angle of from 20° to 40° with the radial line. Since any variation in the head or tail-race level militates against efficient working, it is advisable to regulate the supply level at the wheel by means of an adjustable sluice, so as to keep this constant. As usually constructed, the diameter of the wheel lies between $9\sqrt{H}$ and $10\sqrt{H}$, where H is the lift in feet. The peripheral speed is usually about 8 feet per second.

Fig. 283 shows one of a set of six wheels installed in 1901 at Schelling.

woude for the purpose of maintaining the level of the Amsterdam Canal. The plant is required to lift over a range of heads varying from ·5 foot to 8 feet, and to discharge 1,500 cubic feet per second against a normal head of 1·8 feet at 4·5 revolutions per minute.

Under normal conditions the vane angle at entrance is 24°, and at exit 48°. The diameter is 28 feet, the diameter of the circle to which the vanes are tangent being 5·5 feet. An important feature of the installation is the movable breast float B which is fitted in front of each wheel. This breast is pivoted on a horizontal axis at its lower end, and is free to take up a natural and unrestrained position in the current of discharging water. It is constructed of heavy planking and angle iron, and the comparatively high efficiency of the plant is attributed largely to the action of this float. The overall efficiency varies from 40 per cent. under 1·6 feet head to 74 per cent. under 6·6 feet head.

Art. 183.—The Screw Pump or Archimedean Screw

has in the past been largely used in Holland, and works with advantage against heads not exceeding 10 feet, having an efficiency equel to that of the scoop wheel. The pump consists of an inclined shaft carrying one or more helices of considerable diameter, which rotate with small clearance in a closely-fitting tube or open semi-circular channel connecting head and tail-water.

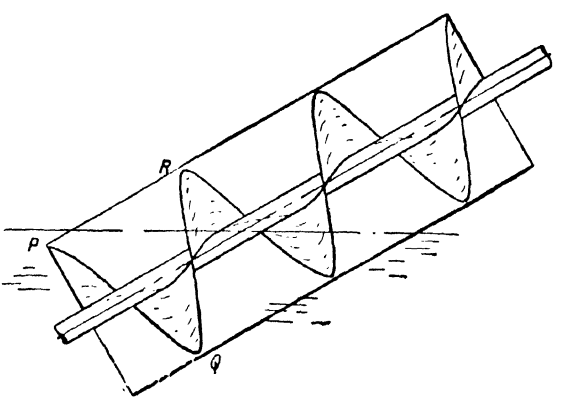


Fig. 284.—Archimedean Screw.

The angle of inclination of the shaft to the horizontal is so arranged as to be less than that of the helical surface, so that water on being admitted to the bottom of the tube always tends to run down this surface. Thus (Fig. 284), a particle of water admitted at P tends to flow to the point Q. By a rotation of the axis portions of the helix from Q to R successively adopt the same position as Q relative to the axis. As they do so the water tends to flow into these positions and is thus passed along the screw, finally emerging at the top into the head-water. The most advantageous angle for the helix is found to be between 30° and 40°.

In the more modern form of screw pump the screw itself is similar to a screw propeller, but usually carries about six blades. This is carried on a horizontal or vertical axis and works with small radial clearance in a cylindrical chamber through which flow takes place. In such a pump, installed some few years ago in connection with the Chicago main drainage scheme, the screw has a diameter of 13 feet 6 inches, and when rotating at fifty-two revolutions per minute, has a capacity of 23,000 gallons per minute against 8·5 feet head.[1]

Although both screw pumps and scoop wheels are capable of high efficiencies they are cumbrous, and run at inconveniently low speeds, and are in almost every case being replaced by the centrifugal pump.

Art. 184.—The Reciprocating Pump.

The oldest type of reciprocating pump is the bucket pump illustrated diagrammatically in Fig. 285. Originally devised as a lift pump, it was fitted with a hollow bucket or piston surmounted by a valve, and with a foot valve to prevent escape of water on the down stroke.

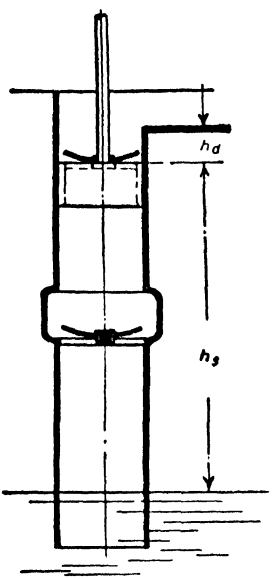


Fig. 285.—Bucket or Lift Pump.

On the up stroke of the pump a partial vacuum is produced below the bucket, and the pressure of the atmosphere acting on the free surface in the supply reservoir produces a flow up the suction tube in virtue of this difference of pressure.

If h_s = suction head in feet of water for any given position of the bucket.

π_a = atmospheric pressure (in feet of water).

Then, neglecting frictional losses and the effect of acceleration, the pressure head on the under side of bucket $= \pi_a - h_s$ feet. This has its minimum theoretical value when $\pi_a - h_s = 0$, *i.e.*, when an absolute vacuum is produced below the bucket, and the maximum possible suction head is thus equal to π_a, or 34 feet approximately. Practically, owing to the resistance of the suction valve, leakage at joints and past the bucket, and to the liberation of dissolved air at low pressures, this head is impossible of attainment, a suction head of 24 feet being only maintained with difficulty, and the maximum value of the suction head,

[1] For a sketch of this pump see the *Mechanical Engineer*, March 20, 1908, p. 367.

i.e., the distance from supply level to level of bucket valve with the bucket in its highest position should not exceed this value.

If h_d = delivery head on pump in feet, *i.e.*, height between bucket and

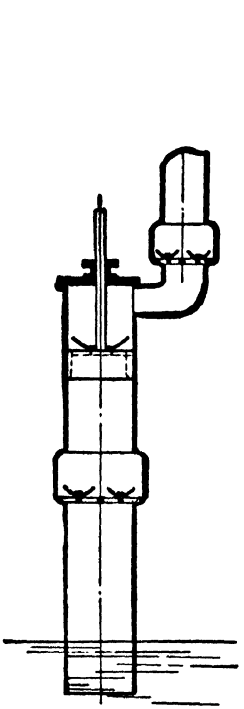


FIG. 286A.—Single-acting Force Pump.

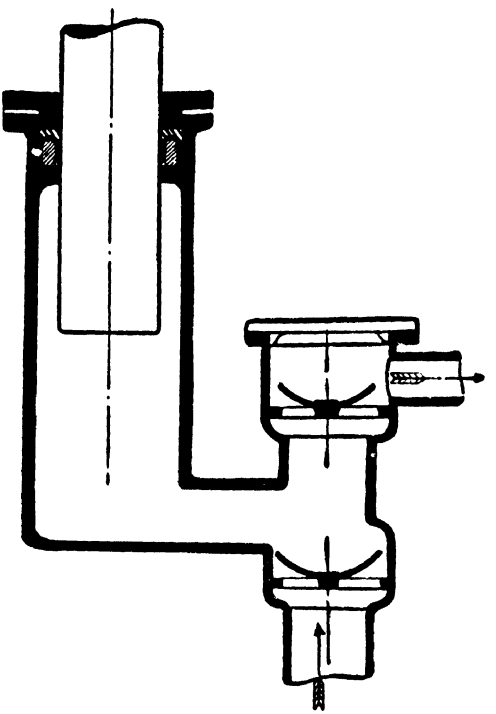


FIG. 286B.—Single-acting Plunger Pump.

outlet, the pressure head on upper side of bucket $= \pi_a + h_d$ feet.

$\therefore$ Total head on bucket

$$= \pi_a + h_d - (\pi_a - h_s) \text{ feet}$$
$$= h_d + h_s = H \text{ feet},$$

where H = total lift of pump.

On the up stroke, then, water is lifted out at the top of the pump and the tension of the pump rod is given by $W\ H\ A$ lbs., where

A = area of bucket in square feet.

W = weight of 1 cubic foot of water = 62·4 lbs.

(This neglects the resistance of valves and passages.) On the down stroke water escapes through the valve, from the under to the upper side of the bucket, so that all the work is done on the up stroke.

By the addition of a closed top fitted with a stuffing box through which

the bucket rod works, the machine becomes a force pump for delivering water under pressure (Fig. 286A). The provision of a back pressure valve on the delivery side of the pump is now necessary to prevent the backward escape of the water on the down or idle stroke. As thus arranged the pump may be either vertical or horizontal.

As pressures are increased, the bucket type of pump is found to be unsatisfactory on account of the difficulty in keeping the bucket and its valves tight and of detecting any leakage, and this led to the adoption of the *Plunger Pump* (Fig. 286B). Here the bucket is replaced by a plunger of

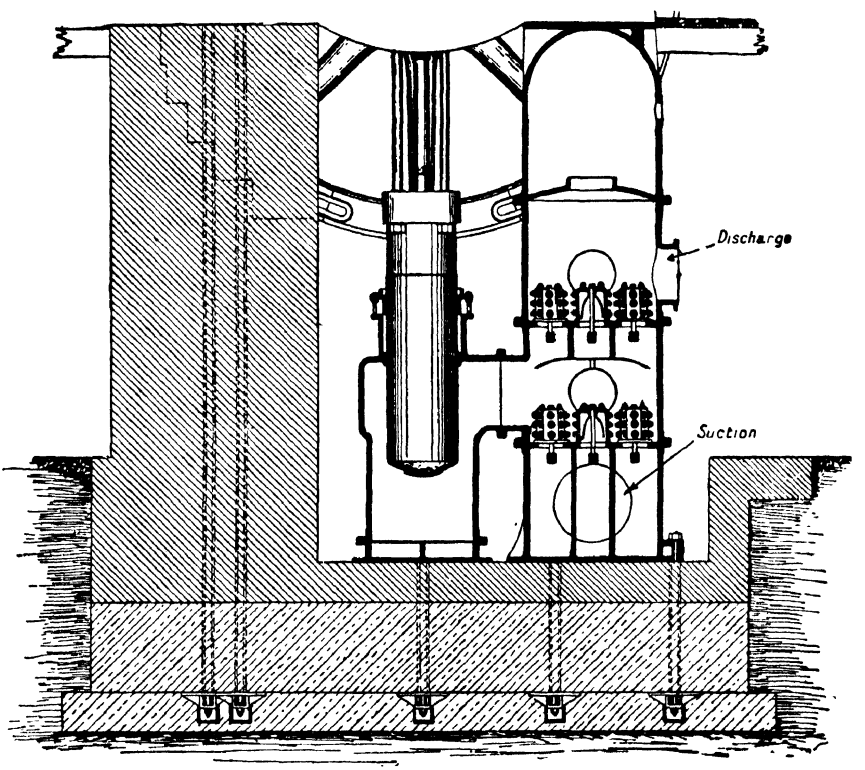


FIG. 287.—Steam Driven Single-acting Plunger Pump.

uniform section, all the packing being supplied from the outside and being easily replaced and examined. In this type of pump, which may be either vertical or horizontal, work is done on both strokes, suction taking place on the out-stroke of the plunger, and delivery by plunger displacement on the in-stroke. The latter in general necessitates the greater

work, and as will be noted the ram or plunger is in compression during this stroke.

If L = length of stroke we have :—

Work done on suction stroke $= A \{\pi_a - (\pi_a - h_s)\} W L$ foot lbs.

$= W h_s A L$ foot lbs.

Work done on delivery stroke $= W h_d A L$ foot lbs.

A separate valve box may now contain both suction and delivery valves, and these become more accessible for examination and repairs.

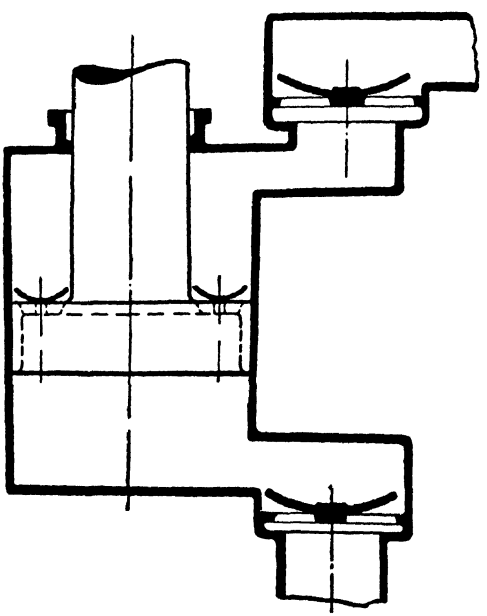


Fig. 288.—Bucket and Plunger Pump.

Fig. 287 shows a section of such a pump having a plunger diameter of 32 inches, and a stroke of 5 feet. The valves for this pump are situated on the sides and top of small hexagonal prisms instead of being arranged on a single plane surface. The pump is directly driven by a steam engine and makes 40 strokes per minute against 153 feet head.

So far, each type considered has suffered from the disadvantage that it only delivers water on alternate strokes. This difficulty is, however, overcome in the "Bucket and Plunger" pump (Fig. 288). Here the piston rod of the ordinary bucket pump is enlarged to form a plunger of about half the area of the bucket which it carries.

Suction now takes place on the up stroke, and, if a and A are the areas of plunger and bucket, a volume of water $= A L$ is drawn into the pump, while a volume $(A - a) L$ is displaced through the delivery valves. On the down stroke the plunger displaces a volume $A L$, and in consequence this volume passes to the upper side of the bucket. Of this the volume $(A - a) L$ remains in the annular space between the barrel and the plunger, while the remaining volume $a L$ is displaced through the delivery valves.

Thus for equal deliveries on the two strokes:—

$$(A - a) L = a L$$

$$\therefore A = 2 a.$$

But if h_s and h_d are the mean suction and delivery heads, we have:—

Work on up stroke $= W \{ (A - a) h_d + A h_s \} L$ foot lbs.

Work on down stroke $= W \{a h_d\} L$ foot lbs.

$\therefore$ For equality:—

$$(A - a) h_d + A h = a h_d$$

$$\therefore \quad A (h_d + h_s) = 2 a h_d$$

$$\therefore \quad \frac{a}{A} = \frac{h_d + h_s}{2 h_d},$$

which approximates to $\frac{1}{2}$ as h_d becomes large compared with h_s.

Where the pump is vertical, and the weight of plunger line great, this requires modification if the forces acting at the upper end of the plunger are

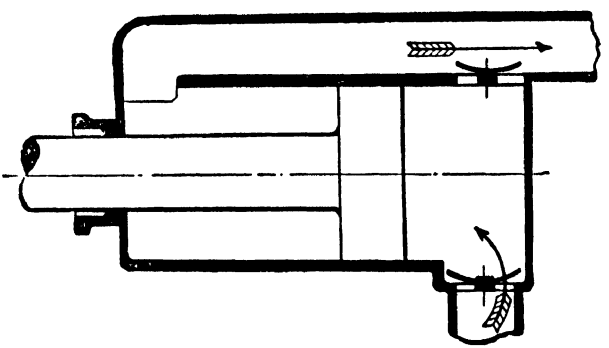


FIG. 289.—Piston and Plunger Pump.

to be equal on both strokes. In this case, if W_P = weight of plunger line:—

Force to be exerted on up stroke $= W \{ (A - a) h_d + A h_s \} + W_P$ lbs.

Force to be exerted on down stroke $= W \{ a h_d \} - W_P$ lbs.

$\therefore$ For equality, $2 W_P = W \{ (2a - A) h_d - A h_s \}$

$$= W \{ 2 a h_d - A (h_d + h_s) \}$$

$$\therefore \quad a = \frac{W_P}{W h_d} + \frac{A (h_d + h_s)}{2 h_d}.$$

The bucket valves may be eliminated while retaining the advantage of uniform delivery or uniform work on both strokes, by the type of construction indicated in Fig. 289 and known as the *Piston and Plunger* pump. Here a solid piston is used instead of the hollow bucket. Suction takes place on the out-stroke, while at the same time a volume $(A - a)$ L is delivered. On the in-stroke, a volume $L A$ is discharged through

the delivery valve, but of this a volume $= (A - a)\ L$ finds its way to the other side of the piston, and the volume actually delivered $= a\ L$. As before, for equality of delivery on the two strokes, $\frac{A}{a} = 2$. In addition to an equal delivery on each stroke, the further advantage of an equal suction on each stroke may be obtained by duplicating the suction and delivery valves at each end of the barrel as indicated in Fig. 290, each end becoming in effect a separate displacement pump.

This, the *Double Acting* type, may be fitted either as a piston pump, having a packed piston and sleeve, or with a turned plunger working in an unpacked ring, both types being shown in Fig. 290.

For fairly low pressures, up to about 75 lbs. per square inch, either type is good, the piston pump giving a slightly larger displacement for the same floor space. Where clean water is to be pumped, the unpacked plunger type, however, is advantageous on the ground of its reduced friction.

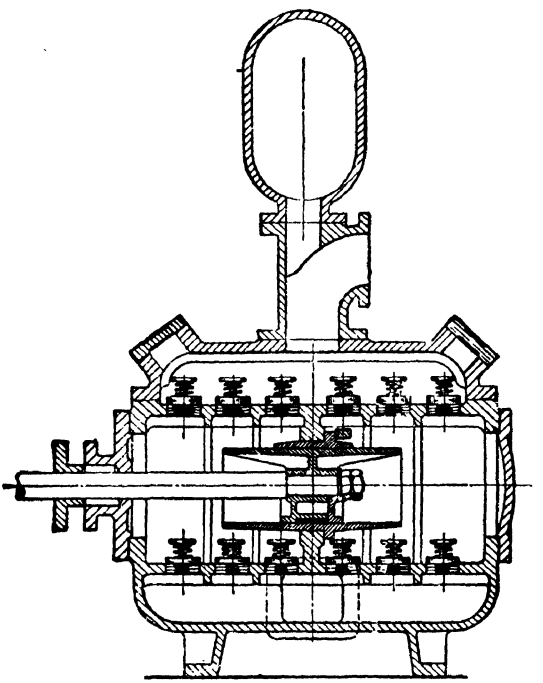


FIG. 290.—Section of 12-inch Duplex Plunger and Ring, or Piston Pump: 10-inch suction; 8-inch delivery; 12-inch stroke. For pressures up to 100 lbs. per square inch.

A short fitting ring is used, easily renewable, and the ease of refitting such a ring compared with that of re-boring a piston barrel renders its use advisable, where the condition of the liquid to be pumped permits. In either case the work done on the two strokes is practically the same.

For very high pressures, or where the water contains gritty matter in suspension, the piston, bucket, or internally packed plunger pump is unsatisfactory, and the type known as the *Double-Acting Outside Packed Plunger* pump becomes more suitable.

Here again the pump discharges a volume $A\ L$ from one end of the

barrel and admits an equal volume (neglecting the volume of the plunger rod if this is used) at the other end of each stroke. Fig. 291 shows a direct steam-driven pump of this type having centre-packed plungers,[1]

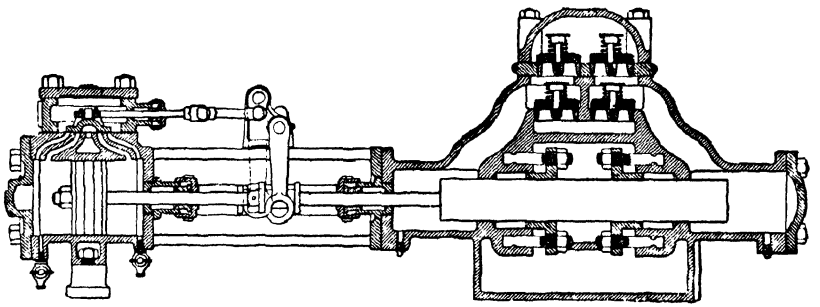


FIG. 291.—Central-Packed Double-Acting Plunger Pump.

and designed for 80 strokes per minute, with 10-inch plunger; 12-inch stroke; 18-inch steam cylinder; 10-inch suction and 7-inch discharge pipes.

Fig. 292 shows an outside end-packed plunger pump of this type, suitable for moderately high pressures, the two plungers carrying cross-heads which are connected by side rods, while Fig. 293 shows a similar

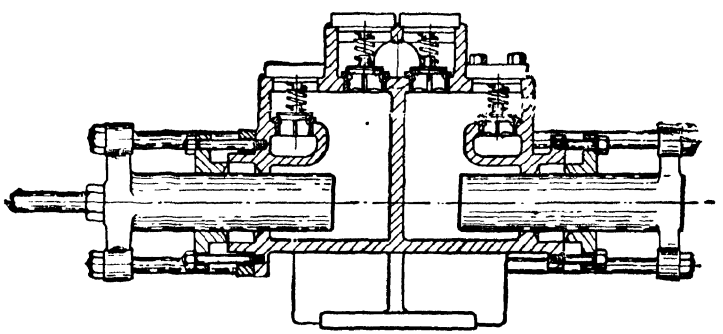


FIG. 292.—Double-Acting Duplex Outside End-Packed Plunger Pump: 6-inch plunger; 12-inch steam cylinder; 12-inch stroke; 5-inch suction; 4-inch delivery; 80 strokes per minute; up to 300 lbs. per square inch.

pump as constructed for the highest pressures. This latter pump has a separate valve box for each end of the cylinder, the construction of the valves (which are 1 inch in diameter) and of the valve box, being shown in Fig. 294.[2]

[1] By courtesy of the Buffalo Steam Pump Company.

[2] Figs. 293 and 294 are inserted by courtesy of the makers Messrs. Henry Berry & Co., Leeds.

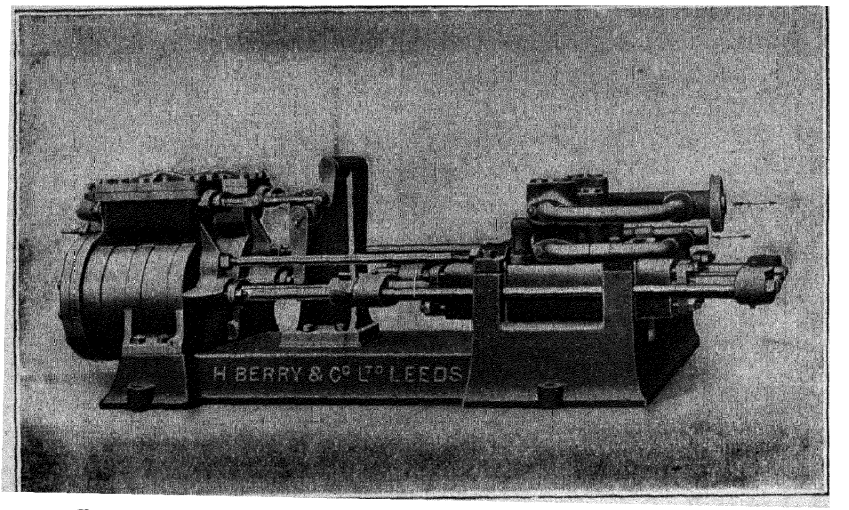


FIG. 293.—Outside End-packed Plunger Pump for pressures up to 1,120 lbs. per square inch.

This type of pump has the advantage that only two glands need packing as against three in the centre-packed type, although the latter type lends itself to a more compact construction.

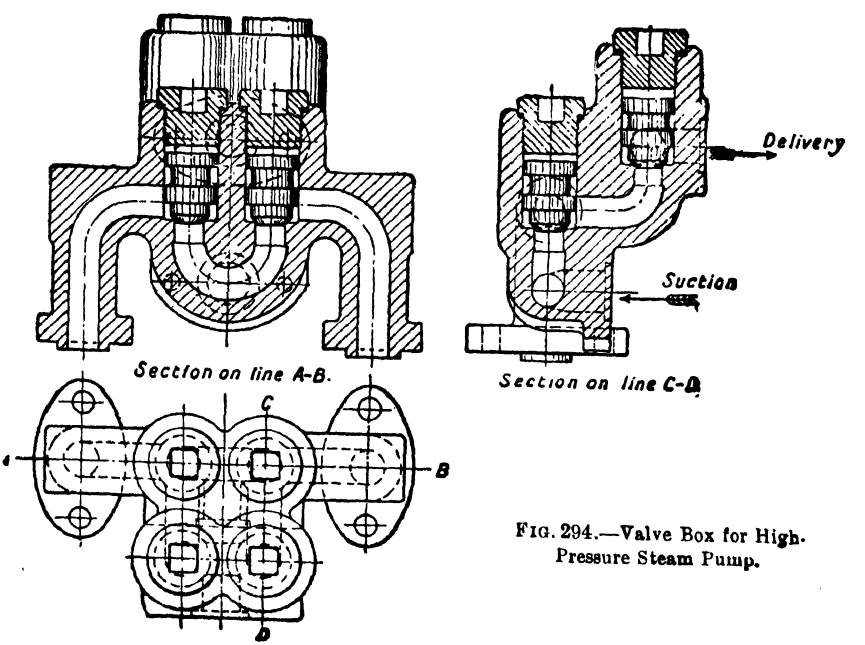


FIG. 294.—Valve Box for High-Pressure Steam Pump.

Deep Well Pumps.—Where pumping operations are necessary in a deep well or bore hole of small diameter, the ordinary lift pump with separate foot valve is at a disadvantage because of the great difficulty which is experienced in raising and replacing this foot valve should it become choked or in any way out of order. To obviate this difficulty the "Ashley" pump, Fig. 295, carries both suction and delivery valves in its bucket, this being readily withdrawn for examination through the rising main.

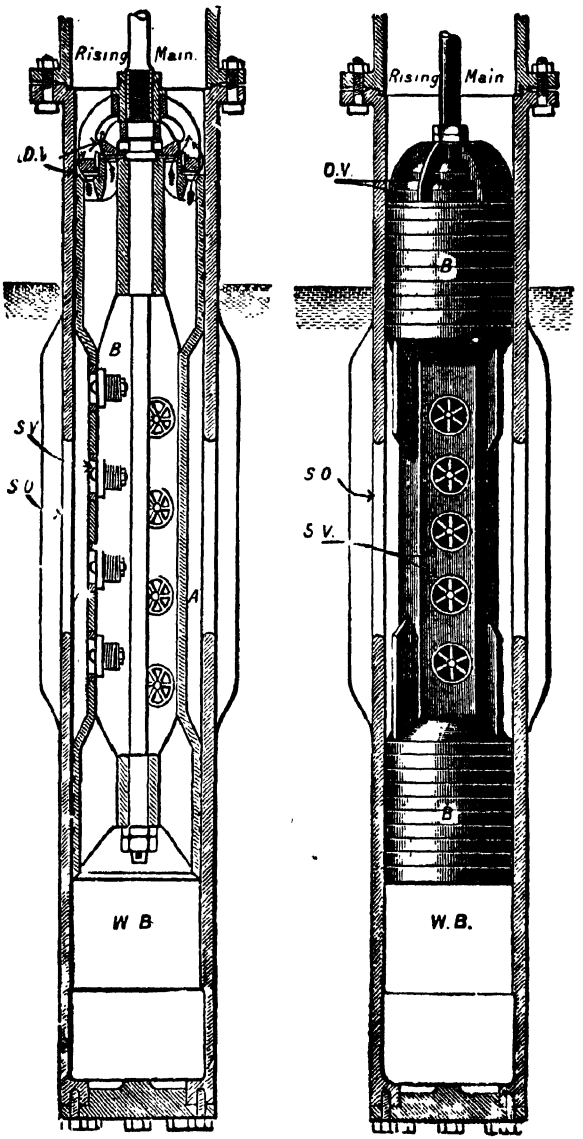


FIG. 295.—Single-Acting "Ashley" Bore-Hole Pump.

The bucket *B*, which is open at its lower end, reciprocates in the working barrel *W.B.*, which is closed at the bottom. The sides of this barrel are pierced by orifices *S.O.* through which the surrounding fluid is admitted to the under side of the suction valves *S.V.* On the upward stroke of the pump a partial vacuum is formed inside the bucket, and water is drawn through these valves to fill the space inside and below the bucket, while the water above the bucket is forced up the rising main. On the downward stroke the delivery valves *D.V.* open, and the water in

the body of the bucket is transferred to its upper side to be pumped to the surface on the succeeding stroke. A double-acting pump of this type is also on the market.

Art. 185.—Pump Valves.

The valve area should be so designed as to allow of a mean velocity not exceeding 4 feet per second. For low pressures these valves usually consist of rubber or composition discs (Figs. 290, 291, 296 *a* and *b*),

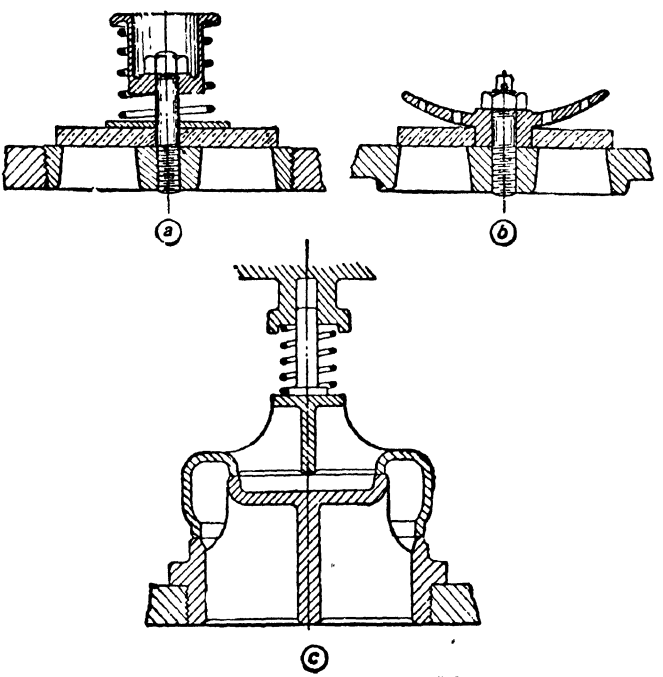


Fig. 296.—Types of Pump Valve.

working against a perforated grid, and are either spring loaded or automatically return to their seats in virtue of their own elasticity, together with the pressure of water above them. For moderate speeds and pressures these are very satisfactory.

At high pressures, however, the discs quickly become indented, and some form of metallic valve becomes essential. Fig. 296 *c* shows a double-beat valve of this type. These valves may be spring loaded, or may return to their seats by their own weight, and so long as the speed is moderate are satisfactory.

The ordinary disc or double-beat valve suffers from the drawback that the kinetic energy of the discharging streams is of necessity dissipated in the shock accompanying the sudden change in the direction of flow. The Haste and the Gutermuth valves are interesting examples of designs intended to obviate this source of loss. The action of the former is obvious from the sketch Fig. 297.

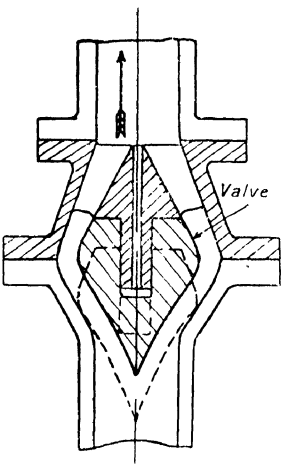


FIG. 297.—Haste Valve.

The Gutermuth valve (Fig. 298) is formed from a single sheet of special bronze, either of the same thickness throughout or, in large valves, having the end forming the valve thickened. The sheets are slightly wider at the coiled end to prevent fouling, and are slipped on to a grooved spindle and clamped to the valve seat cones at the required tension.

The valve is always placed at an angle with the port opening, and the latter thus becomes uncovered with a very small movement of the valve itself, while no such abrupt changes in the direction of the escaping stream are necessitated as in the case of disc or mushroom valves.

The automatic or self-closing valve, while simple and satisfactory for low velocities, possesses several disadvantages which tend to reduce its suitability for high speed work.

In the first place, with automatic suction valves, the difference of

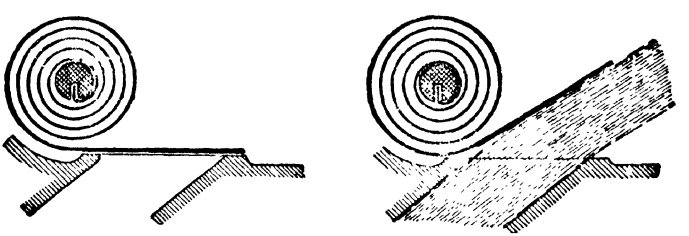


FIG. 298.—The Gutermuth Valve.

pressure below and above must be sufficient to lift the valve, and with a heavy valve this appreciably diminishes the possible suction lift. A light valve, by coming slowly to its seat at the end of the stroke, enables water to leak back into the suction pipe, since, although theoretically the valve is on its seat when the piston is at rest, actually this is not by any means the case except at slow speeds. The same thing applies on

the delivery side. Also considerable shock takes place when these valves close, this shock being due, not only to the valve itself dropping suddenly to its seat, but also to the fact that a large mass of water partakes of the return motion.

The violence of the shock depends on the kinetic energy possessed by the valve and accompanying water at the instant of closing, and will therefore increase with the weight of the valve, with the mass of the accompanying water, and with the maximum distance of the valve from its seat, since the latter factor will determine the velocity of closing.

The mass of water accompanying the valve is found to be proportional to the mass included between the valve and the water level in the corresponding air vessel, so that to minimise this effect, the difference in level between valve and air vessel should be reduced as far as possible, while the lift of the valve should be as small as is consistent with ample inlet and outlet areas.

With self-closing valves, the lift may be reduced either—

(1) by increasing the weight of the valve;

(2) by increasing the spring loading of a light valve;

(3) by limiting the lift by stops;

(4) by increasing the number or diameter of the valves.

The first method, while reducing the lift and giving a quicker closing effect, increases the hydraulic resistance; while the increased weight of the valve is in itself productive of shock. On the other hand, a light valve is more subject to vibration while opening or closing, and this may cause large oscillations of pressure in the pipe line. This tendency to vibration increases with the speed of the pump, and diminishes with an increase in the delivery pressure. On the whole, however, the balance of advantage lies with the fairly light spring-loaded valve.

The method of limiting the lift by stops is decidedly unsatisfactory, as giving rise to oscillations of the valve, and hence of the pressure in the pipe line.

The best method is to increase the effective valve area by an increase in the number or diameter of the valves. In the ordinary disc valve (Fig. 296 *a* and *b*),

If r = radius and l = lift of valve,

the area of valve seat $= \pi r^2$ (neglecting the effect of guides, etc.).

But the effective valve area is the area of the cylindrical surface generated by the perimeter of the valve during its lift $= 2 \pi r l$.

The most effective lift is thus obtained when these values are identical, *i.e.*, when—

$$2\pi r l = \pi r^2$$

$$\therefore l = \frac{r}{2}.$$

It follows that, in order to take advantage of the available valve seat area, it is impracticable to use very large valves, since for a given lift the effective discharge area varies as the diameter of the valves, while the valve weight varies more nearly as $d^{\frac{5}{2}}$. Also if the lift were made equal to $\frac{r}{2}$, this would soon become excessive. Because of this, small valves should be used, with a lift giving a discharge area approaching that of the valve seat, and for high speed pumps modern practice is opposed to the use of disc valves of more than about 3 inches diameter. Where double beat valves are used this may be increased, but in no case should the valve have a lift exceeding $\frac{3}{4}$ inch.

Valve Leakage and Slip.—A further point to be noted is that slip, or leakage past the valves while closing, is proportional to the mean effective opening of the valve; to $\sqrt{h}$, where h is the head on the valve; and to the time of closing, t. Generally we may take t as proportional to $\sqrt{l}$.

If, then, a number of small valves replace a single large valve of the same effective discharge area, the slip will be reduced, since to get the maximum discharge effect the lifts of the valves must in every case be proportional to their diameters, and this lift, and therefore the time of closing, will be greater with the larger single valve.

Thus, whereas the slip past n valves of diameter d is proportional to $n\,h^{\frac{1}{2}}\,d^{\frac{5}{2}}$, that past a single valve of the same effective discharge area, and, therefore, of diameter D where $D = d\sqrt{n}$, is proportional to $h^{\frac{1}{2}}\,d^{\frac{5}{2}}\,n^{\frac{5}{4}}$, so that by increasing the number of valves to n, the leakage is reduced in the ratio $\frac{n}{n^{\frac{5}{4}}} = \frac{1}{n^{\frac{1}{4}}}$.

E.g., where $n = 4$, the leakage is equal to $\frac{1}{4^{\frac{1}{4}}} = \cdot 707$ times that found with a single large valve.

Art. 186.—High Speed Reciprocating Pumps.

Owing to the length of time necessary for an automatic disc valve to close, and to the irregularities in its action produced by inertia effects of the water in the supply and delivery pipes, a high rotative speed is

the delivery side. Also considerable shock takes place when these valves close, this shock being due, not only to the valve itself dropping suddenly to its seat, but also to the fact that a large mass of water partakes of the return motion.

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Art. 186.—High Speed Reciprocating Pumps.

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difficult in a large reciprocating pump fitted with this type of valve, and it becomes necessary to use a long stroke, slow rotation pump, giving ample time for the valves to come to rest at the end of each stroke. With

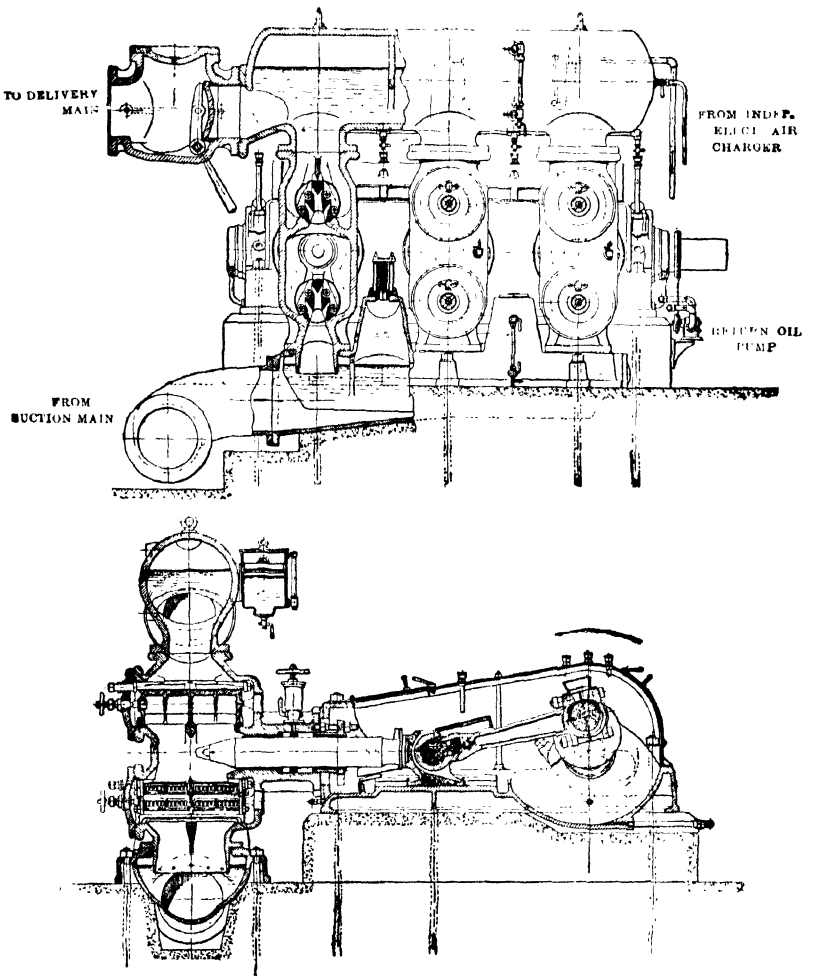


FIG. 299.—Gutermuth 3-Throw Pump ; 6¾-inch diameter ; × 16-inch stroke ; × 180 revs. per min. ; 1,000 gallons per min. ; 750 ft. head.

this type of pump, and until comparatively recently, the maximum attainable piston speed was about 100 feet per minute at about sixty revolutions per minute.

By paying special attention to the design of valves and water passages

it has, however, become possible to adopt much higher speeds, even with automatic valves.

Types of recent high-pressure pumps with automatic valves which operate successfully at speeds up to 1,500 r.p.m. are described by Towler in *Proc. Inst. Mech. E.*, Vol. 137, 1937. With a stroke of 0·75 inches the piston speed is 187 feet per minute.

One modern type of high-speed pump which has proved itself capable of excellent results is the Gutermuth (Fig. 299).[1] The illustrations show a sectional plan and elevation of one of a pair of three-throw pumps of this type, having plungers $6\frac{3}{4}$ inches diameter by 16 inches stroke and delivering 1,000 gallons per minute against 750 feet head at 180 revolutions per minute. Each pump is driven from a 275 H.P. three-phase motor through a flexible coupling. The design is very compact, the pump bodies being mounted on a suction air vessel of cylindrical form, which also acts as a bedplate. On this air vessel is cast a branch flange for connecting up to the suction main. On top of one of the pump bodies is bolted a delivery air vessel, having flanged branches to connect to the pump bodies, and with the end flanged to connect to the retaining valve on the delivery main.

Forced lubrication is fitted throughout, while the two delivery air vessels are supplied by an independent electrically-driven two-stage air charger of the Reavel type, the compressor being capable of compressing 5 cubic feet of free air per minute to a pressure of 350 lbs. per square inch.

The valves, which are of the Gutermuth type and are perhaps the most interesting part of these pumps, are contained in cylindrical bronze valve seats, each pump having one set for suction and one for delivery. These are held in place and tightened by means of wedges which are readily accessible when the valve covers are removed.

The sectional end view of Fig. 299 shows very clearly the straight and unobstructed passages offered to the water in this type of pump.

Frictional losses are thus low; the lightness of the valve and its small opening tend to reduce shock on closing; and the possible speed of rotation is correspondingly increased.

The high speed at which these pumps can be run, as a direct result of the valve action, permits of a design which takes up considerably less floor space than the ordinary slow running pump, the actual overall length being, in this case, 11 feet 8 inches × 8 feet 6 inches. It also allows the pump to be direct connected to the motor, which, though adding somewhat to the expense of the latter, does away with transmission gears which are

[1] By courtesy of the manufacturers, Messrs. Fraser & Chalmers, Ltd.

usually noisy and troublesome, and which, under the most favourable circumstances, in a pump of this size, would reduce the efficiency by from 8 per cent. to 4 per cent.

For very high speeds, however, the automatic valve, even though well designed, becomes unsatisfactory, and mechanically operated valves are necessary.

By their substitution uncertainty as to the exact time of closing is avoided, more uniform closing is effected with less accompanying shock, and by this means, and by careful design of the valves, the speed has been increased until piston velocities of 600 feet per minute at 300 revolutions per minute are now easily obtained with almost entire absence of shock

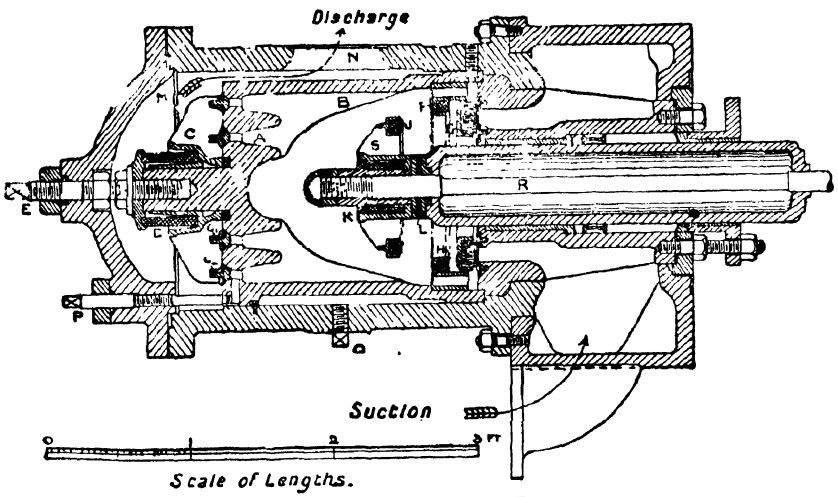


FIG. 300.—Riedler Express Pump

This increase in speed permits of the pump dimensions being reduced for the same duty; gives a smaller delivery per stroke, and hence reduces the liability to shock; while the possibility of a more uniform flow in both supply and discharge pipes tends to the same end.

Fig. 300 shows the construction of the pump barrels, rams and valves of a type of high-speed pump designed by Professor Riedler, this particular pump being designed to deliver 375 gallons per minute against a head of 500 feet. It is a two-throw pump having rams $6\frac{3}{8}$ inches diameter by 9 inches stroke; running at 200 revolutions per minute and driven by an electric motor of 75 B.H.P. running at 500 revolutions per minute.

The outer end A of the working cylinder B forms the seating of the delivery valve, which consists of two annular brass rings, C_1 and C_2, let

into a gun-metal frame *C*. This valve is kept up to its seat by means of the cylindrical indiarubber spring *D*, the compression of which may be adjusted by means of the bolt *E*, while leather sealing rings are also provided to ensure its efficient action.

The inner end of the working cylinder carries a gun-metal guide ring *F*, in which works the annular suction valve *G*. This valve is constructed with a wooden face let into a channel section annular brass ring, and its range of opening is limited by the rubber ring *H*, let into the guide ring *F*.

The ram *R* passes through the suction valve, and at its inner end carries a buffer stop *S*, which mechanically closes the suction valve at the end of the outer or suction stroke. Shock at impact is minimised by the provision of cylindrical rubber springs at *J* and *K*. A series of removable brass liners *L* between the ram and the buffer permit of the latter being adjusted so as to take up any wear in the buffer itself or in the suction valve.

The delivery valve is thus automatically regulated, while the suction valve is mechanically operated only so far as affects its closing at the end of the suction stroke, this valve being both opened and closed on the out-stroke of the ram. On the in-stroke, water is delivered against the resistance of the delivery valve into the annular chamber *M*, from which it flows away through the pipe *N* into an air vessel and away to the discharge. The chamber *M* is fitted with a starting valve at *O*, 2 inches in diameter, so that in starting the pump the pressure may be relieved and the pump started light. The small spindle valve *P* allows of water passing from the chamber *M* into the working barrel, so that if for any cause sufficient water cannot enter through the suction valves, more water may be allowed to enter through the valve *P*, so as to enable the pump to run without shock.

A small compound air pump, having a high pressure plunger $1\frac{1}{2}$ inches diameter by 1·8 inches stroke and a low-pressure plunger 3 inches diameter by 2·7 inches stroke is provided, and is worked directly from the crank shaft of the main pump, exhausting air from the suction air vessel and discharging it into the delivery air vessel.

Should the air supply in the suction air vessel be insufficient, more may be admitted through a small valve; while should the air pump deliver too great a supply, the surplus air can be let out of the delivery air vessel.

Tests carried out on such a pump as described,[1] showed a combined

[1] By Mr. John Morris. See a paper on the "Unwatering of the Aciddu Colliery." Trans. Inst. of Mining Engineers, Vol. 30, part 2, p. 131.

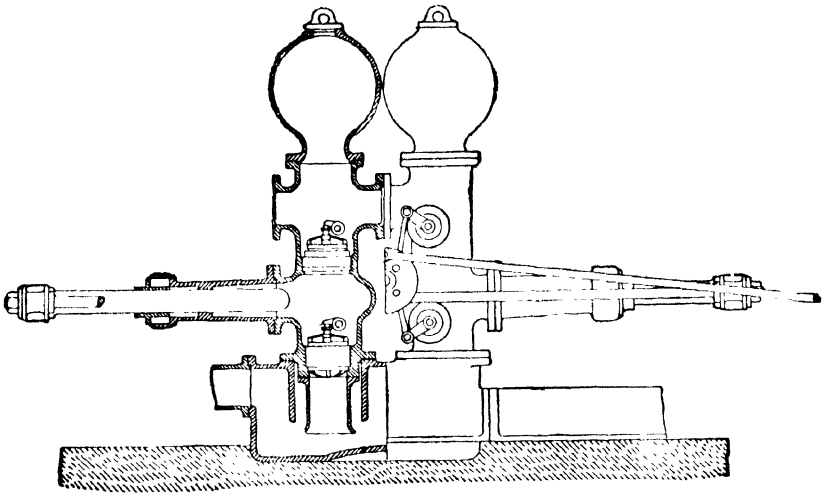


FIG. 301.—Riedler Express Pump.

efficiency of pump and motor of about 85 per cent., with a discharge coefficient of ·92.

For still higher speeds, the pump is designed with suction and delivery

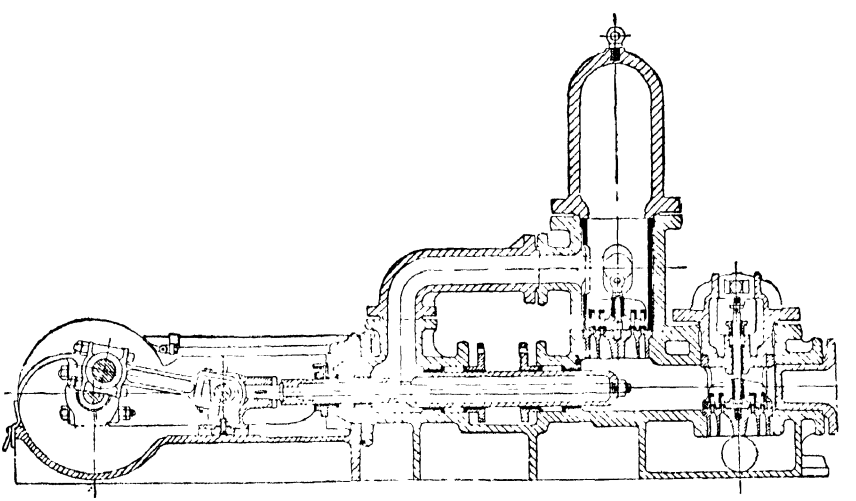


FIG. 302.—Oddie-Barclay High Speed Ram Pump.

valves mechanically operated from a wrist-plate driven by an eccentric on the main shaft, as shown in Fig. 301.

Fig. 302 snows a section through an Oddie-Barclay high speed ram

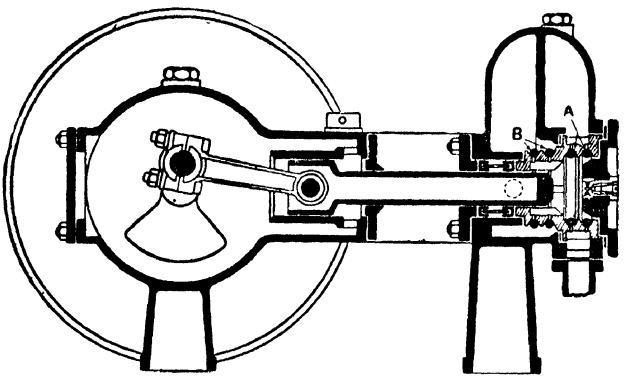


Fig. 303.—"Pulsometer" High Speed Ram Pump.

pump, having mechanical control to both valves. Tests of such a pump, having a differential ram 5 inches and $7\frac{1}{16}$ inches diameter by 9 inches stroke, when running at 150 revolutions per minute showed a mechanical efficiency ranging from 82 to 86 per cent. as the head increased from 620 to 1,050 feet, with a discharge coefficient of ·96.

Fig. 303 shows a neat form of high-speed pump [1] in which the valves consist of rubber rings *A* and *B* fitting into peripheral grooves around the valve cages. The lift is extremely small and speeds up to 500 r.p.m. are possible.

Art. 187.—Hydraulic Reciprocating Pumps.

Where a high pressure water supply is available, the hydraulically operated reciprocating pump has been extensively adopted for pumping water for domestic or other purposes and for pumping sewage. Such a pump is illustrated in Fig. 304.[2] This is driven by two double-acting hydraulic cylinders placed one on each side of the pump body casting to which they are attached, the two plungers *P P* and the pump rod *R* being connected to the same crosshead. The pump is operated by means of a piston valve in the valve chest *V*, this valve being worked from the crosshead by means of the tappet rod *T* and lever *L* shown.

Art. 188.—Displacement Curves.

Discharge Coefficient.—In the theoretically perfect pump the action would be simple. On the suction stroke the delivery valves would be

[1] By the Pulsometer Engineering Co., Ltd.

[2] By courtesy of the Hydraulic Engineering Co., Ltd., Chester.

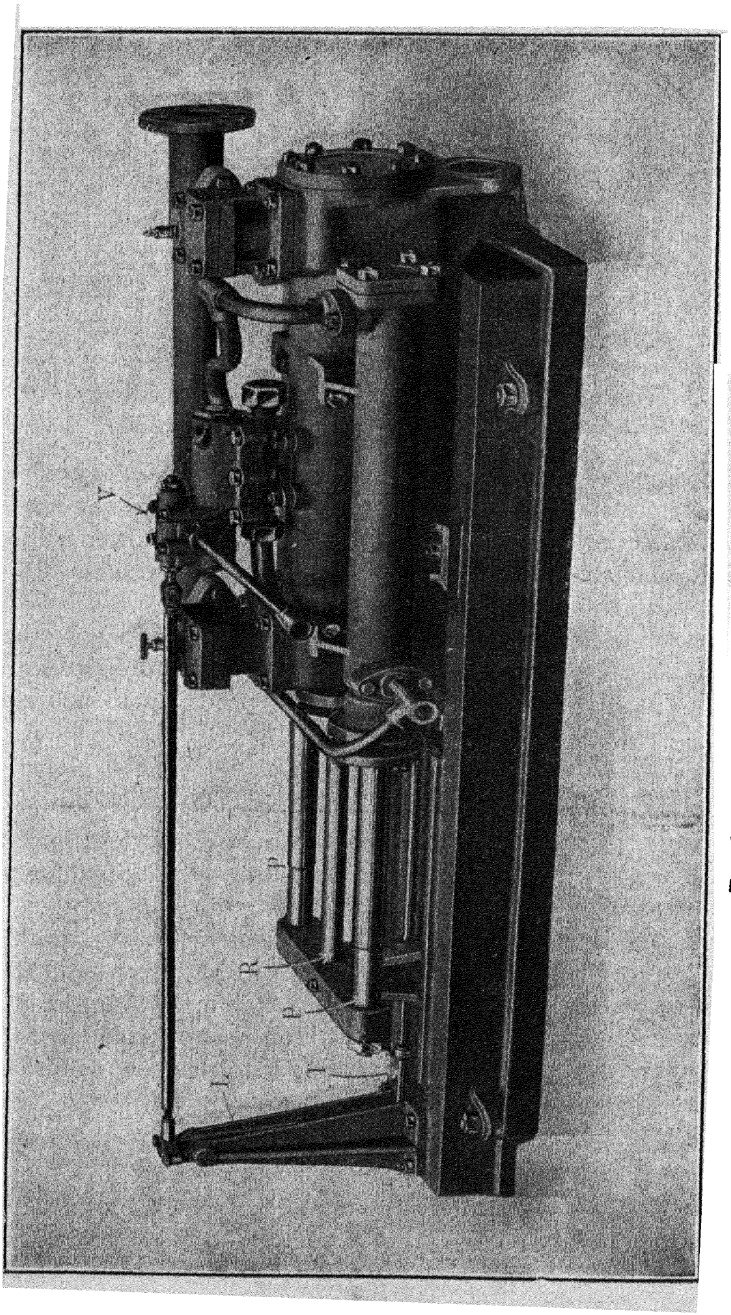


FIG. 304.—Hydraulically operated Reciprocating Pump.

tightly closed while the suction valves would open, admitting sufficient water to entirely fill the barrel. At the commencement of the delivery stroke these valves would immediately close, while the delivery valves would open, and a volume of water equal to the plunger displacement would be discharged. In practice, however, owing to the fact that the valves do not instantaneously close, and because of leakage past the plunger, etc., the volume delivered is not in general equal to the plunger displacement. The ratio $\frac{\text{actual}}{\text{theoretical}}$ discharge, termed the **discharge coefficient**, varies with the type, speed, and condition of the pump, but with moderate speed pumps in good condition and of good design lies between ·94 and ·99. With high-speed pumps the modifying effects of the inertia of the suction column may be such as to give a discharge greater in volume than the piston displacement, the excess amounting in exceptional cases to as much as 50 per cent. This effect will be considered in further detail at a later stage.

If a curve, having piston velocities as ordinates be plotted on a time base, the area included between the curve, the base line, and any two ordinates, will measure to some scale the volume displaced by the piston in the corresponding interval of time. The curve so obtained is termed a displacement curve. Thus, if $O\ Q$ and $Q\ P$ represent the crank and

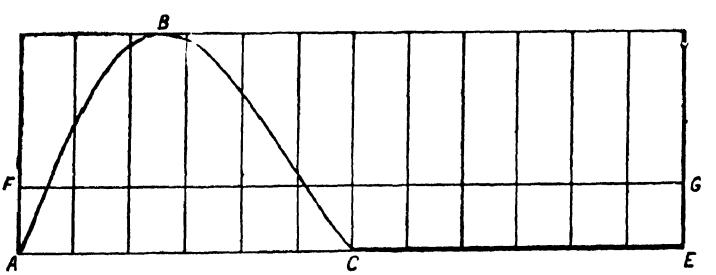


FIG. 305A.—Displacement Curve for Single-Cylinder Single-acting Pump.

connecting rod of a pump for a given piston position, and if $P\ Q$ be produced to meet the perpendicular $O\ C$ to the line of centres in C, then $O\ C$ represents the corresponding piston velocity to the scale on which $O\ Q$ represents $\omega\ r$ the velocity of the crank pin. If a series of such points as Q are taken at equidistant intervals on the crank circle, and the corresponding values of $O\ C$ found, the displacement curve may then be plotted as in Figs. 305A–D. Here Fig. 305A represents the curve for a single barrel single-acting pump having a connecting rod four cranks in

length, $A\ C$ representing the time to perform half, and $A\ E$ to complete a revolution.

The area $A\ B\ C$ now represents the displacement of the pump per revolution, while $C\ E$ represents the idle stroke.

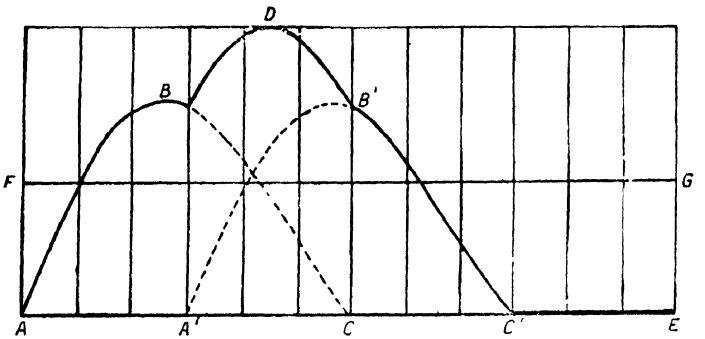


FIG. 305B.—Curve for Pair of Single-acting Cylinders with Cranks at Right Angles.

If $F\ G$ be drawn parallel to $A\ E$, so that $A\ F$ represents the mean velocity of the piston, the area $A\ F\ G\ E$ = area $A\ B\ C$.

Similarly, Fig. 305B represents the displacement curve $A\ B\ D\ B'\ C'\ E$, for a pair of single-acting pumps having cranks at right angles and

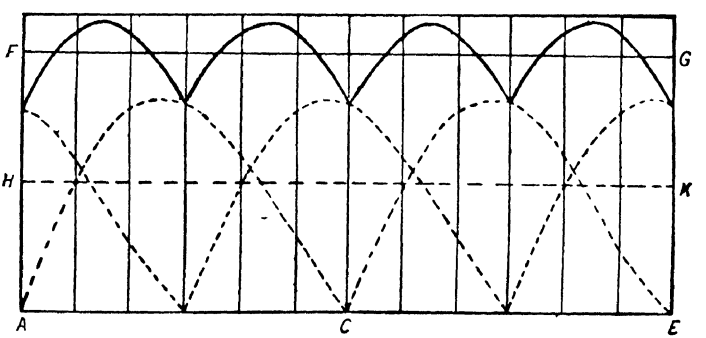


FIG. 305C.—Curve for Pair of Double-Acting Pumps with Cranks at Right Angles.

drawing from and delivering into a common main. Here $A\ B\ C$ and $A'\ B'\ C'$ are the respective curves for each piston, and these are compounded by adding ordinates so as to give the displacement curve. The area $A\ F\ G\ E$ is now equal to the sum of the areas $A\ B\ C$ and $A'\ B'\ C'$, or to the whole area $A\ B\ D\ B'\ C'$.

Again, Fig. 305C, obtained in a similar manner, shows the curve for a pair of double-acting pumps having cranks at right angles, $F'\ G$ again

representing the mean velocity line. Here H K represents what would be the mean velocity line for a single double-acting pump.

Similarly, Fig. 305D represents the curve for a three-throw pump having cranks at intervals of 120°.

It will be noted that as the frequency with which the pumps discharge per revolution increases, the range of velocities in the discharge and

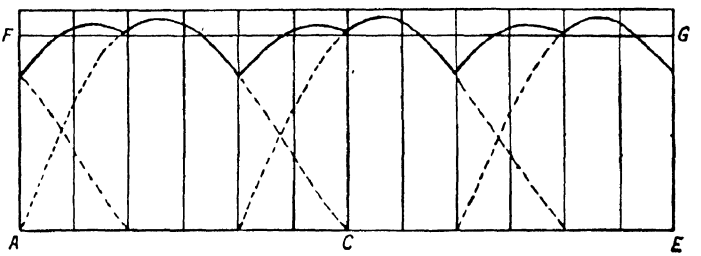


FIG. 305D.—Displacement Curve for Three-Throw Single-Acting Pump with Cranks at 120°.

suction pipes decreases, and also the rate of change of this velocity, *i.e.*, the acceleration, and the smoothness of working in consequence increases.

The following table indicates how this velocity changes :—

Type of Pump.	Ratio of Maximum to Mean Velocity in Discharge Pipes.
Single cylinder, single-acting	3·24
Two single-acting cylinders, cranks at right angles .	2·17
Single cylinder, double-acting	1·62
Two double-acting cylinders, cranks at right angles .	1·11
Three-throw pump, crank at 120° { single-acting .	1·09
Three-throw pump, crank at 120° { double-acting .	1·05

As will be readily understood, the varying velocity and acceleration in the supply and discharge pipes—particularly in the supply pipes—produce a tendency to water hammer.

So long as this hammer is prevented, either by suitably enlarging the pipes ; by reducing the maximum piston velocity or acceleration ; or by the provision of air vessels on suction and delivery pipes, the action of a pump may be predicted very accurately from theoretical considerations

Once water hammer is set up so many factors combine to influence the result, and the subject presents such difficulties of treatment that, except in simple cases, no satisfactory attempt can be made to deal with the matter analytically. This is, however, less important, in that water hammer is not, under any conditions, admissible in a pump, and while its effect may be impossible to predict with any degree of accuracy, save in the most simple cases, the steps necessary to prevent its occurrence are well understood.

The following demonstration may be relied upon as giving results which are substantially correct so long as the pump is working without shock.

Art. 189.—Variations of Pressure in the Cylinder of a Single Cylinder Pump without Air Vessels.

For the water in the suction pipe to follow and maintain contact with the piston throughout the suction stroke, this column of water must receive an acceleration at the commencement of the stroke, the magnitude of the acceleration being given by—

$$a \frac{A}{a_s} \text{ f.s.s.}$$

where a = piston acceleration in f.s.s.
A = piston area in square feet.
a_s = area of suction pipe in square feet.

The force necessary to give this acceleration can only be produced by a difference in pressure at the two ends of the suction pipe, and varies as the mass of water in the pipe and as its acceleration. Expressed symbolically we have—

Force necessary to produce acceleration $= \frac{W}{g} l_s a_s . \frac{A}{a_s} a$ lbs.

If the pressure difference at the two ends of the pipe corresponding to this force is p' lbs. per square foot—

$$p' a_s = \frac{W}{g} l_s A a \text{ lbs.}$$

$$\therefore \quad p' = \frac{W}{g} l_s \frac{A}{a_s} a \text{ lbs. per square foot.}$$

If at the same instant the piston velocity is v f.s., for continuity of flow we have $v_s = v \frac{A}{a_s}$; and the loss of pressure due to friction in the supply pipe corresponding to this is given by:—

$$p'' = \frac{W f l_s v_s^2}{2 g m} = \frac{W f l_s A^2}{2 g m a_s^2} v^2 \text{ lbs. per square foot.}$$

$\therefore$ Total difference of pressure at the inlet and outlet of the suction pipe is given by:—

$$p' + p'' = p = \frac{W}{g}\,\frac{A}{a_s}\,l_s \left\{ a + \frac{f A}{2\,m\,a_s}\,v^2 \right\} \text{ lbs. per square foot.}$$

Or, expressed as a head "h" in feet of water,

$$h = \frac{A\,l_s}{g\,a_s} \left\{ a + \frac{f A}{2\,m\,a_s} \cdot v^2 \right\} \text{ feet,}$$

where a is positive or negative according as the piston is being accelerated or retarded.

Now the head actually available to produce this flow is strictly limited, the higher pressure—at the pipe inlet—being that corresponding to its depth of immersion together with that of the atmosphere, while the lower limit of pressure is theoretically that corresponding to an absolute vacuum, although in practice it is impossible to obtain this degree of exhaustion in the pump. If the level of the pump is above that of the suction reservoir so as to give a suction lift of h_s feet (Fig. 306), the available head is correspondingly reduced, and has a maximum theoretical value of $(\pi_a - h_s)$ feet, where π_a is the height of the water barometer.

Thus, for continuity of contact between piston and water on the suction stroke, we must have:—

$$\pi_a - h_s > \frac{A\,l_s}{g\,a_s} \left\{ a + \frac{f A}{2\,m\,a_s}\,v^2 \right\} \text{ feet.} \qquad (1)$$

Should this condition not be satisfied the piston leaves the water at some point—usually at the beginning of the stroke, since here a has its

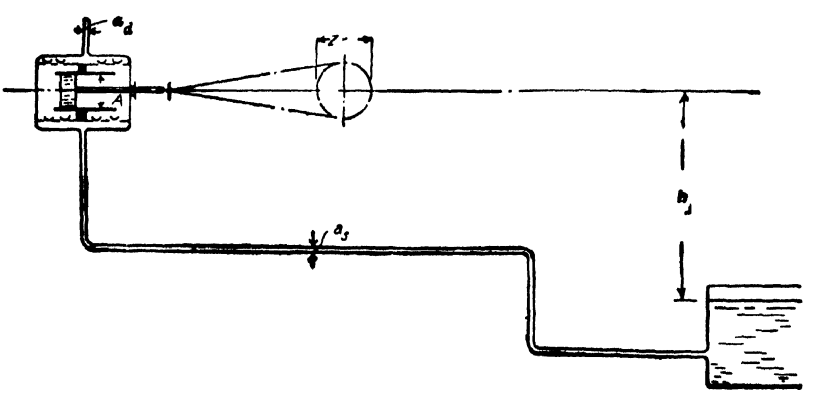


FIG. 306.

maximum value—the action being termed **Separation or Cavitation.** So long as this state of affairs exists, the pressure behind the piston is uniform, being that of water vapour at suction temperature, and in consequence the water flows along the pipe with an acceleration which is uniform, except for the increasing effect of friction as its velocity increases. The piston acceleration is, however, continuously diminishing, so that the water overtakes the latter at some point before the completion of the stroke, the meeting being usually accompanied by considerable shock, while pressures considerably in excess of those corresponding to the statical head are produced.

Separation may also occur between the delivery column and the piston during the second half of the delivery stroke, when the piston is being retarded. Thus if $-a$ is the magnitude of this retardation, the head necessary to produce the retardation in the delivery column is given by $-\frac{A}{a_d}\frac{l_d}{g}a$ feet, so that to prevent separation during this stroke we must have:—

$$h_a + h_d > \frac{A}{a_d}\frac{l_d}{g}\left\{-a - \frac{f A}{2 m a_d} v^2\right\} \qquad (2)$$

Note.—Since during the latter half of the stroke the piston is being retarded ($-a$) is a positive quantity.

Where the pump is driven from a shaft rotating with uniform angular velocity ω radians per second, by means of a crank of radius r, assuming the connecting rod to be of infinite length:—

$$\begin{cases} a = \omega^2 r \cos\theta & \text{f.s.s.} \\ v = \omega\ r \sin\theta & \text{f.s.} \end{cases}$$

where θ is the crank angle, measured from the inner dead centre, while with a connecting rod of length l these expressions become:—

$$\begin{cases} a = \omega^2 r \left\{\cos\theta + \dfrac{r l^2 \cos 2\theta + r^3 \sin^4\theta}{(l^2 - r^2 \sin^2\theta)^{\frac{3}{2}}}\right\} & \text{f.s.s.} \\ v = \omega\, r \left\{\sin\theta + \dfrac{r \sin 2\theta}{2\sqrt{l^2 - r^2 \sin^2\theta}}\right\} & \text{f.s.} \end{cases}$$

giving a the maximum value $\omega^2 r\left\{1 \pm \frac{r}{l}\right\}$ according as the crank is on the inner or outer dead centres respectively (*i.e.*, according as $\theta = 0°$ or $180°$).

Neglecting, for simplicity, the effect of the obliquity of the connecting rod, and therefore assuming the piston to have simple harmonic motion, the maximum velocity is given by ωr, while the maximum acceleration $= \omega^2 r$, and therefore for a given displacement the acceleration is com-

paratively reduced by making the stroke of the pump large and the angular velocity of the crank, or the number of strokes per minute, low. This enables a higher piston speed to be adopted, since frictional effects are usually small compared with those due to acceleration.

Substituting for a and v in terms of the crank angle θ, equation (1) becomes :—

$$\pi_a - h_s > \frac{A}{a_s}\frac{l_s}{g}\left\{\omega^2 r\cos\theta + \frac{f\omega^2 r^2 \sin^2\theta\, A}{2 m a_s}\right\}$$

$$> \omega^2 r \frac{A}{a_s}\frac{l_s}{g}\left\{\cos\theta + \frac{f r \sin^2\theta\, A}{2 m a_s}\right\}$$

giving

$$\omega = \sqrt{\frac{\frac{g a_s}{r A l_s}\{\pi_a - h_s\}}{\cos\theta + \frac{f r \sin^2\theta\, A}{2 m a_s}}} \qquad (3)$$

as the critical velocity, above which separation will occur on the suction stroke.

Similarly, substituting in equation (2) we obtain :—

$$\omega = \sqrt{\frac{\frac{g a_d}{r A l_d}\{\pi_a + h_d\}}{-\cos\theta - \frac{f r \sin^2\theta\, A}{2 m a_d}}} \qquad (4)$$

as the velocity, above which separation occurs on the delivery stroke.

Here θ is in every case measured from the beginning of the stroke, so that cos θ in the latter expression is negative.

