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TECHNICAL ELECTRICITY

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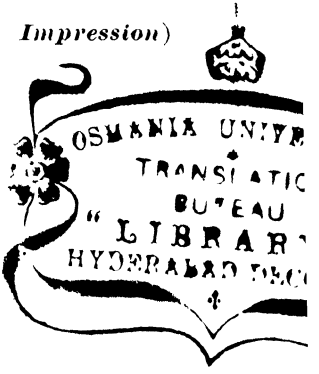
LATE PRINCIPAL OF THE MUNICIPAL TECHNICAL SCHOOL, SMETHWICK

Fourth Edition (*Eleventh Impression*)



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PREFACE TO THE FIRST EDITION.

THE appearance of a new elementary text-book on the subject of Electricity seems to necessitate some explanation. In this case the work is, as far as the scope permits, a somewhat unusual combination of scientific principles and engineering practice—a combination which aims at spanning the gulf which too often divides pure theory and practical engineering. The authors are confident that this will meet with the approval of engineers, teachers, and students, and will be of considerable service.

The main object of the book is to give the reader a thorough working acquaintance with the elements of practical electrical engineering, and to base his knowledge, from the outset, on *sound scientific principles*. To this end, no previous knowledge of the subject is assumed, and a brief but sufficient account of all the necessary theory is given *in a rational and scientific way*, which will neither conflict with the deas of the pure Physicist, nor result in the practical student having to discard his fundamental ideas as his studies lead him into the higher parts of the subject.

The difficulty encountered by beginners with the various units and systems of units has been borne in mind throughout. In the first chapter the leading principles of mechanics and the mechanical units are dealt with, and along with the latter are placed the corresponding electrical units of energy and power, thereby avoiding, in the mind of the student, the too common confusion which arises at a later stage of the work. In the electrical sections, too, the absolute electrostatic and electromagnetic units and the practical units are always mentioned together and their relationships indicated, so that students may experience no difficulty in problems necessitating a change from one system to another. In short, the extreme care which has been devoted to "units" throughout the book will, it is hoped, lessen considerably the average student's difficulties in this connection.

Considerable attention has been devoted to the construction and use of *modern* laboratory and commercial instruments, the calibration of ammeters and voltmeters, lamp-testing and the like. In all cases the scientific principles have been kept in the foreground, and at the same time a great deal of space has been given to such very practical subjects as house-wiring, general distribution, etc. The instruments and accessories dealt with are *thoroughly up-to-date* and *among the best of their class*. The practical working and upkeep of the apparatus described have been fully commented upon. The leading rules of the Institution of Electrical Engineers have been incorporated throughout the book.

H. T. D.
R. W. H.

PREFACE TO THE FOURTH EDITION.

THE demand for a Fourth Edition of this book indicates that the original purpose, outlined in the Preface to the First Edition, has met with approval: in fact the work is now so well known to teachers and students that little further need be added in the way of explanation here.

The specific aims, the essential plan and the general treatment of the subject matter of this edition are identical with those of its predecessors, but brief attention may be drawn to the following points.

The book has been entirely re-set, the employment of a larger page enabling many improvements to be made in subject matter, diagrams, type, etc.

A number of sections throughout the book have been modified, extended, or entirely re-written on the lines of modern theory and in conformity with modern engineering practice.

A large quantity of new matter has been incorporated, more particularly in connection with Electrostatic and Magnetic Theory, Lines of Force and Induction, Testing the Magnetic Properties of Iron, True, International and Legal Units, Applications of Kirchhoff's Laws, Electrical Instruments and Testing, Metallic Filament and other Lamps, Illumination, Electromagnetic Theory, the Magnetic Circuit, Generators, Armature Winding, Armature Reaction, Motors, Boosters, Alternating Currents, Alternators and A.C. Motors, Central Station Equipment, Telegraphy and Telephony, "Wireless," etc.

The characteristic features of the book—*e.g.* the great care devoted to sound scientific theory, to accurate definitions and to the various units—are maintained and emphasised in this new edition. Much attention has been given to electrical and magnetic calculations. The book contains numerous worked examples of a practical character and also problems of a similar type to be worked by the student.

Over 400 diagrams (as against 270 in the first edition) of a kind best suited to aid the student in obtaining a clear understanding of the principles and practical applications are inserted throughout the book. The more important symbols recommended by the International Electro-Technical Commission have been adopted.

Incidentally, no apology is needed for the introduction at quite an early stage of the ideas of permeability, hysteresis, etc., and the determination of the magnetic properties of iron which are, in most books, relegated to the later chapters: these terms and tests are now in daily use, the student will repeatedly encounter them in his reading, and experience shows that an early elementary treatment is alike possible and desirable, minute details of theory and experiment being postponed to a later stage.

The book is intended for first and second year students in electrical engineering at the Technical Schools and Colleges and at the Universities, for candidates for the various professional examinations, for students of Physics, and for shift engineers and test-room staffs. For these it is believed that a careful and systematic study of the book will furnish a sound scientific basis, an accurate insight into fundamental facts and principles and a knowledge of practical applications from which they will be able, profitably, to proceed to the more specialised branches of the subject.

Physics and Engineering students alike should find the book of considerable service in their endeavours to span the gulf which in the past has too decisively divided "Magnetism and Electricity" and "Electrical Engineering," and neither the pure Physicist nor the practical Engineer can really afford to tolerate that gulf to-day. If the teacher of Physics could but work on the assumption (perhaps wrong of course) that many of his students were destined to become Electrical Engineers, and if the teacher of Electrical Engineering could but imagine (perhaps equally wrongly) that many of his students were destined for the domain of Physics and investigations into the yet unknown tracks of physical science, the students themselves would be all the better for the misunderstanding.

Our best thanks are due to Prof. W. Cramp, D.Sc. of Birmingham University, for valuable suggestions and criticism in connection with certain portions of the book; much also is due to the suggestions criticism and help of intimate local colleagues and teacher friends in various parts of the country. Thanks are also due to Mr. Sydney

Rentall, M.I.E.E.—the genial and respected editor of *Electricity*—to Mr. F. Crowter of the Chloride Electric Storage Co., and finally to various firms—too numerous to mention here but whose names are given in connection with their specific manufactures—who have most willingly, and in some cases at no small trouble, supplied blocks for the printing of diagrams.

We are also greatly indebted to the Examining Board of the City and Guilds of London Institute for permission to reproduce the various examination questions of the Institute throughout the book.

R. W. H.
H. T. D.

July 1922.

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PART I.—DYNAMICS.

CHAPTER I.

DYNAMICAL DEFINITIONS AND UNITS.

1. "Unit" and "Measure."—To ascertain the magnitude of any quantity, it is necessary to select some definite quantity of the same kind as a unit, and then proceed to find how many times the unit is contained in the quantity to be measured. Thus every physical quantity is expressed by a phrase consisting of two factors: (1) the "measure" or numerical portion, and (2) the "unit"; *e.g.* 3 yards (unit = 1 yard), 6 pounds (unit = 1 pound), etc.

If the magnitude of the unit be changed, the measure of a given quantity will change *inversely*. Thus the distance 3 yards may be expressed as 9 feet (unit = 1 foot), 108 inches (unit = 1 inch), etc., from which it is clear that a decrease in the value of the selected unit results in a corresponding increase in the measure of any chosen quantity, and *vice versa*. In deciding, then, upon an arbitrary unit, it is imperative that attention should be paid to the average magnitudes of the quantity involved, so that the measures in daily use may be neither too large nor too small. Further, the unit selected should be such that its magnitude is not liable to temporary or permanent change, and a really satisfactory system will be one in which some definite and simple relation exists between the units of various kinds and in which reduction from one unit to another of the same kind can be carried out by simple arithmetical operations. The reader will better realise the force of these statements as his studies proceed.

2. Fundamental Units: The F.P.S. System.—All physical quantities are derived from three fundamental units, viz. **length, mass,** and **time**; these are truly fundamental, for their definition in terms of any quantity simpler than themselves is impossible.

The British unit of length is the *yard*, but the most convenient one for scientific purposes is the *foot*, which is defined as one-third of a yard, the latter being the distance at 62° F. between two lines on a particular bronze bar deposited at the Standards Office of the Board of Trade. For greater distances the mile (1,760 yards), the furlong (220 yards), and other multiples are employed, while in specifications

relating to telegraph operations the terms "**nautical mile,**" and "**telegraph naut,**" are used to designate a distance of 2,029 yards or 1·1528 statute miles of 1,760 yards. Smaller lengths are expressed in inches or fractions of an inch, a common one used in connection with the diameters of wires being the **mil** ($\frac{1}{1000}$ inch). The square foot and cubic foot are accepted as units of area and volume respectively. A **circular mil** is the area of a circle of diameter one mil.

The *pound avoirdupois* is the British mass unit, and is defined as the quantity of matter in a platinum cylinder deposited at the offices of the Board of Trade. The one-sixteenth part of it is termed an ounce, and the one-seven-thousandth part of it a grain. As multiples, the stone (14 pounds), the hundredweight (112 pounds), and the ton (2,240 pounds) are in daily use.

Noon is the precise instant when the sun reaches its highest altitude, and an *apparent solar day* is the interval of time from one noon to the next. These are unequal in duration; but a *mean solar day* is obtained by taking the sum of the lengths of all the apparent days in a year and dividing by the number of days in the year. This mean solar day is again divided into $24 \times 60 \times 60$ equal parts each termed a *second*, which is adopted as time unit.

The system of units based on the foot, pound, and second is known as the **British,** the **foot-pound-second** or **F.P.S.** system. The inconvenience of it is apparent.

3. Fundamental Units: the Metric and C.G.S. Systems.—In the French or Metric system the unit of length is the *metre*, defined originally to be the 10,000,000th part of the earth's meridian through Paris from the Pole to the Equator, and represented by the distance at 0° C. between two marks on a platinum bar deposited in the French archives. Multiples and submultiples are obtained by repeatedly multiplying and dividing by ten. The one-hundredth part of the metre, termed the **centimetre,** is the length unit adopted in scientific work. As units of area and volume the square centimetre and cubic centimetre are employed; but the true metric area unit is the are (100 sq. metres), and the true volume unit the litre (1,000 cubic centimetres).

The metric unit of mass is the *kilogramme* but a more convenient one is the **gramme,** defined as the mass of a cubic centimetre of distilled water at its temperature of greatest density (about 4° C.), or as the one-thousandth part of the mass of a platinum cylinder (the kilogramme) deposited in the French archives. Multiples and sub-multiples of the gramme are obtained by repeatedly multiplying and dividing by ten.

A mean solar second is the metric time unit.

The system of units based on the centimetre, gramme, and second is known as the **centimetre-gramme-second** or **C.G.S.** system, and is almost universally adopted for scientific purposes

4. Metric Tables and Conversion Table.—The multiples and submultiples of the metric units are indicated by the following prefixes:—

Kilo	Hecto	Deka	Unit	deci	centi	milli
1000	100	10		$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$

Note also: **Meg** or **Mega** = 1,000,000, *e.g.* megohm = 1,000,000 ohms; **micro** = $\frac{1}{1000000}$th, *e.g.* microvolt = $\frac{1}{1000000}$ volt. [**Myria** = 10,000.]

Tables.

1. Length.

1 kilometre (Km.) = 1000 metres
1 hectometre (Hm.) = 100 ,,
1 dekametre (Dm.) = 10 ,,
metre (m.)
1 decimetre (dm.) = $\frac{1}{10}$ metre
1 centimetre (cm.) = $\frac{1}{100}$,,
1 millimetre (mm.) = $\frac{1}{1000}$,,

2. Mass.

1 kilogramme (Kg.) = 1000 grammes
1 hectogramme (Hg.) = 100 ,,
1 dekagramme (Dg.) = 10 ,,
gramme (*g. or* grm.)
1 decigramme (dg.) = $\frac{1}{10}$ gramme
1 centigramme (cg.) = $\frac{1}{100}$,,
1 milligramme (mg.) = $\frac{1}{1000}$,,

3. Common Areas.

(10^2 =) 100 sq. mm. = 1 sq. cm.
,, 100 sq. cm. = 1 sq. dm.
,, 100 sq. dm. = 1 sq. m.
etc.

4. Common Volumes.

(10^3 =) 1000 c.mm. = 1 c.cm.
,, 1000 c.cm. = 1 c.dm. (1 litre)
,, 1000 c.dm. = 1 c.m.
etc.

Exercise.—Write out the metric "capacity" table (unit = 1 litre).

Conversion Table.

Units of	British to C.G.S.	C.G.S. to British.
1. Length. .	1 inch = 2·54 cm.	1 cm. = ·3937 inch
	1 foot = 30·4797 cm.	1 metre = 39·37 inches
	1 mile = 1609·3 metres	1 kilometre = ·62138 mile
2. Mass . .	1 grain = ·0648 gramme	1 gramme = 15·432 grains
	1 ounce = 28·3490 grm.	,, = ·035274 ounce
	1 pound = 453·59 ,,	,, = ·0022046 lb.
3. Area . .	1 sq. inch = 6·4515 sq. cm.	1 sq. cm. = ·155 sq. inch
4. Volume .	1 cub. inch = 16·387 c. cm.	1 c.cm. = ·061 c. inch
	1 cub. foot = 28316 ,,	1 litre = 61·027 c.inches

In Fig. 1 the upper portion represents a scale of three inches, and the lower portion a scale of eight centimetres, the first of which is divided into ten millimetres.

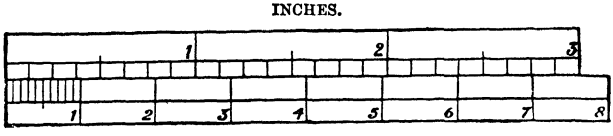

Fig. 1.

5. Advantages of the Metric System.—

(1) The ratios of the various units are all powers of ten, hence the cumbersome multiplications and divisions necessary in reducing from one unit to another in the British system are entirely avoided, and reduction is effected merely by the proper arrangement of a decimal point. Thus: 4 Km. 3 Hm. 2 Dm. 7 m. 1 dm. 6 cm. 3 mm. = 4,327·163 metres = 432,716·3 cm. = 4,327,163 mm.

(2) Being a decimal system, it is uniform with our ordinary arithmetical notation, and one general "table" takes the place of many in the British system.

(3) The various units bear a simple relation to each other. Thus, knowing that 1 c.cm. of water at 4° C. = 1 grm. (approx.), the mass of water can be stated at once if the volume be known (*e.g.* 12 kilolitres of water = 12,000 kilogrammes). In the F.P.S. system inconvenient constants must be remembered, *e.g.* that 1 quart = 69·278 cub. inches, 1 cub. ft. of water = 62·4 pounds, etc.

(4) It is international in character, and almost universally in use among civilised nations.

6. Velocity.—Velocity is rate of motion, and, when uniform, is measured by the distance travelled in unit time. If the rate of motion is not uniform, the velocity at any instant is the distance that would be passed over in unit time if the velocity were to remain constant for that period and equal to that at the particular instant considered. A body has unit velocity if it traverses unit distance in unit time. Hence—

The **F.P.S.** unit is **one foot per second.**	The **C.G.S.** unit is **one centimetre per second.**

7. Acceleration.—Acceleration is rate of change of velocity. Thus if a body be moving at any particular instant with a velocity of 10 ft. per second, and if, one second later, its velocity is 13 ft. per second, the increase in velocity *in that second* is 3 *ft. per second*, and the acceleration is stated to be 3 *ft. per second per second.* Similarly, if in one second the velocity changes from 20 ft. per second to 16 ft. per second, the negative acceleration or retardation is 4 ft. per second

per second. A heavy body placed some distance above the ground and allowed to fall freely from rest, will at the end of the first, second, third, etc., seconds have acquired velocities of 32·2, 64·4, 96·6, etc. ft. per second (approximately); thus the increase in velocity each second is 32·2 ft. per second, or *the uniform acceleration of the falling body is 32·2 ft. per second per second.* Employing the C.G.S. length unit, this is equivalent to 981 cm. per second per second. A body has unit acceleration if its velocity changes by unity in unit time. Hence—

The **F.P.S.** unit is **one foot per second per second.**	The **C.G.S.** unit is **one centimetre per second per second.**

8. Momentum.—The momentum of a body is a quantity measured by the product of its mass and its velocity. If m is the mass in pounds of a moving body, v its velocity in feet per second, and M its momentum,

$$M = mv \text{ (F.P.S. units).}$$

If m be in grammes and v in centimetres per second the momentum M will be expressed in C.G S. units. A body has unit momentum if the product of its mass and its velocity be unity. Hence—

The **F.P.S.** unit is the momentum of **one pound mass moving with a velocity of one foot per second.**	The **C.G.S.** unit is the momentum of **one gramme mass moving with a velocity of one cm. per second.**

9. Force and its Absolute Units.—The whole science of Dynamics is based on three **Laws of Motion** stated by Newton in 1687 and enunciated as follows, the original wording being somewhat altered, the ~~better~~ to meet our present requirements:

Law I. A body continues in its state of rest, or of uniform motion in a straight line, except in so far as it be compelled by impressed force to change that state.

Law II. The rate of change of momentum is proportional to the impressed force, and takes place in the direction of the straight line in which the force acts.

Law III. To every action there is an equal and opposite reaction.

The first law provides us with a definition of force. **Force is that which alters, or tends to alter, a body's state of rest, or of uniform motion in a straight line.**

The second law leads to a means of measuring force; thus let P be

the force acting on a body, m its mass, and f the acceleration produced by the force; by the second law—

$$\begin{aligned} \text{P} &\propto \text{rate of change of momentum,} \\ &\propto \text{rate of change of } mv, \\ &\propto m \times \text{rate of change of } v, \\ &\propto m \times f, \end{aligned}$$

i.e. $\text{P} = kmf$ where k is a constant.

Now let us agree that if $m = 1$ and $f = 1$ the particular force acting shall be our unit force. Putting P, m and f each equal to unity, $k = 1$ and the statement above becomes—

$$\text{P} = mf.$$

i.e. (No. of units of force) = (No. of units of mass) × (No. of units of acceleration).

Clearly, then, *unit force* may be defined as:

(1) *That force which acting on unit mass produces unit acceleration.*

(2) *That force which acting for one second on unit mass originally at rest generates in it unit velocity.*

(3) *That force which acting on any mass for one second gives to it unit momentum in the direction of the force.* This is evident, for in one second the momentum in the direction of the force is mf, and if this be unity, P is unity.

Hence—

The **F.P.S.** unit is the **poundal,** defined as **that force which acting on one pound mass for one second gives it a velocity of one foot per second.**

The **C.G.S.** unit is the **dyne,** defined as **that force which acting on one gramme mass for one second gives it a velocity of one centimetre per second.**

One poundal = 13,825 dynes.

10. Mass and Weight: the Gravitational Units of Force.—The mass of a body is the quantity of matter in it, and is therefore the same everywhere. **The weight of a body is the force with which it is attracted by the earth,** and as this attraction varies from one locality to another, being greater at the Poles than at the Equator and greater at the sea-level than above or below that level, it is clear that the weight of a body is not the same at all places. At the centre of the earth the *weight* of a body would be *nil,* for the earth would not attract it; but its *mass* would be the same as at any other place.

It has been proved that providing the resistance of the air be removed, all bodies will fall from rest at a height above the ground through equal vertical distances in equal times. Hence their velocities at any instant must be the same, and therefore their accelerations must be the same. This acceleration, which is constant for all bodies at the same place, is called the *intensity of gravity*, and is denoted by the letter g. The value of g differs at different localities, being about 32·255 ft. per second per second or 983·1 cm. per second per second at the Poles, and 32·091 ft. per second per second or 978·1 cm. per second per second at the Equator. For calculation purposes, g in England may be regarded as equal to 32·2 ft. per second per second or 981 cm. per second per second.

In this country, then, a mass of one pound falls vertically with an acceleration of 32·2 ft. per second per second, and the force producing this acceleration is the body's weight. But a force of one poundal produces in a mass of one pound an acceleration of one foot per second per second. Hence a **force equal to the weight of a pound is equivalent to 32·2 (*i.e.* g) poundals.**

Similarly a mass of one gramme falls with an acceleration of 981 cm. per second per second, and the force acting on it is its weight. But a force of one dyne produces in a mass of one gramme an acceleration of one cm. per second per second. Hence **a force equal to the weight of a gramme is equivalent to 981 (*i.e.* g) dynes.**

The poundal and the dyne are termed "*absolute*" units of force inasmuch as they are based on the fundamental units and their value is the same everywhere. The "*weight of a pound*" and the "*weight of a gramme*" are called the *F.P.S.* and *C.G.S.* "*gravitational*" *units of force* respectively; their value is affected by the intensity of gravity. Gravitational units may be converted into absolute units by multiplying by the value of g, and absolute into gravitational by dividing by g. Thus—

m pounds weight $= mg$ poundals ($g = 32{\cdot}2$ in England).

m grammes weight $= mg$ dynes ($g = 981$ in England).

11. Work and its Units.—

A force does work when its point of application moves in the direction in which the force acts; work is done against a force when the point of application moves in the opposite direction to that in which the force acts.

The idea enunciated clearly involves two factors, *force* and *distance*, and in fact the measure of the work is the product of the force and the distance through which the point of application moves in the line of action of the force;

i.e. **Work = Force × Distance.**

If the actual displacement be inclined to the direction of the acting force, "the distance moved in the line of action of the force" refers to the projection of the displacement on the line of action of the force. Thus if a force P (Fig. 2) acts in the direction A B, and the point of application moves from A to C, the work done by the force is P × A D.

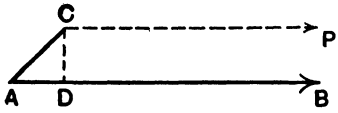

Fig. 2.

From the definition it is apparent that if the distance moved be zero, no work is done however great the applied force. A stone resting on a table, a man supporting a weight without moving it, or pushing a heavy body which he is unable to move, are examples of force applied but no performance of work. Further, if the point of application moves in a direction perpendicular to that of the force the latter does no work, for the point D (Fig. 2) coincides with A, and A D is zero; thus no work is done *by gravity* when a body moves along a horizontal plane.

An absolute unit of work is done by the absolute unit of force in moving its point of application unit distance. Hence—

The **F.P.S.** unit is the **work done by one poundal in moving its point of application one foot.** It is called a **foot-poundal.**

The **C.G.S.** unit is the **work done by one dyne in moving its point of application one centimetre.** It is called an **erg.**

1 ft.-poundal = 421,390 ergs.

The erg being very small, three additional work (or energy) units are employed in electrical engineering, viz.:

(1) The **Joule** = 10 million (*i.e.* 10^7) ergs;

(2) The **Watt-hour** = 3,600 joules = (3,600 × 10^7) ergs;

(3) The **Kilowatt-hour** = 3,600,000 joules = (1,000 × 3,600 × 10^7) ergs; the last named being the legal supply unit fixed by the Board of Trade (B.O.T. unit or B.T.U.). These will receive exact definition in Chap. X. Still another unit—the *calorie*—is largely used in the science of Heat; it is equal to 42,000,000 ergs.

The F.P.S. gravitational unit of work is the **foot-pound,** defined as **the work done in raising one pound mass vertically one foot.** If m pounds mass be raised vertically h feet, the work done is mh foot-pounds. Since "one pound weight" = 32·2 poundals, it is clear that a foot-pound of work = 32·2 foot-poundals.

The C.G.S. gravitational unit of work is the **gramme centimetre,** which is **the work done in raising one gramme mass vertically one centimetre.** Since "one gramme weight" = 981 dynes, a gramme centimetre of work = 981 ergs.

Owing to the smallness of the gramme centimetre, another unit—the *kilogrammetre*—is more frequently employed; it is the work done in raising one kilogramme vertically one metre. Since 1 Kg. = 1,000 grm. and $1m = 100$ cm., the kilogrammetre = (1,000 × 100) = 100,000 grm. cm.

12. Power or Activity and its Units.—By the **"power"** of an agent, say a dynamo or motor, is meant its **rate of doing work,** *i.e.* **the amount of work it can do in unit time.**

The F.P.S. absolute unit is the power of an agent which performs one foot-poundal of work per second.

The C.G.S. absolute unit is the power of an agent capable of performing one erg of work per second. This being very small, two additional power units are employed in electrical engineering, viz.: the **Watt** and **Kilowatt** (1,000 watts). The watt is equivalent to 10,000,000 (*i.e.* 10^7) ergs per second, and since the joule = 10^7 ergs, **the watt may be defined as the power of an agent which performs one joule of work per second** (Chap. X.).

A **"Horse-power"** is the British gravitational power unit. **An agent capable of performing 550 foot-pounds of work per second or 33,000 foot-pounds per minute is said to be of one horse-power.** Thus if the actual horse-power of a motor be four, it is able to do 2,200 foot-pounds of work per second.

The metric gravitational unit is the **Force de cheval.** It is equivalent to 75 kilogrammetres per second or about 542 foot-pounds per second, and is therefore somewhat less than the British h.-p.

The important relation between the h.-p. and Watt may thus be established:

1 lb. = 454 grm. (approx.); 1 ft. = 30·5 cm. nearly.

∴ 1 pound weight = (454 × 981) = 445,000 dynes approx.

Now—

1 foot-pound = work in overcoming 1 pound weight through 1 foot,

" " = work in overcoming 445,000 dynes through 30·5 cm.,

" " = 13,570,000 ergs nearly;

∴ 550 foot-pounds = (13,570,000 × 550)

= (746 × 10^7) ergs.

Hence 1 h.-p. = 550 foot-pounds per second = 746 × 10^7 ergs per second.

= 746 watts, since the watt = 10^7 ergs per second.

Thus **1 h.-p. = 746 watts;**

∴ 1 watt = $\frac{1}{746}$ h.-p.

and **1 kilowatt = $\frac{1000}{746}$ = 1·34 h.-p.**

The distinction between "work" and "power" should be carefully noted. Power is rate of doing work, and is therefore of the nature of work divided by time; *it involves a time unit*: *e.g.* 1 h.-p. = 550 foot-pounds *per second*; 1 watt = 10^7 ergs *per second*, etc. Stated algebraically—

$$\text{Power} = \frac{\text{Work}}{\text{Time}} \text{ and Work} = \text{Power} \times \text{Time}.$$

Thus the "watt" and "kilowatt" are units of power; but the "watt-hour" and "kilowatt-hour," being products of "rates of working" and "time intervals," are work units.

13. Energy.—Energy is capacity to do work; it is measured in work units: foot-pounds, joules, calories, watt-hours, kilowatt-hours, etc.

While a body is in motion it can do work, *i.e.* it possesses energy. Energy in this form is termed *kinetic energy*.

A mass raised to a height above the earth is endowed with energy, for by a suitable arrangement it can, in changing its position, raise another body from the ground and thus do work. The energy of a body due to its position relative to surrounding bodies, or due to the positions of its parts, is called *potential energy*.

In distorting or coiling a spring work must be done in changing the relative positions of its parts; the distorted spring possesses energy (potential) and can do work in returning to its normal position. Work must be done in charging a body with electricity, and the charged body is endowed with energy (potential), for it can do work in attracting other bodies to it, or a spark may be obtained from it, in which case the energy is partly converted into heat energy, part is utilised in setting up the ether vibrations which constitute light, and part gives rise to sound. A piece of coal possesses energy (potential) due to its tendency to combine with oxygen, for when the combination takes place heat, light, and mechanical work are obtained therefrom. Kinetic energy is possessed by a moving cannon ball, and a considerable amount of work can be done in the arrest of its motion.

Owing to the tendency of zinc to combine with sulphuric acid, a piece of zinc in sulphuric acid resembles the coal and the elevated mass previously mentioned. They possess energy (potential), and when the combination occurs heat energy is the result, or by a suitable arrangement part of the energy may be utilised in driving a current of electricity, which in turn may be made to drive a motor and to do mechanical work. In all cases, whether we speak of electrical energy, chemical energy, radiant energy, gravitational energy or heat, it must be remembered that the energies are of the same essential character, and are measured in the same work units.

It is proved in works on Mechanics that if m be the mass of a moving body in pounds and v its velocity in feet per second, its kinetic energy is given by the expression:

$$\textbf{K.E.} = \tfrac{1}{2}\,mv^2 \textbf{ ft.-poundals} = \tfrac{1}{2}\frac{mv^2}{g} \textbf{ ft.-pounds } (g = 32{\cdot}2).$$

If the mass be in grammes and the velocity in cm. per second,

$$\textbf{K.E.} = \tfrac{1}{2}\,mv^2 \textbf{ ergs} = \tfrac{1}{2}\frac{mv^2}{g} \textbf{ grm. cm. } (g = 981).$$

Again, if m be the mass of a body in pounds and h its height above the ground in feet, its potential energy is given by—

$$\textbf{P.E.} = mgh \textbf{ ft.-poundals} = mh \textbf{ ft.-pounds } (g = 32{\cdot}2).$$

If m be in grammes and h in centimetres,

$$\textbf{P.E.} = mgh \textbf{ ergs} = mh. \textbf{ grm. cm. } (g = 981).$$

It is clear from these expressions that if a heavy body and a light one strike an obstacle with equal velocities the heavy one will penetrate to a greater depth, for having greater kinetic energy owing to its greater mass it will do more work in coming to rest. Similarly if a leaden bullet and a wooden one of the same size be projected through the air with equal initial velocities, the leaden one will travel to a greater distance, for it will do more work in coming to rest, and therefore overcome the retarding force due to the air through a greater distance.

14. The Principle of Conservation of Energy.—In simple language this may be enunciated as follows:

The total energy of the universe is constant: energy cannot be created and cannot be destroyed; it can only be changed from one form into another.

Thus we realise the true function of a dynamo or any other machine. It does not create energy, but is merely a device for transforming one form into another more suited to the purpose in view; and in the transformation a certain amount is degraded—not destroyed—but converted in the bearings, etc., into heat energy, which is less available for useful purposes.

EXAMPLES.

1. *Find the number of dynes in a poundal.*

1 pound = 453·59 grm. 1 foot = 30·48 cm.

1 poundal acting on 1 pound for 1 second gives a velocity of 1 ft. per second.

∴ 1 poundal acting on 453·59 grm. for 1 second gives a velocity of 30·48 cm. per second.

Hence the momentum generated by 1 poundal in 1 second

$$= 453{\cdot}59 \times 30{\cdot}48 = 13{,}825 \text{ C.G.S. units.}$$

But the momentum generated by 1 dyne in 1 second = 1 C.G.S. unit.

∴ **1 poundal = 13,825 dynes** (Art. 9).

2. *Find the number of ergs in a foot-poundal.*

1 foot-poundal = work done by 1 poundal in moving 1 foot
= work done by 13,825 dynes in moving 30·48 cm.
= 13,825 × 30·48 = 421,390 ergs (approx.).

1 foot-poundal = 421,390 ergs (Art. 11).

3. *Find the number of foot-pounds in a kilowatt-hour.*

By Art. 12, 1 kilowatt = 1·34 h.-p. = (1·34 × 550) foot-pounds per second.

Now—

Work = Power × Time in seconds.

∴ 1 kilowatt-hour = (1·34 × 550) × (60 × 60) foot-pounds.

∴ **1 kilowatt-hour = 2,653,200 foot-pounds.**

4. *How many* 16 *candle-power lamps taking* 3·5 *watts per candle can be run from an* 18*-kilowatt machine ?*

Watts absorbed by each lamp = 3·5 × 16 = 56.

Output of dynamo in watts = 18 × 1,000 = 18,000.

$$\therefore \textbf{Number of lamps} = \frac{\textbf{18000}}{\textbf{56}} = \textbf{321}.$$

5. *An electric car whose velocity is* 10 *miles an hour is brought to rest in* 12 *yards when the brakes are applied. If the mass of the car be* 10 *tons, find the average retarding force due to the brakes.*

Mass = 10 tons = 22,400 pounds (m).

Velocity = 10 miles per hour = 52,800 feet per hour

$$= \frac{52800}{3600} = \frac{44}{3} \text{ feet per second } (v).$$

$$\therefore \text{K.E.} = \frac{1}{2}\frac{mv^2}{g} \text{ foot-pounds} = \frac{1}{2} \times \frac{22400}{32} \times \frac{44}{3} \times \frac{44}{3}$$

= 75289 foot-pounds (approx.).

Hence work done against brakes = K.E. destroyed = 75,289 foot-pounds, *i.e.* retarding force × distance in feet = 75,289.

$$\therefore \textbf{Average retarding force} = \frac{\textbf{75289}}{\textbf{12} \times \textbf{3}} = \textbf{2,091·36 pounds weight.}$$

NOTE.—For simplicity g is here taken = 32 feet per second per second.

EXERCISES I.

The letter (B.) is put after questions taken from the late "Lower" or Second Stage Examinations in Magnetism and Electricity of the Board of Education. The letters (P.), (O.), (H.) denote respectively the Grade 1, Grade 2, and Final (late Preliminary, Ordinary and Honours) Examinations in Electrical Engineering of the City and Guilds of London. Questions taken from other examinations (the Universities, the Institution of Electrical Engineers, etc.) are fully indicated.

1. Give a list of the prefixes employed with metric units, showing by how much they increase or decrease the value of the unit. Explain the advantages of the metric system.

2. Distinguish between "mass" and "weight." Explain how the weight of a substance as determined by a pair of scales is the same everywhere, while it will vary if a spring balance be employed.

3. Define the various absolute and gravitational units of force, work, energy, and power.

4. Mention the quantities of which the following are the units: watt, dyne, joule, calorie, kilowatt, horse-power, kilowatt-hour, poundal, watt-hour, foot-pound, erg, force de cheval.

5. Why can a leaden bullet be fired further than a piece of wood of the same size and shape? (P.)

6. If a man supports a weight of 10 lb. for 17 minutes, does he do any work? If so, how much? (P.)

7. Why is energy expended in bending or coiling a spring? (P.)

8. What are foot-pound, poundal, dyne and kilogrammetre? How are they related? (O.)

9. Explain numerically the relations between the dyne, the erg, the joule, the kilogramme, the British h.-p. and the Metric h.-p. (O.)

10. Define force, work, energy, and power. Is a kilowatt a unit of work or a unit of power? Of what quantity is a kilowatt-hour the unit?

11. If a 16 candle-power lamp takes 4 watts per candle, how many such lamps can be run from a 16 unit (kilowatt) machine?

12. How many foot-pounds of work will be done in 20 minutes by a motor working at 4 brake h.-p.? (P.)

13. How many 16 candle-power lamps taking 4 watts per candle can be run off a dynamo giving 7 electrical h.-p.? (P.)

14. How many (electrical) h.-p. are needed to drive a dynamo supplying 120 16 candle-power lamps if the lamps absorb $3\frac{3}{4}$ watts per candle?

15. An arc lamp takes 10 amperes at 50 volts, which is equivalent to 500 watts and gives a mean candle-power of 550; how many candles per h.-p. is this equivalent to?

16. Determine: (*a*) the number of foot-pounds in a joule; (*b*) the number of gramme centimetres in a foot-pound; (*c*) the number of watts in a "force de cheval."

17. Prove:—(*a*) that a B.O.T. unit = 3,600,000 joules; (*b*) that a calorie = 3·968 watt-hours; (*c*) that a B.O.T. unit = 1·34 h.-p. hour.

18. Define one h.-p. If 1 foot = 30·48 cm., and 1 pound = 453·6 grm., calculate the number of ergs equal to one foot-pound at a place where the acceleration of gravity = 981 cm. per second per second. Lastly, calculate the number of ergs per second equal to one h.-p. (O.)

19. Find the h.-p. required to keep a car weighing 10 tons moving on the level at 7 miles an hour if the resistance to traction is 25 pounds per ton.

PART II.—ELECTROSTATICS.

CHAPTER II.

GENERAL PRINCIPLES.

15. Electrification by Friction: the Two States of Electrification.—The ancients were acquainted with the fact that pieces of amber and jet possessed, when rubbed, the property of attracting light bodies, and it is from the Greek name *ēlektron* (amber) that our word "electricity" is derived. It is now known that, with proper precautions, all substances suitably rubbed will attract to some extent such light articles as pieces of paper, bran, cork, pith, etc.; glass rubbed with silk, sealing-wax with flannel, and vulcanite with fur, do so in a marked degree, and particularly the last named. A substance endowed with this property is said *to be electrified, to be excited, to be charged, to possess a charge* or *to be in a state of electrification*, and the unknown mysterious agent which is the cause of this state is named "*electricity*." Bodies not endowed with the property mentioned are said to be *neutral*.

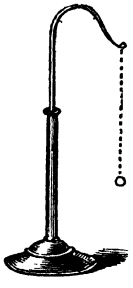

Fig. 3.

If a glass rod which has been rubbed with silk be brought near a ball of elder pith suspended by a dry silk thread from a suitable stand (Fig. 3), the pith-ball will be attracted to the rod, and, after being in contact for a second, will be strongly repelled, from which it may be surmised that the ball becomes charged on touching the rod, and that when a neutral body takes a portion of the charge from an electrified one the two no longer attract, but repel. Again, if a vulcanite rod which has been rubbed with fur be placed in a stirrup suspended from the stand by a dry silk thread, and a second vulcanite rod also rubbed with fur be brought near, the suspended one will be repelled; but a glass rod rubbed with silk will attract it. Similarly, if the electrified glass be suspended, a second glass rod excited with silk will repel it, but the vulcanite rod rubbed with fur will produce attraction. Repeated

experiments of this character with various substances lead to the conclusions:

(1) *That there are two states of electrification.*

(2) *That bodies in similar states of electrification repel.*

(3) *That bodies in unlike states of electrification attract.*

It has been agreed to call the state of electrification exhibited by glass rubbed with silk, "*positive*," and to say that the glass is "*positively*" *charged*; it has further been agreed to name the state of electrification of vulcanite rubbed with fur, "*negative*," and to say that the vulcanite is "*negatively*" *charged*. Hence:

(1) *Positively charged bodies repel each other.*

(2) *Negatively charged bodies repel each other.*

(3) *A positively charged body and a negatively charged one attract.*

So far we have not concerned ourselves with the rubber; but experiment proves that if a vulcanite rod be fitted with a fur cap to which is attached a dry silk thread, and if on electrifying the rod, the thread and not the fur be held in the hand, on separating, the vulcanite is negatively charged and the fur positively charged. Further, if, before separating, the two be brought together near an uncharged pith-ball, the latter is not affected in the least, from which it may be gathered that the two electrifications are equal, and are neutralising each other. Numerous tests all lead to the same conclusion, viz. *that both states of electrification are established at the same time and to an equal degree.*

Whilst speculation and theory are scarcely profitable at this stage of the reader's progress, a suggested explanation (really a slight modification of Franklin's theory) may be of service as satisfactorily explaining to a beginner many well-known phenomena. It may be conceived, for example, that the vulcanite rod and the fur, when neutral, are each imbued with a definite amount of "electricity." On rubbing the two together, it may be further conceived that a certain amount passes from the rod to the rubber; thus the rod will have a "deficit," and may be said to have a "negative charge," the fur will have a "surplus," or a "positive charge," and since the surplus on the rubber has come from the rod, the two "charges" will be equal and capable of neutralising. Of course, since the terms Positive and Negative have been fixed arbitrarily, it is possible that what is termed by consent a positive charge may be the deficit and the other the surplus, but there is no means of settling the question. It should be noted, too, that according to this theory *there are not two kinds of electricity, but two states of electrification, i.e.* two electrical conditions, one due to a surplus, the other to a deficit of electricity.

To harmonise with modern thought, the original "one fluid" theory of Franklin needs much modification, because: (1) it regards electricity as a material fluid;

(2) it ignores the function of the medium surrounding the charged body. Modern theory, on the other hand, justly abandons the idea of a material fluid, and regards the medium as the seat of the energy, attributing to it a condition of strain to which the electrification of the surface of the charged body and the energy associated therewith are due. It is, however, interesting to note that save in the meaning of the terms Positive and Negative, the modified Franklin theory above approaches somewhat the latest *electron theory*, which regards an atom as consisting of a positive nucleus surrounded by small negative corpuscles or electrons, a negatively charged body as one containing a surplus of (negative) electrons and a positively charged body as one with a deficit of (negative) electrons. The even partial explanation of many electrical phenomena on strictly modern lines would, however, only confuse the reader at this stage of his progress.

The section of our subject which treats of electricity at rest and of the measurement of electric charges is called "*Electrostatics.*"

16. Conductors and Insulators.—In the infancy of the science it was discovered that, holding the bodies in the hand, some (*e.g.* glass, wax, vulcanite, sulphur, etc.) could readily be electrified by friction, whilst others (notably the metals) exhibited not a trace of electrification. The difference is due to the fact that the first group are "insulators," the second "conductors." At this stage *resistance* may be defined in a general way as that property of a body which opposes the flow of electricity. **An insulator is a substance possessing such enormous resistance that practically it does not permit electricity to flow through it; a conductor is a substance of such small resistance that it readily allows electricity to flow through it and to pass from it to other bodies.** Consider, then, a glass rod, one end held in the hand, the other rubbed with silk. The end rubbed becomes positively electrified, and the "surplus" remains there; for the rod, being an insulator, does not permit the electricity to flow along it to the hand, from whence it would readily pass to earth, since the human body is a good conductor. Similarly, the "deficit" on the vulcanite rod cannot be made up by a flow of electricity from the earth, because vulcanite is an insulator. A brass rod held in the hand and rubbed with india-rubber becomes positively charged, but the electricity immediately passes along it and down the body to the earth. By fitting the brass with a vulcanite or glass handle it can be electrified in the usual way.

The best conductors are the metals of which annealed silver is pronounced: amongst the good insulators are *dry* air, *dry* glass, paraffin wax, mica, vulcanite, shellac, india-rubber, sulphur, sealing-wax, silk, etc.

17. Changes in Conducting and Insulating Powers: Conductors in Practice: Practical Insulation.—The conducting and insulating powers of most bodies are influenced by temperature. Good

conductors (metals) become worse conductors as their temperature is raised, *i.e. the resistance they offer to the flow of electricity becomes greater.* Insulators, on the other hand, decrease considerably in insulating power when heated; *i.e. the resistance they offer to the flow of electricity becomes decidedly less,* and some, if strongly heated, may even become good conductors. For example, a rise of 10° F. increases the resistance of copper by about 2 per cent., and decreases the resistance of gutta-percha to nearly one-half its original value; a rise of 20° F. increases the copper resistance by about 4·2 per cent., and decreases the gutta-percha resistance to about one-fifth of its initial value.

Alloys do not vary in conducting power to the extent that pure metals do for the same temperature change. **Manganin,** an alloy consisting of 84 per cent. of copper, 12 per cent. of manganese, and 4 per cent. of nickel, increases slightly in resistance from 0° C. to about 35° C. and then decreases, but the variation is so small that it may be neglected in practice; hence it is largely used as a standard in testing. **German silver,** an alloy consisting of 4 parts of copper to 1 part of zinc and 2 of nickel, and **platinoid,** consisting of 59 per cent. of copper, 25·5 per cent. of zinc, 14 per cent. of nickel, and 1·5 per cent. of tungsten, also vary very little with temperature. **Carbon** becomes a better conductor, *i.e.* decreases in resistance when heated; but if the heating be continued beyond a certain limit the resistance begins to increase again (Chap. X.).

In practice **copper** and **aluminium** are mainly used as conductors of electricity; the latter has a higher resistance (volume resistivity) and lower tensile strength than copper, but in certain circumstances its lesser specific gravity renders it more suitable. Owing to their high resistance and small variation with temperature, alloys are largely used as standard resistances for testing purposes. For overhead lines hard drawn copper is employed but aluminium is coming into more extended use and, where great tensile strength is required, **bronzes** composed of copper with tin, silicon, etc., are often used.

As a practical insulator **shellac** is often employed. It is obtained from an insect which deposits its eggs on certain trees in India; these are placed in bags, heated, and the liquid squeezed out, which latter forms crude shellac. Shellac dissolved in methylated spirits produces shellac varnish; it is a very good insulator but does not withstand the action of oil. As an insulating protection for electric wires and cables, **vulcanised india-rubber,** which consists essentially of pure rubber treated with sulphur at a high temperature, is largely employed; it lasts longer and is not so much affected by temperature changes as pure rubber, but owing to the action of the sulphur upon the bare metallic conductor, a layer of pure rubber must be interposed between them.

In the manufacture of switches, fuses, etc. (see Chap. XXIII.), **porcelain** is a usual insulator, but it is easily cracked and tends to collect moisture. **Slate** and **marble** are invariably met with as

insulating material for distribution boards, switch boards, etc. (Chap. XXIII.), but in such cases care must be taken that the slate is free from metallic veins. In dynamo electric machinery, **mica** and various modifications of mica are extensively used as insulating materials. The mineral mica consists essentially of a double silicate of potassium and aluminium; it can be manufactured in very thin layers, is influenced but slightly by temperature changes and does not absorb moisture, which rapidly lessens insulation. **Micanite** and **megomit** are mechanically stronger than mica; they consist of mica sheets fixed together by insulating cement, and by subjection to intense heat can be prepared in any convenient form for practical purposes. **Micanite cloth** and **micanite paper** consist of mica cemented to cloth and paper respectively. **Presspahn,** which is pure wood fibre with an oily glaze, is also largely used for insulating purposes. **Bitumen** is much used for surrounding electric cables when laid; it can be poured round them at a temperature not much higher than that of boiling water. **Vulcanised fibre** is a well-known insulator; it is composed of vegetable fibres, chemically treated, and compounded together under very great pressure. **Ambroin** and **isolit** are insulators of recent introduction, the former being made from fossil copal mixed with certain silicates, and the latter being a special kind of *papier maché.*

Just as the displacement of air, say from one place to another, necessitates the existence of a difference in air pressure, so electrical pressure is needed to transfer electricity, and all insulators become conductors if the pressure be high enough. In practice, electrical pressure is measured in "volts" and the following table shows the "voltage" required to pierce the insulators mentioned, each being ·001 inch thick: *the greater this voltage the greater is said to be the* **dielectric strength** *of the insulator.* The volt is defined later.

Breakdown Pressures of certain Insulators.

Micanite plate . .	1,012 volts	Oiled asbestos . .	320 volts
Micanite paper . .	467 ,,	Red fibre. . . .	307 ,,
Micanite cloth . .	439 ,,	White Bristol board .	240 ,,

18. Electrostatic Induction.—Consider an insulated body C (Fig. 4) in the centre of the room, and let a positive charge be given to it. It is believed that this gives rise to a pressure extending right throughout the insulating medium (air), but diminishing at first rapidly and then slowly as we recede from the body C to the earth-joined bodies in the room; that in consequence the insulating medium is strained until it sets up an opposing influence which exactly balances the forward pressure due to the charged body. Taking,

then, any two points A and B (Fig. 4), the pressure at A exceeds the pressure at B; this pressure difference tends to drive electricity from A to B, but the medium sets up an opposing influence which balances the forward pressure, so that the insulating medium is strained, but there is no "flow." Now let the conductor (Fig. 5) be placed so as to connect A and B. The pressure difference tends to drive electricity from A to B, and the conductor being unable, like an insulator, to set up an opposing influence, permits the flow; thus electricity flows from A to B until uniform pressure exists over the whole conductor. Clearly, then, the end A will have less than its normal amount, *i.e.* a negative charge, and B will have an equal surplus, *i.e.* a positive charge. Remembering, too, how the pressure varies as we travel from C outwards, it will be apparent that the displacement will be greater the nearer the conductor is to the charged body.

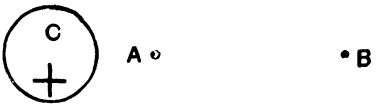

Fig. 4.

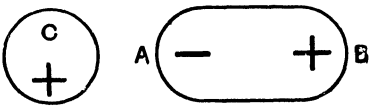

Fig. 5.

If C be negative, its pressure is below that of the earth, *i.e.* the walls, etc., of the room. Thus the pressure *rises* as we recede from it, being therefore greater at B than at A. On placing the conductor in position, electricity flows from B to A; B in this case exhibits a negative charge, and A an equal positive charge.

These actions are referred to as "*electrostatic induction*"; C is called the inducing body, A B the body acted on inductively, and the insulating medium which transmits these influences, itself playing a vitally important part in the action, is known as the dielectric.

By interposing a solid dielectric such as glass, wax, mica, etc. (or a liquid dielectric), between C and the conductor A B, the inductive effects will be still more marked, for they permit the influence to take place through them better than air, and are said to have a higher *specific inductive capacity*. It has been indicated that the nearer the two bodies are together the better is the result, and *the introduction of these solid dielectrics is (electrically) equivalent to shortening the air space*; thus a piece of mica 5 cm. thick is equivalent to an air thickness of 1 cm.; its specific inductive capacity is 5, that of air being taken as unity (Art. 31). Specific inductive capacity is also called the "dielectric constant."

We have seen that a light and suitably suspended conductor is attracted by an electrified body (Art. 15). Prior to attraction inductive displacement occurs, the nearer side of the conductor exhibiting an electrification opposite to that of the electrified body, the remote side an electrification similar to that of the charged body, and attraction ensues between the unlike charges facing each other.

Thus induction always precedes the attraction between a charged and uncharged body.

19. Potential.—If an isolated positively charged conductor be put in metallic connection with the earth, *its "surplus" flows to earth* and the body becomes neutral; this is because its electrical pressure is above that of the earth, and the fact is expressed by saying that *it is at a higher potential than the earth*; or, taking the latter as our zero, it is said to be at a *positive potential.* After the momentary flow the potential of the body is zero.

On earthing an isolated negatively charged conductor, *electricity flows from the earth* until its "deficit" is made up and the body becomes neutral; this is because its electrical pressure is below that of the earth; *it is said to be at a lower potential* than the latter, or at a *negative potential.* After the flow the potential of the body is zero.

Again, if two points in a dielectric at different potentials (*e.g.* A and B in Fig. 4), or two conductors at different potentials, be put in conducting communication, *electricity will flow from the one at the higher potential* and the flow will continue until equality of potential is established; from which it follows that if the flow is to be maintained, means must be devised for keeping up the potential difference (abbreviated P.D.). This is the function performed by batteries and dynamos.

The reader must guard against a likely error here. If the two vessels of Fig. 6 contain water as indicated, on opening the stop-cock the flow takes place from A to B until the two attain the same water level. *The initial quantity of water in* A *may be considerably less than that in* B; it is the respective water levels which settle the direction of flow. Similarly if two like charged conductors be joined *it is their respective potentials which settle the direction of flow, not the amount of their charges,* and the potentials depend on the sizes of the bodies and on the medium as well as on the actual charges (Art. 25).

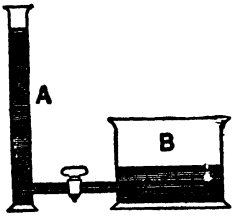

Fig. 6.

At this stage we may summarise in a general way as follows: **The potential of a body is its electrical pressure above or below that of the earth; it is the electrical condition which settles whether the body will give electricity to the earth or another body, or get electricity from the earth or another body when they are joined;** *a body is at a positive potential if electricity flows from it to the earth regarded as the zero of potential, it is at a negative potential if electricity flows from the earth to it, and it is at zero potential if there is no flow; similarly a body* A *is at a higher potential than another one* B *if electricity flows from* A *to* B *when they are joined, and they are at the same potential if there is no flow between them.* The

mathematical aspects of potential and its *exact* definition are dealt with in the next chapter.

20. Potential due to Neighbouring Charges.—Referring again to Fig. 5, experiment proves that when the conductor A B is earthed, the positive charge at B disappears and a greater negative charge appears at A. This must be the result of electricity flowing to earth, hence *before earthing*, A B *must have been at a positive potential, although it had a negative charge at one end and a positive one at the other.* This positive potential is due to the influence of the positive charge on C; it is spoken of as an induced potential, since it is due to inductive influence.

Further, when A B is earthed *it is reduced to zero potential, but it exhibits a negative charge at the end* A. In this case the negative potential due to its acquired negative charge is exactly cancelled by the induced positive potential due to C.

Finally, if A B be disconnected from the earth and the charged body C be *then* removed, A B exhibits a negative charge at all points of its surface *and is at a negative potential* due to its acquired negative charge.

Fig. 7 represents these points graphically, the vertical distances denoting potentials; distances above the horizontal denote positive potentials, distances below the horizontal negative potentials and the horizontal line denotes zero potential. The figure is, of course, diagrammatic only. Note that the potential of C is *lowered* when A B is earthed.

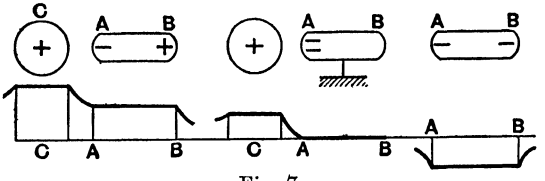

Fig. 7.

If C be negatively charged, the end A exhibits a positive charge and B a negative one. On earthing, the negative charge at B disappears and a greater positive charge appears at A. This must be the result of electricity coming from the earth; hence *before earthing*, A B *must have been at an induced negative potential due to* C, *although it had a positive charge at one end and a negative one at the other.* When A B is earthed *it is at zero potential, but it has a positive charge at* A. Here again the positive potential due to its own positive charge is exactly cancelled by the induced negative potential due to C. When A B is disconnected from the earth and C is *then* removed, A B is positively charged *and is at a positive potential* due to this positive charge.

21. Electric Field. Lines of Force.—An Electric Field is the space surrounding an electrified body (or system of electrified bodies) within which the influence of the charge (or charges) extends. When considering the forces in the field it is spoken of as a **Field of Electric Force;** when concentrating on the phenomena associated with induction it is often spoken of as a **Field of Induction.**

A free, small, positive charge placed at any point in the field will be urged by a definite force in a definite direction, which latter is indicated by the line of force passing through the point in question. **A line of force is a curve such that the tangent at any point gives the direction of the electric force at that point.** The positive direction of a line is the direction in which a free *positive* charge tends to move: in some of the diagrams which follow arrows indicate the positive direction of the lines of force.

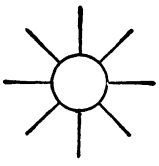

Fig. 8.

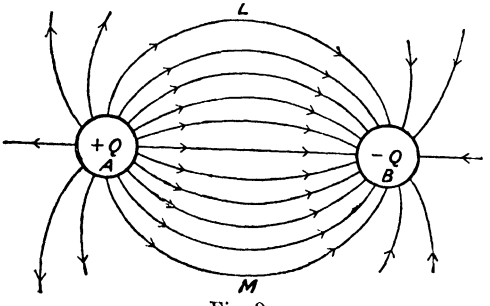

Fig. 9.

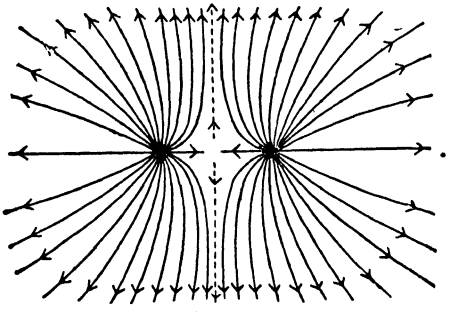

Fig. 10.

It has been mentioned that the insulating medium surrounding an electrified body is *strained.* In more advanced theory it is necessary to distinguish between "lines of force" (which really deal with the **stress** in a field) and what are called "lines of induction" (which deal with the **strain**), but it is not necessary at this stage, for *in an air medium* (which we are considering) *lines of force and lines of induction are coincident.* (Art. 28).

Fig. 8 shows the lines of force for a positively charged sphere in the centre of a large room. Fig. 9 represents the case of two equal conducting spheres with equal and opposite charges, whilst Fig. 10 depicts the case of two equal positive point charges. Fig. 11 shows the lines in the case of two positive charges in the ratio

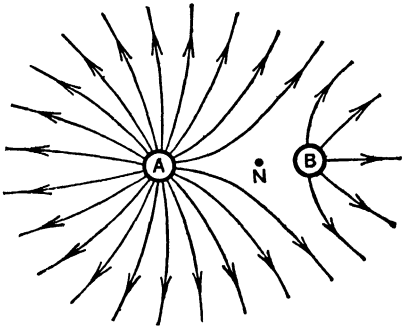

Fig. 11.

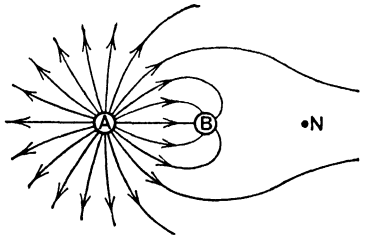

Fig. 12.

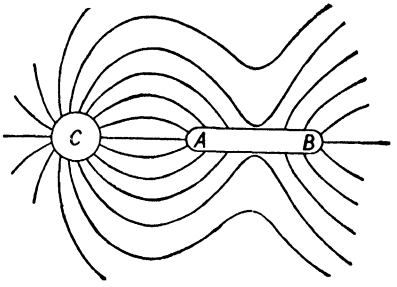

Fig. 13.

4 : 1 (A = + 20, B = + 5), and Fig. 12 gives the corresponding case of A = + 20 and B = − 5. Fig. 13 represents the case of a charged sphere C acting inductively on an insulated conductor A B : the "induced charges" at A and B are equal and opposite (it will be noted that the number of lines ending on A is drawn equal to the number proceeding from B), but *each is less than the inducing positive charge at* C, the balance being on the walls, etc., of the room. Fig. 14 shows the lines in the case of a positively charged sphere A inside an insulated hollow conducting sphere B but not situated at the centre of the latter. The *distribution* of the charge on the inside of B is irregular but outside it is uniform. Further, as will be seen from the lines in the figure, in cases such as this, *where* B *completely surrounds the inducing charge* A, the induced charges on B are not only equal but *each is equal to the inducing charge on* A.

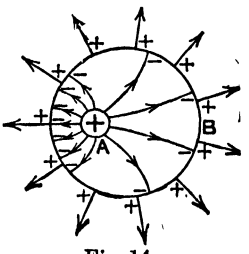

Fig. 14.

(See Arts. 23, 29).

In all cases the lines start from a positive charge and end on a negative charge somewhere. It should also be noted that lines of force *do not pass through a conductor* but end on its surface. It may be mentioned too that the field occupies the entire space surrounding the charges and is not confined to one plane as the illustrations must, of necessity be.

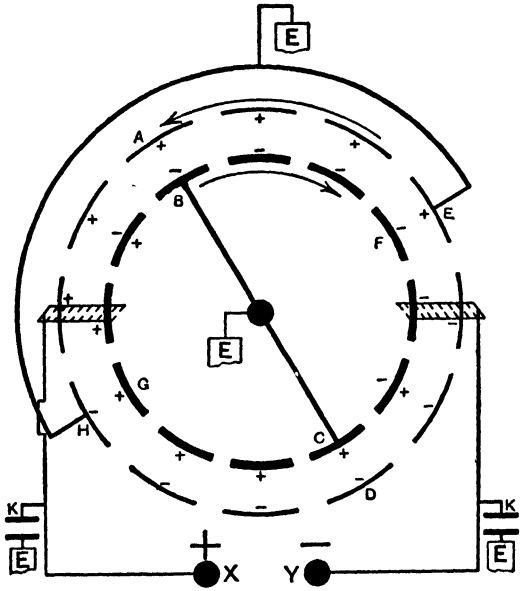

Fig. 15.

In order to explain the fundamental facts of attraction and repulsion and of inductive action, Faraday imagined these lines of force to be, so to speak, real connections between the positive and negative charges, and further, that each line tended to contract longitudinally, whilst lines proceeding in the same direction tended to repel each other laterally. Thus longitudinal contraction will account for the attraction between unlike charges, and lateral repulsion will afford an explanation of the repulsion between like charges.

22. The Wimshurst Electrostatic Induction Machine.—So far our "charges" of electricity have been obtained by the simple friction of suitable substances, or by the simple inductive action of a charged body on an initially uncharged conductor; but to quickly obtain large charges some form of electrical machine must be employed, the most commonly used type being, perhaps, that of Wimshurst. Fig. 15 will serve to explain the action, while Fig. 16 depicts the actual machine.

The machine consists essentially of two vulcanite or shellac varnished glass plates capable of rotation in opposite directions about a horizontal axis. On the sides of the plates remote from each other are fixed a number of metal sectors; in Fig. 15 the sectors on the front plate are represented by the inner broken circle, those on the back plate by the outer and thinner broken circle. Standing at each side is a U-shaped conductor, each carrying on the inside two rows of sharp

points (called the combs), facing the plates; these are connected to the "dischargers" or "poles" of the machine X Y and to the inside coatings of two condensers K K, the object of the latter being to increase the capacity and to render the accumulation of heavy charges

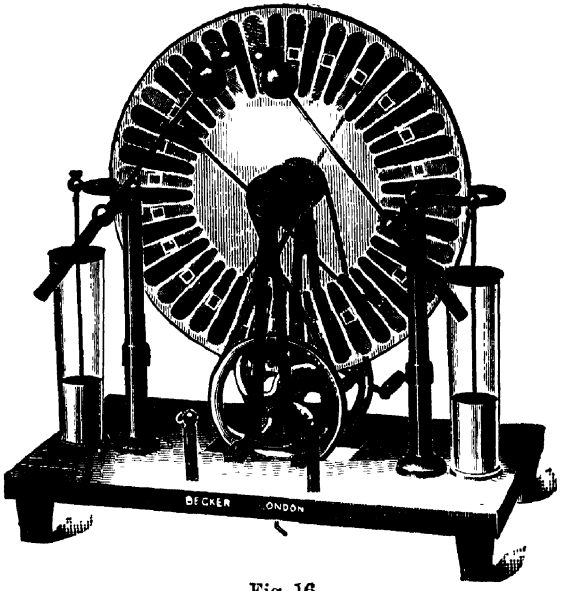

Fig 16.

possible (Art 29). Lying across the front plate, at an angle of about 45° with the combs, is a metal rod B C, terminating in wire brushes, which graze the sectors as the plate rotates. Across the back plate and, roughly speaking, at right angles to B C is a second metal rod E H, terminating in brushes; for convenience this is drawn outside in the figure

Disregarding for a moment the "earthing" of the rods B C and E H, and imagining the sector A to have a slight + *ve* charge given to it, the action of the machine may be described as follows:—A acts inductively on the conductor B C, causing the sector B to acquire a — *ve* charge and C a + *ve* one. Concentrating our attention on the front plate, B moves to the right with its — *ve* charge, and on reaching F acts inductively on E H, making the sector E on the back plate + *ve* and H — *ve* Leaving F, the front sector moves into the comb on the right, where it again acts inductively, causing the points to become + *ve and the discharger* Y — *ve*. The density of the charge at the points is so great (Art. 23) that particles of air and dust in the

vicinity are attracted, become charged by contact, and are then repelled, carrying the + *ve* charge with them: thus the comb loses some of its charge, and the + *ve* particles coming in contact with the — *ve* sector partly neutralise its charge. Further, the air is **ionised,** *i.e.* has + *ve* and — *ve* charged particles called **ions** produced in it (see *Advanced Text-Book of Magnetism and Electricity*) and the — *ve* ions are attracted to the comb thus neutralising its + *ve* charge whilst the + *ve* ions are attracted to the sector thus neutralising its — *ve* charge. Thus the comb loses its charge leaving the discharger Y — *ve*, and the sector comes out of the comb uncharged. When the front sector reaches C it acquires a + *ve* charge, when it reaches G it acts inductively on the back plate, causing H to become — *ve* and E + *ve*, and when it enters the comb on the left, actions similar to those already explained take place, the result being that *the discharger X becomes* + *ve*, and the sector comes out uncharged. At B the sector becomes — *ve*, and the action is repeated.

Turning to the back plate, the sector E moves to the left with its + *ve* charge. At A it acts inductively on the front plate, making the sector at B — *ve* and that at C + *ve*; it then enters the comb, with the result that *X acquires a further* + *ve charge.* On reaching H the sector in question becomes — *ve*, at D it acts inductively on C B, making C + *ve* and B — *ve*: it then enters the comb, with the result that Y *acquires a further* — *ve charge* and the action is repeated. *Thus the rotation of both plates results in* X *acquiring a strong* + *ve charge and* Y *a strong* — *ve one.*

In the above we have assumed that a small charge is given to A; in practice the small difference of potential between different parts is usually sufficient to start the action.

23. Miscellaneous.—The following additional points in the general principles of electrostatics may be briefly referred to:—

(1) When a conductor is charged *the charge is entirely on the outside.* Thus if a metal can be insulated and positively charged the inside will show no signs of electrification, but the outside will be positively charged. If an electrified ball be lowered into an insulated metal can, touch the inside, and then be removed, the ball and the inside of the can will show no signs of electrification, but the outside of the can will show the charge. In both cases, however, *the outside of the can, the inside of the can, and the air inside except near the mouth are all at the same potential* from which it follows that *there is no electrical force inside a hollow conductor if there are no charged bodies inside.* Of course if there is a charged body hanging inside matters are different: thus in Fig. 14 the inside has an induced negative charge and there is an "up gradient" of potential from the inside of B to the positively charged ball. The student should reason these experimental facts out from the conception of lines of force (and induction).

(2) The *distribution* of the charge on a conductor depends on its shape (also on

on a cube it is greatest at the corners, next at the edges; on a disc it is greatest round the edge. If sharp points project from the outside of a charged conductor the charge tends to accumulate at the points and the conductor loses its charge (Art. 22). Put in general terms *the "density" of the charge on an isolated charged body is greatest where the curvature is greatest* (but the *potential* is the same at all parts).

EXERCISES II.

1. Define "conductor" and "insulator." Arrange the following substances in the order of their conducting powers, putting the best conductor first: annealed copper, glass, platinoid, mica, rosewood, dry air, manganin, lead, gutta-percha, water, india-rubber, porcelain, paraffin wax, iron. (See Appendix)

2. Define "potential" and give examples of—(1) a body with a positive charge and at zero potential; (2) a body exhibiting both charges and at a negative potential; (3) a body with a negative charge and at zero potential.

3. State clearly the evidence for the opinion that there are two kinds of electrification.

4. Write a short essay on the one-fluid theory of electricity, and explain in accordance therewith (1) the positive, (2) the negative electrification of a conductor by contact. Explain also the discharge of (1) a positively, (2) a negatively electrified conductor.

5. A charged conductor B is used to induce a charge on a conductor A.

(*a*) What is the reason for breaking the earth contact *before removing the inducing body* B? What would happen supposing B were removed first?

(*b*) Will it make any difference in the final state of A if it had originally a charge of its own?

(*c*) Will it make any difference in the final state of A if it is earthed *before* bringing B near it?

6. What do you understand by an *electric field* and a *line of force* (*electric*)? Sketch the lines of force for:—

(*a*) Three equal like charges at the corners of an equilateral triangle.

(*b*) Two equal like charges at the ends of the base, and an equal unlike charge at the apex of the triangle.

(*c*) A charge of + 20 three inches from a charge of − 10.

7. Describe the construction and action of the Wimshurst induction machine.

CHAPTER III.

QUANTITATIVE RELATIONS AND UNITS.

24. Quantity of Electricity: Units of Quantity.—The degree of electrification of a "charged" body is ascertained by the force the body is capable of exerting in virtue of its "surplus" or "deficit." Consider two positive charges placed at a certain distance apart, and therefore repelling each other. Experimentally it can be proved that if one of the charges be increased 2, 3, 4, . . . times, the force of repulsion is increased 2, 3, 4, . . . times, and if, with one charge doubled, the other be quadrupled, the repulsive force is increased eight times. Various tests lead to the conclusion that *the force between two charges varies directly as their product*; stated algebraically, if Q_1 and Q_2 denote the charges and f the force between them, $f \propto Q_1 \times Q_2$.

Again, if the distance between the two charges be increased 2, 3, 4, . . . times the force is found to decrease to $\frac{1}{4}$, $\frac{1}{9}$, $\frac{1}{16}$, . . . of its initial value, whereas if the distance be reduced to $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, . . . the force is increased 4, 9, 16, . . . times. This establishes a second law—viz. that *the force between two charges varies inversely as the square of their distance apart*; stated algebraically, if d be the distance, $f \propto 1/d^2$.

Remembering the part played by the medium, the reader will be prepared for the statement that *the force depends on the nature of the insulating medium.* Combining these facts, the force between the two charges considered may be expressed thus:—

$$f = \beta \frac{Q_1 \times Q_2}{d^2},$$

β being a constant depending on the medium and on the units we adopt. Now the electrostatic unit charge is so taken that with Q in these units, f in dynes, and d in cm., the factor β is unity **if the charges are in air,** *i.e.* **in air,** the formula is:—

$$f = \frac{Q_1 \times Q_2}{d^2} \text{ dynes.}$$

If $d = 1$ cm., $f = 1$ dyne, and Q_1 be equal to Q_2, Q_1 and Q_2 are each unity. Hence:—

The C.G.S. electrostatic unit of quantity is such that if placed one centimetre in air from an equal and like quantity it repels it with a force of one dyne. It has no name.

The **practical unit** is the **coulomb**; it is equal to 3,000,000,000, *i.e.* 3×10^9 electrostatic units.

The coulomb is defined later (Chap. IX.).

Another quantity unit known as the "C.G.S. electromagnetic unit" is employed in current electricity; it is equal to 3×10^{10} electrostatic units; thus the coulomb is $\frac{1}{10}$ of the electromagnetic unit. The "**electron**" $= 4{\cdot}65 \times 10^{-10}$ e.s. unit $= 1{\cdot}55 \times 10^{-20}$ e.m. unit $= 1{\cdot}55 \times 10^{-19}$ coulomb.

For any other medium the factor β must be introduced. β is really equal to $1/k$, where k is the specific inductive capacity (for air k is taken as unity). Hence the more general formula is:—

$$f = \frac{1}{k}\frac{Q_1 \times Q_2}{d^2} \text{ dynes.}$$

To be exact the above law is only applicable to *point charges.*

Clearly the ratio of the force between two charges in air to the force between the same two charges at the same distance in a medium x measures the specific inductive capacity of x.

25. Potential: Units of Potential.—In Art. 19 "potential" has been defined in a general way; we proceed now to the principle upon which the actual measurement of potential is founded—viz. its relation to work and energy.

We have seen that in practice the earth is taken as the zero of potential, because, owing to its size none of our experiments are likely to alter its potential. The mathematical zero of potential is, however, *at an infinite distance*, for, at infinity, the force due to any electrified body must be *nil*, and to be exact we must, in defining the potential unit, adopt this zero. It should be mentioned, however, that *the P.D. between two points* is the same whether the practical or the mathematical zero be taken, and in practice it is only with P.D.'s that we are concerned.

Let A (Fig. 17) be a positively charged body, no other charge being near; the potential falls from its positive value at A to zero at all points at an infinite distance. Imagine D to be a mathematical point with a unit positive charge. To bring this unit charge from infinity to C work must be done against the repulsion of the charge on A, and when the

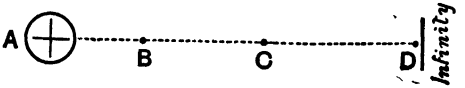

Fig. 17.

unit is at C it possesses potential energy equal in amount to the work expended in bringing it there. Clearly more work must be done in bringing it to B, and still more in bringing it to A.

The work expended in these operations depends on the repulsive force, which again depends on the medium, on A's charge, and on the distance—in short, on the way in which the potential changes from A outwards; and naturally some connection must exist between the potential and the work. *If w_1 ergs of work be done in moving the positive unit from infinity to* C, *w_1 electrostatic units is the potential at this point due to the charged body* A; if w_2 ergs of work be expended in moving the unit to B, w_2 electrostatic units is the potential at B, and *the P.D. between* B *and* C *is* $(w_2 - w_1)$ *electrostatic units.*

Finally, w_3 electrostatic units is the potential of the body A if w_3 ergs of work be done in moving the positive unit up to it. If the body is negatively charged the same reasoning holds, but in this case the unit would do work itself in moving up under the attraction of A.

This leads to the following definition:—*The potential at any point in the field surrounding a charged body is represented numerically by the work done on or by a positive unit in moving from infinity, regarded as the zero of potential, to the point in question; the P.D. between two points is represented numerically by the work done on or by a positive unit in moving from one point to the other.* Hence:—

The potential at a point is one C.G.S. electrostatic unit if one erg of work is done on or by a positive unit charge in moving from infinity to that point. It has no name.

The **practical unit** is the **volt**; it is equal to $\frac{1}{300}$ of an electrostatic unit.

The volt is defined later (Chap. IX.).

Another potential unit, known as the "C.G.S. electromagnetic unit," is employed in current electricity. One electrostatic unit is equal to 3×10^{10} electromagnetic units; hence the volt is equivalent to $\frac{1}{300} \times 3 \times 10^{10}$, *i.e.* 10^8 electromagnetic units.

It can be proved that if Q electrostatic units be put on a sphere of r centimetres radius completely surrounded by a medium of specific inductive capacity k, the potential of the sphere in electrostatic units is Q/kr, and the potential at a point r_1 centimetres from its centre is Q/kr_1. If the medium be air, k is unity, and these expressions become $\frac{Q}{r}$ and $\frac{Q}{r_1}$ respectively. If there are several charges, the potential at a point is the algebraic sum of the potentials due to each.

Clearly the ratio of the potential of a charged body in air to its potential with the same charge in a medium x measures the specific inductive capacity of x.

26. Capacity: Units of Capacity.—The capacity of a body is measured by the quantity of electricity which must be given to it to raise its potential by unity. Hence:—

A conductor has a capacity of one C.G.S. electrostatic unit if the electrostatic unit quantity raises its potential by one electrostatic unit. It has no name.

The **practical unit** is the **farad; a conductor has a capacity of one farad if a charge of one coulomb raises its potential by one volt.**

One farad $= 900{,}000{,}000{,}000 = 9 \times 10^{11}$ electrostatic units.

One microfarad $= \frac{1}{1000000}$ farad $= 9 \times 10^{5}$ „ „

Another capacity unit is employed in current electricity; it is known as the "C.G.S. electromagnetic unit," and is equal to 9×10^{20} electrostatic units; thus the farad is $\frac{1}{10^9}$ electromagnetic unit, as may also be seen from the relation—

$$1 \text{ farad} = \frac{1 \text{ coulomb}}{1 \text{ volt}} = \frac{\frac{1}{10} \text{ electromagnetic unit}}{10^8 \text{ electromagnetic units}}$$
$$= \frac{1}{10^9} \text{ electromagnetic unit.}$$

If Q denotes the charge on a conductor of capacity C, and if in consequence of this charge the potential has been raised from zero to V, it is clear that—

$$\mathbf{C = \frac{Q}{V}}, \quad i.e. \text{ Capacity} = \frac{\text{Quantity}}{\text{Potential}}; \quad e.g. \text{ Farads} = \frac{\text{Coulombs}}{\text{Volts}}$$

$$\mathbf{Q = CV}, \quad i.e. \text{ Quantity} = \text{Cap.} \times \text{Pot.}^1; \quad e.g. \text{ Coulombs}$$
$$= \text{Farads} \times \text{Volts}:$$

$$\mathbf{V = \frac{Q}{C}}, \quad i.e. \text{ Potential} = \frac{\text{Quantity}}{\text{Capacity}}; \quad e.g. \text{ Volts} = \frac{\text{Coulombs}}{\text{Farads}}.$$

If two conductors having capacities C_1 and C_2 be joined together and charged, they acquire a common potential V; if Q_1 and Q_2 denote their respective charges:—

$$Q_1 = C_1V \text{ and } Q_2 = C_2V$$
$$\therefore \mathbf{Q_1 : Q_2} = C_1V : C_2V = \mathbf{C_1 : C_2};$$

that is, *the total charge divides between them in the ratio of their capacities.*

We have stated in Art. 25 that if Q electrostatic units be given to a spherical conductor of radius r centimetres surrounded by a medium

of specific inductive capacity k, the potential of the sphere is equal to $\frac{Q}{kr}$ electrostatic units. If C denotes its capacity in this medium, its potential is also equal to $\frac{Q}{C}$ electrostatic units; hence $C = kr$. With air as medium k is unity, and this reduces to $\mathbf{C = r}$; hence *the capacity of an isolated spherical conductor in air is, in electrostatic units, numerically equal to the radius in centimetres.* Employing practical units:—

$$C = \frac{r}{9 \times 10^{11}} \text{ farads} = \frac{r}{9 \times 10^{5}} \text{ microfarads.}$$

With another dielectric in place of air these expressions must be multiplied by the value of k for that dielectric.

Clearly the ratio of the capacity of a body in a medium x to its capacity in air measures the specific inductive capacity of x.

27. Potential Energy of a Charge.—When a conductor is charged work is done in straining the medium, and the latter is endowed with energy; when the field is made to disappear by, say, taking a spark from the conductor, the medium does an equal amount of work in returning to its normal condition; this work appears partly as heat and sound and part is utilised in setting up the ether vibrations which constitute light. An expression for this energy may be developed thus: Imagine a conductor initially without charge, and picture the charging process to be accomplished by bringing up successive positive units from infinity until the potential is raised from zero to a value V electrostatic units. The average potential during the process is $\frac{V}{2}$, and this represents by the definition of Art. 25 the average work, in ergs, in putting on a unit charge. If the total charge communicated is Q electrostatic units, the total work done in charging is $Q \times \frac{V}{2}$, *i.e.* $\frac{1}{2}QV$ ergs. This represents the work which can be obtained in discharging; hence, employing electrostatic units, it may be said that the energy W of the charge or discharge or the energy of the field surrounding the conductor is given by:—

$$\mathbf{W} = \frac{1}{2}\mathbf{QV} \text{ ergs}$$

$$= \frac{1}{2}CV \times V = \frac{1}{2}\mathbf{CV^2} \text{ ergs}$$

$$= \frac{1}{2}Q\frac{Q}{C} = \frac{1}{2}\frac{\mathbf{Q^2}}{\mathbf{C}} \text{ ergs.}$$

If Q be in coulombs and V in volts, then since Q coulombs are equal to $Q \times 3 \times 10^9$ electrostatic units and V volts are equal to $V \times \frac{1}{300}$ electrostatic units:—

$$W = \left\{ \frac{1}{2} QV \times 3 \times 10^9 \times \frac{1}{3 \times 10^2} \right\} \text{ ergs}$$

$$= \frac{1}{2} QV \times 10^7 \text{ ergs}$$

$$= \frac{1}{2} QV \text{ joules, since one joule} = 10^7 \text{ ergs.}$$

Hence with Q *in coulombs,* V *in volts, and* C *in farads, the energy, as calculated from the three relations just established, will be in joules.* In this case, since one joule = ·7375 foot-pound,

$$W = \left\{ \frac{1}{2} Q \text{ (coulombs)} \times V \text{ (volts)} \times \cdot 7375 \right\} \text{ ft.-pounds}$$

$$= \frac{QV}{2 \cdot 712} = \frac{CV^2}{2 \cdot 712} = \frac{\cdot Q^2}{2\,712 \times C} \text{ ft.-pounds.}$$

The reader may here be briefly warned against the common and erroneous idea that electricity is a form of energy. From the relation $W = \frac{1}{2}QV$ it will be seen that pressure (V) is necessary before electricity has the power to do work. If the electricity of a body is at a different potential to that of neighbouring bodies it is endowed with energy, but if no difference exists there is no energy. The reader should think this out in detail for himself.

28. Field Strength. Unit Field.—It has been agreed that the intensity of an electrostatic field be measured by the force (in dynes) on a unit positive charge. **Unit electric field** may thus be defined as **that field which exerts a force of one dyne on a unit positive charge placed in it,** and a field of intensity F units, is one which exerts a force of F dynes on a positive unit charge.

From the above it follows that the intensity of the electric field at distance *d* cm. from a point charge + Q e.s. units, or at a distance *d* cm. from the centre of a uniformly charged sphere having + Q e.s. units (*d* being greater than the radius of the sphere), is given by

$$\mathbf{F = \frac{Q}{d^2} \text{ e.s. units} \quad \textit{or} \quad F = \frac{Q}{kd^2} \text{ e.s. units,}}$$

the medium in the first case being air, and, in the second, one of specific inductive capacity *k*.

Strictly speaking every point in a field has its line of force and therefore the number of lines falling on a given area is *infinite.* In the theory of electricity, however, it is convenient to imagine a definite number of lines proceeding from a given charge and this leads to

another definition of field intensity. Thus imagine a charge of Q electrostatic units in a medium of specific inductive capacity k and imagine a spherical surface of radius d cm. drawn in the field, the charge Q being at the centre; the intensity of the field at any point on the surface of this imaginary sphere is Q/kd^2 C.G.S. units. Now imagine that $\frac{4\pi Q}{k}$ lines emanate from the charge Q (or in other words that $\frac{4\pi}{k}$ lines emanate from unit charge) in this medium; these are called **Maxwell lines of force.** These lines all pass through the surface of the imaginary sphere, and since the area of this spherical surface is $4\pi d^2$, it follows that

$$\left.\begin{matrix}\text{Number of Maxwell Lines of}\\ \text{Force per unit area}\end{matrix}\right\} = \frac{4\pi Q}{k \cdot 4\pi d^2} = \frac{Q}{kd^2}.$$

But the last expression gives the intensity of the field in electrostatic units; hence:—

(1) The intensity of a field in C.G.S. electrostatic units is represented numerically by the number of Maxwell lines of force per unit area (perpendicular to their direction): thus a field of intensity F has F lines of force per unit area.

(2) **Unit field is that which has one Maxwell line of force per unit area.**

In air k is unity and the number of Maxwell lines of force from a charge Q *is* 4πQ *and the number from unit charge is* 4π. This 4π thus arbitrarily introduced finds its way into many formulae as a consequence of this first choice.

Consider now the inductive action in the field so that our lines are now referred to as lines of induction. Imagine that 4πQ lines emanate from the charge Q *whatever the medium* (*i.e.* 4π lines emanate from unit charge); these are called **lines of induction.** These all pass through the surface of the imaginary sphere; hence

$$\left.\begin{matrix}\text{Number of Lines of Induction}\\ \text{per unit area}\end{matrix}\right\} = \frac{4\pi Q}{4\pi d^2} = \frac{Q}{d^2} = k\left(\frac{Q}{kd^2}\right).$$

But Q/kd^2 gives the number of Maxwell lines of force per unit area; hence

$$\left.\begin{matrix}\text{Number of Lines of Induction}\\ \text{per unit area}\end{matrix}\right\} = \left\{ k \times \begin{matrix}\text{Number of Lines of}\\ \text{Force per unit area,}\end{matrix}\right.$$

i.e. Induction = $k \times$ Field

or $N = kF$.

In air k is taken as unity, and the number of lines of induction, and the number of lines of force, are the same, *i.e.* for air, lines of force and lines of induction are coincident (Art. 21).

Yet another convention—**Faraday Lines**—is met with in electrical theory: Q Faraday lines emanate from a charge Q, and therefore one Faraday line from unit charge (see Figs. 11, 12). Faraday lines are the most useful in electrostatic theory, but they are of less importance to the electrical engineer.

To be scientifically accurate in these numerical relations **tubes** of force and induction, etc., and **not lines** of force, etc., should be dealt with: in electrical engineering, however, it is customary to speak of "lines" (see *Advanced Text-Book of Magnetism and Electricity*).

We would again remind the student that $4\pi Q/k$ lines of force, $4\pi Q$ lines of induction and Q Faraday lines emanate from a charge Q: in air, since k is unity the lines of force and lines of induction are each $4\pi Q$. Further, since $N = kF$, the *specific inductive capacity k may be defined as the ratio of Induction to Field* ($k = N/F$).

EXAMPLES.

1. *Compare the forces between two spheres charged to the same potentials* (1) *in air,* (2) *in a medium of specific inductive capacity k.*

If Q_1 and Q_2 be the charges on the spheres in air, and d be their distance apart, the force f_1 between them in air is given by—

$$f_1 = \frac{Q_1 \times Q_2}{d^2}.$$

When in the medium, the potential of each becomes $\frac{1}{k}$ of what it was in air (Art. 25); hence the charge on each must be increased k times to bring the potentials up to the initial values, *i.e.* the charges must be kQ_1 and kQ_2 respectively. If the spheres were in air with these charges the force between them would be—

$$\frac{kQ_1 \times kQ_2}{d^2},$$

but as they are in the medium the force is (Art. 24) $\frac{1}{k}$ of this; thus if f_2 be the force in the medium,

$$f_2 = \frac{1}{k} \frac{kQ_1 \times kQ_2}{d^2} = k \frac{Q_1 \times Q_2}{d^2}.$$

Thus the force is *k times greater* in the medium *when the potentials are the same.* When *the charges are the same* in air and in the medium, the force is by Art. 24 *k times greater in the air.*

2. *Two small spheres, each of mass m grammes, are suspended from a point by threads, each l cm. long. They are equally charged, and repel each other to a distance of 2d cm. If g = 981, express the charge on each in electrostatic units, electromagnetic units, and coulombs.*

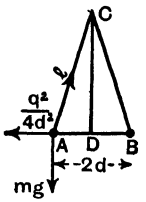

Fig. 18.

The forces on the sphere A (Fig. 18) are:—

(1) The weight, mg dynes.

(2) The repulsion, viz. $\frac{q^2}{(2d)^2}$ dynes.

(3) The tension of the thread,

and these are in the directions indicated.

An examination of the figure will show that the triangle CDA has its sides parallel to these forces; hence the forces are proportional to the sides of this triangle to which they are parallel. Thus,

$$\frac{\frac{q^2}{4d^2}}{mg} = \frac{\text{DA}}{\text{CD}} = \frac{d}{\sqrt{l^2 - d^2}},$$

$$\therefore q^2 = \frac{4d^3mg}{\sqrt{l^2 - d^2}}, \text{ i.e. } q = 2d\sqrt{\frac{dmg}{(l^2 - d^2)^{\frac{1}{2}}}} \text{ e.s. units.}$$

In electromagnet units the charge is $q/(3 \times 10^{10})$, and in coulombs $q/(3 \times 10^9)$.

EXERCISES III.

1. Two equally charged spheres repel each other when their centres are half a metre apart with a force equal to the weight of 6 milligrammes. Find the charge on each in electrostatic units. (B.)

If the charge on one of the spheres be doubled and the distance apart be also doubled, find the force in dynes between them.

2. Charges of 1, 2, 3, and − 4 units are placed at the corners of a square taken in order. If the length of each side is 2 cm., find the potential at the middle point of the side joining 1 and 2. (B.)

If the surrounding medium be paraffin wax instead of air, what will be the value of the potential? Express the results in electrostatic units and in volts (k for wax = 2·3).

3. What is meant by saying that the potential of a conductor is + 20 electrostatic units? What is meant by the statement that the potential of a body is − 20 electrostatic units? How much energy is expended in carrying a charge of 30 electrostatic units from a place where the potential is − 10 electrostatic units to a place where it is + 35?

4. Does the energy of an electric charge depend upon the magnitude of the charge only? If not, upon what other circumstance does it depend? Find an expression for its magnitude, and criticise the statement that "electricity is a form of energy."

5. Two conductors have capacities in the ratio of 5 to 6, and charges in the ratio of 216 to 125. Compare the heats produced by discharging them (*i.e.* compare the energies).

6. A conductor of capacity 10 microfarads is charged to a potential of 85 volts; determine the charge in coulombs. A second conductor is raised in potential to 3,000 volts by a charge of $\frac{1}{1000}$ of a coulomb; determine its capacity in microfarads. A third conductor of capacity 20 microfarads is charged with $\frac{1}{10000}$ of a coulomb; determine its potential in volts. Finally express the three results in the corresponding C.G.S. electrostatic units.

7. Estimate in "joules," "foot-pounds," and "B.O.T. units" the energy stored on each of the conductors of Question 6.

8. A conductor has a capacity of 5 microfarads, and it can be charged by a battery to a potential of 100 volts. It is repeatedly charged and discharged 30 times in one minute. Calculate its "horse-power" and the total work given out in the minute.

9. Two small spherical pith-balls, each one decigram in weight, are suspended from a point by threads 50 cm. long, and are equally charged so as to repel each other to a distance of 20 cm. Find the charge on each in electrostatic units ($g = 980$). (B.)

10. A small brass sphere is charged with 30 units of positive electricity, and is then made to touch another equal sphere having a charge of 10 units of negative electricity. Find the force exerted between the charged spheres when they are separated by a distance of 10 cm.

11. Two charges of + 10 and − 10 units are placed at two corners of an equilateral triangle of 10 cm side. Find the magnitude and direction of the resultant force acting on a charge of + 10 units placed at the third corner of the triangle.

12. A, B and C are three conductors equal in all respects. A is charged, made to share its charge with B and afterwards to share the remainder with C—both B and C being previously without charge. The three are now separately discharged. Compare the quantity of heat resulting from each discharge with what would have been produced by the discharge of A before any sharing of its charge (*i.e.* compare the energies). (Univ.)

CHAPTER IV.

CONDENSERS AND CAPACITY.

29. Principle of Condensers.—Consider an insulated metal plate A (Fig. 19) joined by a wire to the positive pole of an electrostatic induction machine. On working the machine electricity flows to A until the plate acquires the limiting potential V of the pole at the time of the experiment; when this condition is reached no further charge can be accumulated on A, for electricity will not flow between two bodies whose potentials are the same. It is, moreover, evident that the larger the plate the greater the capacity, and therefore the greater the charge necessary to bring it up to the potential of the machine.

Take, now, a second insulated metal plate B, and with A still at potential V let B be placed parallel to, and say four inches from it. Inductive displacement occurs (Art. 18), electricity being urged to the side of B remote from A; thus the near side exhibits a negative charge, the far side a positive charge, and the uniform potential of B is $+ v$ say, this being somewhat less than V in magnitude.

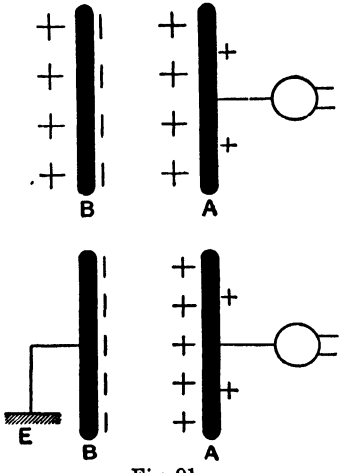

Fig. 91.

Now consider the effect of B on the potential of the plate A. The actual potential of the latter is the resultant of: (1) the free positive potential V due to its own charge, (2) the induced negative potential due to the negative charge on B, and (3) the induced positive potential due to the positive charge on B. The two latter opposing influences nearly counteract each other; but the negative charge being somewhat nearer than the positive has the advantage, and therefore on the whole *the potential of the plate* A *is weakened a little.* Clearly, then, an additional charge can now be given to it from the machine until its potential is once more equal

to V as at first. Clearly also the nearer B is to A the more marked will be the lowering of the latter's potential.

If B be earthed by touching it with the finger, electricity will flow out of it until its potential is reduced to zero. B has now a negative charge only, and, as a matter of fact, *a greater negative charge than in the case considered above*, for, when earthed, more electricity flows out of it to reduce its potential to zero (thus leaving a greater negative charge) than previously existed as a positive charge on the far side of the plate. The increased negative charge reacting on A lowers the latter's potential, and as there is no positive to counteract its influence it is evident that *the potential of* A *will be considerably weakened*, and, in consequence, a much greater charge must be given from the machine to raise the potential to its original value V—in short, *the capacity of* A *is considerably increased.* To summarise:

The potential of an insulated charged conductor is weakened and its capacity is increased when an insulated uncharged conductor is brought near, and the effects are enormously increased if the latter be earthed.

Such an arrangement of conductors separated by an insulator is called a **condenser**: the conductors are termed the *coatings*, and the insulator the *dielectric* of the condenser.

We have previously indicated that glass, wax, mica, vulcanite, shellac, etc., allow inductive influence to take place through them better than air (*i.e.* they have a higher specific inductive capacity), from which it follows that with one of these as dielectric the effects enumerated will be still more marked. Further, not only is the capacity of the insulated plate increased by the introduction of these substances, but owing to their greater mechanical rigidity there is less tendency to rupture and loss of charge by sparking between the plates—a result which might ensue with air as dielectric when the coatings are strongly charged and at a suitable distance apart. For these reasons solid dielectrics of glass, wax and mica are employed in practical condensers.

In the case considered, the charges on B are each less than the inducing charge; when earthed more electricity flows to earth than is represented by B's positive charge, *but the total quantity flowing out of* B *is less than the charge on* A. In the spherical condenser of Art. 31, since B entirely surrounds A, the induced charges are each equal to the charge on A, and on earthing an amount represented by B's positive charge only goes to earth, *i.e. the outflow from* B *is equal to the charge given to* A. To practical condensers consisting of large plates separated by very thin dielectrics, the same remark may be applied. Regarding, then, the two coatings of such a condenser as a single system, there is no accumulation of electricity on the whole, since B gives out as much as A gains (Art. 32), *but there is a heavy charge on* A *nevertheless.*

30. Capacity of Condensers: Electric Absorption.—The capacity of a condenser is measured by the quantity of electricity which must be given to it to establish unit P.D. *between its coatings.* Hence—

A condenser has a capacity of one C.G.S. electrostatic unit if the electrostatic unit quantity produces a P.D. of one electrostatic unit between its coatings.

The practical unit is the **farad; a condenser has a capacity of one farad if a charge of one coulomb produces a P.D. of one volt between its coatings.**

$$\text{One farad} = 9 \times 10^{11} \text{ electrostatic units.}$$

$$\text{One microfarad} = \frac{1}{10^6} \text{ farad} = 9 \times 10^5 \text{ electrostatic units.}$$

When a condenser is charged some time elapses before the coatings reach their maximum P.D., the charge meanwhile gradually "soaking," so to speak, into the dielectric; and similarly if a charged condenser be allowed to stand for a time, the P.D. will be found to have diminished (this is, in part, due to leakage). Again, when a condenser is discharged by momentarily connecting its coats by a thick wire, additional smaller discharges can be obtained after a time (termed residual charges). These phenomena are due to what is termed "electric absorption," and emphasise the all-important part played by the dielectrics.

On charging, the solid dielectric is strained to a greater extent than would be an air dielectric, for it sets up a less opposing influence to the forward pressure of the machine (just as an elastic cord offers less opposition to a stretching force than a stiff cord), thus permitting the charge, as it were, to partially soak into the dielectric; and as long as this condition exists the dielectric will be charged even if the coats be removed. Solid dielectrics are not only very much strained, but they tend to retain this condition of strain; when discharged they do not recover themselves at once but do so gradually, and therefore after a time a slight charge appears on the coatings. In air condensers electric absorption is practically absent.

Clearly the capacity of a condenser will be affected by the phenomena referred to above, and any experimental determination will depend on the time of charging; for these reasons *the capacity is often defined as measured by the instantaneous quantity required to produce between the coats a P.D. equal to unity.*

31. Specific Inductive Capacity.—The fact that solid (and liquid) dielectrics permit the transmission of inductive effects better than air has been referred to, and in Art. 18 it is stated that their introduction between the inducing charge and the conductor is equivalent electrically to a shortening of the air space. **If d centimetres denote the thickness of an air dielectric, and kd centimetres the thickness of a dielectric x which is equivalent to it, k measures what has been called the "specific inductive capacity" or "dielectric constant" of x.**

In Arts. 26 and 29 it is explained that the capacity of a conductor is affected by the nature of the surrounding dielectric; it depends, in fact, upon the amount of strain in the dielectric when a given P.D. is established. The greater the strain to which the dielectric is

subjected the greater is the charge accumulated, and hence the greater is the capacity of the conductor. **If the capacity of a conductor be increased k times when it is embedded in another dielectric x in place of air, k measures the specific inductive capacity or dielectric constant of x.**

The preceding may be put into a more convenient form for experimental purposes. Imagine two precisely equal condensers, one with air the other with x for dielectric, and let their capacities be compared by methods to be explained later. **The ratio of the capacity of the condenser with dielectric x to that of the equal air condenser gives the specific inductive capacity, or dielectric constant of x,** *i.e.*—

$$\frac{\text{Capacity of condenser with dielectric } x}{\text{Capacity of equal air condenser}} = \left\{\begin{matrix}\text{Specific Inductive}\\ \text{Capacity of } x\end{matrix}\right\} = k;$$

$$\therefore \left\{\begin{matrix}\textbf{Capacity of condenser}\\ \textbf{with dielectric } x\end{matrix}\right\} = k \left\{\begin{matrix}\textbf{Capacity of an equal}\\ \textbf{air condenser}\end{matrix}\right\}.$$

The above is usually given as the definition of specific inductive capacity or dielectric constant. (Note also Arts. 24, 25, and 28 however.)

From what has been said it follows that the capacity of a condenser depends on: (1) the size of the plates—the larger the plates the greater the capacity; (2) the distance between the plates—the less the distance the greater the capacity; (3) the specific inductive capacity of the dielectric—the greater the value of k the greater the capacity.

Table of Specific Inductive Capacities.

Mica .	= 6·64	India-rubber .	= 2·5
Glass .	= 6 to 9	Gutta-percha .	= 2·462
Ebonite .	= 3·45	Paraffin wax .	= 2·3
Shellac .	= 2·95 to 3·73	Air . . .	= 1

32. Methods of Charging Condensers.—Our condenser of Art. 29 was charged by earthing one coat B and connecting the other A to the positive pole of the machine. If C and V denote respectively the capacity and potential of A, both in electrostatic units, the charge accumulated (Q) and the energy of this charge (W) will be given by—

$$Q = CV \text{ electrostatic units.}$$

$$W = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C} \text{ ergs.}$$

With C *in farads and* V *in volts,* Q *will be in coulombs and* W *in joules.*

The effects enumerated in Art. 29 will be considerably increased if the potential of B, instead of being zero, be made to take a negative value. To accomplish this both plates must be insulated, A joined to the positive pole at potential + V and B to the negative one at potential − V (approx.). The P.D. between the coatings is V − (− V), *i.e.* 2V, and the "charge" and "energy" expressions become—

$$Q = 2CV$$

$$W = \tfrac{1}{2}C\,(2V)^2 = 2CV^2, \text{ etc.},$$

i.e. double the charge and four times the energy obtained by charging in the usual way.

In practical electricity condensers are usually of large capacity, and are charged by means of batteries; in connection with this the following experiment is instructive: B_1 (Fig. 20) is a battery, its poles being connected to a large capacity condenser C through two galvanometers of the reflecting ballistic type, the characteristic property of which is that the first swing is proportional to the quantity of electricity which passes through in any momentary discharge. On arranging the key K_1, as indicated by the dotted line, the coats become charged to the respective potentials of the positive and negative poles of the battery, the galvanometers are momentarily deflected, *and their first swings show that the quantity passing through* G *to* A *is equal to that which passes through* G_1 *from* B.

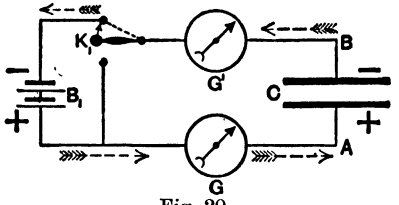

Fig. 20.

If the battery be now cut out of the circuit, and the condenser coatings be joined by moving K_1 to the other stud, the galvanometers will show that the condenser becomes discharged by electricity flowing from A to B through the connecting wire. By varying the number of cells in the battery B_1, estimating the charges (Q) from the galvanometer indications, and measuring the P.D.'s (V) between A and B by an electrostatic voltmeter (Chap. V), it can be proved that the ratio $\frac{Q}{V}$ is constant. In other words, *the capacity of a given condenser is constant*;* it can only be altered by changing one or more of the three factors mentioned in Art. 31.

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* The reader must here distinguish between the *capacity of a condenser*, which is obtained by dividing the charge on the coat A *by the P.D. between the coats* A *and* B, and the *capacity of the coat* A, which is obtained by dividing the charge on A *by the actual potential of* A. In addition to the three factors mentioned in Art. 31, the capacity of A is affected by the electrical condition of B; it is only numerically the same as the "capacity of the condenser" *when* B *is earthed*, for in this case the actual potential of A will be the P.D. between the coats, since B is at zero potential.

If V be the E.M.F. (Art. 108) of the battery in volts and C the capacity of the condenser in farads, the charge given to A is CV coulombs and the energy is $\frac{1}{2}$ CV^2 joules, *i.e.* $\frac{CV^2}{2{\cdot}712}$ foot-pounds, etc. (Art. 27).

33. Standard Condensers and Condensers in Practice.—A convenient type of standard condenser for testing purposes is constructed of alternate sheets of tinfoil (thin lines in Fig. 21) and paraffined paper or mica (thick lines), the odd conducting sheets being bunched together and connected to one terminal A, the even numbers being joined to terminal B; the arrangement is thus equivalent to two large plates separated by a thin and good dielectric. They are frequently of $\frac{1}{3}$ microfarad capacity, this being approximately the capacity of one knot of the Transatlantic cable.

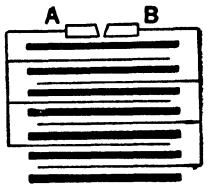


Fig. 21.

An adjustable condenser for telegraph work is shown diagrammatically in Fig. 22. Each consists of tinfoil and paraffined paper, the total capacity of the seven condensers being 7$\frac{1}{4}$ microfarads. Their B coatings are joined to terminal C of the box; the A coatings of four of them can, by inserting plugs, be connected to the brass piece D, while the corresponding coats of the remainder can similarly be put in connection with E. With section D capacities from ·25 to 3·75 microfarads can be obtained with increments of ·25 microfarad; section E varies from ·5 to 3·5 microfarads, the increments being ·5 microfarad. The total 7$\frac{1}{4}$ microfarads can be used by inserting all the plugs and joining the brass pieces D and E.

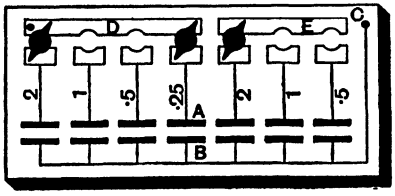


Fig. 22.

For experimental work in statical electricity the *Leyden jar* is a convenient condenser. It consists essentially of a glass jar coated to a certain height both inside and outside (also the bottom) with tinfoil. A metal rod passes through the stopper, its upper end terminating in a ball and its lower end carrying a metal chain which rests on the inside tinfoil.

Every aerial telegraph line is a condenser of small capacity, the conductor forming one coating, the ground and neighbouring earth-joined bodies the other coating, and the intervening air the dielectric; *e.g.* a copper wire ·224 inch diameter and 30 feet above the ground has a capacity of about ·016 microfarad per mile. Concentric and submarine cables are condensers of decided capacity, the surface of the copper conductor being one coat, the water or metal sheath the other, and the insulating material—gutta-percha, india-rubber, diatrine, vul-

canised bitumen, air-paper, etc.—the dielectric. The following values may be quoted:—

Cable.	Dielectric.	Capacity.
Glasgow - Belfast submarine telephone cable from Port Kail to Donaghadee.	Gutta-percha.	·275 m.f. per knot.
Transatlantic cable between Ireland and Newfoundland.	Gutta-percha.	·408 m.f. per knot.
Ferranti high-tension concentric electric light cable.	Compressed brown paper and black wax.	3675 m.f. per mile.
British Insulated Wire Co's. high-tension electric light cable.	Paper.	·31 m.f. per mile

34. Capacity of a Spherical Condenser.—If a conducting sphere A (Fig. 23) of r_1 centimetres radius be concentric with another conducting hollow sphere B of r_2 centimetres radius, the space between being occupied by a dielectric of specific inductive capacity k, it can be proved that the capacity of the condenser thus formed is given by—

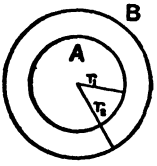


Fig. 23.

$$C = k\frac{r_1 r_2}{r_2 - r_1} \text{ electrostatic units} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$= k\frac{r_1 r_2}{9 \times 10^{11}(r_2 - r_1)} \text{ farads} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$= k\frac{r_1 r_2}{9 \times 10^{5}(r_2 - r_1)} \text{ microfarads} \quad . \quad . \quad . \quad (3)$$

Proof of the Formula.—Let the outer sphere B (Fig. 23) be connected to earth and let the inner sphere A be insulated and have a charge + Q e.s. units communicated to it, thus inducing − Q e.s. units of charge upon the inner surface of the outer sphere.

The potential at all points *inside* the outer sphere due to the outer sphere's charge only $= -Q/kr_2$ (Art. 25).

The potential at all points of the inner sphere due to the inner sphere's charge only $= +Q/kr_1$.

Hence the *resultant* potential of the surface of the inner sphere $= \frac{Q}{kr_1} - \frac{Q}{kr_2} = \frac{Q}{k}\left(\frac{r_2 - r_1}{r_1 r_2}\right)$.

That is:— $$V = \frac{Q}{k}\frac{r_2 - r_1}{r_1 r_2} \text{ e.s. units.}$$

But since the outer coating is at zero potential, this is the *P.D. between the coatings*, and we have seen (Art. 30 and footnote, page 42) that the capacity of a condenser (C) is obtained by dividing the charge on the coating A by the P.D. between the coatings A and B. Thus:—

$$\mathbf{C = \frac{Q}{V} = k\frac{r_1 r_2}{r_2 - r_1}\ e.s.\ units,}$$

and for an air condenser

$$C = \frac{r_1 r_2}{r_2 - r_1}\ \text{e.s. units.}$$

Consider now a spherical air condenser, say, the *external* radius of the coating B of which is r_3 cm. Let B be *insulated* and the condenser charged, in which case + Q e.s. units will appear on its outer surface. The potential of A will be $\frac{Q}{r_1} - \frac{Q}{r_2} + \frac{Q}{r_3}$ and the potential of B will be $+\frac{Q}{r_3}$. Thus the *P.D. between the coatings* will be $\frac{Q}{r_1} - \frac{Q}{r_2}$ and the *capacity of the condenser*, defined as above, will be $\frac{r_1 r_2}{r_2 - r_1}$ as before.

35. Capacity of a Parallel Plate Condenser.—Let A square centimetres denote the surface area of either of the plates of Fig. 19, t centimetres the distance between the plates, and k the specific inductive capacity of any dielectric *completely filling* the space between them; the capacity can be shown to be—

$$\mathbf{C = k\frac{A}{4\pi t}}\quad \text{electrostatic units} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$= k\frac{A}{4\pi \times 9 \times 10^{11} \times t}\ \text{farads}$$

$$= k\frac{A}{4\pi \times 9 \times 10^{5} \times t}\ \text{microfarads.}$$

If the measurements be expressed in inches, then, since 1 inch = 2·54 centimetres, the capacity *in microfarads* becomes—

$$C = k\frac{A \times 2{\cdot}54 \times 2{\cdot}54}{4\pi \times 9 \times 10^{5} \times t \times 2{\cdot}54}$$

$$= k\frac{2{\cdot}25\ A\ (\text{square inches})}{10^{7} \times t\ (\text{inches})} \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

If, instead of filling the entire space between A and B, the dielectric

of S.I.C. k be a slab of thickness d cm., the remaining space being air, the capacity is given by—

$$\mathbf{C} = \frac{\mathbf{A\ (sq.\ cm.)}}{4\pi\left\{(t-d)+\frac{d}{k}\right\}} \text{ electrostatic units} \quad . \quad (6)$$

Proof of the Formula.—Let us reconsider a spherical condenser of inner radius r_1, which is exceedingly large, separated from the outer sphere by a radial distance t, the medium having a specific inductive capacity k. Since $r_2 = r_1 + t$, we have:—

$$C = k\frac{r_1(r_1+t)}{t} \text{ e.s. units}$$

$$= k\frac{r_1^2 + r_1 t}{t} \quad ,,$$

Now, if r_1 be imagined infinitely great, so that any small portion of the sphere is practically plane, and t is small, $r_1 t$ may be neglected, and the expression becomes practically:—

$$C = k\left(\frac{r_1^2}{t}\right) \text{ e.s. units.}$$

Multiply numerator and denominator by 4π, and we get:—

$$C = k\frac{4\pi r_1^2}{4\pi t} \text{ e.s. units,}$$

the numerator, $4\pi r_1^2$, being the area of the whole inner sphere. Since this is true for the whole, it will also be true for any part; hence if A sq. cm. be the area of any part—practically plane—

$$\mathbf{C = k\frac{A}{4\pi t}\ e.s.\ units,}$$

and for a parallel plate air condenser $C = \frac{A}{4\pi t}$ e.s. units.

Formula (6) may be derived by the following reasoning: the actual air-thickness is $(t - d)$ cm., and the slab of thickness d cm. and S.I.C. k is equivalent to an air-thickness of $\frac{d}{k}$ cm. (Arts. 18 and 31). Thus the *total equivalent air-thickness* is $\left\{(t-d)+\frac{d}{k}\right\}$ and the capacity is therefore—

$$C = \frac{A}{4\pi\left\{(t-d)+\frac{d}{k}\right\}} \text{ e.s. units.}$$

36. Capacity of a Submarine or Concentric Cable.—If the length of the cable be l centimetres, the radius (or diameter) of the inner conductor r_1 centimetres, the outer radius (or diameter) of the insulating material r_2 centimetres, and k the specific inductive capacity of the insulation, the capacity of the cable is given by—

$$C = k \frac{l}{2 \log_e \frac{r_2}{r_1}} \text{ electrostatic units}$$

where e = the base of the Napierian logarithms = 2·71828.

Common logs of base 10 can be converted into Napierian logs by multiplying by 2·3026. Hence:

$$\mathbf{C = k \frac{l\ (cm.)}{2{\cdot}3026 \times 2 \log_{10} \frac{r_2}{r_1}}} \text{ electrostatic units} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Further, since $\frac{r_2}{r_1}$ is a mere ratio, r_2 and r_1 may be in any length unit, provided that the same be used for both. Employing practical units:

$$C = k \frac{l\ (cm.)}{9 \times 10^{11} \times 2{\cdot}3026 \times 2 \log_{10} \frac{r_2}{r_1}} \text{ farads;}$$

i.e.

$$\mathbf{C = k \times \frac{2{\cdot}413}{10^{13}} \times \frac{l\ (cm.)}{\log_{10} \frac{r_2}{r_1}}} \text{ farads} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

$$= k \times \frac{2{\cdot}413}{10^{7}} \times \frac{l\ (cm.)}{\log_{10} \frac{r_2}{r_1}} \text{ microfarads} \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

EXAMPLES.

1. *Determine the capacity of one knot of telegraph cable of which the following particulars are known: Diameter of copper conductor* = ·16 *inch; outside diameter of gutta-percha* = ·40 *inch; S.I. capacity of gutta-percha* = 4·03; $\log_{10} 2$ = ·30103.

l = 2,029 yards = (2,029 × 3 × 12 × 2·54) centimetres

k = 4·03

$$\log_{10} \frac{r_2}{r_1} = \log_{10} \frac{40}{16} = \log_{10} \frac{10}{4} = \log_{10} 10 - \log_{10}(2)^2$$

$$= \log_{10} 10 - 2 \log_{10} 2 = 1 - {\cdot}60206$$

$$= {\cdot}39794.$$

Substituting in the above formula, $\mathbf{C = {\cdot}45}$ **m.f. per knot.**

2. *Two cables of equal length have wires of* 80 *and* 100 *mils. diameter and total diameters of* 120 *and* 180 *mils. respectively. What is their relative electrostatic capacity?* $\log 2$ = ·30103; $\log 3$ = ·47712. (*C. & G. Honours.*)

Let l = length of cables, C_1 and C_2 = their respective capacities,

r_1 and r_2 = internal and external diameters of insulation of the first cable,

r_3 and r_4 = internal and external diameters of insulation of the second cable

Then—

$$\frac{C_1}{C_2} = \frac{k \times \dfrac{l}{2 \log_{10} \dfrac{r_2}{r_1} \times 2{\cdot}3026}}{k \times \dfrac{l}{2 \log_{10} \dfrac{r_4}{r_3} \times 2{\cdot}3026}} = \frac{\log_{10} \dfrac{r_4}{r_3}}{\log_{10} \dfrac{r_2}{r_1}} = \frac{\log_{10} \dfrac{180}{100}}{\log_{10} \dfrac{120}{80}}$$

$$= \frac{\log_{10} \frac{9}{5}}{\log_{10} \frac{3}{2}}$$

$$\log_{10} \tfrac{9}{5} = \log_{10} 9 - \log_{10} 5 = \log_{10} 3^2 - \log_{10} \tfrac{10}{2}$$

$$= 2 \log_{10} 3 - \{\log_{10} 10 - \log_{10} 2\};$$

$$\therefore \log_{10} \tfrac{9}{5} = {\cdot}95424 - \{1 - {\cdot}30103\} = {\cdot}25527.$$

$$\log_{10} \tfrac{3}{2} = \log_{10} 3 - \log_{10} 2 = {\cdot}47712 - {\cdot}30103 = {\cdot}17609.$$

Hence

$$\frac{C_1}{C_2} = \frac{{\cdot}25527}{{\cdot}17609} = \mathbf{1{\cdot}4.}$$

37. Condensers in Parallel, Abreast or in Battery.—This arrangemen is indicated in Fig. 24; it is equivalent to forming one large condenser, the area of whose coatings is equal to the sum of the areas of the coats of the individua condensers. Hence if C be the joint capacity, C_1, C_2, C_3 the separate capacities

$$C = C_1 + C_2 + C_3.$$

Fig. 24.

If the condensers are alike, say each of capacity C_1, the joint capacity is 3 C_1, and if there are n condensers the combined capacity, being as before denoted by C, then C is equal to nC_1.

Again, if V be the P.D. between the coatings, the charge given to each is C_1V and the total charge is CV, *i.e.* $n\,C_1V$.

Finally, the energy of each is $\frac{1}{2} C_1V^2$ and the accumulated energy of the battery is $\frac{1}{2} CV^2$, *i.e.* $n \frac{1}{2} C_1V^2$.

Summary of n equal Condensers in Parallel.

1. Joint capacity = n times the capacity of one condenser.
2. Total charge = n times the charge of one condenser used alone.
3. Total energy = n times the energy of one condenser used alone.

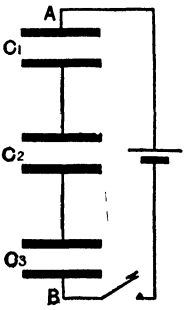


Fig 25.

38. Condensers in Series or in Cascade: Mixed Arrangement.—A series or cascade is indicated in Fig. 25. Since the outflow from one condenser passes to the next, the charge Q on the positive coating of each must be the same and equal to that communicated to the first condenser from the battery. If V be the P.D. between A and B, V_1, V_2, V_3 the P.D.'s between the coatings of the separate condensers, C_1, C_2, C_3 the individual capacities, and C the joint capacity,

$$V = V_1 + V_2 + V_3$$
$$= \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}.$$

But $$Q = CV \therefore V = \frac{Q}{C}.$$

Hence $$\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}.$$

$$\therefore \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.$$

If the capacities are the same, say each equal to C_1, the above expression becomes $\frac{1}{C} = \frac{3}{C_1}$ *i.e.* $C = \frac{1}{3} C_1$. Similarly if there are n equal condensers each of capacity C_1, the joint capacity is $\frac{1}{n} C_1$.

Again, since the condensers are alike, the P.D. between the coatings of each must be the same, and equal to $\frac{1}{n}$ of the total P.D. V, *i.e.* the P.D. for each, is $\frac{V}{n}$; thus the charge on each is $C_1 \frac{V}{n}$, and the total charge $n \times C_1 \frac{V}{n}$, *i.e.* $C_1 V$. This is equal to the charge which one condenser would receive from the battery if used alone.

Finally the energy of each is $\frac{1}{2} C_1 \frac{V^2}{n^2}$, and the accumulated energy of the whole cascade is $n \times \frac{1}{2} C_1 \frac{V^2}{n^2}$, *i.e.* $\frac{1}{n} \times \frac{1}{2} C_1 V^2$. This is only $\frac{1}{n}$ of the energy of a single condenser used alone.

Summary of n equal Condensers in Series.

1. Joint capacity $= \frac{1}{n}$ of the capacity of one condenser.
2. Total charge = same as on one condenser used alone.
3. Total energy $= \frac{1}{n}$ of the energy of one condenser used alone.

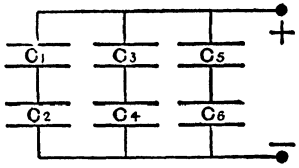


Fig. 26.

If six capacities are arranged, two in series and three rows in parallel (Fig. 26), the capacity of the first row obtained from the relation $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$ is $\frac{C_1 C_2}{C_1 + C_2}$. Similar expressions are obtained for the remaining rows, and the total capacity is clearly—

$$C = \frac{C_1 C_2}{C_1 + C_2} + \frac{C_3 C_4}{C_3 + C_4} + \frac{C_5 C_6}{C_5 + C_6}.$$

Exercise.—Write down a list of all the capacities it is possible to obtain with three condensers of 1, 2, and 3 microfarads respectively (*C. & G.—O.*).

39. Determination of the Capacity of a Cable or Condenser.—Fig. 27 illustrates a good method if the cable is not too long. In it B is a convenient battery, A a standard condenser, E V an electrostatic voltmeter (Art. 41), and C the cable, one end a of which is insulated and left "free," the other end b being also insulated, but with a little removed for connecting purposes. For the present the reader must simply regard E V as similar in principle to a condenser —one system, however, being movable, the amount of which motion measures the P.D. applied in volts.

(1) Place the switch S on stud c, thus charging A and E V to the potential of the positive pole of B, and note the reading V_1 volts of E V.

(2) Move the switch to stud d; the charge is shared with C, and the common potential becomes V_2 volts indicated by E V.

Now let Q coulombs denote the charge from the battery, C farads the capacity of A, C_x farads the capacity of C, F_1 farads the capacity of the voltmeter at V_1 volts, and F_2 its capacity at V_2 volts; then in the first case $Q = (C + F_1)V_1$, and in the second case $Q = (C + C_x + F_2)V_2$. Hence—

$$(C + F_1)V_1 = (C + C_x + F_2)V_2$$

$$\therefore C_x = \frac{C(V_1 - V_2) + F_1V_1 - F_2V_2}{V_2}.$$

F_1 and F_2 may be taken from data supplied with the voltmeter.

If the capacity of E V be neglected, we have $CV_1 = (C + C_x)V_2$.

$$\therefore \frac{C_x}{C} = \frac{V_1 - V_2}{V_2}.$$

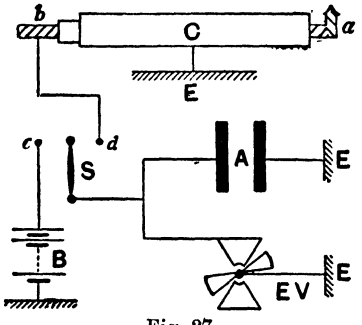


Fig. 27.

In testing the capacity of a short length of cable, it is customary to coil it in a metal tank containing water, leaving only the ends outside; the tank and water correspond to the "earth" in Fig. 27.

The specific inductive capacity k of the dielectric of a cable may be found by finding, as above, the capacity C farads of a known length l cm., and applying the relation of Art. 36 (No. 8).

The capacity of any condenser may be determined as in Fig. 27, the condenser taking the place of the cable.

The specific inductive capacity of any solid or liquid insulator may be found by constructing two suitable condensers exactly alike, except that one has air and the other the given substance for dielectric, comparing their capacities, and applying the third definition of Art. 31.

The capacity of a condenser may also be readily determined by the aid of a ballistic galvanometer. The condenser of unknown capacity C_x farads is charged by a cell of E.M.F. V volts, then discharged through the galvanometer, and the first swing d_1 is observed. This is repeated with the standard condenser of capacity C farads. If d_2 be the first swing—

$$\frac{\text{Charge on the Condenser}}{\text{Charge on the Standard}} = \frac{d_1}{d_2}; \quad \therefore \frac{C_xV}{CV} = \frac{d_1}{d_2}, \text{ } i.e. \text{ } C_x = \frac{d_1}{d_2}C.$$

40. The Cambridge Capacity Testing Set.—This is a portable testing set for the ready determination of capacities ranging from ·01 to 10 microfarads. The principle is that of the ballistic galvanometer method of the preceding section: Fig. 28 shows the construction, and explains the action. The battery is connected to the terminals + and — on the right, and the capacity under test is joined to the terminals X on the left. The two-way switch on the left can bring into circuit either the unknown capacity at X (as shown in the figure) or a standard condenser of ·1 microfarad capacity contained in the instrument. The ballistic galvanometer which is of a special type known as the "unipivot" (Chap. XII.) is shown in the centre. The resistance coils S on the right are joined across the galvanometer as shown: they are provided with five contact studs, and according to the stud on which the lever is placed, the galvanometer deflection, in a test, must be multiplied by 1, 5, 10, 50, or 100.

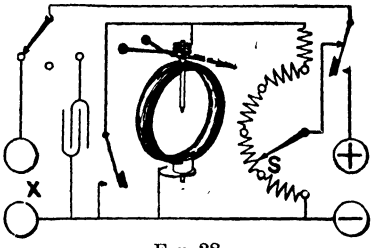


Fig. 28.

With the standard condenser in circuit the key on the right is depressed (thus charging the condenser) and then released: the condenser thus discharges through the galvanometer. The lever S is adjusted until a suitable large deflection is obtained on thus charging and discharging. Let d_1 be the deflection, and S_1 the multiplier (according to the position of S). The unknown capacity is then brought into circuit by means of the two-way switch on the left and the test repeated. Let d_2 be the deflection, and S_2 the multiplier.

Then—

$$\frac{\text{Capacity under test}}{\cdot 1 \text{ m.f.}} = \frac{d_2 S_2}{d_1 S_1} \quad \therefore \text{ Capacity under test} = \frac{d_2 S_2}{10 d_1 S_1} \text{ m.f.}$$

The key just to the left of the galvanometer is merely a short-circuiting key to bring the pointer of the galvanometer quickly to rest. The whole is contained in a neat teak box, the switches, etc., inside being manipulated by projecting knobs. The "multiplying part" due to the position of S will be better understood after reading Chap. XII.

A Few Additional Capacities for Reference.

The following are merely inserted for reference. The capacities are in electrostatic units, and "lengths" in centimetres. For the method of converting the logarithms to common logarithms of base 10 the student should refer again to Art. 36. For "proofs" see *Advanced Text-book of Magnetism and Electricity.*

(1) Isolated wire, length l, radius r:—

$$C = \frac{l}{2 \log_e \frac{l}{r}}.$$

(2) Two parallel isolated telegraph wires, distance d apart, and each of radius r:—

$$C = \frac{1}{4 \log_e \frac{d}{r}} \text{ per unit length.}$$

(3) Telegraph wire, radius r, height h above the ground:—

$$C = \frac{1}{2 \log_e \frac{2h}{r}} \text{ per unit length.}$$

EXAMPLES.

1. *One of the Ferranti mains between Deptford and London has a capacity of* 3 *microfarads. What is the work stored in it in ergs if the inner and outer conductors are charged to a P.D. of* 10,000 *volts?* (*C. & G.—O.*)

By Art. 27, if C and V be the capacity and P.D. in electrostatic units,

Work or Energy = $\frac{1}{2}CV^2$ ergs.

Since 1 farad = 9×10^{11} electrostatic units,

1 microfarad = $\frac{1}{10^6}$ farad = 9×10^5 electrostatic units;

$\therefore$ 3 microfarads = $3 \times 9 \times 10^5$ electrostatic units = C.

And 1 volt = $\frac{1}{300}$ electrostatic unit;

$\therefore$ 10,000 volts = $\frac{10000}{300} = \frac{100}{3}$ electrostatic units = V;

$\therefore$ Energy = $\{\frac{1}{2} \times 3 \times 9 \times 10^5 \times \frac{100}{3} \times \frac{100}{3}\}$ ergs

$= \{1{\cdot}5 \times 10^9\}$ **ergs.**

2. *A conducting sphere of diameter* 6 *is electrified with* 105 *units; it is then enclosed concentrically within an insulated and unelectrified hollow conducting sphere formed of two hemispheres of thickness* $\frac{1}{4}$ *and internal diameter* 7. *The outer sphere is then put to earth. Determine the potential of the inner sphere before and after the outer sphere is earthed.* (*B.*)

Before earthing we have a negative charge induced on the inner surface and a positive charge on the outer surface of the outer sphere, and since the latter *completely surrounds* the inducing charge the induced charges *are each equal to the inducing charge*, viz. 105 units; hence (Art. 25), if V be the potential of the inner sphere,

$$V = \frac{105}{3} - \frac{105}{3\frac{1}{2}} + \frac{105}{3\frac{3}{4}} = \mathbf{33}.$$

After earthing we have, since the + 105 units on the outer sphere have gone to earth,

$$V = \frac{105}{3} - \frac{105}{3\frac{1}{2}} = \mathbf{5}.$$

EXERCISES IV.

1. An insulated conductor placed at some distance from other conductors receives a positive charge. Is its potential altered when another conductor uninsulated and uncharged is brought near to it? Give reasons for your answer. (B.)

2. Describe the construction of a condenser of moderately large capacity, and state how you would compare it with another condenser of which the capacity was known. (B.)

3. An electrified body is brought into the neighbourhood of (*a*) an insulated conductor and (*b*) an earth-connected conductor. Describe exactly the effect on the potentials of the electrified body and of the unelectrified conductors in each case. (B.)

4. How would you measure the static capacity of a length of open telegraph line? (O.)

5. What is meant by the electrostatic capacity of an insulated wire, and how may it be measured? (O.)

6. A condenser of 6 m.f. capacity is joined in "cascade" to a condenser of 3 m.f. capacity; what is the capacity of the combination? (H.)

7. What is meant by charging Leyden jars (condensers) in cascade? Three Leyden jars, whose capacities are $\frac{1}{2}$, 1, $1\frac{1}{2}$, are joined in cascade; what is the capacity of the combination? (B.)

8. If you had two samples of insulated wire, the diameters of the wires and gutta-percha covering being known, how would you calculate the relative electrostatic capacities of the two wires? (H.)

9. A condenser having a capacity of 2 m.f. is connected to two terminals maintained at 2,000 volts; how much work is taken from the terminals, and how much can be got out of the condenser again? (O.)

10. Two condensers are exactly alike, except that one has air and the other glass for dielectric. Equal charges are given to the two condensers. In which is the energy of the charge greater? (B.)

11. An insulated sphere of 2 cm. radius is connected by a long, thin wire with another insulated sphere of 6 cm. radius, which is surrounded by a third sphere of 8 cm. radius concentric with it. The connecting wire passes through a small hole in the third sphere, so as not to touch it. All the spheres are conductors. Calculate the capacity of the two connected spheres. (B.)

12. Calculate the size of plates that must be separated by a layer of air ·01 in. in thickness, so that the air condenser may have ·0001 m.f. capacity. (O.)

13. Calculate the capacity in microfarads of a condenser composed of two plates each $1\frac{1}{2}$ m. long, 65 cm. wide, placed parallel to one another at a distance ·2 mm. apart. 1 f. $= 9 \times 10^{11}$ electrostatic units. (O.)

14. A Leyden jar consists of two concentric spheres of 5 and 6 cm. diameter respectively, the intervening space being filled with air. The outer sphere is uninsulated, the inner is charged with 20 units of electricity. How much work is done when the inner sphere is put to earth? (B.)

15. Three equal similar condensers are connected (1) in series, (2) abreast, and in each case the set is charged as fully as can be by the same machine. Compare the heats produced by completely discharging the two sets. (B.)

16. You have two Leyden jars of the same kind of glass; the coatings of one measure each 1 sq. ft, and the glass is $\frac{1}{10}$ in. thick; the coatings of the other measure each 3 sq. ft., and the glass is $\frac{1}{5}$ in. thick. They are charged to the same P.D.'s. Compare their charges and the heats produced by discharging them. (B.) (Use the formulae of Art. 35.)

17. How would you combine four condensers, each of capacity 1 m.f., so as to produce a capacity of ·75 m.f.?

18. A copper wire ·02 in. in diameter is insulated by a covering of gutta-percha ·01 in. in thickness; what thickness of gutta-percha would be required to insulate a copper wire ·03 in. in diameter in order that its capacity may be the same as that of the first wire? (H.)

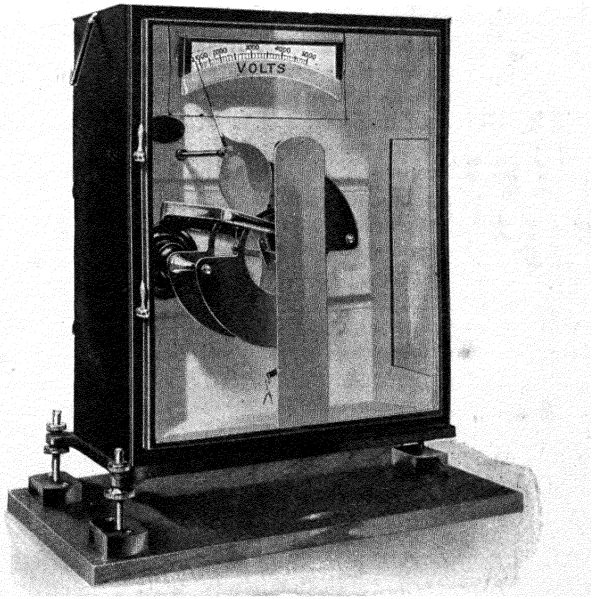
19. The capacity of 1 mile of copper wire 50 mils in diameter covered with gutta-percha to a thickness of $62\frac{1}{2}$ mils is ·29 m.f.; what is the capacity of a wire 100 mils diameter covered to a diameter of 390 mils?

log 3·5 = ·5440680, log 3·9 = ·5910646.

CHAPTER V.

ELECTROSTATIC MEASURING INSTRUMENTS.

41. The Kelvin Vertical Electrostatic Voltmeter.—An instrument designed for the measurement of P.D.'s in practical units (volts) is termed a voltmeter; in the present chapter only those based on electrostatic principles will be considered.



The Kelvin Vertical Electrostatic Voltmeter, which, in common with all electrostatic measuring instruments, depends for its action on the mutual attraction between two conductors at different potentials, is represented in Fig. 29. It consists of two fixed brass plates, each of the shape of a double quadrant, placed parallel to, and in conducting communication with, each other, but thoroughly insulated from the other parts of the apparatus. Between these is a thin light aluminium

vane resting at its centre on a knife edge, and capable of swinging in a vertical plane exactly midway between the fixed conductors.

The upper point of the vane carries a fine aluminium pointer which moves over the scale of the instrument, while the lower end carries a projection and knife edge, to which various controlling weights supplied with the instrument may be attached. Two stops, seen projecting from the back of the case, are placed so as to curtail the movements of the vane and prevent damage to the pointer, and any inconvenient oscillation of this moving system is prevented by causing the pointer to rub against an insulated horizontal checker, the latter being manipulated by the checker handle outside the instrument.

As a protection against air currents and the influence of electrified bodies, the whole is enclosed in a metal case provided with a glass front and a glass window at the side. The movable vane is, through the medium of its central knife edge, in conducting communication with the case and with a terminal attached thereto; the fixed insulated conductors are connected to a second terminal thoroughly insulated from the case. The whole instrument rests on three levelling screws, and is provided with an insulating base with spirit-level attached.

If the movable vane and the fixed plates be respectively connected to two points at different potentials they become charged to this P.D. and the electrostatic attraction between them tends to draw the vane completely within the fixed conductors, thus increasing the capacity of the system. If Q denotes the charge given to either, C the capacity and V the P.D., Q is equal to C V; in other words, the quantity is proportional to the P.D., so that if the latter be doubled, the charge on each will be doubled also. But since the force between two charged bodies is proportional to the product of the charges, doubling the charge on each will increase the force fourfold; hence *the force of attraction is proportional to the square of the P.D.* This force is opposed by the particular weight attached to the lower extremity of the vane, and clearly the latter will come to rest in such a position that these two opposing influences are equal and balance each other.

In one type of instrument the scale is graduated from 0 to 60, the value in volts of each division depending on the controlling weight employed. Three weights are supplied; with the smallest one each division reads 50 volts, so that the maximum reading is 3,000 volts; with the intermediate weight *in addition* each division represents 100 volts, the maximum being 6,000 volts, and *with all three weights* the reading is 200 volts per division, the maximum voltage being in this case 12,000. In the type shown in Fig. 29 the ranges are 1,000—5,000, 2,000—10,000, and 4,000—20,000 volts.

In practice one of the conductors to be joined to the voltmeter is often earthed; in such instances the earthed main should be attached to the terminal in contact with the metal case and vane, for the case will then be at zero potential the same as the human body, and there will be no danger should it be accidentally touched.

It should be observed that the attractive force for a given P.D. is the same, and in the same direction, whether the vane is at a higher or lower potential than the fixed conductors; hence voltmeters of this type can be employed either for continuous pressures or pressures rapidly alternating in direction. Once calibrated for continuous pressures, they read alternating pressures (virtual volts) direct. (See Chap. IX.)

In transit from place to place, damage to the instrument is prevented by easing the vane from its bearings, passing the right-hand stop through its upper wing, and placing its knife edge in slots provided for the purpose.

The following "**general warning**" by the makers is of vital importance when the instrument is in use on high-pressure systems: "*Never open the case to change the weights nor touch the terminals to connect or disconnect without being sure either that the dynamo is not running or that both the conductors leading to the voltmeter are safely disconnected from its circuit.*"

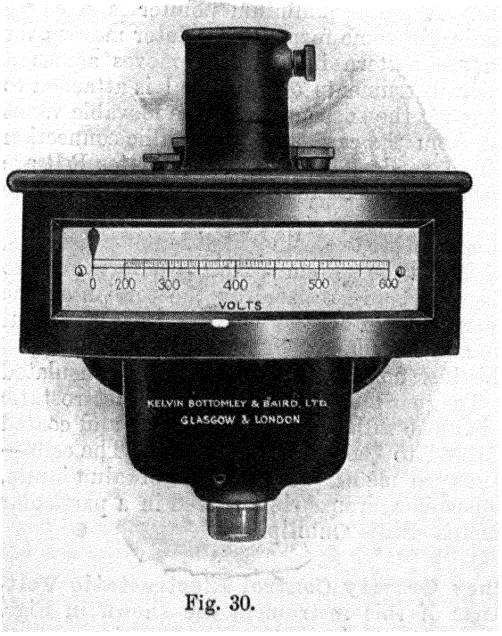


Fig. 30.

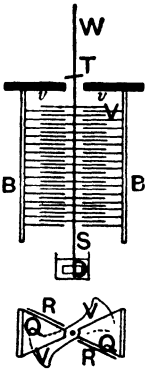


Fig. 31.

42 The Kelvin Multicellular Electrostatic Voltmeter.—This is a modified form of the preceding, specially designed for so multiplying the electrostatic effects that smaller P.D.'s can be measured. Separate instruments, varying in range from 20 to 600

volts, are constructed. The general appearance of the latest pattern is depicted in Fig. 30, the essential parts being shown diagrammatically in Fig. 31.

The fixed system consists of two sets of fifteen cells or boxes supported by a vulcanite plate *v*. These cells are formed of triangular brass plates Q fixed equidistant and parallel to each other on two brass supports B B, the two sets being in metallic connection, but thoroughly insulated from the other parts of the apparatus.

The moving system consists of fifteen light aluminium vanes V fixed parallel and at equal distances apart on a vertical spindle S, and capable of rotating about this vertical axis. The upper end of this spindle passes through a hole in the case, and is attached to a small coach spring T which is suspended by a platinum iridium wire W from a torsion head at the top of the instrument, while the lower end of the spindle carries a disc D which, moving in oil, prevents undue oscillation of the moving system and renders the instrument "dead beat"—an efficient uniform "damping" device in this case, since the whole of the disc rests continually immersed in the oil, no part being at any stage withdrawn therefrom. A light aluminium pointer is fixed to the upper part of the spindle; in some forms this pointer moves over a horizontal scale, but in the pattern indicated it moves across a vertical scale. In the earlier instruments one terminal is attached to the metal case, and therefore *via* the suspension to the movable vanes V; the other is insulated from the case, and in metallic connection with the insulated cells. Two vertical brass repelling plates R R are fixed to the lower side of the cover, and limit the movements of the system V.

On connecting the two terminals to two points at different potentials, the electrostatic attraction causes the system V to move forward into the cells, the motion in this case being resisted by the torsion of the suspending wire. The remarks made on the working of the vertical voltmeter apply also to the multicellular instruments.

In the latest form of this instrument the case is entirely insulated from the working parts. To guard these parts against electrostatic effects, the whole interior is lined with insulating material and coated with tinfoil, which is connected to the movable system. The cells—ten in number—are cast in type metal, the vanes are of aluminium, and the suspension is of phosphor bronze ribbon aged in a particular way. The range can be increased by "multipliers."

43. The Ayrton-Mather Gravity Control Electrostatic Voltmeter.—The essential parts of this instrument are shown in Figs. 32-34.

The fixed system consists of three part-cylindrical conductors A, B, C, of polished and gold-plated brass fixed to the plate D and forming two cylindrical boxes or cells. The movable part consists of two part-cylindrical plates E, F of polished aluminium fixed to a horizontal

axle G pivoted in jewelled screws, and capable of moving about this axis into and out of the cells. The front of the axle G carries a pointer which stands at zero on the scale when the moving system is completely outside the fixed as indicated by the dotted lines in Fig. 32. To the frame which carries the moving part a horse-shoe magnet is attached, its length being parallel to the axis G and one of its poles N over-lapping the other S, thus leaving a narrow air space between them.

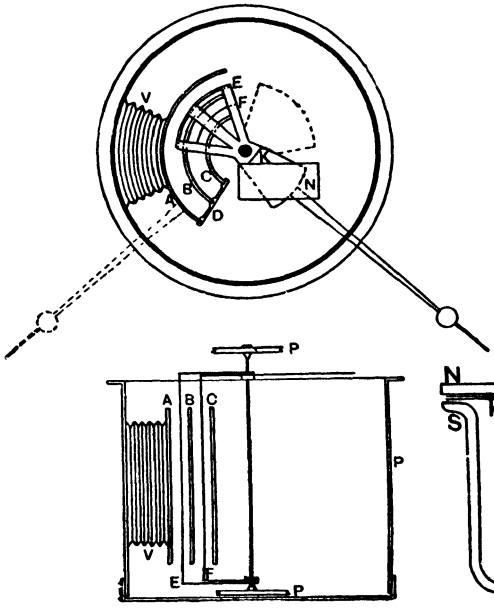


Fig. 32. Fig. 33.

A sector-shaped copper piece K, which is fixed to the movable part, passes across the air space between the poles when a deflection occurs; the reader will learn later that in such cases currents of electricity are developed in the copper, their direction being such as to oppose the motion, although they do not lessen the final deflection; in this way undue oscillation of the moving system is prevented, and the instrument rendered dead beat. In this voltmeter the cells and the vanes are both thoroughly insulated from each other, and from the outer containing metal case; they are put in conducting communication with the two terminals through fuses which protect the parts in case of any short circuit or abnormal rise in voltage. The terminals are insulated from the case by being fixed in a vulcanite block, and the screws are equipped with vulcanite heads as a further precaution for

safety. A weight attached to an arm on the axle furnishes the controlling influence which balances the electrostatic attraction when the two systems are connected to two conductors at different potentials; this attractive force is proportional to the square of the P.D., and the instrument can be used with equal accuracy on direct or alternating pressure circuits.

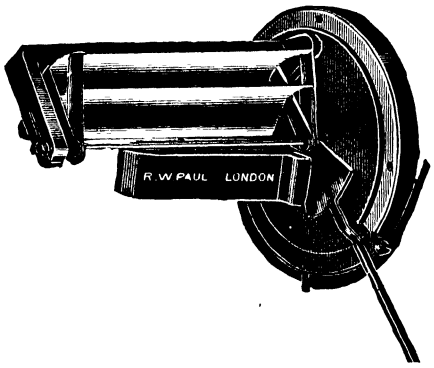


Fig. 34.—Movable Part and Damping Arrangement.

44. The Addenbrooke Low-Pressure Electrostatic Voltmeter.—This is a laboratory instrument for the measurement of small P.D.'s, direct or alternating; its working parts are depicted in Fig. 35.

The movable system consists of two brass crosses placed as indicated, and insulated from each other by mica. The whole is suspended by a phosphor bronze strip, through the medium of which the upper cross is in conducting communication with the metal case and a terminal attached thereto. The lower cross is insulated from the case, and connected to an insulated terminal.

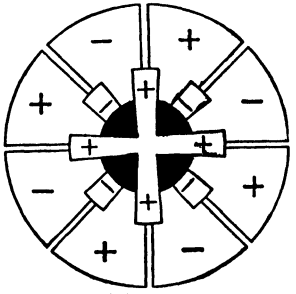


Fig. 35.

The fixed system consists of two discs of brass each divided into eight sectors, the latter being held by eight supporting pillars. One of these discs is fixed above and the other below the moving system; in the figure the upper disc is removed. Four alternate sectors are insulated from the case joined together and to the lower cross, and therefore in metallic connection with the insulated terminal; the remaining sectors are joined to the case, to the upper cross, and to the terminal in connection with it.

Clearly, on joining the terminals to two points at different potentials, say the upper cross to the higher potential, the systems become charged to the relative potentials indicated, and the moving part will be deflected in a counter-clockwise direction. The controlling influence is the torsion of the phosphor bronze strip; the latter carries a mirror, and the deflection is read by means of a beam of light reflected therefrom to a scale.

45. Increasing the Range of Electrostatic Voltmeters.—In the chapter dealing with the transmission and distribution of electrical energy the reader will learn that to secure economy in the copper conducting mains high alternating pressures are essential; in actual practice 10,000 to 20,000 volts has been reached, and abroad 130,000 volts has been attained. In high-pressure systems voltmeters are generally put on the low-pressure side of transformers (Chap. XX.); but should it be desirable to connect them direct to the mains instruments of the electrostatic type are undoubtedly the best, and methods for increasing their range call for attention. The plan, due to Professors Ayrton and W. Peukert, is given diagrammatically in Fig. 36. A B C D and E are five condensers, say of equal capacity, joined in "series" between the two mains, maintained at a P.D. of say 10,000 volts; thus the difference in potential between the coatings of each condenser is 2,000 volts. The voltmeter V is joined across one condenser as indicated; clearly its reading, multiplied by five, will give the total pressure, or it may be graduated to read the total pressure direct. The condensers must each have a *very large capacity compared with* V, otherwise the joint capacity of A and V in parallel will much exceed that of A alone, thereby upsetting the simple voltage ratio mentioned above. The arrangement also reduces the tendency to sparking between the two systems which may arise in consequence of any unusual rise in voltage.

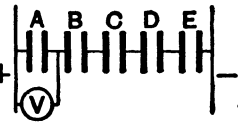


Fig. 36.

46. Advantages of Electrostatic Measuring Instruments.—These may be briefly summarised as follows:—

(1) Being constructed of two disconnected systems electrostatic voltmeters have no "way through"; hence they take no current and waste no energy.

(2) Since their action does not depend on the passage of a current through them they are free from errors due to variation in temperature, and in magnetic field.

(3) They are not affected by stray lines from dynamos or other external magnetic fields.

(4) They act with either direct or alternating pressures, and the law of the instruments is such that when calibrated with continuous pressures they read "alternating pressures" direct. (See Chap. IX.)

(5) They are absolutely independent of periodicity and wave form. (See Chap. IX.)

(6) In laboratory work where battery cells are under test or where the experiment is such that the constancy of the cells is essential, an electrostatic measuring instrument is frequently desirable, for taking no current, it neither alters the P.D. to be measured nor causes "polarisation" (Chap. XIII.) in the battery.

47. A few Errors and their Prevention.—(1) Unless special care be taken, external electrified bodies considerably affect the instruments. The object of the conducting case is to screen the parts from external electrification; but it has been discovered that the mere rubbing of the glass face to remove dust causes sufficient electrification to produce errors. To avoid this Messrs. Ayrton and

Mather coat the glass with a conducting varnish of transparent gelatine, glacial acetic acid, sulphuric acid, and anti-sulphuric enamel.

(2) In some types sparking between the systems may arise if the voltage suddenly increases to an unusual degree. Some manufacturers test their instruments to more than double the normal pressure. In addition to fuses, some are provided with a spark gap, which permits the transit of a spark when the pressure increases to a certain value. Messrs. Ferranti have overcome the difficulty by placing in series with the instruments liquid resistances such that the contained liquid boils away and breaks the circuit immediately any unusual rise in pressure occurs.

(3) *A Defect* :—The lower parts of the scales of these voltmeters are somewhat crowded and the higher parts very open. To remedy this Professor Ayrton suggests a condenser *in series* with the instrument so that the total P.D. divides between them *inversely* as their capacities ($V = Q/C$). When the moving part is only slightly within the fixed, the capacity of the voltmeter is low, and thus the volts on it will be relatively high. When the moving part is considerably within the fixed the capacity is higher, and therefore the volts on it relatively lower. Thus the effect will be to spread out the readings in the lower parts of the scale and to compress the readings in the higher parts. Of course the instrument must be calibrated to give the correct total voltage.

48. Electrometers and Electroscopes.—The electrostatic measuring instruments which have been described have originated from earlier instruments, used in the investigation of statical electricity, and termed *electrometers*.

Lord Kelvin's early form of **Quadrant Electrometer** consists of four quadrants of metal, *a*, *b*, *c*, *d* (Fig. 37), being really four quarters of a flat cylindrical box. Each quadrant is supported on an insulating pillar, but *a* and *c* are connected together by a wire, and so are *b* and *d*. A light needle of aluminium, *n*, of paddle shape, is suspended by a fine fibre so as to hang inside the cylindrical box, just clear of the quadrants.

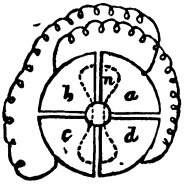


Fig. 37.

When everything is at the same potential the needle hangs symmetrically, as shown in the diagram. If the needle alone is charged to a high potential it still remains as shown, but if the adjacent quadrants are then connected to a source of P.D. the needle will tend to move from the higher potential quadrants to the lower. In such a case it can be shown that the deflection, read by means of a mirror and scale, is proportional to the potential of the needle and to the potential difference between the two pairs of quadrants. If, therefore, the needle be kept at a *constant high* potential, the deflection will be a measure of the *potential difference between the two pairs of quadrants*, and if one pair be earthed (zero potential) and the other pair connected to an electrified body, the deflection will be a measure of the *common potential of the body and the quadrants* to which it is joined. If the needle and one pair of quadrants are connected together, and are at a much higher potential than the other pair, then obviously the law becomes that the deflection will be proportional to the *square of the potential difference between the two pairs of quadrants.*

The formula for the instrument is:—

$$\theta = K\left(V - \frac{v_1 + v_2}{2}\right)(v_1 - v_2)$$

where θ = deflection, K = a constant, V = potential of needle, v_1 and v_2 = the potentials of the two pairs of quadrants. From this the statements above immediately follow. Thus if V be big the term $(v_1 + v_2)/2$ may be neglected and we get:—

$$\theta \propto V(v_1 - v_2).$$

If one pair of quadrants be earthed, say $v_2 = 0$ we have:—

$$\theta \propto Vv_1.$$

Finally, if $v_1 = V$ we get (from the first expression):—

$$\theta \propto \left(\frac{V - v_2}{2}\right)(V - v_2),$$

i.e. $$\theta \propto (V - v_2)^2$$

Some forms of the electrometer are very complex in construction. The needle is connected to a Leyden jar, which is charged by means of a small induction machine, called a *replenisher*, and its charge and potential are noted by another gauge electrometer working on what is termed the *attracted disc* principle, observations on this latter instrument indicating when the needle must be recharged. The whole is very elaborate and costly and is now comparatively little used.

