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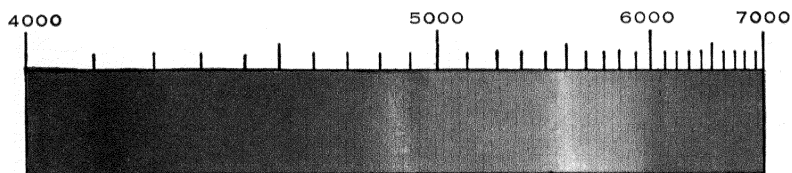
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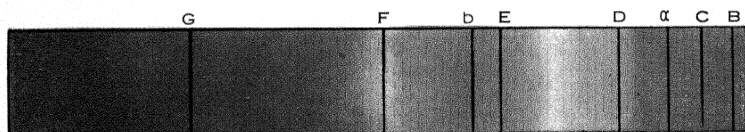
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BY

R. A. HOUSTOUN, M.A., D.Sc.

LECTURER ON NATURAL PHILOSOPHY IN THE UNIVERSITY OF GLASGOW
FORMERLY EXAMINER IN PHYSICS IN THE UNIVERSITIES OF
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PREFACE

THE present book treats of the subject of Physics up to the Intermediate or First Science Standard of the different universities, and if certain portions are omitted, it may also be used as a text-book in schools by pupils preparing for the Higher Certificate of the Oxford and Cambridge Schools Examination Board, the London University Matriculation, and similar examinations. The part of it dealing with Light was published in a slightly different form in 1925, the sections on Electricity and Magnetism and on Heat in 1928, the section on Dynamics and Properties of Matter in 1929, and the book has now been completed by the inclusion of the section on Sound. I have endeavoured to make the treatment as clear and attractive as possible, and owing to the courtesy of the authorities I have been able to add a collection of questions from representative public examinations. Some matter not hitherto found in books of this type has also been added in order to bring the treatment up to date and meet the changing requirements of teachers.

I am indebted to Dr. G. E. Allan for criticising the sections dealing with Dynamics and Properties of Matter and Sound and reading the proofs of these sections, and to Dr. R. C. Gray for criticising the section on Electricity and Magnetism, reading the proofs, and verifying the examples of this section. I am also indebted to Messrs. Jas. F. Shearer and A. J. Younger for reading the proofs and verifying the examples of the sections on Light and Heat respectively, and to Drs. John Thomson and David F. Martyn for further assistance with the proofs. With the experience of so many friends behind it I hope the book will prove a safe guide and lighten the labours of students.

R. A. HOUSTOUN.

February, 1930.

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CONTENTS

(The numbers give pages)

PART I

DYNAMICS AND PROPERTIES OF MATTER

CHAPTER I

UNITS AND MEASUREMENT

Units of Length and Mass, 3. Unit of Time, 4. Dimensions of a Unit, 5. Example, 5. Vernier, 5. Vernier Callipers, 7. Micrometer Screw Gauge, 7. Spherometer, 8. Proof of formula, 9. Examples, 9.

CHAPTER II

KINEMATICS

Divisions of the Subject, 10. Speed, 10. Speed Time Diagram, 11. Composition and Resolution of Velocities, 12. Examples, 14. Acceleration, 14. Uniformly Accelerated Rectilinear Motion, 15. Special Cases, 16. Example, 16. Falling Bodies, 16. Examples, 17. Definition of Acceleration, 18. Uniform Motion in a Circle, 18. Example, 20. Simple Harmonic Motion, 20. Example, 21. Parabolic Motion, 21. Range and Time of Flight of a Projectile, 22. Examples, 22.

CHAPTER III

STATICS

Parallelogram of Forces, 24. Example, 26. Triangle of Forces, 26. Polygon of Forces, 26. Example, 27. Law of Moments, 27. Example, 29. The Lever, 29. Equilibrium of Parallel Forces, 30. Examples, 32.

CHAPTER IV

THE LAWS OF MOTION

Momentum, 33. Newton's Laws of Motion, 33. The First Law of Motion, 34. The Second Law of Motion, 35. Examples, 35. Mass and Weight, 36.

The Third Law of Motion, 36. Conservation of Momentum, 37. Example, 37. Body on Inclined Plane, 37. The Simple Pendulum, 38. Example, 39. Centrifugal Force, 40. Centrifugal Force on Vehicles, 40. Example, 41. Elastic Impact, 42. Examples, 43.