Putting $\pi_a = 34$ feet, and writing $\theta = 0$ in equation (3) :—

$$\omega = \sqrt{\frac{g a_s}{r A l_s}\{34 - h_s\}} \qquad (5)$$

is the limiting speed at which separation will occur at the commencement of the suction stroke. Since $\omega = \frac{2\pi N}{60}$ (where N = revolutions per minute), this becomes :—

$$N = \frac{30}{\pi}\sqrt{\frac{g a_s}{r A l_s}(34 - h_s)}. \qquad (6)$$

With a finite connecting rod of length l, we have :—

$$N = \frac{30}{\pi}\sqrt{\frac{g a_s}{r A l_s}\cdot\frac{\{34 - h_s\}}{\left\{1 + \frac{r}{l}\right\}}}. \qquad (7)$$

The action may be shown graphically as follows :—

In Fig. 307, $O\,O'$ represents the atmospheric pressure line, and,

assuming simple harmonic motion, ordinates drawn to the straight line $A\,H\,A'$ represent the head necessary to accelerate the water column in the suction pipe. Then $G\,A = G'\,A' = \frac{\omega^2\,r\,l_s}{g} \cdot \frac{A}{a_s}$. Vertical ordinates, set off from $A\,H\,A'$ as base line, to the curve $A\,B\,A'$, represent the heads necessary to overcome frictional resistance, zero at the ends, and having a maximum value $H\,B = \frac{f\,l_s\,A^2}{2\,g\,a_s^2} \cdot \frac{\omega^2\,r^2}{m}$, at the middle of the stroke.

The vertical ordinates of the area included between $O\,O'$ and $A\,B\,A'$ then

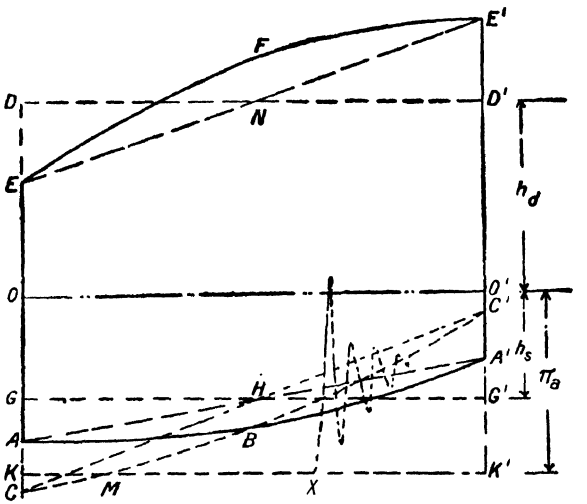


FIG. 307.

give the differences of head between the two ends of the suction pipe due to friction and inertia.

If now $K\,K'$ be drawn at a distance below $O\,O'$, representing the barometric head $\pi\,a$ feet, the ordinates of the curve $A\,B\,A'$, measured from $K\,K'$ as base line, give the effective pressures in the suction chamber expressed in feet of water.

If the speed be increased so that $C\,G$ represents the head necessary to produce the initial acceleration, separation will occur at the commencement of the stroke, since the available head is now insufficient to give the necessary acceleration. At M the head becomes sufficient both for this purpose and to overcome frictional losses, while at some point X, the acceleration of the water remaining approximately constant while that of the piston is continually diminishing, the water overtakes the piston and knocking

occurs. Up to this point the pressure behind the piston is constant, and has a negative value equal to $O\ K$ in feet of water. At the shock, violent oscillations of pressure are set up as represented by the dotted lines and the most that theory can do in this case is to indicate the maximum pressure to be expected and the means of reducing this.

$E'\ F\ E$ represents the pressure line for the delivery side of the pump, $E'\ D'$ being the head required for the initial acceleration of the delivery column, and $F\ N$ the friction head at mid-stroke.

In a high-speed pump, if the delivery head be small, it is possible for the

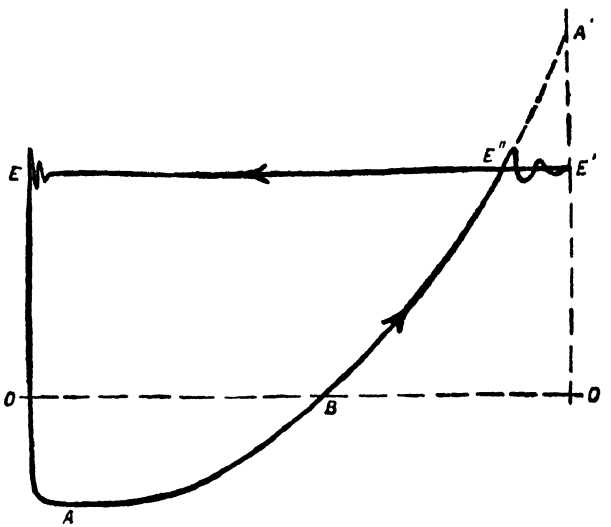


FIG. 307A.—Pump Diagram showing delivery during Suction Stroke.

drop in pressure $D\ E$ due to retardation of the delivery column at the end of the stroke to be so great as to make the pressure at E less than that at A' on the suction side of the cylinder. If this occurs, both suction and delivery valves will, lift and discharge will take place directly from the suction to the delivery side of the pump.

The same action may occur if the suction pipe is very long, even with an air vessel on the delivery pipe. Under such conditions a diagram similar to that of Fig. 307A is obtained, the pressure at E'' on the suction stroke becoming equal to the delivery pressure.

The coefficient of discharge now becomes greater than unity. Its value may be deduced since the area $A'\ E''\ E'$ represents the work done in pumping a volume of water v cubic feet against a head $h_d\ (= O\ E)$ feet.

The area $A\ E''\ E'$ thus represents $62{\cdot}4\ v\ h_d$ foot lbs. $\therefore\ v = \dfrac{\text{area } A\ E''\ E'}{62{\cdot}4\ h_d}$.

The scale to which this area is to be measured is given by the product of the scales to which ordinates of the diagram represent heads in feet and abscissæ represent pounds of water.

The Coefficient of discharge $C_d = 1 + \dfrac{v}{V}$ where V is the piston displacement per stroke.

Assuming simple harmonic motion, the area $A'\ E'''\ E'$ may be calculated analytically. In more complicated cases, it is preferable to measure it by planimeter.

Where an actual diagram is to be estimated (Fig. 307A), the suction head curve $A\ B\ E''$ may be produced by hand with fair accuracy to A', and the area then measured.

Since, at the speed at which separation occurs, water hammer raises the pressure towards the end of the suction stroke by an abnormal amount, it is to be inferred that about this speed a sudden increase in the discharge coefficient will take place. This inference is justified by the results of experiments by Professor John Goodman,[1] the increase in this coefficient at the knocking speed in his pump, varying from 58 per cent. with a delivery pressure of 10 lbs. per square inch to 5 per cent. with a pressure of 70 lbs. per square inch.

Although this increases the capacity of the pump, yet, as might be expected, its efficiency is lowered, due to the increased losses by shock. The experiments showed that in this particular pump an increase in the discharge coefficient from 1·059 to 1·517, was accompanied by a decrease of 10 per cent. in the efficiency.

ART. 190.—RISE IN PRESSURE FOLLOWING SEPARATION IN A PUMP.

During separation the accelerating force on the suction column $= W\ a_s\ \{\pi_a - h_s - h_f - h_t\}$ lbs., where h_f is the head necessary to overcome friction and therefore depends on the velocity, and h_t is the pressure in the cylinder due to vapour tension and to air leakage.

Neglecting the effect of friction, the acceleration in pipe line

$$= \frac{g\ W\ a_s\ \{\pi - h_s - h_t\}}{W\ l_s\ a_s} = \frac{g\ (\pi_a - h_s - h_t)}{l_s} \text{ f.s.s.}$$

$$\therefore \text{ Acceleration of water surface in cylinder } = g\ \frac{(\pi_a - h_s - h_t)}{l_s} \cdot \frac{a_s}{A} \text{ f.s.s.}$$

If the water overtakes the piston at a distance x feet from the beginning

[1] "Proc. Inst. Mech. E.," 1903.

of the stroke, the time taken by the water surface to cover this distance under its constant accelerating force must equal that taken by the piston to cover the same space.

The former of these times is given by $T = \sqrt{\dfrac{2\,x\,l_s\,A}{g\,\{\pi_a - h_s - h_t\}\,a_s}}$

since, with uniform acceleration, $T = \sqrt{\dfrac{2 \text{ space}}{\text{acceleration}}}$.

Assuming *S.H.* motion, the time occupied by the piston to cover the same space is given by $\dfrac{\theta}{\omega}$ seconds,

where $x = r\,(1 - \cos\theta)$ (Fig. 308)

or $\theta = \cos^{-1}\left(1 - \dfrac{x}{r}\right)$.

$\therefore$ Equating these times :—

$$\frac{1}{\omega}\cos^{-1}\left(1 - \frac{x}{r}\right) = \sqrt{\frac{2\,x\,l_s\,A}{g\,\{\pi_a - h_s - h_t\}\,a_s}}$$

$$\therefore \quad 1 - \frac{x}{r} = \cos\omega\sqrt{\frac{2\,x\,l_s\,A}{g\,(\pi_a - h_s - h_t)\,a_s}},$$

an equation from which x may be obtained by trial. For purposes of calculation h_t may be taken as zero. An attempt to include the effects of friction and of a finite connecting rod greatly complicates the result unless h_f be taken to be constant and to have a value corresponding to the mean velocity during the period of separation. By this means a close agreement with experimental results is obtained.

The above formula has been applied to the case of a pump described by Professor John Goodman[1] and having the following dimensions :—

$\dfrac{A}{a_s} = 1{\cdot}83$; $l_s = 63$ feet; $r = {\cdot}25$ feet.

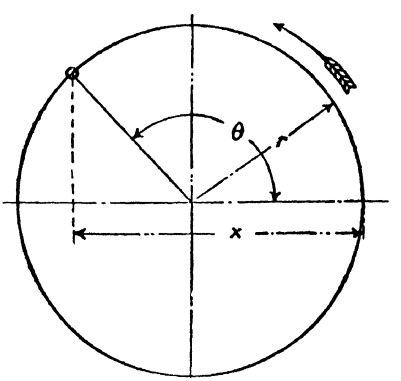


FIG. 308.

In this particular experiment the pressure in the suction valve chamber was measured, the loss of head due to height of suction, to friction, and to air tension, being approximately 8·0 feet, making—

$$(\pi_a - h_s - h_f - h_t) = 26{\cdot}0.$$

[1] "Proc. Inst. Mech. Engineers," 1903, part 1 (p. 143).

The revolutions per minute were 70, making $\omega = \frac{2\pi \times 70}{60} = 7{\cdot}33$.

The equation then becomes:—

$$\cos 7{\cdot}33 \sqrt{\frac{126 \times 1{\cdot}83}{32 \times 26}} \cdot x = 1 - 4x$$

or

$$\cos 3{\cdot}86 \sqrt{x} = 1 - 4x$$

giving a value of $x = {\cdot}47$ feet;

i.e., knock takes place at $\frac{{\cdot}47}{{\cdot}50} = {\cdot}94$ of the stroke.

The indicator diagram, of which Fig. 309 is a copy, shows this to occur at about ·95 of the stroke.

The velocity of the water column immediately before impact may be deduced from the equation

$$\text{velocity} = \sqrt{\frac{2 \times \text{acceleration} \times \text{space covered under this acceleration}}{\text{acceleration}}} \text{ f.s.}$$

And since this effective space $= x \cdot \frac{A}{a_s}$, this gives

$$\text{velocity} = \sqrt{\frac{2g(\pi_a - h_s - h_f - h_l)}{l_s} \cdot x \frac{A}{a_s}} \text{ feet per second.}$$

while the piston velocity $= \omega r \sin\theta$

$$= \omega r \sqrt{1 - \cos^2\theta}$$

$$= \omega \sqrt{\left(2 - \frac{x}{r}\right) x r} \text{ feet per second.}$$

After impact the mean velocity of the supply column becomes equal to that of the piston multiplied by $\frac{A}{a_s}$.

$$\therefore \text{ Change of velocity at impact} = \left\{ \sqrt{\frac{2g(\pi_a - h_s - h_f - h_l)}{l_s} \cdot \frac{xA}{a_s}} - \frac{A}{a_s} \omega \sqrt{x r \left(2 - \frac{x}{r}\right)} \right\}$$

$$= v_x \text{ feet per second.}$$

If this change of velocity be assumed to take place instantaneously, the increase of pressure due to water hammer is given by $63{\cdot}7\, v_x$ lbs. per square inch (p. 235). In the example previously considered, taking $x = {\cdot}47$, we have:—

Change of velocity at impact

$$= \sqrt{\frac{64 \times 26 \times {\cdot}86}{63}} - 1{\cdot}83 \times 7{\cdot}33 \times \sqrt{{\cdot}12 \times {\cdot}12}$$

$$= 4{\cdot}76 - 1{\cdot}61$$

$$= 3{\cdot}15 \text{ feet per second.}$$

$\therefore$ Water hammer pressure $= 3{\cdot}15 \times 63{\cdot}7$ lbs. per square inch
$= 201$ lbs. per square inch.

Owing to the elasticity of the pipe line and cylinder walls the pressure would be expected to be somewhat less than this, and exterpolation from the curves showing the results of the experiments under consideration shows that the actual pressure at this speed would apparently vary from about 115 lbs. per square inch with a delivery pressure of 20 lbs. per square inch, to about 195 lbs. per square inch with a delivery pressure of 60 lbs. per square inch. This is a result which might have been inferred, since directly the delivery pressure is exceeded, the delivery valve opens, and the maximum pressure is reduced by an amount which cannot be deduced quantitatively.

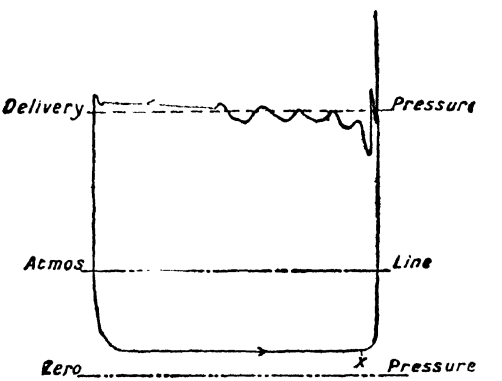


FIG. 309.

It is, however, to be expected —and this is verified in practice —that this reduction will be greater as the delivery valve area is increased, and as the delivery pressure is reduced, since the latter reduces the pressure necessary to produce flow through the valves, while the former ensures a more easy delivery.

ART. 191.—EFFECT OF THE ELASTICITY OF THE SUCTION COLUMN.

While the formulae already obtained enable the pressure on the piston to be calculated with fair accuracy for the greater part of the stroke, it is found that the pressures actually obtained towards the end of the suction stroke, even when separation does not take place, are usually much greater than those calculated in this manner. So far, it will be noted, any effect which the elasticity of the water column may have in modifying this pressure has been neglected. Because of this elasticity, however, the change of velocity and of pressure at the open end of the pipe will lag behind that at the piston by the time necessary for a pressure wave to traverse the pipe, *i.e.*, by a time $\frac{l}{4{,}700}$ seconds, and if the acceleration at the plunger end at a given instant be a, the difference in velocity at the two ends will be $a \times \frac{l}{4{,}700}$ f.s.

Thus, assuming *S.H.* motion, when the piston comes to rest the velocity at the open end will still be $\frac{\omega^2 r l}{4{,}700} \cdot \frac{A}{a_s}$ f.s., while the mean velocity in the pipe will be half this. In virtue of this velocity the column possesses a store of kinetic energy which, on stoppage of the piston, is transformed into energy of strain, and which in consequence gives rise to a rapid rise in pressure, of the nature of water hammer.

Actually this occurs before the end of the stroke at a point where the retardation due to the piston becomes equal to $\frac{4{,}700\, v}{l}$, v being the velocity of flow in the pipe at this instant, and the resultant rise in pressure, assuming a rigid pipe line, will equal $63{\cdot}7\, v$ lbs. per square inch. As the retardation and velocity can both be calculated in terms of θ, the position

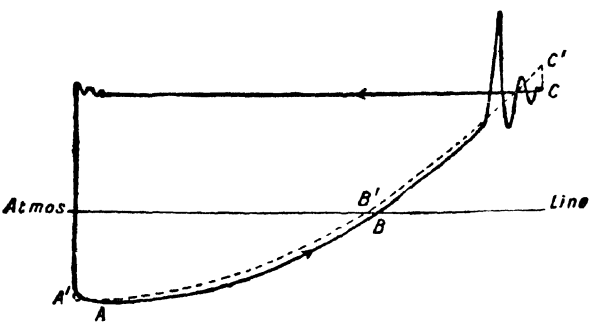


FIG. 310.—Pump Diagram showing the effect of the elasticity of Suction Column. Air Vessel on Delivery side only.

of the piston when this occurs, and the magnitude of the shock, can be readily calculated.

The effect of the elasticity of the suction column is therefore to modify the shape of the diagram as shown in Fig. 310, where the dotted line $A'\ B'\ C'$ represents the theoretical curve, neglecting the effect of elasticity, and $A\ B\ C$ is the actual curve.

In the preceding investigations, the effect of loss of energy due to the sudden enlargement of section of the stream on entering the pump barrel, and to valve resistances, has been neglected. In general, however, this will be comparatively small, except where the suction pipe is very short, in which case the difficulties already outlined cease to exist.

Even with a long suction pipe these may be considerably minimised, if not entirely removed, by the provision of an air vessel—or vacuum vessel as it is sometimes termed, since the pressure of the contained air is less than that of the atmosphere—on the suction side of

the pump. The effect of such an air vessel will now be considered. Its action is as follows:—During the first part of the stroke the pressure behind the piston is reduced and water flows out of the air vessel. The flow along the suction pipe is thus reduced, as is the acceleration of the whole mass of water. This reduces the frictional resistance in the suction pipe, while the pressure behind the piston is increased both on this account and because of the reduced acceleration. At the same time tendency to separation and to water hammer at the end of the stroke is reduced, if not entirely eliminated; the discharge coefficient becomes unity, or slightly less than unity; and the pump diagram approximates

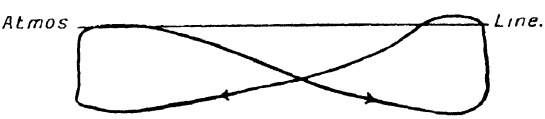


FIG. 311.—Pressure Diagram from Suction Air Vessel of Reciprocating Pump.

more and more nearly to the rectangle given by a perfect pump, as the sizes of the air vessels are increased.

A typical indicator diagram taken from the suction air vessel of a double-acting pump is shown in Fig. 311.

ART. 192.—AIR VESSEL APPLIED TO A SINGLE OR DOUBLE-ACTING SINGLE CYLINDER PUMP.

Effect of Air Vessel placed on Suction Side of Pump.

Let a_s, v_s, and a_s represent the area of suction pipe between supply reservoir and air vessel, the velocity and the acceleration in this pipe respectively.

a_v, v_v, a_v represent ditto, in the air vessel itself.

A, V, a represent ditto, in the pump cylinder.

h_a represent the pressure of air in air vessel, in feet of water.

π represent the atmospheric pressure, in feet of water.

h_v = height in feet, of water level in air vessel above centre line of pump.

During the first part of suction stroke water will flow out of air vessel into pump, and for continuity of flow:—

$$v_s a_s + v_v a_v = V A$$

$\therefore$ Differentiating, we have $a_s a_s + a_v a_v = A a$ (1)

If, as is usual, the air vessel is placed near to the pump, so that the pressure at the junction of air vessel and suction pipe may be taken as

substantially equal to that in the suction chamber we have, denoting this pressure by h_j feet of water, neglecting frictional losses in the supply pipe, and simply considering the acceleration effect:—

$$(\pi - h_s - h_j)\, a_s\, W = \frac{a_s\, W\, l_s\, a_s}{g}$$

$$\therefore \quad h_j = \pi - h_s - \frac{l_s\, a_s}{g} \tag{2}$$

Also, considering the flow down the air vessel:—

$$h_a + h_v - h_j = \frac{h_v\, a_v}{g}^{1}$$

$$\therefore \quad h_j = h_a + h_v - \frac{h_v\, a_v}{g} \tag{3}$$

Substituting for a_v in terms of a_s from (1) we get:—

$$h_j = h_a + h_v - \frac{h_v}{g}\left\{\frac{A\, a - a_s\, a_s}{a_v}\right\} \tag{4}$$

Substituting for h_j from (2), this becomes:—

$$\pi - h_s - \frac{l_s\, a_s}{g} = h_a + h_v - \frac{h_v}{g}\left\{\frac{A\, a - a_s\, a_s}{a_v}\right\} \tag{5}$$

$$\therefore \quad a_s\left\{\frac{a_s}{a_v}\cdot\frac{h_v}{g} + \frac{l_s}{g}\right\} = \pi - h_s - h_a - h_v + \frac{h_v\, A\, a}{a_v g}$$

$$\therefore \quad a_s = g\,\frac{\left\{\pi - (h_s + h_a) + h_v\left(\dfrac{A\, a}{g\, a_v} - 1\right)\right\}}{\dfrac{a_s\, h_v}{a_v} + l_s} \tag{6}$$

as compared with its value $\frac{A\, a}{a_s}$, without air vessel.

Neglecting, for the time being, the variation in h_a and h_v with a variation in the piston acceleration a, it is evident from (6) that for a_s to be affected as little as possible by a variation in a, the term involving a must be as small as possible. This indicates that a_v, the sectional area of the vessel, should be as large, and h_v as small as possible—a deduction which is verified in practice.

When the pump is working, the water surface h_v undergoes cyclical variations in height, h_a assuming corresponding values. The connection between the two may be obtained on the assumption that the air follows

[1] This assumes the air vessel to be of uniform diameter to its junction with the suction pipe. If, as is more usual, the junction is made through a smaller pipe of area a_1, this formula becomes $h_a + h_v - h_j = \frac{h_v\, a_1}{g}$, where $a_1 = a_v \frac{a_v}{a_1}$.

the law (pressure × volume = constant), for if H_v = height of top of air vessel above centre line of pump :-

$$h_a \{H_v - h_v\} = \text{constant} = K$$

$$\therefore \quad h_v = H_v - \frac{K}{h_a} \tag{7}$$

$$\text{or } h_a = \frac{K}{H_v - h_v} \tag{8}$$

The mean height h_v may be adjusted by admitting more or less air into the air vessel, and this adjustment should be made until h_v is as low as is consistent with no air being drawn over into the suction chamber as the level varies.

With a single-acting pump, the total cyclical variation in volume in the air chamber is about ·56 of the piston displacement per stroke.

Since the fluctuation in level diminishes with an increase in cross-sectional area, an increase in a_v has the further advantage that it permits the mean working value of h_v to be reduced.

Again, substituting in (6) from (8), it appears that while the fluctuation in a_s corresponding to a given variation in h_v, diminishes as H_v increases yet this effect is small compared with that of an increase in a_v. In effect then, the area of the vessel is of much greater importance than its length, and for a given volume, the ratio $\frac{\text{area}}{\text{length}}$ should be as large as possible.

EXAMPLE.

$A = 1{\cdot}0$ square foot. $\quad l_s = 30$ feet.

$a_s = \ {\cdot}25$ square foot. $\quad h_s = 9{\cdot}5$ feet.

$a_v = 1{\cdot}00$ square foot. $\quad H_v = 4$ feet.

Length of stroke = 1·0 foot. No. of revolutions = 100 per min.

When the pump is standing let $h_v = 6$ inches. We then have $h_a = (34 - 10) = 24$ feet absolute head,

$$\text{so that } 24 \{4 - \tfrac{1}{2}\} = \text{constant} = K$$

$$\therefore \quad K = 84.$$

Assuming the total fluctuation of volume in the air vessel to be equal to ·56 times the delivery per stroke, this gives a total fluctuation equal to ·56 cubic feet, and therefore a fluctuation in level of ·56 feet.

The acceleration in the supply pipe corresponding to any value of the piston acceleration may now be obtained from equation (6).

$$\text{Thus } a_s = \frac{32{\cdot}2 \left\{34 - 9{\cdot}5 - \dfrac{84}{4 - h_v} + h_v \left(\dfrac{a}{32{\cdot}2} - 1\right)\right\}}{{\cdot}25\, h_v + 30} \text{ f.s.s.}$$

It only remains to substitute for h_v, and if, as indicator diagrams show to be usually the case, we assume that this has its minimum value shortly after the piston begins its suction stroke, we may obtain the acceleration, at this point by putting $h_v = [\cdot 5 - \cdot 28] = \cdot 22$ feet in the above expression, and by writing

$$a = \frac{4\,\pi^2 \times 100^2}{3{,}600} \times \tfrac{1}{2} = 54{\cdot}8 \text{ f.s.s.}$$

$$\text{Then } a_s = \frac{32{\cdot}2\left\{24{\cdot}5 - \dfrac{84}{3{\cdot}78} + \cdot 22\left(\dfrac{54{\cdot}8}{32{\cdot}2} - 1\right)\right\}}{30{\cdot}055}$$

$$= 2{\cdot}62 \text{ f.s.s.}$$

as compared with its value $\omega^2 r \times 4 = 219{\cdot}3$ f.s.s. without air vessel.

The maximum acceleration will in general be found to occur at about ·2 of the suction stroke.

Evidently a further increase in the size of air vessel, or an increase in the length of suction pipe will reduce the value of a_s still further, and with a suction pipe of any considerable length its value approximates very sensibly to zero. In such a case the flow along the pipe is sensibly constant, and the velocity is equal to the discharge in cubic feet per second divided by the area of the pipe. If this assumption be made, calculations relating to the necessary size, etc., of the air vessel are considerably simplified, as will be shown later.

Modifying Effect of Friction and Kinetic Losses in Suction Pipe.—Taking the total difference of head between supply reservoir and piston as being given by

$$\frac{v_s^2}{2\,g}\left(1 + \frac{f\,l_s}{m}\right) + \frac{l_s\,a_s}{g} \text{ feet,}$$

this may be written as:—

$$\frac{a_s}{g}\left\{\frac{r}{2}\,\frac{\sin^2\theta}{\cos\theta}\,.\,\frac{A}{a_s}\left(1 + \frac{f\,l_s}{m}\right) + l_s\right\} \text{ feet.}$$

$$\left(\text{Since } a_s = \omega^2\, r\cos\theta\,\frac{A}{a_s}\right.$$

$$\left.\text{and } v_s^2 = \omega^2\, r^2 \sin^2\theta\,\frac{A^2}{a_s^2} = a_s\, r\,\frac{\sin^2\theta}{\cos\theta}\,.\,\frac{A}{a_s}\right)$$

$$\therefore\quad \pi - h_s - h_j = \frac{a_s}{g}\left\{l_s + B\,\frac{\sin^2\theta}{\cos\theta}\right\} \text{ feet.}$$

The preceding equations now become:—

$$h_j = \pi - h_s - \frac{a_s}{g}\left\{l_s + B\,\frac{\sin^2\theta}{\cos\theta}\right\} \qquad (2')$$

$$h_j = h_a + h_v - \frac{h_v a_v}{g} \qquad (3')$$

(neglecting frictional losses in air vessel as being comparatively small).

On substituting for a_v in (3′) in terms of a_s from (1), as before, we have, equation (4) :—

$$h_j = h_a + h_v - \frac{h_v}{g}\left\{\frac{A\,a - a_s\,a_s}{a_v}\right\} \qquad (4')$$

and substituting in this equation for h_j from (2′), on reduction :—

$$a_s = \frac{g\left\{\pi - (h_s + h_a) + h_v\left(\frac{A\,a}{g\,a_v} - 1\right)\right\}}{l_s + \frac{a_s\,h_v}{a_v} + B\frac{\sin^2\theta}{\cos\theta}} \qquad (6')$$

By substituting in (4′) for a_s from (2′) we get, on reduction :—

$$h_j = \frac{\pi - h_s + \frac{a_v\left(l_s + B\frac{\sin^2\theta}{\cos\theta}\right)}{h_v\,a_s}\left[h_a + h_v - \frac{h_v\,A\,a}{a_v\,g}\right]}{1 + \frac{a_v}{h_v\,a_s}\left\{l_s + B\frac{\sin^2\theta}{\cos\theta}\right\}} \text{ feet,}$$

from which, on assuming values for h_v, a curve similar to Fig. 307, showing the pressure on the piston for a series of values of θ may be plotted.

In practice, with single-barrel double-acting pumps, suction air vessels of from 1·0 to 3·0 times the capacity of the pump per revolution are found to give good results, the necessary volume increasing with the rotative speed of the pump and with the length of suction pipe.

Art. 193.—Air Vessel on Delivery Side of Pump.

Here exactly the same reasoning applies, except that now flow into the air vessel will take place during the first half of the delivery stroke, and flow out of the vessel during the second half when the water column in the delivery pipe is being retarded.

Considering the latter part of the stroke, for continuity of flow :—

$$a_d\,v_d - a_v\,v_v = A\,V$$

$$\therefore \quad a_d\,a_d - a_v\,a_v = A\,a.$$

Also h_d now corresponds to $(h_s - \pi)$ in the case of the suction vessel.

∴ Equation (2) becomes :—

$$h_j = h_d + \frac{l_d\,a_d}{g} \text{ (neglecting friction);}$$

while (8) becomes :—

$$h_j - h_v - h_a = \frac{h_v a_v}{g}$$

$$\therefore \quad h_d + \frac{l_d a_d}{g} = h_v + h_a + \frac{h_v a_v}{g}$$

$$= h_v + h_a + \frac{h_v}{g}\left\{\frac{A a}{a_v} - a_d\right\}$$

$$\therefore \quad a_d = \frac{g\left\{h_v + h_a - h_d + \frac{h_v}{g}\frac{A a}{a_v}\right\}}{\left\{l_d + \frac{a_d h_v}{a_v}\right\}}$$

But without air vessel, so long as separation does not occur :—

$$a'_d = a \frac{A}{a_d}$$

$$\therefore \quad a'_d - a_d = \frac{1}{l_d + \frac{a_d}{a_v} h_v}\left\{\frac{A a l_d}{a_d} - g\,(h_d - h_a - h_v)\right\}$$

An examination of this equation indicates that the change in the retardation in the delivery pipe, and therefore in the pressure in the pump, due to the provision of the chamber, increases with an increase in a_v, and since h_v is in general small compared with h_d, diminishes with an increase in h_v. It follows that the mean level in the air vessel should be reduced to the lowest practicable limit, and that h_a should be as large as practicable, *i.e.*, the air supply should be maintained so as to keep the water-level as low as is consistent with the air vessel still containing some water at the end of the delivery stroke.

The modifying effect of frictional resistances may be examined as in the case of the suction air vessel. In general, owing to the fact that the delivery head is much greater than the suction head, and that the water at high pressure is able to dissolve an increased volume of air, the air in the vessel is gradually absorbed, so that either some device must be fitted for renewing the supply, or an air vessel of such dimensions must be fitted as will enable the mean level to be maintained fairly constant over long periods of working. In practice, with a single-barrel double-acting pump, the delivery air vessel usually has a volume equal to from six to nine times the pump displacement per revolution, depending on (increasing with) the speed of rotation and the length of delivery pipe. Here, again, an increase in the area of the vessel is of greater advantage than an increase in its length.

ART. 194.

If it be assumed that the velocity of flow along the suction and delivery pipes is sensibly uniform, and equal to the mean velocity of the pump plunger, multiplied by the ratio of areas of plunger and pipe, the volume of water entering the air chamber per stroke may be determined with fair accuracy. Thus, if the ordinates of the displacement curves $A\,B\,C\,D\,E$ (Fig. 312) represent piston velocities, and if $A\,F$ represent the mean

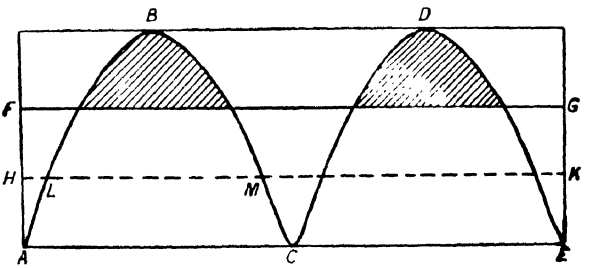


FIG. 312.

piston velocity, on multiplying the vertical scale by $\frac{A}{a_d}$ the curve would give the velocities of flow along the delivery pipe if no air vessel were fitted, $A\,F$ measuring the mean velocity.

Thus the ratio of each of the sectioned areas to the whole area $A\,F\,G\,E$

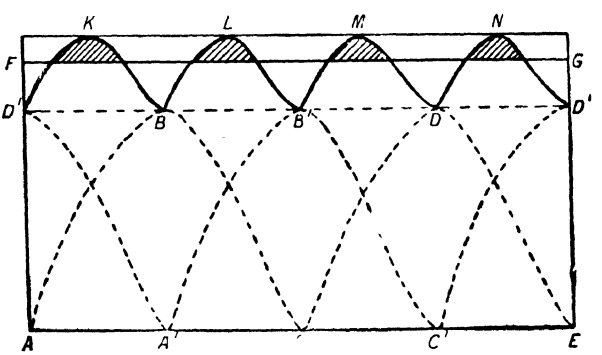


FIG. 313.

gives the ratio of the volume entering the air vessel per cycle to the total discharge per revolution. This applies to a double-acting pump. In a single-acting pump the volume discharged will be given by the area $A\,H\,K\,E$, where $A\,H = \frac{1}{2}\,A\,F$, while the volume entering the air vessel will be given by the area $L\,B\,M$. In the case of a double-acting duplex pump, having cranks at right angles, Fig. 313 represents the state of affairs.

Here the curve $D'\ K\ B\ L\ B'$, etc., formed by adding the ordinates of the two single velocity curves $A\ B\ C$, $A'\ B'\ C'$, etc., gives the velocity of the combined discharge, without air vessel. The area $A\ E\ G\ F$ represents the discharge per revolution, and each of the sectioned areas represents to the same scale the volume entering and leaving the air vessel per cycle.

These areas may be calculated or measured by planimeter. The following table gives the proportion of the whole discharge per revolution which enters and leaves the air vessel per cycle.

Type of Pump.	Proportion of Water entering the Air Vessel per Cycle.	
	Assuming an infinitely Long Connecting Rod.	Ratio $\frac{\text{Connecting Rod}}{\text{Crank}} = 4 : 1$.
Single-acting pump . . .	·55	·562
Two single-acting pumps with cranks at right angles.	·35	·365
Double-acting pump . . .	·105	·115
Two double-acting pumps with cranks at right angles.	·0105	·0106
Three-throw pump with cranks at 120° (Single-acting)	·0109	·0111
Three-throw pump with cranks at 120° (Double-acting)	·0029	·003

For satisfactory working the volume of the delivery air vessel should be from forty to sixty times the volume of water entering it per cycle, this proportion increasing with the speed of rotation and the length of delivery pipe.

On the suction side the volume should be from ten to thirty times the volume entering per cycle, this proportion also increasing with the speed and with the length of suction pipe.

To Summarise.—An air vessel on the suction side of a pump reduces the maximum acceleration and the range of velocities in the supply pipe, and thus, besides reducing friction losses, reduces the fluctuations of pressure in the suction chamber, and therefore the liability to water hammer or separation. The steadying effect becomes more pronounced as A, a, l, are increased and less pronounced as a_1 is increased. The larger the volume of the chamber the greater is its effect, an increase of sectional area

being of more value than an increase in length. Also, for efficient working, the water-level in the air vessel should be adjusted by adjusting the air pressure, until as low as is consistent with water remaining in the vessel during the first part of the suction stroke. For the air vessel to be as effective as possible in preventing shock it should be so situated as to provide an effective air cushion to the entrant water. Fig. 314 *a*, *b*, and *c*,

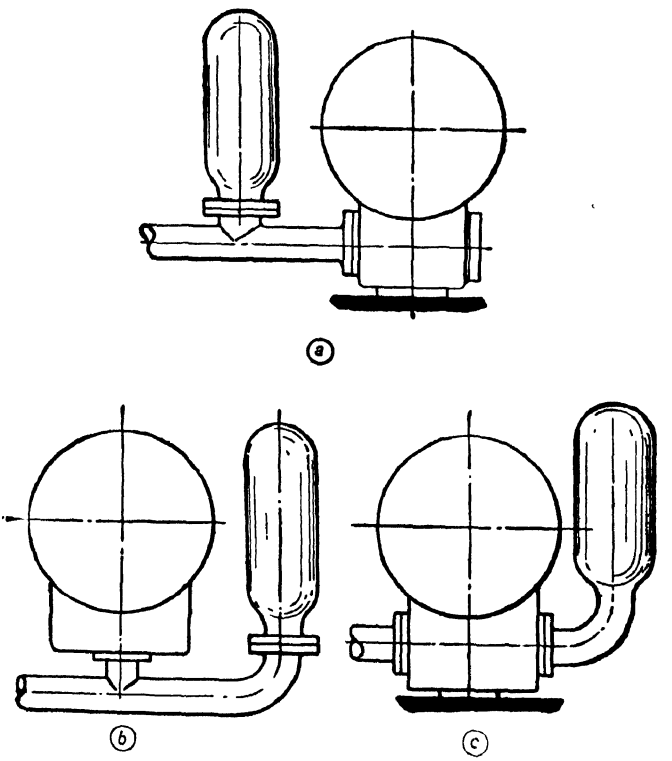


FIG. 314.—Suction Air Vessels for Reciprocating Pumps.

shows various methods of applying the air vessel. Of these (*a*) is very little good in preventing shock. (*b*) is good except that the provision of a right-angled bend at the entry to the suction chamber is inadvisable. The best method is that illustrated at (*c*), since here the air vessel is in a direct line with the suction pipe, while discharge either from the air vessel or supply pipe takes place directly into the suction chamber without the intervention of bends or elbows of any description. It may be premised that the air vessel should always be placed so that a sudden

retardation of the suction column may produce a direct flow into the vessel, without the intervention of any acute bends or contracted passages.

Precisely the same reasoning applies to the delivery air vessel, and here again, for efficient working, the water-level should be adjusted by adjusting the air pressure, until as low as is consistent with water remaining in the vessel throughout the whole cycle.

Art. 195.—Air-charging Devices.

One of the simplest devices for maintaining the level in an air vessel is the air injector of Messrs. Wippermann and Lewis. In this apparatus, which is represented diagrammatically in Fig. 315, the small cylindrical chamber *A* is connected to the pump barrel by means of a small pipe with regulating cock, and at its upper end carries a valve-box *B* containing air inlet and outlet valves, which is in turn connected to the air space of the air vessel *V*.

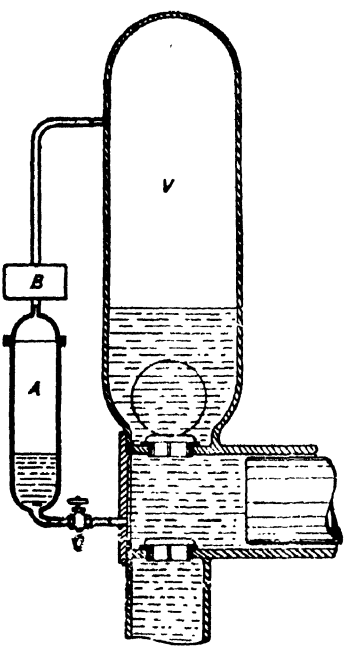


Fig. 315.—Wippermann Air Injector.

On the suction stroke in the main pump this chamber is partially emptied, the air is drawn in through the inlet valve. On the delivery stroke of the pump, water is forced into the chamber *A* and its contained air is driven through its outlet valve into the air vessel *V*.

The amount of water entering and leaving *A* per cycle, and therefore the air entering *V*, may be accurately adjusted by means of the regulating cock *C*.

Another simple device consists of a small compound air pump, worked by the main pump shaft, which draws air out of the suction chamber, where it tends to accumulate, and pumps it into the delivery chamber.

Art. 196.—Efficiency of the Reciprocating Pump.

When dealing with considerable pressures, the reciprocating pump if well designed and working at a fairly slow speed is capable of an efficiency of up to about 90 per cent.

Since a large proportion of the loss is due to mechanical friction and is therefore approximately independent of the pressure, this efficiency falls off rapidly as the working head is reduced, so that in general, for heads below about 100 feet, the centrifugal pump becomes the more efficient. The piston pump, however, has the advantage of being positive in action, is not so liable as the centrifugal pump to lose its water, and has an efficiency which, to a larger extent than in the case of the latter type, is independent of speed.

Art. 197.—Positive Rotary Pumps.

This type of pump forms the connecting link between the centrifugal and the reciprocating piston pump. Like the former, its motion is rotary

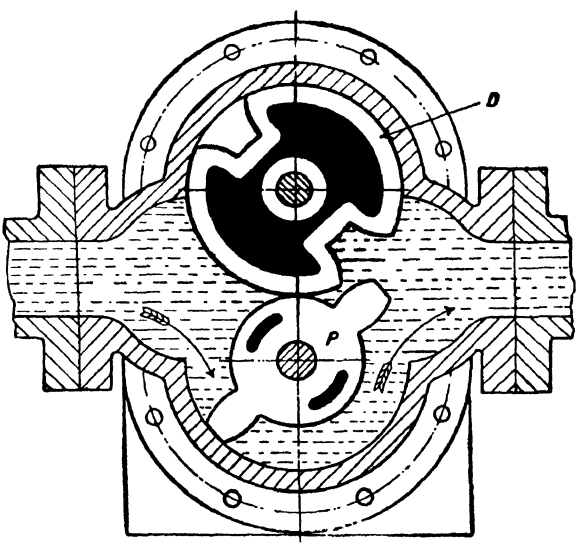


Fig. 316.—Drum Pump.

and its delivery practically continuous and free from vibration, while, like the latter, its action is positive and it will work well over a large range of speeds. Its great drawback lies in the difficulty of keeping the rotating pistons tight against each other and against the pump casing, the wear which invariably occurs leading to considerable leakage and loss of efficiency.

In spite of this, the convenience of the method of driving, the fact that no valves are required, and the steadiness of working, render the pump valuable in many instances, and more particularly where viscid liquid is

to be handled. A typical machine of this class, the Drum Pump, is illustrated in section in Fig. 316.[1] Here P is the revolving piston, which carries two projecting ribs, these gearing into suitable slots on the revolving drum D. The piston and drum are geared together by external gear wheels, and at each rotation of the piston a volume of liquid $= 2\pi R A$ cubic feet is delivered, where R = mean radius of the projecting rib and A = its projected area on a diametrical plane. The pump is built in sizes to deliver up to about 700 gallons per minute.

Examples.

(1) Sketch the displacement curve for a five-cylinder single-acting pump having cranks at 72° and, assuming zero acceleration in the pipe line, determine the proportion of the delivery per revolution which enters the air vessel per cycle.

Answer. ·0013.

(2) Assuming no air vessels, determine the critical speeds at which separation will occur on the suction and delivery sides respectively of the following pump :—

Cylinder diameter	= 10 inches.	Length of connecting rod	= 3 feet.
Cylinder stroke	= 18 inches.	Length of suction pipe	= 40 feet.
Diameter of suction pipe	= 5 inches.	Length of delivery pipe	= 250 feet.
Diameter of delivery pipe	= 4 inches.	Suction lift	= 10 feet.
		Delivery lift	= 180 feet.

Answer. { On suction stroke 21·7 revolutions per minute.
On delivery stroke 20·7 revolutions per minute.

(3) Assuming the pump of example (2) to run at forty revolutions per minute, determine at what point in the stroke shock will take place, and assuming all connections, etc., to be rigid, and neglecting the effect of the opening of the delivery valves, determine the maximum hammer pressure then produced.

Answer. 99·3 per cent. of stroke.
Hammer pressure 834 lbs. per square inch.

(4) Determine the discharge coefficients for a pump of 6-inch stroke, having a ratio $\frac{\text{connecting rod}}{\text{crank length}} = \frac{4}{1}$; a ratio $\frac{\text{piston area}}{\text{suction pipe area}} = 1{\cdot}83$; length of suction pipe = 63 feet, when running at fifty revolutions per minute, and when the delivery pressure is respectively

(*a*) 5 lbs. per square inch.
(*b*) 10 lbs. per square inch.

[1] By courtesy of the Drum Engineering Company, Bradford.

Also (*c*) for the same pump when fitted with a suction pipe 36 feet long and when running at sixty revolutions per minute against a head of 11·5 feet.

Answer. (*a*) . . 1·15.
(*b*) . . 1·01.
(*c*) . . 1·10.

(5) A double-acting pump, cylinder diameter 10 inches, stroke 18 inches, is fitted with an air vessel on the suction side 15 inches diameter, and 4 feet 6 inches long measured from its point of junction with the suction chamber. The length of suction pipe is 150 feet, its diameter is 5 inches, the suction lift, measured to the level of the suction chamber, is 10 feet, and air is admitted to the vessel until the mean working level is 4 feet from the top, the fluctuation in level being 2 inches. Determine the acceleration in the suction column at the beginning of the out-stroke—ratio of connecting rod to crank length being 4 : 1—when running at eighty revolutions per minute.

Answer $\alpha_s = \cdot 206$ f.s.s.

CHAPTER XX.

Centrifugal Pumps—Types and Construction—Theory—Balancing of End Thrust.

ART. 198.—CENTRIFUGAL PUMPS.

UNDER low heads the efficiency of the reciprocating pump falls off considerably, and when in addition very large quantities of water are to be handled its excessive dimensions render it expensive to construct and to instal. When working against a variable head, it also suffers from the disadvantage that its speed cannot be increased to any large extent to enable it to deliver a larger quantity of water as the head is reduced.

Under such conditions, with heads ranging from about 6 feet to 100 feet, the centrifugal pump having a single impeller is on all accounts most suitable, giving as it does a good efficiency—up to about 85 per cent. in the modern type of pump—along with moderate dimensions, simple construction, ease of installation and maintained high efficiency under continuous working conditions.

In the latter respect its freedom from valves gives it an advantage over the reciprocator, whose valves, glands, and packing rings need to be frequently overhauled if the efficiency is to be maintained, this advantage being still more pronounced where the liquid pumped contains gritty matter in suspension.

A further advantage is involved in its continuous and even discharge and the consequent freedom from shock in the delivery pipe line.

The invention, in 1875, by Professor Osborne Reynolds, of the modern form of high lift pump, having an efficiency equal to that of the old type of low lift pump, opened out a new field of application for the centrifugal pump, and of recent years this has been to an increasing extent invading the province of the reciprocator, its efficiency increasing with improvements in design, until at the present time heads of upwards of 1,500 feet may be overcome with efficiencies of from 75 to 80 per cent. In a pump designed for such work the water passes through a sequence of impellers mounted in series on the same shaft, the increase in head taking place in stages.

A further advantage of the centrifugal pump consists in the possibility

of adapting it to the high speeds of rotation common in the case of the electric motor or steam turbine, its even torque rendering it particularly well fitted for such a purpose, and the general tendency towards the extended use of such motors has of recent years given a great impetus to the development of the pump.

The extent of this adaptability may be realised from experiments by M. Rateau,[1] who, using a pump having a single impeller of 3·15 inches diameter, and rotating at 18,000 revolutions per minute, obtained an efficiency of approximately 60 per cent. when pumping against 863 feet head.

Indeed, to such an extent have the possibilities in design responded to the demands made upon them in recent years, that the centrifugal pump promises to replace the reciprocator to an ever-increasing extent in every class of work, except where the volume of water to be handled is very small in comparison with the working head.

For mine drainage, the motor-driven centrifugal has the advantage of requiring no expensive emplacements; while for elevator work this motor-driven pump, with an automatically controlled rheostat to regulate the discharge, has an advantage in that since the delivery pressure cannot increase to any large extent, no bye-pass is required. With a reciprocating pump this bye-pass is essential and involves a loss of energy by leakage.

Applied to dredging operations, the centrifugal pump is capable of removing sand, gravel, or clay broken up by a water jet or mechanical agitator, the amount of solid varying up to 40 per cent. of the volume handled. With a mixture containing 15 per cent. of gravel or 35 per cent. of fine sand, efficiencies of about 45 per cent. may be attained.

In its essentials the pump may be looked upon as a reversed inward radial or mixed flow turbine, having the wheel vanes driven in the opposite direction to that of rotation in the case of the turbine, and discharging outwards.

If the pump be filled, the rotation of the wheel produces a forced vortex in the contained water, with a consequent increase of pressure in an outward radial direction and a tendency to outward flow. If the speed of rotation is sufficiently high, this increase in pressure becomes more than sufficient to balance the statical pressure of the delivery head and flow takes place. A partial vacuum is thus produced at the centre of the wheel and water is forced up through the supply pipe by atmospheric pressure to take the place of that discharged outwards by centrifugal action.

[1] *Engineer*, March 7, 1902 (p. 23).

ART. 199.—TYPES AND CONSTRUCTION OF CENTRIFUGAL PUMPS.

The whole object of a centrifugal pump, as indeed of any pump, is to increase the pressure of the water which it handles, and where, as in the case of a centrifugal pump, the water is delivered from the impeller with a considerable velocity, the degree of efficiency to which the machine may attain depends very largely on the extent to which the kinetic energy of

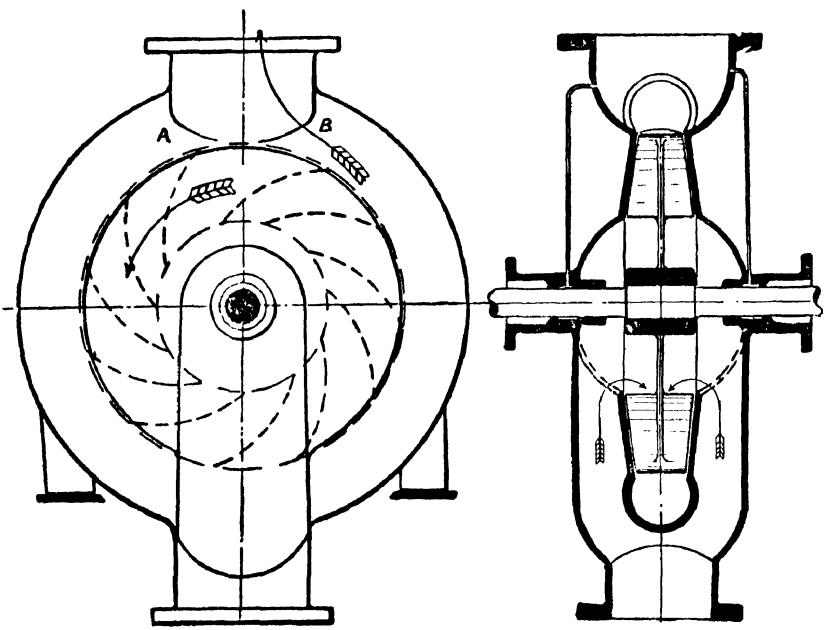


FIG. 317.

discharge from the impeller may be converted into pressure energy before the water enters the delivery pipe line.

Any device having this end in view should be designed so as to reduce the velocity of discharge from the impeller gradually and without shock or eddy formation, to that of flow along the discharge pipe.

In some pumps no attempt is made to do this, and the water is simply allowed to discharge into a small chamber surrounding, and concentric with the wheel, out of which chamber the discharge pipe is led (Fig. 317). Since each of the vanes is continuously discharging, the quantity passing a section of the collecting chamber will increase continuously from a section at *A* just past the discharge pipe, to be a maximum at the section

B just before reaching this pipe. The velocity of whirl in this chamber will thus vary from *A* to *B*, and can only at one particular section correspond with that at discharge. The result is, that practically the whole of the kinetic energy at discharge is dissipated in shock and eddy production.

This may be avoided to a certain extent by designing the chamber so as to have a sectional area which increases uniformly from *A* to *B*, a cutwater being placed as shown at *A* (Fig. 318), so as to ensure the whole flow being at once discharged from the chamber.[1] This is termed a volute chamber and in earlier pumps was usually so designed as to give a uniform velocity of whirl in the chamber, equal to about ·4 times the velocity of whirl on leaving the impeller.

Here again, however, the velocity of whirl on leaving the wheel is always much greater than that of flow in the volute, so that there is still a loss

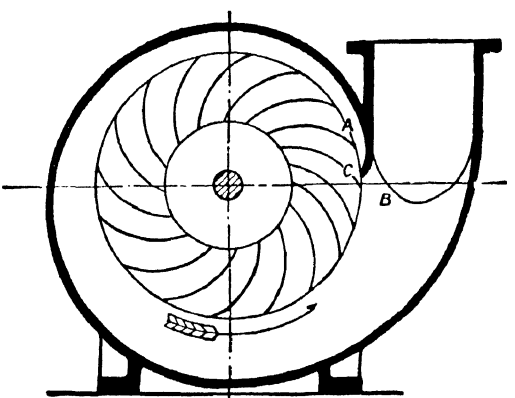


FIG. 318.

by shock due to the impact of the high velocity water leaving the vanes, on the more slowly moving water in the volute chamber. If uncorrectly proportioned the efficiency of the volute chamber is low, in some cases as low as 15 per cent. If correctly designed, however, it is quite a reasonably efficient energy conversion device and modern pumps, especially when working over a range of operating conditions are reverting to this type rather than the guide vane type. For a rational design of such chambers a paper by the Author (*Proc. Inst. Mech. E.*, 1913, p. 519) may be consulted.

[1] This cutwater, if designed so as almost to touch the impeller, tends to cause considerable vibration when the pump is working. Generally the working is found to improve, and the efficiency not to suffer unduly, by the provision of a generous amount of clearance at this point.

Whirlpool Chamber.—In the arrangement of collecting chamber suggested by Professor James Thomson and known as the vortex or whirlpool chamber, the impeller is surrounded by a casing which may be looked upon as a volute chamber of uniformly increasing area superposed upon a circular chamber concentric with and of considerably larger diameter than the wheel. Fig. 319 shows such a vortex chamber. In the concentric portion of the casing, the water on leaving the wheel is free to adopt its own manner of motion which approximates to that of a free vortex. In this vortex the pressure increases outwards, theoretically following the

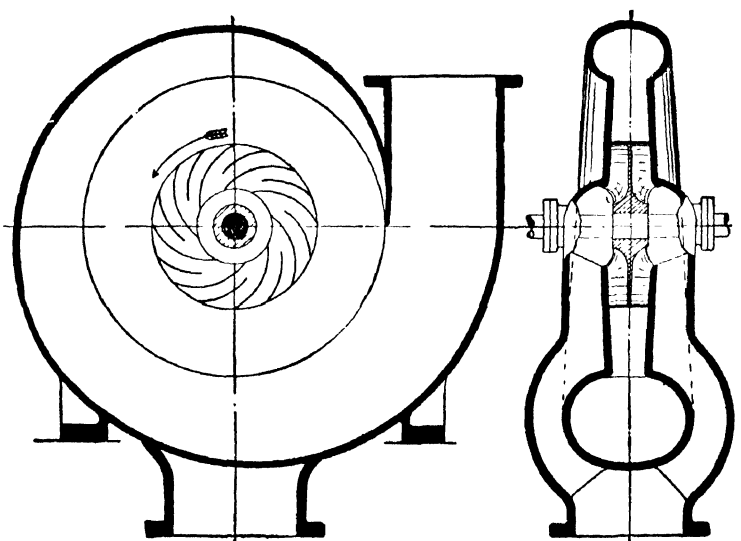


FIG. 319.

ordinary free vortex law. Uniform discharge then takes place around the circumference of the vortex chamber, through the gradually increasing volute passage. The great drawback to this device is that to get a very efficient chamber, the dimensions become excessive—the efficiency increasing with the radius—and, in consequence, it is seldom adopted save in a modified form. In this form it is very general.

Even with this chamber, however, the efficiency of transformation is greatly diminished owing to the instability of diverging motion and the consequent loss of head in eddy formation, and the efficiency actually obtained does not in general exceed about 40 per cent. of the theoretical.

Guide Vanes.—The tendency to instability of motion and the heavy losses due to shock may be largely prevented by the introduction of fixed

guide vanes around the impeller, as shown in Fig. 320,[1] having angles so designed as to receive the water without shock on leaving the wheel, and to direct this by gradually diverging passages, either into a vortex chamber, or directly into the collecting volute from which it is taken by the discharge pipe. In the latter case the pressure change takes place entirely in the guide passages themselves. The angle a which the guide vanes make with the circumference of the discharge circle is calculated exactly as in the case of the inlet vanes of a turbine, and, where the pump is required to

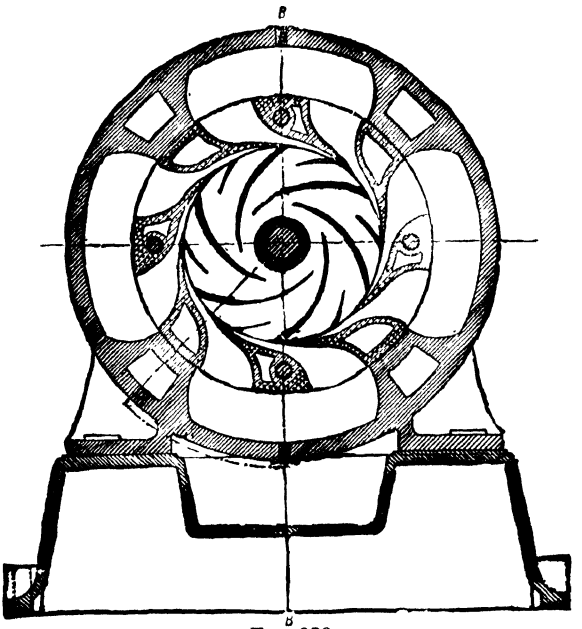


FIG. 320.

work under variable conditions, should be suited to the discharge at which the maximum efficiency is desired. Thus fitted, the pump becomes in every essential a reversed turbine, and is commonly known as a "turbine" pump. Where the guides deliver into a vortex chamber they should be designed so as to follow the curvature of the stream lines in free vortex flow with the discharge, and with the tangential velocity at the entrance to the guides, obtaining under normal conditions of working. The ring of guide vanes in such a pump is known as the *diffuser ring*. Under favourable circumstances such an arrangement is capable of converting up to 75 per cent. of the kinetic energy at discharge into pressure energy.

[1] By courtesy of the Buffalo Steam Pump Company

Where the conditions are very variable, however, the guide—or diffuser ring—may easily prove an actual source of loss by shock rather than of efficiency. Under such circumstances the guides are preferably omitted, the most suitable type of pump being one fitted with a moderate vortex chamber and volute, this both on account of its greater adaptability to varying circumstances, and of its cheaper construction.

Types of Centrifugal Pump.

Centrifugal pumps may be divided into three classes:—

(1) Pumps having a single impeller with open vanes, and discharging directly into a volute casing or vortex chamber.

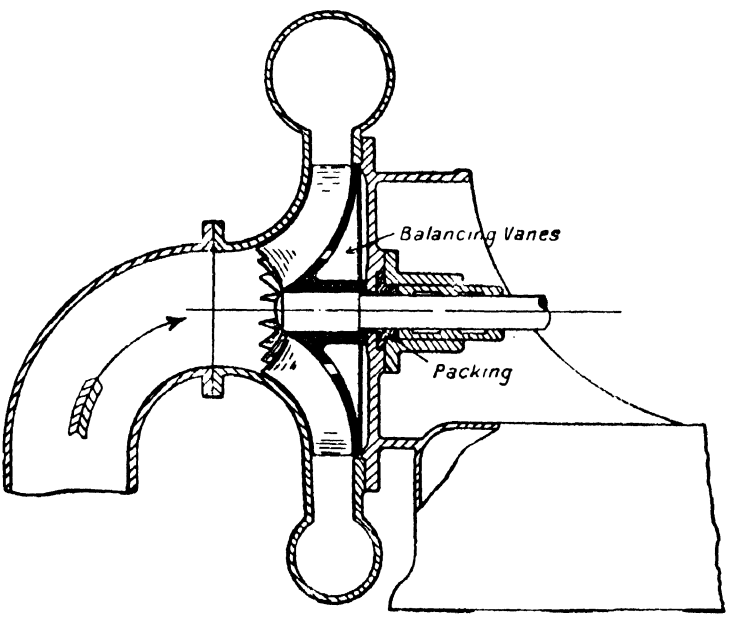


Fig. 321.—Open Vaned Centrifugal Pump with Balancing Vanes.

(2) Pumps having a single impeller with encased vanes, and either discharging as above, or fitted with a diffuser ring between impeller and vortex chamber.

(3) Compound pumps, which are invariably fitted with encased vanes and with diffuser rings with or without the addition of a vortex chamber.

Fig. 321 shows an example of the first type, having an axial inlet on one side only of the impeller. Here the water is deflected into a radial direction by the conical disc on which the vanes are formed.

Fig. 317 shows a pump having vanes open on both sides, with a suction inlet on each side of the wheel. This type has the advantage of being approximately in balance as regards axial thrust on the shaft, while the single-inlet type necessitates provision being made for balancing any such thrust. On the other hand, the single-inlet pump is particularly convenient for situations where a vertical shaft is permissible, and, as denoted in the figure, lends itself to a very compact and simple type of construction.

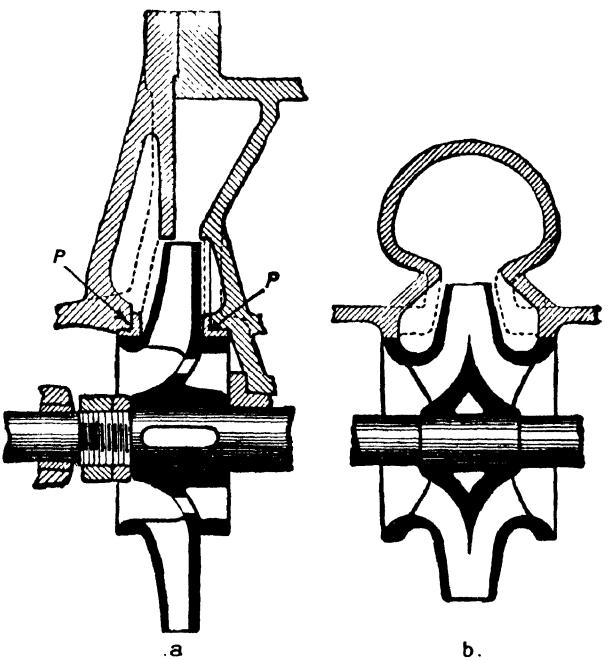


Fig. 322.—Single and Double Inlet Encased Impellers with Vortex Chamber and Volute.

The open-vane pump is, however, subject to considerable and indeterminate loss by slip or leakage of water between the pump casing and impeller blades, and while this may be reduced by making the clearance at these points as small as possible, the presence of gritty matter in suspension in the water causes rapid wear, and the slip may then become excessive. Furthermore, the disc friction accompanying the rotation of such open vanes over the surface of the casing is considerably greater than when the vanes are enclosed at the sides by discs or shroudings, so that on every count the latter encased type of pump is preferable. Such a pump, Fig. 322 *a* and *b*, may have either single or double suction inlet, and

will thus require balancing for end thrust or not, just as does the open vaned pump. The possibilities in this direction are somewhat greater than with the latter type.

It is very suitable for heads between 30 and 80 feet, but must, for high efficiency under high heads, be fitted with guide vanes outside the impeller ring. As thus constructed, efficiencies up to about 80 per cent. may be obtained. Free circulation of water behind the impeller and into the suction space is usually prevented by the introduction of a brass packing ring at P (Fig. 322).

Where the working head exceeds about 100 feet, the single impeller

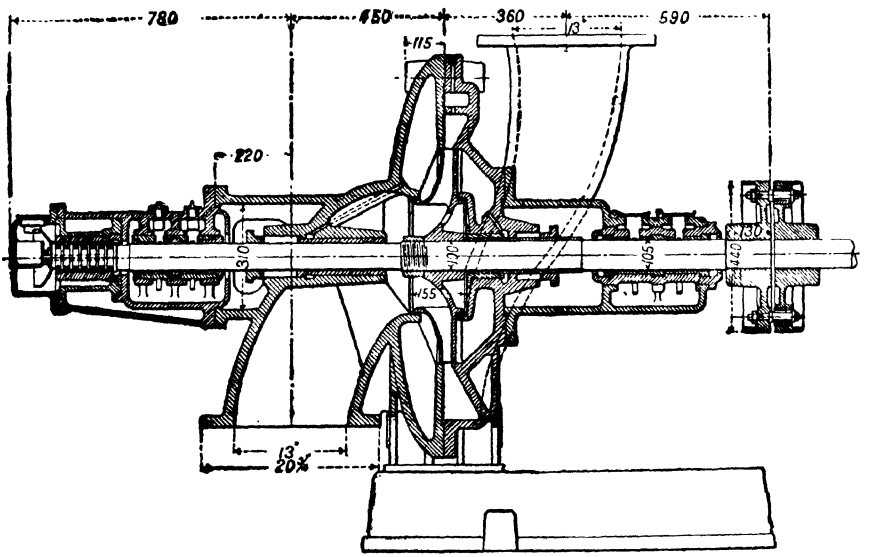


FIG. 323.—Worthington Single Impeller High Lift Pump.

pump as usually constructed falls off rapidly in efficiency owing to the necessary high speed of rotation and the consequent excessive frictional and eddy losses. It may be compounded, and so made suitable for such work by mounting a number of impellers in series in separate chambers and on the same shaft, each taking the discharge from its predecessor and raising it through a fraction of the whole head depending on the number of impellers used. For this to be done with fair efficiency, it is essential that as far as possible the kinetic energy of discharge from each wheel be converted into pressure energy before entering the next chamber, and this renders the use of efficient volute chambers or of guide or diffuser vanes on the discharge side essential.

Where the water to be pumped contains grit, as is usual for example in colliery workings, the leading edges of such vanes are apt to be badly worn or bent from their original position, in which case the efficiency of working is seriously affected. To obviate this difficulty a design of single impeller high-lift pump, in which these multiple vanes are absent, has been evolved

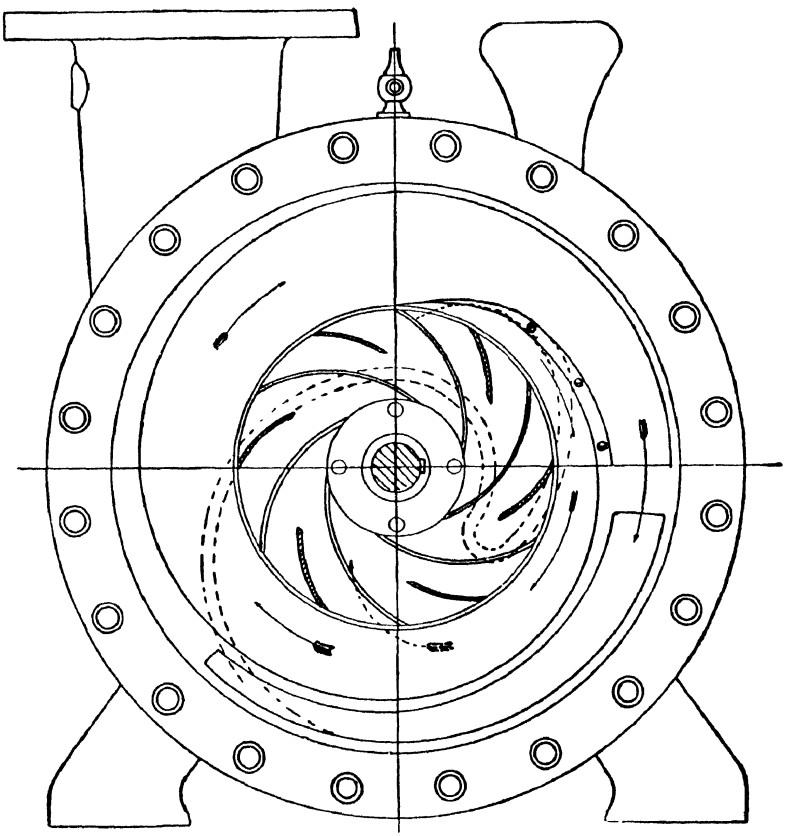


FIG. 324.—Worthington Single Impeller High Lift Pump.

by the Worthington Pump Co., and is illustrated in Figs. 323 and 324. In this pump, designed to deliver 2,500 gallons per minute at 1,170 revolutions per minute against 300 feet head, the combined efficiency of pump and motor attained, on test, a maximum value of 73·8 per cent. the corresponding pump efficiency being approximately 80 per cent.[1]

[1] *Engineering*, Feb. 5, 1909. By courtesy of Messrs. The Worthington Pump Co.

Some modern types of construction of the compound high-lift pump are indicated in Figs. 325 to 333.

Fig. 325 shows the construction adopted by Messrs. Sulzer Bros. Here the impellers are mounted in pairs back to back, the flow through these

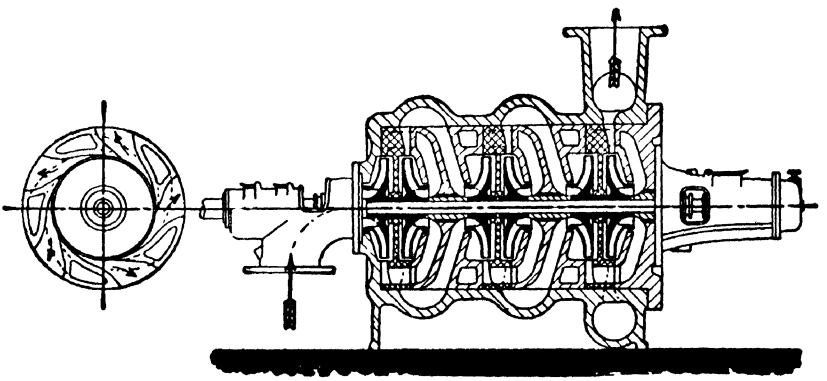


FIG. 325.—Sulzer Sextuple Compound High Lift Pump.

being in opposite directions. By this construction each pair of impellers is in balance as regards end thrust. The pump is fitted with diverging guide passages, curved vanes, and a vortex chamber, while Fig. 326 shows the type of water-sealed stuffing box and water-cooled bearing adopted by the makers.

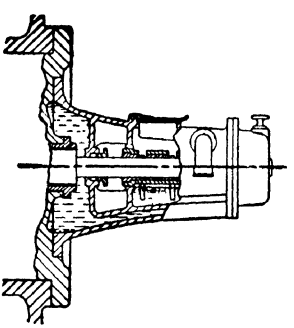


FIG. 326.—Water-Sealed Stuffing-box for Sulzer Pump.

The Buffalo high-lift pump (Figs. 327 and 328) is built on somewhat similar lines also having impellers mounted in pairs, while in the Mather-Reynolds pump (Fig. 329), constructed with a double inlet to each propeller, the guide passages and curved vanes are retained, while the vortex chamber is omitted. The construction is thus simplified without seriously affecting the efficiency, a quadruple pump of this type giving efficiencies up to 75 per cent. when delivering 1,000 gallons per minute against 320 feet head.

