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THE METHOD OF
DESCARTES

THE METHOD OF DESCARTES

A STUDY OF THE *REGULAE*

BY

L. J. BECK

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PREFACE

'The magnificent philosophical work done in the seventeenth century was due not so much to its being a century of genius—*car ce n'est pas assez d'avoir l'esprit bon, mais le principal est de l'appliquer bien*—as to its being a century in which the Cartesian rules of method were attentively observed.' R. G. COLLINGWOOD

To look through the catalogue of a library, or even to scan one's own bookshelves, must reinforce the melancholic conviction that it is an inevitable part of the price of greatness, in philosophical as in other fields of literary achievement, that posterity should pay the doubtful tribute of taking upon itself the task of explaining to the rest of the world what the great really meant, on the presumption doubtless that greatness in speculative matters is perfectly, and even normally, compatible with a certain muddle-headed obscurity and a marked inability to express oneself clearly. There will even be found braver spirits who assume less reverentially the task of explaining what these eminent philosophers ought to have meant. Descartes is, of course, in no way exempt from this posthumous form of tribute; a bibliography of books and articles on Cartesian philosophy would itself require a considerable volume. Yet it was Descartes who likened philosophical commentaries to the troubled waters found in pitchers and rivers compared to the clear spring water of the original text. Despite his assertions to the contrary, Descartes is not perfectly clear and the reader will, I think, agree that he is not the easiest of philosophers to understand, and that, for better, for worse, many of the Cartesian writings require detailed explanation for the student of philosophy. One weak but invincible excuse for a new book on Cartesian philosophy lies in the firm belief that, as this volume is far from being the first, so it will certainly not be the last, in the long series of works which claim, with no little justification, to interpret the thought of Descartes.

The purpose of this study is quite limited. I am attempting to set down an account of the method of Descartes with particular reference to his early work, the *Regulae ad directionem ingenii*. The reasons for my choice of the comparatively unknown *Regulae* as the fundamental text of my commentary

are sufficiently explained in my introductory chapter. Descartes is perfectly right in his contention that the real meaning of a great thinker must be sought and found in his own writings. My purpose here will be fulfilled if I have not clouded the issues but have helped my readers to understand more completely the text of the *Regulae* and, as a corollary, the text of the *Discours de la Méthode*. If the investigation of the Cartesian doctrine of method has sometimes carried me beyond the limits of the *Regulae*, I have tried to follow as closely as possible Descartes's own order of exposition in this work, rule by rule. Inevitably problems have arisen which would have called for the discussion of doctrines expounded by Descartes in his metaphysical treatises. These doctrines have been noted, and then deliberately left aside, in the belief that any deserved stricture of incompleteness will be condoned by my pursuit of a limited objective.

As far as was possible, I have quoted texts from Descartes to support my interpretation. These have been translated from the *Œuvres complètes*, edited by Adam and Tannery (the references indicated by the abbreviation AT. followed by volume, page, and lines). My footnotes would have been shorter if I had not chosen to print in full all the important passages translated in the text itself. It was, I felt, asking too much of the reader to expect him to look up the original words of Descartes in the twelve volumes of the *Œuvres*, even if these were conveniently available. Yet it is important in such a work that the whole of the evidence should be put forward, in order to enable the reader to judge for himself of the correctness of my interpretation; and that evidence is ultimately the text of Descartes. My aim therefore is primarily one of exposition and interpretation and any criticism tries, as its main purpose, to disentangle the essential of Descartes's methodological doctrine and restate it within his own assumptions. It does not follow that I regard his argument as above criticism.

It is, I imagine, universally agreed that the study of Descartes is an essential part of philosophical studies. John Locke was definitely of this opinion, and his notes on a *Méthode pour bien étudier la doctrine de Mr. des Cartes* are still interesting. He writes in his *Journal* on 7 March 1678:

'L'ordre qu'on doit suivre pour etudier avec utilité les ouvrages de Mr. des Cartes, et de ceux qui les ont commentez ou ecrit sur la doctrine est de commencer d'abord par la lecture de sa Methode. Parcequ'en meditant sur le recit qu'il nous y fait des progres de ses estudes on apprend d'une part à eviter les fautes qu'on commet d'ordinaire en etudiant la philosophie del escole et que l'on s'instruit d'ailleurs de la conduite quil faut tenir pour decouvrir la verité dans tout ce qui regarde la philosophie.' The advice is still instructive and sound. I ought perhaps to add that the immediately following paragraph of the *Journal* is ominous. Locke has no hesitation in advising the philosophical novice against the reading of the first commentary ever written on Cartesian methodology, the *Commentaire ou Remarques sur la Méthode de Mr. Descartes*, of Père Poisson, published in 1671. Locke notes, harshly but not injustly, that 'ce commentaire est rempli de tant de choses inutiles au sujet quelles sont plus capables d'embarrasser que dinstruire dans les commencements et quil est plus avantageux de ne les pas voir que de les lire.' I have attempted to keep this criticism in mind when writing my own commentary on the method of Descartes.

To some extent my great indebtedness to the writings of others may be measured by the frequency with which I refer to them. But these passing references represent only a small proportion of those from whom I have learned most. To have had the privilege of attending the lectures of Professors Gilson and Laporte at the Sorbonne, and those of the late Professor Joachim at Oxford, is to have incurred a debt which it is impossible to estimate for oneself or to acknowledge to others. To have discussed the text of Descartes with the late Professor J. A. Smith was in itself a revelation of the correct way to read a philosophical writer; I do not pretend to have learnt fully the lesson but I feel able to recognize the ideal. The printed works of Professors Gilson and Laporte have proved invaluable. Mr. J. L. Austin of Magdalen College kindly lent me his detailed notes of Professor Joachim's lectures. The greater part of what I have learnt about Descartes is due to the instruction and writings of these great scholars and philosophers, and even when my interpretation differs from theirs, my obligation does not end. I must also

express my gratitude to the Warden of Merton and to Mr. W. H. Walsh of Merton who have read through my manuscript. They have tried to improve my style and have offered many helpful suggestions. I have been helped by their encouragement and have profited, I hope, from their criticisms, but for many of the shortcomings which, despite their help, still remain in this work, I alone am responsible.

L. J. B.

MERTON COLLEGE, OXFORD
20 *February* 1951

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I

THE *REGULAE*

§ 1. *The* Regulae *and the* Discours

A FEW years after the death of Descartes, persistent rumours were circulating in the learned circles of France and the Low Countries that amongst the manuscripts left unpublished was a treatise on Logic. Baillet, the first biographer of Descartes, reports that

> Father Rapin, for instance, had said that Descartes had begun a Logic which he had never finished, but that a few fragments of the treatise still remained in the possession of one of his disciples and were entitled 'Erudition' . . . a vigorous search was made amongst his papers but there was nothing found bearing the title of 'Erudition', nor yet anything which could be looked upon as a Logic, with the single exception of his 'Rules for the guidance of the mind in the search after Truth', rules upon which a most excellent Logic may be modelled and which undoubtedly contain a considerable portion of his method.[1]

Baillet goes on to give his opinion that the *Regulae* are a fuller and more detailed exposition of the Cartesian method than that of the *Discours de la Méthode* which he considers to be a mere summary of the main methodological principles, a view which Descartes himself had expressed more than once.[2] Yet, on the whole, the method of Descartes has been studied, with a few notable exceptions, in the exposition of the *Discours*. It is not surprising that this should have happened. Clerselier himself, friend and disciple of Descartes, to whom Chanut left the responsibility of publishing the manuscripts of the philosopher, did not attach much importance to the *Regulae* of which he had the original in the writing of Descartes, and was quite willing to hand over the task to any scholar desirous of undertaking the editing of what he clearly considered odd

[1] l'Abbé Baillet, *La Vie de Monsieur Des-Cartes* (Paris, 1691), i. 282.

[2] Cf., for example, the letter to P. Vatier (AT. i. 559[13–16]), 'Je vous dirai premièrement, que mon dessein n'a point été d'enseigner toute ma Méthode dans le discours où je la propose.' See below, p. 172.

fragments, of much less importance than the letters which would reach a wider public.[1]

One can hardly blame Clerselier for thinking first and foremost of what would attract more easily a wider public and thus add to the fame of his already famous master. The example of this master would have justified his choice in leaving aside the incomplete and somewhat arid Latin text. In the *Discours* he had said: 'If I write in French, which is the language of my own country, in preference to Latin, which is that of my teachers, it is because I hope that those who make use of their natural reason without prejudice will be better judges of my opinions than those who believe only in the writings of the ancients.'[2] Later, in the preface to the *Principia*, he reiterated the same conviction as to the type of readers he had in mind: 'I have observed', he writes, 'on examining the nature of many different minds, that there are very few so dull-witted or slow of understanding that they are incapable of understanding sound opinions or even acquiring all the highest sciences, if they were but trained in the right way.'[3] There can be little doubt that Descartes thought that the average intelligent man and, he would have added, woman[4] was perfectly capable of reading and understanding his works on philosophy, and this meant not only the *Discours* but the *Meditationes* and the *Principia Philosophiae*. In a certain measure this praiseworthy ambition may seem to have been realized. The *Discours*, first published in 1637 (some sixty

[1] Cf. the prefaces of Clerselier which are published in vol. v of AT, especially the third preface, p. 651. 'Le bon M. Clerselier', as Bayle called him, was personally more interested in the theological implications of the Cartesian doctrine. He did, however, lend or show the manuscripts to his close friends. Malebranche must certainly have seen them. The Père Poisson had certainly read the *Regulae*, see his *Commentaire ou Remarques sur la Méthode de Mr Descartes* (1671), pp. 73 and 76.

[2] AT. vi. 77^{24-30}.

[3] In the author's letter to the translator (AT. ix (Part II), 12^{17-23}). The same view is found in the *Regulae*, IV, 'quod etiam experientia comprobatur, cum saepissime videamus illos, qui litteris operam nunquam navarunt, longe solidius et clarius de obviis rebus judicare, quam qui perpetuo in scholis sunt versati' (AT. x. 371^{21-25}).

[4] He writes in a letter to Père Vatier, a few months after the publication of the *Discours*: 'ces pensées ne m'ont semblé être propres à mettre dans un livre, où j'ai voulu que les femmes mêmes puissent entendre quelque chose' (AT. i. 560^{22-25}). See also what Baillet says in *La Vie*, ii. 500, and the charming study by Adam, *Descartes, Ses amitiés féminines*, Paris, 1937.

odd years before the posthumous publication of the *Regulae*) deserves perhaps the somewhat dubious title of the first book of 'popular' philosophy, at least in the history of modern thought. Written in a clear and flowing French, almost devoid of the technical terminology of philosophy, dedicated as it were in its opening words expressly, if somewhat maliciously, to all men of 'good sense', it gained a wide circle of readers and a celebrity which has now lasted more than three centuries. Yet there may be some justification in saying that the celebrity which Descartes achieved is, to some degree, akin to that attributed to Spinoza by the sharp tongue of Voltaire: he is 'moins lu que célèbre'.[1] Despite the apparent success of the *Discours*, both in the learned circles of Europe and in the drawing-rooms of Paris, Descartes himself seems to have guessed that he had not perhaps quite managed to make himself clear and for this reason began to write another work intended for the general public, the dialogue called *La Recherche de la Vérité*.

It has become almost traditional, even amongst able exponents of Cartesian thought, to talk of the admirable clarity of the style and simplicity of the exposition to be found in the *Discours*. But this simplicity and singleness of design and argument is more apparent than real. Descartes's account of his youth and studies, which is intended to give the reader a picture of the writer's mental development, is misleading. As Dr. Roth has pointed out, 'the reader of the Discourse is under a psychological illusion, for which indeed the author is by no means responsible. The book was published in 1637, and the reader is prone to look upon the whole of it, apart from the first chapter, as recounting the thoughts and aspirations of 1637. But the fact is that the major portions of the book, and those its most famous and most significant, really go back many years earlier.'[2] The plan itself of the *Discours*

[1] Or as G. Gadoffre writes in his introduction, '*Le Discours de la Méthode* est plus respecté que véritablement compris. On l'admire de loin', and he goes on to show how this has come about in a convincing discussion from which I have taken several points. Yet even the *Discours* has suffered a partial eclipse of favour. There was no new edition of the work in France from 1724 to 1824.

[2] L. Roth, *Descartes's Discourse on Method*, p. 24. The author points out that a higher criticism of the text 'may possibly show that the first three chapters embody a written text of early date, just as some of the fifth chapter probably absorbed material already prepared for a different purpose'. See also a similar view on the composition

reveals a somewhat arbitrary sequence of ideas. A short introduction explains in an inadequate manner the obligation of the author to introduce his scientific essays by a biographical account of his studies which we are invited to consider 'merely as a history, or, if you will, as a tale'. Some general considerations on the sciences are followed rather abruptly by an account of scientific methodology and the famous enunciation of the four rules of the method. The brief account of the principal rules of the method does not lead, however, to a discussion of methodology and the various applications of the method. We are led into a discussion of morals in Part III, formulated, it is true, in a set of three quite sensible maxims but which cannot be said to have any logical connexion with the rules of method. The next chapter is a metaphysical discussion of the arguments for the existence of the soul and God, involved in the search for a criterion to establish 'the truth and certainty of a proposition', while the following chapter is devoted to various discussions on physics, physiology, and psychology, in fact, it is a disjointed summary of the topics which the author treated in the ill-fated volume *Le Monde*. Part VI, after a first transitional paragraph, produces an almost repetitive justification of the author's reason for publishing his scientific essays, followed by some general thoughts and considerations, not 'very specific' as Descartes himself admits, concerning the progress and advancement of the sciences. And although the structure of the *Discours* seems polished and well-knit together to the casual reader, whose way is smoothed over the more jerky transitions by the easy, almost conversational tone and style, which will always make the *Discours* a masterpiece of elegant and clear philosophical writing, it should be stated, and the uninitiated warned, that the homogeneity and coherence of the most famous of Cartesian writings is far from evident and its apparent simplicity and clearness constitute a formidable snare and delusion for the unwary. The masterly commentary of Professor Gilson should in itself constitute an adequate warning of the difficulties of the *Discours*.[1]

of the *Discours* in an article by G. Gadoffre in the *Revue de synthèse*, Jan.–June 1948, p. 11.

[1] Cf. E. Gilson, *Discours de la Méthode, texte et commentaire*, Paris, 1930. How

However that may be, it is not difficult to appreciate the misjudgement of the relative values of the *Discours* and the *Regulae*, which latter work was first published in 1701. The contrast between the two works is striking since the *Regulae* are unfinished and imperfect in more senses than one. Of the thirty-six Rules which were originally planned in the mind of their author, only twenty-one were in fact composed and of these the last three contain merely the text of the Rules without any of the exposition and commentary which follows in the others.[1] But this obvious imperfection is allied with other imperfections in the content of the work itself which seem to be inherent in the thought of its author. There are many conflicting, if not irreconcilable, statements which seem to spring sometimes from carelessness or hasty composition and at other times to arise from indecision and confusion in the author's mind. Propositions are sometimes crudely affirmed and later entirely modified, as though Descartes was unaware of the radical nature of his change of position. There are often repetitions which can only charitably be accounted for by careless drafting and lack of revision; there are different versions of the same argument to be found, which the author does not seem to have tried to combine or conciliate.[2] The *Regulae* are only, in fact, a major fragment of a book of which we have the first part, the beginning of the second part, and nothing else. There are practically no indications which would usefully allow one to reconstruct the missing parts. The second part of the *Discours* which contains the rules of method is even more general than the *Regulae* themselves and, if one of the contentions of this study is correct, it is the *Regulae* which are needed to understand fully the second part of the *Discours*.

It will be agreed by most readers of the *Regulae* that such criticisms as have been outlined in the previous paragraph

many readers of the *Discours* notice the definition of *Mathesis Universalis* given in the paragraph immediately following the four precepts? See below, p. 36.

[1] Rule 12 gives the plan of the work as originally thought out: cf. AT. x. 428^{21}–429^{13}.

[2] The most obvious case is the repetition of Rule 6 (AT. x. 384–7) in a generalized and shortened form in Rule 11 (ibid. 409–10). See also the AT. editor's note on Rule 8 (ibid. 485–6) on the double version incorporated in the text. The lengthy autobiographical passage at the end of Rule 4 (ibid. 374–9) does not fit into the context and is probably an afterthought; see below, p. 191.

are obviously true but of small value in themselves. It must be always borne in mind that Descartes did not himself pass the text as fit and ready for publication: on the contrary, if we are to take literally his remark at the end of Rule 4, he wrote them entirely for his own personal benefit so that when, through old age, his memory was less active and less retentive, 'he might easily recall things from this note book' and that 'with this relief to his memory, he might apply himself to other studies with greater ease of mind'.[1] Such words must blunt any carping criticism on purely formal grounds.

When the literary nature and historical fate of the *Regulae* and the *Discours* are compared, it is not perhaps so astonishing to find that such an eminent commentator of Descartes as Hamelin, while admitting the value of the *Regulae* as a 'sure and abundant source' of information for the study of Cartesian methodology, should definitely subordinate the *Regulae* to the *Discours*, 'the latter book being the text, the former merely the commentary of the method'.[2] Or that Adam, the editor of the complete letters and works of Descartes, should compare them as 'rough draft' to 'masterpiece', the former being in this case the *Regulae*. At least both these great scholars praised and made considerable use of the *Regulae* in explaining or clarifying the doctrine of Descartes. But it is possible to take large numbers of the never-ceasing editions, with introduction and commentary, of the *Discours de la Méthode* without finding more than a passing reference, sometimes not even that, to the text of the *Regulae*—the famous commentary of Professor Gilson is of course an outstanding exception. It is almost true to say that the generally accepted, authoritative exposition of Descartes's method was based, by a sort of tradition, on the account given in the second part of the *Discours* and it was equally traditional that the student of philosophy should be referred to this same work as containing the complete and final account of the Cartesian doctrine of method. Not that the importance of the posthumous work had entirely escaped notice. Professor Kemp Smith in his study of Cartesian method has written that 'for a more adequate treatment of it we must look to

[1] AT. x. 379⁷⁻¹⁴.

[2] *Le Système de Descartes* (2nd edn., Paris, 1921), p. 49.

his earlier and less famous work, *Regulae ad directionem ingenii*',[1] which was in fact what Boyce Gibson had done a few years earlier in two articles in *Mind* which still are one of the best studies of the text, 'For a clear understanding of the principles of Descartes's natural logic, and the exact bearing of the four rules given in the *Discours*, we must turn to the *Regulae ad directionem ingenii*.'[2]

The first part, consisting of twelve Rules, is complete in itself and, as far as can be judged, would have been the most original and most important section of the projected work. Even if we accept Adam's description of 'a rough draft', it is precisely this character which permits the reader to observe Descartes's doctrine of method in the making, and to avoid some of the pitfalls of the more polished writings, above all that of a style so lucid as to be dangerous where the clearness of the stream conceals its depth. Sometimes Descartes may wander from the point, repeat himself, or slip into rather glaring inconsistencies. These very defects often show up the difficulties inherent in the method itself, more or less successfully concealed in the polished brevity of the second part of the *Discours*. Further, the *Regulae* set before the reader, in more detail, the Cartesian doctrine of the relation of philosophical and mathematical methodology and the theory of the application of mathematical thinking to philosophical problems. It may be agreed that, by this study, one will not have taken a roundabout and tedious road, instead of a short cut, to an understanding of the 'Cartesian revolution' in the history of human speculation. It will probably be found that the *Regulae* and the *Discours* do not differ in their essential doctrine of method and that the former contains fundamentally

[1] *Studies in the Cartesian Philosophy* (1902), p. 19.

[2] The articles are in *Mind*, N.S. vii (1898), 145–58, 332–63. Professor Rivaud in 'Études sur Descartes' (a special number of the *Revue de métaphysique et de morale*, 1937) states that 'Seuls, quelques historiens d'esprit plus pénétrants [he then mentions Boutroux, Milhaud, Meyerson, and Brunschvicg] ont tenté de mieux comprendre le dessein de Descartes, et utilisé avec profit les textes des *Regulae*, où Hamelin et M. E. Gilson ne veulent que des essais encore informes, largement dépassés par l'exposé du *Discours*' (p. 36). The list is much too restricted, the inference perhaps unfair, but it is still true to say, as Professor Keeling says in a note to his Henriette Hertz Lecture to the British Academy in 1948, that 'the detail of the *Regulae* has not yet been explored, correlated, and above all illustrated by reference to his own reasonings and explanations in one and another special science'.

nothing more than the doctrine expressed succinctly in the four Rules of the *Discours*; what is added consists, in the main, of supplementary rules whose object is to offer practical advice and suggestions on the application of the rules, rather than to elaborate new rules. In some respects, as will be noted, Descartes appears in the *Regulae* to be more under the influence of the traditional Aristotelianism than in the *Discours* and his maturer works, but his own theory of method has already taken shape and begun to develop. Some of his fundamental ideas, of which his later works are the development, obtain their fullest expression in the *Regulae*. A sober appreciation of the value of this work does not necessarily drive one to agree with Victor Cousin that it surpasses both the *Discours* and the *Meditationes* 'in lucidity of treatment' or to accept the rather dithyrambic words of Millet that 'this treatise is of all existing treatises on Logic, without even excepting the *Organon* of Aristotle or the *Logic* of Hegel, the most profound and the most wonderful'.[1] It is wiser to follow the advice given in *Piers Plowman*: 'parum lauda, vitupera parcius'; Descartes has suffered much from the non-observance of such advice. It can be argued, with some likelihood of general assent, that any serious attempt to understand, and criticize, and perhaps in the end reject, the Cartesian doctrine and the method underlying both its invention, exposition, and application, must first begin by a study in some detail of the method as expounded in the *Regulae*, the original and fundamental text of Cartesian methodology. The Cartesian 'revolution' is primarily a revolution in method.

§ 2. *The Text of the* Regulae

There can be no doubt as to the authenticity of the *Regulae*. The full evidence is available in the Adam and Tannery edition of the complete works. There is equally little point in retracing the history of their first publication at Amsterdam in 1701 in a volume entitled *Opuscula posthuma physica*

[1] Cf. *Mind*, vii. 149. Millet's work, *Descartes, sa vie, ses travaux, ses découvertes avant 1637*, Paris, Didier, 1867, is important, and should, moreover, be considered as a remarkable achievement, given the state of Cartesian studies, and even the state of the texts, at the time of its publication.

et mathematica.[1] The original manuscript which was once in the possession of Clerselier, and which Baillet certainly had in his hands when writing *La Vie*, is lost. But we know of at least two other copies, one which had belonged to Leibniz and is still in existence (or was before 1939) in the library of Hanover and another which was used by the editors of the *Opuscula posthuma* which has disappeared.[2] There are but few variations between these sources of the text and the necessary corrections do not involve any elaborate textual criticism.

The date of the composition of the *Regulae* is more controversial. Baillet, the first biographer, who saw the text written by Descartes himself, does not in fact attribute any date to their composition. He merely uses them to comment on certain ideas which Descartes had in the forefront of his mind when a young man but he does not imply that the *Regulae* were composed in that particular epoch of Descartes's life which he is describing. There is no direct reference in the letters or writings of Descartes which help to fix a date or even delimit a period of his life, except one general phrase in a letter to Mersenne of 15 April 1630 where he writes, 'at least I think that I have found a means of proving metaphysical truths in a more evident way than one can give a demonstration in Geometry. This is my own opinion, for I do not know whether I can persuade others to accept it. During the first nine months which I have been in this country, I have worked at nothing else.'[3] The allusion is too vague to justify any sure conclusion. The text of the *Regulae* is not much more enlightening. Some biographical indications are given. We

[1] Published by P. and J. Blaeu. There was another edition by the same publishers in 1704 and the text has a few variants from that of the earlier edition. The variants on examination all appear to be due to printer's errors which have crept in.

[2] Cf. AT. x. 486–8. The manuscript, belonging probably to Jean de Raey, which was used for a Flemish translation and for the first printed edition in *Opuscula posthuma*, is referred to by Adam as A. The manuscript which Leibniz bought from Schüller is referred to as 'the Hanover MSS.' or H. Neither was in the handwriting of Descartes, despite Leibniz's statement on H. For the history of the publication of the *Regulae*, the various editions and conjectures on the text, see AT. x. 351–7, Gouhier's edition of the text (Paris, Vrin, 1930), and G. Le Roy's edition and translation (Paris, Boivin, 1933).

[3] AT. i. 144[14–19]. On 18 July of the previous year he writes to P. Gibieuf, '. . . je me réserve à vous importuner lorsque j'aurai achevé un petit traité que je commence' (ibid. 17[6–8]).

are told in Rule 2, where Descartes is discussing the use of syllogistic reasoning: 'We ourselves are glad that in earlier days we had training in the scholastic way; but now that we are released from the bond which held us to the words of the Master, and since, being of more mature age, our hand is no longer subject to the cane, &c.'[1] In Rule 4, after describing at some length the story of his study of the mathematical sciences and of how this study led him to the notion of a general science, he goes on to say: 'Conscious as I am of my inadequacy, I have however determined that in my enquiry into the nature of things I shall follow constantly such an order as will require me to begin always with what is most simple and most easy and I will never proceed farther until it seems there is nothing further to be done in the first set. That is the reason why up to the present moment I have studied, as best I could, universal Mathematics . . .'[2] The most we can prudently infer from these passages is that Descartes is no longer a very young man and that he has devoted some years, after leaving La Flèche, to mathematical studies, all of which we know in more detail from other sources. The absence of any firm indication from Descartes, or from his friends and correspondents, leaves the problem a matter of conjecture. It is possible to determine with some degree of probability limits which allow us to fix a period of years, if not a definite year. For the years 1629–50 we possess full information in Descartes's letters as to his writings, and since there is no indication that he is engaged on the *Regulae*, or anything of the kind, it is reasonably safe to assume that they were written before 1629. Amongst the large number of historians of philosophy who have discussed the date of this work, not one of any weight places the date of composition later than 1629. On the other hand, there is a strong presumption that Descartes had not sufficient leisure to compose the *Regulae* till the winter of 1628. We have his word that he found his stay in Paris from 1625 to 1628 quite unfavourable for study and writing, and in March 1629 he went

[1] AT. x. 364^{7-11}. M. de Corte in *Archives de Philosophie*, xii, ii, pp. 112–18, calls attention to certain passages of Rule 8 which may be taken to refer to the famous 'x Novembris, cum plenus forem Enthusiasmo et mirabilis Scientiae fundamenta reperirem' (cf. AT. x. 179).

[2] AT. x. 378^{26}–379^{4}.

to Amsterdam, spending a few months of the winter somewhere in the north of France. The years previous to 1625 had been spent in almost continual travel.

It is difficult to summarize all the arguments and counter-arguments which have been put forward by eminent scholars who have attempted to fix a more definite date. Millet had suggested, in his early work on the youth of Descartes, that the winter of 1628, when Descartes spent a few months in the north of France, was the moment when the *Regulae* were first drafted.[1] Adam agrees with this date and the majority of Cartesian scholars have generally accepted this view.[2] Professor Gilson argues for an earlier date but his arguments are, as he admits, far from conclusive.[3] Another set of views considers that the latest possible date is 1628 but presupposes that certain parts of the *Regulae* may have been drafted at a date earlier than 1628 and inserted, with or without modifications, in the final draft which would then have been written in the winter of 1628 at the latest.[4] This last suggestion would at least account for certain of the more obvious repetitions and inconsistencies. For the purposes of this study we may assume that the date of the composition of the *Regulae* is before 1629 and that the winter of 1628 is a plausible hypothesis which fits into the known facts of Descartes's life.

§ 3. *The Plan of the* Regulae

The *Regulae* are, as has been already pointed out, unfinished. The original plan of the author was to write a

[1] Cf. Millet, *Descartes . . . avant 1637*, pp. 154–62.

[2] Cf. AT. x. 486–8. Amongst those who agree with this date, within a few months or so, may be mentioned Hamelin, *Le Système de Descartes*, pp. 46–47; G. Cohen, *Écrivains français en Hollande dans la première moitié du XVII^e siècle* (Paris, 1920), p. 418; Gibson, *Mind*, 1898, p. 148; J. Chevalier, *Descartes* (Paris, 1921), pp. 58–59; L. Brunschvicg, *Les Étapes de la philosophie mathématique* (Paris, 1922), p. 107; Keeling, *Descartes* (1934), p. 13. Professor Bréhier in his *Histoire de la philosophie*, ii, 1, p. 47, suggests the period 1626–8 but does not develop his reasons for this theory.

[3] He suggests that the *Regulae* go back to an earlier date, which he assigns tentatively to the period when Descartes was writing the *Studium bonae mentis*, somewhere then about 1623. Cf. *Études sur le rôle de la pensée médiévale* (Paris, 1930), p. 279. Jungmann, in his *Descartes: Eine Einführung in seine Werke*, p. 231, suggests that Rules 4–7 and 13–21 (dealing with aspects of *Mathesis Universalis*) are earlier in composition than the more general Rules 1, 2, and 8–12 inclusive.

[4] Cf. the more elaborate theory of Sirven in *Les Années d'apprentissage de Descartes* (Paris, Vrin, 1928), pp. 352–8.

treatise, for his own use but presumably also with a view to publication, on 'Rules for guiding our mind in its search after truth'.[1] It was to have consisted of three books, each book containing twelve Rules, that is, thirty-six Rules in all. Twenty-one only of these Rules were written and the three last are merely bare enunciations. The first book, the first twelve Rules, are precepts which are supposed to prepare the mind 'for fixing distinctly' before it and 'scrutinizing with keenness' propositions which are 'simple' in themselves or else are directly deduced from such simple propositions. The first twelve Rules are, to use Boyce Gibson's term, a propaedeutic to facilitate the exercise of our reason. The second book deals with problems which can perfectly well be understood *qua* problems, even though we do not in fact know their solution. It is therefore necessary to set down certain rules which will aid us to disentangle the complications and reduce them to the ordinary method of deduction already explained in the first book. The third book, in Descartes's intention, was to deal with cases where the meaning of the problem itself was obscure owing to the complex nature of the data on which the mind must work in its process of reasoning.[2]

Such was the plan. To the man who will follow these rules and 'exercise himself in them a long time',[3] Descartes promises with every assurance that he will see in a short time 'that he has made greater progress than those who apply themselves to particular problems and that he has not only obtained all that others desire to know but that he has climbed to heights of knowledge beyond any level they can even hope

[1] This is the title given by Baillet, who had seen the original Latin manuscript, and who translates it as 'Règles pour la direction de l'Esprit dans la recherche de la Vérité' (*La Vie*, i. 282). He mentions the title again later (ii. 282) in identical terms, except that there is no capital letter for 'esprit'. The reading, usually adopted, of A is *Regulae ad directionem ingenii*, whereas H has *Regulae de inquirenda veritate*. Adam reproduces a note of Leibniz, who states that Clerselier had shown him a 'discours de Mons. des Cartes de la recherche de la Vérité' (AT. x. 208). Perhaps Baillet's version is correct: the combination recalls the full title of the *Discours de la Méthode*, 'pour bien conduire sa raison et chercher la vérité dans les sciences'. It may be noted that 'Inquisitio veritatis', 'Recherche de la vérité', or similar phrases occur frequently in the works of Descartes. See, amongst other passages, in the *Regulae* (AT. x. 360^{24}; 371^{3}), in the *Discours* (AT. vi. 3^{2}, 31^{20}), and in the *Principia* (AT. viii. 5^{22}).

[2] For more details on the plan of the *Regulae* see below, pp. 148–9 and p. 207.

[3] Rule 12, cf. AT. x. 430^{2-4}.

to attain'.[1] This tone of easy confidence which Rule 1 seeks to instil in the mind of the reader reflects the indubitable belief of the author that the various and differing data studied in the various sciences and branches of human knowledge were, in their essential structure and form, logically the same as those studied in mathematics. The object of Cartesian methodology is to extend the method used in the mathematical sciences to all other branches of knowledge, including, of course, metaphysics and the other philosophical sciences. There is the same easy tone of assurance in the *Recherche de la Vérité*.[2]

> But [says Descartes] in order that my grandiose scheme may not fill your minds with an astonishment so great that all credence may vanish, I warn you that my undertaking is not as difficult as might be imagined. For those branches of knowledge, which are not beyond the capacities of the human mind, are all linked by a bond so marvellous and are capable of being deduced from one another by inference so necessary that it is not essential to possess much skill or dexterity in order to discover them, provided that, beginning with those that are most simple we learn gradually to raise ourselves to the highest. That is what I shall attempt to show you in this work by a series of reasonings so clear and yet so ordinary, that everyone will be able to judge for himself that if he has not observed the same things, it is merely because he has not cast his eyes in the right direction, nor fixed his mind on the same considerations as I have, and that I do not deserve any more praise for this than does a casual passer-by who accidentally discovers under his feet some rich treasure which many have diligently sought for during a long period without any success.[3]

Apart from the divine mysteries which Descartes's faith, courtesy, and innate sense of worldly prudence urge him to leave aside, there are to be no inviolable secrets which the mind of man cannot unlock provided no passion is allowed to cloud his mind, no prejudice permitted to poison his judgement, and he does not falter before the complexity of the problem.

[1] Rule 1, cf. AT. x. 361^{23-25}.

[2] It is not possible to fix the date of this work, published posthumously in the same volume as the *Regulae*. Adam's conjecture dating the composition of the dialogue around 1641 is plausible (cf. AT. x. 531–2). E. Cassirer, in an article 'La Place de la *Recherche de la vérité par la lumière naturelle* dans l'œuvre de Descartes' (*Revue philosophique*, 1939) would place its composition later, towards the end of Descartes's life, and suggests that it was composed for Queen Christina of Sweden.

[3] Cf. AT. x. 496^{21}–497^{14}.

II

THE UNITY OF THOUGHT AND SCIENCE

§ 1. *Knowledge and Method*

NOWHERE in his writings has Descartes given a more lucid and emphatic account of his conception of philosophy than in the 'author's letter to the translator' which serves as preface to the French translation of the *Principia Philosophiae*, a preface which has been aptly described as a manifesto against the philosophy of the Schoolmen.[1] This conception presupposes explicitly as its logical basis the fundamental Cartesian principle of the unity of all science or human knowledge. 'Aristotelian scholastics held that the certainty of mathematics, deriving from its purely formal character, was unattainable in sciences having concrete existents for their data, and the method of each science should answer to, and vary with, differences in the materials it investigated.'[2] The Cartesian plan, revolutionary in its time, was to demonstrate that the different sciences were but parts of a single science, in the metaphor of the preface to the *Principia*, branches of the same tree. Putting aside the Aristotelian and Scholastic view of the specific diversity of the sciences, Descartes postulates the unity of a universal science, and this unity, he considers, would show itself 'by the continuous application of a single and identical method to all the various sciences, for the possibility and practicability of its application in common would imply that the sciences together are naught else but the very unity of human reason itself.'

The theme of the first Rule expresses this fundamental conception, formulated in the most general terms. It states: 'The ultimate aim of all our studies ought to be to guide our mind so that we may pass solid and true judgements on everything that comes before us.'[3] The apparently innocuous

[1] Cf. AT. ix (Part II), 2–15. The expression is from Gilson, *Commentaire*, p. 160.

[2] Cf. Keeling, *Descartes*, Henriette Hertz Lecture, p. 3.

[3] 'Studiorum finis esse debet ingenii directio ad solida et vera, de iis omnibus quae

terms of this formulation are made more definite and decisive in the explanatory exposition which follows where Descartes attacks the Aristotelian, and Scholastic, distinction between the arts and the sciences, based on the famous opening chapter of the *Nicomachean Ethics*, which had become the *locus classicus* of all commentaries on the subject of this distinction.[1] He begins his attack on the distinction by contrasting the two terms. To learn 'an art', such as that of agriculture, involves necessarily the acquisition of some special bodily skill, some physical *habitus* implying training, practice, and the building up of certain reflexes. It is clear that, given the normal span of human life, no single individual, however gifted, can master all the arts, or even a small proportion of the arts. Whether or not we find it strange, as George Crabbe did, that men who guide the plough should fail to guide the pen, it is obvious that hands trained to guide the plough become ill-fitted to play a piano concerto. But, Descartes maintains, it is not the same in the case of science. To learn a science in no way depends upon, or entails, any specialized training of the body, no building up of coordinated muscular or nervous reflexes. Science consists entirely in the cognitive use or exercise of the faculties of the mind; the power of knowing, the *vis cognoscens*, is absolutely distinct from, and in no way directly dependent upon, the body or anything bodily: it is a spiritual power and it retains its spiritual character, its self-identity in whatever field it is applied or whatever the knowledge in which it finally issues. However, and to whatever degree it may be conditioned in its exercise by the body, it is one and the same in its own sphere of activity and suffers no more differentiation from the objects to which it is applied than the light of the sun is affected by the differences amongst the objects on which it shines. It is to be regarded as a 'natural light' and 'intellectual

occurrunt, proferanda judicia' (AT. x. 359^{5-7}). A definition of *ingenium* is given in Rule 12: 'proprie autem ingenium appellatur, cum modo ideas in phantasia novas format, modo jam factis incumbit' (ibid. 416^{8-10}). *Ingenium* is generally translated into French, with Descartes's approval, as *esprit*. Cf. versions of the *Discours* (AT. vi. 2^{20-21}) 'Je n'ai jamais presumé que mon esprit . . .' with (ibid. 540) 'plus esse in me ingenii . . .'. There are degrees and variety of 'ingenium'; in the *Entretien avec Burman* Descartes speaks of those endowed with 'ingenium mathematicum' (AT. v. 176).

[1] Cf. *Nic. Ethics*, A. 1, 1094 a_{1-b10}.

vision', a natural power of discerning or discriminating the true from the false, and is variously described by Descartes as 'human wisdom', 'good sense', or the 'natural light of reason'.[1]

The learning of a science, then, is not analogous to the learning of an art or a craft. The knowledge of one truth, far from diminishing or destroying the power of learning other truths, facilitates and increases our ability to learn. All the sciences, indeed, are so intimately interconnected and related, so bound together that it is far easier to acquire them altogether than to pursue the study of one of them in isolation from the others. The sciences are nothing but 'universal wisdom' applied to various subjects, nothing but the illumination thrown on the various and diverse fields of the knowable by a single spiritual light.[2] Thus we must reject the distinction accepted in Thomistic philosophy between the various kinds of knowledge based on the essential diversity of knowable objects, this diversity leading by discursive reasoning to different orders of abstraction, entailing different levels of intelligibility and different degrees of certainty.[3]

There is to be, in Descartes's view, a single kind of knowledge, an identical order of abstraction, one level of intelligibility, one kind of certainty, and one single method to obtain

[1] In Rule 1 all these terms are used: *humana sapientia, bona mens, lumen rationis naturale*. On the general use of these words and their relation to the 'bon sens' of the *Discours* (which de Courcelles translated by *bona mens*) see the *Commentaire* of Gilson, pp. 81–82.

[2] Cf. AT. x. 361[11–13].

[3] Cf. the opinion of St. Thomas: 'Virtutes intellectuales sunt circa diversas materias ad invicem non ordinatas, sicut patet in diversis scientiis, et artibus, et ideo non invenitur in eis connexio quae invenitur in virtutibus moralibus', *Summa Theol.* Ia IIae, 65, 1, ad 3am. The moral virtues are connected because they all depend on Practical Wisdom which depends in turn on them. Sirven, in his *Années d'apprentissage*, pp. 163–6, calls attention to several passages in the works of Suarez, which Descartes certainly knew, closely resembling the doctrine of the *Regulae*; in the *Disputationes Metaphysicae*, Suarez concludes a discussion: 'et potuit moveri ex eo quod *omnes scientiae videntur ita inter se connexae, ut nulla sine aliis possit perfecte tradi*' (ii. 493*a*). The italics are mine. It should be noted that Suarez is here summarizing the views of one Antonius Mirandulanus who held that 'tantum esse unam scientiam', and does not accept the view. It should be noted further that the view rejected by Suarez merely argues for a single *habitus*. The Cartesian unity is more fundamental, it is the unity of mind itself. See also the remarks of P. Garin, in *Thèses cartésiennes et thèses thomistes*, Paris, 1931, who points out that the Jesuit authors were not wholly unsympathetic towards such a view, certainly less than St. Thomas.

this certainty, which will give us the unimpeachable guarantee of truth.

If then we desire to search for and discover the truth of things, we must make it the one and only object of our studies to increase the 'natural light of reason' within ourselves. Whatever we may be studying, the one and identical power of knowing, of intellectual discernment, is at work; the increase of this power, its development and perfecting, are, in a large measure, the whole object of the rules of method laid down in the *Regulae*. Descartes does not exclude the various and legitimate results, desirable and valuable in themselves, which may be obtained from the study of the sciences: the increase of the comforts of life, to which he was himself quite sensible; the alleviation of human suffering; the prolongation even of the term of mortal existence (when he was 42 years old, death could rob him, in his own words, 'of the hope of more than a hundred years'); the solution of this or that particular problem which may hold an intense intellectual pleasure; the vision of truth which may be the supreme and unalloyed pleasure possible in human experience, the calm joy of the *βίος θεωρητικός*. But all these, and other, special ends and rewards of study are, for present purposes, side issues. To set their achievement before ourselves as the end and purpose of knowledge would be to endanger the success of the main design. We must not think of them while we are studying nor must we study in order to secure them. We must try to think only of the 'natural power of reason', the 'good sense', the power of intellectual discernment, which is at work in all the sciences, in all departments of knowledge. It is this which we have to cultivate, to strengthen and to increase. 'We must, in studying, think solely of increasing the natural light of reason, not with a view to solving this or that scholastic problem, but in order that, in all the happenings of life, our intellect may show our will what alternative to select.'[1]

This counsel calls for two comments. It will be noted, first, that, according to Descartes, the same 'good sense' is the indispensable condition, not only of the discovery of truth, but also for the reasonable conduct of life, and that, in his

[1] AT. x. 361[18–21].

terminology, the identity of the natural light of reason covers not only the theoretical and speculative sphere but also the moral sphere. In Kantian language, reason is identical both in its pure and practical uses. Secondly, his view presupposes that judgement, whether in its pure or practical use, is an assent or dissent, an act of will. But in the *Regulae*, apart from the passage under comment, Descartes seems to hold a different theory of judgement, or perhaps it is more exact to say that he does not appear to have formed any definite doctrine of his own and takes, or tends to take, for granted the accepted view of the then prevalent Scholastic philosophy. In Rule 3, for instance, belief in matters of religious faith or generally on matters involving obscure topics are said to be an act 'not of the mind but of the will'.[1] Descartes would not, presumably, at this point of his philosophical development, have attributed to the will judgements in which we assent to what we know or certainly apprehend. In Rule 12 he distinguishes between the faculty of 'the intellect by which it sees and knows things' and that faculty 'by which it judges, affirming or negating'.[2] There is obviously a gap between this doctrine and that formulated in the *Meditationes* and the *Principia*, where Descartes holds that the act of judgement is the product of two distinguishable faculties of the mind. In the latter work, for instance, we are told that 'sensing, imagining and pure conceiving, are just modes of perception; but desiring, affirming, denying, doubting, are different modes of willing'.[3] In reply to Regius, who divides 'understanding into perception and judgement', Descartes asserts, in the *Notae in Programma*, 'I however saw that, over and above perception, which is required in order that we may judge, there must needs be affirmation, or negation, to constitute the form of judgement and that it is often possible for us to withhold our assent, even if we perceive a thing. I referred to the act of judging, which consists in nothing but assent,

[1] AT. x. 370^{19-22}. [2] AT. x. 420^{16-18}.

[3] 'Nam sentire, imaginari, et pure intelligere, sunt tantum modi percipiendi; ut et cupere, aversari, affirmare, negare, dubitare, sunt diversi modi volendi' (AT. viii. 17^{19-25}). Cf. also the *Meditationes*, IV (AT. vii. 58^{20-25}). On the problem in general see Gilson, *La Liberté chez Descartes* (Paris, Alcan, 1913), pp. 236–85, and two articles of Laporte in the *Rev. métaph. et morale*: 'Le Libre Arbitre et l'attention chez St Thomas' (1932, pp. 213–15) and 'La Liberté chez Descartes' (1937, pp. 124 f.).

that is in affirming or negating, not to the perception of the understanding but to the determination of the will.'[1] In dealing here with the doctrine of method, it is not necessary to enter into fuller details of the Cartesian theory of judgement; it is sufficient for the present purpose to note the intermediate theory found in the *Regulae*. A corollary should be added. The fact that Descartes, when writing the *Regulae*, had not yet formulated to himself the doctrine of judgement found in his mature works, leads, however, to other minor differences of doctrine. As we have noted, his mature doctrine maintains that the act of judgement is the product of two distinguishable faculties of the mind, or two modes of mind's being. For it involves (*a*) the passive acceptance or recognition of a content, that is, the apprehension of certain states or modifications of consciousness; and (*b*) the active assent or dissent, affirmation or negation of the ideas apprehended. The passive acceptance is attributed to understanding: the active assent or dissent to will. Strictly speaking, the latter doctrine recognizes these two faculties only, the active and the passive, as constituting the human mind. In the same principle, just quoted, he states that 'all the modes of thinking, of which we are aware in ourselves, may be referred to two general modes, of which the one is perception, or the operation of the understanding, and the other volition, or the operation of the will'.[2] Descartes, following a well-known Aristotelian doctrine, insists that 'passion' and 'action' are two aspects of the same change. Apprehension, for instance, is a passion in respect to mind, an action in respect to the source of the mental modification.[3] But this does not affect the present distinction for, in respect of the mind, apprehension

[1] AT. viii (Part II), 363^{12-20}.

[2] AT. viii. 17^{19-25}. 'Omnes modi cogitandi, quos in nobis experimur, ad duos generales referri possunt: quorum unus est perceptio, sive operatio intellectus, alius vero volitio, sive operatio voluntatis.' He sometimes uses *intellectus* to cover, in a wide sense, all ideas, perceptions, or apprehensions; sometimes he uses it, in a narrow sense, as the totality of our clear and distinct ideas. When, for instance, the will is said to be 'wider' than the intellect, *intellectus* means the totality of clear and distinct ideas. Spinoza calls attention to this ambiguity in his criticism of the Cartesian conception of free-will in the Scholium to *Ethica*, ii. 49.

[3] Cf. *Traité des passions*, xix: 'Et bien qu'au regard de notre âme, ce soit une action de vouloir quelque chose, on peut dire que c'est aussi en elle une passion d'apercevoir qu'elle veut' (AT. xi. 343^{17-20}).

is a passion and volition is an action. Certain texts in the *Regulae* would seem to indicate that the indeterminate doctrine on the nature of judgement to be found there shows itself also in a certain wavering of doctrine relative to the activity and passivity to be found in the distinctive modes of thinking. We are told, for instance, in Rule 12, that 'the power of knowing is at one time passive, at another active and resembles sometimes the seal and sometimes the wax'.[1] This view would appear, at first sight, to be at variance with Descartes's later conviction that mind is purely passive in knowing, governed by its objects, whereas will alone is the active principle. We must remember, however, that the antithesis between activity and passivity is relative, as the text from the *Traité des passions* indicates, and further that, in Rule 12, Descartes is discussing not the conative aspect of mind but the cognitive aspect, and therein distinguishing the power of knowing in its various functions relative to perception, memory, and imagination. As will be seen in the course of this study, it is relative to these functions that the terms active and passive can be used.[2]

Two points emerge from the first Rule, both of which are suppositions which govern the whole doctrine of method and, it must be added, on both these points there is little information in the text of the *Regulae*. The first point to be noted is Descartes's insistence on the unity of thought and science. 'All the sciences united are nothing but the human understanding.'[3] In all true knowledge the activities of the mind are one and the same. This insistence on the singleness of the *vis cognoscens*, the power of knowing, is worded in such a way as to imply that all the data within a given science, as

[1] AT. x. 415^{23-25}.

[2] In a letter to Regius, Descartes points out: 'qui vero putant perceptionem dicendam esse actionem, videntur sumere nomen actionis pro omni reali potentia, et passionem pro sola negatione potentiae: ut enim perceptionem putant esse actionem, ita etiam haud dubie dicerent in corpore duro receptionem motus, vel vim per quam admittit motus aliorum corporum, esse actionem; quod recte dici non potest, quia passio isti actioni correlativa esset in movente, et actio in moto' (AT. iii. 455^{4-12}).

[3] AT. x. 360^{7-8}. K. Jaspers, in *Descartes und die Philosophie* (Berlin, 1948), says: 'Wenn also alle Erkenntnis dasselbe ist, dann gibt es nur *eine* Methode, die Universale Methode; dann muss *eine* Wissenschaft alles Wissen tragen. Es ist mathesis universalis' (p. 42). This last sentence may lead to misunderstanding. See below, p. 284.

all the domains of the different sciences, are 'single' or 'one' because, from the careful examination of any item of true knowledge, we can by abstraction discover the universal nature of the mind which constructed it. Hence the *vis cognoscens* can be said to be 'one' or 'single', but hence also the task of discovering the nature and limits of this power of knowing becomes one with the task of discovering the method of guiding our mind so that we may pass 'solid and true judgements' on all matters which may come before our consciousness. The importance of this supposition for the general doctrine of method is manifest; it contains within itself the germ of the whole Cartesian revolution. Science is one because it consists in one single law of evidence, because it is determined by the human mind in which the 'natural light of reason' is always, and in all circumstances, self-identical, and because there is no difference of kinds of knowledge from the point of view of intelligibility. The method itself is not a mere means for elaborating a system of co-ordinated knowledge. On the contrary, method 'expresses the innermost essence of mind and the problem of method is therefore identical with the problem as to the nature and limits of knowledge. Since in the method we have a complete analysis of the mind, in determining that method we necessarily also determine the measure and scope of mind.'[1] If science is one and its unity is the unity of mind itself, then method is itself the mind at work, the unravelling of the very processes of thought. It is natural that Hegel, a stern critic of Cartesian philosophy, should praise Descartes as the philosopher who 'began the whole subject entirely from its very beginning and constituted afresh the foundations on which philosophy rests and to which, after a hundred years, it is forced to return'.[2] When Descartes argues in a later Rule that we can know nothing until we know what is mind itself, since the knowledge of all things depends on our knowledge of mind itself, and knowledge of mind itself does not depend on knowledge of things, he is laying down the principle of all the idealist logics which are to follow. And, like all idealist theorists, he must necessarily argue that the proof of this

[1] N. Kemp Smith, *Studies in Cartesian Philosophy*, pp. 23–24.
[2] *Vorlesungen über die Geschichte der Philosophie*, II. i.

primordial supposition cannot lie in any particular argument or partial inquiry but that its truth, or falsity, must be found in the whole system of his thought, in the method itself. This is a fair demand to make.

In making his distinction between the sciences and the arts, Descartes explicitly posits a distinction between mind and body, the former depending upon the cognitive exercise of the mind, the latter upon the exercise and disposition of the body. No explanation is given of what mind is, what the body is, and how body is informed by mind or how they act or interact upon each other. There is here a second supposition which we are asked to accept. In Rule 12 he gives a sketchy account of his theory of the relations of the knowing subject, or the mind, and the body. He makes it clear that the account is not at all detailed, for want of space, so he says, and does not attempt to hide from the reader the hypothetical nature of his theory. He writes with unwonted naïvety: 'do not believe that the facts are so, unless you want to. But what prevents us following these suppositions, if it appears that they do no harm to the truth but only make it more clear?'[1] He is content to put his theory forward, not as a proved doctrine but merely as an hypothesis which, if the reader provisionally accepts it, will facilitate his understanding of the *Regulae*. We recognize the hypothesis as the doctrine elaborated in the later works, notably in the *Meditationes*.

We are asked to accept the well-known distinction between thought and extension, between mind and matter, between soul and body. But having asserted this distinction, involving

[1] AT. x. 412$^{6-10}$. It would be beyond the scope of this present study to describe the Cartesian theory of interaction, which is both elaborate and, in a large measure, original. The general doctrine is scattered throughout his works, but it would be easy to reconstitute his views. Cf. the *Meditationes*, VI (AT. ix. 59), *Principia*, iv. 197 (AT. viii. 321); *Traité des passions*, i. §§ 12 and 13 (AT. xi. 336–9) for some characteristic passages. But above all it is important to refer, as Descartes himself does, to the *Dioptrique* and the *Météores*: for the problem of vision and perception in general, to the former, especially *Discours*, I, II, IV, and V. In the *Dioptrique* Descartes uses the reflection of light as the *analogatum princeps* for his whole theory of nervous action. The diagrams clearly indicate that, on his view, the nervous actions produced by the object of vision are to be considered as no more than extensions into the brain of the lines of the light-rays falling upon the optic nerve or retina. This analogy can clearly then be transferred, along with the pineal gland theory, to all sensations. He is attempting a 'geometrical' analysis of a physiological problem.

complete independence, of body and mind, the Cartesian doctrine is equally firm in asserting their intimate union in man's nature. I am not merely lodged in the body, according to the Platonic metaphor, 'like a pilot in a ship', but I form, as it were, 'a single whole with it'.[1] Descartes affirms categorically in a letter to Regius that the soul or mind is united to the body 'really and substantially and not merely by its place and disposition'.[2] We are told in the *Meditationes* that the sensations of 'hunger, thirst, pain, are nothing but confused modes of self-consciousness which arise from the union, and, so to say, the blending, of mind and body'.[3] In a similar manner Descartes emphasizes the distinction between a purely mental or intellectual joy and the sense of animal pleasure which is a confused consciousness involving the co-operation of the bodily mechanism:[4] generally, in his treatment of the emotions, he insists on the importance of the 'animal spirits' as causing them and finally determining their variety.[5] It would be possible, but otiose, to multiply the number of references which show that Descartes's doctrine involves not only a distinction between soul and body but also their co-operation and mutual interaction. It was left to Malebranche to put forward explicitly the doctrine of occasionalism.

Descartes's account of the physiological mechanism involved in sensation, memory, imagination, and voluntary movement is highly ingenious; his psycho-physical doctrine of the emotions is interesting and important for a history of psychology. But, and this remark is almost a commonplace in Cartesian studies, Descartes nowhere offers, or seriously attempts to offer, an explanation of how mind acts on body or body acts on mind. The union of soul and body itself is 'a primitive notion', like existence, number, duration, or extension and thought.[6] It is a fact; it is 'no reasoning or analogy which shows us why the mind, which is immaterial, can set the body in motion, but the most simple and evident experience', an experience which we cannot refuse to

[1] *Meditationes*, VI (AT. vii. 81^{4-5}).
[2] Cf. Letter to Regius (AT. iii. 492).
[3] *Meditationes*, VI (AT. vii. 81^{11-14}).
[4] *Principia*, iv, § 190 (AT. viii (Part I), 316–18).
[5] Cf. *Traité des passions*, i, §§ 27, 37 (AT. xi. 349 and 357).
[6] Cf. Letter to Princess Elizabeth (AT. iii. 664–5).

accept.[1] For our purpose in this study we must note that in treating of the relation of soul and body, by 'soul' he means mind, that is, self-consciousness,[2] and that he assigns to body, that is, to extension and matter, or the purely mechanical conditions which on his theory determine the phenomena of the extended or corporeal world, everything short of self-consciousness. Thus all phenomena are bodily phenomena and explicable on purely mechanical principles except those which express a rational soul or mind, or self-consciousness in the sense of attentive awareness of experiences *qua* experiences of a self. Hence the famous, one might almost say notorious, Cartesian doctrine that animals are automata, highly complex machines. Since animals have no rational souls, no self-consciousness, no mind, they have, strictly speaking, no consciousness at all; their cries and their movements are merely the mechanical reactions by which the disposition of their organs, which is in fact what constitutes the animal *qua* animal, responds to various stimuli. But this 'deadly and murderous opinion', as the gentle Henry More described it, does not apply only to brute animals; the human body too is a highly complex machine.[3] Much that a modern psychologist would consider, or should rightly consider, as part of the subject-matter of his science is ascribed by Descartes without hesitation to the mechanical laws of motion and extension pure and simple. Not only the beating of the heart, the digestive and reproductive functions, breathing in sleep, but also walking and singing in a waking subject, provided the actions take place without his attention being directed upon them, are enumerated and classed as 'motions in us which are not at all dependent on the mind'.[4]

[1] Cf. Letter to Arnauld (AT. v. 222 5–18) and the interview with Burmann: 'Hoc explicatu difficillimum: sed sufficit hic experientia, quae adeo clara est, ut negari nullo modo potest' (AT. v. 163).

[2] Cf. *Meditationes*, II (AT. vii. 26–27). In the *Synopsis* Descartes changes the expression 'animae immortalitate' to 'mentis immortalitas'. The Cartesian assumption might be expressed in the words of Bergson: 'This book affirms the reality of spirit and the reality of matter. . . . It is, then, frankly dualistic.' (Introduction to *Matter and Memory* (English trans. of Paul and Palmer), p. vii. I do not propose in this present study to discuss 'the myth of Descartes'.

[3] Cf. *Meditationes*, VI (AT. vii. 84–85); *Principia*, iv, § 203 (AT. viii (Part I), 325–7); *Traité des passions*, i, § 6 (AT. xi. 330–1).

[4] Cf. *Resp. ad IV Obj.*, 'Plurimi vero ex motibus qui in nobis fiunt, nullo pacto a mente dependent.'

The difference between the Cartesian view and more modern views is not merely a matter of terminology. It must be remembered that everything which the Cartesian doctrine assigns to the 'body' is to be thought of as explicable completely and exhaustively on mechanistic principles; mind, on the other hand, together with all mental phenomena, appears in the theory in total severance from the rest of sense-experience. Mind is always, so to speak, the full-blown self-conscious independent entity. There is consequently no room for psychical dispositions, modes of half-consciousness, even as mere psychological hypotheses or devices of explanation. The human body is 'nothing but a statue or machine made of earth'.[1]

§ 2. *Perception and Imagination*

A consequence of the radical distinction between body and soul, outlined briefly in the preceding section, is that a peculiar turn is given to the problem of the relation of the four faculties, to use this word in a wide meaning, which cover the operations of the knowing subject, that is, sense, imagination, memory, and intellect. Of these intellect alone is capable of grasping truth: the other three play a merely subsidiary role in our knowing. According to the *Regulae*, and the doctrine does not vary in the later works, that in us which knows in the strict sense of the word (*per quam res proprie cognoscimus*) is 'a spiritual power' and accordingly is in no way a bodily act or function or dependent essentially on bodily conditions. It may 'apply itself' to sense, imagination, or memory; it may co-operate with them, use them and their products; it may 'attend to' sensations, images, memories. But whatever element of knowing there is in sense-perception must be attributed to the intellect alone, and the same is true in imagining and remembering. When the activity of mind is 'pure', when it acts alone, it can be said to be a spiritual power and we can be said to know (*intelligere*) and the mind is then called the intellect (*intellectus*).

[1] *Traité de l'homme* (AT. xi. 120⁴–5). See also pp. 201–2 in the same treatise which uses the simile of a watch, 'les mouvements d'une horloge', to describe the functions of the body, a simile used later by Leibniz. The difference between a 'live' and a 'dead' body for Descartes is merely that of a watch which is wound up and a watch which is run down. Cf. *Traité des passions*, § 6 (AT. xi. 330).

Sense, imagination, and memory are functions then of the body. It is in fact, according to Descartes, only as such that there is anything distinctive about them, for the element of knowing which is present in them as they occur in man, a being who is composed of body and soul, is due to the single spiritual power and is one and the same in character throughout all its manifestations. In order therefore to characterize sense, imagination, and memory we must abstract this common element, the spiritual power, and direct our analysis to that which is specific and different in each of them. If we consider them in their specific characteristics, they are bodily functions and therefore, in so far as they are necessarily linked with parts of the body, they can be located in a given part of extension. In other words, this or that singular sensation is a change produced in accordance with the physical laws governing changes in extension and motion; this or that image, of memory, is likewise produced in accordance with the same physical laws.

Thus 'all our external senses, in so far as they are part of the body, sensate only by being acted upon'.[1] Despite the fact that we can direct our sense organs to particular things in the external world, as, for instance, when we look deliberately at a given spot on the floor, thereby manifesting activity in the sense of 'local movement', sensation in itself is a change in which we are passive. The change is a change in the sense-organ, therefore a physical change; it produces an impression, the sense-organ being passive or receptive; there is a change of shape produced by the object on the sense-organ. The twelfth Rule affirms quite starkly: 'Just as the shape which is on the surface of the wax is changed by the impression of the signet, so, literally and really so and not merely metaphorically, the external shape of the sentient body is changed by the object.'[2] Further, when the organ of sense is stimulated, or 'moved' in Descartes's terminology, the change of shape in the sentient body is communicated to some other

[1] Rule 12: 'Proprie tamen sentire per passionem tantum' (AT. x. 412^{16-17}).

[2] Rule 12: 'Neque hoc per analogiam dici putandum est; sed plane eodem modo concipiendum, figuram externam corporis sentientis realiter mutari ab objecto, sicut illa, quae est in superficie cerae, mutatur a sigillo' (ibid. 412^{19-22}). In the *Regulae* Descartes constantly uses the word 'figura'—which is here translated as 'shape'—and not the equivalent of 'image' used in his works written in French.

part of the body, that part which is named 'common sense'.[1] This transference does not imply any movement of a real entity from one part of the body to another; the principle of this movement is, in Descartes's own simile, the same as that by which the point of a pen moves when it traces characters on a piece of paper; it is simultaneously and of necessity a movement not only of the lower end of the pen, but of all the parts of the pen and the pen as a whole. The peripheral organ of sense communicates the shapes (Descartes uses the expression 'figuras vel ideas') to the 'common sense' which impresses the shape on the imagination or fantasy. The latter faculty is 'a true part of the body',[2] situated in the brain and having the bodily characteristic of extension so that shapes or images are in the strict sense alterations of shape, possessing a given determinable size. The 'common sense' forms these images as a signet or seal forms shapes in a piece of wax. And since the imagination is a genuine part of body, its several parts can assume various shapes so that it can hold in itself and retain a multiplicity of shapes. In so far then as imagination fulfils the function of retention of shapes or images, the faculty is called memory. The *Discours* summarizes the same doctrine, telling us 'what must be understood by the "common sense" in which these ideas are received, by the memory which conserves them, by the imagination which can change them in diverse ways and compose out of them new ideas'.[3] At this level, imagination and memory are common to man and beasts.

It is when Descartes passes to the problem of how the intellect functions in connexion with sense, imagination, and memory that the difficulties of this mechanistic physiology and psychology become more apparent. The problems of physiology do not concern us. It might be pleaded in

[1] The expression 'sensus communis' was derived by the Schoolmen from the Aristotelian *κοινὴ αἴσθησις*, cf. Bonitz, *Index Arist.* 20b16. Scholastic texts and definitions are to be found conveniently in Gilson's *Index Scolastico-Cartésien*, pp. 265–6.

[2] 'Hanc phantasiam esse veram partem corporis', Rule 12 (AT. x. 414^{20}). Further references in which the subject is treated in more detail are: *La Dioptrique*, *4e Discours*, 'Des sens en général' (AT. vi. 109^{10-14}); the diagrams in the *5e Discours* of the same work (ibid. 128–9); *Traité de l'homme* (AT. xi. 176^{26}–177^{13}) and in a letter to Mersenne of 1641 (AT. iii. 361).

[3] 5e Part. (AT. vi. 55^{19-22}).

extenuation that at the moment of composing the account summarized above from Rule 12, the author had not yet made his anatomical researches by vivisection.[1] But it is difficult to work out even a plausible psychological doctrine. We are told that the intellect, the spiritual power, which is one (*unica*), receives impressions from the 'common sense' simultaneously with the fantasy or imagination and 'applies itself' to those shapes or images which are preserved in the memory or forms new shapes in the imagination. Descartes goes on to say that 'in all these different operations, this power of knowing sometimes suffers action, sometimes acts, imitating now the wax, and now the signet'. But whereas in the case of the 'common sense' forming the shapes in imagination he had insisted that the relation between the signet and the wax was not to be taken as a mere analogy but 'literally and really', he here adds that 'this comparison must however be taken merely as an analogy, for there is nothing in corporeal things in the least like the power of knowing'.[2]

It should be remembered that in this Cartesian theory of the mind and its faculties there is much that is new but also much that is old, and the marriage makes for ill-assorted bedfellows. The new appears in the radical dichotomy of body and soul, of matter and mind, of extension and thought and in the consequent denial of the Aristotelian, and Scholastic, doctrine of the substantial union of body and soul, the soul being both vital principle of the body and conscious principle of sense-perception. The theory of innate ideas, implied by Descartes, is new and runs directly counter to the Scholastic adage *nihil est in intellectu nisi prius fuerit in sensu*. The key doctrine of the unity of the sciences is new, with its consequent denial of the hierarchy of kinds of knowledge and degrees of certainty based on the essential diversity of the objects of knowledge. The Cartesian psychology is partially new, partially old. Descartes does not

[1] In a letter to Mersenne in 1632 Descartes writes: 'J'anatomise maintenant les têtes de divers animaux pour expliquer en quoi consiste l'imagination, la mémoire etc.' (AT. i. 263).

[2] 'In quibus omnibus haec vis cognoscens interdum patitur, interdum agit, et modo sigillum, modo ceram imitatur; quod tamen per analogiam tantum hic est sumendum, neque enim in rebus corporeis aliquid huic simile invenitur' (AT. x. 415[22–27]).

incorporate in his own view the complicated and subtle *psychologia rationalis* of the Thomists. He does not admit the unconscious function of the *intellectus agens* which uses the *phantasmata* of the imagination to generate the *species intelligibilis expressa* or *verbum mentale* in the *intellectus possibilis*. He rejects the Scholastic doctrine of abstraction, as he rejects in metaphysics the doctrine of matter and form, and of act and potency, whose advocates he describes in the *Regulae* as trying to tie a knot in a bulrush. But he does not offer a detailed account of his own views on abstraction and this gap remains a serious deficiency. Yet, despite these wholesale rejections, Descartes seems to have kept, perhaps unconsciously or unwillingly, much of the prevailing psychological doctrines which he learned at La Flèche. In this instance he seems to have partially adopted, and partially rejected, the doctrine of *phantasmata*, intermediaries between the perceiving mind and the material or perceptible object, and he does not seem to have integrated them into his theory.[1] In the Scholastic doctrine, the *phantasmata* are neither material nor intelligible in their nature; they may be said to partake of the dual nature of the substantial unity which is man.[2] Descartes, however, considers that they are genuinely and purely corporeal in nature, while not necessarily linked to any particular corporeal substance. They are corporeal motions and in themselves not cognitive. This consequence of the dichotomous distinction between body and soul, assumed in the

[1] 'They [*φαντάσματα, species intentionales*] survive, I think, beyond question in the *Regulae*'. Joachim, *Logical Studies*, p. 116, n. 1. Strictly speaking there are two *species*, one *impressa*, and the other *expressa*. The former is 'elaborata ab intellectu agente; recepta ab intellectu possibili'; the latter is formed by the *intellectus possibilis* from the species which it gets from the *intellectus agens*. Gassendi defended the Scholastic doctrine and based his whole theory of perception on the existence of these *species*; cf. *Opera* (Lyons, 1658), i. 441–9. Perhaps he identified them in some way with the εἴδωλα of Democritus. In the *Dioptrique*, Descartes speaks contemptuously of 'toutes ces petites images voltigeantes par l'air, nommés des *espèces intentionnelles*, qui travaillent tant l'imagination des Philosophes' (AT. vi. 85^{25-27}). See also the *Resp. ad VI Obj.* (AT. vii. 437^{11-19}).

[2] 'Les phantasmata ne sont ni matériels ni intelligibles' (Gilson, *Études sur le rôle de la pensée médiévale* (Paris, 1930), p. 22). In his later works Descartes inclines to a similar view. He writes, for instance, to Mersenne, in 1641: 'C'est en un autre sens que j'enferme les imaginations en la définition de la pensée, et en un autre que je les en exclus, à savoir: *Formae sive species quae esse debent in cerebro ut quid imaginemur, non sunt cogitationes, sed operatio mentis imaginantis sive ad istas species se convertentis, est cogitatio*' (AT. 361^{9-15}).

Regulae, has an important effect on his theory of method. The 'power of knowing' does not strictly include all forms of what are generally considered to be cognitive activities of the soul. In so far as it is purely 'spiritual', it excludes sensating, imagining, and remembering; in so far as it 'knows' or 'understands', it is said to act alone. The corporeal motions involved in sense-perception, imagining, and remembering can only be said to be cognitive in so far as they are accompanied, or completed, by an act of intellectual apprehension. In the words of Rule 12, the *vis cognoscens* 'is one and the same which, when together with imagination it applies itself to the common sense, is said to see, to touch, etc.; when it applies itself to the imagination alone, in so far as it possesses diverse impressions, is said to remember; if it turns to imagination in order to create new impressions, is said to imagine or conceive; and finally, if it acts alone, is said to understand'.[1] There is thus, from the very outset of the method, a division in the kinds of cognitive experience; the one being 'pure', reflective, and internal, the other, mixed, sensitive, and, as involving corporeal motions, external. This division is again noted in Rule 12 where Descartes says: 'We experience whatever we perceive by sense, whatever we hear from others, and generally whatever comes to our understanding, either from outside or from its reflection upon itself.'[2] The doctrine of method, especially in the early rules of the *Regulae*, is mainly concerned with the 'pure' use of the *vis cognoscens*, in which alone Rule 8 places 'truth or falsity'.[3] This emphasis on the 'pure' use of mind may lead, as a matter of exposition, to a neglect on the part of Descartes, equally reflected in any commentator, of the role played by, for instance, the imagination in the mathematical sciences as elaborated in Rule 14, or the role of sense-perception in the physical sciences.[4]

[1] AT. x. 415^{27}–416^{4}. The same point is made in the *Notae in Programma*: 'Postea, inter species perceptionis, non enumerat, nisi *sensum*, *reminiscentiam*, et *imaginationem*: unde colligi potest, eum nullam intellectionem puram, hoc est, intellectionem quae circa nullas imagines corporis versetur, admittere' (AT. viii (Part II), 363^{20-24}).

[2] 'Experimur quidquid sensu percipimus, quid ex aliis audimus, et generaliter quaecumque ad intellectum nostrum, vel aliunde perveniunt, vel ex sui ipsius contemplatione reflexa' (AT. x. 422^{25}–423^{1}). This recalls Locke's 'Ideas of Reflexion'.

[3] 'Veritatem proprie vel falsitatem non nisi in solo intellectu esse posse' (AT. x. 396^{3-4}).

[4] On the subsidiary use of the imagination, see below, pp. 217 ff. and 224, and on the role of experience generally, pp. 239 ff.

III

THE FIELD OF KNOWLEDGE

§ 1. *Reflection*

THE opening argument of the *Regulae* progresses on the following lines. All the sciences embody one and the same 'human wisdom'; they all express one and the same human intelligence at work. As will be shown in later Rules, this intelligence, in so far as it functions in accordance with its nature, cannot fail to think properly. If then we desire to think properly, to acquire knowledge, the one essential condition must be that our thinking should be the thinking of the human intelligence functioning in accordance with its own nature. But in order to secure this essential condition, we must eliminate all that is not 'pure thought', that is, all that is dependent on bodily functions, and then reflect upon and study the nature of the pure spiritual power, the *vis cognoscens*. In so doing we shall implicitly determine the power, limits, and structure of the human mind and human knowledge. For to reflect upon the working and logical procedure of the intellect in so far as it follows its own nature in the process of thinking is to reflect upon the conditions of truth. The intellect cannot, in so far as it follows its own nature, fail to think truly, and the truth of the various branches of knowledge is simply the structure of the intellect, a structure which reflection alone can lay bare to us. As Malebranche was to sum it up, 'to reach truth, it is enough to make oneself attentive to the clear ideas which each finds in himself'.

But the method is not merely an instrument for the discovery and construction of knowledge; it is also an exposition of the nature of mind itself and an analysis by reflection of mind at work in discovering and constructing knowledge. A complete account and analysis of method will be *ipso facto* a complete account of the fundamental nature and processes of human knowledge. As all knowledge depends on the nature of this spiritual power, the *vis cognoscens*, it follows

that by a study, involving reflection, of this power and its processes, of the data with which it operates in thinking, of the method by which it acquires and uses these data, we shall arrive at certain truth and knowledge. The only essential condition of reaching 'true and solid judgements', the ideal end of the method according to Rule 1, is to be endowed with the power of reasoning. It would be an error to assume that the method itself is some esoteric species of mystic meditation, reserved only to those few initiated into the art of reflective meditation. Nothing could be more remote from the explicit intentions of the author of the *Regulae* and the *Discours*, and any such esoteric interpretation would run counter to his life-long convictions and would indeed make nonsense of much of the methodological doctrine to be discussed in later chapters of this study. One of the primary aims of the Cartesian method is a practical one; the practice of the method is meant to be productive of results in the whole field of knowledge. The success or failure of the method itself may, on critical examination, be logically bound up with the soundness of its epistemological and metaphysical suppositions but it is vitally important, in attempting to understand the general *schema* of the method, to bear in mind that the practical aim is more often than not uppermost in its author's mind. Descartes does not consider the task an easy one. He is quite aware of the difficulties inherent in the analytic reflection and synthetic construction which are to be the fundamental means suggested for our use. But he clearly considers it a task which is of primary importance and not at all outside the range of our normal capacities. In Rule 8 he explains this point at some length. He insists, repeating previous declarations of a similar nature, that 'there can be no more useful inquiry than that which tries to determine the nature and extent of human knowledge'. He goes on to say that

> this inquiry should be undertaken at least once in his life by everyone who has the slightest love of truth since, in pursuing this inquiry, the true means of knowledge and the whole method come to light. Nothing seems to me to be more fatuous than the behaviour of those men who boldly argue about the secrets of nature, the influence of the heavens on things below, the prediction of future events, and such like matters,

without ever having asked themselves whether human reason is capable of solving such problems. Nor should it seem such a tiring and difficult matter to determine the limits of understanding, of which we are directly conscious, when we often do not hesitate to pass judgement on matters outside us and quite alien to us. Nor is it such a tremendous task to attempt to grasp in thought all things comprised in this whole in order to discover how they fall singly into our mental awareness.[1]

We are given an analogy in Rule 8 to illustrate his point. Suppose a man wished to learn some craft, as, for instance, that of a blacksmith, but is entirely destitute and has no tools of any kind. He would be obliged to find some large piece of stone to use as an anvil and to take another piece of stone as a hammer. Pieces of shaped wood would have to serve as tongs or pincers. With these primitive tools he could then set about making tools of a more suitable and durable kind which, once made, would allow him to fabricate the articles used by a smith. Such is the process which the method attempts to inculcate. We must first forge ourselves the tools for our work, that is, we must train the instrument of research, our human reason, before attacking any major problems of a speculative nature.

The method then is open to all men and, Descartes would have added, even to women. The *bons sens* of the *Discours*, the *bona mens* of the *Regulae* are 'the reasonings of each individual with reference to the affairs in which he is personally interested, where', says Descartes, 'I should find more truth than in the reasonings of a man of letters in his study on speculative matters of no practical import.'[2] Anticipated in this opinion by Montaigne, he firmly believed that right thinking is the perquisite and privilege of no class of scholars in particular but the common gift of all reasoning men, the only condition being the rule of evidence, the *veritas index sui*, and the knowledge and practice of a suitable method to discover and exploit properly the results of the vision of truth.[3] Provided that a man possesses the criterion

[1] AT. x. 397^{27}–398^{16}.

[2] *Discours*, Part I (AT. vi. 9^{30}–10^{4}).

[3] Cf. the advice of Montaigne to Diane de Foix in *Essais*, i, chap. xxv: 'On a grand tort de la [philosophie] peindre inaccessible, et d'un visage refrogné, sourcilleux et terrible: qui me l'a masquée de ce faulx visage, pasle et hideux? Il n'est

of evidence, clear and distinct ideas, and practises the right method there is nothing, in general, that another man can know that he 'is not himself capable of knowing, provided that he applies his mind to it in a proper way'.[1] And Eudoxe, in the *Recherche de la Vérité*, recommends strongly that our study should begin always by a study of the nature of reason itself, which is the instrument of truth itself.[2]

A note of warning is perhaps necessary here. Statements such as those quoted in the preceding paragraph, and the more famous phrases of the *Discours*—'good sense is the best distributed thing in the world', 'naturally equal in all men', &c.—may easily lead to an over-emphasis which would leave Descartes open to the criticism that he must have been palpably blind or singularly lucky in his friends or acquaintances. Or again, over-emphasis may lead to the doubtful lyricism of a recent panegyric: 'What revolutionary force there is in Descartes's appeal to the omnipotence of reason! . . . Descartes proclaims the freedom of mind, the equality of mind, the interdependence of all which is the basis of fraternity. Liberty, equality, fraternity, these are the very principles of the French Revolution; and these very principles are found for the first time in Descartes.'[3] Perhaps this is the case, but it must not be thought that Descartes, while admitting the equality of all men in theory, considered as beings endowed with reason, does not take into account, as a matter of experience and fact, the natural inequalities of mind which are patent to any observant man. This inequality is so obvious that it would be futile to labour the point here if the discussion did not throw a certain light on Descartes's terminology which has, in turn, some importance for his doctrine of

rien plus gai, plus gaillard, plus enioué, et à peu que ie ne die follastre.' (Cf. edit. Lefèvre, 818, i. 242.) On Montaigne's influence on Descartes, see Brunschvicg, *Descartes et Pascal, lecteurs de Montaigne* (Neuchâtel, 1945), pp. 95–133.

[1] AT. x. 396[16–18].

[2] Ibid. 505. It may be noted that Epistemon (the name is borrowed from Rabelais), the 'erudite scholar', is treated more harshly in this dialogue than Polyandre, the 'man of the world'.

