UNIVERSAL
LIBRARY

OU_154600

UNIVERSAL
LIBRARY

OSMANIA UNIVERSITY LIBRARY

Call No 530 1/C34D Accession No. 61/0227

Author Cassirer, Ernest

Title Determinism and indeterminism in modern physics

This book should be returned on or before the date last marked below

DETERMINISM AND INDETERMINISM IN MODERN PHYSICS

Renouveler la notion de cause, c'est transformer la pensée humaine.

TAINE

Published on the Louis Stern Memorial Fund

Other Yale University Press Books by the Same Author

An Essay on Man

The Myth of the State

The Problem of Knowledge

The Philosophy of Symbolic Forms

Volume I: Language

Volume II: Mythical Thought

Determinism and Indeterminism

IN MODERN PHYSICS

Historical and Systematic Studies of the Problem of Causality

BY ERNST CASSIRER

Translated by O. Theodor Benfey

With a preface by Henry Margenau

New Haven: YALE UNIVERSITY PRESS, 1956

London: Geoffrey Cumberlege, Oxford University Press

Copyright © 1956 by Yale University Press.
Printed in the United States of America by
Vail-Ballou Press, Inc., Binghamton, N.Y.
All rights reserved. This book may not be
reproduced, in whole or in part, in any form
(except by reviewers for the public press),
without written permission from the publishers.
Library of Congress catalog card number: 56-10870

Dedicated to Malte Jacobsson

Contents

	Translator's note	viii
	Preface by Henry Margenau	ix
	Foreword by the author	xxi
	PART 1	
	Historical Introduction	
1.	The "Laplacean Spirit"	3
2.	Metaphysical and Critical Determinism	11
	PART 2	
	The Causality Principle in Classical Physics	
3.	The Basic Types of Statements in Physics: Statements of the Results of Measurements	29
4.	Statements of Laws	37
5.	Statements of Principles	45
6.	The General Principle of Causality	58
	PART 3	
	Causality and Probability	
7.	Dynamic and Statistical Conformity to Law	73
8.	The Logical Character of Statistical Statements	89
	PART 4	
	The Problem of Causality in Quantum Theory	
9.	The Foundations of Quantum Theory and the Uncertainty Relations	109
10.	The History and Epistemology of the Concept of the Atom	139
	PART 5	
	Causality and Continuity	
11.	The Principle of Continuity in Classical Physics	155
12.	The Problem of the "Material Point"	175
13.	Concluding Remarks and Implications for Ethics	197
	Bibliography	214
	Index	217

Translator's Note

ERNST CASSIRER'S *Determinismus und Indeterminismus in der modernen Physik* originally appeared in Sweden as Part 3, Volume 42, of the *Göteborgs Högskolas Årsskrift* for 1936. It was available as a separate issue from Elanders Boktryckeri Aktiebolag, Göteborg, 1937.

Harmon M. Chapman of the Department of Philosophy, of New York University, and Joseph Stein of Belmont, Massachusetts, laid the groundwork and prepared the first draft for this translation. When Henry Margenau of Yale University suggested that I undertake the task of completing the translation, he gave me this draft. I owe him thanks not only for this proposal but also for much intellectual stimulus through correspondence and for his careful reading of the translation and many suggestions for improvement.

I wish to acknowledge the help I obtained from discussions of Cassirer's German work in *The Philosophy of Ernst Cassirer* edited by P. A. Schilpp. Several chapters in this volume contained helpful translations of sections of Cassirer's *Determinismus*. I am grateful to Professor L. Arnold Post of Haverford College for helpful criticism and the translation of the Kepler passage in the second chapter; to Gilbert F. White, formerly President of Haverford College, for his encouragement, and to the staff of the Yale University Press for unfailing help in all problems connected with the preparation of the book. My thanks to my wife, Rachel T. Benfey, for creating the atmosphere in which such a work can be carried through, cannot adequately be expressed.

My share in the preparation of this book I dedicate to my father.

O. T. BENFEY
Chemistry Department
Earlham College
Richmond, Indiana

September 1956

Preface by Henry Margenau

Eugene Higgins Professor of Physics and Natural Philosophy, Yale University

DESPITE the scope and depth of his philosophic concerns, Ernst Cassirer never lost interest in the problems of exact science, for he knew that his strength lay in the precision of the insights which science yields. Such an awareness is manifest in all his writings but receives its clearest expression in this book on determinism, wherein his competence as a philosopher combines with his interest in physics to produce, by inversion as it were, a work of unique interest to physics itself. The book was born in exile: written in German but published in Sweden, where Cassirer had found asylum. As his love for America grew, he began to regret that the book was not more widely known, and he made the initial arrangements for its translation into English.

At the same time, only a few months before his death, he asked the present writer to collaborate with him in the preparation of a bibliography and a final chapter on developments concerning the causality problem after 1936. This request was eagerly accepted. Unfortunately, however, the writing did not reach a stage at which the result could carry Cassirer's benediction, even though the bibliography and outline were completed and approved at the time of his death. In view of this it seems proper that the text of his work should be left intact and a summary of this recent material on the problem of causality should be turned into a preface. This is being done with the approval of Mrs. Cassirer, whose interest in seeing the book translated and edited has been unflagging and encouraging.

Physical science has made great strides in the two decades since the publication of this work in German, and a treatise on one of the crucial problems of its methodology would normally be expected to be out of date at this time. Quite the contrary is true, and for two reasons.

The fountainheads of the stream of discoveries in current physics are still to be found among the brilliant insights gained in the 1920's. What has happened since constitutes a tremendously successful application of the basic principles then learned, a massive search with copious means for new phenomena and facts. Most of these were readily explained by

the principles of quantum mechanics and represented further evidence for the philosophic view of which Cassirer wrote. Some phenomena, especially those connected with small regions of space and very high energies, present aspects which may call for modifications of that view. But every tentative departure from it has hitherto created difficulties far greater than those motivating the departure. In a sense, then, recent physics is an exploitation, a development, and a vast proliferation of the methodological resources contained in the creation of the quantum theory, and if certain outstanding achievements like nuclear fission and fusion rank higher in scientific esteem than the formulation of the Schrödinger equation and the uncertainty principle, it is because of the pragmatic appeal and the extrascientific consequences of these successes, which, in a deeper stratum, are linked directly with the basic insights discussed in this book. This is one reason for its continued timeliness.

The other reason lies in the character of the book itself. It was ahead of its day; its thesis was revolutionary and radical, not, like so many philosophic commentaries, a wordy echo of the scientists' own pronouncements. For at a time when every physicist spoke of the uncertainty principle as a restrictive injunction on the process of measurement, as an outgrowth of Newtonian dynamics which changes little except that it injects uncontrollable errors, Cassirer saw more deeply and perceived a basic change in the meaning of reality. And again, by taking a large and embracing view of causality, he showed that the causal controversy which raged at the time was not itself of crucial importance but was an outgrowth of a more fundamental issue, and that the controversy was to be resolved in a manner different from that commonly suggested. In effect he said, addressing the physicists: Your interpretation of uncertainty is a halfway concession. You cannot continue to use classical models in your reasoning and make your peace with the new doctrine by admitting errors of observation forever beyond human control. You must grant that the very concepts of "physical system" and of "physical state" have changed. By going to this deeper level of analysis, you can retain what most philosophers have regarded as causal description.

Perhaps it was the radical character of Cassirer's proposal that caused it to be ignored by physicists at large, caused it to be put on the shelf with other philosophic comments which sounded odd and vaguely provocative; for what physicist in that breathtaking period had the leisure to examine the writings of philosophers who, more likely than not, had misunderstood what the physicists were saying? But Cassirer had not misunderstood. In this book the scientific story is masterful, and the arrangement of the facts in the perspective of history is illuminating even to the person who knows and uses them. Indeed, it seems unfortunate that

Cassirer's voice has had so little response in the area of physics, because time may well show that it has been prophetic. His view has stood the test of two decades and enjoys greater popularity today than when it was propounded.

It behooves the present writer to record here that he is an advocate of this view, since it agrees with his own early writings. In saying this, I am aware of an indebtedness resulting from encouragement received through the reading of this book and through Cassirer's own kindly council.

This preface attempts two things. First, it endeavors to compare and contrast Cassirer's conception of the terms "causality" and "determinism" with others now current. Second, it calls attention to developments which have occurred since the publication of the book. On some of these the author had occasion to commit himself, and I shall try to be his reporter. Others must be mentioned to make the account complete, and Cassirer's presumable reaction to them will be stated.

Some authors, for example, Born and Reichenbach, make a subtle distinction between causality and determinism. Born [1] defines determinism as postulating "that events at different times are connected by laws in such a way that predictions of unknown situations (past or future) can be made." On the other hand, "causality postulates that there are laws by which the occurrence of an entity *B* of a certain class depends on the occurrence of an entity *A* of another class, where the word 'entity' means any physical object, phenomenon, situation, or event. *A* is called the cause, *B*, the effect." Apparently the attempt here is to distinguish between a cosmic condition of general lawfulness, a condition in which human prediction (or postdiction) becomes possible if sufficient knowledge is at hand, and a more particularized connection linking certain designated items in our experience, the latter being called causality. Cassirer makes no such distinction; in fact he needs none. Defining determinism in a manner similar to Born he regards as causes and effects precisely those events or situations which the lawfulness of nature permits the scientist to predict. Prediction always involves a starting point in the form of a known situation; it relates a known to a predicted situation; whichever member of this related pair temporally precedes the other is the cause, the later is the effect. By suitably adjusting the meaning of the loose terms "event," "situation," etc., Cassirer resolves the awkward distinction and simply makes determinism the doctrine which asserts the universality of causal relations. Hence the difference vanishes.

As to the meaning of causality, or the principle of causality, current literature is so rich in divergent interpretations that a mere catalogue of meanings would require a volume. Some of the ideas frequently en-

1. Max Born, *Natural Philosophy of Cause and Chance*, Oxford, 1949.

countered are dissected in Russell's rewarding little paper "On the Notion of Cause." [2] Here our concern will be limited to a review of a very few salient conceptions of causality, conceptions that appear to dominate the present thinking of scientists.

There is first of all the rather unphilosophic one which, in the words of Jeffreys,[3] says, *"precisely similar antecedents lead to precisely similar consequences."* This thesis sets up a straw man which Jeffreys, along with many others, takes great delight in destroying. To quote him, "no two sets of antecedents are ever identical; they must differ at least in time and position. But even if we decide to regard time and position as irrelevant (which may be true but has no justification in pure logic), the antecedents are never identical. The planets have never even approximately repeated their positions since astronomy began. The principle gives us no means of inferring the accelerations at a single instance, and is utterly useless. Further, if it was to be any use, we should have to *know* at any application that the entire condition of the world was the same as at some previous instance. This is never satisfied . . ." And so Jeffreys concludes somewhat redundantly that the principle "may conceivably be true in some sense, though nobody has succeeded in stating clearly what this sense is. But what is quite certain is that it is useless."

Cassirer would doubtless concur in this criticism; he would not, however, accept the definition. He might take issue with the assertion that nobody has succeeded in stating clearly the sense of the principle of causality; part of this book is devoted to that task, and many examples of earlier statements are reviewed in it. To be sure, the reader may look in vain for a succinct vest pocket formulation: Cassirer prefers to deal with all issues in their historical settings. But he will find Cassirer's meaning, and it is not in terms of an invariable conjunction of observable antecedents and observable consequences. The emphasis is on conception rather than observation, on constructs rather than percepts. Lenzen [4] more nearly speaks Cassirer's language when he emphasizes in the very beginning of his discourse that "Causality is a relation within the realm of conceptual objects. The relation of cause and effect refers to conceptual events regardless of the relation of the latter to reality. In the prescientific stage of experience causality is attributed to an intuitively given world which confronts an observer. In the sophisticated stage of

2. Bertrand Russell, "On the Notion of Cause, with Applications to the Free-Will Problem," reprinted in Feigl and Brodbeck, *Readings in the Philosophy of Science* (New York, Appleton-Century-Crofts, 1953), pp. 387–407.

3. H. Jeffreys, *Theory of Probability*, Oxford, 1939.

4. V. F. Lenzen, *Causality in Natural Science*, Springfield, Ill., Charles C. Thomas, 1954.

science causality must be attributed to a model which the scientist constructs out of concepts."

A doctrine often identified or confused with causality is a certain theory concerning the action of force. There is the age-old distinction between action at a distance and contact action, and with it the supposition that some forces, like gravitation, exhibit the former, while others, like a mechanical push, exemplify the latter kind of action. Now to some of the ancient philosophers, notably Aristotle, it seemed that action at a distance was a mysterious phenomenon which violates the law of causality, and so it comes about that this law has been identified with *contact action.* This is quite clearly the case in Hume,[5] whom Cassirer quotes as saying "Whatever objects are considered as causes or effects, are *contiguous,* and nothing can operate in a time or place, which is ever so little removed from those of its existence. We may therefore consider the relation of contiguity as essential to that of causation."

In this primitive form the thesis of contact action has grown foreign to modern physics, which has become increasingly demechanized. But an offshoot from it, the concept of the propagating field, has replaced the cruder idea and is sometimes regarded as an essential component of causality. A body need not act precisely where it is; it can send out an agent, possibly an immaterial agent called a field, and achieve effects at distant points and at later times through the mediation of this field. All physical fields which have thus far been studied and are well understood travel with finite velocities, the fastest among them being the electromagnetic field. Accordingly an influence at a distant point, arising simultaneously with a given event, cannot be caused by that event.

One of the most striking consequences of the special theory of relativity exposes the velocity of light as an upper limit to all possible energy transfers. In the early philosophic discussions of relativity this fact loomed large, and it is not surprising in view of the Aristotle-Hume indoctrination that fields, pre-eminently the electromagnetic field of the special theory, became the quintessence of causality. Reichenbach,[6] for example, speaks of the principle of action by contact (clearly in the enlarged sense which includes fields) as

> formulating a well-established property of causal transmission: the cause has to spread continuously through space until it reaches the point where it produces a certain effect. If a locomotive starts

5. D. Hume, *A Treatise of Human Nature,* Bk. I.

6. H. Reichenbach, *The Rise of Scientific Philosophy,* University of California Press, 1951

moving, the individual cars of the train will not follow immediately, but by intervals; the pull of the locomotive has to be transferred from car to car until it finally reaches the last car. When a searchlight is turned on, it will not immediately illuminate the objects to which it is directed; the light has to travel through the space in between, and if it did not move at such a high speed, we would notice the time required for the spreading of the illumination. The cause does not affect distant objects instantaneously, but spreads from point to point by contact.

This contact action view of the causal principle enjoys some currency in certain special developments of recent physics, where the term "causality condition" is taken to mean nothing more than the relativity requirement which limits the speed of a wave packet (representing the probability distribution of particles) to the speed of light c and says, in effect, that a cause at one point at time t cannot produce an effect at another point, a distance r from the first, at a time earlier than $t + r/c$.

On this formulation, causality is nothing more than a fragmentary version of the special theory of relativity, and one would find it difficult to justify the use of the elaborate designation "principle" as a label for it. Cassirer, of course, did not accept the contact action view, nor was it shared by many philosophers preceding him. Cassirer thought in the tradition of Laplace and Kant, in whose eyes the pinning of a metaphysical principle (for example, causality) to a specific feature of physical theory would have been inappropriate.

Indeed, it seems to me that there are few philosophers and scientists today who would regard the discovery of influence velocities greater than c—unlikely as it may be—as tantamount to an abrogation of the principle of causality. Or, conversely, the failure of causality which is said to have occurred through the establishment of quantum uncertainties is not a contradiction of action by contact in the relativity sense. Hence there must be other interpretations which this summary has not yet uncovered.

Suppose we eliminate from the preceding definition the assumption that fields must travel with finite velocities and permit indefinitely rapid actions. It then becomes a sort of continuous action view, asserting that causality is a pervasive connection between different events in space and time, the events being linked by any kind of scientific laws. Pressure and volume changes of a gas, which are presumably simultaneous, are thus causally related. Likewise, the causal nexus would extend through a chain of events such as this: appearance of a cloud in the sky—darkening of the sun—lowering of the temperature on the earth—people putting on coats, etc. Or another, perhaps more scientific: emission of light from a star—

propagation of an electromagnetic wave through space—absorption of light by a metal—ejection of an electron, and so on.

The events composing these chains form a sequence of continuous action; we know precisely, in terms of visible or postulated agencies, how the appearance of a cloud leads to the obscuring of the sun, how this in turn brings about a lower temperature, how this makes people uncomfortable and induces them to put on their coats. The other sequence can be traced in a similar manner; the events it connects, while occurring in widely different places and in totally different objects, are linked in a manner the details of which are understood. If there were gaps in this understanding, missing links in the chain of action, the term "causal" would not be applied to it.

While one may speak of that meaning of causality which these examples illustrate as continuous action in time and space, this phrase adverts to little more than relatedness by scientific agents and therefore makes causality tautologically equivalent to scientific understanding. Continuous action too, is a variant of the Humean doctrine of invariable sequence, refined by the inclusion of connective agencies between the members of that sequence. This interpretation of causality is large and generous, enjoys favor chiefly in the nonphysical sciences and, of course, in everyday language; it is the stock-in-trade of lawyers and biologists. However, it is difficult to formulate with precision, and the difficulty resides mainly in the circumstance that the view at issue places no restrictions upon the location and the kinds of events which are connected into a single chain. The emission of light can take place on Sirius, the absorption can take place in some photocell on earth, and the photocell may control some mechanical action at a distant point. The only supposition is that the effect is not earlier than the cause.

Physics, while at times espousing the continuous action view (often without being aware of its difference from the one to which we are now turning), is partial to another meaning of causality, one first clearly formulated by Kant and Laplace. To describe it we first note that every physical system is described in terms of *states* which change in time. For example, the state of a body undergoing thermal changes may involve its temperature, its volume, and its pressure, and these variables are said to be variables of state, or variables defining a state. Another physical system, called an elastic body undergoing deformations, has states which are defined through stresses and strains; an electromagnetic field (which is also a physical system in our sense of the word, for physical systems need not be material!) assumes states specifiable by an electric and a magnetic vector. Common to all these instances is the supposition that the variables of state, however defined, change in time in a manner conforming

to certain equations which are ordinarily called laws of nature. Future states are therefore predictable if a complete present state is known. A prior state of a given system is called the cause of a later state, the later state the effect of a former.

The principle of causality, in this sense, asserts the existence of a determinate temporal continuum of evolving states, all referable to the same physical system. Between a given cause and a given effect there is an infinity of other causes of the same effect, though only one cause at one time. The advantage of this view, which I shall designate as the doctrine of *unfolding states,* resides in its greater logical precision and in the uniqueness it confers upon the causal relation. When a cloud appears in the sky, that cause (in the former sense) has *many* effects at a given later time, one of which is a lowering of the temperature; however, when an elastic body has a given distribution of stresses and strains, a *single* definite distribution at a specified time is its effect. The continuous action view permits many causal chains, the unfolding state view only a single train of evolution.

The simplest physical system, indeed the one for which a causal theory of the latter type was first developed, is the moving particle. Its states are pairs of variables: positions and momenta; the law of nature governing their evolution is Newton's second law. The latter is a differential equation of the second order requiring two constants of integration in its complete solution. Position and momentum of the particle at a fixed time can serve as constants of integration and therefore determine the solution at all times.

States and laws of a causal theory must always have this internal affinity. The states must be so chosen that they provide the information demanded by the initial conditions that make the solution of the law complete. It follows from this circumstance that the definition of states in a causal theory cannot be arbitrarily altered without requiring corresponding changes in the laws of nature, and a change in a law will generally necessitate a redefinition of the state of a system.

Newton's theory of the motion of particles is the prototype of all causal description. The connection it prescribes, in a fundamental epistemological sense, between states of a particle at different times is a model of the causal relation. Understandably, though regrettably, the particular laws and states entailed by Newton's theory, and not its general formalism, have come to be regarded as the essential elements of causality. This misunderstanding, or at any rate this inflexible identification of states, has led to the belief that quantum mechanics is no longer a causal discipline. Let us recall the important details: Newton's law was found to fail for atomic particles, and Schrödinger succeeded in replacing it by

a new equation. But that equation did not have solutions specifiable by the old positions and momenta. Heisenberg discovered through his uncertainty principle that these variables had furthermore lost their soundness as universally observable quantities, for they cannot both be known with precision. This seemed to spell the doom of causality, because the states it involved are neither theoretically significant nor available by observation.

It is not idle, however, to ask the question whether Schrödinger's law selects or is compatible with states in terms of other variables, and whether these variables permit a causal description in the last, more formal sense of our principle, in the sense of unfolding states. That is in fact the case; only it happens to be the misfortune of these variables, and of the states which they define, to be somewhat strange and elusive when judged from Newton's familiar standpoint. They turn out to be probabilities.

It is the last thesis, the unfolding state interpretation of causality, which Cassirer affirms and examines for its endurance under the stress of the quantum revolution in physics. His conclusion is in essence the one suggested in the last paragraph, and it was reached after marshaling all the evidence available in 1936 that bore on it. The question we shall consider in the remainder of this preface is whether the last twenty years have brought forth significant new evidence for or against the stated conclusion.

Two current developments have bearing on this question: one is connected chiefly with the names of De Broglie and Bohm and attempts what its proponents call a causal interpretation of the quantum theory; the other is a theory of quantum electrodynamics in which time is permitted to "flow backward," its author being Feynman. By inverting the stream of time, such a theory seems to confuse the causal relation and therefore needs to be considered in this context. We first turn to the De Broglie-Bohm contributions.

The quantity which plays a central role in quantum theory is the famous state function, ψ. Its meaning and its behavior are completely understood on the mathematical side. Equally clear is its relation to empirical observations so long as these are regarded as statistical aggregates; but the connection between ψ and a *single* observation—if there is a connection—is open to a variety of interpretations. Let me first state the facts concerning which there remains no doubt.

For the simplest physical system, ψ is a function of position and time, of x and t. When $\psi\ (x,t_1)$ is given at any definite time t_1 the fundamental equation of quantum mechanics (Schrödinger's, or Heisenberg's, depending on the representation) allows $\psi\ (x,t_2)$ to be predicted for any later time t_2. Thus, if ψ were truly descriptive of the state of the physical

system to which it refers, causality in Cassirer's sense would hold in the new discipline.

But unfortunately, ψ is not related to the kind of things one ordinarily regards as state variables in the older physics. By itself it refers to nothing that can be measured with ease. Only when squared, in the form $\psi^*\psi$, does it correspond to simply measurable quantities: indeed $\psi(x)^*\psi(x)$ represents the probability density that the system shall be found near the place designated by x. This probability itself does not generally develop or unfold itself in accordance with any simple equation and is, therefore, not strictly regulated by a causal law, although ψ is. The crucial point is that neither ψ nor $\psi^*\psi$, when known with any accuracy, permits the prediction of what happens in a single observation; these quantities speak only of distributions within aggregates. A single observation is no longer determined by a causal law.

Hence arises the obvious question: Is quantum mechanical description only a halfway scheme, something which needs to be supplemented in order to deal more adequately with individual observations? Or is it an ultimate sort of discipline—not perfect in the sense of needing no corrections, but final in its renunciation of all facilities for describing individual events? If the first question is affirmed, physics is obliged to look for hidden variables not now included in the quantum theoretical formalism, variables that will bring single observations within its range of predictive competence. If the second, then the probabilities $\psi^*\psi$, or perhaps ψ itself, are the last determinants of physical happenings. In that case the maxim rejected by Einstein would be true; God would "play dice."

Von Neumann has given good arguments in favor of the second alternative; most physicists are at present convinced of the correctness of these arguments and prefer the view it entails. Cassirer tended to accept it on the evidence he had available, which was less complete than it is now. According to this view ψ has no more to do with a single event than temperature has to do with the motion of one particle of gas. But while there is a higher level theory in terms of which the motion of a particle in a gas can be described, there is none that regulates single events in quantum mechanics.

De Broglie [7] and Bohm [8] have tried to affirm the first alternative, have looked for a theory capable of regulating single observations, and tend to regard quantum theory in its present form as incomplete. De Broglie's proposal came twenty years ago; he has declared it a failure and has now

7. L. de Broglie, *La Physique quantique restera-t-elle indeterministe?* Paris, Gauthier-Villars, 1953.

8. D. Bohm, in the *Physical Review, 85,* (1952), 166, 180.

become rather critical of the work of Bohm, who has carried his endeavors beyond the early efforts of de Broglie. Bohm's analytically competent efforts seize upon a well-known connection between the Hamilton-Jacobi equation of classical physics and the Schrödinger equation; the latter is split into a pair of real equations, one of which can be used to define the individual motion of a particle. Its path, however, is disturbed by an unconventional force field which arises from the presence of the particle itself. This field, overlooked in classical physics and implied but not recognized in quantum theory, is responsible for the erratic behavior of single events and forces the physicist to treat events statistically so long as he ignores it.

Bohm speaks of his hypothesis as "the causal interpretation" of quantum mechanics. In Cassirer's sense (and most others) this is a misnomer, for the former interpretation is causal, too. The only difference is this: In the theory under discussion the *state variables* of the system, conceived mechanistically, are subject to causal laws; in the usual theory the *ψ-function* obeys a causal law. Hence, a better distinction would be in terms of *mechanical* and *statistical* causality.

The status of the causal-mechanistic interpretation is not easy to appraise. It leads to all the old results and to some new difficulties.[9] Certainly, it has encountered no insurmountable obstacles. But until it achieves something which the simpler statistical theory cannot produce, or until one of its hidden variables exposes itself to view, there is perhaps not sufficient ground to discuss its philosophic bearing. If it should become established, Cassirer's thesis, far from being abrogated, would be strengthened.

In most versions of the causal principle the cause is temporally anterior to the effect; the future does not determine the past (although, of course, if the future were known, causal laws could be invoked to infer the past and the present). In Feynman's theory, time reversals occur, upsetting the usual causal pattern. The theory of electrons, when suitably developed, is obliged to consider paths on which the electron moves backward in time, and these paths can be regarded as tracks of positrons moving conventionally. The details are complicated and need not be given here. I believe that careful analysis shows this theory to be entirely in keeping with the causal doctrine Cassirer presents.[10]

The last twenty years have seen a great rise to prominence, in the

9. See, for instance, H. Margenau, "Advantages and Disadvantages of Various Interpretations of the Quantum Theory," *Physics Today, 7* (1954), 6. See also the *Journal of the Washington Academy of Sciences, 44* (1954), 265.

10. H. Margenau, "Can Time Flow Backwards?" *Philosophy of Science, 21* (1954), 79.

literature on the philosophy of physics, of Bohr's principle of complementarity. As modestly stated by its author, the principle affirms that nature can be described with equal justification in two ways: with the use of ψ-functions and in terms of individual events. The first method entails a renunciation of our propensity to picture the details of physical events as taking place in space and time but allows a statistical-causal representation of experience; the second aids and abets our desire for models but destroys causality *as applied to events*. Bohr believes this to be a fundamental trait of our description of experience in the physical realm; furthermore, he is convinced that both methods are necessary as avenues to reality.

This relatively sober statement can be and has been made into a basic formula for bifurcation, for a philosophic dualism pervading all fields. It has been invoked as testimony for the irreconcilable contrast between facts and values, between the scientific and artistic approach to life—when pushed far enough, it can be employed to legislate any unresolved difficulty into a norm and thus to arrest proper inquiry.

Cassirer mentions complementarity with approval (pp. 115, 212). In the concluding paragraph of this book he seems to suggest that the physical wave-particle dualism is a facet of the multiplicity of perspectives to which a large part of the *Philosophy of Symbolic Forms* is devoted. Nevertheless, his careful analysis shows a measure of apprehension in his avowal of this principle. He sees uncertainty not merely as a human limitation but as affecting the very meaning of physical reality (p. 128), and he shows a disposition to say that whatever satisfies causal laws is to be taken as defining physical reality. This clearly detracts from the significance of individual events as determinants of physical reality, and it emphasizes the probabilistic aspects of quantum theory, perhaps to the limit of denying complementarity. At any rate, I deem it proper to report that after the publication of this book Cassirer was definitely opposed to the metaphysical inflation of the complementarity idea and to the unprincipled proliferation it has occasionally seen in such fields as psychology and sociology.

Foreword by the Author

THE PREFACE to a book need not confine itself to a discussion of contents; it may and should include a personal word as well. I would like, therefore, to introduce this book by confessing that a personal motive first led me to write it. During the last years I have felt ever more urgently the desire and obligation to take up again those problems from which my philosophical work began. My first systematic work, which appeared under the title *Substanzbegriff und Funktionsbegriff* in 1910,[1] dealt with the problem of the logical structure of the concepts of mathematics and science. It sought to disclose a unified methodological tendency in the systematic content and history of mathematics and physics and to establish its epistemological significance. For this purpose the "factum of science" as it existed at the turn of the century was used as a basis. Classical physics was accepted as valid and uncontested. Relativity and quantum theory were still in their beginnings, and it would have been precipitate to seize on these as points of departure for a purely epistemological analysis. For this reason I refrained from dealing with them and sought to establish and develop my fundamental epistemological thesis without them.

This restriction, however, could not be maintained indefinitely. With the new developments in theoretical physics, the displacement of its epistemological center of gravity became increasingly evident. It also became ever more apparent that from the philosophical point of view the questions which now pressed in from all sides could not be avoided, although certain hasty "speculative" conclusions had to be rejected. This state of affairs gave rise to the essay I published in 1920 under the title *Zur Einsteinschen Relativitätstheorie, erkenntnistheoretische Betrachtungen.*[2] This essay, however, was concerned only with the changes and additions which modern physics had in the meantime undergone as a result of the special and general theories of relativity. No account was taken of the powerful and dangerous "explosive" (*Sprengkörper*) of

1. English trans., W. C. and M. C. Swabey, *Substance and Function,* Chicago, 1923 (cited in the text below by this English title).

2. Translation included as supplement to *Substance and Function* under the title "Einstein's Theory of Relativity Considered from the Epistemological Standpoint."

quantum theory, as Planck once called it. But now that the effects of this explosive are becoming increasingly evident and extend over the whole field of physics, it becomes more and more imperative to investigate their historical origins and their systematic foundations. The first notes for this book were made in the desire to fulfill this demand. Initially they were not intended for publication but solely for clarifying my own viewpoint and for the critical testing of the basic epistemological thesis from which I had started.

I realize that there may still be good reasons for questioning if the time is opportune for the publication of this book. To the present status of the problem one might apply with some justification the words which Schiller used at the end of the eighteenth century concerning the relation between science and transcendental philosophy: "Enmity be between you! It is too early to unite; only if you separate in your search will the truth be known." This separation is more than ever to be enjoined when the new physics in its theoretical aspect is still, so to speak, *in statu nascendi,* when it is still continually struggling to secure its basic concepts and to fix their meaning. Eddington suggested in his well-known book *The Nature of the Physical World* (1928) that the following warning be posted over the entrance gate to the new physics: "Structural Alterations in Progress—No Admittance except on Business." And the doorkeeper was to be particularly instructed under no circumstances to admit "prying philosophers." There are certainly many physicists even today who will concur with this suggestion. But it lies embedded in the very essence and in the task of philosophy that it cannot indefinitely heed such prohibitions. It is not merely out of curiosity that philosophy concerns itself again and again with what goes on behind the partitions and enclosures of the separate sciences. If it did not do so it could not meet the demands of its own ideal: the ideal of methodological analysis and of the establishment of epistemological foundations. For this reason philosophy must time and again step over the boundaries which have been erected between the particular sciences, however indispensable they may be for the progress of science and for a correct and sound division of labor. For theory as such, these boundaries must not become barriers. Theory has to step over them, in spite of the danger that conflicts and boundary disputes may result. I wanted this book to be understood and accepted in this sense. I was not at all interested in viewing physics from the outside or in instructing it from above. What I have tried to do primarily is to prepare the ground for a common inquiry. Only by such a common effort and by continued mutual criticism can the answer to certain basic questions of the new physics finally be found, questions that are generally agreed to be as yet far from their ultimate solution.

The fundamental viewpoint, in accordance with which I have dealt with these problems, does not differ essentially from that of my *Substance and Function*. This viewpoint is, I believe, still justifiable. Indeed, I think I can now justify it better and formulate it more precisely on the basis of the developments in modern physics than I formerly could. I hope it will be clear from the contents of this book that I was not guided by the intention to preserve my own "point of view" under all circumstances when confronting these developments. I was not interested in remaining "in the right" with respect to the new physics. To remain "in the right" in this way would appear to me to be of very questionable merit, for the critique of knowledge has always to orientate itself anew as science advances. Modern physics has been enriched and deepened by the new and more precise formulation of its basic concepts. Epistemology cannot and should not keep itself immune from these developments; it must always be ready to revise its own presuppositions. Thus there is naturally a great deal in my earlier investigations which I would not maintain today in the same sense or which at least I would establish differently. Only in its fundamental tendency, which is expressed more in the form of the general question than in particular answers, do I still hold to my former position.

This leads me to another point which I would like to discuss briefly in order to anticipate a possible objection and a possible misunderstanding. When my essay "Einstein's Theory of Relativity" appeared, there were many critics who agreed indeed with the conclusions I had drawn from the development of the new physics but who supplemented their agreement with the question whether as a "neo-Kantian" I was permitted to draw such conclusions. The present volume will probably be exposed in still greater degree to such questions and doubts. I believe, however, that criticisms of this type arise from a misconception of the essence and historical tendency of "neo-Kantianism" as envisaged by the founders of the Marburg School, Hermann Cohen and Paul Natorp. In his article "Kant and the Marburg School" [3] Natorp expressly declared that it had never been the intention of the Marburg School to hold fast unconditionally to Kant's teaching or to wish to do so. "Talk of an orthodox Kantianism," he emphasized, "was never justified. With the further development of the school it has lost even the remotest appearance of justification. . . . The only reason why one might wish to go back to Kant would be in order to pursue the eternal problems of philosophy in the direction of the fundamental insights which philosophy has ineradicably won through him, and consequently with the increase in depth achieved through him. . . . A poor student of Kant who understands otherwise!" It is evident that Natorp's relation to Kant is in no way different from the relation between

3. *Kant-Studien, 17* (1910).

a modern physicist and Galileo or Newton, Maxwell or Helmholtz. He rejects all dogma, appealing to Kant himself, who constantly stressed that in philosophy there must not be a "classical author." So also the ties which connect me with the founders of the Marburg School are not loosened and my debt of gratitude to them is not diminished when in the following investigations it turns out that the epistemological examination of the principles of science leads me to results different from those set forth in Cohen's *Logik der reinen Erkenntnis* (1902) or in Natorp's *Die logischen Grundlagen der exakten Wissenschaften* (1910).

I cannot let this book be published without another personal word. In dedicating it to Malte Jacobsson I want to express my gratitude to him for the interest he has long taken in my philosophical work and for the genuine cordiality with which he received and advised me last year as I entered a new academic circle. My heartfelt thanks are also due the administration of Gothenburg University, its former rector, Professor Bernhard Kalgren, and the academic council for the high honor which they bestowed on me by calling me to Gothenburg and for the testimony of personal trust which they thereby gave me. This book owes its existence to many others whose names cannot here receive detailed mention, for without the cordial reception which I have met in my new position and the encouragement which was given me from all sides, I would hardly have found the inner tranquillity and the courage to complete this manuscript.

I also wish to thank the librarian and staff of the Gothenburg City Library for their constant readiness to supply me with the necessary literature. Let me here remark that since the manuscript was completed in April 1936 subsequent literature could not be systematically employed but only mentioned in occasional references. My warm thanks go to Dr. Manfred Moritz for his kind help in reading proof.

ERNST CASSIRER

Gothenburg
December 1936

PART ONE

Historical Introduction

CHAPTER 1

The "Laplacean Spirit"

"Rest, rest, perturbed spirit!" HAMLET, I. v. 181

IN THE INTRODUCTION to his *Théorie analytique des probabilités* Laplace envisages an all-embracing spirit possessing complete knowledge of the state of the universe at a given moment, for whom the whole universe in every detail of its existence and development would thus be completely determined. Such a spirit, knowing all forces operative in nature, and the exact positions of all the particles that make up the universe, would only have to subject these data to mathematical analysis in order to arrive at a cosmic formula that would incorporate the movements both of the largest bodies and of the lightest atoms. Nothing would be uncertain for it; future and past would lie before its gaze with the same clarity. The human mind may be seen as the copy, though weak, of such a spirit when one considers the completeness to which it has brought astronomy, but it will certainly never reach the perfection of its original. No matter how great the effort to approach it, human understanding will always remain infinitely far behind.

I begin with this picture of the Laplacean spirit, not because I consider this introduction logically appropriate or as particularly suitable psychologically, but for exactly the opposite reason. In all the discussions of the general problem of causality which have arisen from the present state of atomic physics, the Laplacean spirit has played an important if not a decisive role. The defenders as well as the attackers of the causality principle of classical physics seem to be agreed at least in this respect, that this picture may be taken as an adequate expression of the problem, that one may use it without hesitation in order to clarify the nature of a strictly deterministic view of the world. The following considerations will seek to show in detail why I cannot share this opinion. However, before I begin it might be advantageous to look at the history of the problem, for only such a historical survey can explain the significance which

the Laplacean cosmic formula has attained in the present epistemological and scientific discussions of the concept of cause.

For Laplace himself the idea of this formula was hardly more than an ingenious metaphor by means of which he sought to make clear the difference between the concepts of probability and certainty. The idea that this metaphor should be endowed with a wider meaning and validity, that it should be made the expression of a general epistemological principle, was, to my mind, quite foreign to him. This transition occurred in a much later period, and its date can be established quite definitely. In his famous speech "Über die Grenzen des Naturerkennens" (1872) Emil du Bois-Reymond lifted the Laplacean formula out of its long oblivion and placed it at the focal point of epistemological and scientific discussion. This speech attracted wide attention and exercised a very powerful influence. Half a century later W. Nernst in his article "Zum Gültigkeitsbereich der Naturgesetze" praised the "engaging eloquence" with which du Bois-Reymond had depicted the efficacy of the Laplacean cosmic formula.[1] This eloquence, however, created certain hazards. It provided a fragile yet inviting basis on which certain basic problems of philosophical and scientific knowledge were dealt with, not to be analytically clarified but to be brought to a quick and final though thoroughly dogmatic conclusion.

This conclusion had a positive and negative aspect. It claimed to fix once and for all the permanent, unalterable form of all scientific knowledge. At the same time, however, it regarded this very form as an insuperable limit. For du Bois-Reymond elevated scientific knowledge far above all accidental, merely empirical bounds. Within its own sphere he endowed it with a kind of omniscience. But this exaltation is only the precursor of its fall. From the heights of the strictest, most exact knowledge it is dashed into the abyss of ignorance, an ignorance from which nothing can deliver it, for it is not temporary and relative but final and absolute. If it were possible for human understanding to raise itself to the ideal of the Laplacean spirit, the universe in every single detail past and future would be completely transparent. "For such a spirit the hairs on our head would be numbered and no sparrow would fall to the ground without his knowledge. He would be a prophet facing forward and backward for whom the universe would be a single fact, one great truth." And yet this one truth would present only a limited and partial aspect of the totality of being, of genuine "reality." For reality contains vast and important domains which must remain forever and in principle inaccessible to the kind of scientific knowledge thus described. No enhancement or intensification of this knowledge can bring us a step nearer to

1. *Naturwissenschaften, 10* (1922), 492.

the inner mysteries of being. Our knowledge dissolves into nothingness as soon as we leave the world of material atoms and enter the world of the "spirit," of consciousness. Here our understanding ends; for even with perfect, "astronomically exact" knowledge of all the material systems of the universe, including the system of our brain, it would still be impossible for us to comprehend how material being can give rise to the enigmatic appearance of consciousness. Accordingly the demand for "explanation" not only cannot be fulfilled here—strictly speaking it cannot even be raised: *ignorabimus* is the only answer that science can give to the question of the essence and origin of consciousness.

In the last decades of the nineteenth century the problem as thus put by du Bois-Reymond exercised a strong influence both on metaphysics and on the theory of scientific principles. Of course the attempt was made to escape from the radical consequences he had drawn. There was no ready surrender to the apodictic dogmatic conclusion of du Bois-Reymond's speech. But there seemed to be no doubt that here an important and pertinent problem had been raised with which epistemology and science had to wrestle using every power at their disposal. Even the neo-Kantian movement, which began in the early seventies almost at the time of du Bois-Reymond's speech, did not at first alter the situation substantially. Otto Liebmann, one of the first to call for a "return to Kant," develops his analysis of the causal problem along exactly the same lines. For him too the Laplacean formula is the complete and perfectly valid expression of what he likes to call the "logic of facts." If we begin with a "hypothetical absolute universal intelligence," he explains, "then for this intelligence the whole universal process, which to us is dispersed in infinite space, would be given even in its minutest detail as a timeless universal logic *sub specie aeternitatis*. This would then be the complete logic of facts in objective universal reason; and Spinoza would have been right, albeit in a sense that could not have been completely clear to him, since he died a decade before the publication of Newton's *Principia* and a century before the appearance of Laplace's *Mécanique céleste*." [2] One can see from this that the Laplacean formula is as capable of a scientific as of a purely metaphysical interpretation, and it is precisely this dual character that accounts for the strong influence it exercised. This influence becomes fully understandable when the total intellectual climate of the period is considered in which du Bois-Reymond's speech was made. It was the period of controversy over materialism, when philosophy was confronted with the crisis of deciding whether to accept the guidance of scientific thought, which seemed to lead inevitably to a strictly mechanistic view of nature, or to maintain and defend its own

2. O. Liebmann, *Zur Analyse der Wirklichkeit* (2d ed. Strassburg, 1880), p. 205.

position over against the scientific view, granting to the "spiritual" a different and special status. It was here that du Bois-Reymond's speech took place, interpretable as a resolution of doubt and a way out of the dilemma. For it appeared to do justice to both claims, to satisfy in a certain sense the demands of materialism as well as those of systems having a place for the spiritual. Materialism and mechanism could find satisfaction in du Bois-Reymond's definition of science, for in this domain their basic maxims were not only recognized but set up as the sole and exclusive standard. "For us there exists nothing but mechanical knowledge," du Bois-Reymond emphasized, "no matter how miserable a substitute it is for true knowledge, and accordingly only one true form of scientific thought, that of mathematical physics." On the other hand, however, this form was rejected in regard to intrinsically transcendental problems. The scientist has to give up once and for all the idea of investigating these problems, leaving the way open for others to attempt purely speculative solutions. Thus the radical advocates of materialism as well as its bitterest opponents could appeal with equal right to du Bois-Reymond's basic thesis: the former, because they found enunciated in it the identity of scientific with materialistic, mechanistic thought, the latter because in addition a reality was assumed which was in principle inaccessible to scientific thought and which remained as a dark and impenetrable residue.

At this juncture we immediately find ourselves faced with a question whose significance reaches far beyond the particular situation out of which du Bois-Reymond's speech arose. A systematic connection becomes evident which will become confirmed again and again in the course of our inquiry: the answer that an epistemology of science gives to the problem of *causality* never stands alone but always depends on a certain assumption as to the nature of the *object* in science. These two are intimately interconnected and mutually determine each other. We can never understand the significance and basis of the causal concept of a given period or of a given tendency in scientific thought unless we ask about the concept of physical "reality" that is presupposed, unless we apply the lever at this point. Later on I will try to show that this is the case with modern quantum mechanics also, that in the "crisis of the causal concept" which seems to distinguish it we have far more to do with a critical transformation, a new version of the concept of object. For the moment I will content myself with making clear this relation in the implications of du Bois-Reymond's theory of scientific knowledge. Since the causal claim in this theory is extended beyond all limits of empirical applicability, since it is connected in its expression and definition with the assumption of an infinite spirit, reality too recedes to an

inaccessible distance. It is removed beyond all real comprehension, beyond all grasp, by the fundamental theoretical means of attaining knowledge. In spite of all our understanding, all the refinements and improvements of our physical instruments of cognition, we do not advance a step further, we only enmesh ourselves ever more tightly in the net of our own concepts. For according to du Bois-Reymond, unknowability by no means starts when we enter the realm of the spirit, of consciousness. It is fundamentally the same whether we ask about the essence of consciousness or of the material world and its basic constituents, the atoms. Even the Laplacean spirit, possessing complete knowledge of all particles, together with all their positions and velocities, would not be helped the slightest by this knowledge in understanding the "essence" of mass and force. "Nobody who has gone a little deeper in his reflections," declares du Bois-Reymond, "mistakes the transcendental nature of the obstacle. All the advances of science have not been able to assail it, and all future ones will be unavailing against it. We shall never improve our knowledge of what here, where matter is, 'haunts space.' For even the Laplacean spirit would in this respect be no wiser." [3]

The mode of argument which du Bois-Reymond employs in all his discussions now appears clear and undisguised. At first glance it is strange and almost incomprehensible. For what could be more odd than an approach by which the very principles and bases of scientific knowledge are branded as unknowable, by which concepts such as matter and force, which are nothing but instruments of scientific knowledge, are made into something spectral and ghostlike, mysteriously "haunting space"? And yet science only succumbs, in this strange mode of argument, to the same fate that it shares with all forms of symbolic knowledge. At a very advanced stage of knowledge, indeed at one of its peaks, a process repeats itself which we can trace back to the first attempts at comprehending the universe. Wherever we attempt to analyze the various symbols which have facilitated a "comprehension" of the universe, of nature as well as of "spiritual reality," we come upon this dualism in the interpretations of the basic means on which this comprehension rests. *Language* and *image* are the first means the human mind creates for this comprehension. Through them alone is it able to divide, distinguish, and control the "always equably flowing stream" of happening (*Geschehen*). It is, however, just these *means* of control that now acquire an existence of their own, a reality and significance through which they then turn on the human mind and bring it under

3. "Über die Grenzen des Naturerkennens," *Reden* (Ser. 1, Leipzig, 1886), p. 114.

their dominion. The instrument begins to take on, as it were, a life of its own. It is hypostatized and in this hypostasis it becomes an independent, distinctive, and self-directing force which holds man in its sway. The farther we press back to the origins of language and myth, the more clearly does this fundamental character of language and image symbols become evident. The symbolic turns into the magical: magic based on word and image forms the basis for all magical knowledge and for all magical control of reality.[4] However curious and paradoxical it may seem, not even the most abstract symbol is free from this compulsion to be visualized, and consequently free of the urge toward "reification." It too has to struggle constantly against the danger of being substantialized and hypostasized. At the moment when it succumbs to this danger, the process of attaining knowledge undergoes a curious reversal. The principles, the "first" of knowledge, become the "last," become that which knowledge seeks to grasp but which at the same time withdraws from knowledge more and more and finally threatens to retreat indefinitely. The symbols are divested of their immediate "magical" character, but they are still infected with the character of the mysterious, the "unknowable." Still more sharply and undisguised than in the lecture of Emil du Bois-Reymond, this conclusion is drawn in the book of his brother, the mathematician Paul du Bois-Reymond, *Über die Grundlagen der Erkenntnis in den exakten Wissenschaften,* which purports to show how every attempt of physics to grasp and describe reality is from the beginning doomed to failure. Every such attempt only serves to teach us anew "how impenetrable are the walls of our intraphenomenal prison." "Our thinking, which wears itself out attempting to advance in an area of misty uniformity, is as if lamed and does not take a single step forward. We are encased within the housing of our perceptions and are as it were born blind for what lies outside. We cannot even get a glimmer from the outside, for a glimmer would already be like light, and what in the real corresponds to light?" [5]

For some time now modern physics has stressed with increasing emphasis that such a fundamental position is no longer binding and possible for it. It has given up the presupposition on the basis of which the cognitive ideal of the Laplacean spirit was conceived. It denies the possibility of grasping all physical happening by reduction to the motions

4. For more detailed elaboration I must refer here to my *Sprache und Mythos,* Studien der Bibliothek Warburg, *6;* English trans., Susanne K. Langer, *Language and Myth,* New York, 1946. Also *Philosophie der symbolischen Formen,* 3 vols. Berlin, 1923–29; English trans., Ralph Manheim, *The Philosophy of Symbolic Forms,* 3 vols. New Haven, 1953–57.

5. Ed. Tübingen (1890), Sec. 8.

of simple mass points. And it has rejected still more decisively the further consequence which du Bois-Reymond attached to the universal formula of Laplace. "His *ignorabimus,*" a modern investigator declares, "has no other significance for us than the sober knowledge that the mathematician has of the impossibility of squaring the circle and of other similar problems, which in being brought to their proper form are at once settled and annulled." [6] It certainly did not require the new quantum mechanical formulation of concepts in order to arrive at this purely *epistemological* insight. It was already attainable on the basis of classical physics and could be established with reference to the presuppositions of the latter.[7] In general it may be said that from the point of view not only of the study of physical phenomena but also of logic and epistemological analysis the idea of the Laplacean spirit gives rise to grave doubts. Careful scrutiny reveals that the idea is made up of disparate elements. For how shall we think the condition fulfilled on which the foresight of the Laplacean spirit depends? How should he have obtained the complete knowledge of the initial positions and velocities of all particles? Does he attain to this knowledge in a human or in a "superhuman" fashion, in an empirical or in a "transcendental" manner? In the first case the same conditions apply that hold for our own empirical knowledge. Measurements would have to be performed and definite physical instruments employed. But it is impossible to see how anything but a relative knowledge could be reached and secured in this way. Precision of measurement could never be increased beyond a certain limit, and the employment of physical apparatus would render the results dependent on the nature of the apparatus and determinable not absolutely but only in relation to it. This difficulty can only be avoided if we ascribe to the Laplacean intelligence not merely a mediate but an immediate, an "intuitive," knowledge of the initial conditions. But with this turn the whole problem we are facing slips out of our hands, and in the end dissolves into nothingness. For an intelligence equipped with such an intuitive knowledge would be spared all the pains of mediate inference and calculation. It would not need to "conclude" from the present to the past or future. It would possess, in one single, undivided act, a complete comprehension, an immediate intuition of the whole time series and of its infinite extent. Thus in the idea of the Laplacean spirit two heterogeneous and mutually incompatible tendencies combine and interpenetrate. Expressed in Kantian terminol-

6. R. von Mises, "Über das naturwissenschaftliche Weltbild der Gegenwart," *Naturwissenschaften, 18* (1930), 892.

7. I myself attempted such a proof from the standpoint of classical physics in *Substance and Function.* See esp. pp. 122 ff., 165 ff.

ogy, it contains at the same time the notion of a "discursive" and that of an "intuitive" understanding: a discursive understanding which is restricted to the form of mediate apprehension and of "calculation," and an intuitive understanding which can dispense with all calculation because it proceeds from the "synthetic universal" (the intuition of a whole in its totality) to the particular, that is, from the whole to its parts. On more careful epistemological analysis, therefore, the ideal of scientific knowledge which Laplace delineated and which du Bois-Reymond further amplified and embellished resolves itself into an idol. The limit which the human mind in its advancing scientific knowledge is supposed to approach continually, vitiates itself even as a limit. For merely positing it hypothetically, if strictly considered, leads to a contradiction. We must formulate the ideal and principle of scientific knowledge differently and from a new point of view if the principle is to be logically coherent and empirically useful, applicable to the procedure of "actual" physics and to its formation of concepts.

CHAPTER 2

Metaphysical and Critical Determinism

OUR DISCUSSION so far has not brought us in any sense to the real core of the problem underlying the Laplacean formula. For Laplace himself this formula was only an incidental, ingenious *aperçu,* a thought-experiment which was boldly outlined and developed in its consequences, the true grounds of which, however, were left in darkness. And yet the Laplacean formula has a much greater significance. It is nothing less than the complete expression, the pregnant summary, of that world view from which sprang the great philosophical systems of the seventeenth century, the systems of classical rationalism. We must therefore turn back to this basis if we want to take the Laplacean formula seriously, if we want to appreciate adequately its import and its intellectual motives. Long before Laplace, Leibniz had formulated very precisely the conception that was at the basis of his thought. In fact, he had even created the characteristic symbol that incorporated this conception. "That everything is brought forth through an established destiny," so runs an article by Leibniz,

> is just as certain as that three times three is nine. For destiny consists in this, that everything is interconnected as in a chain and will as infallibly happen, before it happens, as it infallibly happened when it happened. . . . Now each cause has its specific effect which would be produced by it, if it were alone; when it is not alone, there arises out of the concurrence a definite infallible effect . . . in accordance with the measure of the forces, and this is true not only when two or ten or a thousand, but even when infinitely many things work together, as indeed actually is the case in the world. Mathematics . . . can elucidate such things very nicely, for everything in nature is, as it were, laid off (*abgezirkelt*) in number, measure, and weight or force. If, for example, one sphere meets another sphere in free space and if their sizes and their paths and directions before collision are known, we can then foretell and

> calculate how they will rebound and what course they will take after the impact. Very simple laws are followed which also apply, no matter how many spheres are taken or whether objects are taken other than spheres. From this one sees then that everything proceeds mathematically—that is, infallibly—in the whole wide world, so that if someone could have a sufficient insight into the inner parts of things, and in addition had remembrance and intelligence enough to consider all the circumstances and to take them into account, he would be a prophet and would see the future in the present as in a mirror.[1]

Again it is clearly evident that the formulation of the causal law and of determinism, as given here by Leibniz, is connected most intimately with his concept of reality. Indeed it is nothing but a paraphrase of precisely this concept. Leibniz' concept of reality rests on two different basic presuppositions, but for him these have completely permeated each other, forming an inseparable unity. The concept is as much mathematical as it is metaphysical, and it is the one because it is the other. Leibniz' determinism is metaphysical mathematicism. The same infallibility that discloses itself in the rules of mathematical thought and inference must obtain in nature, for if nature did not possess this infallibility it would be inaccessible to mathematical thought. In this mode of argument there is expressed the characteristic subjective fervor that inspired the first founders and champions of classical rationalism. Philosophy and science are at one in this fervor. Kepler and Galileo here talk the same language as Descartes and Leibniz. It was that first exuberant enthusiasm, the intoxication, as it were, with the newly founded and established mathematical knowledge, that coined the language. The causal demand of Kepler and Galileo, of Descartes and Leibniz expressed no more than the conviction of the identity of mathematics and nature. Mathematical thought is no mere empty game of the intellect; it does not move in a narrow world of self-made concepts. On the contrary, it touches the very foundations of reality. At this point thought and being are in immediate contact and accordingly even the distinction between finite and infinite understanding vanishes. For the privilege of the infinite, the divine intellect consists in this, that it knows things not because it considers and observes them from the outside but because it itself is the very ground of their being. Thought grasps being because and insofar as it creates being; and precisely this original act of creation is determined by the basic concepts of mathematics, by mag-

1. Leibniz, "Von dem Verhängnisse," *Hauptschriften zur Grundlegung der Philosophie*, ed. E. Cassirer and A. Buchenau (Leipzig, 1924), *2*, 129.

nitude, number, and measure. These concepts, far from being mere copies of reality, are the true originals, the eternal and unchangeable archetypes of being. On this presupposition rests Kepler's doctrine of "cosmic harmony" which was the stimulus and life-giving principle of his whole astronomical thinking, of his empirical and mathematical studies. Mathematics and astronomy are for Kepler the two wings of philosophy. Through them the human mind rises to a level where it first becomes aware of its true likeness to God, because it can think God's thoughts of creation after him: "From eternity, God the creator had mathematical entities as archetypes with himself in a superlatively simple and divine abstraction even from the quantities themselves materially considered." [2] The same thought permeates Galileo's presentation and justification of the Copernican system. In his *Dialogo supra i due massimi sistemi del mondo* Galileo emphasizes that as far as mathematical knowledge is concerned there is no fundamental, no qualitative difference between the human and the divine intellect. With reference to the extent of knowledge the latter infinitely surpasses the former, but the degree of certainty in mathematical knowledge is the same for God and man. For it is inconceivable that the maximum of knowledge to which we attain in strict mathematical proof can be further increased.[3]

In the case of Leibniz, however, there is joined to this viewpoint of the founders of the new mathematical science still another conception which gives his determinism its own characteristic stamp. While Leibniz once remarked that his entire metaphysics is mathematical, the converse of the proposition also holds, namely that his conception of mathematics has a metaphysical basis. For the strictly mathematical interconnection we discern in the phenomena and seek to establish in exact science is for Leibniz only a derived result. In order to understand it fully we must probe to the last grounds of being, to the simple substances. In these we grasp the real meaning and the origin of the determination of natural events. Mathematics and mechanics can be con-

2. "Creator Deus mathematica ut archetypos secum ab aeterno habuit in abstractione simplicissima et divina ab ipsis etiam quantitatibus materialiter consideratis." Kepler's annotation to the second edition of his *Mysterium cosmographicum* (1621), in *Opera,* ed. Frisch, *1,* 136. Kepler is suggesting that the mathematical formulas found in astronomy were always present in the mind of God, not as ordinary abstractions in terms of quantity but on a higher divine level of abstraction (I am indebted to Professor L. A. Post for the translation and interpretation. *Translator*).

3. Galileo, *op. cit.,* National ed., *7,* 129. For further discussion of this passage—which is historically significant, since it played an important role in the prosecution of Galileo—cf. my *Individuum und Kosmos in der Philosophie der Renaissance,* Studien der Bibliothek Warburg, *10* (Leipzig, 1927), pp. 171 f.

tent with tracing the connection between causes and effects in the corporeal world, in the space-time order. But this space-time universe never offers us more than derivative forces, which we must trace back to their ultimate origin in fundamental forces if we really want to understand them. These fundamental forces no longer belong to the world of appearance, but to the world of simple substances, the world of monads. The world that science describes and for which it coins its concepts of extension and motion, of force and mass, is not absolute but relative; it is a phenomenon, but one which is not an empty counterfeit (*Schein*) —rather it is founded in the monads and is hence a *phaenomenon bene fundatum.*

Thus for Leibniz genuine determinism is shifted to another and deeper stratum. Every simple substance is endowed with an original power of representation—that is, it possesses the power to generate out of itself and progressively unfold the whole of its phenomena. Throughout this generating and unfolding process the strictest necessity holds. Substance *is* for Leibniz nothing else than this necessary law of self-unfolding, and the strict concept of causality must be restricted to this alone. Causality does not express any relationship between particular substances, for a direct influence, an *influxus physicus* from monad to monad, is precluded in Leibniz' system. Causality only signifies a connection to be assumed in the simple substance itself between its original force and the products or effects produced by it. And that no gap, no break in the chain of cause and effect, can occur here, is immediately evident from the fact that such a break would destroy at once the identity of substance. What guarantees this identity, in fact what really first defines it, is the strictness of the causal connection. The being of substance is not static and motionless but is inwardly moving and dynamic. Substance *is* only insofar as it changes, insofar as it passes continually from one state to another. Here that which persists resides solely in the law of transition, which governs the whole sequence of these progressions and always remains the same. Hence substance as fundamental force is likened by Leibniz to the "universal member" of certain algebraic series, the member that specifies the rule according to which the series progresses from one element to the next, while the "derivative force" corresponds to the particular members of the series. "That a definite law persists which comprehends in itself all future states of the subject which we regard as identical—it is precisely this that constitutes the identity of substance." [4]

4. Cf. in this connection the correspondence between Leibniz and de Volder, especially Leibniz' letter of Jan. 21, 1704; *Philosophische Schriften,* ed. C. J. Gerhardt (Berlin, 1875–90), *2,* 261 ff.; *Hauptschriften, 2,* 334 ff.

We can now understand why for Leibniz the strict and unbreakable determinism of occurrence is an inescapable consequence and an unconditional demand. If the causal bond were broken at any point in the world of occurrence, reality and substantiality would collapse. An occurrence not determinable by the law of the series and not derivable from it would no longer fall within the series. It could not be attributed to any definite subject and hence would remain suspended, as it were, in the void. But this void would at once signify the death of all knowledge. The "principle of sufficient reason" must be applicable everywhere and in all strictness, for there is no true being, no objective reality, except in the form of *unity*. "Whatever is not truly *one* being is not even truly a *being*." [5] Thus for Leibniz determinism is the final and ineradicable foundation of all metaphysical knowledge. Upon it rests every judgment, not only concerning the causal nexus, but also concerning being and reality in general.

This compact and dogmatic theory of being and occurrence was suddenly invaded by the skepticism of Hume. And again it is the change in the concept of object that precipitates a change in the concept of cause. Hume directs his doubt primarily at the rationalistic formulation of the principle of sufficient reason, but he can shake this thesis only by depriving it of its ontological basis. For Hume there is no longer that "being" upon which Leibniz based his contention of universal determinism. Hume knows no world of "simple substances" that generate the phenomena out of themselves and therefore are the final principles for the explanation of phenomena. For him the real resolves itself into simple perceptions. In these perceptions, if anywhere, we must seek for the justification of the causal principle. But when we now question this presumably competent tribunal, we find at once that no answer is forthcoming and that none is to be hoped for. In vain do we seek for any kind of original for causality; in vain do we look for an "impression" which corresponds to the idea of necessary connection. Hence the alleged idea resolves itself into a phantasmagoria, a mere product of the imagination. We can trace the process by which the imagination brings forth this singular product, but the knowledge provided does not bring us a single step nearer the answer to the basic problem as to the objective significance and validity of the causal concept.