The foregoing arrangement of impeller, however, necessitates complicated and tortuous connecting passages, and a simpler construction is obtained where, as shown in Figs. 330 to 333, impellers, each having a single inlet, are used. This system has the further advantage that any odd number of impellers may be used. On the other hand, with the single

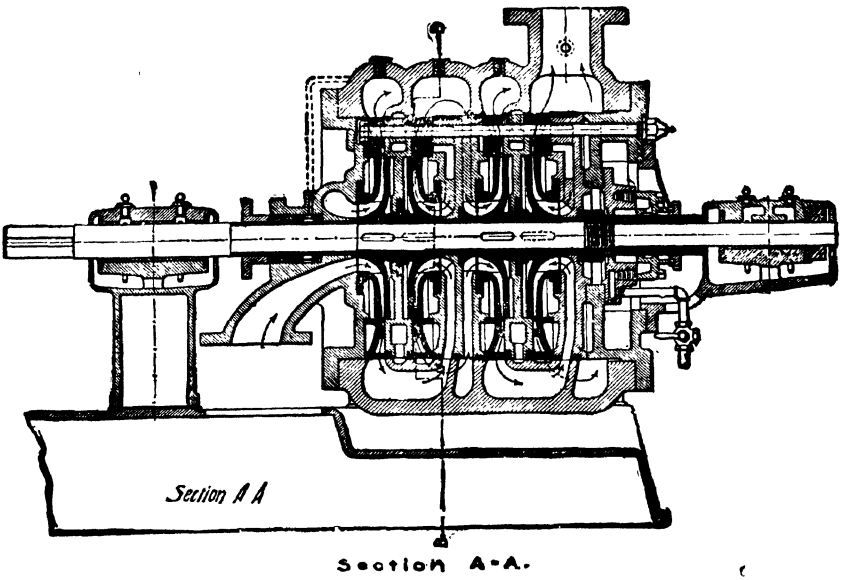


FIG. 327. -Sectional Elevation of Quadruple Compound Buffalo High-Lift Pump.

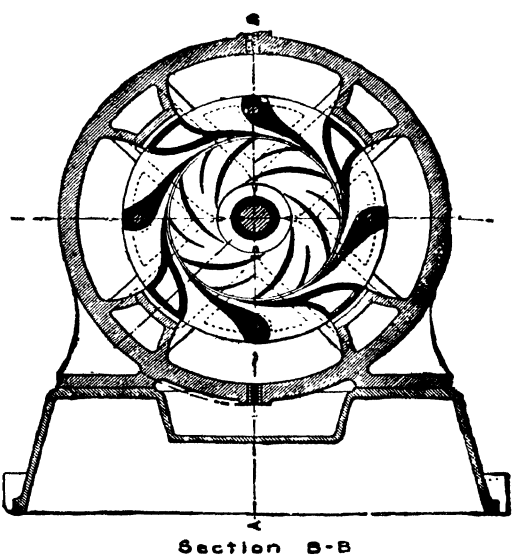


FIG. 328.—Sectional End Elevation of Buffalo High-Lift Pump.

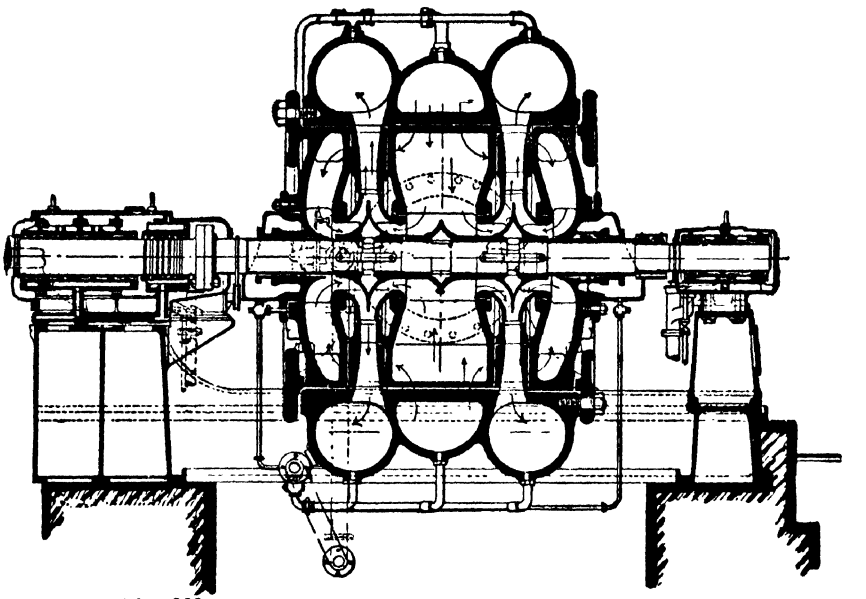


FIG. 329.—Mather-Reynolds High-Lift Pump with Double Inlet.

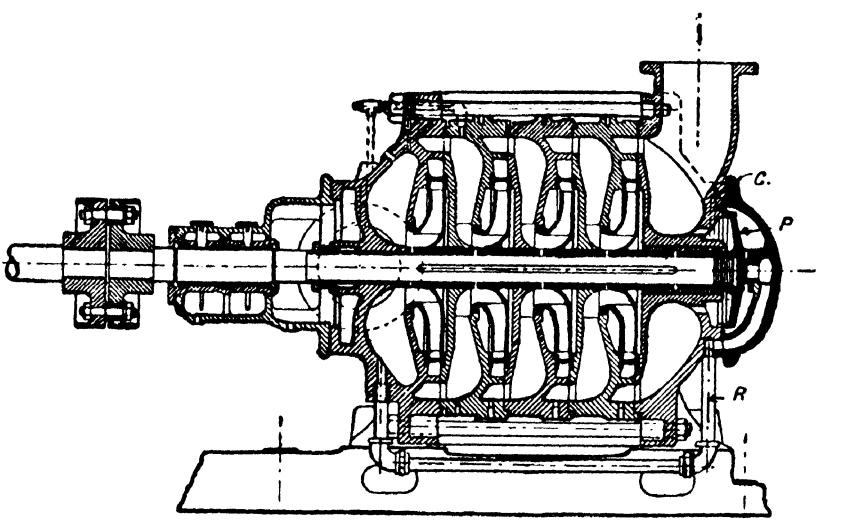


FIG. 330.—Mather-Reynolds Quadruple Turbine Pump.

inlet, end thrust becomes serious and special devices must be adopted to overcome this difficulty. These will be considered in detail in Art. 209.

With any of these types of pump, efficiencies of between 72 and 80 per

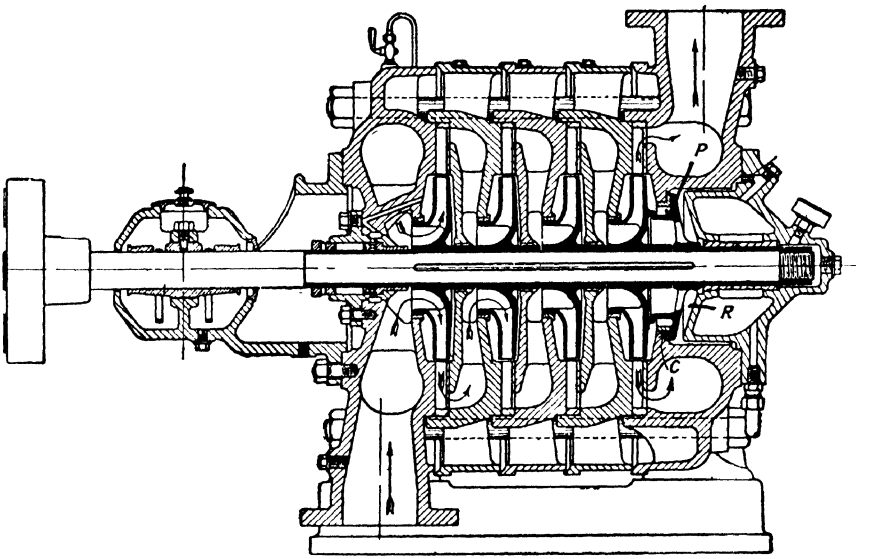


FIG. 331.—Holden and Brooke Quadruple High-Lift Pump.

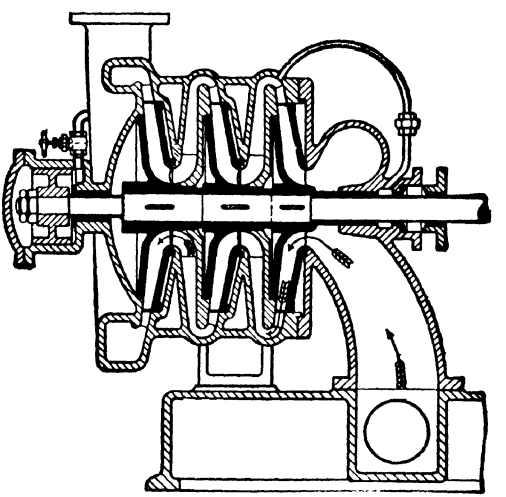


FIG. 332.—Rateau Triple High-Lift Pump.

cent. may be obtained, while they are all capable of being constructed so as to deal with heads up to about 600 feet, the head in each chamber usually varying from 60 to 100 feet.

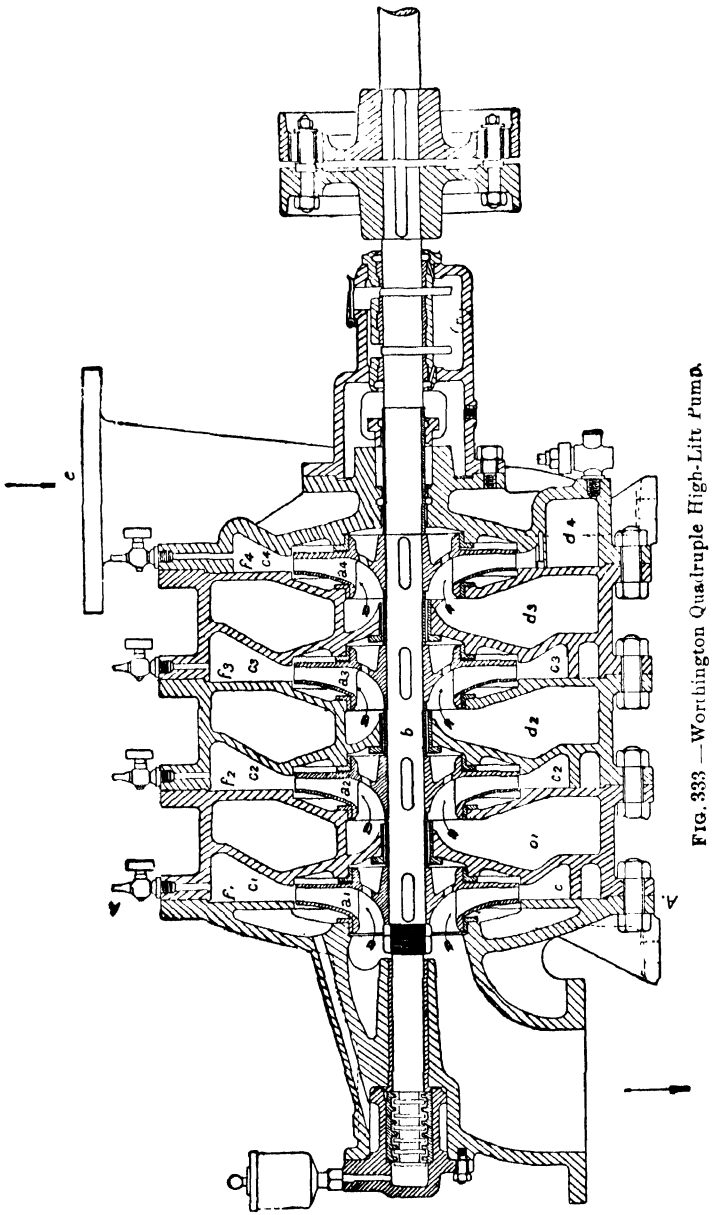


FIG. 333.—Worthington Quadruple High-Lift Pump.

The maximum speed of the impeller is limited by the fact that at extreme speeds cavitation is set up, and the pump will not fill itself.

Leakage from stage to stage of a high-lift pump is prevented by brass packing rings surrounding the shaft between each pair of chambers, while to prevent corrosion of the pump spindle this is usually protected by a brass sleeve.

Admission of air on the suction side of such a pump is to be guarded against with the greatest care, as being productive of inefficiency and of considerable shock in the pump casing and delivery pipes.

Art. 200.—Suction and Delivery Pipes.

The area of the suction pipe is commonly made equal to the discharge area of the impeller. A foot valve and strainer should always be fitted to this pipe, the area through the valve being not less than half that of the pipe. No part of the pipe should be above the pump inlet in order that the formation of air pockets may be prevented, while when the pump is circulating water through a pipe line forming a syphon, an ejector or air pump should be fitted to the highest point of the pipe, for priming the pump and for the removal of any accumulation of air.

The suction lift should be as small and as direct as practicable, and should never, if possible, exceed 20 feet. Where hot liquids are to be handled, the vapour pressure reduces the possible suction lift, the effect of temperature in the case of water being as follows:—

Temperature of water, degrees Fahr. .	60°	100°	140°	180°	212°
Maximum theoretical suction lift . .	33·0 ft.	31·4 ft.	26·7 ft.	16·7 ft.	0

Practically the limit is reached much before this because of the liberation of air when the pressure is reduced nearly to that corresponding to the saturation temperature, and because of the increasing importance of slight air leakage at joints, and at the stuffing boxes of the pump spindle at these low pressures. In no case should the suction lift exceed two-thirds of the values given above, while where hot liquids are to be lifted it is advisable to place the pump below the supply level.

The above remarks may be taken as applying with equal force to the reciprocating pump.

When parallel, the area of the discharge pipe should, in general, be not less than that of the suction pipe, and may with advantage exceed this area where the working head is high. It is, however, very advantageous to have a main discharge pipe of greater area than the

outlet from the collecting chamber of the pump, the connecting pipe having a gradual divergence. By this means a partial conversion of kinetic into pressure energy is obtained in this connecting pipe.

With heads exceeding about 100 feet, a non-return valve should be fitted on the delivery side, while when delivering into a rising main of great length an air vessel should be fitted on the same side of the pump to avoid shock on starting and stopping.

This is specially advisable with a belt-driven pump, where the vibration of the belting may produce variations of considerable magnitude in the angular velocity of the shaft under normal working conditions. In a belt-driven quadruple high-lift pump under the author's observation, making 1,400 revolutions per minute and discharging through 600 feet of 4-inch piping, against a total head of 143 feet, the pressure in the last collecting chamber varied from 122 to 164 feet. One successful combination of non-return valve and air vessel is illustrated in Fig. 334.

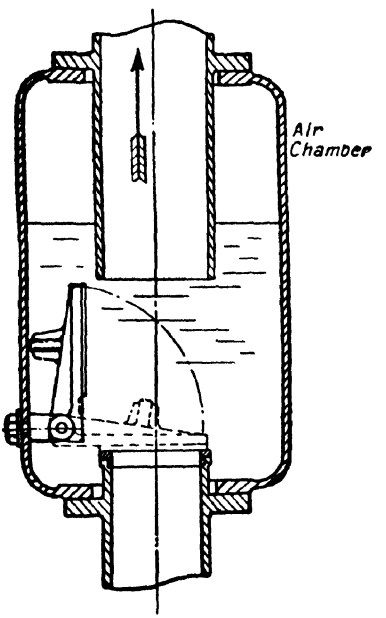


FIG. 334.—Delivery Valve with Air Vessel for High-Lift Centrifugal Pump.

Self-charging Device.—Since a centrifugal pump will not begin to lift unless first charged with water, if fixed above supply water level provision must be made for priming or flooding the pump casing from an auxiliary pressure supply or for withdrawing the air by means of an ejector or auxiliary air pump.

ART. 201.—GENERAL THEORY OF CENTRIFUGAL PUMP.

In the following discussion of the theoretical considerations governing the design of the centrifugal pump, the symbols used are the same and have the same meaning as in the case of the turbine (p. 502).

Thus the suffix (1) refers to the water just before entering the impeller.
(2) refers to the water just after entering the impeller.
(3) refers to the water discharging from the impeller.

while u_2 = peripheral velocity of impeller at entrance.
u_3 = peripheral velocity of impeller at exit.
β = vane angle at entrance.
γ = vane angle at exit. (Fig. 335.)

In this discussion it is assumed throughout that the pump runs full at all speeds within its working limits, the theory ceasing to apply if any action of the nature of cavitation take place. The further assumptions are made that each particle of water, immediately before entering the wheel, is moving radially, so that its initial velocity of whirl w_2 is zero, and also that all particles of water on leaving the impeller have the same velocity and are moving in directions which make the same angle with the tangent to the periphery at the point of discharge.

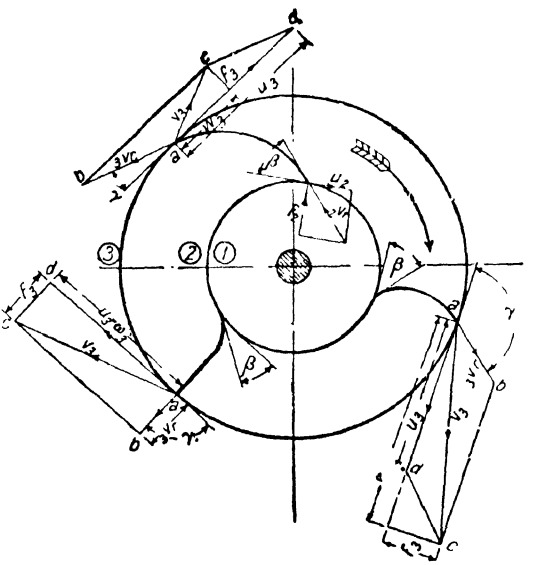


FIG. 335.—Velocity Diagram for Vanes of a Centrifugal Pump.

Form of Vanes.—Just as in the case of the turbine all shock at entrance to the vanes is to be avoided, and, assuming radial flow at the entrance, this gives as a necessary condition for entry without shock. (Fig. 335.)

$$f_2 = u_2 \tan \beta.$$

The relative velocity of water and vane at entrance is then given by

$$_2v_r = f_2 \operatorname{cosec} \beta = \sqrt{f_2^2 + u_2^2}.$$

If the angle β does not satisfy the above condition, there will be loss by shock at entry. The magnitude of this may be approximately calculated, for the relative velocity of water and vane in the direction of rotation before entering the wheel is u_2, while the relative velocity in the same direction after entry is $f_2 \cot \beta$. The loss of head due to this change in relative velocity is then approximately equal to

$$\frac{(u_2 - f_2 \cot \beta)^2}{2g} \text{ feet.}$$

The greatest source of loss in the pump, as compared with the turbine

is, however, due to rejection of kinetic energy in the discharge from the impeller, for while this is comparatively unimportant in the turbine, it becomes of the greatest importance in the pump, since it is when leaving the wheel that the water is moving with its maximum velocity. At the best, only a portion of this energy of discharge can be recovered by means of a vortex chamber or guide vanes, and it is therefore advisable to reduce the velocity of discharge to as low a point as is compatible with efficiency in other directions. This may be done by curving the vanes backward at their tips, so as to make the discharge angle γ (Fig. 336) less than 90°. The relative velocity of water and vane at discharge then has a tangential component in the opposite direction to that of the wheel's rotation, this component increasing as γ is reduced, the result being that the absolute velocity of discharge is reduced. Thus, for example, in Fig. 336 the triangles of velocity are drawn for the cases in which the vanes are respectively radial, curved forward at exit, and curved backward at exit. In each case the same value has been adopted for f_3, the velocity of flow at exit, and for u_3 the peripheral velocity of the vanes, while $\overline{a\ c}$ represents the direction and the absolute velocity of discharge.

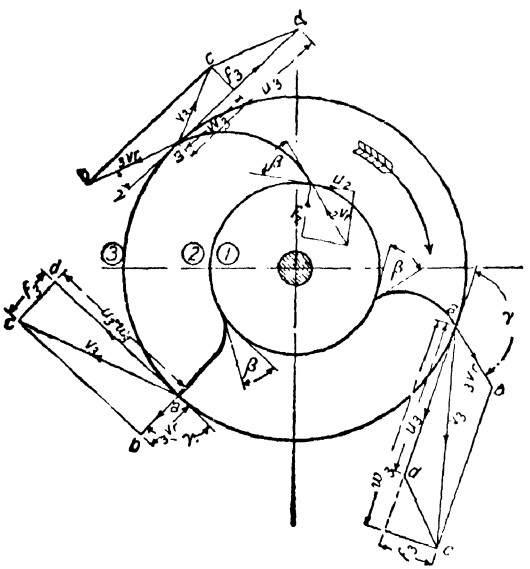


FIG. 336.—Velocity Diagram for Vanes of a Centrifugal Pump.

A comparison of the diagrams will indicate how this diminishes as the angle of backward inclination of the vane tips increases.

As will be seen later, however, the necessary peripheral speed of the wheel for pumping against a given head increases as γ is reduced, and this causes the frictional losses, which vary approximately as ω^3, to increase.

This backward curvature of the vanes offers a further advantage in that it gives passages of more uniform cross section. This reduces the diver-

gence, and thus the tendency to instability of flow, which is one of the important factors in reducing the efficiency of the pump as compared with the inward flow turbine. Since the relative velocity of flow increases outwards, the passages would in all probability be most effective if designed so as to be convergent outwards to suit this increased relative velocity. This view is borne out by the results of a series of experiments carried out by Mr. J. A. Smith, of Melbourne,[1] in which instantaneous photographs of the flow through the impeller of a pump model while freely discharging into the atmosphere, indicate that at a certain velocity the water tends to leave the leading face of a passage as indicated in Fig. 337, which is reproduced from his paper. If discharging under pressure it is evident that the empty space shown in these passages would be occupied by dead water and would be the source of considerable loss in eddy production. The successive curves 1, 2, and 3, mark the boundaries of the stream with increasing velocities.

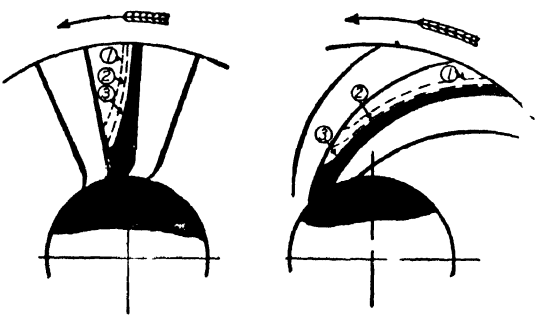


FIG. 337.

In practice, the purpose for which the pump is designed determines the value of γ, which may have any value from 15° to 90°, so that in general the water on leaving the vanes has a comparatively high absolute velocity.

As is apparent from Fig. 336, at discharge,

$$f_3 \cot \gamma = u_3 - w_3,$$

while the relative velocity of water and vane is given by

$${}_3v_r = f_3 \operatorname{cosec} \gamma.$$

Work done on Pump.—The turning moment on the shaft, equivalent to the change per second in the angular momentum of the water passing through the wheel

$$= \frac{W}{g} Q \{ w_3 r_3 - w_2 r_2 \} \text{ foot lbs.}$$

$$\therefore \text{ Work done on water per second} = \frac{W}{g} Q \{ w_3 r_3 - w_2 r_2 \} \omega \text{ foot lbs.}$$

[1] See *Engineering*, December 5, 1902. This view is also borne out by the results of experiments by C. B. Stewart, Bulletin of University of Wisconsin. No. 173, 1907.

And since the initial tangential velocity of the water, before coming under the influence of the impeller is zero, this becomes

$$U = \frac{W\,Q}{g} \cdot w_3\, r_3\, \omega = \frac{W\,Q}{g}\, w_3\, u_3 \text{ foot lbs. per second.}$$

$$= \frac{w_3\, u_3}{g} \text{ foot lbs. per lb.}$$

This is quite independent of all losses in eddy formation, shock, and friction in the wheel passages, and if to it be added the work done against the friction of the impeller on the surrounding water, against the mechanical friction of the bearings, and that absorbed in pumping water which may leak from the discharge side to the suction side of the impeller, the result will give the power required to drive the pump. The work done against these resistances may be experimentally obtained by driving the pump at the required speed of rotation with the discharge valve closed so as not to deliver any water, and by noting the power then absorbed. This will, however, be somewhat high owing to the fact that leakage round the impeller will be greater than under normal running conditions, and also that power will be absorbed, owing to the viscosity of the water, in maintaining a rotation of the water in the eye of the pump and in the collecting chamber surrounding the wheel. Also, in all probability, in maintaining such a series of currents as indicated in Fig. 338.

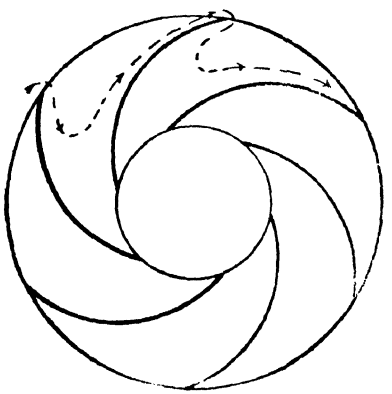


FIG. 338.

Energy obtained from Pump.—The useful work done by a pump is the product of the weight of water handled and the height through which this would be lifted provided there were no losses of head in suction and delivery pipes.

If H is the difference of level between suction and discharge reservoirs, H_f the friction loss in suction and delivery pipes, and $\bar{v}$ the velocity of flow along the discharge pipe, the energy obtained from the pump per lb. of water $= H + H_f + \frac{\bar{v}^2}{2\,g}$ foot lbs.

If pressure gauge or manometer readings be taken on the suction and delivery pipes at the same level immediately before and behind the pump

the suction reading will be $-\left(H_s + H_{fs} + \frac{\bar{v}_s^2}{2g}\right)$ feet, and the delivery reading will be $H_d + H_{fd} - \frac{\bar{v}_d^2}{2g} + \frac{\bar{v}^2}{2g}$ if the section of the pipe is increased after passing the manometer, and will be $H_d + H_{fd}$ if it remains of the same cross section. In either case if $\bar{v}_s = \bar{v}_d$ the "manometric head" will be given by $H + H_f + \frac{\bar{v}^2}{2g}$ foot $= H''$.

It is worth noting that this is greater than the head H' obtained by adding the dead lift H and the friction head H_f. In the average pump $\bar{v}$ may range up to 8 feet per second, so that in a low lift pump the term $\frac{\bar{v}^2}{2g}$ should not be neglected.

Efficiency of Pump.—Manometric Efficiency.—Neglecting losses due to disc friction, mechanical friction, and slip, we have seen that the work done on the water per lb. is $\frac{w_3 u_3}{g}$ foot lbs. The ratio of the energy H'' obtained from the pump per lb. of water, to $\frac{w_3 u_3}{g}$ is sometimes termed the **hydraulic efficiency**, but more correctly the **theoretical manometric efficiency** η'.

$$\therefore \quad \eta' = \frac{g H''}{u_3 w_3} = \frac{g H''}{u_3 (u_3 - f_3 \cot \gamma)}$$

Hydraulic Efficiency.—The work actually done on the water per lb. is given by $\frac{u_3 w_3}{g} + \frac{L_h}{W Q}$, where L_h represents the total loss of energy in foot lbs. expended in overcoming hydraulic resistances in the pump itself, and the ratio

$$\eta_h = \frac{H''}{\frac{w_3 u_3}{g} + \frac{L_h}{W Q}}$$

is more correctly termed the hydraulic efficiency.

Actual Working Efficiency.—The actual working efficiency is the ratio of the energy obtained from the pump to the work done on the pump shaft per lb. of water. If N be the number of revolutions per second; Q the volume per second in cubic feet; and T the turning moment on the shaft in feet and lbs.,

$$\eta = \frac{Q W H''}{2 \pi N T} = \frac{H''}{\frac{w_3 u_3}{g} + \frac{L_h + L_m}{W Q}}$$

where L_h and L_m are respectively the hydraulic and mechanical losses in

the pump. From this result it appears that as the discharge Q is diminished the actual efficiency diminishes more rapidly than the manometric efficiency $\frac{g\,H''}{w_3\,u_3}$ which indeed has a value usually about ·5 when the discharge Q and therefore the actual efficiency is zero. Under normal conditions as to head, speed, and discharge, the calculated manometric efficiency is, however, not widely different from the true working efficiency, the ratio of the latter to the former ranging from about ·85 in a pump with recurved vanes and an inefficient collecting chamber, to about 1·0 in a pump with radial vanes and an efficient vortex chamber or diffuser, so that a knowledge of the probable manometric efficiency guided by a knowledge of the performance of somewhat similar pumps, enables the working efficiency to be predetermined with a fair degree of accuracy.

Change of Pressure in Passing through Pump.—As will be clear from what has already been said, the increase in pressure during the passage of the water through the pump must be such as to balance the statical head together with the head necessary to overcome frictional resistances and that equivalent to the kinetic energy of flow along the suction or discharge pipes.

Where the water, on leaving the wheel, is allowed to make the best of its way to the discharge pipe without the provision of a volute, vortex chamber, or guide vanes, the K.E. of discharge is entirely dissipated in shock, and the full pressure change takes place in the impeller. Where provision is made for gradually reducing the velocity of the discharging water by one of these devices, a further increase in pressure takes place after leaving the impeller but before leaving the pump casing, while if a diverging discharge pipe is used, as is often the case a further increase in pressure takes place in this pipe.

The magnitude of these changes in pressure will now be considered.

(*a*) **Change of Pressure in Passing through the Wheel.**—The absolute velocity of a particle of water at any point in the wheel may be resolved into two components, one of whirl with the wheel with a velocity $\omega\, r$, and the second of flow parallel to the vanes with relative velocity v_r. This latter velocity is evidently that which the water would have if the same volume were passing with the wheel at rest. The total difference of pressure at any two different radii is thus compounded of the differences due to—

(1) Rotation in a forced vortex with angular velocity ω.

(2) Outward flow parallel to the vanes with velocity v_r.

In the forced vortex we have, considering the points (2) and (3):—

$$\frac{p'_2}{W} - \frac{\omega^2 r_2^2}{2g} = \frac{p'_3}{W} - \frac{\omega^2 r_3^2}{2g}$$

$$\therefore \quad \frac{p'_3 - p'_2}{W} = \frac{\omega^2 (r_3^2 - r_2^2)}{2g} = \frac{u_3^2 - u_2^2}{2g}; \qquad (1)$$

while due to relative outward flow we have—

$$\frac{p''_2}{W} + \frac{{}_2v_r^2}{2g} = \frac{p''_3}{W} + \frac{{}_3v_r^2}{2g}$$

$$\therefore \quad \frac{p''_3 - p''_2}{W} = \frac{{}_2v_r^2 - {}_3v_r^2}{2g}$$

$$= \frac{f_2^2 + u_2^2 - f_3^2 \operatorname{cosec}^2\gamma}{2g}. \qquad (2)$$

Summing these and writing $p_3 = p'_3 + p''_3$, etc., we get the total difference in pressure between the inlet and discharge edges of the vanes,

i.e.,
$$\frac{p_3 - p_2}{W} = \frac{u_3^2 + f_2^2 - f_3^2 \operatorname{cosec}^2\gamma}{2g}. \qquad (3)$$

EXAMPLE.

A pump, 1 foot diameter at inlet, 2 feet diameter at outlet. 6 inches broad at inlet, 4 inches at outlet, discharges 5·0 cubic feet per second, when making 200 revolutions per minute. Determine the rise in pressure in passing through the wheel, and hence, neglecting all frictional losses, the head pumped against. Assume $\gamma = 25°$ and neglect the effect of the vane thickness.

Here $Q = f_2 \times \pi \times \frac{1}{2} = f_3 \times 2\pi \times \frac{1}{3}$

$$\therefore \quad f_2 = \frac{10}{\pi} = 3{\cdot}18 \text{ f.s.} \qquad f_3 = \frac{15}{2\pi} = 2{\cdot}386 \text{ f.s.}$$

Again, $u_3 = \dfrac{2\pi \times 1 \times 200}{60} = 20{\cdot}96$ f.s.

and $\operatorname{cosec}^2\gamma = 5{\cdot}6$

$$\therefore \quad \frac{p_2 - p_3}{W} = \frac{(20{\cdot}96)^2 + (3{\cdot}18)^2 - (2{\cdot}386)^2 \times 5{\cdot}6}{64{\cdot}4} = 6{\cdot}48 \text{ feet.}$$

$\therefore$ rise in pressure, $p_2 - p_3 = 6{\cdot}48 \times 62{\cdot}4 = 404$ lbs. per square foot.

Head pumped against $= 6{\cdot}48$ feet.

(*b*) **Change of Pressure in Volute Chamber.**—If $\bar{v}$ be the velocity of whirl in the volute and if it be assumed that the loss of head at impact of the high velocity stream leaving the impeller is $\dfrac{(v_3 - \bar{v})^2}{2g}$ feet, the theoretical gain of pressure is

$$\frac{v_3^2 - \bar{v}^2 - (v_3 - \bar{v})^2}{2g} = \frac{v_3\bar{v} - \bar{v}^2}{g}.$$

Differentiating and equating to zero, shows that the maximum gain occurs

when $\bar{v} = \frac{1}{2} v^3$ and then equals $\frac{1}{2} \cdot \frac{v_3^2}{2g}$ so that the theoretical maximum efficiency is 50 per cent.

(*c*) **Volute with Diverging Discharge Pipe.**—The theoretical gain is now

$$\frac{v_3^2 - \bar{v}^2 - (v_3 - \bar{v})^2 + K\bar{v}^2}{2g}$$

where $\frac{K\bar{v}^2}{2g}$ is the gain due to conversion of kinetic energy in the discharge pipe.

The above expression reduces to

$$\frac{2 v_3 \bar{v} - \bar{v}^2 (2 - K)}{2g}$$

which is a maximum when $(2 - K)\bar{v} = v_3$. The value of K depends on the angle of divergence of the walls and on the ratio of areas of the volute exit and the delivery main. When these are known its value may be determined from the data of Art. 45. If for example $K = \cdot 75$, $\bar{v} = \frac{4}{5} v_3$ and the gain becomes $\frac{4}{5} \frac{v_3^2}{2g}$. If $K = \cdot 66$ the gain becomes $\frac{3}{4} \frac{v_3^2}{2g}$.

With such a device a smaller volute chamber is required and the possibilities of increased efficiencies are great.

(*d*) **Change of Pressure in Vortex Chamber.**—Here the water on leaving the vanes forms approximately a free vortex; the pressure increases as the velocity diminishes towards the outside of the chamber, and part of the kinetic energy of discharge may thus be recovered as pressure energy.

If the suffix (3′) now refers to the outside of the vortex, we have, neglecting changes of level between (3) and (3′):—

$$\frac{p_{3'} - p_3}{W} = \frac{v_3^2 - v_{3'}^2}{2g}, \qquad (5)$$

giving the gain of pressure head in the chamber on the assumption of true vortex motion, with no eddy losses.

But if r_3 and r'_3 are the inner and outer radii of the chamber we have $\frac{v_3'}{v_3} = \frac{r_3}{r_{3'}} = c$ (say)

$$\therefore \quad \frac{p_{3'} - p_3}{W} = \frac{v_3^2 (1 - c^2)}{2g} = \frac{(w_3^2 + f_3^2)(1 - c^2)}{2g} \text{ feet of water.} \qquad (6)$$

The theoretical efficiency of the vortex chamber may be taken as $(1 - c^2)$, the following table indicating how this varies with an increase in the radius of the chamber.

Value of $\frac{r_{3'}}{r_3}$	1·00	1·25	1·50	1·75	2·00	2·50	3·00
Theoretical efficiency of vortex chamber	0	·36	·556	·673	·750	·840	·889

Experiments by Stanton[1] on a pump having a vortex chamber 18 inches in diameter, showed a chamber efficiency of about 39 per cent. with either a 7-inch or an 11-inch wheel.

The actual gain in pressure is thus considerably less than that obtained in the ideal case, and is given more nearly by—

$$\frac{K\,(w_3^2 + f_3^2)\,(1 - c^2)}{2\,g} \text{ feet} \qquad (7)$$

where K varies probably from ·4 to ·55, depending on the form of the vortex chamber.

(*e*) **Change of Pressure in Guide Vanes.**—In Stanton's experiments on a pump fitted with guide vanes and radial impeller vanes, the guide efficiency varied from 59 per cent. to 70 per cent., while with back curved vanes this efficiency varied from 47 per cent. to 52 per cent. With a third wheel of 12 inches diameter having curved vanes and working under a head of 65 feet, the guide passages had an efficiency of 75 per cent.

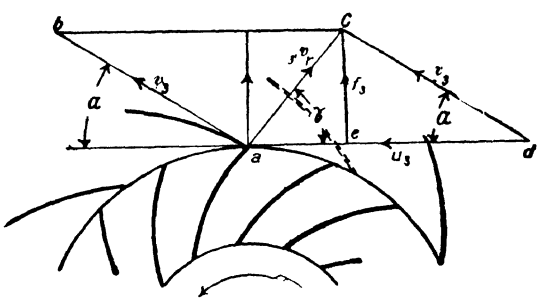


FIG. 339.

In each case, with curved vanes, the vane angle at inlet was 15°, at outlet 30°, guide angle $a = 8°$. Vane thickness ·05 inch. Four guides were fitted, and the number of impeller vanes was varied from twelve to twenty-two without any appreciable effect on the efficiency.

An examination of Fig. 339 shows that the correct value of a is given by—

$$\tan a = \frac{c\ e}{e\ d} = \frac{c\ e}{a\ d - a\ e}$$

$$= \frac{f_3}{u_3 - f_3 \cot \gamma}.$$

Since the guide passages are rectangular in section the guide vanes should diverge at an angle of about 11° (Art. 45) for maximum efficiency. Under these conditions, with a designed so as to prevent shock at entry to the guide vanes as far as possible, about 75 per cent. of the kinetic energy of discharge is converted into pressure energy.

The total gain of pressure head in the pump is then equal to:

$$\frac{k\,v_3^2 + u_3^2 + f_2^2 - f_3^2 \operatorname{cosec}^2 \gamma}{2\,g} \text{ feet}$$

where k has a maximum value of about ·75.

The general effect of the curvature, length, and dimensions of entrance of the guide passages, and of the curvature and angle of delivery of the return passages to the eye of the impeller, are shown in the curves of Fig. 340.[1]

Head and efficiency curves are given for four different designs of guide and return passages, the same impeller of standard shrouded type being

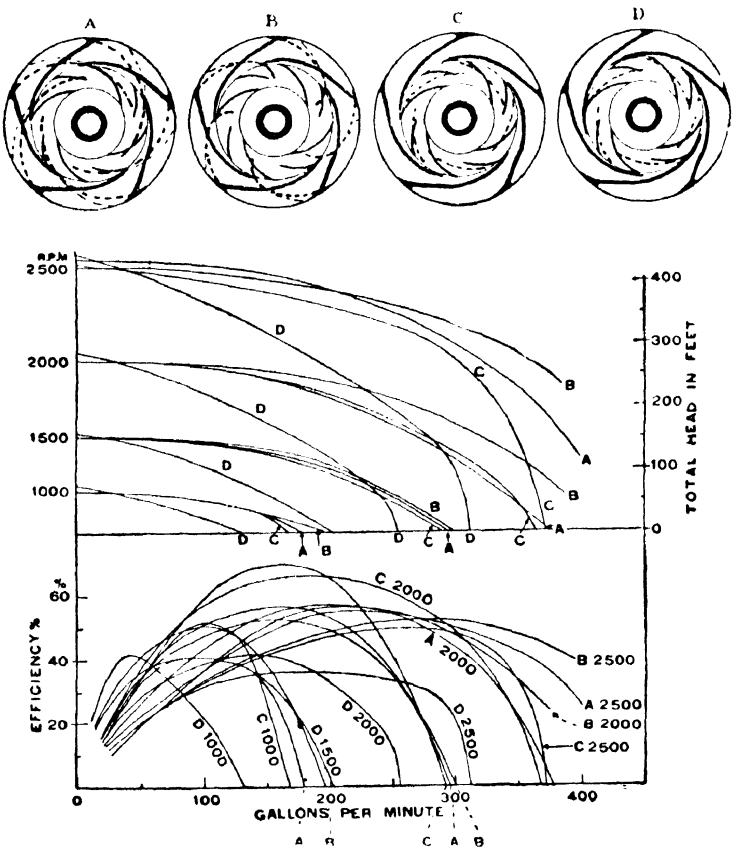


FIG. 340.—Effect of Guide and Return Passages of Different Form.

used in each case. Design *A* has short guide vanes with wide opening on the radial line. The return passages are of similar curvature. Design *B* has similar guide vanes, but the return passages have a short curve becoming radial near the eye of the impeller. Design *C* also has similar guide vanes and the return passages are similar in curvature

[1] Proc. Inst. Mech. Eng., 1912, No. 1, p. 18.

to those of A, but are not extended to the outer periphery of the return chamber. In design D the opening on the radial line is only one-half that in A, B, and C. The return passages are similar to those in C. The head and efficiency curves are given for speeds of 1,000, 1,500 and 2,000 revolutions per minute, and show that design D is much inferior due to throttling at the throat of the guide passages. B is superior to both A and C in generation of head, and superior to A in efficiency especially at low speeds, but is inferior to C in efficiency except at high speeds. At these speeds great loss of energy results on the eddying which takes place in the collecting chamber of C which is prevented by the guides forming the continuous passages of design B. Design B is most usually adopted as best meeting average conditions. Design C, which approaches more nearly to the conditions of a single chamber pump, gives considerably higher maximum efficiency at speeds of 1,500 and 2,000 revs., due to less power being absorbed by friction against the walls of the guide passages, and also to tangential motion imparted to the water as it enters the subsequent impeller by the tangential curvature of the return passages.

Art. 202.—Manometric Efficiency under different Working Conditions.

If the sum of the dead lift, H, and of the friction loss H_f in suction and delivery pipes be called H', the total gain of pressure in the pump must be $H' + \frac{\bar{v}^2}{2g}$ feet, where $\bar{v}$ is the velocity of flow along the discharge pipe. Thus, neglecting hydraulic losses in the impeller :—

$$\frac{u_3^2 + f_2^2 - f_3^2 \operatorname{cosec}^2\gamma}{2g} + \left\{\begin{matrix}\text{increase in pressure} \\ \text{after leaving impeller}\end{matrix}\right\} = H' + \frac{\bar{v}^2}{2g} \text{ feet} \quad (9)$$

and we have :—

$$\eta' = \frac{u_3^2 + f_2^2 - f_3^2 \operatorname{cosec}^2\gamma + k\, v_3^2}{2\, w_3\, u_3}$$

$$= \frac{u_3^2 + f_2^2 - f_3^2 \operatorname{cosec}^2\gamma + k\, v_3^2}{2\, u_3 (u_3 - f_3 \cot \gamma)} \quad (10)$$

where k has the value indicated in the preceding article for the type of pump under consideration.

The various cases will now be considered in greater detail, and as, except for very low lifts, $\frac{\bar{v}^2}{2g}$ is small in comparison with H' while $\bar{v}$ is

approximately equal to f_2, the foregoing expressions will be simplified by writing

$$\frac{u_3^2 - f_3^2 \operatorname{cosec}^2\gamma}{2g} + \left\{\begin{matrix}\text{increase of pressure}\\ \text{after leaving impeller}\end{matrix}\right\} = H' \text{ feet} \qquad (11)$$

and

$$\eta' = \frac{u_3^2 - f_3^2 \operatorname{cosec}^2\gamma + k v_3^2}{2 u_3 (u_3 - f_3 \cot\gamma)} \qquad (12)$$

(1) **Pump without Special Provision for Utilizing the Kinetic Energy of Discharge from the Impeller.**—Here $k = o$, and if the pump be working so that water enters without shock,

$$\eta' = \tfrac{1}{2} \cdot \frac{u_3^2 - f_3^2 \operatorname{cosec}^2\gamma}{u_3 (u_3 - f_3 \cot\gamma)} \qquad (13)$$

while

$$2g H' = u_3^2 - f_3^2 \operatorname{cosec}^2\gamma \qquad (14)$$

so that

$$u_3 = \sqrt{2g H' + f_3^2 \operatorname{cosec}^2\gamma} \qquad (15)$$

If $\gamma = 90°$, *i.e.* with radial vane tips.

$$\eta' = \cdot 5 - \frac{f_3^2}{2 u_3^2}$$

a value which is always less than 50 per cent.

If γ is very small $\operatorname{cosec}\gamma = \cot\gamma$ (approx.) and we have

$$\eta' = \frac{u_3 + f_3 \cot\gamma}{2 u_3} \text{ (approx.).}$$

Since $f_3 \cot\gamma$ or $f_3 \operatorname{cosec}\gamma$ approximates more nearly to u_3 as γ diminishes, the theoretical efficiency in this case approximates to a value unity.

Usually f_3 is taken as some definite fraction, commonly from $\frac{1}{4}$ to $\frac{1}{8}$ of $\sqrt{2gH'}$, and the following table shows how η' varies with γ for these values of f_3.

γ	90°		60°		30°		15°	
	$f_3 = \frac{1}{4}\sqrt{2gH'}$	$f_3 = \frac{1}{8}\sqrt{2gH'}$	$\frac{1}{4}\sqrt{2gH'}$	$\frac{1}{8}\sqrt{2gH'}$	$\frac{1}{4}\sqrt{2gH'}$	$\frac{1}{8}\sqrt{2gH'}$	$\frac{1}{4}\sqrt{2gH'}$	$\frac{1}{8}\sqrt{2gH'}$
$u_3 \div \sqrt{2gH'}$	1·03	1·008	1·04	1·01	1·12	1·03	1·39	1·11
η' . . .	·47	·49	·53	·53	·65	·61	·79	·70

Effect of a Variation in γ.—Equation (12) indicates that as γ is diminished the efficiency is increased, and this is in general borne out by the results of experiment. As previously pointed out, however, this gain in theoretical efficiency is to a certain extent counterbalanced by the fact that since a diminution in γ necessitates an increased speed of rotation for pumping against a given head, this involves increased frictional losses.

Thus experiments by Parsons on two 14 inch impellers, one having $\gamma = 90°$ and the other $\gamma = 25°$ (approx.), showed that the second was about 1·16 times as efficient as the first. In each case $f_3 = \frac{1}{7}\sqrt{2\,g\,H'}$, and an examination of the table on p. 640 shows that the theoretical manometric efficiencies in the two cases would be approximately ·49 and ·63, the ratio of these being 1·28.

Again, since :—

$$H' = \frac{1}{2g}\{u_3^2 - f_3^2 \operatorname{cosec}^2\gamma\}$$

it follows that the necessary peripheral speed increases with the working head, so that, because of increased frictional losses at these high speeds, it might be inferred that the higher efficiencies are to be expected with comparatively low working heads—a view which, in the ordinary type of single impeller pump, is borne out in practice. The minimum permissible value of γ increases with the working head, and while for heads of about 10 feet it may be as low as 15°, it increases to about 25° with 30 feet head, and for heads of upwards of 60 feet is not generally less than 35°. In the modern types of high-lift pump, however, great care is taken to polish every part of the impeller so as to reduce friction losses to a minimum, and under such circumstances γ may be reduced to as low as 20° against heads of from 90 to 120 feet with excellent results as regards efficiency.

Although a high efficiency is to be aimed at in the design of a pump, this is not the only factor which may affect the most suitable value of γ. From (15) we have

$$f_3 = \sin\gamma\sqrt{u_3^2 - 2\,g\,H'}, \tag{16}$$

and if b_3 feet is the width of the impeller at the discharging periphery, and n the number of vanes each of effective thickness t feet, the area of the wheel passages at the periphery is

$$A_3 = 2\,\pi\,r_3\,b_3 - n\,t\,b_3\operatorname{cosec}\gamma,$$

while

$$Q = f_3\,A_3 = A_3\sin\gamma\sqrt{u_3^2 - 2\,g\,H'}. \tag{17}$$

On differentiating equation (17) with respect to H', we finally get:—

$$\frac{d\,Q}{d\,H'} = -\frac{g}{Q} A_3^2 \sin^2\gamma$$

and, therefore, other things being equal, $\frac{d\,H'}{d\,Q}$ will have its least value when $\sin\gamma$ has its greatest value, *i.e.*, when $\gamma = 90°$; that is to say, the change in delivery head corresponding to a change in the volume delivered by the pump, will be least when radial tipped vanes are used.

This is of importance in the case of a high-lift pump used for boiler feed purposes where the quantity pumped may be varied, but where the delivery pressure is required to remain approximately constant, and for this purpose radial vanes are most suitable. The same reasoning applies to the case of pumps for elevator work, where the delivery pressure is to be approximately constant.

Also, where a pump is required for dry dock or similar work, the possibility of obtaining a large increase in Q as H diminishes with the emptying of the dock, renders the radial vane type most suitable, for although the increased volume necessitates a largely increased power, this is in general unimportant as compared with the reduction in the time necessary to empty the dock.

Where a pump is electrically driven, however, this large increase in power with a reduced head involves the danger of overloading the motor, and this is more particularly the case when induction motors are used and where speed variation is in consequence impossible. In such a case backward curved vanes are essential, for, since the horse power is proportional to $H'\,Q$ and therefore to

$$H'\,A_3 \sin\gamma \sqrt{u_3^2 - 2\,g\,H'}$$

we get, on differentiating,

$$\frac{d\,(H'\,Q)}{d\,H'} = A_3 \sin\gamma \left\{ \frac{u_3^2 - 3\,g\,H'}{\sqrt{u_3^2 - 2\,g\,H'}} \right\}$$

This expression diminishes with γ, and indicates that the rate of increase of the horse-power with a diminution of head is less the smaller the value of γ.

Again, with a pump initially designed to work against a certain head, if the vanes are radial, the possible diminution in speed is very small, but increases as the backward curvature of the vanes increases. With radial vanes, indeed, the pump ceases to lift altogether when the speed falls slightly below that corresponding to normal working. It follows that where the working head cannot be accurately predetermined, the pump

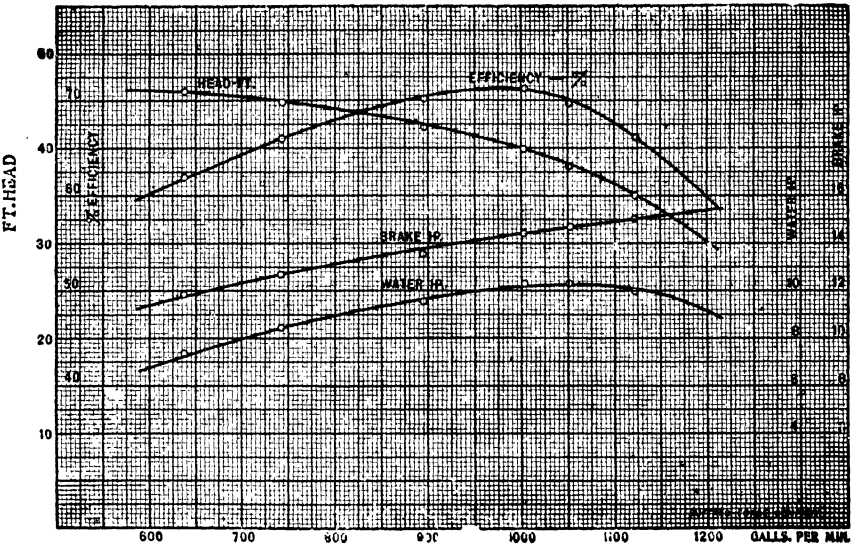


FIG. 341.—Characteristic Curves for Single Low-Lift Centrifugal Pump with Recurved Vanes.

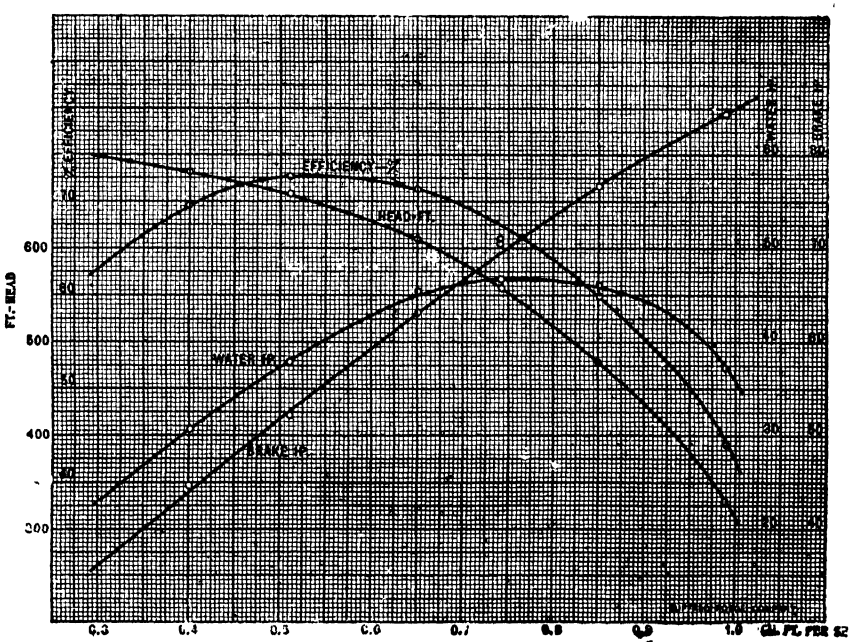


FIG. 342.—Characteristic Curves from 4-Stage Quadruple High-Lift Pump with Radial Vanes.

with recurved vanes has a great advantage in virtue of its greater adaptability to suit varying conditions without serious loss of efficiency.

If, on installing a pump, the head is greater than that for which the vanes were designed, less than the rated quantity will be delivered, or in an extreme case the pump will not lift at all. Generally a slight increase in speed will rectify this.

If, on the other hand, the designed head is greater than the actual, the delivery will be increased, and the engine or motor may be overloaded. This may be rectified either by reducing the speed or, and in general preferably, by slightly throttling the discharge, so as to increase the head artificially. In either case, the possibilities of perfect adjustment are much greater with recurved vanes.

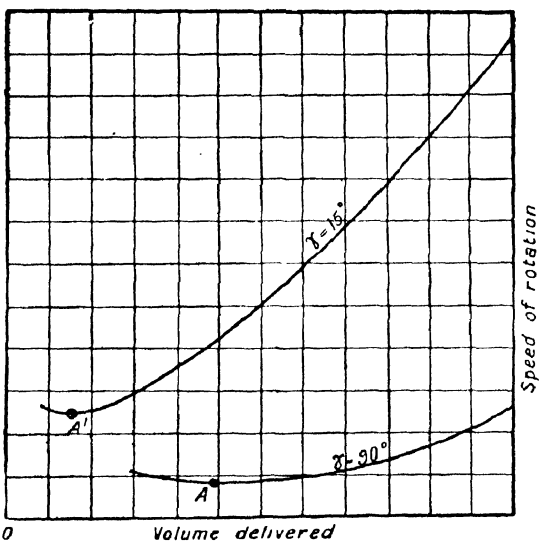


FIG. 343.—Curves showing the influence of the discharge angle γ on the volume delivered under constant head.

These points are well brought out by a comparison of Figs. 341 and 342, which show the characteristic curves from a low-lift pump having recurved vanes with an angle γ approximately 30°, and a high-lift quadruple pump with radial vanes, the speed being constant in both cases Thus, in the pump with radial vanes, a 10 per cent. diminution in head about the point of maximum efficiency is accompanied by a 23 per cent. increase in B.H.P., while the same percentage decrease in the second pump only involves a 7½ per cent. increase in B.H.P.[1]

On the other hand, the corresponding increase in the volume delivered is 31·3 per cent. with radial vanes as against only 11·7 per cent. in the other pump. A glance at the two curves shows that the efficiency falls off much more rapidly with varying conditions when the vanes are radial.

Fig. 343 indicates approximately the effect of a variation in γ on the speed (angular velocity) required to deliver a given volume of water,

[1] See H. R. Lupton, "Pumping Machinery," *Jour. Inst. C.E.*, No. 4, Feb. 1949, p. 291.

where the head is constant. Here the lowest points A and A' of each curve indicate the minimum volume the pump will lift, and the speed below which pumping will not take place. Obviously the minimum speed is increased and the minimum quantity diminished by any recurvature of the vanes.

Pump used for Circulating Purposes.—Where a centrifugal pump is used for circulating water through the tubes of a surface condenser or of a cooler, and where the actual height of lift is small, the resistance to flow, and therefore the head against which the pump works, varies approximately as the square of the velocity of flow. In such a case the suction and delivery pipe line is often arranged so as to form a syphon, in which case the whole work of the pump consists in overcoming frictional resistances.

Here, putting $H' = \frac{f L \bar{v}^2}{m}$ in equation (16), p. 641, we get:—

$$f_3 = \sin \gamma \sqrt{u_3^2 - \frac{f L \bar{v}^2}{m}}$$

and since $\bar{v}$ is proportioned to f_3 for all speeds,

$$\therefore \quad f_3 \propto u_3 = B\, u_3 \text{ for all speeds.}$$

Equation (13) now becomes:—

$$\eta' = \frac{1 - B^2 \operatorname{cosec}^2 \gamma}{2\,(1 - B \cot \gamma)},$$

so that the hydraulic efficiency is independent of the speed of rotation. Since this discussion neglects frictional losses in the wheel which increase with the speed, the actual efficiency will then diminish as the speed increases.

(2) **Pump with Whirlpool Chamber.**—The same general considerations apply to the case of the pump fitted with vortex chamber or guide vanes, as to the simple pump, though modified to some extent quantitatively.

Where a whirlpool chamber is fitted we have, from (9) on equating the gain of pressure head in the pump to the head pumped against, and assuming $\bar{v} = f_2$:—

$$K\,(w_3^2 + f_3^2)\,(1 - c^2) + u_3^2 - f_3^2 \operatorname{cosec}^2 \gamma = 2\,g\,H'.$$

Putting $w_3 = u_3 - f_3 \cot \gamma$, we get

$$K\,(u_3 - f_3 \cot \gamma)^2\,(1 - c^2) + u_3^2 - f_3^2 \operatorname{cosec}^2 \gamma + K f_3^2\,(1 - c^2) = 2\,g\,H', \tag{18}$$

from which f_3 may be found in terms of the peripheral speed and the head.

The efficiency η' is thus equal to—

$$\frac{\left[u_3^2 \{ K\,(1 - c^2) + 1 \} + f_3^2 \operatorname{cosec}^2 \gamma \{ K\,(1 - c^2) - 1 \}\right] - 2\,K\,u_3 f_3 \cot \gamma\,(1 - c^2)}{2\,u_3\,(u_3 - f_3 \cot \gamma)} \tag{19}$$

Putting $K = 1$, *i.e.*, neglecting losses in the whirlpool chamber, this reduces to—

$$\eta' = \frac{u_3^2 (2 - c^2) - f_3^2 c^2 \operatorname{cosec}^2\gamma - 2 u_3 f_3 \cot\gamma (1 - c^2)}{2 (u_3^2 - u_3 f_3 \cot\gamma)}.$$

Assuming $f_3 = \frac{1}{4} u_3$, the following table traces the variation in this efficiency with a change in γ in the ideal cases when $K = 1$, and when $K = \cdot 5$.

$\frac{r_3'}{r_3}$	Efficiency.							
	$\gamma = 90°$.		$\gamma = 60°$.		$\gamma = 45°$.		$\gamma = 30°$.	
	$K = 1$.	$K = \cdot 5$.	$K = \cdot 1$.	$K = \cdot 5$.	$K = 1$.	$K = \cdot 5$.	$K = 1$.	$K = \cdot 5$.
1·00[1]	·469	·469	·535	·535	·584	·584	·661	·661[1]
1·25	·660	·564	·695	·618	·734	·658	·783	·737
1·50	·764	·615	·793	·663	·815	·698	·852	·772
1·75	·826	·647	·848	·694	·863	·724	·890	·795
2·00	·867	·670	·883	·710	·895	·733	·915	·806
3·00	·941	·700	·943	·745	·955	·769	·963	·830

From these results it is evident that with an efficient whirlpool chamber there is very little advantage to be gained by giving the vanes any considerable backward curvature. Moreover, as these efficiencies do not take into account friction losses which increase as γ diminishes, the actual advantage is less than appears from the table.

(3) **Pump with Guide Vanes or Diffuser Ring.**—Here the increase of head in the guide passages will be given by $\frac{k (w_3^2 + f_3^2)}{2 g}$ where k has a maximum value of about ·75.

The total gain of pressure head in the pump is now equal to

$$\frac{k (w_3^2 + f_3^2)}{2 g} + \frac{u_3^2 + f_2^2 - f_3^2 \operatorname{cosec}^2\gamma}{2 g} = H' + \frac{\bar{v}^2}{2 g}.$$

Assuming $\bar{v} = f_2$ and putting $w_3 = u_3 - f_3 \cot\gamma$, we get :—

$$k (u_3 - f_3 \cot\gamma)^2 + u_3^2 - f_3^2 \operatorname{cosec}^2\gamma + k f_3^2 = 2 g H' \qquad (20)$$

or $2 g H' = u_3^2 (k + 1) + f_3^2 \operatorname{cosec}^2\gamma (k - 1) - 2 k u_3 f_3 \cot\gamma$.

[1] No whirlpool chamber.

The efficiency η' of the pump is thus given by—

$$\eta' = \frac{{u_3}^2 (k+1) + {f_3}^2 \operatorname{cosec}^2 \gamma (k-1) - 2 k u_3 f_3 \cot \gamma}{2 u_3 (u_3 - f_3 \cot \gamma)} \qquad (21)$$

Giving k the value ·75 and taking $f_3 = \frac{1}{4} u_3$, we get the following values of η' :—

γ	90°	75°	60°	45°	30°
η'	·867	·875	·884	·896	·921

Here again, any large reduction in the value of γ, by necessitating an increased speed of rotation, is likely to reduce rather than increase the overall efficiency.

For the guides to be efficient it is essential that a sufficient number of guide vanes be used efficiently to direct the mass of water, four being about the minimum ; that the areas of the guide passages at the exit from the wheel should be proportioned so as to keep the velocity at entrance, the same as on leaving the wheel, and that all guide curves should be smooth and gradual.

Since the whole object of the guides is to neutralise the evil effects of a high velocity of discharge, their relative effect will be greatest where this velocity is greatest, *i.e.*, with radial pump vanes when delivering against a high head, and will be least with recurved vanes and a low head. For this reason, and since large percentage variations of head are more likely to occur where this is low, the diffuser ring is seldom fitted for heads under about 80 feet.

The gain in overall efficiency attainable by its use naturally varies considerably, but may be taken as between 10 per cent. and 20 per cent.

Art. 203.—Compound Multiple Chamber Pump.

Where a number "n" of impellers mounted on the same shaft are used in series so as to form a compound high-lift pump, the impeller diameters, and vane angles are made the same for each chamber, so that each impeller gives a total head $= \frac{H'}{n}$ feet. If, in the foregoing theory then,

H' be replaced by $\frac{H'}{n}$, the deductions also apply to the compound pump.

The work done on the shaft per lb. of water, neglecting frictional losses, is now given by $\frac{n\, w_3\, u_3}{g}$ foot lbs. and the manometric efficiency η' by $\frac{g\, H'}{n\, w_3\, u_3}$.

while neglecting friction the power required to drive the pump $= \frac{W\, Q}{550}\, \frac{H'}{\eta'}$ B.H.P.

$$= \frac{W\, Q}{550\, g}\, n\, w_3\, u_3 \text{ B.H.P.}$$

Art. 204.—General Equation for Pump.

From equation (11), p. 640, we have

$$2\, g\, H' = u_3^2 + k v_3^2 - f_3^2 \operatorname{cosec}^2 \gamma$$

for any pump, and since $v_3^2 = w_3^2 + f_3^2$

$$= (u_3 - f_3 \cot \gamma)^2 + f_3^2$$

the foregoing relationship can be written—

$$2g\, H' = A\, u_3^2 + B\, u_3 f_3 + C f_3^2$$
$$= a\, N^2 + b\, N\, Q + c\, Q^2.$$

Where N = revs. per min.; Q = discharge; and where A, B, C, a, b, c, are constants for any particular pump. It follows that if the speed, the discharge, and the head be measured for three different speeds, discharges, or heads, the values of these constants may be obtained and the discharge calculated for any other speed or head and *vice versâ*.

Art. 205.—Peripheral Speed of a Pump.

If H' = total head pumped against, including friction head, we have

$$\eta' = \frac{g\, H'}{u_3\, (u_3 - f_3 \cot \gamma)},$$

so that with a perfect pump in which all losses were negligible, the peripheral velocity of the vanes at discharge would be given by

$$u_3\, (u_3 - f_3 \cot \gamma) = g\, H'.$$

and with radial vanes we should have $u_3 = \sqrt{g\, H'}$. As γ diminishes the peripheral speed increases, while any diminution in efficiency naturally necessitates a higher peripheral speed again, so that actually we have $u_3 = k\, \sqrt{g\, H'}$, where k depends upon γ; upon the value adopted for f_3; and upon the type of pump. In practice it is usual to make f_3 from [·2 to ·3] u_3, the co-efficient increasing from about ·21 when $\gamma = 15°$ to ·29 when $\gamma = 90°$, while k is given a value between 1·2 and

1·8, increasing from about 1·2 in the case of a well designed pump with whirlpool chamber or guides and with a value of $\gamma = 90°$, to about 1·8 in the case of a pump having a volute chamber only, and a value of $\gamma = 15°$.

This makes $f_3 = \frac{1}{4}\sqrt{2\,g\,H}$ (approx.). In the case of a pump dealing with a large head in a single chamber this, however, gives an excessive value of f_3, and in such a case the value may be reduced to as little as $\frac{1}{8}\sqrt{2\,g\,H}$.

Art. 206.—Speed at which Pumping Commences.

Even if a centrifugal pump be primed, there will be no flow through the delivery pipe until the pressure difference through the wheel, which depends entirely on the speed of rotation, is sufficiently great to overcome the total head of lift, the pump until then merely sustaining a stationary column of water in this pipe.

If the suction lift is H_s feet, the pump will not maintain its charge without a foot valve on the suction pipe, unless the speed is sufficiently great to cause a pressure difference of more than H_s feet between the pump inlet and outlet.

Assuming uniform rotation with no flow through the pump, the pressure difference in the forced vortex becomes—

$$\frac{p_3 - p_2}{W} = \frac{u_3^2 - u_2^2}{2\,g} \text{ feet, } f_2 \text{ and } f_3 \text{ being zero.}$$

$\therefore$ For the pump to remain charged when once filled we must have—

$$\frac{\omega^2\,(r_3^2 - r_2^2)}{2\,g} > H_s \text{ feet,}$$

while for delivery to take place we must have—

$$\frac{\omega^2\,(r_3^2 - r_2^2)}{2\,g} > H \text{ feet.}$$

Once free flow is established, the state of affairs changes. A further increase of pressure is now necessary to overcome frictional resistances, and in the case of a pump not fitted with vortex chamber or guides this necessitates the speed for free delivery being slightly greater than is given above. A reduction of the speed below this limit will cause pumping to cease.

With a whirlpool chamber, or guide ring, however, once flow is instituted part of the kinetic energy of flow through the wheel is converted into pressure energy, so that the speed may be reduced below that necessary to initiate flow, without pumping being stopped.

Owing to viscosity moreover, even though the pump be not actually discharging, the water in the vortex chamber will be affected by the rotation of the impeller, and will, to a certain extent, form a free vortex with the pressure greatest and the velocity least at the outside. This reduces the speed for impending delivery by an amount which depends on the design and construction of the vortex chamber, and for the calculation of which insufficient experimental evidence is available. Experiments [1] on a series of pumps having radial vanes with impellers 8·813″ outside and 5·375″ inside diameter, and with a vortex chamber 11·375″ diameter, showed that when on the point of impending delivery this served as a true vortex chamber, the value of K (p. 636) being about ·5.

In all probability this effect would not be so pronounced in a pump having a proportionately larger vortex chamber.

Art. 207.—Size of Pump for a Given Discharge—Similar Pumps—Proportions of Pumps.

For a given speed of rotation and a given radius r_3, the difference in pressure between inlet and outlet, and therefore the head pumped against, increases with the difference between the inner and outer radii of the impeller r_2 and r_3. Since, too, the proportional effect of disc friction ($\propto r^5$, see Art. 76), diminishes rapidly as the radius diminishes, the inner radius should, for efficiency, be made small. This may be accomplished, keeping the discharge constant, by increasing the velocity of flow f_2, but with a large suction head a limit to this maximum velocity is soon reached, and in practice it is usual to make the inner radius from $\frac{1}{3}$ to $\frac{1}{2}$ the outer ($r_3 = 3\ r_2$ to $2\ r_2$), the former value being preferable.

Again, since the head pumped against is approximately proportional to u_3^2, *i.e.*, to $\omega^2\ r_3^2$, a given head may be obtained either by an increase in ω or in r. But $Q \propto f_3\ r_3\ b_3$, and assuming f_3 to be proportional to u_3, we have :—

$$Q \propto u_3\ r_3\ b_3,$$
$$\propto \omega\ r_3^2\ b_3,$$
$$\therefore \frac{H'}{Q} \propto \frac{\omega}{b_3}.$$

It follows that for large values of H', in order to avoid excessive values of ω, the value of b_3 should be comparatively small. This is borne out in practice, where the breadth b_3 of the impeller at the rim is usually

[1] "Bulletin of the University of Wisconsin," No. 173. Vol. III. No. 6, p. 447. By C. B. Stewart.

proportional to the radius and has a value ranging from $\frac{1}{5}\ r_3$ to $\frac{1}{10}\ r_3$, the former value applying to low-lift, and the latter to high-lift pumps.

Putting $b_3 \propto r_3$, we have $Q \propto \omega\ r_3^3$

$$\therefore \quad \frac{H'}{Q} \propto \frac{\omega}{r_3} \propto \frac{u_3}{r_3^2} \propto \frac{u_3^2}{r_3^2\ u_3}.$$

While this is not strictly true in the case of an actual pump, yet it shows that in every case for a given value of H' and of Q, there is a definite value of r for maximum efficiency, this value, however, depending on the relative magnitude of the frictional losses.

As in the case of a reaction turbine, the characteristic curves for a given pump apply to any similar pump of different size, so long as

(1) The peripheral speeds are proportional to $\sqrt{H'}$;

(2) The revolutions are proportional to $\frac{\sqrt{H'}}{S}$;

(3) The quantity discharged is proportional to $S^2\sqrt{H'}$;

where S is the ratio of similar linear dimensions of the two pumps.

Art. 207a.—Specific Speed of Centrifugal Pumps—Variation of Efficiency with Specific Speed.

The specific speed of a centrifugal pump is defined as the speed at which a geometrically similar model reduced in size so as to deliver 1 c.f.s. under a head of 1 foot under conditions of maximum efficiency, would require to operate.

Since in a pump and its model $Q \propto r^2\sqrt{H}$ and $N \propto \frac{\sqrt{H}}{r}$.

$$\therefore \quad \frac{N_m}{N} = \sqrt{\frac{H_m}{H}} \cdot \frac{r}{r_m} = \sqrt{\frac{H_m\ Q}{H\ Q_m}} \left(\frac{H_m}{H}\right)^{\frac{1}{2}}$$

$$= \sqrt{\frac{Q}{Q_m}} \cdot \left(\frac{H_m}{H}\right)^{\frac{1}{2}}.$$

If $Q_m = 1$ and $H_m = 1$, N_m becomes the specific speed N_{sq} so that

$$N_{sq} = \frac{N\sqrt{Q}}{H^{\frac{3}{4}}} [1]$$

Experience shows [2] that with pumps of normal design maximum efficiencies are attained with values of N_{sq} between 80 and 240, and the preliminary design of a pump should aim at a specific speed if possible within this range. Smaller specific speeds involve comparatively large diameters and passages of small widths accompanied by high frictional losses, while higher speeds involve small radii and inadequate guidance of the water.

[1] If Q be taken in gallons per minute the value of N_{sq} is 19·34 times as great as if Q is in f.s.

