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MONOGRAPHS ON THE PHYSICS AND CHEMISTRY OF MATERIALS

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WILLIS JACKSON H. FRÖHLICH N. F. MOTT

MONOGRAPHS ON THE PHYSICS AND CHEMISTRY OF MATERIALS

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DIELECTRIC BREAKDOWN OF SOLIDS

BY

S. WHITEHEAD

DIRECTOR OF RESEARCH

THE ELECTRICAL RESEARCH ASSOCIATION

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PREFACE

It rarely falls to the lot of an author to write two books on the same scientific subject, separated by no long period, in which the material common to both is greatly in the minority. Generally speaking, it has proved possible only to incorporate from the earlier book (*Breakdown of Solid Dielectrics*, Ernest Benn, London, 1932) sections on thermal breakdown. Some general background from the earlier book has also been referred to, and it has been presumed from this and other early literature that the reader is aware of the more elementary ideas (for example, that breakdown means the temporary or permanent loss of normal insulating properties), but otherwise the present volume is quite new.

Experimental scientists and engineers have been preoccupied with the subject since the beginning of electrical studies. Twenty-odd years ago, when the present author's interest began, a good deal was known in a practical way and the composite nature of the phenomena was realized. However, no one properly understood how fundamentally different were the various types of breakdown. Indeed it was not until the general theory of the solid state had advanced, and until theoretical and experimental workers co-operated closely, that a fairly exact analysis and classification became possible. One of the objects of this book is to demonstrate the value of this co-operation in what had hitherto been regarded as an engineering subject. If this object be achieved, it will compensate for the author's bias in favour of work with which he has had some connexion and for the arbitrary choice of the standard and relative amounts of the physics and engineering brought together to make the book.

If it had been possible, preference would have been given to the rationalized M.K.S. system, but the labour of transposing and verifying so much original material was too great. Accordingly the theoretical demonstrations are mainly in the Gaussian system, as generally used by theoretical physicists, and

the practical formulae are usually expressed in the units in which the data are commonly given, even where some rather odd combinations result. In a rather similar way some liberties have been taken with the standardized, but not yet universally followed, lists of electrical symbols. For this reason all symbols, with the pages of first appearance, are listed by chapters and defined at the outset.

The author wishes to express his indebtedness to Professor H. Fröhlich, on whose work the theoretical portions so greatly draw, for his help at all stages in the preparation of the book, and his very detailed advice on the introduction of concepts from modern theoretical physics. A similar indebtedness is acknowledged to his former colleague Dr. A. E. W. Austen, whose work similarly figures in the book, for his advice and help throughout the work. Valuable comments on the manuscript are also acknowledged from Mr. R. Davis (National Physical Laboratory), particularly on the work emanating from his laboratory. Assistance is gratefully acknowledged from many of the author's colleagues, particularly Mr. Church, Mr. Garton, Mr. J. H. Mason, Dr. Pelzer, and Mr. Thomas, and from his former colleague, Mr. Tooke, in the collection of published information.

The kind permission to reproduce original material from the authors mentioned in the text is acknowledged. Permission for the reproduction of illustrations is similarly acknowledged from the following bodies:—the Institution of Electrical Engineers, the Royal Society, the Physical Society, the Editors of the *Physical Review*, Messrs. Taylor and Francis, Ltd. (*Philosophical Magazine*), Messrs. Ernest Benn, Ltd.

Finally, grateful acknowledgement is due to the Council of the Electrical Research Association, and in particular to the late Sir Clifford Paterson, for encouragement and facilities to write this book and to quote from the Association's reports.

S. W.

LONDON
October 1950

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SYMBOLS

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a = lattice constant 18
B = rate of loss of energy from electrons . . 42
B_D = the above on account of dipoles . . . 67
B_P = the above for the pure substance . . . 67
b = from $b^2 = \tau_D B_D \tau_P B_P$ 67
C_1 = constant proportional to number of energy levels in interval $\mathbf{k}T$ in conduction zone . . . 42
C_2 = as above for imperfection levels . . . 42
C_0 = h.f. capacitance 96
c = velocity including $\mathbf{c}$, velocity of light . . 24
c = concentration of a solid solution . . . 57
c_0 = $10^{-11}/36\pi = 8{\cdot}05 \times 10^{-14}$ 6
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d = thickness 23
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i = current 97
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$2\Delta V$ = energy difference in electron volts of excited or trapped electrons below conduction zone . 30, 41
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b = short semi-axis of spheroid 148

b = radius of stem of defect . . . . p. 152

curl = vector function of vector u having components

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c = long semi-axis of spheroid 148

$c_0 = 10^{-11}/36\pi = 8{\cdot}05\times10^{-14}$ 116

D = thickness of sound dielectric 153

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d = thickness of defect 153

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CHAPTER IV

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E	= electric strength of gas	203
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e	= base of napierian logarithms	—
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x = void thickness 203
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δ = loss angle 189
ϵ = permittivity 205
θ = epoch [illegible] 177
θ = temperature rise 229
θ_0 = constant in exponential function e^{θ/θ_0}. . . 205
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CHAPTER I

METHOD OF TREATMENT

Two dangers beset a writer on the present subject. He may provide neither enough physics to interest the physicist nor enough engineering to interest the engineer. On the other hand, dielectric breakdown cannot be treated as a branch of pure physics and, if treated entirely as engineering, there would be no room for the physical basis which enlivens the subject. The progress in the last 20 years is a good story, and this book tries to tell it. Sometimes the story is from physicists and sometimes from engineers. The aim has been to give enough of the contributions of both physicists and engineers so that original papers or other data need not be referred to except for detailed studies and for specific designs where the properties of particular materials or particular mathematical solutions are necessary.

The story is told from the point of view of the author and is therefore incomplete. Disregarded facts or theories may later prove to be crucial. The selection made has to be justified by the evidence. Not very much more than 15 years ago intrinsic breakdown was first realized and measured by von Hippel on alkali halide crystals, followed on other crystalline and amorphous materials by colleagues and associates of the author, such as Austen, Davis, and Thomas, leading to the numerous results of later workers. At the same epoch von Hippel introduced the concept of electronic breakdown, and not long before the recent war Fröhlich elaborated this concept into a complete and quantitative theory. Full justice has not perhaps been done to this achievement, by which electric strengths were calculated from crystal data without any disposable constants to give agreement with experiment to some 10–30 per cent. In showing this remarkable confirmation Austen and Whitehead indicated that the theory could not, however, explain other results such as those of Thomas on varnish. Fröhlich, then working on the matter, soon after brought out his theory of amorphous breakdown, which today is still being examined experimentally. These

discoveries form the main subjects of Chapter II, together with some historical background and the main experimental evidence considered to relate to intrinsic breakdown. The theoretical calculations which are compared with experiment are given in enough detail for those familiar with the subject to fill in omitted steps, but the mathematical demonstrations can be ignored, if desired, because they are prefaced by the qualitative forms of the argument. Experimental techniques are mentioned only briefly because details interest rather few specialists who would read the original papers in any event. Experimental results are detailed enough to assess the evidence supporting theory and the observed facts which have not so far been explained—in particular the (as yet unresolved) problem of the failure, with no obvious reason for it, to attain the intrinsic electric strength.

The general principles of thermal instability have been known for a long time. In Chapter III it is only necessary, therefore, to recapitulate the theory in a more complete form, stating and proving certain generalizations hitherto deduced from particular examples. Some extension has also been made to partial and local breakdown. The extended theory is needed in other chapters.

It is believed that the topic of breakdown by discharges has not previously been summarized so comprehensively as is attempted in Chapter IV. This type of breakdown was known qualitatively in earliest times, but its quantitative study did not really begin until less than 20 years ago, when experimental methods became more refined. This type of breakdown may prove to be the most important in practice. Accordingly it has been treated at some length and some of the experimental technique has been included. Most of the work described has been completed or published in the past 10 years. Much more research is needed.

Electro-chemical breakdown in Chapter V is perhaps an application of physical chemistry. It is ill defined in that it comprises a multitude of processes whereby the material or part of it changes its chemical constitution. With so wide a field it

was decided only to attempt the description of one or two typical instances which have been fully worked out.

Chapter VI suggests some general principles in the application to practice of the scientific discoveries given in earlier chapters. Although necessarily over-simplified it makes some case for the direct utility, as opposed to 'background' knowledge, of the researches undertaken.

It can be no criticism that breakdown in gases, *per se*, is ignored. No short account is possible or useful. It is believed that one or more later volumes in the present series will deal with gaseous discharges. The omission of liquids follows from the declared aims for there is no real theory of the liquid state. Breakdown in liquids could readily become a compendium of unrelated observations. Fortunately the use of liquids is restricted to a few typical situations and in most of these solid dielectrics play a major role. The mechanism of liquid breakdown does not seem so important. This may well be the result of ignorance. Even so, the subject provides a future not a present task.

Another subject has been omitted with regret, namely, the use of thin dielectrics. Throughout the book it will be evident that the less solid dielectric in a design the better. The practical emphasis comes from the need to use as thin an insulating barrier as is safe. In addition great advantages should come from using very thin layers of dielectric separated by conducting layers. Unfortunately constructions have not been realized where the specific effects of thinness have an influence. This influence is not apparent in the known interleaved constructions which may rather be said to avoid some of the disadvantages of excessive thickness. It was therefore thought necessary to await some practical achievement.

Principles and their discussion form the declared aim. Accordingly a great field of discovery, development, and information has been omitted which is essential to an engineer in his actual practice. Detailed facts are here paramount and could not be compressed into the present compass. The number of different insulating materials now available, for

each of which a case can be made that it is the most appropriate for at least one particular use, runs into three figures. Several industries are concerned in their production and use. It is questionable whether possibly the best approach to insulation engineering practice is not by way of the equipment in which the insulation is to be used, granted a fundamental background to which the present study may contribute.

Chapters, sections, and sub-sections are numbered in accordance with the decimal system but the equations in them are numbered by chapters. The universal constants **c** (velocity of light), **e** (electronic charge), **h** (Planck's quantum), **k** (Boltzmann's constant), and **m** (electronic mass) are printed in heavier type. Other symbols are fully listed by chapters with the pages where they first appear, in order to obviate too frequent definitions. Where possible practical electrical units are used so that thermal magnitudes are usually expressed in the electrical system. Each chapter has its own numbered bibliography of references. A complete bibliography has not been attempted because the book itself does not pretend to be complete.

CHAPTER II

INTRINSIC BREAKDOWN

2.1. The experimental concept of intrinsic electric strength

THE considerable experimental and theoretical structure described in the present chapter stands on the foundation of the real and observable nature of what is here called intrinsic breakdown. The foundation is therefore one of experimental fact. It is the aim of the present section to show how this fundamental concept developed and was established by experiment.

In the early study of what was called the 'disruptive electric strength' of solid dielectrics, its variations with conditions external to the dielectric, such as the nature of the electrodes, ambient medium, thermal dissipation and the like, led to the view that the electric strength was rather indefinite a quantity analogous to the contemporary concept of the fatigue strength of materials. The determination of thermal breakdown, considered in Chapter III, and the possibility of its fairly easy avoidance in laboratory experiments by thin and cooled specimens, introduced the conception of what was called 'electrical' breakdown. In fact, the impression is sometimes given even in very recent papers that the non-thermal breakdown of solid dielectrics is necessarily of a homogeneous type. However, the so-called electrical breakdown usually varied with thickness, ambient medium, and nature of electrodes, as well as physical conditions such as temperature. Concentration of electric stress at electrode edges received attention. The hypothesis was proposed that a proportionality between the electric strength and the inverse square root[1] of the thickness was evidence of the influence of this concentration. Kirchhoff's solution for a finite electrode[2] lends some colour to this hypothesis. A similar result can also arise from thermal breakdown. This caused confusion, particularly since rounding the edges of electrodes did not remove, although it sometimes reduced, the variation

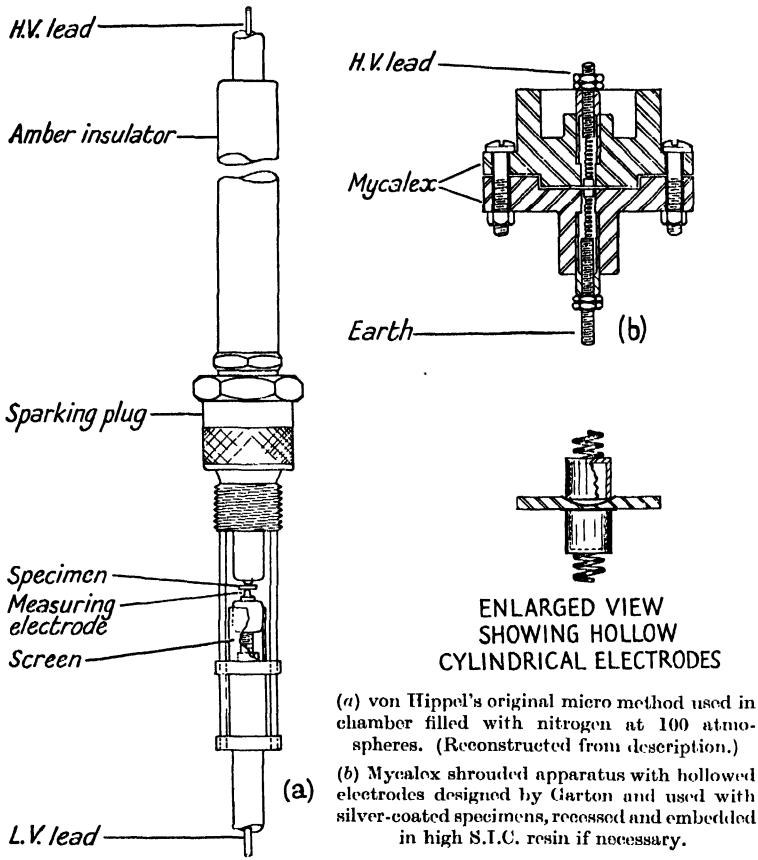
with thickness. Nevertheless when greater care was given to avoiding discharges in the external medium from the electrode to the dielectric and with the use of transient voltages where extraneous effects are reduced, the non-thermal electric strength was found to a greater extent to be independent of thickness, nature of electrode, and form of applied voltage. Thus arose the concept of what is here called the intrinsic electric strength, but is also called 'physical' breakdown, true electric strength, and, at one time by Rogowski,[3] 'electric tensile strength'; the last term found little favour because the mechanical tensile strength is considerably more variable a quantity. This experimental concept was of an electric strength characteristic of the dielectric itself and independent of other factors except its physical state. In a comparatively recent subject, it is unwise to be dogmatic. Nevertheless, it is true to say that with suitable precautions it is possible to determine a value of the electric strength of most dielectrics which is independent, over substantial ranges, of the particular experimental circumstances other than temperature and pressure.

By the use of rounded edges and other devices it is not difficult to avoid appreciable local increases of electric stress. The avoidance of premature discharges in the surrounding medium is difficult. If F_s, ϵ_s, and σ_s are respectively the electric strength, permittivity, and conductivity of the solid dielectric, and F_m, ϵ_m, and σ_m the corresponding quantities for the medium in contact with the dielectric, then for d.c. $\sigma_m F_m$ should exceed $\sigma_s F_s$, for a.c. with a dielectric medium $\epsilon_m F_m$ should exceed $\epsilon_s F_s$, for a.c. with a semi-conducting medium $\sigma_m F_m$ should exceed $\omega c_0 \epsilon_s F_s$ (where ω is 2π frequency and c_0 is $10^{-11}/36\pi$), if the electric strength of the solid is to be attained before the medium breaks down.† It is therefore desirable to increase F_m and ϵ_m or σ_m. Air is unsatisfactory in all ways. Austen[4] estimates that sharp-edged electrodes in air are unsuitable for high sensitivity power factor tests above 50 kV/cm. This does not necessarily

† For a.c. the condition is strictly $F_m \epsilon_m \sec \delta_m > F_s \epsilon_s \sec \delta_s$, where δ is the loss angle. This equation reduces to those given, for the particular cases of δ small and δ nearly $\frac{1}{2}\pi$.

apply to breakdown, since some dielectrics resist discharges better than others and the duration of the applied field has a major influence. Thus the mere occurrence of discharges is not crucial as was thought when the 'electro-camera' technique (Gemant,[5] Thomas[6]) was developed. As will be seen in Chapter IV, although the intensity of these discharges rises at first fairly rapidly with increase of voltage, yet the sensitivity of the mode of detection may partly determine the voltage at which the discharges appear to begin. A value higher than Austen's is therefore appropriate for breakdown, but only a rough estimate can be made, namely, that electrodes in air are unsuitable for intrinsic breakdown strengths if the electric strength is much above $0{\cdot}1 \times 10^6$ V/cm.

Although F_m and ϵ_m are much greater for oil, it is doubtful whether electrodes in oil are suitable for electric strengths greater than of the order of $0{\cdot}5 \times 10^6$ V/cm. It has been suggested that degassed oil does not vary in electric strength with pressure, but oils in normal condition increase in strength as the pressure rises. Inge and Walther[7] took the view that this increase was not more than 2·5 times for a pressure of 10 atmos., thereafter changing slowly; they criticized Marx[8] who recommended pressures of this order. Such pressures are used in pressure-type H.V. cables, but a rather greater improvement is obtained. Thomas[9] used xylene at higher pressures (40 atmos.), obtaining probably satisfactory results on synthetic cellulose materials at 2 to 3×10^6 V/cm.; the technique was rather laborious and limited. von Hippel[10] employed nitrogen or other gases at 100 atmos. pressure applied sometimes to a medium or guarding ring of beeswax or other wax solidified under pressure and sometimes directly. His co-workers have followed his technique, but it has been mainly applied to crystals of fairly low electric strength, less than 2×10^6 V/cm., to partial breakdown in divergent fields, and to surface breakdown where the conditions are less severe. Fig. 1 illustrates von Hippel's apparatus as described in 1931, together with a recent miniature apparatus for recessed specimens (*vide infra*). Subsequently von Hippel[103] has also described recessed specimens and techniques for high



(*a*) von Hippel's original micro method used in chamber filled with nitrogen at 100 atmospheres. (Reconstructed from description.)

(*b*) Mycalex shrouded apparatus with hollowed electrodes designed by Garton and used with silver-coated specimens, recessed and embedded in high S.I.C. resin if necessary.

FIG. 1. Electrodes for small specimens.

strengths. The miniature apparatus in Fig. 1, designed by Garton, has been used with a solidified ambient medium.

Increase of the dielectric constant of the medium suffers from the disadvantage of the chemical activity, limited temperature range or high viscosity of suitable liquids: high viscosity leads to air bubbles which discharge. Nevertheless, alcohol in xylene has been used by Inge and Walther[7] and aniline by Grunewald,[11] but it is doubtful whether electric strengths much above 10^6 V/cm. can be measured by this means. An exception is in impulse tests where Standring[12] used glycerine. If otherwise

permissible, water can be used with impulses and Standring obtained on mica the highest electric strengths (15×10^6 V/cm.) previously observed, employing distilled water as medium.

Increase of the conductivity of the medium has much more flexibility. Nitrobenzene and castor oil mixtures offer a wide range of conductivity with little change in electric strength, which is about $0{\cdot}1 \times 10^6$ V/cm. Similarly chlorinated diphenyl with nitrobenzene or alone can be used up to 300° C. (Thomas,[9] Hackett and Thomas[13]). Tricresyl phosphate is also suitable where its chemical activity permits (Perlick[14]). These are the principal among many other liquids referred to in the literature (see Perlick for a review) and it is probable that electric strengths up to 6×10^6 V/cm. may so be measured, and possibly higher in special instances. An ingenious development of this principle is mentioned by Saito.[15] He deposited metal electrodes on the dielectric by sputtering, which, with evaporation, is itself a well-known technique: but Saito arranged a diffuse marginal zone round the periphery. This diffuse zone varied from conducting to semi-conducting and relieved the stress near the edge. Success was claimed with d.c. The validity of the technique does not seem to have been independently confirmed and it is only applicable to certain materials. In addition, a thin film is rather easily burned away by a small current as in the Mansbridge type capacitor.[16]

Where it is feasible the best technique is to reduce the stress on the medium by suitably shaping the specimen. At an early stage recessed specimens were advised by the Electrical Research Association[17] and adopted by the British Standards Institution,[17] but the technique was not sufficiently developed then to attain the intrinsic electric strength except in the lower ranges. Similarly, though Schumann[18] and his co-workers used lens-shaped specimens, they did not measure the intrinsic electric strength. Glass in blown bulbs offers the obvious opportunity for slow gradation of thickness so that the ambient medium is removed to a region where the field strength is relatively low (Moon and Norcross,[19] Austen and Whitehead[20]), but the heat treatment may be undesirable. The electrodes used

have been either mercury, deposited metal, or carbon. With other materials different procedures are necessary. Recesses in mica have been ground and polished (Hackett and Thomas[13]) and used with deposited metal or carbon. Alkali halides have been similarly treated (Austen and Whitehead[20]), but more consistent and reliable values are claimed by Austen for ground but not polished surfaces, mentioning that polished surfaces of these crystals are liable to have a few deep scratches which do not appear to be present in unpolished specimens. Recesses can be pressed in varnish films (Thomas and Griffith[21]) without great difficulty, but with chain polymers such as polythene and vinyl products (Austen and Pelzer,[22] Oakes[23]) a carefully controlled moulding process is necessary, and even then the orientation of the chains is a problem. The difficulties of producing a suitable recess are sometimes almost insuperable, as was found by Austen and Pelzer with oriented paraffins. Presuming that a satisfactory recess can be produced, an ambient medium of suitable electric strength is needed in which the specimen is immersed to avoid flashover. Petroleum ether, transformer and other oils, and *iso*-pentane have been used, the last for low temperatures, in addition to liquids mentioned earlier. In the usual plane specimen the recess reduces the field outside the recess compared with that at the vertex according to the ratio of the thicknesses. This ratio must therefore be great enough so that when the field at the vertex reaches the electric strength of the dielectric, the field outside the vertex is still low enough to be consistent with a field in the ambient medium less than the electric strength of the latter. Fig. 2 shows an arrangement used for potassium bromide (Austen and Whitehead[20]). The apparatus with recessed specimens in Fig. 1 has recently permitted the measurement at very low temperatures with a suitable ambient medium of the highest electric strengths yet observed at the time of writing (on polyvinyl alcohol).

With very thin dielectrics, discharges are not a difficulty if the breakdown voltage is reduced to the neighbourhood of the minimum breakdown voltage of the air or other ambient medium. The use of a vacuum to increase the breakdown

voltage of the ambient space has been tried (e.g. Mundl,[24] Krassin[25]) but without great success: the maintenance of an adequate vacuum is laborious and scarcely possible for some materials without degassing treatments which would often be undesirable.

Intrinsic breakdown is defined by the exclusion of other forms of breakdown and its occurrence is therefore difficult to verify by simple direct observations of accompanying phenomena. It has rather to be inferred from a variety of evidence. It is indeed

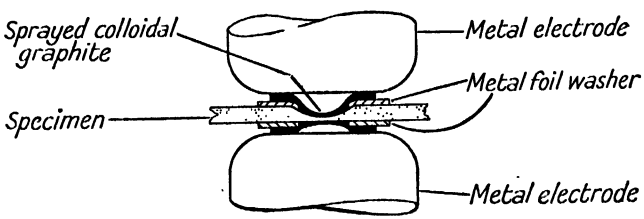


FIG. 2. Recessed specimen with electrodes.
(Austen and Whitehead.)

essential that breakdown should take place within the region of the most intense field and it has been shown (Whitehead[26]) why breakdowns initiated by discharges in the immersion medium frequently occur in weaker parts of the field, but the correct location of the breakdown is by itself insufficient evidence of intrinsic breakdown. An often more stringent test is whether the electric strength is independent of the thickness, but this does not hold for very thin specimens and cannot easily be applied to some materials. If the ambient medium is increased in electric strength, conductivity, or dielectric constant, without altering the breakdown voltage, the absence of injurious discharges in the medium is probable. Unless deliberately retarded, intrinsic breakdown is a rapid phenomenon depending on a critical field strength over a sufficient region. In general, therefore, the same electric strength should be obtained with constant voltages as with alternating or transient voltages provided the crest values of the voltages are compared. This is subject to certain exceptions which will only be appreciated later, together

with possible electrode effects. The net result is that if all the preceding tests yield normal results then the breakdown may confidently be regarded as intrinsic. An abnormal result in certain tests does not necessarily invalidate the experiments because intrinsic breakdown is not yet fully explored. The highest value of electric strength ultimately attained is significant, but this is conditioned by the interpretation of 'ultimately' and by those errors which give a spuriously high electric strength. Errors of the latter type have occurred in the past, but should be rare in modern technique: roughly speaking, they have mainly been associated with errors in voltage measurement (e.g. the inclusion of potential variations in the electrodes), errors in the thickness, and insufficient energy in the source of potential. Space charge effects have been proposed as a possible cause of spuriously high electric strengths and, although considered later, it may be said here that this possibility has no wide acceptance.

2.2. Earlier theories of intrinsic breakdown

It has already been mentioned that probably the earliest approach to electrical breakdown was by analogy with the mechanical failure of metals. Little progress could be made until the lattice structure of crystals was studied. In a manner similar to the attempts made to define the mechanical stability of the structure, Rogowski[3] attacked the problem of finding the electric field necessary to distort the structure sufficiently for it to lose its stability. Taking a simple cubic ionic crystal such as rock salt, he assumed that the ions occupying the lattice-points were retained in equilibrium by inverse square Coulomb forces and Born repulsion forces. He found the field strength necessary to cause displacements of such magnitude that the restoring forces would be on the point of diminishing. The amount of this displacement corresponded to distortions much greater than those observed, and the critical field strengths were of the order of 10^8 V/cm., that is, some 100 times higher than experiment indicated. A similar discrepancy occurs when analogous considerations are applied to mechanical strength, with the additional difficulty in the electrical case that there is normally a

discrete discharge path on breakdown. To overcome these difficulties, Horowitz[27] invoked a further development in mechanical theory, namely the Griffith principle that breakdown begins at microscopic cracks or scratches. If a crack has a semi-elliptical section and its contents, more or less gaseous, are ionized, then a calculable amount of energy is stored from the dielectric field. The crack may be supposed to be prevented from spreading by a surface energy similar to surface tension. Horowitz was then able to calculate the field at which the crack became unstable and obtained more reasonable values for the electric strength as well as a reason for the discrete discharge path. However, no experimental evidence has been adduced as to the existence and influence of these cracks upon electrical breakdown and no absolute calculation could be made of the electric strength owing to the number of parameters for which values had to be assumed unsupported by direct evidence.

Another suggestion to meet the difficulties of Rogowski's calculation was to ignore the condition of instability of the electric forces and to consider the magnitude of the displacement. Smekal,[28] for instance, claimed two transitions in dilatation or displacement for rock salt, one associated with mechanical and the other with optical behaviour. If the corresponding displacements were substituted in Rogowski's equations for displacement in an electric field, more reasonable values of the critical field strength were deduced. Such speculations are only of historical interest in the light of present knowledge of solid structure.

Somewhat allied to these ideas were those which ascribed breakdown to the mechanical stresses which arise from an electric field. Undoubtedly cases of the mechanical failure of dielectrics occur, but are limited to mechanically weak or soft dielectrics and to circumstances where local concentrations of stress are found, as with pointed electrodes or when a large electrode is supported by a small area of contact with the dielectric on which the mechanical force is concentrated (Austen and Pelzer[22]). However, general hypotheses of this kind have been put forward in connexion with impulse breakdown. For

example Lehmhaus[29] suggested mechanical disintegration for breakdowns in less than 10^{-7} sec. on the front of a steep wave. The idea seemed to be held that the rapid application of the mechanical stress due to the electric field was like an impact and caused brittle failure. The difficulty is that the stresses involved are far too small as compared with those concerned in impact failure except in a rather limited range of materials or with special hypotheses as to the distribution of charge.

Another approach by analogy was on the basis of ionic escape from a local field. The suggestion was made by Rogowski,[30] Whitehead,[31] and others that if the electric field causes a relative displacement of ions in a polar structure or a molecular distortion then the amplitude of thermal motion superimposed might give an effective displacement sufficient to lead to disintegration as in Lindemann's early hypothesis for melting. This view was considered in a more serious manner by Ganger.[32] He adopted with Rogowski the Coulomb and Born forces but considered an isolated polar pair in a regular structure of lattice constant d. It is then easy to show that, in absence of thermal motion or electric field, the equilibrium displacement is $(\beta d^8)^{1/m-2}$ where β and m are the parameters in the expression $\beta e^2 d^8 x^{-m}$ for the Born force (e = charge, x = displacement). The critical displacement beyond which the restoring force falls is

$$(\tfrac{1}{2}m\beta d^8)^{1/m-2}.$$

The difference between these two displacements should be the thermal displacement at melting which Ganger considers to be $0{\cdot}085d$. From this equation he finds m to be 9·635. He next assumes the thermal displacement to be proportional to the absolute temperature T, i.e. aT.† Since at temperature T the equilibrium displacement is aT, the displacement which determines the critical field strength is $(\tfrac{1}{2}m\beta d^8)^{1/m-2} - aT$. Thus the electric strength is

$$e\{(\tfrac{1}{2}m\beta d^8)^{1/m-2} - aT\}^{-2} - \beta e d^8\{(\tfrac{1}{2}m\beta d^8)^{1/m-2} - aT\}^{-m}. \quad (2.1)$$

Calculated values should be too high at low temperatures and are so dependent on m that little reliance can be placed on them.

† One presumes a is $0{\cdot}085d$ divided by the melting-temperature.

In any event these over-simplified pictures are of limited interest and, considered quantitatively, suffer from the objection that the thermal modes of oscillation are quite different from the field-induced distortion, which makes a negligible contribution to the polarization. The hypothesis would imply an impossibly high energy loss unless the elements suffering displacement fail, in some way, to make any contribution to the specific heat. Furthermore the field-induced displacements are ordered, whereas melting is a phenomenon of disorder. The introduction of a condition from melting is thus inconsistent.

Electric strength received some measure of attention from physical chemists at the time when interest lay in showing that various physical properties had a chemical significance when expressed per molecule of a substance: but dielectric breakdown was not susceptible to this treatment. Vorobiev[33] revived the method of qualitative correlation with physical constants and chemical composition without any very significant result. Böning[31] used concepts from colloid theory in what was essentially a space charge hypothesis, namely that there were bound and free ions. The free ions moving under the action of the electric field left a bound ion space charge which increased the field strength locally and so reduced the apparent breakdown below the true electric strength. The latter was considered to be a constant of the material and observed only with impulsive voltages. Similarly, the effect of temperature was supposed to come from the change of viscosity with temperature. Certain of these ideas are of interest, but the theory gives qualitatively incorrect results in many important instances and truly quantitative calculations cannot be made because the theory only applies to the variation and not to the magnitude of the electric strength.

This leads to the next group of theories which start to a greater or less degree from the analogy of ionization by collision in gases. Rogowski[3] first confined himself to a development of the Griffith principle in which an electron (or ion) might be accelerated through the depth of a surface crack to ionize a lattice element or to remove an ion from its equilibrium position. The energy required was the quantity which he denoted as

the lattice energy, of the order of several electron volts. The hypothesis could not be pursued quantitatively because there was no evidence of the cracks which were the seat of breakdown. Later Rogowski[30] considered field-induced ionization yielding a field-dependent conductivity which made the current unstable at a certain field strength: in this he was followed by other workers who postulated various expressions for the variation of conductivity with field strength. Since they did not elaborate the mechanism producing these variations, there is no need to deal further with them here.

The hypothesis of self-induced ionization analogous with ionization by collision in gases was naturally attractive and had been invoked by many writers to explain increases of conductivity with field strength as found, for example, by Poole,[35] Schiller,[36] Whitehead and Nethercot.[37] The latter only proposed this hypothesis to explain stable variations and considered the instability inherent in breakdown as outside its scope: furthermore the non-linear behaviour appeared to be a function of the moisture present as an impurity. It is well known that self-induced ionization by ions of one type cannot lead to instability without the interposition of a supplementary factor. Joffe[38] supposed that this factor was the ability of the dielectric to sustain a critical current density. The reasons for this criterion are not very clear, but it seems to be established that a rather similar phenomenon occurs in atmospheric lightning and something of the kind may well occur in shorter gaseous discharges. The conditions for the concentration of a discharge which are present with gases are absent in solids. On the other hand, the structure of a solid becomes unstable if any large number of ions are removed. Be this as it may, Joffe worked out a theory of breakdown on the basis of a critical current density in a manner analogous with Townsend's theory of ionization by collision for electrons. He simplified the calculation by taking the ionization coefficient as the quotient of the field strength by the potential drop necessary to produce a new ion. To determine this potential drop he supposed a characteristic ionic binding energy and a frictional dissipation of

energy by a fast moving ion along its path. The final result for the ionic density n was

$$n = n_0 e^{-\alpha/T} P(e^{V/P} - 1)/V, \tag{2.2}$$

and if V is the breakdown voltage

$$V \simeq (\alpha P/T) + \text{const.} \tag{2.3}$$

Here n_0 is the number of conduction ions at low temperature; $e^{-\alpha/T}$ is the variation of the number of these ions with temperature T based on thermal equilibrium between two energy levels. P is the potential drop required by a moving ion to gain an energy P_0 to free another ion and, accounting for the frictional dissipation, P is approximately $P_0 + (A/F^m)$ where A is a constant depending on the friction, m is a constant, and F is the mean field strength. Joffe's theories are of considerable interest, although there are too many disposable constants for theoretical completeness. If he postulates the acceleration of massive ions a substantial electronic conductivity would precede his mechanism while if the ionizing agents are electrons, his calculation of their motion is invalid, as will be seen later. Finally the variation of electric strength with temperature and thickness which he deduces is not found experimentally.

Electrons are prevented from escaping from a metal by the contact potential difference or thermionic work function which imposes a potential barrier. This means that the electrons inside the metal can move from one position to another, provided only small changes of energy are involved; but cannot cross the surface unless they obtain sufficient energy to scale the potential barrier, such as by absorption of radiation or at a high temperature. Electrons bound in dielectrics are similarly situated except that the retaining potential barriers are so close as to form a potential well, restricting the electron to a limited space, sometimes to the immediate neighbourhood of an atom. Now an electron can be regarded as a wave and, as such, may be reflected back and forth within the potential well to produce standing or stationary waves. These waves are analogous to those which appear when a section of transmission line, short-circuited at both ends, is excited by a high impedance (parallel)

current generator or a zero impedance (series) voltage generator, at a frequency for which both ends are natural modes. The short-circuit prevents the escape of energy from the section into the rest of the line, but only if of low enough resistance, i.e. capable of complete absorption or reflection. Just as a short-circuiting connexion is of finite resistivity so also a potential barrier is only capable of reflecting wave energy at a finite rate. Thus, unless the potential barrier is very thick, it does not reflect completely, and a travelling wave will escape just as standing waves on a short-circuited section of a line will give rise to travelling waves outside the section if the short-circuit is imperfect. A travelling wave corresponds to a moving stream of electrons. The calculation of the rate of escape through a thin barrier has been given by Fowler and Nordheim, and is analogous to the calculation for a transmission line and to that of the anomalous transmitted ray in metallic reflection. With a metal the surface barrier is so thick that normally very few electrons escape, and the same is true of the potential wells in dielectrics where the thickness is usually of the order of atomic spacings, say, 1 to 10×10^{-8} cm. If a strong external field F is applied, the original potential distribution is distorted in the first place by superimposing a decrease of potential, i.e. a potential gradient, in the direction of, and equal to, the field. Thus if the height of the potential barrier was V, this jump in potential would be annulled in at least a distance V/F, which would become the maximum base thickness of the distorted barrier. A strong field therefore facilitates electron escape which in metals is known as cold emission in an electric field. The analogous phenomenon in dielectrics will occur in fields of, say, 10^6–10^8 V/cm., seeing that the potential barriers are of the order of one or more eV, with the atomic spacings already noted. Zener[41] has calculated this for a regular one-dimensional lattice of constant a (in 10^{-8} cm.) and a potential barrier of energy W or V (in eV) in a field of F (V/cm.) and obtains

$$\begin{aligned} \text{rate of escape} &= (\mathbf{e}Fa/\mathbf{h})\exp\{-\pi^2\mathbf{m}aW^2/\mathbf{h}^2\mathbf{e}F\} \\ &= 0{\cdot}243 \times 10^7 Fae^{-3{\cdot}3\times 10^6 V^2 a/F}. \end{aligned} \tag{2.4}$$

Assuming a value for a of 3 ($\times 10^{-8}$ cm.) and 1·3 (eV) for V, values which might apply to a crystal like mica, the rate of escape would be

$$0{\cdot}73 \times 10^{7} F e^{-1{\cdot}7 \times 10^{7}/F}.$$

The corresponding current will be proportional to the rate of

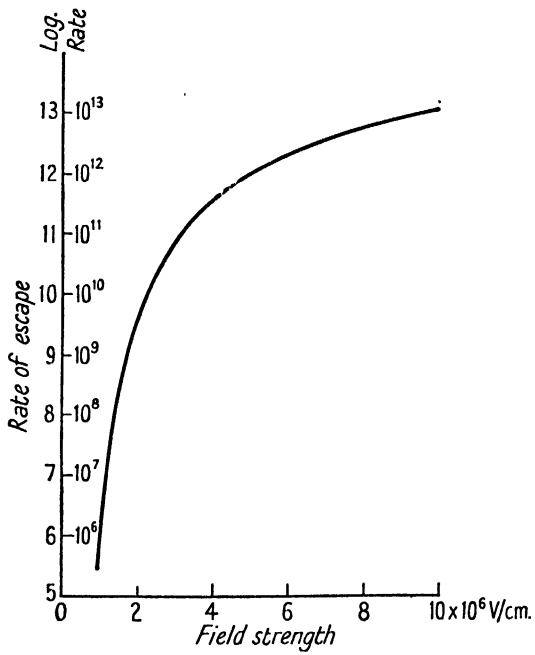


FIG. 3. Rate of escape of electrons from a potential barrier in a unidimensional lattice (Zener's equation).

escape and its increase with increasing field strength is certainly catastrophic as in Fig. 3. Nevertheless, there is something of a dilemma because the rate of increase of current with field strength begins to diminish in a region, say 10^{7} V/cm., which is in the neighbourhood of observed electric strengths. Thus if the multiplying constant gives high breakdown currents in the region of most rapid increase the computed electric strength will be rather low, while if the multiplying constant is less and excessive currents are transferred to a higher region of field strength, the rate of increase of current is diminishing.

Physically the theory accounts for the escape of electrons from a lower filled zone but does not explain what happens thereafter. The electrons may well be captured and fall again ultimately into the zone from which they escaped. This might provide an energy loss mechanism by the absorption of energy from the field for escape and the release of thermal energy by lattice oscillations on return, but would not be associated with breakdown. For these and other reasons the present view is that Zener's theory provides a reason for the existence of some free electrons previous to breakdown, and may possibly explain pre-breakdown currents, but does not explain the mechanism of breakdown. Nevertheless the possibility of local concentrations of electrons freed in this manner provides a basis for hypotheses as to the mechanical and indirectly electrical effects of such concentrations. These hypotheses have not so far been quantitatively pursued and seem related to defects and partial breakdown.

Ionic escape by an activation process seems quite possible. Austen[4] mentions this when considering possible mechanisms for an increase of conductivity at very high field strengths. He remarks that a high applied stress will reduce the activation energy in the direction of the field on account of the superimposed potential gradient, and gives a formula which may be regarded as an approximate upper limit on certain elementary hypotheses. Consider an excess ion or a defect ion, i.e. an ionic hole or vacant lattice-point, in an ordered structure. The singularity is kept in place by a potential barrier produced by the fields due to the neighbouring ions of the structure. In accordance with the theory of Pelzer and Wigner, it has a finite chance of escape within a certain time interval due to thermal oscillation but, if an ion, is likely ultimately to be trapped at a corresponding point in the ordered structure. If, owing to the electric field, the potential barrier is reduced in the direction of the field the ions and holes will tend to be attracted in that direction, and this drift will increase markedly as the reduction of the barrier by the field becomes an appreciable proportion of its normal height. One way of arriving at Austen's formula is

by considering two adjacent corresponding points in the structure separated by the unit difference a which is in the direction of the field. If U is the mean value of the potential barrier, the individual barriers are $U \pm \frac{1}{2}\mathbf{e}aF$, where F is the field and $\mathbf{e}$ the ionic charge. The thermal activation theory gives the rate of escape in opposite senses as $A \exp\{-(U \pm \frac{1}{2}\mathbf{e}aF)/\mathbf{k}T\}$, where A is a constant of the order of the frequency of the thermal oscillations at the absolute temperature T together with a factor taking into account the number of singularities (presumably little affected by the external field) and $\mathbf{k}$ is Boltzmann's constant. Thus the mean rate at which ions or holes move from one stationary or trapped position to the adjacent point in the direction of the field is the difference between the rates in opposite senses

$$A\{e^{-(U-\frac{1}{2}\mathbf{e}aF)/\mathbf{k}T} - e^{-(U+\frac{1}{2}\mathbf{e}aF/\mathbf{k}T)}\}.$$

This can be summed over all possible values of U, yielding

$$Ae^{-U_0/\mathbf{k}T}(e^{\frac{1}{2}\mathbf{e}aF/\mathbf{k}T} - e^{-\frac{1}{2}\mathbf{e}aF/\mathbf{k}T}) \sum_{n=0}^{n=\infty} e^{-n\mathbf{e}aF/\mathbf{k}T}$$
$$= 2Ae^{-U_0/\mathbf{k}T} \sinh(\tfrac{1}{2}\mathbf{e}aF/\mathbf{k}T)/(1-e^{-\mathbf{e}aF/\mathbf{k}T}),$$

presuming $U = U_0 + n\mathbf{e}aF$.

If $\mathbf{e}aF/\mathbf{k}T$ is fairly small compared with unity, this becomes

$$Ae^{-U_0/\mathbf{k}T} \sinh(\tfrac{1}{2}\mathbf{e}aF/\mathbf{k}T)/(\tfrac{1}{2}\mathbf{e}aF/\mathbf{k}T).$$

When $\mathbf{e}aF/\mathbf{k}T \ll 1$, that is for normal linear conduction, the expression reduces to $Ae^{-U_0/\mathbf{k}T}$. Accordingly, the variation of conductivity σ with field strength becomes

$$\sigma = \sigma_0 \sinh(\tfrac{1}{2}\mathbf{e}aF/\mathbf{k}T)/(\tfrac{1}{2}\mathbf{e}aF/\mathbf{k}T). \qquad (2.5)$$

This is the equation given by Austen, and is also found for somewhat analogous circumstances in the theory of semi-conductors. Applying this formula to mica and assuming that the a.c. conductivity at low fields was mainly due to this ionic activation, Austen found that a value for a of 7×10^{-8} cm. was needed to yield about 100 per cent. increase in σ at 10^6 V/cm. a.c. He further showed that any observed changes must increase much less above 10^6 V/cm. than according to the formula. It therefore seems unlikely that action of this kind applied to dielectric

crystals can account for other than currents immediately preceding breakdown, and there is no evidence that it occurs. The theory can be applied to impurities such as moisture, where recent evidence suggests the importance of activation phenomena, but the initial non-linear effects so far observed have usually involved a linear increase of conductivity with field strength, whereas the present hypothesis requires that the first non-linear term should contain the square of the field strength. Nevertheless, the hypothesis is important because the mechanism described is one expected to occur if ionic behaviour is important, not only in ionic crystals but in homopolar crystals, chain molecules, and even amorphous materials. The evidence of the analysis so far made would suggest that instability of a breakdown type could hardly arise except with excessive pre-breakdown currents.

The preceding remark might appear supererogatory because the activation theory has not actually been proposed as an explanation of breakdown but analogous, less authentic theories have been so proposed. As an example, a theory given by Eisler[39] is described since, although proposed for liquids, it could be applied to solids, and many of its concepts have been mentioned, though not evaluated, in this connexion by others. Eisler supposes a normal liquid electrolytic conductivity and a field-induced conductivity, which would contribute most of the conductivity in the region of breakdown. He considers the field-induced conductivity to resemble the escape of electrons through a potential barrier in a strong field as in Zener's theory already described. This yields a term $e^{-b/F}$ in the probability of escape where b is a constant and F is the field strength. It is not clear on what grounds the same result can be justifiably used if it concerns ions, since with heavy particles the mechanism is only associated with nuclear fields. Taking into account ionic mobility and other factors, Eisler arrives at an empirical multiplier $e^{c\theta}$, where θ is the temperature rise. Furthermore, he supposes that the temperature θ (at least presumably in the breakdown path where the ionization is most intense) is proportional to the electrical energy dissipated FId, where I is the

current and d the thickness. From this, omitting for simplicity a number of elaborations, the following expression is obtained, at any rate for a small thickness

$$I = AFe^{cFId-(b/F)}. \tag{2.6}$$

This expression leads to a turning-point on the current-voltage curve, that is a point of electrical instability in a quasi-gaseous discharge. The point of instability of equation (2.6) occurs when $cFId$ is unity, i.e. when

$$F = b/(1+\log AcF^2d). \tag{2.7}$$

Without going into the suggested evaluation of the electric strength F from this equation, which is said to give an order of 10^6–10^7 V/cm., there are difficulties apart from those already indicated in the basic conceptions. Observe that, at instability, the current is increasing rapidly with small changes in voltage, so that it is difficult to see how the initial currents due to this mechanism could be observable as suggested by Eisler. Furthermore, the variation of F with c is relatively slow, so the temperature of the breakdown path intervenes suddenly at breakdown, although the properties of the path do not materially affect the electric strength or the physical circumstances prior to breakdown. The constant A, referring to electrolytic conduction, should presumably be an exponential function of the ambient temperature, which would mean a decrease of electric strength with rise of ambient temperature, immediately limiting the scope of the theory in practice. As in the case of all the other theories so far treated, absolute calculations of electric strength have not been attempted. Ionic hypotheses of breakdown as at present elaborated may therefore be discounted as quantitative physical theories.

2.3. Electrons in dielectrics

The aim of the present section is to describe the elementary features of the solid state† which are relevant to dielectric

† Among other sources reference may be made to the following: Hume-Rothery, *Atomic Theory for Students of Metallurgy*, Institute of Metals, 1946; Seitz, *The Modern Theory of Solids*, McGraw-Hill Book Co., New York, 1940; Mott and Gurney, *Electronic Processes in Ionic Crystals*, Clarendon Press, 1940.

breakdown and to apply these notions to the emergence of free high-speed electrons which have the possibility of producing more free electrons or ions by their action on the solid.

The reader is presumed to be familiar with the structure of the atom comprising a charged nucleus and planetary electrons. Acquaintance is also presumed with the quantum theory in so far as it includes the theory of a quantized state of a system in terms of Planck's quantum of action $\mathbf{h}$; and the theory that a system or particle may only change from one such stationary state or level to another by the emission or absorption of a whole number of quanta $\mathbf{h}\nu$ where ν is the lowest frequency associated with the phenomenon. The elaboration of half-quantum numbers and of electron spin will be mentioned when necessary. It is also convenient to make use of the duality between particles and waves as occasion serves. A free particle of mass m and velocity v has a momentum mv and energy $\frac{1}{2}mv^2$. It may be represented as a wave whose wave-length λ is $\mathbf{h}/mv$. A particle regularly reflected back and forth between limits can be represented as a standing wave of defined frequency and a regular stream of particles can be represented as an ordinary travelling wave. Conversely a wave, as of light, of frequency ν, has an energy $\mathbf{h}\nu$ and momentum $\mathbf{h}\nu/c$ if c is the velocity. The wave number is frequently used instead of the momentum. A sinusoidal wave can be represented by the function $\sin 2\pi\nu t$ so that $2\pi\nu$ is an angular velocity. If the wave has a certain velocity then, if λ is the wave-length so determined, $2\pi/\lambda$ is the number of radians described per cm. by the angular velocity. This wave number for actual waves is often denoted by w. The wave number or momentum coordinate of the particle is also $2\pi/\lambda$, i.e. $2\pi mv/\mathbf{h}$, and denoted by K. The energy E can be expressed in terms of K for a particle since, from the foregoing, E is $\mathbf{h}^2K^2/8\pi^2m$. The particles here involved are usually electrons for which $\mathbf{m}$ is known, though sometimes the relativistic increase of mass with velocity must be taken into account. The laws of reflection of particles and waves are known to be similar. Two particles or a wave and a particle can be considered to collide in a similar manner as regards changes of energy and momentum.

Equally the particles as waves can be considered as being scattered by the collision or interaction with other waves. In solids the statistical averages for a large number of electrons are important and the concept of scattering can be readily appreciated by making the intensity of the scattered wave in a particular direction a measure of the probability of finding an electron deflected by collision in that direction. The term interaction can be applied to collisions between 'particles', interference between 'waves', or scattering of waves or particles; its more general connotation is that of any mutual influence by which changes of energy, momentum, frequency, or wave number can be induced.

One application of the wave concept has been given in section (2.2) in connexion with Zener's theory. In the same section the concepts of potential or energy barriers and wells were introduced. There seems no need to say more except that this concept can be applied to ranges of stationary or stable states of systems in general and is of particular and easy application to quantized states where energy barriers can obviously be interpreted in terms of the quantum conditions already mentioned. As an example, if an electron is 'bound' to or 'trapped' by an ion or structure, it is held in that state by an energy barrier and must normally acquire appropriate quanta of energy before it is released.

In a monovalent gaseous element the extra-nuclear electrons are arranged in a series of shells round their respective atoms according to the atomic number and the place of the element in the Periodic Table. The outer shells largely determine the spectroscopic and chemical properties of the element. The outermost shell may be wholly or partly complete. If there are only a few electrons in the shell they can be shared with an element with a nearly full shell but with a corresponding defect of electrons, yielding a diatomic compound. In a solid the molecules are brought very close together. In a crystal the molecular features may be lost and the atoms or atomic groups occupy the vertices of a regular lattice structure. The inner shells of electrons will remain associated with their individual

atoms. The outer electrons will come under the influence of the atoms at neighbouring lattice points so that their association with a given atom may be still less close than in a gaseous molecule. In a gas the un-ionized atom as a whole is neutral, but the electron shells cannot completely screen the nuclear field for close approaches. In solids this is more strongly in evidence, particularly since the atomic system of nucleus and electron shells may be distorted by the field of its neighbours so as to exhibit a polarization or separation of the mean centres of positive and negative charge. Accordingly, excess electrons can more readily become attached to an atom or atomic group at a lattice-point. At other points there will be a corresponding defect of electrons. Thus it may be conceived that there are a number of electron orbits which can, with a slight perturbation, embrace more than one atom, even though excess and defect of electrons may result. At the same time there are orbits which are associated with a single atom, although strongly influenced by neighbouring atoms. If these orbits are only partly occupied, and if the orbits of the two types interlace (having little difference of energies), then an electron may fairly readily pass from one to the other, from association with one atom to an embracing orbit, thence, or directly, to association with another atom, and so forth.

The foregoing is a crude mechanical picture to indicate that the treatment of solids cannot be based on isolated molecules as in gases. Thus a given energy level of an isolated molecule will split into two levels in the case of two interacting molecules. These two levels have nearly equal energy if the distance between the two molecules is large, but they separate as the distance decreases. Generalizing this picture, the periodic arrangement of n molecules will lead to n energy levels. Clearly, if n is a large number the total spread of these levels should be independent of the number of molecules. We are therefore led to the theory of Bloch that there exist zones (Brillouin Zones) of energy states in which the number of states is proportional to the number of unit cells. The Exclusion Principle is now introduced by which only one electron may occupy a given state or

level after allowance has been made for the fact that two electrons of opposite spin may occupy the same level. It follows that the discrete energy levels in isolated molecules are replaced by zones of energy levels in a solid. In a simple case each zone will contain two states per unit cell, remembering the duality introduced by electron spin.

If there is any choice the lower energy levels will be filled, the preference depending on the existence of a statistical thermal equilibrium. The latter may be postulated because interaction

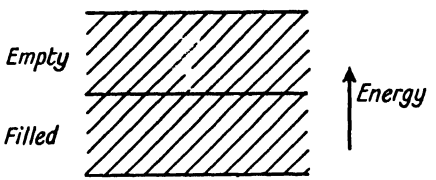


FIG. 4. Zones of alkali metal with odd number of electrons per unit cell.

between the electron and its neighbouring atoms yields means for exchanging energy. Interaction between electrons will not affect the equilibrium unless they are supplied with or deprived of energy by some external means. Consider now the lowest zone of an alkali metal. There is one atom and one electron, but two levels or states per unit cell. The situation is represented in Fig. 4, where the lower half of the zone is completely filled but the upper half is empty. This would be the probable condition at absolute zero. The electrons can readily accept small amounts of energy either thermal in origin or from an external electric field, since there is a half-zone of continuously distributed energy levels open to them. These are also distributed throughout the specimen. An applied field, as mentioned in the last section, necessarily gives a preferential energy exchange in its own direction so that the electrons may freely drift in the direction of the field and will only be impeded by irregularity of the lattice structure due to thermal motion, dissimilar atoms, or defects. Metallic conduction obtains. The electrical resistance in pure metals is low and, since mainly due to thermal motion, increases with temperature. Irregularities such as dissimilar

atoms in alloys increase the resistance in an overwhelming manner in the high resistance alloys, where the resistance may be negligibly affected by temperature.

If there is an odd number of electrons per unit cell there must be at least one zone where the same arguments apply as to the alkali metals; this zone should be the uppermost. If there is an even number of electrons per unit cell then it is possible for all the places in some zones to be filled. The significance of this depends on the disposition of the zones. The propinquity of the

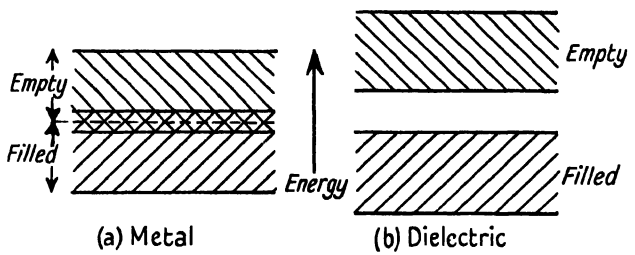


FIG. 5. Zones with even number of electrons per unit cell.

atoms in a solid makes it possible for the energy levels of different order, or the degrees of association in a unit cell, to be rather closely spaced compared with the variation of energy level of a given order from one unit cell to another throughout the specimen. That is, adjacent zones may overlap as in Fig. 5 (*a*). If this is the case the distinction between the two zones is immaterial; precisely the same arguments apply as have been used in connexion with unit cells having an odd number of electrons, and it may be concluded that metallic conduction will take place. On the other hand the adjacent zones may not overlap. If now the lower zone is completely filled as in Fig. 5 (*b*) by an even number of electrons per unit cell, metallic conduction cannot occur because the electrons are bound to their configuration. Motion is only possible if electrons can be expelled from their zone. If the gap between the zones is of the order of 1–5 eV transitions can only become probable at an impossibly high temperature or by some powerful external agency. The material will then be a dielectric. A monovalent element, such

as solid hydrogen, can be a dielectric if the conditions indicated are fulfilled.

Should the gap in Fig. 5 (*b*) be only of the order of millivolts then electrons could escape from the filled to the empty band, i.e. to the conduction level, under the influence of thermal interchanges of energy at ordinary temperatures. A substance of this kind would be a dielectric at low temperatures, when the electrons were unable to escape in any appreciable numbers, but its conductivity would rise with temperature until it became a conductor at high temperatures. It would be called an electronic semi-conductor, but interest must be directed in the present work to conduction in imperfect dielectrics. When a lattice-point is occupied by a 'foreign' or dissimilar atom as in solid solution, the field near this atom will differ from that at other similar lattice-points and the electron energy levels associated with the atom will also differ. One or more intermediate levels will probably be introduced into the gap in Fig. 5 (*b*). It is likely, in this instance, that any 'imperfection' level will lie near one or the other zone limits. There is less interest in occupied levels which lie near the lower zone than those just below the upper zone because levels near the lower zone represent as effective a binding as in the lower zone itself. Accordingly, attention will be confined to levels just below the upper zone. A foreign atom which differs mainly in valency from the atom it replaces provides an excess or defect of loosely bound electrons. That is, there are filled or empty levels just below the upper zone. An excess atom will also have its peculiar energy levels which can be estimated by superposing the field due to the excess atom upon the field of the normal lattice. The absence of an atom has a rather similar effect. In these cases also the isolated 'imperfection' levels so formed will frequently be near the upper zone if due to single atoms or small groups of atoms. Dislocations and other defects can be considered in an analogous manner and are subject to the general conclusion that the 'imperfection' levels will probably be near the upper zone unless the defects are so serious that an appreciable number of them would greatly modify the properties of the structure.

The term 'imperfection' will now be used to include foreign or 'impurity' atoms, excess or absent atoms, dislocations, mosaic structure, defects and all departures from perfect crystallinity, since the term 'defect' has sometimes a more specific meaning.

It is not uncommon to find that the imperfection levels arise from similar causes in a given specimen or substance and tend therefore to be rather concentrated round a level displaced from the upper zone by a certain difference ΔV electron volts. Even should this be only roughly the case the idea of a mean energy difference ΔV is useful in a first approximation to the effect of

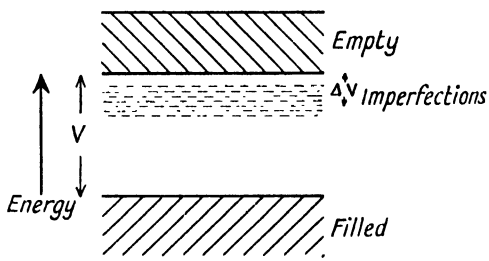


FIG. 6. Imperfect crystal.

these isolated levels. They are indicated diagrammatically in Fig. 6, where V is of the order of a few volts and ΔV may be of the order of millivolts, or of a fraction of a volt, or an appreciable fraction of V. At normal temperatures electrons can escape from the imperfection levels into the conduction zone, so that although the conductivity may tend to disappear at absolute zero it will rise with temperature, attaining a magnitude depending on the number of imperfection levels, i.e. upon the imperfection of the structure. An elaboration of this picture will later be needed, because electrons can be excited by rise of temperature to levels above those characteristic of the imperfections at low temperature.

In the crude mechanical picture of a solid initially drawn the electrons in loose association with atoms could not, in a sense, always be sure to which atoms they belonged. In the conduction zone, which is, say, more than half-empty, the electrons can acquire small amounts of energy from the field and pass from

atom to atom since there are adjacent levels unoccupied. This is the easier if all the atoms are similar and similarly situated. In other words, if the lattice gives rise to an ideally periodic field conduction electrons can pass through this field without loss of energy. On a simplified theory this would be true of a pure metal at absolute zero when all the atoms are the same and disposed in a perfect stationary regularity. At higher temperatures the lattice oscillates and the instantaneous field or potential distribution is no longer regular. This leads to the result that the transition from one level to an adjacent level or from one elementary cell to another has a finite probability in time. This implies energy dissipation. An alternative view is that an electron of given energy state can gain and lose multiples of a quantum of oscillation energy. If an electron gains energy from a superimposed electric field it will lose some of this in oscillation quanta, and in equilibrium the energy derived from the field would nearly all appear as oscillation energy, that is the electrical energy dissipated in resistance heats the metal. It is impossible to identify an electron in this connexion, nor is there any need since only the energy and momentum distributions and their variations are required. Thus we can speak of the probability of finding an electron with a given energy and momentum. The problem then becomes identical with that of the scattering of the electrons, viewed in terms of their undulatory properties, by the waves which correspond to the lattice oscillations.

In dielectrics the same considerations apply to the electrons which find their way into the conduction levels. Since the atoms in a dielectric are usually dissimilar, the variation of potential is greater than in a metal and more complicated. Even in a simple polar crystal, rock salt, as instanced by von Hippel,[10] simple periodicity can only be approached along the principal axes. If the lattice constant (distance between consecutive similar lattice-points) is a then the chlorine (−) and sodium (+) atoms are arranged in a unit cell as in Fig. 7. In a direction other than a cube edge (100), face diagonal (110), or body diagonal (111), the potential, although periodic, will be

complicated. Along the axes it is simply periodic, but the steepness varies as in Fig. 8. Thus there will be a tendency for an electron to follow certain directions in a crystal. Furthermore, in the direction of the greater potential hills and valleys the oscillations of the structure can impede the progress of the electron,

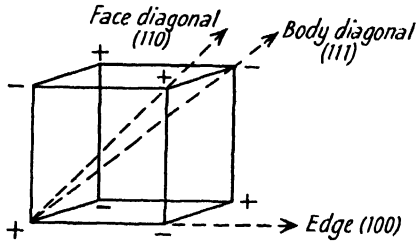


FIG. 7. Principal directions of polar cubic lattice.

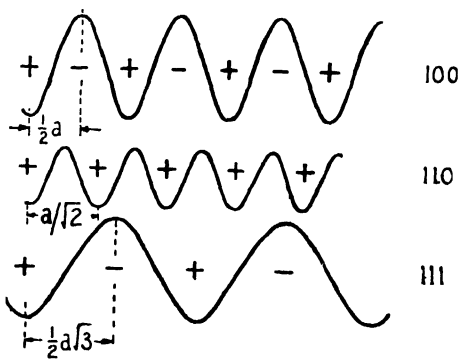


FIG. 8. Potential contours in principal directions (von Hippel).

even if it has the requisite energy to traverse the hills, and make the loss of energy to the lattice much more important than in a metal. An electron in a dielectric does not find it difficult to lose all its free energy and become effectively trapped.

The motion of an electron is much more complicated than in a metal, apart from the fact that it usually requires to have a fair amount of free energy if it is to have an independent life for any length of time. For example, the electron polarizes the dielectric in its neighbourhood so that it produces a kind of bow-wave and wake of polarization as it moves. If the electron

passes rapidly past a polarizable element in a time short compared with the natural period of the medium, then this effect is greatly diminished. (See Fröhlich and Pelzer.[40]) In a somewhat similar fashion most other features peculiar to dielectrics are diminished if the electron is fast, e.g. with an energy of the order of 1 eV or more. Accordingly, while there is as yet no exact theory of slow electrons in solids,† for fast electrons in dielectric crystals the hypothesis of scattering by the thermal oscillations of the structure and by lattice imperfections may approximately be applied. It will be appreciated that the escape of electrons from the lower zone will leave that zone incomplete so that conduction becomes possible and will take the form of the motion of empty levels or the migration of electron 'holes'. These have the properties of positive electrons in their motion in an electric field. This type of conduction has been neglected in breakdown mechanisms mainly, it is thought, because very high fields would be required. Reference has already been made to imperfections where there is a defect of electrons. These can give rise to 'hole' conduction. Its influence on dielectric properties has only recently been considered. There is at present no evidence that it would seriously modify the treatment of breakdown.

It is now to be inquired how electrons can escape from the lower zone to the 'conduction' zone. If the gap is small, thermally excited transitions can occur, but since 1 eV divided by Boltzmann's constant is over 10,000° C., thermal electrons will be rare with dielectrics, although they are a characteristic feature of semi-conductors which become conductors at elevated temperatures. The difficulty of thermal transitions is reduced by the existence of the intermediate or imperfection levels of Fig. 6 so that a series of stepping stones are provided between the two zones. These intermediate levels are rarer the more perfect the crystals, and even with amorphous substances they will be ineffective below a certain temperature in facilitating escape from the filled zone, depending on the distance of the imperfection levels from the ground state zone in Fig. 6.

† A recent paper by Fröhlich, Pelzer, and Zienau[40] gives a theory for slow electrons in polar solids.

Electrons may be excited from lower levels by the absorption of radiation or when electrons or other particles with sufficient energy approach so closely as to make an inelastic collision. The lower limit of the necessary energy is given by the limit of the X-ray absorption band and, therefore, can be found with some accuracy. In section (2.2) Zener's theory for the escape of bound electrons in a strong field was described. Though unsatisfactory as a theory of breakdown it provides an additional mechanism for the appearance of free electrons.

The foregoing suggests that in a dielectric there will always be some free electrons either by the effect of the distribution of thermal energy and intermediate 'imperfection' energy levels at temperatures above absolute zero, or by the effect of a strong electric field, or by absorption of radiation as in photo-conductivity. There may also be a number of loosely bound electrons which require only fairly small amounts of energy to make transitions. Photoelectric or 'cold' emission from the cathode can also supply some electrons. These electrons are dispersed in the solid structure, the oscillations of which interact with them. If numerous they may interact with one another. The net result has some features in common with electrons in a gas. These may collide with neutral molecules or with one another. If the field strength is low, the electrons and neutral molecules will be more or less in thermal equilibrium and the electron cloud will drift or diffuse in the direction of the field with a constant velocity without gaining other than a negligible amount of energy from the field, which appears as a very slight general increase of temperature. When the field becomes stronger, the electrons are able to abstract a more important amount of energy from the field. If the analogous process continues far enough in solids, there will be a considerable number of electrons, with energies greater than the X-ray absorption limit. These electrons, for reasons already given, are likely to expel electrons from the lower zone of energy levels, i.e. the ground state electrons. The electrons so freed have a similar chance of acquiring sufficient energy from the field for further ionization.

This takes the theory as far as an analogy with the Townsend theory of electrons in gases and ionization by collision, in which non-exciting collisions between electrons and neutral molecules are replaced by interaction between electrons and lattice oscillations, which is analogous to scattering or collision; collisions between electrons are replaced by an analogous interaction between electrons; 'inelastic' collisions between electrons and molecules are replaced by the excitation of a ground state electron to a conduction or intermediate level by the close approach of a fast electron. The conception of a mean free path and a mean time between collisions or a relaxation time can also be applied to solids. On the other hand, in gases, ionization by collision of quite appreciable magnitude can precede at a much lower field strength the ultimate instability and electron avalanche associated with a gaseous discharge at normal pressures. Solids behave rather differently. As has been shown, the electron moves as a wave through the strong and complicated fields of the massive ions of the structure so that it tends to move along certain preferred directions. Referring to Fig. 8 for an ideal polar crystal like rock salt, von Hippel notes that the potential variations should be steepest along an edge (100) where alternate positive and negative ions are most closely spaced. The variation is less along the body diagonal (111) where the ions, still of alternate sign, are spaced $\sqrt{3}$ times the edge spacing, and the variations are much less along a face diagonal (110) where the ions are of the same sign and spaced at $\sqrt{2}$ times the (100) spacing. Hence von Hippel concluded the face diagonal to be a preferred direction and the body diagonal a secondary direction. A better approach is by remembering that the longer the free path the more energy can be acquired from the field and, without going into details, the free path is likely to be longer in directions of low ionic density. Furthermore, in directions which pass near similar lattice-points, which are occupied by similar ions, transitions will take place more easily between these like energy states and either the free path can be considered as longer or the interaction between the electron and the lattice as less; both of these hypotheses lead

to the acquisition of a higher energy from the field by the electron.

von Hippel[42] and his co-workers have examined discharge paths with care in a number of elegant experiments. The method of divergent fields (pointed electrodes) and limitation of current in magnitude and duration (i.e. a technique of partial breakdown) were used which render difficult the interpretation of results on theories which relate to uniform fields. In addition, the degree of correlation of direction of discharge path with crystal orientation leaves something to be desired. Nevertheless, the existence of preferred directions has been definitely established. The action of the field in accelerating electrons and the greater transparency of the solid to electrons with a higher energy, within a certain range, tend to segregate the fast electrons into groups. In addition, if ionization of the structure is at all concentrated in a locality, the structure is thereby weakened and further ionization or excitation of electrons is facilitated. All these features combine to produce an electron avalanche if any considerable number of electrons at any point ionize the structure, so that there is no need to invoke some distinct mechanism in order to render unstable the ionization process in solids. In gases the need for such an additional agency together with the time factor led to the transformation of the Townsend theory of ionization by collision into the modern concepts of gaseous discharge processes.

The discussion of the present section has been necessarily qualitative because some of the quantitative studies of the possibilities of stable energy distributions among electrons have not yet been made. However, it is hoped that a demonstration has been made of the generally agreed features of electrons in solid dielectrics leading to the hypothesis of the presence of initial electrons in a dielectric exposed to a strong field and of the acceleration of these electrons by the field until ionization of the structure occurs. If this ionization is of adequate magnitude an electron avalanche will form, producing a discharge path in the dielectric. This discharge path constitutes breakdown for present purposes. The question as to whether it is a

high or low resistance breakdown will be considered elsewhere in so far as it is of importance. In the next section the conditions in which large numbers of fast electrons can be produced by field-acceleration will be studied in a more quantitative manner.

2.4. Electronic breakdown: Fröhlich's theories of amorphous and crystalline breakdown

It is now possible to give a more detailed exposition of the theory of electronic breakdown. First will be described the basic concepts and the reason for preferring the developments due to Fröhlich. The details of the latter's main theories of amorphous and crystalline breakdown will then be given in two sections each prefaced by a qualitative account followed by the mathematical derivation of the quantitative formulae.

As shown in section (2.3) von Hippel[10] was the first to consider the behaviour of electrons in the actual fields in crystals. He assembled the evidence that breakdown was electronic in character and took the form of an avalanche arising from a self-increasing stream of electrons. His criterion that free electrons should multiply was that they should gain more energy from the field than they lost in stimulating oscillations of the solid structure. They would therefore continue to be accelerated until they freed more electrons by ionization.

The further development of this theory by von Hippel and his co-workers, particularly Seeger and Teller,[43] differs from the development due to Fröhlich.[44] von Hippel considered in what circumstances the energy gained from the field by *any* electron in passing from one lattice point to the next in the direction of the field could be so great that this energy could not all be lost in stimulating oscillations of the structure, remembering that the energy must be lost in multiples of a quantum of the residual ray frequency. The residual ray frequency corresponds to that of the oscillations in question.