[3] From a speech by the leader of the French Communist party, Maurice Thorez, in the Sorbonne in May 1946 (quoted by H. Lefebvre in his *Descartes* (Paris, 1947), p. 42, who approves of the *Discours* as a revolutionary manifest). I do not think that Gouhier, in his *Pensée religieuse de Descartes*, Paris, 1924, makes out a strong case for an 'aristocratic metaphysic' but if truth lies at the extremes of interpretation, there is more plausibility and evidence in Gouhier's thesis (cf. pp. 300–7).

method. As has been already noted above,[1] he often uses the word *ingenium*, translated into French as *esprit* and conveniently translated into English as 'mind', to cover the whole of conscious activities, the thinking substance, as contrasted with and opposed to the extended substance, matter.[2] Sometimes, however, he uses the term in contradistinction to the word *raison*, in which mind is wider than reason, comprehending, as the definition in the *Reply to the Second Objections* puts it, 'all the operations of will, intellect, imagination and of all the senses'.[3] It is the reason which is common to all men and the reason which is equal in all men, if considered as the faculty of judgement, the power to make judgements. But something more is needed to ensure true knowledge and science. Not only the reason needs method and practice in methodological inquiry but there is also need to train and equip the auxiliaries of reason, the senses, the imagination, and the memory. It is only by a systematic and co-ordinated training of all the various powers of man's spiritual nature that method can produce a 'good mind', which is not the same as 'good sense'. Both the *Regulae* and the rules of the *Discours* cover this field of training.

It is abundantly clear also from the writings of Descartes that he did not consider that all minds were equally gifted and fit to undertake all types of study. Writing to Mersenne in 1638, he attacks a 'sieur Petit' who had written three notebooks of objections against the fourth part of the *Discours* and would have been better employed, according to Descartes, in questions of physics, 'rather than in problems of ethics and metaphysics, which are not at all his business, and the truth of which problems can only be grasped by very few people, although everyone likes to take it upon himself to judge of these matters and the most ignorant people are capable of talking much on these topics, talk which has some

[1] Cf. above, p. 14, n. 3.

[2] Cf. Gilson, *Commentaire*, pp. 86–87, who also quotes two texts from Clauberg's *Defensio Cartesiana*: 'Longe vero aliud est judicandi facultatem in omnibus aequalem pronunciare, aliud aequali virtutum et recti de rebus judicii mensura cunctos donare' and 'judicii aequalitatem Cartes tribuerat omnibus, non animarum'. It is interesting to note that both Descartes and, later, Clerselier, the translator of the Objections, are aware of a possible ambiguity in the terms *mens*, *anima*, *esprit*. Cf. the Latin in AT. vii. 161 and the French, AT. ix. 125, of def. VI.

[3] Def. I of *Resp. ad II Obj.* (AT. vii. 160[8–10]).

appearance of truth to those who do not examine it closely'.[1] When asked his opinion of the *De Veritate* of Herbert of Cherbury, he writes later in the same year: 'I find several good things in it *sed non publici saporis*; for there are few people capable of understanding metaphysics.'[2] He points out in another letter that 'without more intelligence than the common run of people, one cannot hope to accomplish anything very extraordinary in regard to the human sciences',[3] and in the discussion with Burman he makes several times the point that science, particularly the mathematical branches, require a special bent of mind which has also need of training and exercise.[4] It is in the mathematical sciences particularly that such training is valuable and there too that the analysis of the mind at work is easier to observe and reflect upon.

§ 2. *The Ideal of the Sciences and Mathematics*

There is a passage in the *Discours* which cannot fail to strike the reader as a characteristic summary of the novel approach to philosophical method disclosed in an almost casual biographical tone. The passage occurs in the second part, immediately following upon the formulation of the four rules of method.

> These long chains of reasoning [Descartes writes], all simple and easy, which geometers are wont to use to reach the conclusions of their most difficult demonstrations, had led me to imagine that all those things which fall within the cognizance of man are mutually related in the same way, and that, provided we refuse to accept anything as true which is not so and always preserve the order necessary for the deduction of one truth from another, there is nothing so far removed from us that we cannot reach to it or so recondite that we cannot discover it. And I had little trouble in discovering which objects it was necessary to begin with, for I knew already that it must be with the simplest and those most easy to know. When I considered also that of all those who have hitherto sought truth in the sciences, it has been the

[1] AT. ii. 145^{3-9}. [2] Ibid. 596^{20-23}. [3] Ibid. 347^{28-30}.

[4] Cf. *Entretien avec Burman*: 'Omnes autem homines ad eam apti non sunt, sed requiritur ad id ingenium mathematicum, quodque usu poliri debet' (AT. v. 176). Some, too, are 'in Mathesi ingeniosi qui tamen in physicis rebus *et similibus* infeliciores sunt' (ibid., p. 177). Firmat is judged as a man who 'sait des Mathématiques, mais en Philosophie, j'ai toujours remarqué qu'il raisonnait mal'. Letter to Mersenne, 1641 (ibid. 328^{13-14}).

mathematicians alone who have been able to find any demonstrations, that is, any certain and evident reasons, I did not doubt that they had carried out their inquiry by similar means. . . . But for all that, I had no intention of attempting to master all the particular sciences which are commonly called mathematics. I observed that, although their objects are different, they all agree in considering only the various relations or proportions which exist amongst those objects. I thought that it would be better if I considered these proportions only in their most general form, without referring them to the objects otherwise than in so far as these would aid the knowledge of their proportions.[1]

From this programme, set by Descartes for himself and his readers, the new science of *Mathesis Universalis* is to be evolved. It is clear that from the outset of his intellectual life, Descartes had been struck, as he tells us himself, in his study of mathematics, 'by the certainty of its proofs and the evidence of its reasonings' and genuinely surprised that nothing more substantial had been erected on foundations 'so solid and so firm'. The interest awakened at La Flèche was given new impulse by the friendship of Beeckman.[2] Descartes was fully aware of the latest developments of the comparatively new science of algebra and the works of mathematicians such as Peletier, Oughtred, Vieta, and Clavius.[3] As early as 1619 he considered himself on the way to formulating a new science which would revolutionize mathematics, that is, the application of the algebraic method to geometry.[4] But the *Discours* is suggesting a much wider application, a much more decisive step forward in the unification of the sciences. For the idea of the unity of all the sciences is now linked to

[1] AT. vi. 19⁶–20⁷. In the time of Descartes, as in the medieval universities, *Mathematics* was a label which covered a range of subjects, including arithmetic, geometry, mechanics, optics, hydraulics, and the science of harmony or music. Thus Descartes's *Compendium Musicae*, written in 1618, is a mathematical treatise.

[2] The fragments of Beeckman's *Journal* are to be found in AT. x. 46–78.

[3] Clavius, called the 'Euclid of his century', was a Jesuit. His *Algèbre* was published in 1609 and his commentary on Euclid two years later. Adam (AT. xii. 23) thinks Descartes may have read these works while at college. He must certainly have become acquainted with them by 1619 since he uses the 'signes cossiques' at that date. Descartes's knowledge of Vieta's work is more controversial, cf. AT. xii. 212 ff.

[4] Cf. the letter to Beeckman in 1619: 'Et certe, ut tibi nude aperiam quid moliar, non Lulii *Artem brevem*, sed scientiam novam tradere cupio, qua generaliter solvi possint quaestiones omnes, quae in quolibet genere quantitatis, tam continuae quam discretae, possunt proponi' (AT. x. 156⁷–157³). Beeckman wrote in the margin of this letter: 'Ars generalis ad omnes quaestiones solvendas quaesita.'

the extension of the method used in mathematical thinking to the whole field of knowledge. As a recent writer on Kant has pointed out when discussing the Cartesian contribution to the history of human thought, 'if knowledge of the real must be in the end mathematical, it follows that the real, in so far as it is knowable, must be susceptible of mathematical treatment'.[1]

The second Rule in the *Regulae* makes only a general point. It states: 'We ought to study exclusively subjects which our minds seem competent to know with a certainty which is beyond doubt.'[2] We have then first to analyse a body of knowledge which is demonstratively certain and find out how and why this certainty arises and how we can come to possess it and demonstrate it to others.

When, however, we read the exposition which enlarges upon this Rule, it becomes quite clear that, in Descartes's view, there are few certainly knowable objects of study and that, in fact, of the sciences existing in his day, 'arithmetic and geometry alone are free from any taint of falsity or uncertainty'.[3] We are therefore led to the conclusion 'not indeed that arithmetic and geometry are the only sciences to be studied, but only that in our inquiry for the right way to attain truth, we should occupy ourselves with no object about which we cannot arrive at a certainty equal at least to that reached in the demonstrations of arithmetic and geometry'.[4] The reasons for this selection are twofold. The first is the absence, as compared with the other non-mathematical sciences, of what is called vaguely the taint of falsity or uncertainty. This characteristic of certainty and infallibility is linked with that of self-evidence, the *veritas index sui*, arising out of the nature of the subject-matter itself. We are not asking a question as to the nature of truth itself. 'As to what Truth is', Descartes says in a letter to Mersenne, 'I

[1] T. D. Weldon, *Introduction to Kant's Critique of Pure Reason* (1946), p. 12. Gilson, in his *Commentaire*, p. 214, remarks on the passage of the *Discours* quoted in this paragraph that it is a 'formule très discrète mais qui marque en réalité le pas décisif accompli par la pensée cartésienne: tout ce qui est susceptible de connaissance vraie est, par définition, susceptible de connaissance mathématique'.

[2] 'Circa illa tantum objecta oportet versari, ad quorum certam et indubitam cognitionem nostra ingenia videntur sufficere' (AT. x. 362^{2-4}).

[3] Ibid. 363^{18-19}.

[4] Ibid. 364^{5-9}.

have never had any doubt at all, conceiving it to be a notion so supremely clear that it is quite impossible to be ignorant of it. There are indeed a number of ways of testing a balance before using it but we should find no way of learning what truth is, were it not naturally known to us.'[1] We know what truth is in knowing what the true is. And, moreover, as the *Discours* points out, 'there is one, and only one, truth about any matter, so that if a man discovers this truth, he can be certain that he knows all that can be known about the subject. . . . Hence a child, who has done an addition sum according to the rules of arithmetic, can be assured that he has discovered, so far as concerns the sum he is studying, all that the mind of man could possibly discover.'[2]

The second reason for this exclusiveness lies in the contention that in the mathematical sciences, in geometry for instance, we are accustomed to the successful solutions of highly complex problems and to clearly defined demonstrations of conclusions. This success is achieved in the complex problems by the use of long chains of reasonings, each single set of reasoning being itself simple and easy to follow. Each step in the demonstration is, as it were, as simple, straightforward, and evident as the addition done by the child, and the steps are so ordered that one is led, easily and inevitably, from one to another. At the outset the particular problem in which we are interested may appear complex and obscure and its solution may appear remote. But by taking short and simple steps, every one of which is absolutely safe, and by keeping to the inevitable logical sequence of the order, the geometer succeeds in the end in achieving a demonstration which is absolutely lucid, convincing, and, in retrospect at least, easy.

§ 3. *The Certainty of Mathematics*

The student of Descartes who has even a casual knowledge of the more widely read works, the *Discours de la Méthode*, the *Meditationes*, and the *Principia Philosophiae*, might justifiably be puzzled by the difference of value attached to the certainty of mathematical truths in the *Regulae*, as summarized

[1] Letter of 1639 (AT. ii. 597^{2-6}).

[2] AT. vi. 21^{9-13}. The Latin text varies slightly from the French and runs: 'puer qui primas tantum Arithmeticae regulas in ludo didicit'.

in the preceding paragraph, and that given, for instance, in the *Meditationes*. The upshot of the first Meditation, which deals with 'things as to which we may doubt', is that there is nothing in our experience absolutely free from doubt until we arrive at the fundamental intuition of the self-conscious self. No other truth is so beyond suspicion that it can serve as a true and unshakeable foundation for metaphysical thinking, that is, as a self-evident datum on which the edifice of certain knowledge can be securely built.

In the *Regulae*, mathematical propositions are exempt from any suspicion of doubt and $2+2 = 4$ is quoted as an instance of a self-evident truth. In the *Recherche de la Vérité* mathematical truths are not subjected to examination and their truth is taken to be evident and accordingly beyond doubt. But in the first Meditation even the truths of arithmetic are shown to be open to doubt. The doubt itself may be, as Descartes says, 'very slight': it may only be 'a metaphysical doubt which leads me to question that $2+2 = 4$': and 'no sane person ever seriously doubted that there is a world, that men have bodies, and such like'.[1] Still, there is possibility of doubt. It is just possible that there is an all-powerful malignant demon whose purpose is to deceive, and this bare possibility is sufficient to throw doubt on all our opinions and beliefs, a doubt which is sufficient to prevent us from accepting them as a sound basis on which to construct a body of certain and true knowledge. There is not nevertheless any major inconsistency or fundamental discrepancy between the doctrine of the *Regulae*, or the *Recherche de la Vérité*, and the doctrine of the *Meditationes*, or the *Discours*. In the latter works Descartes is concerned with the principle of methodical doubt: *de omnibus dubitandum*. In the *Discours* the doubt itself does not turn on the actual intuition of the mathematical datum or on the subsequent demonstration but on our memory of the chains of reasoning which would otherwise have seemed completely demonstrative. It is in the *Meditationes* alone that the hypothesis of the 'malignant demon' is used to make it possible to doubt the very intuitions on which all mathematical reasoning is and must be based. Since he

[1] Cf. *Meditationes*, I (AT. vii. 36^{24-26}), *Discours*, IV (AT. vi. 32^{3-9}), *Principia*, i. 5 (AT. viii. 6^{8-24}), and the text quoted from the *Synopsis* (AT. vii. 16^{1-3}).

cannot find a reason, Descartes must make a reason for doubting.

The problem which the *Regulae* aims to solve is that of stating what rules we must observe if we are to recognize the certain knowledge we possess, and having recognized it without any danger of self-deception, how we may develop certain knowledge from it. The rules which we are searching for, that is, the method, are found by a reflection upon the human mind in its natural and unperverted state, by a study of its procedure when it functions 'purely' and when its natural power of grasping truth is not corrupted and hampered by prejudices. In the *Meditationes*, Descartes is at work constructing a system of knowledge which shall be absolutely certain. It is in accordance with the doctrine of the *Regulae* that he sets out in the *Meditationes* to find an indubitable self-evident datum which will serve as the foundation of the truths to be demonstrated. And again, in accordance with the *Regulae*, he insists upon a thorough examination of all our experience, of all the opinions which we may have hitherto held. These opinions must be thoroughly tested and every one of them in respect to which there is the least ground of doubt must be rejected as provisionally false. The main object of this universal doubt, the sceptical review, is to purge the mind of the prejudices which pervert its natural capacity of discerning the truth or, in other words, to restore our mind to its normal or unprejudiced state in which it cannot fail to recognize the experience of self-evident truth if, and when, it has this experience.[1]

But though, for methodological purposes, on the hypothesis of the 'malignant demon' it is possible to have a metaphysical doubt of the certainty of mathematical demonstrations and the data on which these reasonings are based, yet these data and demonstrations are not in themselves necessarily false. Whatever can be doubted is not necessarily false. As a question of method, the *Principia* states, 'it will be useful to reject as false all those things of whose existence we can imagine the least doubt, so that we may discover with all the more clearness which ones are absolutely true and most easy to know.'[2] Because we can entertain what was called in the

[1] See below, p. 293.

[2] *Principia*, i. 2 (AT. viii. 5[1–24]).

Meditationes 'a slight doubt', we are thereby justified in rejecting mathematics as dubious and possibly false, by a methodical doubt based on an hypothesis external to mathematics itself. At the level of the discussion in the *Regulae*, we are not concerned with the metaphysical foundations of our knowledge nor with a criticism of the actual power of knowing itself. We are concerned with knowledge at the level of science. The metaphysical doubt of the *Meditationes* is not due to the nature of the object which is put before the mind nor does it correspond on our part to any real psychological experience of a cognitive nature. It is due to our volitional activity. It is by a purposive act of will that we refuse our assent to the self-evident data of mathematics and this is possible because, strictly speaking, ideas in themselves are neither true nor false. It is only in judgement, when they are affirmed or negated, that they become true or false, and judgement, on the Cartesian view, is dependent on an act of will.

The apparent divergence of views between the *Regulae* and the *Meditationes* will not lead to any confusion if it is remembered that the possibility of rejecting mathematical truths is not based in the latter work on the nature of the self-evident data of mathematics but only becomes plausible on the supposition made voluntarily, and methodically, that we might be the victims of a gigantic divine hoax in a dream world. Descartes was forced to point out to Hobbes that the reasons for doubting used in the first Meditation were not 'true', as admitted by his opponent, but merely possessed 'verisimilitude'. He goes on to say that his purpose in using such reasons for doubting was 'partly because I intended to reply to them in the following Meditations and partly in order to show the strength of the truths which I propound afterwards, since they cannot be shaken even by such metaphysical doubts'.[1] Gassendi was also misled like Hobbes, and he is told much the same: 'when we are attempting to discover what can be known by the human mind with absolute certainty, it is clearly quite unreasonable to refuse to treat some objects as doubtful, and even reject them as false, so that we may become aware that some other objects, which

[1] *Resp. ad III Obj.* (AT. vii. 171[21]–172[7]).

cannot be rejected by a similar method, are for this very reason more certain and in actual fact better known by us'.[1]

Knowledge of mathematical truths and demonstrations is not only distinct from mere opinion and belief but must also be distinguished from the appearance of 'knowing' which may be due to nothing more than our ability to memorize and recapitulate facts and propositions. One might easily, for example, remember all the theorems of Euclid, and their demonstrations, without having any geometrical knowledge whatever. We should not be geometers any more than we could say 'though we had mastered all the arguments of Plato and Aristotle but were not yet capable of passing a solid judgement on these matters, that we were philosophers; the knowledge we should have acquired would not be that of a science but of history.'[2] And we know from the *Discours* that Descartes did not attach much importance to the study of history. Knowledge is an intellectual insight into the materials presented and organized in and through memory. Memory remains a means, an essential means, to knowledge. If therefore we keep strictly to this conception of knowledge and review the then existing branches of so-called knowledge or sciences, it becomes clear on his view that none of them except the mathematical sciences merit the title of subjects 'which our minds seem competent to know with a certainty which is beyond doubt'. Woven into the texture of all the others are opinions, conjectures, dubious propositions and conclusions, and probably, or at least possibly, errors and false principles. The mathematical sciences alone then give us truth unmixed with falsity, certainty free from doubt. They alone give us a genuine field of knowledge on which we can exercise our minds.

Their superiority is due to the fact that they, and they alone amongst the sciences, deal with 'an object so pure and

[1] *Resp. ad V Obj.* (AT. vii. 351^{6-11}). In the *Disquisitio Metaphysica*, published in 1644, Gassendi addresses Descartes in these terms: 'Demonstrationum tuarum ars est velut equus trojanus, e quo illae prodeuntes nullum non praesidium quantumvis munitum expugnent.' *Opera Gassendi*, iii. 279 (Lyons, 1658).

[2] Rule 3 (AT. x. 367^{19-23}). Descartes's attitude to 'History' is disdainful. See his remarks in the first part of the *Discours* (AT. vi. 5^{19}–7^{10}). L. Lévy-Bruhl, in 'The Cartesian Spirit and History' (*Philosophy and History*, Oxford, 1936, p. 193), makes the important point that 'the method of authority which he [Descartes] was attacking, has the closest connexions with history'.

simple that they presuppose nothing which experience renders uncertain, and they consist entirely in the rational deduction of consequences'.[1] The data then are absolutely simple and precise; the mathematical sciences, arithmetic and geometry in point of fact, consist entirely in the logical development of these data by an inference from self-evident data to conclusions linked and interconnected at each stage by steps which are themselves self-evident. Accordingly these sciences presuppose nothing which depends intrinsically on experience, that is, on sense-experience, and their certainty is not therefore subject to empirical verification. The mathematical sciences must be considered, at least provisionally, as the only complete and perfect examples of 'science' or genuine knowledge and are therefore the only actual sciences which fulfil the Cartesian ideal of knowledge because in them alone is included nothing but the inevitable results of the functioning of the power of knowing, the *vis cognoscens*, in its own distinctive and natural activity. The *vis cognoscens* is, as will be seen, precisely the twofold power of apprehending the pure and simple objects, the data, and of passing from one simple object to another through a chain of inferences.

The conclusion of Rule 2, already quoted above,[2] appears obvious in the circumstances. We are not asked to restrict our studies to arithmetic and geometry but, in our search for a method which will ensure certain knowledge, we must direct our attention only to those matters where we can be sure to obtain a certainty equal at least to the certainty obtained in mathematical studies. The field of knowledge excluded by this imperative condition is clear: all the sciences or pseudo-sciences based on mere opinion, conjecture, or, of course, traditional prejudice, and, further, all knowledge so-called, however systematic in appearance, which is based in fact on memory and due to our capacity to recapitulate or remember, even should this knowledge be concerned with objects pertaining to the mathematical sciences. There is an interesting letter of Descartes in which he sets down for Hogelande his views on this point. He writes that he

> is accustomed to distinguish two things in mathematics: the history of this science and the science itself. By history, I understand all that has

[1] Rule 2 (AT. x. 365^{17-21}).

[2] Cf. above, p. 38.

already been discovered and which can be found in books. But by the science itself, I understand the skill to resolve all problems and further to discover by one's own means everything that can be found out by the human mind; when one has this skill, one is not greatly interested in the thought of others and one has really the right to be called *αὐτάρκης*. . . . But no one will ever show himself to be an *αὐτάρκης* mathematician unless his mind has a great natural aptitude for the subject and, in addition, this has been cultivated by long exercise.[1]

If we are not asked to restrict our studies only to arithmetic and geometry, we may well ask what, if anything, is this other science or these other sciences which have a certainty at least equal to the certainty obtained in the mathematical sciences. No indication is given in Rule 2 itself but in Rule 4 Descartes states that he was far from satisfied with the writers and books on mathematical subjects which he had read and studied. He expresses himself in somewhat surprising terms:

> nothing could be more foolish in fact than to busy oneself with bare numbers and imaginary figures in such a way as to appear contented with such trifles and so to use these superficial demonstrations, which are discovered more often by chance than by skill and are a matter more of the eyes and the imagination than of the intellect, that in a way one ceases to make use of one's own reason.[2]

These remarks had been prefaced by the observation that 'I should not think much of these rules [of the *Regulae*] if they had no use save for the solution of the inane problems with which the calculators and geometers are accustomed to pass their hours of leisure.'[3] But this apparent rejection of ordinary arithmetic and geometry, so scornfully expressed in Rule 4, does not imply any denial or cancellation of the views expounded in Rule 2. The precept of this rule remains valid: the study of arithmetic and geometry has definite value as a propaedeutic introduction to right thinking; these sciences are useful as a disciplinary learning. It should be noted that Descartes's scorn is not directed against the mathematical method but against the object of the mathematical sciences in so far as this object is considered as an end in itself, worthy

[1] Letter of 8 Feb. 1640 (AT. xii, supplement, pp. 2–3).
[2] AT. x. 375^{3-10}.
[3] Ibid. 373^{26-28}. 'Logistae' and 'Geometrae' are the gentlemen of leisure.

of study for itself. The defects in the conception of 'ordinary' or 'vulgar' mathematics are such that it is impossible to base a completely adequate method on a study of their processes and we must, in due course, pass on to a higher stage where a study of 'genuine' mathematics, a science of order and measurement, will give us an adequate picture of the power of knowing working out the chains of reasoning. At this stage the object of mathematics will have no particular importance for the very good reason that the science of order and measurement is not restricted to any specific object or set of objects. It is from this study, which will be a study of 'Universal Mathematics', that we can see that the excellence of mathematical demonstration is not dependent on any peculiarity of the subject-matter with which it deals, but on the fact that its material lends itself, far more easily than the material of any other science, to the demands of strict method.[1] It exemplifies better the perfect demonstration, the simplicity, clearness, and distinctness of the data, the logical sequence in the progress from the data to the conclusion, and the completeness of the survey which rounds off the whole process. It exemplifies the mind at work in right thinking. In one sense the method is a description of true mathematics and the method can only be learnt by training oneself to think mathematically.

[1] Cf. below, pp. 194 ff.

IV

INTELLECTUAL INTUITION

§ 1. *The Twofold Function of the Intellect*

THE guidance already given in Rule 2 is continued with more detail in the following rule. We are told: 'In the subjects we propose to study, we must inquire, not what others have taught, nor what we ourselves conjecture, but what we can clearly and evidently see or what we can infer with certainty; for that is the only way in which knowledge is acquired.'[1]

Knowledge, or so-called knowledge, based on probable opinion, conjecture, or memory, is excluded from the list of subjects which we are to study in our search for a method adequate to our power of knowing. Our reflection upon the certainty of mathematical demonstration will lead us, according to Descartes, to conclude that this certainty depends on the observance and practice of two main conditions for right and proper thinking. In all mathematical, and therefore in all certain, demonstration we must first start from self-evident data and, secondly, we must make sure that every step in our deductive progress from the data must itself be self-evident. No conclusion must be affirmed as true unless it follows in an uninterrupted sequence of self-evident steps from the initial vision of self-evident data.

To these two fundamental conditions of the certainty of demonstration, as illustrated and derived from mathematical demonstration, correspond two functions of the intellect, the *vis cognoscens*, which express its proper nature and which, in so far as the intellect functions properly, are exempt from the possibility of error. These two functions are first, *Intuitus*,

[1] AT. x. 366^{11-14}: 'Circa objecta proposita, non quid alii senserint, vel quid ipsi suspicemur, sed quid clare et evidenter possimus intueri, vel certo deducere, quaerendum est; non aliter enim scientia acquiritur.' Cf. the *Discours* (AT. vi. 18^{16-23}). In a letter to Pollot in 1642 Descartes pays tribute to the intelligence of his friend Princess Elizabeth in these terms: 'Je fais bien plus d'état de son jugement que celui de Mrs. les Docteurs, qui prennent pour règle de la vérité les opinions d'Aristote plutôt que l'évidence de la raison' (AT. iii. 577^{14-17}).

intellectual intuition which is the power of apprehending the self-evident data, and, secondly, *Deductio*, deduction or inference, which is the power of moving, by an uninterrupted sequence of self-evident steps, from the data to their consequents. Thus these two functions of the intellect are the primary conditioning factors in all knowledge. They are the expressions of the very nature of the human intellect and no abstract theory of method can teach the intellect to perform them. They are presupposed in all knowledge and a theory of method can do no more than facilitate the right exercise of them. 'Unless our intellect were already able to use them, it could not understand any of the precepts of the very method itself, not even the simplest precept.'[1] The first task then is to attempt to understand the true functioning of the human mind in its primary and simplest operations.

§ 2. *A Problem of Terminology*

In attempting to understand the meaning of *intuitus*, it is perhaps as well to begin with the word itself. We are warned by Descartes himself and told not to be disconcerted by his 'new use' of the term; he adds, not very helpfully, a general rider to the effect that he will pay no attention to the way in which the word had been previously used in Scholastic philosophy, since his own theory is 'wholly different' and that he will only take notice of the etymology of the Latin words which he may wish to use in order to express his own meaning.[2] The Latin word *intuitus* is not in fact much used by Descartes in his later works.[3] Nor was it a term of common usage in the language of the Schoolmen. The hint given in Rule 3 concerning the etymological use of words should be noted since there is in the Cartesian doctrine an obvious

[1] Rule 4 (AT. x. 372^{20-22}).

[2] Cf. Rule 3 (ibid. 369^{1-10}).

[3] In the *Resp. ad II Obj.* he uses it when he is arguing that the necessary link between thinking and existence in the *Cogito* is not a syllogistic inference but is immediately apprehended 'simplici mentis intuitu' (AT. vii. 140^{23}). The French renders this as 'une simple inspection de l'esprit' (AT. ix. 110) which recalls the 'solius mentis inspectio' of the *Meditationes*, II (AT. vii. 31^{25}). Descartes seems hesitant in his usage in a letter of Mar. 1637 where he writes: 'on acquiert peu à peu une connaissance très claire et si j'ose ainsi parler intuitive' (AT. i. 353). In a letter to Regius in 1641 he uses the expression 'intellectio et visio in eadem mente' (AT. iii. 455^{4}).

parallel between *intuitus* and *visio* which is quite deliberate on the part of its author.

It may appear somewhat pedantic to raise here the question of a suitable translation for *intuitus* when common usage may well be said to have consecrated the straightforward rendering of the Latin by the English derivative 'intuition', used in all the more current translations and in many excellent studies of Cartesian philosophy. There are, however, some drawbacks in this apparently obvious translation. It is clear that, for Descartes, *intuitus* is the source of all true knowledge, a faculty or power common to all men without distinction, coextensive with the right and proper use of *bona mens* or the *bon sens*. But, as Professor Kemp Smith pointed out nearly fifty years ago,

> the word 'intuition', by keeping bad company, by mixing with the self-styled 'intuitional moralists' has got itself a bad name. When we speak of intuition nowadays, we think of something unusual, of some special faculty in the individual and that is just the very opposite of what Descartes means by it. Our intuitions are not an aristocracy with a pedigree other than the mass of knowledge that is supposed to come to us in an ordinary or common way.[1]

But even if the word has not kept bad company since these words were written, and on the whole it might be argued that, in common with a great many other terms dear to popular psychology, it has continued a rake's progress, it must be admitted, and the admission taken into account, that the word 'intuition' has become closely associated with doctrines differing entirely from the Cartesian. It has come, at least in popular phraseology and psychology, to mean some mysterious and rare power of mind not given to all men to possess, although women are often said to have some special privilege in the matter, a power of mind which is described as able to penetrate and grasp realities in a manner different from, and superior to, the ordinary powers of understanding. Whatever may be the truth behind this or similar descriptions of the power and extent of intuition, it is clearly not what Descartes means to convey by the word *intuitus*. There are therefore good reasons for avoiding the simple use of the word 'intuition' in translating *intuitus*.

[1] *Studies in Cartesian Philosophy*, p. 34.

But one has every sympathy with the translators who have ignored these drawbacks when an alternative translation is sought for. The real meaning of Descartes might perhaps be more adequately conveyed, at least with less risk of misunderstanding due to the association of entirely different doctrinal contexts, by the use of some such phrases as 'immediate apprehension', 'intellectual insight', or 'intellectual vision'. Descartes himself chose on one occasion 'la lumière naturelle' as an adequate rendering in the French language.[1] But it will clearly not do in English. The advantage of conforming to what has nevertheless become the traditional translation may perhaps be advantageously joined with an adjectival indication of the strictly intellectual nature of the function called *intuitus* by adopting the phrase 'intellectual intuition' as the normal term of translation. It must, of course, be remembered that Descartes himself, in the *Regulae* and in his other works too, does not restrict himself to the word *intuitus* but uses indifferently *visio*, *lumen naturale*, *simplex mentis inspectio*, and other terms as synonymous and equally apt to describe the first fundamental function of the *vis cognoscens*.

§ 3. *Intellectual Intuition*

Descartes sometimes uses the term *intuitus* broadly to express any and every manifestation of the essential nature of the intellect; that is, any and every act of apprehension in which we attain to complete certainty by the 'natural light of reason' may be called an *intuitus*. The most simple and elementary truths, the primary data, as also the most complex tissue of truths, the object grasped by systematic reasoning, all owe their certainty to the same characteristic mark of self-evidence which they possess for the knowing mind and, as self-evident truths, they are the objects of *intuitus* or intellectual intuition. Such a wide use of the term would seem to indicate that the

[1] Cf. Letter to Mersenne, 1639: 'la lumière naturelle ou *intuitus mentis* auquel seul je tiens qu'on doit se fier'. Locke, of course, uses the expression 'intuitive knowledge' in his *Essay*, iv. 2. 1, and Leibniz uses both 'intuition' and 'cognitio intuitiva'. The expression 'intellectual intuition' is used by Joachim as the equivalent of the Aristotelian *νόησις* in *Logical Studies*, p. 26. It is perhaps hardly necessary to add that the use of this expression has nothing to do with Kant's *Intellectuelle Anschauung* as defined in the *Critique of Pure Reason*, B. 72.

fundamental activity of the *vis cognoscens* is simple, single, and one.

On the other hand, in the first two Rules which we have discussed we are given to understand that the activity of the intellect manifests itself in a twofold functioning and these two functions together express the essential nature of the human mind. Rule 2 states that 'there are two ways by which we arrive at the knowledge of things',[1] and Rule 3 speaks of 'all those operations of the mind by which we are able, without any fear of being deceived, to arrive at a knowledge of things'.[2] At the end of a discussion on the objects of intellectual intuition, the 'simple natures' as they are called in the *Regulae*, we find the conclusion that 'mankind has no road to certain knowledge open to it except that of self-evident intellectual intuition and necessary deduction',[3] and deduction itself is described once as 'a supplementary method of knowing',[4] so that it would appear from these texts that there are two methods of knowing open to the human mind, each capable of arriving at true knowledge, at least conjointly if not separately.

The matter is, however, further complicated by the introduction of another process, called 'enumeration' or 'induction' which, according to Rule 7, is required for complete knowledge in addition to intellectual intuition and deduction.[5] It would therefore seem, on the surface at least, that there were not two but three operations of the mind to be considered.[6] It is preferable, before answering the question, to begin by considering each of the operations of the mind separately; we shall then be in a better position to decide to

[1] AT. x. 364^{26}–365^{3}: 'Notandum est, nos duplici via ad cognitionem rerum devenire, per experientiam scilicet, vel deductionem.'

[2] Ibid. 368^{9-13}: 'Hic recensentur omnes intellectus nostri actiones, per quas ad rerum cognitionem absque ullo deceptionis metu possimus pervenire: admittuntur tantum duae, intuitus scilicet et deductio.' I am not following Adam and Tannery in the reading of the last word of this passage. See below, p. 84, n. 3.

[3] Ibid. 425^{10-12}: 'Nempe nullas vias hominibus patere ad certam cognitionem veritatis, praeter evidentem intuitum et necessariam deductionem.'

[4] Ibid. 369^{18-20}: 'Hic alium adjunximus cognoscendi modum.'

[5] Ibid. 388^{18-19}: 'Hic praeterea enumerationem requiri dicimus ad scientiae completum.'

[6] Rule 8 speaks of the need for great care 'in distinguendis et examinandis illis tribus cognoscendi modis' (ibid. 396^{2-3}). But the three modes in this instance are sense, imagination, and intellect and not the modes of demonstration.

what extent these various operations of the mind are to be considered as separate and distinct functions. There is a sound piece of advice in Rule 8 which recommends that the problem should be considered in its double aspect, 'either relative to us who are capable of knowledge or relative to the objects themselves which can be known'.[1] That is to say we can either study the operation itself in its relation to the *vis cognoscens* or the objects which are the specific data of its cognitive processes. They should be studied separately and the results correlated in one synthesis.

To begin then by a study of the operation itself in relation to the power of knowing. Descartes starts his exposition by a negative description:

> By *intellectual intuition* I understand, not the fluctuating assurance of the senses nor the fallacious judgement which expresses an arbitrary composition of the imagination, but the conception which arises in an unclouded and attentive mind so readily and distinctly that there can be no doubt concerning the object of our understanding. Or, what comes to the same thing, intellectual intuition is the conception, free from doubt, of an unclouded and attentive mind and comes from the light of reason alone.[2]

The description is a curious mixture of psychological and epistemological elements. The first point made in this characterization of intellectual intuition is that it does not include all the cognitive activities of the human mind. As has been noted earlier in this study,[3] it excludes sense-perception, imagination, and memory. The intellectual intuition by which I intuit, for instance, the necessary implication of self-consciousness and existence, expressed in the *Cogito, ergo sum*, is immediate and direct as is indeed sense-perception; as if, for instance, I perceive the black dot over the *i* in the word 'is'. But there is an important difference in that the assurance obtained in the case of sense-perception is, from the very

[1] Cf. AT. x. 398^{24-25}. Rule 12 has the same advice: 'ad rerum cognitionem duo tantum spectanda sunt, nos scilicet qui cognoscimus et res ipsae cognoscendae' (ibid. 411^{3-4}).

[2] 'Per *intuitum* intelligo, non fluctuantem sensuum fidem, vel male componentis imaginationis judicium fallax; sed mentis purae et attentae tam facilem distinctumque conceptum, ut de eo, quod intelligimus, nulla prorsus dubitatio relinquatur; seu, quod idem est, mentis purae et attentae non dubium conceptum, qui a sola rationis luce nascitur' (ibid. 368^{24-29}).

[3] See above, pp. 20–30.

nature of sense-perception, subject to fluctuation: it will depend to some degree on the lighting of the room, the strength of the colour of the ink which is variable in relation to time, temperature, &c., on whether I am tired or not, with any consequent effect of this fatigue on my eyesight. My eyesight may be affected by my liver's being sluggish or not, for in the former eventuality I may well be 'seeing spots'. In a word, the assurance of sense-perception is linked essentially with, and consequently must vary with, the state of my body.

Intellectual intuition, on the other hand, is a purely spiritual activity, an activity of the understanding alone, exercised on a special kind of object. It is an intellectual 'seeing', a *visio*, and has a certainty peculiar to itself. To see intellectually, to intuit the truth, to see, for instance, that self-consciousness and existence are necessarily implied in each other, is to see once for all in a manner which leaves no room for change of view or doubt of any kind. To intuit a truth is to know that truth with absolute certainty and infallible assurance, and this certainty and assurance spring positively from the very nature of the mind itself, the natural light of reason. The two main characteristics of intellectual intuition are therefore, first, the purely spiritual or intellectual nature of the operation itself and, secondly, the absolute certainty and assurance which accompany it.

In the definition of intellectual intuition quoted above, the purely spiritual nature of *intuitus* was emphasized not only by contrast with the 'fluctuating assurance of the senses' but also by contrast with the 'fallacious judgement which expresses an arbitrary composition of the imagination', *male componentis imaginationis*. This seems to mean that a judgement in which we link together ideas simply because, and only in so far as, these ideas are associated in our imagination is likely to lead us into error. Descartes explicitly declares in Rule 12 that 'the intellect can be stimulated by the imagination or, on the contrary, the intellect can act on the imagination'.[1] There can therefore be, for instance, an action of the intellect on the imagination when we fix our attention on images already formed or when we form new images. And it is clear that Descartes believes, with some justification, that at a certain

[1] AT. x. 416[17–18].

level of mathematical thinking, the use of the imagination is of great value and even sometimes necessary. He writes to Princess Elizabeth that 'the soul can only be conscious of itself by pure understanding; body, that is to say extension, can also be known by the understanding alone but it can be better known by the understanding helped by the imagination'.[1] Rule 14, moreover, deals at some length with the use which can be made of imagination by intellect and we are told how we can 'use our imagination, employing not pure intellect, but the intellect as aided by the images depicted in the imagination'.[2]

But the concern here is not with the right use of imagination, but rather with its misuse. For the action of the imagination on intellect can be and often is vicious and leads to arbitrary and rash judgements. It may lead the intellect into accepting pseudo-demonstrations where the imagination may see the problem as solved, when in fact it is not solved, due perhaps to the vividness and force of the ideas associated by the imagination, this vividness and force of the association being taken for a necessary logical connexion. By the use of figures, pictorial representations, &c., the mathematicians might exhibit to us 'a great number of truths and conclusions drawn from certain consequences. But they do not seem to make it sufficiently plain to the mind itself why these things were so and how they had discovered them.'[3] For in themselves images have not their own guarantee of truth, their own evidence, and they cannot therefore give to the mind that absolute certainty and assurance which are the *sine qua non* of true knowledge and are the characteristics of intellectual intuition. In a paragraph preceding the definition now under discussion Descartes speaks his mind on the question of mixing probable views with truth:

> I lay down the rule [he says] that we must refrain completely from ever mixing conjectures with our statements on the truth of things. This warning is of no little importance. The fact that we find nothing

[1] Letter of 28 June 1643 (AT. iii. 691^{21-26}). See also his remark to Mersenne in a letter of 1639: 'Car la partie de l'esprit qui aide le plus aux Mathématiques, à savoir l'imagination . . .' (AT. ii. 622^{13-14}). See below, p. 217.

[2] 'Ut vero aliquid etiam tunc imaginemur, nec intellectu puro utamur, sed speciebus in phantasia depictis adjuto' (AT. x. 440^{28}–441^{1}). See below, p. 224.

[3] Rule 4 (AT. x. 375^{6-9}).

in current philosophy which is evident and certain enough to be beyond controversy has no better explanation than that philosophers, not content with the recognition of what is clear and certain, dare to assert obscure and badly understood theories which they have arrived at by probable conjecture.[1]

The Cartesian conception of truth does not admit of degrees of truth. There is no intermediary between certain knowledge and ignorance. Rule 2 makes this point in definite terms:

All science is certain and evident knowledge. The man who has doubts on many matters is no more learned than the man who has never given any thoughts to these same matters. In fact the former will appear less learned if he has formed wrong opinions on some matters. It would be better not to study at all than to occupy oneself with matters so difficult that, owing to our incapacity to distinguish true from false, we were obliged to consider the doubtful as certain. In suchlike cases any hope of increasing our knowledge is exceeded by the risk of decreasing it. Accordingly, on the above principle, we reject all merely probable knowledge and make it a rule to trust only what is completely known and incapable of being doubted.[2]

It is the same doctrine which forms the first Rule in the *Discours*: 'The first was never to accept anything as true which I clearly did not know to be such; that is to say, carefully to avoid precipitancy and prejudice and to accept nothing more in my judgements than what was so clearly and distinctly presented to my mind that I could have no occasion to doubt it.'[3]

§ 4. *Some Characteristics of Intellectual Intuition*

According to Rule 3, intellectual intuition is a conception so easy and so distinct that no doubt remains whatever as to that which we conceive of when we truly have knowledge in the strict sense of the term. It is 'a conception formed by the pure and attentive mind: a conception free from doubt, drawing its origin solely from the light of reason and more certain, because more simple, than deduction itself, although deduction

[1] AT. x. 367^{24}–368^{2}. [2] Ibid. 362^{5}–^{16}.

[3] AT. vi. 18^{16}–^{23}. Gilson, in his *Commentaire*, p. 201, describes the doctrine that truth is reserved for the type of knowledge to be found only in mathematics as 'la véritable révolution cartésienne dans l'ordre de la pensée'.

cannot fail to be correct when the deducing intellect is human'.[1] We have already seen what is meant by 'pureness' of mind. It excludes the use of the powers of the mind which are essentially dependent on bodily functions: that is, it excludes the use of sense-perception, imagination, and memory.

But the mind must not only be *pure*; it must also be *attentive*. It must concentrate and focus its attention on the object presented to it and, in the singleness of this intellectual intuition, must comprehend every essential element in the object and nothing but these essential elements. The introduction of such a psychological factor as that of attention in what is primarily a treatise of scientific and philosophical method should not unduly surprise the reader. It cannot be repeated too often, since this aspect is so often slurred over or ignored, that the aim and purpose of the *Regulae*, as indeed of the *Discours*, is to give not merely a logical account of the methodological principles of science and philosophy, but a doctrine of method which will enable us to think straight and achieve positive results, and therefore the aim is fundamentally a practical one and the advice concerned with practical details. As Descartes wrote to Mersenne, when discussing the title of the *Discours de la Méthode* as it was finally called, method 'consists more in practice than in theory'.[2]

The importance of this factor of attention is brought into prominence throughout the *Regulae*, and in many of the other works of Descartes. Rule 9 is almost entirely devoted to a discussion of its importance in undertaking a complex problem of science or philosophy. It is there divided into 'perspicacity', by which intellectual intuition is enabled to grasp single objects with clearness and distinctness, and 'sagacity', by which deduction is helped in linking the correct facts in the chains of reasoning.[3] These two are different aspects of

[1] 'Mentis purae et attentae non dubium conceptum, qui a sola rationis luce nascitur, et ipsamet deductione certior est, quia simplicior, quam tamen etiam ab homine male fieri non posse supra notavimus' (AT. x. 368^{18-22}). Descartes had noted above (365^{5-7}) that it is impossible to infer wrongly unless the mind is *less* than human.

[2] Letter of Mar. 1637: 'elle [la méthode] consiste plus en Pratique qu'en Théorie' (AT. i. 349^{21}).

[3] Cf. the title to Rule 9 (AT. x. 400^{13-15}): 'Oportet ingenii aciem ad res minimas et maxime faciles totam convertere. . . .' The expression 'acies mentis' occurs in Rules 5 and 12. See below, p. 170.

the same factor. Its function can be compared with the use we make of our eyes when we are said to fix them upon some object or some particular spot in the landscape. Just as a man who tried to take in with a single glance all the pictures hanging, for instance, in one of the rooms of the National Gallery would have no more than a vague and confused vision of the pictures, so a man who in his study disperses his mind over a variety of problems would achieve little or nothing in the way of solid knowledge. He would merely obtain a few confused ideas, a few vague opinions on diverse topics. It is to avoid this dispersion of effort that attention is needed. We are not told in the *Regulae* what is this factor of attention or how it functions, and the general account given in the *Passions* is so closely linked with the curious physiological theory of the pineal gland that it has no interest as such.[1] In so far as it functions in conjunction with 'pure' thought, its function is to distinguish the object, or group of objects, in the field of knowledge, turning towards the chosen object what Descartes calls 'the edge of the mind', *acies mentis*, as we fix our eyes on an object which we may wish to examine. It is clearly then a factor involving an act of will insomuch that we must desire and will to fix our minds on this rather than that object. Our minds must be directed properly at the outset of our inquiry and this direction is dependent on our will to do so. Are not, after all, the *Regulae* themselves written for 'the directing of the mind', *ad directionem ingenii*? If we are to accept the doctrine that the power of reason as such is equal in all men, then the divergence and diversity of views and opinions must arise from a misuse of this power of reason when ill-directed, not by its nature but by will. To pay attention, in the intellectual field, is to fix the mind on the consideration of certain objects of thought. It is by fixing the mind voluntarily on 'the more simple things', by excluding the irrelevant elements, that we are able, other things being equal, to grasp the evidence which is the source of certainty and the basis of the development of true knowledge. Descartes is merely saying that, if we do not look, we shall not see; if we do not pay attention,

[1] Cf. *Traité des passions*, §§ 41–43, especially the last chapter, 'comment l'âme peut imaginer, être attentive, et mouvoir le corps' (AT. xi. 359–61).

we shall not see the evidence, even if the evidence be there. This would perhaps seem a triviality if the history of human errors had not shown that neglect of this primary condition of research had produced, during the centuries, a large crop of absurdities.

The two great obstacles to the attentive mind are suggested in the first rule of the *Discours*, quoted above, where we are recommended 'to avoid carefully precipitancy and prejudice'. We are warned about the former several times in the *Regulae* and told to be careful not 'to make judgements which are hasty and groundless',[1] and told not to copy people 'who are in such a hurry in their investigations that they apply only a blank mind to their solution, without having settled beforehand by what sign they will be able to recognize what they are looking for, if perchance it should occur'.[2] As Descartes remarks, it is rather like an errand-boy who rushes away without knowing what he has to fetch or where he has to go. Such behaviour is so obviously stupid that there is no point in insisting further. There is certainly no need to develop, as Clauberg does in his *Defensio Cartesiana*, the twelve causes of precipitancy, or for that matter the ten causes of prejudice. This second vice of the mind is castigated by Descartes more fiercely than the first, which is often found allied with youthful enthusiasm, whereas prejudice is often found in those who preen themselves with pompous airs of learning and wisdom. The reader of the *Principia* will recall that three whole sections are devoted to the eradication of 'the first and principal of our errors' and it will be noted that most of these prejudices arise because 'in the first years of life the mind was so clearly linked to the body that it only applied itself to those ideas through which it became aware of objects which affected the body',[3] that is, the mind as such was not working 'purely' but was cluttered up with confused ideas derived from sense and imagination. We acquire prejudices when we

[1] Rule 2 (AT. x. 365^{14}).

[2] Rule 13 (ibid. 434^{17-24}). Enumeration, as a preliminary survey, is meant to remedy this.

[3] Cf. *Principia*, i. 71–74. The discussion with Gassendi, who approved of Descartes's 'determination to rid his mind of prejudices', throws light on the importance Descartes attached to the whole question. See *Resp. ad V Obj.* (AT. vii. 257–8 and 278–9).

are young and as we grow up we find it almost impossible to forget them, if we are even aware that we have them. These prejudices are invariably linked and associated with words and, if we do not pay attention, we reason with these words and arrive at purely verbal solutions while discussion degenerates into mere logomachy. The best cure for prejudice is the methodical doubt which puts all opinions in the melting pot of suspicion until they have been tested and found to be of genuine metal.

Since the practice of methodical doubt removes prejudice, our aim should be to accustom the mind to careful and patient analysis of the data so as to acquire an habitual attitude of mind which will result in our feeling that we can fix our attention on the data relevant to the problem which we are investigating; and that means, in the words of the *Discours*, 'I was conscious that my mind was becoming gradually habituated to clearer and more distinct ideas'.[1]

The definition of what constitutes the clearness of an idea implies the attentiveness of the mind to the object presented to it. The *Principia* states: 'I term that clear which is present and open to an attentive mind.'[2] To perceive anything clearly means that it must be 'present' to the mind, 'open to the mind', and the mind must be 'attending' to it, just as objects of vision are said to be 'clear' or 'to be seen clearly' when, placed without anything to disguise or hide them before the eyes of someone who looks at them, they produce of necessity a sufficiently strong and manifest impression upon them. The definition goes on to state that that can be called distinct which is 'so precise and separated from all others that it contains within itself absolutely nothing but what is clear'. To perceive anything distinctly means, in addition to all that is implied in clear perception, that one has nothing before the mind but what is directly relevant to the clear idea. The relation between the clearness and distinctness of ideas is that, whereas an idea may be clear without being distinct, a distinct idea is always a clear idea and an idea which is

[1] *Discours*, Part II: 'Outre que je sentais en la pratiquant, que mon esprit s'accoutumait peu à peu à concevoir plus nettement et plus distinctement ses objets' (AT. vi. 21^{21-24}). The Latin translates 'plus nettement' as 'clarius' and adds the words 'ingenii mei tenebras dissipari'.

[2] *Principia*, i. 45 (AT. viii. 22^{3-6}).

completely clear is always distinct.[1] If an idea is only partially clear, it is because some of the elements essential to its clarity are missing, that is to say, it is confused in so far as these elements are absent. The confused idea is always necessarily an 'incomplete idea' as opposed to the 'adequate idea', to use the terminology of Spinoza, and the latter contains in itself 'all the marks and properties of a true idea, so far as that is considered in itself without relation to its object'.[2] For Spinoza, like Descartes, find the positive quality 'of the true idea in its clarity and distinctness, which depend upon the simplicity or logical coherence of its content—i.e. upon the fact that every true idea is either simple or, if complex, a deductive or implicatory linkage of simples, by which they cohere to form a genuine unity or whole'.[3]

The definitions of the *Principia* refer to the characteristics of clearness and distinctness as attached to perceptions or ideas and not to the nature of the operation as such by which these ideas are apprehended, although it is clear from the explanations given in these definitions that these two aspects of a same whole, the act of perceiving a clear and distinct idea, depend on one and the same principle. The readiness and distinctness, which were stated to be the characteristic of a pure and attentive mind in the definition of intellectual intuition quoted in the preceding section, express rather the psychological result of an intellectual intuition upon the subject than the logical cause which is the clearness and distinctness of the perceptual content. But the psychological aspect cannot be divorced from the logical aspect in considering these characteristics of clearness and distinctness, which constitute the criterion, the hall-mark of what Rule 3 calls 'the evidence and certainty of intellectual intuition', *intuitius evidentia et certitudo*. Attempts to force this divorce began with Hobbes and Gassendi. Leibniz made the criterion of clearness and distinctness one of his principal points of attack, arguing that this criterion was of no value as an index of truth unless we had a criterion to judge clearness and

[1] *Principia*, i. 46 (AT. viii. 22^{15-17}) and *Resp. ad II Obj.* (AT. vi. 147^{9-11}).

[2] *Ethica*, ii, def. 4 (ed. Gebhardt, ii. 85).

[3] Joachim, *Spinoza's Tractatus de Intellectus Emendatione*, p. 97. The Cartesian doctrine is contrasted with that of Spinoza on pp. 91–98.

distinctness, and all Descartes had given us were standards which were purely psychological and not, as Leibniz demanded, logical. These and similar criticisms have been repeated by writers on this philosophical topic down to the present day.

It is not necessary, for the purpose of this study, to enter into a full and detailed discussion of the controversy. But in attempting to estimate the right balance between the psychological and logical elements in this question, some light will be thrown on the method of Descartes which is essentially, in its author's mind, the way to acquire truth and is therefore the practical application of a criterion, which can be fully understood not only if it is considered *qua* practical but also *qua* practised. Gassendi, in his objections to the third Meditation, pointed out that very often men died for opinions which they held, knowing quite well that others were equally ready to die for opinions diametrically opposed. Potential martyrs from opposite sides perceive clearly and distinctly the opinions which they hold and are prepared to die for. In his view, Descartes ought 'to propound a method which will direct us and teach us when we are deceived and when not deceived, in cases where we think that we clearly and distinctly perceive something'.[1] To which Descartes replies that he agrees that 'the question is not so much one of establishing the truth of the rule but of finding a method to discern whether we are deceived or not when we think that we perceive something clearly. But I contend that this has been accurately dealt with by me in its place where I first put aside all prejudices and then enumerated all the principal ideas, distinguishing those which were clear from the obscure or the confused.'[2] It might appear that Descartes is avoiding the main issue in not giving a proof of this criterion and in not setting down a method for devising the really clear and distinct from the apparently clear and distinct. But he does think that he is giving a straight answer to the question, as can be seen from his remarks in the *Regulae* and in other works and letters. There is, and there must be, a psychological element in the process of setting up this criterion and the method

[1] *Resp. ad V Obj.* (AT. vii. 279^{13-17}).
[2] Ibid. 361^{24}–362^{4}.

itself is based on this necessity. The removal of prejudice, mentioned in his reply to Gassendi, is essential to the establishment of an open mind; that is, a mind in which the ideas were confused (not reduced to the simplest but, on the contrary, prejudged) would not be in a position to apprehend the truth, even if it were presented to it. Prejudice can prevent the mind from seeing properly, that is, clearly and distinctly, and therefore it must be removed by the methodological doubt, and this removal is one step in the method of attaining clear and distinct ideas. 'To make an idea "open" to the mind, to render known its elements, and to see how the necessary connexion of its direct and interpretative contents follows from them is thus the purpose of Descartes's basic methodological precept whereby "we reduce involved and obscure propositions step by step to simpler ones", until we come to an intuition of the simplest ones of all.'[1]

The removal of prejudices, which will help towards the openness of mind which conditions all true knowing, does not remove all difficulties. Descartes was well aware of other dangers and snares which might block, or at least hinder, the intellectual vision or 'seeing' through which alone clarity and distinctness could bring conviction and certainty, the latter being the important factor. There are plenty of texts in his works which give instances or illustrations of these difficulties. In the *Discours*, immediately announcing the 'general rule', he adds a warning: 'I might take as a general rule that all the things which we very clearly and distinctly perceive are true; but there is some difficulty in determining exactly which we distinctly conceive.'[2] And, in the *Notae in Programma*, he remarks:

> This rule, that all that we can conceive can also exist, although it is my rule and it is true each and every time it is an instance of a clear and distinct idea, in which the possibility of a thing is conceived, because God can make all things which we clearly perceive to be possible; this rule, I repeat, should not be rashly usurped because it happens easily that one thinks that one understands something properly

[1] Cf. A. Gewirth, 'Clearness and Distinctness in Descartes', a careful study of this problem in *Philosophy*, xviii (1943), p. 30. The quotation in the text is from Rule 5.

[2] 4e Part. (AT. vi. 33^{20-24}). In the *Resp. ad VII Obj.* Descartes notes: 'soli prudentes recte distinguunt inter quod ita percipitur et id quod tantum videtur vel apparet' (AT. vii. 462^{1-3}).

which nevertheless because of some blinding prejudice one does not understand at all.[1]

There is, in Cartesian methodology, no automatic or mechanical rule to be laid down which will ensure, despite human weaknesses, the perception of the truth. Leibniz may well object later that the Cartesian rules of clearness and distinctness are insufficient because they are psychological precepts and not logical, and their value is therefore subjective and not objective, and that there is no need to discourse at length on prejudices and passions as psychological causes of error which must be removed by methodological doubt; what is needed, in his view, is a logical doctrine with 'evident marks' which can infallibly designate whether or not ideas or propositions are clear and distinct.[2] Rightly or wrongly, the Cartesian method insists on the double point of view: it is necessary to prepare the mind to see the truth and this can only be achieved by following certain rules which of necessity deal with psychological factors. The logical factor is not neglected and the analysis recommended by Leibniz has its parallel in the *Regulae*. But ultimately the mind, face to face with the fact, can only recognize the fact to be so; truth in the last analysis is its own evidence, *veritas index sui*, and the mind is driven back, in Leibniz's own terminology, to the *experientia interna idearum*.

Open or unprejudiced, attentive and not dispersed, such are some of the characteristics of the mind when operating in its pure state. It operates thus free from prejudices and obscurities and truth is its natural product; the mind has a natural instinct for truth. Descartes writes to Mersenne: 'I distinguish two kinds of instinct; one belongs to us in so far as we are men and is of a purely intellectual nature: this is the natural light or *intuitus mentis* on which I hold that we should alone rely; the other instinct is in us in so far as we are animals and consists of a certain natural impulsion for the conservation of our bodies. . . .'[3] It should perhaps be added

[1] AT. viii (Part II), 351^{28}–352^{6}.

[2] Cf. Leibniz's statement: 'Itaque principium illud toties decantatum: quicquid clarè distinctèque percipio, verum est, nobis parum prodest, quamdiu non satis exhibentur indicia clari et distincti' (ed. Gerhardt, iv. 328) and the texts quoted in Couturat's *La Logique de Leibnitz* (1901), p. 203, n. 2.

[3] Letter of 1639 (AT. ii. 599^{5-11}).

that this natural propensity to truth should be taken in the Scholastic sense of truth as the natural end of the power of knowing, rather than in the sense which Hume would give to such an expression.[1]

[1] Cf. St. Thomas, *Summa Theologica*, I, art. 62, 8 ad 2m.: 'Intellectus enim non potest non assentire principiis naturaliter notis.'

V

THE OBJECT OF INTELLECTUAL INTUITION

§ 1. *The Simplicity of Intellectual Intuition*

We are told that intellectual intuition is solely dependent on the natural light of reason and is more certain, because more 'simple', than the other function of the *vis cognoscens*, deduction or inference. Leaving aside for the moment the question of the relationship between intellectual intuition and deduction, what does Descartes mean by the simplicity of *intuitus*? In Rule 3 we are given some examples of intellectual intuition but little explanation of what is meant by the simplicity of the act. 'Thus', says Descartes, 'anyone can perceive by intellectual intuition that he exists, that he is thinking, that a triangle is bounded by only three lines, a sphere by a single surface, and so forth.'[1] The examples are the usual ones to be found in Descartes. They recall the examples of the intuitions of the 'heart' given by Pascal: 'The heart feels that there are three dimensions in space and that numbers are infinite; reason then demonstrates that there are no two square numbers, one of which is the double of the other. Principles are felt, propositions are inferred, both with certainty although in different ways.'[2] But the similarity is only partial. For Pascal, the certainty of the reasons 'which the reason knows not', depends ultimately on the feeling of intimate security which accompanies the intuition; for Descartes, the infallibility of intellectual intuition depends not only on the nature of the act but also on the nature of the object intuited. There is not merely a psychological element in certainty, but also a logical element which comes from the object. The simplicity of intellectual intuition arises also from the simplicity of the object which is intuited.

[1] AT. x. 368[22–25].

[2] Cf. *Les Pensées* (edit. minor, Brunschvicg), p. 282. The real difference of view becomes even more apparent in the following *Pensée*: 'Le cœur a son ordre, l'esprit a le sien, qui est *principe* et *démonstration*; le cœur en a un autre.' It is perhaps

The doctrine of the *Regulae* would not seem to imply that the simplicity of the act of intellectual intuition does primarily mean that what it comprehends is absolutely simple, at least in the sense that the object is genuinely single. The act is simple in the sense that the object is apprehended not part by part, or successively, but in the sense that the apprehension is as a whole and simultaneous. Rule 11 states: 'Two things are necessary for intellectual intuition. First the proposition which is intuited must be clear and distinct; secondly it must be apprehended as a whole, at the same time and not successively.'[1] It follows then, in the words of Rule 12, that we must be in error 'if we judge in regard to a simple nature, that we know it in part only.'[2] The doctrine seems to be that in the case of the object of intellectual intuition, the simple nature, to give it the name used by Descartes in the *Regulae*, we must either know it or not know it; it is either wholly present and completely open to our intellectual inspection or not at all. What then is the simple nature?

§ 2. *'Simple Natures'*

One of the foremost of Cartesian scholars has been able to say recently that 'Descartes' theory of simple natures, and indeed the detail of his method generally, are far from having yet received from commentators the attention they merit'.[3] Amongst those who have made a study of the doctrine there has been a tendency to dismiss it as worthless or contradictory. In one instance, after a careful and detailed study, one critic finds that the simple natures are only 'simple' in very different senses of the term and that the only sense of simplicity

hardly necessary to add here that 'le cœur' does not mean what it is generally thought to mean in one of the most abused of all quotations.

[1] AT. x. 407[15–18]: 'Ad mentis intuitum duo requirimus: nempe ut propositio clare et distincte, deinde etiam ut tota simul et non successive intelligatur.'

[2] Ibid. 420[23–25]: 'Qua de ratione evidens est nos falli, si quando aliquam ex naturis istis simplicibus a nobis non cognosci judicemus.'

[3] S. V. Keeling, *Descartes*, British Academy Lecture, 1948, p. 6. The term 'simple nature' is used almost exclusively in the *Regulae*. See by the same author 'Le réalisme de Descartes et le rôle des natures simples' in the *Rev. méta. et morale*, 1937, pp. 63–99. In his *Rationalisme de Descartes*, p. 104, Laporte disagrees with Keeling's statement that Descartes 'semble avoir oublié l'existence des natures simples' and gives several quotations to prove his point. The reference given by Laporte to the *Entretien avec Burman* is wrong; it should be AT. v. 160.

which covers all the various natures is nominal and superficial, so that the doctrine as a whole is valueless.[1] In another attempt, equally detailed, to correlate the doctrine set out in Rule 6, called for reasons of exegetical expediency 'the epistemological aspect', and the doctrine set forth in Rule 12, called 'the ontological aspect', a critic states that 'we can hardly escape the conclusion that there is quite a lot to be said for giving simple natures a wide berth'.[2] Discouraging as this may seem there is no alternative for anyone attempting to understand the methodological principles of Descartes but to try to make sense of the doctrine of simple natures.

There is not anywhere in the *Regulae* a set definition of what is meant by a simple nature. But 'simple natures' are discussed at some length both in Rule 6 and Rule 12 and in both discussions we are given examples of simple natures. There is an important text in the latter Rule which gives us several instances of simple natures.

> We therefore assert [it states] that, in relation to our knowing, single things should be considered differently from the way in which we should consider them in their real nature. Thus, for instance, if we consider a body as having extension and figure, we shall indeed agree that from the point of view of the thing itself it is one and simple. For, from that point of view, we cannot regard it as compounded of corporeal nature, extension, and figure, since these elements have never existed apart from each other. But relatively to our understanding we can call it a compound made out of these three natures, because we have thought of them as separate before we could judge that all three could be found in one and the same subject. Hence we shall here treat of things only relatively to our understanding and its awareness of them and shall only call those simple which are known, and known so clearly and distinctly that they cannot be analysed into others more distinctly known. Examples are figure, extension, motion, etc.; all the others we conceive to be compounded out of these.[3]

[1] Cf. W. A. Merrylees, *Descartes* (Melbourne University Press, 1934), especially chap. X.

[2] J. Hartland-Swann, 'Descartes's "Simple Natures" ' in *Philosophy*, xxii (1947), 139–52, a well-documented and carefully argued discussion of this topic. Another critical study of value is in the *Archives de philosophie*, xii, cahier ii, 'Les Natures simples chez Descartes' by J. M. Le Blond.

[3] AT. x. 418[1–19]. The whole of this doctrine is curiously similar to that of Locke in the *Essay* (especially chap. 2 of Part II). This similarity has been noted by writers on Locke: see Aaron, *John Locke*, p. 11; Gibson, *Locke's Theory of Knowledge*, p. 211; and Charlotte Ware, 'The Influence of Descartes on John Locke', *Revue*

A distinction then must be drawn between the 'single thing' considered as an existent and the 'single thing' considered from the point of view of our knowledge or thought. A body having extension and figure, a shaped extended thing, is a single thing, something one and simple. But it is complex from the point of view of our understanding since it is compounded of 'natures' which, though they cannot exist in isolation, must yet be thought of as separate before we can judge that they are found together in one and the same subject. And, from this point of view, a simple or single thing may be taken to mean a thing which we know so clearly and distinctly that it is impossible for the mind to divide it into constituents which can be more distinctly known than the whole.

It is as well perhaps to anticipate at this point an apparent inconsistency between the doctrine set forth in this passage and that which we shall find in other passages of the *Regulae*, where Descartes says that the simplicity of simple natures consists in the fact that they can be apprehended completely as separated.[1] He states in the passage under discussion that figure and extension are so united that they cannot be apprehended the one without the other. He means (*a*) that we cannot have an intellectual intuition of figure without having one of extension, but also (*b*) that we can clearly distinguish the one from the other. We can fix our attention, the psychological factor conditioning the pure activity of mind, on each of them separately, but it does not imply that the object of our attention exists as a separate entity. We need not have first the intellectual intuition of extension in order to have that of figure but we can have a simultaneous intuition of both.

From the logical point of view a simple nature is an ultimate constituent of knowledge, an indivisible element of thought which is analytically irreducible and, considered from the point of view of the certainty of human knowledge, it can be called in Hamelin's phrase 'an atom of evidence'.[2] But this

internationale de philosophie, No. 12, 1950, who uses the Locke collection of papers now in the Bodleian Library. Locke was in Holland from 1683 to 1689 and may well have seen de Raey's manuscript copy of the *Regulae*.

[1] Cf. Keeling, 'Le Réalisme de Descartes', *Rev. méta. et morale*, special volume, 'Études sur Descartes', 1937, p. 76, n. 3.

[2] Cf. *Le Système de Descartes*, p. 86. See below, p. 79.

indivisibility and irreducibility must not be understood in a purely psychological sense. The clearness and distinctness which manifest the essential guarantee of its simplicity must be understood as applying only in the case of the right use of the power of knowing. It may happen, Descartes informs us, that

> it is easier to be aware of several of them [simple natures] compounded together, than to separate them one from the others. I can know what a triangle is, even though I had never conceived that in that piece of knowledge was contained knowledge of an angle, a line, the number three, figure, extension, etc. But that does not prevent me from saying that the nature of the triangle is compounded of all these natures and that they are better known than the triangle, since they are the things which we understand in it.[1]

However, the simple nature must not be taken to be a mere abstract universal or generality. It is a genuine element of knowledge, the most elementary member of the 'chains of reasoning' which constitute the body of scientific knowledge. The distinction we are asked to make in Rule 12 is between simple natures as they really are and as they are separately present to thought. In the second case, in the order of knowledge, simple notions are ordered in series of which the primary terms are called 'primitive notions' or 'absolute' terms.[2] These primitive notions or absolute terms are not isolated and separate real entities. They only subsist for the mind which is undertaking the analysis of the real; they subsist as such only for the mind which intuits them. They can be considered from the methodological point of view as the 'ultimate limits of formal analysis'. But as simple natures it would appear that Descartes understands them to be genuine characters of reality which are simple in so far as they are universal. We may, for instance, say that figure is the limit of extension, where the notion of limit is an abstract universal, more general than that of figure since there are limits of duration, limits of motion, &c. But the notion of

[1] Rule 12 (AT. x. 422^{9-18}).

[2] The 'absolute' term is defined in Rule 6. Cf. AT. x. 382^{9-16}. See below, p. 163. For the use of 'primitive notion', cf. a letter to Princess Elizabeth in 1643: 'Premièrement, je considère qu'il y a en nous certaines notions primitives, qui sont comme des originaux, sur le patron desquels nous formons toutes nos autres connaissances' (AT. iii. 665^{9-12}).

limit is not more simple than the notion of figure; on the contrary, the notion of limit is not simple at all for it is a compound of many simple natures derived by abstraction from them and not explicable, except equivocally, by them all.

To return to the description of the simple natures. We are given a partial inventory in Rule 12. They fall into three main groups or classes.[1] The first group is 'purely intellectual', the second group 'purely material', and the third group is 'common' to both intellectual and material beings. The first group is attributable to minds, that is, to beings who have the power of grasping them by the natural light of reason without any help or aid from corporeal sensation or imagination. In other words, they are apprehended directly and intuitively. Examples of this first group are: we know 'what knowing is', 'what doubting is', 'what willing is'. The second group can, on the other hand, only be known as characterizing physical bodies. Figure, extension, movement are examples. Our intellectual intuition of them is aided and facilitated by imaginative and even sensuous presentations since these natures can, in a confused way, be both perceived and imagined. The third group is called 'common' since they are attributable indifferently to corporeal things and to minds alike; for instance 'existence', 'unity', 'duration' which can be attributed to material or spiritual things—a soul can 'exist', a body can 'exist', &c. In this class are also included those 'common' natures which are, as it were, relations connecting together other simple natures, links or principles of connexion on which the conclusions of our inferences depend. An instance of these latter is 'two things which are identical with a third thing are identical with one another', or 'any two things which cannot be identically related to a third thing must differ from one another'. The 'common' effects of intellectual intuition may be known either by the inspection of the pure intellect or by the intellect in so far as it apprehends the images of material things. The corresponding privations and negations, so far as these latter are apprehended by us, must also be included among simple natures. Thus, corresponding to my certain knowledge, through

[1] Cf. AT. x. 419^{6}–420^{13}.

intellectual intuition, of what 'existence', 'duration', 'motion' are, there can be equally certain knowledge of what 'nothing' is, of what an 'instant' is, of what 'rest' is, and so forth.

This threefold division, with its examples, is not without its difficulties of interpretation but there is yet another list of examples of simple natures, given in Rule 6, which must be taken into consideration,[1] although it is tempting, as some commentators of Descartes do, to take the easier road of discussing the doctrine in terms of its formulation in Rule 12 alone. These examples are the characteristics of being 'independent', of being 'a cause' or 'simple', 'universal', 'one', 'equal', 'like', 'straight' which are all described as simple natures. Opposed to them, yet 'participating in the same nature or at least sharing in it to some degree',[2] are their correlations, being 'dependent', being 'an effect', or 'composite', 'particular', 'many', 'unequal', 'unlike', and 'oblique'. Their relativity varies in degree whereas the simple nature cannot vary. A straight line cannot be more or less straight; a line is either straight or not straight.

It must be admitted that the discussions in the *Regulae* which treat of simple natures are not amongst the clearest texts of Descartes. There is a faint hesitancy in the expression of the views and a certain ambiguity of which Descartes does not seem to be aware. He often speaks of these objects of intellectual intuition in language which suggests that these simple natures are concepts. A great deal of justificatory text might be put forward to support the conclusion arrived at by Boyce Gibson that 'a simple nature is really a concept, but it can only be affirmed as simple by means of a proposition'.[3] This conclusion is rightly pointed out to be important for the problem of Cartesian deduction and in one sense the simple nature must be considered as a concept. But it is also true, as a critic has not failed to observe, that the close analysis by Boyce Gibson of the simple nature in terms of the relationship between intuition and deduction is worked out only in the context of a theory of knowledge and does not attempt

[1] Cf. ibid. 381–2.

[2] Ibid. 382³⁻⁴: 'quod eamdem quidem naturam, vel saltem aliquid ex ea participat'.

[3] Cf. Boyce Gibson, *The Philosophy of Descartes*, p. 162, and the criticism of J. Hartland-Swann in *Philosophy*, xxii (1947), 141.

to square the interpretation with the examples given by Descartes, or, it might be added, with the ontological language frequently used of these natures.

In the title of Rule 6, where the last-mentioned set of examples is found, Descartes calls the simple natures 'the most simple of things'[1] and in the context of the same Rule he speaks of 'pure and simple natures',[2] an expression which is constantly used in Rule 12 conjoined with 'simple things and simple natures'. In this latter Rule, in a paragraph which follows on the discussion of 'common notions', he proposes to distinguish two faculties of understanding, the one 'whereby it sees and knows things' and a second whereby 'it judges by affirming and negating'.[3] There are, Descartes seems to be saying, certain simple natures which symbolize in letters the language of reality, an alphabet or, as Leibniz was later to name it, a *characteristica universalis*. The simple natures are then characteristics recognizable in all bodies and in all minds, or all existents whether corporeal or spiritual. They are universal, as is clear from the examples, but yet in some sense they have ontological status, they are simple entities which are the fundamental constituents or elements of all bodies, of all minds, of all that exists. To use Dr. Keeling's description, they are 'ontal elements' which in some way are actually constitutive elements of extra-mental reality and there is a correspondence between the concepts which are the principles of our knowledge and give form to our knowledge, and the ontal elements.[4] And if we do not wish to go as far as this eminent commentator, we may suppose that the minimum interpretation of Descartes's words leads to the conclusion that these natures, as simple and singular entities, if they do not exist in the full ontological sense of this term, at least subsist. They may be said to confront the mind and await its recognition. They are not fictitious creations of the

[1] 'ad res simplicissimas ab involutis distinguendas' (AT. x. 381^{2}).

[2] He says that 'naturas puras et simplices' are few in number (ibid. 383^{11-12}). In the next sentence the transition is made to the phrase 'prima et maxime simplex propositio' (ibid. 383^{21-22}) and the next paragraph discusses in the same line of thought 'obvias veritates' (ibid. 384^{13}).

[3] Ibid. 420^{16-17}: 'Quod facile ostendetur, si distinguamus illam facultatem intellectus, per quam res intuetur et cognoscit, ab ea qua judicat affirmando vel negando'.

[4] Cf. Keeling, *Descartes*, p. 236, n. 1.

mind itself; they are not mere figments of mind inserted in or imposed upon an alien reality. The mind's recognition of them is direct and immediate, a knowledge by direct acquaintance, a simple act of intellectual vision or 'seeing'. This direct vision or simple apprehension is the only genuine knowledge and the only way we can grasp the truth of things and is consequently the one indispensable prerequisite of all judgement and inference. Judgement and inference are but the combining and visioning, if the term can be allowed, of what we know by direct vision of the simple natures. In judging and inferring, knowledge, if we attain any or possess any, is more derivative and indirect and, in consequence, of less certainty.

It will not help to clarify this rather obscure doctrine of Descartes by comparing it with, or making it the precursor of, the Kantian doctrine of the Categories of Understanding. Dr. Keeling may be right in calling the doctrine of simple natures 'preparatory' to the Kantian doctrine, but his qualification of the comparison is such that the relation between the two doctrines rests on too slender an analogy to be worth developing. For,

> whereas Kant's 'Copernican hypothesis' effectively prevents us ever from knowing whether our cognition represents the structure of ultimate fact (and cannot therefore be taken as being knowledge of extramental reality), it is of the very essence of Cartesian epistemology (viz. of the theory of representative perception, enforced by God's certification of what is 'clear and distinct') to maintain a correspondence between the *a priori* innate principles and concepts that give form to our knowledge, and the simple natures actually constitutive of extramental reality.[1]

It must be agreed that Descartes is not an 'epistemological idealist'.

There is perhaps more resemblance to the Thomistic doctrine. To the vision of the simple natures in the act of intellectual intuition would correspond the vision of the *quidditas* of things by the *simplex apprehensio*, the primary and simple act of the *intellectus*.[2] And *intuitus*, in the same

[1] Ibid., p. 264, n. 4.

[2] Cf. *De Véritate*, 1, a. 12, in corp.: 'Nomen ergo [intellectus] duplici sensu accipi potest. Uno modo secundum quod se habet ad hoc tantum a quo primum nomen

way as the *simplex apprehensio*, would give infallible and certain knowledge since, in the words of St. Thomas,

> it cannot be deceived *per se* in regard to the essence of any object of thought, though it may be deceived *per accidens* in relating thought-objects in judgement [*vel componendo, vel dividendo*] and reasoning. And hence intellect cannot err in judgements which appear evident from the apprehended essences of the terms compared as in the case of the apprehension of the first principles. . . . But intellect may be deceived *per accidens* in apprehending the essences of the composite thought-objects: not indeed from any defect on the part of the body, since the intellect is not a corporeal faculty, but from the judging processes which intervene in the definition of a composite object. . . . But in the apprehension of simple objects, where there is no composition, we cannot fall into error but fail altogether to apprehend them (or apprehend them as they are).[1]

The similarities of expression are striking and the doctrine of the simplicity of the act of knowing, the simplicity of the object, the infallibility and certainty of the consequent knowledge, the possibility of error in the judgement, all have a definite Cartesian ring about them and might tempt a commentator to transpose the Cartesian doctrine of simple natures on to the conceptualistic plane of the Scholastic theory.

But no such interpretation would fit the doctrine of the *Meditationes* and the *Principia* and, to restrict the present inquiry, it cannot be accorded with the account of inference given in the *Regulae* or even with what is said about the objects of intellectual intuition in other passages of the same work.

impositum fuit, et sic dicimur proprie intelligere cum apprehendimus quidditatem rerum, vel cum intelligimus illa quae statim nota sunt intellectui, notis rerum quidditatibus, sicut prima principia quae cognoscimus cum terminos cognoscimus.'