CHAPTER V

FRICTION AND CENTRES OF GRAVITY

Friction, 45. Laws of Friction, 46. Coefficient of Friction, 46. Kinetic Friction, 47. Inclined Plane with Friction, 47. Example, 47. Rolling Friction, 48. Centre of Gravity, 48. Particular Cases, 48. Example, 51. Experimental Determination of Centre of Gravity, 51. Centroid, 51. Stability, 52. Examples, 53.

CHAPTER VI

WORK. MACHINES. ENERGY

Work, 55. Units, 56. Example, 56. Machines, 56. Efficiency of a Machine, 59. Example, 59. Wheel and Axle, 59. Systems of Pulleys, 60. Activity, 61. Example, 61. Energy, 61. Kinetic Energy, 61. Example, 62. Potential Energy, 62. Potential Energy of a Stretched Spring, 63. The Principle of the Conservation of Energy, 64. Examples, 65.

CHAPTER VII

ROTATIONAL MOTION. GRAVITATION

Angular Momentum. Kinetic Energy of Rotation, 66. Moment of Inertia, 66. Example, 67. Couple, 68. Dynamics of Rotation, 68. Examples on Rotational Motion, 68. Compound Pendulum, 69. Law of Gravitation, 70. Verification of Law of Gravitation, 71. The Cavendish Experiment, 72. Density of the Earth, 72. Examples, 73.

CHAPTER VIII

FLUID PRESSURE

States of Matter, 74. Hydrostatics, 74. Water maintains its Level, 74. Pressure at a Point, 75. Pascal's Vases, 76. Example, 77. Pressure inside a Liquid the same in all Directions, 77. U-Tube, 78. Thrust on an Area, 79. Example, 79. Centre of Pressure, 80. Example, 80. Transmission of Fluid Pressure, 81. The Cartesian Diver, 81. The Hydrostatic Paradox, 82. Hydraulic Press, 83. Examples, 84.

CHAPTER IX

ATMOSPHERIC PRESSURE. BOYLE'S LAW

Torricelli's Experiment, 85. Pascal's Experiments, 85. The Mercury Barometer, 86. Example, 87. The Aneroid Barometer, 87. Standard Atmo-

spheric Pressure, 89. The Magdeburg Hemispheres, 89. Watt's Hydrometer, 90. Boyle's Law, 91. Accuracy of the Law, 93. Short Mercury Column, 93. Diving Bell, 94. Height of the Homogeneous Atmosphere, 94. Example, 95. Force exerted by a Jet, 95. Torricelli's Theorem, 97. Examples, 97.

CHAPTER X

PUMPS

Air Pump, 99. Calculation of the Theoretical Vacuum, 100. Manometer, 100. Toepler Pump, 101. Hyvac Pump, 102. Filter Pump, 103. Compression Pumps, 103. Common Suction Pump or Lift Pump, 103. Force Pump, 104. Example, 105. Siphon, 105. M'Leod Gauge, 105. Examples, 106.

CHAPTER XI

PRINCIPLE OF ARCHIMEDES. SPECIFIC GRAVITY

The Principle of Archimedes, 107. Experimental Illustration, 108. Example, 108. Buoyancy Correction, 109. Specific Gravity, 109. Hydrostatic Balance, 110. Specific Gravity Bottle or Pyknometer, 111. Variable Immersion Hydrometer, 112. Nicholson's Hydrometer, 113. Stability of Ships, 114. Examples, 115.

CHAPTER XII

PROPERTIES OF MATTER

Hooke's Law, 116. Elasticity, 117. Shearing Strain, 117. Pure Change of Volume, 118. Example, 118. Young's Modulus, 118. Example, 120. Viscosity, 120. Distinction between a Solid and a Liquid, 121. Molecules and Atoms, 121. Molecular Forces, 122. Surface Tension, 123. Capillarity, 124. Measurement of Surface Tension, 125. Example, 127. Further Examples of Surface Tension, 127. Excess of Pressure inside a Soap Bubble, 128. Diffusion of Gases, 128. Diffusion of Liquids, 129. Osmotic Pressure, 130. Absorption of Gases by Liquids and Solids, 131. Examples, 131.

PART. II

HEAT

CHAPTER I

TEMPERATURE AND THERMOMETERS

Temperature, 135. Thermometers, 136. Construction of a Thermometer, 137. Thermometric Scales, 137. Examples, 139. Historical, 139. Verification of Thermometer Fixed Points, 139. Stem Correction, 141. Example, 142. Thermometers for Special Purposes, 142. Examples, 143.