Fröhlich, whose theories we shall here adopt as bases, does not, however, assume that electrons always move in the field direction and are gradually accelerated. He takes account of the scattering, rather as in ordinary conductivity, and considers the

rate of change of electronic energy on the average over many collisions. As a necessary condition for breakdown this rate must be positive for at least some of the electrons. He was thus led to consider the circumstances in which a group of fast electrons having a given mean energy would tend to have a net gain or loss of energy in their average motion. If they gained energy, breakdown would ensue provided an adequate supply of electrons with the appropriate energy could be postulated. In amorphous materials the concept of electron temperature can be taken as replacing that of energy. The minimum field in which the electron energy or temperature increases more and more rapidly is the electric strength, according to Fröhlich. His values will therefore be lower, in general, than those calculated by von Hippel and his co-workers. Fröhlich, in fact, selects the group of electrons which, in his view, mainly determines the phenomena and finds that for this group the dissipation of energy to the solid structure at breakdown is not necessarily the maximum value. von Hippel and his co-workers adopt a hypothesis in which the energy dissipation has the most unfavourable value. The breakdown conditions can be summarized as follows.

von Hippel: (i) Electrons move in the field direction and gain energy proportional to the field strength by direct acceleration. (ii) Energy gain must exceed loss for electrons of all energies.

Fröhlich: (i) Electrons frequently change direction; the net energy gain is proportional to the square of the field strength as in conductivity. (ii) Net energy gain must exceed loss for some energies, including some smaller than a certain value.

It follows from the previous discussion of electron energy states that in a perfect crystal at all temperatures and in imperfect crystals or amorphous substances at low temperatures, there will be very few electrons in the imperfection levels, which are isolated levels intermediate between the zone of bound electrons and the free zone. They will be too few to interact with one another and will only interact with the solid structure. As the temperature increases, the number of electrons trapped

at imperfections in intermediate levels in imperfect crystals and amorphous substances also increases. At a high enough temperature these electrons may be sufficiently numerous to interact with one another, forming a species of electron gas phase in the solid, though the electrons are still trapped at the imperfections, since these form isolated levels. The situation changes continuously, but if the major portion of the change is confined to not too wide a range of temperature, it is possible, rather arbitrarily, to define a critical temperature T_c below which the 'crystalline' hypothesis may be adopted as a first approximation and above which the 'amorphous' hypothesis applies. This temperature T_c cannot be determined very exactly. If the imperfection energy levels were grouped around a level V electron volts above the 'bound' zone or the ground level of 'trapped' electrons, from either of which electrons could be excited, then it might qualitatively be said that T_c would be associated with the condition that ne^{-V/kT_c} should give a high enough density of excited electrons, where n is the number in the ground state. It is legitimate to infer that the more imperfect the structure the lower will be T_c: it is analogous to the temperature in electronic semi-conductors at which conduction takes place from the motion of excess or defect electrons. This type of conduction should occur with dielectrics above T_c but with certain special features which will later be described. Apart from these considerations, the existence of a critical temperature will be assumed and the calculations for low and high temperatures will be given separately.

2.4.1. Imperfect crystals or amorphous materials at higher temperatures

The concept of relaxation time must first be explored. Consider a fast electron moving through the solid very much as if through a gas. When it passes near enough to a massive ion, the electron will lose some of its energy to the ion and excite Debye oscillations or may even set free an electron by yielding enough energy to bring the excited electron into the conduction band. The average time between two collisions may be called a

relaxation time analogously with the average time to describe a free path in gases. Consider next an electron which makes a transition from an energy level associated with one ion to a similar level in a neighbouring ionic system under the influence of the external field. The probability that such a transition should occur in a second defines the inverse of a relaxation time. This conception can apply to electrons in isolated intermediate levels as regards thermal transitions and (with some artificiality) to conduction electrons imagined to linger near an ion. Consider next the electron as a wave which is scattered by the oscillations of a lattice structure. The scattering corresponds to an attenuation of momentum and energy, the rate of which defines a relaxation time. Lastly consider the dielectric as a macroscopic structure. It reacts with a measurable slowness to the change of electrical conditions depending on its natural frequencies. This again defines a quite different relaxation time which is short if the natural frequencies involved are in the infra red, i.e. the oscillation or rotation spectrum, but may be quite long if dipoles are concerned, i.e. the 'relaxation' spectrum.†

If the electrons in the conduction and in the imperfection levels interact so that they exchange energy with one another more rapidly than with the solid, they will acquire a certain temperature T different from the temperature of the solid T_0. This arises because the field transfers energy to the free electrons which they share with the other electrons. This energy both kinds of electrons must, in equilibrium, transfer to the lattice. Such an energy transfer can only occur if $T > T_0$.

For convenience of calculation and for rather more exactitude a slightly modified representation of intermediate levels must now be introduced as compared with Fig. 6. In ionic crystals lattice imperfections may come from excess or defect of ions or electrons, e.g. interstitial atoms, F centres, and dissimilar atoms with valencies greater or less than normal, but amorphous substances may comprise a large variety of defects some of which have been previously enumerated. If the imperfections are numerous we must consider electrons trapped in their neighbour-

† See Fröhlich, *Theory of Dielectrics*, Clarendon Press, Oxford, 1949.

hood which may be excited by the acquisition of thermal energy or otherwise to more loosely trapped positions. That is, there is a ground level for these isolated or imperfection electrons, which will be above the ground level corresponding to the filled zone of the normal structure. Just below the conduction level will lie the levels of the loosely trapped electrons which have been excited from the 'trapped' or 'isolated' ground level, i.e. the normal imperfection level. For convenience the spread of

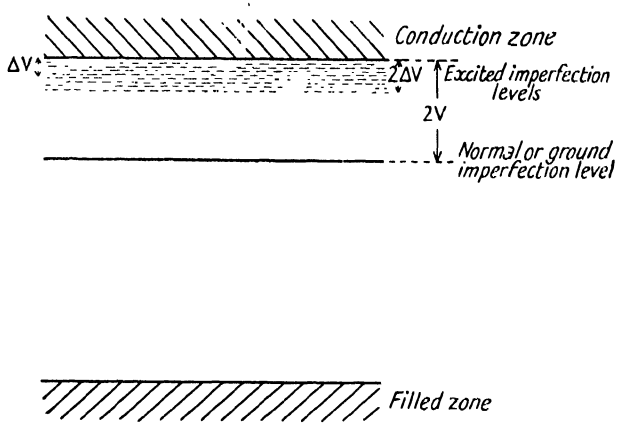


FIG. 9. Electron levels in imperfect crystal or amorphous solid.

the excited or higher imperfection levels is taken as from the limit of the conduction level to $2\Delta V$ below it, leaving the mid-point of the band of levels at ΔV below the conduction zone as before. Similarly for convenience the ground level of the imperfection electrons is supposed to be $2V$ below the conduction level and is of the order of 1 eV. This is illustrated in Fig. 9 and the theory of semi-conductors may be applied to find the distribution of electrons among the levels.

The number N_1 in the conduction band will be much less than the number N_2 in the imperfection levels. If there were no free electrons at absolute zero then it might be supposed that there were N electron levels all filled. At a temperature T there are N_1 free electrons and N_1 vacant places. The chance that an electron is among N_1 is proportional to (N_1/N) times the chance of a vacant place, i.e. N_1^2/N^2 if N is large. Similarly the chance

of occupying a level is $(N-N_1)/N$, or unity if N is large. The probability of N_1 free electrons is thus proportional to N_1^2/N^2 and must also be proportional to the probability derived from entropy, namely const. $\times e^{-2V/\mathbf{k}T}$. Thus N_1 is $C_1 e^{-V/\mathbf{k}T}$, where C_1 does not change greatly with the temperature. The number N_2 in the excited imperfection levels will be those in energy levels from $2V$ to $2(V-\Delta V)$ above ground level, and N_2 may be similarly shown to be approximately $C_2 e^{-(V-\Delta V)/\mathbf{k}T}$ where C_2 changes only slowly with temperature. Accordingly

$$N_1/N_2 = (C_1/C_2)e^{-\Delta V/\mathbf{k}T}. \tag{2.8}$$

C_1/C_2 is now roughly the ratio of the number of energy levels in the interval $\mathbf{k}T$ in the conduction zone to the number of imperfection levels in a similar interval. If the former number obeys the laws of free electrons then the number per unit volume is $(4\pi/3)(2\mathbf{m}\mathbf{k}T/\mathbf{h}^2)^{\frac{3}{2}}$, so that if n_2 is the number of imperfections per unit volume per interval of $\mathbf{k}T$,

$$C_1/C_2 \simeq (1/n_2)(\mathbf{m}\mathbf{k}T/\mathbf{h}^2)^{\frac{3}{2}} \simeq 10^{19}/n_2. \tag{2.9}$$

If there are only about $\mathbf{k}T/\Delta V$ imperfections per 1,000 atoms or ions in an interval $\mathbf{k}T$ then $n_2 \simeq 10^{19}\mathbf{k}T/\Delta V$. If T is of the order of 500° K. this would give

$$C_1/C_2 \simeq \Delta V/\mathbf{k}T.$$

This result means that N_1/N_2 would mainly be determined by $e^{-\Delta V/\mathbf{k}T}$, and would be small since ΔV is usually much greater than $\mathbf{k}T$ if the imperfections are permanent and important features of the structure. The much more numerous trapped electrons will be mainly responsible for the loss of energy to the solid structure, in contradistinction to the gain of energy from the field which is by way only of the conduction electrons.

The conduction electrons acquire an energy denoted by A from the field. They share this energy with the trapped electrons in imperfection levels, if these are numerous enough to interact strongly. There is thus a definable electron temperature T which will exceed that of the dielectric T_0. In consequence of this an amount of energy denoted by B will flow from the electrons, mainly from the trapped electrons, to the solid

structure. Now the energy gained will be proportional to the square of the field strength F and to the number of conduction electrons N_1. The energy lost will be proportional to the number of trapped electrons N_2. Thus A/N_2 will be proportional to F^2N_1/N_2, i.e. to $F^2e^{-\Delta V/kT}$, while B/N_2 will be shown later to have a finite upper limit as the electron temperature T increases. The energy balance can therefore be represented as in Fig. 10. When $F = F'''$ equilibrium will be reached at T_m''; at a higher

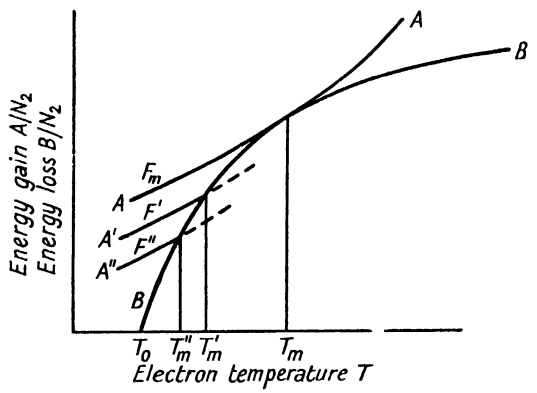


FIG. 10. Thermal balance and instability of electrons in amorphous solids (or imperfect crystals).

field strength F' equilibrium is still possible at T_m'; but with the field strength F_m the equilibrium at T_m is unstable, and in any higher field equilibrium will no longer be possible and the electrons will gain at an increasing rate an excess of energy over that lost. Thus F_m so defined will be the limit at which stability is possible and defines the minimum breakdown field. It will clearly decrease as the temperature T_0 of the solid is increased. This qualitative analysis will now be elaborated into a quantitative treatment.

If the relaxation time of the conduction electrons is τ_1, and if $\mathbf{e}$ is the electronic charge, $\mathbf{m}$ is the electronic mass, and F is the field strength, the rate of gain of energy A is

$$A = \mathbf{e}^2\tau_1 N_1 F^2/\mathbf{m}. \tag{2.10}$$

The conduction electrons are derived from the ground state by

one or other of the mechanisms for exciting electrons already mentioned. Therefore, as previously shown, $N_1 = C_1 e^{-V/\mathbf{k}T}$, and so

$$A = (\mathbf{e}^2 \tau_1 C_1 F^2/\mathbf{m}) e^{-V/\mathbf{k}T}. \tag{2.11}$$

In considering the loss of heat to the structure by the N_2 trapped electrons it is sufficient to consider transitions involving only a single quantum $\mathbf{h}\nu$ of a natural frequency ν. Goodman, Lawson, and Schiff[47] have worked out the probable rate of transitions from imperfection levels in semi-conductors to the conduction band in the particular case of a level associated with an isolated 'donator' atom, that is an imperfection due to an excess, defect, or displaced atom which can be represented by the superposition of the field due to a single atom on the field of the regular structure. They conclude that a transition associated with the absorption of two quanta has a probability reduced by a factor of the order of 10^6. The number of transitions per second of an electron from an energy level E to one of $E+\mathbf{h}\nu$ is the product of the transition probability $P(E)$, the average number of quanta of oscillation energy at T_0 given by the Planck function $(e^{\mathbf{h}\nu/\mathbf{k}T_0}-1)^{-1}$, and the probability $f(E)$ that the level E is occupied. Similarly, the number of reverse transitions per second is $P(E)(1-e^{-\mathbf{h}\nu/\mathbf{k}T_0})^{-1} f(E+\mathbf{h}\nu)$. Thus the net transfer of energy from the N_2 trapped electrons to the lattice by this mechanism is

$$B = \mathbf{h}\nu \sum P(E)\{(1-e^{-\mathbf{h}\nu/\mathbf{k}T_0})^{-1} f(E+\mathbf{h}\nu) - (e^{\mathbf{h}\nu/\mathbf{k}T_0}-1)^{-1} f(E)\}. \tag{2.12}$$

If we assume an elementary statistical equilibrium among the isolated levels $f(E+\mathbf{h}\nu)/f(E)$ will be of the order of $e^{-\mathbf{h}\nu/\mathbf{k}T}$, whence

$$\begin{aligned} B &= \mathbf{h}\nu \sum P(E) f(E) (e^{\mathbf{h}\nu/\mathbf{k}T_0}-1)^{-1} (e^{\mathbf{h}\nu/\mathbf{k}T_0} e^{-\mathbf{h}\nu/\mathbf{k}T}-1) \\ &= (\mathbf{h}\nu/\tau_2) N_2 (e^{\mathbf{h}\nu/\mathbf{k}T_0}-1)^{-1} (e^{\mathbf{h}\nu/\mathbf{k}T_0} e^{-\mathbf{h}\nu/\mathbf{k}T}-1). \end{aligned} \tag{2.13}$$

This is deduced by replacing $P(E)$ by an average value $1/\tau_2$, where τ_2 is the relaxation time of the system formed by the trapped electrons, and by remembering that $\sum f(E)$ is the number of isolated levels N_2. But

$$N_2 = C_2 e^{-(V-\Delta V)/\mathbf{k}T}. \tag{2.14}$$

Thus

$$B = (\mathbf{h}\nu/\tau_2)C_2(e^{\mathbf{h}\nu/\mathbf{k}T_0}-1)^{-1}e^{-(V-\Delta V)/\mathbf{k}T}(e^{\mathbf{h}\nu/\mathbf{k}T_0}e^{-\mathbf{h}\nu/\mathbf{k}T}-1). \quad (2.15)$$

From (2.11) and (2.15) the equation for energy balance

$$A = B$$

can be written

$$(\mathbf{e}^2\tau_1 C_1 F^2/\mathbf{m})e^{-\Delta V/\mathbf{k}T} = (\mathbf{h}\nu/\tau_2)(e^{\mathbf{h}\nu/\mathbf{k}T_0}-1)^{-1}C_2(e^{\mathbf{h}\nu/\mathbf{k}T_0}e^{-\mathbf{h}\nu/\mathbf{k}T}-1). \quad (2.16)$$

If C_1, C_2, τ_1, and τ_2 vary only slowly with temperature, the left-hand side, which corresponds to the energy gained from the field, rises more and more rapidly with the electron temperature T as T increases, while the right-hand side, corresponding to the energy lost to the lattice, rises less and less rapidly with T as T increases. Thus for lower values of the applied field F equilibrium is possible as in Fig. 10 at a value of T slightly in excess of T_0. When F is greater, no equilibrium is possible because the thermal dissipation cannot overtake the acquisition of energy from the field. At a certain value F_m there will be a limiting unstable equilibrium at T_m. This defines an electric strength analogous with the theory of thermal instability studied in Chapter III and obtained mathematically by applying the Jacobian condition for the limit of real solutions of equation (2.16). This, in the present instance, is the condition of tangency of the two sides of the equation, i.e. of curves A and B in Fig. 10, namely

$$(\mathbf{e}^2\tau_1 C_1 F^2/\mathbf{m})(\Delta V/\mathbf{k}T_m^2)e^{-\Delta V/\mathbf{k}T_m} = (\mathbf{h}\nu/\tau_2)(e^{\mathbf{h}\nu/\mathbf{k}T_0}-1)^{-1}C_2\, e^{\mathbf{h}\nu/\mathbf{k}T_0}e^{-\mathbf{h}\nu/\mathbf{k}T_m}(\mathbf{h}\nu/\mathbf{k}T_m^2). \quad (2.17)$$

Dividing equation (2.16) by equation (2.17) and taking the first approximation in $\mathbf{h}\nu/\Delta V$, which has already been supposed to be small, gives

$$\frac{\mathbf{1}}{T_0}-\frac{\mathbf{1}}{T_m} = \frac{\mathbf{k}}{\Delta V}. \quad (2.18)$$

Thus at breakdown the inverse of the electron temperature differs from the inverse of the temperature of the solid by an amount depending only on the imperfect structure, namely the average difference of energy between the imperfection levels and

the conduction band. This is analogous to the result in the theory of thermal instability (Chap. III) that the temperature rise of the solid at breakdown is often a constant of the material.

Substituting T_m for T in equation (2.16) and making the same approximation

$$F_m = \left\{\frac{\mathbf{m}h\nu}{\mathbf{e}^2\tau_1\tau_2}\frac{C_2}{C_1}\frac{1}{(e^{h\nu/kT_0}-1)}\right\}^{\frac{1}{2}}\left(\frac{e^{h\nu/\Delta V}-1}{e}\right)^{\frac{1}{2}}e^{\Delta V/2kT_0}. \qquad (2.19)$$

Fröhlich[45] uses the expression $(h\nu/\Delta V)\{1-(h\nu/\Delta V)\}^{(\Delta V/h\nu)-1}$ in place of $(e^{h\nu/\Delta V}-1)/e$, but both are near enough equivalent to $h\nu/e\Delta V$. Owing to the relatively slow variation of the other factors, roughly speaking,

$$\log F_m = \text{const.} + (\Delta V/2kT_0). \qquad (2.20)$$

This, incidentally, gives the same decrease with increase of temperature as occurs in thermal instability if the conductivity varies with temperature in the manner encountered in semiconductors and electrolytes, as indeed is to be expected since the same formulae are involved but with different connotations. The greater the energy interval between the imperfections and the conduction band, the greater is F_m and the higher the temperature at which this type of breakdown emerges. The greater the number of imperfections the greater is C_2/C_1, but as F only varies with $\sqrt{(C_2/C_1)}$ this effect is unimportant compared with the condition that there should be enough trapped electrons to interact with one another. An artificial excess of free electrons should presumably reduce the electric strength, but it is not clear how this excess could be maintained or how it would affect the interaction assumed. The shorter the relaxation times the greater is the electric strength. This can be deduced *a priori* because longer relaxation times in a system lead to a greater absorption of energy.

If the relaxation time τ_1 and C_1/C_2 vary only slowly with temperature, the conductivity σ may be deduced as

$$\begin{aligned}\sigma &= \mathbf{e}^2\tau_1 N_1/\mathbf{m} = (\mathbf{e}^2\tau_1/\mathbf{m})C_1 e^{-V/kT} \\ &= \sigma_0 e^{-V/kT}e^{V/kT_0}, \end{aligned} \qquad (2.21)$$

where σ_0 is the conductivity in weak fields when $T = T_0$.

If equation (2.16) is divided by equation (2.19),

$$e^{(\mathbf{h}\nu/\mathbf{k})(T_0^{-1}-T^{-1})}-1 = (F^2/eF_m^2)e^{-\Delta V/\mathbf{k}T}e^{\Delta V/\mathbf{k}T_0}(e^{\mathbf{h}\nu/\Delta V}-1).$$

But $(\mathbf{h}\nu/\mathbf{k})(T_0^{-1}-T^{-1})$ and $\mathbf{h}\nu/\Delta V$ are small, whence

$$\frac{1}{T_0}-\frac{1}{T} = \frac{\mathbf{k}}{\Delta V}\frac{F^2}{eF_m^2}e^{-\Delta V/\mathbf{k}T}e^{\Delta V/\mathbf{k}T_0}. \tag{2.22}$$

At low field strengths when F^2 is much less than F_m^2, T does not differ greatly from T_0 so that $e^{-\Delta V/\mathbf{k}T}e^{\Delta V/\mathbf{k}T_0}$ may be taken as unity and

$$\frac{1}{T_0}-\frac{1}{T} = \frac{\mathbf{k}}{\Delta V}\frac{F^2}{eF_m^2}.$$

From (2.21)

$$\log(\sigma/\sigma_0) = (V/\mathbf{k})(T_0^{-1}-T^{-1}).$$

Thus

$$\log(\sigma/\sigma_0) = (V/\Delta V)(F^2/eF_m^2). \tag{2.23}$$

When the field approaches breakdown and $F \simeq F_m$,

$$e^{-\Delta V/\mathbf{k}T}e^{\Delta V/\mathbf{k}T_0}$$

is equal to e from equation (2.18), so that the conductivity σ_m at breakdown is given by

$$\log(\sigma_m/\sigma_0) = (V/\Delta V)(F^2/F_m^2) \simeq V/\Delta V. \tag{2.24}$$

The difference between equations (2.23) and (2.24) lies in the factor e. Owing to the difficulty of conductivity measurements at fields above 1/5th of the electric strength, the two approximations may be regarded as effectively similar for practical purposes.

An alternative theory of the increase of electronic conductivity with field strength due to Frenkel[48] may here be mentioned. After rejecting explanation by the 'tunnel effect' already discussed in Zener's theory, Frenkel applies a theory of thermal ionization facilitated by reduction of the potential barrier or binding energy by the external field analogous to Schottky's theory of thermionic emission in strong fields. He deduces the reduction of the potential barrier for an electron associated with an isolated atom to be $2\sqrt{(\mathbf{e}^3F)}$ so that

$$\sigma = \sigma_0 e^{\sqrt{(\mathbf{e}^3F)}/\mathbf{k}T}, \tag{2.25}$$

where σ_0 is the conductivity in weak fields. No experimental confirmation of this law has been traced, but the theory is

mentioned partly for completeness and also because Frenkel, in order to account for the large spatial dimensions of the potential barrier, has to bring in the influence of other atoms. The manner of doing this leads naturally to the hypothesis of intermediate isolated energy levels much more numerous than the free electrons, which is an essential feature of Fröhlich's theory.

2.4.2. Perfect crystals or low temperatures

According to Fröhlich the trapped electrons are too few to interact in perfect crystals or at low temperatures and so *a fortiori* are the conduction electrons. We may therefore ignore the trapped electrons and consider the energy balance of the conduction electrons with respect to gain of energy from the field and loss of energy to the solid structure. This gain and loss are denoted as before by A and B respectively. The variation of the relaxation time τ of the electrons will be important since it controls both processes. As in section (2.4.1), A is proportional to $F^2\tau$ (F = field strength). The loss of energy B to the lattice will increase with the number of collisions with the lattice vibrations. In fact B will be shown to be approximately proportional to probability of collision, or the inverse of the relaxation time τ_0, at absolute zero, i.e. when the lattice oscillations are entirely derived from the electrons. There is now no longer any question of an electron temperature; it is necessary to refer instead to the probable energy E of the individual electron. The solid is more transparent to the faster electrons, and it will be shown that τ is proportional to $E^{\frac{3}{2}}$, while τ_0 is proportional to $E^{\frac{1}{2}}$, whence

$$A \propto F^2E^{\frac{3}{2}}, \qquad B \propto E^{-\frac{1}{2}}.$$

The energy balance of section (2.4.1) must be replaced by one referred to E instead of T, as in Fig. 11. For given values of field, F'', F', F_m there are corresponding equilibrium values of electron energy E'', E', J for which the energy gained from the field is all lost to the structure. The equilibrium energy decreases as the field increases. An electron with less than the

equilibrium energy will lose more energy than it gains: it will slow down and eventually be trapped. An electron with more than the equilibrium energy will increase in energy, for $A > B$, and will therefore cause ionization. Thus self-increasing ionization is only possible by the appearance of electrons with more than the equilibrium energy. The probability of the appearance of an electron with more energy than J, the ionization potential or X-ray absorption limit, is very remote because an electron

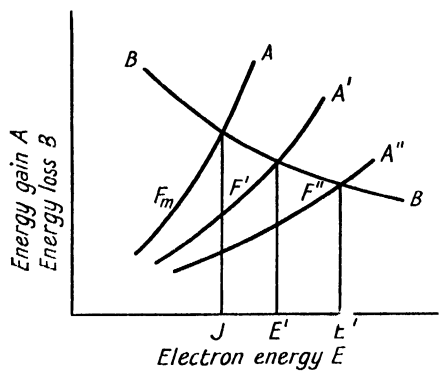


FIG. 11. Equilibrium and instability of fast electrons in crystals.

would lose its energy in inelastic collisions before J was reached. Thus J is the highest value of equilibrium energy which any electron has a chance of securing. Therefore the field F_m, for which J is the equilibrium energy, is the lowest field in which self-increasing ionization is reasonably possible and is therefore the minimum breakdown field according to Fröhlich. At lower fields such as F' or F'' there would be no electrons with energies of E' or E'' and above so that the conduction electrons would be slowed and tend to disappear.

Since the number of conduction electrons does not enter directly, and since the dissipation of energy to the solid depends upon the lattice vibrations, rise of temperature will increase the electric strength because it tends to stimulate the lattice vibrations, thus increasing the dissipation of energy and decreasing the gain of energy by impeding the motion of the electrons.

We now proceed to a quantitative demonstration. The conduction electrons can interact with the longitudinal vibrations of the structure because the polarization therewith associated is non-solenoidal, whereas transverse displacements give a solenoidal polarization. An inexact but perhaps more traditional explanation is that the polarization due to longitudinal vibrations gives a resultant flux across a closed surface in the solid and so may be replaced by a distributed electrification which can interact with the electrons: transverse vibrations give no resultant flux and no equivalent electrification. The interaction between the electron and the lattice vibrations consists in the emission or absorption by the electron of a vibration quantum. If E and K are respectively the energy and wave number (momentum coordinate) of the electron,

$$E = \mathbf{h}^2K^2/8\pi^2\mathbf{m},$$

and if ν and w are respectively the frequency and wave number of the vibrations, then E changes to $E \pm \mathbf{h}\nu$ and K to $K \pm w$ at each 'collision', while the scattering angle or change θ in the direction of K (i.e. of the electron) is $2 \arcsin w/2K$. It follows that only the shortest waves (w large) need be considered and for longitudinal waves the highest frequency is probably of the same magnitude as that of the transverse waves which correspond roughly to the residual rays. The total energy of the system of electrons and vibrations will be the sum of terms of the type $(n+\frac{1}{2})\mathbf{h}\nu + E$. Using a perturbation theory, Fröhlich then shows that the probability P' of emission of a quantum is

$$P' = \left(\frac{2\pi\mathbf{e}^2}{a^3w}\right)^2 \frac{1+n}{2M\mathbf{h}\nu} \frac{\partial}{\partial t} \frac{\sin^2\xi t}{\xi^2}, \tag{2.26}$$

$\mathbf{h}\xi/2\pi = (n'-n)\mathbf{h}\nu + E' - E =$ energy change accompanying quantum change.

The probability P of absorption is

$$P = \left(\frac{2\pi\mathbf{e}^2}{a^3w}\right)^2 \frac{n}{2M\mathbf{h}\nu} \frac{\partial}{\partial t} \frac{\sin^2\xi t}{\xi^2}, \tag{2.27}$$

where a is the lattice constant in a polar crystal, M is the reduced ionic mass, i.e. twice the harmonic mean of the anionic

and cationic masses, and the angle α between K and w is

$$\cos\alpha = -(w/2K)\pm(8\pi^2\mathbf{m}/\mathbf{h}^2)(\mathbf{h}\nu/2Kw). \tag{2.28}$$

Let K_x be the momentum in the direction of the field F and let ΔK_x be the mean change per collision. The rate of gain of momentum from the field is

$$\frac{\partial K_x}{\partial t} = \frac{2\pi\mathbf{e}F}{\mathbf{h}}.$$

From the probability of a change of energy by one quantum

$$K_x/\tau = -\sum \Delta K_x(P+P'), \tag{2.29}$$

where τ is the relaxation time of the electrons defined, as previously discussed, on an attenuation basis. Assume that n has a constant value equal to its mean as given by the Planck function $(e^{\mathbf{h}\nu/\mathbf{k}T}-1)^{-1}$. Replace the sum by an integral with respect to an infinitesimal w between the limits of $2^{\frac{1}{2}}\pi/a$ $(= w_0)$, the shortest wave in view of atomic distances, and $4\pi^2\mathbf{m}\nu/\mathbf{h}K$ $(= w_1)$, then

$$\frac{1}{\tau} = \frac{1}{\tau_0}\left(1+\frac{2}{e^{\mathbf{h}\nu/\mathbf{k}T}-1}\right) \tag{2.30}$$

if

$$\frac{1}{\tau_0} = \frac{\pi}{8\times 2^{\frac{1}{2}}}\frac{\mathbf{e}^4\mathbf{h}}{\mathbf{m}^{\frac{1}{2}}Ma^5\nu E^{\frac{1}{2}}}. \tag{2.31}$$

If E_0 is the energy which corresponds to the lowest wave number (i.e. at absolute zero)

$$E_0 = \frac{\mathbf{h}^2}{8\pi^2\mathbf{m}}(\tfrac{1}{2}w_0)^2 = \frac{1}{16\times 2^{\frac{1}{2}}}\frac{\mathbf{h}^2}{\mathbf{m}a^2} < E. \tag{2.32}$$

If ϵ is the dielectric constant measured at a frequency high enough to avoid dipolar effects and ϵ_0 is the square of the refractive index in the near infra red, $\epsilon-\epsilon_0$ in the present model will represent the polarization contributed by the ionic displacement so that it is permissible to put†

$$\epsilon-\epsilon_0 = \mathbf{e}^2/2\pi Ma^3\nu^2,$$

whence

$$\frac{1}{\tau_0} = \frac{2^{\frac{1}{2}}\pi^2\mathbf{h}\nu(\epsilon-\epsilon_0)\mathbf{e}^2}{8a^2\mathbf{m}^{\frac{1}{2}}E^{\frac{1}{2}}}. \tag{2.33}$$

† See Fröhlich, *Theory of Dielectrics*, Clarendon Press, Oxford, 1949.

The mean energy transferred to the lattice per second per electron is

$$B = \mathbf{h}\nu \sum (P' - P) \tag{2.34}$$

which can be evaluated by similar approximations to give

$$B = \frac{\mathbf{h}\nu}{\tau_0} \frac{4K^2}{w_0^2} \int_{w_1}^{w_0} \frac{dy}{y}$$

$$= \frac{\pi \mathbf{e}^4}{Ma^3} \sqrt{\left(\frac{2\mathbf{m}}{E}\right)} \log \gamma = \frac{2^{\frac{1}{2}} \pi^2 \nu^2 \mathbf{e}^2 (\epsilon - \epsilon_0)}{E^{\frac{1}{2}}} \mathbf{m}^{\frac{1}{2}} \log \gamma, \tag{2.35}$$

$$\gamma = w_0/w_1 = (\mathbf{h}K/4\pi^2 \mathbf{m}\nu)(2^{\frac{1}{2}}\pi/a) = (2^{\frac{1}{2}}\nu a)^{-1}(2E/\mathbf{m})^{\frac{1}{2}}$$

$$= (4/\mathbf{h}\nu)\sqrt{(EE_0)}, \tag{2.36}$$

on substituting for K in terms of E and for $a\sqrt{\mathbf{m}}$ in terms of $\mathbf{h}/\sqrt{E_0}$ from (2.32).

From equation (2.8) of the previous section, the mean gain of energy A from the field per electron

$$A = \mathbf{e}^2 \tau F^2/\mathbf{m}$$

$$= \frac{8a^2 E^{\frac{1}{2}} F^2}{2^{\frac{1}{2}} \pi^2 (\epsilon - \epsilon_0) \mathbf{h}\nu \mathbf{m}^{\frac{1}{2}}} \left(1 + \frac{2}{e^{\mathbf{h}\nu/\mathbf{k}T} - 1}\right)^{-1}. \tag{2.37}$$

Thus the gain of energy is proportional to $E^{\frac{1}{2}}$ while the loss of energy is proportional to $E^{-\frac{1}{2}}$. For any given value of the field F there is an energy E for which $A = B$, and the loss and gain of energy are equal. An electron with lower energy will lose it to the structure and become trapped or fall into the category of slow electrons. An electron with higher energy will gain energy on balance so that the fast electrons will be pulled out and accelerated by the field as already described. Thus a stable current due to fast electrons should not be possible.

If an attempt is made to find a normalized energy distribution among the electrons, Fröhlich states[50] that no equilibrium can be reached in however weak a field, unless ionization processes are taken into account. This would mean that the possibility of equilibrium is determined by fast (ionizing) electrons. According to von Hippel[51] and the elaboration of his views by Seeger and Teller,[43] the mean free path or relaxation time of a slow electron will diminish as its energy rises, and will pass through a minimum when the energy is in the region of $\mathbf{h}\nu$ since it will

then excite the lattice vibrations, giving a kind of energy barrier. von Hippel and his co-workers consider, therefore, that if any electron can be accelerated between collisions to an energy greater than this, some will be able to reach an energy sufficient to ionize the structure because even consecutive collisions with emission of a quantum will still give a net gain of energy, while many collisions will be quite elastic. This is expressed in von Hippel's first elementary formula

$$\text{electric strength} = F = Ch\nu/ea, \tag{2.38}$$

where C is an undetermined constant. Fröhlich's deduction that only the fast electrons need be considered in assessing stability seems to render it immaterial whether slow electrons (e.g. those just freed by collisions) can yield fast electrons. If this is correct, Fröhlich's analysis seems to dispose of the alternative theories mentioned. The variation of electric strength with temperature and the effects of residual ray frequency, crystal constants, foreign atoms, and macro-structure—confirmed so accurately by experiment—mainly result from Fröhlich's calculation of thermal dissipation, which has not seriously been questioned. Thus any inaccuracy in the breakdown condition would only affect absolute magnitudes. These would scarcely be influenced other than in a minor degree by the alternatives to Fröhlich's theory just discussed (cf. Callen[104]).

Although the development of a distribution of fast electrons from slow electrons seems inadmissible it is quite feasible to postulate the existence of a certain number of fast electrons by the photo-electric absorption of radiation, by the tunnel effect (cold emission), or by chance long free paths, and if their energies exceed the intersection of the curves of energy absorption and dissipation, Fig. 11, they will gain in energy, ionize the lattice, and so multiply into an electron avalanche. However, it is almost impossible to envisage an electron with an energy much above that needed to ionize the structure, which is given, as shown earlier, by the X-ray absorption limit J electron volts: before the energy exceeded J it would be shed by inelastic collisions. In weak fields as in Fig. 11 the energy E, for which

$A = B$, is high, and there would be no electrons with energies in excess of the equilibrium energy. As F increases, the equilibrium energy for $A = B$ falls, and when the equilibrium energy is below J then there should be an appreciable number of electrons above the intersection and breakdown could occur. The minimum electric strength is thus given by that value of the field F for which electrons with an energy J show no net loss or gain of energy. Thus from equations (2.35) and (2.37), putting $E = J$,

$$A = (\mathbf{e}^2F^2/\mathbf{m})\tau_0\{1+2(e^{\mathbf{h}\nu/\mathbf{k}T}-1)^{-1}\}^{-1} = B$$
$$= (\nu/\mathbf{h}\tau_0)32\times 2^{-\frac{3}{2}}\mathbf{m}Ja^2\log\gamma.$$

Whence

$$F = 2^{-\frac{3}{2}}\pi^2(\epsilon-\epsilon_0)\mathbf{e}\nu^{\frac{3}{2}}(\mathbf{mh})^{\frac{1}{2}}(Ja)^{-1}\times$$
$$\times\{1+2(e^{\mathbf{h}\nu/\mathbf{k}T}-1)^{-1}\}^{\frac{1}{2}}\sqrt{[\log\{(4/\mathbf{h}\nu)\sqrt{(JE_0)}\}]}. \quad (2.39)$$

Substitute $J = \mathbf{hc}/\lambda_0$, where λ_0 is the wave-length of the first maximum of the ultra-violet absorption band; replace a by $V^{\frac{1}{3}}$, where V is the molecular volume; replace ν by $\mathbf{hc}/\lambda$, where λ is the wave-length of the residual rays; insert values for the universal constants and assume that

$$\log\{(4/\mathbf{h}\nu)\sqrt{(JE_0)}\} = 6{\cdot}4,$$

as is approximately true for alkali halides. The following equation is then obtained:

$$F = 1{\cdot}64\times 10^5V^{-\frac{1}{3}}(\epsilon-\epsilon_0)\lambda_0\lambda^{-\frac{3}{2}}\{1+2(e^{\mathbf{h}\nu/\mathbf{k}T}-1)^{-1}\}^{\frac{1}{2}}\text{volts/cm.} \quad (2.40)$$

Here λ_0 is in Ångström units and λ in microns.

In the high temperature or amorphous case the electric strength fell with rise of temperature mainly because the number of conduction and excited trapped electrons increased with rise of temperature and so increased the energy acquired from the field by the electrons as a whole. Here we are concerned with the stability of the probable energy of an individual group of electrons. As the temperature increases the relaxation time decreases, the dissipation of energy to the lattice oscillations increases and the gain of energy from the field decreases. Accordingly the electric strength increases with increase of temperature, but this will only be considerable at temperatures

where the residual rays are stimulated, that is, at temperatures approaching the Debye temperature of the dielectric.

The preceding applies to a simple crystal such as an ideal alkali halide. With more complicated crystals,[49] account can be taken of the multiple nature of the frequencies ν by summation over all the wave numbers and frequencies. B and $1/\tau$ now become sums. This leads to the formula

$$F = 5\times 10^4 (Z/Vn)^{\frac{1}{2}}\lambda_0 \left[\sum_{r=1}^{r=n} \{(\epsilon_r - 1)/\lambda_r\}\{1+2(e^{h\nu r/kT}-1)^{-1}\}\times \right.$$
$$\left. \times \sum_{r=1}^{r=n} \{(\epsilon_r - 1)/\lambda_r^2\}\log\{(4/h\nu_r)\sqrt{(JE_0)}\}\right]^{\frac{1}{2}} \text{V/cm.} \quad (2.41)$$

Here ϵ_r is the term in the dielectric constant associated with the frequency ν_r and the wave-length λ_r, Z is the number of atoms per molecule, and n the number of infra-red frequencies. The general features remain the same as for the simpler example, and it has not been possible in practice to consider other than the one or two strongest infra-red bands.

2.5. Magnitude of the intrinsic electric strength

The intrinsic electric strength, it may be recalled, was for some time considered to be a physical constant of the dielectric, but on the basis of any theory of electronic breakdown some variation with temperature must occur and on Fröhlich's theory the variation is characteristic. In accordance with section (2.4), in the low-temperature or 'crystalline' region the intrinsic electric strength should increase with increase of temperature, and in the high-temperature or 'amorphous' region the intrinsic electric strength should fall as the temperature rises. Now the increase at lower temperature depends upon $e^{-h\nu/kT}$. Strictly ν is the frequency of the longitudinal waves which are mainly short, but the values are not known precisely. Accordingly the frequency of the transverse waves is taken which can be obtained from residual ray measurements. In any event $h\nu/k$ usually corresponds to a high temperature so that $e^{-h\nu/kT}$ is small. The variation of electric strength with temperature will therefore also be small except when ν is low, i.e. when the 'Debye temperature' of the material is low enough to come

within the normal temperature range. This is largely confined to the alkali halides. On the other hand, the decrease at high temperatures is exponential in character and much greater. Thus usually the electric strength is nearly constant up to a critical temperature T_c and falls rapidly at higher temperatures. It is therefore possible to define a characteristic electric strength as the value at T_c when it should theoretically have a maximum value. Only in a certain group of materials will the maximum be at all pronounced; in most instances the value can be taken at any temperature below T_c. In most of the dielectrics of practical interest, with the exception of the alkali halides, the temperature T_c is either near the melting-point for more ordered structures or very low for amorphous substances, although in the truly solid range in a few cases such as polythene. Thus usually the characteristic electric strength may be taken either at liquid air or lower temperatures or at some convenient temperature such as room temperature or rather higher, because even if, in the latter case, the maximum value was at a higher temperature the difference in the electric strength would be small, except with a few substances.

If Fröhlich's theory be adopted, imperfection energy levels should not affect the electric strength until sufficiently numerous to interact with one another. In the low-temperature region defects in the structure will mainly influence the electric strength by scattering. This reduces the gain and increases the loss of electronic energy by reduction of the relaxation time and so increases the electric strength analogously to the increase of electrical resistance by structural defects. Now the structure must not be too much disturbed or the critical temperature T_c will be decreased and the substance will pass into the high temperature region with a lowered electric strength. Accordingly, the greatest effect on the characteristic or maximum electric strength should occur when the defects do not disturb the general features of the structure. This happens in solid solutions of the same salt of two equivalent metals since the process involves merely the replacement of one atom by another having the same valency but a different atomic number. As the

replacement goes on the electric strength will first rise owing to the disorder and then fall towards the electric strength of the second pure substance when replacement is complete. This is illustrated by some experiments of von Hippel,[51] Fig. 12, on the solution of RbCl in KCl and also by the experiments of von Hippel and Lee[52] on NaCl with an admixture of AgCl. In equation (2.37) the gain of energy A is proportional to F^2 (F is the

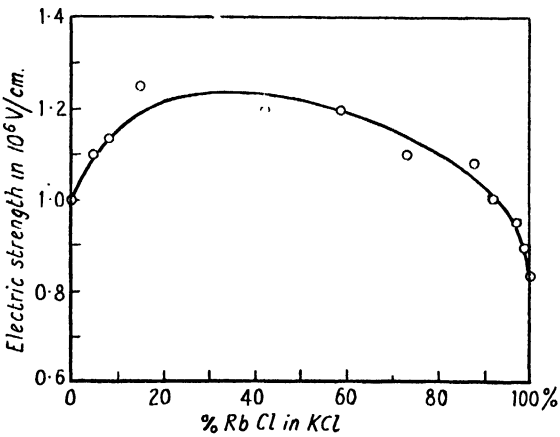


FIG. 12. Electric strength of a mixed crystal.
(Von Hippel.)

electric strength). In (2.4.2) the relaxation time τ was taken as depending solely on scattering by thermal oscillations. By analogy with the theory of the resistance of metals, an additional scattering due to foreign atoms can be obtained by replacing τ_0^{-1} by $\tau_0^{-1}+\tau_c^{-1}$, where τ_c is the relaxation time caused by the foreign atoms. Since the chance of meeting a foreign atom is proportional to their density τ_c will be inversely proportional to the concentration c. From equation (2.35) in a pure crystal the loss of energy $B \propto 1/\tau_0$; from equation (2.37) the gain of energy $A \propto F^2\tau$. Accordingly, for the pure crystal

$$F^2 \propto (\tau_0 \tau)^{-1}$$

and for the solid solution

$$F_c^2 \propto \tau^{-1}(\tau_0^{-1}+\tau_c^{-1}). \tag{2.42}$$

A theoretical estimate of the effect of solid solution made in this

way has come to within 30 per cent. of the observed effect. Since this effect is no large proportion of the total value, which has itself a considerable experimental dispersion, the agreement is reasonable.

Both experimentally and theoretically it may be concluded that the intrinsic electric strength is not much affected by small amounts of impurity present in solid solution. Similar considerations apply to ionic defect or excess which, if very dilute, will not much affect the electric strength. The same applies *a fortiori* to electrons expelled from bound levels, since breakdown does not depend on the number of electrons but on their dynamic equilibrium. Inge and Walther[53] have shown that irradiation by X-rays or ultra-violet light capable of greatly increasing the number of free electrons has no effect on the electric strength of rock salt. Goldman and Wul[54] have confirmed this and state that an effect of light on the electric strength of stained crystals is due to secondary factors. Walker[55] finds some decrease in the electric strength of polythene by irradiation with X-rays, but it is very doubtful whether his results refer to intrinsic breakdown, and other evidence is against the existence of a valid effect presuming the structure to be unaltered (e.g. Oakes'[23] experiments with gamma rays). von Hippel and Alger[103] agree that the electric strength of crystals is unaffected by electrons trapped at '*F*' centres and unaffected by prior irradiation; but they say that exposure to intense X-radiation during application of stress causes some decrease.

Macro-structure may be similarly approached. Since the discharge should take the nearest preferred direction, the orientation of the crystallite where the discharge is situated is immaterial within the limits of deviation commonly encountered. This was verified by Austen and Whitehead[20] who showed with mica that differences in the length of the X-ray diffraction arc, which is a measure of the relative tilting of the crystallites, did not affect the electric strength. An amorphous region between two crystalline regions should have a higher electric strength, provided the dielectric constant is the same. Austen and Whitehead[20] compared quartz and silica, confirming the increase with

disorder if the lower dielectric constant of silica were taken into account on the basis of Fröhlich's formulae, in which the electric strength is proportional to $\epsilon - \epsilon_0$. von Hippel and Maurer[56] at lower temperatures found an increase from quartz to fused silica, independent of any correction for dielectric constant. In no case is a considerable decrease of dielectric constant to be expected in intermediate amorphous regions so that, in general, these will not be weaker than the crystalline regions.

Some experiments on polythene are interesting in this connexion. Austen and Pelzer[22] measured the electric strength of specimens fairly slowly cooled which were shown to possess a spherulitic structure in the crystalline regions. These were connected together by amorphous regions, but the degree of crystallinity was probably high in the experiments of Austen and Pelzer which were confined to room temperatures and below. The size of the crystals was estimated at about 10^{-5} cm. Oakes[23] examined polythene which was cooled rather more rapidly, but which appeared to have a similar structure. Bird and Pelzer[57] measured polythene which was slowly cooled and showed clearer evidence of spherulites without signs of distortion by internal strain. They also measured rapidly cooled polythene where the crystals were probably sub-optical, since the specimens were more transparent and gave no resultant double refraction. Fig. 13 shows the close agreement of all the results. Over the temperature range below 0° C. the degree of crystallinity was probably similar but the size of the crystallites seems certainly to have differed.

Colloidal solution of impurities should not affect the electric strength unless the colloidal particles are large enough to concentrate the electric fields over an adequate volume. This will be considered in more detail later, but it may be stated that an appreciable size is needed (see Austen and Whitehead[20]). This is rather confirmed by Oakes's[23] result on polythene that mild oxidation, which would be highly dispersed, had no significant effect on the electric strength, whereas an organic anti-oxidant which presumably forms large aggregates of high permittivity caused an appreciable though small reduction. The conclusion

is that the microscopic defects, from which it is so difficult to render materials free, will not usually affect the electric strength provided the degree and type of crystallinity is preserved. The electric strength may therefore be regarded as characteristic of a substance in a practicable state of purity and uniformity, albeit one which is often difficult to achieve.

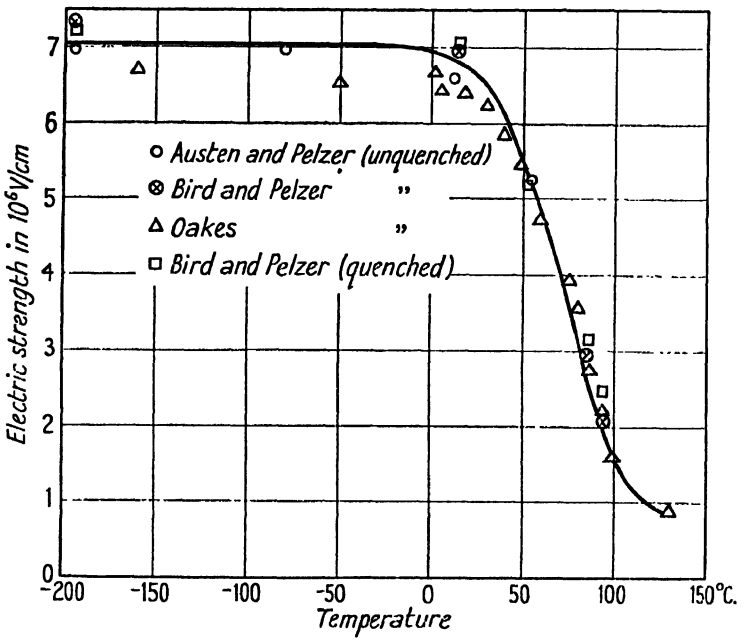


FIG. 13. Electric strength of polythene.

Alkali halides

It is not certain that the maximum values for these crystals have been measured because there is a substantial variation with temperature in the low-temperature region owing to the low Debye temperature. Accordingly, the temperature of measurement is rather critical and the temperature at which the amorphous region begins is not known with much certainty, if at all. The values quoted are those at about room temperature. A number of substances have been tested by von Hippel[51,56] and his co-workers, and a study of KBr has also been given by Austen and Whitehead:[20]

Electric Strength of Alkali Halides in 10^6 *V/cm.*

Substance	NaF	NaCl	NaBr	NaI	KCl	KBr	KI	RbCl	RbBr	RbI
			0·8			0·83†				
Experiment	2·4	1·5	1·0	0·8	1·0	0·8	0·6	1·35	0·5	0·5
Theory		1·5	1·3	1·4	0·89	0·73	0·63	0·84	0·54	0·47

† See Austen and Whitehead: the remaining values are from von Hippel,[51] who quotes recently[103] a value of 0·75–1·1 (mean 0·88) $\times 10^6$ V/cm. for KBr with the most advantageous electrode system.

The calculation of the theoretical values was made by Fröhlich on the hypothesis that the field was in the 100 direction. In a crystal such as rock salt, the electric strength should be least in the 110 direction. However, the breakdown path does not necessarily follow the field, so that it is still somewhat obscure as to whether the resolved field in the direction of easiest breakdown or the critical field strength in the direction of the applied field, or some intermediate value should be taken. Von Hippel only offers indirect evidence on this matter and has suggested that the observed electric strengths are not much affected by direction. Nevertheless, the agreement between theory and experiment is good, particularly when it is borne in mind that there are no disposable constants in the formula, a feature in which Fröhlich's theory is unique and impressive.

Mica

Whereas with the alkali halides it is easy to take a single residual ray frequency, with mica there are two values of about equal probability, namely wave-lengths of about 10μ and 20μ. These lead to a theoretical upper limit of the electric strength for muscovite of 15×10^6 V/cm. A considerable literature exists on the experimental electric strength of mica which has been summarized by Whitehead,[58] Perlick,[14] Hackett and Thomas.[13] The most acceptable values for muscovite seem to be those of $10–11 \times 10^6$ V/cm. from Hackett and Thomas (0·02–0·07 mm.); Standring[12] gives values up to 14×10^6 V/cm. with impulse voltages on specimens similar to those of Hackett and Thomas; Austen and Whitehead[20] quote these values, and the extrapolation of their curve for thin specimens would give 11×10^6 V/cm. at 0·001–0·01 mm.; von Hippel[51] gives 10×10^6 V/cm. for a thickness of 0·03 mm.; a value of $9·4 \times 10^6$ V/cm. for 0·05 mm.

thickness was found by Alexandrow and Joffe.[59] Thus theory and experiment show a reasonable accord, bearing in mind the difficulties both of experiment and calculation.

The d.c. electric strength of stained ruby muscovite, according to Hackett and Thomas, was significantly greater than for clear ruby, i.e. $11–12 \times 10^6$ V/cm. Impulse values did not differ significantly, and strengths up to nearly 14×10^6 V/cm. have been given by Standring. The lattice spacings in the direction of the field were slightly greater for the stained mica, but only by about 0·1 per cent., which is immaterial and in the wrong direction. There is no certain explanation but clearly the solid solution which yields the staining is not deleterious. On the other hand, according to Hackett and Thomas, stained light-brown muscovite gave a slightly lower value than clear ruby, while the electric strength of clear green muscovite was only about 7×10^6 V/cm.: the experiments on clear green mica were very limited.

The phlogopite micas give variable and rather unsatisfactory electric strength results; for example, according to Hackett and Thomas, silver amber, medium amber, and dark amber have electric strengths of about 4×10^6 V/cm., 3×10^6 V/cm., and $3{\cdot}5 \times 10^6$ V/cm. respectively. Some evidence is advanced by these authors that the lower electric strength is due to a lower ionization potential for phlogopite, but the rather doubtful nature of the experimental evidence and the lack of reliable data as to the physical constants, make the matter uncertain quantitatively, although the explanation given is qualitatively probable. Synthetic mica resembles phlogopite more than muscovite, and according to Pelzer its electric strength is rather higher than 6×10^6 V/cm. at room temperature, but the development of synthetic mica is still in rather an early stage.

Natural mica suffers from many macroscopic, indeed visual, defects. Its cleavage surfaces can also be quite irregular on a microscopic scale, having a stepped contour. Anomalously low values are frequently obtained. This is outside the scope of the present section as the problem becomes one of selection. In controlled experiments it is possible to select specimens

so as to obtain consistent and repeatable values of electric strength.

Quartz

Austen and Whitehead[20] have given values at room temperature for quartz of $6{\cdot}7 \times 10^6$ V/cm. in the direction of the optic axis and $5{\cdot}4 \times 10^6$ V/cm. for fused silica. The dielectric constant of the quartz was 4·6 and of the silica 3·9 so that, if the electric strength is proportional to $\epsilon - \epsilon_0$, the electric strength of a hypothetical quartz with a dielectric constant of 3·9 would be $4{\cdot}7 \times 10^6$ V/cm., i.e. lower than the amorphous form. However, measurements by von Hippel and Maurer[56] suggest that the critical temperature for fused silica is below 0° C. so that the maximum electric strength would occur at lower temperatures; they give a value of about 7×10^6 V/cm. for this. On the other hand, they found a fairly considerable increase of electric strength of quartz with temperature so that the highest value observed, also about 7×10^6 V/cm., may not be the maximum. At room temperature their value for quartz is less than that of Austen and Whitehead, while it is higher for silica. Fröhlich's calculation gives a possible range $3{\cdot}3$–$7{\cdot}4 \times 10^6$ V/cm. owing to uncertainty as to the optical properties. Accordingly, the situation is not entirely resolved, but the agreement between theory and experiment is as good as the available physical data permit.

Hydrocarbon chains and derivatives

An ordered, oriented chain compound has a structure similar to that of a non-polar crystal. A non-polar crystal differs in the present connexion from a polar crystal by the fact that an elastic wave does not give rise to electric polarization, the interaction between an electron and the structure depends upon the interaction between an electron and a neutral atom. The modes of oscillation of the lattice cover a wide spectrum, unlike the polar crystal where the important modes have nearly the same frequency. However, since the interaction potential is still independent of the electron energy E, the dependence of the relaxation time τ on E should be similar if a collision with a lattice wave is involved, whence τ is proportional to $E^{\frac{1}{2}}$. The

rate of gain of energy from the field will be $\mathbf{e}^2F^2\tau/\mathbf{m}$ as before. If the transfer of energy from the electron to the lattice wave is $\mathbf{h}\nu$, as for a polar crystal, then to a first approximation the rate of loss of energy B will be proportional to $E^{-\frac{1}{2}}$, as for a polar lattice, ignoring the logarithmic term in the polar crystal equation which has little influence. The same condition for breakdown can obviously be applied, and as previously the electric strength F will be given by

$$\mathbf{e}F = (B\mathbf{m}/\tau)^{\frac{1}{2}} \quad \text{when} \quad E = J \text{ (ionization potential).} \quad (2.43)$$

Fröhlich proposes the approximation $\mathbf{h}\nu/\tau_0$ for B at low temperatures, where τ_0 is the relaxation time at absolute zero, if ν is an average over the various oscillation frequencies of the lattice. This gives

$$\mathbf{e}F_0 = (\mathbf{m}\mathbf{h}\nu)^{\frac{1}{2}}/\tau_0.$$

If l_0 is the mean free path

$$l_0 = \tau_0(2E/\mathbf{m})^{\frac{1}{2}} = \tau_0(2J/\mathbf{m})^{\frac{1}{2}},$$

whence

$$\mathbf{e}F_0 = (2J\mathbf{h}\nu)^{\frac{1}{2}}/l_0. \quad (2.44)$$

Since J has values up to 10 eV, while $\mathbf{h}\nu$ is from 0·01–0·1 eV and l_0 of the order of 100 Ångströms, F_0 will be of the order 10^6 V/cm. similar to polar crystals and also will increase with temperature, since τ will decrease with temperature while B will vary much less. This analysis by Fröhlich[60] refers naturally to the low-temperature range.

There is no evidence of important oscillations in paraffin chains which involve relative displacement of ions of opposite sign, and as the dielectric constant seems of the same order as the refractive index, there are few important infra-red oscillations. Paraffin may thus be treated as a non-polar crystal if the chains are ordered and it is possible to calculate the order of the electric strength as just described, except that Fröhlich has proposed an elaboration based on the hypothesis that the perturbation energy does not depend upon the wave number w of the lattice vibrations, and that the oscillation frequency is proportional to w_1. The opposite is true in both instances with

polar crystals. Fröhlich's[60] calculation gives, if w_0 is the value of w for the shortest waves,

$$\left.\begin{aligned} B &= 24\pi^2 E\mathbf{m}\nu/w_0^2\mathbf{h}\tau_0 \\ \mathbf{e}F_0 &= (2\pi\mathbf{m}/w_0\mathbf{h}\tau_0)(6J\mathbf{h}\nu)^{\frac{1}{2}} \end{aligned}\right\}. \qquad (2.45)$$

If the chain length exceeds the mean free path, the end is a kind of defect, and decrease of chain length will increase the electric strength by multiplication of defects as already explained. If the chain is very short compared with the mean free path, the potential anomaly at the end will increase the effective mass of the electron, so here again the shorter the chain the greater the electric strength. As far as can be seen, the former process is the only one likely to enter in practice because materials with a very short chain length would be liable to behave as amorphous bodies to which, as has been shown, a different mechanism applies.

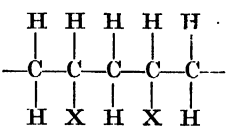
Austen and Pelzer[22] give values of $2–2{\cdot}4\times10^6$ V/cm. for pure paraffins $C_{22}H_{46}$, $C_{26}H_{54}$, $C_{29}H_{60}$, but the results were too variable for an effect of chain length to be observed. In addition, doubt attached to the validity of the results due to the possible formation of crystals too small to be tested as single crystals and large enough to prevent homogeneity. It may here be mentioned that Oakes[23] found little effect of molecular weight and presumably chain length in polythene. More consistent values of $2–2{\cdot}5\times10^6$ V/cm. for ordinary paraffin wax were also regarded as too low. Weber[61] obtained electric strengths of $2–4{\cdot}5\times10^6$ V/cm. for, presumably unoriented, paraffin wax, with a highest mean value of $3{\cdot}7\times10^6$ V/cm. Davis[62] gives values for the impulse electric strength of cable compound, paraffin wax, and Seekay wax as 1·9, 1·4, $1{\cdot}15\times10^6$ V/cm. respectively. Austen and Pelzer, in experiments on oriented paraffin wax, found it difficult to prepare satisfactory specimens oriented at right angles to the field, where the electric strength should be higher than parallel to the field. This conclusion follows from the effectively shorter relaxation time and mean free path at right angles to the chains, in accordance with the views previously discussed. A satisfactory result of $4{\cdot}5\times10^6$ V/cm. was obtained for paraffin wax with the

chains oriented parallel to the field. Paraffin wax as an impregnant, in capacitor dielectrics, for example, tends to be oriented parallel to the field, presumably because the normal direction of flow when soft is at right angles to the field and the orientation is normal to the flow. The highest stress concentrated on thin layers of impregnant would be the mean stress times the ratio of permittivities. From results on cable dielectric (quoted under Cellulose) an upper limit of about 5×10^6 V/cm. for compound could be derived from impulse tests, but this is merely speculation.

From what has been said about chain length it seems that for fairly long chains the effect of the ends should not be great, so that all materials with a similar chain and no dipolar groups should have a similar electric strength. Accordingly, polythene, which is composed of very long paraffin chains, should have an electric strength similar to that of paraffin wax. From Fig. 13 it may be said that Austen and Pelzer,[22] Oakes,[23] and Bird and Pelzer[57] obtained very similar values for polythene all of which lie in the range $6{\cdot}5$–$7{\cdot}3 \times 10^6$ V/cm. Austen and Pelzer concluded that their specimens were largely oriented at right angles to the field and the orientation in Oakes's experiments may have been similar. The quenched specimens of Bird and Pelzer, although having the same degree of crystallinity, must have had a random orientation with respect to the breakdown field. The absence of significant differences may mean that orientation is not as important as supposed. Conversely, in quenched polythene, the small spherulites may yield only a short length of breakdown path where the chains are parallel to the field. The electric strength would then depend mainly on the remainder of the path and approach more closely a value appertaining to chains at right angles to the field. If this were true then the higher electric strength of polythene as compared with paraffin wax oriented parallel to the field could be ascribed, in accordance with theory, to the relative orientations in the specimens tested.

For chlorinated polythene Oakes[63] found the highest values at low temperature, as in the case of amorphous materials. The highest electric strength was about $11{\cdot}5 \times 10^6$ V/cm. Similarly,

Austen and Pelzer found that the low temperature electric strength of amorphous polyvinyl chloride-acetate ('Vinylite') was also much greater than for polythene, namely $10–15\times10^6$ V/cm. with a mean of about 12×10^6 V/cm. Apart from its definitely amorphous character, this material is dipolar, since on one side of the chain half of the radicles are either chlorine or acetyl irregularly distributed.



X is H for polythene.

X is Cl or —O—C(=O)—CH_3 for polyvinyl chloride-acetate.

The two alternative radicles are present in unequal proportions according to the degree of chlorination. Now although the chain structure is different in polystyrene, which again is a hydrocarbon, the electric strength, according to Austen and Pelzer and also Oakes,[63] is still of an order similar to polythene, since maximum values of $6–7{\cdot}5\times10^6$ V/cm. were measured, and no great increase at low temperatures occurred. Polystyrene, as used, was considered to be amorphous. If so, the increase of electric strength of polyvinyl chloride-acetate at low temperatures is due to dipolar radicles such as chlorine or acetyl, although argument by elimination is rather incautious here.

The presence of dipoles provides an additional scattering agency so that the relaxation time is decreased. Equation (2.43) can be easily modified to

$$\mathbf{e}F = \left\{\mathbf{m}\left(\frac{1}{\tau_p}+\frac{1}{\tau_D}\right)(B_p+B_D)\right\}^{\frac{1}{2}}, \tag{2.46}$$

where the subscript p refers to the pure paraffin, i.e. the oscillation of non-polar chains, while the subscript D refers to the dipoles, i.e. dipolar oscillation but not rotation, since the temperatures will be too low. B_p can be estimated as in equation (2.45). If now F_D is $(\mathbf{m}B_D/\tau_D)^{\frac{1}{2}}$

$$F = \{F_p^2+F_pF_D(b+b^{-1})+F_D^2\}^{\frac{1}{2}}, \tag{2.47}$$

where $b^2 = \tau_D B_D/\tau_p B_p$.

From (2.45), taking the value of τ at absolute zero

$$B_p \tau_p = (24\pi^2 \mathbf{m}/\mathbf{h}^2 w_{0p}^2) J \mathbf{h}\nu_p.$$

As regards B_D and τ_D, Fröhlich[64] has used a method of calculation similar to that already given for a polar crystal, since dipolar oscillations will clearly resemble those of a polar crystal as they give rise to polarization in a similar manner. Thus

$$\frac{1}{\tau_{0D}} = \frac{8\pi}{3} \frac{\mathbf{e}^2 e_1^2 N}{\mathbf{h}\nu_D M} \sqrt{\left(\frac{\mathbf{m}}{2E}\right)},$$

M = resultant mass of dipole oscillator,

e_1 = effective ionic charge,

N = number of dipoles per cm.^{3},

ν_D = frequency of dipolar oscillation.

In polar crystals equation (2.31) gave τ_0 for the case there relevant when E is greater than $(\mathbf{h}^2/16 \times 2^{\frac{1}{2}} \mathbf{m} a^2)$. When E is less than this limit the expression for τ_0 in polar crystals becomes

$$\frac{1}{\tau_0} = \frac{2\pi \mathbf{e}^4}{\mathbf{h}\nu M a^3} \sqrt{\left(\frac{\mathbf{m}}{2E}\right)}.$$

If N is $1/2a^3$, i.e. one dipole per cube of side $2^{\frac{1}{3}}a$ then τ_{0D} only differs from τ_0 by a factor of 2/3. Similarly

$$B_D = (\mathbf{h}\nu/\tau_{0D}) \log \gamma,$$

$$\gamma = 4J/\mathbf{h}\nu_D.$$

The effective dipolar wave-length λ_D $(= \mathbf{c}/\nu_D)$ is below 20 microns while λ_p is of the order of 10 microns as may be verified from the Raman effect and calculations on the analogy with the diamond. Thus $\mathbf{k}T$ will be less than either $\mathbf{h}\nu_D$ or $\mathbf{h}\nu_p$ and the approximation should be justified. It leads to

$$b = \frac{\mathbf{h} w_{0p}}{2\pi} \left(\frac{\nu_D \log \gamma}{\nu_p \, 6\mathbf{m} J}\right)^{\frac{1}{2}}. \tag{2.48}$$

J can be found from the ultra-violet absorption limit λ_0, about 3,000 Ångströms, w_{0p} is taken as $2\pi(3n/4\pi)^{\frac{1}{3}}$, i.e. $0{\cdot}6 \times 10^8$ cm.$^{-1}$, if n is the number of atoms per cm.^{3}, while γ becomes $4\lambda_D/\lambda_0$.

Thus $$b^2 \simeq \lambda_D^{-1} \log 13\lambda_D \quad (\lambda_D \text{ in microns}). \tag{2.49}$$

The quantity e_1 can be eliminated if the dielectric constant due to the dipoles is known, since

$$(\epsilon-\epsilon_0)_D = \tfrac{1}{2}e_1^2 N/M\nu_D^2.$$

So finally

$$F_D = 1{\cdot}4\times 10^8(\epsilon-\epsilon_0)_D(\lambda_D)^{-\frac{3}{2}}(\log 13\lambda_D)^{\frac{1}{2}} \text{ V/cm.} \quad (2.50)$$

Consider now polythene and the vinyl copolymer or chlorinated polythene. From Austen and Pelzer, and also Oakes, F_D is about 7×10^6 V/cm. It seems likely that λ_D is of the order of 16 microns for the C—Cl dipolar oscillation. For the copolymer ϵ is about 3·6 and ϵ_0 is 2·6, so that $\epsilon-\epsilon_0$ is about unity, whence F_D is about 5×10^6 V/cm. Again, b is 0·57 approximately. The electric strength of the copolymer should therefore be, from (2.47),

$$F = (49+81+25)^{\frac{1}{2}}\times 10^6 \text{ V/cm.}$$

$$= 12{\cdot}5\times 10^6 \text{ V/cm.}$$

The calculation for chlorinated polythene would lead to a similar though slightly lower result. This agree very closely with the values of the order of 12×10^6 V/cm. actually measured on the dielectrics in question. It may therefore be concluded that dipoles raise the electric strength, but this is only true if maximum values are compared, that is at low temperatures, because dipoles as well as amorphous structure can cause temperature dependence which is otherwise absent owing to the high Debye temperature of materials other than alkali halides. In addition, only the scattering effect of dipoles has been taken into account. If, however, they modify the structure by decreasing the periodic variations of potential in the lattice, which increases the mean free path and the relaxation time, quite different effects might be expected.

Unfortunately, there is little certain evidence on allied structures. For example Oakes[53] found a high electric strength for acrylic resin, perspex, at low temperatures, namely about $13{\cdot}5\times 10^6$ V/cm. This might perhaps be explained on the same basis as for chlorinated polythene and vinyl copolymers. Polyisobutylene gave about 6×10^6 V/cm. according to Oakes. This again confirms that the inclusion of certain radicles is more

important than the crystalline-amorphous contrast. Further confirmation has come recently from measurements on polyvinyl alcohol, which has given the highest electric strength so far measured.[105]

Glass and ceramics

There is a considerable literature on the electric strength of glass, but a great deal is inconsistent and uncertain, since essential test conditions are not stated. No attempt will therefore be made to repeat the information set out in the books by Littleton and Morey,[65] Semenoff and Walther,[1] and Whitehead.[26] Authors, however, all agree that the electric strength of glass is constant at lower temperatures, but the critical temperature is doubtful. Moscicki[66] gave 2×10^6 V/cm. for alkali glass; Inge and Walther[67] gave 3×10^6 V/cm. for cover glass below a critical temperature of about 100° C.; while Perlick[14] gave $2{\cdot}6 \times 10^6$ V/cm. for Jena cover glass, probably the same as the preceding; Moon and Norcross[19] found a value of $3{\cdot}2 \times 10^6$ V/cm. below 40° C. for lead glass; Rochow[68] gave $3{\cdot}4 \times 10^6$ V/cm. for some Jena glasses. Standring[12] gives a number of values for the impulse electric strength of glass; for window glass $1{\cdot}7 \times 10^6$ V/cm. if breakdown occurs on or after the crest and $2{\cdot}8$–$4{\cdot}7 \times 10^6$ V/cm. for breakdown on the front of the wave. Both super-cooled toughened glass and borosilicate 'pyrex' glass gave $1{\cdot}7 \times 10^6$ V/cm. while annealed glass gave $2{\cdot}2 \times 10^6$ V/cm. with breakdowns on or after the crest. More recently von Hippel and Maurer[56] found a value of about $3{\cdot}8 \times 10^6$ V/cm. for soda-lime glass below about −50° C. Austen and Whitehead[20] determined a value of $5{\cdot}1 \times 10^6$ V/cm. total mean and $6{\cdot}0 \times 10^6$ V/cm. as the mean of maximum readings on specimens of a lead glass. Keller[102] criticized previous results on the ground of the heating due to pre-breakdown currents. He proposes $6{\cdot}2 \times 10^6$ V/cm. at room temperature for Thuringian glass, mainly soda-lime-alumina with a little Fe and Mg. His criticisms were inapplicable to the experiments of Austen and Whitehead and of Standring. There may be little real differences between the intrinsic electric strengths of most glasses.