[2] See L. F. Moody, "Mechanical Engineering," N.Y., April 1921, p. 241. Also A. F. Sherzer, "Eng. News Record," June 30, 1921, p. 1114.

Art. 208.—Remarks on Theory of Pump Design.

None of the fundamental assumptions made in the foregoing discussion are accurately true. Thus the assumption of radial motion immediately before entering the impeller is certainly incorrect since viscosity must cause some whirl before the wheel is reached. This has the effect of increasing the value of the inlet angle β for entry without shock. Experiment tends to show that the loss due to shock at entry is a minimum when β is calculated on the assumption that w_2 is between ·40 and ·45 u_2.

The assumption that all particles of water on leaving the impeller have the same velocity v_3 and are moving in paths making an angle γ with the tangent to the periphery, or that $w_3 = u_3 - f_3 \cot \gamma$ for all particles, is also incorrect as appears from the results of experiment. A measure of the error involved may be obtained by comparing the actual and theoretical manometric efficiencies, for if from the energy expended in driving a pump, that necessary to overcome disc and mechanical friction be deducted, the remainder should equal $u_3 w_3 \div g$ foot lbs. per lb. of water. If the ratio of the measured manometric head H_m to this quantity be termed the experimental manometric efficiency, the ratio of the theoretical and experimental manometric efficiencies will be a measure of the ratio of the true w_3 to that given by the foregoing formula.

The following results from tests of a pump by Mr. Parsons [1] have been chosen to illustrate this point. The details of the pump are as follow:— $r_3 = 9{\cdot}25$ inches; $r_2 = 4{\cdot}625$ inches; $b_2 = b_3 = 5{\cdot}75$ inches; $\gamma = 15°$; $\beta = 40°$; 8 vanes, assumed $\frac{3}{8}$ inch thick for purposes of calculation.

Experiment.	H_m ft.	Q c.f.s.	Revs. per min.	H.P. less Disc and Mech. Friction.	H.P. Output.	Manometric Efficiency.		Calculated / Observed Eff.
						Experimental.	Calculated.	
1	14·67	2·70	392	7·68	4·50	·586	·561	·958
7	14 70	4·01	402	10·43	6·69	·642	·586	·915
11	14 80	4·68	406	11·83	7·85	·664	·609	·917
14	17·20	3·20	427	10·88	6·24	·573	·564	·983
18	17·40	4·01	432	12·90	7·90	·625	·590	·945
22	17 60	4·68	436	13 61	9·35	·687	·615	·895
							Mean ...	·935

In these experiments the mean power necessary to overcome the friction of the pump bearings and driving belt and the disc friction of the pump was found to be 1·37 H.P., and this value has been used in deducing the values in column 5.

Again experiments on an experimental pump, $r_3 = 4{\cdot}41$ inches; $r_2 = 2{\cdot}188$ inches; $b_2 = b_3 = 1{\cdot}125$ inches; $\gamma = 90°$; $\beta = 18°\ 50'$, at the

[1] "Proc. Inst. C. E.," Vol. 53, p. 271.

Madison University [1] give the following values for the ratio of calculated to observed manometric efficiency.

Series 1.	24 vanes,	area of waterways	increasing	·910
Series 2.	24 ,,	,,	constant	·910
Series 3.	12 ,,	,,	increasing	·875
Series 4.	12 ,,	,,	constant	·845
Series 5.	6 ,,	,,	increasing	·75
Series 6.	6 ,,	,,	constant	·68

From these and similar results it appears that w_3 is always less than is given by the formula $w_3 = u_3 - f_3 \cot \gamma$. This is probably due partly to the fact that a dead water space which is not utilised for discharge is formed on the rear side of each vane, thus increasing the radial component f_3, and partly to the fact that only those layers of water near to the driving edge of the vane are discharged parallel to its tip. Thus the true mean values of f_3 and of cot γ are both greater than the apparent values. It also appears that the ratio of the true to the calculated value of w_3 increases with the number of vanes and hence as the guidance of the water becomes more perfect, but diminishes as the width of the passages is reduced, probably because the dead water space then forms a proportionately larger part of the whole discharging area.

It also diminishes slightly as the discharge increases, and as the angle γ increases.

The following may be taken as approximate values of the ratio, in pumps of normal design:—

		$\frac{w_3 \text{ experimental.}}{w_3 \text{ calculated.}}$
Pump upwards of 20 in. diameter;	12 or more vanes—	
	γ from 15° to 30°	·97
	,, 30° to 60°	·95
	,, 60° to 90°	·92
Pump 10 in. to 20 in. diameter;	12 vanes—	
	γ from 15° to 30°	·93
	,, 30° to 60°	·90
	,, 60° to 90°	·87
,, ,, ,,	8 to 12 vanes—	
	γ from 15° to 30°	·90
	,, 30° to 60°	·87
	,, 60° to 90°	·84

[1] "Bulletin of the University of Wisconsin," No. 173. 1907.

Pump 10 in. to 20 in. diameter; 6 vanes—

γ from 15° to 30° . .	·81
„ 30° to 60° . .	·78
„ 60° to 90° . .	·75

The values point to the absolute necessity for providing a sufficient number of wheel vanes where a diffuser ring is fitted, and where, in consequence, serious loss by shock may occur if the guide vane angle is designed to suit what may be a fictitious direction of outflow from the impeller.

Art. 209.—Balancing of End Thrust.

End thrust on a pump shaft having one or more impellers with single axial inlets is due to a number of causes.

(*a*) To the difference of pressure at a given radius on the two sides of the vane shrouding, due to the fact that while the water in the impeller is rotating in a forced vortex, that outside the shrouding is in a state of comparative rest, and therefore exists under different pressure conditions. This is by far the most important factor in producing end thrust.

(*b*) To the fact that if the discharge diameters of the shroudings in a double cased pump are equal, there is an unbalanced pressure on the portion of the shrouding opposite to the inlet opening. This produces a thrust in the direction of inflow and so tends to balance (*a*).

(*c*) Since the water is taken in axially and diverted radially, it suffers a change of momentum in an axial direction, and this change of momentum can only be produced by an axial force transmitted through the shaft to the impellers.

Thus if $\bar{v}$ is the velocity of axial flow through the supply passages; Q the discharge per second; and n the number of impellers, this change of momentum takes place n times in the pump, and the total end thrust on the shaft due to this cause is given by—

$$n \cdot \frac{62{\cdot}4\, Q}{g} \cdot \bar{v} \text{ lbs.}$$

This thrust also acts in the direction of inflow.

Apart from the system of arranging the impellers in pairs placed back to back, each pair thus being in balance, four methods of balancing are in use:—

(*a*) By radial balancing vanes mounted on the rear face of the shrouding (Fig. 321).

(*b*) By making the diameter of shrouding on the entrant side greater

than that on the exit side, the difference of area being just sufficient to balance the pressure difference (Fig. 332).

(*c*) By a rotary balance piston or disc keyed to the pump shaft, one side being exposed to delivery pressure, and the other to the pressure on the suction side of the pump (Figs. 330, 331, 332).

(*d*) By relieving the pressure over a certain area behind the shrouding by means of a rotating balance ring (Figs. 322A, 323, and 333).

Whichever of these methods is adopted, with the exception of (*c*), the wheel will only be in perfect balance at one speed and with one rate of delivery, and in such cases it is necessary to arrange a thrust block to take the end thrust caused at starting or stopping and in the case of a variable load.

These methods of balancing will now be considered in further detail :—

(*a*) **Radial Balancing Vanes.**—Since it is impossible to keep the joint between the outer periphery of the shrouding and casing tight, the space behind the shrouding will be full of water normally at delivery pressure, and since the mean pressure in the wheel is considerably less than this, the resultant effect in a single shrouded wheel (Fig. 321) will be an axial thrust of considerable magnitude, in the opposite direction to that of axial flow.

If a series of holes be made through the disc, as indicated in Fig. 321, the pressure at this radius on both sides of the shrouding will be equalised, and there will be a constant circulation of water past the rim of the impeller, behind the shrouding to the zone of low pressure, and out to the working side of the shrouding.

While the axial thrust is then largely balanced, this leakage is productive of inefficient working, and the method, though often adopted in turbine practice, is inadvisable.

If, however, shallow radial vanes are fitted to the rear face of the shrouding, the water in the clearance space is forced to rotate with vortex motion, and in consequence its pressure diminishes from the outside, where it has a definite value, to the inside, following approximately the forced vortex law. The degree to which it deviates from this law depends on the amount of side clearance between the balance vanes and the pump casing, any increase in this clearance, by reducing the mean angular velocity, tending to increase the mean pressure behind the shrouding. By suitably regulating the clearance and the radial length of the balance vanes, the whole, or any portion of the axial thrust, may thus be balanced.

For example, in the wheel shown in Fig. 321, the pressure in the wheel itself varies at different radii owing, firstly, to the production of a forced vortex, and, secondly, to the necessity of maintaining an outward flow. In the clearance space to the right this outward flow is absent, so that the difference of pressure on the two sides of the shrouding at a radius r is given by the pressure necessary to maintain this flow, and if p_r represent this pressure intensity in pounds per square foot

$$\frac{p_r}{W} = \frac{p_2}{W} + \frac{{}_2v_r^2 - {}_rv_r^2}{2g}. \tag{1}$$

This assumes the law of pressure variation in outward flow to be the same as in inward flow and the theory is to this extent unsatisfactory. Allowing for this, we have :—

$$\frac{p_r}{W} = K\frac{p_2}{W} + \frac{K\,{}_2v_r^2 - {}_rv_r^2}{2g}, \tag{2}$$

where K is a co-efficient, varying probably from about ·7 in the case of radial vanes to ·85 with vanes having a delivery angle of 30°.

Now $${}_2v_r = \sqrt{f_2^2 + u_2^2}$$

And ${}_rv_r = f_r \operatorname{cosec}\theta$, where θ = angle made by the vanes at this radius with the tangent to the corresponding circle.

$$\therefore \quad \frac{p_r}{W} = K\left\{\frac{p_2}{W} + \frac{f_2^2 + u_2^2}{2g}\right\} - \frac{f_r^2 \operatorname{cosec}^2\theta}{2g}. \tag{3}$$

The axial force F may be obtained by dividing the disc into a series of concentric elements, and by obtaining the relative velocity at the mean radius of each element by this method or by the graphical construction of p. 514. The unbalanced force on each element may then be calculated by an application of equation (2) above, as shown on p. 515, and the sum of these, taken over the whole area, gives the resultant axial force.

Equating this to the unbalanced pressure on the inlet side of the shrouding produced by the axial change of momentum of the water, and to the unbalanced pressure on the remaining annulus of width $r_3 - R_0$, on fixing R_1 the required outer radius R_0 may be determined.

Where balancing vanes are fitted they should not be too deep and

should work with the minimum possible clearance over the face of the casing. Such vanes, however, in any case greatly increase friction losses, and in view of this and of the impossibility of obtaining exact balancing by their use, are seldom fitted on modern pumps.

(*b*) Here (Fig. 332) the diameter of the shrouding on the side remote from the entrance is reduced so as to make the total area of the shroudings approximately equal.

The clearance space pressures are thus balanced, and the excess of pressure on the inlet shrouding due to the greater pressure at the outer radius of the runner tends to balance the pressure in the opposite direction due to momentum changes. This method also is impossible of exact calculation, and is therefore only suitable when used in conjunction with a thrust block or balance piston to take up any minor unbalanced thrust.

(*c*) Where a balance piston is used (Fig. 332), leakage past this piston is slight, and by adjustment of a valve on the pressure pipe *P*, which adjustment may be automatic, the pressure may be adjusted while running to suit any condition of working.

In the pumps shown in Figs. 330 and 331, the shrouding area which is exposed to the discharge pressure from each impeller is considerably greater on the side removed from the inlet, and consequently the nett pressure is from right to left. To counterbalance this a balance piston or disc *P* is fitted, exposed to pressure water from the last impeller on its left hand face. This works normally with small clearance over the fixed casing at *C*. When the pressure to the right becomes equal to the full discharge pressure it is more than sufficient to counterbalance the end thrust on the impellers, the spindle moves to the right, the clearance between piston or disc and casing is increased, and the pressure water escapes into the low pressure chamber *R*, relieving the end thrust on the piston. The regulation is perfectly automatic, the spindle taking up a position in which the clearance is just sufficient to maintain the necessary pressure in the space to the left of the balance disc. In view of the simplicity and automatic nature of this device it promises to become general on all modern high speed pumps.

Fig. 344 shows a refinement of this device as fitted to the Mather-Reynolds high lift pumps. The balance disc has two active throttling surfaces floating between two fixed seatings, with very slight axial clearance. Pressure water passing through the holes *h* into the chamber *m* escapes past the large throttling surface of the balance disc into the chamber *K* and from there past the small throttling surface into the escape pipe *P* leading to the

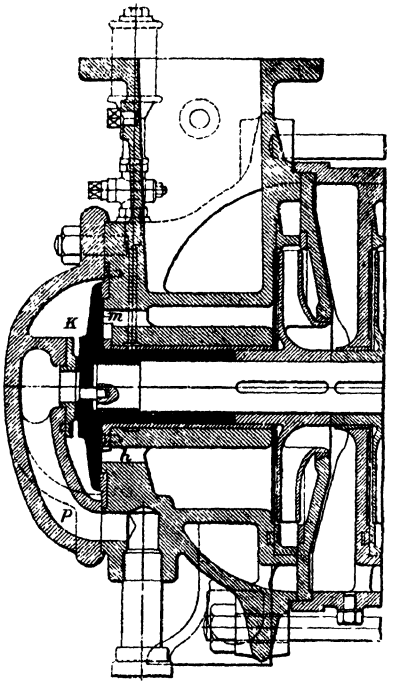


FIG. 344.—Compound Differential Balance Disc for High Lift Pump.

suction side of the pump. The leakage with this type of differential balance disc is appreciably less than with the single-faced disc.

(*d*) In this method of balancing (Figs. 322*a*, 323, and 333) the outer diameter of the balance ring is made slightly less than that of the impeller at the packing ring *P*, while a series of holes through the shrouding equalises the pressure in the interior of the balance ring and of the impeller.

This is then approximately in balance, the slight reduction in the diameter of the balance ring creating an excess pressure towards the inlet, which counterbalances the change of axial momentum of the supply water.

Brass packing rings are fitted at *P* (322 *a*) to prevent leakage of pressure water from the clearance space into the balance ring.

ART. 210.—EXAMPLES OF THE DESIGN OF CENTRIFUGAL PUMPS.

As an example of the application of the foregoing formulae, the main points in the theoretical design of one or two types of pump will now be considered.

(1) A pump—50 feet working head—is fitted with a whirlpool chamber whose radius is twice that of the impeller, and whose efficiency is 50 per cent. The pump is to deliver 5 cubic feet per second. Determine its leading dimensions—its speed of rotation—its hydraulic efficiency, and the probable H.P. necessary to drive it.

To begin with, we will assume that since the whirlpool chamber is fairly efficient and the head high, the value of γ may be taken as fairly high—say 60°—so as to keep down the speed and friction losses. Also assume $f_3 = \frac{1}{8}\sqrt{2\,g\,H'}$.

Then $f_3 = \frac{8{\cdot}02}{8}\sqrt{50} = 7{\cdot}09$ f.s.

Now substituting ·5 for c and ·5 for K in equation (18), (p. 645), we get—

$$\cdot 5\,(u_3 - 7\cdot09 \cot 60°)^2 \times \tfrac{3}{4} + u_3^2 - 50\cdot2 \operatorname{cosec}^2 60° + 25\cdot1 \times \tfrac{3}{4}$$
$$= 100\,g = 3{,}220$$
$$\therefore \quad u_3^2 - 2\cdot232\,u_3 - 2{,}378 = 0,$$
$$\text{or } u_3 = 49\cdot9 \text{ feet per second.}$$

Since $\sqrt{g\,H'} = 40\cdot1$, this makes $u_3 = 1\cdot245\,\sqrt{g\,H'}$.

Next taking $b_3 = \frac{r_3}{10}$ we have $A_3 = 2\,\pi\,r_3\,b_3 = \frac{\pi\,r_3^2}{5}$,

and since $Q = A_3 f_3 = \frac{\pi\,r_3^2}{5} \times 7\cdot09$,

we have $r_3^2 = \frac{25}{7\cdot09\,\pi} = 1\cdot122.$

$\therefore \quad r_3 = 1\cdot06$ feet $= 12\frac{3}{4}$ inches.

$\therefore \quad b_3 = 1\cdot275$ inches.

This neglects the effect of the vane thickness. Since the vanes reduce the effective discharge area by $b_3\,n\,t_3 \operatorname{cosec}\gamma$, when the number and thickness of vanes has been decided upon the above value of b_3 must be increased in the ratio

$$\frac{2\,\pi\,r_3}{2\,\pi\,r_3 - n\,t_3 \operatorname{cosec} 60°}$$

Thus, taking eighteen vanes, each having a thickness at the tips of $\frac{1}{4}$ inch, this ratio becomes

$$\frac{2\,\pi \times 12\cdot75}{2\,\pi \times 12\cdot75 - \frac{18}{4 \times \cdot8660}} = \frac{80\cdot1}{74\cdot9} = 1\cdot07$$

and the true breadth $b_3 = 1\cdot275 \times 1\cdot07 = 1\cdot365$ inches.

Again $\omega = \frac{49\cdot9}{1\cdot06} = 47\cdot0$ radians per second.

$$\therefore \quad N = \frac{60\,\omega}{2\,\pi} = 449 \text{ revolutions per minute.}$$

Assuming $r_3 = 2\cdot5\,r_2$, this makes $u_2 = \frac{49\cdot9}{2\cdot5} = 19\cdot95$ f.s., while if the vanes be broadened out towards the centre so as to keep the velocity of flow down to say 10 feet per second, this makes $\tan\beta = \frac{f_2}{u_2} = \frac{10}{19\cdot95} = \cdot5012$.

$$\therefore \quad \beta = 26° 37'$$

The manometric efficiency, $\frac{g\,H'}{u_3\,(u_3 - f_3 \cot\gamma)}$,

is equal to

$$\frac{1{,}610}{49\cdot9\,(49\cdot9 - 7\cdot09 \times \cdot5774)} = \frac{1{,}610}{2{,}282} = \cdot705.$$

Probably mechanical and hydraulic frictional resistances will combine to reduce this efficiency to about ·65.

Taking this value, the work done on the pump shaft per second

$$= \frac{62{\cdot}4\ Q\ H'}{{\cdot}65} \text{ foot lbs.}$$

$$\therefore \text{ H.P.} = \frac{62{\cdot}4\ Q\ H'}{550 \times {\cdot}65} = \frac{62{\cdot}4 \times 5 \times 50}{550 \times {\cdot}65} = 43{\cdot}6.$$

The speed at which the pump begins to lift is given by

$$u_3^2 - u_2^2 = 2\ g\ H = 3{,}220$$

or

$$\omega^2 (r_3^2 - r_2^2) = 3{,}220$$

(2) Compound high-lift pump, fitted with guide vanes and vortex chambers having a pressure conversion efficiency of 75 per cent., to deliver 5 cubic feet per second under 300 feet head. Assuming a lift of 50 feet in each of six chambers; $\gamma = 60°$, $f_3 = \frac{1}{8}\sqrt{2\ g\ H'}$; and considering each chamber as a separate pump, we have, as in example (2), $f_3 = 7{\cdot}09$ f.s. Then, taking $b_3 = \frac{r_3}{10}$, we get, as before, $b_3 = 1{\cdot}275$ inches, or, allowing for the same vanes, $b_3 = 1{\cdot}365$ inches, while $r_3 = 1{\cdot}06$ feet. We now have the factor k of equation (20) (p. 646) equal to ·75, and this equation becomes:—

$$\cdot 75\ (u_3 - 4{\cdot}09)^2 + u_3^2 - 66{\cdot}95 + 37{\cdot}6 = 3{,}220.$$

$$\therefore\ u_3^2 - 3{\cdot}51\ u_3 - 1{,}850 = 0.$$

$$\because\ u_3 = 44{\cdot}76 \text{ f.s.}$$

This makes $u_3 = 1{\cdot}116\sqrt{g\ H'}$.

Also $N = \frac{60\ u_3}{2\ \pi\ r_3} = \frac{60 \times 44{\cdot}76}{2\ \pi \times 1{\cdot}06} = 403$ revolutions per minute, while since $\tan a = \frac{f_3}{u_3 - f_3 \cot \gamma} = \frac{7{\cdot}09}{44{\cdot}76 - 4{\cdot}09} = {\cdot}1744$, this gives a guide vane angle $a = 9° 54'$.

Assuming $u_3 - f_3 \cot \gamma$ or w_3 to have ·95 times its theoretical value, p. 653, this makes $\tan a = {\cdot}1834$ and $a = 10° 24'$.

In this case the manometric efficiency $\frac{g\ H'}{u_3 (u_3 - f_3 \cot \gamma)}$

$$= \frac{1{,}610}{44{\cdot}76 \times 40{\cdot}67} = \frac{1{,}610}{1{,}822} = {\cdot}884.$$

Frictional losses would probably reduce this to about ·80, so that the H.P. would be equal to

$$\frac{62{\cdot}4 \times Q \times n\ H'}{550 \times {\cdot}80} = \frac{62{\cdot}4 \times 5 \times 300}{550 \times {\cdot}80} = 213.$$

Art. 210A.—The Axial Flow Pump (Fig. 344A).

In this type of pump, as its name indicates, the flow is parallel to the axis, and pressures due to centrifugal action are not involved. The pump is, in effect, a reversed Jonval turbine (p. 462). It consists essentially of an impeller, with usually about four vanes of the propeller type, into which the water flows axially. The vane angles α at entry are so designed as to give tangential inflow. The impeller is followed by a ring of fixed guide vanes with divergent passages, whose entry angle γ is designed to give tangential entry. In passing through the impeller the velocity and kinetic energy are increased and the water entering the guide vanes has its velocity reduced and in part converted into pressure energy. Typical velocity diagrams are shown in Fig. 344B, where u = velocity of impeller vanes, v_r = relative velocity of water and vanes, f = axial velocity and v = absolute velocity at exit from the impeller.

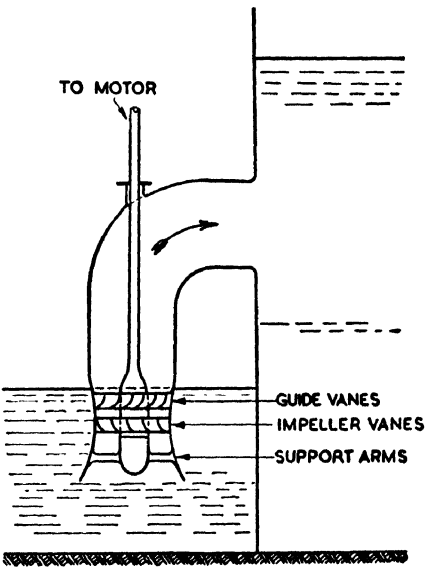


Fig. 344A.—Axial Flow Pump.

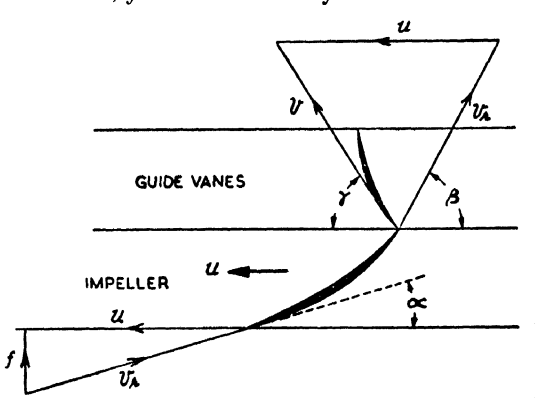


Fig. 344B.

Since the efficiency depends essentially on the efficiency of conversion of the kinetic energy of discharge $v^2/2g$ into pressure in the guide vane passages, it cannot be expected to be quite as high as that of the normal centrifugal pump.

Owing, however, to its simplicity of construction and installation, it has practical advantages where large volumes are to be handled, and where, as in fen drainage systems, etc., the head and volume are reasonably constant.

Since u at any point is proportional to the radius, for entry without shock

the angle α must vary with the radius. In practice [1] the impeller is designed as though composed of a series of annular rings. The head developed at each annulus is computed, and the resultant head is the weighted average of the head developed at each radius.

ART. 210B.—MODEL TESTS OF CENTRIFUGAL AND AXIAL FLOW PUMPS.

The conditions to be satisfied in model tests of rotary pumps are identical with those of tests on turbines (p. 531).

It has been suggested [2] that model tests using air as the fluid may be useful and cheap in providing data for the design of large water pumps. Such models need be of very light construction and easily modified. Neglecting the effect of viscosity, the delivery of Q cusecs of water under a head h feet of water, will, at the same speed, give Q cusecs of air under a head h feet of air. The available results of experiments show quite a close agreement between air tests on the model and water tests on the pump.

[1] See "Propeller Pumps," O'Brien and Folsom, *Am. Soc. Mech. E.*, Summer Meeting, 1934, p. 35. George Reproduction Co., 500 Sansome Street, San Francisco. Also Sparnhake, *Trans. Am. Soc. M.E.* (Hyd-56-1), Vol. 56, April, 1934, p. 225.

[2] "Some Experiences of the Use of Scale Models in General Engineering," R. W. Allen, Brit. Ass., Section G, 1938.

CHAPTER XXI.

Other Types of Pumping Machinery—Water Hoisting—The Hydraulic Ram—Hydraulic Air Compressor—The Jet Pump—The Injector Hydrant—The Air Lift Pump—Reversed Air Lift Pump as Air Compressor—Humphrey's Gas Pump.

ART. 211.—WATER HOISTING FROM MINES.

THE method of direct hoisting of water in large tanks has come very rapidly into favour of recent years in the anthracite region of Pennsylvania for mine drainage purposes.

The system has the advantages that in general, with the exception of the tanks, and the guides used for keeping these vertical while hoisting, no new machinery is needed, the only cost being that due to extra wear and tear of the hoisting engines and to the steam used while hoisting. Further, the whole of the operating machinery is on the surface and free from the danger of being flooded, while no underground steam pipes, with the accompanying losses by condensation and the danger of damage by a slip of the roof, are necessary.

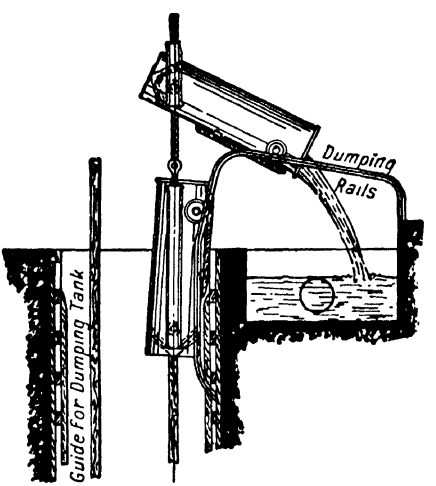


FIG. 345.—Method of End Dumping in direct Water-hoisting Plant.

Cylindrical hoisting tanks are now general, these having a couple of butterfly valves in the bottom placed at an angle of 45° (Fig. 329).

The tanks may discharge at the top of the shaft either by overturning or by automatic opening of the bottom valves. The former method is preferable as rendering more rapid manipulation possible. With tanks of

1,500 gallons capacity, the capacity of such a plant ranges up to about 750,000 gallons per day of twelve hours.

Art. 212.—The Hydraulic Ram.

The hydraulic ram, which owes its conception as a practical machine to Montgolfier (about the end of the eighteenth century), is an apparatus devised to utilize the kinetic energy of a moving column of water to pump up part of this water to a height greater than that of the supply head. In its simplest form, the ram consists of an inclined supply pipe

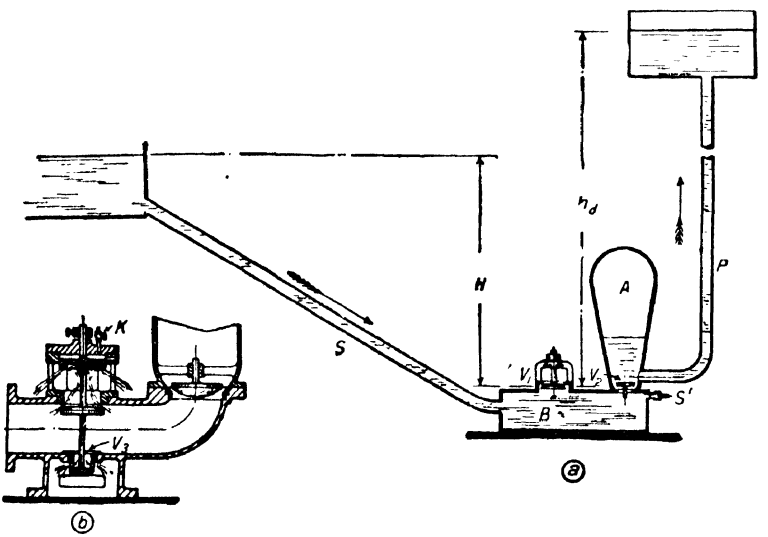


Fig. 346.—Types of Hydraulic Ram.

S (Fig. 330 a), terminating in a valve box B. This valve box is fitted with a waste valve V_1, opening inwards, and a discharge valve V_2 opening outwards and delivering pressure water into an air vessel A, from which it is delivered in a steady stream by the discharge pipe P.

The action of the ram is as follows: The waste valve being opened, water is free to escape, and flow is set up along the supply pipe. The velocity of flow increases under the influence of the supply head until the dynamic pressure on the under side of the valve becomes sufficiently great to overcome its weight. The valve now closes rapidly and the supply column suffers a consequent retardation which gives rise to a rapid increase of pressure in the valve box until this pressure becomes sufficiently great to open the delivery valve.

Water then escapes through this valve into the air vessel, compresses the air, and flows away along the rising main.

As soon as the momentum of the supply column is destroyed the delivery valve closes, the water below the valve partaking of the backward motion thus instituted. This motion, once set up, can only be checked by a reduction of pressure in the valve box below that corresponding to the statical head and consequently the pressure in the valve box is reduced rapidly until at some instant the waste valve reopens, and the whole cycle of operations is repeated. Fig. 347 shows a typical pressure diagram from the valve box of such a ram throughout one complete cycle of operations.

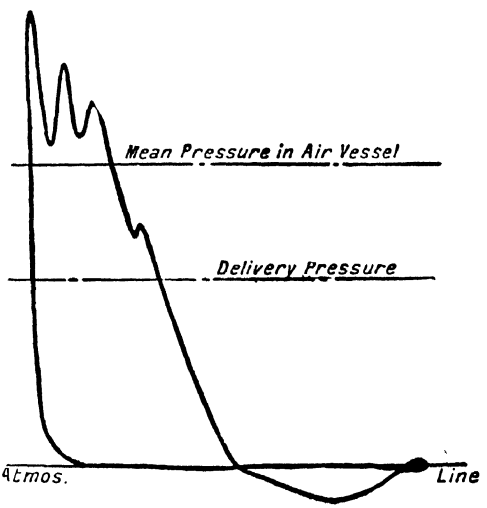


FIG. 347.—Pressure Diagram from Valve Box of Hydraulic Ram.

The idea that the waste valve reopens because the pressure due to the statical head of the supply column is insufficient to keep it to its seat is quite erroneous. In general, the valve will open however light it may be, since the pressure in the valve box is reduced below that of the atmosphere by the reflux action of the water on its rebound. In practice the waste valve is usually made to weigh from ·30 lbs. to ·45 lbs. per square inch of area, so that a statical pressure corresponding to 1 foot head would be sufficient to prevent any opening.

Advantage is taken of this reduction of pressure to keep the air vessel charged, by the introduction of a snifting valve at S_1 (Fig. 346 *a*), air being drawn through this valve into the valve box B when the pressure falls below that of the atmosphere.

The whole cycle, which may only take a fraction of a second to complete, may be divided into four periods, during which the waste valve is respectively opening, wide open, closing, and closed.

Since the force tending to open the valve is equal to its weight together with a much larger force due to pressure differences on its two faces, and since the acceleration of the valve will therefore diminish as its weight

increases, the time of opening will tend to increase slightly as the weight increases, and will also increase with the travel of the valve. For all practical purposes, however, the opening may be considered as instantaneous.

The period during which the valve is full open depends entirely on the time necessary to produce the required velocity of flow along the supply pipe, and will therefore increase with the weight of the valve, and with the ratio of length of supply pipe to supply head.

Thus let :—

H = supply head in feet.
h_d = delivery.
l = length of supply pipe.

a = area of supply pipe in square feet.
a_v = effective discharge area of waste valve.
= actual discharge area multiplied by the co-efficient of discharge.

Then on the assumption of instantaneous opening of the waste valve, the velocity of flow in the supply pipe after t seconds is given by

$$v = c \frac{a}{a_v} \left\{ \frac{1 - e^{-\frac{2c}{k} \cdot t}}{1 + e^{-\frac{2c}{k} \cdot t}} \right\} \text{ ft. per sec.}$$

(equation 12″, p. 244).

where $c = \sqrt{\dfrac{2gH}{1 + \dfrac{fl}{m} \cdot \dfrac{a_v^2}{a^2}}}$; and $k = 2l \dfrac{a_v}{a} \cdot \sqrt{\dfrac{1}{1 + \dfrac{fl}{m} \cdot \dfrac{a_v^2}{a^2}}}$:

Experiments[1] show that this formula gives results in close agreement with those obtained in practice.

The weight of the valve is then adjusted so as to give v a value which, for most efficient working, is approximately $\cdot 4 \sqrt{h_d}$ feet per second.

The time which the valve takes to close cannot be calculated with any pretension to accuracy. It will evidently depend largely on the form of the valve body and of the valve box, and will increase as the lift of the valve, and its weight, increase.

The total time during which it remains off its seat will thus increase with the delivery head and with the length of supply pipe, and will diminish as the ratio $H \div l$ increases, also increasing with its weight and lift.

The time during which it is on its seat increases with the distance from valve seat to delivery air chamber, and increases as the delivery head increases, since the first of these factors regulates the time taken by the reflex pressure wave to reach the waste valve, and the second regulates the time at which the delivery valve closes, and hence the time of initiation of this reflex wave.

[1] Harza. Bulletin of the University of Wisconsin. No. 205, p. 211.

Experiments on a ram having a drive pipe 85·3 feet long with a supply head of 8·15 feet, show the following approximate results:—

h_d ft.	Proportion of cycle during which valve is		
	Wide open.	Closing.	Closed.
10·3	·58	·11	·31
19·3	·78	·09	·13
50·5	·88	·07	·05

Efficiency of Ram.

The efficiency of the ram may be considered from two points of view. If q is the volume delivered by the ram, and Q that escaping through the waste valve, H being the effective supply head, and h_d the effective delivery head measured from the level of the waste valve and including the friction in the delivery pipe line, the total input of energy to the ram is $(Q+q)\,H$, and the total output is $q\,h_d$. The ratio

$$\frac{q\,h_d}{(Q+q)\,H},$$

known as D'Aubuisson's efficiency ratio, then gives the efficiency of the ram as a machine.

The ram may, however, be looked upon as a hydraulically-operated pump, actuated by a volume Q under head H and utilising the energy of this supply to lift a volume q. As this q is initially at a height H, the additional energy given to it in the ram is simply $(h_d - H)$ foot lbs. per lb., and from this point of view the ratio

$$\frac{q\,(h_d - H)}{Q\,H}$$

gives the efficiency of the plant as a whole. This expression is known as Rankine's formula for efficiency.

Although it is difficult to justify the idea of entire separation of the water pumped from the operating water, yet, for the sake of comparison with other types of pump, the latter formula, which gives results consistently lower than the former, will be adopted in this connection.

Whenever the efficiency of a ram is given, it should be stated which formula is used.

Sources of Loss.—These are due to :—

(1) Leakage at waste valve.

(2) Resistance of valves and of supply pipe.

(3) Eddy production due to sudden changes of section.

(4) Loss of resilient energy.

Leakage from the waste valve increases with the time during which it is off its seat; with the lift and area of the valve; and with the velocity of efflux; all of which, with the exception of the valve area, must be increased with an increase in the delivery head.

The area of the valve is usually made from two to four times that of the supply pipe, an increased area with a correspondingly reduced lift tending to efficiency in working.

This is the most serious of all the sources of loss, and usually accounts for between 15 and 25 per cent. of the total energy received.

To enable it to be reduced as far as possible, the weight and travel of the waste valve should be adjustable to suit any given conditions of working. It should be noted that this loss is not necessarily least when the time during which the valve is open is reduced to a minimum, for with a given supply and delivery head there is a certain valve travel below which the pressure will not exceed the delivery head. Increasing the travel increases the velocity of efflux and the time of opening, and therefore the leakage, but at the same time increases the pressure and therefore the proportion of energy entering the air chamber. This goes on up to a certain point, which can only be determined experimentally, where the increased leakage losses counterbalance the proportional gain of energy, and which gives the most efficient working lift.

This point is brought out in Fig. 348 which shows a series of efficiency curves obtained by the author from a small hydraulic ram of the type shown in Fig. 346 *a*, having a supply pipe 4 feet 6 inches long and $1\frac{1}{4}$ inches diameter, and working under a uniform head of 4 feet 6 inches, the waste valve being $1\frac{1}{2}$ inches diameter. From these curves it is evident that although with low delivery heads the efficiency increased with an increase in the number of beats of the waste valve per minute, as the delivery head was increased the speed for maximum efficiency rapidly diminished.

(2) and (3) Valve resistances are approximately independent of head, while loss by shock and frictional losses in supply and delivery pipes increase as the velocity, and therefore as the delivery head, increases.

(4) The loss of energy due to resilience, being proportional to the square of the pressure of the water at the instant of closing the delivery valve, will vary as the square of the delivery head, and also with the mass of water affected, and will therefore increase with the length of the supply pipe and with the ratio $l \div H$.

Evidently, then, the most economical working is to be expected with a ram in which the delivery head is low and in which the ratio $l \div H$ is small. While the latter factor is not essential for fairly efficient working,

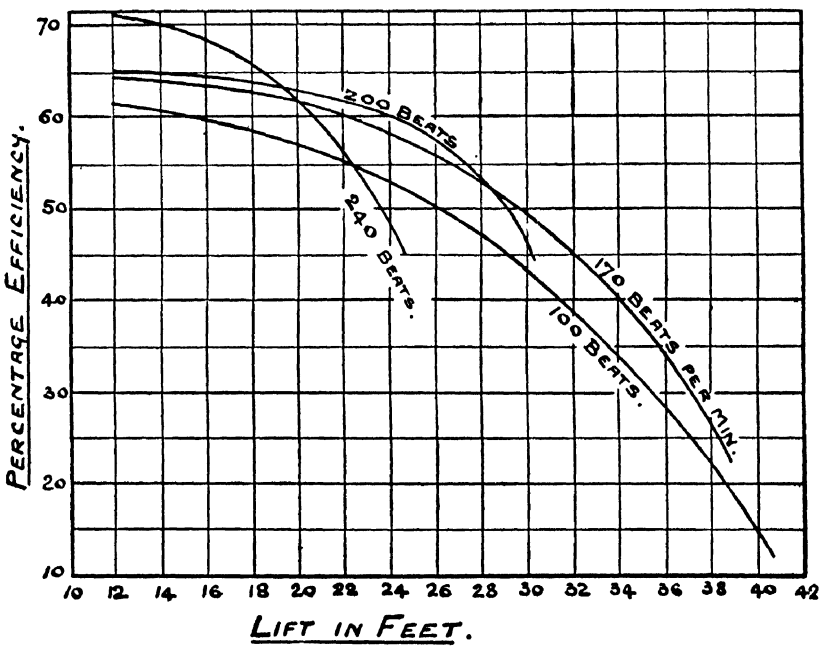


FIG. 348.—Efficiency Curves for Hydraulic Ram working under a constant head of 4·5 feet.

it is advisable where possible that this ratio should not exceed 2·5. However, where necessary this may be largely exceeded, and a supply pipe length of 1,000 feet with a ratio $l \div H = 25$ is well within the limits of everyday practice.

The delivery head may be anything up to about 250 feet and the supply head anything above 18 inches, but the ram becomes very inefficient as the ratio of delivery to supply head becomes great.

The ram will work with this ratio as great as 30 to 1, but under such circumstances has an efficiency not exceeding about 20 per cent. With lower delivery heads, up to about four times that of the supply, the ram

will transform up to 75 per cent. of the energy in the supply reservoir into useful work.

Rankine gives the efficiency in terms of the ratio $\frac{h_d - H}{H}$ as being equal to

$$1{\cdot}12 - {\cdot}2\sqrt{\frac{h_d - H}{H}},$$

and while this cannot be looked upon as being generally true, it indicates how rapidly the efficiency falls off as $\frac{h_d}{H}$ increases. The curves of Fig. 348 also bring out this fact very clearly.

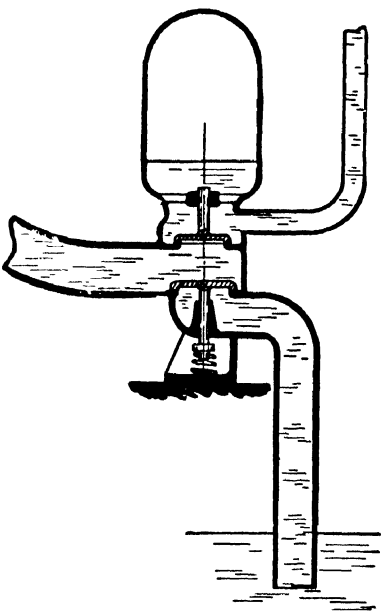


FIG. 349.

The simple type of ram already described gives excellent results where the diameter of supply pipe does not exceed about 4 inches. With larger sizes the shock caused by the sudden closing of the waste valve becomes excessive, and though various devices have been adopted to prevent this, none of them have proved satisfactory as applied to the ordinary ram. One such device is illustrated in Fig. 346 *b*. Here an air cushion, regulated by the air-cock K, is provided for the waste valve, but although this effectively prevents shock, it also prevents any high degree of efficiency being obtained. This is clear if it is remembered that the velocity of efflux of the waste water is increasing the whole of the time that the valve is closing and has its maximum value immediately before the valve comes to its seat, so that leakage during this portion of the cycle is more important than at any other time. Slowness of closing is thus particularly detrimental as the valve approaches its seat, and in fact the more quickly the valve reaches its seat after once beginning to close, the less will be the consequent loss of energy in the waste water expressed as a proportion of the whole kinetic energy of the column. A further drawback to the device lies in the fact that because of this leakage it becomes impossible to pump against a head greater than about six times the supply head.

In the above example the upward pressure of the water on the valve

while closing is increased by the provision of a deflector attached to the valve spindle, this deflector also serving as the piston of the air buffer. An auxiliary waste valve V_3, mounted on the main valve spindle as shown, opens with the main valve, and allows any gritty material or pebbles to escape.

If necessary, the hydraulic ram may be situated well above the tail water level, the waste valve then discharging into a closed chamber which communicates with the tail water by means of a suction tube. The suction head produced in this discharge pipe then helps to increase the velocity of flow on opening the waste valve. The valve must now, however, be aided by means of a spring so as to be approximately balanced under this suction head. Fig. 349 shows diagrammatically an

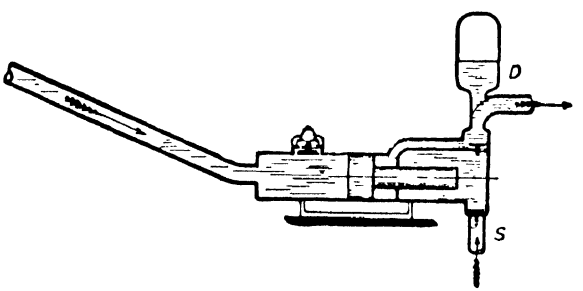


FIG. 350.—Hydraulic Ram for pumping clean water by means of a larger supply of dirty water under a lower head.

arrangement of this type devised by Decœurs, which has given very good results.

Fig. 350 shows the hydraulic ram as arranged for utilizing a large supply of impure water for pumping a smaller supply of clean water. Here S and D indicate the clean water suction and delivery pipes.

In place of the differential plunger pump shown, a flexible diaphragm is sometimes used to divide the waste valve box from the clean water supply and delivery valve box, the vibration of this diaphragm under the action of the ram serving the same purpose as that of the plunger in the previous sketch. As thus constructed, the clean water may be lifted against practically any head though the efficiency is only low.

A simpler device, but one not so certain in its action, consists in the provision of a pipe leading the clean water under a head about $\frac{1}{3}$ that of the supply head into that part of the valve box remote from the supply pipe. A check valve prevents flow out of the valve box along this pipe, and a charge of clean water is drawn into the valve box as the pressure

falls at each beat. The action may be regulated with considerable nicety by the provision of a regulating valve on this secondary supply pipe.

It was not until the invention of the hydraulic engine of Mr. Pearsall that the ram attained its highest development and became a really efficient machine for successfully handling large volumes of water. In general terms, this only differs from the ordinary ram in that its valve is opened and closed by mechanical means, this enabling a cylindrical balanced valve to be used, and the periods of the various portions of a cycle to be regulated to suit any given conditions of working.

Fig. 351 shows the general arrangement of the machine. Here *A* is the supply pipe and *B* the cylindrical waste valve, which is operated by the valve rod *D*, and which allows water to escape by the ports *C*. On closing this valve water enters the chamber *E* without shock and drives out before it the contained air, through a valve regulated by the wooden float *F*. When the water reaches a certain height the valve at *F* closes, and the pressure in *E* rises until sufficient to lift the delivery valves at *G*. The small remaining volume of compressed air and the water then enter the air vessel *H*, from which the water is led away along the delivery pipe *T*. A second air chamber *S* is sometimes fitted, but is not essential. The valve *B* is now opened, the rush of water out of *E* and down the supply pipe is followed by its closure, and the cycle of operations is repeated as before.

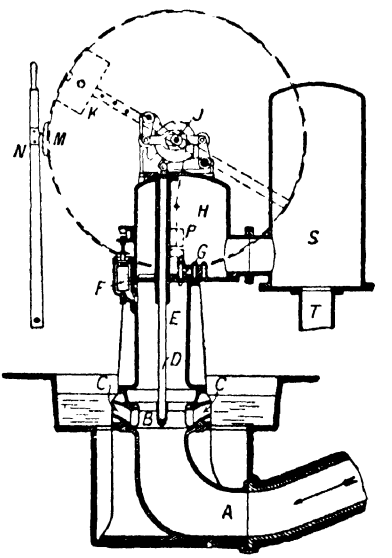


FIG. 351.—Pearsall's Hydraulic Engine or Ram.

The method of working this valve is ingenious. The shaft *J* carries a pendulum *K*, which swings through an arc of about 240°, and also a cam so proportioned as to divide the time of a swing of the pendulum into two parts suitable for the flow and delivery parts of the cycle. This cam regulates the motion of the valve rod.

In order to maintain the swing of the pendulum against friction, a crank on *J* is coupled to a piston in the small single-acting cylinder *P*, into which air is admitted from the air vessel at each double stroke, so as to give a slight impetus to the pendulum at the middle of its swing.

The engine is stopped and started by the bar *N*, which, when held against the pendulum, engages with the ratchet *M* when this begins to descend. This stopping takes place immediately before the end of a working stroke, when the main valve is closed and when the flow of water in the main pump has for the moment ceased. On releasing the ratchet, the ram at once engages on a normal stroke.

As thus constructed, the machine is capable of dealing with practically any quantity of water, and works as noiselessly as a pumping engine. The smoothness of its working as compared with that of the ordinary type of ram may be inferred from Fig. 352, which shows a diagram taken from the valve box of a ram of this type.

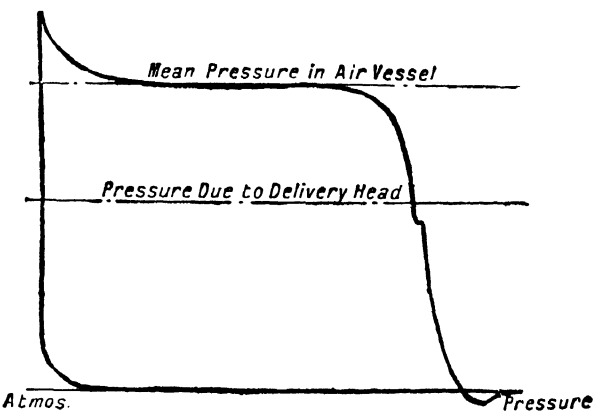


FIG. 352.—Pressure Diagram from Pearsall's Hydraulic Ram.

Recent enquiries (1945) indicate that no machines of this type have been built for a good many years. While it may now be considered as obsolete, historically it is of much interest.

Hydraulic Ram for Air Compression.—With slight modifications the hydraulic ram may be adapted for use as an air compressor. Thus in the Pearsall ram the chamber *E* is enlarged so as to hold the whole volume of air compressed in a single stroke, while the valve at *F* is modified so as to confine this air, and the delivery valves at *G* are slightly modified. The cylinder *P* is also now worked by pressure water from the air vessel *E* instead of by compressed air. As thus constructed, efficiencies of upwards of 80 per cent. may be obtained.

Fig. 353 shows a type of air compressor on somewhat similar lines

designed by M. Sommellier and used in the work on the Mont Cenis Tunnel. Here the inlet valve V_1 and the waste valve V_2 are mechanically driven, and are coupled together. When the valve V_2 is opened the water in the compressor finds its own level and air at atmospheric pressure is drawn in through the air valve V_4. V_2 is then closed and V_1 opened, allowing pressure water to flow along the pipe A, and in virtue of its pressure and momentum to compress the air in the chamber B, whence it passes through the delivery valve V_3 into the air reservoir. V_1 is then closed, V_2 opened, and the cycle of operations repeated as before. Working under a head of 85 feet, this compressor delivers air at a pressure of

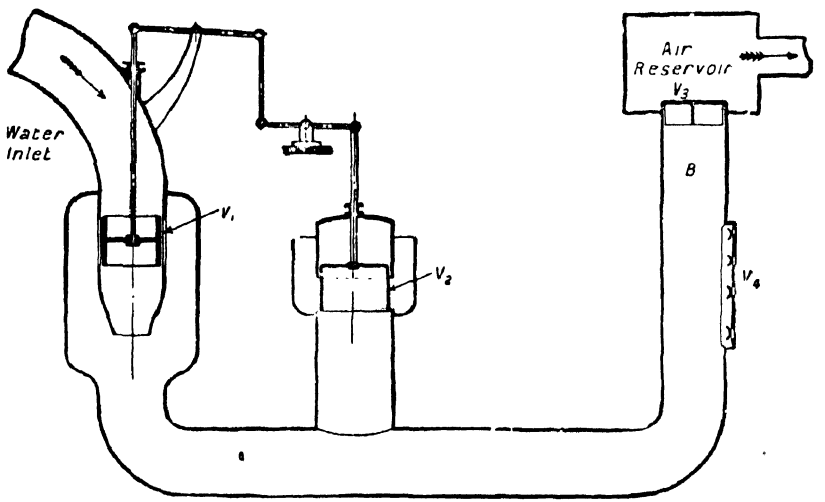


FIG. 353.—Hydraulic Air Compressor.

75 lbs. per square inch. The machine is, however, somewhat cumbrous for the amount of work which it is capable of performing.

ART. 213.—THE JET PUMP.

The fact that the pressure energy of a water supply may be converted into kinetic energy, with a consequent reduction of pressure, is taken advantage of in the type of jet pump devised by Professor James Thomson (about 1852). The pump, as usually constructed, is illustrated in Fig. 354. Here water from the source of supply is led through the converging passage P, its pressure diminishing as its velocity increases, and is finally discharged into the delivery pipe through the diverging passage D. It follows that at the section J, where its velocity is greatest, its pressure

may become considerably less than that of the atmosphere. A chamber surrounding J is connected to the supply of water to be pumped, by the suction pipe S. The reduction of pressure in this chamber is then accompanied by a flow of water along the suction pipe, which, meeting the high pressure jet, is carried forward as a combined stream into the diverging discharge pipe D. Here its kinetic energy is partially reconverted into pressure energy, which is utilized in overcoming the head to be pumped against.

Let h_1 = height of supply head above jet in feet.
„ h_s = height of jet above suction supply „
„ H_d = height of delivery head above jet „

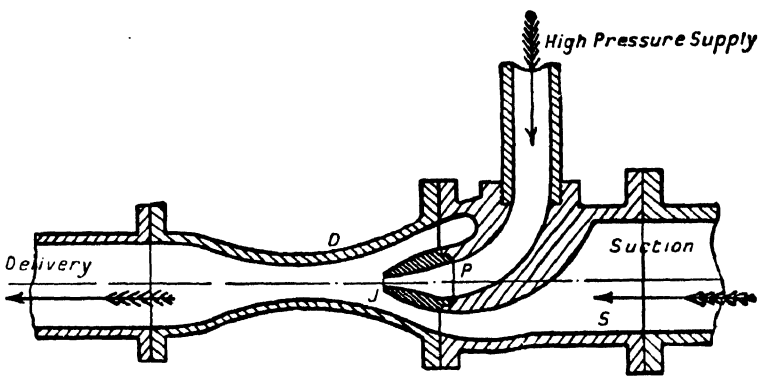


FIG. 354.—Jet Pump.

Let $h_d = \left\{\frac{p_d}{W} + \frac{v_d^2}{2g}\right\} = H_d +$ losses between point D and delivery.
„ a_1 = area of nozzle in square feet.
„ a_s = area of annular suction pipe in plane of nozzle, square feet.
„ a_d = area of mixing chamber at throat, square feet.
„ Q = volume passing per second, v = velocity, and p the pressure in lbs. per square foot at the point denoted by a suffix.

On the assumption that the sides of the mixing chamber are sensibly parallel from the nozzle until the jets have attained a common velocity and pressure at D, so that, neglecting friction, the sides of the chamber exert no force on the water, we may apply the equation of momentum, which now becomes:—

$$\frac{Q_d v_d}{g} - \left\{\frac{Q_s v_s}{g} + \frac{Q_1 v_1}{g}\right\} = \frac{p_1 a_1 + p_s a_s - p_d a_d}{W}$$

$$\therefore \quad \frac{a_d v_d^2 - a_s v_s^2 - a_1 v_1^2}{g} = \frac{p_1 a_1 + p_s a_s - p_d a_d}{W}$$

$$\therefore \quad a_1 \left(\frac{p_1}{W} + \frac{v_1^2}{2g}\right) + a_s \left(\frac{p_s}{W} + \frac{v_s^2}{2g}\right) - a_d \left(\frac{p_d}{W} + \frac{v_d^2}{2g}\right) = \frac{1}{2g} \{ a_d v_d^2 - a_s v_s^2 - a_1 v_1^2 \}$$

$$\text{or} \quad a_1 h_1 - a_s h_s - a_d h_d = \frac{1}{2g} \{ a_d v_d^2 - a_s v_s^2 - a_1 v_1^2 \} \qquad (1)$$

Again, for continuity of flow, we have:—

$$\left.\begin{aligned} a_1 v_1 + a_s v_s &= a_d v_d \\ \text{or} \quad Q_1 + Q_s &= Q_d \end{aligned}\right\} \qquad (2)$$

From these equations, when the dimensions of the pump and the heads are given, any two unknown velocities, and hence quantities, may be determined. For example, if Q is given, Q_s and Q_d may be determined. If, in addition, we assume that the pressure across the mixing chamber immediately in front of the nozzle is uniform, and equal to p_j (an assumption which is only true so long as both streams are parallel), we have

$$\left.\begin{aligned} h_1 &= \frac{v_1^2}{2g} + \frac{p_j}{W} \\ -h_s &= \frac{v_s^2}{2g} + \frac{p_j}{W} \end{aligned}\right\}$$

$$\therefore \quad \frac{v_1^2}{2g} - h_1 = \frac{v_s^2}{2g} + h_s$$

$$\text{or} \quad v_s^2 = v_1^2 - 2g(h_1 + h_s). \qquad (3)$$

Introducing this value of v_s in (1) we get:—

$$a_1 h_1 - a_s h_s - a_d h_d = \frac{1}{2g} \{ a_d v_d^2 - (a_1 + a_s) v_1^2 \} + a_s (h_1 + h_s)$$

$$\text{or} \quad (a_1 - a_s) h_1 - 2 a_s h_s - a_d h_d = \frac{1}{2g} \{ a_d v_d^2 - (a_1 + a_s) v_1^2 \}. \qquad (4)$$

While, by substitution in (2):—

$$a_1 v_1 + a_s \sqrt{v_1^2 - 2g(h_1 + h_s)} = a_d v_d. \qquad (5)$$

From equations (3), (4) and (5), if the areas of the passages and the various heads are given, the velocities v_1, v_s and v_d, and thus the quantities Q_1, $Q_s + Q_d$, may be determined.

Example.

Thomson's Jet Pump.

$h_1 = 40$ feet.	$a_1 = \cdot 2$ square feet.
$h_s = 15$ feet.	$a_s = \cdot 4$ square feet.
$h_d = 10$ feet.	$a_d = \cdot 6$ square feet.

[1] Negative sign because $\left(\frac{p_s}{W} + \frac{v_s^2}{2g}\right)$ is negative if h_s is positive.

Determine Q_1, Q_s and Q_d, and also the pressure $\frac{p_d}{W}$ at the throat of the mixing cone and $\frac{p_j}{W}$ in the plane of the orifices.

From equation (4) we have :—

$$-\{\cdot 2 \times 40 + \cdot 8 \times 15 + \cdot 6 \times 10\}\frac{64\cdot 4}{\cdot 6} = v_d^2 - v_1^2;$$

$$\therefore v_1^2 - v_d^2 = 2{,}790. \quad \text{(i)}$$

Again, from (5),

$$(\cdot 6\, v_d - \cdot 2 v_1)^2 = \cdot 16\,\{v_1^2 - 64\cdot 4 \times 55\}$$

$$\therefore v_1^2 + 2\, v_1\, v_d - 3\, v_d^2 = 4{,}718. \quad \text{(ii)}$$

Substituting for v_d^2 in (ii) from (i) :—

$$v_1^2 + 2\, v_1 \sqrt{v_1^2 - 2{,}790} - 3\,(v_1^2 - 2{,}790) = 4{,}718.$$

$$\therefore v_1^2 - 1{,}826 = v_1 \sqrt{v_1^2 - 2{,}790}.$$

Squaring both sides, we get, on reduction :—

$$862\, v_1^2 = 3{,}333{,}000$$

$$\text{or } v_1^2 = 3{,}866$$

$$\therefore v_1 = 62\cdot 2 \text{ feet per second.}$$

Substituting this value in (i) :—

$$v_d = \sqrt{3{,}866 - 2{,}790} = 32\cdot 8 \text{ feet per second;}$$

while from equation (3) :—

$$v_s = \sqrt{3{,}866 - 3{,}542} = 18\cdot 0 \text{ feet per second.}$$

$$\therefore Q_1 = \cdot 2\, v_1 = 12\cdot 4 \text{ cubic feet per second.}$$

$$Q_d = \cdot 6\, v_d = 19\cdot 7 \text{ cubic feet per second.}$$

$$Q_s = \cdot 4\, v_s = 7\cdot 2 \text{ cubic feet per second.}$$

Again, since $\frac{p_j}{W} + \frac{v_1^2}{2g} = h_1$:—

$$\frac{p_j}{W} = 40 - 60\cdot 1 = -20\cdot 1 \text{ feet of water.}$$

while since $h_d = \frac{p_d}{W} + \frac{v_d^2}{2g}$:—

$$\therefore \frac{p_d}{W} = 10 - 16\cdot 7 = -6\cdot 7 \text{ feet of water.}$$

The actual height through which the water may be forced by the pump is less than the value h_d given by $\left\{\frac{p_d}{W} + \frac{v_d^2}{2g}\right\}$ feet, because of the loss of energy by eddy formation in the diverging discharge pipe, and may amount to between ·6 and ·7 h_d.

This loss of energy is proportional to v_d^2, and will therefore increase—since the necessary value for v_d increases—as h_d increases. Consequently,

since the total work done in pumping is proportional to $h_d + h_s$, while this loss depends only on h_d, the efficiency will increase as h_s is increased at the expense of h_d.

It follows that with a given total lift, the suction head should be increased as far as possible—up to about 22 feet—at the expense of the delivery head. This conclusion is borne out in practice.

The efficiency of the pump is given by—

$$\eta = \frac{Q_s (H_d + h_s)}{Q_1 (h_1 - H_d)}.$$

This efficiency is of necessity low, since the action depends on the mixing of two streams moving with different velocities, and hence involves considerable loss by shock. *E.g.*, in the numerical example considered on p. 677, the efficiency (assuming $H_d = \cdot 7\ h_d$) is given by—

$$\eta = \frac{7 \cdot 2\ (7 + 15)}{12 \cdot 4 \times 33} = \cdot 387.$$

Actually, frictional losses reduce the efficiency still further, and the maximum efficiency attained in practice, even with a pump placed at delivery level, is about 25 per cent. This is increased to about 30 per cent. where, as when used for delivering a high velocity jet of water for fire purposes, the necessity for converting the kinetic energy of the jet into pressure energy is absent.

For continuous pumping and drainage operations where a fair pressure supply is obtainable, and where the volume to be lifted and the working head are small, the method offers the advantages of simplicity and low first cost, while practically no attention is required. Unless the supply head is large compared with the lift, the ratio $\frac{Q_s}{Q}$ is, however, very small—often so low as $\frac{1}{5}$.

The principle of the steam injector, as fitted for boiler feed purposes is identical with that of the jet pump. The preceding equations, however, need to be modified, since the streams of fluid on impinging are not of equal density, although they become so on condensation of the high velocity steam jet.

The principle of the jet pump has been applied in an intensifier for raising the pressure of a large quantity of low pressure water by means of a small supply at high pressure, the delivery pipe D leading directly into the cylinder of the intensifier. While not economical from an energy standpoint, the simplicity and low first cost of the apparatus render it very suitable for such work where its use is only occasional.

Art. 214.—The Injector Hydrant.

Since the chief loss of energy in the ordinary jet pump is due to shock at the collision of the two jets, it would appear that the total loss might be diminished by diminishing the velocity of the high pressure, or increasing that of the low pressure jet in stages, instead of at a single impact.

This method has been applied with success by Mr. Greathead in the

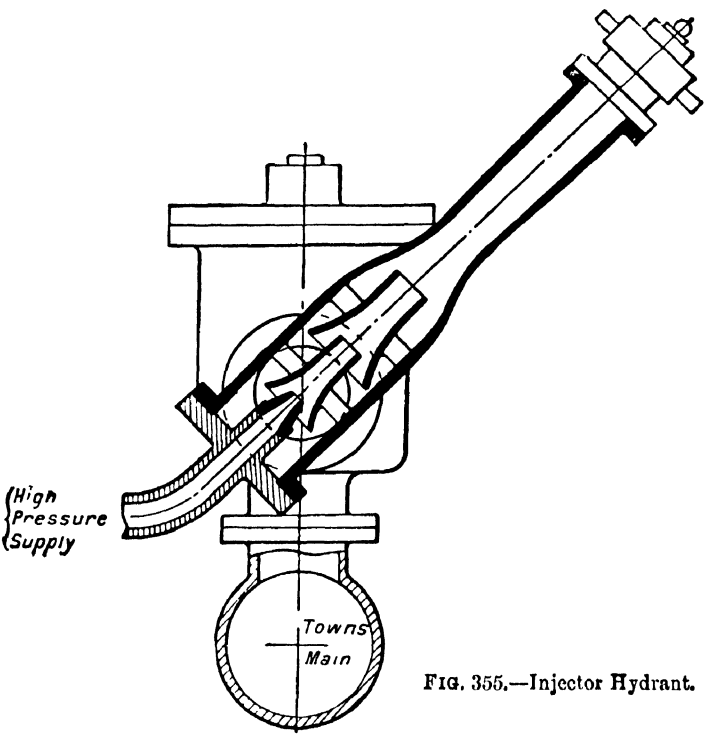


Fig. 355.—Injector Hydrant.

construction of his injector hydrant (Fig. 355), which is adapted for fire extinguishing purposes where a continuous supply of high pressure water is available, as is the case near the pipe line from an hydraulic power station. By itself, the water in the power main is of insufficient volume to have any appreciable effect on a fire, but when used with an injector hydrant in connection with a low pressure main the advantages of the system are very great. As exemplifying the effect of a small jet of high pressure water in increasing the height of the main jet, it is stated that while an ordinary $1\frac{1}{8}$-inch hydrant supplied from a main at 40 lbs. pressure will

give a stream about 50 feet high, the same when reinforced by a $\frac{3}{8}$-inch jet of water at 750 lbs. per square inch will lift a slightly greater (about 15 per cent.) volume of water from the main and will deliver this as a jet 85 feet high.

The efficiency of the hydrant from an energy point of view ranges from 28 per cent. to 33 per cent.

The following table, given by Mr. Greathead[1], shows the quantity of high pressure water, at 700 lbs. per square inch, required to deliver a jet of 150 gallons per minute through a 1-inch nozzle, through a height variously estimated to be from 75 to 84 feet, and requiring a head at the nozzle of 100 feet. Here allowance is made for 200 feet of $2\frac{1}{2}$-inch hose, the resistance of which is equivalent to 50 feet head.

Low Pressure Supply.		High Pressure Supply.
Lbs. per square inch.	Feet.	Gallons per minute.
60	139	3·7
50	115	10·9
40	92	18·1
30	69	25·2
20	46	32·4
10	23	39·6

Art. 215.—The Air Lift Pump.

Among other devices for pumping liquids against a large head, that known as the air lift pump is worthy of notice. Invented probably by Carl Löscher about 1797, the system fell into comparative desuetude for many years, and has only recently been revived and improved. In view of its increasing use, and of its adaptability to many difficult cases of pumping, it is worth while considering the system somewhat in detail. Briefly, the method consists in sinking an open vertical pipe with its lower end submerged in the liquid to be raised, and having its upper end arranged to discharge into a reservoir at the required height. Air from a compressor is then forced through a smaller air pipe into the submerged opening of the lift pipe or rising main. The air bubbles, rising through the water in the lift tube, so reduce the specific gravity of the mixture, and therefore the weight of the column, that the excess pressure at the

[1] "Proceedings Institute Mechanical Engineers," 1879, p. 364.

base of the column, due to the external water pressure, becomes sufficiently great to force the mixture above the supply level and out of the top of the

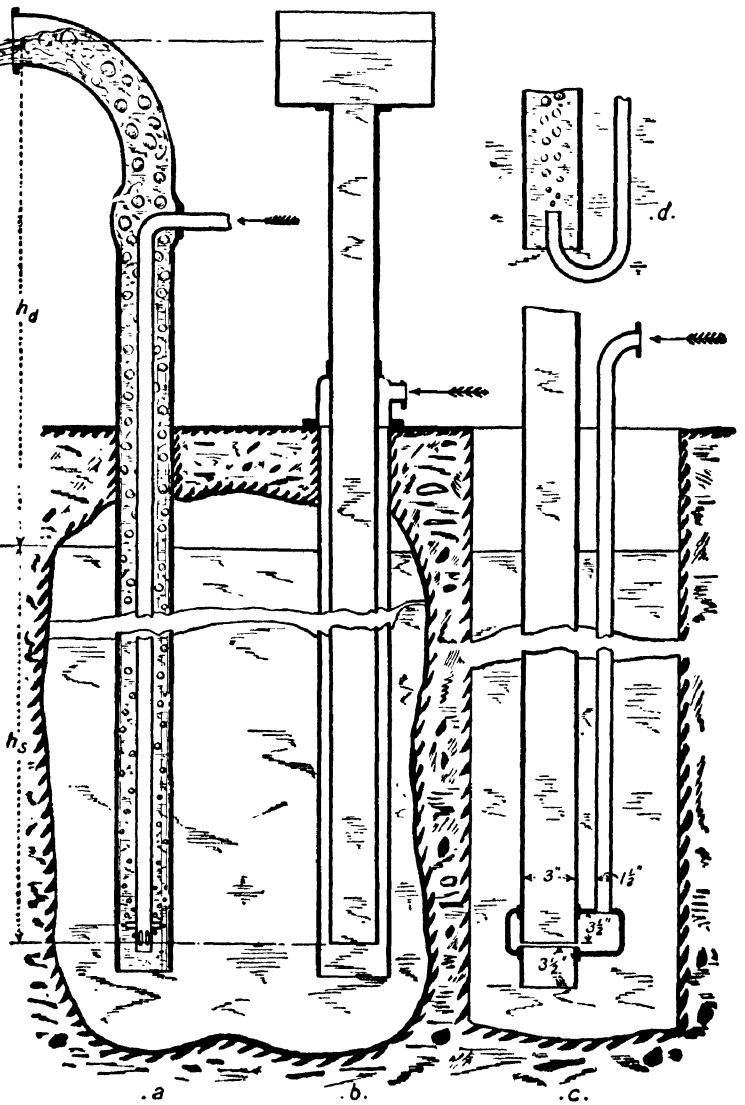


FIG. 356.—Air Lift Pumps.

pipe. This excess pressure increases with the depth of submersion of the pipe, and the latter must therefore be regulated to suit the height to which water is to be lifted.

In general, the depth of submersion h_s (Fig. 356) is made from (1·5 to 2) times the lift h_d, so that the total length of lift pipe = (2·5 to 3) h_d. As the depth of immersion is reduced, the relative air consumption increases and the efficiency diminishes. As will be shown later, a further increase in h_s tends to more efficient working, and is advisable where this may be obtained without great expense in deepening the bore hole.

To keep down frictional losses, the velocity in the lift tube, calculated on the volume of water discharged, should not exceed 5 feet per second. The efficiency of the system, as calculated from the ratio of the work done in lifting water through a height h_d, to the indicated work in the air compressor engine cylinder, is generally between 25 and 30 per cent.—this allowing for a compressor efficiency of 75 per cent.—but under favourable conditions may rise to 45 per cent.

There are three methods of arranging the pipe lines in a well or bore hole.

(1) The central air tube system (Fig. 356 *a*), in which the air pipe is suspended in the centre of the lift tube.

(2) The annular air tube system, in which the space between the lift tube and the bore hole is used as the air line (Fig. 356 *b*).

(3) The side-by-side system, in which the air and lift tubes are carried down the well side by side (Fig. 356 *c*).

The first of these systems has the disadvantage that the hydraulic mean depth of the water passage in the lift tube is reduced by the air tube, being given by—

$$\frac{\pi (R^2 - r^2)}{2\pi (R + r)} = \frac{R - r}{2} = \frac{D - d}{4},$$

where D and d are the internal diameter of the lift tube and the external diameter of the air tube respectively.[1]

This leads to increased frictional losses and so to diminished efficiency. The system is, however, very suitable for application to a small bore hole of suitable dimensions, since the only additional expense is the provision of the comparatively small air pipe, while any alteration in the length of this, to suit different conditions of working, is a simple matter. In general this will be found more advantageous than the second system, the chief advantage of the latter lying in its possibilities of more effective air distribution.

Where the well or bore hole is of large diameter, the side-by-side system has many advantages in virtue of its accessibility, simplicity, and flexibility.

[1] With a plain tube of the same sectional area the hydraulic mean depth is given by $\frac{D_1}{4}$, where $D_1 = \sqrt{D^2 - d^2}$.