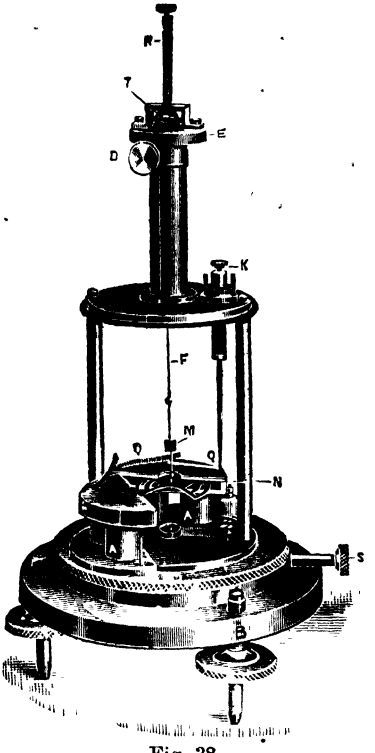


Fig. 38.

In a very neat form, lately introduced by F. Dolezalek, the whole is of more slender construction, the needle being of thin, silvered paper, and the suspension a fine quartz fibre. In order to charge the needle, the quartz fibre is coated with a little calcium chloride, which is always moist in air and a conductor, thus furnishing a path over the surface of the fibre to the needle. With the needle at 100 volts, quite a large deflection can be obtained when the quadrants are connected to the poles of a single primary cell. Fig. 38 shows the instrument with the case removed.

The principle of the **Attracted Disc Electrometer,** mentioned above, may be just stated here. In Fig. 39, P P is an insulated plate which can be raised or lowered by a micrometer arrangement. A A B B is a metal box, the bottom of which has a small square piece C in the centre (the movable plate) separated

rom the remainder B B (the guard ring) by a cut all round. C is suspended by spring from the top of the box and, normally, it lies above the level of the guard ring B B. When the two systems are at different potentials C is attracted by the plate P P, and the latter is raised or lowered until C is just on a level with he guard ring B B. If M grm. be necessary to make C sink to the level of the guard ring, the P.D. V between the two systems is given by—

$$V = d\sqrt{\frac{8\pi M g}{S}} \text{ e.s. units,}$$

where d = distance apart of the plates in cm., S = the area of C in sq. cm., and g = intensity of gravity. Although a fuller treatment of this electrometer annot be given here the student would do well to read up the "theory and pplication" in the *Advanced Text-Book of Magnetism and Electricity.*

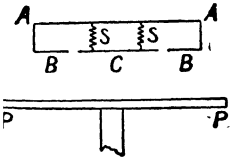


Fig. 39.

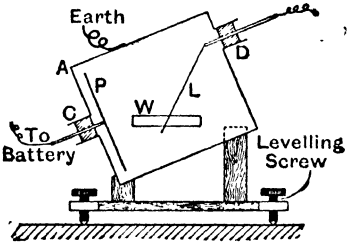


Fig. 40.

A simple type of **Electroscope** for elementary experiments in statical electricity consists of a tin box fitted with two opposite sides of glass and a metal od passing vertically through the cover of the box, its upper end terminating in

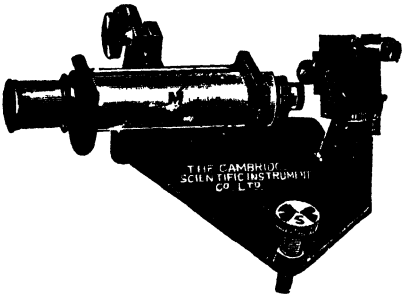


Fig. 41.—General view of Tilted Electrometer.

disc and its lower end carrying two gold leaves which normally hang side by de. The metal rod is insulated from the case. The action depends on the fact hat *the leaves will diverge if there is a P.D. between them and the case.* Thus if he case be earthed (zero potential) and the disc be connected to, or brought ear, a charged body the leaves will diverge and the amount of divergence will

indicate whether the potential is strong or weak. For other applications the student should refer to the *Advanced Text-Book of Magnetism and Electricity.*

A modern type due to C. T. R. Wilson for experimental work on the conduction of electricity through gases is shown in Fig. 40. A is a brass box provided with a window W through which the movement of the single gold leaf L can be observed and measured by means of a microscope (and scale). P is a brass plate joined to a battery and therefore charged. The insulation at D and C is as perfect as possible (say sulphur or quartz). L is joined to the insulated system into which the "current" under examination is flowing. As the voltage of this system changes L is attracted towards P moving about 300 or 400 eye-piece divisions per volt. Thus knowing the capacity C of the electroscope and (from the deflection) the "change of voltage (V) per second" the current flowing into the system is known ($Q = C V$ and current is "quantity flowing per second"). The reader will understand these points better as his studies proceed. Fig. 41 shows one type of the instrument complete.

EXERCISES V.

1. Describe a good form of voltmeter for use on an alternating current system. (O.)

2. What are the special advantages and disadvantages of electrostatic voltmeters as compared with current voltmeters? Describe in detail some form of electrostatic voltmeter that is suitable for use in a central station. (O.)

3. Explain carefully what is meant by specific inductive capacity. How would the indications of an electrostatic voltmeter be affected by immersing the whole instrument in oil which completely fills it? (B.)

4. Define potential, capacity, and specific inductive capacity. Describe any nstrument which may be advantageously employed for the measurement of differences of potential. (Univ.)

5. Describe the attracted disc electrometer, and show how it can be used to measure differences of potential in absolute electrostatic units. Define this absolute unit of potential. If the trap door is 5 cm. in diameter, is 3 mm. from the under plate, and is at 1,200 volts potential difference, what force of attraction would you expect between them? (Univ.)

PART III.—MAGNETISM.

CHAPTER VI.

GENERAL PRINCIPLES.

49. Natural and Artificial Magnets.—Although a few of the simpler properties of a magnet have been known from very early times, accurate knowledge of the subject, as in electricity, is comparatively recent.

Magnetic phenomena are in general easy to observe, and constantly present themselves, without being sought, in many evident natural effects. Thus the mineral known as "*Lodestone*" attracts to itself particles of iron, and if suitably suspended, tends to assume a definite direction in space. If a piece of steel or iron be rubbed with a lodestone, some of the magnetism *appears* to be communicated to the steel or iron, and such a piece then acts precisely as did the lodestone itself, though the effect may not be permanent when the piece is removed from the neighbourhood of the lodestone. The extent to which the property is thus retained depends upon the hardness of the steel or iron, and will be more fully considered later; but it may be noted that a piece of hard steel thus magnetised will retain the property practically unaltered for an exceedingly long time. Such a piece of steel is then termed a permanent magnet, and this property once discovered all investigations relating to magnetism are naturally carried out with artificial magnets of suitable shape, rather than with the natural mineral.

It may be noted here, too, that magnetism does not stand alone, but is related to electricity in a most intimate way, though the discovery of the relation and its connecting laws form a branch of science of quite recent development. In consequence of this connection it will be seen that a magnet can be constructed without the necessity for possessing a magnet to begin with, and further, the artificial magnet may be made of much greater strength than any natural lodestone.

50. Magnetic Bodies.—A body is termed magnetic when it is capable of being affected by a magnet. Faraday was the first to state that all bodies are to a certain extent affected by magnets, but that

they are of two kinds, viz. *paramagnetic* (or, as they are more usually termed, *magnetic*) bodies which are attracted, and *diamagnetic* bodies which are repelled. Few substances appear to be magnetic, and of these the only ones of practical importance are *iron*, *nickel*, and *cobalt*.

Diamagnetism is of little importance to the electrical engineer, as the effect is small and only apparent in the case of a few substances. For practical purposes the study of the magnetic properties of iron and its modifications is all that is necessary.

Modern theory classifies bodies into *ferromagnetics* (iron, nickel, cobalt, and perhaps manganese), *paramagnetics* (feebly attracted by a magnet), *diamagnetics* (repelled). The reasons for this classification cannot be given here.

51. Magnet Poles.—The attracting power of a magnet appears to be concentrated at definite regions near the ends, and these definite regions are called "**poles.**" These poles are not alike in their properties, as is shown by the fact that if two similar magnets be taken, say in the form of bars, and each be placed in a stirrup supported by a thread so as to be free to turn, each will come to rest in a definite direction which is nearly north and south, *the one end of each always pointing towards the north and the other end always towards the south.* If, now, we mark the ends pointing northwards N and N′ (they are known as the *north poles* of the magnets), and the other ends (known as the *south poles*) S and S′, then N and N′ will be found to repel when brought near each other; so also will S and S′, while N and S′ or N′ and S will be found to attract. The directive power of the free magnet is, however, merely another illustration of the attraction and repulsion mentioned above, since, as will be seen later, the suspended bar is in the neighbourhood of another large magnet, the earth, and behaves accordingly. The fundamental rule of which the above effects are manifestations, is—*like poles repel each other and unlike poles attract each other.*

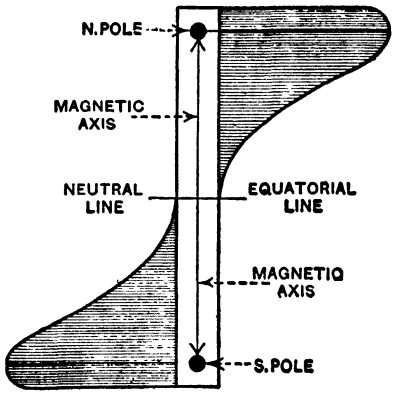


Fig. 42.

Further investigation shows in connection with this, that the two poles of a magnet are always equal. That is, the repulsion between the two N poles of the bars, when these are at a definite distance apart, equals the repulsion between the two S poles at the same distance. Hence the magnetic condition may be regarded much as the production of the two electrostatic charges is regarded: that is,

when bars are magnetised both north and south polarities are produced at the same time, and these magnetic poles are always of equal strength, but the phenomena differ from the electrostatic case inasmuch as *we can never isolate our magnetic poles as we can our charges.*

The distribution of magnetism along a bar magnet may be experimentally studied quite easily by apparatus of the following description. Let the bar magnet be lying upon a table and a small iron ball be suspended above it by a thread from a delicate spring balance. Then the ball is held in contact with the bar by the attraction of the magnet and the reading of the balance, when the latter is raised until the ball is detached, gives the pull needed to overcome the attraction. By this means a measure of the attraction exerted at different points along the bar can be obtained. This attraction is found to be a maximum near the two ends and zero at the centre. The graphical result of such an experiment is shown in Fig. 42.

This experimental fact, that the attraction of a magnet appears to proceed mainly from the ends and is zero at the centre, might lead one to think that if the magnet were broken at the centre one half would contain N magnetism, and the other S magnetism; but such is not the case. Every separate piece of a broken magnet possesses its two poles and, in fact, it *must most carefully be borne in mind that it is impossible to produce a magnet having only one pole.*

52. Constitution of Magnets.—Further investigation indicates that a magnet actually consists of a vast number of minute molecular magnets, and the apparent concentration at the ends is due to the fact that the magnetic molecules may be considered as arranged as in

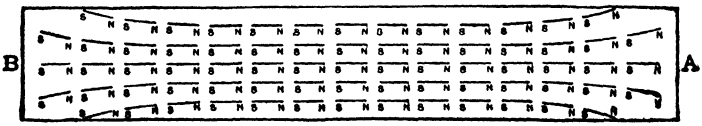


Fig. 43.

Fig. 43. Neutralisation occurs except near the ends where there is N pole at A and a S pole at B. Clearly from the figure if the magnet be broken a N pole will appear on the left of the break and a S pole on the right, *i.e.* each piece will be a complete magnet.

53. Induced Magnetism.—The attraction known to exist between any permanent magnet and a piece of previously unmagnetised iron or steel needs some explanation. *Magnetic induction is the name given to the power that a magnetic pole possesses of inducing in a neighbouring piece of magnetic material a magnetic condition, such that at the end nearer to the magnet pole unlike polarity is produced, and at the more distant end like polarity.*

Thus in Fig. 44 the magnet A B induces in the magnetic material C D a S pole at C and a N pole at D. When this effect is produced, however, we have the condition necessary for attraction between the poles B and C, and between A and D, while there is repulsion between A and C, and B and D. On account of the effect of the varying distances of the poles the resultant is an attraction between the two, though not following the law of attraction between simple poles given in the next chapter. Hence the body moves, or tends to move, toward the magnet.

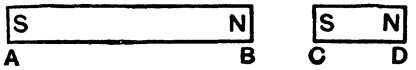


Fig. 44.

It is, then, to be noted that *induction always precedes the attraction of a previously unmagnetised body.* If two *permanent* magnets are near each other there is an attraction or repulsion between the poles already existing, depending upon the way in which the magnets are presented to each other. Thus in Fig. 45 B is attracted by D, while it is repelled by C. If one magnet be much weaker than the other, then the final result may be rather curious. The magnet A B is endeavouring to magnetise C D in the reverse condition to its existing, magnetism, just as would be the case if C D were previously not magnetised. If C D be small and weak, then, on the approach of B, the inductive effect of B may presently overcome the existing magnetism of C D, and thus the repulsion will change to attraction. Such a reversal of magnetism can easily be produced by approaching a fixed compass-needle to the pole of a powerful dynamo fieldmagnet, and the effect must be remembered in all cases where instruments containing weak magnets, *not free to move,* are taken into the neighbourhood of much more powerful magnets.

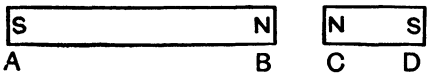


Fig. 45.

54. Demagnetising Effect of Magnetic Poles.—The preceding explains the demagnetising effect of a magnet on itself. Thus if N S (Fig. 46) be a permanent magnet and O any point in it, then at O there will be a magnetic force acting in the direction from N towards

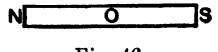


Fig. 46.

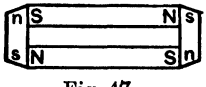


Fig. 47.

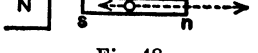


Fig. 48.

S, due to the poles of the magnet. This tends to magnetise, by induction, the material at O, so that southern polarity is towards N and northern polarity towards S, *a distribution of magnetisation exactly*

opposite to that of the magnet. This demagnetising effect is reduced by arranging bar magnets in pairs as shown in Fig. 47, with soft iron "keepers" across the poles. The north pole of the magnet, for example, induces a south pole in the part of the keeper in contact with it, and the effect of this at any point of the magnet is practically equal and opposite to the effect of the magnet pole. Further, these keepers close the magnetic circuit and render the system independent of external disturbances.

Again let Fig. 48 represent a bar of iron lying off the north pole of a magnet, and therefore magnetised by induction as indicated. If an isolated *north* pole be **imagined** free to move at a point O inside, it will be urged to the right, but the induced end poles in the bar will tend to urge it towards the left: *hence the force on the north pole at* O *is reduced* owing to the demagnetising effect of the induced poles *n* and *s*.

Further, to fix ideas, imagine the pole at O to be a unit north pole (Art. 70) and picture it to be *in a cavity of such a shape that the magnetism on the walls of the cavity has no effect whatever on the unit pole* (the kind of cavity will be explained later) so that we have only to consider the effect of the poles N, *n* and *s*. Let H be the force on the unit pole *towards the right* due to N (and this would be the force on it if the iron were absent) and let *h* be the force on it *towards the left* due to the poles *n* and *s*: then the actual force H_1 on the unit pole is given by $H_1 = H - h$, *i.e.* it is less than H owing to the effect of the induced poles. **More exact details of this will be given later.**

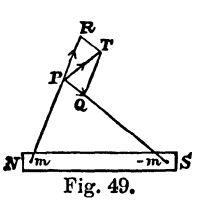


Fig. 49.

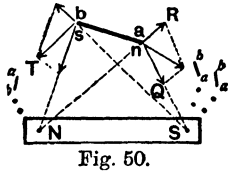


Fig. 50.

55. Magnetic Field.—*The region in the neighbourhood of a magnet within which magnetic material will experience magnetic effect is termed a* **magnetic field.** Strictly speaking, any one magnet has a field of infinite extent; but practically the effect dies away so quickly when the distance from the magnet is increased that the region where the magnetic force is appreciable is very limited, and it is to this region that the term magnetic field is applied. If an isolated N magnetic pole (Fig. 49) be placed at P in the magnetic field due to a large magnet N S, then this single pole will be attracted by S—the S pole of the large magnet—and repelled by N, and these two forces will have a resultant P T as shown in the figure. If the single N pole be free to move, it will naturally be urged in the direction of this resultant,

but as it moves its position relatively to the poles N and S, the resultant will vary in direction from place to place, and thus the free single pole will be urged along a curve and will ultimately reach the S pole of the large magnet. Thus a free N pole would be urged to describe a certain curve starting from the N end of the large magnet and finishing at the S end. Such a curve is termed a **line of force,** and the whole of the space surrounding a magnet—that is, the magnetic field—can be mapped out into these lines of force. Their directions for a bar magnet are shown by the fourth diagram in Fig. 52.

If instead of a single N pole we place a small pivoted compass *a b* in the field of the magnet N S (Fig. 50) then the N pole *a* will experience a force of repulsion due to N and one of attraction due to S and the resultant force on it will be as indicated by the diagonal of the parallelogram on the right. Similarly the resultant force on the S pole *b* is as indicated by the diagonal *b*T. Thus pole *b* tends to move in the direction of *b*T, whilst pole *a* tends to move in the direction of the other diagonal. The net result is that the compass sets in a definite direction, and if it is very small this direction is tangential to the line of force through the small compass: this furnishes a method of mapping out the lines of force (Art. 56). To summarise:—**A line of force is a line along which an isolated N pole would travel if free to move in a magnetic field and it is such that the tangent at any point gives the direction of the resultant force at that point.** Fig. 52 illustrates various cases of lines of force.

So far we have considered only the space outside the magnet—the "external field" as it is called—and we have seen that the lines of force pass from the north pole through the field to the south pole (positive direction of lines). The magnet itself constitutes what is sometimes called the "internal field" and the lines entering the south pole are continued through the magnet itself finally reaching the north pole from which we supposed them starting. These internal lines are strictly *lines of induction* (a line of induction gives the direction of the induction). Thus each line forms a closed circuit, *i.e.* when a line of induction C D A (Fig. 51) leaves the magnet at A it is continued as a line of force A B C in the air, and when this passes from the air to the magnet it is continued into the line of induction. *In air lines of force are also lines of induction* so that the closed circuits, both the parts in air, and the parts in the magnet may (more exactly) be spoken of as lines of induction throughout. These points will be dealt with later: for

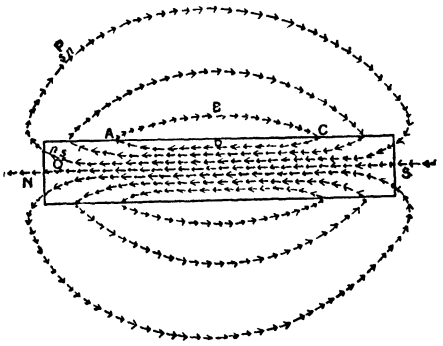


Fig. 51.

the present we are only concerned with the *forces in the external field of a permanent magnet in air* and therefore with the *lines of force in air* (which, however, are also lines of induction *in air*) as previously defined and as shown in

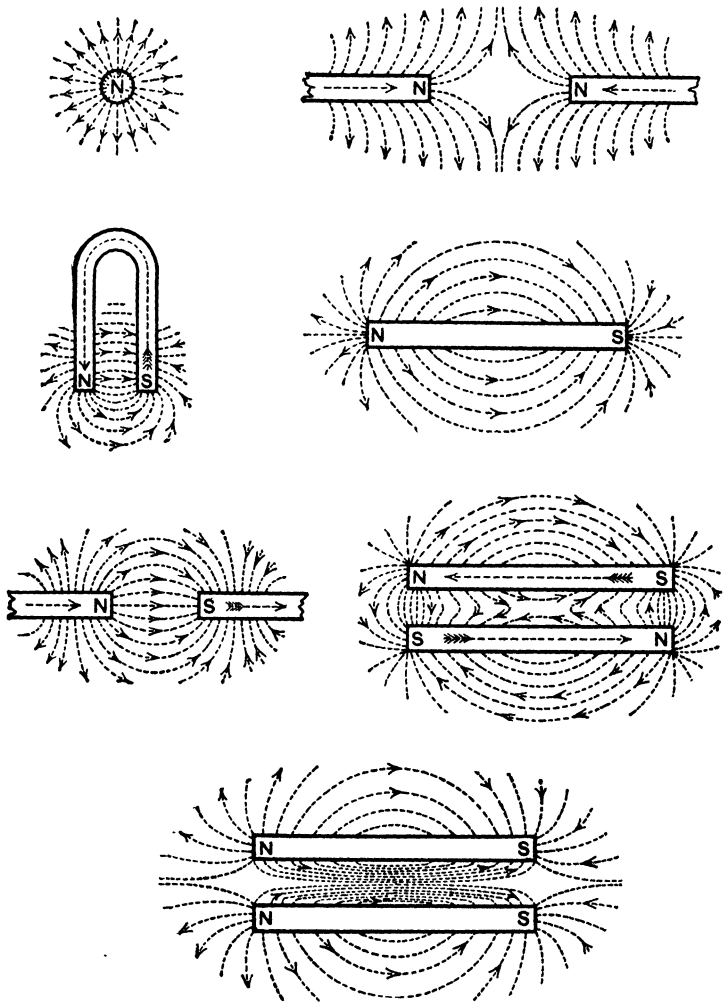


Fig. 52.

Fig. 52. Longitudinal contraction and lateral repulsion of lines (Art. 21) afford an explanation of the attraction and repulsion between poles.

56. Tracing Lines of Force.—The compass needle method of tracing lines of force referred to in Art. 55 is shown by the very small compass needles $a\,b$ on the extreme right and left of Fig. 50. The bar magnet N S is placed on a sheet of drawing paper, the small compass $a\,b$ is placed near say the N pole of the bar magnet and a dot is made opposite each end of the compass. The latter is then moved until the south pole b comes over the dot which has just been made opposite the north pole a and the new position occupied by the pole a is marked. This is continued, and finally a line drawn freehand through the several points gives the line of force required. Thus by starting from various positions the character of the field is determined.

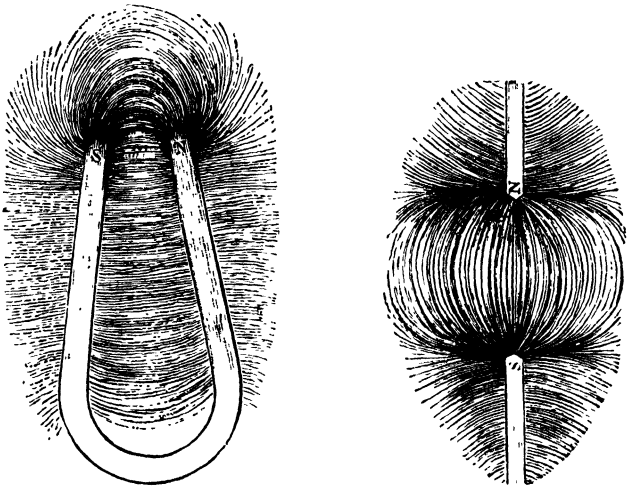


Fig. 53.

Magnetic fields may also be mapped by means of iron filings. The magnet (or combination of magnets) is placed on the table and covered by a thin plate of glass over which is placed a sheet of paper. Iron filings are then sifted through muslin on to the paper, the glass being gently tapped so as to assist the setting of the filings. Each filing becomes by induction a small magnet and therefore sets along a line of force just as the very small compass does. Fig. 53 depicts the results obtained by this method in the case of (1) a horse shoe magnet without its keeper ; (2) two bar magnets end on with unlike poles facing each other.

57. Magnets with more than one Pair of Poles and Magnets with no Poles.—It is possible to so magnetise a bar that poles may be distributed along its length, somewhat as in Fig. 54, in which the magnet appears to possess five poles. Such magnetism may be produced in a strip of steel by suitably

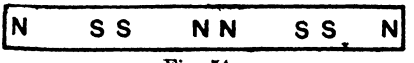


Fig. 54.

touching with a permanent magnet, or better, by an electromagnetic arrangement to be described later. Such poles which appear at places other than the ends of the magnet are termed **consequent poles.**

If a long thin bar magnet be bent into a circular arc until the two ends ultimately come together, the whole will form a ring, and there will be no evidence externally of the existence of poles at all. If a complete ring magnet be cut anywhere a N pole will appear on one side of the cut and a S pole on the other. Such a closed circular magnet will retain its magnetism very perfectly since there is no resisting air portion for the lines of force to pass through in completing their circuit. A ring of iron can be best converted into a ring magnet by the electro-magnetic method described later.

58. Permanence of Magnetism.—A bar of magnetic material once magnetised may not retain the property indefinitely. The rate at which the substance loses the magnetic property depends upon several properties of the substance. *Perfectly soft iron will not retain magnetism at all, while hard steel will retain its powers practically for ever*, provided it is of suitable shape and not exposed to vibration or very high temperature. **The power of retaining magnetism is termed retentivity.** The retentivity depends upon the form of the magnet as well as upon its composition, and also upon the temperature, and it is affected by the mechanical circumstances in which the magnet is placed.

59. Effect of Form.—The more nearly the magnet is formed into a closed metallic circuit the more permanent the magnetism: hence the employment of keepers across the poles of horse shoe magnets and as shown in Fig. 47 for bar magnets.

60. Effect of Composition.—This has already been referred to in the extreme cases of very soft iron and very hard steel. For intermediate degrees of hardness the retentivity is also found to vary between zero and, practically, permanence. Obviously for magnets intended always to be of constant strength the material to use is hard steel, while for magnets which are needed for certain purposes where the magnetism must grow and die away as quickly as possible, as in various magnetic mechanisms, the best material to select is the softest iron.

The addition of other substances to iron or steel affects considerably the magnetic properties of the alloy. Thus permanent magnets are improved if the steel contains from 5 to 8 per cent. of *tungsten*: high grade carbon steels are better than low grade carbon steels for this purpose, and recent work shows that the presence of about 4 per cent. of *molybdenum* is even more effective than tungsten. Again, there is the striking case of the *non-magnetisable alloy* known as *manganese steel* (15·2 per cent. of manganese): another alloy, viz. 80·15 per cent.

of iron, ·8 per cent. of carbon, 5·01 per cent. of manganese, and 14·04 per cent. of nickel is practically unmagnetisable. Finally, in what are called the *Hensler alloys* certain alloys of *non-magnetisable substances and manganese* become magnetisable almost equivalent to cast iron.

61. Effect of Temperature.—Any magnet raised to bright red heat loses the whole of its magnetism, nor can it be given any further magnetic powers until it is cooled below a certain temperature. That is, *magnetic bodies become non-magnetic when raised above a certain limiting temperature,* which is different for different substances. This **critical temperature,** as it was termed by Hopkinson, is between 700° C. and 900° C. for iron and steel, differing for different specimens. When dealing with lower ranges of temperature such as atmospheric changes, etc., the temperature has practically no effect.

62. Effect of Mechanical Surroundings.—*Vibration* undoubtedly much affects the magnetic properties of a substance, and will either weaken or strengthen a magnet according to its condition. If it is placed in a strong magnetic field, and previously unmagnetised, vibration will enable it to take up more than would be the case without it; while if, after being magnetised, the material is removed from the magnetic field altogether, or placed in a reversing field, then vibration will accelerate the loss of magnetism. This may be summed up by saying that *vibration will help the prevailing tendency of the magnet; if it is tending to become stronger by induction, vibration will assist the growth of magnetism; if it is tending to become weaker when alone, vibration will accelerate the loss.*

A practical instance of the effect of vibration in causing the acquisition of magnetism is known to every mechanical engineer in the strong magnetic properties acquired by steel drills, particularly when drilling cast iron. Another instance is the magnetism acquired by steel ships during construction.

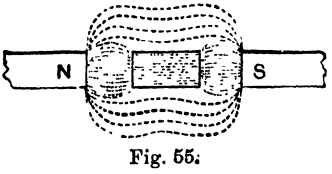


Fig. 55.

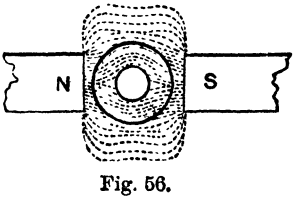


Fig. 56.

63. Preliminary Ideas on Permeability. Magnetic Screens.—Fig. 55 shows the lines when a bar of soft iron is placed between the N and S poles of two bar magnets. The iron is, of course, magnetised, and the lines crowd into it as if they found it easier to

go through iron than through air. Iron, in fact, is a better conductor of lines than air or, as we say, is more "*permeable*," or has greater "**permeability**" than air. **The permeability of a substance may be defined at this stage as its conducting power for lines as compared with air.** Thus if there were 5 lines per sq. cm. (H) in the air space before the iron was placed there, and 4,000 lines per sq. cm. (B) in the iron, the permeability (μ) of the iron would be said to be 800 ($\mu = B/H$), neglecting the demagnetising effect of the induced poles.

Fig. 56 shows the effect of placing a thick cylinder of iron between the poles: it screens the space inside from the influence of the magnets, for the lines keep to the iron and do not pass across the central air space. Galvanometers are sometimes protected from external magnetism by surrounding them by a thick cylinder of iron in this way.

64. Lifting Power of Magnets.—This depends upon two things, (*a*) the strength of the magnet poles, (*b*) the shape of the magnet. By taking an exceedingly strong magnet and suitably shaping the poles so as to concentrate the magnetism upon the armature which is to be attracted, exceedingly large attractive forces can be produced. In some experiments by Joule a force a little larger than 200 lb. per square inch of pole face was obtained. It is found that if the magnet has considerable cross section there is a tendency for the magnetism to exist mainly on the outside surface, and the interior is not affected to any high degree. In order to render the magnetisation more uniform throughout the section, and thus to increase the lifting force, magnets have been constructed of thin sheets of steel instead of solid blocks. Such a magnet is said to be *laminated*.

Practical use of the holding power of magnets is now to be seen in many engineering works. For example, instead of using hooks for lifting large sheets of iron when transporting them from one machine to another, an electromagnet (Art. 66) hanging from a crane is used; this is energised by a current when required, and thus a large thin sheet, which is a difficult thing to grip in the ordinary way, may be easily carried. Again, as a means for holding small iron articles on the bed of a machine while various operations, such as drilling or milling, are being performed, the electro-magnetic vice is exceedingly useful. The track brake on electric trams is another illustration.

65. Rapid-acting Magnets.—When an electro-magnet is required to quickly change its strength, that is to rapidly acquire or lose magnetism, the laminated construction is adopted. Such a magnet is built up of thin soft-iron sheets. Magnets intended to be energised by alternating currents, as in some forms of arc lamps and in transformers, are also laminated for the same reason.

66. Construction of Magnets.—Given one magnet, any number may be made from it by taking advantage of the properties of induction mentioned above. Thus by stroking a bar of steel from end to end with one end of a permanent magnet the bar becomes magnetised as indicated in Fig. 57, *the end of the bar where the rubbing magnet*

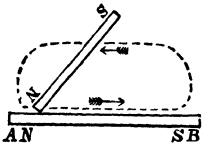


Fig. 57.

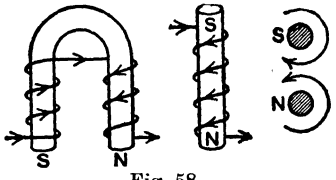


Fig. 58.

leaves it being of opposite polarity to the rubbing pole. There are modifications of this method, but all such have given place to the electro-magnetic method. Mere contact with a large electro-magnet, such as the field magnet of a dynamo, is sufficient to induce strong magnetism in a hard steel bar. Or the bar required to be magnetised may be inserted in a spiral of wire through which a current can be sent at will, and the effect produced may be regulated by controlling the strength of the current. This will be more fully attended to in the section on electro-magnetism later, but it may be mentioned here that, looking at the end of the bar, *if the current in the coil be counter-clockwise in direction, that end will be a north pole; if it be clockwise, that end will be a south pole* (Fig. 58).

67. Mechanical Results of Magnetisation.—It has been shown by experiment that an iron rod lengthens when magnetised, provided that the magnetising force has not exceeded a certain limit, but if the force be greater than this the bar becomes shorter. An allied phenomenon is that known as the **Villari Reversal.** If an iron bar be weakly magnetised the effect of a pull on the bar is to increase the magnetism, but if the bar be strongly magnetised the effect of the pull is to diminish the magnetism. There are other developments in this connection, but they cannot be adequately treated here.

68. Magnetic Condition of the Earth.—For the purposes of the electrical engineer this need only be dealt with very briefly.

The earth is really a huge magnet, one pole—the north magnetic—being in the vicinity of Boothia Felix in the north of North America, and the other pole—the south magnetic—in the southern hemisphere, Lat. 72° 25′ S., Long. 155° 16′ E. Thus it is that a compass needle comes to rest along the lines of the earth's field, *i.e.* nearly north and south, the one end always pointing towards the north and the other end towards the south. The vertical plane containing the magnetic axis of the needle is termed the *magnetic meridian*, and the angle between the magnetic and geographical meridians is called the variation or **declination.**

In the case of a *freely suspended* magnetic needle, the needle not only sets in the magnetic meridian, as above, but *one end dips*. The angle between the horizontal plane and the magnetic axis of a freely suspended needle, or of a proper "dip needle" able to move in the magnetic meridian, is termed the inclination or **dip.** In the northern hemisphere the N pole "dips" towards the north, in the southern hemisphere the S pole "dips" towards the south: at the earth's "magnetic equator" the dip is zero, and at the magnetic poles it is 90°.

The essential measurements to be made at any place are (1) the declination, (2) the dip, (3) the total intensity or total force on unit pole. Now let T denote the total intensity (Fig. 59), and let it be resolved into a horizontal component H and a vertical component V. Clearly $d°$ is the angle of dip, and :—

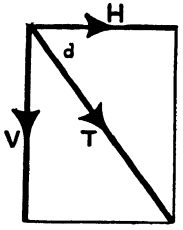


Fig. 59.

$$H = T \cos d, \qquad \therefore T = H/\cos d.$$
$$V = T \sin d\,; \; T^2 = V^2 + H^2\,; \; V/H = \tan d.$$

For practical convenience H and the dip are measured and T found from the second relation above. The Greenwich values are :—Declination = 15° 6·3′ W., Dip = 66° 51′, H = ·18534, T = ·4695, V = ·4318. These values are not constant, but are *gradually* changing year by year; they have also daily and yearly regular changes and sudden fluctuations accompanying "magnetic storms," etc.

Since what we term the N pole of a suspended magnet is attracted by the N magnetic pole of the earth, it follows that the real magnetic property resident at the N magnetic pole of the earth is S magnetism, and at the S magnetic pole of the earth N magnetism.

Thus, if a piece of soft iron be placed in the earth's field at London, inclined at the angle of dip in the magnetic meridian, the bar will have induced in it a *N pole at its lower end and a S pole at its upper end.* If the bar be placed horizontally in the meridian *the end pointing northwards will become a N pole.*

So, too, a *vertical* bar of soft iron in the northern hemisphere will have induced in it a *N pole at the bottom and a S pole at the top.* In the southern hemisphere the reverse will hold—the bottom will be a south and the top a north.

EXERCISES VI.

1. What is the exact difference—magnetically speaking—between a piece of brass, a piece of soft iron, and a piece of lodestone?

2. What are the reasons for the assertion that a magnet cannot be produced which has only one pole?

3. Explain precisely what happens when a piece of soft iron is attracted by a bar magnet brought into its neighbourhood.

4. State clearly what is understood by a *line of force* in a magnetic field. How may the direction of any one such line be obtained practically?

5. State exactly how the following magnets may be constructed: (*a*) A bar magnet with S poles at each end and a N pole at the centre; (*b*) A ring magnet with no poles at all evident; (*c*) A ring with a N pole at one point and a S pole diametrically opposite.

6. If a permanent magnet is desired to be very strong and very lasting in its properties, of what material and shape should it be made?

7. Explain the effect of the following upon the strength of a magnet: (*a*) Alteration of temperature; (*b*) Mechanical vibration.

8. What connection has been discovered between the length of a rod of iron and its magnetic condition?

9. In connection with the magnetic condition of the earth define the following: Total magnetic force, horizontal component, dip, declination. Which of these are connected, and how?

CHAPTER VII.

MAGNETIC UNITS AND THEORY.

69. Law of Magnetic Attraction and Repulsion between Isolated Poles.—Experiment shows conclusively that if two like, but possibly unequal, poles be imagined isolated, and each concentrated at a point, *the repulsive force exerted between them varies as the product of the strength of the poles and inversely as the square of the distance between them.*

The actual force between the like poles of two real magnets does not obey this law because: (1) the poles are not concentrated at points; (2) the poles are not isolated, and the second pole of each exerts an influence. By making the magnets very long and thin the result is, however, approximated to very closely.

Before this law can be used for calculations a system of units must be adopted in terms of which magnetic quantities can be expressed. The following units have been found convenient:

70. Unit Pole.—A pole is said to have unit strength when, if placed one centimetre away in air from another like and equal pole, the two will mutually experience a repulsive force equal to one dyne. The symbol m will be used for the strength of a magnet pole in terms of this unit.

From the law of Art. 69, using the unit defined above, the force between two poles of strength m_1 and m_2 distant d centimetres apart in air, is given by the expression—

$$f = \frac{m_1 \times m_2}{d^2} \text{ dynes.}$$

The more general formula is $f = \frac{1}{\mu} \frac{m_1 m_2}{d^2}$ where μ is the permeability of the medium: for air μ is taken as unity (Compare with Art. 23 but note that whilst k is practically constant for a given medium μ is by no means constant).

71. Magnetic Moment.—The moment of an ideal magnet is defined as being the product of the strength of one of the poles—either will do, since they are always equal—into the length of the

magnet. **In the practical magnet the moment is the product of the pole strength into the distance between the poles,** which will usually be a little less than the length of the bar. Thus, using M for magnetic moment, we have

$$\mathbf{M} = ml$$

if l be the distance between the poles in centimetres. (See Art. 75.)

72. Intensity of Magnetisation.—This is obtained for any magnet by dividing the magnetic moment of the magnet by its volume, and thus it is an expression for **the magnetic moment per unit volume.** Hence if V be the volume and a the cross-sectional area,

$$\textbf{Intensity (I)} = \frac{\mathbf{M}}{\mathbf{V}} = \frac{ml}{al} \text{ (approx.)} = \frac{m}{a}.$$

From this it will be seen that the magnetic intensity is also the **pole strength per unit area of the pole face.**

73. Field Strength.—A magnetic field is said to be of unit strength at any point if a unit pole placed there is acted on with a force of one dyne. The symbol H will be used to denote the strength of a field, thus indicating that unit pole in the field would be acted on with a force of H dynes. Hence if a pole of m units is placed in a field of strength H, the resultant force on the pole is Hm dynes.

The intensity of the field at distance d cm. from an isolated N pole of strength m is therefore given by the expressions:—

$$\mathrm{H} = \frac{m}{d^2} \text{ units} \quad or \quad \mathrm{H} = \frac{m}{\mu d^2} \text{ units}$$

the medium in the first case being air and in the second one of permeability μ. Further the field at a point due to a number of poles is found by the parallelogram law as indicated in Fig. 49.

As in electrostatics (Art. 28) it is convenient to picture a definite number of lines of force proceeding from a given pole. Imagine an isolated N pole of strength m **situated in air** and further imagine a spherical surface of radius d cm. drawn in the magnetic field, the pole m being at the centre: the intensity of the field at any point on the surface of this sphere is m/d^2 units. Now imagine that $4\pi m$ lines of force emanate from the pole of strength m **in air** (in other words 4π lines from unit pole). These all pass through the surface of the sphere and as the area of this surface is $4\pi d^2$ sq. cm. it follows that

$$\text{Number of Lines of Force per unit area} = \frac{4\pi m}{4\pi d^2} = \frac{m}{d^2}.$$

But this last expression also gives the intensity of the magnetic field at the sphere in C.G.S. units: hence:—

(1) $4\pi m$ lines of force radiate from a pole of strength m **in air** and 4π lines of force from unit pole **in air.**

(2) The intensity of a field is represented numerically by the number of lines of force per unit area (perpendicular to their direction): thus a field of strength H has H lines of force per unit area.

(3) **Unit field has one line of force per unit area.**

The 4π introduced in this way is found in a large number of formulae as a consequence of this first choice. Note that in electrostatic theory the number of Maxwell lines of electric force from a charge Q **in air** was taken to be $4\pi Q$ (the equivalent of Faraday lines is not used in magnetic theory).

The unit magnetic field is sometimes called a **"gauss"**: thus a field of strength H units may be spoken of as a field of H gausses.

74. Magnetic Potential and Magnetic Potential Difference.—Before proceeding with this section the student should again read Art. 25 of Chapter III.

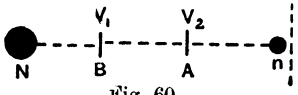


Fig. 60.

Let N (Fig. 60) be an isolated north pole, and imagine n to be a unit north pole at an infinite distance. To bring n from infinity to A, work must evidently be done against the repulsion of N, and to bring the unit pole to B clearly more work must be done. With a given pole N, the work in these cases depends on the positions of the points in question, and such work is taken as a measure of what is termed the **magnetic potential** at these points. Thus, if V_2 ergs of work be done in moving the unit north pole from infinity to A, V_2 C.G.S. units is the potential at this point A; if V_1 ergs of work be expended in moving the unit north pole to B, V_1 C.G.S. units is the potential at the point B, and the difference in magnetic potential between A and B is $V_1 - V_2$ C.G.S. units.

If N be replaced by a south pole the same reasoning holds, but in this case the unit north pole would move up under the attractive force.

Hence *the magnetic potential at a point is defined as measured by the work done, in ergs, in moving a unit north pole from infinity to that point, and the difference in magnetic potential between two points is defined as measured by the work done, in ergs, in moving a unit north pole from one point to the other.* Thus **the magnetic potential at a point in a field is 1 C.G.S. unit if one erg of work be done in moving a unit north pole from infinity to that point, and the magnetic potential difference between two points is 1 C.G.S. unit if one erg of work be done in moving a unit north pole from one point to the other.**

75. Action of a Magnet in a uniform Field.—*If the intensity is the same in magnitude and direction at every point of a magnetic field the latter is said to be a uniform field.* In such a field the lines of force will be parallel straight lines. The earth's magnetic field over fairly extended areas is uniform.

Let m be the pole strength of the magnet N S (Fig. 61), H the intensity of the uniform field, and α the angle the magnet makes with the direction of the field. The force on each pole is mH; these constitute a couple the moment of which is obtained by the product of one of the forces and the perpendicular distance S T between them; thus

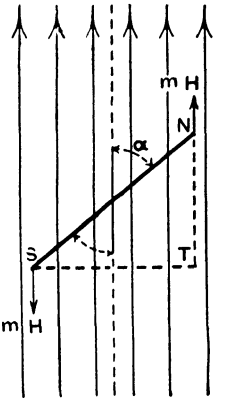


Fig. 61.

$$\text{Couple on NS} = m\text{H} \times \text{ST} = m\text{H} \times \text{SN} \sin \alpha$$
$$= m \times \text{SN} \times \text{H} \sin \alpha.$$
$$\therefore \textbf{ Couple on NS} = \textbf{MH} \sin \alpha.$$

The effect of this couple is to rotate the magnet until it lies parallel to the direction of the field, in which position the couple vanishes since $\alpha = 0°$, and sin α is therefore zero. Clearly the couple has its maximum value when the magnet is at right angles to the field, in which case $\alpha = 90°$, sin $\alpha = 1$, and the value of the couple is M H. Further, if H = 1 the value of the couple is M: hence **the moment of a magnet is numerically equal to the moment of the couple acting on the magnet when placed at right angles to a field of unit strength.**

Of course before coming to rest the magnet will oscillate backwards and forwards about its equilibrium position like a pendulum. In connection with this it should be noted that a double swing, *i.e.* a "to and fro" movement is called a *vibration* whilst a single swing from one extreme position to the other is called an *oscillation.*

76. Oscillation of a Magnet in a Uniform Field.—The time in which a magnet will execute a complete vibration when displaced from the mean position in a uniform field and allowed to swing can be expressed by the following formula—

$$\mathbf{T = 2\pi\sqrt{\frac{K}{HM}}}, \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

where
T = time of a complete vibration in seconds.
K = moment of inertia of magnet.*
M = magnetic moment of magnet.
H = strength of the uniform field.

---

* This depends only on the mass, size and shape.

K (rectangular bar) $= w\left(\frac{l^2 + b^2}{12}\right)$; K (cylindrical bar) $= w\left(\frac{l^2}{12} + \frac{r^2}{4}\right)$;

where w = mass (grm.), l = length, b = breadth, r = radius (cm.).

If this is expressed differently in order to give the number of complete vibrations per second we have—

$$T^2 = (2\pi)^2 \frac{K}{MH},$$

and since n the number of vibrations per second is equal to $\frac{1}{T}$,

$$n^2 = \frac{1}{(2\pi)^2} \frac{MH}{K} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

This formula must be taken as established. It is easily proved, but the proof would be out of place here.

From equation (2) it is at once seen that the square of the number of vibrations a given magnet will make in a magnetic field depends directly upon the strength of the field and upon its own magnetic moment, and inversely upon the moment of inertia of the magnet.

Hence, if this same magnet be transferred from field to field, since M will be practically constant and K will be constant, *the ratio of the squares of the number of vibrations in each field in a given time will be the ratio of the strengths of the fields.* Further, as we are dealing with comparisons, *single swings* and not complete vibrations may be counted. Finally, if two magnets have equal moments of inertia and if they be caused to vibrate in turn in the same field, the ratio of the squares of the number of vibrations in equal times will be the ratio of the magnetic moments.

77. To Compare Two Magnetic Fields.—Take a small suspended magnet and let it oscillate in each of the two fields H_1 and H_2. Count the number of vibrations (*or* the single swings) executed per minute by the magnet in each field and square these numbers; then—

$$H_1 : H_2 :: {n_1}^2 : {n_2}^2.$$

There is one source of error which must be carefully avoided here—that is, the effect of the earth's field, which is always present.

If neither of the above two fields is the earth's field it must be remembered that we are then dealing in the one case with $H_1 \pm$ earth's field and in the other with $H_2 \pm$ earth's field, and thus the procedure must be as follows :—

(*a*) When in the earth's field H_0 only, let the number of swings be n_0 per minute.

(*b*) When in the earth's field plus that of first magnet H_1, let the number of swings be n_1 per minute.

(*c*) When in the earth's field plus that of second magnet H_2, let the number of swings be n_2 per minute.

Then— $H_1 + H_0 : H_0 = n_1^2 : n_0^2$

and $H_2 + H_0 : H_0 = n_2^2 : n_0^2$,

that is $\frac{H_1}{H_0} + 1 = \frac{n_1^2}{n_0^2}$;

$$\therefore \frac{H_1}{H_0} = \frac{n_1^2}{n_0^2} - 1,$$

and so $$\frac{H_2}{H_0} = \frac{n_2^2}{n_0^2} - 1.$$

Hence $$H_1 : H_2 = \frac{n_1^2}{n_0^2} - 1 : \frac{n_2^2}{n_0^2} - 1$$
$$= n_1^2 - n_0^2 : n_2^2 - n_0^2.$$

Further, H_1 and H_2 can each be obtained in terms of the earth's field H_0, which may be assumed to be known.

If the earth's field be *opposing* the fields to be compared, we have—

$$H_1 - H_0 : H_0 = n_1^2 : n_0^2$$
$$H_2 - H_0 : H_0 = n_2^2 : n_0^2,$$

which, as above, leads to the final expression—

$$H_1 : H_2 = n_1^2 + n_0^2 : n_2^2 + n_0^2.$$

78. To Compare Two Magnetic Moments.—If two magnets are supplied and the ratio of their magnetic moments is desired, then each may be caused to oscillate in turn in the same field H. If the moments in question are M_1 and M_2 and the swings per minute n_1 and n_2 respectively,

$$M_1 : M_2 = n_1^2 : n_2^2.$$

H in this case can usually be the earth's field. In this w assume that the magnets have equal moments of inertia K.

79. Field due to a Small Magnet.—This will be investigated for two positions only, viz. at a point on the axis of the magnet produced and at a point in the plane bisecting the magnet at right angles. "Field" is of course measured by "force on unit N pole."

To simplify the problem it is assumed that the length of the magnet is small and even negligible in comparison with the distance of the selected points from the magnet. Thus the results would not hold for points close to the magnet.

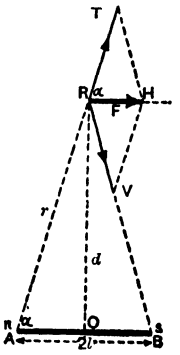


Fig. 62.

Case I.—*To find the force on unit pole at* R (Fig. 62).

To simplify the formulae *the length of the magnet* A B *will be denoted by* 2*l in this and in Arts.* 80 *and* 81.

$$\text{Force at R due to A} = \frac{m}{r^2} \text{ (repulsive).}$$

$$\text{,, ,, ,, B} = \frac{m}{r^2} \text{ (attractive).}$$

Denoting these forces by the lengths R T and R V, completing the parallelogram and drawing the diagonal R H, this latter represents the resultant force F. From the figure it is clear that the triangles ABR and RHT are similar, hence :—

$$\frac{\text{R H}}{\text{R T}} = \frac{2l}{r}, \quad i.e. \frac{\text{F}}{\frac{m}{r^2}} = \frac{2l}{r}$$

$$\therefore \text{F} = \frac{2ml}{r^3} = \frac{\text{M}}{r^3}$$

$$\therefore \text{F} = \frac{\text{M}}{(d^2 + l^2)^{\frac{3}{2}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

If l is exceedingly small compared with d, l^2 may be neglected and we get—

$$\mathbf{F} = \frac{\mathbf{M}}{d^3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

In this case the magnet is said to be **"broadside on"** to the point R.

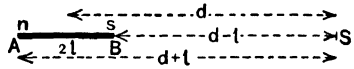


Fig. 63.

Case II.—*To find the force on unit pole at* S (Fig. 63).

$$\text{Force at S due to A} = \frac{m}{(d + l)^2} \text{ (repulsive).}$$

$$\text{,, ,, ,, B} = \frac{m}{(d - l)^2} \text{ (attractive).}$$

Hence the resultant attractive force F is—

$$\text{F} = \frac{m}{(d - l)^2} - \frac{m}{(d + l)^2} = m \left\{ \frac{1}{(d - l)^2} - \frac{1}{(d + l)^2} \right\}$$

$$= m \left\{ \frac{(d + l)^2 - (d - l)^2}{(d - l)^2 (d + l)^2} \right\} = \frac{4mdl}{(d^2 - l^2)^2}$$

i.e.

$$\text{F} = \frac{2\text{M}d}{(d^2 - l^2)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

If l be exceedingly small and therefore l^2 negligible :—

$$\mathbf{F} = \frac{\mathbf{2M}}{d^3} \quad \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

In this case the magnet is said to be **"end on"** to the point S. From (4) and (6) it is apparent that the field due to a small "end

on" magnet is twice the field due to the same magnet "broadside on" at the same distance.

80. Action between two Small Magnets in the Earth's Field.—This will be investigated for two positions only.

CASE I.—*When the first magnet* A B *is at right angles to the earth's field, and "broadside on" to the second magnet* C D (Fig. 64). A B *is fixed whilst* C D *is suspended and therefore deflected.*

Let the small magnet C D be deflected through an angle θ from the direction of the earth's field. If H be the strength of the earth's field, and m' the pole strength of C D, the force on each pole of C D due to the earth is m'H. These two forces constitute a "couple" tending to rotate C D into the meridian, and since the moment of a couple is given by the product of one of the forces and the perpendicular distance between them, we have—

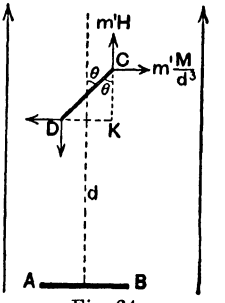


Fig. 64.

$$\text{Restoring couple due to the earth} = m'\text{H} \times \text{D K}.$$

Again since C D is small, the force on each pole due to the deflecting magnet A B may be taken as $m' \frac{\text{M}}{d^3}$, where M is the moment of A B (Art. 79, Case I.). These constitute a couple tending to set C D at right angles to the meridian; hence as above we have—

$$\text{Deflecting couple due to magnet A B} = m' \frac{\text{M}}{d^3} \times \text{C K}.$$

Since these two couples balance each other—

$$m' \frac{\text{M}}{d^3} \times \text{C K} = m'\text{H} \times \text{D K}, \quad i.e. \frac{\text{M}}{\text{H}} = d^3 \times \frac{\text{D K}}{\text{C K}}$$

$$\therefore \frac{\mathbf{M}}{\mathbf{H}} = d^3 \tan \theta \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

If the more accurate expression involving the half length of the magnet be used in stating the forces due to A B (*i.e.* (3) instead of (4) in Art. 79) the expression becomes—

$$\frac{\mathbf{M}}{\mathbf{H}} = (d^2 + l^2)^{\frac{3}{2}} \tan \theta \quad \ldots\ldots\ldots\ldots\ldots \quad (8)$$

CASE II.—*When the first magnet* A B *is at right angles to the earth's field and "end on" to the second magnet* C D (Fig. 65). A B *is again the "fixed" magnet.*

As before we have :—

Restoring couple due to the earth $= m'H \times DK$.

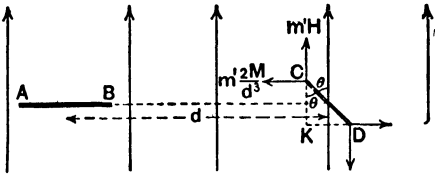


Fig. 65.

Again the force on each pole of C D due to A B is, in this case, $m' \frac{2M}{d^3}$ (Art. 79, Case II.) ; thus—

$$\text{Deflecting couple due to magnet A B} = m' \frac{2M}{d^3} \times CK.$$

Hence as before :—

$$m' \frac{2M}{d^3} \times CK = m'H \times DK, \quad i.e. \frac{M}{H} = \frac{d^3}{2} \times \frac{DK}{CK}$$

$$\therefore \frac{\mathbf{M}}{\mathbf{H}} = \frac{\boldsymbol{d^3 \tan \theta}}{\mathbf{2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

If the more accurate expression (Art. 79 (5)) be used for the forces due to A B, the above becomes :—

$$\frac{\mathbf{M}}{\mathbf{H}} = \frac{(d^2 - l^2)^2}{2d} \tan \theta \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

81. Applications.

(1) *To compare two magnetic moments.*—Let the first magnet of moment M_1 be placed say "broadside on" to a compass needle as in Fig. 64 and let θ_1 be the deflection. Let θ_2 be the deflection produced by the second magnet of moment M_2 when similarly placed at the same distance. Then since :—

$$\frac{M_1}{H} = d^3 \tan \theta_1 \text{ and } \frac{M_2}{H} = d^3 \tan \theta_2$$

$$M_1 : M_2 = \tan \theta_1 : \tan \theta_2.$$

(2) *To compare the earth's field at two places.*—If at a place A where the earth's field is H_a a magnet "broadside on," say, produces a deflection D_a, while at a place B where the earth's field is H_b the deflection is D_b with the *same* magnet at the same distance, we have—

$$\frac{M}{H_a} = d^3 \tan D_a, \text{ and } \frac{M}{H_b} = d^3 \tan D_b$$

$$\therefore H_a : H_b = \tan D_b : \tan D_a.$$

(3) *To determine a magnetic moment* (M) *or the earth's magnetic field* (H).—If H be known the experiment outlined in (1) above may be used for the actual

determination of M, the distance (d) and deflection (θ) being carefully noted, and similarly if M be known, H may be determined. If *neither* H *nor* M be known, they may be found as indicated in Arts. 87 and 98.

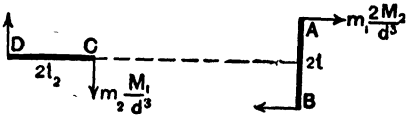


Fig. 66.

(4) *Problem:—Two very small magnets are laid under constraint as shown in Fig.* 66. *Find the couple acting upon each due to the other.*

Let m_1 $2l_1$ M_1 = pole strength, length and moment of A B.
,, m_2 $2l_2$ M_2 = ,, ,, ,, ,, ,, C D.

Couple on C D:—Force on each pole $= m_2 \frac{M_1}{d^3}$

$$\therefore \text{ Couple } = m_2 \frac{M_1}{d^3} \times 2l_2 = \frac{\mathbf{M_1 M_2}}{d^3}.$$

Couple on A B:—Force on each pole $= m_1 \frac{2M_2}{d^3}$

$$\therefore \text{ Couple } = m_1 \frac{2M_2}{d^3} \times 2l_1 = \frac{\mathbf{2M_1 M_2}}{d^3}.$$

82. Magnetic Induction. Lines of Force and Induction.—To fix ideas, we will take, in the first place, a purely imaginary case which will serve the double purpose of explaining the necessary points and bringing out the similarity between magnetic theory and the electrostatic theory dealt with in preceding chapters.

Imaginary Case of an Isolated Pole in a Medium of Indefinite extent and of Permeability μ.—Consider two poles of strength m and m_1 situated d cm. apart in a medium of permeability μ; the force between them is $mm_1/\mu d^2$, and if m_1 be a unit pole the force $m/\mu d^2$ on it measures the strength of the field in C.G.S. units at d cm. from the pole m. Thus the **field intensity** (H) depends on the strength of the pole, on the distance and *on the medium.*

Now it is convenient to have a quantity which depends only on the pole and on the distance, *i.e.* which is the same at distance d from the pole m *whatever the medium*, and this quantity is called the **magnetic induction** (B). Clearly from the preceding it must be taken as μ times the field intensity (*i.e.* $B = \mu H$) for the induction at distance d cm. from the pole m becomes $\mu \times m/\mu d^2$ or m/d^2, *i.e.* it depends upon m and d *but not upon the medium.* To summarise: for a point at distance d cm. from a pole of strength m in a medium of permeability μ:—

$$H = \text{Field Intensity} = \frac{m}{\mu d^2}$$

$$B = \text{Magnetic Induction} = \mu H = \frac{m}{d^2},$$

and if the medium be air ($\mu = 1$) both H and B are measured by the same expression, viz. m/d^2.

[Compare with electrostatic theory, viz. $F = Q/kd^2$ and $N = kF = Q/d^2$: for air $k = 1$ and both F and $N = Q/d^2$.]

Imagine now that the lines are chosen so that at any point *the number of lines per unit area (perpendicular to their direction) measures the induction* at that point: these are called *lines of induction*. Picture a sphere of radius d cm. drawn in the medium, the pole m being at the centre. The induction at any point of the sphere is m/d^2, so that m/d^2 lines of induction pass through unit area. But the area of the sphere is $4\pi d^2$, so that the total number of lines passing through the surface is $m/d^2 \times 4\pi d^2$, *i.e.* $4\pi m$. **Thus the number of lines of induction emanating from the pole m is $4\pi m$, and this is so whatever the medium.** The number of lines of force is $1/\mu$ of the number of lines of induction. In air ($\mu = 1$) the lines of induction and lines of force *become identical*, so that **in air the number of lines of force emanating from the pole m is $4\pi m$** as stated in Art. 73.

83. Practical View of Magnetising Force and Magnetic Induction: Permeability and Susceptibility.—The student will better realise the connection between the preceding section and the present practical treatment as his studies proceed. In the last section we dealt with a single (impossible) pole in a medium of *indefinite* extent of permeability μ: we come now to the *practical* case of a medium of *finite* dimensions of permeability μ placed in a magnetic field, and the main object is to get a connection between B the magnetic induction, I the intensity of magnetisation, and H the intensity of the field.

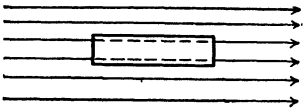


Fig. 67.

Fig. 67 represents a rod of brass lying in a uniform field of strength H, *i.e.* a field which has H lines of force per square cm. (perpendicular to their direction) and which exerts a force of H dynes on a unit north pole.

As usual the arrows in the field indicate the direction in which a free N pole would be urged by the magnetic forces. Hence the field shown corresponds to a N pole lying somewhere to the left, and a S pole lying somewhere to the right. Brass is dia-magnetic, hence its presence scarcely affects the magnetic field. The lines now in the region occupied by the brass were there before, and, in fact, the brass is practically as air so far as the field is concerned. If, however, the brass be now removed and its place filled by a piece of iron or steel of the same size and shape, induced magnetism will make its appearance in the block (Fig. 55), a large extra number of magnetic lines will be produced inside it, and, at the ends where these extra lines emerge into the air or pass from the air into the block, magnetic poles make their appearance, that on the left being a S pole and that on the right a N pole. Suppose the strength of each pole so produced to be m

units, then from what has gone before there will be $4\pi m$ lines passing through the iron from the induced S pole to the induced N pole. In their passage through the block or bar these lines will be in the direction of the lines of the original field and thus, *unless there is any disturbing influence,* there is now, passing along the bar, of which the cross-section is A sq. centimetres, a total number of lines equal to $\mathbf{A\,H} + 4\pi m$; the first lot are due to the original field (and would be present if the bar were removed) while the second lot are due to the effect upon the bar. These lines measure the **total magnetic induction** or **magnetic flux** (ϕ) in the iron. The number of lines *per square centimetre* in the air space, viz. H; measures what is termed the **magnetic** or **magnetising force.**

There is, however, a disturbing influence which renders the magnetising force *in the bar* less than the intensity H of the original field and in consequence lessens the total magnetic induction $A\,H + 4\pi m$. This disturbing influence is of course the demagnetising effect of the poles of the iron (Art. 54). The magnetising force *in the iron* is really the resultant of the force H due to the field (acting from left to right in the figure) and the force h, say, due to the end poles of the iron (acting from right to left), *i.e.* it is H_1 where H_1 may be taken to be $H - h$ (Art. 54). For the remainder of this section, however, the demagnetising effect will be neglected and the magnetising force in the iron taken as H due to the original field only.

It is customary to reduce the expression to **flux density** (more often called the **magnetic induction**) or *lines of induction per unit area* the symbol for which is, of course, B. Hence dividing the expression for the total flux by A, the sectional area of the specimen, we get $B = H + 4\pi \frac{m}{A} = H + 4\pi I$, where I is the intensity of magnetisation: clearly also the lines of induction per unit area in the iron (B) is the sum of two lots, viz. H lines per unit area due to the field and $4\pi I$ lines per unit area due to the magnetisation (sometimes called lines of magnetisation).

The ratio of B to H is called the **permeability** (μ) of the iron, whilst the ratio of I to H is called the **susceptibility** (κ) of the iron: thus:—

$$\mu = \text{Permeability} = \frac{\text{Magnetic Induction}}{\text{Magnetising Force}} = \frac{B}{H}, \quad \therefore B = \mu H$$

$$\kappa = \text{Susceptibility} = \frac{\text{Intensity of Magnetisation}}{\text{Magnetising Force}} = \frac{I}{H}, \quad \therefore I = \kappa H,$$

and further:—

$$\mu = \frac{B}{H} = \frac{H + 4\pi I}{H} = 1 + 4\pi\kappa, \qquad \therefore \kappa = \frac{\mu - 1}{4\pi}.$$

We would again remind the student that to be exact the magnetising force *in the iron* (say H_1) and not its value before the iron is there (H) should be used in the above, and that H_1 is less than H (and $\therefore$ B is less) owing to the demagnetising effect of the induced poles. One case of magnetisation, however, in which the demagnetising effect can be absolutely removed is that of the closed iron ring. If an iron ring be wound over with an electric circuit, H can be easily calculated as will be seen later, and as there are no poles there is no demagnetisation so that this is the proper value of H to be used, and $B = H + 4\pi I$ gives the flux density *exactly*. Further, in the case of bars, if the length be about 500 diameters the demagnetising effect of the poles may be neglected and the magnetising force in the iron taken as identical with that (H) of the original field.

A more simple and practical way of regarding the above result is to consider H and B only, regarding them as cause and effect. In a certain portion of space occupied by a magnetic field there are H lines per sq. cm. when air is the medium, or any other non-magnetic substance, whereas, if the air be replaced by iron or steel, the number of lines per sq. cm. instantly rises from H to some other number B. Then **the ratio of the number present in the material to the number present originally in the air is the permeability** μ (neglecting the disturbances above mentioned).

84. Exact Definitions of the Preceding Quantities.—To apply to the present investigation the exact definition of the intensity of a magnetic field, viz. *that it is measured by the force in dynes on a unit pole*, we must imagine a cavity in the material in which to place the unit pole. A cavity of any shape will not do, however, for the walls of the cavity will exhibit magnetism and exert an influence on the unit pole, so that the force on the latter will not be H_1, *i.e.* the force due merely to the combined action of the original field and the end poles of the specimen.

Consider, however, a cavity such as is shown at *a* (Fig. 68), viz. a long indefinitely thin tunnel in the direction of magnetisation. The sides of the tunnel will exhibit no magnetisation, the poles at the ends of the tunnel will be weak and far away from the centre, and will not appreciably affect a unit pole put there; thus the field intensity at the centre of the tunnel will give the actual value of the magnetising force H_1 in the specimen. Hence **the magnetic or magnetising force H_1 inside the specimen is measured by the force in dynes on a unit north pole placed at the centre of a long and indefinitely narrow tunnel in the direction of magnetisation.**

Fig. 68.

We can also define the induction, B, in a manner similar to the above. Consider, for example, an indefinitely thin crevasse cut across the specimen (Fig. 68 *b*), its plane faces being perpendicular to the direction of magnetisation, and of course infinitely large compared with the distance between them, and imagine a unit north pole in the crevasse. It can be shown that the force on the unit pole due to the walls of the cavity is $4\pi I$. In addition, there is the force H_1

defined above. Thus the total force on the unit pole is $H_1 + 4\pi I$, and this measures the induction B. Hence **the induction or flux density B is measured by the force in dynes on a unit north pole placed in an indefinitely narrow crevasse with its parallel faces at right angles to the direction of magnetisation.**

The relations of Art. 83 may now be more precisely written:—

$$B = H_1 + 4\pi I,$$

$$\mu = \frac{B}{H_1} = 1 + 4\pi \frac{I}{H_1} = 1 + 4\pi\kappa.$$

85. Molecular Theory of Magnetism.—It is generally agreed now that the molecules of a magnetised body are themselves magnets. The growth of magnetism in such a body under external influence might then be due to one or other of two effects. The one view is that each molecule acquires magnetism under the influence of some external magnetic influence, and thus the whole bar gains magnetism because each molecule of the bar is gaining magnetism. The other view, *and the one generally accepted*, is that the molecules are *always* magnetised, but lie more or less in all directions at random when the bar as a whole appears unmagnetised; but when magnetic induction is brought to bear, these molecules, while not gaining in strength, tend to rotate so that they all point in a definite direction—their north poles one way, their south poles the other. The restraining force which prevents them from all pointing one way on the application of the least magnetising force, is due to the action of each molecule on its neighbours. As the influence becomes stronger so more and more become rotated until, when all point in the proper direction, the bar is magnetised to saturation point and no further magnetising force can increase the magnetic effect.

This is a brief outline of the modern view regarding Weber's theory, and certainly the observed facts are in substantial agreement with the theory. For a full treatment of the molecular theory (and electron theory) of magnetism the student should refer to the *Advanced Textbook of Magnetism and Electricity*.

EXAMPLES.

1. *A uniformly magnetised bar magnet* 10 *cm. long, having a moment of* 200 *C.G.S. units, is placed in a horizontal position, with its axis in the magnetic meridian. A small compass needle placed at a distance of* 10 *cm. east of the centre of the bar is observed to be in neutral equilibrium. Find the horizontal intensity of the earth's field.* (B.)

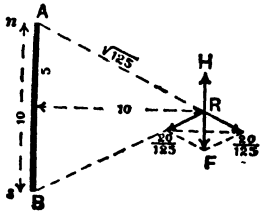


Fig. 69.

The conditions are shown in Fig. 69, and evidently the field at R must be

equal and opposite to H for the needle to be in equilibrium. Thus, from first principles—

$$H = 2\left(\frac{20}{125} \cdot \frac{5}{\sqrt{125}}\right)$$

since the pole strength $= \frac{200}{10} = 20$, and the term $\frac{5}{\sqrt{125}}$ is obtained by resolving the forces along the meridian;

$$\therefore \mathbf{H} = \frac{200}{\sqrt{(125)^2}} = \mathbf{\cdot 143}.$$

See Art. 79 (Case I.), viz. $F = M/AR^3 = H$ in this problem.

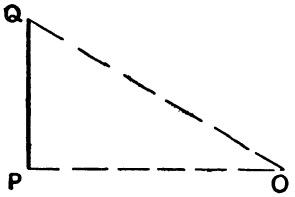


Fig. 70.

2. Field due to Vertical Bar Magnet.—*Find the field at* O (Fig. 70), *due to a vertical bar magnet* P Q *with its lower end level with* O, (*a*) *neglecting the upper pole*, (*b*) *taking the upper pole into account, and assuming the* N *pole to be at the bottom.*

(*a*) If we call the strength of the pole m, we have

$$f_1 \text{ at } O = \frac{m}{OP^2}.$$

(*b*) Taking both into account, we have

$$f_2 = \frac{m}{OP^2} - \frac{m}{OQ^2} \cos \angle POQ$$

$$= \frac{m}{OP^2} - \frac{m \,.\, OP}{OQ^3} = m\left[\frac{1}{OP^2} - \frac{OP}{(OP^2 + PQ^2)^{\frac{3}{2}}}\right].$$

If P Q be very long, then the second term will be negligible in practice.

3. *A small compass needle makes* 10 *oscillations per minute under the influence of the earth's magnetism. When an iron rod* 80 *cm. long is placed vertically, with its lower end on the same level with and* 60 *cm. from the needle and due (magnetic) south of it, the number of oscillations of the needle is* 12 *per minute. Calculate the strength of pole of the iron rod,* (1) *neglecting,* (2) *taking account of, the influence of the upper end* (B.)

Here we first note that the induced magnetism will be such as to make the lower end of the iron rod a N pole. As the rod is due south of the needle, the field of the rod strengthens the earth's field.

Thus, to find the actual field we have—

$$\frac{\text{Field at O}}{\text{Earth's field}} = \frac{12^2}{10^2} = 1{\cdot}44.$$

We know the earth's field to be 0·18.

Thus, since $\frac{H + f}{H} = 1{\cdot}44, \qquad \therefore f = 0{\cdot}44H.$

That is, $f = \cdot 079.$

CASE I.—*Assuming f due to lower pole only.*

$$\frac{m_1}{(60)^2} = \cdot 079, \qquad \therefore\ m_1 = \mathbf{285\ units.}$$

CASE II.—*Assuming f due to both poles.*

Noting that OQ = 100 cm.

$$f = \frac{m_2}{(60)^2} - \frac{m_2 \cdot 60}{(100)^3} \text{ from equation in Ex. 2 above;}$$

$$\therefore\ \cdot 079 = m_2\left(\frac{1}{(60)^2} - \frac{60}{(100)^3}\right) = 0{\cdot}000218 m_2,$$

$$\therefore\ m_2 = \frac{0{\cdot}079}{0{\cdot}000218} = \mathbf{363\ units.}$$

In this case the answers are not at all in agreement, because the length of the magnet is not such that the effect of the upper pole is negligible.

EXERCISES VII.

1. What is the force exerted between a pole of strength 4 units 2 cm. away from an unlike pole of strength 10 units? Give the answer both in dynes and pounds.

2. A bar magnet, of which the cross-section is 1·5 cm. and ·4 cm., and of which the length is 15 cm., is magnetised until the pole strength = 8 units. What are the values of (*a*) the magnetic moment, approximately (*b*) the intensity of magnetisation?

3. If the bar magnet of the last question be placed in a uniform magnetic field of strength 2 units, what will be (*a*) the force upon each pole of the bar, (*b*) the twisting moment on the bar, when it is at right angles to the field?

4. How many conventional lines of force will emerge from the N pole of the bar magnet in Ex. 2?

5. If the same steel bar magnet of Ex. 2 be allowed to oscillate freely in the field of Ex. 3, calculate the number of oscillations which the bar will execute per second. (Specific gravity of steel = 8.)

6. A bar magnet allowed to swing in one field executes 10 complete swings in 3 sec. When removed to another field there are 15 complete swings executed in 2 seconds. Compare the strengths of the fields.

7. Explain clearly what is understood by *permeability* and *susceptibility*. What kind of difference in the value of these would you expect in the case of iron, brass, and air?

8. Explain briefly the molecular theory of magnetism. How does the theory explain the condition of *saturation*?

9. What magnetic quantities are denoted by the symbols B, H, μ, and how are they related to one another? (O.)

10. Two short bar magnets, the moments of which are 108 and 192, are placed along two lines drawn on the table at right angles to each other. Find the intensity of the magnetic field due to the two magnets at the point of intersection of the lines, the centres of the magnets being respectively 30 and 40 cm. from this point. (B.)

11. Solve the following:—(*a*) If a pole of strength 12 repels another pole of strength 6 with a force of 8 dynes, what is the distance apart? (*b*) If the strength of the first pole be doubled, and the distance apart be quadrupled, what will be the force between them?

12. Define the pole strength and the magnetic moment of a simple magnet.

A bar magnet, whose magnetic moment is M and length $2l$, is placed with its centre at a distance r from a magnetometer. The direction of the axis is at right angles to the magnetic meridian and passes through the centre of the magnetometer. Find an accurate formula giving the deflection of the magnetometer.

If the magnet is turned through one right angle, keeping its centre fixed, find the condition that the magnetometer should be at a neutral point. (Univ.)

13. If two short magnets of equal moment are placed with the line joining their centres along the axis of one and perpendicular to the axis of the other, calculate the intensity (magnitude and direction) of the field at the middle point of the joining line. (Univ.)

14. Prove that the magnetic force exerted by a short bar magnet at a point A on the line passing through its centre and perpendicular to its axis is the same as the force exerted at a point on the axis the distance of which from the centre of the magnet is $\sqrt[3]{2}$ times the distance of A from the centre. (B.)

15. A magnet 10 cm long is placed in the magnetic meridian, the north end of the magnet being to the south. The field due to this magnet just counterbalances the earth's field (·18 C.G.S. unit) at a place 35 centimetres from the centre of the magnet (along the axis produced). Find the strength of each pole of the magnet. (B.)

CHAPTER VIII.

MAGNETIC MEASUREMENTS.

86. Law of Inverse Squares.—In this chapter the methods of experimental investigation and verification of the various effects mentioned in the two preceding chapters will be discussed. It has been stated that the force exerted between two isolated poles in air varies inversely as the square of the distance between them (Art. 69). These conditions cannot be exactly realised in practice, but certain results based on the truth of the law can be tested, and the law thus indirectly verified.

A classical form of apparatus for the proof of this law is known as *Coulomb's torsion balance*, but as it forms no part of the equipment of a modern laboratory it need not be described here.

1. *Proof of Gauss.*—In Chapter VII. we have proved that the force on unit pole due to an *end on* magnet is twice the force due to the same magnet placed at the same distance in the *broadside on* position, and *in our proof we assumed the truth of the law of inverse squares.*

Now, in Fig. 71 let D_1^0 be the deflection of a small needle produced by a magnet "end on" to it. For simplicity, let each pole of the needle be of unit strength; let F_1 be the force on each pole due to the magnet, and H the force on each pole due to the earth. Thus, since the needle is at rest,

Deflecting moment
= Controlling moment

$$F_1 \times nK = H \times OK$$

$$F_1 = H \times \frac{OK}{nK}$$

$$F_1 = H \tan D_1.$$

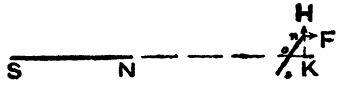


Fig. 71.

Let the magnet now be placed "broadside on" to the needle at the same distance, and let F_2 be the force on each pole of the needle due to it and D_2 the angle of deflection. Then, as above, we have—

$$F_2 = H \tan D_2;$$

that is,
$$F_1 : F_2 = \tan D_1 : \tan D_2.$$

But if the law of inverse squares be true F_1 is twice F_2, and therefore

tan D_1 will, *if the law be true*, be equal to twice tan D_2. Experiment shows that the tangent of the deflection produced by a magnet end on *is* twice the tangent of the deflection produced by the same magnet broadside on at the same distance; hence we are justified in assuming that the basis of our investigation—the law of inverse squares—is true. Gauss started with the assumption that the force exerted between two ideal poles varied inversely as the n^{th} power of the distance, and then carefully investigated mathematically the ratio of the forces exerted in the end on and broadside on positions under this assumption. He proved that the ratio of the forces would be n. *Experiment* shows that the ratio is 2; hence the deduction is at once made that n is equal to 2.

The Magnetometer.—The instrument used for such purposes and known as the magnetometer is shown in Fig. 72. The suspended magnet, which is itself small, carries a long pointer moving over the scale. The disturbing magnet may be placed either broadside on or end on along the graduated bars extending both ways. The arms are much longer in proportion than is shown in the figure. Other types are in use.

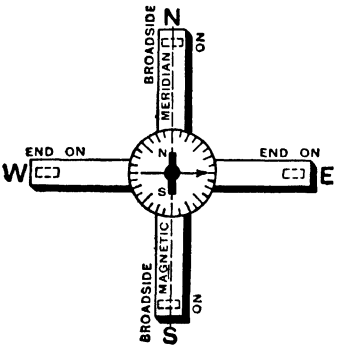


Fig. 72.

2. *Proof by Oscillations.*—Another experimental proof may be mentioned, viz. that due to the employment of an oscillating magnet. The conditions governing the oscillation of a magnet in a uniform magnetic field have been discussed in Art. 76; and evidently, if a suspended magnetic needle be caused to swing when in the field due to a fixed magnet and the earth, the time of an oscillation will involve, amongst other things, the fundamental law of action between the poles. If the needle be set swinging at different distances from the fixed magnet, since the earth's field is unaltered, any alteration in the time of swing will enable calculations to be made involving the effect of alteration of distance upon magnetic force, and thus the law can be verified. Thus, in an actual experiment the needle made 18 swings per minute under the earth alone, 38 per minute when the N pole of a long magnet was 4 in. to the south of it, and 25 per minute when the N pole was 8 in. from it. Hence (Art. 77)—

$$\frac{\text{Force at } 4''}{\text{Force at } 8''} = \frac{38^2 - 18^2}{25^2 - 18^2} = \frac{3{\cdot}7}{1} = 4 \text{ nearly.}$$

That is, at twice the distance the force is one quarter; and this is the law of inverse squares. With a *longer* and *thinner* magnet the result would have been nearer the true law.

87. Measurement of Field Strength and of the Moment of a Magnet.—In the preceding chapter we have seen how to *compare* one field strength with another and how to *compare* two magnetic moments. It is now necessary to deal with the method for actually *measuring* these quantities in any case.

From the investigation given in Art. 76 it is evident that by observing the time of vibration of a magnet when suspended in a field and substituting the value found in the equation, an equality can be obtained of the type—

$$\mathbf{H\,M = \text{some number.}}$$

From the investigation given in Art. 80 it is readily seen that by observing the deflection of a small needle placed in the field H by this same magnet of moment M and substituting the value of the deflection in the equation, an equality may be obtained of the type—

$$\frac{\mathbf{M}}{\mathbf{H}} = \mathbf{\text{some number.}}$$

Hence by taking the two equations—

$$\mathrm{H\,M} = a \text{ and } \frac{\mathrm{M}}{\mathrm{H}} = b$$

and solving, it is easy to find the value of both H and M; thus—

$$\mathrm{M\,H} \times \frac{\mathrm{M}}{\mathrm{H}} = ab, \therefore \mathbf{M} = \sqrt{ab},$$

$$\mathrm{M\,H} \div \frac{\mathrm{M}}{\mathrm{H}} = \frac{a}{b}, \therefore \mathbf{H} = \sqrt{\frac{a}{b}}.$$

The actual form of these two equations is—

$$\mathrm{H\,M} = \frac{4\pi^2\,\mathrm{K}}{\mathrm{T}^2} \quad \ldots\ldots\ldots, \quad (1)$$

and—

$$\frac{\mathrm{M}}{\mathrm{H}} = d^3 \tan\theta, \text{ or } \frac{d^3 \tan\theta}{2} \quad \ldots\ldots\ldots \quad (2)$$

and the experiment is carried out as follows:—Take the magnet of which the moment M is desired and measure its moment of inertia K; in the case of a bar magnet this is given by—

$$\mathrm{K} = \frac{(\text{Mass})\,(l^2 + b^2)}{12},$$

where l and b are the length and breadth respectively of the bar magnet.

Then suspend the magnet in a stirrup in the field of which we desire to know the strength. Allow the magnet to oscillate and measure the time T in seconds of a complete vibration; then all the numbers for equation (1) are known.

Next arrange the bar magnet in the unknown field as explained in Art. 80, so that a very small needle is deflected by it, then measure both d and θ; hence the numbers for equation (2) are known, and thus both H and M can be calculated.

The experiment is commonly carried out for the purpose of finding H at any place, in which case the general solutions obtained from the above equations are—

$$\mathbf{H} = \frac{2\pi}{\mathbf{T}}\sqrt{\frac{\mathbf{K}}{d^3 \tan \theta}}, \text{ and } \mathbf{H} = \frac{2\pi}{\mathbf{T}}\sqrt{\frac{\mathbf{2K}}{d^3 \tan \theta}},$$

the magnet in the deflection experiment being broadside on in Case 1 and end on in Case 2. A worked example is given at the end of the chapter.

88. Determination of the Magnetic Properties of Iron.—As has been already noted, iron is selected for use whenever magnetic effects of considerable magnitude are desired, and so general has this use become in the construction of dynamos, motors, transformers, and the like that the measurement of the magnetic properties of iron is now of fundamental importance. So important is this to the electrical engineer and so often will he meet references to the methods in his reading that an early *elementary* treatment is most desirable.

It will be seen in the section dealing with the magnetic properties of electric currents that there is a particular case in which an electric current may be used to produce magnetic fields of which the strength can be easily calculated with all the accuracy necessary for practical purposes. The case in question is when the wire carrying the current is coiled in the shape of a long, thin, hollow tube termed a *solenoid*, the condition for accurate calculation being that the length of the tube must be very large compared with its diameter; and there is a variation in the shape of the solenoid which is possible which makes an easy formula apply with all necessary accuracy—that is, when the solenoid or long tube is bent round, forming a ring, until the ends meet. These two forms are analogous to the long and very thin bar magnet, which has been mentioned in Chapter VI., and the closed-ring magnet which has been noted as having no poles.

Without going into the matter fully at this stage, as it is discussed later in the section on electro-magnetism, it may be stated here that the lines of force which will be produced in the interior of such a tube, when filled with air or non-magnetic material, is given by the expression—

$$\phi = \frac{4\pi}{10} \times \frac{a}{l} \times (\textbf{ampere turns}),$$

where ϕ = total lines produced,

a = sectional area of solenoid,

l = length of solenoid,

ampere turns = total turns × current in amperes (Art. 105).

Thus, from the above, it will be seen that if we write, as usual, H for the lines per sq. centimetre, then since

$$\phi = Ha, \text{ and since } \frac{4\pi}{10} = 1{\cdot}257,$$

the above rule becomes—

$H = 1{\cdot}257 \times$ (ampere turns per centimetre of solenoid).

We are thus provided with a means for producing a field of any requisite strength, since *the lines produced per unit area only depend upon the ampere turns.*

It has been pointed out that the introduction of an iron core leads to the production of a much larger number of lines than existed before, and if this number can be readily measured the determination of the magnetisation curve (*i.e.* a curve connecting I and H or B and H) and the permeability, etc , of a piece of iron becomes simple enough.

A well-understood property of an electric circuit, which also will be more fully dealt with later, is that if the number of magnetic lines passing through a closed electric circuit be altered, then a current is produced in the circuit. Hence if the circuit includes a galvanometer the alteration in the magnetic lines will produce a motion of the galvanometer needle *and from this motion the number of lines altered may be deduced.*

The apparatus needed for determining the magnetisation curve, etc., of a piece of iron when formed into a closed ring can now be dealt with in an elementary manner.

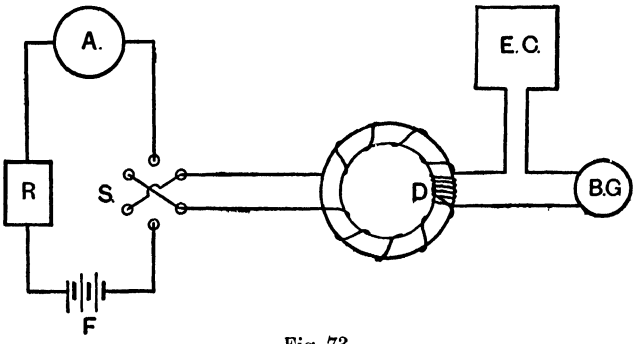


Fig. 73.

89. The Ring Method.—In Fig. 73 the current from a battery F can be caused to flow in either direction at will round a coil which is wound over the iron ring under test. The current may be controlled by a resistance R, reversed by a switch S and measured in amperes by an ammeter A.

Hence the lines which would flow through the coil if there were no

iron present can be at once *calculated* by means of the formula of the preceding section, *i.e.* H is known.

An additional coil of a few turns only is also wound round a portion of the ring at D, and this extra coil is connected to a galvanometer, which is of a special kind known as a ballistic galvanometer, and marked B. G. Included in the circuit of this galvanometer is also an additional piece of apparatus E. C., which is used merely for determining the actual value to be attached to any particular scale reading of the galvanometer needle. This piece of apparatus thus used for *calibration* consists of a coil of known dimensions and number of turns which can be quickly rotated by hand in the earth's field, and since the number of lines per square centimetre due to the earth's magnetism is exactly known, the deflection produced by the rapid twisting of the coil through 180° gives us a means for knowing that each degree of deflection means an alteration of so many magnetic lines through the circuit which includes the galvanometer. Hence the experiment is carried out as follows, after the above calibration.

A known current passing round the coil is suddenly reversed. If there were no iron in the coil it would follow that H lines of force per unit area would be reversed too, where $H = 1{\cdot}257 \times$ (amp. turns per cm.), and this would produce a deflection of the needle. As iron is present, however, the number of magnetic lines actually reversed is much larger than H per unit area, being, in fact, a number which in the preceding chapter we called the flux density and denoted by B. Hence from the first swing of the needle we estimate B; and as H is calculated, both quantities are known and can be plotted together to form a B H *curve.* A series of readings is obtained by altering the current to different values before reversal.

Even at this stage the reader may be able to follow the above in a little more detail.

Imagine the coil E C to be horizontal, the axis about which it is to be turned being in the magnetic meridian. For simplicity we will consider E C and the coil D *each to consist of one turn only.* If V be the vertical component of the earth's magnetic field and A sq. cm. the area of E C, then since there are V lines per sq. cm. the total magnetic lines passing through the coil E C will be V A. If the coil be now rotated through 180° the lines through the coil will change from V A to zero during the first 90° of rotation and from zero to V A during the second 90° of rotation, *i.e.* the total change in the lines is 2 VA. Let d_1 be the galvanometer deflection (first swing): then :—

A deflection d_1 means a change of 2 V A lines

$$\therefore \quad \text{,,} \quad 1 \quad \text{,,} \quad \text{,,} \quad \text{,,} \quad \text{,,} \frac{2\,V\,A}{d_1} \quad \text{,,}$$

Now pass a known current i amperes through the coil which is wrapped round the ring. Let B be the magnetic induction in the ring and a sq. cm. the cross sectional area: the total lines through the ring and coil D is therefore Ba. Now reverse the current i so that the Ba lines are reduced to zero and then put in

again in the opposite direction, *i.e.* the total change in the lines through D is $2\,B\,a$. Let d_2 be the galvanometer deflection (first swing of BG). From the calibration above, this means a change of $\left(\frac{2\,V\,A}{d_1} \times d_2\right)$ lines: hence:—

$$2\,B\,a = 2\,V\,A\,\frac{d_2}{d_1}, \qquad \therefore\ B = V\,\frac{A}{a}\,\frac{d_2}{d_1},$$

and as all the terms on the right are known (V = ·43), B is determined. The corresponding magnetising force H is also known from the value of i and the turns per cm. of the magnetising coil (H = 1·257 × amp. turns per cm.). Further, I is found from the relation $B = H + 4\pi I$, μ from the relation $\mu = B/H$ and κ from the relation $\kappa = I/H$. As already indicated, a series of corresponding values of B and H is obtained by altering the current i before reversal.

It may be mentioned that if the coil EC has n_1 turns and D has n_2 turns, the expression $2\,V\,A$ becomes $2\,V\,A\,n_1$, and $2\,B\,a$ becomes $2\,B\,a\,n_2$: the student will understand this later.

90. The Magnetometer Method.—This is much in use, too, for purposes of magnetic measurement, and has the advantage over the ring method that the specimen may be in the form of a long, thin wire, and this is quite separate from the coil.

The magnetometer is merely a small magnet hanging in the earth's or some other uniform field, and carrying a mirror in order that deflections may be accurately measured.

As in the ring method, the magnetising force is supplied by a solenoid, through which a current of any magnitude may be sent, and which may be reversed if desired.

This solenoid is placed vertically, and the thin wire under test may be placed within it, with its upper pole —when magnetised—on a level with the magnetometer needle. The wire should be very long and thin, the length, say, being equal to 500 diameters. The arrangement is shown in Fig. 74, M being the magnetometer and S S the solenoid.

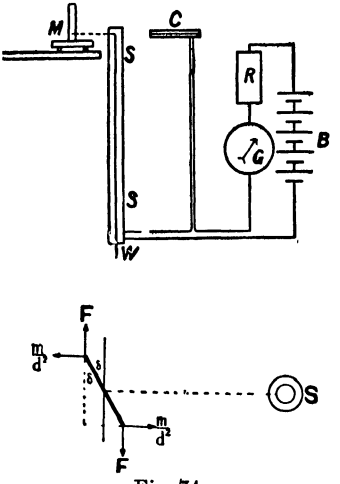


Fig. 74.

The current supplied can be measured by the indicating instrument G, and can be regulated by the resistance R; the reversing arrangement is not shown.

Since the coil is vertical, there is a field inside directed along the axis, even when no current is flowing, due to the vertical component V, of the earth's field. In order to allow for this, an outer solenoid is slipped over the one shown, and a

current is kept constantly flowing through this of such strength as to produce a field *equal* and *opposite* to the vertical field of the earth, thus neutralising the latter.

If this correcting coil were not used, the same result can be attained by remembering that H the magnetising force will be given by

$$1{\cdot}257 \times (\text{ampere turns per cm.}) \pm \text{V or } \frac{4\pi ni}{10} \pm \text{V},$$

i being the current in amperes (Art. 105) and n the turns per centimetre of the solenoid. V is approximately equal to ·43 unit. The plus sign is used if the vertical component assists the solenoid, and the minus sign if it opposes the solenoid.

In this method B is measured, not by a ballistic galvanometer, but by deducing the strength of the induced poles from the effect upon the magnetometer needle. Thus if F denote the strength of the field in which the needle is hanging, and if m be the pole strength induced in the wire, then, neglecting the effect of the lower pole, we have $\frac{m}{d^2} = \text{F} \tan \delta$, when d is the distance between the pole and the needle and δ is the deflection of the needle.

Then, as in Art. 72, $\qquad m = \text{I}a, \ \therefore \ \text{I} = \frac{m}{a};$

$$\therefore \ \text{I} = \frac{\text{F}d^2 \tan \delta}{a},$$

d, a, and δ are easily measured, and F, if it is a field different to that of the earth, can be measured by the method given in Art. 87, and thus I is known. Then, remembering that $\text{B} = \text{H} + 4\pi\text{I}$ (Art. 83), we have the two quantities I and B which we plot with H to obtain the magnetisation curves.

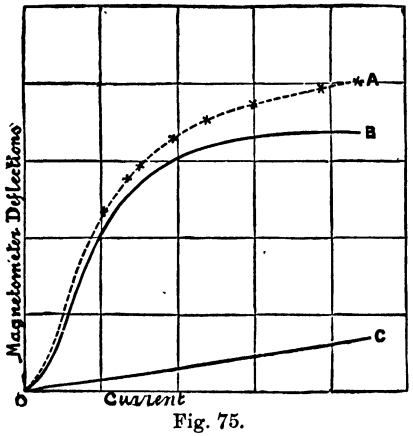


Fig. 75.

It is important to note, in this experiment, that the solenoid will affect the needle, without an iron core, since it acts as a weak magnet on its own account. This may be dealt with in two ways: (*a*) by the method of using a compensating coil, as shown at C in the diagram, which, being in series with the solenoid, is so placed by trial that it always has an equal and opposite effect to the empty solenoid; or (*b*) by drawing a curve of the total effect upon the needle

O A in Fig. 75, and then drawing the line O C for the effect of the coil alone upon the needle and taking the difference of the two; the curve O B is the true curve, giving deflections due to the specimen.

Since the deflections of the needle are proportional to B, and the magnetising force H is proportional to the current, the *shape* of the curve can be at once obtained by plotting deflections vertically and the currents horizontally, without troubling to make the ordinates true values of B and the abcissae true values of H.

To sum up in this connection, three curves may be drawn: (*a*) Deflections and currents plotted together; (*b*) I and H plotted together, in which case the ratio I/H anywhere on the curve is the *susceptibility*; (*c*) B and H plotted together, in which case the ratio B/H anywhere on the curve is the *permeability*.

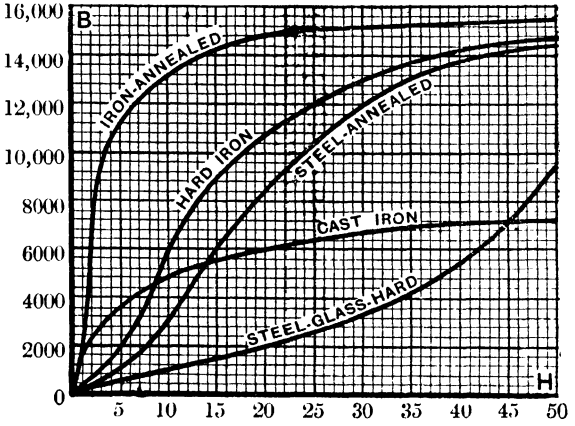


Fig. 76.

91. Magnetisation Curves.—Fig. 76 gives the B H curves for various samples of iron and steel, H being plotted horizontally and B vertically. With a magnetising force of only 2·5 the soft iron (annealed) gives a value of B of about 8,000, the cast iron about 3,000, the hard iron about 1,000, and the glass-hardened steel about 400. When H has the value 10 the soft iron curve is becoming horizontal, *i.e.* the iron is approaching saturation, but this does not take place with the steel and the hard iron until H is from 40 to 45. When H is 50 the soft iron, hard iron, and steel are all practically saturated, but the glass-hard steel is still rising, and its value of B is only equal to that in soft iron under a magnetising force of about 2·6. The I H curves would be somewhat similar.

There are three different stages indicated by a complete magnetisation curve drawn to a suitable scale:—

(*a*) A short initial stage when the magnetising force is small, in which a change in H produces only a very small change in B, and where the permeability is small. This part of the curve is better seen in Fig. 77. For *very small values of*

H the curve is a straight line inclined to the horizontal so that $\mu = B/H$ is constant.

(*b*) A stage where the curve is rising rapidly, *i e.* where a small change in H produces a large change in B, and where the permeability is increasing rapidly to a maximum value. Since $\mu = B/H = \tan \theta$ (Fig. 77), the permeability has its maximum value at C in that figure.

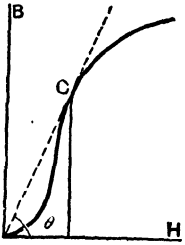


Fig. 77.

(*c*) The saturation stage or nearly horizontal part of the curve. Here a large increase in H produces *comparatively* little effect on B, and the permeability drops to quite low values.

92. Variations in Permeability.—To indicate how μ may alter with the flux, Figs. 78 and 79 may be consulted. In weak fields (Fig. 78) μ rises rapidly in the case of soft iron to a maximum, and then falls; in strong fields (Fig. 79) μ for all the magnetic bodies falls with the flux except in the case of manganese steel, for which μ is practically constant. The variation with temperature has been mentioned in Chapter VII., and is shown in Fig. 80. With a small

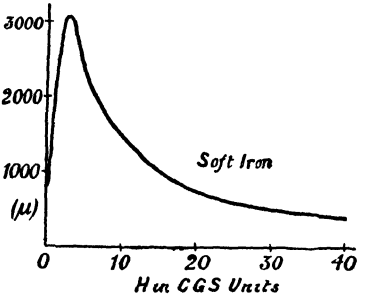


Fig. 78.

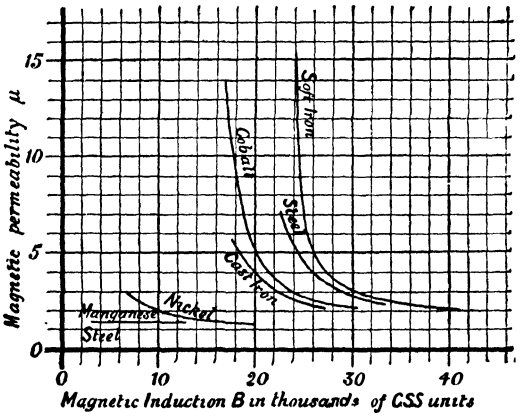


Fig. 79.

magnetising force μ increases with the temperature, at first slowly, then suddenly, and finally drops suddenly to unity at about 785° C.,

the critical temperature for iron. With a strong magnetising force μ steadily falls, and then drops rather suddenly at about 785° C. as before.

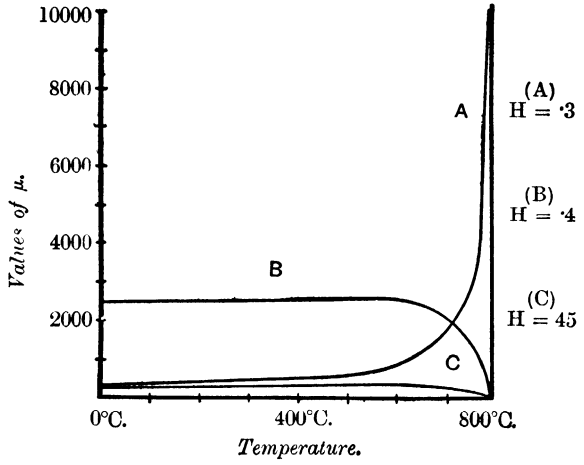


Fig. 80.

93. Carrying out a Cycle of Magnetisation.—If the currents allowed to flow round the ring in Art. 89 or round the bar in Art. 90 are gradually increased, thus producing an increase of H, rising, say, by equal steps, we find that B rises quickly at first, then more slowly, and finally, when the condition of magnetic saturation has been reached, no further increase in current will cause appreciable increase in B.

If we now commence to lessen the current flowing, the flux produced will lessen too, but when the current is reduced to zero, as at the beginning of the experiment, the flux will not fall to zero; some will remain. This remaining flux is termed the **residual magnetism.** If, further, we then commence to send the current round the ring in the reverse direction, it will have to reach some particular value before the previous flux will be quite removed; this particular value of H is termed the **coercive force** of the specimen.

Increase of the reverse current beyond this amount will cause flux in the reverse direction; but, as before, after the reverse current reaches a certain amount the flux will remain stationary, indicating saturation in the reverse direction. If, now, we lessen the reverse current by steady steps, the reverse flux will lessen too; but, as before, when the current has reached zero value again the flux will not be zero, but some will remain.

If, again, the current be reversed into the old direction, and then steadily increased by steps, the reverse flux will lessen, become zero,

and it too will then go back into the old direction, and so on, until the current has again its full value in the first direction and the flux has again reached a maximum. Such increase of current and flux from zero to a maximum in one direction, and then back through zero to a maximum in the other direction, and finally back again through zero to the first maximum, is spoken of as a **magnetic cycle.**

94. Hysteresis.—The effect which is brought into prominence when we find that, although the current causing flux is brought to zero, the flux itself has not returned to zero, is termed **hysteresis.** That is, **hysteresis is the lagging of the magnetic flux produced behind the magnetic force (or magnetising force) producing it.**

Fig. 81 shows such a cycle plotted out to scale. H measured to the right is due to current flowing in one direction, while H measured to the left is due to the reversed current.

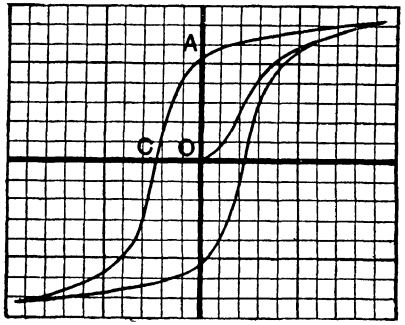


Fig. 81.

So, too, the magnetic flux measured upwards is that due to the direct current, while that measured downwards is that due to the reversed current.

Residual Magnetism.—The intercept O A in the diagram shows the magnetic flux remaining when the current is reduced to zero, and this flux is termed the *residual magnetism* —that is, O A is the residual magnetism to scale.

Coercive Force.—So, too, the intercept O C shows the amount of magnetic force needed to just remove all the residual magnetism, and this, as has been mentioned above, is termed the *coercive force.*

The whole loop is termed the **hysteresis loop,** and it can be shown that *the area of the figure is proportional to the work wasted in the iron due to the rapidly changing magnetic condition:* in fact in the B H case of Fig. 81 if the area is estimated in terms of B and H *and then divided by* 4π, the result is the waste of energy (in ergs) per cubic centimetre per cycle, and if I H curves be plotted, the area, estimated in terms of I and H, is the waste of energy (in ergs) per cubic centimetre per cycle. The narrower the loop the less work wasted in hysteresis. Naturally all such wasted work means heat developed in the iron undergoing magnetisation, and it is obviously to the advantage of the engineer to choose such iron that the work wasted shall be as small as possible, at any rate in all cases where iron is subjected to fluctuating magnetic forces. That is, he desires that the iron under test should give a narrow curve or loop, and it is to this end that

magnetic testing of iron supplied for electrical purposes is a daily necessity. It is a consequence of this hysteresis that the curve which begins at O, is never again re-traced.

95. Commercial Tests.—The importance of the determination of the magnetic properties of iron, etc., in electrical engineering practice has led to the introduction of several commercial instruments and methods for the purpose; thus we have Ewing's Hysteresis Tester, Hopkinson's Bar and Yoke, and Ewing's Double Bar and Yoke, Ewing's Permeability Bridge, Drysdale's and Thompson's Permeameters, etc. Space will permit only of a brief reference to one or two here.

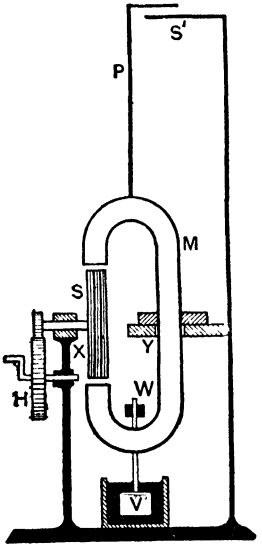


Fig. 82.

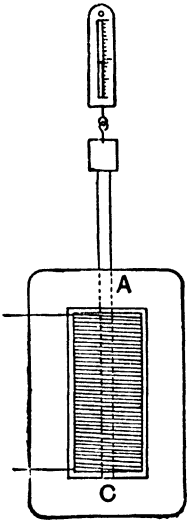


Fig. 83.

(1) EWING'S HYSTERESIS TESTER.—This is shown diagrammatically in Fig. 82. M is a permanent magnet pivoted at Y so that it can move in a plane at right angles to the plane of the paper; it carries a pointer P which moves over the scale S', a control weight W, and a vane V which, moving in oil, renders the instrument dead beat. The specimen S is pivoted at X and can be rotated between the poles of M by means of the hand wheel H. As the specimen rotates it is magnetised by the field of M, the lag in magnetisation causing it to drag the magnet M after it, thereby producing a certain deflection of P over the scale S'; the greater the lag the greater the deflection, so that the latter is proportional to the hysteresis loss in the specimen. The instrument is calibrated by means of a specimen of known hysteresis supplied with it.

(2) THOMPSON'S PERMEAMETER.—This is shown in Fig. 83. The rod under test is a good fit at A, where it passes through the wrought-iron yoke, and it

also makes good contact with the yoke at C. The rod carries a magnetising coil, so that the magnetising force H is known. With the current passing the contact at C is broken by a pull on the spring balance, in which case it can be shown that

$$B = 1317\sqrt{\frac{P}{S}} + H,$$

where P is the pull in pounds and S the area in square inches of the surface of contact at C; thus B is determined.

(3) HOPKINSON'S BAR AND YOKE.—The bar to be tested carries a magnetising coil and is fixed into a heavy soft iron yoke (Fig. 84). A small coil S encircles the middle of the rod and is connected to a ballistic galvanometer, and the first swing d is observed when the magnetising current is reversed. From this first swing the magnetic lines in the rod and therefore B the lines per sq. cm. is known, as indicated in Art. 86. Hence, knowing H, the necessary quantities are determined.

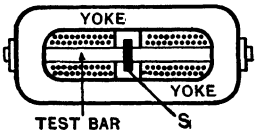


Fig. 84.

(4) HOPKINSON'S DIVIDED BAR AND YOKE.—The bar to be tested is made in two parts and is fixed into an iron yoke as before. It carries a magnetising coil and a small coil S joined to a ballistic galvanometer. With a magnetising current i passing, the left-hand half of the rod is suddenly pulled out whereupon the coil S is suddenly ejected by means of a spring. The lines through S are thus suddenly removed and from the first swing d of the ballistic galvanometer the total lines in the bar and therefore B the lines per sq. cm. is determined. The magnetising force H is known from the known magnetising current and the turns per cm. of the magnetising coil. Thus the quantities B, H, I, μ, and κ are determined.

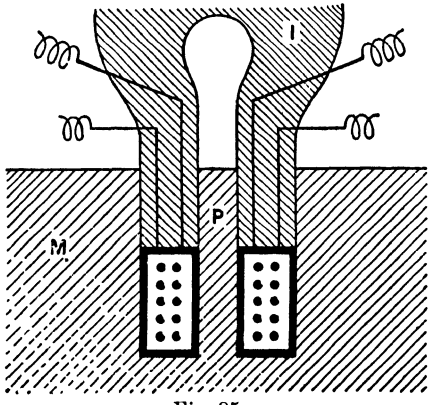


Fig. 85.

(5) DRYSDALE'S PERMEAMETER.—This enables tests to be carried out on castings and forgings in bulk so that the material has not to be made into rings or bars. By a special drill a hole is bored in the material M leaving a central projection P (Fig. 85). An iron plug I is fitted into this hole. It carries a bobbin which fits on P and the bobbin carries a magnetising coil and a secondary coil, the latter being joined to a ballistic galvanometer. The experiment can be carried out as in the preceding ballistic methods.

96. De-magnetisation.—Should a piece of iron have been magnetised, and yet be required for experiment with all its magnetism removed, the process of de-magnetisation must be carried out.

The readiest way in which to perform this is to subject the specimen to a magnetising force which is at least as great as any to which the piece has been previously subjected, and then, while causing this current to reverse at a rapid rate, to gradually decrease its value to zero.

97. Magnetic Traction.—The elementary facts relating to this have been given in Chapter VI. Theory shows that the law of the tractive force or pull (P) between two magnet faces of area A sq. cm. is given by the expression—

$$\mathbf{P} = \frac{\mathbf{B^2 A}}{8\pi} \text{ dynes,}$$

where B is, as usual, flux density in lines per square centimetre. Experiments carried out on this subject quite support the expression given above.

98. Experiment to determine the Moment of a Magnet (M) and the strength of the Earth's Field (H).—The magnet had the following dimensions :—

$l = 15{\cdot}28$ cm. ; $b = {\cdot}65$ cm. ; *wt.* $= 95{\cdot}34$ grm.

Hence K (being rectangular magnet) $= 95{\cdot}34 \dfrac{l^2 + b^2}{12} = 1858.$

(1) *Deflection Experiment.*—The magnet was placed *end on* to a small needle, and the deflection θ was $10{\cdot}5°$; also the distance between the centre of the magnet and the centre of the needle was 40 cm. Using the relation—

$$\frac{\mathrm{M}}{\mathrm{H}} = \frac{\left\{d^2 - \left(\frac{l}{2}\right)^2\right\}^2}{2d} \tan\theta$$

we have by substitution— $\dfrac{\mathrm{M}}{\mathrm{H}} = \dfrac{(1600 - [7{\cdot}64]^2)^2}{80} \tan 10{\cdot}5°.$

$$\therefore \mathbf{\frac{M}{H} = 6371{\cdot}4.}$$

(2) *Oscillation Experiment.*—On suspending the magnet and allowing it to oscillate, the time taken per complete vibration—taking the mean of a large number—was 19·5 seconds. Now—

$$\mathrm{H\,M} = \frac{4\pi^2 \mathrm{K}}{\mathrm{T}^2},$$

$$\therefore \mathbf{H\,M} = \frac{4\pi^2(1858)}{(19{\cdot}5)^2} = \mathbf{192{\cdot}93};$$

$$\text{and } \mathrm{H\,M} \times \frac{\mathrm{M}}{\mathrm{H}} = 192{\cdot}93 \times 6371{\cdot}4 = \mathrm{M}^2,$$

$$\therefore \mathbf{M = 110{\cdot}87};$$

$$\text{and } \mathrm{H\,M} \times \frac{\mathrm{H}}{\mathrm{M}} = 192{\cdot}93 \times \frac{1}{6371{\cdot}4} = \mathrm{H}^2;$$

$$\therefore \mathbf{H = {\cdot}174.}$$

Note that in Art. 80 the length of the magnet is $2l$: here it is l, hence the appearance of $l/2$ in the calculation.

99. Determination of Dip.—Ideally we require a needle balanced absolutely at its centre of gravity on perfect pivots, uniformly magnetised, and accurately placed so that the vertical plane containing the needle is in the magnetic meridian of the place. Such a needle would indicate the correct angle of dip.

In practice the supported needle is arranged in an instrument like a theodolite and termed a **Dip Circle**, so that the readings of both ends can be read by microscopes on a vertical circular scale. The vertical scale can be rotated about a vertical axis, and its position, at any stage, can be read off on a horizontal scale. Fig. 86 depicts the *Kew Dip Circle*.

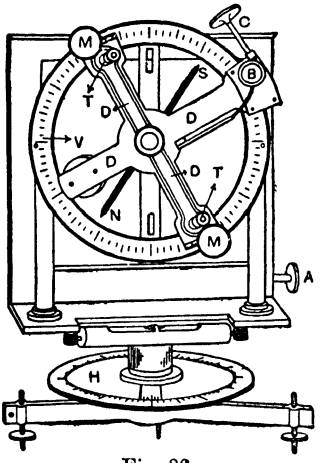


Fig. 86.

V = Vertical Graduated Circle. H = Horizontal Graduated Circle. MM = Microscopes for Reading Scale and Verniers. TT = Eyepieces for noting the ends of the Needle NS by Cross Wires. A = Screw for Raising and Lowering Needle Supports B = Clamp for Fixing the Movable Arms D. C = Screw for Accurate Adjustment.

Several errors may exist, and these must be eliminated in taking an observation :—

(1) *The zero line of the vertical circle may not be truly horizontal.* This is corrected by repeating the readings when the vertical circle is turned through 180° over the horizontal scale.

(2) *The needle may not be pivoted at the centre of the vertical scale.* This is corrected by reading both ends of the needle.

(3) *The magnetic axis of the needle may not coincide with the line joining its ends.* This is eliminated by reversing the needle in its bearings.

(4) *The needle may not be suspended at its centre of gravity.* If the centre of gravity be along the neutral line off to one side, the method in (3) above will correct. If the centre of gravity be nearer one end, the needle must be remagnetised, the poles being reversed.

(A) To Obtain the Direction of the Meridian.—If the vertical circle be at right angles to the meridian, the needle will be acted upon only by the vertical component of the earth's field, and the needle, if the instrument be perfect, will point absolutely vertically, as the horizontal component is resisted by the pivots when so arranged. The operation is then as follows :—

The vertical scale is turned, to be as nearly as possible at right angles to the expected meridian, and adjusted till the lower end of the needle is at zero, and the horizontal circle is read. The needle is reversed in its bearings and a new reading of the horizontal circle taken, when the lower end is at zero again. Two similar readings are then taken for the top of the needle, thus giving four readings on the horizontal scale in all. The vertical circle is finally turned through 180°,

and four more readings on the horizontal obtained. If the arithmetical mean of the eight readings be θ, and the vertical circle be rotated through 90° from this, the needle will lie in the plane of the meridian.

(B) To obtain the Dip.—With the instrument set up as above, a reading is taken for each end of the needle. The needle is reversed in its bearings and two more readings taken. The vertical circle is then turned through 180° and the four readings repeated. Finally, the needle is reversed in polarity and the eight readings repeated. The mean of the sixteen readings gives the true value of the dip.

100. The Compass.—Some properties which are desirable in a compass are :—(*a*) Permanence and intensity of magnetism in the magnetic system. (*b*) Steadiness under shock and vibration, which necessitates a long, natural period of vibration when disturbed. (*c*) Coincidence of the magnetic axis of the suspended system and the line joining the N and S points of the card. (*d*) Quick damping of any vibration. The magnetic moment should be large, because the effective field on a warship, say, is a small fraction of the earth's true horizontal field (sometimes only about 0·2 of it), and the time of vibration should be large in order that it shall never synchronise with the rolling of the ship. The magnets should be short, or they will induce magnetism in neighbouring correcting pieces of soft iron. Kelvin's Compass and the Liquid Compass most nearly satisfy these conditions.

(1) *Kelvin's Compass.*—This consists of eight small parallel needles, suspended by silk threads from an aluminium ring, which carries the card, the weight being supported by a sapphire centre. The small needles have a greater magnetic moment for their weight than a single bar would have, and the moment of inertia of the system is made the same about all diameters. The period of vibration is something over half a minute. This compass meets all the conditions fairly well, except that vibrations are not rapidly damped.

(2) *Liquid Compass.*—This is now the standard for many vessels. The card, which is of mica, is immersed in a vessel containing a mixture of water and alcohol, expansion being provided for by an elastic diaphragm. Such a compass is very satisfactory ; it has good directive power and great damping, but there is sometimes trouble with the quadrantal correctors. (Art. 101.)

101. Errors and Corrections in the Ship's Compass.—Magnetisation by the inductive action of the earth's magnetic field has been dealt with in Chapter VI., and the student should again note these facts before proceeding with this section.

Similar effects apply to the case of an iron or steel ship, so that such a ship on being launched is more or less a *permanent* magnet depending on her position during construction, whilst subsequently the masses of iron will be constantly subject to the earth's inductive action, such *transient* magnetism varying with the whereabouts and position of the ship at the time of observation.

In practice, matters are more intermingled and complex than is indicated above. The iron or mild steel constituents of a ship consist, as a rule, of three classes of magnetic material, viz. hard iron, soft iron, and iron neither hard nor

soft (intermediate iron), any one bar of iron or mild steel possessing, usually, all three properties. The *hard iron* retains *permanent magnetism*. The *soft iron* is magnetised by the earth's field according to the position of the ship at any instant, and such magnetism is determined solely by the ship's direction and position on the globe; this is termed *temporary magnetism*. The *intermediate iron* retains some of its magnetism for a time, and thus the amount in it at any given instant depends not only on the position of the ship at the time, but partly on the history of the ship's movements; this is termed *sub-permanent magnetism*.

Practically, then, the following situation arises:—The ship is a weak permanent magnet due to the hard iron present, and the strength depends upon the direction in which she was built and upon her subsequent experiences. There is, in addition, a magnetic effect which gradually changes with time and vibration, heavy seas and the like, and which is due to the intermediate iron present. Finally, there is an instantaneous effect depending upon the position of the ship at any instant, and due to soft iron present.

Let, then, such a ship be turned through 360° ("swinging the ship"), starting with the bow pointing magnetic north, and remaining on an even keel all the time. The compass deviations due to the above effects may be briefly classified in an elementary manner as follows:—

(1) **Semicircular Error.**—Consider a **vertical mass of soft iron** situated as in Fig. 87, and magnetised by the earth so that the upper end (supposed on a level with the compass) is a south pole. The effect on the compass is *nil* in Fig. 87, but, as the ship swings round, the effect becomes a *maximum* at B (Fig. 88), *nil* at C, a *maximum* at D, and again *nil* at A; hence this is referred to as *semicircular variation*, and the error could, in this case, evidently be corrected by suitably placing a vertical bar of soft iron on the opposite side of the compass ("Flinders' bar").

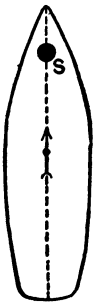


Fig. 87.

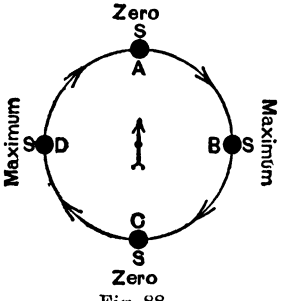


Fig. 88.

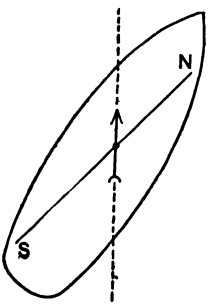


Fig. 89.

A little consideration will convince the student that semicircular variation may also be caused by **longitudinal and transverse masses of hard iron,** due to permanent and subpermanent magnetism. Thus in Fig. 89, if N S be the permanent magnetic axis of the ship, say (due to its position during construction), then, whenever the ship is in such a position that this line lies in the magnetic meridian the deviation effect on the compass is *nil*, but in all other

positions of the ship a deviation is produced; hence the error is semicircular. A diagram will readily indicate that the error due to *transverse hard iron* is also semicircular, vanishing when the ship's head is east and west (Fig. 90). Semicircular errors due to hard iron are corrected by placing magnets fore and aft in the binnacle and by others athwartships.

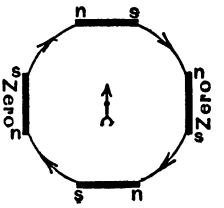


Fig. 90.

(2) **Quadrantal Error.**—Consider a **horizontal mass of soft iron,** situated as in Fig. 91 and magnetised by the earth as shown. As the ship swings the effect on the compass is evidently *nil* in the positions A, C, E, G, and a *maximum* in the positions B, D, F, H; hence this is referred to as *quadrantal variation.* Clearly the effect of D (Fig. 91) is opposite to that of B, so that if B be the disturbing mass the error could evidently be eliminated by suitably placing a soft iron mass D in the adjacent quadrant (*i.e.* so that $\theta = 90°$). In practice, however, the correction is made by means of two soft iron spheres, one on each side of the compass. The variation due to these is evidently quadrantal, and it may, therefore, be made to cancel the other.

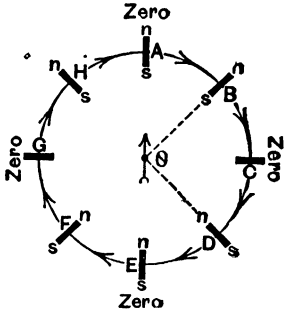


Fig. 91.

(3) **Heeling Error.**—In addition to the above, which are found when the ship is on an even keel, there is an important error due to *heeling*; this is due mainly to vertical hard iron or intermediate iron, to vertical soft iron beneath the compass, and to horizontal soft iron. The error is mainly corrected by adjustable vertical magnets under the compass card. That part of the heeling error due to horizontal soft iron is corrected by the quadrantal spheres.

EXAMPLE.

A magnet 10 *cm. long, with poles of unit strength, is freely suspended in a horizontal uniform magnetic field of intensity* ·18 *dyne; find the moment of the couple tending to restore the magnet to its position of rest when deflected in a horizontal plane through* 30° *from that position.*

We know Twisting Moment = H M sin θ,

and $M = ml = 1 \times 10 = 10$ units,

$H = \cdot 18$ dyne and $\theta = 30°$,

$\therefore$ **Moment of couple** $= 10 \times 0{\cdot}18 \times \sin 30° =$ **0·9 C.G.S. unit.**

Further, *to find the work done during this twisting.* We notice that the N pole has been moved a net distance *up* the field $= \frac{l}{2} - \frac{l}{2} \cos 30°$, and the S pole has been moved *down* the field by an equal amount. Now the force exerted on each

pole, urging the N pole down the field and the S pole up is given by $mH = 1 \times \cdot 18 = \cdot 18$ dyne. And this has been overcome in each case through a distance

$$= 5 - 5 \cos 30^\circ = 5 \times \cdot 866$$
$$= \cdot 67 \text{ cm.}$$

Hence **work** $= \cdot 18 \times \cdot 67 \times 2 = \cdot 2412$ **erg.**

This is the work done during the whole rotation of 30°.

N.B.—The attention of the student cannot be too often drawn, in this connection, to the fact that when any twisting moment or couple of constant amount is allowed to operate through an angle θ,

Work done = Couple × Angle in circular measure.

EXERCISES VIII.

1. Explain the nature of the proof devised by Gauss for the law of action between two magnet poles.

2. State what operations must be carried out to enable you to measure both the moment of a given small magnet and the strength of a given uniform field.

3. Sketch the magnetisation curves you would expect for (*a*) air, (*b*) soft iron, (*c*) hard steel.

4. From the magnetisation curve of Example 3 draw a fresh curve showing the connection between μ or B/H and B.

5. Define cycle of magnetisation, residual magnetism, coercive force. In what practical magnetic applications are cycles of magnetisation to be met with?

6. A bar magnet has poles of strength 4 units and a length of 5 centimetres. The bar is placed in a uniform magnetic field of strength ·5 with its axis parallel to the field. The bar is then rotated slowly through 90°. Draw a curve showing the varying twist necessary to produce this rotation and deduce the work done.

7. What is meant by the statement that the permeability of iron is (say) 800. Is the permeability of a given specimen of iron constant? If not, on what does its variation depend? (B.)

8. A closed soft-iron ring 100 cm. mean circumference and 5 sq. cm. cross-section is wound uniformly with 200 turns of insulated wire. Supposing that you have found the following relations to exist in iron of this quality:

B	10,200	12,000	13,700
μ	2,000	1,500	1,000

calculate the current in amperes at which the total flux of magnetic lines is 65,000. (O.)

9. Calculate the number of ampere turns of excitation required to magnetise up to 14,000 lines per sq. cm. a soft-iron ring 20 in. in mean diameter made of round iron 1 in. thick ($\mu = 800$). (O.)

Note.—The last two problems may be worked from data given in the preceding chapters. They may, however, be postponed until the details of Chapter XVII. are mastered.

PART IV.—CURRENT ELECTRICITY.

CHAPTER IX.

FUNDAMENTAL PRINCIPLES AND DEFINITIONS.

102. Effects of an Electric Current.—In Art. 19 it has been explained that in order to secure a continual flow of electricity from one point to another some conducting path must be provided between them and means must be devised for maintaining the two points at different potentials, in which case electricity will continue to flow from the higher to the lower potential. With the sundry methods whereby a P.D. can be maintained at the ends of a conductor, the reader will become acquainted later; for the present we merely wish him to realise that this state of affairs is attainable. In all cases the conductor acquires certain new properties attributed to this mysterious agent "flowing" along it; technically we refer to it as a "current" of electricity in the conductor, and the branch of our subject dealing with the phenomena is called "*current electricity*" or "*electrodynamics.*"

The existence of a current of electricity is known by various effects which it produces, *e.g.*—

(1) A compass needle suitably placed near the wire is deflected, and if the wire be coiled round a bar of soft iron the latter is converted into a magnet. These are known as the *magnetic effects* of the current, and are utilised in defining the C.G.S. electromagnetic unit current, in current measurements by galvanometers and ammeters, in voltmeters, dynamos, electric bells, etc.

(2) If the wire conveying the current be severed, the ends soldered to two suitable metal plates, and these placed some short distance apart in various liquids, the majority of the latter are decomposed, one of the products of the decomposition appearing at one plate, and another at the second plate. This is known as the *chemical effect*, and is utilised in defining the practical unit current, in accumulators, copper refining, electroplating, electrolytic production of alkalies, etc.

(3) The conductor through which the current flows becomes heated. This is referred to as the *heating effect* of the current, and is utilised

in the working of ammeters and voltmeters, in electric lighting, electric welding, in the construction of electric furnaces, etc.

In magnetism we are concerned with magnetic fields and lines of magnetic force *at rest* and in electrostatics with electric fields and lines of electric force *at rest* the reader will learn later that in current electricity we are really concerned with the *coexistence* of the above two fields *while in a condition of relative motion.*

103. Meaning of Current Strength.—Consider a pipe A B through which water is steadily flowing in the direction A to B. Some idea of the strength of this water current may be conveyed by a statement of the quantity of water entering A, leaving B, or passing any section of the pipe in a definite time, say one second; in short, *the strength of the current may conveniently be defined as the rate of flow of water through the pipe.* Accepting then this definition, the total quantity flowing past any section in a given time is obviously obtained by multiplying the current strength by the time in seconds; stated algebraically—

$$\text{Quantity} = \text{Current Strength} \times \text{Time in Seconds}$$

$$\text{Current Strength} = \frac{\text{Quantity}}{\text{Time in Seconds}}.$$

Again, when the pipe is quite full, the quantity entering A per second must be equal to the quantity leaving B or passing any section of the pipe in that time, however uneven the bore may be; *that is, the current strength is the same at all parts of the pipe.* Clearly, too, the pressure at A is greater than at B, and falls in value as we proceed from A to B; that is, whilst the current strength is the same at all parts, *the pressure falls in the direction in which the current is flowing.*

These elementary ideas have their electrical analogies. "**Current Strength**" is defined as **the quantity of electricity passing any section of the conductor per second,** *i.e.* as the **rate of flow of electricity in the circuit,** *and the total quantity which passes in a stated period is given by the product of the current strength and the time in seconds;* further, *the current strength is the same at all parts of a simple conductor, but there is a fall of potential in the direction of the current.*

104. Units of Current Strength and Quantity.—The C.G.S. electromagnetic unit of current is explained in Chapter XVII.; briefly we may mention here *it is that current which flowing in a circle of* **1** *cm. radius exerts a force of* 2π *dynes on a unit pole at the centre.* For practical purposes 1/10 of this unit is taken and is called an **ampere;** clearly *in the above case the ampere would exert a force of* $\cdot 2\pi$ *dyne on the unit pole.*

The C.G.S. electromagnetic unit quantity is the quantity conveyed by the electromagnetic unit current in one second (it is 3×10^{10} elec-

trostatic units). For practical purposes 1/10 of this is taken and is called a **coulomb**: *it is the quantity conveyed by a current of one ampere in one second* (it is 3×10^9 electrostatic units). The smallest quantity or natural unit of electricity, viz. the **electron**, is $1{\cdot}55 \times 10^{-19}$ coulomb.

105. The Ampere and its Board of Trade Definition.—In Art. 102 we have mentioned the fact that many liquids are decomposed by electricity. The process is termed *electrolysis*, the liquid is called the *electrolyte*, and the containing vessel the *voltameter*. The metal plates by which the current enters and leaves the liquid are termed the *electrodes*; that by which it enters is the *anode*, that by which it leaves is the *kathode*; the constituents which are liberated at the electrodes are called the *ions*.

A typical example is afforded by copper sulphate ($CuSO_4$). On passing a current through the liquid by means of copper electrodes, *metallic copper is deposited in a thin layer on the kathode*, the action being expressed by the equation $CuSO_4 = Cu + SO_4$. The SO_4 attacks the copper anode, forming $CuSO_4$; some also joins with the water (H_2O), forming sulphuric acid (H_2SO_4) and liberating oxygen, thus: $SO_4 + H_2O = H_2SO_4 + O$. Again, the oxygen may combine with the copper, forming copper oxide, and this oxide may dissolve in sulphuric acid, forming copper sulphate. In the majority of cases the action at the anode is of the complex character indicated, but if the plate be in such a condition that it is readily attacked by the SO_4, copper is merely taken from it to form $CuSO_4$, and the loss in weight of the anode is equal to the gain of the kathode. Numerous experiments have established the facts—

(1) That one ampere in one second deposits ·0003293 gramme of copper;

(2) That the chemical action is proportional to the current strength, *e.g.* if the current be doubled, twice the amount of copper will be deposited in the same time;

(3) That the chemical action is proportional to the time the current passes.

No. (1) above furnishes a convenient practical definition of the ampere, viz. **the ampere is that steady current which flowing through a solution of copper sulphate deposits ·0003293 gramme of copper on the kathode in one second.**

From the facts enumerated it will be evident that a current of I amperes flowing for t seconds will deposit ($·0003293 \times I \times t$) grammes of copper; that is—

$$·0003293 \times I \times t = \text{increase in kathode} = w$$

$$\therefore \mathbf{I} = \frac{w}{\mathbf{·0003293} \times t}$$

an expression from which, knowing w in grammes and t in seconds, an unknown current of I amperes may be found.

The method above is that adopted by the Standards Committee of the Board of Trade in its practical definition of the ampere. In the specification silver nitrate is recommended and the resolution really settles that **an unvarying current which, when passed through a solution of nitrate of silver in water, deposits silver at the rate of ·001118 of a gramme per second be taken as a current of one ampere.**

106. Practical Quantity Units: the Coulomb and the Ampere-hour.—The coulomb has been defined as *the quantity of electricity transferred by a current of one ampere in one second*; it is sometimes referred to as the **ampere second.** If I denotes the current strength in amperes, t the time in seconds, and Q the total quantity which has passed in coulombs, $Q = It$ and $I = Q \div t$.

Since one ampere in one second liberates ·0003293 grm. of copper and ·001118 grm. of silver, the **coulomb** is practically defined as **that quantity of electricity which liberates ·0003293 gramme of copper from a solution of copper sulphate, or ·001118 gramme of silver from a solution of silver nitrate.**

Another quantity unit termed the **ampere-hour** is employed. **An ampere-hour is the quantity of electricity conveyed by a steady current of one ampere flowing for one hour;** but one ampere in one hour will transfer 3,600 coulombs; hence—

$$\text{One ampere-hour} = 3{,}600 \text{ coulombs}$$
$$= 360 \text{ e.m. quantity units.}$$

107. Electro-chemical Equivalents.—The C.G.S. or absolute electro-chemical equivalent of an ion is the amount of the ion liberated by the electromagnetic unit current in one second, *i.e.* by the electromagnetic unit quantity; *the ampere electro-chemical equivalent of an ion is the amount of it liberated by one ampere in one second,* i.e. *by one coulomb*; thus ·003293 and ·0003293 are the absolute and ampere electro-chemical equivalents of copper respectively.

Table of Ampere Electro-chemical Equivalents,
i.e. *grammes per coulomb.*

Hydrogen	=	·00001044	Copper	=	·0003293
Oxygen	=	·00008291	Iron	=	·00029
Potassium	=	·0004053	Nickel	=	·0003043
Sodium	=	·0002383	Zinc	=	·0003387
Silver	=	·001118	Lead	=	·001073

If z denotes the electro-chemical equivalent of the ion, our formulae of Arts. 105 and 106 may be put in the general forms—

$$(1)\ w = zIt. \qquad (2)\ w = zQ. \qquad (3)\ I = \frac{w}{zt}. \qquad (4)\ Q = \frac{w}{z}.$$

108. Potential Difference (P.D.) and Electro-Motive-Force (E.M.F.).—In Part II. Potential has been defined as that electrical condition which determines the direction in which electricity will flow, and it has been shown that the P.D. in electrostatic units between two points is represented numerically by the work done in ergs in the transference of the electrostatic unit quantity from one point to the other. Similar reasoning applies to the present branch of the subject. As a simple illustration, consider two points A and B in a conductor through which a current is flowing in the direction A to B, and in which, therefore, A is at the higher potential. The conductor A B is heated, and by the law of conservation of energy this heat must have been produced at the expense of an equivalent amount of energy which has disappeared from the electric circuit; we have in short an *energy transformation* between the two points, electric energy being transformed into heat energy. Imagine, now, that the electromagnetic unit quantity (*i.e.* 10 coulombs) passes from A to B and that w ergs is the energy, which in this case is *subtracted* from the electric circuit, appearing as heat; the P.D. between the two points is w electromagnetic units of potential. If the unit quantity is *forced* from a low to a high potential work must be done on it, and energy is *added* to the electric circuit; but in either case—

The P.D. in electromagnetic units between two points is represented numerically by the amount of energy transformed in ergs when the electromagnetic unit quantity passes between the two points. Hence—

The P.D. between two points is one C.G.S. electromagnetic unit if the energy transformed is one erg when the electromagnetic unit quantity passes.

The practical unit is the **volt**; it is equal to 10^8 electromagnetic units and $\frac{1}{300}$ of the electrostatic unit. One microvolt $= \frac{1}{1000000}$ volt.

Since the volt is 10^8 electromagnetic units, if the P.D. between two points is one volt the energy transformed will be 10^8 ergs when the electromagnetic unit quantity passes, and therefore $\frac{1}{10}$ of 10^8, *i.e.* 10^7 ergs, when one coulomb passes; but one joule is equal to 10^7 ergs. Hence—

The volt is that P.D. which must exist between two points in order that the energy transformed may be one joule when one coulomb of electricity passes.

A practical definition of the volt is dealt with later (Art. 111).

"**pressure**" and sometimes the "**voltage**" between the two points. Another term of constant use in current electricity is "**Electro-Motive-Force**" (**E.M.F.**). To illustrate its meaning consider a battery on "open circuit," *i.e.* with the outside circuit disconnected; imagine an electrostatic voltmeter joined to its terminals and that the reading is 50 volts; this measures the total pressure it is capable of generating, and this total pressure is the E.M.F. of the battery.

If the outside circuit be switched on the voltmeter reading will fall by an amount depending on circumstances; imagine the reading is now 48 volts; this measures the terminal P.D. under present conditions or the volts used in driving the current through the external circuit, the other two volts being used in getting the current through the battery on account of the resistance of the latter. Thus *the E.M.F. of a battery is the total pressure it is capable of generating, and is measured by the P.D. at its terminals on open circuit*; when the current flows the terminal P.D. may be anything according to circumstances, *but it is always less than the E.M.F.* If E volts be the E.M.F., *e* volts the terminal P.D., and V be the volts used up in the generator—

$$\mathbf{E} = e + \mathbf{V}, \quad \therefore e = \mathbf{E} - \mathbf{V}, \quad \text{and } \mathbf{V} = \mathbf{E} - e.$$

The preceding demonstrates what for practical purposes may be regarded as the distinction between P.D. and E.M.F.; *the latter term should be used when referring to the total pressure generated (measured by the P.D. at the terminals on open circuit), the former when referring to the pressure between two points in the circuit.* Scientifically, one corresponds to a reversible, the other to a non-reversible energy transformation, but this is beyond the reader at this stage.

109. Resistance and Conductance.—*Resistance may be defined in a general way as that property of a body which opposes the flow of electricity through it;* good conductors have small resistance, insulators have enormous resistance. With a given P.D. the less the resistance the greater will be the current, and *vice versa*; it is from this that a precise definition of resistance is obtained.

Consider two points A and B in a conductor between which there is no generator of E.M.F. (such, for example, as a dynamo or battery), and imagine a steady direct P.D. to exist between the two points; a certain steady direct current will be flowing between them. If, now, the P.D. be altered in magnitude the strength of the current will also be changed, but experimentally it can be proved that *if the temperature of the conductor be kept constant, the ratio of the P.D. to the current is constant;* this constant is called the "**resistance**" of the part A B of the conductor; stated algebraically—

$$\frac{\text{Potential Difference}}{\text{Current}} = \text{a Constant} = \text{Resistance},$$

or

$$\frac{\mathbf{E}}{\mathbf{I}} = \mathbf{R}.$$

If E and I be each unity, R is unity. Hence—

A conductor has a resistance of one C.G.S. electromagnetic unit if a P.D. of one electromagnetic unit applied to its ends causes a current of one electromagnetic unit to flow through it.

The practical unit is the **ohm**; **a conductor has a resistance of one ohm if a P.D. of one volt applied to its ends causes a current of one ampere to flow through it.**

$$\text{One ohm} = \frac{1 \text{ volt.}}{1 \text{ amp.}} = \frac{10^8 \text{ e.m. units}}{\frac{1}{10} \text{ e.m. unit}} = 10^9 \text{ electromagnetic units.}$$

One megohm = 1,000,000 ohms. One microhm = $\frac{1}{1000000}$ ohm.

Elaborate experiments have been performed with the object of ascertaining the precise dimensions of a column of mercury having a resistance of one ohm, and the results of these lead to the following practical definition:—**The ohm is the resistance of a column of mercury 106·3 cm. long, 1 sq. mm. in cross-section, and of mass 14·4521 grammes at the temperature of melting ice (0° C.).** Special coils of platinum silver wire have been constructed, having at 16·4° C. a resistance of one ohm, and these are deposited at the Board of Trade Standardising Laboratory. An older unit of resistance, constructed by the British Association Committee for the Advancement of Science in the years 1863-4, and known as the B.A. unit, is equal to ·9866 ohm.

Conductance may be defined as *that property which a body has for conducting electricity;* good conductors have large conductance, and insulators have small conductance. Conductance is, in fact, the converse of resistance, and is measured by the reciprocal of it; thus a wire of resistance R ohms has a conducting power of 1/R units of conductance.

The unit of conductance is the **mho.** A body of resistance one ohm has a conducting power of one mho; hence—*The mho is the conductance of a column of mercury* 106·3 *cm. long,* 1 *sq. mm. in cross-section, and of mass* 14·4521 *grammes at the temperature of melting ice.* It is suggested that the unit of conductance be named the "**siemens.**"

110. Another Definition of the Volt and Ampere.—From the relation $\frac{E}{I} = R$, referred to in the preceding paragraph, it is clear that if I and R be each unity, E is unity; hence, *unit electromagnetic P.D. may be defined as that which is necessary to drive the electromagnetic unit current through the electromagnetic unit resistance.* Similarly, **the volt is that electrical pressure which if steadily applied to a conductor whose resistance is one ohm will produce a current of one ampere.**

Finally, if E and R be each unity, I is unity; hence, *unit electromagnetic current will flow in a wire of resistance one electromagnetic unit if the applied P.D. is one electromagnetic unit*, and **an ampere will flow through a wire of resistance one ohm if the applied P.D. is one volt.**

111. Summary of the Practical Units of Current Strength, Resistance, and Potential Difference. The True, International, and Legal Units.—To further help the student, the practical units already dealt with are summarised below and classified into what are known as *true*, *international*, and *legal* practical units, for the student will encounter these terms in his reading. Any slight difference between the true, international, and legal units is, of course, neglected for practical and calculation purposes.

(1) *Current Strength.*—The **true ampere** is $\frac{1}{10}$ of the C.G.S. electromagnetic unit. It is that current which, flowing in a circle of 1 cm radius, exerts a force of $\cdot 2\pi$ dyne on a unit pole at the centre: it is also that current which a P.D. of one *true* volt will cause to flow in a wire of resistance one *true* ohm. The **international ampere** is that steady current which, flowing through a solution of silver nitrate, deposits ·001118 grm. of silver on the kathode in one second: it was intended to be a *practical realisation* of the true ampere of $\frac{1}{10}$ absolute unit, but *to be exact* it is *just a very little less* than the true ampere (about ·025 per cent.). The **legal ampere** is the current which gives a certain reading on a standard ampere balance (Chapter XII.) kept at the offices of the Board of Trade: the balance was standardised by the deposition of silver.

(2) *Resistance.*—The **true ohm** is 10^9 C.G.S. electromagnetic units of resistance. It is the resistance a wire must have in order that a P.D. of one *true* volt may cause a current of one *true* ampere to flow through it. (As the student will see later, it is also the resistance of a wire if the heat produced in it in one second, when one *true* ampere flows, is one joule.) The **international ohm** is the resistance of a column of mercury 106·3 cm. long, mass 14·4521 grm. (cross-section 1 sq. mm.) at 0° C.: it was intended to be a *practical realisation* of the true ohm of 10^9 absolute units, but, *to be exact*, it is *just a very little bigger* than the true ohm. The **legal ohm** is the resistance at 16·4° C. between the terminals of a "standard ohm" kept at the offices of the Board of Trade: this standard was, however, checked by the mercury column.

(3) *Potential Difference.*—The **true volt** is 10^8 C.G.S. electromagnetic units. It is the P.D. between two points if the energy transformed be one joule when one *true* coulomb ($\frac{1}{10}$ e.m. unit of quantity) passes: it is also the P.D. which will cause a current of one *true* ampere to flow through a resistance of one *true* ohm. The **international volt** is the P.D. which, when applied to a wire of resistance one *international* ohm, will cause a current of one *international* ampere to flow, and it may be taken as 1/1·0183 of the E.M.F. of a Weston cell at 20° C. (Art. 145). *To be exact*, it is *just a very little bigger* than the true volt. The **legal volt** is ·01 of the P.D. which produces a certain deflection on a standard electro-

static voltmeter kept at the offices of the Board of Trade : this is intended to be the same as the international volt just as in the other cases above.

Needless to say, in the sections which follow, the true, international, and legal practical units are taken as identical.

112. Preliminary Ideas on Alternating Currents.—The subject of alternating currents is dealt with in Chapter XX., but, as the student will constantly encounter them in his reading, a brief reference to one or two points may be given even at this stage.

The current derived from an ordinary cell or battery is what is known as a continuous current; that is to say, so long as the external circuit is closed, the current flows continuously in one direction. There are, however, currents which do not flow in this way but periodically reverse, flowing for a certain time in one direction and then reversing, flowing for another period of time in the opposite direction; such are called **alternating currents.** Referring to the hydraulic analogy, we may say continuous currents correspond to the flow of water along a line of pipes fed by a force pump, while on the other hand alternating currents correspond to the flow of water along pipes leading at one time from a force pump, and at another from a suction pump, the water being alternately forced into the pipes and then sucked from them.

When an alternating current is produced in a circuit the E.M.F. continuously changes ; it rises to a maximum, then dies away to zero, being followed immediately by a reversed E.M.F., which also rises to a maximum and dies away to zero, and so on. If, now, at any particular moment the E.M.F. is at a certain instantaneous value, and is just going to commence a certain set of variations, then the time which elapses between this instant and the moment when the E.M.F. has the same value and is going to commence an identical set of variations, is called the **Period** of the current, and the number of periods accomplished in one second is called the **Frequency,** the maximum value of the E.M.F. being called the **Amplitude.** The exact meaning of these terms can best be seen from Fig. 92.

Suppose time is measured along O X, and E.M.F. along O Y. Then for a certain alternating current the curve showing the relation between the variation of voltage and time will be O A B C D E. . . . Volts measured above O X correspond to the direct E.M.F.; volts measured below O X correspond to the reversed E.M.F.

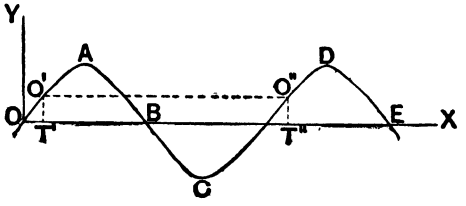


Fig. 92.

We may select any arbitrary point on the curve, such as O'; then, counting time from the instant when the E.M.F. was last zero, we see that at time O T', the voltage was O' T'. Drawing O'O parallel to O X, we also see that at time O T'' the voltage was O'' T'' = O' T', and since when the voltage was O' T' and O'' T'' the same set of variations

was commencing, we see that by definition T′ T″ represents what we have called the *period*. Evidently the period would also be represented by B E. The *frequency* would then be the number of such time intervals as T′ T″ contained in one second, and the volts corresponding to the maximum points A, C, and D, etc., would be the *amplitude*.

The currents accompanying these periodic electromotive forces are also periodic; they rise to a maximum, fall to zero, reverse, rise to a maximum, fall to zero, and so on; and the frequency of the E.M.F. is also that of the current. There is, however, this important difference. The alternating currents do not exactly keep time with the alternating electromotive forces, so that the maximum current is not obtained when the instantaneous value of the E.M.F. is a maximum, but occurs a little before or after it according to the nature of the circuit.

If an inductive resistance (Chapter XVI.) is included in the circuit, the maximum current will occur *after* the E.M.F. has reached its maximum value; the current is then said to **Lag**. On the other hand, if there is capacity in the circuit the maximum current will occur *before* the E.M.F. has reached its maximum value; the current is then said to **Lead**. These terms may readily be understood from Fig. 93, which shows a curve of alternating E.M.F., V V V . . . , and the curve of alternating current with inductance in the circuit. The lag, or lead, is best expressed in terms of the period of the alternating E.M.F. Thus if the maximum current occurred $\frac{1}{8}$ period after the maximum E.M.F., we should say the lag was $\frac{1}{8}$ of a period; and similarly with the lead. The lag and lead are sometimes spoken of as the **Phase Difference.**

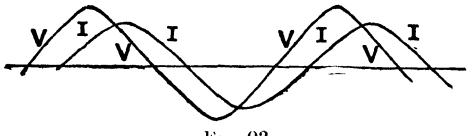


Fig. 93.

The frequencies in general use are—for electric railways 25 and for electric lighting and power 50.

Alternating currents and pressures are measured in what are termed **virtual amperes** and **virtual volts** respectively and these virtual values may be shown to be equal to the maximum values divided by $\sqrt{2}$, *i.e.* the maximum values are equal to the virtual values multiplied by $\sqrt{2}$. *A.C. voltmeters and ammeters read virtual values* so that if, for example, an A.C. voltmeter in an A.C. circuit reads 200 volts the pressure is really fluctuating between $(200 \times \sqrt{2})$, *i.e.* 282·8 volts first one way and then the other. Stated differently, *the virtual values are* ·707 *of the maximum values* (the *mean* value for a half period is ·637 of the maximum value). The reason for this method of measurement in virtual values will appear later.