Theoretically it is rather to be expected that if glass were crystalline the electric strength should be similar to quartz, allowing for any difference in the dielectric constant. The electric strength of amorphous glass should therefore be greater, again allowing for change in dielectric constant. In general, the figures of Austen and Whitehead, and Keller, the highest observed, are in fair accord with the figures of the former for silica and quartz, but the value of 7×10^6 V/cm. by von Hippel and Maurer for quartz and fused silica would suggest higher values for glass than they or others have observed. It would be expected that lead glass should have a higher electric strength than soda-lime glass, but the difference should not be very great. The subject must thus be left in some doubt, but in view of the rather wide range of uncertainty in the calculation of the electric strength of quartz already mentioned, it may be said that the more reliable results on glass agree with theory as to order of magnitude and are probably in correct relative order for different glasses.

There is also a considerable literature on ceramics, but the tests have been mainly of an industrial character affected by the test conditions. Furthermore, electrical ceramics are indefinite, both in composition and in structure, which is liable to lack homogeneity. It is only possible to quote the order of results at room temperature which is $0{\cdot}5 \times 10^6$ V/cm. (see Standring,[12] Krassin[25]). Beldi[69] has published results on the impulse and a.c. electric strength of porcelain from which the critical temperature seemed to be about 100° C. Below this the electric strength was of the order of 10^6 V/cm. on specimens about 1 mm. thick.

Resins

The term resin is here used in the somewhat restricted sense of complicated amorphous organic materials which are used in liquid form either pure, mixed, or in solution, and form amorphous homogeneous solids by suitable treatment. Admittedly, shellac has been found to have a certain structure according to researches at the Lac Institute, but not of a kind to affect the situation as regards electric strength. Oakes's[63] value of about $13{\cdot}5 \times 10^6$ V/cm. at low temperatures for acrylic resin has already

been mentioned. Thomas and Griffith[21] studied a number of resin varnishes in which the specimens were obtained by casting on metal. The critical temperature for these materials is low, and it is by no means certain that the maximum electric strength has been observed. However, the values quoted at −200° C. presumably approximate thereto.

Material	*Mean electric strength*	*Maximum readings*
Clear baking oil varnish . .	8×10^6 V/cm.	9×10^6 V/cm.
Black baking bituminous varnish.	$6{\cdot}5 \times 10^6$ V/cm.	8×10^6 V/cm.
Phenol formaldehyde 'formite' varnish	$7{\cdot}5 \times 10^6$ V/cm.	9×10^6 V/cm.
De-waxed lemon shellac varnish .	$7{\cdot}5 \times 10^6$ V/cm.	$8{\cdot}5 \times 10^6$ V/cm.

The first three included drying oils so that the final material contained the polymerized oxidation products of linseed, china-wood, and similar oils. The basis of the first was a mixture of natural resins with some phenol-formaldehyde resin, the second bitumen and synthetic resin, while the third was entirely synthetic of the phenol-formaldehyde type. The fourth material was shellac with waxes removed and as pure as is commercially usual. It is noted, however, that the values in the table are very similar. A theoretical calculation cannot be made, but a consideration of the formulae already developed confirms that no great difference is to be expected. What is perhaps a little surprising is that the values are high, but this is in accord with the results on acrylic resin and long-chain polymers already discussed.

On account of the low value of T_c, other measurements on resins are not of the greatest significance since they have been made usually at room temperatures and above. For the sake of information, these values are compared with those of the preceding varnishes at the same temperature. Although the relative order will not be the same as at low temperatures, it is reasonable to suppose that similar materials with similar electric strengths at room temperature will have at least a similar order of magnitude at low temperature, unless there is reason to believe that the intermediate energy levels are very different.

Electric Strength of Some Resins at Room Temperature

Material	*Author*	*Electric strength in 10^6 V/cm. at about 20° C.*
Copal varnish	Thomas[9]	2·3–3
Alloprene	,,	1·5
Colophonium resin	Gutin and Sackheim[70]	2·5
Varnish (oil) cloth	Standring[12]	1·3
Phenol formaldehyde (moulding powder with filler)	,,	0·9 (Impulse voltages)
,, ,, ,, (resin-bonded paper)	,,	1·9 (Impulse voltages)
,, ,, ,, (pure resin)	,,	2·3 (Impulse voltages)
Synthetic resin (formaldehyde)	Perlick[14]	0·8
Clear baking oil varnish	Thomas and Griffith[21]	3
Black bituminous ,,	,,	2·5
Phenol formaldehyde varnish	,,	4·5 5
Shellac	,,	3·5
Bitumen	Standring[71]	1·0 Impulse voltages
Methyl methacrylate	Oakes[63]	6

Perlick's value was obtained on varnish paper and the impregnating and testing technique is not above suspicion. The low value for alloprene (chlorinated rubber) may be legitimate but, for reasons given earlier, chlorination usually increases the electric strength. Both alloprene and neoprene are liable to defects of a macroscopic order. In the work of Thomas and Griffith on the more standard resin varnishes the effect of these defects was eliminated and, *inter alia*, it was shown that they could be diminished in number by filtration. Accordingly the value for alloprene may not be valid.

Rubber

For present purposes rubber must, it seems, be regarded as amorphous and has not been measured in a reliable manner over a sufficient range of temperature for the maximum value to be established. Thomas[9] has quoted $2{\cdot}15\times10^6$ V/cm. and Standring[12] a value of $1{\cdot}3\times10^6$ V/cm. on thick specimens 2–4 mm. of high electrical grade ebonite at 20° C. Perlick[14] gives $0{\cdot}92\times10^6$ V/cm. at 0–60° C., together with $0{\cdot}6$–$0{\cdot}9\times10^6$ V/cm. for soft

rubber and 1×10^6 V/cm. for Para rubber at 20° C.; but these values all seem too low.

Cellulose

In the form of fibres the air spaces must be filled or discharges therein determine breakdown. If so filled, the electric strength is usually determined wholly or mainly by the impregnant. The dielectric constant of cellulose is about 7 while that of varnish, wax, or oil, particularly the latter, is much less so that the stress on the cellulose is relieved. Davis[12,62] and his co-workers quote $1{\cdot}2\times10^6$ V/cm. for oil impregnated paper and $0{\cdot}8\times10^6$ V/cm. for impregnating oil with impulse voltages. Clearly the heterogeneity influenced breakdown. For cable dielectric Davis has quoted up to 2×10^6 V/cm. and actual impregnated paper cables subjected to 200 lb./sq. in. pressure have given an impulse electric strength up to $1{\cdot}6\times10^6$ V/cm. Cellulose products have not been measured over a range of temperature adequate to establish the highest electric strength, but Thomas[9] quotes $2{\cdot}3\times10^6$ V/cm. for commercial cellulose acetate, $3{\cdot}2\times10^6$ V/cm. for the pure material and also for cellulose triacetate. The reduction of electric strength by plasticizers, as in the commercial material, is explicable on the grounds of the macroscopic heterogeneity introduced by their presence.

2.6. Effect of temperature

In section (2.4) a characteristic feature of Fröhlich's theory of electronic breakdown was seen to be the existence of two regions. Below a critical temperature T_c the electric strength increases with temperature according to a formula which in the simplest instance is

$$F(T) = F(0)[1+\{2/(e^{h\nu/kT}-1)\}]^{\frac{1}{4}}, \qquad (2.51)$$

where $F(T)$ is the electric strength at an absolute temperature T and ν is the frequency of the residual rays. In addition F for ionic crystals is proportional to $\epsilon-\epsilon_0$, the variation of which with temperature is usually small but may be considerable with dipolar substances over certain ranges of temperature.† Above

† See Fröhlich, *Theory of Dielectrics*, Clarendon Press, Oxford, 1949.

T_c the electric strength decreases with temperature as a first approximation according to the law

$$\log F = \text{const.} + (\Delta V / 2\mathbf{k}T), \tag{2.52}$$

where ΔV is the mean energy difference required to expel an excited trapped electron into the conduction band. Disorder in a crystalline material reduces T_c but increases the electric

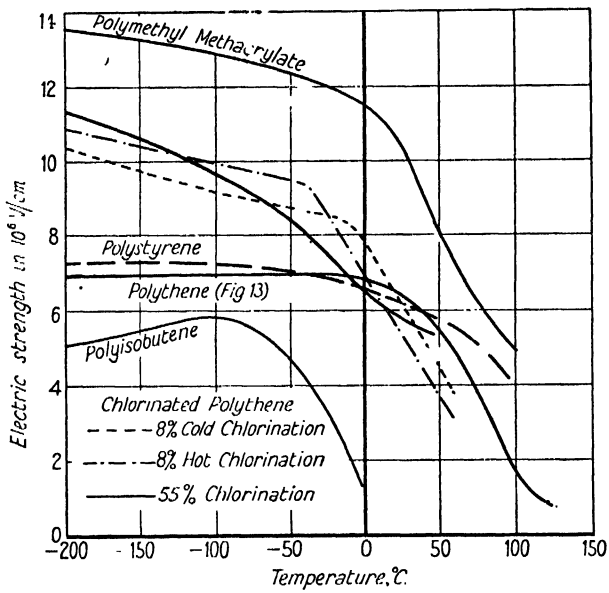


FIG. 14. Effect of chlorination on electric strength of polythene and comparison with other polymers.
(Oakes.)

strength below T_c. Thus the electric strength of a range of basically similar materials, but with increasing degrees of disorder due either to defects, disorientation, or increasing proportions of foreign atoms, would vary according to Fig. 14. This figure has been taken from some experiments by Oakes[63] on polythene with different degrees of chlorination and forms an interesting confirmation of the theoretically predicted effect of foreign atoms. A very similar result may be derived from the experiments of von Hippel and Maurer[56] on solutions of

AgCl in NaCl. Keller[56] has replotted their values so as to obtain curves like those of Fig. 14 but corresponding to successive replacement of some of the Na atoms by Ag. Thus qualitative confirmation has been found both in a simple polar crystal and a long-chain polymer.

Except with the alkali halides ν is large and the variation with temperature is small. Usually, therefore, the electric strength should be fairly constant up to T_c, above which it should fall sharply. This, in fact, is the phenomenon generally encountered in results of electric strength measurements but frequently cannot be ascribed to the foregoing causes. For example, in the work of Whitehead and Nethercot,[37] considered later in Chapter III, the phenomena of Fig. 14 were exhibited with decreasing amounts of moisture present. Nevertheless, the low-temperature breakdown was almost certainly due to external discharges, and the high-temperature breakdown was due to thermal instability. Indeed until recently many, and even now some, workers in this field are inclined to assign the decrease of electric strength to thermal instability in circumstances where it is highly improbable, sometimes clearly impossible.

In spite of the liability to be misled by apparent experimental confirmation, there is a body of definite evidence in support of Fröhlich's hypothesis. In the low-temperature or crystalline region a considerable and characteristic increase of electric strength with temperature is to be expected with alkali halides and has in fact been found by Austen and Whitehead[20] and Buehl and von Hippel.[72] Fig. 15 shows their results compared with theory. There are discrepancies of some magnitude between the sets of experimental results and between experiment and theory, and some rather different figures have recently been published by von Hippel and Alger,[103] but both the sign and the order of the variation is correctly given by an absolute theoretical calculation without disposable constants. The effect should be much less with mica and this was found by Hackett and Thomas[13] as in Fig. 16, where below 150° C. the agreement for muscovite with the full lines calculated for the two main residual ray frequencies is as close as either experimental error or uncertainty

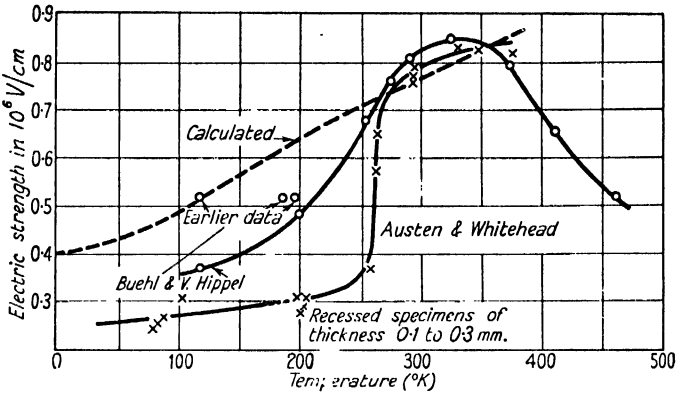


FIG. 15. Electric strength of potassium bromide.

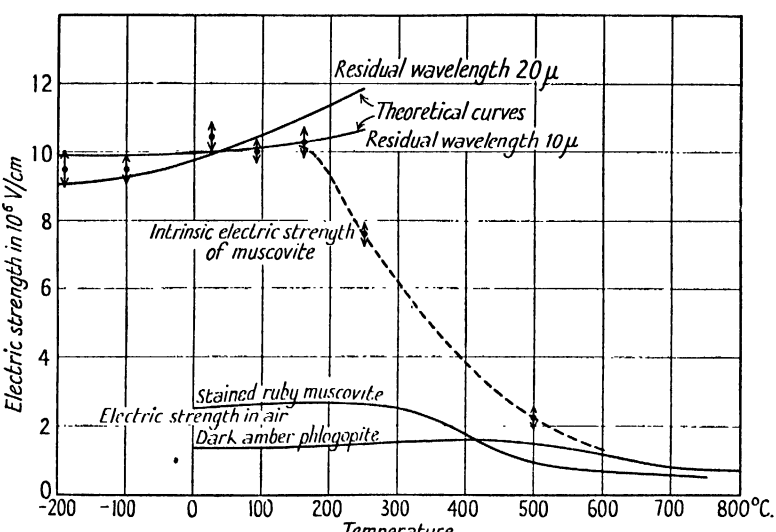


FIG. 16. Electric strength of mica.
(Hackett and Thomas.)

as to physical constants permits. Both with potassium bromide and mica there is a subsequent fall of electric strength with temperature, but this has not definitely been associated with Fröhlich's high-temperature theory.

It follows from the theory already given that at low temperature the amorphous form should have a higher electric strength

(allowing for changes in dielectric constant) than the crystalline form, but that the latter should maintain or increase its electric strength with increase in temperature while the former should fall in electric strength so that the situation could be reversed at higher temperatures. This is illustrated by results of von Hippel and Maurer[56] on quartz and silica in Fig. 17. There are some discrepancies, both those already noted with the results of Austen and Whitehead[20] and also the fact that the increase of

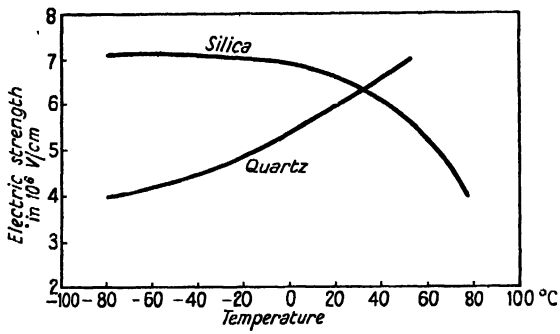


FIG. 17. Variation with temperature of fused and crystalline quartz.
(von Hippel and Maurer.)

the electric strength of crystalline quartz is greater than can easily be accounted for, since theoretically it should not be much greater than that estimated for mica: but the qualitative confirmation of the theory cannot easily be disputed.

A similar confirmation comes from the work of Austen and Pelzer,[22] Oakes,[23] Bird and Pelzer[57] on long-chain polymers. The electric strength of polythene should theoretically increase slightly with increase of temperature. The authors mentioned found that it was substantially constant. They used polythene with a definite spherulitic structure and with large and small crystallites. At low temperatures the degree of crystallinity was probably similar, but the chains would almost certainly be oriented in a random direction with reference to the electric field with quenched polythene, but might have been partially oriented in the unquenched specimens. However, all the tests concurred as closely as could be expected. Fig. 13, already quoted, shows

that above 30° C. there is a large and pronounced fall of electric strength with temperature in accordance with Fröhlich's high-temperature theory. On the other hand, the crystallinity changes as in Fig. 18, derived from the density and specific heat (Oakes[23]), and there is also a change in the rheological properties. Thus other effects may occur in addition to the multiplication of the structural defects postulated by Fröhlich. The magnitude

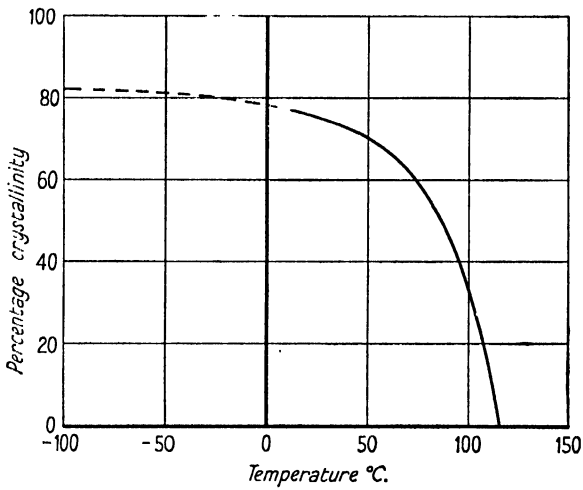


FIG. 18. Variation of crystallinity with temperature for polythene. (Oakes.)

of the fall of electric strength is that required by Fröhlich's hypothesis. Austen and Pelzer studied also the copolymer of vinyl chloride and acetate which, as already noted, has a constitution similar to that of polythene except that some hydrogen atoms are replaced by chlorine or the acetyl group. The copolymer specimens were amorphous and showed, Fig. 19, the expected feature that the electric strength of the copolymer exceeded that of polythene at low temperatures but decreased with temperature and was rather less than polythene at high temperatures. This concurs with the observations of Oakes[63] on chlorinated polythene, quoted earlier. In the previous section quantitative agreement with the relative electric strengths was obtained on the hypothesis of the effect of the dipolar groups in the copolymer. The decrease with temperature can be ex-

plained either on the hypothesis of the presence of dipoles or by the effect of the amorphous nature because both lead to a similar form of equation. Recent work in the E.R.A. laboratory[105] on polyvinyl alcohol implies that the presence of dipoles is the determining feature. Apart from poly-isobutene (Oakes[63]), the

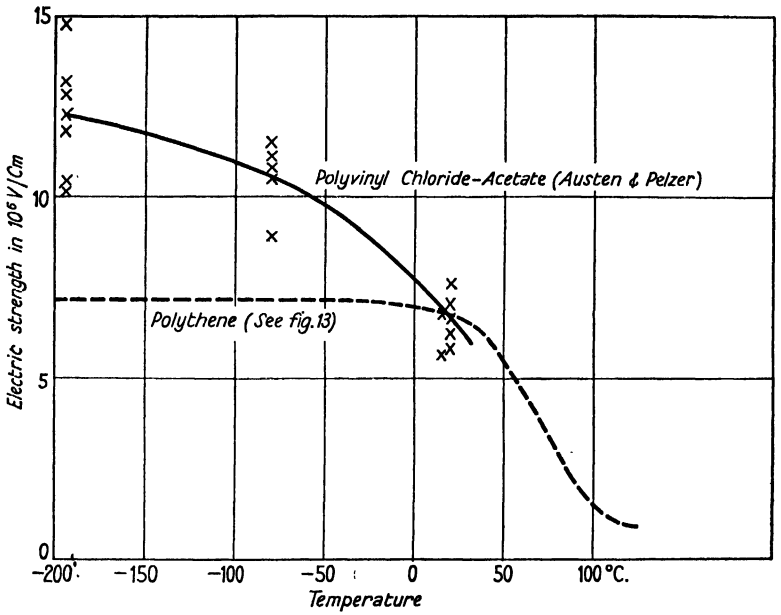


FIG. 19. Comparison of polyvinyl chloride acetate with polythene.

electric strength of a polymer if dipolar decreases from low temperatures but is constant if non-dipolar.

Fig. 20 shows a measure of quantitative agreement between von Hippel and Maurer's experiments on soda-lime glass and theory based on the assumption that $\Delta V/\mathbf{k} = 3{,}400°$. As already explained, there is some doubt as to the experimental results which do not agree with some others on glass. Thomas and Griffith, Fig. 21, found qualitative agreement with Fröhlich's theory for amorphous substances in experiments on varnishes which are known to be dipolar and amorphous; the structure found for shellac is immaterial for present purposes, and studies of gas and vapour permeability show that there

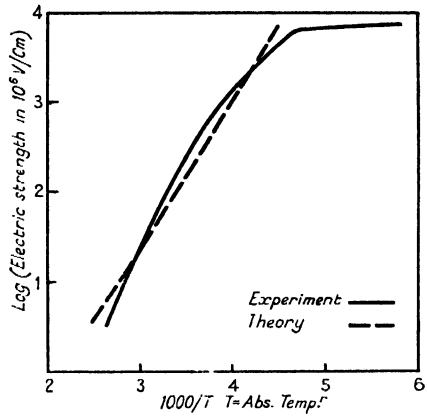


FIG. 20. Comparison of Fröhlich's theory with von Hippel and Maurer's experiments on glass.

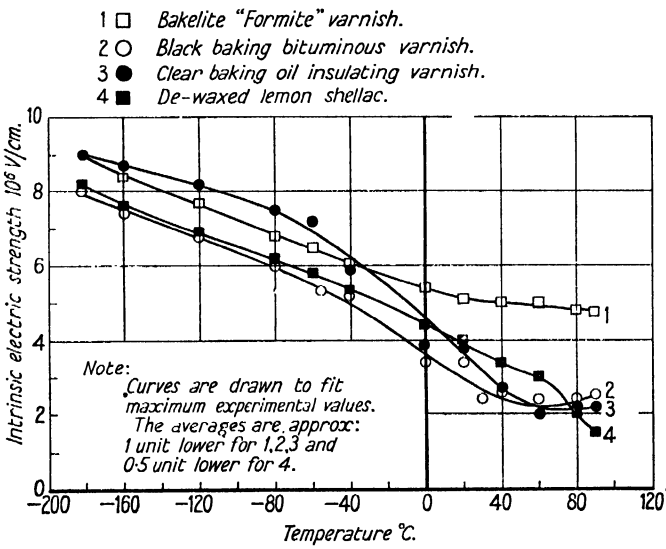


FIG. 21. Intrinsic electric strength of resin varnishes. (Thomas and Griffith.)

must certainly be an adequate number of 'trapping centres' from which the existence of imperfection energy levels should follow. The same authors observed an increase of conductivity with field strength which is rather slower than according to

the square of the field strength as in equation (2.23). From consideration of the variation of conductivity of black baking varnish with temperature, see equation (2.21), Thomas and Griffith consider V to be 2 eV; the nearest approximation in applying equation (2.52) to the observed variation of electric strength gives ΔV as 0·06 eV; the electric strength at room temperature was about 2×10^6 V/cm. From this it may be deduced from equation (2.23) that the logarithm of the ratio of the conductivity at $0{\cdot}5 \times 10^6$ V/cm. to that at $0{\cdot}25 \times 10^6$ V/cm. at room temperature should be about unity; the experimental value is 2, which is a significant comparison since the experimental uncertainty is great. The difference is of the right sign since it is to be expected that there will be contributions to the conductivity other than the electronic processes described, such as those due to the ionic activation mentioned in section (2.2).

The preceding deals with the more easily explained results on variation with temperature. The literature contains an amount of experimental data more difficult to explain. The results of Moon and Norcross on glass, Fig. 22, on the basis of which they suggested three regions of breakdown may be due to experimental dispersion and be explicable on the low- and high-temperature hypotheses. The fall of electric strength of mica above a temperature, depending on the variety, as found by Hackett and Thomas (Fig. 16) arises to some extent from changes in structure because at certain temperatures mica loses some of its water of crystallization and shows marked dimensional and mechanical changes. The critical temperature for electric strength is less than the temperature at which such changes occur but is related thereto. Fröhlich's high temperature theory plays a part, but the qualitative evidence is that the structural changes have a major influence. Ultimately, thermal instability sets in, so that over the whole range for mica at least some four mechanisms may operate. Similarly, the final fall of electric strength found by Thomas and Griffith for shellac cannot consistently be taken as the fall associated with Fröhlich's hypothesis and is not due to thermal instability but probably comes from the development of gross defects in softening. It

must be remembered that the changes involved in the approach to melting cannot for the purposes of dielectric breakdown, be considered merely as the development of amorphous character except possibly in certain mineral dielectrics. Usually softening produces defects in which may be expected a longer mean free path and higher conductivity. A macroscopic defect with a

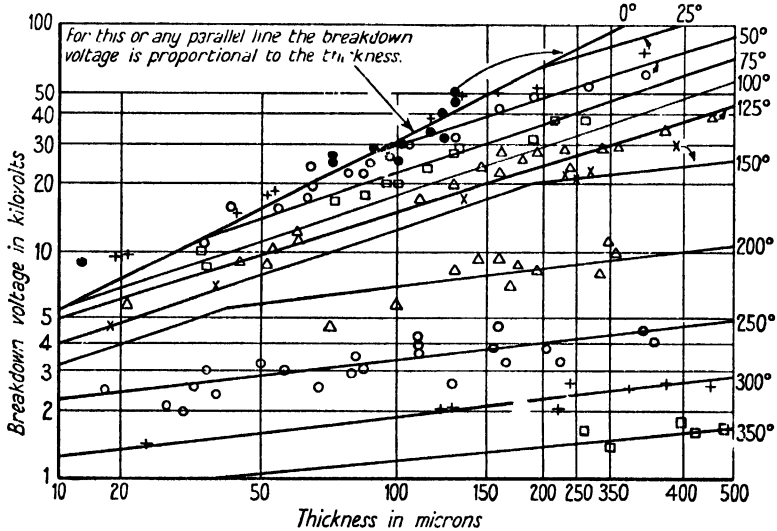


FIG. 22. Variation of breakdown voltage of hollow lead glass spheres (G–I glass) with temperature and thickness.
(Moon and Norcross.)

high conductivity will be attracted to the strongest part of the field which, if locally uniform, will provide a stable position. In this way defects can make a bridge, as do fibres in liquids, so that a channel is formed where the mean free path is very long and the electric strength is low. Again, and probably more frequently in practice, local decomposition or evolution of gas may occur which will introduce types of breakdown other than intrinsic breakdown, although the change may be continuous and difficult of definition.

2.7. Effect of thickness

In section (2.4) the criterion for breakdown was taken to be the condition where electrons with an energy slightly below the

ionization potential J of the structure would essentially gain energy from the field and so liberate further electrons. This process requires a certain amount of time which will decrease as the field strength is raised. If the dielectric is so thick that the great majority of the fast electrons must spend a time in the dielectric greater than the minimum necessary to gain the requisite energy then the period of acceleration merely contributes to the time lag of breakdown and the electric strength is unaffected. If, however, the dielectric is very thin it may be necessary to increase the field strength beyond the value for thick specimens in order that the electrons may be more rapidly accelerated and gain an energy greater than J before they are lost to the electrode. Fröhlich[73] has studied the energy distribution among electrons in a dielectric with some precision, but the order of the effect of thickness can be indicated by a simple calculation[74] which does not involve the detailed properties of this distribution. Admittedly Fröhlich's basic theory presupposes the chance appearance of fast electrons, but we are here concerned with the variation with thickness and not absolute values of electric strength.

It may be assumed that the average electron velocity is small compared with $\sqrt{(2J/\mathbf{m})}$ which is the velocity v required for ionization. Next assume that the electrons may be divided into two groups, those which make collisions and for which the chance of reaching the energy J is low and those which do not make any collisions and gain energy at a high rate. The average rate will be the rate for electrons which make no collisions multiplied by the ratio of their numbers to the total. Since the average time required to reach J is the inverse of the rate, since the ratio of the numbers is the probability W that the electron does not collide, since also the time required by a non-colliding electron to gain J by acceleration in a field F is $\mathbf{m}v/\mathbf{e}F$, then the average time required for the production of a number of fast electrons should probably not exceed

$$\mathbf{m}v/\mathbf{e}FW = (\mathbf{m}v/\mathbf{e}F)e^{\mathbf{m}v/\mathbf{e}F\tau}, \tag{2.53}$$

since

$$W \simeq e^{-\mathbf{m}v/\mathbf{e}F\tau},$$

where τ is the mean relaxation time of the electrons as in section (2.4).

Now the average time spent by the electrons in a dielectric of thickness L is determined by the average drift velocity, which depends on the mean relaxation time of electrons which have the average energy. Strictly speaking, this differs from τ, which refers to fast electrons, but Fröhlich and Mott have shown that fast and slow electrons have relaxation times of the same order. Thus, assuming equality of relaxation times, the average time spent in the dielectric is

$$\mathbf{m}L/\mathbf{e}F\tau, \quad \text{if} \quad L \gg \text{mean free path.} \tag{2.54}$$

Equating (2.53), the time required, to (2.54), the time available, gives

$$L = v\tau e^{\mathbf{m}v/\mathbf{e}F\tau}. \tag{2.55}$$

If F_0 is the electric strength for thick specimens, then when

$$L < v\tau e^{\mathbf{m}v/\mathbf{e}F_0\tau}$$

the electric strength increases with decrease of thickness according to

$$\frac{F}{F_0} = \frac{\mathbf{m}v}{\mathbf{e}F_0\tau}\frac{1}{\log(L/\tau v)}. \tag{2.56}$$

From equations (2.30) and (2.31), assuming one infra-red frequency, τ is given by

$$\frac{1}{\tau} = \frac{1}{\tau_0}\left(1+\frac{2}{e^{\mathbf{h}\nu/\mathbf{k}T}-1}\right),$$

$$\frac{1}{\tau_0} = \frac{\pi}{8\times 2^{\frac{1}{2}}}\frac{\mathbf{e}^4\mathbf{h}}{\mathbf{m}^{\frac{1}{2}}Ma^5\nu J^{\frac{1}{2}}},$$

and this should be true both of the low- and high-temperature regions to the order of approximation here considered.

Austen and Whitehead[20] studied the effect of thickness on thin specimens of mica obtained by splitting and then tearing; the thickness was measured optically. Fig. 23 shows their results. If a single residual infra-red frequency is chosen, as in section (2.6) Fig. 16, so as to give the best agreement with the observed temperature variation of the electric strength of muscovite and if the same values of the other physical quantities

are assumed as were used by Fröhlich[49] to find a lower limit for F_0 as in section (2.5), then $v\tau = 0{\cdot}5\times10^{-6}$ cm. and

$$\frac{F}{F_0} \simeq \frac{5{\cdot}4}{\log(L/0{\cdot}5\times10^{-6})}. \tag{2.57}$$

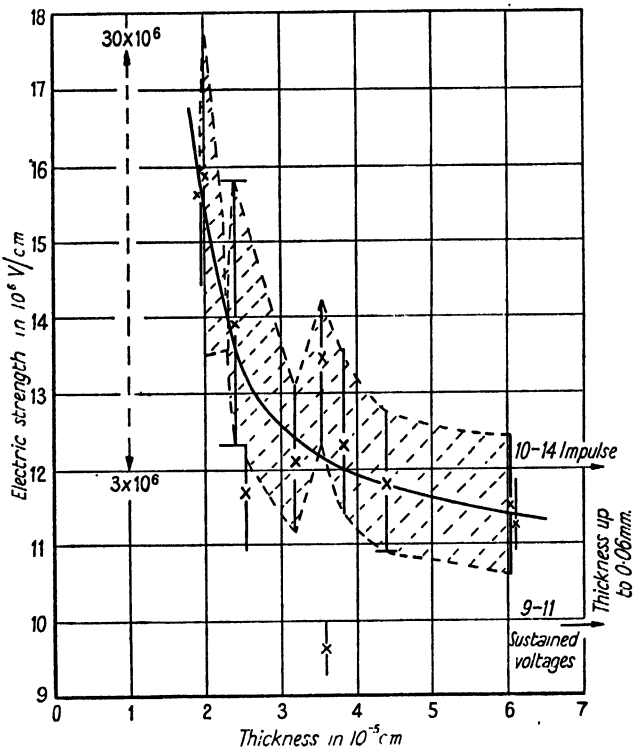


FIG. 23. Electric strength of clear ruby muscovite mica.
× indicates mean value.
Vertical lines indicate range of dispersion.
(Auston and Whitehead.)

For $L = 5\times10^{-5}$ cm., F/F_0 should be about 1·2, while for a thickness of 2×10^{-5} cm., F/F_0 should be about 1·5. Experimentally F/F_0 was 1·6 for $L = 2\times10^{-5}$ cm. The agreement as to order is correct, and is all that can be expected.

Recently Plessner[75] has measured thin evaporated films of various materials. CaF_2, NaF, KBr showed an increase of

electric strength as in the following table of Plessner's experimental results which are here compared with calculations based on Fröhlich's theory.

Material	*Thickness*	10-*sec. pulses*	10^{-4}-*sec. pulses*	*Theoretical ratio*
CaF_2	6×10^{-5} cm.	$2{\cdot}8 \times 10^6$ V/cm.	$4{\cdot}6 \times 10^6$ V/cm.	1·9
	10^{-5} ,,	2·8 ,,	6·4 ,,	
NaF	$4{\cdot}4 \times 10^{-5}$,,	2·6 ,,	5·7 ,,	1·7
	10^{-5} ,,	3·2 ,,	7·1 ,,	
KBr	10^{-4} ,,	0·75 ,,	2·1 ,,	2·6
	10^{-5} ,,	1·6 ,,	4·8 ,,	

The discrepancy between theory and experiment according to this calculation is now of substantial magnitude. With silica and polystyrene no significant effect of thickness was found, which was ascribed by Plessner to the shorter mean free path in amorphous substances. Whatever may be the merit of this argument, an effect is to be expected at 10^{-5} cm. on the theory given unless the mean free path has an inordinately low value. The variation with pulse duration either introduces a new phenomenon or renders suspect the results, particularly those with 10-sec. pulses. If the results with 10^{-4}-sec. pulses be accepted a qualitative confirmation of the theory may be deduced.

There are a number of results from von Hippel and others which also yield qualitative confirmation of the hypothesis. A rather indirect confirmation arises from experiments on the effect of impurities introduced as discrete particles which, though small, are not atoms or ionic groups which can enter into solid solution. For example, Austen and Whitehead[20] found that metallic gold introduced as a colloidal solution into glass did not affect the electric strength. It was improbable that the linear dimensions could exceed 2×10^{-5} cm. and if assumed to be spherical the maximum field would not be increased by more than 3 times due to flux concentration. At a distance of 10^{-5} cm. from the surface the field would be only 1·25 times the mean field. An increase of field of this magnitude over so short a distance would be insufficient to affect breakdown on the hypothesis

just discussed. In the experiments of Gutin and Sackheim the introduction of graphite, which forms much larger particles, caused a fall of electric strength in colophonium. It may be rather speculative to adduce also the results of Schulze[76] on the discontinuous currents in sulphur and solid argon in very thin films stated to be 2×10^{-7} cm. thick. Even if the fields of 10^9 V/cm. claimed to have been applied were actually achieved, it is fairly clear that the films were not truly insulators. Nevertheless, it seems probable that high fields could be applied and the association of the currents with the ionization potential is significant on the hypothesis considered.

A good deal of work has been done on very thin oxide films but the extremely variable results and the electrolytic nature of thin films renders it difficult to bring such results within the ambit of the present discussion as, for example, in the experiments of Guntherschulze and Betz[77] on thin films of oxides of Al, Bi, Nb, Ta, Ti, and Ce. Fields of $9{\cdot}5$–11×10^6 V/cm. were applied. Lomer's[107] electric strength values for aluminium oxide regularly increase from 10^{-5} to 10^{-6} cm. The work of Betz[78] on films nominally of Ta_2O_5 or Al_2O_3 over a range claimed from 3–60×10^{-6} cm., but uncertain actually below 10^{-5} cm., indicated in general that films of different effective thicknesses could not be compared, but occasionally a definite increase of electric strength with decrease of thickness was found.

Hackett and Thomas[13] found increases of electric strength of phlogopite micas with decrease of thickness at thicknesses much greater than are relevant to Fröhlich's theory. There are, in fact, a number of results, which cannot be explained with entire satisfaction. If the mean free path of the electrons is of the order of, say, 10^{-6} cm. interesting, indeed surprising, results might be expected with layers much thicker than some monomolecular layers which have been prepared. The difficulty must then be faced of defining the surface of separation between dielectric and electrode. At high fields electrons will escape from a metal surface so that the dielectric may be exposed to an electron cloud. There are serious experimental difficulties in replacing metal electrodes by artificial electron clouds but Krassin[25] claims

to have measured dielectric breakdown to an electron cloud and to have observed an increase of electric strength compared with conventional electrodes.

On the available experimental evidence it is only possible to say that Fröhlich's theory appears satisfactory in those instances where there is independent evidence that his assumptions are reasonably applicable, but quantitative agreement is not close. The results which contradict Fröhlich's theory are, in general, not very well authenticated.

As a matter of theoretical interest it may be noted that equation (2.56) leads to a minimum breakdown voltage V

$$V = e\mathbf{m}v^2/\mathbf{e} = 2e.J \quad \text{when} \quad L = e\tau v, \tag{2.58}$$

where $e = 2{\cdot}7183$. This means that the minimum breakdown voltage is 5·4 times the ionization potential or X-ray limit. In the case of mica already quoted V would thus be of the order of 50 volts for a thickness of $1{\cdot}4 \times 10^{-6}$ cm. This is outside the range of valid experiment, nor can too much theoretical significance be attached. It is interesting to observe that the minimum breakdown voltage for solids is less than for gases. This is to be expected because the ionization potential is less for solids than for gases, and with the latter a second factor is required to cause instability, apart from the provision of electrons beyond a critical energy which is the criterion in Fröhlich's theory for solids.

2.8. Time effect and impulse breakdown

A number of workers agree that in conditions where intrinsic breakdown may be expected the electric strength is unaffected by time of application of the electric field down to periods of the order of a microsecond. This ignores changes associated with much longer periods, e.g. seconds or more, which will be mentioned in later chapters. Many experiments have been made with standard 1/5 and 1/50 impulses and chopped waves by Davis, Standring,[12,62,71] and their colleagues using specimens and conditions where external discharges were minimized. A number of their results have already been quoted. Some are not directly comparable with sustained voltage tests because thick specimens

were used with the possibility, sometimes actual evidence, of macroscopic defects in the manufacture of the specimen. Where the experiments were comparable, similar and sometimes higher values of electric strength were found. A comparison of similar mica sheets may be quoted between tests with a 1/5 standard impulse and tests with d.c. (see Austen and Whitehead[20]).

Muscovite (Ruby) Mica

Type	*Thickness*	*Electric strength d.c. in 10^6 V/cm.*		*Ratio impulse to d.c.*	
	mm.	*mean*	*max.*	*mean*	*max.*
Clear	0·02	10·15	10·7	1·28	1·26
	0·04	10·3	10·8	1·16	1·25
	0·06	9·75	10·0	1·08	1·50
Stained	0·02	11·6	12·0	0·99	1·17
	0·04	10·5	11·5	1·05	1·17
	0·06	10·6	11·5	0·80	1·00

If the differences are regarded as significant—they are barely so—then they may be ascribed to the rather greater ease with impulse voltages in avoiding electrode effects or the effect of imperfections in the specimens. Confirmation that the electric strength at a microsecond should be the same as at longer times is provided in fairly recent times by Borel,[79] Buss,[80] Draeger,[81] Inge and Walther,[82] and Lehmhaus.[29] It must be admitted that increases of, say, 10–20 per cent. from 100 μs. to 1 μs. are still observed on occasions without a definitely established explanation. Variations of this order in intrinsic electric strength are only just significant.

The results of Plessner,[75] already mentioned in the preceding section, indicate a decrease of electric strength with increase of pulse duration in tests with repeated pulses. The influence of pulse duration increased with decrease of thickness, but not consistently; silica and polystyrene which showed no significant effect of thickness showed no duration effect. Keller[102] suggests that there is an abnormal increase in the electric strength of glass for single impulses lasting less than a millisecond. Further confirmation must be awaited before these results can be assessed.

In considering short times a distinction must be made between breakdown when the voltage is still steeply rising and breakdown when the voltage has reached or passed its crest and is only varying slowly. For example, Lehmhaus states that the electric strength increases with increasing steepness of wave front for breakdown on the front of the wave if the time of stress is less than about 10^{-7} sec. Rather surprisingly, he states that the electric strength for breakdown on the tail of the impulse decreases as the steepness of the front increases. A number of other authors quote an increase of electric strength with breakdown on the front of a very steep wave. Borel concludes that there is an increase in electric strength when the total time integral of the applied voltage is less than a certain minimum. More recently, von Hippel and Alger[103] in tests where the rate of application of voltage was varied have found an unusual decrease of electric strength with KBr when the time to breakdown was about 10^{-5} sec. When the voltage was applied more rapidly the electric strength rose, increasing by 50 per cent. when the time to breakdown had fallen to 2×10^{-8} sec.

If there is in fact a finite time lag in solid breakdown, overvoltages will occur with steeply rising waves. Buss quotes figures of $1{\cdot}5 \times 10^{-8}$ sec. and 7×10^{-9} sec. for the collapse of voltage when breakdown begins. There may be a 'statistical' time lag in addition to a 'formative' time lag, but it has been shown by Inge and Walther[53] that the time lag is not affected by the number of free electrons produced by irradiation with X-rays or ultra-violet light. Time lags due to partial breakdown will be considered in the next section. It need only be remarked here that the existence of a formative time lag in a uniform field implies that partial breakdown will occur when the field exceeds the intrinsic electric strength but is of duration less than the time lag. The evidence both as to the magnitude and nature of the time lag is obscure, and all that it seems possible to say is that the total time lag does not exceed the order of 10^{-7} sec. by any great amount and is probably less with the thin specimens usually tested which are not more than 0·1–0·5 mm. thick.

In the theory of electronic breakdown the existence of some

free electrons due to absorption of radiation, thermal agitation, or crystal imperfection is assumed. Increasing their number beyond a certain value should affect neither the electric strength nor the time lag, so that the 'statistical' time lag which is characteristic of gases is less probable. The experiments of Inge and Walther support this. Owing to the ease with which electrons in a solid are trapped in imperfection energy levels, it should not be easy to remove free electrons or electrons trapped in deep potential wells by the application of an electric field. Nor is it clear that 'cleaning-up' of conduction electrons is possible as in gases. Thus pre-stressing should have little effect, and although Lehmhaus quotes an increase in electric strength of 6 per cent. by pre-stressing, the magnitude is insignificant. In a crystal one might suppose that the minimum time lag is that required to accelerate an electron, which makes no collisions with the lattice, to a velocity corresponding to the ionizing energy. If the latter is, say, 10 eV, the electronic velocity required is $1{\cdot}88\times10^{8}$ cm./sec. The time required to reach this velocity in a field F V/cm. is $1{\cdot}07\times10^{-7}/F$. Thus if F is of the order of 10^{7} V/cm., the time lag t_0 so calculated is about 10^{-14} sec. The quantity t_0 refers only to an electron with a free path not less than about 10^{-6} cm. According to Fröhlich's theory of the effect of thickness, the time interval during which the average electron is expected to have at least one suitable free path is $t_0\, e^{t_0/\tau}$, where τ is the relaxation time of the electrons. Now τ lies between 10^{-13} and 10^{-15} sec. so the time lag on this basis may be as much as $e^{10}\times10^{-14}$, i.e. 2×10^{-10} sec.

In imperfect crystals and amorphous bodies energy gained by the conduction electrons from the field is shared with electrons trapped at imperfections which can lose energy to the solid structure by interaction. It may be expected, therefore, that production of electrons with high energies will be slower. Simpson[83] has developed a theory based upon the temperature rise of the electrons above that of the solid. Below the breakdown field thermal stability is possible, but at or above the breakdown field the conditions are unstable and the electron temperature will rise indefinitely. The rapidity with which this

occurs depends upon the effective specific heat of the electrons. Simpson points out that the specific heat of the conduction electrons is small compared with that of the trapped electrons. If the total energy U is that required to raise the trapped electrons to the excited levels

$$U = (2V-\Delta V)C_2(T)e^{-V/\mathbf{k}T+\Delta V/\mathbf{k}T}. \tag{2.59}$$

The specific heat S is given by dU/dT so that

$$\mathbf{k}T^2S \simeq (2V-\Delta V)(V-\Delta V)C_2(T)e^{-V/\mathbf{k}T+\Delta V/\mathbf{k}T}. \tag{2.60}$$

As in section (2.4), $2V$ is the potential barrier between the ground state of trapped and conduction electrons; ΔV is the average trapping energy of excited states; T is the 'electron' temperature; $C_2(T)e^{-V/\mathbf{k}T+\Delta V/\mathbf{k}T}$ is the number of trapped electrons where $C_2(T)$ varies rather slowly with temperature. If A is the energy per second gained from the field and B is that lost to the solid structure

$$A-B = S\,(dT/dt). \tag{2.61}$$

Both A and B can be found from the equations of section (2.4) so that the variation of electron temperature with time can be determined. By analogy with the similar theory of thermal breakdown, Chapter III, the time to breakdown can be considered as the time for the electron temperature to become infinite, i.e.

$$t = \int_{T_0}^{\infty} S\,dT/(A-B), \tag{2.62}$$

where T_0 is the temperature of the solid assumed constant. When the field is exactly equal to the breakdown field (ratio β is equal to unity), t is infinite but becomes finite with a small over-voltage. Simpson deduces that

$$t = \mathbf{k}I/G\mathbf{h}\nu, \tag{2.63}$$

where

$$G = \mathbf{k}\mathbf{h}\nu/\{\tau_2(2V-\Delta V)(V-\Delta V)(e^{\mathbf{h}\nu/\mathbf{k}T_0}-1)\},$$

$$I = (e/\beta^2)\int_0^{\infty} dx/\{e^x-(ex/\beta^2)\},$$

τ_2 = relaxation time of trapped electrons given in section (2.4).

A calculation has been made for soda-lime glass with $\beta = 1{\cdot}005$ or 0·5 per cent. over-voltage, $T_0' = 300°$ K., $\Delta V/\mathbf{k} = 3{,}400°$, $V = 0{\cdot}5$ eV, breakdown field $= 10^7$ V/cm., $\tau_1 =$ relaxation time of free electrons $= 10^{-13}$–10^{-15} sec.; the other quantities necessary are given in section (2.4). The result is that the time for breakdown should lie between 5×10^{-10} and 5×10^{-12} sec. Comparing this with the previous calculation for crystals, the upper limit does not differ greatly, but for most values of τ_1 the amorphous time lag here calculated will be something like an order of magnitude greater. It is in fact of the order of τ_2.†

These very short theoretical time lags do not contain any statistical time lag associated with the emergence of suitable free electrons in the appropriate part of the field. This may not matter, because the very short relaxation times of electrons in solids, the lower ionization, binding and trapping energies, and the greater density as compared with gases, all favour the transitory occurrence of free electrons. More important is the fact that these theoretical time lags refer to the beginning of an electron avalanche and not to a stage where it is necessarily observable. An observable effect in 10^{-10}–10^{-12} sec. in a specimen 0·1 mm. thick presumes a rapidity of propagation of 10^8–10^{10} cm./sec. It is known that the velocity of propagation of a gaseous discharge is greater than that of the electrons in the avalanche and a similar mechanism can apply to solids, but the speeds are inconveniently greater. The theories only determine the time required for the appearance of a number of fast electrons with high energies and exclude the time required for the propagation of the electron avalanche, so that the thickness naturally does not enter. It would be very difficult to observe the theoretical process because the times are so short and the electric effects would be akin to a polarization wave and therefore difficult to distinguish from displacement, polarization, and absorption associated with the sudden application of a high field. The average velocity of the free electrons is $\mathbf{e}F\tau/2\mathbf{m}$ and of the order of $0{\cdot}9 \times 10^{15} F\tau$ (F in V/cm.) so that if F is 10^7 V/cm. and

† Fröhlich and O'Dwyer[106] have shown that the intervention of electrons trapped in deep wells would be much slower.

τ is 10^{-13}–10^{-15} sec. this velocity is from 10^{7}–10^{9} cm./sec. For 0·1 mm. this yields a time interval of 10^{-9}–10^{-11} sec. Remembering the probability factor for long free paths it may be considered that 10^{-9} to 10^{-10} sec. is more likely. This approaches closer to the observed figures, for example 10^{-8} sec. for 1 mm. thickness is not inconsistent with experiment.

It is unwise to pursue the matter further, except to say that the experimental evidence is inadequate. Rather specialized techniques will be necessary to explore the problem. The first stage of the theoretical breakdown process is probably impossible to observe electrically and the later stages of the avalanche are also very difficult on account of the large capacitance currents which may involve a discrimination of some orders of magnitude greater than 10^{6} to 1, e.g. detection of a discharge current of 1 μA in the presence of a capacitance current of 100 A. In addition, absorption effects which are time dependent will be extremely difficult to eliminate. Capacitance or displacement currents can be discriminated from electronic currents because the former depend on field change and the latter on field magnitude, but this method has only a limited application to absorption currents. Indirect observation by optical or other means with rapid pulses may be more fruitful. For completeness it must also be remarked that surface polarization or space-charge, if a real phenomenon, would introduce an over-riding time effect, in fact would lead to frequency dependence which has so far been ignored in the foregoing. This hypothesis, so often raised and so often contradicted, will be considered in the next section.

No mention has so far been made of mechanical effects due to the sudden application of a field. These are known to occur in partial electrical breakdown and where there are gross defects or mechanical damage. Otherwise it is not easy to see how the stresses in an electric field can cause breakdown except by a hypothesis based on cracks or local charges. A rapid rate of mechanical loading brings mechanical failure within the brittle region, but the mechanical strength is no less than in the plastic range. Accordingly, if electric breakdown is no higher with

rapidly applied fields, as shown, there is no reason to suggest that mechanical effects should enter with impulse breakdown. It has been put forward in favour of the mechanical hypothesis by many workers that there is evidence of a minimum energy in impulse breakdown or rather a minimum value of the time integral of the square of the applied voltage. The evidence in favour of this is uncertain, and it could equally apply to a given observable amount of damage—impulse failure is very difficult to detect so that breakdown with slight damage might escape notice—to thermal breakdown, or to breakdown by deterioration caused by discharges or by electro-chemical action.

2.9. Conductivity, polarization, and space charge

From the time of Curie up to the fairly recent work of Joffé and his co-workers, the hypothesis has been put forward that the true conductivity of a dielectric is exhibited either after a very short application of stress, before any space charge or electrode polarization has time to develop, or after a very long time of application when, adopting the converse theory, the polarization has either disappeared or reached an equilibrium. Modern advances in the theory of dielectric properties have demonstrated that the majority of the work mentioned was based on a misunderstanding of absorption currents and the phenomena of dielectric relaxation. It is now recognized that there exist in substantially all dielectrics, relaxation and oscillation mechanisms of a dipolar, ionic, molecular, or electronic nature which give rise to a time-dependent conductivity and a frequency-dependent dielectric constant and loss angle without the production of internal electric fields, except of a local nature.† If only the conductivity is observed, it is easy to express the effect as a polarization, which, nevertheless, has no physical significance and is only a mathematical device. This was demonstrated several years ago by Hartshorn[84] and certain features have recently been elaborated by Whitehead.[85] Thus if a voltage V is suddenly applied to a dielectric specimen with high-frequency capacitance C_0, anomalous relaxation charac-

† See Fröhlich, *Theory of Dielectrics*, Clarendon Press, Oxford, 1949.

teristic $f(t)$, and ultimate leakance g, the current i is

$$i = C_0 V \times \text{unit transient} + C_0 V f(t) + gV. \qquad (2.64)$$

Neglecting the transient, this can be expressed as

$$i = V(g + C_0 f(0))[1 - \{C_0(f(0) - f(t))/(g + C_0 f(0))\}]. \qquad (2.65)$$

The second term could be regarded as a developing polarization potential and the 'true' conductivity supposed to be $g + C_0 f(0)$. Alternatively in (2.64), $C_0 V f(t)/g$ could be imagined as a disappearing but assisting polarization with g as the 'true' conductivity. Similarly, if the specimen is charged for a very long time at V so that the current becomes gV (at $t = 0$) and is then short-circuited for a period t_0, after which one electrode is insulated, the current flowing must be zero, i.e.

$$0 = -VC_0 f(t) + \int_{t_0}^{t} \{g + C_0 f(t-u)\}(dv/du)\, du. \qquad (2.66)$$

Here v is the potential which appears on the insulated electrode and has the appearance of a polarization potential analogous to the kind observed in electrolytes. It is fairly safe to conclude that all linear superposable variations can be ascribed to the known relaxation and oscillation mechanisms so that in breakdown attention can be confined to non-linear phenomena. This also means that the majority of the attempts to associate purported effects of frequency on breakdown in alternating fields with polarization phenomena may also be discounted, and a closer examination of the results generally reveals evidence of discharges, electrode effects, thermal instability, or electrochemical deterioration.

The presence of these known relaxation mechanisms and other causes of anomalous currents renders the interpretation of conductivity measurements a matter of some difficulty. Suppose a relatively low field be applied to a dielectric with a normal dielectric constant of about 3, then at a time when the anomalous current corresponds to a change of equivalent dielectric constant, polarization, or absorption of 0·1 per cent. per sec., this anomalous current is as great as the conduction current due to a resistivity of the order of 4×10^{15} ohm-cm.

If the total anomalous charge involved were 1 per cent. of the normal dielectric displacement and were associated with a relaxation time of 4 secs., then up to 4 secs. the anomalous current would indicate a spurious resistivity less than that quoted. Even if the relaxation time were only about 10^{-4} sec. then, for a millisecond after the application of the voltage, the spurious resistivity would still be less than 4×10^{15} ohm-cm. Most solid dielectrics have at least two groups of relaxation times, one in the region of 10^{-4}–10^{-6} sec. and one in the region greater than 1 second. Usually there are a number of intermediate relaxation times. The net effect, therefore, is to limit consideration not only to non-linear phenomena, since the normal relaxation mechanisms are linear, but also to currents which correspond to a resistivity very substantially less than the low-voltage resistivity of the dielectric in question.

A good deal of work has been carried out in the U.S.S.R. on the polarization hypothesis. The very interesting experiments on various kinds of dielectric polarization are not here relevant because they deal really with dielectric displacement in the Maxwellian sense which only enters indirectly into breakdown. Similarly, Nasledow and Scharawsky,[86] Tutschewitz,[87] Courtenser and Ignatowitsch[88] decided that the polarization voltages in ceresin, paraffin, and ebonite respectively were proportional to the applied field. For the reasons already stated these are therefore explicable by known dielectric mechanisms which do not actually involve any polarization potential. The same is true of the results of Joffe[38] and his collaborators at low field strengths where this proportionality exists so that no definite evidence of polarization is afforded. Some misunderstanding may have arisen because it was indeed correct of Inge (see Chap. III) to use in thermal breakdown the final conduction $(V-P)/R$, where V is the applied potential, P the polarization potential, and R the 'true' resistance, because the earlier larger currents were mainly anomalous displacement currents which did not contribute any energy dissipation and therefore did not affect the thermal breakdown in which she was interested. The reality of P and R did not enter into the question. Of more

importance are the connexions proposed between the increase of conductivity and the variation of polarization potential with applied field (see also Nakanisi[89]). A closer examination of some of the experimental results suggests some doubt. For example Sinjelnikov and Walther[90] quote a polarization potential for mica which first increased linearly with field strength, as already discussed, then reached a maximum of about 4 kV. at $5–6 \times 10^6$ V/cm. and then fell. In the region of the maximum the conduction current rose rapidly and apparently breakdown occurred at less than 10^7 V/cm. Considering the thickness of the specimen, probably of the order of 0·001 cm., it is difficult to see how the polarization potential could be distinguished as a separate phenomenon from the non-linear increase of conduction current which naturally can always be explained in terms of a variation of polarization potential if the 'true' conductivity is chosen to suit the experimental results. It has already been shown that substantially higher fields have been applied to mica without the changes in conductivity given by Sinjelnikov and Walther.

It is also clear that polarization potentials of the magnitude described can only easily arise due to space charges. Even so, an extremely high concentration of charge would be necessary seriously to modify the high fields of intrinsic breakdown. It must be admitted that these fields are capable of drawing electrons from the electrodes. This has been demonstrated in studies of the electric strength with electrodes separated by a vacuum, and there seems no reason why the electrons should not readily pass into the dielectric and become trapped there or induce ionic defects or excess by processes already discussed. Guntherschulze[77] and his co-workers claim to have established the existence of considerable concentrations of free or feebly bound electrons in thin oxide films which have been exposed to a strong field and subsequently discharged. Furthermore, they state that electrons drawn from the cathode can penetrate these films, which are no large number of molecules thick, and issue with energies of the order of 20–30 eV. This is not in accord with Fröhlich's theories, since according to him an electron with an energy in excess of the ionization potential of the lattice

would be very rare and very transitory, provided the thickness of the dielectric exceeded the mean free path, about 10^{-6} cm., as was the case in the experiments mentioned. The evidence, therefore, in support of the polarization potential hypothesis is generally either consistent with other known mechanisms or, in the form stated by its authors, introduces theoretical incompatibilities as regards an influence on breakdown. Some direct evidence of space charges has been adduced. von Hippel and Alger[103] have put forward space charges in explanation of some of the effects of temperature, electrode material, and duration of stress mentioned earlier. More evidence is needed to assess the validity of the space charge hypothesis.

A number of mechanisms for the increase of conductivity with field strength have already been explored, but it is useful to summarize them. Electronic conductivity due to the escape of electrons through a potential barrier V eV in a field F has been shown by Zener[41] to give a variation proportional to $Fe^{-\text{const.}/F}$. Frenkel,[48] inserting the effect of the field on the potential barrier, obtains a probability of ionization in unit time given by

$$\nu_0 \exp[-(4\pi/\mathbf{h})\sqrt{\{2\mathbf{m}(V-2\sqrt{\mathbf{e}^3F})\}}(1/\mathbf{e}F)\sqrt{(V^2-4\mathbf{e}^3F)}],$$

where ν_0 can be identified with the frequency of an ionizing radiation, i.e. the X-ray limit. Calculation shows that outside a small range of field strength the current would be either negligible or impossibly great so that it is doubtful whether the phenomenon is observable and certainly does not seem to have been observed although recently a claim has been made to this effect (see ref. 35). Frenkel rejected the mechanism for actual breakdown. Next there is the thermal activation theory of the escape of ions. This has been applied by Frenkel to trapped electrons to give a conductivity proportional to $e^{-\sqrt{(\mathbf{e}^3F)}/\mathbf{k}T}$, but it is rather difficult to see how Frenkel's hypothesis can yield an appreciable current because, owing to the low inertia and relaxation time of electrons, other effects such as electronic mobility would be expected to control the phenomena. Austen's[4] application of the activation hypothesis to ions is intrinsically

more probable and gives a conductivity proportional to

$$\frac{\sinh(\frac{1}{2}eaF/kT)}{\frac{1}{2}eaF/kT}.$$

Guntherschulze[77] suggests alternative forms of variation of ionic current in oxide films which would be compatible either with activation or with the tunnel effect and says that only the former applies to electrons. His idea is that electronic conductivity precedes the ionic conductivity which is due to slipping in a polar crystal of the negative ion lattice over the positive ion lattice. Activation is rather unlikely with electrons and the tunnel effect very unlikely with heavy ions. As regards amorphous solids, it has already been shown that Fröhlich's theory of interaction between trapped electrons leads to a variation of conductivity with field strength according to

$$\exp\{(V/\Delta V)(F^2/eF_m^2)\}$$

(see equation (2.23)), where V is the potential barrier of trapped electrons, ΔV refers to the energy barrier of the average excited trapped electron, and F_m is the intrinsic electric strength.

It has become the habit to refer to the early measurements of Poole[35] on non-linear conduction. He found an exponential variation of conductivity with field strength, but at that early stage it is unlikely that the results referred to the intrinsic properties of the structure. The experimental evidence on pure crystals is rather negative. Joffe[38] and his co-workers and also von Hippel[91] and his colleagues have all published conductivity results showing variations with field strength in support of their various theories. The majority of these results indicate a rapid increase of pre-breakdown current, but unfortunately this does not occur in the same field strength range with different experimenters; the range seems determined by the magnitude of the electric strength observed in the tests which often could not have been the intrinsic electric strength. The observations of Austen, Thomas, Hackett, and Whitehead suggest that it is very difficult to remove the influence of stray leakage and impurities in measuring field-dependent currents in crystals. It is to be noted here that Austen's activation hypothesis is applicable

to crystal defects and not inconsistent with the qualitative characteristics of the general experimental findings. With amorphous solids or defective crystals at higher temperatures, the results of von Hippel and Maurer[56] on soda-lime glass and those of Thomas and Griffith[21] on varnish films seem in some quantitative accord with Fröhlich's theory of amorphous substances. Some support is also afforded by the measurements on soda-lime glass by Turner and Lewis[92] using saw-toothed pulses of 160 microseconds duration. However, tests of the latter kind require stringent proof that their validity is not impaired by the intervention of absorption or other anomalous currents.

There is a body of evidence, as indicated by Hartshorn[93] and by Whitehead and Nethercot,[37] that impurities such as moisture cause the conductivity to increase with field strength. As a first hypothesis the activation theory might be applied to the trapped imperfection ions, the binding energy of which can be estimated from work on diffusion in solids (see Thomas and Gent[94]). There remains the difficulty that to a first approximation the experimental dependence on field seems to be an odd function of field strength whereas, according to the activation theory, an even function should apply. The impurities are often only partially ionized, for example Garton[95] has shown the water molecule in cellulose to be still capable of rotation. Thus both activation of the ions from the structure and activation with respect to the interaction between ions of opposite sign may be involved, yielding an ordinary mixed function in qualitative accord with experiment.

Lastly, mention may be made of some interesting observations on the fluctuating character of currents in dielectrics in strong fields. Hawarth and Bozorth[96] suggest the analogy of the shot effect in electron emission and calculate that the number of elementary charges released to the electrodes in a single fluctuation varies exponentially with field strength from one to about 10^6 at fields just below breakdown. Schulze[76] found discontinuities in the current passing through very thin films of sulphur and solid argon, and states that these discontinuities occur at fields in the same numerical relation as the ionization potentials

of the structures. A number of workers have mentioned qualitatively the unsteady nature of currents observed just prior to breakdown. At present the evidence is not definite enough for these fluctuations and discontinuities to be described as an intrinsic property of the solid independently of the experimental conditions or measuring technique. It is, nevertheless, easy to conceive of discontinuous currents either as partial electron avalanches, analogous to the fluctuating character of the corona discharge in gases, or as arising from the transmission, with more or less multiplication, of electrons drawn from the electrode by the high field; to these the shot effect would naturally apply.

2.10. Partial breakdown

Probably the most important example of partial breakdown as an experimental method was the technique of von Hippel[42] and also of Inge and Walther.[97] von Hippel, Lass, Davisson, and others limited the current so that the destructive effects of the discharge were localized in order to show up the discharge path. Even in an initially uniform field, distortion by the arrested breakdown process would produce a divergent field, but the applied field was usually divergent, so that these techniques apply in general to breakdown in a non-uniform field. In thicker specimens discharge paths appeared which failed to bridge the electrodes, branching could be observed and the propagation of the discharge could be considered as slowed, although the initial current instability associated with a path might still be as rapid. The limitation of the current was achieved either by the use of feebly conducting electrodes such as slate and ivory (cf. the stone electrodes used by Wagner and others much earlier) or by the internal resistance of the potential generator or by both together. In Davisson's experiments[42] small electrodes were used to obtain a radial field. Inge and Walther used pointed electrodes and limited the energy available for the discharge by impulsive voltages where again a succession of partial breakdowns is possible. In addition to these methods, some electrodes, such as a sphere pressed against a plane surface, yield surface discharges. For present purposes such experiments can

only be regarded as providing confirmatory evidence about the orientation of discharge paths. Apart from this Lichtenberg figure technique, care was usually taken to prevent discharges in the medium by one or other of the methods given in section (2.1). The interpretation of results always needs care because it is very difficult to avoid ambient discharges with pointed electrodes. The field is greatest in the neighbourhood of the contact with the dielectric, the place where, to avoid discharges, the stress should be alleviated as far as possible.

The general features of the orientation of discharge paths have been given in section (2.3) and, as stated by von Hippel, they confirm in a general manner the hypothesis of electronic breakdown. More recently Davisson[42] made a detailed study of the preferential orientation of breakdown paths usually in a radial field. This also applies when a discharge path changes direction in passing from one crystallite to another or at a characteristic plane within a crystal. A number of partial breakdown paths could be obtained on a single specimen. In the theory of electronic breakdown, the preferred direction would appear to be that in which the loss of energy to the lattice from a conduction electron is least in relation to the gain of energy from the field. According to von Hippel this direction is a compromise between greater distance between lattice-points and less height of potential barriers. In Fröhlich's theory, considering equation (2.39), the determining factor appears to be $(\epsilon-\epsilon_0)\nu^{\frac{1}{2}}/a$, where $\epsilon-\epsilon_0$ is the dielectric constant associated with the main longitudinal polarization wave of frequency ν and a is the spacing of the ionic planes in the crystal. The oscillation modes of lower frequency are important, provided that the Debye temperature of all the modes is above the range of temperatures considered. These considerations seem to lead to a preference in ionic cubic crystals for the 110 or 100 direction. Davisson thought that the preferred direction should be random near absolute zero when the natural modes of oscillation have little influence, should have the preferred direction 100 at higher temperatures, changing to 111, then to 110, and finally any direction xxy in a bisector plane. In calculating the electric strength on Fröhlich's theory, it has not

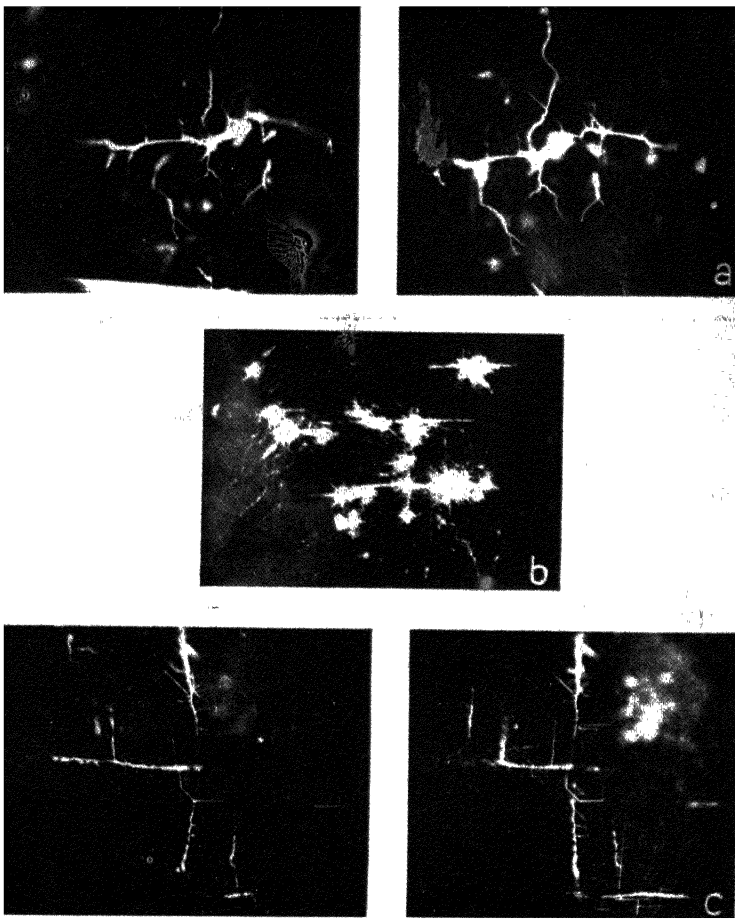


FIG. 24. Breakdown paths by technique of partial breakdown. (Davisson.)

(*a*) (100) and random directions in NaCl at $-180°$ C. Field in (110) direction.
(*b*) (100) path directions in NaCl at $-180°$ C. Field in (100) direction.
(*c*) (110) path directions in NaCl at 20° C. Field in (100) direction.