Thus the mechanical view of nature as taught in the physics of Galileo and Kepler and in the philosophy of Descartes and Leibniz loses all

5. Leibniz to Arnauld: "Je tiens pour un axiome cette proposition identique, qui n'est diversifiée que par l'accent, sçavoir que ce qui n'est pas véritablement *un* estre, n'est pas non plus véritablement un *estre*." *Philosophische Schriften, 2,* 97; cf. *Hauptschriften, 2,* 223.

objective footing. It is incomprehensible how mathematics could secure it such a footing, for mathematics only deals with relations of ideas, which have nothing whatever to do with real connections such as cause and effect, with connections between matters of fact. A necessity *of* the real and *in* the real remains therefore incomprehensible to us. When we analyze the basic phenomenon on which the mechanical explanation of nature is built, we find that precisely this phenomenon contains no shadow of a real explanation. What we are told by the only source of certainty—that is, by perception—of the process of elastic impact, for instance, is merely that two masses approach each other, rebound after collision, and proceed in opposite directions. At no time can we discern how "out of" the one the other results. If the two billiard balls at the instant of collision were to exchange their colors instead of altering their direction of motion, the one would be just as much and as little understandable as the other. It is only constant experience that causes the latter to appear more understandable than the former; it is only the force of habit that deludes us with the semblance of insight. We cannot escape this habit, we can never give up the "belief" in causality, for without it there would be no theoretical and not even a practical possibility of orientation. But we overstep the limits which have been set for human knowledge as soon as we attempt to give the belief itself any kind of objective basis. A "foundation" of the causal idea that claims to be anything but a description of its psychological genesis remains therefore in every case a mere illusion. The circle of subjectivity cannot be pierced by us at any point. All allegedly objective inference rests in the end on a mere feeling, for which no further ground can be given. " 'Tis not solely in poetry and music, we must follow our taste and sentiment, but likewise in philosophy. When I am convinced of any principle, 'tis only an idea which strikes more strongly upon me. . . . Objects have no discoverable connexion together; nor is it from any other principle but custom operating upon the imagination, that we can draw any inference from the appearance of one to the existence of another." [6]

At this point Kant's critical revolution occurs. It confirms Hume's analysis in its full extent, even to its rejection of every foundation of the causal principle in rationalistic metaphysics, every derivation of it from "mere concepts." A *particular* causal connection at least can never be derived in this fashion and proved necessary. For such knowledge we are once and for all referred to empirical data that can inform us only of what happens, not of *why* it occurs or must occur. But Kant takes his argument still further, for he applies it also to the causal con-

6. Hume, *A Treatise of Human Nature,* Bk. I, Pt. III, sec. 8.

cept in general. "How anything can be altered, and how it should be possible that upon one state in a given moment an opposite state may follow in the next moment—of this we have not, *a priori,* the least conception. For that we require knowledge of actual forces, which can only be given empirically, as, for instance, of the moving forces, or what amounts to the same thing, of certain successive appearances, as motions, which indicate [the presence of] such forces." [7] Hume therefore maintained rightly that through reason we do not grasp in any way the possibility of causality—that is, the possibility of relating the existence of a thing to the existence of something else necessarily posited by the former.

But if this much is once conceded, what remains of the alleged apriority, of the universality and necessity of the causal concept? How can this concept be universal if it cannot be demonstrated for "things in general"? And how can it be considered necessary if, in order to give it a definite concrete content, we must obtain this content from experience which is always accidental? Can Hume's conclusion be evaded after we have granted all its premises? Must we not concede that the necessity which we ascribe to the causal principle is only imaginary, that it is a mere illusion produced in us by long habit? [8] To all these objections no answer can in fact be found—unless we fundamentally alter the formulation of the question. If we ask how one thing can be the cause of another, we again fall hopelessly victim to Hume's skepticism. For according to Kant we cannot conceive *a priori* so much as the possibility of how a thing can be changed at all, let alone how it can have an effect on another thing. But Kant's initial starting point and his first requirement consist precisely in this, that the critical question is to be addressed not immediately to things but rather to knowledge. It is this turn, this methodological transformation, that he strives to express and to fix in his basic conception of the transcendental. "I entitle *transcendental* all knowledge," he explains, "which is occupied not so much with objects as with the mode of our knowledge of objects insofar as this mode of knowledge is to be possible *a priori.*" [9] The transcendental analysis of the causal concept, which Kant sets over against the psychological analysis of Hume, applies, therefore, not immediately to the existence of things and to their mutual interdependence but only to the form of

7. Kant, *Kritik der reinen Vernunft* (2d ed.), p. 252 (ed. Cassirer, *3,* 186); English trans., N. K. Smith, *Immanuel Kant's Critique of Pure Reason* (London, Macmillan, 1929), p. 230.

8. Cf. *Prolegomena,* sec. 27; English trans., J. P. Mahaffy and J. H. Bernard (London, 1889), p. 69.

9. *Kritik der reinen Vernunft* (2d ed.), p. 25 (ed. Cassirer, *3,* 49); trans., p. 59.

our knowledge of things, to the form of objective knowledge. If we inquire into the "inner nature" of things, if we ask with Leibniz how a "simple substance" produces the succession of its different states, the causal principle thus understood in its transcendental sense leaves us completely at a loss. It refuses to give us any information as to how, within the absolute substances, the particular states are connected with and mutually condition one another. The causal principle does not hold for "things in general," for that type of being which is dealt with in rationalistic metaphysics and ontology, but only for "objects of possible experience." It is nothing but a definite instruction which serves "to spell out phenomena, that we may be able to read them as experience." [10] "I therefore easily comprehend the concept of cause, as a concept necessarily belonging to the mere form of experience . . . but I do not at all comprehend the possibility of a thing generally as a cause, because the concept of cause denotes a condition belonging not at all to things, but to experience." [11] According to Kant, therefore, the causal principle is valid for "nature" only insofar as we define nature, *materially* regarded, as the complex of all the objects of experience, and *formally* regarded as the existence of things insofar as their existence is determined in accordance with universal laws.[12]

But does not this deduction of the causal principle amount to a mere circular proof? Does one understand causality by showing that only through its presupposition can appearances be ordered under laws and thereby comprehended in a system or a synthetic unity? The reproach of circularity would indeed be justified if we were to take the concept of understanding in the same sense as that which was employed in the dogmatic systems of metaphysics. It is, however, precisely this *ideal* of understanding that Kant rejects and strives to replace by another. The concepts of "orderliness according to law" (*Gesetzlichkeit*) and "objectivity" (*Gegenständlichkeit*) in Kant's theory are strictly correlative, but their mutual connection is not circular. For they are not analytically equivalent; one is the presupposition of the other, the synthetic "condition of its possibility." Only insofar as it is possible to fit appearances into an order according to law is it possible to refer them to an object. Judgments of experience never derive their objective validity from the immediate cognition of the object, but solely from the universal validity of empirical judgments.[13] With reference to his deduction of the "analogies of experience" Kant remarks:

10. *Prolegomena*, sec. 30; trans., p. 71.
11. *Ibid.*, sec. 29; trans., p. 70.
12. Cf. *ibid.*, secs. 14–16; trans., pp. 49–51.
13. *Ibid.*, sec. 19; trans., p. 54.

> As to the mode of proof of which we have made use in these transcendental laws of nature . . . an observation has to be made which must likewise be of very great importance as supplying a rule to be followed in every other attempt to prove *a priori* propositions that are intellectual and at the same time synthetic. Had we attempted to prove these analogies dogmatically . . . that is to say . . . from concepts . . . all our labor would have been wasted. For through mere concepts of these things, analyze them as we may, we can never advance from one object and its existence to the existence of another or to its mode of existence. But there is an alternative method, namely, to investigate the possibility of experience as a knowledge wherein all objects—if their representation is to have objective reality for us—must finally be capable of being given to us. . . . For lack of this method, and owing to the erroneous assumption that synthetic propositions, which the empirical employment of the understanding recommends as being its principles, may be proved dogmatically, the attempt has, time and again, been made, though always vainly, to obtain a proof of the principle of sufficient reason.[14]

Here is formulated the central doctrine of that new "critical determinism" which alone Kant aims to advocate and defend. This determinism says nothing as to the ground of things; in fact, it does not even refer directly to empirical things as such. It is rather a principle for the formation of empirical concepts, an assertion and a prescription as to how we should grasp and form our empirical concepts in order that they may discharge their task—the task of the "reification" (*Objectivierung*) of phenomena. If our concepts of causality fulfill this demand, it is futile to seek another justification, an allegedly higher dignity for them. For what we call the "existence" of things—if we interpret this concept in its "critical" sense—is certainly not a being that obtains *prior* to all experience and independent of its conditions. It is nothing but the thought of a possible experience in its absolute completeness.[15] The kind of objective truth that we may ascribe to the causal principle is thus strictly determined, but at the same time strictly limited. Against Hume, Kant maintains that a merely subjective derivation of this principle can never do justice to its meaning. Causality is an assertion about natural events, not about the mere succession of ideas. Reduction to the latter is impossible, if only because, as closer analysis reveals, Hume's seemingly

14. *Kritik der reinen Vernunft* (2d ed.), pp. 263 ff. (ed. Cassirer, *3*, 193 ff.); trans., pp. 237 f.

15. *Ibid.*, p. 523 (ed. Cassirer, *3*, 352); trans., p. 442.

merely psychological thesis contains a "transcendental" thesis without which the former cannot be made intelligible. Hume had to presuppose an objective regularity at least in the play of our cognitive powers if not in the forces of nature. Habit and custom, memory and imagination are, according to him, the true sources of causality. But all these in fact signify nothing else than psychological universal concepts which Hume opposes to the universal concepts of physics. In order to be able to form these universal concepts there must be a constancy in the psychical happenings. Memory and imagination must operate according to rules. They must act not in a haphazard but in a definitely uniform manner if the emergence of the causal idea is to be made understandable. The mechanism of association is in this respect no less a problem and no more easily solved than the mechanism presupposed by science. Here at any rate, according to Hume, strict determinism prevails: imagination and understanding cannot escape the constraint of association and the force of habit. But Kant goes one step further. He argues against Hume that this psychological determination cannot be made the ground of objective determination, since it already presupposes the latter. If there was not in general a constancy in "outer" occurrence, it would be inconceivable how there could come about that constancy in "inner" occurrence to which Hume holds. "If cinnabar were sometimes red, sometimes black, sometimes light, sometimes heavy . . . my empirical imagination would never find opportunity when representing red color to bring to mind heavy cinnabar." [16] It is obvious from this that Hume's skepticism is caught in a circle, that in its demonstration it has had *im*plicit recourse to a circumstance that it must *ex*plicitly deny.

In summarizing the foregoing considerations two consequences important for our subsequent inquiry stand out.[17] On the one hand it is evident that after the decisive advance attained through Hume and Kant in the analysis of the causal problem, it is no longer possible to regard the causal relation as a simple connection between things, or to prove or disprove it in this sense. Such a formulation of the problem has been rendered obsolete by Hume and Kant; at least it cannot claim to bring the scientific sense and use of the causal concept adequately to expression. That it is nonetheless by no means completely discarded either in philosophical or in physical discussion, but recurs again and again, should not surprise us when we reflect that the customary use of the causal con-

16. *Ibid.* (1st ed.), p. 100 (ed. Cassirer, *3*, 613); trans., p. 132.

17. A complete historical analysis of Kant's theory of causality was not intended here; only a few salient features of significance for the modern discussion have been mentioned. For a more complete analysis I refer the reader to earlier expositions: *Das Erkenntnisproblem, 2,* and *Kant's Leben und Lehre,* Berlin, 1918.

cept, the use in daily life, is almost entirely restricted to this meaning. In all our practical dealings with nature, in all our actions, we are always concerned to operate upon nature in the particular sense of effecting definite changes in the world of things, of letting one state of things pass over into another. This standpoint accordingly tends to intrude involuntarily even into our theoretical investigations, into our investigations in critical epistemology. And yet, as deeper analysis shows, it can provide us with only a first and insufficient formula that does not suffice in any way for an exact definition and for a genuine scientific legitimization of the causal concept. The things of common sense, of daily experience, which we so readily and unthinkingly take as causes lose this claim for the most part as soon as strictly scientific knowledge turns to them and considers them *sub specie* of its own basic concepts. At least it then appears that what this experience customarily takes for *one* thing, for an undivided whole subsisting in itself, is in truth very heterogeneous and multiform, an exceedingly intricate composite, not so much of things (*Dingen*) as of conditions (*Bedingungen*). We must follow these conditions one by one and resolve them into their elements, if we want to attain to genuine, scientifically useful causal judgments.

Hume and Kant are quite aware of this circumstance, but they of course also continue to talk the language of daily life. This occurs above all when they discuss the causal connection in concrete examples. In these instances Hume, in line with his general methodological approach, attempts to hold as closely as possible to the sphere of the immediately familiar, of the "given" in sense perception. He sets up the thesis that between cause and effect there must be an immediate, as it were a visible and tangible, connection, that they must be linked through their direct contiguity in space and their immediate sequence in time. Only through such a "contiguity" can the process of association, on which the formation of the causal concept rests, attain validity. "Whatever objects are consider'd as causes or effects, are *contiguous;* and nothing can operate in a time or place, which is ever so little remov'd from those of its existence. We may therefore consider the relation of Contiguity as essential to that of causation." [18] Kant too speaks not infrequently as though it were sufficient, in order to define a causal connection, to observe the different states of one and the same thing in their simple succession and as though from this observation we could ascertain immediately the earlier state as the cause of the later. If, for example, we watch a ship being carried downstream, we cannot arbitrarily alter the order of the perceptions that we have of it at different times. The perception of it downstream always follows the perception of it upstream,

18. *A Treatise of Human Nature,* Bk. I, Pt. III, sec. 2.

and this objective sequence, which differs from mere subjective apprehension, is determinable only if one state is regarded as the cause and the other as the effect. It is apparent, however, that from the standpoint of a scientific clarification of causality this example is an inadmissible simplification. In order to connect the two positions upstream and downstream, it is not sufficient to note merely their temporal sequence. We must ask about the forces with which we are here dealing. If we dismiss from the notion of these forces all connotations of thing or substance, we will obtain ultimately certain universal laws—the law of gravity, the laws of hydrostatics and hydrodynamics, etc.—which we regard as determining the movement of the ship. These laws are the real components of the assumed causal connection, but in order to formulate them exactly, physics has to employ its own symbolic language, which is far removed from the language of "things."

A further consequence to be drawn from the foregoing considerations concerns the problem from which we began, the problem of the universal formula of Laplace. This formula now moves into a new context. It ceases to be merely a particular case and becomes a typical case by which we can study the character and the formation of certain metaphysical concepts. The Laplacean formula does no more than establish the empirical employment of a given concept and then extend this employment into the unconditioned in that it pushes this employment beyond the limits always present in experience and frees it finally from all empirical restrictions. But this represents precisely that form of the "transition to the limit" whose justification is questioned and denied by critical analysis. One of the main sections of the *Critique of Pure Reason,* the transcendental dialectic, is devoted to showing the contradictions and subreptions into which reason is plunged by this procedure. These contradictions are considered in the case of the concept of the universe, as well as of the concepts of the soul and of God. Yet, according to Kant, we are involved—in this transition to the limit—not in a mere error of thought or argument but in an almost inescapable deception. It is this deception that forces us to place alongside the transcendental logic as a logic of truth, another logic—the logic of illusion. The latter must aim to serve "as a critique of understanding and reason in respect of their hyperphysical employment." [19] It is clear at once that this "hyperphysical" employment of a principle which seemingly has a purely physical content, is the real distinguishing mark of the Laplacean formula.

The establishment of this formula follows throughout the process of

19. *Kritik der reinen Vernunft* (2d ed.), p. 88 (ed. Cassirer, *3,* 88); trans., p. 100.

dogmatic concept formation that Kant described in the first pages of the *Critique of Pure Reason.* Reason begins with principles whose employment in the course of experience is unavoidable and at the same time sufficiently sound to be acceptable. It then raises these principles ever higher, to apply to ever more remote conditions, and finally arrives at propositions which overstep all possible empirical employment and yet appear so innocent that common human reason accepts them here also. In so doing, however, it plunges into darkness and contradiction, for it is now driven to consequences that submit to no "touchstone of experience." "The battle ground of these endless disputes is called metaphysics." If we want to avoid these insoluble problems and these hopeless controversies, we have no other recourse than to take the question back to its original domain and to confine it strictly within those bounds. The causal problem must be grasped as a problem of "discursive," not of "intuitive," understanding, of a finite and not of an infinite intellect. If this finite intellect is limited, this limitation implies by no means a merely negative, but rather a positive, characterization. It indicates not merely an accidental and external barrier but the necessary condition of its effectiveness and fertility. The barrier may not be regarded as a mere impediment that we have to surmount and leave behind us as rapidly as possible. It rather delimits the domain in which alone our thinking and knowing find fulfillment, in which alone they win concrete significance.

The restriction of the understanding to the conditions and limits of possible experience is therefore at the same time its sole realization. From this standpoint, from the standpoint of "critical determinism," the fault of the Laplacean formula can now be definitely indicated. This formula removes the restriction of causal knowledge but surrenders at the same time its realization, its only possible empirical actualization.[20] To be sure, a concept such as that of the Laplacean spirit is thus *logically* unassailable, insofar as it contains in itself no immediate contradiction. But it remains nevertheless an empty concept, a concept to which no empirical object corresponds and which cannot be used even as an ideal, as a methodological maxim or guide-line for our knowledge. As a pure thought-object, as a noumenon, it may be allowed; but in that case it remains—to use the language of the *Critique of Pure Reason*—a noumenon in negative, not in positive understanding. We can conceive of an intelligence which, instead of being confined to the laborious form of our causal inferences and thus led in succession from one phenomenon

20. Concerning "restriction" and "realization" cf. esp. *Kritik der reinen Vernunft* (2d ed.), p. 185 (ed. Cassirer, *3,* 146 f.); trans., p. 186.

to another, comprehended with one glance the totality of phenomena, the entirety of all possible states of the universe. Such a conception, however, is of no use to us. Through it we ascertain no particular "intelligible object." On the contrary the sort of "understanding to which [such an object] might belong is itself a problem. For we cannot in the least represent to ourselves the possibility of an understanding which should know its object, not discursively through categories, but intuitively in a non-sensible intuition." [21]

The Leibnizian conception of destiny as a thoroughgoing, at once mathematical and metaphysical, determination of world events has thus lost its significance and its possibility of application. It is now seen to be but the inadmissible hypostasis of an ideal of pure reason. Such hypostases are alluring, for it appears as though with their aid we could reach to the end, the completion, of knowledge. But precisely this seeming completion turns out to be a delusion. A formula such as that of Laplace appears to converge the scattered rays of our empirical causal knowledge onto one focal point, but this point is found on closer examination to be merely a *focus imaginarius.*[22] The simple path which metaphysical determinism thought it could follow, by a simple transgression of its limits from the world of the conditioned to the world of the unconditioned, rests accordingly on a false self-gratification of reason. "The expedient of removing all those conditions which the understanding indispensably requires in order to regard something as necessary, simply through the introduction of the word *unconditioned,* is very far from sufficing to show whether I am still thinking anything in the conception or perhaps rather nothing at all." [23]

With this I will break off these historical considerations. They were not undertaken for their own sake, but with the purpose of providing a guide for the following systematic investigation. This investigation does not deal with the causal problem as such but with the causal problem of physics. For the latter only critical, not metaphysical, determinism can provide a point of departure. The question of whether outside the realm of physics metaphysical determinism possesses an independent significance will not initially be raised here. In what follows the causal problem is to be restricted exclusively to its epistemological significance; it is to be understood as a question of the methodology of physics. For the purposes of this inquiry we do not need to go beyond that version of the causal principle which is presented in the critical studies of Hume and Kant. On the other hand, however, we cannot rest content with this

21. *Ibid.*, p. 311 (ed. Cassirer, *3,* 222); trans., p. 272.
22. Cf. *ibid.*, p. 672 (ed. Cassirer, *3,* 441); trans., p. 533.
23. *Ibid.*, p. 621 (ed. Cassirer, *3,* 410); trans., p. 501.

abstract and general formulation but must trace the causal concept everywhere to its definite applications and to the particular forms which it thus takes on. In these particular forms its characteristic use and limitation finally come into clear evidence—a use and limitation prescribed not arbitrarily but by the problem of physical knowledge, by the ideal of empirically objective understanding.

PART TWO

The Causality Principle in Classical Physics

CHAPTER 3

The Basic Types of Statements in Physics: Statements of the Results of Measurements

IF, in accordance with the results of the Humean analysis, we do not regard the causal concept as an empirical concept in the sense that it belongs immediately to the given and presents itself as the direct copy of a simple impression or perception, and if on the other hand we cannot grasp it as an *a priori* concept in the sense that it contains an unconditionally certain and necessary statement concerning all objects in general, there remains only the possibility of taking it as a condition which attaches not to things but only to experience. The principle of causality expresses then nothing concerning the metaphysical essence of things, nothing concerning the substantial inside of nature; it presents solely the answer to the question of how, from that which occurs, one may arrive at a determinate experiential concept.[1] But with such a general "instruction" our inquiry is by no means at an end. The task now presents itself of observing how the instruction works out *in concreto,* how in the structure and progress of physical investigation it wins an independent existence. What path does physics take and what methodological guidelines does it follow in order not merely to describe vaguely what occurs when the concepts of our daily experience and ordinary speech are used, but to derive from what occurs a truly determinate, an exact empirical, concept? And what role does the causal principle play in this procedure?

It appears as if this question permitted of a simple solution: we should only have to analyze the actual procedure of physics, which lies open to us in its history, to interrogate the "fact" of physical science, in order to obtain information concerning methodological principles underlying it. But here we encounter a new difficulty. For when we turn to the succession of great scientists who have contributed to building up classical physics and who through their continued cooperative effort have

1. Cf. above, pp. 17 ff.

erected this structure, we find how little agreement there is concerning our problem. None of them was content simply to consummate his accomplishments as an experimenter and theoretician of physics, but each sought also to understand and to justify these achievements epistemologically. When engaged in this task, however, they appear not infrequently to forsake and forget completely the common ground on which they stand. They separate off in different "directions" and "schools" which hardly appear to be held together by the bond of a common task. Galileo and Kepler, Huyghens and Newton, Robert Mayer and Helmholtz, Kirchhoff and Mach—all have reflected on the specific character of physical explanation and they have attempted to discover and define the principle underlying it. But these definitions have no common denominator. They differ from one another in important and essential features. On the other hand, in its steady advance toward knowledge physics has not been impeded or diverted from its goal by these differences and divergences. Its practice displays a peculiar inner constancy, an immanent methodological coherence, which finally always asserts itself, even when judgments concerning this practice are widely divergent. We must therefore start with this practice and examine it directly. If, instead of following the theories *about* physics, we follow rather the process of the formation of physical concepts, we may perhaps expect and hope to discover certain basic principles which are invariant with respect to the different epistemological systems of reference.

Before we enter upon this task, however, we must take note of another and more general state of affairs. No small part of the difficulty with which current discussion of the causal problem in physics has to contend rests, I am convinced, on the fact that epistemology in general has not yet distinguished sharply and clearly between the *different types of physical statements*. There exists a gap here which is felt again and again and which obscures insight into the articulation and methodological structure of the system of physics. The modern development of mathematics and mathematical logic has taught us how necessary such considerations pertaining to the "theory of types" are in the different realms of knowledge. It is well known that the "fundamental crisis" into which mathematics fell had its origin in the formation of the concepts of the theory of "sets" and that one of its basic deficiencies lay in the fact that in the formation of classes certain conditions were overlooked which, as it later turned out, are indispensable for a correct definition of a set. In his theory of "types" Russell sought to remove the deficiencies to which this error gave rise and which led to the paradoxes of set theory. Russell insisted that elements may not be arbitrarily combined into classes but that the validity of such a combination must be decided

in accordance with definite criteria. It was shown above all that it is not permissible to treat elements and classes as objects of like order, that they are heterogeneous with respect to each other and "belong to different spheres (*sphärenfremd*). Only by requiring that the members of a class may not belong to different types and by the complete construction of a definite "hierarchy of types," by distinguishing between individuals on the one hand, and classes of the first level, the second level, etc. on the other, were the antinomies of the theory of sets successfully avoided.[2]

I shall not pursue the development of this theory. I use it here only to point out that physical statements confront us with a similar problem. For they, too, as becomes evident when they are considered and analyzed epistemologically, by no means all belong to the same level of knowledge. Here also sharp distinctions exist, in fact a kind of heterogeneity that is often overlooked or little heeded simply because we are accustomed to subsume all physical judgments without further distinction under the wider concept of experience. The validity of this subsumption is not questioned, for the *reference* of all physical statements to experience is in fact a common and essential element in them all. But precisely this reference is not to be obtained through a simple summation of particular data. It demands on the contrary the interaction of factors belonging to different types and orders of knowledge, which accordingly must be carefully distinguished.

As the first form of physical judgments there occur those statements which may be designated as "statements of the results of measurements" (*Mass-Aussagen*). They represent the first step in that decisive transition which leads over from the world of the given to the world of scientific knowledge, from the world of sense to the world of physics. The transformation of immediate perceptual data into those determinations into which the concepts of measure and number enter is the condition for every physical judgment. It is constitutive for the "sense" in which this judgment must be accepted and in accordance with which it is to be understood and interpreted. It is evident that this first definition of "sense" already contains in itself an abundance of logical and methodological problems. That empiricism which believed that it could explain physical statements simply as immediate reflections of sense data, as

2. For further concerning Russell's theory of types cf. his *Principia mathematica* (Cambridge, 1910), *1*, 39 ff., 168 ff.; and *Introduction to Mathematical Philosophy* (2d ed. London, 1920), pp. 135 ff. For the notion of *Sphärenfremdheit* and *Sphärenvermengung* cf. R. Carnap, *Der logische Aufbau der Welt* (Berlin, 1928), secs. 29, 30.

copies of certain impressions, does not need further refutation today. The development of physics in the last decades has shown us more sharply than ever before how little this simple theory of reproduction suffices. A long distance separates the first statements of the results of measurements in physics from what is given us in a simple observation and from the language used to describe this observation. In order to estimate this distance and to overcome the antithesis, thought processes of a highly complex nature are required, the establishment of whose form constitutes a distinct and independent epistemological task in each individual case. In the following inquiry we may by-pass this task, for its solution belongs to the problem of the formation of physical *concepts,*[3] whereas we will be dealing essentially with the question of the formation of physical *judgments.* In order to arrive at a verdict here we do not need to go behind the physical statements of the results of measurements, for these constitute in a sense the basic stuff out of which the physicist constructs his world, the simple elements of that reality which he seeks to delineate and to determine in his judgments.

The transition from immediate sense apprehension to experimental observation and its systematic exploitation has resulted, if considered purely extensively, in an extraordinary enrichment of our knowledge. This transition has freed us from accidental limitations of our physical organization and from bondage to our particular sense organs. One might attempt to regard this extension as marking the essential advance and the real significance of physical investigation. In his *Principles of Psychology* Herbert Spencer sought to establish the thesis that all our physical instruments are nothing but "artificial extensions of the senses," just as all our levers and screws, our wedges and lathes are artificial extensions of our limbs. "The magnifying glass adds but another lens to the lenses existing in the eye. The crowbar is but one more lever attached to the series of levers forming the arm and hand. And the relationship which is so obvious in these first steps, holds throughout." [4] The most important achievement of physical measuring instruments would therefore be the fact that they enable us to remove the restrictions which are imposed on us by our organic constitution, by the nature and structure of our sensory tools. As a psychophysical process, perception is subject to basic psychophysical laws, in particular the Fechner-Weber law

3. For further concerning this problem see my earlier exposition in *Substance and Function,* chap. 4. Of more recent literature on the subject reference should be made esp. to R. Carnap, *Physikalische Begriffsbildung* (Karlsruhe, 1926) and to Hermann Weyl, "Philosophie der Mathematik und Naturwissenschaft," *Handbuch der Philosophie,* ed. A. Bäumler and M. Schröter (Munich and Berlin, 1927), *2,* 106 ff.; English trans., O. Helmer, Princeton, 1949.

4. *Principles of Psychology* (London, 1870), *1,* sec. 164.

concerning the absolute threshold for sensing stimuli and the threshold determining the least noticeable change in stimulus. The advance to physical observation seems to emancipate us from this condition. The "sensibility" of our senses is supplanted by the sensibility of our physical apparatus, and the latter appears capable of almost unlimited amplification. At a distance of 10 cm. the human eye can just distinguish two lines $\frac{1}{40}$ mm. apart, whereas with the microscope, separations as small as $\frac{1}{7,000}$ mm. are still discernible. The human ear can discern a time difference of $\frac{1}{500}$ sec. between two electric sparks, whereas by means of a rotating mirror optical measurements of time are possible down to 10^{-8} sec.[5] But this sharpening and augmenting of our sense perceptions, this broadening of our whole horizon of reality, is nevertheless only one and by no means the sole determining motive of physical experimentation and theory. No less important and essential is another impulse which appears at first sight to be directly opposed to the first. For over against the broadening of our world picture, which occurs in physics, there comes about also a highly significant concentration. The new material crowding in from all sides leads by no means to a loosening of the form but rather to a form that is much sharper and more rigid. To the growing extensive range of knowledge there corresponds an ever stronger intensive penetration and mastery. Insofar as these two aspects not merely preserve an equilibrium, but to the extent that the latter proves to dominate the former—insofar, that is, as the tendency to concentration, as organizing principle, penetrates and forms the manifold of particular observations—to that extent does the basic form of physical knowledge obtain. Every step toward diversification is at the same time a demand for a new unification, and points the way to such unification. Here, however, the world of physics differs from the world of the senses, in a direction directly opposite to that which was first considered. The world of the senses appears to fade more and more as the physical theory progresses; it appears to lose all its original fullness and color. In comparison with this fullness and color the physical world, the world of the physical concepts of measure and law, seems like a "world of shadows."[6] Its determinateness and its efficacy, however, rest precisely on this purely symbolic character. Whereas sensuous description generally goes no further than simply to place the different "attributes" of an object next to each other, physical description is never content with such a loose conglomeration. It presses on from "aggregate" to "sys-

5. Cf. O. Wiener, *Die Erweiterung der Sinne*, Leipzig, 1900; also E. Mach, *Erkenntnis und Irrtum* (Leipzig, 1905), pp. 144 f.

6. An expression used by A. S. Eddington. See the introduction to his book *The Nature of the Physical World* (New York, 1928), p. xiv.

tem." It strives to reduce the abundance of particular phenomena and particular properties to a small number of basic determinations, from which it attempts to derive them.

This reduction, this peculiar, almost violent condensation, is a typical feature of the formation of all physical concepts and judgments. In the system of universal mechanics as it was presented first in grandiose outline by Newton and as it was subsequently elevated to the logical model and pattern of all explanations of nature, this process even then appeared to be on the verge of final completion. The description assumed the simplest conceivable form. By assuming the definitions and axioms as set forth by Newton in his *Principia,* one came into possession of the fundamental framework for every construction of scientific knowledge. For the forces present in nature are here measured by accelerations which they give to individual mass points, and these accelerations are functions of the distances between the masses, and of their relative velocities. Thus through the specification of a few magnitudes the present state of a physical system, as also its past and future, its being and destiny, were determined.

When physics turned from the description of mechanical phenomena to those of electromagnetism, a new problem arose. A new physical reality now appeared, the reality of the "field," whose states could not be reduced simply to the motions of mass particles. As early as in the basic law which Coulomb formulated (1785) for electrostatic attraction and repulsion and which was formally set up precisely after the pattern of the Newtonian law of gravity, there appeared a new characteristic quantity. The formula of Coulomb, $K = f\frac{e_1 e_2}{r^2}$, enables us to calculate the force (K) with which two electrical charges attract or repel each other. The more this new magnitude became the center of discussion and the more carefully it was analyzed, the more clearly physics discerned the way along which it was finally led from the older substance theory of matter to the modern field theory. In the meantime a dualistic notion first prevailed: "matter" and "field" were in a relation of mutual interaction, but each of them followed its own laws. Later an attempt was made to overcome this duality. Matter was no longer regarded as an independent existence, but instead the attempt was made to derive all its characteristics from field laws and thus to understand matter as an "offspring of the field." [7] Here again physical description

7. Concerning this transition from the substance theory of matter to the field theory see Hermann Weyl's exposition, which is equally illuminating historically and systematically, *Was ist Materie? Zwei Aufsätze zur Naturphilosophie,* Berlin, 1924.

and theory showed that marvelous gift of concentration. For in order to establish the state of the field and to determine its propagation, the knowledge of a few simple magnitudes is sufficient, the magnitudes of the electrical and magnetic field strengths. If now to these magnitudes those of the gravitational field are added, the prospect arises of comprehending with a relatively small number of data, with the knowledge of a few characteristic magnitudes, the totality of natural occurrences and of determining them in their smallest detail.

In this sense we may designate the physical statements of the results of measurements as "statements of the first level" and we thereby imply that all other physical judgments have reference to them and have to be verified through them. In order to avoid a possible misunderstanding, however, we must add at once an important restriction. The relation between these "first" statements and the statements of "higher order" must not be misunderstood in the way the purely spatial image suggests. On the contrary, all the statements of physics are determined through one another, they mutually condition and support one another, and their specific "truth" is due precisely to this mutual interconnection. This reciprocal interweaving and bonding constitutes one of the basic features of the system of physics. Within it therefore there is no proper substantial carrier, nothing that *per se est et per se concipitur.* There is only a functional coordination in which all the elements, all the determining factors of physical truth, uniformly participate. At the "lower" levels accordingly the "higher" levels are already implied and in a definite sense are presupposed. If we choose a spatial analogy for the structure of physics, we must not liken this structure to a pyramid resting on a broad base of immediately given and independent "facts," rising gradually from this and ending in a highest point, perhaps in a simple "cosmic formula." For this would involve overlooking the mutual interconnection and forgetting that "everything significantly factual is already theory." There would always be the possibility of imagining the higher layers removed without destroying the bottom layer or even altering it essentially. But we will attempt to show in detail later why such an assumption is untenable and impracticable for every part of physical knowledge. Physics accordingly is to be compared not to a pyramid but insofar as we regard any spatial symbol as adequate and permissible, to the "well-rounded sphere" with which Parmenides described his "universe": εὐκύκλου σφαίρης ἐναλίγκιον ὄγκῳ μέσοθεν ἰσοπαλὲς πάντηι. This cannot and must not prevent us, to be sure, from ascribing to it a determinate structure, a higher and a lower order of elements; but this logical distinction of the elements must not be interpreted as suggesting the possibility of their actual separation, their independent existence.

With this methodological reservation the statements of the results of measurements may indeed be designated as the alpha and omega of physics, its beginning and end. From them all its judgments take their departure and to them they must all lead back again. The system of physics obviously is not to be understood as a mere aggregate or simple sum of these statements. As a system it is rooted in specific elements of form, whose discernment and clarification constitute an independent task of logical analysis. These elementary forms, however, find their genuine fulfillment only in the concrete determinations provided by the statements of the results of measurements. Only through the mediation of the latter can the concepts and judgments of physics refer to an object and thus attain to objective significance and validity. What physics calls an "object" is nothing ultimately but an aggregate of characteristic numbers. No "entity" and no "property" can be defined except by stating such characteristic numerical values. Insofar as we determine in this way the pressure, volume, and temperature of a gas, the potential or kinetic energy of a system, the electric or magnetic field strength, we have in these determinations precisely what physics understands by its various objects; we do not need to posit objects as sundered beings-in-themselves behind these determinations. What characterizes the statements of the first level and distinguishes them sharply from all others is the feature of individuality which they possess. They all pertain to a definite here and now; they all refer to a definite point of space and time to which, so to speak, they are bound. In field theory the description of the universe consists, to use a term of Hilbert's, in "here-thus relations." The "here" is represented by the space-time coordinates, the "thus" by the values determining the state of the system.[8] At once, however, we observe that we must go a step further, for what is merely individual can indeed make up the content of a definite item of knowledge, but it can by no means exhaust the sense of what we understand by "physical knowledge." In no realm, let alone in the realm of physics, is "knowledge" but an aggregate of individual data, of statements concerning a here and now. Knowledge is concerned with the way these data mutually condition each other and how they are interconnected. To attain such knowledge, new and specific means are required, to the consideration of which we now turn.

8. Cf. Weyl, *Was ist Materie?*, pp. 42 f.

CHAPTER 4

Statements of Laws

IN THE PRECEDING CHAPTER we have tried to point out that an essential feature and an essential logical gain in the passage from the world of sense to the world of physics consists in the fact that an increasing compression of knowledge accompanies its growing extension. Only by means of this compensation is it possible for knowledge to preserve in its almost limitless expansion a compact structure and determinateness. The same characteristic dual process manifests itself in all further developments occurring within physical thought. Every expansion of knowledge due to the steady increase of observational data and the refinements of measuring instruments meets with a corresponding characteristic simplification. The richer the empirical material, the more easily and readily does it allow itself to be fitted into a few great basic forms. Multiplicity here is not disparateness; it is the demand for a new unification and it is what first makes this unification possible. As the profusion of properties presented to us by the perceptual world contracts to definite values of measurement and number which fall under certain classes and thereby permit of comprehensive review, so when these values themselves are considered, the possibility of further condensation becomes apparent. For they do not stand by themselves; they are not elements of reality isolated and actually cut off from one another.

Even in the natural philosophy of antiquity the demand arises not to think of the basic elements out of which the cosmos is built up as separated from one another and as it were "chopped off with an ax." Rather it is maintained that in physical reality, "all is in all" (ἐν παντὶ πάντα), that there is no strictly separated particular existence, but that everything has a share in everything else (οὐδὲ χωρὶς ἔστιν εἶναι ἀλλὰ πάντα παντὸς μοῖραν μετέχει). Anaxagoras who made this notion the basic principle of natural explanation, thought of being in strictly qualitative and substantial terms.[1] His assertion of this universal sharing therefore had to be grasped

1. Anaxagoras, Frs. 6, 8, ed. H. Diels, *Die Fragmente der Vorsokratiker, griechisch und deutsch* (Berlin, 1951–52), 46B; English trans., K. Freeman, *Ancilla to the Pre-Socratic Philosophers,* Cambridge, Harvard University Press, 1948.

in the conception that in every apparently separate existence all the elements of being are really contained and are present in a sense as pieces. Every so-called particular thing is a *panspermie,* it contains in itself all the seeds out of which the universe as a whole is made up, so that flesh is not only flesh, but at the same time blood and bone, and blood and bone are simultaneously also flesh.[2] This interfusion of all the elements of being had to give way to another conception when science liberated itself from the naive schema of things and substance. The postulate of "all is in all" could no longer be satisfied by an intermixing of either things or original qualities, but only by way of an intermixing of laws. The statements of laws now provide the single and only allowable way of joining the particular to the whole and of uniting the whole with the particular, and thus of establishing that "harmony" between them which is the real aim of all scientific knowledge.

Since the Renaissance, since Kepler and Galileo, Descartes and Leibniz, scientific and philosophic thought has devised in the mathematical concept of function the ideal means of satisfying this demand. In it a universal form is given, into which new contents may continually flow without ever bursting it, indeed, without so much as altering its essential features. With Galileo it was the times and distances of fall, with Kepler distances and velocities, with Huyghens lengths and periods of the pendulum, with Boyle, Mariotte, and Gay-Lussac the volume, pressure, and temperature of an ideal gas, that were comprehended through this form and determined in their mutual relations. Even in the transition to wholly new groups of phenomena and despite the complete change in the theoretical presuppositions of scientific description, this universal and fundamental *modus* of physical statements persisted unaltered. The partial differential equations with which Maxwell described the states of an electromagnetic field and their interdependence are not essentially different in their structure from the Lagrangean equations of motion which provide the basic framework of classical mechanics.

Once thrown into this form, the phenomena of nature have been made fast in "enduring thoughts," in thoughts whose duration often extends far beyond the occasion which first instigated the setting up of the particular forms. Thus Fourier, for instance, in his *Théorie analytique de la chaleur* (1822) advanced a theory of heat conduction which was built completely on the idea that heat may be regarded as a liquid which flows back and forth between different bodies.[3] The mathematical de-

2. See further my presentation of Greek philosophy in *Lehrbuch der Philosophie,* ed. Max Dessoir (Berlin, 1925), *1,* 59 ff.

3. Concerning the content and the historical development of Fourier's theory of heat see E. Mach, *Die Principien der Wärmelehre* (Leipzig, 1896), pp. 82 ff.; cf.

scription which he gave of the facts of heat transference on the basis of this idea, however, proved to be completely independent of his particular hypothetical presuppositions. His mathematical account viewed the phenomena as results of purely geometrical relations and was not restricted accordingly to any special assumption about the nature of heat. Fourier's basic equations could thus be preserved in the transition to the modern kinetic theory. Only in certain extreme cases, as with extremely rarified gases, did they need to be modified or supplanted by other determining factors.

In the face of this situation W. Nernst once remarked about Fourier's formulas that they possessed in a certain sense "eternal values," which could not have come to the consciousness of their creator. This observation of Nernst appeared in an article in 1922,[4] and a few years later Fourier's formulas experienced a further resurrection when Heisenberg made them the point of departure for the new quantum mechanics and thus applied them to a domain very far removed from that in which they first proved themselves.

This independent significance of formulas, their indwelling "sagacity" (*Spürkraft*), is one of the most remarkable and fascinating problems in the epistemology of science. But behind all this there stands the really decisive question: how does the transition between the various types of physical statements take place and how is it to be understood logically? How are those further judgments derived from the statements of the results of measurements which assert an exact relationship between different groups of measurements and which in general we designate as "statements of laws"?

The traditional answer usually given to this question is simple, but precisely the simplicity makes it useless. The transition from statements of the results of measurements to those of laws is said to come about and to be justified through "inductive inference" so-called. But when we inquire further about the essence of this inductive inference, it turns out that we do not receive the same answer from any two logicians. The problem of induction has become more and more the chief stumbling block for the whole of epistemology and for the philosophy of science. Around the middle of the nineteenth century, to be sure, there was a time when it appeared as though the problem could be solved by a kind of inversion. Though induction could not be reduced to a basic form of logical thought, all logical thought could be reduced to induction. But

---

also F. Auerbach, *Entwicklungsgeschichte der modernen Physik* (Berlin, 1923), pp. 92 ff.

4. "Zum Gültigkeitsbereich der Naturgesetze," *Naturwissenschaften, 10* (1922), p. 489.

the logic of John Stuart Mill, who advocated this thesis, failed to achieve its purpose despite all efforts. Precisely the most determined and consistent followers of empiricism found themselves compelled to leave the path taken by Mill, for it became clear that it never came into contact with actual scientific thought and in particular with the thought of exact physics. Mill's famous five canons of induction (Method of Agreement, Method of Difference, Joint Method of Agreement and Difference, Method of Residues, and Concomitant Variations) are all set up and worked out with considerable ingenuity, but as canons of induction they have remained at bottom only methodological fictions. Actual inductive thinking and inference moves in quite different channels from those mentioned.

Besides, Mill's inductive logic entangled itself in that circle which Hume had already discovered and sharply criticized. Mill assumed as the basis of induction the proposition that the course of nature is uniform, and he regarded this proposition as the fundamental principle or general axiom of induction. On the other hand, however, he declared that just this generalization which leads us to the conviction of the uniformity of nature is itself a kind of induction—and indeed an induction not of the clearest and most transparent kind.[5] Later theories of inductive inference by which we get from particular to general, from facts to laws, sought to avoid the circle, but found themselves compelled to give up a genuine vindication of the objective validity of the inference. The question as to the *quid juris* was silently dropped and was suppressed and supplanted by psychological considerations. "Inductive logic," therefore, is itself far more a riddle than a consistent and definite solution, and we cannot take recourse to it, if we want to understand the transition from physical statements of the first order to those of the second order, from statements concerning measurements to statements of laws.

At the beginning of modern science Galileo had raised a decisive objection to the thesis of Mill that a physical law is nothing more than an "aggregate of particular truths." If this were so, argued Galileo, universal judgments concerning reality would be either impossible or useless. They would be impossible if the series of particular instances under observation were infinite, for such a series cannot be exhausted by enumeration (*per enumerationem simplicem*). They would be superfluous if the series were finite, for in that case we could satisfy ourselves by making our observations on each member separately.[6] In effect, therefore, Mill's conten-

5. J. S. Mill, *A System of Logic, Ratiocinative and Inductive* (7th ed. London, 1868), Bk. III, chap. 3, pp. 342 f.

6. Galileo, *Opere*, ed. Alberi, *12*, 513.

tion that "all inference is from particulars to particulars" is proclaimed, at least in the realm of exact physics, as inadmissible. The mere "Here-thus" that is contained in the particular statements of the results of measurements undergoes a characteristic transformation in the statements of laws: it is converted into an "If-then." And this "If-then," this hypothetical judgment, *If x, then y,* does not merely combine particular quantities which we consider as belonging to and localized in definite points in space-time, but pertains rather to whole classes of magnitudes, classes which consist generally of infinitely many elements. Since, according to Russell's theory of types, classes are of a nature completely different from that of their elements and may not be thought of as mere aggregates of elements,[7] it is impossible to sustain the notion of Mill that a statement of law is nothing but "a comprehensive expression, by which an indefinite number of individual facts are affirmed or denied at once." For the question always remains open as to what right one has to say anything at all about "an indefinite number of individual cases," if each one has not been previously tested and examined.

It is evident, however, that the physical *experiment* which provides the proper and solely legitimate basis for all statements of laws does not entail such a comprehensive examination but instead actually precludes it. The claim which the experiment inherently makes is precisely that it may attach to the established datum which in and for itself refers and is limited to a specific here and now, an inference free of this restriction and transferable to other points in time and space. An experiment which sought to describe only the processes going on in a given laboratory and only at the instant of reading certain instruments would obviously have no methodological value. It would merely designate a singular case which could not be inserted into the chain of continuing physical observations and inferences. This insertion requires the presupposition that what is established in a particular experiment may be transferred from place to place, from instant to instant, that we may in a certain sense displace it at will without altering anything in the nature or the truth of the established datum. The questionable and precarious inference from some cases to many and from many to all does not occur here at any point. For in connection with what the experiment tells us we do not merely infer from a here to a not-here, from a now to a not-now, but instead we consciously go beyond the viewpoint of the mere here and now. This represents not an extension within the space-time realm but in a certain sense an abrogation of the whole realm. Advance is made in

7. Cf. on this point, for example, Carnap, *Der logische Aufbau der Welt,* sec. 37.

a new dimension, and it is this change of dimension that distinguishes the statements of laws from the mere statements of the results of measurements.

Of all the great scientists it was Maxwell who most clearly discerned this transition to another level, this peculiar μετάβασις εἰς ἄλλο γένος, contained in every strict formulation of a law of nature, and who gave the most pregnant expression to the epistemological problem that lurks there. Maxwell saw in this transition the essential import and content of the general principle of causality. He formulates this principle as follows: "The difference between one event and another does not depend on the mere difference of the times or the places at which they occur, but only on differences in the nature, configuration or motion of the bodies concerned." [8] This formulation contains the presupposition that it is possible to extract such a "nature" of bodies in general out of the continuous "stream of becoming," out of the Heraclitean flux of occurrence—that we can characterize "matter" by properties and relations invariant with regard to changes of particular positions in space and time. The numerical values corresponding to these positions must not therefore enter explicitly into the functional equations by means of which we combine the different classes of measured quantities. If, for example, the relation which holds between the pressure, volume, and temperature of an ideal gas depended on the absolute values of space-time coordinates and were to change with these, the fact that it would be expressible at particular places by the formula of Boyle and Mariotte or that of Gay-Lussac, would obviously be insignificant. It would only be a statement of the results of measurements, and no longer a statement of a law.

In an article on mechanics, Painlevé gave a definition agreeing in principle with Maxwell of what we are to understand by a law of nature and in general by physical causality. "Lorsque les mêmes conditions sont réalisées, a deux instants différents, en deux lieux différents de l'espace, les mêmes phénomènes se reproduisent transportés seulement dans l'espace et le temps." If, for example, on any day in Paris or Versailles one gram of zinc foil is put into one liter of sulfuric acid, then assuming that the conditions of temperature and pressure are the same, the gram of zinc will take the same time to dissolve and will yield the same salt.[9] Against the validity of this formula the objection has at times been raised that it is too broadly framed. A nature that did not obey this formula, so it is said, would not for that reason necessarily cease to be describable in terms of any laws whatsoever. It is conceivable, argues

8. *Matter and Motion* (New York, 1878), p. 31.

9. Paul Painlevé, "Mécanique," in H. Bouasse *et al.*, *De la Méthode dans les sciences* (Paris, Alcan, 1909), pp. 371 f.

Schlick, that regular measurements of electronic charges might yield values which rose and fell 5 per cent, at times in seven hours and again in seven hours and then in ten hours, without our being able to discover the slightest cause for this behavior. We should have to reconcile ourselves finally to such a fluctuation; we could only accept it simply as an ascertained fact.[10] If, however, as Schlick himself emphasizes, no investigator ever made the attempt to find laws corresponding to such a hypothesis, this must presumably have a deeper reason, a reason which is somehow connected with the total form of our experience and which therefore goes back if not to a metaphysical at least to a methodological characteristic of experience. And such a connection can in fact be demonstrated. For in order to apply notions of law of the type indicated by Schlick, in order to be able to refer them to empirical happening, we should in the first place have to possess a knowledge of what phase of the world-process we happened to be in. We should have to possess accordingly not only relative but absolute space and time determinations. Through the structure of the universe here assumed we would thus come into conflict with a certain structure of knowledge, we should be led to an assumption which violated certain universal conditions governing the empirical establishment of coordinates of space and time. If one follows Schlick's objection through to the end, it follows, as far as I can see, not that Maxwell's criterion is dispensable or invalid, but only that by itself it is not sufficient. It must be supplemented with another criterion, a criterion already supplied by Leibniz in his *principe de l'observabilité*.[11] J. C. Maxwell, moreover, appears to have possessed a clear awareness of this methodological connection, for although throughout *Matter and Motion* he holds in general to the orthodox Newtonian concepts of absolute space and time, he nevertheless appends to the above-mentioned explanation of causality a noteworthy observation strongly reminiscent of relativity. For he remarks that "any one . . . who will try to imagine the state of a mind, conscious of knowing the absolute position of a point will ever after be content with our relative knowledge." [12]

10. Cf. M. Schlick, "Die Kausalität in der gegenwärtigen Physik," *Naturwissenschaften, 19* (1931), 148.

11. Concerning this "principle of observability" see below, pp. 125 ff.

12. *Matter and Motion*, p. 30. In *Substance and Function* (pp. 249 f.) I referred to the significance of Maxwell's "principle" for the general theory of induction and causality. I do not find, however, that in recent discussions of the subject its significance has been fully recognized and appreciated. The only exception here appears to be Keynes, who in his treatment of the problem of induction in his *Treatise on Probability* again takes up the basic notions of Maxwell. He sees in the "principle of the uniformity of nature" nothing more than the expression of the methodological conviction that the mere difference of position in space and time can

The transition from the individual statements of the results of measurements to the general statements of laws constitutes an integrating factor in the process of physical knowledge, but it is by no means its culmination. For the statements of laws also possess an intrinsic organization, an organization into higher and lower order, which must be studied and established. This task too is not to be performed by a mere expansion of the basis of knowledge, but demands in its turn a change of viewpoint. It demands a new intensive penetration of the material of knowledge as given in the laws. Here that unremitting labor sets in which Hilbert characterized as the "lowering of the foundations" (*Tieferlegung der Fundamente*) in which he recognizes the real task of every science. The synthesis now does not refer any more to particular facts or to definite classes of facts, but comprehends entire domains of physical knowledge, domains which we are accustomed to indicate by such expressions as is said to be purely relative, and a principle is demanded which will in- mechanics, optics, and electrodynamics. The separation of these domains deed confirm this separation but which will also explain it in terms of an over-all unity. The propositions which establish this over-all unity belong once more to a new type of physical knowledge, which we now have to consider.

---

contain no causal element. "Mere position in time and space cannot possibly affect, as a determining cause, any other characters." "A generalisation which is true of one instance must be true of another which *only* differs from the former by reason of its position in time or space." Cf. J. M. Keynes, *A Treatise on Probability* (London, Macmillan, 1921), pp. 255 f.

CHAPTER 5

Statements of Principles

PHYSICAL research is never satisfied with gathering together observed facts into laws and putting these laws into a definite order *alongside* each other. Physics asks how one law comes forth out of another; it seeks a rule by which thought may be guided from one law to the next. This guidance represents a new fundamental form of induction which is not to be put on the same level or confused with that inductive process which leads "from particulars to particulars." It was such an induction that enabled Maxwell in his theory to undertake the consolidation of light and electricity which was later confirmed experimentally by Heinrich Hertz. By such an induction Schrödinger, setting out from optical facts and relying on Fermat's principle of "least time," could build up a new form of wave mechanics. The clue to this kind of audacious transition is not to be found so much in the facts of the particular domains as in the equations in which the whole structure of the domain becomes manifest. From the form of these equations and from the constants appearing in them, Maxwell was able to discern immediately the identity of "essence" of light and electricity.

We can trace back the operation of the same logical principle to the very first beginnings of exact physical inquiry. The first condensation of facts into laws had hardly occurred when there arose the other, no less strong impulse to combine laws into principles and to comprehend them as special cases of these principles. In mechanics the history of the "virtual principle" provides an example of this development. In the hands of Archimedes, Galileo, and Stevin statics had attained to the formulation of certain laws of equilibrium for particular cases, such as the lever, the pulley, and the inclined plane. But only with d'Alembert and Lagrange was a principle set up, the principle of "virtual displacement," which not only gathered all these different static phenomena under one viewpoint but also with one stroke bridged the gap between statics and dynamics, subjecting static and dynamic phenomena to one and the same rule. A real proof of this rule cannot be achieved merely by putting to-

gether particular instances. "We are thus forced to regard the principle, consisting of equations which leave particular cases quite undetermined, as a rule whose consequences as a matter of fact are in accordance with experience." [1]

This basic characteristic appears no less clearly in the development of that principle which was destined to complete the transition from mechanics to general physics. If one follows the historical development of the principle of conservation of energy as presented, for example, by Mach in his *Die Geschichte und die Wurzel des Satzes von der Erhaltung der Arbeit* (1872), a factor becomes noticeable which even Mach willingly recognized despite his radical "empirical" tendency, and which he emphasized with his characteristic objectivity and lack of prejudice. At the time the energy principle came to the fore, its experimental proof was by no means established. This fact stands out with peculiar sharpness and distinctness in Robert Mayer's derivation and reasoning. Whereas with Helmholtz the theoretical and with Joule the empirical and experimental interest was dominant, Robert Mayer from the beginning took another path that was characteristic of him. He became convinced of the truth of his new principle not through extensive experimental investigations but on the basis of a single observation, an observation, moreover, which belonged not to the realm of physics but to that of physiology. As ship's doctor he had to perform bleedings in Java, where his attention was attracted to the much stronger redness of venous blood. Mayer attributed this phenomenon, in agreement with Lavoisier's theory that animal heat springs from a process of combustion, to the fact that in southern climates, where the heat loss of a body through radiation to its environment is less, the process of combustion that produces the animal heat must also be less intense. Mayer felt himself "inspired as it were" by this observation; it revealed to him at one stroke a completely new world.[2] This has been regarded not infrequently as a flaw in his way of thinking. It has been contended that Mayer neither proved his principle theoretically nor derived it inductively, but arrived at it only intuitively, hence "not by scientific methods." [3] Such an estimate, however, rests upon a wholly inadmissible limitation of what is scientifically allowed. Mach's judgment here demonstrates considerably more justice and freedom. He remarks that "hardly ever has a scientist taken a more important and comprehensive view," and he attributes it to the powerful "instinct for form" which

1. A. Voss, "Die Prinzipien der rationellen Mechanik," *Encyklopädie der mathematischen Wissenschaften, 4,* Pt. 1, 57.

2. Cf. Robert Mayer's letter to Griesinger, June 16, 1844; *Schriften und Briefe,* ed. J. J. Weyrauch (Stuttgart, 1893), p. 213.

3. L. Königsberger, *Helmholtz* (Braunschweig, 1902), *1,* 87.

guided Mayer.[4] But how a mere instinct could produce an insight pertinent to and fundamental for physics remains an unsolved problem.

The truth is that Robert Mayer, in his way of thinking and investigation, by no means stands apart from the circle of the great scientists. And it would be utterly abortive to attempt to explain this way of thinking solely in psychological terms and to regard it so to speak as an outflow of a personal idiosyncracy. Whenever the transition to "principles," to a new type of physical knowledge, is involved, we find this way of thinking operative and successful. "He who cannot realize," remarked Goethe in his critique of the Baconian method of induction, "that a single instance is often worth a thousand and embraces them all . . . will never be able to promote anything that will give joy and be of use to himself or to others." He contrasts Galileo's way of thinking to the "Verulamic method of dispersion" (*Zerstreuungsmethode*) and, referring to Galileo's development of the theory of the pendulum from the swinging church lamps in the Dome at Pisa, he praises him for having proved in early youth that "to the genius one case may stand for a thousand." "Everything in science depends on what we call an *aperçu,* on becoming aware of that which really lies at the basis of phenomena. Such a discovery is infinitely fruitful." [5] It was on such an *aperçu* that Robert Mayer built. He himself once said that he found the new theory because he strongly recognized the need for it.[6] This need sprang from a universal methodological demand, for a thoroughgoing unity of the physical view of the universe.

In elucidating the typical form of physical principles and the epistemological problem latent in it I will limit myself to one example, that of the *principle of least action.* In its application to individual cases this principle was known in the ancient world. Hero of Alexandria used it in order to derive the laws of reflection of light rays. He derived them from the assumption that the path of light must be the shortest possible. The problem appears in a broader and profounder form when we come to Fermat, who, from the assumption that light in passing from one point to another always takes the shortest possible time, was able in the most simple way to derive the law of refraction. The mechanical significance of this action principle was first recognized by Leibniz, who discussed it both in an essay on dynamics in 1689 [7] and in his scientific correspond-

4. Mach, *Principien der Wärmelehre,* pp. 248, 252.

5. *Zur Farbenlehre* (historischer Teil); Weimar ed., Pt. II, *3,* 236, 246 f.

6. Mayer's letter to Griesinger, Dec. 5 and 6, 1842; ed. Weyrauch, p. 194.

7. The essay, which remained a fragment and was not published by Leibniz himself, is now to be found in *Leibniz' Mathematische Schriften,* ed. K. I. Gerhardt (Berlin and Halle, 1849–63), Pt. II, *2,* 281 ff.

ence.[8] Leibniz sets up the thesis that in all mechanical processes the quantity of action—which is to be measured either by the product of the mass, the distance traversed, and the velocity of the displaced body, or alternatively by the product of motive force and time—will be a maximum or minimum. The principle of "least action" became generally known only when Maupertuis in 1756 attempted to make it the basis of a new proof of God, a proof which he regarded as the only exact and convincing one. But quite apart from this purely metaphysical application of the principle, its formulation betrayed such a lack of clarity and precision that it was useless for any definite application in mathematical physics.[9] Only through a series of papers by Euler did it acquire a more precise form and an exact physical significance. It was Lagrange, however, who first presented a complete and accurate statement of the principle.[10] Since then, in the various forms in which it was employed by Gauss, Hamilton, Jacobi, and Dirichlet, it has dominated all the subsequent development of mechanics. In the form of the "Hamiltonian principle" it has in fact become its fundamental theorem.

In this development the question of a metaphysical basis for the principle of least action was more and more lost from view. Lagrange declared in his "Mécanique analytique" that the applications which Maupertuis made of his principle are so specific that they were incapable of demonstrating the truth of a general principle; also that there was something vague and arbitrary about them which could only cast suspicion on the rigorous validity of the principle. He then proposed his own precise formulation, though he went on to add that he had no intention of putting forward a metaphysical principle but only of giving expression to a simple and general result of the laws of mechanics.[11] But even though exact science now consciously held itself aloof from all hasty metaphysical

8. See particularly the letter that König published in 1751, a publication that subsequently led to the well-known conflict with Maupertuis and the Berlin Academy. Concerning the genuineness of the letter compare Helmholtz' statements in his "Zur Geschichte des Prinzips der kleinsten Aktion," *Wissenschaftliche Abhandlungen* (Leipzig, 1887), *3,* 249 ff.; see also Louis Couturat, *La Logique de Leibniz* (Paris, 1901), note 16, pp. 577 ff. Apparently the letter was not, as König thought, directed to the Basle mathematician Hermann but belongs to the correspondence with Varignon. For further concerning this see Gerhardt in *Sitzungsberichte der Akademie der Wissenschaften zu Berlin,* June 23, 1898; and my edition of Leibniz' *Hauptschriften, 2,* 24.

9. Maupertuis' formulation was first presented in 1744 to the Paris Academy and discussed later in the Proceedings of the Berlin Academy and in his "Essai de cosmologie," *Oeuvres* (Lyon, 1756), *1,* 3 ff. See further my *Erkenntnisproblem* (2d ed.), *2,* 424 ff.

10. Concerning Lagrange's derivation see Helmholtz (above, n. 8), *3,* 257 ff.

11. "Mécanique analytique," *Oeuvres,* ed. Serret (Paris, 1888), *11,* 261 f.

conclusions, it may be noted that the methodological and epistemological universality of this principle became ever more clear and convincing. The principle entered an entirely new phase of its development when, in his *Studien zur Statik monozyklischer Systeme* and shortly afterward in his article "Über die physikalische Bedeutung des Prinzips der kleinsten Wirkung" (1886), Helmholtz freed the principle from its restriction to the mechanics of ponderable bodies and proclaimed it a physical postulate of completely universal validity. At this time Helmholtz considered the whole process of nature as comprehended under the energy postulate and the action principle. "All that occurs," he says in his Academy speech on the history of the principle of least action, "is described by the ebb and flow of the eternally undiminishable and unaugmentable energy supply of the world, and the laws of this ebb and flow are completely comprehended in the principle of least action.[12] This principle is thus placed alongside the law of the conservation of energy as a complementary principle; indeed, it proves to be superior to the latter, for as Helmholtz shows, the energy postulate can be derived from the action principle but the latter cannot be derived from the former. Helmholtz concludes from this that the range of validity of the principle has grown far beyond the limits of mechanics, and it is to be regarded as highly probable that it embraces electrodynamics and thermodynamics as well. "In any case," he summarizes his observations, "the universal validity of the principle appears to me to be so far assured that it may claim considerable importance as a working hypothesis and as a guideline in the effort to formulate laws for new classes of phenomena." [13]

The high heuristic expectations which Helmholtz attached to the principle were fulfilled amazingly quickly. In 1915 in a comprehensive historical and systematic survey of the action principle Planck declared that one would be guilty of one-sidedness to contend that it is still to be regarded as specifically mechanical.[14] He views it rather as the highest physical law which contains the four world coordinates in a completely symmetrical arrangement and thus proves to be the "crown of the whole system of physics." [15]

12. The lecture was not printed during Helmholtz' lifetime and did not appear even among his *Wissenschaftliche Abhandlungen.* It was published later by Adolf Harnack in his *Geschichte der Königlich preussischen Akademie der Wissenschaften zu Berlin* (Berlin, 1900), *2,* 287.