113. Preliminary Ideas on Polyphase Currents.—This term is applied to cases in which we have more than one circuit with an alternating current in each, these currents not rising and falling in step with each other. Thus, for example, we may have one pair of wires—lead and return—conveying alternating current, and in addition we may have a separate pair of wires—lead and return

—conveying another alternating current, and this second current be such that when the first is a maximum the second is just reversing—that is, it has the momentary value zero; or, put in the form explained above, the two currents differ in phase by 90° or $\frac{\pi}{2}$ or a quarter of a period, and are said to be in *quadrature*. Such a double circuit would be spoken of as a **two-phase** arrangement, and in practice the two currents would be tapped from the same source of E.M.F.—the alternator—which, however, is only a matter of convenience.

So, again, we may have three separate circuits each conveying a separate alternating current, and the three currents related to each other in such a way that each one differs in phase from the next by one-third of a period or $\frac{2\pi}{3}$. Such an arrangement is spoken of as a **three-phase** system, and in practice here, too, all three currents would be produced from the same machine, by suitable construction.

In practice it is true, for purposes of economy in the line wires, two-phase systems may be used in which there are not four wires but three, two being amalgamated; and so with three-phase systems, which may employ three wires instead of the six. This practice of using a common return, as it were, does not interfere with the conditions stated above. At this stage we cannot enter into the conditions or arrangements of such systems.

EXAMPLES.

1. *How many coulombs and how many ampere-hours will be required to deposit* $\frac{1}{4}$ *lb. of silver from a solution of silver nitrate? What steady current in amperes will be required if the experiment is to be of four hours' duration?*

By Art. 107 $$Q = \frac{w}{z}.$$

$$w = \tfrac{1}{4}\text{ lb.} = \frac{453{\cdot}6}{4}\text{ grammes} = 113{\cdot}4\text{ grammes},$$

$$z = {\cdot}001118\text{ grammes per coulomb.}$$

Hence— $$Q = \frac{113{\cdot}4}{{\cdot}001118}\text{ coulombs} = \mathbf{101{,}431\ coulombs}\text{ nearly.}$$

Again— $$\textbf{Ampere-hours} = \frac{\text{Coulombs}}{3600} = \frac{101431}{3600}.$$

$$= \mathbf{28{\cdot}2}.$$

Finally— No. of amperes × No. of hours = No. of ampere-hours

$$\therefore \textbf{No. of amperes} = \frac{\text{No. of ampere-hours}}{\text{No. of hours.}}$$

$$= \frac{28{\cdot}2}{4} = \mathbf{7{\cdot}05}.$$

2. *A dynamo of resistance* ·02 *ohm, when running on open circuit, gave a P.D. at its terminals of* 100 *volts. It was then joined to mains the total resistance of which, lead and return, was* ·06 *ohm, and the current in the circuit was found to be* 50 *amperes. Calculate*: (a) *Volts used in driving the current through the dynamo*; (b) *P.D. at dynamo terminals*; (c) *Volts used in the mains*; (d) *P.D. at the consumer's end of the mains*; (e) *Total resistance in the consumers' circuit.*

By Art. 109, $\frac{E}{I} = R$; $\therefore E = IR$.

Hence—

Volts used in dynamo = Current in dynamo × Res. of dynamo
= 50 × ·02 = **1 volt.**

By Art. 108, $e = E - V$;

i.e. P.D. at terminals = 100 − 1 = **99 volts.**

Again—

Volts used in mains = Current in mains × Res. of mains
= 50 × ·06 = **3 volts**;

$\therefore$ P.D. at consumers' end = 99 − 3 = **96 volts.**

Finally, since $R = \frac{E}{I}$,

Total resistance in consumers' circuit $= \frac{\text{P.D. for that circuit}}{\text{Current in it.}}$

$= \frac{96}{50} =$ **1·92 ohms.**

EXERCISES IX.

1. Explain the meaning of the terms ohm, volt, coulomb, ampere, megohm, microfarad, milliampere. (H.)

2. Define shortly electromotive force, resistance, and current. (P.)

3. Write down the terms work, force, power, quantity, as headings, and then arrange the names of the following units each under its proper heading: Watt-kilowatt, ampere-hour, horse-power, kilowatt-hour, dyne. (P.)

4. How may the strength of an electric current be measured by means of a copper voltameter? (B.)

5. Explain the term "electro-chemical equivalent." If 3 amperes deposit 4 grm. of silver in 20 min., what is the electro-chemical equivalent of silver? (B.)

6. How many coulombs and how many ampere-hours will be required to deposit 1 lb. of copper from a solution of $CuSO_4$? What steady current in amperes would be required if the experiment were to continue for 25 hr.?

7. A current of 2·5 amperes is passed through a solution of $CuSO_4$ for 1 hr. 50 min.; determine the amount of copper deposited. How long would this current have to pass in order to deposit 4·9 grm. of copper?

8. How many pounds of silver and how many pounds of nickel would be deposited by 1,000 ampere-hr. ?

9. A cable 100 yd. long carries a current of 300 amperes ; if the P.D. between its ends is 2·5 volts, what is the total resistance of the cable, and what is its resistance per yard? What would be the P.D. if the current were reduced to 200 amperes ?

10. A pair of cables each 100 yd. long supply 350 amperes current to a distributing box, and the "drop" (in volts) in the cables is 5 volts. What would be the "drop" if the current were reduced to 225 amperes? (P.)

11. If an increase in the resistance of a circuit by 10 ohms causes the strength of the current to decrease from 5 to 2, find the total resistance of the circuit after the change. (B.)

12. A dynamo of resistance ·015 ohm when running on open circuit gives a P.D. at its terminals of 110 volts. If it is connected to mains of total resistance ·065 ohm and the current is 65 amperes, determine (*a*) volts used in the dynamo, (*b*) P.D. at terminals, (*c*) P.D. at consumers' end of mains, (*d*) volts used in the mains, (*e*) total resistance in the consumers' circuit.

13. The potential at the terminals of a battery falls from 10 to 6 volts when a resistance of 10 ohms is connected between its terminals. What is the resistance of the battery? (H.)

14. An arc lamp taking 10 amperes at 50 volts is connected across supply mains at 110 volts with a regulating resistance in series with the lamp. How many ohms must this resistance be? Determine in "ergs," "joules," and "B.O.T. units" the energy subtracted from the electric circuit, and which appears as heat in the regulating resistance, if the 10 amperes continue to flow through it for 3 hr. ? What would be the cost of the energy thus absorbed by the resistance at 6*d.* per B.O.T. unit?

15. Define the following in connection with alternating currents: Frequency, amplitude, phase difference, period, and distinguish between the average and the virtual value of an alternating current.

16. In an alternating-current circuit the voltmeter reads 150 volts, while the ammeter reads 70 amperes. It is known that the current lags $\frac{\pi}{6}$ behind the pressure. Plot the pressure and current curves to scale in their correct relative positions.

17. If there are three alternating currents, such that the second lags $\frac{1}{3}$ period behind the first and the third lags $\frac{1}{3}$ period behind the second, show by plotting and measurement that the *algebraic* sum of the three currents is always zero.

CHAPTER X.

QUANTITATIVE RELATIONS.

114. Ohm's Law.—This important law has already been given in connection with the definition of resistance in Art. 109; briefly it may be thus enunciated—

When the temperature of a conductor is kept constant, the ratio of the steady direct P.D. between any two points (no generator of E.M.F. being between them) to the resulting steady direct current is constant. In symbols the law may be expressed by the equation—

$$\frac{E}{I} = R,$$

where E is the P.D., I the current, and R a constant, the value of which is termed the resistance of the part of the conductor considered. For purposes of calculation it is convenient to remember three forms of the equation; thus from the relation—

$$\frac{\mathbf{E}}{\mathbf{I}} = \mathbf{R}, \textit{ i.e. } \frac{\text{Potential Difference}}{\text{Current}} = \text{Resistance} \quad \ldots\ldots\ldots \quad (1)$$

$$\mathbf{E} = \mathbf{IR}, \textit{ i.e. } \text{Potential Difference} = \text{Current} \times \text{Resistance} \quad \ldots\ldots \quad (2)$$

and—

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{R}}, \textit{ i.e. } \text{Current} = \frac{\text{Potential Difference}}{\text{Resistance}} \quad \ldots\ldots\ldots\ldots \quad (3)$$

It will be evident that given any two of these quantities the third may be determined from these relations. With regard to "units," if E and R in (3) be expressed in C.G.S. electromagnetic units, I will be obtained in C.G.S. electromagnetic units; if E be in volts, and R in ohms, I will be given in amperes, and similar remarks apply to the two preceding equations.

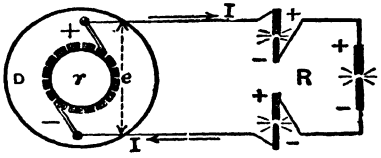


Fig. 94.

The Law of Ohm is equally applicable to the whole electric circuit, and to any portion thereof.

Imagine a dynamo D (Fig. 94) of internal resistance r ohms, supplying a current of I amperes as indicated. If E volts denote the E.M.F. of the dynamo, *i.e.* the total pressure generated, and which is utilised in driving the current through the *entire* circuit, e volts its terminal P.D., *i.e.* that portion of the total pressure utilised in driving the current through the *external* circuit of resistance R ohms,

$$\mathbf{I} = \frac{\textbf{E.M.F.}}{\textbf{Total Resistance}} = \frac{\mathbf{E}}{r + \mathbf{R}},$$

$$\mathbf{I} = \frac{\textbf{Terminal P.D.}}{\textbf{External Resistance}} = \frac{e}{\mathbf{R}},$$

$$\mathbf{I} = \frac{\textbf{Volts used in dynamo}}{\textbf{Resistance of dynamo}} = \frac{\mathbf{E} - e}{r}.$$

If the dynamo be employed in driving a motor, charging accumulators, or work of electrolysis generally, a back pressure of e_1 volts, say, is set up in opposition to the E.M.F.; in such cases the "resultant" or "effective" pressure is $E - e_1$ and—

$$\mathbf{I} = \frac{\textbf{Total Effective Pressure}}{\textbf{Total Resistance}} = \frac{\mathbf{E} - e_1}{r + \mathbf{R}}.$$

Frequently in an electric circuit several E.M.F.'s, say E_1, E_2, E_3, are working in the *same direction*, as, for example, when cells are joined in "series"; in such cases the total E.M.F. is the sum of the individual E.M.F.'s, and—

$$\mathbf{I} = \frac{\text{E.M.F.}}{\text{Total Resistance}} = \frac{E_1 + E_2 + E_3}{r + \mathbf{R}},$$

r *being the sum of the internal resistances.* If there are n equal E.M.F.'s in "series," say each of E volts,

$$\mathbf{I} = \frac{n\mathbf{E}}{r + \mathbf{R}}.$$

115. Laws of Resistance: Effects of Temperature and Molecular Change—

(1) *The resistance of a conductor is directly proportional to its length;* thus 1 yard of copper wire of 20 B.W.G. has a resistance of ·02 ohm; hence lengths of ·5 yard, 2 yards, and 50 yards will have resistances of ·01 ohm, ·04 ohm, and 1 ohm respectively.

(2) *The resistance of a conductor is inversely proportional to its area of cross-section;* thus if one wire has double the area of cross-section of another of the same material, and equal lengths be taken, the thick one will have half the resistance of the thin one.

(3) *The resistance of a conductor of given length and area of cross-section depends on its material;* thus a piece of platinum of given dimensions has 6·022 times the resistance of a piece of silver (annealed), and 5·724 times the resistance of a piece of copper (annealed) of the same dimensions.

(4) *The resistance of a substance depends on its molecular condition, density, purity, hardness, etc.* A decrease in the density of copper has been shown to increase its resistance. Wires subjected to mechanical strain have been found to be hardened, and of increased resistance. The resistance of an alloy is always greater than that of the substances forming it; thus the resistance of a centimetre cube of aluminium (*i.e.* its specific resistance or resistivity) is 2·665 microhms, that of iron being 9·065, whilst an alloy of iron with about 5 per cent. of aluminium has a resistivity of about 70 microhms per centimetre cube. The resistance of metals is diminished by annealing, the effect being more marked in pure metals than in the majority of alloys; but one particular alloy of iron with 14 per cent. of nickel, 5 per cent. of manganese, and 2 per cent. of aluminium shows a pronounced decrease, its resistivity changing from 89 to 48 microhms per centimetre cube on annealing.

(5) *The resistance of a substance depends on its temperature.* This has been referred to in Art. 17; the following are the main facts:—

(*a*) METALS.—These increase in resistance when heated, and decrease when the temperature is lowered. If R_t denote the resistance of a wire at $t°$ C. and R_o its resistance at 0° C.,

$$R_t = R_o (1 + \alpha t + \beta t^2),$$

where α and β are constants for the same wire but slightly different for different materials. For calculations the simpler relation—

$$\mathbf{R_t = R_o (1 + \alpha t)}$$

may be employed; α is known as the **temperature coefficient,*** and may be defined as *the increase in unit resistance per unit rise in temperature* (0° C. *to* 1° C.). In the case of pure metals α may be taken as equal to ·0038; for mercury its value is ·00075 nearly. The variation of resistance with temperature is indicated by the curves in Fig. 95.

It has long been surmised that at the absolute zero of temperature (−273·7° C.) the resistance of all metals will be *nil*, this being based on the assumption that the laws of variation of resistance continue to hold at very low temperatures. Dewar, however, found that at −250° C. the decrease in the resistance of platinum on cooling was *not so marked* as at −200° C., and Dr. Harrison's experiments appeared to show that at −253° C. the resistance of iron *was somewhat greater* than at −191° C. On the other hand, in some recent experiments, Kamerlingh Onnes finds that at the temperature of liquid helium the resistance of certain pure metals is less than one-thousand-millionth of the values at 0° C., which is

* See Appendix.

considerably less than if it fell in proportion to the absolute temperature. Further research is required in this direction.

(*b*) ALLOYS.—Most alloys increase in resistance with temperature, but not to the extent that pure metals do. **Manganin** (Art. 17) increases in resistance from 0° C. to 35° C., after which its temperature coefficient becomes negative, but the variation is so small that it may be neglected. For **German Silver** Art. 17) the coefficient is only ·00044, or about $\frac{1}{5}$ that of the pure metals. **Platinoid** (Art. 17) has $1\frac{1}{2}$ times the resistance of German silver, and its temperature coefficient is only ·00021. **Platinum Silver,** in the proportion one of platinum to two of silver, has a coefficient of ·00027. **Resista,** an alloy of iron with 15 per cent. of nickel and 5 per cent. of manganese, has a resistance equal to four times that of German silver, and a coefficient of ·00109.

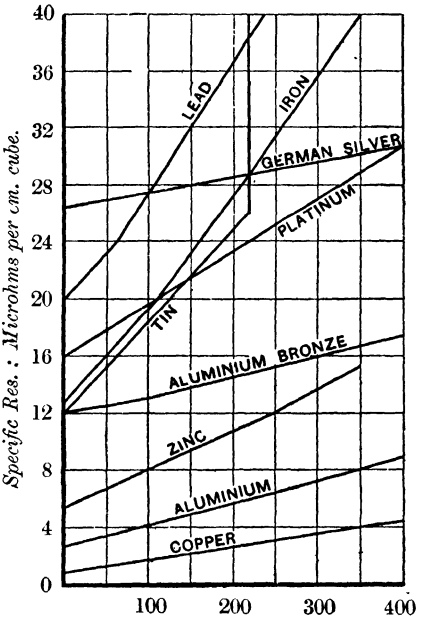


Fig. 95.

(*c*) CARBON, ELECTROLYTES, AND INSULATORS.—These decrease in resistance when heated—*i.e.* they possess negative coefficients. The cold resistance of the carbon filaments of glow lamps is from 1·6 to 2·4 times the hot resistance under their full voltage. In a recent test, an ordinary carbon-filament lamp was found to have a cold resistance of 600 ohms. As the voltage was increased the resistance *fell to a minimum value* of 288·2 ohms, when the applied pressure reached 75 volts; it remained stationary until the pressure was 90 volts, and then *slowly increased* to 293 ohms at 140 volts. The resistance of gutta-percha at 0° C. is about 24 times, and that of Hooper's vulcanised india-rubber about four times their respective resistances at 24° C. (Art. 17). **Zirconia** (an oxide of the alkaline metal zirconium) is a non-conductor at ordinary temperatures, but becomes a conductor at high temperatures; this is utilised in the Nernst lamp.

As already indicated (Art. 17) copper is mainly used as a conductor in practice, but *aluminium* is now coming more to the front. Its cost is lower and it is very light: for equal conductances, although its diameter must be greater, its weight is only about half that of copper. In overhead (bare wire) transmission schemes, its bigger cross-section than copper (for equal conductance) makes it offer more resistance to wind pressure. It allows much more snow, etc., to collect on it. Its greater diameter for equal conductance puts it behind copper for underground mains, etc., where it must be insulated with rubber, etc., owing to the increased

cost of the latter. Acids too have an effect on it which debars its use in certain places. Its tensile strength also is only about half that of hard drawn copper, and this is frequently a drawback (where greater tensile strength than that of hard drawn copper is needed *bronzes* are frequently employed). The soldering of aluminium too presents a certain difficulty.

116. Formulae for Calculations.—The first three laws of Art. 115 may be expressed by the equation—

$$\mathrm{R} = \rho \frac{l}{a} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

where R is the resistance of the conductor, l its length, a its area of cross-section, and ρ a constant depending on the material, and termed its "**specific resistance**" or "**resistivity.**" If l be one centimetre and a one square centimetre, R is equal to ρ; hence: *The specific resistance of any material is the resistance of a piece of it one centimetre in length and one sq. centimetre in cross-section,* i.e. *the resistance of a cube of one centimetre side.** Frequently ρ is expressed in terms of the "*inch-cube*" and "*mil.-foot*" (*i.e.* resistance of one foot of the material of circular section and diameter one mil.). With regard to units—

If ρ be in microhms per cm. cube, l must be in cm., a must be in sq. cm., and R will be in microhms.	If ρ be in ohms per inch cube, l must be in inches, a must be in sq. inches, and R will be in ohms.	If ρ be in ohms per mil. foot, l must be in feet, a must be in circular mils., and R will be in ohms.

If the wire be of circular section of radius r and diameter d, a is equal to πr^2 ($\pi = 3{\cdot}1416$) or $\cdot 7854 d^2$; hence substituting in (1)—

$$\mathrm{R} = \rho \frac{l}{\pi r^2} = \rho \frac{l}{\cdot 7854 d^2} \quad \ldots\ldots\ldots\ldots\ldots \quad (2)$$

An extension of these relations enables the resistances of two wires to be compared with ease. Let R_1, ρ_1, l_1, a_1, and d_1 apply with their usual meaning to the first wire, and R_2, ρ_2, l_2, a_2, and d_2 to the second; then—

$$\frac{\mathrm{R}_1}{\mathrm{R}_2} = \frac{\rho_1 \frac{l_1}{a_1}}{\rho_2 \frac{l_2}{a_2}} = \frac{\rho_1 \times l_1 \times a_2}{\rho_2 \times l_2 \times a_1} = \frac{\rho_1 \times l_1 \times (d_2)^2}{\rho_2 \times l_2 \times (d_1)^2} \quad \ldots\ldots\ldots \quad (3)$$

In making comparisons by means of (3) the warning given as to the units need not be followed; thus l may be in feet and d in centi-

* See Appendix.

metres, *provided that both lengths are in feet and both diameters in centimetres.*

The cross-section is often given in terms of the mass and density (mass in unit volume) of the conductor. If w be the mass and δ the density, the volume of the wire is clearly $\frac{w}{\delta}$; but the volume is equal to $a \times l$; hence—

$$al = \frac{w}{\delta} \quad \text{and} \quad a = \frac{w}{\delta \times l} \qquad \ldots\ldots\ldots\ldots \quad (4)$$

substituting (4) for a in (1)—

$$\mathrm{R} = \rho \frac{\delta l^2}{w} \qquad \ldots\ldots\ldots\ldots \quad (5)$$

Extending (4) to the comparison of two wires of mass w_1 and w and density δ_1 and δ_2, (3) becomes—

$$\frac{\mathrm{R}_1}{\mathrm{R}_2} = \frac{\rho_1 \times l_1 \times \frac{w_2}{\delta_2 l}}{\rho_2 \times l_2 \times \frac{w_1}{\delta_1 l_1}}; \qquad \therefore \frac{\mathrm{R}_1}{\mathrm{R}_2} = \frac{\rho_1}{\rho_2} \times \frac{\delta_1}{\delta_2} \times \frac{l_1^2}{l_2^2} \times \frac{w_2}{w_1} \qquad \ldots\ldots \quad (6)$$

and if the wires be of the same material and density—

$$\frac{\mathrm{R}_1}{\mathrm{R}_2} = \frac{l_1^2}{l_2^2} \times \frac{w_2}{w_1} \qquad \ldots\ldots\ldots \quad (7)$$

In dealing with the "dielectric" or "insulation" resistance of a cable it must be remembered that the *thickness* of the insulating covering corresponds to the "length" in the above expressions, and that a long cable gives a *greater sectional area* of insulating material (and therefore has *less* insulation resistance) than a short one. If l be the length of a cable, ρ the specific resistance of its dielectric r_2, and r_1 the external and internal diameters (or radii) of the insulation, it can be proved that the insulation resistance R is given by—

$$\mathrm{R} = \cdot 366 \times \frac{\rho}{l} \times \log \frac{r_2}{r_1} \qquad \ldots\ldots\ldots \quad (8)$$

Taking two cables of lengths l_1 and l_2, insulated with the same material and having equal internal and equal external diameters,

$$\frac{\mathrm{R}_1}{\mathrm{R}_2} = \frac{l_2}{l_1} \qquad \ldots\ldots\ldots\ldots \quad (9)$$

i.e. the insulation resistances are *inversely* as the lengths. Thus, if the insulation resistance of a cable is 400 megohms per mile, half a mile will have a resistance of 800 megohms, and two miles a resistance of 200 megohms; *if the insulation resistance of a cable of length n knots be* R *megohms, the insulation resistance per knot is* nR *megohms.*

In stating the diameters of wires, a common unit employed is the *mil.* ($\frac{1}{1000}$ inch). A circle of one mil. diameter is said to have an area of **one circular mil.**; a circle of two mils. diameter has an area of 2^2—*i.e.* four circular mils.; thus the area in circular mils. is given by the square of the diameter in mils.

Again, the thickness of a wire is often settled by a number termed its "gauge"; thus 20 B.W.G. (Birmingham Wire Gauge) refers to a wire ·035 inch in diameter; 16 S.W.G. (Standard Wire Gauge) is a wire of ·064 inch diameter The larger the "gauge" number the smaller is the diameter (see Appendix). The French Gauge is an exception.

The conductors of submarine and electric light cables are formed of stranded wire on account of the greater pliability and less liability to complete fracture than in the case of solid cores; thus 7/16 cable refers to one whose conductor is composed of seven-stranded wires, each of 16 gauge.

The resistivity ρ, as previously defined, is called the **volume resistivity.** Sometimes *the resistance of a wire of unit length having unit mass and being of uniform cross-section* is given: this is called the **mass resistivity.** The relation between the two depends on the density and on the units employed.

117. Resistances in Series and in Parallel.—If several resistances r_1, r_2, r_3 . . . be arranged in series (Fig. 94), the total resistance R is clearly their *sum*—

i.e. $$R = r_1 + r_2 + r_3 \quad \ldots\ldots \quad \ldots\ldots \quad \ldots\ldots \quad (1)$$

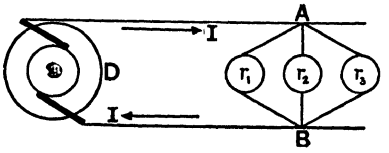


Fig. 96.

Consider, now, a number of resistances (say three) joined in parallel, as indicated in Fig. 96, and let E denote the P.D. between A and B. Then—

$$\text{Current in } r_1 = \frac{E}{r_1}. \qquad \text{Current in } r_2 = \frac{E}{r_2}.$$

$$\text{Current in } r_3 = \frac{E}{r_3};$$

$$\therefore \text{ Total current} = \frac{E}{r_1} + \frac{E}{r_2} + \frac{E}{r_3}.$$

Let R denote the joint resistance of r_1, r_2, and r_3;

$$\therefore \text{ Total current} = \frac{E}{R}.$$

Hence
$$\frac{E}{R} = \frac{E}{r_1} + \frac{E}{r} + \frac{E}{r_3};$$

$$\therefore \frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} \quad \ldots\ldots\ldots\ldots \quad (2)$$

If the resistances are each equal to r_1 (2) becomes

$$\frac{1}{R} = \frac{3}{r_1}, \qquad i.e.\ R = \frac{1}{3} r_1.$$

Similarly, the joint resistance of n equal resistances in parallel is $\frac{1}{n}$ of the resistance of one of them.

In the case of two conductors in parallel—

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} = \frac{r_1 + r_2}{r_1 r_2};$$

$$\therefore R = \frac{r_1 r_2}{r_1 + r_2} \quad \ldots\ldots\ldots\ldots \quad (3)$$

It should be observed that whilst the *conductor resistance* of two cables in series is $r_1 + r_2$, r_1 and r_2 being the individual conductor resistances, the *insulation resistances are really in parallel;* thus if R_1 be the insulation resistance of A (Fig. 97), and R_2 that of B, the total insulation resistance R is given by—

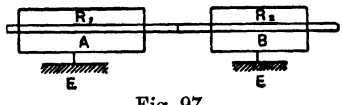


Fig. 97.

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}; \qquad \therefore R = \frac{R_1 R_2}{R_1 + R_2} \quad \ldots\ldots\ldots\ldots \quad (4)$$

We have previously indicated that the current is the same at all parts of a simple (series) circuit, however the parts differ in resistance. In a parallel arrangement, however, *the current divides directly as the conductances, and therefore inversely as the resistances;* thus, in Fig. 96:—

$$\frac{\text{Current in } r_1}{\text{Current in } r_2} = \frac{1}{r_1} \div \frac{1}{r_2} = \frac{r_2}{r_1} = \frac{\text{Resistance of } r_2}{\text{Resistance of } r_1}.$$

EXAMPLE.

Three wires, A, B, C, *of* 2, 4, *and* 6 *ohms resistance respectively, are arranged in parallel, and the total current passing is* 22 *amperes. Find the joint resistance and the current in each wire.*

If R = joint resistance—

$$\frac{1}{R} = \frac{1}{2} + \frac{1}{4} + \frac{1}{6} = \frac{6+3+2}{12} = \frac{11}{12}, \quad \therefore R = \frac{12}{11} = \mathbf{1\tfrac{1}{11}\ ohms.}$$

Again—

Current in A = $\frac{6}{11}$ of 22 amperes = **12 amperes**
,, ,, B = $\frac{3}{11}$ of 22 ,, = **6** ,,
,, ,, C = $\frac{2}{11}$ of 22 ,, = **4** ,,

118. More Complicated Divided Circuits: Kirchhoff's Laws.—The joint resistance and the currents in the various branches of a divided circuit can readily be found by the above method, provided there are no cross connections such as are indicated in Fig. 99. In this and other more complex arrangements applications of **Kirchhoff's Laws** enable the solutions to be obtained.

Kirchhoff's two laws are as follows:—(1) *In any network of wires carrying currents the algebraic sum of the currents meeting at any point is zero.* (2) *In any closed path (or mesh) in a network the sum of the E.M.F.'s acting in that path is equal to the sum of the products of the resistances of, and currents in, the various parts of the path.*

The first law presents no difficulty; thus, to take a simple case (Fig. 98), it is clear that $I = I_1 + I_2 + I_3$, and therefore

$$I - I_1 - I_2 - I_3 = 0.$$

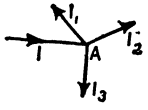


Fig. 98.

Current flowing *to* the point A is given the positive sign, and current flowing *from* A the negative sign in writing down the algebraic sum.

The second law will be best understood by considering a special example: to simplify matters we will make use of a suggestion due to Maxwell, known as the "imaginary cyclic current device."

EXAMPLE.

Five resistances, A B, B C, C D, D A, B D, *of* 2, 4, 5, 6, *and* 8 *ohms respectively, are joined as indicated in Fig.* 99, *and the points* A *and* C *are connected to a cell of E.M.F.* 1 *volt, the total resistance in the path* AEC *being* 10 *ohms. Find the currents in the various branches and the resistance of the network between* A *and* C.

In each mesh, or compartment, we assume an *imaginary* current (x, y, and z), each flowing in the same direction (clockwise). Applying Kirchhoff's law to the compartment on the left, we get:—

$$2x + 8(x - y) + 6(x - z) = 0.$$

In A B the current is x and the resistance 2, so that the product of current and resistance is $2x$: in B D the current is x *clockwise for this compartment and* y *in the opposite direction, i.e.* the *clockwise* current is $(x - y)$, and the resistance is 8, so that the product is $8(x - y)$: similarly for D A the product is $6(x - z)$: the E.M.F. in this compartment is zero: hence the law gives the equation above. Collecting terms, we get :—

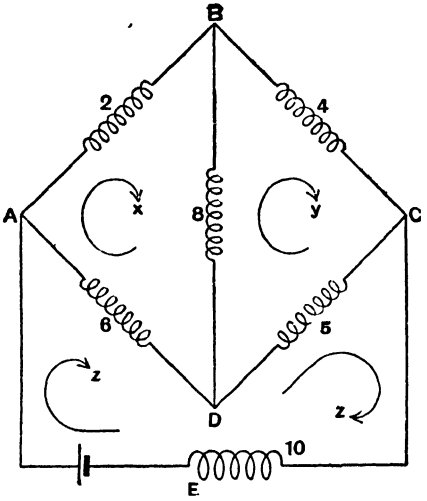


Fig. 99.

$$8x - 4y - 3z = 0 \quad \ldots\ldots\ldots\ldots \quad (1)$$

Similarly, with the compartment on the right :—

$$4y + 5(y - z) + 8(y - x) = 0,$$

for in this mesh the *clockwise* current in D B is $(y - x)$, and in C D it is $(y - z)$. Collecting terms :—

$$-8x + 17y - 5z = 0 \quad \ldots\ldots\ldots \quad (2)$$

Finally, for the bottom compartment—

$$10z + 6(z - x) + 5(z - y) = 1,$$

for in this mesh the E.M.F. is 1 volt. Collecting terms :—

$$-6x - 5y + 21z = 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

By solving the equations (1), (2), and (3) the various currents and the joint resistance are obtained : thus :—

Eliminating x from (1) and (2)—

$$13y - 8z = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

Multiplying (1) by 3 and (3) by 4—

$$24x - 12y - 9z = 0$$
$$-24x - 20y + 84z = 4$$

Adding $\quad -32y + 75z = 4 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$

Eliminating y from (4) and (5) $\quad z = \frac{52}{719}$.

Substituting in (4) $\quad y = \frac{32}{719}$.

Hence from (1) $\quad x = \frac{355}{719}$.

Thus the total current is 52/719 amperes, the current in B D is $(x - y)$, *i.e.* 3·5/719 amperes in the direction B to D, the current in A D is $(z - x)$, *i.e.* 16·5/719 amperes, and so on.

The joint resistance of the network formed by A B, B C, A D, D C, and B D is found thus :—

Let R = joint resistance,

$$\therefore \text{R} + 10 = \text{total resistance.}$$

Now :—

$$\text{Total current} = \frac{\text{E.M.F.}}{\text{Total resistance}}. \quad \therefore \frac{52}{719} = \frac{1}{R + 10},$$

i.e. $R = 3{\cdot}82$ ohms.

This section is perhaps somewhat tedious, but the student should try to understand it, for so many problems can be solved by this method which cannot be readily solved in any other way.

119. Electrical Energy.—It has been explained that whenever a P.D. exists between two points in an electric circuit an *energy transformation* occurs between them. When a current flows along a copper wire, energy is subtracted from the electric circuit appearing as heat in the wire; when a current flows through the filament of a glow-lamp, energy is again subtracted from the electric circuit, part appearing as heat in the filament, part being dissipated as heat communicated to the air and bodies near, and part being utilised in setting up the ether vibrations which constitute light. Passing a current through a voltameter results in the transformation of electrical energy into heat energy and into the potential energy of chemical separation, whilst passing it through a motor gives a transformation into heat and mechanical work. In all cases the electrical energy which disappears is equivalent to the new forms into which it is transformed.

In the previous chapter it has been shown that if the P.D. between two points is one volt, and one coulomb of electricity passes, the energy transformation is 10^7 ergs; this is adopted as the unit of work or energy in electrical engineering, and is called a **joule**; hence—

A joule is the work done (or energy transformed) between two points in a circuit when the P.D. between the points is one volt, and one coulomb of electricity passes; *it is equal to* 10^7 *ergs or* ·7375 *foot-pound.*

If the P.D. be E volts, and one coulomb passes, the work done is E joules; if Q coulombs pass, the work is EQ joules; if the Q coulombs be transferred by a current of I amperes flowing for t seconds, Q is equal to It and the energy transformation is measured by EIt joules; hence—

$$\textbf{Joules} = \mathbf{EI}t = \textbf{Volts} \times \textbf{Amperes} \times \textbf{Seconds} \quad \ldots\ldots (1)$$

$$= \mathbf{I^2R}t = (\textbf{Amperes})^2 \times \textbf{Ohms} \times \textbf{Seconds} \quad \ldots\ldots (2)$$

$$= \frac{\mathbf{E^2}}{\mathbf{R}}\, t = \frac{(\textbf{Volts})^2 \times \textbf{Seconds}}{\textbf{Ohms}} \quad \ldots\ldots (3)$$

By (1) above, if the pressure be one volt, the current one ampere, and the time *one hour, i.e.* 3,600 seconds, the energy transformation will be 3,600 joules. This is another energy unit called a **watt-hour**; hence—

(1) **A watt-hour is the work done (or energy transformed) between two points in a circuit when the P.D. between the points is one volt and one ampere flows continuously for one hour.**

(2) **A watt-hour is the work done between two points in a circuit when the product of the pressure in volts, the current in amperes, and the time in hours is equal to unity.**

One watt-hour = 3,600 joules = 2,654 ft.-pounds nearly.

From the definition it follows that if T be the time *in hours*—

$$\textbf{Watt-hours} = \textbf{EIT} = \textbf{Volts} \times \textbf{Amperes} \times \textbf{Hours} \quad \ldots\ldots (4)$$

$$= \textbf{I}^2\textbf{RT} = (\textbf{Amperes})^2 \times \textbf{Ohms} \times \textbf{Hours} \quad \ldots\ldots (5)$$

$$= \frac{\textbf{E}^2}{\textbf{R}}\textbf{T} = \frac{(\textbf{Volts})^2 \times \textbf{Hours}}{\textbf{Ohms}} \quad \ldots\ldots (6)$$

A still larger unit of energy has been chosen as the legal one by the Board of Trade; it is called a **kilowatt-hour** or **Board of Trade unit** (B.O.T. unit) or sometimes the "Kelvin," and is equal to 1,000 watt-hours, 3,600,000 joules, or approximately 2,654,000 foot-pounds. Clearly—

(1) **A B.O.T. unit is the work done (or energy transformation) between two points in a circuit when the P.D. is one volt and one ampere flows continuously for 1,000 hours.**

(2) **A B.O.T. unit is the work done between two points in a circuit when the product of the pressure in volts, the current in amperes, and the time in hours is equal to 1,000.**

From the definitions it follows that—

$$\left\{\begin{matrix}\textbf{No. of B.O.T.}\\ \textbf{units}\end{matrix}\right\} = \frac{\textbf{No. of watt-hours}}{\textbf{1000}} = \frac{\text{EIT}}{1000} = \frac{\text{I}^2\text{RT}}{1000} = \frac{\text{E}^2\text{T}}{\text{R} \times 1000}. \quad (7)$$

EXAMPLE.

A feeder has a resistance of $\frac{1}{20}$ ohm, and the current it carries averages 50 amperes. How many B.O.T. units does it absorb in 24 hours, and what will the lost energy cost the station at 2d. per unit?

$$\text{No. of B.O.T. units absorbed} = \frac{\text{I}^2\text{RT}}{1000} = \frac{50 \times 50 \times \cdot 05 \times 24}{1000} = 3;$$

$$\therefore \textbf{Cost at } 2d. \textbf{ per unit} = 6d.$$

120. Electrical Power or Activity.—This has been defined in Chapter I. as "rate of doing work"; stated algebraically:

$$\text{Power} = \frac{\text{Work done}}{\text{Time in Seconds}} \text{ and Work} = \text{Power} \times \text{Time in Seconds.}$$

In the preceding section it has been shown that the work done in t seconds when a current of I amperes flows under a pressure of E volts is EIt joules. If E and I be each unity, the work per second is one joule; this is adopted as the unit of electrical power or activity and is called a **watt**; hence—

(1) **A watt is the power in a circuit when the P.D. is one volt and one ampere is passing.**

(2) **A watt is the power in a circuit when work is being done at the rate of one joule or 10^7 ergs or ·7375 foot-pound per second.**

(3) **A watt is a rate of working equivalent to $\frac{1}{746}$ h.-p.** (Art. 12).

Putting t equal to unity in (1), (2), and (3) of Art. 115 we obtain the corresponding expressions for the watts rate of working in a circuit, viz.:

$$\textbf{Watts} = \textbf{EI} = \textbf{Volts} \times \textbf{Amperes} \quad \ldots\ldots\ldots\ldots \quad (1)$$

$$= \textbf{I}^2\textbf{R} = (\textbf{Amperes})^2 \times \textbf{Ohms} \quad \ldots\ldots\ldots\ldots \quad (2)$$

$$= \frac{\textbf{E}^2}{\textbf{R}} = \frac{(\textbf{Volts})^2}{(\textbf{Ohms})} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

For purposes of calculation it may be mentioned that carbon filament glow-lamps absorb from $2\frac{1}{2}$ to 4 watts per candle; a common value is 3·5 or 3·75. A small Nernst lamp taking half an ampere at 100 volts gives about 34 candle-power and absorbs about 1·8 watts per candle. Modern metallic filament lamps take $\frac{1}{2}$ watt per candle.

The **kilowatt** is another unit of power, and is equal to 1,000 watts; hence—

(1) **A kilowatt is the power in the circuit when the P.D. is one volt and 1,000 amperes is flowing or when the P.D. is 1,000 volts and one ampere is flowing.**

(2) **A kilowatt is a rate of working equivalent to $\frac{1000}{746}$, *i.e.* 1·34 h.-p.**

(3) **A kilowatt is the power in the circuit when the product of the pressure in volts and the current in amperes is equal to 1,000.**

The power of a dynamo is generally expressed in kilowatts; thus a machine capable of supplying 40 amperes at 2,000 volts has a power of 80,000 watts or 80 kilowatts and is called an 80-unit machine. Neglecting losses of energy in the transformation, the h.-p. necessary to drive this dynamo is $\frac{80000}{746}$ or 80 × 1·34, *i.e.* 107·2 h.-p. Clearly the h.-p. required varies with the current output. On open circuit the power necessitated will only be that which is required to over-

come friction; when the current is switched on the product of current and pressure increases and, therefore, so also does the h.-p. required to drive the machine.

Since the number of watt-hours is given by the product of the watts and the hours, **a watt-hour may be defined as the work done in one hour when the power in the circuit is one watt.** Similarly, **a kilowatt-hour is the work done in one hour when the power is one kilowatt.** We would again remind the reader that a "watt-hour" and a "kilowatt-hour" are units of *work* or *energy*, not units of *power*.

EXAMPLES.

1. *A compound wound dynamo producing a terminal P.D. of* 150 *volts is used to charge* 60 *storage cells, each having an E.M.F. of* 2·2 *volts and a resistance of* ·001 *ohm. If the leads joining the dynamo and cells have a resistance of* ·2 *ohm, what will be the current generated?* (O.)

Total back pressure of battery = $60 \times 2{\cdot}2 = 132$ volts,

$\therefore$ Effective pressure = $150 - 132 = 18$ volts,

Resistance of battery = $60 \times {\cdot}001 = {\cdot}06$ ohm,

$\therefore$ Total resistance = ${\cdot}06 + {\cdot}2 = {\cdot}26$ ohm.

Hence— $$\textbf{Current} = \frac{18}{{\cdot}26} = \textbf{69·23 ampere.}$$

2. *In a water-power plant the dynamo, which produces a fixed P.D. at its terminals of* 120 *volts, is* 300 *yards away from the house. The usual load consists of* 200 100*-volt* 35*-watt glow-lamps. What size leads should be employed if the resistance of a cubic inch (inch cube) of copper is* ·66 *microhm?* (O.)

$$\text{Watts} = \text{EI}, \qquad \therefore \text{I} = \frac{\text{Watts}}{\text{E}}.$$

Hence— $$\text{Current in each lamp} = \frac{\text{Watts absorbed by each lamp}}{\text{P.D. for each lamp}}$$

$$= \frac{35}{100} = {\cdot}35 \text{ ampere};$$

$\therefore$ Current for 200 lamps in parallel = $·35 \times 200 = 70$ amperes,

i.e. Current in leads = 70 amperes

and Volts used in leads = $120 - 100 = 20$;

$\therefore$ Resistance of leads = $\frac{20}{70} = \frac{2}{7}$ ohm.

Again— $\text{R} = \rho\frac{l}{a}$, $\therefore a = \frac{\rho \times l}{\text{R}}$;

ρ = ·66 microhm = $\frac{{\cdot}66}{10^6}$ ohm per inch cube,

l = 600 yards (go and return) = (600×36) inches,

$\text{R} = \frac{2}{7}$ ohm;

$$\therefore a = \frac{{\cdot}66 \times 600 \times 36 \times 7}{10^6 \times 2} = \textbf{·0499 square inch.}$$

3. *A pair of feeders each half a mile long have to deliver* 100 *kilowatts at* 440 *volts. What cross-section must they have so that the loss in them may not exceed* 5 *per cent. of the power delivered? One cubic inch (inch-cube) of copper has a resistance of* $\frac{2}{3}$ *microhm.* (P.)

$$\text{Permissible loss in feeders} = \left(\frac{5}{100}\text{ of } 100\right) \text{ k.w.} = 5{,}000 \text{ watts},$$

$$\text{Total watts delivered} = 100 \times 1000 = 100{,}000,$$

$$\therefore \text{ Current in feeders} = \frac{100000}{440} = \frac{2500}{11} \text{ amperes.}$$

Again—

$$\text{Watts} = I^2R, \qquad \therefore R = \frac{\text{Watts}}{I^2};$$

Hence—

$$\text{Resistance of feeders} = \frac{\text{Watts absorbed by feeders}}{(\text{Current in feeders})^2}$$

$$= \frac{5000}{\left(\frac{2500}{11}\right)^2} = \frac{121}{1250} \text{ ohms.}$$

Again—

$$R = \rho\frac{l}{a}, \qquad \therefore a = \frac{\rho \times l}{R};$$

$$\rho = \left(\frac{2}{3} \times \frac{1}{10^6}\right) \text{ ohms per inch cube,}$$

$$l = 1 \text{ mile} = (1760 \times 36) \text{ inches,}$$

$$\therefore a = \frac{2 \times 1760 \times 36 \times 1250}{3 \times 10^6 \times 121} = \textbf{·43 square inch.}$$

4. *If three resistances* r_1, r_2, r_3 *are joined in parallel, what is the resultant resistance? If* 75 *yards of* $\frac{7}{16}$ *cable are in parallel with* 50 *yards of* $\frac{19}{20}$, *what is the joint resistance? A single No.* 16 *wire has a resistance of* ·8 *ohm per* 100 *yards; a single No.* 20 *a resistance of* 2·75 *ohms per* 100 *yards; a stranded conductor has* 3 *per cent. more resistance than a solid conductor of the same cross-section.* (P.)

(*a*) See Art. 117.

(*b*) 100 yards of single No. 16 has a resistance of ·8 ohm; hence 75 yards has a resistance of $\frac{75}{100}$ of ·8, *i.e.* ·6 ohm;

75 yards of solid conductor having 7 times the cross-section of a single No. 16 has a resistance of $\frac{1}{7}$ of ·6, *i.e.* ·0857 ohm;

$\therefore$ 75 yards of $\frac{7}{16}$ cable has a resistance of $\frac{103}{100}$ of ·0857, *i.e.* ·088 ohm.

In the same way it can be proved that the resistance of 50 yards of $\frac{19}{20}$ cable is ·074 ohm.

Hence—

$$\frac{1}{R} = \frac{1}{·088} + \frac{1}{·074}, \qquad \therefore \textbf{ R = ·04 ohm nearly.}$$

5. *A dynamo feeds* 1,000 16*-candle glow-lamps. What current must the dynamo supply if the difference of potential at its terminals is* 200 *volts, and each lamp absorbs* 3·6 *watts per candle?* (Bd. of Education, Stage 2.)

Determine also: (1) *the electrical h.-p. required;* (2) *the watt-hours and B.O.T. units supplied in* 10 *hours;* (3) *the cost of the energy at* 4*d. per B.O.T. unit.*

Watts absorbed by 1 lamp = 3·6 × 16 = 57·6

$\therefore$ Watts absorbed by 1,000 lamps = 57·6 × 1,000 = 57,600.

Now— $\text{Watts} = EI, \qquad \therefore I = \frac{\text{Watts}}{E}$;

$\therefore$ since the lamps are in parallel and the P.D. is 200 volts,

$$\textbf{Total current} = \frac{57600}{200} = \textbf{288 amperes.}$$

Again—

Since 746 watts = 1 h.-p., **h.-p. required** $= \frac{57600}{746} = \textbf{77·2,}$

Energy in **watt-hours** = watts × hours = 57,600 × 10 = **576,000**

One B.O.T. unit = 1,000 watt-hours,

$\therefore$ **No. of B.O.T. units** $= \frac{\text{watt-hours}}{1000} = \frac{576000}{1000} = \textbf{576.}$

Finally—**Cost of 576 units at 4d. per unit = £9 12s.**

6. *Current is required along a street* 2,000 *feet long at the rate of* 1 *ampere per* 10 *feet of frontage. Give sectional area of conductor so that the difference of pressure between any two lamps shall not exceed* 2 *volts, the current being supplied midway. The resistance of* 1 *foot of wire* 1 *mil. in diameter is* 10 81 *ohms.*

Length of half the street = 1,000 feet,

$\therefore$ Current taken by each half $= \frac{1000}{10} = 100$ amperes.

Again, since 1 ampere is taken off in each length of 10 feet, the *mean* or *average* current in each half-street will be $\frac{100}{2}$, *i.e.* 50 amperes. The greatest P.D. (which may not exceed 2 volts) will exist between the first and last lamps, and this will be given by the product of the average current and resistance between these lamps: hence—

$$50\ R = 2, \qquad \therefore R = \frac{1}{25}\ \text{ohm.}$$

Now— $R = \rho\frac{l}{a}, \qquad \therefore a = \frac{\rho \times l}{R}$;

$l = 2 \times 1{,}000 = 2{,}000$ feet ("go" and "return"),

$\rho = 10{\cdot}81$ ohms per mil.-foot,

a = cross-section required in *circular mils.*

Substituting and solving: $a = 540{,}500$ circular mils.

$= \textbf{·42 square inch.}$

EXERCISES X.

Series A.

1. A battery of 10 accumulators, each of 2 volts E.M.F. and $\frac{1}{100}$ ohm resistance is joined in series with a resistance of 40 ohms. What is the current?

2. A 10-ampere arc lamp is supplied by a battery of 45 Grove cells in series. How many coulombs will be supplied in an hour? What will be the total weight of zinc consumed in the battery, and what will be the cost of it at 2s. per kilogramme?

3. A 37/12-stranded copper cable has a copper cross-section of ·314 sq. in. What is the resistance of 1 mile of this cable, and what will be the drop in volts along it when it is carrying a current of 300 amperes?

4. A group of 400 100-volt 60-watt lamps is fed by a pair of mains from a dynamo 200 yd. distant. Find (1) the total current required; (2) the cross-section of the cable required at 1,000 amperes per sq. in.; (3) the resistance of the pair of mains; (4) the pressure at the dynamo terminals at full load.

5. A cable 400 yd. long is required to carry a current of 100 amperes with a drop of 20 volts. What should be the section of the copper?

6. Two lamps of 100 and 150 ohms each when running are put in parallel with each other, and the pair is put in series with a lamp of 100 ohms. What E.M.F. will be needed on the system in order that it may consume 250 watts?

7. A dynamo is used to feed 80 lamps, each of 100 volts and 48 watts. The distance of the dynamo from the lamps is 300 yd. The current density in the cable is 1,000 amperes per sq. in. What is the section of the cable?

8. In the above problem, what is the resistance of the cable lead and return? And what must be the P.D. at the dynamo terminals at full load? Resistance of the copper cable = ·66 microhm per inch cube.

9. What must be the section of a manganin wire to have the same resistance as a 16 S.W.G. copper wire of the same length? Specific resistance of copper per cm. cube is 1·57 microhms, and of manganin 47·5 microhms.

10. Two lamps A and B of 30 and 60 ohms resistance are joined in parallel, and the pair is put in series with a lamp C of 40 ohms and connected across mains at 100 volts pressure. Find the current in each lamp.

11. A hot-wire voltmeter has a resistance of 320 ohms, and is connected to mains at 120 volts pressure. Find (*a*) the current taken, (*b*) the watts absorbed, (*c*) the B.O.T. units taken in 24 hours, (*d*) the cost of the energy absorbed at 5d. per unit.

12. A tramway motor is fed at 400 volts pressure, and takes an average current of 30 amperes. How many watts does it absorb, and how many B.O.T. units per hour?

13. Two arc lamps, each taking 10 amperes, are fed in series from mains at 110 volts. What is the cost of each lamp per hour at 6d. per B.O.T. unit?

14. A glow-lamp fed at 200 volts takes ·3 ampere. How many such lamps can be run for an expenditure of power equal to 1 h.-p.?

SERIES B.

1. Define an ampere, 10,000 volts, one megohm, a kilowatt, 40 B.O.T. units, and 100 h.-p. (P.)

2. Define a dyne, watt, horse-power, joule, B.O.T. unit. (P.)

3. Define the terms watt, joule, coulomb, B.O.T. unit, as applied to electric measurement. Distinguish between work and power. Is a foot-pound a unit of work or power? (O.)

4. What effect have changes of temperature on the electrical resistance of metals, insulating materials, and liquids? (O.)

5. Prove that the joint resistance of two resistances joined in multiple arc (parallel) is the product of the two divided by their sum. What is the capacity F of two condensers F_1 and F_2 when joined in cascade (series)? (H.)

6. What is a megohm? What is a microhm? If three lengths of cable, having respectively an insulation resistance of 300, 400, and 500 megohms, be joined in a continuous length, what will be the insulation resistance of the whole cable? (O.)

7. The total insulation of a telegraph line between stations A and C is 5,000 ohms, and the total insulation between station A and an intermediate station B is 7,000 ohms. What is the total insulation of the section B C? (H.)

8. Four 200-volt glow-lamps of 5, 8, 16, and 32 candle-power respectively are connected in parallel on a 200-volt circuit. Make a diagram showing the connections, and find the current passing through each lamp, as well as the current required by the four (efficiency of lamps 3·5 watts per candle). (P.)

9. Calculate the resistance of each of the lamps in Question 8, also the combined resistance when joined in parallel. (P.)

10. Calculate the resistance of one mile of copper conductor, having a cross-sectional area of ·137 square inch. The resistance of an inch cube of copper between opposite faces is ·66 microhm. (P.)

11. A group of 25 16 c.-p. 100-volt lamps, each taking ·55 ampere, are to be run by a dynamo 50 yards away. What must be the cross section of the cable if the drop in volts is not to exceed half a volt? The resistance of an inch cube of copper may be taken as ·66 microhm. (P.)

12. A group of 10 60-watt 110-volt lamps is situated 90 feet from the distributing board. What must be the diameter of the wires so that the fall of pressure between board and lamps shall be 1·1 volt? (P.)

13. Compare the resistances of two copper wires, one of which is 4 metres long and weighs 7·5 grammes, and the other is 5 metres long and weighs 4·2 grammes. (B.)

14. A dynamo maintaining a constant pressure of 220 volts between its terminals supplies a power of 18,000 watts to a house 200 yards away. What must be the cross-section of the copper of the leads, so that not more than 4 per cent. of the power may be wasted in them? Resistance of an inch cube of copper may be taken as ·66 microhm. (P.)

15. Current is required along a street 2,000 feet long at the rate of 1 ampere per 10 feet of frontage. Give the sectional area of conductor so that the difference of pressure between any two lamps shall not exceed two volts in the following two cases:

(a) The current being supplied at one end.

(b) The current being supplied midway.

The resistance of a copper bar 1,000 feet long and 1 sq. inch in cross-section may be taken at ·008 ohm. (O.)

16. A glow-lamp takes a current of ·55 ampere when connected to leads at 120 volts pressure. Find (*a*) the hot resistance of the lamp; (*b*) the watts absorbed; (*c*) the energy taken in 1 hour in joules; (*d*) the watt-hours taken in 50 hours; (*e*) the B.O.T. units taken in 100 hours; (*f*) the cost of the energy absorbed at 5d. per unit.

17. At 6d. per Board of Trade unit how much does it cost for electric energy to keep 10 lamps, each giving 15 candles, glowing for 5 hours? (P.)

18. How many electric h.-p. are needed to drive a dynamo supplying 120 16-candle lamps if the lamps take half an ampere each, and are supplied at 120 volts? (P.)

19. How many h.-p. is required to drive a current of 100 amperes through a resistance of $7\frac{1}{2}$ ohms? (P.)

20. A single cell of a secondary battery delivers 400 ampere-hours at an E.M.F. of 2 volts. Assuming 1 h.-p. = 746 watts, calculate the energy reckoned in foot-pounds of the discharge. (P.)

21. How many electric h.-p. would be required to light a factory in which 1,500 glow-lamps, each taking ·3 ampere at 220 volts, and 40 5-ampere enclosed arcs, two in series, are used at one time? (P.)

22. If a glow-lamp takes $3\frac{1}{2}$ watts per candle, and a gas burner gives 3 candles per cubic foot of gas burnt per hour, what must be the price of a Board of Trade unit so that the cost of electric lighting may be the same as that of gas lighting with gas at 2s. 9d. per 1,000 cubic feet, the cost of lamps, fittings, wiring, etc., being excluded? (P.)

23. In a street is a pair of mains each ·2 square inch in section (resistance ·12 ohm per 1,000 yards). The demand is at the rate of one 30-watt 100-volt lamp per yard. Find the distance between the feeding points so that the maximum variation of pressure between any two consumers shall not exceed 3 volts.

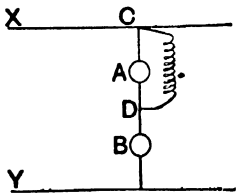


Fig. 99*a*.

24. The mains X and Y have a P.D. of 110 volts. B is a 50-volt 120-watt lamp, and A is a 60-volt 30-watt lamp. What is the resistance of a coil that must be joined from C to D so that each lamp may get correct current at the proper voltage? Determine the watts absorbed by this coil and the cost of the energy taken by it in 100 hours at 6d. per B.O.T. unit.

25. What is meant by the resistance of a conductor? Say how resistance varies with length and cross-section, and calculate the resistance of 1,000 yards of copper wire a tenth of an inch in diameter. (Resistance of an inch cube of copper is two-thirds of a microhm.) (P.)

CHAPTER XI.

MEASUREMENT OF RESISTANCE.

121. Principle of the Wheatstone Bridge.—The general principle underlying this convenient and accurate method of measuring resistance will be gathered from Fig. 100. The current from the battery B, with the conductor arrangements as indicated, divides at A, part flowing through the upper branch in the direction ADC, and part through the lower branch in the direction AEC. Since the fall of potential from its value at A to that at C is the same in both branches, it is clear that selecting any intermediate point D on one of them, there must exist somewhere on the other path a point having the identical potential of the chosen one D.

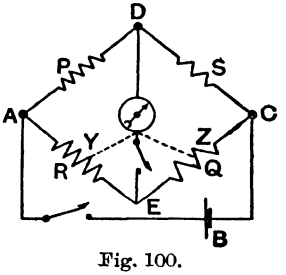


Fig. 100.

If E be this second point, no current will flow between D and E if a conducting path be put between them, and if this path includes a galvanometer the latter will not be deflected. Joining Y to D will result in a current *from* Y *to* D, for Y being nearer to the high potential end A of the lower branch than the point E, is at a higher potential than the latter and therefore at a higher potential than D; metallic connection between Z and D will produce a flow *from* D *to* Z, since the latter is below E and therefore below D in potential.

From this to the numerical relations is but a short step. As D is at the same potential as E, the P.D. between A and D is equal to that between A and E, and the P.D. between D and C is equal to that between E and C; hence:—

$$\frac{\text{P.D. between A and D}}{\text{P.D. between D and C}} = \frac{\text{P.D. between A and E}}{\text{P.D. between E and C}}$$

i.e.

$$\frac{\text{Current in A D} \times \text{Res. A D}}{\text{Current in D C} \times \text{Res. D C}} = \frac{\text{Current in A E} \times \text{Res. A E}}{\text{Current in E C} \times \text{Res. E C}}$$

But the current in A D is the same as that in D C, and the current in A E is identical with that in E C since none flows through the galvanometer; hence—

$$\frac{\text{Resistance A D}}{\text{Resistance D C}} = \frac{\text{Resistance A E}}{\text{Resistance E C}};$$

or, denoting the four resistances by the letters P, Q, R, and S as in the figure,

$$\frac{P}{S} = \frac{R}{Q}.$$

By the aid of this relation an unknown resistance P can be determined provided that: (1) R, Q, and S are known resistances, *or* (2) the resistance of one of the adjacent conductors, viz. R or S, is known and the *ratio* of the other two is also known.

122. The Metre Bridge Pattern.—In its simplest form, Fig. 101, this consists of three thick bars of copper of negligible resistance fixed to a board and provided with two gaps for the insertion of the resistance to be measured and a standard resistance with which it is compared. A straight, hard uniform wire of fairly high resistance (platinoid, German silver or platinum iridium), 1 metre in length, joins the two copper end pieces and has behind it a scale divided into 1,000 equal parts. By means of a slider contact can be made at any point on the wire, the exact position of which contact is indicated by a pointer attached to the slider and moving over the scale.

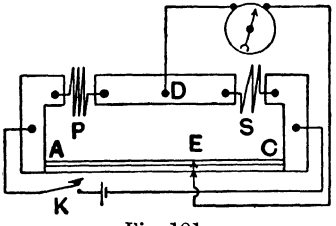


Fig. 101.

The instrument is provided with binding screws for the battery and galvanometer connections; the former includes a key K so that waste of battery power may be avoided. The entire connections are shown in Fig. 101, in which P is the unknown resistance and S a standard known resistance of somewhat similar magnitude. The experiment consists in (1) closing K and (2) moving the slider along the wire until a point of contact E is reached at which the galvanometer is not deflected. With this condition realised,

$$\frac{\text{Resistance P}}{\text{Resistance S}} = \frac{\text{Res. A E}}{\text{Res. E C}} = \frac{\text{Length A E}}{\text{Length E C}};$$

for since the wire is uniform, resistance is proportional to length.

Hence—

$$\text{Res. of P} = \frac{\text{Length A E}}{\text{Length E C}} \times \text{Res. S.}$$

Example.—If S = 5 ohms, A E = 410 divisions, E C = 590 divisions,

$$P = \frac{410}{590} \text{ of } 5 = 3{\cdot}47 \text{ ohms.}$$

123. Errors in the Measurement of Resistance by the Metre Bridge.—Briefly, the sources of error in metre-bridge work may be stated to be—

(1) Lack of uniformity in the wire. To correct for this the wire must be calibrated so that the ratio Resistance A E/Resistance E C is known.

(2) Resistances of the end-pieces too large and unequal to be neglected. These are determined experimentally as equivalent to so many divisions of bridge wire, and these divisions are added to the respective sections into which the slider divides the wire each time a test is made.

(3) The non-coincidence of the pointer with the edge of the slider which makes contact with the wire. This error is eliminated by interchanging the coils and taking the mean of the two results.

(4) Variation of the resistance of the coils used owing to temperature changes during the experiment.

(5) The thermo-electric effects* (Seebeck and Peltier) due to the junctions of dissimilar metals. These are eliminated by using a "reversing" key in the battery circuit, and balancing with the current in opposite directions in the bridge.

124. The Post Office Box Pattern.—Referring to Fig. 100, when a balance is obtained $\frac{P}{S} = \frac{R}{Q}$; hence $\frac{R}{P} = \frac{Q}{S}$. This form of the equation will best suit our present requirements, and the better to show the connection between the theoretical arrangement and the actual apparatus, it will be assumed in this section that S is the unknown resistance.

The Post Office Box consists of a number of coils of known resistance arranged so as to form three arms of a Wheatstone Bridge, the fourth consisting of the coil whose resistance is to be determined. The method in which the coils are fixed and manipulated is shown in Fig. 102. Their ends are attached to solid brass blocks separated from each other by conical gaps, into which conical brass plugs can be inserted. By inserting a plug that particular resistance is cut out of circuit, for the current will pass from one block to the next through the plug; removing a plug necessitates the current going through the coil, thus adding that resistance to the circuit. The coils are doubled upon themselves in the manner indicated so as to eliminate the effects of inductance (Chap. XVI.).

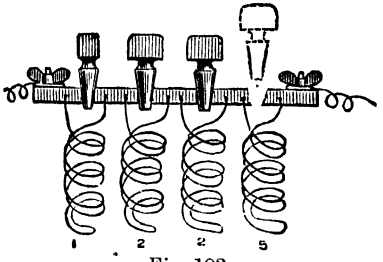


Fig. 102.

* See Chapter XV.

A plan of the box and its connections is shown in Fig. 103. The arms P and R are known as the *ratio* arms; each consists of three coils of 10, 100, and 1,000 ohms resistance. Q is called the *rheostat* arm, and consists of a series of coils whereby resistances ranging from 1 to 10,000 ohms can be obtained. S is the unknown resistance. The lettering of the box is identical with Fig. 100, and the reader would do well to make a careful comparison.

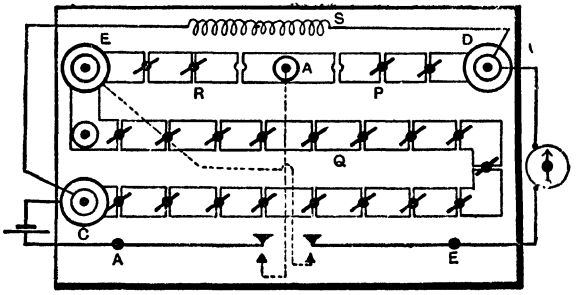


Fig. 103.

The manipulations will be best realised by considering the following actual test of a manganin wire:

I. Ratio arms: R = 10 *ohms*, P = 10 *ohms*.

Since R is equal to P, Q must be equal to S to secure a balance. On taking 2 ohms out of Q the galvanometer deflection was to the *right*, and 3 ohms gave a deflection to the *left*; hence—

S lies between 2 and 3 ohms.

II. Ratio arms: R = 100 *ohms*, P = 10 *ohms*.

Since R is equal to 10 P, Q must be equal to 10 S for a balance, *i.e.* Q must be between 20 and 30 ohms. On taking 23 ohms from Q the deflection was to the *right*, and 24 ohms gave a deflection to the *left*; hence—

S lies between 2·3 and 2·4 ohms.

III. Ratio arms: R = 1,000 *ohms*, P = 10 *ohms*.

In this case Q must be equal to 100 S, and therefore must be between 230 and 240 ohms. It was found that 237 ohms gave no deflection; hence—

S = 2·37 ohms.

Should two consecutive resistances in the third step still produce deflections in opposite directions, the true value may be found by inter-

polation. To take an example: If 237 ohms gave a deflection of 30 divisions to the right, and 238 gave 40 divisions to the left, a balance would be obtained if Q could be made equal to $237 + \frac{30}{70}$, *i.e.* 237·428 ohms; in this case the value of S would be 2·37428 ohms.

If the unknown resistance is *very large*, R must retain its value, 10 ohms throughout, and P must be made equal to, say, 1,000. R being $\frac{1}{100}$ of P, Q must be $\frac{1}{100}$ of the unknown to secure a balance. Thus, if 1,208 from Q gives no deflection, the value of S is 120,800 ohms.

In testing the conductor resistance of a telegraph line the remote end of the line is put to earth, the near end joined to terminal D, and terminal C is also earthed. This latter terminal and the far end of the line, being at equal potentials, are virtually connected; hence the test may be performed in the manner previously indicated.

125. Laboratory Method of Measuring a High Resistance.—The Wheatstone Bridge is unsuitable for the measurement of very high resistances of the order of a megohm, inasmuch as the conditions for sensitiveness must be violated; the following is a good laboratory method capable of giving accurate results:—

B (Fig. 104) is a convenient battery, R a resistance of 100,000 ohms made up of resistance boxes in series, G a high-resistance galvanometer, K a well-insulated key, and X the high resistance to be measured. With the lever on the upper stud *a* the deflection D_1 of the galvanometer is obtained. A known resistance *r* is now introduced, X being cut out of circuit by moving the lever to the stud *b*. The resistance R_1 (also *r* if necessary) is adjusted, the total R between C and D being kept constant, until a deflection D_2 of somewhat similar magnitude to D_1 is obtained. If *e* be the P.D. between C and D, e_1 the P.D. between C and E, I_1 the current in the galvanometer in the first case, and I_2 the current in it in the second case,

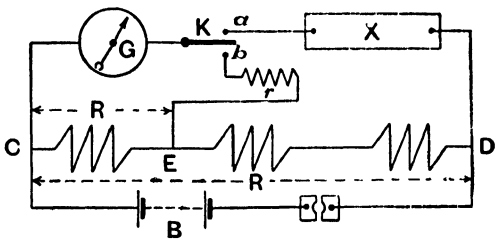


Fig. 104.

$$I_1 = \frac{e}{G + X}; \qquad I_2 = \frac{e_1}{G + r};$$

$$\therefore \frac{I_1}{I_2} = \frac{e}{e_1} \cdot \frac{G + r}{G + X},$$

i.e.

$$\frac{D_1}{D_2} = \frac{R}{R_1} \cdot \frac{G + r}{G + X},$$

in which X is the only unknown.

The **resistance of the insulation of a cable** may be found by the method outlined above. The cable is coiled up in a metal tank containing water, the ends only being outside, and these are well insulated to prevent leakage. One end of the metal core is joined *via* the key (stud *a*) to the galvanometer, the other end of the core being left "free," and the metal tank, which through the water is in contact with the outside of the insulation, is joined to the end D of the high resistance R; thus the insulation takes the place of X in Fig. 104. A slight modification is indicated in Fig. 105. Both ends of the cable are brought together at A and joined to the key K, a metal plate A_1 being joined to D. To avoid surface leakage the potential of the cover just below the core A is raised to the same potential as the core: this is done by the wire *w* as indicated, and it is clear that no leakage current can pass through the galvanometer, the latter only indicating the current flowing through the insulation. It is usual to immerse the cable for 24 hours and to measure the resistance "after one minute's electrification."

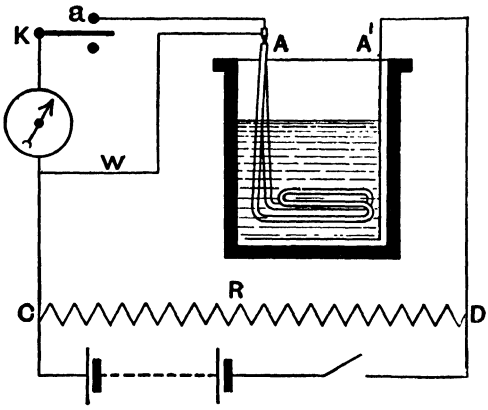


Fig. 105.

The **specific resistance of the dielectric** may now be found by the relation of Art. 116, viz.—

$$\left\{\begin{matrix}\text{Insulation resistance}\\ \text{of length } l \text{ inches}\end{matrix}\right\} = \left\{\cdot 366 \times \frac{\rho}{l} \times \log \frac{r_2}{r_1}\right\} \text{ohms.}$$

126. Portable Testing Sets: Testing Indoor Electric Wiring.—The principle underlying the action of **Evershed's Moving Magnet Set** will be gathered from Fig. 106, *and although this apparatus has been largely superseded by the newer types which follow, the student will still meet with it in practice and the "theory of action" is instructive.* It consists of two sets of coils S and P so arranged that their axes are perpendicular to each other. At their centre is pivoted a soft-iron needle, to which is attached a pointer *p* moving over the scale of the instrument.

The generator, consisting of a small magneto machine, is connected to the terminals G G of the **ohmmeter,** whilst the insulation or conductor resistance to be measured is placed across the terminals L E, *i.e.* in series with the coils S, which are of low resistance and known as the *current coils* of the instrument. The other coil P is joined straight across the generator terminals and is termed the *pressure coil*; it is not in itself of high resistance, but has in series with it a large

resistance in the form of an *exciting coil* (shown at C), one function of which is to magnetise the soft-iron needle when the current passes. The generator is contained in a separate box, and should be placed three or four feet from the ohmmeter in making a test, otherwise the needle may be affected by stray lines from it.

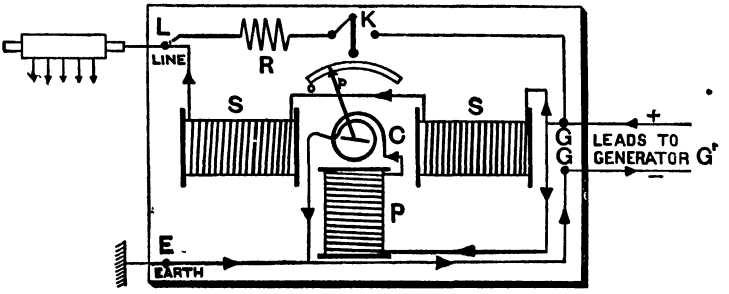


Fig. 106.

On working the machine, the current from it divides at the upper terminal G on the right, part flowing through P, and part through S and the unknown resistance; the former tends to set the needle in one direction, viz. along the axis of P, whilst the latter tends to set it along the axis of S. The indication of the instrument will thus depend upon the relative effects of the currents in P and S, and since the currents in the two branches are inversely proportional to their resistances it is evident that the deflection of the needle will depend on the ratio of the resistances of the two circuits.

With a very small unknown resistance the larger portion of the current passes through the coils S and the pointer sets towards zero on the scale: if the unknown be very large as compared with the pressure-coil circuit, the larger portion of the current passes in this circuit and the pointer moves towards the other end of the scale; thus with unknown resistances between zero and infinity the pointer takes up some intermediate position on the scale more and more to the left the lower the resistance. The ohmmeter is calibrated so that its indications give the unknown resistance direct in ohms or megohms.

Each ohmmeter is fitted with a two-contact switch K, whereby a resistance R (termed a shunt) may be put in parallel with the current coils and the range of the instrument thereby altered. They have ranges from 100 ohms to 50 megohms.

Evershed's Megger is an improved form of testing apparatus in which the generator and the ohmmeter are both contained in one box, and in which the permanent magnet of the generator produces the magnetic field in which the moving system which carries the pointer is deflected. As the reader will learn later, just as a magnet is de-

flected by a current, so a current carrying wire placed in a magnetic field is urged in a definite direction depending on the direction of the current and of the field, and naturally the movement will depend upon the *strength* of the current and of the field.

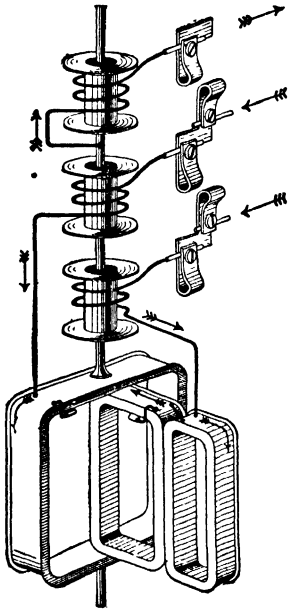


Fig. 107.

The moving system consists of two coils inclined to each other, but arranged so that they must move together as shown in Fig. 107. These are placed in the magnetic field of the generator magnet N S, and each encircles a soft-iron ring so as to concentrate the field. One coil P (Fig. 108) is joined straight across the generator terminals, and therefore carries a current proportional to the terminal P D of the generator. The other coil C is in series with the resistance under test and therefore moves with a force depending upon the current through the unknown resistance, *i.e.* upon the leakage current. The actual position taken up by the combination is therefore a compromise, depending upon the ratio of the currents in the coils, *i.e.* depending upon the ratio of the applied P D to the leakage current, in other words, upon the value of the unknown resistance.

Fig. 109 gives the internal connections of the instrument; this is simply Fig. 108 in greater detail, including a "guard plate" G and its connection, which is of use in eliminating errors due to leakage across the surface of the insulation under test. The instruments are of two varieties known as "variable pressure" and "constant pressure" instruments respectively, and they are of various ranges from 1,000 ohms to 2,000 megohms; they are primarily intended, however, for in-

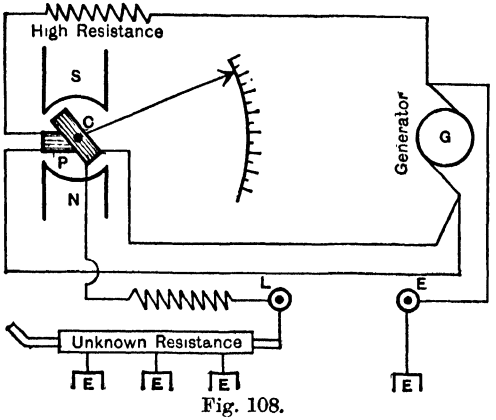


Fig. 108.

sulation (high) resistances. The third coil shown in Fig. 107 is a "compensating coil," the object of which (together with the shape of the S pole of Fig. 109) is to obtain a better divided scale: the reason for this cannot be given here.

A modified form known as the **Bridge-Megger** can be used either as a Megger with a range up to 40 megohms or as a Wheatstone Bridge for ordinary values.

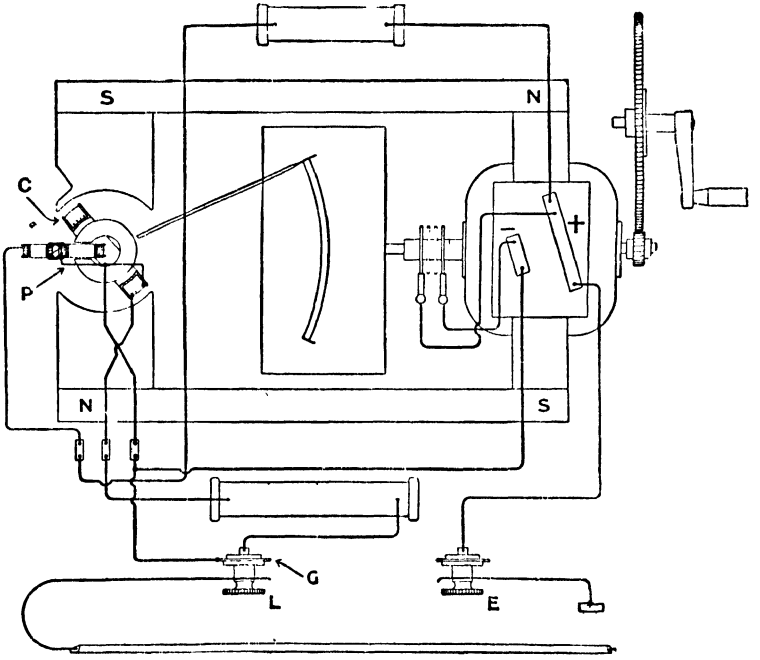


Fig. 109.

The **Cambridge "Omega" Testing Set** works on the same principle as the preceding. The generator and ohmmeter are both contained in the one case, but their magnets are independent of each other. The two moving coils practically cover the whole surface of the iron drum (which is situated between the pole pieces) around which they move, so that the whole of the magnetic flux is utilised at all times. Fig. 110 shows the ohmmeter portion and Fig. 111 the connections for taking a high resistance test, the two moving coils being marked D and C respectively. From the preceding the student will be able to understand the action.

These sets are well adapted for the testing of the internal wiring of

a building. To ascertain the **insulation to earth of any main,** one end of it is joined to terminal L, the other end being *free*, and terminal E is earthed (Fig. 106); with *all switches and fuses on and the lamps removed*, and the generator worked to produce the necessary pressure, the instruments give the insulation resistance between the main and earth direct. To determine the **insulation resistance between two mains** the end of one is joined to L, that of the other joined to E, the remote ends of both being left free, all switches and fuses are "on," and all lamps are removed as in the case above. In testing the **insulation resistance of the whole installation,** all switches and fuses must be "on," all lamps and fittings must be in place and connections to the apparatus arranged as in Fig. 106. The student should make diagrammatic sketches to illustrate these tests.

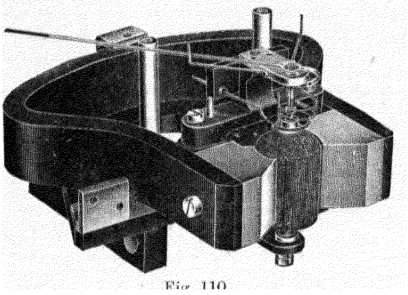


Fig. 110

In carrying out the above tests it is essential that the pressure employed be two or more times the pressure to which the installation when working will be subjected; it should never fall below that value, for defective sections may stand with ease a low pressure in testing, but break down utterly when the working pressure is applied, and moreover electrostatic effects at high pressures differ widely from those when the pressure is low. This point is emphasised in the following rules on minimum insulation resistance laid down by the Institution of Electrical Engineers:—

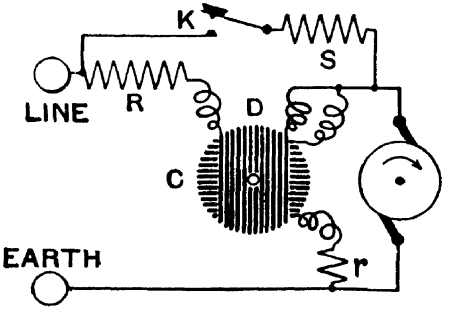


Fig. 111

(1) *The insulation resistance to earth of the whole or any part of the wiring must, if tested previously to the erection of fittings and electroliers, be measured with a pressure not less than twice the intended working pressure, and must not be less in megohms than "30" divided by the number of "points" under test. For this purpose the "points" are to be counted as the number of pairs*

of terminal wires from which it is proposed to take the current either directly or by flexibles to lamps or other appliances.

(2) Current must not be switched on until the following test has been applied to the finished work :—*The whole of the lamps having been connected to the conductors and all switches and fuses being "on," a pressure equal to twice the working pressure must be applied and the insulation resistance of the whole or any part of the installation must not be less in megohms than "25" divided by the number of lamps. When all lamps and appliances have been removed from the circuit, the insulation resistance between conductors must not be less in megohms than "25" divided by the number of lamps.* No motor, heater or other appliance may be connected to the supply of electrical energy unless the insulation of the parts carrying the current measured as above is greater than one megohm from the frame or case.

127. Cable Faults.—The cable faults with which we shall briefly deal are—

(1) A breakdown in the insulating covering only, the conductor remaining intact.

(2) A complete breakdown in both conductor and insulating covering.

(3) A complete breakdown in the conductor, the insulating covering remaining intact.

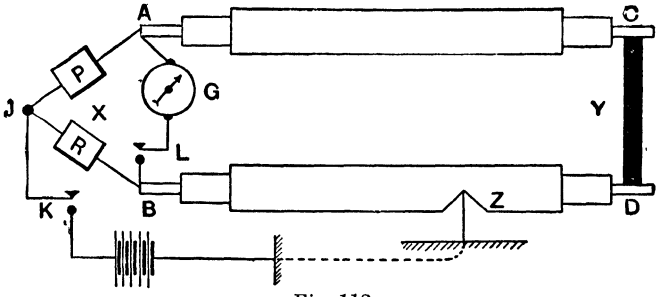


Fig. 112.

(1) *Loop Test for Class* 1.—Let B D (Fig. 112) represent the cable connecting two distant stations X and Y, and in which a partial earth exists at Z. At Y the cable in question is "looped" to a second one A C, also running between X and Y, by a conductor C D of negligible resistance. The free ends of the cables are connected at the test station to two adjustable resistances P and R; a sensitive galvanometer G joins A and B through the key L, whilst the junction J is connected through K to the battery, the other pole of which is earthed.

The arrangement is thus equivalent to a Wheatstone Bridge, the

arms being P – R — A C D Z and Z B. If P and R be now adjusted, so that on closing K and then L the galvanometer is not deflected, we have—

$$\frac{P}{R} = \frac{\text{Res. A C} + \text{Res. D Z}}{\text{Res. Z B}},$$

hence—

$$\frac{P+R}{R} = \frac{\text{Res. A C} + \text{Res. D Z} + \text{Res. Z B}}{\text{Res. Z B}},$$

$$\therefore \frac{\text{Res. Z B}}{\text{Res. A C} + \text{Res. D B}} = \frac{R}{P+R},$$

i.e.

$$\text{Res. Z B} = \frac{R}{P+R} \times r,$$

where r = the resistance of the two cables in series. Thus, knowing the resistance per mile of the cable, the distance B Z corresponding to the resistance B Z is ascertained. With identical cables—

$$\frac{\text{Res. Z B}}{\text{Res. A C} + \text{Res. D B}} = \frac{x}{l},$$

where x = distance Z B, and l = total length of the two cables in series; substituting—

$$x = \frac{R}{P+R} \times l.$$

This method is of service in cases where a fault occurs on one of a pair of electric-light feeders, for the remote ends may be coupled at the feeding points, and the test performed at the supply station. The test is independent of the resistance of the fault, but the E.M.F. of the battery must be sufficiently high to overcome any counter-pressure due to polarisation at Z.

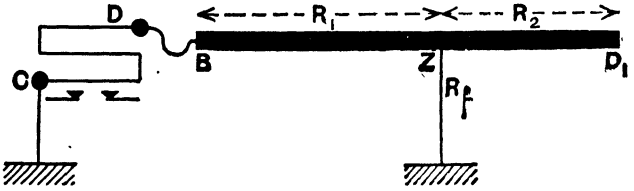


Fig. 113.

Another method may be briefly indicated. With the end D′ of the cable "free" (Fig. 113), the end B is joined to terminal D of the P.O. Box, terminal C being earthed, and the resistance between B and earth is measured: if this be x ohms, then $x = R_1 + R_f$, where R_f is the resistance of the fault. With the end

B "free," the test is repeated at the other end of the cable, *i.e.* the resistance between D' and earth is measured: if this be y ohms, then $y = R_2 + R_f$. Now:

$$x = R_1 + R_f \quad \ldots\ldots \quad (1)$$

$$y = R_2 + R_f \quad \ldots\ldots \quad (2)$$

$$\therefore x + y = R_1 + R_2 + 2R_f,$$

i.e.

$$R_f = \frac{(x + y) - (R_1 + R_2)}{2} = \frac{x + y - R}{2},$$

where $R = R_1 + R_2$ = resistance of the whole cable (this is known). Substituting this value of R_f in (1) and (2), the values of R_1 and R_2 and, therefore, the distances from the ends to Z are determined.

If B D' is too long to admit of the test being applied at both ends, as above, the following modification may be adopted. With D' free, the resistance between B and earth is measured: this gives x as in (1) above. With D' earthed the resistance between B and earth is measured; if this be y, then—

$$y = R_1 + (R_2 \text{ and } R_f \text{ in parallel}),$$

i.e.

$$y = R_1 + \frac{R_2 R_f}{R_2 + R_f}.$$

Now—

$$R = R_1 + R_2 \quad \ldots\ldots \quad (1)$$

$$x = R_1 + R_f \quad \ldots\ldots \quad (2)$$

$$y = R_1 + \frac{R_2 R_f}{R_2 + R_f} \quad \ldots\ldots \quad (3)$$

From these R_1 can be obtained, and, therefore, the distance B Z.

2. *P.O. Box Test for Class* 2.—If the broken end of the wire makes a good "earth," the free end of the cable may be joined to terminal D of the P.O. Box, terminal C being earthed; the resistance of the conductor from the free end to the break is then measured in the usual way, and the distance to the fault ascertained.

3. *Insulation and Capacity Tests for Class* 3.—The localisation of this fault with much degree of accuracy is difficult. With the remote end of the cable "free," one method is to measure the insulation resistance R from the free end to the break, and to apply the relation of Art. 116, viz.—

$$R = \cdot 366 \times \frac{\rho}{l} \times \log \frac{r_2}{r_1},$$

from which, knowing ρ, r_1, and r_2, the distance l may be found.

A second method is to measure the capacity (Art. 39) C farads from the free end to the break, and to use the relation of Art 36, viz.—

$$C = k \times \frac{2 \cdot 413}{10^{13}} \times \frac{l\,(\text{cm.})}{\log \frac{r_2}{r_1}},$$

from which, knowing k, r_1, and r_2, the distance l may be found.

128. Measurement of Low Resistances.—For the measurement of very small resistances, such as, for example, that of a dynamo or motor armature section, one of the best methods is that shown diagrammatically in Fig. 114. A C is a standard low-resistance wire of uniform cross-section, joined to an accumulator B and a variable resistance r. The unknown resistance X and a standard low resistance of somewhat similar magnitude S are connected in series, and the two put in parallel with the standard wire; in the case of an armature section, it must, of course, be detached by unsoldering before connecting as indicated.

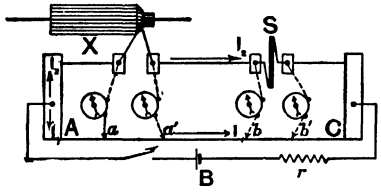


Fig. 114.

A sensitive galvanometer has one terminal joined to one end of the unknown, and by means of a movable contact, a point a is found on the wire, such that the galvanometer is not deflected. This is repeated at the other end of the unknown, and at both ends of S, as shown by the dotted lines in the figure; clearly then—

$$\text{P.D. at the ends of X} = \text{P.D. between } a \text{ and } a',$$

and

$$\text{P.D. at the ends of S} = \text{P.D. between } b \text{ and } b';$$

$$\therefore \frac{\text{P.D. at the ends of X}}{\text{P.D. at the ends of S}} = \frac{\text{P.D. between } a \text{ and } a'}{\text{P.D. between } b \text{ and } b'}$$

i.e.

$$\frac{I_2 \times \text{Res. X}}{I_2 \times \text{Res. S}} = \frac{I_1 \times \text{Res. } a\,a'}{I_1 \times \text{Res. } b\,b'} = \frac{\text{Length } a\,a'}{\text{Length } b\,b'};$$

$$\therefore \text{X} = \frac{\text{Length } a\,a'}{\text{Length } b\,b'} \times \text{S}.$$

129. Evershed's Ducter for Low Resistance.—This instrument (Fig. 115) is similar in general principle to the Megger. The two coils P and C are fixed upon an axle with their planes making an angle with each other. P is wound as a *pressure coil*, and is joined to the ends of the unknown resistance X; it therefore *carries a current proportional to the P.D. at the ends of* X. One end of C is connected to the left-hand side of S, the other end being joined through a resistance R to the right-hand side of S. The battery E is used to send a current through S and the unknown resistance X; thus *the current picked off by* C *depends upon the total current flowing through* X, and the ratio of the current in P to that in C depends therefore *upon the resistance* X. The coils P and C are in the field of a strong magnet N S.

When the currents pass the combination is deflected, each coil

trying to occupy the same position, and as they move together, the actual position taken up depends upon the ratio of the currents in the coils, *i.e.* upon the value of the unknown resistance X.

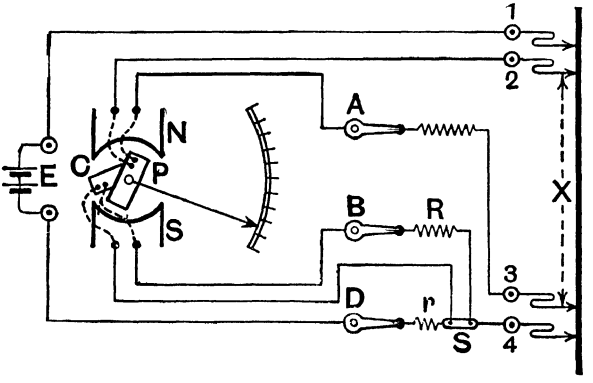


Fig. 115.

The instrument has a standard range of from 10 microhms to five ohms, the variation being effected as shown in Fig. 116, which gives the right-hand side of Fig. 115 in greater detail. The five-way switch K controls the three five-way contacts A, B, D. Contact A varies the resistance in the coil P circuit according to the range. D joins the battery and unknown X to the correct resistance S, at the same time bringing in an additional resistance *r* if necessary. B connects the coil C circuit through a suitable resistance R to the particular resistance S to be used. The scale is graduated 0-500 microhms, and the reading must be multiplied by 1, 10, 100, 1,000, 10,000, according to the position of K (and therefore A, B, D) in making the test. Fig. 117 depicts the method of dealing with armature coils; in such cases the final result must, of course, be multiplied by the number of circuits in parallel.

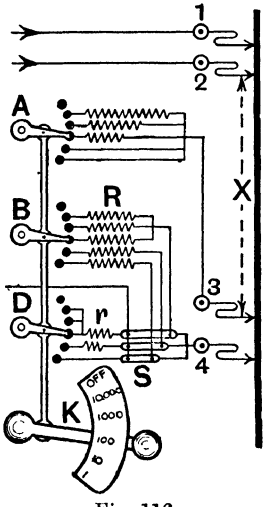


Fig. 116.

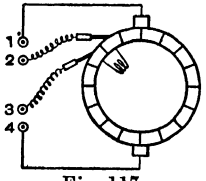


Fig. 117.

130. Ammeter and Voltmeter Methods of Measuring Resistance.—Since resistance is defined as the ratio of P.D. to current, several resistance tests may be performed by the combined use of an ammeter and voltmeter or its equivalent. The *hot resistance of a glow-lamp*, for example, may be determined by inserting an ammeter in series with the lamp, and a voltmeter across the lamp terminals. If I be the ammeter reading, v the voltmeter reading, and r the resistance of the voltmeter under these conditions, the current taken by the latter is $\frac{v}{r}$, and therefore the current in the lamp is $I - \frac{v}{r}$; hence—

$$\text{Res. of lamp} = \frac{\textit{P.D. at its terminals}}{\text{Current in it}} = \frac{v}{I - \frac{v}{r}}.$$

A modification affords a ready test of the *internal resistance of a battery or cell*, such as, for example, an accumulator. The experiment consists of three steps:

(1) A reflecting galvanometer, with a large resistance (100,000 ohms) in series with it, is connected to the poles of the accumulator and the deflection d_1 observed.

(2) The poles of the cell are now joined to a second circuit, consisting of an ammeter and variable resistance, and the latter is adjusted till a current of, say, 10 amperes is registered. While this current is flowing the galvanometer deflection d_2 is read.

(3) The accumulator and ammeter circuit are removed, a Clark standard cell (E.M.F. = 1·433 volts) put in series with the galvanometer and its high resistance, and the deflection d_3 observed.

A deflection d_3 is produced by a pressure of 1·433 volts;

$\therefore$ a deflection d_1 is produced by a pressure of $\frac{d_1}{d_3} \times 1{\cdot}433$ volts,

and a deflection d_2 is produced by a pressure of $\frac{d_2}{d_3} \times 1{\cdot}433$ volts.

Again, if E = E.M.F. of a cell,

and e = Terminal P.D. when a current I flows,

$E - e$ = Volts used up in the cell,

and $\text{Resistance of cell} = \frac{E - e}{I}$.

Now, in Case 1, the cell is practically on open circuit, so that $\left\{\frac{d_1}{d_3} \times 1{\cdot}433\right\}$ volts is its E.M.F. E; the terminal P.D. in Case 2 is $\left\{\frac{d_2}{d_3} \times 1{\cdot}433\right\}$ volts, and neglecting the small portion taken by the galvanometer, 10 amperes may be taken as the current in the accumulator; hence—

$$\text{Res. of cell} = \frac{\frac{d_1}{d_3}1{\cdot}433 - \frac{d_2}{d_3}1{\cdot}433}{10}$$

$$= \left\{\cdot 1433\,\frac{d_1 - d_2}{d_3}\right\} \text{ ohms.}$$

It should be mentioned in passing that the resistance of a cell depends upon the current output, decreasing as the current increases; and polarisation, too, affects many resistance determinations.

The *resistance of the armature of a dynamo* may be measured by a combination of the above two methods. The armature is put in series with an accumulator and an ammeter, and the current I is observed; the P.D. V at the brushes is obtained by a high-resistance galvanometer, as in the last case; the armature resistance is given by $V \div I$.

131. Measurement of Liquid Resistance.—The most important case is that of an electrolyte, and an obvious difficulty arises in the fact that this polarisation at the electrodes sets up a back E.M.F., which, with ordinary methods, would appear as a resistance, and, moreover, the polarisation itself is not constant. In practice the electrodes are invariably of platinum coated with "platinum black," and the test is carried out in various ways.

(1) KOHLRAUSCH'S METHOD.—This is a metre bridge method, the electrolyte being in one gap and a standard resistance in the other. Current is supplied by the secondary of an induction coil, and as this is rapidly alternating in direction, the opposite polarising effects at the electrodes neutralise each other. Balance is obtained by means of a telephone, the balancing point being that for which the sound in the receiver is a minimum. The condenser of the induction coil should be removed, otherwise the currents in the two directions will not be equal, and polarisation will occur.

(2) FITZPATRICK'S METHOD.—A better method, that of Fitzpatrick, is to supply continuous current from a battery as usual, having also the ordinary Wheatstone bridge connections, including the galvanometer, but to pass the current first through a commutator, which is rapidly rotated by a motor and continually reverses the current flow, while at the same time the motor, by means of another commutator, continually reverses the galvanometer connections, so that the swings, if any, are all in one direction. The galvanometer circuit is closed a little after the battery circuit and opened a little earlier to avoid inductive effects.

(3) STROUD AND HENDERSON'S METHOD.—This is another Wheatstone Bridge method, and will be understood from Fig. 118. The resistances S and Q are made equal and very large. The resistance P is that of a tube of the liquid under test. The resistance R is partly that of another tube of the liquid exactly like the first, except that it is much shorter, and partly made up by an adjustable resistance. This adjustable resistance, r, is altered until the galvanometer shows no deflection, in which case the resistance r equals that of a column of the liquid equal to the difference in length of the two tubes. Thus, although continuous current is employed, since the polarisation effects are the same in both arms of the bridge, their effect on the result is eliminated.

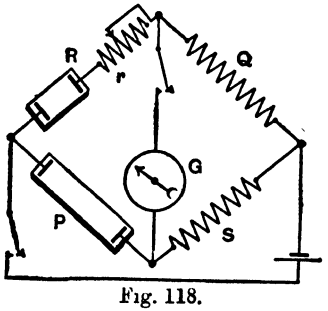


Fig. 118.

EXAMPLES.

(1) *On a 200-volt circuit, the current passing through a bank of lamps is found to be 12 amperes, but on inserting a length of cable in series with the lamps the voltage across the lamp terminals is found to be 195. Suppose that it is permissible to earth the circuit for a short period, how could you test the ohmic resistance of a lightning conductor within easy access of this lighting circuit?* (O.)

$$\text{Resistance of lamps} = \frac{\text{P.D.}}{\text{Current}} = \frac{200}{12} = 16\tfrac{2}{3} \text{ ohms.}$$

$$\text{Current in lamps when cable is in series with them} = \frac{195}{16\frac{2}{3}} = 11{\cdot}7 \text{ amps.}$$

$$\therefore \text{Resistance of this cable} = \frac{\text{Volts used in it}}{\text{Current in it}} = \frac{5}{11{\cdot}7} = {\cdot}43 \text{ ohm.}$$

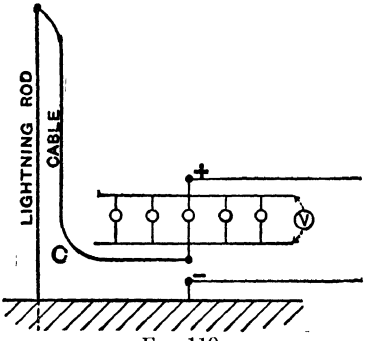


Fig. 119.

Connections are now made as in Fig. 119, where C is the cable whose resistance we have just determined, joined as indicated to the upper end of the lightning conductor and to the lamps; the negative main of the lighting circuit is put to earth.

Let V = P.D. for the lamps now;

$$\therefore \text{Current in lamps} = \frac{V}{16\frac{2}{3}}.$$

Volts used in cable C and lightning conductor = (200 − V);

$$\therefore \text{Resistance of cable C and lightning conductor} = \frac{200 - V}{\frac{V}{16\frac{2}{3}}},$$

$$\text{and resistance of lightning conductor} = \left\{ \frac{200 - V}{\frac{V}{16\frac{2}{3}}} - {\cdot}43 \right\} \text{ ohms.}$$

The method does not allow for the fact that the resistance of the lamps varies with the current passing through them, neither does it correct for the chemical disturbances which may arise with continuous currents at the earth connections.

(2) *To determine the insulation resistance of a pair of live mains the following tests were made, using a voltmeter of* 10,000 *ohms resistance:—*

P.D. measured between the mains = 216 *volts.*
,, ,, ,, + *main and earth* = 196 *volts.*
,, ,, ,, − *main and earth* = 10 *volts.*

Find the insulation resistance of each main from earth, and also the insulation of the whole system. (O.)

(α) *Voltmeter between* + ve *main and earth.*

Let x = insulation resistance in ohms between + *ve* and earth ;
Current in voltmeter = $\frac{196}{10000}$ = ·0196 amp. ;
Current going through insulation to earth = $\frac{196}{x}$ amp. ;

$\therefore$ Total current leaving + *ve* main = $\left(\frac{196}{x} + \cdot 0196\right)$ amp.,

And total current entering − *ve* main = $\left(\frac{196}{x} + \cdot 0196\right)$ amp.

Let y = insulation resistance in ohms between − *ve* and earth ;

$\therefore$ P.D. between earth and − *ve* main = $\left(\frac{196}{x} + \cdot 0196\right) y$ volts.

And $\left\{\begin{matrix}\text{P.D. between}\\ \text{mains}\end{matrix}\right\} = \left\{\begin{matrix}\text{P.D. between + ve}\\ \text{and earth}\end{matrix}\right\} + \left\{\begin{matrix}\text{P.D. between earth}\\ \text{and − ve}\end{matrix}\right\}$

$$216 = 196 + \left(\frac{196}{x} + \cdot 0196\right) y \quad \ldots\ldots \quad \ldots\ldots \quad \ldots\ldots \quad \ldots\ldots \quad (1)$$

(β) *Voltmeter between* − ve *main and earth.*

Current in voltmeter = $\frac{10}{10000}$ = ·001 amp. ;

Current going through insulation to − *ve* = $\frac{10}{y}$;

Total current entering − *ve* = $\left(\frac{10}{y} + \cdot 001\right)$ amp. ;

$\therefore$ Total current leaving + *ve* = $\left(\frac{10}{y} + \cdot 001\right)$ amp.

$\therefore$ P.D. between earth and + *ve* = $\left(\frac{10}{y} + \cdot 001\right) x$.

Hence— $$216 = 10 + \left(\frac{10}{y} + \cdot 001\right) x \ldots \quad . \quad . \quad . \quad \ldots \quad \ldots \quad (2)$$

Solving (1) and (2) we get—

x = **10,000 ohms.** y = **510 ohms.**

Insulation resistance between + *ve* and − *ve.*

$= x + y$ = **10,510 ohms.**

NOTE :—**The above example is important as outlining a method of measuring the insulation resistance of a pair of live mains.**

EXERCISES XI.

1. Describe the Wheatstone Bridge method of measuring resistance, and explain how a fault in the insulation of a cable may be located. (B.)

2. Give a diagram of the Post Office form of Wheatstone Bridge, showing the connections. (P.)

3. Sketch the connections of a Wheatstone Bridge, showing how it is connected to the resistance to be measured. (O.)

4. You are asked to determine the relative electrical values of samples of insulated copper wire. How would you proceed to determine the copper resistance and insulation resistance respectively? (H.)

5. Why should the leakage resistance of an installation not be measured by a 1-volt cell even when a sufficiently sensitive galvanometer is available? (P.)

6. How would you test the insulation in a building you had wired? (P.)

7. What is the construction of an ohmmeter? (O.)

8. Describe with sketches some form of accurate portable testing set used for measuring the insulation resistance of an electric-light installation. (P.)

9. Having wired one floor of a house, how would you test the insulation to ascertain whether it complied with the specification? About what insulation resistance should the wiring of one floor of an ordinary house possess? (P.)

10. Having found that a fault has occurred in one main, how would you localise the exact spot? (H.)

11. The total insulation resistance of a telegraph line 100 miles long is found to be 5,000 ohms, and the total insulation resistance of the line up to a point 60 miles from where the measurement is made is found to be 8,000 ohms. What is the insulation resistance per mile of the remaining 40 miles of line? (H.)

12. The conductor resistance of a well-insulated telegraph line with further end to earth is 2,000 ohms; if there were a fault at its centre of 1,000 ohms resistance, to what would the resistance of the line be reduced? (O.)

13. A cable, whose conductor resistance when perfect is 1,000 ohms, becomes faulty, and the resistance measured from the nearer end when the further end is insulated is found to be 900 ohms; when the further end is put to earth, then the resistance measured from the nearer end is found to be 820 ohms; determine the position of the fault. (H.)

14. Explain the methods you would employ for the measurement of (1) a resistance of the order of one megohm; (2) one of the order of 10,000 ohms; (3) one of the order of 10 ohms; and (4) one of the order of ·001 ohm.

15. A battery of six cells in series, an ammeter and a key are arranged in circuit. The poles of the battery are also connected to the terminals of a high-resistance voltmeter. The voltmeter registers 12·4 volts before, and 10·6 volts after, the ammeter circuit is completed, the ammeter indicating 15 amperes. What is the resistance of each cell of the battery? (P.)

16. Describe the arrangement of a set of resistance coils to act as a Wheatstone Bridge. How would you modify the arrangement if a bridge were required for testing very low resistance? (P.)

17. The wiring of a house on being tested for insulation is found to be defective. Describe in detail how you would proceed to locate the faults, supposing several to exist. (P.)

CHAPTER XII.

MEASUREMENT OF POTENTIAL DIFFERENCE, ELECTROMOTIVE FORCE, AND CURRENT STRENGTH.

132. The Potentiometer.—One of the best methods of comparing and measuring electromotive forces and potential differences is that known as the "**potentiometer**" method, for it possesses the three manifest advantages: (1) that it is a "zero" method, *i.e.* one in which no deflection has to be observed; (2) that in testing batteries no current is taken from them, and thus their true E.M.F.'s unaffected by polarisation are obtained; and (3) that the resistances of the galvanometer and battery tested do not enter into the calculation since no current passes through them. The method is, moreover, a valuable one for the accurate measurement of current and resistance, and for the verification and calibration of ammeters, voltmeters, etc.; it has been employed by Lord Rayleigh in the comparison of the E.M.F.s of standard cells.

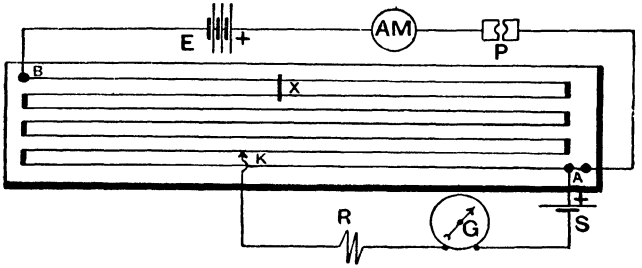


Fig. 120.

The potentiometer of Fig. 120 consists of seven uniform manganin or platinum-iridium wires each one metre in length joined in series by thick copper bars, the whole being attached to a board, and fitted with binding screws at the ends as indicated; scales graduated in millimetres are placed alongside the wires, the graduations commencing from the end A. The potentiometer wire is connected in series with an ammeter A M (for the detection of any current variation during the experiment), a plug key (P), and a battery (E) of three good

accumulators, the positive pole of the latter being joined to the end A of the wire. A convenient resistance for the potentiometer wire is about two ohms per metre.

The positive terminal of a Latimer Clark Standard Cell S (E.M.F. = 1·433 volts at 15° C.) is connected to the end A, the negative terminal being joined through a resistance R and a sensitive galvanometer G preferably of the D'Arsonval type (Art. 136) to the sliding contact K placed 143·3 centimetres from the end A.

The P.D. between the points A and K produced by the battery E *tends* to send a current through the lower branch in the direction A S G R K, whilst the E.M.F. of the standard cell *tends* to drive a current in the opposite direction in this branch, viz. in the direction K R G S A.

By altering the current in the potentiometer it can be arranged that the P.D. between A and K (given by the product of the current and the resistance of this portion of the wire) is equal to the E.M.F. of the cell S, in which case no current will flow in the lower branch and the galvanometer will not be deflected. This is attained by means of the slider X, consisting of a short piece of stout wire held firmly on the potentiometer wires by means of a piece of lead; the position of X is altered, thus varying the total resistance of the wires till no deflection is indicated: finally R is removed for greater sensitiveness and X again slightly changed if necessary till there is absolutely no movement of the galvanometer. Clearly the P.D. between A and K is now 1·433 volts (assuming the temperature of the cell S to be 15° C.); and since the distance A K is 1,433 millimetres, each millimetre of the wire gives a P.D. of ·001 volt.

(1) *Measurement of Small E.M.F.'s.*—In determining, say, the E.M.F. of a Leclanché cell (or, in this case, any P.D. up to about 5½ volts) S is removed, the Leclanché cell substituted, its positive pole being joined to A, and, X remaining fixed, the sliding contact K is adjusted for no deflection; if the distance between A and the balancing point be 1,451 millimetres, the E.M.F. of the Leclanché is clearly ·001 × 1451, *i.e.* 1·451 volts.

(2) *Measurement of High E.M.F.'s.*—If the P.D. to be measured is a large one, say of the order 200 or 250 volts, and therefore quite beyond the range of the potentiometer, a high resistance (10,000 ohms or more) is connected to the two points between which the P.D. is required and the fall of potential down a known fraction of this resistance is measured in the manner explained above; from the value thus determined the total P.D. is readily found,

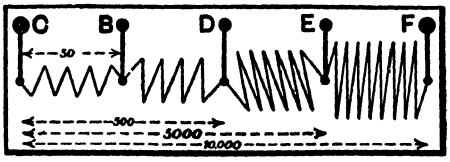


Fig. 121.

since fall of potential is proportional to resistance. A convenient form of "**volt-box**," as it is termed, is shown *diagrammatically* in Fig. 121; the resistances of the various sections are—

C B = 50 ohms.	C E = 5,000 ohms.
C D = 500 „	C F = 10,000 „

Thus, if the P.D. between C and B measured by the potentiometer be 1·25 volts it is clear that the total P.D. between C and F is—

$$\frac{10000}{50} \times 1{\cdot}25, \textit{ i.e. } 250 \text{ volts.}$$

The arrangement of the connections for making the test is shown in Fig. 122.

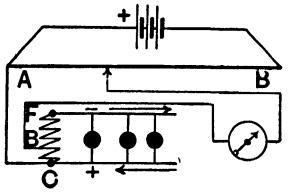


Fig. 122.

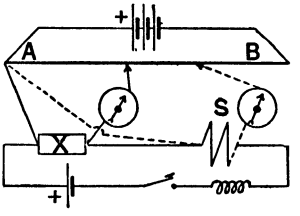


Fig. 123.

(3) *Measurement of Resistance.*—The resistance X (Fig. 123) to be measured is put in series with a standard resistance S, an accumulator, and an adjustable resistance. The P.D. at the ends of the unknown is balanced on the potentiometer, as in the case above. This is repeated with the standard, and—

$$\frac{\text{P.D. at ends of X}}{\text{P.D. at ends of S}} = \frac{\text{(Current in X) (Res. of X)}}{\text{(Current in S) (Res. of S)}} = \frac{\text{Res. of X}}{\text{Res. of S}}$$

i.e.

$$\text{Res. X} = \frac{\text{P.D. at ends of X}}{\text{P.D. at ends of S}} \times \text{Res. S}$$

or

$$\text{X} = \frac{\cdot 001 \times d_1}{\cdot 001 \times d_2}\,\text{S} = \frac{d_1}{d_2}\,\text{S},$$

where d_1 and d_2 are the distances in mm. from A to the sliding contact at the balances. If X is very large the P.D. and resistance for a known fraction of it must be measured and the total calculated.

(4) *Measurement of Current Strength.*—To accomplish this, the current to be measured is sent through a standard low resistance of ·1, ·01, or ·001 ohm. according to circumstances, and the P.D. at the ends of this standard is determined in the usual manner; the P.D. thus ascertained, divided by the resistance of the standard, gives the current value required. It will be evident to the reader that this

method permits of the measurement of very large currents; thus with the potentiometer of Fig. 120, $5\frac{1}{2}$ volts can be accurately measured, and this P.D. at the ends of a standard resistance of, say, ·0005 ohm corresponds to a current of $5\frac{1}{2}/{\cdot}0005$, *i.e.* 11,000 amperes. It is in this way that the large currents employed in copper refining can be measured.

(5) *Measurement of Power.*—The power absorbed by, say, a lamp is determined as follows. A standard low resistance is put *in series* with the lamp, and the P.D. at the ends of it is measured: this P.D. divided by the resistance of the standard gives the current. A volt box is connected *across the lamp terminals* and the P.D. on a known fraction of this is measured: from this the full P.D. on the box and therefore on the lamp is found. The power absorbed by the lamp is given by the product of the current in the lamp and the P.D. at its terminals.

A second method for potentiometer experiments may here be briefly referred to. In it the slider X of Fig. 120 is dispensed with, and, with the standard cell in circuit, *the contact* K *is adjusted for no deflection*: the P.D. between A and K = the E.M.F. of the standard cell = 1·433 volts. The standard cell is now removed, the E.M.F. to be measured is substituted and the contact moved to some point L for no deflection: the P.D. between A and L = the E.M.F. under test. Thus:—

$$\frac{\text{Required E.M.F.}}{1{\cdot}433} = \frac{\text{P.D. between A and L}}{\text{P.D. between A and K}} = \frac{\text{Length A L}}{\text{Length A K}}$$

$$\therefore \text{Required E.M.F.} = 1{\cdot}433 \times \frac{\text{Length A L}}{\text{Length A K}}.$$

133. The Crompton Potentiometer.—At the hands of Mr. Crompton the potentiometer has reached a state of perfection which renders it an invaluable instrument for refined tests of potential difference; Figs. 124 and 125 give diagrammatic views of the connections, the former being merely a simplified sketch of the latter, the better to illustrate the principle.

The potentiometer consists of a wire divided into fifteen segments of equal resistance (about 2 ohms); fourteen of these are formed into coils, their ends being connected to the numbered studs shown at E on the right, the fifteenth being a stretched wire lying over a scale about 25 inches in length divided into 1,000 equal parts; thus the coils are permanently protected, and should the wire become worn or injured a new one may be substituted.

In series with the potentiometer coils and wire is an adjustable resistance; this is shown at G and G_1 on the left, and is for the purpose of altering the P.D. at the ends of the fifteen segments so as to secure a balance with the standard cell in circuit; it corresponds to X and the back two wires of Fig. 120. C is the sliding contact, provided with a spring contact and micrometer adjustment for refined tests. The accumulator is connected to the terminals A, the galvanometer to terminals B, the standard cell to terminals I, any unknown E.M.F.'s to the terminals 2, 3, 4, 5, and 6, and the double contact switch at the centre

enables any of these E.M.F.'s to be put into the galvanometer circuit. The reader should carefully examine these connections and compare them with those of Fig. 120.

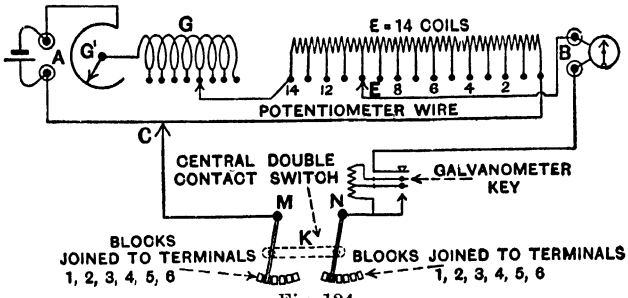


Fig. 124.

Working with one accumulator it is customary to standardise the instrument so that the total P.D. at the ends of the potentiometer coils and wire is 1·5 volts. To secure this, the revolving arm at E is placed on stud 14, and the sliding contact C at division 330 on the scale; the centre switch is now set on I, thus

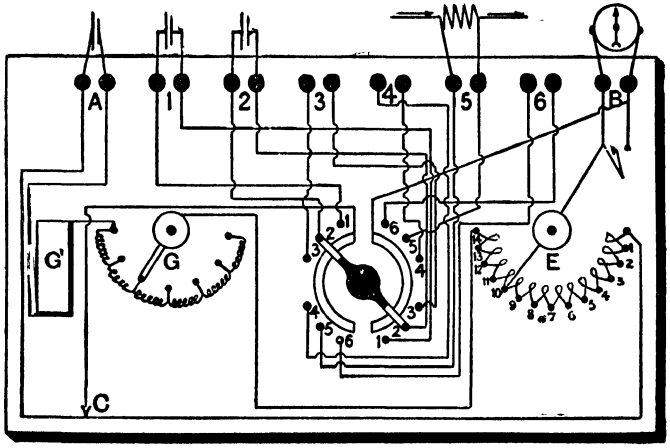


Fig. 125.

bringing the standard cell into the galvanometer circuit, and the resistances in series with the coils and wire are adjusted till the galvanometer is not deflected. Clearly the P.D. for *each* coil and for the wire is now 1·433/14·33 or ·1 volt, and the P.D. for *each* of the 1,000 divisions is ·0001 volt.

The E.M.F. to be measured is now brought into the galvanometer circuit by moving the centre switch to the corresponding studs. With C on the extreme right the revolving arm at E is adjusted till two successive contacts give deflec-

tions in opposite directions: E is now placed on the stud of lower value and C adjusted for no deflection. If this condition is realised with the arm E on stud 10 and the slider C on division 646, the E.M.F. under test is—

$$\{(10 \times \cdot1) + (646 \times \cdot0001)\} \text{ volts, } i.e. \ 1{\cdot}0646 \text{ volts.}$$

The galvanometer key (Fig. 124—not shown in Fig. 125) completes the circuit through two resistances which are short circuited in succession as the key is depressed. The construction of the latest Crompton is identical with the preceding, but G, G^1 and the accumulator terminals A are on the right of the central switch, whilst E, the galvanometer terminals B, and the galvanometer key are on the left.

134. Galvanoscopes and Galvanometers.—In Art. 102 it has been mentioned that if a compass needle be placed in the neighbourhood of a current-carrying wire the former will, under suitable conditions, be deflected in a direction depending on that of the current, and, naturally, by an amount depending on the current strength; instruments in which this property is utilised for the detection of currents are termed galvanoscopes, more accurate forms designed for current measurement being known as galvanometers.

Fig 126.

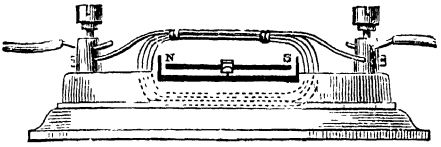


Fig. 127.

The simplest type of galvanoscope consists of a magnetic needle with a wire placed above it and in the meridian, as shown in Fig. 126. To increase the sensitiveness of this arrangement the wire may be formed into a coil with the needle at its centre (Fig. 127); an application of the *right-hand rule* (Chapter XVI.) will reveal the fact that in such a case all the currents both above and below the needle are urging it in the same direction, and thus a weak current will be better able to produce a deflection than if the simple arrangement of Fig. 126 be followed.

In both these cases, however, it will be observed that the action of the earth on the needle is opposing that of the current, for the former is striving to set it in the meridian, whilst the latter is endeavouring to set it at right angles thereto, from which it follows that to secure greater sensibility the action of the earth must be eliminated. This is more or less accomplished by employing instead of a single needle an **"astatic" pair;** that is, a combination of two needles of equal length, strength, etc. (or equal "moment"), fixed parallel with unlike poles adjacent, and with their magnetic axes in the same vertical plane, and in which therefore the turning effect of the earth on one is exactly cancelled by the equal turning effect in the opposite direction on the

other. The two methods of winding such an arrangement so as to form an "**astatic galvanometer**" are shown in Fig. 128; by applying the right-hand rule it will be seen that both needles are urged in the same direction.

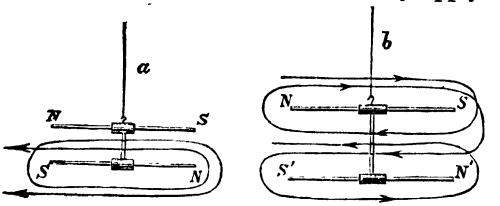


Fig. 128.

The construction of a perfectly astatic pair is a matter of extreme difficulty, but even if partially astatic only, a very sensitive galvanometer is produced. It may be mentioned here that the "lamp and scale" method of reading galvanometer deflections is now extensively adopted, for a very small and almost unreadable deflection of the magnet or other moving system produces a considerable deflection of a spot of light along a scale.

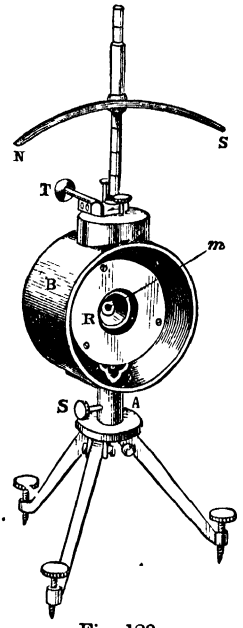


Fig. 129.

135. The Kelvin Mirror Galvanometer.—This instrument, originally devised by Lord Kelvin for submarine cable work, is shown in Fig. 129. It consists of a coil of wire wound to a resistance of 2,000 to 10,000 ohms according to the work for which the galvanometer is required, at the centre of which is suspended by a single cocoon silk fibre a small concave mirror *m* carrying at the back three or four tiny magnets with like poles adjacent. The reel R on which the coil is wound and the suspended system are enclosed in a brass chamber B fitted with a glass window in front. The short suspension and confined air chamber combine in making the instrument to a certain extent "dead beat," *i.e.* when a current passes, the needles move to their final position without undue oscillation, and when the current ceases they move fairly steadily to their zero position. A "controlling" magnet N S is fixed on a vertical rod as shown, and is for the double purpose of setting the "spot of light" on the scale and altering the sensibility of the galvanometer.

Consider, for example, the galvanometer so placed that the coil, needles, and control magnet lie in the meridian, the control having its north pole northwards. Clearly the earth and control magnet are exerting *opposing* influences on the needles, so that the magnetic field in which the latter hang is weak and the moving system will be readily

deflected by a small current in the coils. Lowering the control will still further increase the sensitiveness of the arrangement, and finally a position will be found in which the earth's field is exactly cancelled and the system will be astatic. Raising the control will decrease the sensibility of the galvanometer, and this decrease will be even more marked if the control be set with its south pole northwards, in which position it will *assist* the earth and create a strong field in the vicinity of the needles.

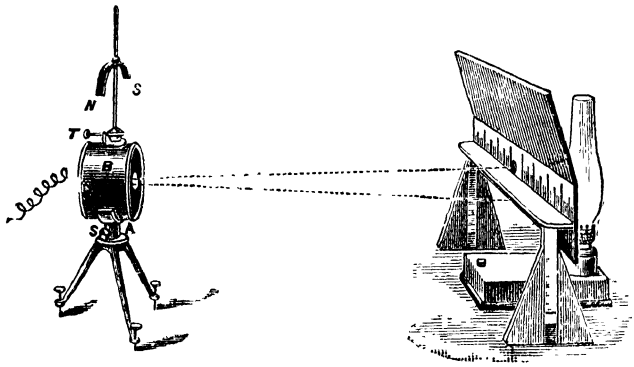


Fig. 130.

The arrangements for reading the deflection are shown in Fig. 130. Light from the lamp passes through a slit below the scales (one form of which contains 720 divisions of half a millimetre each), falls on the concave mirror *m*, and is thence reflected along the upper line, producing a well-defined image of the slit on the scale. When a current passes, the magnets and mirror are deflected, the reflected beam moves through double the angle through which the mirror itself turns, and the spot of light travels a considerable distance along the scale. The law of the galvanometer may be taken to be $I = Kd$, where d is the scale deflection and K a constant for the galvanometer. Currents of the order one-millionth of an ampere can be detected and measured. An electric lamp invariably takes the place of the oil lamp of Fig. 130.

It may here be briefly mentioned that galvanometers are often wound with two coils, one consisting of few turns, the other consisting of many turns of wire. A moderately strong current is passed through the former, but a weak current through the latter coil, for the greater the number of times it is carried round, the greater will be its effect on the needle. Of course, the coil of many turns must necessarily be of fine wire to avoid the instrument approaching unwieldy dimensions. Similar remarks apply to electromagnets, bells, indicators, etc.

The **figure of merit** of a galvanometer in which the deflection is given by a pointer moving over a scale of degrees is usually defined as the current in

amperes necessary to produce a deflection of 1°; in the case of a reflecting galvanometer it is taken to be the current necessary to produce one millimetre deflection on a scale placed one metre from the mirror of the galvanometer. The **sensibility** is the deflection caused by 1 micro-ampere.

136. Moving-coil Galvanometers.—Except, perhaps, for high-class laboratory work these galvanometers have largely superseded the Kelvin instruments in recent years. The form invented by MM. Despretz and D'Arsonval is shown in Figs. 131, 132.

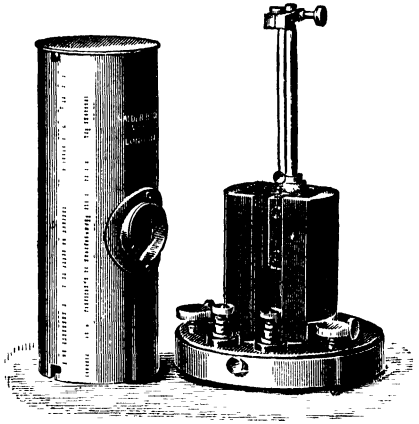


Fig. 131.

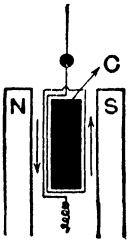


Fig. 132.

The rectangular movable coil consists of a number of turns of fine wire well soaked in insulating varnish, and suspended between the poles of a permanent horse-shoe magnet by wires or phosphor bronze strips which also serve to conduct the current to and from the coil. The lower wire is attached to a small spring, and the upper suspension to the torsion head at the top of the instrument; in this way the coil is maintained firmly in its normal position, viz. with its plane along the lines of force of the field between the magnet poles. A cylindrical piece of soft iron C is supported within the coil, and by concentrating the lines results in an intense field in the space at each side in which the vertical wires of the coil move.

When a current passes in the coil, the latter tends to set itself so as to enclose as many lines of force as possible, *i.e.* it tends to set with its plane at right angles to the field between the poles, and this motion is resisted by the controlling couple furnished by the torsion of the suspension, the coil in consequence taking up an intermediate position in which the deflecting and controlling couples balance each other. The induced pressures developed in the coil when it moves in the magnetic field oppose the motion and render the instrument dead beat.

In some types the coil is wound upon a conducting frame of aluminium or silver; in such cases the induced currents (eddy currents) developed therein still further oppose the motion and increase the dead-beat action of the galvanometer. Further, when the current ceases any oscillation of the coil may be prevented by merely connecting for a moment the galvanometer terminals by a wire of low resistance, the damping being as before produced by induced currents.

The *direction* of deflection depends on the direction of the current, and may be determined by the following **"left-hand rule,"** due to Fleming:—*Place the thumb and the first two fingers of the left hand mutually at right angles; select the vertical wires in one of the gaps, hold the middle finger in the direction of the current, and turn the hand so that the forefinger points in the direction of the lines of force,* i.e. *from* N *to* S; *the thumb will give the direction of motion.* Thus with the current passing as in Fig. 132, the left-hand side of the coil moves "out of the paper," and the right-hand side into it.

The law of this type of galvanometer may be expressed by the relation—

$$I = G\alpha,$$

where I is the current, α the deflection, and G a constant for the galvanometer.

The more recent type of moving-coil galvanometer devised by Messrs. Ayrton and Mather is shown in Fig. 133. The permanent magnet is of the shape of a nearly complete cylinder—a very narrow air gap only existing between the poles. In this gap hangs the long, narrow, rectangular coil, mounted in a thin silver tube; thus the "damping" is very efficient. Coils of various resistances (3, 14, 95, and 325 ohms) are manufactured, any of which may be substituted to comply with the conditions of any specific experiment. The coil has no iron core, its shape and the narrow air gap rendering such unnecessary.

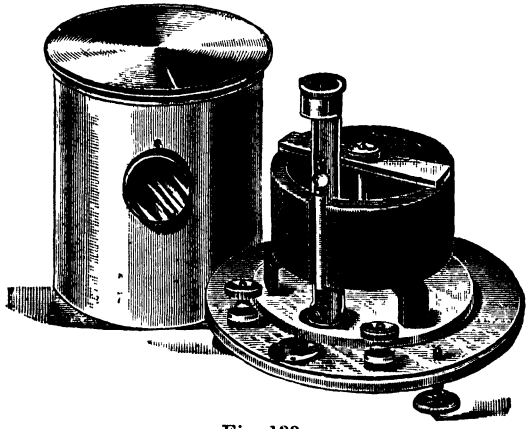


Fig. 133.

The advantages of the moving-coil galvanometer are :

(1) It is independent of the earth's magnetic field, since no sensitive magnetic needle is employed.

(2) The field in which the coil hangs is so strong that external magnetic fields due to dynamos, etc., do not affect the readings *to any appreciable extent.*

(3) It is remarkably "dead beat" in its action, and is not so sensitive to vibration in its vicinity as those previously described.

(4) It may be set up in any convenient position.

137. The Ballistic Galvanometer and the Tangent Galvanometer.—*Ballistic Galvanometers*, which may be either of the moving-magnet or moving-coil type, are instruments constructed for work with currents of short duration, *e.g.* induced currents and condenser discharges. In them "damping" must be as small as possible, and for what exists a correction must be made. Further, the moving system must have a large moment of inertia (*i.e.* must be heavy), so that practically the total discharge passes before the system begins to move. In such cases it can be proved that the total quantity Q which passes through the galvanometer is given by—

$$Q = \frac{Ht}{\pi G} \sin\frac{\alpha}{2} \quad or \quad Q = \frac{ct}{2\pi hA}\alpha$$

the former applying to a ballistic of the moving-magnet type, and the latter to one of the moving-coil type. In these expressions H = strength of earth's field, t = time of swing of the suspended system, G = a constant for the galvanometer, c = a constant for the suspension, h = field in which the coil hangs, A = face area of coil, and α = first angular swing. Thus in the first case we can write $Q \propto \sin\frac{\alpha}{2}$ and in the second case $Q \propto \alpha$: in most cases, for reflecting types, we can write $Q \propto d$, where d = scale deflection (first swing). To be exact a correction should be made for any damping.

The *Tangent Galvanometer* is not extensively used in modern laboratories. Essentially it consists of a circular coil of a few turns of insulated wire with a *small* magnetic needle suspended at the centre. A light aluminium pointer attached to the needle enables the deflection to be read on a horizontal scale graduated in degrees. The needle is very small, so that the magnetic field due to the current in the coil may be assumed uniform throughout the space occupied by the needle and equal to the field at the centre of the coil. In working with the instrument, the coil is first set in the magnetic meridian so that the needle and coil are in the same vertical plane; the current to be measured is then passed through the coil and the angle of deflection read off from the scale.

In Chapter XVII. it is shown that the force on unit pole at the centre of such a coil when a current I absolute units passes is

$\frac{2\pi nI}{r}$ dynes, where n is the number of turns and r the mean radius in cm. If the current be I amperes the force on unit pole will be $\frac{\pi nI}{10r}$ dynes, and on a pole of strength m, $\frac{2\pi nI}{10r} \times m$ dynes.

Referring now to Fig. 134, it will be seen that the needle is at rest (deflection θ) under the influence of two couples —a deflecting couple due to the current and a controlling couple due to the earth. Equating these couples we have—

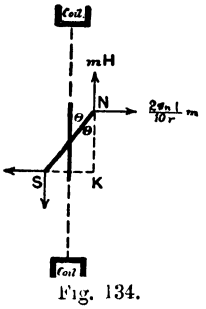


Fig. 134.

$$\frac{2\pi nI}{10r} \times m \times NK = mH \times SK;$$

$$\therefore I = 10H \cdot \frac{r}{2\pi n} \cdot \frac{SK}{NK}$$

$$= \mathbf{10H}\frac{r}{\mathbf{2\pi n}}\mathbf{\tan}\ \theta \text{ (amperes)},$$

H being the horizontal component of the earth's magnetism. The factor $\frac{r}{2\pi n}$ (or its reciprocal $\frac{2\pi n}{r}$) is called the *constant* of the galvanometer; it is the same wherever the instrument is used. The factor $H\frac{r}{2\pi n}$ is called the *reduction factor*; denoting it by K,

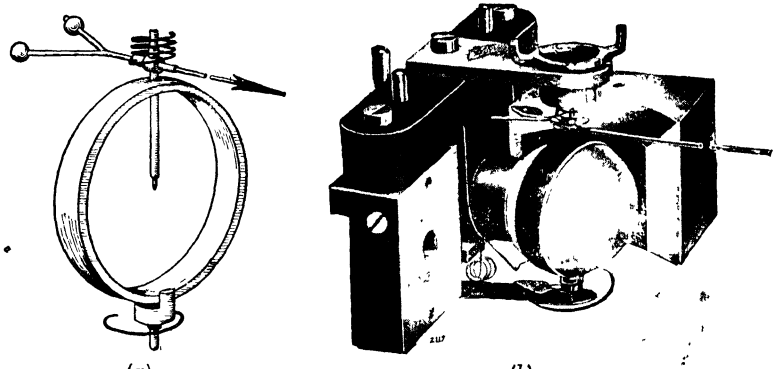
$$I = 10K \tan \theta \text{ (amperes)}.$$

The reduction factor of a tangent galvanometer may be found by passing a current for a suitable time through the galvanometer and a copper sulphate voltameter in series. The current is obtained from the relation $I = w/(0003293 \times t)$, where w = mass in grammes of copper deposited, and t = time in seconds. The reduction factor is then given by $K = 1/10 \tan \theta$, where θ = average deflection. In the same way the constant K say, for a reflecting galvanometer, in the relation $I = Kd$ may be determined. Frequently reflecting galvanometers are *calibrated*, *i.e.* a curve is plotted showing the relation between current and deflection so that the *actual current* corresponding to a given deflection is known.

138. Other Galvanometers.—A brief reference to one or two types may be given here :—

The *Cambridge Unipivot Galvanometer* consists of a circular coil which swings round a spherical core of soft iron, the latter being fixed between the pole pieces of a permanent horse-shoe magnet. The iron core is drilled down to the centre where a jewel cup bearing is fixed to support the coil, the latter being provided with a vertical spindle the bottom point of which rests in the jewel cup

(Fig. 135*a*). The controlling force is supplied by a spring attached to the top of the coil. The current is passed into the coil through this spring and out of it through a flexible connection at the bottom. The pointer and counter weights are attached to the top of the coil. Fig. 135*b* shows the essential details of the working parts, one of the pole pieces being removed so that the coil, method of supporting the cores, etc., may be distinctly seen.



(*a*) (*b*)

Fig. 135.

The *Duddell Thermo-Galvanometer* is primarily intended for the detection and measurement of very small alternating or varying currents, *e.g.* such as are met with in telephone circuits and in the receiving aerials of wireless telegraphy. A loop of silver wire L hangs by a quartz fibre in between the poles N, S of a permanent magnet (Fig. 136). The lower ends of the loop are attached to pieces of bismuth and antimony respectively, the bismuth and antimony being in contact at the bottom. Below this junction is situated the heater, which is a filament of wire or a quartz fibre platinised. When the current passes through the heater, part of the heat developed is radiated to the bismuth-antimony junction; the result is (see Chapter XV.) that another current is set up in the suspended loop in the direction bismuth to antimony through the junction of these two, and the loop is deflected. The deflection is read by means of the mirror M, and a lamp and scale; it is proportional to the square of the current when the heater is central under the junction.

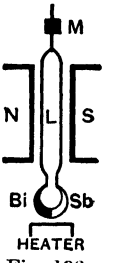


Fig. 136.

The *String Galvanometer* consists of a thin glass fibre CC (Fig. 137), with a film of silver deposited on its surface, stretched vertically between the poles of a very powerful electro-magnet. When a current passes down the string it is deflected at right angles to the magnetic field, *i.e.* "out of the paper" in Fig. 137: when the current passes up the string it is deflected "into the paper." The movement is observed by means of a microscope passing through a hole drilled in the pole pieces or by throwing an enlarged image of the string on to a screen by

an arc lamp and projection lantern. The instrument is largely used in investigations on the action of the heart: it is known that all muscular movements are accompanied by variations in electrical potential and therefore by the production and variation of current in a suitable sensitive receiving apparatus. This galvanometer is suitable for the purpose, and the curves obtained by it in the *Electro-Cardiographic Apparatus* of the Cambridge Instrument Co. give reliable information regarding the heart's action, and it illustrates one of the modern advancements in clinical pathology.

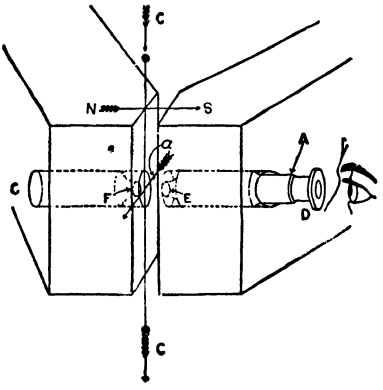


Fig. 137.

139. Shunts and Shunting. —For the purpose of decreasing the sensibility of a galvanometer and preventing damage to the instrument it is frequently necessary to pass only a portion of the current through it; this is effected by connecting the galvanometer terminals by a wire termed a "**shunt**" (Fig. 138).

If I be the total current, I_1 the portion in the galvanometer, I_2 the portion in the shunt, G the galvanometer resistance, and S the resistance of the shunt—

$$\frac{I_1}{I_2} = \frac{\frac{1}{G}}{\frac{1}{S}} = \frac{S}{G}.$$

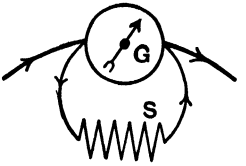


Fig. 138.

Hence— $$\frac{I_1}{I_1 + I_2} = \frac{S}{G + S};$$

that is— $$\mathbf{I_1 = \frac{S}{G + S}(I_1 + I_2) = \frac{S}{G + S} I} \quad \ldots\ldots\ldots\ldots\ldots (1)$$

and— $$\mathbf{I = \frac{G + S}{S} I_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Thus from (1) we learn that the galvanometer current is determined by multiplying the total current by the factor $\frac{S}{G + S}$; the second relation shows that the total current is obtained from the galvanometer current by multiplying the latter by the factor $\frac{G + S}{S}$. This factor is

called the "**multiplying power**" of the shunt, and is frequently denoted by the letter m; thus—

$$\frac{G+S}{S} = m \quad \ldots \quad (3)$$

$$\therefore \quad \mathbf{S = \frac{G}{m-1}} \quad \ldots \quad (4)$$

Should it be desirable to send $\frac{1}{10}$ only of the current through a galvanometer, the multiplying power of the necessary shunt must be 10; substituting this value of m in (4) we find that $S = \frac{1}{9} G$, *i.e.* the shunt must have a resistance equal to $\frac{1}{9}$ of the galvanometer resistance. Similarly $\frac{1}{100}$ of a current will flow in a galvanometer if its shunt has $\frac{1}{99}$ of its resistance, and $\frac{1}{1000}$ of the current will pass through the galvanometer if the shunt resistance be $\frac{1}{999}$ of the galvanometer resistance.

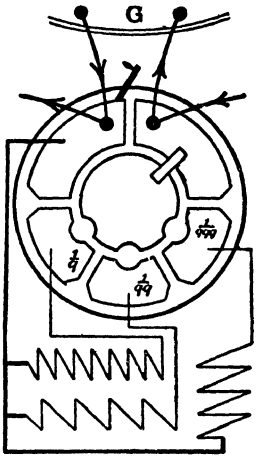


Fig. 139.

A shunt box designed to meet these requirements is shown in Fig. 139, from which it will be gathered that by inserting a plug in any one of the three holes, any desired coil may be put in parallel with the galvanometer. It is obvious that such a shunt box can only be used with the particular galvanometer for which it was constructed.

It should be observed that the insertion of a shunt *reduces the total resistance* of the circuit by an amount—

$$G - \frac{G\,S}{G+S} = G - \frac{G}{\frac{G+S}{S}} = \mathbf{G - \frac{G}{m}} \quad \ldots \quad (5)$$

This measures the additional resistance which must be put in the main circuit to keep the total resistance and total current as before. In "*Constant Total Current*" *shunts* the insertion of a shunt coil throws automatically this requisite resistance into the circuit.

140. Siemens' Electrodynamometer.—It will be obvious that the galvanometers of Arts 134-137 cannot be employed in the measurement of alternating currents; Fig. 140 illustrates the **Siemens' Electrodynamometer,** in which the direction of deflection is independent of that of the current, and which therefore can be employed with either direct or alternating currents.

The action of the instrument depends on the following laws first enunciated by Ampère:

(1) *Parallel currents flowing in the same direction attract, but if flowing in opposite directions they repel.*

(2) *Two currents whose directions meet at a point attract if both flow towards or both flow away from the point of meeting; they repel if one flows towards and the other away from this point.*

(3) *The force of attraction or repulsion between two portions of parallel currents is directly proportional to the product of the current strengths and to the lengths of the portions, and inversely proportional to the distance between them.* (Chapter XVI.)

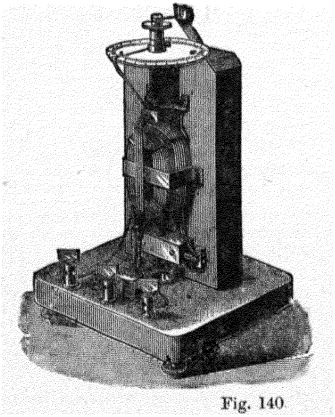
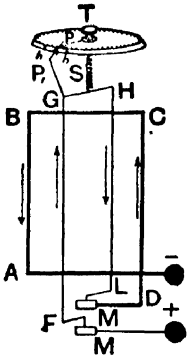


Fig. 140



The electrodynamometer consists of two coils, one of which A B C D is fixed, the other F G H L being movable and suspended by means of a silk thread with its plane perpendicular to that of the other. The spiral S has one end attached to the movable coil and the other to the torsion head T; the latter carries a pointer P, which moves over a graduated scale. M M are mercury cups arranged one above the other into which the lower ends of the moving coil dip, and through the medium of the upper cup the two coils are put in series. A pointer P_1 is also attached to the movable coil, its range of deflection being limited by the two stops, *h h*. The instrument is set up so that the plane of the moving coil is perpendicular to the meridian, in which position the earth will have no tendency to deflect it.

When a current passes, the mutual attractions and repulsions tend to set the moving coil parallel to the fixed one, and the torsion head and pointer P are turned in the opposite direction to the deflection until the pointer P_1 is again at zero. If I be the current and θ the angle through which P is turned,

$$\text{Couple between the coils} \propto I^2,$$
$$\text{Couple due to torsion} \propto \theta;$$

and since these balance each other,

$$I^2 \propto \theta, \textit{ i.e. } I \propto \sqrt{\theta},$$
$$\therefore \mathbf{I = K}\sqrt{\theta},$$

in which K is a constant which may be determined experimentally by passing a known current and observing the value of θ. In most types there are two fixed coils, one consisting of a few turns of thick wire for use with strong currents, the other of many turns of fine wire for use with weaker currents.

Reversing the current will not alter the direction of deflection, for it will be reversed in *both* coils. With alternating currents which vary in strength from instant to instant the average value of the stress between the coils measures the mean of the squares of the instantaneous values of the current; hence the effective current is proportional to the square root of the angle θ.

141. The Kelvin Current Balances.—The general principle of these will be gathered from Fig. 141. In them there are four fixed coils P Q R S placed horizontally, and between these there are two movable ones A B arranged at opposite ends of a light balance beam, the latter being hung at its centre by a number of very fine copper wires which also serve to lead the current to and from the movable coils. With current passing as indicated it is clear that the beam will tilt upwards on the right, the left-hand side falling.

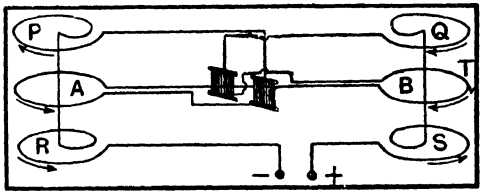


Fig. 141.

A horizontal scale is fixed to the beam, and by means of a suitable sliding weight which can be moved along the scale by means of cords and a corresponding counterpoise placed in the trough T on the right, the beam can be brought to its initial position and the current calculated from the position of the sliding weight and the known constants of the instrument. Each sliding weight has its own particular counterpoise, the latter being so constructed that it keeps the beam horizontal

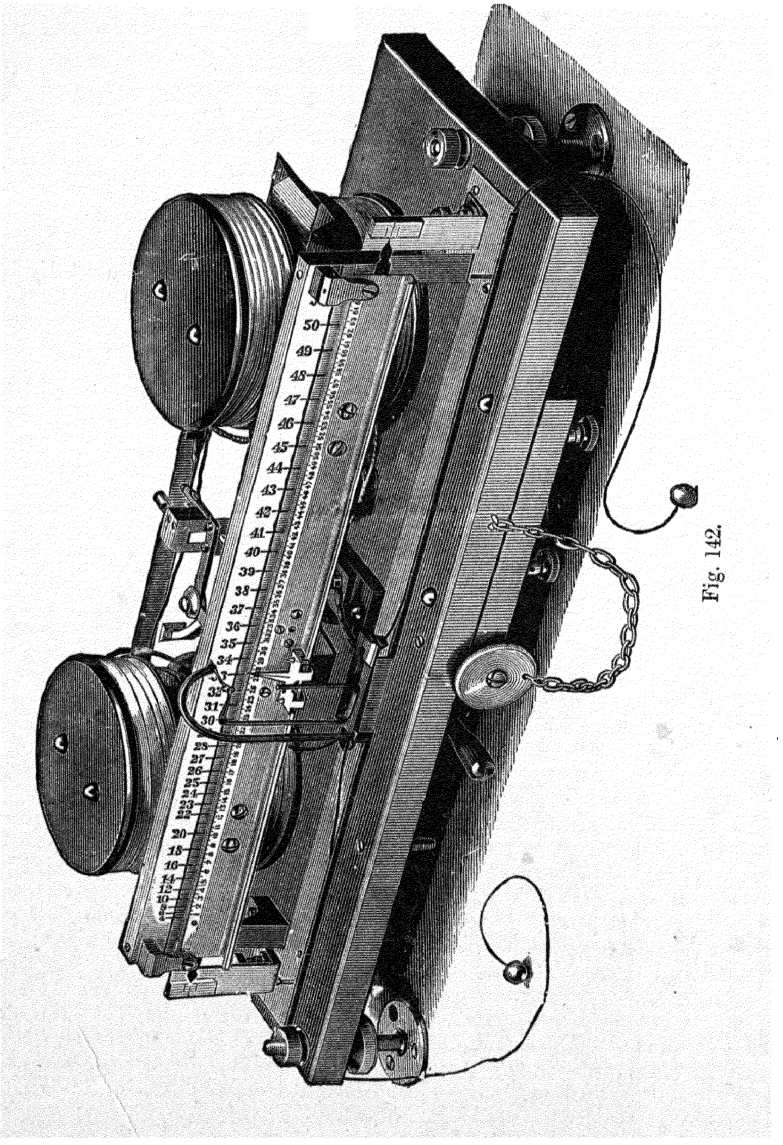


Fig. 142.

when no current is passing and the sliding weight is at zero on the scale.

Each instrument is fitted with two scales, a fixed one of large divisions and a fine scale of equal divisions which is used when great accuracy is required, each number on the fixed scale being double the square root of the corresponding number on the fine scale. Thus, if in the test the pointer is not exactly below one of the divisions on the fixed scale, the fine-scale reading is taken; if this latter be 301, then since $\sqrt{301} = 17{\cdot}35$, the true reading of the fixed scale is 34·7. Fig. 142 shows one form of the Kelvin Balance with the case removed.

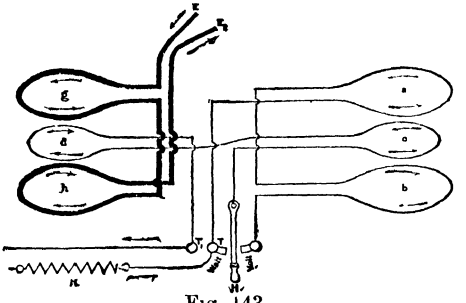


Fig. 143.

The **Composite Balance** is shown diagrammatically in Fig 143; in it the fixed coils on the left are composed of thick copper ribbon capable of carrying large currents, and are attached to terminals at the back separate from the fine-coil terminals at the front. To use the instrument as a *centi-ampere balance* the circuit is connected to the terminals in front, the switch turned to "volt," thus putting the movable and fixed fine coils in series, and with one or other of the weights provided and marked V W_1, V W_2, V W_3, balance is obtained as previously indicated and the current calculated. To use the instrument as a *voltmeter* the same method is adopted, a special resistance being included in the circuit and the volts calculated from the constants supplied.

To use the instrument as a *wattmeter* the thick coils are inserted in the main circuit, the fine-wire circuit is joined between the leads through a special resistance, the switch is turned to "watt," thus putting the *moving* fine coils only in this circuit, and one or other of the weights marked W W_1, W W_2, W W_3 are used; clearly in this case the force between the thick fixed coils and the moving system is proportional to the product of the total current and pressure between the leads, *i.e.* proportional to the watts, which latter are calculated from the constants provided.*

EXERCISES XII.

1. Sketch the details of any good form of instrument for measuring the strength of an electric current. (O.)

2. Explain the construction and action of Siemens' electrodynamometer, and show why the current is proportional to the square root of the scale reading. (O.)

* **Fig. 143** is reproduced here (by permission) from Munroe and Jamieson's *Pocket Book of Electrical Rules and Tables* (Messrs. Griffin).

3. Describe a suspended coil galvanometer. Under what conditions is such a galvanometer dead beat? (B.)

4. Distinguish between a "dead beat" and "ballistic" galvanometer. Explain how dead-beatness may be obtained in a galvanometer (1) with moving needle; (2) with moving coil. (Hons. B.)