PLATE II

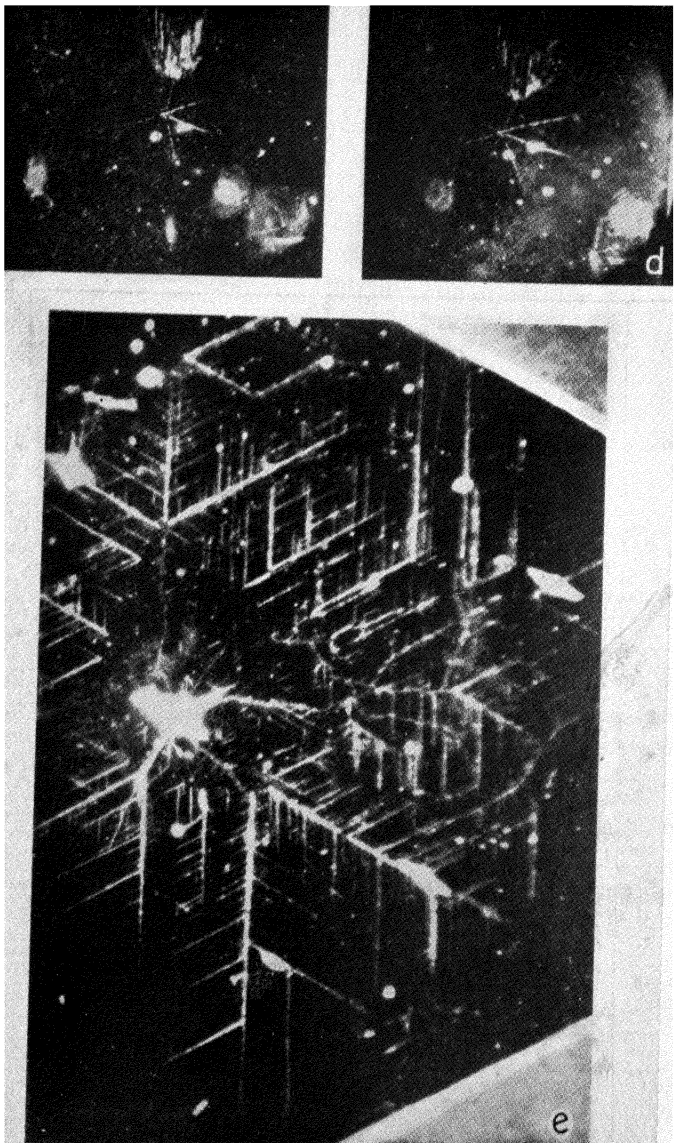


FIG. 24 (*continued*).

(*d*) Star formation in LiF at 150° C.

(*e*) (110) path directions on surface of fluorite. Field in (111) direction.

been found possible to allow for more than one, or at most two, natural frequencies: thus a random direction near absolute zero changing to 100 and 110 can be understood, but the final change to a random direction in a plane applies to temperatures where electronic breakdown has not definitely been observed. The change 100, 111, 110 is difficult to substantiate. A study of Davisson's results on alkali halides suggests that experiment supports the change—random–100–110—to a greater extent than the more elaborate schedule. Davisson goes into details about the effect of substituting analogous atoms and of impurities, the pursuit of which space does not here permit. It may be remarked that if the residual rays are of high frequency, the effect of temperature should not be so marked. This was found with the cubic crystal sphalerite with 110 breakdown paths. Tetragonal crystals seemed to give 001 breakdown paths, the hexagonal crystals break down along the axes, the trigonal crystals calcite and quartz gave 54° and 42° respectively to the C-axis while orthorhombic sulphur gave 100 paths. More complicated results seemed to be obtained with baryta and anhydrite. Beautiful photographic records have been secured in work of this kind and Fig. 24 (Plates I and II) illustrates patterns obtained by Davisson.

In Fröhlich's theory the condition for breakdown is critical, and when the field strength falls below the critical value the probability of the existence of fast electrons diminishes very rapidly. Accordingly, if an electron avalanche has begun in a region of high field strength, additional reasons must be sought for its propagation into regions of low field strength. In very early work the term 'electrostatic cutting edge' was used and the problem of maintaining the field concentration at a partial breakdown still deserves study. The amount of energy gained from the field by the conduction electrons is very great, yet from preceding sections it is clear that the production of fast electrons is a much more rapid process than the heating of the solid, so that any thermal ionization will lag behind the electronic discharge. We can, however, envisage the propagation of thermal instability, discharges in macroscopic defects left by

the electronic discharge and mechanical action enhanced by the heated discharge path. A concentration of electrons in a discharge path will tend to accelerate free electrons in the neighbourhood. The absence of an effect from increasing artificially the number of free electrons argues against the importance of this electron concentration. It is necessary that the discharge path be conducting and the concentration of field at its tip sufficient to yield a local field exceeding the intrinsic electric strength over a distance of at least about 10^{-4} cm. Self-propagation of a partial breakdown after the passage of a transient voltage is unlikely, except possibly by thermal or mechanical means in particular cases.

A good deal of experimental evidence with divergent fields has been affected by discharges in the ambient medium. This applies to observations with transient voltages by Draeger[81] on ceramics, Inge and Walther on glass and rock salt,[82] Shinohara[98] on various materials. In all these instances there was evidence of external discharges, and in the present state of knowledge it is unwise to draw conclusions therefrom as to intrinsic breakdown. Consideration of this work will be deferred to a later chapter. The remaining experimental evidence as to the mechanism of discharge is incomplete. For example, does the electron avalanche necessarily involve complete breakdown, or must it attain a certain current, or must a return stroke pass through the channel of presumably ionized atoms formed by the avalanche? It does not seem that electric strength tests have been made with an electrode capacitance much less than 1 $\mu\mu$F for an applied voltage of 1,000 V. If, now, the longest breakdown time quoted is taken as 10^{-7} sec., a current of at least 10 mA is possible without drawing current from the source of potential. Seeing that this is a lower limit, and that values could have been assumed leading to currents several orders of magnitude greater, it appears that currents are available of the same magnitude as those which yield a low-resistance or arc-like gaseous discharge. von Hippel states that in his 'micro-method' the external current could be limited to micro-amperes. In conjunction with the work of his colleagues and others on partial breakdown with

very high resistance electrodes, this might be taken to mean that breakdown requires something more than the transient current of an electron avalanche, which could be supplied by discharge of the local capacitance, but that any 'follow-up' current need only be small. The idea of a return stroke would seem to derive support from von Hippel's observation that the discharge channels start at the anode and branch towards the cathode. However, Walther and Inge[99] claim that both electron clouds and tree-like discharge channels, 'dendrites', originate from the negative electrode. On this von Hippel[10] has observed that the use of an asymmetrical divergent field discounts any polarity effect, but by analogy with gaseous discharge theory this objection is only valid if the field is so divergent that a partial discharge (cf. 'corona') precedes complete breakdown (cf. 'spark'). According to Walther and Inge, dendrites, at least at low temperatures, are the source of the electron clouds and the channels are formed by heating due to the heavy current density in the stem of the 'tree'. It should be mentioned that Walther and Inge used impulses with time constants of the order of 10^{-2}–10^{-4} sec., compared with which the time required for the development of an electron avalanche is negligible. The rather scanty evidence of the discontinuous nature of pre-breakdown currents (cf. section (2.9)) suggests that electron avalanches of a restricted character can occur without permanent impairment of the dielectric properties. Similarly, the energy available from that stored in the specimen seems ample for an electron avalanche without drawing a further supply from the source during the discharge but, as was previously mentioned, the specific heat of the solid structure is much greater than that of electrons so that the energy required to damage it would be of a higher order of magnitude.

It may tentatively be suggested that the electron avalanche is a very transient phenomenon, which requires comparatively little energy and little total charge, although the instantaneous current may be fairly high. Behind it is left a channel containing a number of ionized atoms or lattice points. The energy levels in any such channel will be seriously blurred so that conduction

may well be possible as explained earlier. Conduction along the channel may cause permanent damage, perhaps by a process of ionic detachment analogous to melting, with the expenditure of a much greater amount of energy than for the electron avalanche and at a much slower rate. The channel will remain in its ionized condition for a moderate period because the diffusion of electrons thereto will be a slower process than the development of an electron avalanche from fast electrons accelerated by the field. Partial breakdown may be due to the stifling of an avalanche. This can arise from a drop in field strength in a non-uniform field. It might be due to an increase in effective electric strength in an unfavourably oriented crystallite so that the easiest breakdown path is inclined to the field, the resolved component of which in the breakdown direction is less. A similar increase in effective electric strength could arise from the unfavourable orientation of chain molecules, or to an amorphous or disordered region. Partial breakdown may also occur because the potential conductivity of the channel is not fully developed so that it becomes a thread of amorphous dielectric or even restores itself. This may be due to limitation of the energy or the current derivable from the source of potential, to limitation of the duration of the applied field, to the configuration of the field, e.g. to the distorting effect of the space charge left behind by the avalanche. It is still an open question as to whether there exists a critical intensity of electron avalanche and the experimental evidence does not justify speculation.

Somewhat associated with partial breakdown are the effects of mechanical stress and macroscopic defects. In an early paper Weicker[100] reported a sudden decrease of the electric strength of porcelain when the elastic limit was approached, but it was not established that he secured intrinsic breakdown. Saito[15] found an increase of electric strength of rock salt with compression up to a certain point, above which the electric strength fell, together with a decrease of electric strength under tension. Akahira and Gemant[101] studied the effect of mechanical stress on glass, mica, rubber, cellulose nitrate, and cellulose (cellophane). In agreement with Saito, they found an increase of electric strength up

to a certain point with compression and a decrease with tension both for uniform and ball-indentation mechanical stresses; bending was found to have no effect. An exception was rubber which showed an increase of electric strength with a uniform tensile stress. According to Fröhlich, the electric strength is inversely proportional to the lattice constant to a first approximation so that the electric strength should rise with compression and fall with tension, but the magnitude of the changes reported in the preceding experiments seems too great to be accounted for quantitatively. Mechanical stress may be expected to favour intermediate energy levels which may increase or decrease the electric strength according to the way in which the critical temperature between crystalline and amorphous breakdown is altered in relation to the temperature of test. It would rather be expected that both tension and compression would favour the development into complete breakdown of the cracks and channels caused by partial breakdown, but tension would be more damaging. Thus if similar defects were always present an apparent increase of electric strength with compression could be explained. Near the elastic limit in crystals, when the structure is seriously distorted and slipping occurs, a fall in electric strength is to be expected.

Some structural defects have already been considered such as solid solution, microscopic colloidal solution, amorphous regions, displacement and disorientation of crystallites, excess and defect of electrons, dipolar and other scattering centres, the accumulation of defects at high temperatures. Macroscopic defects where discharges can occur in a weaker medium will be considered in a later chapter. It will be appreciated that defects which do not give rise to what are usually known as internal discharges, but which, nevertheless, provide the intrusion of a weaker medium, will reduce the electric strength. Channels where the electronic free path is greater or where the atoms or ions are more readily ionized will also reduce the electric strength. A similar reduction occurs when there is a field concentration at a defect, provided it extends over an adequate volume; distances of the order of 10^{-4} cm. seem to be necessary. Some experiments of Gutin and

Sackheim[70] on colophonium resin may be mentioned in which mechanical defects such as cracks were claimed to be obviated by the addition of transformer oil. The admixture of colloidal graphite or lead borate glass particles caused a reduction of electric strength which can be explained by field distortion, but quartz particles had little effect. From the absence of effect of admixtures in non-uniform fields, the influence of discharges at the electrodes is suspected.

The problem of abnormally low values of electric strength remains unsolved although it is so important in practice. Ideas may be formed from the preceding discussion which are valid in particular cases, but direct experimental evidence is scanty. Most workers with experience in intrinsic breakdown tests will recall the occurrence of low electric strengths in specimens apparently identical, by all the usual methods of physical examination, with the remaining specimens which yield normal results. Investigation is hampered by the fact that the breakdown generally destroys the evidence of abnormality. The methods of partial breakdown have not so far afforded a technique of arrested breakdown which might facilitate study of these special instances. Existing methods of investigation are thus laborious. Nevertheless, some significant facts have emerged. There is the great difference in the experimental scatter in intrinsic breakdown tests on different materials; for example mica shows much more scatter than polythene. There is the great difference found by Austen and Pelzer in the behaviour of paraffin wax according to its orientation and the much more erratic results obtained even with transparent and optically good specimens of paraffins than on the similar long-chain hydrocarbon polymers. The study, so far merely preliminary, of mechanical forces is to be taken into account. The concentration of future effort on this problem is recommended and it is concluded with regret that the more extensive discussion which the subject deserves is at present too speculative to be worthy of inclusion.

REFERENCES QUOTED IN CHAPTER II

1. SEMENOFF and WALTHER. *Die physikalischen Grundlagen der elektrischen Festigkeitslehre* (Springer, 1928), p. 126.
2. COCKROFT. *J.I.E.E.* 1927, **66**, 385.
3. ROGOWSKI. *Arch. für El.* 1927, **18**, 1 '3. *E. u. M.* 1926, **44**, 599.
4. AUSTEN. *J.I.E.E.* 1945, **92**, part i, 3 .
5. GEMANT. *Zs. für technische Physik*, 1931, **12**, 250. *World Power*, July 1935, **24**, 8.
6. THOMAS. *World Power*, A g. 1935, **24**, 87; Sept. 1935, **24**, 125.
7. INGE and WALTHER. *E. . M.* 1934, **52**, 243.
8. MARX. *E.T.Z.* 1929, **50**, 41.
9. THOMAS. *World Power*, Sept. 1935, **24**, 125. E.R.A. Reports L/T 57, 1933; L/T 76, 1937.
10. VON HIPPEL. *Zs. für Physik*, 1931, **67**, 707; **68**, 309; 1932, **75**, 145.
11. GRUNEWALD. *Arch. für El.* 1923, **12**, 79. *E.T.Z.* 1924, 1084.
    —— and MARX. *Mit. der Hermsdorf-Schomberg Isolatoren G.m.b.H.* 1925, p. 672.
12. LACEY and DAVIS. *C.I.G.R.E., Compte Rendu*, 1939, vol. iii, no. 304.
    STANDRING. *J.I.E.E.* 1941, **88**, part ii, p. 360. E.R.A. Report L/T 109, 1941. *Surge Phenomena*, published by E.R.A., 1941, ch. 16.
13. HACKETT and THOMAS. *J.I.E.E.* 1941, **88**, part i, 295.
14. PERLICK. *Diss. Technische Hochschule* (Berlin, 1934).
15. SAITO. *I.E.E. Japan*, 1936, **56**, 1036.
16. This type is nowadays made with evaporated metallic films as in the Whiley process.
17. E.R.A. Report L/S 3, 1935, *Directions for determining the electric strength of dielectrics*. B.S. 234: *Ebonite for Electrical Purposes*.
18. SCHUMANN. *Zs. für Technische Physik*, 1925, **6**, 439.
19. MOON and NORCROSS. *J. Franklin Inst.* 1929, **208**, 705. *Trans. A.I.E.E.* 1930, **49**, 755. *Arch. für El.* 1933, **27**, 827.
20. AUSTEN and WHITEHEAD, S. *Proc. Roy. Soc.* A, 1940, **176**, 33.
21. THOMAS and GRIFFITH. *J.I.E.E.* 1942, **89**, part i, 487.
22. AUSTEN and PELZER. Ibid. 1946, **93**, part i, 525.
23. OAKES. *Proc. I.E.E.* 1948, **95**, part i, 36.
24. MUNDL. *Arch. für El.* 1925, **15**, 320.
25. KRASSIN. *Physikalische Zeitschrift der Sowjetunion*, 1935, **9**, no. 5, 449.
26. WHITEHEAD, S. *Dielectric Phenomena*, vol. iii, *Breakdown of Solid Dielectrics* (Benn, London, 1932), p. 109.
27. HOROWITZ. *Arch. für El.* 1927, **18**, 555.
28. SMEKAL. Ibid. p. 525. *Zs. für Elektro-Chemie*, 1929, **34**, 472.
29. LEHMHAUS. *Arch. für El.* 1938, **32**, 281.

30. Rogowski. *Arch. für El.* 1924, **13**, 155.
31. Whitehead, S. *Dielectric Phenomena*, vol. iii, *Breakdown of Solid Dielectrics* (Benn, London, 1932), p. 284.
32. Ganger. *Arch. für El.* 1938, **32**, 401.
33. Vorobiev. *J. Tech. Physics, U.S.S.R.* 1940, **3**, 73.
34. Böning. *Arch. für El.* 1936, **20**, 88 and 503; 1937, **21**, 25. *Kolloid Zeitschrift*, 1940, **92**, 136; 1941, **94**, 31; 1941, **95**, 181.
35. Poole. *Phil. Mag.* 1921, **42**, 488. *See also* Gorelik and Dmitriev, *Journ. Techn. Fiz. U.S.S.R.* 1948, **18**, 333.
36. Schiller. *Ann. der Physik*, 1927, **83**, 137. *Zs. für technische Physik, Jahrbuch*, 1928. *Arch. für El.* 1927, **17**, 600.
37. Whitehead, S., and Nethercot. *Proc. Phys. Soc.* 1935, **47**, 974.
38. Joffe. *Physics of Crystals*, McGraw-Hill, 1928. Sinjelnikov and Kurtchakov, *Comptes Rendus, Acad. of Science U.S.S.R.* (1926). Kobeko, Kurtchakov, and Sinjelnikov, *Papers of the Physical-Technical Laboratory, Leningrad*, 1927, **5**. Joffe, Sinjelnikov, and Kurtchakov, *Publications of the Massachusetts Inst. of Technology*, May 1927.
39. Eisler. *Zs. für Physik*, 1932, **79**, 266.
40. Fröhlich and Pelzer. E.R.A. Report L/T 184, 1947.
Fröhlich. *Nature*, 1946, **160**, 255.
Fröhlich, Pelzer, and Zienau. *Phil. Mag.* 1950, **41**, 221.
41. Zener. *Proc. Roy. Soc.* A, 1934, **145**, 523.
42. von Hippel. See 10; also *Zs. für Physik*, 1936, **98**, 580. See Lass, ibid. 1931, **69**, 313. Davisson, *Phys. Rev.* 1946, **70**, 685.
43. Seeger and Teller. Ibid. 1939, **54**, 515; 1939, **56**, 352.
44. Fröhlich. *Physical Society, Reports on Progress in Physics*, 1939, **6**, 411.
45. —— *Proc. Roy. Soc.* A, 1947, **188**, 521. E.R.A. Report L/T 153, 1945.
46. Mott and Gurney. *Electronic Processes in Ionic Crystals* (Oxford, Clarendon Press, 1940).
47. Goodman, Lawson, and Schiff. *Phys. Rev.* 1947, **71**, 191.
48. Frenkel. *J. Tech. Physics, U.S.S.R.* 1938, **6**, 685.
49. Fröhlich. See 44, also *Proc. Roy. Soc.* A, 1937, **160**, 230; 1939, **172**, 94. *Phys. Rev.* 1939, **56**, 349.
50. —— *Proc. Roy. Soc.* A, 1947, **188**, 532.
51. von Hippel. See 10, also *Ergebnisse exakt. Naturwissenschaft.* 1935, **14**, 79.
52. —— and Lee. *Phys. Rev.* 1941, **59**, 824. See also von Hippel, *Zs. für Physik*, 1934, **88**, 358.
53. Inge and Walther. *Arch. für El.* 1934, **28**, 729.
54. Goldman and Wul. *Physikalische Zeitschrift der Sowjetunion*, 1935, **8**, 369.
55. Walker. *J. Applied Physics*, 1941, **12**, 215.

56. VON HIPPEL and MAURER. *Phys. Rev.* 1941, **59**, 820.
KELLER. *Physica*, 1948, **14**, 15.
57. BIRD and PELZER. E.R.A. Report L/T 194, 1948. *Proc. I.E.E.* 1949, **96**, part i, 44.
58. WHITEHEAD, S. *Report Brit. Association*, 1936. See *World Power*, 1936, **26**, 72.
59. ALEXANDROW and JOFFE. *Physikalische Zeitschrift der Sowjetunion*, 1932, **2**, 527.
60. FRÖHLICH. E.R.A. Report L/T 104, 1940.
61. WEBER. *Arch. für El.* 1933, **27**, 511.
62. DAVIS. *J.I.E.E.* 1942, **89**, part ii, 52. *Surge Phenomena*, published by E.R.A. 1941, ch. 17.
63. OAKES. *Proc. I.E.E.* 1949, **96**, part i, 37.
64. FRÖHLICH. E.R.A. Report L/T 112, 1940, and *Proc. Roy. Soc.* A, 1941, **178**, 493.
65. LITTLETON and MOREY. *Properties of Glass* (John Wiley and Sons, New York, 1933, and Chapman and Hall, London).
66. MOSCICKI. *E.T.Z.* 1904, **25**, 527.
67. INGE and WALTHER. *Arch. für El.* 1926, **17**, 433; 1928, **19**, 257; 1930, **24**, 259; 1932, **26**, 716.
68. ROCHOW. *Arch. für El.* 1925, **14**, 361.
69. BELDI. *Brown Boveri Review*, 1931, **18**, 17–.
70. GUTIN and SACKHEIM. *J. Tech. Physics, U.S.S.R.* 1935, **2**, 493, and 1935, **3**, 641.
71. STANDRING and CHAPPELL. E.R.A. Report L/T 133, 1944.
72. BUEHL and VON HIPPEL. *Phys. Rev.* 1939, **56**, 941.
73. FRÖHLICH. *Proc. Roy. Soc.* A, 1947, **188**, 532.
74. ——— Ibid. A, 1941, **178**, 493. E.R.A. Report L/T 113, 1940.
75. PLESSNER. *Proc. Phys. Soc.* 1948, **60**, 243.
76. SCHULZE. *Ann. der Physik*, Series 5, 1934, **19**, 878.
77. GUNTHERSCHULZE and BETZ. *Zs. für Physik*, 1934, **91**, 70; 1934, **92**, 367.
78. BETZ. *Zs. für Physik*, 1932, **82**, 644.
79. BOREL. *C.I.G.R.E., Paper 202* (Paris, 1939).
80. BUSS. *Arch. für El.* 1934, **28**, 55.
81. DRAEGER. Ibid. 1932, **26**, 597.
82. INGE and WALTHER. Ibid. 1930, **24**, 88; 1930, **29**, 259. WALTHER and INGE, *C.R. Acad. of Sciences U.S.S.R.* 1934, **2**, 68.
83. SIMPSON. E.R.A. Report L/T 178, 1947.
84. HARTSHORN. *J.I.E.E.* 1926, **64**, 1152.
85. WHITEHEAD, S. *Trans. Farad. Soc.* 1946, **42** A, 66.
86. NASLEDOW and SCHARAWSKY. *Ann. der Physik*, 1929, **3**, 63; 1930, **5**, 429; 1930, **6**, 574.
87. TUTSCHEWITZ. Ibid. **6**, 622.
88. COURTENSER and IGNATOWITSCH. *Ann. der Physik*, 1931, **10**, 681.

89. NAKANISI. *J.I.E.E., Japan*, 1936, **56**, 909.
90. SINJELNIKOV and WALTHER. *Zs. für Physik*, 1927, **66**, 786.
91. VON HIPPEL. *Phys. Rev.* 1939, **54**, 1096.
92. TURNER and LEWIS. *Nature*, 1947, **159**, 334.
93. HARTSHORN and RUSHTON. *J.I.E.E.* 1934, **75**, 631; 1938, **83**, 315; 1935, **77**, 723.
94. THOMAS and GENT. *Proc. Phys. Soc.* 1945, **57**, 324.
95. GARTON. *J.I.E.E.* 1940, **86**, 369.
96. HAWARTH and BOZORTH. *Physics*, 1934, **5**, 15.
97. INGE and WALTHER. *Zs. für Physik*, 1930, **64**, 830. *Arch. für El.* 1930, **24**, 259; 1931, **25**, 21.
98. SHINOHARA. *Hokkaido Imp. University Engineering Memoirs*, 1934, no. 3, 157.
99. WALTHER and INGE. *J. Tech. Physics, U.S.S.R.* 1938, **5**, 335.
100. WEICKER. *Mit. der Hernsdorf-Schomberg Isol. G.m.b.H.* 1926, p. 707, *E.T.Z.* 1926, **47**, 177.
101. AKAHIRA and GEMANT. *Arch. für El.* 1933, **27**, 577.
102. KELLER. *Physica*, 1948, **14**, 475, and note in publication.
103. VON HIPPEL and ALGER. *Phys. Rev.* 1949, **76**, 127.
104. CALLEN. *Phys. Rev.* 1949, **76**, 1394. Note also FRÖHLICH and SEITZ. *Phys. Rev.* 1950, **79**, 526,
105. BALL. In course of publication in *Proc. I.E.E.*
106. FRÖHLICH and O'DWYER. E.R.A. Report, L/T 219.
107. LOMER, *Nature*, 1950, **166**, 191.

CHAPTER III

THERMAL BREAKDOWN

3.1. General principles

WHEN an electric field is applied to a solid dielectric there are always some currents which generate heat. Conduction currents from ionic dissociation are of this type. Electrons moving in the electric field have already been shown to lose energy to the solid structure. Currents due to ionic or dipolar changes not involving the transport of charged particles to the electrodes must also generate heat in an alternating field because these changes have a finite relaxation time which is evidence of dissipation in a system. Even the oscillations beyond the present electromagnetic spectrum comprise damping, which means the generation of heat. These currents are often known as 'loss' currents, and the heat produced is known as the energy loss in dielectrics. The energy losses are much greater in alternating than in constant fields because the energy loss in relaxation phenomena such as dipolar motions, the most important in solid dielectrics, depends upon the rate of change of field strength. The energy loss increases with the field strength, and if high enough the dielectric specimen may be unable to dissipate the heat from the regions where it is generated. The temperature will rise and breakdown must ensue because the intrinsic electric strength falls ultimately with increase of temperature, and finally the dielectric itself either decomposes or becomes a dissociable electrolyte.

Stated thus baldly, there is little recondite about thermal breakdown. It has been known from early times. Skinner mentioned it in 1904, Miles Walker dealt with it in *The Theory of Machine Design* in 1912, while Everest, in a discussion at the Institution of Electrical Engineers in 1913, described a test based upon thermal instability, which he called the highest maintained electric strength test, and which found its way, after 1918, into specifications. These engineers and their colleagues viewed thermal instability either as an incident in the design of

a particular piece of equipment or as a means of comparing materials by a test which yielded a practical function of its electrical and thermal properties, more particularly energy losses and thermal resistivity. Much later were studied the physical generalizations and the interesting mathematical solutions of certain simple cases.

First must be stated the general equations which determine a stable distribution of temperature and electric potential. Let θ be the temperature at any point in a dielectric specimen, ϕ be the electric potential, k the thermal conductivity, ϵ the complex dielectric constant $\epsilon'-j\epsilon''$, σ the electrical conductivity or $c_0\omega\epsilon''$ ($c_0 = 10^{-11}/36\pi$); let ω be $2\pi\times$frequency; for d.c. $\omega = 0$ and ϵ' is associated only with a transient current without heating effect. The distributions of potential and temperature in the dielectric are given by two equations. The first of these expresses the fact that in thermal equilibrium there is no gain of heat by an element of volume so that the sum of the heat generated in it and the net heat flowing into it is zero. Now the heat generated per volume element is the scalar product of the voltage gradient and the current. If ϕ is the potential, the voltage gradient is $d\phi/dn$ (dn is a line element in the direction of greatest variation of ϕ), or $\operatorname{grad}\phi$ in vector notation, the current is $j\omega(\epsilon'-j\epsilon'')c_0\, d\phi/dn$ or $j\omega(\epsilon'-j\epsilon'')c_0 \operatorname{grad}\phi$, so that the heat generated is $\omega c_0\epsilon''|\operatorname{grad}\phi|^2$, where $|\operatorname{grad}\phi|$ is the r.m.s. amplitude or modulus of $\operatorname{grad}\phi$. The r.m.s. value takes into account variation with time as with alternating voltages. With d.c. $\omega c_0\epsilon''$ becomes the conductivity σ. The heat flowing into a volume element is the integral of the heat flux, i.e. the vector $k\, d\theta/dn$, over the surface and is equal to the scalar divergence or accumulation† of $k\, d\theta/dn$ (i.e. $d(k\, d\theta/dx)/dx+d(k\, d\theta/dy)/dy+d(k\, d\theta/dz)/dz$) per volume element. Accordingly the first equation, that of thermal continuity, may be written

$$\operatorname{div}(k\operatorname{grad}\theta)+\omega c_0\epsilon''|\operatorname{grad}\phi|^2 = 0. \qquad (3.1)$$

† The term 'divergence' really means a dispersion or negative accumulation. However, the convention of sign used here makes the divergence an accumulation, since in common parlance thermal and electrical continuity are generally referred to as absences of accumulation of heat and charge.

Similarly, the second equation, that of electrical continuity, expresses the continuity of the current, the divergence of which is zero, for no electricity is generated. Since the current is $j\omega c_0 \epsilon \operatorname{grad} \phi$ the second equation is

$$\operatorname{div} \epsilon \operatorname{grad} \phi = 0. \tag{3.2}$$

To those unaccustomed to vector notation the unidimensional case with a single coordinate z should be familiar, namely

$$\frac{d}{dz}\left(k \frac{d\theta}{dz}\right) + \omega c_0 \epsilon'' \left(\frac{d\phi}{dz}\right)^2 = 0$$

and

$$\frac{d}{dz}\left(j\omega\epsilon \frac{d\phi}{dz}\right) = 0.$$

From these two general equations of continuity, it is easy to establish the first generalization concerning thermal breakdown, namely that the breakdown voltage and the inverse square root of the electrical conductivity will vary in a similar manner with the ambient temperature or other like physical parameter, provided the conductivity is independent of the field strength. Let V be the total potential difference applied to the dielectric and σ_0 be the conductivity (a.c. or d.c.) at a temperature equal to the fixed temperature of the ambient medium, and let

$$u = \omega c_0 \epsilon''/\sigma_0, \qquad u' = \omega c_0 \epsilon/\sigma_0, \qquad v = \phi/V;$$

then the continuity equations become

$$\operatorname{div}(k \operatorname{grad} \theta) + \sigma_0 V^2 u (\operatorname{grad} v)^2 = 0, \tag{3.3}$$

$$\operatorname{div} u' \operatorname{grad} v = 0. \tag{3.4}$$

The limit of thermal stability corresponds to the limit of real and continuous solutions to these equations of which the second is non-dimensional. If, now, σ_0 does not enter into the boundary conditions, which usually involve a fixed temperature over one surface or a linear relation between the electrode temperature and the fixed ambient, then the limiting condition will contain $\sigma_0 V^2$ but no other function of σ_0 or V, provided the limit is not a discontinuity involving V. In practice any discontinuity is usually a temperature of decomposition, since a discontinuity involving V would imply a non-thermal type of breakdown.

This means that the limiting breakdown voltage for thermal instability will vary with ambient temperature as the inverse square root of the effective conductivity varies with temperature, provided, of course, that the dielectric properties are independent of field strength.† This relation can readily be

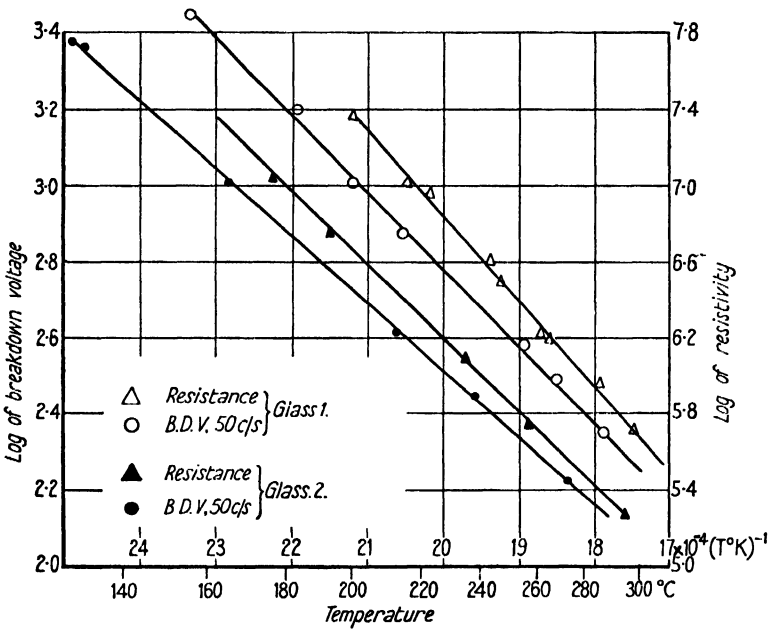


FIG. 25. Variation with temperature of thermal breakdown voltage and resistivity.
(Inge and Walther.)

checked experimentally because the boundary conditions do not have to be very rigidly controlled and need not be known quantitatively. If $\log V$ and $\frac{1}{2}\log\rho$ (ρ = resistivity or inverse of effective conductivity) are plotted against ambient temperature on the same diagram, two parallel lines should be obtained. Inge and Walther[1] obtained a wide range of such comparisons, of which only two are illustrated in Fig. 25. Work on this aspect is very adequately summarized by Semenoff and Walther in

† If the ambient temperature is eliminated between (3.3) and (3.4) the result gives $\sigma_0 V^2$ as a function independent of the ambient temperature.

their book *Die physikalischen Grundlagen der elektrischen Festigkeitslehre* (Springer, 1928). Thermal breakdown at high frequencies is now well known and allowed for specifically in design but, in passing, reference is made to Becker[2] who describes some experiments at high frequency analogous to those of Inge and Walther at lower frequencies.

The next generalization relates to the difference between a.c. and d.c. in that with the former a finite maximum stable temperature is possible, whereas with the latter it is theoretically infinite. Rather paradoxically this result is associated with the severer thermal conditions with a.c. as compared with d.c. Returning to equation (3.2), this is equivalent to saying that the current density I normal to an equipotential surface is continuous or that the current through a tube of force or of current flow is constant and proportional to $-j\omega c_0 \epsilon \operatorname{grad} \phi$ at any point. With d.c. this means that I is $-\sigma \operatorname{grad} \phi$ so that where the conductivity increases the voltage gradient decreases, if I is unaltered, as will be true for a given total current through the specimen. The dielectric is thus relieved of stress where it has a high conductivity, that is, where it has a high temperature. Conversely, with a.c. ϵ is nearly equal to ϵ' since ϵ'' and the loss angle are relatively small for good dielectrics: ϵ' varies comparatively little with temperature. Thus, with a.c., $\operatorname{grad} \phi$ varies little with temperature and the dielectric is not relieved of stress by a local rise of temperature.

Consider now equation (3.1). With d.c. $\omega c_0 \epsilon''$ is σ so that at the hottest part of the dielectric as θ and σ increase $\sigma \operatorname{grad} \phi$ remains unchanged from (3.2) but $\operatorname{grad} \phi$ decreases. Thus $\sigma |\operatorname{grad} \phi|^2$ decreases, and so also must the arithmetical magnitude of $\operatorname{div} k \operatorname{grad} \theta$. This means that the temperature gradient is more uniform where the dielectric gets hotter, so that the temperature can increase indefinitely while preserving thermal continuity. It may therefore be deduced that as the limit of stability is approached the temperature of the hottest portion in certain cases can become indefinitely large, although limited to an indefinitely small region. In practice, breakdown occurs before the dielectric gets very hot, but theoretically there is no

inherent mathematical limit to the maximum temperature. With a.c. if the temperature rises too high, the dielectric may become a semi-conductor in which case the behaviour is the same as for d.c. Conversely, if the loss angle remains small grad ϕ changes little, by reason of (3.2), so that $\omega c_0 \epsilon'' |\text{grad}\,\phi|^2$ increases as ϵ'' and the temperature increase, and so does the arithmetic value of div k grad θ. Thus at the hottest part the temperature gradient must change more and more sharply. Consider a case where the hottest portion is that adjacent to a thermally insulated surface. At the surface grad θ is necessarily zero, and as the temperature increases grad θ must change to a greater and greater negative value towards the interior. This means that the temperature will be constant for a short distance and then fall more and more sharply. Thus indefinite increase in temperature would require a discontinuity in the temperature adjacent to, but not at, the surface. This is inconsistent with stability. Accordingly, at the limit of stability the maximum temperature with a.c. approaches a finite limiting value, although theoretically this limit can have any value according to the properties of the dielectric. In practice, the limiting temperatures are not very high for the dielectrics usually involved, as will later be appreciated.

The third generalization to be established is the existence of a finite maximum thermal voltage for an indefinitely thick specimen and that this voltage depends only on the physical properties of the dielectric and the initial temperature. Consider now, for simplicity, a large plane parallel slab of dielectric with electrodes on opposite surfaces through which must pass any escaping heat. The hottest portion will be in the middle. The equation of electrical continuity becomes $j\omega c_0 \epsilon\, d\phi/dz = -I$, where I is the current density. The equation of thermal continuity becomes

$$\frac{d}{dz}\left(k\frac{d\theta}{dz}\right)+\omega\epsilon'' c_0\left(\frac{d\phi}{dz}\right)^2 = 0.$$

Substituting I, this can be written

$$\frac{d}{dz}\left(k\frac{d\theta}{dz}\right)+\frac{j\epsilon''}{\epsilon} I\frac{d\phi}{dz} = 0.$$

If z is measured from the central plane, then at $z = 0$, $\phi = \frac{1}{2}V$, where V is the applied voltage. If the thickness of the slab is $2d$, then when $z = d$, $\phi = 0$. Thus

$$I(\phi - \tfrac{1}{2}V) = \int_0^z \frac{j\epsilon}{\epsilon''} \frac{d}{dz}\left(k\frac{d\theta}{dz}\right) dz$$

or, substituting for I,

$$-(\phi - \tfrac{1}{2}V) = \frac{1}{j\omega c_0 \epsilon} \frac{dz}{d\phi} \int_0^z \frac{j\epsilon}{\epsilon''} \frac{d}{dz}\left(k\frac{d\theta}{dz}\right) dz,$$

$$\int_{\frac{1}{2}V}^{0} (\phi - \tfrac{1}{2}V)\, d\phi = \int_d^0 \frac{1}{\omega c_0 \epsilon} \int_0^z \frac{\epsilon}{\epsilon''} \frac{d}{dz}\left(k\frac{d\theta}{dz}\right) dz dz.$$

Evaluating the left-hand side and substituting $u = d\theta/dz$

$$\tfrac{1}{8}V^2 = \int_d^0 \frac{1}{\omega c_0 \epsilon} \int_0^u \frac{\epsilon}{\epsilon''} d(ku)\, dz$$

$$= \int_d^0 \frac{1}{\omega c_0 \epsilon}\left[\frac{ku\epsilon}{\epsilon''} - \int ku \frac{d(\epsilon/\epsilon'')}{dz} dz\right]_0^u dz. \tag{3.5}$$

With d.c. $\epsilon = -j\epsilon''$, $\omega c_0 \epsilon'' = \sigma$, so that

$$\tfrac{1}{8}V^2 = \int_d^0 \frac{ku}{\sigma} dz = \int_{\theta_a}^{\theta_m} (k/\sigma)\, d\theta,$$

where θ_m is the maximum temperature, i.e. at $z = 0$, and θ_a is the temperature at $z = d$. At the limit of stability θ_m tends to infinity, so that the breakdown voltage will tend to be given by

$$\tfrac{1}{8}V^2 = \int_{\theta_a}^{\theta_m \to \infty} (k/\sigma)\, d\theta. \tag{3.6}$$

Consider equation (3.5) with a.c. When $z = 0$, $u = 0$. If the dielectric is thick, then at large values of z both u and $d(\epsilon/\epsilon'')/dz$ are small, so that the integrand of the integral within the bracket is small when $z = 0$ and when z is great. Furthermore

$$\int_0^u ku\{d(\epsilon/\epsilon'')/dz\}\, dz \simeq \int_{\theta}^{\theta_m} k(\epsilon/\epsilon''^2)(d\epsilon''/dz)\, d\theta.$$

It has already been shown that θ_m is limited to a value which in practice is not very high, and $d\epsilon''/dz$ for thick specimens necessarily has its greatest values where their temperature approaches, but not too closely, θ_m; but, on the other hand, ϵ'' increases rapidly as θ increases. For all these reasons it is possible to conclude that in practice this integral can be neglected without altering the order of the result. This will be verified later in particular cases. Thus, if only the order of magnitude in practice is wanted,

$$\tfrac{1}{8}V^2 \simeq \int_{\theta_a}^{\theta_m} (k/\omega c_\eta \epsilon'')\, d\theta.$$

Now the limiting voltage for thermal stability or the thermal breakdown voltage is the maximum value of this integral which, in turn, is a function only of the electrical and thermal properties of the material, the surface and central temperatures. As the thickness of the slab is indefinitely increased the thermal conditions at the surface will have less and less effect. Excluding the case of perfect thermal insulation where stability is impossible with any finite voltage, in the limit the thermal insulation of the dielectric itself will greatly exceed any at its surface. Thus with a very thick dielectric θ_a may be taken as the fixed ambient or initial temperature without serious error. This means that V is determined by the value of θ_m which makes the integral a maximum, that is, the breakdown voltage has an upper limit, independent of the thickness, equal to the value obtained when the surfaces of a plane slab are maintained, by perfect cooling, at the constant ambient temperature θ_a. In order to obtain the order of this maximum thermal voltage, we can use the expression

$$V^2 = \int_{\theta_a}^{\theta_m \text{ or } \infty} (8k/\sigma)\, d\theta, \tag{3.7}$$

provided σ is correctly interpreted as the d.c. or a.c. conductivity. In practice the precise value of θ_m does not make much difference to V. In addition, the dielectric cannot exceed its critical temperature θ_c for decomposition or loss of mechanical

properties. It is therefore better to use the formula

$$V^2 = \int_{\theta_a}^{\theta_c} (8k/\sigma)\, d\theta, \tag{3.8}$$

where $\sigma \simeq \omega c_0 \epsilon \tan\delta$ for a.c. if ϵ is the permittivity and δ is the loss angle.

A formula, recurring in later sections, can be deduced both from (3.7) and more generally from dimensional analysis, namely, that if $\sigma = \sigma_0 e^{\theta/\theta_0}$, then

$$V^2 = qk\theta_0/\sigma_0, \tag{3.9}$$

where q is a number and σ_0 the conductivity at the ambient temperature.

If the slab of dielectric had been thermally insulated on one side, then the maximum thermal voltage would be $\frac{1}{2}V$. If the variation with temperature of k can be neglected, div k grad θ is $\nabla^2\theta$, which has invariant properties. Thus it may be deduced that for any uni-dimensional arrangement with one face thermally insulated the maximum thermal voltage will be of the order of $\frac{1}{2}V$. Particular cases will be illustrated later, but on *a priori* grounds the precise shape of the boundaries of a very thick dielectric cannot greatly matter provided the thermal conditions at these boundaries are similar. It must be remembered that heat flow parallel to the electric field is postulated. With lateral heat flow the thermal breakdown voltage can be raised indefinitely as the lateral cooling is improved.

The following table gives values of the maximum thermal voltage for a number of materials with an assumed ambient temperature of 20° C. for d.c. and a.c. of 50 cyc./sec.

Maximum Thermal Voltages at Room Temperatures

Material	*Voltage in* 10^6 *V* d.c.	a.c.
CRYSTALS:		
Quartz perpendicular to axis	12,000	
parallel to axis	66	
Mixed crystals		20
Impure		2·2
Mica Muscovite	24·0	7–18
Ammonium alum	160	
Rock salt	38	1·4

Maximum Thermal Voltages at Room Temperatures (contd.)

Material	*Voltage in* 10^6 *V* d.c.	a.c.
Calcite	9	
Silver halides	about 0·15	
CERAMICS:		
H.F. steatite		9·8
L.F. steatite		1·5
Porcelain		
High grade		2·8
Low grade		0·4
Glass		
Borosilicate		4·8
Soda		1·9
Mycalex		2·0
ORGANIC INSULANTS:		
Paper Capacitor tissue		3·5–4
Greaseproof		1·35
Cotton		0·7
Sulphate		0·6
Impregnated tissue		0·8–3·6
Impregnated cotton		1·3
Paper under high gas pressure		1·3
Varnish paper—high grade		1·3
low grade		0·15
Ebonite		1·45–2·75
Varnish Cambric		0·05–0·25
Synthetic polymers		
Loaded phenol-aniline-formaldehyde		0·75
Phenol formaldehyde		0·2–1·4
Mica-filled phenol-formaldehyde		0·7
Aniline-formaldehyde		0·15
Plasticized ethyl cellulose		1·5
,, cellulose acetate		1·3
,, polyvinyl chloride		0·1–0·2
Polyvinyl acetate		0·9
Polyvinyl formaldehyde		1·5
Polythene (commercial)		3–5
		0·05 (1 Mc./s.)
Polystyrene		5
		0·05 (1 Mc./s.)
Acrylic resins		0·3–1

The usefulness of this table is limited, but the maximum thermal voltage is a significant function of the properties of the material,

it has a quantitative basis and permits a logical comparison of materials from the present aspect. It also indicates the conditions in which thermal breakdown may be important.

With good designs thermal instability will be limited for d.c. to special instances, in fact general thermal instability is probably impossible except at high temperatures. For power frequency voltages the consideration of thermal instability applies mainly to high-voltage equipment, usually in excess of 50–100 kV, except at abnormally high temperatures. Owing to the fact that the loss angle (power factor) of a dielectric rarely falls so rapidly with frequency as to approach an inverse law, thermal instability is of much greater importance at high frequencies. Many good dielectrics such as mica, long-chain hydrocarbons, ceramics, show only a slow variation of loss angle with frequency, which means that the maximum thermal voltage will vary roughly as the inverse square root of the frequency. Thus at frequencies beyond 1 Mc./s. the voltage limitation is normally thermal if solid insulation has to be continuously applied to conductors, subject, of course, to the effect of the deteriorating mechanisms described in later chapters.

Disregarding many practical limitations, there is a temptation to speculate about the ideal thickness of a layer of dielectric, namely that thickness in which the application of the maximum thermal voltage produces a field equal to the intrinsic electric strength. Remembering that intrinsic electric strengths are fairly well comprised in the range 10^6–10^7 V/cm. and that significant maximum thermal voltages might perhaps be considered as lying in the range 10^6–10^7 V for power frequencies, there appears to be little reason to use continuous layers of dielectric more than, say, 1–10 cm. thick. This thickness has more significance than the figures on which it is based, since the same factor of safety applied both to the intrinsic electric strength and the maximum thermal voltage would leave the thickness unchanged. It is interesting to note that in modern designs 1–10 cm. represents the upper range of thickness employed for continuously applied insulation, although important reasons other than those suggested have intervened in practice.

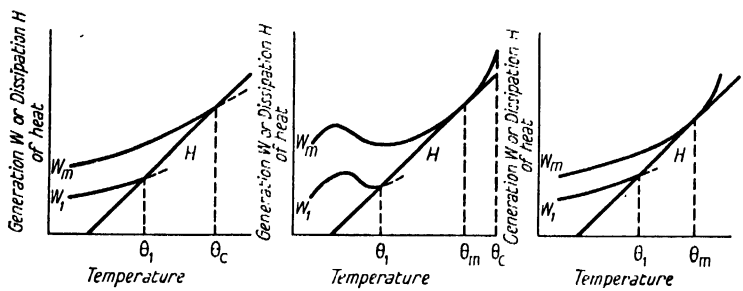
3.2. Typical solutions

The general equations of the previous section have, in principle, a general type of solution:

$$k\,\mathrm{grad}\,\theta+\sigma|\mathrm{grad}\,\phi|^2 = \mathrm{curl}\,u+\mathrm{const.}, \qquad (3.10\,\mathrm{a})$$

$$\epsilon\,\mathrm{grad}\,\phi = \mathrm{curl}\,v+\mathrm{const.}, \qquad (3.10\,\mathrm{b})$$

where u and v are vector functions of position chosen to suit the boundary conditions. However, general solutions are of little



(*a*) Breakdown without prior instability. (*b*) Instability and breakdown. (*c*) Normal Instability.

FIG. 26. Balance and instability in thermal breakdown.

practical interest, so that particular solutions will be given for those typical cases which have been studied.

Thin Specimens

The physical principles are most easily perceived by considering first a thin sheet of dielectric; the opposite extreme of the very thick slab has already been treated. Let the temperature of the sheet be θ, neglecting edge effects; let the generation of heat in the dielectric due to the applied voltage be W per cm.^{3} and the dissipation from the surface be H per cm.^{2} Then the thermal breakdown voltage is the applied voltage for which the equation $Wd = 2H$ (d = thickness) fails to have a real solution. If $\partial Wd/\partial\theta$ is always less than $2\partial H/\partial\theta$ then the limit is given by $Wd = 2H$ and $\theta = \theta_c$ the temperature of decomposition. Otherwise, and more usually, the limit is given by

$$Wd = 2H \quad \text{and} \quad \partial W/\partial\theta = (2/d)\partial H/\partial\theta. \qquad (3.11)$$

Fig. 26 illustrates the equations as thermal balance diagrams

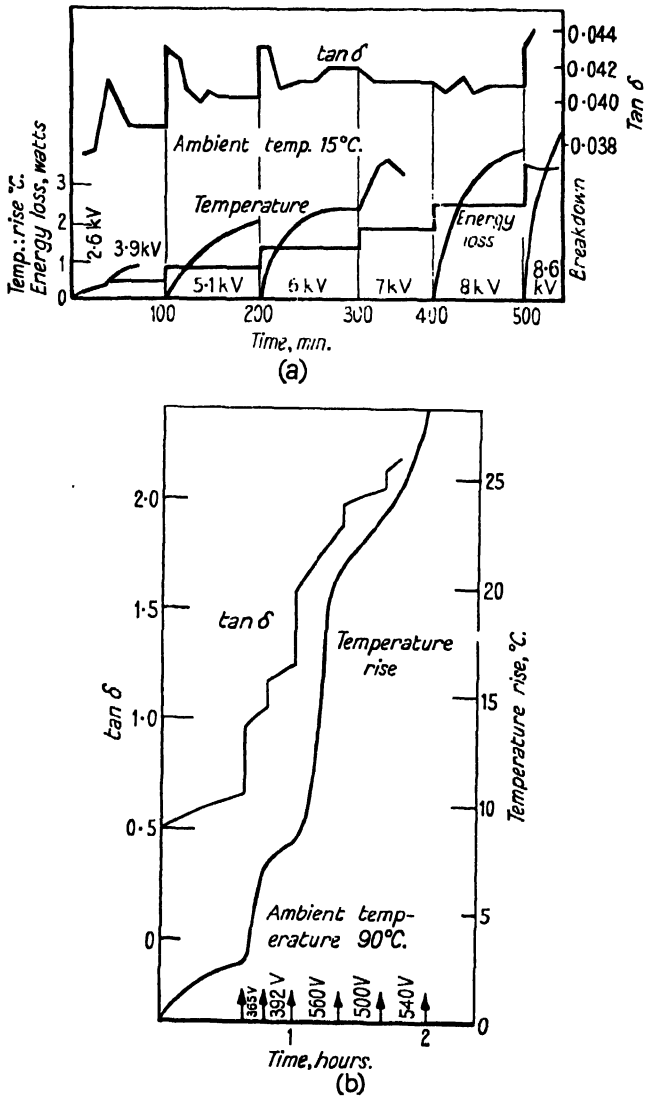


FIG. 27. Thermal instability of cellulose acetate.
(Whitehead and Nethercot.)

and it is noted that, with an energy loss curve such as Fig. 26 (*b*) the limiting temperature for stability cannot be less than the temperature which corresponds to a minimum in *W*. Fig. 27

shows stable and unstable temperatures in tests[3] under controlled conditions where H was known and W was measured electrically during the test, so that at stable points the electrical and thermal estimations of energy losses could be checked. The latter is, of course, the basis of the calorimetric method of measuring energy losses, although in practice the rate of rise of temperature, knowing the thermal capacitance of the system, gives quicker results.

It has been shown by Austen[4] that the variation of conductivity or the energy loss per cm.^{3} divided by the square of the field strength varies very slightly with field strength for homogeneous dielectrics up to the order of at least one-fifth the intrinsic electric strength. However, with moist dielectrics, with subsidiary discharges and under less carefully controlled conditions, variations of conductivity with field strength occur in much weaker fields.[3] Even here it is usually possible to express the conductivity as the product of a function of temperature and a function of field E. The thermal dissipation H is usually proportional to the temperature rise above ambient to a first approximation. Thus the equations (3.11) become

$$dE^2\psi(E)\phi(\theta) = 2H = 2\lambda\theta, \tag{3.12a}$$

$$dE^2\psi(E)\phi'(\theta) = 2\lambda. \tag{3.12b}$$

The function $\phi(\theta)$ is very frequently either e^{θ/θ_0} or $e^{-a/(\theta+\theta_a)}$, where θ_a is the ambient temperature in °K., i.e. $\theta+\theta_a$ is the absolute temperature of the dielectric. It is easy to show that in the first case the maximum stable temperature rise θ_m is θ_0 and in the second

$$\theta_m = \tfrac{1}{2}a\left\{1-\frac{2\theta_a}{a}+\sqrt{\left(1-\frac{4\theta_a}{a}\right)}\right\}. \tag{3.13}$$

It has first to be noted that θ_m depends only on the variation of conductivity with temperature and not upon the actual magnitudes, and secondly that θ_m is not very great. With poor dielectrics θ_m may be as low as 10° C., with most industrial dielectrics it does not exceed 50–100° C., so that only a limited temperature rise due to dielectric loss is permissible.

The thermal breakdown voltage V_m or electric strength E_m is

given by $V_m = E_m d$ and

$$E_m^2 \psi(E_m) = 2\lambda\theta_m/\phi(\theta_m)\, d. \tag{3.14}$$

Where mathematical instability does not occur because the dissipation rises more rapidly with temperature than the energy losses, it is only necessary to substitute θ_c, the temperature of decomposition, for θ_m. If $\phi(\theta)$ is e^{θ/θ_0} and $\psi(E)$ is σ_0, a constant, i.e. the conductivity at ambient temperature,

$$V_m^2 = 2\lambda\, d\theta_0/e\sigma_0, \tag{3.15}$$

provided $\theta_0 < \theta_c$, since $\theta_m = \theta_0$ for instability and $\theta_m = \theta_c$ for decomposition without prior instability. If P_i $(= d/2k)$ is the internal thermal resistance, k being the thermal conductivity, and if P_e $(= 1/\lambda)$ is the external thermal resistance, (3.15) may be written as

$$V_m^2 = 4\theta_0 P_i k/e\sigma_0 P_e. \tag{3.15 a}$$

If $\psi(E)$ is σ_0 and $\phi(\theta)$ is $e^{-a/(\theta+\theta_a)}$,

$$V_m^2 = (\lambda\, da/\sigma_0)[1-(2\theta_a/a)+\sqrt{\{1-(4\theta_a/a)\}}]e^{2/[1+\sqrt{\{1-(4\theta_a/a)\}}]}. \tag{3.16}$$

Provided the conductivity is independent of field strength V_m is proportional to the square root of the thickness for thin specimens but tends, as has already been shown, to an asymptotic constant value for thick specimens. When the conductivity increases with field strength, Whitehead and Nethercot[3] have shown, particularly for moist materials, that the increase is often linear and, in fact, for the present purposes $\psi(E)$ may roughly be taken as bE and $\phi(\theta)$ as e^{θ/θ_0}. In these circumstances

$$V_m^3 = 2\lambda\, d^2\theta_0/eb. \tag{3.17}$$

More complicated functions encountered in practice are given in the paper referred to and Fig. 28 therefrom illustrates the agreement between calculation and experiment in sustained tests to which this analysis applies. Different functions are obtained with discharges but are considered separately, since thermal instability is usually local if initially due to discharges. Among the photomicrographs of cellulose acetate of Fig. 29 (Plate III) (Whitehead and Nethercot), (*a*) is a breakdown due to discharges, but just prior to general thermal instability so

that local instability probably occurred. The blackening, however, was not due to charring as is shown by (*b*) taken under immersion in a medium of similar refractive index. The appearance of (*a*) is often considered evidence of thermal instability but this is not the case as may be seen by comparing (*b*) with (*c*),

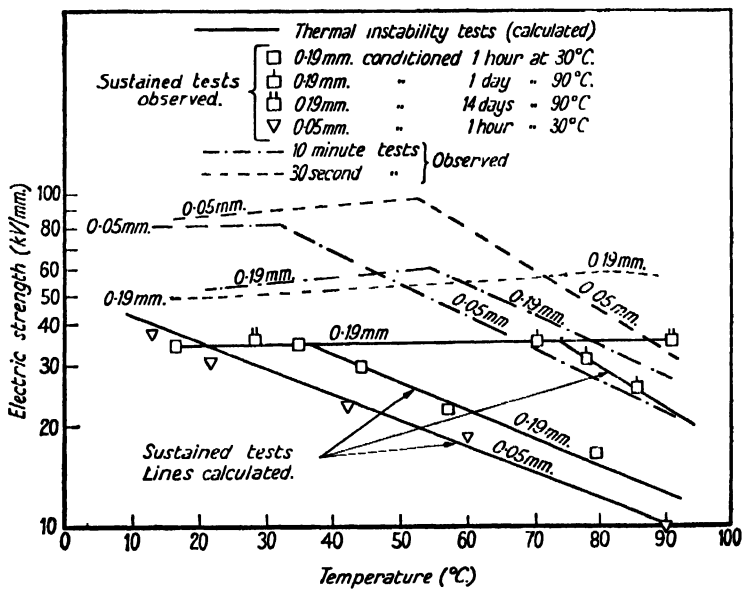


FIG. 28. Thermal breakdown of cellulose acetate.
(Whitehead and Nethercot.)

a true thermal breakdown and (*d*) a pure discharge breakdown, all photographed under immersion. The black margin is the deposited carbon electrode, scraped away near the breakdown.

Thick Specimens

The effect of the temperature distribution in thick specimens is best approached through the unidimensional case of a plane slab wherein the external thermal resistance is P_e, for example with Newton's law of cooling P_e would be the inverse of the coefficient hitherto denoted by λ. In a unidimensional case the condition for electrical continuity becomes merely the continuity of the constant current density I so that the limiting

solution is required of the equation

$$\frac{d}{dz}\left(k\frac{d\theta}{dz}\right)+\frac{I^2\epsilon''}{c_0(\epsilon'^2+\epsilon''^2)\omega}=0. \tag{3.18}$$

The second term varies greatly, from I^2/σ with d.c. to $I^2\epsilon''/c_0\epsilon'^2\omega$ for normal dielectrics with a.c. A general solution is difficult so that attention will be limited to these two extremes corresponding to d.c. and a.c. It is noted that the heat generated between the mid-plane of the slab and one surface must flow through the electrode, whence with d.c.

$$\tfrac{1}{2}IV=\theta_1/P_e, \tag{3.19}$$

where θ_1 is the electrode temperature rise above ambient and V is the applied voltage. Equation (3.18) becomes

$$\frac{d}{dz}\left(k\frac{d\theta}{dz}\right)+\left(\frac{\theta_1}{\frac{1}{2}VP_e}\right)^2\frac{1}{\sigma}=0, \tag{3.20}$$

yielding

$$\frac{\frac{1}{2}\theta_1 d}{\frac{1}{2}VP_e}=\int_{\theta_1}^{\theta_m}\frac{k\,d\theta}{\sqrt{\left(\int_{\theta}^{\theta_m}(2k/\sigma)\,d\theta\right)}},$$

where d = thickness and θ_m = maximum temperature. If k is nearly independent of temperature

$$\frac{\theta_1}{\frac{1}{2}V}\frac{P_i}{P_e}=\int_{\theta_1}^{\theta_m}\left\{\int_{\theta}^{\theta_m}(2k/\sigma)\,d\theta\right\}^{-\frac{1}{2}}d\theta, \tag{3.21}$$

where P_i = thermal resistance of one-half the dielectric slab.

Another equation is that previously quoted (see (3.6))

$$(\tfrac{1}{2}V)^2=\int_{\theta_1}^{\theta_m}(2k/\sigma)\,d\theta. \tag{3.22}$$

The limit of stability or the thermal breakdown voltage V is given by the limiting solution of these equations for θ_m and V, that is, when $\theta_m=\theta_c$, the decomposition temperature, or when the Jacobian function† vanishes if this occurs when $\theta_m<\theta_c$.

† If $u_1,u_2,\ldots,u_n$ be n functions of n variables $x_1,x_2,\ldots,x_n$, the determinant

$$\begin{vmatrix}\partial u_1/\partial x_1, & \partial u_1/\partial x_2, & \ldots & \partial u_1/\partial x_n\\ \partial u_2/\partial x_1, & \partial u_2/\partial x_2, & \ldots & \partial u_2/\partial x_n\\ \ldots & \ldots & \ldots & \ldots\\ \partial u_n/\partial x_1, & \partial u_n/\partial x_2, & \ldots & \partial u_n/\partial x_n\end{vmatrix}$$

Adopting the latter hypothesis, which is more interesting, Fock[5] has obtained parametric solutions which will here be given in the form used by Whitehead and Nethercot. Let $\sigma = \sigma_0 e^{\theta/\theta_0}$, then from (3.15 a) the breakdown voltage V_m for a thin specimen of the same material has been shown to be $2\sqrt{(\theta_0 P_i k/e\sigma_0 P_e)}$

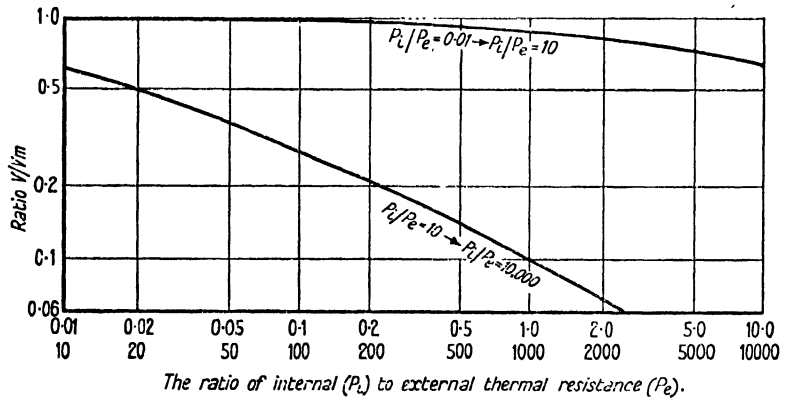


FIG. 30. Effect of internal temperature gradient (i.e. thickness) on the thermal breakdown voltage V with d.c. V_m is the breakdown voltage when the internal temperature gradient is ignored.

whence it may be shown that

$$\left(\frac{V}{V_m}\right)^2 = \frac{2\exp(\frac{1}{2}\sin 2\alpha - \alpha\cos 2\alpha)}{\sec^2\alpha + (\alpha/\sin\alpha)\sec^3\alpha} \tag{3.23}$$

where α is given by

$$P_i/P_e = \sin\alpha\sec^3\alpha(\alpha + \sin\alpha\cos\alpha). \tag{3.24}$$

Thus the ratio of the breakdown voltage V for a thick specimen to the breakdown voltage V_m of a specimen with the same thickness and external thermal resistance, but with a high thermal conductivity so that it behaves thermally as a thin specimen, is a universal function, Fig. 30, of the ratio of the internal to the external thermal resistance of the original thick specimen. This result is convenient because the effect of thickness can be introduced as a correction on V_m which is easy to calculate.

is the Jacobian function of $u_1, u_2, \ldots, u_n$. If it vanishes for particular real values of x_1, etc., which satisfy the equations $u_1 = u_2 \ldots = u_n = 0$ then these values form a limiting real solution of the equations.

Although in principle limited to an exponential variation of electrical conductivity Fig. 30 will give a good approximation for most forms encountered in practice if the correction is small. Increase of thickness resembles the addition of external thermal resistance, since the limiting factor tends to be the thermal stability of the hottest layers which remain thin because the temperature gradient must be increased in these layers to remove the heat fast enough. Provided also the field strength does not change greatly in the specimen, which is the case unless P_i/P_e is fairly large, it will be shown later that the correction function V/V_m is not greatly affected if σ varies with field strength. As has already been seen, it is easy to take such a variation into account with thin specimens, and the present analysis enables thick specimens to be also treated provided P_i does not exceed the order of P_e so that the correction is still fairly small.

The maximum temperature θ_m is

$$\theta_m = \theta_1 + 2\theta_0 \log \sec \alpha. \tag{3.25}$$

Unless P_i is greater than P_e, α is small and θ_m does not differ greatly from θ_1, which in turn will not differ greatly from θ_0 the value for thin specimens. Thus, until the thermal resistance of the dielectric becomes a determining factor, the temperatures at the limit of stability will not be very high so that the observations already made still apply. However, when P_i greatly exceeds P_e, α increases towards $\frac{1}{2}\pi$ and θ_m will ultimately tend to indefinitely large values. Equation (3.23) becomes indeterminate when α is $\frac{1}{2}\pi$, but if $\theta_m = \infty$ is substituted in equation (3.22) it makes the breakdown voltage equal to $2\sqrt{2}\sqrt{(k\theta_0/\sigma_a)}$, where σ_a is the conductivity at the temperature θ_1. This result is true both of a specimen with its faces held at constant temperature and of a very thick specimen. In both instances θ_1 becomes the ambient temperature. When θ_m exceeds θ_c mathematical instability no longer occurs because the material decomposes before the present analysis applies and the alternative form $\theta_m = \theta_c$ must be used, but this makes little difference to the final result. The occurrence of 'infinite temperature' due to increasing the thickness of the specimen or cooling the surface

is a characteristic of d.c. where the field strength can vary considerably through the specimen.

Fock[5] has also worked out the case when σ has the electrolytic form $\sigma_0 e^{-a/(\theta+\theta_a)}$. This is much more complicated and only the final result will be quoted, namely that the breakdown voltage V is found by eliminating α and x from the following equations:

$$\frac{a}{\theta_a}\left\{1+\cos^2\alpha\frac{\psi(\alpha,x)}{\phi(\alpha,x)}\right\}=\frac{1}{x+2\log\sec\alpha},$$

$$P_i/P_e=\sin\alpha\sec^3\alpha\{\phi(\alpha,x)/H(\alpha,x)\},$$

$$V/\sqrt{(2ka/\sigma_0)}=e^{\frac{1}{2}x}\tan\alpha/xH(\alpha,x), \tag{3.26}$$

where $H(\alpha,x)=e^{\frac{1}{2}x}\tan\alpha/x\{f(x+2\log\sec\alpha)-f(x)\}^{\frac{1}{2}}$

$$f(x)=\int_0^x e^x\,dx/x^2, \qquad \phi(\alpha,x)=2\int_0^\alpha \cos^2\alpha H(\alpha,x)\,d\alpha,$$

and $$\psi(\alpha,x)=2\int_0^x\{x/(x+2\log\sec\alpha)\}^2H(\alpha,x)\,d\alpha.$$

The preceding treatment for d.c. based on Fock's analysis applies also approximately to a.c. when the loss angle is great near instability or where only a small correction for thickness is wanted. In all other circumstances the best approximation is based on the fact that the loss angle is usually small and that the permittivity does not vary greatly with temperature. For a.c., if ϵ''^2 may be neglected in comparison with ϵ'^2, the basic equation becomes

$$\frac{d}{dz}\left(k\frac{d\theta}{dz}\right)+\frac{\epsilon''I^2}{\omega c_0\epsilon'^2}=0. \tag{3.27}$$

Again, if V is the voltage applied to the specimen

$$V=\frac{I}{c_0\omega j}\int_{\theta_1}^{\theta_m}\frac{dz}{\epsilon'-j\epsilon''}=\frac{I}{c_0\omega j}\left[\int\frac{dz}{\epsilon'}+j\int\frac{\epsilon''\,dz}{\epsilon'^2}\right]. \tag{3.28}$$

Thus the amplitude of V is $(I/c_0\omega)\int dz/\epsilon'$ to the same approximation. A variation of permittivity with temperature may thus be taken into account by equation (3.27). A higher accuracy is possible in which ϵ'^2 is replaced by $\epsilon'^2+\epsilon''^2$ in equation (3.27) which is solved, giving a value for θ_m and also for $dz/d\theta$ as a

function of θ, whence a value for I can be determined from (3.28). The forms actually given in the equations are themselves suitable for the successive approximation methods which are generally needed in the numerical computation of these more accurate solutions.

These solutions are justified neither by their importance nor practicability, and only the first solution will be given in which ϵ' is taken as independent of temperature so that I is $\epsilon' c_0 \omega(V/d)$, where d is the thickness. Equation (3.27) then becomes, if σ is the a.c. conductivity and E the mean field strength,

$$\frac{d}{dz}\left(k\frac{d\theta}{dz}\right) + \sigma E^2 = 0. \tag{3.29}$$

This is the equation treated by Dreyfus[6] and Berger,[7] but it is convenient to generalize the results (as has been done by Gemant and Whitehead[8]) to include a possible variation of σ with field strength

$$\left.\begin{aligned} -k\left(\frac{d\theta}{dz}\right)_{\theta=\theta_1} &= \sqrt{\int_{\theta_1}^{\theta_m} 2k\sigma E^2\, d\theta} = \theta_1/P_e \\ \text{where } \theta_m \text{ and } \theta_1 &\text{ are respectively the mid-plane and electrode temperature rises above ambient,} \\ \tfrac{1}{2}V &= \int_{\theta_m}^{\theta_1} k\left\{\int_0^{\theta_m} 2k\sigma\, d\theta\right\}^{-\frac{1}{2}} d\theta. \end{aligned}\right\} \tag{3.30}$$

The thermal breakdown voltage is the greatest value of V which permits a real solution for θ_m and V of these equations and is obtained by applying the condition that the Jacobian function of the two equations vanishes, provided that the value of θ_m so found does not exceed the temperature of decomposition θ_c. In the latter event the condition $\theta_m = \theta_c$ applies. Make now the usual simplifying assumptions for a.c. that the thermal conductivity k is constant and that σ is $\sigma_0\psi(E)e^{\theta/\theta_0}$, where σ_0 is the value of σ at the ambient temperature. Then V is given by

$$\sigma_0 V^2\psi(E) = (8\theta_0 k)\eta^2 \exp -\{(2P_e/P_i)\eta \tanh\eta + \log\cosh^2\eta\} \tag{3.31}$$

together with the Jacobian condition which reduces to $\partial V/\partial\eta = 0$,

$$\text{i.e.} \qquad \{(P_e/P_i)+1\}(\tanh\eta/\eta)+(P_e/P_i\cosh^2\eta) = 1/\eta^2. \tag{3.32}$$

For thin specimens η is $\sqrt{(P_i/2P_e)}$, which gives the previous equation, namely

$$\sigma_0 V_m^2 \psi(E_m) = 2\theta_0 d/eP_e.$$

(See (3.14) and (3.15).) From (3.31), (3.32), and the preceding equation

$$\frac{V^2\psi(E)}{V_m^2\psi(E_m)} = \frac{4eP_e k}{d}(\text{function of } P_i/P_e),$$

whence $$V^2\psi(V)/V_m^2\psi(V_m) = \text{function of } P_i/P_e. \qquad (3.33)$$

Thus if the conductivity varies with the stress in a manner which can be represented by a power law, the ratio of the breakdown voltage for a thick specimen to that for an otherwise similar specimen, but of high thermal conductivity so that it behaves thermally as a thin specimen, is a function of the ratio of the internal to the external thermal resistance of the thick specimen. Further, if the effect of field strength is not very great, the ratio is independent of the properties of the dielectric as represented by the coefficient θ_0. This function is shown in Fig. 31, and is analogous and similar to the function for d.c. in Fig. 30. More complicated variations of σ with field strength are considered by Gemant and Whitehead, but are of specialized interest. Dreyfus and Berger considered the effect of heat generated outside the dielectric as in a machine, transformer, or cable.