[1] *Summa Theol.* I. 85, a. 6, in corp. Cf. also *Contra Gent.* iii, c. 108: 'Proprium autem objectum intellectus est quidditas rei. In cognitione igitur intellectus deceptio accidere non potest, si puras rerum quidditates apprehendat.' The doctrine of St. Thomas is based on that of Aristotle and he often refers in this connexion to the text of the *De Anima*, 430b27: 'ὁ δὲ νοῦς οὐ πᾶς, ἀλλ' ὁ τῶν τί ἐστι κατὰ τὸ τί ἦν εἶναι ἀληθής, καὶ οὐ τὶ κατά τινος'. Cf. also *Meta.* 1051b24. The doctrine of the infallible intuition of 'simple ideas' (cf. Rule 3, AT. x. 368^{14-22}, Rule 8, AT. x. 399^{14-16}, and Rule 12, AT. x. 420^{14}–421^{2}) is repeated by Spinoza in the *De Intellectus Emendatione* (ed. Gebhardt, ii. 24–25). See also below, p. 244, n. 3.

§ 3. *'Simple Propositions'*

It has already been noted that the simplicity of the act of intellectual intuition does not, in Descartes's view, mean that what it apprehends is simple, at least in the sense that the object is genuinely single.[1] The act must be simple in the sense that it excludes partial or successive apprehension. If we turn to the examples given in Rule 3 we recall that we can apprehend by intellectual intuition that we exist, that we are thinking, that a triangle is bounded by three straight lines, and so forth. It would therefore seem, from these examples, that the simplicity of the object 'does not exclude inner distinction; for it is the immediate, but necessary, cohesion of two elements or two constituent ideas'.[2] It would seem to be clear from the examples that the object of the intellectual intuition is two elements linked in an immediate and necessary connexion, that is, the content of the intuition is an immediate necessary nexus of *implicans* and *implicatum*, a proposition such that *x* immediately and necessarily involves *y*. The self-evident datum therefore which is apprehended in the act of intuition is an immediate nexus of two elements: *x* immediately necessitating *y* or, in another formula, 'since *x*, therefore *y*'. As Joachim sums it up:

> The self-evident datum which Descartes calls 'a simple idea' or 'a simple proposition', is a hypothetical judgement so formulated that the antecedent immediately necessitates the consequent, though the consequent need not reciprocally involve the antecedent. The elements in the content of an 'intuition' cohere by the immediate necessity which binds consequent to antecedent in an hypothetical judgement of the kind explained. But the content *as a whole* is grasped intuitively, or immediately, as an indubitable self-evident *datum*.[3]

It is only on some such interpretation that we can make sense of the passage in which Descartes speaks of the objects of intellectual intuition, the self-evident data, the simple natures, as

[1] See above, p. 65.

[2] H. H. Joachim, *The Nature of Truth*, 2nd edit., pp. 70–71. See also by the same author, *Logical Studies*, p. 45, n. 1.

[3] *The Nature of Truth*, 2nd edit., p. 71. In Rule 12, Descartes points out that the nexus does not involve reciprocity: 'Denique notandum est, plurimarum propositionum, quae necessariae sunt, conversas esse contingentes' (AT. x. 422[2–6]).

'simple propositions', *propositiones simplices*, or judgements, *enuniationes*.[1]

Again, in Rule 3 we are given as further examples of the objects of intellectual intuition that *Cogito, ergo sum*, that $2+2=4$, that $3+1=4$, that $2+2=3+1$. We must understand Descartes to mean, for instance in the case of the *Cogito, ergo sum*, which is in the *Meditationes* the indubitable primary datum from which the reconstruction of all knowledge must start, that the certain knowledge that my self-consciousness involves necessarily my existence, although expressed verbally in the form of a mediate judgement or inference, is not an inference in the sense that it logically depends on prior truths. Nor is it an inference in the sense that there is any illative movement from ground to consequent within the judgement. This point has been made repeatedly by Descartes:

> Our recognition that we are thinking things is a primary notion which is not the conclusion. When I say 'I think, therefore I am or exist', I do not deduce existence from self-consciousness (*cogitatio*) by a syllogism: for that would involve previous knowledge of the major premiss, 'everything which thinks, is or exists'; this is a universal truth which I learn afterwards from the inseparability of existence and self-consciousness in my own case. 'I think, therefore I am' is really the recognition of something as self-evident by the simple intuition of the mind.[2]

But the self-evident thing (*res per se nota*) which the mind apprehends, in a single and simple act, is the necessary and immediate coherence of two elements; that is to say, this self-evident datum, like all the objects of intellectual intuition, is then a simple proposition in which is contained the necessary but immediate coherence of two ideas. That which is apprehended, clearly and distinctly, is not the content of a single idea in an illative relation to another single idea; it is the

[1] See, for example, Rule 3, where he speaks of 'illas quidem propositiones' (AT. x. 370^{10}), and again: 'At vero haec intuitus evidentia et certitudo, non ad solas enuntiationes, sed etiam ad quos libet discursus requiritur' (ibid. 369^{11-13}). Rule 5 states: 'si propositiones involutas et obscuras ad simpliciores gradatim reducamus' (AT. x. 379^{17-19}).

[2] *Resp. ad II Obj.* (AT. vii. 140). Spinoza, unlike Kant and many less illustrious critics of Descartes, does not fall into the gross misinterpretation of the authors of the Second Objections, but the full significance of the famous *intuitus* is not correctly expressed in the formula 'ego sum cogitans' which Spinoza uses in *Des Cartes Principiorum Philosophiae*, Part I, Prolegomenon. See below, p. 106.

relation of two constituents in, to use an Hegelian term out of its context, the relation of two constituents in an immediate concrete identity.

In Rule 12 the doctrine, especially in its examples, might lead one to assume an atomic view of the simple nature, a view which fits ill into the Cartesian doctrine of inference and, if applied to the ontology of the *Meditationes* and *Principia*, makes fair nonsense of the fundamental doctrine of substance. A critic has pointed out that 'the "ideas" of the *Meditationes*, unlike the intuitions of the *Regulae*, were not all suited to discrimination into absolute terms representative of simple atomic elements, and any attempt to effect such a discrimination would have meant compiling utterly useless catalogues of every cognitional facet of perceived objects'.[1] This must be admitted, with the proviso that the intuitions of the *Regulae* will not fit into any discriminatory scheme any more than the 'true and immutable' natures of the later works. Descartes is not only rejecting the Scholastic doctrine of universals but also refusing to set up, if such an anachronism can be allowed, a monadic doctrine. It is not possible to discuss here the ontological bearing of the doctrine of simple natures, for this would entail a study of the doctrine of substance and its modes and his treatment of 'universals';[2] it would entail entering upon the argument of the *Meditationes* and *Principia*, whereas our commentary is linked primarily to the *Regulae* as the fundamental document for Cartesian methodology.

But it may be added that even in Rule 12, where Descartes seems to incline to an atomic view of the simple nature, there are comments which indicate the presence of a less separatist view. He notes, for instance, that 'no corporeal idea can be imagined of such a kind as to represent to us what knowledge is, what doubt is, what ignorance is, etc.'[3] Or again he says

[1] J. Hartland-Swann, *Philosophy*, xxii. 146.

[2] To treat these topics adequately would involve a lengthy excursus into the metaphysical doctrines of Descartes, which will be treated in another volume. That Descartes is maintaining a doctrine of universals quite different from the Scholastic view can be seen in his tart answer to Gassendi: 'Sed quantum ad essentias quae clare et distincte cognoscuntur, qualis est ea trianguli alteriusve cujusvis figurae Geometricae, facile cogam te ut fatearis illarum ideas, quae in nobis sunt, a singularibus non esse desumptas' *Resp. ad V Obj.* (AT. vii. 380^{16-22}). For Descartes's views on 'universals' see *Principia*, i, 48, 58, and 59 (AT. viii (Part I), 22^{27}–23^{23}, 27^{15-18}, 27^{19}–28^{17}).

[3] AT. x. 419^{12-14}.

of figure, extension, movement that 'they are known only to be in bodies',[1] and of existence, unity, duration that 'they are attributed indifferently now to corporeal things and now to spirit'.[2] What we apprehend, then, in intellectual intuition is something more than a corporeal idea, something which is strictly expressible in terms of a proposition 'that knowledge-is-*x* or -*y*', that 'this-mind-endures', &c. The list of simple natures, or rather the partial catalogue, is further added to in the same Rule by the 'common notions', the *vincula* of knowledge, the principles of connexion which we have already mentioned, and these principles of connexion cannot be considered as aught else but propositions. Despite some ambiguous expressions, there is no reason to consider that Descartes's doctrine implies that simple natures are mere atoms of thought, to which correspond atoms of reality. They can be considered 'atomic' in the modern sense of atomic. The simple natures enclose and engender a number of relations which may be of an infinite complexity, the total constituting the whole of reality and the whole of knowledge. Amongst those who have written on this point there is some degree of unanimity in their different shades of expression. Boyce Gibson notes that

> the simple intuited nature is indeed distinct in its own character, but may stand in an external relation to other intuited natures and form with them a body of knowledge which is none the less systematic for being compounded of detachable units. Distinctness, as an attribute of simple natures, does not mean that no one can ever influence, or be influenced by, any other, but simply that its relations are not inseparable from it, and consequently that it is neither more nor less true whether it stands by itself or in a larger context.[3]

It is not the doctrine of Descartes that simple natures are apprehended as actually separate; they are apprehended as essentially distinct. If they are to be thought of as 'clear', that is without confusion, then this clarity must result from a complete analysis, but it is equally necessary for a full understanding of them that they should be known also as elements or constituents of the whole, which is itself the object of a

[1] 'Quae non nisi in corporibus esse cognoscentur' (AT. x. 419^{18-19}).
[2] Ibid. 419^{21-22}.
[3] Boyce Gibson, *The Philosophy of Descartes*, p. 158.

new intuition of the mind. 'The absolutely simple intuition of the *Regulae*', says M. Serrus, 'can only be such because it is at the source of inference and the result of it, and it must be presupposed in relation to the mind.'[1] The method of Descartes definitely presupposes both analysis and synthesis. The 'atom of evidence', then, which was borrowed from Hamelin's phrase, must be understood as 'simple' only in so far as it is the *minimum cognoscibile* for the knowing mind, but as *minimum cognoscibile* it is a whole of the nexus and what is connected; it is, to continue borrowing phrases, an implication with its *implicans* and *implicatum*, a simple proposition.

The logical simplicity of the simple nature does not imply that the analysis cannot in fact be pushed any farther. The 'atom of evidence' is not the ultimate point of the process of division and abstraction, in the sense that any further division is psychologically impossible. But it is the ultimate point of the process of analysis into clear and distinct ideas. Beyond this point knowledge becomes confused and indistinct. Thus, for instance, we can push our analysis beyond the notion of 'figure' and discover that 'figure is the limit of an extended thing'.[2] But we should be mistaken if we concluded that the notion of 'limit' is 'more simple than the notion of figure' since the former is only a confused notion, 'composed of a number of natures wholly diverse, of which it can only be predicated in an ambiguous way'.[3] The 'evidence' of these simple natures is direct and immediate, as one would expect from the objects of intellectual intuition. They are 'known *per se*'[4] and no difficulty is to be expected in recognizing them; our close attention is only required 'in separating them

[1] C. Serrus, *La Méthode de Descartes* (Paris, 1933), p. 13. See also the view of J. Wahl, 'Notes sur Descartes' (special number of *Revue philosophique*, 1937, p. 371): 'Descartes est si fort en faveur des séparations que, quand il représente une union, il insiste sur le fait que cette union est non pas une idée distincte sans doute, mais une idée distinguée de celles qui la composent . . . une idée innée est un jugement.' The same conclusion is arrived at by L. Brunschvicg: 'L'intuition cartésienne est, ou tend à être, une intuition, non de choses, mais de la pensée. Or la réalité de la pensée consiste dans un acte. Cet acte est d'abord l'acte de jugement' (special number of *Rev. méta. et morale*, 1937, p. 5). Miss S. Brown is very wide of the mark in her discussion in the *Travaux du IX^e Congrès International de Philosophie*, i (1937), 15, when she concludes on logistic grounds that 'no Cartesian *datum* is a proposition'.

[2] AT. x. 418[22].

[3] Ibid. 419[3–5].

[4] Ibid. 420[14–15] and at 425[21–25], 'quia per se sunt satis notae'.

one from the other and examining them separately with a resolute and keen inspection of the mind'.[1]

The various inventories, lists, or classifications of 'simple natures' to be found in the *Regulae* do not constitute a table of *genera* and *species*. The 'simple natures' cannot be defined except in terms of themselves or other simple natures; in the first case, they are undefinable or absolute. If they are arranged in different classes the division is based on their essential characteristics, which enables us to distinguish them one from the other and thus arrange them in an orderly manner. There is not much point in compiling a list of them, except to illustrate a doctrinal point by some examples. In Rule 6 Descartes points out that he does not wish to burden our memory with long lists and elaborate classifications. The 'pure and simple natures' are 'not so much to be kept in the memory as to be discovered by a sort of mental acuity' and accordingly 'we must look for something which will so train our mind that it will apprehend these connected series immediately whenever it needs to do so'.[2] Earlier, in the same Rule, he had noted that 'all facts can be arranged in certain series, not indeed in the sense of being referred to some ontological genus, as is used by the Philosophers in their categorial classification, but in so far as certain truths can be known from others'.[3] The basis of the Cartesian order which permits us to arrange these 'natures' in series is that of their logical dependence in the deductive series. As such, simple natures are not more general than composite natures; they are not *genera* which contain composite natures as 'inferiors'. Descartes is thinking in terms of mathematical classification where such considerations have no meaning: the 'straight-line' is not more general than the 'triangle', the 'angle' is not more particular than the 'straight-line'. The relationship between the triangle, straight-line, and angle is not to be expressed in terms of genus and specific difference. This conception of the relationship of the elements of knowledge must inevitably govern the Cartesian conception of the deductive process in which the simple natures enter as links of the implicatory sequences. The rejection of the Aristotelian

[1] AT. x. 425[22-24].
[2] Ibid. 383[26]–384[4].
[3] Ibid. 381[9-13]. See below, p. 132.

and Scholastic method of classification, essentially based on a graduated hierarchy of concepts, involves also the rejection of all subsumptive deduction and its first principle *dictum de omni, dictum de nullo*—in a word, the rejection of syllogistic reasoning.

The three classes of simple natures given in Rule 12 seem, on the Cartesian dualistic assumption, to constitute what Descartes himself would have called an 'adequate enumeration', since they cover all the possible cases.[1] The 'intellectual' natures are called 'pure' because they are known by the natural light of reason without any intervention of corporeal images. The second class, the 'purely material' natures, can only be known 'in bodies' and the objects represented by these can only be apprehended by a corporeal image derived ultimately from sense perception. The third class, the mixed or 'common' natures, so called because they can be attributed to corporeal and spiritual things alike, 'can be known either by the pure understanding or by the understanding intuiting the images of material things',[2] depending in each case on whether the common natures are to be considered in the context of spiritual or corporeal natures. Descartes inserts the warning in Rule 6 that 'there are only a few pure and simple natures which are not to be apprehended in dependence upon something else but are to be intuited primarily and through themselves, either in experiences themselves or by light innate in us'.[3] The point of this warning seems to lie in the emphasis laid on the need of experience in the majority of the cases where we are seeking to discover simple natures. It must be understood as referring to the second class of simple natures, that is the 'purely material' natures, and, at least partially, to the third class of 'common natures'. As given in experience, clearly in this instance sense-experience, the simple natures are only apprehended in a confused way since they are given as composite together with other simple natures. In Rule 12 Descartes tells us that we must not

[1] See above, p. 70.

[2] 'Et quidem hae communes possunt vel ab intellectu puro cognosci vel ab eodem imagines rerum materialium intuente' (AT. x. 419^{29}–420^{2}).

[3] 'Paucas esse duntaxat naturas puras et simplices, quas primo et per se, non dependenter ab aliis ullis, sed vel in ipsis experimentis, vel lumine quodam in nobis insito, licet intueri' (ibid. 383^{11-14}).

attempt to explain or define simple natures; we must only endeavour to see them clearly and distinctly. Simple natures, he repeats, are *per se notae* and our main task, obviously here in the physical sciences, is to elicit them from sense-experience by careful analysis of the composite data therein presented to the mind. 'There is no one so stupid', he notes, 'as not to perceive that when he is seated he is in some way different from himself when he is standing. But not everyone separates with equal distinctness the nature of position [*situs*] from the rest of what is contained in that thought, or is able to assert that nothing is then altered save position.'[1] The simple natures, then, must be elicited, in the case of 'material natures', from the confused data of sense-experience. They are derived by analytic reduction of the composite but, once derived, they are known *per se* as clear and distinct: they are thus the ultimate principles of explanation in the physical sciences.[2]

[1] Cf. AT. x. 425^{24}–426^{3}.

[2] See below, pp. 239 ff.

VI

THE NATURE OF INFERENCE

§ 1. *Intellectual Intuition and Deduction*

In pursuing his inquiry into the power and limits of the human intellect, the instrument for the acquisition of knowledge, Descartes is attempting to formulate the conditions of certain demonstration which is, in his view, to be modelled upon mathematical demonstration as exemplifying the perfect type of proof. The preliminary and partial conclusion of this inquiry was that the mind should first start from self-evident data and, secondly, progress continuously to its conclusions through a series of steps, each of the steps being self-evident. On the assumption that, in so far as it functions in accordance with its own nature, the human mind cannot fall into error, he argues that, in virtue of its nature, the mind can apprehend the necessary and immediate implication of the simple elements of a self-evident datum. It can apprehend or 'see' that, for instance, '2 added to 2 is 4' or that 'self-consciousness involves the existence of a self-conscious subject'. This is the mind in its function of intellectual intuition. But, further, the mind can move from intuition to intuition, from the clear and distinct vision of the self-evident to another equally clear and distinct self-evident, progressing along a chain of truths, each link of which is a necessary or self-evident link, until the whole chain lies clear before it and it can grasp the last link as the necessary consequent of the first. This is the first function of the mind to which Descartes gives the name *deductio* and which must now be examined in more detail.

'There are two ways by which we arrive at knowledge of things, namely, either by experience or by deduction',[1] is the first statement on this subject of deduction or inference which occurs in the *Regulae*. By 'experience', in the context of Rule 2, it is clear that he means nothing more than the right

[1] 'Nos duplici via ad cognitionem rerum devenire, per experientiam scilicet, vel deductionem' (AT. x. 364^{26}–365^{3}).

intuitive use of the intellect, since even sensation can give rise to indubitable intuition, provided that the intellect restricts itself to the 'pure' act of vision, without adding anything to, or subtracting anything from, the datum. Rule 12 makes this point clear when it states that 'it must be noted that no direct experience can ever deceive the intellect, if it restricts its attention accurately to the object presented to it'.[1] We may therefore extend our knowledge either by experience of isolated instances of self-evident data or by the rational development of the implications of the self-evident data. The second process, deduction, which is defined as 'the pure illation of one thing from another',[2] is as certain and infallible as intellectual intuition. We may circumstantially fail to draw out fully all that was logically implied in the datum from which the deductive process began but, as we have already noted, no human intellect which is normal can err in passing from self-evident link to the next self-evident link, according to the identical process which occurs in the demonstrations in the mathematical sciences. In these sciences it is merely a question of following out the logical implications of simple, abstract, absolutely certain data. If we apprehend these data at all, we must apprehend them wholly as they are: if we progress logically from these data, the mode of our advance cannot be at fault since the advance itself is dependent intrinsically on the self-evidence of the link between the data. Rule 3 lays down that 'we shall here take note of all those operations of our intellect by which we are able, without any fear of deception, to arrive at a knowledge of things. Now I admit only two, namely, intuition and deduction.'[3] So far

[1] 'Intellectum a nullo unquam experimento decipi posse, si praecise tantum intueatur rem sibi objectam' (AT. x. 423^{1-3}). Rule 8 had stated: 'atqui de rebus tantum pure simplicibus et absolutis experientiam certam haberi posse dicentur suo loco' (ibid. 394^{12-14}).

[2] 'Illationem puram unius ab altero' (ibid. 364^{5-6}).

[3] 'Hinc recensentur omnes intellectus nostri actiones, per quas ad rerum cognitionem absque ullo deceptionis metu possimus pervenire: admittunturque tantum duae, intuitus scilicet et inductio' (ibid. 368^{9-13}). Both the manuscripts A and H have the reading *inductio* and Adam follows this reading. It seems, however, preferable to adopt the reading *deductio* (which is done by Buchenau, Gouhier, and Le Roy in their separate editions), and to consider *inductio*, which is in fact crossed out in H, as a blunder of the copyist. In Rule 4 (AT. x. 372^{16-17}), in which Descartes obviously refers back to this text, he calls the two acts of intellect *intuitus* and *deductio* and the same words are used in another back reference in Rule 9 (AT. x. 400^{16-17}). The

then deduction takes its place alongside of intellectual intuition as one of the two acts by which we achieve knowledge with absolute certainty. To understand in the strict sense means then to apprehend, or to infer, or both; to apprehend or to infer is to understand. It would therefore appear that one of these original functions of the intellect gives us certainty of the simple data and the other guarantees our advance from the data to the conclusion or conclusions. The twofold activity of mind manifests itself in the two functions of intellectual intuition and deduction which thus express the fundamental nature of the *vis cognoscens*.

The distinction between these two aspects or functions of mind's activity is discussed in several passages of the *Regulae*. In Rule 3 deduction is described as 'a mode of knowledge' which is added to that of intellectual intuition, and there is a suggestion that this mode of knowledge is supplementary to the basic activity of direct and immediate vision. The reason given is that 'many things are known with certainty, though they are not themselves evident, but only deduced from true and known principles by the continuous and uninterrupted activity of mind which sees each link clearly'.[1] Deduction, therefore, is of a supplementary kind in that it depends on intellectual intuition in so far as its certainty is based on simple data which can only be apprehended by intellectual intuition; and, further, the act itself of deduction, the illation from one datum to another, depends on the apprehension that one datum implies the other, that is, it depends on intellectual intuition. So great is this dependence that, as Rule 8 warns us, 'if in the matters we are in the course of examining, we come to a point which our understanding cannot grasp intuitively, we must stop short there. We must not try to examine the rest but should spare ourselves superfluous trouble.'[2] Deduction is distinct from intellectual intuition in

matter is complicated, however, by Descartes's use of the expression *enumeratio sive inductio* which will be dealt with later; see below, p. 134. In his *Études sur Descartes* (Paris, 1935), i. 113, n. 1, Laberthonnière agrees that the reading *inductio* may be an error, due to distraction on the part of Descartes, or the copyist, but he adds the point that 'peut-être aussi Descartes a-t-il cru pouvoir donner à ce mot le sens d'*illatio*. Nous verrons plus loin qu'il appelle l'énumération une induction en tant justement qu'elle est une suite de déductions.'

[1] AT. x. 369[22–26].

[2] Ibid. 392[10–14].

that first it involves a certain movement or succession which is not found in the act of intellectual intuition and, secondly, its certainty depends to some extent on memory. 'Deduction does not require an immediately present evidence such as intellectual intuition has; its certainty is rather conferred upon it in some way by memory.'[1] There is a cautionary note: Descartes adds 'in some way' (*quodammodo*) to his qualification of the role of memory in the deductive process, which would imply that in some cases of deduction the role of memory may be attenuated, or perhaps dispensed with altogether, as in the case of conclusions which are directly and immediately derived from self-evident data, or first principles, in which the original datum, or what might be called the premiss, is held, together with the conclusion, in a single grasp of attention.

Descartes seems to assert in these passages, as indeed in several other passages, that there is a clear severance of the mind's immediate activity as intuitive vision of the simple nature from its discursive or illative movement. He seems to allot these activities severally to the apprehending of simple self-evident data on the one hand and to the tracing and memorizing of their linkages and connexions on the other. This impression might reasonably be deepened and confirmed for a reader who follows the general trend of the Rules which follow upon Rule 8, which was quoted in the preceding chapter. Rule 9, in very general terms, may be described as guidance on how to train and develop our innate power of intellectual intuition. Rule 10 professes to show us how to develop our power of deduction, how we may train ourselves to see the logical implications of many apparently disconnected data, and how to aid our deductive ability by the use of enumeration. Rule 11 is concerned principally with the combined use of intellectual intuition and enumeration while Rule 12 sets out to show us how to use the power of intellectual intuition and the power of deduction in such a way that their combination may provide us with a sure method of acquiring knowledge. The arrangement of the various Rules seems prima facie to be accommodated to a sharp division between the two powers of the *vis cognoscens*.

[1] AT. x. 370[4–9].

§ 2. *The Nature of Deduction*

The reader of the *Regulae* who had turned, as suggested above, to the various Rules, would probably have noticed that Descartes himself was quite aware that his expressions could easily lead the reader to assume a sharp dichotomous division between the operations of the mind, and that, in Rule 11, he takes pains to correct the impression which he may have made in his attempt to give a full account of their working. There are, both in Rule 11 and other Rules, a number of passages which show that a less ingenuous and simple conception of the relation between intellectual intuition and deduction was in his mind. The somewhat crude analysis of the *vis cognoscens* into two separate and quasi-independent functions, which would imply a correlative division of the real into simple natures and links (*vincula*), into independent terms and independent relations, does not in the least fit into the general scheme of the Cartesian method. It contradicts in a flagrant manner the foundation on which his whole theory of method is built, for it runs counter to the principle of the unity of the sciences and of the human intellect. 'We must not fancy', states Rule 12, 'that one kind of knowledge is more obscure than another, since all knowledge is of the same nature throughout and consists solely in the combining of things which are self-evident.'[1] And, as we have already noted, the certainty of deduction depends on intellectual intuition in a double way, both for the apprehension of the data and for the apprehension of the linkages or connexions between the data. Throughout the *Regulae* this doctrine is affirmed and reaffirmed as though the author feared that his reader might easily lose sight of it or misunderstand the point. 'This evidence and certainty of intellectual intuition are required not only for propositions but also for all inferential reasoning.'[2] In arriving at the conclusion of an inferential

[1] Ibid. 427^{27}–428^{2}.

[2] Rule 3. 'At vero haec intuitus evidentia et certitudo, non ad solas enuntiationes, sed etiam ad quodlibet discursus requiritur' (AT. x. 369^{11-12}). Haldane and Ross, in their translation, *Works of Descartes*, i. 7, use the phrase 'enunciations of propositions', and Le Roy uses 'simples affirmations' for *enunciationes*. The term is used, however, in the same way as *propositio*, that is, 'oratio in qua est verum et falsum'. See, for instance, the text of Toletus in Gilson's *Index Scolastico-Cartésien*, p. 95.

reasoning such as '2 and 2 yield the same sum as 3 and 1 do', we must apprehend by direct and immediate intuition not only that '2 and 2 = 4' and that '3+1 = 4', but further that the conclusion is a necessary logical consequence of these two propositions. But the apprehension of these links is itself an apprehension effected by intellectual intuition, for 'whenever we have immediately inferred one fact from another, they have already been reduced, if the inference was evident, to a intuition'.[1] The hypothetical clause is of no importance to our point, for if the inference were not evident, there could be no real inference in the Cartesian sense. The important phrase is that we are told that, in the case of immediate inference, deduction is reduced to a true intuition. This unequivocal statement of Rule 7 is reaffirmed in Rule 11 where it is stated that 'the simple deduction of one thing from another is effected by means of an intellectual intuition'.[2] Again, in Rule 12 the same doctrine is set forth: 'it is clear that the mind's intuition extends to the apprehension first, of all simple natures, secondly, of their necessary connexions one with the other, and lastly, of everything else which the intellect experiences with precision as being either in the intellect itself or in the imagination'.[3] The doctrine put forward in these passages can but indicate that the view of Descartes, whatever his hesitancy, is that, in essence and fundamentally, deduction is identical with intellectual intuition of which it is ultimately the full development or, as Bradley might have said, the necessary self-development. It may be properly argued, as Joachim points out, that 'there is a sense in which Deductio is, or collapses into, immediate cognizance',[4] or, as another Cartesian scholar remarks, 'deduction, then, is not the source of a special kind of knowledge but simply the process by which intuition extends so as to take in the complex that at first sight appears to lie outside its sphere'.[5] The con-

[1] 'Nam quaecumque una ex aliis immediate deduximus, si illatio illa fuerit evidens, illa ad verum intuitum jam sunt reducta' (AT. x. 389^{15-17}).

[2] 'Simplicem vero deductionem unius rei ex altera ibidem diximus fieri per intuitum' (ibid. 407^{13-14}).

[3] Ibid. 425^{14-18}.

[4] *Logical Studies*, p. 45, n. 1. Hamelin, in his *Système de Descartes*, 2nd edit., p. 82, notes: 'Par conséquent, tirer une conséquence, c'est encore "voir", et la déduction est essentiellement une intuition.' Hamelin, in a few terse pages, puts forward an interpretation of Descartes's doctrine along the lines I have followed.

[5] N. Kemp Smith, *Studies in Cartesian Philosophy*, p. 33.

sideration of other texts on the relation of deduction and intellectual intuition definitely confirms the fundamental identity of the twofold activity of mind.

In Rule 3 we are told that 'those propositions which are immediately inferred from first principles may be said to be known at one time by intellectual intuition, at another time by deduction, according to the point of view taken. The first principles themselves, however, can only be said to be known by intellectual intuition and the remote conclusions by deduction.'[1] An example of a proposition immediately inferred from first principles is given in the preceding sentence. The instance is the oft-repeated one '2+2 = 3+1', because '2+2 = 4' and '3+1 = 4'. If we lay stress on the movement of the inference and emphasize the successive apprehension of the knowable elements; if we pay attention to the advance of the intellect from the self-evident data to the consequence, then we call this type of knowledge deduction. But if, on the other hand, we think of the movement as subordinated to, and concentrated in, the immediate apprehension of the connected and articulated knowable as a whole, then we call our knowledge intellectual intuition. In a simple straightforward case like the instance used above, the conclusion, when we have grasped it, is itself invested with the clarity and distinctness which characterize the self-evident truth. It is an act of intellectual vision. The procedure is called deduction if we lay stress on the movement of illation by which we develop the articulated content—in this instance, 2+2 = 3+1 conceived as flowing logically by a continuous movement of thought from 2+2 = 4 and 3+1 = 4. It is called intellectual intuition if we lay stress on the immediate apprehension of the articulated content in which the illative movement culminates and rests. The matter is summed up in Rule 11:

> As for deduction, if we are thinking of how the process works, it does not appear to occur simultaneously but involves a certain movement on the part of the mind when it infers one thing from another. We were justified therefore in distinguishing in that Rule [i.e. in Rule 3] deduction from intellectual intuition. But if we wish to consider deduction as something completed, as we did in Rule 7, then it no longer signifies a movement but rather the completion of a movement. Hence

[1] AT. x. 370[9–15].

when the deduction [i.e. the content deduced] is simple and manifest, we suppose it to be seen by intellectual intuition, but when it is complicated and involved we take a different view.[1]

In a sense the distinction seems to be based on an external characteristic in so far as 'the point of view' of the observer, who can consider deduction either as a movement of illation or as a completed process, does not obviously constitute an essential or intrinsic characteristic of the activity itself. It would appear that intellectual intuition and deduction were mainly distinguished through their psychological aspect. There is, however, more in this distinction than a mere matter of emphasis on our part. There are, Descartes maintains, certain basic truths, knowable and known by immediate apprehension alone, and it is these self-evident data which are the fundamental basis of all knowledge. There are also, at the other extreme, certain propositions which are described as 'remote' and which are logical consequences of the self-evident data and are linked to them by 'the long chains of reasoning'. The apprehension of these conclusions, in so far as they remain 'remote', is, and remains, illative. It is the reconstruction of the chain of reasoning which allows us not to 'see' but to infer their necessary dependence on the immediately known self-evident data. Somewhere between these two extremes, varying according to the capabilities and acquired knowledge of the individual mind, lies an intermediary stage in which the 'knowing' is both immediate and mediate, while the object of the 'knowing' shows itself both as self-evident and inferentially necessitated. This apparent paradox arises because in this intermediate case we may consider the 'knowing' as either intuitional or inferential, depending upon whether we fix our attention on the immediacy of the self-evidence of the data or on their inferential necessity, and, in certain cases, we can fix our attention on either aspect of what we grasp as a single whole. It is of course obvious that Descartes is only considering the most favourable cases of deduction, where the connexions of the data are uncomplicated and the necessary links and their complete articulation can be grasped in a single act of mind. The change of emphasis is a change of focus of attention within the act

[1] AT. x. 407^{18}–408^{5}.

which must nevertheless remain attentive to data, links, and conclusions.

There are obviously more complicated cases in which the mind, humanly limited in its power of attention, is not able to hold together in one single vision all the multiple stages of the deductive process. There are cases in which the illative movement, owing to the multiplicity and complexity of the steps, cannot be merged or concentrated in an immediate apprehension of the whole content in all its complex articulation. A truth may not be in itself evident and of such a nature as never to be apprehended immediately as self-evident. It may nevertheless be the necessary consequent of an unbroken chain of self-evident truths, a multiplicity of truths, and our thought may be able to pass through this network of truths with unbroken movement. Every step of the process of reasoning is grasped by an act of intellectual intuition, and the final step, the conclusion, apprehended in the same manner is true without any possibility of doubt. Yet the conclusion, that is, the necessary link or connexion between the last and the first step, or the necessary dependence of the consequent on the logical interconnexions of all its self-evident grounds, is not present to the mind with the self-evidence of an intellectual intuition. Descartes himself considers this case as 'parallel to the knowledge by which we recognize the connexion of the last link of a long chain with the first. Even though we cannot embrace in one and the same act of vision all the intermediate links on which that connexion depends, we know it, provided we have seen successively all the links and their several connexions and provided we remember that each successive link, from first to last, was connected with its neighbour.'[1] In such a case our knowledge is called deduction and it is distinguished from intellectual intuition because it is a genuine movement or succession and also because its certainty depends on memory.

The discussion may appear somewhat circular in that we have come back on our tracks to the dichotomous view of

[1] 'Non aliter quam longae alicujus catenae extremum cum primo connecti cognoscimus, etiamsi uno eodemque oculorum intuitu non omnes intermedios, a quibus dependet illa connexio, contemplemur, modo illos perlustravimus successive, et singulos proximis a primo ad ultimum adhaerere recordemur' (ibid. 369^{26}–370^{4}).

the operations of mind. Descartes considers that the intermediate stage of a science or particular branch of knowledge is, from different points of view, the object of both intellectual intuition and deduction, and sometimes even he seems to imply that this is also the case of the extremes, namely, that first principles require deduction and the remote consequences require intuition. It must be remembered that even in the privileged instance of the indubitable primary datum of the *Cogito*, that which is clearly and distinctly apprehended is the relation of two constituents in a single whole, though this is not an illative process in the ordinary sense of the word.[1] On the whole, Descartes tends to emphasize the distinction between the two operations of the *vis cognoscens*. Both, however, form necessary correlative elements in the total activity which emerges as genuine knowledge, yet within this total activity they remain separate, each still being what it was, or was supposed to be, when it constituted knowledge by itself, instead of co-operating with the other and constituting knowledge only in this co-operation. Both functions are necessary to give a complete account of the act of knowledge but both must be taken to mean each separately and severally and the other. To know, for instance, that the equality of $3+1$ and $2+2$ is a necessary consequence of $3+1=4$ and $2+2=4$, the intellect must apprehend that $2+2=4$ and that $3+1=4$. It has also to deduce the equality of $3+1$ and $2+2$ and, finally, it has to apprehend that the conclusion is a necessary consequence of the premisses. Deduction, then, stands apart from, and is proportionately distinguished from, intellectual intuition in the degree that the immediacy of the latter cannot be retained. The distinction does not always lie in the simplicity or complexity of their objects, since the object of intuition is, in one sense, a complexity, but rather in the degree of complexity which, in the case of the deduction involving 'remote' conclusions, may necessitate the use of memory in order to retain and preserve the certainty of the first or some intermediary truth and its necessary connexion. It is clear then that the methodological problem which arises from this fact is to reduce, or entirely remove, from deduction the dependence

[1] See above, p. 76.

on memory which conditions its functioning in the more complex cases and the solution to this problem is found by Descartes in his methodological principle of enumeration by which the progressive steps or series of steps of a deduction are reviewed. The object of this revision is 'to learn to pass so quickly from the first to the last' that we arrive at 'a simultaneous vision of the whole process'. The ultimate aim and final goal of the method is to merge the whole series of elements and links into a single articulated system of truth which is the object of a single act of intellectual vision. Before, however, considering the full implications of this bold and sweeping theory, it is necessary first to consider the question of the links or connexions in and through which the systems of truth are articulated.

§ 3. *'Necessary Connexions'*

There is a passage in the *Regulae* in which Descartes discusses, in a summary manner, the links between the self-evident truths, between the elements or steps of the deduction. These links he discusses under the name of *conjunctio necessaria*, the 'necessary connexion'. In Rule 12 he distinguishes the various classes of simple natures. All of these simple natures, as we have seen, figure, extension, motion, duration, knowledge, doubt, &c., are known *per se* and are wholly free from falsity. He then goes on to argue that they are joined together or compounded with one another and this conjunction must be either necessary or contingent. 'It is necessary', he says, 'when one is implied in the other concept in a confused sort of way so that we cannot conceive either of them distinctly, if we judge them to be disjoined from one another.'[1] This is, at a first reading, a somewhat surprising definition but it fits the general scheme of the Cartesian deduction. It means this. Figure, for instance, is necessarily conjoined with extension, or motion is necessarily conjoined with duration, for it is manifestly impossible to conceive a figure devoid of all

[1] 'Necessaria est, cum una in alterius conceptu confusa quadem ratione ita implicatur, ut non possimus alterutram distincte concipere, si ab invicem sejunctas esse judicemus' (AT. x. 421$^{5-8}$). This passage 'a parfois déconcerté les commentateurs', notes Laporte in his excellent paper on 'L'Idée de liaison nécessaire chez Descartes,' in *Travaux du IX^{e} Congrès de Philosophie*, ii. 12. See also by the same author, *Le Rationalisme de Descartes*, pp. 94–95 and 102–3.

extension or a movement without any duration. We are given another example. If we say that $4+3 = 7$, then 'this composition is necessary for we do not conceive the number 7 distinctly, unless in our conception of it we include in a confused sort of way the number 3 and the number 4'.[1] Or another instance: if Socrates says that he doubts, it follows necessarily that he knows as a *minimum cognoscibile* that he doubts. He knows also that something can either be true or false, for all these notions are necessarily conjoined to the nature of doubt.

From the examples given one may gather that necessary connexion, in Descartes's use of the term, is akin to logical implication in the sense that one concept presupposes another, figure presupposes extension, movement presupposes duration, &c. But the text contains also the disconcerting statement that a necessary connexion is based on a logical inclusion which is apprehended in a confused sort of way, *confusa quadam ratione*, and the examples suggest that the number 7 is not conceived distinctly. Both statements require some explanation.

The use of the expression 'in a confused sort of way' would seem to indicate that Descartes is here concerned with a distinction of reason, *distinctio rationis*, as opposed to a real distinction, *distinctio realis*.[2] Figure, for instance, is necessarily conjoined with extension for it is impossible to conceive a figure devoid of all extension. Yet it must be admitted that 'to conceive a figure' has some meaning and a geometer can study the nature and properties of a figure, a triangle, for instance, without considering the nature of extension as such. If I, as a geometer, study the nature and properties of the triangle, I do this, says Descartes, by a process of abstraction,

[1] 'Ita etiam si dico quattuor et tria sunt septem, haec compositio necessaria est: neque enim septenarium distincte concipimus, nisi in illo ternarium et quaternium confusa quidem ratione includamus' (AT. x. 421[11–15]). Sirven justly remarks: 'Remarquons actuellement que Descartes semble se souvenir, peut-être un peu trop à son point de vue, de l'affirmation des Coïmbrois: *Etiam naturas simplices sub cognitionem confusam actualem cadere posse*.' *Les Années d'apprentissage*, p. 402, n. 3. See also the quotation from the Coimbricenses in Gilson's *Index scolastico-cartésien*, no. 313, pp. 200–1.

[2] In terms of the Scholastic distinctions, *distinctio realis major* is between *res* and *res*, *minor* between *res* and its *modus*; *distinctio rationis ratiocinatae* implies a *fundamentum in re; distinctio rationis ratiocinantis* is *sine fundamento in re*.

per abstractionem intellectus, that is to say, 'by turning my thought away from a part of that which is comprised in the wider idea, in order to apply my thought in a better manner and to make it more attentive to the other part. This is the case when I consider a figure, without thinking of the substance or of the extension of which it is the shape.'[1] In all abstraction, one of the terms is omitted without thereby being denied.[2] But an idea in which some of the elements, essential to its complete clarity, are missing is only partially clear, and that which is only partially clear is apprehended in a confused sort of way. The confusedness of the idea is due to the fact that certain elements are missing and the total composite is therefore incomplete; this incompleteness is caused by abstraction on the part of the apprehending mind. Two ideas are necessarily conjoined when they can only be separated by abstraction; they are not *really* distinct, that is, they cannot be posited independently one of the other; they cannot be conceived one without the other, except by a mental abstraction which conceives the whole imperfectly by fixing the attention on one of the elements to the exclusion of the other. In the case of a whole in which the elements are necessarily linked, the distinction made is a distinction of reason, a *distinctio rationis*. 'This distinction', notes Descartes in the *Principia*, 'is made obvious from the fact that we cannot have a clear and distinct idea of a substance if we exclude such an attribute; nor can we have a clear idea of one of the attributes if we separate it from the other.'[3] The *confusa quadam ratione* is therefore merely indicative of the result of distinguishing mentally that which is really one. It is the Scholastic *distinctio rationis ratiocinatae* which has necessarily a *fundamentum in re*.

Descartes must therefore be understood to be arguing that

[1] Letter to P. Gibieuf, 1642 (AT. iii. 475^{1-15}). Descartes adds later in the same letter: 'Mais l'idée d'une substance étendue et figurée est complète, à cause que je la puis concevoir toute seule et nier d'elle toutes les autres choses dont j'ai des idées' (ibid. 475^{18-23}).

[2] Cf. Rule 14: 'Item si agamus de figura, putemus nos agere de subjecto extenso, sub hac tantum ratione concepto, quod sit figuratum' (AT. x. 446^{3-5}). See also *Resp. ad II Obj.* (AT. vii. 149^{9-11}).

[3] *Principia* i. 62 (AT. viii. 30^{10-14}) and the letter to de Launay, 1641: 'Lorsque nous concevons une chose sans l'autre, cela se fait seulement par une abstraction de l'esprit, ou bien à cause que les choses sont véritablement diverses' (AT. iii. 420^{30}–421^{4}).

the simple natures, known *per se*, such as movement, duration, and figure, are the elements or implicates of wider conceptual wholes and it is these wider conceptual wholes which are clear and distinct and constitute the *res per se notae*. These wholes are in a sense completely and adequately known; the only qualification being that, as is noted in the *Reply to the Fourth Objections*, strictly speaking only God knows that He has complete and adequate knowledge of all things.[1] But, as Descartes goes on to point out to Arnauld, it is one thing for knowledge to be entirely adequate, which would presuppose a certainty guaranteed by a revelation of God, and quite another matter for our knowledge of these simple natures to have sufficient adequacy to enable us to see that we ourselves have not rendered it inadequate by intellectual abstraction. And he concludes: 'In a similar way, when I have said that a thing must be comprehended in a *complete* manner, I did not mean that the comprehension of it must be adequate, but merely that we have a knowledge of it sufficient to allow us to know that it is complete.'[2] Within these limitations, our knowledge of these conceptual wholes, which might in so far as they are complete be called 'concrete', is the necessary condition of our knowledge of the elementary simple data. There are, in other words, degrees of abstraction to which correspond degrees of simplicity. Movement is 'concrete' relatively to its speed or direction; it is 'abstract' relatively to the body which is in movement.[3] Through the various degrees of 'concreteness', following in order the necessary connexion of the ideas, the mind can arrive methodically at the ultimate term, or terms, of the whole deductive chain, at the *maxime absolutum* of Rule 6 which is *primo* and *per se* apprehended independently of all others. The line of thought then we should follow, basing ourselves on the implications of this doctrine, would lead us to the recognition of at least two clear and distinct

[1] *Resp. ad IV Obj.* (AT. vii. 220^{10-11}). [2] Ibid. 221^{11-14}.

[3] Cf. a letter sent to Mersenne for Hobbes in 1641: 'Quand j'ai dit que le mouvement était à la détermination du mouvement comme le corps plan est à son plan ou à sa surface, je n'ai point entendu par là faire comparaison entre le mouvement et le corps comme entre deux substances, mais seulement comme entre *deux choses concrètes*, pour montrer qu'elles étaient différentes de celles dont on pouvait faire l'abstraction' (AT. iii. 355^{18}–356^{2}). The italics are mine.

ideas, two self-evident data, complete and 'concrete', two really simple natures, namely, *cogitatio* and *extensio*.[1] Each of these can only be in and by itself, but they are the logical basis of explanation for a series of other ideas, less complete, and accordingly less 'concrete', linked by necessary connexions in a network of interconnected ideas. This network of necessarily connected ideas, or implicates, constitutes a conceptual system[2] and it would follow that, given two fundamental ideas, thought and extension, there would be two systems, the one of matter or extension, the physical world, and the other of mind or thought, the spiritual world. In both cases, the self-evidence of each system is the obverse of an equally essential logical coherence deploying itself in a deductive network. In themselves these systems do not communicate except in the quasi-mysterious union of body and soul, and the ideas which express their nature are not dependent on any other idea, except the most general notion of 'being'. They are, in the fullest sense, primitive notions. The physical world is, however, transparent to our intellectual intuition, as, of course, it is to our sense-perception, through *ideae adventitiae*, and to our imagination through *ideae factae vel fictitiae*. We can have a clear and distinct conception of the physical world because, and in so far as, our thought inevitably distinguishes therein certain features and definite characters and, in distinguishing them, is driven by logical illation from one to the other and forced to recognize these characters as essential and integral elements of the intelligible whole.

The other disconcerting remark which was noted was to the effect that if we say that $4+3=7$, then this composition is necessary 'for we do not conceive the number 7 distinctly, unless we include in a confused sort of way the number 3 and the number 4'. Yet, as many critics have not missed the opportunity of pointing out, $2+2=4$ and $3+1=4$ are

[1] At least two, since Descartes adds a third, the union of body and soul. Cf. a letter to Princess Elizabeth in 1643 (AT. iii. 665^{9-24}). I have attempted to give some account of the system *Cogitatio* and its relations to its implicates, or in more Cartesian language *substantia cogitans* to its *modi*, in an article 'Cogitatio in Descartes', in the special number of the *Rivista di filosofia neoscolastica*, 1937, pp. 41–52.

[2] Laporte, in *Le Rationalisme de Descartes*, p. 113, speaks of these ideas as 'des assemblages, eux-mêmes imbriqués dans des autres assemblages plus vastes'. The word suggests mere aggregates, whereas they must be coherent logical wholes.

almost monotonously quoted by Descartes as examples of the objects of intellectual intuition and it is difficult to see at first sight why 3+4 = 7 should not be a similar instance. Whereas it seems to be used, in this passage of Rule 12, as an instance of the composition of simple natures which is necessary because the distinct conception of 7 involves confusedly the conception of 3 and 4. It seems, say the critics, impossible to maintain that 3 and 4 are simple natures and 7 is not. On the other hand, if 7 is a simple nature, it is admittedly such because its distinct conception, which is also its essential definition, includes the conceptions of 3 and 4. This can hardly mean anything less than the recognition of a movement within, and at the same time constitutive of, the clear intellectual apprehension of the definition of the number 7. What then can be this movement within the intellectual intuition itself? The answer, in Descartes's own terms, is [a movement] from the implicit, contained in the apprehension, to the explicit; from that which is contained in the original content of the intuition but which, as he explains to Mersenne,[1] 'I had not paid attention to', that is, 'to which I had closed my attention *per abstractionem mentis*'. The simplicity of the idea, or simple nature, according to the point of view from which we consider it, does not exclude inner distinction but in some cases this inner distinction may not be apparent. When this implicit distinction becomes explicit, there is a true movement of thought within intuition itself. Potential knowledge becomes actual knowledge, although Descartes himself would have refused to use such phraseology. Deduction, in such a case, is the explicitation of the intellectual intuition. This is the sense of the *ergo* in the great intuitions of Cartesian metaphysics. It is equally the point most ignored by the critics of Descartes, from Hobbes to the present day. It must always be borne in mind that, in the intentions of its exponent, the logical process involved in the Cartesian deduction is 'a pure illation of mind which, in the simplest

[1] 'Sed quod ex Idea Innata aliquid eruam, quod quidem in ea *implicite* continebatur, sed tam prius in ipsa non advertebam, ut ex Idea Trianguli, quod ejus tres anguli sint aequales duobus rectis, aut ex Idea Dei, quod existat, etc., tantum abest ut sit petitio principii, quin potius est, etiam secundum Aristotelem, modus demonstrandi omnium perfectissimus, nempe in quo vera rei definitio habetur pro medio.' Letter of 1641 (AT. iii. 383$^{13-20}$). My italics.

cases, is not distinguishable from the immediate intuition of a necessary nexus within a content'.[1] Contained in the simple nature of the intellectual intuition are the seeds which logically grow into the deductive science, as the whole of metaphysics springs from the *Cogito*.

[1] Joachim, *The Nature of Truth*, 2nd edit., p. 71, n. 2. Or as Brunschwicg notes: 'Il faudra seulement demander à la déduction de tendre en quelque sorte vers l'intuition, en se resserrant et en se concentrant' ('Études sur Descartes', *Rev. méta. et morale*, 1937, p. 3).

VII

CARTESIAN DEDUCTION AND THE SYLLOGISM

§ 1. *'The Perfect Method of Demonstration'*

In the letter to Mersenne, quoted in the last paragraph of the preceding chapter, Descartes defends his theory of deduction against the objection that it would involve a *petitio principii* and adds that, far from being such a paralogism, it is 'the most perfect method of demonstration, even according to Aristotle, namely, the method in which the true definition of a thing is used as a middle term'. The perfect demonstration is a movement along 'a chain of reasoning', one of Descartes's favourite metaphors,[1] within a network of necessarily connected links. Each link is a self-evident truth known in an intellectual intuition, and the movement from link to link, or rather the connexion of the second link with the first, when the movement has led the mind to the second, must be self-evident. Such is the elementary sketch of deduction which Descartes usually sets before his readers when writing in a summary and popular manner, as, for instance, in the *Discours*, and it should be noted, as a caution, that in some of these summary accounts simplification is pushed to the limit almost of real misrepresentation and the 'continuous and uninterrupted act of thought' which constitutes the vital activity of the mind in the deductive process is passed over.[2] Yet it is this movement which he considers as characteristic of the most perfect form of demonstration and, in principle, identical with the perfect demonstration, the ἀπόδειξις, as Aristotle conceived it. The perfect demonstration which sets out to show that α must be ω, has to resolve the interval between α and ω into a continuous succession of self-evident links, so that α immediately involves β, β involves γ, γ involves

[1] See, for instance, in the *Discours*: 'Ces longues chaînes de raisons' (AT. vi. 19^{6-7}); or Rule 3: 'non aliter quam longae alicujus catenae' (AT. x. 369^{26-27}).

[2] As, for example, in the 'summary of all that has to be observed in order to philosophize correctly' in *Principia*, i. 75 (AT. viii (Part I), 38–39).

δ, &c., until there is nowhere, within the stretch from α to ω, a single link missing, or, to change the metaphor to another often used by Descartes, until there is no step to be traversed by the mind which is not itself a self-evident step.

It would have been decidedly more helpful if Descartes had developed more fully his theory of demonstration and if he had said where it agreed with, or differed from, the perfect ἀπόδειξις of Aristotle.[1] For he suggests, in this letter to Mersenne, as he does on other occasions, that his own theory is a development of the Aristotelian doctrines, which he presumably learnt at La Flèche, while at the same time making it quite patent that he disagrees fundamentally with the syllogistic logic, based at least in theory on the *Prior* and *Posterior Analytics*, which was the *Dialectica* of the Scholastics. Unfortunately he never does enlarge upon this comparison nor even develop his own doctrine. Perhaps it was because, as one commentator remarks, he is in fact not interested

> in the explanation of deduction, but in its liberation from the uncertainties of memory and its absorption into intuition itself. The process by which this transformation is practically accomplished is that of mental *enumeration*, whereby the mind, by persistently repeating over to itself the various steps in the deductive process, gradually acquires the power to pass so rapidly along the series of steps that the passage is quasi-instantaneous.[2]

In a sense it is true that Descartes is more interested in the process which he calls *enumeratio sive inductio*, which will be discussed in the next chapter, than in giving any details of his theory of deduction or in setting down in systematic form laws or principles of deductive reasoning. In fact, he quite clearly does not believe that it is possible, or desirable, to reduce deductive reasoning to any formal methods of reasoning

[1] 'The true definition of a thing being used as a middle term' might be taken as suggesting that each step is, so to speak, an ἄμεσον διάστημα. We should have to remember that 'Aristotle holds that even a mediated connexion, if not recognized as such, is the object of opinion and not of knowledge'. G. R. G. Mure, *Aristotle*, p. 209, n. 2. In his discussion of the συλλογισμὸς τοῦ τί ἔστιν, Hamelin comments: 'On pourrait dire que la démonstration est la définition *déployée* et que la définition est la démonstration concentrée.' *Le Système d'Aristote*, 2nd edit., p. 253. It is at this point that the Aristotelian and Cartesian methods of demonstration tend to meet; 'in qua vera rei definitio habetur pro medio', as Descartes would have been told at school.

[2] Boyce Gibson, *Mind*, N.S., vol. vii, no. 26, p. 151.

and he rejects the formulas of the traditional logic which, according to his view, might be supposed to have been invented to eliminate the need for attention, perspicacity, and all the other psychological requisites of intellectual intuition and deduction, in order to substitute for them a quasi-mechanical mental device. In all mental activity, Rule 10 warns us, 'we must be careful lest our reason should take a holiday while we are examining the truth of anything'.[1] The essential object of the method is to train the mind by careful and experimental use of the rules. These rules, if observed and practised, will perfect our intellect and sharpen our natural ability, our *ingenium*. They will create, through constant usage, a habit of mind which will enable one to pass more quickly and surely through the links of the demonstration, hold the links in one's mind with more sureness, and work thus towards the final aim of the demonstration, the collection and concentration of the links in a single act of intellectual vision. But all this is dependent on the constant practice of careful, attentive thought and not on the mere understanding of a theory of the precepts of methodology. Descartes is quite frank and sincere about this in the *Discours*: the practice 'of passing onward step by step from what is easy to the more difficult, will benefit more than my instructions'.[2]

§ 2. *Criticism of the Syllogism*

Descartes's attitude to the syllogistic method of reasoning, as practised by the Scholastics of his day, is mainly a negative one when it is not frankly hostile. He admits with condescension in Rule 2 that 'these weapons of the Schoolmen, so well suited for polemics'—and Descartes, unlike his friend Arnauld, detested polemics—are suitable for exercising the wits of youth and that it was definitely better for their minds to be moulded by some sort of opinion than that they should be left to their own devices. Otherwise these young people might 'stray into some abyss' and the path which they are taught to follow, 'even though it may diverge from the truth, is prob-

[1] AT. x. 406[9–10].

[2] Part VI (AT. vi. 72[5–7]). See also the injunction of Rule 1: 'Cogitet tantum de naturali rationis lumine agendo' (AT. x. 361[18–19]).

ably safer in that it has the approval of prudent people'.[1] It has, in a way, the same function to fulfil as the provisional code of ethics in the *Discours*. He also admits that syllogistic argument may have some use at least as a formal method of exposition to present in cogent form truths which have already been discovered. He uses, somewhat reluctantly, the syllogistic form in the *Reply to the Second Objections*, where the 'arguments demonstrating the existence of God and the distinction between soul and body' are drawn up *more geometrico demonstratae* which he describes earlier as 'something appended' in the synthetic style.[2] But it is made clear that this synthetic method, although it may force even the hostile and obstinate reader to give his assent to the argument and the body of the doctrine, is not as satisfactory as what is called the method of analysis, which is that used in the *Meditationes* and which is 'the best and truest method of teaching',[3] because the synthetic method of the syllogism 'does not show the way in which the matter taught was discovered'. In the *Regulae* Descartes is less polite and less diplomatic than to P. Bourdin, but he probably speaks his mind more fully: 'The ordinary dialectic (*vulgarem Dialecticam*) is utterly useless for those who wish to investigate the truth of things. Its only possible use is to explain sometimes more easily to others the truths we have already discovered; hence it should be transferred from Philosophy to Rhetoric.'[4]

This contemptuous rejection of the syllogism as a means of discovering any new truth is based on the view that all syllogistic reasoning is sterile and useless. It is even useless for deducing a conclusion from a truth already known since the human mind is perfectly capable of so deducing without any need or knowledge of formal laws. 'As for those other mental operations', we are told in Rule 4, 'which Dialectics tries to

[1] AT. x. 363^{21}–364^{9}.

[2] 'Pauca quaedem synthetico stilo hic subjungam' (AT. vii. 159^{13-15}).

[3] Cf. AT. vii. 156. Yet Descartes writes in a different tone to P. Bourdin: 'Me non improbare syllogismos, nec eorum veterem formam deturpare, satis judicant mea scripta, in quibus semper iis usus sum, cum fuit opus' (*Resp. ad VII Obj.* (AT. vii. 544^{4-6})). The AT. text has 'earum' which is an obvious error.

[4] Rule 10 (AT. x. 406^{21-26}). See also the *Discours*, Part II: 'Je pris garde que, pour la logique, ses syllogismes et la plupart de ses autres instructions servent plutôt à expliquer à autrui les choses qu'on sait, ou même, comme l'art de Lulle, à parler sans jugement de celles qu'on ignore, qu'à les apprendre' (AT. vi. 17^{15-20}).

direct by making use of these prior ones [i.e. intellectual intuition and deduction], they are quite useless here or rather they should be considered as obstacles, since nothing can be added to the pure light of reason which does not, in some way, obscure it.'[1] Moreover, those who uphold this formal logic 'cannot formulate any valid syllogism which yields a true conclusion, without having first obtained its matter, that is, unless they have already found out the very truth, which they deduce in that syllogism. Whence it is clear that they themselves learn nothing new from a formula of this kind.'[2] And this charge made in Rule 10 is repeated in Rule 13, where Descartes argues the same point that 'in teaching the forms of the syllogism, their terms or matter is assumed to be known'.[3] He argues again in Rule 14 that

> in every train of reasoning we arrive at a precise knowledge of the truth merely by comparison. Here is an instance: all *A* is *B*, all *B* is *C*, therefore all *A* is *C*. Here we compare with one another that which we are searching for and that which is given (*quaesitum et datum*), namely *A* and *C*, in respect of the fact that each is *B*, etc. But because, as we have often pointed out, the forms of the syllogism are of no use in apprehending the truth about things, it would be better for the reader to reject them altogether and to conceive that all knowledge whatsoever, other than that which consists in the simple and pure intellectual intuition of the single object, is acquired by the comparison of two or more things with each other.[4]

It is always the same point that Descartes is making: the

[1] AT. x. 372^{22}–373^{2}. I am following Lachelier's emendation of the text.

[2] 'Nullum posse Dialecticos syllogismum arte formare, qui verum concludat, nisi prius ejusdem materiam habuerint, id est, nisi eamdem veritatem, quae in illo deducitur, jam cognoverint. Unde patet illos ipsos ex tali forma nihil novi percipere' (AT. x. 406^{16-20}).

[3] 'Ad syllogismorum formas tradendas, eorumdem terminos, sive materiam cognitam esse supponunt' (AT. x. 430^{12-15}). Locke, in the *Essay*, iv. 16, 6, makes this general criticism of the syllogism: 'This way of reasoning discovers no new proofs, but is the art of marshalling and ranging the old ones we have already.' He calls it 'the art of fencing' with the little knowledge we may have.

[4] 'Adeo ut in omni ratiocinatione per comparationem tantum veritatem praecise cognoscamus. Ver. gr., hic: omne A est B, omne B est C, ergo omne A est C; comparantur inter se quaesitum et datum, nempe A et C, secundum hoc quod utrumque sit B, etc. Sed quia, et saepe jam monuimus, syllogismorum formae nihil juvant ad rerum veritatem percipiendam, proderit lectori, si illis plane rejectis, concipiat omnem omnino cognitionem quae non habetur per simplicem et purum unius rei solitariae intuitum, haberi per comparationem duorum aut plurium inter se' (AT. x. 439^{19}–440^{5}).

syllogism is on his view useless because nothing new can be discovered by its means. The truth must be known first and, if so, then the syllogism is a mere rhetorical device of exposition. It will be noted that he always emphasizes the formal aspect of syllogistic reasoning and refers often, as in the last quotations from the *Regulae*, to the *syllogismorum formae*. It is the formal aspect of Scholastic logic which Descartes is most anxious that his readers should discard and reject as sterile. His reason is clear for he is directly attacking the idea of a science of method which assumes, or is said by him to assume, that it can give rules and precepts for reasoning which are independent altogether of the nature of the content about which the reasoning occurs. This runs counter to the fundamental precepts of the method which he is outlining and in Rule 10 he lays bare the roots of his antagonism. The formal logicians of the Schools 'dictate certain formulas of argument which lead to a conclusion with such necessity that, if reason entrusts itself to them, even though it loses interest and does not pay a careful and close attention to the inference, it is able nevertheless in the meantime to arrive at a sure conclusion by virtue of the form'.[1] And of course if the mind does 'take a holiday', *ferietur*, then obviously it may occur that formal reasoning can lead to absurdities for 'it is not difficult', Descartes writes to Mersenne, 'to build absurd principles from which we can deduce truths which have been learnt elsewhere'.[2] Descartes's opponents on this issue were also aware of the axiom *ex falso sequitur quodlibet*. But it is not merely the possibility of such absurdities which is in the forefront of Descartes's mind when he repeatedly urges his readers to reject formal syllogistic reasoning. His preoccupation is a deeper one. He is very anxious to distinguish his own method, which he will qualify later as a *vera Logica*, and this method has, as its corner-stone, the concrete experience of the intellectual intuition, the apprehension of the particular truth in its own self-evidence. He is anxious to distinguish

[1] AT. x. 405^{25}–406^{2}.

[2] Letter of 1641: 'Il n'est pas malaisé de bâtir des principes absurdes dont on puisse conclure des vérités qu'on a apprises d'ailleurs. Comme si je disais: *omnis equus est rationalis, omnis homo est equus, omnis homo est rationalis*' (AT. iii. 339^{25}–340^{2}). The Schoolmen would have made a distinction on the basis that *ex falso non potest per se sequi verum*, but merely *per accidens*.

this method from the formalistic doctrine that the syllogism is conclusive *vi formae* and the intellect can by virtue of the form arrive at a conclusion as proved and true, without any need to investigate the evidence of the steps, beyond being assured of the validity of the formal reasoning. The Cartesian method should, if correctly followed, guarantee that the self-evident content of the intuition should *ipso facto* engender the form of the reasoning, which is nothing else but the human mind itself. The form in itself is nothing but the movement of the mind itself, the *ingenii motus*, working out the links of the self-evident data by its analysis and synthesis of the necessary connexions of the ideas. There can be no purely formal element of reasoning which is reducible to a logical formula applicable to all reasoning, independent of the content.

Historians of philosophy have pointed out that the Cartesian revolution in logic is due in no small measure to the rejection by Descartes of the Scholastic logic of extension, or denotation, and the substitution for it of a logic based on comprehension or connotation.[1] This remark is true provided one bears in mind, as Gilson notes, that Descartes himself would not present his case in these terms. The change is not one by which Descartes interprets in comprehension concepts which the Schoolmen interpret in extension. The very objects on which the mind works are changed, for it is not a question of concepts but of simple natures intuitively perceived. It is quite clear that in the Cartesian view the conclusions of a deduction are not extensionally contained in the antecedent propositions, and in the controversy which grew around the *Cogito, ergo sum* we see Descartes affirming and reaffirming this point while rebutting the accusation that his fundamental datum involved syllogistic reasoning. This controversy has already been touched upon in discussing simple natures.[2] A detailed study of the *Cogito, ergo sum* must come in its place in a study of the *Meditationes* in another volume, but it may perhaps be helpful to consider some of the points

[1] Cf. Hamelin, *Le Système de Descartes*, 2nd edit., p. 89: 'C'est donc contre la logique de l'extension et en particulier contre le syllogisme interprété en extension que Descartes s'est élevé.' See also Sirven, *Les Années d'apprentissage*, p. 389; Gilson, *Commentaire sur le Discours*, pp. 184–5.

[2] See above, p. 76 and p. 80, and below, p. 132.

raised in this controversy which directly throw light on the relation of the Cartesian deduction and syllogistic reasoning.

§ 3. *Implicit and Explicit*

By a syllogism Descartes usually means a form of demonstration in which a particular subject is subsumed under a general or universal principle or rule: for instance, a demonstration of the type, 'All self-conscious beings exist, I am a self-conscious being, therefore I exist.' It may be noted that Descartes does not seem aware that this form of argument is implicitly rejected by Aristotle in the *Posterior Analytics* as not constituting a genuine inference at all. However, it is this type of syllogism which Gassendi and the authors of the *Second Objections* assert to be implied in the argument of the *Second Meditation*. In the various answers which Descartes, with considerable patience, makes to his critics, answers which do not always seem to be read with equal patience by some modern critics, he has little difficulty in showing that far from 'that *qua* self-consciousness, I exist' being deduced from the supposed universal major, my knowledge of the latter, if I have it at all, is acquired by illation from my immediate experience that my own self-consciousness implies my own existence. This point he drives home in a letter to Clerselier in which he is referring to the reiterated objections of Gassendi:

> The most considerable error is that this writer supposes that the knowledge of particular propositions should always be deduced from universal propositions, following the order of the syllogisms of the Dialectic. In all this matter, he shows that he knows very little about the way in which truth should be sought for. For it is certain that, in order to discover truth, we should always begin with particular notions, in order to reach general notions afterwards, though reciprocally, after having discovered the general notions, we may deduce from them others which are particular.[1]

Descartes admits to Burman that the major premiss really precedes the conclusion and that the conclusion depends on it, but the important point is that this priority is purely

[1] *Resp. ad Inst. Gassendi* (AT. ix. 205^{25}–206^{5}).

implicit and the conclusion is known before *expressly* and *explicitly*, because 'I only give my attention to what I experience, namely, *I think, therefore I am*, and do not pay any attention to that general notion, *whatever thinks is*'.

Spinoza, expounding the principles of Cartesian philosophy, states that, in his view, the *Cogito, ergo sum* is not the conclusion of a syllogism, is not a conclusion, particular or singular, dependent on a major premiss of a general nature. When, however, he goes on to summarize the Cartesian doctrine in the words, 'and therefore *Cogito, ergo sum* is a single proposition which is equivalent to this proposition *ego sum cogitans*',[2] he goes beyond the doctrine of Descartes at this particular stage of the Cartesian argument. For it seems clear that Descartes, in speaking of the supposed major premiss, does not mean quite the same thing as his illustrious commentator. He says quite plainly that the truth becomes evident because 'of what I experience in myself' or in the words of the *Reply to the Second Objections*, 'the truth indeed is rather that a man learns the universal premiss in that he experiences in his own case that it is impossible for him to be self-conscious unless he is existing'.[3] The experience is a recognition by the simple vision of the mind of a fact,

[1] The whole of this text repays careful study: 'Ante hanc conclusionem: *cogito, ergo sum*, sciri potest illa major: *quidquid cogitat, est*, quia reipsa prior est mea conclusione, et mea conclusio illa nititur. Et sic in PRINCIP. dicit auctor eam praecedere, quia scilicet implicite semper praesupponitur et praecedit; sed non ideo semper expresse et explicite cognosco illam praecedere et scio ante meam conclusionem, quia scilicet ad id tantum attendo quod in me experior, ut, *cogito, ergo sum*, non autem ita attendo ad generalem illam notionem, *quidquid cogitat, est*; nam, ut ante monitum, non separamus illas propositiones a singularibus, sed eas in illis consideramus' (AT. v. 147). It may be noted that in replying to Gassendi, Descartes is not discussing the metaphysical status of 'ideas' or 'notions' but how the mind discovers them and makes use of them in scientific deductions. The 'particular' instance quoted to Burman is a privileged one. Descartes does not hold that we can deduce particular 'existents' from general laws, nor general laws from particulars. The *Cogito* is a privileged instance, for the 'law of evidence'.

[2] *Des Cartes Principia Philosophiae*, Part I (edit. Gebhart, i. 144). On Spinoza's doctrine in the *Tractatus de Int. Emendat.*, and the manner of the composition of the idea as 'deductive' and 'implicatory', see Joachim's *Commentary*, pp. 95–96 and note 2 on p. 95 where the possibility of Spinoza's acquaintance with the manuscripts of the *Regulae* is discussed and a conclusion drawn contrary to that of L. Roth in *Spinoza, Descartes, and Maimonides*, 1924, p. 10. See also D. Borkowski, *Der junge de Spinoza*, p. 292.

[3] '. . . ex eo quod apud se experiatur, fieri non posse ut cogitet, nisi existat' (AT. vii. 140).

a datum, a singular experience of the *vis cognoscens*. Yet it would be difficult to find a more forcible expression than the phrase, 'it is impossible for him to be self-conscious unless he is existing', to emphasize that the simple object of intellectual intuition is itself a necessary implication. The *Cogito, ergo sum* is accordingly, from the logical standpoint, rightly described by Spinoza as 'a single proposition' (*unica propositio*) and this single proposition includes within itself the illation of one discriminated element from the other.

To put the same point from another angle which has already been discussed.[1] The separation of self-consciousness from existence can only take place *per abstractionem intellectus*. The *Cogito, ergo sum* is shown on analysis to be fundamentally a movement of thought from implicit to explicit within the single intuitive whole. Thus intellectual intuition and deduction merge, as it were, into one single living moment of thought. As Rule 7 states: 'Whenever single facts have been deduced immediately the one from the other, they have already been reduced, if the illation is evident, to a true intellectual intuition.'[2] This same rule adds the qualification that in the case of an illation from various and disconnected facts, disconnected at least *quoad nos*, our intellect may not be able to embrace all the facts within a single intellectual intuition and this circumstance calls for a process named by Descartes enumeration or induction. This process he even at times seems to identify with deduction, especially in the later rules of the first part of the *Regulae*.

The syllogism is ultimately rejected by Descartes because, in his view, the perfect deduction is reducible, if the inference is evident, to a true intellectual intuition and this latter is a direct and immediate vision of the truth. Given the proper conditions of attention, &c., there is no need of elaborate formulas; it is merely a question of seeing. Like Spinoza, Descartes does not conceive intellectual intuition, or what the *Ethics* calls *Scientia Intuitiva*, as an immediate knowing

[1] See above, p. 76.

[2] 'Nam quaecumque una ex aliis immediate deduximus, si illatio fuerit evidens, illa ad verum intuitum jam sunt reducta' (AT. x. 389^{14-17}). See Rule 3 (AT. x. 369^{11-17}) and Rule 11 (AT. x. 407^{13-14}). These short notes on the *Cogito* obviously do not meet many of the logical difficulties which can be raised on this point. A full discussion must be reserved for a study taking into account the text of the *Meditationes*.

exclusive of reasoning. To see is, in a sense, to reason: 'the eyes of the mind, with which it sees and observes, are the demonstrations themselves'.[1] And Aristotle also would have agreed that the fundamental principles from which all demonstration must begin are themselves the objects of *νοῦς*, for 'in all demonstrative knowledge the unit is the intuitive apprehension of an unmediable truth'.[2]

[1] 'Mentis enim oculi, quibus res videt observatque, sunt ipsae demonstrationes', *Ethica*, v. 23. Schol. (edit. Gebhardt, ii, p. 296).

[2] '*οὕτως ἐν συλλογισμῷ τὸ ἓν πρότασις ἄμεσος, ἐν δ' ἀποδείξει καὶ ἐπιστήμῃ ὁ νοῦς*', *Anal. Post.* 23, 85 1. Two Aristotelian scholars have pointed out the similarity between the Cartesian *deductio* and Aristotle's apodeictic syllogism. Joachim, in his edition of *De Generatione et Corruptione*, p. xxx, n. 3, says: 'Aristotle's conception of *ἀπόδειξις*, looked at from this point of view, is in principle identical with Descartes's conception of "deductio".' G. R. G. Mure, in his *Introduction to Hegel*, p. 115, n. 2, notes: 'The *Regulae* of Descartes offers a theory of demonstration which entails much the same difficulties as Aristotle's.'

VIII

ENUMERATION

§ 1. *Provisional Summary of the Method*

IT is perhaps well at this point to take stock and make some attempt at recapitulating the various elements of the doctrine of method discussed in the preceding chapters. In the most general terms, the problem of the *Regulae* is to answer two questions: What rules must we observe if we are to recognize the certain knowledge which we possess and recognize it without any danger of self-deception? Moreover, how can we, having recognized this certain knowledge, develop further certain knowledge from it?

The rules which we are in search of, that is, the Method, are found by a careful reflection upon the human mind in its natural and unperverted state, by a study of its procedure when it functions purely and its natural power of grasping truth is not corrupted or hampered by 'prejudices'. The human mind, so far as it functions in accordance with its nature, cannot fail to think truly. If then we wish to acquire knowledge, the one vital condition is that our thinking should be the thinking of the human intelligence in so far as it is functioning in accordance with its own nature. In order to secure this end we must reflect upon and study that nature; that is, we must determine the powers and limits of the human mind and of human knowledge. For to reflect upon the procedure of the human intellect, in so far as it follows its own nature in its thinking, is to reflect upon the conditions of truth. The intellect cannot, if it follows its own nature, fail to think truly. Truth in the various branches of knowledge is simply the structure of the intellect itself and this structure can be laid bare by reflection.

The best way to study the powers and limits of the intellect is to reflect upon its procedure in those sciences where we find it most clearly and perfectly expressed, that is, in the mathematical sciences or in any other sciences which attain to the same demonstrative certainty as the mathematical

sciences. If we reflect upon the certainty of mathematical demonstration, we find that it depends upon the observance of two main conditions. In mathematical, and therefore in all certain, demonstration, first, we must start from self-evident data and, secondly, every step in the progress from the self-evident data must itself be self-evident, and no conclusion must be affirmed unless it follows from an uninterrupted sequence of self-evident data. To these two fundamental conditions of the certainty of mathematical demonstration, there correspond two functions of the intellect in which its nature is expressed and in which therefore there is no possibility of error. The two functions are first, intellectual intuition, the power of apprehending the self-evident data, and, secondly, deduction, the power of moving, by an uninterrupted sequence of self-evident steps, from the data to their consequents. These two operations of the intellect are the primary conditions of all knowledge. They are expressions of the very nature of the intellect. No theory of method can teach the intellect to perform them. For they are presupposed in all learning and a theory of method can do no more than facilitate the right use of them.

By intellectual intuition is meant a conception of 'the pure and attentive mind' and this conception is such that no doubt whatever can remain as to what it is that we conceive. The objects of the acts of intellectual intuition are the 'simple natures', but these simple natures consist of two simple ideas or two elements of an idea in immediate and necessary connexion. A self-evident datum is, in fact, 'a simple proposition' in which *A* necessarily and immediately involves *B*, though the implication need not be reciprocal. What is grasped in an act of intellectual intuition is the immediate necessary sequence of *B* upon *A*. The self-evident datum, which we know in this primary act, might be represented as a hypothetical judgement of the type: 'If *A*, then *B*; if not-*B*, then not-*A*', but not necessarily of the purer type 'If *A*, then *B*; if *B*, then *A*'. Two conditions must be fulfilled for an act of intellectual intuition. The proposition must, first, be apprehended *clearly* and *distinctly* and, secondly, it must be apprehended *all at once* and not successively. To apprehend the elements of a self-evident proposition *clearly* means to have the appre-

hended content present to a mind which is *attending* to it; to apprehend it *distinctly* means to have before the mind precisely the relevant content, neither more nor less. 'Just *A* and *B*, in so far as *A* involves *B*, excluding all irrelevant detail', is the content of an intellectual intuition. When Descartes insists that the mind must be 'pure and attentive', he is stating the conditions under which the act of intellectual intuition must take place so as to give indubitable truth. So the phrase 'clear and distinct perception' sums up the character of the content of an act of intellectual intuition taking place under those conditions. Hence we may describe an *intuitus* as insight into the necessary connexion between two simple ideas, or two elements of an idea; an insight conditioned by attention and exclusion of irrelevant matter, so that its content is clear and distinct.

The second primary operation of the intellect is deduction, and the intellect, which is *vel minimum rationalis*, cannot err in the pure illation of one thing from another. The inference, if the intellect draws it at all, cannot be wrong. Deduction has nothing to do with the traditional subsumptive syllogism of formal logic, in which Socrates is proved to be a mortal because he is an instance of man. Yet deduction does come, in the end, to the same thing as the ideally perfect ἀποδεικτικὸς συλλογισμός of Aristotle. For to demonstrate that *A* is *X*, is to fill in the interval between *A* and *X* so that *A* immediately involves *B*, *B* immediately involves *C*, *C* immediately involves *D*, &c., so that *A* is connected with *X* by a chain of self-evident truths. Deduction involves the use of intellectual intuition in all its stages and is, as it were, a process by which *intuitus* extends itself so as to comprehend within itself the complex. Deduction includes reasonings of very different degrees of complexity. In the simplest cases, it is very near to *intuitus*; or rather, whether we call such simple inferences 'deductions' or 'intellectual intuitions' depends almost entirely on which aspect of the experience we may desire to emphasize. In the more complicated types of reasoning, *deductio* diverges from *intuitus* and is to be identified with a process called *enumeratio* or *inductio*.