13. "Die physikalische Bedeutung des Prinzips der kleinsten Wirkung," *Wissenschaftliche Abhandlungen, 3* (1895), 210.

14. M. K. E. L. Planck, *Kultur der Gegenwart* (1915); now in *Physikalische Rundblicke* (Leipzig, 1922), pp. 117 f.

15. "Die Stellung der neueren Physik zur mechanischen Naturanschauung" (1910), in *Physikalische Rundblicke,* p. 58.

The latest developments of physics, both in general relativity theory and in quantum mechanics, have confirmed this view and added new and surprising proofs. The general theory of relativity set out from the Hamiltonian principle and derived from it the properties which determine the "metric." In this way it arrived at its "generalized law of inertia" which permits it to combine in one expression the phenomena of inertia and gravitation. For every movement of a point under the influence of inertia and gravity the principle now holds that its world-line is always a "geodesic" in the space-time continuum.[16] As regards quantum mechanics, Schrödinger began directly with Fermat's principle and from here effected the transition from Fermat's "geometrical ray optics" to a new "physical wave optics," in which matter itself is explained as a wave process. He bridged the gap between Fermat's optical principle and the general principle which Hamilton had set up for mechanics. The latter was thus generalized so that it could deal with motions of particles and wave processes in closely related fashion and as subject to the same laws. In Schrödinger's development two equations which formerly seemed to refer to wholly different classes of phenomena were fused into one single equation. A complete coincidence between the dynamic laws governing the motion of a mass point and the optical laws governing the motion of a light ray was established.[17] Thus matter has "become" waves, for the nature of a physical object is determined by nothing other than the laws it obeys.

I will not follow the action principle any further in its physical applications. It was intended from the beginning to serve us only as an illustrative example, as a paradigm in which the character of statements of principles, statements of the third order, could be exhibited and studied. At first glance the universality of the action principle seems by no means beyond question. This universality could be attained only at the cost of a circumstance which, from the purely physical point of view, led again and again to difficulties and doubts. For the more universally the principle was conceived, the more difficult it became to specify clearly its proper concrete content. It became finally a kind of Proteus, displaying a new aspect on each new level of scientific knowledge. If we ask what precisely that "something" might be to which the property of a maximum or a minimum is ascribed, we receive no definite and unambiguous answer.

16. Cf. A. Einstein, *Die Grundlage der allgemeinen Relativitätstheorie* (Leipzig, 1916), sec. 15.

17. Cf. Schrödinger, *Vier Vorlesungen über Wellenmechanik* (Berlin, 1928), first lecture; also Louis de Broglie, *Einführung in die Wellenmechanik* (Leipzig, 1929), chap. 3.

The "subject" of the various action principles changes from epoch to epoch. It may have been this iridescent indeterminateness that led so many scientists to doubt the value of the principle altogether. As late as 1837 Poisson called it "only a useless rule." [18] And in fact, a rule that failed to specify a particular object for which it should hold would seem to have little significance. It was found almost impossible to specify what that object was. From Fermat to Leibniz, from Maupertuis to Euler, to Lagrange, Hamilton, Gauss and Helmholtz, in every instance it was sought in a different quarter. For Hero of Alexandria it was the path of light, for Fermat it was the time taken by light to which the property of a minimum was ascribed. For Maupertuis the product of the velocity and the length of the path appeared as the subject for the "economy of nature," whereas Euler in the essay of 1751,[19] in which he sought to improve on Maupertuis' attempt, set out from the assumption that for every motion the temporal mean value of the potential energy will approach a minimum to the extent that the conditions governing the motion permit. In this formulation the general condition, which is decisive in comparing the real with the modified paths, was not yet discovered: the condition, namely, that all these motions must be such that the total energy remains unchanged. By adding this condition Lagrange was able to present an exact formulation of the principle.[20] For him the virtual motions had to satisfy the law of conservation of energy, but no restriction was placed on the times required for the path. In Lagrange's *Mécanique analytique* it is the sum of the products obtained by multiplying the space element traversed by each particle by its mass and the integral of the velocity that is to be a maximum or a minimum. Gauss' principle of least constraint stipulates that in constrained motion the sum of the products of the masses and the squares of the deviations from the path of free motion is a minimum at every instant. In Hamilton's formula, instead of the sum of the kinetic and potential energies, it is their difference, the excess of kinetic over potential energy, that provides the determining quantity. Starting from here Helmholtz designates the magnitude, whose time integral represents Hamilton's action, as the "kinetic potential" and then derives the proposition that the negative mean value of the kinetic potential calculated for equal time intervals in the path of a moving system is a minimum (or for greater distances a limiting value) in comparison with all other

18. Cf. Planck, *Physikalische Rundblicke,* p. 114.

19. "Harmonie entre les principes généraux de repos et de mouvement de M. de Maupertuis," *Akademie der Wissenschaften zu Berlin,* 1751.

20. See also Helmholtz' speech on the history of the principle of least action, in Harnack (above, n. 12), p. 293.

neighboring paths which lead in the same time from the initial to the final position.[21]

Thus there results an extraordinary multiplicity, even a disparateness, in the points of attack. This multiplicity, however, is by no means solely a defect, for it proved to be extremely fruitful for a deeper understanding of the import and methodological character of the principle itself. In every passage to a new field, particularly in the advance beyond the mere mechanics of points, the *definition* of a certain quantity was always demanded which, through its limiting or its critical values, correctly determines the course of a certain group of phenomena. In this respect the problem is analogous to that encountered in applying the energy principle, where in a similar way the equivalents for a definite amount of mechanical work are not immediately given but require that in each domain a certain constant is first found which makes possible the conversion into mechanical energy in accordance with the conservation principle.[22] Here in fact we find a basic methodological characteristic common to all genuine statements of principles. Principles do not stand on the same level as laws, for the latter are statements concerning specific concrete phenomena. Principles are not themselves laws, but rules for seeking and finding laws. This heuristic point of view applies to all principles. They set out from the presupposition of certain common determinations valid for all natural phenomena and ask whether in the specialized disciplines one finds something corresponding to these determinations, and how this "something" is to be defined in particular cases.

The power and value of physical principles consists in this capacity for "synopsis," for a comprehensive view of whole domains of reality. When Helmholtz' essay "Über die physikalische Bedeutung des Prinzips der kleinsten Wirkung" appeared, Heinrich Hertz declared that the task which Helmholtz had set himself in this essay was a discovery. "For in fact it required a discovery to recognize that from such general persuppositions, conclusions so particular, correct, and pertinent can be drawn." [23] Principles are invariably such bold anticipations that justify themselves in what they accomplish by way of construction and inner organization of our total knowledge. They refer not directly to phenomena but to the form of the laws according to which we order these phenomena.

21. *Wissenschaftliche Abhandlungen, 3,* 203 ff.

22. Adolf Kneser in particular has emphasized this point and its epistemological significance. See his article, *Das Prinzip der kleinsten Wirkung von Leibniz bis zur Gegenwart,* Wissenschaftliche Grundfragen, *9,* ed. R. Hönigswald (Leipzig, 1928), esp. pp. 29 f.

23. Heinrich Hertz, *Die Prinzipien der Mechanik,* in *Gesammelte Werke* (Leipzig, 1892–95), *3,* 20; English trans., D. E. Jones and J. T. Walley, *The Principles of Mechanics,* New York, 1899.

A genuine principle, therefore, is not equivalent to a natural law. It is rather the birthplace of natural laws, a matrix as it were, out of which new natural laws may be born again and again. We can now see why a principle is not to be understood as a mere collection of laws. The relation between principles and laws is the same as that between laws and results of measurements. As laws ("statements of the second level") do not arise simply through the summation of individual statements of the results of measurements, but possess as classes an independent significance, a logical being *sui generis,* so also principles as "classes of classes" have a characteristic nature of their own. With them we enter once again a new "dimension."

Strict empiricism, insofar as it rests upon a *sensuous* basis, has continually sought to deny or to remove this peculiar multidimensionality. It believed that it could manage with a single level, the level belonging to immediate sense perception, and it persistently called physical thought back to it. Bacon found it absurd that men should be bidden to climb high towers in order to peer at objects instead of going directly to the things and observing them from their immediate vicinity.[24] In the same way Mill declared that knowledge of individual instances is the only sure knowledge and that it cannot be increased by any logical form in which it may be couched. "I am unable to see," he added sarcastically, "why we should be forbidden to take the shortest cut . . . and constrained to travel the 'high priori road' by the arbitrary fiat of logicians. I can not perceive why it should be impossible to journey from one place to another unless we 'march up a hill, and then march down again.' " [25]

It is possible to give a clear and definite answer to this question. Perhaps in general it would be more advisable to choose Mill's course "from particulars to particulars" and in this way to avoid the laborious and dangerous high road of knowledge. But this would only be possible if the goal to be attained were already established and we knew in advance the place we wished to reach. This knowledge, however, is precisely what we lack. We can only strike out in a definite direction in the hope that it will bring us nearer a certain presumed goal. But we cannot ascertain in advance the exact position of this goal and for that reason the highroad is necessary. It is indispensable for our orientation in the country we have to traverse.

The principles of physics are basically nothing but such means of orientation, means for surveying and gaining perspective. At first they are only valid as hypotheses. They cannot stipulate dogmatically from the

24. "Redargutio philosophiarum," in *Works,* ed. James Spedding and Robert L. Ellis (London, 1857–74), *3,* 581.

25. *A System of Logic,* Bk. II, chap. 3, sec. 3; ed. 1868, pp. 207 ff.

beginning a particular result of investigation. But they teach us how to find the direction in which we have to advance. We must raise ourselves again and again above the level of immediate perceptions, and also above the level of experimental data and of particular laws, in order to gain once more a sure footing *within* all these levels. The outward and upward movement that is here demanded therefore serves the purposes not of any kind of transcendence but only those of a pure immanence—the inner construction and securing of experience.

If we now survey once more the levels of physical statements which we have distinguished and attempt to characterize them purely formally and methodologically, we may say that statements of results of measurements are individual, statements of laws general, and statements of principles universal. It might here be objected that in all three classes the statements are indubitably empirical and hence are not capable of any strict generality and necessity. But this objection rests on an ambiguity in the concept of the empirical itself. To be sure, we cannot claim "absoluteness" or ultimacy for any empirical judgment however high in the order of empirical knowledge it may stand, but must always leave open the possibility that an advance in knowledge may lead us to supplement or correct that judgment. But this by no means implies that within empirical knowledge itself all logical differences are to be leveled and obliterated. If such an obliteration were possible, the very *process* of experience, on which all possibility of correction depends, could not be carried out. For this process cannot go on except in a definite logical rhythm, in the "three-step" of facts, laws, and principles. This rhythm expresses an organization within empirical knowledge, an organization which is an integral part of the concept of empirical knowledge and is indispensable for it.

The fault of dogmatic empiricism does not consist in the fact that it aims to hold fast all knowledge to experience and recognizes experience as the sole criterion of truth, but in the fact that it does not push its analysis of experience far enough and hence stops short with an unclarified concept of it. The reason for this is not infrequently found in a vague assumption of continuity, in consequence of which empiricism refuses to distinguish rigorously the different levels of knowledge so that it can instead develop them one from the other. But this development is deceptive if one tries to understand it as a mere reproduction of statements of the same kind. Somewhere in the course of knowledge we must acknowledge genuine mutations, which lead to something new and independent. Our thinking, also, moves in discrete paths, and for each of these paths a quite particular form of thinking is prescribed. Should we leave one path and pass over to another, should we ascend from statements of the results of measurements to those of laws, or from laws to principles, we may do so

only by means of a "jump" which requires the exertion of a certain amount of mental energy. This jump cannot be avoided, and we should not let ourselves be misled into the attempt to explain it away because of a preconceived epistemology. Occam's "razor" is certainly a valuable, and for the whole of epistemology an indispensable instrument; but if we apply it at the wrong place, we are in danger of severing the natural ligaments and tendons that hold together the organism of knowledge.

A *historical* criterion for the specific character of principles may also be found in the fact that in the development of philosophical and scientific knowledge the attempt is made repeatedly to ascribe to them the highest form of "universality"—that is, to identify them in some way with the general causal principle or to derive them immediately from it. In these attempts it becomes evident again and again why such a derivation cannot succeed—but the tendency to attempt it nevertheless continues unabated. The higher we ascend in the hierarchy of scientific propositions, the harder it becomes to distinguish between these propositions and the summit of the hierarchy, the principle of causality. In the beginnings of modern science, in the foundations of dynamics, it was the principle of inertia that was given the dominant position, and at once attempts were made to fuse it in some fashion with the causal principle. It was declared that a body left to itself *must* persist in its state of rest or of uniform rectilinear motion, because every deviation from this motion would constitute a "groundless" and hence impossible event. Even Leibniz, who referred constantly to the sharp distinction between *vérités de fait* and *vérités de raison,* inclined toward this type of argument. At various places in his writings he falls back on the law of inertia in order to give an example of what is to be understood by a miracle. If God were so to arrange it—he explains in his correspondence with Clarke, for example—that a free body described in the ether a circular path about a fixed center without anything else working on it, then this would be possible only through a miracle. For anything is miraculous that cannot be explained from the nature of bodies, and their nature demands that a free body move off on a tangent from the path of the curve.[26] An utterance of this kind shows how deeply rooted was the postulate of inertia in the philosophical and scientific consciousness of Leibniz' day—a fact that is the more remark-

26. "Briefwechsel mit Clarke," sec. 3; *Schreiben,* sec. 17; *Hauptschriften, 1,* 139; cf. sec. 4; *Schreiben,* sec. 45; *Hauptschriften, 1,* 152; *2,* 218, 281; and the letter to Hartsoeker of Feb. 6, 1711, in *Philosophische Schriften, 3,* 518. Later attempts to regard the law of inertia as an immediate corollary of the general causal principle and to derive it from the latter have frequently been made; see e. g. A. Schopenhauer, *On the Fourfold Root of the Principle of Sufficient Reason,* English trans., Mme. Karl Hillebrand (rev. ed. London, 1897), and E. F. Apelt, *Die Theorie der Induction* (Leipzig, 1854), pp. 60 f., 93 ff.

able when one recalls how paradoxical it had appeared but a few decades before. When Galileo first enunciated it, it seemed to his peripatetic opponents an unheard of and audacious innovation. Even Galileo himself appears to have come to the conviction of its truth and to its general formulation only slowly and by stages.[27]

As regards the action principle, the earlier customary teleological deductions have in general been abandoned by now, especially after emphatic warnings against such false "side glances" on the part of critical thinkers like Heinrich Hertz.[28] Even in the strictest positivist circles, however, the significance of the various maximum and minimum principles has frequently been said to be merely that they establish the postulate of the uniqueness of events (*Eindeutigkeit des Geschehens*) and thus in essence only express a demand indispensable to all causal comprehension. Petzoldt contends, for example, that no theological or teleological speculations can gain the least support either from Euler's conception of the action principle or from the conceptions of Hamilton and Gauss; for all these are only analytical expressions of the fact of experience that natural processes are univocally determined. "Throughout all the propositions in question . . . we discern only the fact of the univocality of occurrences. . . . This fact alone makes science possible; if this fact was not present there could be no talk of natural laws. . . . The principle of least action and the postulates related to it can also be grasped, therefore, within the sphere of their validity, as analytical expressions of the principle of sufficient reason." [29] Mach too shares this view. The principle of least action, he claims, and hence all the minimum principles of mechanics as well assert only that in the cases in question exactly so much occurs as *can* occur under the given circumstances—that is, as determined and uniquely determined by these circumstances.[30] It is evident, however, that in this statement Mach has entangled himself in a circular argument. For if we follow him, the word *can* is not, of course, to be understood rationalistically or metaphysically. The only thing left to us then is to interpret it

27. See further Emil Wohlwill, "Die Entdeckung des Beharrungsgesetzes," *Zeitschrift für Völkerpsychologie, 14, 15* (1883–84).

28. With reference to Gauss' naming of the principle the "principle of least constraint (*Zwang*)" Hertz remarked (*Prinzipien der Mechanik,* p. 38): "There is certainly a peculiar charm in side glances of this kind and Gauss himself emphasized this in justifiable joy at his important . . . discovery. We must confess nevertheless that this charm is merely a play with the mysterious. In all seriousness we do not actually believe in our capacity half silently to solve the riddle of the universe by intimations of this sort."

29. Petzoldt, *Maxima, Minima und Oekonomie* (Altenburg, 1891), pp. 12 ff.

30. *Die Mechanik in ihrer Entwickelung historisch-kritisch dargestellt* (Leipzig, 1904), chap. 3.

purely empirically and hence to reduce it precisely to those propositions whose foundation is here in question. Thus the difference between statements of principles, no matter how general, and the causal principle itself cannot be wiped out. Even the energy principle, which has so well justified itself in its universality, is and remains a particular law of nature whose negation would by no means be tantamount to an abrogation of the causal principle as such. We can arrive at the latter, therefore, even from the level of the universal principles, only by means of a jump, by means of a μετάβασις εἰς ἄλλο γένος that is analogous to those which led us from statements of the results of measurements to statements of laws, and from statements of laws to those of principles.

CHAPTER 6

The General Principle of Causality

THE TRANSITION to the general principle of causality places before us a problem far more serious and difficult than those which faced us during the analogous transitions from facts to laws and from laws to principles. It seems in a sense like a jump into the void. For what do we achieve when we leave behind us all particularity, not only of physical facts but also of physical laws and principles? Do we not land in this way in utter emptiness? Can we say anything whatever about physical reality if such a statement does not have the form of a definite experimental finding or of a definite law which expresses the interdependence between certain classes of measurements? After this has been done and after laws in their turn have been gathered together and unified in principles, is there anything left about which we can ask the question "why?"?

It must indeed be confessed that the causal principle is not a natural law in the usual sense of the words. In this respect Mach is correct in affirming that there is no cause and no effect in nature; for nature is present only once, and those same events to which we refer when we say that under the same circumstances the same consequences occur do not exist in nature but only in our schematic reproduction of it.[1] But what remains of causality if we can understand it neither directly as a statement concerning nature nor as a metaphysical statement concerning the world of absolute things?

The causality principle can be understood only as a "transcendental" statement, referring, not to objects, but rather to our cognition of objects in general (cf. above, p. 17). Instead of being a direct statement about things, it must be viewed as a statement about our empirical knowledge of things—that is, about experience. The fear that because it occupies this intermediate position it might lose something of its objective significance

1. "Die ökonomische Natur der physikalischen Forschung," *Populärwissenschaftliche Vorlesungen* (2d ed. 1892), p. 221; English trans., T. J. McCormack, *Popular Scientific Lectures,* Chicago, 1895.

is allayed by a critical analysis of the concept of object. For this analysis shows that *all* objectivity exhibits the same relational characteristic, that in fact it is based on this characteristic. Experience manifests itself as the main factor defining objectivity; hence the conditions of objectivity prove to be *eo ipso* conditions of the possibility of objects of experience, they provide the conditions for existence of these objects.

Critical determinism, to say the least, knows no other way of establishing a general basic principle of scientific knowledge. It cannot accept such basic principles on trust and faith; it cannot appeal to a mere compulsion of thought, to a necessity grounded in our mental organization. This kind of appeal, which not infrequently has been regarded as the essence of the Kantian *a priori* and the Kantian solution of the causal problem, was quite foreign to Kant. In fact, he energetically rejected it. If the categories are explained, he declared with emphasis, as

> subjective dispositions . . . implanted in us from the first moment of our existence, and so ordered by our Creator that their employment is in complete harmony with the laws of nature in accordance with which experience proceeds . . . the *necessity* of the categories . . . would then have to be sacrificed. The concept of cause, for instance . . . would be false if it rested only on an arbitrary subjective necessity, implanted in us. . . . I would not then be able to say that the effect is connected with the cause in the object—that is to say, necessarily—but only that I am so constituted that I cannot think this representation otherwise than as thus connected. This is exactly what the skeptic most desires.[2]

The causal law, therefore, must not be looked upon as an *idea innata* in which we may trust simply on the basis of its evidence. On the contrary, its validity must be demonstrated for nature, for the things of experience, but at the same time restricted to nature. In the Kantian sense this restriction does not imply that its significance is diminished, for we cannot carry the concept of objectivity beyond the realm of experience. "Our knowledge of the existence of things reaches, then, only so far as perception and its advance according to empirical laws can extend. If we do not start with experience or do not proceed in accordance with laws of the empirical connection of appearances, our guessing or inquiring into the existence of anything will be only an idle pretence." [3]

We have already seen, however, that Kant did not follow to the end the road which he took in his solution of the Humean problem. In the deduction of the causal principle which he gave in the "Analogies of Ex-

2. *Kritik der reinen Vernunft* (2d ed.), p. 167 (Cassirer ed., *3*, 136); trans., p. 174.
3. *Ibid.*, p. 273 (*3*, 199); trans., p. 243.

perience," he directed the question once more to empirical things and phenomena, instead of directing them exclusively to empirical cognition, to the form of experience (cf. above, p. 21). Putting the problem exclusively in this latter sense we may then ask: What is the significance of the causal principle and what new insight does it add to what we have already learned from the foregoing epistemological analysis? What further step is still to be taken and what new insight concerning scientific knowledge is yet to be expected after we have traversed the earlier levels, after we have progressed from statements of the results of measurements to those of laws, and from these to statements of principles?

I would like to give an answer to this question, which at first sight will perhaps seem paradoxical. There is in fact nothing left over, nothing new in principle to be added to the description of the process of cognition and of the epistemological structure of science. What the causal principle signifies—and this is the thesis I want to explain and establish in the sequel—is not a new insight concerning content, but solely one concerning method. It does not add a single factor homogeneous with the foregoing, which could be placed alongside it as a material supplement. With regard to content it does not go beyond what has already been observed; it only confirms it and confers upon it as it were the epistemological *imprimatur*. In this sense it belongs, using the language of Kant, to the modal principles; it is a postulate of empirical thought. And this postulate specifies fundamentally nothing more than that the process, which we have sought to describe in detail, is possible without limitation. It does not maintain that the process of translating data of observation into exact statements of the results of measurements, or the process of gathering together the results of measurements into functional equations by means of general principles, is ever complete. What it demands and what it axiomatically presupposes, is only this: that the completion can and must be sought, that the phenomena of nature are not such as to elude or to withstand in principle the possibility of being ordered by the process we have described. We should understand the causal principle in this sense, and in this sense we will submit it to critical examination in the following pages. For us the causal principle belongs to a new type of physical statement, insofar as it is a statement about measurements, laws, and principles. It says that all these can be so related and combined with one another that from this combination there results a system of physical knowledge and not a mere aggregate of isolated observations.[4]

4. I concur with R. Hönigswald in the view that causality "is not so much a 'law' as rather a 'viewpoint' in order that we may speak of laws"; see his paper "Kausalität und Physik," *Sitzungsberichte der Akadamie der Wissenschaften zu Berlin*, Physikalisch-Mathematische Klasse, *17* (1933), 568 ff. It is evident from this

Before we go on with our investigation, however, we must anticipate a possible objection. Is not this explanation of causality—someone might ask—but an arbitrary nominal definition? Does it exhaust what actual physical knowledge understands by the causal law? In answering this question I will restrict myself for the moment to the realm of classical physics. The new problems that have been raised by quantum theory and modern atomic physics will be discussed later. Even in classical physics, to be sure, there is hardly discernible a common use of language which would unambiguously determine the causal concept as it was employed there. The explanations offered by the individual scientists diverge widely. But the actual procedure of physics followed, nonetheless, a single direction; and from this direction, not from the words of the particular scientists, we can extract its basic viewpoint.[5]

Reflection about this procedure of physics was given the greatest clarity and precision by Helmholtz, and to him we therefore turn. His formulations represent the highest and most mature insights to which classical physics attained in its determination of the concepts of cause and effect. It is strange and striking that in modern discussions of the causal problem the name of Laplace is met with almost constantly, the name of du Bois-Reymond very often, but the name of Helmholtz seldom or never. This strikes me as a palpable shortcoming, for in critical insight and depth there is hardly another scientist of the nineteenth century who compares with Helmholtz. He is the genuinely classical representative of modern empiricism. On the one hand he covered almost the whole range of the natural sciences—he was a physicist, electrochemist, physiologist, and psychologist; and on the other, he was concerned in all his specialized investigations with extending and making precise the evidences for empiricism. In this sense he dealt with the general problem of space, as well as with the problem of the foundations of geometry from a strictly episte-

---

that the general principle of causality need not appear overtly in the presentation of the objective *content* of physics, indeed that it cannot appear there. This circumstance, however, should not be used—as it has sometimes been used—to prove the barrenness and superfluity of the principle. In a discussion of the problem of determinism Eddington argues that modern physics is at last beginning to turn away from determinism and cites as proof the fact that in a work such as Dirac's quantum mechanics the causal principle is nowhere mentioned as a particular axiom. But this argument is obviously untenable. For even within the conceptual world of classical physics it was not customary for the causal principle to appear *explicitly* as a particular theorem, as a part of physical doctrine, because of its above-mentioned methodological character. As such it is missing not only in modern presentations but also in the earlier classical systems—of Newton and Euler, d'Alembert and Lagrange.

5. Cf. above, p. 30.

mological standpoint. Hence if we are looking for a truly representative expression of what classical physics understood by the concept of cause and of the sense in which it used this concept, we may hope to meet with it, if anywhere, in the writings of Helmholtz.

Helmholtz, to be sure, did not set out with a ready-made concept of causality. This concept took shape for him only gradually, in the course of his concrete experimental researches. In his youth he accepted the viewpoint that dominated the whole physics of the epoch. He was a confirmed Newtonian, a strict mechanist. From this standpoint he sketched in the introduction to his lecture "Über die Erhaltung der Kraft" (1847) a complete program for the natural sciences. In substance the supreme theoretical aim of science is to find the unknown causes of occurrences from their visible effects; and this can be achieved with complete satisfaction only if we succeed in reducing all natural phenomena to motions of matter endowed with unalterable forces that depend solely on spatial relations. The task of physical science is thus defined as the reduction of natural phenomena to changeless, attractive, and repulsive forces, whose intensity depends on the distance. The possibility of performing this task is at the same time the condition for the complete intelligibility of nature.

Much broader and freer, much less bound to particular hypothetical presuppositions, is the view of the causal problem which Helmholtz advanced and carefully worked out a generation later in his lecture "Die Tatsachen in der Wahrnehmung" (1878). We know from his letters how intensely the theme of this speech had preoccupied him and what fundamental significance he ascribed to it.[6] Every special assumption concerning the form of natural events now derives from the general statement of the causal law. What we can find unambiguously and as fact without hypothetical interpolations—so he claims—is the fact of orderliness according to law (*das Gesetzliche*) in phenomena. This orderliness is the first product of the conceptual grasp of nature. What we call "cause" can be understood and justified only in this sense, even though in the common use of language the word is employed in a very confused way for antecedent or condition. We cannot achieve more than a knowledge of orderliness according to law in the domain of the real, though the latter is represented, to be sure, merely in the sign system of our sense impressions. Every rightly framed hypothesis in keeping with its factual meaning sets up a law concerning phenomena more general than what has till then been directly observed; it is an attempt to ascend to an orderliness ever more general and comprehensive. How far this attempt will succeed cannot be predetermined; in each particular case we must leave the decision to experience. But the search after ever more general laws is a basic

6. Cf. Königsberger, *Helmholtz*, 2, 246.

feature, a regulative principle of our thought. It is precisely this regulative principle, and nothing else, that we call the causal law. In this sense it is given *a priori,* it is a transcendental law: for a proof of it from experience is not possible. It is true on the other hand, however, that we have no other warrant for its applicability than its success. We could live in a world in which every atom differed from every other and no regularity was perceivable. In such a world our intellectual activity would necessarily come to rest. But the investigator does not reckon with such a world. He trusts in the intelligibility of natural phenomena; and every particular inductive inference would be untenable for him, if this universal trust did not form its basis. "Here only one counsel is valid: Trust the inadequate and act on it; then it will become a fact." [7]

This formulation of the causal principle of classical physics confirms in every particular the results of our former analysis. What Helmholtz demands and what he regards as the necessary and sufficient condition for the validity of the principle is precisely the gradation of knowledge that we attempted to present in detail: the procedure from experimental findings and their exact formulation to ever stricter statements of laws and ever more general statements of principles. In Helmholtz we find no appeal to the Laplacean formula. In order not to confuse and burden his critical analysis with metaphysical issues, it seems that he assiduously avoided this formula, which since du Bois-Reymond's lecture had again moved into the center of discussion. Even the concept of complete prediction of the future from the past plays no decisive role in Helmholtz' considerations. This concept has often been regarded as the essential, even the sole determining factor in the formulation of the causal law. It has been held that prediction is a "sufficient indicator of causality," though, to be sure, one was forced to add that the fulfilling of a prophecy is something ultimate, something not further analyzable.[8] Were this the case, we should have to give up the causal principle whenever physics found itself in a cognitive situation which no longer permitted of absolutely exact predictions within a given sphere. It seems to me that this view radically overestimates the theoretical significance of the concept of prediction. The desire in some sense or other to predetermine the future is certainly one of the most important motives leading us to form series of "causal" inferences. But it is by no means the sole significance, the exclusive conceptual import of these series. If we should exalt this motive to a principle,

7. "Die Tatsachen in der Wahrnehmung" (1878), *Vorträge und Reden, 2* (4th ed. Braunschweig, 1896), 240 ff.; cf. also Helmholtz, *Handbuch der physiologischen Optik, 2* (2d ed. Hamburg, 1896), 591 ff.

8. See M. Schlick, "Die Kausalität in der gegenwärtigen Physik," *Naturwissenschaften, 19* (1931), 150 f.

we should succumb to one of those anthropomorphisms the removal of which is one of the tasks of theoretical physics. Whenever man intervenes in the causal nexus of events, whenever he is concerned to achieve a definite effect—this is, to be sure, only possible insofar as he anticipates this effect beforehand in his thought. He must experiment in thought with the future; he must picture to himself, on the assumption of given actions on his part, how the future will shape itself. Predictions of the future are thus an indispensable factor in all technical mastery of nature; they are a necessary constituent for every practical orientation in the world. But to take this factor for the whole, to view it as exhausting the aim of causal knowledge, can only be attributed to the fact that since the beginning of the technical age we have become all too accustomed to define the significance of concepts exclusively in terms of their use, to confuse their logical with their pragmatic meaning. Bacon has here led the way with his *scientia propter potentiam.* He attacked the fatal practice of starting a science with a purely contemplative part and allowing it to pass only from this to an active part. He demanded the opposite order of procedure and rank.[9] The power of men over nature was to be established by supplanting the old logic with a new one, the "logic of discovery." "Quod in religione verissime requiritur, ut fidem quis ex operibus monstret: idem in naturali Philosophia competit, ut scientia simpliciter ex operibus monstretur." [10]

Modern positivism has adopted this notion completely. It has not only set up the demand of *savoir pour prévoir,* but has finally explained the *savoir* by the *prévoir.* Such a pragmatic limitation, however, is quite foreign to Helmholtz. He advocates a purely theoretical, a genuinely contemplative ideal of physical truth and this ideal is his guide for the analysis of the causal concept. "Any particular fact taken by itself," he once said, "may perchance arouse our curiosity or amazement, or it may be useful to us for practical application. Only the coherence of the whole, however, and precisely because of its conformity to law, yields intellectual satisfaction." [11] The knowledge of this conformity proves and justifies itself by predicting the future; but this prediction together with the technical mastery of nature founded upon it is not the sole and essential significance of this knowledge. Even in classical physics the causal concept was never so narrowly conceived that its use was denied in realms where for any reason the practical possibility of strict prediction did not exist. Such prediction always requires in fact not only the knowledge of

9. *Novum organon,* Bk. II, 4.

10. Bacon, "Cogitata et visa," *Opera omnia* (1694), fol. 593.

11. "Über die Erhaltung der Kraft" (Vortragszyklus 1862), *Vorträge und Reden, I,* 191.

general laws, but also the exact knowledge of initial conditions, a knowledge attainable only within definite limits. For this reason the strict Laplacean ideal enjoyed validity only in certain particular domains, such as astronomy, whereas it lost this validity in fields like hydrodynamics or elasticity theory.[12]

In his well-known discussions of causality Planck advanced, in an initially tentative fashion, a basic definition of the causal concept which stipulated that an event is to be regarded as causally determined if it can be predicted with certainty. He added at once, however, that this simple definition was not adequate. For although the possibility of making correct predictions does provide an infallible criterion for the presence of a causal nexus, we must not infer conversely from this that the former is equivalent to the latter. Such an inference, if taken strictly, would render illusory every employment of the concept. For even in classical physics it is not possible in a single instance really to predict a physical event accurately. From the standpoint of epistemology we are thus confronted with a strange dilemma: a criterion for causality is set up which, when taken in all strictness and followed through to its ultimate consequences, cannot exhibit a single instance of its concrete verification and application.[13]

We can avoid this dilemma if, in keeping with the foregoing methodological considerations, we formulate the principle of causality in all strictness as a propostion concerning cognitions, instead of trying to understand it as one concerning things and events. We must think of it as a guide-line which leads us from cognition to cognition and thus only indirectly from event to event, a proposition which allows us to reduce individual statements to general and universal ones and to represent the former by the latter. Understood in this sense, every genuine causal proposition, every natural law, contains not so much a prediction of future events as a promise of future cognitions. Such a law is "pregnant with the future," as Leibniz liked to express it. But this means primarily that it harbors an abundance of knowledge which only later will unfold itself and come to full fruition.

12. In this connection see P. Frank, *Das Kausalgesetz und seine Grenzen* (Vienna, 1932), pp. 41 ff.

13. Planck, "Die Kausalität in der Natur," *Wege zur Physikalischen Erkenntnis* (Leipzig, 1933), p. 236. That the causal law must be distinguished from the criterion of its applicability has recently been emphasized also by Grete Hermann, *Die naturphilosophischen Grundlagen der Quantenmechanik* (Berlin, 1935), pp. 50 ff. Here it is assumed, however—and to my mind, incorrectly—that for classical physics at least "causality" and "predictability" coincided completely and that quantum mechanics first showed the epistemological necessity of separating the two (Hermann, pp. 11 ff.).

The essence of a prediction is the extrapolation of which a natural law is capable. The range of validity of a natural law is not restricted to those observations from which it was immediately derived but extends beyond this particular set. And its proper confirmation lies in the fact that this extension succeeds, that it leads to phenomena previously unknown and discloses new classes of laws. The temporal direction of this disclosure is not of decisive importance. The way leads not from past to future but rather from a limited domain of knowledge to a wider domain. Thus Maxwell's field equations, for instance, contained implicitly the whole treasure of knowledge which Heinrich Hertz in his experimental work later uncovered. Hertz himself once said that one could not study Maxwell's theory without feeling at times as though an independent life and intellect resided in these mathematical formulas, as though they were cleverer than we, cleverer even than their discoverer, as though they gave us more than was put into them at their inception.[14] This becomes particularly clear and convincing in the case of formulas which not only govern the particular domain for which they were initially set up, but also show themselves valid and applicable to remote, even wholly unrelated, realms. For example, the equations that Fourier set up for heat transmission were of this type.[15] Physics is rich in similar examples, all of which make evident the proper and peculiar productivity of physical thought.

The same thing reveals itself in the demand for simplicity of natural laws—a demand whose purely epistemological interpretation and establishment meets with peculiar difficulties. In order to avoid these difficulties the attempt has occasionally been made to cut the Gordian knot by holding that the demand for simplicity is not capable of strict formulation and must therefore be excluded from the theory of scientific principles. At least in the logical formulation of this theory no place was allotted to the demand, and it was reduced to a purely aesthetic claim incapable of a properly objective foundation.[16] The historical course of physical knowledge, however, speaks against such a view, for in the development of physics the simplicity demand has continually reasserted itself and has proved eminently fruitful.

This process can be traced from the first beginnings of exact science down to the most recent investigations. Galileo and Kepler employed the criterion of simplicity automatically and without deliberation, as yet unencumbered with any kind of epistemological scruples and doubts. They were convinced from the beginning that simplicity can be used as a

14. Hertz, *Über die Beziehungen zwischen Licht und Elektrizität*, in *Gesammelte Werke, 1*, 344.

15. See above, p. 38.

16. This standpoint was adopted, for example, by Schlick (above, n. 8), p. 148.

touchstone of truth. This argument in particular plays an important role in the defense of the Copernican system of the universe. "These Copernican hypotheses," said Kepler, "not only do not sin against the nature of things but help it much more. Nature loves simplicity, loves unity. . . . With the hypotheses which have been used there is no end of postulating orbits: with Copernicus many motions proceed from the fewest orbits." [17] Here was stated for the first time a clear definition of what is to be understood by the methodological ideal of simplicity. The motto *plurima ex paucissimis* expresses the decisive factor: the derivation of a multitude of inferences from the smallest possible number of particular assumptions. Formulations of this postulate by modern thinkers are more cautiously worded, but basically they express the same thing. In his address on the occasion of Planck's sixtieth birthday Einstein remarked, "the system of concepts that is unambiguously coordinated with the world of experience is reducible to a few basic principles, from which the whole system can be logically derived. With every important new advance the scientist finds his expectations exceeded, in that the fundamental laws, under the pressure of experience, become more and more simplified. With amazement he beholds the apparent chaos fitting into a sublime order that is to be attributed not to the dominion of his own mind but to the constitution of the world of experience." This shows how strongly and how rigidly the simplicity principle is still held in modern physics and how unshaken is the belief in its objective significance.[18] We must therefore distinguish between the merely pragmatic claim of the principle and its purely methodological claim, its claim to truth. If we now look back to our preceding considerations, we can understand and establish one feature of this methodological claim. We saw that with every transition to a new type of physical statement a double process is involved: an extensive enlargement of knowledge and a corresponding intensive compression. Every amplification leads at once to a previously unknown concentration. Only by virtue of this concentration is it possible to master the new material

17. *Mysterium cosmographicum* (1596), chap. 1; *Opera,* ed. Frisch, *1,* 113.

18. Of modern scientists Hermann Weyl, in particular, has stressed the significance of the demand for simplicity. "The assertion that nature is governed by strict laws," he holds, "is devoid of all content, if we do not add the statement that it is governed by mathematically simple laws . . . one cannot but admit that this working principle of simplicity has stood the test well. The astonishing thing is not that there exist natural laws but that the further the analysis proceeds, the finer the details, the finer the elements to which the phenomena are reduced, the simpler—and not the more complicated, as one would originally expect—the fundamental relations become and the more exactly they describe the actual occurrences." H. Weyl, *The Open World* (New Haven, Yale University Press, 1932), pp. 40 f.

that is pouring in and incorporate it into the form of physical knowledge. Thus simplification has already begun when we gather together data from the world of sense into physical concepts. And this simplification becomes greater and greater the higher we ascend in the hierarchy of these concepts, and the farther we proceed in the direction of statements of laws and of principles. The history of the least-action principle showed us, for example, how by its means equations for the motions of quite distinct and highly complex mechanical systems could be condensed into a single variation principle and be derived from it. In this respect the simplicity demand arises from and corresponds to the clearer and more fundamental demand of "axiomatization" advanced by Hilbert. "I believe," says Hilbert, "that whatever can become an object for scientific thinking submits, as soon as it becomes ready for the formation of a theory, to the axiomatic method and hence indirectly to mathematics." [19] The mathematical formulation of physical judgments therefore provides a condition and warrant for the progressive simplification of these judgments. It permits and suggests the reduction of particular propositions to propositional functions, and of propositional functions to doctrinal functions; [20] and thus it secures a continually progressing "lowering of the foundations."

This lowering, however, will be misconstrued if the principle of simplicity is given a merely economical interpretation. It is not by any means true that in passing from one type of physical statement to another intellectual effort is saved. On the contrary this passage always requires a considerable *expenditure* of intellectual energy. Only through this expenditure can the "jump" be made from the level of measurements to the level of laws and from here to the level of principles.[21] Mach's theory attempts to avoid this jump or to explain it away by reducing the higher levels to the lower, maintaining that laws and principles are nothing but collective statements. But laws are class concepts, and classes according to a basic theorem of the theory of types can never be construed as mere aggregates of elements. Hence physics too will become involved in contradictions if it does not carefully avoid the confusion of levels, which consists in putting a mere sum of particulars in the place of that intensive unity which signifies a law.

According to its logical intention a law is different from and more than a mere record or catalogue of particular instances. But according to Mach a rule for derivation, a formula, a law, "does not in the least have any more objective value (*sachlichen Wert*) than the particular facts

19. "Axiomatisches Denken," *Mathematische Annalen, 78* (1917), 415.

20. Concerning the theory of doctrinal functions see esp. the discussions of Cassius J. Keyser, *Mathematical Philosophy* (New York, 1922), Lecture 3.

21. See above, pp. 54 ff.

taken together. Its value lies solely in the convenience of its employment. It has an economic value." [22] When we follow it further, however, this purely subjective and pragmatic interpretation becomes involved in contradictions that are difficult to resolve. For if the task of physical inquiry really consisted in attaining a complete knowledge of all particular facts, how then in the discharge of this task could physical inquiry "save" itself work of any kind? How could it break off prematurely the process of traversing all the particular instances and acquiesce in a general formula? Could this alleged generality be anything but a superficiality, a tarrying with a vague and arbitrary assumption unsupported and unjustified by any factual observation? In trying to dodge this consequence, some kind of objective significance must always be admitted for the simplicity of the formula, and thus it cannot be understood merely in a subjective psychological sense. If it is held, for example, that by means of such a formula a progressive fitting of thought to the facts is achieved,[23] then the order, the articulation, the systematic interconnection of facts is again treated as something objective which thought in a certain sense may strive after and to which it can adapt or fit itself. Constancy and simplicity are thus attributed to natural phenomena themselves, instead of merely to our representations. In the words of Kant, subjective association is explained through objective affinity.

In order to secure a satisfactory formulation and a genuine analysis of the concept of simplicity, the differences in type between the various kinds of physical statements must in every case be carefully observed. We can never ascribe simplicity to particular physical statements, not even to a whole class of such statements. For simplicity is a predicate pertaining to the total system. This system is closed in the sense that every intrusion in any one of its parts is communicated at once to the whole and makes itself logically perceptible throughout. How it affects the whole, however, depends essentially on the place where the lever is applied. Not infrequently it happens that at a certain place in the system of physical knowledge a change is introduced which from the standpoint of that particular region proves to be not a simplification but a very far reaching complication. If the total system is not affected by this complication or if it becomes more self-contained and more easily surveyed, the simplicity demand is satisfied. For this demand refers not to a particular layer of the conceptual and cognitive apparatus of physics, but to the interrelation and interlocking of all its individual parts. As soon as these

22. Mach, *Die Geschichte und die Wurzel des Satzes von der Erhaltung der Arbeit* (Prague, 1872), p. 31.

23. See Mach, Über Umbildung und Anpassung im naturwissenschaftlichen Denken," *Populär-wissenschaftliche Vorlesungen,* pp. 237 ff.

parts are isolated from each other, therefore, the question as to whether or not a given change within the physical system leads to a genuine simplification can no longer be answered strictly and unambiguously. There arise, for example, differing estimates according to whether we demand that the physical *processes,* which we coordinate with the contents of sensation and with the immediate data of observation as their conceptual correlates, be as simple as possible, or whether we insist on simplicity of the physical *axioms.* The introduction of Riemannean geometry that occurred in general relativity theory appeared at first as a complication. For Euclidean as contrasted with non-Euclidean geometry undoubtedly had the advantage of simplicity. But this apparent disadvantage on the part of the latter is outweighed and more than compensated for by the circumstance that the fundamental law describing events and the definitions correlating physical concepts with data of experience took on a particularly simple form as a result of introducing Riemannean geometry.[24] According to the principle of mutual limitation, statements of the results of measurements and statements of laws and principles mutually depend on each other. The demand of simplicity, therefore, can never be directed at one of these constituent parts alone. It specifies rather that all these three parts must be so attuned to one another that the condition *plurima ex paucissimis* is satisfied, that the greatest possible number of phenomena is comprehended by the fewest possible determining factors and exactly described by them. The possibility of this determination is and remains a problem; but in the principle of causality this problem is raised to a postulate. That this postulate determines the whole structure of classical physics and was the decisive impulse in its thinking is unmistakable. We now face the question whether physics can continue to trust this impulse and be guided by it, or whether, because of the new tasks with which it is confronted, physics will be compelled to transform the concept of cause in essential respects, if not to depart from it altogether.

24. That the demand of "simplicity" may be directed either to the axiomatic part of physics or to the correlations relating phenomena to physical concepts and hence to the descriptive part, has been strikingly demonstrated by R. Carnap, "Über die Aufgabe der Physik," *Kant-Studien, 28* (1923), 90 ff.

PART THREE

Causality and Probability

CHAPTER 7

Dynamic and Statistical Conformity to Law

ACCORDING to strict positivism, the relation between theory and experience is such that theory is not only an expression of experience but also a copy of it, a copy, that is, of what is given in immediate sense data. The truth of theory consists in its fitting the facts as closely as possible: and no adaptation can be more complete than that which adds nothing new or foreign to the facts but reproduces them as faithfully as possible. Theories are memory aids; they provide an inventory of the given. And this inventory obviously changes with each new element that it records. A genuine constancy is neither to be expected nor to be hoped for from experience. Experience constantly alters its form as the circle of the given widens, as, with continuing observation, ever new material presses in. This growth is purely an epigenesis; the elements are introduced from without and are added singly one to the other, piece by piece.

In opposition to this view, the critical theory of knowledge takes a radically different fundamental approach. It defines experience not through its material but through its form—and it seeks to show that this form is once and for all bound to definite conditions. "Our reason," Kant argues against Hume, "is not like a plane indefinitely far extended, the limits of which we know in a general way only, but must rather be compared to a sphere, the radius of which can be determined from the curvature of the arc of its surface—that is to say, from the nature of synthetic *a priori* propositions—and whereby we can likewise specify with certainty its volume and its limits." [1] The synthetic *a priori* propositions give experience a fixed form and a definite limit, but they do not prescribe this form and limit with reference to any specific content of experience. In this respect they permit a continual expansion, but one that does not run over into the indeterminate. No matter how far experience may reach out and extend over new realms, it is nonetheless certain of always finding itself in these realms: its own structure and its characteristic structural principles.

1. *Kritik der reinen Vernunft* (2d ed.), p. 790 (ed. Cassirer, *3*, 514); trans., p. 607.

Kant believed he could completely map out the system of these principles. For this project he leaned on classical logic and classical mechanics. Just as he patterned the categories after the table of judgments, so he patterned the synthetic principles after Newton's laws of motion—that is, the law of inertia, the law of the proportionality between change of motion and moving force, and the law of the equality of action and reaction. In accordance with this basic schema, the construction of "the metaphysical foundations of science" was carried out. All genuine science requires, according to Kant, a "pure" part; and it is a necessity with regard to method which cannot be avoided to work out this pure part separately and in complete detail "so that we may strictly determine what reason can accomplish by itself and where it begins to need the aid of principles of experience." [2]

We know today why this pure part could not fulfill the task that Kant put to it. It was too closely bound to a specific form of science, which classical rationalism held to be the plainly rational form. Kant was certain that all rationality was enclosed within a definite area which was determined on the one side by the axioms of Euclidean geometry and on the other by those of Newtonian physics. Hence if these axiomatic conditions were dropped, all the rational aspects would appear to lose their force. The conclusion, however, would only be valid if modern physics had simply abandoned these presuppositions without replacing them by others. But in fact no such elimination has occurred. Physics has not given up the rational aspects; it has merely defined them differently and in this redefinition has generalized them radically and freed them from accidental limitations.

When in the face of the new factual material and the new theoretical tasks which it was facing, physics extended and transformed its conceptual apparatus, it did not simultaneously give up its general character and structure. It only made evident the fact that this structure is to be thought of not as rigid but as dynamic, that its significance and efficacy do not rest upon its substantial rigidity established once and for all but precisely upon its plasticity and flexibility. The *a priori* that can still be sought and that alone can be adhered to must do justice to this flexibility. It must be understood in a purely methodological sense. It is not based on the content of any particular system of axioms, but refers to the process whereby in progressive theoretical research one system develops from another. This process has its rules, and these rules provide the presuppositions and foundation for what we may call the "form of experience."

2. Kant, *Metaphysische Anfangsgründe der Naturwissenschaft,* in *Werke,* ed. Cassirer, *4,* 371.

A mere comparison of the various empirical theories shows us that their succession is not characterized by lack of law or by arbitrariness but by a methodological continuity. Later theories do not simply push aside and repudiate earlier ones but incorporate their content. Einstein's theory of gravitation came about not through the overthrow of the Newtonian system of astronomy but from a development of it that allowed the newer theory to include within the area of explanation certain phenomena which the older theory could not deal with. Thus, for example, the movement of the perihelion of Mercury was regarded no longer as a special case or anomaly to be explained by special assumptions introduced *ad hoc* but as a general property of the elliptical orbits of planets, strictly deducible from the theory, even though the movement in the case of other planets is too small to be directly observed. Similarly, the equality of the gravitational and inertial mass of a body, which had long been known and established as a fact of experience, was placed in a completely new light conceptually by the general relativity theory. As Einstein expressed it, this equality was not only registered but interpreted.[3] In this continual oscillation the essential character of the dynamic form is established and verified as a form that not merely receives and incorporates new material passively but seeks the new material, and because it seeks it, it is able to form and organize it.

I have begun with these general observations, so that they may serve us as a guide in understanding that transformation of the causal schema which began in recent physics. This transformation too is not accidental but occurs under the pressure of experience. To this pressure, however, there corresponds a counter pressure from the side of theory; and action and reaction here also tend toward a state of equilibrium. Whether this equilibrium has as yet been attained in modern physics is uncertain, but as a task, a general methodological objective, it is definitely established. Even the gradual emergence and development of the problem point unmistakably in this direction. If we ask what particular problems and phenomena gave rise to this new development in thought, we are directed to the questions that have found theoretical expression in the second law of thermodynamics. This law arose primarily out of quite definite, particular experiences. Sadi Carnot's fundamental essay, *Sur la Puissance motrice du feu* (1824), which paved the way for the second law, dealt with a clearly limited special problem arising from matters of technical interest. The essay investigated the mechanical capacity of heat and established the fact that in every instance it involves the transition of heat from a warmer to a colder body. In introducing the concept of

3. A. Einstein, *Über die spezielle und die allgemeine Relativitätstheorie* (Braunschweig, 1920), sec. 19.

the reversible cycle for determining the work maximum of heat, Carnot fashioned a new theoretical instrument which proved eminently fruitful in subsequent developments. The general significance of the problem, however, first became apparent when Clausius introduced the basic concept of entropy and enunciated the law that the entropy of the universe tends toward a maximum. We are not considering here those inferences from the Clausian thesis which have led to the well-known philosophical speculations concerning the heat death of the universe. Far more significant than these speculations, which not infrequently end in obscure metaphysical conclusions, are the new epistemological criteria latent in the thesis of Clausius.

This second law and the theoretical considerations connected with it confronted scientific thought with a fundamental difference within the realm of natural events, a difference that had found no adequate expression in the system of classical mechanics. The opposition between reversible and irreversible processes now came clearly and definitely into view. A direction was indicated within the totality of natural phenomena, a definite tendency which was carried unalterably within it. This basic fact received exact formulation in Clausius' formula for the increase in entropy, but it was still far from being intelligible in the real meaning of the term. For this to happen it would have to be derived from the basic principles of physics, or at least to appear clearly in accord with them. There seemed to be no way, however, from these principles to the second law. If we hold to the general presuppositions of classical physics, we must acknowledge the law in its factual validity without being able to deduce it theoretically, in fact without even being able to represent it properly. The differential equations of classical mechanics do not give us a point of contact or origin, for according to them the course of any occurrence can be reversed simply by giving the velocities of the individual mass points the opposite sign. Similarly, in Maxwell's equations of electrodynamics no factor is discoverable which would lead to that characteristic determinateness of direction in natural events which is expressed by the fact that energy is dissipated. The entropy law thus came to be a kind of irrational remainder, a foreigner and intruder in the securely articulated system of classical mechanics and electrodynamics.

Precisely at this point there set in the independent reaction of theory, a conceptual contribution as significant for epistemology as for physics. It was the work accomplished by the kinetic theory of gases and linked chiefly with the name of Ludwig Boltzmann. Boltzmann succeeded in removing the strangeness and paradox from the entropy law. Being one of the most consistent representatives of classical mechanics, he felt that he must attempt to incorporate within mechanics this phenomenon of

onesidedness in natural events and to make it intelligible through the fundamental concepts of mechanics. Boltzmann's undertaking succeeded by giving a new interpretation to the preference of nature for a given state. A reversal of events is consistent with mechanical principles and with the principle of the conservation of energy, but Boltzmann showed why such a reversal would be exceedingly improbable. "The fact that in nature entropy tends toward a maximum shows that in all interactions of real gases (diffusion, heat conduction, etc.) the individual molecules interact in accordance with the laws of probability." [4] Gibbs had preceded Boltzmann in interpreting the second law as a law of probability and had based the new science of statistical mechanics on this interpretation.[5] Boltzmann gave it exact form in his law that entropy is proportional to the logarithm of the probability ($S = k \log W$).

This approach, however, did not solve the riddle epistemologically but only reiterated it more emphatically. For Boltzmann's solution was successful only by introducing a new kind of physical conformity to law and by giving it equal rank with "dynamic" laws. The probability laws on which he based the kinetic theory of gases, however, do not have the same epistemological quality and "dignity" that had previously been ascribed to the laws of nature. For every natural law had been regarded as possessing the essential property of an indwelling necessity, a necessity that excluded every exception. But precisely this property would have to be surrendered, if one were to go over to mere probability laws. An event no matter how improbable is still not an impossible event; not only can it occur, but it will in general occur one day, if we but extend our observations over a sufficiently long period of time.

It is evident from this that in viewing the entropy law as a probability law there had been introduced into the very concept of law itself a dualism wholly foreign to its original meaning. This dualism is illuminated by Maxwell's "demon," who, without violating a single dynamic law, without acting counter to the laws of mechanics or the principle of conservation of energy, could reverse the direction of world events and thus abrogate the law of entropy. The new problems that here confront us arise from the fact that in the probability approach of the kinetic theory of gases we have to deal no longer with statements concerning the behavior of individual particles, but solely with the general effect for the gas as a whole of all the separate motions of the molecules. We restrict ourselves from the beginning to certain mean values, such as the mean

4. L. Boltzmann, *Vorlesungen über Gastheorie, 1* (Leipzig, 1896), sec. 9, p. 60.

5. See further Boltzmann's essay "Statistische Mechanik," *Populäre Schriften* (Leipzig, 1905), pp. 345 ff.; and M. von Laue's article "Statistische Physik," *Handwörterbuch der Naturwissenschaften, 9* (2d ed., Jena, 1934), 537 ff.

velocity, the mean number of collisions, the mean free path of the molecules, from which nevertheless the total behavior of the gas can be exactly deduced and definite assertions concerning its pressure, density, specific heat, etc. derived. The fruitfulness of the kinetic theory of gases is thus evident, but the epistemological problem which it harbors remains unsolved. For the very fact that it is possible to make such far-reaching assertions concerning a physical whole while renouncing knowledge about its individual parts represents a paradox from the standpoint of particle mechanics and implies a transformation of the cognitive ideal that had previously guided it.

With Boltzmann this transformation had not yet become clearly evident. For the task he set himself was to establish complete harmony between mechanics and thermodynamics by making the second law intelligible in terms of the presuppositions of mechanics. Even after introducing the concept of probability, therefore, the spheres of application for statistical and dynamic laws were separated for Boltzmann in a simple fashion. The statistical procedure was applied solely to the formulation of initial conditions, whereas the further course of events was regarded as dominated completely by strict dynamic laws, the laws of conservation of energy and of momentum during molecular collisions. The onesidedness stipulated in the second law could not be established from the equations governing the motions of the molecules, for these do not alter when we change the sign of the time. This onesidedness, therefore, was to be sought solely in the initial conditions. But this statement is not to be understood as implying that for every case it had to be specially assumed that these initial conditions were certain particular ones rather than their equally possible opposites. Instead it sufficed to make "a unified assumption concerning the initial constitution of the mechanical picture of the universe," from which it followed with logical necessity that every time bodies acted on each other there ensued a transition from an ordered, improbable state to an unordered, probable one.[6]

If we follow the development of the kinetic theory of gases, it becomes evident that the epistemological ideal which Boltzmann had before him has not been completely fulfilled. For the different constituent elements that went into the theory could not be held together without some friction. Although the empirical fruits of this theory have been extraordinary—in that it succeeded, for example, in determining exactly the relation between the specific heats, the relation between the coefficients of heat conductance and viscosity, etc.—progress in the treatment of principles did not keep up with the empirical advance. Certain difficul-

6. See Boltzmann, *Vorlesungen über Gastheorie, 2* (1898), sec. 87, pp. 251 ff.

ties and obscurities continually appeared at the foundations of the theory that could only be laboriously removed by introducing special hypothetical assumptions. The *H-Theorem* of Boltzmann appeared destined to bridge the gap between the general principles of mechanics and the second law of thermodynamics, insofar as in it the irreversibility of certain mechanical processes was proved. But it was precisely this theorem that was subjected to grave objections in principle (such as the "reversal objection" of Loschmidt and the "recurrence objection" of Zermelo). The special statistical innovations from which Boltzmann set out in the construction of his theory thus retained a precarious and not strictly demonstrable character.

The inner difficulties of the theory came distinctly to light when the attempt was made to transfer it to the new realm of radiation. The law of equipartition of energy which in other areas had justified itself so eminently, as in the derivation of the Dulong-Petit law of atomic heats and in the theory of the Brownian movement, now suffered final shipwreck.[7] The difficulties that arose could only be resolved when Planck in 1900 set up his elementary dynamic law for the vibrations of an ideal oscillator. By means of this law the inconsistencies with experience, in which the equipartition law of classical mechanics was entangled, were resolved by one stroke; but the resolution of these inconsistencies was only successful because, through the assumption of the "elementary quantum of action," something foreign was admitted into the system of basic presuppositions. Before we go into these beginnings of quantum theory, however, we must consider another attempt, significant for epistemology, to answer the question of the relation between dynamic and statistical conformity to law. I refer to the attempt made by Franz Exner in his *Vorlesungen über die physikalischen Grundlagen der Naturwissenschaften.*[8] This attempt is of special significance in the development of recent quantum mechanics, inasmuch as Schrödinger in his inaugural address in Zürich in 1922 formulated his own basic view with reference to it.

The second law of thermodynamics and the consequences arising from it had introduced scientific thought to a fundamental antithesis between

7. I shall not enter here into further detail as it is not essential for the general problem but will refer the reader to von Laue's brief but comprehensive discussion in his article "Statistische Physik" (above, n. 5).

8. Vienna, 1919. The text of this inaugural address appeared later in *Naturwissenschaften, 17* (1929), 9 ff., and still later in English translation in the collective volume, *Science and the Human Temperament,* trans. James Murphy (London, 1935), pp. 107 ff.

"reversible" and "irreversible" processes. In admitting this antithesis, science did not have to surrender its claim to methodological unity, but it could no longer hope to sustain the claim in the same way and with the same means it had previously employed. If this unity was to be maintained, the uniformity which had prevailed in classical mechanics and which energetics still sought to preserve and enforce was lost once and for all. When Planck in his address at Leiden in 1908 developed his thesis of the unity of the physical world view, he did not fail to point out that the antithesis between reversible and irreversible processes is in a certain sense unbridgeable, that with a better right than any other, it might be made the most fundamental basis of division for all physical processes, and that it will presumably play the leading role in the physical world view of the future.[9]

But whenever science is confronted with a fundamental difference of this kind in the content of empirical reality, it is also faced with a general methodological problem. All scientific thought is dominated and guided by two opposing tendencies that are engaged in a continual process of mutual adjustment. The demand of "specification" is the counterpoise to the claim of "homogeneity." The struggle between these two cannot be decided purely objectively from the nature of the object. It is a dissension and competition that belongs not so much to the nature of things as to scientific reason itself. In this sense homogeneity and specification were introduced into Kant's Critique of Reason, not as constitutive principles, pertaining to the knowledge of objects but as regulative principles, as maxims of scientific inquiry. They are subjective principles that derive not from the constitution of objects but from the interest of reason. According to Kant, therefore, it is not surprising that one investigator should have a great interest in multiplicity while for another the interest in unity should be dominant. A true conflict between the two goals does not exist. Strife arises between them only when, being in truth nothing more than logical rules of pedagogy, they are mistaken for pronouncements of metaphysical wisdom. In the actual course of empirical research the two tendencies are not only compatible but also interdependent; they stand to one another not in a relation of opposition but in one of complementarity: "this . . . manifests itself also among students of nature in the diversity of their ways of thinking. Those who are more especially speculative are, we may almost say, hostile to heterogeneity, and are always on the watch for the unity of the genus; those, on the other hand, who are more especially empirical are constantly endeavoring to differentiate nature in such manifold fashion as almost to extinguish the hope of

9. Planck, *Wege zur physikalischen Erkenntnis,* p. 15.

ever being able to determine its appearances in accordance with universal principles." [10]

On the basis of this distinction Exner's attempt to formulate anew the concept of natural law contains a speculative or, better, a purely methodological characteristic. Exner did not lean on new empirical facts; and considering the prevailing state of research at the time, he could hardly have found an adequate justification in the merely factual state of scientific knowledge. What moved Exner was above all his interest in reason, which caused him to object to the indissoluble dualism of dynamic and statistical laws. A way out of this dualism did not seem to be available so long as one held to the current view, so long as one regarded dynamic laws as the proper and indispensable foundation of all genuine scientific knowledge. For it seemed impossible to carry through the idea that statistical laws can be viewed as merely provisional ones replaceable later by laws of dynamics: theory had shown that for all those phenomena subsumed under the second law—that is, for all irreversible processes—the statistical approach was indispensable. Therefore if unity was again to be secured, it could be so only by reversing the procedure: statistical law must be regarded as the comprehensive genus, a concept of higher order than that of dynamic laws and including them as a special case. This was the thesis advocated and defended by Exner in a new and original way. We must follow his deductions in detail, for they touch everywhere on the basic epistemological problem and are thus of fundamental significance for its clarification.