P_i increases with the thickness and P_e must be reduced the smaller the temperature rise allowed to the electrodes. Thus in the limit the breakdown voltage for an indefinitely thick specimen is the same as for a specimen of any thickness if the surfaces are maintained at a constant temperature, provided ψ is unity or varies little with field strength. This feature was noted earlier when the breakdown voltage for very thick specimens was evaluated in a generalized manner. In the present specific case η is 1·2 for very thick specimens and

$$V\psi^{\frac{1}{2}}(E) = 1{\cdot}88(k\theta_0/\sigma_0)^{\frac{1}{2}},$$

hence $$V = 1{\cdot}88(k\theta_0/\sigma_0)^{\frac{1}{2}} \quad \text{if} \quad \psi(E) = 1. \qquad (3.34)$$

This compares with $2\sqrt{2}(k\theta_0/\sigma_0)^{\frac{1}{2}}$ for d.c. Both are particular

cases of (3.9) to which (3.8) is a sufficient approximation for comparisons between different materials.

The maximum temperature rise θ_m above ambient is given by

$$\theta_m = \theta_0\{(2P_e/P_i)\eta \tanh \eta + \log\cosh^2\eta\}. \qquad (3.35)$$

Numerical evaluation shows that θ_m varies from θ_0, when

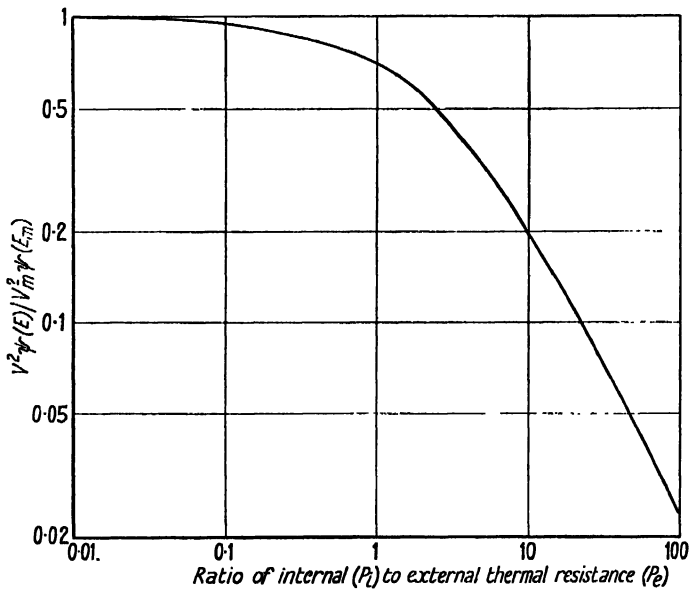


FIG. 31. Effect of internal temperature gradient (i.e. thickness) on the thermal breakdown voltage V with a.c. V_m is the breakdown voltage when the internal temperature gradient is ignored.

$\eta = \sqrt{(P_i/2P_e)}$ and the dielectric is thin, to $1{\cdot}187\theta_0$, when $\eta = 1{\cdot}2$ and the dielectric very thick. Thus irrespective of any variation of conductivity with stress, the maximum stable temperature rise changes little with thickness. Accordingly the observations already made on this subject for thin specimens apply also to thick specimens in alternating fields.

If the dielectric is stratified, that is, if it is divided into a number of thin slices by very thin conducting layers, which are held at potentials such that the field strength in all the layers is the same, then the conditions here assumed for a.c. are true for d.c. also and the same formulae are valid. This applies to

the well-known flat capacitor roll with a thin solid dielectric such as paper, or the stacked capacitor using mica for instance. It is generally accounted an advantage to obtain a uniform electric stress, but this is not the case here because, with an unstratified dielectric passing direct current, the hotter portions with higher conductivity are relieved of stress by the colder portions where the conductivity is less, so that the generation of heat is thereby reduced. The actual breakdown voltage of a capacitor roll is Vt/d, where V is taken from the preceding analysis and t is the thickness of the unrolled dielectric. Thus for a thick roll the capacitor breakdown voltage tends to vary inversely as the thickness of the roll. The capacitance per cm.^{2} is $c_0\epsilon d/t^2$. With a.c. σ_0 is $\omega c_0 \epsilon \tan\delta$. Thus the volt ampere rating per cm.^{2} for a thick roll is given by

$$\frac{V^2 c_0 \epsilon \omega}{d} = \frac{3{\cdot}53 c_0 \epsilon k\theta_0 \omega}{d\omega c_0 \epsilon \tan\delta} = \frac{3{\cdot}53 k\theta_0}{d\tan\delta}. \tag{3.36}$$

Thus the volt amperes per cm.^{2} of the flat surface is inversely proportional to the thickness of the roll but independent of the dielectric thickness, permittivity, and frequency.[9]

3.3. Concentric cylinders

If r is the radius of the thin cylindrical element under consideration, a the internal radius, and b the external radius, the fundamental equation is

$$r\frac{d}{dr}\left(kr\frac{d\theta}{dr}\right)+\frac{I^2}{\sigma} = 0, \tag{3.37}$$

where $2\pi I$ is the total current across a cylinder of radius r, which must be constant through the specimen. If $z = \log r/a$ equation (3.37) becomes

$$\frac{d}{dz}\left(k\frac{d\theta}{dz}\right)+\frac{I^2}{\sigma} = 0, \tag{3.38}$$

which for d.c. is identical with Fock's equations and may similarly be solved, leading to results identical with those already given for uniform fields with d.c. if the ratio P_i/P_e is correctly interpreted. In practice the conditions which apply to a.c. or to a stratified dielectric are of more interest and only

these solutions will be given. By comparison with the uniform field already studied, the main conclusions apply generally to unidimensional fields.

With a.c. it may be assumed that the field is distributed according to a constant permittivity, whence

$$\frac{1}{r}\frac{d}{dr}\left(kr\frac{d\theta}{dr}\right)+\sigma_0 E^2\frac{b^2}{r^2}e^{\theta/\theta_0}=0 \tag{3.39}$$

if σ is $\sigma_0 e^{\theta/\theta_0}$. If z is $\log r/a$, if k is independent of θ, and if E is the field strength at the outer surface,

$$k\frac{d^2\theta}{dz^2}+E^2\sigma_0 b^2 e^{\theta/\theta_0}=0 \tag{3.40}$$

and the boundary conditions are that $d\theta/dz = 0$ when $z = 0$ and that $k(r\,d\theta/dr) = -b\theta_1/P_e$ when $r = b$, i.e. $k\,d\theta/dz = -b\theta_1/P_e$ when $z = \log b/a$. This means that the solutions are the same as for a uniform field if P_e is replaced by P_e/b and $\frac{1}{2}d$ by $\log b/a$. The quantity P_i/P_e for a uniform field is $\frac{1}{2}d/kP_e$ transforming, by substitution, to $b\log(b/a)/kP_e$, which is P_i/P_e correctly for the cylinder. Again E in the uniform field becomes Eb in the cylinder so that the breakdown voltage V which is Ed in the uniform field becomes $2Eb\log b/a$ or twice the cylindrical breakdown voltage. Thus all the previous results for a uniform field apply, provided P_i/P_e is correctly expressed, but the breakdown voltage must be halved. This follows also from the physical consideration that no heat is lost from the internal surface of the cylinder. In the limiting case of a very thick cylinder, or when the external surface is cooled to a constant temperature

$$V = 0{\cdot}94(k\theta_0/\sigma_0)^{\frac{1}{2}}, \tag{3.41}$$

as given by Dreyfus[6] and Berger.[7]

In a stratified dielectric such as that in a condenser bushing or a cylindrical capacitor roll where the field must be constant throughout, both for d.c. or a.c. the fundamental equation is

$$\frac{1}{r}\frac{d}{dr}\left(kr\frac{d\theta}{dr}\right)+\sigma_0 E^2 e^{\theta/\theta_0}=0 \tag{3.42}$$

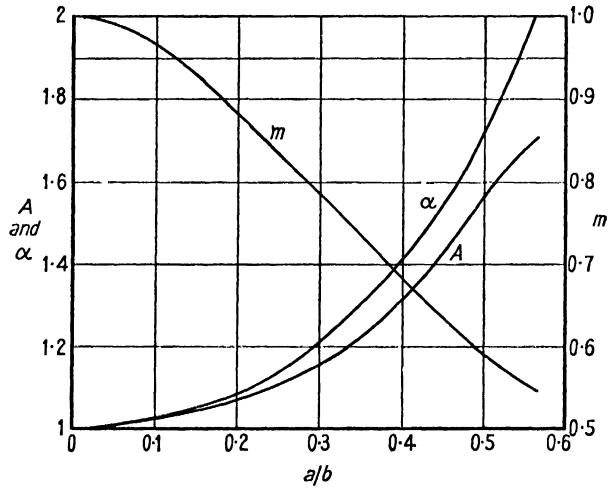
with the same assumptions. Dreyfus[6] and Goodlet[10] have assumed that the outer surface is maintained at a constant

temperature, i.e. $P_e = 0$, but a more detailed treatment has been given by Whitehead,[9] whose results are quoted here. The equation is easily integrated by substituting $\log r/b$ for the independent variable r and $2\log(r/b)+(\theta/\theta_0)$ for the dependent variable θ. Let

$$m = \left(\frac{\alpha+1}{\alpha-1}\right)^{\frac{1}{2}}\left(\frac{a}{b}\right)^{\alpha} \tag{3.43}$$

where

$$\alpha^2 = 1+(\tfrac{1}{2}a^2E^2\sigma_0 e^{\theta_m/\theta_0}/k\theta_0). \tag{3.44}$$



A, α and m as functions of a/b.

FIG. 32. Parameters for cylindrical stratified dielectric.

Then the equations for equilibrium are

$$-\alpha\sqrt{\{1-[2m/(m^2+1)]^2\}} = \tfrac{1}{2}(P_i/P_e)(\theta_1/\theta_0)-1, \tag{3.45a}$$

$$\sqrt{(\tfrac{1}{2}b^2E^2\sigma_0 e^{\theta_1/\theta_0}/k\theta_0)} = 2\alpha m/(m^2+1). \tag{3.45b}$$

Since the field E is constant, P_i has to be interpreted as b/k and the ratio b/a enters separately. The Jacobian condition for the limit of stability is

$$1 = \left\{\left(\frac{2m}{1+m^2}\right)^2+\frac{P_i}{\alpha P_e}\left(\frac{1-m^2}{1+m^2}\right)\right\}\times$$
$$\times\left\{1-\frac{1-m^2}{1+m^2}\left(\frac{\alpha}{\alpha^2-1}-\tfrac{1}{2}\log m^2\{(\alpha+1)/(\alpha-1)\}\right)\right\}. \tag{3.46}$$

The values of m and α are given in Fig. 32 as functions of a/b

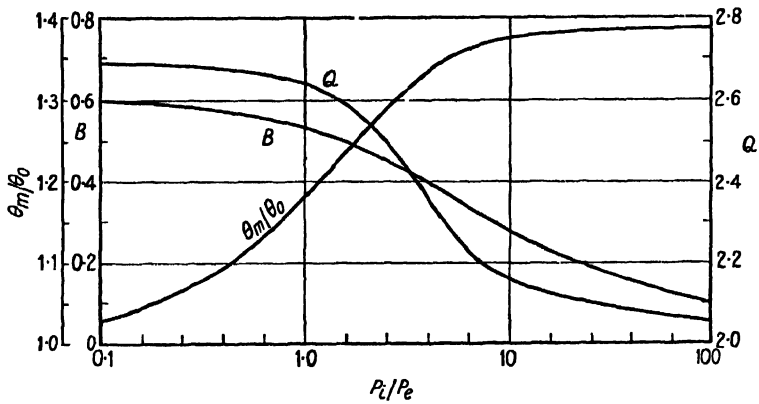
from which E and $(b-a)E$ can be calculated in any given case.

If the external surface is kept at a constant temperature then, if σ_0 is the conductivity at the ambient temperature, $\theta_1 = 0$ and $P_c = 0$, whence

$$E(b-a) = \{2\alpha m/(m^2+1)\}\sqrt{\{2k\theta_0(b-a)^2/\sigma_0 b^2\}} \qquad (3.47)$$

and

$$\frac{1+m^2}{1-m^2} - \frac{\alpha}{\alpha^2-1} = \tfrac{1}{2}\log\left(m^{-2}\frac{\alpha+1}{\alpha-1}\right). \qquad (3.48)$$



B: Numerical factor for breakdown voltage.
Q: Conductance at breakdown/Conductance at initial temperature.

FIG. 33. Parameters for cylindrical stratified dielectric.

The variation of $2m/(m^2+1)$ or A is given in Fig. 32, and it will be appreciated that the result, previously obtained by Dreyfus, does not differ greatly from the earlier result for the unstratified dielectric. If a/b is small, i.e. if the dielectric is thick or the internal radius small, then in accordance with Dreyfus[6] and Goodlet[10]:

$$Eb = \sqrt{(2k\theta_0/\sigma_0)}, \qquad (3.49)$$

which means that the breakdown voltage Eb is about 1·51 times that for the unstratified dielectric.

More generally, if a/b is small, but the external surface is not held at a constant temperature,

$$Eb = B\sqrt{(P_i/P_e)}\sqrt{(2k\theta_0/\sigma_0)}, \qquad (3.50)$$

where B is shown in Fig. 33 as a function of P_i/P_e. When P_i/P_e is great, B is $\sqrt{(P_e/P_i)}$ in accordance with equation (3.49). When P_i/P_e is small the situation is analogous to that of a thin plane specimen; B is $e^{-\frac{1}{2}}$ and

$$Eb = \sqrt{(2b\theta_0/\sigma_0 eP_e)}. \tag{3.51}$$

The maximum temperature rise θ_m may also be calculated from α and varies between θ_0 when the thermal resistance is outside the dielectric to $2\theta_0 \log 2$ or $1{\cdot}4\theta_0$ when the thermal resistance of the dielectric is the main factor. Fig. 33 gives θ_m/θ_0 when a/b is small. As previously, the temperature rise for stability is quite limited.

If the stratification is achieved by some means such as suitable axial dimensions of interleaved metal foil (as in condenser bushings) or some external mechanism, then $E(b-a)$ or Eb is the breakdown voltage which has the familiar property of the already established asymptotic limit as the dielectric thickness is increased. In a capacitor roll, however, the capacitor electrodes form interleaved spirals so that the interleaving is of alternate polarity and the breakdown voltage is Et where t is the thickness of the unwound dielectric: t is, of course, small compared with b. Thus the property that Eb tends to be independent of b means that the product of breakdown voltage and total capacitance tends towards an upper limit. With a.c. this means that the volt-ampere rating of the capacitor per cm. length on a thermal basis has an upper limit. For the cylindrical roll a very simple result is obtained for a.c.

$$\text{limiting volt amps./cm.} = 2\pi\theta_0 k/\tan\delta \tag{3.52}$$

where δ is the loss angle at ambient temperature, if the permittivity is constant. This is not at present of great practical interest: a typical figure for impregnated paper, 200 VA/cm., would not be approached in practice. A large flat capacitor roll or stack where the breadth was λ times the thickness would have a limit for the volt-amperes per cm. length of the order of $3{\cdot}53\lambda k\theta_0/\tan\delta$, so that it is not until the breadth is more than twice the thickness that the advantage would seem to lie with the flat roll. Similarly, the maximum permissible dielectric

stress in the flat roll will not usually much exceed that for the cylindrical roll with the same capacitance until the breadth of the flat roll exceeds about twice the thickness.

The insulated cable forms another practical example of a cylindrical dielectric, but is complicated by the heat generated in the conductor, more than one conductor in a circular sheath, shaped conductors and the variation of conductivity with field strength, and other factors. An account of how approximate methods can be applied to take into account these variations from the ideal cases already treated has been given by Brazier.[11] It is not repeated since abbreviation would not be satisfactory.

3.4. Lateral cooling

K. W. Wagner,[12] who was the first to study experimentally the thermal nature of localized breakdowns, considered the instability of a hot section from which heat escaped into the surrounding dielectric. He used semi-conducting electrodes which both limited the current so that the breakdown process could be protracted and also impeded the flow of heat into the electrodes which were of low thermal conductivity. It is therefore of some interest to consider the idealized case when the electrodes are thermally insulating so that the flow of heat is at right angles to the electric field, which will be everywhere uniform although the current density will vary over the cross-section. A little consideration will show that the equations deduced for a cylindrical stratified dielectric will apply, in particular those equations which have been deduced when b/a, i.e. the ratio of external to internal diameter, is great, since in the present instance the hottest portion of the dielectric will be a filament at the centre of the cross-section. Thus from (3.50)

$$E = B\sqrt{(P_i/P_e)}\sqrt{(2k\theta_0/\sigma_0 b^2)} = B\sqrt{(2\theta_0/P_e\sigma_0 b)}, \qquad (3.53)$$

where E is the thermal electric strength; k, θ_0, and σ_0 refer to the same dielectric properties as hitherto; b is assumed to be the radius of the cylindrical surface of the disk specimen whence heat escapes through a thermal resistance P_e, while B is the function of b/kP_e or (P_i/P_e) given in Fig. 33. Clearly the electric

strength is independent of thickness and inversely proportional to the square root of the electrical conductivity. These were the main factors brought out by Wagner (see [12] and [20]), the second of which is a particular example of a general law (see section 3.1). Independence of thickness must obviously apply whenever the heat flow is at right angles to the electric field. There is no need further to consider the solution given because in a homogeneous dielectric it is limited to rather special conditions which in present practice do not often arise. Wagner proved experimentally the occurrence of instability in apparently secure conditions. From the analyses previously given, his result is now known to be general, namely that the hottest part of the dielectric immediately preceding instability may be well below the maximum working temperature of the material.

Another model which has been proposed is that of a semiconducting filament the resistance R of which falls with temperature according to $e^{-\theta/\theta_0}$. The dissipation of heat from such a filament is complicated since it cannot be treated as a line source as the temperature must be finite. Thus equilibrium never occurs until the control exercised by the cooled dielectric surface comes into play. It is possible to consider that the loss of heat from the filament is proportional to its temperature rise according to a thermal resistance P which has to be evaluated (as in the analogous example of a cable in short-circuit conditions) according to the time intervals involved. These assumptions lead to equations similar to those for a thin dielectric except that the electric strength E is constant,

$$E^2 = R\theta_0/Pe \quad \text{and} \quad \theta_m = \theta_0. \tag{3.54}$$

3.5. Defects and partial breakdown

A filament bridging the electrodes is somewhat rare, except as the result of earlier breakdown processes. Therefore it is relevant to inquire whether the increased field at the tip of a conducting defect, penetrating part of the dielectric wall, can cause thermal instability. In order to fix ideas, suppose that the d.c. conductivity and also the a.c. conductivity or specific energy losses can be expressed as $\sigma_0 e^{\theta/\theta_0}$, where θ is the tempera-

ture rise above ambient while σ_0 and θ_0 are constants. Let the dielectric constant ϵ and the thermal conductivity k both be independent of temperature. Then, for any unidimensional case such as a uniform field, concentric cylinders, or concentric spheres,† the maximum thermal voltage V_m for a thick dielectric, that is when the thermal resistance of the dielectric greatly exceeds the thermal resistance of one electrode, is $\frac{1}{2}q\sqrt{(k\theta_0/\sigma_0)}$‡ if one electrode is thermally insulated, or $q\sqrt{(k\theta_0/\sigma_0)}$ if both electrodes are cooled; where q is $2\sqrt{2}$ for d.c. and 1·88 for a.c. It follows, therefore, that if the thermal resistance is mainly within the dielectric, the thermal breakdown voltage is largely independent of shape so that a conducting inclusion adjacent to an electrode will not reduce the thermal breakdown voltage provided it does not seriously diminish the thickness of the dielectric wall. If the inclusion is in the body of the dielectric, or is of shape and composition such that heat is only feebly conducted to the cooled electrode, then the thermal breakdown voltage from the inclusion may be reduced to the value corresponding to a thermally insulated electrode. This reduction is 50 per cent. if the electrodes are both cooled, but there is no reduction if one electrode is thermally insulated in normal service. In general, the design of electrical insulation provides a factor of safety considerably in excess of 2 with respect to thermal breakdown. Again, if an electrode is well cooled, the thermal breakdown voltage is independent of thickness until the latter is so low that the thermal resistance of the dielectric is of the same order as that of the electrode. This means that penetration of the dielectric wall must proceed very far before thermal instability is produced by reduction of insulation thickness and still further to cause intrinsic breakdown. For all these reasons it appears possible to dismiss as inherently improbable the view that the

† It may readily be shown that V_m is the value of V for concentric spheres for which the following equation reaches the limit of real solutions:

$$k(d^2\theta/dx^2)+E^2\sigma e^{\theta/\theta_0} = 0, \quad \text{if} \quad E = V/a \quad \text{and} \quad x = a^2/r,$$

where $\theta = 0$ when r is great, i.e. $x = 0$; $d\theta/dx = 0$ when r is a, i.e. $x = a$. This is the same equation as for a uniform field in a thick dielectric or as for concentric cylinders when the ratio of radii is great.

‡ Equation (3.9).

concentration of field at or by a conducting defect can induce thermal instability and progressive breakdown.

A defect is, however, usually a source of energy dissipation independently of the dielectric. It will be shown in Chapter IV that if discharges occur the change in electrical energy appears as a generation of heat when averaged over a suitable period. It will also be shown that if these discharges produce in a void a conducting layer which short-circuits the discharges, then when the latter have disappeared the energy dissipated in the conducting layer is of the same order of magnitude as that dissipated by the discharges. The question arises as to how much heat must be generated at the defect seriously to lower the thermal breakdown voltage. A rigorous solution even of the simplest case is extremely laborious, but the order is not so difficult to find. It is easy to show that the heat in the defect will have an effect when the temperature rise which it produces is comparable with that from dielectric losses alone when thermal instability is approached.

With d.c. there is the difficulty that the temperature and conductance of the dielectric become theoretically infinite at thermal instability, but a practical result can be deduced by observing how thermal instability is approached. Consider a plane slab of dielectric, one electrode of which is thermally insulated; adjacent to this electrode is a defect in the form of a hemispherical intrusion into the dielectric of radius a. Let the thickness of the dielectric be d. If $d \gg a$ the temperature distribution θ when an electric field is applied is given by the solution of

$$\operatorname{arc\,tanh}\sqrt{(1-e^{-\theta/\theta_0})} = \operatorname{arc\,tanh}\sqrt{(1-e^{-\bar{\theta}/\theta_0})}\{1-(r\cos\phi/d)(1-(a^3/r^3))\}, \quad (3.55)$$

where $\bar{\theta}$ is the maximum temperature at the thermally insulated electrode; r and ϕ are polar coordinates from the centre of the sphere with $\phi = 0$ normal to the electrodes. The maximum thermal voltage V_m for this case is

$$V_m^2 = 2k\theta_0/\sigma_0. \quad (3.56)$$

$\bar{\theta}/\theta_0$ can be found as a function of V/V_m, where V is the applied

voltage. Similarly the power dissipated by the current flowing from the hemisphere can be calculated as W and plotted against V/V_m. Suppose now the hemisphere is a heat source of strength

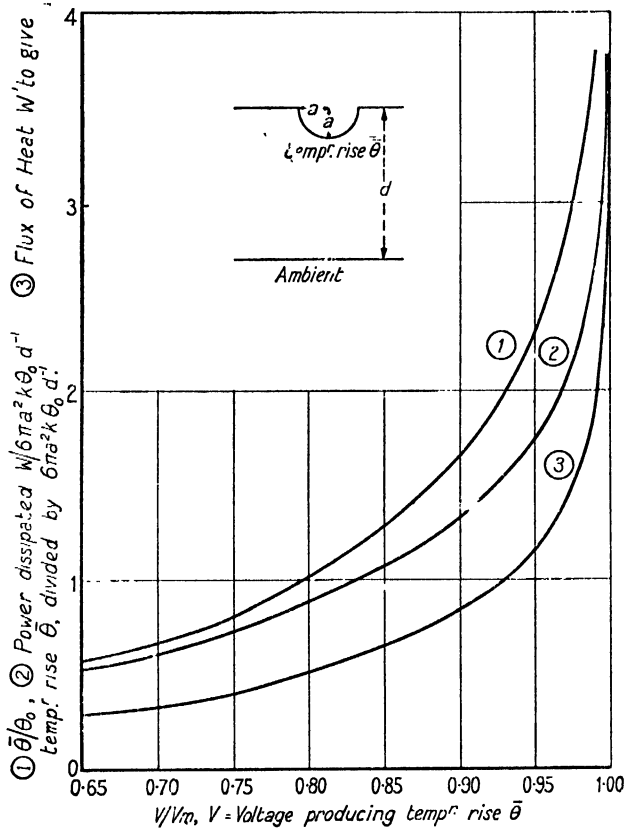


FIG. 34. Approach to thermal instability at a hemispherical defect with d.c.

W' and that this produces a temperature $\bar{\theta}$ at the surface when no electric stress is applied. Now $\bar{\theta}$ is also produced at the surface by a voltage V so that W' can be plotted against V/V_m, where V is the voltage which produces the same maximum temperature as W' in absence of the field. The results are plotted in Fig. 34. It seems that from $0{\cdot}95V_m$ onwards the temperature and energy consumption rise very rapidly. In

round figures if the heat generated in the hemispherical defect is about $12\pi a^2 k\theta_0/d$ or if the temperature of the hemisphere is raised by, say, $4\theta_0$ then the breakdown voltage will be appreciably lowered.

With alternating voltages the maximum stable temperature rise for a thick dielectric is finite and given by $1{\cdot}19\theta_0$ with the preceding assumptions. Accordingly, the same order applies both to d.c. and a.c., except that the energy dissipation in the defect must be some 3 or 4 times greater with d.c. than with a.c. to lower the thermal breakdown voltage by a comparable amount. This is not unexpected because the increase of d.c. conductivity with enhanced temperature tends to relieve of stress the portion of dielectric affected, whereas with a.c. there is no such relief, since the field distribution is largely independent of the in-phase conductivity or energy losses and is determined by the permittivity which changes little with temperature. Since d.c. is at present of less importance in practice, and since the behaviour is analogous with d.c. and a.c., attention will mainly be confined to a.c.

A question arises as to whether the foregoing applies to intrusions of a pointed nature where the field concentration is exaggerated while the volume of the intrusion remains appreciable. If the electrode adjacent to the intrusion is thermally insulated there is clearly no effect of field concentration on the thermal breakdown voltage which for a thick dielectric will have a value independent of shape. Thus the preceding conclusion holds good that the reduction due to field concentration cannot exceed a decrease of 50 per cent. and is zero if one electrode is thermally insulated. However, a given energy dissipation per unit volume or surface may yield a higher additional temperature rise with a pointed rather than a rounded defect. To test this, consider a plane dielectric with a prolate hemispheroidal intrusion in one surface and suppose that a uniform flow of heat U per sq. cm. of surface passes from the surface of the hemispheroid into the dielectric. Let the shorter semi-axis of the spheroid be b and the longer axis be c. Let θ be the temperature of a ring on the surface of the spheroid parallel to the base formed by the

electrode and distant z from it; then $k\theta/U\sqrt{(c^2-b^2)}$ is a numerical function of z/c depending upon the eccentricity of the elliptical section. Fig. 35 shows two curves, one where the ratio of breadth to length ($2b/c$) of the elliptical section is 1·732 and the other where the ratio of breadth to length is only 0·6325. Even

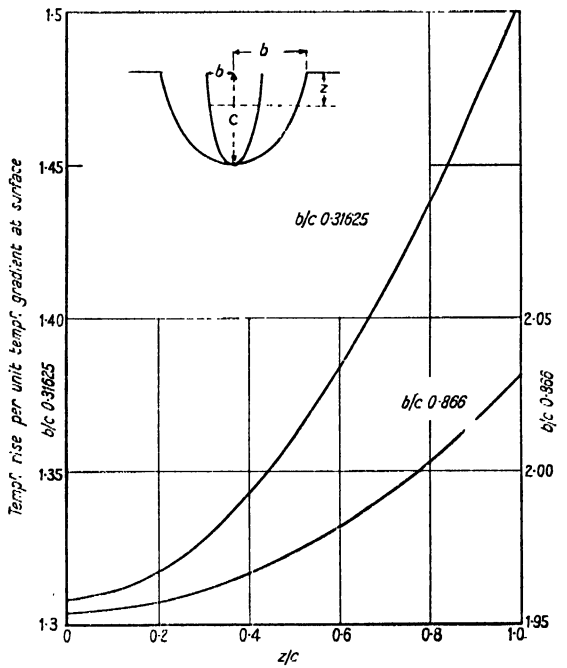


FIG. 35. Distribution of temperature rise with uniform flux of heat across a hemispheroidal surface.

for the more eccentric spheroid, the variation of temperature is not very great. Furthermore, with a pointed or cylindrical intrusion, the energy dissipated is unlikely to give a uniform surface flux of heat but one which would give a more uniform temperature distribution. Accordingly, it is unlikely that the shape of a defect will have any influence in reducing the thermal breakdown voltage either because of the field concentration or because the energy dissipated in the defect causes an excessive temperature at its tip.

Consider now the amount of energy dissipated in the defect

sufficient to produce a local temperature high enough to affect the thermal breakdown voltage. Referring to the hemispheroids of the last paragraph the thermal resistance or ratio of maximum temperature rise to total flux of heat over the surface is $1/5{\cdot}125ck$ and $1/2{\cdot}154ck$ for the less eccentric and more eccentric respectively. The thermal resistance of a hemisphere of radius c is $1/2\pi ck$, so that it will make little difference to the order of the result if a hemisphere of the same volume is adopted, which is much easier to calculate. If the source of heat generates W watts per cm.^{3} of the defect, then the condition that this should materially decrease the thermal breakdown voltage is

$$Wc^2/3k \simeq \theta_0. \tag{3.57}$$

If, instead of being rounded, the defect is thin in the direction of the field, a similar result may be deduced. Consider a concentric cylinder system of electrodes, as in a cable or bushing where the ratio of outer to inner diameter is e, the ratio where an increase of the inner diameter will just increase the surface electric field. If the defect covers a fraction λ of the circumference of the inner electrode, supposed thermally insulated, if the axial length is l and the thickness d, the thermal resistance from the void is about $1/2\pi\lambda lk$ and the volume is $2\pi\lambda ald$. Then the condition for an effect on the thermal breakdown voltage is

$$adW/k \simeq \theta_0. \tag{3.58}$$

These equations will be useful in the next chapter where the possible significance of W will be studied. Here consideration will be limited to heat generated in a conducting defect either of decomposed dielectric or a conducting film over the surface of a void. Consider, as before, a plane slab of dielectric of thickness d with a hemispherical defect of radius a on one face. If the plane and hemisphere are equipotentials, then the voltage v at any point is approximately the electrostatic solution

$$v/V_m = 1-(r\cos\phi/d)\{1-(a^3/r^3)\}. \tag{3.59}$$

V_m = impressed voltage, r, ϕ = polar coordinates from the centre of the hemisphere with $\phi = 0$ normal to the electrodes. Roughly speaking, the current through any element of the hemispherical

surface is that which flows through the capacitance between the surface beyond the element and the opposite electrode. By this means the energy dissipated in an imperfectly conducting film on the surface can be found. Calculation gives

$$\text{energy dissipation} = \left(\frac{\rho}{t}\right)\left(\frac{\epsilon^2\omega^2}{2\pi}\right)V_m^2\left(\frac{a^4}{d^2}\right)\frac{2}{3}\frac{10^{-22}}{144},$$

ϵ = dielectric constant, $\omega = 2\pi \times$ frequency,

ρ = resistivity of the film, t = thickness of the film.

If V_m is the maximum thermal voltage, V_m^2 is $3{\cdot}53k\theta_0/\sigma_0$, where σ_0 is the a.c. conductivity. If $\tan\delta$ is the loss angle of the dielectric at ambient temperature, then σ_0 is $\epsilon\omega\tan\delta(10^{-11}/36\pi)$. Whence

$$\text{energy dissipation} = \left(\frac{\rho}{t}\right)\frac{\epsilon\omega k\theta_0}{\tan\delta}\frac{a^4}{d^2}\frac{3{\cdot}53\times10^{-11}}{12}.$$

Assuming that the equipotential boundaries are also isothermals, then the energy dissipation to produce a temperature rise at the hemisphere of θ_0 is $3\pi a^2k\theta_0/d$.† Such a temperature rise will appreciably reduce the thermal breakdown voltage. Accordingly, the nature of the defect necessary to affect the breakdown voltage is given by

$$\frac{\rho}{t}\frac{\epsilon\omega k\theta_0}{\tan\delta}\frac{a^4}{d^2}\frac{3{\cdot}53\times10^{-11}}{12} = \frac{3\pi a^2k\theta_0}{d},$$

whence $$\rho = (\tan\delta/\epsilon\omega)(td/a^2)3{\cdot}2\times10^{12} \text{ ohm-cm.} \qquad (3.60)$$

At 50 cyc./sec., ω is 100π. Let $\tan\delta$ be 10^{-3}, ϵ be 3, and d/a be 10. Then ρ becomes $3{\cdot}4\times10^7t/a$ ohm-cm. This would yield practicable values for conducting films since 10^7 ohm-cm. is found with some carbonized insulants. If t is 10^{-6} cm., and a is 0·1 mm., ρ would be 3,400 ohm-cm., which is not unreasonable for some electrolytic products. At higher frequencies solid carbon could satisfy the conditions. Adopting the same values for d/a and t/a, and assuming a value of 10^{-4} for $\tan\delta$, as with hydrocarbons at 5 Mc./s., ρ would be $3{\cdot}4\times10^{-3}$ ohm-cm., a possible value for carbon in a coherent form. The conclusion

† The difference from equation (3.57) is due to the fact that the adjacent electrode was there supposed thermally insulated: here it is an isothermal.

for the rounded void or defect is therefore that rather special conditions are necessary for thermal instability at power frequencies, but there is no difficulty at radio frequencies.

Consider next a breakdown path of resistivity ρ like the stem of a tree, length l and radius b, but ending in a spherical knob of radius a. Suppose the heat, generated by the flow through the stem of the capacitance current from the knob, passes into the dielectric by way of the knob. The capacitance of the knob is ϵa cm. and its thermal resistance is $1/2\pi ka$. Accordingly, if a temperature rise θ_0 is to be produced by the heat generated in the stem by the capacitance current which corresponds to the application of a voltage to the knob equal to the thermal breakdown voltage, then it can readily be calculated that

$$\rho = 1{\cdot}6 \times 10^{12} \frac{b^2}{al} \frac{\tan \delta}{\epsilon \omega} \text{ ohm-cm.} \qquad (3.61\text{ a})$$

If the stem is not solid, but is a hollow cylinder of thickness t, then

$$\rho = (\tan \delta / \epsilon \omega)(bt/al) 3{\cdot}2 \times 10^{12} \text{ ohm-cm.} \qquad (3.61\text{ b})$$

which is very nearly the same as the equation for the hemispherical defect and, as far as order is concerned, the same conclusions apply, namely that propagation at 50 cyc./sec. requires special conditions but is easy at radio frequencies. When there is a tree with branches as in Robinson's theory of cable breakdown or as in surface tracking (see Chap. IV), the capacitance or conduction current from the branches to the surrounding dielectric is concentrated in the stem of the tree which may readily be propagated at all frequencies by thermal instability if the branches are extensive enough. The problem still remains as to the propagation of the stem beyond the branches as stated by Brazier[11] when the dielectric wall has been sufficiently penetrated. It is possible that the increased stress coupled with an enhanced loss angle at the higher stress, due to discharges or other mechanism, reduces the thermal breakdown voltage so that self-propagation can occur. This has been proposed for thermal breakdown in thin specimens.

Lastly, consider a void with a conducting film where the void

is thin (thickness = d) in the direction of the field but of appreciable area A normal thereto. The capacitance of the void to the opposite electrode will be of the order of $A\epsilon 10^{-11}/36\pi D$ farads and the thermal resistance will be about D/kA where D is the effective thickness of the dielectric from the void to the opposite electrode. It may then be deduced that when the thermal breakdown voltage V_m is applied, the capacitance current flowing through a film of thickness t on the surface of the void will heat the surface to a temperature θ_0 capable of appreciably reducing the thermal breakdown voltage if the resistivity ρ of the film is

$$\rho = 5.10^{13}(\tan\delta/\epsilon\omega)(Dt/d\sqrt{A}) \text{ ohm-cm.} \qquad (3.62)$$

It is possible to suppose that $Dt/d\sqrt{A}$ is 10^{-6} and $\tan\delta/\epsilon\omega$ of the order of 10^{-7} at 50 cyc./sec. which gives $\rho = 5$ ohm-cm. Thus at power frequencies breakdown by the heating of a carbon film is possible, though rather unlikely, but there is no difficulty with films of higher resistivity. At higher frequencies a great decrease of thermal breakdown voltage may readily occur.

Finally it may be noted that the most severe conditions arise when the defect forms a small volume of deteriorated material where the conductivity is only just high enough for the conductance to be comparable with the permittance. Thus if, for some reason, a defect of this kind arises, local thermal instability takes place. This may result in a greatly increased conductivity which, as has been shown, may prevent further instability. Accordingly, partial thermal breakdowns due to local defects are not improbable and evidence shows that they are not infrequent in practice.

Thermal instability is thus unlikely to arise due to a local concentration of field or of heat flow. The defect must itself generate heat to cause thermal instability except where special conditions exist. A conducting defect of a tree-like form may be thermally unstable in the stem at all frequencies. Neither the stem nor a breakdown path without branches nor a rounded defect will propagate itself by thermal instability at power frequencies except in rather special conditions, but propagation

at radio frequencies occurs readily. At power frequencies partial thermal breakdowns are to be expected and indeed are found.

3.6. The time factor in thermal breakdown

The thermal breakdown voltage so far considered corresponds to the limit of thermal instability and to breakdown after a theoretically infinite time of application of the electric field. If a greater voltage is applied, the temperature will rise to the decomposition value θ_c in a finite time. With a thin dielectric and with a simple variation of conductivity with temperature such as the exponential type hitherto considered, it is easy to evaluate the time to reach θ_c since the rate of increase of temperature is equal to the difference between the heat generated and that dissipated divided by the thermal capacity S. If the conductivity is also a function $\psi(E)$ of the field E then it is a matter of simple integration to show that the time t to reach θ_c, i.e. to breakdown, if a voltage V is applied to a specimen of minimum breakdown voltage V_m, is[3]

$$t = 2SP_c \sum_{n=0}^{n=\infty} e^{n+1} \left\{ \frac{V_m^2 \psi(V_m)}{V^2 \psi(V)} \right\}^{n+1} \left[\frac{n!}{(n+1)^{n+1}} - e^{-(n+1)\theta_c/\theta_0} \left\{ \frac{1}{n+1} \cdot \left(\frac{\theta_c}{\theta_0}\right)^n + \frac{n}{(n+1)^2} \left(\frac{\theta_c}{\theta_0}\right)^{n-1} + \ldots + \frac{n!}{(n+1)^{n+1}} \right\} \right]. \tag{3.63}$$

If $\psi(x)$ is a power of x, then the terms containing V_m and V are of the form $(V_m/V)^m$. Thus the relation between proportional over-voltage, i.e. ratio of applied voltage to minimum breakdown voltage, and time for breakdown is the same for all ambient temperatures. Therefore if curves of the logarithms of the applied voltages are plotted against ambient temperatures then the curves characterized by different constant times for breakdown will be parallel. This is illustrated in Fig. 28 from Whitehead and Nethercot[3] where the curves of minimum breakdown voltage (infinite time of application), 10 minutes to breakdown and one curve for 30 seconds to breakdown are all parallel; in the other 30 second curve a non-thermal breakdown mechanism was present.

Equation (3.63) may usually be approximated by putting $n = 0$ in the second member so that

$$t = 2SP_c\Big[\sum_1^\infty e^n(n-1)!\,(V_m^2\psi(V_m)/nV^2\psi(V))^n - \\ -(V_m^2\psi(V_m)/V^2\psi(V))e^{1-(\theta_c/\theta_0)}\Big]. \quad (3.64)$$

If θ_c is large and ψ is unity, then the second member of equation (3.64) can be neglected and

$$t = 2SP_c\sum_1^\infty (n-1)!\,(V_m^2/nV^2)^n e^n, \quad (3.65)$$

which was deduced by Wagner[12] for the form of breakdown postulated by him and discussed in the last section. Another approximation is

$$(V/V_m)^2 = 2cSP_c(1-e^{-\theta_c/\theta_0})/t. \quad (3.66)$$

This elementary treatment indicates the general features which have been verified in a number of experiments, but quantitative determination of breakdown time is difficult owing to the indefinite nature of the breakdown condition with unstable temperature distributions.

When the specimen is thick enough for the temperature distribution in the dielectric to be of importance, theoretical solution is difficult. Some estimates were made by Goodlet, Edwards, and Perry,[13] and subsequently calculations were made, using a differential analyser by Copple, Hartree, Porter, and Tyson.[14] In both papers a uniform field was assumed and the outer surfaces of the dielectric slab were supposed to be maintained at the constant ambient temperature. Utilizing the symbols of section (3.2), the problem reduces to the solution of the non-dimensional equation

$$\frac{d(\theta/\theta_0)}{d(t/2SP_i)} = \frac{d^2(\theta/\theta_0)}{d(z/d)^2} + (3{\cdot}53V^2/V_m^2)e^{\theta/\theta_0}, \quad (3.67)$$

where d = total thickness, V = applied voltage, V_m = minimum thermal breakdown voltage. When $V = V_m = 1{\cdot}88\sqrt{(k\theta_0/\sigma_0)}$ the temperature distribution is just stable and attained after an indefinite period. When V exceeds V_m the distribution is

unstable. The boundary conditions are that $\theta = 0$ everywhere when $t = 0$, $\theta = 0$ at all times when $z = 0$ or $z = d$. It is possible to determine θ as a function of t and z, but it is only the central temperature, θ in Fig. 36, which is really of interest since thermal instability begins at the hottest point. Thus the authors mentioned have calculated the variation of θ with time for applied voltages of magnitude $0{\cdot}75V_m$, $1{\cdot}07V_m$, and $1{\cdot}5V_m$,

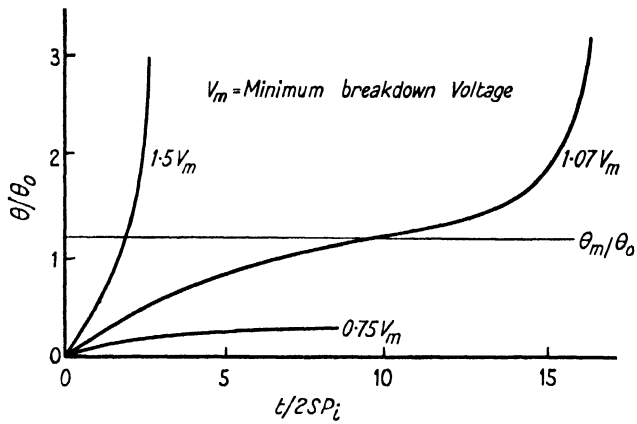


FIG. 36. Calculated variation of highest temperature. (Copple, Hartree, Porter, and Tyson.)

i.e. 25 per cent. under-voltage, 7 per cent. and 50 per cent. over-voltages respectively. The results are shown in Fig. 36 and indicate that even when the applied voltage is only 7 per cent. above the thermal breakdown voltage the temperature ultimately rises rapidly. It is observed that a good deal of time is consumed in attempting stability or reaching an inflexion in the temperature/time curve near the limiting temperature for stability θ_m. For 7 per cent. over-voltage this time is of the order of $20SP_i$. Now SP_i represents the internal thermal time constant. Since the thermal resistivities of most dielectrics lie between about 200 and 1,500 thermal ohm-cm., and since the specific thermal capacitances lie in a range around (but mostly below) 1 joule/cm.^{3} it is clear that $20SP_i$ is a long time unless the dielectric is very thin, when the earlier analysis applies since the external thermal resistance is now important. It may

be deduced, therefore, that in practice thermal instability takes some time to develop unless a considerable over-voltage is applied. This is confirmed by experimental curves given in the same paper, reproduced in Fig. 37, which closely resemble those found by Whitehead and Nethercot on thin specimens. Once the temperature starts rising unstably it does so rapidly and

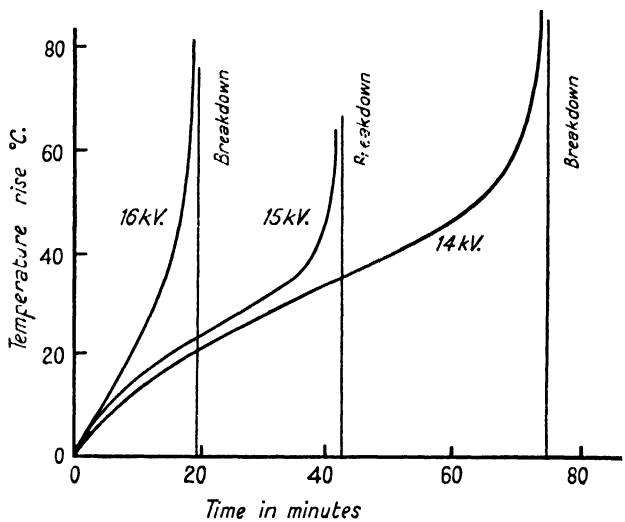


FIG. 37. Observed temperature variation.
(M.-V. Research Dept.)

breakdown can appear suddenly, but only after a considerable total period of application of voltage. Besides the differential analyser, solutions have been obtained in series of functions and by numerical methods as by Grunberg, Kontorovich, and Lebedev[15] but do not affect the conclusions already given. Solutions of this type of problem are of academic interest rather than utility since the hottest portion behaves near enough as a thin specimen.

3.7. Thermal breakdown in practice

The range of application and the confirmatory evidence for the principles of thermal breakdown have been given in earlier chapters in relation to specific principles enunciated. At industrial

frequencies, designers in general pay appropriate attention to thermal instability. In addition to the references already made, a fairly large body of earlier evidence on bushings is given in the previous book on this subject by the present author[21], a summary for d.c. was made by Moon[16], and a recent example is the work of Zalessky[17]. With cables the evidence of Brazier[11] has already been mentioned. At these lower frequencies, high voltage equipment is in general the type affected. At high frequencies the energy losses are much greater and their thermal effect is a major problem which has led on the one hand to a great improvement in dielectrics for high-frequency purposes and has stimulated, on the other hand, the use of pulse techniques. The latter have the advantage that high stresses of short duration can be withstood by dielectrics, and if the ratio of active to inactive time is small, the effective values of the continuous harmonic frequencies into which the repeated pulse may be resolved are low enough to avoid thermal breakdown. Conversely, however, there are repeated transients which impose sometimes a greater strain than is imagined. For example, the contact breaker capacitor in an electric ignition system is effectively exposed to high-frequency voltages of some magnitude from the resolution of the repeated damped oscillations applied. Owing to the higher conductivity of the dielectric, e.g. paper, at high frequencies and the construction of the capacitors, thermal instability is a greater danger than would be usually thought to apply to a low voltage d.c. system. Nevertheless, thermal instability only occurs in practice when other forms of deterioration have reduced the quality of the paper dielectric.

There are definite limitations to the application of thermal breakdown as it is understood here. It cannot occur with stresses of very short duration. Consideration of the time factor in the preceding section shows that for durations of the order of milliseconds or less the applied voltage would have to be impossibly high compared with the impulse ratios actually observed, i.e. the ratio of the impulse to the sustained breakdown voltage. An exception is in a special case such as the effect of certain types of impulse on rock salt above 350° C.

where Walther and Inge[18] found that V^2T was constant, where V is the breakdown voltage and T the duration of the impulse. This result is consistent with the previous section, since thermal absorption would greatly outweigh thermal dissipation in short intervals. Similarly, it has been shown that the propagation of a partial breakdown by thermal instability requires special conditions. Very often thermal breakdown is said to be the end process of a breakdown mechanism when in fact what is meant is that a feebly conducting path between the electrodes is transformed into a highly conducting path by the energy developed which extends or intensifies the damage. Similarly, in a number of papers, thermal breakdown is stated to have occurred in circumstances which render it doubtful. In some of the work of Moon and Norcross[19] on glass, quantitative comparison of calculated thermal breakdown voltage with measurement is needed to establish the thermal breakdown of thin specimens with electrode arrangements of substantial dimensions. It is often thought that sufficient evidence is the decrease of electric strength with temperature at the same rate as the decrease of the square root of the electrical resistivity and proportionality of breakdown voltage to the square root of the thickness. The latter relation can be associated with other mechanisms while the former is only true when the appropriate thermal conditions are fulfilled in the manner already described. These conditions require some care if they are to be reproduced with the accuracy required for a crucial test.

Both in laboratory experiments and in industrial practice it is difficult to be sure that thermal breakdown has occurred or to determine the limit of stability. The physical appearance of the breakdown path can only provide confirmatory evidence. The observation of the temperature variation of the hottest part of the dielectric is adequate but difficult to achieve. Nevertheless, there is the alleviating feature that the temperature cannot rise too high. Except in certain cases with d.c., the temperature rise cannot exceed a value of the order of θ_0, the parameter in an assumed variation of conductivity e^{θ/θ_0}. Since θ_0 is not usually very large, there is no need to allow the

temperature to rise very high to know that thermal instability will ensue. A less accurate but more practical method is to observe the variation of the conductivity, i.e. the current with d.c. or the loss angle with a.c. The direct current, except with thick specimens, and the a.c. loss angle in nearly every instance can only rise to not more than about 2 to 3 times the initial value if stability is to be maintained. The most satisfactory test of thermal breakdown is to calculate the breakdown voltage and obtain agreement with experiment. While difficult in industrial practice, it can and has been done, as already mentioned, in controlled laboratory tests, but all the conditions, particularly the thermal conditions, must be carefully observed. It is also to be noted that with d.c. it is the irreversible conduction current from which the conductivity must be determined, because it is only this current which generates heat. Accordingly, absorption and so-called 'polarization' currents which do not contribute to the thermal balance-sheet must be ignored.

Since the variation of conductance has been mentioned, it is convenient to summarize the formulae for the ratio Q of the conductance or loss angle at the limit of stability to the corresponding initial values, i.e. the maximum stable proportionate rise of conductance or loss angle, assuming the dielectric initially to have everywhere the ambient temperature.

(a) Thin specimens d.c. and a.c.

$$Q = e. \tag{3.68}$$

(b) Thick plane or cylindrical specimens d.c.

$$Q = (\alpha/\sin\alpha)\sec\alpha\, e^{2\alpha\cos^2\alpha/(\alpha+\sin\alpha\cos\alpha)}. \tag{3.69}$$

α is a function of P_i/P_e given by equation (3.24).

There is no finite limit of stability in the limiting case $\alpha = \frac{1}{2}\pi$ for very thick or well-cooled specimens.

(c) Thick plane specimens a.c. or a.c. and d.c. if stratified

$$Q = \eta^{-1}\tanh\eta\cosh^2\eta\, e^{2P_i\eta\tanh\eta/P_i} \tag{3.70}$$

η is a function of P_i/P_e given by equation (3.32).

Limit of $Q = 2{\cdot}27$ (very thick).

(*d*) Cylindrical stratified dielectric a.c. or d.c. (see Fig. 33 when a/b is small)

$$Q = e \text{ (thin)}, \qquad Q = 2 \text{ (very thick)}. \tag{3.71}$$

The experimental investigation of thermal instability demands the elimination of edge discharges as in the measurement of intrinsic electric strength (see Chap. II), but less rigorous

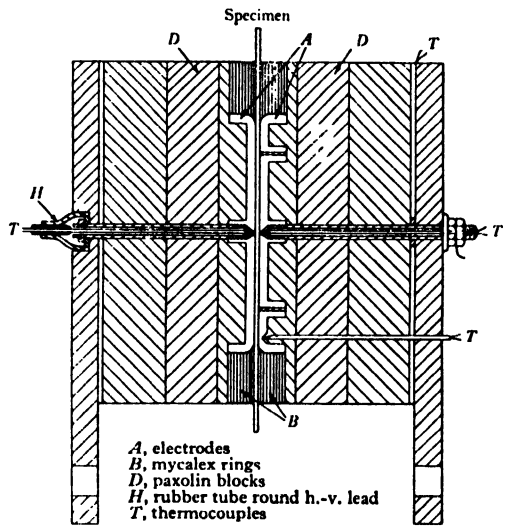


FIG. 38. Apparatus for thermal instability tests.
(Whitehead and Nethercot.)

measures may generally be used because the thermal electric strength cannot exceed the intrinsic electric strength and must usually be much less for accurate study. Apart from this and from the means of detecting instability already discussed, the main desideratum is to control and measure the thermal conditions. Inge and Walther and others who have used their technique have employed either a foil electrode backed by a further slab of the material under test or, following Wagner, special electrodes with a fairly high thermal resistivity. In the highest maintained electric strength test thermal insulating packing of a standard type is required. Whitehead and Nethercot[3] used electrodes backed by a known thermal resistance as in Fig. 38 with thermal insulation to reduce lateral heat flow.

Most of the workers mentioned have used the device of flat resistance elements which may be inserted in place of the dielectric or between the dielectric and the electrodes in order to calibrate the thermal dissipation. In the measurement of energy losses by the thermal method this technique has been greatly developed, but the final choice in this connexion has been a calorimeter method, sometimes a double balanced calorimeter, which does not reach necessarily a constant temperature and so does not apply to thermal breakdown.

REFERENCES QUOTED IN CHAPTER III

1. INGE and WALTHER. See summary of their work on thermal breakdown in SEMENOFF and WALTHER, *Die physikalischen Grundlagen der elektrischen Festigkeitslehre* (Springer, 1928).
2. BECKER. *Arch. für El.* 1936, **30**, 411.
3. WHITEHEAD, S. and NETHERCOT. *Proc. Phys. Soc.* 1935, **47**, 974.
4. AUSTEN. *J.I.E.E.* 1945, **92**, part i, 373.
5. FOCK. *Arch. für El.* 1927, **19**, 71.
6. DREYFUS. *Schweizer Elektrotechnischer Verein Bull.* 1924, **15**, no. 7, p. 321 and no. 12, p. 577.
7. BERGER. Ibid. 1926, **17**, no. 2, p. 37. *E.T.Z.* 1926, **47**, 665.
8. GEMANT and WHITEHEAD. *Phil. Mag.* 1939, **37**, 582.
   GEMANT. *J. Franklin Inst.* 1939, **228**, 79.
9. WHITEHEAD, S. *Phil. Mag.* 1939, **37**, 276.
10. GOODLET. *Proc. Cambridge Phil. Soc.* 1933, **29**, 401.
11. BRAZIER. *J.I.E.E.* 1935, **77**, 104.
12. WAGNER. *Trans. A.I.E.E.* 1922, **41**, 283. *Sitzungsberichte der Akad. der Wissenschaft, Phys.-Math. Klasse*, 1922, **29**, 438.
13. GOODLET, EDWARDS, and PERRY. *J.I.E.E.* 1931, **69**, 695, 727, and 1195.
14. COPPLE, HARTREE, PORTER, and TYSON. Ibid. 1939, **85**, 56. See also JAEGER, *J. Sci. Inst.* 1950, **27**, 226.
15. GRUNBERG, KONTOROVICH, and LEBEDEV. *J. Tech. Physics, U.S.S.R.* 1940, no. 3, **10**, 199.
16. MOON. *Trans. A.I.E.E.* 1931, **50**, 1008.
17. ZALESSKY. *E. u. M.* 1935, **53**, 112.
18. WALTHER and INGE. *J. Tech. Physics, U.S.S.R.* 1938, **5**, 335.
19. MOON and NORCROSS. *J. Franklin Institute*, 1929, **208**, 705.
20. WAGNER. *Arch. für El.* 1948, **39**, 215.
21. WHITEHEAD, S., *Dielectric Phenomena*, vol. iii, *Breakdown of Solid Dielectrics* (Benn, London, 1932).

CHAPTER IV

BREAKDOWN CAUSED BY DISCHARGES

4.1. Early recognition

THE earliest history of electrical science records the disruptive effect of the lightning discharge in penetrating considerable thicknesses of insulating materials during its passage to earth. This is an example of dielectric breakdown due to an external discharge. The same occurred with the laboratory spark when high enough voltages were artificially produced. For this and other reasons solid dielectrics were first used only as mechanical supports. The main insulation was air, considered to be the ideal dielectric, as in fact it then was. The deleterious effects of limited discharges such as the glow, brush, or corona in addition to the unstable spark were soon recognized on account of the action of discharges at the edges of the metal attachments to insulating supports. To this day it is impracticable always to avoid corona on the insulators of outdoor gear. As a consequence, the action of external discharges on solid dielectrics is a commonplace of electrical engineering. A certain measure of external discharge may be tolerated with some materials for a limited period.

A rather similar phenomenon is well known as 'high-voltage tracking'. This refers to conducting paths over the surface of the dielectric when the surface separates electrodes between which a high voltage is maintained. It is necessary to distinguish between what is here called 'high-voltage' tracking, which essentially involves a discharge to start it, and 'low-voltage' tracking, where the conducting path may arise from thermal decomposition at discontinuities in a semi-conducting surface without the intervention of a macroscopic discharge; indeed it may occur at voltages less than the minimum breakdown voltage of the ambient medium.

The importance of internal discharges was not appreciated until continuously applied solid insulation was used. Insulation of this kind was first exposed to higher voltages in bushings,

transformers, and cables. Precursors of the 'hissing' test for bushings were in early use among designers, and this is an audible test for discharges. Attention was given to the oil-filled high-voltage transformer where the solid insulation was considered to provide barriers to prevent the formation of bridges of impurity across the oil, which was thought to be the main insulant. Consequently, the deleterious action of discharges was rather confused with breakdown of the oil. In cables it appears that the impregnant in paper insulation was first introduced to prevent loss of insulating properties by ingress of moisture. It was found fairly early that faulty impregnation was a weakness in circumstances where very little moisture could enter. The effect of internal voids in increasing energy loss was fairly soon realized, as has been described in the present writer's previous book on the breakdown of solid dielectrics. Emanueli (1926) developed the pressure cable impregnated with thin oil on the hypothesis that the voids, which formed more easily with more viscous impregnants at normal pressure, were the limiting factor in cable breakdown. He was not very clear as to what happened in the voids. About the same time the term ionization began to be used for variation of power factor with stress, but without detailing the nature of the ionization invoked. In 1932 Gemant and Philipoff[2] described the incidence of discharges by means of a model circuit and a spark gap, indicating the discrete and sudden nature of the discharges and the influence of the time constants of the circuits. By 1933 the existence of discrete internal discharges was known and a discharge detection bridge was described by Tykociner, Brown, and Paine.[3]

In 1935 Robinson[4] described a reasonably complete mechanism for the eventual breakdown of a cable due to discharges. He realized the discrete nature of the gaseous discharge in voids and that with alternating voltages the periodic variation of voltage would cause the discharges to repeat each cycle, whereas with a constant unidirectional stress a discharge would occur on application and on removal but not necessarily in between. He distinguished between 'cores', which were fine channels

through the thickness of a paper, and 'trees' which arose from surface action between layers of paper and round their edges. The dependence of propagation of the discharge channel upon the existence of transverse stress was a main feature of his theories. Brazier,[5] at the same time, described the thermal instability of the stem of a tree-like discharge channel and stated that this instability became self-propagating when the dielectric had been sufficiently penetrated by the stem of a tree formation. Arman and Starr,[6] pursuing parallel inquiries in association with Robinson and Brazier, described the instantaneous nature of the discharge and developed the Discharge Bridge which used, as a means of detection, the higher frequencies produced by the rapidity of the phenomena. Some quantitative deductions were also made. Quinn,[7] at a later date, incorporated the dielectric under test in a tuned circuit and used an amplifier and oscillograph to detect the oscillations excited by the discontinuous nature of the discharges. The work of J. B. Whitehead[8] and his co-workers gave experimental support to these findings, although, until a fairly late date, he tended to ascribe the discharges to electrical instability in the impregnant, oil or compound, and dealt in detail with problems such as the movement and drainage of the impregnant which are of a specific industrial interest rather outside the scope of the present book.

It has already been shown that intrinsic breakdown in a liquid is difficult to determine.[9] Unless special precautions are taken, the discharge in the liquid is more usually associated with the presence of a gas bubble or the evolution of gas after the preliminary stages. If the discharge develops into arc-like conditions, the current is carried by a gaseous discharge. It seems, therefore, safe to conclude that when internal discharges occur in a solid dielectric even if impregnated with a liquid, the agency which counts towards breakdown in the majority of instances is a gaseous discharge. Attention will therefore be confined to gaseous discharges.

The early recognition of edge discharges, surface flashover, and breakdown beginning at a surface has already been mentioned.

Study was pursued mainly along two lines corresponding to certain practical interests. Engineers were concerned with the effect of external discharges which occurred at frequent intervals over long periods at working voltages and therefore at normal stresses in the solid dielectric; these were anything from 1/20th to 1/100th of the intrinsic electric strength or even less. Interest was therefore concentrated on deterioration due to discharges. Conversely, laboratory workers were concerned to reduce the effect of discharges so as to measure the intrinsic electric strength as described in Chapter II. They tended, therefore, to study the effect of discharges when measures, already taken, had increased the voltage at which the edge discharge occurred to a value corresponding to a high stress in the solid dielectric, often approximating to the intrinsic electric strength. Even in industrial electric strength tests, where no very great reduction of edge discharges is achieved, the stress in the solid dielectric can reach 1/5th of the intrinsic electric strength.

It is not proposed to repeat the details of the earlier work, but to incorporate findings of general significance in the account which follows. The method of treatment will follow the scheme of exposition adopted by recent workers such as Austen, Thomas, Hackett, and Davis, with whom the writer has been associated. The work of Davis on the problem of the effect of discharges on impulse breakdown with few repetitions of the impulse must be considered separately since principles elsewhere valid cannot easily be applied.

4.2. Gaseous discharges

To aid in understanding later sections of the present chapter, a summary of the properties of gaseous discharges here relevant will be given. Townsend discovered ionization by collision and showed that a suitable factor additional to field acceleration of electrons could yield an unstable self-increasing ionization. The suitable factor proved a difficult problem. It is generally agreed, and has been adequately documented recently by Loeb and Meek,[10] that a gaseous discharge begins as an electron avalanche. In long sparks such as the lightning discharge, an electron

avalanche or leader stroke traverses the gas, often in steps, leaving behind an ionized channel through which the heavy current return stroke passes. In short sparks the return stroke cannot easily be distinguished as a separate phenomenon and in the very short discharges of this chapter the primary discharge alone need be considered. An electron avalanche can be short-lived and fail to discharge the gap: it is necessary that it should bridge the gap with a highly conducting channel. According to Meek,[11] the condition for complete instability is provided by the existence of an adequate field transverse to the discharge channel and photo-ionization in the gas. According to Bruce[12], a measure of transverse current, either from more diffuse ionization round the main channel or from displacement currents due to the field change caused by the advancing discharge, is required to yield a sufficient current to produce a transition in the discharge to arc-like conditions. Since it follows from Meek's hypothesis that the current must also attain certain proportions, although these vary with the conditions, it is possible to conclude that a low-resistance discharge cannot occur with too low a current or current density. Rogowski[13] considered the limit for low resistance conditions to be a current density of 10^{-4} A/cm.^{2} Bruce mentioned values of the order of 0·5 A for arc-like conditions but has later quoted lower figures. Two methods of describing the terms 'arc' and 'glow' are current. The first pays attention to phenomena at the cathode. In an 'arc' the cathode fall of potential is low, 10–20 V, and ions from the cathode appear in the gas: in a 'glow' the cathode fall of potential is higher, e.g. 100 V, and the excited molecules are gaseous. The second method is concerned with phenomena in the gas and 'arc' or 'arc-like' refers to discharges of field strength very low compared with the initial value and high current density and luminosity, whereas in a 'glow' the luminosity and mean current density are low but the field strength is high, some hundreds or thousands of V/cm. The second method is usually pertinent to the high-voltage phenomena here considered and will be adopted. In the 'spark' a low-resistance or arc-like channel develops through the gas but is not sustained

long enough to be stabilized by the effect of the electrodes or external circuit.

There is a good deal of evidence[14] that discharges in a gas bounded by insulating material occur at potentials of order similar to those required with metal electrodes and, provided there is adequate charge available on the dielectric surfaces, have much the same properties in a transitory régime as discharges between metal electrodes. Therefore a spark discharge should neutralize the surface charges near the ends of the discharge. High surface resistivity may limit the area discharged, and surrounding areas will tend subsequently to restore the potential: this will be referred to later. Apart from this some experiments have shown the potential remaining after discharge to be as low as are electrode potentials, 20–50 V. Elsewhere it has been stated that the remanent field after discharge may be that of a glow, say 500 V/cm. to some kV/cm. Other workers, such as Dawes and Humphries[15] who exposed large areas of dielectric to discharges, say that the remanent potential is some 50–80 per cent. of the potential required to start the discharge. With small voids the balance of evidence, particularly that with which the writer has been associated (see Mason[16]) favours a low remanent potential, not more than 100 V or so, or 10–20 per cent. of the discharge voltage.

In a fairly uniform field a discharge usually develops into a spark if enough energy is available. Fig. 39 shows the potential necessary in air and hydrogen, and indicates a certain minimum potential below which no discharge occurs. For air the minimum is 300–400 V when the product of pressure in mm. Hg and spark length in cm. is 0·5–0·8. At atmospheric pressure the corresponding spark length is of the order of 0·001 cm., provided other fundamental phenomena such as photo-effects do not intervene. For spark lengths from, say, 1 mm. upwards, in air at 20° C. and 760 mm. Hg

$$V = g_0 x + a\sqrt{x} \quad \text{(Toepler, Stephenson, Bruce[17])}, \qquad (4.1)$$

where x = spark length in cm., $g_0 = 24{\cdot}75$ kV (crest)/cm., $a = 5{\cdot}95$ kV/cm.$^{\frac{1}{2}}$

Once the electron avalanche has formed in favourable conditions, the propagation of the discharge is rapid, probably at a rate of the order of 10^7 cm./sec. in air and possibly 10^8 cm./sec. in hydrogen. After a discharge has died out due to exhaustion of the available electrical energy, the ionization decays rapidly. This is a complicated subject, but it is sufficient here to say that

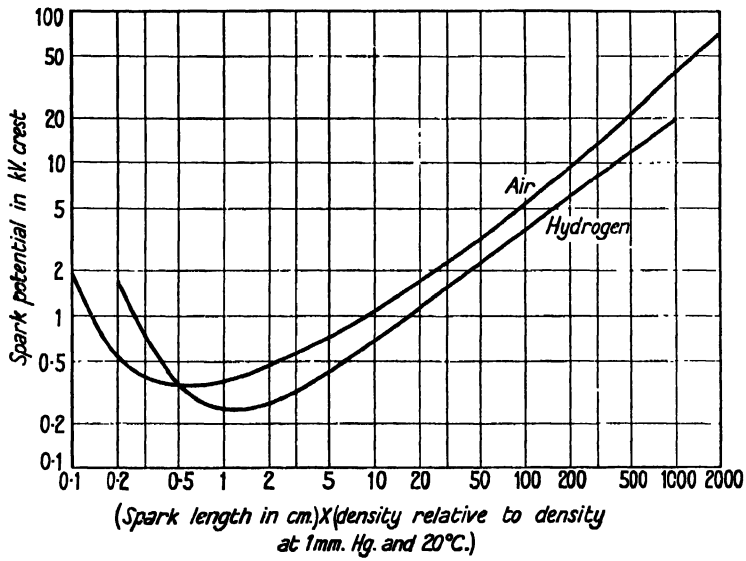


FIG. 39. Spark potential as a function of spark length and density - Paschen's Law.

the decay time will not exceed the order of a micro-second in the present circumstances. The electronic processes in a gas are much slower than in the electronic breakdown of solids where the relaxation time of the electrons is of the order of 10^{-14} sec.

The term corona has been used by engineers to mean a discharge which does not apparently bridge the electrodes, is visible or audible, and can continue without forming a visible arc or requiring a large mean current from the source of potential. On the basis of the views here adopted, corona may comprise a diffuse discharge or glow, but one of its characteristic appearances, intermittent 'brushes', may be due to a large number of

intermittent sparks which fail to bridge the electrodes and end in the gas or on an insulating surface. It is believed that low-resistance conditions are developed along the whole channel of some of these sparks if they end on a dielectric surface or along part of the channel if they end in air. If so, then large transient currents can flow, although the mean current may be very low. The 'dark' discharge, which in any event takes place beyond the luminous region, is sometimes called 'dark corona'.