The simplest case is that in which from one or two self-evident simple propositions, we move to another proposition

which follows immediately from our data or datum and which, when we have inferred it, we recognize as being self-evident. Thus, to use one of Descartes's examples already quoted, the recognition that '2+2 = 4' is an *intuitus* and so is the recognition that '3+1 = 4'. It is *deductio* to infer the necessary consequence that '2+2 = 3+1', and yet the grasp of this truth, i.e. the grasp of it as a consequent of the data, is an *intuitus*. In a simple case like this, the conclusion, when we have grasped it, is itself invested with the character of a self-evident truth. The procedure is called *deductio* if we lay stress on the movement of illation whereby we develop the articulated content, that is 2+2 = 3+1 conceived as flowing from 2+2 = 4 and 3+1 = 4. It is called *intuitus* if we emphasize the immediate grasp of the articulated content, in which the illative movement culminates and rests. We may think of an inference like $A \therefore B$, $B \therefore C$, $C \therefore D$, $D \therefore E$, hence $A \therefore E$, as typical of a Cartesian deduction. Each step is apprehended in an act of intellectual intuition. The movement of thought is deduction. In so far as the mind could retain and gather before itself, in a single undivided vision, all the single steps of the whole process, the completed deduction would be an intellectual intuition.

In more complex cases, the inference presupposes an ordered grasping of the many self-evidents, from which the grasp of the mediate truth is ultimately to emerge. To this preparatory grouping Descartes gives the name of *induction* or *enumeration*: and he occasionally extends these titles to inference as a whole [cf. *Regulae*, vii, xi, xii]. But even here the logical character of the inference remains the same. The 'deduction', 'induction', or 'enumeration', is always the illative movement from a content or contents apprehended to another content which follows by direct logical necessity from the first. And it is a mere accident due, e.g., to the limitations of our memory, that the movement does not always issue in an intuition.[1]

§ 2. *Enumeration*

The word 'enumeration' in a Cartesian context almost inevitably calls to mind the fourth Rule of method set forth in the second part of the *Discours*. This fourth Rule states: 'Lastly, I resolved to make throughout such complete

[1] Joachim, *The Nature of Truth*, 2nd edit., p. 71, n. 2.

enumerations and such comprehensive reviews that I could be sure that I had left nothing out.'[1] The rule of enumeration, or of revision as it is sometimes called, is designed to meet the cases of deduction which are more complicated than the instances which we have been considering hitherto. These, and other similar cases, were of such a nature as to be apprehended ultimately *uno eodemque intuitu*, in the single intellectual vision of an intuition.

But there are more complicated cases in which the mind is unable to grasp and hold the multiple stages of the process together and in which the illative movement, precisely owing to the multiplicity of the steps, cannot be concentrated or merged in an immediate grasp of the whole content of the deduction in all its complex articulation. A truth may be not self-evident in itself and, as such, never capable of being apprehended immediately as self-evident; but it may be the necessary consequent of an unbroken chain of self-evident truths, or of a multiplicity of chains of self-evident truths, a network of chains, so to speak, through which our mind can pass with unbroken movement. Every step or link of the reasoning is grasped by immediate intuition and the conclusion is true without possibility of doubt or suspicion of error. Yet the conclusion, that is, the necessary connexion between the last and first steps, or the necessary dependence of the consequent on the interconnexions of all its self-evident grounds, is not present to our mind with the immediate certainty and self-evidence of an intellectual intuition. We are confronted with a deduction which while reducible in theory and principle to an *intuitus*, is nevertheless not reducible in fact. Descartes illustrates this, in Rule 3, by comparing the situation to our recognition of the connexion between the first and last links of a long chain which, because of its length, we cannot see all at once. Though we cannot see all the intermediate ones, we cannot really doubt the connexion between the first and last links, provided, of course, that we have seen and checked the

[1] AT. vi. 19^{3-5}. 'Et le dernier, de faire partout des dénombrements si entiers, et des revues si générales que je fusse assuré de ne rien omettre.' The Latin translation renders 'partout' by a gloss, 'tum in quaerendis mediis, tum in difficultatum partibus percurrendis' (ibid. 550), which, as Gilson points out, indicates that the precept applies also to the practice of the second rule of the *Discours*. See below, p. 119.

connexion between each successive pair of links and provided equally that we can remember what we have seen on our first inspection.[1]

The only, but major, difficulty in a case like this lies in the length of the chain and the number of the links and linkages which are welded together to form it. When, in order to set out the results of our analysis in their logical order, we try to collect together all the links which we have discovered in the process of reasoning, and when we try to arrange them so that our knowledge shall take the form of a continuous and necessary movement of illation, in which the necessary connexion of every pair of links from first to last is self-evident, we may get a chain which exceeds the span of our intuitive apprehension. Consequently, we are forced to rely to some extent upon our memory and herein lies the major difficulty since memory is inherently unreliable. We cannot, in the strict sense of the term, 'know' the conclusion; we cannot know the necessary conjunction of the last link to the first unless we can deduce it; and to deduce it means to arrive at it by passing from link to link of the whole chain with 'a continuous and unbroken movement of thought'.[2] Now this is obviously impossible if even one link in the chain is missing, or is perceived in a confused way, as may well occur if we are forced to rely upon memory.

In order to meet this difficulty which arises only where complex deductions are made, a process which is called enumeration, or sometimes induction, is required which will assist the memory and remedy its possible deficiencies. That is to say, in the cases where the data do not form a single chain but form many interconnected groups of facts or a network of chains, there is need for an additional methodical operation to ensure accuracy and certainty. The fourth Rule of the Discours and the seventh Rule of the *Regulae* are formulated by Descartes expressly to offer an efficacious remedy to the failings of memory in all types of complex deductions. In a word, the remedy suggested is that we

[1] AT. x. 369^{26}–370^{4}.

[2] 'Per continuum et nullibi interruptum cogitationis motum' (ibid. 369^{25}). These words of Rule 3 occur also in the formulation of Rule 7 and it is the problem of enumeration which is the connecting link between these two rules. See p. 117, n. 2.

should rethink the various proofs of the long chain of deductive links. Such repetitive thinking will, so we are told, facilitate and strengthen our power of illation and make us more capable of deducing with greater rapidity and thus bring longer sections of the deductive chain within our intuitive span, with the final result that our reliance upon memory will be reduced to a *minimum*. It is not suggested that the role of memory can, by such a practice, be totally eliminated. All we can hope to achieve is to shorten the time required for the illation so that, leaving as little as possible to memory, the whole chain of links and linkages may be seen simultaneously by intellectual intuition.[1] Descartes clearly considers that our knowledge, *qua* knowledge, gains directly in proportion as the discursive movement falls within the span of a single act of intellectual intuition. With this aim in view, he lays down that 'in order to complete our knowledge of any subject, we ought to survey, by a continuous and absolutely unbroken movement of our thought, all the matters that are relevant to our inquiry, all of them together and each singly, and to include them within a sufficient and well-ordered enumeration'.[2] This is the title of Rule 7.

The wording of this precept is very general and vague. It would appear that Descartes is merely advising us to take care to look over all the various items which form part of our scientific inquiry so as to ensure that each and all the parts are present; that no essential item is missing; and, having made this methodic check, we may be reasonably certain that our memory has not been at fault and that we may count the result of our inquiry as knowledge. Doubtless the advice to make a mental inventory when dealing with complicated problems is sound advice and fewer mistakes would be made were conclusions checked in the light of a recapitulation of

[1] 'Ut *fere* nullas memoriae partes relinquendas, rem totam simul videar intueri (AT. x. 388^{5-6}). My italics. The identical words are used in Rule 11 (ibid. 409^{6-7}).

[2] 'Ad scientiae complementum oportet omnia et singula, quae ad institutum nostrum pertinent, continuo et nullibi interrupto cogitationis motu perlustrare, atque illa sufficienti et ordinata numeratione complecti' (ibid. 387^{10-13}). Baillet, who quotes this passage in his *La Vie*, ii. 405, begins: 'Pour se perfectionner dans une science . . .'. 'Perlustrare' is, I think, best translated as 'survey'. Perhaps the use of this word suggested to Descartes the substitution of 'continuo quodam imaginationis motu' in the body of Rule 7 (AT. x. 388^{3}) for the phrase which occurs in the formulation of the rule and in an earlier passage of the rule (ibid. 387^{21}).

their grounds, but if that was all that Rule 7 meant as a principle of methodology, it would be of little value to the scientific researcher. Leibniz would have every reason to criticize it as a banal truism and to write the acid comment that 'those who give us methods, give us splendid precepts but they do not tell us the way to follow these precepts'.[1] But, as is so often the case when the reader of Descartes is faced with the apparently simple, almost ingenuous text, there is more in the doctrine of enumeration than meets the casual eye. Few points of Cartesian doctrine have given rise to such divergent interpretations, but all these interpreters of Descartes are agreed that the doctrine of enumeration is a key-point in his theory of demonstration. It has been variously described as 'a means of verification, a process which permits the elimination of uncertainty brought by memory into the continuous deduction'. It has been considered as a complementary process to the two main activities of the intellect and called 'an analogical induction'. It has been stated that it has none of the experimental or empirical characterisics of the Baconian theory of enumeration but is 'a deduction in actual process, not yet completed and concentrated'. It has been described as 'a judgement which condenses in a single relation a sum of relations and organizes them' as a whole.[2] All these descriptions have their value and are in different ways true accounts. But to a reader attempting to amalgamate them in a single simple account, Descartes does not by his text help very much to elucidate the matter. He does not give any firm definition of what he means by enumeration and in his discussions of the process, mainly in Rule 3 and Rule 7, he appears to shift his point of view almost to the degree of inconsistency and to hide his thought behind vague generalities which are capable of multiple

[1] Letter to Galloys, 1677 (*Phil. Schriften*, edit. Gerhardt, vii. 21). He says with equal acerbity: 'Et parum abest ut dicam similes [the Cartesian rules] praecepto Chemici nescio cujus: Sume quod debes et operare ut debes, et habebis quod optas' (ibid. iv. 329). These quotations are taken from Couturat, *La Logique de Leibniz*, pp. 94–95.

[2] These are typical summaries of the opinions found, in the order of the quotations, in R. Hubert, *Rev. méta. et morale*, 1916, pp. 489–516; L. Liard, *Descartes*, p. 30; Hamelin, *Le Système de Descartes*, p. 73; Hannequin, *Études d'histoire des sciences et d'histoire de philosophie*, i. 229. See also Sirven, *Les Années d'apprentissage*, pp. 378–9, and Chevalier, *Descartes*, p. 186.

interpretation. It is just as though he was applying the famous dictum of the *Cogitationes Privatae*: 'Scientia est velut mulier: quae, si pudica apud virum maneat, colitur; si communis fiat, vilescit'.'[1] Whether his intention was to be deliberately obscure, because the theory was not very clear and definite in his own mind, or whether the obscurity is merely the natural expression of this inward uncertainty, the result is the same, and to set down Descartes's doctrine of enumeration as consistently as possible, it is necessary to fill the gaps by conjecture. The examples and applications of the rule are not of themselves very helpful.[2] It should also be noted that the third part of the *Regulae*, which was never written, was meant precisely to deal with the problem of the reduction of 'imperfect questions' or problems to 'perfect questions' or problems and would thus have dealt with the type of deductions where enumeration is most essential.

If we begin by turning to the rule of enumeration as enunciated in the *Discours*, it is clear that enumeration covers two separate cases, to use a vague term. Descartes says he resolved to make 'complete enumerations' and 'comprehensive reviews' and, as the Latin translation makes even clearer, these should not be regarded as identical processes.[3] The 'comprehensive reviews' seem to refer to cases in which we are faced with a long but straightforward linear deduction and in which the length of the chain, necessitating the intervention of memory, is the main difficulty to be overcome by the new procedure. But it seems also that by enumeration he sometimes means something like the setting-out, in a particular order, of the conditions on which the solution of a *quaestio* or problem will depend, that is, a definite arrangement of antecedent data and premisses, a process therefore which takes place in the initial stages of a scientific inquiry. These aspects of enumeration are obviously diverse in their application

[1] AT. x. 214^{4-5}.

[2] The examples are to be found in Rule 7 (AT. x. 390^{9-24}); in Rule 8 (ibid. 393^{22}–396^{25}); in Rule 12 (ibid. 411^{3-16}) and in Rule 13 (ibid. 430^{23}–431^{27}).

[3] 'Ac postremum, ut tum in quaerendis mediis, tum in difficultatum partibus percurrendis, tam perfecte singula enumerarem et ad omnia circumspicerem, ut nihil a me omitti essem certus' (AT. vi. 550). The Latin text was revised by Descartes himself. The translation of 'in quarendis mediis' as 'seeking methods of solution', by Boyce Gibson in his article in *Mind*, N.S. vii. 343, expresses exactly the meaning.

and it is better to discuss them apart, before attempting to consider yet other meanings.

§ 3. *The Role of Memory*

The meaning and function of enumeration as 'a comprehensive review' is perhaps the easiest and most obvious of Cartesian usages. It is, less fortunately, the only meaning given to the term by those who criticize the urbane author of the *Discours* on his face-value statements. Descartes is attempting to eliminate, as far as is possible, from the deductive process any dependence on memory which must condition the process at least in its early stages and must, to that extent, detract from the certainty of the conclusion. The immediate deduction is easily reduced to intellectual intuition but, as Rule 11 remarks, when the deduction is 'complex and involved, we give it the name of enumeration or induction because it cannot be apprehended as a whole at one and the same time by the mind and its certainty depends partly on memory, which must retain our judgements on the various matters under enumeration in order that a single judgement may be inferred from the whole collection'.[1] Memory in this case has been justly compared with a scaffolding with which the complete evidence of knowledge can dispense once the structure of intuition is completed.

Descartes's views on memory are to be found scattered throughout the *Regulae*, as indeed throughout his other works and letters, and there is little, if any, systematic coherence about the doctrine which can be constructed piecemeal out of them. We are told, naturally enough in view of what has already been stated, that to have a good memory counts as one of the perfections of the human mind.[2] But we are also warned that we must be extremely careful to avoid 'lapses of memory' which is 'weak and liable to fail us'.[3] And this last warning is accompanied by the practical advice, from Rule 11, to run over all the various steps 'repeatedly in my mind, until I pass so quickly from the first step to the last that for

[1] AT. x. 408^{4-10}. The word 'judicia' in the text is not used here in the full meaning of judgement but rather as equivalent to 'propositio'.

[2] Cf. Rule 12 (AT. x. 410^{8}) and *Discours*, Part I (AT. vi. 2–3).

[3] Cf. Rule 12 (AT. x. $417,^{10-12}$) and Rule 11 (ibid. 408^{23-24}).

all practical purposes nothing is left to memory and I seem to view the whole all at the same time'. Descartes, in fact, criticizes Galileo because his arguments are so long, and so full of digressions that one tends to lose the trend and it is impossible to remember the first steps. Further practical advice is given in Rule 16 where we are advised that 'as a general rule, nothing that need not be kept continuously in mind ought to be committed to memory, if we can note it down on paper'. The object of this recommendation is to economize on our power of attention, necessarily limited in the human mind, which is thus relieved from a superfluous burden. And earlier in the same rule we are recommended to use as far as possible *brevissimas notas*, the very briefest symbols, in order that 'after distinctly examining each step in accordance with Rule 9 ['to give the whole of our attention to the most insignificant and most easily mastered facts'], we may be able, as Rule 11 tells us to do, to traverse them all with an extremely rapid movement of our thought and include as many as possible in a single intuitive vision'. The examples of this symbolism, designed as a mnemonic aid, are from mathematical problems, when we are told to use *a*, *b*, *c*, &c., to express magnitudes already known; *A*, *B*, *C*, &c., for those unknown; 1, 2, 3, 4 for the purpose of making clear their number and appended to them to mark 'the number of relations to be remarked in them'. Thus the formula $2a^3$ is equivalent to 'double the magnitude which is symbolized by the letter *a* and which contains three relations'.[1] Now all this may be excellent and highly practical, and we must not forget that the method is set down as a practical guide. But there are equally many warnings about the fallible nature of memory, which, as we have seen, was excluded, with sense and imagination, from the 'pure' use of the *vis cognoscens*.[2]

The Cartesian doctrine of memory does, however, allow for a distinction, not explicitly mentioned in the *Regulae*, between a 'sensible memory' and an 'intellectual memory'. The first, which is the memory in the normal use of the term, is a mixed faculty in that it depends both on the purely

[1] Cf. Rule 16 (AT. x. 455–6). See below, p. 182.
[2] See above, pp. 26–27.

spiritual element of the soul and also on the body where it is dependent on various impressions left on the brain.[1] Baillet tells us, on the authority of the *Studium Bonae Mentis*, of which he had the manuscript before him, that Descartes, 'over and above this memory which depends on the body, recognized another altogether *intellectual* memory which was dependent on the soul alone'.[2] This latter was not capable of variations in the same way as the sensible memory varies according to individuals, some being possessed of a 'good' memory, others of a 'bad' memory. In the discussion with Burman we find further details on this point. Descartes is reported as saying:

> I do not deny intellectual memory; on the contrary, it is a fact. When, for instance, I hear the word R-E-X signifying the supreme power, I commit it to memory and this takes place surely through intellectual memory since there is no affinity between these three letters and their meaning from which I could draw out the latter, but I remember by intellectual memory that these letters denote this meaning. In truth then, this intellectual memory is more concerned with universals rather than particulars and accordingly we cannot remember by its means all our particular facts.[3]

This recollection, therefore, which would be a memory of

[1] Cf. *Traité de l'homme* (AT. xi. 177^{19}–178^{14}).

[2] Baillet, *La Vie*, tome II. v, chap. ix, p. 66, reproduced in AT. x. 201. Baillet refers in the margin to article 6 of the fourth *Discours* of the *Dioptrique*, which AT. reproduce in their text as article 9. These articles refer to the Latin version and it is Baillet's reference which is correct. There can be no doubt about Descartes's doctrine on the two memories. See, for instance, a letter to Mersenne, 1640: 'Mais, outre cette mémoire, qui dépend du corps, j'en reconnais encore une autre, du tout intellectuelle, qui ne dépend que de l'âme seule' (AT. iii. 48^{27-29}); a letter to Huygens, 1642: 'Car je trouve en nous une mémoire intellectuelle, qui est assurément indépendante du corps' (AT. iii. 580^{16-18} and Roth, 182^{46-47}); and the correspondence between Arnauld and Descartes in 1648, where the former speaks of a 'duplex memoriae vis necessario admittenda' (AT. v. 186^{25}), in reply to which Descartes agrees: 'Duplicem memoriae vim admitto'; one of which powers is qualified as 'a quadam reflexione intellectus sive memoriae intellectualis' (ibid. 192–3). There is a curious Scholastic tang about his remark in a letter to Mersenne, 1640: 'La mémoire intellectuelle a ses espèces à part, qui ne dépendent nullement de ces plis' (AT. iii. 84^{22}–85^{2}). 'Ces plis' are the creases in a piece of paper, used as a simile to illustrate how a physically extended brain retains the traces of sensible memory.

[3] *Entretien avec Burman*, in the discussion on the *First Meditation* (AT. v. 150). Descartes does not explicitly refer to the well-known distinction between μνήμη and ἀνάμνησις which is to be found in St. Augustine and the Scholastics (cf., for example, St. Thomas, *Summa Theol.* I, q. 78, a. 4, in corp. for the distinction between *memoria* and *reminiscentia*).

clear and distinct ideas, might be argued to be exempt from all confusion, and the divine guarantee, posited in the doctrine set forth in the *Discours* and the *Meditationes*, could be extended to this type of memory without any risk of the guarantee covering or sustaining the illusions to which sensible memory, by its reliance on the body, is necessarily prone at every instance. 'It would not be self-sufficient', notes Boyce Gibson, 'and stand beyond the need of a guarantee, for we are not perpetually aware of what we remember, and in any case God is the only source and principle of the duration in which memory persists. But it would be free from irruptions of prejudice: and if prejudice should enter in and confuse the issue . . . it would be due to our faulty judgment.'[1]

On the other hand, Descartes points out in a letter that 'in purely intellectual matters, there is no remembering properly so called'.[2] It seems, on the whole, that the distinction between sensible memory and intellectual memory is not really one of memory in the sense that there is one kind of memory, linked with psycho-physical modifications, which is concerned with particular images and another kind of memory, purely spiritual in nature, which is concerned with universal ideas. It is more than probable that the expression 'intellectual memory' is adopted from the current terminology of the Schoolmen who did not maintain a separate faculty called intellectual memory, distinct *re aut essentia* from the intellect itself.[3] Descartes himself gives us no account of the act of intellectual memory or the faculty it might be supposed to be a manifestation of, except the vague mention of certain special 'species' it operates with. He suggests, in a letter to Arnauld, that the psycho-physical mechanism of sensible memory may be the occasion of the intellectual memory of the soul.[4] This theory is obviously fraught with all the difficulties inherent in the dualistic doctrine of body and soul.

[1] *The Philosophy of Descartes*, p. 317.

[2] Letter to X, 1641: 'De rebus vero pure intellectualibus, nulla proprie recordatio est', and he adds, 'sed prima vice qua occurunt, aeque bene cogitantur ac secunda; nisi soleant nominibus quibusdam alligari, quae cum corporea sint, de ipsis etiam recordamur' (AT. iii. 425^{11-16}).

[3] See the text from the Coimbrincenses on 'Memoriam intellectivam non esse facultatem re aut essentia ab intellectu distinctam' in Gilson's *Index Scolastico-Cartésien*, p. 177.

[4] Letter of 1648 (AT. v. 219^{19}–220^{9}).

What does emerge from his views is that the act of intellectual memory is an act of reflection on ideas already cognized as clear and distinct, and there would seem to be this difference between the original act of intellectual intuition and the second or reflective act that in the latter case we recognize the ideas as already known. There cannot be any question of knowing them better or more clearly and distinctly. *Qua* acts of intellectual intuition, they cannot differ in value of certainty. They differ, so to speak, as acts of intellectual discovery and rediscovery of the same ideas. The very possibility of the act of rediscovery arises from the fact that we have turned our attention away from the ideas in attempting to follow another set or series of ideas, related logically to the 'forgotten' ideas. In making the final summing-up or verification of the whole series of ideas which form the deductive chain under review, we return to the 'forgotten' ideas, and, at that moment, revive the clarity and distinctness of the original intuitive act. But this act of revivified intuition is no more dependent on the body than the original act. It is again an act of the *vis cognoscens* and thus merits the name of 'pure memory'. The physical aids, such as symbols, which must depend on sensible memory are merely means to the evocation of the act of pure memory, the reflective act of vision which recaptures the primal certainty of the original act.

In so far as enumeration is to be considered as 'a comprehensive review', it may be properly described as a verification of the logical steps already traversed in the deductive process. Its object is to enable us, possibly with the help of mnemonic symbols which will be discarded when they have served their purpose, to traverse a series of necessary connexions rapidly enough to embrace the whole series in a single intuitive vision. To achieve this it may be necessary to repeat the whole process several times, checking and rechecking the various links and linkages of the chain until the individual intuitions are grouped and apprehended in a single act of intellectual intuition which assures their truth-value. In a sense, it is an attempt to exclude the extra-logical factor of duration by reducing the linear time-series of deductions to the essential unity of a *praesens evidentia*. It can obviously only be used in the more simple cases of straightforward

linear deduction, and the Cartesian instances come not surprisingly from mathematical deductions. Enumeration then is the simultaneous apprehension, at the limit, of a collection of simple necessary connexions which have been united by a reflective act redefining, as it were, their clarity and distinctness to an attentive mind which has already worked its way once through the series in a partially disjointed manner. It can be objected that the identification between the deductive series when enumerated and the intellectual intuition which apprehends the unity of the *enumerata* is more psychological than logical in that it results from an attempt to eliminate or at least diminish the conditioning factor of duration. It would therefore be an identity in terms of the *tota simul* characteristic of intuitive thought and not in terms of the other characteristic of simplicity of content. This is true, yet it would be a sufficient identity to give enumeration, *qua* completed act, the same certainty as intellectual intuition, for certainty is a psychological fact rather than a logical fact. But Descartes is also aware that the attempt to eliminate the factor of duration is an ideal to be set as a methodological principle and that in fact the instantaneity is in most cases a limit which may not be attainable. In Rule 7 he notes that 'whenever single facts have been immediately inferred the one from the other, they have already been reduced, if the inference was evident, to a true intuition'. This is the ideal case but, he goes on,

> if we infer a consequence from various and disconnected propositions, often our intellectual capacity is not able to embrace them all in a single intuition; in which case we must be content with the certainty of this operation. In the same way, we cannot, in a single act of vision, distinguish all the links of a very long chain, but if, nevertheless, we have seen the connexion of each link with the one that precedes it and the one that follows it, then we are entitled to say that we have seen how the first is connected with the last.[1]

This text is strange in that Descartes talks of 'disconnected' (*disjunctis*) propositions as making a chain, but the general sense of the difference of certainty between the act of intellectual intuition and that obtained by enumeration is clear. If we are not able to survey the deductive process or demonstration as a whole, it will necessarily rest on grounds which we

[1] AT. x. 389[14–25].

have not got before the mind. Enumeration as a comprehensive review is intended to reduce the number of 'unknown' grounds, that is, grounds of which we are not presently aware.

§ 4. '*Complete Enumerations*'

It was noted at the end of the second paragraph of this chapter that enumeration was also used to cover a definite, or methodical, arrangement of antecedent data and premisses, a process which must obviously take place in the initial stages of a scientific inquiry. In this sense then enumeration is a kind of logical classification for the determining and limiting of problems. Rule 7 gives us this account of the process: enumeration is

a scouting (*perquisitio*) of everything relevant to the problem before us; a scouting so careful and accurate that *we may conclude from it with manifest certainty that we have left nothing out which ought to have been included.* Hence, whenever we have used it, even if we fail to find the object of our search, we have at least advanced in knowledge to this extent, namely, that we apprehend with certainty that the object in question *could not possibly have been discovered by any method known to us.* And if, as often happens, *we have been able to survey all the methods which are available to man for the investigation of that object*, we may boldly assert that the knowledge of it is beyond the reach of any human mind.[1]

Enumeration then is, in this sense, an operation of the mind which should normally occur *before* the process of deduction, whereas the process of 'comprehensive review' must of necessity occur *after* the deductive chain is completed. In a way it is like setting out what Aristotle calls an ἔκθεσις. The problem we are set is limited by the object of our inquiry, and enumeration is the complete preliminary investigation of the object. Rule 7 gives some examples of this type of survey. The first problem is the comparatively simple one of the construction of a perfect anagram by the transposition of the letters of a name. It is sufficient to adopt an order for the transposition of the letters which have to be

[1] AT. x. 388^{25}–389^{7}. The italics are mine. The corresponding passages are: 'ut ex illa certo evidenterque concludamus nihil a nobis perperam fuisse praetermissum . . . nulla via a nobis cognita potuisse inveniri . . . vias omnes, quae ad illam hominibus patent, potuerimus perlustrare.'

used in making the anagram, in such a manner that the same arrangements of letters are never twice handled in the operation. By splitting up the complete set of transpositions into a number of definite classes we can see immediately 'where there is best hope of finding what is sought. In this manner the task is often not tedious but merely child's play.'[1] Another example is taken from mathematics. We wish to demonstrate that the area of a circle is greater than the area of all other figures of equal periphery. There is no need to consider all figures; it is quite sufficient to demonstrate the relationship to certain chosen figures.[2] If, for instance, we wish to prove that the rational soul is not corporeal, it is not necessary to make an exhaustive list of all corporeal things; it is quite sufficient to class all corporeal things in certain specified groups and then set out to show that the rational soul has nothing in common with any of the groups.[3]

Another example of enumeration is given in Rule 8 which Descartes himself describes as *omnium nobilissimum exemplum*. The problem here is that of examining the conditions and limits of human knowledge.[4] If we apply our minds methodically, that is, in a methodical survey of the essential data of the problem, we should discover first that nothing can be known prior to the intellect itself, since all knowledge depends upon the intellect and not conversely. Having then understood this fundamental priority of the human intellect in the problem, we should next proceed to make a complete survey of all the 'instruments of knowing', that is, the various faculties of the soul. We should classify these faculties and in so doing we should find that, besides

[1] AT. x. 391^{18-28}.

[2] Ibid. 390^{18-24}. In a letter to Mersenne in 1638, Descartes gives an instance from the theory of numbers: 'Comme, en la règle que donne Euclide pour trouver les nombres parfaits, il fait examiner tous les nombres qui suivent de l'unité en proportion double, jusques à ce qu'on trouve un, duquel ôtant l'unité, le reste soit un nombre premier; au lieu qu'il devait donner un moyen pour excepter tous ceux qui, étant diminués d'une unité, ne deviennent pas nombres premiers' (AT. ii. 254^{19}–255^{5}). That is, for instance, multiples of 4, 16, 64, 256, &c., which on subtraction by 1 are divisible by 3.

[3] AT. x. 390^{13-18}.

[4] Ibid. 395^{17}–396^{25}. See also the survey of the total implications of knowledge, both as regards things known, that is, as possible objects of knowledge, and as regards the faculties of knowing in the subject in Rule 12 (ibid. 411–25) and the three classes of 'simple natures', cf. above, pp. 70 and 81.

that of understanding, there were only two other faculties, those of sense and imagination (memory being included under this latter). The problem would then be reduced to that of examining and distinguishing these three instruments of knowledge and their relations to truth and falsity. We should then discover that, in the strict sense, truth and falsity can only be a matter of the understanding and therefore our task would be directed to a study of the understanding and its proper functions. In other words, we should be undertaking the study coextensive with the task of the *Regulae*. An analogous instance is to be found in the second part of the *Traité des Passions* where it is a question of the number and order of the passions. In article 52 Descartes notes that the various objects which stimulate our external senses do not do so because of their own diverse natures but because of the diverse ways in which they can harm or help us or at least affect us in some important degree. The normal and habitual way our feelings act, or react, is such that we desire those objects which our natural instinct prompts us to desire because the objects are of some utility, or give some pleasure, to us and, contrariwise, we reject those objects which instinct prompts us to regard as harmful or painful. Accordingly, says Descartes, 'in order to enumerate them [the passions of the soul], it is only necessary to examine, in their order, the various ways of importance to us in which our senses can be moved by their objects: and I shall here make an enumeration of all the principal passions according to the order in which they may be found'.[1]

The *First Meditation*, which is concerned with 'all the things which may be brought into doubt', must also be considered as an example of a preliminary survey of an analogous nature, in which not all the detailed knowledge I may possess is examined but the classified sources of my knowledge,

[1] Cf. AT. xi. 372^{10-28}. Descartes seems to have in mind the distinction between 'complete enumerations' and 'comprehensive reviews', for he speaks in article 69 of 'faisant une revue sur toutes celles que j'ai dénombrées' (AT. x. 380^{4-5}). He writes to Mersenne about Herbert of Cherbury's *De Veritate*, which he has just read in French, and he adds this comment at the end: 'Ses Zététiques sont fort bons pour aider à faire des dénombrements dont je parle en la page 20 [of the *Discours*, i.e. the fourth precept]; car, lorsqu'on aura dûment examiné tout ce qu'ils contiennent, on pourra s'assurer de n'avoir rien omis' (AT. ii. 599^{13-16}). The second form of analysis was given the name of *Zetetic* analysis by Vieta.

senses, imagination, memory, &c., are reviewed under general headings. The second paragraph itself, with its metaphor that 'the ruin of the foundations brings down of its own accord the rest of the superstructure',[1] is almost an enunciation of the principle of methodical enumeration.

We are given another example of what is called 'an exact enumeration' in the seventh *Discours* of the *Dioptrique*, which work we are told on the title-page to consider as one of the 'essais de cette MÉTHODE'. This is again an illustration of enumeration as a setting-out, in a logical form, of the conditions on which the solution of a problem will depend. In this *Discours* Descartes is attempting to explain various means which may be of use in perfecting human vision. In the previous chapter he had discussed how human vision functions. The problem before him is to discover what kind of lenses, what shapes, what arrangement of lenses, &c., are most suitable for the new and improved instruments he intends to construct. He begins his inquiry with these words:

> Now that we have sufficiently examined how vision takes place, let us briefly summarize and set before ourselves the conditions required to perfect it, in order that, considering what provision Nature has already made to satisfy each of these conditions, we may be able to make an exact enumeration of all the improvements that still remain to be supplied by craftsmanship. One may reduce everything, that is to say, anything it is indispensable to consider in this respect, to the following three chief heads: namely, i, the objects of vision; ii, the interior organs which receive the actions of these objects; iii, the exterior organs [that is, the transparent parts of the eye itself and the medium of vision between the eye and the object] which dispose these actions so that they are able to be received properly.[2]

He then takes these three heads one by one, considering the relevant distinctions in each, as, for instance, proximity or remoteness, accessibility or inaccessibility, of the objects of vision, and eliminates from his inquiry such hindrances and obstacles to the perfecting of vision as are clearly beyond the power of craftsmanship to remove or even diminish.

[1] 'Suffossis fundamentis, quidquid iis superaedificatum est sponte collabitur' (AT. vii. 18^{11-13}). Mersenne wished Descartes to change his wording to 'erutis fundamentis' but could not persuade him to do so. (Cf. AT. iii. 268^{3-8}.)

[2] *La Dioptrique* (AT. vi. 149–50).

It would appear from all these examples, which are specifically referred to as examples of 'enumeration' by Descartes himself, that enumeration, in one use of the term, is a preparatory marking-out of the field of knowledge in which a proposed investigation of some particular problem is presently to take place.[1] The object of the process is to make certain that we shall take into consideration whatever, within the territory or domain of the particular branch of knowledge, may be connected with, or relevant to, our subsequent research. As a scientific procedure it can be described as a complete investigation of all matters that concern directly the problem under review. It is a prereview of the problem, if such a paradox can be allowed. As such it is 'a kind of natural classification, at once adequate and complete, but which may be abbreviated to any extent so long as brevity is consistent with thorough accuracy of treatment. It is, however, no mere collection of instances: so long as the various heads under which we seek to bring our discussion do not exhaust the series of possible divisions, there can be no question of enumeration in Descartes's sense of the word.'[2] Thus, in Descartes's own example, the division of triangles into acute-angled, right-angled, and obtuse-angled triangles, must be considered as 'a complete enumeration' in the phraseology of the *Discours*, because it is, by its very nature, a complete and exhaustive survey of the particular field of triangles.

It may not perhaps be too difficult to see what Descartes wishes to achieve by this presimplification of problems and, considered as a piece of practical advice, there is little to criticize in the methodological precept. But it is more difficult to see how, on the Cartesian theory of the *vis cognoscens* and its functions, this preliminary survey is possible at all. This review or inventory of 'all the matters that are relevant to the problem which has been raised',[3] as Rule 7 describes it, seems to imply logical powers on the part of the human

[1] Somewhat similar to what Aristotle calls ἔκθεσις, the setting-out of the terms in an argument so as to make clear to the mind a particular instance of the class denoted by the middle term. Cf. *Prior Anal.* i. 6, $28^{a}23$.

[2] Boyce Gibson, *Mind*, N.S. vii. 153.

[3] 'Eorum omnium quae ad propositam aliquam quaesitionem spectant, tam diligens et accurata perquisitio' (AT. x. 388^{25-27}).

intellect for which he has not allowed in his account of the intellect or the power of knowing. The doctrine of the *Regulae*, and Descartes's own practice in early works such as *La Dioptrique* and later works such as the *Traité des passions*, obviously imply that such inventories, or surveys, are possible and even in some cases necessary, if we are to attain certainty in our scientific research by method and not merely by chance. If we consider the examples and small fragments of doctrine on this point, we find that two conditions seem to be required, both admittedly general and abstract, for these types of enumeration. The enumerations should be 'sufficient' and 'ordered'. To demand that an enumeration should be sufficient, is merely to ask that it should be adequate to safeguard us against the omission of what will be essential for our subsequent inquiry.[1] As Gilson points out, the rather vague epithet of 'sufficient' seems to be used purposely in order to indicate that there is no general rule which determines *a priori* the degree of strict and exhaustive classification which any particular enumeration may ensure. The degree would then vary according to the nature of the problem under review. Further, the surveys must be 'ordered', that is, the grouping or enumeration must be conducted on a methodological principle of some kind. We are told that what kind of principle is used 'depends upon each man's judgement' and will accordingly vary.[2] It would seem that any principle will do, provided the requisite completeness and adequacy are secured. If all the facts relevant to a particular problem are arranged in the best possible order, that is, some order which is based on a principle, they will, so Descartes thinks,

> be reduced, for the most part, to definite classes, out of which classes it will suffice to examine carefully either one sample, or some one item in a single case, or certain things rather than others, and so at least we shall never uselessly cover the same ground twice. This course of action has so many advantages that often many items, thanks to a proper methodical arrangement, can be gone over in a short time and with great ease, though at first their number appeared tremendous.[3]

[1] 'Sufficienti et ordinata enumeratione' (ibid. 387^{13}). See Gilson, *Commentaire*, p. 212.

[2] 'Hic autem ordo rerum enumerandarum plerumque varius esse potest atque ex uniuscujusque arbitrio dependet' (AT. x. 391^{12-14}).

[3] Ibid. 391^{4-11}.

Although Descartes uses the word 'classes' in this passage and enumeration has been described as a method of logical classification, it must not be taken that this classification has any similarity with the Scholastic doctrine based on *genera* and *species*. Rule 6, which is said to reveal 'the chief secret of the method', begins by informing us that 'all facts can be arranged in certain series, not indeed in the sense of being referred to some ontological genus, as is used by the Philosophers in their categorial classification, but in so far as certain truths can be known from others'.[1] The Tree of Porphyry is the subject of biting sarcasm in the *Recherche de la Vérité* on the part of Eudoxe who describes the method of classification used in the Schools as a *βαττολογία*, which leaves us as ignorant at the end as we were when we began our inquiry.[2] The reason for the attack, repeated in various forms, is always the same. The classification of the Schools is based, as in the case of the syllogism, on the extension of the concepts, whereas the classification of concepts, if attempted at all, must be based, on the Cartesian view, on the comprehension of concepts. We must, then, understand that this survey or analysis of the constitutive elements of a problem must be based on their comprehension. If then the various elements are arranged in different groups or classes, it is because we have discovered their essential characteristics, that is, their simple natures, and this permits us to distinguish them one from the other and thus distribute them in a methodical manner. If we wish to exclude a given element from some group or class, it is quite sufficient to discover that there is an essential incompatibility between the nature of this element and the simple natures which form the group or class in question. This must obviously be the work of intellectual intuition which alone can apprehend the essential charac-

[1] 'Monet enim res omnes per quasdam series posse disponi, non quidem in quantum ad aliquod genus entis referuntur, sicut illas Philosophi in categorias illas diviserunt, sed in quantum unae ex aliis cognosci possunt' (AT. x. 381^{9-13}). See above, pp. 80 and 106.

[2] 'Tandemque omnes hasce egregias quaestiones in meram Battologiam, quae nihil illustraret et in prima nos relinqueret ignorantia' (AT. x. 516). Montaigne shared Descartes's view on this point: 'je sçais mieulx que c'est qu'*Homme*, que je ne sçay que c'est qu'*Animal*, ou *Mortel*, ou *Raisonnable*. Pour satisfaire à un doubte, ils m'en donnent trois; c'est la teste d'Hydra.' Quoted in note in AT. x. 515, from the *Essais*.

teristics or simple natures. But, it must be noted, intellectual intuition is, in one sense, an act of experience, that is, it is an act of the internal experience which occurs in a given moment and under certain well-defined conditions and therefore it would be true to say that enumeration, considered as a preliminary survey of a problem, allows for a certain experiential aspect of the method. It should also immediately be added that intellectual intuition is a purely spiritual act, or, in other words, the experience is strictly internal and reflective. But in this limited sense of experience, which must exclude any data derived from sense-experience, the method may be said to allow for an 'experience' of the immediate data of consciousness, given, or not given, according to the particular state of mind of the individual researcher. It is this factor which would explain that the order to follow in an enumeration may, in certain cases, depend on the will of the individual. We should then have to say that it is not the logical powers of the *vis cognoscens* which vary but merely the ability of each individual to use these powers properly, that is, to fix his attention directly on the essential elements of the problem. Such a view is consistent with the beginning of the *Discours* and the 'need of a method'.

IX

DEMONSTRATION

§ 1. '*Enumeratio sive Inductio*'

ENUMERATION depends in a certain measure on the subjective point of view of the researcher who is tackling the problem. It depends also in a large measure on the nature of the problem and on the nature of the data which the mind has to deal with in its scientific inquiry. This means that the functions of enumeration become more complicated in proportion to the complication of the data involved in any particular problem. In certain cases of complicated deduction, the role of enumeration, or induction as it is sometimes called, is different from that discussed in the previous chapter; it is no longer a review of the various steps in a demonstration nor is it a preliminary survey of the grounds of a demonstration or solution of a problem. This new aspect of enumeration is discussed by Descartes in Rule 11 at the beginning of the exposition.

> This [he says] is a good opportunity to explain more clearly what has been said about intellectual intuition in Rules 3 and 7. For, in one passage, we contrasted it with deduction. But, in another passage, we contrasted it with enumeration, which we defined as an inference gathered together from many disconnected antecedents taken together; and there we asserted that the simple deduction of one thing from another takes place through direct intellectual intuition.

He then goes on to explain what may have seemed to the reader an inconsistency.

> Two conditions must be satisfied if there is to be intellectual intuition: namely, the proposition must be first apprehended clearly and distinctly and secondly it must be apprehended all together and not part after part. Now a deduction when (as in the exposition of Rule 3) it is being considered as something to be made, does not seem to take place all together; on the contrary, it involves a certain movement of mind, a movement by which the mind infers one thing from another. Hence, in Rule 3, we rightly distinguish deduction from intuition. But if, as in the exposition to Rule 7, we are considering a deduction as already

made, then the term no longer signifies a movement but the end of a movement. Hence, when the deduction [i.e. the content deduced] is simple and manifest, we suppose it to be seen by intellectual intuition, but when it is complicated we take a different view. To such a complicated and involved deduction we give the name of enumeration or induction because it cannot be comprehended whole and all together by the intellect. On the contrary, its certainty depends in a sense on memory; for our memory must retain the judgements which we have passed on the several parts of the proof [i.e. the parts specified in the enumeration], if we are to draw a single consequence from all these parts taken together.[1]

Descartes goes on to point out that, if we are to understand how these two operations aid and complete each other, all these distinctions have to be made, for by aiding and completing each other, these two operations 'seem to grow together into a single operation'.

Much of this is familiar ground already covered in previous chapters. Descartes is, however, drawing our attention to a further point. He is pointing out that in the complicated processes of reasoning we are forced to rely upon our memory, as we were to a certain extent in those cases of deduction where the length of the chain was the obstacle to reducing the links to the single bond of an intuitive vision. When the deduction is complicated and involved, we are forced to lean more heavily upon our memory because the data and steps of the demonstration are too heterogeneous and too intricate to be included, and yet kept distinct, within the limited span of a single intellectual intuition. In such a case some sort of methodical grouping, tabulating, and ordered numbering of these data and steps is required to prevent any of them slipping from our memory or becoming blurred and confused through disarrangement. The aim and purpose of enumeration or induction in these cases is to arrange and group data and steps of the demonstration, data already known and steps already deduced, so as to facilitate their retention in memory.

In Rule 17 Descartes discusses again the complicated and involved argument. He states there that 'in every problem that is to be solved by deduction there is one way which is

[1] AT. x. 407^{8}–408^{10}: I only reproduce the closing phrase: 'in qua judicia de singulis partibus enumeratis retineri debet, ut ex illis omnibus unum quid colligatur'.

plain and direct' and 'others which are more difficult and indirect'.[1] He goes on to point out, referring back to the passage from Rule 11 just quoted, that when we fix our attention on the interdependence of the links of a demonstration, and keep our attention fixed so that we do not lose the thread of the continuous illation of one from the other, apprehending thus the connexion of the first to the last, then we review the problem in a direct manner. But if, on the other hand, we are dealing with an involved demonstration in which the problem is to find, given certain extremes, intermediaries by 'the inverse process of reasoning', then the methodological principle is to treat 'the unknown as though they were known'.

It must be recalled that, on the Cartesian view, every possible subject of scientific research is, in principle, fully intelligible. The structure of reality, in its double aspect of *cogitatio* and *extensio*, is fully rational and forms a complete system of interrelated necessary connexions; the subject of scientific knowledge is a whole of self-evident elements interconnected according to a definite logical order. It would therefore seem justifiable to assume on such an integrally rationalistic view that, provided we are able to discover the primal self-evident data—and removal of prejudice, proper attention, and the 'pure' use of the intellect are supposed to guarantee this—provided we advance from these data in the correct logical order, that is, without breaking the continuity of the movement of thought by omitting one of the necessary links; provided, in a word, we observe the proper conditions for the exercise of intellectual intuition and deduction, then we should be able to reconstruct the whole series of elements of the real and ultimately arrive at a complete knowledge of reality itself, *qua* rational whole and unique science. Descartes encourages such an assumption inasmuch as for him everything can, in principle again, be known with the same absolute certainty. 'The knowledge of one thing', he says, 'is never more obscure than that of another. All subjects of knowledge are of the same order, consisting always only in

[1] 'In omni quaestione per deductionem resolvenda quamdam esse viam planam et directam . . . caeteras autem omnes esse difficiliores et indirectas' (AT. x. 459^{26}–460^{3}) and 'ignota pro cognitis supponendo' (460^{21}).

the conjunction of things which are self-evident.'[1] It is this rationalistic belief in the unity of science and the rationality of the real which is, in a sense, the σπερματικὸς λόγος of Cartesian doctrine and provokes M. Maritain's anger as constituting a usurpation of angelic privileges. But, it must be added, all this is so only in principle, for there is no evidence to show that Descartes is prepared to argue that the human mind in fact, or any particular human mind, is capable of mastering all subjects of inquiry and exhausting all their possible connexions and interconnexions so as to apprehend them *qua* one rational whole. He is definitely more modest than Spinoza in his claim for human knowledge.[2] It is this recognition of the limited capacity of the human mind which lies behind his distinction, in Rule 17, of the simple, direct instance and the complicated and involved instance of reasoning.

The simple or direct case, we know, is when we have a linear line of implication, leading directly from the self-evident datum to the final consequent which ends the reasoning process by completing the solution of the specific problem on which we were engaged. The whole problem, as solved, then resolves itself, element by element, under our analysis into its self-evident constituent simple natures and the necessary links implied in these natures. The deduction is simple and direct and can, given the requisite comprehensive review, be apprehended as a whole. But it may occur that, even in a linear inference from a simple datum, that is, in following through a line of direct linear implication, we may find that our advance is checked, either by the nature of the data, or by our own inability to apprehend the connexions between the data within the particular problem or branch of knowledge which we are studying. For although we may, by an enumeration in the sense of a preliminary survey, define

[1] Rule 12 (AT. x. 427^{27}–428^{2}): 'Nullas rerum cognitiones unas aliis obscuriores esse putandas, cum omnes ejusdem sint naturae, et in sola rerum per se notarum compositione consistant.'

[2] Cf. L. Meyer's preface to *Renati Des Cartes Principia Philosophiae* (Gebhardt, i. 132): 'ex Cartesii mente tantum dici, quod aliquibus in locis reperitur, nempe *hoc aut illud captum humanum superare.* Neque enim hoc ita accipiendum, ac si ex propia sententia talia proferret noster Author'. Meyer adds that, in Spinoza's view, the difficulties which Descartes found could be overcome by the use of other methods than those which Descartes himself employed.

and delimit the particular problem which we are attempting to solve, we may find that the linear implicatory series cuts across the boundary or limits of another branch of science and may thus involve us in other implicatory series not foreseen in the original setting forth of the problem. A biologist, for instance, may be studying some problem which falls strictly within his own particular sphere, that is, the biological sciences (on the somewhat hazardous supposition, for the present instance, that it is possible to define strictly the sphere of biology or any other science). In the course of his investigation the biologist may suddenly come upon certain facts which remain 'obscure' to him, since the 'link' which he has before him has no known and directly-connected relation with any biological fact; it may, for instance, involve some chemical factor which, *ex hypothesi*, is unknown to him *qua* biologist.[1] But it may also equally well occur that, even within the limits of a particular branch of knowledge, there will not be, or not appear to be, any single direct line of implication, or at least, if such a direct line exists, it may be hidden and obscured by the complicated nature of the demonstration. Instead then of deducing our consequent, the next link in our chain of reasoning, from an antecedent which is in fact a simple link, we may find that the antecedent is composed of many co-ordinate links, that is, of more than one simple link, each of these links depending upon a different series of logically connected elements.

When this occurs, it may not be a case, says Descartes, 'of deducing some one fact from a single simple element (this can be done without rules, as we have already pointed out), but of disentangling with skill some single fact which is conditioned by a number of facts which all involve each other'.[2] In such cases, where the whole difficulty lies in the com-

[1] Cf. *La Géométrie*, II (AT. vi. 390^{9-15}), where Descartes instances 'La Spirale, la Quadratrice, et semblables qui n'appartiennent véritablement qu'aux Mécaniques et ne sont point du nombre de celles que je pense devoir ici être reçues, à cause qu'on les imagine décrites par deux mouvements separés et qui n'ont entre eux aucun rapport qu'on puisse mesurer exactement'. Leibniz was critical of Descartes on this limitation of the geometrical domain: cf. Laporte, *Le Rationalisme de Descartes*, pp. 197–200.

[2] Rule 12 (AT. x. 429^{19-25}): 'Non quidem ex una re simplici unum quid deducendo (hoc enim sine praecepto fieri posse jam dictum est), sed unum quid ex multis simul implicatis dependens tam artificiose evolvendo.'

plexity of the implicatory series, enumeration or induction is essential as an auxiliary to the deductive argument, and when it is so used Descartes speaks of it as 'an illation drawn from many and disjointed things',[1] or describes it as 'a conclusion which is drawn by induction',[2] using the word *inductio* as synonymous with *enumeratio* in this type of auxiliary process. From these last expressions it would appear at first sight that enumeration was to be considered as a special, albeit auxiliary, method of proof, applicable, in the words of Rule 7 which precede the phrases, when the demonstration involves 'a long series of transitions from ground to consequent'. Such an interpretation is fostered by a passage in the same Rule which tells us that 'we must further note that by adequate enumeration or induction is only meant that method by which we may attain surer conclusions than by any other kind of proof, with the exception of simple intellectual intuition. But when knowledge of something cannot be reduced to intellectual intuition, we must throw aside all syllogistic fetters and use induction which is the only method left to us, but which is a method in which we can have complete confidence.'[3] Here it seems to be suggested that demonstration is reduced to two kinds, intellectual intuition and adequate enumeration or induction. But the confusion is not a doctrinal one but merely a certain carelessness of drafting due to overemphasis on the important role played by enumeration in complicated and involved deduction.

There is an important example of the use of enumeration in Rule 8, in the paragraphs preceding the example already quoted,[4] the inquiry into the conditions and limits of human knowledge. Suppose, Descartes says, someone who is 'a pure mathematician' sets out to determine in the science of optics (as he would call it, dioptics) what is called the 'line of

[1] Rule 11 (AT. x. 407^{12-13}): 'Illationem ex multis et disjunctis rebus collectam.' See also Rule 7 (AT. x. 389^{8-25}).

[2] Rule 7 (AT. x. 390^{23-25}). 'Per inductionem . . . concludatur' and at line 9 of the same page: 'Probare per enumerationem'.

[3] 'Per sufficientem enumerationem sive inductionem, nos tantum illum intelligere, ex qua veritas certius concluditur, quam per omne aliud probandi genus, praeter simplicem intuitum; ad quem quoties aliqua cognitio non potest reduci, omnibus syllogismorum vinculis rejectis, superest nobis unica haec via, cui totam fidem debeamus adhibere' (ibid. 389^{8-14}).

[4] See above, pp. 127–8. The present instance is found at AT. x. 393^{22}–395^{16}.

refraction' (the *linea anaclastica*), that is, the line in, or at, which parallel rays from a given object of vision are refracted in such a manner that they all, after being refracted, intersect one another and are focused in a single point. This pure mathematician, applying Rules 5 and 6 (the method of analysis and synthesis there prescribed),[1] will, or should, see at once that the determination of this line depends upon the proportion or ratio (*a proportione*) which is observed by each and all the parallel lines between their respective angles of incidence and angles of reflection.[2] But he will also remark that the discovery of this proportion requires a knowledge of physics and cannot be determined by the pure mathematician. Our supposed inquirer, who incidentally is warned that 'he can learn nothing from philosophers' to help him, should then break off his inquiry since the problem is insoluble by pure mathematical reasoning. But if we widen our choice and make the inquirer not a mere mathematician, but someone who has as his aim the study of more than 'one special science', then he will be able to extend and complete his analysis until he reaches the simplest link in the whole implicatory sequence in question. Our researcher 'will find that the proportion between the angles of incidence and the angles of refraction varies in accordance with the changes produced in these angles by differences in the [physical] character of the media'.[3] These changes will be found to depend upon and to be determined ultimately by 'the manner in which the ray penetrates the entire transparent body'.[4] Further, it will become clear that 'knowledge of this mode of penetration presupposes that the nature of light be known'.[5] Lastly, he

[1] See below, pp. 161–71 and 234–5.

[2] On the wide use of the word 'proportio', see Gilson's *Commentaire*, pp. 217–18. In the *Dioptrique* (AT. vi. 211–14) Descartes states that the proportion between two straight lines serves as the common measure of all the refractions which can be caused by the difference between the two media through which the rays of light pass. See also G. Milhaud, *Descartes savant* (1921), chap. v, 'Les Travaux d'optique de 1620 à 1629'.

[3] AT. x. 394^{22-24}: '. . . propter varietatem mediorum'.

[4] 'Pendere a modo, quo radius penetrat per totum diaphanum' (ibid. 394^{25-26}). The word 'diaphane' was used as a noun in seventeenth-century French.

[5] Ibid. 394^{26}–395^{2}. The problem of refraction is discussed in the second *Discours* of the *Dioptrique* but the general nature of light is treated in the first, 'De la Lumière'. The 'totum diaphanum' would seem to be the 'very subtle and fluid matter which extends continuously from the stars to us and fills the pores of

will find that knowledge of the nature of light presupposes knowledge of what 'a power of nature' is in general. With this presupposition he has reached the ultimate link in the whole demonstration and having obtained an intellectual intuition of this absolute link, he will now, in accordance with the rule of synthesis, return link by link in the reverse order through the complete implicatory series of the demonstration. Then, adds Descartes, 'if he finds himself, already at the second step [of the synthesis], unable to perceive the nature of light, he will, in accordance with Rule 7, *enumerate* all the natural forces in order that he may understand what light is, learning its nature, if not otherwise, at least by *analogy* (of which later) from his knowledge of one of the other forces'.[1] It is unfortunate that the promised explanation is never given; it would probably have been part of the third section of the *Regulae*.

What Descartes seems to have in mind is some form of 'an argument by analogy', by which we can arrive at a notion of what light is by analogy with a known force of nature which it is supposed to resemble. The pure mathematician is held up in his demonstration because he arrives at a link which cannot be determined by linear deduction from his own data which are purely mathematical. The physicist can continue, but it may occur that the nature of the action of light may escape him to the extent that he is unable to apprehend this nature as a clear and distinct idea. He will, however, understand that the nature of light implies the more simple notion of 'a natural power'. There is, then, necessarily a common element between the notion of light and that of natural power in general; that is to say, the action of light is the action

the grosser bodies' (AT. vi. 87^{2-5}). It is a sort of subtle matter of, and in, which light is the movement. It is then the *unspecified analogue* which, in a previous simile, Descartes had compared to the stick by the movements of which a blind man feels the objects which may be on his path (ibid. 13–22). See also the small fragment *De Refractione* amongst the *Excerpta* (in AT. xi. 639–40), where Descartes criticizes experimental results meant to establish the angle of refraction. He would have added, if Leibniz is correct in his claim about the genuineness of the *Annotationes*: 'Ex experientia etiam evidenti, sed ad examen rationis non satis expensa, falsum saepe concludi' (AT. xi. 654).

[1] 'Atque si statim in secundo gradu illuminationis naturam non possit agnoscere, *enumerabit*, per regulam septimam, alias omnes potentias naturales, ut ex alicujus cognitione saltem per *imitationem*, de qua postea, hanc etiam intelligit' (AT. x. 395^{6-10}). The italics are mine. Haldane and Ross translate 'per imitationem' as 'by analogy'

of one of the natural powers. In so far as the physicist is aware of the action of natural powers in general, he can argue by analogy from the more generally known to the comparatively unknown. The analogy is not, of course, empirical, based on external characteristics apprehended extensionally. In a sense such knowledge must be based on an hypothesis, similar to the hypothesis used in mathematics which posits an unknown term as known in order to continue a demonstration. This is in fact the essential recommendation of Rule 17, and this is possible because 'we recognize the dependence of the unknown terms in the investigation on those that are known to be such that the former are determined by the latter'.[1] It is a similar use of analogy that Descartes has in mind in Rule 14, where he is discussing the part imagination can play as an aid to thinking. He states that whenever one unknown fact is deduced from a fact already known, it does not follow that a new entity has been discovered but merely that the whole of our knowledge has been extended in such a manner that we now see that what was sought for participates in one way or another in the nature of the known datum. A blind man, who has been totally blind from birth, cannot be brought by reasoning to see 'the true ideas' of the colour which we derive from sense-perception. But if a man had perceived through his senses only the primary colours, he would be able to construct the ideas of the other intermediate colours 'from the similarity of their likeness to the others by a sort of deduction'.[2] This 'sort of deduction' is the kind sometimes called by

(i. 24), and both Le Roy (p. 73) and Gouhier (p. 49) write 'par analogie' in their translations. Descartes does not, to my knowledge, use 'imitatio' in this sense in any other work.

[1] 'Nos agnoscere eorum, quae in quaestione sunt ignota, talem esse dependentiam a cognitis, ut plane ab illis sint determinata' (AT. x. 460^{25}–461^{1}). Descartes writes to Mersenne in 1638 apropos of the problem of refraction and certain criticisms of Roberval. He thinks that he has given a 'demonstration', as far as this is possible, before giving a demonstration of 'les principes de la Physique par la Métaphysique' which he hopes to do one day (i.e. in the *Meditationes*) and he adds: 'Mais d'exiger de moi des démonstrations géométriques en une matière qui dépend de la Physique' is an impossible demand (cf. AT. ii. 141–2).

[2] '. . . ex aliorum similitudinem per aliquam deductionem effingat' (AT. x. 438^{18}–439^{1}). In *Resp. ad II Object.*, Descartes notes: 'non verisimilius mihi videtur, quam si diceretis nullam nos audiendi habere facultatem, sed ex sola colorum visione in sonorum notitiam devenire: plus enim analogiae sive paritatis fingi potest inter colores et sonos, quam inter res corporeas et Deum' (AT. vii. $136^{20–25}$). See below, pp. 218 and 244.

the *Regulae* enumeration or induction, and its nature is akin to an argument by analogy.[1]

§ 2. *Summary of the Rules of Demonstration*

If we now consider the various and varying interpretations of the meaning of 'enumeration' which were mentioned earlier in this study,[2] it can be fairly asserted that the descriptions were in fact true but did not allow for the different senses of the term in the *Regulae*. For enumeration or induction is considered by Descartes sometimes as a preliminary survey or setting out of the grounds of a demonstration. Sometimes it is considered a verification of the logical steps already traversed in the deductive process. Sometimes it is considered as a form of deduction based on the enumeration of all possible alternatives or on analogous instances and akin to the actual deduction itself when the implicatory sequence is grounded on a complex and disjointed set of data. All these accounts of enumeration are to be found, together with examples, scattered up and down the length of the *Regulae*. The diversity of opinion on the part of commentators merely reflects the diversity and sometimes the needless confusion of Descartes's own statements on this matter. It is, however, possible, without straining the sense of the texts, to fit together a more or less consistent picture of the rules governing demonstration and to give an account of the relations of intellectual intuition, deduction, and enumeration which holds together.

We think then of a typical Cartesian deduction as set forth in some such form as '*A* implies *B*, *B* implies *C*, *C* implies *D*, *D* implies *E*, hence *A* implies *E*'. Each step has been apprehended by intellectual intuition. The continuous movement of thought itself through the necessarily linked steps of the sequence is a deduction. In so far as the mind could retain, and gather before itself in an undivided and single *conspectus*, the single steps of the whole process, the completed deduction would be an intellectual intuition and would possess the self-evidence of immediate presence, the *praesens evidentia*

[1] A. Gewirth notes that 'the justification given in the *Regulae* for the analogical employment of the material simple natures to "express all the differences of sensible things" is a frankly methodological one, and itself exhibits an empirical orientation' (*Journal of the History of Ideas*, ii. 196, n. 2).

[2] Cf. above, p. 118.

which is essential to the primal and fundamental act of knowing. In other cases the mind's certainty of the conclusion of the demonstration must depend to some extent on memory, and accordingly, in the more complex cases when the data do not form a single chain of linear deductions but many interconnected groups of linear deductions or what Descartes calls a network of chains, enumeration is required to assist, and in a sense eliminate as far as possible, memory by arranging and preparing the data and by introducing and ordering them so as to facilitate the unbroken movement of illation. Partly owing to the importance of the role played by this preliminary grouping, and partly perhaps in order to have a distinct name for these complex inferences which do not issue in a single act of intuition; partly also by careless use of terms, Descartes sometimes calls the whole complex inference an enumeration or induction. Deduction, as we have noted many times, is distinguished from intellectual intuition in so far as it involves a movement on the part of the mind as it infers one thing from another. In so far as deduction covers only the movement of illation, the act of inferring one fact from another, it does not cover the whole function of enumeration, since it does not include the preliminary survey of the grounds of the inference. Considered as the completion of the movement and not as the movement of illation; considered, in other words, as the apprehension of the articulated content in which the illation completes itself, deduction falls within intellectual intuition. If the movement as a whole, the total demonstration, cannot be apprehended *tota simul* by the mind, because it is too complex and involved, then deduction becomes enumeration. In other words, if we consider deduction as covering all that is involved in the continuous movement of illation, from the preliminary survey of the grounds to the final review of all the steps of the demonstration, then enumeration can be taken to be a special case of deduction, that is, a complex and involved deduction which, owing to its very complexity, is not reducible to intellectual intuition. Whether or not this irreducibility is merely *de facto* or *de jure* does not seem to enter into account but it would seem appropriate to assume that the complexity which is the distinctive feature may well be relative and variable.

Such would seem to be the Cartesian doctrine of enumeration and the best interpretation to set upon those passages where Descartes speaks of intellectual intuition and enumeration as aiding and completing each other so that the two operations seem 'to grow together' into a single operation by virtue of a movement of thought which has an intellectual intuition of one set or series of facts and yet can at the same time pass to another set of facts. 'To prove by enumeration' does not then mean that enumeration is an entirely distinct method of proof which differs in logical character from deduction. It is, like deduction, an implicatory development of intellectual intuition which provides the seeds of growth; it is an indispensable auxiliary or supplement to deduction in the more complicated and involved demonstrations, where we are forced to rely heavily upon memory because the links of the demonstration are too intricate and too heterogeneous to be included and kept distinct within the limited span of an intellectual intuition. It is not, by itself, a form or mode of demonstration at all, since its function, auxiliary to that of deduction, is to group, arrange, and classify premisses which have already been intuited, or links in the argument which have been deduced, so as to aid their retention in memory. We do not acquire new data nor do we acquire new premisses of demonstration by this process since we merely arrange or rearrange data or links already acquired. Enumeration then covers the auxiliary devices of demonstration by which either we recapitulate data and steps of a demonstration already completed, or else we survey the ground to be covered in a demonstration in order to select the relevant data for the adequate solution of some particular problem; or else it attempts to integrate the elements of one series of deductive facts under the formula of a higher series, thus linking disjointed series in a more complex series.

It may be noted that the formulation of the doctrine of enumeration raises several questions which must be left over until later. Enumeration as preliminary survey would clearly seem to be a process of analysis whereas as verification or integration, it is, on Descartes's own assertion, a synthetic process. It is also difficult to distinguish the logical from the psychological delimitations which separate the various forms

of demonstration. In the case of the preliminary survey, the sufficiency and adequacy, which were laid down as the essential characteristics of enumeration, do not seem to imply more than a delimitation of the lacunas in our knowledge, in itself a variable factor, and these characteristics must be delineated in the most general and abstract terms. We are supposed to mark the general character of the group or class and this general description is all that need represent it in our enumeration. The 'scouting' over the particular field of knowledge, however careful and accurate we may make it in searching for methods of solving the problem, may lead to nothing more than a disjunctive separation of the groups in the most general of terms. The examples produced by Descartes, as, for instance, the preliminary survey relative to the proof that the rational soul is not corporeal, shows clearly that the survey tends to become the most general of classifications. It is perhaps for this reason that Descartes shows a prudence and modest reserve in his estimate. 'It is', he says in Rule 7, 'only by the help of enumeration that we can pass a true and certain judgement on any subject whatever to which we apply our minds, so that nothing can escape us altogether but we seem to know something about everything.'[1]

[1] AT. x. 388[20-24].

X

THE ESSENTIALS OF METHOD

§ 1. *The Rules of Method*

THE theme of discussion in this study has been dictated by an attempt to describe the 'instruments of knowledge', the *vis cognoscens* in its functions as source and stream of clear and distinct ideas, and this course has led to an account of demonstration and the functions of intellectual intuition, deduction, and enumeration. It has not been possible to follow exactly the various Rules in the order in which they appear in the *Regulae* and it is necessary to return upon some of the earlier Rules. Our starting-point was the Cartesian assertion that the 'natural light of reason' was in itself infallible in so far as it functioned in accordance with its proper nature as 'a pure and attentive mind'. But it was also noted that the 'natural light of reason' might become confused so that we might mistake false for true or we might suppose ourselves to have a clear and distinct vision of the truth when we have no such thing. 'Prejudices', as they were generically termed, might fill our mind and pervert our natural 'good sense'. Such prejudices might arise, for instance, from the uncritical acceptance of the data of sense or imagination; they might equally well be due to studies conducted in an unmethodical way, from ill-advised and ill-directed attempts to solve various problems before we have cleared up the simpler questions on which their solutions logically depend. Hence we require 'easy and certain rules' to secure the right use of the *vis cognoscens* so that we can never mistake false for true; we require 'easy and certain rules' to guide us in the construction of our deductions, so that we may extend our clear insight over the whole field of possible knowledge by certain and graduated steps. Only on these conditions can we pass, with requisite security, in an uninterrupted movement of thought from the simplest self-evident data in logical order to the complex truths. Such rules constitute a method.

Rule 4 states shortly and emphatically: 'A Method is

necessary for investigating the truth of things.'[1] This brief enunciation is followed by a passage full of Cartesian malice and irony.

> People [he says] are possessed by such a blind curiosity that they often lead their minds through unknown roads, without any reasonable hope of success, but merely willing to risk the chance that what they are looking for may lie there; much as if someone, who is stupidly anxious to find a treasure, were to wander aimlessly up and down the streets, hoping to find something which had been dropped by a passer-by. This is the way nearly all chemists, most geometers, and many philosophers pursue their studies. I do not deny that, on occasions, they may be lucky enough to find something true. But I do not consider them for that reason hardworking but merely lucky. But it is far better never to think about discovering the truth at all than to do so without a method. For it is quite certain that haphazard studies and vague reflections of this kind only confuse the natural light and blind our mental powers, as those who have become so used to working in the pitch dark weaken their eyesight so much that they cannot afterwards support the sunlight. All this is confirmed by experience, for how often we see those who have no book-learning at all showing a sounder and clearer judgement than those who have spent all their time in the Schools.[2]

He would have agreed with Swift that 'Method is good in all things. Order governs the world, The Devil is the author of confusion.'

The general principles of the method itself are set forth in Rules 5 and 6, which are to be taken together, Descartes says, with Rule 7, where the question of enumeration is discussed.[3] Rules 9, 10, and 11 give detailed instructions on the carrying out in practice of the general principles of method: Rule 9 deals with ways and means of practising and developing the attentiveness, &c., required for intellectual intuition; Rule 10 deals with the practice and development of deduc-

[1] 'Necessaria est Methodus ad rerum veritatem investigandam' (AT. x. 371^{2-3}).

[2] Ibid. 371^{4-25}. See also the letter to Villebressieu, 1631, where Descartes tells of the famous public discussion with de Chandoux, in the presence of Cardinal de Bérulle—probably the first time he mentioned his method to other than his most intimate friends: 'Ce fut là que je fis confesser à toute la troupe ce que l'art de raisonner peut sur l'esprit de ceux qui sont médiocrement savants et combien mes principes sont mieux établis, plus véritables et plus naturels' (AT. i. 213^{8-11}).

[3] AT. x. 392^{1-7}. We must study Rules 5, 6, and 7 together because '**pariter** omnes ad methodi perfectionem concurrunt'. The rest of the treatise down to Rule 12, I surmise, merely works out 'in particulari quae hic in genere complexi'. Rule 7 has already been discussed in the last two chapters.

tion (including enumeration in the complex implicatory sequences); Rule 11 with the practical ways of combining intellectual intuition and deduction together so as to utilize them advantageously in scientific research.[1] Rule 8 stands somewhat apart from others and discusses the special cases where problems involve 'insolubles'. Rule 12, the longest of all, deals with the various aids to intellectual intuition and how to use sense, imagination, and memory. Rule 13 returns to enumeration and the use of the analytic method in breaking up problems. It is the first of what Boyce Gibson aptly names 'the principles of Economy', applied to the need of mental economy and the avoidance of wasting time on unessentials. Rule 14 attempts to show how the process of abstraction can be simplified by reduction of problems to their purely quantitative form while Rule 16 urges the same principle by advocating time-saving and memory-aiding notations. Rules 18 and 20 continue the same line of thought while Rule 15 gives a practical application of the use of imagination. Rule 17 consists of a summary of the doctrine of enumeration in the sense of an inference by analogy. Rule 4 itself splits into two parts. The first part deals with the necessity and purpose of a method in general and the second, which in the Hanover MS. was separated, wrongly but understandably, from the main text, breaks off into an autobiographical passage where Descartes is led to a short account of his new science of order and measure. Such in the broadest outlines are the rules of method.

It may be asked: how do these Rules compare with the better-known rules of the *Discours*? Roughly speaking, Rules 2 and 3 of the *Regulae* correspond to the first Rule set forth in the *Discours*. The fourth Rule of the *Discours* corresponds to the doctrine of enumeration as explained principally in Rules 7 and 11 of the *Regulae*. The second and third Rules of the *Discours* are practically the same as Rule 5 of the *Regulae*, since this last combines in its enunciation both of the essential features of the Cartesian method, analysis and synthesis. It may perhaps help to set out in parallel columns the texts of the *Regulae* and the *Discours*, in the latter case

[1] 'Haec [Rule 11] explicat quo pacto hae duae operationes se mutuo juvent et perficiant' (ibid. 408[11–17]).

using the Latin translation to mark the points of similarity. The texts of the *Discours*, which are to be found in AT. vi. 550, offer no difficulty, but in the case of the *Regulae* there is often more than one text to appose.

Regulae	*Discours de la Méthode*
Circa illa tantum objecta oportet versari, ad quorum certam et indubitatam cognitionem nostra ingenia videntur sufficere. (Rule 2, AT. x. 362^{2-4}.)	Primum erat, ut nihil unquam veluti verum admitterem nisi quod certo et evidenter verum esse cognoscerem; hoc est, ut omnem praecipitantiam atque anticipationem in judicando diligentissime vitarem; nihilque amplius conclusione complecterer, quam quod tam clare et distincte rationi pateret, ut nullo modo in dubium possem revocare.
Circa objecta proposita, non quid alii senserint, vel quid ipsi suspicemur, sed quid clare et evidenter possimus intueri, vel certo deducere quaerendum est: non aliter scientia acquiritur. (Rule 3, 366^{11-14}.)	
Tota methodus consistit in ordine et dispositione eorum ad quae mentis acies est convertenda, ut aliquam veritatem inveniamus. Atque hanc exacte servabimus, si propositiones involutas et obscuras ad simpliciores gradatim reducamus, et deinde ex omnium simplicissimarum intuitu ad aliarum omnium cognitionem per eosdem gradus ascendere tentemus. (Rule 5, 379^{15-21}.)	Alterum, ut difficultates quas essem examinaturus, in tot partes dividerem, quot expediret ad illas commodius resolvendas. Tertium, ut cogitationes omnes quas veritati quaerendae impenderem, certo semper ordine promoverem: incipiendo scilicet a rebus simplicissimis et cognitu facillimis, ut paulatim et quasi per gradus ad difficilliorum et magis compositarum cognitionem ascenderem; in aliquem etiam ordinem illas mente disponendo, quae se mutuo ex natura sua non praecedunt.
Ad scientiae complementum oportet omnia et singula, quae ad institutum nostrum pertinent, continuo et nullibi interrupto cogitationis moto perlustrare, atque illa sufficienti et ordinata enumeratione complecti. (Rule 7, 387^{10-13}.)	Ac postremum, ut tum in quaerendis mediis, tum in difficultatum partibus percurrendis, tam perfecte singula enumerarem et ad omnia circumspicerem, ut nihil a me omitti essem certus.

A casual inspection of these parallel sets of rules shows that the methodological doctrine is essentially the same. The cautious judgement of Boyce Gibson that 'the same points are insisted on in each case and the spirit of the two methods is essentially the same', should be reasserted, with Roth, to state that 'the fundamental doctrine of the *Regulae* and the

Discourse is the same. We should not speak then of two stages of Desçartes's doctrine of method, and still less of two different doctrines. His primary conception remained unaltered from the earliest times of which we have record, and the rules of the *Discourse* are its classic, as they are its earliest, expression.'[1] But this earliest, that is to say, first published, expression of the new method that was to revolutionize and reorientate all the sciences and philosophy, left many of its first readers with a feeling of keen disappointment. The very simplicity of the rules, or perhaps we may now say with some justification, their apparent simplicity, seemed to lead these readers to conclude that they were in fact banal and commonplace. Pascal could dismiss the whole matter with his *Descartes inutile et incertain*. But both friendly and hostile critics, Hobbes, Gassendi, Fermat, Huygens, and later Leibniz, all seem agreed in the opinion that the precepts of the *Discours* could not be a full expression of the method. Lipstorp, as Baillet tells us, in his *Specimina Philosophiae Cartesianae*, thought that the true methodological principles were to be found in the *Geometriae*—a view which has been shared by modern commentators like Brunschvicg, again with some justification. Gassendi thought that the *Meditationes* were in fact the real exposition of the method and made a summary of them which he entitled *Logica Cartesii*.[2] The more ardent disciples hinted that Descartes had only expounded part of his doctrine and that he had other more advanced secret principles which were reserved for an esoteric audience. Leibniz, who had read the manuscript of the *Regulae*, was in a better position to write, in his *Notata quaedam circa vitam et doctrinam Cartesii*, that Descartes 'has not expounded in his writings [what took place on the 11 November 1620] nor

[1] Cf. Boyce Gibson, *Mind*, N.S. vii. 158, and L. Roth, *The Discourse on Method*, pp. 64–65. In Roth's work there is an interesting parallel table between the rules of the *Discours* and the rules of method of Francis Bacon: cf. pp. 69–70. Gilson notes, *Commentaire* on the *Discours*, p. 196, that 'la méthode a donc été dès le début, quant à l'essentiel, ce qu'elle devait toujours demeurer'.

[2] Baillet, *La Vie*, i. 281–2. The work of Lipstorp was published, in two volumes, by Elzevier in 1653. Baillet, who mentions Gassendi in the margin, gives the reference 'tom. i, oper. pag. 65, 66'. The *Opera* were published at Lyons in 1658. Gassendi, unlike Descartes, did not despise historical erudition. The *Syntagma Philosophicum complectens Logicam*, &c., published posthumously in vols. i and ii of the *Opera*, contains a short history of logical doctrines.

has he published his method but he has only written about it and decided to give examples of its use. Those, therefore, who are too satisfied with what he has published are greatly mistaken in thinking that they know what is his method.'[1]

Leibniz is right up to a point in saying that Descartes had 'only written about' his method. It is, in almost identical words, what Descartes himself said to Mersenne, to Vatier, and to Huygens, the last finding only the Italian phrase *più saporita* adequate to express his enthusiasm.[2] Yet despite these protestations of his modest purpose and aim in setting down the precepts of the method, essentially and fundamentally they contain the full methodological principles of Cartesian philosophy, and the *Regulae* are only a more elaborate and detailed account of the same principles with more practical aids to help towards their use. As we read the declaration of the *Discours*, that 'my purpose here is not to teach the method which everyone should follow in order to use his reason in a right manner but merely to show in what way I have endeavoured to use my own',[3] we should keep in mind the ironical smile on the portrait now hanging in the Louvre, painted by Franz Hals.

§ 2. *The Definition of Method*

Rule 4 gives us a definition of method. 'By method', Descartes says, 'I mean a set of certain and easy rules such that anyone, who obeys them exactly, will first never take anything false for true and secondly, will advance by an orderly effort, step by step, without waste of mental effort, until he has achieved the knowledge of everything which does not surpass his capacity of understanding.'[4] If we do not in fact

[1] Cf. Gerhardt, iv. 310.