The validity of strictly universal scientific laws was not denied by Exner, but he declared them problematical. In the case of the kinetic theory of gases we know that the laws do not possess absolutely exact, but only statistical, validity, that they yield only mean values derived from a large number of observations. Is it different with the other laws, with the laws of classical physics? Does the formula set up by Galileo for the free fall of bodies really hold always and everywhere? How could we decide this, since our experience can always cover only the average of a phenomenon over a considerable time span? Perhaps the acceleration is not really constant, as Galileo had assumed. It is conceivable at any rate that the acceleration shifts very rapidly about a mean value and that therefore the motion of a falling body in minute intervals of time is not uniformly but irregularly accelerated. In this case a falling body would move not in a straight line but by jerks in a zigzag path. It is possible that although we formerly believed we were dealing with

10. *Kritik der reinen Vernunft* (2d ed.), pp. 682 f. (ed. Cassirer, *3*, 448); trans., p. 540. Cf. *Kritik der Urteilskraft*, Intro., sec. 5 (ed. Cassirer, *5*, 251).

natural laws that had absolute validity, actually we are dealing only with laws of averages which lose their validity in sufficiently minute ranges of time and space. Dogmatic assertion at least is here no more justified than dogmatic negation: hence we must leave the question open if we are not to impose arbitrary restrictions upon research. It certainly remains true that research cannot dispense with such hypothetical assumptions as those concerning causality or forces, and that we owe the greatest advances and the finest discoveries to the able selection of these assumptions; "but for this very reason their origin must not be forgotten and they themselves must not be made a part of nature." [11]

We, too, are not aiming to make the laws of dynamics a part of nature —for the question as to conformity to law, as we repeatedly had to emphasize, is to be leveled not directly at the things and processes of nature but at the *cognition* of nature. What does Exner's problem then mean, when we take it exclusively in this latter sense? Shall cognition be denied the right and the possibility of setting up hypothetically universal and exact laws, of drawing definite conclusions from the assumption of these laws, and of testing the results by experiment? This was in fact the only way by which Galileo and with him the whole of classical physics believed it possible to arrive at strict laws and to demonstrate their empirical truth.

We have already emphasized that Galileo was not acquainted with an induction in the sense of John Stuart Mill and that he definitely rejected the inference from several to "all" observations.[12] And concerning the difficulty, which is here expressed by Exner, this, too, was anticipated by him in a certain sense. He seeks to invalidate it, however, by explaining that his demonstrations would by no means lose their significance if it were to be shown that bodies in nature do not fall with strictly uniform acceleration, inasmuch as these deductions were based merely on hypothetical and not on "assertorial" statements.[13] Consequently,

11. F. Exner, *Vorlesungen über die physikalischen Grundlagen der Naturwissenschaften* (Vienna, 1919), Lectures 86, 87, pp. 647 ff.

12. Cf. above, p. 40.

13. Galileo to Carcaville, *Opere,* ed. Alberi, *7,* 156 f.:

"I argue *ex suppositione,* imagining a motion toward a point, which, starting from rest, would accelerate, increasing its velocity in direct ratio to the increase in time: and from such a motion I conclusively demonstrate many phenomena. I then add that, if an experiment were to show that these phenomena were found to be verified in the movement of naturally falling bodies, we could affirm without error that this is the same movement which was defined and supposed by me; if not, my demonstrations, based on my supposition, would lose none of their force and conclusiveness; just as the conclusions demonstrated by Archimedes concerning the spiral are in no way prejudiced by the fact that there is found in nature no moving body which has such a spiral motion."

Galileo requires absolute exactness for his laws only in the sense that he demands precise hypotheses as starting points, but not in the sense that they are to be considered as collective descriptions of all individual cases, of all individual statements of position and time measurements. This could be expressed, somewhat paradoxically, by saying that Galileo's equations claim to be correct not because they apply always and everywhere, and because this "always" and "everywhere" has been experimentally demonstrated by him, but rather because strictly speaking they never apply anywhere. They are based on ideal cases, not on immediately given, empirically real ones. And all those laws which have been established by classical physics, following the example of Galileo, are of the same nature. Even Mach has entirely acknowledged this historical and phenomenological situation, despite the fact that his own theory of knowledge does not possess the means to justify it. He says that "all general physical concepts and laws, the concept of the ray, the laws of refraction, Mariotte's law, etc., etc. are obtained through idealization. In this way they take on that simplified form . . . which makes it possible to reconstruct and therefore understand any fact even of a complex nature by means of a synthetic combination of these concepts and laws." [14]

This admission, from Mach's point of view, is of course somewhat problematical. In view of the fact that according to him a rule for derivation, a formula, or a law, has "absolutely no more factual validity than the individual facts combined," [15] it appears difficult to understand how, in establishing such a rule, we can ignore even a single individual case, or how we can progress toward idealizations that in the end must always appear as violations of the actual facts. This problem, however, offered no difficulty for Galileo because he obtained the particular only by the superposition of general laws, by what he called the "resolutive and compositive method." For him, therefore, the particular is not originally given as an isolated datum but is rather something determinable only as the intersection of general relations. Galileo's equations for falling bodies show that they do not claim to be statements about the actual fall of bodies, for air resistance, which is indispensable for the determination of the latter, is entirely omitted. However, Galileo does not consider this a flaw in his equations; he sees in it rather an incentive to go a step further in the determination and to represent the factor of air resistance by new and equally exact laws. In this he is fully aware that in the end only experience can decide on the validity of this entire process, of the possibility of progressively determining the particular by means of the general. However, this proof by experience for which

14. Mach, *Erkenntnis und Irrtum* (Leipzig, J. A. Barth, 1905), p. 189.
15. Cf. above, p. 68.

he looks and which he acknowledges as binding is not the one which Exner presupposes in his presentation and critique. Exner is wrong when he declares that Galileo arrived at his laws of falling bodies in a purely inductive manner as far as this was possible by means of experimental methods in his day, whereas Newton, availing himself of Galileo's inductive investigation, used the theory of gravitation as a hypothetical basis for the explanation of the phenomena of falling bodies.[16] The relationship between the hypothetical and the factual, the deductive and inductive, is exactly the same with Galileo as with Newton. Newton considerably extended the sphere of application of Galileo's laws by incorporating entirely new phenomena such as lunar and tidal movements. But Galileo's method did not have to be altered. This method was already determined in the first decisive experiments of Galileo. The experiments on the inclined plane served for the verification of Galileo's laws not for their discovery, which had taken place in an entirely different manner. They belong to the process of verification, which is to be strictly distinguished from the ordinary form of induction, the *inductio per enumerationem simplicem.*

It might be contended, however, that all these historical considerations do not touch the core of the problem pointed up by Exner. His question was not of a historical but entirely of a systematic nature. We admit that the most valuable results were attained by classical physics through its own method, the method of hypothetical deduction: but must we forever adhere to this particular method of research? Is it not rather advisable to remember that other ways are also possible and practicable and that the time may come in which physics will see itself definitely forced to take the step from dynamic to statistical laws? As we have seen, Exner also does not want to deny the fruitfulness of classical methods; what he doubts and questions is merely their finality. And at this point it must be allowed that this finality cannot be assured by *a priori* and speculative considerations. If in any branch of physics new facts are discovered of which it can be shown and made probable that they cannot be represented with conceptual clarity by the previous method of superposing strict dynamic laws, this would undoubtedly give rise to a new problem that epistemology should not be permitted to ignore but which instead it would have to take into serious consideration. Later on we shall have occasion to ask whether we are faced with such a situation today—whether the problems of the new atomic physics do not require a reversal of the current path of research and, in a sense, a departure from it. Exner, however, does not appeal to such empirical considerations. His line of questioning, as we have seen, arose from

16. Exner, *Vorlesungen,* p. 651.

quite different, purely methodological, viewpoints. It sprang from the desire to bring the laws of nature back to one common denominator and to abolish the dualism between dynamic and statistical laws. However, it is clear that this "interest of reason," although in itself entirely justified, has no weight as empirical proof. Whether and in what sense it can be satisfied can be decided only on the basis of the existing status of research. And here the demand of homogeneity emerges as equally justified with that of variety; the logical precept *entia non sunt multiplicanda praeter necessitatem* is complemented and, as it were, kept in check by the other, equally valid *entium varietates non temere esse minuendas.*[17]

Should it turn out that in the transition from the problems of macrophysics to those of microphysics the prevailing conformity in the formulation of natural laws cannot be upheld, we should undoubtedly have to take this circumstance into account. But this would by no means be tantamount to a renunciation of the classical conception of law; it would merely be a more precise logical determination of it, in other words not an abrogation but a specification. Exner's observations are important and interesting because they anticipate certain problems which subsequently became especially clear and acute in the development of quantum mechanics, but his method of proof is not sufficient for a definite decision on these problems. The factum of classical physics cannot be removed or upset by the presentation of mere possibilities. This factum is a single continuous proof of the ability of physical science to attain an ever more complete conception of the special situation or even of the particular instance by means of the formulation of strict laws, even if these are at first only hypothetically valid. It is of course obvious that this development is not of an even or steady nature but that it is subject to impediments and reversals. From the point of view of classical physics, however, these impediments constitute not barriers to research but rather important and indispensable incentives. Whenever there appears difficulty in determining the particular through general laws, whenever the subsumption of the factually given under the hypothetical rule is not immediately successful, a new formulation of concepts, designed to overcome the difficulty, takes place. The apparent gap becomes closed, and each such closure represents another step in the direction of specialization of the laws of nature. In this way, for example in the study of the behavior of gases, we see the progression from Boyle and Mariotte's law $vp = const.$ to Gay-Lussac's law $VP = RT$, and finally to Van der Waals' equation of state $(P + \frac{a}{V^2})\ (V - b) = RT$. The latter equa-

17. *Kritik der reinen Vernunft;* cf. above, p. 80.

tion, by adding the constants *a* and *b*, introduces two new parameters, by means of which the behavior of real and ideal gases can be distinguished. In the same way it is necessary to introduce new constants, such as the dielectric constant, when the behavior of gases in the presence of electric or magnetic fields is studied. And each such introduction of a new factor brings about a closer approximation to reality. A different, more direct grasp of reality is not possible within the framework of physical knowledge and of the mode of formation of concepts in physics. All this demonstrates clearly that not the mere accumulation of experiments and observations but the progressively increasing accuracy of the tools of research is the important consideration. When physics repeats certain lines of experimentation, this repetition serves the purpose of increasing accuracy. The experiment is to be freed from all accidental circumstances that were a disturbing factor in the beginning; it is to be purged of possible experimental errors. Once this task is accomplished the point is reached for the investigator where "one instance represents a thousand," and he can without hesitation take the step from the individuality of statements of the results of measurements to the universality of statements of laws.

Opposed to this procedure, Exner's question of whether we can ever be quite certain that the actual event in nature corresponds to what we presuppose and determine in our general laws loses its point. Against Planck's contention that only strictly dynamic laws satisfy the requirements of our urge for knowledge while every statistical law is basically inadequate because it offers an indefinite answer in place of a definite one, Exner counters that "nature does not ask whether or not man understands it; also we are not to construct a nature that satisfies our understanding but instead we must accept nature as it is given us." [18] The epistemological problem, however, consists precisely in determining how far a "nature" is "given" us at all, and what is meant by being "given." By "nature" is meant an objective order of phenomena—and it is understood and presupposed in this sense by Exner. He emphasizes that even when we turn from absolute to statistical order this conception of nature is not changed, because probability cannot be interpreted in a purely subjective sense. A probability that is merely the expression for a psychological phenomenon, for the extent of our subjective expectation, would be neither useful nor fruitful in science. "The actual confirmation of the results of the probability calculus in experience proves that chance is entirely independent of man and of his knowledge, that it must be something objectively given in nature. Otherwise it would not be possible . . . for physical laws to be derived on the assumption of

18. *Vorlesungen,* p. 697.

contingency." [19] But here we are back at the same problem—for with what justification can we claim that nature is not an utter chaos, but subject to a definite regularity expressible in our objective statistical laws? A *proof* for this can never be established if nature is taken as an absolute entity that does not ask whether or not it is understood. One always becomes entangled in antinomies and pseudoproblems, unless one is determined to carry through the "Copernican revolution" in all strictness. Nature certainly asks no questions—for the function of questioning is restricted to knowledge. It is knowledge which asks whether and to what extent it is possible to find an objective order and determination in phenomena. All its individual concepts are but partial expressions for this basic problem. If it turns out that physical research by starting out from general hypothetical presuppositions is able to link them together in such a way as to result in an increasingly complete knowledge of individual phenomena, we have all that we can demand and look for as strictly dynamic conformity to law. If then all measurements and experimental tests which we are able to perform within our given limits in the realm of macrophysics actually bear out the strict correctness of Galileo's equations—as is assumed and presupposed by Exner—then all foundation is taken away from the supposition that things are different in those realms into which our observations do not penetrate. Naturally such a supposition remains logically possible but at the same time it would remain empty unless it can be supported by definite empirical reasons. Such reasons have not been offered by Exner, since he was solely concerned with demonstrating the *relative* justification of the statistical approach. But an actual occasion for such an approach is presented only when we deal with phenomena which show the nature of a "collective." This occasion was provided in the kinetic theory of gases, which, according to its atomistic assumptions, was concerned with mass phenomena, the detailed investigation of which is neither possible nor necessary for the establishment of the desired laws. But to demand the extension of this viewpoint over the whole of nature and over all the problems of the macrocosm finds no justification here. As a matter of fact the statistical conception in Exner's presentation of nature only enjoys a kind of shadow existence. It is not contested that in most realms the dynamic approach is adequate and that we are indebted to this approach for the most important advances in our knowledge of nature; it is merely claimed that the indispensability of this approach does not necessarily follow and that we must always reckon with the possibility that behind the strict causality there may lie only a statistical regularity. Therefore statistics still remains somewhat in the background; it is comparable to the Epi-

19. *Ibid.*, p. 667.

curean gods who do not dwell in the world itself but have their home in the empty spaces between the worlds. Exner emphasizes that the semblance of exact uniformity may perhaps disappear if, instead of measuring time in seconds, we used time units of billionths of a second, or even less.[20] Since, however, such refinement of instruments of observations is denied us, the question, when considered and judged empirically, ends with a *non liquet:* it points to certain possibilities of asking physical questions without claiming to anticipate a definite answer, either positive or negative.

20. *Ibid.,* p. 657.

CHAPTER 8

The Logical Character of Statistical Statements

THE CONSIDERATIONS connected with the second law of thermodynamics lead to an important transformation in the world view of physics, not only in a material but also in a formal sense. The difference between reversible and irreversible processes does not only concern events as such—it indicates that the conceptual representation of events is subject to two different basic types of law. In all processes which are related to the second law a transition to a type of statement becomes necessary that is foreign to the system of classical mechanics. It cannot be denied that this transition, this acknowledgment of statistics as an indispensable and equally valid factor in scientific knowledge, contains not only a physical but also a profound epistemological problem. However, the fact that this problem was usually expressed in a form which did not do justice to its intrinsic content has impeded and harmed its development and its deeper understanding. With statistics a new method and a new instrument of description was introduced into science, an instrument which has proved usable and eminently fruitful even in those areas where the existing classical methods failed. However, instead of comprehending the significance of this new method and of appreciating its particular logical character, instead of allowing it its due position in the systematic totality of physical determination, exactly the opposite path was chosen. Dynamic and statistical laws were not regarded as two complementary methods and directions, as two different modes of description; they were instead opposed as the "determined" and the "undetermined." Thus the new problems that were introduced by the second law and by the development of quantum theory were discussed under the heading of "indeterminism," a title which gives rise to the most dangerous equivocations. It seemed to open the doors to a *liberum arbitrium indifferentiae,* a state of freedom which was hardly distinguishable from caprice.

Such conclusions can only be combated by an analysis of the simple and sober meaning which physics attaches to its statements of probability. It is true that epistemology has a difficult and insecure footing here. The definition of the probability concept, as given in Laplace's classical theory and as adopted by virtually all textbooks on the theory of probability is now almost universally considered inadequate. The explanation of the so-called *a priori* probability as the quotient of the number of favorable cases to the number of equally possible cases ends in a circle: the expression "equally possible" has in the end no meaning other than the expression "equally probable." [1] Thus at this point a new foundation had to be attempted, which for the time being does not seem to have been brought to an entirely satisfactory conclusion. At any rate through this development certain basic features become increasingly clear and precise by means of which the character of scientific probability statements can be clearly defined and limited within the boundaries of definite conditions. It may be said generally that from an epistemological point of view the concept of probability can never supplant or supersede the concept of objective truth, because it is founded on the latter and contains the latter as an integral factor. It has repeatedly been emphasized that every probability is a logical function of certain truths and exists only relative to them.

C. S. Peirce, in his article "Chance and Logic," says: "To speak of the probability of an event, without naming the condition, really has no meaning at all." [2] Even the clearest and most consistent attempt in recent years to set up a purely subjective theory of probability, that of Keynes in his *Treatise on Probability* adheres entirely to this supposition. Keynes defines probability as a "rational belief" and he regards its numerical value as a measure of a definite "rational expectation." But the adjective "rational" is designed to preclude any confusion with purely subjective caprice. "Probability," emphasizes Keynes,

> may be called subjective. But in the sense important to logic, probability is not subjective. It is not, that is to say, subject to human caprice. A proposition is not probable because we think it so. When once the facts are given which determine our knowledge, what is probable or improbable in these circumstances has been fixed objectively, and is independent of our opinion. The

1. For a critique of the classical definition of probability see R. von Mises, *Wahrscheinlichkeit, Statistik und Wahrheit* (Vienna, 1928), pp. 61 ff.; trans. J. Neyman, D. Scholl, and E. Rabinowitsch, *Probability, Statistics and Truth* (New York, 1939), pp. 98 ff.

2. Quoted from E. Kaila, "Die Prinzipien der Wahrscheinlichkeitslogik," *Annales Univers, Fennicae Aboensis, Turku* (1926), pp. 32 ff.

> Theory of Probability is logical, therefore, because it is concerned with the degree of belief which it is *rational* to entertain in given conditions, and not merely with the actual beliefs of particular individuals, which may or may not be rational. . . . When we argue that Darwin gives valid grounds for our accepting his theory of natural selection, we do not simply mean that we are psychologically inclined to agree with him . . . We believe that there is some real objective relation between Darwin's evidence and his conclusions, which is independent of the mere fact of our belief, and which is just as real and objective, though of a different degree, as that which would exist if the argument were as demonstrative as a syllogism.[3]

Consequently, "probability" can never be defined except by referring it to a definite set of propositions to which truth value is ascribed. Without the declaration of such an objective system of reference, the statement of probability loses its meaning. As Keynes emphasizes, the statement *"a* is probable" is just as incomprehensible per se as *"a* equals" or *"a* is greater" unless we have further reference. This leads to the rejection of that purely negative explanation and justification of probability statements which is based upon the so-called principle of insufficient reason, a view which leads for example to the conclusion that if A and C are two totally unknown objects, the probability of A being identical with C is exactly equal to the probability that the opposite is true, or expressed numerically, is equal to $\frac{1}{2}$. Accordingly, cases concerning which we are equally ignorant could be designated as "equally possible," a statement which in its full strictness is valid only if we know absolutely nothing about which of the distinguishable cases will occur.[4] The remark that, as Leslie Ellis expressed it, mere ignorance is no ground for any inference whatever (*ex nihilo nihil*) decides against this principle of insufficient reason or of "indifference," as it has also been called.[5]

We have to go a step further in order to understand the character of statements of *physical* probability. It is not sufficient to interpret these

3. Keynes, *A Treatise on Probability,* pp. 4 ff.

4. This view is presented in recent literature for instance by K. Stumpf. See his essay "Über den Begriff der mathematischen Wahrscheinlichkeit," *Sitzungsbericht der Königlich Bayerischen Akademie der Wissenshaften, philosophisch-historische Klasse* (1892).

5. Ellis, *On the Foundations of the Theory of Probabilities,* 1842 (quoted by Keynes, p. 85). For a critique of the principle of insufficient reason cf. esp. J. von Kries, *Die Prinzipien der Wahrscheinlichkeitsrechnung* (Freiburg i. B., 1886), and *Logik* (Tübingen, 1916), pp. 595 ff.

statements in a subjective sense, even if this subjectivity is limited by the additional condition of rationality. The question can never be directed merely at our expectation of the event; it must be directed at the event itself. It has been said that the physical probability of an event "can depend only upon the circumstances which influence its happening, and never upon the degree of our knowledge." [6]

The application of probability assertions to empirical events can only be made intelligible, therefore, by an objective interpretation, for this alone can clarify the question as to the *quid juris* of these assertions, their validity with reference to the objects of experience.[7] Within the natural sciences the subjective theories have therefore been pushed more and more into the background. Their place has been taken by the attempt to interpret the theory of probability objectively as statements of the relative frequency of certain events within a series of such events. It cannot be denied that in this way the problem becomes considerably more serious and complicated in an epistemological sense and that it leads to all the difficult questions about the determinability of probability assertions.[8] But this complication cannot be avoided. In fact it is indispensable for characterizing and establishing the use of probability statements in physics. These problems do not arise as long as we remain solely within the realm of the probability calculus, for this calculus does not differ materially in its logical structure from other mathematical disciplines. The calculus of probability originated in the seventeenth century as a branch of pure, or to use S. Cantor's expression, "free" mathematics. Though many and varied fields have been gradually apportioned to it, there can be no doubt that their particular characteristics do not de-

6. M. von Smoluchowski, "Über den Begriff des Zufalls und den Ursprung der Wahrscheinlichkeitsgesetze in der Physik," *Naturwissenschaften, 6* (1918), 253 ff.

7. It is exactly for this reason that I find it impossible to see in statistical statements, instead of judgments concerning objects and actual relations, merely subjective or "reflective" opinions "about" the object. Thus Fritz Medicus, for instance, in his *Die Freiheit des Willens und ihre Grenzen* (1925), in which the attempt is made to determine "what credit is to be ascribed to modern physics for the philosophical conception of the idea of freedom," declares that statistical thoughts are nothing but "fabrications of subjective reflection." "Statistics treats everything as object, although nothing is grasped by it as object. There is in it no knowledge of reality: statistical laws always remain in the realm of the abstract, in the realm determined by mere reflection. Statistical judgments are not 'objectively' valid; they are not valid relative to an objective reality, but only relative to a picture created by intellectual operations" (*ibid.*, pp. 98 f.). If this conclusion is correct, there would be no justification, it seems to me, for any application of statistics to natural phenomena: for such an application always demands an "objective validity."

8. Cf. H. Reichenbach, "Kausalität und Wahrscheinlichkeit," *Erkenntnis, 1* (1930–31), 158 ff.

stroy its methodological uniformity. Leibniz was the first thinker who strongly emphasized this methodological character, and who demanded a strict analysis of probability statements, which according to him would add a new branch to logic. He held that this branch has its own rules and principles and that it must also have its own symbolism and algorism.

These basic ideas can be traced back to Leibniz' first scientific and logical sketches, which led him to the problem of the theory of combinations. What lends this theory its fundamental philosophical significance is, according to Leibniz, the fact that it is founded on a single basic concept, the concept of order, and that it is on this concept that all our knowledge is based—the knowledge of the ideal as well as of the actual, of the necessary as well as of the accidental. Thus Leibniz, even in the very first establishment of the basic rules of the theory, pointed to certain problems of physics to which these rules may some day be applied.[9]

The probability calculus retains its uniform character despite the wide range of applications of which it is capable, applications which as early as the seventeenth century included games of chance as well as such questions as population statistics and life annuities. The probability calculus is a purely mathematical discipline insofar as it proceeds from definite hypothetical starting points in accordance with strict rules—rules which yield results as certain as those in other branches of mathematics. Its purpose is to determine from the given probabilities in a number of initial collectives the probabilities in a collective derived from the initial ones, a task that can essentially be reduced to four basic operations, designated as selection, mixing, partition, and combination of collectives.[10]

The step toward empirical probability statements confronts us with new and difficult epistemological problems. However these problems are quite analogous to those which result whenever exact concepts are applied to empirical reality. Consequently, as far as I can see, they do not give rise to the expedient proposed by several investigators of constructing a new logic of probability emancipated from the principles and

9. It almost sounds like a prophecy of the kinetic theory of gases and of statistical physics when the 20-year old Leibniz in his paper "De arte combinatoria" (1666) writes: "Through these combinations it is not merely that geometry can be enriched by infinite new theorems, but (if it is indeed true that great things are composed of small, whether you call them atoms or molecules) this is then the only way of penetrating the secrets of nature" (*Opera philosophica,* ed. J. E. Erdmann, Berlin, 1839–40, p. 19).

10. See von Mises, *Wahrscheinlichkeit, Statistik und Wahrheit,* esp. pp. 30 ff.; English trans., pp. 40 ff.

demands of "classical logic," in which, for instance, the rule that every statement must be either true or false would lose its validity. There is no need to seize on such a desperate measure as long as any other way is available. It seems to me that of the modern theorists of probability von Mises has offered the simplest and most consistent solution. Von Mises bases his theory on the concept of the collective, which he defines by definite axiomatic requirements. He emphasizes that such "synthetic definitions" are indispensable whenever a scientific theory is to be constructed. We can never confine ourselves here within the circle of the immediately given, nor can we simply take our concepts out of the vague vernacular of daily life. It is necessary that an exact demarcation, a fixation and "rationalization," take place which points the way to each use of the concept and to each application to special cases. Generally speaking, von Mises defines the concept of the collective by means of two postulates: the postulate of the existence of a "limiting value of the relative frequency," and the so-called postulate of "randomness," the principle of the "impossibility of a gambling system." Let us consider their general methodological character, without for the time being going into further detail concerning contents. When von Mises explains a collective as being a mass phenomenon, or a repetitive process, an extended sequence of individual observations appearing to justify the assumption that the relative frequency of the occurrence of each particular observed feature tends toward a definite limiting value, he is emphasizing that such a collective is not an empirical object but an idealized conception similar to that of the sphere in geometry or of the rigid body in mechanics. Furthermore a rational theory can be built up only on the basis of such idealized concepts, which are particularly indispensable for the exact sciences. By means of this precise definition of the initial concept, the ambiguity constantly occurring whenever a statistical observation is confused with an inexact observation is prevented. A collective as such contains no inaccuracy within itself; rather it consists of a series of intrinsically exact observations. In any case statistics can only begin where univocal and precise observations are available. It is true that concrete statistics never lead to numerical series appearing immediately and exactly as collectives in the sense of the ideal concept here indicated; this, however, does not matter. What does matter is solely the discovery of those applications of probability theory based on this concept that are possible in the realm of empirical events.[11]

11. *Ibid.*, pp. 8 ff., 16 ff., 111 f.; English trans., pp. 8 ff., 20 ff., 206 ff. Cf. esp. "Über kausale und statistische Gesetzmässigkeit in der Physik," *Erkenntnis, 1* (1930–31), 189 ff.

It is obvious that this epistemological justification of statistical statements does not deviate materially from the path already indicated for the whole of exact science by Galileo. It has to start with the process which Galileo designated by the expression *mente concipio,* and which he used, for instance, in formulating his law of inertia.[12] It is just as impossible to find a collective in immediate experience as to find a "body independent of outside influences," but there is a significance in the search for empirical facts that approximate the requirements contained in both concepts. These facts can then be subsumed under the ideal, hypothetically established concept—that is, they can be determined by means of the conditions deducible as consequences from it. If this is granted, the relationship between induction and deduction is not significantly altered when we go from the dynamic laws of classical physics to statistical laws, as happens, for example, in the kinetic theory of gases. Leibniz, in his *Dissertatio de stilo philosophico Nizolii* objected to the view of Nizolius that all general statements must be capable of being traced back to statements about individual things or events. He distinguishes between general statements of a purely collective character and those to which we have to ascribe a distributive character. Propositions of the form "*X* implies *Y*" are, according to Leibniz, always of the latter type; they do not claim that each and every *X* which has been found or is yet to be found has the attribute *Y,* but that a universal functional relation exists between *X* and *Y,* that wherever *X* is given, *Y* necessarily follows. He deduces from this distinction between the distributive sense of universal statements and their purely collective sense that induction also, when it exceeds that purely subjective expectation which we can ascertain even in animals, when it is to be considered as a form of scientific inference and conclusion needs definite rational basic assumptions, and that it would lose all support without these *adminicula rationis.*[13]

However paradoxical it may seem at first glance, it can yet be said that this required distributive universality is by no means absent from statistical statements; in fact it is essential for their full understanding, because the designation "collective," which is admittedly connected with every statistical statement, refers to its object and not to its logical validity. The objects of statistical statements are mass phenomena or recurring

12. Cf. above, p. 82, n. 13.

13. Cf. Leibniz, *Philosophische Schriften,* ed. Gerhardt, *4,* 160 ff. In modern logic this sharp differentiation between the purely collective and the distributive universality is particularly emphasized by E. Husserl. He declares that "the forms 'A' and 'all A' are not identical in meaning; their difference is not of a 'purely grammatical' nature. They are logically different forms, expressing essentially different meanings." *Logische Untersuchungen* (Halle, 1913–21), *2,* 148.

events, but it does not follow that the statement as such consists of a direct description of what is observed in these phenomena. Here also there must take place at some point that μετάβασις εἰς ἄλλο γένος which leads us to the new type of statement, the statements of laws; here also our thought must rise from one discrete path to another by a kind of jump.[14] The level of mere particular statements must be abandoned, and must be replaced by some general, indeed universal assertion. In the probability theory of von Mises this step is designated by his axiom of the limiting value. The so-called law of great numbers receives a sharply defined meaning only through this axiom. Von Mises points out that this law, which was first promulgated by Jacob Bernoulli and further developed by Poisson in his *Recherches sur la probabilité des jugements* (1837), is so far ambiguous in that it appears at first as a purely arithmetical theorem, strictly provable mathematically but containing no statement concerning the course of a series of events. When we want to pass from this theorem to reality, the law of great numbers must be put into a new form; it must be expressed in the form that in certain series of events the relative frequency of occurrence of an event approaches a definite limiting value as the number of observations increases indefinitely. One can attempt, in constructing a probability theory, to make it independent of this particular form; but the removal of the axiom of limiting value is only possible if another assumption is put in its place which likewise contains a rational, a methodological, demand and not a directly observed fact.[15]

The hypothesis of a probability function, on which Reichenbach, following Poincaré, attempts to base the applicability of probability laws to the things of reality, also points essentially in the same direction.[16] Reichenbach vacillated in the interpretation and epistemological justification of this hypothesis. In his first publication he not only emphasized that this hypothesis is not purely a statement of experience; he did not even hesitate to designate it a "metaphysical principle of scientific knowledge." It is true that the particular form of probability function must be determined by experience; but that such a statement can be made at all about

14. Cf. above, p. 55.

15. Thus, for instance, Karl Popper, *Logik der Forschung* (Vienna, 1935), pp. 128 ff., attempts to replace the "axiom of limiting value" by the "methodological decision never to explain the effects of reproducible regularities as being based on an accumulation of accidents."

16. Cf. H. Reichenbach, "Philosophische Kritik der Wahrscheinlichkeitsrechnung," and "Die physikalischen Voraussetzungen der Wahrscheinlichkeitsrechnung," *Naturwissenschaften, 8* (1920), 46 ff., 146 ff. See also "Wahrscheinlichkeitslogik," *Sitzungsbericht der Berliner Akademie, physikalisch-mathematische Klasse* (1932), pp. 476 ff.

an unlimited number of cases can never be taught us by experience. To the hypothesis of causal connection of events we must add another principle, concerning the ordered distribution in time and space; however, from a logical and transcendental viewpoint both principles are alike in character, being general "conditions of the possibility of experience." [17] In Reichenbach's later writings a fundamentally different foundation is attempted. Here the determinability of probability assertions is based on the principle of induction, and an "inductive determinability" is spoken of. Reichenbach sees no objection to the circle which threatens to entangle the argument. He declares that such a circle only becomes embarrassing when we adhere to the presuppositions of classical logic, which, however, are unable to do justice to the peculiar mode of thinking required by probability. Reichenbach is led to this position because by this time he has replaced the point of view of idealism by that of positivism. According to him the latter view permits statements about the "limit" only insofar as they are translatable into statements about the convergence of the process near the limit, and since nowhere in the process of approximation can there be reference to "events existing and proceeding of themselves," such a statement cannot be considered valid for the limit either. But with this concession to strict positivism Reichenbach again comes dangerously close to precisely that "psychologism" which his theory is designed to oppose. He states that "when we are asked why we believe in probability laws, we can only answer (there being no logical justification for these laws) that we cannot believe otherwise." [18]

The problem, however, cannot be solved by reverting to "common sense" as the proper source of knowledge. It would bring us back to the time before Hume, for his opponents, the founders of the philosophy of "common sense," constantly confronted him with just this argument. "If there are certain principles," says Reid, "which the constitution of our nature leads us to believe, and which we are under a necessity to take for granted in the common concerns of life, without being able to give a reason for them—these are what we call the principles of common sense; and what is manifestly contrary to them, is what we call absurd." But Hume had anticipated this method of proof and had clearly formulated the intrinsic logical sense of his question. "My practice, you say, refutes my doubts. But you mistake the purport of my question. As an agent, I am quite satisfied in the point, but as a philosopher who has some share

17. *Der Begriff der Wahrscheinlichkeit für die mathematische Darstellung der Wirklichkeit* (dissertation, Leipzig, 1916), pp. 26, 45 ff., 61 ff.

18. See Reichenbach, "Kausalität und Wahrscheinlichkeit," *Erkenntnis, 1* (1930–31), 169 ff., 187.

of curiosity, I will not say scepticism, I want to learn the foundation of this inference." [19]

The question which is here raised retains its full import even when we pass from dynamic to statistical conformity to law, when we exchange the viewpoint of strict causality for that of probability. For even pure probability assertions are invariably supported by some assumptions of constancy which may be expressed differently in the various theories, being at times designated by the introduction of a probability function, at times by the demand of a strict limit to the relative frequency of occurrence of certain observable indices of events. Without the hypothesis of such a constancy, however defined in a particular case, the law of great numbers would lose all footing and objective importance. It is therefore impossible to base probability assertions on the principle of induction, unless to the latter is ascribed in turn not only a collective but also a distributive universality. Among modern logicians of probability theory this state of affairs has been emphasized most sharply by Johann von Kries. He declares: "it is evident that whenever we want to establish the admissibility of one conclusion by basing it on another and the latter by basing it in turn on a third and so on, we are bound, after a longer or shorter regression, to arrive at final assumptions, for which such a reduction no longer exists, for the validity of which some radically different principle has to be drawn on. . . . The probability which we utilize for any empirical knowledge of objective significance will therefore have to be based upon the intellectual value of conformity to law—that is, upon the principle of homogeneity." [20]

But this appears to lead to the most fundamental and difficult question, the question which has time and again occupied the critique of knowledge and which has not infrequently caused it to deviate from the sure path of analysis. Is it still possible and is it meaningful to speak of objective laws once we have entered the realm of accidental events? Is not conformity to law on the part of the accidental a contradiction in terms, is not an accident precisely that which by definition precludes any orderly determination or regulation? This question can only be answered if a sharp distinction is made between the various meanings of the term "accident." In common usage this word is a veritable chameleon that can take on any color. But this multiplicity of meanings does not disappear even when we pass to theoretical considerations and scientific usage. "Accident" is invariably a relative and negative concept, the

19. T. Reid, *Inquiry into the Human Mind* (2d ed. Edinburgh, 1765), chap. 2, sec. 6; Hume, *Enquiry Concerning Human Understanding*, Pt. IV, sec. 2. Also cf. Kant's rejection of the derivation of the causal principle from a "compulsion of thought"; see above, p. 59.

20. Von Kries, *Logik*, pp. 409, 426.

understanding of which requires the positive concept to which it refers implicitly, and it is this reference that continually changes as we enter different spheres of thought.

In order to make this quite clear we can start out from the contrast between the mythical world view and the world view of empirical and theoretical science. Starting from the latter, and accepting its categories and principles as the solely and universally valid norms of truth, we then look upon the mythical world as a texture of the most peculiar and improbable accidents. It is a world within which no entity has its definite place and no event its firm and regular course. A "thing" can not only change its place freely but also be in different places at the same time; it can continually transform itself, assuming now this form, now that, and these forms need not be connected by any fixed rules whatever.

There seem to be no determinations or restrictions here; it is still possible for "all to become all." If, however, we now replace the standpoint of the theoretical and scientific world view by the characteristic mythical way of looking at existence and events, the picture is transformed in a remarkable manner. The functions seem to be exactly reversed—that which appeared incomprehensible, merely accidental from the former point of view, is transformed into the opposite when we change the point of reference. In innumerable cases in which empirical scientific knowledge waives any definite explanation, and which accordingly it acknowledges as accidental, mythos imperially demands an explanation. For mythos there can be no accidental event in the course of human existence; for example, no accidental failure of an enterprise, no accidental sickness, and no accidental death. Everything has to be strictly determined; but it is determined not by general principles underlying events, as expressed in the sciences by "natural laws," but by individual purposes and forces servile to them. Magical and demonic forces rule and penetrate all events and there is nothing that can elude them. This magical causality therefore penetrates much further into detail than the causality of theoretical science; it leaves nothing to pure accident; it demands a definite cause for every case of illness or death, to be found in some individual act of will, in some antagonistic sorcery worked on the person. Here we find, in a certain sense, instead of an absence of all causal explanation, a hypertrophy, a superabundance of the causal impulse, and only the character of the causes differentiates between mythical and theoretical thinking, in that mythical causality is based on intentions and acts of will as contrasted with rules and laws.[21]

This magical, mythical form of causality was declared invalid by

21. For further on this subject see my *Mythical Thought,* Vol. 2 of *Philosophy of Symbolic Forms.* Also cf. the abundant material in L. Lévy-Bruhl, *La Mentalité primitive,* Paris, 1922.

Greek science and classical Greek philosophy. The seemingly mythical explanations were rejected as arbitrary and accidental. They are both, not because they deny the causes but because they consider them as changing from case to case. They invent a particular cause for every particular event. In place of this mythical inventing, Greek philosophy demands a definite and universally valid form of thought, an objective law, grasped and formulated in thought in order then to be recognized in nature. Only at this point does nature cease to be the stage and battlefield for arbitrary demonic forces; nature becomes an order, a cosmos. This concept of cosmic causality was first speculatively established and developed in the thought of the Pythagoreans. But its real fulfillment and maturity was reached in Greek atomism, which thus not only in its contents but also in its form steps to the forefront of all scientific explanation of the universe. Nothing in the world happens "at random" (*μάτην*), but everything by virtue of a definite logical "necessity" (*ἐκ λόγου τε καὶ ὑπ' ἀνάγκης*). The necessary is juxtaposed to the accidental, as the general to the particular and the definitely established to the phantastic and arbitrary. But the development of this line of thought by no means ends here, for now we have to concern ourselves with the distinction between the various forms of necessity, and with the task of referring each one to its proper domain. Aristotle's objection to the system of atomism, is that it has not made this distinction properly, and that owing to this deficiency it has not attained to a complete and truly satisfying explanation of nature. In order to prove this, Aristotle had to develop a new definition of "accident" and "necessity," grasping the relation between the two in an entirely different way. "Accidental" is, according to Aristotle, that which does not follow from the essence of a thing; this essence, however, is determined by the form, the *οὐσία* by means of the *εἶδός*. An attribute is accidental (*συμβεβηκός*) when it is not inherent in the subject, when its presence or absence does not jeopardize the existence of the subject. And within the realm of events we have to consider as accidental everything that may in itself be determined but which owes its determination not to the form but to mere matter. Whenever in nature we encounter any deformity, or wherever the form, instead of appearing in utter perfection, appears to be blemished, we have to ascribe this to the resistance of matter. The *causa materialis* consequently always remains accidental insofar as it is in opposition to that ultimate and "real" causality which can be found exclusively in the *causa formalis* and the *causa finalis*. Matter therefore appears as necessary and accidental at the same time; necessary because its being determines all its effects; accidental because this being as such is incomplete, because it is merely a factor which has no meaning or reality by itself. Matter is the source of natural necessity

(ἀνάγκη) as well as of chance (αὐτόματον or τύχη). The ultimate and true ground lies in the final causes, whilst matter must always be considered merely as an accompanying cause, as a συναίτιον, to which nothing but a hypothetical necessity can be ascribed (ἐξ ὑποθέσεως ἀναγκαῖον).

Another shift in the use of concepts and in the contrast between the accidental and the necessary takes place in modern philosophy. Furthermore, the specific change in meaning corresponds to the general tendency distinguishing modern from ancient philosophy. The problem is no longer confined to natural philosophy and metaphysics; it is brought into and, in a sense, restored to the field of epistemology. This thought process had already reached its conclusion with Leibniz; for him the difference between the necessary and the accidental serves not so much to separate the types of being from each other as to designate the fundamental forms of knowledge in their characteristic significance and to distinguish them from each other. The concepts of the accidental and the necessary now appear each as a specific characteristic of definite types of truth: the *vérités nécessaires* stand over against the *vérités contingentes*. The former can be understood and established from purely logical considerations; for their establishment they require no other principle than the postulate of identity and contradiction. For they are purely hypothetical statements, containing an apodictic "If-then," but no assertorial statement about a particular, definite reality. They therefore do not express a particular feature of the actually given world; they are valid rather for the totality of possible worlds. As soon, however, as we take the step from the possible to the actual, the contrast between the accidental and the necessary becomes evident, because the existence of the empirical world does not rest on its logical necessity and can never be adequately explained by it alone. It is based upon a selection which the divine intellect has made among the possible worlds and this selection is determined not by logical but by teleological considerations. This teleological principle—*le principe du meilleur* as Leibniz calls it—therefore constitutes the ground of the actually given world; and it is therefore at the same time the principle for all those accidental truths which are based on it. Natural laws, as distinguished from logical and mathematical laws, are therefore contingent truths. But this contingency in no sense contradicts their thoroughgoing definiteness and certainty or the possibility of their rational establishment. In fact the relation is exactly the reverse; the accidental truths are so far from being unfounded that the principle of sufficient reason is to be regarded as their proper source and their characteristic epistemological principle. The principle of contradiction is the law of the possible, the law of the essences which originate in the divine intellect; the principle of sufficient reason is the law of the ex-

istent, as it results from the "selection of the best." In order to understand this existent, we always have to go back to its ground. The *principium rationis sufficientis* is, therefore, according to Leibniz, as much a principle as a postulate; he frequently expresses it as *principium reddendae rationis.* A rational basis has also to be found for everything accidental—that is, everything that cannot be arrived at analytically from logical and mathematical principles; this and nothing else is the meaning according to Leibniz of the principle commonly designated as the "causal principle." [22]

It is evident that Leibniz has set himself a new, important methodological task, but also that his solution does not come up to his expectations. He wished to consider the distinction between the accidental and the necessary from a purely epistemological viewpoint; he sought to base it on an analysis of the concept of truth, and to derive it purely from this concept. However, it was the theory of principles itself which led him back to the basic problems of metaphysics. The difference between necessary and accidental truths lies in the fact that the former have their source in the divine intellect, the latter in the divine will. This intermingling of epistemological and metaphysical considerations was obviated by Kant, who established the principle that not an improvement but a distortion of the realms of knowledge results when their boundaries are allowed to run into each other. Once more the distinction between contingency and necessity has to take on a new form. It does not refer to the world as such, and can consequently not be reduced, as Leibniz thought, to the difference between possible worlds and the actual world. Its origin must be exhibited in the system of empirical knowledge and it must be demonstrated as a major factor in this system. Whether the world has existed from eternity or whether it arose out of a chance act of creation, whether an absolutely necessary being exists—these are questions that are rejected. Knowledge is not equal to them, and they would soon entangle reason in insoluble antinomies. Like all other categories, the categories of necessity and contingency cannot be extended beyond the limits of experience but must themselves be understood and defined as one of the conditions of the possibility of experience. But now there follows a peculiar consequence. If we take the general principle of causality into consideration, we can regard it with equal justification as accidental or necessary, de-

22. "There are two principles of all reasoning, namely the principle of contradiction . . . and the principle of giving a reason . . . to the effect that for every truth a reason can be given, or, as they commonly say, nothing happens without a cause. This principle is not wanting in arithmetic and geometry, but it *is* wanting in physics and mechanics." *Philosophische Schriften,* ed. Gerhardt, *7,* 309.

pending on the point of view which we choose. It is necessary because every individual empirical statement is based on it, and because it precedes all empirical judgments as a synthetic judgment *a priori*. On the other hand, it is accidental because the whole of experience to which it refers and on which it has to base its justification is not given in any other way than purely factually. The principle of causality can be demonstrated as a *conditio sine qua non* for the fact of mathematical science; but this fact must be presupposed, though it cannot be proved to be absolute or plainly necessary. This paradoxical state of affairs is clearly formulated by Kant. He points out that if we wish to consider as necessary only that which can be attested by an apodictic proof, provided this proof is intuitive, no necessity can be ascribed to the causal principle. For our reason indeed

> establishes secure principles, not however directly from concepts alone, but always only indirectly through relation of these concepts to something altogether contingent, namely, *possible experience*. . . . Thus no one can acquire insight into the proposition that everything which happens has its cause, merely from the concepts involved. It is not, therefore, a dogma, although from another point of view, namely, from that of the sole field of its possible employment, that is, experience, it can be proved with complete apodictic certainty. But though it needs proof, it should be entitled a *principle*, not a *theorem*, because it has the peculiar character that it makes possible the very experience which is its own ground of proof, and that in this experience it must always itself be presupposed.[23]

I return now from this historical digression to the proper theme, to the question of the logical character of probability statements. The historical considerations were designed to show only that the concepts of "accident" and "order according to law" do not stand in a relation of contradictory opposition, as is frequently supposed. Such an understanding of the concepts is not to be found in Aristotle, Leibniz, or Kant. They all endeavor rather to show them in a complementary relationship, although at the same time they emphasize their sharp dissimilarity. It may be asked whether such a relationship of mutual supplementation is also valid for the use made of probability concepts in physics. As long as this use is not unambiguously defined, it is difficult to answer this question. But when we take those fields into consideration, in which the concept of

23. *Kritik der reinen Vernunft* (2d ed.), p. 764 (ed. Cassirer *3*, 499); trans., p. 592.

chance enjoys a clearly defined significance and a precise application, it can be shown that here also it is in no instance in direct opposition to the causal concept as such. In the kinetic theory of gases this situation is evident in its most simple and transparent form. Here the behavior of molecules on collision is based upon the classical laws of impact, the laws of conservation of energy and of momentum, to which are added statistical assumptions concerning the number of collisions, the mean free path, the mean velocity of the molecules, etc., etc. From these assumptions the laws of gas pressure, of viscosity, of diffusion, etc. are calculated with precision, although, to be sure, only hypothetically; their final verification rests in every case on experience. Circumstances seem to be somewhat different with the various machines dependent on chance. But they all so far agree that out of trifling or perhaps infinitesimal inaccuracies during the course of the process, large finite differences are obtained in the end result. It is therefore possible actually to define the concept of chance through the principle "small causes, large effects," a formulation which makes it immediately obvious that it would be quite wrong to speak of chance as falling outside the scheme of causality as such. Smoluchowski says in this connection that "chance is the name for a particular type of causal relation. It is said ordinarily that an event y depends upon chance, if it is a function of a variable cause (possibly also unknown as to value, or purposely ignored) or partial condition x of such a kind that its occurrence depends upon a very small variation of x." However it is further shown that this commonly accepted conception of chance is not suitable as a foundation for an accurately defined concept of probability. "One can only then speak of a mathematical probability law $W(y)$, concerning a quantity y, when the causal relation $y = f(x)$ in addition to the above mentioned property has another special property, namely that the distribution of y, at least within certain limits, is independent of the type of distribution function $(\phi(\chi))$ which determines the relative frequency of x." [24]

From all this we can conclude that "causality" and "probability," "order according to law," and "accident," not only can but must exist side by side when we want to determine an event as completely as possible. In classical physics causality refers essentially to the knowledge of the course of the event, and probability to the knowledge of its initial conditions. From the two combined there arise the theorems of statistical mechanics which, as Boltzmann for instance emphasizes, are

24. Smoluchowski, "Über den Begriff des Zufalls und den Ursprung der Wahrscheinlichkeitsgesetze in der Physik," *Naturwissenschaften, 6* (1918), 253 ff.; concerning the various types of "machines dependent on chance," cf. von Mises, *Wahrscheinlichkeit, Statistik und Wahrheit,* pp. 142 ff.; trans., p. 247.

strict consequences of the assumptions made; in their application to events in nature, however, they are the prototype of a physical hypothesis.[25]

If one wishes, as von Kries did, to differentiate the two types of laws which are thus obtained as "nomological" and "ontological" laws, it becomes clear that the two nowhere contradict each other, that no factual or methodological conflict exists between them. The only requirement which restricts the probability approach, but which already follows from the general determination of scientific knowledge and hence needs no special formulation, is that this approach must be nomologically permissible—that is, not contrary to a known law of nature.[26]

The characteristic difference between probability laws and dynamic laws nevertheless persists, but on the other hand it becomes clear how the two interweave and how only in this way the universal form of "order according to law" arises. Besides, the methodological difference which here appears is by no means confined to statistical mechanics; it has an exact analogy in Newton's mechanics. The latter also must acknowledge a realm of contingency in addition to the realm of necessity, and from this distinction serious epistemological problems and difficulties arise. Newton fully realized that the equations he established are by no means in a position to answer all the questions put to us by the structure of the universe. Newton's law of gravitation determines what will happen under the presupposition of a certain distribution of masses in space; however, it does not give any information concerning the state of this distribution. Newton concludes, in the face of this relative uncertainty, that we have to introduce at this point a different kind of ground; he takes refuge in teleological considerations. The fact that the orbits of the different planets are nearly in the same plane and that their movements follow the same direction, and that the same is true of the satellites which accompany the individual planets: all this cannot have "merely mechanical" causes. This marvelous arrangement of the solar system could only arise from the dominion of a wise and powerful being. Here we have to go back to an immediate divine decree: *Deus corpora singula ita locavit.*[27]

It was Kant's ambition to extend the boundary at this point. His *Allgemeine Naturgeschichte und Theorie des Himmels* is expressly intended to find an answer for Newton's problem remaining strictly within the limits of "mechanical causes." But Kant, in his critical period, did not revert to this hypothesis about the origin of the universe; he now measured with a far stricter methodological and empirical yardstick, in

25. "Statistische Mechanik," p. 361.

26. Cf. von Kries, *Logik,* pp. 623 ff.

27. Newton, "Scholium Generale," *Principia,* ed. T. Le Seur and F. Jacquier, *3* (Geneva, 1742), 672 f.

the face of which the former mode of proof collapsed. It may be said generally that since Galileo, classical physics has renounced the task of answering any question "why?" and that it is only by virtue of this renunciation that it has been able to solve the specific task which it has undertaken. It no longer seeks to know *why* events occur but only *that* they occur and *how* they occur, and it is satisfied with establishing definite laws for both the nomological and the ontological structure of the world. In field theory, also, the field appears ultimately as a reality *sui generis* whose constitution can only be described and no longer explained—that is, referred to something more ultimate. We encounter such limitations of the question "why?" on all sides; but they are no barrier to but rather the condition for the determination which science is seeking.

PART FOUR

The Problem of Causality in Quantum Theory

CHAPTER 9

The Foundations of Quantum Theory and the Uncertainty Relations

THE QUANTUM THEORY, as has been repeatedly emphasized, stands in far greater contrast to classical physics than does the general theory of relativity. The latter, in spite of its fundamental transformation of the concepts of time and space, can be incorporated without great difficulty into the mode of thought of classical physics. Planck says of it that it has brought classical physics to its consummation, as it were, in that by the amalgamation of time and space, it has also united the concepts of mass and energy and those of gravity and inertia in a single higher point of view. Of the quantum theory, however, he remarks that it had the effect of a dangerous foreign explosive which has already caused a gaping rift throughout the entire structure. "The introduction of the quantum hypothesis therefore does not represent a modification, as was the case with relativity theory, but a breach in the classical theory." [1]

Yet it took a considerable time before this effect of the hypothesis was fully realized. Its first introduction did not take on the form of a revolutionary act. It was not presented as a declaration of war against classical physics, but merely as a calm development taking place in one of its branches. When Planck turned to the question of the distribution of energy in the normal spectrum of heat radiation, he had no intention of overthrowing the structure of classical physics; rather he sought to strengthen and develop it by bridging the gap between two fundamental fields which till then could not be united in a satisfactory manner. Starting out from the thermodynamic laws of thermal equilibrium, he sought an explanation, based on thermodynamic principles, of electrodynamic phenomena, of the laws of emission and absorption of light and heat radiations. However, this unification only succeeded after the introduc-

1. "Physikalische Gesetzlichkeit" (1926); *Wege zur physikalischen Erkenntnis,* pp. 169 f.

tion of the hypothesis of the elementary quantum of action, by the assumption that the interchange of energy between oscillators takes place only in integral multiples of a definite quantity ϵ. This requirement broke with the basic presuppositions of classical theory, according to which the energy of a wave spreads continuously in space; Planck was manifestly concerned to keep this discrepancy at a minimum compatible with experience. In his *Lectures on the Theory of Heat Radiation* he states: "In setting up the dynamic laws we shall proceed in a rational manner in such a way that we make the admittedly necessary deviation from the laws of classical electrodynamics as small as possible. For this reason we adhere completely to classical theory, considering the influence of a field of radiation on an oscillator." [2] Planck stays within this general tendency of thought when, in developing his "second theory," he drops the assumption of quantized *absorption* entirely and only requires that *emission* takes place discontinuously and only when the energy U of the oscillator amounts to an integral multiple of the elementary quantum $h\nu$.[3]

Even in 1912 Planck, in the preface to the second edition of his lectures, still expressly warned against too rapid a generalization of the basic conception of quantum theory, for nothing could be more harmful to the healthy development of a new hypothesis than an overstepping of its boundaries. For this reason he was always in favor of making the connection of the quantum hypothesis with classical dynamics as close as possible and of breaking the barriers of the latter only when the facts of experience left no other choice.

The further development of the theory, however, soon went beyond this initial limitation in which its originator attempted to keep it. In Bohr's theory both emission and absorption always take place in quanta. Then a development ensued which was most significant and indicative from a methodological point of view: what was originally the quantum *law* became the quantum *principle*. Its applications were no longer confined to individual branches of physics, however extended; it came to be understood and used rather as a general point of view, as a postulate of scientific knowledge. In a short time this change, this transformation of the quantum hypothesis from a "statement of law" to a "statement of principle" was completed. Totally different and apparently heterogeneous groups of phenomena are brought under this principle: the theories of heat radiation—of the photo-electric effect and of specific heats—are brought together, and interrelated in a new manner by this

2. *Vorlesungen über die Theorie der Wärmestrahlung* (3d ed. Leipzig, 1919), p. 148.

3. Concerning Planck's "second theory," cf. A. March, *Theorie der Strahlung und der Quanten* (Leipzig, 1919), Sec. 2, chap. 3.

principle. When Einstein in 1905 introduced the concept of the light quantum he designated it as a "heuristic point of view concerning the emission and transformation of light." This viewpoint was at first chosen from purely theoretical considerations; it was experimentally tested, and confirmed in detail ten years later by Millikan.[4]

How quickly this development of thought progressed and how irresistibly it encompassed every field of physics is shown by the enthusiastic exuberance with which Sommerfeld describes the world picture of quantum theory in the preface to the first edition of his *Atombau und Spektrallinien* (1919). "What we today can hear from the spectra is a veritable atomic music of the spheres, a carillon of perfect whole-number relations, an increasing order and harmony in multiplicity."

And yet in this seemingly so harmonious system strong dissonances were soon noticed. The steady and linear progress which is evident in the development of classical physics from Kepler and Galileo to Newton, and then from Newton to Lagrange, is absent in quantum theory. It could not attain such a simplicity and directness for the simple reason, among others, that it was constantly threatened by a dualism in principle. The quantum requirement broke through the system of classical mechanics and electrodynamics; but it was nevertheless forced to continue using this system. This dual position constitutes from the beginning an epistemological paradox. In order to overcome or alleviate it, it became necessary at the very least to attempt a definite connection between the two languages which physics now had to employ continuously and side by side. General instructions had to be offered establishing the rules for the use of the two languages and clarifying the manner in which the concepts of quantum theory and those of classical mechanics and electrodynamics were to be combined and harmonized. Classical theory as such had to be modified in essential features, because it was unable to render any account of the basic empirical facts of atomic physics—the fact of the stability of atoms and of the existence of sharp spectral lines—and because it even seemed to be entirely irreconcilable with them. But it was just as impossible simply to sacrifice its laws, which within macrophysics had been tested and confirmed by a wealth of observations. Bohr's "correspondence principle" attempted to show the way out of this dilemma; it became, in a sense, Ariadne's thread in the labyrinth of quantum theory. The relation between the interpretations of radiation phenomena by the electromagnetic theory of light and by quantum theory is here established by showing that in the case of moderately

4. For details see A. J. W. Sommerfeld, *Atombau und Spektrallinien* (4th ed. Braunschweig, 1924), pp. 42 ff.; 3d ed. English trans., H. L. Brose, *Atomic Structure and Spectral Lines* (London, 1923), pp. 39 ff.

large quantum numbers the expressions governing the intensity and polarization of spectral lines become more and more interchangeable, and with sufficiently large quantum numbers approximate one another asymptotically. The correspondence principle, as Bohr emphasizes, is an expression for the endeavor, despite the basic opposition between the postulates of quantum theory and classical theories, to utilize by suitable reinterpretation every feature of the classical theories in the development of the quantum theory.[5]

Accordingly, the contrast between the classical viewpoint and that of quantum theory was not to be obliterated, nor could the chasm between the two be properly bridged. But by means of the correspondence principle a methodological, heuristic maxim was set up which was to guide the course of research and instruct it to use the different types of laws in such a manner as to lead to a unified description of natural events.[6]

The same endeavor gave rise to the establishment of Ehrenfest's "adiabatic hypothesis." Here also the requirement was made primary that the stationary states of an atomic system—excepting the phenomena of radiation—should be calculated as far as possible according to the laws of classical mechanics. The latter, in conformity with the principle of "adiabatic invariance" should hold not only for isolated systems but also when external influences acted on the system, provided only that the influence takes place infinitely slowly. The "adiabatically invariant" quantities appeared as those which were immediately "quantizable"; it was therefore necessary to find such quantities in the classical laws of motion in order to establish the connection between the different basic types of law.[7]

With this the immediate requirements of investigation in physics appeared to be satisfied for the time being. But it is understandable that this conception could not serve indefinitely, because, judged from an epistemological point of view, it bears much more the earmarks of a compromise than those of a final solution. We certainly do not have a satisfactory situation when definite concepts are accepted for the time being only to be subsequently corrected through restrictions to certain

5. N. H. D. Bohr, "Atomtheorie und Mechanik" (1925), in *Atomtheorie und Naturbeschreibung* (Berlin, 1931), p. 24.

6. For the history and formulation of Bohr's correspondence principle I refer to the concise presentation by H. A. Kramers, "Das Korrespondenzprinzip und der Schalenbau der Atome," *Naturwissenschaften, 2* (1923), 550 ff. Also cf. H. A. Kramers and H. Holst, *The Atom and the Bohr Theory of Its Structure* (London, 1923), pp. 139 ff.