5. Describe the construction of a potentiometer for comparing electromotive forces, and show how it can be used for measuring large electric currents. (H.)

6. Why are shunts for sensitive galvanometers usually made $\frac{1}{9}$, $\frac{1}{99}$, $\frac{1}{999}$ of the resistance of the instrument? Of what material should such coils be made? Give reasons for your answers. (P.)

7. A galvanometer of 500 ohms is shunted with a coil of 100 ohms. What external resistance must be added so that the total resistance remains unaltered? Obtain a formula which will enable the result to be determined. (H.)

8. A battery of very low resistance is connected to a galvanometer of high resistance, and a deflection of the needle is produced. A shunt is now connected across the galvanometer, but the deflection remains practically unaltered. Explain exactly why this is the case. (O.)

9. Determine the resistance of a shunt which when joined to a galvanometer of resistance 3,663 ohms will result in $\frac{1}{34}$ of the total current passing through the galvanometer. Determine also (1) the joint resistance of the galvanometer and shunt; (2) the external resistance which must be added when the shunt is applied, so that the total current may be unaltered.

10. The resistance of a shunted galvanometer is 75 ohms, that of the shunt being 100. A certain deflection of the galvanometer is obtained when the resistance in the rest of the circuit is 2,000 ohms. Find what additional resistance must be inserted that the galvanometer deflection may remain the same when the shunt is removed. What is the multiplying power of the shunt?

11. A circuit contains a battery of 1 ohm resistance, a reflecting galvanometer of 4 ohms, and other conductors of 2 ohms resistance. The galvanometer deflection is 100 divisions. What will the deflection be (if deflection is proportional to current) when the galvanometer is shunted with a 4-ohm coil? (B) What is the multiplying power of this shunt?

12. A galvanometer of 4 ohms resistance is in a circuit where the total resistance is 80 ohms. The galvanometer is then shunted with a 4-ohm coil. Find the ratio of the current in the galvanometer before and after it is shunted.

CHAPTER XIII.

PRIMARY CELLS AND CHEMICAL EFFECTS OF CURRENTS.

142. Simple, Galvanic, or Voltaic Cell.

(1) If a plate of common zinc (Zn) be placed in dilute sulphuric acid (H_2SO_4) a violent action ensues, the zinc being eaten away, zinc sulphate ($ZnSO_4$) formed, and hydrogen gas (H) evolved; the chemical action is represented by the equation—

$$Zn + H_2SO_4 = ZnSO_4 + H_2.$$

On the whole *energy is liberated during the process,* appearing as heat in the solution.

(2) If the zinc be now amalgamated (*i.e.* its surface coated with mercury) and replaced in the dilute acid, no action will be observed.

(3) If a plate of copper be put in the acid, again no action will be noticed.

(4) If the copper and amalgamated zinc be placed side by side but without contact in the acid, no action will be observed; but if they be connected outside by a wire (Fig. 144), it will be found that (*a*) the zinc is eaten away and zinc sulphate formed, (*b*) hydrogen gas appears *at the copper plate,* and (*c*) *a current of electricity flows in the circuit in the direction copper to zinc in the connecting wire, zinc to copper in the liquid.*

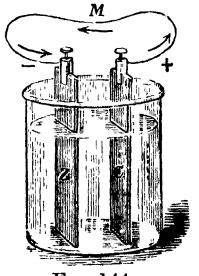


Fig. 144.

Such an arrangement is known as a simple, galvanic, or voltaic cell; the copper plate is at a higher potential than the zinc, and is called the *high-potential plate,* the portion of it outside the liquid being termed the *positive pole* of the cell; the zinc is known as the *low-potential plate,* the portion of it outside the acid being termed the *negative pole.* In the outside circuit the current *naturally* flows from the high-potential copper to the low-potential zinc; inside the cell the energy of the chemical action previously referred to is utilised in *forcing* the electricity from the low to the high potential. The chemical action is, in fact,

analogous to that of a pump lifting water from a lower to a higher level, from which position the water would naturally run down again, doing work in virtue of the energy conferred upon it. Thus in the cell the consumption of the zinc really furnishes the energy which maintains the current in the circuit.

Investigations relating to the alteration of the freezing point, boiling point, and vapour pressure of water produced by dissolving acids and salts therein have led to the conclusion that dilute solutions are dissociated into "ions," and further it is supposed that these ions are electrically charged; thus in acidulated water there are a large number of dissociated hydrogen and oxygen ions, the former positively, the latter negatively, charged.

Consider now the zinc and copper plates in the acidulated water, *but not connected.* The zinc attracts the negative oxygen ions within a very narrow film round about it, the final result being that its potential is lowered by an amount e_1, say, below the outer surface of this film. The acidulated water being a conductor has the same potential throughout. The copper also attracts the negative oxygen ions within a very narrow film, so that its potential is, say, e_2 below that of the outer surface of this film; the attraction is, however, less than in the case of the zinc, so that e_2 is *less* than e_1, and *the copper is therefore at a higher potential than the zinc.* The P.D. between the zinc and the copper is clearly $e_1 - e_2$, and this measures the E.M.F. (E) of the cell.

When the plates are connected by a wire the condition of equilibrium is upset, but there is a perpetual process towards readjustment, so that the P.D. in the film at the zinc is maintained equal to e_1, and that at the copper equal to e_2 (neglecting the influence of the hydrogen there). Now, however, there is a fall of potential in the liquid from the outer surface of the zinc film to the outer surface of the copper film equal to V, say, and a potential fall in the wire equal, say, to e; clearly (Fig. 145)—

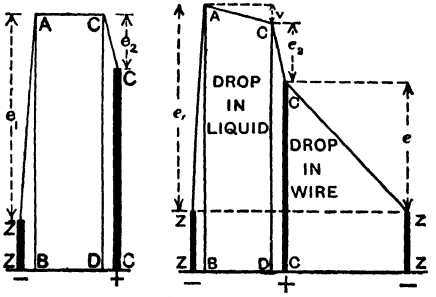


Fig. 145.

$$e_1 = V + e_2 + e,$$

$$\text{i.e. } e_1 - e_2 = e + V;$$

$$\therefore E = e + V$$

$$V = E - e$$

$$e = E - V,$$

as was stated in Art. 108. The chemical action at the zinc *forces* the electricity from the low-potential zinc to the high-potential side of the film; from thence it naturally flows round the circuit to the zinc again.

According to the theory thus set forth in brief outline, *the E.M.F. of a cell is due to the different tendencies of the two plates to combine with oxygen, and to this should be added the tendency of the ions to diffuse through the liquid owing to differences in concentration.* Fig. 145 represents (not to scale) the potential slopes referred to above, the vertical distances denoting potentials, and A B and C D the outside surfaces of the films at the zinc and copper respectively.

143. Local Action and Polarisation.—Common zinc contains many impurities, such as iron, lead, arsenic, etc.; these, together with the zinc, being in contact with the acid give rise to a number of local currents all over the surface of the plate, the result being that the latter is consumed without any advantage being gained therefrom. This, termed **"local action,"** is prevented by *amalgamating* the plate, *i.e.* coating its surface with mercury. The latter dissolves the zinc, forming a uniformly soft amalgam, which covers up the impurities; as the zinc is consumed the impurities fall to the bottom of the cell. Local currents between portions of the plate differing in hardness are also prevented by this device.

In Experiment 4, Art. 142, it was found that hydrogen bubbles *appeared at the copper plate* when the poles of the cell were connected. Their appearance there is perhaps best explained to a beginner by the hypothesis of Grotthüs, pictorially represented in Fig. 146. The upper row shows the arrangement of the molecules prior to connecting the poles, the lower row after the connection is made. Alternate separations and recombinations take place, until finally the hydrogen of the molecule on the right is liberated at the copper plate. The dissociation theory gives a more modern explanation: the positive hydrogen ions "travel" towards the copper, and the negative ions towards the zinc. This deposition of hydrogen on the copper is termed **"polarisation"**; it weakens the current and spoils the cell in two ways:

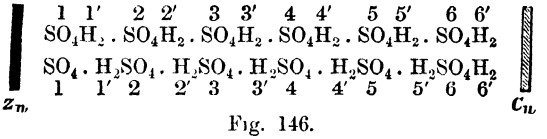


Fig. 146.

(1) The gas has a large resistance.

(2) A back E.M.F. greater than e_2 is set up, so that the main E.M.F., which (Art. 142) is equal to $e_1 - e_2$, is greatly diminished.

The modern primary cells are mainly devices for the elimination of polarisation.

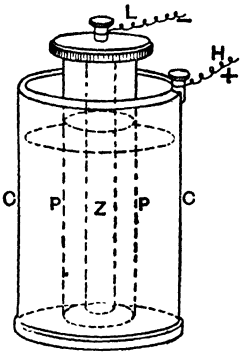


Fig. 147.

144. Daniell's Cell.—This cell (Fig. 147), introduced by Professor Daniell in 1836, consists of an outer cylindrical copper vessel, constituting the high-potential plate and positive pole of the cell. The vessel contains a strong solution of copper sulphate used as the "*depolariser*," *i.e.* the substance which prevents polarisation. In this stands an unglazed earthenware pot (a "porous" pot) containing dilute sulphuric acid and a rod of amalgamated zinc, the latter constituting the low-potential element and negative pole of the cell. To

the upper portion of the copper cylinder a perforated shelf is attached, carrying crystals of copper sulphate; these are covered by the solution, and by gradually dissolving maintain the strength of the latter. In the porous pot the action, as in the case of the simple cell, is represented by the equation—

$$Zn + H_2SO_4 = ZnSO_4 + H_2.$$

Alternate separations and recombinations now take place according to the hypothesis of Grotthüs until the porous pot is reached, where the hydrogen meets the copper sulphate with the result—

$$H_2 + CuSO_4 = H_2SO_4 + Cu.$$

Alternate separations and recombinations again take place thus—

→ Porous Pot	H_2 | SO_4Cu	SO_4Cu | SO_4Cu	Copper Pot
	H_2SO_4 | $CuSO_4$	$CuSO_4$ | Cu	

until, finally, copper is deposited on the copper cylinder; thus hydrogen does not appear at the copper plate and polarisation is prevented. Frequently zinc sulphate is employed in place of sulphuric acid; in such cases the actions are as follows:—

$$\mathbf{Zn} + ZnSO_4 = \mathbf{Zn}SO_4 + Zn: \quad Zn + CuSO_4 = ZnSO_4 + Cu.$$

The student should also consider the action of the cell from the point of view of the dissociation theory: the hydrogen and copper are positive ions which "travel" towards the copper under the influence of the electric field: the sulphions are negative.

The Daniell's cell is scentless, develops no poisonous fumes, has a constant E.M.F. and fairly constant resistance, and for these reasons has been much employed in Telegraphy where small and constant currents are required. Its E.M.F. is only low, however (1·1 volt), and its resistance somewhat high compared with certain other types of cells.

145. Grove's Cell and Bunsen's Cell.—Grove's cell, like the preceding, is a two-fluid one, and was invented by an English electrician, Grove, in 1839. The zinc is bent in the form of a U, and is placed in dilute sulphuric acid. Standing in the bend of the zinc is a flat porous pot containing strong nitric acid (HNO_3) and a plate of platinum, the acid acting as depolariser, and the platinum as the high-potential plate. The chemical reactions are—

$$Zn + H_2SO_4 = ZnSO_4 + H_2$$
$$H_2 + 2HNO_3 = 2H_2O + 2NO_2 \text{ (nitric peroxide)}.$$

Thus hydrogen does not appear at the platinum, and there is no polarisation. The nitric peroxide gas (NO_2) is very soluble in strong nitric acid; hence it does not gather upon the face of the high-potential plate.

The Grove's cell is very compact in form and is more powerful than the Daniell, its E.M.F. being about 1·9 volts and its resistance fairly low (·14 ohm for a pint cell); hence it is suitable for the deliverance of strong and fairly constant currents. The use of platinum, however, makes the cell expensive, and the red fumes of nitric peroxide are disagreeable and injurious.

Bunsen's cell, the production of a German in 1843, is a modification of Grove's, differing therefrom somewhat in shape, and in the fact that the expensive platinum is replaced by a rod of cheap carbon; the liquids employed and the chemical reactions are identical with those just mentioned.

146. Poggendorff's Bichromate and the Chromic-acid Cell.—This is a single-fluid cell, the fluid in question being made by mixing bichromate of potash (3 oz.), sulphuric acid (3 oz.), and water (1 pint), or corresponding quantities according to the size of the cell. The mixture is contained in a glass bottle, provided with a vulcanite cap to the under side of which are fixed two vertical parallel carbon plates metallically connected to one terminal. Between these carbons is a plate of zinc attached to a brass rod by means of which it may be lowered into the liquid, or withdrawn therefrom when the cell is not in use; the zinc is joined to a second terminal, and constitutes the negative pole of the cell. The chemical reactions prevent polarisation, but a waste product, chrome alum, is formed which crystallises on the plates and impairs the cell. To avoid its formation chromic acid is now invariably used instead of potassium bichromate, in which case the action is represented by the equations:—

$$3Zn + 3H_2SO_4 = 3ZnSO_4 + \mathbf{3H_2}.$$

$$\underset{\text{Hydrogen.}}{3H_2} + \underset{\text{Chromic acid.}}{2H_2CrO_4} = \underset{\text{Chromic oxide.}}{\mathbf{Cr_2O_3}} + \underset{\text{Water.}}{5H_2O}.$$

$$\underset{\text{Chromic oxide.}}{Cr_2O_3} + \underset{\text{Sulphuric acid.}}{3H_2SO_4} = \underset{\text{Chromium sulphate.}}{\mathbf{Cr_2(SO_4)_3}} + \underset{\text{Water.}}{3H_2O}.$$

The E.M.F. of this cell is about 2 volts, and its resistance low; but the current it gives soon begins to fall off. For laboratory work where a strong current is required for a short time this cell is convenient. It should be mentioned that a complete change of colour from red (chromic acid) to green (chromic sulphate) indicates that the solution has had its day and should be renewed.

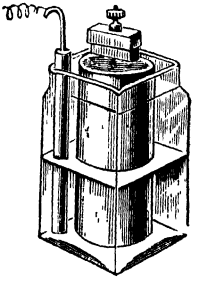


Fig. 148.

147. The Leclanché Cell.—The low-potential element of this cell (Fig. 148) consists of a rod of zinc placed in a solution of ammonium chloride (sal-ammoniac) contained in a glass vessel. A rod of carbon forms the high-potential element; this is placed in a porous pot, and is surrounded by broken carbon and black oxide (dioxide) of manganese. In many types the carbon is fitted with a cap of lead carrying the terminal; this, however, is conducive to the production of white lead, so that a better

device is to fix the terminal to the carbon direct. The chemical reactions are as follows:—

$$\underset{}{Zn} + \underset{\substack{\text{Ammonium}\\\text{Chloride.}}}{2NH_4Cl} = \underset{\substack{\text{Zinc}\\\text{Chloride}}}{ZnCl_2} + \underset{\text{Ammonia.}}{2NH_3} + H_2. \quad \ldots\ldots \quad (1)$$

$$H_2 + \underset{\substack{\text{Black}\\\text{Oxide of}\\\text{Manganese.}}}{2MnO_2} = \underset{\substack{\text{Brown}\\\text{Oxide of}\\\text{Manganese.}}}{Mn_2O_3} + H_2O \quad \ldots\ldots \quad (2)$$

The hydrogen, however, is liberated quicker than the MnO_2 can use it up, so that after a short time polarisation sets in and the current falls off. If allowed to rest for a few minutes the MnO_2 performs its work as per equation (2), and the cell in consequence regains its strength; thus Leclanché cells are admirably adapted for intermittent work, such, for example, as bell-ringing and telephone calls.

To maintain the efficiency of the cell it should be placed in a cool, dry situation, a little water added occasionally to compensate for the evaporation of the liquid, and at intervals a little sal-ammoniac. The latter has a tendency to crystallise out, and to "creep" up the sides of the glass vessel—a result which can be prevented to a great extent by coating the upper portion of the inside of the jar with paraffin wax. The deposit of crystals on the zinc is also objectionable, for the resistance of the cell is thereby increased. In recent years *siebrosal* has come into use with Leclanché cells for the elimination of this "creeping" of the salts.

The advantages of this cell are—

(1) It is not expensive, and does not emit objectionable or injurious fumes.

(2) It will last for many months with very little attention.

(3) With a little rest any accumulation of hydrogen becomes oxidised, and the cell recovers its power; it is therefore suited to intermittent work.

(4) There is practically no "action" when the cell is not in use.

The disadvantages are—

(1) It quickly polarises, and is therefore useless for continued work.

(2) The creeping of the salts, and the formation of white lead in some cells.

The E.M.F. of a Leclanché is about 1·5 volts, but its resistance may amount to several ohms when a porous pot is employed.

148. The Agglomerate Leclanché.—This type dispenses with the porous pot, as will be seen from Fig. 149. A mixture consisting of 40 parts of manganese dioxide, 55 parts of carbon, 3 parts of shellac, 2 parts of potassium sulphate, and a little sulphur is heated to a high temperature, thoroughly mixed, and by hydraulic pressure formed into a compact mass. Two blocks of this agglomerate are attached by

india-rubber bands to a rod of carbon, forming the high-potential element. A rod of zinc forms the low-potential element, and the exciting fluid is sal-ammoniac. The resistance of the cell is less than that of the preceding; the chemical action is identical with that of the ordinary type.

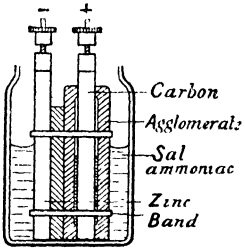


Fig. 149.

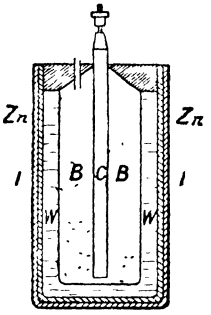


Fig. 150.

149. Dry Cells.—Nearly all the various forms of so-called dry cells are modifications of the Leclanché in principle.

The *E.C.C. type* (Fig. 150) consists of a zinc cylinder, next to which is a paste W composed of plaster of Paris, flour, zinc chloride, sal-ammoniac, and water. Adjoining this is a second paste B of carbon, manganese dioxide, zinc chloride, sal-ammoniac, and water. C is a rod of carbon, forming the high-potential element of the cell. The whole is covered with a case of millboard, is sealed with pitch, and is provided with a vent for the escape of gas. The E.M.F. is about the same as that of an ordinary Leclanché, and the internal resistance often less than ·5 ohm.

The *Hellesen Cell* consists of two cylinders of zinc, the inner one being perforated and lined with paper. Between these is a paste consisting of sal-ammoniac, plaster of Paris, and gum tragacanth. In the centre of the cell stands the carbon rod, around which is the depolariser, composed of manganese dioxide, plumbago, and sal-ammoniac. The whole is sealed with pitch, and provided with a gas vent. The internal resistance varies from ·2 to ·7 ohm, according to the size of the cell.

The main points in favour of dry cells are: (1) they are compact and portable; (2) they may be placed in any convenient position; (3) they require very little attention; (4) they are more cleanly than the ordinary type of Leclanché.

It should be noted in passing, that the E.M.F. of a cell depends on the materials employed in its construction (and the temperature), but it does *not* depend on the size of the plates or their distance apart.

150. Clark and Weston Standard Cells.—The Board of Trade pattern of the Latimer Clark Standard Cell is shown in Fig. 151. The containing vessel is a small test-tube about 2 cm. in diameter and 4 or 5 cm. deep. Mercury is placed in the bottom of the tube and forms the high-potential element of the cell. Above this is placed a mixture of mercurous sulphate and saturated zinc sulphate solution in the form of a thick paste, and above the paste saturated zinc sulphate solution is added. The low-potential element is a rod of zinc supported as indicated in the figure. Contact with the mercury is made by means of a platinum wire protected by a glass tube. The whole is sealed with marine glue coated with sodium silicate. The E.M.F. of this cell is 1·433 volts at 15° C., and falls in value with rise of temperature; its E.M.F. at $t°$ C. is—

$$E_t = 1{\cdot}433 \ \{1 - {\cdot}00077\ (t - 15)\} \text{ volts.}$$

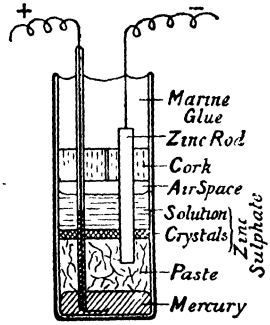


Fig. 151.

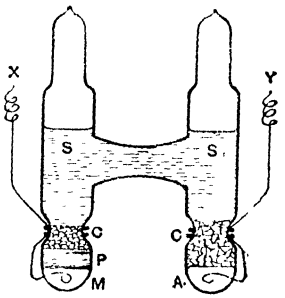


Fig. 152.

In the **Weston Cadmium Cell** (Fig. 152) mercury (M) is the positive pole, an amalgam of mercury and cadmium (A) the negative pole, cadmium sulphate (C and S) the electrolyte, and mercurous sulphate paste (P) the depolariser. (C = crystals of, and S = a saturated solution of, cadmium sulphate.) Expressed in international units the E.M.F. of the Weston is 1·0183 volts at 20° C., and at any temperature $t°$ C.:—

$$E_t = E_{20} - {\cdot}0000406(t - 20) - {\cdot}00000095(t-20)^2 + {\cdot}00000001(t-20)^3.$$

151. Current Calculation.—Battery Efficiency.

I. Single Cell.—If E denote the E.M.F. and e the terminal P.D., both in volts, r the internal resistance and R the external resistance, both in ohms, and I the current in amperes, it is clear that—

$$\mathbf{I = \frac{E}{r + R} = \frac{e}{R} = \frac{E - e}{r}} \quad \ldots\ldots\ldots\ldots \quad (1)$$

II. Series Arrangement (Fig. 153).—In this grouping the negative pole of one cell is connected to the positive of the next. If there are n cells each of E.M.F. E volts and resistance r ohms, the combined E.M.F. is nE volts and the total internal resistance nr ohms; hence, if I amperes be the current—

$$\mathbf{I} = \frac{n\mathbf{E}}{nr + \mathbf{R}} \quad \ldots\ldots\ldots\ldots\ldots \quad (2)$$

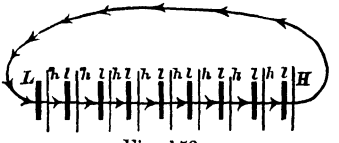


Fig. 153.

Extreme Cases.—(*a*) If the external resistance be very large compared with the internal, the latter may be neglected, and $\mathrm{I} = n\frac{\mathrm{E}}{\mathrm{R}}$; this is n times the current that one cell would give.

(*b*) If the external resistance be very small compared with the internal, the former may be neglected and $\mathrm{I} = \frac{\mathrm{E}}{r}$; this is identical with the current that a single cell would give.

Thus a series grouping suggests itself when the external resistance is very large.

III. Parallel Arrangement (Fig. 154).—In this all the low-potential plates are joined together, forming as it were one large plate, all the high-potential plates being similarly connected. Since the E.M.F. of a cell does *not* depend on the *size* of the plates, *the E.M.F. of such a group is merely that of one cell*, viz. E.; *the internal resistance is, however, 1/n that of one cell*, viz. *r/n*, and the external current I is given by—

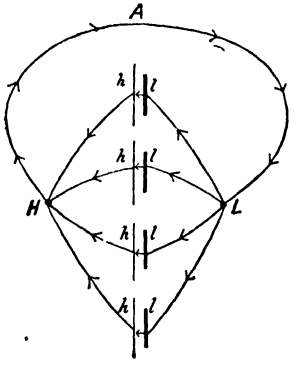


Fig. 154.

$$\mathbf{I} = \frac{\mathbf{E}}{\frac{r}{n} + \mathbf{R}} \quad \ldots\ldots\ldots\ldots\ldots \quad (3)$$

Extreme Cases.—(*a*) A very large external resistance will result in a current equal to $\frac{\mathrm{E}}{\mathrm{R}}$, *i.e.* the same as that given by one cell.

(*b*) With a very small external resistance I is equal to $n\frac{\mathrm{E}}{r}$; this is n times the current given by one cell.

Thus a parallel arrangement lends itself to a low external resistance.

IV. Mixed Arrangement (Fig. 155).—With the notation of the previous groupings—

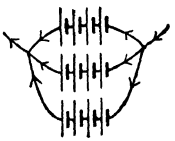


Fig. 155.

(*a*) E.M.F. of each row = 4E.
Resistance of each row = $4r$.

(*b*) E.M.F. of 3 rows in parallel = 4E.
Resistance of 3 rows in parallel = $\frac{4r}{3}$.

Hence—

$$I = \frac{4E}{\frac{4r}{3} + R}.$$

Similarly, if there are n cells in series per row and m rows in parallel, the external current is given by—

$$\mathbf{I = \frac{nE}{\frac{nr}{m} + R}} \quad \ldots\ldots \quad (4)$$

V. Battery Efficiency.—The *efficiency* of a battery is defined as *the ratio of the power in the external circuit to the total power developed.* If the external circuit does *not* contain a voltameter or motor the whole of the power outside appears as heat, and, if r be the *total* internal resistance of the battery—

$$\textbf{Efficiency} = \frac{I^2R}{I^2(R + r)} = \frac{\mathbf{R}}{\mathbf{R} + r} = \frac{\textbf{Terminal P.D.}}{\textbf{E.M.F.}}. \quad (5)$$

This is clearly nearer *unity* (*i.e.* 100 per cent.) the smaller the value of r.

If the battery has to do useful mechanical or chemical work the efficiency *of the system* refers to the ratio of the power spent *in this useful work* to the total power developed. If e be the back E.M.F. of the voltameter or motor, the power spent in the useful work is shown in Art. 156 to be eI watts, the watts supplied are E I, and—

$$\textbf{Efficiency} = \frac{eI}{EI} = \frac{e}{E}.$$

152. Two Important Problems.

(1) *Determine the arrangement of a given number of cells so that the greatest current may be produced in a given external resistance.*

By (4) Art. 151,

$$I = \frac{nE}{\frac{nr}{m} + R} = \frac{nmE}{nr + mR}.$$

Since $n \times m$ is equal to the total number of cells, the numerator nmE is constant;

hence for the maximum current *the denominator* $nr + m\mathrm{R}$ *must be as small as possible.* Now :

if $nr + m\mathrm{R}$ is a minimum,

$(nr + m\mathrm{R})^2$ will be a minimum ;

i.e. $n^2r^2 + 2nmr\mathrm{R} + m^2\mathrm{R}^2$ will be a minimum ;

i.e. $(nr - m\mathrm{R})^2 + 4nm\mathrm{R}r$ will be a minimum.

But the last factor $4nm\mathrm{R}r$ is constant; hence the expression will have a minimum value when $nr - m\mathrm{R}$ has its least value, *i.e.* when—

$$nr - m\mathrm{R} = 0,$$

or

$$\mathrm{R} = \frac{nr}{m}.$$

The expression $\frac{nr}{m}$ is equal to the total internal resistance ; **hence to secure the greatest possible current the resistance of the battery must be made as nearly as possible equal to that of the external circuit.**

Find the arrangement for maximum current in the case of 24 *cells each of resistance* 4 *ohms, the external resistance being* 6 *ohms.*

$$mn = 24 \quad \therefore m = 24/n$$

Now $\frac{nr}{m} = \mathrm{R}$ *or* $nr = m\mathrm{R}$, *i.e.* $4n = 6m$

$$\therefore n = \frac{6m}{4} = \frac{6 \times 24}{4n}, \quad i.e.\ n^2 = 36 \text{ or } n = 6.$$

The arrangement required is, therefore, 6 cells in series per row, and 4 rows in parallel.

(2) In (3) and (4) of Art. 151 *the cells and batteries in parallel were of equal E.M.F.'s,* but different methods of solution must be adopted when **cells and batteries of unequal E.M.F.'s are joined in parallel.** The method of solution in such cases will be best understood from a particular example.

A battery of 10 *volts and internal resistance* ·5 *ohm is connected in parallel with one of* 12 *volts and internal resistance* ·8 *ohm. The poles are connected by an external resistance of* 20 *ohms. Find the current in each branch.*

Let Fig. 156 represent the details. Taking the top battery and the outside circuit, and applying Kirchhoff's law (Art. 118) we get :—

$$20\mathrm{I} + \cdot 5\mathrm{I}_1 = 10 \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Taking now the bottom battery and the outside circuit we get :

$$20\mathrm{I} + 8\mathrm{I}_2 = 12 \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Dividing (1) by ·5 and (2) by ·8 :—

$$40\mathrm{I} + \mathrm{I}_1 = 20 \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

$$25\mathrm{I} + \mathrm{I}_2 = 15 \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

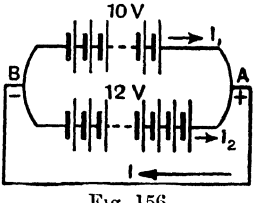


Fig. 156.

Adding (3) and (4) and remembering that $\mathrm{I}_1 + \mathrm{I}_2 = \mathrm{I}$, we get :—

$$66\mathrm{I} = 35, \quad \therefore \mathbf{I} = \frac{\mathbf{35}}{\mathbf{66}} \textbf{ ampere.}$$

Substituting this in (1)—

$$5I_1 = 10 - \frac{700}{66} = -\frac{40}{66}, \quad \therefore \mathbf{I_1 = -\frac{80}{66}\ ampere,}$$

and substituting in (2)—

$$\cdot 8I_2 = 12 - \frac{700}{66} = \frac{92}{66}, \quad \therefore \mathbf{I_2 = \frac{115}{66}\ amperes.}$$

Thus the current through the stronger battery is 115/66 amperes, of which 35/66 ampere flows through the external resistance and 80/66 amperes flow *back through the weaker battery.*

153. Fundamental Definitions in Chemistry.—The chemical effects of a current have been referred to in Chap. IX., and certain numerical relations have been established; before proceeding further the following technical terms and definitions should be noted:—

(1) An *atom* is the smallest quantity of any element which can be made to enter into or be driven from a chemical compound.

(2) A *molecule* is the smallest particle of a substance which is capable of separate existence.

(3) The *atomic weight* of an element is the weight of its atom compared with that of hydrogen, which is taken as unity. Modern chemistry refers atomic weights to that of oxygen taken as 16, in which case the atomic weight of hydrogen is 1·008. For our purpose, however, approximate figures are sufficient.

(4) The *valency* of an element is the number of atoms of hydrogen which one atom of the element is capable generally of displacing or replacing. The atoms are said to be monovalent, divalent, trivalent, tetravalent, etc., according as they are capable of displacing one, two, three, four, etc., atoms of hydrogen.

(5) The *chemical equivalent* of an element is the mass of it which is equivalent chemically to unit mass of hydrogen; *it is obtained by dividing the atomic weight by the valency.* Thus, considering our familiar friend—

$$Zn + H_2SO_4 = ZnSO_4 + H_2,$$

we see that one atom of zinc has displaced two atoms of hydrogen, and, therefore, the valency of zinc is *two.* Further, the atomic weight of zinc is 65 (approx.); hence 65 grammes of zinc are equivalent from a chemical point of view to 2 grammes of hydrogen, and therefore 32·5 grammes of zinc are chemically equivalent to 1 gramme of hydrogen, *i.e.* the chemical equivalent of zinc is 32 5 (approx.). The chemical equivalents of copper, oxygen, silver, and chlorine are 31·5, 7 94, 107·12, and 35·18 respectively.

(6) The *electro-chemical equivalent* of a substance is the amount of it liberated by unit current in unit time (see Art 107).

154. Faraday's Laws of Electrolysis.

(1) *The amount of chemical decomposition which takes place in a given time in a cell or voltameter is proportional to the total quantity of electricity which passes in that time;* thus if w denote the mass of the

particular ion liberated in time t seconds, and Q the quantity of electricity which has passed—

$$w \propto Q \propto It$$

or

$$w = zIt$$

in which I is the current and z the electro-chemical equivalent (see Art. 107).

(2) *If the same current flows through several electrolytes the masses of the ions liberated are proportional to their chemical equivalents.* Thus if the current from 20 Daniell's cells in series be passed through four voltameters in series containing respectively $CuSO_4$, H_2O, $AgNO_3$, and HCl, it will be found that while 1 gramme of hydrogen is liberated 31·5 grammes of copper, 7·94 grammes of oxygen, 107·12 grammes of silver, and 35·18 grammes of chlorine will be liberated in the voltameters, 31·5 grammes of copper will be deposited in *each* cell, and 32·5 grammes of zinc will be used up in *each* cell of the battery. If the cells be arranged in two rows in parallel, 10 cells in series per row, only half the external current traverses each cell, and therefore the chemical action per cell will be half the chemical action in the voltameters; thus while 1 gramme of hydrogen is liberated in the voltameter, $\frac{32{\cdot}5}{2}$ grammes of zinc will be used up per cell.

155. Back E.M.F. in Electrolysis.—When an electrolyte is decomposed by electricity, the separated ions possess potential energy and tend to reunite, thus setting up an opposing E.M.F. In a water voltameter, for example, the back pressure is about 1·5 volts, and clearly for continued electrolysis the battery E.M.F. must exceed this value. It is a matter of simple proof that in such cases the back E.M.F. is given by the relation—

$$E = \frac{HJz}{10^8} \text{ volts,}$$

where E = back E.M.F.,

z = E.C.E. of the ion deposited per C.G.S. electromagnetic unit quantity,

J = mechanical equivalent of heat = $4{\cdot}2 \times 10^7$ ergs,

H = heat produced when 1 grm. of the ion in question combines with the other ion.

We have previously referred to the electrolysis of copper sulphate, the electrodes being copper plates *in such a condition that they are readily acted on by the sulphion* SO_4; in such cases the chemical actions are:

$$\text{(Kathoe)} \quad CuSO_4 = Cu + SO_4 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

$$\text{(Anode)} \quad Cu + SO_4 = CuSO_4 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

Equation (2), it will be observed, is similar to that of Experiment 1, Art. 142. As in that example *energy is liberated*, and further, as in the simple cell, *a forward E.M.F. is the result.* Equation (1) is the converse of (2); it represents a condition in which *energy is absorbed*, and, like the polarisation of the high-potential plate in the simple cell, a *back E.M.F. is the result.* These E.M.F.'s

are equal, each being about 1·26 volts, and cancel each other; hence *when copper sulphate is subjected to electrolysis, the electrodes being copper plates which can readily be acted on by the* SO_4, *there is no back E.M.F. of any importance.* The only counter E.M.F. which need be mentioned in the case we have just considered is a very slight one, due to a difference in concentration in the electrolyte, the copper solution becoming slightly stronger near the anode, and weaker near the kathode as electrolysis continues.

156. Watts expended in Chemical Decomposition.

Let E = E.M.F. of battery, I = current passing,

e = Back E.M.F. of voltameter, R = resistance of voltameter,

x = Watts spent in chemical decomposition = power stored in the liberated ions.

Then—

$$I = \frac{E - e}{R} \text{ (neglecting all resistance except voltameter).}$$

Power supplied = EI watts,

Power spent in heat = I^2R watts,

Power spent in chemical work = x watts;

$$\therefore EI = I^2R + x,$$

i.e

$$x = EI - I^2R \quad . \quad . \quad (1)$$

$$= EI - I\frac{E - e}{R}R$$

$$= EI - EI + eI$$

$$= eI \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

We have, therefore, the simple result that *the watts spent in chemical decomposition are equal to the product of the back E.M.F. in volts and the current in amperes.*

EXAMPLES.

1. *A copper sulphate vat has a resistance of* ·014 *ohm and a polarisation E.M.F. of* ·3 *volt. The total current required is* 1,000 *amperes. Find the total watts supplied.*

By the above, watts supplied $= I^2R + x = I^2R + eI.$

$= (1000^2 \times \cdot014) + (\cdot3 \times 1000).$

$=$ **14,300 watts.**

2. *What is Faraday's Law regarding electrodeposition? How much caustic soda is produced per ampere hour, and how much lead, silver, and mercury (from mercurous nitrate) would be deposited per ampere hour? One coulomb evolves, say,* ·0104 *milligramme of hydrogen, and the atomic weights of sodium, lead, silver, and mercury are respectively* 23, 207, 108 *and* 200. (O)

(*a*) See Art. 154.

(*b*) Valency of sodium = 1 ∴ its chemical equivalent = 23.
,, ,, lead = 2 ∴ ,, ,, ,, = 103·5
,, ,, silver = 1 ∴ ,, ,, ,, = 108.
,, ,, mercury (ous) = 1 ∴ ,, ,, ,, = 200.

Now 1 coulomb liberates ·0104 m.g. of H, ∴ 1 ampere hour (3600 coulombs) liberates (·0104 × 3600) = 37·44 m.g. of H.

Hence by Law 2, Art. 154—

(1) Amount of sodium liberated per ampere hour
$$= \mathbf{37{\cdot}44} \times \mathbf{23} = \mathbf{861\ m.g.}$$

(2) Amount of lead liberated per ampere hour
$$= \mathbf{37{\cdot}44} \times \mathbf{103{\cdot}5} = \mathbf{3875}\ \text{,,}$$

(3) Amount of silver liberated per ampere hour
$$= \mathbf{37{\cdot}44} \times \mathbf{108} = \mathbf{4043}\ \text{,,}$$

(4) Amount of mercury liberated per ampere hour
$$= \mathbf{37{\cdot}44} \times \mathbf{200} = \mathbf{7488}\ \text{,,}$$

Caustic soda = NaHO.

Atomic weights of Na, H, and O are respectively 23, 1, and 16 (say); hence in 40 grm. of caustic soda there are 23 grm. of sodium, 1 grm. of hydrogen, and 16 grm. of oxygen.

In the problem the 861 m.g. (·861 grm.) of sodium will combine with water, forming caustic soda; clearly then—

23 grm. of sodium form 40 grm. of caustic soda;
∴ 1 grm. ,, ,, forms $\frac{40}{23}$,, ,, ,,
and ·861 grm. ,, ,, $(\frac{40}{23} \times \cdot 861) = \mathbf{1{\cdot}497\ grm.\ of\ caustic\ soda}$
$= \mathbf{1{\cdot}5\ approx.}$

EXERCISES XIII.

1. Describe the Leclanché cell and add some hints as to how to maintain it in working order. (P.)

2. Describe the construction of a Leclanché cell, and state what are its advantages and disadvantages. Under what circumstances would you employ Leclanché cells? (P.)

3. Describe three different types of primary batteries, and state what kind of work each type is most suitable for. (P.)

4. What is essential in a galvanic battery in order that an electromotive force may be set up and a steady current generated? (O.)

5. Define the terms "in parallel," "in series," "on open circuit," and give sketches in illustration. (P.)

6. Explain "local action" and "polarisation," and show how these are avoided in Daniell's cell.

7. Ten voltaic cells each of internal resistance 3 ohms and E.M.F. 2 volts are connected--(1) all in series; (2) all in parallel; (3) in two series of five each, the similar ends of each series being connected together; (4) in five series of two each. The external resistance in each case is 20 ohms; find in each case (1) the total current, (2) the current in each cell, (3) the terminal P.D.

8. A line of 2,300 ohms is required to be worked by Daniell's cells (in series), whose resistance is 10 ohms per cell. What number of cells would be required to produce 25 milliamperes? Give the formula. (O.)

9. Two dry cells each have an E.M.F. of 1·5 volts and an internal resistance of ·3 ohm. How would you couple them up so as to give the greatest current through a wire having a resistance of ·4 ohm, and what would be the value of the current? (P.)

10. If there are 20 cellseach of resistance 2 ohms, and if the external resistance be 12 ohms, what arrangement will give the strongest current? If the E.M.F. of each cell be 2 volts, determine the strongest current.

11. A battery is required to supply a power of 200 watts to an external resistance of 2·4 ohms. The E.M.F. of each cell is 2 volts and its resistance ·4 ohm. Find the least number of cells required.

12. A battery of 12 equal cells in series screwed up in a box being suspected of having some of the cells wrongly arranged is put into circuit with a galvanometer and two cells similar to the others. Currents in the ratio of 3 to 2 are obtained according as the introduced cells are arranged so as to work with or against the battery. What is the state of the battery? (B.)

13. If 10,000 kilocoulombs are passed through cells arranged in series containing solutions of Cu_2Cl_2, $Hg_2(NO_3)_2$, $CuSO_4$, H_2SO_4, and NaCl, how much copper, mercury, hydrogen, and caustic soda can be obtained? (O.)

14. In what time will 200 amperes produce 4 grm. of caustic soda from a solution of sodium chloride (NaCl)?

15. Find the H.P. that will provide 100·2 amperes at a pressure of 57·4 volts to an electric furnace. If the furnace worked for 2 hours 53 minutes, find the horse-power hours of energy supplied. (A horse-power hour is the energy supplied by a horse-power in one hour.)

16. A battery consists of 12 similar cells (equal E.M.F.'s and resistances) in series, the total internal resistance being 27 ohms. The outside resistance is 40 ohms and the terminal P.D. 6 volts. Find (1) the current, (2) the E.M.F. of each cell, (3) the "lost" volts in each cell, (4) the P.D. between the terminals of each cell.

CHAPTER XIV.

SECONDARY CELLS OR ACCUMULATORS.

157. Principle of Secondary Cells: The Planté Plate Type.—If acidulated water be decomposed by a current of electricity, the electrodes being platinum plates, and if, when the action has continued for some time, the battery be disconnected and the electrodes joined by a wire, it will be found that the voltameter gives for a short period a current in the direction anode to kathode through the connecting wire. This is a direct consequence of the facts referred to in Art. 155, and was employed by Ritter in the construction of the earliest secondary batteries.

In commercial secondary cells lead plates are employed. Consider two such plates, A and K (Fig. 157), immersed in dilute sulphuric acid (ten parts by weight of water to one of acid), and let a current of electricity be passed through the voltameter from A to K. The acidulated water will be decomposed into its constituent gases, hydrogen and oxygen, the latter appearing at the anode A, the former at the kathode K. The oxygen at the anode combines with the surface lead forming the dark brown peroxide of lead (PbO_2); the hydrogen at the kathode mostly rises to the surface, so that this plate remains in the metallic state.

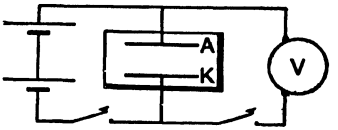


Fig. 157.

When the "*charging*" process, as it is termed, has continued for some time, let the battery be disconnected and the plates A and K joined through a voltmeter; the instrument will indicate about 2 volts at first, a current will flow through the outside circuit in the direction A to K, and, decreasing gradually, will after a time cease. On examining the plates it will be found that the peroxide has disappeared, and that both plates have lead sulphate ($PbSO_4$) formed on them, with traces of the monoxide PbO.

The charging process may now be repeated by passing a current through the voltameter in the direction A to K. The nascent oxygen at A will convert the $PbSO_4$ (and any PbO there) into PbO_2, while the nascent hydrogen at K will reduce the products there to the metallic state; thus the electrodes are again in their "formed" condition, viz. PbO_2 at A, Pb at K.

Such an arrangement is termed a **secondary cell, storage cell,** or **accumulator;** the *anode* is called the *positive plate*, and the *kathode* the *negative plate* of the cell. It should be particularly observed, how-

ever, that there is no accumulation or storing of electricity; fundamentally, it is again the transformation of electrical energy into the potential energy of separated ions, *i.e.* into "chemical potential energy"; and when the cell gives a current the energy transformation is merely reversed.

In the preceding it was mentioned that on the first "charge" the plate K remained more or less in its initial condition, and as such it does not readily combine with the sulphion SO_4 to form $PbSO_4$ at discharge. This defect is eliminated by adopting the "alternate" charge process introduced by Planté. The current is first passed through the electrolyte in the direction A to K, with the result that A is peroxidised; it is then reversed, in which step the oxygen at K forms PbO_2 there, while the hydrogen at A reduces the existing PbO_2 to porous spongy lead. This operation is several times repeated, with the final result that the last anode has a thick coating of dark brown lead peroxide, while the last kathode is coated mainly with metallic lead of a greyish colour in a porous spongy condition, which is readily acted on by the SO_4 during discharge.

This process is technically known as the "*forming*" of the plates; it is a lengthy business, which, however, can be accelerated by subjecting the plates to a preliminary treatment as follows: The plates are first annealed in steam, then placed for several hours in a hot bath of water and nitric acid, and finally allowed to dry in the air. This treatment renders the surfaces more porous and susceptible to the electro-chemical actions, and hastens the formation. By allowing periods of rest between the reversals, the charge and discharge actions become more efficient.

In modern cells Planté positives are largely used, these being *separately* formed in "forming baths" with dummy negatives, thus reducing the time and expense entailed by the alternate charge method. The effective area is increased by providing the plates with ridges and cavities.

158. Chemical Changes during Discharge and Charge.—Consider a charged storage cell of the Planté type; the anode (or positive plate) is coated, as we have seen, solely with the peroxide PbO_2, while the kathode (or negative plate) is entirely coated with spongy metallic lead.

(1) *Chemical Changes during "Discharge."*—In discharging the current is taken from the cell in the direction positive plate to negative plate in the *outside circuit;* hence, *in the cell,* hydrogen will appear at the positive plate and oxygen at the negative plate. The hydrogen at the positive combines with the peroxide there with the following results—

$$PbO_2 + H_2 = \mathbf{PbO} + H_2O \quad \ldots\ldots \quad (1)$$

$$PbO + H_2SO_4 = \mathbf{PbSO_4} + H_2O \quad \ldots\ldots \quad (2)$$

The oxygen at the negative acts on the lead according to the equations—

$$Pb + O = \mathbf{PbO} \quad \ldots\ldots \quad (3)$$

$$PbO + H_2SO_4 = \mathbf{PbSO_4} + H_2O \quad \ldots\ldots \quad (4)$$

(2) *Chemical Changes during "Charge."*—In charging, *i.e.* passing a current from the positive plate to the negative plate in the liquid, oxygen is liberated at the former and hydrogen at the latter plate. The oxygen at the positive combines with any oxides there, producing higher oxides, and particularly PbO_2; thus—

$$PbO + O = \mathbf{PbO_2} \quad \ldots\ldots \quad (5)$$

while the $PbSO_4$ is converted into PbO_2 according to the equation—

$$PbSO_4 + O + H_2O = \mathbf{PbO_2} + H_2SO_4 \quad \ldots\ldots \quad (6)$$

The hydrogen at the negative combines with the products there, reducing them to the metallic state and forming water and sulphuric acid thus—

$$PbSO_4 + H_2 = \mathbf{Pb} + H_2SO_4 \quad \ldots\ldots \quad (7)$$

$$PbO + H_2 = \mathbf{Pb} + H_2O \quad \ldots\ldots \quad (8)$$

It will be observed that the actions represented by equations (6) and (7) increase the density of the electrolyte, while those represented by (2) and (4) weaken it. In modern accumulators the specific gravity of the acid is a good test of the condition of the cells; when fully charged the specific gravity ranges from 1·205 to 1·215, and when discharged from 1 17 to 1·19.*

159. The Faure Pasted Type and its Chemical Changes.—The lengthy and expensive formation of the Planté plates was improved upon in 1880 by Camille A. Faure, who, prior to charging, coated the plates with a paste of red lead (Pb_3O_4) and sulphuric acid, the adherence of the paste to the lead being assisted by a covering of paper. The chemical changes may be thus summarised—

(1) *Preliminary Chemical Action.*—On placing the pasted plates in the acid, direct chemical action ensues according to the equation—

$$Pb_3O_4 + 2H_2SO_4 = \mathbf{PbO_2} + \mathbf{2PbSO_4} + 2H_2O \quad \ldots\ldots \quad (1)$$

Thus, *prior to charging* both plates contain the peroxide PbO_2 and the sulphate $PbSO_4$.

(2) *Chemical Changes during "Charge."*—When the charging is in progress, the oxygen at the positive plate converts the sulphate into peroxide thus—

$$PbSO_4 + O + H_2O = \mathbf{PbO_2} + H_2SO_4 \quad \ldots\ldots \quad (2)$$

* The actual figures vary with the type of cell and are given by the makers.

The hydrogen at the negative plate reduces the peroxide and the sulphate to spongy lead as follows—

$$PbSO_4 + H_2 = \mathbf{Pb} + H_2SO_4 \quad \ldots\ldots\ldots\ldots \quad (3)$$

$$PbO_2 + 2H_2 = \mathbf{Pb} + 2H_2O \quad \ldots\ldots\ldots\ldots \quad (4)$$

Thus, as before, the positive is coated with PbO_2 and the negative with spongy metallic lead. Further H_2SO_4 is formed as per equations (2) and (3); hence the density of the electrolyte increases during the charge.

(3) *Chemical Changes during "Discharge."*—These are identical with those explained in Art. 158, and are represented by equations (1) to (4) of that section. In the discharge it will be observed that the acid becomes weaker.

Frequently the preliminary treatment consists in coating the anode (positive plate) with the red lead and sulphuric acid paste, and the kathode (negative plate) with a paste made of the monoxide PbO (litharge); when immersed in the acid the preliminary chemical action at the latter plate is—

$$PbO + H_2SO_4 = \mathbf{PbSO_4} + H_2O \quad \ldots\ldots\ldots\ldots \quad (5)$$

after which the chemical changes are as explained above.

The main drawback to the Faure invention proper was the imperfect adherence of the paste to the plates; this led to the introduction of the *Sellon-Volckmar* plates, which, being constructed in the form of grids, more effectively secured the paste. Thus accumulators follow in general two specific types: (1) the Planté or naturally "formed" cell, and (2) the Faure or pasted grid cell. Owing to the heavy service which is now required of accumulators, cells are frequently of a composite character, having Planté positives and pasted negatives.

Modern cells must often be capable of being discharged *in one hour* without injury (this means, of course, heavy currents), while Central Station batteries must often give 50 per cent. in excess of the one hour discharge rate, so that pasted positives, owing to their liability to disintegration (falling of paste), are discarded in favour of Planté positives for such purposes. Pasted negatives do not suffer from the disadvantages inherent in pasted positives, and are quite satisfactory for these high rates. The type of pasted negative known as the "box" negative (Art. 163) and others somewhat similar are well adapted for these high rates of discharge.

160. General Remarks on Accumulators.—The current to be taken from an accumulator depends, amongst other things, upon the size of the plates. To obtain large currents with plates of reasonable dimensions the arrangement shown in Fig. 158 is adopted. This depicts a thirteen plate cell, the positive consisting of six positive plates connected together, and the negative consisting of seven negative plates similarly connected together. To keep the internal resistance as low as possible the individual plates are placed very near each other.

To prevent internal contact between the plates, glass tube separators or ebonite forks are suspended between them. Another form of separator introduced by the Chloride Co. is shown in Fig. 159. This consists of a thin sheet of wood (slightly larger than the plates) supported by two slotted wooden rods or dowels which in turn are suspended from the tops of the plates by india-rubber rings. The wood is specially treated so as to remove from it substances which would be deleterious to the plates, and at the same time the fibre is rendered very porous. Such separators thoroughly prevent internal contact and short circuit, lessen the attention required by the battery, and maintain the capacity. Other types of patent sheet separators are in use.

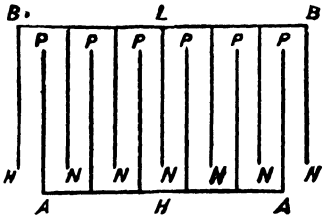


Fig. 158.

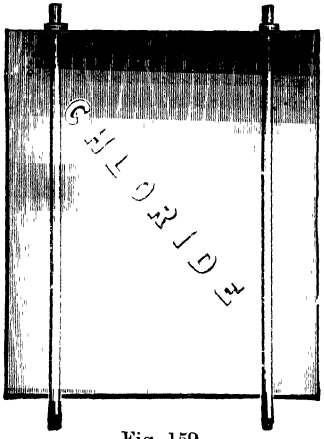


Fig. 159.

The nest of plates is contained in a vessel of glass or ebonite or in a teak box lined with thin sheet lead, the latter being employed for ship lighting, railway lighting, and allied purposes. Ample space must be left between the plates and the bottom of the containing vessel to contain any deposit which may fall from the plates. The various methods devised for this are mentioned in Art. 162, the modern practice, with cells in glass boxes, being to suspend the plates from the edges of the boxes.

Particulars of a few modern cells will be gathered from the following table :—

No of Plates.	Weight with Acid.	Dimensions.	Max. Charging Rate.	Discharge.		Box.
				Amp.	Hours.	
13	210 lb.	18″ × 12″ × 14″	72	114	2	Lead-lined
				51	7	wood.
21	282 lb.	22″ × 12″ × 14″	60	105	5	Glass.
				84	7	
				70	10	
31	1123 lb.	22″ × 15″ × 19″	300	600	3	Lead.
				240	12	
41	3000 lb.	30″ × 24″ × 36″	1200	2000	1	Lead-lined
				700	7	wood.

For the comprehension of the above table, a few preliminary remarks may be necessary at this stage. The *maximum discharge current* for an accumulator

depends on the type of cell, and on its dimensions; it is always specified by the makers, and its value should never be exceeded. Thus in one type of E.P.S. cell the maximum discharge rate is about $4\frac{1}{2}$ amperes, in another 8 amperes per positive plate; the latter value for the cell in question works out to about 1 ampere per 14 square inches of positive plate (estimating both sides).

The **capacity** of a cell depends on its size and "formation," and is measured in ampere hours; thus if a cell has a capacity of 704 ampere hours, and the greatest discharge current as stated by the makers is 64 amperes, it will be able to give this current for 11 hours. Experiment shows that, up to a 10 hour discharge rate, the capacity increases as the discharge current decreases; at lower discharge rates (more hours) than this the capacity is, for all practical purposes, constant. This fact will be gathered from the table; thus taking the 31-plate cell the capacity is 2,880 ampere hours with a discharge current of 240 amperes, but only 1,800 ampere hours when the discharge is 600 amperes.

In a general way the ampere hours given out vary from 85 to 90 per cent. of the ampere hours put in; thus if a cell becomes discharged in 5 hours with a steady output current of 18 amperes, 90 ampere hours have been given out, and 100 to 106 ampere hours will be necessary in the recharge. Taking general values for a few well-constructed accumulators, from 2 to 3 ampere hours may be "stored" per pound of lead. For traction cells the value is about 5 or 6 ampere hours per pound.

In all ordinary cases a cell should be deemed to be "discharged" when its E.M.F. falls to about 1·85 volts; with an E.M.F. below this value a cell should not be worked, otherwise the formation of a large amount of hard sulphate will render the recharge difficult and tedious, will be conducive to buckling, and will considerably shorten the life of the cell. At a one hour discharge rate the voltage *may* be allowed to drop to 1·75.

The *energy efficiency* of an accumulator is given by the expression:—

$$\text{Energy Efficiency} = \frac{\text{Watt hours given out at discharge}}{\text{Watt hours put in at charge}}$$

and is of the order 65 to 75 per cent. The *quantity efficiency* is given by:—

$$\text{Quantity Efficiency} = \frac{\text{Ampere hours given out at discharge}}{\text{Ampere hours put in at charge}}$$

and is of the order 85 to 90 per cent.: it may reach 95 per cent. if discharge immediately follows charge.

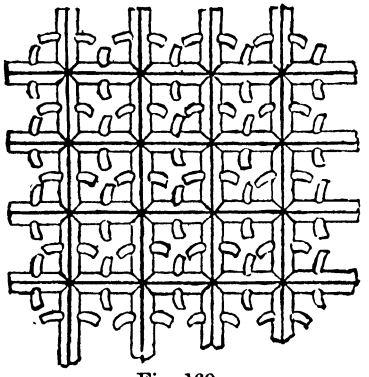


Fig. 160.

161. The E.P.S. Accumulators.—E.P.S. cells are either composite or of the Faure type; the following constructional details apply to the latter. The negative plates are grids of lead, of which there are many patterns to suit various purposes. Fig. 160 was one form used with the K or high

discharge rate cell; the apertures ($\frac{3}{16}$ in. square) had four or eight claws projecting from the sides so as to give a good hold to the paste; Fig. 161 is another pattern employed with the P cell for lighting and tramway work. The paste for the negatives is made of pure well-ground litharge and pure sulphuric acid in water, the specific gravity of the latter being 1·2 when cold. After filling, the plates are placed in racks, and allowed to dry and harden in air.

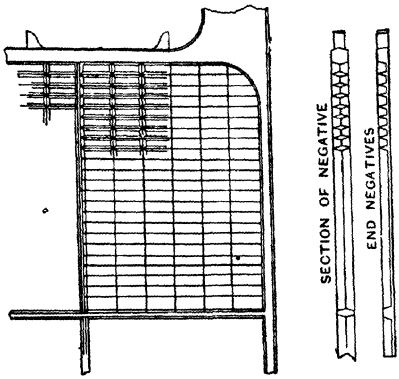


Fig. 161.

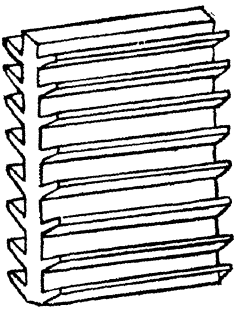


Fig. 162.

The positive plates are grooved horizontally, as shown in outline in Fig. 162, the grooves being filled with a paste of pure red lead and pure sulphuric acid in water of specific gravity 1·1 when cold. After drying, the positives and negatives are assembled as previously indicated (Fig. 158), placed in dilute sulphuric acid of specific gravity 1·2, and formed by passing a current through the electrolyte from the positive plate to the negative, when the former becomes peroxidised and the latter coated with spongy lead. A weak charging current of about 1 ampere per 36 square inches of positive plate is at first employed, and permitted to flow continuously for from four to five days; it is then doubled in strength, and passed for a period of about twelve hours; and finally a few hours at the full rate is given, thus bringing the first charge to a close. The cell is next slowly discharged till its voltage is about 1·85, and the forming is completed by a second charge more rapid than at first.

162. The Hart Accumulator.—The pastes of this cell are identical with those of the preceding, but certain features in its construction and fitting are worthy of consideration. The positive grid consists of horizontal ribs rolled and upturned at the edges, so as to form a secure holding for the active material. Vertical ribs at distances of 1 in apart connect these horizontal ones, and these, together with a solid

diaphragm in the centre, result in the production of a strong plate well calculated to withstand any buckling tendency, provided the maximum rate of discharge specified by the makers is not exceeded. The negative is a special cage-like grid with rectangular openings (staggered) at both sides, and horizontal binding ribs. Planté positives are also supplied with these cells.

It is essential to provide means whereby accumulator plates may be raised 2 to 12 inches from the bottom of the containing vessel, for metallic waste from the plates falls to the bottom, and if this provision be not made, local action, internal contact, and short circuiting would ensue. Lugs cast on one set of plates, and supporting blocks of wood placed in the bottom of the cell, were two of the means adopted.

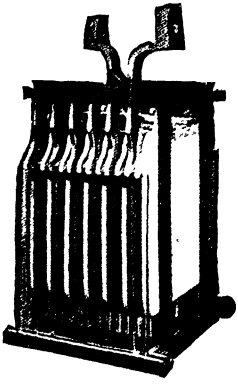


Fig. 163.

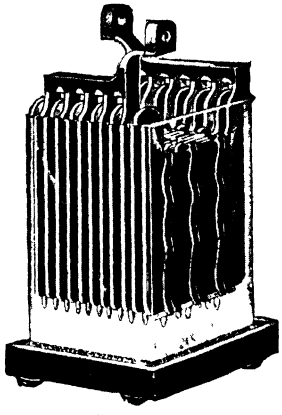


Fig. 164.

A method employed with the Hart cell was novel and effective. Referring to Fig. 163, it will be seen that a frame (teak) surrounded the sections, the top bars being removable to permit of the insertion of the plates. The bottom bars at two opposite sides of this frame were provided with grooves in which stood two vertical glass slabs, and the plates were fitted with projecting shoulders, which rested on the upper edges of the glass side slabs. The modern practice, however, is to hang the plates on the edges of the glass boxes (Fig. 164).

Each plate is separated from the next by glass tube and patent sheet separators, and the cells are connected together by a special non-corrosive terminal. The bolt of this terminal is tapered to suit a corresponding taper in the lead lugs, so that connection is obtained not only by the lug face surfaces, but also by the bolt itself, a proper fit being secured by previously drawing the terminals together by a gun-metal master bolt. The whole connection is made of an alloy not readily attacked by acid fumes, and by its use a very low resistance is obtained.

163. The Chloride Accumulator.—The positive plates of these cells are constructed as follows: A plate with circular holes is cast, the material being an alloy of lead with antimony, the latter giving the plate greater strength. Pure lead in the form of a tape, about $\frac{7}{16}$ in. in width, is passed through gimpers, which rib and sever it into definite lengths. These are rolled into coils and pressed into the circular apertures in the plate, the whole being next subjected to a pressure of over 70 tons in a hydraulic press. Plates thus constructed are fixed as anodes in a voltameter with temporary kathodes and a current passed for several days, with the result that the liberated oxygen produces on them a large amount of lead peroxide.

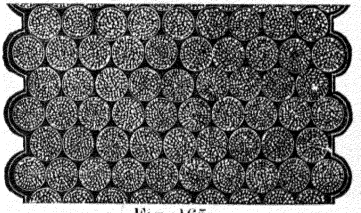


Fig. 165.

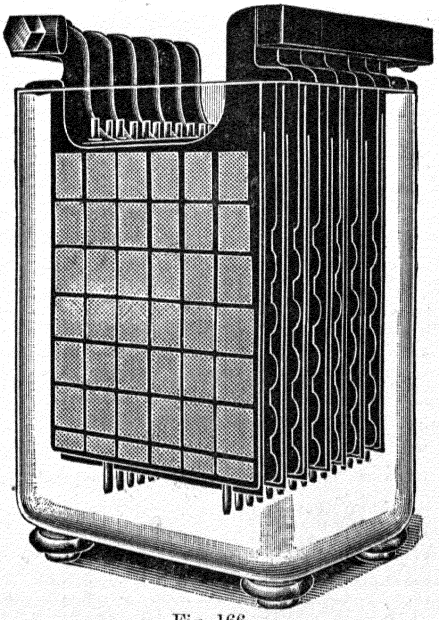


Fig. 166.

The general appearance of a portion of the positive plate is depicted in Fig. 165.

The negative plate consists of an antimonial perforated lead cage in two halves, the intervening space forming pockets which act as receptacles for the active material. In the construction of this plate, perforated sheet lead is cut to shape and laid in a mould and an antimonial lead frame is cast on to it. This forms a half plate on which a corresponding half is placed after the active material has been inserted and the two halves are then rivetted together by passing under a press specially designed for the purpose. The paste is made by a combination of raw materials which, after reduction, produces spongy lead in a highly porous form.

The above is known as the "Box" negative and is well adapted for the high discharge rates now necessary in central station work. In the new cell the plates are provided with projecting lugs by which

they hang on the edge of the containing glass vessel, and the positives and negatives are separated by thin sheets of wood (Fig. 159). The new cell is shown complete in Fig. 166.

164. D.P. Accumulator.—The accumulators have positives of the Planté type, the plates being of ribbed design with horizontal binding ribs, the construction resulting in a large superficial area, free circulation of the electrolyte throughout the plates, and a tough adherent skin when "formed." A portion of the positive plate is shown in Fig. 167.

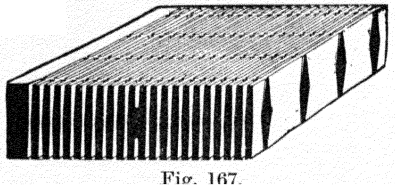


Fig. 167.

In the negative (known as the "Sponge" type) the active material is so constituted as to resemble a sponge in consistency, with a tendency to expand on charge, thus making complete effective contact with the lead grid. The whole is encased in a perforated shield which prevents the active material from falling away and at the same time allows the electrolyte free circulation and access to it. The general appearance of the negative is shown in Fig. 168, the perforated shield being shown apart on the right.

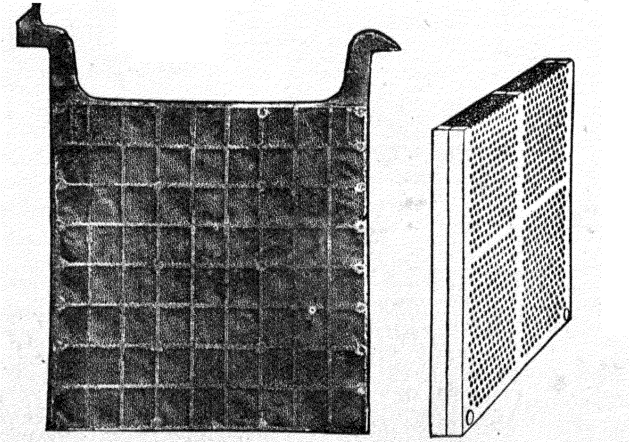


Fig. 168.

The plates hang on the edges of the containing vessel, and glass tube separators are used between them. In the older type the sections rested on teak bearers in the bottom of the containing vessel.

165. Practical Hints on the Charging, Maintenance, and Care of Accumulator Batteries.

(1) The battery room must be dry and well ventilated to permit of the escape of the evolved gases. Since acid fumes vigorously attack metal, all metal fittings and connections in the room must be protected by acid-resisting enamel.

(2) The thorough insulation of a battery is essential. Large cells should stand firmly on sawdust contained in shellac-varnished wooden trays, which in turn should rest on glass oil insulators of a type similar to that of Fig. 169, in which the lower cup carries some form of non-evaporating oil. With capacities below 600 ampere hours, the trays may be dispensed with, discs of lead or rubber being placed on the insulators to prevent slipping, and for circuits below 200 volts solid glass insulators may be used. If cells stand on wooden shelves, the latter must be coated with acid-resisting paint.

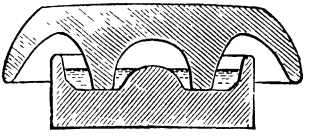


Fig. 169.

(3) In the case of a new battery the acid must not be put into the cells until all arrangements for charging are complete, otherwise the formation of lead sulphate will impair the efficiency of the cells. The acid must be of the purest, of specific gravity about 1·2,* and must reach to a height of not less than ½ in. above the plates. The dynamo employed must be either separately excited or shunt wound. If a series machine were joined to a battery whose E.M.F. exceeded the T.P.D. of the machine, the battery would discharge in such a way as to prevent the field from being built up in the direction necessary to produce a charging current; hence such machines are unsuitable for charging purposes. With a shunt machine the battery would discharge through the shunt coil in the proper direction, thus increasing the magnetisation and the E.M.F. (see Chapter XVIII.). If only a compound wound dynamo is available its series coil should be disconnected. The dynamo E.M.F. should be 2·6–2·75 volts per cell.

(4) On first charging, the dynamo must continue its supply for from 40 to 50 hours, and the battery must not be used until the acid in each cell has turned milky and gas is freely evolved from both plates, nor until the specific gravity of the acid has ceased to rise. At the conclusion of the charge the battery must be switched out of circuit before the dynamo is stopped, so as to prevent the battery from damaging the armature coils by discharging through them.

(5) The maximum rate of discharge as specified by the makers should not be exceeded, otherwise the intense chemical action and heat produced will result in unequal expansion of the plates, excessive formation of hard sulphate, loosening of the paste, and bending or buckling of the plates—the last named producing internal short circuits, and eventually spoiling the cells. For like reasons accumulators should never be short circuited outside, for their internal resistance is so low (of the order ·001 ohm, depending on the number and size of plates and their distance apart) that heavy discharge currents will flow, and sulphating, disintegration, and buckling will follow. Normally, cells should not be worked

facing page 332 *Acme Photo*

HARRY EMERSON FOSDICK

Russell H. Conwell, who founded Temple University with funds secured from public speaking, declared that if you want to be an orator as a man you must speak your piece as a boy. Harry Emerson Fosdick followed this formula. At Colgate University he was in the thick of debating and oratory. He praises his public-speaking experience at his alma mater: "I never can be sufficiently grateful for the fact that in my undergraduate days public speaking was regarded as one of the most important enterprises on the campus and we were rigorously disciplined and drilled in it."

after the E.M.F. has fallen to 1·85 volts and the specific gravity to about 1·18,* or the plates will become excessively sulphated. A drop to 1·75 volts may be permitted when the discharge rate is at one hour.

(6) Accumulators should be kept well charged; they should on no account be permitted to stand for any length of time in an uncharged condition, or the amount of hard sulphate will become excessive. When not in use short chargings should be given occasionally, say weekly or fortnightly, till the plates gas freely.

(7) Any evaporation of the electrolyte must be compensated for by the addition of pure water. Glass spray arresters should be placed over the plates; these will retard evaporation, and during charge will collect the drops of acid, permitting them to fall back into the cell. Preferably water should be distilled.

(8) If a cell in a battery does not gas freely at the end of the charge, or has a low voltage or specific gravity, it should be disconnected during the discharge, and then joined up for the next charge Another method is to charge this cell alone by a small *booster dynamo* (milking booster).

(9) In private installations it is the custom with many contractors to instal a battery of such a size that it is capable of running half the total lights for 12 hours, this being the maximum period (over the week-end) during which the battery will probably be worked without recharge. Should the battery installed be too small to meet the demand, it is not advisable to put a new battery *in parallel* with it, as has, on occasions, been done, for the total load will be very unequally distributed between them. A better plan is so to arrange that the load is divided between them in proportion to their capacities and to recharge them separately.

(10) Records should be kept of the input and output of batteries, and of the voltage and specific gravity of the cells.

(11) NOTE.—We have referred to the injurious formation of hard sulphate when a battery is over-discharged, or allowed to stand in an uncharged condition. This must not be confused with the ordinary sulphate formed during normal discharge.

We have seen that the positive pole of the dynamo must be joined to the positive pole of the battery in charging. If any doubt exists as to which is the positive of the machine, it may be found by using an electrolytic cell or by *pole-finding paper*, the latter being paper chemically treated, which changes colour at one pole when moistened and placed between the leads.

166. Automatic Switch.—Should a breakdown occur in the dynamo employed in charging a battery of accumulators, the latter may injure itself and the armature coils by discharging heavily through them. Fig. 170 diagrammatically represents an automatic switch designed to eliminate this.

A B is a rod of iron capable of moving about an axis at C; it carries two projecting arms K, K, and a coil of wire G, as indicated. D E is

* These values vary with the type, but are stated by the makers.

a fixed horseshoe-shaped piece of iron, which is converted into a magnet when a current flows in the coil F. The arrangements of F and G are such that they develop opposite poles on the left of the figure. When the E.M.F. of the dynamo has risen sufficiently high to charge the accumulators, the current in F will have magnetised D E to such an extent that the left-hand side of A B will be raised by attraction, thus bringing K down into the mercury and closing the accumulator circuit; and, further, the current now flowing in G will so magnetise the end A as to assist in keeping this circuit closed. Should the dynamo E.M.F. fall below the requisite value, the currents will be less, the attraction of D E will be overcome by the pull of the spring S, and the end A will fall, thus raising K and breaking the accumulator circuit. The lever is adjusted to act at the right moment by means of the weight W and spring S.

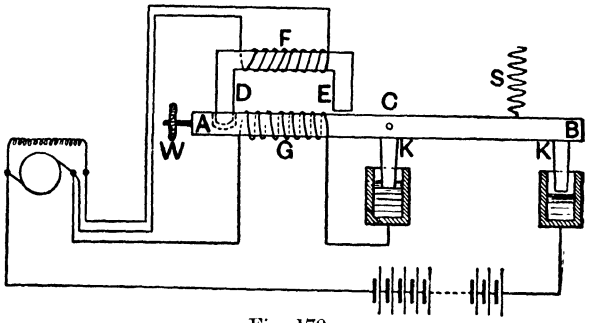


Fig. 170.

167. The Edison Accumulator.—The positives are perforated steel tubes, nickel plated, and filled with alternate layers of nickel hydroxide and metallic nickel in thin flakes: the tubes are clamped into contact with a steel frame nickel plated. The negatives consist of nickel plated steel frames provided with rectangular holes in each of which is a perforated nickelled steel pocket carrying the active material which is powdered iron oxide and graphite. The whole is immersed in a 21 per cent. solution of caustic potash with a little lithium hydrate, hard rubber separators being placed between the plates to prevent internal contact. The containing vessel is of nickel-plated sheet steel.

The energy efficiency is about 60 per cent., and the quantity efficiency about 80 per cent. The E.M.F. is about 1·33 volts: this drawback from a traction point of view is, perhaps, compensated for by the increased strength and lesser weight of the plates. For electromobiles, train lighting, etc., the cell is better than the lead cell, but for stationary central station work, etc., the lead cell has the advantage. Fig. 171 depicts the cell partly " cut away " to show the construction.

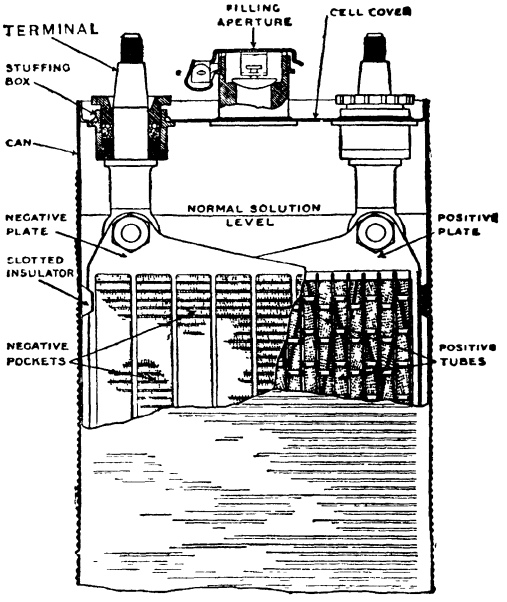


Fig 171.

168. Experimental Investigations.—Fig. 172 shows for a Chloride cell e variation of the E.M.F. during "charge." Commencing at the value indicated, the curve rises rapidly at first and then more gradually, showing that the E.M.F. remains between 2·1 and 2·2 volts for some time. Once again the curve shows a rapid rise, which is continued until 2·5-2·75 volts are reached, at which stage gas is violently evolved. The

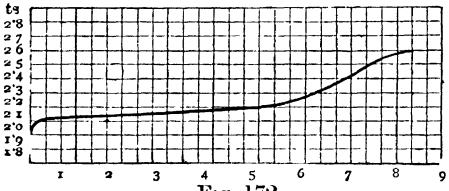


Fig. 172.

e in E.M.F. towards the end of the arge has been attributed to the increase the density of the acid in the pores of e active materials, and to the presence the separated ions, hydrogen and ygen.

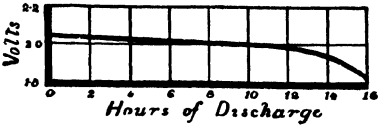


Fig. 173.

If a cell be permitted to commence its charge immediately the charging is included the E.M.F. falls rapidly to about 2·3 volts, and then drops gradually

to about 2·1 volts. If, however, an interval of rest intervenes the starting E.M.F. will be a little over 2 volts, and the discharge curve will be as indicated in Fig. 173. The fall in volts during the period of rest has been attributed to the creeping of the strong acid referred to above, out of the pores of the plates.

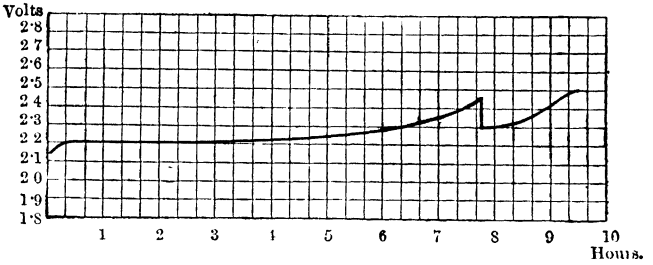


Fig. 174.

The usual method of recharging a battery is to maintain the charging current at approximately constant value, to effect which the voltage per cell is gradually increased as the charging proceeds. This is clearly indicated in Fig. 172, which shows the rise of voltage on recharge at the normal rate. If the cell is recharged at a slower rate (for an equal time), the final voltage will be lower; this is shown in Fig. 174, where the rate is reduced after 7¾ hours. Under certain circumstances, such as recharging the battery in an electric vehicle from the public supply mains, the method of charging at constant potential instead of at constant current may be used, although it is somewhat unusual, as, at the commencement of the charge, the current may be several times the value of that employed at constant current.

When the cell is being discharged, the drop in voltage depends on the rate at which the discharge is taking place, as shown in Fig. 175. It will be noticed that with the higher rates of discharge, not only is the fall in voltage more rapid than

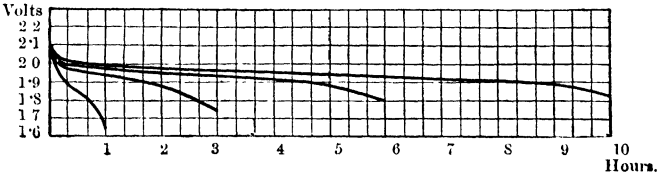


Fig. 175.

at the lower rates, but it can be safely continued to a lower potential than is advisable at the rates occupying a longer period of time. When a cell is discharged at a high rate in proportion to its capacity, although its voltage will drop rapidly, directly the discharge is ended the voltage on open circuit immediately rises, and in a short time will probably reach the same value as before the high rate commenced.

We have referred to the marked decrease in the capacity of an accumulator as the discharge rate increased (Art. 160). Fig. 176 gives a series of discharge

curves obtained by Mr. Hibbert in his tests on the Edison accumulator, from which it will be gathered that though a fall in capacity is shown with increased discharge rate, it is small compared with certain other types of cells. Thus a cell with a discharge rate of 30 amperes gave about 80 per cent. of its normal ampere-hours when the discharge was 200 amperes.

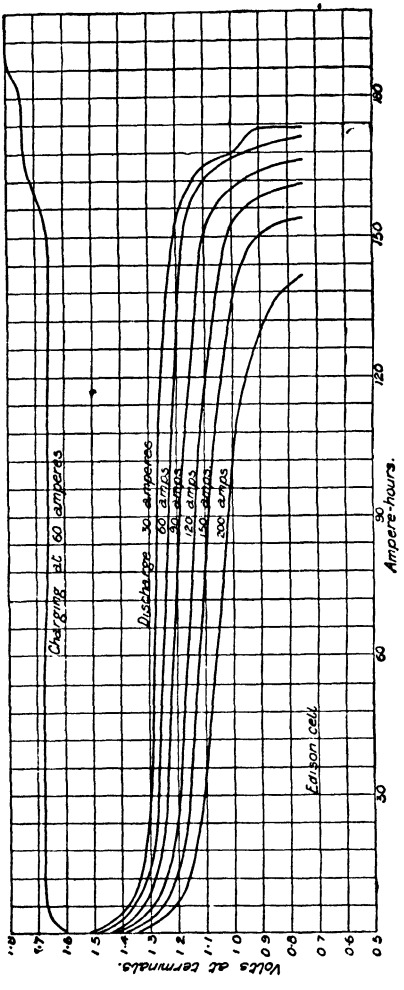


Fig. 176.

Examination of the positive plate at various stages of the discharge and when the discharge is complete has revealed the fact that below the sulphated surface there still remains a supply of peroxide which plays no part in the chemical changes. This is due to the fact that the sulphate formed acts as a shield to the material below and prevents the electrolyte from reaching it.

From a series of recent experiments by D. Tommasi it appears that the rate of formation of an accumulator is affected by *light*, negative plates being formed more quickly when exposed to light, and positive plates more quickly when shielded from light. Again, heating a cell during discharge has been shown to increase its capacity; thus with a particular cell manufactured by the Hagen Company the capacity increase was 3 per cent. per degree between the temperatures 14° C. and 45° C. With the Edison accumulator the increase is smaller, perhaps of the order ·26 per cent. per degree Centigrade.

169. Advantages and Disadvantages of Accumulators.—The very many practical applications of accumulators in lighting, traction, etc., the important part they play in assisting and relieving the generators, and their employment as pressure regulators, are points too lengthy to be discussed fully here. Briefly, their advantages and disadvantages as compared with primary cells may be stated to be as follows:—

A. ADVANTAGES.

(1) They have a low resistance and high E.M.F., and can therefore supply large currents.

(2) When "run down" they can be recharged and brought back to their initial condition.

(3) They are of service in "heavy" electrical engineering for lighting, traction, etc., where primary cells are absolutely useless.

(4) In practical telegraphy where a large number of cells are employed they are really cheaper and work more satisfactorily than primary cells.

B. DISADVANTAGES.

(1) Their initial cost is somewhat high.

(2) To maintain them in good order careful and constant attention is necessary.

(3) Their efficiency is only low.

(4) Unless care be exercised they are subject to the following diseases: sulphating, buckling, disintegration, and short circuiting.

(5) Their weight renders them not very portable for laboratory purposes.

In recent years the employment of accumulators in telegraph and telephone working, private lighting sets, train lighting, motor-car ignition, X-ray working, etc., has been much extended, but perhaps the most important application is in modern central station practice.

In such practice it happens that the demand for current during a comparatively short period out of the 24 hours (the "peak" period) reaches a point when all the machinery in the station would be necessary to meet the demand, thus leaving no "spares" in case of breakdown and perhaps uneconomically loading the last unit put into service to meet the peak load. By installing a battery of suitable size, however, the generators can be assisted during the peak period, and a larger current be supplied to the mains than could be provided by the dynamos alone. At certain other periods the load may fall below a point at which it would be economical to run an engine, for dynamos and steam engines running at small load have a low efficiency and the coal consumption per useful B.O.T. unit supplied may increase four-fold or even more when machines are only slightly loaded. In such cases the supply could be taken from the battery, thus enabling the machines to be shut down.

The constant aim of the central station engineer is to improve the "LOAD FACTOR" (*i.e.* the ratio of the actual demand for a given time to the maximum demand maintained for the same period) by endeavouring to arrange that the running machinery shall be operating as near as may be at full load. It will be evident that the load factor will be improved by arranging the machinery to charge the battery during periods of light load, the battery then assisting during the peak, and being again charged when the peak load goes off.

In cases where the load varies considerably from minute to minute, as on a tramway system where the starting of the cars and the gradients cause sudden peaks in the load, batteries are operated in series with an automatic reversible booster which, by discharging the batteries when the peaks come on, and charging them when the peaks go off, relieves the generating plant of all fluctuations

and enables it to run at constant load; this results in economy of coal and necessitates less generating plant to handle a given load. A battery really acts like a BUFFER between the generators and the network, smoothing matters by taking up all irregularities and sudden fluctuations. Charging boosters are described in Chapter XIX.

The demand for current in a district some distance from the station may reach a point when the existing cables cannot carry it satisfactorily. In such cases a battery may be installed in a sub-station near the district. This will often be found to be cheaper than laying down extra cables from the station, and being charged and discharged just as if it were in the station itself, the load factor will be improved as indicated above.

The employment of accumulators as pressure regulators on multiple wire systems is indicated in Chapter XXII.

NOTE.—It is advisable to have an hydrometer floating in a cell with a distinct mark on the level indicating the correct charged specific gravity, and this should be repeatedly transferred from cell to cell. Further the cells should often be tested for correct voltage individually by means of a small cell-testing voltmeter (0 – 3 volts) with two flexible spear connections. In the *Cadmium test*, a cadmium rod enclosed in a perforated rubber tube is placed in the electrolyte and the voltmeter is joined to the cadmium and the negative plate. The cell is fully charged when the cadmium is ·15 to ·2 volt *negative* to the plate, and fully discharged when the cadmium is ·25 to ·27 volt *positive* to the plate. This is a good test when, as sometimes occurs, the T.P.D. is not a sufficient guide to the exact state of the plates.

EXERCISES XIV.

1. Give a short historical account of the development of the storage battery and describe carefully some modern form. (B.)

2. Give a general account of the chemical changes which occur in a storage cell during charge and discharge. (B.)

3. What is an accumulator? What is its normal E.M.F.? How would you determine whether it was fully charged or not? (O.)

4. How is an accumulator made? What are the various precautions that should be adopted in the use of accumulators and what happens if these precautions are neglected?

5. Sketch and describe some good form of accumulator and explain how you would ascertain whether it was charged or discharged. About how much current may be taken from an accumulator per square foot of positive plate and what occurs if the accumulator be discharged at a much higher rate? (P.)

6. In starting to charge a set of accumulators by means of an engine and dynamo, what precautions would you take after starting the engine before finally closing the connection between the dynamo and accumulators? (P.)

7. Describe and draw a switch which automatically connects a dynamo with a battery of accumulators when the E.M.F. of the dynamo is high enough to charge the cells, and automatically disconnects them before the E.M.F. falls too low for the accumulators to begin to discharge back through the dynamo. (O.)

8. A dynamo has to charge 50 accumulators in series. What E.M.F. ought the dynamo to have? When the battery has been discharged, what will be the P.D. at its terminals?

9. A battery of accumulators was charged for a period of 6 hours by a steady current of 40 amperes, the mean P.D. employed being 126 volts. The battery was then made to supply a constant current of 42·4 amperes, and in 5 hours it became discharged. If the mean E.M.F. during discharge was 100 volts, what was the efficiency of this battery?

10. A 60 cell battery is to be charged at 40 amperes from 230 volt mains. The charging E.M.F. per cell at the beginning is 2·1 volts, and 2·5 volts at the end. A rheostat is in circuit between battery and mains. Find the value of this resistance (1) at the beginning, (2) at the end of the charge.

11. Describe with sketches one important type of accumulator (lead), and explain the chemical changes during "charge," "discharge," and "rest." In what way, if any, are the E.M.F., life, internal resistance, capacity and efficiency affected by the rate of charge and discharge, and by the specific gravity of the electrolyte? (Univ.)

12. Describe the main chemical changes that occur in a lead accumulator cell during charge and discharge respectively.

A battery has a capacity of 800 ampere hours when discharged at the 10 hour rate, 560 ampere hours when discharged at the 3 hour rate. What will be the capacity at the 4 hour rate? (Univ.)

CHAPTER XV.

HEATING EFFECTS, LIGHTING, AND LAMPS.

170. The Laws of Heating Effects of Currents.—The two heat units chiefly employed in electrical engineering are the "**calorie**" and the "**joule.**"

A calorie is the quantity of heat required to raise the temperature of one gramme of pure water from $9\frac{1}{2}°$ C. to $10\frac{1}{2}°$ C.

1 calorie = 4·2 joules.

A joule is the amount of heat produced in one second in a resistance of one ohm by a current of one ampere.

1 joule = ·24 calorie (approx.).

The foregoing definition of the calorie is in conformity with Rowland's work on the mechanical equivalent of heat, *i.e.* the number of units of energy which is equal to one calorie; practically, however, the unit may be taken as *the heat necessary to raise one gramme of water one degree Centigrade in temperature.* Formerly the definition specified 0° C. to 1° C., which leads to the relation that 1 calorie is equal to 4·223 joules.

We have seen that whenever a potential difference exists between two points in a circuit an *energy transformation* occurs between them. In the case of a battery, the poles of which are joined by an ordinary conductor, the whole of the energy supplied by the former is transformed into heat, partly in the battery and partly in the conductor. Should the external circuit contain a voltameter or motor (and therefore a back E.M.F.), a certain amount of the energy is utilised in chemical or mechanical work, the balance again appearing, however, as heat in the circuit. The transformation of electrical energy into heat always takes place when a current flows in a conductor; the laws relating to the development may be thus established—

Consider a current-carrying wire; let E denote the P.D. and I the current, both in electromagnetic units, and let t denote the time in seconds during which the current flows; then—

E = energy (in ergs) appearing as heat when unit electromagnetic current flows for one second.

$\therefore$ EI = energy (in ergs) appearing as heat when I electromagnetic units flow for one second.

And EIt = energy (in ergs) appearing as heat when I electromagnetic units flow for t seconds.

Now the mechanical equivalent of heat is $(4{\cdot}2 \times 10^7)$ ergs per calorie; hence if H be the heat in calories in the case above, $(4{\cdot}2 \times 10^7)$H ergs will represent the total energy appearing as heat in the conductor. Clearly, then,

$$(4{\cdot}2 \times 10^7)\mathrm{H} = EIt;$$

$$\therefore \mathrm{H} = {\cdot}24\frac{EIt}{10^7}.$$

If the P.D. be E volts and the current I amperes, then, since E volts are equal to $(E \times 10^8)$ electromagnetic units and I amperes are equal to $(I \times \frac{1}{10})$ electromagnetic units, the expression above becomes—

$$\mathrm{H} = {\cdot}24\frac{E \times 10^8 \times I \times \frac{1}{10} \times t}{10^7} = {\cdot}24EIt.$$

Hence when a current of I amperes flows for t seconds between two points of a conductor where the P.D. is E volts, the heat produced, in calories, is given by the expressions—

$$\textbf{Heat in calories} = {\cdot}24\,EIt = {\cdot}24I^2Rt = {\cdot}24\frac{E^2}{R}t,$$

R being the resistance of the conductor in ohms. Again, since one joule is equal to ·24 calorie, the heat produced in joules is given by the expressions—

$$\textbf{Heat in joules} = EIt = I^2Rt = \frac{E^2}{R}t.$$

Any of the above may be taken as the mathematical statement of Joule's Law relating to the heating effects of a current; it is customary to take the second, which embodies the facts that—

(1) The heat produced is proportional to the square of the current strength.

(2) The heat produced is proportional to the resistance.

(3) The heat produced is proportional to the time the current flows.

Each of these statements has been repeatedly verified experimentally.

171. Rise in Temperature of Conductors.—Joule's Law refers to the *heat produced* in a conductor, and though the *rise in temperature* certainly depends on the amount of heat developed, sundry other factors exert an influence, thus rendering the treatment of temperature elevation by no means of a straightforward character. It is obvious that were equal amounts of heat communicated to two wires A and B of the same material, the former being short and thin, the latter long and thick, the rise in temperature of A would be greater than that of B, since the quantity of matter to be raised in temperature is smaller in A than in B. Again, each wire would be losing heat by radiation from its surface; experiment shows that, other things being the same, the greater the surface area the greater is this loss, and, further, the radiation is affected by the nature of the surrounding gas and its pressure, and by the character of the surface; black bodies, for example, emitting radiation more freely than white or transparent ones.

Yet another factor, the capacity for heat of the material (*i.e.* the heat required to raise its temperature by unity) exerts an influence; thus, if the same current flows through equal pieces of platinum and copper, the heat developed in the former is about seven times that in the latter, since the specific resistance of platinum is about seven times that of copper (annealed in both cases), but the temperature elevation of the platinum is *more than* seven times that of the other, for its capacity for heat is only about three-fifths that of an equal volume of copper.

Consider now a wire through which a current is flowing. The production of heat is accompanied by a rise in temperature until finally a steady condition is reached when *the heat lost per second (by radiation, conduction, and convection) is exactly equal to the heat gained per second.* By Newton's Law of Cooling, if T° C. be the elevation of temperature of the wire above the enclosure, the heat lost per second is proportional to T°, and experiment proves it to be proportional also to the surface area of the wire. If d cm. be the diameter of the wire its circumference is πd cm., and if l cm. be its length, its surface area is πdl sq. cm.; hence—

$$\text{Heat lost per second} \propto \pi dl\text{T}$$
$$\text{,, ,, ,,} = a\pi dl\text{T calories,}$$

where a denotes the "emissivity" of the material, and may be defined as the heat (in calories) radiated in one second from unit area when the temperature difference between the body and the enclosure is 1° C.

Again, if I be the current in amperes and R the resistance of the wire in ohms,

$$\text{Heat gained per second} = \cdot 24\text{I}^2\text{R calories.}$$

Hence— $$a\pi dl\text{T} = \cdot 24\text{I}^2\text{R,}$$

i.e. $$a\pi dl\text{T} = \cdot 24\text{I}^2\rho \frac{l}{\cdot 7854 d^2},$$

where ρ is the specific resistance of the material;

$$\therefore \text{T} = \cdot 097 \frac{\rho}{a} \frac{\text{I}^2}{d^3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Thus—

(1) With a given current the elevation of the temperature does not depend on the length of the wire.

(2) For the same current the elevation of temperature is directly proportional to the specific resistance and inversely proportional to the cube of the diameter (hence the high temperature of very thin wires).

(3) The elevation of temperature is proportional to the square of the current strength.

From (1) by transposing—

$$\text{I}^2 = \frac{\text{T}a d^3}{\cdot 097\rho}; \quad \therefore \text{I} = \sqrt{\frac{\text{T}a}{\cdot 097\rho}} \cdot d^{\frac{3}{2}},$$

or $$\mathbf{I} = \mathbf{A}d^{\frac{3}{2}}, \text{ where A} = \sqrt{\frac{\text{T}a}{\cdot 097\rho}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

To be exact, expressions (1) and (2) are only applicable to *bare* conductors.

Knowing, then, for such a conductor, the diameter d, the *maximum permissible temperature* (and therefore T), and the other constants included in A, the *maximum current allowable* may be calculated from the above relation.

In practice, current at the rate of 1,000 *amperes per square inch* of cross-section of the conductor is customary. For small conductors, however, this may with safety be exceeded, but for large conductors (and therefore large currents) it is rather excessive; hence it is that the Phœnix Fire Office demands a less current density than 1,000 amperes per square inch if the total current exceeds 100 amperes. In fact, the absurdity of adhering to the above rule has long been recognised, and the new "**maximum current rule**" of the I.E.E. involves current densities so chosen as to permit of a *temperature rise* of 20° F. (11·1° C.) in the case of rubber insulated conductors. (50° F. or 27·7° C. for paper or fibre insulation.) This permits of larger current densities for small conductors, but means smaller current densities for large conductors; thus, for $\frac{3}{20}$ cable the rule gives a current density of 4,000 amperes per sq. inch, but for $\frac{37}{14}$ only about 930 amperes per sq. inch (see Appendix).

172. Particulars of Fuse Wires.—In practice it is customary to protect circuits and apparatus from injury and premises from the possibility of fire should an excessive current flow by introducing "fuse wires" of suitable material and dimensions, which melt and break the circuit when the current exceeds a predetermined value. The exact current at which a fuse should be calculated to blow is to a great extent settled by the views of the engineer in charge; in a general way from 75 to 100* per cent. overload may be allowed. The more common materials employed as fuses are tin, lead, alloy of tin and lead, copper and aluminium, and the current required to produce fusion may be determined from the relation $I = Ad^{\frac{3}{2}}$ of Art. 171 if the constant A and diameter d be known in any case; or, knowing A and the current I at which the fuse must give way, the diameter d of the required fuse may be ascertained. Some values for A are:—tin 12·8, lead 10·8, copper 80 (see Appendix).

Experiments have recently been carried out by Messrs. Schwartz and James, one object being to determine whether the law $I = Ad^{\frac{3}{2}}$ connecting the fusing current I and the diameter d could be relied upon for any considerable range; their results seem to indicate that although perhaps sufficiently accurate for most practical purposes, the true connection is not thus expressed. Experimenting with tinned copper wire fixed horizontally, the following general expressions were deduced:

Fusing Currents.	Diam. (d) in cm.	Diam. (d) in inches.
1. Between 1 and 10 amperes.....	$I = 821{\cdot}3\ d^{1{\cdot}195}$	$I = 2501\ d^{1{\cdot}195}$
2. Between 10 and 100 amperes...	$I = 1768\ d^{1{\cdot}403}$	$I = 6537\ d^{1{\cdot}403}$

* This may be exceeded if the normal current is below 10 amperes.

Much diversity of opinion exists as to the best fuse material under given conditions. Taking all factors into consideration, for large currents and for high pressures copper is perhaps one of the best materials to use, because (1) for a given fusion current the diameter is less than one of either tin or lead, hence there is less molten metal to contend with; (2) being harder, it is not so easily stretched or scratched, hence it is more reliable; (3) it does not give off an excessive amount of vapour, which latter assists "arcing" between the terminals; (4) it can be easily soldered to terminals when necessary. The main drawbacks to the use of copper are (1) its cost; (2) it maintains a dull red heat often under normal load, and often becomes quite hot for some time prior to actually melting; (3) the particles into which it breaks up on fusion sometimes remain hot for some considerable period.

For ordinary currents and low pressures tin is satisfactory as a fuse, for it has a comparatively low melting point, and in consequence is frequently recommended by engineers for branch circuits in internal wiring.

Strip zinc is also satisfactory if a fuse with a fair "time-lag" is required, *i.e.* one which will not immediately melt on the smallest overload. Strip copper is good for *very large* currents.

A fuse should be attached to a support of fireproof material (usually glazed porcelain), and should be at least $1\frac{1}{2}$ in. in length for fusion currents of from 1 to 10 amperes, otherwise a greater current than the calculated one will be required to melt it, for its temperature elevation will be retarded by the loss of heat by conduction through the terminals.

As the reader will learn later, when the circuit is opened by the blowing of a fuse, there arises the possibility of serious "arcing" between the terminals; and this is enhanced by the presence of vapour from the molten metal. Many methods have been devised to counteract this; in the Ferranti fuse the terminals are made to drop instantly into separate compartments of non-conducting oil, when the fuse gives way; in another type the fuse is enclosed in a tube, and packed round with non-conducting powder, preferably of a material which fluxes with the metallic oxide at the temperature of the break; in yet another the fuse is enclosed in a tube open at both ends, in which case, at break, the air expands and rushes out, thereby extinguishing the arc. One or two specific types of fuses are described in Chapter XXIII.

173. Construction of the Carbon Glow-lamp.—Allied to the heating effects of a current and the consequent rise in temperature of the conductor is the artificial production of light by electrical means. A platinum wire through which a current passes may be so raised in temperature that it emits light; but platinum is unsuitable for commercial lamps because: (1) it is very expensive; (2) the temperature at which it produces light is too near its melting point; (3) it does not emit light radiations so extensively as a black body at the same temperature. In practice carbon has been found to be a suitable conductor; but carbon *in air* is readily burnt if heated to the extent necessary; hence in the commercial glow-

lamp *carbon in the form of a filament is heated in a globe from which the air has been exhausted.*

The manufacture of the modern carbon filament lamp may be briefly outlined as follows :—

Pure cotton wool is dissolved in chloride of zinc, forming a combination of the consistency of treacle. This is squirted through small circular holes into a vessel containing alcohol, in which solution the threads are permitted to remain for several hours, with the result that they are hardened and freed from zinc chloride, after which they are treated with cold water for eight or nine hours to completely clear them of alcohol. The threads are next wound on a carbon former so as to have the loop form required in the finished lamp, the "former" and its threads are packed with powdered carbon in a carbon crucible, and the whole is heated in a furnace to a temperature of about 1,000° F., the filaments, which are now practically pure, hard, and brittle carbon, being allowed to cool slowly prior to removal.

Small pieces of platinum wire, the ends of which are tiny tubes, are now threaded and pinched with pliers on to the ends of the filaments, which are next fixed in a special holder, the latter providing a short-circuiting connection between the legs of the filaments, and the whole is immersed in benzine. On passing a current across one junction, thence across the short-circuiting connection and the other junction, the heat produced at the junctions decomposes the benzine and causes the deposit of carbon there, thus effecting a sound electrical connection between the filaments and the wires.

Each filament is next placed in a chamber which has first been freed from air and then filled with benzine vapour, and a current is passed through it. The object of this, termed **flashing**, is to secure on the heated filament a deposit of carbon from the vapour and absolute uniformity in the filament, for any thin portions will naturally be more heated and carbon will be deposited there to a greater extent than at thicker places ; it is further contended by some makers that flashed filaments are more durable, less liable to sublime and blacken the bulb (Art. 175), and work more satisfactorily than unflashed ones.

The filaments are next placed in the bulbs and the platinum wires sealed into the glass by the aid of the blow-pipe flame. The bulbs are then exhausted, first by means of a powerful air pump and then by a mercury pump, a gradually increasing current is passed through the filaments until they give out more than their normal candle-power, at which stage all the absorbed gases will be driven therefrom, and finally the bulbs are sealed by the aid of the blow-pipe flame.

The quality of the vacuum is next tested in a dark room by placing the lamp on an insulated conducting plate connected to one terminal of a transformer which raises its potential to from 2,000 to 4,000 volts ; if the lamp be perfect there is no effect, but if the bulb glows the vacuum is at fault. This brings the manufacture of the lamp to an end save for the fixing of the cap and the tests of efficiency and candle-power.

Sometimes the final exhaustion of the bulb is assisted chemically. Prior to pumping, phosphorus dissolved in alcohol is placed in the tube projection of the bulb. When pumping is over the tube is sealed at a short distance from the bulb, the phosphorus is then vaporised, thus combining with any oxygen, and finally the bulb is sealed where it forms into the tube, *i.e.* at the "pip."

174. Lamp Cap and Holder.—The "bayonet" cap, shown in Fig. 177, consists of a cylindrical brass collar which is fixed to the top of the lamp, the platinum ends of the filament being connected to two brass pieces which are coplanar with the top of the collar and well insulated therefrom and from each other by vitrite. The collar carries two small side pins of brass which fit into side recesses in the lamp holder.

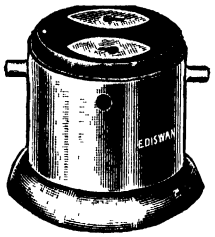
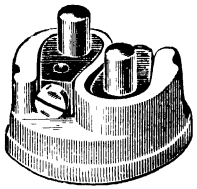


Fig. 177.



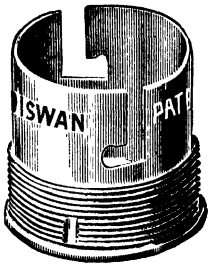


Fig. 178.

The S insulator bayonet holder is depicted in Fig. 178, the essential portion being shown removed from its containing case. The circular base with the S-shaped ridge is of glazed porcelain. The two connecting wires leading to the lamp pass through two holes (not shown) in the porcelain base, and by means of screws are attached to the two brass blocks situated in the recesses of the ridge. Each block contains a cavity in which is a spiral spring and projecting brass plunger, the latter being held out at its greatest distance by the force of the spiral spring, its wider base inside, however, preventing it from being completely expelled. When the side pins of the cap (Fig. 177) are forced into the side recesses of the holder, the two plungers press firmly on the two brass contact pieces in the cap, and conducting communication is thus established between the connecting wires and the filament.

In connection with this type of holder it may be claimed that: (1) it is simple in construction, (2) it is easily manipulated and wired, (3) its plungers are well separated from each other and the outer metal case, (4) the insulating material is incombustible, (5) it is not expensive and has stood the test of time. Its main drawback lies, perhaps, in the fact that its plungers, having rounded ends, make contact at a small area only and thus a more than necessary amount of heat is developed there; plungers with flat ends are preferable.

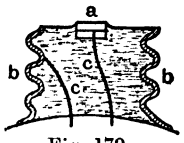


Fig. 179.

Fig. 179 shows a screw lamp end: there is one central contact, and the screw collar forms the other: similarly the holder has one central contact and a screw collar forms the other.

175. General Remarks on Carbon Filament Glow-lamps.—*The British Light Unit is the light of a sperm candle $\frac{7}{8}$ inch in diameter weighing six to the pound and burning* 120 *grains per hour.* For laboratory work standard candles are now rarely used. The *Methven standard* (a coal-gas flame from an Argand burner, the light passing through an aperture in a screen), the *Hefner Alteneck standard* (a flame from burning amyl acetate), the *Vernon Harcourt standard* (a flame from a burning mixture of air and pentane vapour), and the *Carcel standard* (burning colza oil) are amongst those most frequently employed. For a description of these the student should refer to some advanced text-book on Light.

The efficiency of a lamp is generally expressed by stating *the number of watts absorbed per candle-power given by the lamp*, but more correctly, perhaps, this might be called its "inefficiency," since the less the number of watts absorbed per candle-power the greater really is the efficiency of the lamp as a light producer. *The number of candles emitted per watt absorbed* is therefore a better conception; hence—

$$\textbf{Efficiency} = \textbf{Candles per watt} = \frac{\textbf{Candle-power of lamp}}{\textbf{Watts absorbed by lamp}}.$$

Thus if a lamp takes 64 ampere at 100 volts and its candle-power is 16, the total watts absorbed are $\cdot 64 \times 100$, *i.e.* 64, the watts per candle are 64/16, *i.e.* 4, and the efficiency in candles per watt is 16/64, *i.e.* 25. Clearly a lamp taking $2\frac{1}{2}$ watts per candle will be of higher efficiency than one taking 4 watts per candle. From 3 to 3·75 watts per candle are absorbed by lamps commonly employed.

If a lamp be worked at a higher voltage than its normal, it of course absorbs more watts (EI), but at the same time its candle-power is increased, the latter to a greater extent than the former, so that the watts absorbed per candle are reduced and the efficiency is increased. Working at increased pressure, however, shortens the life of the lamp; hence renewals are frequent. On the other hand, if a lamp be worked at a lower voltage than its normal its candle-power is so reduced that the watts per candle are increased and therefore the efficiency is reduced; but the lamp will have a longer life and renewals will not be frequent.

Glow-lamp makers classify their wares into: (1) High efficiency lamps to work at $2\frac{1}{2}$ to 3 watts per candle; (2) Standard medium lamps to work at $3\frac{1}{2}$ to $3\frac{3}{4}$ watts per candle; (3) Long duration low efficiency lamps to work at 4 (or over) watts per candle. From an economical point of view the choice of the lamp depends on circumstances. In cases where energy is cheap, as, for example, on board ship, or where coal is cheap, it will perhaps be most economical to spend on energy and save on lamp renewals; hence long duration low efficiency lamps suggest themselves. If on the other hand energy is expensive it is, perhaps, better to save on energy and spend on lamps; hence high efficiency lamps may be employed. It may be mentioned, however, that high efficiency lamps ($2\frac{1}{2}$ watts per candle) cannot hold out against sudden fluctuations of pressure to the extent that those of low efficiency can.

The higher the temperature to which the filament is raised the greater will be the light emitted and the efficiency; but there is a limit to the temperature elevation possible. In the first place, when carbon is strongly heated in a vacuum sublimation sets in, *i.e.* the solid carbon is converted into vapour without passing through the liquid condition; hence the blackening of the bulbs in all lamps, and

more so in those with indifferent filaments; in the second place, if heated until the melting point of platinum is reached the lamp is at once destroyed.

The blackening of the bulb cuts off a marked percentage of the light, and sublimation results in the surface of the filament acquiring an increased ability to radiate heat, so that its temperature cannot rise so high; hence the candle-power is lowered, the watts per candle increased, and the efficiency decreased.

Again, if the filament be an uneven one, the thin parts, being heated more than the thicker portions, sublime the most, become still thinner, and finally the filament severs. When blackening becomes very marked the lamp should be replaced by a new one.

As previously indicated, economical distribution demands high pressure, and high-pressure lamps are now essential. Taking the previous example of a 16 c.p. lamp taking 64 watts and imagining a 200-volt circuit, the resistance of the lamp would have to be $200^2/64$, *i.e.* 625 ohms. To secure this resistance in a globe not of serious dimensions a very thin filament is necessary, and it must be fairly long; this is the essential feature in, and the difficulty attending the construction of high-pressure incandescent lamps.

A lamp does not emit light equally in all directions, and the term "candle-power" is frequently used to denote the c.p. in the most favourable direction for the lamp. In the case of glow-lamps the "mean horizontal candle-power" (M.H.C.P.) is invariably referred to: this is the mean of the candle-powers in all directions in the horizontal plane passing through the centre of the light. It is, however, becoming customary (particularly with arc lamps) to refer to the "mean spherical candle-power" (M.S.C.P.): this is the mean of the candle-powers in **all** directions. The M.S.C.P. of a carbon filament lamp is about ·82 of the M.H.C.P. (For metallic filament vacuum lamps it is about ·75, and for the half-watt lamps ·85.)

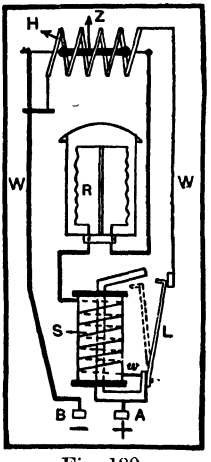


Fig. 180.

The *useful* life of a lamp is the time, in hours, taken for the c.p. to fall to 80 per cent. of its original value (this is referred to as the *smashing point*), but frequently price lists give the *total* life. The useful life of a good English carbon filament lamp ($3\frac{1}{2}$-$3\frac{3}{4}$ watts per candle), never over-run, is from 700 to 1,000 hours.

176. The Nernst Lamp.—In Chapters II. and X. we have referred to the fact that many substances are insulators at ordinary temperatures, but become conductors when their temperatures are sufficiently raised; this property of *zirconia*, an oxide of the alkaline metal zirconium, *thoria*, an oxide of thorium, and certain other metallic oxides, is utilised in the Nernst lamp.

The glower Z (Fig. 180) consists of a thin rod of zirconia mixed with the oxides of thorium, yttrium, and erbium, its length in some cases being from $\frac{1}{2}$ to 1 in., and diameter $\frac{1}{32}$ in. This glower is surrounded by a heater H consisting of a porcelain spiral wound with platinum wire.

Owing to the marked decrease in the resistance of the glower when heated, and its sensitiveness to any change in pressure and current, a steadying resistance R of thin iron wire in a containing vessel of hydrogen is put in series with it. Iron is selected for this purpose owing to its high specific resistance, and also for the fact that at a particular stage a big rise or fall in the terminal P.D. produces practically no change in the current, since the resistance changes in the same ratio; a hydrogen atmosphere is used to prevent oxidation.

In series with R and Z is a solenoid S, fitted with a bent iron core; L is an iron lever, which serves to break the heater circuit at the correct instant.

The action of the lamp is as follows: The current enters at the positive terminal A, passes along the lever L and wire W to the heater H, returning to the negative terminal B by way of the wire W_1. At this stage no current passes through the glower circuit, since the glower, being cold, is a non-conductor. The current renders the heater (porcelain and wire) red hot, and these radiating to Z so raise its temperature that in a second or two it becomes a conductor. Current now passes from A to the solenoid S through the wire *w*, thence to R and Z, and back by W_1 to the negative terminal B. The bent iron rod is magnetised, the lever L attracted (dotted lines), and the heater thus cut out of circuit, the whole current now passing through Z, rendering the latter luminescent.

Unlike the glow lamp filament the glower is surrounded by air. The efficiency is high, of the order ·6 candle per watt, and owing to the high resistance of the glower, glowers for high pressure lamps can be readily constructed. The complications—heater, magnet, ballast—are, however, not ideal, the glower after some time becomes crystalline in structure, and the efficiency is then reduced, and finally unexpected failure in some form often results in a comparatively short life (400—500 hours).

177. Metallic Filament Lamps.—The main drawbacks to the carbon filament lamp are its large power consumption, and the fact (associated with its negative temperature coefficient) that it is sensitive to pressure fluctuations; hence many attempts have been made to discover a satisfactory substitute, which could be heated to a high temperature, which would have a longer "useful life," and for which the power consumption and sensitiveness to changes in voltage would be considerably less than in the case of the carbon filament. As a result the Nernst lamp appeared in 1897, and was followed in 1902 by the **Osmium lamp** as a commercial article. The filaments of the latter were made of metallic osmium. Their candle-power was high (25 and upwards), and they absorbed about 1·6 watts per candle; but the lamp has given place to newer types.

In 1905 the **Tantalum lamp** was placed upon the market, and although its power consumption exceeds that of certain other metallic

filament lamps, it has special points in its favour, and very soon won a reputation. The filaments are made of pure drawn tantalum wire, their power consumption is from 1·5 to 2 watts per candle, and a pleasing white light of high candle-power is produced. As the metal, unlike carbon, increases in resistance when heated, the lamp is not affected by voltage variation to the extent that the carbon filament lamp is.

Owing to the low specific resistance, however, a very long filament is necessary, and special methods must be adopted for enclosing such in a vacuum globe of ordinary dimensions. Fig. 181 shows one of the devices employed. A central glass stem carries two sets of hooks as indicated, and the tantalum is supported in the form of a continuous filament on these hooks; this method, with slight modifications, is adopted in general with metallic filament lamps.

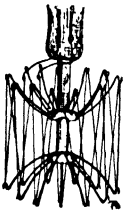


Fig. 181.

The advantage of tantalum over osmium is that the former is stronger, and higher voltage lamps (250 volts) are possible. The tantalum filament is, in fact, one of the strongest; the lamps can be suspended in any position, are well able to withstand vibration, and can therefore be employed with

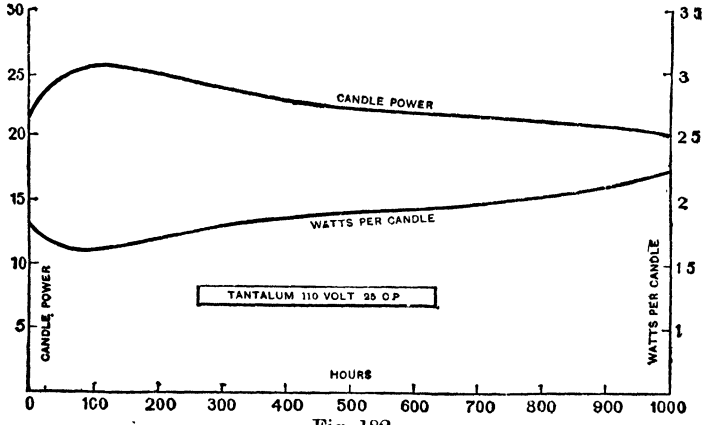


Fig. 182.

success in workshops, for electric railways, ship-lighting, etc., traction tantalums at 2 watts per candle burning five in series on a 600 volt circuit having outlived 2,000 hours.

Fig. 182 gives the result of a "c.p." and "watts per candle" test on a 110 volt, 25 c.p. tantalum lamp for a run of 1,000 hours. Fig. 183 depicts similar tests on 16 c.p. carbon lamps. Although the tantalum has points in its favour, it also has recently given place to newer types.

Metallic Filament Lamps of Later Introduction.—The **Osram, Corona, Wotan, Mazda,** etc., have filaments of the metal **tungsten.**

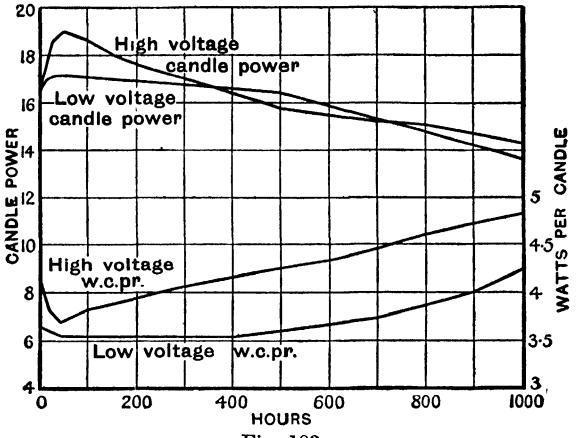


Fig. 183.

These filaments can be raised to a high temperature so that their candle-power covers a wide range (up to 1,200 c.p.), their power consumption is only of the order 1·1 to 1·4 watts per candle, their average life is over 2,000 hours, and they can be used at various pressures from low values to 260 volts. Owing to the positive temperature coefficient the lamps display less variation in candle-power and watts, due to changes in voltage, than the carbon filament lamp does.

Originally the filaments were very delicate, and difficulty was experienced in the fixing of the long filaments in globes of reasonable dimensions. Improvements in manufacture have, however, overcome the difficulties, and the modern tungsten lamp is now a commercial success for either direct or alternating current circuits, and for both indoor and outdoor illumination.

The methods which have been adopted in the manufacture of tungsten filaments may be briefly outlined as follows:—

I. The Coating Process.—A fine carbon filament is heated by means of an electric current in an atmosphere of tungsten chloride or oxychloride, and hydrogen. The latter reduces the volatile metallic compounds, and the metal is deposited on the carbon filament. A further application of heat causes the tungsten and carbon to combine, forming tungsten carbide, which is then reduced and the carbon abstracted by heating in an atmosphere of steam and hydrogen; a pure tungsten filament is the result.

II. The Paste Process.—Tungsten oxide and an excess of zinc dust are heated together, the zinc is afterwards dissolved by hydro-

chloric acid, and the reduced metallic tungsten remains in the form of a dark powder. This is made into a paste by mixing with caramel or gum tragacanth, and is then squirted through a very fine orifice so as to form filaments. These are next heated under exclusion of air, and a current passed through them in an atmosphere of hydrogen, with the result that the filaments "sinter," the gas attacking the constituents of the binding agents but not the metal, so that finally a filament of pure tungsten remains.

III. The Colloidal Process.—Tungsten electrodes are placed under water and an arc is formed, thereby producing the "colloidal" state. The finely divided tungsten powder is then mixed, if necessary, with binding agents to produce the proper consistency, and finally "squirted," and subsequently treated as outlined above.

IV. The Tungsten Wire Process.—The search for a *ductile* tungsten filament has now met with success. These are drawn tungsten wires which retain their ductility and are strong mechanically. The ductile wire is bright, tough, and steel-coloured, the tensile strength increases as the drawing proceeds, and the drawn and annealed wires result in *robust* lamps having all the good points of the earlier tungsten filaments which they are rapidly superseding.

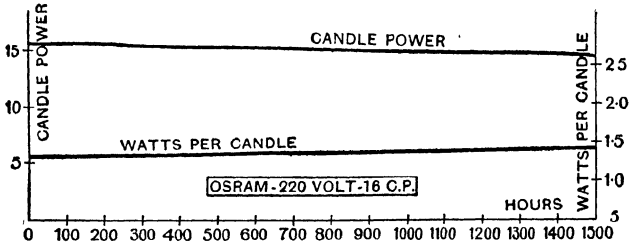


Fig. 184.

Osram lamps are supplied with either squirted or drawn wire filaments. Fig. 184 depicts a recent test on a 220 volt 20 watt 16 c.p. Osram on an alternating current circuit. *Corona lamps* are fitted with annealed tungsten wire filaments. *Mazda lamps* have wire filaments drawn from a block of pure tungsten.

In the **tungsten half-watt lamp** the filament is in an atmosphere of nitrogen (or argon). When a metal filament is run at a very high temperature it tends to evaporate (this blackens the bulb), and the temperature at which this sets in is lower if the filament is in a vacuum than if it is in a gas: hence the introduction of gas into the bulb, nitrogen or argon being chosen as they have no chemical effect upon the hot filament. Thus it is that the temperature can be raised about 500° F. above that of an ordinary tungsten (vacuum) lamp, and at such a temperature the light is whiter in colour and approaches sunlight.

Convection currents in the gas tend to cool the filament; to counteract this it is wound in a close coil, which results in adjacent turns heating each other and less surface being exposed to the cooling effect of the gas. The bulbs have comparatively long necks, so that the lamp sockets, etc., are a fair distance from the high-temperature filaments, and any tungsten which does evaporate is carried there by convection, so that any blackening mainly occurs in the *upper* portions of the lamps.

The intrinsic brilliancy is about eight times that of a tungsten (vacuum) lamp, and from 15 to 20 times that of a carbon filament lamp. In some makes the ratio of the M.S.C.P. to the maximum horizontal candle-power is about ·85. They can be run on voltages from 25 to 260, the watts range from 15 to 1,500, and the c.p. is about twice the watts absorbed (·5 watt per candle): the life is about 1,000 hours, and various types give up to 3,000 c.p. The lamps are not only suitable for indoor lighting, but in places they are replacing arc lamps for street, railway, and factory lighting, etc.: over arc lamps they have several advantages.

178. The Electric Arc: General Remarks.—If two carbon rods in contact, end to end, form part of a suitable continuous current circuit, and if when the current is flowing the carbons be drawn apart to a distance of $\frac{3}{16}$ or $\frac{1}{4}$ inch, a luminous "arc" will be formed between them, the arc constituting a conducting path from one carbon to the other. After a time the ends of the carbons become luminous, and more so than the arc; in fact, about 85 per cent. of the light is due to the positive carbon (*i.e.* the one joined to the positive lead or positive of generator), 10 per cent. to the negative carbon, and 5 per cent. to the arc. The positive carbon assumes a crater-like form, as indicated in Fig. 185, the temperature of which is from 3,500° to 4,000° C.; the end of the negative becomes gradually pointed, incandescent matter being carried to it from the positive, and its temperature is about 2,500° C. The rate of consumption of the positive carbon is about twice that of the negative; hence for the former carbons about twice the cross-sectional area are employed.

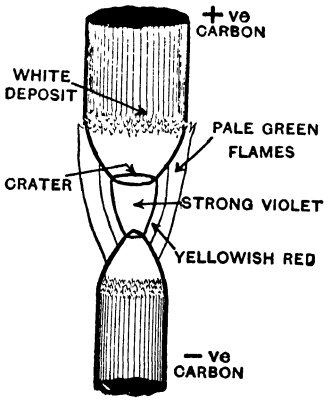


Fig. 185.

Modern theory indicates that when the carbons are separated the space becomes conducting owing to *ionisation* of the air at the high temperature, and that the negative carbon emits a constant stream of

electrons, which, bombarding the positive carbon, keep the latter very hot.

Experiment shows that in the case of continuous current arcs, a P.D. of about 44 volts is necessary; they will not act at all with a pressure below 38 volts, and a frequent allowance is 50 volts for a 10-ampere arc. A considerable P.D. is found to exist between the positive carbon and a point slightly below the crater, and in comparison a small P.D. exists between this point and the negative carbon. To account for this it has been suggested that the volatilisation of the carbon in the crater results in the establishment of a back pressure of 37 to 38 9 volts in a manner similar to the back pressure consequent upon the separation of the ions in electrolysis, the result being, of course, that the applied pressure has to overcome this in addition to the ohmic resistance (of the order $\frac{1}{5}$ to $\frac{1}{2}$ ohm) of the arc.

The P.D. necessary to maintain an arc has been found by Mrs. Ayrton to be given by the following relation—

$$E = \left\{a + bL + \frac{d + eL}{I}\right\} \text{ volts,}$$

where a, b, d, and e are constants, L the length of the arc in mm., and I the current in amperes; the values of the constants were found to be as follows:—

$$a = 38{\cdot}9,\ b = 2{\cdot}07,\ d = 11{\cdot}7,\ e = 10{\cdot}5.$$

From the relation given above it follows that: (1) an increase in the current causes a decrease in the voltage if the length of the arc remains the same; (2) a decrease in the length of the arc causes a decrease in the voltage if the current remains the same. In connection with (1) it may be mentioned that as the current increases, the voltage decreases at first rapidly and then more slowly; the crater also increases, and when this continues until the crater walls extend to the sides of the carbon, the "hissing" of the arc sets in, at which stage a sudden fall in voltage (about 10 volts) occurs, and beyond which no further decrease in voltage takes place. Hissing also occurs when the arc is first started, and when its length falls below a certain value; it is probably due to the combustion of carbon in air, which is alternately sucked in and expelled from the arc.

The positive carbon is uppermost, so that the crater may throw its light downwards, a certain percentage, however, being stopped by the negative carbon. Owing to the high temperature the c.p. is high in certain directions (see later)—of the order 1,000 +, so that the power consumption may be given as of the order ·5 to ·8 watt per candle, and the efficiency of the order, say, 1·25 candles per watt. The M.S.C.P., however, is of the order 700, so that the power consumption is of the order 1·4 *watts per mean spherical candle-power.* A more perfect crater is formed if a cored carbon, *i.e.* one with a core of softer carbon, be used as positive.

Arc lamps can be arranged either in series or in parallel. For lighting railway stations and similar places where all the lights are required at once, or in cases where glow-lamps are not used, it is customary to run them in series, the dynamo, of high E.M.F., being arranged to keep a constant current flowing, in which case no part of the cable carries more than, say, 10 amperes, the current required for one lamp. If, on the other hand, the lamps (say 12) be put in simple parallel, the cable between the dynamo and the first lamp must be of sufficient section to carry 120 amperes, and the remainder of it in proportion; thus heavier and more costly cable will be necessitated.

Again, to ensure satisfactory working of an arc, the proper voltage must be maintained at its terminals, and the current must be kept as uniform as possible. It will be clear, however, that if lamps be connected in parallel between mains at the required voltage for each (say 50 volts), any variation in the length of an arc will result in great instability, since the arcs themselves form the only resistance. To avoid this it becomes necessary when single lamps are in parallel to use a higher pressure than that for one lamp, and to place in series with each a steadying resistance to use up the extra volts, in which case any change in the resistance of the arc will not materially affect the whole path. Thus, if the mains be at 60 volts, 10 volts must be disposed of; and assuming 10-ampere lamps, the steadying resistance with each must clearly be $\frac{10}{10}$, *i.e.* 1 ohm. The watts spent in this resistance will be 100, and the watts spent in the lamp will be 500; thus instability is avoided by wasting $\frac{1}{6}$ of the power.

Sometimes two lamps are put in series on a 100-volt circuit without a steadying resistance, but the arrangement is not an excellent one. Two in series on a 110-volt circuit and four in series on a 220-volt circuit, steadying resistances in each case, are customary. If a large number are in series, steadying resistances are not necessary, since one acts as such to another; but in such cases automatic cut-outs are necessary, so that if a lamp fails an alternative path is opened out, along which the current continues to flow to the others.

The internal mechanism of an arc lamp generally includes two coils—one in series with the carbons, termed the "series" coil, the other acting as a shunt thereto, and termed the "shunt" coil, the combined effect of the two being to keep the arc of proper length. Such a lamp is suitable either for parallel (*i.e.* constant pressure) or series (constant current) circuits. In a lamp intended for parallel working *only* and connected directly to the mains, the shunt coil might be dispensed with, for its effect would always be the same; the series resistance previously mentioned, however, varies the P.D. at the shunt terminals, and renders it effective in governing properly. Further, a lamp with a shunt coil only may be used on a series (constant current) circuit (Art. 180).

An alternating current arc lamp takes 30 per cent. more watts per candle, the lower efficiency being due to the following facts:—

(1) Since the upper and lower carbons act alternately as positives and negatives, about half the light is thrown upwards. (2) Since each carbon is heated as a positive for short intervals only, neither can attain the same temperature as the positive crater with a continuous current. A minor drawback is the humming noise the alternating arc gives rise to, due partly to molecular vibration in the metal parts, and partly to the alternate heating and cooling of the gases and vapours as the current rises and falls and is reversed. In such lamps the carbons (both cored) are equally consumed, equal in diameter, and both become pointed, and all iron parts are laminated to eliminate the effects of eddy currents (Chap. XVIII.). The minimum E.M.F. required is about 30 volts.

In connection with the alternating arc, however, it may be mentioned that in parallel working "choking coils" take the place of the regulating resistances of the direct current arc; a choking coil adjusts the pressure without dissipating as much energy as a resistance coil does.

As the name suggests, an **enclosed arc** is one formed in a compartment from which the air is more or less excluded; it is often done by enclosing the carbons in a smaller globe inside the usual one, the latter being, moreover, specially designed to attain the object in view. When the arc is established, the carbon combines with the air in the inner globe and forms carbon monoxide; this fills the globe, preventing further access of air, and, being a non-supporter of combustion, the chief loss of the carbons is due to volatilisation; hence it is that they last longer than those of the open arc. The carbons do not become conical since there is no burning, but remain more or less flat. This flattening would be a decided disadvantage were it not for the fact that the carbons are much further apart than in the open arc, for it would result in the lower one obscuring more light from the upper. The arc is frequently 12 mm. in length, and the pressure at the lamp terminals about 80 volts.

The advantages of the "enclosed" over the "open" arc may be summarised thus:

(1) The carbons last longer; new ones are necessitated about once in 100 to 200 hours, as against 10 to 20 hours for the open arc.

(2) A longer arc and higher P.D. is possible; hence single lamps can be better adapted to parallel working with existing pressures, *e.g.* single lamps with *small steadying resistances* can be run in parallel on 110-volt circuits.

(3) For indoor work there is, perhaps, a greater element of safety with enclosed arcs, since there is less fire risk from sparks and falling carbon.

(4) Owing to the longer arc the width of the zone of light, in which the illumination is practically a maximum, is greater, and uniform illumination is spread over a large area. In the open arc $\frac{8}{10}$ of the

light falls between the angles 28° and 48° from the horizontal; an enclosed arc gives $\frac{8}{10}$ of its light between the angles 25° and 60°.

The disadvantages of this type as compared with the open arc are—

(1) It absorbs about 2·2 watts per spherical c.p., and has, therefore, a less efficiency.

(2) The employment of the inner globe naturally results in a greater absorption of light.

(3) The arc moves about over the flat ends of the carbons, producing a certain amount of unsteadiness; the employment of two opalescent glass globes, however, greatly eliminates the effect of this.

In some recent types the outer globe is omitted, the single globe being of opalescent glass to diffuse the light. The lamp is also less efficient than the "open" type, but the light is more pleasing and excellent for interior lighting. The inverted open type used to illuminate a white ceiling and the room by reflection therefrom is, perhaps, its chief rival for this purpose as far as arc lamps are concerned.

179. Functions of Arc Lamp Mechanism.—The duties of the mechanism of an arc lamp are—

(1) To bring the carbons together (if they are not already in contact) when the current passes, and then to separate them (or "strike" the arc) by a definite amount.

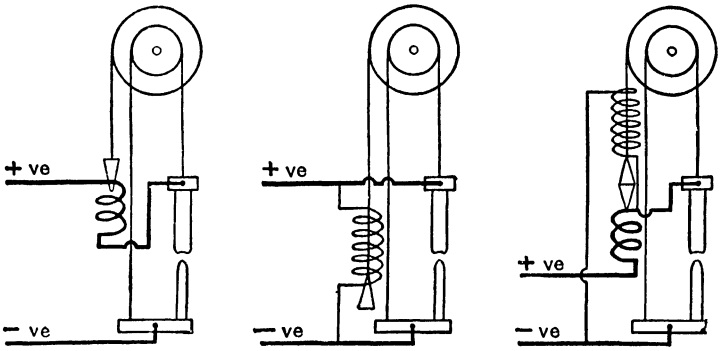
(2) To lessen the distance between them (or "feed" the arc) should they separate too much, and increase the distance should they come too near.

To carry out these operations various devices in addition to series or shunt coils or both—brake wheels, clutches, clockworks, and small motors—are employed in modern lamps.

180. The Regulating Action of the Series and Shunt Coils.—Fig. 186 (diagrammatic) will serve to explain the action of a series wound lamp (*a*), a shunt wound lamp (*b*), and a differentially wound lamp (*c*). The carbons are suspended from a pulley in such a way that a clockwise rotation brings them together whilst a counter-clockwise rotation moves them apart; this pulley rotates with another from which a soft iron core is hanging.

(*a*) Series wound Lamp (Fig. 186*a*). This is fitted with one coil only in series with the carbons and *is suitable for arc lamps intended for parallel (constant pressure) working.* When no current is passing *the carbons are in contact.* When the current passes the coil attracts the iron core thus producing a counter-clockwise rotation of the pulleys and separating the carbons. Equilibrium is established (with the arc of proper length) when the pull on the iron core on the one side is

balancing the weight of the carbons, etc., on the other side. As the carbons are consumed, the arc gets longer, its resistance increases, the constant P.D. causes a less current to flow, the pull on the iron core becomes less and the carbons move towards each other until equilibrium is once more established with the arc of proper length and the correct current passing.



(*a*) Series-wound. (*b*) Shunt-wound. (*c*) Differential-wound.

Fig. 186.

(*b*) SHUNT WOUND LAMP (Fig. 186*b*). This is fitted with a shunt coil only and *is suitable for arc lamps intended for series (constant current) working.* When no current is passing *the carbons are separated.* When the current is switched on, it passes through the shunt coil, the iron core is attracted and the carbons brought into contact. Current now passes through the carbons, and therefore less current through the shunt coil, so that the attraction of the latter on the iron core is reduced and the carbons are separated until equilibrium is established with the arc of proper length. As the carbons burn away, and the resistance there increases, the constant current gives an increased P.D. at the terminals, so that a greater proportion of current passes through the shunt coil, the core is attracted and the carbons brought nearer together until equilibrium is again established with the arc of proper length and the correct P.D. on the lamp.

(*c*) DIFFERENTIALLY WOUND LAMP (Fig. 186*c*). This is fitted with both a series and a shunt coil and *it is suitable for either series or parallel working.* Imagine a constant current circuit and the carbons in contact. On switching on the current, the latter mainly goes through the lesser resistance series coil; this coil attracts the iron core, and the carbons are separated. More current now goes through the shunt coil so that it also attracts the iron core tending to bring the carbons together. Equilibrium is established, with the arc of

proper length, when these two opposing influences exactly balance each other. The student should modify the explanation to fit the case of a constant pressure circuit.

The series lamp tends to maintain a constant current and is suitable for use on a constant pressure supply: the shunt lamp tends to maintain constant voltage and is suitable for use on a constant current circuit: the differential lamp combines the properties of both and is suitable for use on either constant voltage or constant current circuits.

Additional devices employed to secure the smooth and efficient working of arc lamps will be gathered from the descriptions which follow.

181. The Crompton S. Type Arc Lamp.—This, a good example of an up-to-date efficient lamp, is of the brake wheel and differential type (*i.e.* two solenoids opposing each other), and is illustrated in Figs. 187 and 188.

Referring to Fig. 187, L is a rocking lever pivoted at P and carrying near its ends two vertical soft-iron cores C and D, so arranged that they can move freely up and down inside the solenoids A and B. One end of the brake chain H is fixed to the lever at E, and after passing twice round the brake wheel Z is fixed at its lower end to the flat spring N. To the brake wheel Z are attached two drums (seen on the right of it in the figure) insulated from each other and from the wheel but turning with it, the whole being carried in the bearing straps S S.

One end of the flexible copper cord P is attached to one drum, its other end being fixed to the upper carbon holder X, which latter is insulated by mica from its carrier T. The upper end of another flexible copper cord R is attached to the second drum, its lower end being fixed to the carrier rod which is attached to the negative carbon holder V. The cords P and R are wound in opposite directions on the two insulated drums, so that any movement of the latter will, according to the direction of the movement, draw apart the carbons or cause them to approach each other.

Thus looking at the drums from the right-hand side of the figure, a slight turn in a clockwise direction will result in the upper carbon being raised and the bottom one lowered; a slight turn in a counter-clockwise direction will lower the upper carbon and raise the bottom one; hence in the first case the carbons will be separated, while in the second case they will be brought together.

From the base plate, extending downwards, are two brass tubes W for guiding the positive carrier T and the negative carrier rod, which latter moves in the tube on the right. The guide tubes are negative, *i.e.* they are in conducting communication with the lower carbon, but their upper ends are thoroughly insulated from the base plate by mica washers, and their lower ends from the steel globe rods by ebonite insulators. A dashpot K is provided to secure perfectly smooth and steady working.

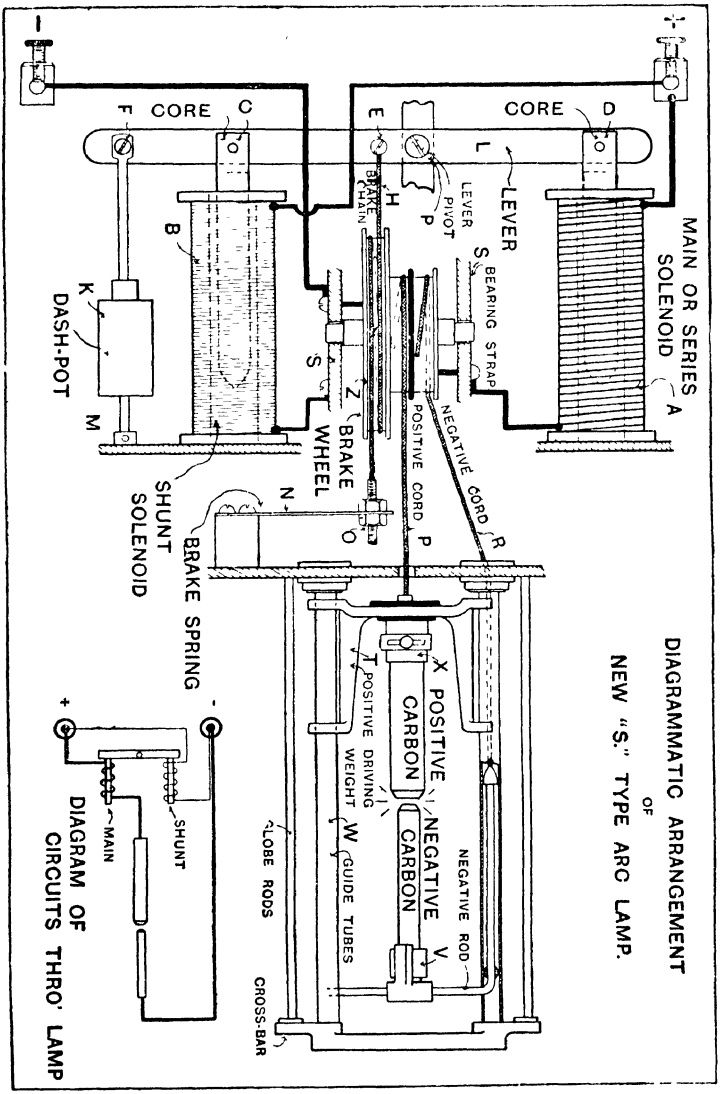


Fig 187.

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One end of the main or series coil A of thick wire is joined to the positive terminal of the lamp, the other end being attached to the bearing strap S on the right, from which a spiral connection leads to the drum which carries the positive cord P. Another spiral connection leads from the drum which carries the negative cord R to the bearing strap S on the left, from which a connection passes to the negative terminal of the lamp. The spiral connections between the bearing straps and drums cannot be seen in the figure.

One end of the fine wire shunt coil B is joined to the positive terminal, the other end being connected to the strap on the left, from whence it communicates with the negative terminal. The reader should carefully compare these connections with the circuit diagram and he will realise that the two are identical.

Imagine now that the carbons are in contact and that the lamp is on a constant current circuit. The current enters at the positive terminal, passes round the series coil A, thence to S on the right, to the drum and the positive cord P by way of the spiral connection, thence through the carbons and the negative carrier rod to R and the other drum, from whence it passes to S on the left and to the negative terminal.

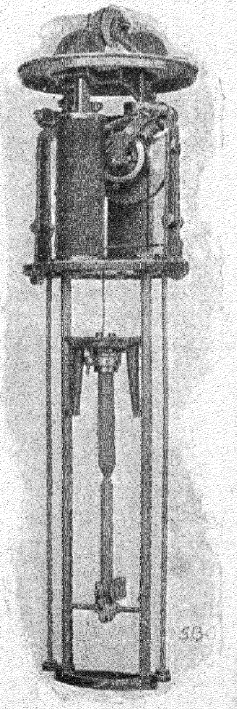


Fig. 188.

On the passage of the current, as indicated, the core D is sucked down into A, thereby tilting the lever L, raising E, tightening the brake chain H, and giving a slight turn to the brake wheel and drums in a clockwise direction viewed from the right, so that the carbons are separated and the arc struck. As the carbons burn away and the length and resistance of the arc increase, the constant current gives an increased potential difference at the terminals; hence a greater portion of the current passes through the shunt coil so that it attracts C, until, at the right moment, the brake wheel is released, and, turning in the opposite direction, brings the carbons nearer together, thus permitting

the lamp to feed. In practice this feeding of the carbons is practically continuous, and the arc is kept at a constant length.

It will be observed that prior to the striking of the arc (*i.e.* when the carbons are in contact) the resistance of the series coil and carbons is small compared with that of the shunt coil, so that practically the whole current takes the series coil path at first. It may be mentioned, too, that since both carbons are fed forward, the position of the arc relative to the containing globe is constant and the lamp is termed a "**focussing lamp.**"

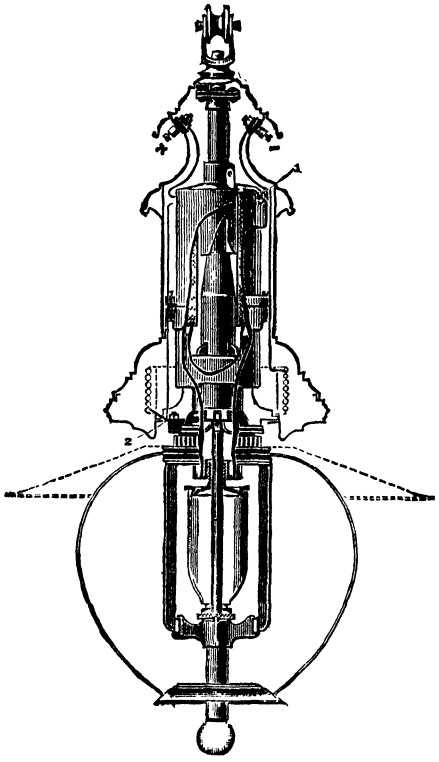


Fig. 189.

These lamps can be arranged to burn in parallel, in series, or in series parallel on alternating or continuous current circuits. Continuous current lamps are supplied for 5, 10, 12, and 15 amperes, and alternating current ones for 10, 12, and 15 amperes; the latter can be run three in series on 100 volts.

In our explanation we have assumed the lamp to be one of a series on a constant current circuit. As an exercise the reader should slightly modify the explanation to fit the case of a constant P.D. being maintained at the lamp terminals.

182. The Jandus Enclosed Arc Lamp. — This lamp is shown in Fig. 189, a diagrammatic view of the regulating mechanism, which is of the clutch type, being given in Fig. 190.

Referring first to Fig. 189, the bottom of the outer globe rests on a brass gas-check pan, between which and the china knob at the very bottom of the lamp is a spiral spring, which ensures close contact between the globe and pan, thereby preventing access of air from without. The top of the outer globe is fixed to the lamp and sealed by asbestos washers. It should be observed that the gas-check pan is sufficiently yielding to permit of the escape of the enclosed gases when

the internal pressure, consequent upon the heat of the arc, is sufficiently great to depress the spring a little.

The inner globe is fixed at the bottom to the support which carries the lower carbon, and the upper end of this globe carries a metal cover provided with a hole which allows the upper carbon to move freely through it. When fresh carbons are inserted, the upper one projects about $1\frac{3}{4}''$ into this globe. It will be noticed that, since the bottom carbon is fixed, the lamp is not a focussing one, and the arc will move downwards as the carbons are consumed; owing, however, to the slow rate of consumption this does not matter so much.

The controlling mechanism will be understood by reference to Fig. 190. On a thin tube is wound a series coil S, one end of which is joined to the positive terminal of the lamp, the other end being in conducting communication with the centering contact rings R R, which serve to lead the current into the upper carbon. There are in all six rings arranged round the positive carbon, permitting it to move freely between them and giving a smooth and frictionless contact.

The negative carbon is connected through a steadying or regulating resistance R to the negative terminal of the lamp. The same thin tube carries a shunt coil wound in the opposite direction, so as to oppose the series coil; this for simplicity is not shown in the diagram.

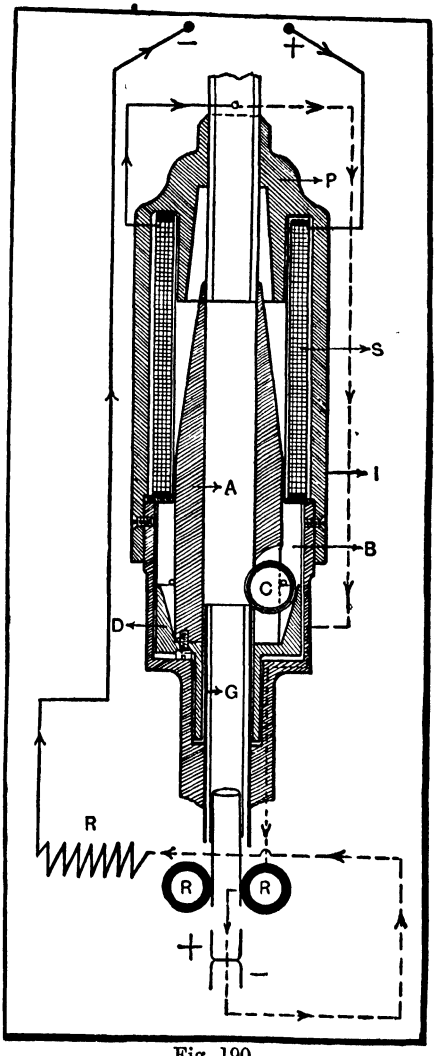


Fig. 190.

Outside the coils is an iron tube I, at the top of which is a soft-iron

pole-piece P of the peculiar shape indicated. Attached to the bottom of I is a chamber B, in which moves the stout soft-iron tube or armature A, the lower end of which is fastened to an iron dish D, the inner wall of which is inclined as shown. The armature A is sloped at its upper extremity to correspond with the shape of the pole-piece P, while its lower end is grooved in three places, each groove containing a clutch ring C. G is a tube which guides the positive carbon.

When the current passes through the series coil, A and D move upwards owing to the attraction between A and P; the sloping side of D naturally causes the rings C to press forward, upon which they clutch the upper carbon holder, raise it, and strike the arc. Should the arc become too long, its resistance so far increases that the increased current in the shunt partially cancels the effect of the series, and A and D fall a little—the result being that the clutch rings ease their grip, and the positive carbon falls. The construction is such that the motion of D inside B provides an efficient dashpot.

183. Automatic Cut-out for Arc Lamps in Series.—We have mentioned the fact that in series working, arc lamps are provided with cut-outs such that, should a lamp fail, an alternative path is opened out along which the current continues to flow to the others. The *principle* of a cut-out which introduces an equivalent resistance is shown in Fig. 191. The shunt coil S of the lamp has in its circuit an electromagnet E, opposite the end of which is an armature held off by the spring *p*. Should the arc become too long, the increased current through S so energises the electromagnet that the armature is attracted against the pull of the spring *p*, and, making contact with C, opens the alternative path A R C B, including the resistance R, which is equivalent to that of the lamp. The current through this path also energises E, thus maintaining the contact at C.

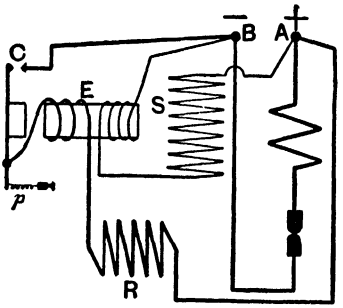


Fig. 191

184. The "Excello" Flame Arc Lamp.—The essential features wherein this new "flame" arc differs from the ordinary are:—

(1) The arc is formed in a magnetic field which serves the double purpose of blowing it out into the shape of a flaming fan (hence the name) and extinguishing it altogether when necessary.

(2) Long thin carbons are employed, and these, instead of being end to end, are inclined at a slight angle to each other, their tips being fed forward into an inverted porcelain cup (termed the "economiser") in which the flaming arc is formed. The carbons have mineralised cores.

Remembering that a current-carrying wire when placed in a magnetic field is urged in a direction depending on the directions of the current and of the field, the reader will have no difficulty in realising that an arc will be similarly affected; with a field not exceeding a certain value the arc is bulged out; with a field stronger than this limiting value the arc is ruptured.