[2] Cf. Letter to Mersenne, 1637: 'Car je ne mets pas *Traité de la Méthode* mais *Discours de la Méthode*, ce qui est le même que *Préface ou Avis touchant la Méthode*, pour montrer que je n'ai pas dessein de l'enseigner, mais seulement d'en parler' (AT. i. 349^{16-20}) and again in a letter to X, May 1637: 'une méthode générale, laquelle véritablement je n'enseigne pas' (ibid. 370^{10-11}), and similar expressions in the letter to P. Vatier (ibid. 559^{13-20}) and to Huygens (Roth, 34^{23-28}).

[3] AT. vi. 4^{7-10}.

[4] 'Per methodum autem intelligo regulas certas et faciles, quas quicumque exacte servaverit, nihil unquam falsum pro vero supponet, et nullo mentis conatu inutiliter consumpto, sed gradatim semper augendo scientiam, perveniet ad veram cognitionem eorum omnium quorum erit capax' (AT. x. 371^{25}–372^{4}).

understand things which we are capable of understanding, then the fault must be laid at the door of want of method and misuse of our natural powers of reason. Two points in this definition should be noted: the need to apply strictly the 'rule of evidence', the criterion of clear and distinct ideas which can only be apprehended by the 'pure' use of the *vis cognoscens*, and the need to advance from this original apprehension in an orderly manner. Descartes points out that he has already, in the earlier Rules, made it clear that no real knowledge can be acquired except by intellectual intuition and deduction, the primary and simple operations of the mind. Consequently, he adds, 'unless our mind were already able to employ them, it would not be able to understand any of the precepts of the method itself, not even the simplest of them.'[1] The quasi-objection contained in this comment does suggest that the definition of the method adds little or nothing to what we already know from previous discussions. Since it is of the very nature of the *vis cognoscens* to apprehend the primary datum or simple nature, and in this apprehension to recognize it as an indubitable truth with its own self-guaranteeing evidence, and since it is equally of the nature of mind *qua* deductive to pass in a continuous movement from self-evident step to self-evident step, and these twofold operations constitute the whole activity of mind as a purely spiritual power, what in fact can a method mean which proposes to institute rules to teach us to use them properly? The rules are designed, Descartes replies, to train and educate the mind in these operations. They are meant to facilitate the task but they are in no sense substitutes for the right use of reason. A knowledge of these rules does not provide an alibi for careful attention. The rejection of the syllogism and all formal reasoning was essentially based on the view that such reasoning *qua* formal was sterile and could not help in any way to the discovery of the truth. The 'easy and certain rules' are not therefore a set of formal principles which, once learnt by heart and mechanically applied, will automatically yield truth. The rules are guides to practice and are an expression of the mind's own working. By practice we can increase our powers of discernment and vision. By practice we can train

[1] Ibid. 372[20–22].

ourselves to pass rapidly from step to step of the demonstration until we can grasp each and all the steps in a single act of intellectual vision. By practice we can arrange and classify data on which the mind is set to work; all this in an orderly manner so that the mind will work and develop its knowledge under the best possible conditions. The rules of method are in fact the description of the proper working of the mind in its operations of intuiting, deducing, and enumerating. It is not enough to have a good mind; one must learn to use it properly.

The method will thus help to cultivate the 'primary seeds of useful thought' which are to be found in all human minds which themselves have something almost divine in them.[1] And the author of the rules of method is so much impressed with the prospect of possible results of their use and their unique value as an instrument of scientific and philosophical research that he adds, without any false modesty: 'I am prepared to believe that the greater minds of yore had some knowledge of this [method], led to it by nature alone.' We are not, however, here concerned with tracing in the history of philosophy any unconscious anticipations of the rules of Cartesian method. If we assume that the four rules of the *Discours* express the essential rules of method, and are the comprehensive definition of the method, then still we have left as our task the study and commentary of the second and third precepts of the *Discours* or, in terms of the *Regulae*, the Study of Rules 5 and 6.

[1] AT. x. 373^{8-9}. The expression 'prima cogitationum utilium semina' is echoed again in 376^{12-13}: 'prima quaedam veritatum semina humanis ingeniis a natura insita'. See also the *Discours*: 'certaines semences de vérité qui sont naturellement en nos âmes' (AT. vi. 64^{4-5}), and the curious passage in the *Cogitationes Privatae*, where he compares poets and philosophers and then concludes: 'sunt in nobis semina scientiae, ut in silice, quae per rationem a philosophis educuntur, per imaginationem a poetis excutiuntur magisque elucent' (AT. x. 217^{20-22}). In the French translation of the *Principia Philosophiae*, there is a passage which does not appear in the Latin edition and must doubtless have been added by Descartes when he revised the translation of the Abbé Picot. It reads: 'Nous quitterons sans peine *tous* les préjugés *qui ne sont* fondés *que* sur nos sens et ne nous servirons que de notre entendement, parce que c'est en lui *seul* que les *premières notions* ou idées, *qui sont comme les semences des vérités que nous sommes capables de connaître*, se trouvent naturellement' (AT. vii (Part II), 65). The expression 'scientiarum semina' is used by the Schoolmen: see texts from the *De Veritate* of St. Thomas in Gilson's *Index Scolastico-Cartésien*, pp. 152–3. See below, p. 193. The expression would appear to be in linear descent from the σπερματικοὶ λόγοι of the Stoics.

XI

THE RULES OF ANALYSIS AND SYNTHESIS

§ 1. *Order and Thinking*

We must learn to think in an orderly fashion. This precept, or some such similar injunction, is repeated throughout the *Regulae*. It would be a true, if somewhat over-simple, summary of the principal lesson which the author wishes to inculcate. A reasonable person would not object to such a precept but would naturally tend to ask what was meant by 'thinking in an orderly fashion' or rather what is meant by 'order' when applied to thinking. The short exposition of Rule 5, which only contains twenty-four lines, is devoted entirely to an exhortation on the importance of an exact observance of the principles of orderly thinking, as laid down in the Rule, for in this 'alone lies the sum of all human endeavour'.[1] We cannot dispense with the exact observance of these principles of order in the pursuit of truth any more than we would dare to enter the labyrinth without the thread which guided Theseus. We are warned against the tendency common to many scientists, who tackle the more complicated and involved problem before they have solved the primary and fundamental difficulties which underlie these problems. But not only scientists are at fault. Philosophers too are prone to neglect experience and the observation of the simple and obvious. They 'imagine that truth will spring from their brain, like Minerva from the head of Jupiter'. We must start from what is simplest and easiest; this has been urged again and again in the *Regulae* but it cannot be too often repeated or given too much emphasis.[2] Does 'order', then, merely mean beginning with the simplest and easiest?

[1] 'In hoc uno totius humanae industriae summa continetur' (AT. x. 379^{22-23}). Cf. also Rule 13, 'tantum operae pretium esse' (AT. x. 438^{3}).

[2] Cf. Rule 2 (AT. x. 364^{13-21}) and Rule 3 (AT. x. 368^{25-27}).

The enunciation of Rule 5 shows that there is another aspect of order. The text runs thus:

> The entire method consists in the ordering and arranging of the materials on which the mind's attention must be concentrated, if we are to discover truth. The exact observance of the method will be secured if (i) we reduce involved and obscure propositions step by step to more simple ones; and (ii) if we start from our intellectual intuition of the simplest propositions and endeavour, by retracing our path through the same steps, to work our way up to the knowledge of all the others.[1]

The order, it will be noted, is twofold and corresponds to the two Rules of the *Discours* which already have been set side by side with Rule 5 of the *Regulae*. We may therefore restate this twofold aspect of 'order' in the actual words of the *Discours*, which formulates these precepts:

> Secondly, to divide up each difficulty, which I have examined, into as many parts as possible and as seemed requisite for its proper solution.
>
> Thirdly, to think in an orderly manner, beginning with the objects which are the simplest and easiest to understand and working my way, step by step, to knowledge of the most complex; assuming an order to exist even amongst those objects which have not a natural sequence of their own.[2]

It is the generally accepted usage amongst writers on Cartesian philosophy to describe the second rule of the *Discours* as 'the rule of analysis' and the third rule is then described as 'the rule of synthesis'. On the basis of this traditional nomenclature, Rule 5 of the *Regulae* would be described as 'the rule of analysis and synthesis'. It should be remarked here that Descartes does not use this particular terminology in the *Regulae*. He uses the words 'analytic' and 'synthetic' elsewhere to denote the twofold aspect of this method and gives a general account of the main differences between these aspects in the *Reply to the Second Objections*, where he declares that 'the method of demonstration is twofold, one being analytic, the other synthetic'.[3] Analysis shows the true way

[1] AT. x. 379^{15-21}. The Latin text has been given on p. 150. 'Ex omnium simplicissimarum intuitu' means, I suppose, 'the simplest of all' and not 'of all the simplest'. See below, p. 205.

[2] AT. vi. 18^{24}–19^{2}.

[3] 'Demonstrandi autem ratio duplex est, alia scilicet per analysim, alia per synthesim' (AT. vii. 155^{21-22}). See also a letter to Mersenne, 1638, where Descartes

by which a thing is methodically discovered and derived *tanquam a priori*, that is, as effect from cause; synthesis, on the contrary, proceeds in the opposite direction, *tanquam a posteriori*, that is from effect to cause. The use of these terms, however, does not appear to have been thought out by Descartes to any great extent and in so far as he does use them, with great latitude of meaning, he seems to be following what was the traditional usage in the Scholastic manuals of the time which he studied at La Flèche.[1]

It is particularly the word 'analysis' which is used somewhat loosely. To be sure of the sense in which it is being employed, it is almost always necessary to look at the context. Sometimes Descartes uses 'analysis' in the sense of the second rule of the *Discours*, that is, to signify the order followed in resolving complicated and involved problems into an intelligible combination of simple elements which are clearly seen and self-evidently known. It is in this sense that the *Meditationes* as a whole is a single problem resolved by the analytic method. The multiple data of knowledge are broken down until the first and simplest of all data is discovered, the *Cogito, ergo sum*. But he sometimes used the word

speaks of 'la démonstration *analytice* et *synthetice*' and adds farther on: 'je l'avais trouvé [la démonstration de la Roulette] la première fois *analytice*; et depuis, pour ce que j'ai vu qu'il n'en avait su faire le calcul, je l'ai expliqué après *synthetice*' (AT. ii. 394^{3} and 400^{16-18}). The French text of the *Resp. ad II Obj.* is more explicit: 'La manière de démontrer est double: l'une se fait par l'analyse ou résolution, et l'autre par la synthèse ou composition' (AT. ix (Part I), 121). Hobbes, in the *De Corpore*, chap. vi, § 1, gives a similar definition. He states that 'omnis methodus . . . est vel compositiva, vel resolutiva' or partly one and the other: 'Et resolutiva quidem *Analytica*; compositiva autem *synthetica* appellari solet.' Analysis, therefore, for Hobbes corresponds only to the divisive operation of Descartes's method and synthesis to the deductive aspect. Analysis for Descartes includes both.

[1] See, for instance, the extract from the *Institutionum dialecticarum libri octo* of Fonseca, which was printed for the use of La Flèche: 'Naturae ordo duplex est, generationis et perfectionis. . . . Sub hoc genere [generationis] duo potissimum continentur, *σύνθεσις* et *διαίρεσις*, hoc est compositio et divisio, seu compositionis ordo et divisionis. Compositionis ordo est, cum a partibus integrantibus ad totum integrum. . . . Divisionis ordo dicitur, cum a superioribus ad inferiora distribuimus. . . . Sub ordine perfectionis, duo itidem, quemadmodum sub generationis, praecipue comprehenduntur *ἀνάλυσις* et *συναγωγή*, hoc est resolutio et collectio, seu resolutionis ordo et collectionis. Resolutionis ordo est, cum a toto integro ad partes integrantes procedimus. . . . Collectionis vero est, cum ab inferioribus ad superiora conscendimus'; quoted by Sirven in *Les Années d'apprentissage*, pp. 368–9. See also the definition of Eustachio di Santo Paulo: 'Methodus sive ordo resolutionis quae dicitur analysis. . . . Quoniam vero analysi, id est divisioni opponitur synthesis, id est compositio', in Gilson's *Index Scolastico-Cartésien*, pp. 182–4, and his *Commentaire*, pp. 189–90.

'analysis' in a more restricted sense to apply to geometrical analysis, which is a particular application of the general analytic method of regression from the complex to the simple. In order to discover this order it is necessary that we should, *qua* plane geometers, have at our disposal all the lines which are essential to the construction of the particular problem under consideration; otherwise it would be impossible to know how and in what relationship the lines conditioned one another. In order to resolve the problem, therefore, we must consider the problem as if it were already resolved 'and give names to all the lines which seem necessary to its construction, whether they be known or unknown'.[1] Then the order imposed by the rule of analysis becomes applicable and we can work out their mutual dependence until we have found a way of expressing 'the same quantity in two ways: that is what is called an equation'.[2] Analysis also covers, for Descartes, the whole of what is now called analytical geometry and represents the application of his method of measuring geometrical curves by means of coordinates which allow geometrical relations to be expressed in terms of algebraical equations. In Rule 5 we are concerned mainly with the first sense of the term, which has to be broadened to include the use proposed in the 'Universal Mathematics' sketched in the latter half of Rule 4.

The statement that the second precept of the *Discours* is traditionally named 'the rule of analysis', and the third precept 'the rule of synthesis', while a fair statement of common practice, may perhaps tend to over-simplify the exposition of Descartes's thought. In point of fact both in his enunciation of the Rules and his comments on them there seems to be a little confusion and, perhaps significantly, the distinction between the two processes is much less clear-cut and tidy.

[1] *La Géometrie*, I. 'Ainsi, voulant résoudre quelque problème, on doit d'abord le considérer comme déjà fait et donner des noms à toutes les lignes qui semblent nécessaires pour le construire, aussi bien à celles qui sont inconnues qu'aux autres' (AT. vi. 372^{10-14}). See below, pp. 179 ff. Compare with the definition given in Book XIII of Euclid's *Elements*, probably the work of Pappus: '*Ἀνάλυσις μὲν οὖν ἐστι λῆψις τοῦ ζητουμένου ὡς ὁμολογουμένου διὰ τῶν ἀκολούθων ἐπί τι ἀληθὲς ὁμολογουμένον.*' Quoted by A. Robert in 'Descartes et l'analyse des anciens', *Archives de philosophie*, vol. xiii, no. 2, p. 227.

[2] Cf. *La Géométrie*, I (AT. vi. 372^{19-20}). For Descartes's contribution to analytical geometry see Milhaud, *Descartes savant* (1921), pp. 124–48.

The third precept of the *Discours* begins with the fundamental demand that we should think in an orderly manner, and this order obviously applies with equal force to the second precept, the rule of analysis, which is supposed to discover the order which the synthetic process will reconstruct in the reverse order. Hamelin notes that if one uses algebraic methods, as Descartes does, all the propositions being reciprocal or, in other words, the equalities being reversibles, then there is no real need for synthesis at all since the analytic method in fact completes both the discovery of the simple natures and their definitive order.[1] Yet the third precept seems to lay down precisely this order in the discovery of these simples. It is perhaps with this point in mind that Gilson, in his *Commentaire*, speaks of 'the rule of method, called that of analysis, which consists in the third precept of the method'.[2] How far is the traditional account of the precepts of the *Discours* mistaken? Or is the possible confusion due to some inherent doctrinal point which prevents any distinction of analysis and synthesis from remaining a complete dichotomy? The answer is best given after an account of the two processes, assuming in the meantime the more traditional view.

§ 2. *The Rule of Analysis*

We may grant that it is sound advice that we should start from the simplest and easiest. Descartes is aware of the proverb of his own time, *entre fait et dit a moult*. His readers may legitimately ask how we are to reduce these involved and obscure propositions to more simple ones. How can we map our way through the various regions of thought so as to be able to recognize our route in the regressive march? How, for instance, shall we know when we have attained the set goal and reached 'the simplest proposition'? The question is all the more legitimate in that Descartes himself grants that 'the order is often so obscure and complicated that not everyone can follow it'.[3] Error, therefore, is not only possible but likely, 'unless', he adds, 'we observe carefully what is laid down in

[1] *Le Système de Descartes*, pp. 56–57, and Sirven, *Les Années d'apprentissage*, pp. 190–1.

[2] *Commentaire*, pp. 189–90.

[3] AT. x. 380[18–19].

the following rule', that is, in Rule 6. There we find the description of what is meant by 'the ordering and arranging of materials' and the rules presumably to execute this ordering. Descartes admits that 'it may not contain anything very new' but it is nevertheless called 'the chief secret of the method' and is said to be of greater utility than any other rule in the *Regulae*.[1] The 'chief secret' is formulated in the following terms:

> If we are to distinguish the most simple things from those that are complex, and if we are to advance from the former to the latter in due order, we must proceed as follows: in any series of things [that is, in any sequence of truths] which we deduced directly one by one from another, we must observe which term is the simplest of all, and how the other terms are related to that simplest term, namely, whether they are [as compared with one another] more, or less, or equally removed from it.[2]

Presented with a set of involved and obscure propositions, we must then attempt to proceed, step by step, until we reach a term which, in the series in question, is the most simple. In Rule 6 Descartes makes it quite clear that the method of this reduction of the complex is gradual, that the analysis (and the same is true of synthesis) proceeds step by step (*gradatim* or *per easdem gradus* is continually used to describe the process). But it is also clear from the last sentence of the text quoted above that the ultimate goal or aim of the analytic regression is therefore not the simple *qua* simple but the simple 'relatively' to the other terms of the series; the simple, in other words, is the limit of a gradually regressive movement from the more to the less relative.

It is as well to note immediately that 'the series' does not signify that we are to consider that things or facts can be arranged in a conceptual classification similar to that adopted by the Schoolmen from the Aristotelian *κατηγορίαι τοῦ ὄντος*,

[1] 'Praecipuum artis secretum' (AT. x. 381^{8}, 382^{17}).

[2] 'Ad res simplicissimas ab involutis distinguendas et ordine persequendas, oportet in unaquaque rerum serie, in qua aliquot veritates ex aliis directe deduximus, observare quid sit maxime simplex, et quomodo ab hoc caetera omnia magis vel minus, vel aequaliter removeantur' (ibid. x. 381^{2-6}). The Latin is not very clear and I have paraphrased and expanded a little. Baillet, in his *La Vie*, ii. 405, omits Rule 6 from his summary of the *Regulae*. The omission seems to have escaped the notice of the editor of the AT. edition who refers to Rule 6 what is clearly a summary of Rule 5. Cf. AT. x. 478.

the *praedicamenta* of the manuals. What is meant is that all things or truth are to be considered *seriatim* in so far as knowledge of one can be derived, by deduction, from knowledge of another. The series, therefore, is not a static ontological classification based on genus and specific difference but an implicatory sequence of antecedent and consequent in which the important and decisive factor is the logical relation of one to the other. The series is essentially a deductive series. There is undoubtedly a certain community of expression between the principle of analysis as described by Descartes and the *praeceptum dividendi* which was one of the commonplaces of the Schools, but apart from the common object which both admitted in the need for clear and precise thought as the condition of every *resolutio* of a problem, the resemblance does not go much farther.

Analysis is sometimes described as a special form of deduction. When it comes to a question of practice, the never-forgotten aim of the *Regulae*, we are advised to begin our analyses by avoiding the investigation of the more difficult matters and to try to become familiar with the handling of a number of implicatory series which involve the dependence of the complex on the simple. We are told

> to collect, without making any selection, the first truths which present themselves to us and then, step by step, to see whether any others can be deduced from these, and again others from those just deduced, and so on, in order. When we have done this, we should attentively think over the truths which we have discovered and carefully note the reasons why we have been able to discover some more easily than others and which these are, and thus, when we have to tackle some definite problem, we shall be able to judge what previous questions it is best to settle as a preliminary point.[1]

The type of practice Descartes has in mind is shown by an example later on. If we note that 6 is twice 3, we may go on to ask what is twice 6, and we find the answer 12. We may then proceed to double 12, that is 24, and then double again, that is 48. We can easily deduce that there is the same proportion or ratio between 3 and 6 as there is between 6 and 12, and likewise 12 and 24 and so on and hence arrive at the

[1] Ibid. 384^{12-21}. Rule 11 repeats in a summary and generalized form the doctrine set forth here and in the rest of the Rule (cf. ibid. 409–10).

conclusion that the numbers 3, 6, 12, 24, 48, &c., are in a continued proportion. This may seem childish, but we may grasp from an attentive study of these series what 'is the form involved in all problems which can be propounded about the proportions or habitual ratios of things and the order in which they should be investigated: and this discovery embraces the whole science of Pure Mathematics'.[1]

The point of the example lies in careful attention to the relation of the series in respect of its simplicity and easiness. For, taking the same example, we note that it is no more difficult to find the double of 6 than of 12; and further, that when we have found a proportion between any two numbers we can find innumerable others which have the same proportion. There is therefore no increase of difficulty if we have to find three or four more numbers because each has to be found separately. We also notice that when two numbers, such as 3 and 6, are given, it is easy to find a third in continued proportion, that is, 12. But if we were given the two extremes 3 and 12, it would not be so simple to find the mean proportional, 6. If we ask ourselves the reason, we see that the difficulty in the latter case, that of the mean proportional, arises from the fact that we have to attend to the two extremes at the same time in order to find the new ratio by dividing the previous proportion existing between them. The difficulty would be greater if the given extremes were 3 and 24 and we had to determine one of the intermediate proportionals, in this case 6 and 12; for the number of factors to which we had to pay attention would have increased yet more. If we proceed and suppose 3 and 48 to be the given extremes, we should have three mean proportionals, 6, 12, and 24. But if we proceeded by splitting the difficulty and establishing the mean proportional between 3 and 48, that is 12, we should then be able to seek for the mean proportional between 3 and 12, and the mean proportional between 12 and 48; that is, we should have reduced the problem to a lower stage of our research. We should have it down to a simpler stage of the deduction. But analysis does not always take the form of a deduction as illustrated above in what is called 'a practice analysis'. It is in the discovery of the simple that the main work of analysis lies.

[1] AT. x. 385[1-4].

Rule 6 gives us an important indication of how we are to consider this problem. It points out that the terms are not to be considered as solitary or isolated 'natures' but must be compared one with the other, relatively 'to their dependence in knowledge one upon the other', that is, as forming part of implicatory series or sequences.[1] All these terms may be distinguished, in respect of the kind or degree of their dependence, as absolute or relative terms. In each series there is a first term on which the others logically depend, and this term is independent of the others; there is a second term, dependent on the first and independent of the rest, though they depend upon it. The dependence is, in other words, unilateral and not reciprocal. This leads us to the following definitions.

A term is said to be 'absolute' (*absolutum*) if 'it contains the nature under investigation in its pure and simple form'.[2] This term can be applied to whatever is considered as being 'independent, cause, simple, universal, one, equal, like, straight, and so on'.

A term is said to be 'relative' (*respectivum*) which, 'while it participates in the same nature, or at least shares the same nature to a degree which allows us to relate it to the absolute term and to deduce it from that term by an implicatory series, involves something else in its concept which I call relativity'.[3]

[1] Cf. ibid. 381^{19}, '. . . naturas solitarias'. What Descartes means by 'naturas' in this context is made clear in 382^{25}–383^{1}, where he points out that he has put 'cause' and 'equality' amongst the absolute terms 'quamvis eorum natura sit vero respectiva'. See also Rule 14 (AT. x. 440^{10-20}). Sirven, in *Les Années d'apprentissage*, pp. 369–71, draws attention to the similarity of this use of 'natura' to that of Francis Bacon and quotes passages of similar import from the *Instauratio Magna* and the *Novum Organum*. See also Milhaud's brilliant essay, 'Descartes et Bacon', in *Descartes Savant*, pp. 213–27, where the doctrine of Rules 5 and 6 is compared to the Baconian theory of method; and L. Roth, *Descartes's Discourse on Method*, pp. 52–71. Gassendi was aware of certain similarities between Descartes and Bacon. In the *Syntagma Philosophicum* (i. 90) he remarks: 'Logica Cartesii recte quidem Verulamianae imitatione ab eo exorditur, quod ad bene imaginandum, prava praejudicia exuenda, recto vero induenda vult.' But he adds that Descartes, in order to attain true knowledge, does not apply his method to the objects *per se ac in se*, but rather he begins from his own mind and thoughts, and, adds Gassendi, 'id facere profecto minus apposite quam superior videtur'.

[2] 'Absolutum voco, quidquid in se continet naturam puram et simplicem, de qua est quaestio' (AT. x. 381^{22-23}).

[3] 'Respectivum vero est, quod eamdem quidem naturam, vel saltem aliquid ex ea participat, secundum quod ad absolutum potest referri, et per quamdam seriem ab eo deduci; sed insuper alia quaedam in suo conceptu involvit, quae respectus appello' (ibid. x. 382^{3-7}).

Examples of such terms are whatever is considered as dependent, effect, composite, particular, &c.; in short, the antitheses of the absolute terms.

But it must be added that 'absolute' and 'relative' are themselves relative terms. A term may be absolute from one point of view and relative from another—the point of view is always that of knowledge. A term may also be 'more' and 'less' and 'most' absolute or conversely 'more' and 'less' relative. For instance, the universal may be considered more absolute than the particular because 'its essence is simpler'. Yet, from another point of view, it can be said to be more relative than the particular because it depends on particulars for its existence. Relatively to individuals, species is something absolute but, in antithesis to genus, it is a relative term. The antithesis between the terms, the point of contrast, is always the order of *consequentiarum contextus*, the order of logical implication in the series. It is essentially the order *quoad nos*, that is, the absolute and relative as known to us, not as existent. Descartes notes: 'For though philosophers may make cause and effect correlative terms, we may even find here that if we wish to know what the effect is, we must first get to know the cause and not conversely.'[1] If *A* is the cause of *B*, then *A* and *B*, the existing cause and effect, are correlative; but the knowledge of *B* presupposes the knowledge of *A*, the cause, in the implicatory series.

The rule of analysis, as regressive process, demands that we should work towards the most simple or absolute which is also, in the words of the third precept of the *Discours*, 'the easiest to understand'. This is not, however, to be taken as a purely psychological definition implying that most simple or absolute terms require less effort on our part in order to discover them. On the contrary, the first principles of science are far from easy to discover and the avowed purpose of the Cartesian method is to smooth the way for the novice, as far as it is possible to help and give guidance in such matters. There is no short route to knowledge nor a vade-mecum in the search for truth which has ready-made answers. The 'easiest to understand', the most simple, is to be understood in the Scholastic sense of the term *notior*, as Descartes uses it

[1] Cf. AT. x. 383[5–6].

in the *Reply to the Fifth Objections*: 'one thing is not proved to be better known than another because it appears to be true to more people but solely because to those who know both things, as they may well do, it appears to be prior in knowledge, and more evident and certain'.[1] A term is 'more known' in proportion as it is required for knowledge of other terms; the 'more known' is that term on which knowledge of others depends in order that they should become known; 'the most known' of a series is that term upon which all other terms of the series in question depend in order to be known. We must bear in mind the point made in Rule 12 in the question of 'simple natures': 'In relation to the order of knowledge, single things must be taken in an order different from that in which we should consider them in so far as they really exist in themselves.'[2]

There are other instances of absolute and relative terms in Rule 6. 'Equal' or 'similar' is correlative to another 'equal' or 'similar' and need not be prior to an 'unequal' or 'dissimilar'. But, argues Descartes, the knowledge (and here he presumably means the definition) of 'unequal' or 'dissimilar' presupposes the knowledge of 'equal' or 'similar' respectively; the latter terms are then prior to the former as terms in the implicatory series since they are what Aristotle would have called τῷ λόγῳ πρότερα, the logically prior, and accordingly, in Descartes's language, less relative. The distinction between absolute and relative is purely epistemological. It should be noted therefore that 'an absolute term' will not necessarily be 'a simple nature'. It may occur in a highly complex problem, what Descartes often tends to call a 'difficulty', that the analysis will stop short of an absolute term which is itself the object of an intellectual intuition, that is to say, a simple nature, clear and distinct in itself and therefore indefinable in terms of any other nature. In such a case,

[1] The Latin text of the *Discours* is 'et cognitu facillimis' (AT. vi. 550). See *Resp. ad V Obj.* (AT. vii. 384^{12-16}). The Scholastic usage is exemplified in a text of the Coïmbricenses in Gilson's *Index Scolastico-Cartésien*, p. 203: 'Ea nobis notiora dicuntur, quorum cognitio ad aliorum intelligentiam requiritur.' A similar distinction, but slightly different doctrine, is found in St. Thomas, *Summa Theol.* I–IIae, q. 14, a. 5, in corp.: 'Procedere enim a causis in effectus, est processus compositivus; nam causae sunt simpliciores effectibus. Si autem id quod prius est in cognitione, sit posterius in esse, est processus resolutorius.'

[2] AT. x. 418^{1-3}.

the absolute term is only relatively absolute and we are justified in stopping our analysis short, provided, of course, we know that the relatively absolute term in question can be explained by other more simple and ultimate terms. When we 'know' a simple nature, we intuit it directly and completely and there can be no question of analysis involved which would imply knowledge of its parts. Only the *maxime absolutum* is a simple nature, because knowledge of it does not presuppose knowledge of something else.

If we turn back to the example of the *linea anaclastica*, the problem of the determination of the line of refraction, we have another example of what Descartes means by analysis. The problem, it will be remembered, was to find the line in which parallel rays are refracted in such a manner that they all intersect one another and are focused in a single point. The first result of the analysis was the conclusion from it that the ratio between the angles of incidence and reflection determines the line. This, then, is the first step in the regressive analysis from the composite to the simple or from the relative to the absolute. But this proposition is itself composite and relative. On further analysis, it is found that the ratio between the angles of incidence and reflection varies and this variation depends on the nature of the transparent media which produce differences in the angles. This once again is a composite and relative proposition. The analysis must pass on to the nature of light which penetrates the transparent media, but it cannot stop at this point for we are still dealing with the composite and relative: the nature of light, to be fully understood, presupposes that we know what a power of nature is in general, and that is the limit-term of the regressive analysis, the ultimate term and the *maxime absolutum*. This absolute term of natural power or force is absolute as compared to the second term of the series, light, because it stands to it in the relation of a general or universal to a particular. The other terms, within the physical sphere, such as heat, magnetism, light, &c., all share in the 'nature' which is common to them all and present to them all, though not necessarily in the same degree. This power of nature, *potentia naturalis* in general, is, of course, the first term in the progressive synthesis which will reconstruct step by step the implicatory series.

The same method of analysis is to be found in 'the essays in method' appended to the *Discours*, in the *Dioptrique*, and the *Météores*. In the first, Descartes begins by determining in what the action of light consists and discovers that light is a movement comparable to other 'natural movements'. From this he concludes that it is a movement comparable, in the laws which it follows, to the laws of movement in general. Consequently, when meeting with certain kinds of objects, rays of light are reflected similarly to the way a ball bounces back from a solid surface; rays of light penetrate into transparent media as a ball into liquid but, like a ball, they are diverted from their original trajectory and the degree of diversion, or reflection, will depend on the nature of the media which the ray of light penetrates. So the analysis proceeds, driving back to the more general and simple. It is interesting to note what Descartes says in his foreword to the 'Index of the principal problems which are explained in the Dioptrics'. He warns his readers that 'those who look at the indexes of books in order to pick out the matters which they desire to look at and thus save themselves the trouble of reading the rest of the book, will not get any satisfaction out of this book: for the explanation of the problems, which are found therein, depend almost always on what is said in the preceding problem, and often also on the succeeding problem, so that it is impossible to understand them properly unless one reads with care the whole text'.[1]

Analysis consists, then, essentially in a reduction of a problem (which Descartes usually calls a *quaestio*) by a regressive and gradual division until we reach a term which is *maxime absolutum*. There is, however, another and wider sense of analysis which proceeds not from the known to the known but from the unknown to the known on the hypothesis that the unknown is the known. This new type or development of analysis must be discussed in relation to the Cartesian conception of *Mathesis Universalis*.[2]

§ 3. *The Method of Synthesis*

From the discovery of the *maxime absolutum* can begin the

[1] Cf. 'Avertissement' to the 'Table des principales difficultés' in the *Dioptrique* (AT. vi. 486).

[2] See below, pp. 194 and 220.

constructive synthesis, that is the arrangement of the facts discovered by analysis, in such an order that they will be successively more relative, more concrete terms of the implicatory series which will issue as the solution of the problem. The terms of the series, except the first or most absolute, are all relatives. We are given to understand by Descartes that the concept of a relative term includes one or more elements, inclusive of the 'nature' which was the precise definition of the absolute term in the series.[1] This concept of a relative term includes, besides this logical participation in the nature of the absolute term, other elements or determinants and, if we are to conceive these elements correctly, we must take account of these additional elements as well as the nature of the absolute term in which they participate as relative. Even the least relative of the relative terms in a series must contain in its conception an element or determinant which restricts the pure and simple nature of the absolute term, and this element or determinant is itself a *respectus* or relation, connecting the term to the elements of a different series. Thus, in the synthetic progress, as the series grow in complexity 'so those relative are farther removed from the absolute terms in proportion as they contain more elements of relativity, subordinate the one to the other'.[2] Accordingly, as the synthesis is built up, there is a gradual addition of new elements; every new element is a relative term which links the term with a new series, that is, with a new implicatory series which contains other absolute terms. The reverse process of analysis, dividing as it were the elements or determinants, breaks the relations with the diverse implicatory series involved through the *respectus* and progresses until it arrives at the absolute term of one particular series.

In order, therefore, to get an orderly series in which the successive terms advance methodically and gradually, synthetically from the simplest to the most complex, or analytically from the most complex to the simplest—in both cases the problem is the same—certain rules must be observed. It

[1] There is some similarity between this view and that of Aristotle's on the relation of the middle term and the definition of the major term, λόγος τοῦ πρώτου ἄκρου. Cf. *Post. Anal.* 99a, 21–29.

[2] 'Quae respectiva eo magis ab absolutis removentur, quo plures ejusmodi respectus sibi invicem subordinatas continent' (AT. x. 382⁹⁻¹²).

is not enough to note vaguely the continuity and number of the various terms in the series. The number of the terms must be observed exactly so that the number of illative acts involved in the total implication is known. Relative terms can only be known by inference from simple or absolute terms, that is, from relatively more simple or absolute terms in the series under consideration. The inference itself may be immediate or dependent on two or more acts of inference. In the latter case the number of the acts of inference must be noted in order that we may know whether the relative terms are separated from the primary or simple term by a greater or smaller number of steps. For, in every case, it is the connexion of ground and consequent which gives rise to the series of objects which are being investigated, and it is to this logical connexion that every series must be reduced.[1] In the case of synthesis, which is in essence a deductive reconstruction, the important role of enumeration, checking and rechecking the various steps, is obvious.[2] We should expect the fourth precept of the *Discours* to follow upon the third as in the *Regulae* Rule 7 follows on Rule 6, so that in both cases the rule of enumeration would follow after the rule of synthesis.

It may not be possible to attack a problem directly, especially when the series involved is not one of linear deduction. It may be necessary to look for the privileged instance, through the solution of which we may be able to extend by a generalization the formula discovered in the instance to similar or analogous problems. We may not be able, for instance, to discover the value of the sum of the angles in a given polygon. But if we have noticed that a polygon, either this particular one or any other, is composed of a number of triangles, we can reduce our problem to that of the value of the interior angles of a triangle. Once we know the formula that the three interior angles of a triangle are equal to two right-angles, we can proceed to solve the problem of the polygon. The method of solution is always the same. First the analysis, the 'resolution', of the complex into the most

[1] Cf. ibid. 383^{21-26}.

[2] The answer to the question on p. 145 is that enumeration as a preliminary survey is analytical and as a verification of a completed deduction is synthetic.

simple of terms in a given series. We must always, as far as possible, select from amongst the terms before us, that is, from the results of our analysis by which we have divided or decomposed the complex, that term which is the most simple and which contains the 'nature' under investigation in its pure and simple form, as, for instance, the triangle is the most simple of plane figures bounded by straight lines. This pure and simple term, which may or may not be 'a simple nature', will be the absolute term of the series. Inside the other terms, which will be all more or less relative, we must disentangle and classify, at least provisionally, the various *respectus* or determinants in regard to the absolute term. Since each of these *respectus* is that by which a term is connected with another, and different, implicatory series and its absolute term, we are dealing in fact with a multiplicity of coincident series. We are forced then to study these various *respectus* included in each relative term of our original series and the various coincident series which they severally imply. When we come to reconstruct them, as when we divide them, our primary aim is the arrangement of these terms in their proper order of logical subordination with reference to our own implicatory series and its absolute term.[1]

If we ask how we are to select the most simple from amongst the various terms of the series, or how we are to disentangle the various determinants inside the series, preliminary conditions of all syntheses, we find always the same cautious advice. We must practise; and above all we must practise the art of observation and classification until we have developed our natural power of 'sagacity'.[2] There is no other alternative, no short-cut, especially no purely formal law.

[1] Cf. Rule 6 (AT. x. 382^{3-16}). I assume that the text is sound, that is, 'quos omnes distinguendos' (in l. 12) is not a mere mistake for 'quae omnia distinguenda'.

[2] 'Cum quadam *sagacitate* reflectere' (ibid. 384^{7-8}), and at the end of the same Rule 6: '. . . ab attente reflectentibus et *sagaciter* disquirentibus posse inveniri' (387^{6-8}). The need for practice is a constant theme in the *Regulae*. See, for instance, Rule 11, which refers directly to the analytic method: 'ad mutuam simplicium propositionum dependentiam reflectendo, *usum* acquiramus subito distinguendi, quid sit magis vel minus respectivum, et quibus gradibus ad absolutum reducatur' (409^{11-15}), and Rule 12 (428^{23-27}). When it is a question of 'ingenium', Descartes says, in the *Entretien avec Burman*, 'ut autem illud excoli possit, opus est *Scientia mathematica*, et ea non ex libris, sed ex ipso *usu* et arte hauriri debet' (AT. v. 176). The italics are mine, except for *Scientia mathematica*. See above, p. 56.

The rule of enumeration will give us guidance in the preliminary work of surveying the field, help us to eliminate irrelevant and superfluous matters, and direct our 'attention' to the vital links in the connexions of terms. But, in the last resort, our knowledge depends on the overriding primary factor of 'seeing' that which is most simple and absolute in the series, which, if it is a simple nature, can only be grasped in the clear and distinct vision of intellectual intuition. The same sagacity is required, *mutatis mutandis*, to grasp the links of the deductive chains of reasoning in which we reconstruct the ordered series. The rules of method, it must be repeated, do not, nor are they intended to, replace mental effort and concentration on our part. Analysis is essential as the true *ars inveniendi*, the art of discovering the data which are the ground and starting-point of all demonstration, according to the old formula, ἀνάλυσις ἐστιν ἀποδείξεως εὕρεσις.[1] But it cannot be taught by any set of formal rules. We are finally dependent on the ability of the natural light of reason and the education of a *bona mens* which, avoiding waste of time and fruitless mental wandering, can grasp the essentials of the problem, separate out its component elements, and attach itself to the clear and simple datum. Obviously this needs experience and continual practice, as well as a certain innate flair and mental ability. We can thus 'acquire the habit of distinguishing at a glance what is more or less relative, and what are the steps by which a relative is related to something absolute'.

[1] Geminus, quoted by Hamelin, *Le Système de Descartes*, p. 57.

XII

THE SECRET OF THE METHOD

§ 1. *'The Secret of the Method'*

One can easily sympathize with those early Cartesians who, despite their enthusiastic devotion, could not but express their feeling of disappointment at the exiguity of the rules of method as expounded in the *Discours de la Méthode*. The outspoken criticism of Leibniz, who had the great advantage of having read the slightly more ample account of the *Regulae*, may well find an echo in the mind of the modern reader. The expectation created by the announcement of 'the chief secret of the method' seems exaggerated. The exposition of the rules of analysis and synthesis, even in the fuller version of the *Regulae*, seems meagre fare, even if we take into consideration some of the subsidiary rules of procedure which are intended to aid us in the practice of this twofold fundamental process. Is there a real 'secret' which the text is meant to hide from the curious and yet reveal to the initiated? In the last few years much has been written and spoken of the cautious and secretive character of Descartes and much play made with the famous *larvatus prodeo* of the *Cogitationes Privatae*.[1] Although he did not himself write an autobiography, the large element of autobiographical notes in his writings and the comparatively large extent of his surviving correspondence make the character and life of Descartes, in this respect only, like the character and life of Hume, an unfailing source of discussion and controversy. But however interesting and instructive in itself such a study of the character of the author of the method may be, it would throw little light on the method. It is as well to content ourselves with the view expressed by Descartes himself to Regius, who had written to inform his master that certain people believed that Descartes did not in fact publish his real opinions and even

[1] 'Ut comoedi, moniti ne in fronte appareat pudor, personam induunt: sic ego, hoc mundi theatrum conscensurus, in quo hactenus spectator exstiti, larvatus prodeo' (AT. x. 213^{4-7}).

held privately views opposed to his published works. Descartes replied:

> Those who suspect me of writing in a manner contrary to my views on whatsoever subject, do me the greatest injustice. If I knew who those people were, I could not help considering them as my enemies. I admit that there are certain moments when it is prudent to be silent and not publish the whole of one's thought. But I consider that only a vile and dishonest man would, without strong pressure, write something contrary to his own opinion and try to persuade his readers of this view.[1]

The statement is not without its twists and ambiguities but we may take it to mean that the written account of the method does in fact state the whole doctrine. Whatever secret there may be, it does not lie in anything but the obscurity and brevity of the exposition itself.

There can be no doubt at all that Descartes himself attached the greatest importance to these rules of method. It is also clear that, in his view, the whole of his scientific and philosophical doctrine was based on his use of the methodological principles. It is equally indisputable, even to the student beginning his acquaintance with the history of philosophy, that the philosophy of Descartes, whatever it may owe to the great medieval Schoolmen both in vocabulary and doctrine, was nevertheless the beginning of a new line of thought and the opening of a new era of philosophical speculation. Hegel, who was not usually inclined to bestow the gift of originality on his fellow philosophers, speaks of Descartes as the true originator of modern philosophy 'who started the whole subject afresh from the beginning and constituted anew the groundwork on which all philosophy is based'.[2] This groundwork, according to Descartes himself, lies in the method, and the method is what he claims as his original contribution. It should be added, as a matter of record, that Descartes himself often puts in a modest disclaimer when the originality of his doctrine is praised. The sixth part of the

[1] Letter to Regius, 1645 (AT. iv. 256[12–19]).

[2] *Vorlesungen über die Geschichte der Philosophie*, iii. 1. i (Berlin, 1844, xv. 301). Hegel does not give any real account of the Cartesian method, which he merely describes as 'essential even now in mathematics'. He would appear to have read Descartes mainly through Spinoza's account in the *Principia philosophiae Cartesii*, and ignores entirely the *Regulae*.

Discours, which cannot fail to attract the reader by its deep humanity and candour, declares:

> As regards those opinions which are truly my own, I do not apologize for their novelty, inasmuch as I feel sure that, if their reasons are well pondered, they will be seen to be so simple and so conformable to common sense that they must appear less extraordinary and less paradoxical than any other opinions which may be held on the same subjects. Nor do I pride myself in the least that I am the first to discover them but only state that I have accepted them, not because they have, or have not, been held by others but solely because my reason has persuaded me of their truth.[1]

This view represents a whole attitude of mind which is often expressed in his letters and is neatly summed up in his advice to Beeckman, an easily disgruntled and jealous friend:

> Many indeed may know the same thing although none of them has learnt it from another, and it is ridiculous to try to distinguish what is yours and what is someone else's, when it is a question of knowledge, as you are attempting to do, with the same accuracy as if it were a question of land or money. What you know is entirely yours, even though you may have learnt it from someone else.[2]

The originality of Descartes's philosophy might be said to lie in the attempt to treat the whole of human knowledge as one vast all-embracing science, capable of being treated on the lines of mathematical method and thinking, and there would be good reason to assume that the secret of the method lay in the expansion of the principles to the science of universal order or *Vera Mathesis*. In a sense, such an assumption can be justified but, and here we must attach their full import to Descartes's own words, the existence of this vast all-embracing science is dependent itself on the application of the essential rules of method. These, we were told, are the rules of analysis and synthesis and accordingly in these rules, at least in germ, must lie the new

[1] AT. vi. 77^{3-14}.

[2] Letter of 1630 (AT. i. 159^{13-18}). Wittgenstein, in his *Tractatus Logico-Philosophus*, p. 27, shows a similar indifference: 'How far my efforts agree with those of other philosophers I will not decide. Indeed what I have here written makes no claim to novelty in points of detail; and therefore I give no sources, because it is indifferent to me whether what I have thought has already been thought before me by another.'

and distinctive features which, at least in the intention of their author, will develop into this universal science of *Vera Mathesis* or *Mathesis Universalis*. If these rules are, then, the key to the whole method, it is advisable to attempt to summarize them before proceeding, to recapitulate the doctrine of method on the verge of its transformation into the ultimate science of order.

The method of analysis and synthesis, as set forth in Rules 5 and 6, would appear to amount, in brief, to this: In every problem or scientific inquiry we must attempt to transform an involved and obscure proposition into an intelligible combination of elements which are simple and are, therefore, clear and distinct and self-evidently known to intellectual intuition. This is the process of analysis. We must then attempt to reconstitute the original problem, which was complex and obscure, in terms of the simple and self-evident elements discovered by analysis. Analysis therefore conditions the process of reconstruction which is called synthesis. The method of analysis begins by positing the problem in all its complexity and obscurity. The problem is then divided into as many parts as are thought necessary by a preliminary survey or enumeration. This preliminary survey may have to be repeated on successive parts of the first ordering or classification until the problem is laid out in its provisional series of deductions. The deductions follow in the prescribed order of effects to causes, culminating in the intellectual intuition of the simple self-evident. The reverse process is followed in synthesis. The result of the synthetic method is to reproduce the problem as it was posited before the simplifying process of analysis, but the problem is now seen as a clear and distinct series of deductive links. Synthesis, in other words, does not start from a problem but from a result established by analysis, a result which gives in its generality and simplicity the solution of the problem. The *analytic* process proceeds then in successive movements of *enumeration*, *deduction*, and *intellectual intuition*; the *synthetic* process proceeds in successive movements of *intellectual intuition*, *deduction*, and *enumeration*. It is hardly necessary to add that, in the case of analysis, enumeration is to be understood as the preliminary survey, a form of classificatory analysis, and that, in the case of

synthesis, enumeration is the verification of the steps of the deduction.

One is struck by the fact that the major emphasis is laid by Descartes on the method of analysis. The emphasis is shown particularly in the insistence that the method of analysis is essentially a means of discovery of truth, whereas the method of synthesis is chiefly a means of explicitation and demonstration of whatever is already known. At the end of the *Reply to the Second Objections* Descartes puts this view forcibly. He has been asked, and has agreed under protest, to propound the arguments and demonstrations of the *Meditationes*, which are set forth in that work in analytical order, in 'geometrical fashion', and he points out that this method is a synthetic procedure. 'It was this synthesis alone', he adds, 'which the ancient Geometers used in their writings, not because they were entirely ignorant of analysis but, in my opinion, because they thought it so valuable that they wished to keep it to themselves as an important secret.'[1] The synthetic method then is clearly akin in its deductive process to the syllogism, and Descartes notes that it uses 'definitions, postulates, axioms, theorems'. The perfect syllogism, in the form $A = B$, $B = C$, therefore $C = A$, implies in point of fact the first axiom of Euclid: *τὰ τῷ αὐτῷ ἴσα καὶ ἀλλήλοις ἐστὶν ἴσα*. Synthesis functions after analysis has laid bare the principles, the self-evident truths. The analytic method does not supersede the synthesis; it needs synthesis, once the resolution of the problem has been completed, but the fundamental moment of the method lies in the analysis which alone can discover and reveal the self-evident truths.

§ 2. '*An Exercise in Method*'

Professor Keeling has pointed out that the method of Descartes cannot be appreciated from a consideration of its general and abstract formulation. 'Thus Descartes himself insisted that we should study, say, his *Géométrie*, which is an 'exercise in this method', not primarily for the sake of acquiring mathematical knowledge, but in order to understand and appreciate what he regarded as the method of *all* science, and

[1] AT. vii. 156[17–20].

of which this work was an illustration.'[1] If we follow this sound piece of advice we notice the link between the doctrine of the *Regulae*, especially that of the six last rules, viz. Rules 16 to 21, and the opening pages of the *Géométrie*.

Rule 16 states that 'when we come across matters which do not require our attention at the moment, although they are necessary for our conclusion, it is better to represent them by the briefest possible symbols'.[2] On the third page of the *Géométrie* is a small section devoted to the use of symbols, and in particular the use of algebraic signs in geometry.[3]

Rule 17 advises us 'that when a problem is put forward for discussion, we should run over the problem in a direct course, neglecting the fact that some of its terms are known, others unknown'.[4] On the fourth page of the *Géométrie* we find the recommendation that when faced with a problem, we should consider it as done 'and give names to all the lines, known or unknown, which are necessary for its construction'.[5]

Rule 18 lays down that to achieve the purpose of the previous rule, 'only four operations are required, addition, subtraction, multiplication, and division'.[6] The second paragraph of the *Géométrie* contains an identical enumeration.[7]

Rule 19 states that 'employing this method of reasoning we must find out as many magnitudes as we have unknown terms, these latter treated as though they were known . . . for this will give us as many equations as there are unknowns'.[8] The *Géométrie* contains a similar injunction: 'one must find

[1] Henriette Hertz Lecture, 1948, *Descartes*, p. 6, n. 2. In a letter to Mersenne, 1636, written just before the publication of the *Discours*, he refers to this latter as 'une partie de ma Méthode', and of the *Géométrie* he writes: 'je tâche à donner une façon générale pour soudre tous les Problèmes' (AT. i. 339^{26} and 340^{10-12}).

[2] 'Quae vero praesentem mentis attentionem non requirunt, etiamsi ad conclusionem necessaria sint, illa melius est per brevissimas notas designare' (AT. x. 454^{10-12}).

[3] Cf. AT. vi. 371^{4-15}, 'Comment on peut user des chiffres en Géométrie'.

[4] 'Proposita difficultas directe est percurrenda, abstrahendo ab eo quod quidam ejus termini sunt cogniti, alii incogniti' (AT. x. 459^{6-8}).

[5] Cf. AT. vi. 372^{10-14}.

[6] 'Ad hoc quatuor tantum operationes requiruntur, additio, substractio, multiplicatio et divisio' (AT. x. 461^{12-13}).

[7] Cf. AT. vi. 369^{8-12}.

[8] 'Per hanc ratiocinandi methodum quaerendae sunt tot magnitudines . . . quot ad difficultatem directe percurrendam terminos incognitos pro cognitis supponimus: ita enim tot comparationes inter duo aequalia habebuntur' (AT. x. 468^{21-25}).

as many equations as there are lines which have been assumed as unknown'.[1]

Rule 20 states that 'when we have obtained our equations, we must carry out such operations as we have omitted, taking care never to multiply when we can divide'.[2] The *Géométrie* advises that in treating these equations, 'we should use division whenever this is possible and thus we will obtain infallibly the simplest terms to which the problem can be reduced'.[3]

Finally, Rule 21 states that 'in the case of several equations of this kind, we should reduce them all to a single equation, namely, one whose terms do not occupy so many places in the series of magnitudes which are in continued proportion'.[4] The *Géométrie*, on the fifth page, says that the equations should be reduced to one, or an equation of equations.[5]

The identity of doctrine, and even of verbal expression, is striking, and the remainder of the *Géométrie* provides an illustration of the actual working of the more fundamental rules of method. It is not an easy work to read, especially to the non-mathematician. Even some of the mathematicians of 1637 found the going extremely hard and confessed their partial inability to understand the treatise. This difficulty and obscurity is deliberate on the part of its author. Before the publication of the *Discours*, and the accompanying 'essays in method', Descartes warns Mersenne that few people will be able to understand the *Géométrie* and he repeats the same sentiments to Plempeius later in the year:

I know, he says, that my Geometry will have very few readers, for I have left out all those things which I assumed to be known by others and have tried to compress many things into a few words, just touching upon certain points and leaving out others altogether. It demands not only readers who are well versed in matters already known in Geometry

[1] Cf. AT. vi. 373[1–2].

[2] 'Inventis aequationibus, operationes, quas omisimus, sunt perficiendae, multiplicatione nunquam utendo, quoties divisioni erit locus' (AT. x. 469[2–4]).

[3] Cf. AT. vi. 374[16–19].

[4] 'Si plures sint ejusmodi aequationes, sunt omnes ad unicam reducendae, nempe ad illam cujus termini pauciores gradus occupabunt in serie magnitudinum continue proportionalium, secundum quod iidem ordine disponendi' (AT. x. 469[6–9]).

[5] Cf. AT. vi. 373[7–9].

and Algebra but also readers who are hard-working, sharp-witted, and very attentive.[1]

The same warning is repeated in the *Avertissement* to the work itself. This is not the whole story, for a personal element, half vanity, half spite, is behind Descartes's deliberate attempt to make the *Géométrie* a difficult book to read and understand. But however instructive and interesting such a study might be, it is unnecessary here to recount the rivalry and misunderstanding, fostered artfully or artlessly by the tireless Mersenne, which embittered the relations of Descartes and men like Roberval. Nor is it necessary to decide whether Fermat arrived at the idea of analytic geometry before Descartes. Attention may be called to a letter, written a year after the publication of the *Discours*, where Descartes opens his mind to a correspondent who had pleased him by a sympathetic and constructive criticism of the *Géométrie*. In this letter he declares that

> in the case of the tangents, I have only given a simple example of analysis, taken indeed from a rather difficult aspect and I have left out many of the things which could have been added so as to make the practice of the analysis more easy. I can assure you, nevertheless, that I have omitted all that quite deliberately, except in the case of the asymptote, which I forgot. But I felt sure that certain people, who boast that they know everything, would not miss the chance of saying that they knew already what I had written, if I had made myself easily intelligible to them. I should not then have had the pleasure, which I have since enjoyed, of noting the irrelevance of their objections.[2]

The warning is clear but it does not affect in any major degree an attempt to consider the *Géométrie*, not in the detail of its mathematical proof and validity, but merely as an important example of the application of the analytical method.

The third of the 'essays in method' is divided into three books, and it is mainly in the first book that Descartes explains the method he proposes to use. The problem which Descartes sets out to solve is a theorem already discussed by Euclid and Apollonius, and named the 'problem of Pappus'.

[1] Letter of 3 Oct. 1637 (AT. i. 411[12–20]). In the letter to Mersenne, written on 25 May of the same year, Descartes writes: 'car je vous dirai bien que je appréhende qu'il ne se trouvera que fort peu de personnes qui l'en endront' (AT. i. 377[12–14]).

[2] Letter to De Beaune, 1639 (AT. ii. 511[18–20]).

It can be stated in the following terms: The problem is to find a curve traced out by a point which is so situated that if lines are drawn from it, making fixed angles with three or more fixed lines, the segments of the lines cut off from the point should have a certain ratio to each other.[1] Or it may be formulated as: Given any number of straight lines, A, B, C,..., K, A', B', C',..., K', fixed in position, the problem is to find the locus of a point, from which are drawn to each of these straight lines, other straight lines, fixed in direction, such that, if the segments intercepted between the point and the lines $A, B, C,..., K, A', B', C',..., K'$ be $\alpha, \beta, \gamma,..., \kappa, \alpha', \beta', \gamma',... \kappa'$ respectively, the product $\alpha\beta\gamma,..., \kappa$ has a given relation or ratio to the product $\alpha'\beta'\gamma',..., \kappa'$. Thus in the case of three given lines, the rectangle of the segments should equal the square of the third; in the case of four lines, the rectangle of one pair should equal the rectangle of the other pair; in the case of five lines, the parallelepiped of three should have a given ratio to the rectangle of the other two and so on. Our concern is not with the final solution of the problem which can be found in the *Géométrie* but with the methodological principles utilized by Descartes in the solution.

He begins by *a preliminary survey* of the most general kind; in the particular instance, of all geometrical problems. This survey is stated in the terms that 'all problems in geometry can easily be reduced to such terms that a knowledge of the length of certain straight lines is sufficient for the construction [of a problem]'.[2] He then proceeds from this most general enumeration of the type of data or simple natures' which will be encountered in the problem—it will be remembered that, in Rule 6, 'straight' was given as an instance of 'simple nature'—to a further enumeration, based on Rule 18, of the possible operations which can be applied to these data, that is, the fundamental operations of arithmetic, addition,

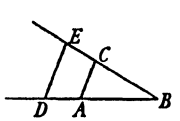
[1] Cf. *La Géométrie*, I (AT. vi. 377–9). On the problem of Pappus and the mathematical proofs see Milhaud, *Descartes savant*, pp. 124–48; Brunschvicg, *Les Étapes de la philosophie mathématique*, 3rd edit., pp. 116–23, and Haldane, *Life of Descartes*, chap. xvii. Tannery, in AT. i. 225, formulates the problem more modernly as 'Étant données $2n$ droites trouver le lieu d'un point tel que le produit de ses distances à n de ces droites soit dans un rapport déterminé au produit de ses distances aux n autres'. There are excellent things also in Liard, *Descartes* (1882), pp. 35–63.

[2] *La Géométrie*, I (AT. vi. 169[4–7]).

subtraction, multiplication, division (and extraction of roots which may be considered as a species of division). He then explains how by treating a line as 'unity' the operations of multiplication, division, and root-extraction can be applied to geometry 'for the sake of greater clearness'. An exact parallelism is established between arithmetical operations and geometrical constructions. Two line segments can be multiplied by laying off a unit line and one of the factors on one side of a triangle beginning at the vertex and laying off the other factor on the other side. A line parallel to the line segment that joins the end point of the unit line to the end point of the second factor cuts off a segment equal to the product of the lines.[1] Square roots can be extracted by means of a semicircle erected on the line, whose root is required, increased by unity, and the root is the length of the perpendicular drawn to the point where the unit segment joins the line. The preliminary survey has thus simplified the operations and determined the nature of the data.

But there is yet another simplification to be performed. We are reminded of the use of *brevissimae notae* as aids to rapid deduction and of the final enumerative verification which may lead to the intellectual grasp of the whole deductive series. Rule 16 is brought into play. There Descartes points out that he is making abstraction of numbers and geometrical figures and 'everything else'. He gives as his reason that he desires that 'those parts of the matter which contain the essence of the difficulty should always remain distinct and not get mixed up with numbers that tell us absolutely nothing'.[2] It is thus possible to note at each stage and compare the various stages of the deduction from the

[1] If *AB* is taken as 'unity', and it is required to multiply *BD* by *BC*, join *A* and *C* and draw *DE* parallel to *CA*, then *BE* is the product of *BD* and *BC*. To divide *BE*



by *BD*, join *E* and *D* and draw *AC* parallel to *DE*, then *BC* is the result of the division. See below, p. 205, n. 1, where the assumption of a 'unity' is advocated by Rule 14 as a means of reducing a magnitude to a manifold.

[2] AT. x. 456[2–4].

point of view of the simplicity and complexity of the equations. The *Géométrie* proceeds to apply the technique of *brevissimae notae* which will enable Descartes to trace at each step the connexion between the *quaesitum* and the datum which is the essential requirement of the order demanded by Rule 6. What is introduced is the use of letters to denote the quantities involved in the problem, the method of algebra. Thus if we wish to add the lines *BD* and *GH*, we call one *a*, the other *b*, and write $a+b$. Subtraction is represented by $a-b$; ab represents multiplication; a/b means *a* divided by *b*; a^2 that *a* is multiplied by *a*, &c. The square root of a^2+b^2 is to be written $\sqrt{(a^2+b^2)}$; if I wish to extract the cube root of $a^3-b^3+ab^2$, then it should be written $\sqrt[3]{(a^3-b^3+ab^2)}$ and so on.[1] In so doing we are beginning to liberate that science, 'known by the barbarous name of Algebra', from 'the vast array of numbers and inexplicable figures which overwhelm it' and beginning to allow it to display, according to Rule 4, 'the clearness and simplicity which ought to characterize a mathematical science'.[2]

The next step in the *Géométrie* is a direct application of Rules 17 and 19 which treat of the method to be adopted when the problem under discussion is involved and intricate. In such a case 'the whole of the device consists in treating the unknown as though they were known',[3] and there should be no difficulty in achieving this since it is assumed that we

[1] Cf. *La Géométrie*, I (AT. vi. 371^4–372^9). For the cube root Descartes writes in fact $\sqrt{(C \cdot a^3-b^3+abb)}$. He uses also a^2, a^3, a^5, &c., to represent the respective powers of *a*, but he also uses *aa* to represent a^2 indifferently. The algebra of the *Regulae* is different from that of the *Géométrie*, and in the latter work the simplification of the 'signes cossiques' has progressed considerably. He does not as yet use Recorde's symbol = for equality, invented by its author 'bicause noe 2 thynges can be moare equalle'.

[2] Cf. AT. x. 377^{4-9}, and *Discours*, Part II (AT. vi. 17^{27}–18^5). 'Pour l'analyse des anciens et l'algèbre des modernes . . . on s'est tellement assujetti, en la dernière, à certaines règles et à certains chiffres, qu'on en a fait un art confus et obscur, qui embarrasse l'esprit, au lieu d'une science qui le cultive.' And a few pages further on 'il fallait que je les expliquasse par quelques chiffres les plus courts qu'il serait possible; et que, par ce moyen, j'emprunterais tout le meilleur de l'analyse géométrique et de l'algèbre, et corrigerais tous les défauts de l'une par l'autre' (ibid. 20^{18-24}).

[3] 'Totum hujus loci artificium consistet in eo quod, ignota pro cognitis supponendo' (AT. x. 460^{20-21}). This is from Rule 17, but Rule 19 in fact lays down the precept in its final form. See above, p. 177, n. 8. In Rule 17 the reading *proportionum* from the Hanover MS. (at l. 6) fits better the general sense than the reading from A adopted by the editors of AT., i.e. *propositionum*. Hamelin, in

recognize the dependence of the unknown terms in question on the known terms to be of such a nature that the former are determined by the latter. The following instructions are given by the *Géométrie*, specifying naturally the type of problem with which the treatise is dealing and its specific data, lines.

If we wish, then, to solve any problem, we must, in the first place, assume the solution as already effected and give names to all the lines which seem necessary to its construction, to those that are unknown as well as to those that are known. Then, without making any distinction between known and unknown lines, we must survey the difficulty in the order which shows most naturally the mutual relations between these lines, until we have found a way to express a single quantity in two ways. This is called an equation, since the terms of one of these two expressions are together equal to the terms of the other. We must find as many such equations as there are assumed to be unknown lines. If so many equations are not found, having made an exhaustive inventory of everything involved, it is clear that the problem is not entirely determined, and accordingly we may choose arbitrarily lines of known length for all the unknown lines to which corresponds no equation.[1]

The simplification of the problem must be pushed still farther. We must, in the case of several equations, 'reduce them all to a single one'.[2] The instruction of Rule 21 is repeated and developed in the *Géométrie*:

If there are several equations, we must treat each of them in order, either considering it alone or comparing it with the other equations, so as to obtain a value for each of the unknown lines; and we must so combine them that there remains a single line [unknown] which is equal to some known line or else whose square, cube, fourth power, fifth power, sixth power, &c., is equal to the sum or difference of two or more quantities, of which one is known while the others are composed of mean proportionals between unity and this square, or cube, or fourth power, &c., multiplied by other known lines. This I may write:

$$z = b$$

or $$z^2 = -az + b^2$$

or $$z^3 = az^2 + b^2z - c^3$$

or $$z^4 = az^3 - c^3z + d^4,$$ &c.

Le Système de Descartes, pp. 79–80, quotes, on the authority of Ravaisson, a text which I have not found in Descartes but which summarizes well the importance of this precept: 'Dans l'analyse on déduit de l'inconnu en traitant l'inconnu comme connu et le connu comme inconnu.'

[1] *La Géométrie*, I (AT. vi. 372^{10}–373^{2}).

[2] 'Sunt omnes ad unicam reducendae' (AT. x. 469^{6-7}).

That is to say, z, which I take to be the unknown quantity, is equal to b; or the square of z is equal to the square of b, diminished by a multiplied by z; or the cube of z is equal to a multiplied by the square of z, diminished by the cube of c; and so on for the other equations.

In a like manner, all unknown quantities can be expressed in terms of a single quantity, whenever the problem can be constructed by circles and straight lines and also by conic sections, or even by some other line of degree not greater than two [i.e. of the third and fourth degree].[1]

Descartes adds that he will not explain in more detail lest he should deprive us 'of the pleasure of mastering the problem' ourselves and also of the advantage of training our minds by working it out for ourselves which is, in his view, 'the principal benefit to be derived from this science'.[2] Whatever the pedagogic advantage to be derived from a detailed study of the solution of the problem of Pappus, it is extraneous to the major interest of the method of which this particular solution furnishes a brilliant example. There is, however, in the third book of the *Géométrie* a passage which is a kind of footnote on the doctrine of Rule 21. Descartes says:

We can thus express the roots of all equations, up to those of the fourth power, by the rules which have been explained above. As a result I do not know that anything further is to be desired in this matter. For the nature of these roots is such that they do not allow of a simpler expression nor can they be determined by any construction which is on the whole more general and more simple. It is true that I have not as yet given the reasons on which I base my argument in order to assert whether something is possible or not. But, if one pays attention to the method which I use, all the problems which are considered by geometers are reducible to one single kind of problem, that is to find the value of the roots of some equation, and one can well judge that it is not difficult to make an enumeration of all the ways in which these

[1] *La Géométrie*, I (AT. vi. 373^{2}–374^{5}). There is a slight inversion between the order of the *Regulae* and that of the *Géométrie*. Rule 21 which fits this passage is preceded by Rule 20 which prescribes the preferential use of division over multiplication (cf. above, p. 178), in reverse order to the *Géométrie* (374^{16-17}). Sirven, basing himself on Rabuel's commentary, suggests that Rules 22–23 and 24 not completed, might be filled from the *Géométrie*, in the suggested texts, AT. vi (444^{11}–454^{7}), (458^{8}–464^{16}), and (464^{17}–475^{2}).

[2] *La Géométrie*, I (AT. vi. 374^{5-10}).

roots can be found, an enumeration quite sufficient to demonstrate that the chosen one is the most general and most simple.[1]

The *Géométrie* reproduces clearly the fundamental characteristics of the analytical method and illustrates the reduction of a problem to its most general and most simple form. Whether or not it was a new idea to apply algebra to the solution of geometrical problems does effect the value of the illustration. Descartes's expressed desire to reform geometry by the use of algebraic methods is certainly governed by a wider consideration, namely, his desire to extend his method to an even more general science. The *Géométrie* is a good illustration of what Descartes means by 'the simplification of problems'. The representation of a curve is simplified by relating it, by putting it 'into proportion' with the lengths of two straight lines, than which, says Descartes, we can find, in the field of continuous magnitudes, no other objects 'more capable of being distinctly represented', since 'straightness' is a simple nature or absolute. Descartes saw that curved lines can be expressed algebraically in terms of an equation involving the perpendicular distance of a point to two perpendicular lines of reference. A point in a plane can be uniquely determined when its distances (usually called x and y) from the lines of reference (the coordinates, as Leibniz was to name them) are known. With this knowledge, an equation can be formulated, $F(xy) = 0$, which expresses a property true of every point of the curve. For every curve, some relationship can be shown to exist between the coordinates of a point anywhere on it. A curve might also be defined in terms of an equation involving x and y. And when two or more curves have the same coordinates or lines of reference, their points of intersection may be found by the common solution of their equations. A curve has then been simplified by reduction to an equation which is ultimately soluble by two straight lines: simple natures which are proper objects of intellectual intuition.

The rest of the *Géométrie* is a continuation of the procedure of analytical simplification, linked to the solution of the problem of Pappus and the required locus of a point set therein.

[1] *La Géométrie*, III (AT. vi. 475[3–21]).

Book II deals with 'the nature of curved lines'. All curves are to be classified into two kinds, geometric and mechanical. The first kind is that generated by the intersection of two straight lines which move parallel in a constant manner to two given directions in such a way that the rate parallel to one direction is commensurable with the rate parallel to the other direction. If the ratio is not commensurable, the curve is called mechanical. The point is, for Descartes, that the latter kind cannot be expressed in an equation: it is what is called 'transcendental' by Leibniz. But 'all the points of those curves which we may call geometric' must have a definitely known relation to all points of a straight line and 'this relation must be expressed by means of a single equation'.[1] The fundamental concept of analytical geometry is formulated in this definition. The survey continues by a classification of geometrical curves. The Cartesian grouping into genres does not quite fit into modern mathematical terminology. In short, curves of the first and second degree belong to the first genre, those of the third and fourth degree belong to the second genre, those of the fifth and sixth to the third genre, and so on. The methodological point to be kept in mind is that 'no matter how we may conceive a curve to be described, provided it be one of those curves which I have called geometric, it is always possible to discover in this manner an equation determining all its points'.[2]

Descartes then turns to an amplification of the solution of the problem of Pappus which he had sketched in Book I. The procedure is again characteristic of the method. The *cognita* or data from which the deduction must begin are 'designated'. 'First', says Descartes, 'I have shown that when there are only three or four lines, the equation which determines the required points [i.e. the equation of the *quaesitum*, the required locus] is of the second degree.'[3] The curve therefore belongs to the first genre. Where there are not more than eight given lines, the equation is at the most biquadratic; the curve will therefore belong to the first or second genre. Where there are not more than twelve given lines, the

[1] *La Géométrie*, II (AT. vi. 392^{21-25}).
[2] Ibid. 395^{19-24}).
[3] Ibid. 396^{23-25}) '. . . ne monte que jusqu'au carré'.

equation is of the sixth degree or lower and accordingly the required curve belongs to the third or a lower genre. Descartes is now in a position to determine the series of analyses which he must undertake, and having stated the various possibilities he can sum up in the words: 'It follows that there is no curve which is measurable and geometrical [i.e. no geometric curve whose equation can be obtained] that may not be used for some number of lines.'[1] It will be noted that Descartes makes his enumerative classification in accordance with the rule of proceeding from the simple to the complex; the continuity of the enumeration is assured by a serial deduction which enables the requisite equation to be formulated for each genre.

Descartes then proceeds to the synthetic reconstruction of the problem, starting from the enumerated totality of the lines posited by the *quaestio*. It is impossible to summarize the various demonstrations here; they are no more than the bare essentials of the proof or, to use a metaphor of Descartes, the construction is given as an architect gives his specifications, leaving the actual manual labour to the masons and carpenters. He arrives at the conclusion that when, in the problem of Pappus, there are four given straight lines, the locus is a conic section or curve of the first genre; when there are five straight lines, the locus is a curve of the second genre, and so on.[2] He then continues with a general method for the equation of lines that are normal and tangent to a curve and a discussion of certain 'Ovals' useful in the 'theory of Catoptrics and Dioptrics'. A hint is dropped at the end of Book II that the method used for dealing with curves that are described on a plane surface can be applied to all curves which can be conceived of as generated by the regular movement of the points of a body in three-dimensional space, 'by dropping perpendicular lines from each point of the curve under investigation upon two planes intersecting on right-angles'. The hint was taken, amongst others, by the translator into Latin of the *Géométrie*, Van Schooten. The theory is thus extended or generalized to solid geometry.

The third book is mainly devoted to considerations on the nature of the roots of equations and reads more like a treatise

[1] *La Géométrie*, II (AT. vi. 397^{18-20}).
[2] Ibid. 396^{20}–397^{8}).

on algebra than on geometry. But the methodological preoccupations of its author are to the forefront. We are warned at the very beginning that 'we should always choose carefully the simplest curve that can be used to solve the problem', but that 'the simplest' is not merely 'the easiest described', nor the one that yields the easiest demonstration or construction, but it is 'the one of the simplest class which can be used to determine the required quantity'.[1] In turning to the general nature of equations, he recommends that all terms should be taken to one side of the equation and equated to zero. The *resolutio* then proceeds to the nature and number of the distinct roots or values of the unknown quantity. This leads to the proposition that the number of roots is the same as the number of degrees. Positive and negative roots are then analysed and the famous 'Rule of Signs' is given to determine the number of positive and negative roots that any equation can have: 'An equation can have as many positive roots as it has changes of sign from + to − or from − to +; and as many negative roots as the number of times two + signs or two − signs follow each other.'[2] A method is then given for 'diminishing' the number of 'dimensions' of an equation, when one of its roots is known.

The conclusion gives an admirable summary of the purpose and method of Descartes in this 'essay in method'.

> It was not my intention [he writes] to compile a large volume. I am attempting rather to cram much into a few words, as one can judge from what I have done, if it is considered that, while reducing all the problems of one class to a single construction, I have at the same time given a method of transforming them into an infinity of others, and thereby of solving each one in an infinite number of ways. Furthermore, having constructed all plane problems by the cutting of a circle by a straight line, and all problems of solids by the cutting of a circle by a parabola, and finally all those problems, which are more complex by a degree, by cutting a circle by a curve but one degree higher than the parabola, it is only necessary to follow out the same method in order to construct all problems, more and more complex, *ad infinitum*. For in the matter of mathematical progressions, when the first two or three terms are given, it is not difficult to find the others. And I hope that our nephews

[1] *La Géométrie*, III (AT. vi. 442$^{9-17}$).

[2] Ibid. 446$^{6-9}$. Descartes uses the old terminology 'vraies et fausses racines'.

will be grateful, not only for the things which I have explained, but also for those which I have purposely left out so as to leave to others the pleasure of discovery.[1]

[1] *La Géométrie*, III (AT. vi. 485[5–26]). Cf. the summary of De Schooten in the *Argumentum* of the third book of his commentary: 'E quibus tandem colligere licet, quod, postquam omnia Geometriae Problemata ad usum quasi Problema revocata fuerint, quod est, ut quaeratur tantummodo longitudo quarundam linearum rectarum, quae alicujus aequationis sint radices, reductisque ad eandem constructionem, quae ejusdem generis existunt, tradita simul sit via eadem resolvendi' (*Renati Des Cartes Geometria* (Frankfort, 1695), p. 281).

XIII

THE SCIENCE OF ORDER

§ 1. '*Mathesis Vulgaris*'

THE Cartesian conception of method was derived, as Descartes himself tells us in the autobiographical passages of the *Regulae* and *Discours*, from a personal dissatisfaction with the philosophical and scientific doctrines which he was taught and, more positively, from a careful reflection on and practice of the procedure of the mathematical sciences. He had been impressed, he tells us, by the self-evident certainty of their demonstrations and the seeming facility by which the most abstruse and involved problems were solved. The *Regulae* and the *Discours* are, in theory, nothing more than a setting-out of the principles to which the mathematicians owed their striking achievements: striking, above all, if compared to the confusion and diversity prevalent amongst the metaphysicians. Impressed, as Kant was to be, by these achievements, Descartes attempted to formulate these principles in their most general and abstract terms and in so doing thought that he could constitute a general theory of method. His interest in the mathematical sciences is accordingly a secondary one. He is interested in them in so far as they provide the best example of the methodological principles as applied and worked out to successful conclusions. His enthusiasm for his own demonstrations and their use in the *Géométrie* is limited. In the course of his life after 1637 he showed a singular lack of interest in possible developments of the geometry which was to be such a fertile source of inspiration to succeeding generations of mathematicians. It would even appear that, in writing this work, he was in possession of technical procedure of a mathematical nature which he did not bother to use or communicate to his readers because he could not, at that time, fit this use into a rational account of his methodological principles.[1] Descartes speaks his mind in a letter describing

[1] Cf. Brunschvicg, *Les Étapes de la philosophie mathématique*, 3rd edit., p. 115, and the conjectures of Tannery in AT. i. 75.

his intentions in publishing the *Discours* and the three treatises which were attached to it.

I want to tell you that the whole plan of what I propose to publish on this occasion is to prepare the road [for his treatise on physics] and see how the land lies. I am putting forward for this purpose a general method, which I do not in fact give instruction in, but which I attempt to prove by the three following treatises, attached to the above-mentioned *Discours*. I discuss, in the first, a subject in which Philosophy and Mathematics are mixed; in the second, a pure problem of Philosophy, and in the third, a pure problem of Mathematics. In these treatises, I can state that I have avoided speaking of anything, except those things which can be known by the power of reasoning, because, with this exception, I believed that I had no knowledge; with the result that it seems to me that people can judge my use of a method by which I could just as well explain a quite different kind of problem, granted circumstances under which I might make the necessary experiments and the time to consider them. Beyond all this, in order to show that my method extends to all problems, I have inserted some metaphysical, physical, and medicinal problems into the opening *Discours*.[1]

The subsequent writings of Descartes confirm entirely this expression of opinion.

In the Hanover MS. of the *Regulae*, the second half of Rule 4 is transcribed after the enunciation of Rule 21.[2] There is doubtless good reason to restore it to what is considered its proper place, despite the strange autobiographical break in the argument which it introduces. Yet the introduction of the consideration of 'Universal Mathematics' logically links it to the rules leading up to the general science of order and measure. The major part of this section of Rule 4 is taken up with a criticism of 'common mathematics'. In attempting to set down the principles underlying the reasoning used in the mathematical sciences, Descartes found a serious defect in most of the mathematical writings of his time. They exhibited a certain 'number of truths' and drew 'conclusions from evident consequences'. But they fell short in one vital matter: 'they did not make it clear to the mind itself why these things were so and how they had discovered them'.[3]

[1] Letter to a friend of Mersenne in 1637 (AT. i. 370^{7-28}).