7. Further details in P. Ehrenfest's "Adiabatische Transformationen in der Quantentheorie und ihre Behandlung durch Niels Bohr," *Naturwissenschaften, 11* (1923), 543 ff.

conditions and thus alone really justified and certified in their application. The quantum rules which had to be applied in the older quantum theory resembled in a certain sense a *deus ex machina.* They were heteronomous rules that could not be based on the classical principles. From the latter standpoint, Bohr's frequency condition, despite its indispensability and its extraordinary fruitfulness, remained a mere expedient which one faced without any real theoretical comprehension. It retained, employing an expression used by Hilbert in a different connection, something of a *Verbotsdiktatur.* It was postulated of certain magnitudes in the older theory that they had to be "quantizable," that they could only have integral values. The question remains whether it is possible to grasp this prohibition not only in its "what" but also in its "why," not only in its ὅτι but also in its διότι. It is at this point that the work of the new quantum mechanics begins. Schrödinger, in constructing his wave mechanics, aimed explicitly at bringing the quantum rules closer to an objective understanding instead of letting them appear as mere external expedients. He sought a method for the development of quantum rules entirely free from the necessity of arbitrarily postulating that any particular magnitude had to be an integer. What was demanded was a uniform structure of physics embracing ordinary mechanical phenomena (in which quantum conditions played no appreciable role) as well as the typical quantum phenomena. This goal was attained in a surprising manner by reducing the physical problem in question to a mathematical one. The differential equation set up by Schrödinger can only yield a single-valued, finite, and continuous solution (as can be proved generally from the theory of differential equations) when the parameters appearing in it have certain discrete values. These "eigenvalues" thus give us the particular discrete values of a system, without our having to revert to any special assumptions whatever. Thus Bohr's stationary states were explained as the characteristic frequencies of oscillation of Schrödinger's waves.[8]

From the point of view of the new wave mechanics the question of "quantization" is now seen in a new light; the older theory and its formulation and interpretation of quantum conditions appeared only as a first approximation.[9] The requirement that the wave functions be single valued and finite now furnishes the desired διότι of the quantum rules. The latter need therefore no longer penetrate the world of physics from "outside" (θύραθεν) like Aristotle's νοῦς; they appear understandable and justified from within. In this respect a clear and steady progress is ap-

8. Cf. E. Schrödinger, *Vier Vorlesungen über Wellenmechanik* (Berlin, 1928), Lecture 1; English trans., *Four Lectures on Wave Mechanics,* London, 1928.

9. Cf. for instance L. de Broglie, *Einführung in die Wellenmechanik,* chap. 17.

parent when we follow the development from Planck's law of radiation, to Bohr's model of the atom, and thence to the new quantum mechanics. That the theory has not fully reached its goal and that it has not spoken its last word seems today universally agreed, but it is equally certain that the still existing difficulties can be solved only by further research. We are here entirely concerned with the discovery and establishment of "special laws of nature," which, to use Kant's phrase, are "accidental to our reason," and about whose validity, therefore, only empirical observation can decide.

It is an entirely different question whether, and in what manner, the significance of the causal principle has changed because of this shift from the classical conception to that of quantum theory, whether this principle has become invalid or empty for scientific knowledge. An answer to this question is not possible until the sense of the causal law itself has been determined, until it is stated what is and what is not meant by it. Our entire preceding investigation was directed at this problem. We have tried to show that the customary formulation of the causal principle as expressed by Laplace's formula, wholly fails to do justice to its contents or its previous classical application. The objections which have been raised (rightly, we believe) against this formula are accordingly by no means objections against the principle of causality. We find the essential significance of the causal relation, if interpreted in a critical rather than in a metaphysical sense, to be that it contains a statement not immediately about things but about experience, by which and in virtue of which alone things, as objects of knowledge, can be given us. It expresses something about the structure of empirical knowledge; it determines the individual phases of the path by which empirical knowledge strives toward its goal, the goal of knowledge of objects. Our problem, accordingly, can only be whether this determination of means and ends has suffered an essential alteration through quantum mechanics and its results; whether science must henceforth change its direction and radically alter its methodology.

In answering this question I shall not consider the uncertainty relations for the time being; their significance will be discussed later. Unquestionably they have uncovered a characteristic and decisive feature of the mode of observation and cognition of quantum theory; but this feature seems to me to concern not only the structure and articulation of the physical system itself but even more the basis on which it is founded. These relations are not concerned with that hierarchy of types which our previous analysis attempted to present in detail, but rather with the first step, that of statements of the results of measurements, within which a particular, previously unnoticed set of problems presents itself. The

uncertainty relations, as expressed by Planck, deal with the uncertainties which arise on transferring an occurrence from the world of sense into the world picture of physics or vice versa, an uncertainty which, it is claimed, can never be entirely eliminated, no matter to what extent methods of measurement will be refined. But the uncertainty thus obtained neither justifies nor favors those far-reaching consequences for metaphysics or our *Weltanschauung* which are often derived from it. Schrödinger once differentiated between a conservative and a revolutionary viewpoint within quantum theory. The former assumes that every law of nature, even if we can only formulate it statistically, is based on a strictly dynamic law; the latter makes the assumption that the supposedly necessary connection determinable in natural events is merely the result of innumerable elementary occurrences which take place entirely at random. One can choose between the two points of view, since a definite decision can be neither arrived at through experience nor obtained through rational deduction.[10]

But even when one turns to the statements of the avowed revolutionaries of modern physics, it becomes obvious that they in no way encourage the metaphysical consequences frequently drawn from them. For none of these revolutionaries wanted to dispense altogether in their actual physical procedure with conformity to law of events; rather they ask how they may express and establish this conformity unobjectionably under the conditions of our observation of nature. Bohr certainly did not deny the validity of the causal concept as such; he emphasized repeatedly that it is indispensable for every interpretation of results of measurements. He designates as an illusion the idea that the basic concepts on which the classical physical theories depend can ever become superfluous when describing physical experience. He merely objects to the view that it is still possible in the sense of the classical theories to connect directly and, as it were, to amalgamate the causal and the space-time descriptions of phenomena. They supplement each other but cannot be used simultaneously or embraced in one view. In consonance with the nature of quantum theory we must be satisfied with conceiving the space-time representation and the demand of causality as complementary but mutually exclusive features of the description of experience, symbolizing the idealization of the possibilities of observation and of definition respectively.[11] Heisenberg expressed himself much more strongly,

10. Cf. Schrödinger, "The Law of Chance," in *Science and the Human Temperament,* English trans., J. Murphy (London, 1935), pp. 40 ff.

11. Cf. Bohr, "Das Quantenpostulat und die neuere Entwicklung der Atomistik" (1927), in *Atomtheorie und Naturbeschreibung,* pp. 34 ff.; see also the advance report introducing this collection (1929), pp. 10 ff.

for he actually declared, on first introducing the uncertainty relations, that through these relations the causal principle was proven invalid.[12] But that this rough denial is leveled against a certain narrow formulation of the causal principle and not against its universal meaning is shown by the further development and interpretation which Heisenberg has given to the physical principles of quantum theory. For here Bohr's basic concept of complementarity is adhered to completely. According to Heisenberg quantum theory has to choose between two possibilities: either it can maintain the space-time description (but must then accept the peculiar indefiniteness of the concept of "observation" as characterized by the uncertainty relations), or else it has to be satisfied with a mathematical scheme which cannot be interpreted as a simple interrelationship of things in time and space—in which case, however, it is in a position to reestablish causality in its own rights.[13] The statistical interpretation of quantum mechanics by Born and Jordan also insists that a deterministic theory, according to which the course of events within a closed system is entirely determined by the state of the system at time $t = 0$, can no longer be maintained; such a theory, although conceivable without contradiction, would lack any possible application, because the initial condition can never be exactly determined. In this sense the causal law becomes "empty"; in principle, physics is indeterministic and thus a matter of statistics. But this deduction is promptly supplemented by Born's declaration that statistical statements are thoroughly strict statements; the probabilities themselves, as he emphasizes, are by no means indefinite; they are strictly determined by the formalism of the quantum theory.[14]

This state of affairs can be clarified from a different angle when the role played by the principle of energy conservation in the construction of quantum mechanics is taken into consideration. In the course of classical physics it was this principle which proved more and more to be the prototype of strictly causal thought and inference. The representatives of strict energetics tend to merge the concepts of energy and causality; they claim that the causal concept has its only actual fulfillment and adequate physical representation in the concept of energy. Thus W. Ostwald, for instance, says: "Since all occurrences consist of spatial or temporal changes in energy, this form of the causal law is in a certain

12. *Zeitschrift für Physik, 43* (1927), 197; see below, pp. 123 ff.

13. Heisenberg, *Die physikalischen Prinzipien der Quantentheorie* (Leipzig, 1930), pp. 47 ff.; English trans., C. Eckart and F. C. Hoyt, *The Physical Principles of the Quantum Theory* (Chicago, 1930), pp. 62 ff.

14. M. Born, "Quantenmechanik und Statistik," *Naturwissenschaften, 15* (1927), 240 ff.; cf. *Elementare Quantenmechanik,* Vol. 2 of *Vorlesungen über Atommechanik,* chap. 6.

sense exhaustive. . . . In . . . more definite form the causal law states that 'nothing occurs without an equivalent transformation of one or more forms of energy.' " [15] The coincidence of the first law of thermodynamics with the general principle of causality can be contested on epistemological grounds; however, it is undeniable that causal thinking constitutes an essential factor in "energetic" thinking. If therefore the latter is adhered to, the former is implicitly included and acknowledged. But this adherence is also characteristic for quantum mechanics. In the development of quantum theory doubts as to the strictness and the general validity of the energy principle have been expressed at times, and individual investigators were ready to sacrifice this principle. This sacrifice, however, has been shown to be neither necessary nor possible. On the contrary quantum mechanics has maintained the principles of conservation of energy and momentum and has taken them over without change. These principles seem to be part of the basic scaffolding of every exact description of nature; they belong among the invariants which are independent of any special form adopted by the system of physical description. It is precisely the theoretically most important and fruitful explanations of certain basic phenomena of quantum theory that are based on these principles. In the theory of the photoelectric effect as well as in that of the Compton effect the conservation of energy and momentum are postulated as valid for the impacts between light quanta and electrons and for the resulting exchange of energy. In recent discussions of this problem which, as far as I can see, have not reached a definite conclusion, there certainly has been some mention of the view that perhaps the principle of the conservation of energy is only statistically significant, so that deviations in individual cases are possible.[16] If, however, one considers the present form of quantum mechanics, the change in the application of the energy concept appears to be limited to the fact that according to quantum theory the question as to the determination of the energy state of a system at a given moment appears much more complex than before, and that there is even an assumption of the existence of states, to which no definite energy corresponds at all. But the validity of the energy principle as such is not questioned here; it underlies the statistical presuppositions of quantum mechanics.

Still another circumstance should warn us from the very beginning

15. W. Ostwald, *Vorlesungen über Naturphilosophie* (Leipzig, 1902), p. 296.

16. The conjecture, expressed among others by Bohr, that exceptions to the principles of conservation of energy and momentum may occur in an individual atomic process, so that these principles can be considered only statistically valid, i.e. valid for a large number of individual atomic processes, could not be confirmed by experimental tests. Concerning the present status of this question cf. N. Bohr and P. A. M. Dirac, *Nature, 137* (1936), 298, 344.

against the uncritical use of the term "indeterminism," which occurs so frequently in popular discussions. Schrödinger, in one of his recent presentations on the present state of quantum mechanics,[17] emphasizes that although in general, quantum mechanics deals with probability statements, on the other hand, these statements are distinguished by a singular exactness. When, for example, the energy of a Planck oscillator is measured, the probability of finding a value between E and E' is zero, unless the range between E and E' includes one of the values $\frac{1}{2}\ h\nu$, $\frac{3}{2}\ h\nu$, $\frac{5}{2}\ h\nu$, $\frac{7}{2}\ h\nu$, . . . All values other than odd multiples of the "model constant" $\frac{1}{2}\ h\nu$ are excluded. Schrödinger adds that this statement certainly does not suffer from an excessive lack of precision; on the contrary, it is much more accurate than an actual measurement can ever be. And the increase in precision of quantum laws and principles is as great as that of the statements of results of measurements. Thus the Hamiltonian principle, in its classical form, required only that a certain integral be a minimum, without fixing the numerical value of this minimum, while the quantum theory adds the condition that this value must be an integral multiple of Planck's quantum of action. One can refer to a basic lack of precision in the statements of quantum theory only so long as one presupposes that statistical statements are necessarily "inexact" statements. In reality they are strict statements referring, however, not to an individual thing or event but to definite collectives.[18] When the theory of radioactive decay declares that for every substance there exists a certain probability that an atom chosen at random will decompose within a randomly chosen interval of time; when it is shown that there is a definite exponential law governing the decay; when accordingly the "decay series" of various radioactive families are established—then these are all extremely precise conclusions, even though they say nothing about the fate of the individual atom and the precise instant of its decomposition. There is here no thought of relinquishing causality, for causality in principle has nothing to do with "fate" but simply and solely with law. Thus in quantum theory also this problem should be understood exclusively in this, its solely "critical" sense. The thesis of indeterminism has occasionally been expressed in the form that the path taken by a single electron is not prescribed by scientific laws but that instead the electron may "choose" between certain limits, and that this choice is determined, as it were, by "nature's game of dice." [19]

17. See *Naturwissenschaften, 23* (1935), 807 ff.

18. See above, p. 94.

19. In this connection cf. Hugo Bergmann, *Der Kampf um das Kausalgesetz in der jüngsten Physik* (Braunschweig, 1929), p. 55.

But a real indeterminism, truly worthy of the name, can only be arrived at if we insist in going a step farther, when the attack, instead of being leveled at the determinateness of the individual event, is leveled at the determinateness of the laws which we consider as governing the event. "Nature's game of dice" then assumes a different form; it would no longer be concerned with a decision about the path of an individual electron; rather it would have to deal with the question whether nature, in a given moment, could apply this or that law, whether it can change its laws from case to case in a manner uncontrollable by our knowledge. However, it is obvious that the very formulation of the question in this sense is an inadmissible anthropomorphism. It is a false hypostasis when we think of nature as a subject existing by itself, as an active or passive "thing." We are subject to this error, regardless of whether we consider nature as an independent agent capable at certain moments of making a choice [20] or whether we consider it as subject to strict and unalterable laws from which it cannot escape. In strict physical terminology "nature" is nothing but an aggregate of relations, of laws; and to such an aggregate, to such a pure form, the category of active or passive is not applicable. Thus for a decision about the requirement of determinism the lever must be applied at a different point. This requirement also must not be applied immediately and primarily to the material event, but rather to the "form" of scientific knowledge, to the structure of its basic concepts and principles. The definiteness, the logical determination of these concepts and principles would not be nullified if it should become evident that an event within atomic physics can no longer be represented by dynamic laws of the classical type but only through statistical laws.

Eddington, in his defense of the thesis of indeterminism, once made a witty comparison. He likened the change which has taken place in the thinking of modern physics to the economic change which arose from the fact that many countries in recent years have come to put their money and currency on a new basis. In the same way as it became apparent, that a change in monetary policy does not necessarily entail a shattering of the economy and of commercial credit, so the truth value of physical statements need not suffer on account of their transformation. The validity of the strictly dynamic laws cannot be maintained any longer—but it has become clear that they can be dispensed with while still fulfilling the function of physics in the totality of knowledge. Hitherto, the dynamic laws were considered as the proper and necessary foundation in the formation of physical concepts. They were

20. "In Brussels in 1927 Dirac said: 'At certain moments nature makes a choice' " (Langevin, *La Notion de corpuscules et d'atomes,* Paris, 1934, p. 33.)

> the gold stored in the vaults; [statistical laws were] the paper currency actually used. But everyone still adhered to the traditional view that paper currency needs to be backed by gold. As physics progressed the occasions when the gold was actually produced became rarer, until they ceased altogether. Then it occurred to some of us to question whether there still was a hoard of gold in the vaults or whether its existence was a mythical tradition. The dramatic ending of the story would be that the vaults were opened and found to be empty. The actual ending is not quite so simple. It turns out that the key has been lost, and no one can say for certain whether there is any gold in the vaults or not. But I think it is clear that, with either termination, present-day physics is *off the gold standard.*[21]

The parallel which is here drawn can in fact be used to illustrate the epistemological problems to which the development of modern atomic physics has led. It seems to me that no indeterminist, no matter how radical, will be willing to introduce a pure paper currency into the system of physical knowledge. Statements of physics may bear a purely symbolic character; but for the totality of these symbols some attachment to reality, some *fundamentum in re* will always have to be demanded. Without the fulfillment of this requirement the truth-value, the logical credit of physics, would be doomed once and for all. Consequently, when the scientific character of theoretical physics is to be maintained, there can be no question of altogether waiving all "backing" for its concepts and judgments; the question can only be, where are we to look for it? Modern atomic physics also must establish certain ultimate, objectively valid basic principles, on which it founds the actual contents of knowledge. But it no longer formulates these contents in the manner customary to classical physics. The hope of ever successfully reducing all physical statements to the one type of classical dynamic law seems to dwindle more and more. Instead of looking for such a reduction, we must be ready to acknowledge statistical laws as a particular and fundamental type of physical statement. They must be evaluated as nonderivable, equally basic and equally valid elements of knowledge. The backing for our physical judgments has thus been shifted; but it is in no sense abolished. It now consists in that which proves invariant in the face of this shift. There must be some Archimedean point, some secure basis, immune to all uncertainty, if the construction of modern physics is to succeed.

21. A. S. Eddington, *New Pathways in Science* (New York, Macmillan, 1935), pp. 81 ff.

Bohr and Heisenberg also have repeatedly emphasized the necessity for such a basis. In his concise and comprehensive survey of the development of quantum mechanics [22] Heisenberg submits that even in modern physics the observer's procedure as well as his apparatus have to be discussed in accordance with the laws of classical physics, for otherwise there is no problem for physics to consider. Within the measuring apparatus all events are regarded as determined in the sense of classical theory; this provides the necessary presupposition for believing that what has happened can be unambiguously inferred from the result of measurement. Heisenberg goes on to say that the general procedure of quantum mechanics consists of ascribing to every quantity in classical mechanics, such as the momentum or energy of the electrons, a corresponding matrix. Then, in order to go beyond a mere presentation of the empirical situation, these matrices must be combined according to laws, in the same way the corresponding quantities in classical mechanics are combined by the equations of motion. On the basis of this procedure it is thus possible to prove in quantum mechanics those laws which were the foundation of classical mechanics, and particularly to maintain the constancy in time of energy, momentum, and angular momentum. Another essential precision and constancy is obtained by maintaining the invariability of certain fundamental quantities and by presupposing this invariability in all theoretical descriptions of natural events. A definite system of universal constants of nature is established; thus the speed of light, the charge or mass of an electron, the mass of a proton, etc. are considered as absolutely definite quantities that always have the same value. It is in these determinations—to continue Eddington's metaphor —that the real "gold reserves" of modern physics are to be found. And the fact that quantum theory has not and could not deviate from this standard is plainly evident from the fact that it is bound to this methodological ideal in origin and development. The very result upon which it is founded consisted in the discovery that there is a universal constant of nature which leaves its stamp on every natural occurrence, whatever its detailed constitution might be. All considerations and experiments of quantum theory presuppose the universal validity of this constant whose numerical value invariably remains the same, no matter by how many methods it is verified. All particular certainties as they are given for instance in Planck's law of radiation, in the Balmer series of the hydrogen spectrum, in the formula for the atomic heat, etc. always lead back to this general certainty. The elementary quantum of action constitutes,

22. Nobel Lecture, 1933; see the collective volume *Die moderne Atomtheorie,* lectures delivered on acceptance of the Nobel prize in Stockholm (Leipzig, 1934).

as it were, the fixed frame, into which all statements of quantum theory are fitted; and the security and firmness of this frame alone ought to be sufficient to protect the indeterminism of the theory against those speculative interpretations to which it was exposed in the transition from physics to general conclusions concerning man's *Weltanschauung*.

It is obvious that the uncertainty relations cannot and do not aim to transgress this frame of "quantum theoretical determinism," for their actual meaning and content can only be understood where the firm structure of quantum theory is presupposed as given and universally valid. So little do the uncertainty relations waive the assumption of strict laws of nature that they actually give directions as to how to arrive at, and how to formulate, these laws in order to make them conform to the conditions of our empirical knowledge. In his first introduction of the uncertainty relations, Heisenberg starts from the consideration that all statements in physics have a relative character in that they can express the state of the observed object never "in itself" but merely in relation to the means of observation used. The limitation resulting from this may be disregarded as long as we remain within the realm of macrophysics; but it attains decisive importance as soon as we turn to the phenomena of microphysics. For here we are dealing with an order of magnitude in which the above-mentioned circumstance can no longer be neglected. It is this dependence on instruments of observation which prevents us in principle and once and for all from making statements which exceed a certain measure of precision. The most important and also epistemologically significant conclusion drawn by Heisenberg from these circumstances is the fact that this dependence sets a limit not only to our experimental technique but also to the formulation of physical concepts. Whenever we formulate a physical concept, we must be mindful of this limit if the concept is not to be empty or ambiguous. When, for instance, we speak of the position of an electron, we must indicate definite experiments for determining this position. The electron could be illuminated and observed under a microscope. It then turns out that the shorter the wave length of the light used, the more precise will be the determination of position. But on the other hand, the shorter the wave length chosen, the more strongly the "Compton effect" becomes noticeable: the light which hits the electron and is reflected or deflected by it changes the momentum of the electron. The electron receives a recoil which becomes greater the greater the frequency—that is, the smaller the wave length chosen. From this it can be seen why it is basically impossible to measure simultaneously the position and the velocity of an electron with any desired accuracy. The more accurate the measurement of position, the more inaccurate will be that of velocity,

and vice versa. If we designate the uncertainty in the measurement of position as $\triangle x$ and that in the measurement of momentum as $\triangle mv$, then the product of these two quantities can never be reduced below a certain value which is of the order of magnitude of the elementary quantum of action.

If we want to clarify the epistemological significance of the uncertainty relation, we must differentiate between the state of affairs here described and the conclusions attached to them by Heisenberg. The latter interprets the relation in such a manner as to imply a direct denial of the causal law. He maintains that "in the precise formulation of the causal law: 'If we have exact knowledge of the present, we can determine the future' not the consequence but the antecedent is wrong. As a matter of principle, we cannot come to know the present in all its determinative factors. . . . Since all experiments are subject to the laws of quantum mechanics (and thus to the equation $\triangle x \, \triangle mv \geqq h$) the invalidity of the causal law is definitely established by quantum mechanics." [23] What we object to in this argument is only the one assertion that here we have a truly "precise formulation of the causal law." Heisenberg's demonstration is leveled exclusively against a certain form of the causal principle which we found to be subject to grave theoretical faults, even without considering the uncertainty relations. If, however, we put in place of this inexact formulation the critically refined form as given in classical physics by von Helmholtz,[24] if we express the demand of causality merely by the general requirement of conformity to law, then Heisenberg's uncertainty relations no longer constitute an exception. In the very same place where Heisenberg pronounces invalid the causal principle in the form presupposed by him, he declares that in all cases where there are relations in classical theory between quantities that actually can be measured precisely, the corresponding exact relations remain valid in quantum theory also (that is, the principles of conservation of momentum and energy). In fact the very derivation and proof of the uncertainty relations, as given by Heisenberg, furnish evidence that there is no intention of contradicting causal thought and inference as such. For the theory of the Compton effect, on which this derivation is based, makes very definite use of precisely this type of thought. It is true that it does not apply to the mechanism of the interaction between electrons and photons in the sense of introducing definite forces acting between particles of matter and light quanta and describing the temporal process of the observed events in the usual manner of mechanics. Even

23. W. Heisenberg, "Über den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik," *Zeitschrift für Physik, 43* (1927), 197.

24. See above, p. 61.

the directions assumed by the particle and the photon after collision cannot be determined accurately. However, the mechanical laws of particle collisions are applied to the collisions between light quanta and electrons in such a way that the final results are determined on the one hand by the principles of the conservation of energy and momentum, and on the other by Planck's relation between energy and frequency.[25] The basic postulate of quantum theory is thus connected in a definite and characteristic manner with the principles of conservation of energy and momentum, both of which, as has been emphasized previously, must be considered as pure and typical causal principles. From this interconnection it is absolutely clear that the uncertainty relations have by no means demolished the bridge between quantum theory and classical physics, or between the former and the causal principle. Thus they may correct the formulation of the causal principle but they cannot simply deny or overthrow its content. We may now ask what this correction consists of, and what its epistemological significance might be.

We arrive at a clear answer to this question if we consider on the one hand the meaning of the uncertainty relations, and on the other the logical form of the causal law. The latter is expressible as a simple hypothetical statement, which can be stated in the form: *If x, then y.* What can be said about this formulation or its validity if an uncertainty, or a "fallacy" has crept into its first member, into *x?* Traditional logic answers that this fallacy does not necessarily imply the falsehood of *y;* much less is the universal validity of the relation as such affected by it. According to well known logical rules the sentence: *Posita conditione ponitur conditionatum* can lead by contraposition to the sentence: *Sublato conditionato tollitur conditio,* but not to *Sublata conditione tollitur conditionatum.* Aristotle expresses this in the form that from correct premises nothing erroneous can be deduced but that the incorrectness of a premise does not result in the incorrectness of the conclusion, as long as one regards the *that,* not the *because* of the latter (πλὴν οὐ διότι ἀλλ' ὅτι).[26] When therefore, the uncertainty relations show that certain causal judgments in physics contain a false premise, they still say nothing about the form of the hypothetical conclusion. The statement *if x, then y* is not rendered invalid; it is merely claimed that in order to make it useful or applicable in physics, each individual case has to be studied, in order to see whether the quantities which we have included are capable of being measured accurately. In other words, the problem does not concern

25. For further details on this subject see for instance I. Frenkel, *Einführung in die Wellenmechanik* (Berlin, 1929), chap. 1, sec. 4.

26. Aristotle, *Analytica priora* 2.2.

the causal relation as such, but only the places for the independent variables in this relation: the values for the variable x must be "permissible" values, in order to give the causal relation a definite unambiguous meaning. From the point of view of physics, however, only those values are permissible that can be determined by a certain mode of measurement which must be accurately stipulated. Only through this limiting condition does the "causal principle" attain a physically comprehensible significance—and its legitimate application remains confined to this condition.

The general principle thus established is not new, insofar as its purely epistemological content is concerned. It has a long and significant history in the development of physics. Newton included this principle in the basic methodological rules which he designated as *Regulae philosophandi.* It is here expressed in the form that for the explanation of natural phenomena physics is to admit only such causes as are "true" and sufficient to explain the phenomena: *causas rerum naturalium non plures admitti debere, quam quae et verae sunt et earum phaenomenis explicandis sufficiant.*[27] At first glance the demand of *vera causa* appears to be a logical overstipulation. But it contains a clear and pregnant meaning; it declares fictitious every alleged cause which cannot be somehow experimentally proven, whether directly or indirectly. Since that time this Newtonian demand for verification has been the methodological maxim of all physical research. But Newton's own example proves that the establishment of this principle by no means assures a consequent compliance with it. For the question as to which quantities are to be considered as strictly observable requires in every case a careful investigation. The law of inertia was undoubtedly for Newton a genuine and universally valid natural law; it is the axiom upon which his entire mechanics is based. Yet if the form which he gave to this principle is examined, one finds that it only assumes a comprehensible physical meaning if one introduces absolute space as that reference system in relation to which a body left to itself shall move uniformly in a straight line. Absolute space here figures as *vera causa;* it is true, Newton points out, that it is not directly empirically given, but certain experiments, such as for instance the well-known rope experiment, can be named, by means of which the existence in certain cases of absolute or merely relative motion can be distinguished. Leibniz contested this assertion; he considered absolute space purely fictitious and consequently claimed that it must not be included in the formulation of the laws of nature. And in connection with this critique he comes to pronounce explicitly the principle of "observ-

27. *Philosophiae naturalis principia mathematica,* 3.

ability" (*principe de l'observabilité*), at first in a general form,[28] and to treat it as a fundamental methodological principle of physics.[29] But in the subsequent development of physics, governed entirely by Newton's authority, this principle remains for a time completely in the background. Not until the last decades of the nineteenth century did anyone dare express doubts about Newton's formulation of the principle of inertia. Carl Neumann, Mach, Streintz, and Ludwig Lange proposed various formulations designed to bring the law of inertia into agreement with the principle of observability. The attempt was made to indicate a spatial "inertial system" and an "inertial time scale," of concrete physical significance, expressible in definite observable and measurable quantities.[30] A really universally valid and epistemologically satisfactory solution of the problem was however only arrived at by the general theory of relativity; and this solution in turn is based on the express realization that the law of causality has significance as a statement about the world of experience only when by causes and effects there appear in the last analysis only observable facts.[31]

It was the same basic methodological idea which led Heisenberg to the establishing of his uncertainty relations. But in fact there was here a new task to be solved, since meanwhile, because of the transition to atomic physics, the problem had assumed a new form. Planck once rightly pointed out that a decision on whether a physical quantity is observable in principle, or whether a certain question is meaningful in physics, can never be arrived at a priori but always and solely from the point of view of a given theory, the difference between the different theories consisting precisely in the fact that according to one theory a certain quantity is observable in principle, or a certain question is physically meaningful, while according to the other it is not.[32] Heisenberg's uncertainty relations have attacked the problem from the standpoint of quantum theory, and for the first time answered it precisely. They show that the general principles of quantum theory make it once and for all impossible to measure simultaneously and with any desired precision, certain pairs of quantities, such as the position and the momentum of

28. See Leibniz on Clarke, *Hauptschriften,* ed. Cassirer and Buchenau, *I,* 188; further details in my *Substance and Function,* p. 376.

29. Cf. *Nouvelles Lettres et opuscules inedits,* ed. A. Foucher de Careil (Paris, A. Durand, 1857). "Herculinum illud argumentum, quod ea omnia quae sintne an non sint a nemine percipi potest, nihil sunt."

30. For the epistemological significance of these attempts at definition, cf. my *Substance and Function,* pp. 172 ff.

31. Einstein, *Die Grundlage der allgemeinen Relativitätstheorie,* sec. 2.

32. "Das Weltbild der neuen Physik" (1929), *Wege zur physikalischen Erkenntnis,* p. 199 f.

an electron or the electric and magnetic field strengths within a small space. In this way the application of the causal principle in quantum theory is subjected to a condition which did not immediately affect classical physics, and which the latter could ignore within its own realm, the realm of "macrophenomena." However, the introduction of this limiting condition by no means constitutes the abolition of the causal principle; it merely indicates that this principle requires a renewed analysis with every transition to a new realm of science if it is to remain physically usable and fruitful. Only such an analysis can prevent the stealthy introduction in the formulation of the causal principle of elements which are in principle unobservable, and which would thus render its application illusory. Of all the various explanations of the causal concept offered in the *Critique of Pure Reason,* perhaps the most precise and most satisfying is the one in which it is said that the concept represents nothing but a direction for the formulation of definite empirical concepts. "That everything that happens has a cause cannot be inferred merely from the concept of happening in general; on the contrary, it is this fundamental proposition which shows how in regard to that which happens we are in a position to obtain in experience any concept whatsoever that is really determinate." [33] The uncertainty relations have added something new and epistemologically highly significant to this observation. They have shown that in the transition to new sets of problems in scientific knowledge, the old concepts cannot simply be taken over but require in each case a fresh determination and interpretation in order to be applicable without contradiction. Thus even when we adhere to the general causal principle, we must first consider, in progressing to the phenomena and problems of atomic physics, how this principle can be applied under the changed conditions so as to do justice to its general function—so that it teaches us how to obtain a definite empirical concept of what occurs. This goal cannot be forcibly attained by means of a dogmatic assertion; we must be directed by experience itself when establishing empirical concepts. If experience shows that there are definite, precisely definable limits to our observation, then we may not arbitrarily transgress these limits in our empirical formulation of concepts. Rather we must give our causal judgments such a form as to remain compatible with them, so as to respect the new conditions which are now indicated for the application of causal thinking. This, it appears to me, was the way adopted by quantum mechanics. It has by no means renounced the requirement of determination; but it had to arrive at new conceptual means in order to do justice to it and, by means of it, to penetrate successfully into the newly discovered

33. *Kritik der reinen Vernunft* (2d ed.), p. 357 (ed. Cassirer, *3,* 249); trans., p. 302.

factual realm. Schrödinger's wave equation, Heisenberg's square array, Born and Jordan's matrix mechanics, Dirac's "q-numbers," all these are conceptual means by which a strict coordination of observable quantities is to be made possible. In this sense Heisenberg established a causal law of quantum mechanics which states that if at any one time certain physical quantities are measured as accurately as in principle possible, there exist quantities at any other time whose value can be precisely calculated, or in other words for which the results of measurement can be precisely predicted.[34] Determination is thus re-established; but it is only valid for that which is known in quantum mechanics as a "pure case," for those measurements which, according to the principles of quantum theory, are not capable of any increase in precision.

Even when closer scrutiny thus shows that it is not poison but helpful medicine which is administered to the causal concept through the uncertainty relations—when these, correctly understood, lead to a clarification rather than the abolition, of the concept in that they complete it by means of the additional demand of "observability"—we are still by no means at the end, but only at the beginning, of the problem. For the difficulty returns from a different angle; instead of dealing with the formulation of the causal principle, it now assails the definition of the physical concept of reality. And here we appear to jump from the frying pan into the fire. What do we gain by putting the pure cases of quantum mechanics into relation with each other in a manner entirely corresponding to classical ideals: what advantage is it to advance from statements of the results of measurement to strict statements of laws, if the former nonetheless remain tainted with a fundamental uncertainty, and if, accordingly, the transposition of data from the world of sense to the world of physics and vice versa can no longer be accomplished in the manner hitherto considered possible and reliable? And it appears that this conclusion has actually to be accepted. Heisenberg expresses this by saying that in the description of atomic phenomena a line has to be drawn between the observer's measuring apparatus and the object of observation. On both sides of this line, on the one which leads to the observer as well as on that which contains the object of observation, all processes are sharply determined: on this side by the laws of classical physics by which the measuring apparatus is to be described, on the other by the differential equations of quantum mechanics. But at the line itself there appears an uncertainty, since the influence of the means of observation on the object to be observed must be considered as a not completely controllable disturbance. It is in such a fundamentally uncontrollable manner that for instance the speed of an electron is changed in Compton recoil by the

34. *Die physikalischen Prinzipien der Quantentheorie,* p. 45.

mere fact of observation because of the light rays necessarily used for the purpose. This means according to Heisenberg, that we must renounce finally and not just temporarily the idea of objective occurrences in space and time independent of all observation. "The hope of hitting upon objective occurrences in space and time by means of new experiments is probably no better founded than the hope of finding the end of the earth somewhere in the unexplored parts of the antarctic." In this train of thought Heisenberg sees a synthesis of physics and philosophy and a rapprochement and reconciliation between their respective formulations of problems; for philosophy has at all times pointed out the fundamental difficulties attendant on the division of the world into subject and object.[35]

But how does quantum mechanics influence the development of the subject-object problem, and what contribution has it to make toward this development? The answer to this question is certainly not simple; it can only be arrived at on the basis of a thorough systematic as well as historical analysis. Such an immediate interaction between problems of physics and of epistemology may at first appear strange. There was a time not so long ago when both physicists and philosophers adhered to the thesis that physics has to carry through its concrete scientific task completely independent of all epistemological considerations. It was declared that science could never be based on any universal concept other than healthy common sense, and that the latter forced it to hold to a certain naive realism. Thus it need not bother about any epistemological subtleties and had better avoid the path of their discussions, which are more likely to be harmful than beneficial. But even within the framework of classical physics this conception became untenable. A decisive change took place with thinkers like Mach, Helmholtz, and Heinrich Hertz. For Mach, physical and epistemological considerations are so closely interwoven that no sharp dividing line can be drawn between them, so that it is almost impossible to indicate where the one ends and the other begins. And Helmholtz never considered his epistemological work as a mere side line of his scientific research; rather he declared that the establishment of the presuppositions of science is a common task of physics and philosophy that no era can escape with impunity. But on the other hand it was precisely Helmholtz' teaching which contained a characteristic trait hinting at a difference in the conceptions of classical and atomic physics. Helmholtz was the last great representative of the mechanical viewpoint, and though he by no means presupposed it dogmatically, it stands as the

35. *Ibid.*, p. 49; trans., p. 65; cf. *Wandlungen in den Grundlagen der Naturwissenschaft* (6th ed. Leipzig, 1945), pp. 13 ff.; see also Heisenberg's paper "Prinzipielle Fragen der modernen Physik," in *Neuere Fortschritte in den exakten Wissenschaften* (Leipzig and Vienna, 1936), pp. 91 ff.

background of all his epistemological considerations and gives them their characteristic coloring. From this standpoint matter becomes the fundamental reality which has to be described in our scientific concepts of nature. This description can never be considered as a direct copy; rather it bears a purely symbolical character. What matter actually and essentially "is" can only be expressed in the sign language of our senses. The sign thus becomes for Helmholtz the basic concept of scientific epistemology. At the end of his *Physiologische Optik* Helmholtz summarizes his considerations as follows: "The main thesis of the empirical standpoint is that sense impressions are signs for our consciousness, the interpretation of which is left to our intellect." [36] In this way a firm connection between knowledge and reality is established as well as a thoroughgoing correspondence between the two, in which, however, a strict dualism is contained. For however much we may extend and refine the sign system of our concepts, it always remains merely a mediate expression of reality. The objectively real is presupposed as something persisting and substantial, but in its substantial existence it cannot fit into the sign language of our concepts of nature. This language is merely able to reproduce relations between individual phenomena but not the general substratum underlying them. The presupposition of the necessity of such a substratum, of a hard core of reality to which the relations adhere, is a heritage of the mechanistic viewpoint. Physical phenomena are completely known when wholly reduced to phenomena of motion; but these are not understandable without the assumption of some material "carrier." The latter constitutes, as it were, a rigid, self-existent nucleus, which our knowledge encounters and before which it must come to a halt. It has to acknowledge this "carrier" without ever being able to comprehend it completely. The light of knowledge can continually expand and it can illuminate reality ever more clearly and accurately, but it can, as it were, only touch it from the outside instead of truly penetrating it. For no matter how far the beam of light stretches forward, in back of it there will always remain a final dark remainder which eludes us. The substantialism essential to the mechanical viewpoint thus always contains a trait of agnosticism: the "unknowable" becomes a presupposition of knowledge itself.

I have attempted elsewhere to show at length how this "substantialistic" conception underwent a gradual change—how the concept of substance was increasingly pushed back and finally displaced by the pure concept of function. In these considerations I confined myself to the development of classical physics and its contemporary situation,[37] but

36. *Handbuch der physiologischen Optik,* pp. 947 ff.
37. Cf. *Substance and Function,* especially chap. 4.

they could have been formulated much more briefly and precisely had I had the general relativity theory and modern atomic physics before me at the time. Today this process of thought has been brought to a secure and apparently final conclusion. Hermann Weyl in his publication *Was ist Materie?* expresses his conviction that substance has today fulfilled its role in physics. "The claim of this idea, metaphysically conceived by Aristotle, to express the essence of real matter—the claim of matter to be substance incarnate—is unjustified." This becomes obvious in the transition to field theory. For the field is a complex of pure effects of, or pure relations between, "lines of force" which are no longer necessarily tied to a material substratum but which determine physical events, as it were, by their free mutual interaction. In field theory, according to Weyl, the space-time continuum has in a sense taken over the role of substance: "The 'this,' which is qualitatively not characterized but to be given only by individual reference is not a hidden carrier in which the qualities inhere but the *here-now* of the particular space-time locus." The transition, however, was gradual. The older field theory, in the form in which it furnished, for example, the basis for Lorentz and Abraham's electronic theory, still represents the electron as a rigid sphere, with whose spatial elements the electric charge is rigidly connected. But gradually the dynamic functional viewpoint wins here also; mass, particularly according to the results of the general relativity theory, no longer appears as a self-sufficient entity but merges into the concept of electric charge.[38]

In the face of this development the general epistemological situation in physics is also completely changed, and we cannot do justice to this change except by a reformulation of the physical concept of object. No path any longer leads back to that "naive realism" which used to be recommended as the natural and adequate standpoint for physics; physics is by necessity directed to a re-examination, to a strict critical analysis of both its conceptual procedure and its procedure of measurement. The uncertainty relations are but a single, though particularly significant, step in this process of re-examination. One of the essential results of this process consists in the reversal, in a sense, of the relationship between the concepts of object and law. The concept of law is now regarded as prior to that of object, whereas it used to be subordinate to it. In the substantialistic conception there used to be a definitely determined entity, which bore certain attributes and which entered, with other entities, into definite relations expressible by laws of nature. In the functional viewpoint, by contrast, this entity constitutes no longer the self-evident starting point but the final goal and end of the considerations: the *terminus a quo* has become a *terminus ad quem.* We no longer have absolute,

38. H. Weyl, *Was ist Materie?,* pp. 17 f., 41 f.

completely determined entities, from which we can immediately read off the laws and to which we can attach them as their attributes. What in fact constitutes the content of our empirical knowledge is rather the totality of observations which we group together in definite orders and which, in accordance with this process of ordering, we can represent by theoretical laws. The extent of the dominance of these laws marks the extent of our objective knowledge. Objectivity or objective reality, is attained only because and insofar as there is conformity to law—not vice versa. Thus it follows that we cannot speak of physical entities except under the conditions of physical cognition—both the general conditions and those particular ones which hold for their observation and measurement. The entity has thus forfeited its absolute fixation; it is, so to speak, drawn into the process of physical knowledge and can only be regarded as the limit which this process approaches but never completely reaches. From being absolutely determined it has become determinable without limit. Yet precisely this apparent barrier proves to be an important, fruitful, and entirely positive determination. For the supposed "dark nucleus" of existence, to which skepticism could constantly point and in which it flourished, has now disappeared. The entities of physics, its empirical objects, are of course never completely given, because they are never completely determined; but on the other hand they no longer threaten us as a mysterious impenetrable absolute to whose last roots we cannot reach. For the attribute of their empirical and theoretical determinability is now included in their definition; this determinability constitutes physical entities, instead of merely expressing an accidental and individual feature of them. We do not simply read off the laws from the objects—we condense into laws, and thus into objective statements, the empirical data available through observation and measurement. Apart from this reality there exists for us no other objective reality to be investigated or sought after.

In modern quantum theory this state of affairs has received a new and rather strange confirmation. The quantum theory appears at first sight to require, more than any other theory in physics, a strict and "rigid" substance concept from which it cannot free itself. For its basic problem is that of atomic processes; and the very concept of the atom was always considered the perfect example, the prototype of substantialism. For it is precisely the stability of the atom which constitutes the basic problem of atomic physics: and how can this stability be accounted for theoretically when the presupposition of substantialism is discarded, when atoms are no longer regarded as rigid spheres? This, beyond doubt, represented a difficult task—but it is precisely the progressive solution of this problem, logically as well as physically, that is so significant for

modern quantum theory. From the very beginning it could not accept the older interpretation of the atom as indivisible. It had to jettison the "simplicity" of the atom; instead it had to regard it conceptually as a most complex structure. The atomic concept changes from a simple "thing concept" to a relational and systemic one. We have seen that the new basic viewpoint in physics puts the concept of law ahead of the concept of thing; what a thing is can only be described by referring to the laws governing it. The very first laws that could be established for the structure of the atom proved to be analogous to the laws of planetary motion, both in structure and appearance; Bohr's theory of the Balmer series could be developed in strict analogy with Kepler's laws of planetary motion.[39] It thus follows at once that the problem of the stability of the atom has taken on a much more complex character and that it could no longer be attacked adequately with the same simple means that had been satisfactory before. In classical theory, atoms could actually be regarded as the primary elements of things, as the smallest building blocks of nature. The task then merely consisted in establishing certain simple basic laws for the behavior of these elements, for the collisions of atoms, sufficient for the explanation of physical events. This classical solution of the problem could not be accepted by quantum theory. For the latter the atom invariably represents not a simple entity but a most complicated system of forces. In order to understand this system as a stable whole certain basic assumptions are required by which its relative equilibrium can be assured. And these assumptions are not based upon the rigidity or the absolute hardness of the atom but are derived by presupposing the quantum postulate. Thus it was the recourse to a principle, not the assumption of an irreducible thinglike attribute, that rendered possible the solution of the problem. Bohr declared: "We have to demand of the atom a stability of a character entirely foreign to mechanical theory. Thus the mechanical laws admit of a continuous variation of possible motions which is quite incompatible with the detailed properties of the elements." And along this path he was led to the establishment of the basic postulates of his theory of atomic structure: to the assumption that an atomic system has a certain number of stationary states to which in general a discrete series of energy values corresponds, and whose essential stability is evidenced by the fact that any change in the energy of the atom is due to a transition of the atom from one stationary state to another.[40] Bohr's theory replaces the picture of the atom as a thinglike entity perceived by our senses,

39. This analogy is strictly carried through, for instance, in Sommerfeld's *Atombau und Spektrallinien,* chap. 2, sec. 4.

40. "Atomtheorie und Mechanik," pp. 19 f.

by the characteristic frequency condition, according to which the change in energy of an atomic system is related to the frequency of a light wave by a definite equation, $h\nu = E_1 - E_2$. Its success in calculating the energy levels of the hydrogen atom, so that this condition led to the hydrogen spectrum in exact accordance with Balmer's formula, provided empirical proof for the fruitfulness of the theory.

Thus the first step, a step of fundamental importance, was made. It was Bohr's structure of the atom which first permitted the establishment of a solid foothold in the realm of atomic physics. It was the "Archimedean point" which was badly needed. An extraordinary concentration of existing knowledge was accomplished; the derivation of Balmer's equation, of Rydberg's constant, and of Ritz' combination principle was achieved. The construction of the periodic system of elements was now made possible in the simplest and most comprehensible manner by setting the number of electrons in the atom equal to the atomic number, that is equal to the number designating its position in the periodic system. And yet, in the first formulation of Bohr's theory, a factor remained which was not fully clarified theoretically and which was bound to give rise to epistemological questions. For certain "states" were assumed without any indication being given as to how they could be ascertained empirically. In the stationary orbits the electron moves continuously, without radiating: as Sommerfeld phrases it, "according to quantum theory, the electron is, as it were, immunized against radiation." But how is this immunization compatible with the basic requirement of observability, since radiation is the only way we can obtain knowledge of the electron? Here was a gap which was increasingly felt as time went on. Bohr himself, in his later work, expressly acknowledged this gap, declaring that according to the fundamental postulates originally assumed, such features of the "mechanical picture" as frequency and shape of the electronic path are not accessible to direct observation. The purely symbolical character of these features, he adds, can hardly be more strongly emphasized than by the fact that in the normal state no radiation takes place, although even there, motion is ascribed to the electron.[41] What is particularly significant here from an epistemological point of view is the fact that we are led to exactly the dilemma that we face again and again throughout the history of physics. In the stationary orbits of the electron something is posited to which nothing empirical corresponds. Once more we seemed committed to an obscure realm of "being," inaccessible in principle to our observation; the substantial "core" again claims precedence over that which is empirically available.

Again there was no way out of the difficulty except by analyzing and

41. *Ibid.*, p. 23.

reformulating the problem. Instead of complaining that we cannot penetrate to the "inside of nature," we have to realize that for us there is no other "inside" than that which is revealed through observation and analysis of phenomena. It is this process of empirical knowledge that determines the horizon of objective existence—a horizon which of course alters its position with every new advance in science. It is to the credit of the new quantum mechanics that it observes this methodological requirement far more strictly and consistently than had been the case before. In its line of questioning it desists from representing what happens within the atom and from illustrating it by means of some intuitive model. It limits itself to those quantities such as the frequencies and intensities of spectral lines that are open to observation. The theory is no longer founded on assumptions about the substantial "interior" of atoms, but rather on functional relations between observable magnitudes. On the basis of this viewpoint Heisenberg's square array was developed. It represents a clear, sharp, and mathematically precise form for whatever can be asked about the constitution of the atom. It provides a method by which the functional relations that alone are available to us can be given precisely and completely. Statements of the electron's position, its period of rotation, or the shape of its orbit no longer appear; the reality of the electron is not to be based on, is not even to consist of, anything but the radiation laws that we can establish. Schrödinger's wave mechanics also does not differ, in principle, from matrix mechanics. For Schrödinger also it is clear that no definition of physical existence is or ever will be possible except through the medium of physical laws. That electrons actually exist and that they follow definite paths means, as Schrödinger himself expressly says, nothing else than that those laws are valid which we deduce from cathode ray and other experiments.[42] No physical reality exists for us except the one that is mediated to us by physical measurements and by the determination of laws based on them, which are objective because of this relation.

It is thus obvious that we cannot transgress the boundaries of theoretical concepts and the results of experimental measurements; here as

42. "Conceptual Models in Physics and their Philosophical Value," in *Science and the Human Temperament,* p. 123. It must be pointed out that this observation is not limited to "physical" objects, but that it is equally valid for the "things" of "daily experience." For the latter are by no means "immediately given," as is supposed by some tendencies in positivism, but are the result of orderly connections according to laws; they are the result of a "logical construction." Of modern logicians, Russell formulated this most clearly. According to him, the things of experience are no "hard data"; rather they are open to, and in need of, analysis. "Things are those series of aspects which obey the laws of physics" (*Our Knowledge of the External World,* New York, W. W. Norton, 1929, Lecture 3).

always we cannot jump beyond our own shadow. But to designate this as a lack in our physical knowledge we would first have to show exactly what is lacking, and what this absolute knowledge that is supposed to be hidden from us would consist of. The very attempt at such an exposition would entangle us in contradictions. The question here is analogous to that of the absolute determination of space and time, in connection with which Maxwell once said that no one would complain of ignorance once he had tried to imagine the condition of an intelligence possessing knowledge of the absolute position of a point.[43] It must be admitted that in this formulation of the concept of objective knowledge we are in a sense moving in a circle, though it is by no means a vicious one. It consists in the first place of the fact that we cannot clearly separate statements of laws and principles from experimental observations, that we cannot demonstrate the latter in their merely factual "givenness," independent of all theoretical assumptions. Thus, for instance, the limitation imposed on our statements of the results of measurements which is expressed in the uncertainty relations, is only valid relative to the quantum principle and to the general formalism of quantum mechanics. Should an empirical reason ever arise for relinquishing this principle and replacing it by some other basic assumption, the question of observability would then also appear in a different light. This constitutes just that mutual interlocking of the individual factors of knowledge, which we saw to be an indispensable condition of our knowledge of nature. The quantum theory confirmed from a new angle the truth of Goethe's words: "Everything significantly factual is already theory." Epistemology, long before physics, had hit upon the vicious circle consisting in the circumstance that when we examine the correctness of knowledge, we invariably have to employ those means whose validity is to be proved and established by this critical examination. There seems to be no way out of this dilemma, and on first examination it looks as if any attempt at a "critique of reason" is illusory. Lotze, in his *Logik,* presents the problem by saying that

> the content of truth can only be determined when thought reflects on its own nature by constantly measuring and testing its particular results against the general laws governing its activity. To be worried about the circle thus created is not only useless since its inevitability is now obvious, but also superfluous since the harm suspected from that quarter can never . . . become noticeable. . . . Since, then, this circle is inevitable, we have to enter it. We must first attempt to determine what knowledge, in its widest sense, may mean, and

43. See above, p. 43.

> what relationship is conceivable between a knowing subject and the object of its knowledge in the light of the more general conceptions concerning the influence of any one element on another.[44]

These words are equally valid for physical and logical knowledge. And it may be that the most important contribution of the uncertainty relations is that in pointing emphatically to this circle they taught us at the same time that we must enter it.

The danger which seems to threaten the physical object concept disappears when one has learned to face it; relativity does not remain a mere blemish but becomes a positive force, on which all the fruitfulness of physical knowledge is built.[45] I have previously attempted to express this state of affairs, and perhaps I may be permitted to repeat that formulation here, since it was confirmed, and apparently illuminated from a new angle, by the problems of modern atomic physics:

> All this doubt and suspicion disappear . . . when we reflect that what appears here as the uncomprehended residuum of knowledge really enters into all knowledge as an indispensable factor and necessary condition. To know a content means to make it an object, by raising it out of the mere status of givenness and granting it a certain logical constancy and necessity. Thus we do not know "objects" as if they were already independently determined and given as *objects,* but we know *objectively,* by producing certain limitations and by fixing certain permanent elements and connections within the uniform flow of experience. The concept of the object in this sense constitutes no ultimate limit of knowledge. . . . The object marks the logical possession of knowledge, and not a dark beyond forever removed from knowledge. The "thing" is thus no longer something unknown, lying before us as a bare material, but is an expression of the form and manner of conceiving. What metaphysics ascribes as a *property* to things in themselves now proves to be a necessary element in the process of objectification. . . . In this connection the peculiar changeableness is explained, that is manifest in the content of scientific concepts of objects. According as the *function* of objectivity, which is unitary in its purpose and nature, is realized in different empirical

44. H. Lotze, *Logik* (2d ed. Leipzig, 1880), pp. 492 f., 525; English trans., Bernard Bosanquet, *Logic,* 2 vols., Oxford, 1887.

45. Concerning the "Circle of physical investigation" cf. the excellent presentations of E. Wind in *Das Experiment und die Metaphysik,* Tübingen, 1934; with the systematic conclusions which Wind draws from this circle concerning the relationship between physics and metaphysics, however, I cannot agree.

material, there arise different concepts of physical reality; yet these latter only represent different stages in the fulfillment of the same fundamental demand. Merely this demand is unchanging, not the means by which it is satisfied. . . . In this sense, the critical theory of experience would constitute the *universal invariant theory of experience*. . . . Just as the geometrician selects for investigation those relations of a definite figure, which remain unchanged by certain transformations, so here the attempt is made to discover those universal elements of form, that persist through all change in the particular material content of experience. . . . The goal of critical analysis would be reached, if we succeeded in isolating in this way the ultimate common element of all possible forms of scientific experience; i.e. if we succeeded in conceptually defining those moments, which persist in the advance from theory to theory because they are the condition of any theory.[46]

46. *Substance and Function,* pp. 303 f., 268 f. With this viewpoint based entirely upon general epistemological considerations, and on material taken exclusively from classical physics, cf. the program of modern theoretical physics, as presented for instance by P. A. M. Dirac in his *Principles of Quantum Mechanics* (Oxford, Clarendon Press, 1930), p. v: "The formulation of these laws requires the use of the mathematics of tranformations. The important things in the world appear as the invariants . . . of these transformations . . . The growth of the use of transformation theory, as applied first to relativity and later to the quantum theory, is the essence of the new method in theoretical physics."

CHAPTER 10

The History and Epistemology of the Concept of the Atom

FROM THE TIME of the discovery of the atomic concept in Democritus' system this concept has belonged among the surest foundations of scientific knowledge. It resists all objections that have been leveled against it, objections that found their first comprehensive systematic presentation in the physics of Aristotle, and it has proved its ability constantly to absorb new empirical material. But this empirical fertility of the concept, which becomes increasingly apparent and unmistakable as knowledge progresses, stands in a strange and hardly comprehensible contrast to the estimation bestowed on it throughout the history of epistemology. For here we never find a uniform conception and evaluation. According to the various epistemological schools of thought the criteria applied vary, and so do the results obtained by means of these criteria. Estimates of the objective significance of the atomic concept run through almost the entire gamut of possible evaluations. Every kind of praise and abuse has been heaped on the concept. At times it appears as a necessary condition, at other times as a menace for empirical knowledge; sometimes its philosophical establishment, and sometimes its abolition, is called for. Some consider it the *ens realissimum* of physics, the complete and sufficient expression of physical reality; others see in it a mere fiction or at best an economical artifice of thought. For a time the doubts arising seemed to be pushed back by the wealth of constantly new empirical confirmations; yet they have never been entirely silenced and today they seem to draw new nourishment from the problems of quantum theory.

But here also we cannot hope to overcome these doubts unless we first decide the question of the meaning of the "reality" of atoms, and in what sense this question can be posed and answered. At this very point begin the first ambiguities which have divided physicists and epistemologists

into two rival camps. The decision varies according to whether one grasps the object concept in its "realistic" or "idealistic," in its "critical" or "positivistic" sense. Mach's critique of atomism was rightfully opposed by Ludwig Boltzmann as being based in the last analysis on a *petitio principii.* If one accepts Mach's explanation of reality, if one considers it as nothing but the totality of the given, of the simple elements of sensory perception, then one must necessarily decide against the atomic concept. But Boltzmann points out that this viewpoint is not only inadequate in defining physical objects but utterly fails in the explanation of common "things." These also contain a constructive factor which cannot be reduced simply to sense data. In rebuttal of Mach's position that atoms and molecules are merely artificial constructs, or "economical symbolizations of physico-chemical experience" which, however, should not "impress" the physicist,[1] Boltzmann remarks that one could say the same of the fixed stars, since the assumption that they are gigantic bodies millions of miles away is also not forced upon us by visual perception as such; hence here also one could be accused of constructing a whole world of imaginary things beyond the world of our sensory perceptions.[2]

Energetics went still further in the battle against atomism. In Ostwald's lectures on natural philosophy Robert Mayer is designated as the "demigod" who with mighty fist smashed the world of classical physics, the world of matter and of atoms. The ultimate goal of science is now presented as the task of establishing a world view consisting purely of energy concepts, without the use of the concept of matter.[3] If this goal were reached, the atomic concept would appear to have outlived its usefulness; physics could relegate it to scholasticism. "Scholasticism or energism—that is the choice," is the way Helm summarized his view of the contrast between energetics and atomism.[4] The essential advantage of the energy concept and the conservation law was that they made it possible to circumvent the elementary processes and with them the

1. Cf. E. Mach, *Die Analyse der Empfindungen und das Verhältniss des Physischen zum Psychischen* (2d ed. Jena, 1903), p. 207.

2. *Vorlesungen über Gastheorie, I,* 9. The same argument can also be found in modern physical epistemology. Eddington says: "An electron is no more (and no less) hypothetical than a star. . . . When we discuss the reality of the physical world and the entities which constitute it, we have no reason to discriminate between the macroscopic and the microscopic entities. It is to be treated as a whole. . . . If the physical world is an inference, stars and electrons are inferential; if the physical world exists, stars and electrons are real" (*New Pathways in Science,* p. 21).

3. *Vorlesungen über Naturphilosophie* (1902), pp. 163 f.

4. G. F. Helm, *Die Energetik nach ihrer geschichtlichen Entwicklung* (Leipzig, 1898), p. 366.

atoms and the forces to which they are subject. Besides energy we need no other elements in the world, for whatever we know about the world, we know about energy.[5] In his well-known address at the Science Congress, in Lübeck (1895), Ostwald went so far as to brand as an absurdity, not far removed from pure nonsense, the assumption that in a particular chemical compound, such as that of iron and oxygen, the elements continue to exist and merely take on different properties.[6] Later on, however, under the pressure of new experimental data appearing from all sides, he modified his viewpoint considerably and acknowledged the empirical proofs for the existence of atoms. He declared that, by the isolation and counting of gaseous ions by J. J. Thomson, by the agreement of the Brownian movement with the demands of the kinetic theory and so on, the experimental proof for the atomistic nature of matter was furnished; atomism has now advanced from a mere hypothesis to the level of a well-grounded scientific theory.[7] Thus the goal was attained for which Boltzmann constantly struggled in his sharp methodological critique of energetics.[8] But the actual misunderstanding of which Ostwald was guilty in his challenge against atomism lay in a different quarter. He denied atomism because he saw in it the most dangerous support of scientific materialism. The fate of this materialism seemed to him to be indissolubly tied to that of atomism; the one seemed to stand and fall with the other. But precisely this assumption in the long run proved untenable. It became evident that the atomic concept was not only independent of that of matter, but that after having hitherto been considered the mainstay of materialism and mechanism, it was destined to instigate the truly scientific and not speculative victory over the mechanical conception of nature. This development had not been anticipated by the opponents of atomism. It is true that Helmholtz in his Faraday lectures of 1881 pointed out that Faraday's discoveries, in particular his law that in electrolysis chemically equivalent quantities of matter always carry equal quantities of electricity, lead to the conclusion that not only matter but also electricity has an atomistic structure and is divided into definite elementary quanta.[9] Helmholtz immediately

5. G. F. Helm, *Die Lehre von der Energie historisch-kritisch entwickelt* (Leipzig, 1887), p. 56.

6. W. Ostwald, *Die Überwindung des wissenschaftlichen Materialismus* (Leipzig, 1895), pp. 12 f.

7. W. Ostwald, *Grundriss der allgemeinen Chemie,* 4th ed. Leipzig, 1909.

8. Cf. esp. L. Boltzmann, "Ein Wort der Mathematik an die Energetik," *Wiedemann's Annalen, 57* (1896); *Populäre Schriften* (Leipzig, 1905), pp. 104 ff.

9. "On the Modern Development of Faraday's Conception of Electricity," Faraday Lecture, 1881, *Wissenschaftliche Abhandlungen* (3 vols. Leipzig, 1882–95), *3,* 69.

pointed to the fundamental significance of this view: he designated it as perhaps the most surprising result of Faraday's investigations. However the ultimate, and epistemologically most remarkable, result was attained through Planck's law of radiation. For here the roles were exchanged: the enemy that was resisted so violently and stubbornly by energetics penetrated its own ranks. By proving that the energy of radiation of each frequency can only be emitted and absorbed in multiples of a definite elementary quantum, the atomistic character of energy itself was maintained, and only in this way was the key supplied that opened up the world of "atomic physics."

The epistemological problem of the "reality" of the atom thus entered a new stage and became more significant and urgent than ever before. But an immediate solution of the problem has not been offered by quantum theory either. Rather, as time goes on, all the old contradictions appear again. While the theory initially made use of definite pictures from which it attempted to read off intra-atomic events, the use of such models has been abandoned by modern quantum mechanics. The latter declares expressly that in relation to the microcosm every attempt at "visualization" is doomed to failure.[10] It is bound to fail because through the uncertainty relations the problem of space-time descriptions has become considerably more acute: for an isolated object such a description no longer exists. Instead all statements are tied to the interaction between the object on the one hand and the observer or his apparatus on the other. Modern atomic physics, according to Heisenberg, deals with the nature and structure not of atoms but of the events which we perceive when observing the atom. The process of observation cannot be simply objectified; its results cannot be turned immediately into real objects.[11]

Thus we are again confronted with the old dilemma, the choice between subjectivism and objectivism, between phenomenalism and realism. But before we try to come to any decision, we have to pose again the general question as to the sense in which an objective reality of atoms can be spoken of. It can hardly be doubted that the purely nominalistic solution, as offered by Mach, is no longer tenable or usable, for it cannot explain the essential features of the atomic hypothesis, or its eminent fruitfulness. If the atomic concept is really nothing but an

10. Cf. for instance Schrödinger, "Conceptual Models in Physics": "Despite the immeasurable progress which we owe to Bohr's theory, I consider it very regrettable that the long and successful handling of its models has blunted our theoretical delicacy of feeling with reference to such questions. We must not hesitate to sharpen it again."