The quality and colour of the light depend upon the constitution of the carbons. By mixing therewith certain metallic salts various colours* from white to red are obtainable, but experiment has shown that for maximum efficiency "ruddy yellow" or "sun ray" light is to be preferred. The carbons are consumed quicker than in the ordinary arc; hence longer ones are necessary, and for the purpose of reducing their resistance (and consequent drop of volts in the lamps) the carbons are fitted with a wire alongside the core.

The economiser fulfils one or two important functions. The flame is very sensitive to air currents, and the economiser protects it from draughts. A limited supply of oxygen must be furnished to the arc, and the economiser limits the supply. Finally the surface of the economiser becomes coated with a white metallic oxide which serves as an admirable reflector.

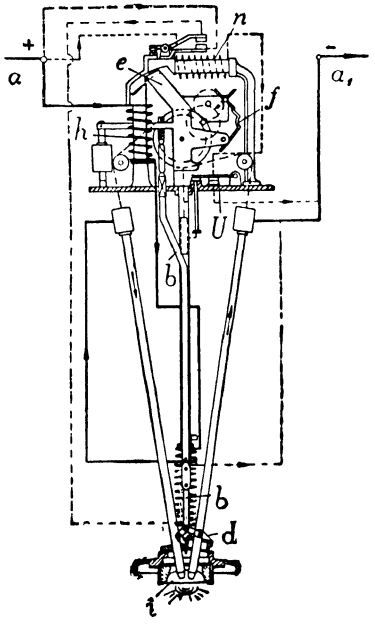


Fig. 192.

The working details of the lamp will be gathered from Fig. 192, in which the double lines inclined downwards represent the carbons whose points come near each other in the economiser *i*. The cores of the vertical "series" solenoid *h* and of the horizontal "shunt" solenoid *n* are firmly connected together, and between them swings the armature *e* connected to the detent *f*; *d* is a slider connected to the rod *b*, and as will be seen from the figure any movement of *e* will result in a movement of *b* and *d* and one of the carbons.

Briefly the action of the lamp is as follows: the carbons are initially separated. On closing the circuit the current first flows from the + terminal *via* the dotted line to the shunt coil *n* and thence to the

* With *lime*, *calcium fluoride*, golden yellow; *barium*, *cerium fluoride*, white; *strontium*, pink.

— terminal, with the result that the armature *e* is drawn over towards the right, the rod *b* raised, and the carbons brought together by means of the slider *d*. Current now passes from the + terminal through the series coil *h*, thence to the series turns on the blow magnet just above the economiser, upwards (firm line) to the positive carbon, thence to the negative carbon and to the — terminal, the result being that *e* is now drawn to the left, the carbons separated, and the arc struck.

When the distance between the carbons becomes too great, the shunt coil has a greater effect on *e* than the series coil, the armature moves towards it, the detent *f* is released, and the carbons feed forward. When the carbons are in their lowest position and cannot feed further, the shunt coil *n* is automatically cut out by the switch U, the series coil *h* suddenly draws *e* over towards the left, thereby separating the carbons as far as possible and extinguishing the arc.

For pressures exceeding 240 volts an additional extinguishing device is provided. At the moment of cutting out the shunt coil *n* as above, a second shunt circuit is brought into action, current passing downwards (*via* the dotted line on the extreme left) to the coil *b* on the blow magnet, thence upwards (*via* the dotted line on the right) to the — terminal; this so increases the strength of the blow magnet and of the field in which the arc is formed that the latter is at once extinguished.

The lamps can conveniently be run at 50 volts each. Their efficiency is about 3 to 5 spherical candles per watt, and it is stated that the maximum c.p. is over 3,000. The construction results in entire absence of shadows, and they appear unequalled for the lighting of streets and open spaces. For alternating currents the above construction is modified, and a differentially wound motor lamp produced for periodicities, from 40 to 100 per second.

185. The Jandus Regenerative Flame Arc Lamp.—This is an enclosed arc lamp with a chemically treated positive carbon. This latter consists of a carbon core of star-shaped section, the interspaces between the rays of the star being filled with a paste formed of a mixture of calcium tungstate and calcium fluoride with salts of sodium and potassium. The carbons, end to end, *the positive being the lower*, are enclosed in a cylindrical clear glass vessel provided with side tubes giving free communication between the top and the bottom, and the cylinder is further surrounded by an opalescent glass globe, the whole being arranged in an airtight enclosure.

Flame arc carbons, if burning in an ordinary enclosure, give a dense white deposit on the sides of the glass which almost entirely cuts off the light after a few hours' run. In this case, however, the heat of the arc produces a strong upward draught of heated gases which sweep downwards through the side tubes and pass upwards again through the central glass cylinder; thus the glass cylinder is swept by a rapid current of hot gases which prevents deposits thereon, and this rapid circulation of gases and chemical vapours up the cylinder, down the

tubes, and up again through the arc regenerates the chemical vapours therein and intensifies the arc. The success of this regeneration is

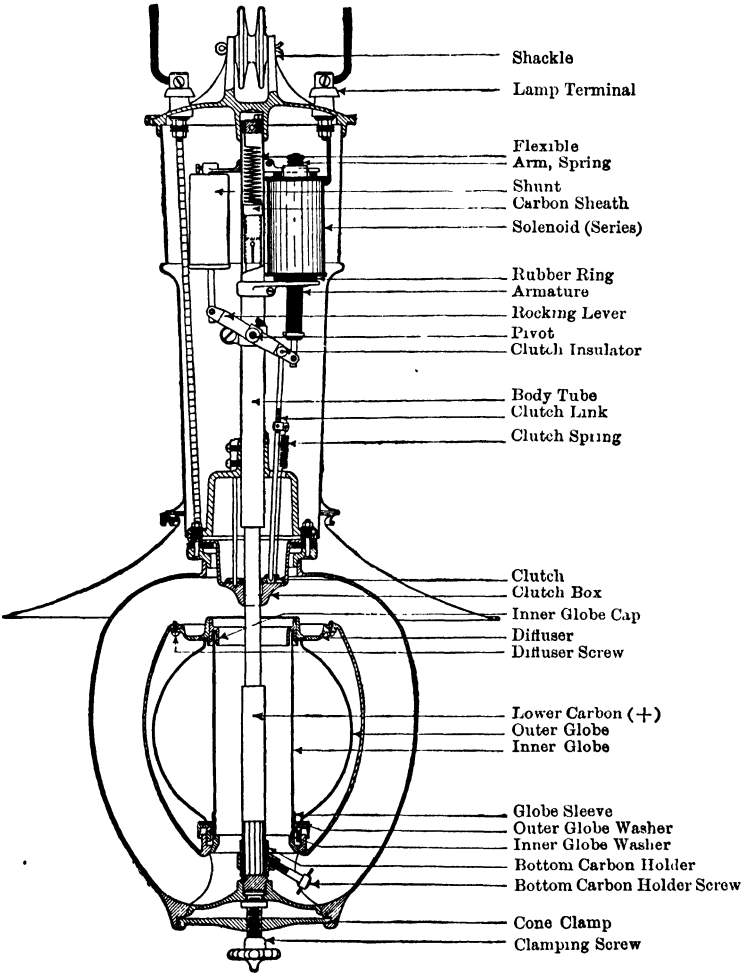


Fig. 193.

seen in the fact that only 15 grains of chemicals are consumed per hour as against several ounces for the open flame arc.

The construction and action of the lamp will be gathered from Fig. 193. The bottom positive carbon is fixed, while the upper negative

carbon slides in the central tube vertically above it. The current, entering the positive terminal, descends by an insulated lead to a connecting bolt making contact with the outer globe frame of aluminium. From this it enters the lower carbon, passes to the upper carbon, thence through a flexible connection to the series solenoid and the negative terminal. The series solenoid is connected to a rocking lever, the tilting of which raises the upper carbon through a simple clutch, thereby striking the arc. A shunt solenoid acts upon the opposite end of the lever in opposition to the series solenoid, and regulates the arc length. The efficiency is 2·8 spherical candles per watt.

186. The Cooper Hewitt Mercury Vapour and Westinghouse Silica Lamps.—The Cooper Hewitt Lamp consists essentially of a glass vacuum tube with a mercury electrode at one end and an iron electrode at the other end, the light being due to the incandescence of mercury vapour between the electrodes, in the form of a luminous column filling the tube.

To light the lamp it is necessary to tilt the tube, and Fig. 194 shows how this is done automatically. On switching on, the current flows through the tilting solenoid M, cut-out J, substitutional resistance N, and through series resistance R back to line. The current in M causes the tube to tilt, and a parallel path is provided through the mercury which flows in a continuous stream between the electrodes; this current flowing also through the inductive resistance S in series with the tube operates the cut-out J, and the tube returns to its normal position by gravity. Meanwhile the incandescent mercury vapour produces a luminous arc throughout the tube, the light being emitted in uniform intensity from every part of it. Red rays being absent from these lamps, special reflectors coated with a layer of rhodamine, a rosy-coloured aniline dye, have been introduced to compensate. The power consumption is about ·5 watt per candle.

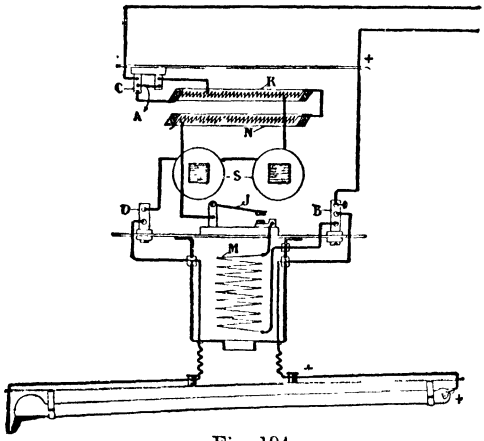


Fig. 194.

In the Westinghouse Silica Lamp, the tube is of transparent quartz and both electrodes are of mercury. The power consumption is about ·22 watt per candle.

187. The Bastian Type Mercury Vapour Lamp.—This, another form of mercury vapour lamp, is shown in Fig. 195. The lamp proper *a* consists of a glass tube of the shape indicated containing mercury (dotted lines) and mercury vapour. Red rays being practically absent

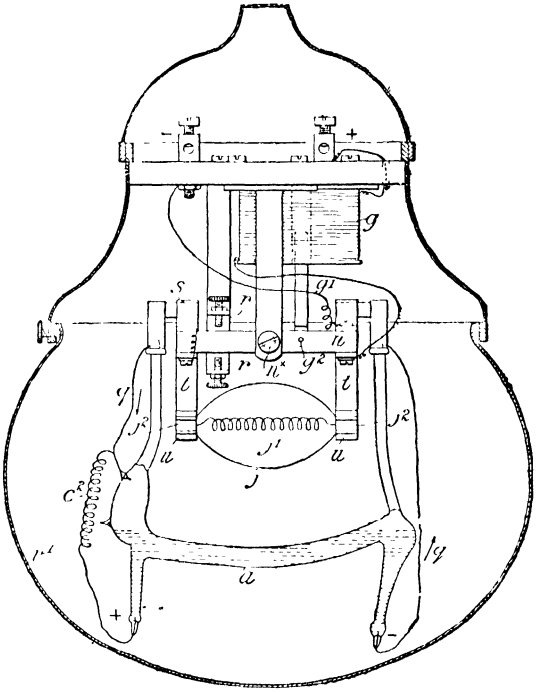


Fig. 195.

from the light of this lamp, a carbon filament lamp *j* is arranged in series with it; *j* is under-run (*i.e.* run at a red heat), so that its emanations are particularly rich in red rays; and since the globe *v* is of opalescent glass the two lamps cannot be distinguished from the outside, and the combination produces an excellent white light. For street lighting, *j* might be dispensed with and greater efficiency would result. C_2 is a starting resistance which, as will be seen presently, is automatically cut out when the "arc" is formed. The lamp *a* is attached to the beam *n*, which latter carries a vertical iron plunger g_1 capable of moving up and down in the solenoid *g*. The upper part of the lamp is provided with a bell-shaped copper cover.

When no current is passing the arrangement is as indicated. On closing the circuit the current enters at the positive terminal, passes round g to the lamp j, thence through C_2 to the positive end of a, through the mercury to the negative end, and thence to the negative lamp terminal. The plunger g_1 is thus attracted and the lamp a tilted, with the result that the mercury leaves the negative end of a and a short arc is formed. The pressure of the mercury vapour set up by the arc forces the mercury into the left-hand bulb, and this short circuits and cuts out C_2, the arc now extending throughout the entire tube. The efficiency of this vapour lamp is about 2·5 c.-p. per watt. Fig. 196 pictures the latest form, with double S burner and steadying resistances; it takes ·4 ampere at 220 volts.

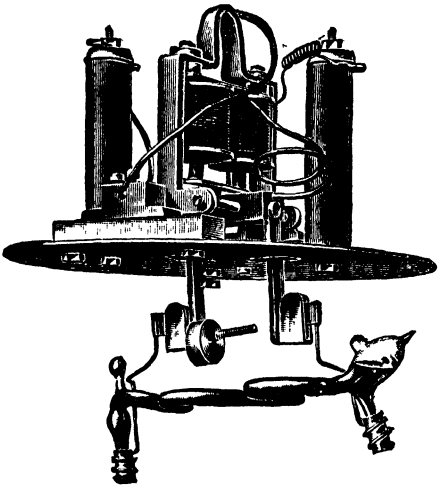


Fig. 196.

188. The Pointolite Lamp. Other Lamps.—The "Pointolite" lamp is an arc lamp with one or two tungsten electrodes, the arc being

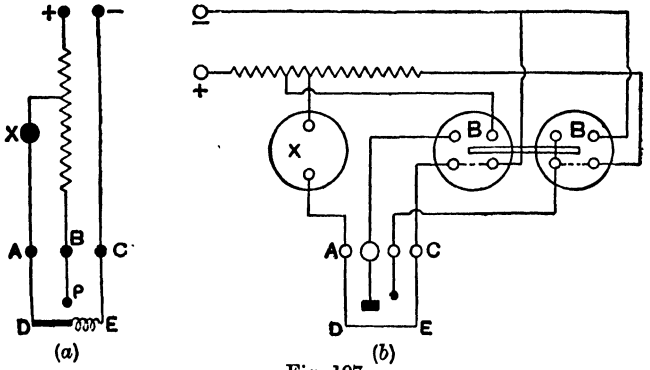


Fig. 197.

formed in a partial vacuum in a glass bulb similar to that of an ordinary glow lamp. Fig. 197*a* diagrammatically represents one

arrangement. The lamp has three terminals A, B, C. The terminal B is connected to the positive electrode P which is a small globule of tungsten. D E is a tungsten wire filament wound into a spiral at one end, the straight portion carrying a tube composed of tungsten and refractory metallic oxides and termed the *ioniser*.

The push button X is first closed, thus sending the current along D E. This heats the ioniser and when it attains the temperature at which its ionising power is effective in making the space between P and D E conducting, X is released and the current leaps across the gap between P and the coil. The tungsten electrode P then moves away from the spiral to a position opposite the ioniser, the arc now being formed between the two.

The electrode is mounted on a thermal expansion device which moves it along the ioniser. The nature of the arc necessitates the employment of a ballast resistance as shown in the figure. The globule P becomes brilliantly incandescent, practically the whole of the light coming from it.

In another type there are two tungsten electrodes and four terminals: the arc is struck as before between the ioniser and a small tungsten globule and then by a change over switch a new one is struck between the latter and a larger tungsten plate electrode: in the first arc the small globule is positive to the ioniser but in the second it is the negative electrode, the plate electrode being the positive.

The method of working will be gathered from Fig. 197*b*, where X is the starting switch and B B the change over switch. The figure shows the position for the first arc with X released, the globule positive and the ioniser negative. When B is "changed over" the top contacts are connected and the globule becomes negative and the plate positive. The voltage is of the order 45 and the 1000 c.p. lamp takes ·5 watt per candle. It is almost a "point source" lamp.

Magazine Flame Arc Lamps.—As flame arc lamp carbons are rapidly consumed "magazine" lamps have been placed on the market containing up to six pairs of carbons. In one type a second pair is brought automatically into use when the first pair is used up. In another type the carbons are arranged so that the arc is first formed between a pair which projects further than the others, then when these have burnt until they are shorter than the next pair the arc transfers to the latter and so on.

Magnetite Arc Lamp.—This originally used copper as the positive electrode but now an alloy is used, black oxide of iron mixed with salts of titanium, chromium, etc., being used as the negative electrode. The light is emitted from the magnetite vapour formed and is a very near approach to daylight. The power consumption is of the order ·5 watt per candle. The negative carbon is the upper one.

189. Illumination: Intensity of Illumination of a Surface.—Light is a form of energy: it is that part of the radiation from a hot body represented by waves in the aether between the wave lengths ·000076 cm. (red) and ·000037 cm. (violet).

The intensity of illumination at a point of a surface is measured by the amount of light falling per second on unit area of the surface in the neighbourhood of the point. The laws relating to this are as follows:—

(*a*) The intensity of illumination of a surface depends (directly) on the illuminating power or intensity of the source of light.

(*b*) The intensity of illumination of a surface *varies inversely as the square of the distance* of the surface from the source of light.

(*c*) The intensity of illumination when the light does not fall normally on the surface varies as the cosine of the angle of incidence of the light (the angle of incidence is the angle between the direction of the incident light and the normal to the surface).

The first law is obvious. The truth of the second law may be gathered from the following:—Let P be a luminous point giving out light equally in all directions, and imagine a sphere A of radius a surrounding P, the latter being at the centre. If Q be the total light per second from P the intensity of illumination of A will be $Q/4\pi a^2$. If A be replaced by B of radius b the intensity of illumination of B will be $Q/4\pi b^2$: hence:—

$$\frac{\text{Illumination of A}}{\text{Illumination of B}} = \frac{Q}{4\pi a^2} \Big/ \frac{Q}{4\pi b^2} = \frac{b^2}{a^2},$$

i.e. $$\frac{\text{Intensity of illumination of A}}{\text{Intensity of illumination of B}} = \frac{(\text{Distance of B from P})^2}{(\text{Distance of A from P})^2}$$

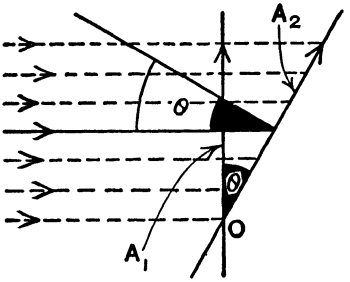


Fig. 198.

or *the intensity of illumination is inversely as the square of the distance from the source.*

The truth of the third law will be seen from Fig. 198 where a parallel beam of light is considered for simplicity. Let Q be the light falling per second normally on the area A_1 of a screen, then the intensity of illumination I_1 of A_1 is Q/A_1. Imagine the screen now tilted about the point O so that the area on which the light falls is now A_2: the intensity of illumination I_2 of A_2 is Q/A_2. Hence:—

$$\frac{I_2}{I_1} = \frac{Q}{A_2} \Big/ \frac{Q}{A_1} = \frac{A_1}{A_2} = \frac{A_2 \cos\theta}{A_2} = \cos\theta,$$

$$\therefore I_2 = I_1 \cos\theta,$$

i.e. *the intensity of illumination when the light is incident at an angle θ is equal to the intensity of illumination when the light falls normally multiplied by cos θ.*

Both laws (2) and (3) are true for "point sources," and are applicable in practice if the dimensions of the source are small compared with the distances from the source. The inverse square law does not hold for a parallel beam.

The British unit of intensity of illumination is the **candle foot,** defined as the intensity of illumination, produced by a source of one candle-power at a distance of one foot, the light falling normally. Another unit is the **candle metre,** defined as the intensity of illumination, produced by a standard candle at a distance of one metre: from the preceding laws it follows that, since a foot is ·305 metre, the candle foot is $1/(\cdot 305)^2$, *i.e.* 10·75 candle metres, and a candle metre is ·093 candle foot. Still another unit is the **lux,** defined as the intensity of illumination at a distance of one metre from a Hefner lamp: since the "candle" is 1·11 hefner units and the foot is ·305 metre, it follows that the candle foot is $1{\cdot}11/(\cdot 305)^2$, *i.e.* 11·93 or 12 luxes approximately, and the lux is ·08 candle foot approximately. (The lux is given by some writers as identical with the candle metre.)

The following are approximate intensities of illumination in candle feet:—Lecture rooms, theatres, etc. 1 to 3, living rooms 1 to 2, writing tables and general reading 3 to 5, drawing offices, etc. 5 to 8, main streets ·05, and side streets ·03. In indoor lighting a good deal of light comes by reflection from the walls, ceiling, etc.: in fact this may be from 2 to 4 times the light received direct depending on whether the wall papers, etc., are light or dark.

190. Illuminating Power or Intensity of a Source of Light.—*The illuminating power of a source of light is measured by the intensity of illumination which it produces on a screen at unit distance, the light falling normally.* The unit is an arbitrary one, viz. the **"candle"** or **"candle-power"** (Art. 175). The illuminating power of the *Harcourt pentane lamp* is 10 standard candles and in fact *an international candle-power is now defined as an illuminating power equal to one-tenth of that of the Harcourt pentane lamp.* The *Hefner lamp* is about ·9 standard candle and the French *Carcel lamp* is about 9·62 standard candles. The student should note that "candle-power" refers to the illuminating power of a source, whilst "candle foot" refers to the intensity of illumination of a surface.

Two important results follow from the definition given above:—

(*a*) Let c.p. be the illuminating power (candle-power) of a source of light, then c.p. is the intensity of illumination it produces at unit distance (light falling normally). Hence to develop an expression for the intensity of illumination produced by this source on a surface at distance d, the angle of incidence being θ, we have:—

$$\frac{\text{Illumination at distance } d \text{ (light normal)}}{\text{c.p., } i.e. \text{ illumination at unit distance (light normal)}} = \frac{1}{d^2},$$

$$\therefore \text{ Illumination at distance } d \text{ (light normal)} = \frac{\text{c.p.}}{d^2}.$$

Hence if I be the intensity of illumination produced by this source at distance d when the angle of incidence is θ:—

$$\text{I} = \frac{\text{c.p.}}{d^2}\cos\theta \text{ (candle feet)}.$$

(*b*) Imagine a source A at distance a from a screen, the light falling normally: the intensity of illumination of the screen is given by I = (c.p. of A)/a^2. Suppose A is now removed and another source B substituted, and let its distance b from the screen be adjusted until it produces the *same* intensity of illumination as above: then I = (c.p. of B)/b^2. Hence:—

$$\frac{\text{c.p. of A}}{a^2} = \frac{\text{c.p. of B}}{b^2}, \quad i.e. \quad \frac{\text{c.p. of A}}{\text{c.p. of B}} = \frac{a^2}{b^2} = \frac{(\text{Distance of A})^2}{(\text{Distance of B})^2},$$

i.e. *the illuminating powers (candle powers) of two sources of light are directly proportional to the squares of the distances they must be placed from a screen in order to produce the same intensity of illumination.* This is the principle used in **photometry** (measurement of the candle-powers of various lamps), for clearly if B above is a standard of known c.p., the c.p. of A is determined.

The student must not confuse the above with the **inverse square law** in connection with the illumination of a surface at different distances from a source.

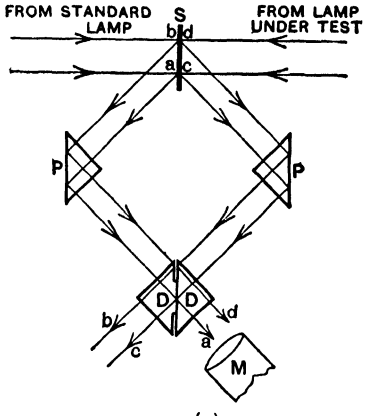
The **mean horizontal candle-power** (M.H.C.P.) of a lamp is the mean of the candle-powers in all directions in a horizontal plane passing through the centre of the light.

The **mean spherical candle-power** (M.S.C.P.) is the mean of the candle-powers in **all** directions from the centre of the light: it is therefore that c.p. uniform in all directions, which would give the same total illumination as the particular lamp in question.

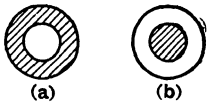
The **mean hemispherical candle-power** (M.H.S.C.P.) is the mean of the candle-powers in all directions below the horizontal plane through the centre of the light (this is the *lower* M.H.S.C.P.; sometimes the *upper* M.H.S.C.P. is referred to).

The **spherical reduction factor** of a lamp is the ratio (M.S.C.P.)/(M.H.C.P.), *i.e.* it is the number which the M.H.C.P. must be multiplied by in order to obtain the M.S.C.P. The reduction factor for a carbon filament lamp is about ·82 and for a metallic filament lamp about ·75.

191. Photometers.—An instrument for comparing and measuring the illuminating powers of different sources of light is called a *photometer*. One type—the **Bunsen photometer**—is described in Chapter XXI., *and the student should read this before proceeding further.*



(a)



(*a*) Left side of S brighter.
(*b*) Right side of S brighter.

(*b*)

Fig. 199.

The **Lummer-Brodhun photometer** consists of a milk-white screen S, Fig. 199(*a*), two totally reflecting prisms P P and two prisms D D placed base to base, the one on the left being ground down round the edge so that the two are really in contact only over a circular area round about the centre.

Any light from the left, which, after reflection at S and P falls on the central parts of D D in contact, passes through to the telescope M (e.g. *a*) whilst any light from the left which falls on the outer ground parts of D D is reflected again and does not enter M (e.g. *b*). Again, any light from the right which after reflection at S and P (on the right) falls on the central parts of D D in contact passes straight on and does not enter M (e.g. *c*), whilst light from the right which falls on the other part of D is again reflected and enters M (e.g. *d*).

Thus, on looking through M the central part of the field of vision is illuminated by light from the left of S and the outer part by light from the right of S and the appearance may be as indicated in Fig. 199(*b*). The distances of the two lights from S are adjusted until the two parts appear the same, in which case the two lights are producing equal intensities of illumination at S and

$$\frac{\text{c.p. of lamp}}{\text{c.p. of standard}} = \frac{(\text{distance of lamp from S})^2}{(\text{distance of standard from S})^2}$$

so that the c.p. of the lamp is determined.

For other photometers—the Rumford, Joly, Flicker, Trotter, etc.—and for descriptions of the standard lamps the student must refer to some advanced text-book on Light.

192. Determination of the M.H.C.P., the M.S.C.P., and the M.H.S.C.P. of Lamps. Polar Curves or Light Distribution Curves.—Imagine a glow-lamp at the centre of a table and let its c.p. be determined *in various horizontal directions* on all sides of it: the mean of all these candle-powers gives *the mean horizontal candle-power* (M.H.C.P.). This statement requires a little modification if the candle-powers are vastly different in different horizontal directions. Another method of finding the M.H.C.P. is to use a "spinner": this rotates the lamp about a vertical axis at about 150 revolutions per minute and the c.p. horizontally of this rotating lamp, measured in the ordinary way is really the M.H.C.P.

In determining the M.S.C.P. of a glow-lamp in the laboratory, it is customary to select, say, six vertical planes passing through the lamp, and to measure *in each plane* the candle-power in the various directions L A, L B, etc. (Fig. 200). Taking now the direction L A, the mean of the candle-powers in this direction is estimated; this is repeated for each of the other directions, and these mean values are used in plotting the curve L A′ B′ C′, etc., which is termed the "mean vertical polar curve" (Fig. 200).

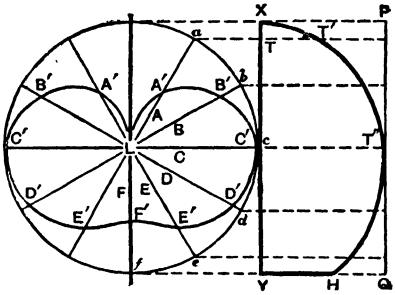


Fig. 200.

In this L A′ represents the mean of the candle-powers in the direction L A for the six vertical planes, L B′ the mean candle-power in the direction L B, etc. Selecting now the right-hand side, say, of this polar curve, a circle is drawn touching it, and the vectors are produced to meet the circle in *a*, *b*, *c*, etc. These latter points are next projected on the tangent X Y, and distances are marked off along the dotted perpendiculars equal to the candle-powers along the corresponding radii; thus T T′ = L A′, etc. On joining these points the curve XT′T″H is obtained; in it *c*T″ is (say) the M.H.C.P., and it is matter of simple proof that—

$$\frac{\text{M.S.C.P.}}{\text{M.H.C.P.}} = \frac{\text{Area of figure XT′T″H Y}}{\text{Area of rectangle X P Q Y}}.$$

i.e.

$$\frac{\text{M.S.C.P.}}{c\text{T}''} = \frac{\text{Area of figure XT′T″H Y}}{c\text{T}'' \times \text{X Y}}$$

$$\therefore \text{M.S.C.P.} = \frac{\text{Area of figure XT′T″HY}}{\text{Length X Y}},$$

i.e. the M.S.C.P. is given by the *height* of the rectangle on the base X Y which has an area equal to the figure X T′T″H Y.

Similarly the *mean hemispherical candle-power* (lower) is given by:—

$$\text{M.H.S.C.P.} = \frac{\text{Area H T''}cY}{\text{Length }cY}.$$

In the case of an arc lamp, it is usual to carry out the tests in one vertical plane only. Fig. 201 shows the polar diagram for the arc, and proceeding as in the case of the glow-lamp explained above it follows that:—

$$\frac{\text{M.S.C.P.}}{\text{Maximum candle-power (L L)}} = \frac{\text{Area of figure X T Y}}{\text{Area of rectangle X P Q Y}}$$

i.e.

$$\text{M.S.C.P.} = \frac{\text{Area of figure X T Y}}{\text{Length X Y}}$$

as indicated above. The M.S.C.P. of an ordinary arc is approximately equal to $\frac{1}{2}$ (horizontal c.p.) + $\frac{1}{4}$ (maximum c.p.).

Fig. 202 gives the distribution curves for (*a*) a flame arc; (*b*) a Nernst lamp with horizontal filament (full line —in plane perpendicular to the filament: innermost dotted line—in plane of the filament: middle dotted line—mean values) and (*c*) a half-watt tungsten lamp (G.E.C.) fitted in lantern with special reflector for outdoor lighting. Reflectors do not of course increase the total flux of light but they modify the distribution.

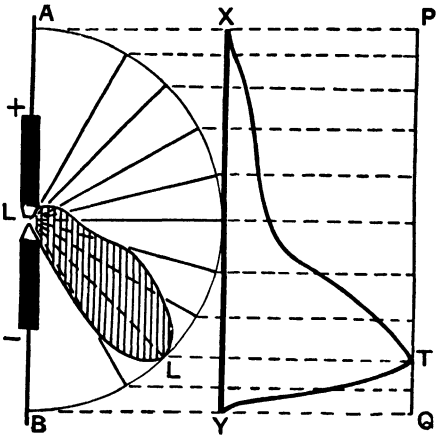


Fig. 201.

193. Other Terms used in Illumination.—The total light emitted per second by a source is called the **luminous flux.** One unit—the **spherical candle-power**—is the flux from a source giving one candle-power in all directions. Another unit—the **lumen**—is the flux along unit solid angle from the above source; hence a spherical candle is equal to 4π lumens. Another unit—the **spherical hefner**—is used: it is the flux from a source giving one hefner unit in all directions and is equal to about ·9 of a spherical candle. Again, the lumen is sometimes defined as the flux along unit solid angle from a source giving, not one c.p., but one hefner unit in all directions: in this case the spherical hefner is equal to 4π lumens. The total amount of light

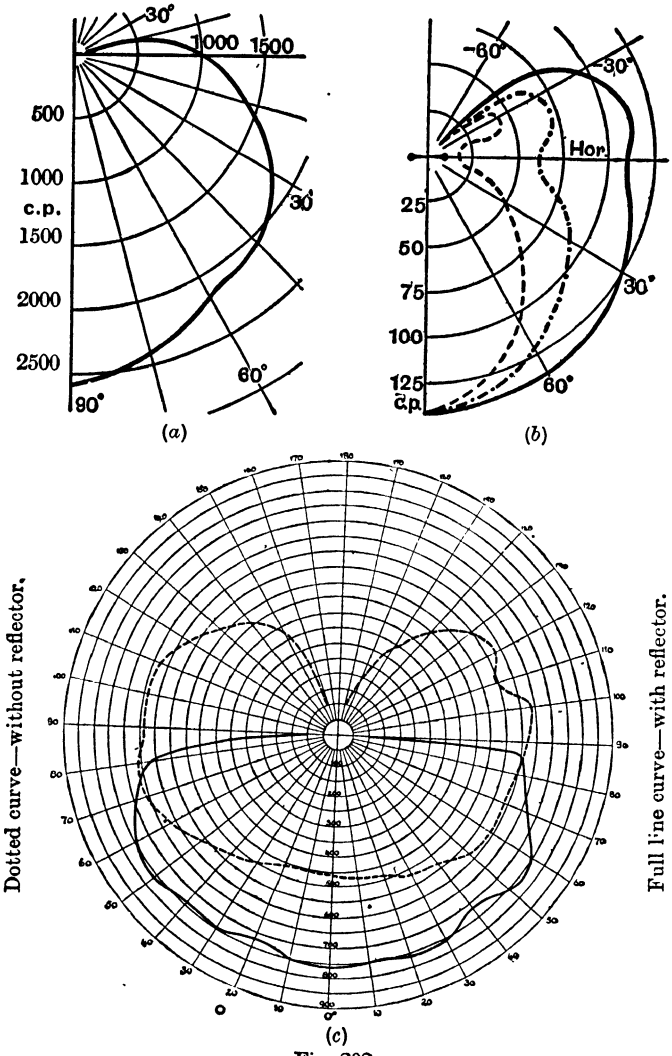


Fig. 202.

emitted in one hour by a source of one spherical candle-power is called a **candle hour**: similarly a **lumen hour** is the light which passes in one hour with a flux of one lumen. **Intrinsic brightness** is the candle-power emitted per unit area of a source (or of a reflecting surface) normal to its surface.

194. Horizontal Illumination.—Let L be a lamp situated at a height h (Fig. 203) and giving the same c.p. in all directions. The intensity of illumination at O is c.p./h^2 and, at P it is given by:—

$$\text{Intensity of illumination at P} = \frac{\text{c.p.}}{d^2}\cos\theta = \frac{\frac{\text{c.p.}}{h^2}\cos\theta}{\cos^2\theta}$$

$$= \frac{\text{c.p.}}{h^2}\cos^3\theta.$$

But c.p./h^2 is the intensity of illumination at O so that $\cos^3\theta$ is the factor which the intensity of illumination at O must be multiplied by to obtain the illumination at any other point: $\cos^3\theta$ is called the **"illumination reduction factor."**

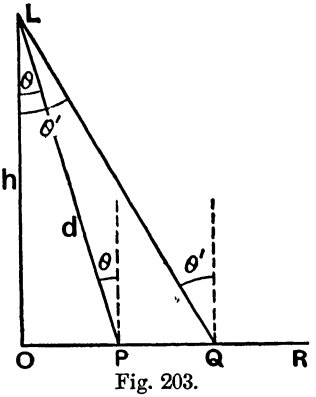


Fig. 203.

The preceding indicates the method of finding the illumination at various points on a horizontal plane and of plotting *horizontal illumination curves*. Thus if L be a lamp giving 50 c.p. in all directions and fixed 8 feet above the point O, the illumination at O is $50/8^2$, *i.e.* ·78 *candle foot*. If P be 2·5 feet from O, then tan θ is 2·5/8 = ·3125 so that $\theta = 17°\ 21'$ (from tables) and cos θ is ·9545: hence the illumination at P = ·78 × (·9545)^3, *i.e.* ·67 *candle foot*. If Q be 5 feet from O then tan θ' is 5/8 = ·625 so that $\theta' = 32°$ (about) and cos θ' is ·848: hence the illumination at Q is ·78 × (·848)^3, *i.e.* ·47 *candle foot*.

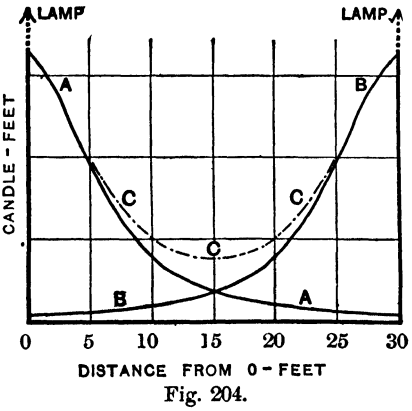


Fig. 204.

This is repeated for other points, R, etc., and finally a curve is plotted with distances as abscissae and candle feet as ordinates. The general shape of such a curve is shown in Fig. 204 (curve A). If a similar lamp be similarly placed some distance from the first a similar curve is obtained for it (curve B) and the total illumination at any point will be obtained by adding the ordinates of the curves: the dotted curve C gives the total illumination.

Another method of solution is as follows:—the intensity of illumination at P is (c.p. cos θ)/d^2. Now d^2 is $(8^2 + 2{\cdot}5^2) = 70{\cdot}25$ and cos θ is $8/\sqrt{70{\cdot}25}$. On substituting, the illumination at P is determined.

If the c.p. of the lamp is different in different directions then the c.p in the various directions L P, L Q, etc. must be used in the calculation

195. Choice of a Lamp.—This depends on so many circumstances that *actual experience with various types under requisite conditions is the best and truest guide.*

Problems on comparative lamp costs may be worked from the point of view of **lamp hours** or **candle hours.** Thus as a simple example take the case, say, of a 16 c.p. lamp taking 30 watts lasting 450 hours and costing 1/6 (evidently pre-war). The cost of energy is, say, 4d. per unit: then:—

$$\text{Cost of energy per hour} = \left(\frac{30}{1000} \times 4\right)\text{d.} = {\cdot}12 \text{ pence.}$$

$$\text{Cost of renewals per hour} \doteq \frac{18}{450}\text{d.} = {\cdot}04 \text{ pence.}$$

$$\therefore \text{ Total cost } \textbf{per lamp hour} = {\cdot}16 \text{ pence.}$$

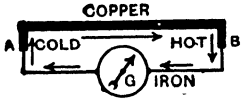
Now the c.p. is 16

$$\therefore \text{ Total cost } \textbf{per candle hour} = \frac{{\cdot}16}{16} = {\cdot}01 \text{ pence.}$$

By working out these costs for various types of lamps the reader will be able to decide (?) what is best to do under given circumstances. In a general way if a definite number of lamps are to be purchased the "lamp hour" decision is better, but if a contractor is dealing with a big room where the number of lamps depends on the candle-power required, the "candle hour" is better.

196. The Establishment of an E.M.F. Thermally.—This branch of our subject—known as **Thermo-Electricity**—is not of vital importance to the electrical engineer, but a brief outline of the essential points is advisable.

(1) Fig. 205 represents pieces of copper and iron wire joined together at their ends A and B, and G is a low resistance mirror galvanometer included in the circuit. If both junctions be initially at 0° C., and the junction B be then gradually heated, a current will flow in the circuit in the direction indicated, viz. copper to iron through the hot junction, and iron to copper through the cold junction. This current will increase in strength until the hot junction is at a temperature of about 270° C. (different for different specimens of iron and copper), at which stage the maximum current will be flowing.



SEEBECK
Fig. 205.

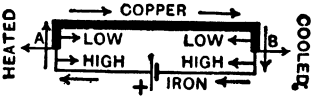
If heating be continued, the current will decrease in strength, and will be zero when the hot junction is at 540° C. On heating still further the junction B, the current will increase again, but *it will be reversed in direction*, i.e. it will flow from iron to copper through the junction B, and from copper to iron through the junction A. Currents produced in this way by heating junctions of different metals are known as *thermo-electric-currents*, and were discovered by **Seebeck** in 1821. The phenomenon of *inversion* of the E.M.F. was discovered later by Cumming.

If both junctions of the copper-iron couple be initially at 10° C., say instead of at 0° C., there will again be maximum current when the hot junction is at 270° C., but there will be zero current and reversal in this case when the hot junction is at 530° C. The temperature of the hot junction at which maximum current flows is a constant for a given couple, and is known as the **neutral temperature** for that couple. The temperature of the hot junction at which there is zero current and reversal is a variable one, being always as much above the neutral temperature as the cold junction is below it; thus, if the cold junction be at T° C. and the neutral temperature be θ° C., there will be inversion of current when the hot junction is at a temperature $\{\theta + (\theta - T)\}$° C.

Let the two junctions of a couple be at T° C. initially, and then let one of them be raised in temperature by a *very small* amount dT; if dE be the corresponding small E.M.F. generated, the ratio $\frac{dE}{dt}$ is known as the **Thermo-Electric Power** of the two metals at the temperature T. If one junction of a couple be at T_1° C. and the other at T_2° C., then, *provided that one of these is not above and the other below the neutral temperature* θ, the total E M.F. is given by the product of the thermo-electric power at the mean temperature $\frac{1}{2}(T_1 + T_2)$ and the difference in temperature $(T_2 - T_1)$. If T_1 be *below* and T_2 *above* the neutral temperature θ, the actual E.M.F. in the circuit is found thus:—

Total E.M.F. = (E.M.F. with junctions at T_1 and θ) − (E.M.F. with junctions at θ and T_2).

(2) Consider now the arrangement indicated in Fig. 206. Delicate experiments show that with the current passing in the direction of the arrow, heat is *absorbed* at the junction B and *generated* at the junction A, and by comparing Figs. 205 and 206 it will be seen that that particular junction is cooled, which must be heated in order to give a thermo-electric current in the same direction as the battery current. This discovery was made by Peltier in 1834, and it is known as the **Peltier Effect.**



PELTIER
Fig. 206.

The explanation of the Peltier Effect lies in the fact that contacts of dissimilar metals give rise to potential differences. At B (Fig. 206) the current traverses a junction where there is a *rise in potential*; it therefore gains energy, which is absorbed as heat, and a cooling effect results. At A it traverses a *fall of potential*, gives out energy as heat, and a heating effect results.

(3) In 1856 Professor Thomson discovered a phenomenon in unequally heated conductors somewhat similar to the Peltier Effect, and the name **Thomson Effect** has been given to it. In an unequally heated copper wire the hotter parts are at a higher potential than the colder parts; hence if a current passes along such a copper wire from "cold to hot," heat is absorbed, but if it passes from "hot to cold" heat is evolved. Of course on the whole the Joule heating effect predominates.

In the case of iron, the colder parts are at the higher potential, and the result is opposite to that in copper. In lead the Thomson Effect is *nil*.

EXERCISES XV.

1. Of what material should fuses for 200-volt circuits be made, and what length of break should be allowed for a 5-ampere fuse? State in what way the current at which the fuse will melt depends on (*a*) the length, (*b*) the diameter of the fuse wire.

2. Describe shortly the construction of a glow-lamp. Why does the filament become hotter than the platinum leading-in wires? Why does the globe of a bad lamp get very hot? (P.)

3. In what way does the construction of a 200-volt lamp differ from that of a 100-volt lamp? What are the relative advantages of the two kinds? (P.)

4. What are the advantages and disadvantages of running incandescent lamps at high efficiency? (O.) State the circumstances under which it is economical to work glow-lamps above their normal voltage. (O.)

5. State the conditions under which the electric arc can be formed and maintained between two carbon electrodes, and describe carefully the difference in shape, temperature, and rate of consumption of the two electrodes. (B.)

6. Discuss the nature of the relation between the current passing through an arc lamp, and the difference of potential between the carbons. (B.)

7. A single arc lamp is supplied with current from a pair of mains at a pressure of 100 volts. What other apparatus is necessary in addition to the lamp itself (*a*) when the current is direct, (*b*) when it is alternating? (P.)

8. Why are glow-lamps generally arranged in parallel, while arc lamps are frequently joined in series? (P.)

9. Describe with sketches any form of clutch or brake mechanism for an arc lamp. (P.)

10. What is an enclosed arc lamp? What are its advantages and disadvantages as compared with the open type? (P.)

11. Describe the construction and action of the Nernst lamp, and state its advantages and disadvantages. (B.)

12. A 90-volt 8 c.p. glow-lamp in series with a resistance is placed across 100-volt constant pressure mains. On applying a voltmeter across the lamp terminals to ascertain whether the pressure is correct the light diminishes. Explain exactly how this occurs. (P.)

13. A wire of resistance r connects A and B, two points in a circuit, the resistance of the remainder of which is R. If, without any other change being made, A and B are also connected by $(n-1)$ other wires, the resistance of each of which is r, show that the heat produced in the n wires will be greater or less than that produced originally in the first wire according as r is greater or less than $R\sqrt{n}$. (B.)

14. The E.M.F. of a battery is 18 volts and its resistance 3 ohms. The P.D. between its poles when they are joined by a wire A is 15 volts, and falls to 12 volts when A is replaced by another wire B. Compare the resistances of A and B, and the amounts of heat developed in them in equal times. (B.)

15. If electric energy for heating purposes be supplied at 2d. per unit, find the cost of boiling a quart of water in 5 minutes, and the mean power required. Initial temperature of water 15° C.; efficiency of kettle 90 per cent. (P.)

16. Describe some good type of photometer, and explain how you would determine the c.p. of a lamp by it.

17. Define *intensity of illumination, illuminating power or intensity of a source, luminous flux, intrinsic brightness* of a surface, *spherical candle-power*, and the *spherical reduction factor for a lamp.* Explain how you would determine the M.S.C.P. of an arc lamp, and show how the curve of illumination on the surface of the ground below the lamp may be determined.

18. Explain the action of the three types of arc lamps, viz. series wound, shunt wound, and differentially wound, saying for what purpose each is specially suited.

CHAPTER XVI.

GENERAL PRINCIPLES OF ELECTROMAGNETISM.

197. Magnetic Field due to a Current: Action of a Current on a Magnet.—Although some of the more elementary properties of **electro-magnetism** have been already mentioned in connection with their application, these properties will be again included in their proper order on account of their great importance when investigating electrical effects from first principles.

The history of that part of electrical science which is known as electro-magnetism dates from 1820, when Arago discovered that iron filings in the neighbourhood of a current-bearing conductor were influenced by it in such a way as to suggest that *the conductor was surrounded by a magnetic field.* The magnetic effects of a current may easily be shown by a series of simple experiments:—

(1) Bore a hole in a piece of cardboard, fix in a horizontal position, pass a wire vertically through it, and let a strong current flow in the wire. Sprinkle iron filings on the cardboard and gently tap the latter. The filings arrange themselves in concentric circles round the wire as their common centre; these indicate the lines of force in the magnetic field due to the current.

(2) Arrange that the current is flowing *down* the wire. Move a small compass needle round the wire and note in which direction the north pole points. In this case it will be found that the north pole points as indicated by the arrows in Fig. 207 (*a*), *i.e.* the positive direction of the lines of force is *clockwise.* Repeat with the current flowing *up* the wire: the north pole will move in the opposite direction, *i.e.* the positive direction of the lines is *counter-clockwise* (Fig. 207 (*b*)).

(*a*) (*b*)

Fig. 207.

(3) Place a compass needle on the table and hold a wire *above* and parallel to it, as shown in Fig. 208. Pass a current through the wire (*a*) from south to north, (*b*) from north to south, and note in which direction the north pole of the needle is deflected; it will be found that in (*a*) the north pole of the needle moves towards the

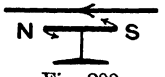


Fig. 208.

west, and in (*b*) towards the *east*. Hold the wire *below* the needle and again pass a current (*c*) from south to north, (*d*) from north to south; it will be found that in (*c*) the north pole of the needle is deflected towards the *east* and in (*d*) towards the *west*.

The first experiment above was originally due to **Arago** and the last to **Oersted**. The results may be obtained from the following rules which the reader should verify from the experiments:—

(1) AMPÈRE'S RULE.—*Imagine a man swimming in the circuit in the direction of the current and with his face towards the needle; the north pole of the needle will be deflected towards his left hand.*

(2) RIGHT-HAND RULE.—*Hold the thumb of the right hand at right angles to the fingers. Place the hand on the wire with the palm facing the needle and turn the fingers in the direction of the current; the thumb will point in the direction in which the north pole will be deflected.*

From (*a*) and (*d*), or (*b*) and (*c*), of experiment (3) above, it follows that if a compass needle be placed at the centre of a coil of several turns of wire, the plane of the coil being set in the meridian, and a current be passed, the currents in all the wires above and below the needle will tend to deflect it in the same direction and thus a weak current is enabled to produce a deflection: this is utilised in galvanometers, etc. (Arts. 134, 135, etc.).

(4) Bend a copper wire into a circle of about 10 inches diameter and fix to a horizontal sheet of cardboard as shown in Fig. 209. Pass a strong current and obtain the lines of force by filings as before. Set the coil with its plane in the magnetic meridian, place a compass needle at the centre of the coil and note the direction in which the north pole is deflected. This gives the direction of the magnetic field at the centre of the coil; in Fig. 209 the positive direction is away from the observer.

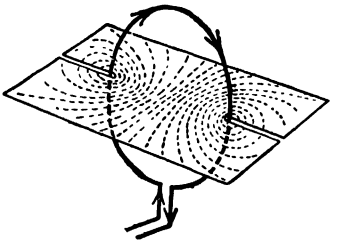


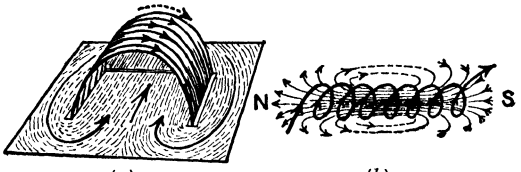
Fig. 209.

(5) Reverse the current and repeat the experiment. The lines of force will be as before, but the direction of the field will be reversed.

The direction of the field at the centre of a circular coil carrying a current may be obtained from the rules given above, but the following is also convenient:—*Looking at the face of the coil, if the current is clockwise, the positive direction of the lines inside the coil is away from the observer; if the current is counter-clockwise, the positive direction is towards the observer.*

(6) Make a solenoid by winding insulated copper wire on a glass or cardboard tube and fix in a horizontal sheet of cardboard as shown in Fig. 210. Pass a current and obtain the lines of force by filings and the direction of the field inside and outside by a compass. Reverse the current and repeat.

An examination of Fig. 210 (*a*) will show that the magnetic field in the case of a solenoid carrying a current resembles the magnetic field of a bar magnet. The magnetic lines leave one end of the solenoid, pass through the outside field and enter the other end, completing their circuit through the solenoid itself. In fact, Fig. 210 (*a*), for example, corresponds to a bar magnet, the near end, *at which the current is circulating clockwise*, corresponding to the *south pole*, and the remote end, *where the current is circulating counter-clockwise*, corresponding to the *north pole*. Fig. 210 (*b*) will serve to emphasise these facts.



(*a*) (*b*)

Fig. 210.

(7) Ampère constructed an apparatus in which a single coil of wire, the ends of which made contact in cups of mercury, was suspended so as to be free to rotate about a vertical diameter. When a current passed in the coil it was affected by the earth's magnetic field, and set itself at right angles to the magnetic meridian, *the face at which the current flowed counter-clockwise being towards the north.* Again, if a solenoid be suitably suspended, and a current be passed, it will tend to set itself in the meridian, *the end where the current flows counter-clockwise being towards the north, and the end where it flows clockwise towards the south:* by bringing a magnet near it can be verified that *the former is a north end and the latter a south end.*

From the results of the above experiments the following rules for the polarity of a current-carrying solenoid are deduced:—

(1) Ampère's Rule.—*Imagine a man swimming in the circuit in the direction of the current and with his face towards the inside of the solenoid; his left hand will be towards the north end of the solenoid.*

(2) Right-Hand Rule.—*Hold the thumb of the right hand at right angles to the fingers. Place the hand on the solenoid with the palm facing the inside and turn the fingers in the direction of the current; the thumb will be towards the north end of the solenoid.*

(3) End Rules.—*Look at the end of the solenoid; if the current is* **counter-clockwise** *that end is a* **north,** *if it is* **clockwise** *that end is a* **south.**

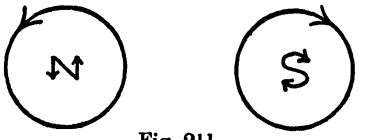


Fig. 211.

The last rule is important, and the drawing of Fig. 211 will assist the student to remember it. If a bar of iron be placed inside a solenoid it is converted into a magnet when the current passes, the polarity of the iron being the same as that of the solenoid: such magnets are termed *electro-magnets.*

198. Effect of a Magnet on a Current-bearing Conductor.—Just as a conductor carrying a current exerts a very definite influence on neighbouring magnets, so also does a magnet exert a reciprocal influence on neighbouring current-carrying conductors. In fact the phenomena are connected in much the same way as the action and reaction of mechanical forces.

When our coil surrounded the needle, the latter moved simply because the resistance offered to its motion at the point of support was very small compared to that offered by the coil, which would usually be fixed in a wooden framework. If, however, the needle had intentionally been made heavy, or had been artificially restricted in its motion, and if also the coil had been suspended so that rotation of it could easily have been effected, then the coil would have moved and the needle remained at rest; for any couple action which is called into play by the electrical forces tending to cause rotation of the needle in any one direction, will invariably and necessarily be accompanied by an equal couple action tending to cause rotation of the coil in the opposite direction. Fig. 212 gives the direction in which a north pole N tends to move round the current-carrying wire A, and the direction in which A tends to move round N.

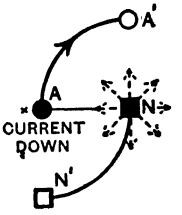


Fig. 212.

This motion of a conductor in a magnetic field may be examined a little more in detail. In Fig. 213, A is the cross-section of a straight wire placed at right angles to the magnetic field due to the poles N and S, *i.e.* at right angles to the plane of the paper; the lines at B and C indicate the general direction of the lines of force of the field.

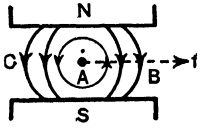


Fig. 231.

Imagine now that a current passes *upwards* in the wire; the circle about the wire indicates the general direction of the lines of force of the field due to the current. It will be seen that on the side C the two sets of lines are in the same direction and therefore repel each other, whilst on the side B the two sets are in opposite directions and therefore cancel each other. The result is that the wire is acted on by a force in the direction indicated, *i.e.* towards the right; the tendency is, therefore, either for the conductor to move towards the right, or for the field to move in the opposite direction. Of course the *actual* field between N and S is the resultant of the two component fields above, and is indicated in Fig. 214.

An examination of Figs. 212-214 will show that *the direction in which a conductor carrying a current, and placed as indicated in a magnetic field, tends to move* is given by the **Left-Hand Rule** due to Fleming stated in Art. 136 (see also Fig. 214). For convenience we repeat it here:—**Hold the thumb and the first two fingers of the**

left hand mutually at right angles; place the forefinger in the direction of the lines of force of the field in which the conductor is situated (N to S), and turn the hand so that the middle finger points in the direction of the current; the thumb will indicate the direction of motion of the conductor.

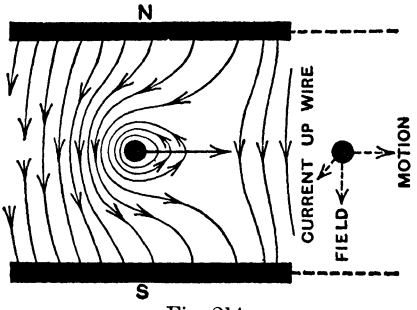


Fig. 214.

199. Action of One Current-bearing Conductor on Another.—If we place two flat spirals of wire a short distance apart, with their axes in the same straight line, and pass currents round them, then (provided they have suitable freedom of movement) they are found to approach or recede from one another, thus indicating respectively attraction or repulsion, according as the currents circulate round them in the same or in opposite directions. Bearing in mind that the current renders each spiral practically a magnet, and also the fundamental law of electromagnetic polarity, it is easy to see that these actions are in accordance with the laws of attraction and repulsion between two magnets.

Ampère demonstrated in the case of straight circuits the following laws (see Art. 140):—

(1) *Two* PARALLEL *currents attract or repel one another according as they flow in the same or in opposite directions.*

(2) *Two* NON-PARALLEL *currents attract one another if both approach or both recede from the point of meeting of their directions, while they recede one another if one approaches and the other recedes from that*

(2) 1.

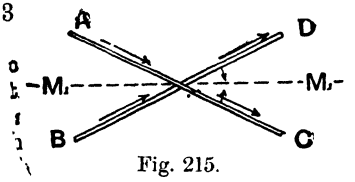


Fig. 215.

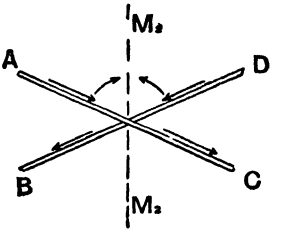


Fig. 216.

Thus if A C and B D are two conductors carrying currents which are flowing in the directions indicated, in Fig. 215 from A to C and B to D, and in Fig. 216 from A to C and D to B, then the conductors

will be urged to move so that the currents in both of them will tend to flow in the same direction. Thus in Fig. 215 the conductors will be attracted to some mean position M_1M_1, and in Fig. 216 to some mean direction M_2M_2.

The experiments whereby Ampère's laws are established merely consist in having two circuits, one fixed and the other movable, placing them in various relative positions, and passing currents in sundry directions. One form is indicated in Fig. 217; the movable coil is bent in the form of two rectangles in such a way that the current flows through them in opposite directions (thus nullifying the effect of the earth on the apparatus), and the coil is suspended from the mercury cups C, C; on bringing the second current-carrying circuit into various positions the laws may be verified.

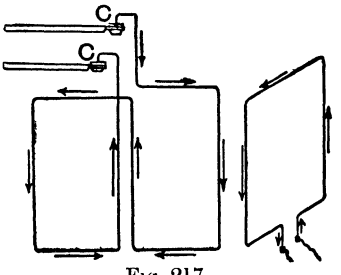
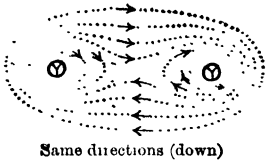


Fig. 217.

Fig. 218 gives the field in the case of two parallel wires carrying equal currents in the *same* direction (downwards), and Fig. 219 the field about two parallel wires carrying equal currents in *opposite* directions. It is evident from these lines of force that there will be attraction in the first case and repulsion in the second case, for "lines" tend to *contract*, and "lines proceeding in the same direction" tend to *repel*.


Same directions (down)
Fig. 218.

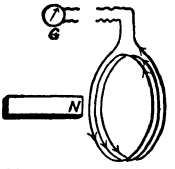
Opposite directions
Fig. 219.

A further consequence of this association of a magnetic field and a current is the phenomena known under the name of **induction.**

200. Induced E.M.F.'s and Currents.—In 1831 Faraday showed that **whenever the magnetic flux through a circuit is changed an E.M.F. is developed in the circuit,** such E.M.F. **lasting only while the change is taking place;** E.M.F.'s and currents produced in this way are spoken of as **induced** E.M.F.'s and currents respectively. The more rapid the change the bigger is the induced E M.F. and current.

201. Induction of Currents by Magnets.—If a magnet be moved in the neighbourhood of a coil in such a way that the number of magnetic lines passing through the coil is altered, then a current is produced or *induced* in the coil so long as the alteration lasts. If the magnet be fixed and the coil be moved the same thing occurs. A simple experiment will illustrate this :—

Fig. 220 shows a coil connected to a galvanometer. On quickly inserting the N pole of a magnet the galvanometer is momentarily deflected, showing that a current is induced in the coil, and from the direction of the deflection it can be deduced that the current is *counter-clockwise* looked at from the side approached. On withdrawing the N pole the induced current is in the opposite direction, viz. *clockwise*. On inserting the S pole the induced current is *clockwise* : it is *counter-clockwise* when the S pole is withdrawn. It can also be shown that the more quickly the magnet is moved the greater is the deflection and, therefore, the induced current.



Magnet approaching Coil
Fig. 220.

From the preceding we learn that when the north pole is inserted the induced current has such a direction that the face of the coil approached is a north face, *which, therefore, tends to oppose the motion of the magnet.* When the north pole is withdrawn the induced current has such a direction that this face is a south face, which, therefore, *tends to draw the magnet back again,* and so on. Hence we have the important law known as **Lenz's Law,** viz. **the direction of the induced current (and E.M.F.) is such that it tends to oppose the motion or change which produces it.**

202. Induction of Currents by Currents (Mutual Induction).—This also can be illustrated by a simple experiment :—

In Fig. 221 A B diagrammatically represents a *coil* of wire joined to a galvanometer ; it is referred to as the *secondary circuit.* C D represents a *coil* in series with a battery and key ; it is referred to as the *primary circuit.*

Now start a current in the primary in the direction C to D and a momentary deflection of the galvanometer will ensue, showing that a current is induced in the secondary in the direction B to A, *i.e.* opposite or **inverse** to the primary current ; it should be noted that starting a current in the primary means **increasing** the flux in the secondary, for the flux in the primary naturally reaches over to the secondary circuit.

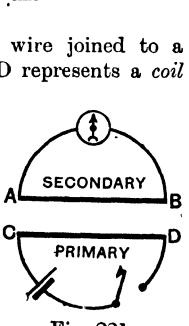


Fig. 221.

Switch off the primary current, and a momentary deflection of the galvanometer will follow, indicating a secondary current in the direction A to B, *i.e.* in the same direction as, or **direct** to, the primary current.

and it should be noted that stopping the primary current means **decreasing** the flux in the secondary.

Similarly, *increasing* the primary current or *moving it nearer* to the secondary circuit results in an *inverse* current; *decreasing* the primary current or *moving it away from* the secondary circuit results in a *direct* current.

From the preceding we arrive at another important law, viz. **an increase in the flux results in an induced inverse current, whilst a decrease in the flux results in an induced direct current** (**I**ncrease – **I**nverse : **D**ecrease — **D**irect). This is quite in agreement with Lenz's Law, *e.g.* if the primary current flows from C to D and the primary is moved nearer the secondary, the induced secondary current flows from B to A; these two parallel currents, being in opposite directions, repel each other, so that the tendency is to drive the primary back again. Further, by considering the flux, it will be clear that the flux due to the induced current is always such as to oppose the change in the flux due to the primary. It should also be noted that not only does the primary act upon the secondary, but that the secondary reacts upon the primary; hence these effects are known as **mutual induction.**

203. Self-induction.—We have already seen that the growth of a current means the growth of a magnetic field, also that an altering magnetic field near a conductor produces a current in the conductor; hence it is not surprising to find that this magnetic field which grows with the current *acts upon the current itself*; that is, it induces an E.M.F. in the original conductor acting in such a way as to oppose the growth of the current. Hence, when we start a current in a wire, the current produces a field, this reacts upon the current by setting up an *opposing E.M.F.* which delays its growth, the result being that it takes time to cause a current to reach its full value in the circuit. Similarly, when the current in a circuit is broken, the field is destroyed, and an induced *current* in the *same* direction as the one cut off is established. These effects are known as **Self-induction.**

It should be noted that the same rules apply here as in Art. 202. Thus, when we start or increase the current in a circuit (flux *increasing*) the induced E.M.F. is *inverse*: when we stop or decrease the current (flux *decreasing*) the induced current is *direct.*

From the preceding it will be clear that self-induction behaves in a circuit like *inertia, e.g.* when we try to produce a current in a circuit this self-induction or inertia tends to choke the current back, and when we try to stop the current this self-induction tries to make it keep on.

As a consequence of these facts, it is found that when we complete an electrical circuit containing a battery, an electromagnet, and the necessary connecting wires, the growth of the current is opposed *and there is no spark on make*; but when the circuit is opened by separat-

ing two conductors as usual, a heavy spark may be obtained, this being due to the storage of energy in the magnetic field, which on release forms the so-called **extra current** in the same direction as the one cut off, and this in turn gives rise to the *spark at break*. In the case of a big magnet, such as the field magnet of a shunt dynamo, this discharge at break may be most dangerous, and any circuit containing large magnets should not be suddenly opened unless absolutely necessary, and then only with suitable switches.

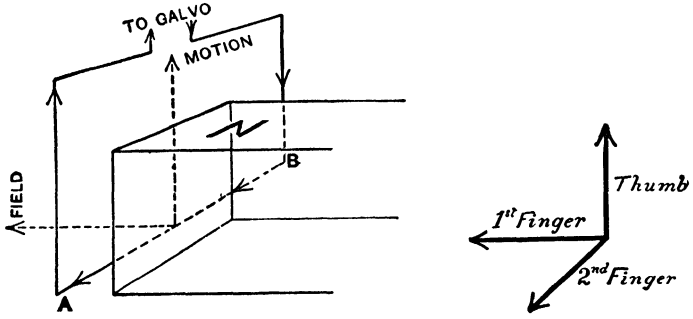


Fig. 222.

204. Fleming's Right-hand Rule.—If a wire A B, forming par of a closed circuit be moved, say, upwards in front of the north pol of a magnet, as shown in Fig. 222, the induced current will by Lenz' law be in the direction B to A (clockwise, *i.e.* S face opposite N). I such a case as this, where the wire is moving at right angles to itsel and the field, **Fleming's Right-hand Rule** is a convenient one fo finding the direction of the induced current in the wire, viz.:—

Place the thumb and the first two fingers of the right han mutually at right anlges: let the forefinger point in the direc tion of the lines of force (N to S) and the thumb in the directio of motion of the wire: the second finger will point in the direc tion of the induced current. It is clear that this rule is quite i accordance with Lenz's law.

205. Properties of an Electric Circuit.—It will now be clear that it i necessary to know three things about an electric circuit in order to make calcu lations about current production in it, viz.—**resistance, capacity,** and **induc tance** (this being the general name for self and mutual induction). The first c these, as has already been explained, depends merely upon the area and lengt of the wire, the material of which it is made, and its temperature. The othe two, however, are not constants for any particular circuit, but depend upon it circumstances. Those which affect the capacity have been already dealt with and the treatment of those which affect the inductance will be given in the nex

chapter. For the present we may note that *capacity and inductance are only of any importance when the current is varying.* With a *constant* current circuit we need only bother about the resistance.

EXERCISES XVI.

1. A bar magnet is caused to oscillate, and a thick sheet of copper is then placed beneath it. Describe and explain the effect.

2. A wire is coiled on a glass tube with a right-handed twist from one end to the middle, and a left-handed twist from the middle to the other end. A current is passed through the wire. How will a compass needle behave if moved inside the coil from one end to the other?

3. A coil of wire carrying a current is placed in a strong field between the N and S poles of a horse-shoe magnet. What will happen when (*a*) the coil is so placed that its clockwise circulating side is next the N pole; (*b*) when the anti-clockwise aspect is nearest the N pole?

4. A flat coil of wire, the ends of which are connected to a sensitive galvanometer, is (1) moved up towards the north end of a long bar magnet; (2) threaded on the bar; (3) moved along it to the south end; (4) removed to a distance. What indications does the galvanometer give during these operations? (B.)

CHAPTER XVII.

ELECTROMAGNETIC UNITS AND THEORY.

206. Magnetic Field of a Current.—Let A B (Fig. 223) represent a wire carrying a current; then, considering a very small element, *ab*, the intensity of the field at *c* due to the current in that element is—

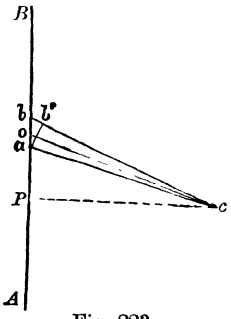


Fig. 223.

1. *Directly* proportional to the strength of the current in A B.

2. *Directly* proportional to the distance *ab'*, that is to the apparent length of the element *ab* as seen from *c*. (The line *ab'* is drawn at right angles to the line *co*, which joins *c* to *o*, the middle point of *ab*.)

3. *Inversely* proportional *to the square of the distance* of the element *ab* from *c*, that is to co^2.

Hence, if H denote the intensity of the field at *c* due to the element *ab*, and I the strength of the current in absolute units in AB, we have

$$H \propto \frac{I \cdot ab'}{(oc)^2} \quad or \quad H \propto \frac{I \cdot ab \sin a}{(oc)^2} \quad or \quad H \propto \frac{I \cdot ab \cos \theta}{(oc)^2},$$

where *a* denotes the angle *Poc*, and θ the angle *Pco*. As a matter of fact the absolute electromagnetic unit current is so chosen that with I in these units the above expressions become *equalities* (Art. 211).

207. Magnetic Field due to a Long Straight Wire carrying a Current.—To obtain this we must divide the wire into elements as above and sum up the effects due to each element: the result of this summation for any point gives the total intensity, at that point, due to the current in the conductor taken as a whole.

In the case of a finite conductor (Fig. 224) it can be shown that the field at P due to a current I absolute units in A B is given by:—

$H = \frac{I}{d}(\sin\theta_2 + \sin\theta_1)$. In the case of an **infinitely long conductor,** $\theta_1 = \theta_2 = \pi/2$, and therefore $\sin\theta_1 = \sin\theta_2 = 1$: hence:—

$$H = \frac{2I}{d} \text{ units} \quad or \quad H = \frac{2I}{10d} \text{ units},$$

the current in the first case being in absolute units and in the second case in amperes.

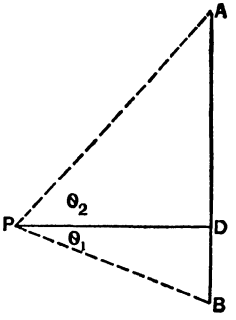


Fig. 224.

Illustration.—A unit pole 5 cm. from a very long wire carrying 20 amperes would be acted on by a force $\frac{2 \times 20}{10 \times 5} = \cdot 8$ dyne, *i.e.* the field at this distance from the wire is ·8 unit (gauss).

If this unit pole were carried once round the wire the work done would be force × distance, *i.e.* $\cdot 8 \times 2\pi 5$, *i.e.* 8π ergs. But $8\pi = 4\pi \times 2$, *i.e.* it equals $4\pi \times$ current in absolute units, and what is true in this numerical case is true in all such, i.e. *the work done in carrying unit pole once round a long straight wire carrying a current* I *absolute units is* 4π *times the current,* i.e.

$$\text{Work} = 4\pi I.$$

This is merely mentioned here; it will be dealt with later.

208. Magnetic Field at the Centre of a Circular Current.—Consider the conductor A B to be looped into a circle as shown in Fig. 225, and let us determine the intensity of the magnetic field at *c*, the centre of the circle.

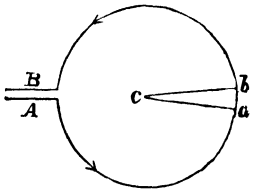


Fig. 225.

Considering the action of the element *ab*, the intensity of the field is $\frac{I \times ab}{r^2}$, where I is the current in absolute units in A B and *r* the radius of the circle. Now, since each element of the conductor is similarly placed relatively to *c*, the intensity of the field at *c* due to the conductor A B taken as a whole is $\frac{I \times AB}{r^2}$, that is $\frac{I \times 2\pi r}{r^2}$ or $\frac{2\pi I}{r}$. If the coil consists of *n* turns, sufficiently thin to be regarded as coincident this must be multiplied by *n* and we get:—

$$H = \frac{2\pi nI}{r} \text{ units} \quad or \quad H = \frac{2\pi nI}{10r} \text{ units},$$

the current in the first case being in absolute units and in the second case in amperes.

EXAMPLE.

Find the current in amperes which must flow in a ring 20 *centimetres in diameter in order that the field at the centre shall be just equal to the horizontal component of the earth's field*—i.e. 0·18 *unit.*

$$H = \frac{2\pi nI}{10r} \quad \therefore I = \frac{10Hr}{2\pi n},$$

$$i.e. \quad I = \frac{10 \times \cdot 18 \times 10}{2 \times 3 \cdot 14 \times 1} = 2 \cdot 86 \text{ amperes.}$$

209. Magnetic Field at any Point on the Axis of a Circular Current.—If we extend the preceding result to the case where the point is not at the centre of the coil but anywhere on the line through the centre at right angles to its plane, it is readily shown that:—

$$\text{Field at the point} = H = 2\pi nI \frac{r^2}{(r^2 + x^2)^{\frac{3}{2}}} \text{ units,}$$

where n = number of turns, I = current *in absolute units*, r = radius of ring in cm. and x = distance in cm. of the point along the axis from the plane of the coil; when $x = 0$ this of course reduces to $2\pi nI/r$ as in Art. 208.

EXAMPLE.

If 5 *amperes pass through a ring* 3 *cm. in radius, find the force on unit pole on the axis of the ring, but* 4 *cm. away from its plane.*

$$f = 2\pi \times \frac{5}{10} \times \frac{9}{(9 + 16)^{\frac{3}{2}}} = \cdot 23 \text{ dyne}$$

and H = ·23 unit (gauss) is also the strength of the field at the point in question.

210. Magnetic Field in the Case of a Solenoid.—A further extension of the results of Art. 209 shows that the intensity of the field at the centre of a long thin closely wound solenoid is given by:—

$$H = \frac{4\pi SI}{l} \text{ units} \quad or \quad H = \frac{4\pi SI}{10l} \text{ units,}$$

the current in the first case being in absolute units and in the second case in amperes: S = total number of turns and l = length of solenoid in cm. The field is practically uniform except near the ends and is *half the above value at the ends.* Taking the second expression, since $4\pi/10 = 1 \cdot 257$ and S/l = turns per cm. = n, say, we have:—

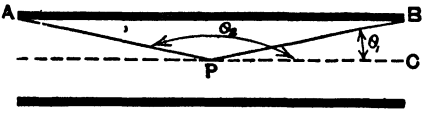
$$H = 1 \cdot 257nI = 1 \cdot 257 \text{ (ampere turns per cm.).}$$

A more complete formula for the solenoid is as follows:—The field at P (Fig. 226) is really:—

$$H = \frac{2\pi SI}{l} (\cos \theta_1 - \cos \theta_2)$$

where I *is in absolute units.* If the solenoid be *very long and thin* and P be well removed from either end, practically $\theta_1 = 0$ and $\theta_2 = \pi$, so that (cos θ_1 − cos θ_2) = 2 and :—

$$H = \frac{4\pi SI}{l} \text{ (as above).}$$



∠ CPB = θ_1 and ∠ CPA = θ_2.
Fig 226.

At the extreme end (say at C) of a very long solenoid $\theta_1 = \pi/2$ and cos $\theta_1 = 0$, whilst θ_2 is practically π and cos $\theta_2 = -1$, so that $H = \frac{2\pi SI}{l}$, *i.e.* half the intensity at points well inside. Fig. 227 gives approximately the variation of the field in the case of the solenoid.

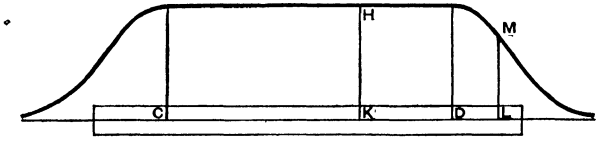


Fig. 227.

A complete proof of the formula for the field of a solenoid demands an application of the calculus (see *Advanced Text-Book of Magnetism and Electricity*), but an approximate proof may be given if we assume a result given in Art. 216 (see also Art. 207), viz. that *the work done in carrying unit pole once round a wire carrying a current* I *absolute units is* 4πI *ergs.*

Let Fig. 228 represent a portion of the solenoid. Imagine unit pole moved along the path A B C D A (Fig. 228): the work done is 4πI for *each turn* of wire. If there are n turns per unit length, the total turns in the part considered are $n\overline{AB}$ and the work done is $4\pi I \times n\overline{AB}$.

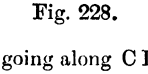


Fig. 228.

The work in going along B C and D A is *nil*, since these paths are at right angles to the lines of force. The work in going along C D is negligibly small compared with that in going along A B, for the lines of force which are crowded into a small space inside are spread out throughout the whole field outside, so that the force along C D, and therefore the work, is negligible. The work in going along A B is H × $\overline{AB}$, where H is the intensity inside; hence

$$H \times \overline{AB} = 4\pi I \times n\overline{AB},$$
$$H = 4\pi nI.$$

But $n = S/l$, where S is the total turns in the solenoid, and l the length: hence :—

$$H = \frac{4\pi SI}{l} \text{ units} \quad \textit{or} \quad H = \frac{4\pi SI}{10l} \text{ units,}$$

I being in absolute units in the first case and in amperes in the second case.

EXAMPLE.

A solenoid has 1,500 *turns, carries a current of* 10 *amperes, and its length is* 1 *metre. Calculate the magnetic field at the centre and the total "flux" if the cross-section be* 3 *square cm.*

$$H = \frac{4\pi SI}{10l} = \frac{4 \times 3{\cdot}1416 \times 1500 \times 10}{10 \times 100} = 188{\cdot}5 \text{ units}$$

$\therefore$ No. of lines per sq. cm. = 188·5 (gauses),

i.e. Total flux = 188·5 × 3 = 565·5 lines (maxwells).

211. Absolute Electromagnetic Unit of Current Strength.—From the preceding we see that, considering a single turn of wire of radius r cm. carrying a current of I absolute units, the force on unit pole at the centre is given by the expression—

$$f = \frac{2\pi I}{r} \text{dynes.}$$

If I and r be each unity, $f = 2\pi$; hence:—**The absolute unit current is that which flowing in a coil of unit radius exerts a force of 2π dynes on unit pole at the centre.**

Again, since 2π cm. is the circumference of a circle of radius 1 cm., if we consider the force due to *unit length* of the coil we find that it is one dyne. Hence—

The C.G.S. electromagnetic unit current is that which in a wire one centimetre long bent into an arc of one centimetre radius acts on unit pole at the centre with a force of one dyne.

The practical unit of current strength is the **ampere** (Art. 105), which is equal to $\frac{1}{10}$ of the C.G.S. electromagnetic unit. One milliampere = $\frac{1}{1000}$ of an ampere.

Since "quantity" is equal to the product of "current strength" and "time in seconds," unit current flowing for unit time will result in the transfer of unit quantity. Hence—

The C.G.S. electromagnetic unit of quantity is the quantity conveyed by the electromagnetic unit current in one second.

The practical unit is the **coulomb; the coulomb is the quantity conveyed by one ampere in one second.** It equals $\frac{1}{10}$ e.m. unit.

212. Force exerted on a Conductor carrying a Current in a Magnetic Field.—In the last chapter we called attention to the fact that since action and reaction are equal and opposite, if a current acts upon a magnet with a certain force, the magnet reacts upon the current with the same force.

An expression for the *magnitude* of the force acting on the conductor

may be obtained as follows. The intensity h of the field at c (Fig. 223) due to the element ab is:—

$$h = \mathrm{I} \times ab \times \sin a \times \frac{1}{r^2}$$

where $oc = r$ (Art. 206).

This expression for the intensity of the magnetic field at c gives the force which a unit north pole would experience if placed at that point. It therefore gives the force which a unit north pole at c would exert on the element ab. But the strength of the magnetic field at o, due to a unit north pole at c, is $1/r^2$. Hence the force exerted on the element ab (length l say) in a magnetic field of intensity $1/r^2$ is $\mathrm{I} \,.\, l \sin a \,.\, 1/r^2$; that is, in a field of intensity H the force exerted on the element ab of the conductor A B carrying a current I is

$$\text{Force} = \mathrm{I\,H}\, l \sin a.$$

If the conductor is straight and the field uniform, then this result may be applied to a conductor of finite length. Further, in the case of a conductor of length l, placed *at right angles* to a uniform field of strength H, we have (since $\sin a = 1$)—

$$\text{Force} = \mathrm{I\,H}\, l \text{ dynes} \quad \textit{or} \quad \text{Force} = \frac{\mathrm{I\,H}\, l}{10} \text{ dynes}$$

the current being in absolute units in the first case and in amperes in the second case. The direction in which the conductor tends to move under this force is of course given by the *left-hand rule* of Arts. 198, 136.

Illustration.—If a straight wire 20 cm. long, carrying a current of 15 amperes, be placed at right angles to the earth's horizontal field ·18 unit, the force tending to move the wire at right angles to the current and to the field (left-hand rule) is $\frac{15}{10} \times \cdot 18 \times 20 = 5 \cdot 4$ dynes.

213. Work Done in Displacing in a Magnetic Field a Circuit carrying a Current.—For simplicity consider a straight conductor of length l cm. placed at right angles to a uniform field of strength H units and carrying a current of I e.m. units; the force on the conductor is $\mathrm{I\,H}\, l$ dynes. If the conductor moves as indicated in Fig. 229 through a distance x cm., the work done is $\mathrm{I\,H}\, l x$ ergs; but $l \times x$ is the area swept out, and $\mathrm{H}\, l x$ is, therefore, the total number of magnetic lines (ϕ) cut by the conductor during the displacement; hence—

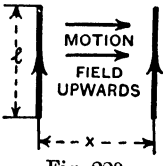


Fig. 229.

$$\text{Work} = \mathrm{I}\phi \text{ ergs} \quad \textit{or} \quad \text{Work} = \frac{\mathrm{I}\phi}{10} \text{ ergs}$$

the current being in absolute units in the first case and in amperes in the second case.

Illustration.—If a wire carrying 30 amperes moves in a field so as to cut 10,000 lines, the work done is $\frac{30}{10} \times 10{,}000 = 30{,}000$ ergs.

214. Magnitude of the Induced E.M.F. in a Circuit.—In the preceding chapter it is indicated that the induced E.M.F. (and current) when the flux through a circuit is changed, depends upon the "rate of change," *i.e.* it is bigger the faster the "change": the actual "law" may be developed mathematically as follows:—

Let P Q (Fig. 230) be a copper bar capable of sliding along the parallel copper rails S P and R Q, and let B be a battery joined to S and R. *Let the arrangement be in air*, and let H denote the intensity of the vertical field, supposed uniform. An application of Fleming's Left-Hand Rule to P Q will show that P Q will tend to move in the direction indicated; let it move through a small distance dx in a small time dt (the student must not be alarmed at this—dx merely means *a small distance* and dt *a small time*).

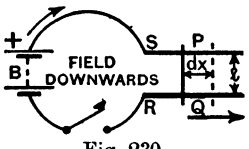


Fig. 230

Then since H is the number of lines per sq. cm., and $l \,.\, dx$ is the area swept out, l being the distance between the rails, $l \,.\, dx \,.\, \mathrm{H}$ is the number of lines cut during the motion, and if I be the current in the circuit in absolute units the work done is—current × lines cut—*i.e.* $\mathrm{I} \,.\, l \,.\, dx \,.\, \mathrm{H}$, *i.e.* $\mathrm{I} \,.\, d\phi$, where $d\phi$ denotes the *small change in the number of lines* through the circuit B S P Q R B (Art. 213). The energy equivalent to this is derived from the battery. The total energy supplied by the battery in time dt is $\mathrm{E\,I}\,dt$, and the energy spent in heat is $\mathrm{I^2\,R}\,dt$; hence:—

$$\mathrm{E\,I}\,dt = \mathrm{I^2\,R}\,dt + \mathrm{I}\,d\phi$$

$$\therefore\ \mathrm{I} = \frac{\mathrm{E} - \dfrac{d\phi}{dt}}{\mathrm{R}},$$

and the E.M.F. (E) of the circuit is therefore *opposed* by an *induced E.M.F.* equal to $d\phi/dt$, *i.e.* if e be the induced E M.F.:—

$$e = -\frac{d\phi}{dt}.$$

Thus the induced E.M.F. in absolute units is measured by the rate of change of the number of lines threading the circuit: in other words the induced E.M.F. in absolute units is measured by the number of lines changed per second: hence, if the flux varies at the rate of 10^8 lines per second, an induced E.M.F. of 1 volt will be produced.

The above result has been derived from the very special case shown in Fig. 230 in which there is an original current in the circuit, but even in a circuit with no original current an E.M.F. is induced if the flux be changed in any way, and the above law applies.

EXAMPLE.

Through a single coil of wire of resistance 2 ohms magnetic lines are varying at the rate of one million per second. What is the current flowing in the circuit during the variation?

$$e = \frac{10^6}{10^8} \text{ volts} = \cdot 01 \text{ volt}, \quad \therefore \text{ I} = \frac{\cdot 01}{2} = \cdot \mathbf{005} \text{ ampere.}$$

For calculation purposes it is well to remember that since $e = -d\phi/dt$ we may, neglecting sign, write—

$$\text{Induced E.M.F.} = \frac{\text{Change in flux}}{\text{Time in seconds}} \text{ (e.m. units)}$$

$$= \frac{\text{Change in flux}}{10^8 \times \text{Time in seconds}} \text{ (volts).}$$

Further, if there are n turns in a coil and the flux through the coil is changing, *each* acquires the above E.M.F., and since the several turns are in series the total E.M.F. is n (Change in flux)/Time in seconds. The product of the flux through a coil and the number of turns is called the *effective flux* or the *linkages*; hence—

$$\text{Induced E.M.F.} = \frac{\text{Change in effective flux}}{\text{Time in seconds}}$$

i.e.

$$e = \frac{\text{Change in linkages}}{\text{Time in seconds}} \text{ (e.m. units)}$$

$$= \frac{\text{Change in linkages}}{10^8 \times \text{Time in seconds}} \text{ (volts).}$$

EXAMPLE.

A coil of wire is connected to a galvanometer, the resistance of the coil and galvanometer being 200 and 400 ohms respectively. The coil is moved in the field, and at a given instant there are 20,000 lines through it, whilst $\frac{3}{4}$ of a second later the number is 2,000. If there are 100 turns in the coil, find the average E.M.F. and current during this period.

Effective flux or linkages at the beginning of the time in question = 100 × 20,000 = 2,000,000, and at the end of the time = 100 × 2,000 = 200,000; hence change in effective flux = 1,800,000, and

$$\text{Average induced E.M.F.} = \frac{18 \times 10^5}{10^8 \times \frac{3}{4}} = \cdot 024 \text{ volt,}$$

and

$$\text{Induced current} = \frac{\cdot 024}{200 + 400} = \cdot 00004 \text{ ampere.}$$

A useful expression for the induced E.M.F. in a *straight* conductor

when cutting across lines in a magnetic field may be developed. Considering the straight wire P Q (Fig. 230) :—

$$e = \frac{d\phi}{dt} = \frac{l \,.\, dx \,.\, \mathrm{H}}{dt} = \mathrm{H}l \,\frac{dx}{dt} = \mathrm{H}lv \text{ (e.m. units)},$$

where v is the velocity of the moving conductor, for it is the distance travelled (dx) divided by the time (dt). Hence *the E.M.F. induced in a straight wire l cm. long moving sideways at a velocity of v cm. per second across a magnetic field of intensity* H *is* :—

$$e = \mathrm{H}lv \text{ (e.m. units)} = \frac{\mathrm{H}lv}{10^8} \text{ volts}.$$

EXAMPLE.

A railway carriage is travelling at the rate of 75 *kilometres per hour. If the axle is* 150 *cm. long and the vertical component of the earth's field is* ·41 *unit, find the E.M.F. between the wheels at opposite ends of the axle.*

$$\mathrm{H} = \cdot 41 \text{ unit}; \qquad l = 150 \text{ cm.}; \qquad v = \frac{75 \times 1000 \times 100}{60 \times 60} \text{ cm. per sec.}$$

$$\therefore\ e = \frac{\cdot 41 \times 150 \times 75 \times 1000 \times 100}{10^8 \times 60 \times 60} = \cdot 0013 \text{ volt.}$$

215. Magnetic Circuit.—It is convenient here to deal with the modern treatment of magnetic effects due to permanent and electromagnets. This treatment arises from a recognition of the fact that magnetic flux proceeds in a closed path just as does the electric current. Thus the flux from a bar magnet, as we have seen earlier, passes out of the N pole, and, returning through the air, enters the S pole, finally reaching the point from which it started.

The **flux** (ϕ) when produced by an electromagnet, needs an agent to produce it, just as current needs E.M.F., and the name given to the exciting agent is **magnetomotive force** (M.M.F.). The amount of flux produced by a certain magnetomotive force depends upon a property of the magnetic circuit analogous to resistance in the electric circuit, and the name given to this property is **reluctance (R).** The law used for magnetic circuits, analogous to Ohm's Law, is due originally to Rowland, and may be stated thus :—

$$\textbf{Magnetic flux} = \frac{\textbf{Magnetomotive force}}{\textbf{Reluctance}};$$

i.e. $$\phi = \frac{\text{M.M.F.}}{\mathbf{R}} \text{ corresponding to } \mathrm{I} = \frac{\text{E.M.F.}}{\mathrm{R}}.$$

216. Magnetomotive Force. The magnetomotive force in any magnetic circuit is measured by the work (ergs) done on unit magnetic pole moved once round the circuit.—Consider first a long straight wire carrying a current I absolute units: the force on unit pole at distance d cm. from the wire is $2I/d$ dynes. If the unit pole be moved once round the wire at this distance in the direction against this force, the work done will be force × distance, *i.e.* $\frac{2I}{d} \times 2\pi d$ or $\mathbf{4\pi I}$ **ergs.** In words—*the work done in carrying unit pole once round a long straight wire carrying a current is equal to* 4π *times the strength of the current in absolute units:* this work is a measure of the power to magnetise, *i.e.* it is a measure of the **magnetomotive force** (see Illustration, Art. 207).

Consider the next simple case used in practice, viz. that of a solenoid bent round to form a closed ring. The field intensity inside is $\frac{4\pi S I}{l}$, where I = current in absolute units, S = total number of turns and l = mean circumference in cm., and this is the force on unit pole inside in dynes. If a unit pole be moved once round this mean circumference thus looping through all the coils once, the work done will be $\frac{4\pi S I}{l} \times l$, *i.e.* $\mathbf{4\pi S I}$ **ergs,** and this work is a measure of the **magnetomotive force.** (Note that it is simply the $4\pi I$ above multiplied by the number of turns looped (S).)

The next case used in practice is that of the long thin straight solenoid. Here the field inside is $\frac{4\pi SI}{l}$, and, if we assume this uniform for the whole length, the work done in moving unit pole inside from one end to the other is $\frac{4\pi S I}{l} \times l$, *i.e.* $4\pi S I$ ergs. This really measures *the magnetic potential difference between the ends of the solenoid.* To move the pole *round the whole circuit* we have to encounter the return air path, but, as indicated in Art. 210 (small type), the work for this part is negligible in comparison. Hence for the cases used in practice the statement which follows may be made :—

The work done in carrying unit pole once round the magnetic circuit, thus looping all the coils once, is equal to $4\pi S I$, where I is the current in absolute units, and S the number of turns on the coil, and this measures the magnetomotive force, *i.e.*

$$\text{Magnetomotive force} = 4\pi S I \text{ units, } or \frac{4\pi S I}{10} \text{ units,}$$

the current in the first case being in absolute units, and in the second case in amperes: the latter can be written :—

$$\text{M.M.F.} = 1{\cdot}257\ S I = 1{\cdot}257 \text{ (}total\text{ ampere turns),}$$

The reader must not confuse "magnetic force" and "magnetomotive force." The former is measured by "**force** on unit pole," the latter by "**work done** on unit pole, in moving it *once round the whole magnetic circuit.*" In fact, M.M.F. is numerically equal to magnetic force multiplied by *l*.

EXAMPLE.

A solenoid consists of 1,000 *turns, is* 50 *cm. long, and carries* 10 *amperes. Determine* (1) *the magnetomotive force;* (2) *the magnetic force at the centre;* (3) *the total flux if the area of cross-section is* 4 *sq. cm.*

Magnetomotive force = 1·257 × total amp. turns
= 1·257 × 10 × 1,000
= **12,570 units** (gilberts—Art. 218);

Magnetic force = 1·257 × amp. turns per cm.
$= 1{\cdot}257 \times \frac{10 \times 1000}{50}$
= **251·4** dynes.

Now 251·4 = lines per sq. cm. (gausses—Art. 218);
∴ Total flux = 251·4 × 4 = **1005·6** lines (maxwells—Art. 218).

217. Reluctance.—The reluctance (or magnetic resistance, as it is sometimes called) of a piece of material depends upon its length, the area of cross-section, and the permeability (μ). Thus it is very much like the resistance of a conductor, but whereas we have $R = \frac{\rho l}{a}$ for the electrical resistance, ρ being the specific resistance of the material, we have in this case $\mathbf{R} = \frac{l}{a\mu}$, l and a in both formulae being the length and area of section of the piece considered; thus ρ corresponds to $\frac{1}{\mu}$.

Just as in the electric circuit the total resistance depends upon the sum of a series of different resistances, so in the magnetic circuit the total reluctance consists of a series of separate reluctances. Thus in the case of the field-magnet circuit of a two-pole dynamo the total reluctance consists of the reluctance of the armature plus the reluctance of the double air gap (one each side), plus the reluctance of the two pole-pieces, plus the reluctance of the limbs, plus that of the yoke. Each of these separate pieces has usually to be separately calculated, hence the total reluctance of a circuit is generally written—

$$\mathbf{R} = \frac{l_1}{a_1\mu_1} + \frac{l_2}{a_2\mu_2} + \frac{l_3}{a_3\mu_3}, \text{ etc.,}$$

there being as many terms as there are separate portions of the magnetic circuit.

It remains only to point out that the flux ϕ has the same meaning as we have given it in magnetic calculations; that is, it is *the total lines passing through the circuit*, and thus $\phi = \mathbf{B}a$, where B is the flux density, and a the area of the circuit in square centimetres.

218. Formula for Magnetic Circuit Calculations.—For the solenoid the magnetomotive force is given by $\frac{4\pi S I}{10}$, where I is the current in amperes, and S the number of turns on the solenoid; hence

$$\text{Flux} = \frac{\text{Magnetomotive force}}{\text{Reluctance}},$$

$$\phi = \frac{4\pi S I a\mu}{10l} = 1{\cdot}257\,\frac{S I a\mu}{l}.$$

And from this— $$S I = \frac{10}{4\pi}\cdot\frac{\phi l}{a\mu};$$

i.e. **Ampere turns** $= {\cdot}8\frac{\phi l}{a\mu}$.

If the magnetic circuit consists, as usual, of more than one piece, that is, if all parts have not equal permeability, we may write the complete expressions—

$$\phi = \frac{4\pi S I}{10\left(\frac{l_1}{a_1\mu_1} + \frac{l_2}{a_2\mu_2} + \quad\right)}$$

and **Ampere turns** $= {\cdot}8\phi\left(\frac{l_1}{a_1\mu_1} + \frac{l_2}{a_2\mu_2} + \quad\right)$.

EXAMPLES.

1. *Calculate the ampere turns needed to send a flux of* 40,000 *lines through a closed iron ring formed of square iron of* 2 *cm. side, bent into a circle of outside diameter* 22 *cm.*

Here— $a = 2 \times 2 = 4$ sq. cm.,

$l = \pi \times$ mean diameter $= \pi \times 20$ cm.,

$$B = \frac{\text{Total flux}}{a} = \frac{40000}{4} = 10{,}000.$$

Turning up permeability tables for soft iron, when B = 10,000 we find $\mu =$ 1,900 nearly.

Hence— $$\text{Amp. turns} = 0{\cdot}8 \times \frac{40000 \times \pi \times 20}{4 \times 1900}$$

$$= \mathbf{264}.$$

Further, suppose we cut the ring across with a saw and separate the two halves by 1 mm., thus causing an air gap of that amount in two places diametrically opposite.

We have now— $a = 4$, $l_1 = \pi \times 20$, $l_2 = {\cdot}2$,

$\phi = 40{,}000$ as before;

Then— $$\text{Amp. turns} = {\cdot}8\left[\frac{40000\,(20\pi)}{4 \times 1900} + \frac{40000 \times {\cdot}2}{4 \times 1}\right],$$

μ for the air gap being 1.

This gives, Amp. turns = **1,864.**

It is important to notice these figures. To cause the flux in the iron alone 264 ampere turns only will suffice, but when we make an air gap even so small as 2 mm., then the additional ampere turns to cause the flux to pass it equal 1,600, making a new total of 1,864. This effect of air gaps must be carefully remembered. The following example will further illustrate this and bring in an additional point *in practice*, viz. the question of **leakage of lines.**

2. *An iron ring, 20 cm. mean circumference and 4 sq. cm. cross-section, has an air gap cut in it ·5 cm. wide. The flux for the gap is required to be 25,000. If the permeability of the iron be 100, find the ampere turns required if the leakage coefficient be 1·3.*

Owing to **leakage of lines** in practice, the maximum flux to be generated must be bigger than the flux necessary for the gap. For the present we may simply say that *the maximum flux to be generated in the iron* must be $25000 \times 1{\cdot}3$, *i.e.* 32,500, since the definition of the coefficient of leakage (u) is:—

$$u = \frac{\text{Maximum flux generated}}{\text{Useful flux}},$$

i.e. maximum flux $= u$ (useful flux), and useful flux in practice means the flux in the gap. All this will be understood later: the point is that in this problem we want a maximum flux of 32,500 in the iron to get a flux of 25,000 in the gap. Of course this leakage occurs all along the iron (Fig. 231), but to simplify the problem we assume that the flux *for the whole iron circuit* is the maximum, viz. 32,500. Now:—

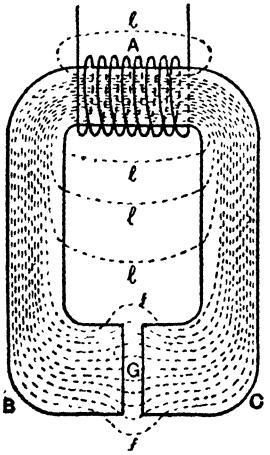


Fig. 231.

(1) *Ampere turns for the iron.*

$$\text{Ampere turns} = \cdot 8\phi \frac{l_1}{a_1\mu_1}$$

$$= \cdot 8 \times 32500 \times \frac{20}{4 \times 100} = 1300.$$

(2) *Ampere turns for the air gap.*

$$\text{Ampere turns} = \cdot 8\,\phi \frac{l_2}{a_2\mu_2}$$

$$= \cdot 8 \times 25000 = \frac{\cdot 5}{4 \times 1} = 2500.$$

$\therefore$ Total ampere turns = 3,800.

There is still another point we have neglected, viz. **"fringing,"** *i.e.* the spreading out of the lines at the air gap so that the cross-section of the gap is really more than 4 sq. cm. A method of dealing with this is to add to the pole face area a strip all round of width equal to half the length of the gap, and to take this bigger area as the effective area (a_2) of the gap. The electromagnet shown in Fig. 231 will help the student to grasp the preceding: *lll* = leakage lines, *ff* = fringing.

The following **magnetic circuit units** have been suggested, but they are not extensively used:—

Magnetic Field ..... **Gauss**
Magnetic Induction or Flux (ϕ) ..... **Maxwell** (formerly the **Weber**)
Magnetomotive Force (M.M.F.) ..... **Gilbert**
Reluctance (**R**) ..... **Oersted**

$$\text{Maxwells} = \frac{\text{Gilberts}}{\text{Oersteds}}.$$

219. Further Treatment of the Magnetic Circuit.—Owing to the importance of the matter dealt with in the last four sections, the following additional simple treatment may be added. Consider the simplest case of an iron ring wound closely round with wire carrying a current (Fig. 232). The intensity H of the field inside is 4πS I/l, where I is in absolute units, and l is the mean circumference, *i.e.* the total length of the magnetic circuit. The flux density B is μH, *i.e.* $4\pi\mu$S I/l, where μ is the permeability of the iron, and the total magnetic flux ϕ is Ba, *i.e.* $4\pi a\mu$S I/l, where a is the sectional area of the iron. Hence:—

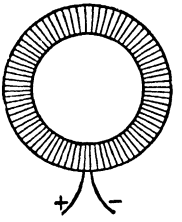


Fig. 232.

$$\text{Flux } (\phi) = \frac{4\pi a\mu \text{S I}}{l} = \frac{4\pi \text{S I}}{\frac{l}{a\mu}} \text{ (as before).}$$

Now 4πS I is evidently $\frac{4\pi \text{S I}}{l}$ multiplied by l, *i.e.* it is "force on unit pole" multiplied by "the length of the complete magnetic circuit," *i.e. it is the work done in moving unit pole once round the complete magnetic circuit,* and this measures the *magnetomotive force.* The quantity $l/a\mu$ is the magnetic resistance or *reluctance,* and just as in the electric circuit, we have:—

$$\text{Current} = \frac{\text{Electromotive force}}{\text{Resistance}}, \text{ i.e. } \text{I} = \frac{\text{E.M.F.}}{\text{R}},$$

so for this magnetic circuit we have:—

$$\text{Flux} = \frac{\text{Magnetomotive force}}{\text{Reluctance}}, \text{ i.e. } \phi = \frac{\text{M.M.F.}}{\mathbf{R}}.$$

220. Coefficient of Self-Induction (L) of a Circuit.—When a current flows through a coil it produces a flow of magnetic induction through the coil. This flow of induction is proportional to the current when the permeability of the surrounding medium is constant. In this case if ϕ denotes the flow of induction through the coil, and I the absolute current in the coil, then

$$\phi = \text{LI} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

where L is a constant. This constant is the *coefficient of self-induction*

of the coil. If I be unity, ϕ is numerically equal to L; thus **the coefficient of self-induction of a circuit is numerically equal to the flow of induction when absolute unit current passes.**

Clearly *a circuit has a coefficient of self-induction of one C.G.S. unit if the flow of induction be unity when the unit electromagnetic current passes.* The practical unit is the **henry,** which is equal to 10^9 C.G.S. units; thus *a circuit has a coefficient of self-induction of one henry if the flow of induction be* 10^9 *when the unit electromagnetic current passes, and therefore* 1/10 *of* 10^9, i.e. 10^8, *when a current of one ampere passes.*

From the preceding sections it will be clear that ϕ above really refers to "linkages" in practice. As already indicated we obtain the number of linkages by multiplying the flux through a coil by the number of turns upon the coil: thus if a solenoid has 200 turns and the flux through it is 40,000 lines then the linkages are 40,000 × 200 = 8,000,000. Thus a more convenient definition of L for practical purposes is as follows:—**the coefficient of self-induction of a circuit is numerically equal to the linkages of the circuit when absolute unit current is flowing through it.** *This divided by* 10^9 *will give the coefficient of self-induction in henrys.*

If the flow of induction or flux is changing owing to the current changing then the *rate of change* of flux depends upon the *rate of change* of current. In fact from (1) above we can write $\frac{d\phi}{dt} = L\frac{dI}{dt}$, where $\frac{d\phi}{dt}$ is rate of change of flux, and $\frac{dI}{dt}$ rate of change of current.

But the induced E.M.F. (e) is given by $-\frac{d\phi}{dt}$ (Art. 214): hence:—

$$e = -L\frac{dI}{dt} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

or $e = -$ **L (Rate of change of Current).**

The negative sign merely means that it is an *opposing* E.M.F. when the current is growing, that is, when the rate of change of current is positive, and is a *direct* E.M.F. when the current is decreasing.

Clearly if $\frac{dI}{dt}$ is unity, L is numerically equal to e: thus **the coefficient of self-induction of a circuit is numerically equal to the induced E.M.F. round the circuit due to unit rate of change of the current in it.** Hence:—

A circuit has a coefficient of self-induction of one C.G.S. unit when current increasing at the rate of one e.m. unit per second brings on an opposing E.M.F. of one e.m. unit.

A circuit has a coefficient of self-induction of one henry when current increasing at the rate of one ampere per second brings on an opposing E.M.F. of one volt.

One henry = 10^9 C.G.S. units.

221. Coefficient of Self-Induction of a Solenoid.—Let the solenoid have S turns, its length be l cm., and its sectional area a sq. cm., and let the current flowing be unity (absolute).

We have— $$\text{Flux} = \frac{\text{M.M.F.}}{\mathbf{R}} = \frac{4\pi \text{ S I}}{\mathbf{R}}$$

$$= \frac{4\pi \text{ S}}{\mathbf{R}} \text{ since I is unity;}$$

$$\therefore \text{ Number of linkages} = \text{Flux} \times \text{Number of turns}$$

$$= \frac{4\pi \text{ S}}{\mathbf{R}} \times \text{S}$$

$$= \frac{4\pi \text{ S}^2}{\mathbf{R}};$$

i.e. $$\text{L} = \frac{4\pi \text{ S}^2}{\mathbf{R}} \text{ absolute units} = \frac{4\pi \text{ S}^2}{10^9 \mathbf{R}} \text{ henrys,}$$

or since $$\mathbf{R} = l/a\mu$$

$$\text{L} = \mu \frac{4\pi \text{ S}^2 a}{l} \text{ absolute units} = \mu \frac{4\pi \text{ S}^2 a}{10^9 l} \text{ henrys.}$$

Illustration.—If a solenoid has 200 turns and is 20 cm. long and its area is 4 sq. cm., we have—

$$\text{L} = \frac{4\pi (200)^2 \times 4}{10^9 \times 20} = 0{\cdot}001 \text{ henry,}$$

μ being 1 for air core.

222. A few Practical Effects of Self-Induction.—As has been noted in Art. 205, the property of inductance of a circuit is only of importance when the current is changing its value. This occurs in a continuous current system only when the pressure is first applied or when it is removed.

We have seen that when a current is started in a circuit an induced inverse E.M.F. is set up which delays its growth. The way in which a current grows in a circuit with resistance and inductance is indicated by the following **Law of Helmholtz:**—

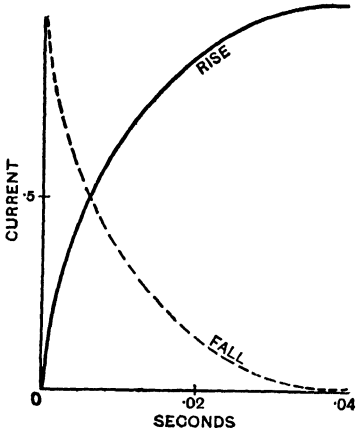
$$\text{I}_t = \frac{\text{E}}{\text{R}} \left(1 - e^{-\frac{\text{R}t}{\text{L}}} \right)$$

where I_t is the current at the end of time t from the instant of closing the circuit R the resistance, L the coefficient of self-induction, E the E.M.F. in the circuit, and e the base of the Napierian logs (2·71828). On plotting this relation as a curve connecting I and t, it is easily seen that the current rises at first quickly and then more slowly, taking some time to reach its final value. The time actually taken depends fundamentally upon L, and in the case of large electromagnets may be several minutes. In the formula, E/R is, of course, the maximum current.

The law for the decay of a current when a circuit is broken is expressed thus :—

$$I_t = \frac{E}{R} e^{-\frac{R}{L}t}.$$

This assumes that the circuit is broken instantaneously, but in practice this is not the case: we cannot, however, enter further into detail here. Fig. 233 gives the general shapes of the rise and fall curves in a circuit containing resistance and inductance.



R = 2 hms.
L = ·02 henry.

Fig. 233.

The quantity $\frac{L}{R}$ is known as the **time constant**: it is, from the first equation, the time that it takes the current to grow to $\left(1 - \frac{1}{e}\right)$ of its maximum value (e = base of Napierian logs = 2·71828).

Some idea of the magnitude of the self-inductance effects may be gathered from applications of formula (2) of Art. 221, viz.—

Induced E.M.F. (e)
= − L (Rate of change of Current).

Illustrations.—(1) Using practical units, in a circuit for which L = 0·5 henry and in which current is growing uniformly at the rate of 2 amperes per second, then $e = -0{\cdot}5 \times 2 = -1$ volt. If the rate of growth of current were quicker, as, for example, if the current were growing at the rate of 2 amperes in $\frac{1}{100}$ of a second, then—

$$e = -0{\cdot}5 \frac{2}{{\cdot}01} = -100 \text{ volts.}$$

(2) If the rate of dying away of current be taken, and if it be very rapid, then the forward E.M.F. will be very large. For example, if the current were dying away at the rate of 2 amperes in $\frac{1}{1000}$ second, then—

$$e = -0{\cdot}5 \times \frac{-2}{{\cdot}001} = 1{,}000 \text{ volts.}$$

Hence the necessity—as mentioned earlier—for exercising *great care when interrupting the current in the case of circuits possessing much inductance*—for example, powerful electromagnets.

223. Coefficient of Mutual Induction (M) of Two Circuits.—The mathematics of mutual induction are very similar to those of self-induction, so that it may be very briefly dealt with here.

The coefficient of mutual induction of two circuits is numerically equal to the linkages of one circuit when absolute unit

current flows in the other. *This divided by* 10^9 *will give the coefficient of mutual induction in henrys.*

Again this coefficient M also appears in the E.M.F. relation similar to that for the self-induction, *i.e.* e (**in either coil**) $= -$ **M** (**Rate of change of current in other coil**), and if the rate of change of current be unity $e = -$ M: thus:—**the coefficient of mutual induction of two circuits is numerically equal to the induced E.M.F. round one circuit due to unit rate of change of the current in the other.** Hence:—

Two circuits have a coefficient of mutual induction of one C.G.S. unit when current changing in one at the rate of one e.m. unit per second results in an induced E.M.F. of one e.m. unit in the other.

Two circuits have a coefficient of mutual induction of one henry when current changing in one at the rate of one ampere per second results in an induced E.M.F. of one volt in the other.

One henry $= 10^9$ C.G.S. units.

Coefficient of Mutual Induction of two Solenoids.—We shall only deal with two solenoids so associated that there is no magnetic leakage, *i.e.* all the lines pass through both coils.

If S_1 and S_2 be the turns respectively, and if the first be the primary and the second the secondary, then:—

$$\text{Flux} = \frac{4\pi S_1 I}{\mathbf{R}} = \frac{4\pi S_1}{\mathbf{R}} \text{ if I be unity,}$$

$$\therefore \text{ Linkages for secondary} = \text{Flux} \times \text{Turns} = \frac{4\pi S_1}{\mathbf{R}} \times S_2,$$

i.e. $$M = \frac{4\pi S_1 S_2}{\mathbf{R}} \text{ absolute units} = \frac{4\pi S_1 S_2}{10^9 \mathbf{R}} \text{ henrys,}$$

where, of course, $\mathbf{R}$ = reluctance $= \frac{l}{a\mu}$.

A few Practical Effects of Mutual Induction.—A simple illustration will suffice:—Suppose, for example, in an induction coil M is 4 henrys, and the primary current changes at the rate of one ampere in $\frac{1}{5000}$ of a second, then the induced E.M.F. in the secondary circuit is given by—

$$e = -\text{ M (rate of change of primary current)}$$

$$e = -4 \frac{1}{\cdot 0002} = -20{,}000 \text{ volts.}$$

Evidently very high voltages are obtainable from such a device.

The two cases in which this is of practical use, viz. induction coils and transformers, are discussed in Chapter XX.

The curves depicted below will be of service to the students in working out problems on the "magnetic circuit" in practice. Frequently a problem does not give all the necessary data: these can invariably be obtained from the curves.

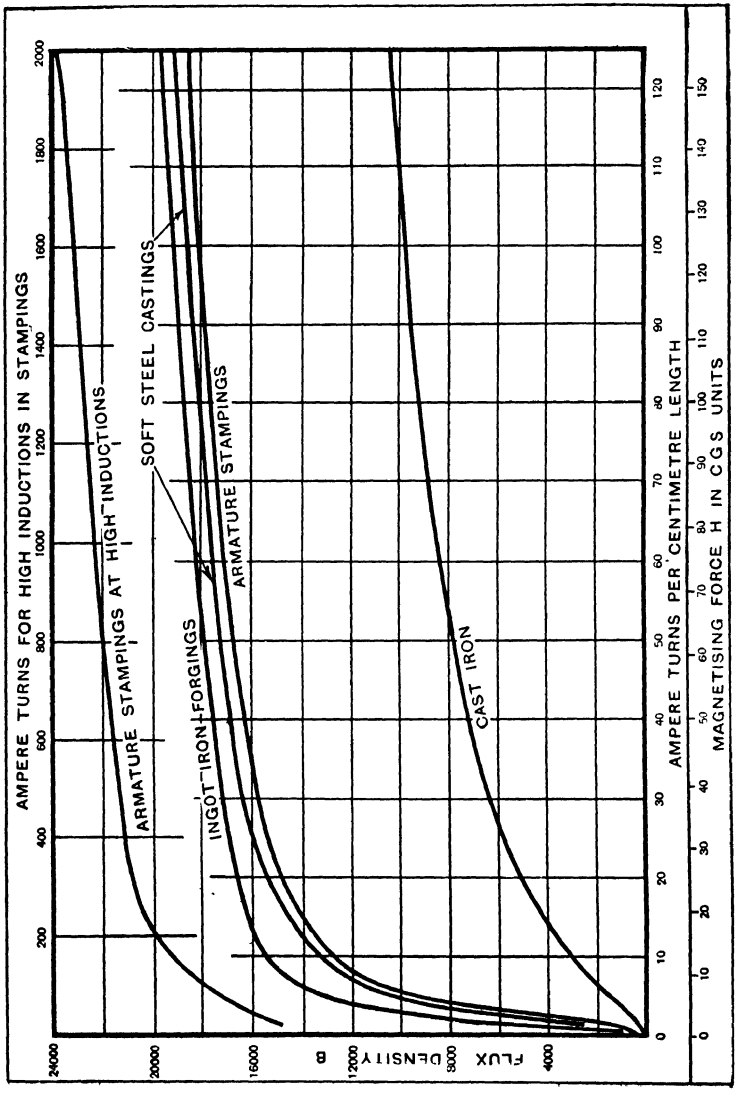


Fig 238a.

EXERCISES XVII.

1. You have to make a long coil of wire such that when 10 amperes flow through it the magnetic field at the centre shall be 1,000 times as strong as the earth's horizontal field. (H.) How many turns per cm. must be put on the coil? H = ·18. (O.)

2. You are required to wind over a brass tube, 500 cm. long and 9 cm. external diameter, with one layer of covered wire 1 mm. diameter over the covering. What length of wire will you require? What will be the strength of the magnetic field at the centre of the axis of such a helix when a current of 1 ampere flows through the wire? (O.)

3. A solenoid consists of 1,500 turns, has a length of 1 metre, and carries 10 amperes. Determine (1) the magnetomotive force; (2) the magnetic force; (3) the total flux if the cross-section be 3 square cm.

4. A circular coil traversed by a current is placed horizontally in the earth's field. What is the nature of the force acting upon the coil, and how does it vary with (1) the strength of the current; (2) the strength of the field; (3) the number of turns of wire on the coil; (4) the size of the wire; (5) the radius of the coil? (B.)

5. A wire, 20 cm. long, carrying a current of 5 amperes, is placed in the earth's field at right angles to the lines. If the earth's field be ·18-unit, find the force tending to move the wire.

6. What is the inductance of a coil of 200 turns wound on a paper tube 25 cm. long and 5 cm. radius?

7. A primary coil of 100 turns and then a secondary coil of 50 turns are wound on an iron ring of mean diameter 30 cm. and radius of cross-section 1 cm. If $\mu = 800$, find the inductance of each.

8. If 250 volts are impressed on a circuit of resistance 10 ohms and inductance 4 henrys, what is the final current, and what is it at the end of $\frac{1}{200}$ second, and 1 second?

9. Why is it dangerous to lift the brushes of a shunt-wound dynamo when shutting down? State what would occur if you were to lift the brushes (*a*) while a fair load of lamps is being supplied; (*b*) when no current is being supplied to the external circuit. (O.)

10. Calculate the ampere turns needed to send a flux of 65,000 lines through an iron ring of outside diameter 20 cm. and sectional area 3 sq. cm. ($\mu = 1{,}120$). What would be the ampere turns if the ring were cut into two halves, and the two halves separated to a distance of 2 mm.?

CHAPTER XVIII.

DYNAMOS,

224. Introduction.—The discovery that currents could be induced in circuits by altering the magnetic flux through them led in time to the introduction of the dynamo and the commercial applications of electricity. Faraday commenced the construction of such machines in 1831, and following him were Pixii, Sturgeon, Werner Siemens, Pacinotti, Gramme, and others.

To produce useful currents in this way we must have a powerful magnetic field, coils arranged so as to cut the field, and suitable collecting arrangements to convey the current to the place where it is needed.

The machine as a whole is called a **dynamo.** The magnet producing the field is termed the **field magnet,** and the coils cutting the field are grouped in what is called the **armature.** In the early days the field magnets consisted of permanent magnets, but now they are always electromagnets, and the current used to *excite* them is often obtained from the machine itself. We have noted that the relation concerned is:

$$\text{E.M.F. (volts)} = \frac{\text{Rate of alteration in magnetic lines}}{10^8}.$$

Thus we can obtain an E.M.F. by steadily increasing the flux, or by steadily decreasing it; but since we can neither increase nor decrease the flux indefinitely, all machines work by first increasing, then decreasing, and so on alternately. Such an oscillation of fluxes is conveniently obtained by using a rotating coil, and in practice this is the form always adopted. *Hence, the E.M.F. is proportional to the speed of rotation of the armature, to the number of turns of wire upon it, and to the magnetic flux in the space where the armature rotates.*

In this chapter the *Fleming right-hand rule* is most suitable for determining the direction of the induced current; in certain diagrams current flowing towards the reader is indicated by a "dot," current flowing away from the reader by a "cross."

Some writers use "dynamo" as a general term including motors, using the term "**generator**" for *a "dynamo" which converts mechanical energy into electrical energy,* and, of course, "**motor**" for *a "dynamo" which converts electrical energy into mechanical energy.*

Dynamos (*i.e.* generators) may deliver continuous (direct) current to the external circuit (D.C. or C.C.) or alternating current (A.C.): the latter are termed "**alternators.**"

225. Principle of the Dynamo.—Suppose a rectangular coil of wire, as A B C D in Fig. 234, to be capable of rotation about an axis X X between the poles N and S of a powerful magnet. Imagine the rotation is such that A B is rising, and C D is falling. Using the Fleming right-hand rule we see that current will tend to flow in A B in the direction of the arrow, and in C D in the reverse direction, but such that, taking the circuit as a whole, the currents help each other. In such a case the end P of the coil will be the positive pole, and Q the negative pole, and the outside current will flow from P to Q.

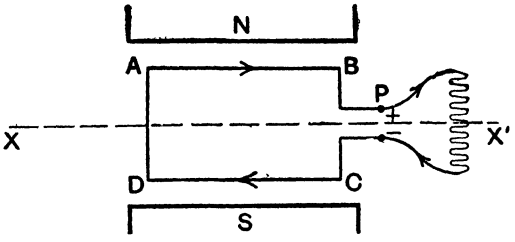


Fig. 234.

As the rotation proceeds the coil will presently be vertical, and at that instant the number of lines of force embraced by it will be a maximum, *but the rate of cutting will be zero, and so also will be the induced E.M.F.*

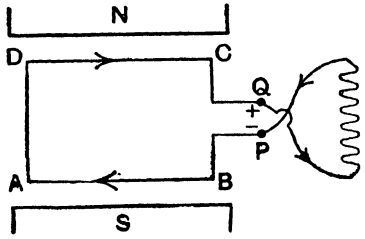


Fig. 235.

A little later the side A B will be descending past S, and C D will be ascending past N; thus the currents will be reversed, and will now be as in Fig. 235; hence the end P, which was formerly the positive pole, is now the negative pole, and Q is now the positive pole, and the outside current is from Q to P.

Thus carrying the coil right round through a complete revolution we see that *the current reverses every time the coil passes through the vertical position*, that is, twice in each revolution, and the current produced is an **alternating** one (A.C.). To make contact with the coil, and still allow it to rotate, an arrangement is used as shown in Fig. 236*a*, and known as **slip rings.** A metal brush rests upon each ring and serves to lead the current, which is still alternating, to and from the external circuit.

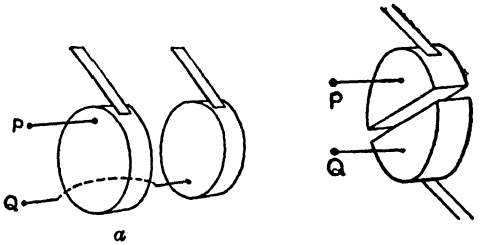


Fig. 236.

If we wish to collect the current in such a way that reversal never occurs *in the outside circuit* we may modify our collecting arrangement, as shown in Fig. 236 *b*, introducing the **split-ring commutator.** In this case one ring is used split into two halves, and the brushes are diametrically opposite instead of being side by side. If the brushes are so placed that *the gap in the ring passes under them when the coil itself is vertical*, the current in the external circuit will go up and down in value but will not reverse; it will be *unidirectional, i.e.* a continuous or **direct** current (D.C.). This device automatically arranges that the positive end of the coil is always connected to the same end of the external circuit, but it must be remembered that no contrivance prevents the current *in the coil or armature itself* from reversing.

Thus the essential point in practice is, that alternating current dynamos or alternators are fitted with slip rings and deliver alternating currents to the outside circuit, whilst continuous or direct current dynamos are provided with commutators, and deliver, to the external circuit, currents always flowing in one direction.

In practice the coil (or coils) is wound upon an iron core, and Fig. 237 shows a direct application of the preceding. The core consists of an iron cylinder with two slots cut along it at opposite ends of a diameter. A wire is wound round and round in the slots, the two ends being joined to a two-part commutator (for D.C), or to two slip rings (for A.C.). Such simple machines may still be met with in the form of magneto generators for ringing up on telephone circuits, in magneto systems of ignition for petrol engines, and in small dynamos used for exploding mines.

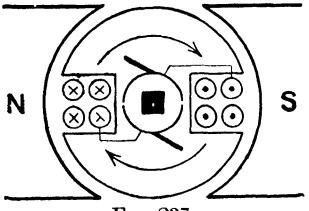


Fig. 237.

In this "shuttle" armature of Fig. 237 the coil ends are joined to a *two-part commutator*, so that the external current flows in one direction; it is, however, not of uniform strength, but is intermittent, rising and falling in magnitude. If now the armature be modified by cutting two additional slots in the core midway between the existing ones and placing therein a coil similar to the other, it is evident that the E.M.F. in one coil will be a maximum when it is zero in the other, so that, by suitably connecting the coils, as will be explained later, and using a *four-part* commutator, *the external current will become more constant in magnitude.* It is evident, too, that this condition will be even better realised by having a large number of coils uniformly distributed round the core, suitably connecting them as described later, and using a commutator divided into a corresponding large number of parts, bars, or segments; this is the method adopted in modern (D.C.) armatures.

226. Preliminary Ideas on General Construction.—A number of armature types have been introduced from time to time, which may all be included in four general classes. These are (1) **Ring Armatures,** (2) **Drum Armatures,** (3) Disc Armatures, and (4) Pole Armatures. Disc and Pole armatures are, however, no longer used.

In the ring pattern the core is a ring or hollow cylinder of soft iron, and each turn of wire passes along the outside and then returns through the inside of the ring. Any wires thus returning through the central hole cut practically no lines of force, do not produce E M.F., and are therefore *idle wires*. Ring armatures are, in consequence, now rarely employed; but since the action is easy to follow, the working of such an armature is given first (Art. 227), as it helps to an understanding of general armature behaviour.

The drum is the pattern usually employed. The core is an iron cylinder or drum, and each wire passes along the outside of the drum in a direction parallel to the shaft, crosses the end of the drum to a certain position and returns from that position along the outside of the drum: thus each conductor on the outside is joined across the end to a return conductor on the outside, and there are no idle wires returning through the interior.

The introduction of iron into the core or central portion of the armature is designed to cause the total flux due to the field magnets to be concentrated where it is cut by the coils wound on the core, and it is to be again noted that when the core is partly hollowed to allow for the driving shaft, ventilation, etc., practically none of the flux goes through the holes: it keeps within the material of the iron.

The introduction of the iron leads, however, to a result of importance, which has to be carefully allowed for, viz. the production of *eddy currents*. The fact that we are rotating a large conductor (iron) in the magnetic field necessarily tends to produce current *in the iron* as well as in the wires surrounding it. Such internal currents circulating locally in a mass of metal are termed eddy currents. From the laws we have already investigated, such currents will tend to flow at right angles to the field and at right angles to the motion; that is, *parallel to the shaft*. To reduce this effect as much as possible armatures are now made up, not of solid iron, but of thin charcoal iron plates, or discs, or "stampings," which are all punched out to the same shape and size and threaded upon the shaft or otherwise arranged to rotate with it. The surfaces of separation, often increased by varnish, oxidisation of the surface, or thin paper attached to one side, offer considerable resistance to a current tending to move *parallel* to the axis, but offer no appreciable resistance to the magnetic flux which is passing through at right angles to the axis. Such **lamination** has been referred to in the section on magnetism.

As indicated above the core discs (in smaller machines) are threaded upon the shaft: each disc has holes cut in it, and these are arranged in line forming horizontal channels through the core for ventilation

purposes. In big machines the core discs really become rings, the driving power being communicated from the shaft through a non-magnetic three- or four-armed **hub** or **spider**, which suitably grips all the plates and compels them to turn with the shaft. Larger and stronger spiders are used for very large machines.

With regard to the **field magnets,** dynamos are either *two-pole* or *multipolar* machines. In the case of the former one pole-piece is of N polarity and the other of S polarity, but the actual shape of the magnet itself is very different in different cases: all such differences in detail are unimportant at this stage however. The constituent parts of a two-pole electromagnet causing the field can be seen from Fig. 238, in which, *p*, are the **pole-pieces,** C the **limbs,** which carry the magnetising or **field coils,** B the **yoke,** and F the **tunnel** or space in which the armature revolves. The armature itself always consists, as we have seen, of a mass of soft iron which when wound with wire only just clears the surface of the pole-pieces, the small clearance left between the two surfaces being called the **air-gap,** and since even a small gap means many extra magnetising coils, this gap must be kept as small as is mechanically convenient. As the magnet is frequently built up of separate pieces of iron, great care must be taken that all joints are good, as even a fair joint between two pieces introduces considerable extra magnetic reluctance.

In the case of multipolar machines there may be any even number of poles, and these are spaced alternately N and S round the field. In modern machines there are frequently, in addition, **interpoles** (Art. 238) for purposes of better regulation. Stampings of sheet-iron known as **pole shoes** are often used on the poles so as to reduce the eddy currents which tend to flow when toothed armatures are employed; in some machines the entire field poles are laminated for this purpose. Pole shoes also spread the flux over a bigger area, lessen the reluctance and give improved wave-form of E.M.F. Fig. 239 shows a typical four-pole field magnet. The four poles (or field cores) project inwards from the circular yoke: each carries a magnetising coil (or field coil), one of which is shown in the figure, and each terminates in a pole shoe as indicated.

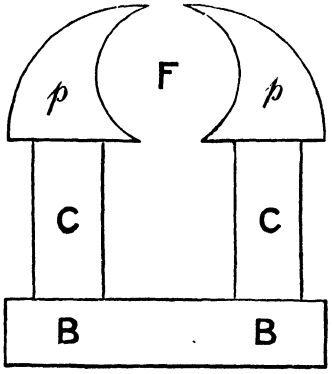


Fig. 238.

The materials used in the construction of the field magnets are wrought iron, cast iron, cast steel, ingot iron, and stampings. Wrought iron has a large permeability, but the small amount of residual magnetism it retains prohibits its *exclusive* use for self-excited machines;

it is often, however, used for the limbs (or field cores) which should be as small as is convenient so as to economise copper in the field windings. Cast iron is of low cost but small permeability and is not extensively used except for small machines. Cast steel, although of fairly high cost, is now frequently employed in large machines; it has a large permeability resulting in decreased weight of limbs, less copper in the coils, and neater machines generally, and it retains a large amount of residual magnetism.

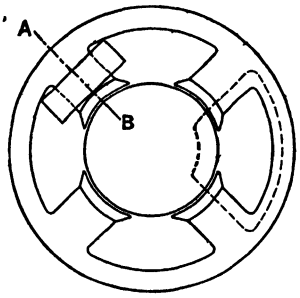


Fig. 239.

The **commutator** is a rigid, self-contained, cylindrical structure which rotates with the armature. It is built up of wedge-shaped bars of copper—drawn or drop forged—insulated from each other and surrounding parts by mica: the bars are connected to the armature coils. **Slip rings** for alternators are mainly of copper, brass, or steel. **Brushes** are chiefly of copper gauze or carbon. **Bearings** and **pedestals** for supporting the shaft, and the necessary oiling arrangements, are practically the same as those employed in other machinery.

227. The Bi-polar Ring Armature.—The armature core consists of an iron ring (or cylinder) built up of sheet iron stampings and wound uniformly with insulated copper wire the beginning and end of the coil being soldered together so that the coil is endless as shown in Fig. 240. This armature is capable of rotation between the pole-pieces N and S of a powerful magnet. To make contact between the coils and the external circuit we will imagine at present that each outside wire has a small piece of insulation removed from it and that two brushes A and B are so fixed as to graze the bare parts of the wires as the armature rotates. It will be remembered that in Fig. 234 we had zero E.M.F. and reversal when the coil was vertical, *i.e.* its plane at right angles to the field from N to S, and it will be noted that here the brushes are so placed as to

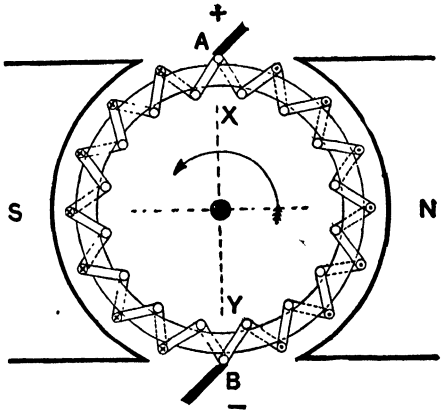


Fig. 240.

touch a coil when it is vertical: the line X Y at right angles to the line joining the poles N and S is called the *geometrical neutral axis.*

Imagine now that the armature is rotating counter-clockwise. An application of the Fleming right-hand rule to the outer or active wires under the S pole on the left will show that these wires have currents induced in them flowing *from front to back,* whilst the active wires under the N pole on the right have currents induced in them flowing *from back to front* (the inside wires are idle wires, *i.e.* are merely "returns"). An examination of the figure will now reveal the following facts:—(1) the induced currents in the coils *on both sides* flow upwards from B to A, and therefore (2) in the external circuit the current flows continuously from A (positive brush) to B (negative brush). The two halves of the ring therefore form two equal sources of E.M.F. each in parallel with the other, the currents from each half uniting at A, flowing through the external circuit from A to B and dividing into the two halves again on reaching B. Fig. 241 will diagrammatically represent this.

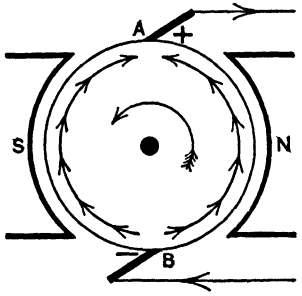


Fig. 241.

In practice of course the brushes do not make contact with the coils themselves but with the bars of a commutator to which bars the coils are joined (Fig. 243). The separate copper bars of the commutator are assembled together so as to constitute a cylinder which is fixed to the shaft and rotates with the armature. The construction is diagrammatically represented in Fig. 242 which shows a portion of the commutator with a brush resting

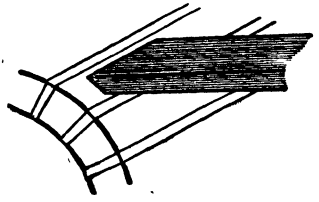


Fig. 242.

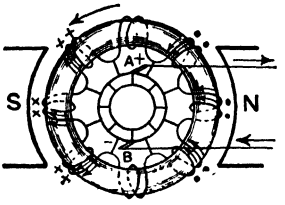


Fig. 243.

on a bar. These commutator bars are thoroughly insulated from each other and from the shaft, and, as stated, the coils of the armature are connected to them: clearly when a brush touches a bar it is electrically the same as touching the coil joined to that bar. Every coil turn (*i.e.* the active conductor and its return) may be joined to a bar in which case the number of bars on the commutator would be equal to the number of conductors (Z) on the armature. On the other hand,

every second or every third or every fourth, etc., complete turn may be joined to a bar so that we may say the necessary number of bars is Z/n, where n is a whole number which is a factor of Z. In Fig. 243 there are 16 conductors, the winding is bunched into 8 coils each of 2 turns and therefore 2 active conductors, and the junctions of the coils are joined to commutator bars, so that there are 8 bars on the commutator ($n = 2$).

Incidentally it may be mentioned that in practice the armature current distorts the field and moves the reversal positions from the geometrical neutral axis in the direction of rotation and the brushes must be set fofward accordingly: this is dealt with later.

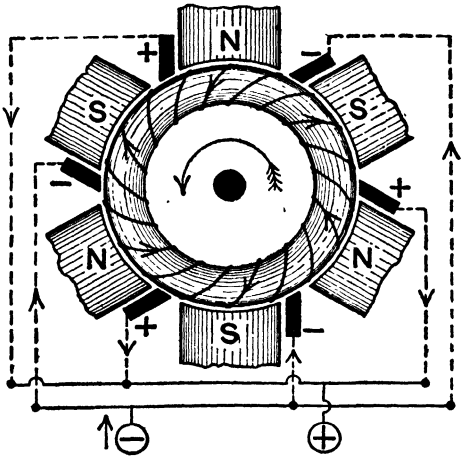


Fig. 244

228. The Multipolar Ring Armature.— Fig. 244 shows a ring armature wound in the same way as that of Art. 227 but rotating in a six-pole field. An application of Fleming's right-hand rule to the outside active conductors under the various poles will indicate that there are not two paths in parallel in the armature as in the previous case but that there are *six*. Fig. 245 shows the paths of the currents inside. Current flows *from two paths* to the point X_1, *from two paths* to X_2, and *from two paths* to X_3, whilst current flows from Y_1 *to two paths*, from Y_2 *to two paths*, and from Y_3 *to two paths*. Hence brushes must be placed at $X_1 X_2 X_3$ and joined to one terminal of the machine which will constitute the positive terminal, and brushes must be placed at $Y_1 Y_2 Y_3$ and joined to another terminal of the machine which will constitute the negative terminal. In an armature of this type if there are p poles there are p paths in parallel in the armature and p

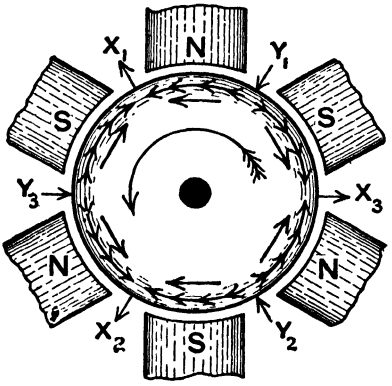
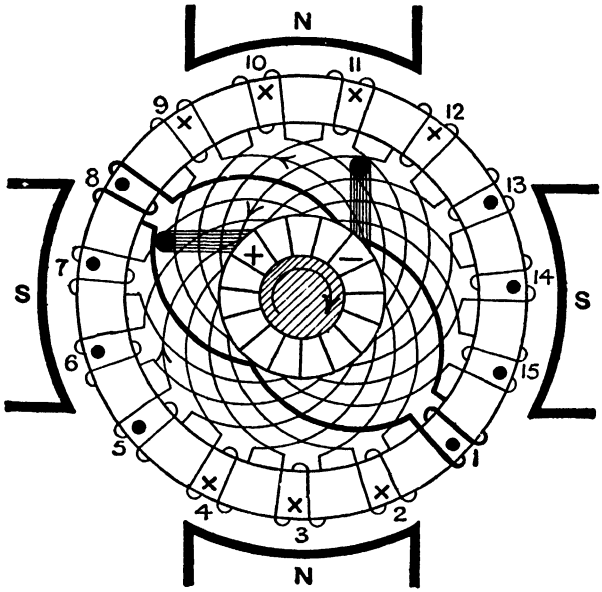


Fig. 245.

brush sets are necessary: it corresponds to the (simplex or single) "lap" method of winding of a multipolar drum armature as will be seen later.

The windings of a multipolar ring armature may be so arranged, however, as to give **two paths in parallel through the armature** (and therefore render two brush sets only necessary) **whatever the number of field poles:** this corresponds to the (simplex or single)



Paths:

$$-\left|\begin{matrix}1, 8, 15, 7, 14, 6, 13, 5\\9, 2, 10, 3, 11, 4, 12\end{matrix}\right|+$$

Fig. 246.

"wave" method of winding of a multipolar drum armature as will be seen later. In this case instead of joining one coil to the next adjacent coil, a coil under a north pole is put in series with a coil similarly placed under the next north pole and then this is joined to a coil similarly placed under the next north pole and so on until we have been once round the armature and come to the coil next to that from which we started and so on. Such a winding is shown in Fig. 246. An application of the Fleming right-hand rule will reveal the fact that there are two paths through the armature and that

two brush sets only are necessary and these 90° apart for a four-pole machine.

The first type of multipolar ring is sometimes called a *parallel* or *multiple circuit* ring armature, and the second a *series* or *two-circuit* ring armature.

229. The Bi-polar Drum Armature.—The drum or core consists, as usual, of thin soft iron stampings. The circuit of any one conductor is now starting from a bar of the commutator, passing along the surface of the drum, crossing the back end, and thence along the surface of the drum again back to another bar of the commutator. There are various methods of winding drum armatures: but the following are simple illustrations which will answer our present purpose :—

Fig. 247 will serve to explain in brief one method of winding a bi-polar drum armature carrying 16 conductors. The 16 wires are spaced uniformly along the outside of the iron drum, and cross connections are made as indicated, the continuous lines denoting the connections at the front, and the dotted lines those at the back of the armature.

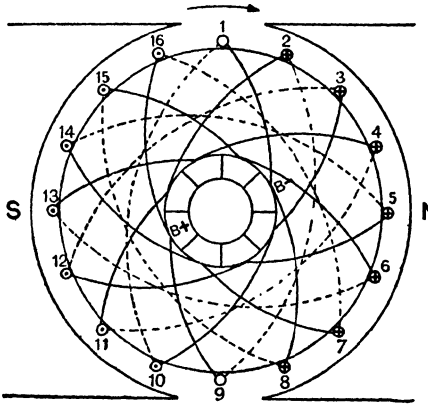


Fig. 247.

From the front end of conductor 1 a connection is led *via* a commutator segment to the front end of conductor 8. Between 1 and 8 there are *six* conductors; adding one to this we get what is called the *pitch* (seven in this case), and this pitch will be adhered to for the whole winding. Thus the back end of conductor 8 is connected to the back end of conductor $8 + 7 = 15$, and the front end of 15 is joined *via* the adjacent segment to the front end of a conductor which is seven ahead of it—viz. to the front end of conductor 6. The back end of 6 is joined to that of $6 + 7 = 13$, and the front of 13 to the front of 4 *via* the next commutator segment. This is repeated, and finally the back end of 10 is joined to the back of 1, and a closed winding is obtained.

The brushes make contact as indicated with the commutator bars to which conductors 1 and 8 and conductors 9 and 16 are connected. An application of the right-hand rule will show that, with the armature revolving as indicated, the currents in the conductors on the left are flowing from back to front, whilst those in the conductors on the right

are flowing from front to back. Thus *in the armature circuit* the current has two paths, viz.:

(1) B — along 1 — 10 — 3 — 12 — 5 — 14 — 7 — 16 — B +

(2) B — along 8 — 15 — 6 — 13 — 4 — 11 — 2 — 9 — B +,

whilst in the external circuit the current flows from B + to B —. As in the case of the bi-polar ring, so here there are two paths in parallel in the armature. In all cases the pitch must be such that no two connected conductors come under the same pole at the same time.

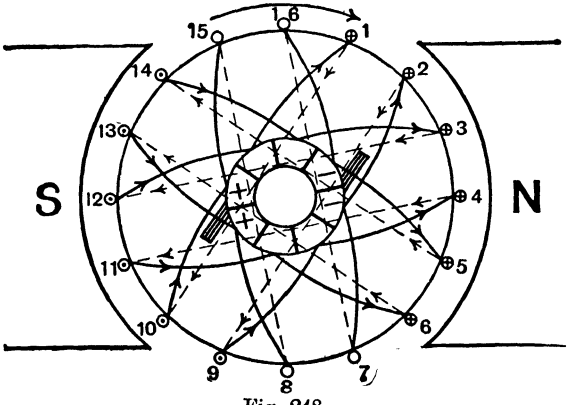


Fig. 248.

The student should note that in the above the pitch is always taken in the same direction (say forward) both at the back and at the front of the armature; thus in Fig. 247 each conductor at the back and at the front is connected to a conductor which is seven *ahead of it.* In the example chosen there are an *even* number of commutator bars (eight), and the pitch (seven) is the same at both ends of the armature. If the construction involves an *odd* number of bars, the pitches at the back and front differ from each other by two, but *both pitches are taken forward* as above. Thus, with eighteen conductors and nine bars, the back pitch would be *nine* forward and the front pitch *seven* forward.

Fig. 248 depicts the armature of Fig. 247 wound on another method in which the pitch is in one direction at the front and in the opposite direction at the back so that if one pitch be called *positive* the other would be called *negative.* Using, however, the expressions "forward" (direction of rotation) and "backward," it will be seen in Fig. 248 that the pitch is taken *forward at the back of the armature and backward at the front.* The pitches differ by two, the back pitch being

7 *forward* and the front pitch 5 *backward*. With eighteen conductors and nine bars the back pitch would be 9 *forward* and the front pitch 7 *backward*. There are two parallel paths in the armature as in the previous case.

The first winding above, viz. Fig. 247, is sometimes called the **wave** method, and the second (Fig. 248) the **lap** method. *There is really, however, no essential difference, electrically, between lap winding and wave winding in the case of a bi-polar machine, but the difference is important in the case of multipolar machines.* Further, a little consideration will indicate that the greatest number of commutator bars for a drum is Z/2, where Z is the number of conductors: the number may be Z/2*n*, where *n* is a whole number and 2*n* a factor of Z.

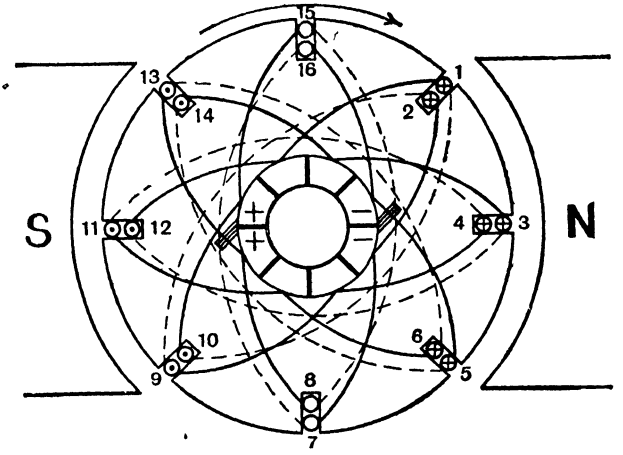


Fig. 249.

One drawback to both of the above is that the full pressure of the machine exists between adjacent conductors in turn. This is eliminated by arranging the conductors in two layers, as shown in Fig. 249, which is a repetition of the winding of Fig. 248. In this, it should be noted that a loop is made by an outer conductor in one layer joined to an inner conductor in another layer, and so on right round.

Bar Winding and Former Winding.—The conductors may be either composed of a number of wires laid up together, or, when large currents are to be generated, they may be made of stout strips of copper, which form a **bar winding.**

In practically all cases now **former winding** is resorted to. That is, the coils are wound on a frame beforehand, all to an exact shape, they are taped, dipped, and baked for insulation, and then can be assembled together on the surface of the iron core in such a way that the ends of the coils all come out correctly placed for

connection by soldering to the ends of the commutator bars. The whole is balanced both electrically and mechanically, and minimum waste of material has been incurred.

Ventilation, etc.—The core is usually not solidly built together, but at intervals a gap is left which is maintained by radial corrugated pieces of brass permitting free passage of air from the central openings in the core to the coils outside on the surface. This ventilates the armature, and higher current densities can be employed with such devices than without.

In all cases where conductors are fixed upon the surface of the drum considerable mechanical force is exerted between the conductor and the magnetic field in which it is revolving, and the attachment between the conductor and the drum must be sufficiently strong to resist this. This attachment is provided by means of pegs which fit into the surface of the drum, and act as drivers to the conductors. Further, binding wire is wrapped tightly round the armature (over mica) at various parts and soldered, thus securely holding the coils.

Slotted Cores.—Instead of constructing the drum of plain circular stampings, each disc may have a number of slots cut out of its periphery, and the plates are all assembled so that the slots are in line. Thus the complete drum has its surface cut up into alternate slots and teeth running in the direction of its length. The windings are then placed in the slots and are frequently kept in place by a strip of wood fitting into a dovetail near the top of the slot. Hence there can be no slip between the conductors and the drum, and, further, the air-gap may be made much smaller, since nothing protrudes beyond the surface of the drum.

In such an arrangement as this the flux does not pass down into the slot carrying the conductor. An obvious question then arises as to the production of E.M.F. by the conductor which is in the slot. The answer is that, as the drum turns, the flux snaps across the slots from tooth to tooth, in so doing causing E.M.F. in the conductor.

A disadvantage of the toothed armature is that the teeth tend to set up eddy currents in the pole pieces, but this can be practically avoided by making the distance between the teeth at their ends not more than 2 to $2\frac{1}{2}$ times the air gap. Fig. 250 gives a case in which this condition holds, and it will be seen that the lines spreading from the teeth are uniform over the pole face. In Fig. 251 the distance between the teeth is greater than the value given above; this results in areas A where there are few lines, and as these move across the pole face, the field variation causes eddy currents in the pole pieces, which must in consequence be laminated.

Fig. 250.

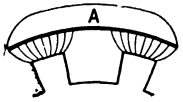


Fig. 251.

End Connections.—In modern armatures the ends of the coils are not carried across the actual ends of the armature as shown in the preceding figures, but the connections are invariably made on a cylindrical extension of the armature surface.

The end connections are well insulated from the supporting extension, and special devices are employed to protect them against the action of centrifugal force,

230. Multipolar Drum Armatures.—There are two general methods of winding these, viz. **lap winding** and **wave winding.** In lap winding the front pitch and the back pitch are in opposite directions so that if the back pitch be called *positive*, the front pitch must be called *negative*: in wave winding the front and back pitches are of the same sign. In (simplex or single) lap winding there are as many paths in parallel in the armature as there are field poles: in (simplex or single) wave winding there are two paths in parallel whatever the number of field poles may be. In lap winding the number of brush sets required is equal to the number of poles: in wave winding two brush sets only are necessary, although a number equal to the number of poles may be used.