In a divergent field such as that near a pointed electrode or a small sphere, and when the available energy is limited, as when a sheet of solid dielectric is interposed, characteristic discharge channels are formed. With maintained alternating voltages a number of these patterns are superposed, owing to the variation of the potential which extinguishes and reignites the discharges, and to the reversal of polarity. With unidirectional impulses, single patterns may be seen and recorded photographically. Fig. 40 (Plate IV) shows characteristic examples (both when the spheres are positive and negative) of these patterns which are known as Lichtenberg's figures. Recently† Thomas[18] has found that the pattern presumably of bound charges may be impressed on a hard dielectric such as a gum or resin and then thermally developed in relief by softening the dielectric. The relief is fixed by re-hardening. Fig. 41 (Plate V) shows for comparison patterns thus obtained. The general formation of the discharge, known also as a pole brush, is that of a number of trees radiating from the point. The stem of a tree carries the current of all the branches. The current is sufficient to cause the transition to a low-resistance channel which is highly luminous. The low-resistance channel extends the electrode and discharges occur from this channel to the vicinity. Remote from the electrode the field is less intense, so that the branches are only able to propagate themselves for a short distance before the current falls and the branch ends in a high-resistance or glow discharge. Normally the branches are shorter the farther from the electrode. The

† Reference to the same method as that of Thomas was made by J. W. Swann (*Proc. Roy. Soc.* 1897, **62**, 38), but no details were given and until lately the point remained unnoticed.

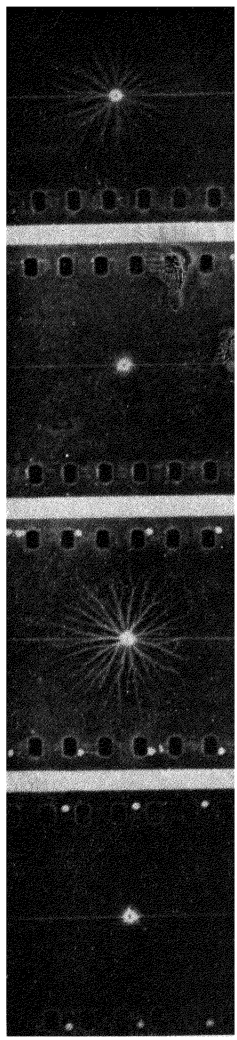


FIG. 40. Lichtenberg figures with klydonograph using 3/32 in. diameter sphere against plane.

PLATE V

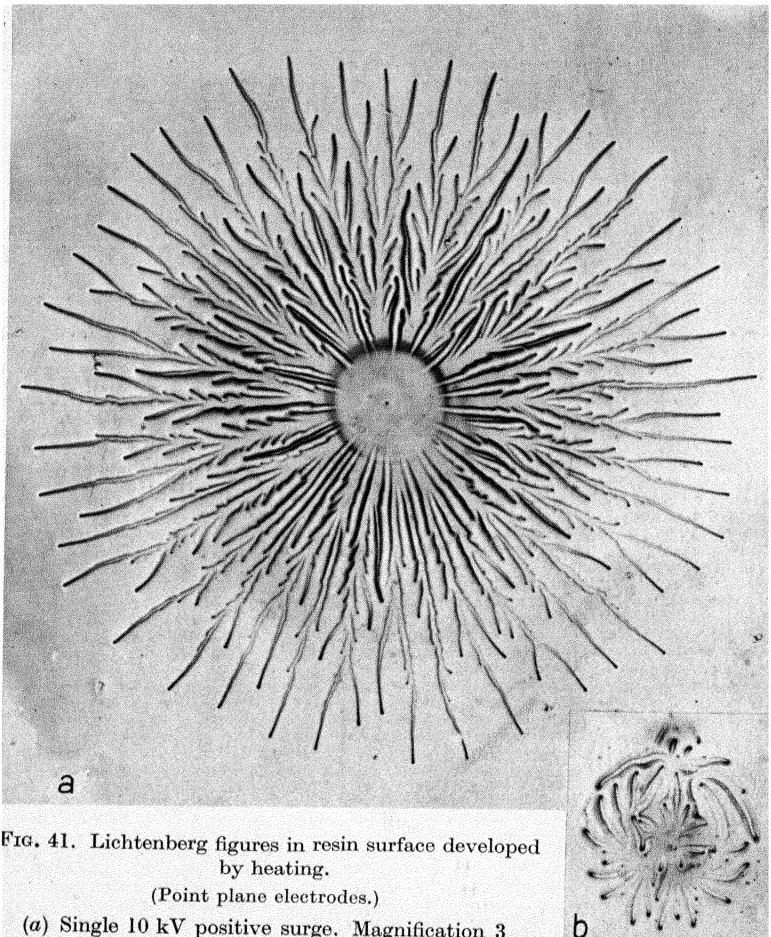


FIG. 41. Lichtenberg figures in resin surface developed by heating.

(Point plane electrodes.)

(*a*) Single 10 kV positive surge. Magnification 3 diameters.

(*b*) Single 10 kV negative surge. Magnification 3 diameters.

stem itself ends in a number of branches for similar reasons. The positive point attracts electrons from weaker parts of the field, producing long stems with few branches because the electrons are not repelled from the stem. Breakdown from a positive point occurs at a much lower voltage than from a negative point, so that the positive figure is brighter and stronger for the same voltage. The negative point and stems therefrom repel electrons forming numerous short branches, so that the pattern is difficult to distinguish. Lichtenberg figures have been extensively used in electrical engineering for surge recording but with mixed success. Their theory is complicated and still obscure. It is not the intention to deal with so extensive a subject, but it is hoped that mention may clarify references made later.

4.3. Mechanism and detection of internal discharges

4.3.1. Discharge sequences

The foregoing considerations support generalization of the hypothesis, used by Arman and Starr[6] and Gemant and Philippoff[2] and adopted by Austen and Whitehead[19] on specific experimental evidence, that an internal discharge involves the partial or complete discharge of the capacitance of a void in a time very short compared with that required to charge the capacitance by variation of the applied voltage, except at the highest frequencies. Accordingly, an ideal representation is provided by Fig. 42 (*a*), where C_c represents a gas-filled hole in a solid dielectric, C_b the solid dielectric in series with it, and C_a the remaining dielectric. If now the gas in the hole breaks down, the

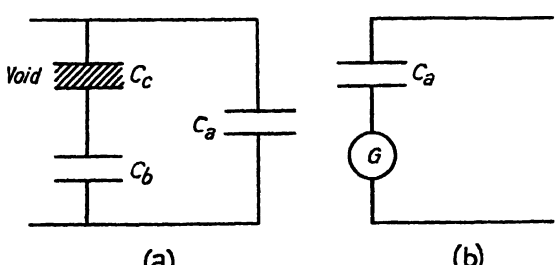


FIG. 42. Equivalent circuit of internal discharge.

voltage across C_c will fall by ΔV in a time not more than 10^{-7} sec. when the discharge is extinguished (the frequency spectrum is found to extend to the region of 100 Mc./s.). This is equivalent, except with applied potentials of very high frequency, to connecting in parallel with C_c a generator of zero impedance with a voltage $V-\Delta V$, where V is the instantaneous potential across C_c, in series with a switch closed instantaneously. The voltage across C_b and C_c falls by ΔV, and to restore it a charge $\Delta V C_b C_c/(C_b+C_c)$ must be drawn from C_a as far as is possible. If $C_b \ll C_c \ll C_a$, the charge to be restored is approximately $C_b \Delta V$, and the action is nearly equivalent to Fig. 42 (*b*), where G is the zero impedance generator of a Heaviside step function $-\delta V$ and $C_a\, \delta V = C_b\, \Delta V$. Although this idealization is often obscured in practice its analysis serves as a useful basis for understanding the phenomena.

Let the externally applied voltage be V_t, then a voltage $V_t C_b/(C_b+C_c)$, or $V_t C_b/C_c$ approximately, is applied to C_c. This is denoted by V_t'. When V_t' reaches the discharge voltage V of C_c, the potential difference falls by ΔV. If V_t' is now increased by ΔV, i.e. V_t by $\Delta V C_c/C_b$, a discharge again occurs, and so forth, until V_t' no longer increases by a sufficient amount. If now V_t' falls, it must fall by an amount $V'+V-\Delta V$ below the previous value at which a discharge occurred before the potential difference across the void, previously $V-\Delta V$, becomes $-V'$, at which a discharge can occur of opposite sign. Consider the case where $\Delta V = V$, that is, the void is completely discharged and where $V = V'$, that is, the discharge voltage is unaffected by polarity. Fig. 43 then shows the incidence of discharges when a sinusoidal potential is applied, and $V_t' = V_0 \sin \omega t$. The discharges tend to occur first in those quadrants where the voltage is increasing in arithmetical value, extending over the whole cycle as the voltage is increased, causing more discharges to occur per cycle. It would appear from Fig. 43 (*c*) that discharges would begin when the crest value of the voltage exceeds the discharge voltage, and that four discharges would then occur per cycle. Furthermore, if a discharge fails to occur at the appropriate epoch a voltage will always appear across the void at some subsequent

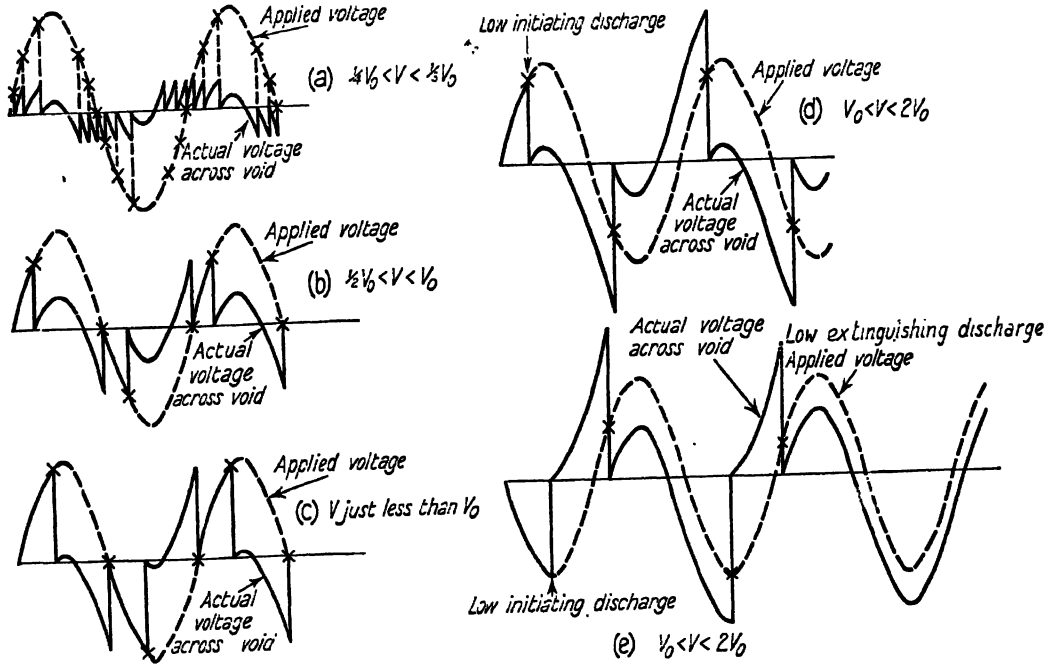


FIG. 43. Ideal discharge epochs.

epoch large enough to produce a discharge in normal circumstances and to renew the sequence. From Fig. 43 (*d*) it is to be seen that if a discharge occurs by chance within $(2V_0 - V)$ of the crest of the applied voltage wave so that the voltage change from the preceding crest exceeded V, then discharges at the rate of two per cycle can thereafter occur provided the crest value of the applied voltage exceeds one-half the discharge voltage. If a discharge fails to occur at an appropriate epoch, a voltage exceeding the normal discharge voltage will reappear, if the void persists in its failure to discharge, until the conduction in the dielectric restores the normal distribution of charge over the void; this interval will be of the order of the relaxation time of the dielectric, and thereafter no normal discharges can occur. On the other hand, if the initiating discharge occurs at an abnormally low voltage, then this may happen again. If so, then as in Fig. 43 (*e*), the effect of such a discharge is to extinguish the phenomenon since the voltage on the void will not thereafter attain the normal discharge voltage. Thus while discharges can occur at two per cycle once they have been started, yet even discharges which are unaffected by polarity are liable to be extinguished by the same kind of cause which started them. This uncertainty and instability can clearly arise from the sudden appearance of a polarity effect—that is, of a difference between positive and negative discharges.

Consider first the case when both the positive and negative discharge voltages exceed the amplitude of the voltage applied to the void which is the fraction of the total applied voltage deduced from Fig. 42 (*a*). Suppose, as in Fig. 44 (*a*), that the applied voltage yields an amplitude of 20 units while the discharge voltages are 21 units positive and 25 units negative; let some special circumstance produce an initial discharge at an abnormal voltage of 14 units positive. The first negative discharge will cause the second positive discharge to occur earlier in the cycle, which in turn will cause the second negative discharge to occur later, which makes the third positive discharge still earlier. This process continues until no negative discharge occurs, after which no further discharges are normally

possible. Thus if the discharge voltage exceeds the amplitude of the applied voltage, discharges are unlikely to persist and have not been observed to do so. The minimum stable rate of 2 discharges per cycle is therefore infrequent.

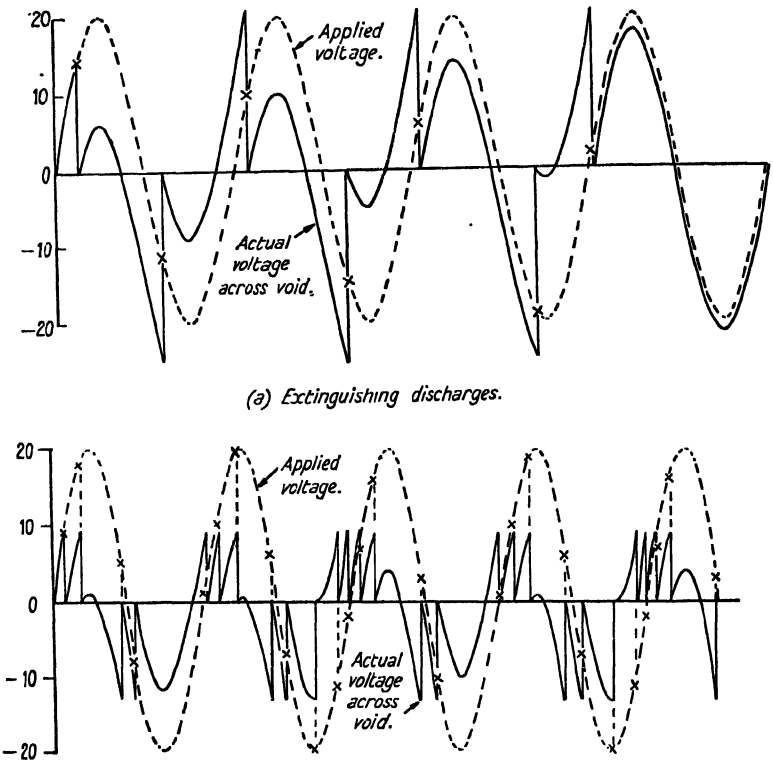


FIG. 44. Asymmetrical discharges.

Fig. 44 (*b*) shows what happens when the positive and negative discharge voltages are 9 and 13 units respectively, while the amplitude of the applied voltage remains at 20 units. In this instance the positive discharges get later in the cycle and the negative discharges earlier, until an additional negative discharge appears which yields an additional positive discharge with the last positive discharge earlier in the cycle than before:

the process then repeats so that cycles with 3 negative and 4 positive discharges alternate with cycles of 2 negative and 3 positive discharges. This alternation of n and $n+1$ discharges per cycle is characteristic of the phenomenon when the positive and negative discharge voltages differ, which seems likely in most instances.

The general features of the mathematical analysis of the foregoing are as follows. Let V^+ and V^- be the positive and negative discharge voltages respectively, and let V be the

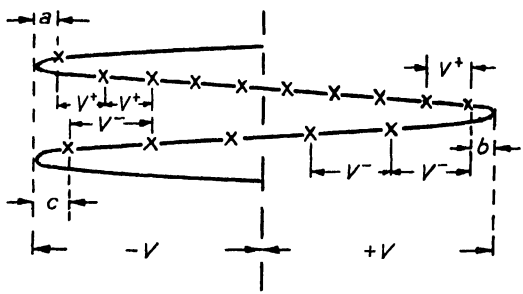


Fig. 45. Numerical relations of discharge epochs.

amplitude of the applied voltage after dividing by the ratio of potential division. Then from Fig. 45

$$2V = a+n^+V^+ +b = b+n^-V^- +c, \tag{4.2}$$

where n^+ is the number of positive discharges in a half-cycle and n^- is the number of negative discharges in the consecutive half-cycle. Again

$$c < V^-, \quad a < V^-, \quad \text{and} \quad b < V^+. \tag{4.3}$$

Thus

$$n^+ \quad \text{or} \quad n^+ +1 = \text{the integer next below } 2V/V^+ \tag{4.4}$$

according as $a+b <$ or $> V^+$, and similarly for n^-. For stability $a = c$, whence

$$n^+V^+ = n^-V^-. \tag{4.5}$$

Thus for stability V^+ and V^- must be in an integral relationship and the applied voltage V must at least exceed one-half the product of the lower discharge voltage by the L.C.M. of the numbers of positive and negative discharges in consecutive half-cycles. If $V^+ = V^-$ stability will always occur as the equations can always be satisfied by $n^+ = n^-$ and $a = c$. More usually

V^+ and V^- are not in a suitable integral relationship with each other and V; accordingly $a \neq c$ and there is a discharge precession. It is easy to see that $a > c$ or $a < c$ according to the alternative solutions necessary for n^+ and n^- and the residues $(a+b)$ or $(b+c)$. If $a > c$, positive discharges will get earlier in the cycle and negative discharges will get later until either there is an additional positive discharge or one less negative discharge: this depends on the nature of the alternative solutions for n^+ and n^-. In the former instance the first negative discharge will jump back earlier in the cycle; in the latter instance the last negative discharge will have jumped back earlier; in both instances the precession will then repeat as before. If $c > a$ the converse permutations will apply. If the discharges leave a residual voltage, the phenomena are more complicated but similar principles apply. Reference is made to papers by Austen and Whitehead[19] and Austen and Hackett[20] for other details.

Although symmetrical discharges can be stable with a range of epochs, yet in practice certain average epochs are to be expected. Thus with an even number of discharges per half-cycle, one discharge will take place at a voltage zero, since from Fig. 43 this will always occur when the phenomenon starts or restarts and it is also the mean configuration of the range. If the number of discharges per half-cycle is odd, the mean configuration is when zero voltage occurs halfway between the two adjacent discharge epochs. The instantaneous applied voltages at which discharges occur in one half-cycle are given by

$$\lambda V_k = \lambda V_0 \sin\theta_k = V(k - \tfrac{1}{2}n), \tag{4.6}$$

which may be generalized when there is a residual voltage to

$$\lambda V_k = \lambda V_0 \sin\theta_k = V\{1 + (\Delta V/V)(k - 1 - \tfrac{1}{2}n)\}, \tag{4.7}$$

where n = number of discharges per half-cycle, $k = 1,\ldots,n$, $\theta = \omega t$, V = discharge voltage, ΔV = voltage drop on void due to discharge, $\lambda = C_b/C_c$.

The uncomplicated behaviour just described sometimes occurs particularly with slowly varying voltages, as has recently been shown in some detail by Gross[59]. It also forms a useful basis for

calculations of the effects of discharges because these must be simplified to be practicable. Usually the phenomena are much more complicated. There are frequently a number of voids and a single void may not behave like a capacitor with metallic electrodes. Mason[16] has shown, that in a void in polythene only the surface near the root of the discharge is affected so that with voids above a small size an undischarged area remains. Transverse discharges and leakage between the undischarged and the discharged areas affect the sequence of discharges and Mason has indicated agreement with experiment on this hypothesis. The matter is further complicated by the effect of the non-uniform distribution of charge in determining the actual site of the next discharge. The tendency of discharges to congregate about voltage zeros but to extend into the preceding quarter-cycle much less readily is in general accord with the theory given. Further detail would be premature.

4.3.2. Detection and study of discharges

The effect of a discharge resembles an increase in capacitance and the incidence of the discharges depends upon change of voltage. If a constant voltage is applied one or more discharges may occur during its application, but during its maintenance they will only happen as the voids recharge by dielectric conduction. The relaxation times of dielectrics with respect to irreversible conduction, namely the product of dielectric constant, resistivity in ohm-cm. and $10^{-11}/36\pi$, are very long. Accordingly, the phenomena are usually studied with variable voltages which may be sinusoidal, recurrent pulses of various forms, linearly varying voltages with or without superimposed a.c. and a number of similar wave forms some of which have been described by Whitehead.[21] Most work has been carried out with sinusoidal voltages. Since the discharges are associated with only small magnitudes of charge, it is necessary to filter out the normal components of current and voltage so that the discontinuous phenomena can be studied separately. One of the most convenient and most frequently used methods of doing this is the Schering Bridge, Fig. 46.

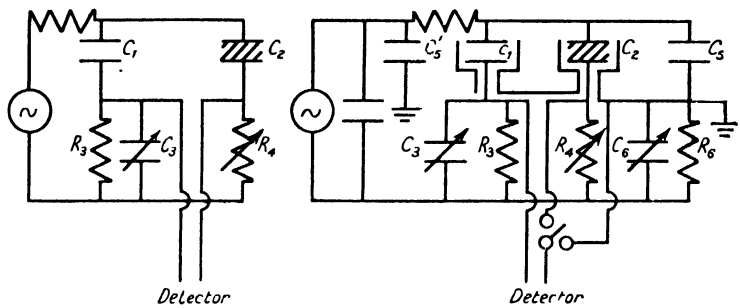
C_2 is the specimen under test and, as a result of the discharge, a fall of voltage suddenly occurs of value q/C_2, where q is the charge neutralized by the discharge. This voltage decays according to a relaxation time τ, where

$$\tau = C_1 C_2 (R_3 + R_4)/(C_1 + C_2) = R_3 C_1 = R_4 C_2$$

for Fig. 46 (*a*) and

$$\tau = R_3 C_1 C_5/(C_2 + C_5) \tag{4.8}$$

for Fig. 46 (*b*). Only a small proportion of the high total voltage



(*a*) Simple bridge. (*b*) Screened bridge with Wagner earth.

FIG. 46. Schering bridge detector.

can usually be permitted across L.V. arms such as R_3, so $R_3 C_1$ will usually be small, and τ is of the order of, say, 10^{-5} sec. If an amplifier and oscillograph are used which respond to frequencies from zero to well beyond $1/\tau$ then a pulse is recorded which rises as steeply as the upper frequency of the amplifier permits to a value q/C_2, multiplied by the overall gain of bridge and amplifier. The pulse decays according to $e^{-t/\tau}$. If the amplifier has an upper frequency limit much less than $1/\tau$ the output will be proportional to the time integral, i.e. to $\tau q/C_2$, and will take the form of a pulse of duration corresponding to a half-cycle of the upper frequency. With Fig. 46 (*a*) a differential amplifier is necessary: with the Wagner earthing device of Fig. 46 (*b*) a differential amplifier is advantageous in reducing harmonics and the effect of certain stray capacitances but is not theoretically necessary as the detector points are near earth potential. It must be remembered that stray capacitances to earth of the detector arms must be kept small, and that with

respect to the pulse due to the discharge, only the capacitance circuit counts. For this reason double screening is desirable for ease of calibration by a pulse generator if a differential amplifier is not used. Measurements of this kind are attended with some difficulty, and reference should be made to the literature[20] on the subject. Simpler arrangements are possible if the apparatus can be calibrated *in situ* by a pulse generator (see [22]).

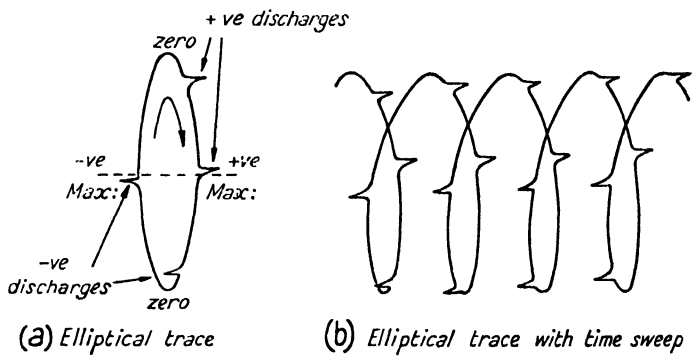


FIG. 47. Oscillograph trace methods.

In order to observe the epochs of the discharges, the bridge or filter is not completely balanced, and the output of the amplifier comprises the discharge pulses superposed on a more or less sinusoidal voltage. If now a voltage $\frac{1}{2}\pi$ radians out of phase with the supply voltage is applied to the second pair of deflecting plates (or coils) of the oscillograph, an ellipse will be obtained by the compounding of the sine waves. On this ellipse the discharge pulses will be superposed as in Fig. 47 (*a*). The ellipse is described at constant peripheral speed for each cycle so that the epochs are distinguishable. Capacitative unbalance of the bridge tilts the ellipse, phase angle unbalance broadens it, so generally the slight unbalance is solely of phase angle to give a horizontal or vertical ellipse. Positive discharges have pulses to the right and negative discharges to the left. When drawn out by a moving film or other time sweep to separate the cycles, a trace such as Fig. 47 (*b*) is obtained or even a series of V's.

The theoretical relations were illustrated by Austen and

Whitehead[19] using a discharge model. Fig. 48 (Plate VI) clearly shows the precession of the discharges in the consecutive cycles *abcd*, when an additional negative discharge appears and causes the positive discharge to return to near its original epoch so that the process repeats in *a'b'c'd'*. With actual dielectrics the phenomena are more complex. Fig. 49 (Plate VII) shows a record obtained with extruded polythene cable.[20] Small discharges first appeared and increased in frequency. Eventually the larger kind of discontinuity appeared. In this record the cable conductor was positive over the up-strokes and negative over the down-strokes and the cycles are numbered 1 to 46. A sequence of large discharges always started on or near the positive crest, e.g. cycle 22+, because the amplitude was approximately equal to the discharge voltage for this class of discharge. A discharge occurred in the ensuing negative half-cycle 22—. The following positive discharge occurred in 23+ earlier than in 22+ and the discharge in 23— was later than that in 22—. During the following negative half-cycle 24— the voltage did not become sufficiently negative with respect to its value at the preceding positive discharge to enable a negative discharge to take place. Similarly, no positive discharge occurs in 25+. Another sequence began on 29+ and extinction occurred by the mechanism described after 3 positive and 2 negative discharges or 2 positive and 1 negative discharge. Fig. 50 (Plate VIII) shows discharges in a cylindrical petroleum jelly impregnated paper capacitor for low-voltage service. The precession of discharges is observable although detailed analysis of consecutive cycles is uncertain. The double trace is due to the superposition of a trace derived from a pure sine wave applied to the detector terminals, which demonstrates the harmonic distortion and also the sign of the phase angle unbalance of the bridge.

By observing the magnitude of the pulse δV, the charge neutralized, $q = C_a \delta V$, can be found which is also equal to $C_b \Delta V$, where C_b is the capacitance of Fig. 42 and ΔV is the discharge voltage less the residual voltage. If a void behaves in a fairly constant manner, which is perhaps more likely with small voids,

further analysis is possible.[20] By observation of the epochs of the discharges an estimate can be made of the discharge voltage multiplied by the potential division factor $(C_b+C_c)/C_b$. Let the residual voltage be zero and the observed difference of discharge epoch voltages be V then

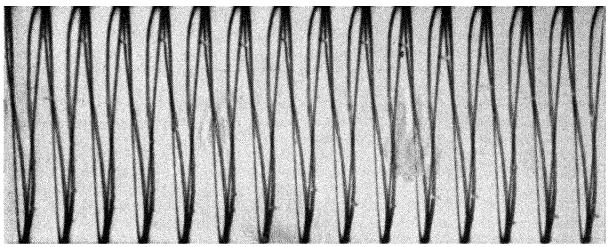
$$q = C_b \Delta V \quad \text{and} \quad V = (C_b+C_c)\Delta V/C_b \simeq C_c \Delta V/C_b$$

or

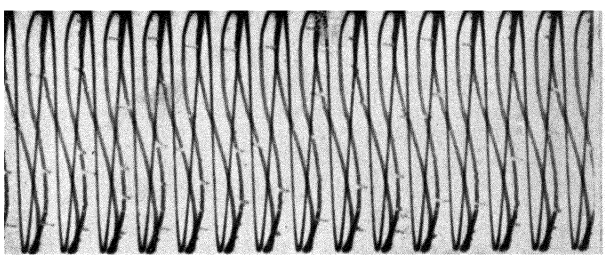
$$C_c(\Delta V)^2 = qV \quad \text{and} \quad q/V = C_b^2/C_c. \tag{4.9}$$

Now, if the pressure and composition of the gas in the void can be assumed, ΔV is related to the thickness of the void x according to the curve of spark potential against spark length for the gas. Again $C_c = A/4\pi x$ cm. and $C_b = Af(x) \simeq \lambda A/(d-x)$, where d is the thickness of the dielectric and λ a non-dimensional multiplier. Thus it is possible to deduce x and A. In other words both the area and the thickness of the void can be found. In non-uniform fields the position of the void must be assumed, but this is not so difficult as it seems because there are often reasons for assigning the voids of most importance to a particular part of the dielectric. If no absorption or evolution of gas occurs with rise of temperature, the breakdown voltage of the void will be unaffected by temperature according to Paschen's law, because the linear dimensions will change little unless the dielectric softens. With the more stable forms of dielectric the void pressure depends on the conditions of formation and it is often possible to deduce this from the manufacture of the dielectric. If equalization of pressure occurs at any stage of the process it is easy to determine the void pressure from the ambient conditions. In a surprisingly large number of instances the assumption of atmospheric pressure is an adequate approximation.

For many purposes such as study of the inception or other critical voltages associated with discharges, it suffices to observe a measure of the magnitude and frequency of the discharges, without determining their epochs.[20] For such observations a suitable output meter such as a crest voltmeter of appropriate time-constant may be substituted for the oscillograph. There is freedom of choice as to the acceptance band of the detector



$0{\cdot}25\times10^{5}$ V/cm r.m.s.



$0{\cdot}33\times10^{6}$ V/cm r.m.s.

FIG. 50. Discharges in impregnated paper capacitors.
Sensitivity on original trace, width 5·5 cm., was 670×10^{-12} coulombs/mm.

and of the bridge or filter. In particular, the latter need not necessarily have a short time-constant decreasing the energy available for the detector. A discharge in a dielectric specimen may be represented as a generator of Heaviside unit impulses δV in series with the capacitance C_0 of the dielectric. Fig. 51 (*a*) shows a bridge which is equivalent to Fig. 51 (*b*) if the source is of high impedance. Fig. 51 (*b*) also represents essentially any simplified relaxation type of filter where C is the capacitance

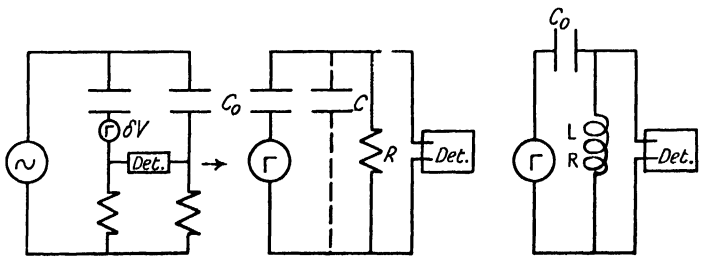


FIG. 51. Equivalent circuits for discharge detection.

of the measuring circuit. Similarly any simplified tuned circuit type of filter can be represented by Fig. 51 (*c*) for the present purposes. If the detector includes a band-pass amplifier of frequency limits $\omega_1/2\pi$ and $\omega_2/2\pi$, Whitehead[23] has shown that the output $W(t)$ from the circuit of Fig. 51 (*b*) will be, for unit amplification,

$$W(t) = (\lambda/\pi)e^{-t/R(C+C_0)}[\mathrm{Si}(\omega_2 t - 0{\cdot}61\pi) - \mathrm{Si}(\omega_1 t - 0{\cdot}61\pi)],$$

where λ is the ratio of potential division of the input circuit including the input impedance. Band-pass characteristics are presumed to be obtained by crowding the natural frequencies of the detector circuit towards the edges of the band. If the relaxation time $R(C+C_0)$ is much greater than π/ω_1 or $\pi/(\omega_2-\omega_1)$ and if $\omega_1/\omega_2 < 0{\cdot}5$ the crest value W_m which may be measured is

$$W_m \simeq \lambda\{0{\cdot}59 - (1/\pi)\mathrm{Si}[5{\cdot}06(\omega_1/\omega_2) - 1{\cdot}92]\}. \qquad (4.10)\dagger$$

† $\mathrm{Si}\,x$ is the sine integral, i.e. $\int_0^x x^{-1}\sin x\,dx$; see Jahnke and Emde, *Tables of Functions* (Teubner, Leipzig and Berlin, 1938).

The equivalent crest value of the first circuit noise is rather uncertain as it depends upon what constitutes the observable crest. To obtain the order it will be sufficient to assume a crest factor of $\sqrt{2}$ (the most recent values quoted do not differ very greatly). The crest value of the output due to first circuit noise will thus be

$$N = \sqrt{[4\mathbf{k}T(C+C_0)^{-1}\{\arctan R(C+C_0)\omega_2 - \arctan R(C+C_0)\omega_1\}]},$$

T = absolute temperature.

Since $2\pi/(\omega_2-\omega_1)R(C+C_0)$ must be small the value of δV which gives the same crest output as the first circuit noise is

$$\left.\begin{aligned} \delta V = \frac{2\sqrt{(\mathbf{k}TR\pi)}}{RC_0}\sqrt{\left(\frac{\omega_2-\omega_1}{\omega_1\omega_2}\right)}\times \\ \times[1{\cdot}857 - \mathrm{Si}\{5{\cdot}06(\omega_1/\omega_2) - 1{\cdot}92\}]^{-1}, \\ \text{minimum } \delta V \simeq 1{\cdot}35\sqrt{(\mathbf{k}TR\pi)}/RC_0\sqrt{\omega_2}, \\ \text{when} \qquad \omega_2 \simeq 3\omega_1. \end{aligned}\right\} \quad (4.11)$$

The greater R and ω_2 the better, but R cannot exceed the resistance of the first valve, and a very broad band amplifier is difficult of construction, for example, a high overload capacity is needed. Next, if the high voltage applied to the dielectric is alternating (ω') then $RC_0\omega'$ must be small to avoid excessive voltages on the detector. Lastly the set noise instead of first circuit noise may impose a limit which may then be taken as an equivalent input of, say, 10^{-7} volt, giving

$$\delta V = (1+C/C_0)\pi 10^{-7}[1{\cdot}857 - \mathrm{Si}\{5{\cdot}06(\omega_1/\omega_2) - 1{\cdot}92\}]^{-1} \text{ V}.$$

The practical implications of this analysis have been considered by Austen and Hackett,[20] who draw the conclusion that, if $RC_0\omega'$ must not exceed a value such as 0·01 to avoid excessive detector voltage and if ω' is 100π, then

$$\begin{aligned} \delta V &= 17{\cdot}4\times 10^{-10}\sqrt{(\omega'/C_0\omega_2)} \text{ V} \\ &\simeq 0{\cdot}17\times 10^{-10}/\sqrt{C_0} \text{ V}. \end{aligned} \quad (4.12)$$

For capacitances about 0·003 μF the sensitivity would be limited by set noise and δV would be about 0·3 μV. If the applied voltage is constant so that RC_0 need not be limited as when

alternating, then the minimum δV is inversely proportional to C_0 instead of $\sqrt{C_0}$ as in equation (4.12).

With the tuned circuit of Fig. 51 (*c*) a fairly narrow band-pass amplifier is an obvious choice since there is no advantage in making the band much wider than the resonance width of the input circuit. With this arrangement transients appear as shown by Whitehead[23] which a voltmeter would not indicate. Neglecting these, he shows that the output $W(t)$ for unit amplification comprises an oscillation of the mid-band frequency $\omega_0/2\pi$, of which the envelope of the amplitude varies according to

$$W(t)/\delta V = \frac{e^{-\omega_0 \sin\alpha t'}}{2\pi}(\pi + 2\mathrm{Si}\,\tfrac{1}{2}\Delta\omega t')\left(1 - \frac{2\omega_0 \sin\alpha}{\Delta\omega}\arctan\frac{\Delta\omega}{2\omega_0\sin\alpha}\right) + \\ + \pi^{-1}\arctan\left(\frac{\Delta\omega}{2\omega_0\sin\alpha}\right)\frac{\sin\tfrac{1}{2}\Delta\omega t'}{\tfrac{1}{2}\Delta\omega t'},$$

where

$$\Delta\omega = \omega_2 - \omega_1, \qquad t' = t - (2\pi/\Delta\omega), \qquad \sin\alpha = 1/2Q,$$

$$Q = \text{magnification of input circuit} = L\omega_0/R.$$

$W(t)$ has two maxima at $t' = 0$ and $t' = 3\pi/\Delta\omega$, but the first is the greater, except when $\Delta\omega/2\omega_0 \sin\alpha$ is 50 or more. Thus the maximum W_m may be taken as

$$W_m = \left\{\frac{1}{2} - \left(\frac{1}{2x} - \frac{1}{\pi}\right)\arctan x\right\}\delta V, \tag{4.13}$$

where $x = Q\Delta\omega/\omega_0$. This is plotted as curve *A* in Fig. 52, from which it is clear that there is little advantage in increasing $Q\Delta\omega/\omega_0$ beyond, say, 2. The crest output for first circuit noise may be taken as

$$N = \sqrt{\left(\frac{\mathbf{k}TR\omega_0}{\pi}\right)\left[\log\frac{1+(x/4Q)}{1-(x/4Q)} + \right.} \\ \left. + 2Q\left(2\arctan x + \arctan\frac{2x}{1+16Q^2-x^2}\right)\right]^{\frac{1}{2}}.$$

Thus

$$\text{Minimum } \delta V = \sqrt{(\mathbf{k}TR\omega_0 Q/\pi)}/f(x, Q) = \sqrt{(\mathbf{k}T/\pi C_0)}/f(x, Q), \tag{4.14}$$

where

$$f(x, Q) = Q^{\frac{1}{2}}\left\{\frac{1}{2} - \left(\frac{1}{2x} - \frac{1}{\pi}\right)\arctan x\right\} \times$$

$$\times \left[\log\frac{1+(x/4Q)}{1-(x/4Q)} + 2Q\left(2\arctan x + \arctan\frac{2x}{1+16Q^2-x^2}\right)\right]^{-\frac{1}{2}}.$$

The quantity $f(x, Q)$ is also plotted on Fig. 52 in curve B for $Q = 2$ and curve C for $Q = 5$ or 10, from which it is seen that

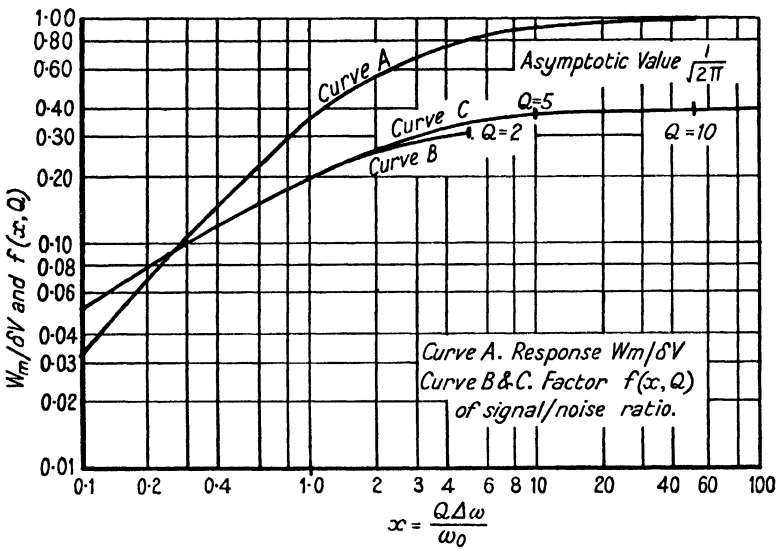


FIG. 52. Response and noise factors of tuned circuit with band-pass amplifier.

Q need not exceed about 2 to obtain most of the possible sensitivity. The practical implications have been considered by Austen and Hackett with the following results. When C_0 exceeds 0·2–0·4 μF, δV will be about 0·15–0·4 μV, depending on how nearly the optimum conditions can be approached. For smaller capacitances δV is of the order of 1 to $2\times10^{-10}/\sqrt{C_0}$ as compared with $0{\cdot}17\times10^{-10}/\sqrt{C_0}$ for the capacitative coupling. The apparent lower sensitivity of the tuned circuit must be offset against the voltage limitations of the capacitative coupling and the greater ease of constructing a tuned amplifier with output meter. The pulse which reaches the latter from the band-pass amplifier has a duration of $4\pi/\Delta\omega$ so that no very short charging time

constant is needed. Accordingly, circuits of the type of Fig. 51 (c) are now generally used with a mid-band frequency of 10–200 kc./s.

Discharges give rise to energy loss which should therefore be observable in power factor measurements, and this was indeed the earliest method by which they were detected. Curves showing a power factor increasing with voltage were often called 'ionization' curves, although the nature of the ionization was not clearly understood until fairly recently. Referring to Fig. 42, if C_c discharges, causing a voltage drop ΔV, and if the discharge voltage is V, the energy dissipated is approximately

$$C_c \Delta V(V - \tfrac{1}{2}\Delta V),$$

provided C_a and C_c are much greater than C_b. There is then a small net reduction of total voltage on the combination of approximately $C_b \Delta V/C_a$. To restore the voltage from the source of potential requires an energy approximately $C_c V \Delta V$. If $\Delta V = V$, that is if there is no remanent voltage, the energy $C_c V^2$ derived from the source is twice that dissipated, namely $\frac{1}{2}C_c V^2$. The balance remains as the potential energy of the redistributed charge which is usually given back at a discharge in the next half-cycle, which occurs at a reduced voltage. With two discharges per half-cycle, if one occurred at $(C_c/C_b)V$, the other would occur at zero, so that the mean energy per discharge for the two discharges would be $\frac{1}{2}C_c V^2$. To illustrate this feature, which is rather confusing, Fig. 53 shows a more complicated example such as might occur in practice.† As previously, let $V_0 \sin \omega t$ be the fraction of the total applied voltage which is actually applied to the void. Let the positive and negative discharge potentials of the void be respectively V^+ and V^-. Suppose $V_0 : V^+ : V^-$ to be in the ratio 6:4:5, and the remanent potentials after discharge to be zero. Fig. 53 shows the variation of potential across the void in absence of discharges and also when discharges occur in accordance with the theory propounded. The curves are similar to those of Fig. 43. The first line under Fig. 53 shows the individual energies $q\,\delta V$ taken

† The details of this illustration are taken from a note by Pelzer.[21]

from the supply. These have various signs since, according to the epoch of the discharge, energy may be withdrawn from, or yielded to, the supply. The second line under Fig. 53 shows the individual energies dissipated in the discharge, which are essentially positive and either $\frac{1}{2}C_c V^{+2}$ or $\frac{1}{2}C_c V^{-2}$. If C_c is taken as a unit then the sum of both lines comes to the same value 90 over a complete series, but over part of the series the energy drawn from the supply may differ considerably and irregularly from

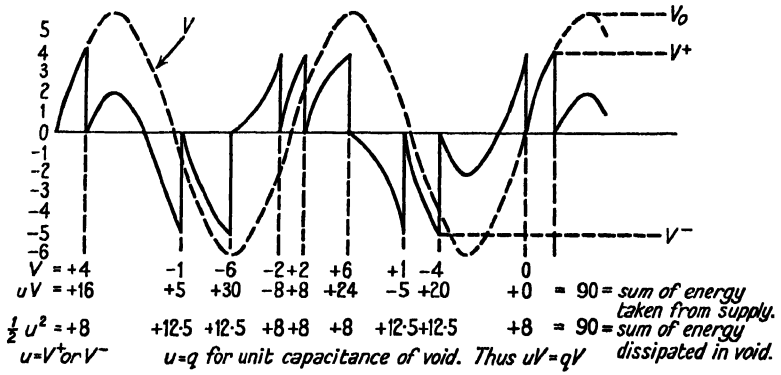


FIG. 53. Energy changes in discharge sequence.

the energy actually dissipated. If the insulation leakage were considerable instead of being negligible as has been assumed, the energy dissipated by leakage of the distributed charge would make a contribution, but this would alter the discharge sequence, as has been mentioned earlier in connexion with Mason's work.

Arman and Starr correctly assumed the discharge current to approach a Heaviside transient and determined the component of fundamental frequency as $-(2\omega\Delta V/\pi)C_c\cos(\omega t-\theta)$, where θ is the epoch of discharge and the applied voltage is $V_0\sin\omega t$. However, what they called the void capacitance C_c should be the series capacitance C_b owing to the increased charge on C_b due to its rise in potential as already shown. Correcting this, then, with two discharges per cycle, as supposed by Arman and Starr[6] the energy loss per cycle would seem to be

$$W = 2C_b\Delta V\, V_0\sin\theta.$$

Now if the discharge occurs at an epoch when the voltage on the void is V, then $V_0 \sin\theta$ is $(C_c/C_b)V$ so that

$$W = 2C_c V\,\Delta V,$$

which is correct for the energy drawn from the source for two discharges at $\pm(C_c/C_b)V_0$, but incorrect if expressed as an energy loss of $C_c V\,\Delta V$ per discharge. When, however, $V_0 \sin\theta$ is correctly summed over all the *four* discharges which would actually occur in this case, namely two at zero voltage and two at $\pm(C_c/C_b)V_0$, then the more nearly correct value of $\frac{1}{2}C_c V\,\Delta V$ per discharge would be found, although this is only right if $V \simeq \Delta V$. If now the loss per cycle is taken as

$$W = 2C_b \Delta V \sum_1^n V_0 \sin\theta_k,$$

where n is the number of discharges per half-cycle, the result should be nearly correct for symmetrical discharges. Inserting equation (4.7),

$$W = 2C_c V\,\Delta V \sum_1^n \{1+(\Delta V/V)(k-1-\tfrac{1}{2}n)\}$$

$$= 2C_c V\,\Delta V n\{1-\tfrac{1}{2}(\Delta V/V)\}. \qquad (4.15)$$

If there is no remanent voltage, $\Delta V = V$ and

$$W = nC_c V^2$$

or $\frac{1}{2}C_c V^2$ per discharge since there are n positive and n negative discharges. The loss angle δ is given by

$$\tan\delta = \frac{1}{\pi}\frac{W}{V_0^2 C_a} = \frac{2C_c}{C_a}\frac{nV\,\Delta V}{\pi V_0^2}\left\{1-\frac{1}{2}\frac{\Delta V}{V}\right\}.$$

Suppose n is large so that $n \simeq 2V_0/V$ then

$$\tan\delta = (C_c/C_a)(4\Delta V/\pi V_0)\{1-\tfrac{1}{2}(\Delta V/V)\}$$

$$= (C_c/C_a)(2V/\pi V_0) \quad \text{if} \quad \Delta V = V. \qquad (4.16)$$

Austen[20] has noted that in a trace of the type obtained with the oscillograph bridge $V_0 \sin\theta_k$ is roughly proportional to the displacement along the long axis of the basic ellipse of Figs. 47 and 54. ΔV represents the displacement from the normal trace due to the discharge pulse and is at right angles to the axis. Accordingly, $\Delta V \sum V_0 \sin\theta_k$ is roughly proportional to the area

between the distorted and the normal traces. This enables the loss to be directly evaluated from complicated records. It is rather convenient that a similar result applies to a normal continuous loss. Referring to Fig. 46 for nomenclature, if the ellipse is due to unbalanced loss in C_2 (i.e. C_a) the minor axis of the ellipse is given by the quadrature detector voltage u which is $V_0 \sin\delta R_3 C_1 \omega$. The major axis is proportional to V_0 as before. The area of the ellipse is thus $\pi V_0^2 \sin\delta\, R_3 C_1 \omega$ to the

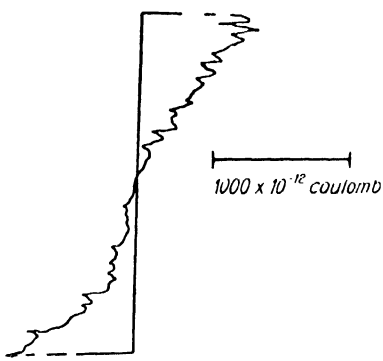


FIG. 54. Single record of discharges.
(Davis, Austen, and Willis Jackson.)

same factor of proportionality. Since the loss per cycle is $\pi V_0^2 C_2 \sin\delta$, it is correctly given by the area apart from the factor $R_3 \omega (C_1/C_2)$.

It has already been shown that when the amplitude of the voltage normally applied to the void exceeds one-half the discharge voltage, intermittent discharges can occur if there is a chance mechanism to start them. A small loss associated therewith may occur. When the amplitude reaches the discharge voltage, stable discharges will appear, and there will be a rapid increase of energy loss and of loss angle in this region. When the voltage is increased beyond the region of inception of maintained discharge sequences, the energy loss will thereafter rise linearly with voltage, but $\tan\delta$ will tend to decrease linearly with voltage. Austen and Hackett[20] have worked out the case of the air gap between a metal electrode and a glass plate measured by Dawes and Humphries.[15] Some of the assumptions made in

the foregoing simplified treatment did not apply, and allowance was made for them. Fig. 55 shows that remarkably good agreement was obtained on the hypotheses that the permittivity of the glass plate was 10, the discharge voltage was given by the

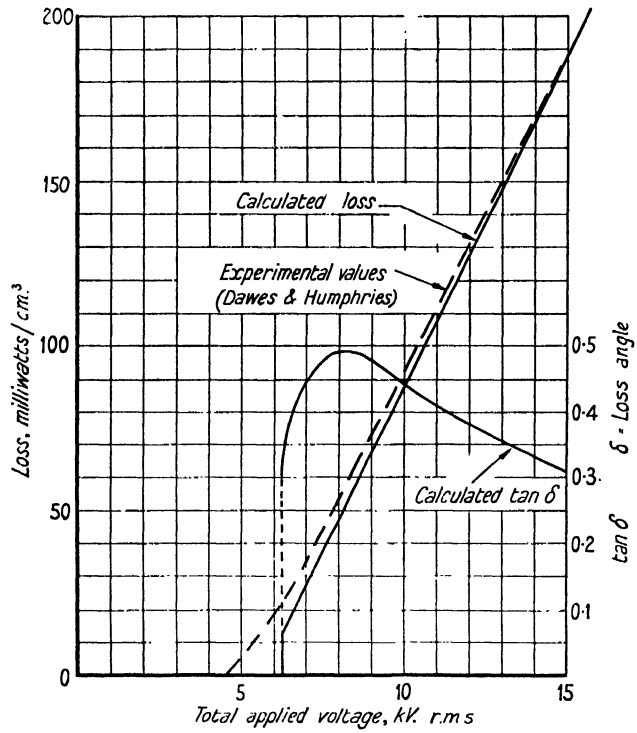


FIG. 55. Comparison of theory and experimental values for energy loss due to discharges.
(Austen and Hackett.)

Toepler–Stephenson–Bruce formula and $\Delta V/V$ was taken as 0·8 as deduced from an oscillogram by Dawes and Humphries, and as suggested from Thomas's work (*vide infra*). The variations exhibited, though more relevant to a later section, are characteristic of discharge phenomena when a few well-defined voids are present. In practice the variations with voltage are blurred owing to the number of elements involved. Nevertheless, some confidence in the order of these calculations can be derived from

an example given by Davis, Austen, and Willis Jackson[25] in which the discharge behaviour of polythene cable specimens was studied at 50 cyc./sec. and the losses per cycle computed, losses too small to distinguish at 50 cyc./sec. by direct measurement. Assuming the same mechanism, the losses at U.H.F. should have been easily discernible in one case and the comparison of energy loss measurements on two specimens at 600 Mc./s. yielded good confirmation.

Specimen	*Abnormal Energy Loss in Joules/Cycle*	
	50 cyc./sec.	600 Mc./s.
A	4×10^{-4}	$3{\cdot}4\times10^{-4}$
B	$> 0{\cdot}06\times10^{-4}$ $< 0{\cdot}3\times10^{-4}$	$0{\cdot}5\times10^{-4}$

From the foregoing it would seem that the loss angle should respond rapidly to the onset of discharges. However, the magnitude of the loss angle associated with any particular class of void is usually small, so that the decrease of $\tan\delta$ with rise of voltage tends to produce a rather indefinite inception region if there is a distribution of voids. Typical loss angle curves associated with discharges are shown in Fig. 56.

The oscillographic methods described show immediately the ignition of voids within their sensitivity limits, but for rapid, sensitive tests the high-frequency discharge detector with a crest-type output meter or an aural indicator such as a loudspeaker is often preferred. With d.c. the first discharges will repeat at a rate of the order of the relaxation time of the dielectric and will appear as separate 'clicks'. When the voltage increases and more voids discharge, with some discharging more rapidly, a continuous sound or reading is obtained. With a.c. the indications are usually nearly continuous at an early stage owing to the integrating properties of the detector for discharges repeated at a rate of several per cycle. Three typical cases occur frequently, namely a sharp rise at a well-defined inception voltage, an initial increase due to discharges of low intensity with a subsequent fairly rapid and substantial rise, and lastly a rather slow increase. These are illustrated in Fig. 57. The majority of instances so far studied fall into the first category.

Discharges are visible in transparent or translucent dielectrics, but the radiation therefrom is feeble. It can be increased by increasing the rate of repetition by increasing the frequency. In favourable circumstances this forms a valuable method of

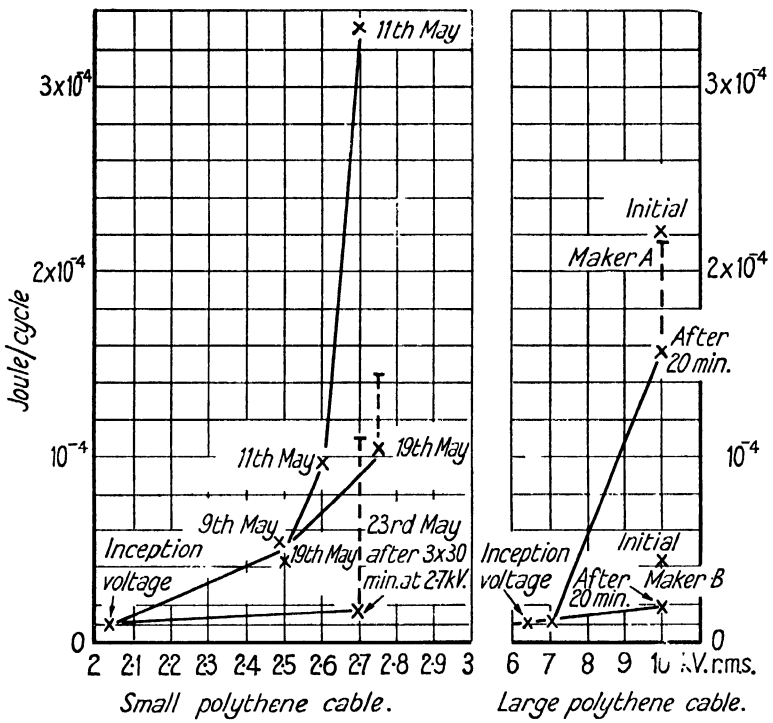


FIG. 56. Energy loss due to discharges. Variation with time and continued application of 50 c./s. stress. The contribution due to loss angles at the highest voltages is shown by the vertical dotted line.

(Davis, Austen, and Willis Jackson.)

test since the location of the discharges is apparent. Fig. 58 (Plate IX) shows photographs of discharges, for which the writer is indebted to Dr. Konstantinowsky (Messrs. Standard Telephones and Cables), obtained by J. H. Gosden with the core of a concentric polythene cable, i.e. the conductor and dielectric but no sheath or covering, when a high radio-frequency voltage was applied. Visual observations are most sensitive in darkness after dark adaptation has developed.

One of the earliest ways of detecting discharges was by ear as in the 'hissing' test and the practice of listening for corona. The practical difficulties and uncertainty of such methods are fairly clear.

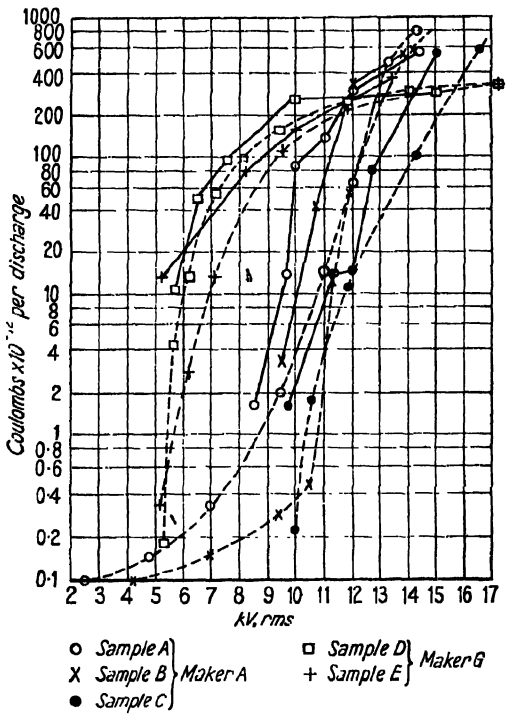
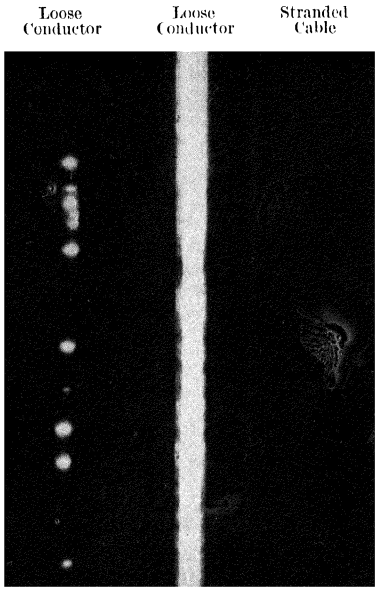
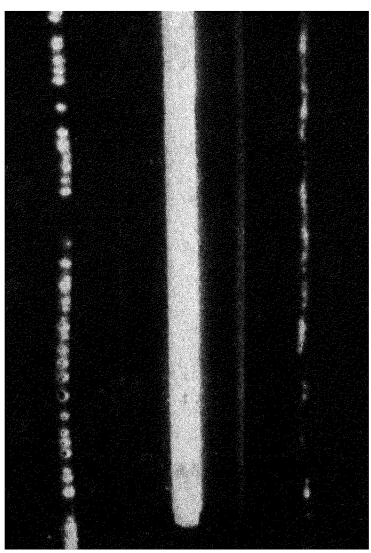


FIG. 57. Variation of discharge magnitude in polythene cables with applied voltages.
(Davis, Austen, and Willis Jackson.)

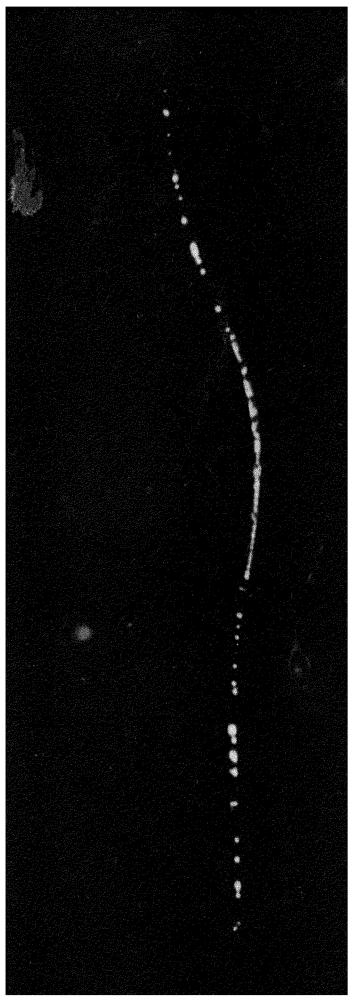
It is now possible to estimate the relative sensitivities of the various methods, which may conveniently be compared in terms of the minimum detectable change of electric charge on a specimen with the stated capacitance. To arrive at these values the theoretical principles already given have been used. In the aural method it has been shown (see Austen and Hackett[20]) that a discharge repeated 100 times a second and dissipating $1{\cdot}5\times10^{-5}$ W can be just heard at 1 metre, which corresponds to



100 kc./s. 4·5 kV.



100 kc./s. 7·25 kV.



Higher voltage cable 2 Mc./s. 7 kV.

FIG. 58. Discharges in polythene cables.

a sound energy considerably greater than the usual proportion of 10^{-5} of the total. In the visual method a single void discharging 100 times a second and having an energy dissipation of 10^{-5} W is just visible with dark adaptation, a sensitivity which agrees with that deduced from data on a discharge in oxygen: this implies that the proportion of total energy appearing as visible radiation is at least 10^{-4} on the basis of the minimum energy to which the eye responds (see Austen and Hackett).

The table refers to existing technique; for example set noise and first circuit noise will no doubt be possible of reduction by electron multipliers and other more recent circuit elements. A conventional output voltmeter is postulated where appropriate. This is limited by its response to the impulses from a band-pass amplifier, of duration about $4\pi/\Delta\omega$. A varying degree of arithmetical integration of successive impulses is likely to occur in a conventional voltmeter well before algebraic or vector integration occurs in the amplifier. With 10 kc./s. band-width $4\pi/\Delta\omega$ is 200 μs. and the estimation of individual impulses is correspondingly difficult without the loss of sensitivity indicated in a comparison of the first with the second line in the table. Disadvantages of the oscillographic bridge are the loss which occurs in the bridge circuit and sensitivity to interference, but it distinguishes between different types of discharges. The H.F. detector is sensitive, easily constructed, fairly easy to screen, and distinguishes between widely different discharges such as between clicks and rapidly repeated discharges. With large capacitances under test both the bridge and the H.F. detector are reduced in sensitivity so that the aural method, particularly with a microphone, may have advantages and is, in fact, frequently used to detect partial breakdowns in impulse tests in transformers and the like.

4.4. The influence of internal discharges on electric strength

A great deal of evidence is spread through the literature quoted in section (4.1) that internal discharges can and do lead

Relative Sensitivity of Discharge Detectors

Method	*Limiting factor*	*Capacitance* = 0·1 μF			*Capacitance* = 60 μμF		
		Minimum detectable discharge, 10^{-12} *coulomb*			*Minimum detectable discharge,* 10^{-12} *coulomb*		
		2 per cycle	*much lower rate*	*Voltage* V	*2 per cycle*	*much lower rate*	*Voltage* kV
H.F. Detector of Austen and Hackett .	Valve anode noise extraneous noise	16	110		0·17	1·15	
Probable practicable limit of method .	Set noise or circuit noise	0·1			0·006		
Theoretical limit for capacitative coupling	Set noise or circuit noise	0·03			0·0002		
Quinn's† resonant circuit with oscillograph					$\geqslant$ 24	$\geqslant$ 0·24	
Oscillograph Bridge of Austen and Hackett	Sensitivity of amplifier	200	200		2	2	
Theoretical limit	Set noise or circuit noise	about 3	about 3		about 0·5	about 0·5	
Discharge Bridge of Arman and Starr .					estimated 25		
Visual method							
at 50 cyc./sec.					20		4
at 100 kc./sec..					about 0·5		4
Aural method					40		6
Loss angle	Minimum detectable change = 10^{-4}	10^4		600	40		4
	Minimum detectable change = 10^{-3}	10^5			4		4

† Quinn's apparatus is included as a H.F. detector since his circuit resonated apparently between 1 and 10 kc./s.

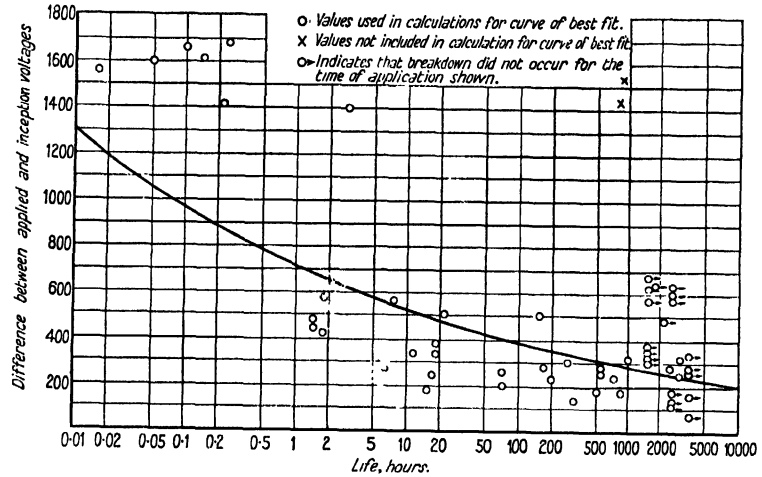


Fig. 59. A.C. voltage-life curve for impregnated paper capacitors.
(Austen and Hackett)

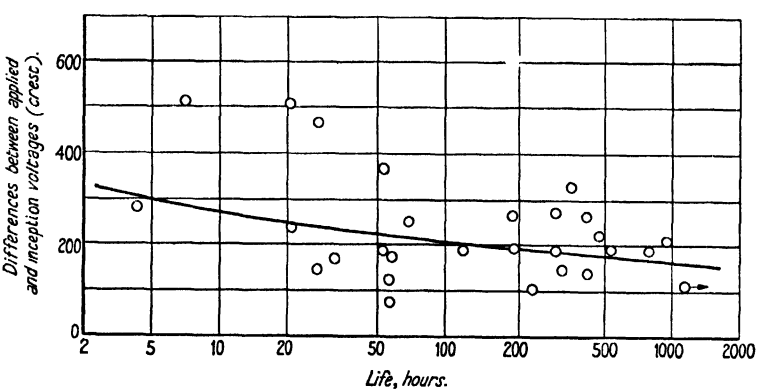


Fig. 60. D.C. voltage-life curve for impregnated paper capacitors.
(Austen and Hackett)

to breakdown. In particular, reference is made to the books by Emanueli[1] and Robinson[4] and to the papers by Robinson[4] and Brazier.[5] These authors dealt with power cables. A quantitative and statistical examination of certain types of radio capacitors was described by Austen and Hackett.[20] The types of capacitor examined were by modern standards imperfectly impregnated and sealed, though satisfactory for the relatively

light service then required. Remembering the high working stresses, the conditions were favourable to discharge breakdown. The life tests summarized in Figs. 59, 60 indicate clearly that the discharge inception voltage, or a value somewhat greater, represented the highest voltage for indefinite operation.

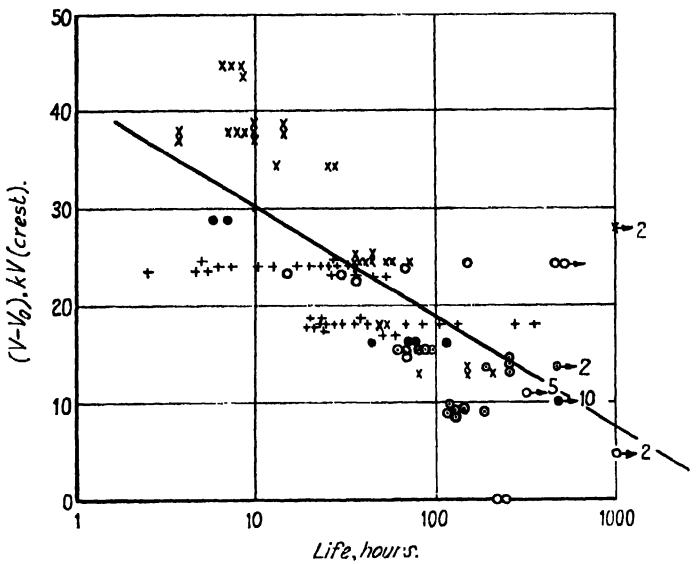


FIG. 61. Life with unidirectional pulses of voltage swing V at 500/sec. for polythene cables.

V_0 = Voltage for stress of 100 kV/cm. at inner conductor.

(Davis, Austen, and Willis Jackson)

× + Small cables from several makers.
● Small cable from one maker.
○ Large cables from 3 makers.
⊙ Large stranded cables from 2 makers.

A lengthy study of polythene cables was reported by Davis, Austen, and Willis Jackson[25]. The life tests were made with unidirectional short pulses repeated at 500 per sec., which in accordance with the previous section would give the same number of discharges as an alternating voltage. Figs. 61 and 62 are plotted from the results. There is a significant reduction of spread when the excess of applied voltage over twice inception voltage ($2V_c$) is used as parameter in place of the excess of the

applied voltage over that required to give 100 kV/cm. at the inner conductor. It was possible to deduce a double regression line (Austen and Pelzer[26]) yielding life in terms of this excess. It was also observed by Austen that discharges below a certain minimum, corresponding to the order of 2×10^{-12} coulombs of charge neutralized, could be neglected and evidence from Davis,

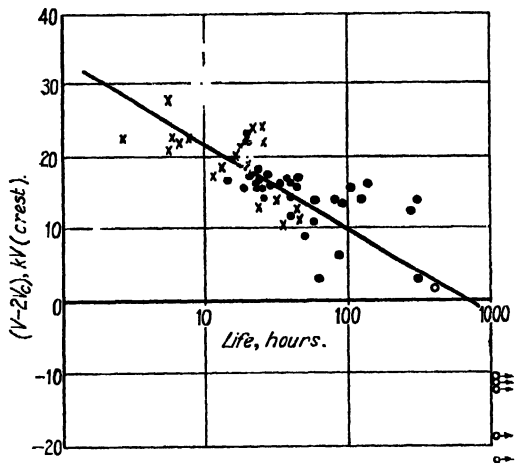


FIG. 62. Life with unidirectional pulses of voltage swing V at 500/sec. for polythene cables (see Fig. 61). V_c = crest 50 c./s. voltage for discharge of 2×10^{-12} coulomb.