CHAPTER II

EXPANSION OF SOLIDS AND LIQUIDS

Expansion of Solids, 145. Coefficient of Linear Expansion, 145. Examples, 146. Measurement of Linear Expansion, 147. Example, 147. Roy and Ramsden's Method, 147. Applications, 148. Stress produced by Change of Temperature, 150. Cubical Expansion, 151. Effect of Temperature on Density, 152. Dilatometer, 152. Example, 153. Weight Thermometer, 153. Absolute Expansion of Mercury, 154. Regnault's Apparatus for Absolute Expansion of Mercury, 155. Hydrostatic Method of determining Coefficient of Expansion, 156. Maximum Density of Water, 156. Thermostat, 157. Examples, 157.

CHAPTER III

EXPANSION OF GASES

Expansion of Gases, 160. Equality of the Coefficients, 161. Charles's Law, 161. Determination of α at Constant Pressure, 162. Determination of α at Constant Pressure. Regnault's Method, 163. Miller's Air Thermometer, 164. Determination of α at Constant Volume, 164. Air Thermometer, 165. Absolute Temperature, 166. The Gas Equation, 167. The Gas Constant, 167. Examples, 168.

CHAPTER IV

CALORIMETRY

Fundamental Experiment, 169. Definitions, 170. Law of Mixtures, 171. Determination of the Water Equivalent of a Calorimeter, 171. Example, 172. Determination of Specific Heat, 172. Example, 173. Law of Dulong and Petit, 174. Latent Heat, 174. Determination of Latent Heat of Water, 175. Determination of Latent Heat of Steam, 175. Convection Correction, 176. Bunsen's Ice Calorimeter, 177. Joly's Steam Calorimeter, 178. British Thermal Unit, 179. Calorific Value of a Fuel, 179. Examples, 179.

CHAPTER V

CHANGE OF STATE

Cooling Curve, 182. Determination of Melting Point, 183. Change of Volume on Solidification, 184. Sublimation, 184. Experiment to determine the Density of Ice, 184. Change of Melting Point with Pressure. Regelation, 184. Boiling, 185. Determination of Boiling Point, 186. Effect of Pressure on the Boiling Point. Franklin's Experiment, 186. Papin's Digester, 187. Evaporation and Boiling, 187. Cooling Effect of Evaporation. The Cryophorus, 188. Examples, 189.

CHAPTER VI

VAPOUR PRESSURE. HYGROMETRY

Vapour Pressure, 190. Determination of Boiling Point of Alcohol, 191. Measurement of Vapour Pressure, 191. Marcet's Boiler, 192. Distillation, 193. Distinction between a Vapour and a Gas, 193. Isothermals for Carbon Dioxide, 194. Dalton's Law, 195. Relative Humidity, 195. Formation of Dew, 196. Example, 197. Regnault's Hygrometer, 197. Wet and Dry Bulb Hygrometer, 198. Other Hygrometers, 199. Rain, Hail, Snow, Hoar Frost, 200. Examples, 201.

CHAPTER VII

THE CONNECTION BETWEEN HEAT AND WORK

The Caloric Theory, 202. Rumford's Experiments, 202. Davy's Experiment, 203. Joule's Experiments, 203. Examples, 204. Searle's Apparatus for determining J , 205. Nature of Heat, 206. Work done by an Expanding Gas, 207. Difference of the Specific Heats of a Gas, 208. To Calculate the Difference of the Specific Heats of Air, 209. Proof that the Kinetic Energy of the Molecules of a Gas varies as the Absolute Temperature, 209. Conservation of Energy, 210. Example, 210. Dissipation of Energy, 211. Adiabatic Change. Adiabatic Equation of a Gas, 211. Example, 213. Liquid Air Machine, 213. Examples, 214.

CHAPTER VIII

CONDUCTION AND CONVECTION

The Transference of Heat, 215. Flow of Heat along a Bar, 216. Fundamental Law of the Conduction of Heat, 217. Example, 218. Ingenhousz's Experiment, 218. Measurement of the Conductivity of a Good Conductor, 219. Measurement of the Conductivity of a Bad Conductor, 220. Conductivity of Liquids and Gases, 221. Conductivity and Sensation, 222. The Davy Safety Lamp, 222. Scale in Boilers, 223. Experiments on Convection, 223. Hot Water Supply, 224. Example on Chimney, 224. Land and Sea Breezes, 225. Trade Winds, 225. Examples, 225.