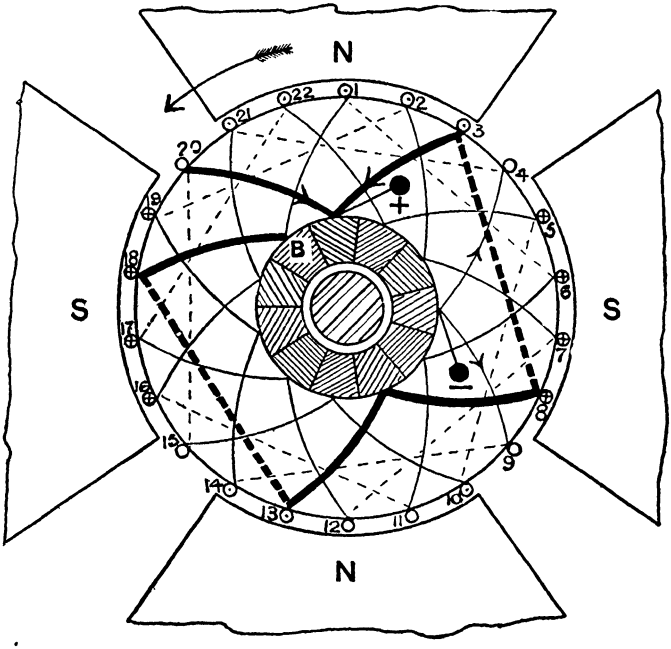


Fig. 252.

Four pole drum (simplex) wave wound. Z = 22. Front pitch = + 5. Back pitch = + 5. Average pitch = 5. Commutator pitch = 5. Number of paths = 2. Single re-entrant.

Fig. 252 shows a four-pole drum (simplex or single) wave winding with 22 conductors. The back pitch is + 5 (or 5 forward say) and the front pitch is + 5 so that the *average pitch* is 5. Starting, say, at the commutator bar B in the figure, a front connection goes to conductor 18: this conductor passes along to the back of the armature and is con-

nected to conductor 13 which is 5 steps ahead of it: this conductor passes along to the front and is connected (via *a commutator bar on the opposite side of the commutator, in this case, to the starting bar*) to conductor 8 which is 5 steps ahead of it. this conductor passes along to the back and is connected to conductor 3: this conductor passes along to the front and is *first connected to the commutator bar next to the one at which we started* and then to conductor 20. The winding is repeated, until finally we arrive at conductor 1 which brings us down to the bar at which we started, and a closed winding is the result.

By applying the right-hand rule it will be found that conductors under the N poles have upward currents, whilst those under the S poles have downward currents, that the current paths in the armature are:—

Negative Brush to | 9 – 14 – 19 – 2 – 7 – 12 – 17 – 22 – 5 – 10 – 15 – 20 | to Positive Brush

4 – 21 – 16 – 11 – 6 – 1 – 18 – 13 – 8 – 3

and that two brush sets only are necessary. (p sets may be used.)

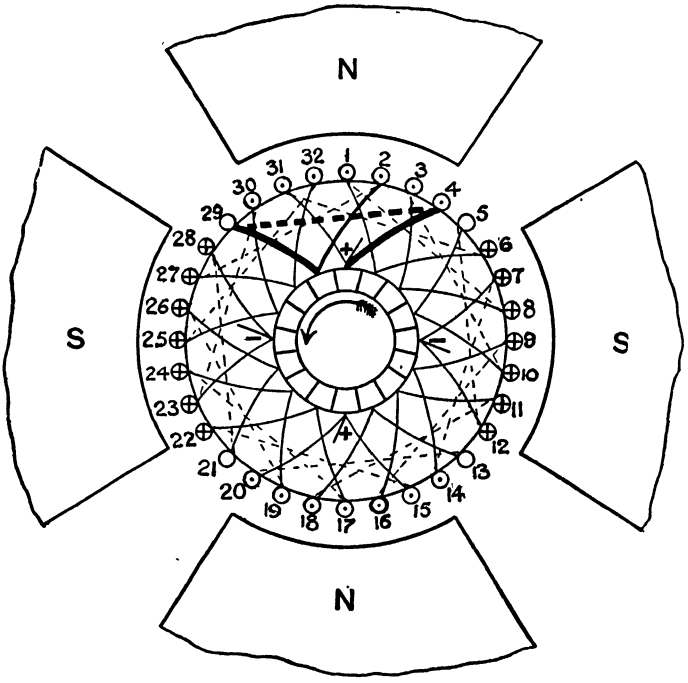


Fig. 253.

Four pole drum (simplex) lap wound. Z = 32. Front pitch = – 5. Back pitch = + 7. Average pitch = 6. Commutator pitch = 1. Number of paths = 4. Single re-entrant.

That portion of a winding which terminates on two commutator bars is called a **winding element.** It will be seen from Fig. 252 that in the case of a *four*-pole drum (simplex or single) wave winding *two* winding elements must be passed through to bring us back to the commutator bar next to the one at which we started. In the case of a six-pole drum (simplex or single) wave winding, the number will be *three*, and in general $p/2$, where p is the number of field poles.

Fig. 253 shows a four-pole drum (simplex or single) lap winding with 32 conductors. The back pitch is + 7 (or 7 forward) and the front pitch is — 5 (or 5 backward), so that the average pitch is 6. Starting, say, at the top commutator bar a front connection goes to conductor 4, this at the back is joined to conductor 29 (pitch + 7) and at the front conductor 29 laps back *first to the commutator bar next to the one at which we started* and then to conductor 2 (pitch — 5). This is repeated until finally we arrive at conductor 31 which brings us down

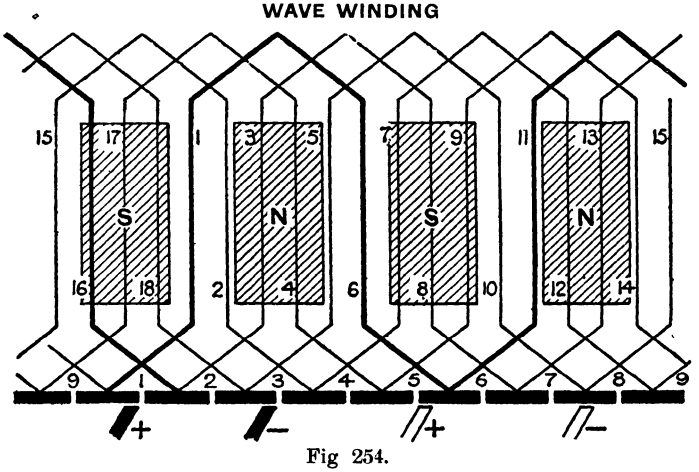


Fig 254.

to the bar at which we started, and a closed winding is the result. By applying the right-hand rule it will be found that four brush sets are necessary and that the current paths in the armature are:—

First Negative Brush to	12 – 5 – 10 – 3 – 8 – 1 – 6 – 31	to Positive Brushes
	7 – 14 – 9 – 16 – 11 – 18 – 13 – 20	
Second Negative Brush to	28 – 21 – 26 – 19 – 24 – 17 – 22 – 15	
	23 – 30 – 25 – 32 – 27 – 2 – 29 – 4	

i,e. there are as many paths in parallel as there are poles.

If we imagine the drum to be cut into two halves by a plane passing along the axis and the surface to be spread out flat, the appearance gives us another method of showing armature windings, and is known as the **developed diagram.** The student should draw the following in the way already indicated (Figs. 252, 253) and then compare them with the developed diagrams given in Figs. 254 and 255.

Case 1 (Fig. 254): Four pole (single or simplex) wave, wound drum. No. of conductors = 18. No. of commutator bars = 9. Back pitch = + 5. Front pitch = + 5. No. of parallel paths = 2.

LAP WINDING

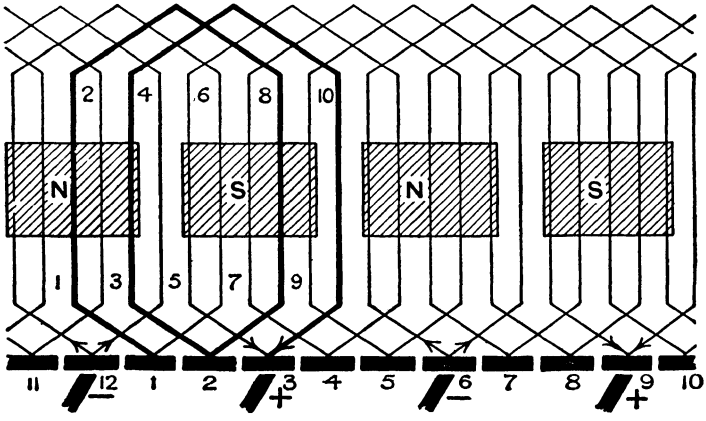


Fig. 255.

Case 2 (Fig. 255): Four-pole (single or simplex) lap wound drum. No. of conductors = 24. No. of commutator bars = 12. Back pitch = + 7. Front pitch = − 5. No. of parallel paths = 4.

In these diagrams the poles are sometimes assumed to be *above* sometimes *below* the conductors (examine Figs. 254, 255, 256).

Yet a third method of showing armature windings is given in Fig. 257: it is called the **radial diagram,** the short radial lines denoting the conductors.

For wave winding if y = average pitch, then $y = \frac{Z \pm 2}{p}$, where Z = number of conductors, and p = number of poles. Both pitches must be odd numbers, so that if y, worked out from the above, is itself odd, then both the front pitch y_F and the back pitch y_B are made equal to the average pitch y. If y be even, then $y_F = y \pm 1$, and $y_B = y \mp 1$. For lap winding both pitches must be odd and differ by 2 so that y is even, and is made nearly equal to $\frac{Z}{p}$: thus $y_F = y \pm 1$ and $y_B = y \mp 1$.

If a winding element starts at commutator bar a and ends on bar b then the number of steps from a to b is called the **commutator pitch**: in Fig. 252 the commutator pitch is 5 and in Fig. 253 it is + 1.

The usual arrangement with slotted armatures has been indicated in Fig. 249, and the number of slots stepped over by a coil is similarly called the **slot pitch.** The commutator and slot pitches are now the main points required by the armature winder.

Lap winding is sometimes called *parallel* or *multiple circuit* winding and wave winding *series* or *two-circuit* winding. The former is suitable for large current output and the latter for large pressure.

231. Multiple and Multiplex Windings: Re-entrancy.—We would warn the student that authors differ considerably in the terms used in connection with the matter dealt with in this section. Further, only a brief and very elementary treatment is possible here.

In a wave winding, if we start at a commutator bar and trace through $p/2$ winding elements (p = number of poles), and if we find that this brings us to the bar next to the one at which we started, the winding is called a **single or simplex wave** winding: this is the case in Figs. 252, 254. If we come to the next bar but one the winding is called a **double wave** winding: if we come to the next bar but two the winding is called a **triple wave** winding.

In a lap winding, if we start at a commutator bar and trace through a winding element, and if we find that this brings us to the bar next to the one at which we started, the winding is called a **single or simplex lap** winding: this is the case in Figs. 253, 255. If we come to the next bar but one we have what is called a **double lap** winding: if we come to the next bar but two we have what is called a **triple lap** winding.

In all the above we are taking the winding *as a whole* to form still a *single closed winding* as in preceding cases.

By these multiple windings the number of paths in parallel in the armature is increased. Thus a single wave winding gives two paths in parallel (Art. 230), but a double wave winding gives *four*, and a triple wave gives *six*: a single lap winding gives p parallel paths (p = number of poles), but a double lap gives $2p$ and a triple lap gives $3p$.

Some writers refer to double and triple windings as above as duplex and triplex windings respectively.

Consider now, say, two single or simplex wave windings like Fig. 252 and call them W_1 and W_2. Picture the conductors of W_1 to be spaced between the conductors of W_2 on the same armature core (*i.e.* conductor 1 is W_1, conductor 2 is W_2, conductor 3 is W_1, and so on), and the commutator bars of W_1 to be spaced between the commutator bars of W_2. We have now an armature with 44 conductors and a commutator with 22 bars, bar 1 belonging to W_1 say, bar 2 to W_2, bar 3 to W_1, and so on. Picture the brushes to overlap at least 2·3 bars: then the windings of W_1 and W_2 will be in parallel between the brushes, and the

number of paths in parallel will be doubled. Such a winding, composed of two *separate and independent* wave windings sandwiched together, is called a **duplex wave** winding. Similarly, two *independent* lap windings arranged in this way is called a **duplex lap** winding. Three *independent* wave or lap windings thus combined is called a **triplex wave** or **triplex lap**: here the brushes must overlap at least 3·3 bars and the arrangement gives three times the number of paths in parallel.

Further, in for example, duplex wave, $p/2$ winding elements brings us to the next bar but one just as in double wave, and in duplex lap a winding element brings us to the next bar but one just as in double lap, and so on; but in duplex, the winding *as a whole* does *not* form *a single closed winding*, although each component winding does so.

Some writers refer to duplex and triplex windings as above (*i.e.* *independent* single or simplex windings sandwiched together) as double and triple windings. Other writers use the terms duplex, triplex, etc. (multiplex) for *any* winding which is not single or simplex.

The formulae for all these windings are somewhat more complex than the formulae for single or simplex windings, and cannot be dealt with here.

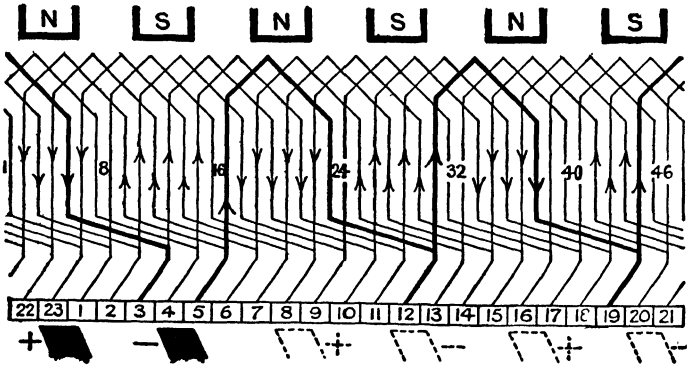


Fig. 256.

Six pole drum {double (or duplex)} wave wound. $Z = 46$. Front pitch = 7. Back pitch = 7. Average pitch = 7. Commutator pitch = 7. Number of paths = 4. Single re-entrant.

Fig. 256 gives a six-pole double wave winding, and it will be seen that there are *four* parallel paths in the armature (a single wave gives *two*), viz. two paths from each of the two bars touched by the broad brushes. In this there are 46 conductors and the front and back pitches are each 7.

Fig. 257 gives a six-pole duplex lap winding (*i.e.* two independent simplex lap windings on the same core), and there are *twelve* parallel

paths (a simplex lap gives *six* in a six-pole machine). The radial lines in Fig. 257 are the 60 conductors: back pitch (outside lines are the back connections) = + 7, front pitch = − 11.

If we start at conductor 1 on an armature and trace the winding, and if we find that all the winding is passed through before we get back and "re-enter" conductor 1, the winding is said to be **single re-entrant**: if we find that when we reach conductor 1 again only half the winding has been traced, the winding is **double re-entrant**: if only one-third of the winding has been passed through, the winding is **triple re-entrant.** According to these definitions, Figs 252-255 are single re-entrant, the double wave of Fig. 256 is single re-entrant, and the duplex lap of Fig. 257 is double re-entrant. Here again different writers use different classifications.

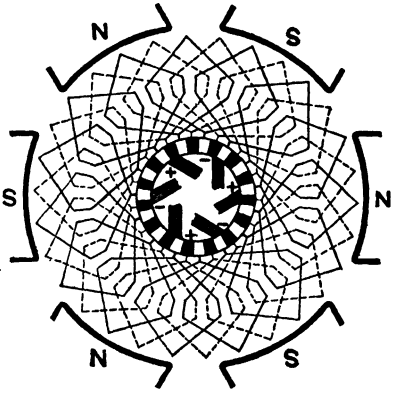


Fig. 257.

Six pole drum (duplex) lap wound. Z = 60. Front pitch = − 11. Back pitch = + 7. Average pitch = 9. Commutator pitch = 2. Number of paths = 12. Double re-entrant.

In lap-wound armatures of large output small P.D.'s may exist between circuits in parallel so that current does not divide equally between them. To remedy this low resistance **equalising connections or rings** of copper are fitted so as to form connections between parts of the armature windings which should be at the same potential.

232. Armature Reaction.—Unless the brushes be correctly placed vicious sparking will ensue between the brushes and commutator, doing damage to both, and considerably interfering with the working of the machine. The circumstances on which this depends are dealt with in this and the following section.

"Armature reaction" is due to the magnetising effect of the currents *in the armature*, and such may be classified into:—(*a*) The **cross-magnetising or distorting effect,** and (*b*) the **demagnetising or weakening effect.** If the brushes touch the coils when they are in the geometrical neutral axis (*i.e.* have no "lead," as we have assumed in Fig. 240), then only (*a*) is present: if the brushes have "lead" (and they *have* in practice), both (*a*) and (*b*) are present. The designer must reduce both as much as possible.

Since an armature is a mass of iron carrying coils upon it, and the coils are delivering current—through the brushes—to the outside

circuit, the iron of the armature is subjected to a magnetising force which is greater as the current increases. If the brushes are at right angles to the axis of the field magnets (*i.e.* on the *geometrical* neutral axis), the core will be magnetised along the line joining the brushes, and therefore at right angles to the magnetisation produced in it by the field magnets (Fig. 258). Hence the *resultant magnetisation* is oblique and becomes the more inclined the larger the current; and since the brushes must (approximately) be on the diameter at right angles to the lines of force through the armature (*i.e.* on the *magnetic* neutral axis), it is therefore found necessary to shift the brushes forwards (*i.e.* give them "lead") to obtain no sparking at the brush contacts. These facts will be apparent on examining a few simple diagrams.

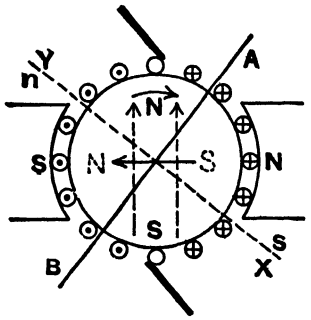


Fig. 258.

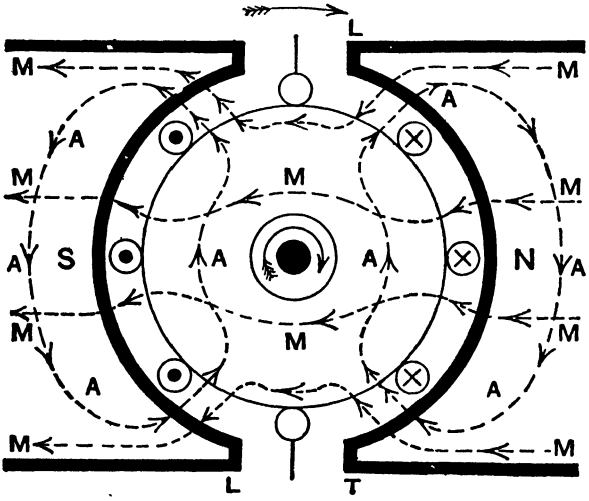


Fig. 259.

Consider first Fig. 258, in which the brushes have no lead: current will flow as indicated. Each of the conductors on the right may be imagined as paired with a conductor on the left. An application of the simple right-hand rule (Art. 197) will show that these currents tend to magnetise the core in the direction of the dotted arrows (*i.e.* at right angles to the magnetisation of the core produced by the

field magnets), N towards the top, and S towards the bottom. Taking now the joint effect of the field magnets and the armature current, it will be seen that the resultant magnetisation is in the general direction X Y, and the brushes should be placed on the line at right angles to this, viz. on A B (*magnetic neutral axis*): in other words, the brushes must be moved forward in the direction of rotation. The angle through which they are moved is called the *angle of lead.* All this is referred to as the cross-magnetisation or distorting effect.

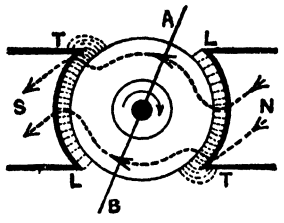


Fig. 260.

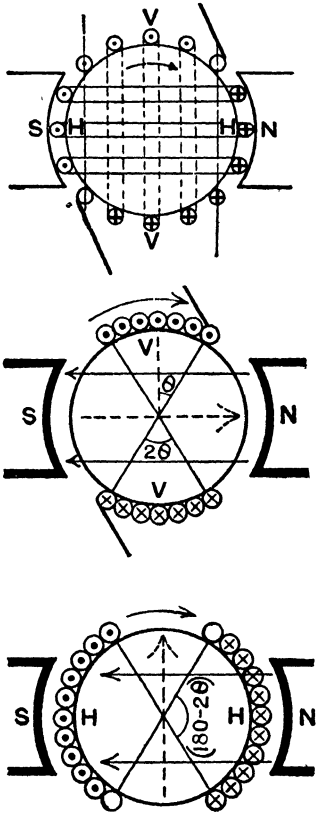


Fig. 261.

The above can also be seen from a consideration of (*a*) the lines due to magnetisation by the armature currents alone, (*b*) the lines due to the fields alone, and (*c*) the resultant of (*a*) and (*b*). Thus in Fig. 259, the lines A give the direction of the flux due to the armature currents alone, and the lines M give the direction of the flux due to the fields alone. It will be noted that at the pole tips T T, where the armature passes out of the poles, the lines A and M are in the same direction, and help each other; whilst at the pole tips L L, where the armature passes into the poles, the lines A and M are in opposite directions, and partly cancel each other. The pole tips T T are called the **trailing pole tips,** and L L the **leading pole tips:** hence we may say that **cross-magnetising action (in dynamos) strengthens the field under the trailing pole tips, and weakens the field under the leading pole tips.** Fig. 260 shows the direction of the resultant flux: the magnetic neutral axis is at right angles to this, and is the same as A B in Fig. 258, *i.e.* moved round in the direction of rotation.

Fig. 262 further illustrates the above in a slightly different ways.

Consider now Fig. 261, where the brushes have been given an angle of lead θ, and it will be found that we have still some cross-magnetising and, in addition, a demagnetising effect. It is convenient in this case, to conceive the whole winding divided into two groups—a vertically-wound belt V V, consisting of the top conductors, which subtend the angle 2θ joined to the corresponding bottom conductors, and a horizontally-wound belt H H, consisting of the conductors under N, which subtend the angle $(180° - 2\theta)$ joined to the corresponding conductors on the left.

The belt H H produces a cross-magnetising or distorting effect as already explained. An application of the simple right-hand rule to the vertical belt will show that it produces a flux in the core *from left to right, i.e.* in the opposite direction to the flux due to the fields; in other words, it produces a demagnetising or weakening effect, and is termed the *demagnetising belt* or *back turns*. The bigger the armature current and the bigger the lead the bigger this weakening effect. Further, the bigger the armature current the more marked will be the weakening of the field in the vicinity of the leading pole tips due to the cross-magnetisation effect. Now it is in the vicinity of these tips that the coils come under the brushes, and if the field be too weak, due jointly to strong cross-magnetisation and demagnetisation, sparkless commutation is difficult to obtain. The designer must keep both cross-magnetisation and demagnetisation small. Methods of dealing with these in practice will appear later.

For simplicity we have assumed the *actual brushes* to be in the magnetic neutral axis: of course, this really means *the coils* which come under the brushes: owing to the shape of the end connections the actual brushes may be opposite the centres of the poles.

The "ampere turns" to which the demagnetising effect is due, is called the "*back ampere turns*," and this must be allowed for in working out dynamo magnetic circuit calculations. Similarly, the "ampere turns" to which the cross-magnetisation is due, is called the "*cross ampere turns*": for simplicity we have taken all the conductors in the angle $(180° - 2\theta)$ as contributing, but strictly it is only those under the poles.

The back ampere turns per magnetic circuit (B.A.T.) is given by:—

$$\text{B.A.T.} = \frac{\theta}{180} Z \frac{I}{a},$$

where θ = lead, Z = total armature conductors, I = total armature current, a = number of armature paths.

233. Commutation: Lead and Sparkless Collection.—We have seen that, owing to the cross-magnetising or distorting effect of armature reaction, the magnetic neutral axis is moved forwards from the geometrical neutral axis in the direction of rotation, and the brushes must be given "lead" accordingly: this is again shown (for a ring armature) by a slightly different method in Fig. 262.

The brushes must not, however, be even in the magnetic neutral axis: they have, we find, to be shifted further on still for sparkless collection, and the reason for this extra (small) shift may be briefly explained as follows:—

We have seen that while two adjoining bars of a commutator are passing under a brush the section of the armature winding included between the two bars, whether ring or drum is immaterial, is short-circuited. This is seen by reference to Figs. 262, 264. Now the coil, while ascending, has had a certain current circulating in it, and when it is free from the brush again it will have another current in it equal to the first, *but in the opposite direction*; thus the short-circuiting position is a transition one between reversal of current. Further, the self-induction of the coil tends to oppose this reversal, *i.e.* it tends to prolong the current in the first direction and oppose the growth in the new direction. Now the occurrence or not of a spark at the edge of *b*, when leaving the brush, depends entirely upon the current flowing in the coil *p* at that particular instant. If the current in the short-circuited coil is exactly equal to the current it has to carry afterwards, there will obviously be no spark. Hence, if we so arrange the position of the brushes that the coil *p* is not in the position of no E.M.F. production when short-circuited, *but is just beginning to feel the effect of the next pole*, then it is possible to arrange that this field which it is just entering shall have destroyed the current which was flowing before, and formed a new one, which is exactly equal to the current it will have to carry when clear of the brush, in which case there will be no spark. It is, then, for this double reason that the brushes are given *lead*. Since any one section of an armature winding is of very low resistance, when it is short-circuited a very small field may cause quite large currents; hence this secondary shift need not be large. The resistance of the section, however, at the instant considered, includes the contact between the brush and the commutator, and this is found to have a great effect on sparking(see below).

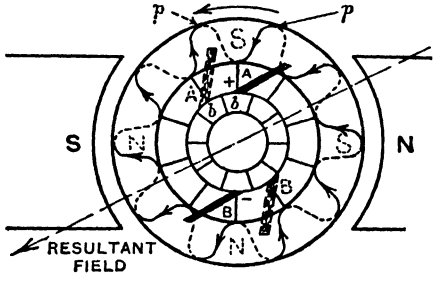


Fig. 262

Such shifting of the brushes to avoid sparking as the current varies, needs attention, and it is desirable to arrange matters, if possible, so that the collection shall be sparkless *at all loads*, and this condition is commonly introduced now into the specification of a machine. This result is accomplished by using carbon brushes, and is much facilitated by remembering that when a magnet is near saturation point a small

alteration in the ampere turns produces a negligible change in the magnetic flux. Hence, if the field be very stiff and near saturation, the demagnetising effect of the back armature turns may be so small as not to cause any appreciable "weakening," and the "shift of the resultant field" may be small, and thus the brushes may be fixed in position.

Amongst the various methods devised for reducing armature reaction and assisting commutation brief reference may be made to the following:—

By introducing an air gap in the pole-pieces as indicated in Fig. 263 the reluctance in the path of the lines A due to cross magnetisation is increased whilst the lines M due to the fields are not appreciably interfered with: thus the effect of armature reaction is reduced.

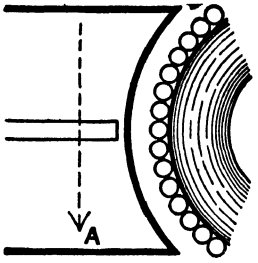


Fig. 263.

A similar effect can be obtained by increasing the length of the air gaps under the pole tips using shorter air gaps under the poles: this increases the reluctance in the path of the cross flux A as above. It is done by suitably shaping the pole-pieces or pole shoes, *e.g.* by cutting off the ends of alternate laminations and by other methods.

Narrow pole faces and strong field excitation (as indicated above) lessen the effect of armature reaction.

The use of a high resistance carbon brush in assisting commutation will be understood from Fig. 264. The coil X is short-circuited by the brush B. Now the current it has been carrying (in the direction of the current in Z) must first be reduced to zero and this will be done all the easier if the contact resistance of B and bb' is big. Again a current must be started in X in the direction of that in Y. Now the contact resistance between B and b is large, and as b moves gradually away from B the area of contact becomes less and the resistance still bigger so that the current from Y tends to turn away from b and into X and b_1 in the direction required as previously indicated.

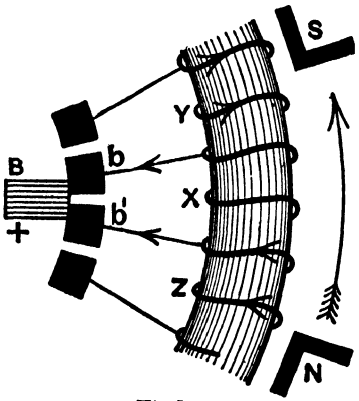


Fig. 264.

The use of **interpoles** or **commutating poles** and of **compensating** or **neutralising windings** are two of the best methods for overcoming armature reaction and are dealt with in Art. 238.

234. Calculation of E.M.F. produced in a Two-pole Machine: Extension to Multipolar Machines.—Referring to Fig. 240—

Let ϕ = total magnetic flux passing through the armature,

Z = total number of external wires connected in series on the armature,

N = number of revolutions per second.

Each external wire cuts ϕ lines *twice* in one revolution, *i.e.* each conductor cuts 2ϕ lines per revolution, and therefore $2\phi N$ lines per second; hence—

$$\text{E.M.F for each conductor} = \frac{2\phi N}{10^8} \text{ volts.}$$

Again, since the brushes are placed diametrically opposite each other, the current has obviously two paths through the armature from brush to brush, viz. one path up one side, and the second path up the other. Further, each path contains $\frac{Z}{2}$ conductors in series forming E.M.F., the two paths being arranged in parallel, and just as the combined E.M.F. of two equal batteries in parallel is the same as that of one battery, so the total E.M.F. in this case is that due to $\frac{Z}{2}$ conductors; hence—

$$\textbf{Average E.M.F. of dynamo} = \frac{2\phi N}{10^8} \times \frac{Z}{2},$$

$$= \frac{\phi N Z}{10^8} \textbf{ volts,}$$

$$= \frac{\textbf{Flux} \times \textbf{Rev. per sec.} \times \textbf{No. of conductors}}{10^8}.$$

For multipolar machines the E.M.F. is calculated by an extension of the fundamental relation above. Thus, it can readily be shown that the general formula for bi-polar or multipolar machines is:—

$$E = \frac{p}{a} \times \frac{\phi N Z}{10^8} \text{ volts,}$$

where p = number of poles, a = number of paths in parallel between brushes, and ϕ = flux from *each* north pole.

The truth of the above for multipolar machines may be readily established. Consider a *four-pole machine wave* or *series wound*. The conductors are divided into two groups each of Z/2 conductors. Since ϕ is the total lines entering the core from one pole, the number of lines cut by a conductor in one revolution is 4ϕ, and therefore $4\phi N$ in one second. The average E.M.F. in a conductor is therefore $\frac{4\phi N}{10^8}$ volts, and in $\frac{Z}{2}$ conductors $\frac{4\phi N}{10^8} \times \frac{Z}{2}$, *i.e.* $\frac{2\phi NZ}{10^8}$ volts, or $\frac{4\phi NZ}{2 \times 10^8}$

volts, where 4 = number of poles (p) and 2 = number of paths in parallel (a).

Consider now a four-pole machine *lap* or *parallel wound*. The Z conductor are joined in four parallel groups each of Z/4 conductors. As before the average E.M.F. in a conductor is $\frac{4\phi N}{10^8}$ volts and in $\frac{Z}{4}$ conductors $\frac{4\phi N}{10^8} \times \frac{Z}{4}$, *i.e.* $\frac{\phi NZ}{10^8}$, *i.e.* $\frac{4\phi NZ}{4 \times 10^8}$ volts and this is the E.M.F. of the machine: here 4 = number of poles (p) and 4 = number of paths in parallel (a).

235. Excitation and Winding of Field Magnets.—The methods may be classified as follows:—

(1) Permanent magnets, used on magneto-electric machines, but not for practical electrical engineering.

(2) Electro-magnets used on direct current machines excited by the machine itself, the residual magnetism of the soft iron being sufficient to start the action.

(3) Electro-magnets which are separately excited, used on alternating current machines, the excitation being by means of an auxiliary direct current dynamo or by accumulators. The auxiliary dynamo may be either quite separate or attached to the shaft of the alternator itself.

Class (2) is divided into three types, differing from each other in the way in which the machine is made to excite itself.

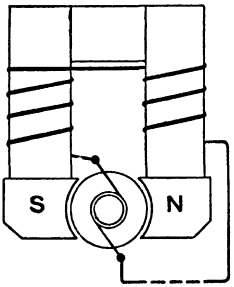


Fig. 265.

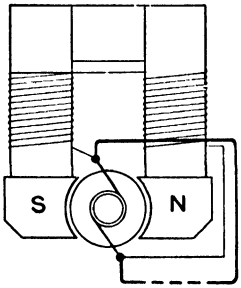


Fig. 266.

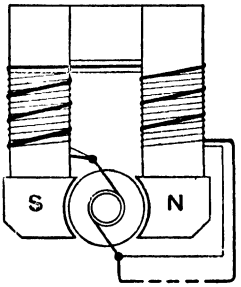


Fig. 267.

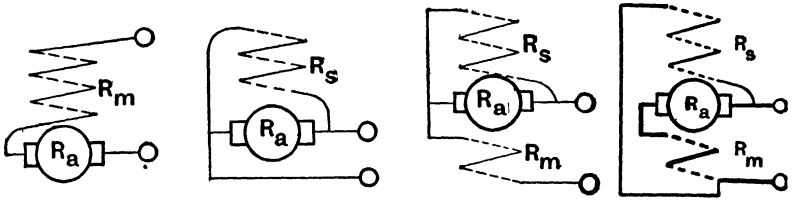
(*a*) *Series Wound Machines* (Fig. 265).—This is the name given to the arrangement when the coils of the armature, the field magnets, and the external circuit are all arranged in series. Thus the total resistance is given by $R = R_m + R_a + R_e$, using m, a, and e to denote the resistance of the series or magnet coils, armature resistance, and external resistance respectively. Series machines were formerly much used for arc lighting, but have now largely given place to the next type.

(*b*) *Shunt Wound Machines* (Fig. 266).—In this form the coils of the field magnet are placed as a *shunt* across the brushes, so that the current from the armature may *either* pass through the external circuit

or through the field coils, the current naturally dividing according to the usual laws for divided circuits when it leaves the + brush. In this case we have three resistances in parallel, the centre one R_a containing the E.M.F., the other branches being R_s and R_e respectively where *s* indicates the shunt coil. These machines are extensively used for lighting purposes, and *always for charging batteries* (Art. 239). Running at constant speed they give a fairly (but not quite) constant voltage in practice.

(*c*) *Compound Wound Machines* (Fig. 267).—The compound machine, as used, consists of a shunt machine with a few series turns in addition. In dealing with the resistances of such a machine, we have in all R_e, R_a, R_s, and R_m, these having the same significance as before. Thus in Fig. 267 (short shunt compound) we have three resistances in parallel *between the brushes*, viz. R_a, R_s, and $(R_m + R_e)$.

The advantage arising from compounding, which is better seen from a study of the characteristic curves in Art. 236, is that the pressure at the terminals can be made to be practically constant, whatever the current being taken. *With the series machine the pressure rises with the current,* whereas *with the shunt machine the pressure falls*: hence a combination, if of suitable proportion, will obviously give *practically constant pressure at all loads.*



Series. Shunt Short-shunt Compound Long-shunt Compound.

Fig. 268.

Fig. 268 diagrammatically represents the three types, the third being known as a *short shunt compound* (Fig. 267) and the fourth as a *long shunt compound.* In the latter the shunt winding is connected across *both* the armature and series coils: thus there are three resistances in parallel *between the terminals*, viz. $(R_a + R_m)$, R_s, and R_e.

If still more series turns be added to a compound wound machine the volts may be made to rise with increase of current, and this may be used to neutralise the effect of drop in the mains when desiring a constant pressure at a *distant* point instead of at the machine terminals; this is termed **"over-compounding."**

The whole principle of self-excitation is briefly that, owing to residual magnetism in the fields, the armature, on starting, has a

small E.M.F. and current developed in it, and the current passing round the fields strengthens them, so that a larger E.M.F. and current are developed, and so on; thus the machine quickly builds up.

Theory indicates that for a series dynamo the resistance of the field magnets should be a little less than that of the armature, both being small compared with the external circuit. For a shunt dynamo the external resistance should be a geometrical mean between that of the shunt coils and that of the armature.

Field magnet coils are usually wound on **formers,** which, when wound, can be slipped over the magnet limb, this being more convenient than winding upon the limbs direct. The windings in a shunt machine must be of high resistance, and are composed of thin wire: in the case of the series machine the wire must be thick. In compound machines the suhnt winding is underneath, and the series coils are placed outside or on top of the shunt coils. Still more modern practice puts them side by side.

236. Characteristic Curves.—To facilitate the study of dynamos Hopkinson proposed the **characteristic curve,** *which shows for any particular speed the relation between the varying quantities "current" and "pressure."*

To plot the characteristic curve for a given machine we run it for about 15 minutes, until it has reached a steady state, and then, keeping the speed constant, we take with an ammeter and voltmeter, the amperes in the external circuit and the volts at the dynamo terminals. The load is then altered and the speed adjusted to what it was before, and readings are again taken. On plotting the volts as ordinates, and the amperes as abscissae, we get what we call the **external characteristic curve,** *i.e. the curve showing the relation between the variation of current in the external circuit and the variation of P.D. at the dynamo terminals.* Fig. 269 depicts the external characteristic (O′ N) for a series machine. In the series machine it will be observed that the volts increase with the amperes up to a certain maximum, after which they fall.

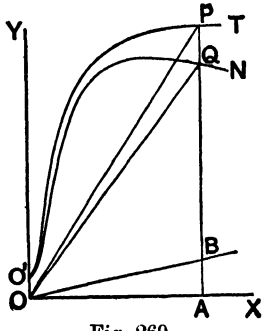


Fig. 269.

Referring to Fig. 269, in which O′ N is the external characteristic for a series machine, we see that when the amperes are equal numerically to O A the external volts are represented by Q A; hence by Ohm's law the resistance in the external circuit is $\frac{Q\,A}{O\,A}$, that is, it is measured by tan Q O A. If now we draw a line O B, such that tan B O A measures the internal resistance of the machine to the same scale as tan Q O A measures the external resistance, we have, when the amperes are repre-

sented by O A, the line A B representing the lost volts in the machine; thus O B may be referred to as the **internal characteristic curve.** Again, since when the amperes are represented by O A the volts in the outside circuit are measured by Q A, and the volts lost in the machine by A B, the total E.M.F. of the machine will be represented by Q A + A B. Making P Q = A B we have the total E.M.F. (when the amperes are represented by O A) represented by P A, and the locus, namely O′ T, gives what is called the **total characteristic curve** of the series machine, *i.e.* the *curve showing the relation between the total E.M.F. and the current supplied.*

The curves for a shunt machine are similarly obtained. The curve A P B (Fig. 270) gives the external characteristic, *i.e.* the curve *connecting terminal P.D. and current in the external circuit:* this is obtained as indicated for the series machine. In the shunt machine it will be observed that the volts decrease as the amperes increase. Now O C is drawn so that tan C O M represents the resistance of the shunt coil so that K L, for example, gives the current in the shunt coil when the P.D. at the terminals is O K. Drawing K Q parallel to O M and making P Q equal to K L we get, when the terminal P.D. is O K (or M Q), K P giving the current outside and P Q the current in the shunt, *i.e.* K Q gives the *total current* when the terminal P.D. is M Q. Now drawing N O so that tan N O M gives the armature resistance we have, when the total current is O M, the terminal P. D. given by M Q and the volts used in the armature given by M N. Making Q R = M N we have when the *total current* is O M, the *total volts* given by M R, and the locus of R, viz. C D, is the *total characteristic curve* of the shunt machine, i.e. *the curve connecting total E.M.F. and total current.*

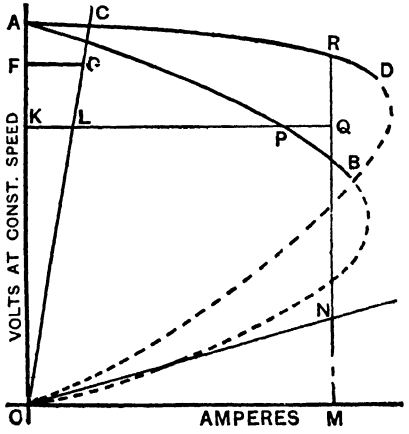


Fig. 270.

For many practical purposes, such as the feeding of glow-lamps, it is desirable that the pressure produced by the dynamo, either at its terminals or at some distant point of the circuit, shall be constant whatever the current. Since a series machine shows increase of pressure with current and a shunt machine decrease of pressure with current, it is obvious that a suitable combination of the two windings will give us either a practically level characteristic or a characteristic which is practically a slowly rising straight line. The former is said

to be a *compound wound* machine, and the latter is said to be *over-compounded.* Fig. 271 shows roughly the combination of the two characteristics, from which the level characteristic of the compound machine can be at once deduced.

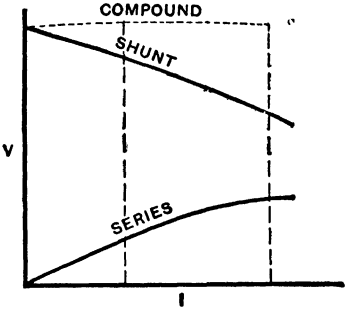


Fig. 271.

237. Voltage Regulation of Dynamos.—We have seen that the compound dynamo is a constant pressure machine (compound winding does not of course compensate for drop of voltage due to heating). Constant pressure can be approximately obtained from a shunt dynamo by either altering the speed suitably or altering the field resistance suitably; but either means a watchful attendant, while the compounding method is automatic.

In order to regulate the pressure of a shunt dynamo effectively by means of the field, a rheostat, termed the *field-winding rheostat*, is placed in series with the field winding (Fig. 272); the more resistance there is inserted in this rheostat, the less the pressure produced, because the current and the ampere turns round the field magnet are lessened, and therefore so also is the magnetic field. The regulation is effected by the attendant with his eye on the voltmeter joined to the dynamo terminals.

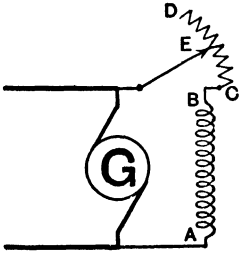


Fig. 272.

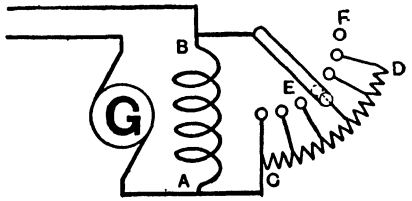


Fig. 273

To regulate the pressure of a series machine by means of the field, a variable resistance is joined *as a shunt to the series coil* on the fields (Fig. 273); it is sometimes called a "diverter." The *bigger this resistance,* the less current is diverted, the more current goes through the series windings and therefore *the higher the voltage:* the voltage will be a maximum if the diverter be disconnected (infinite resistance).

Compound dynamos are fitted with field rheostats (in series with the shunt coils) and sometimes also with diverters (across the series coils) for further regulation.

238. Interpole (or Commutating Pole) Machines, and Neutralising Windings.—We have seen that one of the two effects of the armature current is to magnetise the core approximately at right angles to the main field, giving rise to the oblique resultant field which necessitates lead for the brushes, and that this lead should vary with the current (cross magnetising or distorting effect).

Interpoles are small extra poles, series wound, which are placed halfway between the main poles, as in Fig. 274, and have (*for dynamos*) the polarity shown. These added poles have a strength which is thus proportional to the current delivered, and are so designed as to annul the cross-magnetisation of the armature—in simple language, they tend to produce a N pole in the core, where the armature current tends to produce a S pole, and *vice versa*. Thus by considering the conductors under the top main N pole and those under the left-hand side main S pole and imagining them paired, it will be found on applying the simple right-hand rule to the currents in them that they tend to produce a N pole in the core in the vicinity opposite the intermediate small N interpole (this is the "cross-magnetising" of armature reaction). But this N interpole tends to produce a S pole in the core in its vicinity: thus interpoles counteract cross magnetisation due to armature reaction.

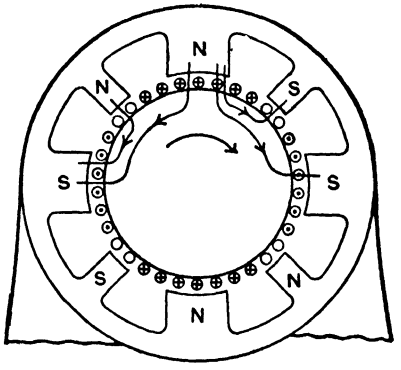


Fig. 274.

Again *in the case of a dynamo* an interpole is of the same polarity as the *next main pole in the direction of rotation*. Now we have seen that a coil undergoing commutation should be *feeling the reversing effect of the next pole*: hence these commutating poles being of the necessary polarity, provide the requisite *reversing fields, i.e.* they assist commutation by helping to reverse the E.M.F.'s as the coils are undergoing commutation. Interpoles thus enable dynamos to deliver widely varying currents, with fixed brushes, quite sparklessly. The same principle applied to large motors enables them to run at very varying speeds without trouble.

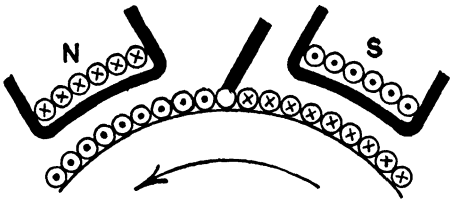


Fig 275.

Another method of neutralising armature reaction and assisting commutation is the employment of **neutralising windings** which are conductors in series with the armature and placed in slots widely distributed in the material of the main pole-pieces (Fig. 275). Considering the current directions shown in Fig. 275, and applying the simple right-hand rule to the armature coils and the neutralising coils on N and S, it will be seen that the two have opposite effects on the armature core: thus the neutralising windings counteract armature reaction just as interpoles do, but the balancing flux is more distributed. Some machines have both interpoles and neutralising windings.

239. Dynamo for Charging Accumulators.—Special consideration has to be given to the choice of a dynamo when it is required to supply a circuit which contains an E.M.F. in addition to that of the dynamo considered. A particularly important case is that arising when accumulators have to be charged. The positive pole of the dynamo must be coupled to the positive pole of the battery, and thus the E.M.F. of the dynamo is opposed by that of the battery, and the charging current is thus due to the difference of the E.M.F.'s, since $I = (E - e)/R$. Now, should by any chance the E.M.F. of the dynamo fall below that of the battery, the current will obviously reverse in direction. Referring to Fig. 268, it will be seen that if we send current into a series dynamo in the reverse direction, *the fields will be reversed in polarity.* (The armature current will also be reversed, and the machine will tend to run as a motor in the *reverse* direction.) Applying the same reversed current to the shunt dynamo (Fig. 268) we see that *the fields will not be reversed in polarity.* (The armature will be reversed and the machine will tend to run as a motor in the *same* direction.) The important point is, however, that *in a series machine the field is reversed in such a case, while in a shunt machine it is not.*

Now the field remains to some extent after the machine is stopped. Hence after such an accident if the series machine were started up as a dynamo, it would send current in the same direction as the cells; and thus at the moment of closing the circuit, serious burning would happen, due to excess of current, while the shunt machine, when started up again as a dynamo, would behave as it did formerly, and no harm would ensue on joining to the battery circuit.

Hence, in practice, shunt machines are always used for cell charging. The machine is speeded up until the volts shown across the terminals are slightly in excess of the battery volts, and then the connecting switch is closed, the engine speed or the field being regulated until the charging current has the proper value.

In order to prevent such reversals as those indicated, which are never desirable, it is usual to fit an automatic cut-out on the switchboard, which will disconnect the battery from the dynamo when the charging current becomes too small and there is danger of reversal (Art. 166).

240. Efficiency of Dynamos.—In every machine there are always certain causes which lead to waste of energy. With dynamos and motors we have mechanical losses due to friction of the shaft in its bearings, to brush friction and resistance due to parts rotating in air. We have also losses represented by heating of the armature and field coils, arising from their resistance, and other losses in the iron due to eddy currents and hysteresis. It is not possible to get as much energy out of a machine as we put into it. Note the following definitions:

$$\textbf{Electrical efficiency} = \frac{\textbf{Watts in the external circuit}}{\textbf{Total watts generated}}.$$

$$\textbf{Commercial efficiency} = \frac{\textbf{Watts in the external circuit}}{\textbf{Total power supplied mechanically to the machine.}}$$

$$\textbf{Mechanical efficiency} = \frac{\textbf{Total watts generated}}{\textbf{Mechanical power supplied}}.$$

From the definitions it follows that:

$$\text{Mechanical efficiency} = \frac{\text{Commercial efficiency}}{\text{Electrical efficiency}}.$$

The commercial efficiency will always be less than the electrical efficiency, because energy is wasted by friction, etc., in the machine. The commercial efficiency of a good large dynamo should be from 92 to 95 per cent.

The *combined efficiency* of a generating set (say, steam engine and dynamo) is given by:—

$$\textbf{Combined efficiency} = \frac{\textbf{Dynamo output in watts}}{\textbf{Indicated H.P. of steam engine} \times \textbf{746}}.$$

TABLE OF EFFICIENCIES.

Let W be the loss in watts in any dynamo due to mechanical friction, hysteresis loss, and eddy currents. Let also e be the terminal P.D. produced by the machine, I the current supplied to the external circuit, R_a, R_m, and R_s respectively the resistances of the armature, the series winding, and the shunt winding. Then we can write the efficiency of each as follows:

Machine.	Electrical Efficiency.	Commercial Efficiency.
Series.	$\frac{eI}{eI + I^2(R_a + R_m)}$	$\frac{eI}{eI + I^2(R_a + R_m) + W}$
Shunt.	$\frac{eI}{eI + \frac{e^2}{R_s} + \left(I + \frac{e}{R_s}\right)^2 R_a}$	$\frac{eI}{eI + \frac{e^2}{R_s} + \left(I + \frac{e}{R_s}\right)^2 R_a + W}$

In the case of the *short shunt compound*, as shown in Fig. 268, the results are—

$$\text{Electrical efficiency} = \frac{eI}{eI + I^2R_m + \frac{(e + IR_m)^2}{R_s} + \left(I + \frac{e + IR_m}{R_s}\right)^2 R_a}.$$

Commercial efficiency = The above with W added to denominator.

Exercise.—Establish by the aid of the diagrams in Fig. 268 the above expressions for the electrical efficiencies.

EXAMPLE.

A gas engine of $9\frac{1}{2}$ *i.h.p. is used to drive a dynamo which maintains a P.D. of* 110 *volts at its terminals when the current generated is* 35 *amperes. If the mechanical efficiency of the gas engine is* 75 *per cent., what is the commercial efficiency of the dynamo?*

The B.H.P. of the gas engine is $\frac{3}{4} \times 9\frac{1}{2} = 7\frac{1}{8}$

$= 7\frac{1}{8} \times 746$ watts $= 5315{\cdot}25$ watts.

Now 35 amperes at 110 volts = 3853 watts;

$\therefore$ Commercial efficiency of machine $= \frac{3850 \times 100}{5315{\cdot}25}$

$= 72{\cdot}4$ per cent.

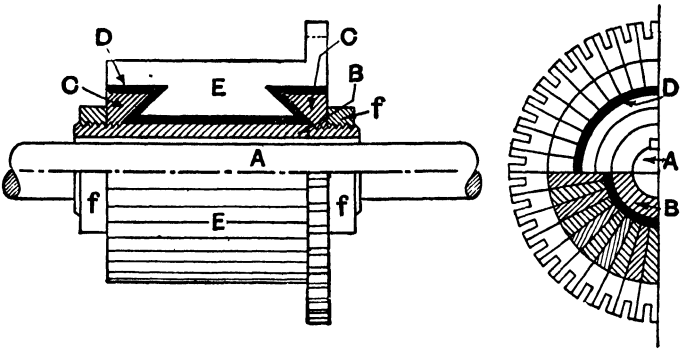


Fig. 276.

241. Commutators, Brushes, etc.—Little further need be said on these points in this book. The commutator is always made up in the form of a rigid cylindrical structure. It is built up of wedge-shaped pieces of copper, drawn or drop forged, and separated from each other by mica strips: the mica should wear at the same rate as the copper. The segments are also insulated from all surrounding parts. Connections ("*risers*") are brought down from the armature conductors and sweated into saw cuts in the commutator bars. A simple arrangement for mounting a small commutator is shown in Fig. 276, where A = shaft, B = metal sleeve, E = commutator bars, D = insulation, C = end rings, and f = adjusting nuts. It will be

noted that each commutator bar dovetails into the projections at each end. Another commutator with shaft and bearings is (through the courtesy of Mr. Frank Broadbent, M.I.E.E., and Messrs. Rentell & Co.*) shown in Fig. 277: the index to parts makes the diagram self-explanatory.

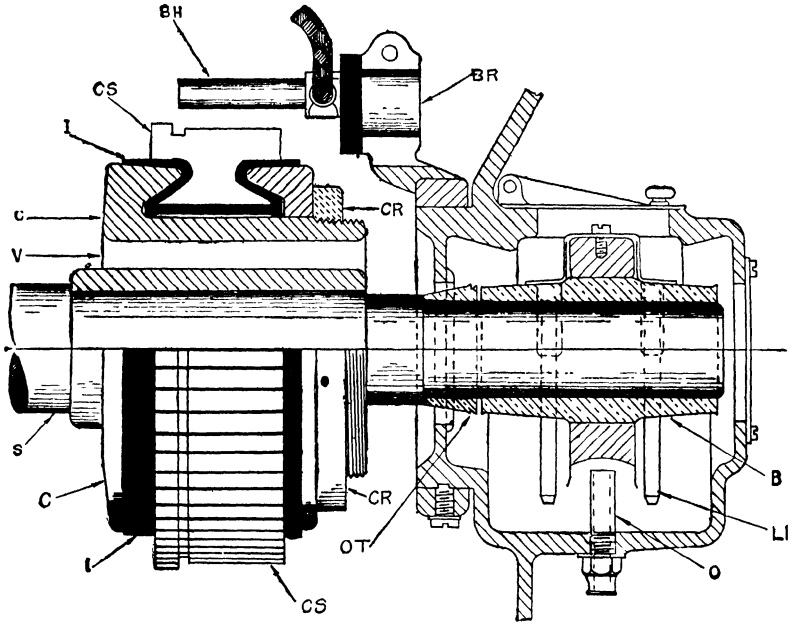


Fig. 277.

B.H.—Brush holder arm.	B.R.—Brush rocker.	C.S.—Copper segment.
I.—Mica insulation.	C.—Com. centre.	C.R.—Clamping ring.
S.—Shaft.	B.—Bearing brass.	O.T.—Oil thrower.
O.—Overflow pipe.	L.R.—Lubricating ring.	V.—Ventilating duct.

Bearings and **pedestals** for supporting the shaft are practically the same as for other machinery. Slots in B enable two lubricating rings to rest upon the shaft. The lower parts of the rings dip into oil and, as the shaft revolves, it carries the rings with it, thus ensuring lubrication of the bearings. The oil thrower on the left causes the oil to be thrown off the shaft, thus preventing it from getting along to the commutator.

* Fig. 277 is from an excellent little practical hand-book "*Dynamo and Motor Attendants and their Machines*," by Frank Broadbent, M.I.E.E.

Formerly copper gauze was always used for brushes, but now **carbon** brushes are almost entirely in favour.

In many machines sparkless collection is insisted upon *at all loads* with *fixed brushes*, and it is found that this is attainable with carbon brushes where impossible with copper (Art. 233).

Carbon is not so good a conductor as copper, and the current density allowable in the two varies from about 170 amperes per square inch for copper to about 40 amperes per square inch for carbon. Also the *contact resistance* is much higher for carbon than for copper. Hence carbon brushes must be larger than copper brushes, and as the area of contact must be larger, the commutator must either be longer or larger in diameter for the use of carbon. Whereas copper brushes are always used inclined to the surface, carbon brushes are always normal, and this in the case of motors permits of running indifferently in either direction. Carbon brushes are usually coppered where they are held by the brush holder, and now there are some varieties in which the carbon is stratified into layers of different conductivity to improve their collecting properties and with some success. A point of importance in connection with carbon brushes is that the contact resistance is not constant, but *decreases* as the current density *increases*.

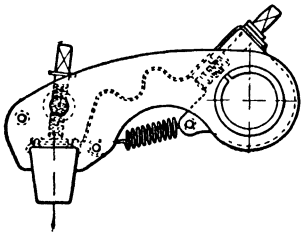


Fig 278.

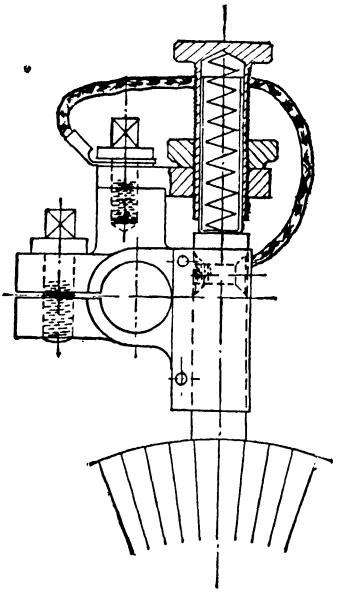


Fig 279.

There are numerous types of **brush holders**, and Fig. 278 shows the "hammer" type, which is in common use. The tapered carbon brush is firmly held in the V-shaped jaw, and a copper ribbon assists in conducting the current from the brush to the holder. The pressure of the brush is regulated by the steel spring. Fig. 279 shows one of the many "box" types: in this one the pressure applied is in a direct line over the brush: this holder has many points in its favour.

Holders are carried by a **brush rocker**, which keeps them correctly

spaced, and enables the lead to be readily adjusted. These rockers are ring structures, which for medium machines are fixed on the bearing concentric with the commutator, and next to the commutator. They are fitted with horizontal arms projecting over the commutator, and to these arms the brush holders are attached (Fig. 277). The arms are insulated from the ring, and alternate arms are joined to the two terminals of the machine. Each arm carries several brushes, *i.e.* except for very small machines, brushes are in "sets." The rocker can, of course, be rotated, thus altering the brush positions on the commutator. In larger machines the ring carrying the brush gear is supported by brackets from the yoke. In still larger machines the brush rocker is supported by an independent ring structure (Fig. 281).

242. Defects in Dynamos.—One of the commonest defects which arises in the actual working of dynamos is that, although the main shaft is run at the required speed, the dynamo partially or wholly fails to give the current for which it was designed. This defect may be due to faults in the dynamo itself, or to the unsuitable nature of the circuit which the dynamo has to feed. Thus, with a shunt machine, failure to generate current may arise from the external circuit having too low a resistance. In an efficient shunt machine the resistance of the field coils is large compared with that of the external circuit, the latter diminishing as the number of lamps switched on in parallel is increased. If the external resistance be very much reduced, the current round the field magnets will be too weak to excite, and the dynamo will fail to work. We may reinforce the field magnets, if they be weak, by switching off the whole of the external circuit from the terminals, and running the machine for a minute or two, so that the whole of the current is in the field magnet coils. The external circuit may then be switched on.

Failure to excite may also be due to bad contacts at the terminals, arising from the latter being dirty or not screwed tight; also to bad contacts along the line. If the connecting wires which couple the brush holders to the field magnet coils, or to the terminals, be loose or broken, or if the brushes be incorrectly placed on the commutator, the dynamo may also fail to excite.

In many cases, however, the inefficient working of a dynamo is due to defective insulation and short-circuiting. Short-circuiting may occur in a dynamo between the coils of the field magnets and the magnet cores, between the field magnet coils themselves, between the armature coils and the armature, between the armature coils, between the several commutator bars, and between the brush holders and the rocker.

To detect a leak between the field magnet coils and the magnet core, we may connect one pole of a 50-volt battery to one terminal of the field coils, and the other pole to a suitable galvanometer. Then, if a leak exists, a wire taken from the other terminal of the galvanometer to any bare iron portion of the field magnets will cause a deflection.

If we suspect a leak between individual coils of the field winding, we calculate what the resistance of the known length of the wire in the coil ought to be, and then determine what the resistance actually is by means of a Wheatstone Bridge.

If the results differ very much, it is a sign of leakage, and the magnets will have to be rewound.

To detect a leak between the armature and its coils, we proceed as for the field magnet coils, taking care that we first remove the brushes from the commutator. If the wire led from the galvanometer to any bare iron part of the armature causes a deflection, a leak exists.

The test for leakage between the individual coils of the armature consists in disconnecting the coils from the commutator bars, and finding the resistance of each coil separately. If the resistance of any one coil differs much from the average of the other coils, we may infer leakage.

A second test for leakage between the armature coils is to investigate the fall of potential down each side of the armature as follows:—Remove the brushes and clamp to one bar of the commutator a lead from the battery clamping the other lead to the bar diametrically opposite. Thus current will flow down both sides of the winding. Take now a sensitive voltmeter and couple one of its leads to the top battery clamp, and with the other voltmeter lead touch each bar in succession, noting the readings in each case; go down each half of the armature in turn in this way, and if the resistances of the coils are all equal, then the increments of pressure should be equal. If abnormal in one of them, that coil is defective.

A practical test is to lift the brushes and run the armature between the field magnets excited as usual; if a coil be short-circuited it will quickly be rendered evident by local heating, since large local currents will circulate.

To detect leakage between the several commutator bars, we disconnect the coils from them, and test each bar against its neighbours by means of a cell and a galvanometer. Any deflection will indicate unsound insulation between the bars.

If a short circuit occurs in the mains, a series dynamo will run very stiffly and require more power to drive it; a shunt machine, on the other hand, will race. With a compound wound machine the speed will be practically unaffected, but excessive sparking will occur, and the machine will heat up. If defective working of the dynamo is noticed, the mains should first be cut out, and the dynamo run on the pilot lamp. If the lamp glows properly, the fault must be in the mains, and these must be examined in detail until the fault is localised.

243. Points in the Working and Care of Dynamos.—To start a dynamo the prime mover is slowly run up to full speed, with the brushes on and the pilot lamp in. When the latter glows brightly the mains may be switched on and the steam stop-valve of the engine opened out to the full. If now excessive sparking occurs, the brushes must be moved backwards and forwards until the correct lead is obtained.

To stop a dynamo the mains are first switched off and the main steam stop-valve slowly closed to reduce the speed. If the brushes are left on the commutator until the machine comes to rest, they may be injured by a back turn of the dynamo after stopping; on the other hand, if they are taken off too soon the pilot lamp may be burnt out by the high E.M.F. generated on breaking circuit. The brushes should not be taken off until the pilot lamp is a dull red, when the breaking of the circuit will cause it momentarily to glow.

After several months' running, or as a result of excessive sparking, the commutator may become very rough, and it must be turned cylindrical by bolting the slide of a lathe to the bed of the engine, which should be run at a suitably low speed. For easy working the commutator must be kept clean and smooth. It is best kept smooth by running the machine with the brushes off, and rubbing with fine emery or glass paper or chalk. The commutator should then be lubricated slightly. Commutators are often ground up in position by a portable emery wheel, which can be bolted to the brush holder. The brushes must be kept making good and light contact. Copper brushes should be inclined at from 45° to 60°, and should be unsupported for about 1 to 1½ in. from their tips in order that they may be elastic. The points should also be filed to the shape of the commutator, and should be of such a width as to have bearing surface on 1½ strips (simplex). The brushes should always be set to touch the correct commutator strips with the machine at rest. If these points are not attended to, sparking at, and rapid wear of, the commutator bars will result. If the sparking is continuous the brushes are probably incorrectly assembled, and they should be moved backwards and forwards on the rocker to see if there is any improvement. If the sparking continues, the brushes must be taken out, filed, and reset. Intermittent sparking will usually arise from a fault in the armature, and the bars affected are easily found if the machine is run slowly ; the connections at the ends of the bars will generally be found to require cleaning and resoldering. Carbon brushes are of course radial.

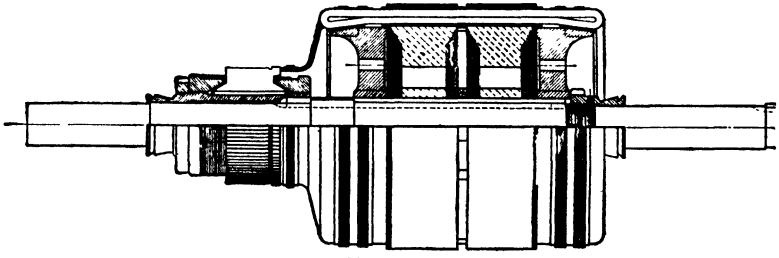


Fig. 280.

244. Brief Description of a Modern Drum and of a Modern Generator.—The drum armature shown partly in section in Fig. 280 is typical of practically all modern methods of construction for medium and even large machines.

Core.—The armature is of the slotted core drum type, and is built up of best charcoal iron stampings, each plate being carefully insulated from the adjacent one.

One special feature of the armature is the method of ventilation, air channels being provided which run horizontally and radially through the core. Winding barrels are provided at each end to support the windings, and are keyed direct to the shaft.

Windings.—The armature is *former* wound, the coils dropping into the insula-

ted channels, and they are secured therein by wooden wedges and by bands of steel wire wound tightly on and soldered.

The coils are all wound alike by the aid of a machine, and are interchangeable.

After winding, the armature is impregnated by the vacuum process which draws the varnish through the windings to the core, and in this way renders it practically damp-proof.

Commutator.—The commutator is of hard drawn copper, each segment being insulated from the next by an annealed sheet of mica; this wears down at the same rate as the copper, and thus prevents sparking, which may occur if the mica projects above the copper after wear, they being termed *high micas* in such a case.

The sections are insulated from the bushes by mica rings, which are tightened down under hydraulic pressure, and finally secured by a nut as shown. It will be noted that each commutator bar is so shaped that it dovetails into the projections at each end.

A dynamo may be driven by any convenient source of power—water turbine, oil, gas, or steam engine, electric motor, steam turbine, etc. The last-named has in recent years come largely to the front as a *most efficient prime mover*, but owing to the high speed, it was, until comparatively recently only used with alternators which, of course, use slip rings and not commutators (it is with the latter that high speed difficulties arise). When continuous current is required, direct driven turbo-alternators may be used and the A.C. transformed to D.C. by means of rotary convertors: this will be explained later. In recent years, however, steam turbines have been employed for the direct driving of large multipolar D.C. generators through a system of speed reduction gear. Fig. 281 will give the reader some idea of the appearance and construction of such a machine used in modern electrical engineering practice. The machine shown is a 1000 k.w. generator, turbine driven through gearing of 8 : 1 ratio.

Field Poles.—The field poles are built up of laminations riveted together and bolted to the circular yoke. The field coils are wound on formers, thoroughly impregnated with varnish by the vacuum process and mounted on the poles. Each coil is provided with two separate terminals fixed to the body of the coil and fitted with two screws to which the stout connections between different coils are attached. The machine is compound wound, the series and shunt coils being on separate bobbins. There are 16 field poles on the type indicated. The machine is fitted with interpoles as shown, the windings of which are bare copper on micanite spools. Compensating or neutralising windings of insulated heavy copper bars are arranged in two layers in slots in the pole faces.

Armature.—The armature is of the laminated slotted drum type lap wound (simplex), the laminations being coated with an insulating compound. The laminations are built up on the rim of a spider which is cast in the form of a wheel. Ventilation ducts are provided in the core and there are air spaces between the core and end plates, thus prevent-

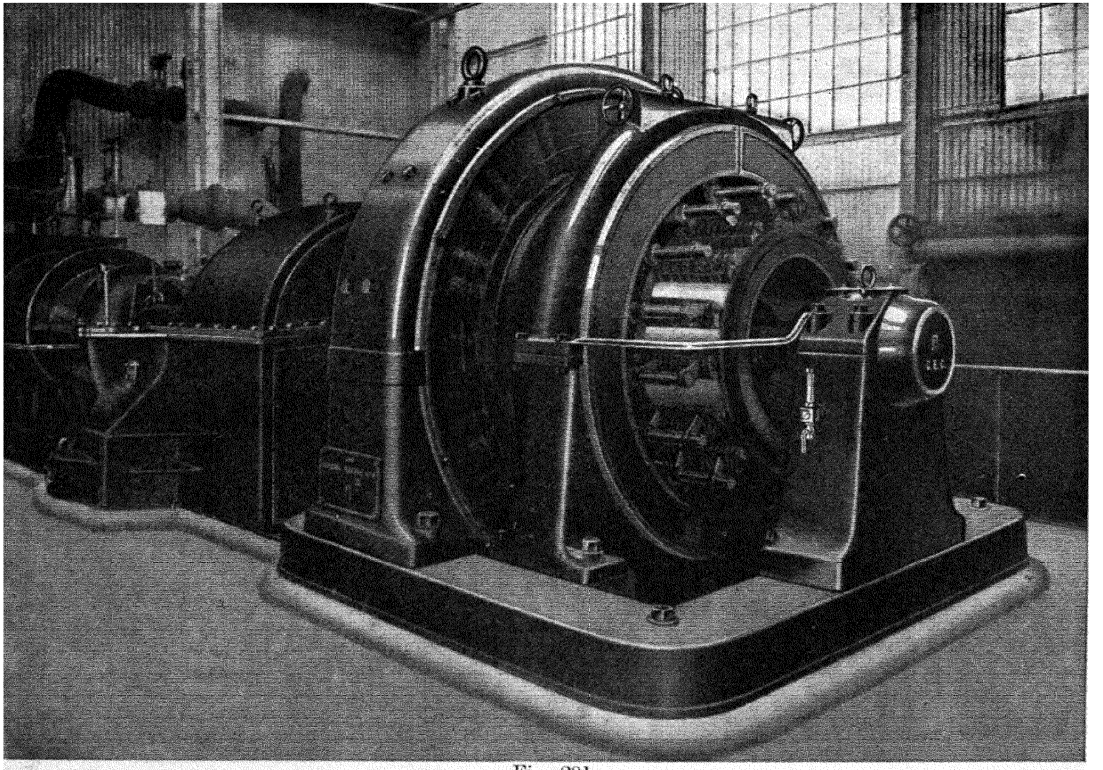


Fig. 281.

ing eddy currents in the cast-iron checks. The winding consists of former wound bars thoroughly insulated before being inserted in the slots. The slot insulation consists of micanite and impregnated leatheroid, and wooden wedges in the top of the slots hold the winding in position. The whole armature is sprayed with insulation when complete. There are 560 active conductors in the armature shown.

Commutation.—The bars are hard drawn copper with mica insulation. The commutator is mounted on a cast-iron ring which is keyed to an extension of the armature spider, and thus differs from many types in that it is independent of the shaft. Equalising connections are provided to give equal distribution of current in the various parts of the windings and of the commutator.

Brushes and Brush gear.—A large number of independent brushes is used as is clearly indicated. Each brush carrying spindle is supported at both ends by arms insulated from a large rocker ring. Each alternate arm is connected to collecting rings of copper which are fixed inside the rocker, and connections are made from these rings (by copper ribbons) to the machine terminals placed at the bottom of the yoke: from these terminals, connections go, of course, in channels to the switchboard. The brush ring is adjusted by means of a hand wheel and rocker spindle. In the type shown there are 16 brush sets.

General.—The shaft is of mild steel, the bearing is of the pedestal type, ring lubricated, and the bedplate is constructed to reduce deflection to a minimum. These details, however, are the same as in most other types of machinery.

The machine depicted is manufactured by the General Electric Co. of Birmingham.

EXERCISES XVIII.

1. What do you understand by a shunt dynamo? What is the difference between it and a compound dynamo? What is the object of compounding?

2. Why is it generally necessary to adjust the position of the brushes according to the current which the machine is producing? Explain the term *lead*.

3. In what combination of magnetic fields does the armature of a dynamo revolve? Is the effect of the combination to cause a weaker or a stronger field than that due to the field magnets alone? State one other effect of the combination in determining the position of the brushes.

4. Explain what is meant by the external and the total characteristic curve of a dynamo. How is the latter deduced from the former? Give the general shape of both for a shunt and a series dynamo.

5. Describe the construction and action of the ring armature, and show hwo the E.M.F. generated may be calculated. (B.)

6. Assuming a curve for the external characteristic of a series machine which gives its maximum P.D. of 80 volts at 100 amperes, deduce the total characteristic, the resistance of the armature being ·5 ohm, and that of the field magnets ·15 ohm.

7. What is Lenz's Law? Three magnets pivoted like compass needles are placed respectively in an ebonite box, a silver box, and an iron box. Investigate all the causes which will bring the magnets to rest when oscillating, explaining carefully what forces are at work in the metal in each case.

8. Find the electrical and commercial efficiencies of a series dynamo, a test of which gave the following results—

P.D. at terminals, 130 volts.
Resistance of dynamo, 0·0514 ohm.
Current in external circuit, 103 amperes.
Power supplied, 20·7 h.p.

9. An engine of 150 h.p. drives a dynamo which sends a current of 25 amperes through an external resistance of 112 ohms. What is the net (or commercial) efficiency of the system?

10. Find the power required to maintain 80 amperes in each of the following systems—

(1) 120 lamps, each needing 40 volts, connected each by leads of 1 ohm in parallel with a dynamo.

(2) Same number of lamps in 10 rows of 12 each, the leads to each row having a resistance of 10 ohms.

11. A generator has an E.M.F. of 69 volts, and a P.D. of 65 volts when the current is 14 amperes. Assuming that the resistance of the generator is constant, what current will be supplied when the E.M.F. is 113 and the P.D. is 102?

12. Describe the chief short circuits that may occur in the working of dynamos. How may they be detected?

13. A dynamo fails to excite. To what may this be due, and how is it remedied?

14. A gramme machine has resistance 0·0025 ohm, and is employed to send current through 30 coils, the resistance of each being ·03 ohm. Assuming the E.M.F. to be proportional to the speed of the machine, and that the watts in external circuit are constant, find the number of revolutions made by the armature when the coils are (1) in multiple arc, (2) in series of 5, if it makes 900 revolutions per minute when all the coils are in series.

15. What are the devices in dynamos to obtain (1) a constant current in one direction, (2) an alternating current, (3) a constant current at varying voltage, (4) a varying current at constant pressure?

16. What points have to be attended to in starting and stopping an ordinary dynamo?

17. It is known that a fault exists in an electric circuit. How would you ascertain whether the fault is in the mains or in the dynamo itself?

18. How are brushes usually made, and what points have to be attended to in fitting them? What arrangement is made so that their position on the commutator can be altered?

19. If instead of taking current from the terminals of a continuous current dynamo, current be supplied to the machine from some other source, what would be the effect on the dynamo? Is this arrangement ever adopted in practice; if so, is it advisable to alter anything?

20. State the differences between a series, a shunt, and a compound dynamo. Why is it not advisable to use a series dynamo for charging storage cells? (B.)

21. A drum armature in a two-pole field contains 150 external conductors and runs at 550 revolutions per minute. Find the total flux passing through the armature which is required to produce an E.M.F. of 115 volts on open circuit. (O.)

22. Calculate the resistance of a gramme armature wound with 144 turns of rectangular wire ·2 × ·21 inch, length of armature core without insulation 12 inches, radial depth 2·5 inches. The resistance of 100 yards of copper rod 1 square inch in cross-section is ·0025 ohm. (O.)

23. A 20-unit (kilowatt) shunt dynamo has a terminal P.D. of 120 volts and an electrical efficiency of 90 per cent. If the armature and shunt coil losses are equal, determine: (*a*) the resistance of the armature; (*b*) the resistance of the shunt coils; (*c*) the current in the shunt coil; (*d*) the current in the armature.

24. An armature revolves at 1,000 r.p.m., the commutator has upon it 120 segments, and the brush is equal in thickness to two bars. During what interval of time is the coil short-circuited?

25. In the coil of Q. 24, during short circuit the total flux change is 200,000 lines. What voltage (reactance voltage) is induced in the coil by the change?

26. The magnet of a bi-polar dynamo is composed of wrought iron cores and yoke, and cast iron pole-pieces; the diameter of the cores is 14 inches; the cross-section of the yoke is equal to that of the cores; the section of the pole-pieces is 400 square inches, that of the armature core is 170 square inches, and the gap area is 430 square inches; the length of the magnetic circuit in the cores and yoke is 80 inches, in the pole pieces 26 inches, in the armature 12 inches, and in the air gaps $1\frac{1}{4}$ inches; the armature flux is 10^7 lines; the factor of leakage is 1·36; the armature is wound with 50 turns; current output 470 amperes; polar embrace 75 per cent. Find the number of ampere-turns required.

CHAPTER XIX.

MOTORS, MOTOR GENERATORS AND BOOSTERS.

245. General Principles.—We have seen that when a wire of length l centimetres carrying a current of I amperes is placed at right angles to a magnetic field of strength H units, the wire is acted upon by a force $HIl/10$ dynes, the force acting at right angles both to the field and the current, and the sense of the force depending upon the direction of the field and current. Under suitable conditions this force produces motion of the conductor, the direction of motion being determined by the *Fleming left-hand rule.*

Now the armature of a dynamo when supplied with current from some outside source is a collection of current-carrying conductors, the field magnets cause the necessary magnetic flux, and thus we should expect that the continuous current dynamo would rotate when supplied in this way; this is so in practice, and the arrangement is termed a **motor.** Any direct current dynamo may be used as a motor if desired, so that we have separately excited, series, shunt and compound wound machines.

Fig. 282 is a diagrammatic representation of a motor with a ring armature, the *field magnets being separately excited.* The current, say from a battery of accumulators, enters at the + brush and divides equally, half flowing down the coils on the right and half down those on the left to the — brush. An application of the left-hand rule will show that the pulls on the outside conductors on the left cause these coils to move downwards whilst the conductors on the right are urged upwards, so that continuous rotation of the armature in a counter-clockwise direction is the result.

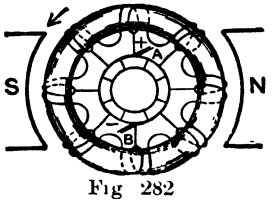


Fig 282

Clearly, if the current entered at the lower brush, the armature would rotate in a clockwise direction, and the same would result if, with the current flowing as indicated, the polarity of the field magnets were reversed. If both the polarity of the fields and the direction of the current were reversed the direction of rotation would not be altered. These statements should be verified by the left-hand rule.

Now this motor, when running, will act as a dynamo, and generate e volts in the opposite direction to E, the volts supplied, as will be seen by applying the *right-hand rule.* The watts supplied = EI, the internal watts wasted = $(E - e)$ I, and therefore the *watts transformed by the motor* = $EI - (E - e)I = eI$ (compare Art. 156); *i.e.*—

$$\left\{\begin{matrix}\text{Mechanical power (in watts)} \\ \text{developed by the armature}\end{matrix}\right\} = (\text{Back E.M.F.}) \times (\text{Armature current}).$$

The **back E.M.F. of a motor** (e) is a very important factor in practice as will be seen later.

246. Changing the Direction of Rotation of a Motor.—*By the aid of sketch diagrams and the Fleming left-hand rule, the reader should verify the facts mentioned in* (*a*) *to* (*c*) *below.*

(*a*) In the case of a separately excited motor, the direction of rotation is altered by altering the direction of the armature current or reversing the polarity of the fields; if both of these be reversed the direction of rotation is unaltered.

(*b*) In the case of a series or shunt motor, if the direction of the current be reversed, both the current in the armature and the field polarity are reversed, and thus the direction of rotation is unaltered. Hence to alter the direction of rotation of a series or shunt motor it is necessary to change *either* the direction of the armature current, *or* that of the current in the fields, *but not both of these;* in practice, it is the custom to change the former by means of a reversing switch.

(*c*) Compound motors are either "Differential Compound," *i.e.* those in which the series winding *opposes* the shunt winding or "Cumulative Compound," *i.e.* those in which the two *help* each other: the former are rarely used and the name "compound" is frequently applied to the latter only. To reverse the direction of rotation of a compound motor in practice, we must *either* (*a*) reverse the armature current *or* (*b*) reverse *those of the series and shunt coils simultaneously.*

To summarise:—to reverse the rotation we must reverse the armature current *or* reverse the field currents, but *we must not reverse both armature current and field.*

247. Dynamo run as Motor—Direction of Rotation.—*By the aid of sketch diagrams such as Figs.* 283-5 *and the Fleming right-and left-hand rules, the reader should verify the facts mentioned in* (*a*) *to* (*d*) *below. Further, he must not confuse the statements in this section with those of the preceding section.*

(*a*) A series dynamo run as a motor turns in the opposite direction to that in which it would have to be rotated as a dynamo to generate a current in the same direction as the motor driving current: hence:—

(*b*) If a series dynamo be run as a motor without any change being made, the motor current will be in the same direction as before in both field and armature (Fig. 283 (*a*) and (*b*)) or it will be in the opposite direction in both field and armature (show this on the diagram) and

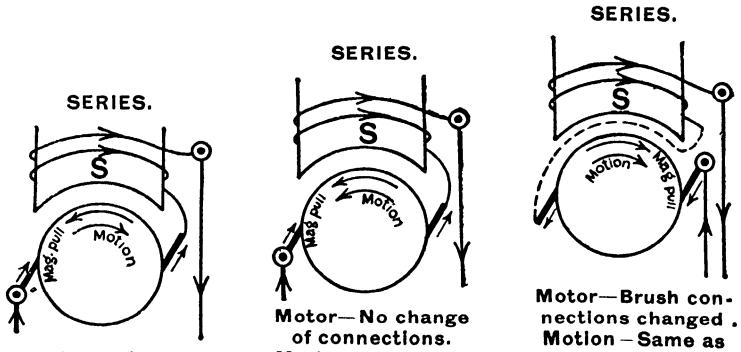


Fig. 283.

the motion of the motor will be opposite to that of the dynamo. *To make the series dynamo run in the same direction when used as a motor,* it is necessary to cross the brush leads (Fig. 283 (*c*)): in this case either the armature current will be opposite and the field the same as in Fig. 283 (*c*), *or* the field will be opposite and the armature current the same, and the motion will be the same in the motor as in the dynamo.

(*c*) If a shunt dynamo be run as a motor without any change being made, then either (1) the armature current is the same and the field altered (Fig. 284 (*a*) and (*b*)) or (2) the armature current is altered and the field the same (show this on the diagram); *hence the motion is the same in the motor as in the dynamo.*

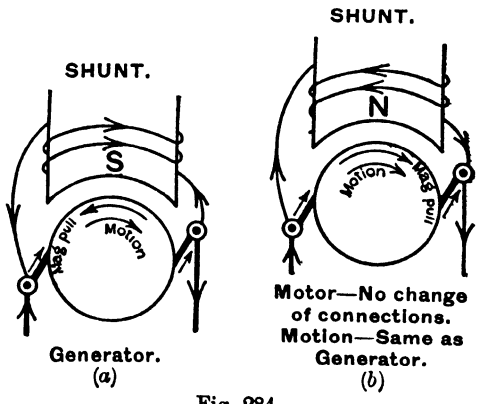


Fig. 284.

(*d*) If a compound wound dynamo be run as a motor with no change

in the connections it is a *differential* motor, *i.e.* the series and shunt coils will be opposing each other as is shown in Fig. 285 (*b*): in this figure the armature current is reversed and **(taking the shunt coil as predominating)**, the field is the same, so that the motion is the same. Better starting qualities will obtain if the series coil connections are reversed, thus making the motor *cumulative* (Fig. 285(*c*)): in this figure the armature current is reversed and **both** series and shunt windings make the field the same, so that *the motion is the same in the motor as in the dynamo.* If the motor current enters at the terminal on the left in Fig. 285(*c*), the armature current will be the same in the motor as in the dynamo, but the fields will be opposite and *again the motion will be the same.*

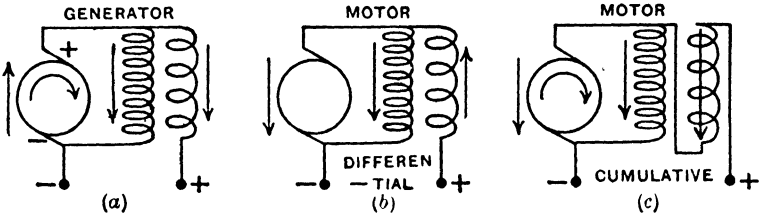


Fig. 285.

248. Series Motor run as Series Dynamo: Electric Brake.—Series motors are used on electric trams (see later). By quickly disconnecting the motors from the supply and joining the terminals through a resistance, they continue to rotate in the same direction (owing to the moving car) as series dynamos, and the kinetic energy of the car is converted into electric energy and dissipated as heat in the resistance, so that the car is quickly "pulled up": this is the **electric brake.**

One important point, however, must be noted. If the motor be merely disconnected from the supply and run in the same direction as a dynamo, then since the dynamo E.M.F. (back E.M.F. of motor) is opposite to the previous motor current, *the current in the fields will be opposite to the current that has been flowing,* so that it will simply wipe out the magnetism and the machine will refuse to excite. Hence, when the motor is disconnected from the supply and run as a dynamo, **the field connections must at the same time be reversed,** so that the polarity of the fields is the same as before, and a braking effect is the result. Both operations are done by the *controller.* The student should make simple diagrams to illustrate these facts.

Another tram brake, applied in addition, is the **magnetic track brake,** *i.e.* an electromagnet on each side of the car is magnetised by the current generated, pulled down on to the rails, and the friction between them and the rails helps to pull up the car. Further in, for example, the Westinghouse type, the track brake is mechanically connected to the ordinary **brake blocks for the wheels** so that when the former is applied it causes the brake blocks to push against the wheels at the same time, thus increasing the braking effect.

249. Preliminary Ideas on Torque and Back E.M.F. in Motors.—It is desirable to have a single word to express the moment of a force about an axis or the product of the magnitude of the force into the perpendicular distance between the axis and the line of action of the force, and the single word used is **torque.** The numbers may be expressed in any units, but the answer must specify the units also, as there is no fixed conventional name for the unit of torque. Thus, if a belt were hanging over a rough pulley with its plane vertical and having a diameter of 3 ft., and if a mass of 40 lb. were hanging from one end of the belt and 50 lb. hanging from the other end, the *torque* on the pulley shaft is obtained thus :

$$\textbf{Torque} = \text{effective pull in lb.} \times \text{radius of pulley in feet}$$
$$= (50 - 40) \times 1{\cdot}5 = \textbf{15 pound-feet.}$$

Or,

$$\textbf{Torque} = \text{effective pull in dynes} \times \text{radius of pulley in centimetres}$$
$$= (50 - 40) \times 454 \times 981 \times 1{\cdot}5 \times 30{\cdot}5$$
$$= \textbf{204} \times \mathbf{10^6}\ \textbf{dyne centimetres.}$$

As already indicated, any machine running as a motor is also acting as a dynamo, producing E.M.F. which opposes the applied E.M.F. (as is readily seen by an application of the right- and left-hand rules), and the current flowing into the machine is due to the *effective* pressure or difference of the two, and thus falls off as the speed of the motor rises (for bigger speed means bigger back E.M.F.), other things being the same.

Thus, if the back pressure of a series motor be e volts, and the applied pressure E volts, the current flowing will be given by $I = \frac{E - e}{R}$ when R is the resistance of the machine ; this is less the bigger the value of e, *i.e.* the greater the speed.

If, then, a constant pressure, say that of a battery of accumulators, be applied to the terminals of a motor at rest, the motor commences to rotate, producing pressure e volts opposing the applied pressure E volts. If the motor has no work to do it will increase in speed until the pressure e nearly equals E, and the current falls practically to nothing.

The work done by a motor in a stated period is given by the product of the *armature* current, the back E.M.F. and the time (Art. 245), and thus equals eIt or $e\left(\frac{E - e}{R}\right)t$ joules for a series motor. Hence, if the motor is clamped so that it cannot rotate at all, the current will be very large, the back E.M.F. (e) will be *nil* and the useful work done will be zero ; if, on the other hand, the motor is allowed to rotate so quickly that e nearly equals E, the current will drop nearly to zero, and again no useful work will be done.

In practical running the motor has always some work to do, even if only against friction, and thus e never quite equals E. The more loaded the motor the greater the current taken, and since the power supplied is obviously E I, while the power used in a motor of the series type is e I, we have an expression for its *electrical efficiency* the ratio $\frac{eI}{EI}$ or $\frac{e}{E}$. Thus the efficiency is unity or 100 % when the motor is allowed to run away until e practically equals E ; the motor is then doing infinitely little work, but doing it at maximum efficiency. It is easy to show

that the *greatest rate of doing useful work is reached when the motor is allowed to run until* $e = \frac{1}{2}E$, *and the efficiency is then* $\frac{1}{2}$ *or* 50 %.

For simplicity we have considered the series motor, but the same principles apply to all, only we must remember that in the shunt and compound motors the current supplied from outside is *not all armature current.* Thus, if E be the pressure of supply at the terminals of, say, a shunt machine, the shunt current is E/R_s, and the armature current is $I - E/R_s$, if I is the total current supplied; hence the work done in time t is $e\,(I - E/R_s)\,t$ joules.

250. Motor Calculations: (1) Horse Power and Torque.—

Let I = armature current in amperes.
e = back E.M.F. of armature in volts.
n = revolutions **per minute** of armature.
T = torque in pound-feet.
ϕ = flux per pole.
Z = number of armature conductors.

Since **"work per minute" = "torque × angle turned through in radians per minute,"** we have:—

$$\text{Work per minute} = (T \times 2\pi n) \text{ foot-pounds,}$$

$$\therefore \text{Horse-power} = \frac{T \times 2\pi n}{33000} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Again, since the horse-power = the mechanical power in watts developed by the armature divided by 746, we have:—

$$\frac{eI}{746} = \frac{T \times 2\pi n}{33000},$$

$$\therefore T = \frac{33000 \times e \times I}{2 \times 746 \times \pi \times n}.$$

Further, *for a bi-polar machine* running as a dynamo we have, with the usual notation (Chapter XVIII.):—

$$e = \frac{\phi n Z}{60 \times 10^8} \text{ volts.}$$

Hence— $$T = \frac{33000 \times \phi n Z \times I}{2 \times 746 \times 60 \times 10^8 \times \pi \times n},$$

i.e. $$\text{Torque} = T = \frac{\phi Z I}{8{\cdot}52 \times 10^8} \text{ pound-feet} \quad \ldots\ldots\ldots\ldots\ldots (2)$$

If the machine be multipolar then $e = \frac{p}{a} \cdot \frac{\phi n Z}{60 \times 10^8}$, where $p =$

number of poles and a = number of armature paths (Chapter XVIII.): substituting as above:—

$$T = \frac{\phi ZI}{8{\cdot}52 \times 10^8} \times \frac{p}{a} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

The expressions (2) and (3) give the *total* torque: the *useful* torque is really less, being proportional to $(I - I_0)$ where I_0 is the no-load current.

Referring again to the expression for horse-power given in (1) above we have:—

$$\text{h.p.} = T \times \frac{2\pi n}{33000} = \frac{\phi ZI}{8{\cdot}52 \times 10^8} \times \frac{p}{a} \times \frac{2\pi n}{33000} \quad \ldots\ldots\ldots\ldots \quad (4)$$

The brake horse-power is less than this owing to various losses: in fact:—

$$\text{b.h.p.} = \textit{useful} \text{ torque} \times \text{angular velocity} \div 33000$$

$$= \frac{\phi Z(I - I_0)}{8{\cdot}52 \times 10^8} \times \frac{p}{a} \times \frac{2\pi n}{33000} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

251. Motor Calculations: (2) Force on a Conductor.—In addition to the symbols already used let r = radius of the armature in feet and f = average pull on each conductor in pounds. From Art. 249 it follows that:—

$$T = Zfr,$$

and substituting this for T in (2) (bi-polar machine):—

$$Zfr = \frac{\phi ZI}{8{\cdot}52 \times 10^8}.$$

Hence for the average pull on a conductor we get:—

$$\text{Force on conductor} = f = \frac{\phi I}{8{\cdot}52 \times 10^8 \times r} \text{ pounds} \quad \ldots\ldots \quad (6)$$

The student should make the necessary slight modification for multipolar machines.

In problems *the current* (i) *per armature conductor* is sometimes given, in which case it must be remembered that $i = I/a$. In these problems the beginner will experience a little difficulty at first with the various units (see Examples, Art. 253).

252. Motor Calculations: (3) Speed.—If E be the E.M.F. applied to the motor, R_a the armature resistance and I_a the armature current (we are using I_a for convenience later) then:—

$$E = I_aR_a + e,$$

$$= I_aR_a + \frac{p}{a} \cdot \frac{\phi nZ}{60 \times 10^8},$$

$$\therefore \frac{p}{a} \cdot \frac{\phi nZ}{60 \times 10^8} = E - I_aR_a,$$

i.e.

$$\text{Speed} = n = \frac{(E - I_aR_a)\, 60 \times 10^8}{\frac{p}{a}\phi Z} \quad \ldots\ldots\ldots\ldots \quad (7)$$

In, for example, a long shunt compound the resistance of the armature *and* th series coil, viz. $R_a + R_m$, should be used for R_a : it is hardly necessary, however to draw attention to all these very slight modifications.

253. Motor Calculations : (4) Examples :—The following worked examples will help the student to grasp the details of the preceding sections :—

EXAMPLES.

1. *In a bi-polar series motor the total magnetic flux was* 10^7 *lines, total conductors* 140, *radius of armature* 6 *in., current in armature* 200 *amperes, revolution* 900 *per minute. Find—*

(1) *The torque in pound-feet and dyne-centimetres ;*
(2) *The average pull in lb. on each conductor ;*
(3) *The power in watts and the h.p.*

We have—

$$T = \frac{10^7 \times 140 \times 200}{8{\cdot}52 \times 10^8} \text{ pound-feet} = \mathbf{328{\cdot}6} \text{ pound-feet ;}$$

also $T = 328{\cdot}6 \times 454 \times 981 \times 30{\cdot}5$ dyne-cm. $= \mathbf{4{,}460 \times 10^6}$ dyne-cm.

Again— $$f = \frac{10^7 \times 200}{8{\cdot}52 \times 10^8 \times {\cdot}5} = \mathbf{4{\cdot}7} \text{ lb. nearly.}$$

This is assuming that the conductors are active all the time. If we take account of the angular space taken by the pole-pieces only, however, and call this angle θ, then only $\frac{\theta}{180}$ Z conductors are active, thus the average force exerted on each is given by f (as above) $\times \frac{180}{\theta}$.

Hence applying this to the case taken above we have, if the polar angle be assumed 135°,

$$f_1 = 4{\cdot}7 \times \frac{180}{135} = \mathbf{6{\cdot}27} \text{ lb.}$$

Now $$\text{h.p.} = \frac{\text{Torque (lb.-feet)} \times 2\pi n}{33000} = \frac{328{\cdot}6 \times 2\pi \times 900}{33000},$$

$$= \mathbf{56{\cdot}3}.$$

$$\therefore \mathbf{\text{Power in watts}} = 56{\cdot}3 \times 746 = \mathbf{42{,}000}.$$

2. *In a 4-pole wave wound motor* Z = 180, $\phi = 2{\cdot}9$ *megalines per pole, current in each armature conductor* = 50 *amperes. Find the torque in kilogram-metres.*

$$T = \frac{\phi ZI}{8{\cdot}52 \times 10^8} \times \frac{p}{a} \text{ pound feet,}$$

$$= \frac{\phi ZI}{61{\cdot}6 \times 10^8} \times \frac{p}{a} \text{ kilogram-metres (prove this).}$$

The motor being wave wound has 2 parallel paths and the total armature current is therefore (50 × 2) amperes : hence :—

$$T = \frac{2{\cdot}9 \times 10^6 \times 180 \times 50 \times 2}{61{\cdot}6 \times 10^8} \times \frac{4}{2},$$

$$= \mathbf{16{\cdot}95 \text{ kilogram-metres.}}$$

We might also proceed as follows:—The field strength under a pole is $(2{\cdot}9 \times 10^6) \div (\text{area}) = (2{\cdot}9 \times 10^6) \div (\frac{1}{4}\pi dl)$ where d = armature diameter in cm. and l = length in cm. (it is a 4-pole machine, hence the $\frac{1}{4}$): thus:—

(*a*) Force on a conductor $= \dfrac{Hil}{10}$ dynes $= \dfrac{2{\cdot}9 \times 10^6 \times 50 \times l \times 4}{\pi dl \times 10}$

$$= \frac{2{\cdot}9 \times 10^6 \times 4 \times 50}{\pi \times d \times 10} \text{ dynes.}$$

(*b*) Torque = force on a conductor × radius × no. of conductors

$$= \left(\frac{2{\cdot}9 \times 10^6 \times 4 \times 50}{\pi \times d \times 10} \times \frac{d}{2} \times 180\right) \text{dyne.-cm.}$$

$$= \left(\frac{2{\cdot}9 \times 10^6 \times 4 \times 50 \times 180}{\pi \times 10 \times 2} \times \frac{1}{981 \times 1000 \times 100}\right)$$

$$= \mathbf{16{\cdot}95 \text{ kilogram-metres.}}$$

NOTE.—Using the expression given in Art. 251 for the force on a conductor we get:—

$$f = \frac{\phi \mathrm{I}}{8{\cdot}52 \times 10^8 \times r} \times \frac{p}{a} \text{ pounds,}$$

$$\therefore f = \frac{(2{\cdot}9 \times 10^6) \times (2 \times 50)}{8{\cdot}52 \times 10^8 \times \dfrac{d}{2 \times 30{\cdot}48}} \times \frac{4}{2} \times (454 \times 981) \text{ dynes,}$$

since d above is in cm., and $\therefore r$ feet $= d\,(2 \times 30{\cdot}48)$,

i.e. $$f = \frac{2{\cdot}9 \times 10^6 \times 2 \times 50 \times 4 \times 454 \times 981 \times 2 \times 30{\cdot}48}{8{\cdot}52 \times 10^8 \times 2 \times d} \text{ dynes,}$$

and this will be found to give the same answer as the expression obtained in (*a*) above.

254. The Shunt Motor.—When the circuit is closed the magnet offers an independent path, and thus may be taken as producing a constant magnetic flux, since, if the pressure of supply is constant, the current through the field is constant, and thus the magnetic flux ϕ is constant too. Hence the shunt motor supplied at constant pressure behaves like a separately excited machine would behave.

If the algebraical expression connecting torque and speed be worked out from the formulae given above, it is found that the following holds:

$$\text{Speed} = \text{A} - (\text{B} \times \text{Torque}).$$

Where A and B are constants, and in practice B, which involves the armature resistance, is much smaller than A, and thus it is usually taken as sufficiently correct to say that Speed = A, or is constant, for all loads. Hence the property that *the shunt motor runs at practically the same speed for all loads.* In consequence this form of motor is convenient for driving machine tools, etc., where such a property is of service.

The student will also realise the truth of the above by examining the formula of Art. 252, viz.:—

$$\text{Speed} = \frac{(E - I_a R_a)\, 60 \times 10^8}{\frac{p}{a}\phi Z}.$$

Since ϕ is constant the drop in speed (since E is constant) from no load to full load is proportional to $I_a R_a$ and since $I_a R_a$ is always small it follows that the speed is practically constant from no load to full load.

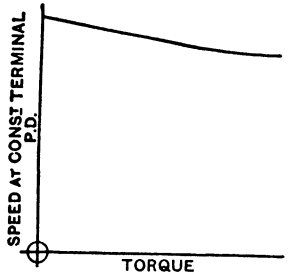


Fig. 286.

The same can be seen from the "mechanical characteristic" of Fig. 286, viz. *the curve showing the relation between speed at constant pressure and torque* for a shunt motor: the curve is only slightly falling.

It will be seen later that in motors the brushes are given a "backward lead" or "lag": the armature reaction weakens the field, *i.e.* makes ϕ in the denominator above *less* as the load increases, and this increases the speed: in other words *armature reaction reduces the drop in speed as the load increases.*

255. The Series Motor.—Since the armature and the field are of low resistance, when the circuit is first closed the current is large—as the back E.M.F. is not produced until some rotation is caused—and therefore the field is very strong, and the current in the armature is also strong, therefore the torque is powerful. This will be clear from the formula of Art. 250, for both ϕ and I are in the numerator, and, therefore, the bigger they are the bigger the value of T. Hence *series motors are always used when a powerful starting torque is required,* as, for example, in electrical trains, trams, and cranes. On the other hand, with a light load the back E.M.F. will be high, and, consequently, the field magnets will weaken considerably, the whole tending to make the motor run at an excessive speed, and do damage in consequence. Hence, *series motors, unless very small ones, should never be used where the load may be altogether removed.*

Again, for the speed of the motor, we may write the formula as follows:—

$$\text{Speed} = \frac{\{E - I_a(R_a + R_m)\}60 \times 10^8}{\frac{p}{a}\phi Z},$$

where R_m = resistance of field coils.

If the load and, therefore, I_a be increased, then, since I_a is also the field current, the flux ϕ in the denominator is greater and *therefore*

the speed is less. At first the increase in ϕ is nearly proportional to the current and, therefore, we have a rapid drop in speed. Later, as the fields reach saturation, the increase in ϕ is less in proportion, and the drop in speed is not so rapid.

Further, if the load be suddenly removed, ϕ drops considerably, the back E.M.F., viz. $E - I_a (R_a + R_m)$, increases, and the motor begins to race as indicated above.

The large torque exerted at low speeds, *i.e.* on starting, and the drop in speed with increasing load, will also be seen from the characteristic of a series motor shown in Fig. 287.

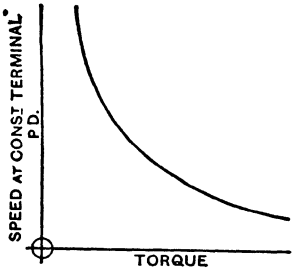


Fig. 287.

256. The Compound Motor.—The point in favour of the **differential** type is that even more constant speeds for all loads can be obtained than with the shunt motor. Owing to difficulties in connection with starting and overloads, it is frequently arranged that the series coil is short-circuited until full speed is reached, and then the short circuit is removed. This type is, however, rarely used.

The **cumulative** type has a characteristic somewhat intermediate between that of a shunt and that of a series motor. It exerts a large starting torque, but if the load be removed the motor *will not race* like the simple series motor, for the action of the shunt keeps the no-load speed within safe limits. The motor is adapted for machinery where the work is thrown on and off suddenly, *e.g.* hoists in mining work, rolling mills, etc.

Over-compounded machines may be made to give a higher speed with increase of load.

257. Motor Starters.—The back E.M.F. in a motor does not reach its full value until the motor has reached its final speed; hence the current taken by a motor at starting would be excessive and would damage the armature if the full line pressure were applied at once to the machine. Thus, for motors of any size a starting resistance should be employed, which is cut out by steps as the speed of the motor increases. There should be sufficient contacts to ensure easy acceleration, and to prevent the current rushes at each step being excessive.

A **series motor starter** is a rheostat with a pivoted arm, placed in series with the machine, and the arm cuts out sections as it is moved over to the full speed position. A simple type is shown in Fig. 288, which is self-explanatory.

Should the supply for any reason be cut off from a motor, it should be impossible to start up again without having the starter in circuit.

For this reason starters are invariably fitted with a little electro-magnet, which attracts and holds on the revolving arm in the "full on" position. Should the supply cease, the electromagnet loses its magnetism, and a spring attached to the arm at once pulls it to the "off" position. The device is known as a "**no-voltage release.**" Frequently there is also another device known as an "**overload release,**" which causes the motor to be disconnected when the current supply becomes excessive. Both these devices are indicated in the next figure.

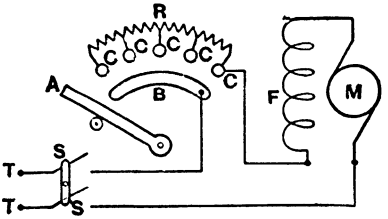


Fig. 288.—Starter for Series-wound Motor.

A, Movable arm. B, Contact strip C C, Contact studs. F, Field winding of motor. M, Armature of motor. R, Starting resistance. S S, Double pole switch. T T, Main terminals.

A **shunt motor starter** is shown in Fig. 289. It will be noted that the rheostat is between the mains and the armature. When the revolving arm is placed on the first contact C **the full pressure is on the fields,** the field circuit being as follows:—M (on left)—E—field coil—D—A—C—revolving arm—sector—B—M (on right) whilst the armature circuit is:—M (on left)—E—armature—F—G—**starting resistance**—C—revolving arm—sector—B—M (on right). As the arm moves to the right, the resistance coils are cut out of the armature circuit and put into the field circuit. When the arm is on G all the resistance is cut out of the armature circuit, but it appears in the field circuit. The insertion of this resistance into the fields does not matter so much as it is small compared with the field coils. By fixing a contact stud near G and joining it to C, then in the full on position the resistance will also be cut out of the fields.

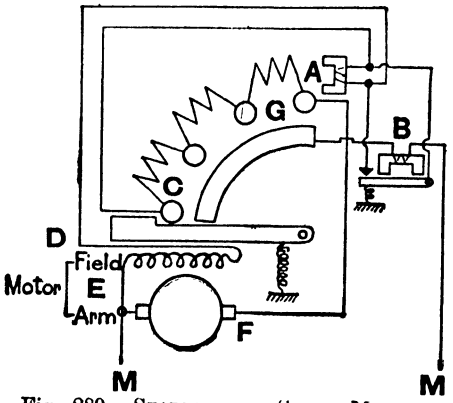


Fig. 289.—Starter for Shunt Motor.

A = No voltage release. B = Overload release. M M = Supply mains.

No-voltage release:—This is shown at A. The electromagnet is energised by the field current and holds on the arm in the "full on" position. Should the field current stop or fall below a certain value

A loses its hold, the arm is pulled back by the spring, and the motor is disconnected.

Overload release:—This is shown at B. The electromagnet is energised by the main current supplied to the motor. Should this current exceed a certain value, the magnet draws up a small pivoted arm, which makes contact with a stud *and short circuits* A: thus A loses its hold in this case also, the arm is pulled back, and the motor is disconnected.

A **compound motor starter** is identical with the above: in a sketch of it, the series coil on the fields would be shown in the line connecting the armature to G.

Liquid starters merely consist of an iron vessel containing salt and water, or soda and water, above which is a pivoted iron plate (the "dipper"), which can be gradually lowered into the solution. The dipper is made the anode, and the iron vessel the kathode in the motor circuit. As the dipper is gradually lowered, the area immersed increases and the resistance decreases, and when completely lowered the two are short-circuited. They are not much used now except perhaps for induction motors.

258. Speed Regulation of Shunt Motors.—(1) An examination of the formula of Art. 252 will again reveal the fact that in the expression for "speed" the flux ϕ appears *in the denominator*; hence it follows that **the speed may be increased by weakening the field, and decreased by strengthening the field.** For this purpose, in the case of a shunt motor, a rheostat is inserted in the shunt winding: the larger this resistance the less the exciting current, the less the field and the greater the speed. By lessening this resistance the field is stronger and the speed less. Fig. 272 shows the arrangement.

(2) A further examination of the speed formula will show that **if a variable resistance r be put in the armature circuit,** the numerator becomes $\{E - I_a(R_a + r)\}\ 60 \times 10^8$. The bigger the value of r the bigger is the *negative* term and, therefore, the smaller is $E - I_a(R_a + r)$ and the lower the speed. Similarly, a decrease in r makes the speed higher.

(3) Another method known as the **multiple voltage control** may be employed. Thus, for simplicity, imagine we have five mains, A, B, C, D, E, the potential differences between A and B, B and C, C and D, D and E being each e volts. Then the shunt coil could be connected to one pair of mains, and the armature to A and B, *or* A and C, *or* A and D, *or* A and E, thus being supplied at pressures e, $2e$, $3e$, and $4e$, and giving speeds *approximately* n, $2n$, $3n$, and $4n$, intermediate speeds being obtained by means of the field rheostat. This method can readily be applied to three- and five-wire systems of supply, but we cannot enter into the many essential details here.

(4) In the "**Ward Leonard**" system an auxiliary motor drives a dynamo *at a constant speed*, and this dynamo supplies current *to the armature* of the main variable speed motor, the field of the latter being excited from the mains. The voltage given out by the dynamo is varied by its field rheostat, and this, of course, is the variable voltage supplied to the motor armature. *Thus the speed of the motor is completely controlled by the shunt regulator of the dynamo.*

259. Speed Regulation of Series and Compound Motors.—The speed may be regulated by the field just as in the case of the shunt motor, but with this difference:—In the series motor the rheostat is put **in parallel with the field** ("diverter"—see Fig. 273). The larger this resistance, the more curent passes through the series field coil, the stronger the firld and the less the speed. By lessening this resistance the field is weaker and the speed greater.

A resistance in series with the armature (and field) can also be used for a series motor as indicated in the case of the shunt motor.

Speed regulation of a compound wound motor is effected by any of the above methods, or a combination, depending on circumstances and the construction of the machine.

260. Series Parallel Control.—A brief reference only can be made to this. In electric trams (series motors), which are repeatedly starting and stopping the speed of the motors is often low, and they would be frequently running with large resistances in series with them, resulting in considerable waste of energy (as heat in the resistances) and low efficiency (effic. = e/E, and e is small at low speeds).

In practice, however, two motors are used, and on starting these are in series together with, of course, a series resistance. The total current taken by the two on starting is only half the current which they would take if in parallel, the torque being the same, and, further, as the back E.M.F. is doubled, the efficiency is nearly doubled. As the speed increases the resistance is cut out by steps as usual. When the resistance is all out the motors are quickly arranged in parallel, and the resistance re-inserted. As the speed rises the resistance is again slowly cut out by steps.

The above is known as the **series parallel system of control**, but all the steps in the various connection changes have not been indicated. They are all done by the **controller**, which is also arranged for braking and reversing.

261. Armature Reaction in Motors: Lag and Sparkless Running.—In dealing with dynamos, we saw that the brushes had, *for two reasons*, to be shifted forward in the direction of rotation, *i.e.* given "lead." Before proceeding further, the student should re-read the sections dealing with this in the preceding chapter.

By diagrams, similar to those in the sections referred to, the student will readily see that in the case of a motor the **distorting effect** is such as to move the line of the resultant field *backwards*, and hence

the diameter of commutation is shifted backwards too. Fig. 290 will also illustrate this. Further, the field under the **leading** pole tips is **increased** while the field under the trailing pole tips is weakened. Hence *one of the causes of lead in a dynamo produces "lag" in a motor.* (This, as with the dynamo, results also in a **demagnetising or weakening effect.**)

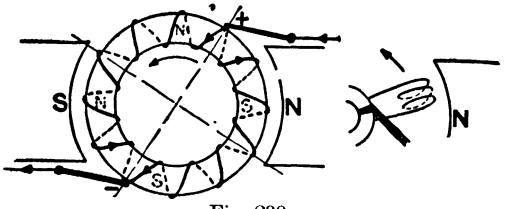


Fig. 290.

Again, as the operation of reversing the current in any coil when passing under the brush is the same whether dynamo or motor, the brush must be further shifted to produce this reversal sparklessly; but, as will be seen from Fig. 290, the direction is again *backwards.* Hence *both reasons causing lead in the dynamo cause lag in the motor.*

Obviously in the case of motors which are intended to run either way at will, there must either be some means for altering the position of the brushes at reversal, or else the motors must be designed so that the lag is exceedingly small, and—if radial brushes are used—the machine can then run practically sparklessly in either direction without the brushes needing any adjustment at all. Of these two options, the latter is the one always adopted, the former needing constant attendance and skilled supervision. Hence, in practice, *radial carbon brushes* are used, and the field magnets are very strong in order that the diameter of commutation shall be practically unaltered in position by armature reaction, no matter what the current may be. Commutating poles and compensating windings are also employed for this end. In motors "interpoles" have *opposite* polarity to the next main pole.

262. Losses in Motors: Efficiency of Motors.—Owing to losses, the efficiency of a motor is a variable quantity depending upon the construction and the load. The losses incurred when running motors are due to (*a*) friction in bearings, brushes, and air; (*b*) hysteresis; (*c*) eddy currents; and (*d*) heating in armature and field windings.

If we assume the friction to be constant at all speeds, the torque due to friction will be constant, and since the power wasted is given by the expression: Waste power $= 2\pi nT$, T being the friction torque and n the revolutions, we may say that the friction losses are proportional to the speed.

Under the magnetic and electrical losses we have the heating of armature and field coils, and loss due to hysteresis. This hysteresis loss may be taken as following the same law as the friction losses, the torque due to it being independent of the speed, and therefore the *power* wasted varying directly as the speed. And, further, there are the losses due to eddy currents in the armature core, armature conductors, and pole-pieces.

There are various methods for practically measuring these losses in the case of a particular motor, and curves may be drawn showing each constituent loss, together with the total loss, and the consequent efficiency of the motor. Large machines are usually more efficient than small ones.

As in the case of dynamos we have :—

$$\textbf{Electrical efficiency} = \frac{\textbf{Electrical h.p. spent in producing motion}}{\textbf{Total electrical h.p. supplied}},$$

$$\textbf{Commercial efficiency} = \frac{\textbf{Output (measured on brake)}}{\textbf{Total power supplied}},$$

$$\textbf{Mechanical efficiency} = \frac{\textbf{Output (measured on brake)}}{\textbf{Electrical power spent in producing motion}}.$$

In the following table the symbols employed are as follows :

E = pressure of supply to terminals of motor,
e = back E.M.F. of motor,
I = current supplied to motor,
W = power lost in friction, etc.,
I_a = current in armature,
I_s = current in shunt coil.

Motor.	Electrical Efficiency.	Commercial Efficiency.
Series	$\frac{e}{E}$	$\frac{Ie - W}{IE}$
Shunt	$\frac{I_a e}{(I_a + I_s)E}$	$\frac{I_a e - W}{(I_a + I_s)E}$

EXAMPLES.

1. *A series motor produces a B.E.M.F. of* 94 *volts, and is supplied with* 100 *amperes at a P.D. of* 100 *volts. If* $\frac{1}{2}$ *h.p. is lost in various frictions, find the brake horse-power and the electrical and commercial efficiencies.*

Here— $$E\,I = 100 \times 100, = 10000$$

$$I^2 (R_a + R_m) = (100)^2 \times \left(\frac{100 - 94}{100}\right),$$

$$W = \frac{746}{2}, = 373$$

$$\therefore \text{B.H.P.} = \frac{10000 - \{(100)^2 \times (\cdot 06)\} - 373}{746}.$$

$$\therefore \textbf{B.H.P.} = \textbf{12·1}.$$

Also :— $$\textbf{commercial efficiency} = \frac{12 \cdot 1 \times 746}{10000}$$

$$= \textbf{90·27 per cent.}$$

and :— $$\textbf{electrical efficiency} = \frac{94}{100} = \textbf{94 per cent.}$$

2. *A motor* 1·3 *ohms resistance develops* ·56 *h.p. when a current of* 10 *amperes is sent through it. The P.D. at its terminals being* 58 *volts, find the electrical and commercial efficiencies.*

The power wasted in the motor is given by I^2R

$$= 100 \times 1{\cdot}3 = 130 \text{ watts.}$$

The power received is 580 watts, therefore the electrical efficiency is—

$$\frac{580 - 130}{580} \text{ or } 77{\cdot}6 \text{ per cent.}$$

Again the power developed is ·56 h.p. = 417·76 watts, and the commercial efficiency is—

$$\frac{417{\cdot}76}{580} \text{ or } 72 \text{ per cent.}$$

Answers: 77·6 and 72 per cent.

263. Motor-Generators.—We have seen that the E.M.F. of, say, a two-pole dynamo is given by the expression $E = \phi N Z/10^8$ volts, from which it follows that, if we were provided with two machines identical in regard to flux and speed, but the one with Z_1 and the other with Z_2 armature conductors, the E.M.F.'s generated would be in the ratio Z_1/Z_2.

Now a dynamo may be driven by any convenient source of power, turbine, gas, steam or oil engine, or electric motor, etc. In the case of the last mentioned, the dynamo and motor armatures may be on the same shaft, so that they run at equal speeds: such an arrangement of motor and dynamo keyed to a common shaft is termed a **motor-generator.**

One application of a motor-generator is that of transforming on continuous current circuits from, say, a higher to a lower pressure with, naturally, a corresponding increase in current strength. As a simple illustration, consider a motor-generator in which the two components are both, say, shunt wound and identical, save that the generator armature has one quarter the number of conductors that the motor armature has. Conceive the motor joined to the supply mains at V volts; in thus rotating as a motor it drives the generator at the same speed, and the latter develops an E.M.F. *equal approximately to* $\frac{1}{4}$ V *volts* with a corresponding increase in current strength.

We have previously indicated that for long-distance transmission, economy in the copper mains demands high pressures, and the reader will now realise how, by the employment of motor-generators at a sub-station, it becomes possible to transmit at a high pressure and then transform to 250 volts, say, which is convenient for lighting purposes. Of course the E.M.F. of the generator may be made greater numerically than the volts supplied to the motor by, for instance, making the number of the generator armature conductors exceed that of the motor.

Owing to unavoidable losses, the *watts* given out by the generator

can never equal or exceed the watts supplied to the motor. If E be the supply voltage, I the current taken by the motor, E^1 the volts developed by the generator, and I^1 its current, and if a be the percentage efficiency of the motor generator—

$$\frac{E^1 I^1}{E I} = \frac{a}{100}.$$

264. Feeder Boosters.—Another application of the preceding is that of a feeder "**booster**," *i.e.* an arrangement for raising or "boosting up" the pressure at the end of a long **feeder** (see Chapter XXII.) thus compensating for the volts lost in the feeder. Thus, in Fig. 291, M is the motor, shunt wound and connected to the mains as indicated, while G is the generator which is series wound, the main current passing through the generator armature and field coils. The speed of the generator is constant and equal to that of the motor. If the load is heavy the lost volts in the feeder will be great owing to the large current passing ($E = I R$), but this large current passing through the generator fields will so excite them that G will develop an E.M.F. which will depend on the circumstances. Thus in the generator a rise in pressure occurs in proportion to the current passing, thus compensating for the volts lost in the feeder. If the load is light the current will be small and so will the lost volts; but the boosting action will be small also, since the generator fields are only slightly excited by the small current passing.

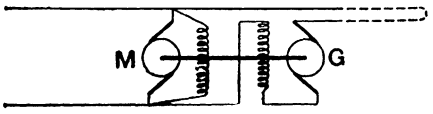


Fig 291.

EXAMPLE.

The pressure in a long feeder falls from 250 *to* 225 *volts and the current in the circuit is* 300 *amperes. By means of a motor generator the pressure is raised to* 250 *volts. If the efficiency of the booster be* 88 *per cent., find the current taken by the motor.*

The generator must develop 25 volts and its current is 300 amperes.

$$\therefore E^1 I^1 = 25 \times 300 = 7{,}500 \text{ watts.}$$

If I be current taken by motor, then, since E the volts supplied is equal to 225—

$$E I = 225 I.$$

Again $$a = 88.$$

Hence—

$$\frac{7500}{225 I} = \frac{88}{100}$$

$$\therefore I = \frac{7500 \times 100}{225 \times 88}$$

$$= \mathbf{37{\cdot}8 \text{ amperes.}}$$

Amongst other types of feeder boosters may be mentioned (1) a series wound generator connected at the station between the positive **bus bar** (see Chapter XXII.) and positive feeder, driven by a *separate* shunt wound motor connected to the positive and negative bus bars, (2) a generator driven by a shunt wound motor, the field of the former being supplied by an "exciter" the field of which is in series with the feeder: the generator armature is of course between the positive bus bar and positive feeder.

265. Battery Boosters.—The application of accumulators to Central Station work has been referred to in Chapter XIV., and the necessity for charging such by shunt generators has been dealt with. In actual practice *it is often necessary to charge the battery at the same time that constant potential is demanded on the bus bars for other purposes*, and it becomes necessary, therefore, either to use a generator of sufficient voltage to charge the battery, and to supply the constant potential required by cutting down the number of cells on the discharge side as charging proceeds (Fig. 292), or to raise the bus pressure for charging by means of a small machine called a **charging booster** (Fig. 293).

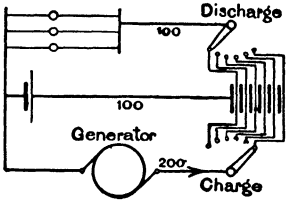


Fig. 292.

The disadvantage of the first method is apparent. Thus if the normal charging rate of the battery be 100 amperes, and while charging is in progress 100 amperes be required for lighting at constant potential, then as the discharge switch is moved to the left to secure constant pressure on that side as charging proceeds, the end cells between the charge and discharge switches would carry 200 amperes, while the body of the battery would carry the correct 100 amperes. The end cells would thus be charged at double rate, with the consequent danger of injury resulting. Attempts have been made to overcome this difficulty by using larger end (or regulating) cells to carry larger currents, and by installing counter E.M.F. cells.

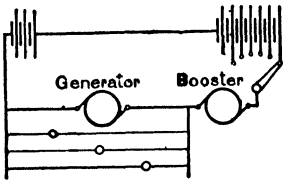


Fig. 293.

The second method referred to, viz. the adoption of a charging booster, is shown in Fig. 293. This consists either of a small shunt wound dynamo driven by a shunt motor, or of a small machine arranged on an extension of the main generator shaft. It will be clear that the main generator may supply the current required at constant potential whether the battery is being charged at the time or not, while the booster adds the necessary voltage for charging all the cells of the battery equally at the same time.

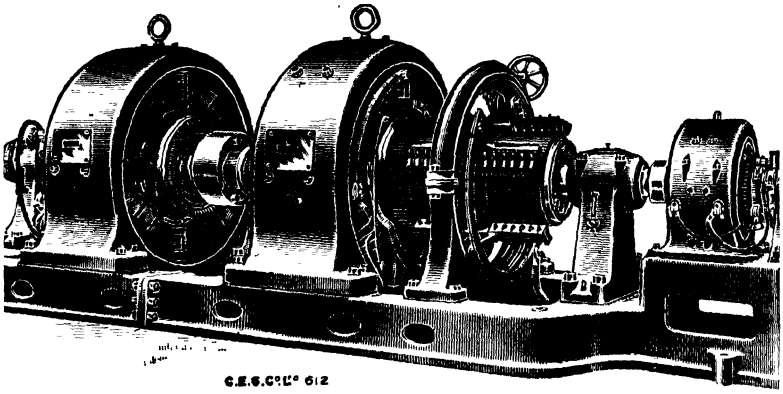


Fig. 294.

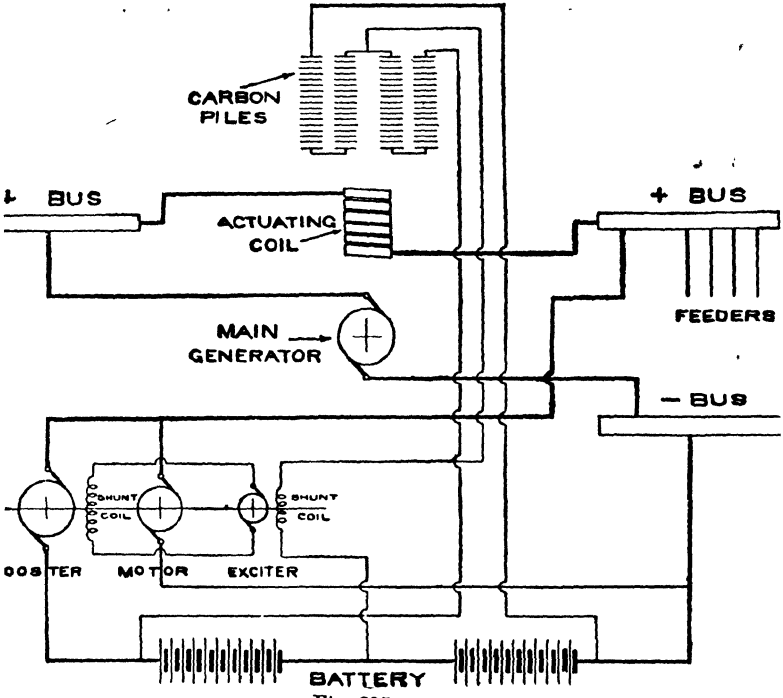


Fig. 295.

266. Automatic Reversible Battery Boosters.—Some classes of loads are subject to rapid variations, such as those met with on electric tramway and railway systems, and also in industrial plants, in which such apparatus as electrically driven air compressers are employed. It is obvious that the generating plant must be large enough to handle the maximum peaks met with, and it follows therefore that during the greater part of the day the plant will be working much below its full output, and therefore uneconomically. If a battery by itself were installed in parallel with the generators, it would assist the generators by discharging should the voltage of the generators begin to fall on account of the load, but as the voltage of the battery itself would also tend to fall when it began to discharge heavily, it is evident that a battery by itself cannot be relied upon to assist in handling peak loads, unless its size be such that it never discharges heavily in proportion to its total capacity.

In such cases **reversible boosters** are employed which automatically increase in voltage in direct proportion as the voltage of the battery decreases, and as these reversible boosters are *in series* with the battery, the voltage of the combination is kept up. **Such an arrangement can be made to relieve the generators of the sudden fluctuations in the external load by discharging all the peaks, and being again charged when the peaks go off.** These boosters are of two main types, viz. (1) exciter-controlled, and (2) differential wound: owing to space we will consider an example of the former only.

The "Entz" reversible booster is made by the Chloride Electric Storage Co., Ltd. It consists of three machines—motor, booster, and exciter—coupled together on one bed plate (Fig. 294), and the connections are shown diagrammatically in Fig. 295. It will be noted that the booster armature is in series with the battery, and that an "actuating coil" carrying the generator current is provided; the function of this coil will be gathered from the details which follow.

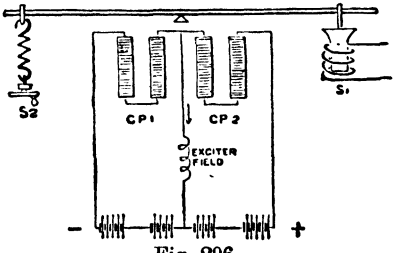


Fig. 296.

Instant reversibility is secured by means of a carbon regulator, shown, with connections, in the upper part of Fig. 295 and in greater detail in Fig. 296. This consists of two sets of piles of carbon discs and a pivoted lever, from one end of which is suspended a soft iron core surrounded by the coil which carries the full generator load; the other end of the lever is attached to a spring which counterbalances the pull of the coil on the soft iron when the desired average load is on the generator.

If now the load on the generator increases above the average, the pull due to S_1 overcomes the pull due to spring S_2, the carbon plates

CP_2 will be compressed and CP_1 released, the resistance of CP_2 will b less and that of CP_1 greater, and current will flow from a portion o the battery in the direction indicated in Fig. 296. The current passing in this direction through the exciter, causes the latter to excit the booster field coil in such a manner as to *add* to the voltage of th battery and cause it to discharge, and the regulation is such that thi discharge plus the average load (on the generator) equals the lin load required.

If the load falls below the average, the pull of S_2 overcomes that o S_1, the piles CP_1 are compressed, the direction of excitation is reversed and the battery receives a charge equal to the difference between th average generator load and the line load at this time.

When the average load is on, and therefore S_1 and S_2 are balancing CP_1 and CP_2 are equal in resistance, and the booster field coil is no excited.

Such an arrangement, therefore, relieves the generating plant of al fluctuations, enabling it to run at constant load.

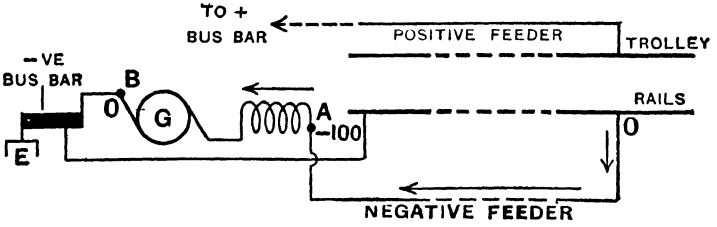


Fig. 297.

267. Negative Booster.—These are used in connection with electric trams their object being not to raise, **but to lower** the potential difference betwee certain points of the rails and earth (the Board of Trade regulation is that n point of the rails must be more than seven volts above the earth). The booste is put in a negative feeder, *i.e.* a feeder connected to a certain point of the rail as indicated in Fig. 297 (the motor and its connections are not shown). Th rotation of the generator results in the terminal A being at a lower potential tha B (in the figure B = 0 and A = - 100), *i.e.* the potential of the station end of th negative feeder is considerably below the rail end and current is therefore draw along the feeder as shown: thus we have less current returning by the rails an the necessary pressure regulations are readily satisfied.

EXERCISES XIX.

1. Prove the following from first principles:—

(*a*) A series dynamo supplied with reverse current will run against th brushes.

(*b*) A series *motor* will run the same way irrespective of the direction of th current supplied.

(*c*) A shunt dynamo supplied with reverse current runs the same way as before.

(*d*) A shunt motor will run the same way irrespective of the direction of current supplied.

2. Briefly indicate the different types of motor and the special work for which they are adapted.

3. A dynamo sends current to a distant motor. The dynamo is driven by an engine of 115 B.H.P. The nett (commercial) efficiency of the dynamo is 90 per cent.; the loss of energy in transmission is 7·2 per cent.; the nett efficiency of the motor is 93 per cent. What power would be available for a line of shafting driven by the motor?

4. The resistance of a motor is 0·7 ohm and the P.D. maintained at its terminals is 89 volts. If the current supplied is 27 amperes, what is the back E.M.F. of the motor?

5. The resistance of a motor is 1·2 ohms and the P.D. 103·5 volts; when the speed is such that the back E.M.F. is 87·2 volts, what is the current through the motor?

6. Why does a motor require a starting resistance in the circuit? Give a diagrammatic sketch of a starter and state the chief points which have to be attended to in the design.

7. Electric energy is supplied at 2d. per Board of Trade unit. Assuming the motors have an efficiency of 82 per cent., what is the cost of the energy used per horse-power hour?

8. A motor drives a pump which lifts water to a height of 118 feet. The motor runs for 9 hours each day and 6 days each week, and 20,000 gallons are to be raised per day. What current will the motor take at 104 volts if the total efficiency of the motor and plant is 55 per cent.? If energy is bought at 2d. per Board of Trade unit, find the cost of pumping water for a year.

9. Calculate the electrical efficiency of a shunt dynamo from the following data:—

Armature resistance	0·1 ohm.
Field magnet resistance	25 ohms.
External current	100 amperes.
Voltage at terminals	100 volts.

10. Give three ways of varying the speed of a motor and point out the advantage and disadvantages in each case.

11 It is required to raise water out of a mine 800 feet deep at the rate of 8,000 gallons per hour. If the combined efficiency of pump and D.C. motor is 58 per cent., and 10 per cent. is lost in pipe friction, what current would be required with 500 volts at motor terminals? (O.)

12. Enumerate the principal sources of waste of power in an electric motor. Current is supplied to a series motor at 100 volts, the resistance of the circuit being ·5 ohm. Determine the power expended in turning the armature when the current is 10 amperes. Determine the current when the power thus expended is a maximum. Compare the values of the electric efficiency in the two cases. (B.)

13. What is the magnitude and direction of the force acting on a straight conductor 10 cm. long placed at right angles to a magnetic field of 50 lines per sq. cm., the current through the conductor being 5 amperes? In what unit is your result expressed? (B.)

14. A series dynamo has an armature resistance of ·4 ohm and a field resistance of ·6 ohm and gives 120 terminal volts at a given speed and a current of 10 amperes. Work out its electrical efficiency as a dynamo and as a motor at that speed and current. (O.)

CHAPTER XX.

INDUCTIVE CIRCUITS AND ALTERNATING CURRENTS.

268. The Induction Coil. Modern Interrupters.—The induction coil is a practical application of the principles of mutual induction, consisting of an apparatus for the purpose of transforming a low P.D. between the terminals of a primary coil into a high P.D. between the terminals of a secondary coil. The two essential points are:—

(1) A given current in the primary coil shall make the linkages—flux × number of turns—in the secondary coil as large as possible.

(2) The current in the primary coil must be made and broken very rapidly.

The construction of the apparatus is shown in Fig. 298, and is, briefly, as follows: Round an inner core composed of a bundle of soft-iron wire is wound a coil consisting of a few turns of very stout wire, termed the *primary* of the coil; this is in series with a break consisting of a spring touching a contact screw, the spring carrying at its upper end a small piece of soft iron, which is attracted by the iron core of the coil when current flows through the primary, and falls back again when the primary current ceases and the iron core becomes demagnetised. Hence, on attaching the battery, an intermittent current passes through the coil, the frequency of the currents depending upon the strength and inertia of the spring.

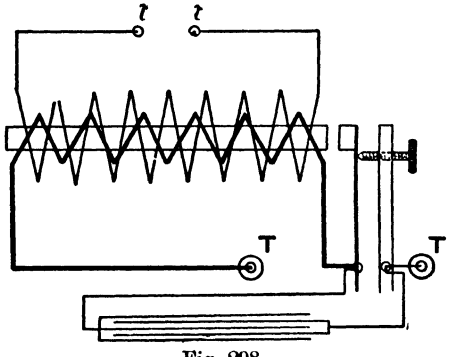


Fig. 298.

Surrounding the primary coil, but insulated from it with great care, is a coil consisting of a very great number of turns of very fine wire called the *secondary*. The ends of this wire are led out of the casing and are attached to terminals on the top of the coil, and usually when the coil is working the secondary circuit is complete, except for a gap, across which it is desired to cause sparks to leap.

Up to this stage in the construction the action may be briefly explained as follows. When the primary current passes (1) *an induced inverse current is developed in the secondary*, and (2) the core is magnetised, the soft iron head and spring are attracted and the primary circuit is broken; when the primary current ceases (1) *an induced direct current is developed in the secondary*, and (2) the core is demagnetised, the spring falls back against the screw, the primary current again starts and the actions are repeated. It follows from this that so long as the primary current is made and broken so long will currents alternating in opposite directions circulate in the secondary.

In practice matters are not so simple as indicated above, owing to the self-induction of the primary circuit itself. When the primary is "made" the resistance is small and the time constant L/R is therefore great; when the primary is "broken" the resistance is great and the time constant relatively smaller. Hence the decay of the primary is quicker than the growth, and therefore the inductive effect on the secondary at "break" is more pronounced than that at "make."

On the other hand, when the primary is broken the "extra current" developed in it in the *same* direction as the primary makes the break less sudden and definite than it would otherwise be (thereby reducing the inductive effect on the secondary), and, sparking across the interrupter, damages the surfaces of contact. To prevent these effects a condenser, made of sheets of tinfoil and paraffined paper, is connected with the primary circuit; one pole of it is connected to the pillar of the screw, and the other to the spring. The action of this condenser may be briefly explained thus. When the current is broken the self-induced current, having to charge the condenser is not able to spark across the break and thus the current is very suddenly broken. Again, the charged condenser at once discharges round the primary but in the opposite direction to that of the primary current and this tends to produce an induced current in the secondary in the same direction as that due to the break: in fact in practice the flux in the core is *reversed* at each break and the induced quantity in the secondary is almost twice that without a condenser. To be exact, the condenser current is really oscillatory, but these oscillations are quickly damped, dying out before the circuit is closed again so that only the first discharge is practically important. Summarising we may say that the secondary induced current at make is comparatively small, while that at break is intensified so that in an ordinary induction coil with a condenser the secondary induced currents are those due to the breaks of the primary current.

The above is only an elementary treatment of the induction coil and more exact details cannot be given here. As indicated the coil produces in the secondary a comparatively small current at a high pressure from a comparatively large current at a small pressure in the primary, and for this the secondary consists of a very large number of turns compared with the primary. Mathematically it

can be shown that $I_2/I_1 = 2M/L_2$ (approximately) where I_2 = secondary current, L_2 = inductance of secondary, M = the mutual inductance and I_1 = primary current at the instant of breaking circuit.

In modern coils the vibrating hammer as a make and break is frequently replaced by other devices giving much more rapid interruptions. One form is the **Wehnelt Electrolytic Interrupter.** This is merely a cell containing as one electrode a large plate of lead immersed in dilute sulphuric acid, the other consisting of the end of a piece of thin platinum wire projecting from the end of a glass tube. This cell is placed in the primary circuit, so that the current leaves by the lead plate, and it is then found that if the pressure of supply exceeds 24 volts or so, the current is rapidly broken and the secondary emits powerful discharges. Another form much used is the **Motor Mercury Interrupter** shown in Fig. 299. In this pattern a jet of mercury is discharged from a tube and caused to strike a toothed wheel which is rotated by a small motor at a high speed. The circuit of the primary coil is completed through the jet; hence when the jet strikes a tooth the primary is made, but when the jet misses the tooth the primary is broken, and the rate of this depends obviously, upon the speed of rotation of the toothed wheel, which is under complete control. Also the teeth are tapered, and thus by either moving the jet or the wheel up or down relatively to each other the proportion of time during which the circuit is made and broken is under control too. This device gives excellent results. The mercury is pumped up by the action of the motor which rotates the wheel.

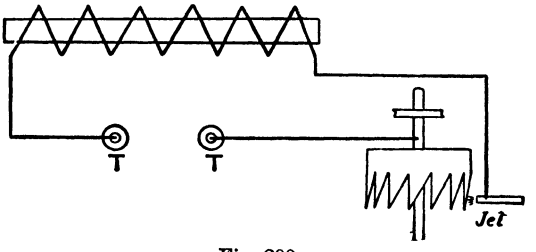


Fig. 299.

269. Alternating Currents and Measurements.—Alternating currents have already been briefly described in Chapter IX., which should be re-read before going on to the remainder of this chapter.

Such currents are due to the application of an E.M.F. which is reversed in direction a great many times per second; these E.M.F.'s are easily produced by an *alternator*, and can be safely allowed to have very high values, *e.g.* alternators may generate pressures which fluctuate between ± 10,000 volts, and more, a pressure which is impossible in a direct current machine. Further, alternating currents may have their pressures altered up or down very easily and cheaply by means of a *transformer*, which is an appliance needing no skilled attendance and having no moving parts. These two reasons—ease and safety of production, and facility for variation of pressure—have led to the extended use of alternating currents on all long-distance transmission schemes (high pressure results in economy of copper in the mains).

When we seek to measure an alternating current we think of the three usual effects—chemical decomposition, magnetic effect, and heating effect. Now alternating current can produce no chemical decomposition, to a first approximation, because the effect of the current when flowing in one direction is undone by the reverse current immediately following, and for the same reason no effect can be produced upon a magnetic needle. On the other hand, a wire will be heated by a current independently of its direction, and thus instruments which depend upon the heating effect can be used for the purpose. So also can those which depend upon the dynamometer principle, since, if the alternating current be passed through both coils, the alternations will have no effect upon the attractions or repulsions produced. In both methods the readings will, with continuous currents, depend *upon the squares of the currents*, and, with alternating currents, *upon a mean square*. As already indicated an electrostatic instrument can be used for alternating pressures.

For practical purposes of A.C. measurements we define **a Virtual Ampere as one which will produce the same heat in a resistance as a steady current of one ampere will produce in the same time,** and it can be shown that if the alternating current fluctuates between the maximum values $\pm$ I amperes, **the virtual value of the current is $\frac{I}{\sqrt{2}}$ amperes, *i.e.* ·707 I amperes.** Again, if I_0 amperes be the virtual value, the maximum value, I, is clearly $\sqrt{2}\,I_0$. Measuring instruments read virtual values, and thus, when an A.C. ammeter reads 100 amperes, the current is really fluctuating between $\pm$ 100 $\sqrt{2}$ *i.e.* between $\pm$ 141·4 amperes.

The truth of the above will be gathered from the following. Consider an alternating current obeying the sine law and fluctuating between $\pm$ 1 ampere. The *average value of the current* is ·637 ampere, *i.e.* ·637 of the maximum value. On the other hand, the heat at any instant depends on the square of the current at that instant, and the *average of the squares of the currents* at every instant is ·499849, the square root of which is ·707. As far as heating is concerned, this A.C. fluctuating between $\pm$ 1 ampere is equivalent to a steady current of ·707 ampere, *i.e.* Virtual Amperes = ·707 × the Maximum Amperes.

Similar remarks apply to the measurement of an alternating E.M.F. **A Virtual Volt is one which, when applied to the ends of a resistance, results in the same heating effect as in the case of a steady pressure of one volt applied for the same time, or which produces the same deflection on an electrostatic voltmeter, and the virtual value of the pressure is ·707 × the maximum pressure.** Similarly an A.C. voltmeter reads virtual values, and thus if it registers 200 volts the pressure is really fluctuating between $\pm$ 200 $\sqrt{2}$, *i.e.* between $\pm$ 282 volts. To summarise:—

$$I_0 = \underset{\text{(virtual amps.)}}{\text{Ammeter Reading}} = \frac{\text{Max. current}}{\sqrt{2}} = \frac{I}{\sqrt{2}}, \qquad \therefore I = \sqrt{2}\, I_0.$$

$$E_0 = \underset{\text{(virtual volts)}}{\text{Voltmeter reading}} = \frac{\text{Max. pressure}}{\sqrt{2}} = \frac{E}{\sqrt{2}}, \qquad \therefore E = \sqrt{2}E_0,$$

and incidentally it may be again mentioned that:—

(*Average* E.M.F. or current) = ·637 (max. E.M.F. or current).

Virtual values are often referred to as "the square root of the mean square" or "root mean square" or "R.M.S." values.

We may note in passing that alternating and rapidly altering currents tend to confine themselves to the surface of the conductor, and this is more marked the higher the frequency. Hence, conductors for such should have a large surface compared with the cross section, a thin flat ribbon being a good pattern to adopt.

270. Law of Current Production with Sine E.M.F.: Impedance and Reactance.—The methods in use for the production of alternating currents are such that it is sufficiently near to assume that the law of variation of pressure, and therefore current, is a **sine law.** That is, the curves sketched in Art. 112 are sine curves, and mathematically the pressure curve is given by the expression $e = \mathbf{E} \sin 2\pi nt$, where n is the frequency, E the maximum E.M.F., and e the instantaneous E.M.F. at time t, while the current curve is given by $i = \mathbf{K}\,\mathbf{E} \sin (2\pi nt - \phi)$, where K is a constant depending upon the nature of the circuit, while ϕ is the angle of lag, which also depends upon the circuit.

Thus when the pressure *in a circuit with resistance and inductance only,* rises and falls in accordance with the sine law, the resultant current rises and falls too, with the same frequency, but the strength of the current depends very much upon the inductance, and it lags behind the pressure by an amount which also depends upon the inductance.

For example, if a pressure be applied which oscillates between 100 volts one way and 100 volts the other way, making 50 fluctuations per second and obeying the sine law, and if the resistance of the circuit be 1 ohm and the coefficient of self-induction 0·1 henry, the current will oscillate between the values 3·18 amperes in one direction and 3·18 amperes in the other, instead of being 100 amperes, as it would be in a non-inductive circuit; and, further, instead of the current going to and fro in time with the pressure, there will be a lag of a little over 88°—that is, it will be *nearly* $\frac{1}{200}$ of a second behind the pressure.

In such cases (circuit with resistance and inductance), the results are obtained by the following laws:—

$$\text{Virtual current in amperes} = \frac{\text{Virtual pressure in volts}}{\text{Impedance}}$$

$$\text{Impedance} = \sqrt{(\text{Resistance})^2 + (\text{Reactance})^2}$$

$$\text{Reactance} = 2\pi \times (\text{Frequency}) \times (\text{Coeff. of Self-Induction}) = 2\pi n \mathrm{L}.$$

Hence in symbols we have:—

$$I_0 = \frac{E_0}{\sqrt{R^2 + (2\pi nL)^2}} = \frac{E_0}{\sqrt{R^2 + 4\pi^2 n^2 L^2}} \quad \ldots\ldots\ldots\ldots (1)$$

and the current lags behind the pressure by an angle whose tangent is measured by (Reactance)/(Resistance), *i.e.* by $2\pi nL/R$,

$$\therefore \text{ tan angle of lag} = \frac{2\pi nL}{R} \text{ or } \phi = \text{lag} = \tan^{-1}\frac{2\pi nL}{R}.$$

Two points may be noted in passing, viz. (1) if $L = 0$ the expression becomes $I_0 = E_0/R$ just as in the case of continuous currents; (2) since the maximum current $I = \sqrt{2}\, I_0$, and the maximum pressure $E\sqrt{\sqrt{2}}\, E_0$, the law, obviously, might also be written:—$I = E/\sqrt{R^2 + 4\pi^2 n^2 L^2}$.

These quantities and their relations are best remembered by the triangle shown in Fig. 300 (*a*); the hypotenuse is the *impedance*, the base the *resistance*, the vertical the *reactance*, and the angle between the hypotenuse and base is the *angle of lag*.

Further, from the preceding:—

$$E_0 = I_0 \sqrt{R^2 + 4\pi^2 n^2 L^2};$$

that is,

$$E_0 = \sqrt{(I_0 R)^2 + (2\pi n L I_0)^2}.$$

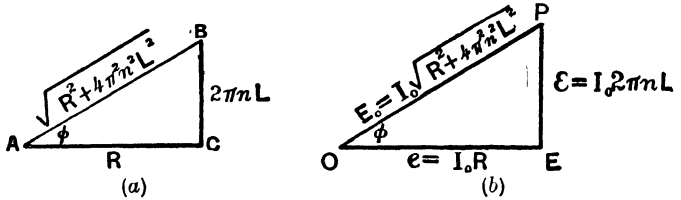


Fig. 300.

This relation is shown in Fig. 300 (*b*), in which the hypotenuse gives the virtual impressed volts E_0 necessary to drive I_0 virtual amperes through a circuit of resistance R and reactance $2\pi nL$. e is the virtual active E.M.F. and ϵ the virtual E.M.F. of self-induction.

If the circuit possesses resistance and capacity but no inductance, it may be shown that the impedance becomes $\sqrt{R^2 + \left(\frac{1}{2\pi nC}\right)^2}$, the reactance $\frac{1}{2\pi nC}$, where C is the capacity in farads, and the law becomes:—

$$I_0 = \frac{E_0}{\sqrt{R^2 + \frac{1}{4\pi^2 n^2 C^2}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

The effect of capacity is to make the current **lead** in front of the pressure and:—

$$\text{tan angle of lead} = \frac{\frac{1}{2\pi nC}}{R} = \frac{1}{2\pi nCR} \text{ or } \phi = \tan^{-1}\frac{1}{2\pi nCR}.$$

If the circuit possesses resistance, inductance and capacity, it can be shown that:—

$$I_0 = \frac{E_0}{\sqrt{R^2 + \left(2\pi nL - \frac{1}{2\pi nC}\right)^2}} \quad \ldots\ldots\ldots\ldots\ldots (3)$$

In this case $\sqrt{R^2 + \left(2\pi nL - \frac{1}{2\pi nC}\right)^2}$ is the impedance of the circuit and $\left(2\pi nL - \frac{1}{2\pi nC}\right)$ is the reactance. If $2\pi nL$ is greater than $1/2\pi nC$ the current lags, but if it is less the current leads. If $2\pi nL = 1/2\pi nC$ the effect of capacity neutralises that of inductance, and the current becomes $I_0 = E_0/R$: that is there is neither lag nor lead, and the circuit is in effect **non-reactive,** although both inductance and capacity are present. When this is so we have the phenomenon known as **electrical resonance.**

From the preceding we may regard *impedance* as the effective resistance encountered by an alternating current: it comprises both the ohmic resistance and the reactance, the latter depending on the inductance, the capacity, and the frequency.