In every case provision must be made for distributing the air evenly and in small bubbles among the water in the lift tube, since experiment shows that the system is then much more effective than where the air bubbles are so large as to fill the tube. The diameter of the bubbles at their initiation should be about $\frac{1}{4}$ inch. Some such arrangement of foot box, as shown in Fig. 356 *c*, is essential for efficient working. Comparative tests show that this type, in which air enters all around the circumference of the lift tube gives efficiencies 20 per cent. greater than that shown in Fig. 356 *d*, in which the air is supplied in a single central jet.

Theory of the Air Lift Pump.

Let V_w = volume of water raised in cubic feet per second.

„ V_m = mean volume of air used, in cubic feet per second, during its passage through the rising main.

„ V_a = volume used in cubic feet per second, at atmospheric pressure, p_a.

„ p_1 = pressure at base of rising main.

„ $h_d + h_s = l$ feet.

Then, assuming isothermal expansion of the air in the lift tube, we have:—

$$V_m = \frac{p_a}{p_1 - p_a}\left(\log_e \frac{p_1}{p_a}\right) V_a \qquad (1)$$

The mean specific gravity of the mixture in the tube $= \frac{V_w}{V_w + V_m}$.

$\therefore$ Head producing flow $= h_s - (h_d + h_s)\frac{V_w}{V_w + V_m}$ feet of water,

$= h_s\left(\frac{V_w + V_m}{V_w}\right) - (h_d + h_s)$ feet of mixture in the lift tube,

$= \frac{V_m}{V_w} h_s - h_d$ feet of mixture.

Equating this to the sum of the friction head h_f, and the kinetic head h_v, we have, on reduction:—

$$V_m = V_w \frac{h_d + h_f + h_v}{h_s},$$

$$\therefore\ V_a = V_w \frac{p_1 - p_a}{p_a \log_e \frac{p_1}{p_a}} \cdot \frac{h_d + h_f + h_v}{h},$$

$$= \frac{V_w (h_d + h_f + h_v)}{34 \log_e \frac{p_1}{p_a}} \cdot \left(\text{since } \frac{p_1 - p_a}{p_a} = \frac{34 + h_s - 34}{34} = \frac{h_s}{34}\right).$$

This relation enables the volume of free air per cubic foot of water to

be determined in terms of h_f and h_v. Writing $h_v = \frac{1}{2g}\left\{\frac{V_a + V_w}{A}\right\}^2$, and putting $h_f = \frac{f(h_d + h_s)\, v^2}{2\, g\, m}$ (where v^2 is the mean square of the velocity in the tube) this latter term may be taken as approximately equal to $\frac{f(h_s + h_d)}{2\, g\, m}\left\{\frac{V_a + 2\, V_w}{2\, A}\right\}^2$, in which case the ratio $V_a \div V_w$ may be directly obtained in terms of h_d, h_s, A and f.

The author has determined the values of f for a mixture of air and water (the friction head being expressed in feet of a column of the mixture in the tube) from the published data of a large number of trials on such pumps.

Making the above assumption as to the value of v, the value of f varies from ·033 in the case of a 3-inch pipe with $v = 12{\cdot}16$ f.s. to ·023 in a 12-inch pipe with $v = 6{\cdot}5$ f.s. In every case f has a value very approximately six times that obtaining for the flow of water alone at the same velocity.

In practice the following approximate values of $V_a \div V_w$ are found to give the best results :—

h_d (feet)	10	20	30	50	100
$V_a \div V_w$	1·0	1·5	2·0	2·5	3·0

Efficiency of the Air Lift Pump.—Assuming isothermal compression, the work done on the air during compression from p_a to p_1 $= p_a\, V_a \log_e \frac{p_1}{p_a}$ foot lbs.

The useful work done by the air in raising V_w cubic feet of water through a height h_d feet $= 62{\cdot}4\ V_w\, h_d$ foot lbs.

∴ Efficiency—

$$\eta = \frac{62{\cdot}4\ V_w\, h_d}{p_a\, V_a \log_e \frac{p_1}{p_a}} = \frac{62{\cdot}4\ V_w\, h_d \times 34 \log_e \frac{p_1}{p_a}}{p_a \log_e \frac{p_1}{p_a} \times V_w\,(h_d + h_f + h_v)}$$

$$= \frac{h_d}{h_d + h_f + h_v} \qquad (2)$$

On the assumption of adiabatic compression from p_a to p_1, the work done on the air $= 3{\cdot}463\ p_a\, V_a \left\{\left(\frac{p_1}{p_a}\right)^{\cdot 29} - 1\right\}$ foot lbs., so that the efficiency

$$= \frac{h_d \log_e \frac{p_1}{p_a}}{3{\cdot}463 \left\{ \left(\frac{p_1}{p_a}\right)^{\cdot 29} - 1 \right\} \{h_d + h_f + h_v\}} \qquad (3)$$

From expression (2) it appears that as h_d is increased from zero (keeping h_s constant) the efficiency will increase until some point is reached at which the (velocity of flow)^2 increases more rapidly than does h_d. This gives the value of h_d for maximum efficiency. In practice it is found that the ratio $h_s \div h_d$ then lies between 1·5 and 2·5, the larger values of the ratio being used with low values of h_d.

The following table gives the results of tests on a plant of the type illustrated in Fig. 356 *c*, carried out by Professor Jossé at Charlottenburg:—

Diameter of lift tube, inches	3 1/16	3 1/16	3 1/16	3 1/16	3 1/16	7 1/2	2 3/4	2 3/4	2 3/4
Diameter of air pipe, inches.	1 9/16	1 9/16	1 9/16	1 9/16	1 9/16	5	—	—	—
Diameter of bore hole, inches	6 1/8	6 1/8	6 1/8	6 1/8	6 1/8	—	—	—	—
Depth of immersion, h_s feet	49·2	49·2	49·2	49·2	49·2	63	69·2	72·9	62·0
Height of lift, h_d feet	24·6	24·6	24·6	24·6	24·6	43·5	50·5	46·7	57·8
V_w Gallons per minute	24·2	68·3	94·0	96·8	94·8	800	47·5	51·1	39·2
V_w Cubic feet per second	·065	·177	·251	·259	·253	2·35	·127	·137	·105
V_a cub. ft. at atmospheric pressure	·127	·309	·739	·842	1·766	6·46	·316	·334	·346
$\frac{V_a}{V_w}$ = volume of air per cub. ft. of water	1·96	1·75	2·94	3·68	7·50	2·75	2·49	2·45	3·30
Velocity of flow at entrance, f.s.	2·54	3·46	4·90	5·07	4·61	7·65	—	—	—
Mean velocity of mixture in tube	5·16	6·64	12·46	14·87	22·80	20·8	—	—	—
Efficiency $\left\{\frac{\text{Water H.P.}}{\text{I.H.P. in compressor cylinder.}}\right\}$	39·8%	44·9%	26·7%	21·3%	10·6%	38·4%	44·5%	32·9%	12·3%

For high efficiency the footpiece should be such as to give an even distribution of the air in comparatively small bubbles through the rising main. Fig. 357 shows a design used in an installation pumping against a head of 239 feet and giving a maximum efficiency of 51·9 per cent.[1] Fig. 358 shows the design used in tests by Dr. J. S. Owens [2] of which the results are given in the following table, and in which efficiencies exceeding 65 per cent. were attained. In these tests the diameter of the delivery pipe was $6\frac{5}{8}$ inches.

Test No.	1	2	3	4	5
Dip, or submergence, in feet	156·0	156·0	156·0	156·0	156·0
Lift, in feet	105·0	105·0	105·0	105·0	105·0
Submergence per cent., *i.e.*— $\frac{\text{submergence} \times 100}{\text{lift} + \text{submergence}}$	60	60	60	60	60
Water raised per minute, cubic feet	76·0	74·0	67·0	82·5	85·4
Volume of free air used per minute in cubic feet	253	244	218	296	254
Volume of free air used in cubic feet per cubic foot of water raised	3·33	3·30	3·23	3·59	3·00
Water horse-power	16·2	15·8	14·35	17·62	18·20
Air horse-power	28·8	27·7	24·8	33·6	29·4
Efficiency	56·3	57·1	57·9	52·5	62·0
Throat-piece No.	1 4·59 in. diam.	1	1	1	2 3·25 in. diam.

Test No.	6	7	8	9	10
Dip, or submergence, in feet	156·0	156·0	158·5	158·5	158·5
Lift, in feet	105·0	105·0	97·7	97·7	97·7
Submergence per cent., *i.e.*— $\frac{\text{submergence} \times 100}{\text{lift} + \text{submergence}}$	60	60	62	62	62
Water raised per minute, cubic feet	73·4	69·2	80·4	80·0	76·0
Volume of free air used per minute in cubic feet	209	199	238	238	222
Volume of free air used in cubic feet per cubic foot of water raised	2·85	2·88	2·96	2·98	2·92
Water horse-power	15·70	14·80	16·04	15·99	15·19
Air horse-power	24·2	22·4	26·63	26·63	24·84
Efficiency	64·9	66·1	60·23	60·05	61·16
Throat-piece No.	2	2	3 3·9 in. diam.	2	3

[1] "Eng. News Record," Vol. 82, p. 1111.

[2] British Association, Section G, 1921.

A very extensive series of experiments carried out by the Westinghouse Air Brake Company in a 6-inch well, 174 feet deep, at Wilderning, Pa., lead to the following general conclusions:—

(1) The rate of delivery of water, and the air consumption per gallon,

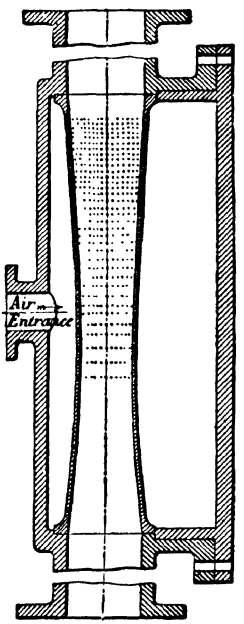


FIG. 357.

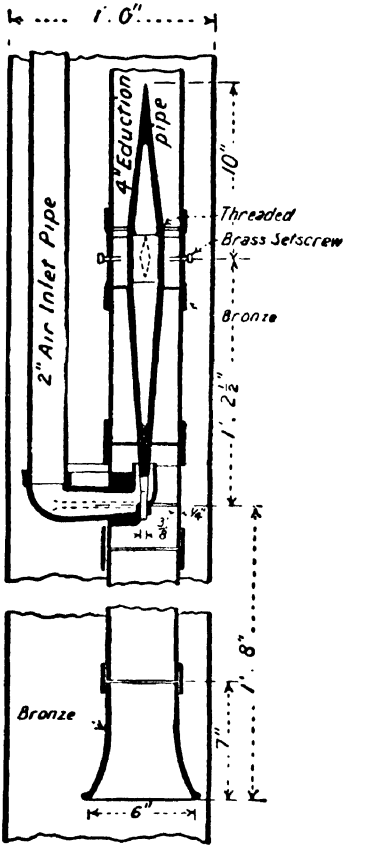


FIG. 358.

with fixed size of discharge pipe, are practically constant for all lifts, provided the ratio of lift to submergence is maintained constant.

(2) With a discharge pipe of given diameter, the delivery decreases and the air consumption per gallon increases as the ratio of lift to submergence increases.

(3) With a fixed ratio of lift to submergence, the air consumption per gallon decreases as the size of discharge pipe increases.

(4) The least air pressure that will give continuous flow is the proper pressure to use. A slightly lower pressure gives intermittent delivery and the amount delivered is much decreased, though the air consumption per gallon is slightly lower than with continuous flow. With pressure higher than just enough to give continuous flow, the delivery is increased somewhat, but the air consumption per gallon delivered is increased in greater ratio; and with further increase in air pressure a point of maximum delivery is reached, beyond which the delivery is decreased in amount. The sound of the discharge is a reliable guide to proper regulation of the air supply.

(5) It appears from (2) that by increasing the submergence, *i.e.* locating the foot piece deeper down in the water, for a given lift, the air consumption is progressively reduced. But as the required air pressure is increased with the greater depth, a cubic foot of air represents greater power. A curve representing the variation of horse-power required per gallon of water delivered, with depth varying, shows that the power first decreases with increasing depth, then reaches a minimum and thence increases. The ratio of lift to submergence at this minimum point may be called the "economical ratio."

(6) For a given size of discharge pipe the economic ratio decreases as the lift increases; *i.e.*, the submergence should be increased in greater ratio than the lift. For a given lift, the economical ratio increases (submergence decreases) as the size of discharge pipe increases.

(7) A tail piece, or projection of the discharge pipe below the air inlet, is essential in starting, as it tends to prevent the air from backing down into the well and rising in the casing outside the discharge pipe.

(8) Anything in the shape of a jet or pipe introduced into the discharge pipe to serve as air inlet has no value, and is, in fact, detrimental by forming an obstacle to the free passage of water.

(9) In starting the pumping the air should be admitted slowly. Pumping will not commence immediately, but after several seconds, perhaps even a minute, water will come with a rush. This is followed by a lull, after which the operation becomes more uniform. The valve can then be opened until continuous flow is obtained.

For deep well pumping, the air lift pump is very suitable, since it can be used in any bore hole of sufficient diameter to admit the necessary pipe lines, and can take advantage of the whole discharging capacity of the hole.

In addition, the absence of moving parts below the surface, the possibility of installing the compressing plant at practically any distance

from the bore hole, and of pumping corrosive liquid or water carrying solid matter in suspension, together with the certainty of operation, give the system very obvious advantages, and in many instances greatly outweigh the disadvantage of moderate efficiency.

With hot liquids, too, the efficiency is augmented, since the volume of

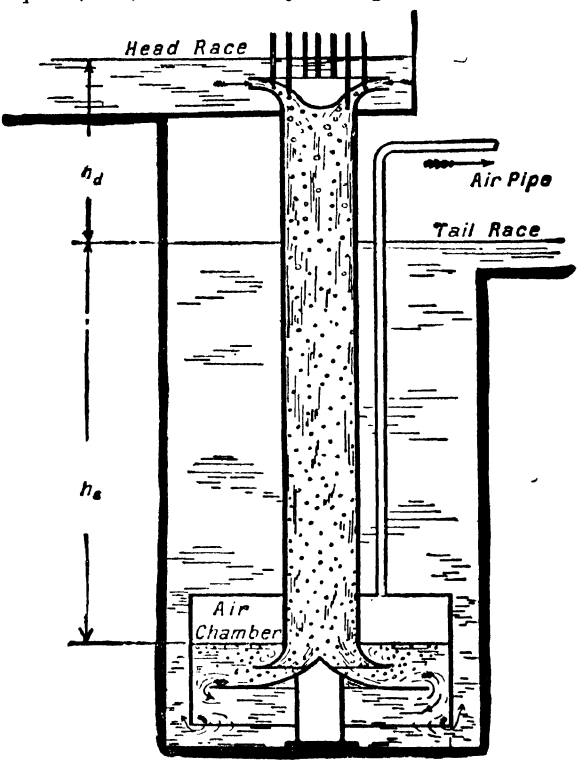


FIG. 359.—Hydraulic Air Compressor.

air in the rising main is increased by the rise in the temperature, while in many instances the aerating effect of the air is an advantage.[1]

ART. 216.—HYDRAULIC AIR COMPRESSOR.

By reversing the action of the air lift pump, and allowing water, under a head h_d, to flow down a vertical pipe of length $(h_d + h_s)$, which has a

[1] Further information on this subject may be obtained from the following papers :—
"Proceedings Institute Civil Engineers," vol. 163, 1905-6, part I., p. 353.
"British Association of Waterworks Engineers," 1903.
Engineer, January 10, 1908, p. 26.
"Proc. Inst. C. E.," vol. 193, 1912-13.

length h_s submerged in the tail race (Fig. 359) a type of air compressor is obtained which is fairly efficient, and has obtained some success from its simplicity of construction.

Water entering at the upper end of the down pipe induces a series of small air jets through suitably placed openings, and, if the velocity is sufficiently great, carries the entrained air to the bottom of the pipe, where its pressure $= 2{\cdot}3\ h_s$ lbs. per square inch approximately. The water is then allowed to escape, while the air is collected in an air chamber surrounding the falling main.

The pressure to which the air may be compressed is thus independent of the supply head and depends solely on h_s. Since, however, the head required to maintain the required velocity of flow increases with h_s, this limits the pressure attainable.

The velocity with which fine bubbles rise through still water is approximately 9 inches per second, and it is essential that the velocity of flow be greater than this.

The volume of air compressed per cubic foot of water used may be determined as in the case of the air lift pump.

Experiments by Webber [1] on an installation compressing about 20 cubic feet of free air per second up to 52–54 lbs. per square inch show a maximum efficiency of 71 per cent., this being attained with a velocity of about 7 f.s.

Much higher efficiencies—up to 85 per cent.—have been claimed from more recent tests,[1] with velocities between 12 and 16 f.s., and with compressions up to 120 lbs. per square inch. The most efficient velocity appears to increase with the compression.

Art. 217.—The Humphrey Gas Pump.

In the Humphrey pump the expansive force following the ignition of an explosive mixture of gas and air is directly utilised to pump water. In its simplest form the pump consists of a combustion chamber C (Fig. 360), fitted with valves A and E for the admission of gas and air and for the exhaustion of the waste products of combustion. A continuation of the combustion chamber forms a suction chamber W with its suction valves V and the delivery main D, which discharges into the elevated tank $E\ T$. The action of the pump is as follows:—A compressed charge of gas and air in C is ignited by an electric spark and expands,

[1] "Proceedings Institute Civil Engineers," vol. 169, p. 500. See also *Engineer*, November 10th, 1911, p. 482, and "Inst. C. E. Engineering Abstracts," No. 9, October, 1921, p. 191.

driving forwards the column of water in the delivery pipe. Expansion proceeds until the pressure falls to or below atmospheric, when the suction valves V and the exhaust valve E open. In virtue of its momentum the delivery column maintains its motion for some appreciable time, during which water is drawn through the suction valves, part of this joining in the motion of the column and part entering the combustion chamber. After a short time the momentum is destroyed and the column, act d upon by the pressure due to the delivery head, begins to return, closing the suction valves and forcing the waste products out through the valve E. This action is continued until the water level rises to that of this valve, which is closed by the impact, and the remaining products are compressed

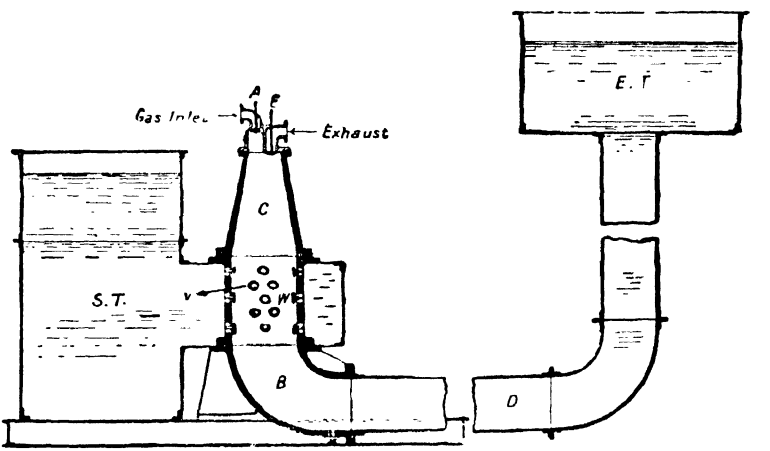


FIG. 360.—Humphrey Gas Pump.

into the space below the inlet valve, until the column is brought to rest. A second outward motion of the column then follows and the pressure falls to atmospheric. At this instant the inlet valves are opened, and the further motion of the column draws in a fresh charge of air and gas. Again the column returns under the elevated tank pressure compressing this charge which is then ignited to start a fresh cycle of operations. The cycle is thus identical with that of the four-cycle gas engine. By a comparatively simple modification of the combustion chamber and its valves the cycle may, however, be made to correspond with the two-cycle engine.[1]

[1] Details of the mechanical construction of these pumps are given in *Proceedings Institute Mechanical Engineers*, 1909, p. 1075. A set of five recently constructed for the Metropolitan Water Board pump 180 millions of gallons per day, with a lift of 25 to 30 feet.

EXAMPLES.

(1) A hydraulic ram uses 50 gallons of water per minute under a supply head of 4 feet, and pumps 5 gallons of this against an effective head of 30 feet. Determine the efficiency of the ram.

Answer. 72·25 per cent.

(2) A ram uses 900 gallons of water per minute under 10 feet head, and pumps 50 gallons of this through 500 feet of $2\frac{1}{2}$-inch piping into a reservoir at a height of 80 feet above the ram. Determine the efficiency, taking $f = \cdot 014$.

Answer. 60 per cent.

(3) The waste valve of a hydraulic ram is 4 inches diameter and is required to begin to close when the velocity of flow past the valve itself is 6 feet per second. Assuming the dynamic pressure on the valve per unit area to be given by $1 \cdot 35 \frac{W v^2}{2 g}$ lbs., where W = weight per cubic foot of water, determine the necessary weight of the valve.

Answer. ·328 lbs. per square inch.

Total weight = 4·12 lbs.

(4) A jet pump placed 8 feet above the suction reservoir and 60 feet below the supply reservoir, lifts its water through a total height of 8 feet 9 inches. Determine its efficiency when delivering 100 gallons per minute and when using 36·5 gallons per minute from the supply reservoir.

Answer. ·257.

(5) An injector hydrant takes 25·2 gallons per minute of high pressure water at 700 lbs. per square inch and delivers 150 gallons per minute at a pressure of 65 lbs. per square inch, the low pressure supply being at 30 lbs. per square inch. Determine the efficiency.

Answer. ·273.

CHAPTER XXII.

The Hydraulic Transmission of Energy—Accumulators—Intensifiers—Friction of Leather Collars for Rams—Wave Transmission of Energy.

Art. 218.—The Hydraulic Transmission of Energy.

Since water is virtually incompressible, if one end of an enclosed column be exposed to the pressure of a moving ram the energy of this will be directly transferred to the other end, the only loss of energy being due to pipe friction. With water of a given pressure intensity, the energy transmitted varies directly as the volume of water flowing per second, so that as the loss due to friction increases as the square of the velocity, the proportional effect of this will diminish as the working pressure increases, and for high efficiency of transmission the working pressure is of necessity high.

In many instances the use of water under considerable pressure as a medium for the transmission of energy from a central power station to a private consumer offers advantages over other methods of power transmission, and this is particularly the case where the power is required to operate machinery in which the action is either—

(*a*) Comparatively slow, but in which a considerable force is required, and particularly where the motion is to be regulated with great precision;

(*b*) Very intermittent, a large force being required at intervals and for a comparatively short time.

It is thus well adapted for the operation of presses, flanging and riveting machinery, lifts, hoists, cranes, and testing machines.

Among other advantages of hydraulic transmission, it may be noted that gearing in the machine is in general unnecessary, the force being transmitted directly from the hydraulic piston; that perfect regulation is easy; that when applied to the direct working of lifts and of hoists a brake is unnecessary and that so long as the velocity of flow is kept low the transmission losses are small—with well-designed pipe lines this loss should not exceed 10 lbs. per square inch per mile.

As compared with electric transmission, it has the further advantage that fire risks are eliminated. Compared with transmission by compressed air it has the advantage that any leakage is easily detected, while under suitable conditions the hydraulic transmission losses are the smaller. Each of the three systems, of course, has its own particular sphere of application. For long-distance work the necessary cost of, and losses in the pipe line would effectively militate against the application of hydraulic transmission and a radius of 15 miles from the central station would appear to mark the limit of its effective use.

In some large towns, of which London, Manchester and Glasgow are notable examples, private customers are supplied by a system of hydraulic mains. The pressure adopted varies from 700 to 1,600 lbs. per square inch, being 750 lbs. per square inch in the City of London, and 1,120 lbs. per square inch at Manchester and Glasgow.

In Manchester there are three pumping stations. In the latest of these the pressure supply is mainly provided by two motor-driven multi-stage centrifugal pumps, one (11-stage) of 45,000 gallons per hour capacity and the other (14-stage) of 66,000 gallons per hour. In the older stations reciprocating pumps are used. The total output of pressure water before the war was about 380 million gallons per year. The total h.p. of the pumping machinery is 6,000.

There is a bore hole at each station and water is lifted by air-lift pumps, giving some 80 per cent. of the total required. The remainder is obtained from the city water supply, at a pressure of about 40 lb. per square inch, and before passing into the storage tanks from which the pumps lift, is utilised to work a direct-acting intensifying pump, which pumps part of the supply direct into the pressure mains.

Each station has two accumulators, each 18 inches in diameter and having a lift of 23 feet, the dead weight, consisting of iron slag, being carried in iron casings 11 feet 3 inches diameter. The dead weight on each accumulator is 129 tons.

One of these is slightly heavier than the other, the heavier being provided with a tappet gear, which automatically cuts off the power supply to the pumps as the ram reaches the extreme limit of its upward stroke.

The three stations are linked together by trunk mains which feed the distributing mains. The aggregate length of the mains which vary in diameter from 6 inches to 3 inches is about 34 miles.

The power is used for operating lifts, hoists, cranes, presses, intensifiers, motors, rotary engines, capstans, riveters, injectors, ejectors, bending and testing machines, shears, punches, wagon tips, moulding machines, etc. The total number of machines supplied is of the order of 3,000, and the number of premises supplied about 550.

The joint flanges are oval, the joint consisting of a bevelled spigot and faucet union securing a $\frac{3}{8}$-inch gutta-percha ring by two bolts. Fig. 361 illustrates the type of pipe joint as adopted for a 6-inch pipe, and as invented by Mr. E. B. Ellington, the maximum and minimum diameter of the flange being 19 inches and $10\frac{1}{2}$ inches respectively. These pipes are laid with the longer axis of the flange horizontal for facility in getting at the bolts.

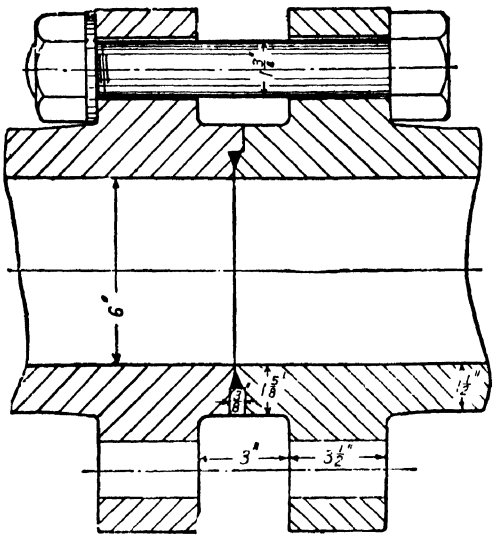


FIG. 361.—Joint for 6-in. Pipe, 1,100 lbs. per square inch.

The main stop valves are balanced both ways by the insertion of a small valve $1\frac{1}{2}$ inch in diameter inside the main valve, the effort required to open the latter only being that necessary to overcome its dead weight. A spring-loaded momentum valve, having a ram $1\frac{1}{2}$ inch diameter, is fitted on either side of every stop valve to minimise any shocks that may occur in the main.

To indicate the condition of the mains and valves, a daily record of the minimum flow during the time the demand is at its lowest (between 11 p.m and 4 a.m.) is kept by means of an automatic electrical recorder. Should this show an abnormal increase in the output for several consecutive nights the mains are tested.

All the pressure water is metered by Kent's high pressure rotary meters before reaching the consumer, some 97 per cent. of the water delivered by the pumps being registered on these meters.

In the London installation the water is taken from the river or from wells, and as it is essential that all deposit should be removed before use, it is allowed to stand for some time in storage tanks. The greater part of the solid matter is deposited here, and the water is then passed through the surface condensers of the engines to a series of filters, in which it is passed first through a layer of broken sponge 18 inches thick, and afterwards through a bed of charcoal. After leaving the filters it is pumped into the clean water tank, from which the main pumps derive their supply

Losses.—The various losses occur :—

(1) At the power station—roughly about 15 per cent.;
(2) In transmission—about 5 per cent.;
(3) In use—about 8 per cent.;

leaving a percentage to be utilized of about 72 per cent.

The last two items, however, vary considerably with the type of machinery, and the energy utilized may vary from 30 to 95 per cent., the latter percentage being obtained with such machines as direct acting coal shoots, where the load, during its descent, may be made to pump pressure water back into the mains.

The losses at the power station are those incidental to the use of reciprocating pumps and accumulators, and are considered in detail in that connection.

Transmission Losses.

Let p = pressure at pipe inlet in lbs. per square inch.
„ d = pipe diameter (supposed uniform) in feet.
„ a = „ area in square feet.
„ l = length of pipe in feet.
„ v = velocity of flow in feet per second.

Then the energy at entrance, per lb. $= \frac{144\ p}{62{\cdot}4} = 2{\cdot}31\ p$ ft. lbs.

$\therefore$ Energy entering pipe per second $= H = 2{\cdot}31\ p \times 62{\cdot}4\ a\ v.$
$= 144\ p\ a\ v$ foot lbs.
$= \frac{144}{550}\ p\ a\ v$ H.P.
$= {\cdot}262\ p\ a\ v$ H.P.[1]

Again the loss of energy in friction per lb. $= \frac{f\ l\ v^2}{2\ g\ m}$ foot lbs.

$\therefore$ loss of energy per second $= H_f = \frac{f\ l\ v^2}{2\ g\ m} \times \frac{62{\cdot}4\ a\ v}{550}$ H.P.

Substituting for v in terms of H we get—

$$H_f = \frac{62{\cdot}4\ a\ f\ l}{2\ g\ m \times 550} \left\{ \frac{H}{{\cdot}262\ p\ a} \right\}^3 \text{H.P.}$$

and putting $m = \frac{d}{4}$; $g = 32{\cdot}2$ this becomes—

$$H_f = {\cdot}635\ f\ l\ \frac{H^3}{p^3\ d^5} \text{ H.P.} \qquad (1)$$

[1] If $p = 750$ lbs. per square inch we thus get the approximate rule that two gallons of water per minute is equivalent to one horse power.

The energy delivered per second-U—

$$= H - H_f = H \left\{ 1 - \cdot 635 f l \frac{H^2}{p^3 d^5} \right\} \qquad (2)$$

$\therefore$ Efficiency of transmission—

$$= \frac{U}{H} = 1 - \cdot 635 f l \frac{H^2}{p^3 d^5} \qquad (3)$$

Differentiating U with respect to H, and equating the result to zero, we get the condition that the maximum horse power may be transmitted.

Expressed algebraically this gives—

$$H^2 = \frac{1}{3} \frac{d^5 p^3}{\cdot 635 f l} = \frac{d^5 p^3}{1 \cdot 90 f l},$$

or

$$H = \cdot 725 \sqrt{\frac{d^5 p^3}{f l}}, \qquad (4)$$

from which, by substitution in equation (2), we have the energy delivered through the pipe, or $U = \cdot 483 \sqrt{\frac{d^5 p^3}{f l}}$ H.P.

$$= \cdot 138 \sqrt{\frac{d^5 h^3}{f l}} \text{H.P.}$$

where h = head in feet at entrance to pipe.

Substituting the value of H from (4) in equation (3), we see that under circumstances of maximum transmission, the efficiency is $\frac{2}{3}$, and that $\frac{1}{3}$ of the energy entering the pipe is absorbed in overcoming friction. On the other hand it is evident that maximum efficiency is obtained when H_f is as small, and p and d as large as practicable.

The point at which it ceases to pay to still further increase the diameter of the pipe line for a given horse power, depends on the relative cost per yard of the pipe line, including excavation, jointing and laying, and of the power production per horse power.[1] In general, however, a size of pipe which allows of a pressure drop of about 10 lbs. per square inch per mile will be found to give most economical results in practice. In modern practice the largest pipes are about 6 inches diameter, the pipe lines being duplicated for large powers.

EXAMPLE.

Let $H = 100$ H.P.

,, $p = 750$ lbs. per square inch.

Assume $f = \cdot 006$ (this varies with the diameter, velocity of flow, and condition of pipe).

[1] See "Proceedings Institute Mechanical Engineers," 1895, p. 353; also *Engineering*, May 22nd and June 5th, 1891.

Then allowing for a drop of 10 lbs. per mile we have

$$\text{Efficiency of transmission} = 1 - \frac{10}{750} \cdot \frac{l}{5{,}280}$$

$$= 1 - \frac{\cdot 635\, f\, l\, H^2}{p^3\, d^5},$$

$$\therefore d^5 = \frac{\cdot 635 \times \cdot 006 \times 5{,}280 \times 10{,}000 \times 750}{750 \times 750 \times 750 \times 10} \text{ feet}$$

$$= \cdot 0358 \text{ feet},$$

$$\therefore d = \cdot 514 \text{ feet} = 6 \cdot 17 \text{ inches}.$$

The loss per mile $H_f = \frac{10}{750} \times 100 = 1 \cdot 33$ H.P.

The efficiency of transmission is then given by:—

Length of pipe in miles . .	1	2	5	10
Efficiency of transmission . .	·987	·973	·933	·867

Also

$$v = \frac{H.P. \times 550}{144\, p \times a} \text{ f.s.}$$

$$= \frac{55{,}000 \times 4}{750 \times \pi \times 144 \times \cdot 2642} \text{ f.s.}$$

$$= 2 \cdot 45 \text{ feet per second.}$$

If $p = 1{,}120$, then for the same fall in pressure per mile we have, for 100 H.P.:—

$$d = \cdot 4377 \text{ feet} = 5 \cdot 25 \text{ inches}$$

while $v = 2 \cdot 27$ feet per second.

The following table then gives the efficiency for various lengths of pipe line:—

Length in miles . .	1	2	5	10	20
Efficiency . . .	·991	·982	·955	·911	·821

The velocity of flow through the pipe should not exceed 4 feet per second, velocities ranging from 2·5 to 4·0 feet per second being usual.

While an increase in the working pressure increases the efficiency of transmission, it also necessitates an increase in the thickness of the pipe

walls, which counterbalances the advantages of the reduced internal diameter. Also the difficulty of preventing leakage at joints increases with the pressure, so that in practice it has not been found advisable to adopt pressures much in excess of 1,100 lbs. per square inch.

Losses in Use.—These are due partly to friction, but, in the majority of hydraulic machines, more particularly to shock at sudden changes of section in valve boxes and supply ports and pipes, and to the necessity in many machines for filling the inlet passages with pressure water before the commencement of each working stroke. The latter loss may be prevented by having separate inlet and outlet passages to the working cylinder, while to prevent the former losses becoming excessive all working velocities must be kept low.

A further loss of energy occurs in some such machines as presses and riveters, where the maximum force which the plunger can exert is only needed for a short portion of its stroke, but where the expenditure of energy is the same as if this were needed for the whole stroke. Also, in the case of a hydraulic crane, if only one lifting cylinder is provided without any special regulating device, the expenditure of energy is the same whatever load, up to the maximum capacity of the crane, be lifted. Certain devices which have been invented to overcome this difficulty will be considered later.

ART. 219.—ACCUMULATORS.

Since the delivery from a reciprocating pump is not uniform and since it is necessary to have some reserve of energy to meet a sudden or abnormal demand, some means of storing pressure energy is a necessary adjunct to the hydraulic power station.

With the high pressures in common use the elevated storage tank is out of the question and the accumulator, devised by Sir W. G. Armstrong, takes its place.

Pressure water from the pumps, then, is not led directly into the supply mains, but first into an accumulator from which it is taken to feed the pipe line.

Stripped of unessentials, the accumulator consists of a vertical cylinder fitted with a weighted ram, whose weight and area is adjusted so as to give the required pressure in the mains.

The energy storage capacity of the accumulator is equal to the potential energy of the lifted ram and weight, and if L is the length of its travel in feet, is given by $L\ W$ foot lbs., or by $p\ A\ L$ foot lbs., where p is the pressure in lbs. per square inch and A is the area of the ram in square inches.

EXAMPLE.

Let $p = 1{,}120$ lbs. per square inch, $L = 23$ feet, $A = \pi \times 9^2 = 81\,\pi$.

Storage capacity $= 1{,}120 \times 81\,\pi \times 23$ foot lbs.

$= 6{,}554{,}000$ foot lbs.

$= \dfrac{6{,}554{,}000}{33{,}000 \times 60} = 3{\cdot}31$ horse power hours.

The accumulator is therefore capable of giving out energy at the rate of 3·31 H.P. for one hour, or 39·7 H.P. for five minutes.

From this example it is evident that the storage capacity is not large and that the main function of an accumulator is not so much to store energy, in the sense that an electric accumulator stores it, as to permit of momentary fluctuations in the rates of supply and demand, or in other words, to act as a flywheel does to a steam or gas engine. It also serves to regulate the delivery pressure, and is usually made to control the motive power automatically. Its efficiency is high, up to 98 per cent. of the energy expended in charging being returned during delivery.

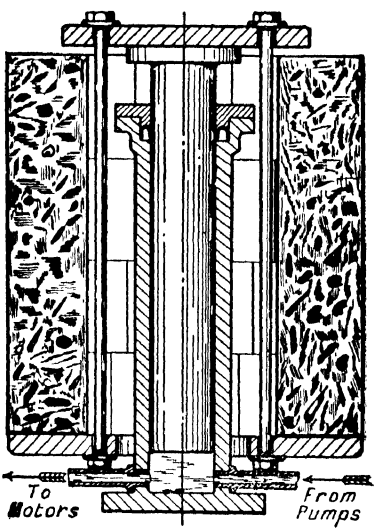


FIG. 362.—Accumulator.

In its most common form the ram carries a platform which is weighted with some heavy material, usually pig iron or iron slag.[1] Fig. 362 shows this type, the weight here being carried in a wrought-iron cistern suspended from the ram platform.

The ram is guided in its travel by a framework not shown in the figure. If the pumps are delivering more water than the motors require, the ram

[1] The following table shows the approximate volume occupied per ton of weighting material :—

Substance.	Cubic Feet per Ton.
Pig iron or wrought iron scrap . . .	6·25
Broken stone	17·2
Clay or earth	18·5
Bricks	22·2

rises, and on reaching the upper limit of its travel moves a stop which, by suitable link connection, causes steam to be shut off from the pumping engines.

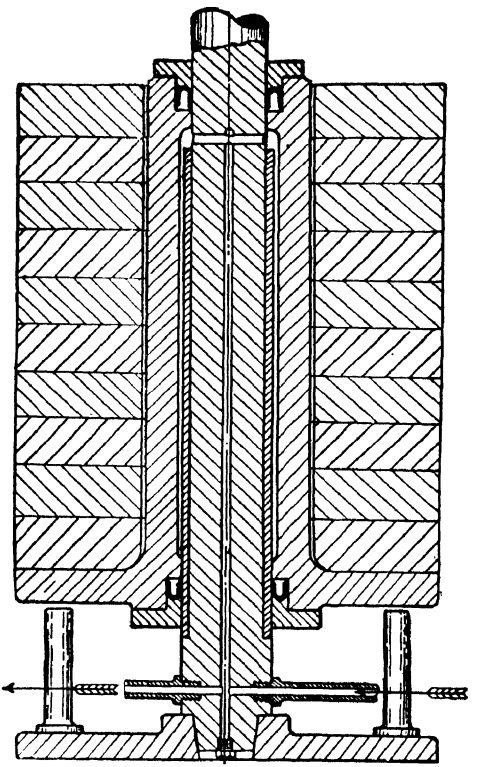


FIG. 363.—Tweddell's Differential Accumulator.

Various modifications of this type have been adopted, the ram in some cases being inverted, fixed and fitted with suitable inlet and outlet orifices, and the loaded cylinder moving vertically.

The differential accumulator of Mr. Tweddell may also be noted. As indicated in the sketch (Fig. 363), this consists of a fixed ram of area A, surrounded over the lower portion of its length by a closely fitting bush of area a.

This bush terminates below the inlet and outlet holes. The ram passes through both ends of the storage cylinder, through glands of area $(A + a)$ and A, and the effective cylinder area exposed to upward pressure is a.

Thus $p\ a = W$, and by making the bush of small thickness, a very large pressure may be maintained by a comparatively small weight.

EXAMPLE.

If the ram diameter = 6 inches and the bush is $\frac{1}{4}$ inch thick, we have $a = 4{\cdot}91$ square inches.

$$\therefore \text{If } p = 1{,}120 \text{ lbs. per square inch,}$$

$$W = p\ a = 1{,}120 \times 4{\cdot}91 = 5{,}500 \text{ lbs.}$$

Since the storage of energy is only small, this type of accumulator is more suitable for use with single machines of the riveter type.

On board ship, and especially for naval purposes, the use of hydraulic

machinery for training the heavy guns, rotating turrets, etc., is very usual, but here, for obvious reasons, the accumulator loaded with a mass of pig iron weighing many tons is quite inadmissible.

In its place the steam accumulator (Fig. 364) is used. This consists of a steam cylinder fitted with a piston and its piston rod or ram, which takes the place of the weight-loaded ram of the ordinary type. Steam

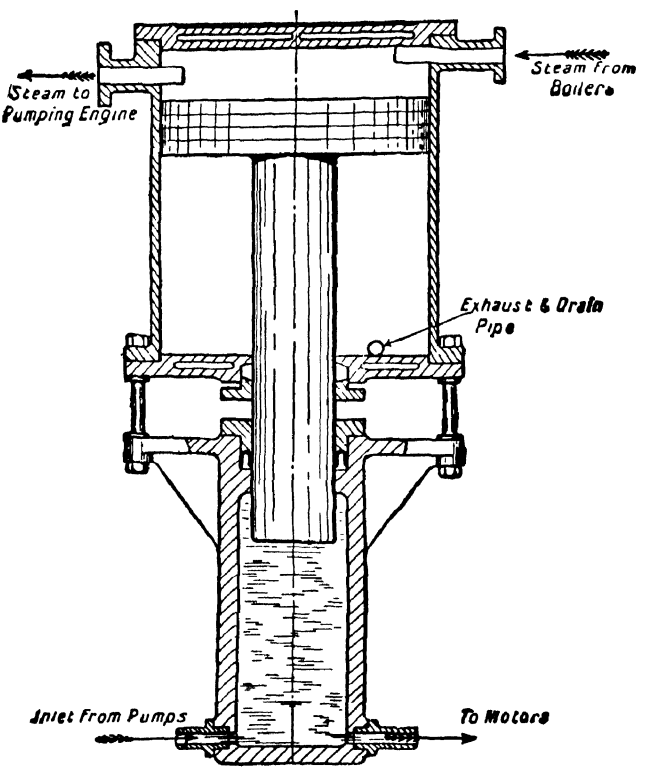


FIG. 364.—Steam Accumulator.

from the boiler is admitted to the upper side of the piston. first passing through a reducing valve which ensures a constant pressure, and with a given steam pressure P, a suitable adjustment of the areas A_1 and A of the steam piston and ram, will enable any required hydraulic pressure, $p = P\frac{A_1}{A}$ lbs. per square inch, to be maintained. The steam supply to the pumping engines is taken through the steam cylinder, and the accumulator piston automatically cuts off this supply on reaching a given height,

and thus stops pumping until the ram descends. The steam port is usually designed so that steam may be cut off gradually and the speed of the pumps gradually reduced as the piston approaches the upper limit of its travel.

A drain pipe is arranged to carry away any water or steam leaking past the piston. The storage capacity is $P\,A_s\,L$ foot lbs., and this type has the advantage that the ram may be placed either horizontally or vertically.

Effect of Accumulator on Working Pressure in Motor Cylinder.—If the motor, piston-area a square inches, derives its supply from the accumulator cylinder alone, then if A square inches = area of ram; a = acceleration of motor piston; a' = acceleration of ram, we have $a' = a\,\frac{a}{A}$.

The force necessary to produce this acceleration is given by

$$F = \frac{W}{g}\,a' = \frac{W}{g}\cdot\frac{a}{A}\cdot a, \text{ lbs.}$$

$\therefore$ Equivalent pressure in lbs. per square inch on ram $= \frac{W}{g}\cdot\frac{a}{A^2}\cdot a$.

$\therefore$ Pressure at entrance to delivery pipe $= \frac{W}{A}\left\{1 - \frac{a}{g\,A}\,a\right\}$ lbs. per square inch.

If a_s = area of this pipe, then $a_s = a\,\frac{a}{a_s}$, and if l is its length, we have putting v = velocity of piston:—

Pressure on piston

$$= \frac{W}{A}\left\{1 - \frac{a}{g\,A}\,a\right\} - \frac{62{\cdot}4}{144}\,\frac{l\,a}{g\,a_s}\,a - \frac{62{\cdot}4}{144}\cdot\frac{a^2}{a_s^2}\,\frac{f\,l}{2\,g}\,\frac{v^2}{m}$$

$$= \frac{W}{A} - \frac{a\,a}{g}\left\{\frac{W}{A^2} + \frac{62{\cdot}4}{144}\,\frac{l}{a_s}\right\} - \frac{62{\cdot}4}{144}\cdot\frac{a^2}{a_s^2}\cdot\frac{f\,l\,v^2}{2\,g\,m} \text{ lbs. per square inch.}$$

With a steam accumulator this becomes:—

$$\frac{P\,A_s}{A} - \frac{a\,a}{g}\left\{\frac{W}{A^2} + \frac{62{\cdot}4}{144}\,\frac{l}{a_s}\right\} - \frac{62{\cdot}4}{144}\cdot\frac{a^2}{a_s^2}\cdot\frac{f\,l\,v^2}{2\,g\,m} \text{ lbs. per square inch,}$$

and since W is now comparatively small, the term $\frac{a\,a}{g}\cdot\frac{W}{A^2}$, which represents the effect of the inertia of the ram, becomes negligible. For this reason a machine supplied through a steam accumulator is not subject to inertia shocks of such magnitude as may be produced by a weighted accumulator.

To prevent inertia shocks becoming dangerous, a relief valve is sometimes placed on the outlet pipe, this being set to blow off at 10 per cent. above normal pressure. The loss due to the leakage which this necessi-

tates may be avoided by the provision, in the place of the relief valve, of a spring-loaded plunger, in which case the kinetic energy of the moving ram is expended in compressing the spring.

Its inertia is, however, an advantage in some cases, as, for example, where fitted to a hydraulic riveter or similar machine. Here the inertia is utilized to increase the pressure at the end of the stroke, the final and sudden impact thus produced causing the rivet to fill up its hole effectively.

Fig. 365 shows a spring loaded plunger with relief valve arranged to open when the rise in pressure exceeds a certain limit. The necessary dimensions of this may be calculated when the size of accumulator and circumstances involved in its use are known. If, for example, the weight of accumulator ram and load is 160 tons; the maximum velocity of fall 10 ft. per minute = ·166 ft. per second; the diameter of ram 9 inches; the normal working pressure 2·5 tons per square inch; the energy of the falling weight to be absorbed by the spring loaded plunger with a 10 per cent increase in pressure.

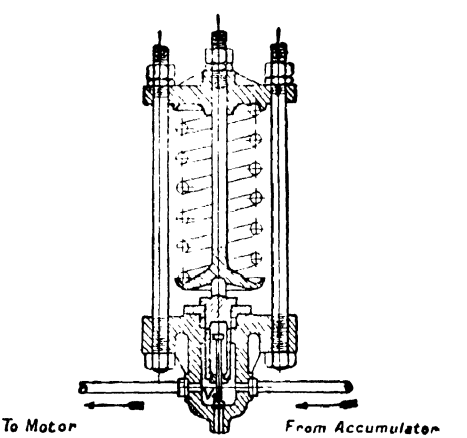


FIG. 365.

Kinetic energy of falling weight $= \frac{160}{36 \times 64{\cdot}4} = {\cdot}069$ ft. tons.

To absorb this amount of energy the ram must fall through a further distance x feet, against the excess resistance due to this increase in pressure, and as the mean excess resistance is 5 per cent. of 2·5 tons per square inch we have

$$\cdot 125 \times 81 \frac{\pi}{4} \times x = \cdot 069$$

$$\therefore x = \cdot 0087 \text{ feet} = \cdot 104 \text{ inches.}$$

$$\therefore \text{vol. of water displaced while coming to rest}$$

$$= \cdot 104 \times \frac{81\,\pi}{4} = 6{\cdot}61 \text{ cub. in.}$$

Making the plunger of the valve 2 inches diameter, the spring would require to be so designed as to allow of a displacement of $\frac{6{\cdot}61}{\pi}$ = 2·1 inches under an increase of pressure from 2·5 to 2·75 tons per square inch. With any greater displacement the under side of the plunger would engage with and would lift the small escape valve V.

Art. 220.—The Hydraulic Intensifier.

Where the main pressure supply is of less intensity than is required to work the hydraulic machinery an intensifier is used In its simplest form, this consists of a ram of area a, carrying a piston of larger area A (Fig. 366). Water from the pressure mains, at pressure p, is admitted behind the piston and compresses the water in the ram cylinder to an increased pressure P where $P = p\frac{A}{a}$.

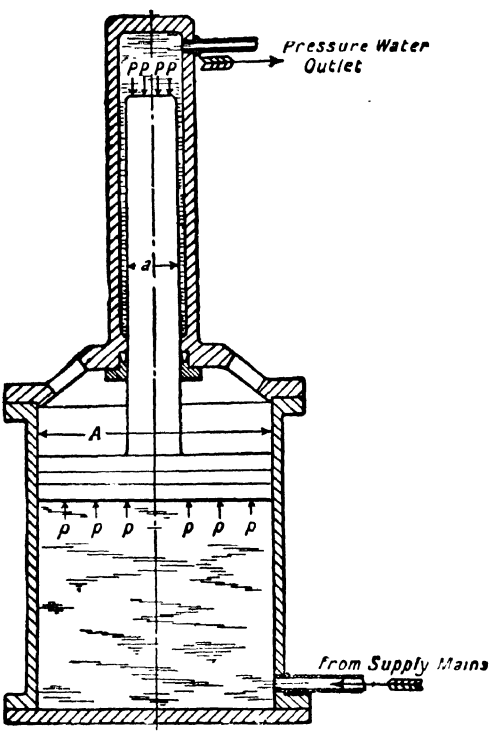


Fig 366. –Hydraulic Pressure Intensifier.

Example.

$\begin{cases} p = 40 \text{ lbs. per square inch,} \\ \text{Piston diameter} = 48 \text{ inches.} \\ \text{Ram diameter} = 8 \text{ inches.} \end{cases}$

$$\frac{A}{a} = 36.$$

$\therefore\ P = 40 \times 36 = 1{,}440$ lbs. per square inch.

This neglects the friction of the packings, and also the weight of the ram and piston. Including these we have

$$p\,A - (w + F) = P\,a$$

$$\therefore\ P = p\frac{A}{a} - \frac{w + F}{a} \text{ lbs.}$$

per square inch. Where $w =$ weight of ram and piston in lbs. $F =$ frictional resistance in lbs.

If on the down stroke communication be made between the under side of the piston and the upper side of the ram, the pressure p' below the piston becomes

$$p' = \frac{w - F}{A - a} \text{ lbs. per square inch.}$$

Various modifications of this type of intensifier are in use, one of these being illustrated in Fig. 367. Here low pressure water is admitted above the hollow ram A, while the intensified water is led away through the small stationary ram of area a

If w is the weight of the outer ram, we now have

$$P = p\frac{A}{a} + \frac{w - F}{a} \text{ lbs. per square inch.}$$

With a single intensifier the supply of high pressure water cannot be made continuous, and delivery only takes place on the in stroke. At the end of this stroke water is admitted to the ram cylinder from the supply mains, while the water below the piston is allowed to escape. This type of intensifier is often fitted to testing machines deriving their pressure supply from towns' mains at a comparatively low pressure.

Where a continuous supply of high pressure water is required, this may be obtained by using two intensifiers placed side by side, each automatically working the valves of the other.

When applied to the working of a hydraulic press, the water escaping from beneath the piston during the down stroke may be utilised to bring the press platform up to its work, and to perform the first part of the compression.[1]

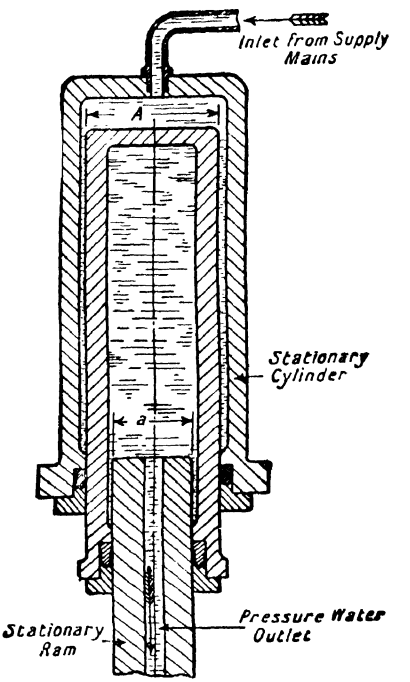


FIG. 367.—Hydraulic Pressure Intensifier.

ART. 221.—FRICTION OF LEATHER COLLARS FOR RAMS.

One or other of the types of packing illustrated in Fig. 368 is commonly used for hydraulic rams or plungers. The material used is leather, and, since the pressure of the water itself forces the packing against the ram, this pressure, and the friction produced, become proportional to the pressure intensity of the water. A very complete series of experiments carried out by Mr. John Hicks on rams $\frac{1}{2}$ inch, 4 inches, and 8 inches diameter, and with pressures up to 6,400 lbs. per square inch, indicate that—

(1) For pressures above 400 lbs. per square inch, friction is directly proportional to pressure intensity.

[1] For further details of valve arrangements, etc., the reader may refer to Blaine's "Hydraulic Machinery," p. 345.

(2) For rams of different diameters exposed to the same pressure intensity, the friction in lbs. is directly proportional to the diameter, and therefore to the square root of the gross load.

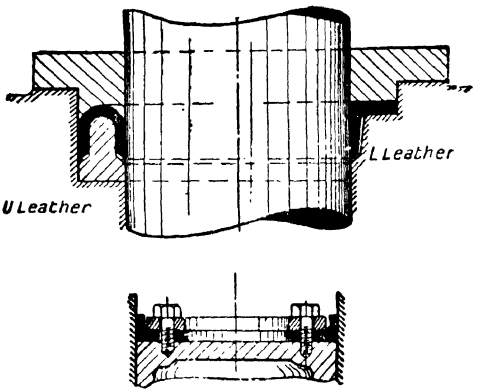


FIG. 368.—Leather Collars for Hydraulic Rams and Pistons.

(3) The depth of the collar does not affect the friction. In several of the experiments the depth was reduced from $\frac{7}{8}$ inch to $\frac{3}{8}$ inch without any appreciable effect on the friction.

The following approximate formulae were deduced from the results of these experiments :

Friction in lbs. $= C \times$ diameter in inches $\times$ pressure in lbs. per square inch.

where $\begin{cases} C = \cdot0471 \text{ for new or badly lubricated collars,} \\ C = \cdot0314 \text{ for collars in good condition and well lubricated.} \end{cases}$

The annexed table gives the frictional resistance expressed as a percentage of the total pressure on the piston, for rams from 2 inches to 20 inches diameter and for pressures exceeding 400 lbs. per square inch :

Diameter in inches		2	3	4	5	6	7	8	10	12	14	16	18	20
Friction, % of total pressure on ram.	Well lubricated	2·00	1·33	1·00	·80	·66	·57	·50	·40	·33	·28	·25	·22	·20
	New or badly lubricated	3·00	2·00	1·50	1·20	·99	·85	·75	·60	·50	·42	·37	·33	·30

For lower pressures, the formulae

$$F \text{ (lbs.)} = p\,d\,\{\ \cdot0467 - \cdot0000139\,p\ \}$$

gives more accurate results, the coefficients here applying to leathers in good condition and well lubricated.

Where the loading is eccentric, as is often the case in hydraulic jacks, etc., these values may however be increased by as much as 100 per cent.

More recent experiments by Prof. Martens, of Berlin,[1] on the packings

[1] *Mechanical Engineer*, September 7th, 1907. The results of more recent experiments *Zeitschrift d. v. d. Ingenieure*, 1921, p. 1016, agree well with these.

fitted to the rams of five testing machines showed the following friction losses :

Pressure (Atmospheres).	Per cent. Loss per Packing.
50	2 — 5
100	1·8 — 3·0
200	1·0 — 1·6

Experiments by S. L. Davis[1] on a 5 inch testing machine ram carrying a U leather at its lower end and working in a copper lined cylinder showed that after a fair amount of service the friction, with increasing loads, varied irregularly from about 5 per cent. to zero, with a mean value of 1·3 per cent., as the pressure was increased from 500 to 4,500 lbs. per square inch. With diminishing loads the friction had a maximum value of 3·6 per cent. at the lowest pressure, with a mean value of ·2 per cent. over the whole range.

For satisfactory working the U leather should have a ring of metal or other material inserted between the flaps, and should as far as possible have a metal backing over its curved portion.

Experience shows that a narrow fitting strip—not above $\frac{1}{2}$ inch wide—is preferable to one which is wider, since, owing to the reduced tendency to bending at the bottom of the U with a shallow collar, the leather is not so liable to crack.

Hemp packing is also used to a limited extent for hydraulic glands. Here the percentage loss in friction decreases with an increasing load, but since the packing must be tightened so as to prevent leakage at the highest pressures to which it may be subjected, the loss at low pressure is probably three to four times that of a leather collar.

Art. 222. Wave Transmission of Energy in a Pipe Line.

If one end of a closed pipe line full of water under a mean pressure p_m be fitted with a reciprocating plunger, a wave of alternate compression and rarefaction is produced, which is propagated along the pipe with the velocity of sound, V_p, in the medium. If the plunger has simple harmonic motion, the state of affairs in a pipe line so long that, at the given instant, the disturbance has not had time to be reflected from its further end, is represented

[1] *Engineering News*, February 11th, 1909, p. 167.

in Fig. 369. The pressure at each point will oscillate between the values $p_m \pm p$ and the velocity between the values $\pm v$, where v is the maximum velocity of the piston. At the instant in question, particles at A, C, E, and G are oscillating to and fro along the axis of the pipe through a distance r on each side of their mean position, while particles at B, D, F, and H are

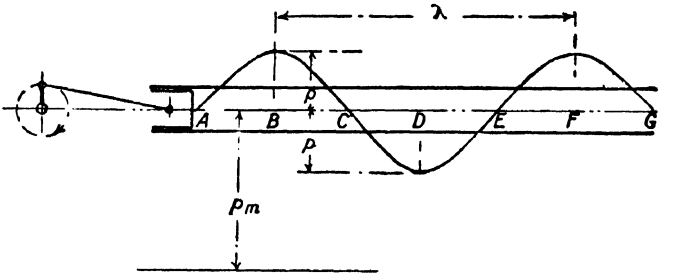


FIG. 369.

at rest. If n be the number of revolutions of the crank per second, the wave-length $\lambda = V_p \div n$ feet.

In a pipe closed at both ends such a state of vibration is reflected from end to end, forming a series of waves of pressure and velocity whose distribution at any instant depends on the ratio of the length l of the pipe to λ. In the cases where l is respectively equal to $\lambda/2$ and λ, stationary waves are

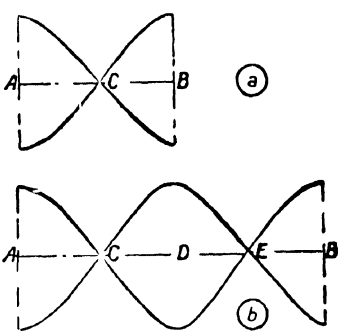


FIG. 370.

produced as indicated in Fig. 370. The excess pressure at a given point oscillates between equal positive and negative values, the range of pressure being given by the intercept between the two curves. The velocity at the points of maximum and minimum pressure, as at A, D, and B, is zero, while at the points C and E, where the variation of pressure is zero, the velocity varies from $+ v$ to $- v$.

In a pipe fitted with a reciprocating plunger at one end and closed at the other the wave system initiated by the plunger will be superposed on this reflected system. Thus if $l = \lambda/2$ or λ, the wave initiated by the plunger will be reflected, and will reach the plunger as a zone of maximum pressure at the instant the latter is completing its in-stroke and is producing a new state of maximum pressure. The pressure due to the reflected wave is now superposed on that due to the direct compression, with the result that the pressure is doubled. The next revolution will again increase the pressure, and so on until the pipe either bursts or until the rate of dissipation of energy due to friction, and to the imperfect elasticity of the pipe walls, becomes equal to the rate of input of energy by the plunger.

On the other hand, if the length of the pipe be any odd multiple of $\lambda/4$, the pressure at the plunger at any instant, due to the reflected wave, will be equal in magnitude but opposite in sign to that primarily due to the displacement of the plunger, and the pressure on the latter will be constant and equal to the mean pressure in the pipe. Except for the effect of losses in the pipe walls and in the fluid column, reciprocation may now be maintained indefinitely without the expenditure of further energy. For any intermediate lengths of pipe, the conditions will also be intermediate and the wave distribution complex.

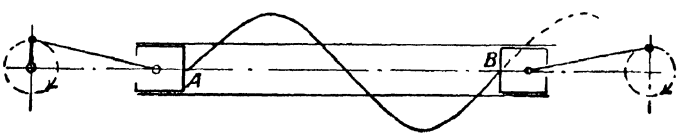


FIG. 371.

Instead of closing the end of the pipe at *B*, suppose (Fig. 371) a piston to be fitted to a crank rotating at the same angular velocity, in the same direction and in the same phase as the crank at *A*. If the column were continued beyond *B*, the movement of the piston would evidently produce in the column a series of waves forming an exact continuation of the wave system between *A* and *B*. There will now be no reflection from the surface of the piston, and, neglecting friction losses, it will take up the whole energy of the waves produced by piston *A*. It is to be noted that the piston *B* may be placed at any point in the pipe, so long as its phase is the same as that of the liquid at the point of connection.

If more energy is put in by the piston *A* than is absorbed by *B*, reflected waves will be formed, and the continuation of the motion will accumulate energy in the system, increasing the maximum pressure until, as in the case of the closed pipe, the pipe will burst.

This may be avoided by fitting a closed vessel, filled with liquid, having a volume large in comparison with the displacement of the piston in communication with the pipe near to the piston. Alternatively, this may be replaced by a spring-loaded plunger. In either case the contrivance acts as a reservoir of energy. If the piston *B* is not absorbing the whole of the energy supplied from *A*, the liquid in this chamber is compressed on each instroke of the piston *A*, to re-expand on the outstroke, and by giving to it a suitable volume the maximum pressures, even when the piston *B* is stationary, may be reduced to any required limits. If perfectly elastic, the reservoir will return as much energy during expansion as it absorbed during compression, so that the net input to the driving piston is only equivalent to that absorbed by piston *B*.

In the case of a pipe (Fig. 370 *b*) whose length is one wavelength, and which is provided with branches at *C*, *D*, *E*, and *B*, respectively one-quarter, one-half, three-quarters, and one wavelength from *A*, if all the branches are closed, stationary waves will be produced in the pipe as previously described.

If now a motor running at the synchronous speed be coupled to the branch at *D*, this will be able to take up all the energy given to the column. The stationary half-wave between *A* and *D* will vanish, being replaced by a wave of motion, while the stationary wave will still persist between *D* and *B*.

Since there is no pressure variation at *C* and *E*, motors coupled at these points, with the remaining branches closed, can develop no energy.

If a motor be connected at any intermediate point, part of the input of energy can be taken up by the motor. The stationary wave will then persist, but be of reduced amplitude between *A* and the motor, the wave motion over this region being compounded of this stationary wave and of a travelling wave conveying energy.

With a motor at the end *B* of the line, not absorbing all the energy given by the generator *A*, there is, in the pipe, a system of stationary waves superposed on a system of waves travelling along the pipe, so that there will be no point in the pipe at which the variation of pressure is always zero. It follows that under these conditions a motor connected at any point of the pipe will be able to take some energy and to do useful work.

In practice a three-phase system is usually employed, as giving more uniform torque and ease of starting. A three-cylinder generator, having cranks at 120°, gives vibrations to the fluid in three pipes, which are received by the pistons of a three-cylinder hydraulic motor having the same crank angles. The mean pressure within the system is maintained by a pump, which returns any fluid leaking past the pistons.

Art. 223.—Theory of Wave Transmission of Energy.

The simple theory of the process is outlined below,[1] on the assumption that the friction loss due to the oscillation of the column in the pipe is directly proportional to the velocity. Where such a viscous fluid as oil is used this is true, but where water is used it may or may not be true, depending upon the velocities involved. If the resistance is equal to $k v^2$ per unit length as with turbulent motion, an approximation to the true result may be attained by choosing such a frictional coefficient k' as will make $k' v = k v^2$ at the mean velocity. At velocities below the critical $k' = 32 \mu \div d^2$ in lbs. per square foot per unit length of the pipe (Poiseuille).

Consider the fluid normally in a plane at x, displaced from that plane through a small distance u, so that its velocity $v = \frac{\partial u}{\partial t}$. The difference of pressure on the ends of an element of length δx, due to the variation in compression along the axis of the pipe, is equal to

$$K \frac{\partial}{\partial x} \left(\frac{\partial u}{\partial x} \right) \delta x,$$

and the equation of motion becomes

$$\rho \frac{\partial^2 u}{\partial t^2} \frac{\pi d^2}{4} \delta x = K \frac{\partial^2 u}{\partial x^2} \frac{\pi d^2}{4} \delta x - \frac{32 \mu}{d^2} \frac{\pi d^2}{4} \frac{\partial u}{\partial t} \delta x,$$

or

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2} - b \frac{\partial u}{\partial t} \quad \ldots \ldots \ldots \ldots \quad (1)$$

where

$$a = \sqrt{\frac{K}{\rho}} \quad \text{and} \quad b = \frac{32 \mu}{\rho d^2}.$$

If b is small compared with $4 \pi n$, where n is the frequency of the vibration, a solution of equation (1) is

$$u = u_o e^{-\frac{1}{2} \frac{b}{a} x} \sin 2 \pi n \left(t - \frac{x}{a} \right) \quad \ldots \ldots \ldots \quad (2)$$

which represents an axial vibration throughout the column, of maximum amplitude u_o at the end where $x = 0$.

At any other point the maximum amplitude is $u_o e^{-\frac{1}{2} \frac{b}{a} x}$ gradually

[1] H. Moss, "Proc. Inst. Mech. E.," November, 1920.

diminishing along the pipe owing to the friction represented by the term b.

The excess pressure p, at any instant and at any point is equal to $K \frac{du}{dx}$

$$= K u_0 e^{-\frac{1}{2}\frac{b}{a}x} \times \frac{2\pi n}{a} \cos 2\pi n \left(t - \frac{x}{a}\right) \text{ approx.}$$

and the maximum excess pressure, $p_{max.}$, at any point,

$$= \frac{2\pi n K u_0}{a} e^{-\frac{1}{2}\frac{b}{a}x} \quad \ldots\ldots\ldots (3)$$

The velocity of a particle at x is equal to

$$\frac{\partial u}{\partial t} = 2\pi n u_0 e^{-\frac{1}{2}\frac{b}{a}x} \cos 2\pi n \left(t - \frac{x}{a}\right),$$

and the maximum velocity,

$$v_{max.} = 2\pi n u_0 e^{-\frac{1}{2}\frac{b}{a}x} \quad \ldots\ldots\ldots (4)$$

The energy transmitted across a given section of the pipe in time δt is equal to

$$\frac{\pi d^2}{4} p v \delta t = \frac{\pi d^2}{4} K \frac{\partial u}{\partial x} \frac{\partial u}{\partial t} \delta t.$$

The mean rate of transmission of energy per second over each stroke of the plunger is thus given by

$$\frac{\pi d^2}{4} \frac{K}{\tau} \int_0^{\tau} \frac{\partial u}{\partial x} \frac{\partial u}{\partial t} dt.$$

where τ is the duration of a stroke, *i.e.*, of a half-cycle.

Writing $2\pi n \left(t - \frac{x}{a}\right) = a, \quad dt = \frac{da}{2\pi n}, \quad \tau = \frac{1}{2n}$;

this becomes

$$\frac{K d^2}{4} \int_0^{\pi} \frac{\partial u}{\partial x} \frac{\partial u}{\partial t} da$$

$$= \frac{\pi}{2} \frac{K}{a} (\pi n d u_0)^2 e^{-\frac{b}{a}x} \quad \ldots\ldots\ldots (5)$$

$$= \tfrac{1}{2} p_{max.} v_{max.} \frac{\pi d^2}{4} = E,$$

and the horse-power transmitted, if the foot be the unit of length, is obtained by dividing expression (5) by 550.

The loss of energy per unit length of the pipe, due to friction, is

$$-\frac{\partial E}{\partial x} = \frac{b}{a} E,$$

and the efficiency of transmission through a pipe line of length l is

$$E_l \div E_o = e^{-\frac{bl}{a}} \quad \ldots\ldots\ldots \quad (6)$$

It should be noted that in any application of these results, it the calculations are in English units,

$$\rho = \frac{w}{g} = \frac{62{\cdot}4}{32{\cdot}2}$$

for water, while the value of μ is to be taken in lbs. per square foot, and the pipe diameter in feet.

From equation (6) it appears that if water at 15° C. is the working fluid ($\mu = 1{\cdot}23 \times 10^{-5}$), and if the pipe diameter is 1 inch (·0833 feet) the efficiency of transmission with different lengths of pipe line becomes :—

l (feet)	1,000	2,000	5,000	10,000
η	·94	·88	·73	·54

from which it is evident that the system is not well adapted for transmission over long distances. Experiments by H. Moss,[1] on a small installation transmitting 28 K W. showed a maximum overall efficiency of about 83 per cent. Here the pipe line was only 6 feet long and ·5 inches diameter, and sperm oil was used as the working fluid.

For a more detailed investigation of the theory, which becomes complex when a complicated pipe system is used, a treatise by the inventor, Mr. Constantinesco, should be consulted.[2]

Examples

(1) Determine the H.P. transmitted through a 6-inch pipe if the velocity of flow is 3 feet per second, and the delivery pressure 1,000 lbs. per square inch.

Answer. 154 H.P.

[1] "Proc. Inst. Mech. Engineers," November, 1923.

[2] "The Theory of Wave Transmission," G. Constantinesco.

(2) If in the preceding question the pressure at the station is 1,120 lbs. per square inch, determine the efficiency of transmission, and also, taking $f = \cdot 01$ the length of the supply pipe line.

Answer. Efficiency = 89·2 per cent.
Length = 4·7 miles.

(3) 100 B.H.P. is required from a hydraulic motor having an efficiency of 75 per cent. The motor is two miles from the generating station and only a single 6-inch pipe line is available. The pressure at the station being 1,120 lbs. per square inch, determine the pressure at the motor, the efficiency of transmission, and the velocity of flow.

Answer. Pressure = 1,088 lbs. per sq. inch.
Efficiency = 97·0 per cent.
Velocity = 2·392 f.s.

(4) Two branch pipes, respectively 4 inches and 3 inches diameter, are supplied from a 6-inch pipe. The pressure at the delivery end of the 4-inch pipe is 700 lbs. per square inch ; that at the end of the 3-inch pipe is 710 lbs. per square inch. These pipes are respectively 500 and 700 feet long. The 6-inch pipe is 880 feet long. Take f throughout as being $= \cdot 01$ and determine the pressure at the inlet to the latter pipe.