[2] That is, Rule 4 (AT. x. 374^{16}–379^{13}). See the note of the editors on pp. 374 and 469.

[3] 'Sed quare haec ita se habeant et quomodo invenirentur, menti ipsi non satis videbantur ostendere' (AT. x. 375^{7-9}).

He realized that the mathematical writers and their works fell short of his own conception of knowledge and science. The mathematical works did not exhibit a logically coherent system of demonstrated truth. His barely concealed contempt for the futility of many of the problems and the childish ingenuity of many of the writers is manifestly the result of personal hope and disappointment in his early studies when he had turned to these works as a refuge from the diverse and conflicting doctrines of the academic textbooks of philosophy. The proofs put forward in these mathematical writings are, in Descartes's opinion, 'superficial demonstrations, which are discovered more by luck than by skill and pertain more to the eyes and the imagination than to the understanding'.[1] The reader may find himself prevented from denying the conclusion, because of some previous admission or because of some extraneous absurdity in which its denial would dialectically involve him. But the so-called demonstration does not include, as its logical link, the real necessary bond of the premiss and the conclusion. Its appeal is a proof by 'seeing' or visualizing as in the manner of the theorems of Euclid. The problems are also useless. 'Nothing is more futile than to devote oneself to the study of bare numbers and imaginary figures as though one's aim in life were the knowledge of such empty trifles.'[2] The point of this criticism seems to be that the study of numbers, or figures, as such, has perhaps some value as a propaedeutic to the more concrete study of the facts of the physical world, or to what Descartes, and Hegel, would have called a philosophy of nature. But, if this study is made an end in itself, then it must be rejected, for it would be a mere study of abstractions. Hence it is that the mathematical sciences tend to become a collection of increasingly abstract and complex problems, of ingenious solutions, of more and more elaborate conventions, both of language and symbolism, of possible technical interest to the professionals but not contributing in the least to real knowledge of the universe. The criticism expressed in Rule 4 is carried on in Rule 14 where Descartes accuses the arithmeticians (*logistae*) and the geometers of a certain confusion of thought. The arithmetician tends to regard his numbers as really dis-

[1] AT. x. 375^{16-19}. [2] Ibid. 375^{13-15}.

tinguishable from every 'subject' by imagination, that is, as having a kind of isolated picturable subsistence, instead of being separable only by the abstracting process of thought. In a similar way the geometer, having conceived 'line' as 'length without breadth', and 'plane' as 'length and breadth', nevertheless proceeds to generate 'plane' from 'line' and 'solid' from 'plane'. He forgets, or ignores, that 'the line, which by flowing generates a plane, is itself a true body', whereas the line which is length without breadth is a mere 'made of body', an abstracted feature, not an isolable element which could constitute anything actual, any picturable or perceptible concrete object.[1]

There is, however, an important reservation to make. The praise of the mathematical sciences in Rule 2, as in numerous other passages of the *Regulae*, is not cancelled out by these criticisms. Descartes himself modifies his strictures in Rule 4 itself and remarks that it cannot be without reason that the earliest philosophers refused to admit to the study of wisdom any one who was not well versed in the study of mathematics. He would clearly, and with some justification, have inscribed on the portals of the Cartesian school the adage of Plato: *Μηδεὶς ἀγεωμέτρητος εἰσίτω*. The defects of the mathematical sciences of his time are not inherent in the sciences themselves but lie rather in the practice of the mathematicians who pervert and distort the true significance and scope of them. But he considers that he can detect signs that this true scope was recognized even in the earliest days of these sciences. 'I seem', he says, 'to recognize certain traces of this true mathematics in Pappus and Diophantus, who, though they were not the earliest of mathematicians, did in fact live many centuries ago.'[2] A contrast is made between the ordinary mathematical sciences, the *Mathesis Vulgaris*, and the *Vera Mathesis*. The latter is the genuine mathematical science which alone fulfils the conditions of simplicity requisite for the right subject of study in order to discipline the mind and prepare

[1] Cf. AT. x. 442–6, especially 446^{10-26}. He emphasizes his point by stating: 'Ipsae enim artes Arithmetica et Geometria, quamvis omnium certissimae, nos tamen hic fallunt.'

[2] AT. x. 376^{21-23}. Pappus of Alexandria (*c.* A.D. 300) and Diophantus of the same town who flourished *circa* A.D. 250 and who is the subject of the riddle-epigram in the Greek anthology. Cf. below, p. 201.

it for philosophical study. The same view is stated in the *Discours* in a passage which is rarely given its full weight and importance, despite the fact that it follows immediately upon the formulation of the four precepts.

I had not much trouble [Descartes says] in finding out which objects it was necessary to begin with, for I already know that it was with the most simple and those most easy to apprehend. When I considered also that of all those who had up to the present sought for the truth in science, the mathematicians alone have been able to succeed in their demonstrations, that is, in giving certain and evident reasons, I did not doubt that it was by similar means that they carried out their inquiries. . . . But it was not my plan to try to master all the sciences usually called mathematics. I saw that, although their objects were different, they all did nevertheless agree in considering nothing but the various relations or proportions of their subject-matter. Accordingly I thought the best way would be to examine these proportions in general, without attaching them to any object in particular, except such as would make it easier for me to know them . . . my object being to apply them the better afterwards to all other objects to which they were applicable.[1]

The decision is the same as that given in Rule 4:

Consequently I saw that there must exist some general science to give an explanation in general of that which gives rise to problems about order and measurement, unrestricted to any special subject-matter. This, I saw, was called 'Universal Mathematics', which is not a recondite designation but one long recognized and currently used, because in this science is everything on account of which the others are called parts of mathematics.[2]

In a word, Descartes is convinced that over and above the particular sciences of mathematics, there exists a science, called *Mathesis Vera* or *Universalis*, which is, as it were, merely waiting to be discovered and had, in fact, been partially discovered by some of the Greek mathematicians.

§ 2. '*Mathesis Universalis*'

The account of *Mathesis Universalis* in the *Regulae* is rambling and spread out over Rules 4, 12, 14, and the subsequent Rules. In one sense the ground has already been covered by the extremely brief and inadequate account of the method

[1] *Discours*, II (AT. vi. 19^{17}–20^{10}). Cf. also the *Entretien avec Burman* (AT. v. 176–7).

[2] AT. x. 378^{4-11}.

used in the *Géométrie*. But our interest, *qua* philosophers, does not lie in the actual mathematical discovery which Descartes had made. It is indeed possible to explain what that discovery was, in comparatively simple terms, by selecting and emphasizing certain features of the patchwork exposition of the *Regulae*, features which are restated more succinctly or confirmed by the corresponding account in the *Discours*. In non-mathematical language, and also as a piece of common knowledge of the history of ideas, it can be stated that Descartes reformed the algebra of his time by the invention of a consistent, simplified, and greatly improved notation. By this reform he was able to give an algebraical formulation to problems concerning the general relations of magnitudes and hence to problems concerning the proportions of magnitudes in general. In order to solve these problems he conceived the idea of 'calling Geometry to the aid of Algebra, imagination to the aid of the pure intellect' as Liard sums it up. The general project may then be described as 'the graphic solution of equations' and the third book of the *Géométrie* is in fact nothing but the execution of this project. Thus the *Géométrie* is not strictly a geometrical treatise in the ordinary sense of the word but a treatise of *Vera Mathesis*, in so far as it is algebra illuminated by an appeal to spatial intuition.

There is, however, danger of serious misunderstanding of the whole of Descartes's position. It can be admitted that the algebraical method, and consequently algebra itself—since pure algebra considered apart from its applications is a mere method—certainly would have played a capital role in the elaborating of the Cartesian *Mathesis Universalis*, if this latter had ever been fully worked out.[1] It is historically quite plausible that the famous discovery of analytical geometry was made in the course of Descartes's work on the graphic solution of equations. But, contrary to what has been stated or implied sometimes by certain commentators, there are no real grounds for asserting, at any rate on the authority of the *Regulae* and the *Discours*, that analytical geometry is the same as the new *Mathesis Universalis*. It is rather a pendant or consequence of it, an incidental result obtained, quite logically, by reversing

[1] Cf. P. Boutroux, *L'Idéal scientifique des mathématiciens* (1920), pp. 104–5; Liard, *Descartes*, 435–63; Sirven, *Les Années d'apprentissage*, p. 440.

the procedure of the new science. For, as will be shown, one of the fundamental ideas in this new science is to facilitate and simplify the solution of problems, expressed in algebraical terms, by an appeal to the intuitable spatial forms: whereas analytical geometry, as described by Descartes, is based on the idea of substituting for the perceptible and imaginable spatial form an algebraical formula or equation which gives the law of its generation. Since, for instance, the position of any point in a plane can be uniquely determined by its distance from the two intersecting axes, or coordinates, in that plane—that is, its distance along lines parallel respectively to those axes—it is possible to express in algebraical formula, in a constitutive equation, the law prescribing the successive positions in the plane through which a point must 'flow' as it generates any required intuitable line. The new science includes analytical geometry, but it is concerned with 'order and measure' and it makes no difference whether 'it be in numbers, figures, stars, sounds, or any other object that the problem of measurement arises'.[1] Instead of calling this new science *Mathesis Universalis* or *Vera*, Descartes might well have named it a *Meta-Mathesis*. But it would be an even more serious misunderstanding to assume, as has been done, that the *Scientia Universalis* of Rule 1, the total explanation of the universe in terms of mechanism, is nothing else but the new science of *Mathesis Universalis*.[2] The latter is used by Descartes to cover a narrower field of knowledge. The warning must suffice without explanation at this stage until some preliminary statement of the doctrine has been completed. It is not possible to present a clear and unambiguous picture of this important feature of Cartesian method. In Rule 13 Descartes rather smugly states that 'almost all controversy would vanish from the midst of philosophers if they were

[1] AT. x. 378[2–4]. See below, p. 199, n. 1.

[2] Descartes uses *Mathesis Universalis* or *generalis* or *Vera* or *pura* in the *Regulae*. In the *Meditationes* V, he uses the expression '*Mathesis pura et abstracta*' but Laporte (*Le Rationalisme de Descartes*, p. 8, n. 5) makes a good case against Brunschvicg's view that there is a change of view between the conception of the *Regulae* and that of the *Meditationes*, the latter work containing a doctrine which tends towards a complete intellectualization of the mathematical sciences. Both Sirven and Laporte criticize Boutroux's unfounded generalization that *Mathesis Universalis* is co-extensive with the universal science set out as an ideal in Rule 1. Cf. Sirven, *Les Années d'apprentissage*, pp. 439–40.

always to agree on the meaning of words'.[1] It is irritatingly unfortunate that he did not follow his own excellent advice in defining what he meant by *Mathesis Universalis* and *Scientia Universalis* so as to leave no room for misunderstanding, which must be put down in this instance to the vagueness of the author and not to any prejudice or obtuseness on the part of the readers. There is, however, little left to his commentator but to attempt to fit together into some sort of pattern the scattered and disparate accounts to be found in the *Regulae*.[2]

In Rule 2 the essence of the Cartesian method was formulated in two requirements which had to be fulfilled by the subsequent rules: we must, in the first place, admit no datum which is not self-evident and, secondly, we must follow the inevitable logical order in the continuous movement of thought from self-evident step to self-evident step in the deductive series, which may be described as a serial intuition. The explicit recognition of these requirements was the result of Descartes's meditation on the procedure of arithmetic and geometry as illustrative of the best examples of the activity of the *vis cognoscens*. The guarded language of Rule 2, relative to the mathematical sciences of his own time, left room for the open criticism of incidental defects of these sciences in Rule 4 and led him to postulate the new *Mathesis Universalis* which would, as it were, transcend their limiting factors and remedy their faults of specialization by its very universality. But what remained always true, over and above the special disciplinary and propaedeutic value of mathematical study, was that the special excellence of the mathematical demonstration was not dependent on any peculiarity of the subject-matter with which it deals but because, even more valuable for the philosopher, its material lends itself more easily than any other to the demands of strict method. For expertness, or sagacity as Descartes would have called it, in the mathematical sciences, as elsewhere, shows itself essentially in the ability to arrive at the constituents of a problem and break them up into their fundamental data, set them out in their proper

[1] AT. x. 434^{2-4}.

[2] The important passages are in ibid. 374^{16}–379^{13}; 438^{12}–440^{27}; 442^{1}–452^{26}; also 428^{21}–300^{5}.

order of relationship as equations, and then construct a synthesis of the whole problem.

In the *Discours* it is pointed out that, in spite of the difference in their subject-matter, the particular mathematical sciences all agree in positing a common object of investigation.[1] All the mathematical sciences, it is argued, are concerned solely with the relations or proportions obtaining within their several subjects of study. Accordingly the field or domain of mathematics, as a general or universal science, would seem to be relations or proportions as such or in general, that is to say, proportions without reference to their special subjects or matters in which they are embodied, abstracted from the various proportional terms which constitute their peculiarity. The science, then, of *Mathesis Universalis* is that science in which the method is directly applied to, and exemplified in, the systematic exposition and solution of all *quaesitiones*, or problems, involving proportions as such and considered apart from and in abstraction from any particular terms. The same doctrine is set out, at more length but perhaps less clearly, in the autobiographical passage in Rule 4. Descartes tells us there that when, after years of reflection on the special sciences of arithmetic and geometry, his thoughts turned to the idea of a universal mathematics, he asked himself what precisely was meant by the term 'mathematics'. In the nomenclature of the seventeenth century not only arithmetic and geometry but a multiplicity of special sciences, astronomy, music (i.e. the theory of harmony), optics, mechanics, and others are reckoned as 'parts' of mathematics. It seems clear that all these branches of study, in spite of the difference in their subject-matters, have some conspicuous and important character in common. Even the merest novice sees at a glance, amongst the problems he encounters, which belong to mathematics. It should be possible, then, to determine what constitutes the mathematical character of an inquiry. The answer is given in these terms:

> problems in which order or measure is examined, and these alone, are referred to mathematical science; but it makes no difference whether such measure has to be investigated in numbers, figures, stars, sounds, or any other special object; that, consequently, there must be a general

[1] *Discours*, Part II (AT. vi. 19^{29}–20^{24}).

science, which sets forth and solves all possible problems relating to order and measure as such, considered in general or free from their restriction to this or that special matter; and that this general science of order and measure is entitled to the name of *Mathesis Universalis*.[1]

The special sciences of mathematics, arithmetic, geometry, astronomy, &c., are called 'parts' of mathematics only because, and in so far as, they treat in part the same subject which, considered as a whole, constitutes the domain of universal mathematics. *Mathesis Universalis* is the science of order and measure, or, in the words of the less technical summary of the *Discours*, it is the science which treats of proportions in general.

We are then to take it that the subject-matter, or *objectum materiale* of this science of universal mathematics can be called 'proportions', 'order', 'measure', or, as Rule 14 adds, 'dimension'. This latter is defined as 'nothing but the mode and aspect according to which some subject is held to be measurable',[2] Dimension, that is, applies to any aspect of a perceptible object in respect to which it is measurable. Descartes goes on to make it clear that dimensions are not only length, breadth, and depth but also weight, in terms of which the heaviness of objects is measured, velocity, which is a dimension of motion, and number, though there cannot be said to be absolute identity between the meaning of this last term and dimension. Order applies to a multitude or manifold and measure applies to continuous magnitude. He goes on to point out that a continuous magnitude can always, at least partially, be reduced to a manifold by 'the help of an assumed unit' and the manifold so obtained can then be so ordered as greatly to facilitate its measurement. It is easy to recognize the embryonic sketch of the method applied in the *Géométrie*. It is interesting to note that in the *Meditationes* Descartes defines the object of pure mathematics as 'those

[1] 'Illa omnia tantum, in quibus ordo vel mensura examinatur, ad Mathesim referri, nec interesse utrum in numeris, vel figuris, vel astris, vel sonis, aliove quovis objecto, talis mensura quaerenda sit; ac proinde generalem quamdam esse debere scientiam, quae id omne explicet, quod circa ordinem et mensuram nulli speciali materiae addictam quaeri potest, eamdemque. . . . Mathesim universalem nominari' (AT. x. 377^{23}–378^{8}).

[2] 'Per dimensionem, nihil aliud intelligimus, quam modum et rationem, secundum quam aliquod subjectum consideratur esse mensurabile' (ibid. 447^{23-25}).

things which pertain to corporeal nature' and he makes it clear, in his reply to objections raised on this point, that he refers only to those aspects of corporeal things which constitute their essential nature *qua* corporeal or material, that is, indifferently considered as to their existence or non-existence and merely in respect to extension and its modes, i.e. the objects of *Mathesis Universalis* in the *Regulae*.[1]

The idea of such a science of pure magnitude, order, and measure had not a very great novelty. M. Gilson produces a Scholastic text which expresses much the same ideas as those outlined above and this same text refers to Aristotle and his Greek commentators as having put forward a similar doctrine on the object of pure mathematics.[2] There is a phrase in *Metaphysics* E which might well have come out of the *Regulae*. 'Geometry and astronomy deal with a certain particular kind of thing while universal mathematics applies alike to all.'[3] A passage in the *Posterior Analytics* 'makes it clear that Aristotle contemplates a science wider than arithmetic or geometry; and a specimen of it is to be found in Euclid's treatment in Book V of the *Elements*, of proportion as existing between *any* kind of magnitudes'.[4] This passage almost certainly refers to the theory of proportion which is applicable to all magnitudes alike, the discovery of which is attributed to Euxodus.[5] It clearly indicates that the Greek mathematicians of Aristotle's day had begun to develop a theory of proportions in general. Aristotle says that the theorem of the Alternation of Proportionals, that is, the proposition, if $A : B = C : D$, then $A : C = B : D$, used to be demonstrated separately (*χωρίς*) of numbers, lines, solids, &c. But alternation is true of all proportionals, in virtue of a

[1] Cf. *Meditationes*, V: 'tum etiam de omni illa natura corporea, quae est purae Matheseos objectum' (A.T. vii. 71^{6-8}) and (80^{9-10}) and the *Resp. ad V Obj.* (A.T. vii. 384^{20-21}).

[2] *Index Scolastico-Cartésien*, p. 167. The text is from the commentary on the *Physics* by the Coïmbricenses, and contains *inter alia* the passage on the object of pure mathematics: 'intelligibilem materiam esse magnitudinem, hoc est lineam, superficiem, corpus, dicique materiam, propter similitudinem sive analogiam quam cum materia prima habet'.

[3] *Metaphysics*, E, 1026^{a} 26–27: 'ἀλλ' ἡ μὲν γεωμετρία καὶ ἀστρολογία περί τινα φύσιν εἰσίν, ἡ δὲ καθόλου πασῶν κοινή'. Cf. also K. 1061^{b} 19.

[4] W. D. Ross, *Aristotle, Metaphysics*, i. 356–7.

[5] Cf. Heath, *Mathematics in Aristotle*, pp. 43–44, and W. D. Ross, *Aristotle, Prior and Posterior Analytics*, p. 525.

common character in them, and in no way depends upon the features in which they are specifically distinct from one another. 'Today, however, the proof is commensurately universal, for they do not possess this attribute *qua* lines or *qua* numbers, but *qua* manifesting this generic character which they are postulated as possessing universally.'[1]

The text of the *Posterior Analytics* was one of those commented upon in detail in the *cursus studiorum* at La Flèche.[2] Whether or not the passage of Book I, Chapter V, quoted above ever occurred to the mature mathematician of 1628 is a vain speculation. In Rule 4 he says:

> We have sufficient evidence that the ancient geometers employed a kind of analysis which they extended to the solution of all their problems, though they jealously kept it secret from posterity. There is, moreover, at the present day a flourishing development of arithmetic (the so-called algebra), which aims at achieving, in regard to numbers, the same [general solution of problems] which the ancients [by their analysis] achieved in regard to figures.[3]

Further on, in the same Rule, he adds: 'Traces of this *Vera Mathesis* are, I think, still discoverable in the works of Pappus and Diophantes'[4] and 'certain men of genius have tried, in the present age, this same art [the genuine science of mathematics]. For it seems to be precisely this art to which they give the barbarous name of algebra.'[5]

But whatever the debt owed by Descartes to the ancient Greek mathematicians, or to his contemporaries, it is in his criticism of their failings and deficiencies that the main interest lies. The *Discours* again provides a summary of the *Regulae*. 'In algebra, as it is commonly taught, one is so much at the mercy of arbitrary rules and symbols, that one seems

[1] *Post. Anal.* i. 5, 74ª 23–25: '*νῦν δὲ καθόλου δείκνυται· οὐ γὰρ ᾗ γραμμαὶ ἢ ᾗ ἀριθμοὶ ὑπῆρχεν, ἀλλ' ᾗ τοδί, ὃ καθόλου ὑποτίθενται ὑπάρχειν*'.

[2] Cf. Sirven, *Les Années d'apprentissage*, p. 33.

[3] AT. x. 373^{12-18}.

[4] Ibid. 376^{21-22}. For Pappus's definition of algebra, see Heath, *History of Greek Mathematics*, ii. 400–1.

[5] AT. x. 377^{2-5}. Hamelin, in *Le Système de Descartes*, p. 37, points out that Viète was mainly responsible for giving to algebra its definitive form and came nearer than any writer before Descartes to a geometrical interpretation of equations. But Descartes does not appear to have read Viète till 1629, and Viète 's'était surtout formé par la lecture de Diophante' (p. 7).

to be learning an obscure and technical art which embarrasses and injures the mind, instead of a science which clarifies and educates it.'[1] Algebra, states Rule 4, is 'wanting in that supreme clearness and facility' which ought to characterize *Mathesis Universalis* as the science directly exemplifying the method.[2] The reason for all this obscurity lies in the fact that both the ancient and modern mathematicians, while attempting to deal with proportions in general as embodied, in the case of the former, in figures, and, in the case of the latter, in numbers, did not fully realize that the proportions have, in their essential nature, nothing to do with figures and numbers. They did not sufficiently analyse the general problems of figure and number and accordingly they did not arrive at a completely general theory of proportions. But 'there is no less reason for abstracting our proportions from the figures treated in geometry, if the problem involves them, than there is from any other kind of matter'.[3] And the same is true of number.[4] By not making this further step in analysis, the other mathematicians restricted their study of proportions to the domain of figure and number. The distinction between their studies and that of *Mathesis Universalis* lies precisely in the fact that this latter treats of proportions in complete abstraction from figure, number, or any other subjects. Its object is proportions *ut sic* and the nature of these proportions as such is identical in whatever subject-matter it may be involved. Proportions as such are all expressible in terms of order, measure, and dimension, and it is of these that the genuine science of *Mathesis Universalis* treats.

[1] *Discours*, Part II (AT. vi. 18^{2-5}). The Latin text is clearer (AT. vi. 549), the variants being: 'à certaines règles et à certains chiffres' to 'certis regulis et numerandi formulis', and 'une science qui le cultive' to 'scientia qua excolatur et perspicacius reddatur'.

[2] Rule 4 (AT. x. 377^{8}): 'perspicuitas et facilitas summa'.

[3] Rule 14 (AT. x. 452^{14-16}): 'hic non minus abstrahendas esse propositiones ab ipsis figuris, de quibus Geometrae tractant, si de illis sit quaestio'. The sense of the whole section demands 'proportiones'.

[4] Cf. Rule 16 (AT. x. 455^{25}–456^{10}) where Descartes notes: 'nos autem hoc in loco non minus abstrahere ab ipsis numeris, quam paulo ante a figuris Geometricis, vel quavis alia'. Thus in attempting to find the hypotenuse of the right-angled triangle whose sides are 9 and 12, instead of writing that it is $\sqrt{225}$, i.e. 15, we should write $\sqrt{(a^2+b^2)}$, thus keeping the two members distinct. We can thus see how the result 'dependeat ex datis' (458^{7-8}).

XIV

THE SOLUTION OF PROBLEMS

§ 1. *The Mathematical 'Shell'*

DESCARTES suggests, in Rule 14, that he would prefer to have a reader who was not an expert mathematician, in fact someone who was not at all accustomed to working out mathematical problems, although he qualifies this by remarking that he would like his reader to be eager and keen to study these sciences. 'For', he adds, 'it is not the case that this part of our method was invented for the express purpose of dealing with mathematical problems but, on the contrary, that the study of mathematics should be undertaken almost uniquely as a means of training in this method.'[1] Yet the non-mathematician, or the reader whose interest is not primarily in the study of mathematical method, may well feel in his study of the *Regulae*, especially in the last section (i.e. from Rule 13 onwards), that the Cartesian method is, in fact, entirely mathematical both in its subject-matter and in its possible applications. The somewhat negative account of *Mathesis Universalis*, consisting, in the main, of a criticism of past mathematical practice and finally asserting a new science whose subject-matter, abstracted from the particularities of the 'parts of mathematics', is order, measure, and dimension, is not very satisfying, and the connexion between this new science and the previously outlined doctrine may be nebulous. If, as Descartes suggests, the expert mathematician may fail to see the wood for the trees, the non-mathematician is in danger of seeing neither the wood nor the trees. At the risk of some repetition of points already dealt with, it is advisable to attempt to link this *Mathesis Universalis*, the science of order and measure, to the general doctrine of method and, in so doing, to let Descartes give the explanation in his own words.

The existing sciences of arithmetic and geometry fell short of the conception of a science which had been formulated at

[1] Cf. AT. x. 442[1–11].

the very outset of the *Regulae*. They failed to fulfil the two essential requirements of starting from self-evident data, transparent to mind and clearly and distinctly apprehended, and also they did not constitute a logically-coherent system of demonstrated truth. Quite apart from the inherent futility of many of the problems discussed in the 'vulgar mathematics', they failed, from the point of view both of intellectual intuition and of deduction, to fulfil the essential condition of *perspicuitas et facilitas summa* which characterizes the apprehension of the data and the logical links of the deduction in true knowledge. They left the evidence 'to the eye or to the imagination, rather than to the intellect'. These sciences, therefore, do not represent the activity of pure mind, the natural reason itself, the natural light, which cannot manifest itself in a confused activity, involving an appeal not only to reason but also to the senses and the imagination. The activity employed in these sciences cannot therefore be identified with the method itself. Descartes admits, in Rule 4, that 'I am prepared to believe that the greater minds of yore had some knowledge of this [method], led to it by nature alone. For the human mind has within it something almost divine, wherein are scattered certain seeds of useful thought, which Nature has implanted in it. However neglected and stifled these may be through harmful studies, they bear fruit of their own accord.'[1] The natural method, the 'inborn principles' of the method are the 'spontaneous fruit of the pure activity of mind'.[2] In the earlier activities of the Greek analysts, these 'seeds of truth', to use the expression of the *Discours*, had to a certain extent ripened and yielded some harvest, but extraneous elements had spoiled their efforts. Purified of their attachment to sense and imagination, abstracted from their

[1] Cf. AT. x. 373^{5-11}. See above, p. 154, n. 1.

[2] 'Spontaneae fruges ex ingenitis hujus methodi principiis natae' (AT. x. 373^{19-20}). The expression *ingenita principia* which occurs in Rule 4, and later on in the *Regulae* (cf. 'mentibus nostris ingenitae', written of similar truths in a letter to Mersenne, 1630, AT. i. 145^{18-19}), refers to those data ('les verités mathématiques' of the letter just referred to) to which the mind is 'attentive'. These data are implanted in the mind: objects 'given' to the immediate apprehension; but being 'given' they are also 'received', and *qua* received, they are determinate modes of mind's own intuitive function. What the mind intuits is one with the intuiting, both as object and mode of it. The doctrine would appear to be the same difficult one found in the *Meditationes*, and akin to that of *νοῦς* in Aristotle.

particularization in figures and numbers, these sciences become the general science of *Mathesis Universalis*, the science of order and measure, which then exemplifies *sub specie puritatis* the activity of mind itself, that is, the method. Now order applies to a multitude or manifold, and measure to continuous magnitude. But, as Rule 14 makes clear, a magnitude can, at least partially, be always reduced to a manifold by assuming a unity, and the manifold so obtained is then subject to ordering.[1] It is, then, in the science of order that the ultimate nature of the method lies. The whole is reduced by analysis till *a solius ordinis inspectione dependeat*. Here lies the link with the 'secret of the method' as laid down in Rule 5. 'The entire method consists in the ordering and arranging of the materials on which the mind's attention must be concentrated, if we are to discover truth. The exact observance of the method will be secured if (i) we reduce involved and obscure propositions step by step to more simple ones; and (ii) we start from our intellectual intuition of the simplest propositions and endeavour, by retracing our path through the same steps, to work our way up to the knowledge of all the others.'[2] The whole method consists in the constant observation of these systems of order; this is the burden of Rule 10,[3] and the final summing-up of Rule 6: 'But though these facts are all so evident that they may seem almost puerile, I can now understand by careful reflection what is involved in all problems, implicating proportions and relations of things and the order in which they should be investigated; and this covers the whole science of pure mathematics.'[4] Reduced, as it were, to a science of order, removed from its mathematical 'shell',[5] the method seems in

[1] 'Sciendum etiam, magnitudines continuas beneficio unitatis assumptitiae posse totas interdum ad multitudinem reduci, et semper saltem ex parte; atque multitudinem unitatum posse postea tali ordine disponi, ut difficultas, quae ad mensurae cognitionem pertinebat, tandem a solius ordinis inspectione dependeat, maximumque in hoc progressu esse artis adjumentum' (AT. x. 451^{24}–452^{6}). The word 'pertinebat' is a conjecture of the editors: the MSS., both A and H, have 'pertineat'. See also the doctrine of the *Géométrie*, above, p. 180.

[2] AT. x. 379^{15-21}. See above, p. 150.

[3] Cf. AT. x. 404^{12-25}.

[4] Ibid. 384^{29}–385^{5}.

[5] 'Integumentum vero dixi, non quo hanc doctrinam tegere velim et involvere ad arcendum vulgus, sed potius ita vestire et ornare, ut humano ingenio accommodatior esse possit' (ibid. 374^{12-15}).

fact to take upon itself, 'to clothe and adorn itself', in Descartes's metaphor, in more suitable terms as a universal means of attaining to true and solid knowledge, and the logical panacea for the solution of all problems, whatever their subject-matter.

All this may be said to hang together, somewhat loosely, but nevertheless connectedly. The picture is not, however, complete. We may let Descartes speak once again. It is in Rule 14 that the complications of the method appear in a somewhat astonishing form. Before touching on them, let us note the following remark:

> Because, as we have often pointed out, the syllogistic forms serve no useful purpose in the apprehending of the truth of things, it will be of benefit to the reader to reject them altogether and to consider that knowledge of every possible kind, except knowledge which consists in the simple and pure intellectual intuition of a single thing, is a matter of the comparison of two things or more, with each other. In fact, the whole business of the human reason consists in preparing for this activity; for when it is an evident and simple operation, we need no aid from method, but there is only need to rely on the light of nature alone in order to apprehend the truth therein given to us. But we must note that the comparisons are only said to be simple and evident when the problem (*quaesitum*) and the datum participate equally in the same nature.[1]

This phraseology, which reminds us of Locke's famous definition of knowledge, recalls equally the methodological principle of the *Géométrie* that all knowledge must be reduced to the equational form and ultimately to one equation, which can be interpreted in terms of a simple nature, the evident and simple object. But the passage also suggests that, while this is possible when the problem and the datum both participate in the same nature, that is, the same simple nature, there may be cases when this is not so. What then are these cases and what change of method is necessitated?

§ 2. *Perfect and Imperfect Problems*

The word *quaestio*, problem, occurs frequently in the later rules of the *Regulae*. Like the words *quaesitum* and *datum*, it is a common word in the Scholastic manuals of the time, as

[1] AT. x. 439^{24}–440^{12}.

is the distinction introduced in Rule 13 between *quaestiones perfectae* and *imperfectae*.[1] The similarity is, for practical purposes, merely one of terminology and there is little parity of doctrine.

In Rule 12 all the possible objects of knowledge are divided into simple propositions and problems.[2] It is of simple propositions that Descartes is talking in the passage from Rule 14, quoted above, when he speaks of 'evident and simple' operations, in which we need no method and can rely on the light of nature alone. There are no precepts to be given for finding simple propositions. This is obvious: there can be no rule for 'having' an intellectual intuition. The most that can be done, as we have already seen, is to remove prejudices of all kinds, and train the *vis cognoscens* to be attentive and sagacious, and this is, in the main, the practical purpose of the first book of the *Regulae* (i.e. Rules 1–12). A well-trained, observant, and attentive mind will be in the best position to apprehend in a single act of vision the clear and distinct data. These rules are, in brief, meant 'to facilitate the use of our reason'. Descartes then goes on to treat of *quaestiones*, problems, which are subdivided into perfect and imperfect problems. Problems fall into two classes, according as their terms are, or are not, completely understood. Rules 13–24 are to treat of problems whose terms are completely understood, even if their solutions are unknown, or not yet known —these are called 'perfect problems'. Rules 25–36 will be devoted to problems whose terms are imperfectly understood —that is, imperfect problems—and will show how they can be reduced to problems of the perfect type. It will, of course, be remembered that the *Regulae* contains only eighteen complete Rules and the last three (19–21) have only their enunciations. Within this framework, it is natural that the problems which are 'perfectly understood' should be

[1] Cf. the texts from Fonseca's *Institutionum dialecticarum libri octo* (quoted by Sirven, *Les Années d'apprentissage*, p. 405, n. 5) and (on p. 406, n. 1) where the division into 'de perfectis et imperfectis orationibus' is referred to in the manuals. On *quaesitum* and *datum*, see the text from Eustachio a Santo Paulo: '*quaenam sit legitima rei propositae divisio et partitio?*' in Gilson's *Index Scolastico-Cartésien*, p. 184.

[2] AT. x. 428^{21}–430^{5}. The meaning of 'quaestio' is also treated in Rule 14 (AT. x. 442^{1-14}) as well as in Rule 13 (AT. x. 430^{6}–438^{7}). The *Discours* calls them 'difficultés'.

practically all encased in a mathematical 'shell' and should deal with matters of arithmetic or geometry and that consequently Rules 13–21 become technically mathematical.[1] The 'imperfectly understood' problems, he notes, are obscure, or may be obscure, both relatively to the terms involved in them and to the conditions relevant to their solutions.

Rule 13 begins by making three rather obvious points. First, 'there must be something which we do not know in every problem, otherwise there would be no point in asking the question'. Secondly, this something must be 'designated in some way or other, for otherwise nothing would push us to investigate it rather than anything else'. And, thirdly, it can only be 'designated' or determined 'by the aid of something which is already known'.[2] In every problem, then, there is something unknown, the *quaesitum*, which is determined by reference to something already known, the *datum* or *cognitum*. In other words we must not conduct our inquiries in a random manner but set up a specific point to be investigated. These conditions are realized in all problems, imperfect as well as perfect. Descartes gives us an example. Suppose the problem which we are studying is 'the nature of the magnet', basing ourselves on knowledge we have acquired of the meaning of 'nature' and 'magnet'. The problem is imperfect but there are known terms in its formulation, and accordingly the *quaesitum* is narrowed and determined so that we restrict our inquiry to a certain type of solution. But the problem may be set in another form. We may be asked: 'What is to be inferred from the set of experiments which Gilbert states that he has carried out, whether these experiments be trustworthy or not?'[3] That is, we are asked to consider these experiments as data, as given facts, and taking them as granted argue from them to the nature

[1] Cf. Rule 14 (AT. x. 440^{21-27}; 441^{21-29}; and 442^{1-15}), and Rule 12 (AT. x. 429^{27}–430^{5}).

[2] 'Primo, in omni quaestione necesse est aliquid esse ignotum, aliter enim frustra quaereretur; secundo, illud idem debet esse aliquo modo designatum, aliter enim non essemus determinati ad illud potius quam ad aliud quidlibet investigandum; tertio, non potest ita designari, nisi per aliud quid quod sit cognitum' (ibid. 430^{17-22}). The second point is repeated at 434^{25}–435^{2}.

[3] The reference is to W. Gilbert, who published his *De Magnete* in 1600, which is often quoted, favourably, by Mersenne. See R. Lenoble, *Mersenne ou la naissance du Mécanisme* (1943), p. 488.

of the magnet. Because we know certain *cognita*, viz. the meanings of the terms 'nature' and 'magnet', and certain data, viz. the experiments carried out by Gilbert, the *quaesitum* is the solution specifically determined by reference to these data and these alone.[1] Our only concern is with the inferential consequences of all these *cognita* and the imperfect problem has now become a perfect problem. The purpose of this procedure is explained in the previous Rule which is a kind of commentary on the same example.

> It is extremely useful to note this point [that is, that knowledge consists in seeing distinctly how simple natures are linked together to form complexes]; each time there is a problem to investigate, most people get stuck from the very outset, because they are not sure which terms ought to be kept in mind and are convinced that they must look for some term of a kind unknown to them hitherto. For instance, if they are asked what is the nature of the magnet, they are once foresee difficulties and obstacles in the investigation and, dismissing from their minds all evident facts, they turn their minds on the most difficult, hoping vaguely that, by wandering about the empty fields of multifarious causes, they will happen upon something new. But the man who knows that there is nothing to know about the magnet, which does not consist of certain simple natures, will not be in doubt about his method of attacking the problem. He will first collect together all the experimental observations which have been made about the magnet, and then he will attempt to deduce from these the complex of simple natures which is necessary to produce all those effects which have been observed in the magnet. Once this has been achieved, he can confidently assert that he has discovered the real nature of the magnet, in so far as the human mind and the experimental observations allow.[2]

The problem should be 'reduced to its simplest terms in accordance with Rules 5 and 6 and resolved into its parts in accordance with Rule 7'.[3] The instance given in Rule 6, the problem of the mean proportional between 3 and 48, is a case of the same kind.[4]

The process, then, is that imperfect problems should be resolved into perfect problems by analysis of the *quaesita* and *cognita*, and that the perfect problems should be further resolved until they become problems concerning order and

[1] Determined therefore by reference to 'experience'. See below, pp. 239 ff.
[2] Rule 12 (AT. x. 427^{6-26}).
[3] Rule 13 (AT. x. 432^{1-3}).
[4] Cf. AT. x. 384^{21}–386^{14} and above, pp. 161–2.

measure, where what is ordered is 'proportions in general'. It is thus that we can meet the repeated demand of the *Regulae* that we must reduce involved and obscure propositions step by step to those that are simpler.

§ 3. *Another 'Exercise in Method'*

Few, except historians of science, would begin, and then persevere in, the reading of the second *essais de cette méthode*, the *Météores*, which follow upon the *Dioptrique* and precede the *Géométrie* in that slim volume of 1637. The interest of the *Météores* is now almost strictly antiquarian. 'The Treatise on Light was eclipsed by Newton's work in optics, and the problems treated in the *Meteors* were dealt with more successfully later on by experimental methods. But though the detail of Descartes' procedure here can no longer be supported, we must never forget that the more general and original conception which inspired it (namely, that natural data can and should be investigated according to the method and presuppositions of his mathematical physics) has been amply vindicated in having made possible the solid advances from his own day to our own.'[1]

If we turn to the eighth *discours* of the *Météores* we find that the problem treated there is that of the '*arc-en-ciel*', the rainbow. Descartes introduces his subject with this comment: 'The rainbow is so noteworthy a wonder of nature, the cause of which has been sought for so eagerly by great minds and yet so little is known about it, that I could hardly choose a better subject to show how, by the use of my method, true knowledge can be got which was not in possession of those who have written on this point.'[2] Voltaire, who had studied this *discours*, adds his own comment in his *Lettres Philosophiques*: 'Up to Antonio di Dominis', he says, 'the rainbow was considered as an unaccountable miracle. This philosopher had guessed that it was a necessary effect of rain and sun. Descartes made his name immortal by applying mathematics to this purely natural phenomenon; he calculated the reflections and refractions in the raindrops and his sagacity

[1] Keeling, *Descartes*, p. 22.
[2] *Les Météores*, VIII (AT. vi. 325[3–9]).

was almost divine.'[1] Perhaps, as in the case of the 'divine seeds of truth', Descartes might have accepted the qualification of 'divine' for that characteristic condition of intellectual intuition, sagacity. Leibniz, as usual, always keen to diminish the originality of his predecessor, accuses Descartes of plagiarism in respect to di Dominis: a charge which Baillet is at pains to deny.[2] Whatever the truth, Leibniz also finds another grief in the fact that Descartes pays no heed to, and does not mention even the name of, François Maurolic. Here his facts are quite wrong since Descartes not only mentions but criticizes and corrects the results of Maurolic in this *discours* of the *Météores*, where he is called Maurolycus.[3] It would appear probable, and the assumption is quite adequate for the purpose of illustrating the method, that it was the work of Maurolic which was used as the basis of discussion in the *Météores* in order to apply the mathematical method, in the same way as one might use the experimental observations of Gilbert on the nature of the magnet.

The theory of Maurolic, based on certain experiments, was *grosso modo* this.[4] The rainbow was formed by rays of the sun, arriving from behind the observer. These sun-rays are broken by the raindrops of the mist facing the observer; they are reflected to the eye of the observer. Two reflections are produced in each raindrop, one on the convex side, the other

[1] *Lettres Philosophiques*, vol. ii (edit. Lanson), 16: letter 'Sur l'optique de M. Newton', p. 45. The work of di Dominis, *De radiis visus et lucis in vitris perspectivis et iride tractatus*, was published in Venice in 1611. Voltaire had certainly read this treatise, but it is not at all certain that Descartes had done so; he does not mention the rather unfortunate archbishop in any of his writings or letters.

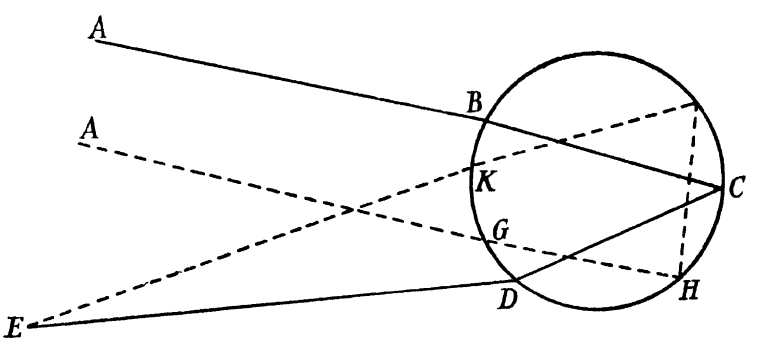
[2] 'On prétend que ce Prélat [de Dominis] avait trouvé la manière dont M. Descartes a expliqué la réflection et la réfraction', says Baillet, *La Vie*, ii. 540, and the reference in the margin gives 'Leibn. Epist. Mf. 1689'. The idea that the rainbow was formed by the reflection of the sunbeam from drops of falling rain was no new idea. It was familiar to the Greeks: see, for example, Aristotle's *Meteorologica*, iii. 2–6, 378^{a} 14.

[3] Cf. *Les Météores*, VIII (AT. vi. 340^{24}), and the comment: 'ce qui montre le peu de foi qu'on doit ajouter aux observations qui ne sont pas accompagnées de la vraie raison'. On the problem in general see G. Milhaud, *Descartes savant*, pp. 118–21, and P. Mouy, 'La Théorie cartésienne de l'arc-en-ciel' (*IX^{e} Congrès International*, 1937), ii. 46–53, where a full account of Maurolic's theories is given.

[4] Maurolic's work, first published in Naples in 1611, was republished in 1613, at Lyons, under the title *Theoremata de lumine*, &c., with notes by P. Clavius, S.J., who was the author of the mathematical textbooks used at La Flèche. Clavius himself died in 1612.

on the concave side, thus forming what he calls the 'interior arc'. On the exterior arc there are further complications due to refraction. The effect of the two phenomena in the interior arc is to return the reflected ray at an angle of 45°, or half the angle of incidence, with the result that, the incident being almost horizontal, the interior arc has an 'altitude' of 45°. This latter is a little less than 45° since the raindrops are slightly lengthened by the force of gravity. In the exterior arc, the reflected ray has an angle of approximately 56° which is five-eighths of a right-angle.

Descartes begins his inquiry in a typical manner. First, a preliminary survey and delimitation of the *quaesitum*. In making the survey he accepts the datum established by Maurolic's experiments, namely, the relation of the sun to the raindrops, and the position of the observer. He then notes that the problem of the rainbow is only a special case of the general *respectus* of the sun-rays and drops of water—cannot we see equivalent phenomena in waterfalls and fountains?—and further that these drops of water are in principle spherical, large and small; he was therefore justified in considering the problem in general. Descartes then goes on to describe how he filled a large round glass bottle ('fiole') full of clear water and reproduced artificially the conditions under which the natural phenomena of iridescence occur. A simplified version of Descartes's original, and much more charming, sketch would be:



The observer's eye is supposed to be at *E*. The rays of the sun fall in the direction indicated by the line *AB*.

Two parts of the sphere, notes Descartes, appear red; the

angles of *DE* and *KE* with the sun-rays being respectively 42° and 52°. Through the interposing of a screen at suitable places, it is possible to determine the trajectory of the rays which form the two arcs: 'The first rainbow [the interior arc] is caused by rays which reach the eye after two refractions and one reflection, and the second [the exterior arc] is caused by other rays which only reach the eye after two refractions and two reflections'.[1] The first step in the analysis has been made: the difficulty has been 'divided' by reducing the problem to one of geometrical optics. The next step is also one of simplification. Descartes replaces his spherical bottle of water by a prism, thus reducing the curved lines to straight lines, and finds that the characteristic angles of the red ray have approximately the same number of degrees as in the previous experiment. The experiments have enabled him to arrive at the causes through the effects and all that remains to be done is to set the whole problem in equations, to 'mathematize' the *cognita* in a general equation. In order to achieve this, Descartes constructs a table of columns, giving ten values of the angle of incidence, then another table, ten times more accurate, for certain values (values between 8,000 and 9,800) of the first table. By means of these tables, with the aid of the law of sinus, and his knowledge of the index of refraction of light between air and water (187/250) which he has established in the *Dioptrique*, he can calculate the value of the angles of incidence, of refraction and total reflection and thus show that the luminous rays have a maximum density around the two values 41° 37″ and 51° 37″ of the total deviation.[2] Maurolic, then, was wrong and Descartes adds: 'this only goes to show how little faith one can have in experimental observations which are not accompanied by true reasoning'.[3] The optico-geometric problem is complete, once the fundamental equation has been fixed. A problem of physics has been simplified to a problem of mathematics and Descartes has given 'an almost perfect example of mathematical physics and, at the same time, without knowing it, laid the foundation-stone of the wave-motion theory'.[4]

[1] *Les Météores*, VIII (AT. vi. 329^{10-15}).

[2] Cf. *Les Météores*, VIII (AT. vi. 337^{5}–340^{6}).

[3] See above, p. 211, n. 3.

[4] P. Mouy, 'La Théorie cartésienne de l'arc-en-ciel', p. 53. The same author notes

Whatever its value or failings as a physical theory, which had to be reformulated by Newton, it is another instance of the application of the method, the breaking down of problems into their simplest expressions. It is interesting to note that the first, and by no means the stupidest, commentator on the Cartesian method was struck by the demonstration, which has just been sketched in its barest outlines. Father Poisson remarks in his *Commentaire*:

> One should note that, in the physical sciences, it is often necessary to make use of a mean, which can be called an *analogue* or *proportion*. M. Desc[artes], for instance, in explaining the Iris or rainbow, considered what refractions the rays of the sun underwent when they fell on a ball of glass; then, having discovered on this rather large ball that the rays fell in different fashions and composed different colours, he had only to apply his reasoning entire to the raindrops in the sky. This ball of glass is a *mean*, which I call an *analogue*, because it is, in fact, by the proportion found between it and the raindrop, and, in truth, by the proportion between the shapes of the objects, that the demonstration is established.[1]

It may now appear a very elementary, almost schoolboy, lesson in scientific procedure, but essentially it represents the fundamental methodology of present-day science.

on p. 51: 'On y voit la marque du génie dans la manière neuve et féconde dont les mathématiques et l'expérience sont associées.'

[1] In his *Commentaire ou Remarques sur la Méthode de Mr. Descartes*, published in Vendôme in 1671. (Quoted by Sirven, p. 431, who adds a similar observation by Berthet.)

XV

THE METHOD IN MATHEMATICS

§ 1. *Sense and Imagination in Reasoning*

In the last chapter the method was summarily described in relation to the solution of problems. Imperfect problems are to be reduced, as far as is possible, to perfect problems. Where this reduction is not found possible, it must be due, Descartes implies, either to want of data—the preliminary survey may be incomplete—or to some inherent limitation of the human mind. We are not told how we may distinguish between these two cases or any other possible detrimental circumstance. Perfect problems should then be further reduced so that they become susceptible of treatment as instances of *Mathesis Universalis*, that is, they should be reduced to problems concerning order and measure. The process of reduction, of analysis, consists then in the classifying, generalizing, and precisely designating or defining the *cognita* and data in relation to the *quaesitum*, until, in the words of Rule 13, 'we no longer suppose ourselves to be studying this or that special subject but recognize that we are concerned solely with the quite general problem of comparing certain magnitudes with one another'.[1] This is obviously a key-point for the recommendation is repeated several times in the subsequent rules.[2] It is the same doctrine that is expressed in the *Discours* in slightly different terms where it is pointed out that the mathematical sciences, in spite of the difference in their subject-matters, all have a common object of investigation since all these sciences are concerned with relations and proportions obtaining within their several

[1] 'Ut non amplius cogitemus nos circa hoc vel illud subjectum versari, sed tantum in genere circa magnitudines quasdam inter se componendas' (AT. x. 431[21–23]).

[2] Cf. Rule 14: 'Maneat ergo ratum et fixum; quaestiones perfecte determinatas vix ullam difficultatem continere, praeter illam quae consistit in proportionibus in aequalitates evolvendis' (AT. x. 441[21–24]).

Rule 17: 'Superiores quatuor regulae docuerunt, quomodo determinatae difficultates et perfecte intellectae a singulis subjectis abstrahendae sint, et eo reducendae, ut nihil aliud quaeratur postea, quam magnitudines quaedam cognoscendae, ex eo quod per hanc vel illam habitudinem referantur ad quasdam datas' (AT. x. 459[10–15]).

subjects of investigation and hence the general or universal science of mathematics is that of relations or proportions as such, abstracted from the various proportional terms.[1] The reader who has turned to the page of the *Discours* where Descartes is explaining these views may notice one of those curious and characteristic twists of position which Descartes often makes when writing about his method. Sometimes he will, almost brusquely, turn from an application of the method, an 'exercise', as he would call it, to a reflection on the methodological principles themselves. This twist is, quite naturally, more common in the scientific essays attached to the *Discours* itself.[2] Sometimes it is the reverse procedure, as here, and the reflection on the practice of the method often conceals an important doctrinal point. Having told us that he decided to study relations or proportions in general without 'supposing' them to be embodied except in those objects which would 'facilitate understanding of them' and enable him to achieve knowledge of proportions in general, he goes on to say that it would be necessary sometimes to study them singly and separately apart from the rest and sometimes to study many of them at once, comprehending them, for this purpose, merely in the memory. All this, if somewhat succinct, is for the reader of the *Discours* (who has already the four precepts of method explained to him) reasonably straightforward. To study 'each in particular', singly and separately, is to submit them to the scrutiny of intellectual intuition. To study 'many of them', is to consider the long chains of deduction which may involve the use of memory and the application of the rule of enumeration, in order to reduce the complex series to a serial intuition.

But, Descartes adds, in order to examine each proportion separately, 'I thought it would be better to suppose them embodied in lines, for I could find nothing simpler, or better adapted to represent the proportions distinctly to my senses and to my imagination. But, in order to keep them in my memory or to survey many together, I thought it best to express them by certain symbols.'[3] He then concludes that

[1] *Discours*, Part II (AT. vi. 20^{1-10}).

[2] See, for instance, the *Géométrie*, III (AT. vi. 475^{11-21}).

[3] 'Je pensai que, pour les considérer mieux en particulier, je les devais supposer

'by adopting this plan, it seemed to me that I should be borrowing all that was best in the analysis of the geometers and in algebra, whilst correcting the defects of each by the help of the other'.

The general methodological import of this passage is clear. The primary object remains the same: to reduce all problems to a general science of relations and proportions and, in order to facilitate this reductive analysis, to adopt a general system of symbolism or notation which can be based either on spatial relations or on numbers, or rather which, in its full development in *Mathesis Universalis*, can be based on numerical relations and apply to figures. The view implies that, after the process of abstraction and generalization has been completed, mathematical relations can be posited as indifferently spatial or numerical, 'so that space can have no adverse effect on the accuracy of our mathematical knowledge'.[1] But there is a further point of doctrine in the text quoted from the *Discours* which is, on the surface at least, somewhat at variance with the views which we have been accepting up to the present. The text speaks of representing 'the proportions to my senses and my imagination', which must be understood to mean that, at least in its spatial aspects, the method recommends the use of pictorial or imaginative representation when studying problems in detail. Yet, it might well be objected, does not this run counter to the view already explained that imagination is a hindrance to true knowledge, and that we must concentrate on training the purely spiritual activity of the *vis cognoscens*?[2] Is this not a complete reversal of the fundamental identity of the pure activity of mind and the method itself? What of Descartes's own criticism of the Logistae and Geometrae whom he accused in Rule 14 of confusion of thought, the first because they tended to regard

en des lignes, à cause que je ne trouvais rien de plus simple, ni que je pusse plus distinctement représentes à mon imagination et à mes sens; mais que, pour les retenir, ou les comprendre plusieurs ensemble, il fallait que je les expliquasse par quelques chiffres' (AT. vi. 20^{14-19}). The Latin text has 'si tantum illas in lineis rectis supponerem' (ibid. 551) and 'characteribus sive notis quibusdam' (ibid.). On the use of symbols, see Rule 16 and above, p. 177. It is clear from Rule 14 (AT. x. 445^{10-11}) that '*chiffres*' or '*character*' means letters: 'utemur characteribus *a*, *b*, *c*, etc., ad magnitudines . . . exprimendas'.

[1] Cf. Boyce Gibson, *The Philosophy of Descartes*, pp. 188–90.

[2] See above, pp. 14–17 and 25–30.

their numbers as really distinguishable from every subject 'by imagination', that is, as having some sort of picturable substance, the second because having conceived line as length without breadth, they proceed by imagination to generate plane from line, &c.;[1] a similar criticism to that of the *Discours*, where the analysis of the ancient geometers is shown to be so 'restricted' to the consideration of figures that 'it cannot use the understanding without overtiring the imagination'.[2] It seems prima facie odd to be recommended to use the imagination 'to represent proportions' at the highest point of the analytical regression. Yet the doctrine of the *Discours* clearly means this and a fuller and more elaborate recommendation in the *Regulae* leaves no doubt as to the meaning to be attached to the words.

Rule 13 is concerned with the necessity, once a problem has been perfectly understood, of removing all superfluous terms, to reduce the problem to its simplest terms, and through an enumeration, that is, preliminary survey, to divide it up into the minutest parts'.[3] Rule 14 is formulated in these terms: 'The same rule [of analysis into the simplest] is also to be applied to the real extension of bodies. The analysis must be set before the imagination by means of simple figures, for thus is it apprehended more distinctly by the understanding.'[4] Descartes begins the exposition to this latter rule by pointing out that when one unknown fact is deduced from another known fact, that does not mean that we discover 'a new kind of being'. We cannot by pure reasoning, by the process of pure analysis, discover a new kind of being, for, in brief, this would demand that we had 'some new sense' or else 'a divine intellect'. If, from what is already

[1] See above, pp. 192 and 202.

[2] *Discours*, Part II (AT. vi. 17^{27-18^4}), and Rule 4 (AT. x. 375^{18-19}): 'et magis ad oculos et imaginationem pertinent quam ad intellectum'. The reader of the *Discours* may have noticed, however, that the author, speaking of himself, says: 'J'ai souvent souhaité d'avoir la pensée aussi prompte, ou l'imagination aussi nette et distincte . . .' (AT. vi. 2^{22-23}), which obviously must be different from the 'imagination' as a corporeal organ, described in Rule 12 (AT. x. 414^{16-24}).

[3] Rule 13 is formulated thus: 'Si quaestionem perfecte intelligamus, illa est ab omni superfluo conceptu abstrahendo, ad simplicissimam revocanda et in quam minimas partes cum enumeratione dividenda' (AT. x. 430^{6-10}).

[4] Rule 14: 'Eadem est ad extensionem realem corporis transferenda, et tota per nudas figuras imaginationi proponenda: ita enim longe distinctius ab intellectu percipietur' (AT. x. 438^{9-11}).

known, we deduce an unknown, the unknown in question can never be a totally distinct 'nature', or character of the real. All that this deductive knowledge amounts to is the expansion of our existing knowledge so that we apprehend that 'the *quaesitum* participates, in one way or other, in the nature of the data given in the proposition'.[1] Thus, Descartes argues, it would be impossible to make a man, blind from birth, perceive the true nature of the colours by mere force of reasoning.[2] We derive our ideas of colours from sense. But a man who had seen the primary colours might perhaps construct for himself 'by a kind of deduction', based on similarity, the complex images of the secondary colours which are blends of the primary.[3] In the same way, he says, if the magnet contained 'a new kind of being' utterly unlike anything our intellect had hitherto perceived, we could never, by a process of reasoning, attain to knowledge of this unknown nature. The immediate corollary of this doctrine is obvious: when we talk of 'new knowledge', it means, in terms of the Cartesian doctrine, some piece of knowledge which is 'new' because it has not been hitherto conceived, and that is the sense to be always attached to the words 'concluding from the known to the unknown' which is the aim of all demonstration.

Descartes, then, is arguing that we cannot by reasoning discover a new kind of being. If our mind is to deduce, it must start from a 'nature' which is already familiar to us, which is known to us as an object presented to us by the senses or represented to us by the imagination. Our reasoning moves within and develops the sensuous or imaginative basis which is given to us. Our mind must start, at least in part, from knowledge which depends upon our senses or our imagination, and throughout its reasoning it is developing and expanding the knowledge from which it starts. Our

[1] 'Ut percipiamus rem quaesitam participare hoc vel illo modo naturam eorum quae in propositione data sunt' (ibid. 438^{16-18}).

[2] Cf. ibid. 438^{18}–439^{1}. The MS. H has in the margin: 'Non absolute verum est hoc exemplum, sed melius non habui ad explicandum id quod verum.' The same example is used by Locke in the *Essay*, ii. 4 (Pringle-Patterson, abridgement, p. 60).

[3] 'Ex aliorum similitudine per deductionem quamdam' (AT. x. 438^{24}–439^{1}). By an argument, therefore, 'per analogiam' or 'imitationem'. See above, pp. 141–3.

knowledge, then, at the end of the analytic and synthetic process is, in the instance of the magnet, 'to perceive with perfect distinctness that combination of already-known beings or natures which would produce the same effects as now appear in the magnet'.[1] It will be remembered that the definition of the *maxime absolutum*, the most absolute term in the deductive series, was 'that term which contains the nature under investigation in its pure and simple form'.[2] All the terms in the implicatory sequence participate in this 'nature'. The logical necessity of the inferential movement from the relative to the absolute term, or vice versa, depends basically on the fact that all these terms have a common nature. The problem is reduced by analysis until the *maxime absolutum* is revealed and its simple nature grasped by intellectual intuition. Synthesis then proceeds to reconstruct the deduction by determining how the consequents are a certain composition or proportion of the common nature or natures. The new knowledge arises in that the relation, or proportion, implicated in the consequent offers a genuine new element of knowledge as compared to the less complex absolute term from which it was in the first place inferred.

But what Descartes is insisting upon is that a sensuous, or at least an imaginative, 'knowledge' is an essential prerequisite of all reasoning; that prior to all deductive reasoning there must be present to the mind knowledge of certain data, or, as he calls them, 'certain kinds of being' or certain 'natures'. It has been evident that the primary rule of all method is that the mind must begin its deduction from an immediately apprehended self-evident datum. In Rule 14, however, instead of the rationalist interpretation which one was justified in attaching to the meaning of this precept, it seems now to be argued, in terms very like those of an empiricist such as Locke, that the starting-point of the deduction is not a 'purely intelligible' datum, and, correspondingly, a pure intellectual act of intuition, but a 'nature' which is 'known' as an object presented to sense or represented as an imaginative idea, and, correspondingly, perceived by an act of 'mixed' knowing,

[1] 'Illam jam notorum entium sive naturarum mixturam, quae eosdem qui in magnete apparent, effectus producat, distinctissime percipiamus' (AT. x. 439$^{7-10}$). See below, p. 247.

[2] See above, pp. 163–7.

that is, not a pure act of the *vis cognoscens*, but an act, partially at least, dependent on the mind as linked to the body and its organs. The reader of the *Discours*, as well as its commentators, may well be excused from not noticing the somewhat striking qualification of doctrine that this view implies—it is mentioned in a few words and the sixth part of the *Discours* assumes it to have been understood.[1] The third paragraph of this last part, which is considered as the *locus classicus* of the Cartesian distinction between general and special physics, turns entirely on this doctrinal point which is taken as known. The reader of the *Regulae* is left in no doubt as to the full import of the meaning in the exposition of Rule 14.

§ 2. '*Entia jam nota*'

All 'already known beings', or natures—'extension, figure, motion, and the like', are given as examples—are recognized by means of the same idea (*per eamdem ideam*) in all the diverse subjects or matters in which they are embodied. We imagine, or picture to ourselves, the shape of a crown, that is, we have the same pictorial image of the shape, whether the crown be silver or gold. 'And there is only one way by which this common idea is transferred from one subject to another, namely, by simple comparison in virtue of which we affirm that the *quaesitum* is, in this or that respect, like, identical with, or equal to, something given.'[2] Apart, therefore, from the intellectual intuition of the single, self-evident solitary nature, all deductive knowledge, in the strictly intellectual sense of the term, is obtained by comparison of two or more things. For instance, says Descartes, in the deductive piece of reasoning: All *A* is *B*, all *B* is *C*, therefore all *A* is *C*—we compare together a *quaesitum* and a datum, namely *A* and *C*, in respect to the fact that each of them is *B*. But, he notes, the comparison must fulfil one condition: it must be 'simple and open' (*simplex et aperta*) if we are to attain truth, and this condition is fulfilled when the things compared share

[1] *Discours*, Part VI (AT. vi. 63^{30}–65^{17}).

[2] 'Atque haec idea communis non aliter transfertur ex uno subjecto ad aliud, quam per simplicem comparationem, per quam affirmamus quaesitum esse secundum hoc vel illud simile, vel idem, vel aequale cuidam dato' (ibid. 439^{15-19}).

a certain nature.[1] That means that we can only arrive at truth by the inferential process if we so arrange the various terms of the *quaestio*, *per enumerationem simplicem*, and then develop the series by analysis so that the *quaesitum* is shown to be equal to, to be put in an equation with, the datum in respect to some nature common to both. The importance of the preliminary survey of the *quaestio* is clear. The whole point of the *perquisitio* demanded by Rule 7 was to cover cases when even in linear inference from a simple datum we might discover that the deductive process was checked, or held up, not only by our inability to grasp the connexions between the various steps but because of the nature of the datum. In a deductive series the nature is contained precisely and perfectly in the absolute term, but it is only contained imperfectly and confusedly in the relative term. Hence when a problem is being investigated in the preliminary survey, which should lead to the analytic division, the data may not be sufficiently clear to allow them to be equated by 'a simple act of comparison'. In terms of *Mathesis Universalis*, we are not yet in a position to set up our 'equations'. The *quaesitum* and the *datum* do not prima facie exhibit a common nature—the common nature, Descartes says, 'does not exist equally in both'. There is not, at first inspection, a known and an unknown which can be equated in respect of a common character. *A* and *C* do not manifest an equality with *B* on simple inspection. They contain the common nature unequally 'involved in certain relations or proportions'.[2] The problem is imperfect.

In such cases we must attempt 'to reduce these proportions in such a way that equality between the *quaesitum* and something else which is known already may be clearly exhibited'.[3] For the perfect problem, as Descartes points out later in Rule 14, involves practically only one kind of difficulty, namely, the difficulty of developing proportions so that 'the equalities, which they enclose and implicitly contain, may be

[1] Cf. AT. x. 440[10–12].

[2] 'Quia natura illa communis non aequaliter est in utraque, sed secundum alias quasdam habitudines sive proportiones in quibus involvitur' (ibid. 440[14–16]). The text should surely read 'utroque', in the *quaesitum* and *datum*.

[3] 'In proportionibus istis eo reducendis, ut aequalitas inter quaesitum et aliquid quod sit cognitum, clare videatur' (ibid. 440[18–20]).

disengaged'.[1] The essential task, in the words of the *Géométrie*, is to set all, *quaesita* and data, in the form of equations. The somewhat obscure argument of the *Regulae* seems to proceed by the following steps. First, nothing can be reduced to equality except 'that which admits of more and less',[2] that is, magnitude. Consequently, when the *quaestio* has been disentangled from any reference to a particular subject-matter, we shall be left with 'magnitudes in general'. Magnitudes in general must be the primary object in our attempt to set up the problem in equational form, that is, entities which exhibit the possibility of being susceptible of 'equality', and which can therefore be the proper objects of *Mathesis Universalis*. But, secondly, we must 'make use of our imagination, employing not only our pure understanding but also the understanding as helped by the ideas depicted in the imagination'.[3] The rest of Rule 14 is taken up by a discussion of the aid which the imagination can give in the study of mathematics, with long examples and a detailed analysis of the meanings of 'extension', 'body', 'dimension', and kindred *entia jam nota*.

It may be noted immediately that there is an important and vital difference between the use of the imagination and senses as supplementary aids to the mathematical sciences and their use as reliable guides to knowledge in other matters. Descartes is not retracting any of the criticisms of the senses and imagination as sources of true knowledge which he made in the *Regulae* and in the *Discours*. It is clear from the exposition to Rule 14 and the following rules that the role of the imagination as an aid to mathematical thinking is a subordinate one, in which the imagination plays a passive part, merely embodying at the dictate of the understanding what sensuous or imaginative 'shapes' the latter may decide to construct. The guiding power, the initiative, lies in every instance with the understanding which projects its own conceptions as sensual constructions. In the case of ordinary matters, the role of the senses and imagination is reversed:

[1] Cf. ibid. x. 441[21–24]. See above, p. 216, n. 2.

[2] 'Nihil ad istam aequalitatem reduci posse, nisi quod recipit majus et minus' (AT. x. 440[21–22]).

[3] 'Ut vero aliquid tunc imaginemur, nec intellectu puro utamur, sed speciebus in phantasia depictis adjuto' (ibid. 440[28]–441[1]).

the senses or the imagination have the initiative: it is they which supply the stimulus and matter for reasoning and, as such, having no rational insight to guide them, they may be the causes of error and confused thinking. The argument in the *Regulae* assumes, on the basis of the discussion in Rule 12, that we can have sensuous or imaginative 'knowledge' of certain fundamental characters of the physical world as we perceive it. The magnitudes in general, which as objects of *Mathesis Universalis* are studied by the *vis cognoscens* and form the elements of the equations, are abstracted and generalized from 'particular species' of magnitude which can be easily depicted in the imagination. But when we have, by analysis, reduced the problem to its simplest form, that is, when we have reduced it to its most perfect formulation in terms of magnitudes in general (by abstracting from the particularity of the various magnitudes), then we should apply the instruction of Rule 14, and transfer the problem to 'the real extension of bodies' and 'by means of simple figures set it before the imagination'. The point being, it would appear, that the problem thus exhibited pictorially to the mind will 'be apprehended more distinctly'.[1]

We are therefore recommended, as it were, to reverse the process which we had followed in attempting to set up the 'equalities' required between *quaesitum* and datum. Having obtained our magnitudes in general, or our proportions, we are then to embody them—or rather reclothe them, since Descartes speaks of the imaginative aids as 'nudas figuras'—in a particular subject-matter, so as to make them objects of imagination. This, Rule 14 tells us, will help the mind to apprehend them 'more distinctly'. And this equally is the doctrine of the *Discours*. Surely, the reader may object, in asking us to reverse the process, Descartes is eating all his own words in the earlier rules of the *Regulae*. One of the major points of the very first rule was that all our endeavour should be to increase the pure 'light of natural reason' as a 'purely spiritual power'. Descartes would reply that there is nothing radically inconsistent between what he said then and what he is advocating now. He could point out that when, in Rule 12, he distinguished four faculties, sense, imagina-

[1] Cf. above, p. 218, n. 4.

tion, memory, and intellect, he had insisted that while intellect alone is 'capable of perceiving truth', the other three have a subsidiary role to play in our knowing.[1] And this is just an instance of their possible help which lies *in nostra industria*. Further in the same Rule he made the point that often what we intuit or intellectually perceive is something material or corporeal, such as relations or proportions between, or properties of, bodies and that, in these cases, it will help our intellect to perceive if we imagine the bodies and their relations or properties; or, again, if we observe a sensible instance, that is, it will help us if our intellect directs itself to, or applies itself immediately upon, the ideas or 'shapes' formed in the organ of sense.[2] This does not imply that we confuse the intellectual act of vision with sensation or imagination, nor the certainty of intellectual intuition with the assurances derived from mere sensation or with a judgement based on an imaginative ground alone. He could recall that he made it perfectly clear that while the imagination could work on the intellect, the intellect could also work on the imagination;[3] that while it is true that ideas may be compounded and compared in imagination, which may lead to falsity, since the composition is arbitrary and irrational (or may equally well be so as not), it is equally true that a composition of ideas may be dictated by the intellect.[4] He had remarked that certain common objects of intellectual intuition could be known either by the inspection of the pure intellect operating alone or by the inspection of the intellect in so far as it was 'seeing' the images of material things.[5] Finally, it had been made clear that the *vis cognoscens* is 'one and the same power, which, when together with imagination it applies itself to common sense, is said to see, to touch, &c.; when it applies itself to the imagination alone, in so far as it possesses diverse impressions, is said to remember; if it turns

[1] 'Solus intellectus . . . qui tamen juvandus est ab imaginatione, sensu, et memoria, ne quid forte, quod in nostra industria positum sit, omittamus' (AT. x. 411^{7-10}). See above, pp. 25–26.

[2] Cf. Rule 12 (AT. x. 415 ff.).

[3] 'Nam cum intellectus moveri possit ab imaginatione, vel contra agere in illam' (AT. x. 416^{17-18}).

[4] Cf. Rule 12 (AT. x. 421–4): 'hanc compositionem tribus modis fieri posse: nempe per impulsum, per conjecturam, vel per deductionem' (424^{1-3}), and the 'male componentis imaginationis judicium fallax' of Rule 3 (AT. x. 368^{15-16}). See above, pp. 53–54.

[5] Cf. AT. x. 419^{29}–420^{2}.

to imagination in order to create new impressions, is said to imagine or conceive; and finally, if it acts alone, is said to understand'.[1] Provided, therefore, it is borne in mind that the initiative lies with the intellect, and that the role of imagination is merely to ease the work of the intellect, principally in the preliminary stages of the construction of the problem, there is nothing inconsistent in the recommendation of Rule 14. The danger lies in arguing from imaginative data, which was the major defect of the 'ancients' in their geometry. Here we are merely using them as subsidiary, practical aids. How this in fact works out is not especially clear in the *Regulae*, and it does not appear very profitable to enter into the detail of the tortuous account of extension, body, &c.

The general point would appear to be this. The object of the science of *Mathesis Universalis* is to study the relations or proportions of magnitudes in general, so as finally to be able to express their order and measurement in the form of equations. The intellect forms its conceptions of these magnitudes in general by a preliminary abstraction from perceived or pictured magnitudes. This is the process of analysis by which we first strip our imperfect problems of all sensuous or imaginative matter and reduce them to perfect problems, by extracting the magnitudes in the imperfect problems from any 'subjects' in which they are embodied or any materials in which they are, so to speak, enwrapped. When this is completed, we can apparently do nothing with them, except reincarnate them in extension and the simplest figures, as straight lines, rectilinear figures, &c. We are clearly warned by Descartes, that throughout the abstract reasoning of *Mathesis Universalis*, the concrete imaginative background must be kept constantly in mind. Extension, in this practice of the method, is defined as something pictured as having length, breadth, and depth. That is to say, as well as the simple nature of extension as intelligible quantity, we are asked to keep in mind constantly the imaginative conception of it. The concretization of the nature of pure extension need not be pushed further; it is not necessary to determine the image by asking whether it is a body, possessed of physical proper-

[1] Cf. AT. x. 415^{27}–416^{4} and above, pp. 26 and 29.

ties other than the spatial ones, or whether it is empty space, that is, the cubic contents of a body. The requirement of Rule 14 is met by the spatial embodiment of magnitudes. The abstracted magnitudes must always be interpreted by reference to the imaginative whole out of which they have been abstracted by analysis. Accordingly, in *Mathesis Universalis*, 'figure' means the imagined solid *qua* solid having shape; 'plane' means the same imagined solid *qua* solid having length and breadth (depth, Descartes points out, being omitted not by negation but by abstraction); 'line' means the same imagined solid *qua* solid having length (breadth being abstracted and not negated); and 'point' means the imagined solid conceived in abstraction from all its characters except that 'of being something existent'.[1] The same process applies in arithmetic, when we have to deal with 'number'. We must then imagine a subject measurable by a multiplicity of units. We 'attend only to the abstracted units' but we must not draw, from our reasoning about the abstracted units, conclusions which are incompatible with the numbered something from which they have been abstracted.[2]

It is not easy to fit this doctrine in with the theory of the *vis cognoscens* as elaborated earlier. We must take it that it is 'one and the same power' which together with the imagination receives the 'shapes' from the common organ of sense, and which, Descartes declares, 'sees, touches, &c.'. So that, given the paucity of information in the *Regulae*, we must assume that the *vis cognoscens* apprehends the pictured image both in the common organ of sense and in the imagination. The imagination's role, as a subsidiary aid to the purely intellectual function of the *vis cognoscens*, is the visualization of spatial proportions, shapes, and figures, generalized images which have been apprehended in the past by the senses as particular proportions, shapes, and figures. The account given in the *Regulae* is too meagre, too full of unanswered questions which need an answer, too suspiciously superficial to allow the commentator to suggest that the role of the

[1] Cf. AT. x. 446^{3-10}. See the criticism of the other definitions by *logistae* and geometers, above, p. 192. They use the generalized images as the basis of their deduction, not as 'aids'.

[2] Cf. AT. x. 445^{23-28}: '. . . in quo res numerata a nostro conceptu exclusa fuisse supponatur'.

imagination in the mathematical sciences is fully plausible, still less coherent with the rest of the doctrine. It would be necessary to enter further into the doctrine of perception and to follow, for instance, the famous analysis of the 'piece of wax' in the *Second Meditation*, which lies outside the study of the method itself.

It is clear, however, that in the 'knowledge' of bodies or material things—knowledge being understood in the strict sense of the term—what is known is the idea of extension, the simple nature, in the terminology of the *Regulae*, which is abstracted, as in the instance of the piece of wax, from all its sensible qualities, its shape, &c., and that this is 'the object of the Geometers' in the *Discours*.[1] As such, as the simple nature, which is *maxime absolutum* in the corporeal world, and, one might say, the simplest datum in the implicatory series of 'body', opposed to 'mind', extension can only be the object of intellectual intuition. *Qua* object of the intellect, it is the idea of a continuous magnitude, devoid of and abstracted from any determinate dimensions, dimension being, in the words of Rule 14, merely the mode of measurability.[2] As such it is thought of as undivided yet divisible, undetermined yet determinable. But it is part of this simple nature 'to be divisible', 'to be determinable', and as 'divided' and 'determined', it can be imagined and sensed. The relation, or proportion, between the two notions, that of intelligible extension and that of imagined (or sensible) extension, is the relation of the indeterminate to the determined. But if this simple nature of extension were not determined, not made more concrete by modes of measurement, lines, planes, figures, points, &c., there could be strictly speaking no mathematical sciences at all. This must be the sense in which the words of Rule 12, that 'figure, extension, and movement are only known in corporeal things',[3] are to be understood

[1] 'L'objet des géomètres' (AT. vi. 36^{5}). The notion of pure extension, which is identical with that of matter, is described in *Le Monde*, V, as 'rien de plus simple, ni de plus facile à connaître dans les créatures inanimées; et son idée est tellement comprise en toutes celles que notre imagination peut former qu'il faut nécessairement que vous la conceviez, ou que vous n'imaginiez jamais aucune chose' (AT. xi. 35^{12-17}).

[2] 'Per dimensionem, nihil aliud intelligimus, quam modum et rationem, secundum quam aliquod subjectum consideratur esse mensurabile' (AT. x. 447^{23-25}).

[3] Cf. AT. x. 419^{18-19}. Descartes is here speaking generally of the simple natures, and contrasting 'pure intellectuales' with 'pure materiales', i.e. those mentioned

and linked to the other injunction that, when we are dealing with matters relating to corporeal objects, 'it is in the imagination, as far as possible, that we should form the idea of it'.[1] When we recall that the imagination is often said by Descartes to be a pernicious influence upon the philosopher, and even on the physicist, it is right to take this advice as formulated first and foremost for the mathematician. Mathematics deals with certain elements only of the modes of extension, and it needs the aid of imagination to set out first the relations and proportions of these modes. Precisely because the mathematical sciences work at a high level of abstraction, the mind tends to tire, to let its attention wander from the *quaesitum*. The imagination can help to relieve this—as can the senses—by the drawing of figures which we do not reason *about* but merely consider as auxiliary to our reasoning on the abstracted properties of, for example, the triangle. The reduction of geometry to analytical geometry, the use of algebraical symbols, the whole process of simplification into equations are merely imaginative aids to the final objective, the apprehension of the proportions and relations of the object of geometry, extension; but this apprehension is itself a pure act of the *vis cognoscens*, an act of intellectual intuition. *Mathesis Universalis*, the method itself, must be understood in a manner which allows for complete abstraction from all that may be a particular technical form ascribed to it when dealing with the objects of the mathematical sciences proper. 'The conception of the method furnished by an examination of mathematical technique is independent of the technique itself.'[2]

here. But 'motion' or 'movement' is not in the same line of argument as 'extension' or 'figure'. See Descartes's distinction in the *Géométrie* between 'lignes géométriques' and 'lignes mécaniques' and the question of the physical sciences in the next chapter which touches on the method as treating 'movement'. See above, p. 147.