CHAPTER IX

RADIATION

Apparatus, 227. Demonstration of Heat Rays, 228. Reflection and Absorption of Heat Rays, 229. Different Kinds of Heat Rays, 231. Experiment with Conjugate Mirrors, 231. Heat Rays in History, 231. Experiments with Leslie's Cube, 232. Ritchie's Apparatus, 233. Prevost's Theory of Exchanges, 233. Newton's Law of Cooling, 234. Specific Heat by the Method of Cooling, 235. Emissivity of a Surface, 236. Stefan's Law, 236. Optical Pyrometry, 237. Examples, 238.

PART III

SOUND

CHAPTER I

WAVE MOTION

Water Waves, 241. Reflection, 242. Refraction, 242. Interference, 243. Diffraction, 244. Stationary Waves, 244. Motion of the Particles, 245. Solitary Wave, 247. Resultant of Two S.H.M.'s in the Same Straight Line, 248. Resultant of Two S.H.M.'s at Right Angles to each other, 249. Blackburn's Pendulum, 251. Examples, 251.

CHAPTER II

PRODUCTION AND PROPAGATION OF SOUND

Source in Vibration, 252. Velocity of Sound, 252. Variation with Intensity, 253. Medium of Transmission, 253. Reflection, 254. Interference of Sound, 254. Diffraction of Sound, 255. Nature of Sound, 255. Calculation of the Velocity, 255. Effect of Temperature, 256. Refraction of Sound, 257. Proof of Formula for Velocity, 257. Sensitive Flame. Galton's Whistle, 258. Sound Ranging, 260. Energy of the Sound Wave, 260. Examples, 261.

CHAPTER III

MEASUREMENT OF FREQUENCY, MUSICAL SCALE

Loudness, Pitch, Quality, 262. Siren, 262. Savart's Wheel, 264. Tuning-fork, 264. Electrical Tuning-fork, 265. Measurement of Pitch of a Tuning-fork, 265. Formula for Falling Plate Method, 266. Stroboscopic Disc Method of Finding Frequency of a Fork, 266. Determination of Frequency by Lissajous' Figures, 267. Beats, 268. Interval between two Notes. Musical Scale, 269. Equally Tempered Scale, 270. Example, 270. Doppler Effect, 270. Examples, 271.

CHAPTER IV

VIBRATIONS OF STRINGS AND COLUMNS OF AIR

Vibrating Strings, 273. Harmonics of a Stretched String, 274. Timbre, 276. Example, 276. Proof of Formula for the Velocity, 276. Melde's Experiment, 277. Resonance, 277. Vibrating Air Columns, 278. Velocity of Sound by Resonance, 279. Example, 280. Organ Pipes, 280. Manometric Flame. Rotating Mirror, 282. Singing Flame, 283. Kundt's Tube, 283. Vibrations of Rods and Plates, 284. Audition, 284. Phonograph. Gramophone, 285. Examples, 286.

PART IV

LIGHT

CHAPTER I

SHADOWS. PHOTOMETRY

Rectilinear Propagation, 289. Pinhole Camera, 290. Shadows, 291. Eclipses, 291. The Standard Candle, 292. The Inverse Square Law, 292. Inverse Square Law; Otherwise, 293. The Rumford or Shadow Photometer, 294. The Bunsen or Grease Spot Photometer, 294. The Wedge Photometer, 295. The Trotter or Perforated Screen Photometer, 295. Photometry, 296. Fraction of Light transmitted by a Glass Plate, 297. Variation of the Illumination of a Surface with the Angle of Incidence of the Light, 298. Experiments in Photometry, 298. To measure the Candlepower of a Lamp in Position, 299. To measure the Illumination of the Surface of a Desk, 299. Test of a Pocket Flash Lamp, 299. Illumination of Rooms, 300. Examples, 300.

CHAPTER II

REFLECTION

Laws of Reflection, 302. Reversibility of the Ray, 303. Image of a Point; Real Image; Virtual Image, 303. Image of a Line, 304. Experiments on Reflection, 304. Rotation of a Plane Mirror, 306. Applications, 306. Pepper's Ghost, 307. Parallel Mirrors, 308. Inclined Mirrors, 309. Examples, 311.

CHAPTER III

REFRACTION

Laws of Refraction, 313. Experiment on Laws of Refraction, 314. Twilight, 315. The Coin and Basin, 315. Ptolemy's Data on Refraction, 315. Image of a Point formed by Refraction at a Plane Surface, 316. Determination of Index of Refraction by the Microscopic Method, 317. Experiments on Image formed by Refraction at a Plane Surface, 318. Multiple Images formed by Reflection at a Glass Plate, 320. Caustic formed by Refraction at a Plane Surface, 321. Experiment on Caustic, 322. Total Reflection, 322. Critical Angle, 322. Experiment on Total Reflection, 323. Experimental Demonstration of Total Reflection, 323. The Fish and the Setting Sun, 323. Totally Reflecting Prism, 324. The Luminous Jet, 324. Two Parallel Slabs, 325. Absolute Refractive Index, 326. Several Parallel Slabs, 326. Atmospheric Refraction, 327. Mirage, 328. Examples, 329.