(5) Determine the maximum H.P. which can be transmitted through a 4-inch pipe two miles long—$f = \cdot 012$—if the inlet pressure = 750 lbs. per square inch. Also determine the pressure at the outlet, and the velocity of flow when this power is being transmitted.

Answer. 56·4 H.P.
Pressure = 500 lbs. per square inch.
Velocity = 4·94 feet per second.

(6) An accumulator has an 18-inch ram and 23 feet lift, and is loaded with 129 tons total weight. Taking friction to account for ·25 per cent. of the total pressure on the ram, determine the H.P. given into the mains if the accumulator falls steadily in three minutes, the pumps delivering 500 gallons per minute in the meantime.

Answer. Total H.P. = 67 + 396 = 463.

(7) A steam accumulator is placed horizontally and has a 48-inch steam piston coupled to a 6-inch ram. The steam piston packings, etc., exert a frictional force of 500 lbs., while the hydraulic packings account for 1 per cent. of the total pressure exerted on the ram. Determine this pressure if the steam pressure is 150 lbs. per square inch.

Answer. 9,486 lbs. per square inch.

(8) The ram of a differential accumulator is 4 inches diameter, and the surrounding bush is $\frac{1}{2}$ inch thick. Determine the accumulator pressure if the supported weight is 10 tons, neglecting the effect of friction.

Answer. 3,168 lbs. per square inch.

(9) The accumulator of the preceding question feeds a riveter, whose ram, at the end of its working stroke, suffers a retardation of 5 feet per second per second. The riveter ram is 5 inches diameter. Determine the pressure on the rivet head at the end of the stroke.

Answer. 39·8 tons.

CHAPTER XXIII.

Hydraulic Appliances and Machines.

Water Meters—Lifts and Hoists—The Hydraulic Jigger—Cranes—Coal Tips—Crane Valves—Hydraulic Jack-Press—Riveters—Lock Gate Machinery—Sluice Gates—Capstans—Hydraulic Transmission Gears—The Hydraulic Recoil Brake—Hydraulic Dynamometers—Hydraulic Transformer.

Art. 224.—Water Meters.

Water meters may be divided into the following classes :—

(1) Low pressure meters.
(2) Inferential meters.
(3) Positive meters.
(4) Meters for waste detection.
(5) Venturi meters.

(1) This type suffers from the disadvantage that all the pressure head of the supply is lost, and that when used for domestic supply it must therefore be placed at the top of the building. It is, however, well fitted for the measurement of small flows. The "Parkinson" meter, which is of

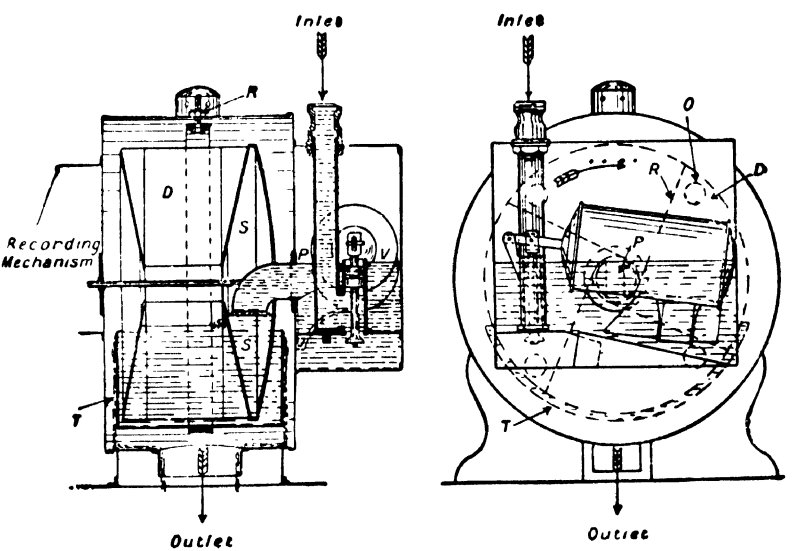


Fig. 372.—"Parkinson's" Low Pressure Water Meter.

this type, is illustrated in Fig. 372, and has given excellent results in measuring the power water delivered by the City of London Hydraulic Supply Company. In this case the meter is applied to measuring the exhaust water from the various machines. Here water from the inlet pipe is maintained at a uniform level in the inlet chamber by means of a float and valve V. This valve is constructed as shown so as to be balanced whatever the pressure in the supply pipe. On leaving this chamber by the pipe P, the water flows into the annular space S formed in the drum D. This drum is hollow and contains four compartments formed by oblique radiating plates R, which overlap each other to the extent of about 90°. Each compartment opens at its inner periphery into the space S, and in turn receives a supply of water. The centre of gravity of the water in the compartment being, on account of its position, to one side of the axis of

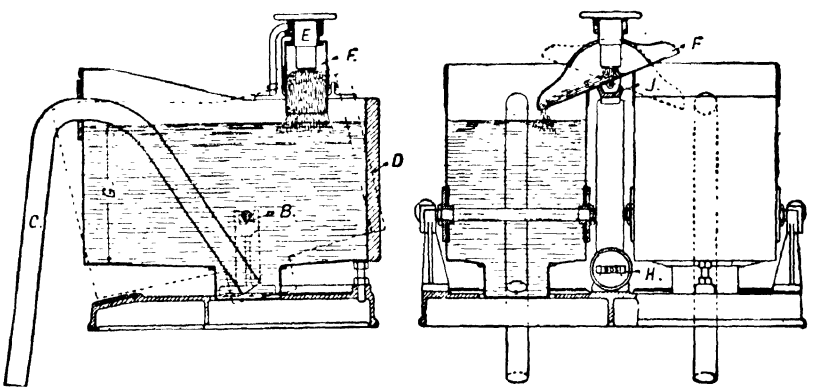


FIG. 373.

the drum, this produces a rotation about the axis and brings the next compartment into communication with the space S. At the same time the outlet O, from the first compartment, which is on the opposite side of the drum to S, is brought by the rotation below the water level in the compartment, and discharges the water into the trough T in which the drum rotates, and from which it flows away to the discharge pipe. The height of the trough may be adjusted as required, and as the volume discharged per revolution depends on the depth of the immersion of the drum, this enables the discharge per revolution to be adjusted. The number of revolutions of the drum, and hence the volume passing the meter, may then be recorded on a suitably engraved dial.

When used for domestic supply purposes, discharge takes place into a cistern from which the water is led over the building. An inlet valve on

the supply pipe—not shown in the sketch—is then opened by a ball valve when the water level in this cistern is lowered.

Another meter of this general type is shown in Fig. 373. The apparatus consists of two tanks of equal size carried on knife edges at *B*. Each tank is fitted at one end with a syphon pipe *C*, and at the other with a weight *D*. The liquid to be measured flows through the inlet pipe *E* along the movable glide channel *F* into whichever tank happens to be in operation.

The weights *D* are so adjusted that until the tanks are full up to the height marked *G*, they remain in a horizontal position, but as the weight of liquid increases by the continued flow, the tanks tilt into the position shown by the dotted line, when the liquid flows through the syphon pipe. After the syphon has been started and the level of the liquid in the tank has fallen sufficiently, the tank tilts back again to its original position, under the action of the weight *D*, the syphon continuing in action until the tank is emptied. As each tank assumes the position indicated by the dotted lines, it suddenly tilts the guide *F* over, so that the new liquid to be measured falls into the other tank, when the same operation is repeated. It will thus be seen that both tanks are filled automatically with fresh liquid, while the measured liquid runs away into a reservoir or other receptacle as required.

The number of times each tank is filled and emptied is registered by the indicator *H*, which is connected with both tanks. When either tank is in a horizontal position, the guide *F* rests on the support *J*, which is of saddle form, so that when the tank is in the act of tipping no influence is exercised either by the weight of the guide or by the pressure of the liquid in the guide, or by the resistance of the counter.

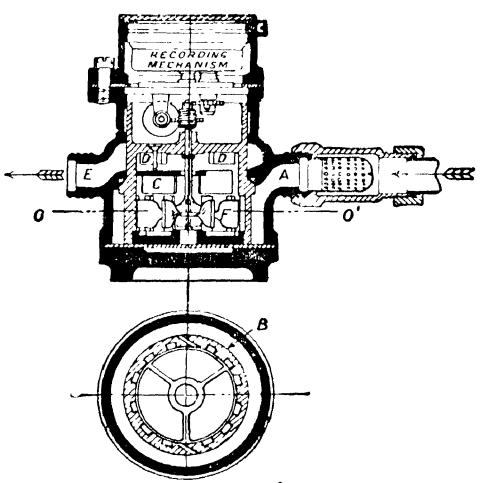


FIG. 374.—Tylor's Inferential Water Meter.

(2) The inferential meter consists simply of a small turbine, through which the whole supply is passed, which drives the recording apparatus. The water is not actually measured, but its volume is inferred from the number of revolutions of the turbine runner or fan, which is the only moving part. The meter must be calibrated

by allowing it to pass a known volume of water per minute, and has the advantage of being small, light, and cheap, and fairly accurate for good speeds of flow. Since, however, there is a limiting velocity of flow, below which the reaction on the runner vanes is insufficient to overcome the friction of the bearings and recording mechanism, it is unsuitable for recording small flows. Moreover, the runner tends to keep on rotating for some short time after the flow of water has ceased, and thus to over-record the flow. Where taps are opened and closed frequently, this action, unless guarded against, may lead to the flow registered being largely in excess of that actually taking place.

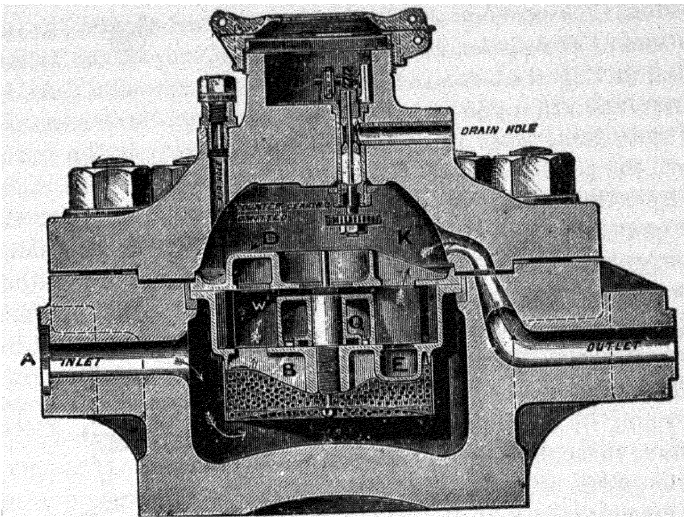
Tylor's Inferential Meter (Fig. 374) is of this type. Here water enters at A, and surrounds the inner casing shown in section at B. It then finds its way through two inclined ports in this casing, and, impinging on the vanes of the fan F, drives this around with a velocity which depends on the flow of water, and which is recorded on the dial worked by its spindle and worm gear, afterwards escaping through the ports D and pipe E. To prevent overrunning, eddy formation is fostered by a series of recesses formed around the inside of the inner casing, and by baffles above the fan at C. To reduce friction at the lower footstep bearing an oil reservoir is provided in the hollow spindle of the fan.

(3) The Positive Meter consists of a small hydraulic engine, either of the rotary or reciprocating piston type. All the water to be measured passes through the cylinder or cylinders of this engine, and its volume is taken as that of the piston displacement.

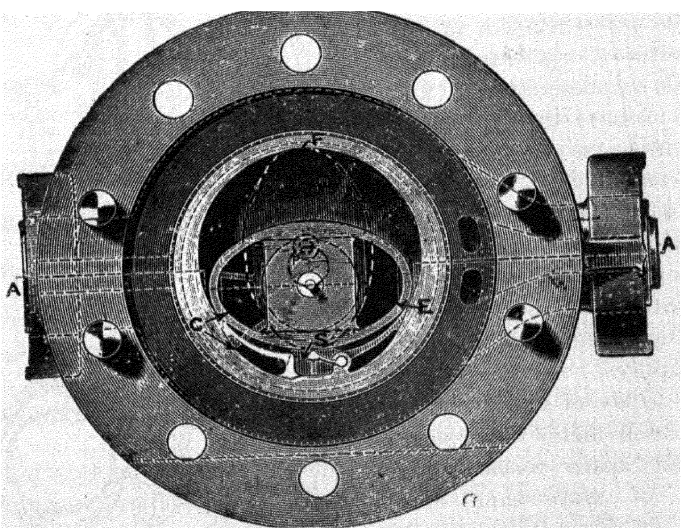
The number of revolutions or strokes of the piston being registered on a recording apparatus, this is easily arranged to record the volume passed.

The rotary type is common in the United States of America, and, as usually made, consists of a casing of gun-metal or vulcanite, in which works a rotary vulcanite piston. This has no means of compensating for wear, and, as thus constructed, is very unreliable for small flows, even when new. After being in use for some time the increased leakage past the piston renders it still less reliable.

The Kent "Uniform" meter (Fig. 375) is one of the best of this type, and here compensation is made for the effect of wear of the rotary piston P by an adjustable metal tongue S. In this meter water is admitted at A and fills the casing around the working chamber B. The vulcanite piston P, elliptical in section, rotates and slides freely, but without play, on the fixed hub Q, which is itself eccentric with respect to the working chamber. The piston carries a central pin which describes a circular path as rotation takes place, and which actuates the recording mechanism.



SECTION THROUGH AA.



PLAN WITH TOP COVER REMOVED

FIG. 375.—"Kent" Positive Water Meter.

The bottom ports at *C* are in free communication with the chamber *B*, and the action of the meter is as follows:—

Assuming the piston to be in the position shown in dotted lines, water is admitted through these ports and the upper port *D*, to the space between the chamber, the piston, and the tongue *S*, and to the interior of the

hollow piston, thus exerting a pressure between the hub and the inside of the inner end of the piston, as well as on the outside of the piston from C to S, and driving the latter round in a clockwise direction. At the end of half a revolution the other end of the piston becomes the driver and so on.

In the meantime water has been filling the space to the left of the piston until the position shown in full lines is reached, when the supply is cut off from the space F, this space is put into communication with the discharge port K by way of the lower port E and the hollow piston, and for the remaining half revolution the water in this space and in the outer end of the piston is discharged through K, the whole cycle of operations then being repeated. The action becomes much more evident if a tracing showing the piston be rotated, keeping its correct relative position, through a whole cycle. This meter is used exclusively in connection with the measurement of pressure water from the power station of the Manchester Corporation, and has given excellent results. For such a purpose this type has the advantage that in case of a breakdown the flow of water is not stopped, the wastage of water thus entailed being of small consequence when compared with the inconvenience caused by a stoppage of the power supply.

Many types of the reciprocating piston meter have been made, but space forbids the mention of more than one example. The "Imperial" meter of Mr. Schönheyder (Fig. 376) consists of three single-acting cylinders fitted with a single hemispherical gun-metal distributing valve V bearing on a vulcanite seat S. The upper

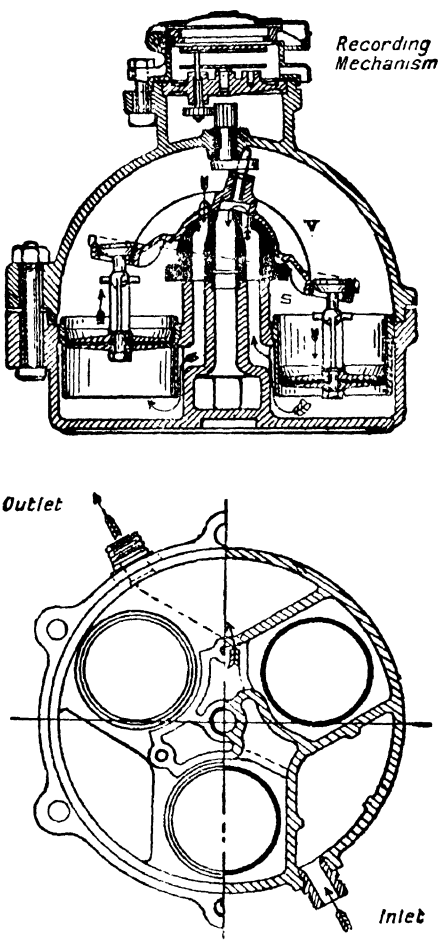


FIG. 376.—Schönheyder Positive Water Meter.

side of each piston is exposed to inlet pressure, and according to the position of the valve the lower side of each in succession is put into communication with the outlet passage. The pistons are thus successively forced down and their contents discharged into the outlet. At the same time one or both of the other cylinders is having its piston raised, water being admitted below the piston.

Discharge is then practically continuous and the machine has no dead centre, while the smallest flow is registered. The positive type has the big advantage over the inferential meter that whereas the correct registration of the former is unaffected by an increase in the friction of the moving parts, such an increase in friction may seriously affect the speed of the latter for a given flow. Its chief drawbacks consist in its liability to water hammer unless worked at a very low speed, and its consequent large dimensions and high first cost.

(4) One of the best types of meter for the determination of the varying flow in a pipe, and hence of leakage, is that of Mr. Deacon (Fig. 377). Here the water flows through a conical tube containing an axial rod which carries a circular disc *D*. Any axial movement of this disc is resisted by a spring, and is recorded on the registering apparatus. The varying pressure on the disc, produced by any variation in the rate of flow, may then be registered, and by suitably calibrating the recording mechanism the position of the rod may be arranged to indicate the flow in gallons per minute.

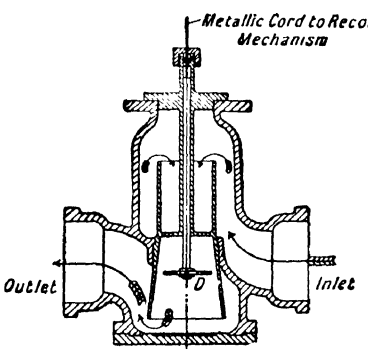


FIG. 377.—"Deacon" Meter.

ART. 225.

The Venturi water meter, invented by Herschel in 1881, and called by him after Venturi because of the experimental work of the latter on the physical properties of diverging tubes, depends in its principles on the truth of Bernoulli's theorem.

It is at once the simplest and, for large quantities of water, the most satisfactory meter yet designed, and simply consists (Fig. 378) of a pipe passing the whole quantity of water to be measured, and fitted with a portion *B C*, uniformly converging to a short parallel throat *C D*, to which it is connected by easy curves. At *D* the pipe again diverges to its full diameter at *E*. The usual proportions of the meter are indicated in the figure in terms of the pipe diameter, experiments showing that an angle of

divergence of 5° 6′ gives the best results in the reconversion of kinetic to pressure energy from D to E. If A and a be the areas of main pipe and of throat, then, as shown on p. 88, with a perfect fluid

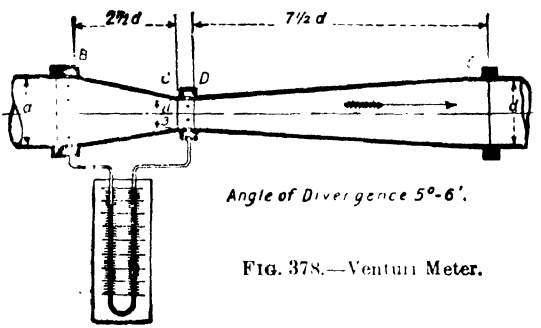


FIG. 378.—Venturi Meter.

$$V_A = \sqrt{\frac{2g(p_A - p_a)}{W\left\{\left(\frac{A}{a}\right)^2 - 1\right\}}}$$

Owing to friction the true velocity is less than this and the discharge is given by

$$Q = C A \sqrt{\frac{2g(p_A - p_a)}{W\left\{\left(\frac{A}{a}\right)^2 - 1\right\}}}$$

Where C is a coefficient of discharge which depends slightly on the diameter and roughness of the pipes and on the velocity of flow.

For any meter $\left(\frac{A}{a}\right)$ is fixed, the ratio often being 9 : 1, so that

$$\sqrt{\frac{2g}{\left(\frac{A}{a}\right)^2 - 1}} \text{ is constant} = k.$$

$$\left.\begin{array}{l}\text{Then the volume in cubic feet} \\ \text{per second through meter}\end{array}\right\} = V_A A = C k A \sqrt{\frac{p_A - p_a}{W}}$$

$$= C k' \sqrt{\frac{p_A - p_a}{W}} \qquad (1)$$

Here $\frac{p_A - p_a}{W}$ or $h_A - h_a$ is the difference in pressure at the throat (1) and entrance (2) expressed as a head in feet of water, and is directly measured in the meter. The constants C and k being determined for the instrument, the volume passing per second may be directly inferred.

Coefficient of Discharge C for the Venturi Meter.—Since friction losses increase more slowly than v^2, the value of C increases slightly with the velocity As shown in Chapter IV., conditions of dynamical similarity indicate that in similar meters the same value of C should be attained for identical values of $v\,d/\nu$, and this is confirmed by experiments on meters passing fluids of widely different viscosities. With very low velocities C may

vary within fairly wide limits, values as low as ·75 and as high as 1·25 being recorded.[1] If the following precautions are adopted the values of C will usually lie between ·97 and 1·00 so long as the pipe diameter exceeds 2 inches.

(1) The velocity in the pipe line should not be less than 1 f.s.

(2) A length of at least 15 pipe diameters should be allowed from any bend or valve on the upstream side of the meter. Any whirl in the water entering the meter increases the value of C.

(3) The absolute pressure at the throat should not be less than about 3 lbs. per square inch.

(4) The pressure connections at throat and entrance should be taken from a series of points—not less than four—around the periphery.

(5) Great care should be taken to ensure that all air is removed from the gauge connections.

The values of C obtained from a meter measuring pulsating flow are less than with uniform flow. If α is the proportional fluctuation of velocity on each side of the mean, C is reduced approximately in the ratio $\sqrt{\frac{2}{2+\alpha^2}}$

For measuring the difference of head $h_A - h_a$, a differential gauge consisting of a U-tube containing mercury (Fig. 378) is often used. In this case, since the specific gravity of mercury is 13·6 the observed difference of level is to be multiplied by (13·6 − 1) = 12·6 to give the equivalent head in feet of water. Where small differences of head are involved a simple inverted U-tube may be used as shown in Fig. 379. Here air under pressure must be supplied to the top of the water columns.

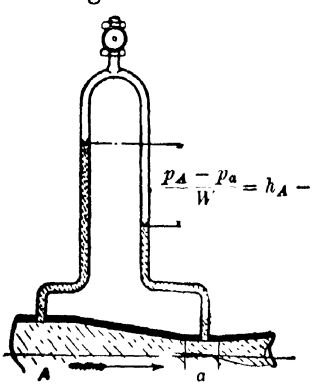


FIG. 379.

When metering any other fluid, such as petrol (sp. gr. $= S_1$), if a U-tube as in Fig. 378 is used, if the heavier fluid has a sp. gr. $= S_2$, the effective head $= h(S_2 - S_1)$.

It is to be noted that the foregoing equations apply *whether the meter be horizontal or inclined*, since if the flow be stopped by closing the outlet valve, the levels of the two columns will become identical. Any subsequent difference of level is simply due to the differences of kinetic energy at points A and a, due to the flow.

[1] Abnormal coefficients of the Venturi meter. Gibson, "Proc. Inst. C. E.," vol. 199, 1914–15, Pt. I.

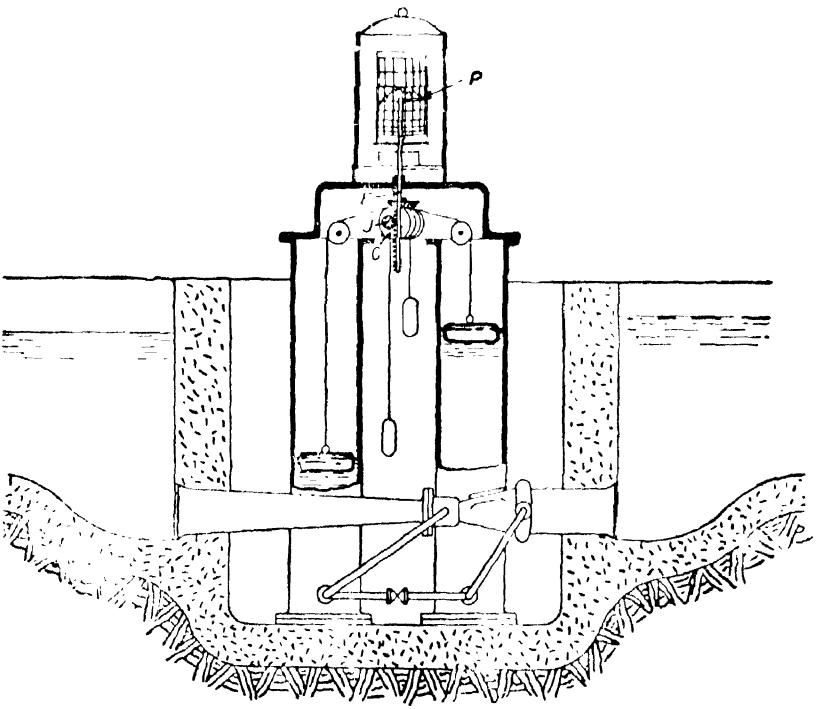


FIG. 380.—Recording Device for Venturi Meter.

It should be noted that the converging portion of the main is the only part really essential to the meter action. The diverging cone simply ensures that the reconversion of kinetic into potential energy shall take place without undue loss. Where the meter is required to measure the flow in either direction, the angles of convergence and divergence are each made equal.

The Venturi meter is often used with a registering device giving an automatic record of the velocity or discharge. One form of such a recorder is shown in Fig. 380. Here two float pipes are connected respectively to the upstream and throat chambers of the tube. Each contains a float, and these are connected by means of wire cords to a differential gear consisting essentially of three bevel wheels A, B, and C of which the motions of the two equal wheels A and C are controlled by the floats, while B is carried on an axis attached to a loose sleeve. Evidently if the wheels A and C revolve in opposite directions owing to the water columns rising or falling together, the wheel B will simply revolve on its axis. Any alteration in the relative level, however, causes the wheel B and the sleeve on which it is mounted to rotate about the axis D, and thereby to produce a vertical movement of the rack E, which gears with a pinion J mounted on the outer end of the sleeve, and which carries the recording pencil P at its upper end. The recorder may be arranged to indicate either the velocity of flow and therefore the discharge at any instant, or the total discharge of the meter. In the former case the pencil traces out a curve on a sheet carried by a vertical drum driven at a constant rate by clockwork, this

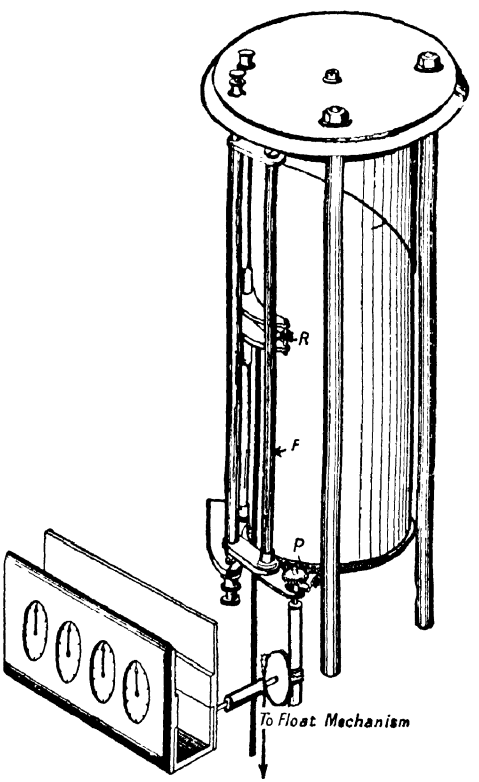


FIG. 381.—Integrating Mechanism for Venturi Meter.

paper being ruled with horizontal lines the distances of which (representing equal increment of velocity) from the zero line, are proportional to $\sqrt{h}$. Where it is required to have an automatic record of the discharge as well as a record of the rate of flow, an ingenious integrating device is used. This consists of an additional drum concentric with the recording drum and rotated uniformly—usually once in 10 minutes—by clockwork. The surface of this drum is in two planes ; one of the full diameter of the drum, and the other of a reduced diameter forming a recessed surface. The boundary curve of the raised surface is a portion of a parabola. The arrangement is shown

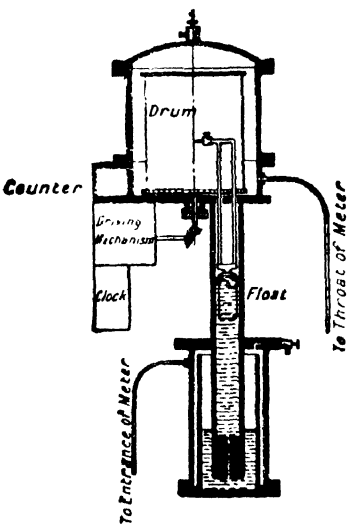


FIG. 382.—Recording Mechanism for Venturi Meter.

diagrammatically in Fig. 381. A roller R receives vertical motion from the float mechanism, and is pressed against the drum by a rocking frame F. This frame carries a pinion P which actuates the counter mechanism, and which does, or does not, gear with the teeth of a spur wheel carried by the drum according as the roller is on the recessed or the raised portion of its surface.

Thus, when the roller is at the highest point it engages only with the raised surface, so that in this position—corresponding to no flow—the counter is not actuated. At the lowest position the roller is entirely upon the recessed surface, and the counter is actuated continuously. In any intermediate position the counter revolves only while the roller is in contact with the recessed surface, so that the fraction of the period of revolution of the

drums during which the counter rotates is proportional to $\sqrt{h}$. Thus since the velocity of flow is also proportional to $\sqrt{h}$, the discharge is summed or integrated, and can be read directly from the counter.

Another type of velocity recorder is shown in Fig. 382, while where a combined velocity and discharge recorder is required a modification of this in which a cast-iron U, having both legs of the same area and containing mercury, may be used. These contain iron floats which carry light racks gearing with pinions, which convey their motion to corresponding racks placed outside the tubes. The racks carry light rods which pass up to the recording mechanism. That connected to the float in the throat chamber regulates the amount of registration by the counter, while the second carries the pencil of the velocity recorder.

Art. 226.—Hydraulic Lifts and Hoists.

Probably in the aggregate more power is used by lifts and hoists than by any other class of hydraulic machinery, and for such work as this, hydraulic transmission is particularly suitable.

Several types of lift are in use, these consisting of modifications of the simple direct-acting or of the suspended type. The former consists of a hydraulic cylinder sunk vertically in the ground; of length slightly greater than the maximum travel of the lift; and fitted with a ram which carries the lift cage as shown in Fig. 383.[1]

Pressure water is admitted below the ram, and thus raises the cage. It follows that the ram, considered as a loaded column, must be of sufficiently large sectional area to support the weight without buckling, and this prevents the use of very high pressures in the ram cylinder.

Example.

If the gross weight to be lifted = 2 tons.
„ pressure in cylinder = 750 lbs. per square inch.
„ lift travel = 80 feet.
We have the area necessary to raise the weight $= \frac{4{,}480}{750} = 6$ square inches.
∴ Ram diameter $= 2\frac{3}{4}$ inches.

a diameter which is obviously too small for a column 80 feet long, to support 2 tons without buckling.

Actually, however, the area of the ram would need to be greater than this in order to overcome friction and to give the necessary acceleration at starting.

[1] By kind permission of the makers.

V_a and V_D, into the space C below the lifting ram, and raises the latter. The weight to be lifted is carried either on the ram table T, or on a side shoe projecting from the ram casing. Screws are provided for supplying the cistern A with water and for allowing of the inlet of air, while a lowering screw permits of the escape of pressure water from the space below the lifting ram into the supply cistern when it is required to lower the load. The lifting ram is usually packed by means of a cup leather, and the pump plunger by means of a single leather ring.

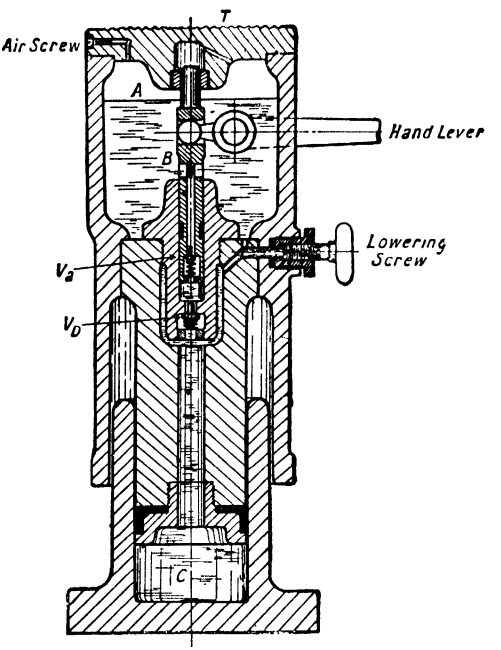


FIG. 400.—Hydraulic Jack.

If m = ratio of travels of lever handle and of pump plunger, and if a and A are the areas of plunger and ram, the theoretical mechanical advantage of the jack $= m\frac{A}{a}$, so that, neglecting friction, the force P to be applied at the lever handle to support a weight W on the ram table is given by $P = W\frac{a}{m\,A}$ lbs.

Thus if $m = \frac{24}{1{\cdot}25} = 19{\cdot}2.$

„ diameter of ram $= 5\frac{1}{2}$ inches
„ diameter of plunger $= 1$ inch
$\Big\}\ \frac{A}{a} = 30{\cdot}25.$

if $P = 50$ lbs., $W = \frac{50 \times 19{\cdot}2 \times 30{\cdot}25}{2{,}240} = 13$ tons.

Actually the weight lifted is less than this because of friction losses. The efficiency of the jack, or the ratio $\frac{\text{actual}}{\text{theoretical}}$ weight lifted by the ram, can only be determined experimentally, and varies largely with the condition and size of the apparatus, as well as with the magnitude and position of the load.

Thus if $a = 2$ feet per second per second, while $F' = 10$ per cent. of the total force on ram, we have

$$\text{area of ram} = \frac{4{,}480}{750}\left\{1\tfrac{1}{16}\right\}1{\cdot}1$$

$$= 1{\cdot}17 \times \frac{4{,}480}{750} = 7 \text{ square inches.}$$

In practice it is usual to make allowance for friction and acceleration

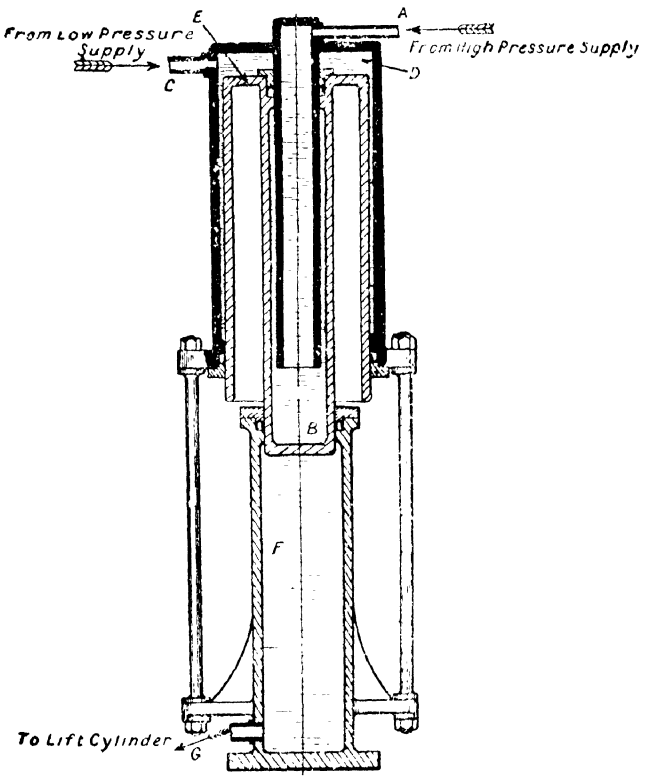


FIG. 384.—Balance Cylinder for Hydraulic Lift.

by making calculations for a load about 25 per cent. in excess of the nominal.

If the area of the ram be increased, the working pressure must be reduced to suit, and for this purpose some form of pressure reducer—in effect a reversed intensifier—is used. Also, since the weight of ram and cage forms a large proportion of the whole load to be lifted, this must be balanced for efficient working.

Since the volume of water displaced by the ram diminishes as the lift rises, the effective weight of the ram, which is its own weight less that of the water displaced, increases. In lifts for large powers the effect of this variation is in general unimportant, and becomes of less importance as the working pressure increases.

EXAMPLE.

Pressure = 250 lbs. per square inch.

W = 5 tons.

Lift = 60 ft.

$$\text{Area of ram} = \frac{5 \times 2{,}240}{250} = 44{\cdot}8 \text{ square inches.}$$

$$\therefore \text{ Difference in apparent weight of ram at top and bottom of its stroke} = \left\{ \begin{array}{l} \text{Weight of a column of water 60 feet long and} \\ 44{\cdot}8 \text{ square inches sectional area,} \end{array} \right.$$

$$= \frac{60 \times 44{\cdot}8}{144} \times 62{\cdot}4$$

$$= 1{,}164 \text{ lbs.}$$

If working pressure = 500 lbs. per square inch, the area of ram = 22·4 square inches and its difference of weight = 583 lbs., a value which is small in comparison with the weight of five tons.

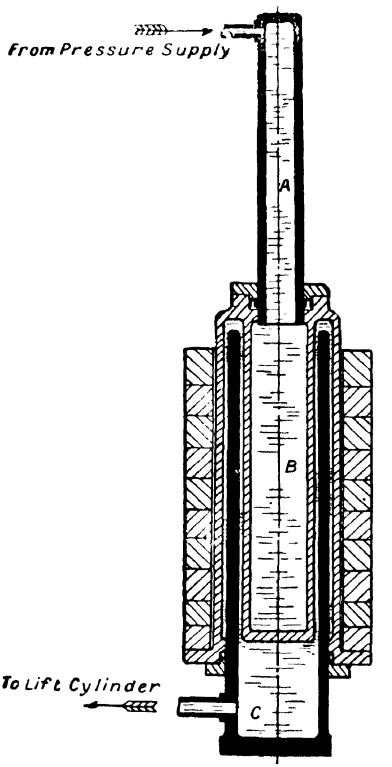


FIG. 385.—Balance Cylinder for Hydraulic Lift.

Various devices have been adopted to overcome these difficulties.

The weight of the ram and cage may be balanced by a counterbalance weight attached to the cage by chains passing over a series of pulleys at the top of the lift shaft, and since, as the lift rises, the length of chain on the balance-weight side of the pulleys increases, this may be made to counterbalance the increasing effective weight of the ram by making the chain of such dimensions that its weight per foot run is half that per foot run of the column of water displaced by the ram. This method suffers from the disadvantage that the upper part of the ram is in tension, and a fracture would cause the cage to be dashed against the top of the shaft. It thus detracts from the otherwise essentially safety features of this type of lift, and also increases the mass to be accelerated at the beginning of the travel.

A second device, which is more common in high-class work, is that of the balance cylinder, one type of which is illustrated in Fig. 384. Here pressure water is admitted to the interior of the hollow ram *B*. The

cylinder D is in communication with an auxiliary low pressure supply, through the pipe C, and a downward pressure on the annulus at E is thus produced, which, together with the weight of this ram, produces a pressure in the cylinder F sufficiently great to balance any required proportion of the weight of the lift ram and cage. The total pressure transmitted to the water in the cylinder F is then the sum of the weight of the ram B and of the pressures on E and on ram B, the former performing the balancing, and the latter lifting the load.

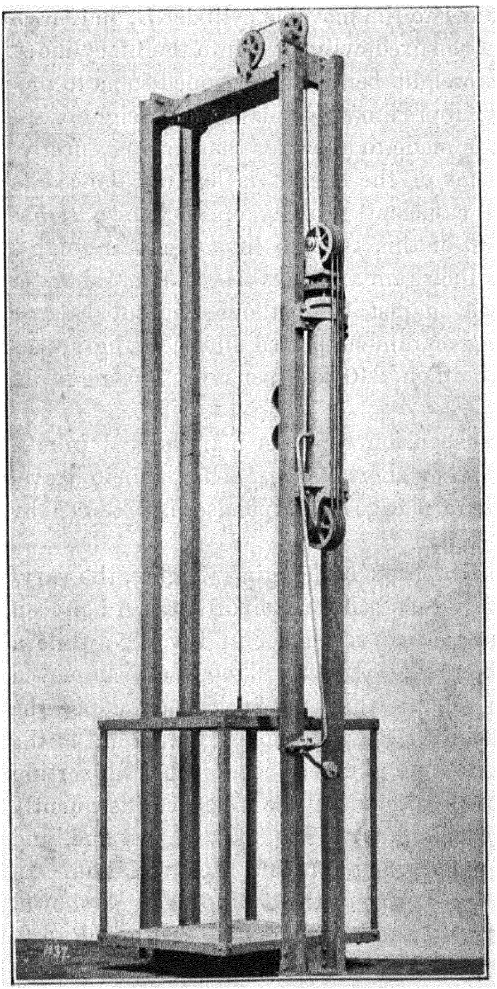


FIG. 386.—Suspended Hoist.

A suitable area of lift ram being assigned, the external diameter of B is calculated so as to give the required intensity of pressure in the cylinder F. The lift cylinder is supplied from F through the pipe G. On the down stroke of the lift, the ram B rises, the balance water is returned to its own supply tank, and the only water rejected is that originally filling the high-pressure ram B. In a lift of this type mentioned by Mr. Ellington, the lift ram was $4\frac{1}{4}$ inches diameter, the lift carrying 7 cwts. with a 70 feet rise. The volume of pressure water at 700 lbs. per square inch was $10\frac{1}{2}$ gallons per trip, as against 43 gallons when working direct-acting and without the balance cylinder.

It will be observed that as the balance ram falls, the pressure on the annulus E increases, due to the increasing head to which it is subjected,

and this to a certain extent counterbalances the difference in effective weight of the lift ram.

A second type of balance cylinder, which has the advantage that all packings are external, is illustrated in Fig. 385. Here pressure water is admitted through the hollow ram A to the movable cylinder B, and forces water at a reduced pressure out of the fixed cylinder C into the lift cylinder. The cylinder B is weighted, this weight being proportioned so as to produce a pressure in C sufficient to approximately balance the weight of the lift ram and cage. No attempt is made to allow for the varying effective weight of the lift ram. Sufficient of the weight is left unbalanced to cause the lift to descend with sufficient rapidity on the down stroke even with an empty cage. A lift of this type, to lift 20 cwt. through a distance of 90 feet, carries a six-inch ram and uses $24\frac{1}{2}$ gallons of water at 700 lbs. pressure per trip, as against 109 gallons without balance cylinder. With the direct-acting or ram-supported lift, hoisting speeds up to 180 feet per minute are common, 240 feet per minute being about the maximum.

The second type of lift—the suspension type—is manipulated from a hydraulic ram having a comparatively short stroke. The requisite travel in the wire rope or ropes by which the cage is suspended, is obtained by multiplying this by means of a jigger.

The weight of the cage may be balanced by hanging weights, the varying immersion of the ram in this case being unimportant. A hoist on this principle, but with unbalanced cage, is shown in Fig. 386, while a balanced lift is shown in Fig. 387. Here two wire ropes are employed for lifting and two for carrying weights which partly counterbalance the cage. As the cage of a suspended lift rises, a portion of the weight of the suspending rope is transferred to the plunger side of the supporting pulley, and the effective weight transferred to the plunger consequently varies throughout the whole of its stroke. Fig. 388 shows one, and Fig. 389 a second method of compensating for this rope variation. In the former a double-balance chain is suspended from the cage as shown, so that if R be the travel of the cage, the length of each chain is $R \div 2$. Let m be the multiplying factor for the jigger; W the weight of unbalanced portion of cage; w the weight of the suspending cable per foot run; w' the weight of each balance chain per foot run.

Then with cage at bottom, the pull on plunger $= m \{ W + w R \}$

" " " top, " " $= m \{ W + w' R \} - w R$

And for these to be equal, $w' = w \left(1 + \frac{1}{m} \right)$.

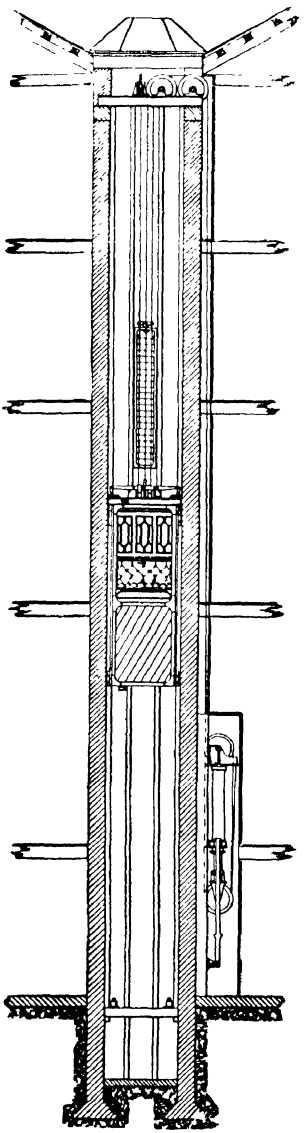


FIG. 387. — Multiple Wire Suspended Lift with Counterbalance Weights.

In the second method, the lower side of the plunger is in free communication with an open standpipe of height h feet containing water. Let d and D be the diameter of standpipe and of plunger, in feet. Then if $h = \frac{R}{m} \cdot \frac{D^2}{d^2}$, the difference of upward pressure on the plunger at the top and bottom of its stroke is $62{\cdot}4 \frac{\pi D^2}{4} \left(h + \frac{R}{m}\right)$ lbs.

For this difference of pressure to counterbalance the difference in the effective weight of cable, *i.e.*, $wR\left(1 + \frac{1}{m}\right)$, we must have

$$w R \left(1 + \frac{1}{m}\right) = 62{\cdot}4 \frac{\pi D^2}{4} \left(h + \frac{R}{m}\right)$$

$$\therefore h = \frac{R}{m} \left\{ \frac{4w(m+1)}{62{\cdot}4\,\pi D^2} - 1 \right\}$$

while $$d = \frac{D}{\sqrt{\frac{4w(m+1)}{62{\cdot}4\,\pi D^2} - 1}}$$

The suspension system has certain advantages in virtue of the cheapness of construction of the shorter ram and cylinder, and does not necessitate the provision of a deep well below the lift shaft to contain the cylinder, which may be fixed horizontally if required. Its drawbacks are due to the inefficiency of the multiplying jigger, and to the serious effects which may follow the rupture of a wire rope. If carefully designed, and frequently examined, the latter contingency should, however, be very remote, while the provision of adequate safety catches and brakes renders this almost as safe as the direct-acting system. The speed of hoisting may be made as great as is convenient. For passenger hoists this is usually about 2 feet per second, and for warehouse hoists up to about

6 feet per second. In modern American practice, express passenger lifts are occasionally run at speeds of 8 to 10 feet per second.

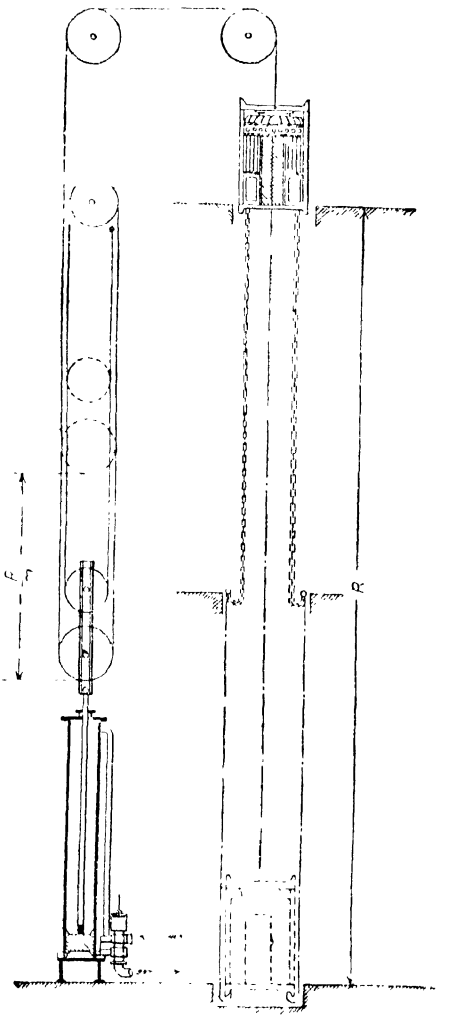


FIG. 388.—Elevator with Chain Compensation for Rope Variation

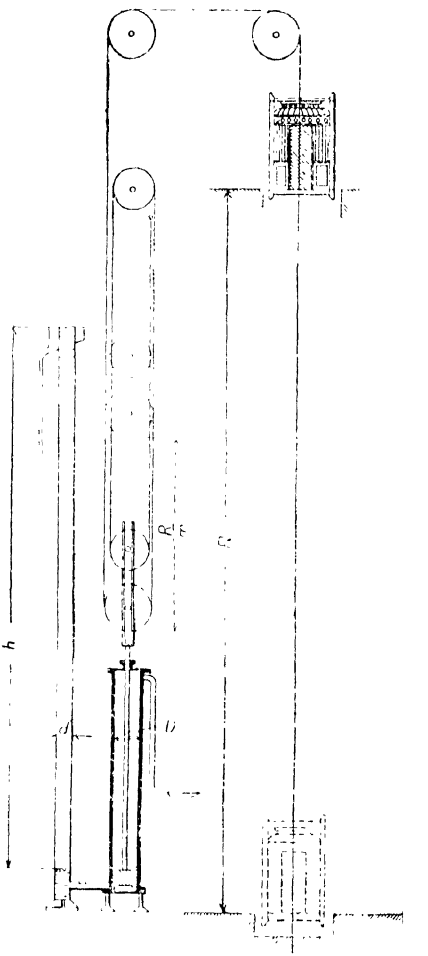


FIG. 389.—Hydraulic Elevator with Water Column Compensation.

For very heavy lifting, such as is necessary in canal lifts, etc., where loads up to 1,000 tons may be carried on a single ram, the direct-acting is the only suitable type.

Art. 227.—Efficiency of the Hydraulic Jigger.

The chain, or wire rope, and pulley multiplying gear known as the hydraulic jigger which is used to multiply the motion of a short stroke hydraulic ram for crane or hoist, as illustrated in Figs. 387 to 389, diminishes the mechanical efficiency of the system to a fairly large extent.

The loss thus introduced increases with the number of multiplications, and its magnitude varies so largely with the size and condition of sheaves, bearings, and rope or chain, that no definite law can be expected to cover each case even approximately.

With ordinary well-designed gearing, having large pulleys, small bearings, and wire ropes in good condition and well lubricated, the efficiency of the jigger may be taken as being approximately given by the following formula (H. Adams)—

$$\eta = \cdot 906 - \cdot 021\, m$$

where m is the number of multiplications of the stroke.

If, then, the friction of packing leathers = 5 per cent. of the total force on the ram, the efficiency of ram and jigger is equal to

$$\cdot 95 \{ \cdot 906 - \cdot 021\, m \}$$
$$= \cdot 86 - \cdot 02\, m$$

This gives the following values of the efficiency:—

m	2	4	6	8	10	15	20
Eff.	·82	·78	·74	·70	·66	·56	·46

Art 228.—Hydraulic Cranes.

Where high-pressure water is available it provides a most convenient means of operating power cranes, and in its safety, adaptability to suit varying conditions, and steadiness of operation, offers many advantages over its rivals—compressed air and electricity.

Such cranes are usually operated by hydraulic jiggers, the various operations of lifting, racking, and slewing being often performed by separate rams and cylinders, each regulated by its own separate valve.

Where the load to be lifted may vary within wide limits, some device is usually adopted to economise water at light loads. For small cranes, up to about two tons, a differential or telescopic ram may be used, the smaller

working inside the larger, which itself works in the pressure cylinder. For light loads the larger is held stationary by locking gear, the smaller ram then doing the lifting. For heavy loads the two rams work together as one.

Such an arrangement is shown in Fig. 390, R_1 and R_2 being the two

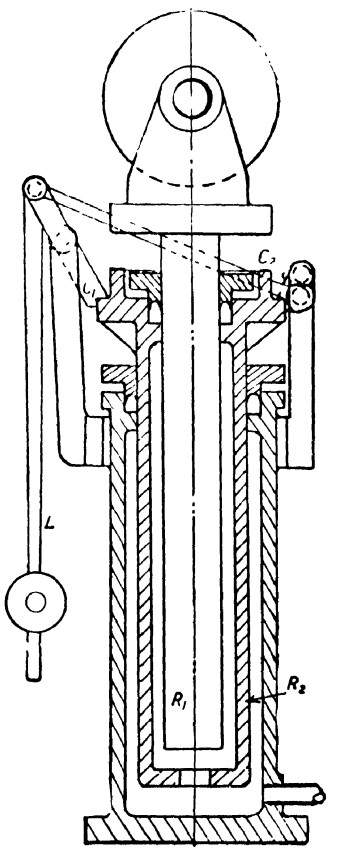


FIG. 390.—Hoisting Cylinder and Rams for Two-Power Crane.

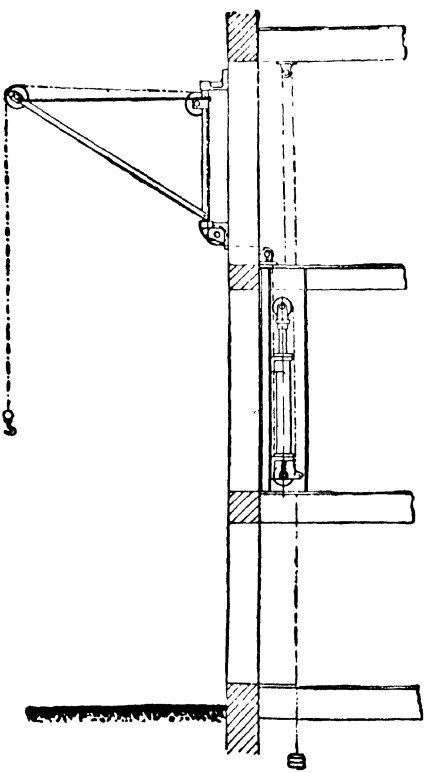


FIG. 391.—Hydraulic Wall Crane.

rams. Under light loads the ram R_2 is held by means of the catches C_1 and C_2 which are actuated by the lever L.

A second arrangement consists of a single ram d carrying a plunger of somewhat larger diameter D. For heavy loads water is admitted only to the larger plunger, while for lighter loads both sides of the plunger are put into free communication with the pressure supply. The forces exerted

in the two cases are then in the ratio of $D^2 : D^2 - d^2$, while the volumes of water used per stroke are in the same ratio.

For loads much above two tons, two distinct cylinders, each having its own valves, pipes, chains, and jib-head sleeves, offer many advantages. Thus for a 10-ton crane, one cylinder to lift $6\frac{1}{2}$ tons and a second to lift $3\frac{1}{2}$ tons would be suitable, the two working in conjunction for loads above $6\frac{1}{2}$ tons.

In Fig. 391 a simple type of hydraulic wall crane, suitable for a ware-

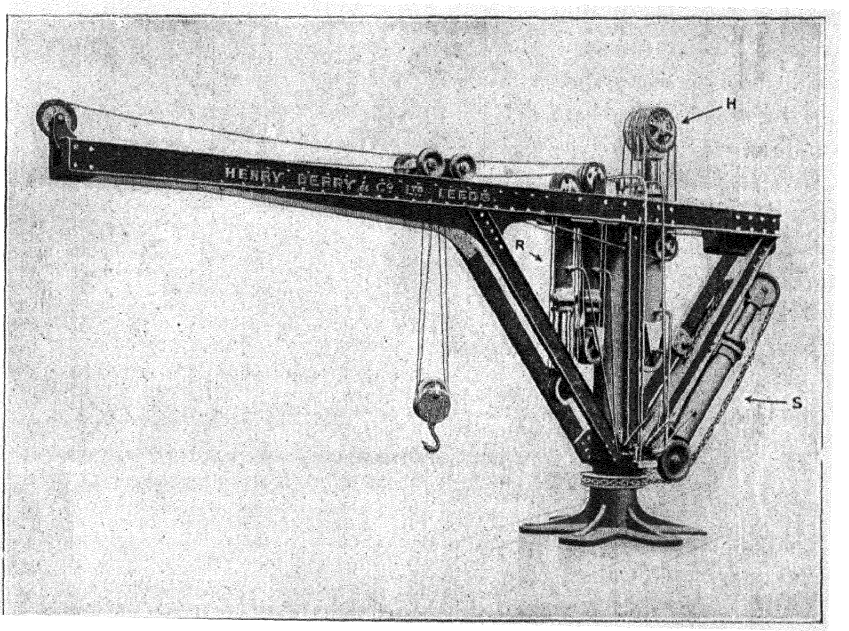


FIG. 392.—Self-contained Hydraulic Crane with separate Racking, Hoisting and Slewing Cylinders.

house, is illustrated. Here hydraulic power is used for lifting only. The jigger may be placed against the wall as shown, or horizontally if preferable, and the controlling valve worked from a hand rope on any floor of the building.

In Fig. 392, a type of hydraulic crane much used in steel works, for removing ingots from the soaking pit to the rolling mill, is illustrated.[1] The crane is self-contained, hoisting, lowering, racking in and out, and slewing being performed by hydraulic power. The latter operation is

[1] By kind permission of the makers

performed by means of chains gearing with a chain wheel fixed on the foundation plate.

There are two racking cylinders R, with a six-fold multiplying motion, one hoisting cylinder H, with an eight-fold multiplication, and two slewing cylinders S, with a two-fold multiplication, the valves being so placed that one man has full control over the whole crane.

The illustration shows details fairly well, and is self-explanatory.

For comparatively small powers and lifts these cranes may be direct-acting, such an one, suitable for loads up to two tons, and for lifts up to about 6 feet, being illustrated in Fig. 393.[1] Here the lifting hook is attached directly to a carriage running along the jib, which is itself raised and lowered by a direct-acting hydraulic cylinder sliding between the cheeks of the mast and working upon a hollow steel ram forming the bottom centre of the crane. For smaller loads and lifts the lifting cylinder may itself be suspended from the travelling carriage, the hook being attached to the ram, and pressure water being conveyed to the cylinder by a walking pipe.

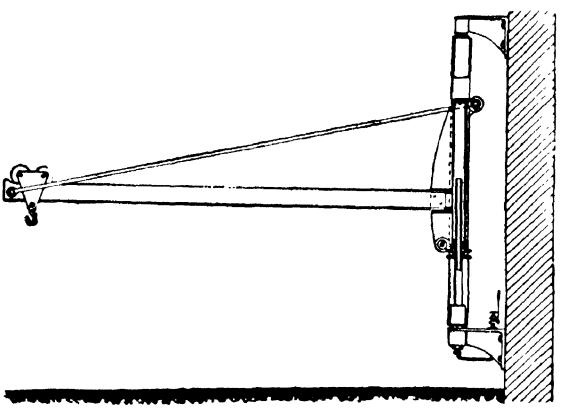


FIG. 393.—Direct-acting Wall Crane.

With the hydraulic crane the speed of the lifting hook may be adjusted up to about 6 feet per second.

Dock Cranes.—Fig. 394 [2] shows the general arrangement of a movable hydraulic luffing crane having a lower power of 15 tons and a maximum power of 30 tons. This crane has a lift of 66 feet and the jib has 39 feet maximum and 20 feet minimum rake.

Fig. 395 shows a type of balanced jib luffing crane,[3] as built for coaling purposes. The jib has an extended end to which are attached the counter-balance weights and tie rods. The lower ends of the tie rods are attached to a travelling crosshead actuated by the rams of the luffing cylinders and

[1] By Messrs. The Hydraulic Engineering Co., Ltd., Chester.
[2] By courtesy of Messrs. Sir W. G. Armstrong, Whitworth & Co., Ltd.
[3] By courtesy of Mr. Arthur Musker, M.I.C.E.

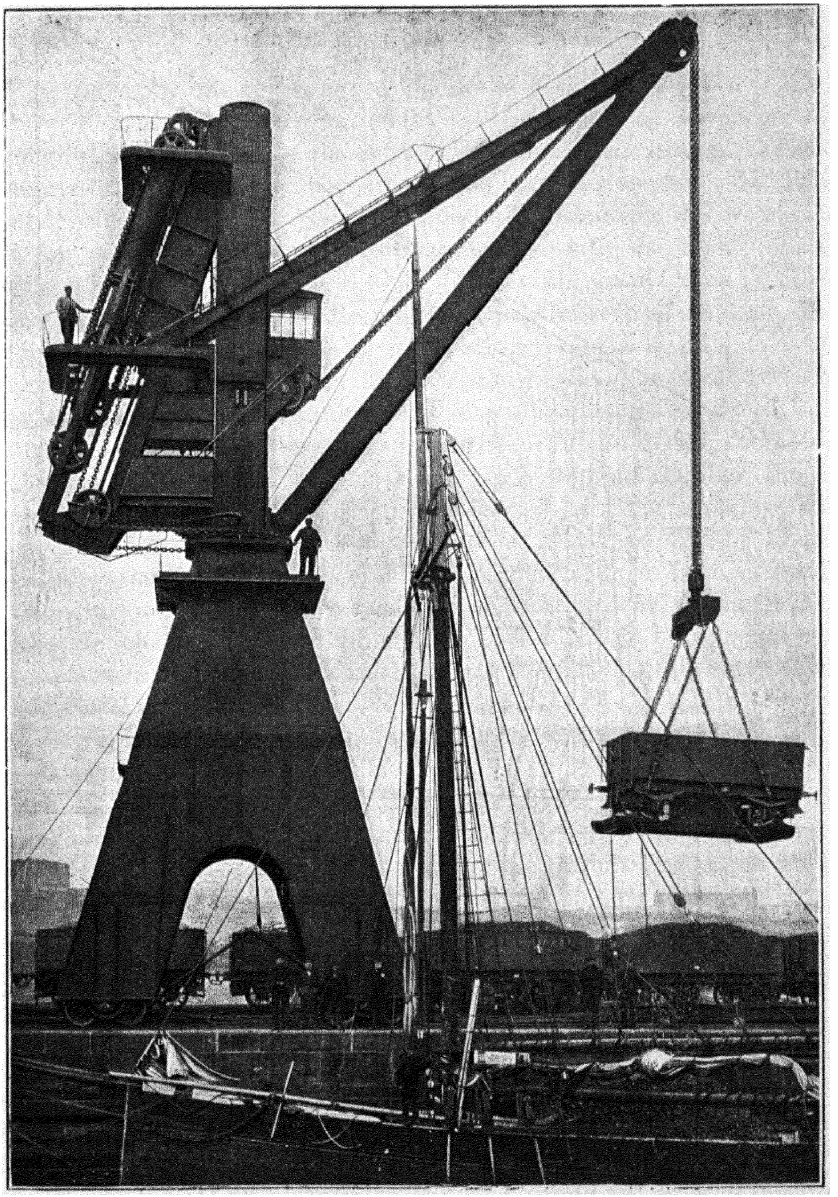


Fig. 394.—30-Ton Hydraulic Lifting Crane

also carry a compensating pulley. The lifting rope or chain passes over this pulley as shown in the sketch, with the result that when the jib

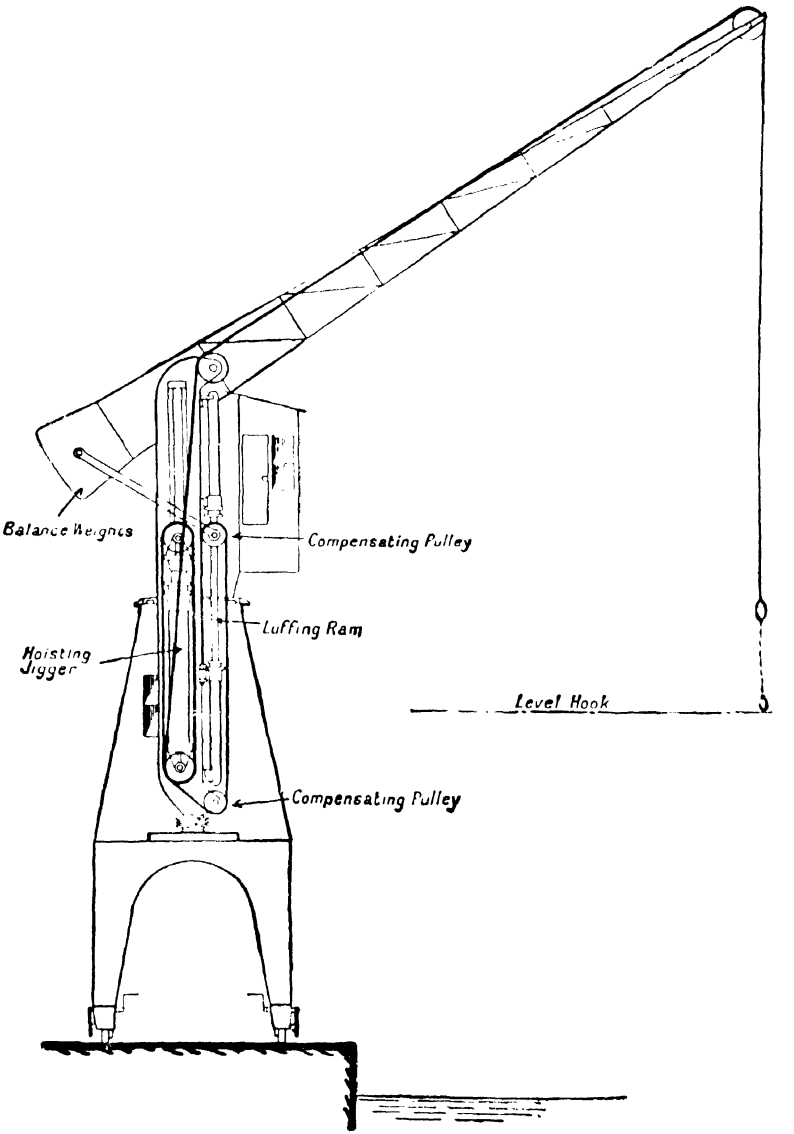


FIG. 395.—Hydraulic Balanced-Jib Luffing Crane.

is luffed inwards the hoisting rope is paid out to compensate for the rise at the point of the jib. By suitably adjusting the stroke of the luffing ram and the inclination of the rods the level of the lifting hook may be maintained constant for all radial positions of the weight.

Art. 229.—Hydraulic Coal Tips.

Fig. 396 shows one of a series of hydraulic coal tips designed to lift a waggon of ten tons through a height of 45 feet and to discharge this through the shoot *S* into the hold of a vessel. The lifting cradle *C* is actuated by four rams, one small and one large one on each side. The smaller rams are for the purpose of partially balancing the weight of the cradle, and are in constant communication with the high-pressure mains. These four rams are each 45 feet long, and press against a cross girder above the cradle, which travels in vertical guides. The cradle is suspended from this cross girder by adjustable bolts, and the tipping cradle rests upon the main cradle and is hinged on the end nearest the dock. The shore end of the cradle for tipping purposes is actuated by means of wire ropes which pass up from the shore end of the cradle on either side to near the top of the tip framing, thence over sheaves. The bight of the rope is carried over guide pulleys and up to the side of the main cradle, where it is passed over a sheave attached to the framing of this cradle. The tipping ropes from each side of the cradle are by this means made out of one rope, and any inequality in the stretch of either end of the rope is adjusted by the bight of the rope traversing over this sheave upon the tip cradle; this rope, which acts the part of two ropes for the tipping cradle, is free to run round the sheaves of the tipping ram and cylinder, as the main cradle lifts or lowers, carrying the tipping cradle with it.

The main cradles are fitted with cross girders above the ordinary height of the top of a wagon, and should the tipping cradle be very rapidly lifted, these girders act as a stop to prevent the wagon being thrown off. There are two cranes on each tip, one capable of working four tons and the other eight tons. All movements of the various appliances are controlled from the elevated cabin on the side of the tip. The point of the shoot can also be lifted or lowered from the same place.

The butt of the shoot is lifted or lowered by means of the main cradle, upon which there are a pair of sliding dogs, which may be pushed out to engage in the shaft which carries the butt. The latter is held, when the proper position is obtained, by means of chains, each attached to one end of the shoot shaft and carried up to the top of the tip framing, thence over sheaves and back down alongside the middle frame of the tip, where there are suitable cleats arranged into which these chains are placed, and there secured by bolts. The point of the shoot is moved by means of wire ropes which pass up to the top

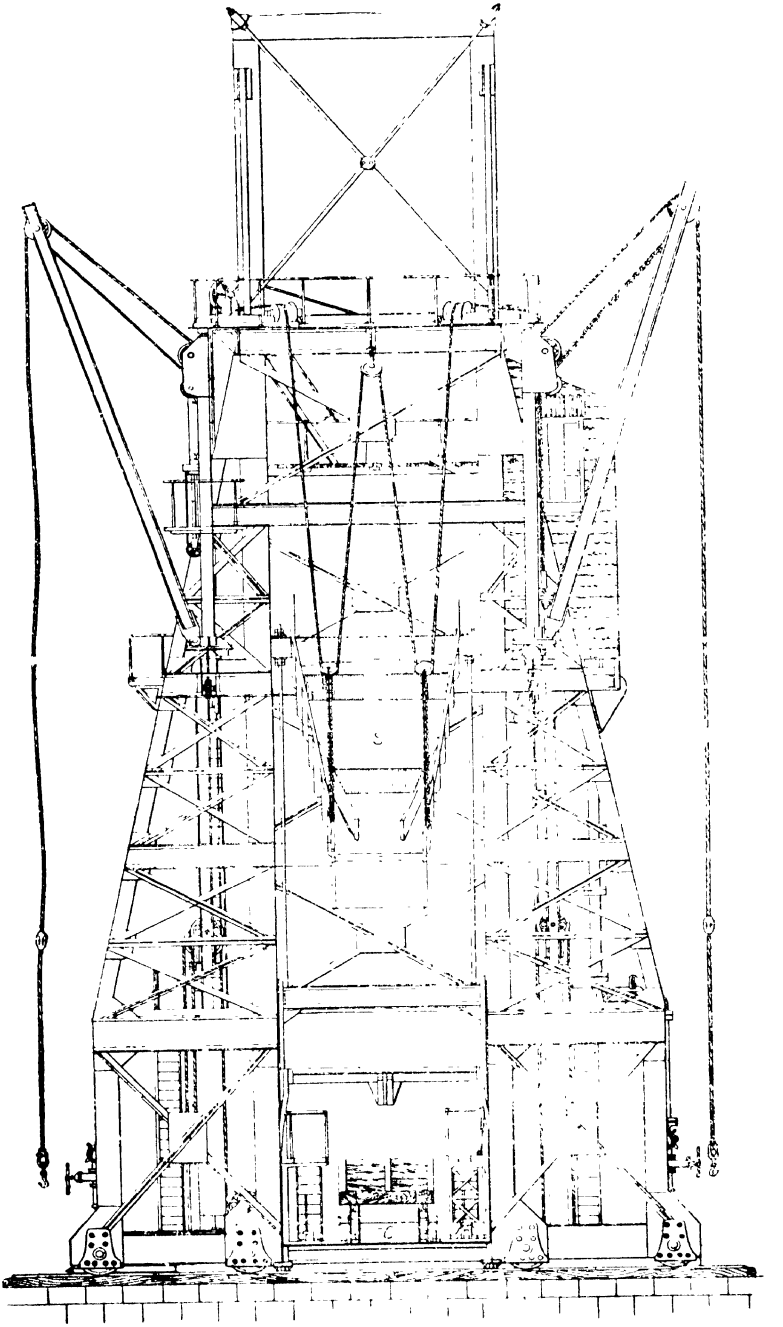


FIG. 396.—Hydraulic Coal Tip.