271. Choking Coils.—In (1) Art. 270 if the resistance of the coil is negligible in comparison with its reactance, the impedance is the same thing practically as reactance. R being assumed zero (Art. 270), the angle of lag is the angle whose tangent is infinite, or 90°. A coil arranged so as to have this property is termed a **choking coil.**

In the path of a *rapidly altering* current a choking coil acts like a very high resistance, while to a *continuous* current its resistance is practically zero. This is because in the first case the inductive action is all-important, and in the second case the inductive action does not matter at all.

Such a coil is frequently placed in the circuit leading from the switchboard to the overhead conductors in a tramway power station, etc. Should lightning strike the overhead wires and attempt to pass along to the generators, the choking coil prevents it from doing so and causes it rather to leap a gap and pass harmlessly to earth, while to

the large steady currents always being supplied to the line the choking coil makes no difference at all.

An A.C. lamp needing so many volts when connected between mains at a higher pressure may be arranged with a choking coil *in series with it*, which reduces the effective pressure to the desired amount without causing any considerable waste (Art. 272).

A simple type of choking coil is shown in Fig. 301. The coil C consists of four layers of wire wound upon an insulating tube T, in which the laminated iron core I can be raised or lowered, thus varying the inductance and reactance. The core is suspended by a steel cord, which passes twice over a pulley and carries a counterpoise. The core can be raised or lowered by the hand wheel HW, a fine adjustment being obtained by a worm and pinion.

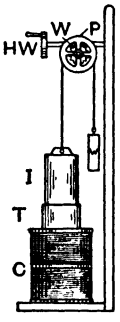


Fig. 301.

With respect to a choking coil the pressure due to its inductive action, being of necessity 90° different in phase from the effective pressure, must be given by the triangle of Fig. 300 (*b*). The pressure usefully used in the circuit is $e = I_0R$, that produced by the choking coil reaction is $\epsilon = I_0 2\pi nL$, and the applied pressure E_0 is that given by the hypotenuse; hence

$$E_0^2 = e^2 + \epsilon^2. \quad \therefore \epsilon = \sqrt{E_0^2 - e^2}.$$

272. Typical A.C. Problems.—The following worked examples will help the student to grasp the details of the preceding sections:—

EXAMPLES.

1. *An alternating pressure of* 100 *volts (virtual always understood) is applied to a circuit of resistance* 0·5 *ohm and self-induction* 0·01 *henry, the frequency being* 50 *cycles per second* (50 ~). *What will be the reading of an ammeter in the circuit, and what will be the lag in time between pressure and current?*

We have $$I_0 = \frac{100}{\sqrt{(0{\cdot}5)^2 + 4\pi^2 (50)^2 (0{\cdot}01)^2}};$$

$$\therefore I_0 = 31{\cdot}4 \text{ amperes (nearly).}$$

$$\therefore \textbf{Ammeter reading} = \mathbf{31{\cdot}4} \textbf{ amperes.}$$

Again $$\tan \phi = \frac{2\pi \times 50 \times 0{\cdot}01}{0{\cdot}5}$$

$$= 6{\cdot}28.$$

$$\therefore \phi = \mathbf{81°} \textbf{ (nearly),}$$

and $$\textbf{lag in time} = \frac{\mathbf{81}}{\mathbf{360}} \times \frac{\mathbf{1}}{\mathbf{50}} \textbf{ second} = \frac{\mathbf{1}}{\mathbf{222}} \textbf{ second.}$$

In this example 31·2 amperes will be the virtual or ammeter reading because the pressure is given in virtual volts. The current will fluctuate between $31{\cdot}2 \times \sqrt{2}$ amperes, first one way and then the other, and it will reach this maximum value $\frac{1}{222}$ second after the pressure each time reaches its maximum value of $100 \times \sqrt{2}$ volts.

2. *A coil of wire of negligible resistance and inductance* ·02 *henry, and a wire of zero inductance and resistance* 12 *ohms are in series. If the impressed E.M.F. be* 130 *volts and the frequency* 40 *cycles per second, determine* (*a*) *the current,* (*b*) *the lag,* (*c*) *the P.D. across the inductive resistance,* (*d*) *the P.D. across the non-inductive resistance.*

(*a*) For the current (using I and E for virtual values for simplicity) :—

$$I = \frac{E}{\sqrt{R^2 + (2\pi nL)^2}} = \frac{130}{\sqrt{12^2 + (2\pi \times 40 \times \cdot 02)^2}} = \mathbf{10\ amperes.}$$

(*b*) The current lags behind the impressed E.M.F. by an angle ϕ such that :—

$$\tan \phi = \frac{2\pi nL}{R} = \frac{2\pi \times 40 \times \cdot 02}{12} = \cdot 417,$$

$$\therefore \phi = \mathbf{22 \cdot 5^\circ} \text{ and } \mathbf{Lag\ in\ time} = \frac{22 \cdot 5}{360} \times \frac{1}{40} = \mathbf{\frac{1}{640}\ second.}$$

(*c*) P.D. (E_1) on the non-inductive resistance is obtained in the usual way, viz. :—

$$I = \frac{E_1}{R}, \textit{ i.e. } 10 = \frac{E_1}{12}, \quad \therefore \mathbf{E_1 = 120\ volts.}$$

(*d*) P.D. (E_2) for the inductive resistance is obtained from :—

$$I = \frac{E_2}{\sqrt{0 + (2\pi nL)^2}}, \textit{ i.e. } 10 = \frac{E_2}{2\pi nL}, \quad \therefore \mathbf{E_2 = 50\ volts.}$$

Note that the relation between the three pressures is represented by the triangle OPE of Fig. 300 (*b*), OP representing 130, OE representing 120, and PE representing 50 ($130^2 = 120^2 + 50^2$). In the case of a *direct* current we would have, of course, $E_1 + E_2 = E$.

3. *An A.C. arc lamp needing* 50 *volts is put across mains at a P.D. of* 80 *volts, and a choking coil is in series with the lamp. The lamp takes* 10 *amperes and the frequency is* 50 *per second. Find* (1) *the pressure produced by the choking coil reaction,* (2) *the self-inductance of the choking coil.*

The solution is given by the triangle of Fig. 300 (*b*). Here OP is the impressed or applied E.M.F. (80), OE is the effective pressure (50) and PE is the pressure produced by the choking coil. Clearly :—

$$PE^2 = OP^2 - OE^2, \textit{ i.e. } PE = \sqrt{OP^2 - OE^2}.$$

$\therefore$ **Pressure produced by choking coil**

$$= \sqrt{80^2 - 50^2} = \mathbf{62 \cdot 5\ volts.}$$

Again, from Art. 270, $E_0 = \sqrt{(I_0R)^2 + (2\pi nLI_0)^2}$, and from Fig. 300, $OP = \sqrt{OE^2 + PE^2}$. Here OP represents E_0, OE represents I_0R, and PE represents $2\pi nLI_0$. But PE = 62·5 volts.

$\therefore$ **Self-inductance of choking coil**

$$= L = \frac{62 \cdot 5}{2\pi nI_0} = \frac{62\ 5}{2\pi \times 50 \times 10} = \mathbf{\cdot 02\ henry.}$$

4. *A certain choking coil of negligible resistance takes a current of* 8 *amperes if supplied at* 100 *volts, at* 50 *periods, per second. A certain non-inductive resistance, under the same conditions, carries* 10 *amperes. If the two are transferred to a*

supply system working at 150 volts, at 40 periods per second, what total current will they take (a) if joined in series, (b) if joined in parallel ?

The value of R for the non-inductive resistance and the value of L for the inductive resistance must first be found. For R we have R = 100/10 = 10 ohms, and for L we have

$$I = \frac{E}{\sqrt{R^2+(2\pi nL)^2}}, \quad i.e.\ 8 = \frac{100}{\sqrt{0+(2\pi nL)^2}} = \frac{100}{2\pi nL}.$$

$$\therefore L = \frac{100}{2\pi \times 50 \times 8} = \frac{1}{8\pi} \text{ henry.}$$

Case (a):—The two in series.

$$I_1 = \frac{E}{\sqrt{R^2+(2\pi nL)^2}} = \frac{150}{\sqrt{10^2+\left(2\pi \times 40 \times \frac{1}{8\pi}\right)^2}}$$

$$= \mathbf{10{\cdot}6 \text{ amperes.}}$$

Case (b):—The two in parallel.

If i_1 be the current for the non-inductive resistance,

$$i_1 = \frac{E}{R} = \frac{150}{10} = 15 \text{ amperes.}$$

If i_2 be the choking coil current,

$$i_2 = \frac{E}{\sqrt{0+(2\pi nL)^2}} = \frac{150}{2\pi \times 40 \times \frac{1}{8\pi}} = 15 \text{ amperes.}$$

But the current i_2 has a phase relationship of 90° with i_1, and the resultant current I_2 is given by :—

$$I_2 = \sqrt{i_1^2 + i_2^2} = \sqrt{15^2 + 15^2}$$

$$= \mathbf{21{\cdot}2 \text{ amperes.}}$$

The total currents in the two cases are therefore 10·6 and 21·2 amperes respectively.

273. Some Peculiarities of Electrical Resonance.—If the inductance and capacity in an A.C. circuit are such that the circuit is in the vicinity of electrical resonance ($2\pi nL = 1/2\pi nC$) peculiar results may follow. To illustrate one case the student should work out the following problem :—"A circuit A, of resistance 5 ohms and inductance 5 henrys, is in series with a circuit B, of resistance 10 ohms and capacity $1/(2 \times 10^6)$ farad: the applied pressure is 200 volts and the frequency 100: find the P.D. at the ends of A and B." The student will find that *although the applied pressure is only* 200 *volts, really dangerous pressures of nearly* 15,000 *volts exist at the ends of* A *and* B *respectively.* Again, if a circuit with resistance and inductance be in parallel with a circuit with resistance and capacity, and the condition for electrical resonance be satisfied, then *ammeters placed in the two branch circuits will give definite readings, but an ammeter in the main circuit will give no reading at all.* (Prove this.) The first phenomenon above is often referred to as *pressure resonance*, and the second as *current resonance.*

274. Power in an A.C. Circuit: Power Factor: Wattless Current.—If at any instant the pressure is e volts and the current i amperes, power is supplied at that instant at the rate ei watts. When e and i are fluctuating and reversing periodically, as in A.C. circuits, and further, are not keeping time together, the power can be proved to be given by the expression:—

$$\text{Power} = E_0I_0 \times (\cos \text{ angle of lag or lead}),$$

i.e. $$\textbf{Power} = \mathbf{E_0I_0 \cos \phi \text{ watts.}}$$

Clearly, $\cos \phi$ is given by the expression:—

$$\cos \phi = \frac{\textbf{Resistance}}{\textbf{Impedance}} = \frac{R}{\sqrt{R^2 + \left(2\pi nL - \frac{1}{2\pi nC}\right)^2}}.$$

Now $E_0I_0 \cos \phi$ measures the *true power* supplied, and E_0I_0 being the product of the voltmeter and ammeter readings (which would be correct for a continuous current) measures the *apparent power*; $\cos \phi$ measures what is termed the **power factor** of the circuit; hence

$$\textbf{True power} = \mathbf{E_0I_0 \cos \phi} = \textbf{Apparent power} \times \textbf{Power factor};$$

$$\therefore \textbf{ Power factor} = \cos \phi = \frac{\textbf{True power}}{\textbf{Apparent power}}.$$

Before the power supplied to an A.C. circuit can be determined from the voltmeter and ammeter readings, the power factor must be known. In practice a wattmeter, suitably designed for A.C., having as few metal parts as possible, will read *the true power* direct, and this furnishes a good method for measuring the power factor, when it is desirable to know it; thus:—

$$\text{Wattmeter reading} = \text{True power};$$

$$\text{Ammeter reading} \times \text{Voltmeter reading} \times \cos \phi = \text{True power}.$$

$$\therefore \textbf{ Power factor} = \cos \phi = \frac{\textbf{Wattmeter reading}}{\textbf{Ammeter reading} \times \textbf{Voltmeter reading}}.$$

Now, suppose the circuit to be so very inductive that the current lags practically 90°, or suppose the circuit to have such a capacity that the current leads practically 90°, then the value of $\cos \phi$ will be zero; but the ammeter will read something and the voltmeter reading will be as b efore; hence the Real power $= E_0I_0 \cos \phi = 0$, and the Apparent power $= E_0I_0$. The current in such a case is said to be a **wattless current**; it flows sometimes with the pressure and sometimes against it, so that the net work done is zero, *except on account of the resistance.* Thus wattless currents do no useful work; but since every circuit possesses resistance, more or less, there is some waste of energy in heating the circuit while allowing such a current to flow. In stationary transformers for A.C. circuits, the inductance is such that when the secondary is open we have *practically* a wattless current in the primary.

275. Alternators—Single and Polyphase.—In one type of alternator the field magnets are stationary and the armature rotates. In another type the armature is stationary and the field magnets rotate. For clearness, the rotating part (armature or fields) is called the **rotor** and the stationary part (armature or fields) is called the **stator.**

Fig. 302 indicates the principle by a machine in which the armature is the rotor and the fields the stator. The multipolar field is excited by a separate direct current dynamo, and the field windings are such that the poles are alternately north and south as indicated. A ring armature is shown, the winding consisting of as many coils as there are poles, and the spacing of the winding corresponding to the spacing of the poles. At the same instant a coil will be passing under an opposite pole to that of its neighbours, so that the E.M.F. induced in it will be in the opposite direction. If, however, the coils be wound in opposite directions and joined as indicated, the extreme ends being connected to two slip rings, they will, at any instant, help each other from one extreme end to the other, but the outside current will be reversed every time the coils pass from pole to pole. Thus the *frequency* will be obtained by multiplying the number of *pairs of poles* by the revolutions per second. Ring armatures are rarely used, drums being preferred. Armatures are, of course, laminated.

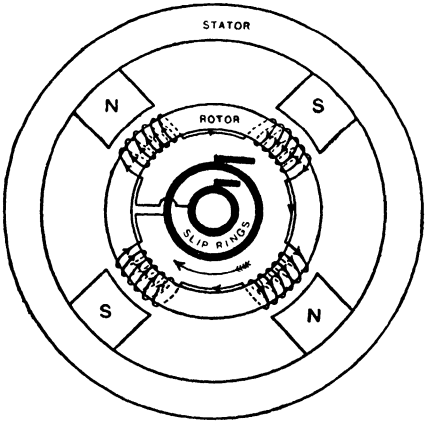


Fig. 302.

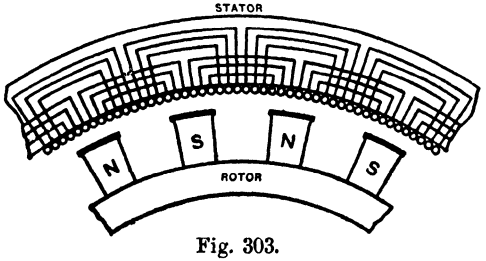


Fig. 303.

Large commercial alternators invariably consist of a set of rotating field magnets (the rotor) and a stationary group of windings on the armature (the stator).

The rotor consists of a heavy flywheel, carrying radial pole-pieces on which are fitted carefully secured former-wound bobbins, these being coupled in series and supplied with current from a direct current

exciting dynamo, either on its own shaft or elsewhere. The exciting current is led into the rotor through two slip rings and the polarity of the poles is alternately N and S. The *frequency* is obtained by multiplying the number of pairs of poles by the revolutions per second.

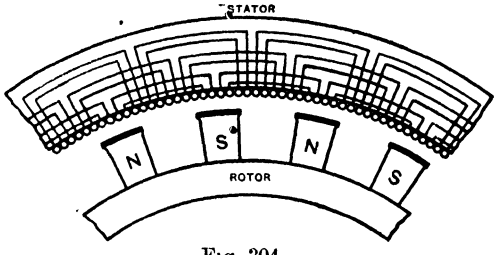


Fig. 304.

The stator consists of sheet stampings, containing slots parallel to the shaft, in which the windings are placed in insulating tubes. The slots are usually wholly or partly closed, and the conductors, together with their insulation, are threaded through them.

In **single-phase alternators** all the conductors are connected in series, and the ends joined to the terminals of the machines. In **two-phase alternators** there are two different sets of conductors, the constituents of one set being spaced half way between the constituents of the other set, as shown in Fig. 303, so that the E.M.F. in one set is a maximum when it is zero in the other; thus there is a phase

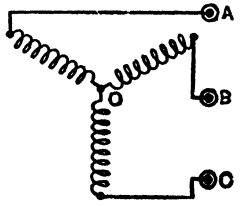


Fig. 305.

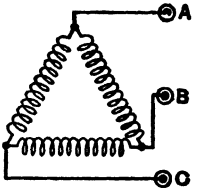
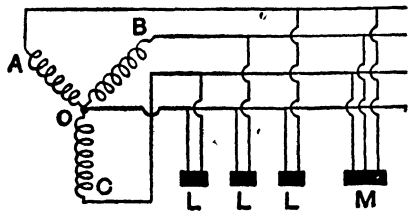


Fig. 306.

difference of 90°, and there are four terminals to the machine. In **three-phase alternators** there are three sets of conductors (Fig. 304), arranged so that, while set (1) is producing the maximum E.M.F., set (2) is producing E.M.F. lagging 120° behind set (1), and set (3) is producing E.M.F. lagging 120° behind set (2), and therefore 240° behind set (1). Instead of six terminals, the starting point of all three windings is usually a common junction; this is often earthed to the shaft of the generator, and the other three ends after forming the windings round the stator are connected to *three terminals* on the machine (Fig. 305). Such a machine is said to be **star wound.** It may also be **mesh** or **delta wound;** this is shown diagrammatically in Fig. 306.

Referring to Fig. 305, the voltage between any two of the star

points A, B, C, is called the "line" voltage whilst that between the centre point O and a star point is called the "phase" voltage. The ratio is:—phase voltage : line voltage = 1 : 1·73 so that if the phase voltage is 100 the line voltage is 173. When both lighting and power are fed from a three-phase (star) circuit it is customary to run a line from the centre point O: the lighting is then run from this fourth line and any of the line wires whilst three-phase motors are fed from the three line wires: this is indicated in Fig. 307. In the delta winding the line and phase voltages are the same.



L = lamps. M = three-phase motor.
Fig. 307.

A good idea of the construction of a modern alternator will be gathered from Figs. 308, 309: the former shows the stator in course of winding, one layer of end turns being complete: the latter shows the rotor with slots milled ready for winding (G.E.C. Birmingham).

276. Alternating Current Motors.—The general principles only of these can be discussed here.

(1) Synchronous Motors.—Just as any direct current dynamo can be run as a motor, the direct current supplied to the armature coils being reversed at the proper instant *by the commutator*, so any alternator can be run as a motor if supplied with alternating current, *but its frequency must keep step with the frequency of the alternating current supplied.* Since the frequency is equal to the number of pairs of poles multiplied by the number of revolutions per second, these motors must first be speeded up by some external means, and when the correct speed is attained they are connected on to the mains. This fact, that the frequencies must keep step, is expressed by saying that the motors must run *synchronously.* Thus, though these motors are efficient, they will not start themselves. When overloaded they fall out of step and stop, in which case the load must be removed and the machines synchronised again.

On polyphase systems, synchronous motors are generally started by means of a small induction motor. In the case of polyphase supply, synchronous motors may also be started by opening the fields and giving polyphase current directly to the armature. In this way eddy currents are set up in the field poles, a starting torque is brought into play, and the armature begins to rotate. When the speed of synchronism is reached the field excitation is put on, and the load may then be applied.

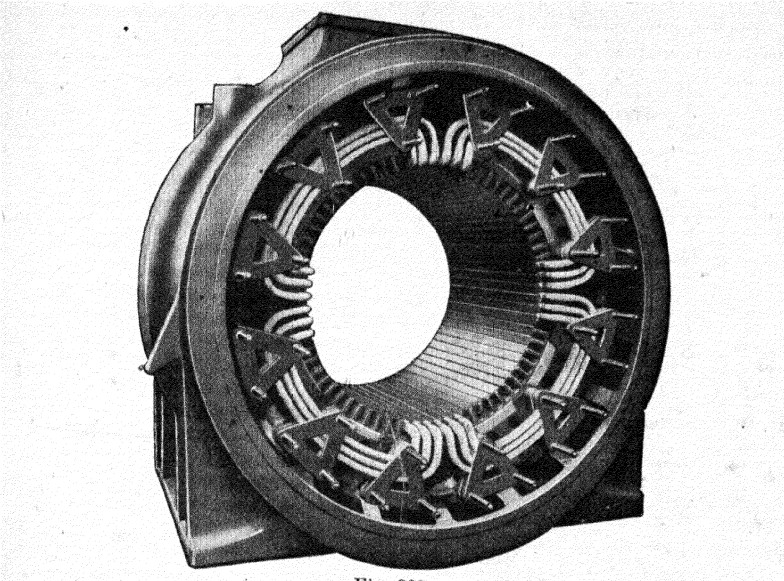


Fig. 308.

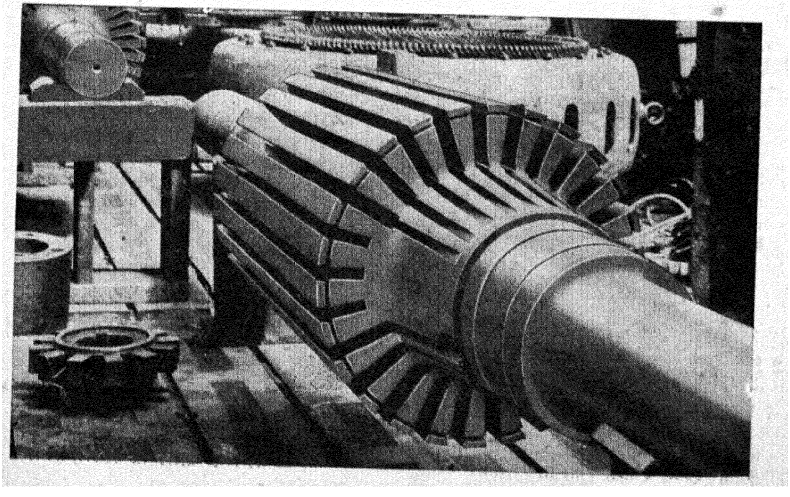


Fig. 309.

(2) Induction Motors.—Consider a stationary iron cylinder wound with two separate coils, as shown in Fig. 310, and coupled to a two-phase alternator. Since the current in one coil will be a maximum when that in the other is zero, the polarity of the ring during the different stages of the passage of the currents will be as indicated. That is, although the cylinder itself is stationary, the poles of the ring rotate and we have what is termed a *rotating field*. The idea may be extended to three-phase currents. This is the principle of the induction motor.

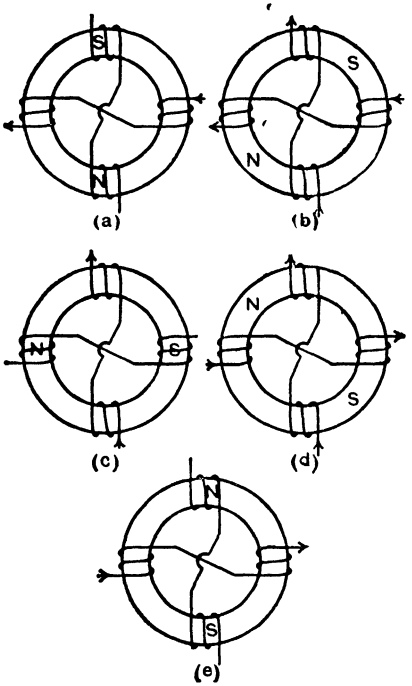


Fig. 310.

The core of the stator is laminated and the coils on it are joined to the polyphase current supply. The core of the rotor is also laminated, the conductors on it being placed in slots and suitably connected. The rotating field of the stator *induces currents* in the conductors of the rotor, with the result that the latter moves in the direction of the rotating field. If the speed of the rotor were equal to that of the field there would be no E.M.F. in it, but the two can never be equal, if only on account of the friction. If F_r be the frequency of the current in the rotor, F_s that of the current in the stator, n_r the revolutions per second of the rotor, n_s the revolutions per second of the field, and P the number of pairs of poles; then—

$$F_s = n_s \times P,$$
$$F_r = (n_s - n_r)P;$$
$$\therefore \frac{F_r}{F_s} = \frac{n_s - n_r}{n_s}.$$

This ratio is known as the **slip.** Since the induced E.M.F. depends upon the rate at which the lines are cut, the E.M.F. and current are proportional to the slip. The torque depends on the strength of the current, hence the torque is proportional to the slip, so that the latter increases when the load is increased. If $n_r = n_s$ the slip is zero, and

there would be neither induced E.M.F. nor current; but the rotor would then tend to stop, would thus get out of step with the field, and the actions would begin again.

There are two types of induction motors depending upon the way in which the rotor is wound: these are:—

(*a*) The squirrel cage type.

(*b*) The slip ring type.

In the former the rotor is slotted and the slots carry copper bars *which are short-circuited at the ends* by being riveted and soldered to copper rings: thus *the bars are not in any way connected to any outside circuit.* In the slip ring type, the rotor windings are brought out to slip rings by means of which resistances can be inserted in the *rotor* circuit for starting and speed regulation purposes.

Large squirrel cage motors take large currents at starting: to reduce this three methods of starting are in use:—

(*a*) Starting resistances inserted in the line.

(*b*) Starting by an auto-transformer (Art. 281).

(*c*) "Star-delta" method for three-phase motors.

In the last case, by means of a special switch, the stator windings are arranged in "star" on starting and then altered to "delta" when the speed has risen sufficiently. When on "star" at start each phase gets only ·58 of the line voltage: when on "delta" each gets the full line voltage.

(3) Laminated Commutator Series Motors.—In Art. 246 it has been pointed out that the direction of rotation of a series or shunt motor is not altered by altering the direction of the current, since it will be altered in *both fields and armature*; hence it is that a series direct current motor may be worked by an alternating current. Since the alternating current passes round the fields, it is clear that they must be laminated as well as the armature. The large inductance of the shunt coil of the direct current shunt motor prohibits its use in this connection.

(4) Repulsion Motors.—In these the stator consists of a laminated electromagnet excited by means of the single-phase alternating current supplied. The rotor is like the armature of a direct current machine; it is fitted with a commutator, but the brushes making contact with the commutator are joined together, short circuiting the armature coils. Thus the arrangement is similar to a transformer, the stator acting as a stationary primary, and the rotor as a secondary capable of rotation. The alternating field in the stator causes current to be induced in the rotor conductors, and the directions of these currents are so adjusted by the short-circuited brushes that the conductors carrying them tend to move out of the field. This result will be better realised by the following considerations:—The induced E.M.F. lags 90° behind the magnetic flux, and the current, owing to the inductance due to the iron core, lags behind the flux considerably. If, therefore, a curve be plotted by taking the product of the

current and the magnetism in the armature, such a curve will graphically represent the torque, and it will be found that the positive loops are less than the negative, thus indicating that the repulsion between the field and rotor exceeds the attraction throughout each cycle.

(5) SPLIT-PHASE MOTORS.—We have seen that the simple single-phase motor is not self-starting, but must be speeded up by some external means. The *split-phase motor* dispenses with this. The stator of the motor has two windings, known as the "starting" and "running" windings respectively. The "starting" winding consists, say, of a large number of turns joined directly across the mains, whilst the "running" winding consists of a few turns in series with a non-inductive resistance, the whole being also across the mains, and therefore in parallel with the starting winding. The different inductances of the two paths will result in two currents differing in phase from the alternating current supplied, and will result in good starting qualities. When synchronous speed is reached, the "starting" winding and the non-inductive resistance are cut out of the circuit and the load applied.

277. Rotary Converters.—We dealt, in Chapter XVIII., with the simple rectangular coil rotated between the poles of a magnet. If such a coil be fitted with slip rings at one end and a commutator at the other, we have a **rotary converter;** though, in practice, these machines are always ordinary dynamos, but with slip rings on the end of the armature, remote from the commutator, connected to points in the armature winding at equal intervals apart. Such a machine may do any of the following:

(*a*) **Supplied with D.C. it will deliver A.C.**
(*b*) **Supplied with A.C. it will deliver D.C.**
(*c*) Supplied with A.C. it will run as a synchronous motor.
(*d*) Supplied with D.C. it will run as a motor.
(*e*) Driven by an engine it will deliver A.C.
(*f*) Driven by an engine it will deliver D.C.
(*g*) Driven by an engine it will deliver both A.C. and D.C.
(*h*) Supplied with A.C. it will do mechanical work and deliver D.C.
(*i*) Supplied with D.C. it will do mechanical work and deliver A.C.

It is important to note that the relation between the voltages is—

$$\text{D.C. volts} : \text{A.C. volts} :: \sqrt{2} : 1 \text{ (if two-phase)},$$

but more commonly it is used as a three-phase machine and we have

$$\text{D.C. volts} : \text{A.C. volts} :: \frac{2\sqrt{2}}{\sqrt{3}} : 1.$$

In fact the voltage and current ratios are as follows, assuming unity power factor and 95 per cent. efficiency:—

Type of Rotary.	A.C. volts / D.C. volts	Current per slip ring / Direct Current
3 phase	·612	1
4 phase	·707	·75
6 phase	·707	·5

A rotary converter intended to be run according to scheme (*g*) above is termed a **double-current generator.**

278. Transformers.—The transformer is another device in which the inductive action of one circuit upon another is employed in practice. In this, too, there are two circuits wound round an iron core—a primary and a secondary—and a typical arrangement is shown in Fig. 311. The transformer is only intended to work, however, with alternating currents, and thus the current in the primary is fluctuating periodically, and so no mechanical interruption is required. The fluctuating current in the primary produces fluctuating flux in the iron core, which, penetrating the secondary, causes E.M F. in the secondary winding proportional to the number of turns contained upon it. Practically we have as follows:

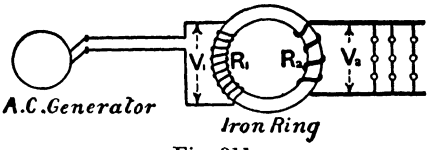


Fig. 311.

$$\frac{\textit{Pressure in secondary}}{\textbf{Pressure in primary}} = \frac{\textit{Number of turns in secondary}}{\textbf{Number of turns in primary}}.$$

But since we cannot gain energy by this transformation, we have, too—

Current in secondary × Pressure in secondary
= Current in primary × Pressure in primary,

or $\mathbf{E_2 I_2 = E_1 I_1}$.

This neglects losses due to heating and magnetic frictions; from this it follows that (approximately):—

$$\frac{\text{Current in secondary}}{\text{Current in primary}} = \frac{E_1}{E_2} = \frac{\text{No. of turns in primary}}{\text{No. of turns in secondary}}.$$

Transformers are used commercially in two ways:

(1) *Step-up Transformers.*—In these A.C. is generated at a low voltage and transformed up so as to be transmitted at a high pressure.

(2) *Step-down Transformers.*—In these A.C. is transformed down from a small current at a high pressure to a large current at a low pressure, ready for use in the lamps, etc.

In (1) the primary has few turns and the secondary many, while in (2) the primary has many turns and the secondary few, the second exactly undoing the effect of the first, neglecting losses.

Referring to Fig. 311, let E_1 be the E.M.F. operating in the primary coil, E_2 that in the secondary coil produced by induction; let V_1 and V_2 be the primary and secondary terminal voltages, R_1 and R_2 the resistances of the coils, N_1 and N_2 the number of turns on each, and I_1 and I_2 the currents in each.

Then, calling $$\frac{N_1}{N_2} = K,$$

we have— $$\frac{E_1}{E_2} = \frac{N_1}{N_2} = K;$$

also— $$E_1 I_1 = E_2 I_2;$$

$$\therefore \frac{I_1}{I_2} = \frac{N_2}{N_1} = \frac{1}{K}.$$

From the figure we see that—

$$E_1 = V_1 - I_1 R_1$$

and— $$E_2 = V_2 + I_2 R_2;$$

$$\therefore \frac{E_1}{E_2} = K = \frac{V_1 - I_1 R_1}{V_2 + I_2 R_2}.$$

$$\therefore K V_2 + K I_2 R_2 = V_1 - \frac{R_1 I_2}{K},$$

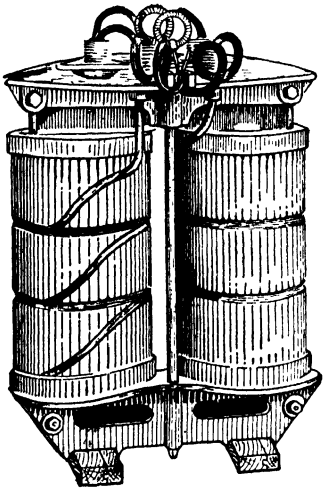
and— $$\therefore \mathbf{V_2 = \frac{V_1}{K} - I_2\left(\frac{R_1}{K^2} + R_2\right)}.$$

This expression gives the secondary terminal voltage in terms of the primary voltage and other constants.

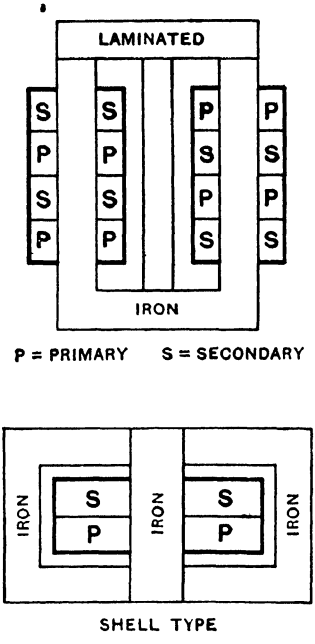
Obviously, since R_1, R_2, and K are constants, the value of V_2 is not a definite fraction of V_1, *but is lessened the more* I_2 *is increased*; that is, the more current is taken from the secondary.

Hence, if we desire the secondary voltage to remain constant, we must arrange matters so that the primary voltage rises somewhat as the secondary load is increased.

279. Transformer Construction.—Types of modern transformers of the *closed magnetic circuit* type are shown in Fig. 312. In older patterns the iron core, like that of an induction coil, did not pass round the outside of the windings or otherwise form a closed iron circuit for the flux; such were called *open magnetic circuit* transformers.



CORE TYPE



SHELL TYPE

Fig. 312.

The primary and secondary coils are frequently intermingled, carefully insulated from each other, in order to avoid magnetic leakage, and it is desirable that a transformer should be so constructed that the iron core may be removed and the windings slipped out without any unwinding of the coils being necessary. This merely means that the iron is in pieces, making good magnetic contact, but that the pieces may be easily removed. A humming noise may be heard frequently from transformers when at work, and they tend to become hot. Hence ventilation must be amply provided for, and many are furnished with an air blast to keep them cool. Cores are of course laminated.

280. Losses in Transformers.—The foregoing is only an elementary treatment of the theory of the transformer, and more rigid investigation is beyond the scope of this book. But it may be noted that some of the losses which have to be considered are as follows:

(1) I^2R *losses* in primary and secondary windings, which are controlled by the size and length of the windings used.

(2) *Losses due to eddy currents* set up in the iron core. This source of loss is combated as far as possible by the employment of thin iron plates or wires; that is, by the adoption of the laminated construction.

(3) *Hysteresis losses* in the iron core. To keep these down special care must be taken in selecting the iron to be used for the purpose.

(4) *Losses due to magnetic leakage*; that is, due to the fact that all the magnetic flux produced by the primary circuit does not pass through the secondary. These are kept low by suitably arranging the positions of the coils.

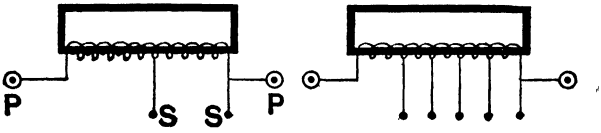


Fig. 313

281. Auto-Transformers.—As will be gathered from Fig. 313, the "primary" winding is mounted on a laminated iron core, and instead of a separate secondary coil, a portion of the turns is tapped off to form the "secondary" circuit. In this way a secondary P.D. lower than the primary supply P.D. is obtained, practically in the ratio of the number of turns tapped off to the total number in the winding.

EXERCISES XX.

1. State the laws concerning the action of one current upon another in its neighbourhood. Illustrate your answer by describing the behaviour of two parallel coils A and B placed side by side, when currents are started and stopped through A, and when A while carrying current is moved towards or away from B.

2. Explain with a sketch the action of an induction coil. What is the function of the condenser connected to the primary circuit?

3. Describe the general construction of a transformer with a closed magnetic circuit. Explain the general principles of its action, and quote the approximate relation between the currents and the E.M.F.'s in the primary and secondary circuits.

4. A bundle of iron wires instead of an iron rod is used in an induction coil. Explain the reason for this. An alternating current is sent for one minute through a solid iron rod and a bundle of insulated iron wires of the same total cross-section. Which will get the hotter, and why?

5. A lightning conductor is usually made in the form of a ribbon. What advantage does this possess over a solid rod of the same cross-section?

6. A choker, having an inductance of 0·05 henry and a resistance of 0·5 ohm, is put in series with a non-inductive resistance of 10 ohms, and an alternating sine E.M.F. of 200 volts, of frequency 50, is applied to the terminals. Find the current in the circuit and the P.D. on the choker and resistance respectively. (O).

7. Enumerate the various losses which occur in an A.C. transformer, and describe the methods employed for minimising the same. (O.)

8. An alternating E.M.F. of 200 (virtual) volts and frequency 50 is applied to a circuit of resistance 10 ohms and inductance ·25 henry. Find (1) the current, (2) the angle of lag, (3) the impedance.

9. A circuit has an inductance of ·02 henry and a capacity of 10 microfarads. Find the frequency so that the impedance may be equal to the ohmic resistance.

10. A transformer gives out 50 amperes at a pressure of 60 volts at the secondary terminals. The windings are 350 primary and 10 secondary. The resistances are 14 ohms primary and 0·012 ohm secondary. Find the volts to be applied to the primary terminals and the coefficient of transformation.

11. A condenser of 4 m.f. capacity is in series with a resistance of 600 ohms and the applied alternating pressure is 1,000 volts. Find (1) the current, (2) the P.D. across the condenser, (3) the P.D. across the resistance.

CHAPTER XXI.

COMMERCIAL MEASURING INSTRUMENTS AND TESTING.

282. Functions of Various Commercial Measuring Instruments.—(*a*) AMMETERS indicate the current strength in amperes in any electrical circuit at any particular instant. The principles involved in their action are: (1) the heating effect of the current (hot-wire instruments); (2) the electro-magnetic effect of the current on a magnet or *vice versa* (moving iron and moving coil instruments); (3) the electro-dynamic attraction and repulsion between two portions of the current (dynamometer instruments).

(*b*) VOLTMETERS indicate, in volts, the P.D. between two points at any particular instant. In addition to the above principles of action, some utilise the electrostatic attractions and repulsions of charged conductors (electrostatic voltmeters, Chap. V.).

(*c*) RECORDING AMMETERS AND VOLTMETERS register on moving sheets of paper the variations, from time to time, in current strength and P.D. respectively.

(*d*) WATTMETERS indicate at any particular instant the rate, in watts, at which energy is being utilised in any part of a circuit (*i.e.* they indicate the "power").

(*e*) RECORDING WATTMETERS register the variations from time to time in the rate at which energy is being utilised in any part of a circuit.

(*f*) ELECTRICITY METERS are of two types, viz.: (1) those measuring the quantity of electricity supplied in a given period, and known as "**ampere-hour**," "**coulomb**," or "**quantity**" **meters**; (2) those measuring the total electrical energy supplied in a given period and known as "**watt-hour**" or "**energy**" **meters**; the former really give the value of the product IT, I being the current in amperes and T the time in hours during which the current passes; the latter measure the product EIT, E being the pressure in volts; but watt-hour meters are generally graduated to read in the larger B.O.T. units (1,000 watt-hours) direct. Again, *on constant pressure circuits* the ampere-hours (IT) are proportional to the watt-hours (EIT) and B.O.T. units; hence the dials of ampere-hour meters are usually graduated to read

directly *at constant specified pressures* the number of B.O.T. units consumed.

According to the principles involved in their action, electricity meters may be further classified into: (1) electrolytic meters, (2) motor meters, (3) clock meters, and (4) thermal meters (maximum demand indicators).

(*g*) MAXIMUM DEMAND INDICATORS are employed in conjunction with ordinary electricity meters, their function being to indicate the greatest current taken by a customer in a given period, and thereby to regulate the price to be levied per B.O.T. unit when the tariff is variable, depending on the maximum demand.

In addition to the above the student will also encounter *power factor* metres, *frequency* metres, and *synchroscopes* in A.C. work.

283. Essential Differences between Ammeters and Voltmeters.—Frequently the same design is utilised in the construction of ammeters and voltmeters, the essential difference being, however, that the former are *low resistance*, and the latter *high resistance*, instruments. Since an ammeter has to measure the current, it must be placed *in* the circuit through which the current passes, and must, of necessity, be of low resistance, otherwise: (1) it would reduce the strength of the current, and result in a decided drop of volts in itself ($E = IR$); (2) it would waste considerable power ($w = I^2R$). Thus, considering the power consumption, if the current be 100 amperes and the resistance of the instrument 1 ohm, the power wasted in it is ($100^2 \times 1$), *i.e.* 10,000 watts; but if the resistance be ·001 ohm the waste of power is only ($100^2 \times \cdot001$), *i.e.* 10 watts.

A voltmeter is connected *in parallel* with that portion of the circuit the P.D. for which is required, and a little consideration will show that its resistance must be high. Consider, for example, a coil of 3 ohms resistance in a circuit through which a *constant current* of 10 amperes is maintained; the P.D. at its ends will be 30 volts. Imagine now that an instrument of only 1 ohm resistance is joined, as a shunt, to the coil in question; the combined resistance of the two in parallel is $\frac{3 \times 1}{3 + 1}$, *i.e.* $\frac{3}{4}$ ohm, and the P.D. at the ends of the coil is reduced from 30 volts to ($10 \times \frac{3}{4}$), *i.e.* $7\frac{1}{2}$ volts, whereas, had the resistance of the instrument been high—say 3,000 ohms—the P.D., as the reader will readily perceive, would have been practically unaltered by its introduction. Again, consider the above instruments joined across mains kept at a *constant P.D.* of 100 volts. The power wasted in the first instrument is $\frac{100^2}{1}\left[\frac{E^2}{R}\right]$ or 10,000 watts, and in the second $\frac{100^2}{3000}$ or $3\frac{1}{3}$ watts; thus, to render the power consumption as small as possible, the resistance of the voltmeter must clearly have a

high value. The actual resistance, in practice, depends on the type of instrument and the range; thus a cell tester for an accumulator may have a resistance of the order 6 to 15 ohms, whilst some voltmeters for 100-volt circuits reach 10,000 ohms and more.

To increase the range and enable large currents to be measured, ammeters are often provided with *specific shunts* which permit a definite fraction of the total current to pass through the instruments. Thus, if an ammeter be so constructed that its maximum permissible current is $\frac{1}{5}$ ampere, a current of 2 amperes may be measured by using with the instrument a shunt having $\frac{1}{9}$ of the ammeter resistance, for in this case only $\frac{1}{10}$ of the current passes through the instrument. Similarly, with a $\frac{1}{99}$ shunt 20 amperes may be measured, and a $\frac{1}{999}$ shunt will permit of the instrument being employed on a 200-ampere circuit. The scales of ammeters with specific shunts are graduated to read the *total* current direct.

To increase the range and enable large P.D.'s to be measured, certain voltmeters are provided with resistance coils *in series* with them. Thus, if the series resistance be equal to that of the voltmeter, and the two be joined to two points in a circuit, one-half of the volts will be spent on the former; hence the P.D. for the voltmeter must be doubled to give the total P.D. between the two points. With a series resistance *double* that of the voltmeter, the P.D. for the former will be *twice* that for the latter, which must, therefore, be multiplied by *three* to give the P.D. between the two points. Similarly, if the series resistance be n times that of the voltmeter, the P.D. for the latter must be multiplied by $(n + 1)$ to give the total P.D., or, if V_1 be the P.D. for the voltmeter, R_1 its resistance, R_2 the value of the series resistance, and V_2 the total P.D. to be measured,

$$\frac{V_2}{V_1} = \frac{R_2 + R_1}{R_1}; \qquad \therefore \mathbf{V_2 = \frac{R_2 + R_1}{R_1} V_1}.$$

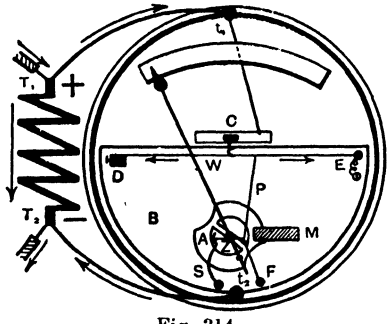


Fig. 314.

Here, again, voltmeters with specific series resistances are graduated to read the *total* pressure direct.

284 Hot-wire Ammeters and Voltmeters. — Fig. 314 gives a diagrammatic view of the hot-wire ammeter manufactured by Messrs. Johnson and Phillips. The brass plate B carries, on the left, a brass clamp D to which one end of the measuring wire W of platinum silver is attached. The other end of the wire is joined to a pillar E fixed on a vulcanite strip but metallically connected to the plate B as indicated. Near

the middle of W a second wire P of phosphor bronze is attached, and this latter is kept taut by having its other end fixed to the insulated pillar F. Not far from the centre of P a cocoon silk fibre Z is attached; this passes round a metal pulley fixed on a pivoted steel spindle mounted in jewels, and is finally attached to a flat steel spring S. The whole is thus subjected to tension, and any slackening of W is immediately taken up by S, resulting in a rotation of the pulley, the spindle of which carries the aluminium pointer.

Mounted on the spindle is an aluminium disc A, which, when the pointer is deflected, moves between the poles of the magnet M of Wolfram steel, thus rendering the instrument dead beat. C is an insulated copper strip in conducting communication with the middle of W by means of a fine spring; this strip is connected to one terminal t_1, the other terminal t_2 being joined to the brass plate B. The terminals are also connected by a shunt T_1T_2 of constantin, which, when the range does not exceed 100 amperes, is placed in the back of the instrument, but which for higher ranges is separate, and may be had to carry several thousand amperes.

The action is as follows: the current to be measured divides at T_1, part passing through the shunt T_1T_2, the remainder flowing from t_1 to C, thence to W, where it again divides, part passing to D and part to E, the two again uniting at B and flowing by way of t_2 to T_2. As a result, the wire is heated and expands, and the sag is taken up by S, thereby deflecting the pointer over the scale. With suitable shunts, any desired fraction of the current may be made to pass through the instrument, which is calibrated by means of a standard instrument or potentiometer to read the total current direct. The metal plate near the wire protects the latter from air currents and ensures uniform heating. The vulcanite to which E is fixed is clamped to B near the centre only; as vulcanite *contracts* when heated, its dimensions can be so arranged as to compensate for the unequal expansion of B and W with equal increments of temperature.

The voltmeters are similar to the ammeters, save that the wire W is thinner, and a resistance of constantin is put *in series* with the instrument in place of the shunt T_1T_2. For ranges up to 400 volts the series resistance is (in some) placed at the back of the instrument; for higher voltages it is contained in a separate case. For 10 volts and upwards, each voltmeter is provided with a fuse.

The advantages of these instruments may be claimed to be: (1) Errors due to the heating effects of the current are absent, since this is the property utilised. (2) They are of service for either direct or alternating currents. (3) The absence of solenoids and iron eliminates the errors of self-induction (Chapter XVI.). (4) They are unaffected by external magnetic fields and by neighbouring conductors, which suggests their suitability for switchboard use.

Excessive sluggishness and drop of volts in the ammeters are lessened by the device of dividing the wire into sections in parallel, instead of allowing the whole

current to flow from one end to the other. In this connection it may be mentioned that the working wire of these ammeters is frequently further sub-divided into equal parallel parts by thin silver foil strips A B C and D E F as shown in Fig. 315, where S is the shunt of constantin. With this device the fall in volts does not exceed ·2 or ·3 even with large currents.

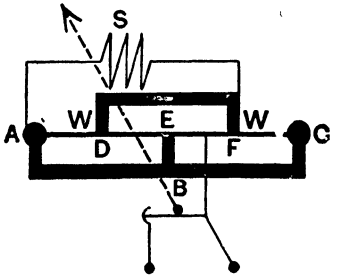


Fig. 315.

The following are the chief defects to which hot-wire instruments are liable:— (1) The fineness of the wire and its high temperature may result in its being fused should the current even slightly exceed the maximum intended. (2) The watts absorbed in some makes are somewhat excessive. (3) There is a certain amount of lag in their readings since the wire takes time to attain its final temperature, and the zero is liable to change. (4) Even in cases where the base or other support which carries the wire is so constructed that its total expansion, due to a given rise of temperature, is equal to that of the wire itself, thus compensating for outside variations of temperature, the wire, being very thin, responds to these external variations first, with the result that the indications gradually change as the temperature of the support creeps up.

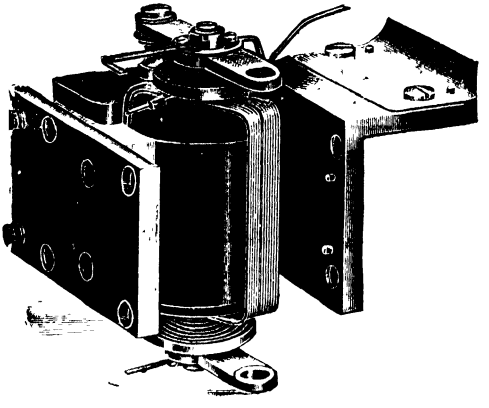


Fig 316.

285. The Weston Moving Coil Ammeters a

The principle of these instruments is identical with that of the D'Arsonval galvanometer. Between the wrought-iron pole-pieces (Fig. 316) of a permanent steel horse-shoe magnet is fixed a cylinder of soft iron, the object of which is to concentrate the lines of force and ensure an intense field in the narrow air gap between it

Embracing the cylinder is the moving coil of copper wire, wound on a frame of aluminium which is pivoted in jewelled bearings, and, of course, moves with the coil. One extremity of the upper spring is attached to one end of the coil, the other end of the spring being fixed to a non-moving part of the instrument. The other end of the coil is similarly attached to a second spring seen in the lower part of the figure. These springs are made of a non-magnetic alloy, and are coiled in opposite directions to cancel the effects of expansion and contraction following temperature variations, the effect on one being neutralised by the opposite effect on the other; they lead the current to and from the coil, and furnish the controlling couple which balances the deflecting couple when a current passes. The coil carries an aluminium pointer, and the whole is enclosed in an iron case to shield the parts from external magnetic fields.

In its normal position the coil lies at an angle of 45° to the line joining the poles, and when a current passes, it tends to set at right angles to the magnetic field so that the lines due to the current may coincide with those due to the magnet. The actual deflection depends on the current strength and on the field; and since the latter is uniform for the region in which the coil swings, the pointer moves over equal distances for equal current increments. The direction of deflection depends on the direction of the current and follows the "left-hand rule"; hence these instruments are not available for alternating currents. Shunts are employed with Weston ammeters. The eddy currents developed in the aluminium frames oppose the motion and render the instruments dead beat.

The voltmeters are similar to the ammeters except that the coils are of finer wire, and in place of the shunts high-resistance coils are inserted in series with the moving coils.

The advantages of moving coil instruments may be stated to be as follows:—(1) Owing to the intense magnetic fields, the number of ampere turns required for the coils is small; hence the moving parts are light, the instruments sensitive, and the watts absorbed small. (2) They are remarkably dead beat. (3) They are free from errors due to hysteresis. (4) In the case of voltmeters the main resistance consists of the series coils, which are made of a material varying very little in resistance with temperature; hence temperature errors are practically negligible.

Changes (with time) in the springs and the magnet, bending of the pointer caused by passing a current in the wrong direction, and bringing the pointer violently against the sides, the fact that external fields exert an influence and that ammeters are to some extent affected by temperature changes, are the main points to be mentioned against moving coil instruments as a class. On the whole, however, they are admirable instruments either for switchboard or laboratory purposes.

The accuracy depends on the magnets being of constant strength: to ensure this they are of tungsten or molybdenum hard steel and are specially "aged."

286. Moving Iron Ammeters and Voltmeters.—These instruments depend for their action on the following facts: (1) the magnetic field inside a solenoid is strongest and most uniform at the centre, but near the ends it is strongest near the wires owing to the leakage of lines through the sides, and as a result of this: (2) a *long* piece of soft iron suspended inside tends to move towards the wires, and (3) a short piece of soft iron tends to move towards the centre. The magnitude of the motion depends upon the strength of the field, and, therefore, upon the current strength; hence the motion may be utilised as a measure of current strength and pressure.

The essential parts of the **Schuckert Gravity Control Ammeters and Voltmeters** are shown in Fig. 317. The solenoids through which the currents pass are placed with their axes horizontal. O I I is a curved plate of thin soft iron extending the full length of the solenoid; it is fixed on a horizontal axle somewhat to one side of the axis of the solenoid. C is a balance arm of copper and P a light aluminium pointer. On the passage of a current, the curved iron is raised against the gravitational pull of the earth so that the curved portion is nearer to the wires, with the result that the system rotates about the axle, thereby moving the pointer over the scale.

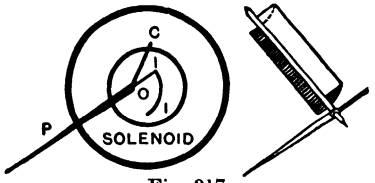


Fig. 317.

The essential parts of **Messrs. Siemens Bros. & Co.'s Instruments** are shown in Fig. 318. Here I is a thin soft-iron plate fixed on a horizontal axle A, which also carries the pointer P and a projection p to the end of which the curved wire W carrying the piston S is attached. The latter moves in the tube T T, thereby forming an "air damper" which renders the instrument dead beat. On the passage of a current through the rectangular coil C C, the iron I is drawn down into the coil, thereby moving the pointer over the scale. The working parts are protected from external magnetic influence by a curved soft-iron shield indicated by X Y Z in the figure.

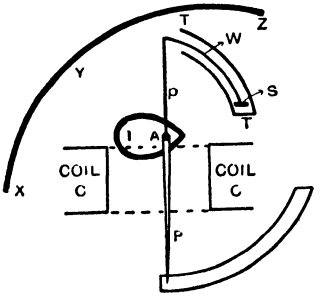


Fig. 318.

The essential parts of **Nalder and Thompson's Repulsion Instruments** are indicated in Fig. 319. The solenoid A has its axis horizontal. B is a fixed rod of iron running the whole length of the solenoid, whilst C is another rod of iron attached to the pointer. When no current is passing C and B are together. When current passes they are magnetised in the same direction and repel each other. Thus

C moves as indicated and the pointer moves over the scale. The instruments are spring or gravity controlled and "damping" is provided for as in Fig. 318 (not shown in Fig. 319).

Many other types of these instruments are in daily use. Some are liable to be much affected by external magnetic influence, and the lower parts of the scales are often of little use; but the chief disadvantage, as a class, is that of hysteresis, which results in the indications, when a current is increasing in value, being less than those for the same current when it is decreasing, and again, with alternating currents, hysteresis and eddy currents render the readings less than the true value if the instrument has been calibrated with direct currents.

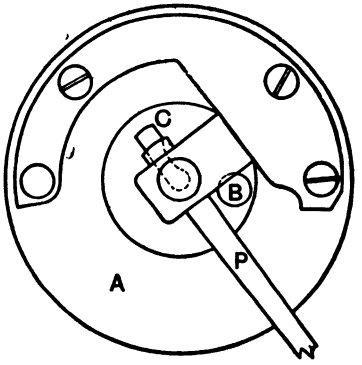


Fig. 319.

287. Dynamometer-Type Ammeters and Voltmeters.—That excellent current measurer, **Siemens' Electrodynamometer,** has already been described in Chapter XII. It can be used for direct or alternating currents, and, since iron is altogether absent, it is independent of "wave form" and "periodicity."

The essential features in the construction of the **Weston Dynamometer-Type Voltmeter** will be gathered from Figs. 320 and 321. In Fig. 320, F C are two fixed fine wire coils placed one on each side of a rectangular block of ebonite V, the latter being provided with a central circular opening in which the moving coil M C works. The moving coil is pivoted between jewelled centres, the upper pivot carrying a light aluminium pointer P, and, as in the Weston instruments of Art. 285, the controlling couple is furnished by two springs, one of which is seen in Fig. 321.

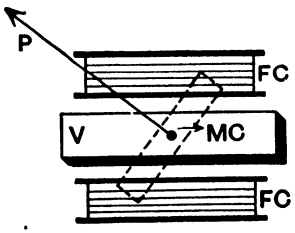


Fig. 320.

The latest form of the instrument is shown standing inverted on its face in Fig. 321. In this the ebonite plate is dispensed with, and the fixed coils, one of which is removed in the figure, are wound on brass forms and bolted to the angle plates as shown. The moving coil carries a light aluminium vane (not shown) fitted with two pistons, the vane and pistons moving with very small and uniform clearance in the air damper box when the coil rotates, thereby making the instrument dead beat. The air damper box is shown in the upper part of

the figure; its bottom lid is removed, and the two pistons visible. As the instrument is standing on its face in the figure, the pointer cannot be seen; it consists of a light aluminium tube carefully balanced by the cross arms seen in the lower part of the figure, and is really a continuation of the foremost cross arm which is shown at right angles to the other. These are excellent D.C. or A.C. instruments.

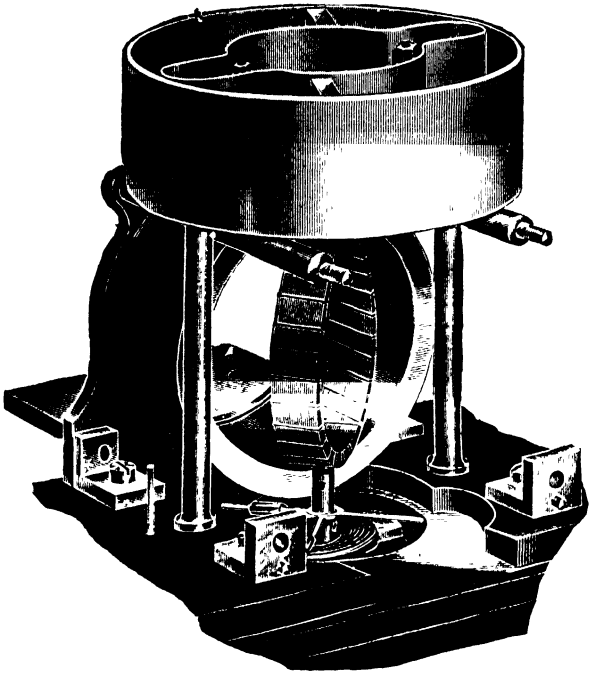


Fig. 321.

288. Siemens' Wattmeter and the Weston Wattmeter. The general principle of **Siemens' Wattmeter** is identical with that of the Siemens' Electrodynamometer (Art. 140). The moving coil C (Fig. 322) is of thick wire and low resistance, and as it is inserted in the main circuit it may be termed the "ammeter" coil; the high-resistance fixed coil V consists of many turns of fine wire, and as it is joined as a shunt to that part of the circuit for which the power consumption is required (a lamp L in figure), it may be termed the "voltmeter" coil.

In the normal position the planes of these coils are at right angles to each other, and the pointers P, p stand at zero. On closing the

circuit, the main current I passes through the moving coil, and a small current proportional to the voltage E at the lamp terminals passes through V. The turning moment is proportional to the product of these currents, *i.e.* proportional to E I or the watts expended in L. When the movable coil is brought back to its normal position by turning the torsion head and its pointer through an angle D°, say, the turning moment is balanced by the torsional moment which is proportional to D°. Hence—

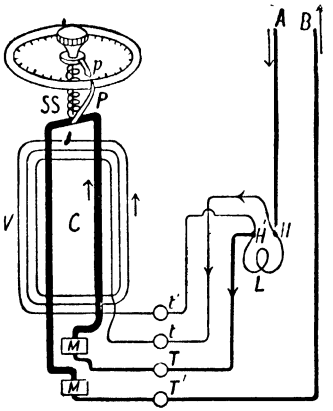


Fig. 322

$$EI \propto D°;$$

i.e. Watts expended in $L = KD$

where K is the constant of the instrument, which must be determined experimentally.

In using the instrument for direct currents, the plane of the moving coil must be perpendicular to the meridian so as to eliminate the action of the earth on it, and the connecting wires must be close together

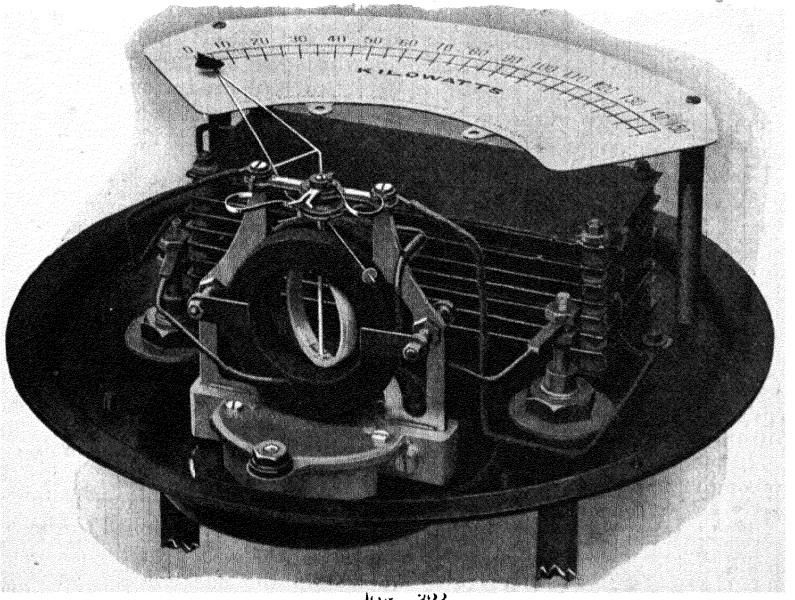


Fig. 323.

otherwise the moving coil may be affected by the currents in them. In the case of wattmeters intended for alternating currents, the framework and supports should be of non-conducting material, since with metal frameworks the eddy currents developed therein would considerably affect the moving coil; and further, the fixed coil should be of two parts capable of adjustment relative to each other so as to vary the field in the vicinity of the moving coil. A series resistance is employed in conjunction with the "voltmeter" coil; this is "double wound" like the coils of a P.O. box so as to eliminate self-induction.

The **Weston Wattmeter** is shown in Fig. 323. There are two fixed coils and one movable coil. The moving system carries two vanes which move in an air chamber which also forms the base for the coil supports. This chamber is seen just below the coils in Fig. 323. The construction is very much the same as the voltmeter of Fig. 321, and the student should have no difficulty in understanding the construction and action.

289. The Bastian Electrolytic Meter.—This meter consists of a glass tube about 2 inches in diameter and 1 foot in length, fitted with a bulb at its lower extremity, the whole containing a dilute solution of caustic soda and a non-freezing mixture; this electrolyte (with a little paraffin oil on the top to prevent evaporation) stands at zero on the scale when the meter is first installed. At the bottom of the containing vessel is a vulcanite box, partly open at the top and bottom, in which are placed two nickel electrodes. The top of the containing vessel carries a porcelain cover through which pass two vulcanite tubes, the latter serving to convey and protect two lead conductors leading from the terminals of the meter to the nickel electrodes in the box. The whole is contained in an outer sheet-iron case provided with a glass window in front.

When the meter is included in an electric circuit, the liquid is decomposed into its constituents, the amount thus electrolysed in a given time being, by Faraday's Law (Art. 154), proportional to the total quantity of electricity which has passed in that time; thus the fall in level of the liquid may be used as a measure of the quantity of electricity in ampere hours which produces that fall. On *constant pressure* circuits, however, the ampere hours (I T) are proportional to the watt hours (E I T) and B.O.T. units; hence the meter is graduated to read B.O.T. units direct at a constant pressure which is specified on it. The liberated gases escape into the atmosphere.

In premises supplied on the two-wire system, the meter is simply put into one of the leads. Premises on the three-wire system require two meters (generally arranged in one containing case); one is in series with the positive lead, and registers the energy supplied to the positive side of the middle wire; the other is in series with the negative lead and registers the energy supplied to the negative side; the total energy supplied is obtained by adding the two meter indications.

The Bastian meter works with small currents, is accurate, inexpensive, simple in construction, and requires little attention. The scale is straight so that the readings may be readily and accurately taken, and when refilling is required it is only necessary to add water up to the zero of the scale. The chief disadvantage is that the drop of potential caused by the instrument itself is greater than in certain other types of meters; but this only occurs when the meter is actually in use, whilst in certain other types (see later) a small current is constantly flowing, and the "loss" in this way perhaps more than balances. Electrolytic meters are used for continuous currents.

290. The Elihu Thomson Motor Meter.—The principle of this meter will be gathered from Fig. 324. The instrument consists of two fixed coils F C of thick wire in series with each other and with one of the leads. Between them is the rotating coil or armature R C consisting of eight coils of fine wire wound on a light non-magnetic frame after the manner of a Siemens' drum. R C is attached to a spindle S which is pivoted in jewelled bearings, and carries at its upper end an eight-part commutator C of silver, the object of the latter being to reverse the current in each coil of the armature at the right instant; the details as to how this is accomplished are given in Chapter XVIII. Two silver or silver-tipped brushes B B press on the commutator and serve to lead the current to and from the armature. In series with R C is a high non-inductive resistance R and a "starting coil" (not shown), the latter being placed within the coil F C on the right. The whole of the circuit, including the armature, non-inductive high-resistance and starting coil, is joined as a shunt between the leads as indicated.

Consider now one of the coils of R C in such a position that its axis is, say, perpendicular to the plane of the paper and, therefore, perpendicular to the axes of the coils F C. When the current passes, this coil tends to place itself with its axis parallel to the axes of the coils F C, in which position it would, of course, tend to stop, were it not for the fact that at the correct instant the current in it is reversed; this applies to each coil, and continuous rotary movement of R C is the result. Resistance to rotation is supplied by means of a Foucault brake (not shown) consisting of a thin disc of copper mounted on the lower part of the spindle S and rotating in a constant magnetic field between the poles of two or more permanent magnets; eddy currents developed in the disc oppose the motion of the latter, thus establishing a drag on the armature, which varies with the speed of the moving parts.

Since the stationary coils F C carry the total current I, and the armature a current proportional to the potential difference E between the leads, the turning effort or driving torque is proportional to the product E I, *i.e.* proportional to the watts. The brake gives rise to a retarding torque proportional to the speed or number of revolutions,

n, per second. Clearly the final speed of the armature is reached when the driving and retarding torques are equal. Hence—

Driving torque = Retarding torque;

$$\therefore \text{E I} \propto n.$$

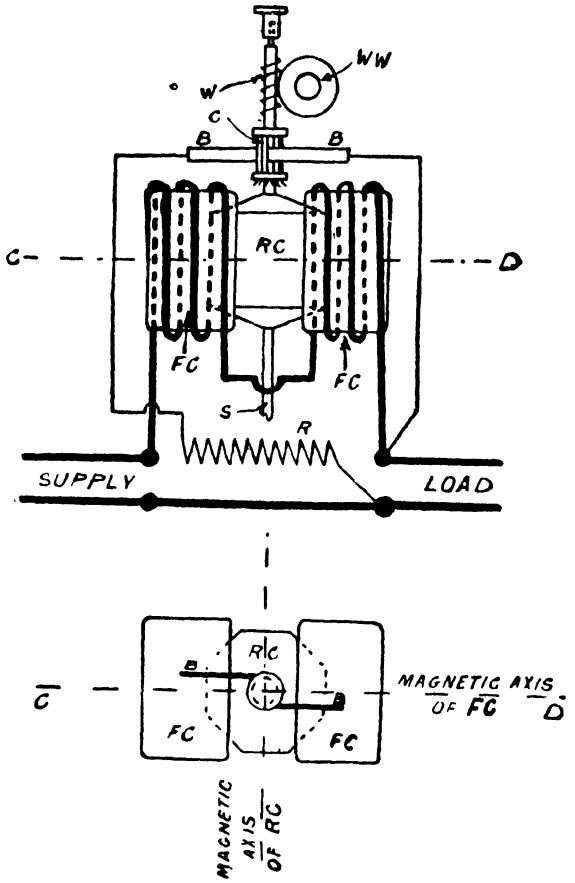


Fig. 324.

If, now, the power consumption be constant, n will be constant: thus, considering a time t seconds, the relation

$$\text{E I } t \propto n t$$

will apply. But E I t measures the energy in joules consumed in t

seconds and $n\,t$ denotes the total number of revolutions in the same time; hence the total number of revolutions made by the armature in a given period may be made to measure, by suitable calibration, the total energy in joules, watt-hours, or B.O.T. units consumed during that period. The meter dials are driven by an endless screw w mounted on the spindle and seen in the upper part of the figure.

It will be evident from the figure that a small current is constantly passing through the armature circuit; this circuit, as previously mentioned, includes a starting coil inside F C on the right, so adjusted that the meter is just on the verge of starting, so that, when the smallest load is switched on, the meter commences to register. Too close an adjustment in this respect may result in the meter "running on the shunt," *i.e.* commencing to register owing to a slight rise in the supply voltage even when the customer has no load.

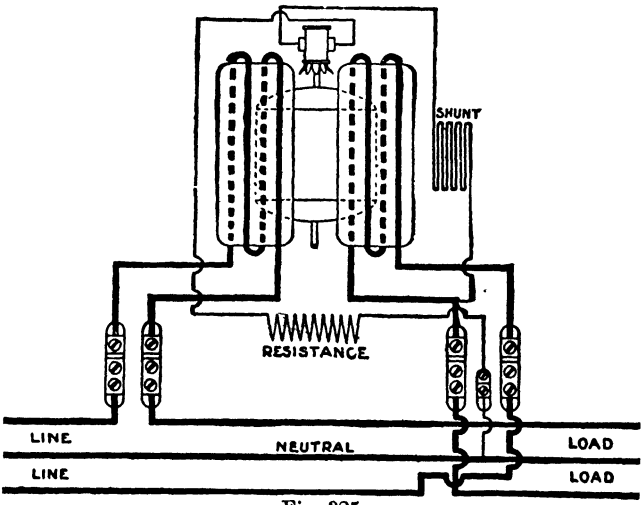


Fig. 325.

Fig. 324 gives the method of connecting the meter on a two-wire circuit. For a three-wire system the instrument is slightly modified as shown in Fig. 325. The fixed coils are disconnected, one being put in series with the positive lead and the other in series with the negative lead, while the armature is joined as a shunt between one of the outers (negative one in the figure) and the middle wire. Considering the left-hand coil, the current in it is that required for the positive side of the system and the current in the armature is proportional to the P.D. on this side (assuming it equal to that on the negative side); hence the driving torque is proportional to the power consumption on the positive side of the neutral wire. Similarly the fixed coil on the right

carries the current used on the negative side, and the driving torque is proportional to the power consumption on this side. An examination of the figure will show that the currents in the fixed coils are in the same direction and tend, therefore, to rotate the armature in the same direction; hence the total driving torque is proportional to the total power consumption on the system.

A little consideration will show that the Thomson meter can be used on direct or alternating current circuits.

291. The Aron Clock Meter.—The earliest form of Aron Meter consisted of two pendulum clocks side by side, one having an ordinary bob and keeping correct time, the other having, as bob, a permanent magnet, below which was placed a solenoid with its axis vertical and through which the main current passed. The two were synchronised so as to keep the same time in the absence of any current in the circuit. Assuming now that the lower end of the magnetic bob is a north pole, and that the current passes through the solenoid in such a direction that its upper end exhibits south polarity, mutual attraction will ensue between the two, the solenoid thus assisting the earth in tending to bring the pendulum to rest, and therefore accelerating its motion; thus the magnetic pendulum will gain upon the other, the rate of gain depending on the strength of the current in the solenoid. This difference of motion was by a suitable contrivance communicated to the counting wheels and dials; the "ampere hours" could thus be registered, or on constant pressure circuits "watt-hours" or "B.O.T. units" could be recorded. The clock required winding up by hand once per month.

In the latest form of this meter the bobs of both pendulums carry coils of thin wire which are in series with each other, the pair being joined as a shunt between the leads. Below these are two fixed solenoids carrying the main current, which is passed through them in such a direction that one pendulum is accelerated, the other retarded, and the difference in motion is registered on the dials. The instrument is mainly employed as an energy meter.

The new instrument is self-winding, and is provided with an automatic reversing gear, the function of which is to reverse the current in the pendulum coils every ten minutes, thereby eliminating errors due to faulty synchronism when there is no load. The modern Aron meter is accurate, registers with small currents, and is available for direct or alternating current circuits.

292. The Wright Maximum Demand Indicator.—This consists of a U-tube, L L (Fig. 326), the extremity of each limb being provided with a bulb, the right-hand limb being further provided with a vertical branch tube R, closed at the bottom. The limbs L L are partly fitted with sulphuric acid, as indicated. Round the left-hand bulb a heating coil H C, consisting of one or two turns of platinoid

CONNECTIONS OF TWO WIRE DEMAND INDICATOR

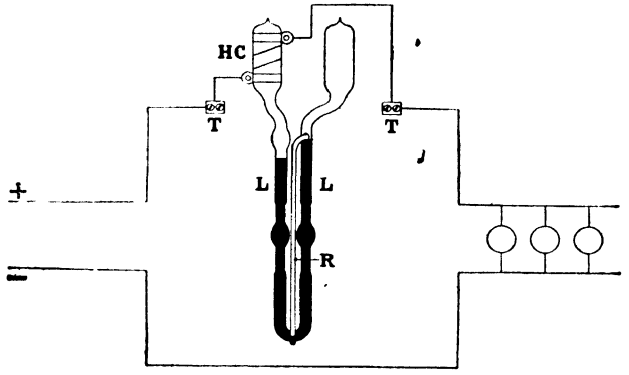


Fig. 326.

DIAGRAM OF CONNECTIONS THREE WIRE DEMAND INDICATOR.

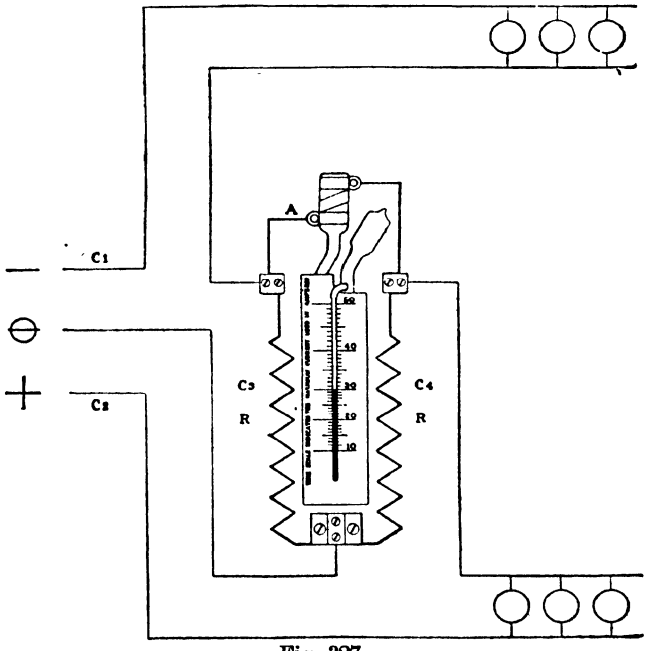


Fig. 327.

strip, is wrapped, and this coil is in series with one of the leads. On the passage of a current, the heat produced in the coil causes the air in the left-hand bulb to expand, thus driving some of the liquid over into R. This overflow liquid remains there, and is not added to until a current greater than the previous one passes; hence the amount of liquid in R may be used as a measure of the greatest current which has passed in a given period. A scale attached to R indicates the maximum current, and also the number of units to be charged for on the higher rate (Chapter XXII.). The indicator is used in conjunction with an ordinary meter, and can be reset by tilting.

Fig. 326 shows the indicator on a two-wire system, while Fig. 327 shows the arrangement for a three-wire circuit. In the latter case it will be observed that the instrument is fitted with three terminals, and two equal resistances R R are added. With the arrangement indicated—

Let M = current in the heating coil
I_2 = current on the positive side
I_1 = current on the negative side
R_1 = resistance of the heating coil
I_4 = current in R on the right.
I_3 = current in R on the left.

Then—P.D. between upper terminals $= M R_1$
P.D. in R on the right $= I_4 R = (I_2 - M)R$
P.D. in R on the left $= I_3 R = (I_1 - M)R$;

$$\therefore (I_2 - M)R + (I_1 - M)R = M R_1$$
$$I_2 R + I_1 R = M R_1 + 2 M R$$
$$R(I_2 + I_1) = M(R_1 + 2 R)$$
$$I_2 + I_1 = \frac{R_1 + 2 R}{R} M$$

i.e. $$I_2 + I_1 = K M$$

where K is a constant. Thus the current in the heating coil, and therefore the indications of the instrument, depends upon the sum of the currents on the two sides of the middle wire.

293. Testing of Commercial Instruments, Lamps, and Machines.—*For full particulars of, and practical hints in connection with, the following tests, the reader should refer to some work on Practical Electric Testing.*

TEST 1.—TO CALIBRATE A DIRECT-CURRENT VOLTMETER BY MEANS OF A POTENTIOMETER.

(1) The potentiometer must first be standardised by the Clark cell, so that each division of the wire denotes a P.D. of ·001 volt.

(2) The standard cell is then removed, and the arrangements of Fig. 328 followed in the case of a voltmeter reading from 0 to 5 volts. The voltmeter V is connected through a rheostat R and switch S to some convenient source of electricity E, whereby readings varying from 0 to 5 volts may be obtained on the instrument. Potential wires are then taken from the positive terminal of the voltmeter to the end A, and from the negative terminal through the galvanometer G to the sliding contact K. Commencing with a small current in V, K is adjusted for no deflection on G, and the P.D. between A and K is calculated from the relation: P.D. in volts = ·001 × length A K in millimetres; this equals the P.D. at the terminals of V, and should therefore agree with the reading of the instrument. The experiment is repeated with various currents through V, and the results recorded thus :—

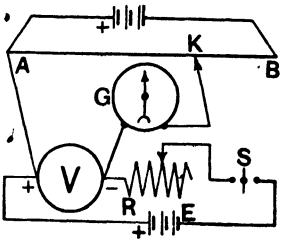


Fig. 328.

Voltmeter reading.	Balance at—	Potentiometer reading.	Correction to Voltmeter.
·5 volt	511 mm.	·511 volt	+ ·011 volt
·75 ,,	742 ,,	·742 ,,	− ·008 ,,
etc.	etc.	etc.	etc.

Finally a "correction curve" should be plotted, "voltmeter readings" being taken as abscissae and "corrections" as ordinates.

In the case of a voltmeter reading above 5 volts, the instrument must be shunted by a coil of very high resistance and the P.D. for a known fractional part of this resistance measured as above, from which the full P.D. at the voltmeter terminals is readily obtained.

TEST 2.—TO CALIBRATE A DIRECT-CURRENT AMMETER BY MEANS OF A POTENTIOMETER.

(1) The potentiometer must be standardised as before.

(2) The standard cell must be removed, and the ammeter put in series with a rheostat, switch, battery of accumulators or other convenient source of electricity, and a standard low resistance, the value of which must be such that the fall of potential down it does not exceed the range of the potentiometer when the maximum current for the ammeter is flowing. Potential wires are now taken from the ends of the standard resistance to the potentiometer, and the fall of potential down it is measured; this divided by the resistance of the standard gives the current in the ammeter circuit, which should of course agree with the ammeter reading. The experiment is repeated with various currents, and results tabulated thus :—

Ammeter reading.	Balance at—	Potentiometer Reading = E.	True Current = $\frac{E}{R}$.	Correction to Ammeter.

Finally a correction curve should be plotted as in Test 1.

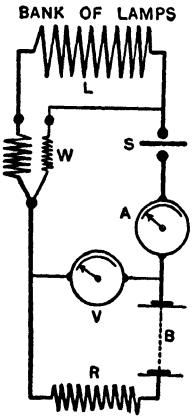


Fig. 329.

TEST 3.—TO CALIBRATE A WATTMETER BY MEANS OF A PREVIOUSLY CALIBRATED AMMETER AND VOLTMETER.

The arrangement of the apparatus is shown in Fig. 329. Here W is the wattmeter under test, A a calibrated ammeter, V a calibrated voltmeter, B a battery of accumulators, S a switch, R a rheostat, and L any convenient resistance, such, for example, as a bank of lamps, in which the power consumption is to be measured. It will be observed that the current coil of W is in series with L, the pressure coil being arranged as a shunt thereto.

The test consists in taking simultaneous readings of A, V, and W with various currents flowing. The true watts are given by the product of the ammeter and voltmeter readings, and these should agree with the indications of W. Correction curves are plotted as in the preceding beats.

TEST 4.—TO TEST A DIRECT-CURRENT ELECTRICITY METER BY MEANS OF A PREVIOUSLY CALIBRATED AMMETER AND VOLTMETER.

The connections are depicted in Fig. 330, M being the meter under test. The resistances R and r are first varied until the correct voltage for which the meter is intended is indicated by V, and say $\frac{1}{5}$ the full current is passing; this current is permitted to flow for a definite time, the indications of A and V being kept constant. The true ampere hours are given by the product of the ammeter reading and the time in hours, the watt hours by the product of the ampere hours and the voltmeter reading, and the B.O.T. units by dividing the watt hours by 1,000; these should agree with the meter indications. The experiment should be repeated with various currents, and finally a correction curve should be plotted as before.

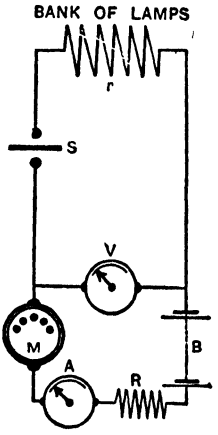


Fig. 330.

TEST 5.—TO DETERMINE THE CANDLE-POWER AND EFFICIENCY OF AN ELECTRIC LAMP.

As already indicated an instrument for comparing the illuminating powers of different sources of light is called a "photometer." A photometer used by the Board of Trade is Bunsen's, which, in its simplest form, consists of a grease spot in the centre of a screen of white paper. When a candle is placed on one side of this screen, the grease spot, being less opaque than the rest of the paper, will appear *darker* than the latter when viewed from the side on which the candle is placed, but *lighter* than the rest when viewed from the side remote from the candle. By placing a candle at one side and a lamp at the other, and adjusting their respective distances, it can be arranged that the grease spot cannot, at either side, be distinguished from the rest of the paper, or at any rate, the two sides can be made to appear alike. When this condition is realised the lights are producing equal intensities of illumination, and the illuminating power of the lamp in terms of that of the candle can be determined from the relation:

$$\frac{\textit{Illuminating power of Lamp}}{\textit{Illuminating power of Candle}} = \frac{(\textit{Distance of Lamp from Screen})^2}{(\textit{Distance of Candle from Screen})^2}.$$

To determine the c.p. and efficiency of a glow-lamp the latter must be put in series with an ammeter A (Fig. 331), switch S_1, adjustable resistance R_1 and a battery or other convenient source of electricity capable of running the lamp at its required voltage, and a high-resistance voltmeter V must be connected across

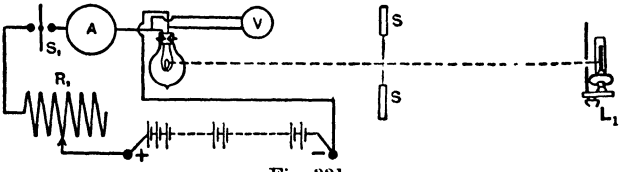


Fig. 331.

the lamp terminals as indicated. The watts absorbed by the lamp at its rated voltage are given by the product of the ammeter and voltmeter readings. The c.p. is found by placing the lamp on one side and a standard light L_1 on the other side of the Bunsen screen S S, adjusting for a photometric balance and applying the relation—

$$\frac{\text{C.p. of lamp}}{\text{C.p. of standard}} = \frac{(\text{Distance of lamp})^2}{(\text{Distance of standard})^2}.$$

Finally the "efficiency" is obtained by dividing the c.p. by the watts absorbed.

It is customary to secure a balance (1) with the plane of the lamp-filament parallel to the screen; (2) with the plane of the filament perpendicular thereto, the mean distance being estimated. Further, as the lights are of different colours, it is advisable to observe the screen through "signal red" and "green" glasses, balance being obtained with each and the mean distance taken.

The method outlined above is applicable, with special precautions, to an efficiency and c.p. test of an arc-lamp. The standard light frequently consists of an over-run glow-lamp of 32 c.p. at normal voltage, in which case a 6-per-cent.

over-run will give a c.p. of about 50, and the colour of the light will be somewhat similar to that of the arc. The arc-lamp is arranged, *not in line* with the standard and screen, but off to one side, a mirror being so arranged as to reflect the beam on to the grease-spot, in which case, at photometric balance, the distance of the arc-lamp is equal to the sum of the distances—(screen to mirror) and (mirror to lamp). In these tests the ratio of the light reflected by the mirror in its given position to the total light falling on it must be known and allowed for, since it is the intensity of the reflected beam which is measured; thus, if only half the incident light were reflected the intensity measured would only be half that of the arc. In a similar way the c.p. of the arc at different angles from the horizontal may be determined by suspending the lamp from the ceiling and suitably arranging the mirror referred to.

According to Gerard the M.S.C.P. of an ordinary arc is got by finding experimentally as above (1) the horizontal c.p.; (2) the maximum c.p., and by adding $\frac{1}{4}$ of the latter value to $\frac{1}{2}$ of the former value. A complete test of the M.S.C.P. may be carried out as explained in Chapter XV.

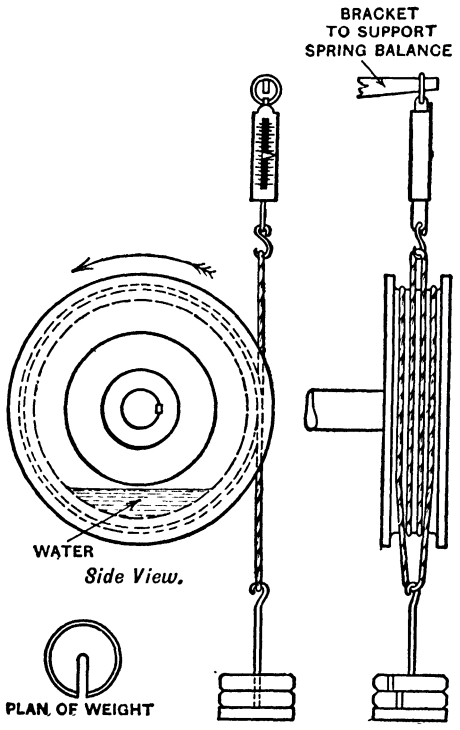


Fig. 332.

TEST 6.—TO DETERMINE THE BRAKE HORSE-POWER AND EFFICIENCY OF A MOTOR.

A double rope passes round the motor pulley as shown in Fig. 332, its upper end being attached to a spring balance which in turn hangs on a bracket, and its lower end carrying weights as indicated: let R feet = radius of pulley + $\frac{1}{2}$ diameter of rope. With the motor rotating as shown, let W_1 = weight (pounds) hanging on the rope, W_2 = reading of spring balance (pounds), and n = revolutions per minute: then:—

$$\text{Torque} = T = (W_1 - W_2)R \text{ pound-feet},$$

$$\therefore \text{B.H.P.} = \frac{2\pi nT}{33{,}000} = \frac{2\pi nR(W_1 - W_2)}{33{,}000} \quad \ldots\ldots\ldots\ldots (1)$$

and all the terms on the right-hand side are known so that the brake horse-power

is determined. By means of an ammeter and voltmeter the power W in watts supplied to the motor is determined from the product of their readings, viz. $W = EI$; in H P this is E I/746. Hence :—

$$\text{Commercial Efficiency} = \frac{\text{Output}}{\text{Input}} = \frac{2\pi n R(W_1 \sim W_2)\cdot 746}{33{,}000\ \text{E.I.}}. \quad \ldots\ldots\ldots (2)$$

A series of readings should be taken up to maximum load, and a curve plotted, showing the relation between B.H.P. and efficiency. In Fig. 332 a water cooled pulley is shown.

EXERCISES XXI.

1. How does a voltmeter differ from an ammeter in construction and use ? What sort of resistance may be given to a voltmeter used with a single accumulator? (P.)

2. Sketch some form of gravity ammeter or voltmeter. (O.)

3. Describe with sketches some form of moving coil voltmeter. What are the advantages of this type? (O.)

4. State the principles on which the Johnson and Phillips ammeter is based. Describe the construction and action of the instrument and mention its specific advantages.

5. What forms of ammeters and voltmeters are likely to be affected by neighbouring current circuits—for instance, those on a switchboard—and what are the forms that are practically unaffected? (O.)

6. Describe the construction and use of a wattmeter. Give sketches. (B.O.P.) What are the important points to be attended to if the wattmeter is to be used on an alternate current circuit? (O.) How would you test the accuracy of a wattmeter at a given load? (B.)

7. What is the function of an electricity supply meter? Mention the difference between a watt-hour meter and an ampere-hour meter.

8. Describe the construction and action of (1) an electrolytic meter; (2) a motor meter, and point out their respective merits and drawbacks.

9. A shop is supplied with electrical energy on the three-wire system through an energy meter of the motor type; show how the meter should be connected with the mains. (P.)

10. Describe the construction and action of a maximum demand indicator, and explain the maximum demand system of charging for electrical energy.

11. How would you test the graduations of a voltmeter reading from 0 up to 5 volts? (B.)

12. Describe an efficiency and c.p. test of a glow-lamp.

13. An electromagnetic voltmeter reads from 0 to 5 volts and has a resistance of 500 ohms. How may the same instrument be adapted to read from 0 to 50 volts with the same scale? (B.)

CHAPTER XXII.

GENERATING STATIONS, DISTRIBUTION AND TRANSMISSION OF POWER.

294. Introductory Note.—In the problem of distribution two special points are involved, viz. the pressure of supply and the distance to which the power must be transmitted, and as we shall see presently, these are not independent, if economy is to be studied.

In practice the various *ranges of pressure* are usually classified as follows:

(*a*) If the pressure does not exceed 250 volts the supply is termed a *low pressure supply.*

(*b*) With pressure between 250 and 650 volts the system is termed a *medium pressure supply.*

(*c*) Between 650 and 3,000 volts the supply is termed a *high pressure supply.*

(*d*) Exceeding 3,000 volts the supply is termed an *extra high pressure supply.*

With regard to the question of distance we may briefly note here that if this be great the power lost in the mains, viz. I^2R, may become great. This loss can be reduced by decreasing R, *i.e.* by using thicker mains, but this may increase the initial cost enormously. The loss can also be reduced by decreasing I, but this means working at higher pressure if the same power (E I) is to be transmitted, and high pressure has certain drawbacks as well as points in its favour. At this stage we may merely mention, in a general way, that when the supply station is near the consumer the pressure of supply is usually the same as that at which the current is generated, since the size and consequent cost of short mains are not very material. When, however, the station is at a distance, the problem becomes more involved and needs careful consideration.

295. The Central Station: (1) Feeding Points, Feeders and Distributors.—The equipment of a D.C. central station usually comprises *boilers* for supplying steam to the engines, *steam engines* for driving the generators, *generators* connected to the *switchboard* which carries all the switches, regulators, measuring instruments, etc., and

from the switchboard cables run to the various districts where the electrical power is required: there is, in addition the *accumulator battery*, which can also be joined to the outside circuit *via* the switchboard, and the necessary regulating and measuring apparatus. An A.C. station is pretty much the same, with of course, certain essential modifications, and most handle both D.C. and A.C. (accumulators work only with D.C.). A few visits to one or two modern stations will be of the greatest value to the student.

The conditions fixed by the Board of Trade include regulations that the pressure shall not vary more than 4 per cent. from the normal stated, and, further, it must be practically uniform throughout the system to ensure uniformity in the fittings.

To realise this, separate conductors, **from which no lamps, etc., are supplied,** are connected to various points of the network, their sole function being to keep the points of connection, termed **feeding points,** at a constant pressure. Such additional mains are termed **feeders.** From these feeding points pairs of **distributors** are run along the streets and the **service mains** are taken from these to the main terminals inside a building: from here the **internal wiring** of the building commences. Thus by these feeders the station engineer can ensure that certain selected points of the network remain at the correct pressure, and this being assured the remainder of the system has to look after itself. In the vicinity of the station a pair of distributors may be supplied direct.

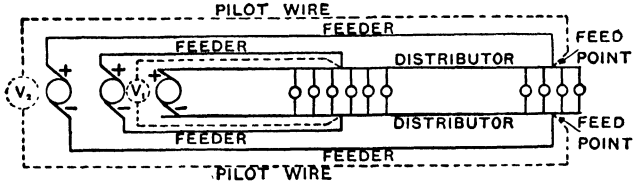


Fig. 333.

In order that the feed-point pressure may be known to the station attendant, special small wires (**pilot wires**) are led back to the station from the feed-points; these wires are connected to voltmeters on the switchboard, and the latter are graduated to read the pressure actually existing *at the distant feed-point.* The attendant has thus merely to see that these voltmeters are kept at the fixed reading. Each pair of feeders may be supplied by its own dynamo, which ensures perfect regulation of each, or several may be run off **bus bars** (see below) from the station, in which case special arrangements have to be made in order to allow the longer feeders to be supplied at a higher pressure than the shorter ones—on account of the greater drop in pressure in them (*e.g.* feeder boosters). Fig. 333 shows diagrammatically a typical system on the lines given above.

296. The Central Station: (2) The Switchboard.—As already indicated, the necessary switches, fuses, automatic cut-outs, ammeters, voltmeters, wattmeters, energy meters for recording the energy supplied to the system, recording ammeters, rheostats for controlling the generator fields, etc., are grouped together in the generating station, usually upon slabs of marble or slate fixed in an elevated position from which all the machines can be readily seen. Such an arrangement is termed the **switchboard,** and the whole supply is controlled from this board.

The generators are coupled (through the necessary switches and other appliances on the board) to large bars of copper arranged in pairs at the back of the board and termed **bus bars.** The feeders are also coupled (through the necessary appliances on the board) to these bus bars. A number of machines in parallel may supply one pair of bus bars, and from this pair of bus bars feeders may pass to different feeding points. Such feeders would all be supplied at one pressure, whereas it is desirable that the longer ones should be supplied at a higher pressure. To do this, regulation cells and feeder boosters are used as already indicated: these also are manipulated at the board.

One method of grouping for a switchboard is to divide the whole board up into panels and to keep the positive panels on one side of the centre and the negative panels on the other side. Another method is to have a panel for each generator, in which case both positive and negative leads from the machine pass to the same panel, and similarly there would be a panel for each pair of feeders. The generator panels are invariably arranged on one side of the centre and the feeder panels on the other side. There is usually a separate panel or section for dealing with the secondary battery and boosters, and in traction boards there is usually a separate Board of Trade panel containing the apparatus necessary for carrying out the tests laid down in the Board of Trade traction regulations.

297. The Central Station: (3) Miscellaneous.—(*a*) The student will have no difficulty in understanding the various types of switches, etc. (slow break, quick break, multiple way, battery regulating, excess current cut-out, minimum current cut-out, reverse current cut-out, time limit devices for cut-outs) met with in central station practice from preceding pages, and the manufacturers catalogues. A few minutes spent in the actual handling and examination of these will well repay the student. Attention may be drawn, however, to the **Field Breaking Switch.**

The danger (owing to the large self-induction) of suddenly breaking the field circuit of a dynamo has been frequently mentioned. It is usual, in consequence, to remove the field-winding from the active circuit, not by breaking it, but by short-circuiting it by a non-inductive resistance, and thus the energy stored in the field is used up in

heating this resistance instead of producing a dangerous and destructive spark.

In the *Siemens switch* one end of the winding is connected to the switch fulcrum while two other blocks are connected, one to the brush and the other to one end of the special resistance, the other end of this resistance being joined to the other end of the field. Further, the handle portion is wide enough to bridge the space between these two blocks, so that the contact with the resistance is made before the shunt is broken. Carbon blocks are fitted to this switch to take the spark which, though minimised, is still formed on cutting out the field.

A better one is the *Cowan-Still switch*, in which the switch is in two halves connected by a spring. The hinge of the switch is connected, as before, to one end of the field, and the two blocks are connected to the bus-bars from which the field is excited. When the main half of the switch is put across to the second contact the other half is still hanging on to the first contact through a resistance equal to the field resistance, and it is only when the switch is pushed quite home that this hanging contact is broken and the operation concluded. Fig. 334 shows in outline the construction of this switch.

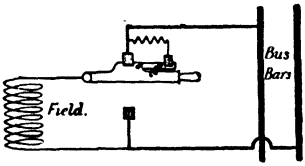


Fig. 334.

(*b*) In transmission schemes it is important to provide suitable arrangements to enable a lightning discharge, striking an overhead conductor, to pass safely to earth without injuring the generators or other appliances. This is usually done by providing one or more air gaps across which the lightning P.D. will leap, while that in use will not. Such devices are termed **lightning arresters.**

Choking coils are frequently fitted between the generator and line, as mentioned in Art. 271, to act as a barrier to the high frequency lightning discharge and compel it to leap a gap and pass to earth.

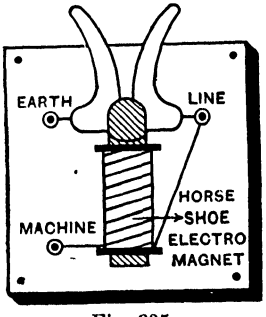


Fig. 335.

Thomson's Lightning Arrester is largely used, and is shown in Fig. 335. A spark gap is made consisting of two metal horns, the gap being narrow at the bottom and wide at the top. An electromagnet carrying the line current is placed below the gap, and connections are as indicated. Should lightning strike the line, it will jump the air gap between the horns and pass to earth rather than pass on to the machines. If the line current follows and sets up an arc, the magnetic field will force the arc upwards and it becomes severed.

Another favourite form is that of the *Wurtz Arrester*, which consists

of a number of small cylinders of special alloy, each separated from the other by a small space. Thus there are many arcs formed simultaneously on discharge of lightning, but the alloy has this special property—*it does not permit a persistent arc.* A row of seven of these is commonly allowed for every 2,000 volts line pressure. Another form, the *Stanley Arrester*, consists of concentric cylinders instead of cylinders arranged in a row.

Condensers are also used as protection against surges on both A.C and D.C. circuits. The condenser depends for its action as an arrester upon the fact that its impedance varies inversely as the frequency. It offers a big impedance to line voltages of normal frequency, but when a high-frequency pressure surge presents itself the impedance falls to a low point and the condenser earths the line for that surge, thus permitting the surge current to dissipate itself over a path of low impedance. A practical type is the **Moscicki Condenser** which consists essentially of a glass tube, the inner and outer surfaces being coated with chemically deposited silver which is finally copper plated.

The **electrolytic arrester** for D.C. circuits consists of aluminium electrodes in a special electrolyte. When a current attempts to pass, an insulating film is formed on one electrode and the resistance of the film increases with the pressure until a certain critical pressure is reached. When this pressure is applied the film breaks down, the surge which has caused the breakdown is discharged, and the film re-forms. Further, on the appearance of high-frequency surges, it acts like the condensers mentioned above, *i.e.* the impedance falls to a low value and the surges discharge.

(*c*) "Grounds" frequently occur on electric mains, and in order to indicate when such grounds become unsafe and need attention, "**ground detectors**" are mounted on the switchboard. One type merely consists of two (or more) lamps in series between the mains, the junction of the two being earthed. If one main has a low resistance ground (which therefore needs attention) the lamp connected to that main will be much less bright than the other. If both mains have high or both low resistance grounds, then the two lamps will glow alike. In this case, if one lamp be disconnected and the remaining lamp is brighter, it indicates a low resistance ground: if the remaining lamp is dim, it indicates a high resistance ground. The student should think this out in detail for himself.

(*d*) Shunt and compound dynamos are often arranged in parallel so that larger currents may be supplied to a pair of bus bars when the load on the station increases. Assume a shunt machine supplying the load and that it becomes necessary to bring in another shunt machine to share the increasing load. The second machine is first speeded up and the voltage adjusted until it is a little higher—about 2 %—than the voltage on the bus bars, and then it is switched on to the bars, the voltage being further regulated if necessary by the shunt regulator

until it takes its proper share of the load. The method is somewhat similar for compound machines save that when the machines as a whole are in parallel between the bus bars *the series coils also are independently in parallel* between one of these buses and a third bus bar known as an **equalising bus bar** (Fig. 336): without this third bar the proper division of load could not be ensured and there would be the possibility of the field being reversed if a reverse current flowed.

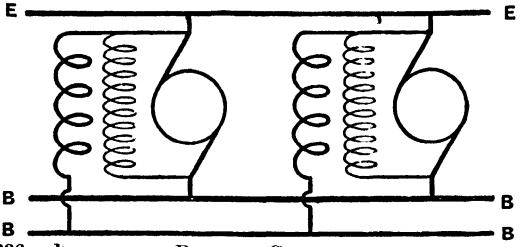


Fig. 336.—Equalising Bar for Compound-wound Generators.
B B, B B, Bus-bars. E E, Equalising bar.

When alternators are put in parallel they must not only have the same voltage and frequency *but they must be in phase:* the operation of bringing them into phase is called **synchronising,** but we cannot enter into details here.

298. Load Curves: Load Factors.—The student should again read Arts. 169, 265 before proceeding further. Fig. 337 depicts the load curves of the Hull Lighting Station for the heaviest and lightest days in one period of twelve months. The upper curve, showing the heaviest load, was obtained on December 18th, and the lower curve, for the lightest load, was obtained on July 2nd.

The actual output of a station in kilowatt-hours during a day divided by the output that could be obtained by working the station at its full capacity the whole day is called the **station load factor.** It should be noted that *rated* capacity is meant: as a matter of fact the boilers may be forced and the engines and generators overloaded for short periods to meet an excessive "peak." Further, this factor should be kept as high as possible for not only are the standing charges (Art. 299) based on it, but practice shows that the cost per kilowatt-hour decreases with increasing load factor.

The actual output of a station in kilowatt-hours during a day divided by the output that could be obtained from the plant *in use* working at its full capacity the whole day is called the **plant load factor.** This factor is, of course, larger than the previous one because its denominator does not include (like the station factor) the "spares" in case of emergency.

The actual output of a station in kilowatt-hours during a day divided by the output that would have been obtained if the maximum demand for that day had lasted the whole day is called the **daily load factor**: it is nearly the same as the plant load factor. Clearly, in Fig. 337, if a horizontal line be drawn across the top of the winter peak, the load factor for the day in question will be given by the area under the curve divided by the area under this horizontal line.

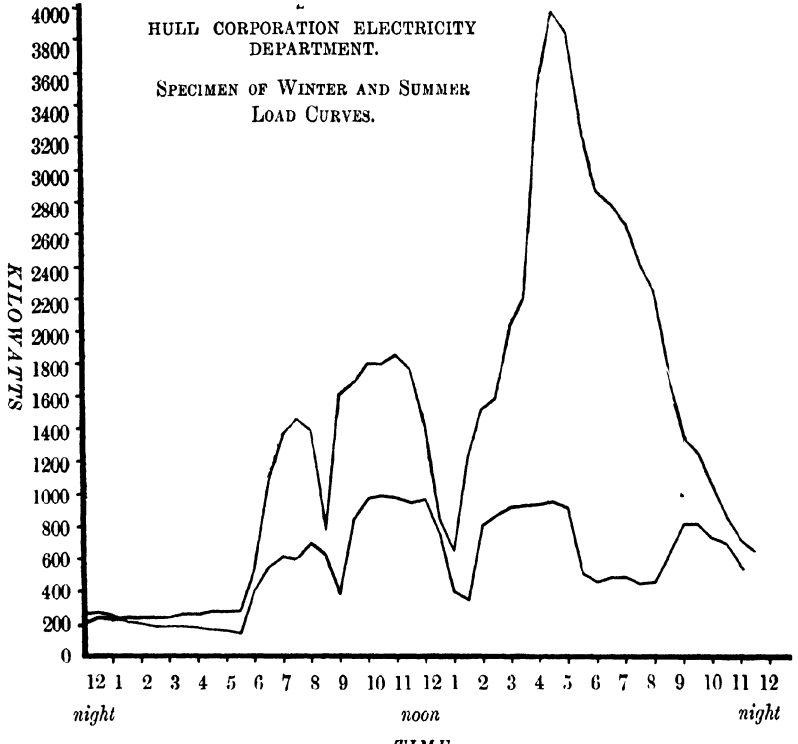


Fig. 337.

299. Systems of Charging for Electrical Energy.

(*a*) THE FLAT-RATE SYSTEM.—In this a uniform price per B.O.T. unit is levied, whether the consumer's load be heavy or light, steady or unsteady, during the day or during the night, for long or for short periods. The advantage of the system is that it is easily understood by the man in the street; the main drawback is that the large short-hour consumer, for whom plant of considerable horse-power must be kept in readiness, does not contribute his just share towards

the expenses of the supply station, whilst the long-hour consumer, taking the same amount of energy, pays more than his due proportion, for less plant is required to meet his demands.

(*b*) THE MAXIMUM DEMAND SYSTEM.—This is a two-rate system devised by Mr. Arthur Wright, of Brighton, in which an attempt is made to distinguish between the good and bad consumer, from the supply company's point of view.

The expenses in connection with a supply station may be regarded as consisting of two separate and distinct parts, viz. :

(1) *Readiness to serve, or "standing" costs, i.e.* expenses incurred by the company in placing itself in a position to provide a certain output, and including interest on the money invested, sinking fund, rent, rates and taxes, repairs, depreciation, etc. (2) *Running costs, i.e.* cost of fuel, oil, etc.* Suppose, now, the maximum output for which the company is prepared is such that a charge of £7 per year per kilowatt will meet the "standing" costs: clearly, then, A, whose maximum demand at any time is, say, one kilowatt, must pay 35s. per quarter towards these costs. Further, let us assume that A runs his maximum demand for one hour each day, or, say, 90 hours during the quarter, so that, during his maximum demand periods, 1,000 × 90, *i.e.* 90,000 watt-hours, or 90 B.O.T. units, will be consumed. Again, let it be agreed that the 35s. be levied on the energy taken during the periods of maximum demand. Dividing 35s. by 90 we obtain 4·6d. per unit as A's contribution towards the "standing" costs. But, in addition, the "running" costs of the station must be met; if 2d. per unit will provide for these, A's quarter's account would be made to comprise a charge of *2d. per unit on all the energy consumed, plus an additional* 4·6*d. per unit for* 90 *units*, or, in other words, 6·6*d. per unit for the first* 90 *units, and* 2*d. per unit afterwards.*

If B's maximum demand be 2 kilowatts he must pay 70s. per quarter towards "standing" costs; assuming that he uses this maximum demand for 90 hours he will consume 2,000 × 90 watt-hours, *i.e.* 180 B.O.T. units, and 70s. divided by 180 gives 4·6d. per unit as before. But B must pay this on 180 units, so that his account would be based on charges of 6·6*d. per unit for the first* 180 *units, and* 2*d. per unit afterwards.*

The reader will now understand such charge expressions as: **"6·6d. per unit for the first 90 hours (1 hour per day, say) of maximum demand, and 2d. per unit afterwards."**

Perhaps the chief drawbacks to the system are: (1) the average customer fails to understand the underlying principle of the charge; (2) it requires a little care on the part of the consumer, otherwise the maximum demand may increase considerably on one special occasion, resulting in a large number of units being charged for on the higher scale; (3) it does not properly distinguish a "day" from an "evening" load.

(*c*) ADDITIONAL TWO-RATE SYSTEMS.—The object of the various two-rate systems which have been suggested and tried, is to more or less encourage "day load" by charging a higher rate for energy consumed during the "peak" or heavy demand period (sunset to 10 p.m., say), a smaller charge being made

* "Wages" are partly "running" costs and partly "standing" costs.

for consumption outside the peak period. In the "*Double Tariff Meter System*" of the *Electrical Company, Ltd.*, this is accomplished by the use of two meters, one—the standard—giving the total consumption, and the other—the rebate—indicating the consumption for a chosen period. In this case, if the tariff be, say, 8d. and 3d., the standard meter reading is charged for at 8d. per unit, and the rebate meter reading at 3d. per unit, the latter account being deducted from the former in drawing up the quarter's bill. In yet another two-rate arrangement a *time switch* is employed in conjunction with the meter, the function of the former being to make the meter run slower during the low-rate period, in which case the total meter indications are charged for on the higher scale. Again, a cut-out switch driven by a self-winding electric clock is sometimes used in conjunction with a meter and demand indicator, its function being to switch the latter in and out at the beginning and end of the peak period, thus definitely settling the maximum demand during this period.

(*d*) PREPAYMENT SYSTEM.—In this, "prepayment" or "money-in-the-slot" meters are employed; the system is fairly extensively used with small consumers.

300. Points in Favour of High-pressure Distribution.—The problem before the electric lighting engineer is to supply a large area with current at a practically constant pressure at all parts of the system, and at all times, and yet to spend as little in copper for the construction of mains as is possible. The full considerations are extremely complicated, and beyond the scope of this book, but we can nevertheless arrive at one or two general conclusions from first principles.

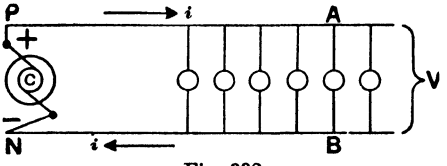


Fig. 338.

Let C be the commutator of a dynamo connected to the distributors P A and N B, the glow lamps being grouped in parallel as shown (Fig. 338). Let V = required voltage for each lamp, i = *total* current required for the lamps, and r = *total* resistance of the leads P A and N B.

Hence the rate at which energy is utilised for the production of light in the lamps = Vi watts. The rate at which energy is wasted in the leads is i^2r watts. The desired object of economy is to keep the rate of waste as small as possible compared with the rate of utilisation for lighting purposes. The possibility of attainment lies in decreasing

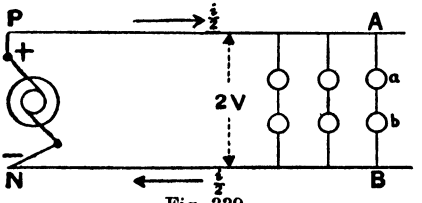


Fig. 339.

i or decreasing r—the latter by using thicker distributing leads. It is apparent, however, that the reduction of r must not be carried beyond a certain limit, otherwise the interest on the extra cost of copper may exceed the cost of the waste prevented.

Consider now the arrangement of Fig. 339, consisting of the same six lamps arranged in three parallel rows of two lamps in series per row. Each lamp requires the same current as before, but as the one current now supplies *two* lamps, the total current output will only be *half* that above, viz. $\frac{i}{2}$. Moreover, since the resistance of each path is double the previous value, it is evident that the voltage must be double, viz. 2V, in order that each lamp may have the requisite current. The rate at which energy is utilised in the lamps

$$= 2V \times \frac{i}{2} = Vi \text{ watts},$$

as before. The watts wasted in the mains

$$= \left(\frac{i}{2}\right)^2 r = \frac{i^2 r}{4} = \frac{1}{4}\, i^2 r \text{ watts}$$

i.e. $\frac{1}{4}$ of the previous value, if the same leads are used. Similarly, if arranged as in Fig. 340, with a pressure = 3V, and a total current = $\frac{i}{3}$ the watts lost would be $\frac{1}{9}$th and so on. *Thus by utilising the same mains, working at increased pressures, and arranging as indicated, less lost watts in these mains would be the result.* The lamps, however, are not independent, for a failure in *a* also cuts out *b* (Fig. 339).

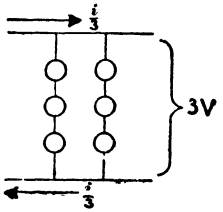


Fig. 340.

Another feature in favour of high pressures in general must be noted. *Since smaller currents are used the leads may be of smaller cross-section, and thus a great saving may be realised in the prime cost of copper.* If the pressure be increased k times, the current (see above) may be k times less, or $\frac{1}{k}$ of its former value. In illustration, consider two systems of distribution transmitting equal power, one employing high pressures, the other low pressures, expressed by the ratio $k:1$. Further, imagine circumstances so arranged that the length of and watts lost in the one system equal those of the other.

Then Watts lost in low-pressure mains $= i^2 r$,

$$\text{Watts lost in high-pressure mains} = \left(\frac{i^2}{k^2} \mathrm{R}\right),$$

where R = total resistance of high-pressure mains.

Hence, since $$\left(\frac{i}{k}\right)^2 \mathrm{R} = i^2 r,$$

$$\mathrm{R} = k^2 r.$$

That is, $$\rho \frac{\mathrm{L}}{\mathrm{A}} = k^2 \rho \frac{\mathrm{L}}{a},$$

where ρ = specific resistance of the conductor,

L = its total length,

A = cross-section of high-pressure mains,

a = cross-section of low-pressure mains.

Thus $$\frac{1}{\mathrm{A}} = \frac{k^2}{a},$$

$$\therefore \mathrm{A} = \frac{a}{k^2}.$$

The cross-section, the weight (and thus approximately the cost) of the copper of the high-pressure system is, therefore, $\frac{1}{k^2}$ that of the other. *Under such conditions as are specified in this example, the weight and cost of copper are inversely as the square of the pressure;* hence the advantages of high pressures. Of course, other cases arise, one of which will be discussed later.

301. The Three-Wire System.—The three-wire system of Hopkinson and Edison is an extension of this idea of efficient and economical distribution, and is represented in outline in Fig. 341. P A and N B are the positive and negative leads respectively, and M M the middle, neutral, or balancing wire. C_1 and C_2 are the commutators of two shunt or compound wound dynamos coupled in series. Each machine is capable of producing the required voltage V for each lamp, so that the total voltage between the positive and negative leads is 2V, as in Fig. 339. Connections and service mains (S) are as indicated

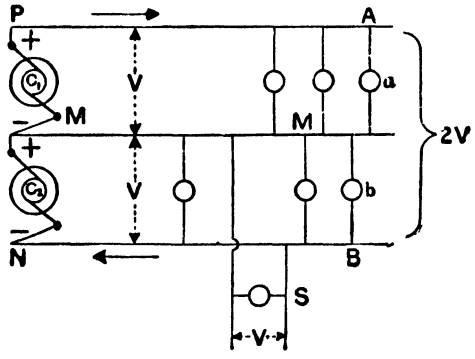


Fig. 341.

The actual current in the middle wire depends on the relative demands on the two sides of it. In cases where the load is not the same on the two sides, the balancing wire will carry either from the dynamos to the lamps or from lamps to dynamos a current equal to the difference between the currents on the two sides of it. If the demand on one side be exactly equal to that on the other, no current will flow along this portion of the middle wire. The case is, therefore, very similar to that of Fig. 339, save that the lamps are independent, *e.g.* cutting out *a* will not influence *b*. In actual practice the service mains should be arranged as far as possible so that the load at any time, on one side, is fairly equal to that on the other, and with this object in view arrangements may be made for moving a portion of a load from one side to the other, should the necessity arise. Thus, in cases of simple lighting, the current in the balancing wire need never be of much magnitude; this wire may, therefore, be of smaller cross-section (in practice about half the cross-section of the other two), and thus the initial cost of this extra wire may be considerably reduced. Should motor work be introduced, however, this no longer holds, the cross-section of the three being invariably the same, for with a motor on, say, the positive side only, the middle wire must carry the same current as the positive lead, and so on. The principal advantages of the three-wire system may thus be briefly summarised:

(1) The balancing wire

(*a*) renders the lamps independent; for, although two lamps are in series in each branch, the failure of any one *a* on the positive side, does not interrupt *b* the one in series with it on the negative side;

(*b*) results in permissible safe pressures in the customers' mains, although higher pressure is employed between the distributors than in the ordinary two-wire system of Fig. 338.

(2) Since double pressure is used, as compared with the case first considered, only half the current is necessary in the distributors, assuming the same number of lamps in each case. Thus, if the positive and negative leads be of like cross-section and length in the two systems, and if the load be equally divided, so that no current flows in the middle wire, we have—

$$\left\{\begin{matrix}\text{Loss of power in}\\ \text{three-wire}\end{matrix}\right\} : \left\{\begin{matrix}\text{Loss of power in}\\ \text{two-wire}\end{matrix}\right\} = \left(\frac{i}{2}\right)^2 r : i^2 r$$

$$= 1 : 4 \text{ (see second case considered in Art. 300).}$$

In this example the cost of copper is greater in the three-wire owing to the additional balancing wire. Its section, however, may be small, as previously explained.

(3) By employing higher pressure in the distributors, and, therefore, less current, these leads may be of smaller cross-section, and thus

a saving in the initial cost of copper is realised. The full question of economy of copper involves many factors which cannot be treated here. The following examples will provide a general idea of the point in question:

302. Approximate Comparisons of the Weights of Copper on Two- and Three-Wire Systems—

CASE I.—Let P denote a simple parallel system, and T a three-wire system of *equal length*. *Further, let the watts lost in* P = *those lost in* T. Let r = resistance of the leads of P, and R = resistance of T. Then for the same lamps the current in T = half the current in P, and equating the lost watts we have:

$$i^2r = \left(\frac{i}{2}\right)^2 \mathrm{R};$$

$$\therefore \mathrm{R} = 4r.$$

Hence the cross-section of the leads of T = $\frac{1}{4}$ the cross-section of those of P.

Now, assuming the middle wire = $\frac{1}{2}$ the cross-section of the outer,

Let w represent weight ($\propto$ cost) of middle copper,

$\therefore 2w$ represents ,, ,, ,, positive copper.

and $2w$,, ,, ,, ,, negative copper,

$\therefore 5w$,, ,, ,, ,, copper of T.

Again, $8w$,, ,, ,, ,, *each* copper of P.

Thus— $\left\{\begin{matrix}\text{Weight } (\propto \text{ cost}) \\ \text{of T}\end{matrix}\right\} : \left\{\begin{matrix}\text{Weight } (\propto \text{ cost}) \\ \text{of P}\end{matrix}\right\} = 5w : 16w$

$= 1 : 3{\cdot}2.$

That is, we have, roughly, a saving of 69 per cent. in weight and rough cost of copper by using the three-wire system.

CASE II.—As a further illustration let us consider the case where *the number of amperes per square inch section of* P = *that of* T, the *lengths* and *loads* being, as before, the same in both. Since half current flows in T the leads of this system will be half the sectional area of the leads of P, *i.e.* R = $2r$, and—

$$\left\{\begin{matrix}\text{Loss of power} \\ \text{in T}\end{matrix}\right\} : \left\{\begin{matrix}\text{Loss of power} \\ \text{in P}\end{matrix}\right\} = \left(\frac{i}{2}\right)^2 2r : i^2r$$

$$= 1 : 2.$$

Again— $5w$ represents weight ($\propto$ cost) of copper of T,

$8w$,, ,, ,, ,, P,

since $4w$ (*i.e.* $2 \times 2w$) represents *each* lead of P.

Thus— $\left\{\begin{matrix}\text{Weight } (\propto \text{ cost}) \\ \text{of T}\end{matrix}\right\} : \left\{\begin{matrix}\text{Weight } (\propto \text{ cost}) \\ \text{of P}\end{matrix}\right\} = 5w : 8w$

$= 1 : 1{\cdot}6.$

That is, we have a saving of about 37 per cent. in weight, and rough cost of the copper by using T, and only half the watts wasted.

In cases where still higher pressures are desired the **five-wire system** has been employed, in which there are four machines coupled in series between the

five cables, the three inner cables being of much smaller cross-section than the two outers. The reader would do well to consider this system on lines similar to those suggested above, when certain specific merits of the system will become apparent. While it has certainly proved successful in Manchester, it possesses, however, one or two marked drawbacks, particularly in regard to pressure regulation, and so the method is not greatly employed.

303. Balancers on Three-wire Systems.—In Art. 262 we have indicated the application of a motor-generator as a *booster* for hoisting up the pressure at the end of a long feeder, thus compensating for the lost volts in the feeder. Allied to boosters are **balancers,** which are of service on multiple-wire systems when the various sections are unequally loaded. Fig. 342 depicts a motor-generator used as a balancer on a three-wire system. The two armatures keyed to the same shaft are similar.

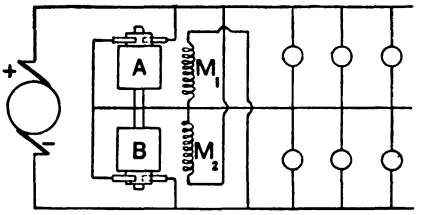


Fig. 342.

If both sides are loaded alike, the field magnets will be equally excited, the machines will have equal P.D.'s across their armatures, and each will run light as a motor. Should, however, the upper side of the neutral become *overloaded,* the P.D. between the positive and neutral will be reduced, the magnetisation of the fields M_2 of machine B will be reduced, and since the weaker the field the greater the speed of a motor, B will increase in speed, will drive A at this increased speed *so that* A *generates E.M.F. and delivers current to the overloaded side.* B, *acting as a motor, takes the excess current from the underloaded negative side of the neutral.*

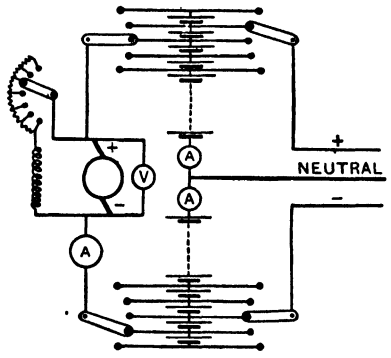


Fig. 343.

Fig. 343 shows one method of arranging a battery of accumulators for the same purpose, the battery E.M.F. being equal to the pressure across the outers. Charge and discharge switches on each half battery permit exact regulation to be maintained, and the current may be supplied by either the dynamo alone, the battery alone, or both in parallel. Various modifications of this are in use.

In both the above a single generator is employed, its voltage being double that required for the lamps.

304. Long-distance Transmission by Alternating Current.—We have seen that since the power supplied to a circuit is E I watts, and the power wasted in a main of resistance R ohms is I^2R watts, it is advisable to keep the current as small as possible (thus reducing the I^2R loss) while transmitting the same power, and that the best method of doing this is to *raise the pressure.* For example, we can transmit 100 amperes at 100 volts, or 1 ampere at 10,000 volts, and in each case the *power* is the same; but in the second case, since the current is $\frac{1}{100}$ of its first value, the loss in the conductor, if the size of the latter remains unaltered, is $\frac{1}{10000}$ of its former value.

Hence, if long distances have to be covered the rule is to transmit at a high pressure, and, since smaller currents flow, to have a thin conductor, which results in economy of copper. A rough rule for the most economical pressure for high-pressure transmission is that suggested by Mr. C. Scott, and is to the effect that **the pressure in thousands of volts should equal one-third of the number of miles over which the energy is to be transmitted.**

Of course, *up to a certain voltage,* we could use high-pressure D.C. generators at the central station, transmit to a sub-station or feeding point, transform to lower pressure by means of a motor generator placed there, and then pass on to the distributing network.

In most cases, however, it is customary to use alternating current for long distances, and the following are the chief methods employed:—

(*a*) Generate A.C. at an ordinary pressure, raise to a high pressure, say 10,000 to 20,000 volts, by step-up transformers, transmit at that pressure to the far end, and then transform down again by step-down transformers, and if D.C. is required pass the A.C. into a rotary converter and produce D.C. by its means. In modern stations for traction purposes it is frequently arranged that the alternators generate direct at 10,000 volts or more, thus saving the first transformer with its attendant losses.

(*b*) Similarly generate high-pressure polyphase current, transmit to a polyphase motor coupled to a D.C. generator at a sub-station, and then pass from the generator to the distributing network, *or* the polyphase current at the sub-station could be "stepped down" by a static transformer, and then the current could be transformed to D.C. by a rotary converter as indicated in (*a*) above.

For very high-pressure transmission alternating current is always selected, because it is at present impossible to make a satisfactory D.C. dynamo to generate at, say, 6,000 volts. The trouble arises in the commutator, which cannot be satisfactorily insulated at these high pressures.

The British Standardisation Rules (1920) quote the following figures:—

(*a*) The standard high-pressure systems for A.C. work are:—2,000, 3,000, 6,000, 10,000, and 20,000 volts.

(*b*) The standard generator pressures are:—for D.C. generators, 115, 230, 460, 525 volts, and for A.C. generators, 440, 550, 2,200, 3,300, 6,600, 11,000 volts.

(*c*) The standard pressures at consumers' terminals are:—for D.C. 110, 220, 440, 500 volts, and for A.C. 100, 200, 400, 500 volts.

305. Advantages and Disadvantages of Various Systems.—These may be briefly stated as follows: Low-pressure systems are simpler than high, but, as has been seen, are not so economical if a large quantity of power has to be transmitted: the use of higher voltage in the three-wire system is justified by its more than covering the cost of the additional wire. With high-pressure currents, however, insulation is more liable to get defective, and if any leak does occur the danger is much increased. Again, continuous current systems have an advantage in that they can be used with accumulators, which allows the main plant to be shut down during the hours of least demand; whereas alternate current machines must be kept continually running, as they will not work accumulators. The continuous current is also, in some respects, more suited for the transmission of mechanical power, and there is less danger than with A.C. On the other hand, much higher pressures, and consequent economy, can be obtained from alternating currents than from continuous currents, and alternating current can be raised or lowered in pressure quite readily and economically by transformers, and changed to direct current where essential.

In general, for small compact districts, continuous current systems at low pressure have the advantage, but for widely scattered districts high-pressure alternating currents should be used.

306. Cable Laying.—The two chief methods are known as "solid" and "draw in."

In the first method the cable rests on porcelain supports in an earthenware trough, an insulating compound being poured round it, and the trough closed by an earthenware cover.

In the second method earthenware conduits are used and the cables are "drawn in." We cannot, however, enter into details here.

307. Economies in an Electrical Undertaking.—The distribution of electrical energy for commercial purposes involves problems of great scientific and technical interest, for not only have we to secure economical transmission of the energy itself, but also that nature, size, and method of laying the mains which will involve the minimum initial outlay, and minimum annual expenditure for repairs and maintenance. The importance of this latter aspect of the problem will be seen when we remember that in most cases of local distribution, and in nearly every case of long-distance transmission, the cost of the mains is the

largest item of the installation, and sometimes is more than the entire cost of the whole of the generating plant, comprising boilers, engines, and dynamos.

We have seen that the power lost in the mains is measured by I^2R watts. Since during the whole period of working of an installation this lost energy will represent monetary outlay without any monetary return whatever, it is obvious that it will have to be considered in relation to the initial cost of the mains in order to determine the best design of conductors for economical working. We have in fact to find the best size of cross-section which gives the minimum loss of energy with the minimum cost of conductors.

This problem was first attacked by Lord Kelvin in 1881, who put forward what is now known as **Kelvin's Law.** It states that "**the most economical size of conductor is that for which the annual interest on capital outlay just equals the annual cost of the energy wasted.**" In other words, the total annual expenditure for interest on the investment and energy lost on the line is a minimum when these two items are equal. The law, however, was afterwards shown to have only a limited application, although it was a good general guide. Various modifications were suggested by Ayrton and Perry, and Forbes in 1885 amended the law to read, "**the most economical area of conductor is that for which the annual cost of energy wasted is equal to the annual interest on that portion of the capital outlay which is proportional to the weight of conductors used.**"

EXAMPLES.

1. *If the distance between the feeding centres be* 400 *yards on a three-wire circuit running two* 200*-volt lamps in series per foot of street, each lamp taking* 0·3 *ampere, what must be the size of the outers if no two lamps are to differ by more than* 2 *volts?*

Obviously the lamps with the greatest pressure will be those at the feed-points, while those with the lowest pressure will be half-way between the feed-points. Since there are two lamps in series between the outers, the maximum drop permissible is 2×2 volts $= 4$ volts.

Current entering by each feeder $= 400 \times 3 \times \cdot3$ amperes,
$= 360$ amperes.

Of this one half goes right, the other half left, and diminishes to zero each way.

Hence **average current in distributors** $= \frac{360}{4} = 90$ amperes;

$\therefore$ Since $IR = 4$ volts and since $R = \frac{4}{I}$,

$$\therefore \frac{400 \times 3 \times 12 \times \cdot00000066}{a} = \frac{4}{90};$$

$$\therefore a = \frac{400 \times 36 \times \cdot00000066 \times 90}{4}$$

$$= \mathbf{0{\cdot}214} \text{ sq. in.}$$

2. *A motor taking current of* 100 *amperes at a P.D. of* 2,000 *volts is run from a dynamo* 8 *miles away. If the loss in the leads is not to be more than* 10 *per cent., what will be the nearest cable of S.W.G., and what will have to be the P.D. at the dynamo?*

Power taken by motor = 200,000 watts.

Since the loss is not to exceed 10 per cent., we see that 90 per cent. of power given out by dynamo = 200,000 watts.

$$\therefore \text{Power given by dynamo} = \frac{10}{9} \times 200{,}000 \text{ watts},$$

and
$$\therefore \text{P.D. at dynamo} = \frac{10 \times 200000}{9 \times 100} \text{ volts}$$
$$= 2222{\cdot}2 \text{ volts.}$$

Hence lost volts = 222 nearly.

Since lost volts = I R,

$$\therefore R = \frac{222}{100} = 2{\cdot}22 \text{ ohms};$$

$$\therefore \frac{\rho l}{a} = 2{\cdot}22 \text{ ohms.}$$

$$\therefore a = \frac{16 \times 5280 \times 12 \times {\cdot}00000066}{2{\cdot}22}$$

$$\therefore a = 0{\cdot}301 \text{ sq. in.}$$

Hence the cable needed is 91/16 S.W.G.

3. *If the power in the last question be transmitted at* 20,000 *volts at the motor end, what will then be the size of the bare solid conductor used if the loss of pressure is to be only* 5 *per cent.?*

Since the power is to be the same as before, the new current will be 10 amperes.

We have
$$\frac{95}{100} V = 20{,}000,$$

$$\therefore \text{V at dynamo} = \frac{2000000}{95} = 21{,}052 \text{ volts.}$$

$$\therefore \text{lost volts} = 1052,$$

$$\therefore I R = 1052,$$

$$\therefore R = \frac{1052}{10} = 105{\cdot}2 \text{ ohms.}$$

$$\therefore \frac{\rho l}{a} = 105{\cdot}2$$

$$\therefore a = \frac{16 \times 5280 \times 12 \times {\cdot}00000066}{105{\cdot}2}$$

$$= 0{\cdot}00634 \text{ sq. in.}$$

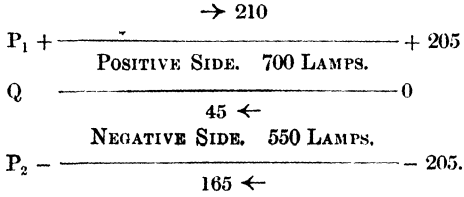
$\therefore$ Suitable solid wire = No. 13 S.W.G.

Note that the ratio of the *weights* of the above cables

$$= \frac{0{\cdot}301}{{\cdot}00634} = 48 \text{ to } 1.$$

4. *The electric lighting of a block of buildings is effected from an engine room* 1,350 *yards distant. The wiring of the block is arranged for the three-wire system, and is fed by a three-wire feeder, the cross-sectional area of the conducting cores of*

which are 0·5, 0·25, *and* 0·5 *square inch respectively. There are* 700 *lamps of* 1(*candle-power in use on the circuits connected with one side of the three-wire system, and* 550 *on the other side. What voltages must be maintained at the generating station ends of the feeder between the positive and negative conductors and the neutral respectively, in order that the pressure between the terminals of each of the distributing circuits may be* 205 *volts? The current required by a* 16 *candle-power lamp may be taken at* 0·3 *ampere, and the resistance of a cubic inch of copper a* 0·66 *microhm.* (O.)



$$\text{Res. of + lead} = \frac{\cdot 66}{10^6} \times \frac{1350 \times 36}{\cdot 5} = \cdot 064 \text{ ohm} = \text{Res. of - lead}$$

$$\text{Res. of middle wire} = \frac{\cdot 66}{10^6} \times \frac{1350 \times 36}{\cdot 25} = \cdot 128 \text{ ohm } (i.e.\ 2 + \text{lead}).$$

Current taken by 700 lamps on the + side = 700 × ·3 = 210 amp.
,, ,, ,, 550 ,, ,, ,, − ,, = 550 × ·3 = 165 ,,
∴ Total current passing along the + lead = 210 amp.
And ,, ,, ,, ,, ,, − ,, = 165 ,,
∴ ,, ,, ,, ,, ,, neutral = 45 ,,
Fall in volts in + lead = 210 × ·064 = 13·4
,, ,, ,, ,, − ,, = 165 × ·064 = 10·56
,, ,, ,, ,, neutral = 45 × ·128 = 5·76
∴ Pot. at P_1 = 205 + 13·4 = 218·4 volts,
Pot. at Q = 0 − 5·76 = − 5·76 ,,
∴ **P.D. between P_1 and Q = 224·16 volts.**
Pot. at P_2 = − 205 − 10·56 = − 215·56
∴ **P.D. between P_2 and Q = 209·8 volts.**

EXERCISES XXII.

1. A dynamo supplies power to mains over an area of 50 square miles. How is it arranged that the dynamo shall work at constant P.D., and also that the pressure in the mains shall be uniform?

2. What is the advantage of low against high-pressure systems of distribution and of continuous against alternating current systems? If you were deriving power from the Niagara Falls and supplying electric energy at a place 500 miles away, what system of transmission would you adopt? State clearly the advantage of your system.

3. Explain fully why it is more advantageous to distribute electric energy at a high than at a low potential.

4. A copper main is already laid, having a resistance of ·47 ohm. It is desired to transmit 5 kilowatts and to deliver it at the distant place at a pressure not less than 90 volts. Which of the following methods would you adopt: (*a*) Send a current at an initial pressure of 117 volts; or (*b*) send a current at 2,500 volts, using a transformer having an efficiency of 86 per cent.?

5. Explain the advantages of the three-wire system of distribution for lighting purposes. (B.)

6. In a central station supplying continuous current on the three-wire system, what arrangements would you adopt to keep the pressure constant at the consumers' lamps? (H.)

7. State the relative advantages and disadvantages of the two following systems of distributing alternate currents: (*a*) transformers in consumers' houses; (*b*) transformers in sub-stations. (O.)

8. When are alternate current systems better than continuous current systems? Give your reasons as fully as possible. (O.)

9. The declared pressure at which current is supplied to houses by mains coming from a central station is raised from 100 to 220 volts. If the percentage loss in the mains is to remain the same as before, by how much per cent. will their carrying capacity be increased when the heating limit has not to be regarded? (O.)

10. A dynamo driven with water-power at a constant speed supplies energy to fifty 60-watt lamps and a 20-B. h.p. motor used for driving a sawmill 1,000 yards away; consider what kind of dynamo and motor you would use, and what electric pressure would be suitable. Calculate approximately the cross-section you would give to the mains, the drop in pressure that would occur, and the power required to drive the dynamo at full load. (O.)

11. A, B and C are three points, each at a distance of 157 yards from the next. At A there is a dynamo, at B a group of 200 incandescent lamps requiring 110 volts and 0·75 ampere each, and at C a group of 250 lamps requiring 100 volts and 0·83 ampere each. Calculate the diameter of the copper conductor that must be used for the lead and return wires between A and B, and between B and C respectively, if the potential difference maintained at the terminals of the dynamo is 112 volts; 1 yard of commercial copper wire, 1 square inch in section, has 0·00002435 ohm resistance. (O.)

12. Current is required along a street 2,000 feet long at the rate of half an ampere per foot run, or in all 1,000 amperes. The voltage drop between any two points of the mains must not exceed 4 volts. The generating station is at one end of the street. State size of main and total weight of copper for the two following cases: (*a*) The station is directly connected to the end of the main; (*b*) the station is connected to the middle of the main by a feeder of such size that 10 volts are lost in the feeder. Include weight of feeder in giving the total copper weight. N.B.—A bar of copper 1 square inch in section and 1 mile long has a resistance of 0·0425 ohm. One cubic inch of copper weighs ·32 pound. (O.)

13. You have in a street a pair of mains, each conductor 0·2 square inch in area (0·12 ohm per 1,000 yards), and the demand for current is at the rate of one 30-watt 100-volt lamp per yard run. Give the distance between the feeding points so that the maximum variations of pressure between any two consumers shall not exceed 3 volts. (O_0)

14. You have to deliver 115 electrical h.p. by means of a continuous current, $1\frac{1}{2}$ miles distant from the generator. The terminal pressure at the generator is 1,000 volts, and the loss due to line resistance is assumed to be $7\frac{1}{2}$ per cent. of the power delivered. Give the size and weight of the conductor required. N.B. —A bar of copper 1 square inch in section and 1 mile long has a resistance of 0·0425 ohm. The specific weight of copper is 8·9. The insulation of the line is perfect. (O.)

CHAPTER XXIII.

INDOOR WIRING.

308. Switches.—In a general way these devices may be divided into two classes—viz. (1) *Main Switches* for use on circuits in which the current exceeds 20 amperes, and (2) *Branch Switches* for currents below this value. Each may be further divided into *single pole* (S.P.), *double pole* (D.P.), and *triple pole* (T.P.) switches according as one, two, or three leads are opened or closed simultaneously. Further, a switch may open a lead in one place or in two; the former is known as a *single break* (S.B.), the latter as a *double break* (D.B.) switch. *Two-way, multiple way, reversing,* and switches for special purposes are also in daily use.

The construction and action of the commoner types of main switches will be readily understood by reference to the manufacturers' catalogues.

The essential parts of an excellent **tumbler** switch, which under the general name of the "**Pivot**" is manufactured by Mr. A. P. Lundberg for a variety of useful purposes, is shown in Fig. 344, the full lines representing the "on," the dotted lines the "off" position, one of the contact arms being removed for the sake of clearness. In the figure, A is the contact arm and B the actuating knob, the two being attached by means of the pin C. The knob rocks upon a fixed fulcrum pin H, the contact arm likewise rocking upon a second fixed fulcrum pin D. The contact arm is furnished with a slot F, which, sliding over the fixed pin D, provides a variable lever action, and allows the movable pin C to pass from right to left, or from left to right between the fixed pins D and H when the switch is turned to the "on" and "off" positions respectively. The movement is controlled by a spiral spring E, which works between the contact arms, one end being attached to a fixed hook J, the other being fixed to the contact arms at G.

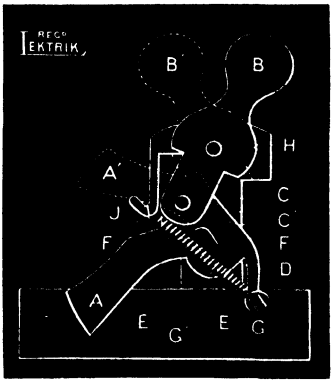


Fig. 344.

The switch has many valuable features consequent upon the construction and the manner in which the various parts are assembled; chief amongst them may be mentioned the marked ease of action, the impossibility of the switch remaining in an intermediate position between "off" and "on," and the effective locking of the contact arms in the latter condition.

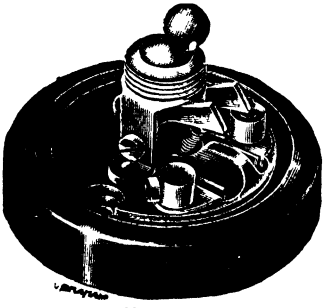


Fig. 345.

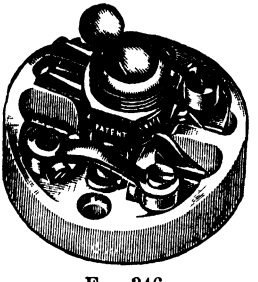


Fig. 346.

(*a*) *Pivot "Single-way."*—Fig. 345 shows the ordinary single-way pattern, fitted with a porcelain base, and a dividing rib of porcelain between the contact springs. Switch covers are of brass or china, the latter being preferable for damp situations.

(*b*) *Pivot "Two-way."*—Fig. 346 illustrates the two-way pattern. With the arrangement indicated the terminal in the front is in metallic connection with the back terminal *on the right* by means of the contact arms which are down; a movement of the knob raises the contact arms on the right, and lowers those on the left, thereby establishing connection between the front terminal and a back terminal *on the left.*

(*c*) *Pivot "Intermediate."*—The connections of this switch are shown diagrammatically at B, Fig. 366. With the knob in one position connection is made between terminals D and F, and between E and G by two sets of contact arms on the rocking-piece; with the knob in another position D is joined to E and F to G by the other two sets of contact arms on the rocking-piece. The utility of this switch is given in Art. 319.

(*d*) *Duplex "Two-way and Off."*—Fig. 347 gives the essential parts of the Lundberg Duplex Two-way Switch, which is provided with an "off" position, the controlling spring D in this pattern being coiled round the pivot C on which the contact arms A and B rock. In the figure, A is in the "on" position joining a front terminal of the switch with the back terminal *on the left*, whilst B is locked "off"; a movement of the knob H into the perpendicular raises A, which becomes likewise locked in the "off" position, whilst a further movement of H to the left brings B down into the "on" position, establishing a connection between the front terminal and a back terminal *on the right.* Thus when one contact is "on" the other is locked "off," an "off" position for both being provided intermediate

between the two; by no possibility can both arms be in the "on" position together.

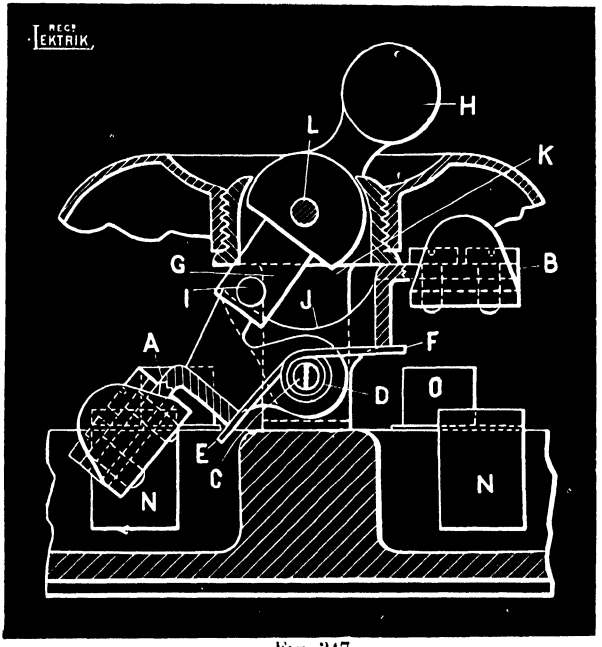


Fig. 347.

(*e*) "*Series Parallel and Off*" *or* "*Whole and Part Circuit.*"—Yet another switch enables lamps to be put in series, in parallel, or off, and allows the whole or part of a bank of lamps to be lit as desired. This is effected by means of a specially designed rocking-piece, somewhat similar to the rocking-piece of the "Intermediate" switch. The switch is fitted with four terminals indicated by the four double circles of Fig. 369. In one position of the actuating knob two terminals are joined, as in Fig. 369*b*, by means of one contact arm on the rocking-piece; in another position of the knob two *pairs* of terminals are joined, as in Fig. 369*a*, by means of two brass strips on the rocking-piece; in a third position the terminals are completely disconnected (Fig. 369*c*), *i.e.* the switch is "off." Wiring diagrams for these special switches are given in Art. 319.

Whatever the construction of the switch may be, certain essential conditions—electrical and mechanical—should be complied with. Briefly these are as follows:

(1) The design should be such as to render the formation of a permanent arc when circuit is broken impossible. To attain this the break must be long, the action of the switch rapid, and the contact arms must be incapable of remaining in an intermediate position between "on" and "off." The actual length of break depends on the pressure, increasing with it.

(2) The conducting portion must be of sufficient cross-section to avoid overheating. The current density should never exceed 800 amperes per square inch for copper and 500 amperes per square inch for brass or gunmetal, whilst at the rubbing surfaces of contact it should not be greater than 250 amperes per square inch.

(3) The base of the switch should be of insulating, incombustible, and waterproof material. For small switches the bases are generally of porcelain, marble and slate being used for larger ones.

(4) The covers should be of incombustible material. In cases where covers are not of insulating material but of metal, the latter must not come into contact with the internal live parts or mechanism; preferably metal covers should be lined with an insulator.

(5) All handles must be so insulated and arranged that live metal cannot come into contact with the hand of the operator.

(6) It should be impossible for any of the internal mechanism to become loose by continued use.

(7) Small switches should not contain a fuse; to put both under the same cover is always more or less a failure.

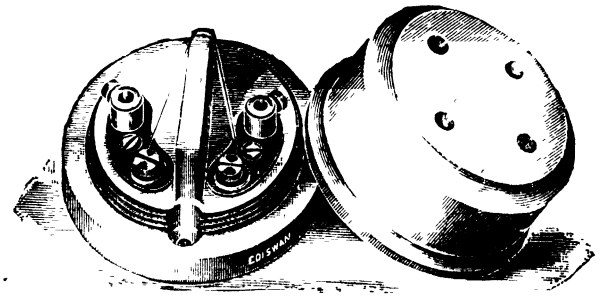


Fig. 348.

309. Fuses.—The main principles have been dealt with in Chapter XV. Fig. 348 shows a type in common use. The two terminals between which the fuse wire is connected are separated by a wall of china, which divides the fuse chamber into two parts and provides a break of substantial length. The cover is supplied with ventilating holes, thus eliminating the possibility of an "explosion" consequent upon the rapid expansion of the air inside when the fuse "blows." For larger circuits the "porcelain handle" or "bobbin" type is largely used. We assume the student familiar with all these however.

In a good fusible cut-out, (1) the base should be of insulating material, incombustible, and waterproof; (2) the cover should be incombustible, and either an insulator or lined with incombustible insulating material; it should be kept quite clear of all internal mechanism; (3) metal parts should not come out at the back; (4) thoroughly good connection should exist between the terminals and the fuse

wire ; there should not be undue heating when the normal current flows, but the fuse should promptly break circuit when the current exceeds the normal by 100 per cent.* ; (5) the extinction of the arc should be prompt ; (6) it is best that the construction be such that the insertion of a fuse of wrong dimensions is well-nigh impossible. Enclosed fuses are strongly recommended.