11. "Kausalgesetz und Quantenmechanik" (1930), *Erkenntnis, 2* (1931), 182.

economic device, a very natural symbol arising unconsciously for a whole complex of sensory elements—elements, however, which should not impress the physicist—then this fruitfulness remains a riddle. For we can readily understand how by such a device the facts already given can be simplified, classified, and brought to order, but not how entirely new facts can be discovered. And it is precisely this function which the atomic concept throughout its history has consistently discharged. It is not to be compared to a map depicting the explored territory completely and concisely; rather it resembles a compass which in the hands of the explorer leads ever and again to distant unknown shores. It was such an advance into previously unexplored territory when Laue, for instance, in his studies on the bending of X rays, succeeded in obtaining the diagrams by which an entirely new key to the structure of crystals was provided, and through which its regular construction out of atoms and molecules was made visible on the photographic film. The wealth of different independent proofs of the granular structure of matter which the last decades have provided is indeed enough to impress not only physics but epistemology as well. The statements resulting from the kinetic theory of gases also took on a more precise form and permitted an increasingly exact verification. It became possible to arrive at quite definite conclusions concerning the properties of the individual particles, their number, their size, and their mass. For instance by means of Avogadro's Number, the number of gas molecules in one cubic centimeter was determined. These are new universal constants to whose knowledge the atomic hypothesis leads—a knowledge enjoying the highest degree of objectivity attainable within science.

The above, however, also shows in what sense we are to understand this objectivity. We are concerned not so much with the existence of things as with the objective validity of relations; and all our knowledge of atoms can be led back to, and depends on, this validity. Here also we can only arrive at a clarification of the problem if we adhere strictly to the proposition that in all scientific knowledge laws precede objects. No object is "given" us other than through laws, and our knowledge of the object goes only as far as it can be designated by definite relations, by exact functional equations. To the same extent that the system of functional equations grows, to that extent the knowledge of the object becomes more precise and takes on a stricter form. The changing character of the atomic concept furnishes a characteristic example of this process. Every new field in which atomism is employed furnishes a new set of conditional equations, which the atom must obey. As the number of such conditional equations increases, the atom so to speak takes on a more concrete form, because an increasingly large number of particular

data of knowledge become united in the concept. But this "concrescence" of determining factors stands in an inverse relation to what we commonly designate as visualizable concretion. The more the conceptual determination progresses, the less it proves possible to fix its results in simple particular visual images. We have to be satisfied with determination by means of laws, and must forego any clarification through models. But true physical objectivity is here not diminished but increased, for it must never be sought in the direction of "naive" things, but rather in the direction of critical laws.

The history of the atomic concept and its applications is one continuous example of this situation. Each new application sets up a new condition. All branches of science take part in this consolidation of the atomic concept, a consolidation, however, that is the exact opposite of its immediate hypostasis, its immediate conversion into an object. The process begins in logic and philosophy of science, and from there it moves to mechanics, to chemistry, electrodynamics, and spectroscopy. And whenever, by means of the atomic concept, a new field of experience is opened up, the atom itself gains, as it were, a different and new appearance. For its essential significance is that it leads not only to new individual results but to new universal aspects of natural events. However, it can only remain adequate for the latter function if it always remains flexible. If in order to make this clear we begin with Greek atomism, we will find that the conditions which the atom has to fulfill and through which its "being" is determined are limited to a minimum. It has frequently been claimed that the introduction of the atomic concept in the system of Democritus had no relation to empirical knowledge, that it took place only out of speculative considerations. But this assumption is not confirmed by exact historical and systematic investigation. It is true that it was not individual observations that led to the formation of the classical atomic concept, and that it was not derived from experimental determinations; but this must not lead to the conclusion, as it did for Baeumker,[12] that Greek atomism in establishing its basic thesis did not permit itself to be influenced by an analysis of objective processes of nature. Such an analysis indeed exists, and was prosecuted with admirable clarity. But it is directed not to individual processes of motion but rather to the basic phenomenon of motion itself. The ground for this phenomenon seemed to be taken away by the Eleatic critique of becoming. The attempt to uphold the reality of motion led to unanswer-

12. See C. Baeumker, *Das Problem der Materie in der griechischen Philosophie* (Münster, 1890), p. 87; cf. also E. Zeller, *Philosophie der Griechen* (6 vols. Leipzig, 1879–1919), *1*, Pt. II, 854. For what follows cf. my presentation of Greek atomism in M. Dessoir, ed., *Lehrbuch der Philosophie* (Berlin, 1925), *1*, 55 ff.

able contradictions, as Zeno's arguments attempted to demonstrate. There was no choice but to put logic ahead of sensory perception and thus surrender all reality of motion. This is where the atomistic idea begins. It accepts the condition of Eleatic logic; it admits that whatever truly deserves the appellation of "being" must be imagined as something permanent, persistent, and unchangeable. But on the other hand it does not wish to forego the basic conditions demanded by empirical observation. The Eleatic condition seems to be compatible with reason, but in face of actual facts it resembles something akin to madness. There is only one way of avoiding both absurdities, the logical as well as the empirical. We must find rational grounds (λόγοι) which harmonize with perception—that is, explain and elucidate the phenomenon of motion instead of destroying it.[13] Such grounds are the atoms and empty space, whose objective reality is thus assured, although they elude every possibility of being perceived by the senses. For not the senses but the understanding is the criterion of reality: what it recognizes as being objectively necessary is at the same time proven in its truth and actuality.

Thus an entirely universal condition was set up, and "the prolegomena for any future physics which can appear as a science" were provided. However, there was at first no means for satisfying the methodological postulate thus set up. A concrete physics could not be based upon the foundation of Greek atomism. For such a physics it was found necessary at the time to go back to the qualitative view of natural events, which finds its consistent presentation and its systematic justification in the physics of Aristotle. As long as the atomic concept remained in the uncertainty of a purely logical conception it had nothing conclusive to oppose to this system of peripatetic physics. The atomistic theory did not cease to exist during the Middle Ages, but it was unable to achieve a philosophically and scientifically satisfactory foundation.[14] A decisive change did not take place until Galileo undertook the problem of a mathematically precise foundation of dynamics. From here on the fate of atomism is indissolubly coupled with this problem. It played an important part with Galileo himself and later with Gassendi in the formulation of the new view of nature, the "mechanical world view." But the latter no longer remains a mere outline, for instead of being based only on the concepts of motion and empty space, it can now be based on the empirical laws of motion and their mathematical formulation. These laws now constitute the firm framework for the structure of atomism. The

13. Cf. Aristotle's account of the evolution of atomism, *De generatione et corruptione* 8. 324^{b}, 35.

14. For further details see Kurd Lasswitz, *Geschichte der Atomistik vom Mittelalter bis Newton,* Hamburg and Leipzig, 1890.

visualizable properties such as the solidity, rigidity, and absolute hardness of atoms are now, though not entirely given up, pushed further and further into the background. They are retained as visual aids, but they no longer determine the basic character of the atomic theory. This theory is concerned with describing not the actual nature of atoms but solely their actions and reactions; and in the general rules for these interactions it sees the conditions for all natural events. Thus in Huyghens, whose work may be considered the high point of kinetic atomism, the synthesis of the concepts of substance and causality was already accomplished. Huyghens explains all natural events as mere changes in position of atoms. In his presentation and establishment of atomism he no longer needs to stress the atom's hardness or rigidity. The stability of nature is no longer guaranteed by such visualizable thinglike representations, but by certain universally valid principles. The laws of conservation of momentum and energy are introduced as the basic principles of mechanics and hence as the rules governing atomic collisions. These mechanical principles are shown by Huyghens to be the basic presuppositions of atomism and the conditions of its feasibility. Lasswitz, in his excellent presentation of Huyghens' atomism, claims:

> The most essential and decisive aspect of Huyghens' work is the fact that he started out not from concepts that were anthropomorphically colored or derived from sense perception, but from mechanical facts which were taken by him as basic laws because they were necessary and sufficient for the unambiguous determination of the motions of bodies, for the calculation of their speeds and directions, when those before the collision were given. It is not that the *vis viva* of moving bodies is retained after the collision because the bodies are elastic; rather the collision takes place in the manner observed in bodies which we designate as elastic, because momentum must be conserved.[15]

This shows what changes have taken place in the nature and properties of the atom: because a new condition is imposed and because it was no longer determined by those theoretical speculative considerations which were of primary importance in classical atomism, but was determined instead by certain empirical and mathematical laws.

With the beginning of scientific chemistry a new development in the atomic concept began. The demand for a strictly corpuscular structure for chemistry was made by Robert Boyle in the seventeenth century. Boyle's *Sceptical Chymist* (1667) argues that only by introducing the atomic concept could the dominion of occult qualities in the realm of

15. Lasswitz, *2*, 369.

chemistry be ended. In this book the step from alchemy to chemistry was in principle taken; it taught that we can understand chemical facts not by the assumption of secret affinities between bodies but solely through structural and dynamic laws governing their smallest particles. But this viewpoint still remained unfruitful, because the necessary quantitative premises for its development were lacking. For the time being, chemistry persisted in the qualitative consideration of nature, which once more was emphasized with characteristic clarity in Stahl's phlogiston theory. But when, through Lavoisier's experiments, the revolution took place—when all chemical facts were brought under the viewpoint of measure, number, and weight—the hour of atomism had struck. Lavoisier points out that all chemical experimentation is based on the principle that an actual equality or equivalence exists between the original substances and those which are produced by chemical reaction.[16] Thus all statements about the atoms that are arrived at from these experiments must ultimately serve to give a definite expression for this equivalence, and to make it theoretically understandable. In Dalton's law of multiple proportions atomism celebrated its first triumph. It was immediately enabled to explain the empirical fact of constant combining weights. These constant equivalent weights found their simplest and most compelling explanation in the assumption that atoms of different elements have a definite weight, characteristic for each element. To this is added Avogadro's hypothesis according to which equal volumes of all gases contain the same number of molecules. On this simple basis the entire subsequent evolution of chemistry has been constructed.[17]

As with Huyghens, the atom is here defined strictly substantially; but all statements concerning its nature ultimately merge into statements concerning quantity—definite numerical relations. All the basic concepts and theories of chemistry, the concepts of valence and types, the concept of the radical as well as the theory of compound radicals—all point in this direction. The conceptual language of atomism in conjunction with the general symbolic language of mathematics now permits that simple and magnificent development in the language of chemistry that is evident in the progress from simple to structural formulas. Stereochemistry, as established by van't Hoff, permitted the elucidation, through a complex construction of formulas, of an entirely new group of facts, the phenomena of stereoisomerism; it threw new light on the distribution of atoms in space. Thus the visual picture of the atom is modified and refined, but all these changes serve only the one

16. A. L. Lavoisier, *Traité élémentaire de chimie* (Paris, 1789), Pt. I, chap. 13.

17. I attempted to demonstrate this evolution from epistemological viewpoints in my *Substance and Function*, pp. 203 ff.

purpose of circumscribing and representing an increasingly large domain of functional relations.

Another new beginning and a fundamentally new orientation of atomism occurred when the electrodynamic theory replaced the substance theory of matter, thus constituting a further step toward the purely functional interpretation of nature. With the discovery of the facts of electrolysis, with the investigations of the nature of cathode rays and the phenomena of radioactivity, the basis for a new theory of matter is laid which ascribes to it a purely electrical constitution. The concept of electric charge is now introduced as the essential basic element of scientific knowledge, and further developments tend more and more toward considering the mass of the electron as merely an "apparent mass," and to merge it completely with the electric charge. Electricity thus displaced matter from its central position; Huyghens' statement, from the standpoint of kinetic atomism, that either the mechanical interpretation must be adhered to or all hope of a rational explanation of nature abandoned, has thus been disproved. Again it is the atomic models which express this revolution in the basic concept most clearly. They are, as it were, the mirror which captures and reflects all the new observational material as well as the progress of theoretical ideas. Herein lies their extraordinary value from an empirical viewpoint, but here also lies their epistemological limitation. Their power resides in their function of expression and representation and not on an independent existence apart from this function. It is one of the achievements of quantum mechanics that it recognizes this state of affairs more clearly than had ever been done before, that it saw the significance of atomic models especially in their heuristic function, and thus clarified their epistemological limitation. Now the express condition was placed on them not to embody any contents which in principle defy experimental confirmation. In 1904 J. J. Thomson, following Lord Kelvin, constructed a model still based on the presuppositions of the classical theory of electrodynamics. The atom here consists of a sphere uniformly filled with a positive charge, within which the electrons either move in circular orbits or oscillate around certain equilibrium positions.[18] The shortcoming of this model was its inability to adjust itself to the new data from experiments made with corpuscular radiations; the scattering of α-particles when passing through thin metal foil could not be explained by Thomson's model. Rutherford's theory (1911) was therefore advanced, according to which the bearer of the greatest part of the atom's mass is a positively charged nucleus, around which a number of lighter negatively

18. Cf. J. J. Thomson, *The Corpuscular Theory of Matter* (London, 1907), pp. 103 ff.

charged electrons rotate. The deflection of the α-particles was explained as a repulsion from the nucleus. Thus an astronomical description of intra-atomic phenomena was obtained. The atom was regarded as a planetary system, and the individual electrons as planets moving around the central body, the nucleus. However, the stability of this model necessitated a separate justification supplied only by Bohr through the quantum postulate. The path of the electron was now fixed by two different equations, one belonging to classical, the other to quantum, mechanics.[19] Only thus could the fundamental fact of spectroscopy, the existence of sharp spectral lines, be explained. This shows how once again the addition of new conditions, whether required empirically or theoretically, lends definite "form" to the atom. Its structure is nothing but the comprehensive expression of these conditions. All major departments of physics take part in its increasingly clear and precise formulation. Mechanics, chemistry, thermodynamics, and electrodynamics all influence this process in the same way, and by thus striving toward a common goal they are united into a methodological unity.

But when contemplating this process in its entirety, one can hardly help feeling a keen disappointment and deep skepticism. For what is the significance of this sequence of pictures which move past us in unending change and kaleidoscopic brilliance? Is all this anything but a curious phantasmagoria? What good is the wealth and constant refinement of pictures if not one of them has a claim to actual, final truth, if when hardly conceived, it is supplanted by another that in turn is subject to the same fate? Again the answer seems to be none other than du Bois-Reymond's "ignorabimus." Never shall we know better than we do today, as du Bois-Reymond expressed it, what "here," where matter is, "haunts space." [20] But *do* we know this better today? Do we not perhaps know it less clearly than ever, and has not quantum mechanics—far from disproving du Bois-Reymond's prophecy—made the reasons for our ignorance better known to us and enunciated them more clearly in the uncertainty relations? This conclusion is indeed unavoidable if we remain within the substantialistic viewpoint: here again substantialism is bound to lead to agnosticism. However, the problem presents itself from an entirely different angle if the functional viewpoint is taken seriously. Then the multiplicity of answers to the question of the "being"—the nature and structure—of the atom is no longer objectionable. For a function *is* only insofar as it unfolds itself in a progressive series of values. It cannot be condensed into a single picture; but it includes all of its

19. For details see Bohr, *Drei Aufsätze über Spektren und Atombau*, 2d ed. Braunschweig, 1924.

20. See above, p. 7.

individual forms under a universal rule; it contains them as different cases of application. Not one of these applications is a final goal with which knowledge can remain satisfied. It is always only a milestone from which our knowledge can find out where it is and how far it has progressed. The fact that the road continues farther and farther ahead, and that we see no end, no *non plus ultra,* need not trouble us, for the truth which we seek, being an empirical truth, can only be that of a process and not of a finally concluded result. If, in this progression, we can move in a single direction and be sure of it, we have in this direction and in it alone the criterion of truth. Thus we need not find an answer to the question of what the atom actually and ultimately is, as long as we are sure that the various forms of determination given us by progressing experimental and theoretical investigation can dovetail and mutually supplement one another. As long as this empirical interconnection is safeguarded, and as long as we are convinced that the progression of atomic models is guided not by mood or caprice but by a definite rule, the atom has all the reality of which it is capable. It is, then, no mere subjective economical device, nor does it possess a substantial thinglike being, a being immediately describable by analogy with ordinary perceivable objects. That kind of description, as opposed to the development of the atomic concept which we have attempted to follow in its individual phases, would indeed appear to be a curious misconception, an epistemological naiveté. For it would force us to abandon the entire progress of knowledge so painfully won; it would throw us back to the very beginning of the path. In Greek atomism a definite visual, tangible form could still be ascribed to the elements which make up matter; indeed the term "form" (ἰδέα) appears to have been the one Democritus chose for his atoms. Here it was not necessary to avoid visualizable descriptions. In order to explain how the differently formed particles are connected and interlock, they can be equipped with any kind of hooks or claws.[21] But there is no room for such naive descriptions in the establishment of mechanics, for here the nature of atoms is determined by the laws and principles of motion, and no attributes but those that directly follow from these principles can be ascribed to them. However, it is not necessary to shun these intuitive aids as long as one is conscious of their limitations. The atoms may be thought of, in a certain sense, as small rigid spheres filling a definite amount of space. Yet this representation collapses as soon as we take the step into field physics: the electrical constitution of matter cannot be visualized as simply as the mechanical one. As far as the quantum theory is concerned, it was evident from the

21. Cf. Simplicius, *De coelo,* 133a, 18 (Zeller, p. 889).

beginning that definite limits are set to every direct space-time description of atomic processes. Niels Bohr in this respect never overestimated the significance of his atomic model; rather he expressed his conviction that the quantum theory does not represent a modification in mechanical and electrodynamic theories describable on the basis of common physical concepts, but a profound failure of the space-time representations by which descriptions of natural phenomena were hitherto attempted.[22] The representations fail—but the relations laid down in the statements of the results of measurements and laws remain in force, and it is these which form the ultimate foundation of physical knowledge.

If we now revert to the starting point of our considerations, we notice a peculiar development in the use and the epistemological appraisal of the atomic concept. When Ostwald, in his lecture at Lübeck in 1895, declared war on atomism, when he announced that we must definitely forego any hope of ever visualizing the physical world by reduction of all phenomena to an atomic mechanics, he at the same time posed the question as to what other means of forming a picture of reality were left to us? "In answer," he declares, "I should like to exclaim: Thou *shalt not* make unto thee any graven image or any likeness of anything! Our task is not to see the world in a more or less clouded or distorted mirror, but to see it as directly as the constitution of our being will possibly allow. The task of science is to put realities, demonstrable and measurable quantities, into definite relation with one another, so that when some are given, others can be deduced, and it cannot be fulfilled by applying some hypothetical model, but solely by the evidence of interdependent relations between measurable quantities." [23] In this methodological postulate Ostwald did not err. The representatives of modern quantum mechanics would no doubt all agree in this respect with his program for scientific knowledge. Ostwald's mistake consisted in accusing the atomic concept as such of a fault which actually concerned only an application of it in the construction of the mechanical world view. Only when the concept is freed from this entanglement does it appear in all its fertility and significance. The merely pictorial attributes vanish, but its use is extended rather than hindered. Epistemologically speaking, modern quantum mechanics represents an important and decisive step in this direction. It avoids the danger of pictorialism by forcing the pictures it uses to limit themselves and hold each other in check. For it never allows one picture alone to hold the stage and to appear as the exclusive representation of the process observed. It elevates the dualism

22. *Atomtheorie und Mechanik*, p. 22.

23. *Die Überwindung des wissenschaftlichen Materialismus*, p. 22.

between the particle and the wave picture to a principle, and thus exerts a strong critique on both representations.[24] In this way the dangers inherent in each purely pictorial description are counteracted by neutralizing the particle representations with those based on the wave picture, and vice versa. The question as to which of these pictures—discontinuous particles or continuous waves—correctly represents reality remains unanswered by quantum theory. All that the latter can and intends to teach us is merely the rule according to which we can establish a purely symbolic correspondence between the two pictures, in such a way that both combined can furnish a precise conceptual expression of the laws valid for the phenomena of atomic physics.[25] We are forced, then, to exercise a certain amount of caution when attempting concrete representations of natural events; we may never ascribe to any of them an absolute value, only a relative one. But this limitation and neutralization leaves no ambiguity and no mere uncertainty as long as we strictly adhere to the new criterion of physical knowledge thus established—as long as insight into strict and precise relations between observable magnitudes is regarded as the only goal.

24. This program was carried through by Heisenberg in his *Physikalische Prinzipien der Quantentheorie.*

25. Cf. for instance I. Frenkel, *Einführung in die Wellenmechanik,* chap. 1, sec. 9.

PART FIVE

Causality and Continuity

CHAPTER 11

The Principle of Continuity in Classical Physics

THE MECHANISTIC VIEWPOINT—the view, that is, that all natural occurrences can be reduced to the motions of infinitesimal mass particles—can be contested and shown to be inadequate on epistemological grounds; but it is an entirely insufficient and inadmissible weapon to contest it on the grounds that mechanistic physics is derived from a materialistic form of thought. This weapon has again and again been used not only in popular discussions but even in refutations which try to appear strictly philosophical. Mechanism is accused of representing an external non-spiritual (*ungeistige*) mode of interpretation, of not having any feeling for the wholeness of natural occurrence, being satisfied instead with the mere comprehension of its parts. However effective this argument may be, it is entirely false and superficial. A glance at the history of the mechanical world view suffices to invalidate this mode of proof. It is essentially and intimately connected with the evolution of idealism and not with materialistic philosophy. Hobbes indeed appears to coordinate mechanism and materialism, and to merge them into each other. Yet even for him the latter is merely a consequence of the former and by no means its intrinsic ground. The basis of mechanism is found in a different place, even for Hobbes. It is to be found in the principles of Galilean dynamics and in Hobbes' own nominalistic and rationalistic logic. As for the actual methodological foundation of the mechanical viewpoint, it is associated with names like Descartes, Leibniz, and Kant. Descartes' *Principia philosophiae* offers the first universal and consistent foundation for mechanism; Leibniz, despite his "spiritualism" never tires of emphasizing that all natural events are to be explained "mathematically and mechanically" and he even goes so far as to consider mechanical reasons the only intelligible reasons within the realm of matter.[1] As for

1. Cf. for instance *Philosophische Schriften*, ed. Gerhardt, *7*, 265.

Kant, he changes mechanism into pure dynamism by reducing matter to forces of attraction and repulsion, but this dynamical construction itself is based on the presupposition that all mathematical observation of nature is necessarily tied to the concept of mass points and the laws which determine their mutual interaction. If it is desired to penetrate to the real reasons for this presupposition, it is necessary to go much deeper, to clarify the historical and systematic conditions from which the mechanical world view has arisen and from which it has drawn its essential force.

Descartes' *Regulae ad directionem ingenii* and his *Discours de la méthode* attempt to establish a strictly universalistic world view, a view which in no sense proceeds from the parts to the whole, but instead from the whole to the parts. Thus the *Regulae* begin by combating the conventional view, which dissects knowledge and believes that "true" knowledge can be attained by fitting its parts together. Knowledge is an indivisible unity, as indivisible as the unity of the intellect from which it springs. All the sciences are merely the human intellect (*humana sapientia*) applied to many and varied objects. There must, therefore, exist a truly universal science, a *mathesis universalis* forming the basis for all the different directions and branches of knowledge. It is from this science, and not from the individual objects of knowledge, that we have to start if we are to gain a firm and unshakable foundation for knowledge. Based on this proposition, Descartes attempted to replace the existing ontological division of knowledge by a purely methodological one. All objects, in order that we may survey them, must be divided into certain classes, but these classes cannot be determined from the viewpoint of "being," only from that of knowledge. The first point of view has hitherto governed the doctrine of categories of the philosophers. It is the second that characterizes the new method and determines its scientific character. The arrangement, organization, and structure of knowledge must be accomplished in such a manner as to furnish a survey of the interrelation of the branches of knowledge, of the manner in which they dovetail into and depend on each other. Only on the basis of this methodological insight can we venture an ontological judgment, or a statement about the constitution and relationship of things.[2] Not the

2. Regula VI: "Haec propositio praecipuum . . . continet artis secretum; monet enim res omnes per quasdam series posse disponi, non quidem in quantum ad aliquod genus entis referuntur sicut illas Philosophi in categorias suas diviserunt, sed in quantum unae ex aliis cognosci possunt."

Rule VI: "This proposition . . . contains . . . the chief secret of method. . . . For it tells us that all facts can be arranged in certain series, not indeed in the sense of being referred to some ontological genus such as the categories employed by Philosophers in their classification, but in so far as certain truths can be known

complex intermingling of things with one another but the complexity of cognitions is the basic theme of Cartesian philosophy. Its principal goal is to furnish the crucial clue that will protect us from getting lost in the labyrinth of particular cognitions that interlace in a way that is quite incomprehensible at first glance. There is no escape from this labyrinth unless we adhere strictly to the instruction always to proceed from the simple to the complex, from the easier to the more difficult. The complexity here taught, far from being a mechanical complexity, is a strictly logical one. We begin with a rule of thought, the rule that we are to put all thoughts in such order that we may begin with the easiest and most understandable problems and proceed, slowly and steadily, in a definite gradation, to the most complex.

From this method of division and subdivision arises the mechanism which governs and penetrates Descartes' philosophy. It requires that all problems that the human intellect poses and all difficulties encountered by it should be separated into their smallest parts, and in this he sees the only way of mastering them.[3] We cannot come to a firm footing in any realm of objects, we cannot describe it by clear and distinct ideas, unless we first determine the "simple natures" out of which it is constructed. These *naturae simplices* are the fundamental and original concepts of each realm of objects; they are the originals and models on which we pattern all other cognitions. There is only a small number of such basic concepts: the concepts of being, number, duration, extension, and motion, and in the realm of spirit those of consciousness or thought.[4] By virtue of this system of primary concepts (*notions primitives*) it is possible to strip the realms of objects of their apparent diversity and heterogeneity and to relate them one to the other. Through the concept of extension physics is based on geometry, and within the latter a further simplification may be attained by reducing the concept of extension to that of number by the method of analytical geometry that Descartes discovered and established on the basis of precisely this primary conception. This is the firm foundation upon which Descartes'

---

from others." *Descartes Selections,* ed. Ralph M. Eaton, New York, Scribner's, 1927, p. 57.

3. *Discours de la méthode,* Pt. 2: "Le second précepte est de diviser chacune des difficultés que j'examinerois en autant de parcelles qu'il se pourroit, et qu'il seroit requis pour les mieux resoudre." *Oeuvres de Descartes,* ed. Adam and Tannery (Paris, L. Cerf, 1897–1909), *6,* 18.

"The second precept was to divide up each of the difficulties which I examined into as many parts as possible, and as seemed requisite in order that it might be resolved in the best manner possible." *Descartes Selections,* p. 17.

4. Cf. letter to Countess Elizabeth, May 21, 1643; *Correspondance,* ed. Adam and Tannery, *3,* 665.

mechanical system of nature is based. "I may be asked," he says in his *Principles,*

> by what method I arrived at the knowledge of the smallest, unobservable particles of matter. To this I reply that I first considered all the clear and distinct concepts of material things that can occur in our mind and that I found none but the concepts of magnitude, figure, and motion, and the rules according to which they affect each other. Since these rules are the principles of geometry and mechanics, I judged that all knowledge of nature available to man must be derived from them alone because all other conceptions we have of sensible things are obscure and confused and therefore cannot serve to give us a true knowledge of these things, but may serve to impede us. Then I examined all the principal differences which can exist among the figures, magnitudes, and motions of different bodies and which, owing to their smallness, cannot be directly observed, and I asked myself what perceivable effects can arise out of their interaction. When I actually discovered such effects in nature, I thought that they might have originated in this manner. Later, however, I drew the conclusion that they must certainly have so originated, because I saw that in the whole realm of nature no other cause capable of bringing them forth could be found.[5]

Leibniz develops with equal clarity the logical and philosophical conditions underlying the mechanical system for the explanation of nature. But in the meantime an important change had taken place in the theory of principles through the step from analytical geometry to infinitesimal calculus, a change which required a clarification and a more accurate determination of this system. Leibniz' analysis of infinity is based on his continuity concept. It is this which henceforth connects and unifies mathematics and natural science. The description of nature in continuous and differentiable functions now becomes the primary postulate; the continuity principle becomes the basic presupposition of all exact knowledge of nature. For Leibniz it is not an empirical principle deducible from, and capable of being reduced to, individual observations. Rather he introduces it as a rational principle, a principle of universal order (*principe de l'ordre général*). As such it is unconditionally valid in the fields of logic and geometry, but the same exceptionless validity must also be granted it in science. For according to the basic presuppositions of Leibniz' philosophy no chasm can exist here. The ideal and the real, truth and actuality, are interwoven and related to one another through

5. *Les Principes de la philosophie,* Pt. IV, article 203; ed. Adam and Tannery, *9,* 321.

a pre-established harmony. All "factual" truths (*vérités de fait*) are thus based on "eternal" truths (*vérités éternelles*) and only explicable by the latter. And it is precisely this connection that is guaranteed by the continuity principle. This principle is not simply a law of nature; rather it is a universal law showing us how we are to find and to grasp natural laws. It is in this sense that Leibniz uses it as the universal "touchstone of knowledge." The laws of impact introduced by Descartes must be wrong because they do not pass this test, because they contradict the principle of continuity.[6] Nothing, perhaps, is more suited to illustrate the sharp opposition between the classical and the modern system of basic physical concepts than the fact that Leibniz places the continuity principle in precisely the same spot which the quantum principle occupies in atomic physics, and that he uses it in exactly the same methodological sense. For him it is a heuristic principle and a general postulate of scientific knowledge.[7] Only on the basis of this postulate can the newly achieved instrument of knowledge, the analysis of the infinite, be made to serve the investigation of nature and to be truly fruitful there.

Leibniz, however, is quite clear that the existence of a nature in which sudden jumplike changes take place is by no means impossible; yet according to him it would be most unsatisfactory logically; it would not be commensurate with the "Creator's wisdom." This teleological and metaphysical argument, however, is by no means the only one given by him for the general validity of the continuity principle. Much more characteristic and significant is another argument, based on a purely epistemological postulate. If the continuity principle were invalid, we should have to recognize that a barrier was imposed on thought by the very nature of things. A refinement of our instruments of observation, an increased accuracy of analysis, would then not necessarily lead to a closer approach to reality; in some cases it might even lead us away from it. However, Leibniz' system, which is based entirely on a thoroughgoing correspondence and unbroken agreement between thought and being, between concept and reality, cannot accept such a possibility. Nature cannot impose any fixed limitation on analysis, since analysis itself is the only means by which we can arrive at clear and distinct ideas

6. Cf. esp. *Mathematische Schriften,* ed. Gerhardt, *6,* 129 ff.; *Hauptschriften,* ed. Cassirer and Buchenau, *1,* 84 ff.; *Animadversion in partem generalem principiorum Cartesianorum,* in *Philosophische Schriften,* ed. Gerhardt, *4,* 375: "Before I come to the special laws of motion given by our author, I shall give a general criterion and, as it were, a Lydian stone, with respect to which they can be examined, which I am accustomed to call the law of continuity."

7. "The law of continuity which I have been using for a long time as a *principle of invention in physics,* and [which I have been using] as a very useful test to check whether some given rules work well." *Mathematische Schriften,* ed. Gerhardt, *4,* 105.

that assure us of the truth of nature. There cannot and must not be a breach here, for it would at once render questionable the rationality of nature.

From this standpoint Leibniz' attitude toward atomism becomes immediately understandable. Not as an empiricist but as a logician of mathematical-physical knowledge does he contradict the atomic hypothesis. Many attempts have been made, in the history of philosophical as well as physical thought, to establish a compromise between atomism and continuity, by formulating the continuity principle in such a manner as to contain merely a demand applying to occurrences and not to being. In occurrences continuous transitions were required, whereas for being, for the configuration of things in space, precise and discrete limits were assumed. But Leibniz was too keen an analyst to overlook the faults in such a compromise. The assumption of atoms, as he explains, always involves a violation of the continuity principle. For if one bases the interaction of atoms on the laws of impact, it follows that in rebounding the change of location proceeds continuously, but at the instant of collision a discontinuous change of velocity of the atoms takes place, since this velocity suddenly changes its sign. Such a change in the direction of the velocity is, however, impossible unless the latter continuously approached and finally reached a zero value.[8] This specific reason is further supplemented by Leibniz by another more general one, stating in essence that it is arbitrary to prescribe an insurmountable limit to our analysis. Such a supposition cannot claim any truly rational basis; rather it springs from the weakness of our imagination and attempts to adjust nature to this weakness. Nature, however, laughs at any such attempts, for it comes from the infinite and proceeds to the infinite.[9] If mathematics, or the new form of analysis discovered by Leibniz, is to be harmonized with nature, one has no choice but to grant full applicability to its basic concepts of the infinitesimally small and the infinitely large. Leibniz was not dissuaded from this attempt by the objections raised by Aristotle against the actual infinite. He writes to Foucher: "I am so much in favor of the actual infinite that instead of admitting that nature abhors it, as it is commonly said, I hold that nature affects it everywhere, in order to point up better the perfections of its author. Thus I believe that there is no part of matter which is not, I don't say divisible, but actually di-

8. On this point cf. Leibniz' correspondence with Huyghens on the subject of atomism, *Hauptschriften*, ed. Cassirer and Buchenau, *2*, 35 ff.

9. "Atoms are the effect of the weakness of our imagination, for it likes to rest and therefore hurries to arrive at a conclusion in subdivisions or analyses; this is not the case in nature, which comes from the infinite and goes to the infinite. Atoms satisfy only the imagination, but they shock the higher reason." To Hartsoeker, Oct. 30, 1710, *Philosophische Schriften*, ed. Gerhardt, *3*, 507.

vided, and therefore the smallest particle must be considered as a world full of an infinity of different creatures." [10]

Thus a general program was established, a program which, however, had to wait a long time for its scientific realization. The union between the new mathematics and physics, in spite of the fact that each has benefited directly from the other, has taken place very slowly if considered from a methodological viewpoint; thus for instance the concept of the coordinate system was introduced into mechanics comparatively late by Maclaurin. It is true that Leibniz analyzed and gave a new form and basis to the fundamental concepts of dynamics; but he did not develop a complete system of mechanics. Newton also proceeds extremely cautiously in the physical application of his method of fluxions; although he tacitly presupposes it everywhere, he prefers to use the older, synthetic methods as developed by ancient mathematics in his explicit presentations. A true penetration did not take place till the time of the great French mathematicians of the eighteenth century, d'Alembert and Lagrange. As empirical investigators they are both Newtonians, but as logicians and methodologists of physics they remained entirely Cartesian. The very title of Lagrange's basic work points to this. Analysis is now definitely to have its place in physics; the Cartesian ideal of simplicity, of reduction to *naturae simplices,* is to have universal validity. A single system of equations is to embrace all static and dynamic phenomena, and the form of these equations is determined by the new analysis of the infinite. In this manner d'Alembert and Lagrange arrived at the principle of "virtual work" which expresses in the simplest conceivable way the general condition of all phenomena of equilibrium and motion. D'Alembert, in the preface to his *Dynamique,* points out that success had been attained long ago in the attempt to apply algebra to geometry, geometry to mechanics, and each of these sciences to physics. But insufficient care had been taken either to reduce the principles of these sciences to the smallest possible number, or to give them the desired clarity. D'Alembert's *Dynamique* and Lagrange's *Mécanique analytique* attempt to rectify this logical fault, and Lagrange's work, particularly, completely realized this ideal. By the establishment of differential equations of motion, immediately deducible from d'Alembert's principle, it achieved the first full unification of physics and infinitesimal analysis and at the same time furnished a model of logical clarity and precision which remained exemplary for all the following period.

Thus the link between the principles of causality and continuity became ever closer and more natural. In classical physics they were

10. *Philosophische Schriften,* ed. Gerhardt, *1,* 416.

henceforth so closely interwoven that it became difficult to distinguish them even conceptually. Kant did not destroy this close relationship; rather he confirmed it and attempted to justify it from a new angle. Concerning the pure concept of the category of causality, Kant indeed maintains that it does not include any immediate relation to, or any direct connection with, the concept of continuity. But for the empirical use of the causal concept a further limiting condition is immediately added. The concept, in order not to remain empty, must refer to and be describable in pure intuition; it must be schematized. This condition requires the consideration of the pure form of time; for the schemata are for Kant nothing but "*a priori* time determinations, according to rules" and according to the order of categories, they pertain to the time series, the time content, the time order, and finally the scope of time (*Zeitinbegriff*) of appearances. Depending on whether or not this demand for schematization is explicitly included in the definition of the causal concept, Kant arrives at two different formulations of it. In the first edition of the *Critique of Pure Reason* there is still found a quite general formulation of the causal principle: "Everything that happens, that is, begins to be, presupposes something upon which it follows according to a rule." [11] Here merely the possibility of relating happening by means of rules is demanded, while no definite assumption about the nature of these rules is made. The demand of causality seems to be equivalent to the demand for order according to law in natural happening, and to confine itself to this. But the proof of the causal law that follows goes a step further, by introducing a connection with time, in designating as the schema of cause and effect the "succession of the manifold," insofar as it is in conformity with a rule. But with this succession, time is again immediately introduced as its condition, and this first condition carries the second one, of continuity, with it. No cause can engender an alteration suddenly,

> but in a time; so that, as the time increases from the initial instant *a* to its completion in *b*, the magnitude of the reality (*b-a*) is in like manner generated through all smaller degrees which are contained between the first and the last. . . . That is the law of the continuity of all alteration. Its ground is this: that neither time nor appearance in time consists of parts which are the smallest [possible], and that, nevertheless, the state of a thing passes in its alteration through all these parts, as elements, to its second state. In the [field of] appearance there is no difference of the real that is the

11. *Kritik der reinen Vernunft* (1st ed.), p. 189 (ed. Cassirer, *3*, 630); trans., p. 218.

> smallest, just as in the magnitude of times there is no time that is the smallest; and the new state of reality accordingly proceeds from the first wherein this reality was not, through all the infinite degrees, the differences of which from one another are all smaller than that between zero and *a*.

If one reflects once more on this evolution of philosophical and physical thought, in which the concepts of causality and continuity are more and more welded together, it becomes understandable how difficult it must have been to rend asunder the bonds holding them together. For this nothing less than the mighty explosive of the quantum theory was necessary. And in the meantime the belief that causality and continuity are indissolubly bound together and interdependent had taken such deep roots that an abolition of this union was considered by the representatives of the new view as an abolition of causality itself. The scission which separated continuity from causality was considered fatal. But such reasoning can hardly be followed through consistently from an epistemological point of view. The constitutive, essential characteristic of causality consists in the general requirement of order according to law, not in instructions as to how this order can be discovered and followed through in detail. If empirical and theoretical reasons show that the requirement of continuity can no longer be satisfied, it is still by no means proven that the causal demand has become untenable. Rather we have to liberate the latter from the limiting conditions which have hitherto interfered with it; we have to formulate it in such a manner that it no longer appears as a continuous bond between the "things" of our sensuous space and "events" in our sensuous time. There can be no doubt that such a liberation is possible from the point of view of a "critical" theory of causality, for the critical causal principle contains no direct statement about the connection between "things" and "events" but rather a statement about the systematic interrelation of cognitions.[12] We now must take into account the circumstance that within these cognitions a new order and classification has taken place, that causality and continuity are no longer in the simple relationship to each other that was presupposed by classical physics. Therefore when exponents of modern atomic physics draw the conclusion that the quantum postulate must of necessity entail the overthrow of the causal principle, they reveal a basically ambiguous position: they themselves still judge from the standpoint of the very epistemological situation which has been supplanted by the quantum theory.

The "crisis of causality" produced by quantum mechanics certainly

12. See above, pp. 17 ff.

persists and is quite serious. But it is a crisis not of the pure causal concept but "in the mode of viewing nature" (*Krise der Anschauung*); it shows us that we can no longer refer this concept, in the customary manner, to the perception of "pure time" or schematize it within the latter. Such schematization has been definitely limited through the advent of the quantum theory. We can no longer combine causality with space-time description, let alone amalgamate the two in the manner of classical physics. From a purely epistemological viewpoint this realization is not surprising, as it had been anticipated prior to the evolution of modern physics by developments in mathematics in the nineteenth century. By that time the "crisis of perception" had come into prominence.[13] Leibniz, in laying the foundation for his infinitesimal calculus, starts with the general problem of tangents. He first considers two points P and P', located on a curve, and lets them approach each other until they finally coincide. In this manner he arrives at the construction of his so-called "characteristic triangle," whose elements are the magnitudes dx and dy. In this transition it must necessarily be supposed that the limiting value sought after actually exists, that the curve at this particular point has a definite direction. In Leibniz' days no doubt of this seemed possible; yet it had of necessity to arise since modern mathematical analysis brought the proof that continuity and differentiability need not coincide, and since Weierstrass even offered the example of a continuous curve which had no definite tangent at any of its points. Therefore, the supposition that one could always and unhesitatingly pass from a "difference quotient" to a "differential quotient," from the value of $\frac{\Delta y}{\Delta x}$ to that of $\frac{dy}{dx}$, was shown to be false, and only valid with certain limitations and reservations. Purely physical observations and deductions likewise lead to the same conclusion.[14] This, however, shatters one of the essential bases on which the edifice of classical analysis as well as that of classical physics rests. It is now shown that "macrostates" do not permit immediate inference to "microstates." Leib-

13. Cf. Hans Hahn's presentation, *Krise und Neuaufbau in den exakten Wissenschaften* (Leipzig and Vienna, 1933), pp. 41 ff.

14. "We shall still remain within the experimental reality if, looking through the microscope, we observe the Brownian movement which agitates every small particle suspended in a fluid. To establish a tangent to its trajectory, we should find a limit at least approximate to the direction of the straight line which connects the positions of that particle in two closely placed successive instants. Now as long as one can make the experiment, this direction varies greatly when one decreases the duration separating those two instants. Hence what is suggested by this study to the impartial observer is once again the function without derivative and by no means a curve with tangent." J. B. Perrin, *Les Atomes* (2d ed. Paris, Alcan, 1913), p. ix.

niz, at times, formulated his continuity principle in such a manner as to demand exactly this analogy. He puts it in the form of an "analogy principle," according to which a complete uniformity of being and occurrence in all realms of reality is to be assumed. No matter what realm we may turn to, we always find in the end that there is nothing fundamentally new and that the rules which we can observe are everywhere the same: *C'est tout comme ici*—as the fool says in the French harlequinade ("Harlequin Empereur de la Lune").[15] Leibniz expressed the same maxim more strongly by saying that the rules of the finite are valid in the infinitesimally small and vice versa: les règles du fini réussissent dans l'infini et, vice versa les règles de l'infini réussissent dans le fini.[16] However the development of quantum theory broke down the empirical and theoretical foundations of this maxim. Physical thought can and may no longer rely on this kind of conclusion by analogy. It has to take into account the possibility that the transition to new realms of objects may demand profound changes, not only in the individual laws but in the general physical presuppositions and forms of thought. The demand for order according to law must be maintained at all times, but the demand, often made in the logic of classical physics, for "uniformity" and "equiformity" in nature must be abandoned.[17] Modern physics finds itself forced to apply at the same time different systems of concepts that are incapable of being reduced one to the other. But the unity of natural knowledge does not demand any such uniformity. It is sufficient that the various systems can be put into definite relation with one another, that we can step from one to another in accordance with a definite rule. Such a rule has been established for the relation between concepts of classical physics and quantum theory, in Niels Bohr's correspondence principle. Here a kind of translation code is given us, which shows us in what way the different

15. Leibniz to Clarke, *Philosophische Schriften,* ed. Gerhardt, *7,* 394; *Hauptschriften,* ed. Cassirer and Buchenau, *1,* 173.

16. Leibniz to Varignon, *Mathematische Schriften,* ed. Gerhardt, *4,* 937.

17. Leibniz expressly made this demand and he considered it a definite advantage of his system that its demands were completely satisfied by uniformity. He writes to C. Wolff: "Apud me magna uniformitate naturae omnia ubique in magnis et parvis, visibilibus et invisibilibus, eodem modo fiunt soloque gradu magnitudinis et perfectionis variant." *Briefwechsel zwischen Leibniz und Christian Wolff,* ed. Gerhardt, Halle, 1860, p. 44. Cf. this with the words of a modern physicist: "What this is actually about is not at all a crisis of determinism but rather a crisis of mechanism which we are trying to use to represent a new field. We are indeed ascertaining the insufficiency, in the microscopic, of those notions and ideas that had been successfully used in the macroscopic, which had been created for its use and its prolonged contact for so many generations. This is what I find far more interesting." Paul Langevin, *La Notion de corpuscules et d'atomes* (Paris, Hermann, 1934), p. 35.

languages may be interconnected and used side by side. Of course it remains true that the translation cannot be a directly literal version; we cannot translate the classical language word for word into that of the quantum theory but must be satisfied with a free rendering of the meaning.

Thus even the question whether it is possible to retain our geometrical concepts in the transition to the description of microcosmic phenomena becomes a difficult problem in quantum theory. Schrödinger declares: "Mathematical space, in the small, has an exceedingly simple structure. No matter how small a dimension is arrived at, the same thing will always be found, except for a similarity transformation. This seems too simple to accommodate the map of actual events." [18] The causal principle, understood in its true generality, demands only that such a "map" exist, without stating anything about its particular form. In investigating nature we must always use some general "orienting net," but we need not decide on the mesh of the net in advance. Each new horizon disclosing itself to us may demand a change of orientation. But the demand for order according to law, the demand for functional determination, is not affected; rather it proves again and again to be the true invariant. This invariant is of course too general to determine the concrete form of natural knowledge completely by itself. Thus Kant rightly insists that the general concept, the "category" of causality, must be specified in a definite sense, in order to be usable and applicable empirically. However, we can no longer seek this specification in the same direction that Kant did; we cannot be satisfied with the mere reference of concepts to the purely sensuous schemata, to the "perceptual forms of space and time." For it is precisely these schemata which have lost their universal significance through the discovery of non-Euclidean geometry on the one hand, and the results of the special and general relativity theories on the other. Transcendental logic can thus no longer be connected with or be dependent on transcendental aesthetics, as was the case in Kant's system. The demanded specialization, indispensable for the empirical use of the causal concept, must now be looked for within the domain of concepts itself. The causal concept is capable of uniting itself in manifold ways with this or that conceptual means, and can amalgamate itself with it more or less firmly. According to the manner and extent of this amalgamation it assumes various forms. Its content may be determined by a connection with the category of substance, or with that of continuity; at times it may be permeated with the continuum of numbers, at other times it may refer to the pure form of discrete numbers.

And whenever it severs one connection in order to enter another, it

18. "Über die Anwendbarkeit der Geometrie im Kleinen," *Naturwissenschaften, 22* (1934), 519.

always seems as if causality itself is endangered. Yet a glance at the history of physics shows that this illusion was always successfully dispelled. The causal principle emerged triumphant from the severe trials to which it was exposed throughout this history; in the end it always rose, like a phoenix, from its ashes. Such a trial occurred when physical thought proceeded to give up the condition of immediate spatial proximity between cause and effect. Today we are perhaps not sufficiently sensitive to the intellectual sacrifice which the surrender of this condition implied. Here a sharp incision was made; the tie connecting the concept of cause with the intuition of continuous space was dissolved. The transfer of effects in space takes place discontinuously: it leaps from one location to another without touching the intermediate space. When this concept of "action at a distance" was first introduced by Newton, it encountered the sharpest skepticism on the part of philosophers and physicists. Leibniz and Huyghens protested against it for analogous reasons. They did not consider it a mere modification of the then existing view of nature, but they feared that it would produce a complete collapse of all explanation of nature. Science appeared to revert to the "occult qualities"; it seemed as if it would have to surrender to mysticism as soon as it abandoned the secure foundation of kinetic theories. Leibniz saw in the assumption of a force acting at a distance nothing but a return to "barbarism," a relapse into the "realm of darkness," [19] and Huyghens also fought against the "absurdity" inherent in such a supposition. He expressed his astonishment at the trouble Newton went to in perfecting the difficult and complicated calculations based upon such a principle.[20] Yet shortly afterward it was this same "absurd" principle which was considered not only admissible but actually necessary. Action at a distance, which previously appeared as a strange anomaly, now became the pattern and prototype of true natural laws. It gradually subjugated all domains of natural events; from mechanics it penetrated into chemistry and general electrical theory. But then a decisive relapse took place. Through the quiet and persistent work of Faraday, far from the rivalry of the schools and free from all physical dogma, the concept of action at a distance was overcome. This victory, however, by no means constituted a return to the older kinetic theories. The laws of the electric field, as established by Faraday and Maxwell, cannot be reduced to the mechanical laws of impact. For even though in the Faraday-Maxwell

19. "Le Royaume des tenèbres." Cf. "Briefwechsel mit Clarke," in *Hauptschriften,* ed. Cassirer and Buchenau, *1,* 208. Cf. esp. the essay "Antibarbarus Physicus," *Philosophische Schriften,* ed. Gerhardt, *7,* 337 ff.

20. "An Leibniz," November 18, 1690, *Mathematische Schriften,* ed. Gerhardt, *2,* 57.

"contact forces" the relation to continuous space is reestablished, the substance theory suffered a fundamental transformation in the transition from mechanical to field physics.[21] Inasmuch then as the causal concept is tied to an altered concept of substance and must refer to the latter, it confronts us once more in a new and maturer form. When scrutinizing this evolution, it becomes clear immediately that the causal concept of classical physics was by no means that simple and completely harmonious structure so frequently represented, but that it embraced a good many problems and a wealth of dialectical tensions.

In order to clarify the actual reasons for these tensions, we have to go back considerably further. Goethe once said that all attempts to solve the problems of nature originate in the conflicts continually arising between thought and observation.[22] In connection with this it can be pointed out that the present problems of the quantum theory also are rooted in a conflict between observation and thought which can be traced back to the first conscious beginnings of the philosophical and mathematical formation of concepts. The first union established between the two types of concepts is represented by Pythagorean number theory. The analyses of mathematics and philosophy are here directly fused with each other; the philosophical concept of truth is related to the mathematical concept of number and is limited to the conditions of the latter. If number and its peculiar basic function of "delimitation" did not exist, there could also be no object of knowledge. No thing would be clear to anybody, according to a Philolaus fragment, neither in its relation to itself nor to others, if number and its essence did not exist. It is number that provides knowledge, that guides and informs everyone about everything that may be doubtful or unknown to him.[23] But in this ultimate foundation of all knowledge, which is to guarantee its definiteness and certainty, a new and profound set of problems soon became visible. This set of problems put its stamp on the entire subsequent development of Greek thought in mathematics and philosophy. It arose in that moment when the impossibility became apparent of penetrating and completely dominating with the nature and essence of whole numbers precisely that realm which primarily concerns mathematical thought: the continuum of numbers. The fact of the "irrational," the fact that there are lengths for which no corresponding numerical values can be found, confronts the fundamental Pythagorean view of the universe with a riddle that appears at first insoluble. Cosmic "harmony" seemed no longer maintain-

21. Cf. above, p. 130.

22. "Der Kammerberg bei Eger," *Naturwissenschaftliche Schriften,* Weimar ed., *9,* 91.

23. Philolaus, Fr. 2; Diels, *Die Fragmente der Vorsokratiker,* pp. 253–4.

able, for the force of number, through which this harmony was to be established, was paralyzed by the problem of irrationals. Something impenetrable, an ἄλογον and ἄῤῥητον, again confronted thought. This was no longer an isolated problem of mathematics but rather a question standing in the very center of scientific and philosophical thought. Plato was so affected by it that in the seventh book of the *Laws* he declared a study of the fact of "incommensurability" to be practically required by all and designated the ignorance on this subject a "deep-rooted, ridiculous and shameful lack of knowledge," for which one should be ashamed, not only on one's own behalf, but on that of all Greeks.[24] The passion, the genuine and profound "pathos," of Greek philosophy and science speaks in these words. Both philosophy and science are conscious of standing at the threshold of a new fundamental problem which may become decisive for the entire form of knowledge.[25]

The way in which mathematical thought attempted to overcome this problem is well known. It necessitated nothing less than the creation of an entirely new set of tools for mathematical analysis. If number was to be able to give an exact and adequate representation of the realm of continuous numbers, it had first to change its own nature. The concept of the discrete number had to be altered in such a way that it could not only be applied to the intuitive continuum—as it presents itself in the basic forms of space and time—but that it could, as it were, exhaust this continuum. This goal appeared to be attained when modern mathematics, by steadily extending and refining its notions, succeeded in transforming the system of "natural numbers" into the system of "real numbers." Only when this point was reached did the breach seem to be healed, the conflict between observation and thought overcome. The set of all points in a line can now be put unambiguously and without exception into relation with the "set of all real numbers." Cantor's foundation of the theory of sets and Dedekind's definition of irrational numbers could be considered as a final solution of the problem, for solely by means of pure thought a continuum was established which was truly commensurate with that of observable magnitudes and which was able to express the latter in all its individual features. Dedekind considers arithmetic and analysis a part of pure logic. He demands that the concept of number be developed as a direct result of the laws of pure

24. *Laws* 7. 819D.

25. At this point the attitude of Greek thought toward the problem of irrational numbers cannot be further amplified. I refer in this connection to the two excellent articles by Otto Toeplitz, "Mathematik und Antike," *Die Antike, 1* (1925), 175 ff., and Heinrich Scholz, "Warum haben die Griechen die Irrationalzahlen nicht aufgebaut?" *Kant-Studien, 33* (1928), 35 ff.

thought, and entirely independently from representations or intuitions of space and time. The fact that such a development is possible, that, by means of logic, we can construct a continuum of numbers, which then can be made a yardstick for our representations of space and time —this fact is considered by Dedekind as the real triumph of analytical thought.[26] The "fundamental crisis" to which mathematics was led in the further extension of the theory of sets certainly showed that dangerous controversial material still remained in the first formulation. Even after the development of the number continuum it could not be denied that a true coincidence between it and the intuitive continuum had not been attained. For number, even in this extension and refinement, is unable to discard its logical basic character of discreteness. In his work on the continuum, in 1918, Hermann Weyl could declare that the great task set by the Pythagorean discovery of the irrational, the task of "grasping mathematically the content, formulable in exact knowledge, of the intuitively given continuum as a totality of discrete 'stages,' is today as much unsolved as ever, in spite of Dedekind, Cantor, and Weierstrass." For the correlation now attained is by no means a real congruence: "it must not be forgotten that in the 'continuum' of real numbers the individual elements are in fact exactly as much isolated from each other, as are whole numbers." [27] From the standpoint of pure number theory all those "extensions," which the number concept had to suffer in order to satisfy the demands made on it on the part of irrationals were always regarded with a certain skepticism. They were considered as a kind of violation of the original character of pure discrete number, as a transformation, admissible as an artificial pragmatic aid but always accompanied by a certain artificiality and arbitrariness. Characteristic of this attitude is Kronecker's well-known saying: "Whole numbers are made by God, everything else is the work of man."

Modern science attempted to solve the conflict between the logical requirements of discrete numbers and the requirements springing from consideration of the continuum of numbers in a different manner. It started out from the basic phenomenon of motion, and by the analysis of this phenomenon gained certain fundamental concepts, which on being pursued further led to an entirely new structure of mathematics. The new analysis of the infinite can be traced back, historically and systematically, to the problems of dynamics. In his endeavor to define the concepts of velocity and acceleration Galileo discovered those new

26. Cf. esp. J. W. R. Dedekind, *Was sind und was sollen die Zahlen?*, Braunschweig, 1887

27. H. Weyl, *Das Kontinuum,* in *Kritische Untersuchungen über die Grundlagen der Analysis* (Leipzig, 1918), pp. 16, 69.

modes of thought which already contained, in embryo, the later methods of analysis of the infinite. And these bonds become closer during subsequent progress. Cavalieri's *Geometria indivisibilium,* Kepler's *Stereometria doliorum,* and Newton's method of "first and last ratios" are the additional characteristic stages in this process of thought. Kepler's work, which became the source of all later cubatures, prescribes a method of determining the volume of bodies by imagining these bodies to be developed by a particular mode of rotation. Newton, from an analysis of uniform and nonuniform motion developed the basic ideas for a universal mathematics of variable magnitudes. This development attained its logical consummation when Leibniz introduced the general concept of function and formulated precisely the concept of the continuity of a function. Thus a domain was designated in which mathematical thought, as it were, directly touches actuality. It is true, as Leibniz explains, that among the entities of nature there is none so constituted as to correspond in all strictness to the requirements contained in our mathematical definitions; nevertheless the actual phenomena of nature are, and must be, so arranged that no actual occurrence ever runs contrary to the continuity law and all other exact rules of mathematics. Were it otherwise, there would be no comprehensible explanation of nature, no insight into its intelligible causes.[28]

By breaking with this presupposition the quantum theory takes this problem, in a certain sense, back to its origin. It is a Pythagorean theory of nature. It claims not only the possibility but the necessity of permeating nature with the concept of discrete numbers, and of understanding nature by means of this concept. But the consummation of this basic view requires a peculiar and most revolutionary step. The presuppositions of the hitherto existing explanation of nature are reversed. When the latter, for the sake of the requirement of continuity, which appears to be directly given and guaranteed by intuition, requires a transformation of all mathematical concepts, then in the new view nature itself will be considered *sub specie* of the discrete numbers and consequently appears in a new and different light. Discrete numbers demand an atomism in phenomena. It was not speculative or mathematical but purely empirical considerations which, in establishing Planck's law of radiation, required this conclusion. But once this consequence was drawn there appeared elsewhere in nature, with ever increasing clarity, those features which could not be accounted for by using the basic concepts of classical physics, the concept of the continuous function and the method of differential equations. Precisely when dealing with the really fundamental

28. Cf. Leibniz, "Réponse aux reflexions de Bayle," *Philosophische Schriften,* ed. Gerhardt, *4,* 568 f.; *Hauptschriften,* ed. Cassirer and Buchenau, *2,* 402 f.

problems of nature, when attempting to understand the stability of the atom and the constancy and determinateness of its properties, one encountered a boundary to classical methods. The latter, when considering the case of radiation, would demand a continuous loss of energy and a simultaneous emission of all spectral lines, completely irreconcilable with the requirement of stability and the basic experimental fact of precise spectral lines. Thus in the phenomena themselves a form of limitation and "particularization" became evident that could only be adequately accounted for by the method of "discrete" numbers—that is, by the quantum rules. But in order to help this interpretation to succeed, the basic concepts of science first had to enter into a novel and at first glance most peculiar interrelation. In the "magic formula," $E = h\nu$, standing at the head of quantum theory, the equivalence of an energy and a frequency is asserted, and thus magnitudes of entirely different dimensions are connected with each other. But the empirical consequences which are drawn from this paradoxical relation show from an entirely different angle that the power of knowledge inherent in discrete numbers consists by no means only in the power of separation, but equally in the power of uniting. Number again satisfies that ideal established for it by the Pythagoreans. It gives "unity to diverse things, and harmony to discordant ones." [29] Planck once said that one of the greatest advantages, not always sufficiently appreciated, of Bohr's atomic theory lies in the fact that even in its most specialized applications it requires no new constant, that it requires only the elementary quanta of mass, of electric charge, and of action, and that with the sole aid of these quantities and of the natural series of integers it accomplishes the complete construction of physics and chemistry.[30]

Analogous considerations arise if the procedure of modern quantum mechanics is contrasted with that of classical mechanics. The latter is based on a methodological principle to which Leibniz once gave a most pregnant and characteristic formulation. In a letter to the mathematician de Volder in which he tried to convince de Volder of the correctness and necessity of his continuity principle, he declared that in investigating nature one may start out from the general premise that things become more understandable, the more precisely they are analyzed and discussed: *quanto res discutiuntur magis, tanto magis intellectui satisfit.*[31] This type of discussion was understood literally by classi-

29. Πολυμιγέων ἕνωσις καὶ δίχα φρονεόντων συμφρόνησις.

30. Planck, "Die Bohr'sche Atomtheorie," Niels Bohr number of *Naturwissenschaften, 11* (1923), 536 ff.

31. Leibniz to de Volder, March 24, 1699. *Philosophische Schriften,* ed. Gerhardt, *2,* 168; *Hauptschriften,* ed. Cassirer and Buchenau, *2,* 288.

cal physics. It required that knowledge never stop before an undivided entity, that it break up every whole into further component parts until it penetrated to the ultimate indivisible elements of natural events. Applied to motion this means that every motion of a material system must be described in such a way that it is constructed out of individual elementary events that are strictly localized—that is, referred to separate infinitesimal elements of space and to their immediate spatial and temporal vicinity. This requirement had to be abandoned by modern mechanics; it has led to the strange conclusion that every individual material point of a system occupies, in a certain sense, at every instant all positions of space simultaneously. Thus in the quantum theory a new concept of wholeness of natural occurrence is formulated. The new mechanics no longer connects the formulation of the laws of motion with sharply localized individual events; rather it seeks to grasp certain pure "basic forms" of natural events, which are to be retained and presented as forms, without dissection into parts. All this shows that in modern physics the relation between continuity and discreteness had to be understood in an entirely different manner than in the earlier scientific systems.