(Davis, Austen, and Willis Jackson)

Hackett, and others confirms that such small discharges do not appear to have much effect on polythene and similar hydrocarbon or mainly hydrocarbon dielectrics. Some illustrations of discharge breakdowns from various sources are given in Fig. 63 (Plate X). See also Fig. 29 (Plate III).

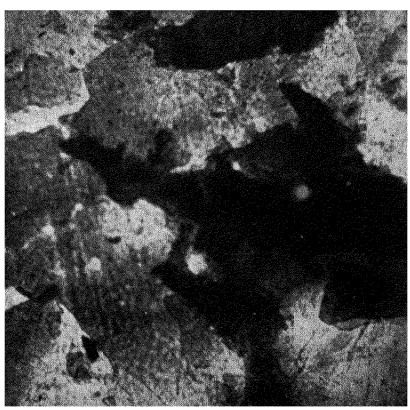
With a dielectric specimen such as a polythene cable, the discharge behaviour is fairly characteristic of the specimen and not much affected by normal treatment such as temperature cycles, voltage application, and handling within reasonable limits. Indeed a sample is sufficiently representative of a batch for sampling tests to be practicable. A similar result for a different reason applies where the discharge inception voltage is of the order of the minimum breakdown voltage for the gas

in the voids. A change in void distribution often does not alter the discharge behaviour not too far removed from the inception region because there will always be some voids in the neighbourhood of the minimum breakdown voltage and these will determine the behaviour. This was found to be the case with some of the radio capacitors mentioned earlier. In other instances, application of voltage may make a considerable difference. For example, in certain well-made oil-filled paper capacitors the discharge voltage was initially some 3 or 4 times the working voltage or even indistinguishable from the breakdown voltage, but the application of so high a voltage for an appreciable period reduced the discharge voltage to a value of some 50 per cent. above the working voltage. These differences are to be expected according to the chemical and physical properties of the dielectric.

The effect of a discharge may comprise electrical, chemical, and thermal actions. It will be shown in the next section that electrical breakdown can occur in a manner apparently unconnected with the chemical deterioration which accompanies it. Direct action by a discharge seems dependent on the development of low-resistance or arc-like conditions. A probable result is the production of a conducting path. If the mean field in the dielectric is high enough the field concentration at the tip of the path may cause intrinsic breakdown in the neighbourhood and the path may be successively extended through the dielectric. From the discussion of intrinsic breakdown in non-uniform fields in Chapter II the simplest form of this mechanism should be limited to high fields and short times. The energy dissipated by the discharge, the chemical and thermal effect of the discharge, and the path it produces may reduce the intrinsic electric strength in a gradual manner so that successive propagation by intrinsic breakdown in this rather more complicated way can apply to considerably lower fields and much longer times than those considered in Chapter II. If the electric strength at right angles to the path is much lower as it often is between layers in a laminated dielectric, transverse breakdown can take place as in Robinson's theories. The breakdown path can then grow



(*a*) ×44 (Mason). (*b*) ×28 (Mason). (*c*) ×45 (Mason).



(*d*) (Austen and Hackett.)

(*a*) Uncarbonized pit at periphery of void in polythene.
(*b*) Partly carbonized discharge channel from void in polythene.
(*c*) Semi-carbonized channel, void between polythene and brass electrode.
(*d*) Carbonized areas with discharge breakdown in impregnated paper.

FIG. 63. Photomicrographs of partial breakdown caused by internal discharges.

in a tree or other extended form, so that thermal instability can occur in a stem where the current is concentrated. With a non-uniform dielectric large local concentrations of field can be built up by differential changes in conductivity or permittivity, and partial breakdown can take place. Thus in a laminated or other suitably non-uniform dielectric, breakdown in low mean fields and at long times can be explained by considerations such as the foregoing and in some instances, for example in cables, there is a good deal of direct evidence.

In other instances the beginning of a breakdown path seems in any event to follow from ionic bombardment or the thermal effect of arc-like conditions. There is some initial difficulty over the latter because from (4.2) the duration of the discharge is of the order of 10^{-8} sec. and almost certainly not less than 10^{-9} sec. which, with a dissipation of charge of 2×10^{-12} coulombs would not yield enough current for a glow-arc transition except on the basis of a limit similar to Rogowski's 10^{4} A/cm.^{2} Ignoring the theoretical problem, it is known that the discharge can sometimes erode the dielectric where it strikes. This has been noted for some time in certain transparent dielectrics but only recent evidence, in course of publication, communicated by J. H. Mason[16] confirms the mechanism quantitatively. If the erosion continues intrinsic or mechanical breakdown will ensue. This mechanism is inapplicable to what is at present believed to be a frequent circumstance, namely where the void does not extend in the direction of the field. Nor does it apply *a fortiori* where the discharge is stifled because the void surface conducts by carbonization or other action or when the void fills up with liquid or solid products. The further propagation of the partial breakdown raises the same difficulties as when partial breakdowns were considered in Chapter II since the supplementary hypothesis of the preceding paragraph are no longer postulated. Neither the present type of partial breakdown nor the analogous type in Chapter II have been adequately studied regarding their influence on complete breakdown.

Chemical actions are associated with longer times. If, in accordance with Thomas,[27,35] the chemical action is mainly due

to the products of the discharge, such as ozone, oxides of nitrogen, nascent hydrogen, and the like, then certain decomposition products are to be expected. In a number of instances, chiefly with organic dielectrics, the decomposition in the neighbourhood of a void has been found consistent with this hypothesis. Furthermore, the use of materials which absorb the discharge products is beneficial, such as the use of unsaturated hydrocarbons to absorb nascent hydrogen and the chemical stabilization methods of Chapter V. This chemical action will make the void conducting but not highly conducting, so that it will still be a source of energy loss. Probably more important from the point of view of thermal instability, to be considered later, is the fact that the products of decomposition can diffuse into the solid dielectric and produce bodies of not inconsiderable dimensions with a higher conductivity or energy loss. This hypothesis is limited to where the products of deterioration are generated in sufficient quantities. In Chapter V consideration is given to self-increasing deterioration by electrolysis, and catalysis must be taken into account. Admittedly self-increasing deterioration has mainly been studied in connexion with d.c. but extreme examples with poorer dielectrics have been observed with a.c. and in other cases it seems the only probable explanation even though direct confirmation is lacking.

It is desirable to give a rather detailed account of the possibility of thermal instability as a result of discharges, both because thermal instability is often the final cause of breakdown and also because it is often invoked without proper thought as to its limitations. It was noted by Austen (Davis, Austen, Willis Jackson[25]) that discharges, e.g. in polythene, tend sometimes to diminish in intensity, but if so the energy loss due to the discharges is replaced by a more permanent energy loss, with a loss-angle not voltage-dependent but of a conductive type. The second energy loss is of the same order of magnitude as that due to the discharges at the same voltage and frequency. This result follows from considerations of a fairly general order. Representing the dielectric by the model of Fig. 42, then if the discharge voltage of C_c is V the energy dissipated per discharge is $\frac{1}{2}C_c V^2$.

If the r.m.s. voltage across C_c, due to the application of U (r.m.s.) to the dielectric, is V_0 then from equation (4.4) the number of discharges per cycle is $4\sqrt{2}V_0/V$ to the next lower integer or two less. The energy loss per cycle due to discharge is thus of the order of $2\sqrt{2}C_c VV_0$ provided the residual voltage on the void is small. If the decomposition due to the discharges yields an equivalent resistance R, the energy loss due to this is $RC_b^2\omega^2U^2$ per second, provided $RC_b\omega$ is much less than unity. The energy loss per cycle is $2\pi RC_b^2\omega U^2$. But $C_b U \simeq C_c V_0$, making the energy loss per cycle $2\pi RC_c^2\omega V_0^2$. The r.m.s. voltage across the void is of the order of $RC_b\omega U$, i.e. $RC_c\omega V_0$. If the discharge ceases when the maximum voltage swing applied to C_c is reduced by R to a value below the discharge voltage V, then $2\sqrt{2}RC_c\omega V_0$ must be rather less than V. Suppose $2\sqrt{2}RC_c\omega V_0 = V$. The energy loss per cycle in the resistance R is then $\pi C_c V_0 V/\sqrt{2}$. From this it follows that the ratio of the energy loss in the conducting film or defect which replaces the discharging void to the energy loss in the original discharges is $\pi C_c V_0 V/4C_c V V$ or 0·79. Although the two kinds of energy loss are therefore of the same order of magnitude in these conditions, in other conditions they may be quite different. The energy loss per second due to discharges is to a first approximation proportional to the frequency, but the conduction energy loss is proportional to the square of the frequency. The discharge energy loss is zero below a certain voltage, increases rapidly in a narrow critical region, and is thereafter proportional to the voltage or, more precisely, $V-V_c$, but the conduction energy loss is uniformly proportional to the square of the voltage. It is possible to analyse practical cases by these properties and examples have already been given.

In Chapter III, section (3.4), it was shown that the distortion of neither equipotentials nor isothermals due to an electrically and/or thermally conducting defect could usually cause thermal instability. But a discharge generates heat, namely $\frac{1}{2}nC_c V^2$ if repeated n times a second. To obtain the order let the field be uniform in the void supposed to have a cross-section A and thickness x, so that C_c is $A.10^{-11}/36\pi x$ farads. Next make the rough approximation that V is Ex, where E is the effective

electric strength of the gas. The heating developed is then $nE^2/(72\pi.10^{11})$ watts/cm.[3] If it is permissible to apply the last result to a hemisphere of radius a, the heating would be $nE^2a^3/(108.10^{11})$ watts. Refer now to Chapter III, section (3.4), whence it can be deduced that this would produce a temperature rise of $nE^2a^2/(k.216\pi.10^{11})$ °C. (k = thermal conductivity), a temperature which must be of the order of θ_0 (specifically $1{\cdot}19\theta_0$) appreciably to reduce the thermal breakdown voltage in a dielectric where the losses vary with temperature θ according to e^{θ/θ_0}. Thus is obtained the equation

$$nE^2a^2 = 2\pi k\theta_0 \times 1{\cdot}19 \times 108 \times 10^{11}. \tag{4.17}$$

The maximum voltage V across the defect or void is Ea, whence

$$nV^2 = 2\pi k\theta_0 \times 128 \times 10^{11}. \tag{4.18}$$

For a dielectric of high thermal resistivity k may be about 10^{-3} so that

$$V = 11.10^4\sqrt{(2\pi\theta_0/n)} \text{ volts}. \tag{4.19}$$

θ_0 varies usually from 25° C. to 200° C. so that if θ_0 is 100° C. then V is $11.10^5\sqrt{(2\pi/n.)}$ Now n will rarely be less than 4 per cycle, which gives 200 per sec. at 50 cyc./sec. whence the highest value of V is about 200 kV for these conditions. If the number of discharges per cycle is increased, then the discharge voltage V to satisfy the preceding condition will fall, since it is inversely proportional to $\sqrt{n}$, but the voltage swing applied to the void is of the order of $\frac{1}{2}nV$, so this voltage will increase. Accordingly, local thermal instability is more likely to occur in large voids not far removed from the inception region. The magnitude of the voltage just calculated makes thermal instability of this character unlikely at power frequencies, but at higher frequencies the situation is different. For example at 200 kc./s., the discharge voltage so calculated would be 5 kV. At 25 Mc./s. V would be 300 volts, i.e. the order of the minimum breakdown voltage for air so that above this frequency any discharge should materially reduce the local thermal breakdown voltage. This calculation assumes conditions favourable to thermal instability, as may be instanced by one of its implications, namely that in the critical condition the charge neutralized, q,

would appear to be $0{\cdot}6k\theta_0/fE_0$, if n is $4fE_0/E$, where f is the frequency and E_0 the crest working stress applied to the dielectric. Such a formula would yield far too low a value for q. For example, if q were taken as the practical limit of about 2×10^{-12} coulombs, the minimum frequency for thermal instability would be about 1 Mc./s. for a working stress of 50 kV/cm. A more realistic calculation by Bowdler[28] for a polythene radio-frequency concentric cable, assuming the heat from the discharge to be applied to the inner conductor and dissipated therefrom over the length of the cable, yields a critical frequency of the order of 50 Mc./s. for a single void with $q = 2\times10^{-12}$, or 3 Mc./s. for one void per cm. length, presuming 4 discharges per cycle per void. Nevertheless, the previous calculation gives the order of frequency at which to expect thermal instability, indicates general features, and leads to the suggestion that the charge neutralized per discharge is likely to have a maximum safe value for a given dielectric operated at a working stress of given magnitude and frequency.

Somewhat different considerations apply to a void in a uniform field E_0 which has an appreciable area normal to the field and is thin in the direction of the field. The capacitance C is $A.10^{-11}/36\pi x$, where A is the area and x the thickness of the void. The thermal resistance is $\lambda d/Ak$, where d is the dielectric thickness to the cooled electrode and λ a factor taking into account the non-uniform heat flow. The number n of discharges per second is $4f\epsilon E_0/E$, where E is V/x and ϵ is the permittivity. If the temperature rise is θ_0,

$$\theta_0 = \tfrac{1}{2}CV^2n(\lambda d/Ak) = (10^{-11}/18\pi)(f\epsilon\lambda/k)V^2(E_0d/Ex)$$
$$= (10^{-11}/18\pi)(f\epsilon\lambda VV_0/k)$$

or
$$VV_0 = 18\pi\times10^{11}(k\theta_0/f\epsilon\lambda). \qquad (4.20)$$

The higher the applied voltage V_0, the thinner are the voids which can produce thermal instability since the higher V_0 the greater is the dielectric thickness d, if it is assumed that d is of the order of the total dielectric thickness, or the greater is the working stress. If $k\theta_0$ is about 0·1 and $\epsilon\lambda$ about 10, then at 50 cyc./sec. VV_0 would be $36\pi\times10^7$ so that thermal instability

would be possible for insulation working at over 100 kV. The charge q neutralized per discharge can be calculated as CV, i.e. $(k\theta_0/2f\epsilon\lambda)(A/V_0x)$. At 50 cyc./sec. q would have to exceed 100 micro-micro coulombs. If A/x is taken as 1 mm., then to yield thermal instability with $q = 2\times10^{-12}$ coulombs, the frequency would have to be about 0·1 Mc./s. if V_0 is 10 kV. This is rather more favourable than the similar calculation for the hemispherical void but not greatly so and, in addition, the less the value of A/x the less is q. It may therefore be concluded that shape is not a strongly determining factor but that, with a void of shape such that the area normal to the field determines the thermal dissipation, thermal instability is possible at power frequencies within certain distinctly limited conditions. This otherwise would seem scarcely possible.

In practice, the thickness of the void or defect will initially be comparable with the thickness of the dielectric only where thin insulation is involved, e.g. in capacitors. Here the external thermal resistance is significant. Provided the area of the defect is so large that lateral conduction of heat is not important, the effect is that of replacing part of the dielectric by the discharge. The energy loss in the latter has a smaller temperature coefficient than that of the dielectric. This raises the maximum stable temperature, but since the absolute magnitude of the loss in the discharge is normally much greater than in the dielectric it replaces, the thermal breakdown voltage is lowered. Breakdown through this cause at power frequencies is more likely in special cases such as capacitors, because in other examples of thin dielectrics there is a great margin between the thermal breakdown voltage in absence of discharges and the working voltage.

The previous paragraph draws attention to another important cause of breakdown, namely, a diffuse deterioration which results in a macroscopic volume of poorly insulating material. If such a volume forms an inclusion of sufficient magnitude in a sound dielectric, the greatest energy loss was shown in Chapter III to occur when the conductance was of the same order as the permittance. Thermal instability within the inclusion could then occur in quite low fields, the lower the thicker the inclusion,

because the material would be of high energy loss. As long as the deterioration continues the energy loss is likely to increase to the maximum, since rise of temperature usually accelerates deterioration. The main difficulty has already been mentioned: which is to account for the amount of chemical action in all cases. Complete analyses of the process of the action has been largely confined to cables and d.c. capacitors, which use rather similar insulation. If sought, similar evidence would be found in many other examples.

It is desirable to attempt some brief summary of the conclusions of this section. Breakdown due to discharges in fairly high fields and rather short times can be satisfactorily explained. Similarly breakdown at high frequencies is easy to understand and the time factor presents little difficulty; thermal breakdown, as follows from Chapter III, can cover a very wide range of times, in the limit it is theoretically infinitely slow. A wide range is also appropriate because the discharging voids themselves can develop on occasions from the innocuous to the dangerous. Long time breakdowns at power frequencies are more difficult. For particular types of dielectric Robinson's theory, erosion and the postulation of suitable defects, or other theories discussed in Chapter III and the present section afford an explanation. Deterioration of a more diffuse character initiated by discharges can lead to electrical or thermal breakdown. Nevertheless, the analyses available are very incomplete. In practice a great deal of insulation has given satisfactory and lengthy service in the presence of internal discharges. The inadequacy of theoretical analysis is illustrated by the fact that the prediction of what is tolerable and what is dangerous is still largely based on experience. One or other of the mechanisms described is capable in principle of yielding an explanation, but until many more instances have been analysed the intervention of further mechanisms remains a possibility.

4.5. External discharges

It was noted in section (4.1) that the deleterious effect of corona upon insulation has been known for a long time. Indeed

in a number of cases standard specifications include clauses designed to ensure the absence of corona. The methods of test specified were simple and insensitive.[29] Accordingly, the extent of the problem was not generally appreciated and systematic investigation is of no very long standing. Certain features of the mechanism are known to be the same as for internal discharges, so that there is no reason to repeat the previous discussion. Fig. 42 for an internal discharge applies also to an external discharge if the void C_c is replaced by the capacitance between the element of surface where the condensed discharge strikes and the electrode, C_b is the capacitance in series with the element of surface, and C_a is the reservoir capacitance of the system. Analogously to internal discharges, the phenomena are intermittent and an elementary discharge, i.e. a single current pulse, may be associated with the discharge of as low as a few $\mu\mu$F (e.g. 3 $\mu\mu$F) according to Thomas[27]. Again, the time constants of these capacitances in absence of a discharge are long so that, as before, the changes of electric charge caused by an elementary discharge will persist into the succeeding cycles with alternating voltages. This was indicated by Wellauer[30] in 1925 and the discussion in Chapter V of the present writer's earlier book[31] (1932) was based on similar hypotheses. Whereas this means that a great many of the deductions made in earlier sections of the present chapter can be used, yet there are certain differences. Neither the capacitance C_c nor C_a is necessarily much greater than C_b, in fact C_b can here exceed C_c. It is not proposed to analyse the circuit behaviour. Experimental studies have been limited and the indications given will suffice together with the statement that the same techniques and criteria for their detection apply to external as to internal discharges. In considering mechanisms of breakdown it has to be kept in mind that external discharges can give rise both to surface breakdown and to penetration of the dielectric and that the discharging space is open to the ambient atmosphere.

From his work on cable breakdown, Robinson[4] developed a method of test and study of insulation by exposing it to discharges from a pointed electrode near the surface and the same

method was followed both previously and subsequently in connexion with Lichtenberg figures and surface-spark-corona, the term used by Toepler.[32] We have seen that Robinson was concerned with the special hypotheses of transverse breakdown, while Lichtenberg figures and surface-spark-corona are not necessarily associated with penetration of the dielectric. Accordingly, surface breakdown as such will be considered in the later section on 'tracking', that is, breakdown which may only affect a thin surface layer of the dielectric.

If the discharge causes chemical deterioration, then breakdown may ensue due to reduction of intrinsic electric strength, thermal instability, or mechanical failure. These forms of breakdown have already been discussed, and chemical attack is a well-known concomitant of these discharges. We need, therefore, only consider the source and nature of the chemical activity. The work of J. B. Whitehead,[8] Goldman and Wul,[33] Paine and Brown,[34] and Thomas,[27,35] covering various types of paper insulation, cellulose acetate, varnish cloth, and rubber, is consistent with the view that the chemical agents are the gaseous products of the discharge. Thomas's experiments seem almost crucial on this point as regards air, since he directly compared the effect of the gaseous products in presence and absence of the actual discharge and found little difference. It seems that ozone and nitrogen oxides are mainly responsible for the chemical effects with nitrogen, oxygen, or mixtures thereof. Exposure of organic insulants to corona in dry air produces substantial quantities of water even with an inert dielectric like polythene. On the other hand, there is a considerable literature devoted to the electronic bombardment of oil which suggests that this aspect is important. A closer analysis reveals no decisive evidence of a factor additional to the nascent gases which may be expected to be formed in the gaseous discharge. The same refers to earlier work on crystalline and mineral dielectrics. The conclusion may be drawn that chemical deterioration with a number of organic dielectrics definitely arises from the action of the gaseous discharge products, but in some instances bombardment by charged particles may be a material factor.

Chemical deterioration is normally a slow process and causes breakdown only after a long period. Breakdown in the presence of discharges can occur after quite short periods. Examination of earlier work yields no evidence to contradict Thomas's assertion that breakdown at other than very long times cannot be directly correlated with chemical attack. Hill[36] in fact says that with cellulose compounds failure owing to chemical attack is after very long times and is primarily mechanical. The deterioration spreads over the whole area exposed to the discharge, and usually slowly penetrates from the surface into the volume of the dielectric. Organic materials also absorb physically the gaseous discharge products, causing deterioration apart from the slower chemical attack. Thus some recovery in dielectric quality is possible after exposure to corona if the conditions favour desorption.[27]

Breakdown is a microscopic phenomenon and can occur at the same applied voltage with varying degrees of chemical attack and the point of breakdown is not characteristically associated with any local peculiarity of chemical action, at any rate such as is observable by ordinary means. Thomas failed to find any warning signs of the onset of breakdown, although his observations were admittedly not highly localized as to place or time, that is, they dealt with the mean properties of an area of the order of 1 cm.^{2} and time intervals of, say, 1 second. The classical work of Rayner[37] showed that the breakdown voltage in presence of discharges is essentially time-dependent and later work has confirmed this on all other materials tested. The mean voltage which has to be applied to a dielectric to cause breakdown in a short time with discharges varies greatly from one dielectric to another. Thomas and others have indicated that if a layer of corona is formed over an organic material it will ultimately break down so that the final breakdown voltage is determined by the field in the air gap and not the field in the dielectric. This must necessarily be true for very long times because the electrical and chemical types of breakdown would be indistinguishable. Inorganic dielectrics and possibly certain oxygen-free synthetic dielectrics may resist the discharge for so

long that the ultimate breakdown cannot easily be ascribed to a given class of phenomena. With these exceptions, it seems possible to say that breakdown due to external discharges is a definable phenomenon, with the property that the field strength in the dielectric has probably not a major influence on the ultimate breakdown voltage extrapolated to indefinitely long times but has a major influence on the time before breakdown occurs when a higher voltage is applied.

In the same way, dielectrics seem to divide themselves into those which are corona-resistant and those which are not corona-resistant. The latter will nearly always break down if the discharges are long enough maintained, but the time required for any particular voltage above the minimum will vary with the dielectric. The breakdown voltage of corona-resistant materials is only affected by some 30 per cent. unless the corona is so intense and so long maintained that the material becomes largely decomposed.

In the present state of knowledge it seems convenient to discuss separately the phenomena which occur in three different regions. At very long times, measured in weeks, months, or years, and where the electric stress in the dielectric is normally very low compared with its intrinsic electric strength, breakdown is a function of the chemical stability of the material with respect to the active discharge products and failure is usually due to macroscopic chemical deterioration.

The second region comprises times from minutes to hundreds of hours if the electric stress in the dielectric is still quite low. For materials which can support a higher electric field the second region extends to longer times. Breakdown is here of a microscopic character. The apparent electric strength of corona-resistant materials does not decrease very markedly in presence of corona as the time of application is increased, possibly some 10–30 per cent. With a non-resistant material, the breakdown voltage falls decisively to, say, one-half or less (Thomas,[27,35] Montsinger,[38] etc.) over the range of times mentioned. As the time of application of voltage grows longer the influence of the form of the electrode becomes less important. This is subject

to the condition that the air gaps are comparable, since for a given thickness of dielectric there may exist a thickness of air gap for which the breakdown voltage is a minimum as suggested in Fig. 64. There is a further condition that the areas of the dielectric exposed to discharges must not differ too widely, or the probability of defects may decrease the electric strength for

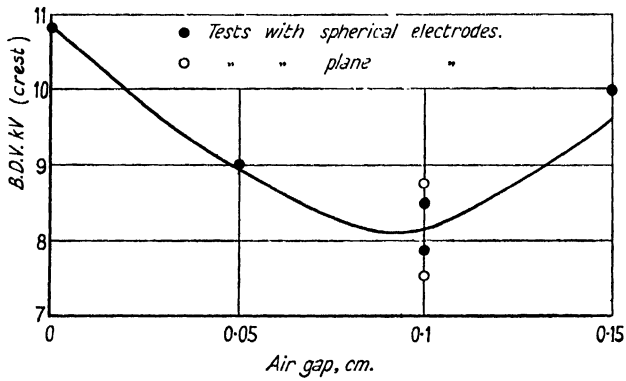


FIG. 64. (Thomas.) Variation of long-time (2 hour) B.D.V. with air gap. Polyvinyl chloride sheet 0·015 cm. thick.

the greater area. Thus the breakdown voltage with a plane discharge layer may be less on this account than when the discharges form round a spherical electrode or at the edge of an electrode in contact with the dielectric, but the mean differences have usually been found to be within the extreme range of dispersion of results. It follows from the present discussion that the ratio of the breakdown voltage at a long time of a few or some hundred hours to the breakdown voltage for a minute or minutes should yield the order of the 'corona resistance' irrespective, to some extent, of the corona-forming electrode system and the thickness of the dielectric. Up to a point this seems to be true, but there is no definite evidence as to how far this ratio is affected by the intrinsic electric strength as distinct from the chemical composition and structure. Corona resistance seems to be least in organic compounds containing oxygen, such as some of the plastic polymers; hydrocarbons without oxygen are better, the replacement of oxygen by nitrogen as in some amino

polymers effects a considerable improvement (compare, for instance, melamine and urea-formaldehyde with phenol-formaldehyde), as also does the replacement of hydrogen by halogens such as chlorine (compare, for instance, neoprene and chlorinated rubber with rubber products); the replacement of carbon by silicon, as in the silicones, tends to transfer materials analogous to organic polymers into the class of mineral dielectrics which as a whole may be regarded as corona-resistant. However, this tentative classification corresponds either to the same order of intrinsic electric strengths or else the latter do not greatly differ, so that no conclusion can be drawn. Corona resistance, the decomposition or softening or melting temperature, and the intrinsic electric strength tend to fall in the same order when these properties are listed.

The third range to be considered is from a fraction of a second to a few minutes where the field strength in the dielectric is high, approaching the region in which may fall the low values sometimes obtained in intrinsic breakdown tests. Little evidence is available from experiments with deliberately controlled air gaps. Information is mainly derived from experiments with rounded electrodes pressed on plane dielectrics or in ill-fitting recesses or with flat electrodes making poor contact with the dielectric or with unprotected edges. In Chapter II reference was made to the variation of apparent electric strength with thickness caused by discharges and to an explanation on the hypothesis that there was a critical field strength at an edge concentration in the medium which caused breakdown. The hypotheses of Schwaiger and Kropp[39] on edge breakdown, and Goldman and Wul[40] on pore breakdown are essentially akin to the preceding theory. This theory yields some rough qualitative descriptions, but is quantitatively unsatisfactory. A similarly qualitative explanation has been given by the present writer[31] of the fact that with rounded electrodes, that is with a gap of variable thickness, breakdown will take place at a certain distance from the vertex. This theory made use of the increase of electric strength with decreasing thickness for thin films of gases and liquids: its semi-quantitative application was thus limited to the very thin films

round the vertex of an electrode with a shape not greatly dissimilar from the dielectric surface. These theories are based on the hypothesis that breakdown of the medium causes breakdown of the solid dielectric, and ignore the essential feature that with the same field in a similarly disposed ambient medium a dielectric with a much higher intrinsic electric strength has a higher short-time breakdown strength. The shorter the time of application the more the breakdown voltage is determined by the electric strength of the solid dielectric as well as by that of the medium, in contrast to long times where the electric strength of the medium is the chief factor in determining breakdown. It is impossible to give quantitative comparisons of general validity for different materials because, with d.c. or voltage of a given frequency, the effect of discharges is greater if the thickness or the permittivity is greater as well as varying with the period of voltage application. It is possible, however, to quote some values to give an idea of the orders of magnitude. The apparent electric strength in air of ruby muscovite mica, say 0·05 mm. thick, with voltage applied for some seconds, may be only about one-tenth of its intrinsic electric strength, although a comparison by Hackett and Thomas[41] gave one-third to one-fourth; but a phlogopite mica will probably only be reduced to one-quarter, or even one-third or one-half, of the intrinsic electric strength. A cellulosic material in similar conditions may be reduced in electric strength to one-third or one-quarter of the intrinsic value. Compressed cellulose has a permittivity approaching that of mica. Experiments with discharges on synthetic polymers have not unfortunately been made on specimens strictly comparable with those from which intrinsic electric-strength values have been deduced, but the reduction appears to be to something of the order of one-third on specimens with thickness of the order of 0·1 mm. With materials of low electric strength such as the alkali halides the reduction of electric strength seems to be less.

The discussion of what we have called the third range largely applies to most of the time-voltage curves derived in engineering tests and therefore to the data on such tests assembled and

analysed in a previous book.[31] We may conclude that tests such as the 'minute' electric-strength test of British standards and the corresponding U.S.A. (A.S.T.M.) tests are in reality tests of corona resistance, i.e. electric-strength deterioration occasioned by external discharges. The work of Montsinger[38] and of many others based on Peek's formula for the time effect on breakdown may similarly be regarded. Montsinger has expressed Peek's formula as:

$$V = V_0\{a+(1-a)t^{-\frac{1}{8}}\}, \tag{4.21}$$

where V = breakdown voltage at time t in minutes,

V_0 = 'minute' breakdown voltage,

a = constant depending on the material and the temperature.

For certain insulation he considers this formula to hold from, say, 1 second to several hours, but, as we have seen, the flattening of the curve so obtained at longer times is due to the extension of the phenomena into what we have called the second region, where breakdown is increasingly and largely determined by the breakdown of the medium and independent to a considerable extent of the conditions in the solid insulation.

In view of the complexity of the phenomena and the limited basic information available, only a brief discussion of the mechanism of breakdown by external discharges is possible. Failure at very long times by chemical deterioration need not be discussed at all since the possible mechanisms are numerous and obvious. If partial breakdown can readily occur, as with laminated dielectrics, the discharge tends to strike at a particular spot on the surface and to be of a condensed or low-resistance type (Fig. 65, see also Mason[16] and Fig. 29). Breakdown of the dielectric begins at the spot which by virtue of the discharge acquires the potential of the electrode. It can be propagated by a tree-like or other extended breakdown path as already examined in connexion with internal discharges.

With other dielectrics, and with electrode arrangements where breakdown is sudden and complete, there is some photographic evidence (Fig. 66, Plate XI) that a condensed discharge strikes

frequently at a certain spot and starts breakdown, but there is little direct confirmation. Theoretically it is possible to say that a spot capable of emitting electrons would facilitate the formation of a condensed discharge. On the other hand, the rather small surface-capacitance discharged by a single discharge raises the same difficulty as with internal discharges of finding a high

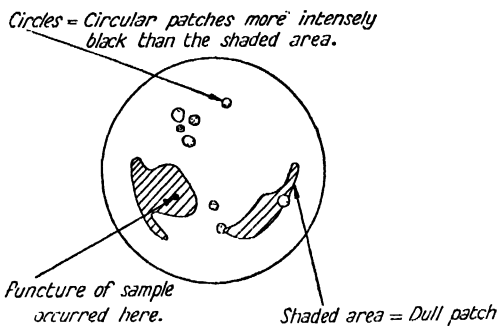
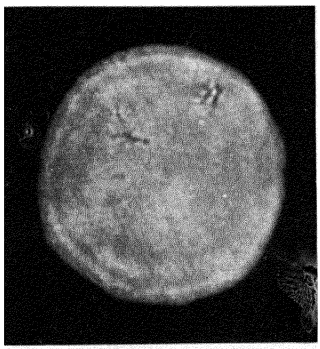


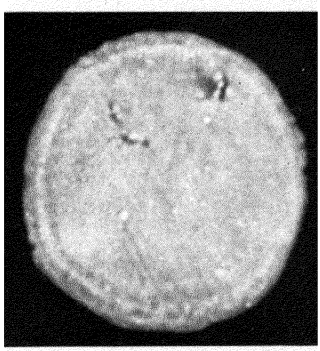
FIG. 65. Appearance of surface of varnish cloth after exposure to corona. (Thomas)

enough localized current to provide a low resistance discharge. The evidence from internal discharges, together with that of Thomas[27] and Hill[42] on external discharges, makes it highly probable that a condensed discharge takes place. Hill speaks of the heating effect of H.F. components, but this is a misconception since the energy arises from the discharge of a capacitance and the appearance of these H.F. components is a manifestation of the rapidity of the discharge. He makes, however, the useful point that phenomena depending on the size of electrode or area of dielectric exposed and ascribed to defects (cf. Whitehead,[31] Holmes[43]) may in reality arise merely from variability of the edge or contact film of the ambient medium. There is an unresolved difficulty, already mentioned in connexion with internal discharges. If the energy dissipated per second is of the order of $\frac{1}{2}nCV^2$, where n is the number of discharges per second which discharge the capacitance C by a voltage-drop V, then both n and C may be expected to decrease as V increases but at unknown rates. Thus, there is no evidence

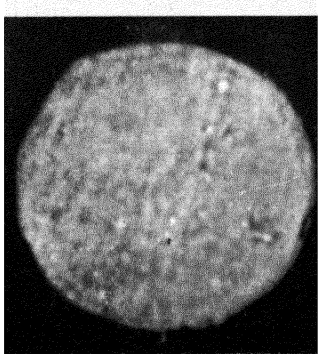


(*a*) Photograph of discharge on 0·010-inch black varnish cloth immediately after switching on applied voltage 7,800 V.

Note: Thread-like dark areas are associated with scoring of the glass surface and not the sample (cf. (*b*) and (*c*)).



(*b*) Photograph of same area taken after $6\frac{1}{2}$ hr. continuous exposure.

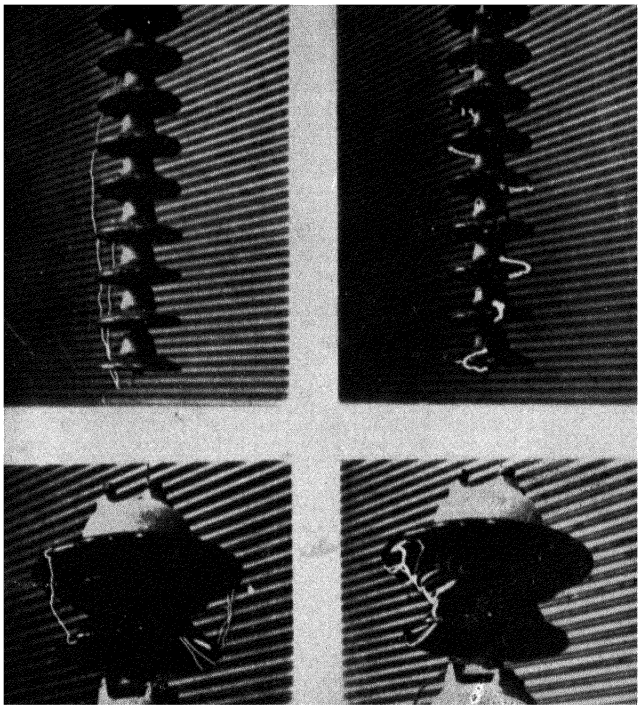


(*c*) Photograph of same area after rotating the glass electrode through 90° with respect to the sample.

Note that the spark concentrations associated with the sample remain in the same position whereas the thread-like dark areas have moved with the glass.

FIG. 66. Surface discharges. (Thomas.)

PLATE XII



(*a*) (*b*)

FIG. 67. Cascading discharges. (Davis, etc.)

(*a*) At minimum flashover voltage.

(*b*) With high over-voltage.

to show whether the most damaging discharge is one frequently repeated, at the minimum breakdown voltage of the gas, or one where the gas film or layer is only just thin enough to provide a discharge repeating at the minimum rate for stability defined in earlier sections.

Presuming that the hypothesis of a repeated condensed discharge at a selected spot is valid, it is easy to explain the beginning of a breakdown path but less easy to understand its propagation except when the mean field in the dielectric is high. The same hypotheses studied for internal discharges can apply. The limitations of these hypotheses already considered are here somewhat alleviated by the facts that the external discharge will not be extinguished by conduction in accumulated discharge products or changes in the gaseous contents, and the external discharge is not necessarily dependent on the field distribution in the dielectric.

If the mean field strength in the dielectric is high enough, the concentration of field at the striking point of a discharge can start a self-propagating intrinsic breakdown, provided the volume where the intrinsic electric strength E_0 is exceeded is large enough and provided the ratio of maximum field E_m to mean field E is not too great. That is,

$$E_m = kE_0, \qquad E_m = k'E,$$

whence

$$E = (k/k')E_0. \tag{4.22}$$

Thus the apparent electric strength E will be a certain fraction k/k' of the true electric strength E_0 if (k/k') is to a first approximation independent of field strength. This assumption implies also that the cross-section of the condensed discharge varies not too greatly with the field strength so that the non-uniform field caused thereby conserves a certain shape. Considering a spheroidal inclusion in the dielectric surface, k' would not exceed the order of 30, and from Chapter II a fairly low value of k would be 3, so that there are *a priori* reasons for considering this mechanism down to mean fields as low as the order of one-tenth the intrinsic electric strength. Over this range the effect of time of application of stress would be similar to its effect on the short

gaseous discharges which initiate breakdown. The decrease of breakdown voltage when pointed electrodes are used instead of a more uniform field should be less with external discharges than with true intrinsic breakdown, if indeed the latter has ever been validly measured with pointed electrodes. These are the kind of conclusions drawn by Inge, Walther, and Wul[44] from their experiments. The effect of time over this range is qualitatively supported by Montsinger.[38] Some further mention of these ideas will be made in connexion with impulse breakdown.

The preceding descriptions and discussion have mainly related to power frequencies. With higher frequencies the time limits of the breakdown ranges indicated will be greatly shortened because discharges will be much more damaging on account of the increased frequency of repetition. In addition, thermal instability as a propagating mechanism for breakdown becomes more probable, indeed inevitable, if the frequency is high enough. Reference is made again to the discussion of internal discharges.

Variation of temperature affects the gaseous discharge only to a limited degree in the temperature ranges normally studied. Chemical action is increased by rise of temperature and thermal instability is facilitated. The intrinsic electric strength may fall with increase of temperature. These considerations can be applied to any particular example in so far as the variation with temperature of the parameters mentioned is known, but clearly no generalizations are possible owing to the very different behaviour encountered in various dielectrics.

4.6. Impulse breakdown

The time required for gaseous breakdown is much longer than for intrinsic solid breakdown. If the electric field is only applied for a very short time, intrinsic breakdown of the solid dielectric should take place even if unprotected edges and small voids are present where gaseous discharges could otherwise develop. While this has often been stated,[44,45,46] there is no really quantitative and direct evidence which can be quoted as free from criticism. Standring[46] obtained intrinsic breakdown using impulse voltages with specimens of thickness much greater than

have been found possible in tests with more sustained voltages. Although he was compelled to take the precautions described in Chapter II, it may be deduced that, except with mica, where the intrinsic electric strength is unusually high, specimens as thin as those generally used in breakdown tests would have exhibited intrinsic breakdown with comparatively minor precautions such as the prevention of flashover. This refers to external discharges, but the same would apply *a fortiori* to internal discharges which are less damaging at short times provided field distortion due to a void can be ignored. It may perhaps be said that breakdown in times from a fraction to the order of a microsecond is unlikely to be affected by discharges, but the upper limit is speculative and depends upon the electric strength of the dielectric and the distribution of the field.

When the time to breakdown is longer, there is a decrease of electric strength, but observations of its magnitude are discordant and hard to analyse. For example, Davis and his colleagues, whose work shows all the signs of care, give results on cables (both polymerized hydrocarbon and impregnated paper) which suggest that the variation over the microseconds range does not exceed the order of, say, 10 per cent.[47] However, the values of electric strength observed did not exceed one-half those normally applying to the materials and were sometimes much less. Explanation in terms of the non-homogeneity of the electric field in a heterogeneous medium was of limited application. Inge[48] and her co-workers have found changes of as much as 2 to 1 in the range of times up to milliseconds. Analogously with section (4.5), Inge,[49] Draeger,[50] and others consider that partial breakdown may take place with impulses in homogeneous materials exposed to discharges or to non-uniform fields. The conditions for discharges were certainly present in the experiments with non-uniform fields. The first effect of the discharge as the duration of stress lengthens may be to provide a field concentration at its tip. From Chapter II it is known that partial breakdown can then occur. If this breakdown is propagated through the dielectric because the mean field is high enough, it is not unreasonable to suppose that the time required is much longer than for intrinsic

breakdown in a uniform field and is also dependent on the mean field, i.e. on the voltage. It is possible in this way to explain some results by Inge and Walther. They state that partial breakdown (see also Draeger) or the 'initial' discharge occurs at a voltage which is only slightly time-dependent. This is apparently associated with the development of a surface spot having a time of formation which is not much reduced by an over-voltage. In rock salt the propagation of the partial breakdown through the dielectric is very rapid and follows the appropriate directions in the crystal, presumably owing to the low intrinsic electric strength of rock salt, so that the voltage for complete breakdown varies little with duration of the impulse. With glass the propagation through the dielectric takes longer and the breakdown voltage increases appreciably as the duration of the impulse is reduced. Inge and Walther[48,49] invoke thermal action for the slowness of propagation in glass, but this is unlikely as a primary phenomenon as has previously been shown. The difference with glass is more likely to be due to its high intrinsic electric strength and to its amorphous character which, incidentally, explains the straight breakdown paths along the field encountered in Inge and Walther's experiments. Electronic breakdown in amorphous materials is associated as in Chapter II with a certain 'specific heat'. In uniform fields it causes only a very short time lag, but this would be increased in non-uniform fields and should be much longer in amorphous substances than in crystals. Unfortunately no quantitative study is available. Ideas of this kind are not inconsistent with more modern work such as that of Davis[46] and his co-workers and of Allibone and Perry,[45] but they have not studied materials with really low intrinsic electric strengths. Furthermore, the rather small variations in recent work[51] (usually not more than 10 per cent., sometimes 30 per cent.) refer to shorter times than those used by Inge and Walther. Nevertheless, provided the voltage on the gaseous discharge space is sufficient to yield a complete discharge in not more than, say, 10^{-7} sec., there is something to be said for the hypothesis that breakdown is conditioned by the intrinsic breakdown of the solid dielectric propagated at a finite

rate by the field concentration in the neighbourhood of a partial breakdown. This rate seems to be less with amorphous dielectrics of high intrinsic electric strength and is voltage-dependent, so that as the time is lengthened the electric strength falls from the intrinsic electric strength for a uniform field towards a lower value corresponding to a non-uniform field. Unfortunately, only rather vague indications can be given as to the amount or rapidity of the variation.

A paper by Austen, Garland, and Lacey[57] gives an analysis of the discharge behaviour to be expected with the wave forms encountered in over-voltages, including repeated impulses. An internal discharge during a unidirectional impulse of given sign produces an abnormal voltage distribution which, from section (4.3), will tend to require a higher voltage to cause a discharge in a succeeding impulse of the same sign and may prevent the recurrence of the discharge. This inhibiting effect lasts for a period corresponding to the time constant of the dielectric which may be minutes (with low grade insulation), hours, or days. Accordingly the reduction of electric strength with number of repeated impulses observed by Davis and his co-workers[46,47,51] may be the reduction, with repetition, in the over-voltage necessary to transform initial damage into complete breakdown. It would underestimate the damage to the dielectric in the usual condition in practice where a maintained alternating voltage is present to exploit damage of a serious enough nature. Rapidly repeated impulses should also be less severe than impulses repeated at intervals which permit the restoration of normal charge distributions. The fact that the variation of electric strength with number of impulses is sometimes similar to the variation with duration of impulse can, on the other hand, be due to the mechanism by which succeeding impulses can extend the breakdown path initiated by the first discharge if the mean field is high. This requires local fields in the intrinsic breakdown region. The abnormal charge distribution due to a discharge will have the reverse effect for a succeeding impulse of opposite sign since a voltage up to twice the normal voltage may be impressed on some parts of the insulation. It is for this reason that

with repeated impulses breakdown sometimes occurs on reversal of polarity. If the impulses are frequently and rapidly repeated then, as far as discharges are concerned, the behaviour is similar to that of an equivalent alternating voltage. A further development is pulsed high frequency, which is the same as continuous high frequency except that the mean number of cycles per second is correspondingly less. With rapid repetition of impulses the longer time regions of breakdown are entered and the considerations of sections (4.4) and (4.5) apply. Reference should be made to a paper by Davis, Austen, and Willis Jackson[25] where these forms of stress are studied in detail in relation to internal discharges.

The variation of breakdown voltage with the duration of its application may be expected to be greatest when the mechanism of breakdown is changing. The change from intrinsic breakdown appears to occur at short times except when special precautions are taken to reduce the external gaseous discharge. The regions of breakdown already described in section (4.5) could probably in each instance be identified with a transition from one mechanism to another if enough were known of breakdown mechanisms. If large discharges occur in a material susceptible to their action and in a peculiarly damaging form, then breakdown due to discharges might be expected to persist until quite short times. The variation of breakdown voltage with duration would then be relatively small for periods greater than those concerned in impulse tests. In the impulse region the breakdown voltage should decrease from a value corresponding to the intrinsic electric strength at very short times to the value associated with discharge action. More than one mechanism of discharge action may apply, so that there may be intermediate transitions. Discharges are always most damaging with a thin dielectric easily penetrated once breakdown has begun. The chance of breakdown is also necessarily greater with gaseous voids or films which expose a large surface to the discharges. The intensity of external discharge which is likely to cause breakdown seems to be of the order of 10^{-10} coulomb or more for these short times.

In unimpregnated paper, the time-dependence of the breakdown voltage is often restricted to times of less than a second and sometimes to less than, say, 50 milliseconds, excluding slow deterioration in 20 or more hours. The increase of breakdown voltage which occurs at shorter times yields an impulse ratio which may exceed that for impregnated paper. This is because the electric strength of unimpregnated paper is much lower at longer times than that of impregnated paper so that there is a greater margin for improvement. On this kind of hypothesis, the ratio of breakdown voltages for two different stress durations in the transition region should be greater the greater is the ratio of the intrinsic electric stress to the stress at breakdown in the solid dielectric (i.e. the apparent electric strength) at the longer time. Outside the transition region the reverse would probably be true since low breakdown stresses are likely to be less dependent on time in the range considered, because the gaseous discharge is more important relative to the properties of the solid dielectric than when the breakdown stress is higher. There is a tendency to a saturation effect on the breakdown voltage.

There is little question that no generalization about impulse or similar ratios is possible unless the time-scale is known of the breakdown mechanisms which can occur. These ratios are naturally here understood to compare the effect of wave forms with the same ratio of total voltage change to crest voltage.

4.7. Surface breakdown

The term surface breakdown here means the development of a conducting channel between two electrodes on a dielectric surface. Surface breakdown may be divided into flashover and tracking. The distinction is not highly scientific and is therefore much less easy to make on paper than in practice. The best course is perhaps to say that flashover essentially involves a gaseous discharge but does not necessarily impair permanently the insulating properties of the surface. Tracking, on the other hand, requires essentially that the conducting channel shall lie wholly or mainly in the dielectric surface which suffers damage

in the process. If the ambient medium is liquid or solid, surface breakdown either takes the form of tracking or else may usually be regarded as a breakdown of the medium. Thus the consideration of flashover can be limited to the atmosphere as ambient medium, although compressed air, nitrogen, argon, freon (CCl_2F_2), and the like are now of importance. The actual evidence on these compressed gases relates to the gaseous discharge so that further specific mention of them will not be made except to note that the electric strength of the gas is increased by pressure, but the energy available in the discharge is also increased with correspondingly greater potentiality of damage to adjacent insulation.

Flashover is a gaseous discharge with more or less interaction between it and the solid insulation. The gaseous discharge has its appropriate properties which lie mainly outside the scope of this book. Reference is made to works on discharges in gases, some observations on flashover in an earlier summary[31] and more recent papers by Inge and Walther,[52] Hippauf,[53] Dowell and Foust,[54] Allibone and Perry,[45] Jones and Lusignan,[55] Davis and his co-workers,[46,56] to mention only a few from an extensive literature. The dielectric surface affects the field in the gaseous discharge if not too remote and, in general, the field distortion has the effect of reducing the flashover voltage. This is associated with surface conduction. Bowdler and Standring[56] have shown that a surface wet from rain yields a considerable reduction in flashover voltage, less with impulses than with power-frequency voltages so that the impulse ratio is thereby increased. When the electric field has a high enough value near the dielectric surface the discharge clings to the surface and with line insulators this is known as cascading (Fig. 67, Plate XII). Owing to the complicated nature of gaseous sparkover with these fairly long gaps and non-uniform fields it is impossible to give a quantitative account, but a number of studies have been made of the fields round insulators and correlation has been attempted with the occurrence or absence of cascading. The margin is small and design is mainly on the basis of experience and type tests, for example wet and dry flashover tests.

Since the flashover of undamaged insulators is due to switching or lightning surges more frequently than to any other cause the behaviour with impulse voltages is important. For higher voltages insulators are provided with protective gaps which spark over in the event of an impulse and so prevent cascading. Allibone and Perry[45] and Bowdler and Standring[56] agree that the time lag associated with the sparkover of the protective gaps in the arrangements tested was greater than that associated with flashover of the insulators. The times quoted for sparkover or flashover are much greater than those considered in Chapter II for the time lag of intrinsic breakdown of the solid dielectric. Bowdler and Standring state that their experimental results suggest flashover to have a minimum time lag, ranging upwards from 3×10^{-8} sec., when the equivalent gap is 4 inches, increasing with the length of the discharge. The sparkover voltage for a protective gap normally rises more steeply than the flashover voltage as the time to breakdown is decreased (Fig. 68). It may be suggested that the dielectric surface reduces the time lag the closer the discharge occurs. As the crest value of the impulse is increased, breakdown occurs earlier in the wave and the time lag must also be shortened as the duration of the impulses decreases. In these circumstances there is a greater probability that the discharge will impinge on the surface. A variation of flashover or sparkover voltage of 20–50 per cent., depending upon the design of insulator and protective gap, polarity, and humidity, may accompany a variation of the time from 1 to 10 microseconds (Figs. 68 and 69). The change from, say, 10 microseconds to the periods involved in continuous power frequencies or d.c. seems to be of the order of 5 to 20 per cent. When breakdown takes place on the front of a steeply rising wave which in standard impulses usually lasts a microsecond or less, both protective-gap sparkover and insulator flashover voltages rise steeply and it is possible for breakdown of the solid dielectric to occur first. In engineering this feature is rather inconvenient as it is not possible to design a protected insulating system in which damage to the solid insulator or its surface is avoided for all impulses, without unduly reducing

the rated sparkover or flashover voltage for normal working conditions.

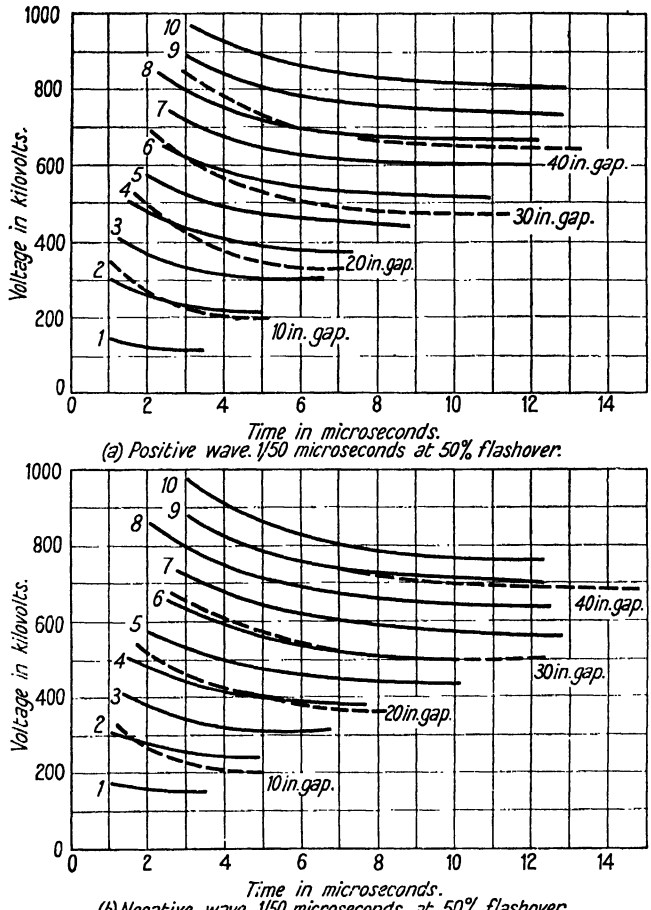


FIG. 68. Flashover of insulators and protective gaps with impulse voltages. (Davis etc.)
(Numbers refer to numbers of units in insulator string)

The deleterious effects of a discharge which impinges on the surface of insulation are well known because the energy is not limited by the dielectric as in the external discharges considered in section (4.6). In an impulse the total energy is limited by the

available charge, which to some extent determines the amount of damage. However, in practice an impinging or cascading discharge initiated by an impulse is liable to be followed by an arc produced by the action of the normal voltage upon the ionized channel left by the discharge. A power arc of this character

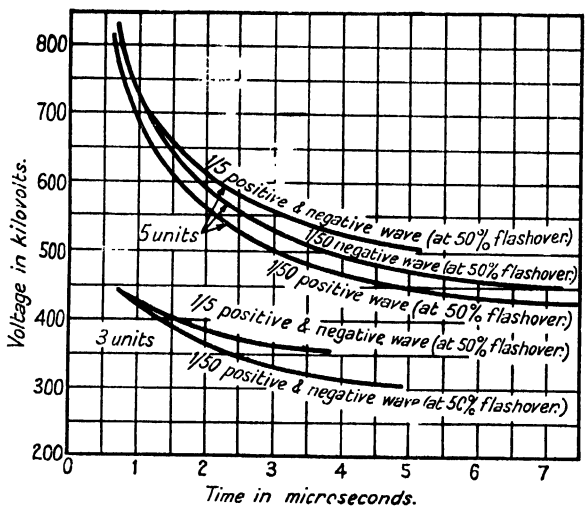


FIG. 69. Flashover of insulators with impulse voltages.
(Davis etc.)

disposes of large amounts of energy, and destruction of the insulation by heat is probable. Further to pursue the subject of flashover would not be justified. Although of great interest, the problems involved are primarily those of gaseous discharges and of the engineering aspects of the intervention and reaction of the solid dielectric, the specific breakdown properties of which enter only in a secondary manner.

In dealing with tracking it is necessary first to distinguish between the high-voltage and low-voltage types. The latter may involve no discharge at all since it can take place at voltages less than the minimum breakdown voltage of the medium. It arises from the presence of conducting paths. These may be due to conducting deposits from atmospheric pollution, including condensation: or the surface may be exposed to salt spray or

coal dust. Alternatively, chemical action may occur at the surface including the diffusion of electrolytically active substances. The conduction over the surface is enhanced by electrolytic action of the applied field. This effect is more pronounced with d.c. but occurs also with a.c. on account of the non-reversibility of the electrochemical actions either because secondary reactions occur or because the products become separated by diffusion. The net effect is the formation of conducting paths because an elementary path will grow in the direction of the field concentration. It may relieve the stress in the laterally adjacent surface. This is less likely to occur where the average surface stress is fairly high: in fact with electrodes of moderate size a conducting path may yield an enhanced lateral stress over part of its length, whence treeing and extensive burning may occur by electrochemical action analogous to the propagation of partial breakdown already discussed (Fig. 70, Plate XIII). When a path carries enough current it will become thermally unstable, resulting in a permanently conducting state. The action is progressive and ultimately a conducting path will bridge the electrodes. No more detailed account is required here since the question is one of the electric and chemical stability of a surface film which is really foreign to the dielectric. (See [58] for a recent summary.)

In high-voltage tracking the conducting path is begun by the impinging of a discharge on the dielectric surface. Since organic insulants are the more subject to tracking, carbonized conducting paths are those most frequently met. Graphite has a positive temperature coefficient of resistance like a metal, but a carbonized path is usually amorphous with a negative temperature coefficient, or comprises very small crystals, the boundaries of which yield also a negative temperature coefficient. A path which bridges the electrodes can therefore be thermally unstable in the manner of the filamentary thermal instability described in Chapter III, section (3.4), and similar equations apply. Accordingly, interest lies in the initiation and propagation of a conducting path and not in its behaviour when formed.

It is desirable first to indicate the order of the current required by a conducting path to reach a temperature at which further

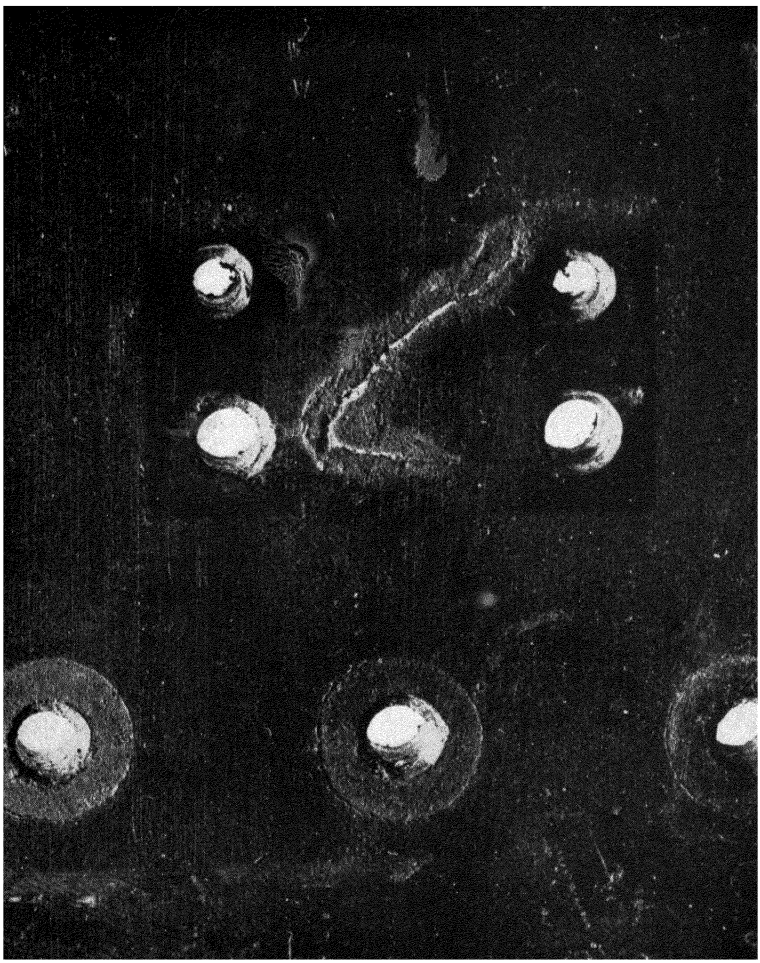


FIG. 70. Low voltage (400 V a.c.). Tracks enlarged laterally on bonded asbestos board with surface pollution by acid.

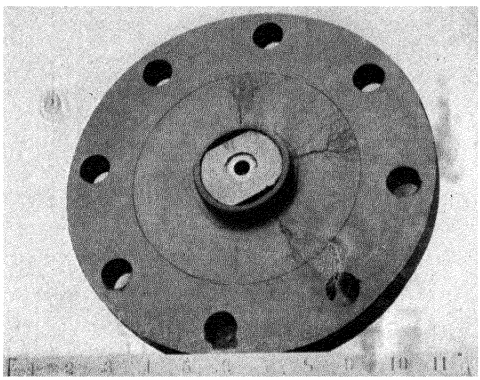
decomposition may occur. Thermal equilibrium between the dielectric and the conducting path is unlikely to be reached, so that the calculation of the heat lost to the dielectric is rather complicated. Since the main interest is in how small a current can cause decomposition, only the heat lost to the ambient medium need be taken into account. Adopting Newton's law, this can be taken as $bl\theta\mu$, where b and l are the breadth and length respectively of the path (in cm.), θ is its temperature rise in deg. Cent., and μ is the thermal dissipation in watts per deg. Cent. per cm.^{2} of surface. If R is the surface resistivity in ohms of the conducting material, and if i is the current flowing therein, the heat generated is Rli^2/b, so that

$$i/b = \sqrt{(\mu\theta/R)}. \tag{4.23}$$

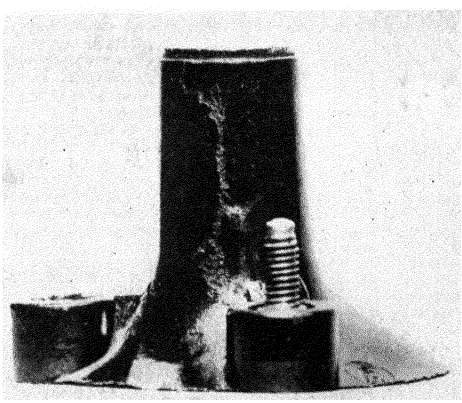
The temperature θ cannot be taken as much less than 200, and μ is about 10^{-3}. Although the resistivity of carbonized insulants is said to reach 10^7 ohm-cm., a carbon track must here be stable enough when heated not to break up without permanent damage to the surface. Thus, we may take for R a value corresponding to a satisfactory deposit, namely about 1,000–10,000, which refers to a track thickness of the order of 100–10 angstroms. These values give a value of 14–4·5 mA/cm. for (i/b). In practice, tracks of less than 0·1 mm. breadth seem infrequently to be of importance, which leads to a value for i of 140–45 μA. From section (4.5) the capacitance associated with an elementary discharge may be as low as 3 $\mu\mu$F and, if repeated 4 times a cycle at 50 cyc./sec. with a voltage drop on discharge of 1,000 volts, this would give a current of 0·6 μA. This calculation, although so speculative, serves to show that a conducting path can only normally be propagated by a discharge current from a small area at high frequencies, or when the voltage so far exceeds the discharge inception voltage that a large number of discharges occur per cycle, or if the discharges take place at very high voltage. In any event, propagation from a very small area is unlikely at power frequencies not greatly removed from the inception region so that a conducting film is necessary to provide larger discharging patches.

If only a small area with a capacitance of a few $\mu\mu$F is at first available a discharging 'spot' or nucleus, near a discharge-forming electrode, where a discharge impinges should move back to the electrode to form a conducting area or path and so provide a connexion with the electrode. There is no great difficulty in this. Should the discharge to the spot occur at a voltage crest, the instantaneous charge transferred to or neutralized at the spot will cause a double voltage to be impressed between the spot and the electrode at the succeeding voltage crest of opposite sign. Without proposing a detailed mechanism, there is clearly a tendency for breakdown to proceed from the spot to the electrode. There seems little difficulty therefore in accounting for the existence of a conducting area or conducting paths from a discharging electrode up to the limit at which discharges can take place. This extension of the electrode coupled with the instantaneous discharge energy, or the mean discharge energy if a large area is involved, is often sufficient to cause breakdown sometimes by mechanical effects and at other times because the margin of safety with respect to neighbouring conductors has been too far diminished. If this does not occur, then we may ignore a regular extension of the electrode, which merely restates the original problem, and consider a discharge-initiated conducting track which has proceeded a certain distance from the parent electrode.

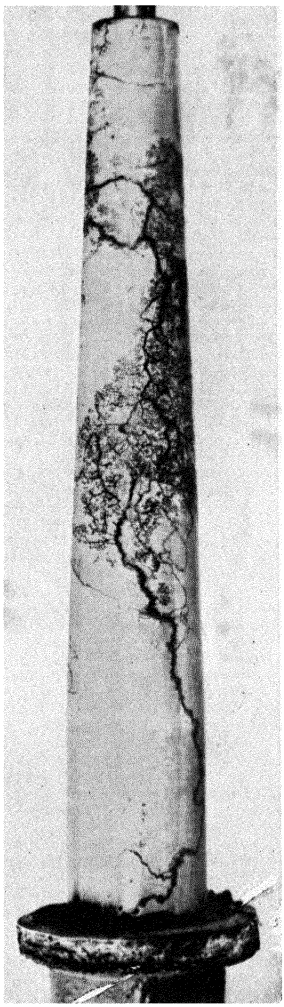
This last problem has a great deal in common with low-voltage tracking, but there is this difference, that propagation can often occur without abnormal pollution of the surface, a condition necessary in low-voltage tracking. A conducting track cannot be propagated by thermal instability of the surface layer surrounding the tip, because the cooling is lateral. However, if the tangential field, i.e. the field parallel to the surface, were strong enough to cause a surface discharge from the electrode, it may be strong enough at a distance from the electrode to cause a glow or corona discharge at the tip of the track by field concentration. The conducting track would then grow in the direction of the field until the electrodes were bridged. This often occurs in practice and is possibly the most usual explanation of



(*a*) Simple tree formation. 5 kV a.c.

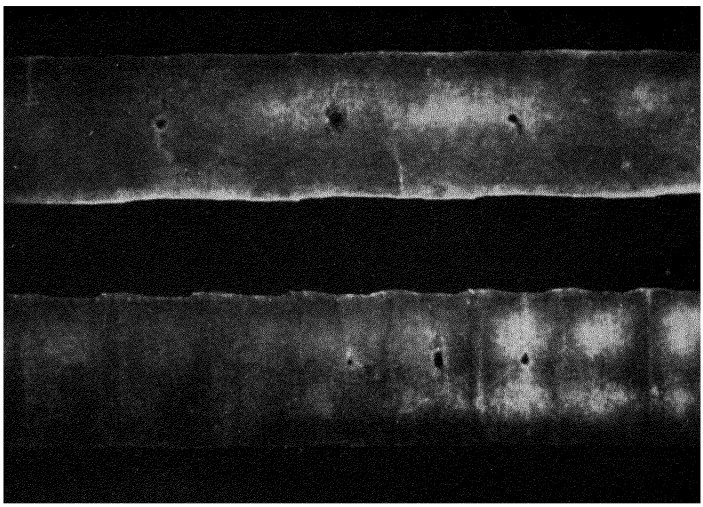


(*b*) Single tracks with some areas of damage. 10 kV a.c.

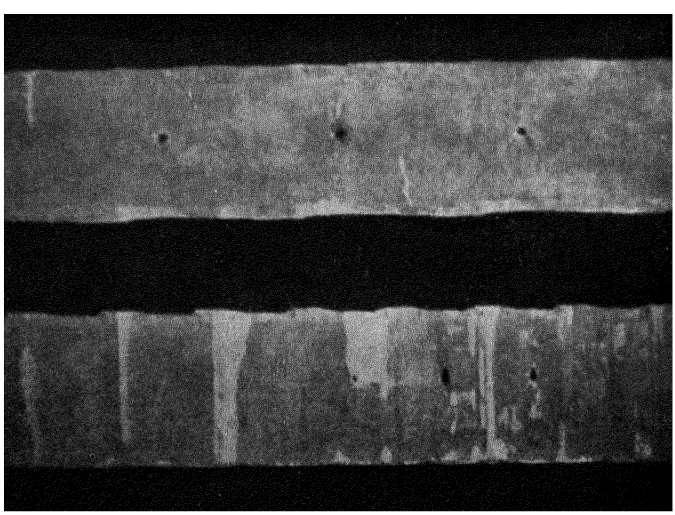


(*c*) Tree formations with areas of damage. Owing to the latter the branches are less developed than in (*a*). 30 kV a.c.

FIG. 71. High-voltage tracking on varnish paper.



(*a*) Fluorescence photograph. Panchromatic plate. Yellow filter. Actual width of paper 1 inch.



(*b*) Photograph in ordinary incandescent light. Panchromatic plate. No filter.

FIG. 72. Chlornaphthalene impregnated capacitors contaminated with rosin. (Church.)

single tracks. A track will not often be straight, as it is sensitive to surface irregularities and contamination. On account of these a limited number of branches may be observed. If the discharge at the tip is only capable of producing a high-resistance track, it will ultimately cease to propagate owing to the voltage drop in the track when the latter is only just able to maintain a small intermittent discharge current. Lateral discharges may, nevertheless, occur until the total current carried by the main track heats it sufficiently to transform it into a low-resistance track. This is precisely the phenomenon of tree-formation, analogous to Lichtenberg's figures, which has already been considered in another connexion. Tree-like conducting paths are fairly frequently observed (Fig. 71, Plate XIV).

If the dielectric surface is feebly conducting, an analogy with the preceding effect may occur without discharges due to lateral conduction from the track, but a fairly high degree of deterioration or contamination is needed, as with low-voltage tracking which is often due to the same mechanism. Consider, for example, a track which collects or discharges over 1 cm. of its length. If the field normal to the track, the lateral surface field is E r.m.s. volts/cm., and if the surface resistivity of the dielectric is r ohms, then the total current entering one end of the track and discharged over its length (ignoring voltage-drop which will be negligible for a conducting track) is $2E/r$ amps. r.m.s. Now E cannot exceed the electric strength of air or a discharge will occur, which is contrary to the hypothesis. Thus the order of E cannot exceed 20 kV/cm. so that the current cannot exceed the order of $4 \times 10^4/r$ amps. r.m.s. If the current is to exceed 0·1 mA, which has been shown to be probably necessary, then r must not exceed 4×10^8 ohms. This is quite a low value and corresponds to a fairly high degree of contamination or deterioration.