[1] Cf. the whole correspondence with Morus for a discussion of this point in vol. v of AT. My reference is to the reply of Descartes to the first letter, 5 Feb. 1649. 'Illud solum quod est imaginabile, ut habens partes extra partes, quae sint determinatae magnitudinis et figurae, dico esse extensum' (AT. v. 270[28–30]). And the letter to Princess Elizabeth, where he explains that 'l'extension, les figures et les mouvements, se peuvent aussi connaître par l'entendement seul, mais beaucoup mieux par l'entendement aidé de l'imagination' (AT. iii. 691[23–26]).

[2] A. Rivaud, 'Réflexions sur la méthode cartésienne' (*Rev. méta. et morale*, 'Études sur Descartes', 1937), p. 55.

XVI

THE METHOD IN PHYSICS

§ 1. *Mathematics and Physics*

A STUDENT in philosophy who was forced, shall we say by the stringent time-limits of examinations, to give some general account of Descartes's philosophy and based his essay on the description of this philosophy as a 'pan-mathématisme', would not—and quite rightly—be considered wide of the mark. The very first Rule, in the *Regulae*, might plausibly be held to justify such a contention. It was argued there that all the sciences are so intimately connected and interrelated 'that it is far easier to acquire them all together than to pursue the study of one of them in isolation from the others' and that the sciences are nothing but 'universal wisdom', 'the illumination' thrown on the various and diverse fields of the knowable by a single spiritual light, the *vis cognoscens*. There was to be, consequently, a single kind of knowledge, an identical order of abstraction, one level of intelligibility, and a single method. Contrary to the doctrine of Plato which distinguished between *διάνοια* and *ἐπιστήμη*; contrary to the Aristotelian, and Scholastic, doctrine of a hierarchy of mathematical, physical, and metaphysical degrees of abstraction and knowledge, the Cartesian ideal might be said to extend the mathematical type of intellection to the whole realm of knowledge. Might not the reproach made by Aristotle against the Platonists of his day be even more cogently directed against the Cartesian ideal: *γέγονε τὰ μαθήματα τοῖς νῦν ἡ φιλοσοφία*·[1] It is not without reason, it might be pointed out, that Descartes noted in the first Rule: 'We have not put this rule first of all without reason',[2] for the idea of this unity of the body of the sciences is logically inseparable from the extension of the mathematical method to the whole domain of knowledge. There are frequent ex-

[1] Cf. *Metaphysics*, *A*, 992ª 32–33.

[2] 'Proinde non immerito hanc regulam primam omnium proponimus' (AT. x. 360$^{22-23}$).

pressions of opinion in the letters of Descartes which would tend to confirm a pan-mathematical interpretation of his method and its universal application, unchanged and invariable at all levels of thinking. He is writing to Mersenne in 1640, and is seriously worried that the violent criticisms of his doctrines by P. Bourdin may result in the entire Jesuit Order becoming open adversaries of Cartesianism. 'I hope', he writes, 'that these religious fathers will not disdain to examine my views on the pretext that they are demonstrated by mathematical reasoning, and that because they [the Jesuits] make a distinction between mathematics and philosophy, their business is to teach the latter rather than the former; for I have treated many subjects which belong purely to philosophy, amongst others all problems relating to Meteors, and nothing, in my opinion, is more desirable than to have mathematical demonstrations in philosophical matters.'[1] Leaving aside the view, strange to modern opinion, that the *Météores* is a philosophical treatise—the Jesuits would have called it cosmology, a philosophical treatise—the desirability of applying the mathematical method to philosophy could not be more clearly expressed. One is tempted to apply the words used of 'Nature' when faced with 'the blow of a hammer', to the whole Cartesian philosophy; Nature, says Descartes, *agit en tout Mathématiquement*.[2]

Yet Descartes also realized that the facility, clearness, and distinctness of mathematical demonstration could not be obtained in the physical and philosophical branches of knowledge to the same degree as was possible in pure mathematics. There must, he admits, be some modification of the method when the subject-matter was changed. The commentator who hopefully wishes to determine and define the detail of this modification has little positive doctrine to build his comment upon, yet it is important to note this modification in order to have a complete picture of Cartesian methodology. Let us take first the method in relation to the physical sciences.

[1] Cf. AT. iii. 173^{10-17}. Descartes considers that 'on voit bien plus de gens capables d'introduire dans les mathématiques les conjectures des philosophes, que de ceux qui peuvent introduire la certitude et l'évidence des démonstrations mathématiques dans les matières de philosophie' (Letter to Huygens, 1635, AT. i. 331^{21}–332^{1}).

[2] Cf. AT. iii. 37^{6}.

It is significant, in a minor degree, that, after the publication of the *Géométrie*, Descartes's own personal interest in purely mathematical problems almost disappeared as compared with his earlier curiosity in this field of knowledge. There is a strange passage in a letter to Mersenne, where, as early as 1638, we find Descartes saying:

> I am grateful to M. des Argues for the attention which he pleases to pay me and his remark that he regrets that I do not desire to study Geometry any longer. But I have only decided to give up abstract Geometry, that is, the investigation of problems which merely exercise the mind; I am doing this purposely in order to have more time to study another kind of Geometry which concerns itself with problems about the phenomena of nature. If he would only consider what I have written about salt, snow, the rainbow, &c. it would soon be seen that my Physics are nothing else but Geometry.[1]

It is clear what Descartes means, namely, that he is applying his mathematical method in the physical sciences and that 'geometry' as opposed to 'abstract geometry' means the application of *Mathesis Universalis* to the phenomena of nature. Can this be done within the limits of the description of this new science as understood from a reading of the *Regulae* and *Discours*?

There is an immediate difference which strikes one between the method in the mathematical and in the physical sciences. Whereas the imagination was affirmed, in the *Regulae* and *Discours* as elsewhere, to be an essential subsidiary element of aid in mathematical reasoning, in the physical sciences its influence must almost always be considered as dangerous and possibly pernicious. In his conversation with the young Burman, relative to certain points in the *Discours*, Descartes points out that 'certain people, who have a gift for mathematics, are nevertheless less happy when they come to study physics. This comes about, not because their reasoning is at fault, but because they have treated mathematical problems, not by reasoning (with algebraical symbols), but by imagining (with geometrical figures); they are only used to this latter method of imagining, and there

[1] AT. ii. 268³–¹⁴. Desargues is the famous architect-geometer of Lyons and a regular correspondent of Descartes from 1638 onwards.

is no place for this in the physical sciences.'[1] Presumably the reason for this interdiction lies in the fact that the relations and proportions, which are the *quaesita* of the physical sciences as such, have no direct relation to the sensuous appearances of the phenomena of nature, i.e. that the secondary qualities as such are no help at all to enable us to 'know' the real nature of the physical world and in fact that, given the propensity of the human mind to accept the 'prejudices' of sense-perception, the sensible qualities, if used analogically to figures in geometry, might serve rather as decoys than aids of reasoning. No functions are therefore to be ascribed directly to the imagination in the application of the mathematical method to the realm of the physical sciences.[2]

Yet the method is to be applied to this realm, as to all realms of knowledge. And in so far as knowledge is obtainable in the realm of physical sciences, the only means available must, since there is no other alternative, be through intellectual intuition and deduction. In one of the letters to Mersenne quoted above, Descartes notes: 'As for Physics, I should consider that I knew nothing about them, were I only able to explain how things might be and were unable to demonstrate that they could not be otherwise; for having reduced it [Physics] to the laws of mathematics, the demonstration is now possible and I think that I can do it within the small compass of my knowledge.'[3] It is clear also from the sixth part of the *Discours* that Physics *qua* science is based on intellectual intuition and deduction. Speaking of the 'principles'

[1] *Entretien avec Burman* (AT. v. 177). This is not very consistent with what he says in the same discussion, apropos of art. 66 of the *Principia*, where he recommends the use of eight 'little balls', 'quibus motus ille demonstretur' and adds 'quamvis auctor ingenium suum imaginationi adeo assuefecisset, vix tamen ea sine globulis concipere poterat' (AT. v. 173). Lord Kelvin would have approved.

[2] The physical sciences are, in one sense, part of Philosophy for Descartes, that is, they are 'Natural Philosophy' or what Hegel would call *Naturphilosophie*. I am not treating here of Descartes's physical theories or their relation to his metaphysics and can therefore ignore, in this study, the controversy on their logical dependence. Cf. a summary of these views in Keeling, *Descartes*, pp. 58–61. I am also, from the restriction of my subject, passing over all discussion of 'the assumptions of fact, ontological and epistemological', which are presupposed by Descartes in his method. 'The onus of demonstration' falls on metaphysics, that is, on the *Meditationes*, and the corresponding parts of the *Discours* which are to be discussed in another study.

[3] Letter to Mersenne, 1640 (AT. iii. 39[1–6]).

of his physics, he declares that they are 'nearly all so evident that it is only necessary to understand them in order to assent to them and although there is not one of them which I do not consider myself able to give a demonstration of', yet to avoid time-wasting controversy, he will not publish them.[1]

It is advisable to let Descartes put the difficulties of the problem as far as possible in his own words, for the relations of the method to the physical sciences is one of many points of Cartesian exegesis where the commentator may listen respectfully to the whisper of the nameless ghost in the Voyage to Laputa. Certain points seem to be made quite emphatically enough to justify their acceptance as true expressions of Descartes's views on the matter.

It may be said, somewhat obviously but vitally, that there is a clear distinction between the applications of the method in the mathematical and physical sciences. The distinction was presupposed in the discussion of the *linea anaclastica* in Rule 8.[2] It will be recalled that it was supposed that 'a pure mathematician' set out to determine the line of refraction and by the method of analysis and synthesis (i.e. the application of Rules 5 and 6) arrived at the conclusion that the determination of the line depended upon the proportion, or ratio, which is observed by each and all the parallel lines between their respective angles of incidence and angles of reflection. But the pure mathematician was unable to discover the angle 'because it was a matter not of mathematics but of physics, and he is forced to stop on the threshold. It will do him no good to try and get this information from the philosophers or to gather it from experience; for this would be against Rule 3.'[3] That is, he would be appealing to what others have said or else relying on something not clearly and distinctly apprehended by intellectual intuition or deduction. We are, in this passage, promised that later it will be shown 'that experience is unambiguous when dealing with the wholly simple and absolute',[4] but this promise was to have been kept

[1] AT. vi. 68^{4-7}: 'les fondements de ma physique . . .'. He is referring to the unpublished treatise, *Le Monde*.

[2] Cf. above, pp. 109–110.

[3] AT. x. 398^{7-10}.

[4] 'De rebus tantum pure simplicibus et absolutis experientiam certam haberi posse dicetur suo loco' (ibid. 394^{12-14}).

in the third, i.e. the unwritten, section of the *Regulae*. A physicist, however, can proceed: some experimental work is involved in the progression of the data: but how experience is involved can be left for the moment. What emerges from this instance is that certain types of problems, dealing, as in this case, with the phenomena of nature, are admitted to be insoluble by purely mathematical methods.

It might be suggested that since the *purely* mathematical method could not be applied when the data involved were phenomena of nature, we should abandon hope of obtaining the certainty requisite for true knowledge when we turn to the physical sciences. Some such view seems to be suggested by a passage in the *First Meditation*.

> We shall not have come to a wrong conclusion if we say that physics, astronomy, medicine, and all the other sciences which depend on the consideration of composite things, are very doubtful and uncertain; while arithmetic, geometry, and other sciences of this kind, which deal only with most simple and most general things, and are not greatly concerned whether they exist in nature or not, contain something which is certain and beyond doubt.[1]

Beyond doubt, except the metaphysical doubt of 'le malin génie', which lies outside the present discussion. It might be argued that, in this text of the *Meditationes*, Descartes is referring to physics, astronomy, &c., with reference to these sciences as understood and practised in his own day and not to what may be called, by contrast, Cartesian physics. But this interpretation will not hold in face of the evidence from the text itself. For just before the quoted passage, he is discussing the 'simple and general things' which are 'true and real', and amongst these things are 'corporeal nature in general' and 'its extension', 'figure', 'quantity, magnitude, number', &c., all of which are 'simple natures' and are consequently possible objects of certain knowledge. We are told, however, that the doubtfulness and uncertainty of the physical sciences arises because we are there dealing with 'composite things' and the composition must be inferred

[1] AT. vii. 20$^{20-27}$; the 'magis simplicia et universalia' in the paragraph above are 'natura corporea in genere', 'ejusque extensio', 'figura rerum extensarum', 'quantitas, sive earumdem magnitudo et numerus'.

to come from a mixture of the 'pure' elements with data derivative from sense-perception. That same doubt is expressed in an oft-quoted letter to Mersenne, written in 1638.

> You ask me [he writes] whether I think what I have written about refraction is a demonstration. I think it is, at least as far as it is possible, without having proved the principles of physics previously by metaphysics, to give any demonstration on this subject... as far as any other question of mechanics, optics, or astronomy, or any other question which is not purely geometrical or arithmetical, has ever been demonstrated. But to demand that I should give geometrical demonstrations of matters which depend on physics is to demand that I should do the impossible. If you restrict the use of 'demonstration' to geometrical proofs only, you will be obliged to say that Archimedes demonstrated nothing in mechanics, nor Vitellion in optics, nor Ptolemy in astronomy, &c., which is not commonly maintained. For, in such matters, one is satisfied that the writers, having presupposed certain things which are not obviously contradictory to experience, have besides argued consistently and without logical fallacy, even if their assumptions are not exactly true.[1]

How are we to reconcile this account of procedure with the view, only two years later expressed to the same correspondent, that such a demonstration is 'now possible', since 'Physics' has been 'reduced to the laws of mathematics'?[2]

Two points in the text of the *First Meditation* call for some comment. The mathematical sciences are there said to be indifferent as to whether their objects, the most simple and general things, 'exist in nature or not'. There is a particular sense given to these words in their context since the existence of God has not yet been proved, and hence the divine guarantee of the veracity of clear and distinct ideas has not been established. In one sense, then, the physical nature of the world lies under the shadow of metaphysical doubt and differs, to that extent, from certain mathematical truths as $2+3=5$ which seem to be true whether we are 'waking or sleeping'. But this partial superiority does not provide the complete answer. In the discussion with Burman, Descartes pointed out that the object of the mathematical sciences

[1] AT. ii. 141^{22}–142^{19}.

[2] See above, p. 223, n. 3.

existed only as 'a possibility' and this was sufficient ground on which to base the whole structure of mathematical deduction. The fundamental data were objects of intellectual intuition and constituted the system of interconnected simple natures derived from extension and its modes. But the physical sciences are essentially concerned with actual extension in space. Let it be granted, through the agency of a truthful God, that we are fully guaranteed against some satanic universal hoax; do the mathematical sciences and the physical sciences fade into one complete deductive system of mathematical reasoning? Descartes's answer would probably be that they do not in fact merge into one physico-mathematical science, but they tend to do so, as, and to the extent that, we amass more knowledge of the phenomena of nature, he would be dogmatic on the point that the ideal is certainly some such science and we can attain to true knowledge only in so far as we approximate to this ideal. This ideal would be fulfilled in a physics which 'was demonstrated in the same way as a geometry', as it was put to Mersenne, or when we were able to describe, in the words of the *Principia*, 'the universe and the whole visible world as if it were a machine, in which there was nothing to consider but the shapes and movements of its parts'.[1] The ideal, in other words, is a mechanistic explanation in all the physical sciences, and mechanism implies the possibility of the application of the mathematical method.

There is, however, a gap between the ideal and the demonstrations which are possible in the physical sciences. The method demands that the only proper objects of knowledge, in the strict sense of the term, should be simple natures and

[1] *Principia*, iv. 188 (AT. viii, Part I, 315^{16-18}): 'Quippe hactenus hanc Terram, totumque adeo hunc mundum aspectabilem, instar machinae descripsi, nihil praeter figuras et motus considerans.' Descartes goes on to explain that he may be thought to have left out 'multa alia', colours, odours, &c., but the following sections deal with their 'reduction'. But in no. 204 he qualifies this ambitious programme: 'Sufficere si de insensibilibus qualia esse possint, explicuerim, etsi forte non talia sint' (ibid. 328 in margin). The French text adds 'et que c'est tout ce qu'Aristote a tâché de faire' (AT. ix. 322). But there is a crescendo in the following sections: 'Ea tamen quae explicui, saltem moraliter certa' (205) and 'Immo plusquam moraliter' (206). P. Duhem, in *La Théorie physique* (Paris, 1914), p. 63, is, I think, right in suggesting that Descartes would have been aggrieved if we were to take his pragmatic concessions too seriously.

relations or proportions between these simple natures which are simple *respectus*. The rule of analysis formally insisted that the process of decomposition should be carried out down to the *maxime absolutum*, until we arrive at the clear and distinct idea. If we cannot, for any reason whatsoever, complete this analysis, then we should, according to Rule 8, give up the inquiry as beyond our human powers. The second point, in the text of the *First Meditation*, may well form the general type of difficulty which calls for an acceptance of the limitations of human knowledge. For, we are told, in the physical sciences we are dealing with 'composite things' and it is perhaps the case that in their 'composition' there is some element, some 'residue', empirical or sensuous, intractable by the mathematical method, spelling bankruptcy to the whole doctrine of *Mathesis Universalis*.[1] It is not the idea of extension itself, in so far as it *also* is the object of the physical sciences as of the mathematical sciences, which offers any obstacle, any unexplained or unexplainable residue. *Qua* simple nature, extension is perfectly clear and distinct. But the physical sciences deal with more than the pure idea of extension; they deal, amongst other things, with motion, that is, what Aristotle called *φορά*, local movement, the only kind of motion recognized by Descartes.[2]

According to the difficult doctrine of substance and modes —which cannot be discussed here within the limits set by a study of the method—motion is only 'a mode of the mobile thing' and not a substance. It has therefore less 'reality' than extension, and is dependent on extension, as relative to absolute term, that is, extension can be conceived without motion but not conversely. It follows, therefore, that whatever may be the logical relationship of extension and motion, the latter being a mode of the former, once it is asserted that extension is really distinct from motion, that is, as conceived by the geometer, the connexion of motion with the extended thing is not one of logical necessity; motion is not deducible from the very nature of extension. It is then necessary to link

[1] I borrow this use of the word 'residue' from G. R. G. Mure's *The Logic of Hegel*.

[2] Cf. the *Physics*, *Δ*, 208a31–32: '*τῆς κινήσεως ἡ κοινὴ μάλιστα καὶ κυριωτάτη κατὰ τόπον ἐστίν, ἣν καλοῦμεν φοράν*.' Descartes says in *Le Monde*, VII, of motion: 'Et moi, je n'en connais aucun, que celui qui est plus aisé à concevoir que les lignes des Géomètres, qui fait que les corps passent d'un lieu à un autre' (AT. xi. 39[28]–40[3]).

these two, substance and mode, by another necessity, and that Descartes finds in God, in the words of the *Principia*, 'the first cause of motion'. The geometrical conception of extension and its relations and proportions expressible in algebraical symbols imply the fatal break between the mathematical world of pure extension and the physical world involving motion and time. The gap could only be covered metaphysically by a *deus ex machina*.

But how does Descartes consider he can close the gap methodologically? For there can be no doubt that he was aware of the problem. In what sense is the method widened by the words of Rule 2: 'It must be noted that we arrive at knowledge of things by a double way, namely, by experience and deduction'?[1]

§ 2. *The Role of Experience*

Of all the great philosophers, none, with the exception, perhaps, of Aristotle, devoted more time to experimental observation than Descartes. Yet it is almost traditionally asserted that the Cartesian doctrine of scientific methodology is one basing its whole claim on the power and use of *a priori* reasoning. Both statements may, of course, be true. Descartes may well have found himself forced in practice to have recourse to purely experimental means in order to establish his theories, while professing a thoroughgoing rationalism when describing his method. He would not have been the first, or the last, philosopher to be discovered with a gulf between his own life and his theory of life.

He is continually referring, in his letters, to various experiments which he has made or would like to make on many and diverse matters. All his biographers from Baillet to Adam give further evidence of his keen and assiduous interest in all types of experimental work, both his own and those of other scientists. In the *Discours*, Descartes tells us of those nine years of travel and study combined during which 'I made a variety of observations and acquired a quantity of experience'.[2]

[1] 'Nos duplici via ad cognitionem rerum devenire, per experientiam scilicet, vel deductionem' (AT. x. 364^{26}–365^{2}).

[2] 'Je faisais diverses observations et acquérais plusieurs expériences' (AT. vi. 29^{17-19}). The Latin text has: 'et multa experimenta colligebam' (ibid. 556).

Baillet tells us of dissection, and vivisection, of embryos:[1] the *Cogitationes circa generationem animalium* was a notable piece of anatomical writing for an amateur.[2] Dogs, cats, rabbits, codfish, mackerel, eyes, livers, hearts all appear as objects of some dissection or other which he speaks of to his correspondents. But his interests are wider than anatomy. In 1631 he is experimenting on the weight of air and, a few years later, on the laws of sound. He experiments on the nature of reflection and the *Dioptrique* and *Météores* abound in references to experimental observation. But whatever may be the difficulties of forming a firm estimate of the character of Descartes, there is no question of an empiricist Hyde cloaked by a rationalist Jekyll. Descartes speaks his mind frankly to Huygens in a letter of 1645:

> What I find strangest of all [he writes] is the conclusion of the criticism which you sent to me, namely, that what will prevent my principles from being accepted by the Schools, is that they have not been confirmed by experience and I have not refuted the reasoning of other philosophers. For I am lost in wonder before the fact that, although I have proved in detail as many facts of experience as there are lines in my writings and although, having given a general demonstration of all the phenomena of nature in my *Principia*, I have explained, in the self-same manner, all the facts of experience concerning inanimate bodies, and in doing so have never had recourse to the commonly accepted Philosophy, nevertheless those who follow this kind of philosophy never get tired of objecting against me that I have not done enough experiments.[3]

Towards the end of his life he writes to the Princess Elizabeth:

> I have dared (but only during the last eight or ten days) to wish to explain the manner in which the animal system develops from the start of its existence; I speak of animal in general, for as concerns man in particular, I should not dare to undertake such an explanation, being short of the required experimental evidence.[4]

[1] Cf. *La Vie*, i. 195–7. Baillet adds: 'Il joignit l'étude de la Chimie à celle de l'Anatomie dès la fin de l'an 1629.'

[2] Cf. AT. xi. 498–538. From a letter to Mersenne, we hear that 'someone, rather weak in mind', accused Descartes of travelling round the villages to see the killings of pigs. Descartes's reply is that anyhow more pigs are killed in a town and that, as a matter of fact, he goes daily to an *abattoir* to get various pieces for dissection (cf. AT. ii. 621^{3-15}). The same letter speaks of 'the sensitive plant' and reflects another side of Descartes's interest, this time in botany.

[3] AT. iv. 224^{21}–225^{7}.

[4] AT. v. 112^{19-25}.

There are frequent references in his letters which echo the phrase of the sixth part of the *Discours* where he speaks of being 'hindered either by the shortness of life or the want of experiments', and further on that 'I noticed, moreover, with respect to experiments, that they become always more necessary the more we advance in knowledge'.[1] All this is quite sincere and expresses a real and profound conviction on the part of Descartes. But it is not obvious at first sight how these views on the need of experiments fit into a view of method which laid all the emphasis on the deductive approach and set as its ideal mathematical demonstration from self-evident principles. Descartes thought himself consistent but he has never set down his views on this point in any detail. His formulas are nearly always the same and nearly always vague. Discussing his treatise *Le Monde* with Mersenne, he tells how he hopes soon to get to the study of 'particular bodies' and that he has already been studying the different natures of oils, spirits (or brandies), common waters, salts, &c., and adds: 'I hope to open the route sufficiently so that in the course of time one can know them all [the substantial natures of particulars], by adding experience to reasoning'.[2] What can experience add to reasoning?

Some indication of the general principle governing the Cartesian approach is given in another letter to Mersenne, where Descartes gives his views on Galileo and his theories. There is irony in the fact that it was Galileo who was right in the famous discussion of 'la loi de la chute des corps', and Descartes who was unable to formulate the law in mathematical terms. However, this is what he writes:

I find, in general, that he [Galileo] philosophizes better than the usual run, and he gets away as much as he can from the errors of the Schools and tries to examine problems of physics by mathematical reasoning. On that point I agree with him entirely and I consider there is no other way to discover the truth. But it seems to me that he is continually wandering from the point and does not explain any matter

[1] AT. vi. 63^{18-20}. Cf. his letter to Huygens in 1638: 'mais parce que j'ai besoin de beaucoup de temps et d'expériences' (AT. i. 507^{13-14}).

[2] Letter of 1632 (ibid. 143^{15-18}): '. . . en ajoutant l'expérience à la ratiocination'. Cf. also the author's letter serving as a Preface to the French translation of the *Principia*: 'si j'avais la commodité de faire toutes les expériences dont j'aurais besoin pour appuyer et justifier mes raisonnements' (AT. ix (Part II), 17^{13-16}).

thoroughly; which goes to show that he has not examined the points in order and, without having considered the first causes of nature, he has merely looked for the causes of certain particular facts, building thus without any foundation.[1]

The criticism reads harsh and too condescendingly magisterial in view of the comparative success and failure of their solutions of the problem of the velocity of falling bodies. The point to notice is that Descartes is once more insisting on the ultimate explanation by mathematical reasoning and the lack of scientific value, as he thought, in detailed explanation of isolated phenomena of nature.

The Cartesian assumption is obvious. If geometrical extension constitutes the ultimate nature of the material world, and movement is an essential mode of this extension, then, while the basic laws of nature are deducible by *a priori* mathematical methods, they can only be deduced as possible. To say that extension and its modes exist requires a justification over and beyond the analytical deduction itself. In other words, the physical sciences need a metaphysical justification in order, so to speak, to set themselves up in complete independence. Before extension and its modes have received the guarantee of real substantiality from a veracious God, they have only a quasi-regulative value as governing the whole deductive series and thus enabling the mind to tend towards the unity of knowledge. But granted this guarantee, which falls outside the scope of methodology, and assuming that one has avoided the reproach of not 'considering the first laws of nature', can the pure deductive process give an adequate account of the physical world?

The answer would appear to be negative. The general laws derivative from extension must be universal. But as is made clear in the *Regulae*, we cannot by reasoning discover a new kind of being and we cannot deduce from the universal anything but the universal. It would therefore follow that the universal laws, deducible from the simple natures which

[1] Letter of 1638 (AT. ii. 380³–16). On the problem of the velocity of falling bodies and Descartes's attempts to solve it see A. Koyré, *Études galiléennes*, II, 'Descartes et Galilée' (Paris, 1939). M. Koyré notes that 'l'attitude mentale de Galilée diffère sensiblement de celle de Descartes. Elle n'est pas purement mathématique; elle est *physico-mathématique*' (p. 74). He also notes that 'en revanche . . . c'est Descartes, et non Galilée, qui a formulé, expressément du moins, le *principe d'inertie*'.

constitute the 'material world', cannot give us knowledge of particular bodies. Nor, on the other hand, can we pass from particular instances to general laws. Rule 12 makes it clear that 'we compound badly', we are in error, 'whenever we judge that it is possible to deduce something general and necessary from a particular and contingent fact'.[1] There seems at first sight little scope left for sense-experience in this rigorously deductive system. The rule of enumeration implies that the collection of facts obtained in and through sense-experience has a certain value as a preparation for scientific work proper. Experience is therefore essential to the preliminary survey of a problem which results in a classification of the *quaesita* and data. It is through the classification of data derived from sense-perception that the problem is 'set out' in such a manner that the particular field of knowledge to be covered can be summed up in a few contrasting alternatives. In a sense, then, experience may be said to set the deductive process going. What are these data which have to be surveyed? We are told in Rule 12 that we know nothing but simple natures and their 'mixture' or composition, and we are warned that it is easier 'to be aware of several of them joined together (*conjunctas*) than to separate one from the others'.[2] In the following paragraph Descartes states: 'those natures which we call compound are known to us either because experience shows us what they are or because we have formed them ourselves'.[3] He goes on to explain that the term 'experience' covers whatever we perceive through the senses as well as what we may hear from others. In the case of the data of problems of physics, we are dealing with the second class of simple natures, 'the purely material' natures, and also partially with the third class of 'common natures' as known 'by the understanding intuiting the images of

[1] AT. x. 424^{25-27}. In the *Notae in Programma* Descartes argues that nothing comes from external objects to the mind through sense-perception but 'motus quosdam corporeos', and that nothing could be more absurd than to think that the 'communes notiones' arise from these motions for, he adds, 'omnes enim isti motus sunt particulares, notiones vero illae, universales, et nullam cum motibus affinitatem, nullamve ad ipsos relationem, habentes' (AT. viii (Part II), 359^{24-26}).

[2] AT. x. 422^{7-11}.

[3] 'Dicimus sexto, naturas illas quas compositas appellamus, a nobis cognosci, vel quia experimur quales sint, vel quia nos ipsi componimus' (ibid. 422^{23-25}).

material things'.[1] This appeal to sense-experience may well appear to run counter to the Cartesian doctrine that sense-perception is confused and obscure, summarized in the description of the *Regulae* as the 'fluctuating testimony of the senses'. But Rule 12 insists that 'no direct experience can ever deceive the understanding, if the understanding restricts its attention precisely to that which is presented to it'.[2] Descartes is not denying that sense-experience is confused or obscure, but he is insisting that error or falsity cannot as such be attached to sense perception; it is only to judgement that the qualifications of truth and falsity can be rightly attributed.[3] Rule 8 makes the same point: simple natures are divided there into spiritual or corporeal or those which are 'at once spiritual and corporeal'. A division is also made between the 'complex and composite natures'. Amongst these latter, 'there are some which the understanding experiences to be such [i.e. complex] before it judges that it can determine something about them; others, however, it compounds itself'. 'But', adds Descartes, 'there can be no falsity save in the last class which are compounded by the understanding.'[4]

Descartes's attitude to sense-perception is fully expressed in Rule 12, mentioned in the preceding paragraph.

> No direct experience can ever deceive the understanding, if the understanding restricts itself precisely to that which is presented to it, in so far as it has the object either in itself or in an image and if it, moreover, does not judge that the imagination faithfully reports the objects of the senses, or that the senses take on the true shapes of things, or finally that external things are always such as they appear to be; for in all these ways we are exposed to error.[5]

[1] See above, p. 81.

[2] 'Intellectum a nullo unquam experimento decipi posse, si praecise tantum intueatur rem sibi objectam' (AT. x. 423^{1-3}).

[3] Descartes, writing to Plempius in 1637, makes this distinction between *sensus* and *ratio*: 'nempe in eo quod cognitio sensus sit apprehensiva et simplex, nullique ideo falsitati obnoxia: cognitio vero rationis sit paulo magis composita . . .' (AT. i. 415^{13-15}). This is of course the well-known Scholastic doctrine that the *simplex apprehensio* cannot be deceived and that truth or falsity lie only in the judgement. St. Thomas puts it thus: 'Verum est tantum in intellectu componente, vel dividente; non autem in sensu, vel intellectu quid est rei concipiente' (*Summa Theol.* I, Q. 16, a. 2). Cf. Aristotle's doctrine in *De Anima*, $430^{a}26$: *'Η μὲν οὖν τῶν ἀδιαιρέτων νόησις ἐν τούτοις, περὶ ἃ οὐκ ἔστι τὸ ψεῦδος. ἐν οἷς δὲ καὶ τὸ ψεῦδος καὶ τὸ ἀληθές, σύνθεσίς τις ἤδη νοημάτων ὥσπερ ἓν ὄντων.'* See above, p. 74, n. 1.

[4] AT. x. 399^{10-16}.

[5] Ibid. 423^{1-18}.

If we *judge* the sense-data to be essentially the same as the objects of the material world, we fall into error, the error of the mind *male componens*. The essential nature of the external world is not, of course, revealed in the confused and composite data of sense-perception. The whole object of methodical analysis is to elicit from these confused and complex ideas the simple natures which are their principles of explanation. But the analysis, in the case of the physical sciences, begins with the composite data given in sense-perception and the first operation to be performed is that of enumeration, the preliminary survey and classification of the data, which determines the limits of the particular *quaestio*. Natures which are complex or composite are known to us either because we experience what they are or because we ourselves compound them, and this alternative is meant to state the difference which arises when either we are faced with obscure and involved *quaestiones*, posited by the composite nature of the data, or when we are dealing with intricate judgements or deductions which the understanding has formed, in the words of Rule 12, 'by impulse, by conjecture, or by deduction'.[1] Sense-experience provides not only the data which set out the possibilities of the scientific field to be covered; it also provides the material with which scientific conjecture or hypothesis can be constructed. The use of enumeration in the sense of an analogical induction clearly implies, when dealing with 'purely material' natures, our use of sense data, at least in the initial steps of the deduction.[2]

Rule 12 gives us an example which indicates what Descartes means by the use of experience. The *quaestio* is 'what is the magnet?' The man who 'reflects' will first 'collect all the observations with which experience can provide him', then 'from these he will try to deduce [by analysis] what mixture of simple natures is necessary to produce all those effects which have been experienced in connexion with the magnet; this once discovered, he can confidently assert that he has discovered the true nature of the magnet in so far as it could be discovered by the human mind and the given experiments'.[3] It will be noted that the final verification of

[1] AT. x. 424[2–3].
[2] See above, pp. 134–43.
[3] AT. x. 427[19–26].

the deduction by enumeration will include not only the checking of the various links and linkages of the deduction itself but also a checking of the composite attained by the deduction with the composite given in the initial experiences. 'There must hence be a necessary connexion not only within the several steps of the deduction but also between the original experienced effects and the mixture of simple natures which, as their explanation, is the conclusion of the deduction.'[1] Another example of the 'determination' of a problem is given in Rule 13, and the 'determination' required is such that 'we shall having nothing more to seek for than that which can be deduced from the data'.[2] The *quaestio* is 'what my conclusion is to be as to the nature of sound from these precise data that the three strings *A*, *B*, and *C* give out an equal sound, where, by hypothesis, *B* is twice as thick as *A*, but not longer, and is kept tight by a weight twice as heavy; while *C*, though no thicker than *A*, is twice as long, but is kept tight by a weight four times as heavy'.[3] The problem is entirely specified and limited by the data enumerated. The quantitative solution is restricted to a limited set of composites. In the physical sciences, then, we are dependent on sense experience for the 'determination' of the problems and the possibility of making our initial hypothesis.

There is nothing inconsistent in all this with the general doctrine of method expounded in the *Regulae*. In Rule 14, as elsewhere, Descartes took pains to point out that we cannot, by reasoning, discover a 'new kind of being'. If, from what is already known, we deduce an unknown, all that this deductive knowledge amounts to is this: the perception that the *res quaesita* participates in this or that manner, in the nature of our data. Descartes, like Locke, would say that it is impossible to argue a man who had been blind from birth into perceiving the colours, 'to discourse into him the ideas of light and colours'.[4] In the same way, if the magnet contained a 'new kind of being', utterly unlike anything which our mind had hitherto 'perceived', we should never by any

[1] A. Gewirtz, in an excellent study of the 'Role of Experience in the Cartesian Method', *Journal of the History of Ideas*, Apr. 1941, p. 198.

[2] 'Adeo ut nihil amplius quaeratur, quam id quod deduci potest ex datis' (AT. X. 431[5–6]).

[3] Ibid. 431[9–15].

[4] Locke, *Essay on Human Understanding*, ii. 4, § 6.

process of reasoning attain to knowledge of this unknown nature. We should need a new sense or a creative mind like that of God. The most the human mind could achieve in such a case would be 'to perceive with complete distinctness that combination of already-known beings or natures which would produce the same effects as now appear in the magnet'.[1] The understanding, then, presupposes, as an essential condition of its deductive movement, an already existent knowledge of 'certain kinds of being', of certain 'natures'. But in the physical sciences, which are concerned with 'purely material' simple natures, we are not in the first instance dealing with 'purely intellectual' data, and the Cartesian doctrine seems to imply that the *sine qua non* of all deductive reasoning is a sensuous or at least imaginative knowledge. If we wish to deduce in the physical sciences, we must start from a nature which is known as an object presented, in a confused manner, to sense or represented in an imaginative idea. The implications of this view have already been discussed.[2] When we consider what are the *entia jam nota*, in a case like that of a magnet, we see that Descartes means such natures as extension, figure, movement, and the like. Yet it is these very common notions which, we are told in the *Notae in Programma*, cannot arise from particular movements, since they have no affinity with them.[3] These common notions are also in Cartesian terminology 'innate'. Clearly, then, they cannot be derived from sense-experience, which is essentially perception of the particular. What, then, can be the role of experience in this matter?

When Descartes is attacking Regius's thirteenth article in the *Notae in Programma* he declares uncompromisingly that

> In our ideas there is nothing which is not innate in the mind or power of knowing, except only those circumstances which point to experience: namely, the fact that we judge that these or those ideas, which we now have present to our thought, refer to certain things external to us, not because these external things transmitted the ideas

[1] AT. x. 439[6–10]. See above, p. 220, n. 1. *The Logic or the Art of Thinking* (1685), Part IV, has an obvious reference to this passage: 'And therefore we are to believe that we have found out all that can be found by human Invention, could we conceive distinctly, that such a mixture of *Beings* and *Natures*, which are known to us, could produce those effects which are known to us in the Loadstone' (p. 177).

[2] See above, pp. 218–22.

[3] See above, p. 243, n. 1.

themselves to the mind through the organs of sense, but because they transmitted something which has given it the occasion, by its innate faculty, of forming these ideas at this time rather than another.[1]

The difficulties inherent in Cartesian dualism appear in the vague reference to 'the occasion' given by the sense-experience to the mind to make ideas explicit which are presumed to be already implicit in the mind. The role of experience would seem to be that of allowing for the reference of ideas to objects and providing at the same time the initial experience by which the mind attends to and thus becomes aware of the implicit ideas of simple natures. The mind does not in fact get the common notions, simple natures, or innate ideas from perception of particulars, i.e. from sense-experience, but it is in and through experience of particular objects that the mind is led to the consideration of the general principles of their explanation, the simple natures from which they are compounded. It is not unlike the role of *ἐπαγωγή* as described in the first paragraphs of the *Posterior Analytics*, by which the mind is brought face to face with the particulars and thereby 'induced' to recognize the universal virtually known.[2] We know beforehand that the sum of the angles of every triangle is equal to two right angles, says Aristotle, but it is only at the very moment that we are led on to recognize this as true that we come to know that 'this figure inscribed in the semicircle' is a triangle. The simple natures, or innate ideas, are not derived from experience, nor are they derivable from experience. They are evoked or elicited 'on the occasion' of sense-experience. But it remains true that deduction from general laws cannot give knowledge of particulars and, on the other hand, experience of particulars, even if it be submitted to rigid control, cannot finally yield knowledge of general laws. Descartes does in fact infer certain general laws about physical phenomena from experiments conducted on rigorously methodical principles. He is aware of and appreciates certain

[1] The 13th article of Regius runs: 'Atque ideo omnes communes notiones, menti insculptae, ex rerum observatione vel traditione originem ducunt' (AT. viii (Part II), 345^{8-9}). The text quoted from the *Notae* is at 358^{25}–359^{5}: '. . . quod ei dedit occasionem ad ipsas . . .'.

[2] Cf. *Post. Anal. A* $71^{a}17$–19: *"Εστι δὲ γνωρίζειν τὰ μὲν πρότερον γνωρίσαντα, τῶν δὲ καὶ ἅμα λαμβάνοντα τὴν γνῶσιν, οἷον ὅσα τυγχάνει ὄντα ὑπὸ τὸ καθόλου οὗ ἔχει τὴν γνῶσιν.*

practical rules of the Baconian method, including the *instantia crucis*. But these laws derived from experiments are merely hypotheses, and knowledge derivative from sense-experience does not constitute 'real knowledge', that is, the knowledge by which we apprehend the fundamental principles of scientific explanation, the premisses of scientific deduction.

Professor Roth describes the method of Descartes in the physical sciences in these words:

> The method is essentially deductive. It is modelled on the conventional conception of the process of geometrical reasoning. It sets out from a few self-evident principles, and from those principles educes consequences which are, as it were, the 'propositions' of science. But propositions are by nature general, and general propositions have only a limited value in the field which we now know to be all-important for Descartes, the field of practical application to the concrete problems of the individual. There is a 'gap', definite and unbridgeable, between the general laws deduced by the help of the method and the particular facts with which we have to deal.[1]

There is much in this description which fits the account given in this study. But there is also much that is misleading. Descartes himself made it perfectly plain that his own method of 'analysis' differed entirely from the 'conventional conception' of the geometers which he called the 'synthetic' method. His method, on the basis of his own claims, is essentially a method of 'discovery' of the simple and easy principles. The synthetic exposition follows upon the actual discovery of these principles. Deduction is both analytic and synthetic. The 'gap' is recognized by Descartes and is bridged by a view which has been mentioned already in this chapter, which takes the discussion beyond the level of methodology to that of metaphysics.

Descartes puts the point himself in the following form in the third part of the *Principia*. The fourth article states:

> Now the principles which I have explained above [*Latin text*: which we have discovered] are so wide and pregnant that many things can be

[1] L. Roth, *Descartes' Discourse on Method*, p. 78. See above, pp. 103–4, the quotations from the *Resp. ad II Obj*. There is also a gap between 'expliquer' and 'prouver', recognized fully by Descartes in the *Discours*, Part VI (AT. vi. 76[20]), and the objections and answers to Morin in Gilson's *Commentaire*, p. 473. Descartes admits that the existence of any particular cannot be proved *a priori*; his contention is that all scientific *explanation* is *a priori*.

deduced from them which we see to be contained in this visible world, and many more even than our mind could survey in thought [*French*: during the whole of our life]. We accordingly make a short review of the principal phenomena of nature, not to use them as the logical grounds for demonstration, for we want to deduce the explanation of effects from causes, and not the explanation of causes from their effects, but merely so that among the numerous effects which we consider may be produced from those causes, we may restrict our mind to the apprehension of some rather than others [*French*: so that we may choose, among an infinity of effects which can be deduced from the same causes, those which we should principally try to deduce from them].[1]

If we take it that sense-experience cannot provide the principles or grounds of the deduction, since it cannot provide the general laws governing the physical world, then the 'wide and pregnant' principles mentioned above must be 'innate'. They are not derived from or created by experience. At the most we can say that they are provoked or elicited by experience, either in the preliminary survey of the problem or at the various steps of the deduction. The deduction itself *qua* necessary connexion of self-evident steps is conditioned purely by the simple natures and their various *respectus*. From this point of view the physical sciences conform in their essential methods to those of the mathematical sciences. The senses aid in the same way as the imagination was auxiliary to the pure use of the mind. But in the material world there is the possibility of an 'infinity of effects all logically derivable from the same causes'. Two or more solutions may be possible and equally compatible with general principles, expressible in mathematical terms, by which we interpret the physical universe. The deduction as such cannot give sufficient grounds to decide between one phenomenon, or group of phenomena, rather than another. God, the veracious guarantor of the principles themselves, could equally well have freely decided that x group should exist rather than y group. Which He chose, out of the multiple possible effects,

[1] *Principia*, iii. 4 (Latin: AT. viii (Part I), 81^{19}–82^{3}; French: AT. ix (Part II), 104–5). In the Latin text Descartes uses the term 'brevem historiam', a short review, which recalls the use of 'historia' by Bacon. The 'effects' will, of course, be 'composites' and their 'nature', composite by definition, is necessarily explained in terms of 'causes' which will be 'simple' or 'relatively simple' natures. What is the specific composition of 'this' or 'that' body is contingent and therefore dependent on experience.

must be decided by experience controlled by reason. The role of experience, then, is to answer the questions set by reason at the outset of the deduction during the preliminary survey of the ground, and at each stage of the deduction when a logical bifurcation is possible. It is thus that the function of enumeration aids not only at the beginning, and in the final verification of the links and linkages, but during the course of the deduction.

The deductive method, then, posits the most simple and general truths or laws, reduced to their simplicity by analysis. Then, as the sixth part of the *Discours* notes, we have 'the principles or first causes of everything that is, or can be, in the world, without considering for this end anything but God who created the world, or deriving them from any source but certain seeds of truth naturally implanted in our souls'. From these first principles we must descend to the explanation and demonstration of particular laws governing the existent world, that is, to the physical domain of particular existents as opposed to the world of possibles, viz. the domain of τὰ μαθηματικά. The *Discours* continues:

> Afterwards, when I wished to descend to the more particular, so many objects of various kinds presented themselves to me, that I did not think it possible for the human mind to distinguish the forms or species of bodies that are upon the earth from an infinity of others which might have been so if it had been the will of God to place them there, or consequently to apply them to our use, unless we rise to causes through their effects and make use of particular experiments.

But Descartes does not consider that his method has broken down and his words are not, as has been said, a confession of failure, even though it may be possible to detect a note of regret and disappointment in the recognition of the limitations of the human mind face to face with the infinite possibilities of creation. He continues:

> Reviewing in my mind all the objects that had been presented to my senses, I venture to assert that I have not observed anything which I could not satisfactorily explain by the principles I had discovered. But I must also confess that the power of nature is so vast and extensive, and these principles so simple and general, that I have hardly observed any particular effect which I did not at once recognize to be deducible from them in more ways than one, and my greatest difficulty is usually

to discover in which of these ways it depends on them. In such a case, I know no other device than to devise certain experiments such that their result is not the same if it has to be explained in one of these ways as it would be if explained in the other.[1]

If we grant that the sixth part of the *Discours* is not an otiose appendix to the whole but is a mature *confessio fidei* by a Descartes who has been through the intellectual stages of the *Meditationes* and the *Principia*, then we can take these views as expressing his full doctrine on the relations between the pure deductive method and sense-experience critically examined. Subject to a metaphysico-theological guarantee of its evidence and certainty, the basis and structure of the physical sciences as a whole are the work of reason, of intellectual intuition and deduction, basing itself on the mathematical pattern and descending in logical steps of implication from homogeneous extension and its modes. The empirical facts, contingent from their very nature as willed by God, must, with their mutual relations, find their explanation and representation within the conclusions of this deductive system. The possibles are, as it were, *a priori* deducible within the system of theoretical or deductive physics. Experience is necessary to decide which of the possibles is the actual. But the key to all the problems of the physical sciences lies in the general laws and principles which are based by Descartes on the view that nature is

> the realization of the simplest conceivable mathematical ideas. I am convinced that we can discover by means of purely mathematical constructions the concepts, and the laws connecting them with each other, which furnish the key to the understanding of natural phenomena. Experience may suggest the appropriate mathematical concepts, but they most certainly cannot be deduced from it. Experience remains, of course, the sole criterion of the physical utility of a mathematical construction. But the creative principle resides in mathematics. In a certain sense, therefore, I hold that pure thought can grasp reality, as the ancients dreamed.

These words express the ideal of Cartesian science, but they were written some three hundred years later by Einstein.[2]

Descartes, therefore, is no follower of Bacon. He does not

[1] Cf. the *Discours*, Part VI (AT. vi. 63^{31}–65^{7}).

[2] A. Einstein, *The World as I see It*, p. 136.

believe, in fact he strongly denies throughout his writings from the *Regulae* onwards, that the fundamental concepts and laws of physics can be deduced from experience by abstraction or generalization. The purpose of induction is to complete the process of deduction which cannot, from the limitations of the human mind, descend to the particularities of the contingent, or to answer questions of a similar nature as the deduction proceeds. The experimental method is justified, as is the use of imagination in the mathematical sciences, as a subsidiary to the pure use of the *vis cognoscens* which grasps intuitively the ultimate mathematical nature of physical laws. While the pure deductive process may not be able to offer any ground or reason for deciding on one set of facts rather than another, out of a range of possible facts, nevertheless 'deduction', in the words of Rule 12, 'is the only way of connecting things together which can leave us in no doubt as to the truth of their synthesis'.[1] It is possible to say this and yet to write in Rule 5, in scornful accusation of the apriorists, 'of those philosophers who neglect experiments and expect truth to rise from their own heads like Minerva from Jupiter's'.[2]

[1] AT. x. 424[19–20]. [2] Ibid. 380[14–16].

XVII

THEORY AND PRACTICE

§ 1. *Mathematics and the* Principia Philosophiae

PHILOSOPHERS who have scientific pretensions have a habit of making themselves look almost as ridiculous as scientists with philosophical theories. The student of Hegel, however great his admiration for that masterpiece, the *Encyclopädie*, cannot help a feeling of bewildered regret when he arrives at the *Zweiter Teil: Naturphilosophie*. It may remove some of the sting to remember that Hegel is reflecting on the science of his own time but, as a friendly but stern critic has noted recently: 'Hegel, it must be admitted, does sometimes deprive himself of this well-grounded excuse for the obsolescence of his dialectic by indulging in what would seem to be an effort to reach scientific conclusions by independent philosophic insight. When he does so the result is sometimes a happy anticipation, but often a baseless construction of fancy.'[1] Now Descartes, quite expressly, is attempting precisely to reach scientific conclusions by independent philosophic insight. Some of the Cartesian theories, scientific theories, might well be called, and were equivalently called not many years after his death, baseless constructions of fancy. Already, in 1686, Leibniz is demolishing the third law of movement and substituting mv^2 for mv. But there is no point in separating what is living or what is dead in the physical theories of Descartes, any more than a like treatment of the *Naturphilosophie* would appeal to the commentator of Hegel. Moreover, much has been excellently written on this subject.[2] But what must be of some concern here is that

[1] G. R. G. Mure, *A Study of Hegel's Logic*, p. 359. Of the three views of a 'Philosophy of Nature' set out by W. H. Walsh in *Reason and Experience*, pp. 66–67, Descartes would have to be classed under the third.

[2] Cf. P. Mouy, *Le Développement de la physique cartésienne* (1934), *passim*; P. Duhem, *Les Origines de la statique*, vol. i; G. Milhaud, *Descartes savant*, chaps. viii to xi; D. Dubarle, 'L'Esprit de la physique cartésienne' (*Revue des sciences philosophiques et théologiques*, Apr. 1937, pp. 213–43); Meyerson, *L'Explication dans les sciences*, vols. i and ii.

Descartes himself has indissolubly linked the value of his method to its application in all the fields of knowledge, the mathematical, the physical, and the philosophical fields alike. We must therefore obtain as much light as possible on its application to these fields and to the physical domain which has been our present concern. The previous chapter collected together what can, at the best, be described as general statements of position. And these statements of position remain fundamentally unsatisfactory since the explanation of Descartes's vital modification of the 'purely mathematical' method lies in a metaphysical theory, of which the truth itself lies outside any methodological discussion. There cannot therefore be any final decision until the metaphysical court of appeal has been reached. Pending this appeal, all that can be attempted at a lower level is to examine minor instances of the practice of the method itself, as applied in the physical realm, and to note, without any attempt at a complete historical account, the attitude of Descartes himself when dealing with the 'imperfect problems' of the phenomena of nature.

There is a curious evolution in the attitude of Descartes to problems dealing with physical phenomena; an evolution which puzzled some of his contemporaries and has since given rise to many speculations on the nature and significance of the change. To do full justice to this point, it would be essential to enter in some detail into the physical theories themselves and to date them, both in the intellectual development of Descartes himself and in the contemporary history of the physical sciences. Within the narrow and somewhat artificial scope of a study of the method, it is only feasible to draw broad outlines which must be dogmatically put forward, assuming in the reader some acquaintance with the history of scientific ideas. Briefly, the evolution appears to run counter to many of the key notions of the *Regulae*, though it emphasizes and develops certain other aspects of the same work. The principle, enunciated already in Rules 2 and 3, that we must start always with the 'simplest and easiest' ideas, that we should build each stage of the deductive process by a continuous and orderly advance from self-evident step to self-evident step, leads inevitably through the rule of analysis to

a theory of *Mathesis Universalis* in which the ideal form and formula of the ultimate 'simple' of deduction were posited as the equation, and 'true knowledge', ideally at least, was to develop what Poincaré called a 'cascade of equations'. Yet even a superficial acquaintance with the *Principia Philosophiae*, which contains the mature expression of Descartes's view on physics, must inevitably drive home the reflection that this work, especially the last two books, contains no mathematical formulae, no equations, and none of the mathematical technique which seems to play such an important part in the earlier works previous to 1630, such as the *Physico-mathematica* and other correspondence with Beeckmann,[1] and even the essays following upon the *Discours* itself. Had Descartes abandoned, at least in so far as the physical sciences were concerned, all pretence of applying the mathematical method to the phenomena of nature, and does his personal desertion indicate a recognition of the bankruptcy of the method itself applied beyond the strictly mathematical field?

To settle such a doubt involves raising issues of very different kinds. Primarily, as the previous chapter has indicated, the solution lies in the Cartesian doctrine, present already in germ in the *Discours*, that the final explanation of physical phenomena must be based on metaphysical propositions: an issue already ruled out of court as beyond the competence of the present inquiry. But this very dependence of the physical sciences on metaphysics, or, in other words, this incorporation of physics into the philosophical sciences, must be recognized, whether it be legitimate or not, to be directly consequent on the fundamental postulate of the method, formulated in the very first rule of the *Regulae*, that all knowledge is one and identical, and that, consequently, all the sciences are one. The negative condition of such a unity implies that there cannot be a complete separation of the various sciences. Whatever differences may exist among the objects, possible or real, abstract or particular, necessary or contingent, we must bear in mind that, in the

[1] Cf. AT. x. 15–169. On this point see Bréhier, *Histoire de la philosophie*, tome II. i. 86–88, and the Latin thesis of G. Milhaud, 'Num Cartesii Methodus tantum valeat in suo opere illustrando quantum ipse senserit?' (Montpellier, 1894), pp. 44–63.

words of Rule 12, 'things must be considered, relatively to our knowledge,' in an order different from that in which we should consider them in their real nature'.[1] Relatively to our knowledge, the sciences must form a single coherent intellectual system, within which must exist an implicatory series of necessary connexions, guaranteed, it is true, externally by the veracity of God, but in itself one and independent. This ideal system of implicatory truths is the ideal 'universal science' and 'all the sciences are nothing else but human knowledge which is always one and the same'.[2] Now this statement from the opening pages of the *Regulae* is followed in the same Rule 1 by the assertion, repeated elsewhere, that 'all the sciences are interconnected and dependent on one another'.[3] From the narrower methodological point of view of the *Regulae*, it is sufficient to base this interconnexion and interdependence of the sciences on the unity and unifying power of knowledge. Such is clearly the point of view dominating the whole discussion of Rule 12 and the simplification of the method in terms of the 'perfect problems' of mathematics.

But it is also possible to consider the problem of the unity of the sciences from the point of view of the structure of the real. Such is the angle from which the *Principia* is written, as indeed the *Meditationes* and, in a hybrid fashion, the *Discours*. At this level, or from this angle, the interest of the explanation must, of necessity, lie, according to the principle of the method, in that which, because 'simplest and easiest' in the logical and, here, the ontological order, constitutes the pivot of all explanation and demonstration, i.e. the existence and nature of God as creator and guarantor of the eternal truths (these latter, epistemologically considered, being 'the simple natures', the 'clear and distinct ideas', the *absoluta*, &c.). The change of emphasis from the mathematical formulation of detailed solutions of particular problems to the universal principles and general descriptive laws, these latter linked deductively to the basic metaphysical truths, is not to be considered as an implicit rejection of the method, but, on the contrary, as the natural evolution of the methodological theory and a logical widening of the field of its application. It has been argued with much learning and some plausibility,

[1] AT. x. 418[1-3]. [2] Ibid. 360[7-8]. [3] Ibid. 361[16-18].

by such eminent commentators as Gilson and Laberthonnière, that Descartes fabricated the vast construction of his metaphysics for the sake of his physics. On the basis and evidence of the *Regulae* at least, it must prima facie be admitted that nothing less than a metaphysical explanation of the physical world could have satisfied the fundamental demand of the method for the unity of all the sciences and allowed for the global completion, in theory at least, of the analytico-synthetic process of deduction.

There is also in the *Principia* no variation in the key-doctrine that true knowledge is ultimately reducible to the use of the *vis cognoscens* in intellectual intuition and deduction, and that, in the case of the physical world, this deduction is possible for the basic principles and laws governing the phenomena, however 'obscure' these latter may be owing to their essentially contingent nature. The very last section of the *Principia* is in direct affiliation with the doctrine of the *Regulae*. Descartes points out that certainty and true knowledge should be held to belong by right 'to all those conclusions, concerning corporeal natures, which have been derived by the principles of mathematics or by other demonstrations as certain and evident, provided they are derived by a continuous deductive series from the first and most simple principles of human knowledge'.[1] He then notes that the deductive method can, of course, only reach conclusions about *saltem generaliora*, but this is entirely consistent with all that has been said. There is no lowering of the standard, perhaps in itself impossible, on which we are to judge of what constitutes scientific knowledge and what is mere conjecture or probable hypothesis. The latter, though 'confirmed' by experiment, can lay no claim to be truly 'scientific' in the Cartesian sense; only in so far as they are integrated into the deductive system of total explanation can they be said to be

[1] *Principia*, iv. 206 (AT. viii. $32^{8}23$–30). I have used also the French version (AT. ix (Part II), 324) which is a little fuller than the original text—the key-words are: 'quo pacto ex primis et simplicibus cognitionis humanae principiis, continua serie deducta sint'. In the first section of the third book of the *Principia*, Descartes notes: 'Inventis jam quibusdam principiis rerum materialium' which were discovered 'a lumine rationis'—we are, in our attempt to explain all the phenomena of nature, to *begin* with the 'most general' and that on which all else depends, i.e. 'totius hujus mundi adspectabilis constructio' (cf. AT. viii. 80^{5}–12).

'known'. It would be possible, but tedious, to show by many quotations that the ideal of science and mathematics did not vary in any important factor during the course of his various writings. The reduction of matter to extension, the rejection of the distinction between matter and space, were meant to lay the foundations of a unique science of 'concrete Geometry', in which the physical laws in their most general forms—and perhaps Descartes would have added, in its most general form, as one single equation expressing the whole physical universe —could be mathematically expressed. It has been said that the generalized theory of relativity 'achieves what the greatest of all rationalists among modern philosophers, Descartes, dreamed of three hundred years ago, the identification of "matter" with "extension"'.[1] Descartes was the sort of person who would not have been surprised by such a compliment: he might well have been offended by the suggestion that he 'dreamed' the clear and distinct equation of matter and extension.

It might be suggested that even if the theory of method was incomplete in the Cartesian system, at least a study of its practice, and the results thereby achieved, might enable one to come to some firmer decision on its heuristic value, at least for mathematicians and scientists. Such a study, however, apart from its intrinsic historical interest, would not provide any criterion to enable a definite judgement of value to be arrived at by the observer. It would not be difficult to collect together and catalogue the more notable deficiencies and errors of Cartesian physics. In almost every branch of the physical sciences (and the biological sciences must be

[1] A. E. Taylor, *Contemporary British Philosophy*, p. 299. I take this quotation from a note in Keeling, *Descartes*, p. 256, who also quotes from Whitehead, *Science and the Modern World*, p. 180, this tribute from a none too favourable witness: 'Descartes, in his distinction between time and duration, and in his way of grounding time upon motion, and in his close relation between matter and extension, anticipates, as far as is possible at his epoch, modern notions suggested by the doctrine of relativity.' I find in Meyerson, one of the greatest historians of scientific thought, the following remarks of Weyl: 'Le rêve de Descartes d'une physique purement géométrique semble se réaliser d'une manière toute naturelle et imprévue.' Meyerson adds that this was not a dream but 'même mieux qu'un programme — le programme éternel de la science de toujours — elle constituait une véritable entreprise, tentée à l'aide de toutes les ressources que le savoir de l'époque, mis au service d'un génie à la fois scientifique et philosophique unique, était susceptible d'offrir', *La Déduction relativiste*, pp. 125–6.

included when speaking of Descartes) there are to be found, and friendly and hostile critics have not failed to note them, numerous examples of error on the part of Descartes, some arising from the acceptance of false data and resulting in false deductions, others from ignoring factors which are vital to the chain of causation, others from just glaring misinterpretation of the problems. But suppose one did point out that the *Dioptrique*, for instance, is vitiated in all its conclusions because Descartes has completely left out of account the phenomenon of chromatic aberration, or that the physical theories of the *Principia* suffer from the fundamental error of defining extension without reference to motion, thus destroying the possibility of a genuine kinematic theory, and so on. There is, without the slightest shadow of doubt, much that should be covered with silence and buried in decent oblivion by all who have the sense not to look merely for the weaknesses of their betters. But what would a catalogue of these errors and failures show? The answer may perhaps be found in another question. What do the undoubted and outstanding contributions of Descartes to almost all the physical sciences demonstrate in respect to the value of the method? The answer surely must be that neither the failures nor the successes of Descartes, *qua* physicist, prove anything which gives a categorical ground for approval or disapproval of the doctrine of the *Regulae* and *Discours*. It would have to be shown, amongst other points of detail, that these failures and successes were due directly to the strict application of the method itself. Such a demonstration would be directly faced with the major difficulty of any commentator on the Cartesian method, that is, with the task, impossible in the eyes of the author of the method, of setting down a formal set of rules, independent of the practice of the method itself. This practice, too, involves such subjective elements as the 'sagacity' and 'attention' of the scientist himself, his *ingenium*, his temperament as experimenter and reasoner, and a multitude of elusive and intangible factors attached, in this case, to the character of Descartes himself.

The 'philosophe au masque' does not reveal himself easily. A patient and sympathetic reading of the works and letters of Descartes, and this at least is possible, helps his reader to

seize upon some of these fleeting traits of character which explain, or diminish the mystery of, those curious blind spots which have prevented Descartes from anticipating, or nearly anticipating, discoveries which the historian of science must attribute to the glory of names such as Galileo, Pascal, Newton, Harvey, Leibniz, and others. One trait may be rapidly outlined since it throws some light on Descartes's own attitude to the role of experiment and observation in the physical sciences and his own predominant interests when engaged in such studies. It has already been noted that, for a philosopher, Descartes is outstanding for the amount of work which he did in actual experimentation. Yet there is a curious paradox in the various efforts which he made in this direction. A man of science devoted to laboratory work would not call him a good experimenter; he is too much interested in the theory or hypothesis which the experiment may or may not verify. As Gilson has pointed out, Descartes is not really interested in facts but only in their explanation.[1] The early meeting with Beeckman and the subsequent anecdote relative to the velocity of falling bodies is typical. Beeckman merely wished to know how bodies, in this instance stones, fall. Descartes's answer is an elegant mathematical solution, incorrect in fact, followed by another equally brilliant demonstration, based on a different assumption, of how stones *might* fall. His attitude in the *Dioptrique* and in the correspondence with Ferrier is typical in another manner. Already at La Flèche, Descartes had probably heard of the discoveries made by Galileo through the use of the telescope. Descartes wishes to draw up a plan for the construction of an even better one, and above all of a more powerful lens. This cannot be done until, he tells us, it is known what happens when light traverses several lenses, until the law of refraction has been established and the problem of the *linea anaclastica* solved. Then only can the plan of the various curves of the lenses be worked out. Descartes works these out but, as one can see, the unfortunate Ferrier is unable to carry out in practice the

[1] Cf. Gilson, *Études sur le rôle de la pensée médiévale* (1930), in the essay on 'Météores cartésiens et météores scolastiques', p. 137: 'La critique si vigilante de Descartes porte beaucoup moins sur les faits que sur leur explication.' Cf. on this point the study by G. Milhaud in *Descartes savant*, pp. 191–213.

difficult requirements set by Descartes. Galileo cannot solve the problem of the *linea anaclastica*; he does not know the law of refraction, but he manages to construct an instrument which gives a substantial magnification. Kepler, slightly more theoretical, knew only of approximations to the law of refraction but his description of the telescope provided a working model for future astronomers. Descartes required the exact measurements for his lenses, and failing this, he lost interest in the whole topic.

If one reads the account of the agreement and disagreement between Harvey and Descartes, it is clear that the latter, while appreciative of the great work done by the English savant, is not really interested in the facts. The *Traité de l'Homme* is really an account of how the body could work under a purely mechanistic set of laws and, in the particular instance of the circulation of the blood, Descartes refuses to accept Harvey's correct account of the systole and diastole of the heart, merely because, even if Harvey had shown his facts to be correct, he had not explained the *reason* which makes the heart contract. In the *Description du corps humain*, Descartes does admit that Harvey's theory might be confirmed by 'a very obvious experiment', and suggests such an experiment on the vivisection of a dog's heart. But he adds: 'Nevertheless this only goes to show that experiments themselves can often furnish the occasion for our being deceived, when we do not sufficiently examine all the causes which they may have.' The theory of Harvey may give an explanation of some of the phenomena but the same effects may proceed from an entirely different cause. In attempting to ascertain whether Harvey's or the Cartesian explanation of the motion of the heart is correct, 'it is essential to consider other experiments which cannot agree both with the one and the other'.[1] The main interest of Descartes, here as in many other instances, lies in the general explanation of the various phenomena, and this explanation must lie in a strictly deductive science, in descending order from causes to effects. Descartes does not, as we have seen, deny the value and necessity of experimentation which is, in a sense, an anticipation of causes by means of effects. But this necessity is due to the com-

[1] Cf. AT. xi. 242[14–20].

plexity of natural phenomena, and this complexity must, in the course of study and by experiment, resolve itself into a rational science. The ideal which Descartes set for the science of nature is that of a fully developed science. The *Principia Philosophiae* set down such a science of nature at a moment when this science was merely, as it is today and may always be, a developing science.[1] A science of nature is perhaps always premature.

§ 2. *The* Dioptrique

The reader of the *Discours*—and this may implicitly mean also the reader of the *Regulae*—is specifically warned by its author that it is only by carefully studying the 'essays in method' which follow the 'introduction' that he will grasp the essentials of the method. The first of these essays, the *Dioptrique*, may then be profitably considered as an application of the method to a problem of physics, and this may perhaps be briefly achieved by taking the argument of the ten *discours* which form its contents as strands in a single demonstration.

One feature of the treatise will strike the modern reader as slightly incongruous when compared to a modern treatise on a similar subject. Nearly two-thirds of the *Dioptrique* deals with a study of the human eye, its normal and abnormal physiology, and we should expect such a discussion to take place not in a treatise on physics but in some text of physiology or ophthalmology. It must, however, be remembered that in 1637 the science of optics, as that of ophthalmology, was in its infancy and consequently much that is common knowledge nowadays was unknown, or known to very few. But, more important, in the methodological treatment of the problem set in the *Dioptrique*, it is essential that, first, nothing should be taken for granted; secondly, since the aim of the treatise is to ameliorate and increase human vision, we must

[1] Meyerson notes, in the *Déduction relativiste*, that 'un physicien de nos jours n'éprouvera certainement aucune répugnance et même, peut-on dire, aucun étonnement (si ce n'est un étonnement d'admiration), en parcourant les *Principes* de Descartes; alors que, si nous essayons de le faire pénétrer dans la *Naturphilosophie* de Hegel, il manifestera le plus complet ahurissement, suivi (ces expressions ne sont pas trop fortes) de dégoût et presque d'horreur', p. 36.

clearly know what takes place in human vision, both in its normal and pathological states, as part of the general investigation. We must not, according to the instruction of Rule 3, suppose any element which is directly relevant to the problem to be acquired either on the basis of hearsay or conjecture. A telescope is built to increase our power of vision; before discussing the best way of constructing a telescope, we must ask ourselves what is in fact this power of vision. In the science of 1637 this is sound method.

Descartes accordingly begins by the general statement that 'the whole conduct of our life is dependent on our senses and of these the sense of vision is the most universal and the noblest'.[1] Hence the primordial role of vision in the acquisition of knowledge makes any possibility of increasing its power a major point of interest to all. The discovery of the telescope is one of vital importance and opens up immense possibilities of new fields of knowledge. Unfortunately, 'to the shame of our science', the invention of this instrument has been made possible merely by chance and luck rather than by deliberate study. Jacob Metius, to whom Descartes attributes the honour of the discovery, was 'a man who had never studied, although his father and brother were mathematicians'.[2] This remark, despite its tone of intellectual snobbery, is not directed at this ingenious inventor, but at the men of science, the group mentioned in Rule 4, 'most chemists, many geometers, and not a few philosophers', who 'in their wanderings are lucky enough to find sometimes some truth'. Which scornful remark was followed by the warning: 'It is far better never to think of investigating truth at all, than to do so without a method.'[3] So it was that on the model of Metius, invented by accident so to speak, that other telescopes were constructed, without, adds Descartes, 'anyone, as far as I know, having sufficiently determined the shape that the lenses ought to have'.[4] The purpose of the treatise therefore is to remedy this want of scientific planning and construction by giving a theoretical account of the working

[1] *La Dioptrique*, I (AT. vi. 81^{3-5}).

[2] Ibid. 82^{2-16}. A note on p. 227 points out that it is the father of Jacob Metius who discovered the approximation $\pi = 355/113$.

[3] Cf. AT. x. 371^{10-16}.

[4] *La Dioptrique*, I (AT. vi. 82^{18-20}).

of the telescope. To construct the best possible telescope, it is necessary to understand the principles on which the telescope works and this implies, on the one hand, a knowledge of the rational principles underlying the function of vision in general and, on the other hand, a knowledge of the laws governing the 'natural power of light' which is the other factor involved. The *quaesitum* of the problem has thus a double aspect and the method of approach in dealing with these aspects varies in the *Dioptrique*. In the first aspect, the search for the determining principle which governs the construction of the telescope, the examination of the 'power of vision', the method of analysis is primarily used in the attempt to lay bare 'in what vision essentially consists'. This analysis covers in fact the major part of the treatise from the third *discours* to the seventh inclusive. The general theory of 'the nature of light' and the laws of reflection and refraction are treated in the first two *discours*; they are treated by mathematical synthesis. The last three *discours*, especially the eighth and the ninth (the tenth, 'on the manner of cutting lenses', is almost entirely practical advice) which are concerned with the construction of the apparatus itself, are the synthesis of the conclusions reached in the two major divisions of the inquiry. It should also be noted that, throughout the whole treatise, no abstract mathematical formulae are used by Descartes. The reason for this policy is made clear in the first *discours*. The practical aim of the *Dioptrique* is to show, by the use of a methodical inquiry, how to construct better telescopes. But those who will undertake this work are likely to be 'artisans', probably therefore incapable of understanding abstract mathematical reasoning. 'I shall therefore try to make myself understandable by everybody',[1] adds Descartes. This has obvious advantages but sometimes leads to a certain difficulty when technical points are described in rather metaphorical terms. Moreover, the need for mathematical expression is partially met by the frequent diagrammatical appeal to the imagination made by the elaborate anatomical

[1] *La Dioptrique*, I (AT. vi. 82^{28}–83^{3}). It may be remembered that, according to Berkeley, the fatal defect of the Cartesian theory was its demonstrability, so that even the blind would have to admit it. (See *A New Theory of Vision* and the *Appendix* in the 2nd edit.)

and geometrical drawings dispersed in the text. The authority of Rule 14 would cover this practice.

The third *discours* treats 'of the Eye' and gives a physiological analysis of the organ of vision, describing in detail the various components and their functions, the retina (which he calls 'the optic nerve'), the iris and pupil and the phenomenon of contraction and dilation, and the aqueous humour. The function of the ciliary muscle is noted with surprising accuracy. The fourth *discours* makes a general point about the nature of sensation which is then applied to the particular sense under discussion. Descartes points out that sensating is not a function of the body but of the soul. The brain is merely the 'seat' or instrument of the soul and all sensation is 'seated' in the brain. This is true of all sensations—a brain injury, remarks Descartes, may destroy sensations mistakenly related to particular members of the body—and they are transmitted as stimuli to the brain by the nerves which are to be conceived as little threads or filaments, enclosed in tubes and agitated by 'animal spirits'. The same means serve to transmit the reflex and motor activities. This general analysis is then applied to the sense of vision and the point is made that we must not imagine that the 'transmission of images' implies that the images themselves are transmitted as such through the nerves. The analysis turns in the fifth *discours* to the question of the images and how they are formed in the eye. There is a wealth of anatomical detail in this section, as well as evidence of careful observation. The reversal of the image on the retina, the function of the black pigment in preventing internal reflection, the part played by the iris are noted with great care. In the following *discours* the general nature of vision is analysed. The act of vision is due to the motion of light, just as the other sensations are due to other forms of motion. The various forms of co-ordination, the sense of situation and distance, are explained by reference to the various habitual alterations of the body of the eye itself, as are also the power of accommodation and other habitual reflexes, and the better-known optical illusions. The seventh *discours* begins thus: 'Now that we have examined sufficiently how the act of vision takes place, let us sum up the matter in a few words and lay down all the conditions which are

required to perfect the vision so that, considering what nature has already provided to meet each, we may make an exact enumeration of everything that craft can add.'[1] The enumeration can be classified under three different heads. The first relates to the objects of vision themselves which can, in some circumstances, be moved to suit the vision, but in other cases are not accessible. The second concerns the 'interior organs' of the eye, that is the nervous structure which relates the retina to the brain—this head falls in the province of medicine. The third head concerns the 'external organs', that is the eye itself and the 'instruments' which may possibly aid or supplement its power of vision. Considering this third group, Descartes enumerates four points which cover 'all the needs' that have to be met by these external organs in order to have good vision.

The first point is that it must be the case that the rays of light, striking on the retina, must emanate from the same point of the emitting object and do not alter in traversing the space between the object and the eye. The motion of light is rectilinear. Secondly, the images formed by the action of the rays are large enough to be recognizable as to their essential figure and shape; the larger the image, the more easily the details are visible. Thirdly, they must be strong enough to stimulate the optic nerve, but the rays should not be so powerful as to cause pain. In other words, there is an *optimum* intensity. Fourthly, as far as possible, the field of vision should be as large as possible in order to include as many images as possible. The points of major interest here are the first two. Nature, according to Descartes, has met the majority of the conditions of good vision. The first is quite adequately met by the complex organization of the human eye, as shown in the previous analysis of its anatomical structure: the vitreous humour is transparent and colourless, the black pigment prevents internal reflection, the lens gives a constant degree of refraction, &c. True, there are occasional pathological variations in the case of myopia or presbyopia (which Descartes does not distinguish from hypermetropia) but these can be corrected by the use of lenses which precisely remedy the defective convergence of the rays due to a faulty focal length

[1] *La Dioptrique*, VII (AT. vi. 147^{16-23}).

of the lens of the eye or the length of the eyeball. As to the second point, the size of the image on the retina depends 'only on three things, namely, the distance between the object and the place where the rays cross which the object emits from its several points towards the base of the eye; then, on the distance between this place and the base of the eye; and finally on the refraction of these rays'.[1] These relative distances can obviously be altered in circumstances in which the object can be handled, or the body easily displaced. The ordinary magnifying-glass provides an illustration of how, by the use of a converging lens, vision becomes possible when the object is beyond the *punctum proximum* which would allow us to distinguish the distinct lines and shapes of the object.

But the problem is different in the case enumerated of the 'inaccessible' object, specifically an object at a great distance, when, owing to the remoteness of the object, the rays from any point of it are sensibly parallel on reaching the eye. The only possible solution is to cause 'the rays which come from the several points of the object, to cross as far as possible from the base of the eye'.[2] The problem is thus theoretically solved. The requisite condition could be met by a long tube which fitted the eye, the tube being filled with water or some other refracting liquid. This, of course, is absurd practically, and the telescope, with one lens serving as the objective, producing the real image of the distant object, and another lens, the eyepiece viewing this image, giving thus the virtual image, is the equivalent, the practical expression, of the instrument which fulfils the condition set by the analysis of the act of vision and its functions. The analysis has shown that the size, or apparent size, of an object of vision depends solely on the angle it subtends at the eye; the function of a telescope must therefore be to increase the angle subtended at the eye by the object, which will have the double result that the object will

[1] 'Pour la grandeur des images, il est à remarquer qu'elle dépend seulement de trois choses, à savoir, de la distance qui est entre l'objet et le lieu où se croisent les rayons qu'il envoie de divers de ses points vers le fond de l'œil; puis de celle qui est entre ce même lieu et le fond de l'œil; et enfin, de la réfraction de ces rayons', *La Dioptrique*, VII (AT. vi. 152^{29}–153^{4}). The phrasing is obscure but the explanation referring to the diagram on p. 139 makes it a little clearer. 'Le lieu' K is the pupil.

[2] '. . . le plus loin qu'il se pourra du fond de l'œil' (AT. vi. 155^{23-24}).

appear larger or magnified and it will also appear nearer. The equation between the size of the apparent object and the angle subtended at the eye by the object completes the analytic regression to a simple and evident proposition. The principle of the construction must start from this fundamental relation.