On the other hand it cannot be denied that in this new formulation the fundamental contrast between the two basic concepts is not entirely wiped out. From the standpoint of content this contrast persists, since physics falls into two sharply defined divisions, the theory of radiation and that of matter. Within the first division all phenomena can be placed in a continuous sequence. If we go from Hertzian waves to infrared rays, to visible light, to ultraviolet light, and finally to the hardest X rays, there is in this progress no sudden jump—we are dealing with a continuous change of frequency. However, the various forms of matter cannot be placed in a continuous progression in this manner; the chemical elements, regardless of all systematic interrelations, remain separated like "islands in the ocean." We are led to the same characteristic difference when we consider the various forms of the new quantum mechanics. With Schrödinger the viewpoint of continuity clearly has the upper hand, whereas with Heisenberg the category of discrete numbers plays the primary and decisive role. With the former, continuous functions, in the form of the wave equation, determine the structure of modern mechanics; the latter introduces an algebra dispensing with any support from the continuum of real numbers, and hence with any possibility of direct visualization, and confines itself to operations with pure symbols. In Heisenberg's theory the classical continuous variables are replaced by matrices—that is, systems of discrete numerical quantities, while wave mechanics takes exactly the opposite step, going from the classical point

mechanics to a pure continuum theory.[32] If we survey the whole of this development and consider its epistemological presuppositions, it must be admitted that those "conflicts between thought and observation" which, according to Goethe, are basic to every real problem of nature are by no means settled; on the contrary they are recognized as such more sharply than ever before. For instead of simply oscillating between the two basic determinations of continuity and discreteness, instead of assigning them to different realms of being, applying the one to radiation and the other to matter, there arises the much more difficult and paradoxical task of permeating each with the other in such a way that the very same event may be grasped from both viewpoints and described in accordance with both.[33] The continuous and the discontinuous, the wave picture and the particle picture, shall in principle be applicable to all physical being; the continuous spreading of light is to be presented and interpreted by means of the corpuscular concept, discrete matter by means of the wave picture. Yet though this conflict becomes more and more acute, it does not have to lead to a skeptical renunciation, to an "ignorabimus"; for always in the past such conflicts have been shown to have not only a negative but also an eminently positive significance; far from erecting insurmountable barriers to knowledge, they prove to be its most important incentives.

32. For further details see Schrödinger's *Abhandlungen zur Wellenmechanik* (Leipzig, 1928), Essay 1.

33. Cf. the explanations of Langevin, "L'Orientation actuelle de la physique," and of Perrin, "La Chimie physique," in the collective volume, *L'Orientation actuelle des sciences,* Paris, 1930.

CHAPTER 12

The Problem of the "Material Point"

IN HIS EPISTEMOLOGICAL DISCUSSIONS on energetics, Boltzmann starts out from the premise that there are two ways of viewing nature. The first, which he calls phenomenological, is content to establish differential equations for each domain of problems and believes that it thus has a complete description of the domain, making unnecessary the introduction of any further hypothetical assumptions. Hence it looks down somewhat disdainfully on the other form of comprehending nature which finds its clearest and most characteristic expression in atomism. It accuses atomism of thinking in terms of mere pictures, instead of using strict concepts. But Boltzmann tries to show that this judgment is based on a delusion. He points out that the differential equations of the phenomenological viewpoint ultimately also require some "thought pictures" on which they are based, and that they could not be arrived at without them. One always has to begin by thinking of a finite number of elements that are then permitted gradually to increase. But what is the use of hiding the fact that it is necessary to imagine a large number of individual elements, when they have to be included in the formation of differential equations? Those therefore who think they can get rid of atomism by the use of differential equations cannot see the forest for the trees. In setting up the differential equations we now have only to make the further assumption that however refined the means of observation become, we can never observe differences between facts and calculated limiting values. It is, however, precisely this assumption that adds something new and unproven to the picture. The possibility is not at all precluded that this picture may best represent the phenomena when there is a certain very large number of points in the manifold, but that deviations begin to occur once more when this number is further increased—and this must always be the case if we assume the existence of a very large but finite number of atoms.

These considerations, which may be found in two of Boltzmann's writings,[1] have never, to my knowledge, received any special attention, in the literature of either physics or epistemology.[2] And yet they contain an important and fruitful problem, as they anticipate a difficulty which became fully noticeable only after physical knowledge had progressed considerably further. If the atomic hypothesis is really taken seriously, then it must indeed appear questionable whether certain transgressions of boundaries, committed without question by classical physics, are permissible. The assumption that we can increase our measurements beyond any limitation of precision is ultimately always based on the presupposition of infinite divisibility. Therefore it contains a continuity requirement that is contradicted by atomism. There exists then from the first a certain conflict between the classical methods based on differential equations and atomism based on the principle of discrete finite numbers. Leibniz clearly recognized this conflict, and for this reason he quite consistently banished the atoms from his physics which he intended to construct as a strict physics of infinitesimals. His principles of continuity and homogeneity required the possibility of unlimited extrapolation—toward the infinitely large as well as the infinitesimally small. But he was unable to furnish a logically compelling reason for this demand; he had to resort to certain teleological assumptions.[3] The adequate expressions for the description of atomic phenomena are in the end not differential quotients but difference quotients. Here too the quantum theory was the first to bring this long hidden problem clearly to light. Continuous and discrete magnitudes are sharply contrasted. Thus in Bohr's theory of the Balmer series of the hydrogen spectrum the path of the electron is determined by one classical and one quantum condition. Together they demand that the electron can only move in certain quantized orbits. And according to Bohr's correspondence principle two frequencies are distinguished, one classical, the other quantum mechanical, the difference between them proving to be that between the difference quotient and the differential quotient.[4] If we adhere to our general viewpoint that order according to law is prior to objectivity, and that the latter is determinable solely through the former, on the basis of this dualism we

1. "Über die Unentbehrlichkeit der Atomistik in der Naturwissenschaft" in *Annalen der Physik und Chemie,* new ser. *60,* reprinted in *Populäre Schriften* (Leipzig, 1905), pp. 141 ff.

2. An exception is R. Hönigswald's article "Zum Begriff des Atoms" in *Festschrift für Paul Natorp* (Berlin, 1924), pp. 178 ff., which emphatically points to the epistemological significance of Boltzmann's discussion.

3. Cf. above, pp. 159 ff.

4. For further details see A. J. W. Sommerfeld, *Atombau und Spektrallinien* (4th ed.), pp. 105 ff., 328 ff.; English trans. (3d ed.), H. L. Brose, *Atomic Structure and Spectral Lines* (New York, 1934), pp. 84 ff., 292 ff.

must expect to find in the "object" of modern atomic physics a new and more complex structure than was presupposed by classical theories. This expectation is fully confirmed on closer scrutiny.

The reality concept of classical mechanics rests on two basic presuppositions, the concept of "substance" and that of space. Matter is *substantia extensa* and in the end it is nothing but that. All its observable attributes must be reduced to this determination. This concept of substance is developed with pregnant precision by Descartes. What do we mean, he asks, when we ascribe to an object a physical being, an "objective" existence? Does this assertion signify only that we think of it as combining various qualities, each describable through the senses? Apparently not, for the mere aggregate of these qualities does not constitute the existence of the object. If we look at some individual thing, say for instance a piece of wax, we can characterize it by means of certain observable attributes, such as its size, its form, its color, its hardness, its odor, etc. Nevertheless the wax is something other than the mere aggregate of these individual properties. For suppose it is now brought close to the fire and melts—we find the properties entirely changed: the color and hardness have disappeared; the odor is gone. Is it nevertheless still the same wax? Nobody doubts it; everybody is convinced that the "being" of the wax survives all these changes. It follows that this "being" can only "be grasped in thought," and cannot be apprehended by the senses: *superest ut concedam, me ne quidem imaginari quid sit haec cera, sed sola mente percipere.* Accordingly it is only the logically definable characteristic of persistence that constitutes the wax as a physical object.[5] With this the general condition is established on which all objectivity rests in the mechanical view of nature. "Objective" denotes a being which can be recognized as the same in spite of all changes in its individual determinations, and this recognition is possible only if we posit a spatial substratum. The entire axiomatic system of classical mechanics is based on this presupposition. Wundt, in an article on physical axioms and their relation to the causal principle (1866) designated as the first axiom the premise that all causes in nature are causes pertaining to motion. He is not satisfied, however, with the mere statement of this premise, but demands for it a logical deduction. This is found in the fact that there is only one case where a thing changes in representation and yet remains the same, and this is motion. In motion the change consists in a mere shift of an object's spatial relation to other objects; the change of locality is thus the only conceivable change of things in which they retain their identity.[6] Wher-

5. Descartes, *Meditationes de prima philosophia* Pt. II, Meditation 2.

6. W. M. Wundt, *Die physikalischen Axiome und ihre Beziehung zum Causalprinzip* (1866), p. 126.

ever the attempt has been made to maintain a strictly mechanical explanation of all physical phenomena, this argument has played a part. Heinrich Hertz, in his principles of mechanics, used it as a virtual definition of the concept of mass. He declares: "A mass particle is an indicator (*Merkmal*) by means of which we can unambiguously correlate a definite point in space at a given time with a definite point in space at every other time." Hertz emphasizes that in determining the concept in this way the immutability and indestructibility of mass particles need no longer be deduced, because they are already contained in the definition.[7] But this definition fails us as soon as we make the transition to field theory. The field is not a "thing"; it is a system of effects (*Wirkungen*), and from this system no individual element can be isolated and retained as permanent, as being "identical with itself" through the course of time. The individual electron no longer has any substantiality in the sense that it *per se est et per se concipitur;* it "exists" only in its relation to the field, as a "singular location" in it. Since Faraday the concept of "lines of force" has replaced and pushed aside the concept of the persisting thing on which classical mechanics was constructed.

This step could not be retracted even in the quantum theory, for which the problem presented itself in a new and more general sense. In quantum theory, as we have seen, the primacy of pure "laws of action" over "laws of being" becomes ever more evident. When it asks questions about structure, it is always aware that the problems are soluble only if they can be reduced to pure questions of law, to questions concerning the intensities and frequencies of spectral lines. In order to answer these questions quantum mechanics created in the matrix calculus a new mathematical instrument, an abstract algebra which holds equal rank with the differential equations of classical physics. Here the transition is accomplished to which the discovery of the quantum of action had pointed from the beginning. Instead of seeking its fulfillment solely in the concepts of substance and in the continuum of numbers, causality is very closely connected with the concept of discrete numbers. It can and must, therefore, forego the attempt to follow the movements of individual particles within the atom in the same visualizable manner that was customary in the classical concept. But this at once forces us to draw a further epistemological conclusion. For what *are* these electrons whose path we can no longer follow? Is there any sense in ascribing to them a definite, strictly determined existence, which, however, is only incompletely accessible to us? Or must we not rather take the opposite path—must we not take seriously the demand that we use the conditions of the possibility of experience—that is, the conditions of accessibility

7. *Prinzipien der Mechanik,* p. 54.

as conditions of the objects of experience? Then there will no longer exist an empirical object that in principle can be designated as utterly inaccessible; and there may be classes of presumed objects which we will have to exclude from the domain of empirical existence because it is shown that with the empirical and theoretical means of knowledge at our disposal, they are not accessible or determinable.

Modern atomic physics seems to have hesitated before it drew this conclusion. For a time it seemed anxious to hold on to the idea of a pointlike existence of electrons and of their motion along fixed orbits, even when it was shown that all these concepts dissolved in its very grasp—when no means of empirical determination remained. But we must not at this point evade the ultimate epistemological consequence. We cannot say that an electron, at a given time, "is" at a certain location, nor that it "possesses" a sharply defined velocity, when this "possession" only exists for the electron itself and not for physical knowledge. We cannot claim a "status" (*Bestand*) exceeding that which can be posited by the means of physical knowledge; for this function of "positing" is the only possible logical basis, the *quid juris* of every status. Possessions that cannot be "realized" in some manner are, from the standpoint of a critical interpretation of objectivity and of knowledge, a contradiction in terms. Kant once summed up this viewpoint as follows: "The objects of experience, then, are *never* given *in themselves,* but only in experience, and have no existence outside it. That there may be inhabitants in the moon, although no one has ever perceived them, must certainly be admitted. This, however, only means that in the possible advance of experience we may encounter them. For everything is real which stands in connection with a perception in accordance with the laws of empirical advance." [8] If therefore the uncertainty relations show us that we can no longer hope to "encounter" an electron at a certain point in space with a sharply determined momentum, then the assertion of empirical reality for this state of affairs loses its basis; it has passed out of "the context of experience." All those peculiar problems that have troubled atomic physics from time to time now automatically disappear. We need no longer ask where an electron might be in the intervening time, when suddenly moved from an inner orbit to an outer one, or vice versa; whether at the instant when it leaves the first orbit, the orbit of its destination is already determined; what happens to radiation if the process were interrupted before the emission of one quantum is completed, etc. There is no answer to these questions—unless the answer is that they cannot be asked in this form because they presuppose a situation which is not empirically definable for us.

8. *Kritik der reinen Vernunft* (2d ed.), p. 521 (ed. Cassirer, *3,* 350); trans., p. 440.

From another angle we can also see clearly that the uncertainty relations, as Laue recently expressed it,[9] set a quite definite limit to all corpuscular mechanics but not to all physical knowledge. Quantum mechanics has emphasized again and again that within it a strictly mathematical schema exists, but that this schema is not to be imagined as a simple interconnection of things in space and time. If this is so, however, some very definite conclusions follow concerning the "individuality" of the elements with which quantum mechanics constructs nature. Schopenhauer declares that space and time are to be regarded as the real *principium individuationis.* In other places also in the philosophical history of the problem of individuation, we encounter this determination frequently. "It is easy to discover," Locke observed, "what is so much inquired after, the *principium individuationis;* and that, it is plain, is existence itself, which determines a being of any sort to a particular time and place incommunicable to two beings of the same kind. Let us suppose an atom . . . existing in a determined time and place; it is evident that, considered in any instant of its existence, it is, in that instant, the same with itself." [10] Conversely it follows, however, that, when an object is no longer determinable by means of a "here" and "now," when it is not denotable as a τόδε τι, its "individuality" can no longer be maintained in the conventional sense, valid for things in space and time. Determinations of space and of time are here, in principle, alike, for "energy" and "time" belong to those "canonically conjugated" quantities whose simultaneous, completely exact, determination is forbidden by the uncertainty relations. In stationary states in which energy has a strictly determined value, the time coordinate of the electron becomes indeterminate, and conversely, every exact time determination results in an uncertainty in the energy. If then we continue to talk about the individuality of single particles, this can only be done indirectly; not insofar as they themselves, as individuals, are given, but so far as they are describable as "points of intersection" of certain relations. If we scrutinize the development of modern quantum mechanics, we will find this assumption fully confirmed. In de Broglie's wave theory of matter and in Schrödinger's wave mechanics the concepts of proton and electron are maintained, but they are defined no longer as "material points" in the sense of classical mechanics, but instead as centers of energy. We may thus continue to talk of the electron as a definite object but it no longer possesses that individuation that could be designated by a simple "here" and "now." Waves are not tied to a single spatiotemporal point; they

9. M. von Laue, "Über Heisenberg's Unbestimmtheitsbeziehungen und ihre erkenntnistheoretische Bedeutung," *Naturwissenschaften, 22* (1934), 441.

10. Locke, *An Essay Concerning Human Understanding,* Bk. II, chap. 27, sec. 3.

enjoy a kind of omnipresence. Each extends through the entire space—which, however, is no longer to be considered as an empirical space but as a configurational space. Particularity, as far as we can grasp and accurately define it, has turned from being the starting point of consideration, to being its result; it arises through the concentration of energy into a definite small portion of space. The charge of an electron is no longer linked to a definite location but is distributed throughout a "charge cloud." The corpuscular character of the electron is abandoned. In the eigenvalues, Schrödinger's wave equation yields definite discrete energy values, and it is they which take the place of the former discrete electron orbits. It now appears that the laws established for stationary atomic states can be reduced to laws of physical wave optics. Such a theory, in which the electron is explained as a "superposition of characteristic vibrations," fills an important epistemological function as well; it is a characteristic and typical expression of that general tendency of physical thought that strives to think that the nature of matter is determined only by certain dynamic relations, and strives to reduce matter entirely to these relations. "Static" concepts need not be abandoned, but they are considered as special cases that may be deduced from general dynamical considerations. Thus in Schrödinger's wave mechanics, Bohr's stationary energy states are interpreted as the frequencies of the characteristic vibrations of waves, whereas Heisenberg's matrix mechanics established a general square array in which stationary states are contained as special cases. The "diagonal elements" of the matrix correspond to vanishing frequencies and accordingly serve as means of representing the magnitudes in the stationary states.

In the current discussion of the problem the validity of this epistemological viewpoint, as far as I can see, has become more and more acknowledged. Admittedly complete agreement among the founders and representatives of modern quantum mechanics seems by no means to have been attained; but it is almost universally conceded that the vulnerable point is the circumstance that the concept of the "material point" requires drastic revision. Von Laue, Schrödinger, and Langevin recently expressed themselves along these lines: they see no way out of the dilemma unless physics entirely does away with the concepts of classical point mechanics and definitely abandons the idea that the electron is a corpuscle—that is, a very small body.[11] Against this standpoint certain objections may be raised, partly on empirical grounds and partly from

11. Cf. Von Laue and Schrödinger's observations in *Naturwissenschaften, 22* (1934), 439 ff.; M. von Laue, "Materie und Raumerfüllung," *Scientia* (Dec. 1933), pp. 402 ff.; P. Langevin, *La Notion de corpuscules et d'atomes* (Paris, 1934), pp. 35 ff.

theoretical considerations. With respect to the former, one may argue that in Wilson's photographs of the paths of α- and β-rays in gases we almost have the separate particles and their paths before our very eyes. In the Wilson cloud chamber it appears that individual particles and their tracks can be made immediately visible. On the basis of this state of affairs and of a series of other "elementary experiments," it was for a time believed that the purely corpuscular nature of cathode rays was placed beyond any doubt. The analogy between these rays and light rays appeared as a superficial similarity which did not have to be followed up. The contrast to light rays, which were always to be considered as purely continuous wave phenomena, seemed to be unbridgeable. However, the further the theory progressed, the more strongly it was convinced of the "dualistic" nature of all radiation. It was no longer possible to juxtapose two different kinds of radiation, one corpuscular and the other of waves; it became necessary to carry out a dual description for every phenomenon of radiation. The "particle picture" had to be complemented with the "wave picture," and vice versa. The absolute significance of the former was thus sacrificed. From a general theoretical standpoint the fact of the indivisibility of an electronic charge has frequently been considered the decisive argument in favor of the view that the electron is a pointlike entity. It is for this reason that Sommerfeld, for instance, refuses "to take literally the charge cloud to which Schrödinger's theory leads." [12] But here also it may be said that the fact that the charge of electrons and protons remains constant by no means demands a substantialistic interpretation. For the constancy of a certain relation is not at all sufficient for the inference of a constant "carrier." "All that we know in matter," says Kant, "is merely relations . . . but among these relations some are self-subsistent and permanent, and through these we are given a determinate object." [13] The indivisibility of charge is just such a self-subsistent and permanent relation, which justifies our speaking of the electron as being a "determinate object"; but it is not sufficient for a substantialization and hypostasis of the electron.

As is well known, the first determinations of the mass, velocity, and electric charge of electrons were made by means of cathode rays obtained in discharges in highly rarefied gases. From the deflections of these rays in electromagnetic fields, the first data were obtained for the

12. A. J. W. Sommerfeld, *Atombau und Spektrallinien. Wellenmechanischer Ergänzungsband* (Braunschweig, 1929), p. 98; English trans. (3d ed.), H. L. Brose, *Wave-mechanics* (New York, 1930), p. 83.

13. *Kritik der reinen Vernunft* (2d ed.), pp. 321, 341 (ed. Cassirer, *3*, 227, 239); trans., pp. 279, 291.

determination of the electronic mass and of the specific charge of the electron, or rather for the ratio of elementary charge to electronic mass. Later determinations of the value of $\frac{e}{m}$, arrived at by entirely different methods, invariably gave results in agreement with these. Thus the electron was recognized as a "universal element of structure of all matter." "Whether it is flowing along slowly in an electric current, or hastening through space at an extremely high rate as a cathode ray, whether it is emitted in radioactive disruption or in a photoelectric process . . . it is always the same physical unit, proving its identity by exhibiting the same charge and the same mass, in particular by keeping the ratio of charge to mass constant." [14] The knowledge that the ratio of charge to mass is the same in all electrons, that it is therefore a fundamental constant, made it possible to regard the electron as the ultimate "building stone" of all matter. Yet the principle that "whatever we know of matter is purely relations," that it is a *substantia phaenomenon* and thus an aggregate of relations, was by no means overthrown but rather strengthened. It was precisely the quantum theory which has again and again sharpened our perception at this point, for all its statements can be understood only as systematic statements, not as statements about the being and behavior of individual atoms and electrons. Quantum mechanics deals with complexes whose separation into "parts" is no longer possible in the manner of classical physics. The further the theory advanced, the more it had to give up the habit of asking *where* the individual electron is at a given instant of time, *how* it is transferred from one state to another, and *why* it performs such a transition. All these questions proved to be inadequate; they proved to be obstacles to theoretical understanding. If, for instance, we look at the development of wave mechanics, then according to de Broglie's initial conception every individual particle is supposed to be connected with a system of "phase waves." These were to "carry along" the particle, so that each and every particle had to possess its own train of waves. But this original interpretation of the carrier wave was subsequently rectified and essentially transformed by de Broglie himself. He declared that the theory of carrier waves that tried to maintain the classical view by seeking to localize the particle at one point of the wave and by ascribing to it an accurately determined motion at every instant encounters serious difficulties. A visualization of the carrier wave in ordinary space proved to be impossible. In its later development it was necessary to adopt far more complicated premises for the theory. To a harmonic system of

14. Sommerfeld, *Atombau* (main vol.), p. 7; English trans., p. 6.

de Broglie waves, proceeding in a given direction, a homogeneous current of uniform material particles of a definite density had to be assigned. Even a property such as velocity could no longer be thought of as being connected with an isolated mass particle, after the fashion of classical physics; rather one had to think of each particle as being bound to a wave, whose group velocity is the expression for the mechanical velocity of the particle.[15]

Heisenberg's development of quantum mechanics also expressly renounces any statements about individual distinguishable particles. Radiation is analyzed as a whole: from the observation of frequencies and intensities of spectral lines certain characteristic quantities are derived, which take the place of coordinates and velocities of electrons. The theory recognizes only light frequencies, while the frequencies of rotation of electrons in their orbits are excluded as unobservable. It confines itself to statements about energies, frequencies, amplitudes, without furnishing any data about coordinates, velocities, or momenta of particles within the atom.[16] If, lastly, one considers the statistical interpretation given to statements of wave mechanics by Born, the picture we obtain is not different in principle. It is true that this interpretation tried to retain the corpuscular character of the electron. It considered the magnitude that Schrödinger took as the measure of the density of electric charge to be the measure of the probability of finding a particle in a given place. From this Born concluded that we can continue to represent matter as before as a picture of moving, pointlike particles (electrons or protons), but he immediately adds that in many cases these corpuscles cannot be "identified as individuals at all," for example, when they unite as a group in an atom.[17] However, what a corpuscle still is

15. For further details see de Broglie, *Einführung in die Wellenmechanik,* chap. 5, sec. 4; chap. 9, sec. 5.

16. Further on Heisenberg's kinematics in *Elementare Quantenmechanik,* ed. Born and Jordan, chap. 1, sec. 3.

17. Born, "Quantenmechanik der Stossvorgänge," *Zeitschrift für Physik, 38* (1926), 803 ff.; cf. *Naturwissenschaften, 15* (1927), 240. The impossibility of delimiting different electrons from one another, and of ascribing to each of them an independent individuality, has been brought into clear light through the evolution of the modern quantum theory, and particularly through the considerations connected with the Pauli exclusion principle. Considered solely from the standpoint of its methodological significance in the construction of the quantum theory, Pauli's exclusion principle is strangely analogous to the general principle introduced into philosophy by Leibniz under the name of *principium identitatis indiscernibilium.* This principle states that there cannot be two objects which completely correspond to each other in every determining characteristic, and thus are indistinguishable except by mere number. There are no things that differ from each other *solo numero;* rather every true difference must be definable as a qualitative difference,

after it can no longer be identified as an individual is very hard to say. Did not classical physics, as we have seen in the case of Heinrich Hertz virtually defines the mass point by this possibility of identification? And the statistical approach points with particular emphasis to the fact that where descriptions of microcosmic phenomena are concerned we can no longer maintain and fulfill the demand for individualization and identification in the same way that appears possible for macroscopic objects. For statistical statements are, in themselves, strict statements, which apply, however, to collectives and not to individual cases, and which result in a determination solely for these collectives and not for a particular member picked from them. If quantum mechanics demonstrates that the possibility of determination does not reach beyond these collectives, then every means is lacking for going further and postulating the existence of an isolated particle. Therefore the statistical character of the premises of quantum theory must not be viewed in an exclusively negative way. It does not state that we are uncertain about location and momentum, about the path and the entire "fate" of the individual electron, and that, whether we like it or not, we have to be satisfied with inexact statistical statements about aggregates; it states rather that we are certain such an individualization no longer has any definite meaning within the realm of atomic physics. When quantum mechanics declares that within its domain only statistical predictions are possible and admissible, it intends to assert that it now considers as its ultimate goal not the determination of individual events but the determination of whole systems of events. Weyl formulates the situation as follows: "The fundamental question . . . is not, as in classical physics: 'What value has *this* physical quantity in *this* particular case?' but rather: 'What are the possible values of the physical quantity A, and what is the probability that it assumes a definite one of these values in a given case?' " [18]

---

a distinction of the attributes and conditions that constitute the object. Cf. Leibniz, *Briefwechsel mit Clarke,* Letter 4, sec. 4; Letter 5, secs. 5, 6, etc. The Pauli principle is, as it were, the *principium identitatis indiscernibilium* of quantum theory. It characterizes every electron within the atom by a definite complex of conditions, by ascribing to it four quantum numbers that completely determine its orbit. Furthermore it states the conclusion that electrons that show no differences in this respect are to be regarded as a single physical entity. In one atom there cannot exist two electrons which have the same four quantum numbers. Two systems of quantum numbers resulting one from the other through an exchange of two electrons, represent a single state.

18. H. Weyl, *Gruppentheorie und Quantenmechanik* (2d ed. Leipzig, 1931), p. 66; English trans., H. P. Robertson, *The Theory of Groups and Quantum Mechanics* (New York, 1932), p. 74.

The renunciation of "individuality" with reference to the physical object—and of the conception that photons and electrons exist as objective individual things—certainly seems to be difficult for physics. But epistemologically it is facilitated by the realization that even when classical physics spoke of "individual" mass points, this expression was always to be taken with a grain of salt—with definite reservations in principle. In no case can we simply assume this "individuality" of the given—that is, of direct sense perception; it must be "defined" in some way—that is, represented in the exact conceptual means of physics, before we can admit it into its system of knowledge. If in the transition to quantum mechanics we have to determine the relation between the whole and its parts differently than before, it must not be forgotten that this relation, even within classical physics, was not taken over naively from the world of the senses, but that its determination always implied a quite definite logical task. The systems of rationalism, in their attempts to establish philosophically the basic concepts of classical physics, have always pointed to this state of affairs. It was particularly Leibniz who clearly saw this problem, and formulated it in explicit clarity. He starts from the consideration that the pair of concepts "whole-part" gives only the appearance of containing an entirely unified thought content. Closer analysis shows that the relations between the whole and its parts are capable of diverse applications and formulations, each of which must be separately designated and examined as to its logical conditions. This examination was to be the theme and task of a special chapter of logic which Leibniz intended to incorporate in his general theory of knowledge under the title *De continente et contento*. This chapter, like the whole *Scientia generalis,* remained a fragment, but even in its fragmentary form it contains the most important and fruitful hints for problems which become evident only much later in the history of mathematics and science. If we dissect the various possible forms of "inclusion," declares Leibniz, it becomes evident that by inclusion (*inclusio*) we may mean a relation of coincidence, of congruence, of causal implication, of logical content, or of the quantitative "larger" and "smaller." In its purely logical sense, inclusion was discussed for instance by Aristotle, whose construction of syllogistics had no other purpose than to show the way in which the final statement is contained in the premises. The theory of the "whole" and the "parts" must be distinguished from this because the whole is invariably larger than the parts, whereas that which contains and what is contained can be equal, as is the case for instance in the so-called "reciprocal" propositions.[19] Every part is contained in the whole, yet not everything that is contained is a part. For instance

19. Leibniz, *Nouveaux essais sur l'entendement humain,* Bk. IV, chap. 17, sec. 8. For further details concerning the doctrine *De continente et contento* see Louis

a point must be considered as being contained in a line, but not as forming a part of it.

For this logical character of the concepts "whole" and "part" the quantum theory furnishes a significant physical example. From the very beginning it had to desist from defining the whole as the sum of its parts; it declared that the whole is more than the sum. A system consisting of two electrons determines, from the point of view of quantum mechanics, the state of these electrons, but the reverse does not follow. A knowledge of the states of the two parts does not determine the state of the joint system, and a derivation of the latter from the former is out of the question.[20] The question, how, within a given whole, the separation into parts may be accomplished and how a certain aggregate may be differentiated and "individualized," accordingly always constitutes a difficult problem for quantum theory. The ordinary method of counting, which presupposes that it is known from the beginning what is to constitute one thing and what two or more things, is here insufficient. Individual things are not separated from each other in as simple a manner as in the sensuous-spatial view; complicated theoretical considerations are thus always required in order to determine precisely what is to be treated as an individual, what is to be counted as a "one." According to the premises chosen, entirely different forms of quantum statistics may arise. The latter cannot simply take over the methods of classical statistics; it has to adopt new methods of statistical "counting." Certain elements which, according to the former interpretation, were considered as separate must now be taken and counted as one, because it developed that they could not be differentiated with our theoretical and experimental means. Thus modern quantum mechanics embraces a plurality of statistical forms whose applicability has to be decided in each individual case on the basis of the particular circumstances associated with the problem. According as we are dealing with electrons or photons, "symmetrical" or "antisymmetrical" eigenfunctions, we have to use one or another of these forms.[21] Here also we see clearly that the determination of the individual, of that which truly figures as "one" being, is not

---

Couturat, *La Logique de Leibniz* (Paris, 1901), pp. 303 ff. In present-day philosophy this hint by Leibniz was first taken up again and extended by Husserl; cf. his "Gedanken zu einer Theorie der reinen Formen von Ganzen und Teilen," *Logische Untersuchungen* (Halle, 1913–21), *2*, 254 ff.

20. Cf. esp. H. Weyl, *Gruppentheorie und Quantenmechanik* (2d ed. Leipzig, 1931), p. 88; English trans., H. P. Robertson, *The Theory of Groups and Quantum Mechanics,* (New York, 1932), p. 92. Also *The Open World* (New Haven, 1932), pp. 55 ff.

21. Concerning the difference between classical (Boltzmann), Bose-Einstein, and Fermi-Dirac statistics see P. Jordan, *Statistische Mechanik auf quantentheoretischer Grundlage* (Braunschweig, 1933), chap. 3.

the *terminus a quo,* but always only the *terminus ad quem* for quantum theory—a result of the theory which cannot be anticipated dogmatically, or from immediate intuition.

Summarizing all these considerations, it will be seen from a new angle that the real difficulties of quantum mechanics are not due to a tendency toward a radical indeterminism, or to a demand that the causal concept be abandoned. The essence of the causal concept remains untouched as long as this essence is grasped in its true universality—that is, defined only by the demand for strict functional dependence. If the individual elements of determination available to quantum mechanics are used in accordance with the general principles of the theory and in keeping with the limits fixed by the uncertainty relations, a functional relationship, precisely definable, will always exist between them. Then the "causal law of quantum mechanics" is valid—that is, the thesis that if at any time certain physical quantities are measured as exactly as possible in principle, quantities will also exist at any other time whose magnitude on being measured can be predicted with precision.[22] It is true that a pictorial description of individual processes in space and time is no longer compatible with this requirement of functional dependence. A mechanism which, in accordance with classical laws, describes the connection between waves and particles and also determines the motion of the particles cannot be found. Dirac for instance maintains: "What quantum mechanics does is to try to formulate the underlying laws in such a way that one can determine from them without ambiguity what will happen under any given experimental conditions. It would be useless and meaningless to go more deeply into the relations between waves and particles than is required for this purpose." [23] But such an attempt is neither demanded nor even looked on with favor by the causal principle. Rather this principle is entirely content with the establishment of strict laws. If we can formulate observable events in mathematical language and thus describe them with precision, the postulate of the "comprehensibility of nature," which the causal principle contains, is fulfilled.

The essential problems posed by quantum mechanics for epistemology thus lie at a different point. They deal primarily not with the category of cause and effect but with the category of thing and attribute, of substance and accident. It appears that we must here carry through a much more far-reaching transformation and relearn much more radically than we had to in the case of the causal concept. It seemed hitherto to be an axiom, not only of classical physics but of classical logic, that the state

22. Cf. above, p. 127.

23. P. A. M. Dirac, *Principles of Quantum Mechanics* (Oxford, The Clarendon Press, 1930), p. 2.

of a thing in a given moment is completely determined in every way and with respect to all possible predicates. This complete determination was considered so certain that it was often actually used as a definition of what we are to understand by the "reality" of a thing. The *Critique of Pure Reason* related the two concepts in this manner and bound them indissolubly together. "Reality" and "complete determination" appear in it as interchangeable concepts. "Every *thing,*" declares Kant, ". . . is subject to the principle of complete determination, according to which if *all the possible* predicates of *things* be taken with their contradictory opposites, then one of each pair of contradictory opposites must belong to it." An object of the senses can be completely determined, however, only if it is compared with all the predicates of appearance, and is represented with reference to them either affirmatively or negatively. Thus if I choose any predicate *x,* then it shall always be unequivocally determined whether some empirical thing observed by me does or does not possess this predicate, and in the former case in what degree the thing can be ascribed to the predicate.[24]

Empiricist and rationalist epistemologists supported this viewpoint and attempted to base the definition of reality on it. It was particularly the spatiotemporal determination which since early times was considered as the true criterion of the "existence" of an empirical object. Thus Natorp declares for instance: " 'Existence' has no other meaning for 'Criticism' (*Kritizismus*) than a thoroughgoing, in no respect incomplete, determinateness of being with respect to space and time. Space and time are the condition of judgment of existence—that is, of the complete determination of objects in experience. . . . To every point in space, . . . that is to be given existentially, there must correspond a point or absolute element of what exists, namely each must correspond to an identical point in existence, i.e., in time." [25] The same basic viewpoint is represented in present epistemology by Schlick, who limits it, however, to temporality, in consideration of the fact that not all objects can be represented spatially. "Definite spatial determination is a property of most realities, but since this is not true in all cases . . . an unequivocal time determination alone is to be regarded as the necessary criterion of reality." [26] If one uses these explanations as a basis for objective reality, it will at once be seen what far-reaching changes in our empirical world picture are demanded by quantum mechanics by

24. Cf. *Kritik der reinen Vernunft* (2d ed.), pp. 599 f. (ed. Cassirer, *3,* 397 ff.); trans., p. 488.

25. P. Natorp, *Die logischen Grundlagen der exakten Wissenschaften* (Leipzig, 1910), pp. 341 ff.

26. M. Schlick, *Allgemeine Erkenntnislehre* (2d ed. Berlin, 1925), pp. 172 ff.

virtue of the uncertainty relations. For now we can no longer define existence as something completely and thoroughly determined. The "state" of a physical system no longer exhibits, according to the language of quantum theory, the same form of spatiotemporal connection which it possessed in classical mechanics. In the latter, all the individual elements of being could be isolated from each other; each particular entity was, in a given instant of time, referred to a quite definite point of space, and "adhered" to it exclusively. Quantum mechanics, on the other hand, demands that we abandon this conception. In the sense of wave mechanics we must think of a particle as being distributed in every instant throughout the whole of space; we have to ascribe a kind of omnipresence to its mass and charge. In the place of sharply defined orbits of the electron there appears an infinitely extended "charge cloud." Thus the possibility disappears of ascribing to the electron a single precisely circumscribed position within intuitive space. It seems to me that in this change in the significance of the concept of the "state" lies one of the most essential paradoxes of quantum mechanics, and one of the most difficult tasks with which it confronts epistemological analysis. In this respect, not in respect to causality, it demands an abandonment of presuppositions which appeared hitherto indispensable for any description of nature. The new "state" concept of quantum mechanics must indeed constitute the real stumbling block in the path of anyone coming from classical physics. Measured by conventional criteria it appears almost to lead to a contradiction in terms. For the state takes on entirely different values, according to whether we describe it in one language or the other, and neither of these languages can claim to define it unambiguously and exhaustively. It presents itself differently to us according to the different standpoints from which we view it, and it is impossible ever to unite the different perspectives in one glance. The particle and the wave pictures must be used side by side, without our ever being able to make them congruent. They are superposed without ever uniting with, or penetrating, each other.

This principle of superposition was placed by Dirac at the head of his presentation of quantum mechanics:

> We must now imagine the states of any system to be related in such a way that whenever the system is definitely in one state, we can equally well consider it as being partly in each of two or more other states. The original state must be regarded as the result of a kind of a *superposition* of the two or more new states, in a way that cannot be conceived on classical ideas. Any state may be considered as the result of a superposition of two or more other

states, and indeed in an infinite number of ways. Conversely, any two or more states may be superposed to give a new state, even also when they refer to different positions of the system in space.

Quantum mechanics forces us, for instance, to conclude that in the world of atoms states may exist for which the concept of energy no longer has a definitely expressible meaning and which we have to consider as being energetically undefined. And this by no means holds for energy alone, but for all other quantities of states as well, for all such quantities which are related to each other through the uncertainty relations, such as the coordinate x and the coordinate of momentum p_x of the electron, or the strengths of the electric and magnetic fields. In the case of each of them we must be prepared for the possibility that in a given state we will not be able to ascribe a sharply defined value to it. This appears the more peculiar if one considers that the respective determination which is here attainable never depends on the object alone but also upon the mode of observation which we choose. The object presents, as it were, a different "face" according to the conditions under which the observation takes place. According to the choice of measuring instruments and the particular use we make of them, we obtain different "pictures" of the event. No single observation can reveal to us the totality of all possible aspects at one time. Through each particular measuring device certain features of events, such as the wave nature or the particle nature of light, become dimmed for us, while others appear in their place. The answer given to us by nature is thus determined not only by nature itself but at the same time by the manner of questioning, and by the chosen instruments of observation. There is no experiment by means of which both the wave and particle nature of light can be demonstrated simultaneously. What a "thing" is in an absolute sense, without the circumstances of observation characteristic of the various series of experiments—for that we no longer receive any answer. If we attempt to force a reply, if we attempt to make the thing into something completely determined, into an *ens omnimodo determinatum,* immediately a remarkable dialectical transformation takes place. In place of an absolutely determined object, a mere shadow, as it were, remains in our hand. If, on the other hand, we accept the empirical conditions under which, according to the premises of the quantum theory, every observation takes place, the whole situation is reversed. The abandonment of absolute determination restores the highest degree of relative determination of which physical knowledge is capable. For if, in the determination of a physical system, we admit only such elements of determination as satisfy the conditions expressed in the uncertainty relations, if we are satisfied

with maximum observations—that is, with the largest number of independent compatible observations—we can bring these into a sharply defined relationship with each other. We can then establish the principle that when a maximum observation of a physical system is made, its subsequent state is completely determined by the result of this observation and this principle can be used as the axiom to express what we are to regard as the "state of a system" in the sense of atomic physics.[27]

All these conceptual transformations inherent in quantum mechanics become entirely clear only when it is constantly remembered that its conceptual terminology is an instrument which was not created for the description of "things" and states but refers to the representation of the behavior of physical systems. We may continue to talk about "things" in the sense of classical mechanics and of macroscopic experience, but we must take great care lest these things become rigid. Thus, for instance, in wave mechanics material bodies become dissolved; what hitherto appeared as a rigid system is transformed into a phase difference that is transmitted from one point in space to another in a given time. Where, in the sense of the classical view, we had *one* thing, existing always in a completely determined "state" and changing its state in a strictly determinable manner, we must now consider aggregates of processes, to each of which we ascribe a definite wave function. In this way we obtain exact statements, but they are statistical only. The question now is: what is the probability of obtaining this or that result from an observation made under definite conditions and at a certain instant? Dirac formulates this as follows:

> When an observation is made on any atomic system that has been prepared in a given way and is thus in a given state, the result will not in general be determinate—that is, if the experiment is repeated several times under identical conditions several different results may be obtained. If the experiment is repeated a large number of times, it will be found that each particular result will be obtained a definite fraction of the total number of times, so that one can say there is a definite probability of its being obtained any time the experiment is performed. This probability the theory enables one to calculate. In special cases this probability may be unity, and the result of the experiment is then quite determinate.[28]

27. Dirac, *Principles of Quantum Mechanics,* secs. 3, 4. Concerning the new "state concept" of quantum mechanics, and the principle of superposition, see especially Arthur March, *Einführung in die moderne Atomphysik* (Leipzig, 1933), chap. 3, sec. 15.

28. Dirac, *Principles of Quantum Mechanics,* sec. 3.

Thus quantum mechanics, unlike classical mechanics, does not lead to the determination of the position of an individual mass point at a given instant; it only provides the probability, for the totality of electrons, that the individuals are found in a certain position at a given time.

If these presuppositions and foundations of the problems of quantum theory are carefully studied, the skeptical consequences frequently drawn from the uncertainty relations fall away. At times their content was formulated by saying that the natural laws themselves are so constituted as to prevent the exact determination of a momentary state. If this is interpreted in the sense that the state exists in all exactness and definiteness but that nature by means of special laws devised stratagems to withdraw it from our gaze for all times, it seems to point to a strange metaphysical riddle.[29] But the metaphysical dilemma is rather a logical and epistemological one. For with what justification can we presuppose such a "state" when knowledge lacks every access to it? Even if the uncertainty relations are described as natural laws, such a designation is hardly appropriate from a methodological point of view. Heisenberg, in *Die physikalischen Prinzipien der Quantentheorie* (1930), sets out by postulating as a natural law the lower limits of precision for the simultaneous knowledge of different variables, and uses this postulate as a starting point for a critique of the concepts of quantum theory. But the uncertainty relations cannot be designated as natural laws in the customary sense; for such laws refer to physical things and phenomena, while the uncertainty relations refer to measurements—that is, to particular forms of obtaining scientific knowledge. They are not categorical statements about the objectively real, but rather modal statements about the empirically possible, about the physically observable. They accordingly do not presuppose a definite object only to determine subsequently that it will never be entirely accessible to our knowledge; rather they contain a new stipulation concerning objects, which we may rightly use as long as we adhere strictly to the limits of the observable. Here, too, we must keep strictly before us the differences in type of physical statements, so as not to become entangled in antinomies.[30] This objection could be

29. The fact that even in the realm of exact physics such interpretations of the indeterminism of quantum theory are not always avoided is shown by a strange utterance of W. Nernst. Nernst claims that modern physics approaches certain theological conceptions, and that a "parallelism which was hardly ever before foreseen," between physics and theology, could be realized. Has not the latter always claimed that cosmic happening was completely determined by the will and decree of God, whereas the human spirit was denied the ability of penetrating this determination into its ultimate details? Cf. Nernst, "Zum Gültigkeitsbereich der Naturgesetze," *Naturwissenschaften, 10* (1922), 495

30. See above, p. 30.

countered by pointing out that one of the most significant results of atomic physics was that it taught us the impossibility of drawing a sharp line between nature itself and our knowledge of nature; the two are inseparably interconnected. But when this conclusion is reached, the rejection of the skeptical consequences becomes all the stronger, for then the difference designated by Aristotle between πρότερον τῇ φύσει and πρότερον πρὸς ἡμᾶς disappears. Then nothing that is not "for us," that is not for physical knowledge in any sense, is any longer in "itself" in nature. The type of determination prescribes limits to the being which we can attribute to natural things, and not the reverse. It is not being, determined in itself, that sets permanent limits to knowledge, with the absolute intrinsic nature of being remaining impenetrable.

The fact that this state of affairs, and consequently the entirely positive meaning and significance of the uncertainty relations, did not at once become apparent in all clarity is perhaps due to the historical course followed by the quantum theory. These relations were based on the picture of the older quantum theory, on which, however, they exerted a limiting and constricting influence. The representation of pointlike electrons was not immediately abandoned; but knowledge about these electrons and their stationary orbits was recognized as impossible by the realization of a limit in principle to observation and measurement. In this way, however, two basically different epistemological systems of reference were introduced which could not exist and function simultaneously for any length of time. If this dualism is abolished, if a strictly unified standard of epistemological judgment and evaluation is set up, then the uncertainty relations lose much of the paradoxical character which they undeniably at first brought with them. What they actually demonstrated was only a certain overdetermination that had remained in the earlier formulations of quantum theory. By embracing statements about quantities which on closer analysis proved to be unobservable, this theory had overstepped the goal that can be set for physical knowledge, and it became necessary to retract this step. In this respect the uncertainty relations have no skeptical but solely a critical meaning. The necessary correction of the physical object concept of the older theory could only then appear as a skeptical abandonment, when this concept was not yet overcome in principle, when the viewpoint was once more assumed which one desired to abandon and which, with the establishment of the uncertainty relations, had already been left behind. We must face squarely the new problems thus created. There seems to be no return to the lost paradise of classical concepts; physics has to undertake the construction of a new methodological path. I do not wish to claim at all that the end of this path is already clearly in sight. But the direction

in which the solution is to be found seems to me clearly recognizable. Physics and epistemology cannot continue to posit a being with the full realization that it contradicts the conditions of physical knowledge. If it appears that certain concepts, such as those of position, of velocity, or of the mass of an individual electron can no longer be filled with a definite empirical content, we have to exclude them from the theoretical system of physics, important and fruitful though their function may have been.

It cannot be difficult to fulfill this condition if we remember the general structure of our physical concepts, when we realize that all these concepts are nothing but predicates of possible judgments. Only experience can decide about the truth value and the objective content of these judgments. In this respect a characteristic connection as well as a characteristic contrast exists between mathematical and physical concepts. One of the most important advances in the modern logic of mathematics lies in the fact that the basic concepts are no longer defined explicitly, but implicitly. A point, a curve, a straight line—these no longer have a firmly determined existence or a definite significance ascribed to them independently of their mutual relations. All these structures do not exist in order, subsequently, to enter into certain relationships; rather it is these relations themselves which determine and completely exhaust the being expressed in mathematical concepts. Likewise concepts like atoms or electrons fully share the logical character of these geometrical concepts. They do not admit of an explicit definition but basically can only be defined implicitly. In this respect there is no difference between the material point and the ideal mathematical point. To such a point also no being in itself can be ascribed; it is constituted by a definite aggregate of relations, and consists in this aggregate. Modern quantum mechanics thus tended more and more to begin not by positing definite realities, which are subsequently brought into relation with each other, but rather by choosing the opposite path. It starts out with the establishment of certain symbols expressing the state and the dynamic variables of a physical system. From these, on the basis of definite axiomatic presuppositions, other equations are derived, and physical consequences drawn from them. At first it is not necessary to dwell on the exact significance of the symbols in a particular case. Only at a later stage of consideration are the representations of the abstract symbols examined—in other words things and attributes are examined which satisfy the rules valid for the interrelationship of the symbols.[31] The difference between mathematical and physical concepts consists solely in the way they are constituted.

31. This treatment appears most clearly in Dirac's presentation of quantum mechanics.

Mathematical concepts can be obtained by construction; we "create" these concepts by means of the conditions which we impose on them, by means of the systems of axioms which they have to satisfy. In physics the place of these logical axioms is taken by the hypothetical formulation of the basic concepts, and by hypothetical deductions.[32] We apply concepts in such a way that through them events are described as completely and uniformly as possible, that through them we may "save" the phenomena. This demand for σώζειν τὰ φαινόμενα goes back to the earliest period of scientific physics.[33] In this way physical concepts are constantly enriched by new material that demands from them a constant capacity and willingness to change but not a complete giving up of their contents. A concept whose implicit definition consists of a set of empirical relations must be stable and flexible at the same time: stable in order that physics may have a definite point of reference, flexible so that it can always reorientate itself and test itself by experience. The concept of the mass point seems to face such a necessity of reorientation. The significance which this concept held in the construction of scientific knowledge can hardly be overestimated, and it resides, as we attempted to show by following its history, by no means only in the physical, but almost as much in the logical, purely methodological realm. Yet a concept such as that of the material point can, in the nature of the case, never be considered as a copy of a physical object; it is a "form," whose meaning and content consists of its theoretical performance, of its ability to lead to simple and strict laws governing phenomena. Every such form has its definite limitation; we must count on the possibility that areas of experience will turn up that it can no longer entirely cover and express. If today the sacrifices which have to be made in this respect seem greater and more difficult than ever before, it nevertheless appears to me that atomic physics has not destroyed the bases on which physical knowledge rests; rather it has made them known more clearly than ever before, in their characteristic individuality and conditional nature.

32. See above, p. 82.

33. Cf. the abundant historical material compiled by Pierre Duhem in his work, Σώζειν τὰ φαινόμενα. *Essai sur la notion de theorie physique de Platon à Galilée,* Paris, 1908.

CHAPTER 13

Concluding Remarks and Implications for Ethics

OUR EARLIER CONSIDERATIONS have moved exclusively within the circles of physics and epistemology. Without looking to right or left, without any speculative side glances or concerns about ethical implications, we have attempted to pursue our goal, which consisted of no more than the logical clarification of the problems of modern physics. Only at the end do we turn to ask what the significance of these problems might be for the whole of philosophical knowledge, for our *Weltanschauung* as a whole.

But if we look back to the results of our analysis of the basic concepts of physics, we realize that we must not expect too much. For the essence of these results consists precisely in this, that they exhort us to methodological restraint and moderation. Kant always insisted on the principle that no augmentation but only a distortion of the fields of knowledge results when we permit their boundaries to run into each other. Such a running together of boundaries occurs when statements about indeterminism in quantum theory are directly connected with metaphysical speculations about the freedom of the will. It is imagined that thus a greater unity between physics and ethics can be achieved and the gulf separating them can be bridged. But does this gulf really present a danger for critically schooled thought? Are we to yield here to the facile tendency of unifying and simplifying all problems—or is it necessary to perceive each set of problems in sharp isolation and to see in each set its own characteristic uniqueness?

It is precisely this uniqueness that lies at the basis of the problem of ethics. There is no competition between physics and ethics in the sense that they both presuppose the same realm of being but fight with each other about its explanation and significance. Ethics would certainly be in a bad way—it would lose all its dignity—if it could maintain its authority and fulfill its particular function in no other way than by keeping a

lookout for gaps in the scientific explanation of nature and taking shelter, so to speak, in these gaps. There it could lead a shadowy existence; ethical freedom would somehow be tolerated in the world but it could not exert any effect, any true power; it could not move anything outward. Yet for the problem of ethics everything depends on this outward influence. Moral freedom must not be limited in its meaning to a mere possibility, an empty potentiality; its meaning and significance consist in its actualization, in its constantly progressing self-realization. For this act of self-realization the merely negative concept of indeterminacy or indeterminability is inadequate; other positive forces, characteristic principles and grounds of determination, are required. These grounds are not to be found on the level of physical explanations of nature, whether those of classical physics or those of quantum mechanics; they cannot even be envisaged on this level. They demand in every case a μετάβασις εἰς ἄλλο γένος, a shift to another realm. No physical indeterminism, however far it is pursued, can save us the transition that is here expected and demanded of us. The problem is not how to break through or in any way to lessen the force of strict laws of nature in the realm of empirical events. The problem is to discover and develop a new viewpoint, to set up a new standard which cannot be reduced to that of empirical causality but which on the other hand is in no sense in conflict with it. Ethics demands that human actions are to be capable of and accessible to a double judgment; they are to be determined as events in time, but their content and meaning is not to be exhausted by this determinism. The insight into the course of these actions, no matter how strictly these may be determined, should not blind us to the other yardstick with which we have to measure them—to the question of their *quid juris,* their ethical value and dignity. This requirement of evaluation can never be grasped and justified in a merely negative way; it must be secured positively. We are not dealing here with an attempt to discover some exceptions to certain general rules; we are dealing with the far more serious and difficult problem of discovering a new rule, a new type of conformity to law, which is to underlie all action that we designate as moral action.

If the history of ethics is followed from its first beginnings, it can be seen that it has always been this basic problem that has occupied and deeply concerned all students of philosophical ethics. Philosophical, as contrasted with religious, systems of ethics are not satisfied with teaching morality; they aim to base morality on rational grounds. But in this attempt at supplying a rational basis (λόγον διδόναι), philosophical thought is immediately faced with two types of causes. In classical pregnancy and clarity this duality is described at one point in Plato's *Phaedo,*

which can justly be considered the true origin of philosophical ethics. What is the reason, Socrates asks in the *Phaedo,* for my sitting here in prison, awaiting the execution of the death sentence, when I was at liberty to escape?

> Should I say first that I am now sitting here because my body is composed of bones and sinews, and the bones are hard and have joints which divide them and the sinews can be contracted and relaxed and, with the flesh and the skin which contains them all, are laid about the bones; and so, as the bones are hung loose in their ligaments, the sinews, by relaxing and contracting, make me able to bend my limbs now, and that is the cause of my sitting here with my legs bent? . . . [But this] should fail to mention the real causes, which are, that the Athenians decided that it was best to condemn me, and therefore I have decided that it was best for me to sit here and that it is right for me to stay and undergo whatever penalty they order. For, by Dog, I fancy these bones and sinews of mine would have been in Megara or Boeotia long ago, carried thither by an opinion of what was best, if I did not think it was better and nobler to endure any penalty the city may inflict rather than to escape and run away. . . . If any one were to say that I could not have done what I thought proper if I had not bones and sinews and other things that I have, he would be right. But to say that those things are the cause of what I do, and that I act with intelligence but not from the choice of what is best, would be an extremely careless way of talking. Whoever talks in that way is unable to make a distinction and to see that in reality a cause is one thing, and the thing without which the cause would never be a cause is quite another thing. (ἄλλο μέν τί ἐστι τὸ αἴτιον τῷ ὄντι, ἄλλο δὲ ἐκεῖνο, ἄνευ οὗ τὸ αἴτιον οὔκ ἄν ποτ' εἴη αἴτιον.) And so it seems to me that most people, when they give the name of cause to the latter, are groping in the dark, as it were, and are giving it a name that does not belong to it.[1]

What from purely physical considerations, from the standpoint of pre-Socratic philosophy of nature, was commonly designated as a cause sinks for Socrates to the level of being merely the occasion (*Veranlassung*) or condition. And Plato intensified this distinction. The true ground of moral or free action cannot be found in the current physical situation, or in what immediately preceded the action in time. It demands a new direction of outlook, a new form of viewing things. That man acts freely who acts with regard to the world of Ideas; and a free subject is the one

1. Plato, *Phaedo* 98c ff.; English trans., H. N. Fowler (London, 1917), pp. 339 ff.

who is aware of these Ideas and in virtue of them can survey the whole realm of phenomena, of spatiotemporal appearances, and can recognize it in all its determinateness. Thus this type of freedom does not limit or declare invalid a single relation or a particular form of determination within the physical world—instead it aims to go beyond the totality of the physical world, which is to be ethically transcended. According to Plato the specifically moral bearing is only attained through such a transcendence, for the Idea of the good lies "beyond being" (ἐπέκεινα τῆς οὐσίας).

But even if this Platonic transcendence is abandoned, if a solution of the ethical problem is sought strictly within the boundaries of scientific knowledge, the twofold aspect remains. Even within a strictly naturalistic ethic, the dualism between freedom and natural necessity cannot be evaded. Spinoza is the first who demanded and carried through this naturalistic justification within modern philosophy. The leading methodological principle on which the structure of his ethics rests demands that we no longer treat human actions as a state within a state. There is only one order and one law underlying happening, as truly as there exists only one being, one all-embracing substance. The concept of purpose must, therefore, be excluded not only from science but also from ethical considerations. It proves in both cases to be merely an *asylum ignorantiae*. Human actions are not to be judged differently from the way we judge mathematical figures such as triangles or circles. We may not make judgments about them by an assumed anthropomorphic standard. They must merely be described, and understood in virtue of this description. But even this purely descriptive method does not abandon the concept of freedom within Spinoza's system. Even Spinoza cannot evade the admission that it is a "different kind of cause" to which we are led when considering human actions. We cannot nor do we wish to withdraw from the dominion of general laws of nature; but these take on a different character when they refer not to the motions of bodies but to our self-conscious activity, to our representing, desiring and willing. Ethical laws are natural laws but they are laws of our rational nature. This expression of the rational nature represents for Spinoza the unification and reconciliation of nature and freedom—the only one there is, and the only one that can rightfully and meaningfully be demanded by ethics. To act freely does not mean to act arbitrarily or without prior decision; it means rather to act in accordance with a decision which is in harmony with the essence of our reason. This essence and with it the specific priority of reason consists of the knowledge of the whole. Reason does not deal with particular things and events; its concern is to understand the whole in its form and essence. Its realm is not that of mere existence

but of pure essence. Out of this essential knowledge springs the true and deepest form of the moral will and moral behavior. It is the *amor dei intellectualis* that is the inner connection between the two. He who is filled with love for and insight into the whole does not succumb to the illusions of the imagination or the incitements of passing and momentary motives. He is "free" insofar as he does not live in the moment and himself does not suffer alteration with the changing moments. He grasps the norm of the universe and determines his actions by this norm as by an unchanging and eternal rule. To act "freely" means for Spinoza merely to act in accordance with this rule. The aim is not to elude all determination but to realize in one's action a quite definite, qualitatively designated form of determination: that one which is derived from the rational law as such. *Libere agere* and *ex ductu rationis agere* are one and the same. It is a waste of effort to seek another concept of freedom and a different concept of morality. The power and compulsion of all merely external particular causes break down in the face of this highest and all embracing form of human willing and human knowledge: "nothing is given in nature which is contrary to this intellectual love, or which could destroy it."

We face a completely different systematic formulation of the relation between freedom and necessity in Kant's development of ethics. Kant directs us back to Plato. His doctrine of morality rests entirely on that distinction which he himself called "classic," the "division of all objects whatsoever into phenomena and noumena." But it is clear to Kant from the start that this division cannot be carried through in a dogmatic-metaphysical sense but only in a "transcendental" sense. The "sensuous world" and the "intellectual world" are thus not posited as two opposing absolute forms of being, and for that reason the necessity for this opposition must be derived from principles of knowledge. It is the particular form of practical knowledge and practical judgment which leads to this opposition as its necessary correlate. Every human action has a double connection with being, since it can and must be subsumed under different norms of judgment. By its temporal origin and its temporal course it is a link in the chain of causes and effects—an element of that nexus which we designate as nature. But its determination within this totality of nature never exhausts its true content and meaning. As our actions belong to the realm of nature, so they also belong to the realm of purposes and must be referred to the systematic unity of this realm and judged in accordance with it. This judgment is governed by entirely different concepts and rules from those on which the intuition of the phenomenon "nature" rests. Though for the latter we may be satisfied to "spell out appearances that we may be able to read them as experience,"

we must in the former case take the step into the intelligible. An action may in all its aspects be spatiotemporally conditioned and therefore determined as a phenomenon of nature. Yet it is never merely the connection with the empirical world of things and facts which comes to expression in it. It points at the same time to the moral subject that plays its part in it and whose character and essence are revealed in it. It is this revelation of the person that Kant designates by the expression of "intelligible character," an expression which summarizes for him the essential aspects of the problems of freedom and ethics. By virtue of this doctrine Kant can remain a strict empirical determinist and can nevertheless assert that precisely this empirical determination leaves the way open for another determination, different in principle, which he calls the determination through the moral law or the pure autonomy of the will. The two are not mutually exclusive in the Kantian system but rather require and condition each other—a situation that can of course only be understood and justified if we strictly hold fast to the position that we are here dealing not with a metaphysical but with a critical-transcendental antithesis. It does not refer to absolute being as such, independent of all conditions of our empirical and our ethical knowledge. It merely establishes the aspect and the horizon of our reality from the standpoint of our basic theoretical concepts and ethical demands. Every rational being, according to Kant, has two points of view from which he can regard himself and recognize laws governing his actions.