It is also interesting to note that the surface resistance may safely be deliberately reduced by a semi-conducting film in order to ensure a correct field distribution which might otherwise be upset by variations of surface conductivity or bound charges. This is exemplified in practice by the stress-controlled insulator. A semi-conducting film, such as a special glaze on a ceramic

insulator, serves also to eliminate surface discharges near metal electrodes, supports, or connectors. It is indeed probable that variations of charge on surfaces, due either to variations of conductivity or to bound ions, are often responsible for potential distributions which lead to breakdown. There is evidence, though not very direct or conclusive owing to the uncertain and variable nature of these phenomena, that breakdown can occur between patches where an abnormal voltage develops. Suppose one patch is held at a certain potential by induction or conduction from an electrode. The breakdown will bring the second patch to the same potential instantaneously. The connexion so effected may be permanent, if the energy is sufficient to damage the surface, and so the insulating margin provided by the surface is reduced. Alternatively, the connexion may be that of a transient discharge. If so, then with alternating voltages the charge neutralized or transferred may at some later epoch cause an abnormal and unfavourable distribution of potential and the discharge will repeat.

Although this account of surface breakdown is necessarily qualitative, its main applications in design and use are not difficult. It is only possible to control the shape of the surface, the tangential or surface stress, and the nature of the material. Indications of the fairly obvious limits of the former have been given and, subject to limitations of field distribution, the shape of the surface is determined by the usual engineering considerations of manufacture, mechanical stress, and the type of atmospheric or other exposure. It is also clear that the properties of the material should include resistance to discharges, chemical stability, and freedom from continuous surface films such as the so-called hydrophobic property. The first feature is the same as that already discussed under internal and external discharges. The two latter features have been the subject of detailed study in applied chemistry. Details of surface treatments also lie outside the scope of this book, but it may be mentioned that they fall mainly into two classes. Firstly, there are the semi-conducting glazes and paints to provide a uniform conductivity low enough for safety but high enough to be unaffected by contamination.

Secondly, there are the highly insulating, discharge resistant, and hydrophobic treatments such as the amino- or silicon resin varnishes or mineral finishes. With these should be included treatments which fill the potential holes such as can arise with exposed ions or dipoles and give hydrophilic properties. Examples of such treatments are the quaternary ammonium compounds recently reported. Circumstances determine the relative appropriateness.

REFERENCES QUOTED IN CHAPTER IV

1. EMANUELI. *High Voltage Cables* (Chapman & Hall, London, 1929). *Congrès International d'Electricité*, Paper 9, Paris (1932).
2. GEMANT and PHILIPOFF. *Zs. für Technische Physik*, 1932, **13**, 425.
3. TYKOCINER, BROWN, and PAINE. *Univ. of Illinois Bulletin*, 1933, **30**, nos. 49 and 50.
4. ROBINSON. *J.I.E.E.* 1935, **77**, 90. *Dielectric Phenomena in H.V. Cables* (Chapman & Hall, London, 1936).
5. BRAZIER. *J.I.E.E.* 1935, **77**, 104.
6. ARMAN and STARR. Ibid. 1936, **79**, 67.
7. QUINN. *Trans. A.I.E.E.* 1940, **59**, 680.
8. WHITEHEAD, J. B. *J. Franklin Inst.* 1937, **224**, 363. *Trans. A.I.E.E.* 1940, **59**, 650 and 715.
   —— Ibid. 1941, **60**, 267.
   —— Ibid. 1942, **61**, 628.
   —— and KOPPER. Ibid. 1945, **64**, 171.
9. WHITEHEAD, S. *Dielectric Phenomena*, vol. ii, *Electrical Discharges in Liquids* (Ernest Benn, London, 1928).
10. LOEB and MEEK. *Mechanism of the Electric Spark* (Stanford University Press, California, and Oxford University Press, 1941). See also LOEB, *Fundamental Processes of Electrical Discharges in Gases* (John Wiley & Sons, New York, Chapman & Hall, London, 1939).
11. MEEK. *Phys. Rev.* 1940, **57**, 722. LOEB and MEEK, *J. App. Phys.* 1940, **11**, 438 and 459. See also 10.
12. BRUCE. *Nature*, 1941, **147**, 805. *Proc. Roy. Soc.* A, 1944, **183**, 228.
13. ROGOWSKI. *Arch. für El.* 1931, **25**, 551. *Physikalische Zeitschrift*, 1932, **33**, 797.
14. DUBSKY. *Trans. A.I.E.E.* 1919, **38**, 537. See also WHITEHEAD, S., *Dielectric Phenomena*, vol. i: *Electrical Discharges in Gases* (Ernest Benn, London, 1926).
15. DAWES and HUMPHRIES. *Trans. A.I.E.E.* 1940, **59**, 701.
16. MASON. *E.R.A. Reports* L/T 192, 1948; L/T 210, 1949; L/T 235, 1950; *Proc. I.E.E.* 1951, **98**, 44.

17. TOEPLER. *E.T.Z.* 1930, **51**, 777.
STEPHENSON. *J.I.E.E.* 1933, **73**, 69.
WHITEHEAD, S. Ibid. 1939, **84**, 408.
BRUCE. Ibid. 1947, **94**, part ii, 138.
18. THOMAS. *Nature*, 1945, **156**, 451.
19. AUSTEN and WHITEHEAD. *J.I.E.E.* 1941, **88**, part ii, 88.
20. —— and HACKETT. Ibid. 1944, **91**, part i, 298.
21. WHITEHEAD, S. *Trans. Farad. Soc.* 1946, **42** A, 66.
22. *E.R.A. Report* Trans./T 191*b*.
23. WHITEHEAD, S. Ibid. M/T 90, 1944.
24. PELZER. Ibid. L/T 207, 1948.
25. DAVIS, AUSTEN, and WILLIS JACKSON. *J.I.E.E.* 1947, **94**, part iii, 154.
26. AUSTEN and PELZER. *Nature*, 1946, **157**, 693.
27. THOMAS. *J.I.E.E.* 1944, **91**, part ii, 549.
28. BOWDLER. At present privately communicated.
29. Air Ministry DTD. 2002; HAAS, *E. u. M.* 1934, **52**, 429; THOMAS, loc. cit. (27).
30. WELLAUER. *Schweizer Elektrotechnische Verein Bull.* 1925, **16**, 365.
31. WHITEHEAD, S. *Dielectric Phenomena*, vol. iii, *Breakdown of Solid Dielectrics* (Ernest Benn, London, 1932).
32. TOEPLER. *Zs. für Technische Physik*, 15 Jan. and 2 Feb. 1921. *Arch. für El.* 1921, **10**, 157 and 455.
33. GOLDMAN and WUL. *J. Tech. Physics, U.S.S.R.* 1934, **1**, 42.
34. PAINE and BROWN. *Trans. A.I.E.E.* 1940, **59**, 709.
35. THOMAS. *E.R.A Report* L/T 191.
36. HILL. *Trans. A.I.E.E.* 1939, **58**, 484.
37. RAYNER. *J.I.E.E.* 1912, **49**, 3.
38. MONTSINGER. *Trans. A.I.E.E.* 1935, **54**, 1300.
39. SCHWAIGER and KROPP. See Kropp, *Arch. für El.* 1933, **27**, 685.
40. GOLDMAN and WUL. *J. Tech. Physics, U.S.S.R.* 1936, **3**, 519.
41. HACKETT and THOMAS. *J.I.E.E.* 1941, **88**, part i, 295.
42. HILL. *Electric Journal*, 1934, **31**, 277 and 324.
43. HOLMES. *J. Franklin Inst.* 1931, **211**, 777; 1932, **213**, 57.
44. INGE and WALTHER. *Arch. für El.* 1931, **25**, 21. *E. u. M.* 1934, **52**, 234.
—— and WUL. *Arch. für El.* 1931, **25**, 597.
45. ALLIBONE and PERRY. *J.I.E.E.* 1936, **78**, 268.
46. STANDRING. Ibid. 1941, part ii, **88**, 360. See also DAVIS and LACEY. *C.I.G.R.E.* 1939, no. 304, Paris; *E.R.A. Reports* L/T 109, L/T 110. *Surge Phenomena*, published by E.R.A. (London, 1941), part iv.
47. DAVIS. *J.I.E.E.* 1942, **89**, part ii, 52; *E.R.A. Report* L/T 106; *Surge Phenomena* (see 46).
48. WALTHER and INGE. *J. Tech. Physics, U.S.S.R.* 1938, **5**, 335.
49. INGE and WALTHER. *Arch. für El.* 1930, **24**, 88 and **25**, 259.
50. DRAEGER. Ibid. 1932, **26**, 597.

51. See 45 also *E.R.A. Reports* L/T 86, L/T 133.
52. INGE and WALTHER. *Arch. für El.* 1932, **26,** 409.
53. HIPPAUF. *Zs. für Physik,* 1933, **82,** 803.
54. DOWELL and FOUST. *General Electric Review,* 1937, **40,** 141.
55. JONES and LUSIGNAN. *Electrical World,* 1930, **96,** 516.
56. BOWDLER and STANDRING, *J.I.E.E.* 1941, part ii, **88,** 443. *E.R.A. Reports* L/T 110 and L/T 116. *Surge Phenomena* (see 46).
57. AUSTEN, GOSLAND and LACEY. *C.I.J.R.E.* Paper no. **339,** 1946 also LACEY, *Proc. I.E.E.* 1949, **96,** part ii, 287.
58. YARSLEY and GRANT. *E.R.A. Report* L/T 177, 1948.
59. GROSS. *British Journal of Applied Physics,* 1950, **1,** 259.

CHAPTER V

ELECTROCHEMICAL DETERIORATION

THIS chapter will mainly concern itself not with breakdown mechanisms as in previous chapters but with processes which ultimately lead to dielectric failure. The actual breakdown is usually due to thermal instability although internal discharges may also precipitate failure. Intrinsic breakdown is possible but for the reasons given in Chapter II progressive loss of insulating properties is more probable, such as when a dielectric slowly becomes a semi-conductor or even a poor conductor. Often the final breakdown is mechanical. Electrochemical deterioration is one aspect of the problem of chemical stability. It has so far been tacitly assumed that dielectrics are chemically definable and permanent, but it is legitimate to include the particular forms of chemical instability to be described, since by not very obvious actions they so often produce gross effects on insulating properties.

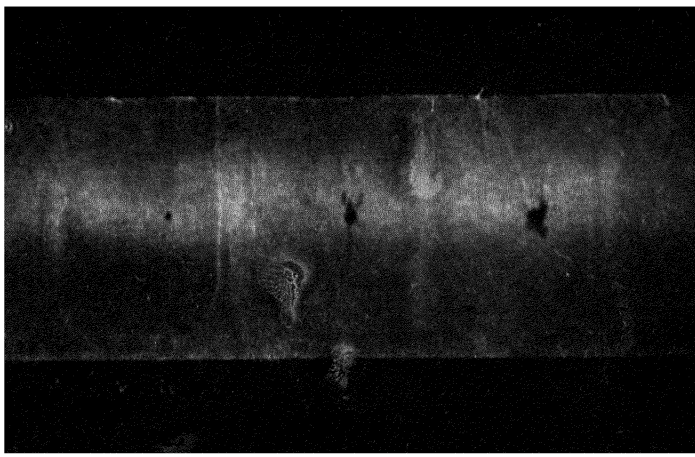
Many of the principles of electrochemical stability are now used in known industrial techniques, yet the detailed effect of traces of active impurities is not so well understood. Recently, this has been worked out for some instances connected with impregnated paper capacitors used with d.c., though similar but reduced effects can occur with a.c. It is proposed to give some account of this both because the capacitor provides an example of a dielectric in conditions favourable for a high working stress and also because the principles are of wider application.

From the work of McLean[1] and his colleagues on chlorinated hydrocarbons, the deterioration associated with these impregnants shows marked polarity effects and depends upon the material and nature of the electrodes as well as upon the chemical nature of the paper. Dissociation arising from the dielectric itself, or from impurities to be discussed later, takes place in organic impregnants in a manner similar to aqueous electrolytes and electrode potentials arise from the discharge of cations and anions. These are of the same order as in aqueous solutions and

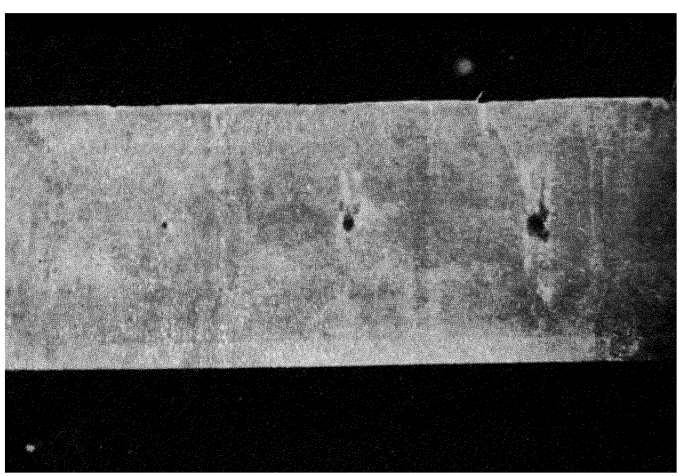
are therefore negligible compared with the applied voltage since the potential drop in the material is the greatly predominant factor owing to the high resistivity. Egerton and McLean and also Berberich and Friedman[2] observed an autocatalytic type of degradation in chlorinated impregnants which they apparently thought to arise from the evolution of hydrogen chloride and the subsequent attack of the acid upon the aluminium electrodes to yield aluminium chloride. Garton and Church,[3] however, consider that the first stage in the deterioration is the chemical reduction of the dielectric by hydrogen ions discharged at the cathode which do not combine readily with one another because the leakage current density is very low. Production of hydrogen chloride would thus be a secondary process. As has been noted in connexion with internal discharges, the action of nascent hydrogen is the more popular theory in the deterioration of impregnating oils in cables. Garton and Church state that the nascent hydrogen effects more or less quantitative reduction of a variety of materials: the magnitude of the action will thus depend upon the total leakage or irreversible charge passed by the dielectric during its life. The products of reduction or hydrogenation may be only mildly harmful in some cases, but even in materials comprised almost entirely of carbon, hydrogen, and oxygen, water or organic acids (particularly fatty acids) are liable to be formed which increase the conductivity and accelerate the action. With chlorinated impregnants such as chlornaphthalene, chlordiphenyl, etc., hydrogen chloride will be formed with the damage already noted to the insulation and to most electrode metals. Importance attaches to nascent hydrogen also on account of the possibility of its fixation by readily hydrogenated materials known as stabilizers. As with impregnating oils, unsaturated hydrocarbons are beneficial and Garton and Church predicted the value of octadecene and similar aliphatic hydrocarbons. In support of their hypothesis, they indicate that with anthraquinone, an early stabilizer, the reduction product oxanthranol is found diffused throughout the dielectric and that stabilization is possible whatever the electrode material, even if platinum. Other stabilizers are certain nitro-aromatic

compounds, maleic anhydride, sulphur, substances in general having a quinoid structure, aromatic azo-compounds and benzil, given by McLean and his co-workers and Berberich and Friedman.

Leakage currents which result in damaging electrolysis, as in the foregoing, or which become large enough to cause serious deterioration directly may be due to the dissociation of the dielectric. Great improvements have been made in industrial insulation in this respect and impurities may be expected to become more and more important. One impurity in capacitors is rosin from soldering operations either in attaching the connexion to the ends of the foil electrodes or in sealing the case. Figs. 72 and 73 (Plates XV and XVI) show d.c. breakdowns, at a voltage much less than the discharge inception voltage, located in patches where rosin has entered the chlornaphthalene impregnant of the capacitor paper. A comparison of the photographs denoted by (*a*) taken with ultraviolet illumination and those denoted by (*b*) in ordinary light shows the convenience of examination by ultraviolet fluorescence. Both chlornaphthalene and rosin have good insulating properties, but a dilute solution of rosin in chlornaphthalene is relatively conducting and also diffuses easily at higher temperatures. The deleterious effect of rosin solutions is greater when the rosin itself is impure, but even with a pure grade, for example 'water-white' rosin, the harmful effects are only reduced by a small amount. Ionization in solution is greater with dielectrics of a higher dielectric constant and diffusion is facilitated if the solvent medium is dipolar in character. Chlornaphthalene has a high dielectric constant and is dipolar. In non-polar impregnants, such as petroleum jelly and ozokerite, the rosin forms small patches owing to reduced diffusion. If the contamination in these patches is concentrated, then breakdown may occur there. For example, Garton and Church[4] quote tests in which this kind of contamination was associated with lives of 11–330 hours, whereas similar capacitors free from this condition withstood a life test for 9,000 hours. This was with petroleum jelly as an impregnant, but similar conclusions were derived from tests on capacitors impregnated with ozokerite wax.



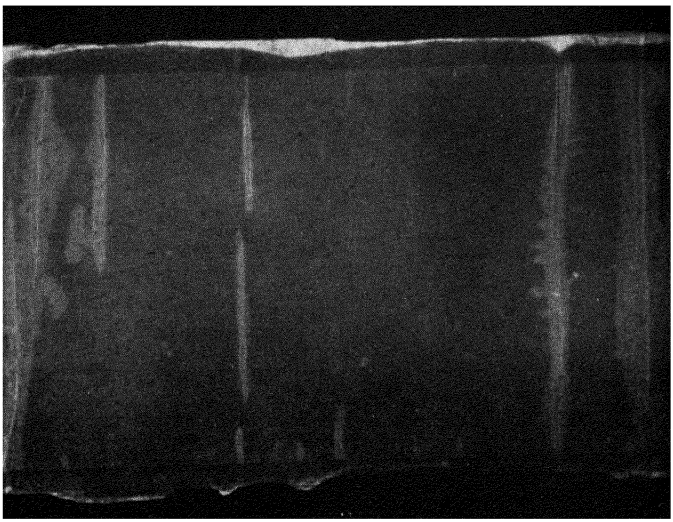
(*a*) Fluorescence photograph. Actual width of paper 1 inch.



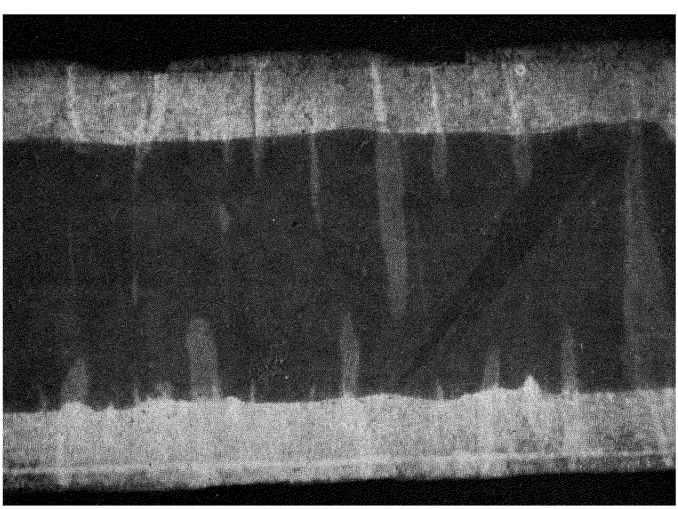
(*b*) Photograph in ordinary light.

FIG. 73. Chlornaphthalene impregnated capacitors contaminated with rosin. (Church.)

PLATE XVII

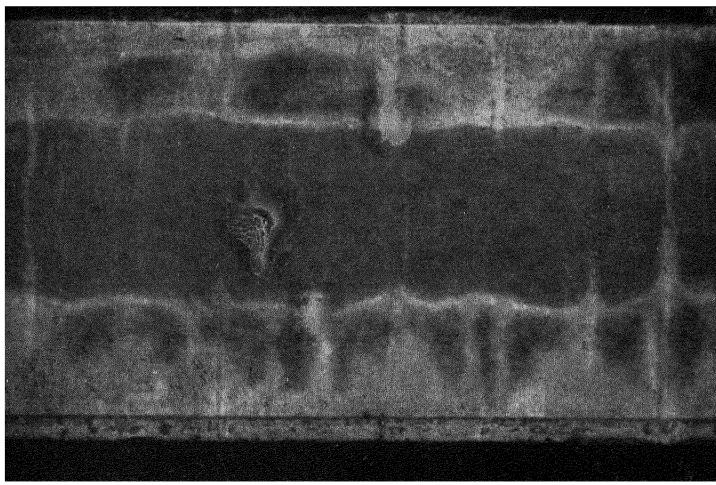


Unaged.

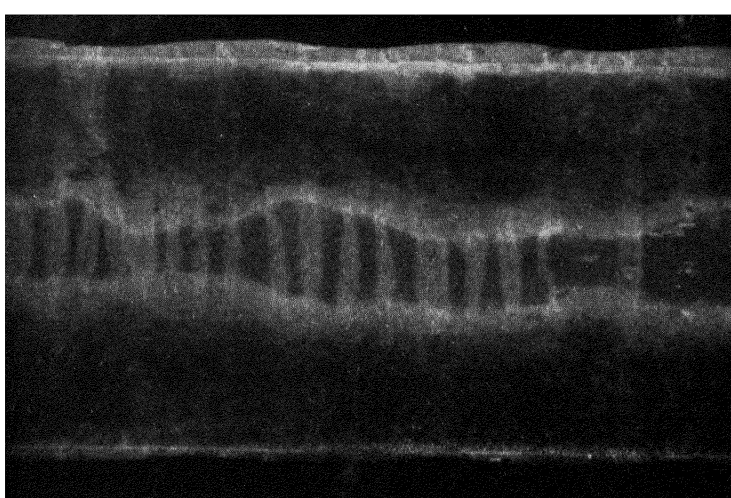


Aged 630 hours.

FIG. 74. Diffusion at 71° C. of ozokerite into chlornaphthalene impregnated capacitors sealed with ozokerite (350 V d.c.). (Church.) Fluorescence photograph.



Aged 1,000 hours.



Aged 1,700 hours.

FIG. 74 (*continued*). Fluorescence photograph.

Contamination by bitumen is less simple. Garton and Church[4] found that when capacitors impregnated with chlornaphthalene (M.P. 93° C.) in cases filled with a wax (M.P. 65° C.) and sealed with bitumen (softening temperature over 100° C.), were tested at 75° C., the bitumen diffused into the wax which penetrated into the dielectric carrying the bitumen with it. Capacitors subject to this form of contamination gave mean lives of 150 and 170 hours respectively for two different makes of paper. When the wax and bitumen were removed and replaced by chlornaphthalene the mean lives became 1,060 and 200 hours respectively, since the bituminous impurity had masked the difference between the two types of paper which were being compared.

A more complicated but interesting example is described by the same authors when paper impregnated with chlornaphthalene (M.P. 93° C.) is sealed in ozokerite (M.P. 80° C.). The two substances yield a eutectic mixture containing 30–40 per cent. chlornaphthalene and this mixture has a melting-point less than 60° C. The mixture was formed when chlornaphthalene and ozokerite were in contact at 71° C., the temperature of the test, and diffused into the impregnated paper (Fig. 74, Plates XVII and XVIII). The ozokerite penetrated by progressive solid-liquid solution. Ozokerite is oxidized by any available oxygen, producing fatty acids or similar substances which ionize in chlornaphthalene but not to any extent in ozokerite itself. Thus, as the eutectic layer advances into the dielectric, the conductivity rises on the front with penetration of small quantities of oxidized ozokerite, reaches a maximum, but then falls behind the advancing front, where there is more ozokerite and less ionization of the impurities. From the experimentally determined conductivity of aged ozokerite mixed in chlornaphthalene, it is possible to calculate the total coulombs/cm.^{2} passed in a given time by a part of the dielectric at a given distance from the initial boundary between the dielectric and the ozokerite. This is displayed in Fig. 75 calculated for a rate of penetration of the eutectic front of 1 mm./100 hours at 71° C. If XY corresponds to a degree of electrolysis which just renders the deterioration visible in

ultraviolet light, the observed form illustrated in Fig. 74 is precisely explained. It begins as a narrow band close behind the advancing front and gradually spreads in both directions ultimately to fill the whole strip of dielectric which is wound into the capacitor roll.

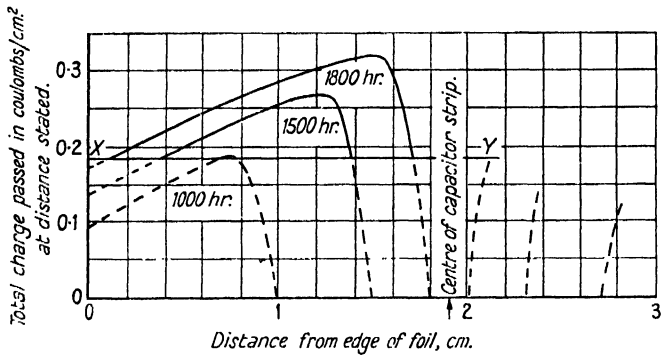


FIG. 75. Progressive electrolytic deterioration in impregnated paper. (Garton and Church)

The disconcerting or devastating effects of moisture are well known. In early days deliquescent materials were sometimes left behind after processing, such as zinc chloride in insulating fibre. Fibrous material unimpregnated and imperfectly sealed or else imperfectly impregnated was not uncommon in early types of electrical insulation. Today impregnation and sealing have reached a high degree of scientific development. Nevertheless, it is not easy in practice to avoid all defects which may admit moisture as water or water vapour and Garton[5] has shown that the ways in which moisture can enter are many. The range of materials which are effectively impervious to moisture is restricted. Metals, some non-metallic crystals, and some amorphous mineral dielectrics possess this property, but most organic dielectrics are subject to molecular diffusion of moisture, although the degree varies greatly. The following table, from data collected or quoted by Thomas and Gent,[6] illustrates this.

It is a matter of some difficulty to exclude water permanently although the rate at which it enters can be very greatly reduced.

Moisture permeability in grammes per hour per cm.² surface per pressure gradient of 1 mm. of mercury per cm. at room temperatures

Wax	10^{-9} *circa*	Rubber	$2–7 \times 10^{-8}$
Thiokol and gutta percha	$2–3 \times 10^{-9}$	Waterproof cellophane	3×10^{-8}
Varnish	$4 \times 10^{-9}–5 \times 10^{-8}$	Benzyl cellulose and cellulose nitrate	up to $1{\cdot}5 \times 10^{-7}$
Polythene	5×10^{-9}	Pure cellulose acetate	3×10^{-7}
Polyvinyl chloride	$1{\cdot}5 \times 10^{-8}$	Cellulose triacetate, Cellulose acetate, Cellophane	$1–2 \times 10^{-6}$
Polystyrene and Polymethyl methacrylate	3×10^{-8}		

The amount of moisture absorbed is also important and varies considerably. For example, the absorption by weight at room temperatures and normal humidities is 1–2 per cent. for varnishes and only of the order of 0·1 per cent. for polystyrene, but fibrous cellulose absorbs about 6 per cent. Both in impregnated insulation and generally, it is desirable to avoid unimpregnated spots and voids. This is the same problem as in the reduction of internal discharges. It is only necessary here to mention a special problem in impregnation with varnish, which is an important material for this purpose. Unimpregnated spots and voids are liable to occur with a normal solvent type of varnish because the solvent vapour is easily trapped in the material. On the other hand solventless types of varnish may contract on polymerization or liberate water of condensation with a similar result. Where this has been largely overcome there remain problems of mechanical properties and compatibility with other materials. Considerable difficulty attends the drying of materials prior to impregnation and/or sealing. Garton[7] has shown that the last traces of moisture are even more difficult to remove from paper than supposed by J. B. Whitehead.[8] In fact, if the paper is too unfavourably disposed, drying is impossible by normal methods. Enough has been said to show that although with sufficient trouble moisture can in certain cases be eliminated and although in many other cases the amount present can be greatly reduced by good practice, it essentially remains a sempiternal problem in industrial insulation.

The electrolytic effects of water in sufficient quantities need

no elaboration and the potentially fatal action on insulation is fairly obvious. It is not perhaps realized that small traces of certain materials such as hydrogen chloride can induce a much higher degree of dissociation in the water itself. This refers to slight traces of the active agent since clearly any substantial quantity of electrolyte is directly dangerous. Careful prescriptions exist as to the neutrality or pH value of insulation. In many materials, perhaps in the majority of dielectrics which absorb moisture, the water molecules on sorption are to a greater or less extent ionized. The conductivity induced by the presence of moisture is thus, in general, very much greater than corresponds to liquid water. In alternating fields a further and important source of energy loss lies in the dipolar rotation (Garton[7]) of water molecules which are bound as regards translatory motion. In Chapters II and III it was recalled that both the conductivity and loss angle of dielectrics which contain a fair amount of moisture tend to increase considerably with rise of temperature and also increase with field strength even if the dry material has constant properties (Hartshorn,[9] Whitehead, S., and Nethercot[10]). The explanation of the variation with field strength is not known with any certainty although it may arise from restricted ionic motion or perhaps from field-induced activation. With large amounts of water, breakdown may occur owing to reactions at the electrodes and secondary reactions in the dielectric which result in the diffusion throughout the material of electrolytes or the dissociation of components of the dielectric itself. Effects similar to the preceding can occur with a.c., although on a much reduced scale as compared with d.c., because rectification can take place either by irreversibility of the reactions or diffusion of the products. With the small amounts of moisture usually involved, breakdown generally occurs by thermal instability as in Chapter III owing to the enhanced energy losses, although discharges may develop from an unfavourable voltage distribution caused by selective deterioration in heterogeneous insulation.

If there are exposed surfaces, active impurities can be attracted with d.c. to the electrode of appropriate polarity with

curious and often disagreeable results. Cases are known where the polarity of the contact wire of electric railways has been changed to diminish corrosion of structures in contact with the earth, but has resulted in severe damage to the insulators of the contact wire. Analogous difficulties still remain a problem in the design of insulators for high voltage transmission with d.c.

Apart from moisture, which has been extensively explored from early days, there are a number of other substances which cause electrochemical deterioration of a rather similar character, such as organic acids derived from the sludging of insulating oils, fatty acids difficult to remove from waxes, anti-oxidants used to protect thermoplastic materials such as polythene during processing at high temperatures, the decomposition products of the plasticizers used with plastic dielectrics. These are additional to the free acids or alkalis formed in the manufacture of some dielectrics. A great deal of industrial development has been devoted to these problems. The trend of it is indicated by the use of less highly refined oils in those cases where unsaturated hydrocarbons diminish electrochemically active products, by the use of plasticizers such as dibutyl phthalate in place of more active substances such as tricresyl-phosphate and by the use of unplasticized or anti-oxidant-free materials.

REFERENCES QUOTED IN CHAPTER V

1. See EGERTON and MCLEAN. *Industrial and Engineering Chemistry* (Industrial Edition), 1946, **38**, 512.
   Also MCLEAN, EGERTON, and HOUTZ, ibid. 1946, **38**, 1110.
2. BERBERICH and FRIEDMAN. *Westinghouse Research Laboratory Scientific Paper*, no. 1257 (29 Aug. 1941). *Industrial and Engineering Chemistry* (Industrial Edition), 1948, **40**, 117.
3. GARTON and CHURCH. *Nature*, 1948, **162**, 301; E.R.A. Report L/T 196, 1948.
4. —— —— Ibid. L/T 190, 1947.
5. GARTON. Ibid. A/T 97, 1945.
6. THOMAS and GENT. *Proc. Phys. Soc.* 1945, **57**, 324.
7. GARTON. *J.I.E.E.* 1940, **86**, 369.
8. WHITEHEAD, J. B. *Impregnated Paper Insulation* (McGraw-Hill Book Co., New York. 1st edition, 1935).

WHITEHEAD, J. B., and GREENFIELD. *Electrical Engineering*, 1934, **53**, 1389 and 1498.

WHITEHEAD, J. B., and HAMBURGER. *Trans. A.I.E.E.* 1928, **47**, 314.

9. HARTSHORN and WARD. E.R.A. Report L/T 48; and *J.I.E.E.* 1935, **77**, 723.

HARTSHORN and RUSHTON. *J.I.E.E.* 1934, **75**, 631; 1938, **83**, 315.

10. WHITEHEAD, S., and NETHERCOT. *Proc. Phys. Soc.* 1935, **47**, 974.

CHAPTER VI

DIELECTRIC BREAKDOWN IN PRACTICE

6.1. General

THE preceding chapters have described the scientific principles on which the study of dielectric breakdown is based. The treatment has been mainly from the point of view that the subject forms one aspect of the solid state. Some types of breakdown have been discovered as a result of engineering studies. On occasions breakdown phenomena have been considered in relation to the engineering needs from which their importance has arisen. Nevertheless detailed examination of the practical application of the theories and basic phenomena of dielectric breakdown is not within the province of this book. In any example of electrical equipment a variety of engineering considerations enter. To isolate the particular aspect here treated is difficult or impossible and to treat all relevant aspects would require a treatise of some volumes on materials and design. Even so it is pertinent to inquire whether any generalizations are possible on the relation between the present investigations and practice, of wider scope than the more detailed connexions which have been discussed when they have arisen.

The theory of intrinsic breakdown provides an upper limit and a target for possible improvement. More important perhaps at the present time is the basis which it affords for examining other phenomena of high stress. As a result of investigations of intrinsic electric strength we know a great deal more about the behaviour of solid dielectrics and those features of the structure which are important in determining the reaction to high voltages and high fields and the electronic, ionic, and other actions which may accompany them. Attention has already been drawn to ignorance about low breakdowns and defects, but it is possible to speculate about them and so to form an idea as to a practicable as well as an ultimate target. The theory of thermal breakdown can be directly applied when the requisite data are available on properties and conditions. The same is true of the

principles of breakdown due to discharges subject to the incompleteness of the analysis already described. Electrochemical and similar deteriorations are, in principle, capable of elucidation where the physical and chemical data are known.

It is desirable to consider these views against the background, described very briefly, of some aspects of the engineering interests concerned. This it is proposed to do according to the rather arbitrary division into working stresses, over-voltages, testing, and future developments.

6.2. Working stresses

When the applied voltage is low, an extremely thin layer of dielectric will support it. Two features prevent the realization of this in practice. In the first place, the dielectric must withstand the mechanical stresses of the structure and if, as is common, these involve a shear stress, too thin a material may yield. In the second place, there is the engineering principle of minimum size. With solid dielectrics, this usually enters by the way of defects which it is uneconomical completely to remove. If the dielectric is too thin, a nearly microscopic defect may cause failure. In paper capacitors it is necessary in practice, except with metallized paper, to allow a few conducting particles per sq. ft. so that with a moderate electrode area it is impracticable to use a single layer of paper. If self-scaling metallized paper is employed, the metallic layer will penetrate the paper in some places. In practice, a minimum of 3 papers is used where a higher degree of security is needed, though 2 layers of paper are found in many types of low-voltage capacitors. For these reasons and the need to provide terminations with adequate clearance and creepage distances, the working stresses in low-voltage equipment are low and have rarely much connexion with dielectric breakdown. The dividing line must be drawn at widely different voltages according to the nature of the equipment. In capacitors the mechanical stresses are low, the design is well protected, and the dielectric is fairly homogeneous so that high working stresses are found at a few hundred volts. Some of these advantageous features apply to cables, but cables must

withstand mechanical conditions and treatment of much greater severity, so that the working stress is important at higher voltages, say above 3–5 kV. Bushings are themselves mechanical supports, and it might be said that electrical features are not of the highest importance until the voltage exceeds 10 kV except in special instances. In outdoor equipment the conditions of installation impose a low working stress, and electrical limitations on the latter probably do not enter until at least 30–50 kV is exceeded. With overhead line insulators breakdown of the solid dielectric can only occur in special circumstances.

Questions of general thermal instability or thermal breakdown appear in practice as the problem of temperature or thermal rating. The principles discussed in Chapter III enable suitable limitations to be placed on the stable temperature rise. It has been noted that if the energy dissipation is mainly in the form of dielectric loss, then the maximum permissible temperature rise may be low even with low ambient temperatures. This is the most direct intervention of the theory of thermal breakdown. It is rather limited. It applies to higher frequencies where the dielectric losses are enhanced, particularly to frequencies above 0·5 Mc./s., although in many types of equipment it applies to audible frequencies. Thermal breakdown is important at low frequencies when very high voltages are used. The criterion provided by the region of the maximum thermal voltage has already been studied and remains the most useful generalization. An alternative, which is rather similar, is to consider the energy capable of being transmitted through a dielectric by an alternating field, sometimes called the Poynting vector, which is often a useful concept in equipment for alternating currents. Lastly, thermal instability applies at high ambient temperatures when the dielectric properties are usually greatly impaired.

Outside these ranges the thermal equilibrium is partly or mainly determined by energy dissipated in the resistance of conductors or in eddy-current or hysteresis losses in magnetic materials. The contribution of this energy dissipation to the total temperature rise for thermal stability may make it so high

that the limit is determined by the maximum temperature advisable for the materials or construction, modified often by the deleterious effect of temperature cycles with varying load or excitation. Frequently these limitations come mainly from the need for mechanical stability. The mechanical properties of dielectrics are more complicated and less manageable than those of metals and their processing is more difficult and critical. In addition, they tend to show flow and/or recoverable creep with a wide range of time constants so that some dielectrics are rarely in real mechanical equilibrium and their dimensions are correspondingly indefinite. For these and other reasons, maximum temperatures are usually low. Typical values† are 50–65° C. for normal low-temperature grades, 70–100° C. for normal high-temperature grades; 120° C. is the limit for most organic insulants, but some halogenated types are possible up to 150° C. and silicone resins are available up to 200° C. and rather higher; 200–300° C. applies to the commoner mineral insulants, above 300° C. is the range of the refractory materials such as glass, ceramics, mica, silica, and the furnace refractories. The permissible temperature rise, where it is itself an accepted limit, has values from 11° C. for some forms of rubber insulation, 20, 30, and 40° C. for common forms of heterogeneous and impregnated insulation, and up to 70° C. for some special types of impregnated or continuously applied insulation. Thermal design consists in applying known forms of calculation and the working stress is often a secondary feature, so no further attention need be paid to general thermal equilibrium or instability.

It follows that the consideration of working stresses may be limited to equipment for fairly high voltages of power frequencies and also for d.c.—with certain exceptions. These exceptions relate to favourable conditions, as with capacitors, where high working stresses may be associated with low voltages, and to special high-frequency dielectrics, for example in radio-frequency cables, where high working stresses are permissible at elevated

† These values are taken from present-day physical knowledge: at the time of writing the British Standard including a temperature classification of insulants is being revised.

frequencies. Some examples of the order of working stress encountered in practice are given in the following table. Some of these values are nominal rather than actual, but this distinction does not affect the order.

Material	*Crest value of working stress*
Paper impregnated with compound or oil at normal pressures	70 kV/cm. a.c.
As above at, say, 200 lb./sq. in. pressure	150–220 kV/cm. a.c.
Wax-impregnated capacitor paper	100–200 kV/cm. a.c. and d.c.
Oil-impregnated capacitor paper	150–220 kV/cm. a.c. and d.c.
Varnish cambric	20–25 kV/cm. a.c.
Varnish paper	20–25 kV/cm. a.c. 40–75 kV/cm. a.c. (exceptionally)
Rubber and polyvinyl chloride	30–40 kV/cm. a.c.
Polythene	50 kV/cm. a.c. H.F. 100–150 kV/cm. a.c.
Porcelain	5–10 kV/cm. a.c. 20 kV/cm. a.c. (exceptionally)
Mica · stacks	150 kV/cm. a.c.
single plates	300–500 kV/cm. a.c.

The term 'exceptionally' refers to materials, normally used in conjunction with oil or a viscous compound, where it is assumed that the latter has temporarily failed. Any more extensive or exact a list would have to take into account the variation of factors of safety according to type of equipment and the different policies adopted by different manufacturers. In spite of its crudity, the list serves to show how small are the working stresses compared with the intrinsic electric strength, the ratio of the former to the latter is indeed from 0·01 to 0·1.

It is likely that electrical limitation of working stresses below 50 kV/cm. (crest) arises usually either from the need for long-term chemical stability (e.g. the avoidance of acid formation or slow oxidation) or from the need to prevent macroscopic discharges. Even though the working stress is high, long-term chemical stability is a limiting factor in a great many applications

of impregnated capacitor paper (see Chapter V) but not in all. In electrochemical breakdown the field strength is rather secondary in that it is determined by the current density with which it is associated. This current density is in turn profoundly affected by small traces of impurities or minor changes in composition.

Under macroscopic discharges are included surface discharges acting indirectly in the following manner. In many designs the area of the high-voltage electrode is small. With a high voltage the surface of the insulation must not be exposed to too high a tangential stress. For example, at an air surface it must not exceed 30 kV crest/cm. and in fact 5–10 kV crest/cm. is not unduly safe. In addition to this, allowance must be made in the majority of instances for the formation of conducting or semi-conducting deposits so that the tangential stress may be still further reduced. Allowance must also be made for sharp edges and other forms of distortion. For all these reasons, a very long surface path is required relative to the thickness, with the result that there is often little or no economy in reducing the thickness to the minimum permissible value. Internal discharges occur in fields of less than 50 kV/cm. but are thought to be rare on account of the limited conditions in which a discharge field could be produced in a void.

The values of working stress from 50–150 or 200 kV/cm. are probably limited by the internal discharges described in Chapter IV. The working stress for polythene could probably be increased to 150 kV/cm. for lower frequencies. Thus a fairly narrow range of 100–200 kV/cm. might be postulated as corresponding to a discharge limitation though this would be reduced in the event of difficulties in sealing or of mechanical changes which impaired the effectiveness of impregnation.

It might indeed be thought that the intrinsic electric strength remains an entirely academic characteristic from the aspect of normal design. This is not really true, even ignoring the occasional but inevitable conditions of undue severity mentioned later. Consider a working stress of 200 kV/cm. which may at present be limited by the incidence of discharges. With a.c. it

is sometimes possible for the total voltage swing to be impressed on part of the insulation which implies that the dielectric must cater for a stress of 400 kV/cm. Account must also be taken of local distortions of field and also of the defects inevitable if a considerable area or volume of dielectric is exposed to a high field. The experiments described in Chapter II have shown the dispersion of electric strength results which cannot be satisfactorily explained. In some instances abnormally low values of this type are less than one-tenth of the supposed true value and occasional values one-third of the true value may be encountered. Thus a factor of safety of from 5 to 10 to take these features into account is possible which would yield a basic upper limit of stress of from 2 to 4×10^6 V/cm. This is in the region of the intrinsic electric strength. Thus no great increase of nominal working stress would, in many instances, require the working stress to be related to the intrinsic electric strength by a factor of safety of not unreasonable magnitude remembering the uncertainties often encountered. Conversely great expense in raising the existing limit might not be justified if intrinsic breakdown, with a factor of safety, gave too narrow a margin of possible improvement.

With d.c. the discharge hazard is very greatly reduced, but a hazard due to long-term electrolysis may arise. Information on long-term behaviour with d.c. at high voltages is, apart from line insulation, rather limited to cables and high-voltage laboratory (or laboratory-type such as industrial radiological) apparatus or medical equipment. In these cases some confidence is felt that with d.c. a stress above double the crest a.c. working stress should be feasible. Opinions have been expressed that a further doubling would not involve too great a risk. If this were the case, then insulation satisfactory for an a.c. working stress of 200 kV/cm. could be operated at 800 kV/cm. with d.c., a field strength related to the intrinsic electric strength by no exaggerated factor of safety. One of the attractions of H.V. transmission with d.c. is in the reduced cost of insulated cables where these are essential as in underwater or underground routes.

Although the preceding arguments are rather hypothetical, yet it can be said that in those existing techniques and designs where the greatest use is made of the properties of the dielectric, further progress will soon involve the reduction of the uncertainty associated with the intrinsic strength of dielectric specimens of practicable size. This is at present a serious difficulty. In some instances gross defects are known and tolerated. Thus the number of conducting inclusions in thin paper is such that at least one layer must be ignored in assessing working stress, and the same applies to layers of insulating liquid in some immersed insulation. Mica plates with gross defects which break down below 500 kV/cm. are often discarded, but this ignores a number which will not withstand a field only moderately more intense, although the true intrinsic electric strength may be $10–15 \times 10^6$ V/cm. The paucity of information on the statistical variability of electric strength and its causes, indicated in Chapter II, must be remedied.

It must, on the other hand, be admitted that a large range of designs should be brought up to the standard of the best. In a cable system the joints and ends are much the more susceptible elements owing to the conditions of installation which render impregnation and sealing more difficult and require the application of insulation of a more heterogeneous character. The use of heterogeneous insulation is a rather widespread weakness. It is now appreciated that the interposition of a solid dielectric in an air gap can sometimes weaken it. Great efforts have been made to increase the electric strength of the gaps between solid barriers. The development of high-pressure gaseous, liquid, or viscous impregnants has been one of the important more recent advances. Nevertheless, from one point of view these might be regarded as palliatives and a more fundamental solution would be the use of a homogeneous dielectric if one could be found with suitable properties.

If homogeneous dielectrics of high quality were available for the various essential constructions, the problem would remain of their association with conductors so as to avoid air-gaps or electrode edges. With cables this might be achieved in manu-

facture and is nearly achieved in existing techniques, but the joints and terminations would remain. If the main conductor can only be applied to part of the dielectric surface then the exposed surface can be sprayed with a conducting film to ensure intimate contact of an electrode, connected to the conductor. Considerable advances have been made in processes such as spraying with metal or graphite, electroplating, evaporation, and sputtering, but these are unfortunately not always possible and often costly. The same principle can be applied to internal surfaces where it is essential to apply one dielectric to another. These internal conducting surfaces can be dimensioned to provide stress control, a principle well known from the capacitor-type bushing. Further development might well produce coatings cheap and suitable for the majority of cases.

There must always be edges which form the boundaries of the surfaces which give the requisite clearances and creepage distances against flashover and tracking. Semi-conducting coatings over the free dielectric surface will often be effective in preventing edge discharges by preventing the development of a high enough potential gradient. The use of these with line insulators is well known, but they are not yet entirely satisfactory for long periods of service. They are also limited to constructions where the clearances are great enough for the potential gradient to be low. In the laboratory the use of shaped electrode recesses has been very successful. These reduce the potential gradient proceeding towards the edge by making the dielectric thicker towards the edge, so that the possible field concentrations in the air are very much reduced. No other treatment, or at most a limited semi-conducting border, may then be necessary. Coatings of graded conductivity are attractive, if capable of simplification, because the conductivity could be adapted to the potential distribution which is to be smoothed out by the coating. Increase of the electric strength of the atmosphere or other ambient medium by high pressure or high vacuum does not differ in principle from the use of impregnants under pressure. These methods are costly and can scarcely be otherwise since the vessel enclosing the compressed or evacuated

medium must itself include another dielectric surface unless it contains the high-voltage conductors involved.

In spite of great advances in some quarters, a strong case can be made for further study and development of constructions by which the dielectric is made homogeneous with electrodes in intimate contact and exposed to uniform fields both over its volume and over its surface, provided the surface field is kept well below the electric strength of the ambient medium.

In considering the additional expense of measures to increase the working stress two cases must be distinguished. If the design operates in a region approaching the maximum thermal voltage (Chap. III), it means that the temperature rise due to dielectric losses is a major or limiting factor and is a function of the dielectric properties since the dielectric may be regarded as thick. Improvement of the insulation to allow a higher working stress will probably not do much more than save the cost of the insulation omitted. Roughly speaking one could afford an increased cost of insulation proportional to the increased working stress. If, however, the temperature rise is determined by the energy loss in the electric or magnetic circuits and by the thermal resistance of the insulation, then increase of working stress also reduces the thermal resistance. This means that the cross-section of metallic conductor may be correspondingly reduced. Where the cross-section of magnetic material is not a limiting factor it might be possible to reduce all the dimensions of the design in conformity with the reduced insulation and conductor sizes. A very much higher cost of insulation could then be justified as compared with the reduction of amount of insulation, because in the theoretical limit the whole cost of the appliance, as well as the insulation, might be reduced in inverse proportion to the increased working stress and this margin applied to the additional expense of insulation. One must nevertheless not overlook the many instances where reduced insulation may need mechanical strengthening elsewhere so that no economy results; but conversely reduction of insulation thickness sometimes eases mechanical problems.

The application of the preceding considerations to raising the temperature limits of insulation is fairly easy to see. The details are more complicated and the optimum advantages are less than in increasing the working stress because higher temperatures bring in other constructional and design problems.

6.3. Over-voltages

First consider over-voltages of a sustained character from persistent faults or abnormal operations. For the reasons given in (6.1) it is desirable to confine attention to what were there described as higher voltages. Only a.c. can be taken into account because high voltage d.c. is at present used in special applications each of which must be treated on its merits. In 3-phase systems with the neutral directly earthed, it is usually thought that the voltage to earth on sound phases should not exceed about 1·5 times the nominal voltage to earth in the event of a fault on another phase. Since about 10 per cent. tolerance should be allowed on the nominal voltage the over-voltage is about 40 per cent. above highest working voltage. In some systems higher abnormal voltages may appear. Values of 2 or 3 times normal seem possible when there is a high impedance in the neutral. The minimum voltage rating in insulated systems is commonly 1·83 times the normal line to neutral voltage. Probably twice the working voltage would cover the majority of over-voltages, other than surges, and a factor of $\sqrt{3}$ would cover most. This conclusion does not introduce any new important factor but reinforces the arguments of the preceding section that with materials now operated at the higher working stresses the region of intrinsic breakdown must now begin to be related to the working stress by means of a factor of safety not unduly great.

If the thermal time constant is long enough, high powers can be applied to or transmitted by equipment for short periods. These high powers can be associated with a low average power, within the rating of the equipment, if repeated at long enough intervals. The commonest form of this method comprises repeated pulses. When the pulses modulate a very high frequency,

only exceptionally can the instantaneous electric field be allowed to attain the values associated with d.c. or low-frequency a.c. If unidirectional pulses are used, the repetition rates hitherto employed are sufficiently rapid for the instantaneous electric field to be limited by the same mechanisms as apply to working stresses with a.c. and d.c. However, in the future and in a few instances in the present, higher voltages must be envisaged with this method of operation. The study of radio-frequency cables referred to in Chapter IV indicated that new features have not so far emerged so that the question need not be further pursued.

The highest over-voltages are those from abnormal surges of brief duration. The commonest types are those encountered on power systems caused by lightning and switching, including the phenomena accompanying faults. In theory, certain switching operations can build up a voltage surge of almost indefinite magnitude. Similarly, some millions of volts could theoretically be applied by a direct lightning stroke. On the other hand the highest voltages are improbable and it is really only necessary to achieve a probable rarity of breakdown such that the expected normal life of the equipment will not be decreased. In addition an attempt is made in insulation coordination to adjust the relative breakdown voltages of items to ensure that those fail first which only involve slightly unpleasant consequences. These aims cannot be said to have been achieved.

In a paper by Austen, Gosland, and Lacey, and a more recent paper by Lacey, already quoted in Chapter IV, an attempt was made to estimate the probable magnitude of the transient voltages due to various kinds of hazard. Interruption of the current at a current zero, which is the instant at which a circuit can most conveniently be broken, does not give rise to serious over-voltages except in a circuit where resonance has already caused an abnormal voltage distribution. Since over-voltage protection will normally have been provided, over-voltages can exist up to the impulse breakdown value of the protective equipment. If the current is interrupted before a current zero in an inductive circuit two opposite peaks, not necessarily equal, of an oscillation at the resonance frequency may occur. Re-striking of the

arc in the circuit breaker may take place, and a series of transients will then be produced by attempted extinctions and reignitions; the main transient will be a large change from a voltage of one sign to a voltage of opposite sign. Maximum voltages up to 8 times normal have been observed, but a more reasonable value is 5 times normal for service voltages up to 20 kV, decreasing as the service voltage increases. The total change of voltage in the transient may be of the order of twice the maximum value: thus equipment rated at 100 kV might well be exposed to a transient voltage change of 500 kV or 1,000 kV. In an arcing fault to earth on an insulated system or during the interruption of a capacitative network, the interruption and restrike of the current may give rise to a rectangular wave of alternating voltage, usually at the supply frequency. The change from a crest of one sign to a crest of the opposite sign may, however, be very rapid, since at least one-half of it can take place in a fraction of a microsecond and the flatness of the wave tops may be such that the greater part of the change may occur in a microsecond or so. Crest values up to times service voltage have been observed, although 3 times seems rarely exceeded. A rapid voltage change of double these values may accordingly take place. A single current pulse in lightning has a front of from 2 to 10 microseconds and a tail up to 60 microseconds, the average resembling a 5/25 impulse wave. Voltage surges from indirect strokes and from direct strokes without flashover conserve this form except that the lengths are greater with more distant indirect strokes. Flashover following a direct stroke to the live conductor (e.g. overhead line conductor) gives a very short tail. Back flashover to the live conductor from directly struck earthed conductors (e.g. tower or earth wire) gives a short-fronted impulse of which the tail may be shortened or stepped if flashover from the line to earth subsequently occurs. The crest of the voltage surge is ultimately limited by the flashover of the conductor affected and the flashover voltage does not increase proportionately with the service voltage. Both the flashover voltage and the probable voltages without flashover depend upon constructional features some of which are only

indirectly connected with service voltage. For example, very high voltages can appear on lines with little or no earthed metal; 5×10^6 V was recorded on a 110-kV wood pole line. It is uneconomic to design against the highest voltages since, although a lightning current of 160 kA has been observed, only about 40 per cent. exceed 20 kA. A crest lightning surge voltage of 6 times service voltage is probably not unreasonable for high-voltage lines in the region up to 100 kV but the ratio is greater for lower voltage systems and less for higher voltage systems—at 400 kV the lightning hazard is probably quite small.

The crest value of an impulse test voltage of the 1/5 to 1/100 type (time constant of front 1 microsecond, tail 5–100 microseconds) according to one formula (proposed for voltages above 45 kV) is $4{\cdot}4V + 50$ kV, where V is the nominal r.m.s. line-to-line voltage in kV, allowing for simplicity 10 per cent. excess of highest working over nominal voltage. Another often used formula is $4{\cdot}5V + 45$ kV for V exceeding 10. If E is the crest value of the nominal working stress in kV/cm. this latter formula means that the transient field strength for testing is

$$5{\cdot}5E\{1 + (10/V)\}.$$

The higher working stresses mentioned in (6.1) will apply mainly to the higher values of V, so that there is not a great deal of difference between the effect of these formulae and the limit of 6 times normal stress suggested for surges in the preceding paragraph.

If the normal crest working stress were 50 kV/cm., the surge stress would attain about 300 kV/cm., and if the crest working stress were 100 kV/cm. or 200 kV/cm. the surge stress would be $0{\cdot}6 \times 10^6$ V/cm. or $1{\cdot}2 \times 10^6$ V/cm. It is not suggested that working stresses so high as 100–200 kV/cm. are other than limited to certain classes of insulation and equipment. However, it is a fair deduction that surge stresses are envisaged in practice which can involve a field strength in the region of the intrinsic electric strength. It may further be deduced that with high-grade insulation which can be operated normally at a high working stress, surge conditions are the most onerous to which

the insulation is expected to be exposed. It therefore appears that the design of the insulation, from the electrical aspect, will be mainly determined by the surge voltage to be withstood and that this will tend to depend upon the intrinsic electric strength. If the surge voltage is suddenly reversed in polarity, the total change of stress may sometimes be imposed on a part of heterogeneous insulation which will bring the maximum surge stress still nearer to or further into the region of intrinsic breakdown. Fortunately the transient is much less onerous than the sustained condition so that the factors of safety considered in (6.2) can be greatly reduced. It is still necessary, even with transient voltages, to include the factor which takes into account the occasional, but apparently at present inevitable, defects which yield low electric strength values. This factor of uncertainty is less than in intrinsic electric strength tests with sustained voltages, but from the evidence quoted in Chapter II and other chapters it is of important magnitude. In considering the physical calculations of Chapter II allowance must be made for the fact that these often correspond to the highest observations selected from intrinsic breakdown tests. The fact remains that the instantaneous stresses envisaged in surge conditions reach values in the region of intrinsic breakdown as we now know it from the assemblage of results obtained in intrinsic breakdown tests.

6.4. Notes on insulation testing

Tests on the insulation of electrical equipment may be divided into type tests, proof tests, and service tests.

Type tests are made on a specimen or sample of specimens representative of others made to the same drawings and specifications. Stability type tests often take the form of the measurement of capacitance, loss angle, insulation resistance, and temperature rise at elevated ambient temperature for long or short periods. Their direct connexion with general thermal stability and power rating has been noted, and it is only necessary to refer to Chapter III. Capacitance and loss-angle tests at high voltage can indicate abnormalities since both these quantities

should be independent of voltage for most normal high-grade insulation up to quite high stresses, provided the temperature remains the same. Mechanisms of stress-dependent conductivity have been considered in Chapter II. This kind of test can also serve to detect internal discharges and has been so used. In this it has to be compared with more specific 'ionization' tests which are now coming into use. The technique and interpretation of these tests were considered in some detail in Chapter IV. Both stability and loss angle tests can be used as measures of manufacturing uniformity and freedom from manufacturing defects, which is also true of the 'ionization' test. The latter is, however, more specialized. Ingress of moisture and electrochemical stability can be studied by means of insulation resistance as in Chapter V.

In contradistinction to these non-destructive tests, there are the breakdown and flashover tests. The latter are fairly straightforward on account of the principles given in Chapter IV, except in the initiation artificially of the unfavourable surface conditions which may be encountered in practice. This is a question of detailed engineering study. It is only possible to expose inexpensive samples to a breakdown test. This, accordingly, is limited to small articles made in large numbers, such as capacitors and mouldings, and to materials or test-pieces supposed representative of the insulation. For materials and test-pieces the most popular method is to use flat electrodes with rounded edges in air. The voltage is either rapidly applied and maintained until breakdown, so that from a number of tests the breakdown voltage can be interpolated, corresponding usually to 1 minute but sometimes to 30 seconds or a few minutes; or else the voltage is raised at a constant rate from zero or a fraction of the expected breakdown voltage (such as two-thirds) so that breakdown will occur in about $\frac{1}{2}$–2 minutes, usually the former. It has been shown in Chapter IV that these tests are mainly a measure of the resistance of the dielectric to edge discharges. The intrinsic electric strength probably affects these tests only in the sense that it may influence the resistance to discharges. Unless the insulation has to withstand discharges of a given

type, this kind of test has no obvious and direct validity and its usefulness is limited to comparison even when discharges form the limiting feature of the ultimate design. Protected electrodes and oil immersion have been used to reduce edge discharges, and the use of moist semi-conducting oil can effect an appreciable reduction. However, the intrinsic electric strength is only attained for low-grade or defective insulation and, in any event, design is at present rarely limited by the intrinsic electric strength. Accordingly, little seems to be gained by this elaboration, except that if the breakdown strength is increased more defects may be found. This view is born of hope rather than experience. It is unpromising when applied to a material or test piece. A breakdown test on an actual item is rather more informative as to defects, provided it can be established that these defects cause, or are inherently associated with, the causes of breakdown in service. This is rarely possible. Nevertheless, relative magnitudes are useful in articles of similar construction and if the breakdown stress is high enough, it can be deduced that the quality of the insulation is also high. A standard breakdown type test has been applied to impregnated capacitors with some success. The dielectric is here used in conditions where a high stress should be supported, and the construction and materials of these capacitors are, on the whole, very similar. The average breakdown stress is measured with a rate of application of voltage of, say, 50 V/sec. to give breakdown in 20–60 sec., and is required to be not less than 12 times the working stress in one class of capacitor. With this class the crest working stress might well be about 50 kV/cm. so that the breakdown stress must exceed $0{\cdot}6 \times 10^6$ V/cm. and in practice breakdown stresses well in excess of 10^6 V/cm. have been observed. Thus an electric strength approaching the ultimate is being measured. In spite of the degree of success attending the test, it fails to detect causes of breakdown of the kind described in Chapter V and in onerous conditions of temperature and/or humidity, these causes may be by far the most important. In addition, the types of capacitors mentioned in Chapter IV as failing by discharges often do not exhibit a performance in service

similar to their behaviour in the breakdown test. Accordingly, even in one of the most favourable instances, the success of the breakdown type test is limited.

Where possible, manufacturers use life tests or voltage–time curves extending to very long times, say 1,000–10,000 hours, as type tests. These are only occasionally practicable on grounds of time, expense, and the facilities for testing an adequate sample. The principles of breakdown here developed have shown the great difficulty in accelerating a life test by increase of voltage. Indeed, it is probable that the rather minor acceleration induced by an over-voltage of 50 per cent. is often all that is practicable. Rather greater success attends acceleration by increase of temperature, humidity, or cycles of heat, cold, dry, and damp. Here again great caution is necessary, since with very many materials the acceleration is meaningless when related to service conditions since it may be so great as to reduce good and bad insulation to the same low level. From Chapter IV it will be appreciated that a life test ought generally to be made with the same type of stress as in service, that is, whether alternating at a given frequency, a pulse type, or constant unidirectional. However, if breakdown is due to discharges, acceleration is possible by the use of the same excursion of voltage as in service, associated with a higher frequency, but not so high as to introduce thermal problems. A first approximation rather suggests that the determining factor is the number of cycles of stress in these circumstances, so that a large number of cycles can in this way be compressed into a short time. Unfortunately there are often practical difficulties in applying this idea.

Proof or factory tests are those applied as a routine to all articles usually by the manufacturer, but sometimes as on-site or after-installation tests. They aim to test uniformity of manufacture and to find fairly gross defects. As they are necessarily simple and must essentially avoid damage to sound insulation, they have little of special interest here.

Service tests may be carried out regularly or irregularly with the object of detecting incipient breakdown so that repair or replacement can be effected at the user's convenience. Even if

destructive of temporarily satisfactory but ultimately unsound insulation, the value remains because a breakdown under test is immeasurably to be preferred to a breakdown in service. The same remarks made in connexion with type tests apply also to service tests of capacitance, loss angle and insulation resistance. These quantities are more informative in service testing because of the long-term deterioration which may have developed. Although there are many other causes of breakdown in service, yet the number of occasions in, for example, electricity supply when breakdown is electrochemical, makes popular the insulation resistance test at a moderate voltage, say 10–20 kV. A d.c. test on an article has always the disadvantage that faulty insulation may be masked by sound insulation in series. Its competitor, the loss-angle test at high voltage, is at present much more cumbersome and is difficult of interpretation. Development is in progress by which these difficulties may be reduced. Another test with heterogeneous insulation is to find the voltage distribution in service. This is valuable, as preceding chapters have indicated, but it is not very easy to make.

Of more interest here are the voltage tests which can be associated with mechanisms of breakdown. Tests with alternating voltage do not exceed the working stress by an order of magnitude. In fact, twice the working stress is a more usual approximation. The highest in general use provide an electric stress for 1, 5, or 20 minutes, which is about 3 times the working stress. In almost every case so high a figure does not apply above 10–20 kV rated voltage. Above this, the severity of the test is reduced, reaching about twice the working stress with a rated voltage of the order of 100 kV, although recently rather over twice the working stress has been applied to 200 kV equipment. From what has already been said, an over-voltage of this magnitude with a relatively short duration will serve mainly to reveal the grosser forms of defect or deterioration.

It is possible, however, to apply a much higher voltage if d.c. is used, as considered in Chapters III and IV and to some extent in II. The equipment needed is of less power capacity since it only has to supply an adequate charging current to keep the

duration of the charging period within reasonable limits. Up to the present the test voltage has usually been kept within twice the crest value of the working voltage. This means that the stress does not necessarily exceed that which may occur transitorily but frequently and regularly in service. Some hazards such as discharges, thermal instability, and the propagation of defects by electrical energy derived from capacitance currents will be greatly reduced by the use of d.c. Test voltages have sometimes been used up to about 7 times the crest value of the service voltage. In some instances, such as cables, there is apparently conclusive evidence that a d.c. test with 5 times the crest working voltage does not damage insulation free from defects, and this probably applies to the majority of homogeneous insulation. The test stress corresponding to the highest working stresses used in practice would then be $0{\cdot}5–1 \times 10^6$ V/cm. The latter approaches the intrinsic breakdown range and should discover the majority of defects, but conversely should not cause permanent damage by reason of weak spots which might conceivably have survived long periods of service. There remains the difficulty that the voltage distribution in heterogeneous insulation is different with d.c. from that with a.c. If the d.c. is sufficiently restricted in duration this does not apply, but the method requires a high charging current. The number of cases in which the change in voltage distribution is serious is rather limited, possibly to gear such as instrument transformers, a few types of bushing, and some switchgear. The objection is not necessarily of major import. Another criticism of d.c. is its relative insensitivity to discharges and defects with abnormal energy loss. At the cost of some additional complexity, suitable detectors can be associated with the test to reveal these features, since they must cause enhanced currents even though transitory and not repetitive in the d.c. test. It may be concluded that d.c. tests with high voltage are attractive but need further development.

Impulse voltages are mainly used for type tests, sometimes for proof tests, and rarely, if ever, in service tests. From what was said in section (6.3), it follows that the kind of impulse test

there mentioned is valuable and often applies a stress as near the intrinsic breakdown region as it is probably safe to go in the present state of knowledge. The normal standard wave form, a front of 1 microsecond time-constant with a tail of 50 microseconds, is distinctly arbitrary. The impulse is sometimes repeated, say half a dozen times, with an impulse of opposite sign added at the end. If the interval between reversal of sign is less than the relaxation time of the dielectric components, then it is sometimes possible, particularly if a discharge occurs in the preceding impulse, for the instantaneous electric stress to be doubled by the last impulse in some parts of the dielectric by impressing a stress corresponding to the change of voltage from one sign to the opposite. It is sometimes found that breakdown, particularly in heterogeneous insulation, occurs on the last impulse of such a sequence. It must also be remembered that if the sign is not so reversed, then the crest value of the test impulse must be halved in comparing it with the type of switching surge which corresponds to a sudden reversal of polarity.

Another difficulty about the standard type of impulse test is the limited energy available, which often renders it impossible to detect a breakdown in insulation, especially if only a partial breakdown. In impregnated insulation a fine channel caused by an impulse may rapidly fill with the impregnant and the products of decomposition may diffuse away or be too inconsiderable to influence a subsequent test at lower voltage. Some consideration has been given to impulses superimposed upon a sustained follow-up voltage of adequate power capacity and to the sudden reversal of a relatively sustained unidirectional voltage provided with sufficient capacitance so that a partial breakdown due to the reversal would be transformed into a complete and detectable breakdown. In the 'flash' or 'jump spark' test the excited transformer or inductor under test yields a spark to produce a surge which, nevertheless, does not exceed the original voltage although the 'follow-up' power is adequate. Study has been applied to sensitive detectors such as those depending on current fluctuations, radiated or transmitted energy, and even aural detectors. Valuable as is the impulse test,

it has many unsolved problems and its beneficial effect on design for normal service is mainly due to its greater though more transient severity compared with the standard types of sustained tests.

6.5. Future developments

The impact of the scientific study of dielectric breakdown on insulation practice has thus been of no inconsiderable order. It has, nevertheless, been largely indirect as regards intrinsic breakdown. There is reason to suppose that the intrinsic electric strength may receive less scientific attention in the future. The main features of fundamental importance seem to be established although there is a considerable field of great potential interest in the theory of the solid state to be explored, particularly with extremes of thinness, temperature, duration of stress, and the like on materials with accurately controlled structures. The rather narrow range within which the intrinsic electric strength may be expected to lie rather militates against its value in fundamental studies and against the industrial importance of trying to improve it. More immediate is the problem of its experimental dispersion and the reasons for low values in apparently satisfactory circumstances. The solution to this problem might not only be of fundamental scientific importance but undoubtedly is required for the further improvement of performance in impulse conditions and will soon be needed for increase in the working stress of the highest grades of insulation.

The theory of thermal instability is now an integral part of design, and further progress depends upon the improvement of the normal stable properties of dielectrics.

The study of breakdown by discharges has been somewhat disappointing in practical outcome like the theory of the gaseous discharge itself. Townsend's principle of ionization by collision, when it first appeared, seemed likely to yield readily a well-ordered theory, but unfortunately the longer the study the greater the complication. In the same way, the basic principles of breakdown by discharges appeared logical and straightforward. More detailed examination has revealed the complication in practice, the obscurity of the breakdown mechanisms, and the

uncertainty of the field of application. In view of the rather limited concentrated study, scientific progress in this field has been creditable. Use of the concept has not, indeed, been hindered so far because technical interest lay less in understanding the nature of discharges than in avoiding them. It is now realized that for some time it will remain uneconomic to eliminate all discharges. In a high-voltage laboratory, for example, it is probably no an exaggeration to say that most of the equipment would have to be derated to one-half or one-third its normal rated voltage to be truly discharge-free. Accordingly, the detailed study of discharges, the way in which they can induce breakdown, and the circumstances in which breakdown becomes improbable are matters of immediate practical importance and considerable theoretical interest.

Chemical stability and electrochemical deterioration enter the realm of physical chemistry and a great deal of patient detailed work is required. Added point but added difficulty may come if power transmission with d.c. at high voltage is introduced on any scale. Powerful weapons are now to hand for overcoming the problem of predetermining the structure and characteristics of new substances and of providing substances with desirable properties. It is not expected that the use of d.c. will be impeded at the outset by insulation difficulties.

It must not be overlooked that dielectric breakdown is only one aspect of the electrical properties of insulating materials. The whole electrical aspect itself is only one of the many which must be taken into account in electrical insulation. A great deal of engineering lies behind the production of even the simplest piece of electrical equipment and changes, desirable and simple as they may appear electrically, may be impracticable on account of other difficulties entailed in their realization. An electrical installation must so often be regarded as a great engineering undertaking in which the part dependent purely on electrical science is quantitatively small, however vital its importance and however fascinating its scientific conception.

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