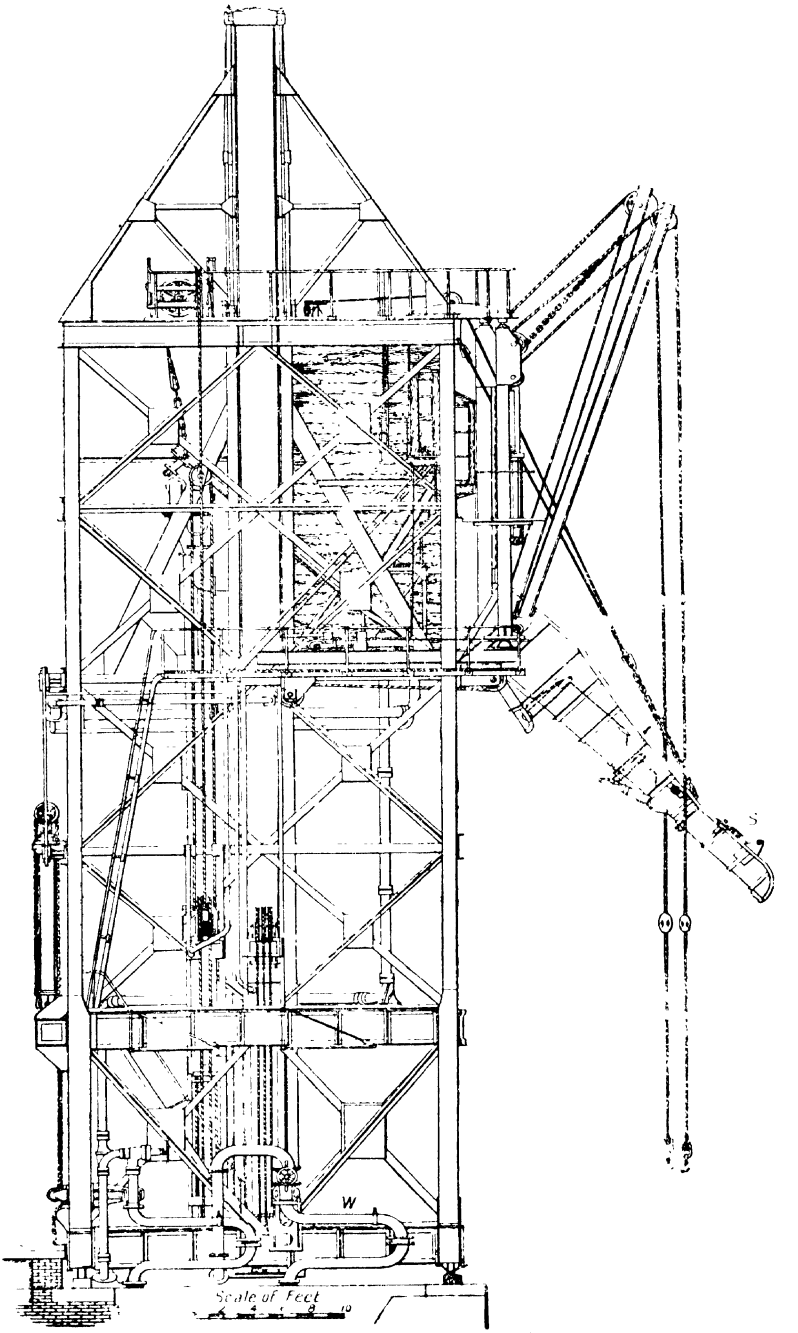


FIG. 396A.—Hydraulic Coal Tip.

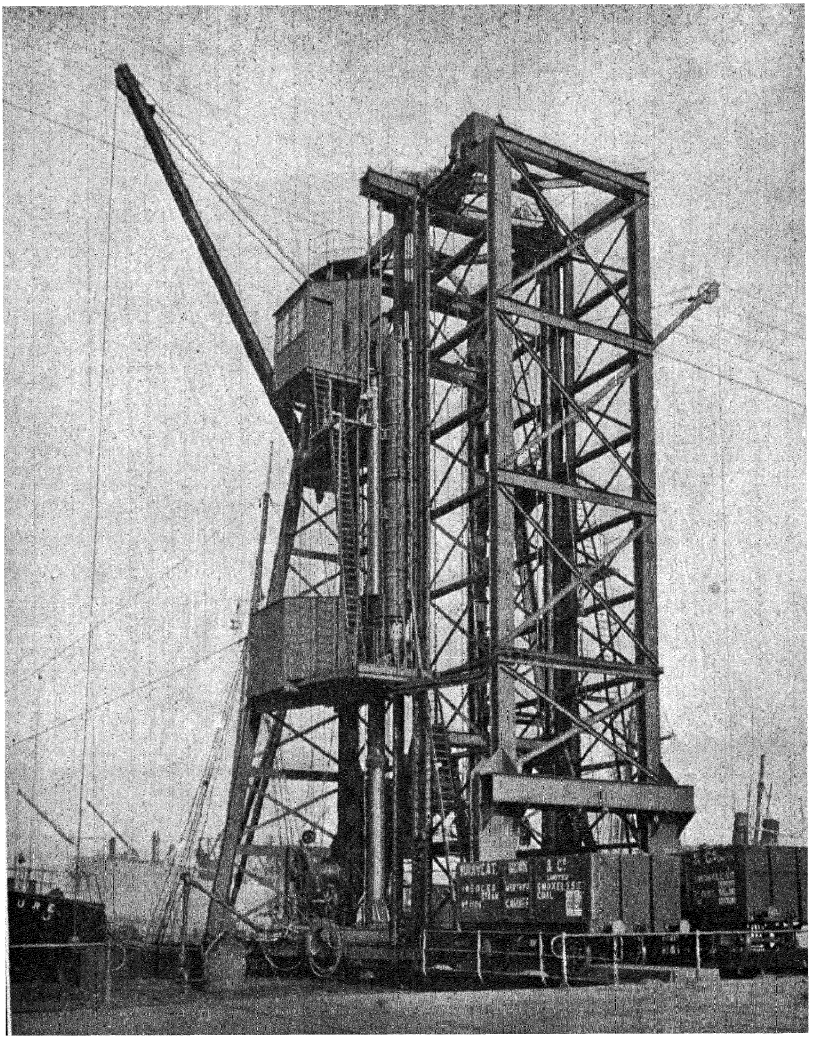


FIG. 397.—30-Ton Hydraulic Coal Tip.

of the tip and thence to a crab winch actuated by three-cylinder hydraulic engines and fitted with brakes for adjusting the shoot point where required.

The pressure water is brought by two tiers of 8-inch mains in a trench beneath the tips. To enable the tips to be moved laterally without uncoupling the pipes, these are fitted with walking pipes, shown at *W*

in the illustration, which have a range of 15 feet, and as the shoots themselves are pivoted and may be radiated in either direction 5 feet from the centre of the tip, this gives an extreme range of 25 feet without changing any water connections.

The capacity of each tip is about 500 tons per hour. Fig. 397 shows one of a series of more recent tips erected by the same makers[1] at Newport, Mon. These are designed for wagons of 30 tons gross weight, the total lift being 50 feet. Hoisting is performed by wire ropes, two hoisting cylinders being provided. The walking pipes for supplying the cylinders are shown on the left hand of the sketch.

The main hoisting cylinder has a ram of 18 inches diameter, with a 25-foot stroke. The auxiliary ram is 9 inches in diameter, and has also a 25-foot stroke, while the tipping ram is 9 inches in diameter, with a stroke of 18 feet. Two cranes, one of 3 tons, and one of 5 tons capacity, are provided for handling anti-coal-breaking gear, having each a lift of 500 feet. They are provided with $10\frac{1}{2}$-inch and $12\frac{3}{4}$-inch rams respectively, both of 25-foot stroke. The slewing-gear of each crane is driven by a ram $6\frac{1}{2}$-inch in diameter and 2-foot stroke. The hoist may be worked anywhere within a range of 200 feet, the traverser gear for working the trucks extending over this distance. The cylinders for operating the traverser gear are fixed in pits beneath the track.[2]

Art. 230.—Hydraulic Crane Valves.

The supply and discharge of water to and from the cylinder of a crane may be adjusted by means of a simple or compound slide or piston valve, as shown in Fig. 398, or by poppet valves directly manipulated by hand. Where pressures are very high and the volume of water large the effort required to actuate such valves becomes excessive and some other arrangement becomes desirable. Such an arrangement[3] is shown in Fig. 399. Here A marks the pressure water inlet, D the connection to hydraulic cylinder, and E the exhaust.

The valves are operated by handle F, connected to rock-shaft G. Throwing F in one direction opens the inlet valve at A; throwing it in the other allows the water to escape from D through outlet at E.

[1] Messrs. Fielding & Platt, Ltd., Gloucester, by whose courtesy the foregoing sketches are available.

[2] These hoists were illustrated and described in a paper read before the Institution of Mechanical Engineers at their summer meeting held at Cardiff in 1906, and for further details of working and for drawings, &c., reference may be made to the proceedings of this Institution.

[3] Messrs. Dewhursts' Engineering Co., Ltd., Sheffield.

The valve stem H is raised by a cam surface cut in the upper side of rock-shaft G. The stem does not directly open the main valve; the upper end of a stem passes through main valve L in a clearance hole as shown, and when raised, it strikes against the bottom of pilot valve K, which it lifts against the pressure of the spring above and against the pressure of the water in chamber B. Chamber B is filled with water leaking from inlet space A, past the easy fit of the piston portion of valve L. As soon as K is raised, the pressure in B drops to that in service

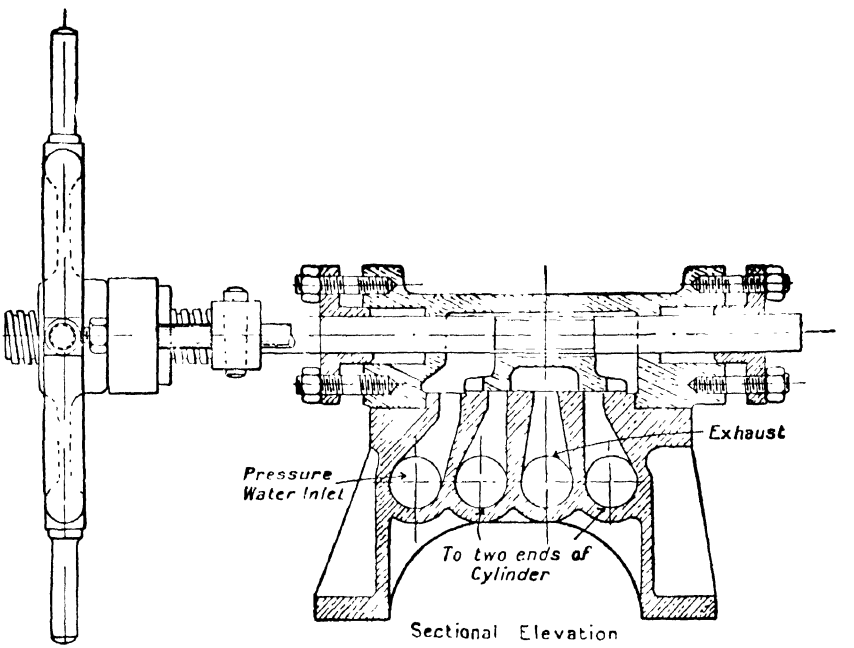


FIG. 398.—Slide Valve for Heavy Hydraulic Machinery.

connection D, since the outlet through the centre of the main valve is much larger than the area through which the leakage escapes from A. Under these circumstances there is an unbalanced pressure on valve L from the water in A, which forces it from its seat on ring C, this unbalanced pressure being due to the fact that the piston portion of main valve L is much larger in diameter than the seat on which it rests in ring C.

In closing, the reverse of this action takes place. Handle F being brought back to its central position, valve stem H, and with it pilot valve K, are lowered until the latter reaches its seat in valve L. Connection

between B and D being thus closed, the leakage from A into B allows the pressure in the latter space to rise until it balances that in A, when the coil spring has sufficient pressure to force valve L down against its seat.

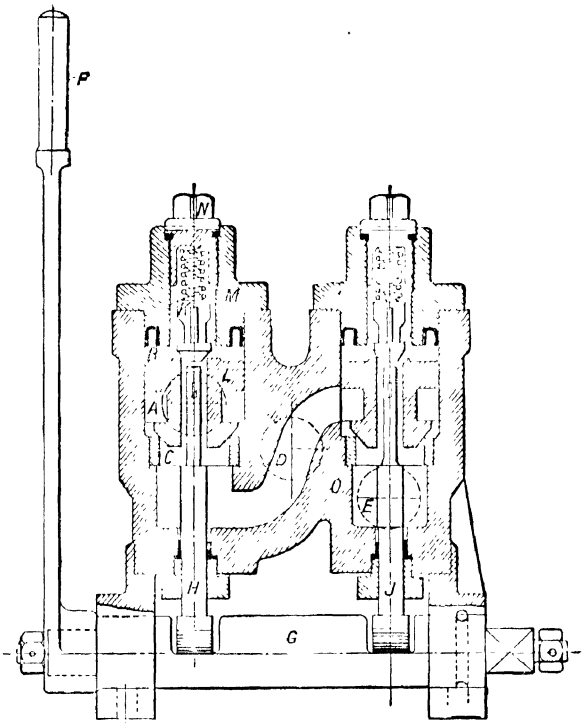


FIG. 399.—Hydraulic Operating Valves with Pilot Valves.

This construction evidently has a number of advantages. The use of the small pilot valve, the only member which has to be lifted against the full pressure of the water, makes the apparatus more easy to handle than would be the case if the whole main valve L had to be raised from its seat against the full pressure. Also there are no sliding movements to be effected through packings under heavy pressure, as the valve stem H is not under pressure until the valve is opened.

There would appear, also, to be little danger of shock from sudden closing of the valves, since the lowering of main valve L to its seat on C is effected by the leakage from A to B, which can be made as gradual as seems advisable, being regulated by the fit of L in its cylindrical chamber in the main casting O.

ART. 231.—THE HYDRAULIC JACK.

For the manipulation of heavy weights by hand the hydraulic jack is of the greatest value. In principle it consists of a Bramah's press on a small scale, and one type of its construction is illustrated in Fig. 400. Here the reciprocation of the hand lever pumps water from the cistern A, through the hollow plunger B, past the suction and delivery valves

V_a and V_D, into the space C below the lifting ram, and raises the latter. The weight to be lifted is carried either on the ram table T, or on a side shoe projecting from the ram casing. Screws are provided for supplying the cistern A with water and for allowing of the inlet of air, while a lowering screw permits of the escape of pressure water from the space below the lifting ram into the supply cistern when it is required to lower the load. The lifting ram is usually packed by means of a cup leather, and the pump plunger by means of a single leather ring.

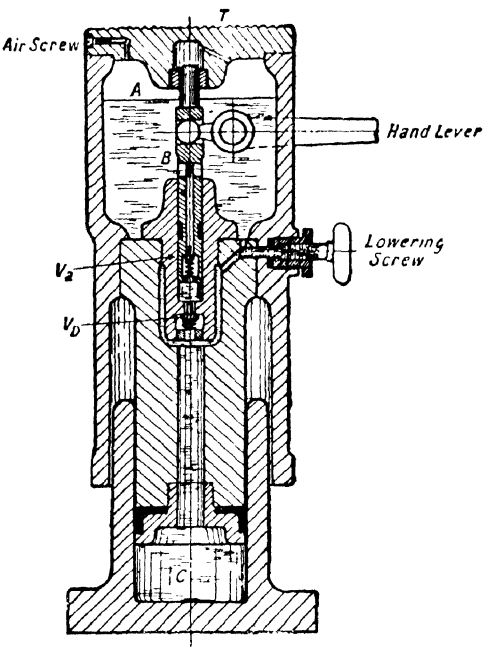


FIG. 400.—Hydraulic Jack.

If m = ratio of travels of lever handle and of pump plunger, and if a and A are the areas of plunger and ram, the theoretical mechanical advantage of the jack $= m\frac{A}{a}$, so that, neglecting friction, the force P to be applied at the lever handle to support a weight W on the ram table is given by $P = W\frac{a}{m\,A}$ lbs.

Thus if $m = \frac{24}{1{\cdot}25} = 19{\cdot}2.$

„ diameter of ram $= 5\frac{1}{2}$ inches
„ diameter of plunger $= 1$ inch
$\Big\}\ \frac{A}{a} = 30{\cdot}25.$

if $P = 50$ lbs., $W = \frac{50 \times 19{\cdot}2 \times 30{\cdot}25}{2{,}240} = 13$ tons.

Actually the weight lifted is less than this because of friction losses. The efficiency of the jack, or the ratio $\frac{\text{actual}}{\text{theoretical}}$ weight lifted by the ram, can only be determined experimentally, and varies largely with the condition and size of the apparatus, as well as with the magnitude and position of the load.

Friction losses are proportionately less as the size of the machine increases, and, owing to the fact that the friction is partly mechanical and independent of the load, the losses diminish proportionately as the load increases.

Eccentric loading largely increases the mechanical friction, and may double the friction losses.

The efficiency of a jack in fairly good order may be taken as varying from about 66 per cent. in the case of a small 3-ton jack, with eccentric loading to about 93 per cent. with a 100-ton jack and central loading, and is approximately constant for loads greater than one-fifth of the nominal capacity of the jack.

Art. 232.—The Hydraulic Press.

Reference has already been made in Art. 8 to the Bramah's press. Its modifications, as applied to such work as cotton baling, boiler-plate flanging, cartridge-case drawing, and heavy forging, are too numerous for detailed mention, and only one or two of such applications will be considered in detail in the present treatise.[1] Fig. 401[2] illustrates a form of press used for flanging operations, &c., and capable of exerting a pressure of 420 tons. In this machine two small rams are installed for lifting the head on the up stroke, while the pressure is applied by one central and two side rams, pressure water being supplied to these in succession as additional force is required. During the idle part of the down stroke, water from the exhaust is allowed to fill the space vacated by the rams. The arrangement of valves by which this is rendered possible is simple and worthy of notice, and is shown in detail in Fig. 402. The main pressure and exhaust valves V_P and V_E each carry a closely fitting piston of greater area than the valve itself, so that if the pressure above the piston and below the valve is equalised the effect of the pressure between the piston and valve is to lift the latter from its seat. A small orifice is provided in each piston, so that under normal conditions the pressure above and below the piston is equal, and pressure keeps the valve on its seat. The auxiliary valves V_1 and V_2 are worked directly from the operating lever, and when open give free communication between the under side of the corresponding main valve and the upper side of its piston.

[1] For further applications of the press the reader is referred to Blaine's "Hydraulics."

[2] By courtesy of the makers, Messrs. Henry Berry & Co., Leeds

The opening of the auxiliary valve thus leads indirectly to the opening of the corresponding main valve. During the idle part of the down stroke both auxiliary valves are closed, a partial vacuum is produced

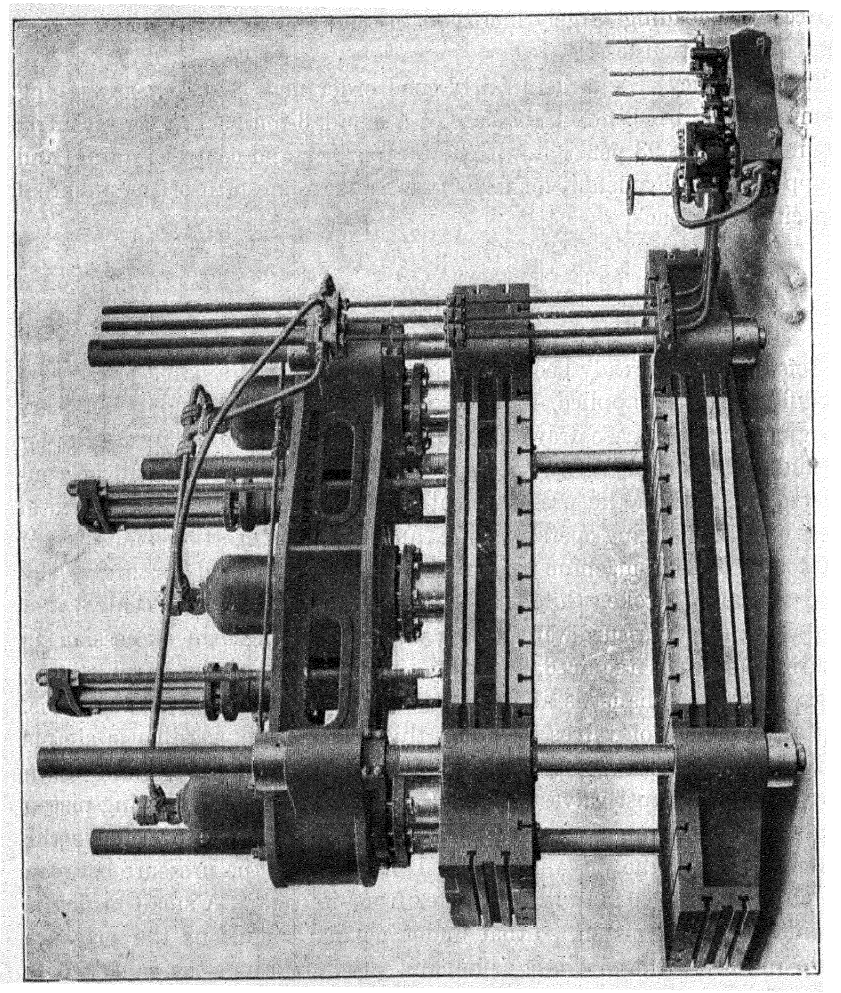


FIG. 401.—Hydraulic Flanging Press. Maximum Pressure 420 tons.

above the exhaust valve, which opens, and water is drawn from the exhaust into the cylinder. When pressure is required, a quarter-turn of the operating lever raises the valve V_1 and thus the pressure valve, and admits pressure water through the passage P_1 to the cylinder. On the working stroke being completed, a half turn of the lever closes V_1 and

opens V_2, thus opening the exhaust valve V_E and putting the cylinder into communication with the exhaust, while a second lever and valve admit

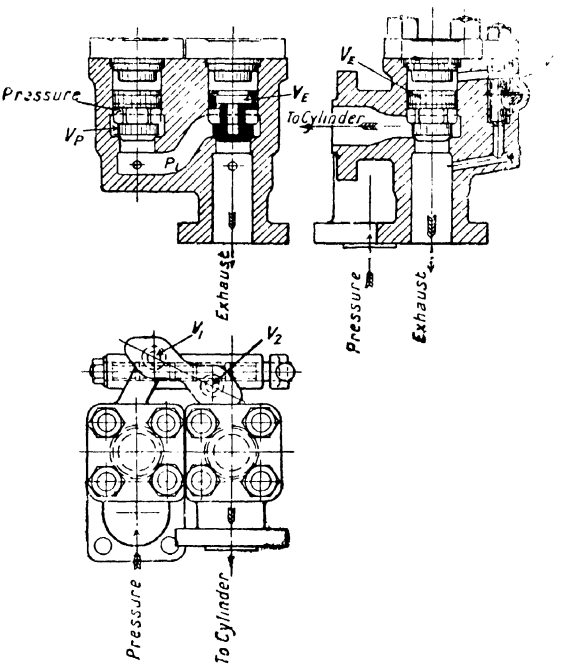


FIG. 402.—Details of 2¼-inch Operating Valve for 420 tons Press.

pressure water to the return cylinders. A smaller press for similar work is illustrated in Fig. 403.

ART. 233.—THE HYDRAULIC FORGING PRESS.

In the production of heavy forgings for large ingots of mild steel, it is essential that every part of the ingot should be equally worked if the resultant forging is to be homogeneous in structure. Where a steam hammer is used, the energy of the blow is absorbed in producing distortion of the outer layers, while the interior is practically unaffected. This disadvantage is overcome by the use of the hydraulic forging press, with its slow and powerful compression, and this is gradually supplanting the steam hammer for the production of very heavy forgings. The principle of the press is the same as that of the ordinary flanging press,

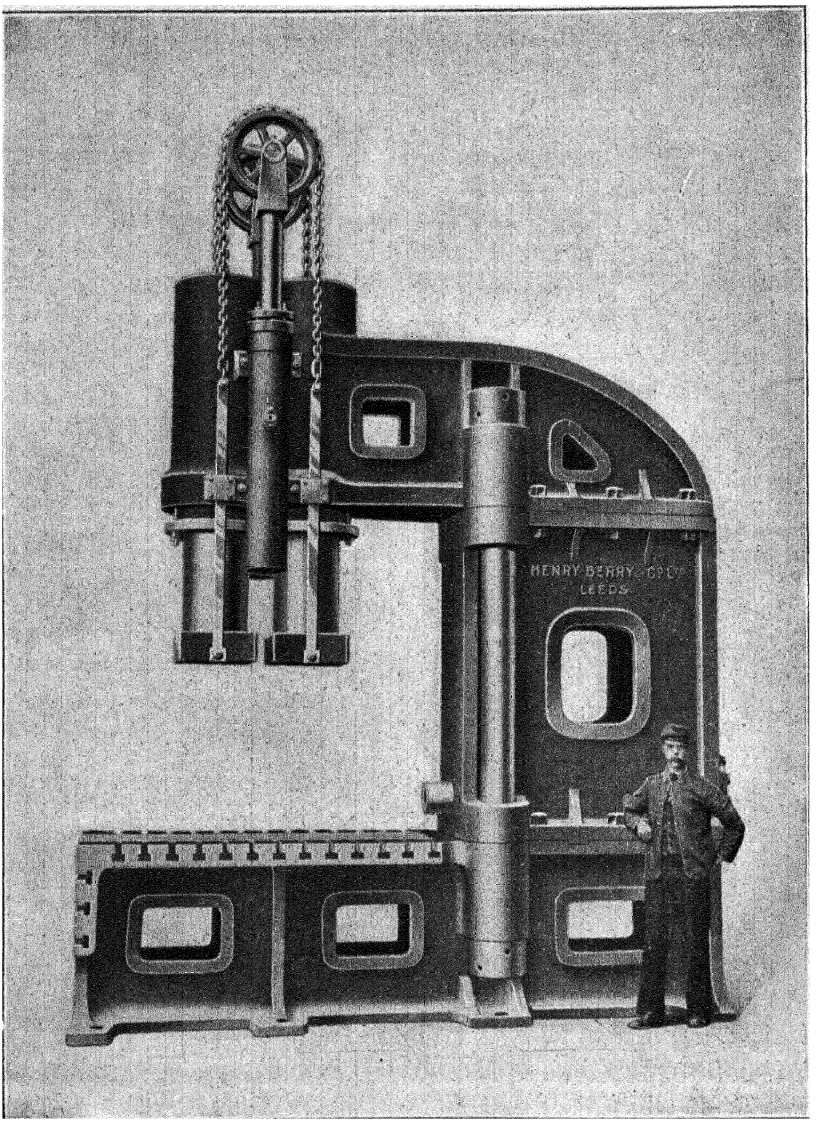


FIG. 403.—Hydraulic Flanging Press.

and for a very complete account of its development and for details of its design, the reader is referred to a paper by R. H. Tweddell.[1]

[1] "Proceedings Institute of Civil Engineers," vol. 117, 1893–4, p. 1

The Allen press works on a very ingenious principle and is illustrated diagrammatically in Fig. 404. Here a pressure accumulator is unnecessary, as are valves in the high-pressure water column. For its operation a low-pressure water supply, at about 200—300 lbs. per square inch, is necessary, and during the idle part of the stroke this follows up the ram, the high-pressure connecting pipe being kept full in the meantime

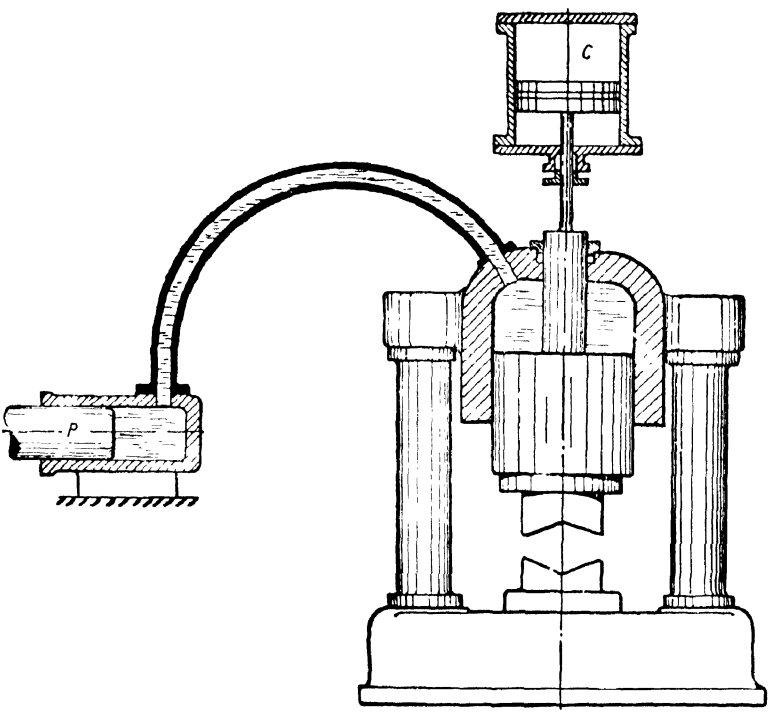


FIG. 404.—Allen's Hydraulic Forging Press.

When pressure is required this supply is cut off and communication is made with the high-pressure pump *P*. This has no valves, so that the ram has a continuous up-and-down motion, the water column simply following the motion of the pump plunger. The inertia of this column thus has a useful effect in increasing the pressure on the ram at the end of the working stroke. A steam cylinder *C* is usually provided for lifting the ram.

A type of forging press to be worked in connection with an accumulator and intensifier is illustrated in Fig. 405. This press is fitted with a differential ram having diameters of 21 inches and 36 inches and supplied

with pressure water at 600 atmospheres. By using each cylinder separately, or the two in combination, three powers having the ratio 1 : 2 : 3 may be obtained, the effective force varying from 1,300 tons with the small ram to 4,040 tons with the two in combination. Two lifting cylinders, using water at 50 atmospheres pressure, exert a constant upward

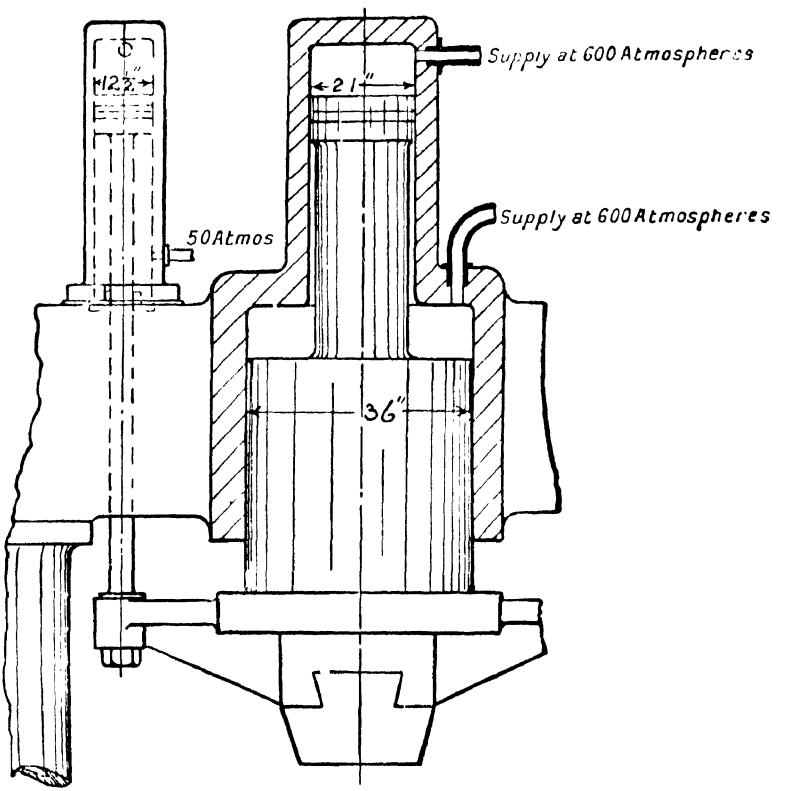


FIG. 405.—Cylinders of 3-power Hydraulic Forging Press for a maximum effort of 4,000 tons.

force of 70 tons, which has been taken into account in obtaining the above values.

As, during the working portion of the stroke of a hydraulic press or similar machine, the ram velocity is required to be only very slow, the diameters of the supply pipes may be very small without appreciable loss of head. A ratio of ram diameter to pipe diameter of about 12 to 1 is usually adopted.

Art. 234.—Steam-Hydraulic Forging Press.

By combining a steam accumulator or intensifier with the hydraulic cylinders of a forging press and arranging steam actuated drawback cylinders to the main ram, a type of forging press is obtained which has been largely adopted of recent years on account of its flexibility, simplicity, and speed of working. Such a machine is illustrated in Fig. 406, which shows a press designed for a full working power of 1,200 tons, and a maximum stroke of 48 inches. At full power the stroke is about 6 inches, and the machine is capable of making from 70 to 80 short strokes per minute.

The press as shown is in direct communication with the steam intensifier S and a water reservoir R shown behind the steam intensifier. The main cylinder C of the press is supplied with water during the idle portion of the stroke from the reservoir through a large filling valve V bolted directly on the top of the cylinder. The water is returned to the reservoir after the pressing operation through the exhaust valve P secured to the top cylinder of the steam intensifier and connected directly to the reservoir.

The movements of the press are controlled by means of a main hand lever carried on a bracket on the steam intensifier, the movement of the press head corresponding with the movement of the main hand lever. The downward movement of the hand lever first of all exhausts the steam from the under side of the drawback pistons contained in the drawback cylinders DD secured to the tops of the columns of the press, thus allowing the press head to fall until the top die reaches the work. During this portion of the stroke of the main ram, the cylinder is filled with water from the reservoir through the filling valve. A further downward movement of the main hand lever raises the main steam admission valve of the steam intensifier which forces high-pressure water from the top cylinder of the intensifier into the main cylinder of the press thus exerting full power on the work. The upward movement of the main hand lever first opens the main exhaust valve of the steam intensifier, allowing the intensifier piston to descend by gravity, and thus releasing the high pressure on the main ram The continued upward movement of the main hand lever then opens the exhaust valve on the top of the steam intensifier and allows the water to return from the main cylinder to the reservoir, the upward movement of the press head being effected by the re-admission of steam to the drawback cylinders, thereby completing the cycle of operations of the press and steam intensifier.

An auxiliary hand lever close to the main hand lever operates a small

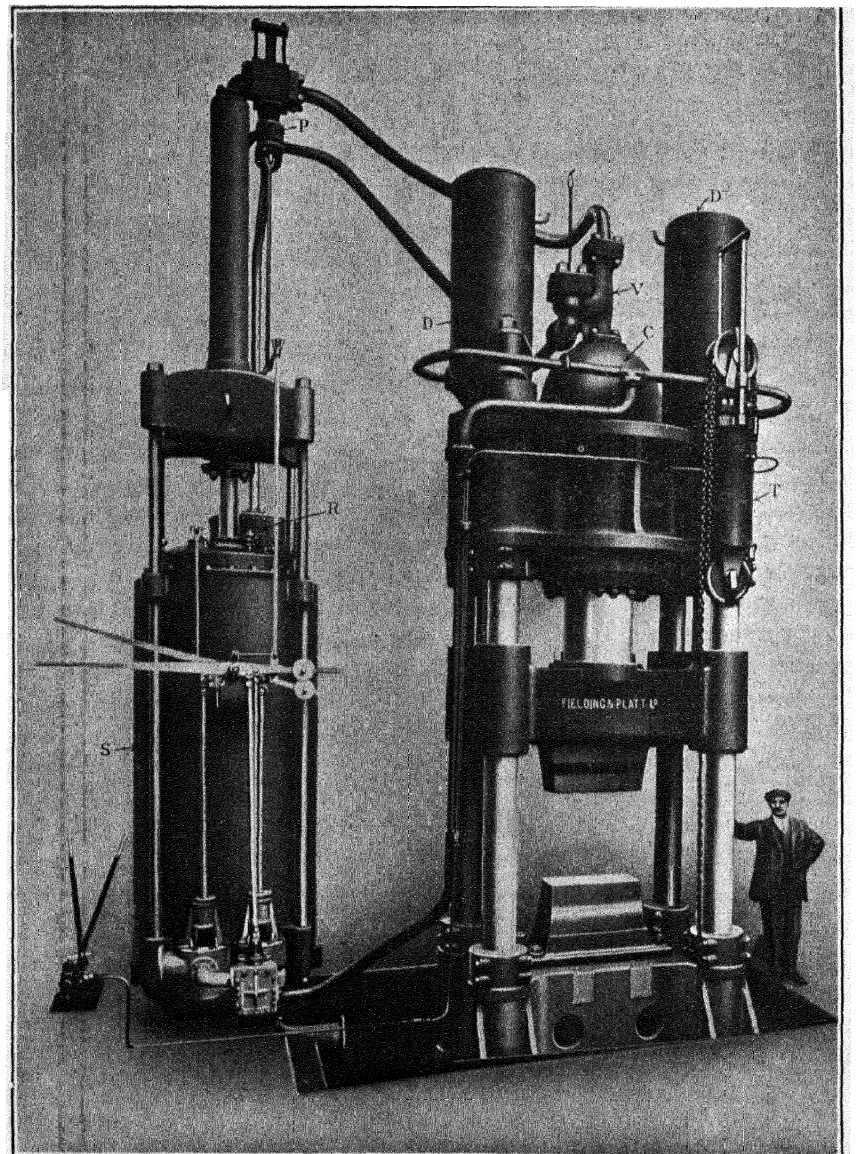


Fig. 406.—1,200-Ton Steam-Hydraulic Forging Press.

steam slide valve which admits constant pressure steam to the drawback cylinders on the press for working the press with short quick strokes for planishing.

A small lever-worked valve at the foot of the steam intensifier controls the movements of a hydraulic turning cylinder T secured to the top table of the press.

Art. 235.—Hydraulic Riveters.

The hydraulic riveter provides another good illustration of the adaptability of the hydraulic machine to workshop processes. Here the problem is to get a fairly large pressure on the rivet during the first portion of the ram stroke, so as to form the rivet head and to clinch the plates, and a final larger pressure of the nature of an impact to cause the rivet to expand and fully fill its hole. The extent to which this is attained in the riveter will be evident from Fig. 407, which represents a typical pressure diagram taken from the cylinder of such a machine, supplied from an accumulator under a pressure of 1,100 lbs. per square inch. Here A B represents the idle part of the stroke during which the ram is being brought up to its work, B C the setting up the rivet and the formation of the head, C D the clinching of the rivet and the closing of the plates, while the sudden stoppage of the heavy accumulator ram is responsible for a further rise in pressure D E above the accumulator pressure, which is depended upon to fill up the rivet hole.

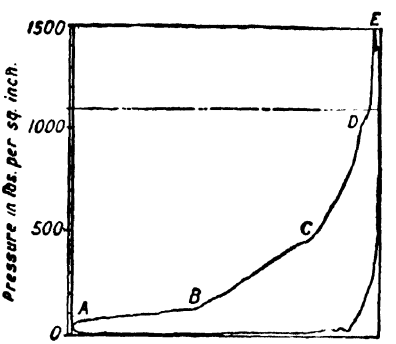


Fig. 407.

Pressure Diagram from Cylinder of Hydraulic Riveter. Accumulator pressure 1,100 lbs. per square inch.

One type of portable riveter is shown in Fig. 408.[1] Here the ram is situated at one end and the riveting is performed at the other end of a lever hinged at C. As thus arranged the machine offers some advantages over the more ordinary type of bear riveter shown in Fig. 409,[1] for work in restricted spaces. In the latter type the operation of riveting is directly performed by the hydraulic ram, the riveter jaws being formed as a single steel casting. In Fig. 410[2] a section of the cylinder and valves of a riveter of the hinged type is shown. Here water is admitted to or discharged from the ram cylinder by the arrangement of valves shown.

[1] By courtesy of the makers.

[2] By the courtesy of the makers, Messrs. Henry Berry & Co., Ltd., Leeds.

Thus a quarter-turn of the regulating lever raises the valve V_S and puts the cylinder into communication with the pressure supply. On the com-

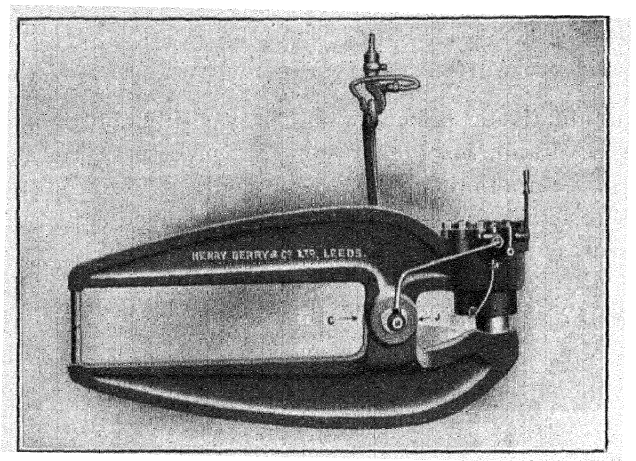


FIG. 408.—Portable Riveter. Hinged Type.

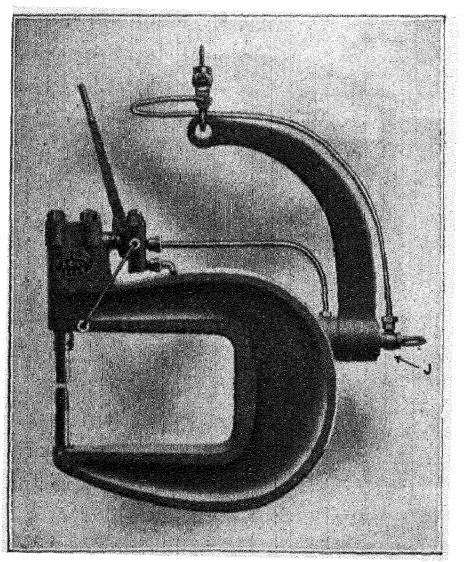


FIG. 409.—Portable Bear Riveter.

pletion of the working stroke a half-turn in the opposite direction closes the valve V_S and opens valve V_D, putting the cylinder into communication with the discharge passages. The main ram is drawn back on its idle stroke

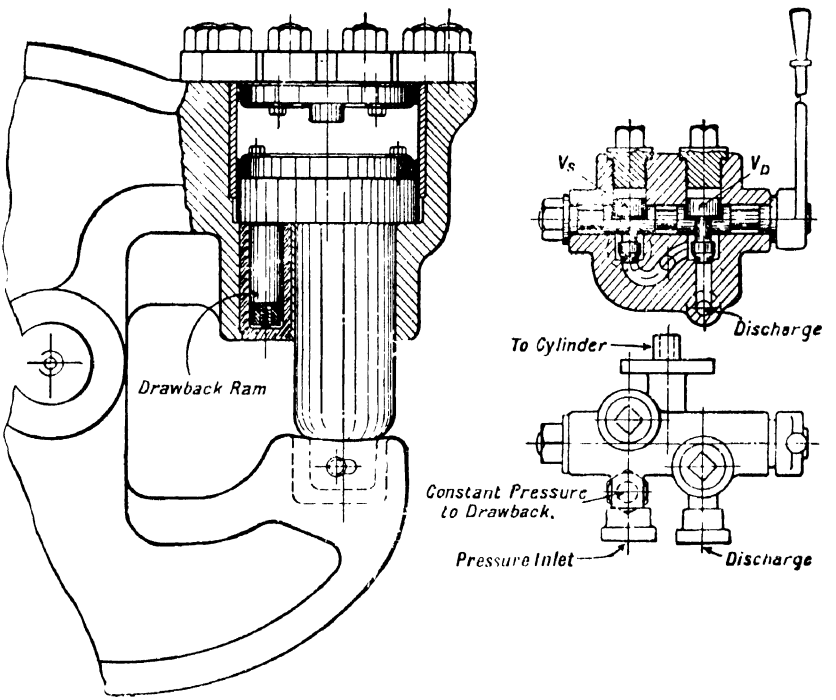


FIG. 410.—Section of Cylinder and Valves of Hinged Type Riveter.

by means of a special drawback ram which is always exposed to supply pressure. The method of packing the rams and the general construction is sufficiently well indicated in the figure.

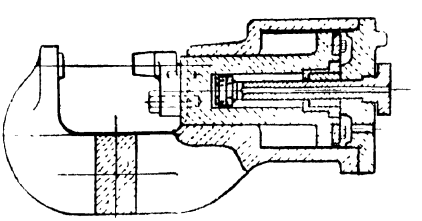


FIG. 411.

In these portable machines pressure water is supplied to the suspending hook by means of a walking pipe, and provision is made for working at a rivet at any inclination, by means of a watertight swivel joint at J (Fig. 408).

Fig. 411 shows a design of fixed-jaw portable riveter which has the advantages of a central drawback ram and a main ram with central guide.

Fig. 412 illustrates a form of fixed riveter suitable for the circumferential joints of large boiler shells. Here both jaws of the bear are fitted with rams, that to the right of the illustration being adjusted to

suit the length of rivet to be handled. The working cylinder, to the left, is fitted with a differential ram so as to allow a larger range of work to

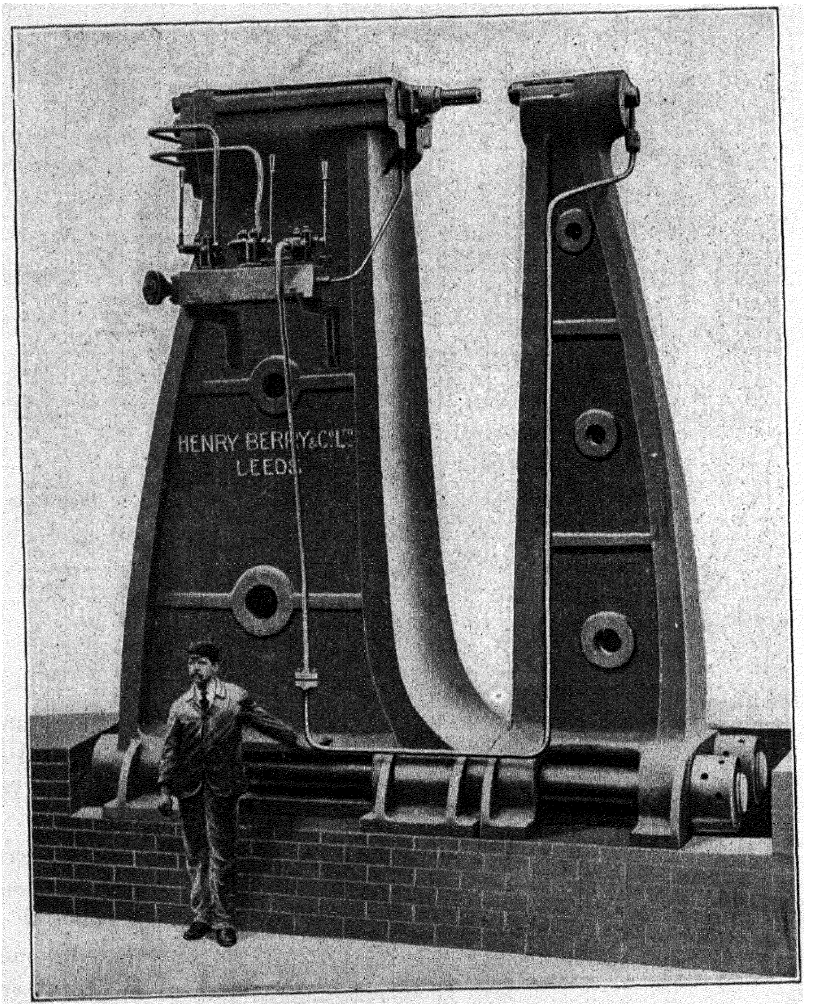


FIG. 412.—Fixed Hydraulic Riveter.

be handled with economy. In this case the pressure supply to each ram is regulated by a separate valve.

ART. 236.—HYDRAULIC LOCK-GATE MACHINERY.

In the majority of modern docks the lock gates or caissons are actuated by hydraulic machinery. In the new "Royal Edward" Dock at

Avonmouth, for example, each leaf of the entrance lock gates is actuated by a direct-acting hydraulic cylinder with piston and rod, the stroke being 12 feet 9 inches.

The cylinder of each machine is placed in a pit at the side of the lock The piston-rod is attached to a steel crosshead frame faced with gun-metal and working between steel guides. The connecting-rod is of steel plate of box section, and is fitted with a gimbal attachment at each end, so that a certain amount of vertical movement can take place in the gate

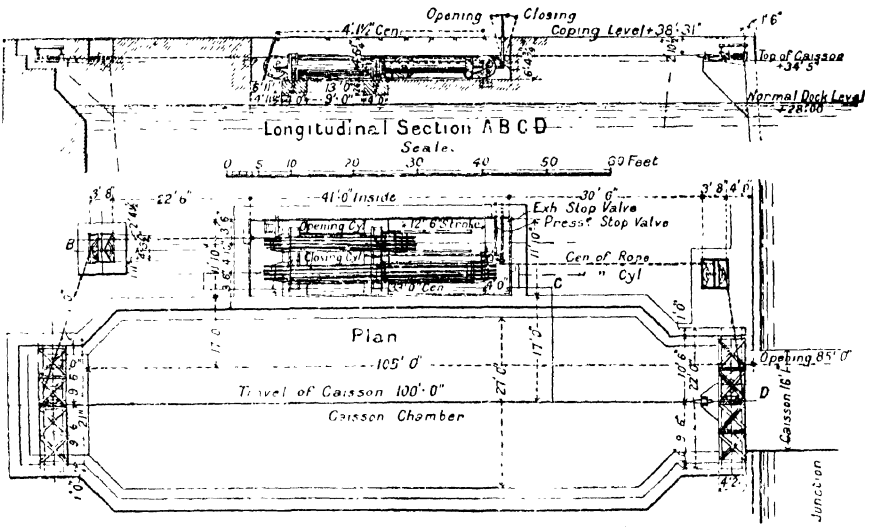


FIG. 413.—Hydraulic Gear for Operating Dock Caisson.

without interfering with the action of the machinery. Control is effected by valves which are operated from quay level.

The hauling machinery for operating each of the caissons installed in connection with the same dock is shown in Fig. 413,[1] and consists of two cast-iron hydraulic cylinders placed side by side. The working pressure is 750 lbs. per square inch. The multiplying ratio is 8 to 1, and the rams give a travel of the hauling rope of 66 feet per minute. One ram is used for opening, and the other for closing the passage. The hauling ropes are of steel wire fixed to an adjustable fast end on each cylinder, carried along the centre line of the caisson under the recess cover, and attached to a bracket projecting from the inner end of the caisson. Automatic cut-off gear is provided to prevent over-winding at either end of the travel.

Hydraulic Sluice Gates and Penstocks.—The sluice gates, 54 inches

[1] Figs. 413 and 414 are by courtesy of the Editor of "Engineering."

diameter, in connection with these caissons are also operated by direct-acting hydraulic rams working under the same pressure, the details of

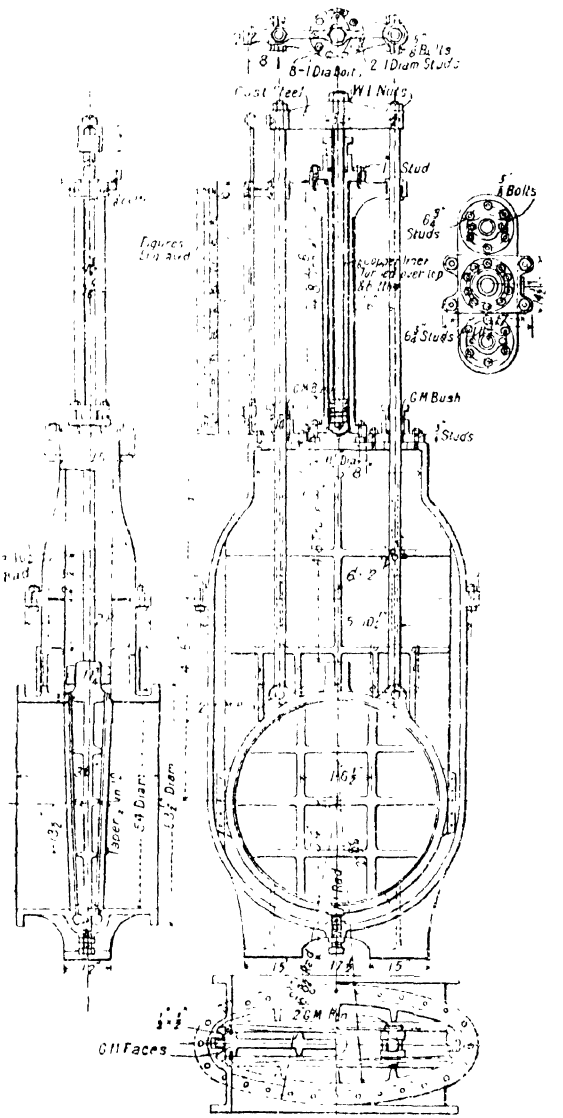


FIG. 414.—Hydraulically Operated Sluice Valve.

construction being shown in Fig. 414. The motion of the rams is regulated from a valve chest with slide valves of gun-metal.

The Johnson Valve.— A type of valve which is being used to an increasing degree in large power installations is illustrated in Fig. 415. The valve has a differential plunger which, in connection with its internal cylinder, forms two operating chambers *A* and *B*. The valve is closed by admitting pressure

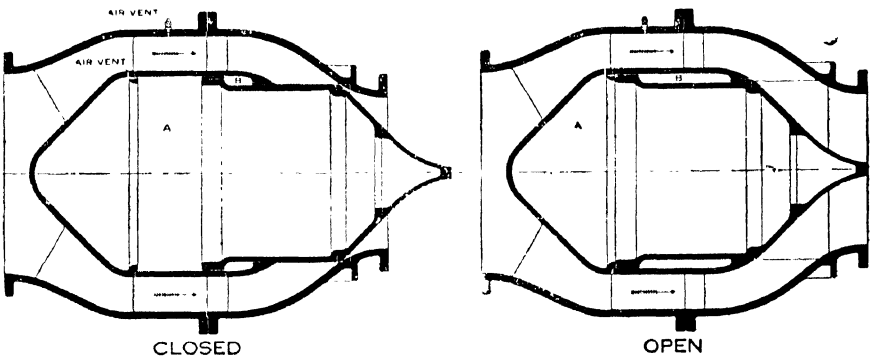


Fig. 415.

water from the pipe line to *A* and exhausting *B* to the atmosphere. It is opened by reversing this operation. Whether open or partially closed the waterway is of easy form and there is little loss in eddy formation.

Art. 237.—Hydraulic Capstans.

Fig. 416, A, B, and C, shows a type of hydraulic capstan driven by a three-cylinder Brotherhood engine.[1] The supply and discharge pipes are led to the engine through trunnions *T T*, about which, on releasing the catch *C*, the whole machine, capstan and engine, may be rotated, thus bringing the engine upwards for inspection or repairs without breaking any joints. The water supply is regulated by means of the treadle *K* which operates the admission valve. With a capstan head 1 foot 6 inches in diameter the hauling speed of such a machine is about 180 feet per minute.

Where the capstan is intended for very heavy work the engine may be connected to the capstan head by speed-reducing gear wheels, and Fig. 417 shows such a machine designed for two speeds exerting a pull of 7 tons at 51 feet per minute and 14 tons at 26½ feet per minute, the working pressure being 700 lbs. per square inch and the diameter of the capstan head 3 feet.

[1] By courtesy of the Hydraulic Engineering Co., Ltd., Chester

Art. 238.—Hydraulic Transmission Gear.

Several devices for transmitting the torque developed at the crank shaft of the engine of a motor car to the driving wheels by hydraulic means have been patented recently. All shock due to changes of gear wheels is thus avoided, while the ratio of speeds of the driving and driven shafts may be regulated with a much greater degree of flexibility than is

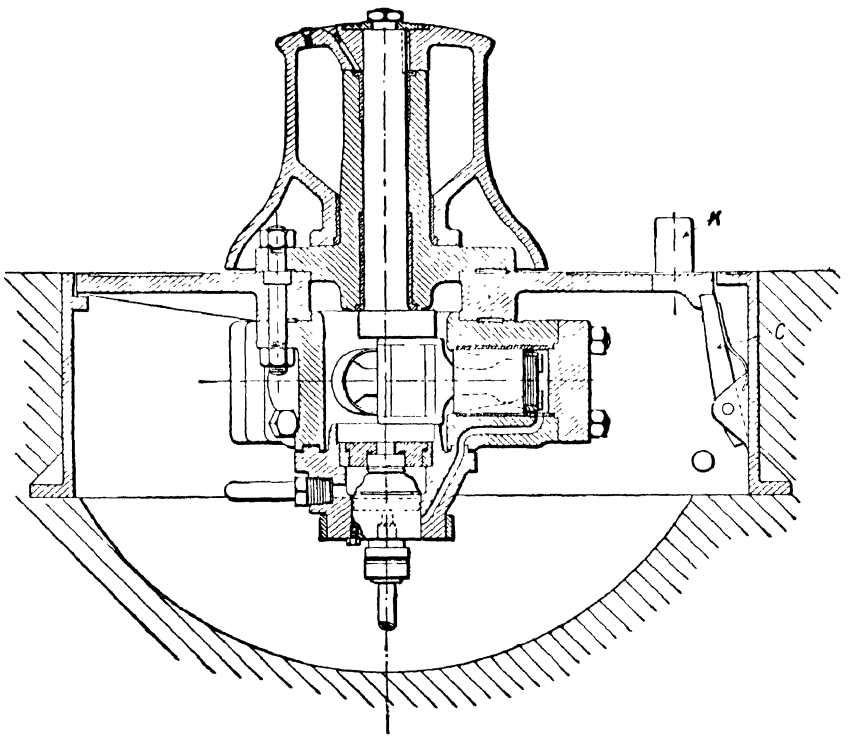


Fig. 416a.

possible with a mechanical drive. One such device, due to Dr. Hele Shaw, is illustrated in Fig. 418. In this a ring R mounted on ball bearings and fixed as regards rotation, carries the crank pins of four pistons working in cylinders A. The ring is eccentric with respect to the centre line of the gear, its eccentricity being capable of regulation as desired. The cylinders A form a single casting keyed to the jaw clutch K which is coupled directly to the engine shaft. The supply of fluid to or from the cylinders is regulated by means of a circular rotary valve V with supply and discharge ports. The eccentricity of the ring R determines

the stroke of the pistons in these cylinders and thus the volume of fluid displaced by them per revolution of the engine.

A second set of four cylinders *B*, of the same size as *A*, are keyed to the jaw clutch *N* which is coupled direct to the driving wheels. The connecting rods of these pistons are coupled to a single crank pin *H* with fixed throw and thus have a constant stroke, while the crank shaft *H* is

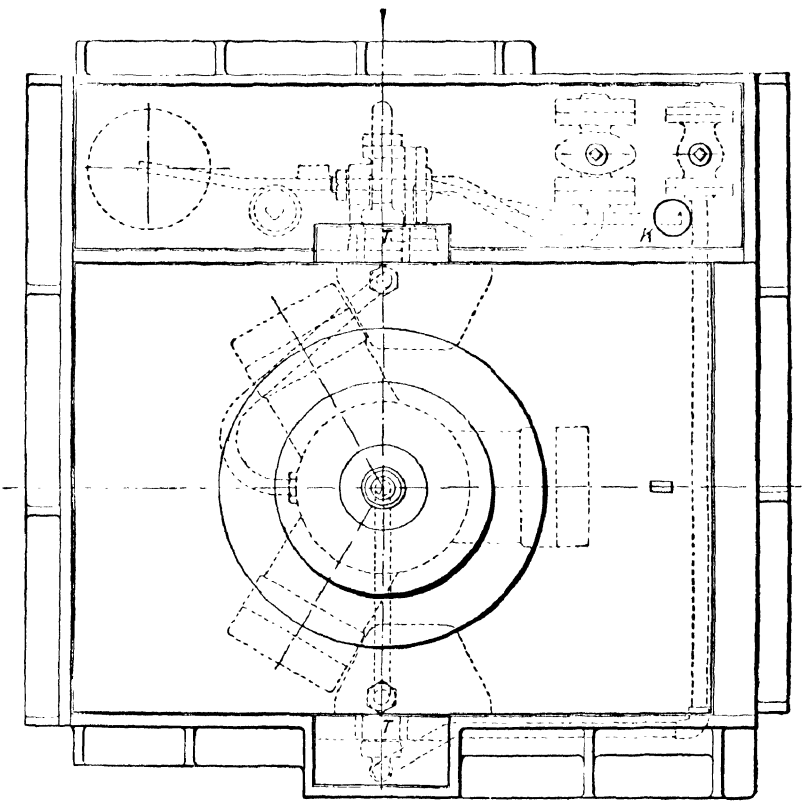


FIG. 416 B.

keyed to one half of the cone clutch *M* but is otherwise free to rotate relatively to the frame of the motor. The compound clutch *M* consists of three parts, one of which is fixed to the frame, one to the shaft *H*, and one to the cylinders *A*, and by suitable manipulation the shaft *H* may be either clutched to the frame or to the cylinder *A*. Assuming these cylinders to be rotating continuously in one direction with the engine, the action of the apparatus is as follows.

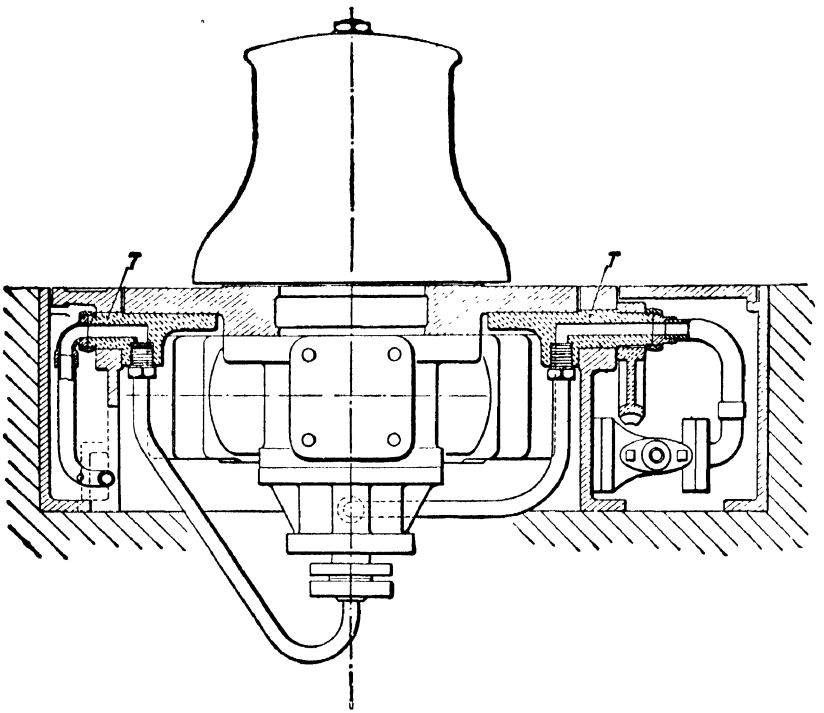


FIG. 416 C.

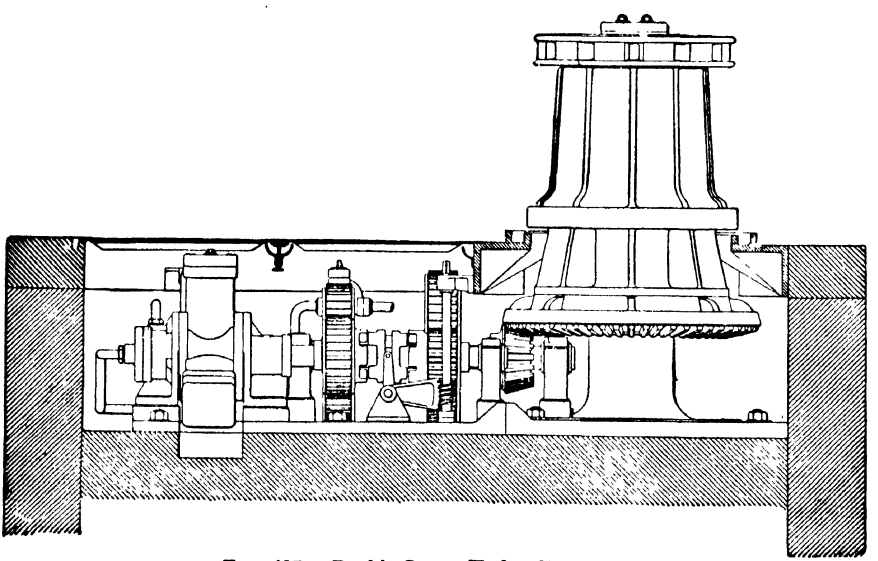


FIG. 417.—Double Power Hydraulic Capstan.

(1.) **Cylinders A and Crank Shaft H both declutched.**—Liquid pumped by pistons *A* is circulated freely through the system along the pipe *S* and through cylinders *B*, causing shaft *H* to rotate idly in its bearings, with a speed which is equal to or less than that of the engine according as the eccentricity of the ring *R* is equal to or less than the throw of the crank *H*.

(2.) **Shaft H Clutched to Frame.**—This shaft being fixed, the cylinders *B* are now constrained to rotate, the speed depending on the eccentricity of ring *R*, and the direction of flow, and therefore that of rotation, being regulated by the position of the rotary valve *V*.

(3.) **Shaft H clutched to Cylinders A.**—If now the eccentricity of *R* be reduced to zero, no circulation of fluid takes place, and the cylinders *A* and *B* are in effect directly coupled together through the medium of an

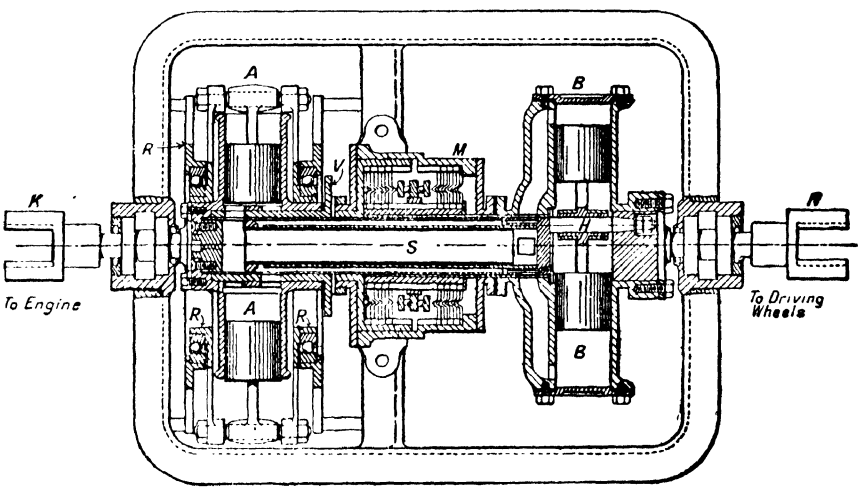


FIG. 418.—Hydraulic Transmission Gear.

incompressible column of liquid and rotate at the same speed. As the eccentricity of *R* is increased, the valve *V* remaining in the reverse position, circulation takes place and cylinders *B* are driven backwards relative to *A* to an extent which depends on the eccentricity. When this eccentricity is the same as the throw of *H* the backward speed of *B* is equal to the forward speed of *A*, and the actual speed of *B* relative to the frame is zero, while by suitably regulating the eccentricity between these two limits any speed lower than that of the engine may be given to *B* and hence to the driving wheels.

By the use of such a device the full power of the engine is available for work on the driving wheels at all speeds of transmission, except in so

far as this is reduced by hydraulic friction losses produced by the circulation of the working fluid. On the direct drive this is of course zero, and it is only at lower speeds that it becomes important. At low speeds it is not likely that the efficiency of transmission will be high, while the difficulties consequent on any leakage of the working fluid are obvious. The chief advantage of the system lies in its extreme flexibility and in the absence of shock on changing speeds.[1]

Art. 239.- The Föttinger Hydraulic Transmitter.

This is a combination of centrifugal pump and turbine forming a reduction gear through which the torque from a high-speed steam turbine can be transmitted to a slow-speed propeller. The principle of the device is illus-

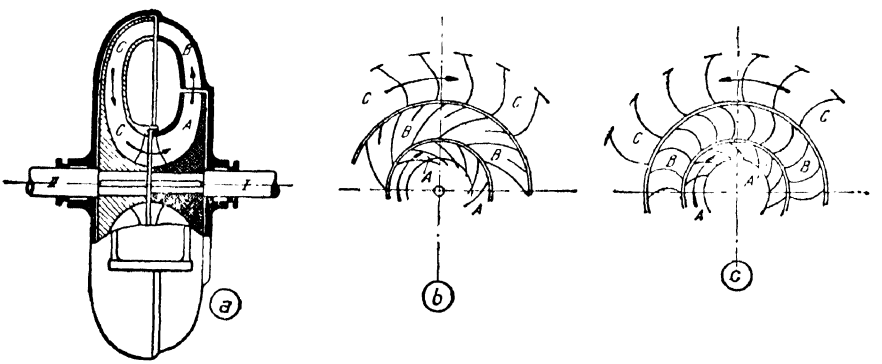


Fig. 420.—Diagrammatic arrangement of Föttinger Transmitter.

trated in Fig. 420 *a*, *b* and *c*. The hydraulic impeller *A* mounted on the driving or primary shaft delivers water into the guide ring *B*, where it is deflected into the turbine wheel *C* mounted on the driven shaft. The water leaving *C* again enters the impeller *A*, either directly or after passing through a small guide wheel, so that it circulates again and again through the system in a series of vortices. Fig. 420 *b* and *c* show diagrammatically the blading as arranged respectively for driving in the same and opposite direction to the primary shaft. In practice a continual circulation of cold water is maintained through the transmitter by an auxiliary centrifugal pump.

Fig. 421 [2] shows the arrangement of such a transmitter. The efficiency is of the order of 90 per cent.

[1] For other types of hydraulic gear, see Hele-Shaw, "Proc. Inst. Mech. E.," No. 7, 1921, p. 843.

[2] By courtesy of the Editor of *The Engineer*. For full details of construction, see *Engineering*, August 15th, 1913, or *The Engineer*, July 7th, 1922.