The first two *discours* approach another side of the central problem. The first deals 'with the nature of light' but Descartes immediately notes that he will not as a matter of fact deal strictly with this question. He is going to presuppose the analysis which was outlined in Rule 8.[1] He uses comparisons—it must be recalled that this work is meant to be understood by 'artisans'—and the analogy is drawn between the effect of light and what occurs when a blind man feels his way by using a stick in order to discover his whereabouts and the objects immediately surrounding him.[2] The stick is like 'a sixth sense' by which he decides which object is a tree, which a stone, or water, or grass, or mud, and so on. So the rays of light determine in us what is yellow, red, or green, and we need not assume that this determination of our sense of vision presupposes the passage of anything material, any more than the resistance or hardness of the bodies touched by the stick of the blind man passed along the stick except in the form of transmitted shock or movement. Light, then, can be considered as a similar shock or movement. The rays of light are, as it were, the lines along which the shock or movement of 'a subtle matter' tends in straight lines; in straight lines, that is, unless modified by changing from one medium to another medium of different density. The second *discours* deals with this change of circumstances and treats of the laws of reflection and refraction, which presuppose at every point the view that light is the tendency of the elements of subtle matter to move in straight lines from the object emitting the luminous rays to the eye. The table of contents indicates the synthetic deduction of the various laws. Working on the analogy of a tennis-ball hit by a racket, he defines the laws of reflection, namely, that the incident ray, the reflected ray, and the perpendicular to the surface lie in one plane and that

[1] Cf. AT. x. 393^{23}–395^{16} and above, pp. 139–41. Descartes does not, of course, refer in the text of the *Dioptrique* to this passage.

[2] The same simile is used in Rule 9 (AT. x. 402^{22-28}).

the angle of incidence is equal to the angle of reflection. The ball is metaphorically hit through a piece of cloth, and the law of refraction, giving as a constant quantity, for any two given media, the ratio of the sine of the angle of incidence to the sine of the angle of refraction, is formulated in a series of the simplest diagrams.

The eighth and ninth *discours* apply the geometrical data to the problem of the right shape of lens required to fulfil the conditions laid down in the analysis of vision. Combining the results of both inquiries, it is now possible to determine the forms which lenses meant to refract light must have, in order to cause the rays falling upon them parallel to their axes to meet in one point. The general conclusion is that the surfaces of such lenses, or rather the cross-section of their surfaces, should have the form of hyperbolas and ellipses, of which a detailed analysis is given in order to enable the 'artisan' to construct the telescope.[1] Such is, in its barest outlines, the demonstration of the *Dioptrique*, despite certain verbal obscurities a masterpiece of scientific clear thinking, eminently readable, and no small justification of the doctrine of method expounded in the introduction, the *Discours de la Méthode*. It is noteworthy that the whole problem is dominated by the physico-mathematical tone of the first two *discours*. The problem of the construction of the telescope becomes, from the very outset, a problem in which the principles, from which a solution is to be sought, are posited in geometrical terms. It is true that, in the *Dioptrique*, these mathematical principles are only called 'assumptions',[2] since he does not give their justification but merely uses them as sufficient for the purpose, limited in scope, set by the nature of the problem. But he had already explained himself on this subject in the *Discours*. There he says:

> If some of the matters of which I have spoken in the beginning of the *Dioptrique* and the *Météores* should, at first sight, shock people because I have called them assumptions, and do not seem to bother about their proof, just have the patience to read these right through and I hope that you will be satisfied. For it seems to me that the reasoning is so interwoven that the last are demonstrated by the first, which

[1] Descartes's practical solution did not allow for the phenomenon of chromatic aberration.

[2] Cf. *La Dioptrique*, I (AT. 83[11–27]).

are their causes, and the first are demonstrated by the last, which are their effects. And let it not be thought that in this I fall into the fallacy which logicians call arguing in a circle, for since experience renders the majority of these effects most certain, the causes from which I deduce them do not so much serve to demonstrate their existence as to explain them; on the other hand, the causes are explained by the effects.[1]

Descartes goes on to explain that he could have shown how these principles were related to the 'first truths', but he had not chosen to do so. The principles, then, become 'assumptions', relatively to the metaphysical truths on which they ultimately depend, but not relatively to the physical sciences themselves. A full demonstration would necessitate the setting-out of the whole implicatory series from the simplest and most evident truths; the *Principia Philosophiae* is an attempt at such an exposition. Physical theories in themselves are only conditionally true; their truth lies in philosophy. 'The method's competency is not limited to excogitating the physics of a logically possible world; his unitary "concrete physics" must be demonstrated true of this actually existing world. Hence the need of a new application of the method, namely, to that, whatever it may be, which can substantiate this claim.'[2]

[1] *Discours*, Part VI (AT. vi. 76[6–22]). On the objection of Morin and the 'circle' see the quotations in Gilson's *Commentaire*, p. 473.

[2] S. V. Keeling, *Descartes*, Henriette Hertz Lecture, 1948, p. 12.

XVIII

THE METHOD IN PHILOSOPHY

§ 1. *Logic and Method*

THE reader of the *Regulae* is left in no doubt that, in the mind of its author, the method is meant to apply in the metaphysical as well as the mathematical and scientific fields of knowledge. The scope of intellectual intuition and deduction includes the metaphysical truths. The examples given, in Rule 12, for instance, include the doubt of Socrates, the *Cogito, ergo sum*, the distinction between body and mind, and the existence of God.[1] The *nobilissimum exemplum* of Rule 8, that of the investigation of the limits of human knowledge, is clearly a matter appertaining to philosophy.[2] Descartes considered himself above all a philosopher and it is a title that is granted to him without question or quibble. Each of the examples quoted from the *Regulae* is in itself a formidable stumbling-block of Cartesian exegesis; there can be no question of discussing these examples here in any detail and mere generalities on the various controversial interpretations would be a waste of time. One can feel a great deal of sympathy with those early philosophers who sought for the 'logic of M. des Cartes' in order to discover the secret of his method for philosophizing, and Gassendi was not far wrong when he thought that the philosophical method was to be found, not in the *Regulae* or the *Discours*, but in the *Meditationes*. For, as it must have been apparent, the *Regulae*, especially in the later rules, speaks only of examples presupposing the knowledge of *res extensa*, quantitative being, capable of measurement, and in no way indicate how the method is applied to *res cogitans*. Yet, it must be repeated, rightly, or, according to Boyce Gibson, wrongly,[3] Descartes considered that his method was a universal one, the method of metaphysics as well as of science, and that the only logic that was necessary for philosophical thinking was

[1] Cf. AT. x. 421^{19}–422^{6}, and above, pp. 60 seq.
[2] Cf. AT. x. 395^{17}–397^{3}.
[3] Cf. Boyce-Gibson, *The Philosophy of Descartes*, pp. 85–86.

that set forth, in its essentials at least, in the two treatises on method.

To some extent, therefore, the logical doctrine of Descartes must be sought for, and expounded from, his account of the method as explained, illustrated, and exemplified in all his works.[1] He has, on the whole, told us what his logical theories were not, rather than what they are. His strongly worded criticism of the formal logic of the Schools will be recalled.[2] We are not surprised that logic is left out of the list of sciences to which the student is recommended, in Rule 2, to devote his early studies. The next paragraph softens the blow by mentioning that the 'probable syllogisms' are good for polemics and sharpening the wits, but the war therein declared against Scholastic logic was never closed in the works of Descartes.[3] There is one criticism in particular which is particularly noteworthy by its constant repetition and by the manner in which the weakness of the syllogistic method thereby exposed is contrasted with the new method of the *Regulae*. The syllogistic mode of reasoning is condemned because, amongst other reasons, it is quite useless as a means for the discovery of truth. It is contrasted with the new method and its failure is emphasized by pointing out that the syllogistic mode of reasoning cannot prevent us from falling into error, from 'supposing that to be true which is false',[4] one of the major requisites of all methodical thinking, nor can it 'arrive at a knowledge which takes in all things'.[5] When one considers the arguments adduced each time to support these and similar charges of uselessness, there is always one major count in the indictment, viz. that Scholastic logic is purely formal. According to this logic, we are told, it is held possible to reach a conclusion *ex vi formae*. More than half of Rule 10 is devoted to a condemnation of this theory. Descartes rejects 'the formulae of demonstration' which are supposed to lead to conclusions with logical necessity, even if one is not attentive. 'Only too true,' Descartes

[1] Locke noted in his *Journal*: 'Il est vrai quil [Descartes] ne traite en nulle autre part de la Logique pretendant que les 4 regles quil a données estant bien pratiquées doivent remplir tout ce quil y a d'utile dans cette partie.' In *Locke's Essay, An Early Draft*, by Aaron, p. 106.

[2] See above, pp. 102–6.

[3] Cf. AT. x. 363^{14}–364^{21}.

[4] Cf. Ibid. 372^{5-7}: 'nihil nimirum falsum pro vero'.

[5] Ibid.

adds; 'the fact, however, is that we often see that truth escapes the bonds meant to imprison it, while those who meant to utilize these bonds get entangled in them.' The conclusion is the well-known phrase: 'the logicians are quite unable to devise any syllogism which has a true conclusion, unless first they have secured the material of which they are to construct it, that is to say, unless they have already ascertained the very truth to be deduced in the syllogism.'[1] Useless, then, for the discovery of truth, formal logic should be transferred from philosophy to rhetoric. It is thoroughly in the Cartesian spirit that the *Logic* of Port Royal notes:

> We naturally walk . . . and in walking, we move regularly with our Bodies. But it would be ridiculous for a *Master of walking* to pretend to tell us, that the Animal Spirits were to be sent into certain Nerves; that such Muscles were to be mov'd; or to teach us to move such joynts, and to set one Leg before another. 'Tis true, Rules might be given for all these things: but that these actions should ever be perform'd by the help of any rules were ridiculous.[2]

But, it has been suggested, suppose, using the same metaphor as the *Logic* of Port Royal, we admit that a knowledge of the laws of mechanics would not help us to walk, could it not be used to relieve us from the task of walking by replacing, to some extent, our effort by a machine which was constructed according to the laws of mechanics?[3] Is it perfectly useless to attempt to fix some of the laws of thought in formulas which would relieve the mind of the more cumbrous processes of deduction?

Given the Cartesian predilection for the mathematical sciences, and above all the aim set in the *Regulae* and carried out in practice in the *Géométrie*, the ideal, i.e. the reduction of a problem by analysis and its synthetic construction in accordance with a constitutive equation, it would not be astonishing to find in his 'logic' some sympathy with a mathe-

[1] Cf. AT. x. 405^{21}–406^{26}.

[2] *Logic* or *The Art of Thinking* (English trans. of Port Royal), 1685, Part III, chap. xvii, vol. ii, p. 80. See also chap. ix of the same part: 'To avoid this vanity which savors so much of Pedantry so unbecoming a generous spirit, we ought to examine the solidity of an argument rather by the light of reason, than by forms' (p. 37).

[3] Cf. an article 'Descartes', by P. Schrecker, in a special number of the *Revue philosophique*, 1937, pp. 336–67.

matical logic similar in spirit to the later developments of, for instance, Leibniz, Jevons, or Russell. But there is one fundamental obstacle to any such *rapprochement* which must be regarded as a major stumbling-block. The very definition of intellectual intuition, *intuitus mentis purae et attentae*, includes within it the vital factor of attention not only to the form of the argument, or its *connexio*, but to the very nature of the content itself.[1] It must be clear that the logic implied in the rules of the method is one of form and content, and that deduction *qua* serial intuition is equally bound by this indissoluble liaison. There would appear to be a loophole in the cases envisaged by Rule 16, which states: 'When we come across matters which do not require our present attention, it is better, even though they are necessary for the conclusion, to symbolize them by very short symbols, rather than by complete figures.'[2] In matters not requiring 'present attention', there would appear to be room for symbolic substitution, and the more complicated and involved the deductive series, the more it would appear feasible and advisable, if only to lighten the task of memory, to substitute symbolic forms for the deductive chains. There are a few indications that Descartes might have developed a part of his method along the lines to be suggested later by Leibniz in his *Ars Combinatoria* and *Characteristica Universalis*. Hamelin suggests that some similar idea was present in the mind of Descartes as he composed the *Regulae* and when he wrote to Mersenne on the problem of a universal language.[3] But the letter does not betray any unqualified approval for the project, which looks 'more admirable at first sight than I found on looking at it more closely'.[4] The detail of Descartes's criticism, while astonishingly appropriate to even more modern attempts at similar projects of reform, is not relevant here. Towards the end of the letter he makes the point that 'the invention of such a language would depend on the true philosophy', 'since', he adds, as Leibniz was to assert later,

[1] See above, pp. 55–61.

[2] 'Quae vero praesentem mentis attentionem non requirunt, etiamsi ad conclusionem necessaria sint, illa melius est per brevissimas notas designare quam per integras figuras' (AT. x. 454^{10-12}).

[3] Hamelin, *Le Système de Descartes*, pp. 60 and 149.

[4] The letter to Mersenne was written in 1629 (cf. AT. i. 76–82).

it would be necessary to 'enumerate all the thoughts of men and put them in order', to analyse all the composite ideas and lay down the clear and simple ideas which would be, as it were, the fundamental word-catalogue of the universal language. Descartes thought, in 1629, that this might be possible, if one could find the proper science and method to discover these fundamental truths. But even then he added that it would presuppose 'a great change in the order of things' and that this world would be 'an earthly paradise', and later he argued that such a task was neither possible nor necessary.[1] Moreover, it could never be used as 'a means of discovery' but merely as a mode of retaining and fixing ideas already discovered. The gap between Descartes and Leibniz is patent in this quotation from Leibniz, writing on his own theory of the *Characteristica Universalis*: 'I hold that the invention of the form of syllogisms is one of the most beautiful which the human mind has made, and even one of the most considerable. It is a kind of *universal mathematics* whose importance is not sufficiently known.'[2]

The *vera logica* which Descartes speaks of as his own to Mersenne is certainly not akin to the ideal later to be proposed by Leibniz and to become known as logistics or symbolic logic. There is, besides, another aspect of the relation between the formal algebra of logistics and 'the method for acquiring truth' which would prevent Descartes from developing a purely mathematical logic from the methodological principles laid down in the *Regulae*. The search for truth, according to Descartes, must begin by the search for those truths which form the body of that doctrine which he calls his method. It is the search for that 'set of rules', certain and easy, so framed that 'anyone who obeys them precisely, will never take anything false for true and will advance, by an orderly expansion, step by step, without waste of mental effort, until he has achieved the knowledge of everything within his capacity'.[3] But it was noted that there cannot be any question of *learning* to use them, *qua* formal automatic

[1] *Principia*, i. 49 (AT. viii. 23^{24-24^7}), which has as its title: 'Aeternas veritates non posse ita numerari, sed nec esse opus'.

[2] Quoted by Russell in *The Philosophy of Leibniz*, new edition, p. 282. Italics in original.

[3] AT. x. 372^{17-22}.

rules, since without them we could not understand what they were all about. It was only the practice of these rules which could help to increase 'the natural light of reason' and our powers of discernment and intellectual vision. The search for these truths is in itself the practical search for truth. The apparent paradox solves itself, if it be realized that a methodology is itself to be judged, not primarily as true or false, valid or non-valid, but as useful, practical, and factually as achieving truth.

In the letter to Mersenne which has already been mentioned, and in which Descartes discusses the *De Veritate* of Lord Herbert of Cherbury, he made it clear that there was a complete difference of outlook between himself and the author of this book. 'He examines what is Truth, whereas for my part I have never doubted what it could be, since it seems to me a notion so transcendentally clear that it is impossible not to know what it is.' And he concludes that 'one can give no logical definition which could help one to know its nature', and as for rules of discovering truth, 'my own rule is the natural light' of reason.[1] The definition of truth cannot therefore lie in some purely formal equation; it cannot be defined apart from the actual awareness or apprehension of it in the act of intellectual intuition. There cannot consequently be a set of abstract rules which trace a path to truth. But the truth apprehended in any act of intellectual intuition lies outside the province of a pure methodology; its guarantee of truth is a self-guarantee, not an extraneous stamp of genuine veracity imposed by conformity to a rule or set of rules. On the contrary, in Descartes's view, it is the method which receives the mark of authenticity from the fact that self-evident truth is obtained in and through its practice. A purely formal logic, isolated by its very definition from all contact with the real, the content, and the only truth, must of necessity become more and more abstract, and in the rarefied atmosphere of empty formulas, of forms of nothing, lead to more and more sterility. Even the logistic of Leibniz was based on an elaborate metaphysical structure and the analytic method of Descartes presupposes and finds its 'truth' only in the metaphysical doctrines of the *Meditationes*. Both these philosophers

[1] Cf. Letter to Mersenne 1639 (AT. ii. 596^{16}–597^{27}).

made considerable additions to both mathematical and scientific knowledge; whether despite or because of their metaphysical basis for logic, it is not here possible or necessary to decide. It still remains to be seen whether the more modern logics will be equally productive of results as important and far-reaching. The method of Descartes is a logic, but it is a philosophical logic whose subject-matter is truth, a truth obtainable only by the actual practice of the sciences, mathematical, physical, or philosophical, with which we happen to be dealing. Expressed as it is in the *Regulae*, or in the four rules of the *Discours*, it takes on a purely normative and disciplinary character and appears to be in principle a practical art, akin therefore to the *dialectica* so viciously attacked by Descartes himself. It is practical, but not formally so, since it is primarily and essentially concerned with the actual investigation of truth itself, with concrete reality.

§ 2. *The Method of Analysis and Synthesis*

It is accepted that Rule 5 of the *Regulae*, embodying the second and third Rule of the *Discours*, contains the pivotal doctrine of the Cartesian method. For the sake of convenience it may be repeated here:

> The entire method consists in the ordering and arranging of the materials on which the mind's attention must be concentrated if we are to discover a truth. The exact observance of the method will be secured if (*a*) we reduce involved and obscure propositions step by step to more simple ones; and if then (*b*) we start from our intellectual intuition into the simplest propositions and endeavour, by retracing our path through the same steps, to work our way up to the knowledge of all the others.[1]

There is, as the admirably succinct wording shows, a twofold *processus* in thought, a combination of analysis and synthesis. We discover the constituent parts of the problem by analysis, and by a discovery of its simplest elements we find the rational solution which the second process, synthesis, enables us to construct in all the complexity of its detail. Now this part of the method, as indeed the whole method, is confessedly based on the procedure of the mathematical sciences. A general doubt may inevitably grow in the mind of the reader

[1] AT. x. 379[15–21]. For Latin text see above, p. 150.

as to the legitimacy of introducing a method, basically mathematical, into the field of metaphysics. Starting from some such doubt, M. Serrus, after a careful investigation of the Cartesian doctrine of method, arrived at the conclusion that this method, which was fruitful in the mathematical sciences, was impotent in the construction of a metaphysical doctrine.[1] Some of the points he raises must be considered as well as the answers suggested by the doctrine of method as elaborated in this study. According to Serrus, the *Regulae* does set forth a methodological view which is unquestionably the method of mathematics. 'In truth, he [Descartes] was the first to become conscious of a mathematical science liberated *ad maximum* from sense-imagination and based on a limited number of principles from which all the rest are logically deducible.'[2] It was Descartes who 'created the science of *order*, in which *pura Mathesis* is summarized, and which certainly represents, with the theory of groups, what quite incontestably is logical in modern mathematics'.[3] The essential factor in the success of the Cartesian method is that it evolved a science of pure quantity and order and thus established the possibility of mathematical analysis.

Before turning to the less favourable comment, it is perhaps as well to hesitate before these words of praise. The doubt from which we began is akin to the very grounds of this encomium. The account of analysis given by Descartes is expressed in terms which suggest that every complex must be resolved by analysis into 'simple' constituents or parts and then, by the inverse process, synthesized by combining those constituents in such an order that their interrelations or connexions, when they are combined together, are manifestly intelligible. But such an analysis of a complex would presuppose that all complexes are in fact susceptible of such treatment, of reduction to simple, isolable units or constituents and are also the result of combinations of such units, sums of units or aggregates. That such a conception is applicable in a science whose object is pure quantity may be granted, but it would be fatal, if the method of analysis

[1] Ch. Serrus, *La Méthode de Descartes et son application à la métaphysique*, 1933, p. 124.

[2] Ibid., p. 33.

[3] Ibid., p. 34.

presupposed that all complexes were in reality mere aggregates of composite units and not genuine wholes. The Cartesian method would, from the outset, be doomed, or liberated—the choice depends on a personal point of view—from any value as a method for metaphysical inquiries. It is no idle pretence to say that Descartes's language at times might give plausibility to some such interpretation. The doctrine of 'simple natures' is not easy to fit into the general picture. The dogmatic assertions of Rule 12 stand out: 'We can never know anything beyond those simple natures and what may be named their mixture and composition',[1] and the expression, used later of '*res copulatae*', is strongly reminiscent of Bacon and his crude view of analysis. We were left with a great deal of guesswork in determining the meaning of *mixtura* and *compositio*. But this much must be accepted: the character of the simple nature is such that its simplicity is *for* the knowing mind only and the mixture or composition is not a mixture of concrete, separate reals or elements in an aggregative whole, but a logical combining of implicatory elements in a whole of knowledge. It may be interesting to treat Descartes as a kind of *präexistent-Kantien*, and to trace in his work the implicit theses of critical philosophy, as does Natorp. He may be, as Hamelin suggests, *un idéaliste qui s'y ignore*. These facets have more verisimilitude than the existentialist of Jaspers or the Marxist of H. Lefebvre.[2] But from Descartes's own words, it is clear that reality is not constructed by the human mind; the mind knows reality. Knowledge itself is built up and constructed from the data apprehended in intellectual intuition, through which act the mind is in immediate contact with the real. It is through this contact with the real that the universal concepts keep their value as indicative of the nature of the real. The thoroughly unsatisfactory account of the process of abstraction leaves this key point vague and ambiguous. It is made clear to us that, in his view, a complex

[1] AT. x. 422$^{7-9}$. See also in the same Rule: 'omnia quae cognoscemus, ex istis naturis simplicibus composita esse' (420$^{10-11}$) and, 'Colligitur tertio, omnem humanam scientiam in hoc uno consistere, ut distincte videamus, quomodo naturae istae simplices ad compositionem aliarum rerum simul concurrant' (427$^{3-6}$).

[2] Cf. Natorp, *Descartes Erkenntnistheorie*, 1882; Hamelin, *Le Système de Descartes*, 2nd edit.; Jaspers, *Descartes und die Philosophie*, 1948; H. Lefebvre, *Descartes*, 1947. M. Sarthe has also claimed Descartes in an article in *Labyrinthe*, no. 14.

whole can only become known to us in terms of the simpler features of the whole. Our knowledge of complex reality is a function of our knowledge of its simple elements and 'their necessary links'. This was the point of the doctrine of 'absolute' and 'relative' terms and their added determinants the '*respectus*'.[1]

It is against this view of analysis and synthesis that the criticism of Serrus is directed, that is, analysis and synthesis as applicable beyond the domain of quantitative reality. The words and examples of the *Regulae*, however generously interpreted, certainly lend colour to such a picture of the scope of the method, even though it must equally be granted that such was not the final interpretation of Descartes himself. The analysis and synthesis, which constitute the essential of the method, result in a number of interconnected implicatory sequences which are called deductions. From the point of view of the synthetic process, each sequence or deductive series consists of terms growing in complexity as the sequence advances, the advance being gradual or step by step. In the language of the *Regulae*, each sequence must contain a 'most simple' or absolute term which embodies 'a simple nature' purely and perfectly. In the succeeding terms this same 'nature' must be again embodied, more or less partially or derivatively, and together with it, added to it, are one or more other features which embody, again partially and derivatively, other 'natures'. The doctrine suggests that it is always by 'subtraction' that the terms yielded by analysis become progressively more simple and it is by 'addition', in the synthesis, that the successive terms become more complex. Given the wording of Rule 18, it would almost seem an understatement to talk of mere suggestion in this context.[2] If, then, in any deductive sequence, x be the simplest term, or *maxime absolutum*, then the next term, a *relativum*, will always contain x plus an addition. If x embodies precisely the simple nature α, so that the character of $x = \alpha$, then, on the view of Serrus, the deductive synthesis means that y, the first relative

[1] See above, pp. 163–6, and below, p. 291.

[2] 'Ad hoc quattuor tantum operationes requiruntur, additio, substractio, multiplicatio et divisio, ex quibus duae ultimae saepe hic non sunt absolvendae' (AT. x. 461[12–14]).

term, contains what is the nature of x plus something else (which we may call β) so that we have the equation $y = (\alpha+\beta)$. If z is y plus some addition (e.g. γ), then $z = (\alpha+\beta)+\gamma$. If this is all that analysis and synthesis mean, then the method cannot deal with any type or kind of complex other than a purely quantitative series, and, indeed, as Kant and Hegel would point out, not even with these.

The above account of analysis and synthesis, as has been already shown, does not, in fact, fully represent Descartes's views.[1] Admittedly, the vagueness of his doctrine of abstraction reflects itself in the imprecision of the doctrine of distinctness and necessary connexions, as one of the ablest of his defenders is forced to assert.[2] The relation between the absolute term, the basis of their distinction, is not merely that of simple and composite; it is also that of universal and particular, as well as that of cause and effect. Now these types of relationship are not reducible to a mere subtractive or additive process. The real object is not a composition or combination of universals. The latter are obtained by abstraction,[3] but they are not really distinct from the things from which they are abstracted. The universal may exist only for knowledge, but knowledge requires that the universal should ultimately be based on intuition of the real. What Descartes is trying to assert, it seems, is that we can come to knowledge of the real world only by knowledge of the 'simple natures', or 'primitive notions' which must underlie its structure. In so far as the real is susceptible of this resolution into 'simples', that is, completely resolvable, our knowledge will in fact be perfect. This is true of the world of mathematics, where number and numerical proportion would appear to satisfy the requisite conditions. Yet it must also be remembered, as became slightly clearer when the application of the method to the physical sciences was discussed, that while the ideal of a deductive science of implicatory sequences was to be kept as the end of scientific thinking, the method did not pretend to deduce the 'factual' as such from universal and simple principles. 'The union of these things [simple natures] is

[1] See above, pp. 175–6.
[2] Cf. Laporte, *Le Rationalisme de Descartes*, p. 482.
[3] Rule 12 (AT. x. 418[21]) 'ex simplicibus ipsis abstrahimus'.

either necessary or contingent', was the vital distinction in Rule 12. 'The union or link is contingent in those cases where the things are linked by no inseparable relation, as, for instance, when we say a body is animate, a man is clothed, &c.'[1] Some facts as such may remain unanalysable, and as such inexplicable, except by the metaphysical explanation of the will of God. In so far as we can be said to 'know' these facts, we know their universal features. I may 'know' what I mean by saying Socrates is a man, that he has a body, that he breathes, &c., although I do not 'know' what really makes him Socrates. That the self-conscious, self-identical *res cogitans* Socrates should have existed is not deducible because it is a contingent fact. We must fall back on experience to find the answer to the question of whether he exists or not.

The *suggestio falsi* lies in the way the *Regulae* describes and illustrates the doctrine of analysis and synthesis. With few exceptions the examples are taken from the mathematical sciences. The last rules, almost entirely dealing with problems relating to magnitudes, cannot fail to give the impression that the object of the method, that is its function as a set of methodological principles, is restricted to dealing with quantitative reality, or a manifold of quantities susceptible in theory of being measured directly or indirectly. Now it is accepted, or was accepted traditionally, that philosophy deals not with quantitative facts but, to keep a formal verbal opposition, with qualitative facts, facts of consciousness or facts appertaining to, or derivative from, knowledge and knowing; 'conscious facts' as opposed to and contrasted with 'material facts'. The method of Descartes would seem to ignore this distinction, while nevertheless, patently and clearly, basing the distinction, for instance, of the *vis cognoscens*, the pure activity of mind, from the mixed activities, sense, imagination, &c., the corporeally-dependent activities, on the radical division of body and soul. Yet—and there can surely be no such need to document this—whatever divergences of doctrine there may be between the *Regulae* and the later works—on the theory of judgement, for instance—there cannot be any grounds for suggesting that the radical distinction between *res cogitans* and *res extensa* is not fully present in the early

[1] Rule 12 (AT. x. 421^3–422^6).

work. The method, however, is explicitly a universal method and it refers, therefore, to both the fundamental types of existence. The insistence on analysis in terms of quantitative analysis may be accounted for, to some extent, by a mere over-emphasis in Descartes's exposition.

It will be remembered that the doctrine of *Vera Mathesis* implied that, in order to solve its problems concerning magnitudes-in-general, the *vis cognoscens* was to be supplemented and aided by the imagination, the function of the imagination being the visualization of spatial properties, proportions, figures, or shapes. It was due to this visualization, this imaginative representation, that the idea or notion of quantity was applied, as it were, to these 'pictures' of external spatial characters and relations. These 'pictures' are due to activity on the part of the body, to corporeal mechanism. But, in principle, the *vis cognoscens* is one and identical in all its activity, and is present in all acts of imagining, as well as in sensing and remembering. The differences which distinguish the various experiences of 'knowing' (in the wide sense as defined in the *Principia*) from one another arise solely from the differences in the objects to which the *vis cognoscens* is variously applied. But when the objects of knowledge, as in the case of the self and God, do not as such depend directly or indirectly on any sense-knowledge or imagination, when, by definition, it is impossible to have a spatial image or awareness of bodily change, what then becomes of the elaborate doctrine of *Vera Mathesis*? There can be no question here of imagining or visualizing magnitudes, since the objects are essentially non-spatial, non-quantitative. There can be no question of setting up equations in terms of measurable quantities since the objects are non-measurable. The objects of philosophy are, by their very nature, to be apprehended by the pure activity of mind.

Such a conclusion does modify the account of *Vera Mathesis*, in so far as it excludes the subsidiary aid of visual imagination in dealing with philosophical matters—and Descartes may well be blamed, then, for the *suggestio falsi* left in his readers' minds by the almost exclusive concentration on the method as applied to the mathematical sciences, with the consequent over-emphasis mentioned above. But, it should

be added, this conclusion is explicitly stated in the *Regulae* in the clearest terms. For while Rule 12, anticipating the auxiliary use of imagination in *Vera Mathesis*, states in the opening paragraphs: 'The understanding indeed is alone capable of apprehending the truth, nevertheless it ought to be aided by imagination, sense, and memory, lest by chance we let slip any expedient that lies within our power';[1] this is modified some pages further on by the assertion: 'we come to the sure conclusion that if the understanding is dealing with matters in which there is nothing corporeal or akin to the corporeal, then *it cannot be helped* by these faculties but, on the contrary, to prevent the understanding being hampered by them, the senses must be banished and the imagination as far as possible emptied of every distinct impression.'[2] *Vera Mathesis*, modified as here indicated, becomes essentially the science of order, but the order is that of logical implication and metaphysical priority, the real order of spiritual natures starting from the original datum of the *Cogito*, the order as illustrated in the analysis of the *Meditationes*. It demands the exercise of the 'pure' faculty of mind; Descartes does not imply that such an exercise is easy or can be switched on or off at will. The insistence in the *Regulae* on the need for the removal of prejudices, for constant attention, sagacity, &c., signifies that a certain ascetic effort of concentration is required and that in this matter, no less than in Hesiod's day, the Gods have placed labour before excellence. The effort required is reflected in the memorable passage at the beginning of the *Third Meditation* which runs: 'I shall now close my eyes, I shall stop my ears, I shall avert my senses, I shall even efface from my consciousness all the images of bodily things, or at least, because that is hardly possible to achieve, I shall hold them to be vain and false; and thus, holding converse only with myself, and considering my own nature, I shall try to obtain little by little a better and more intimate knowledge of myself. I am a thinking being, that is to say, one who doubts, affirms, denies, knows a few things and is ignorant of many, who loves, hates, wills, refuses, who imagines and perceives; for, as I have remarked before, although the things which I perceive and imagine are perhaps nothing apart from

[1] AT. x. 411$^{7-10}$.

[2] Ibid. 416$^{23-28}$; my italics.

me and in themselves, I am nevertheless assured that these modes of thought, which I call perceptions and imaginations, inasmuch as they are modes of consciousness only, exist certainly in me.'[1] Both the effort of concentration and the almost insuperable difficulty of excluding all imagery are described with convincing sincerity. It is not surprising that the phrase *abducere mentem a sensibus* is found coming so frequently from the pen of Descartes.[2] For this negative process must be the condition of any pure exercise of the mind; without it, there can be no philosophical datum, no fundamental equation in the spiritual order.

Decorticated, as it were, from its 'mathematical shell', stripped of most of its technical trappings which have no applicability outside the province of quantitative analysis, the method may seem to contain but the bare outlines of words of sound advice on the necessity of thinking clearly, avoiding precipitancy in our judgements, and setting our ideas out in their proper logical order. Inevitably the complaints of Leibniz occur: 'I almost feel like saying that the Cartesian rules are rather like those of some chemist or other: take what is necessary, do as you ought to do, and you will get what you wanted. Do not admit anything except what is evidently true (in other words, except what you ought to admit), divide the matter into as many parts as is requisite (that is as many as you ought to), proceed in the order (in which you ought to proceed) and make a perfect enumeration (as you ought to).'[3] But how is all this to be achieved? The answer has already been given. Gilson puts it in these words: 'Descartes does not teach us the method but what we must do in order to master

[1] *Meditationes*, III (AT. vii. 34^{12}–35^{2}), the words, 'qui aime, qui hait', and 'the things which I perceive ... "in themselves"' are only found in the French translation (AT. ix. (Part I), 27).

[2] It occurs often in the *Meditationes* (AT. vii), in the *Praefatio*, 9^{25-27}; in the *Synopsis*, 'animas a sensibus abducarem', 14^{20-21}; in the passage quoted above; at the beginning of the Fourth, 52^{23-24}, as well as in the *Responsiones*. The whole of art. lxxiii of the *Principia* is devoted to the difficulty of the 'pure' use of the understanding. Descartes notes that 'omniumque difficillime ad illa attendit [mens], quae nec sensibus, nec quidem imaginationi praesentia sunt' (cf. AT. viii. 37^{5-22}). Lesser spirits will agree with Pascal that 'l'esprit du plus grand homme du monde n'est pas si indépendant qu'il ne soit sujet à être troublé par le moindre tintamarre qui se fait autour de lui'.

[3] *Philos. Schriften* (edit. Gerd, iv. 329) quoted by Laporte, *Le Rationalisme de Descartes*, p. 28, n. 4.

the method. In reality, there exists no *a priori* recipe by means of which to distinguish, in a given problem, the simple from the composite, but one can acquire the knack of it by a kind of progressive training . . . the third precept of the method does not demand that we should admit an abstract truth but rather that we should acquire an intellectual habit.'[1] Leibniz is asking for more; he wishes for some formula equivalent to his own ideal of an *Ars Combinatoria*. Nothing, as we have seen, could be farther from the Cartesian conception, with its Leibnizian presupposition of formalism and the possibility of 'giving reason a holiday'. There is no easy way to truth, no philosophers' stone to convert the dross of ordinary everyday experience into magical wisdom. Philosophical thinking, as all types of thinking, is hard work and we can only make progress by 'cultivating our mind', by practice, by philosophizing. There is no taint of intellectual snobbery in the advice of the *Discours* to those who feel incapable of the effort—Descartes admits that he himself might, but for circumstances of education and reading, have belonged to this class of people: 'There are those', he says, 'who, having sufficient reason or modesty to judge that they are less capable of discriminating truth from falsehood than others by whom they might be instructed, are right in being content with the opinion of such men rather than in searching for better ones for themselves.'[2] The words of Rule 9 are a corollary to this: 'It is true that some men are born with a much greater aptitude for discernment than others, but the mind can be made much more expert for this work [analysis] by art and exercise.'[3]

§ 3. *The Method in the* Meditationes

There is a further point raised by Serrus. He considers that the method of Descartes, while applicable in the mathematical sciences, is not valid in metaphysics. This is not quite a fair statement of Serrus's views, since the point raised is more complex and subtle. There are, he argues, two 'methods' in Cartesian thought: the first, that of the *Regulae*, incomplete,

[1] Gilson, *Commentaire*, pp. 207–8. [2] *Discours*, Part II (AT. vi. 15^{26-31}).
[3] 'Ad quod quidem nonnulli longe aptiores nascuntur, quam caeteri, sed arte etiam et exercitio ingenia ad hoc reddi possunt longe aptiora' (AT. x. 402^{2-4}).

yet 'liberated from the syllogism'; the other, that actually used in the *Meditationes*, in which work Descartes is in fact proceeding according to the traditional mode of syllogistic logic, the logic of the Schoolmen, from self-evident propositions and *a priori* principles, indemonstrable and guaranteed ultimately by the existence of God. If we therefore ask whether Descartes did apply his method to metaphysics, the answer must be given that he did apply it, in so far as he used the Scholastic elements which had survived in it, but that, in so far as the original and specifically mathematical elements of methodology expressed in the doctrine of *Vera Mathesis* are concerned, he did not apply it because this genuine aspect of the method cannot be applied outside the domain of the mathematical, and physical, sciences. In other words, Descartes did not apply *his* method when he came to treat of philosophical questions. It is not suggested by Serrus that Descartes was aware of this change of front. The reason why the method, that is, the original method, is not applicable in metaphysical problems may be summed up in this contention. The method, as used in the mathematical and physical sciences, in what Serrus calls the hypothetico-deductive systems, does not contain assertions about existence, whether necessary or contingent, nor propositions of an attributive character; it is essentially based on a relational logic, and is valid only for relational propositions. These sciences do not posit the problem of the criterion of truth and they are indifferent to the objective truth of their axioms.[1] On the other hand, the method used by Descartes in his metaphysical treatises involves the use of 'general propositions which can only be used in the syllogism'. Descartes set himself 'the impossible task of constructing deductively a system of thought based on the relations which the verb "to be" introduces into the proposition, and even on different senses of the verb "to be" '.[2] 'Syllogistic logic is a logic of classes and not of relation',[3] and the metaphysics of Descartes surrepti-

[1] '1) Les mathématiques ne font jamais intervenir le jugement d'attribution ni le jugement d'existence; 2) elles ne posent pas le problème du critérium de la vérité; 3) elles sont indifférentes à la vérité objective de leurs axiomes; 4) les principes qu'elles admettent ne les intéressent que par les combinaisons des relations qui peuvent en sortir', Serrus, *La Méthode de Descartes*, p. 118.

[2] Ibid.

[3] Ibid., p. 112.

tiously reintroduces the Scholastic logic back into the whole philosophical method. For not dissimilar reasons, Boyce Gibson concludes that 'the metaphysical method is not beholden to the method of scientific logic. As we shall see, Descartes' actual procedure in metaphysics is a scandal to almost every principle of his scientific logic.'[1]

The only way to meet this wide challenge is, as its authors have done in formulating its terms, to take the arguments of the *Meditationes*, not merely in their general structure, but in the actual detail of their demonstration. This involves nothing less than a discussion of the central thesis of Cartesian metaphysics, the *Cogito*, the relation of this basic certainty to the general criterion of clearness and distinctness as the criterion of truth, the arguments for the existence of God and their logical implications, the doctrine of ideas and judgement, the proof of the existence of matter, &c. It is obviously impossible to undertake such a discussion within the scope of this book which is limited to a general study of the method. It might be objected that such a deficiency may well indicate the vanity of attempting to examine the method of Descartes apart from the actual application of it. The pointed truth of this objection lies in the Cartesian contention that method and doctrine are indissolubly bound up together and that the method arises in fact from reflection upon the doctrine. The only justification for a commentary, restricted in principle to the *Regulae*, lies in the *Regulae* themselves which, like parts of the *Discours*, would argue that it was possible, and useful, to lay down certain methodological principles. But, however unsatisfactory this may be here, no adequate answer, short of a detailed examination of the work itself, can be set forth, or even attempted, to the objection that Descartes did not use the 'original' method in the *Meditationes*.

A few points may, however, be noted which throw some light on the alleged relapse of Descartes into the syllogistic form of reasoning which he is, at least verbally, so anxious to reject as sterile. It is vain, we have already seen, to look for any set expression of logical doctrine in his writings. The methodological doctrine, of which he is said to be the innovator, is described variously as a 'logic of relations' or a

[1] Boyce Gibson, *The Philosophy of Descartes*, p. 86.

'relational logic'[1] and is contrasted, favourably, with some form of Aristotelian logic of attribution or predication. Now this 'logic of relations', a phrase entirely remote from Descartes's language, must be taken to refer to the doctrine expounded in Rule 6, where absolute and relative terms are defined.[2] On the basis of these definitions, it was argued by Descartes that the terms of a sequence, all the terms, that is, except the first or 'most absolute' term, are to be called 'relative' (*respectiva*) terms, because the conception of them includes one or more features in addition to the 'nature' which was embodied precisely and simply in the first term. They share in the nature which the first term purely and simply contained and they can be shown to derive from it by a continuous deductive chain. But, Descartes added, they include also *respectus*, determinants, added to the nature which they share with the absolute term. These *respectus* are essentially relative since they are those features which relate the terms to terms other than themselves and which belong to other implicatory sequences. If we ask how these terms and their *respectus* are related, we are told it is by a *connexio necessaria*, a necessary connexion.[3] On this notion of necessary connexion turns the whole theory of deduction in Cartesian philosophy. Now it does not seem clear how the *illatio unius ab altero*, which was said to characterize the movement of the mind in the deductive process, can draw from the simple nature any attribute other than the simple nature itself which, by definition, contains nothing but itself purely and simply. In that case, what kind of necessity could be said to link the *respectus* to its relative term, except some external form of synthesis, which is contrary to the Cartesian view that synthesis merely reconstructs links and linkages which have been discovered by analysis? There cannot be, despite the loose phraseology of Rule 4, any view of the relation between subject and predicate, by which 'the subject contains the predicate'.[4] This difficulty, over-simplified and crudely stated here, has been

[1] The expression is used by A. Hannequin in *Études d'histoire de philosophie*, pp. 208 ff., where he bases his use of it on the doctrine set out in Rule 6. Serrus and Brunschvicg also use it.

[2] See above, p. 163.

[3] See above, pp. 93–99.

[4] 'Et ad veritates ex quovis subjecto eliciendas se extendere debet [methodus]' (AT. x. 374^{8-9}).

met, as far as is possible, earlier in this study.[1] It will not do when dealing with Descartes, as Serrus tends to do throughout his work, to attempt to apply the Kantian distinctions of 'analytic' and 'synthetic' to the propositions enunciated, whether these propositions be taken from the mathematical, physical, or metaphysical fields of knowledge. They are not appropriate for two reasons: the first is that the words 'analytic' and 'synthetic' apply in Descartes's terminology only to the processes of mind involved in the *discursus*; and secondly that, if pushed as to the nature of the link between two terms, he would reply, in Laporte's words, that 'if two things are connected, it is necessary that they should be different without being so'.[2] This phrase, to some perhaps too reminiscent of an Hegelian 'identity in diversity', must be interpreted in the light of Descartes's use of the distinction of reason.[3] What we may think as distinct, is not necessarily so in reality; we are considering the matter *quoad nos*. And it is from the same point of view that the doctrine of analysis and synthesis is expounded. The *connexio necessaria* is apprehended by an act of intellectual intuition, in the same way as the simple natures.[4] There is thus, in the simplest act of mind, the recognition of a necessary nexus between the two elements, the *implicatum* and the *implicans* as well as their *implicatio*. Joachim notes on this point: 'However much Descartes may stress the intuitive character of the recognition—its formal "immediacy", its unitary wholeness—there is an act, or *discursus*, of thought involved. The "content" of this, as of every, *intuitus*, is a "simple proposition"; and the "simple insight of the mind" is an "analytic-synthesis" or a "synthetic-analysis", by which it distinguishes and holds apart, and *eo ipso* unifies and connects by a logical necessity, the two elements (the antecedent and consequent) of the proposition it views as "one" and "whole".'[5] This interpretation of the

[1] See above, pp. 107 ff.

[2] 'La pensée de Descartes, en l'espèce, paraît revenir à ceci: que si deux choses sont "liées", *il faut* qu'elles soient différentes sans l'être', *Études cartésiennes*, ii (9th Inter. Congress of Phil. 1937), p. 10.

[3] See above, pp. 94–95.

[4] Rule 12: 'Atque perspicuum est, intuitum mentis, tum ad illas omnes extendi, tum ad necessarias illarum inter se connexiones cognoscendas' (AT. x. 425[14–16]).

[5] H. H. Joachim, *Logical Studies*, p. 142.

central doctrine of the method may perhaps be said to be a favourable view of Descartes at his best, for, although it is undoubtedly the doctrine which Descartes is putting forward in the *Regulae*, his recognition of the double aspect of every act of intellectual intuition—and deduction is a serial intuition—is not so steadfast and sure that he does not waver at times and assert too sharp a distinction between the analytic and synthetic processes. But his doctrine clearly implies that the primary datum, the *minimum cognoscibile*, is 'a simple nature' which is also 'a simple proposition' and that the essential act of knowing is an act of *intuitus* and *deductio*, while the *illatio* itself is both intellectual intuition and *discursus* at the same time and that there cannot be one without the other. The examples we are given of such acts include the major truths of the *Meditationes*. It is difficult to admit, prima facie, that Descartes reverts to a syllogistic doctrine of subject-predicate inclusion.

There is, moreover, one other consideration which may easily be forgotten, or passed over, but which, despite its simplicity, obviousness, and almost childishness, is paramount in reading Descartes's account of his method. To engage in a controversy as to whether Descartes did or did not apply his method when he turned to the treatment of philosophical problems, might well lead the disputants to accept the common assumption that it was possible, on Descartes's own view, to formulate a set of rules, determinate and formal, which could by their own virtue constitute a logical standard or criterion for judging the validity of the procedure and the truth of the conclusions obtained in and through the application of these rules. Enough has already been said to show that such a formalistic view is exactly the contrary of his doctrine of method; the attack upon the syllogistic mode of reasoning is directed against this very assumption. There cannot be a method which stands or falls on its own merits, considered apart from that of which it is the methodical activity. Method is not an end in itself, having, on its own account, an intrinsic value which can both be declared and valued, as well as an ulterior value as a means to mathematical, physical, or metaphysical knowledge. It is to be judged as having intrinsic value because it achieves its ulterior end,

because we arrive at knowledge by its practice.[1] Further, the method, that is to say, its expression as a set of rules, is based on the 'natural' method. In Descartes's own account, the requirements epitomized in these rules are the result of his reflection on the procedure of the mathematical sciences, but this procedure was studied only for disciplinary purposes, as expressing in a limited degree the activity of the nature of reason itself, the 'natural light'. The study of *Vera Mathesis* is meant to remedy the deficiencies in the existing sciences of geometry and arithmetic. What we are trying to get hold of in and through our study of these sciences is the 'natural' method, the proper procedure of the human reason itself. The rules can only be formulated as a consequence of this reflection, or, as Hamelin puts it, 'there are only general rules after the intuition of the particular instances'.[2] The differentiation of method between the mathematical and the physical sciences arises only after reflection on the difference between the problems set by these sciences. The same is true in the case of the philosophical sciences. The remarks by which Descartes introduces his demonstration 'drawn up in geometrical fashion', in his *Reply to the Second Objections*, show that he was keenly aware of this differentiation. The method which he is asked to use, though suitable in the domain of geometry, 'nevertheless cannot be so conveniently used in the metaphysical matters under discussion', and the primary reason given is the central difficulty, not merely of synthetic exposition, but that of the primary analysis which synthetic exposition presupposes. 'For', he says, 'nothing in metaphysical study causes more trouble than to conceive the first notions clearly and distinctly.'[3] This is not a declaration that this essential requirement of the method cannot be met in dealing with metaphysical matters, still less a confession that he has not been able to apply the method of analysis in

[1] The opening paragraph of the *Regulae* says of 'bona mens': 'Alia omnia non tam propter se, quam quia ad hanc aliquid conferunt, sint aestimanda' (AT. x. 360^{20-22}).

[2] Hamelin, *Le Système de Descartes*, p. 105.

[3] 'Non tamen ad has Metaphysicas tam commode potest applicari' (AT. vii. 156^{25-26}); 'Contra vero in his Metaphysicis de nulla re magis laboratur, quam de primis notionibus clare et distincte percipiendis' (ibid. 157^{6-8}). Descartes tells Burman that 'ad ea quae Auctor in Philosophia scripsit intelligenda, Mathesi non opus est, nisi forte pauca quaedam in Dioptrica, quae mathematica sunt, excipias' (AT. v. 177).

the *Meditationes*.[1] What it does imply is that this apprehension of the 'first notions' cannot be obtained 'except by those who give very careful attention and reflection to them and, as far as is possible, withdraw their minds from corporeal things'.[2] This is the initial lesson of the *Regulae*.

It is probable that Descartes would have agreed that his methodological theory, especially in its relation to metaphysics, remained incomplete, but he would have insistently maintained that the practice of the method would have been fruitful and instructive. He might have pointed to what he had written in Rule 8 on how he proposed that his reader should begin his studies. 'This method', he there writes, 'resembles in fact those devices used in the mechanical crafts, which have no need of any external aid but themselves supply the directions for making their own tools. Thus if a man wished to practise any one of these crafts, as, for instance, that of a blacksmith, and was entirely without tools, he would be forced at first to use a lump of hard stone, or some shapeless lump of iron, as an anvil, take a piece of rock as a hammer, make pieces of wood do service as tongs, and provide himself with other tools according to his needs. Thus equipped, he would not at once have set about forging swords or helmets or other iron articles for the use of others. He would first of all forge hammers, an anvil, tongs, and other tools useful for his own work. This example teaches us that if, at the outset, we have only been able to discover some vague principles which seem rather to be innate in our minds than produced by method, we should not straightaway try to settle the controversies of philosophers or solve the problems of mathematicians by their help. We should first use them to discover with the greatest care all the other things which are necessary for the investigation of truth, all the more so since there is no reason why it should be more difficult to discover these than any of the solutions of problems set in geometry, in

[1] On the contrary, he declares: 'Ego vero solam Analysim, quae vera et optima via est ad docendum, in Meditationibus meis sum secutus' (AT. vii. 156^{21-23}).

[2] 'Non nisi a valde attentis et meditantibus, mentemque a rebus corporeis, quantum fieri potest, avocantibus, perfecte cognoscuntur' (ibid. 157^{12-14}). See above, p. 286, n. 2. There seems to be an echo of Descartes in Newton's declaration in the *Principles*, i. 9: 'We ought to abstract from our senses, and consider things themselves . . .'.

physics, or any of the other sciences.'[1] To which, one feels, he would like to add the malicious note from the following Rule:

> It is a common failing amongst mortals to regard the more difficult things as the fairer, and the majority believe that they have learned nothing when they see a very clear and simple explanation of a fact, while in the meantime they admire the lofty and profound doctrines of the philosophers, even though these, for the most part, rest on grounds that no one has ever sufficiently examined—a kind of mental disease which prefers darkness to the light.[2]

To return to the challenge of those critics who deny that the method employed in the *Meditationes* is in fact that advocated by their author in the *Regulae* and *Discours*. If it is not possible to meet fully and squarely this criticism here in the manner in which the criticisms themselves are formulated, that is, by a detailed discussion of the arguments themselves, some profit may be had from following up a suggestion made by Descartes himself in a letter to Huygens, in which, while warning his correspondent that although whole weeks of reflection are required to meditate on the subjects treated in this work, he says that some advantage may be gained by running through the *Meditationes* 'all in one breath' in order to note the sequence of the argument.[3] Some such attempt should show, however sketchy and superficial it must perforce be, the general lines of the analytical method which Descartes explicitly states that he has applied 'as accurately as possible' in the course of this work. After all, Descartes himself has facilitated the task by writing the *Synopsis*.

The problem, or *quaestio*, is set before us in the very

[1] AT. x. 397^{4-26}. Spinoza, in the *De Intellectus Emendatione*, discussing the point as to whether in investigating the 'best method', there is need of another method, 'et ut secundus methodus investigatur, non opus est alia tertia', &c., uses the same simile as Descartes: 'Sed quemadmodum homines initio innatis instrumentis quaedam facillima, quamvis laboriose et imperfecte, facere quiverunt, iisque confectis alia difficiliora minori labore, et perfectius confecerunt, et sic gradatim ab operibus simplicissimis ad instrumenta, et ab instrumentis ad alia opera et instrumenta pergendo eo pervenerunt, ut tot et tam difficilia parvo labore perficiant: sic etiam intellectus vi sua nativa facit sibi instrumenta intellectualia' (Gebhardt, ii. 13^{30-145}).

[2] Rule 9 (AT. x. 401^{11-18}).

[3] He writes to Huygens in 1640 '— mais je dirai: si ce n'est qu'on prenne la peine de lire tout d'une haleine les cinq premières Méditations . . . et qu'on fasse un abrégé des principales conclusions, afin qu'on en puisse remarquer la suite' (AT. iii. 242^{2-6}).

title-page of the volume published by Michel Soly in 1641, the two essential points of *prima philosophia*, God and the immortality of the human soul, covering the whole range of metaphysical questions which Descartes intended to treat.[1] Now the *First Meditation* deals with the grounds 'on which we may doubt in general of all things, and especially of material objects'.[2] The reason why we are to begin by this doubt is stated in these terms: the usefulness of this universal doubt lies in the fact that 'it delivers us from all prejudice and sets out for us an easy way by which the mind may detach itself from the senses'.[3] All this is familiar ground to the reader of the *Regulae*; the removal of prejudice and sense-perceptions was posited as the prime condition, that is the negative condition, of the pure activity of the *vis cognoscens*, and this pure activity is necessary in order to apprehend the truth or truths discovered in the metaphysical inquiry. Clearly here we have a genuine and correct application of the method. Within the limits of this first step of the inquiry we are led to an analysis of 'the things which can be doubted'. Here we are told that it is not necessary to 'show that the whole of these are false', that is, by taking each opinion, belief &c., individually, but a preliminary classification is to be made—the *perquisitio* of the *Regulae*—the sources and kinds of belief, &c., are first grouped and the test is applied to each group, including the ultimate test of the malignant demon. The result is known: it is the 'darkness' or, as the next *Meditation* begins, 'the deep water' of universal uncertainty.

The problem raised by this universal doubt in its ultimate metaphysical form is not solved until the *Third Meditation*, when the demonstration of the existence of God annihilates the hypothesis of the 'malignant demon' and offers a firm guarantee of the value of human thought. The *Second Meditation* therefore has an intermediary, but essential, role in that the transition from the universal doubt to the certainty of the existence of God is an almost dialectical reversal from doubt to certainty, from imperfection to perfection. The transition

[1] The title gave rise to a great deal of correspondence with Mersenne, who pointed out that it did not correspond to the contents. Descartes put his subject as 'toutes les premières choses qu'on peut connaître en philosophant par ordre' (AT. iii. 239^{6-7}).

[2] Synopsis, AT. vii. 12^{1-2}.

[3] Ibid. 12^{6-8}. Again we have 'ad mentem a sensibus abducendam'.

necessitates an analysis of the nature of the mind. We have, on this point, Descartes's own explanation. In the *Reply to the Second Objections*, he says: 'Since our previous ideas of what belongs to the mind have been completely confused and mingled with the ideas of sensibles, and this was the first and main reason why none of the propositions about God and the soul could be understood with the requisite clearness, I considered that I should be doing something worthwhile if I showed how the properties or qualities of the soul are to be distinguished from the qualities of the body. Many have doubtless held that, in order to understand metaphysical facts, the mind must be abstracted from the senses, but no one hitherto, to my knowledge, has shown how this was to be done. Now the true, and, in my opinion, the only way to do this is found in my *Second Meditation*, but such is the nature of the problem that it is not enough to have considered the matter once; it must be examined for a long time and the examination repeated, if we are to form the opposite habit of distinguishing intellectual from corporeal matters, thus destroying the life-long habit of confusing them. This appeared to me a very good reason for restricting my subject to this point in the *Second Meditation*.'[1]

The analytical method as applied in the *Second Meditation* has a double role. It must push the doubt to the extreme until 'in the use of its privilege of freedom, it assumes that all the objects, of whose existence it has even the slightest doubt, do not exist and recognizes that it is, however, impossible that it does not exist itself'.[2] At the same time, the rejection of all knowledge based on sense-perception must show the value of pure and simple intellectual intuition as the source of certain knowledge. The analysis, then, of the objects of knowledge has, as its first objective, the revelation of the existence of the subject. From the doubt arises the first certainty: 'if I doubt, then I am' indicates the process of the analysis, which then proceeds to a further determination of the nature of this certitude. The beginning of this further step is indicated in the phrase: 'But I do not yet know with sufficient clarity what I am, I who am certain that I am.'[3]

[1] *Resp. ad II Obj.* (AT. vii. 130^{30}–131^{18}). [2] Synopsis (AT. vii. 12^{10-13}).
[3] *Meditationes*, II (AT. vii. 25^{14-15}), and the French version (AT. ix. 19).

Although we are not here concerned with the detail of Descartes's argument, part of his answer to the pönderous subtleties of P. Bourdin gives us a glimpse of this aspect of the analysis. 'He [P. Bourdin] adds: *You know who you are indeterminately and confusedly, not, however, determinately and clearly.* Now the most obvious inference to draw from these words is that there is a way open to further knowledge, because by reflection and attention we are able to bring about that what we knew merely in an indeterminate and confused manner should be apprehended by us clearly and determinately.'[1] This was not Bourdin's conclusion, but we can recognize, at least in Descartes's own view, the next step in the analysis, the passage from the implicit to the explicit, the extension of knowledge described in Rule 14.[2] It is interesting to note that in attempting what might be called the identification of the subject posited in the primary *Cogito, ergo sum*, Descartes applies the same process of analysis as in the *First Meditation*: he passes in review all the various classes or groups of objects of sensible or intellectual knowledge as outlined in the preliminary survey, in case, as he put it curiously, 'I might take some other object in place of myself'.[3] These classes or groups had in the *First Meditation* been objects of doubt; their status is not surreptitiously altered in the second analysis. Bourdin, once again, who wanted to know if he was supposed to pick up again his doffed coat, provoked the following note:

> I shall make use, says Descartes, of a very homely illustration in order to explain the rationale of my method. . . . Supposing a man had a basket of apples and feared that some of them might be rotten and accordingly wished to remove those apples lest they rot the others, how could he do it? Would he not first tip the whole lot of apples out of the basket and then look them over one at a time? Then having chosen those which he saw not to be rotten, he would place them again in the basket, leaving out the rotten ones. So it is with those who have never philoso-

[1] *Resp. ad VII Obj.* (AT. vii. 515^{16-23}).

[2] 'Notandum est, quoties unum quid ignotum ex aliquo jam ante cognito deducitur, non idcirco novum aliquod genus entis inveniri, sed tantum extendi totam hanc cognitionem ad hoc, ut percipiamus rem quaesitam participare hoc vel illo modo naturam eorum quae in propositione data sunt' (AT. x. 438^{12-18}). See above, pp. 107 ff.

[3] *Resp. ad VII Obj.* (AT. vii. 481^{1-27}). The same example is used to cover a similar objection further on (ibid. 512^{16-21}).

phized in the right way, and who have a variety of opinions, collected in their youth, some of which they rightly fear to be false. . . . Now there is no better way of doing this than to reject all of them together as uncertain and false, and then having investigated each one separately and in order, to reinstate only those which we know to be true and beyond doubt. Thus it was no bad course to reject everything at the outset, and then, having recognized that I knew nothing more certainly and more evidently than that *qua* thinking [being] I existed (*quod ego cogitans existerem*), it was right to assert this first. And finally, it was not wrong for me to enquire afterwards, who was it I formerly believed myself to be, not with the intention of believing exactly the same, but in order to reinstate any truths, to reject all false ones, and to keep such as were doubtful for a future examination.[1]

There is one other simile—a favourite one of Descartes's when discussing his method—which may perhaps be quoted here in place of the detail of the analysis which is beyond the scope of the present study. 'Throughout my writings', says Descartes, 'I have made it clear that my method is like the planning of houses by architects. In order to construct solid houses where the soil is sandy, and stone, clay, or any other lasting material is used in the building, they first of all dig ditches and empty out the sand and whatever else rests on or is mixed up with the sand, so that they may fix their foundations.'[2] So, says Descartes, he has thrown out all his beliefs as doubtful, until he found the rock of the *Cogito* on which to rest the foundations of his philosophy. But it does not follow that the architect may not make use of any stones which may be dug up, or even mix the sand with his mortar, if he should need to or find it convenient. Or, as Descartes points out in a similar illustration, if an old shaky house be demolished because its foundations have sunk, the materials may be set aside for use in the new construction, subject to their fitness and durability.

The analysis thus takes us through the series of questions corresponding to the basic question: 'what am I?' A man, an animal, a body, a group of sensations, a thinking thing—until the final certainty of 'I am a thing which thinks'. From the discovery of this essential attribute, the analysis begins

[1] *Resp. ad VII Obj.* (AT. vii. 481^{1-27}).

[2] Cf. *Resp. ad VII Obj.* (AT. vii. 536^{27}–537^{5}). The simile is carried on, somewhat facetiously and ironically, for twenty-five pages of the AT. edition.

again with the question: 'what is a thing which thinks?' Each of the modes 'doubting, understanding, affirming, denying, willing, refusing, imagining, feeling' is submitted in turn to investigation and related to the fundamental identity of thought and the self. The result of the inquiry is that the self is determined as that of a subject characterized by activity and consciousness.[1]

The analysis then takes another turn, indicated by the almost casual expression, 'let us therefore leave the mind to itself once more and grant it every sort of freedom', and is once more turned upon 'the objects that are commonly considered the most distinctly apprehended', to provide as it were a counter-proof of the truth already established. It is a kind of empirical investigation of the value of the criterion implicitly established in the primary certitude of the *Cogito*. The analysis of the piece of wax is too well known to require further mention. The conclusion too is well known: 'Bodies themselves are properly perceived, not by the senses, nor by the imagination, but by the intellect alone', as is this other corollary 'nothing is more easily or more clearly apprehended than my own mind'. The *Third Meditation* reopens the whole inquiry on a higher level, and 'I endeavour', says Descartes, 'to extend my knowledge more widely'.[2] The analysis of the *Cogito* has not been completed fully, either from the point of view of the extent of knowledge, for it has only yielded a limited knowledge of the self as *res cogitans*, or in respect of its ontological value, the value of its limited and imperfect activity. The first inquiry, then, turns to the problem of the extension of certain knowledge. Descartes enumerates the data: on the one hand the certainty of the *Cogito*, and then returning to the result of the analysis in the *First Meditation*, the extreme *causa dubitandi*, the malignant demon. The positive achievement of the *Cogito* is safe but it is useless against the malignant demon, except in its own privileged instance. To stay, therefore, at the level of the *Cogito* is to be thrown

[1] We ought, of course, to bear in mind the striking remark in the letter to Clerselier in answer to Gassendi's replies: 'Où il faut éviter l'équivoque du mot de pensée, lequel on peut prendre pour la chose qui pense et aussi pour l'action de cette chose; et je nie que la chose qui pense ait besoin d'autre objet que de soi-même pour exercer son action, bien qu'elle puisse aussi l'étendre aux choses matérielles, lorsqu'elle le sexamine' (AT. ix. 206[25-31]).

[2] *Meditationes*, III (AT. vii. 35[3-4]).

from *Cogito* to doubt and doubt to *Cogito*. From this opposition (and the Cartesian method again takes on a dialectical tone) arises the opposition of imperfect thought to perfect thought, in which, as Hegel would have said, thought is forced to transcend itself in order to vouch for its own origin and the certainty of its ideas. The fundamental equation must be altered to that used by the *Regulae*: *sum, ergo Deus est*. Once this is established, it would be possible to proceed synthetically from the first proposition of the geometrical demonstration of the *Reply to the Second Objections*, if, as Descartes says, such a procedure would better please the reader. But it is well to remember the advice given in the same *Reply* that nothing, no summary of the argument, can replace the careful reading and meditation of the original text. It is only too easy a task in such a study to incur the reproach made in one of the bitterest letters Descartes ever wrote—to Beeckman—a few months, probably, after the completion of the *Regulae*: 'Water is always water; but it has a different taste when it is drunk at the spring from that which it has when drunk from a pitcher or a river.'[1] Whether pitcher or river, a commentary must always taint the purity of its source.

Within the narrow limits of a synopsis of the *Meditationes*, it is still justifiable to assert that there is a prima facie case in favour of Descartes's unswerving declarations that his method was used in the discovery and formulation of his metaphysical doctrine. Whatever may be the value of the arguments, enumerations, and analyses contained in this work, even a superficial examination reveals that the logical structure is identical with that laid down in the *Regulae* and the *Discours*.

The problem or *quaestio* is clearly stated in the title. We know what we are looking for and consequently we fulfil the primary requirement of stating initially a delimited subject of research. Only thus can we avoid 'the haphazard studies and vague reflections', characteristic, according to Rule 4, of those who do not delimit their problems. The *First Meditation* is the direct application of Rule 2: we must only direct our studies to matters on which sure and indubitable knowledge can be obtained; accordingly we must reject all things 'on which we may doubt in general'. The second paragraph

[1] Letter of 1630 (AT. i. 160^{25-27}).

of this *Meditation* is, as we have already noted, almost an enunciation of the principle of methodical enumeration, the preliminary classification of the data according to the injunction of Rule 7. The object of the *First Meditation* is, then, devoted to this preliminary task, to the critical examination of all our experience, in order to eliminate all opinions which are, or may be, false and thus to purge the mind of all prejudices and restore it to its 'natural state' in which it can grasp self-evident truth and know that it is grasping self-evident truth and cannot be deceived in this knowledge. It is easy to recognize the application of Rule 2; the removal of prejudice is one of the primary conditions of the *inspectio mentis purae et attentae*, of intellectual intuition. The upshot is that there is nothing in our experience absolutely free from doubt. All that we commonly suppose ourselves to 'know' crumbles away. None of it is so beyond suspicion that it can serve as a solid foundation, as the self-evident datum on which the edifice of certain knowledge can be built.[1]

The *Second Meditation* deals with the 'nature of the human mind', the same problem which is given as an example of enumeration, *omnium nobilissimum exemplum* in Rule 8.[2] It begins again by a preliminary survey which is also a comprehensive review of the previous examination. 'I suppose, then, that all the things that I see are false; I persuade myself that none of those things which my fallacious memory represents to me ever existed; I suppose that I possess no senses; I believe that body, figure, extension are but fictions of my mind. What, then, can be considered true? Perhaps nothing at all, unless there is nothing in the world that is certain.'[3] It is from this extremity of doubt that the double analysis begins. It should be noted that the very notion of analysis, as pro-

[1] It is worth noting that Descartes does not claim any originality in this sceptical examination of experience. He is, as he says in the *Resp. ad II Obj.*, 'summarizing, not without a certain distaste, the old arguments to be found in many volumes by the Academics and Sceptics'. But, and this once again emphasizes the *practical* guidance which Cartesian methodology assumes as its main purpose, Descartes insists: 'vellemque ut lectores non modo breve illud tempus, sed menses aliquot, vel saltem hebdomadas, in iis de quibus tractat considerandis impenderent, antequam ad reliqua progrederentur: ita enim haud dubie ex ipsis longe majorem fructum capere possent' (AT. vii. 130^{23-29}).

[2] Cf. AT. x. 395^{17}–396^{25}, and above, pp. 127–8.

[3] Cf. *Meditationes*, II (AT. vii. 24^{14-18}).

pounded in the *Regulae*,[1] lays down that we must start from the 'simplest proposition' and that the 'simplest proposition' is the *maxime absolutum* of the series, i.e. that which logically depends upon no other proposition. The *absolutum* in the metaphysical series must therefore be a self-evident proposition, beyond all possible doubt and requiring no further postulate. The doubt itself carries with it the being of the doubting mind so far and so long as the mind is aware of itself doubting. From this initial step begins the analysis which we have already sketched above as a typical example of the use of the method in metaphysics. The first *maxime absolutum* is the self-evident proposition, or simple nature, the *Cogito, ergo sum*. The indubitable primary datum, from which the reconstruction of knowledge must start, is our certainty that our self-consciousness involves our existence, the *unicum et certissimum fundamentum totius Philosophiae*, as Spinoza was to call it.[2] The analysis then proceeds from the further *quaestio*: What am I? to the final equation of thought and the self as a self-conscious existent.

It is impossible to read through the thirteen paragraphs which contain this analysis without being aware of the conscious methodological pattern that Descartes imposes upon his inquiry; they illustrate the use of *perquisitio* and *divisio* with copybook exactness.[3] The data are outlined and classified as in the *First Meditation*. 'I consider once again what I formerly believed myself to be, before I began this present train of thought.'[4] As in the homely simile mentioned earlier, I tip all the apples, good and bad, out of the basket before beginning to sort them out. I believed myself to be a man. But what is a man? Possible answers split into two major divisions: the notion of man generally accepted by common sense, a vague notion based on an uncritical common experience, and a philosophical definition based on a more or

[1] Cf. Rule 6 (AT. x. 381^{2-6}), and above, pp. 160 ff. Brunschvicg makes the point that 'l'application de la méthode d'analyse à la métaphysique exclut la velléité de déduire quoi que ce soit à partir de principes que le doute interdit de postuler'. 'Mathématique et Métaphysique chez Descartes', *Rev. méta. et morale*, 1927, p. 315.

[2] *Principia Philosophiae Renati Des Cartes*, Prop. IV, Scholium (Gebhardt, i. 153^{15-16}).

[3] The original editions of the *Meditationes*, both that of 1641 and that of 1642, are not paragraphed at all. I am referring to the paragraphs of the AT. edition, that is, vii. 25^{14}–34^{9}.

[4] AT. vii. 25^{19-20}.

less valid criticism of this same common experience, i.e. the definition of man as rational animal. From this division, Descartes not unexpectedly rejects the philosophical definition as requiring further analysis of the meanings of 'rational' and 'animal', analyses more difficult than the first. 'I have not', he adds, 'enough time and leisure to waste in trying to unravel subtleties like these.'[1] He turns then to the other notions, those accepted by common experience, 'which spring up of themselves in my mind and are not inspired by anything but my own nature'.[2] Though confused and interwoven with falsity, they are at least the result of real experience and may thus yield, by analysis, the true nature of the self. There is a further methodological advantage in this choice of common sense data as the material on which the analysis will be conducted. The object of the *Meditationes* is not to inculcate a set of principles or a body of doctrinal conclusions—this would be mere 'history' in the language of the *Regulae*—but to lead its reader through a train of thought which he himself can follow step by step in his own experience. Hence the pedagogical importance of starting from what is common to all men endowed with the natural light of a *bona mens*. The data of common experience may be classified under two major heads of experience: certain experiences are related to what I call a 'body', others are thought to appertain to what I call a 'soul'. The enumeration is shown to be complete and then each division is subjected to analysis. The first head of division is shown to be the wrong answer to the *quaestio*: what am I who doubts? The body in its fundamental structure is part of the sensible world. It is therefore an object of doubt and cannot be placed in equation with the doubting self which is the limit of doubt. There remains, therefore, the second element of the division, the soul. Again, Descartes begins by an enumeration, that of 'the attributes of the soul'. The attributes are found to fall into two groups, those which presuppose the existence of the body, walking, taking food,

[1] AT. vii. 25^{29-31}. See also the *Discours*, Part I (AT. vi. 2^{25-30}).

[2] AT. vii. 25^{31}–26^{2}. In the *Recherche de la Vérité* Polyander, the 'common man' of the dialogue, puts the same point: 'Imo, cum me *hominem* esse, . . . dixi, animum in omnia entia Scholastica, quae ignorabam, et de quibus nunquam aliquid inaudiveram, quaeque, ut existimo, in sola tantum eorum, qui ea invenerunt, Phantasia subsistunt, non intendi' (AT. x. 517).

perceiving, and the solitary attribute of 'thinking' which does not presuppose the body. The first group must be rejected, and we are therefore left with the conclusion that thought alone is inseparable from the soul and the first analysis is complete. The elimination of all that pertains directly or indirectly to the body reproduces, within the limits of a restricted enumeration, the eliminatory analysis of all objects contained in the *First Meditation*. The body is just another object.

The datum now stands as the self equated with a *res cogitans*.[1] The meaning of *cogitatio* leads us to another preliminary enumeration. A 'thinking thing' is a thing which 'doubts, understands, affirms, denies, wills, refuses; that also imagines and perceives'. The analysis proceeds by three steps: the discovery in the attributes of the original element of 'thought', the recognition of the equation of thought and the self, and the determination of the specific characteristics of thought. The uncritical notions accepted by common sense are shown to be based not on a complete misrepresentation of the facts, but on an incomplete and confused analysis of experience. The act of vision and the act of walking regarded as corporeal occurrences are not 'thoughts'; but our feeling and consciousness of seeing or walking are 'thoughts'. It is the mind which feels or is conscious of our seeing or our walking. So one might argue rightly 'I walk, therefore I am', provided one understands, 'I am conscious of', walking or breathing.[2] Common sense does not distinguish consciousness from the object of which it is conscious. The methodical doubt of the *First Meditation* applied once again to the attributes enumerated as commonly accepted as 'thinking', eliminates all the doubtful objects, i.e. those involving corporeal activity of any kind, and isolates the unknown, explicitly unknown, implicitly known, quality which is common, i.e. self-consciousness.[3]

[1] The expression is used in the *Second Meditation* (AT. vii. 28^{20}). The *Discours*, Part IV, says: 'Je connus de là que j'étais une substance dont toute l'essence ou la nature n'est que de penser' (AT. vi. 33^{3-5}), and the sixth Definition of *Resp. ad II Obj.* states: 'Substantia, cui inest immediate cogitatio, vocatur *Mens*' (AT. vii, 161^{24-25}). I am not here concerned with the validity of Descartes's transition from the *Cogito* to the assertion of a persistent identity implied in this expression.

[2] Cf. the letter of Mar. 1638 (AT. ii. 37^{26}–38^{21}) which contains the same analysis as that of the *Second Meditation*. The *Principia*, i. 6 is even clearer (AT. ix (Part I), 7^{24-82}).

[3] The use of 'self-consciousness' may be misleading. It should be noted that

The second step of the analysis consists in the determination of the identity of the self and thought. 'Thought is an attribute that belongs to me: this alone is inseparable from me.'[1] The equation can be posited because the attribute of thought is essential, unique, and exclusive to the self. It is an essential attribute because the self cannot be posited without consciousness which is its constitutive nature; it is unique because all the other attributes are subject to the methodical doubt which cannot weaken the existence of the doubting self and this same imperviousness to doubt renders it the sole datum which remains at the end of the destructive criticism of the notions of common experience. By a series of simplifications consisting of preliminary enumerations, each simplification entailing a more restricted classification of the data, the analysis has been pushed until the attributes have, according to the injunction of Rule 21, 'all been reduced to a single one'.[2] If one desired to keep to the more mathematical language of the *Regulae*, one might say that a value has been obtained for the self; an identity has been obtained. The third step consists of an elaboration of the specific characteristics of thought.[3] The rest of the *Second Meditation* is an empirical investigation of certain criteria implicit in the analyses which led to the *Cogito*. The analysis of 'the piece of wax' is a return upon the data of empirical knowledge, and Descartes attempts to show how the investigation of a particular instance of common experience leads to the same conclusion that he has reached by his general inquiry. We take an object which is 'commonly thought the most distinctly known', one of the bodies which we touch and see, in the instance, a piece of fresh wax. The conclusion is that 'it is the mind alone which perceives it'. But the perception of it is neither an act of touch or sight, nor an act of imagination; it is simply an act of intellectual intuition, an *inspectio solius*

Descartes does not wish to confine it to reflexive consciousness or to consciousness of anything as definitely referred to a self. The consciousness that we are conscious no doubt is a *cogitatio*, but it is not essential to a *cogitatio* that it should be turned upon itself. Cf. *Resp. ad VII Obj.* (AT. vii. 559^{16-23}).

[1] The Latin text is striking: 'Cogitare? Hic invenio: cogitatio est; haec sola a me divelli nequit' (ibid. 27^{7-8}).

[2] Cf. AT. x. 469^{6-7}.

[3] Cf. *Meditationes*, II (AT. vii. 28^{20}–29^{18}). The first definition of *Resp. ad II Obj.* is clearer (cf. AT. vii. 160^{7-13}).

mentis.[1] Starting from the notion of body, we are given a further proof of the central identification of the self and pure thought which is to be the primary datum of his metaphysical doctrine. As Descartes puts it himself: 'I find that I have insensibly reverted to the point I desired.' The reversion is quite methodical.

To continue with this skeleton outline of the *Meditationes*, divesting the arguments of all their solid flesh, would be tedious to the reader who was aware of the many difficult points of interpretation and controversy which have been passed over or ignored. The reader unacquainted with the work would gain a very wrong impression of Descartes's philosophical masterpiece. Enough has probably been said, both in the general survey and more detailed sketch, about the general structure of the first two *Meditationes* to show that the method used by Descartes in working out the argument of these two chapters is the same method which he expounds and advocates in the *Regulae* and the *Discours*. In the *Meditationes* the method is indeed decorticated from its mathematical shell and, from the very nature of its subject matter, it does not need many of the technical devices suggested in the second book of the *Regulae*. But essentially it is the same method as used by Descartes in the 'essays in method', the *Dioptrique*, the *Météores*, and the *Géométrie*. The unity of science, which was the banner raised in the Cartesian revolution against the philosophy of the Schools, depended, in his own words, on 'the continuous application of *a single and identical method* to all the various sciences, for the possibility and practicability of its application in common would imply that the sciences together are nought else but the very unity of human reason'. This was Descartes's unfailing contention: it was also his constant practice.

[1] AT. vii. 31[25].

INDEX OF PASSAGES FROM THE *REGULAE*

This Index is designed to help the student who wishes to read the *Regulae* for himself, or herself, and who may therefore wish to consult this study in order to examine any comment made on some particular passage. The texts of the *Regulae*, with page and line reference to vol. x of the *Adam et Tannery* edition, are in the left-hand column, grouped under the various Rules. The numbers in the right-hand column refer to the pages of this study. It has not been possible to avoid some overlapping of the text references.

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