In practice, the switches and fuses for any particular circuit are grouped together, forming what is termed a **Distribution Board.** Good insulation and freedom from the possibility of fire are secured by constructing the base of slate, free from metallic veins, and by means of distance pieces at the back rendering contact with the wall impossible. To avoid the possibility of short circuit the slab is generally divided, positive and negative connections being mounted on separate slabs.

It must be mentioned in passing that fuses and single-pole switches must *not* be placed on the middle wire in cases where the three wires of a three-wire system are continued into the house, for if the house load on the two sides of the middle be unequal, and the fuse or switch in question be opened, the lesser loaded side will be subjected to a pressure greater than normal, and the lamp filaments may be destroyed. For a like reason, in cases where a T.P. switch is used, the middle one must either *close before* and *open after* the outers or come into action *at the same instant.* In concentric wiring all fuses and switches must be on the inner conductor (Art. 316).

310. Ceiling Roses.—These are devices for establishing connection between overhead leads and lamps suspended by flexible conductors. A convenient and much employed pattern is depicted in Fig. 349. The flexible of the lamp passes through the central opening of the cover, and the two parts then open out, one passing through the upper hole in the I-shaped ridge of china and being attached to the terminal on the right, the other passing through the lower hole in the ridge and being attached to the terminal on the left ; the terminals are thus much relieved of the weight of the lamp, etc. The leads enter through holes in the porcelain base, pass into the metallic tubes of the terminals, and are fixed therein by the screws shown.

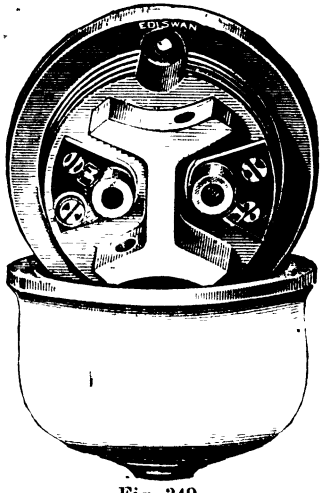


Fig. 349.

* The I.E.E. rules allow 200 per cent. excess if the working current is below 10 amperes.

Full details of the wiring will be gathered from Figs. 362 and 363. Numerous patterns of multiple-way ceiling roses are manufactured, enabling several flexible pendants to be taken therefrom.

Fig. 350 is a plan of, and wiring diagram for, the Simplex New Ceiling Rose for "Loop-in" wiring. The advantages of "looping in" and of this specific ceiling rose are given in Art. 318. The diagram is self-explanatory.

An efficient ceiling rose should comply with the following conditions: (1) The base should be of insulating material, waterproof and incombustible. (2) The cover should be incombustible, and either an insulator or rigid metal kept quite clear of all internal mechanism. (3) The terminals should have some good insulating barrier between them and should not carry the full weight of the lamps and accessories. (4) No metal parts should come out at the back. (5) It should not contain a fuse. The replacing of a fuse in such an out-of-the-way locality is, to say the least of it, inconvenient, while the presence of molten metal and the detached end of a fuse wire amongst the internal mechanism is not desirable. Further, the blowing of the fuse may result in fusion of the flexible and falling of the fittings, to the discomfort and danger of people beneath.

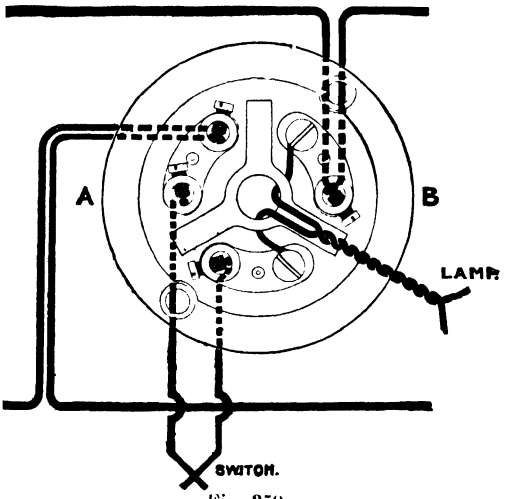


Fig. 350.

311. Indoor Distribution.—House distribution may be carried out by two distinct methods, viz.:

(*a*) The Tree System.

(*b*) The Distribution Board System.

312. The Tree System.—The incoming mains XY (Fig. 351) communicate in turn with the company's main fuses C.M.F., the meter M, the demand indicator I (when required), the tenant's main fuse T.M.F., and the main switch M.S., the two latter devices being under the control of the customer.

In the "Tree" system, a pair of mains extend from the main

switch throughout the building in a vertical direction and in the central part thereof. At each floor sub-mains radiate, and from these again, tappings are taken to the lamps. Where submains and tappings emanate, D.P. fuses should be inserted on the conductors of smaller section. These branch fuses are shown at B.F.; their utility will be gathered from the fact that, if absent, a short circuit in any section might result in the blowing of the main fuse T.M.F., thus putting the whole building in darkness, whilst the presence of suitable branch fuses will cause the faulty section only to be affected. Branch switches S are inserted for the purpose of controlling the lamps in the individual sections.

The disadvantages of the "Tree" system are: (1) a large number of joints must be made; (2) the switches and fuses are in all parts of the building.

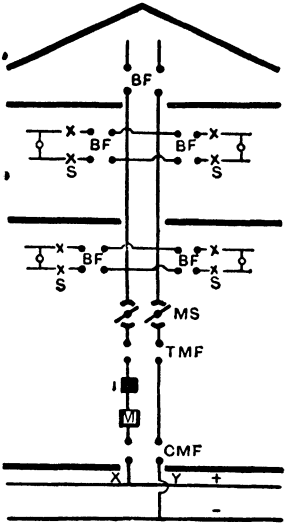


Fig. 351.

313. The Distribution Board System.—Fig. 352 gives in outline the arrangement for a large installation on the three-wire system. B_1, B_2, and B_3 are the positive, negative, and neutral wire "bus" bars respectively of the *main distribution board*, M.D.B., situated in a convenient and accessible position. Considering the left of the figure it will be seen that three sub-circuits emanate (viz. *a*, *b*, *c*), each being controlled by a double pole switch S S and a double pole fuse F F. Each of these circuits terminates in a *sub-distribution board*, S.D.B., fitted with double-pole fuses and single-pole switches. From these boards circuits radiate to *fuse boards*, of which one F.B. is shown, fitted with single-pole fuses, and from these boards connections pass to the lamps, which are controlled by small switches in the usual way. Fig. 353 shows, in outline, a similar arrangement for a smaller installation on the two-wire system. In some cases a combination of the tree and D.B. systems is adopted.

314. Size of Cable.—We have seen that, for a given conductor, the lost volts and the lost energy are dependent on the strength of the current and increase with it, and it is well to have a rough rule, by means of which the size of conductor to carry, without overheating, a certain curr nt can readily be determined. Such rules have already been given in Chapter XV., the one still very commonly employed being the "1,000 amperes per square inch" rule (but see Appendix 6).

With small currents this density may with safety be exceeded, but,

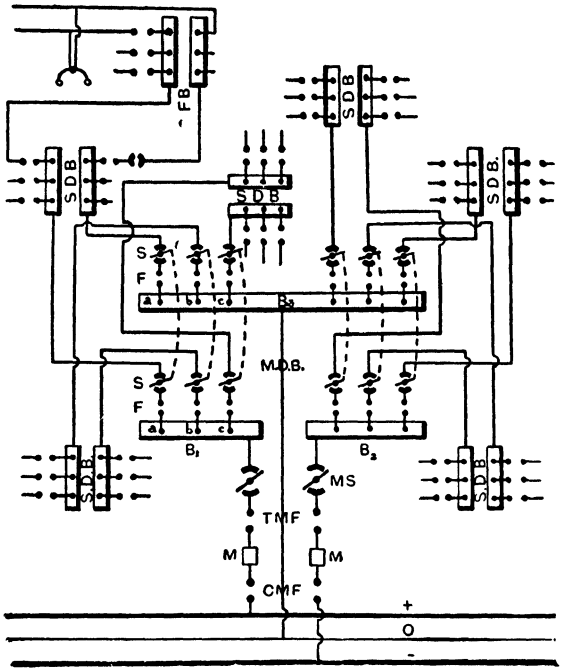


Fig. 352

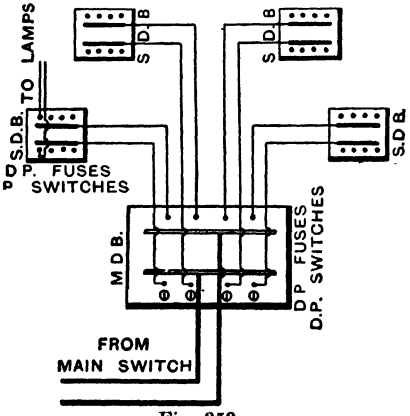


Fig. 353.

with large currents it is somewhat excessive; and in cases where copper conductors are not used, but some metal or alloy of lower conductivity, such as brass, etc., in switches and ceiling roses, it is necessary to use *not more than* 500 amperes per square inch of cross-section. In many installations engineers specify that the drop in volts throughout an entire circuit in a building must not exceed from 2 to $2\frac{1}{2}$ per cent. of the total voltage, and in this case the size of conductor will be determined by the fall of pressure rather than by the heating effect of the current. The I.E.E. rules allow 2 per cent. + 1 volt drop.

315. Running Leads.—The following rules are usually observed in all wiring systems:

1. When the leads run up and down, the leads are to the left, and returns to the right. (Rule remembered as "l to l, r to r.")

2. When the leads run along a wall, the leads are low, the returns are raised. (Rule remembered as in 1.)

3. When the leads run along the ground, the positive lead is to the left, and the negative return to the right. (Rule remembered as in 1.)

316. Systems of Fixing and Protecting Conductors in House Wiring.

1. *The Wood-casing System.*—In this, the positive and negative wires lie in two parallel grooves extending throughout the length of a bar of wood (termed the *casing*), the whole being covered by a thinner bar of wood (known as the *capping*).

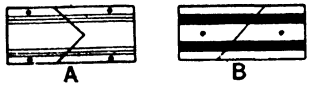


Fig. 354.

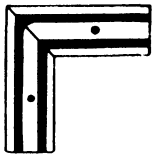


Fig. 355.

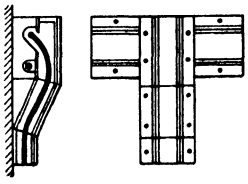


Fig. 356.

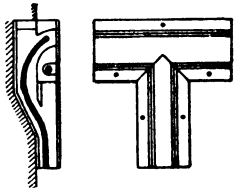


Fig. 357.

Casing and capping are generally of American white wood, which has a straight grain and is easily manipulated. In *surface work* the casing is merely fixed to the surface of the walls, being screwed to wooden plugs therein; in *concealed work* it is buried in the plaster, the capping in some cases being likewise hidden, in others, the capping being arranged flush with the surface of the wall. Prior to erection, the wood is frequently treated with shellac varnish as a protection against damp. Capping should be fixed to casing by screws at distances of not more than 18 inches, and preferably along the outer edges where the width will admit of it (Fig. 354).

In Fig. 354 A and B depict the method of making end-to-end joints n capping and casing respectively, and Fig. 355 shows a flat right-ngled joint. Fig. 356 gives an internal and external view of an *over* Γ joint, whilst Fig. 357 depicts an *under* T joint: the latter necessi-ates the cutting away of the wall to accommodate the bridging over of the wire, and may lead to imperfect workmanship; the former is a ood one, but inartistic if too freely indulged in. At all angles the rooves should be rounded so that the conductor does not take a harp bend.

The advantages of the system may be claimed to be:—(1) Economy in first ost. (2) The conductors are a good distance apart; they can be readily nspected and additional lamps added. (3) The capping can easily be made to armonise with existing decorations.

The chief disadvantages are:—(1) It is not fireproof; in fact, it is highly nflammable. (2) It is unsuited for exposed and damp situations, for it readily bsorbs moisture, and in this condition is really dangerous. (3) It is often ttacked by vermin. (4) Owing to the numerous joints, etc., a vast amount of abour (at a high rate) is entailed in its neat and proper erection, and it does not eadily lend itself to the surmounting of obstacles—*e.g.* cornices, pipes, etc. 5) Special arrangements, not absolutely satisfactory, have to be made for wire rossings, and in this connection it lends itself to defective workmanship. 3) It lacks mechanical strength and insulating properties.

With regard to the leaving of unsightly crevices at the joints of capping and asing, the driving of nails or fixing of screws so close to the grooves as to pierce he insulation, the cutting through of the casing when rounding off the corners f the grooves, the careless driving of huge nails through boards below which the asing runs, the fixing of casing below water-pipes without thoroughly protecting from leakage of water, the bunching indiscriminately of positive and negative ires in the same groove, or even the utilisation of the casing for hat pegs, icture hangers, and physical developers, the best advice that can be given is to uote that of our friend "Punch"—DON'T.

(2) *The Simplex Conduit System.*—Conduit systems consist of the tilisation of metallic tubes with specific metallic accessories. The implex conduits are of steel, stove enamelled or galvanised, and are anufactured in two main types, viz.: (1) Light gauge conduits; 2) Heavy gauge conduits.

In *Ordinary Light Gauge* the conduit has an open longitudinal eam (*i.e.* the edges of the steel strip forming it are brought closely ogether, but are not actually metallically joined), and the ends of the ibes are so formed as to fit accurately into specific junctions, bends, aspection boxes, etc. For ordinary surface work in dry situations ese light gauge socket junction tubes are suitable, for they afford ood mechanical protection and safety from fire, and can be erected at ss cost and in shorter time than other types; but for circuits run in laster or exposed to steam or vapour, and in cases where mechanical

injury is possible, conduits with a closed seam, or conduits without a seam, are necessary. To this end, *Brazed Light Gauge* conduit is constructed; it is similar to the above, but the seam is metallically closed by hand brazing. *Seamless Light Gauge*, yet another pattern, is absolutely without a seam.

In *Heavy Gauge Wireduct* the edges are joined by welding, and the ends of the tubes are provided with a screw thread. *Heavy Gauge Brazed* is similar to the brazed light gauge, but its walls are thicker, and screwed junctions are employed. *Heavy Gauge Screwed Solid Drawn* differs similarly from seamless light gauge.

For high-pressure work the I.E.E. recommend the adoption of a conduit system, the tubing being electrically continuous throughout and effectively earthed (zero potential), so as to eliminate the danger from shock consequent upon a fault leading to the electrification of the tube. In the case of the screwed conduit, continuity is readily provided; with the light gauge socket tubes, the condition is realised by the utilisation of the *contact nipple*—a split screwed bush which, when screwed into any fitting, grips the conduit and secures permanent continuity. Heavy Gauge screwed is, to-day, most extensively used.

In tube systems, the conduits with "draw wires" threaded through them may be erected before the building is completed, and the leads drawn in afterwards by hooking the draw wires to them, or the leads may be erected simultaneously with the conduit; in the first method "draw-in" boxes must be inserted at convenient places.

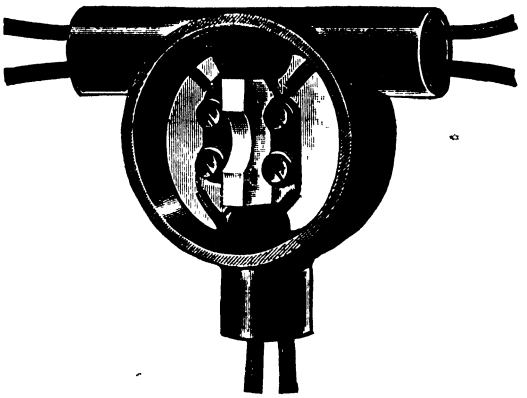


Fig. 358.

A large number of accessories, such as bends, couplings, T jointing, and through-connection boxes, etc., are provided, thus facilitating erection and dispensing with the ordinary process of joint-making in the conductors. Fig. 358 depicts a T connection box.

The advantages claimed for the Simplex System are:—(1) It affords efficient mechanical protection and safety from fire and damp. (2) It involves a minimum of time, labour, and cost in erection. (3) It is light and durable. (4) It admits of perfect continuity and efficient earthing.

The main drawbacks to plain tubing in general are :—(1) The possibility of internal roughness or the "burr" left by a careless wireman in cutting the pipe, injuring the insulation of the wires in "drawing in." (2) The possibility of internal condensation of vapour, working destruction on the insulation and leading to injurious rusting and corroding effects. (3) The possibility of unfortunate results following any marked leakage or electrification of the tube.

(3) *The Bergmann Conduit System and the General Electric Company's Conduit System.*—In general appearance these are similar to the preceding. The Bergmann consists of a brazed steel tube and a complete set of accessories, *but the tube is thoroughly lined with an insulating compound*, from $\frac{1}{16}$ to $\frac{3}{32}$ of an inch thick, the junctions, etc., being likewise insulated. The conduit and fittings are of the screwed pattern and readily admit of perfect continuity and earthing.

Paper insulated steel tubes have for some time been manufactured by the General Electric Co., but no very successful method of lining the fittings with the paper could be devised. This difficulty has now been overcome by coating the interior of the fittings with a mineral compound before enamelling. The initial cost may be reduced by using the unscrewed type of conduit in conjunction with insulated fittings and patent nipples. Screwed conduits can also be obtained.

The specific advantages of these insulated conduits are :—(1) The insulating lining is waterproof, reduces internal condensation and prevents the injurious rusting and corroding effects. (2) The lining provides a smooth interior surface; the absence of "burrs" eliminates damage to the wire insulation in "drawing in." (3) They provide sound electrical and mechanical protection for the wires and freedom from fire risks and arcing between conductors and tube.

(4) *Andrews' Concentric System.*—In concentric systems, the outer tube forms one of the conductors; it is separated from the inner conductor by insulating material.

One form of Andrews' cable consists of an outer copper tube (tinned) within which runs the other conductor, the latter being provided with a covering of vulcanised rubber, and separated from the former by an insulating bedding of impregnated paper. The outer is continuous throughout the installation and is earthed (zero potential), thus rendering a violent shock on coming into contact with it impossible.

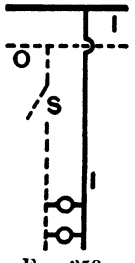


Fig. 359.

Special junction boxes and other accessories are provided whereby the labour item in fitting and erecting is reduced to a minimum.

Perfect continuity of the outer is the feature aimed at in the construction of all accessories, and for this reason, too, all switches and fuses must be placed on the *inner* conductor. Thus, should the outer O (Fig. 359) carry a switch S, the opening of the latter would cause the outer below it to

take the same potential as the inner I, and any one accidentally touching the outer below S would receive a shock.

The wiring of ships is frequently carried out on the concentric system, the outer being earthed by connection with the metallic framework of the ship.

(5) *The British Insulated Wire Company's Twin Cable System.*—In this system both wires are enclosed in a seamless lead tube, which latter is usually earthed. In one form, the two wires are, in section, each half a circle, and each is covered with manilla paper to the requisite thickness; the two are then stranded together with the flat sides facing each other, and paper is wound round the combination. The cable is next allowed to remain in hot insulating oil until the paper becomes thoroughly saturated, after which it is passed through a hydraulic press, by which means the lead covering is fixed on it. The advantages claimed for the system are:—

(1) It is perfectly waterproof, ensures safety from fire, has high insulation properties, and is most durable.

(2) It is small and flexible and can be erected with ease in situations where wood casing and conduit erection are difficult.

(6) *Other Flexible Wiring Systems.*—In one type of *Simplex Flexible*, the wires are covered with rubber and cotton, and the whole "armoured" by tinned iron, the outside diameter of the cable being ·3 inch. In another type the wires are covered with special flame-proof material. In the *Stannos Wires* of Messrs. Siemens Bros. the copper wires are insulated with rubber or unvul, paper taped, and covered with tinned copper sheet. Various patterns of wall clips and other wiring accessories are supplied for erecting these flexible wires. The small initial cost, the ease and simplicity of erection, the rapidity with which the work can be done, and the fact that they can be easily applied to existing houses without interfering with decorations or architectural features to any extent, have given an impetus to flexible systems in recent years.

(7) *Metallic Casing.*—This is more or less a combination of the general idea of wood casing and capping and of metallic conduits. The two metallic portions (Fig. 360) corresponding to the wood casing and capping respectively are each larger than a half cylinder, so that when one is pressed home over the other a very firm connection is obtained. The system claims most of the advantages of the solid conduit systems previously mentioned, whilst the somewhat awkward process of "drawing-in" is eliminated, and the wiring admits of easy inspection just as in the wood casing system. A flexible variety of casing which can readily

Fig. 360.

be made to pass round corners, beams, mouldings, etc. (Fig. 361), has recently been introduced.

It may be mentioned in passing that single wires should not be placed in metal conduits, if alternating currents are employed, for the eddy currents developed in the tube would result in a waste of energy in the form of heat and a lowering of the pressure, thus diminishing the efficiency of the system.

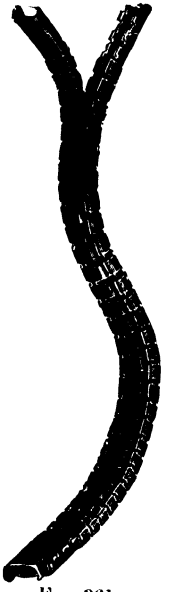


Fig. 361.

317. The choice of a System.—There are numerous other systems, even brief summaries of which space forbids our giving. The particular system to be adopted, or the particular make of conduit to be employed, must of necessity be regulated by many considerations, such as, for example, the amount of money to be spent; whether the work is to be "surface" or concealed"; whether the situation is damp or dry; whether the installation is likely to be subjected to steam, chemical effects or mechanical injury; subsequent extensions, etc. Hard-and-fast rules are in consequence impossible, but the following general ideas may be submitted:

(1) For perfectly dry situations, where rough usage is impossible and where the price is of vital importance, wood casing may fulfil requirements; taken all round, however, it is not an ideal system and is gradually becoming looked upon as the hall-mark of cheap and inferior wiring.

(2) For surface work in private houses or public buildings free from moisture, where the cost must be maintained at a fairly low figure, light gauge conduit, such as the Simplex Ordinary, suggests itself.

(3) For concealed work in plaster or cement, or for surface work subject to steam or vapour (*e.g.* laundries) or mechanical injury (*e.g.* workshops), and where at the same time the cost must be kept as low as possible consistent with the conditions, a brazed or welded tube such *e.g.* as the Simplex brazed, the uninsulated brazed and screwed of the Bergmann System, or the ordinary conduit of the General Electric Co., may be recommended.

(4) For the best work where cost is not the ruling factor, solid drawn screwed conduits of the Simplex type, or insulated tubes such as the Bergmann, etc., should be employed; the last named, used in conjunction with insulated accessories, cannot be improved upon for safety and reliability. Of course flexible systems are very convenient and quite satisfactory for many special cases.

318. Looping in.—This is a term given to a method of connection to lamps, switches, etc., whereby the ordinary process of joint-making in conductors is eliminated.

Fig. 362 depicts two lamps $L L_1$, each controlled by its own switch $S S_1$; clearly *joints must be made at* A, B, C, and D.

Fig. 363 shows the ordinary method of "looping in" applied to the same arrangement. The lower lead is carried down to the switch, bared, and looped into one clamping screw. The upper lead is similarly looped into one ceiling-rose connection. Another wire is run back from the second switch screw to the second ceiling-rose connection.

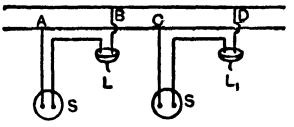


Fig. 362.

Fig. 364 shows the method to be employed with the Simplex ceiling-rose of Fig. 350. In the preceding method, one of the main cables is brought down to the switch; in this method

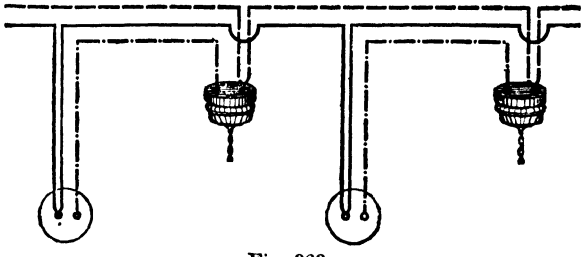


Fig. 363.

it is only necessary to bring down two single wires which carry the current for one lamp only, and not the main current in the circuit.

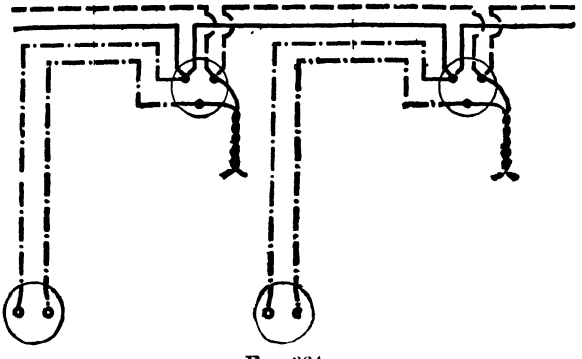


Fig. 364.

These wires leading to the switch being thinner, a smaller conduit down to the switch may be employed; thus a saving is realised alike in wire and in conduit.

319. Wiring Diagrams and Problems.—It is to be regretted that the public are unaware of the many convenient and economical adaptations of electricity in lighting, for these conveniences alone should place it easily ahead of gas. In the following we summarise a few of these adaptations:

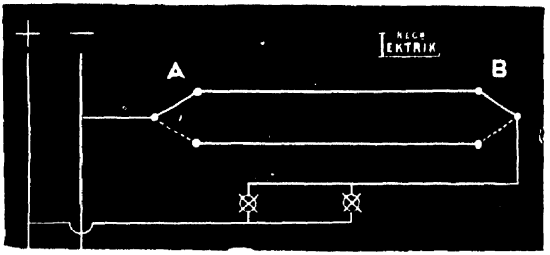


Fig 365.

(*a*) Fig. 365 shows a group of lamps which can be controlled from two points by the use of *two-way* switches A and B; it is employed in cases where it is desirable that the lamps be capable of being turned on or off at either end of a corridor or either the top or bottom of a flight of stairs. A person passing, for example, along a corridor, may turn on the lights at one end on entering, and cut them off at the other end on leaving, the circuit being then ready for the next person passing in *either* direction. The method lends itself to bedroom lighting, switch A, for example, being near the door and B near the bed

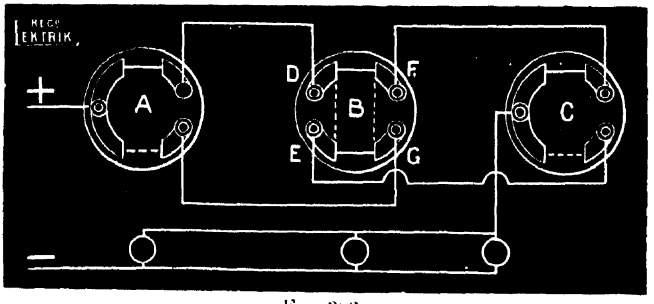


Fig. 366.

(*b*) In the case of long corridors and staircases in hotels, it is often desirable that the lamps be capable of control from intermediate positions as well as from the extreme ends; this is accomplished by the use of *intermediate* switches at the additional controlling points. Fig. 366 shows the arrangement for a three-point control, the continuous lines representing one position of the switches, the dotted lines the other; in the figure the lamps are lit, but a movement of *any one* of the switches will extinguish them, whilst a further movement of *any one* will light them up. As many intermediate switches may be used as there are intermediate points of control.

(*c*) Fig. 367 gives a useful modification of the above. On the left, in the porter's room, say, are placed a *single-way* and a *two-way and off* switch. With both these switches *off* the lamps cannot of course be lit; with the "two-way" *on* and

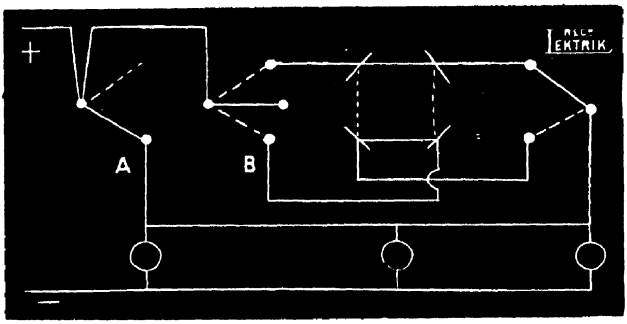


Fig. 367.

the "single" *off*, the circuit can be controlled as in (*b*) above; with the "single" *on* the lamps *cannot be extinguished by any of the other switches*, thus, at times, the housekeeper or porter can have absolute control of the lights.

(*d*) In Fig. 368 we have an arrangement suitable for an economical student who desires to study "Technical Electricity" in bed D is a lamp at the dressing table, C another near the bed; a *two-way* switch B at the door enables D to be put on or off, and irrespective of the position in which this switch is left, the *three-way*

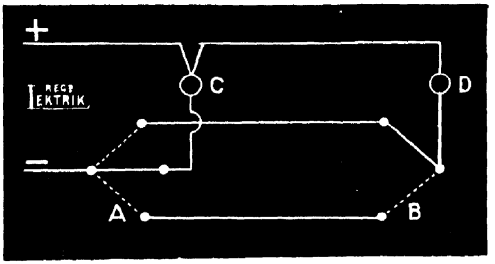


Fig. 368.

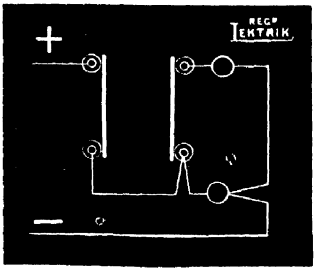


Fig 369*a*.

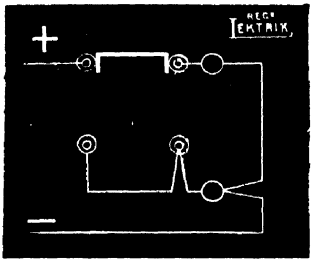


Fig 369*b*.

switch A at the bedside enables *either* lamp to be controlled, but at no time can both lamps be lit together.

(*e*) A useful adaptation of the *series parallel and off* Duplex switch in connection with a two-light electrolier is shown in Fig 369, whereby it is possible to light one or both lights with one switch; Fig. 369*a* gives the two in parallel; in Fig. 369*b* one only is lit, whilst Fig. 369*c* shows both extinguished

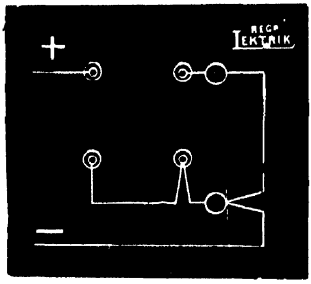


Fig. 369*c*.

(*f*) Figs 370*a*, *b*, *c*, depict the above switch controlling the lamps in a corridor; in the first all lights are on; the second shows how, after a certain hour, every third lamp only is lit, and in the third all lights are out.

(*g*) **Problem.**—*Two three-light pendants* (*a*) *and* (*b*) *are to be controlled by two switches, one intended to light one lamp, the other two lamps in each pendant. When both switches are closed, one lamp in* (*a*) *and two in* (*b*) *light up. Indicate by a diagram the mistakes in the connections and show how you would remedy same* (*C. & G.—Prelim*).

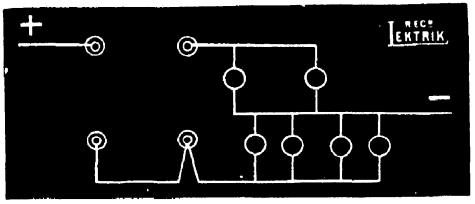


Fig 370*a*

In Fig. 371 P Q R and P_1 Q_1 R_1 denote the three contact pieces on the ceiling-roses, K_1 the switch intended to light one lamp and K_2 that intended to light two lamps in each pendant. If the two lamps of (*a*) be connected to stud Q and the one lamp of (*b*) between P_1 and Q_1 an examination of the full lines will show that, with both switches closed, one lamp in (*a*) and two in (*b*) will light up. If the returns from the two lamps of (*a*) and the single lamp of (*b*) be joined to R and R_1 respectively (dotted lines) the necessary conditions of the problem will be realised.

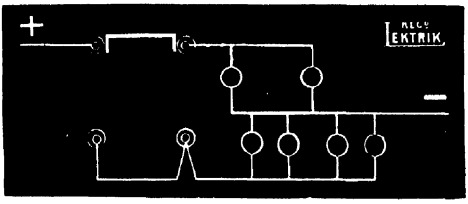


Fig. 370*b*.

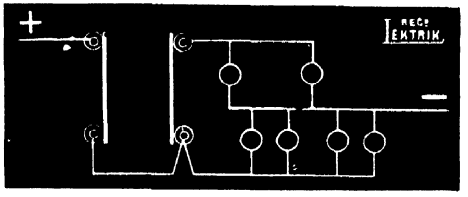


Fig 370*c*.

320. Disposition and Number of Lamps for House and Workshop Lighting.—Of the many approximate rules which have been given, the two following may be taken as typical and useful:

(*a*) *Incandescent Lighting.*—One 25 c.p. lamp 0 ft. from the floor lights a circle 8 ft. in diameter with quite good illumination.

(*b*) *Arc Lighting.*—For workshops, etc., 10 mpere arcs each about 15 ft. above the floor nd 45 ft. apart, give satisfactory illumination.

For the wiring contractor various tables ave been compiled from which the necessary umber of lamps in specified cases may be stimated: thus a general formula is:—

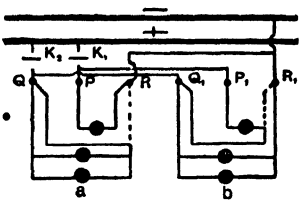


Fig. 371.

$$\text{No. of lamps} = \frac{\text{Floor area in sq. ft.} \times \text{necessary candle feet}}{\text{Utilisation factor} \times \text{Lumens for the lamp used}}.$$

Tables give the necessary illumination in candle feet for various cases—workops, offices, sewing rooms, etc. The utilisation factor—also given in tables for pecified conditions—is the fraction of the light given out by the source which is vailable on the working plane. The "lumens per lamp" for various types of mps working at various pressures is also given in tables. Knowing the total umber of lamps the "spacing" is readily settled.

EXAMPLE.

A four-storied mill is to be lighted with two hundred 16*-candle-power* 200 *volt amps on each floor, and the height of the mill is* 48 *ft. Design a system of mains from the switchboards on the respective floors to the main switchboard—which is* 80 *yards distant from the point where the mains enter the building—such that each floor shall be independent of all the others, and the drop of pressure between any switchboard and the main switchboard shall not exceed* 1½ *per cent. when all the lamps in the building are turned on.* (O.)

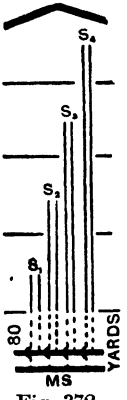


Fig. 372.

Since the height of each room is 12 ft. and the switchboards for each (viz. $S_1 S_2 S_3 S_4$, [Fig. 372) are, say, 5 ft. (usual) above the floor:

(*a*) Length of mains, lead and return, from main switchboard M S to $S_1 = 2\ (240 + 5) = 490$ ft.

If the lamps take 4 watts per candle, the total watts delivered to the first floor $= 16 \times 4 \times 200$. If I be the current, then—

Since the pressure = 200 volts,

$$200\ I = 16 \times 4 \times 200,$$

$$\therefore I = 64 \text{ amperes.}$$

"Drop" allowed between M S and $S_1 = 1\frac{1}{2}$ per cent. of 200 = 3 volts.

Hence since $$R = \frac{E}{I}.$$

Total resist. of mains from M S to $S_1 = \frac{3}{64}$ ohm.

But $R = \rho \frac{l}{a}$; $\therefore a = \frac{\rho l}{R}$;

i.e. $a_1 = \frac{\cdot 66}{10^6} \times \frac{490 \times 12}{1} \times \frac{64}{3}$;

$= \cdot 082790$ sq. in.

= cross-section of mains for first floor.

(*b*) Length of mains, lead and return, from M S to $S_2 = 2\ (240 + 17$ = 514 ft. The current is, as above, 64 amperes, and the resist. $\frac{3}{64}$ ohm, so tha the "drop" is $1\frac{1}{2}$ per cent.

Hence if a_2 be the cross-section—

$$a_2 = \frac{\cdot 66}{10^6} \times \frac{514 \times 12}{1} \times \frac{64}{3};$$

$= \cdot 086845$ sq. in.

= cross-section of mains for second floor.

(*c*) Similarly, length of mains for third floor from M S to $S_3 = 2\ (240 + 29$ = 538 ft., and if a_3 be the cross-section—

$$a_3 = \frac{\cdot 66}{10^6} \times \frac{538 \times 12}{1} \times \frac{64}{3};$$

$= \cdot 09090$ sq. in.

= cross-section of mains for third floor.

(*d*) And, again, length of mains from M S to $S_4 = 2\ (240 + 41) = 562$ ft., an if a_4 be the cross-section—

$$a_4 = \frac{\cdot 66}{10^6} \times \frac{562 \times 12}{1} \times \frac{64}{3};$$

$= \cdot 094955$ sq. in.

= cross-section of mains for fourth floor.

EXERCISES XXIII.

1. Describe, with sketches, the essential features of a switch for inside wo suitable for controlling a 5-ampere circuit, the pressure being 100 volts. (P.)

2. Is it desirable to use fuses on electric light ceiling-roses and switche Give your reasons. (P.)

3. Contrast the advantages of wood casing and of interior conduits for hous wiring. (P.)

4. How would you fix and protect electrical conductors in the following po tions: (*a*) between floor and ceiling; (*b*) running through a wall; (*c*) in a dar cellar? (P.)

5. Describe, with sketches, a system of wiring houses in which the wires r in metallic tubes. State what you consider to be the advantages and disa vantages of the system you describe, and mention any case in which special ca is needed in carrying out this method of wiring. (P.)

6. Describe, with sketches, the method of wiring for a four-light pendant so that one light may be turned on or off both from the top and from the bottom of a flight of stairs with switches at top and bottom, and the other three with a switch at the top only. (P.)

7. Describe, with sketches, the method of wiring and fixing a two-lamp bracket fitting with switches on the wall beneath, so that either of the two lamps can be turned on independently. (P.)

8. What do you understand by "looping in," and what are the special advantages of the system?

9. An electric light installation, the wiring of which is contained in an earthed metallic covering, is supplied from a source having its negative side definitely earthed, and is provided with two main fuses, one protecting the positive and the other the negative bus-bar of a circuit distributing board. It is found that with the negative main fuse withdrawn there is no interruption of supply; but if with this fuse still out, the negative fuse of one of the circuits on the distributing board is withdrawn all the other circuits are interrupted, but this one still lights. State the nature of the fault, and indicate its position by means of a diagram. (O.)

10. A building of four floors has ninety 8-candle-power lamps on each floor: the height of each floor is 18 ft., and the mains run straight up in the middle of the building. On each floor is a passage extending 100 ft. each way from the middle, and in the passage on each side of the middle are five lamps. Opening out of the passage on each floor there are twenty rooms to the front and twenty rooms to the back, each room containing two lamps. Calculate the size of the mains coming from the bottom to the top of the building, of the sub-mains in the passages, and of the lamp leads, on the supposition that when all the lamps are turned on the drop in pressure from the basement to the furthest lamp does not exceed 2 volts, and that the pressure supplied is 100 volts. (O.)

11. A rectangular building consists of five stories, of 12, 12, 11, 11, and 10 ft. in height respectively, and each floor measures 50 ft. by 100 ft., and is to be lit with eighty 16-candle-power lamps, the lamps being distributed evenly over the floors on the two-wire system, and regarded as taking 60 watts each. The main conductors pass up the wall at about the centre of one of the long sides of the building, and 3-ampere-way distributing fuse boxes are to be provided on the floor; the lamp circuits are to be of 3/20 conductor and may be run parallel with either wall, but not diagonally. Calculate the sizes of the mains, and the number of branch circuits, so that the maximum fall of potential when all the lamps are on does not exceed $1\frac{1}{2}$ per cent. (*a*) when the supply is at 100 volts pressure; (*b*) when the supply is at 200 volts pressure. The resistance of 3/20 conductor is 0·27 ohm per 100 ft., and of a cubic inch of copper 0 66 microhm. (O.)

CHAPTER XXIV.

ELECTRIC BELLS, TELEGRAPHS AND TELEPHONES—MISCELLANEOUS.

321. Electric Bells, Relays, Fire Alarms, Indicators.—The construction and action of the **trembling bell** which continues to ring as long as the push is closed will be gathered from Fig. 373. In it L M N R is an electromagnet opposite the poles of which is a soft iron armature I attached to a brass upright K by the steel spring S. The brass screw E works in the bearing H, and its end and the back of the armature, where it touches, are tipped with platinum. One end of the coil is attached to terminal T, the other end being attached to the upright K, whilst terminal T′ is joined to H. The armature I carries the hammer C, which strikes the gong G. On closing the push P, current flows, the armature I is attracted, and C strikes G. When I is drawn forward, the circuit is broken, the cores lose their magnetism, and the armature flies back again, making contact with E, and the action is repeated; thus the bell continues to ring as long as P is closed.

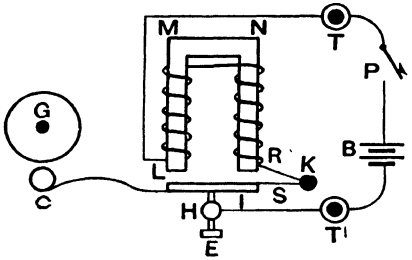


Fig. 373.

The screw E and armature I are tipped with platinum, owing to the fact that a spark, due to the extra current at break, takes place when I is attracted. This tends to oxidise and damage the contact points, and platinum does not readily oxidise or fuse.

By connecting K to T′ and breaking the connection between H and T′ the bell is converted into a **single-stroke bell.** When the push is closed I is attracted, and the hammer strikes the gong. As, however, the circuit is *not* broken the armature *remains attracted.* To ring again the push P must be opened and again closed.

Fig. 374 depicts the **continuous-ringing bell**—*i.e.* one which continues to ring even after the push is opened. The armature carries a projection *p*, on which rests a lever L, which is joined to H. A second contact screw and bearing are joined to a third terminal T_2. The

figure shows the bell in its normal condition. When P is closed, current flows *via* T_3 K S H T_1 and back to the battery, I is attracted and the hammer strikes the gong. When I is attracted the lever L drops from the projection *p* on to C, and even if P be opened a path is still there for the current—viz. B to T_3 K S H L C T_2 and back to B. Thus the bell continues to ring until L is raised from C and placed on the projection *p*.

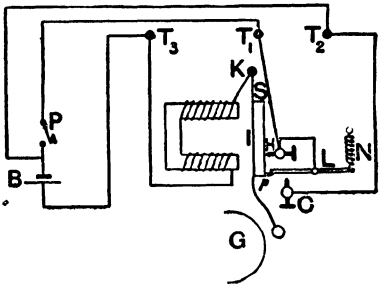


Fig. 374.

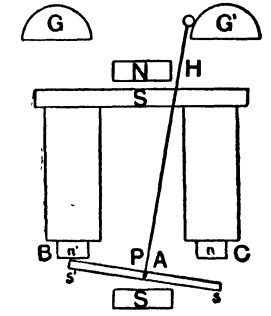


Fig. 375.

Fig. 375 shows the essentials of a **magneto bell** worked with alternating current. The armature A is capable of rocking about P, and carries the hammer H, which strikes alternately the gongs G and G'. N and S represent the poles of a permanent horse-shoe magnet; it is clear that S, acting on the armature, will induce a north pole in the centre thereof and south poles *s* and s_1 at the ends, whilst N, acting on the horse-shoe iron core, will induce a south pole at the bend and north poles *n* and n_1 at the ends. Imagine now that the current flows in such a direction that B becomes a north and C a south; clearly the attraction between *n'* and *s'* will be increased, whilst repulsion will ensue between the poles on the right, the armature will tilt, as shown, and H will strike G'. When the current reverses so also do these actions and H strikes G.

In the case of large buildings, the current passing through a long line wire may be too weak to ring the bell, and in such cases a **relay** is employed. Fig. 376 will render the principle clear. When P is closed current from B passes through the coils of the relay R, and the armature is attracted, thereby bringing the projection *p* in contact with the screw S and closing the local battery circuit on the right, current from B_1 ringing the bell.

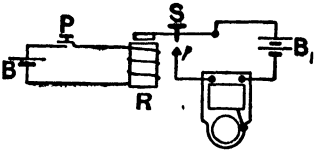


Fig. 376.

Fig. 377 depicts a common type of **thermostat** or **fire alarm.** The vertical rod is a strip of brass and one of iron fixed together, the brass

being on the left in the figure. When the temperature rises the brass expands more than the iron; the compound rod bends inwards, in time making contact at the bottom and ringing the alarm bell. In another type the air in a closed vessel expands by the heat, bulges out the top of the vessel, thereby making contact with a stud and closing the bell circuit. In yet another the chamber is partially exhausted; when the glass front is broken by a person giving the alarm, the air rushes in, forcing a thin metallic plate against a contact screw behind it, thereby closing the circuit and ringing the alarm bell.

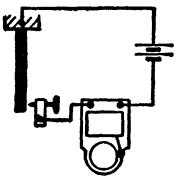


Fig. 377.

Fig. 378 shows a common type of **indicator.** When current passes through the indicator coils the armature is attracted, and the flag attached thereto indicates the quarter from which the summons has come

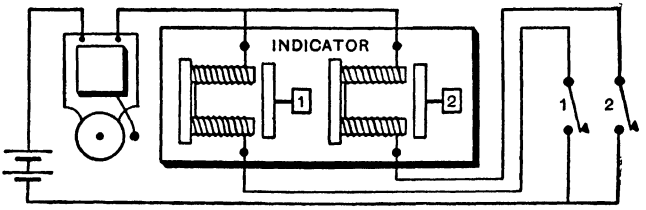


Fig. 378.

322. Circuit Diagrams.—When a number of bells are required to ring simultaneously on the closing of one push they are placed in parallel (Fig. 379A).

Fig. 379B shows how A may ring up B or B ring up A, without ringing the bell at his own end. Fig. 379C shows the same with special switches and a single line of wire between them, the earth being used as "return."

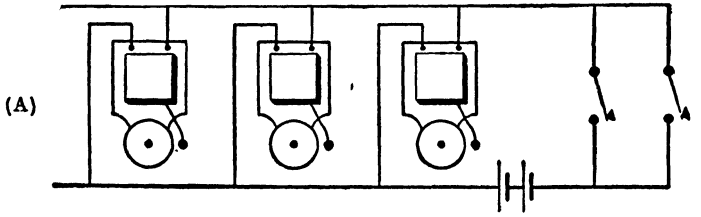


Fig. 379.

Fig. 379D shows the connections for ringing a bell from four different rooms—A, B, C, D—a four-way indicator and relay being employed.

Burglar alarms may be looked upon as merely specific pushes attached to windows or doors, etc., in such a way that when the latter are interfered with

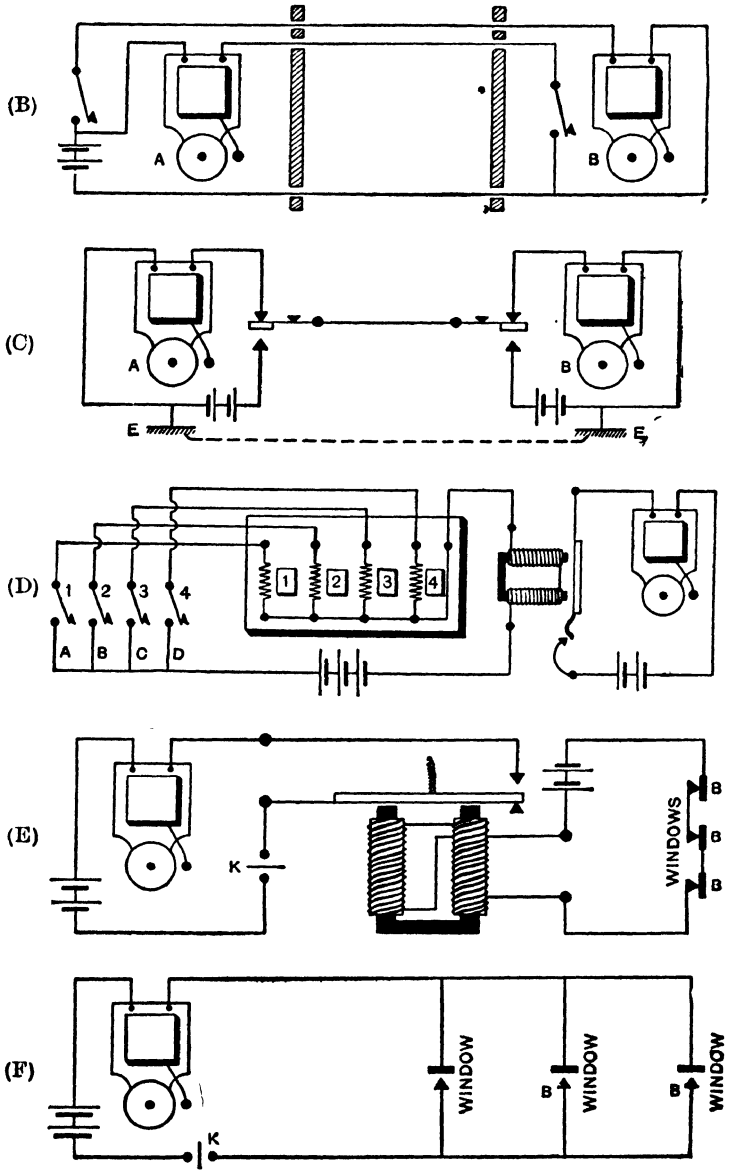

(B)
A
B
(C)
A
B
E
E
(D)
1
2
3
4
A
B
C
D
1
2
3
4
(E)
K
WINDOWS
B
B
B
(F)
K
WINDOW
WINDOW
WINDOW
B
B

Fig. 379.

an alarm bell rings. Fig. 379E shows the **closed-circuit system.** As long as matters are normal, current flows, and the armature of the relay is held down in the position shown, but when B is *opened* by the opening of a window, say, the armature goes up against the stud, closing the bell circuit, and the bell begins to work as a trembler. Fig. 379F shows the **open-circuit system:** when B is *closed* by the opening of the window the bell rings. In the first system if more than one window is to be fitted they are placed in series: in the second system they are in parallel. It is usual to provide a switch K to cut out the alarms during the day.

323. Principles of Telegraphy.—Only the simplest reference to one or two general principles can be given here. For fuller details the student must refer to some modern book on Telegraphy.

Overhead lines are of galvanised iron or copper supported on wooden poles and insulated from them. **Underground lines** are generally in the form of a cable, the separate copper wires being insulated from each other by gutta-percha or they are paper insulated, covered with a lead sheath: they are laid in cast iron pipes or earthenware conduits. **Submarine cables** are composed of an inner strand of copper wires, a gutta-percha envelope of several layers, a covering of tarred hemp, and an outer covering of iron wires covered with hemp. The shore ends are made much stronger than the rest. It will be noted, in the diagrams that follow, that in telegraphy the earth is used as the (so-called) "return."

The *principle* of the **single current key** will be gathered from Fig. 380. In the normal condition the key is held in the position shown by the spring S, and the "line" is therefore connected to R which in turn is in connection with the recording apparatus. When the right-hand side of the key is depressed the circuit is broken at R and the line is connected to B which in turn communicates with the battery.

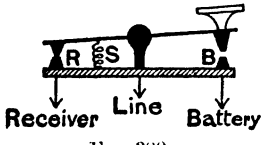


Fig. 380.

In the **Morse System** the two signals which form words, etc., according to a pre-arranged code (the Morse Code) are currents of short and long duration ("dot" and "dash"). One method of receiving them is by means of a **Morse Sounder** which consists of an electromagnet fitted with an armature in such a position that the latter is drawn down upon a stop when the current flows and flies back against another stop when the current ceases; the operator thus listens to the clicks and notices whether the intervals between them are "short" or "long" ("dot" or "dash"). Another method of receiving the signals is by means of the **printing mechanism;** in this the armature is attached to a small inked wheel and under this is drawn by clockwork a long narrow strip of paper. When the armature is not attracted the wheel is clear of the paper but when attraction ensues the wheel comes into contact with the paper and

prints a dot or a dash according as the duration of the current is "short" or "long." Combinations of dots and dashes form letters, etc., according to the Morse Code, *e.g.* A · —, B — ···, C — · — · etc.

A simple circuit is shown in Fig. 381. If the key on the left be depressed it will be lifted from the upper contact and current will flow from the battery on the left to the depressed key, thence to line, to the sounder on the right, to the key and to earth. The line galvanometers inserted at each end (Fig. 381) show the operator, when "sending" if his apparatus is working correctly. If the distance between the stations is great the line current may be too weak to properly work the sounder; in this case a **relay** is employed, *i.e.* the line current actuates an electro-magnet so that it attracts an armature and thus brings a local battery into circuit to operate the sounder. The action of the relay will be gathered from Fig. 381*a*.

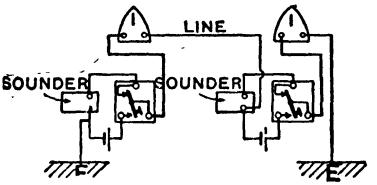


Fig. 381.

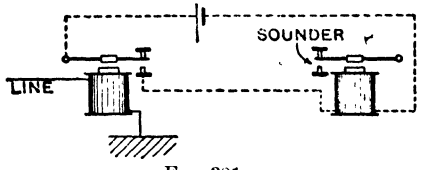


Fig. 381*a*.

324. Principles of Duplex Telegraphy.—In **Duplex telegraphy** it is possible to send two messages simultaneously along the same wire, *i.e.* station A can transmit to station B at the same time as B is transmitting to A. The principle of one method—the **differential duplex**—will be gathered from Fig. 382. The relays M_1 and M_2 are each wound with two equal coils in opposite directions and arrangements are such that, for station A for example, the resistance of coil D_1 *plus* the resistance R_1 is equal to the resistance of coil C_1 *plus* the line resistance *plus* the resistance of coil C_2 and a similar relation holds for station B.

It is almost unnecessary to say that Fig. 382 is diagrammatic only, in order to show the principle: this is particularly so in the winding, etc., of M_1 and M_2. The sounders, etc., which respond when the armatures of M_1 and M_2 are affected are not shown.

If A *only is transmitting to* B then on closing K_1 the current divides into two *equal* parts at X and since C_1 and D_1 are wound in opposition they cancel each other's effect on M_1 which therefore does not respond, but the current passing through C_1, line, C_2, to earth actuates M_2 so that the signal is recorded at B. A similar explanation holds *if* B *only transmits to* A.

Suppose now that A *and* B *are both "sending" together, say each*

sending a "long current." As the two batteries are working "equal and opposite" through C_1, C_2 and line, there will be (practically) no current in this part, but current continues to flow through D_1 and D_2 so that both M_1 and M_2 respond to a "long current," *i.e.* they respond as long as K_1 and K_2 are held down.

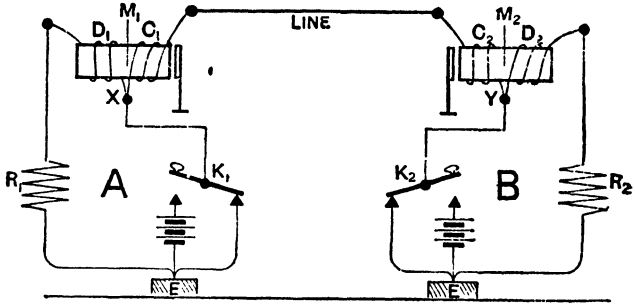


Fig. 382.

Now imagine that, simultaneously, A *sends a "long" and* B *a "short."* They start together say, then, when the key at B opens (key at A still down) the battery at B is cut out so that no current goes from it through D_2 but current now comes in from the line through C_2 so that M_2 continues to respond to the "long" from A. At A, however, there are now equal currents in both C_1 and D_1 from the battery at A which cancel each other's effect on M_1 so that it no longer responds. Thus B's signal to A ceases when K_2 is opened but A's signal to B continues as long as K_1 is closed. The student should think out other cases for himself.

The principle of a second method—the **bridge duplex**—is indicated in Fig. 383. The receiving system is put across two branches of the sending system so that the sending current raises the potential of the two ends equally, and thus the sending current does not affect the receiving system at the sending end. On the other hand, the current coming in has two paths open to it, one of much higher resistance than the other, and thus it divides unequally, and this inequality is caused to actuate the receiving system. This method is shown in Fig. 383. In the figure *m* is the receiver on the left, R is a resistance adjusted so that the resistance between the point *c* and earth is the same whether we consider the circuit through R to earth or the circuit

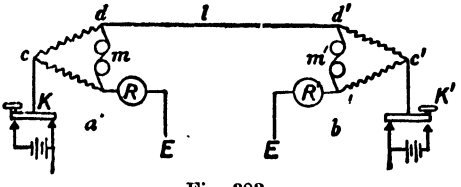


Fig. 383.

through *l* to the distant station to earth. Hence on depressing the left-hand key *m* is unaffected, while *m'* responds. Similarly, when the right-hand key is depressed, *m'* is unaffected, while *m* responds, and similar results will follow when both stations are working. In "quadruplex" four separate messages can be passed along one wire between two stations, *i.e.* two from A to B simultaneously with two from B to A. In what are frequently called "multiplex" more, usually up to six, can be sent, and these may all be in one direction, or some in one direction and the remainder in the opposite direction.

325. Principles of Double Current Working.—If the simple relay be delicately adjusted for weak currents the residual magnetism may cause the armature to "hold on" during the interval between say two "dot" signals, so that the dots become more or less a dash, whilst on the other hand if the adjustment be in the opposite direction the armature may not readily respond and dots may not be recorded. Again the electrostatic capacity of the line tends to delay the first signal (for the "line" must first be "charged") and to prolong the signal when the key is raised (for the line charge comes out at both ends) and further if a second signal be sent while the discharge is taking place it will be partly cancelled and a dash may become a dot or a dot may not be registered. **Double current working** removes these defects to a very great extent.

A special **P.O. Standard Relay** is employed in which the armatures are magnets (they are really iron magnetised by induction by a permanent magnet). When the sending key is depressed current flows in a certain direction through the relay coils with the result that the armatures move in such a direction that the local battery circuit is closed and the sounder is affected (this current is referred to as the "marking" current). When the key is raised another current flows through the relay coils in the opposite direction with the result that the armatures move in the opposite direction and the local battery circuit is broken so that the sounder is "out" (this current is referred to as the "spacing" current).

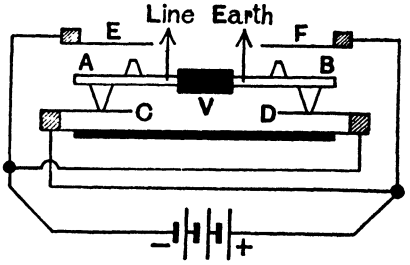


Fig. 384.

The principle of action of one type of **double current key** is shown in Fig. 384. The brass bar A B is carried on the end of the lever of the key: it is really in two halves separated by vulcanite V. In the figure the positive pole of the battery is to line and the negative pole to earth: this is the "spacing" direction so that the distant sounder is unaffected. On depressing the key, A B rises and makes contact

with E and F so that the negative pole is to line and the positive to earth: this is the "marking" direction and the distant sounder is affected. On releasing the key, A B falls to C and D again and the reverse current flows breaking the local battery circuit at the distant sounder.

In another type two batteries are used. When the key is depressed the *negative* pole of one battery is put to line (marking direction) and when the key is released this battery is cut out and the *positive* pole of the second battery is put to line (spacing direction).

"Duplex" is also worked on the double current system.

326. Principles of the Central Battery Omnibus System.—A system of telegraphy largely used in this country is that known as the **Central Battery Omnibus System** (Fig. 385). In this system, which enables a number of the smaller offices to be grouped on one circuit, only one battery is required, and this is located at the head office.

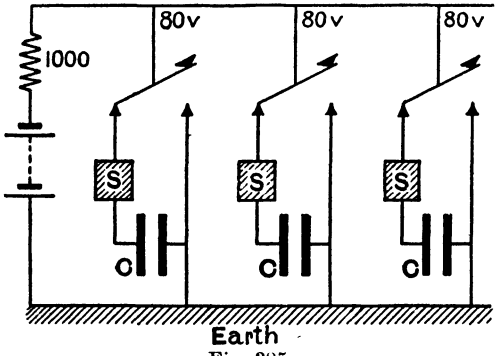


Fig. 385.

Normally the line and condensers are charged to a potential of 80 volts (negative). When any one station depresses its key all the condensers discharge through their respective polarised recorders, thus recording a mark at each office. On the release of the key the condensers again become charged, but this charge, passing through the recorders in a "spacing" direction, leaves the sounders unaffected. The object of the 1,000 ohm resistance at the head office is to ensure a sufficient drop of potential at all stations when any one key is depressed.

327. Miscellaneous.—The **Wheatstone Automatic System** is much used in the British Post Office. The message is sent by passing a perforated tape through the sending machine, the tape being perforated beforehand in a punching machine—a special arrangement of circular holes distinguishing between dots and dashes. The perforated tape may be passed through the sender very quickly, and the receiving gear sends out another slip on which a broken inked line conveys the message by means of a Morse code. A speed of signalling as high as 650 words per minute has been reached with this apparatus, but the usual maximum working speed is about 400 words per minute.

In addition to the above, there are various systems in use by means of which the message is reproduced at the distant end printed in ordinary characters on a tape. One such method is the **Hughes' Type-printing System,** of which the common working speed is about forty words per minute. Another method is the

Bandot System now extensively employed throughout the United Kingdom by which two to six operators are enabled to send messages over a single line. To effect this the station at either end of the line is fitted with a "distributor" consisting of a metallic disc divided into concentric rings which again are divided up into segments. Over these a series of brushes are made to pass. The two distributors are caused to run in unison so that the brushes at either end are passing over corresponding segments at the same time. At the "sending" end each keyboard, consisting of five pianoforte keys, is connected to five segments on the distributor, and positive and negative combinations from the five keys are sent to line when the brushes pass over the segments. These currents, at the "receiving" end are passed on by the revolving brushes to five segments connected with electromagnets, the operation of which brings printing mechanism into play and records the message in Roman capital letters. The system admits of being "duplexed."

In submarine cable telegraphy condensers are used in the line and the **Syphon Recorder** is employed for receiving the messages. This is really a D'Arsonval galvanometer in which the moving coil carries a style which leaves a trace in ink on a moving sheet of paper.

328. Principles of Telephony.—As in Telegraphy, only the simplest reference to one or two general principles can be given here. For fuller details the student must refer to some modern book on Telephony.

A section through the **Bell Magneto-Telephone** is shown in Fig. 386. Here M is a permanent magnet carrying at one end a piece of soft iron S. This forms the core for a coil C which has leads to the "line" and "return." In front of S is fixed a very thin soft iron disc D D. The main body of the instrument is of vulcanite.

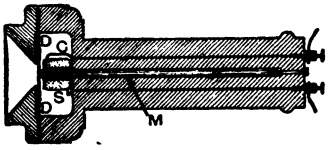


Fig. 386.

The action is as follows: When a person speaks into the orifice the waves of sound cause the disc to vibrate, and as it moves to and from the soft iron core the distribution of the lines of force in the coil is altered. A current is therefore generated in C, the E.M.F. of which is determined by the rate at which the number of lines of force passing through C is changing. At the distant end of the line is a similar instrument, the terminals of which are directly connected to those at the transmitter, so that as the currents generated in C vary with the sound, so also do those received at the distant station. The reverse operation then takes place at the receiver. The current in its coils varying, the field in the neighbourhood varies, and the soft iron piece of the receiver attracts with varying strengths the vibrating disc immediately in front of it. Thus the disc at the receiver copies the movements of the disc at the transmitter. These movements being transferred to the air, the sound also is reproduced. No battery is

needed to work the instrument, as the vibrating plate generates all the current required.

If sound waves are produced near a loose electrical contact, notably in the case of one or more sticks of carbon supported lightly by two fixed blocks of carbon, the vibrations cause the resistance of the point of contact to vary enormously, and thus a battery in series with the loose carbon will send a varying current to line as long as the vibrations continue; this varying current passing through a Bell magneto-telephone will actuate it in the manner described above so that the sound is reproduced. Such an arrangement is shown diagrammatically in Fig. 387. Modern **Microphone Transmitters** act on this principle; they contain granules of carbon between two plates of carbon, one of which is attached to the diaphragm of the transmitter and therefore receives the air vibrations.

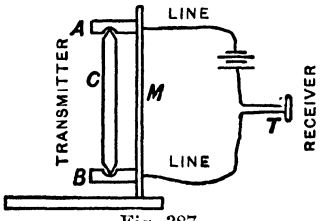


Fig. 387.

In practice the microphone is used as the transmitter and the Bell magneto-telephone is used as the receiver. Further, the microphone is not directly connected to the line as in Fig. 387, but to the primary of an induction coil, the secondary of which is connected to the line; by this device the E.M.F. is magnified in the ratio of the secondary to the primary turns (Art. 278), and thus can overcome a high resistance in the line. Further, in this way the resistance of the microphone may be kept low, and the constant current does not find its way to line but only the varying component.

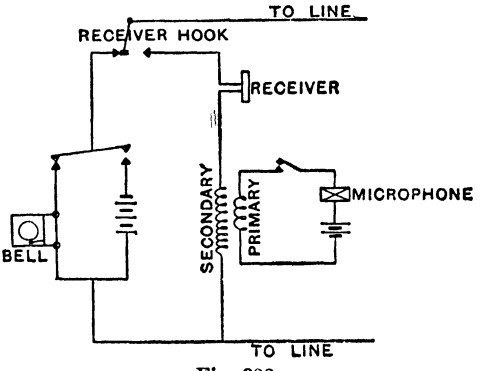


Fig. 388.

A simple circuit is shown diagrammatically in Fig. 388. When the receiver is *on the hook* the circuit is as shown and the "calling up" bell can be rung by current coming in from the line. When the receiver is taken off the hook the latter moves to the other contact and the "speaking" circuit is then complete. A switch is included in the microphone circuit so that the battery is only sending current when the telephone is in use.

Another method of calling up the far end is by means of a magneto

machine which is attached to the instrument. This is shown in Fig. 389; the student will be able to make out the action for himself.

A further method is by means of one battery which is at the exchange (central battery system). In telephone exchanges small electric lamps light up automatically when the instruments are lifted from the hooks, and the exchange then rings up the other person with whom speech is desired connecting the two subscribers together. Other lamps indicate to the exchange operator when conversation is finished so that no ring off is necessary. There are many recent developments in central battery working which lessen the work of the operator and increase the efficiency of the system, but these cannot even be mentioned here.

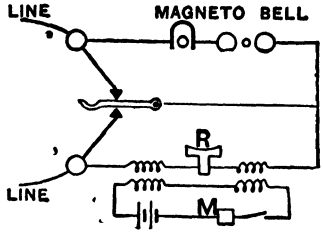


Fig. 389.

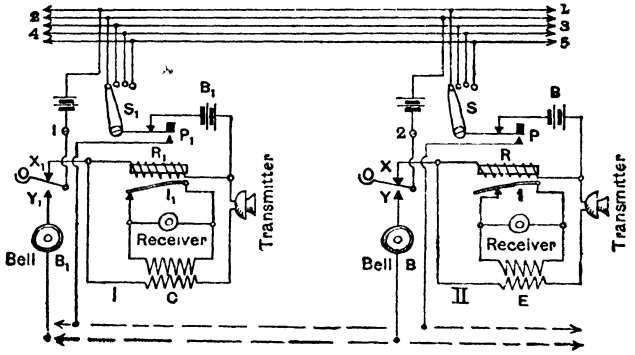


Fig. 390.

329. A Simple Telephone Circuit.—Fig. 390 will serve to explain a simple arrangement for five subscribers with a special device to secure, as far as possible secrecy in conversation. For II to call up I, the selector switch S is turned to contact (1) and P is depressed. This connects the battery of the called instrument through the switch hook O_1 which is held down on Y_1 by the weight of the receiver, to the bell B_1 and back through the common return, P, S and line. To answer the call the receiver at I is taken from the hook (which then rises making contact with X_1) and S_1 is slowly rotated over the studs, until a loud click is heard (in both receivers) which indicates that the stud of the calling instrument has been reached.

The current circuit at this stage is as follows: from battery B to switch S, thence to line (1) to switch hook O_1 contact X_1 coil C, transmitter, battery B, switch S_1 line (2) to switch hook O, contact X, coil E, transmitter and back to B.

In addition a portion of the current flows through the relays R and R_1, the armatures I and I_1 are attracted and the sudden flow of current in the receiver circuit produces the signalling click referred to, thereby indicating that the "calling and "called" circuits are in connection. The conversation can now be carried on in the usual way and a third party will be unable to overhear the conversation for being unable alone to establish a complete circuit, the short circuit across the receiver will not be released, and the receiver will therefore not be affected.

If several instruments happen to be in use at the same time, circumstances may arise which render it possible to overhear if desired. As an exercise the reader should think out the case of A calling up B at the same time that B is calling

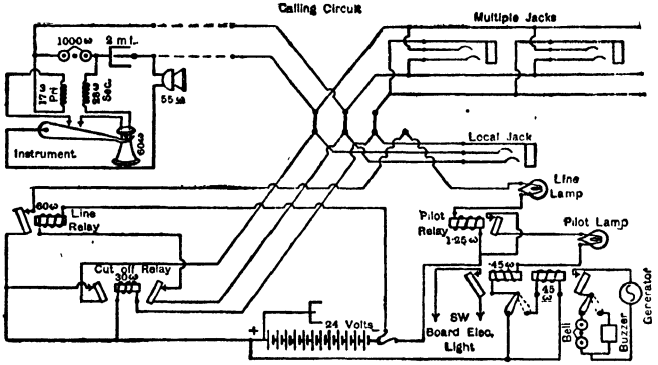


Fig. 391.

330. The Central Battery System.—Only the simplest reference to this can be given here.

(1) *Calling Circuit.*—The usual multiple switchboard is utilised, that is, the internal wiring in the exchange is carried right round the switchboard, and at stated intervals wires are tapped off and terminated on two metallic springs called a jack, thus allowing connection to be made with the line when a plug is inserted. Every line is within reach of any one operator, but the arrangements are such that a subscriber, when he calls, does so in front of one operator only. The outline of the operations will be gathered from Figs. 391 and 392.

When the subscriber takes his receiver from the rest, current flows from the battery to one armature of the cut-off relay, thence to the line, round the subscriber's instrument, along the other line to the second armature of the cut-off relay, thence to a 60 ohm line relay, and back to the battery. The line relay being thus actuated, current flows from the battery to the line relay armature, thence to the 24 volt line or calling lamp, to a 1·25 ohm pilot relay and back to the battery. When the line lamp glows the operator knows that a subscriber is calling, and also to what class of service he is entitled.

The actuating of the 1·25 ohm pilot relay results in current passing through a ·45 ohm night bell relay, a ·45 ohm automatic lighting relay, and a 24 volt pilot

lamp. One pilot lamp is fitted per panel and is common to 60 line lamps: when it glows, it indicates that one of the 60 line lamps on the same panel is calling.

Each of the remaining two relays is now affected. The night bell relay closes a circuit which results in a bell or buzzer being actuated by current from a small alternator. This bell or buzzer is common to the entire exchange, and is only made use of during periods when business is very quiet. It indicates that there is at least one pilot lamp glowing, and therefore at least one line lamp lit. The remaining relay closes the lighting circuit, and thus lights up the switchboard. The reader should make a careful study of these circuits from Fig. 391.

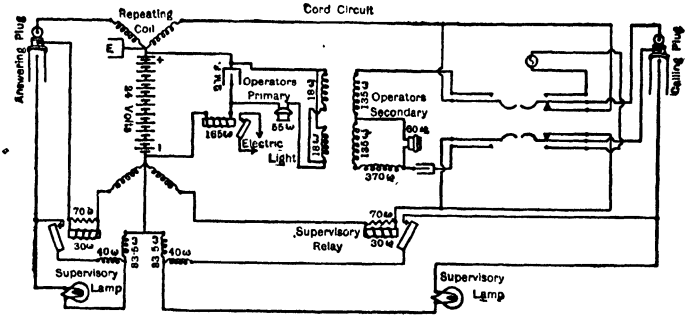


Fig. 392.

(2) *Cord Circuit.*—When a line lamp glows, the operator inserts the answering plug in the jack just below the lamp, and current then flows through the 30 ohm cut-off relay to the bush of the jack, through the cord, the supervisory lamp, and an 83·5 ohm resistance and back to the battery (Fig. 392). Both armatures of the cut-off relay being attracted, it will be found on reference to the calling circuit diagram that all the calling equipment is now cut out, and therefore all relays fall back to their normal position. The line current is now supplied as follows: —From the battery through one leg of the repeating coil to the tip of the plug, through the short spring of the jack and out to the line, thence round the instrument and back to the ring of the plug, through a 30 ohm supervisory relay shunted by a 70 ohm non-inductive resistance, through another leg of the repeating coil and back to the battery. This current actuates the supervisory relay and causes it to shunt the supervisory lamp, and the current through this lamp is thus too small to render it incandescent.

Should the line required be disengaged, the calling plug is inserted into the jack in connection with the particular number required. As the receiver of the subscriber required is still on the rest, no current flows round the line, the supervisory relay in this cord is not actuated and this supervisory lamp therefore glows. The operator thereupon depresses a ringing key and an alternating current flows to the condenser in the instrument, which immediately discharges through the bell coils. When the subscriber called removes his receiver from the rest the supervisory relay is actuated, the lamp is shunted and immediately extinguished. When the conversation is completed and the receivers returned to

the rests both supervisory lamps will again glow. The operator then withdraws both cords from the jacks.

Other details relating to the registration of the calls, the question as to whether the subscriber required is already engaged, and the complicated circuits in connection with junction working are too lengthy to be discussed here.

For fuller details and modern improvements—"manual" and "machine"—the student must refer to some modern book on Telephony.

331. Principles of Wireless Telegraphy.—Here again, only the simplest reference to one or two general principles can be given. For fuller details the student must refer to the *Advanced Text-Book of Magnetism and Electricity* (Chap. XXII.) and to specialised works on the subject. Imagine the prongs of a tuning fork to be pressed towards each other, between the finger and thumb. They are now "charged" with a quantity of energy stored up as potential energy of strain in the material of the prongs. When the prongs are let go this energy is "discharged" by a rapid series of oscillations in each of which there is a transformation of the potential energy of strain into kinetic energy of motion and re-transformation of this kinetic energy into strain potential energy. During each oscillation also there is dissipation of energy, as heat due to molecular friction in the prongs and during the vibration of the fork energy is radiated out into the air as **sound waves.** The *velocity* (v) is 1,120 ft. per second. The *frequency* (n) is the number of complete vibrations in one second, the *period* (t) is $1/n$ and the *wave length* is v/n: the latter is of course the distance the sound travels during one complete vibration. **The student must note that these are waves in the air.**

Picture now a charged condenser, and let its plates be connected by a wire so that the condenser begins to discharge. If the resistance be large the discharge consists of a steady flow from one plate to the other, the charges thus neutralising and the potentials of the plates both becoming zero. If, however, the resistance be below a certain value the discharge consists, not of a steady flow, but of a number of rapid oscillations or surgings to and fro: the first "rush" more than neutralises the opposite charge and charges the condenser in the opposite direction; this is followed by a reverse rush which again "overshoots the mark" and charges the condenser in the same way as it was originally, and so on. Each successive oscillation is weaker than the preceding; thus, after a number of such surgings the discharge is complete and the potentials of the plates equalised. At each oscillation the electrostatic energy of the condenser field is converted into electromagnetic energy accompanied by the dissipation of a small quantity of energy as heat in the conductor (and also by electric radiation as explained below). These oscillations take place very rapidly (*very much* more rapidly than in the case of the tuning fork above) and gradually cease, but the time of each is the same.

As the result of these transformations of energy it is evident that

the portion of the medium involved primarily in the transformations *must undergo a definite periodic variation of state.* This variation of state is propagated with finite velocity throughout the surrounding medium, that is, **it is the origin of a set of electric or electro-magnetic waves** *which travel outward in all directions through the medium with a definite velocity.*

The student must note that these electro-magnetic waves are (like light waves) in the aether, *i.e.* they are not waves in the air like sound waves. Their velocity (v) is 3×10^{10} cm. per second, the same as the velocity of light. In fact **these waves and light waves are identical except as regards wave length and frequency,** the former consisting of much slower vibrations and longer waves than those of light. The wave length of the *visible spectrum* ranges from ·39μ (violet) to ·75μ (red), where μ is the symbol for one micron or ·001 millimetre. In the *ultra violet* waves have been detected as short as ·1μ possessing properties similar to light waves. In the *infra red* waves have for some time been known as long as 61·3μ. Some distance beyond the infra red come the electromagnetic waves dealt with above, the shortest being about 3 mm. (3000μ), whilst those used in wireless telegraphy range from a few hundred feet to several miles. Fig. 393 shows diagrammatically, within the limits of the sketch, the range of ether waves, V to R being the range of the visible spectrum.

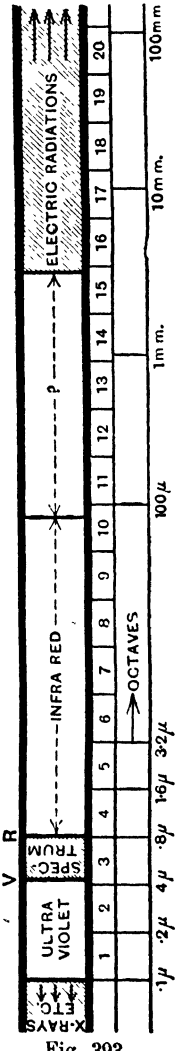


Fig. 393.

Incidentally, it may be mentioned that the frequency (and therefore the period and wave length) depends upon the three properties of an electric circuit already referred to, viz. resistance, inductance and capacity: in fact the *frequency* (n) may be taken as approximately given by the expression :—

$$n = \frac{1}{2\pi}\sqrt{\frac{1}{CL} - \frac{R^2}{4L^2}}$$

where the letters have their usual meaning. The *period* is of course $1/n$, and the *wave length* v/n.

Hertz investigated electric oscillations and waves by means of an apparatus in which the **oscillator, vibrator** or **transmitter** was a dumb-bell shaped brass conductor joined to an induction coil as shown (Fig. 394). When the coil is worked, Aa say, is charged to a higher potential than Bb, then a spark passes at ab, and *the path of this spark*

on account of its comparatively low resistance practically connects Aa and Bb as one conductor, and the potential is equalised by very rapid electric oscillations in this conductor. The cycle of changes that go on in the medium near the vibrator during one complete vibration is that associated with the transformation of electrostatic to electromagnetic energy, the re-transformation of electromagnetic to electrostatic energy, and, during these, a gradual dissipation of energy as heat. This cycle of changes is transmitted out into the medium, and the sequence of states so transmitted constitutes the electric waves. When these waves pass a conductor they tend to reproduce the electrical conditions in it that is, electrical oscillations are induced in the conductor and small sparks may be caused between conductors placed close together.

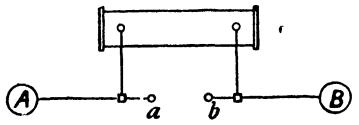


Fig. 394.

Hertz took advantage of this to detect the waves. One form of **detector** or **receiver** used by him was a piece of thick copper wire bent into a circle, but with a small air-gap between the ends of the wire the width of the gap could be adjusted. As the waves passed through this circuit small sparks were observed at the gap, and it was found that the sparks were strongest when the dimensions (and therefore R C, and L) of the detector were such that its period of oscillation as an oscillator was the same as that of the oscillator originating the waves. This is an example of the general principle of **resonance,** and a circuit showing electric resonance is specially effective in detecting electric waves. By the proper adjustment of inductance and capacity it is possible to adjust the period of the detector within wide limits, and so **tune it to resonance** in any particular case. Further, it was found that the gap in the detector must be parallel to the gap in the oscillator to obtain the best results.

For the detection of the waves, a **coherer** the principle of which was discovered by Lodge and utilised by Branley, Marconi, and others may be used. A simple form is shown in Fig. 395. Two brass, copper, or silver discs *d*, *d*, each soldered to the end of a copper wire *w*, are fitted into the glass tube *gg*, and a thin layer of filings rests lightly between the discs. If this coherer is placed in the circuit of a battery and an electric bell, the contacts made by the filings between the discs may be so adjusted that the current passing will not be strong enough to ring the bell. If, however, waves from a distant vibrator fall upon the coherer after this adjustment is made, the contact at once becomes good and the bell in the circuit rings. If the coherer is slightly tapped the contact again fails and the arrangement is again ready to detect the incidence of the waves. In Marconi's coherer the plugs *d*, *d* are o

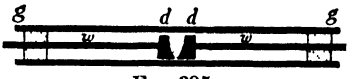


Fig. 395.

silver, the filings are a mixture of nickel and silver, and the tube is exhausted and sealed. In laboratory work a suitable galvanometer may take the place of the bell above. Evidently with the coherer joined to a battery and relay (thence to recording apparatus) and fitted with a tapping device to "decohere" the filings, and further with a Morse or other suitable key in the transmitting circuit we have all the fundamentals of *wireless telegraphy*.

For the various steps in the development of "wireless" from the above, more or less, experimental stages to its present commercial position, some specialised book on the subject must be consulted.

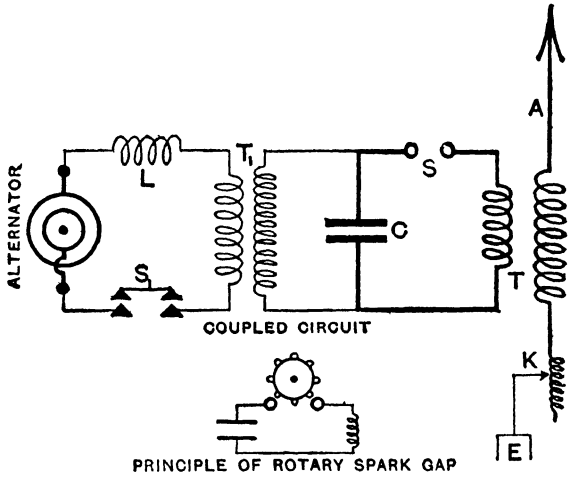


Fig. 396.

One form of transmitting circuit is shown in Fig. 396. The condenser C and spark gap S are in series with one winding of the transformer T, the other winding of the transformer being joined to a long (originally vertical) conductor A (known as the **antenna** or **aerial**) and to earth. The contact K and therefore the inductance in series with the aerial is adjusted until the aerial circuit and condenser circuit are syntonised, *i.e.* until the period of oscillation of the aerial circuit is equal to that of the condenser circuit. The "charging" is effected from the secondary of the transformer T_1, the primary of which is connected to an alternator; this circuit also includes an inductance L and a switch S_1, the latter being used to produce the required "longs" and "shorts" of the Morse code. At each discharge oscillations occur in the condenser circuit; these induce oscillating electromotive forces and currents in the aerial circuit, and the two being syntonised the aerial becomes a powerful source of radiation of electric waves. At

the receiving station the waves encounter a similar aerial and produce electric oscillations in it.

In place of the simple spark gap S a **rotary spark gap** is often employed. A wheel carrying a number of projecting studs is rotated between two fixed studs in the condenser circuit, and arrangements are such that the fixed and moving studs come opposite each other at the instant when the condenser is fully charged, *i.e.* just when the spark is required.

The coherer as a detector has been replaced by others more suitable for the purpose and known as magnetic, thermal, and rectifying detectors. The latter, one type of which consists of a carborundum crystal between two brass plates, depend for their action on the fact that their conductivity is different in different directions, so that an oscillatory current may be partially changed into a direct current by choking down the flow in one direction. These are now extensively used in practice, and Fig. 397 shows the arrangement of a receiving circuit fitted with such a detector.

The aerial circuit is adjusted to syntony with the arriving waves. The other coil of the transformer T and the condenser C_1 form an oscillatory circuit syntonised to the aerial circuit. The detector D in series with a second condenser C_2 is in parallel with C_1 and the connections to the telephone *t* are as indicated. The waves arriving at the aerial set up oscillations in it which induce oscillations in the condenser circuit. This tends to cause currents to flow to and fro through D into and out of C_2, but owing to the property of D mentioned above current in one direction only passes, so that C_2 is charged. This charge in one direction, due to the arrival of a train of oscillations, is given out to the telephone circuit as a single unidirectional current flow. Many such detectors work better if they have a small current flowing through them; hence the insertion of the battery B.

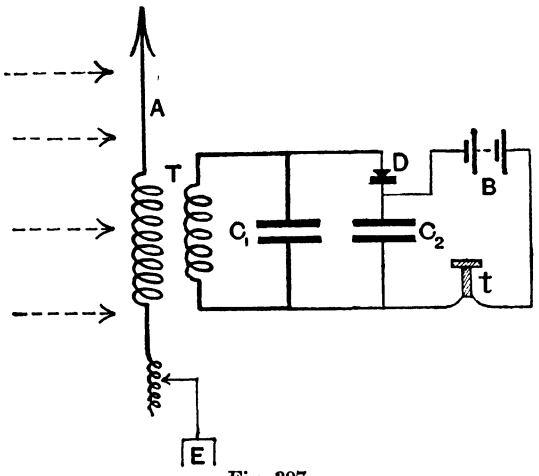


Fig. 397.

Another type of rectifying detector is the **Fleming Valve.** This consists of a 12-volt carbon filament lamp, the filament being surrounded

by (but not touching) a metal cylinder connected to a separate terminal. The filament is joined to a battery of accumulators, and when it glows, *negative ions are projected from the filament to the cylinder*, so that we may say the rarefied gas possesses *unilateral conductivity*, *i.e.* a current can pass in one direction between the plate and filament but not in the other direction. Between the negative terminal of the lamp and the cylinder terminal are connected, therefore, the circuit with the high frequency E.M.F. and the telephones in series, so that the high frequency alternating current is cut down in one direction and travels as a unidirectional current through the telephones as in the preceding case.

The **Lee de Forest Valve or "Audion"** consists of a filament and a metal plate, with a wire grid between them, the three being quite separate from each other. The filament is made to glow by the current from an accumulator battery. A small primary battery with telephones in circuit is joined to the plate and filament, the positive pole of the battery being joined to the plate: thus a small current passes in this circuit. If now an oscillating circuit is connected between the grid and filament the conductivity of the rarefied gas in the preceding circuit is altered, the current in the telephone circuit is altered, and sounds are produced in the telephones. Potentiometers are used for adjusting the various voltages.

Modern **thermionic valves** contain as above a filament (usually tungsten), a metal plate or its equivalent (the anode), and a grid between the two. They are used in various ways and for various purposes, viz. (1) as detectors, (2) as high frequency amplifiers, (3) as audible frequency amplifiers, (4) as high frequency generators, (5) as combinations of the preceding. Further particulars cannot be given here, however, and some standard book on "Wireless" must be consulted.

332. Röntgen or X Rays.—These "rays," discovered by Professor Röntgen in 1895, have now become *an important technical application of electricity*, so that a brief and elementary treatment here is desirable. Attention must first be drawn, however, to a few phenomena which preceded the discovery of Professor Röntgen, and which, perhaps, are of even more far-reaching importance, particularly in the domain of electrical theory, than X rays themselves.

333. Electric Discharge through a Gas at Low Pressure.—Most gases at ordinary atmospheric pressure are good insulators (bad conductors), but their dielectric strength is only moderate however, *i.e.* under suitable conditions they are fairly readily pierced by a spark. If the pressure be increased above atmospheric pressure, they become better insulators, and improve also in dielectric strength. If the pressure be reduced the dielectric strength becomes less, reaches a minimum value at very low pressures, and then begins to increase

334. Kathode Rays: Electrons.—When the condition mentioned in (4) above is reached, certain experimental facts indicated that some emanation must be coming *from the kathode*, and to this the name **kathode rays** was given (Crookes "radiant form of matter" or "fourth state of matter," *i.e.* neither solid, liquid, nor gas). The kathode rays were really discovered by Plücker in 1859, and the main properties may be summarised as follows :—

(1) The rays are shot out normally from the kathode, their direction being in no way connected with the position of the anode.

(2) They travel in straight lines, and cast shadows of objects placed in their path ; in Crookes's experiment a hinged Maltese cross was placed opposite the kathode, and a shadow appeared on the opposite end of the tube.

(3) A body placed in their path experiences a force tending to urge it away from the kathode.

(4) The rays produce heat when they fall upon matter.

(5) They produce phosphorescence. Lead glass phosphoresces blue, soda glass yellowish green. Phosphorescence is also produced in barium platino-cyanide and the rare earths (cerium, lanthanum, etc.).

(6) When the rays strike a solid obstacle, the latter becomes a source of *aether pulses* known as **Röntgen** or **X rays.**

(7) The rays are deflected by a magnetic field, *the direction of deflection being that in which a stream of negatively charged particles would be deflected.* The deflection, under like conditions, is independent of the gas in the tube before exhaustion. The rays are also deflected by an electric field and the direction of deflection is again that in which negatively charged partioles would be deflected.

(8) The rays can "ionise" a gas and render it a conductor (Art. 22, etc.).

Goldstein (1876) regarded kathode rays as waves in the aether, a view long upheld in Germany, but in England, Varley (1871) and Crookes (1876) contended that they were charged particles. Recent work has confirmed the latter view; the rays consist of negatively charged particles of mass about $\frac{1}{2000}$ that of a hydrogen atom, moving with a velocity about $\frac{1}{10}$ that of light, and carrying a (negative) charge equal to the smallest quantity or "atom of electricity" ($4{\cdot}65 \times 10^{-10}$ e.s. units). They are, in fact, **"electrons"** (Art. 15, etc.), identical with the electrons met with in many other phenomena: further, they are not matter as ordinarily understood, but simply "electricity."

335. Röntgen or X Rays.—When the condition mentioned in (4) Art. 333 was reached, Röntgen in 1895 found that a photographic plate lying near the apparatus was affected: he attributed this to some unknown form of radiation coming out of the tube and to this he applied the term **X rays**. It is now considered that **the X rays are aether pulses** (shorter than the wave length of violet light—see

Fig. 393) which originate at a solid substance when it is struck by kathode rays. An early X ray tube is shown in Fig. 399; the kathode on the left is concave and a platinum plate (which may be the anode) is placed at the centre of curvature with its plane at an angle of 45° to the axis; the X rays are given off as indicated by the dotted lines.

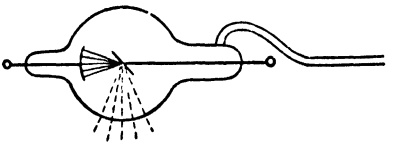


Fig. 399.

The main points about these rays may be briefly summarised as follows:—

(1) The rays are *not* deflected by a magnetic or electric field. This differentiates them from kathode rays; *they are not charged particles.*

(2) They can penetrate a layer of air several feet thick and can pass through many solid substances. The transparency of substances to X rays depends upon the density of the substances; thus lead is practically opaque to the rays, but aluminium is transparent. Soda glass is transparent to them, but lead glass is opaque. Flesh is much more transparent to the rays than bones, and bones are more transparent than, for example, metal; hence their use in surgery to examine the bones, to detect fractures and foreign bodies, etc. (See "radiograph" facing this page.)

(3) They excite fluorescence in many substances, *e.g.* barium platino-cyanide. If the rays fall on a screen coated with this and the hand be interposed, a shadow of the bones appears on the screen, the fluorescence being less here than at other parts owing to the rays being more absorbed by the bones.

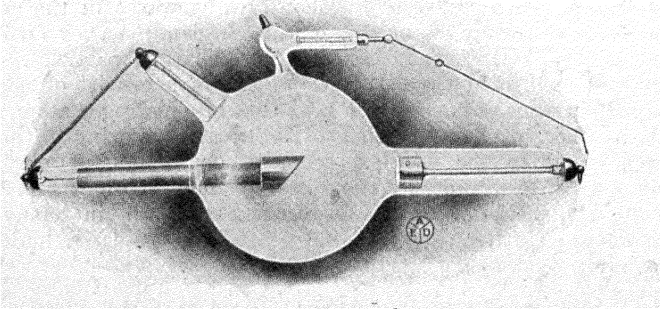


Fig. 400.

(4) The rays are not refracted. Very little trace of regular reflection has been noted. Quite recently, and with difficulty, signs of polarisation have been detected. Sharp shadows due to them indicate rectilinear propagation, and investigations seem to indicate a velocity equal to that of light.

RADIOGRAPH BY J. F. BRAILSFORD, SMETHWICK TECHNICAL SCHOOL.

Compound comminuted fracture of upper arm (gunshot wound in Flanders) showing also numerous particles of bone and bullet.

(5) They ionise a gas through which they pass.

(6) When they fall upon any material they give rise to other rays of a somewhat similar character known as *secondary X rays*.

(7) Recent work on X ray diffraction and crystal structure, etc., confirms the wave (or aether pulse) nature of X rays: the "wave" length is of the order 10^{-8} cm. which is much less than the shortest light wave, viz. 4×10^{-5} cm. (see Fig. 393).

A modern X ray tube is shown in Fig. 400. The kathode on the right is a concave cup of aluminium so arranged as to focus the electrons of the cathode stream on the *anti-kathode* or *target* seen near the centre of the bulb. The target is of tungsten, a metal of high atomic weight: it also acts as anode although another anode is provided as shown. Since kathode rays have a considerable heating effect, the anti-kathode is frequently fitted with a water-cooling (or air-cooling) device (Fig. 400*a*). In the "Coolidge tube" the supply of electrons is increased by using an incandescent kathode.

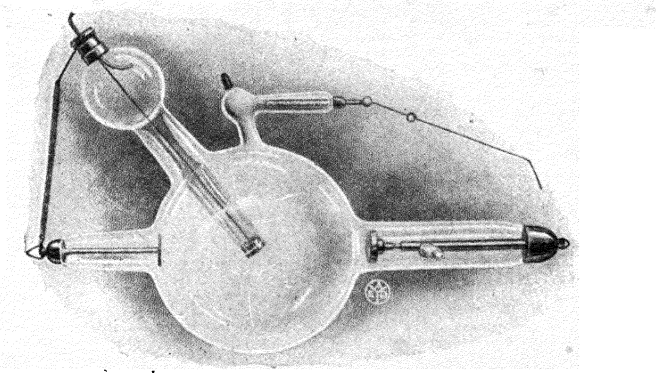


Fig. 400*a*.

When the vacuum is very high the X rays are very penetrating and are spoken of as "hard" rays: with a relatively lower vacuum they are less penetrating and are spoken of as "soft" rays. With prolonged use X ray tubes become more and more exhausted and the rays become harder. A spark gap in parallel with the tube is now used in almost every instance, partly to serve as a safety valve and partly to indicate the condition of the vacuum.

Fig. 401 depicts a compact arrangement of coil, interrupter, and switchboard. In the type shown the primary of the coil (which is seen on the top of the pedestal) is joined to the town supply at 220 volts: the coil is a 16 in. one and the output 80 m a. The interrupter is of the motor mercury type and is fitted with a mechanical

rectifier to suppress inverse currents. The board carries all the requisite appliances for the primary circuit—ammeter, voltmeter, rheostats, main double pole switch, switch for motor, fuses, etc.—and a foot switch is also provided so that the operator may manipulate the apparatus at a distance if necessary.

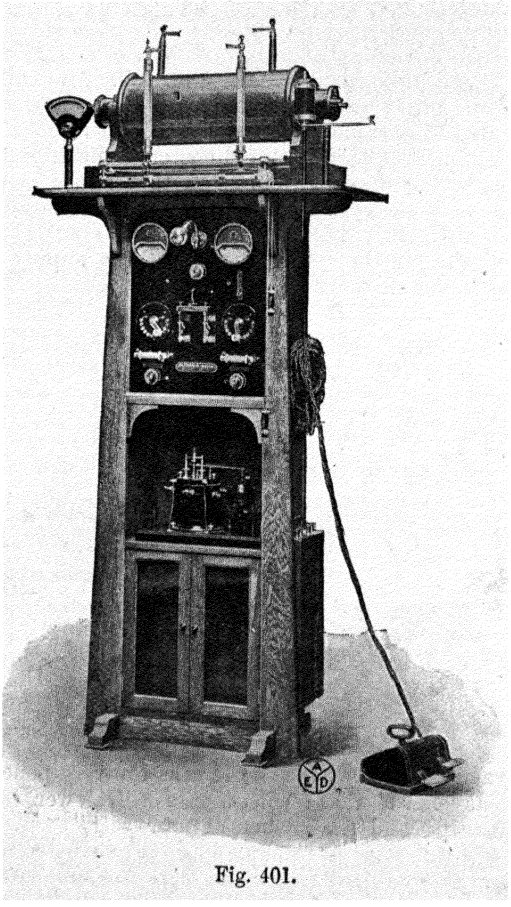


Fig. 401.

trolley wires stretch right across the room from one wall to the opposite wall from which, however, they are thoroughly insulated. The secondary terminals of the coil (seen at the top of the coil in the figure) are connected to these trolleys (*via* a milliammeter, device for the suppression of inverse current, etc.), and at any point of the trolleys, movable connectors (manipulated by insulating handles) can

be taken down to the X ray tube, the positive trolley to the anode and the negative trolley to the kathode. The tube is usually carried by a suitable stand (Fig. 402) which can be moved from place to place and the tube itself arranged in various positions according to the position of the patient under examination. For many examinations a suitable couch is essential, preferably with a support which is transparent to X rays so that the tube may be placed below the patient and the fluorescent screen or photographic plate above the patient if necessary. It is, of course, not necessary for a patient to undress provided the part under examination is not covered by some metal, *e.g.* buttons, belts, corsets, and the like.

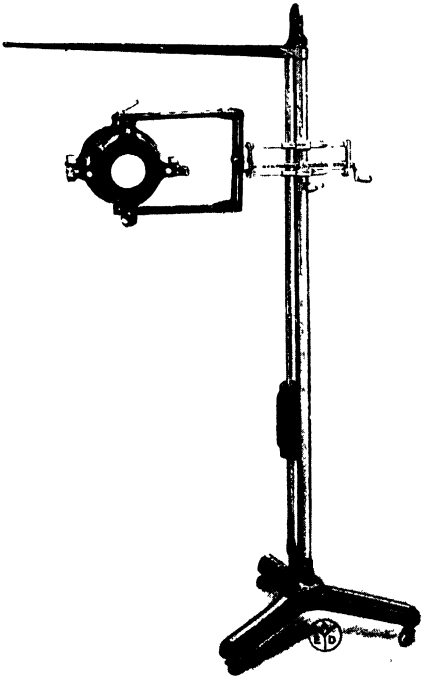


Fig. 402.

Fig. 403 depicts the "dioscope" for vertical screening or upright radiography. The carrier, seen at the back, contains the X ray tube; it can be raised or lowered, and by means of the three handles in the front, it can be moved horizontally, and the aperture through which the rays pass can be adjusted and arranged to be either horizontal or vertical. The patient stands upright in the apparatus, and is examined

by means of the fluorescent screen of willemite seen in the front of the apparatus. The screen may be removed and a photographic plate substituted if a radiograph is to be taken.

The descriptions above are purely elementary and refer to some of the more important parts of the equipment of the X ray Department of the Smethwick Technical School, the apparatus for which was mainly supplied by Messrs. A. E. Dean and Co. of London. Numerous other excellent designs are in daily use in hospitals and private work. The use of high tension transformers (in place of induction coils) with Coolidge tubes and special accessories is rapidly extending.

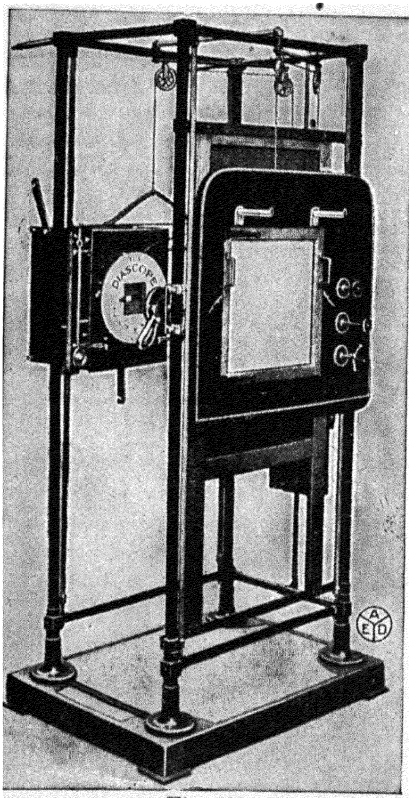


Fig. 403.

X rays have also been successfully applied in dental work and to the cure of rodent ulcer, lupus, etc., but in this latter connection they cannot be played with, for they can also inflict severe injury to the skin, destroy the hair, and so on. In the hands of an experienced operator the patient is now, however, perfectly safe, but the operator himself who is subject to repeated exposure should avail himself of all protective devices. We have seen that lead is opaque to the rays, and in Fig. 402 the tube rests in a lead glass shield provided with an opening through which the rays can pass only in the required direction: other tube shields are made of lead. Lead glass spectacles, full length lead-lined aprons, and leather and rubber lead-lined gloves, etc., are regularly used by the operator of to-day.

APPENDIX.

(1) Specific Resistances, Conductivities, and Temperature Coefficients.

Substance.	Sp. Res. at 18° C. (ohms per cm. cube).	Conductivity at 18°.	Temperature Coefficient.
Silver	$1{\cdot}65 \times 10^{-6}$	$6{\cdot}06 \times 10^{5}$	·0040
Copper (drawn)	$1{\cdot}78 \times 10^{-6}$	$5{\cdot}62 \times 10^{5}$	·0043
Zinc	$6{\cdot}1 \times 10^{-6}$	$1{\cdot}6 \times 10^{5}$	·0037
Iron	$1{\cdot}4 \times 10^{-5}$	$7{\cdot}1 \times 10^{4}$	·0062
Lead	$2{\cdot}1 \times 10^{-5}$	$4{\cdot}8 \times 10^{4}$	·0043
Mercury	$9{\cdot}41 \times 10^{-5}$	$1{\cdot}06 \times 10^{4}$	·0009
Platinum ...	$1{\cdot}16 \times 10^{-5}$	$8{\cdot}6 \times 10^{4}$	·0037
Aluminium ...	$3\,0 \times 10^{-6}$	$3{\cdot}3 \times 10^{5}$	·0038

Manganin (Cu = 84, Ni = 4, Mn = 12) :—Sp. Res. = $4{\cdot}76 \times 10^{-5}$ at 0° C. ; Temp. Coeff. = ·000018.

Platinoid (Cu = 62, Ni = 15, Zn = 22) :—Sp. Res. = $3{\cdot}44 \times 10^{-5}$ at 18° C. ; Temp. Coeff. = ·00021.

Resista (Fe = 80, Ni = 15, Mn = 5) :—Sp. Res. = 12×10^{-5} ; Temp. Coeff. = ·00109.

German Silver (Cu = 4, Zn = 1, Ni = 2) :—Sp. Res. = 3×10^{-5} ; Temp. Coeff. = ·0004.

Note :—Specific Resist. per inch cube = ·3937 × Specific Resist. per cm. cube.

(2). Conductors in the order of their Conducting powers and Insulators in the order of their Insulating powers (approximately).

(*a*) *Conductors* :—Silver, Copper, Gold, Aluminium, Brass, Silicon Bronze, Zinc, Nickel, Phosphor Bronze, Platinum, Iron, Tin, Lead, German Silver, Platinoid, Manganin, Eureka (Copper and Nickel), Constantan, Tantalum, Tungsten, Mercury, Carbon, Acids, Water, Human Body.

(*b*) *Insulators* :—Dry Air, Paraffin Wax, Quartz, Resin, Flint Glass, Sulphur, Micanite, Porcelain, Pure Rubber, Shellac, Silk, Marble, Amber, Vulcanised Fibre, Ordinary Glass, Gutta-percha, Mica, Asbestos, Slate, Hard Wood.

(3) Illustrations of Wire Gauges.—For fuller detail complete tables must be consulted.

British Standard Gauge.		Birmingham Gauge.		American Gauge.		French Decimal Gauge.	
No. S.W.G.	Diameter in Inches.	No. B.W.G.	Diameter in Inches.	No. A.W.G.	Diameter in Inches.	No. F.W.G.	Diameter in Mm.
1	·300	1	·300	1	·2893	1	·6
2	·276	2	·284	2	·2576	2	·7
3	·252	3	·259	3	·2294	3	·8
4	·232	4	·238	4	·2043	4	·9
5	·212	5	·220	5	·1819	5	1·0
6	·192	6	·203	6	·1620	6	1·1
7	·176	7	·180	7	·1443	7	1·2
8	·160	8	·165	8	·1285	8	1·3
...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...
28	·0148	28	·014	28	·0126	18	3·4
29	·0136	29	·013	29	·0113	19	3·9
30	·0124	30	·012	30	·0100	20	4·4

(4) Resistance of Pure Copper Wire per 100 yards.

S.W.G.	Ohms.	S.W.G.	Ohms.	S.W.G.	Ohms.
8	·1195	16	·7478	24	6·324
9	·1476	17	·9762	25	7·654
10	·1868	18	1·328	26	9·449
11	·2275	19	1·913	27	11·39
12	·2831	20	2·362	28	13·98
13	·3617	21	2·990	30	19·9
14	·4784	22	3·905	32	26·25
15	·5904	23	5·316	34	36·17

(5) Approximate Fusing Currents (Sir W. H. Preece).

Current in Amperes.	Tin Wire. S.W.G.	Lead Wire. S.W.G.	Copper Wire. S.W.G.	Current in Amperes.	Tin Wire. S.W.G.	Lead Wire. S.W.G.	Copper Wire. S.W.G.
1	36	35	47	45	13	12	22
2	31	30	43	50	12·5	11·5	22
3	28	27	41	60	11	10	21
4	26	25	39	70	10	9·5	20
5	25	23	38	80	9·5	8·5	19
10	21	20	33	90	9	8	18·5
15	19	18	30	100	8·5	7	18
20	17	17	28	120	7	6	17·5
25	16	15	26	140	6	5	17
30	15	14	25	160	5	4	16
35	14·5	13·5	24	180	4	3	16
40	13·5	13	23	200	3·5	2	15

(6) Dimensions and Safe Currents.

Size of Copper Conductor. S.W.G.	Diameter. Inch.	Sectional Area. Square Inch.	Capacity at 1000 Amperes per Square Inch. Amperes.	I.E.E. Capacity with Rubber Insulation. Amperes.
Flexibles. 35/40	·028	·000616	0·62	1·7
23/36	·036	·00102	1·0	2·6
40/36	·048	·00181	1·8	4·2
70/36	·064	·00322	3·2	6·8
110/36	·080	·005	5 0	9·8
3/24	·047	·0011	1·1	4 5
3/23	·052	·0013	1·3	5·3
1/18	·048	·0018	1·8	7 2
3/22	·060	·0018	1·8	7·2
7/25	060	0022	2 2	8·6
3/21	·069	·0024	2·4	9·5
1/17	·056	·0025	2 5	9 8
7/24	·066	·0026	2·6	10·4
3/20	·078	·0030	3 0	12·0
7/23	·072	·0031	3·1	12·4
1/16	·064	·0032	3 2	12 9
3/19	·086	·0037	3 7	14·8
1/15	·072	·0041	4 1	16·3
7/22	·084	·0042	4·2	17·0
1/14	·080	·0050	5·0	19
3/18	·103	·0053	5·3	20
7/21	·096	·0055	5 5	21
7/20	·108	·0070	7·0	24
7/19	·120	·0086	8·6	28
7/18	·144	·0125	12·5	34
7/17	·168	·017	17	40
19/20	180	·019	19	43
7/16	·192	·022	22	46
7/·068″	·209	·025	25	50
7/15	·216	·028	28	53
19/18	·240	·034	34	59
19/15	360	·075	75	97
19/14	·400	·094	94	113
19/·083″	—	·100	100	118
19/13	·460	·125	125	134
37/15	·504	·150	150	152
37/14	·560	·182	182	172
37/·083″	—	·200	200	184
37/13	·644	·250	250	214
37/·112″	—	·350	350	264
91/·108″	—	·800	800	488
91/·112″	—	·900	900	540
91/·118″	—	1·000	1000	595

ADDITIONAL EXAMINATION QUESTIONS

(from the City and Guilds of London and from University Examinations).

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1. What is an electromagnet? In what way does its magnetism depend on its core? In what way does its magnetism depend on its coil? What are the reasons which determine, in any case, whether the coil should consist of few turns of thick wire or of many turns of thin wire?

2. State Ohm's law, both in symbols and in language. Show how you would apply it to find the relation between the current through the working coil of a shunted ammeter and that in the main circuit.

3. If electric bells are to be made to ring by a magneto-ringer through a line many miles in length they must be wound with fine wire; why is this? Show also why it is that the coils of the ringer itself must also be wound with many turns of fine wire.

4. Distinguish between the electromotive force of a battery and the potential difference between its terminals when working. A hot-wire voltmeter can be used with accuracy for measuring the electromotive force of a storage battery: is the same instrument equally suitable for measuring the electromotive force of a battery of Leclanché cells? If not, why not?

5. State how the resistance of each of the following substances is affected by a rise of temperature:—Copper, manganin, platinum, carbon, platinoid, glass. How would you measure the change of resistance of any metallic conductor when heated?

6. Define *permeability*, explaining fully the several terms you use in the definition. What are the advantages, if any, of using iron of high permeability in the construction of dynamos and motors?

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7. Make a sketch diagram of the three-wire system of supplying current to glow-lamps, including disposition at a station or a sub-station.

8. In what manner do (*a*) the *conductor resistance* and (*b*) the *insulation resistance* of a uniform electric light cable depend upon the length of the cable? A 55-yard length of such a cable when tested gave 0·11 ohm and 10,000 megohms, respectively, for (*a*) and (*b*); find the corresponding values for a mile length of similar cable.

9. A straight wire carrying a continuous current of 200 amperes, lying in a direction north and south, is subjected to a vertical magnetic field the intensity of which is 8,000 lines per square centimetre, downwards. Find the direction and amount of the force to which each centimetre length of the wire is subjected.

10. Enumerate the various kinds of glow-lamps with which you are acquainted, and mention the amount of power per candle normally required for each kind. What considerations would influence you in deciding which type to use in a concert hall where light is required only four hours per week on the average?

11. Define the term *specific resistance* of a metal. A wire made of a certain alloy, of diameter 0·09 inch, has a resistance of 0·047 ohm per yard. Find the specific resistance of that alloy. How many yards of the same material if drawn to a diameter of 0·018 inch would have a resistance of 100 ohms?

12. A series dynamo has its direction of rotation reversed; will it still act as a generator? If it does, explain why. If it does not, explain how it could be made to do so. Give diagrams illustrating your answer.

---

13. Explain the causes to which is due the spark seen on breaking any circuit in which there is an electromagnet at work. Why do not the same causes produce a spark when the circuit is made?

14. Quantity meters and energy meters are used in electric supply systems; show by simple diagrams the essential differences between the two classes. Describe one device which is used in motor meters for improving the accuracy of such meters at low loads.

15. A certain continuous-current amperemeter has a resistance of 0·01 ohm, and may be used for measuring currents up to 2 amperes. Explain how the range of readings may be extended by the use of a shunt, and give the resistance of the shunt that would enable currents to be measured up to 20 amperes.

16. Draw up a list of the various materials that are used as insulators in the construction of dynamos, and describe briefly how their suitabilities to their various purposes should be tested.

17. Define the terms "intensity of field," "permeability," and "ampere-turns." Calculate the intensity of the field at the central point of a solenoid 100 centimetres long, of small diameter, wound with 6,000 turns of wire, traversed by a current of 5 amperes.

18. To what point should accumulators be discharged? What is the suitable density of the acid that should be used in them? How would you test a set of accumulators to discover whether they were fully charged? What is the particular harm that results if accumulators are left long in a discharged state?

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19. Draw up a comparative statement of the advantages and disadvantages of carbon filament glow-lamps and metallic filament glow-lamps.

20. What is an electrostatic voltmeter? Describe the construction and operation of any one with which you are familiar.

21. Under what conditions is it permissible to determine, by means of the indications of an ampere-hour meter, the money value of electrical energy supplied to a consumer? Why do the managers of continuous-current central stations generally prefer such meters to watt-hour meters?

22. Explain what is meant by the terms "series-dynamo," "shunt-dynamo," and "compound-dynamo." Give diagrammatic sketches of the circuits of four-pole machines of each of these kinds. State approximately what you would expect the resistances of each of the different parts of the circuits to be for machines of 50 kilowatts to work at 220 volts.

23. Describe the electrodynamometer as used for measuring currents, and explain why the amount of turning of the torsion head is proportional to the square of the current. May this instrument be used for alternating currents? If it may, why? If it may not, why not?

24. State and explain the principle on which a three-phase non-synchronous motor works.

---

25. Define the terms "one joule," "one kilowatt-hour," "one Board of Trade unit." The price of electrical energy in a certain town is fourpence per Board of Trade unit, and the supply pressure is 200 volts. An electricity energy-meter on this circuit has a resistance of 10,000 ohms in its pressure coil; calculate the cost of the energy wasted in this coil in a quarter of a year.

26. Explain the relations that exist in continuous-current motors, between the direction of current in one armature conductor, the direction of the magnetic field in which it lies, and the direction of the driving force. Why does a shunt-motor, when supplied at constant voltage, run slower if the magnetism of the field is increased?

27. A cell of 1·5 volts electromotive force and 5 ohms internal resistance is connected to two resistances of 100 ohms and 10 ohms respectively in parallel. How much current will pass through the cell and through each resistance?

28. Prove the law that the number of watts wasted in a resistance, when a current is supplied at a constant voltage, is inversely proportional to the resistance. An electromagnet used on a constant voltage of 30 volts has a resistance when cold of 8 ohms. After being on circuit for half an hour the resistance has increased to 12 ohms. Find the rate at which energy is being wasted at first, and when the coil has thus warmed up.

29. Describe some form of amperemeter suitable for a switchboard. State whether the deflection of the pointer will be proportional to the current or not and give reasons for your answer.

30. Describe some simple experiment to show that while the resistance of an iron wire increases with temperature, that of a carbon filament decreases as the temperature is raised.

---

31. What is the relation between the current in a single turn of wire made in the form of a circle of large diameter and the density of the magnetic field (due to the current) at the centre of the circle?

32. Sketch the connections for the experiment of charging a condenser from a battery, and discharging it through a galvanometer. Make a second sketch showing the change necessary if the galvanometer is to show both the charging and the discharging current.

33. What are the chief points of difference between a flame arc lamp and an ordinary arc lamp? To what is the golden colour of the light from some flame arc lamps due?

34. What is a back electromotive force? Give examples of the occurrence of such. Why is no electric work (except heating) ever done by a current without the presence of a back electromotive force?

35. Sketch a motor starting switch with no-voltage and overload release coils, and show its connections to a shunt motor and supply mains.

36. Sketch the form and distribution of the lines of magnetic force in the field surrounding a long straight conductor which carries a current. Give a second sketch showing similarly the lines of force in the field of two parallel straight conductors carrying currents flowing in the same direction.

---

37. Explain the action of the commutator of a continuous current dynamo.

38. Draw up a table of the different classes of liquids, classified according to their power of conducting currents, giving examples of each class. How does a rise of temperature affect the conductivity of the different classes?

39. Two resistances of 5 and 8 ohms respectively are placed in parallel, and in series with this combination is a resistance of 4 ohms. If the potential difference between the extreme terminals is 100 volts, what will be the current in each resistance and the difference of potential between the terminals of each resistance?

40. An overhead transmission line consists of two wires, each 2 kilometres long and 150 square millimetres cross sectional area. If the continuous current flowing in one and returning in the other is 150 amperes, what will be the power in watts wasted in the line? The specific resistance of copper is 1·72 microhms per centimetre cube.

41. Mention the most important insulating materials used in electrical engineering, and discuss their suitability for specific purposes.

42. Describe, with sketches, some form of moving coil ammeter, and explain (*a*) how a uniformly divided scale is obtained, (*b*) how the instrument would be arranged to measure a very large current.

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43. Sketch, approximately to scale, the magnetisation and permeability curves of wrought iron and cast iron.

44. An iron ring of 10 centimetres mean diameter and 1 square centimetre cross sectional area is wound with 100 turns of wire. What will be the value of the magnetic induction in the iron when a current of 3 amperes flows in the wire? Take the permeability of the iron as 800.

45. Describe the construction of a secondary cell, and explain the chemical and electrical changes which occur during charge and discharge.

46. Describe, with sketches and diagram of connections, one form of automatic switch for use in a battery circuit during the process of charging the battery, and explain its action.

47. What are the relative advantages and disadvantages of carbon filament and metal filament glow lamps?

48. Explain how you would proceed to test an incandescent lamp for efficiency. Describe the photometer you would use, and the standard of light you would employ. Why does the light from a metal filament lamp appear whiter than that from a carbon filament lamp?

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49. Suppose you wind additional turns on the field coils of a shunt motor, using the same size wire, what will be the effect on (*a*) the efficiency, (*b*) the speed?

50. Explain what is meant by the terms "series dynamo," "shunt dynamo," "compound dynamo," and state which of these is most suitable for (*a*) charging a battery, (*b*) supplying power to an electric tramway system.

51. Two wires in parallel, which have resistances of 2 and 4 ohms respectively, are connected to the terminals of a battery which has an electromotive force of 6 volts and an internal resistance of 1 ohm. Find the current in each of the resistances.

52. Explain the principle and method of measurement of resistance by the Wheatstone bridge, and give a diagram of connections.

53. Mention the operations which an arc lamp mechanism has to perform, and describe the essential parts of one type of mechanism showing diagram of connections with the supply circuit.

54. Describe some form of incandescent electric lamp, giving figures for volts, amperes, and candle-power, and state how the resistance varies with change of temperature.

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55. A field coil of a dynamo has a resistance of 850 ohms at 20 degrees centigrade. After the dynamo has been running for six hours the resistance increases to 908 ohms. What is the average temperature of the coil under this condition? The temperature coefficient of copper is 0·4 per cent. per degree centigrade.

56. What is "specific resistance"? State the effect of temperature on the specific resistance of the following materials:—German silver, copper, iron, and carbon. What is the effect of temperature on the insulation resistance of the following materials:—Glass, indiarubber, paper as used for the insulation of cables, mica and marble?

57. Define "magnetising force" and "magnetic induction," and state the relation between them. How many ampere turns are required for an air gap 200 square centimetres in area and 5 millimetres long when the magnetic flux is 1 megaline?

58. Describe some form of armature for a continuous current dynamo, stating the material of which each part is made and the reason why that material is used. Why is the armature core made up of a number of thin plates instead of a solid cylinder?

59. What is the "counter" or "back" electromotive force of a motor and on what does it depend? What change takes place in the speed of a series motor if the load is reduced?

60. Describe carefully, with sketches of connections, some form of starting switch suitable for a 220-volt shunt motor of about 10 horse-power.

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61. Two coils, connected in parallel, take a total current of 10 amperes at 230 volts from the supply mains. If the power absorbed by one coil is 950 watts, what is its resistance, and what is the resistance of the other coil?

62. Mention the principles on which the action of different types of voltmeters is based, and state the conditions for which each type is most suitable.

63. How does the presence of self-induction in any circuit affect a continuous current therein (*a*) when the current is switched on; (*b*) while the current is flowing regularly; (*c*) when the current is switched off? What has self-induction to do with the possible danger that there may be in breaking the circuit in which there is a large current?

64. Six hundred horse-power, at 600 volts, is to be delivered at a place two miles distant from an electric generating station. Find the power and voltage of the generator required if the loss in transmission is to be 14 per cent. of the power generated. Also determine the cross-section of the cable required.

65. What considerations determine the proper rating of the capacity of a storage battery? In what way is the capacity dependent upon the rate of discharge? What are the conditions which limit the rate of charging?

66. A compound-wound dynamo is to be used as a compound-wound motor, with the direction of rotation unchanged. What alterations, if any, will be required in the machine? Give full reasons and, if need be, diagrams also, to explain your answer.

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67. A ring-shaped electromagnet has an air-gap 6 millimetres long and 20 square centimetres in area, the mean length of the core being 50 centimetres, and its cross-section 10 square centimetres. Calculate approximately the ampere-turns required to produce a field of strength H = 5,000 in the air-gap. (Assume permeability of iron as 1,800.)

68. Describe any potentiometer you are acquainted with, and show how to use it for calibrating (1) an ammeter reading to 250 amperes, and (2) a voltmeter reading to 600 volts.

69. Explain briefly the principal phenomena of the continuous current arc. How are they affected (*a*) by inverting the electrodes; (*b*) by putting the electrodes horizontal; (*c*) by enclosing the arc in a nearly air-tight vessel?

70. Twelve wires, each having 1 ohm resistance, are joined up to form the edges of a cube. The joints are electrically perfect. Show that the total resistance of this system taken between two corners of the cube on the same edge is $\frac{7}{12}$ of an ohm.

71. State the Law of Helmholtz for the value of currents while growing to their steady value. A battery having 15 volts electromotive force and no appreciable resistance is suddenly switched on to an electromagnet having $1\frac{1}{2}$ ohms resistance and a co-efficient of self-induction of $2\frac{1}{2}$ henries. Find the value that the current will have reached in 5 seconds. [N.B.—$\epsilon = 2{\cdot}718$, $\epsilon^2 = 7{\cdot}388$, $\epsilon^3 = 20{\cdot}08$.]

72. Show that the torque given by a continuous-current motor is proportional to the following three quantities: (*a*) flux emanating from one pole, (*b*) total number of armature conductors, (*c*) current in one armature conductor. If these three things are fixed, will it make any difference to the torque whether the conductors are sunk in slots or bound by binding wires on to a smooth core?

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73. State the relation that exists in an arc lamp (using ordinary carbons) between the length of the arc and the voltage between its poles, if the current is kept constant. Also state how the voltage between the poles varies in an arc of constant length if the current through it is varied. What is the necessary arrangement of the mechanism in those arc lamps that have a shunt electromagnet only, in order that the striking and feeding shall occur properly?

74. What do you know of the maximum demand system of charging for electrical energy? What are the advantages and what the disadvantages of it?

75. A feeder has to supply 100 kilowatts, at 440 volts, at a distance of half-a-mile from the generating station. What must be the cross-section in order that the loss may be exactly 6 per cent. of the power put into the feeder? [N.B.—Work this out on the assumption that the resistance of a bar of copper 1 foot long and 1 square inch in cross-section is 7·9 microhms.]

76. Define the terms:—Watt, ampere, volt, joule. Prove that the product of volts and amperes is equal to the power in a circuit and that one horse-power is equal to 746 watts. [N.B.—One foot = 30·48 centimetres, and 1 pound = 453·6 grammes.]

77. Sketch the form of the curve of magnetisation of a specimen of mild steel, giving numerical scales to your diagram. Give a second sketch showing a corresponding curve for a specimen of very hard steel.

78. Explain how you would calculate the excitation required for the magnetic field of a dynamo. The air gap area of each pole of a smooth-core four-pole dynamo is 300 square centimetres, and the distance from pole face to core is 5 millimetres. How many ampere turns must be used on each pair of poles for air gap excitation alone if the magnetic density in the air gap is 10,000 lines per square centimetre?

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79. A solenoid, 12 in. long, wound on a brass tube $1\frac{1}{4}$ in. external diameter, is coiled with a No. 13 S.W.G. wire, having 10 layers with 115 turns in each layer,

and can carry 5 amperes without undue rise of temperature. Calculate the strength of the magnetic field produced (*a*) at the centre of the coil, (*b*) at the open mouth of the coil. What is the maximum pull you would expect it to exert on a cylindrical rod of soft iron half an inch in diameter and 15 in. long? In what position of this rod will the pull be a maximum?

80. What is the relation between the currents in two parallel wires and the force of attraction or repulsion between them? The leads from an accumulator cell carry 200 amperes and run parallel to each other, 8 centimetres apart from centre to centre. What is the force acting on these wires per metre run?

81. Give a dimensioned hand-sketch of a commutator, and state the appropriate number of its segments for a 4-pole lap-wound armature, having an output of 50 kilowatts at 110 volts at 720 revolutions per minute, and having 288 armature conductors.

82. Describe how you would measure the insulation resistance of the wiring in a building. Give a diagram of connections of the apparatus used for the purpose.

83. Explain the principles and uses of a magnetic blow-out.

84. How would you calibrate a wattmeter, using a Clark standard cell, a potentiometer, a volt box and a standard resistance capable of carrying the maximum current of the circuit for which the wattmeter is required? Give a diagram of connections.

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85. Describe the factory process of insulating the coils that are to be used for the windings of the armatures of tram-car motors or similar machines.

86. What is meant by the terms "hysteresis loss" and "eddy current loss" in the iron cores of dynamo armatures? Why are these cores laminated, and how are the losses affected by the thickness of the plates? In what magnetic and electrical qualities do the silicon-iron alloys now often used for transformer and dynamo core-plates differ from the ordinary iron generally used for the same purposes?

87. An electric kettle has an efficiency of 95 per cent. With current at 3d. per Board of Trade unit, how much will it cost to raise two pints of water from 15 degrees Centigrade to 100 degrees Centigrade? Find the time required for this operation with a current of 2 amperes at 220 volts.

88. Define the terms:—Volt, ampere, watt, joule. Prove that 746 watts is equivalent to one horse-power, and that 736 watts is equivalent to 75 metre kilograms per second.

89. Derive a formula for the electromotive force developed by a continuous current dynamo in terms of the number of poles, flux per pole, number of armature conductors, number of parallel circuits in armature winding, and revolutions per minute.

90. A bar of wrought iron, 1 in. square and 15 in. long, is made into a horse-shoe magnet with centres of poles 3 in. apart. The pole ends and keeper are faced to make good contact. What force is required to tear off the keeper if the excitation is 1,000 ampere turns and the permeability at this excitation is 400?

91. How would you proceed to calibrate an ammeter by a Crompton potentiometer and a standard cell? Give sketches of connections and state what other apparatus is required.

92. Define the term "capacity" as applied to a battery of storage cells. How is the capacity affected by the rate of discharge? What do you understand by the term "sulphating"? How is it caused and how is it remedied?

93. In a 4-pole direct current motor the number of conductors on the armature is 180, and there are 2·9 megalines per pole. What torque, in kilogram metre units, will the motor exert when a current of 50 amperes flows in each conductor?

94. Describe the principle of the potentiometer. Make a neat sketch of the connections for measuring resistance, current, electromotive force and power by means of a potentiometer, standard cell and standard resistance.

95. Explain under what conditions it is possible to get nearly constant speed under varying load at the motor of a power transmission with series wound machines.

96. Sketch a typical curve of the candle-power given at various angles above and below the horizontal by a continuous current arc lamp, and show how the mean hemispherical candle-power is obtained from it. If you were selecting an arc lamp for a given position would you be satisfied to know that it gave the required mean hemispherical candle-power, or would you also take the candle-power at the different angles into consideration? Give reasons for your answer.

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97. A 100 candle-power glow-lamp requires an alternating current of 6 amperes at 60 volts. The supply is at a pressure of 100 volts, at a frequency of 75 complete cycles per second. To give the lamp the right voltage, a choking-coil is to be inserted. Calculate how much self-induction, in henries, this choking-coil must have.

98. Calculate the number of alternating volts needed to drive an alternating current of 10 amperes through a choking-coil having a coefficient of self-induction of $\frac{1}{100}$ henry, the frequency being taken at 80 periods per second.

99. Define impedance and reactance as applied to alternate current circuits, and draw a clock diagram showing the place-relation between E.M.F. and current in a circuit whose impedance is 5 units and reactance 4 units. What is the resistance of the circuit?

100. An alternate current transformer has 200 turns in its secondary winding, the induced E.M.F. in which is 102 volts, the frequency being 100 periods per second. Assume that B will not exceed 4,000 lines per square centimetre. What must be the sectional area of the core?

101. An alternating (virtual) pressure of 100 volts is applied to a circuit containing a resistance of 120 ohms and an inductance ·8 henry. If the frequency be 50 cycles per second, find the virtual current and the angle of lag.

102. A circuit has a resistance of 200 ohms and capacity 20 microfarads. An alternating pressure of 100 volts is applied at a frequency of 70 cycles per second. Find the current and the angle of lead.

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103. Find the current obtained when an alternating pressure of 200 volts is applied to a circuit of resistance ·4 ohms capacity, 20 microfarads, and inductance ·025 henry. The frequency is 60 cycles per second.

104. Write an essay on "armature reaction" in dynamos and motors. (Univ.)

105. Explain the action of "interpoles" and "compensating windings." Why do carbon brushes assist commutation? (Univ.)

106. Define the terms—magnetic force, magnetic flux, flux density, and permeability. Find the ampere turns required to produce a flux of 100,000 lines in a steel ring of 8 square centimetres cross-sectional area and 60 centimetres mean circumference, if $H = 7$ when $B = 12{,}500$. If a saw-cut 2 millimetres wide is made across the ring at one place, find the total ampere turns then required for the same flux. (Univ.)

107. Find the cross-sectional area of copper wire and number of turns in a coil which has to give 7,000 ampere turns with a constant potential difference of 35 volts between its terminals and a loss of power of 105 watts. The temperature of the coil is 56° C., and mean length of one turn is 40 centimetres. (Univ.)

108. Describe the parts of a direct-current shunt motor which are essential to its action. Why is the armature core made of a number of thin disks, while the remainder of the magnetic circuit is usually of solid iron? (Univ.)

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109. What fixes the speed at which a shunt motor runs and the current it takes? How are these affected by inserting a resistance in the field coil circuit? (Univ.)

110. A four-pole shunt motor takes 58 amperes at 440 volts and has an efficiency of 88 per cent. at that load. What is the output in horse-power? If the armature is wound with 123 coils each of 5 turns, lap connected, and has a total resistance of 0·2 ohm, and the flux is 2·75 million lines per pole, what will be its speed at the above load? (Univ.)

111. Make a neat diagram of connexions of all the apparatus required for starting and controlling the speed of a shunt motor. Describe the operation of starting the motor, stating the use of each part of the apparatus during the starting period. (Univ.)

112. Define the following terms:—lumen, candle-power (including spherical and hemispherical), lux, brilliancy, absorption, reflection. (Univ.)

113. Describe a method for measuring the hemispherical candle-power of an arc lamp. (Univ.)

114. Describe the construction of the so-called "half-watt lamp," and give reasons for its high efficiency. (Univ.)

115. A circuit contains in series a resistance of 100 ohms, an inductance of 6·2 henrys, a capacity of 1 microfarad. If an E.M.F. of 1,000 volts at 50 frequency is impressed, what will be the current and the power factor? At what frequency will there be maximum current, and what will be its value? (Univ.)

116. A factory takes single-phase A.C. at 530 volts and 50 frequency. The current is used for power and light. The motors give a mechanical output of 25·5 kw. at a power factor of 0·75, their efficiency being 0·85. The lighting load is 10 kw. at unity power factor. What is the total current taken by the factory, and what is its power factor? (Univ.)

117. Describe and give the theory of a method of testing the insulation resistance of the positive and negative side of a private D.C. installation without interrupting the service. The supply of power is a dynamo on the premises, and there is no connexion to an outside supply. (Univ.)

118. Write an essay on "Armature Windings for Direct Current Machines." (Univ.)

119. Explain briefly the advantages of the multipolar type of construction for dynamos over the two-pole type, and establish a formula for the E.M.F. induced in the armature of a multipolar dynamo. A six-pole lap-wound armature running at 500 revolutions per minute has 760 conductors, and the flux from each field-pole is $7{\cdot}74 \times 10^6$ C.G.S. lines. Calculate the E.M.F. induced in the armature. (Univ.)

120. Calculate the magnetising ampere-turns required for a six-pole machine, given—

Effective length of air-gap, including slots, ·4 cm.

Effective area of pole-face, 800 square cm. Reluctance of rest of magnetic circuit is ·2 of *the whole*.

Flux density in gap, 7,500. (Univ.)

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121. Enumerate the requirements which should be fulfilled by a good form of motor-starter, and give a sketch showing in detail the construction of such a starter for a shunt-wound motor. (Univ.)

122. Explain what is meant by "feeders" and "distributors." A distributor, ·5 square inch in cross-section, is loaded uniformly with ·5 ampere per foot run. What is the maximum permissible distance between two feeding points along this distributor, if the drop from a feeding point to the point of lowest potential is not to exceed 6 volts? Assume the conductivity of copper at the working temperature of the distributor to be $·75 \times 10^{-6}$ ohm per inch cube. (Univ.)

123. Give sketches showing the method of wiring which you would propose to adopt for a building. Explain by a diagram of connections, what arrangement you would adopt for the purpose of enabling a lamp to be controlled independently by either of two distant switches. (Univ.)

124. The maker of a storage battery guarantees its capacity to be 400 ampere-hours at the 10-hour rate. What do you understand by this statement? Show by means of curves how the voltage of a cell varies with the time for different currents of charge and discharge. (Univ.)

125. At what price of electrical energy will the two types of glow lamps here specified be equally economical?

Type of 25 c.p. glow lamp	*Carbon*	*Metal*	
Cost in shillings	1	4	
Watts per c.p.	4	1·1	
Useful life in hours	800	1,200	(Univ.)

126. A motor has its field separately excited by a constant current. The resistance of the armature (allowing for any effect due to brush resistance and field distortion) may be taken as 0·8 ohm. When 125 volts are applied to the brushes the steady speed of the armature is 850 r.p.m., and the armature current is 1·4 amperes. What will be the speed when the terminal pressure applied is 120 volts and the load such that the armature takes 11·2 amperes? In the latter case, if the field strength were increased by 10 per cent., what would you expect the new speed to be? (Univ.)

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127. Explain what is meant by "Electrical Resonance." If a capacity of 10 microfarads is placed in series with a choking coil and the whole circuit supplied with current at a frequency of 50 cycles per second, what value of the inductance of the choking coil will give resonance?

128. Between two terminals with a potential difference of 1,000 virtual volts at 50 frequency are connected in parallel a resistance of 200 ohms, an inductance of 0·5 henry, and a capacity of 20·6 microfarads. Give the currents in each of these circuits, and the total current taken from the terminals and its power factor. Illustrate your answer by a vector diagram.

129. What capacity must be placed in parallel with an inductance of ·05 henry so that there will be no current in the main circuit, the resistances being negligible and the frequency 75 per second?

130. The outer conductors of a three-wire feeder have each 0·25, and the middle conductor has 0·5 ohm resistance. The load is 100 amperes on one, and 80 amperes on the other side. It is desired to have at the feeding-box a total of twice 230 volts. What total voltage must be kept at the home end of the feeder, and how must it be divided between the two sides? (Univ.)

131. State and prove the law of attraction between a magnet and its armature when in contact. If the surface of contact is 5 square inches and the total number of lines through this area is 400,000 C.G.S. units, calculate the force of attraction in kg. (Univ.)

132. Describe carefully how you would determine by the potentiometer method the error of a voltmeter at the reading denoting 110 volts. (Univ.)

ANSWERS.

Exercises I. (*Page* 12).

11. 250 lamps. **12.** 2,640,000 ft.-pounds. **13.** 81 lamps. **14.** 9·65 h.-p.
15. 820·6. **16.** ·7375; 13,825 approx.; 735 approx.
18. 13,563,039; (13,563,039 × 550) ergs per second. **19.** $4\frac{2}{3}$ h.-p.

Exercises III. (*Page* 36).

1. 121 units; 2·928 dynes.
2. In air: 2·553 e.s. units; 765·9 volts. In wax: 1·11 e.s. units; 333 volts.
3. 1,350 ergs. **5.** Heat from No. 1 : Heat from No. 2 = 3·58 : 1.
6. First conductor: ·00085 coulomb; 2,550,000 e.s. units. Second conductor: $\frac{1}{3}$ m.farad; 300,000 e.s. units. Third conductor: 5 volts; $\frac{1}{60}$ e.s. unit.
7.

First Conductor.	Second Conductor.	Third Conductor.
·036125 joule	1·5 joules	·00025 joule
·02664 ft.-pound	1·106 ft.-pounds	·00018 ft.-pound
10^{-8} B.O.T. unit	4×10^{-7} B.O T. unit	$\cdot 69 \times 10^{-10}$ B.O.T. unit

8. ·000016 h.-p.; ·553 ft.-pound. **9.** 89 5 e.s. units. **10.** 1 dyne.
11. 1 dyne parallel to the line joining the two charges. **12.** $\frac{1}{16}$, $\frac{1}{4}$, $\frac{1}{16}$.

ART 38 (*Page* 49).

(*a*) Separately: 1, 2, and 3 m.fs. (*b*) Three in parallel: 6 m.fs. (*c*) Two in parallel: 3, 4, and 5 m fs. (*d*) Three in series: $\frac{6}{11}$ m.f. (*e*) Two in series: $\frac{2}{3}$, $\frac{3}{4}$, and $1\frac{1}{5}$ m.f. (*f*) Two in parallel with one in series: $1\frac{1}{2}$, $1\frac{1}{3}$, $\frac{5}{8}$ m.f. (*g*) Two in series with one in parallel: $3\frac{2}{3}$, $2\frac{3}{4}$, $2\frac{1}{5}$ m.fs.

Exercises IV. (*Page* 53).

6. 2 m.fs. **7.** $\frac{3}{11}$ unit of capacity.
9. $E = QV = CV^2 = 8$ joules. $E_1 = \frac{1}{2}QV = 4$ joules.
10. The air condenser has the greater energy owing to its smaller capacity ($E = \frac{1}{2}Q^2/C$). **11.** 26 e.s. units. **12.** 4·4 sq. in. **13.** ·043 m.f.
14. $13\frac{1}{3}$ ergs. **15.** 1:9. **16.** 2:3; 2:3.
17. Three in parallel with one in series. **18.** ·015 in. **19.** ·314 m.f.

Exercise V. (*Page* 65).

5. 12,500,000 dynes.

Exercises VII. (*Page* 95).

1. 10 dynes / ·00002248 lb. } attractive.

2. (*a*) 120 units of moment. (*b*) $13\frac{1}{3}$ units of intensity.

3. (*a*) 16 dynes. (*b*) 240 dyne cm. **4.** $32\pi = 100{\cdot}5$.

5. T = 14·8 sec., $\therefore n = 0{\cdot}0677$. **6.** $a : b :: 0{\cdot}198 : 1$. **10.** 0·01 unit.

11. $d = 3$ cm., F = 1 dyne.

15. M = 3702·85, or (neglecting l) M = 3858·75 and $m = 185{\cdot}1$ or 192·9.

Exercises VIII. (*Page* 116).

6. 10 ergs. **8.** 4·33 amp. **9.** 2,235 ampere turns.

Exercises IX. (*Page* 128).

5. ·00111. **6.** 1,382,505 coulombs; 384 amp. hr.; 15·36 amp.

7. 5·4136 grm.; 1 hr. 39 min. 34 sec. **8.** 8·87 lb. of silver; 2·41 lb. of nick

9. ·0083 ohm; ·000083 ohm; 1·66 volts. **10.** 3·2 volts. **11.** $16\frac{2}{3}$ ohms.

12. 975 volt, 109·025 volts; 104·8 volts; 4·225 volts; 1·612 ohms. **13.** $6\frac{2}{3}$ ohn

14. 6 ohms; 648×10^{11} ergs; 648×10^{4} joules; 1·8 B.O.T. units; 10·8 pence.

Exercises X. (*Page* 145).

Series A.

1. ·498 amp. **2.** 36,000 coulombs; 545·9 grm.; 13·1 pence.

3. ·133177 ohm; 39·95 volts. **4.** 240 amp., ·24 sq. in.; ·0396 ohm; 109·5 vol

5. ·0475 sq. in. **6.** 200 volts. **7.** ·0384 sq. in. **8.** ·37 ohm; 114·2 vol

9. ·0973 sq. in. **10.** A. $1\frac{1}{5}$ amp.; B. $\frac{5}{9}$ amp.; C. $1\frac{2}{3}$ amp.

11. $\frac{3}{8}$ amp.; 45 watts; 1·08 B.O.T. units; 5·4 pence.

12. 12,000 watts; 12 B.O.T. units. **13.** 3·3 pence. **14.** 12 lamps.

Series B.

6. 127·6 megohms. **7.** 17,500 ohms.

8. ·0875 amp.; ·14 amp.; ·28 amp.; ·56 amp.; 1·0675 amp.

9. 2285·7 ohms; 1428·5 ohms; 714·2 ohms; 357·1 ohms; 187·3 ohms.

10. ·305 ohm. **11.** ·065 sq. in. **12.** ·094 in. **13.** 1 : 2·79.

14. ·092 sq. in. **15.** 1·6 sq. in.; ·4 sq. in.

16. 218·18 ohms; 66 watts; 237,600 joules; 3,300 watt hr.; 6·6 B.O.T. units; 2/

17. 1/6 at 4 watts per candle. **18.** 9·65 h.-p. **19.** 100·5 h.-p.

20. 2,123,324. **21.** 162·2 h.-p. **22.** $3\frac{1}{7}$ pence. **23.** 577 yd. (approx

24. 31·5 ohms; 114 watts; 5/8·4. **25.** 3·055 ohms.

Exercises XI. (*Page* 167).

11. 533,333$\frac{1}{3}$ ohms. **12.** 1,500 ohms.

13. $\frac{7}{10}$ of the distance from the nearer end. **15.** ·02 ohm.

Exercises XII. (*Page* 187).

7. 416$\frac{2}{3}$ ohms. **9.** 111 ohms; 107·73 ohms; 3555·27 ohms.

10. 6,000 ohms; 4. **11.** 70 divisions; 2. **12.** 39 : 20.

Exercises XIII. (*Page* 203).

7. (*a*) $\frac{2}{5}$ amp.; $\frac{2}{5}$ amp.; 8 volts. (*b*) $\frac{20}{203}$ amp.; $\frac{2}{203}$ amp.; 1·97 volts (*c*) $\frac{4}{11}$ amp.; $\frac{2}{11}$ amp.; $7\frac{3}{11}$ volts; (*d*) $\frac{10}{53}$ amp.; $\frac{2}{53}$ amp.; 3·77 volts.

8. 68 cells at 1·1 volts per cell. **9.** In series; 3 amp.

10. 10 cells in series in a row, the two rows being in parallel; $\frac{10}{11}$ amp.

11. 80 cells. **12.** 1 cell is wrong.

13. Copper: 6560·6 grm.; 3280·3 grm. Mercury: 20,747 grm. Hydrogen: 103·8 grm. Caustic soda: 4153·5 grm.

14. 48 sec. (approx.) taking z for Na = ·00024. **15.** 7·7 h.-p.; 22·2 h.-p. hr.

16. $\frac{3}{20}$ amp.; $\frac{201}{240}$ volt; $\frac{27}{80}$ volt; $\frac{1}{2}$ volt.

Exercises XIV. (*Page* 222).

8. 130 volts; 92·5 volts. **9.** 70·1 per cent. **10.** 2·6 ohms; 2 ohms.

Exercises XV. (*Page* 266).

14. Res. of A = 15 ohms. Res. of B = 6 ohms. Heat in A : Heat in B = 5 : 8.

15. ·2499 pence; 1,499 watts (taking 1 quart of water = 2·5 pounds, and 1 pound = 453·6 grm.).

Exercises XVII. (*Page* 296)

1. 14·3. **2.** 1428·7 metres and 12·57 dynes.

3. (1) 18855 units. (2) 188·55 dynes. (3) 565·65 lines.

5. 1·8 dynes. **6.** 0·00157 henry.

7. Primary (*a*) 0·0033 henry, nearly. Secondary (*b*) 0·0008 henry, nearly.

8. Final current = 25 amp. After $\frac{1}{200}$ sec. = 0·31 amp. After 1 sec. = 22·95 amp. **10.** 874 amp. turns, approx.; 7806.

Exercises XVIII. (*Page* 340).

8. Elec. efficiency = 96 per cent. Commercial efficiency = 86·7 per cent.

9. 62·6 per cent. **10.** (1) 4·36 h.-p. (2) 60·05 h.-p. **11.** 38·5 amp.

14. Coils in parallel 104·6 rev. per min. Coils in rows of five 164·4 rev. per min.

21. $8{\cdot}36 \times 10^6$. **22.** $\frac{0{\cdot}069}{4}$ = ·017 ohm (if two pole machine).

23. Resistance (armature) = 0·0359 ohms Resistance (shunt) = 12·96 ohms Current (shunt) = 9·26 amp. Current (armature) = 176 amp.

24. t = ·001. **25.** c = 2 volts. **26.** 17,600 (approx., using curves).

Exercises XIX. (*Page* 364).

3. 89·3 h.-p. **4.** 70·1 volts. **5.** 13·58 amp. **7.** 1·819 pence.
8. 17·27 amp. £42 0s. 5d. per annum. **9.** 87·7 per cent. **11.** 92·8 amp.
12. 1·27 h.-p. 100 amp. 95 per cent. 50 per cent. **13.** 250 dynes.
14. As dynamo, 92·3 per cent. As motor, 91·7 per cent.

Exercises XX. (*Page* 388).

6. 10·537 amp. 166·7 volts on choker. 105·4 volts on resistance.
8. (1) 2·52 amp. (2) 82° 45′. (3) 79·3.
9. $n = 356$. **10.** $k = 35$, $v = 2140{\cdot}95$ volts.
11. I = 1 amp. Condenser P.D = 800 volts nearly. P.D. on resistance = 600 volts nearly.

Exercises XXI. (*Page* 411).

13. Series resistance = 4,500 ohms.

Exercises XXII. (*Page* 430).

4. For economy in copper, the high-pressure system. For economy in running expenses, the low-pressure system. **9.** 120 per cent.
10. Using, say, a dynamo overcompounded to give 200 volts always at the mill and generating at 250 volts, the answers are (allowing 90 per cent. efficiency for both dynamo and motor): Drop = 50 volts, Current = 100 amp. approx., Cross-section = 0·095 sq. in., H.-p. of wheel = 37.
11. 1·3 in.; ·44 in. **12.** (*a*) 4·01 sq. in.; 5145·5 lb. (*b*) 1 sq. in.; 2316·9 lb.
13. 577 yd. (approx.). **14.** ·17 sq. in.; 1,028 lb. (approx.).

Exercises XXIII. (*Page* 452).

10. ·103 sq. in.; ·01 sq. in ; ·001 sq. in.
11. Assuming 200 volt supply and mains reduced after the first three floors, we have:—Mains for 1st, 2nd, and 3rd floors = $\frac{37}{16}$ cable; mains for 4th and 5th floors = $\frac{19}{16}$ cable; 10 circuits in each room, each supplying 8 lamps; proportionate variation for 100 volt supply.

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