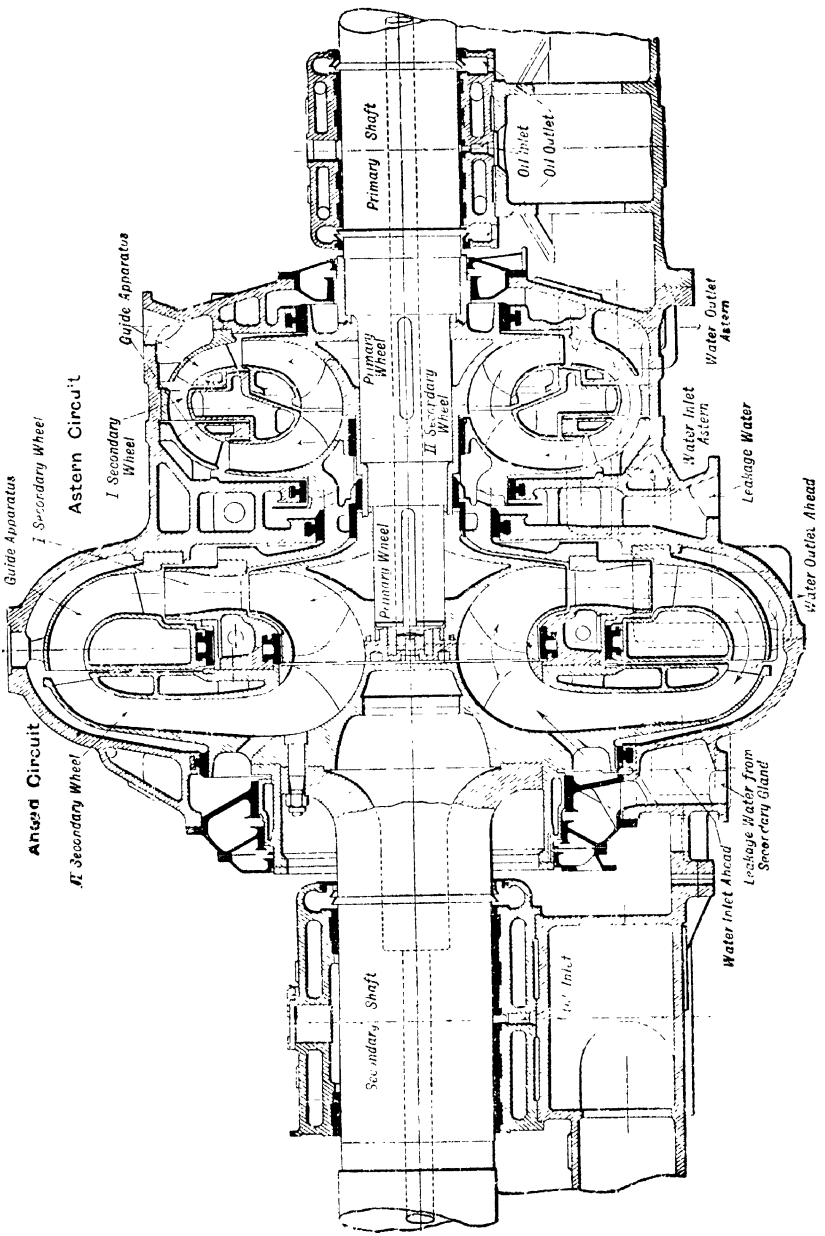


FIG. 421.—Föttinger Transmitter for Ahead and Astern Driving.

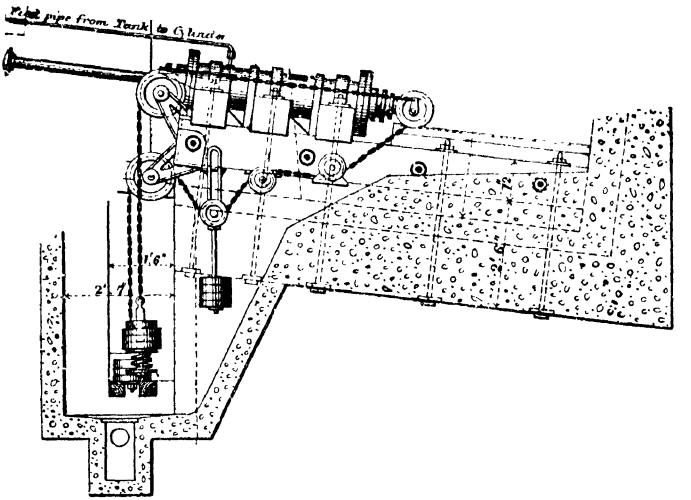
ART. 240.—THE HYDRAULIC RECOIL BRAKE AND BUFFER STOP.

The necessity for some braking apparatus by which the kinetic energy of a heavy body—such as a moving train or of a gun during recoil—might be quickly and safely absorbed without the recoil effect obtained by the use of spring buffers, led to the invention of the hydraulic brake.

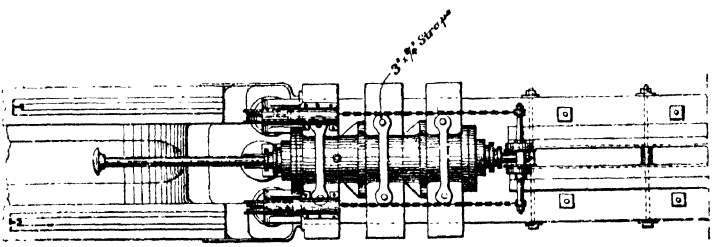
In its simplest form this consists of a cylinder fitted with piston and rod and filled with some liquid, usually oil, water, or glycerine. The two ends of the cylinder are connected, either by one or more small passages formed by holes in the body of the piston itself, or by a bye-pass pipe fitted with a spring-loaded valve or with a throttling valve by which the area may be adjusted. In its simplest form the brake is extensively used as a dashpot for damping the vibrations of governing mechanism and the like.

When used as a buffer stop, the body whose kinetic energy is to be absorbed forces in the piston rod and produces a flow of liquid at high velocity through the connecting orifices. The energy of the body is thus partly transformed into kinetic energy of the liquid, which is dissipated in eddy formation, and partly expended in overcoming the frictional resistances of the connecting passages, together with the mechanical friction of the brake. The whole of the energy is thus ultimately transformed into heat. Since the energy absorbed by the brake is constant for a given mass moving with a given velocity, and is equal to the mean resistance of the brake multiplied by the length of its stroke, it is evident that the pressure in the brake cylinder will have its least maximum value when this pressure, and therefore the resistance, is uniform throughout the stroke, and when in consequence the pressure-displacement diagram forms a rectangle. The brake is therefore preferably designed so as to give as nearly as possible uniform resistance, and since the resistance varies as the square of the velocity of the liquid through the connecting orifices (very nearly), while the velocity of the moving body, and therefore of the piston, varies from a maximum at the instant of impact to zero at the end of the stroke, it is necessary either to make the connecting passages of diminishing area towards the end of the stroke so that the velocity of efflux may remain constant, or to discharge from one side of the piston to the other through a spring loaded valve set to open at the required pressure.

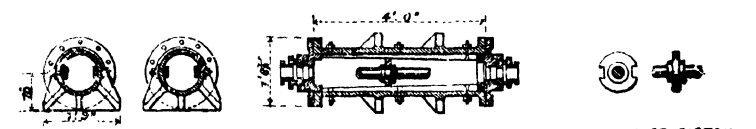
Where passages of constant area are used it is evident that the resistance falls off very rapidly as the velocity diminishes and, *e.g.*, has only one-quarter of its initial value when the velocity is reduced to one-half. In fact, if the resistance were solely measured by the production of kinetic energy in the contained fluid, the body would never be brought



LONGITUDINAL SECTION OF TERMINAL PIT.



PLAN OF HYDRAULIC BUFFER



SECTIONAL ELEVATIONS OF CYLINDER. SECTIONAL PLAN OF CYLINDER. DETAILS OF PISTON.

FIG. 422.—Hydraulic Buffer Stop.

absolutely to rest. Actually, however, the additional resistance in the shape of solid friction at the cup leathers, &c., together with that of returning springs or balance weights, prevent this state of affairs being realised in practice.

The area of the connecting passage may be varied by forming it as a circular orifice through the piston, and allowing this to work over a taper circular spindle fixed longitudinally in the cylinder, the available passage area varying with the diameter of the spindle. A somewhat similar

device is applied to a type of buffer stop adopted by Mr. Langley, two rectangular longitudinal slots cut in the piston body working over two rectangular longitudinal strips which are fixed inside the cylinder, have the same width as the slots, and vary in depth from end to end.

A buffer stop on these lines[1] is illustrated in Fig. 422. The cylinder, of cast iron, is 10 inches diameter and 4 feet long, its thickness varying from $1\frac{1}{2}$ inches at the front to $1\frac{3}{4}$ inches at the back end. The piston has a travel of 3 feet 6 inches, and is a plain disc, $9\frac{63}{64}$ inches diameter, with two waterways 3 inches wide by $1\frac{1}{2}$ inches deep working over two feathers running from end to end of the cylinder. These feathers are 3 inches wide, and are $1\frac{1}{2}$ inches thick at the back tapering to $\frac{1}{2}$ inch at the front end, the area of the waterways varying from 6 square inches at the back to zero at the front end of the cylinder. The clearance between cylinder and piston is equivalent to a further constant area of ·25 square inch. The covers are provided with two screwed glands, the inside gland of cast iron holding the hat leathers in position, while the outer glands of brass, which are merely intended as a stand-by, are packed slackly with hemp. A rubber pad $1\frac{1}{2}$ inches thick is inserted at the back end of the cylinder as a final cushion.

To bring the buffer forward after being pushed in a series of three counter weights are attached to the tail rod of the piston through chains working over pulleys at the front end. These weights must be put in motion gradually to prevent breakage of the chains by shock, and to this end the centre weight is divided into a main weight and a jockey weight, the latter being suspended from a 6-inch pulley working in guides and riding on the centre chain. The jockey weight is then supported on the slack of the chain when the buffer is in the forward position, and when the buffer is driven in is lifted before the main weight, thus taking up the first shock on the chain. For the same reason the side weights are mounted on spring hangers.

The cylinder is automatically fed with water by gravitation through a small pipe fitted with a non-return valve, while a small valve attached to the top of the cylinder is provided for the escape of any air which may accumulate, and for convenience in filling.

A similar brake, described by Mr. Langley, has a $3\frac{3}{4}$-inch piston rod and a piston 12-inch diameter and 4-feet stroke.

The total area of the connecting passages varies from 7·18 square inches at the front to ·38 square inch at the back end. After being driven home the piston is returned by means of counterweights. With this

[1] Described by Mr. P. W. Shaw ("Proc. Inst. C. E.," vol. 165, p. 290)

brake a maximum pressure of 800 lbs. per square inch was obtained when stopping a train weighing about 200 tons, and moving at 8 miles per hour.[1]

In a somewhat similar brake at Strasbourg[2] the piston diameter is 16 inches, and its travel 8·2 feet. A train of 200 tons moving at 8 miles per hour is brought to rest with a pressure of 580 lbs. per square inch, giving an uniform resistance of 52 tons. An automatic valve is fitted to a pipe connecting the two ends of the cylinder so that this may open if the pressure becomes excessive. As an additional precaution against the production of a dangerous cylinder pressure the links supporting the whole stop are designed so as to break under an excessive load.

It will be noted in Fig. 422 that the piston rod passes through both ends of the cylinder, the reason being that where the rod is in compression during the working stroke, if no tail rod is fitted the displacement of rod plus piston increases throughout the stroke, and if the cylinder were originally full of liquid no motion of the piston would be possible. The difficulty can be avoided by the provision of an air space inside the cylinder of volume slightly greater than that of the portion of the rod entering the cylinder during the stroke. Owing to this air space, however, the piston during the first portion of its stroke is engaged in compressing air, with the result that the pressure rises more slowly and the necessary maximum pressure for a given weight and velocity is increased by about 15 per cent. Where the piston rod is in tension during the working stroke the above difficulty is entirely obviated. The cylinder is filled with liquid with the piston run in, and the resistance is now increased by the formation of a partial vacuum behind the piston on its outward stroke.

In a more recent type of compression buffer stop without tail rod,[3] the difficulty is overcome by an escape valve set to blow off at 50 lbs. per square inch, at the front end of the cylinder. A volume of water equal to the displacement of the piston rod is then discharged through this valve to waste per working stroke. The piston is returned automatically to the front of the cylinder by means of pressure water from the town's mains which is supplied to the cylinder through a small back-pressure valve. This does away with all necessity for springs or counterbalance weights and gives a very neat and compact arrangement, the only drawback being due to the fact that town's water is used and that effective lubrication of the working surfaces must be relied upon to prevent corrosion. In

[1] "Proc. Inst. Mech. Eng.," 1886, p. 105.

[2] "Proc. Inst. C. E.," vol. 119, p. 449.

[3] By the Hydraulic Engineering Company, of Chester.

this buffer stop the connecting passages are formed by two longitudinal taper grooves cut in the body of the cylinder $1\frac{1}{2}$ inches wide and varying in depth from $\frac{3}{4}$ inches at the front to zero at the back end of the cylinder, while in addition there are twelve $\frac{3}{8}$-inch holes bored through the body of the piston, which is 12 inches diameter and has a travel of 5 feet. Protection against excessive load is provided by a relief valve loaded to 1,000 lbs. per square inch at the back end of the cylinder.

Recoil Cylinders for Guns.—In the case of a 6-inch quick-firing gun having a weight of recoiling parts equal to 14,800 lbs., when firing a cordite charge of $13\frac{1}{4}$ lbs. and giving a muzzle velocity of 2,150 feet per second to a projectile weighing 100 lbs., the velocity of recoil is found to be approximately equal to 17·5 feet per second. The energy of recoil is about 70,500 foot lbs., and is absorbed by a buffer of 12-inch stroke and $7\frac{1}{2}$ inches internal diameter.

For such a purpose a tension buffer is commonly used, and Fig. 423 illustrates such an one as fitted to a 6-inch gun. The liquid, escaping from left to right through the valve V on the working stroke, passes

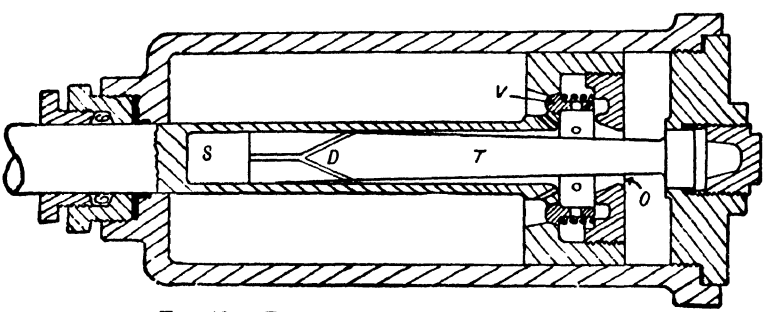


FIG. 423.—Tension Recoil Cylinder for 6-in. Gun.

through the annular passage O, whose area depends on the position of the piston relative to the central taper spindle T. The piston is returned by springs, whose action is buffered near the end of the stroke by the resistance to the flow of liquid from the space S through the holes at D.

Fig. 424 shows a type of compression buffer without tail rod, as fitted to the Canet quick-firing guns. Here the fact that the displacement increases throughout the stroke has been utilized to give a return device. In this buffer, when the piston A is pressed home, the displaced fluid passes through the pipe P and forces home the piston S, in the auxiliary cylinder B, against the resistance of the spring T. When the recoil has been absorbed this spring forces back the piston S, and increases the pressure in the main cylinder to an extent which depends on

the strength of this spring. This pressure, acting on the unbalanced area of the piston rod, forces the main piston forward to the end of the cylinder. In this buffer the orifices are of constant area.

Where communication between the two ends of the cylinder is made through a spring-loaded valve, in order that the pressure should be approximately constant throughout the stroke it is necessary that the lift of the valve should be small, its weight small, and the resistance offered by the spring as constant as possible over the range of lift of the valve.

As a liquid for use in recoil cylinders, Castor oil or Rangoon oil is good and keeps the leathers in good condition. A mixture of four parts of glycerine to one of water is also good, as is a mixture of methylated spirits

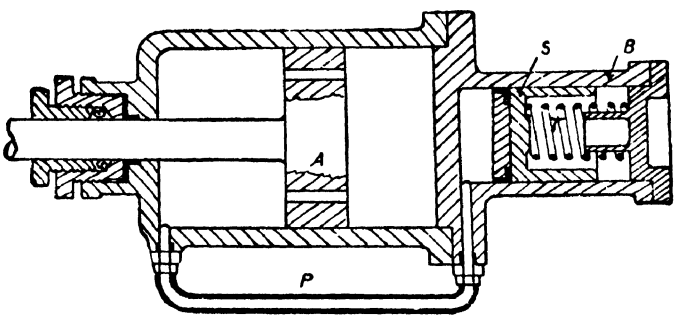


FIG. 424.—Recoil Cylinder for Canet Gun.

66 per cent, water 31 per cent., mineral oil 3 per cent., with 25 grains of carbonate of soda per gallon.

Theory of the Hydraulic Recoil Brake.—Suppose the piston rod to be in compression,

Let A = effective area of piston in square feet, *i.e.*, area of piston minus that of rod and of connecting orifices, if these are through the piston.

a = effective area of connecting orifices.

L = length of piston travel in feet.

l = length of connecting passages.

R = resistance of brake in lbs.

p = excess pressure behind piston in lbs. per square foot.

W = weight of moving body in lbs.

V = velocity of moving body, and of piston if brake is in operation.

v = velocity of fluid through orifices.

If the fluid is water we now have

$$\frac{p}{62{\cdot}4} = \frac{v^2}{2g}\left(1 + \frac{f\,l}{m}\right) = \frac{V^2}{2g} \cdot \frac{A^2}{a^2}\left\{1 + \frac{f\,l}{m}\right\} \tag{1}$$

while if F = resistance in lbs. due to mechanical friction and to effect of counterweights, &c.,

$$R = p\,A + F = \frac{62{\cdot}4\ V^2}{2g}\left\{1 + \frac{f\,l}{m}\right\}\frac{A^3}{a^2} + F. \tag{2}$$

Brake with Uniform Resistance.—With uniform resistance the retardation of the moving body is uniform, and will be given by $\frac{V_1^2}{2L}$ feet per second per second, where V_1 is the velocity at impact, so that the velocity at any point distant x feet from the beginning of the stroke is given by

$$V^2 = V_1^2 - \frac{V_1^2}{L}x,$$

$$= V_1^2\left(1 - \frac{x}{L}\right).$$

By equating the work done on the brake to the loss of K. E. by the moving body we get

$$R\,L = \frac{W\,V_1^2}{2g}$$

$$\therefore \quad V_1^2 = \frac{2\,R\,L\,g}{W}, \tag{3}$$

and

$$V^2 = \frac{2\,R\,L\,g\left(1 - \frac{x}{L}\right)}{W}.$$

Inserting this value in (2) above, we get

$$p\,A + F = \frac{62{\cdot}4\,L}{W}(p\,A + F)\left(1 - \frac{x}{L}\right)\left(1 + \frac{f\,l}{m}\right)\frac{A^3}{a^2} + F,$$

which finally reduces to

$$a^2 = \frac{62{\cdot}4\,L\,A^2\left(1 + \frac{f\,l}{m}\right)\left(A + \frac{F}{p}\right)}{W}\left(1 - \frac{x}{L}\right) \tag{4}$$

giving the passage area for any value of x, when the form of the orifice is known.

If it be assumed that mechanical friction is equivalent to 5 per cent. of the force on the piston, and if the term $\left(1 + \frac{f\,l}{m}\right)$ be taken as unity (*i.e.*, if fluid friction be neglected), this simplifies to

$$a^2 = \frac{65{\cdot}7\,L\,A^3}{W}\left(1 - \frac{x}{L}\right) \tag{5}$$

EXAMPLE.

Buffer stop—piston, 12¼ inches diameter, provided with two slots of total area 4·75 square inches—3¼-inch rod—length of stroke 4 feet. Weight of train = 100 tons.

$$A = \frac{105}{144} = \cdot 728 \text{ square foot}; \quad W = 224{,}000 \text{ lbs.}$$

Equation (5) now becomes

$$a^2 = \frac{65 \cdot 7 \times 4 \times \cdot 386}{224{,}000} \left(1 - \frac{x}{4}\right)$$

$$= \cdot 000453 \left(1 - \frac{x}{4}\right)$$

$$\therefore \quad a = \cdot 02127 \sqrt{\left(1 - \frac{x}{4}\right)} \text{ square feet.}$$

$$= 3 \cdot 06 \sqrt{\left(1 - \frac{x}{4}\right)} \text{ square inches.}$$

Thus when $x = 0$, *i.e.*, at the beginning of the stroke, $a = 3 \cdot 06$ square inches. This gives the effective area of the orifice, and, when the entrant edges are well rounded so as to prevent the formation of a *vena contracta*, this will be the true area. With sharp-edged orifices, however, the orifice area will need to be greater than this in the ratio $\frac{1}{\text{coefficient of contraction}}$, and this will depend upon the form and situation of the orifice.

To avoid errors in the preliminary calculation it is advisable to form the orifices with rounded entrant edges.

Assuming, in the example above, that $C_c = \cdot 65$, we get the following values for a_1, where $C_c\, a_1 = a$:—

x feet	0	1·0	2·0	3·0	4·0
a_1 square inches . .	4·71	4·09	3·34	2·35	0

It is worthy of note that since both the hydraulic resistance and the kinetic energy of the moving body vary as the velocity squared, their relative value is independent of the velocity, so that the piston travel is approximately independent of the initial velocity of the moving body, and depends only on its weight.[1]

[1] In a buffer stop described by Mr. P. W. Shaw ("Proc. Inst. C. E.," 1905–6, Part iii., vol. 165, p. 290), the piston came to dead slow at about 9 inches from the end of its stroke, when resisting a mass of 11 tons, for all speeds of collision from 4 to 12 miles per hour. The details of the brake are as follow :—Cylinder, 10 inches diameter, 3 feet 6 inches stroke piston, 9$\frac{3}{4}$ inches diameter, with two rectangular slots 3 inches wide × 1½ inches deep, working over strips 3 inches wide × 1½ inches tapering to ¼ inch.

Brake with Passages of Constant Area.—In this case we have, neglecting mechanical friction,

$$p \times A = R = \frac{62{\cdot}4\ V^2}{2\,g}\left\{1 + \frac{f\,l}{m}\right\}\frac{A^3}{a^2}$$

$$= k\ V^2 \text{ where } k = \frac{62{\cdot}4}{2\,g}\left\{1 + \frac{f\,l}{m}\right\}\frac{A^3}{a^2}$$

Since this measures the mass × acceleration of the moving body

$$k\ V^2 = -\frac{W}{g}\frac{d\,V}{d\,t} = -\frac{W}{g}\,V\frac{d\,V}{d\,x}$$

the negative sign indicating that the motion is being retarded.

$$\therefore\quad k\ V = -\frac{W}{g}\frac{d\,V}{d\,x}$$

or

$$\int_V^{V_1}\frac{d\,V}{V} = -\int_x^{x_1}\frac{k\,g}{W}\,d\,x$$

The solution of this equation gives :—

$$\log_e\frac{V_1}{V} = -\frac{k\,g}{W}(x_1 - x) = \frac{k\,g}{W}(x - x_1).$$

Writing $x_1 = 0$, so that V_1 is the velocity at the instant of impact, we have

$$\log_e\frac{V}{V_1} = -\frac{k\,g}{W}x,$$

or

$$V = V_1\,e^{-\frac{k\,g}{W}x},$$

giving the velocity corresponding to any position x of the piston. Since $e^{-\frac{k\,g}{W}x}$ never becomes equal to zero, no matter how large the value of x, the velocity cannot become zero until a change takes place in the law of resistance, either on account of the effect of solid friction, or because of the motion of the fluid becoming so slow that the resistance becomes approximately proportional to the first power of the velocity.

On taking mechanical friction into account we have, assuming this to be constant,

$$R = k\ V^2 + F = -\frac{W}{g}\,V\frac{d\,V}{d\,x} = -\frac{W}{2g}\frac{d\,(V^2)}{d\,x},$$

$$\therefore\quad \frac{d\,(V^2)}{d\,x} + \frac{2\,g\,k}{W}\,V^2 + \frac{2\,g\,F}{W} = 0,$$

or $\dfrac{d\,V^2}{d\,x} + b\,V^2 = c,$ where $\begin{cases} b = \dfrac{62{\cdot}4}{W}\dfrac{A^3}{a^2}\left(1 + \dfrac{f\,l}{m}\right). \\ c = -\dfrac{2\,g\,F}{W} \end{cases}$

The solution of this equation is

$$V^2 = \frac{c}{b} + D\,e^{-bx} \quad \text{where } D \text{ is a constant.}$$

Putting $V = V_1$ when $x = 0$ we have $V_1^2 = \frac{c}{b} + D$,

$$\therefore \quad V^2 = \frac{c}{b} + \left(V_1^2 - \frac{c}{b}\right) e^{-bx}$$

giving the velocity after a piston displacement x feet.

This may be written

$$e^{bx} = \frac{V_1^2 - \frac{c}{b}}{V^2 - \frac{c}{b}}$$

or $$x = \frac{1}{b} \log_e \left\{ \frac{V_1^2 - \frac{c}{b}}{V^2 - \frac{c}{b}} \right\} = \frac{2{\cdot}302}{b} \log_{10} \left\{ \frac{V_1^2 - \frac{c}{b}}{V^2 - \frac{c}{b}} \right\} \text{feet.}$$

giving the position of the piston when the velocity has been reduced from V_1 to V feet per second.

Fig. 398 indicates the variation of pressure intensity inside the cylinder with piston displacement in the cases (*a*) where the resistance is uniform; (*b*) where a compression buffer without tail rod, having passages of uniform area and with air compression is used; and (*c*) with a tension buffer with passages of uniform area.

With constant pressure and resistance, the maximum pressure attained is about one-third that in the case of the tension buffer with passages of uniform area.

Art. 241.—The Hydraulic Dynamometer

is certainly the most perfect of all mechanical devices for measuring and absorbing the energy developed by a prime mover at a rotating shaft.

The original dynamometer was the invention of Mr. William Froude, and was modified by Professor Osborne Reynolds.

The Reynolds—or Mather-Reynolds—Dynamometer shown in Fig. 426 consists of a double disc *D*, fixed to the power shaft by set screws or keys and carrying on its outer faces a series of narrow pockets.

These latter are semicircular in section, their plane is inclined at 45° to the axis of the shaft, and they face forwards in the direction of motion.

A casing *C*, carrying a double series of pockets similar to those on the disc, in the same planes but facing in the opposite direction, surrounds the disc and carries a graduated lever *L* from which are suspended the weights comprising the brake load.

The shaft passes through bushed openings in the casing which it fits closely so as to prevent undue leakage.

In the case of an 18-inch brake wheel the wheel itself carries twenty-four pockets on each face, while the casing carries twenty-five similar pockets. These are $4\frac{1}{2}$ inches in radial depth and have an axial width of $1\frac{1}{2}$ inch, the dividing vanes or partitions being $\frac{1}{4}$ inch thick.

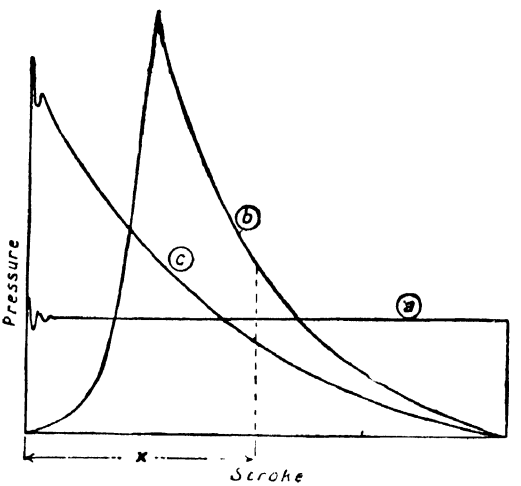


FIG. 425.—Pressure Diagrams from Cylinders of Hydraulic Recoil Brakes.

Provision is made for supplying the pockets with water, and for allowing this to escape after having work done on it in the dynamometer, while a small air vent is provided in the thickness of each vane to admit air from an annular chamber *K* in the casing to the centre of the circle formed when two of the pockets on disc and casing directly face each other. It is to this detail that the brake owes its successful action. As originally designed by Mr. Froude these air vents were absent.

The action of the dynamometer is as follows:—Water admitted to the casing through the flexible rubber tube at *T* finds its way through a series of four holes in the boss *B* into the chamber *E* between the discs, and thence through a series of small holes *F F*, formed in the thickness of the wheel vanes, into the pockets.

To prevent the water being projected into the air holes the radius of the inlet hole circle is made rather greater than that of the air vent circle.

Considering one of the pockets, its contained water is thrown outwards by centrifugal action, and, guided by the circular boundary of the pocket, is projected forwards into a stationary pocket in the casing.

Guided back by this pocket it enters a second moving pocket with increased velocity, is thrown outwards and projected forwards with still greater velocity into a second stationary pocket, and so on. Theo-

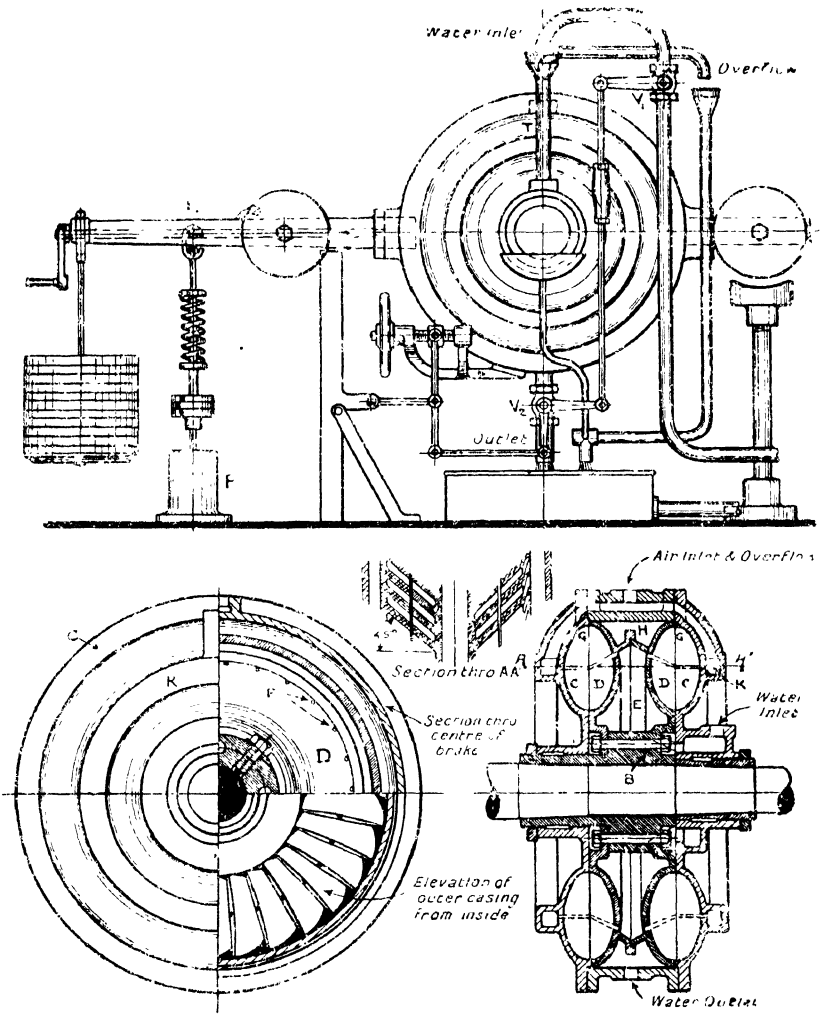


FIG. 426.—Reynolds' Hydraulic Dynamometer.

retically, but for the effect of friction, eddy losses, and the introduction of fresh water, this velocity would increase indefinitely. To produce the change in the moment of momentum about the shaft which occurs at each reversal of each stream, an equal moment must be applied to the

disc and to the casing, and since the friction moment of the side glands also reacts equally on the casing and on the shaft, there is, so long as the shaft is rotating uniformly, an exact balance between the driving moment on the shaft and the resisting moment on the casing. The latter is provided and measured by the resisting moment of the brake load.

A slight side clearance between the outer circumference of the wheel and of the casing at G G permits of the escape of water into the concentric chamber H, which is always full when the brake is working. It escapes from this chamber through the automatically regulated valve V_2.

Since the change of momentum varies directly as the quantity of water in motion, and directly as its velocity, the brake resistance must be varied by varying either the inflow or outflow, a system of levers being arranged as indicated so as to do this if the brake lever rises or falls, and thus to prevent over or under loading. An oil dashpot at P serves to prevent hunting.

The object of the air vents may now be noted. The water in motion in each pair of pockets forms a vortex, and in consequence is at a greater pressure at the outside than at the centre of the pocket. The pressure at the centre may, in fact, in a closed pocket, become less than atmospheric, while in any case there is a tendency for air to accumulate at the points of least pressure, so that in the original type the dynamometer gradually emptied itself of water and became air charged. Further, since with a given speed of rotation the pressure from the inside to the outside of the vortices increases at a fixed rate, any change in the internal pressure is accompanied by a corresponding change in the pressure on the pockets of the wheel and casing, and hence affects the resistance.

By the provision of air vents, however, a constant pressure, equal to that of the atmosphere, is maintained at the centre of the vortices under all normal conditions of working, and for a given speed and water supply the resistance remains uniform.

This type of dynamometer has many advantages, in view of its safety, its accuracy, ease of adjustment, and moderate dimensions.

The resisting moment varies as $A\ v^2\ r$, where A is the combined cross-sectional area of the streams suffering change of direction, v is their mean velocity which is proportional to the angular velocity of the brake and to its radius, and r is the radius of the centre line of the brake pockets.

It follows that in two similar dynamometers rotating with the same angular velocity, and having corresponding linear dimensions in the ratio s : 1, if both brakes are running full we shall have:—

$$A_2 = s^2 A_1;\ v_2 = s v_1;\ r_2 = s r_1;$$
$$\therefore\ A_2 v_2^2 r_2 = s^5 (A_1 v_1^2 r_1),$$

so that the resisting moment, and, therefore the horse-power absorbed, varies as the fifth power of the linear dimensions.

Experiments show that such a brake as illustrated will absorb a maximum of 29·5 H.P. at 100 revolutions, the disc diameter being 18 inches.

It follows that a brake having a disc 24 inches in diameter will absorb 1,100 H.P. at 300 revolutions, while by mounting two or more discs on one shaft the power which may be absorbed is practically unlimited.

The resistance with a given quantity of water varies approximately as the square of the brake speed, so that the brake is specially well adapted for high-speed work. Also, since the resisting moment at the instant of starting is zero and gradually increases with the speed, the brake is well adapted for the testing of internal-combustion engines or steam turbines of the pressure type, while once having been adjusted it requires no further attention no matter how the speed or power of the prime mover may vary within wide limits.

Fig. 427 shows a modern form of the Froude dynamometer.[1] Here the

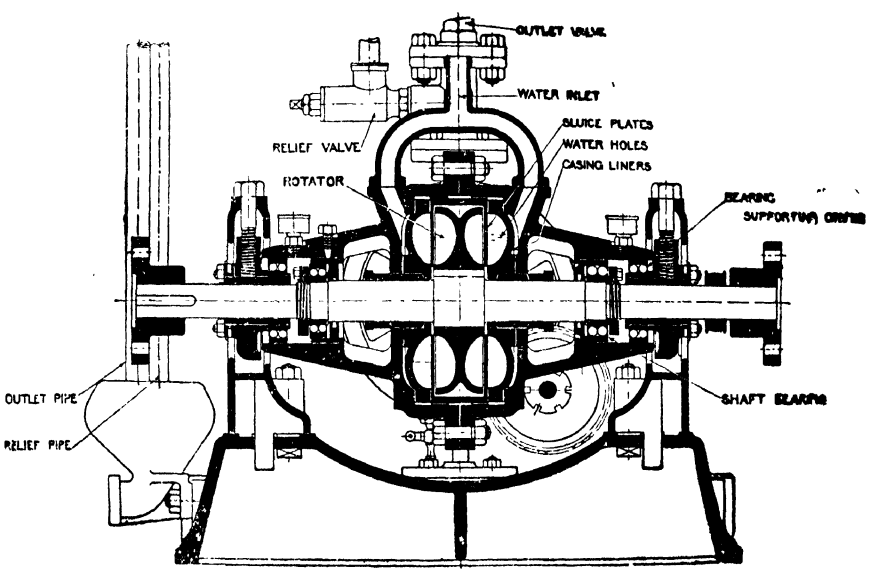


FIG. 427.—The Heenan and Froude Hydraulic Dynamometer.

[1] Messrs. Heenan and Froude, Worcester.

impeller always runs full of water, the pressure at inlet being sufficiently great to prevent the liberation of air at the centre of the vortices. The resistance is regulated by a sliding sluice gate fitted between the fixed and rotating discs, which cuts more or less of the periphery out of action.

A dynamometer to absorb 6,000 H.P. at 300 r.p.m., built on almost exactly the same lines as the Parsons steam turbine, with circulation of water through the various rings of fixed and moving vanes is illustrated and described in *Engineering News*, December 30, 1909, p. 726.

For such high speeds as are common in steam turbines of the impulse type, and for small powers, a simpler type of hydraulic dynamometer gives good results. This consists simply of a series of parallel discs keyed to the power shaft, and rotating with small side clearance between a similar series of stationary discs fixed to the outer casing. As in the previous type of brake, with an uniform speed of rotation the resistance varies with the quantity of water in use, and may therefore be regulated by the opening or closing of the inlet or outlet valve. With a constant quantity of water in the brake the resistance depends on the wetted area, and varies approximately as the 1.8th power of the angular velocity. Assuming it to vary as the square of the velocity, the work done on each face of each disc is given by $2\pi f \omega^3 \int_{R_2}^{R_1} r^4 \, dr$, where f is a coefficient of resistance, probably having a value of about ·0040 with roughened metal surfaces; ω = angular velocity in radians per second; and R_1 and R_2 are the outer and inner radii of the submerged portion of the rotating disc in feet.

On integrating this gives $\frac{2\pi f \omega^3}{5}(R_1^5 - R_2^5)$ foot lbs.

With n discs the work done against the resistance per second (neglecting the effect of the edges of the discs) is then given by

$$2n \times \frac{2\pi f \omega^3}{5}(R_1^5 - R_2^5) \text{ foot lbs.}$$

$$= \frac{4n\pi f \omega^3}{5 \times 550}(R_1^5 - R_2^5) \text{ H.P.}$$

EXAMPLE.

Brake provided with three rotating discs enclosed on both faces. Outer diameter 9 inches, submerged to an inner diameter of 6 inches. Assuming f = ·004, determine the B.H.P. absorbed at 10,000 revolutions per minute.

Here
$$\omega = \frac{2\pi \times 10{,}000}{60}$$
$$R_1^5 = (\cdot 375)^5 = \cdot 007413$$
$$R_2^5 = (\cdot 25)^5 = \cdot 000976$$
$$\therefore \text{ B.H.P.} = \frac{12 \times \pi \times \cdot 004 \times 8\pi^3 \times 10^9 \times \cdot 006437}{5 \times 216 \times 550}$$
$$= 405.$$

EXAMPLES.

(1) A direct acting hydraulic lift has a travel of 40 feet. The cage and ram weigh four tons, and are counterbalanced to the extent of three tons, by hanging weights. The lift is to take a load of three tons, with a maximum acceleration of 2 feet per second per second. The diameter of ram is decided on as being 5 inches. Determine the working pressure in the cylinder, assuming ram friction to account for 5 per cent. of this. If the pressure supply is 750 lbs. per square inch, this being reduced by means of a reversed intensifier of the ordinary type and having a stroke of 4 feet, determine the necessary size of this cylinder and of its ram, assuming friction to cause a loss of 5 per cent. Also determine the relative gain in efficiency of this system as compared with supplying the lift cylinder directly with pressure-water throttled down to the required pressure.

N.B.—The total weight to be accelerated in this case = 4 + 3 + 3 = 10 tons.

Answer.
- Working pressure 554 lbs. per square inch.
- Diameter of reducer cylinder = 15·8 inches.
- Diameter of reducer ram = 13·95 inches.
- Gain in efficiency = 35·4 per cent.

(2) A direct acting hydraulic lift has a travel of 40 feet. The cage and ram weigh four tons, and the lift takes a weight of three tons, with an acceleration of 2 f.s.s. Assuming a ram diameter of 5 inches, and a hydraulic balance cylinder of the type shown in Fig. 361 to be fitted, this being designed so as to balance three-quarters of the weight of the cage and ram when the latter is at the upper limit of its travel, determine the leading dimensions of this balance cylinder, given that the stroke of its ram is 6 feet; that the ram B weighs one ton; and that the annular space D is supplied from a tank whose level is such as to produce a pressure of 30 lbs. per square inch on the annulus E when this ram is at the bottom of its stroke. The supply pressure at A is 1,100 lbs. per square inch.

Answer.
- Outer diameter of ram B = 12·95 inches.
- Outer diameter of stationary inlet ram = 9·0 inches.
- Outer diameter of annulus E = 44 inches.

(3) If in the lift of the preceding example the type of balance cylinder of Fig. 362 is used, determine the necessary diameters of A, B, and C, and the necessary weight of ram B. Take into account the acceleration of B.

Answer.
- Outer diameter of ram C = 12·95 inches.
- Outer diameter of ram A = 9·0 inches.
- Weight of B = 20·2 tons.

(4) If in example (1) the cage and ram form part of a suspended lift, counterbalanced to the same extent as in that question, and operated through a jigger giving a multiplying ratio of 6 : 1, determine the necessary diameter of the cylinder of the jigger if the working pressure = 750 lbs. per square inch; the efficiency of the multiplying mechanism = 75 per cent.; and frictional losses account for 10 per cent. of the pressure on the ram.

Answer. 12·8 inches diameter.

(5) A direct acting crane of the type shown in Fig. 370 is to lift two tons, with a maximum acceleration of 4 f.s.s. The supply pressure is 500 lbs. per square inch. Assuming an efficiency of 75 per cent. determine the necessary size of ram.

Answer. 3·5 inches diameter.

(6) A hydraulic jack has an efficiency of 80 per cent. The diameter of the ram is 8 inches, of the plunger $\frac{3}{4}$ inch. The leverage of the operating lever is 20 : 1. Determine the pressure on the handle when lifting forty tons.

Answer. 49·2 lbs.

(7) A buffer stop is to be designed to stop a train weighing 150 tons when going at ten miles per hour, in 8 feet. The cylinder diameter is $16\frac{1}{2}$ inches, the piston rod being $5\frac{9}{16}$ inches diameter. The thickness of the piston is 4 inches, but the entrant edges of the orifices are rounded so as to make their effective length equal to 2 inches. There are two rectangular orifices, each 3 inches wide by 2·10 inches deep. Determine their effective depth at points where the piston travel is respectively 0, 2, 4, 6 and 8 feet, in order that the resistance may be uniform throughout the stroke. Also determine the magnitude of this resistance and the pressure in the cylinder, assuming that counterbalance weights and friction exert a constant resistance of one ton.

Answer. Resistance 140, 300 lbs. = 62·65 tons.
Pressure in cylinder = 781 lbs. per square inch.

Stroke	0	2 ft.	4 ft.	6 ft.	8 ft.
Effective depth . .	1·30 ins.	1·13 ins.	·92 ins.	·65 ins.	0

(8) If the buffer stop of the preceding example has the same effective piston area with two rectangular connecting orifices each 3 inches wide which together have a combined area of 6 square inches, this remaining uniform throughout the stroke, determine the maximum pressure attained in the cylinder and the length of stroke which would be necessary for the piston to come to rest without touching the end of the cylinder.

Answer. Maximum pressure = 1,320 lbs. per square inch.
Stroke = 22·45 feet.

(9) If the area of the passages in Example 8 were 3 square inches, determine the maximum pressure attained, and the length of stroke before coming to rest.

Answer. Pressure = 5,275 lbs. per square inch.
Stroke = 7·53 feet.

CHAPTER XXIV.

The use of Models of Hydraulic Structures and of Tidal and River Models—Wave Action Models.

ART. 242.

CHAPTER IV deals with the general dynamical conditions to be satisfied if a scale model is to reproduce the behaviour of its prototype. Reference has also been made to the use of models of turbines,[1] of pumps,[2] and of surge tanks.[3]

The present chapter deals more specifically with models of hydraulic structures such as weir and siphon spillways and of river and tidal estuary improvement works.

ART. 243.—WEIR MODELS.

Such data as are available for comparing the discharge over spillway weirs and their models show that even with very small models the scale effect is small.

In experiments by Thoma [4] on a weir of ogee cross section under heads from 2·4 feet to 5·5 feet and on its 1/50 scale model, the weir coefficient for the model was about 2 per cent. greater than that of the full-sized weir.

Experiments in the Manchester laboratory on a series of models of ogee cross section to scales of 1/25 and 1/50, the originals of which were tested at Cornell University [5] under heads from 6 inches to 4 feet, show that the model coefficient is about 1·3 per cent. greater than that of the original when the scale is 1/25 and 2·8 per cent. greater when the scale is 1/50 so long as the head in the model is not less than 0·25 inches.

These models had smoothly finished varnished surfaces. Tests on one of the models with its surface coated with coarse sand showed a slight increase of 1 per cent. in the coefficient.

[1] P. 531.
[2] P. 662.
[3] P. 550.
[4] Trans. Hyd. Lab. of Tech. University, Munich, No. 2, 1928.
[5] Water Supply and Irrigation Paper No. 200, U.S. Geol. Survey. Also Rafter, *Trans. Am. Soc. C.E.*, Vol. 44, 1900.

The foregoing experiments showed coefficients greater in the model than the original, and suggest that writing $Q = K\,b\,h^{\frac{3}{2}}$, the relationship

$$K = K_m\left(1 - \frac{\cdot 00056}{s}\right)$$

where s is the scale ratio (1/25, 1/50), etc., will give the large-scale weir coefficient K with very close agreement, *so long as the smallest head in the model exceeds* 0·25 *inches.*

Experiments by E. Soucek,[1] on a 1/12-scale model, however, have shown a coefficient somewhat less than in the original, so that all that can be said at the moment with any certainty is that so long as upstream conditions are accurately reproduced, the coefficients of the weir and prototype may be expected to agree within about $\pm$ 2·5 per cent.

Art. 244.—Sluice Gate Models.

A comparison [2] between the discharge from the sluice gates and culverts of the Assuan Dam, and from a series of models shows a very close agreement. The scales were 3/100, 2/100 and 3/200. On the average the discharges in the model were slightly less than in the original, but the mean difference between the two was only 0·4 per cent., the greatest difference with the smallest model, being $-$ 4·5 per cent. As a result of these experiments the authors conclude that, for reliable results, the product of the velocity of flow and of the smallest dimension of the orifice should not be less than 0·107 in foot sec. units, and the smallest dimension of the orifice not less than about 1 inch.

Art. 245.—Siphon Spillways.

Much work has been done on models of siphon spillways, and where the model has been of suitable size the results in general show good agreement as regards the coefficients of discharge, except where leakage of air into the crown of the large siphon was known to be taking place.

Experiments in the author's laboratory [3] on two similar models of the type shown in Fig. 125, one four times the size of the other, have given some information as to the minimum size of model of this type, to give reliable results. The smaller model had a throat depth of 1 inch. On testing these the following values of the coefficient of discharge K were obtained.

[1] E. Soucek, "Meter Measurements of Dam Discharge," *Proc. Am. Soc. C.E.*, Part I. Oct. 1942, p. 1352.

[2] Hurst and Watt, *Proc. Inst. C.E.*, 1923–4, Part 2, p. 72.

[3] "Experiments on Siphon Spillways," Gibson, Aspey and Tattersall, *Proc. Inst. C.E.*, Vol. 231, 1930–1, Part I, p. 203.

Head (ft.) .	0·2	0·4	0·6	0·8	1·0	1·5	2·0
K (small model).	0·69	0·72	0·73	0·74	0·745	0·75	0·75
K (large model).	0·77	0·79	0·795	0·80	0·802	0·805	0·806

From these it appears that at similar heads there is quite a large-scale effect as between the two models.

If however values of K are plotted against ($v\,d$) where v is the velocity f.s. and d the depth in feet of the throat, the results lie on one and the same curve.

Corresponding values of K and of $v\,d$ are as follows :—

$v\,d$ (ft.²/sec.)	0·2	0·4	0·6	1·0	1·5	2·0	3·0	3·5
K .	0·686	0·73	0·755	0·778	0·794	0·801	0·806	0·806

From these results it appears that if the model is of such a size that its $v\,d$ value exceeds about 1·8 in foot sec. units the value of K is sensibly constant and may be expected to be the same as its prototype at all heads and therefore for the all-important head at which $h/H = d/D$.

For example, in a proposed siphon of throat depth 2 feet to operate under a head of 16 feet, with a probable coefficient of discharge of say 0·75, the value of v would be $0{\cdot}75\sqrt{2\,g \times 16} = 24$ ft. sec. and the value of $v\,d$, 48 ft. sec. units. If the model scale is 1 : 5, v will be $24/\sqrt{s}$, and d will be $2/s$. Writing $\frac{24 \times 2}{s^{\frac{3}{2}}} = 1{\cdot}8$, s becomes 9·0 and the model should not be made to a scale smaller than 1 : 9.

Making no assumptions as to the value of K, the experiments indicate that close agreement between the model and its prototype may be expected if $d\sqrt{h}$ is not less than about 0·28 in foot second units.

It is to be noted that while such models give excellent results as regards coefficients of discharge when running full, they do not give reliable indications of the time necessary to prime the siphon, since exact conditions of similarity as regards air and water flow cannot be provided in a model working under atmospheric conditions.

Art. 246.—Tidal and River Models.

During the last twenty years or so the value of the river or tidal model as a means for investigating the various problems which arise in the training

or improvement of a river or estuary has become increasingly appreciated, and no such scheme of any magnitude would now be embarked upon without a preliminary model investigation.

Osborne Reynolds, then Professor of Engineering in Manchester University, was the first to put the use of such models on to a scientific basis. In 1885 he made a tidal model of the Upper Mersey which gave results in such accordance with the observed behaviour of the estuary as to lead him to suggest to a committee of the British Association[1] an investigation of a series of models, to different scales, of a hypothetical estuary. In 1888 this investigation was started at Manchester. In 1891 the committee reported to the effect that " by carefully observing certain (stated) precautions the method of model investigation may now be applied with confidence to practical problems. The Civil Engineers of those days were however very conservative and somewhat sceptical and it was not until some forty years later that the values of such investigations became generally acknowledged.

The flow in any river or estuary is essentially turbulent, and one essential condition to be satisfied is that any model shall be of such a size that its Reynolds number $\frac{v\,m}{\gamma}$, where m is the hydraulic mean depth, is in the range which gives turbulent flow.

Experiment (see p. 295) shows that in a straight parallel channel turbulent flow is initiated when $v\,m/\gamma$ is approximately 1,400. *E.g.*, if the hydraulic mean depth is 1 inch (2·54 cm.), since the kinematic viscosity γ of water at normal temperatures is approximately 0·010 in c.g.s. units (p. 12), any velocity exceeding 5·5 cm. per sec. (0·21 feet per second) would give turbulent motion. In a model of any actual river or estuary with its tortuous course and irregular bed, the critical velocity would be appreciably less than this.

Osborne Reynolds, as a result of experiments on models of hypothetical estuaries of simple triangular form to different scales, concluded that for the motion in the model to be turbulent and to be able to produce similar sand formations in each case, the value of h^3e should not be less than $0{\cdot}09\sqrt{\frac{30}{H}}$. Here h denotes the maximum tidal range in the model in feet, H the corresponding range in the estuary and e the ratio of the vertical and horizontal scales. Thus, in a model of an estuary with a tidal range of 30 feet, the model having a horizontal scale of 12 inches to 1 mile (1 : 5,280), calculation shows that a vertical scale of 1 : 200 will just satisfy this criterion.

[1] Reynolds, " On Certain Laws Relating to the Régime of Rivers and Estuaries and on the Possibility of Experiments on a Small Scale," Rep. Brit. Ass., 1887, or Papers on Mechanical and Physical Subjects, Reynolds, Vol. 2, p. 334.

Here $h = 0{\cdot}15$; $e = 26{\cdot}4$. A vertical scale of 1 : 250 would give a criterion of 0·0366, and one of 1 : 150 would give a criterion of 0·28. Actually in a normal estuary the irregularities of the conformation both of sides and bottom cause turbulent flow to be set up at lower velocities than in those for which the criterion was deduced by Reynolds, so that somewhat lower values can safely be used in practice.

Art. 247.—Distortion of Scale.

In some types of hydraulic model it is essential that the horizontal and vertical scales shall be the same. This is the case in models of weirs or sluices or in any model in which a standing wave is produced, since the form of the free surface depends entirely upon the velocity of flow, which in its turn depends only on the head and therefore on the vertical scale, so that if horizontal and vertical distances on the profile are to correspond in model and original the horizontal and vertical scales must be identical.

River and tidal models are almost invariably made with a distorted scale. This is because the horizontal scale ratio has to be large to keep the model within reasonable dimensions. If this same ratio were used for the depths, these, and the corresponding velocities of flow which are proportional to $\sqrt{h}$, would be so small that the motion would be non-turbulent and no motion of the bed materials would be likely. Also in tidal work the tidal range would be too small to be observed with any accuracy.

In practice the vertical exaggeration of scale is usually between about 5 : 1 and 40 : 1.

The distortion of scale means that both lateral and longitudinal gradients in the model are greater than in the river or estuary and at first sight it might appear that this would reduce the usefulness of such models. Actually, however, some such exaggeration of natural scale appears to be natural and rather an advantage than otherwise except in special cases. What in effect is a distortion of scale is usual in nature since small streams in alluvial ground have much steeper side slopes and gradients than large rivers of similar régime in similar ground. In a very large river such as the Mississippi, the Ganges or the Irrawaddy, the maximum depth rarely exceeds 1 : 50 of the maximum width, while in a small stream in similar ground, this ratio will seldom be less than 1 : 5.

Provided then that the side slopes in the model do not exceed the natural angle of repose of the bed material—usually about 30°—the distortion of scale would not appear to be harmful. Where side slopes of this magnitude are involved the model fails to reproduce the actual conditions and the

depths of scour will be less than in nature. The actual slopes in most estuaries and rivers are however very small, and experience shows that the areas over which the angle of repose would be exceeded in the average model is a very small fraction of the whole. In such cases, if thought desirable, a stiffening of the bed materials with an admixture of clay will allow the required side slope to be maintained.

In this general connection it may be noted that observations on Indian rivers and canals [1] in alluvial material have suggested that an exaggeration of scale is essential if similar régimes are to be established. If the bed material is the same in both, Lacey, from theoretical considerations, suggests the relationship, $R/r = (L/l)^{\frac{1}{3}}$ where R is the hydraulic mean depth and L the length.

Experiments by the author and his colleagues on many estuarine and river models, in some of which this relationship was approximately satisfied and in others in which it was decidedly not, have not shown evidence that this does necessarily give the best scale exaggeration.

In a model intended to investigate the effect of such training works as spur dykes or groynes, which cause an intense scour and comparatively steep slopes in their vicinity in the river itself, the distortion of scale should be as small as possible. In general the area of local scour will be exaggerated in the model, owing to the fact that after the material is scoured out to its natural angle of repose any further scour causes more surface material to slip down the scoured incline.

Experiments on the effect of spur dykes on a model of the Rhine [2] having a natural scale of 1 : 200 have shown an excellent agreement with the results observed in the river. Experiments with a vertical exaggeration of 2·5 showed the same maximum depths of scour, but over a somewhat greater area. Experiments on the effect of spur dykes in a model of the Mississippi,[3] with a horizontal scale of 1 : 1,000 and a vertical scale of 1 : 125, showed results which have since been confirmed in the river. The general agreement is said to be very close.

Art. 248.—Time Scale and Velocity Ratio.

In a tidal model the correct reproduction of the tidal wave is an essential factor. Since the velocity of such a wave is proportional to the square root of the depth of water in which it is propagated, corresponding speeds in

[1] G. Lacey, "Stable Channels in Alluvium," *Proc. Inst. C.E.*, Vol. 129 (1929–30), Part I, p. 259. Also Vol. 237 (1933–4), Part I, p. 421.
[2] J. R. Freeman, "Hydraulic Laboratory Practice," p. 219. New York, 1929.
[3] Paper 15, U.S. Waterways Experiment Station, Vicksburg, Jan. 1934.

model and estuary must be in the ratio $\sqrt{h/H}$ of the square roots of corresponding depths. If then the wave is to travel corresponding distances l and L of the model and estuary in corresponding times, these times must be in the ratio

$$\frac{t}{T} = \frac{l}{L}\sqrt{\frac{H}{h}}.$$

This ratio gives the necessary time t of a tidal period in the model corresponding to the time T, which is about 12 hours 25 minutes, in nature. Thus in a model having a horizontal scale ratio of 1 : 5,280 and a vertical scale ratio of 1 : 200, the time ratio would be 1 : 373 and the tidal period 119·8 seconds, so that the equivalent of 1 year's tides would be reproduced in 24 hours.

Corresponding current velocities in estuary and model are in the ratio $\sqrt{H/h}$; corresponding volumes are in the ratio HL^2/hl^2 ; and corresponding volumes per unit time in the ratio $LH^{\frac{3}{2}}/lh^{\frac{3}{2}}$, so that the necessary discharge of any river entering the model can be computed if the discharge of the original is known.

Art. 249.—Bed Materials.

A criticism which has been levelled at the use of tidal and river models is that since the actual bed material is usually a not very coarse sand, unless something of the nature of an impalpably fine powder is used in the model the grain size and textual roughness of the bed will not be reduced in the same proportion as the remainder of the model. The idea that the bed material should of necessity be largely scaled down in size appears to be based on two misconceptions. The first is that the resistance to flow in an irregular channel is appreciably affected by the surface roughness. Actually, in any model of a river or estuary having a sandy bed, the resistance is almost entirely due to eddy formation caused by curves and irregularities in the sides, and by irregularities in the depth whose effect is much greater than that of a change in the textual roughness of the surface itself. Recent experiments by the author on the resistance to flow around an easy bend in the model of a canal of uniform cross section showed no appreciable difference whether the surface was smoothly painted or was coated with coarse sand, practically the whole loss of head being due to eddy formation and interference between the cross currents set up.

Again, in a tidal estuary, the tidal wave, after attaining its maximum range is rapidly damped out by these various resistances as it travels upstream. My colleagues and myself have constructed quite a number of

tidal models, in each of which the size of the bed material has been of the same order as in the estuary, and if the effect of this roughness had been important, the tidal range in the upper estuary would have been reduced relatively much more rapidly in the models than in nature. Actually in every case the modification in the range of the tides, the levels of high and low water, and the duration and velocity of the tidal currents have been reproduced at every point with a high degree of accuracy.

As regards the movement of the bed materials by the scour of the current, the best material is that which reproduces most accurately that of the bed of the river or estuary, and this can only be determined by experiment.

Thus in the preliminary work on a model of the Severn Estuary at Manchester University [1] thirteen bed materials were tested. These consisted of powdered pumice, silica sands, and emeries, of different grain sizes, some finer and others coarser than the sand in the estuary. Two charts of the estuary were available, taken at intervals of seventy-eight years. Moulding to the contours of the first chart and running the model for a number of tides corresponding to seventy-eight years, a comparison was made with the second chart. Broadly speaking, the results showed that so long as the material was not too large or too heavy to be moved by the currents the general conformation of the estuary was the same with all materials. They showed a fairly definite relationship between grain size and density and the heights and depths of the banks and channels, these being inversely proportional to $d^{0.29} \times \rho^{0.26}$ and directly proportional to $r^{0.15}$ where d is the mean diameter of the grains, ρ the effective density in water, and r is a measure of the angularity of the grains, being the mean ratio of the longest to the shortest diameter. The best overall agreement was obtained with a sand having a mean diameter of 0·007 inch, which is about 25 per cent. less than the size of the sand in the estuary.

It is interesting to note that Osborne Reynolds, after trying a number of bed materials, finally decided on a sand of grain diameter 0·0065 inch. He used this in all his models and although the scales were varied over a fairly wide range—the longitudinal scale between the limits 1 : 39,000 and 1 : 12,000, and the vertical scale between 1 : 170 and 1 : 960, he does not remark on any marked discrepancy between the results. Indeed, he does mention the general similarity of the results from the different models.

This view receives some indirect confirmation from the results of experiments at Karlsruhe [2] on the scour at bridge piers, in which an increase in

[1] A. H. Gibson, "Construction and Operation of a Tidal Model of the Severn Estuary," H.M. Stationery Office, 63-78-2, 1933.

[2] J. R. Freeman, "Hydraulic Laboratory Practice," p. 201. New York, 1929.

the size of sand grain from 0·02 inch to 0·06 inch did not result in any appreciable difference in the scour. Further confirmation is afforded by the experiments of Messrs. A. D. D. Butcher and J. D. Atkinson [1] on the scour at the end of the floor of models of the Esna sluice-dam, where identical slopes were formed with models of scales 1 : 100 and 7 : 100, using the same sand in both.

On the other hand, experiments in the Manchester University laboratory [2] on the scour below the toe of an ogee weir in which three grades of sand, pumice and emery were used showed the following results :—

Material.	Specific Gravity.	Mean Diam., ins.	Depth of Scour, ins.	Depth Corrected for Grain Size.	Corrected for Grain Size and Density.
Sand A	2·63	0·0237	1·9	2·3	2·3
,, B	2·63	0·0157	2·4	2·4	2·4
,, C	2·63	0·0093	2·9	2·3	2·3
Emery	3·89	0·0097	2·4	1·9	2·4
Pumice	1·96	0·0162	3·3	3·3	2·7

These show an effect both of size and density. Comparing sands A, B, C and taking the medium sand B as a standard, calculations show that the depth is inversely proportional to $d^{\cdot 44}$. Making this assumption, the corrected depths would appear as in column 5. The relative effect of density is shown in the last two lines. The effective density of emery in water is 2·89 and of pumice 0·96. Calculation shows that if the observed depths are corrected in the ratio $\left(\frac{\rho}{\rho_s}\right)^{40}$ where ρ and ρ_s are the effective densities in water of the material and of sand, these become as shown in column 6.

The agreement is very close except for pumice. Pumice, however, is not a good material for this type of investigation as so much of the finer particles are whirled away in the excessively turbulent flow below the weir.

In river model work, other materials than sand are nowadays often used, such as gilsonite (sp. gr. 1·3) which is a hard brittle bitumen; pumice; powdered lignite and soft coal. The object being to prevent or reduce the ripple formation which is often found on the bed of sand models.

[1] "The Causes and Prevention of Bed Erosion, with special reference to the Protection of Structures Controlling Rivers and Canals." Minutes of Proceedings Inst. C.E., vol. 235 (1932-33, Part I), p. 175.

[2] By C. N. Hsia, M.Sc., Thesis, 1940.

ART. 250.—SILT PROBLEM.

Any model of an estuary in which trouble is caused by the deposition of silt must include this effect. Since the silt is usually brought down by the rivers feeding the estuary, an analysis of the percentage of silt carried in the river-water under various conditions of flow is required, and this percentage is to be added to the river-water supply in the model. Silt samples are also required at various points in the estuary, and on starting up the model sufficient silt is to be added to the estuary water to give the approximate concentration at these points. The colloidal silt brought down by the rivers tends to coagulate and to settle, forming silt banks where the fresh river-water meets the saline sea-water, and this effect should also be reproduced if the material is to be deposited at the same place in the model as in the estuary.

Now corresponding times in model and estuary are in the ratio $l\sqrt{H}/L\sqrt{h}$, whilst corresponding depths are in the ratio h/H, and since corresponding particles of silt in model and estuary should sink through corresponding depths in corresponding times, it follows that the actual rate of fall of a particle in the model should be greater than that in the estuary in the ratio $hL\sqrt{h}/Hl\sqrt{H}(=Lh^{\frac{3}{2}}/lH^{\frac{3}{2}})$. Thus with a value of $L/l = 5{,}280$ and $H/h = 200$, the rate of fall should be 1·9 times as great in the model.

This greater rate of fall can be produced by increasing the size of the particles, either by using silt of greater coarseness or by using some coagulating medium more effective than the salts in sea-water. In the models constructed at Manchester, the colloidal silt from the estuary itself has always been employed and alum solution has been used as the coagulating medium. Experiments are made with samples of sea-water from the seaweed end of the estuary to determine the rate of deposition of the silt, and further experiments are carried out with alum solution to determine the necessary concentration of this to produce the required rate in the model. On starting up the model the water at the seaward end is brought up to the required concentration. Since river-water is constantly being added at the upper end of the estuary and discharged over a spillway at the seaward end so as to maintain a constant mean level, the concentration would gradually get less unless fresh alum solution were added ; sufficient of this is therefore fed in at a constant rate at the seaward end to give the required concentration to the amount of water being discharged over the spillway.

ART. 251.—RIVER MODELS.

In a tidal model, since flow takes place alternately up and down the model on the flood and ebb tides, the longitudinal gradient is settled definitely

by the ratio of the vertical and horizontal scales and is greater than that in the estuary in this ratio. In a river model, however, with flow in one direction only, it is possible to make the gradient independent of the distortion of scale, and a technique has been developed in U.S. and continental laboratories in which the bed-material used is coarser and somewhat less dense than that in the river. Powdered lignite has been found to give good results. At the same time the slope of the bed is increased so that its gradient is relatively greater than that corresponding to the distortion of scale. This is found to prevent the formation of sand ripples which are formed with some combinations of grain-size and velocity of flow. The correct gradient for any size or density of bed-material is determined by experiment.

Much work has been done on river models during the last decade in the U.S. Waterways Experiment Station at Vicksburg, Mississippi, and in continental stations. Here a technique has been evolved based on the validity of the relationship known as DuBoys's law,

$$T = wds,$$

where T denotes the tractive force in lbs. per square foot exerted by flowing water on its bed, w the weight of water per cubic foot, d the depth of the water, and s the slope of the water-surface. Tests on the actual material of the river-bed are made in a flume whose slope can be varied. In the tests the slope is gradually increased, keeping the water-surface parallel to the bed of the channel until general movement of the bed-material occurs. The tractive force then calculated from DuBoys's formula is termed the " critical tractive force " for the material in question. Similar experiments are carried out for the material to be used for the model, which may be of any convenient grain-size.

Water-surface profiles from the river are then examined, and by inserting the observed slopes and the critical tractive force in the formula, the stage at which general bed-movement begins in the river is determined. The product of this depth and the vertical scale gives the stage at which similar bed-movement should begin in the model. Using this depth and the value of T for the model bed-material gives the required slope for the water-surface in the model.

Having chosen the horizontal and vertical scales, the value of the hydraulic mean radius R is computed and the appropriate velocity of flow for the model is deduced from Manning's formula

$$v = 1{\cdot}486\, R^{\frac{2}{3}} S^{\frac{1}{2}}/N,$$

where S denotes the slope and where the constant N is, as a result of experience, taken as being the same for river and model.

Then $$\frac{v_m}{v} = \left(\frac{R_m}{R}\right)^{\frac{2}{3}} \left(\frac{S_m}{S}\right)^{\frac{1}{2}},$$

giving the ratio of corresponding velocities and hence enabling the correct discharge for the model to be calculated.

This method is frankly empirical, but its application appears to have given excellent results in practice. The horizontal scales of the river-models used at Vicksburg are usually between 1 : 1,000 and 1 : 600, and the vertical exaggeration of scale about 10 : 1. The reports published from this station are of great interest as showing the agreement between results obtained from the models and those observed in the river.

Art. 252.—Comparison of Model Results with those Observed in Nature.

On the hydrodynamical side—the reproduction of currents, water-levels, tides and wave-motions—a suitable model is capable of giving results which may fairly be claimed to be accurate within the limits of experimental investigation.

The question as to how nearly quantitative agreement is to be expected regarding the movement of bed-materials, erosion, and deposition, is not so easy to answer. We know that training works, bridge piers, and other structures affect the set and velocity of the currents to an extent and in a manner which is reproduced with close accuracy in a suitable model. Inasmuch as an increased velocity causes scour and a reduced velocity causes deposition, if the bed-material is moved the movement caused by the change will be in the same general direction and of the same kind as in nature, so that it is to be expected that the same kind of configuration will result.

Much depends on how far it is possible in a model to reproduce accurately all the factors involved. Thus in an estuary model, while it is generally possible to reproduce approximately by fans the effect of any prevailing wind, it is impossible to reproduce the effect of gales whose incidence as regards duration and direction is casual, although the general effect of any one gale may be examined. While it is true that in a tidal estuary the scour of the ebb and flood currents is the predominant factor in the maintenance of the deep-water channels, and that over a long period of time where there is no prevailing gale-direction the effect of such occurrences may be expected partially to counteract each other, yet one violent gale may produce changes in an exposed estuary greater than would occur in months or even years of normal ebb and flow. For this reason anything like close detailed agreement between estuary and model over a definite period of years is not probable,

except where the estuary is comparatively sheltered and where the physical features are such as to give rise to well-defined and comparatively strong currents.

The final answer to the question can only be given when we have available the results of many model tests and corresponding results under similar conditions in nature, and until then caution must be used in predicting quantitative results.

As yet there are not a large number of such comparisons available, although they are accumulating gradually. Our own experiments at Manchester on the Severn model showed that in such a case it is possible to find a bed-material which will not only reproduce the general behaviour but will also give results which are in reasonable quantitative agreement with nature.

Much excellent work of this kind has been done of recent years at the U.S. Waterways Experiment Station, Vicksburg. Lt. H. D. Vogel,[1] comparing the results on five stretches of the Mississippi, in which spur dykes have been used to train and deepen the channel, states that the model tests have shown excellent agreement both quantitatively and qualitatively with the behaviour of the river.

The official opinion of the staff of the Experiment Station as to the value of such tests, based on their own experience, is expressed in the following quotation from a recent report :[2] " It has been clearly shown that reliance can be placed on the small-scale hydraulic model to simulate accurately the condition of nature in any particular stretch of river, and that the model can be used as a rational basis for the study of the effects on the river of any improvement works."

It is perhaps in the type of investigation where a comparison of the *relative* effect of different schemes is required that the scale model can be used with most confidence. Thus, in a model for investigating the scour at the toe of a projected spillway, even though conditions are such that the actual depth and length of scour are not reproduced with any great accuracy, the relative merits of different designs in reducing scour are likely to be indicated accurately. The same remarks apply to an investigation of the effect of different schemes of training works in a river or estuary.

I think it is not overstating the case to say that in expert hands such models are capable of giving results whose interpretation enables quantitative values to be predetermined with sufficient accuracy for most practical engineering purposes. Much work is now being done in the hydraulic

[1] " Hydraulic Laboratory Results and Their Verification in Nature." *Proc. Am. Soc. C.E.*, Vol. 61 (1935), p. 57.

[2] Paper No. 15, U.S. Waterways Experiment Station, Vicksburg, Jan. 1934 (p. 57).

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