> First, so far as he belongs to the world of sense, he finds himself subject to laws of nature (heteronomy); secondly, as belonging to the intelligible world, under laws which being independent of nature have their foundation not in experience but in reason alone. . . . The conception of a world of the understanding is then only a point of view which reason finds itself compelled to take outside the appearances, in order to conceive itself as practical, which would not be possible if the influences of the sensibility had a determining power on man, but which is necessary unless he is to be denied the consciousness of himself as an intelligence, and consequently as a rational cause, energizing by reason—that is, operating freely.[2]

Within the limits of this investigation it is neither possible nor necessary to pursue further the history of the concept of freedom and its

2. *Grundlegung zur Metaphysik der Sitten* (1st ed. 1797); English trans., T. K. Abbott, *Fundamental Principles of the Metaphysics of Ethics* (4th ed. London, 1889), pp. 72, 78.

systematic significance. The examples quoted are only to prove one thing, that none of the great thinkers who were deeply conscious of the problems connected with this concept and continuously wrestled with them, ever yielded to the temptation to master them simply by denying the general causal principle and equating freedom with causelessness. Such an attempt is not to be found in Plato or Spinoza or Kant. For all of them freedom did not mean indeterminateness but rather a certain form of determinability: determinability through the pure intuition of Ideas, determinability through a universal law of reason that at the same time is the highest law of being, determinability through the pure concept of duty in which autonomy, the will's self-ordering according to law, expresses itself; these are the basic criteria to which the problem of freedom is brought back. That other form of determination, which occurs in the general laws of nature, is here neither denied nor discarded, but rather presupposed. The new mode of determination which is to be established is not built on the ruins of nature's conformity to law; rather it joins the latter as a correlative and complement. For this reason alone it is most questionable whether, or in what manner, a relaxation or dissolution of scientific determinism can be made useful for the solution of the fundamental problem of ethics. A "freedom" emanating from such a source and based on such a foundation would be a fatal gift to ethics. For it would contradict the characteristic and positive meaning of ethics; it would not leave room for that moral responsibility the possibility and necessity of which ethics aims to prove. Whenever something is "ascribed" to a person in the ethical sense, it presupposes, and is connected with, some type of prior determination on the part of that person. An action which should simply fall out of the causal nexus, which should take place at random without reasons, would stand entirely alone and could not be referred or ascribed to a persisting ethical subject. Only an action "grounded" in some way can be considered a responsible action, and the value ascribed to it depends on the type, on the quality of these grounds and not on their absence. Thus the question of free will cannot and must not be confused with the question of physical indeterminism. The free will whose establishment concerns ethics is incompatible with a dogmatic fatalism; but it is by no means incompatible with a critically conceived and developed determinism. We have seen time and again that quantum mechanics has also by no means abandoned the idea of natural order according to law, but has rather lent it a new form. Thus if the idea of ethical freedom is threatened by natural order, it could not expect any help from quantum mechanics. For the problem under consideration, it makes no difference whether we think of natural events as being governed by strict dynamical laws, or whether

we merely presuppose a statistical regularity. For from the latter standpoint they would also remain determined to such an extent that the supposed "freedom," a *liberum arbitrium indifferentiae,* would not find any refuge there. An action which from a physical standpoint is branded as not entirely impossible but in the highest degree improbable is not an action we can count on in any way in the realm of the decisions of our will. Our ethical decisions would be in a sad state if we had to count on such improbabilities and had to make our decisions dependent on them. The extremely improbable is practically the same as the impossible; the degree of predictability which is left us by quantum mechanics would be entirely sufficient to destroy ethical freedom if the latter, in its conception and its essential meaning, were inconsistent with predictability.

But one of the essential tasks of philosophical ethics consists of showing why such an opposition does not exist, why freedom does not need to be upheld against physical causality, but instead maintains and asserts itself on its own grounds. "Persistence" is not only a physical but also and at the same time an ethical category, although in an entirely different sense. For all truly ethical actions must spring from the unity and persistence of a definite ethical character. This in itself shows us that it would be fatal for ethics to tie itself to and, as it were, fling itself into the arms of a limitless indeterminism. From such a standpoint we would have to evaluate an action more highly, the more it bears the earmark of the arbitrary, the unforeseen, the unpredictable. Yet true ethical judgment runs in exactly the opposite direction. It does not value behavior highly which is capricious, "uncontrollable," and changing from moment to moment; rather it values a course of action that springs from the basic substratum of the personality and is firmly anchored in it. Ethical character is distinguished by the fact that it is not completely determined from the outside, that in its decisions it is not thrown hither and thither by the changing conditions of the moment but remains itself and persists in itself. By virtue of this persistence we can count on such a person; we rely on his remaining true to himself and on his arriving at his decision not by mood and arbitrariness but by an autonomous law, by that which he himself recognizes and acknowledges as right. Schiller sets the foundation of his doctrine of moral freedom on the postulate that there exist ultimately two fundamental theoretical and ethical concepts, before which analysis must come to a halt and acknowledge its limitations. The one is the concept of person, the other the concept of state (*Zustand*). In man, as finite being, the two determinations must necessarily remain different and cannot be reduced to each other. "The person must be his own ground, for the permanent cannot

arise out of change; and thus we have in the first place the idea of the absolute being, grounded in itself—that is, freedom. A state must have a cause; it must follow something, since it does not exist through the person, and it is therefore not absolute; and thus we have in the second place the condition of all dependent being or becoming, viz. time." Accordingly, causality and freedom are as little opposed as are being and time; the structure of our theoretical as well as ethical world depends on the permeation and correct complementation of each by the other.

This complementation is of course not to be understood as suggesting that the two factors are simply to be put side by side, or regarded as parts which may be added together and which, by this addition, produce a homogeneous whole. The synthesis here sought and demanded is entirely a synthesis of different elements, which, however, are not incapable of union, though they are qualitatively disparate. This disparity cannot be overcome or eliminated in any way. Thus it becomes clear from this side as well, that a possible change of the physical "causality concept" cannot directly touch ethics. For however physics may change its internal structure, by abandoning, for example, the concept of the simple mass point or the possibility of strict predictions, the opposition in principle between the physical and ethical world, between the realm of nature and the realm of ethics cannot be bridged. These realms will always confront each other in the same manner, no matter to what immanent transformations of their form we consider them subjected. The problem of freedom and nature remains the same, whether we formulate the general laws constituting nature as dynamic or statistical laws. We are concerned here not with a difference in their thing-content, but with a formal difference, or more precisely a difference in category. We cannot do away with the guiding concept of determination in either case, in the structure of the physical word or of the ethical world. But determination follows different categories in the realm of being and in that of duty (*Sollen*). These categories do not conflict because they belong to entirely different dimensions of consideration. Thus they can never meet in one point; they cannot become identical, nor do they disturb or destroy each other. Equally, they are not distributed over separate special realms of being, but always demand the whole of being, though each from a special "aspect."

The methodological problem here before us is by no means restricted to the relation between "nature" and "ethics"; it has a far more general character. It recurs wherever different determinations and interpretations of meaning confront each other. For instance in the case of religion, its philosophical interpretation was again and again faced with the basic

problem, whether, and in what manner, the religious explanation of events can be brought into harmony with the other, humanly "natural" one. Again and again at this point the conflict between faith and knowledge sprang up; and for faith the miracle seemed to be "faith's dearest child." An event was more securely understood and established in a religious sense; the more it was documented as a miracle, the more it constituted a breach of the general laws of nature. However, in the modern philosophy of religion since Leibniz and Schleiermacher, this interpretation has undergone a considerable change. It no longer questions the validity of a strict and general order according to law in nature; rather it lends the latter a religious character; it sees in it a proof of the divine nature of being. "A miracle," Schleiermacher maintains, "is nothing but the religious term for an event"; it does not contradict the concept of regularity; rather it elevates this regularity as such into the sphere of religion and expresses it in religious terminology

The aesthetic "sense" is constituted in an analogous manner. Art is not an "imitation of nature," nor does it add something entirely foreign to nature by transforming it according to aesthetic ideals. Rather it *discovers* the beautiful in nature by measuring it with a new and independent standard. An idol revered in a shrine can be described according to purely scientific principles, and can be represented according to the concepts and categories of science. In this way it becomes a "piece of nature," subject like any other to physicochemical laws. Yet we know that with all these determinations we do not penetrate to its full meaning. The latter is not exhausted by the mere enumeration of scientific data. It demands other criteria, different in principle. No matter from how many viewpoints we may observe and analyze the marble as a natural object, the result will never divulge anything about its form and the beauty of this form, or about its significance in religious worship as an object of religious reverence. And it is just as impossible to arrive at the characteristic content of the problem of freedom if we strictly adhere to the realm of statements of scientific knowledge. This problem also is a problem *sui generis,* a question which cannot be solved by simple reduction to natural laws but has to be based on an independent type of orderliness according to law, the autonomous orderly structure of the will. If this is kept in mind, it becomes understandable why ethics has nothing to fear and little to hope from the changes in the basic concepts of science which have taken place in modern physics. Now as always it will have to seek and find its own way, a way on which physics can neither confuse nor greatly assist it. Even where, as with Spinoza, ethics was bound to a strict naturalism, this situation was not changed in principle. The distinctive methodological char-

acter of its approach always broke through at some point or other. Regardless then of how the conflict between determinism and indeterminism will ultimately be decided in the realm of physics, one thing is certain, that the decision of ethics cannot be anticipated by it. For ethics, the cry *Hic Rhodus, hic salta* [3] will always be valid at this point. Ethics must pass judgment in its own right on the problem of freedom. It cannot refer the problem to another court, nor can it acknowledge any prejudice in this its most proper basic question. The student of ethics who inquires after the possibility of freedom cannot expect any essential help from physics, as long as he poses his question in the only sense significant for ethics. Even if a solution to the riddle could be offered in the form of some physical indeterminism, he would have to reject it with the words Queen Christina of Sweden is said to have used when she renounced crown and kingdom: *non mi bisogna e non mi basta.*[4]

This state of affairs becomes the more apparent, the more we examine the individual problems to which the indeterminism of quantum theory has led us. The question may be raised as to what constituted the new understanding and the new fundamental view which atomic physics put in the place of the classical physical concepts. It consisted primarily of no longer posing the question *why?* in the same manner and at the same point as before. In Bohr's theory the path of an electron is determined by two different conditions, one classical, the other required by quantum theory. From the interaction of these conditions it develops that the electron can move only in certain designated circles whose radii are in proportion to the squares and whose frequencies are in proportion to the cubes of the quantum numbers; further it was shown that the electron can be raised from an inner to an outer orbit upon absorption of energy, and that it can fall from an outer to an inner orbit by energy emission. But to the question as to the *why?* of these two processes the theory lacks an answer. It merely determines *that* the process occurs and *how* it occurs; it narrows down the process, observed and accepted as empirically given, to certain precisely formulable rules. When it is said

3. Cf. T. James, *Aesop's Fables* (New York, 1848), p. 209. Fable 199, "The Boasting Traveller": "A man who had been travelling in foreign parts, on his return home was always bragging and boasting of the great feats he had accomplished in different places. In Rhodes for instance, he said he had taken such an extraordinary leap, that no man could come near him, and he had witnesses there to prove it. 'Possibly,' said one of his hearers, 'but if this be true, just suppose this to be Rhodes, and then try the leap again.' " *Translator*

4. Cf. Leibniz, "Theodicée": "As a comment on the explanations of such mysteries which appear from time to time, one might cite what the queen of Sweden had inscribed on a medal, with reference to the crown she had abandoned: 'I don't need it, and it's not enough anyway.' " *Philosophische Schriften*, ed. Gerhardt, *6*, 81.

that the electron is bound in no other way than as demanded by these rules, or that it has a certain playground within which it is "free," this is nothing else and nothing more than a metaphysical mode of expression. From this interpretation of freedom as a mere possibility, bounded by natural laws, there is no path toward that reality of will and decision which concerns ethics. To identify the "selection" that an electron is able to make from the set of different quantum orbits—in accordance with Bohr's theory—with "choice" in the ethical sense of that concept would be to succumb to a purely linguistic confusion.[5] For a choice exists only when there are not only different possibilities, but where also a conscious differentiation and a conscious decision is made. To credit the electron with such acts would constitute a severe relapse into anthropomorphism, of which modern physics, insisting more strongly than ever on emancipation from the anthropomorphic elements in the description of nature, cannot and has not become guilty. For modern physics the electron has become so unlike a personal unity that it demands exactly the opposite step from us; the physicist ceases even to consider and treat the electron as an individual in the ordinary sense of the word—that is, as a single thing. Far from favoring any form of personification, modern physics even questions the attempt at mere identification, at retention of even numerical identity of individual electrons. We have seen how the theoretical establishment of quantum statistics in its various forms was led again and again to question a fully determined individuality of electrons. We can no longer isolate two electrons from an electronic grouping and juxtapose them as independent individual entities in the way that used to be considered possible for two material points within the system of classical physics. If we take two electrons which are at first located at two distant points A and B, and which then collide, after which the positions A and B are again occupied each by one electron, then, according to the general considerations and principles of quantum theory it is meaningless to ask whether the same electrons again occupy points A and B or whether they exchanged positions. Such a form of identification not only cannot be attained, but according to the principles of quantum theory cannot even be sought after, and this has been realized and emphasized with ever increasing clarity. A "re-recognition" of an electron is no longer possible in the same manner and with means analogous to those which were possible within the framework of classical physics.[6] Thus here, where

5. Cassirer's word for selection is *Auswahl;* his word for choice is *Wahl.* Hence the linguistic confusion. *Translator.*

6. Cf., for instance, Jordan, *Statistische Mechanik auf quantentheoretischer Grundlage,* chap. 3, pp. 86 ff. Also cf. above, pp. 184 ff.

physical individuality itself becomes questionable, we are further than ever removed from ethical individuality, from the "person" as the subject of autonomous decisions of the will; the distance between the two is not diminished but rather more strongly accentuated.

We find ourselves forced to the same conclusion when we approach the question from the opposite direction, by fixing our attention solely on ethical problems as such. Is it really true that what these problems demand of us stands in apparent or actual opposition to a *certain form* of natural causality? Does not this question extend, when asked with true accuracy and consistency, to the *whole* of this natural causality? Fundamentally only one decision remains to be made: the possibility of uniting "nature" and "freedom" must be either asserted or denied. One is invariably led to a denial, to an insoluble antinomy, when one understands "determinism" in its metaphysical instead of its critical sense. For the moment one does this, causality ceases to be a principle of physical knowledge; it is hypostatized as an independent being, and becomes a metaphysical fate (*Fatum*) with which the human will conflicts and to which it must ultimately acknowledge itself captive. This uncritical metamorphosis of causality into a thinglike necessity or imperative (*Müssen*), a kind of kismet, can be countered only by a change of principle, by a philosophical about-face. It is not sufficient when, in order to avoid or weaken the consequences, one merely attempts a "loosening" which is unable to change the nature of the bond here presupposed. When once the rules of knowledge are changed into a thinglike compulsion, or the norm of causal understanding into a necessity which inexorably forces its form on events, and thus, as it were, puts them in fetters, all subsequent rectification of this fundamental methodological error comes too late. It does not matter in this case whether one thinks of the causality governing nature in the form of strict dynamic laws, or of mere statistical laws. The real fault lies in a different place and must be corrected by methods which are different in principle. If this kind of necessity is once included in the formulation of natural laws, it will make itself no less strongly and imperatively felt when we consider statistical regularities as the basic feature of natural events than when we think of the latter as being governed by strictly deterministic relations in the sense of classical physics. Neither the one nor the other path leaves an opening for that sphere of freedom which ethics claims for itself. To make this clear it is sufficient to cast a glance at the historical development of the problem.

The first contact between the problems of moral philosophy and those of statistics was brought about by the so-called "moral statistics." This discipline, in the formulation given to it by its founder, Quételet, con-

sciously set itself the task of abolishing the frontiers between merely physical and purely moral phenomena, by undertaking to show that both groups of phenomena are subject to exactly the same regularity, and can be described fully and without exception by means of the latter. According to Quételet there exists a physics of human society in exactly the same sense as there is a physics of atoms and of the material bodies made from them. The apparent contradiction in the "what," in the mere substratum of happening, is more than bridged. It is completely abolished by the insight that happenings in both cases are subject to exactly the same regularities. These regularities are of the same character and strictness, whether they concern material or spiritual, physical or ethical phenomena. In this way ethics was in principle included in the circle of calculability, which hitherto seemed reserved to physics. And the only methodologically compelling consequence resulting from this postulate had to be that freedom was lost for both ethics and physics. The numerical laws of statistics seemed to show, in an immediately comprehensible and compelling manner, that in the realm of phenomena of the will also there is no room for any free choice. The law of large numbers obliterates the differences which we are in the habit of making between appearances of nature and appearances of the will, and within the latter, between free and unfree actions. From the standpoint of knowledge, from the standpoint of the historian and the sociologist, these differences must disappear. They only see the never changing chain of causes and effects, through which each individual event and entity is held together with every other in such a manner that it loses its particular meaning, its significance as an individual. Even the ethical values by which we seek to elevate certain groups of actions and to secure them a privileged position are lowered to purely accidental determinations, when viewed from this standpoint. The statistical rule embraces both the good and the bad, and tolerates neither an essential difference of being nor a methodological difference of explanation between them. "Crimes are recurring each year in equal numbers with the same punishments, and in the same proportions," says Quételet, and he seeks to show that this principle refers not only to the type of crime but also to all their apparently accidental external circumstances. Not only will there be, in a given country, a definite number of murders committed annually, but the methods and weapons used will show a relation just as uniform as that which we are able to observe in physical phenomena, such as for instance in the movements of the tides. From this basic view there developed that form of materialistic philosophy of history which sees in the strictly mechanical determination which has to be acknowledged and used for both natural and historical consider-

ations the connecting and conciliatory link between the two. Statistics everywhere took on the role of the true spiritual mediator. Thus Buckle in the introduction to his *History of Civilization in England* (1857), where he pursues Quételet's basic idea further, writes that those who have a steady conception of the regularity of events and have firmly seized the great truth that the actions of men, being guided by preceding causes, are always consistent will no longer be astonished by the fact that this regularity extends to include even the smallest and apparently most insignificant. "Indeed, the progress of inquiry is becoming so rapid and earnest, that I entertain little doubt that before another century has elapsed, the chain of evidence will be complete, and it will be as rare to find an historian who denies the undeviating regularity of the moral world, as it now is to find a philosopher who denies the regularity of the material world. . . . The . . . proofs of our actions being regulated by law, have been derived from statistics; a branch of knowledge which, though still in its infancy, has already thrown more light on the study of human nature than all the sciences put together." [7]

All this demonstrates why the statistical character of the laws of quantum mechanics cannot in itself decide the question of ethical freedom, without the help of quite different considerations. From the same premises offered by physics, entirely dissimilar conclusions can be drawn, according to the rules of the philosophical and epistemological method accepted as valid. From the statistically "given," a path can be followed equally well in the direction of an indeterministic view as in the direction of a strict determinism and mechanism. For even statistical thought, as its history clearly shows, is by no means immune from the danger that its results will at a certain point "turn rigid" dogmatically, that the *law* of knowledge which is established in it undergoes such an interpretation that it appears to the thinking mind as an external, absolute, and compelling entity. As soon as this stage is reached, we find ourselves again caught in the antinomy of the problem of freedom; and this can be countered, as with dynamical laws, only by a radical transformation of the question, by a kind of Copernican revolution. Thus Simmel writes in his epistemological analysis of the concept of historical laws:

> If it is considered a social law that among 10,000 deaths per year there are a definite number of suicides, there has been a misunderstanding. For each one of the suicides under consideration is merely the result of social and psychological forces, or rather of the laws governing these; and the fact that there is a total of so and so many suicides is the result of the action of these laws on a

7. H. T. Buckle, *History of Civilization in England* (New York, 1877), *1*, 24.

> given material, and thus cannot itself be a law. . . . The summation of these cases is a synthesis undertaken by the observer; the fact that it yields this particular result has of course an objective foundation, yet only through the circumstance that each of its elements has one, whereas it would be a faulty circle and a kind of mystical teleology to attempt to deduce from the necessary determination of the result the conclusion that the elements are determined. . . . The history of each individual suicide furnishes the material for this . . . question, but does not answer it, because the question applies not to the level of immediate realities but to those levels in which the more abstract categories are developed from these realities. In the same way, for instance, the geometrical description of crystalline forms and of their systematic arrangement is not concerned, according to this standpoint, with the energies which cause each individual crystal to form.[8]

If one adheres to this fundamental interpretation, he will find that the πρῶτον ψεῦδος, the original fallacy, of the entire causal problem consists of considering laws themselves as a kind of reality and in describing them by predicates which are only applicable to realities. Once this confusion has arisen, there is no way out of the labyrinth. It is useless to attempt to save freedom by putting statistical regularities in place of dynamic laws, for the stumbling block in the way of recognizing and making visible the problems of ethics is not the *type* of law accepted; rather it is the ambiguous and epistemologically insufficient formulation of the concept of law as such.

But in another indirect sense the problems of quantum mechanics can be used to draw a conclusion of general philosophical significance. What modern physics has taught us is the fact that the change of standpoint which we have to make whenever we move from one dimension of meaning to another, whenever we exchange the world of science for that of ethics, art, etc. is not confined to this type of transition alone. The manifold of perspectives which open up before us has its counterpart within the scientific realm itself. Modern physics had to abandon the hope of exhaustively presenting the whole of natural happening by means of a single strictly determined system of symbols. It finds itself faced with the necessity of applying various types of symbols, of schematic explanations to the same event. It has to describe one and the same entity as a particle and as a wave, and must not be deterred in this use by the fact that the intuitive combination of the two pictures proves im-

8. G. Simmel, *Die Probleme der Geschichtsphilosophie* (2d ed. Leipzig, 1905) pp. 104 ff.

possible. When the fundamental task of physical knowledge, the connection of phenomena into firm orders according to law, demands a duality of description, the habits and demands of intuitive representation and understanding must be subordinated to this fundamental requirement. When, even in science, such a superposition of dissimilar aspects is necessary, it will be the more easily understandable that we shall meet such a superposition again as soon as we go outside its realm—as soon as we seek to realize the full concept of reality, which requires the cooperation of all functions of the spirit and can only be reached through all of them together.

BIBLIOGRAPHY

An asterisk before a reference indicates that it was part of Cassirer's intended bibliography in 1945; all other references have been added by Henry Margenau.

BOHR, N. "Development in Quantum Theory," *Nature, 121* (1928), 580.

FRANK, P. *Das Kausalgesetz und seine Grenzen.* Springer, 1932.

*MARGENAU, H. "Probability and Causality in Quantum Physics," *Monist, 42* (1932), 161.

BOHR, N. "Light and Life," *Nature, 131* (1933), 421, 457.

HOPF, E. "On Causality, Statistics, and Probability," *Journal of Mathematics and Physics, 13* (1934), 51.

*MARGENAU, H. "Meaning and Scientific Status of Causality," *Journal of Philosophy of Science, 1* (1934), 133.

VON LAUE, M. "Über Heisenbergs Ungenauigkeitsbeziehungen und ihre erkenntnistheoretische Bedeutung," *Naturwissenschaften, 22* (1934), 439.

BIRKHOFF, G. D., and LEWIS, D. C. "Stability in Causal Systems," *Philosophy of Science, 2* (1935).

*EINSTEIN, A., PODOLSKI, B., and ROSEN, N. "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?" *Physical Review, 47* (1935), 777.

*KEMBLE, E. C. "The Correlation of Wave Functions with the States of Physical Systems," *Physical Review, 47* (1935), 973.

*MARGENAU, H. "Methodology of Modern Physics, I," *Philosophy of Science, 2* (1935), 48.

*——— "Methodology of Modern Physics, II," *Philosophy of Science, 2* (1935), 164.

*RUARK, A. E. "Is the Quantum-Mechanical Description of Physical Reality Complete?" *Physical Review, 48* (1935), 466.

*BOHR, N. "Can Quantum-Mechanical Description of Physical Reality Be considered Complete?" *Physical Review, 48* (1935), 696.

*SCHRÖDINGER, E. "Discussion of Probability Relations between Separated Systems," *Proceedings of the Cambridge Philosophical Society, 31* (1935), 555.

*——— "Die gegenwärtige Situation in der Quantenmechanik," *Naturwissenschaften, 23* (1935), 807, 823, 844.

*HEISENBERG, W. *Wandlungen in den Grundlagen der Naturwissenschaft.* Leipzig, 1936.

*MARGENAU, H. "Quantum-Mechanical Description," *Physical Review, 49* (1936), 240.
*STEBBING, S. L. *Philosophy and the Physicists.* London, 1936.
BIRKHOFF, G. D., and VON NEUMANN, J. "The Logic of Quantum Mechanics," *Annals of Mathematics, 37* (1936).
BOHR, N. *Erkenntnis, 6* (1936), 293.
LENZEN, V. F. "The Interaction between Subject and Object in Observation," *Erkenntnis, 6* (1936), 326.
SCHLICK, M. "Quantentheorie und Erkennbarkeit der Natur," *Erkenntnis, 6* (1936), 317.
*BOHR, N. "Causality and Complementarity," *Philosophy of Science, 4* (1937), 289.
*MARGENAU, H. "Critical Points in Modern Physical Theory," *Philosophy of Science, 4* (1937), 337.
*KEMBLE, E. C. *Fundamental Principles of Quantum Mechanics.* New York, 1937.
*EINSTEIN, A., and INFELD, L. *The Evolution of Physics.* New York, 1938.
*FRANK, P. *Interpretations and Misinterpretations of Modern Physics.* Paris, 1938.
*KEMBLE, E. C. "Operational Reasoning, Reality, and Quantum Mechanics," *Journal of Franklin Institute, 3* (1938), 263.
*EDDINGTON, A. A. *The Philosophy of Physical Science,* Cambridge, 1939.
*LONDON, F., and BAUER, E. *Théorie de l'observation en mécanique quantique.* Paris, 1939.
BOHR, N. *The Causality Problem in Atomic Physics.* Paris, 1939.
LILLIE, R. S. "Biological Causation," *Philosophy of Science, 7* (1940), 314.
*MARGENAU, H. "Metaphysical Elements in Physics," *Review of Modern Physics, 13* (1941), 176.
*JEFFREYS, H. "Epistemology and Modern Physics," *Philosophical Magazine, 32* (1942), 177.
*WITTAKER, E. T. "Some Disputed Questions in the Philosophy of the Physical Sciences," *Philosophical Magazine, 33* (1942), 353.
*JEFFREYS, H. "Probability and Quantum Theory," *Philosophical Magazine, 33* (1942), 815.
ROSENFELD, L. "L'Evolution de l'idée de causalité," *Mém. Soc. Roy. Sci. Liéges, 6,* 1942.
*JEANS, J. *Physics and Philosophy.* New York, 1943.
*REICHENBACH, H. *Philosophic Foundations of Quantum Mechanics.* University of California Press, 1944.
BERGMANN, G. "The Logic of Quanta," *American Journal of Physics, 15* (1947), 397.
BOHR, N. *Newton's Principles and Modern Atomic Mechanics.* Newton Tercentenary Celebration. Cambridge, 1947.
——— *Dialectica, 2* (1948), 312.

DESTOUCHES, J. L. "Quelques Aspects théoriques de la notion de complémentarité," *Dialectica, 2* (1948), 351.

GONSETH, F. "Remarque sur l'idée de complémentarité," *Dialectica, 2* (1948), 413.

BORN, M. *Natural Philosophy of Cause and Chance.* Oxford University Press, 1949.

SCHILPP, P. A., ed. *Einstein, Philosopher Scientist,* Evanston, Ill., 1949. German trans. published by Kohlhammer, 1955.

WEYL, H. *Philosophy of Mathematics and Natural Science.* Princeton University Press, 1949.

MARGENAU, H. *The Nature of Physical Reality.* New York, 1950.

SCHRÖDINGER, E. "What Is an Elementary Particle?" *Endeavor, 9,* No. 35 (1950).

ELSASSER, W. A. "Quantum Mechanics, Amplifying Processes, and Living Matter," *Philosophy of Science, 18* (1951), 300.

KEMBLE, E. C. "Reality, Measurement, and the State of the System in Quantum Mechanics," *Philosophy of Science, 18* (1951), 273.

POPPER, K. R. "Indeterminism in Quantum Physics," *British Journal of Philosophy of Science, 2,* 1951.

BOHM, D. "A Suggested Interpretation of the Quantum Theory in Terms of Hidden Variables," *Physical Review, 85* (1952), 166, 180.

MARGENAU, H. "Physical Versus Historical Reality," *Philosophy of Science, 19* (1952), 193.

POLLARD, WILLIAM G. "The Significance of Complementarity for the Life Sciences," *American Journal of Physics, 20* (1952), 281.

SCHRÖDINGER, E. "Are There Quantum Jumps?" *British Journal of Philosophy of Science, 3* (1952), 109, 233.

DE BROGLIE, L. *La Physique quantique restera-t-elle indeterministe?* Paris, 1953.

LANDÉ, A. "Quantum Indeterminacy, a Consequence of Cause-Effect Continuity," *Dialectica, 8* (1954), 199.

MARGENAU, H. "Advantages and Disadvantages of Various Interpretations of the Quantum Theory," *Physics Today, 7* (1954), 6.

BORN, M. "Continuity, Determinism, and Reality," *Dan. Mat. Fys. Medd., 30,* No. 2 (1955).

FOGARASI, BELA. "Ist der Komplementaritätsgedanke widerspruchsfrei?" *Deutsche Zeitschrift für Philosophie, 2* (1955), 190.

HEISENBERG, W. "The Development of the Interpretation of the Quantum Theory," Niels Bohr volume (New York, 1955), p. 12.

LANDÉ, A. *Foundations of Quantum Theory.* Yale University Press, 1955.

Index

Abraham, M., 131
Acceleration, 170
Accident, 98, 100, 103 f.
Action, 172; at a distance, xiii, 167; contact, xiii, xiv; human, 200, 201; laws of, 178; moral, 198, 199; principle of least, 47 f.; quantum of, 79, 110, 118, 121, 123, 178
Action principle, 48–51, 56, 68
Aesthetics, 166, 206
Aggregate, 119
Agnosticism, 130, 149
Air resistance, 83
Alchemy, 147
Algebra, 161, 173, 178
Alpha-particles, 148, 182
Amplitude, 184
Analogies of experience, 18
Analogy principle, 165
Analysis, 160, 161, 164, 169; logical, 36; mathematical, 169
Anaxagoras, 37
Animal heat, 46
Animals, 95
Antarctic, 129
Anthropomorphism, 119
Antiquity, 37
Apparatus, 121, 128, 142
A priori, 17, 59, 63, 73 f.
Archetypes of being, 13
Archimedean point, 120, 134
Archimedes, 45, 82
Aristotle, xiii, 100, 103, 113, 124, 131, 139, 145, 160, 186, 194
Arithmetic, 169
Art, xx, 206, 212
Association, 20
Astronomy, 3, 13, 65, 75, 149
Atom, 7, 132 f. 178, 180, 183, 191, 195; Bohr theory of, 134; chemical concept of, 146; history and epistemology of, 139 ff.; model of, 114; nucleus of, 148; radioactivity of, 118; reality of, 139, 142, 145; stability of, 111, 172
Atomism, 141, 160, 171, 175, 176; in different fields of knowledge, 143; and energetics, 140; Galileo on, 145; Greek, 100, 144, 145, 150; kinetic, 146, 148; language of, 147; of Lavoisier and Dalton, 147; Mach on, 140; Ostwald on, 151
Autonomy, 202–4
Avogadro, A., 143, 147
Axiom, 70, 74, 177, 196; of limiting value, 96
Axiomatization, 68

Bacon, F., 47, 53, 64
Baeumker, C., 144
Balmer, J. J., 121, 133 f., 176
Bauer, E., 215
Being, 145, 156 f., 177, 194 f., 205; absolute, 202; laws of, 178; and thought, 159
Bergmann, G., 215
Bergmann, H., 118 n.
Bernoulli, J., 96
Birkhoff, G. D., 214 f.
Bohm, D., xvii, xviii, xix, 216
Bohr, N. H. D., xx, 110 f., 113–17, 121, 133 f., 149, 151, 165, 172, 176, 181, 207, 214 f.
Bois-Reymond, E. du, 4–10, 61, 63, 149
Bois-Reymond, P. du, 8
Boltzmann, L., 76, 78 f., 104, 140 f., 175
Born, M., xi, 116, 128, 184, 216
Boyle R., 38, 42, 85, 146

Broglie, L. de, xvii, xviii, xix, 180, 183, 216
Brownian movement, 79, 141, 164 n.
Buckle, H. T., 211

Calculus, 158, 164. *See also* Matrix, Probability
Cantor, S., 92, 169
Carnap, R., 70
Carnot, S., 75
Carrier, 130 f., 182
Carrier wave, 183
Categories, 24, 59, 102, 156, 205
Cathode rays, 135, 148, 182
Causality, 6, 100, 104, 116, 162, 188, 198; Born's definition of, xi, Cassirer's definition of, xi; and continuity, 161, 163; crisis of, 6, 163; critical theory of, 163; "empty," 116; and energy, 116; and fate, 209; and freedom, 204 f.; Heisenberg on, 123; Helmholtz on, 62; Hume on, xiii, 15; Jeffreys on, xii; Kant on, 16–22, 162, 166; and law, 118; Leibniz on, 14, 102; Lenzen on, xii; Maxwell on, 42; mechanical, xix; mythical, 99; not relinquished, 118; Painlevé on, 42; Plato on, 199; Reichenbach on, xiii; Russell on, xii; Spinoza on, 200; statistical, xix; and substance, 146; two types of, 198
Causality condition, xiv
Causality law, of quantum mechanics, 128
Causality principle, 29, 55, 58 ff., 63, 65, 70, 102, 114, 124, 127, 166, 188; and physical axioms, 177
Cavalieri, F. B., 171
Chance, 86, 101, 104; games of, 93
Charge, 43, 121, 131, 148, 172, 182, 184
Charge cloud, 181 f., 190
Chemistry, 144, 146 f., 149, 167, 172
Choice, 118 f., 208, 210
Christina, Queen of Sweden, 207
Clarke, S., 55
Classes, 37, 41, 156; in type theory, 30 f., 68
Clausius, R. J. E., 76
Cognition, 58, 82, 132, 157, 163
Cohen, H., xxiii, xxiv
Collective, 87, 93–5, 118, 185
Combination principle, 134
Combinations, theory of, 93
Combustion, 46
Complementarity, xx, 80, 89, 103, 115 f., 182
Compound, chemical, 141
Compton effect, 117, 122 f., 128
Concepts, 30, 32; primary, 157
Consciousness, 5, 7, 157
Conservation. *See* Energy, Momentum
Constraint, principle of least, 51, 56
Contact forces, 168
Contiguity, 21
Contingency, 102
Continuity, 54, 158, 166, 171, 173; and causality, 161, 163; and measurement, 176; principle of, 155 ff., 160, 172
Contradiction, 101
Coordinate system, 161
Copernican revolution, 211
Copernicus, 13, 67
Correspondence principle, 111 f., 165, 176
Coulomb, C. A. de, 34
Crimes, 210
"Criticism," 189
Crystals, 143, 212
Cubature, 171
Current, 183
Cycle, reversible, 76

D'Alembert, J. le R., 45, 61, 161
Dalton, J., 147
Darwin, C., 91
Decay series, 118
Dedekind, J. W. R., 169
Democritus, 139, 144, 150
Demon, 77
Descartes, R., 12, 38, 155–7, 177
Destiny, 24
Destouches, J. L., 216
Determinability, 132, 203
Determinism, 116, 118, 198, 203, 207, 209, 211; Born on, xi; Cassirer's definition of, xi; critical, 11 ff., 19, 23 f.,

59; Hume on, 20; Leibniz on, 12–15; metaphysical, 11 ff., 24; of quantum theory, 122; scientific, 203
Dielectric constant, 86
Differentiability, 164
Diffusion, 77
Dirac, P. A. M., 61, 117, 119 n., 128, 138 n., 188, 190, 195
Dirichlet, P. G. L., 48
Displacement, principle of virtual, 45
Dualism, 111, 130, 152, 176, 194
Dulong, P. L., and Petit, A. T., 79
Duty, 203, 205
Dynamics, 45, 47, 50, 55, 73, 145, 156, 161 170; classical, 110; Galilean, 155; laws of, 81 f.
Dynamism, 156

Eddington, A. S., xxiii, 33, 61, 119, 140 n., 215
Ehrenfest, P., 112
Eigenfunctions, 187
Eigenvalues, 113, 181
Einstein, A., xviii, 67, 75, 111, 214 f.
Elasticity, 65
Electricity, 45, 141, 148, 167
Electrodynamics, 44, 49, 76, 109, 111, 144, 148 f.
Electrolysis, 141, 148
Electromagnetism, 34
Electron, 180, 182, 187, 194; choice of, 118; Eddington on, 140; individuality of, 186, 208; mass and charge of, 121, 148; nature of, 178 f.; orbits of, 134; path of, 176, 207; and photons, 117, 123; position of, 122, 135; properties of, 182, 191, 195; a rigid sphere, 131; substantiality of, 178
Electrostatics, 34
Elements: chemical, 173; in type theory, 30 f., 41, 68
Ellis, L., 91
Elsasser, W. A., 216
Empiricism, 40, 46, 52, 54, 61, 189
Energetics, 80, 116, 140–2, 175
Energy, 121, 184; and causality, 116; centers of, 180; dissipation of, 76; distribution of, 109; equipartition of, 79; and frequency, 124, 172; indeterminate, 191; intellectual, 68; kinetic, 36, 51; and matter, 140; mechanical, 52; mental, 55; Ostwald on, 116; potential, 36, 51; and time, 180
Energy conservation, principle of, 46, 49, 51, 77 f., 104, 116 f., 123 f., 140, 146
Energy principle, 57, 117
Energy values, 133
Entropy, 76 f.
Epistemology, 9, 30, 39, 55, 60, 65, 76, 84, 90, 92, 101, 117, 124, 134, 136, 143, 151, 163, 175 f., 178, 181, 188, 193, 195; of atom, 139 ff.; critical, 21; and physics, 129; present, 189; scientific, 39, 130
Equations, 195; differential, 38, 113, 171, 175 f., 178; functional, 42, 60, 143; Lagrangean, 38; of quantum mechanics, 128; system of, 161
Equilibrium, 45, 161
Essence, 101, 201
Ethics, 197 ff., 203 f., 209, 212
Euclid, 70, 74
Euler, L., 48, 51, 56, 61
Exclusion principle, 184 n.
Existence, 189
Exner, F., 79, 81 f., 84, 86 f.
Experience, 29, 31, 58, 127, 179; analogies of, 60; and causality, 114; critical theory of, 138; form of, 74; process of, 54; scientific, 138; and theory, 73
Experiment, 41, 86, 123
Experiments, elementary, 182
Explanation, demand for, 5
Extension, 157

Facts, logic of, 5
Faith, 206
Faraday, M., 141, 167, 178
Fate, 118, 185, 209
Fechner-Weber law, 32
Fermat, P. de, 45, 47, 50 f.
Feynman, R. P., xvii, xix
Field, 34, 106; electric and magnetic, 86, 167, 191; electromagnetic, 38, 182; gravitational, 35; propagating, xiii
Field equations, 65
Field physics, 150

Field strength, 36, 127
Field theory, 34, 36, 131, 178
Figure, 158
Fluxions, 161
Focus imaginarius, 24
Fogarasi, B., 216
Force, xiii, 11; Coulombic, 34; derivative, 14; fundamental, 14; Helmholtz on, 62; Kant on, 17, 22; lines of, 131; in Newtonian mechanics, 34
Form, 138; and matter, 100
Formula, cosmic, 3 f., 35. *See also* Laplacean
Formulas: significance of, 39; structural, 147
Foucher, S., 160
Fourier, J. B. J., 38 f., 66
Frank, P., 214
Freedom, 92, 197 f., 200–11
Frequency, 113, 135, 176, 181, 184; continuous change of, 173; of events, 92; and energy, 124, 172
Frequency condition, 113, 134
Function, 38, 130, 137, 149, 171; doctrinal, 68
Galileo, 12 f., 30, 38, 40, 45, 47, 56, 66, 81–4, 87, 95, 106, 111, 145, 155, 170
Gas, ideal, 38, 42
Gases, 36, 77, 85 f., 182; kinetic theory of, 76, 78, 81, 87, 104, 141, 143; rarefied, 39
Gassendi, P., 145
Gauss, C. F., 48, 51, 56
Gay-Lussac, J. L., 38, 42, 85
Genius, 47
Geodesic, 50
Geometry, 70, 74, 94, 157 f., 161, 166; analytical, 157 f.
Gibbs, J. W., 77
God, 48, 55, 105, 170
Gods, Epicurean, 87
Goethe, J. W. von, 47, 136, 168, 174
Gold, 120 f.
Gonseth, F., 216
Gravitation, xiii, 50, 75, 84, 105
Gravity, law of, 22
Habit, 16 f., 20
Hahn, H., 164 n.
Hamilton, Sir W. R., xix, 48, 50, 51, 56, 118
Harmony, cosmic, 13, 168
Heat, 75; animal, 46; atomic, 79, 121; conduction of, 38 f., 66, 77 f.; radiation of, 109 f.; specific, 78, 110
Heisenberg, W., xvii, 39, 115, 121–3, 126, 128 f., 135, 152 n., 173, 181, 184, 193, 214, 216
Helm, G. F., 140
Helmholtz, H. L. F. von, 30, 46, 49, 51 f., 61–4, 123, 129, 141
Heraclitus, 42
Here-thus, 36, 41
Hermann, G., 65
Hero of Alexandria, 51
Hertz, H., 45, 52, 56, 66, 129, 173, 178, 185
Hilbert, D., 36, 44, 68, 113
History, 210
Hobbes, T., 155
Hönigswald, R., 60, 176 n.
Hoff, van't, 147
Homogeneity, 80
Hopf, E., 214
H-theorem, 79
Hume, D., xiii, xv, 15, 17, 19–21, 29, 40, 60, 73, 97
Husserl, E., 95 n., 187 n.
Huyghens, C., 30, 38, 146–8, 167
Hydrodynamics, 22, 65
Hydrogen, 121, 134, 176
Hydrostatics, 22
Hypostasis, 8, 24, 119, 144, 182
Idealism, 97, 140, 155
Idealization, 83, 94
Ideas, 199
Identification, 185, 208
Identity, 101
Idol, 206
Ignorabimus, 5, 9, 149, 174
Image, 7 f.
Imagination, 20
Impact, laws of, 159 f.
Inclusion, 186
Indeterminacy, 198

Indeterminism, 89, 116, 118 f., 122, 188, 193 n., 197, 203 f., 207, 211
Indifference, principle of, 91
Individuality, 180, 186
Individualization, 185
Induction, 39 f., 45, 84, 95, 97 f.
Inertia, 50, 55, 75, 95, 125
Infeld, L., 215
Inference, 39, 41, 63
Infinite, 159–61, 171
Infinity, 158
Instruments, 7, 9, 32, 37, 191
Integers, 172
Intellect: divine, 12 f., 101 f.; finite, 23
Intensity, 112, 135
Intuition, 24, 162, 171
Invariance, principle of, 112
Invariants, 117, 120, 138
Irrationals, 168–70

Jacobi, K. G. J., 48
Java, 46
Jeans, J., 215
Jeffreys, H., xii, 215
Jordan, P., 116, 128
Joule, J. P., 46
Judgments, physical, 31, 35

Kant, I., xiv, 103, 155; a priori, 59; on astronomy, 105; on causality, xv, 16, 18, 127, 162, 166; on critical determinism, 19–21; *Critique of pure Reason*, 22 f.; on discursive and intuitive understanding, 9; on ethics, 201–3; on homogeneity and specification, 80; on matter, 182; on modal principles, 60; on necessity, 103; on objects of experience, 179; on reality, 189; on realms of knowledge, 102, 197; on reason, 73 f.; return to, 5; on science, 74; on simplicity, 69; on special laws of nature, 114
Kelvin, Lord, 148
Kemble, E. C., 214–16
Kepler, J., 12, 13, 30, 38, 66 f., 111, 133, 171
Keynes, J. M., 43, 90
Keyser, C. J., 68
Kirchhoff, G. R., 30
Knowledge: critical theory of, 73; dissection of, 156; empirical, 54, 58, 102, 114, 122, 132, 135; fields of, 197; forms of, 101; Galileo on, 13; intuitive, 9; Kant on, 17; levels of, 31; logical, 137; magical, 8 means of attaining, 7; metaphysical, 15; and nature, 194; objective, 136; organism of, 55; philosophical, 197; physical, 25, 35 f., 44, 60, 137; realms of, 102; relative, 9, 43; scientific, 31, 38, 119, 196; structure of, 43; symbolic, 7; touchstone of, 159
Kries, J. von, 98, 105
Kronecker, L., 170

Lagrange, J. L., 38, 45, 48, 51, 61, 111, 161
Landé, A., 216
Lange, L., 126
Langevin, P., 165 n., 181
Language, 7 f.; symbolic, 22
Laplace, P. S., xiv, xv, 3, 5, 9–11, 22, 24, 61, 65, 90
Laplacean formula, 4 f., 11, 22 f., 63, 114
Laplacean spirit, 3 f., 7–9, 23
Lasswitz, K., 146
Laue, M. von, 143, 180 f., 214
Lavoisier, A. L., 46, 147
Law, 212; causal, 12, 62, 124; and causality, 118; classical conception of, 85; conformity to, 73 ff., 77, 82, 87, 98, 115, 123, 132, 198, 203; dynamic, 115; Galileo on, 40; logical intention of, 68; moral, 202; natural, 42, 53, 58, 66; and object, 131; order according to, 18, 62, 103–5, 162–6, 176, 203, 206, 213; of self-unfolding, 14; social, 211; transcendental, 19; types of, 112
Laws, 45; of being and action, 178; of classical physics, 81; critical, 144; dynamic, 77, 84–6, 89, 95, 105, 110, 119 f., 205, 209; of equilibrium, 45; form of, 52; historical, 211; logical and mathematical, 101; Mach on, 83; mechanical, 124; natural, xvi, 56, 82, 99, 101, 114; nomological and ontological, 105; specialization of, 85;

Laws (*continued*)
statements of, 37 ff., 54, 57, 63, 70, 86, 96, 128, 136, 151; statistical, 78, 81, 84–6, 89, 95, 105, 119 f., 205, 209; universal, 22
Leibniz, G. W. von, 11–15, 18, 24, 38, 43, 47 f., 51, 55, 66, 93, 95, 101–3, 125, 155, 158 ff., 164 f., 167, 171 f., 176, 185 n., 186, 206, 207 n.
Lenzen, V. F., xii, 215
Lever, 45
Lewis, D. C., 214
Liebmann, O., 5
Light, 47, 51, 129, 173, 182, 184, 191; and electricity, 45; electromagnetic theory of, 111; emission and absorption of, 109; and matter, 123; speed of, 121
Light quantum, 111
Lillie, R. S., 215
Locke, J., 180
Logic, 144, 158, 186; classical, 74, 97, 188; of discovery, 64; Eleatic, 145; Hobbes', 155; of illusion, 22; inductive, 40; mathematical, 30; of J. S. Mill, 40; new branch of, 93; of probability, 93; pure, 169; traditional, 124, transcendental, 22, 166
London, F., 215
Lorentz, H. A., 131
Loschmidt, J., 79
Lotze, R. H., 136

Mach, E., 30, 33, 46, 56, 58, 68, 83, 126, 129, 140, 142
Maclaurin, C., 161
Magic, 8, 99
Magnitude, 12, 158
Marburg School, xxiii, xxiv
Margenau, H., 214 ff.
Mariotte, E., 38, 42, 83, 85
Mass: of electron, 121, 131, 182, 195; gravitational and inertial, 75; Hertz on, 178
Mass particles, 178, 184
Mass point, 9, 34, 156, 185, 193, 196, 205
Materialism, 5 f., 141, 155
Mathematicism, 12
Mathematics, 11; ancient, 161; basic concepts of, 12; crisis of, 30, 170; "free," 92; Greek, 168; Hilbert on, 68; history of, 164, 186; Hume on, 16; Kepler on, 13; Leibniz on, 13, 155; logic of, 195; modern, 30, 161; and nature, 12, 160; and philosophy, 168; and science, 158; new structure of, 170; symbolic language of, 147
Mathesis universalis, 156
Matrix, 121, 173
Matrix: calculus, 178; mechanics, 128, 135, 181
Matter, 42, 50, 177, 181; atomic, 141; and electricity, 148; and energy, 140; and field, 34; and form, 100; granular structure of, 143; Kant on, 182; Leibniz on, 155; and light, 123; and radiation, 173; and reality, 130; wave theory of, 180
Maupertuis, P. L. M. de, 48, 51
Maxwell, J. C., 38, 42 f., 45, 66, 76 f., 136, 167
Mayer, R., 30, 46 f., 140
Measure, 11, 13, 31, 147
Measurement, 9, 37, 176, 194; procedure of, 131; refinement of, 115
Measurements: classes of, 58; results of, 115, 121, 128; statements of the results of, 29, 31 f., 35 f., 39, 42, 44, 54, 57, 60, 70, 86, 114, 118, 128, 136, 151; uncertainty in, 123
Mechanics, 44–6, 48, 50, 56, 78, 144, 149 f., 161, 167; classical, 38, 74, 76, 80, 111, 177; Descartes on, 158; idealization in, 94; laws of, 48; Leibniz on, 13; limit to corpuscular, 180; modern, 173; Newton's, 105; Painlevé on, 42; of points, 52; principles of, 79, 146; statistical, 77, 104; system of, 161; and thermodynamics, 78; universal, 34
Mechanism, 5 f., 62, 141, 155, 211
Medicus, F., 92 n.
Memory, 20
Mercury, 75
Metaphysics, 23, 101 f., 114 f., 137; dogmatic systems of, 18; rationalistic, 16, 18
Methods, scientific, 46
Metric, 50

Microscope, 122
Middle Ages, 145
Mill, J. S., 40, 53, 82
Millikan, R. A., 111
Miracle, 55, 206
Mises, R. von, 9, 90 n., 93 n., 94, 96, 104 n.
Model constant, 118
Models, 135, 142, 144, 157; atomic, 148, 150 f.
Molecules, 140, 143
Momentum, 121, 127, 179, 191; angular, 121; of electron, 122; conservation of, 78, 104, 117, 123 f., 146
Monads, 14
Motion, 157 f., 161, 170, 173, 177; absolute and relative, 125; differential equations of, 161; Greek view of, 144; Lagrangean equations of, 38; laws of, 112, 145; planetary, 133; reality of, 145
Myth, 8, 99

Natorp, P., xxiii, xxiv, 189
Naturalism, 206
Nature, 86 f.; constants of, 121; and freedom, 205, 209; hypostasis of, 119; inside of, 135; Kant on, 18, 201; and knowledge, 194; law of, 42; principle of uniformity of, 43; rational, 200; special laws of, 114; uniform course of, 40
Necessity, 100, 102, 201
Neo-Kantianism, xxiii, 5
Nernst, W, 4, 39, 193 n.
Neumann, C., 126
Neumann, J. von, xviii, 215
Newton, I., xvi, 5, 30, 34, 43, 61 f., 74 f., 84, 105, 111, 125, 161, 167, 171
Nizolius, M., 95
Nominalism, 142
Noumenon, 23
Nucleus, 148
Number, 11, 13, 31, 37, 147, 157, 168; atomic, 134
Numbers: continuum of, 166, 168–70, 178; discrete, 166, 171–3, 176, 178; law of great, 96, 98, 210; natural and real, 169; theory of, 168; whole, 170

Object, 59, 143, 177; causality and, 6; classes of, 156; Hume on, 15; knowledge of, 137; and law, 131; physical, 36, 131, 137; realms of, 157
Objectivism, 142
Objectivity, 18, 144, 176 f.
Observability, 128, 134, 136; principle of, 43, 125 f.
Observation, 32, 41, 115 f., 122, 127, 135, 191; single, 46; and thought, 168 f., 174
Observer, 121, 128, 142
Occam, 55
Occult, 167
Ontology, 18, 156
Optics, 44, 50, 181
Orbits, 179, 181, 184, 190, 194, 207
Order, 93; principle of, 158. *See also* Law
Oscillator, 110, 118
Ostwald, W., 116, 140, 151

Painlevé, P., 42
Panspermie, 38
Parmenides, 35
Particles, and waves, xx, 152, 174, 182, 188, 190 f., 212
Pauli, W., 184 n.
Peirce, C. S., 90
Pendulum, 38, 47
Perception, 15, 21, 32
Perrin, J. B., 164 n.
Persistence, 204
Person, 202, 204, 209
Perspectives, xx, 212
Petzoldt, J., 56
Phase waves, 183
Phenomena, saving the, 196
Phenomenalism, 142
Philolaus, 168
Philosophy: Greek, 100, 168; of history, 210; and mathematics, 168; modern, 101; moral, 209; natural, 101; natural of antiquity, 37; and physics, 129; of religion, 206; of science, 39, 144
Phlogiston, 147
Photoelectric effect, 110, 117, 183
Photons, 123 f., 186 f.
Physics, 46, 161, 172; atomic, 111, 120,

Physics (*continued*)
126 f., 129, 142, 152, 159, 179, 185; causal problem in, 30; classical, 29, 61, 64, 70, 81, 84, 104, 106, 164; demechanized, xiii; and ethics, 197; exact, 41; history of, 167; of human society, 210; macro- and micro-, 85, 122; mathematical, 6; methodology of, 24; modern, 165; Newtonian, 74; and philosophy, 129; procedure of, 29; structure of, 35, 74; system of, 49, 195; task of theoretical, 64; and theology, 193 n.; world view of, 115
Physiology, 46
Pisa, 47
Plane, inclined, 45, 84
Planck, M., xxii, 49, 65, 79 f., 86, 109 f., 114 f., 118, 121, 124, 126, 142, 171 f.
Planets, 75, 105
Plato, 169, 198, 201, 203
Podolski, B., 214
Poincaré, H., 96
Point: material, 175 ff., 180 f., 195 f.; mathematical, 195
Poissan, S. D., 51, 96
Polarization, 112
Pollard, W. G., 216
Popper, K. R., 96 n., 216
Position, 122, 127, 135, 195; absolute, 136
Positivism, 56, 64, 73, 97, 140
Prediction, 63–6, 204 f.
Principle: causal, 15, 18, 55, 57 (*see also* Causality); of induction, 40; of least time, 45; of mutual limitation, 70; of uniformity of nature, 43; of virtual displacement, 45. *See also under special names, e.g.* Observability, Reason
Principles, 47; Kant on, 23; modal, 60; physical, 47, 53; regulative, 63, 80; statements of, 45, 50, 52, 54, 57, 63, 70, 136; theory of, 102, 158
Probability, 4, 86, 104, 116, 184, 192; calculus, 86, 92 f.; density, xviii; function, 96, 98; laws, 77, 104, 105; physical, 91; statements, 90, 103, 118; theory, 91, 96
Processes, reversible and irreversible, 76, 80 f., 89
Prolegomena, 145
Propositions, synthetic, 19, 73
Proteus, 50
Proton, 121, 180, 182
Psychologism, 97
Pulley, 45
Purpose, 200
Pythagoreans, 100, 168, 170–2

Q-numbers, 128
Quantization, 113
Quantum, 141 f.; of light, 111, 117. *See also* Action
Quantum conditions, 113
Quantum electrodynamics, xvii
Quantum laws, 118
Quantum mechanics, 50, 61, 65, 79, 85, 192; achievement of, 148; and atomism, 149; and causality, xvi, 114, 163, 188; determination in, 127; differential equations of, 128; Dirac on, 190; and energetics, 117; energy principle in, 116; formalism of, 136; forms of new, 173; Heisenberg on, 121; and individuality, 180; laws of, 123; modern, 117, 142, 151, 172, 187, 195; new, 39, 113 f., 135; and order, 203; paradoxes of, 190; pure cases of, 128; Schrödinger on, 118; and states, 189 f.; statistical interpretation of, 116, 211
Quantum numbers, 112, 184 n., 207
Quantum postulate, 124, 133
Quantum principle, 110, 118, 136, 159
Quantum requirement, 111
Quantum rules, 113, 172
Quantum statistics, 187, 208
Quantum theory, 89, 109, 142, 183; and laws of action, 178; and atomism, 139, 150; causal interpretation of, xvii; causal principle in, 114; and continuity, 176; explosive of, 163; and gold standard, 121; Heisenberg on, 116; history of, 194; and indeterminism, 118, 207; and Leibniz, 165; modern, 132; problems of, 168, 193; Pythagorean in nature, 171; and radiation, 111; structure of, 122; and transformation theory, 138; and wholeness, 173

Quételet, L. A. J., 209, 210

Radiation, 79, 111 f., 134, 171–3, 179, 182, 184. *See also* Heat
Radiation law, 114, 121, 135, 142
Radicals, 147
Radioactivity, 118, 148, 183
Randomness, 94, 100
Rationalism, 11, 74, 186, 189
Rationality, 92
Rays, α- and β-, 182
Realism, 140, 142; naive, 129, 131
Reality, 4, 86, 120, 132; of atoms, 139, 142, 145; change in meaning of, x; classical concept of, 177; and concept, 159; of electron, 135; empirical, 93; full concept of, 213; judgments concerning, 40; Leibniz' concept of, 12; Mach on, 140; mathematical and metaphysical concept of, 12; and matter, 130; objective, 189; Ostwald on, 151; physical, 6, 58, 128, 135, 138, 139; of the physical world, 140; spiritual, 7
Realization, 23
Reason, critique of, 136; Kant on, 73; principle of insufficient, 91; principle of sufficient, 15, 19, 56, 101; scientific, 80
Recoil, 122
Reflection, 47
Refraction, 47, 83
Reichenbach, H., xi, xiii, 96, 215
Reid, T., 97
Relativity, 43, 137 f.; general, 50, 70, 75, 109, 126, 131, 166; special, xiii, xiv, 166
Religion, 205
Renaissance, 38
Research, physical, 45
Responsibility, 203
Restriction, 23
Rhodes, 207
Riemann, G. F. B., 70
Ritz, W., 134
Rope experiment, 125
Rosen, N., 214
Rosenfeld, L., 215
Rotation, 135
Ruark, A. E., 214
Russell, B., xii, 30, 41, 135 n.
Rutherford, E., 148
Rydberg, J. R., 134

Sagacity, of formulas, 39
Schemata, 166
Schematization, 162
Schiller, F., xxii, 204
Schilpp, P. A., 216
Schleiermacher, F. E. D., 206
Schlick, M. 43, 189, 215
Scholasticism, 140
Schopenhauer, A., 180
Schrödinger, E., xvi, xvii, xix, 45, 50, 79, 113, 115, 118, 128, 135, 142 n., 166, 173, 180–2, 184, 214, 216
Science, 158; epistemology of, 39; Greek, 100, 169; history of, 186; mathematical, 103; and mythos, 99; philosophy of, 39, 144; task of, 151
Selection, 208
Sense, common, 97, 129
Sense data, 31
Senses, extension of, 32
Sets, theory of, 30 f., 169
Shape, 135
Sign, 62, 130
Simmel, G., 211
Simplicity, 66–70, 133, 161
Skepticism, 132
Smoluchowski, M. von, 92 n., 104
Sociology, 210
Socrates, 199
Sommerfeld, A. J. W., 111, 133 n., 182
Space, 61, 177; configurational, 181; continuity of, 167; empty, 145; intuitive, 190; mathematical, 166
Space and time, 189; absolute, 43, 125, 136; determinations, 180
Space-time: continuum, 50, 131; description, 115 f., 142, 151, 164, 170
Specification, 80
Spectrum, 111, 121, 134 f., 172, 176, 178, 184; intensity and polarization in, 112
Spectroscopy, 144, 149
Spencer, H., 32
Spinoza, B., 5, 200, 203, 206
Square array, 128, 135, 181
Stahl, G. E., 147

Stars, 140
State, x, xv, 134, 190–2, 195, 204; macro- and micro-, 164; stationary, 112 f., 133, 180 f.; unfolding, xvi
State function, xvii
State variables, xix
Statements: metaphysical, 58; modal, 193; physical, 29–31; 38 f., 54, 69, 120, 122, 193; statistical, 89, 95, 116, 118, 185. *See also* Laws, Measurements, Principles
Statics, 45, 161
Statistics, 73, 87, 89, 94, 117, 185, 187, 204; moral, 209
Status, 179
Stebbing, S. L., 215
Stereochemistry, 147
Stevin, S., 45
Streintz, 126
Structure, 178
Stumpf, K., 91 n.
Subjectivism, 142
Subject-object problem, 129
Substance, 38, 132, 166, 177 f., 188; and causality, 146; and function, 130; Leibniz on, 14; simple, 13, 15, 18
Substantialism, 130, 132, 149
Suicides, 211
Superposition, principle of, 190
Syllogistics, 186
Symbols, 7 f., 120, 173, 195, 212
System, 33 f.; inertial, 126; periodic, 134

Tangents, 164
Teleology, 101, 105, 159, 176
Theology, 193 n.
Theory, conditions of any, 138; electromagnetic, 111; kinetic, 39 (*see also* Gases); and experience, 73; of types, 30 f., 41, 68
Thermodynamics, 78, 109, 117, 149; second law of, 75, 79 f., 89
"Thing," 137, 140, 191
Thomson, J. J., 141, 148
Thought, 157; and being, 159; logical, 39; mathematical, 12; scientific, 40
Tides, 210
Time, 205; and energy, 180; principle of least, 45; pure, 162, 164; reversals of, xix. *See also* Space and time, Space-time
Transcendental, 6, 9, 17, 20, 49, 58, 63, 200
Transformation theory, 138
Triangle, 164, 200
Truth, 168; criterion of, 150; factual and eternal, 159; Lotze on, 136; objective, 90; types of, 101
Types, chemical, 147; hierarchy of, 31, 114; of physical statements, 29 f., 64; theory of, 30 f., 41, 68

Uncertainty, xx, 120, 123, 128, 152
Uncertainty principle, x, xvii
Uncertainty relations, 109, 114, 122–4, 126, 131, 137, 149, 179, 180, 188, 191, 193 f.; and causality, 116, 124, 128; and epistemology, 123, 127; and measurement, 136
Unconditioned, 24
Uniformity of nature, 43, 165
Universal, synthetic, 10
Unknowable, 130

Valence, 147
Variables, hidden, xviii
Variation principle, 68
Velocity, 122 f., 128, 170, 182, 184, 195
Verbotsdiktatur, 113
Verification, 84, 125
Virtual principle, 45
Viscosity, 78
Visualization, 142, 183
Volder, de, 172

Waals, J. D., van der, 85
Wave: equation, 128, 135, 173, 181; function, 113, 192; length, 122; mechanics, 45, 113, 180 f., 190, 192; packet, xiv
Waves, 50, 180; and particles, *see* Particles
Wax, 177
Weierstrass, K. T., 164
Weight, equivalent, 147
Weltanschauung, 115, 122, 197
Weyl, H., 34, 36, 67, 131, 170, 185, 216

Whittaker, E. T., 215
Whole, 186, 200
Wholeness, 173
Wiener, O., 33
Will, 197, 201–3, 206, 209
Wilson cloud chamber, 182
Wind, E., 137 n.
Word, magic based on, 8
Work, principle of virtual, 161
Wundt, W. M., 177

X-rays, 143, 173

Zeno, 145
Zermelo, 79