CHAPTER IV

SPHERICAL MIRRORS

Proof of Formula for Concave Mirror forming a Real Image, 331. Conjugate Points; Focus, 332. Graphical Construction, 333. Alternative Proof of

Formula, 333. Magnification produced by a Mirror, 334. Convention of Signs, 334. Concave Mirror; Virtual Image, 335. Convex Mirror, 335. Focus of a Convex Mirror, 336. Graphical Construction for Convex Mirror, 336. Rules for Solving Problems, 337. The Optical Bench, 338. Experiments on Spherical Mirrors, 339. Tracing the Position of the Image, 340. Table of Results, 341. The Magic Dagger and the Phantom Bouquet, 342. Caustic formed by Reflection at a Concave Mirror, 342. Examples, 343.

CHAPTER V

LENSES

Lenses, 345. Graphical Determination of Position of Image, 346. Formula for Lens, 347. Formula for Magnification, 348. Conjugate Points, 348. Two Lenses in Contact, 349. Determination of Focal Length of Convex Lens, 350. Determination of Focal Length of Concave Lens, 352. More Accurate Method of Determining Focal Length, 353. Combination of Lenses and Mirrors, 353. Tracing the Position of the Image, 354. Table of Results, 355. Refraction at a Spherical Surface, 355. Refraction through a Lens, 357. Formula for Focal Length of Lens, 358. Optical Centre of a Lens, 359. Cylindrical Lenses, 360. Examples, 360.

CHAPTER VI

OPTICAL INSTRUMENTS

Magnifying Glass, 363. Astronomical Telescope, 364. Resolving Power of a Telescope, 366. To measure the Magnification of a Telescope, 367. To measure the Resolving Power of a Telescope, 367. The Galilean Telescope, 367. Terrestrial Telescope; Prism Glasses, 368. Mirror Telescopes, 369. The Microscope, 370. The Sextant, 371. Photographic Camera, 372. Optical Lantern, 373. Cinematograph, 374. Examples, 375.

CHAPTER VII

THE SPECTROMETER AND THE SPECTRUM

Prisms, 376. Minimum Deviation, 376. Image produced by a Prism, 377. Twinkling of Stars, 377. Variation of Deviation with Angle of Incidence, 378. Formula for Minimum Deviation, 378. Determination of Index of Refraction of a Glass Prism by Pin Optics Methods, 379. Spectrometer, 380. Experiments with the Spectrometer, 382. Sodium Flame, 383. Newton's Fundamental Experiment on the Spectrum, 383. Attempt to further decompose a Colour of the Spectrum, 384. Experiment with the Crossed Prisms, 385. Recomposition of White Light, 385. Projection of a Spectrum, 385. Infra-red, 386. Experiment on Heat Rays, 387. Ultra-violet, 387. Experiments on Actinic Rays, 387. Fluorescence, 388. Phosphorescence, 388. The Spectroscope, 389. Kirchhoff and Bunsen Spectroscope, 389. Spectrum Analysis, 390. The Rainbow, 391. Chromatic Aberration, 392. Experiments on Chromatic Aberration, 394. Spherical Aberration, 394. Direct Vision Spectroscope, 394. Examples, 395.

CHAPTER VIII

THE EYE AND COLOUR VISION

The Eye, 397. Experiments on the Resolving Power, Chromatic Aberration, and Spherical Aberration of the Eye, 398. To find the Blind Spot, 399. Defects of Vision, 399. Diopters, 400. Binocular Vision, 401. The Stereoscope, 401. Adding Coloured Lights, 402. The Colour Triangle, 403. Maxwell's Colour Disc, 404. Lambert's Method of Superposing Colours, 404. Newton's Method of Superposing Colours, 404. Mixing Coloured Pigments, 405. The Wratten Filters, 406. Three-colour Printing, 406. Anaglyphs, 406. Colour Vision, 406. Colour Blindness, 407. Examples, 407.

CHAPTER IX

THE NATURE OF LIGHT

Galileo's Experiment, 409. Römer's Method, 409. The Aberration Method, 410. Fizeau's Method, 411. Foucault's Method, 413. Emission Theory, 414. Wave Theory, 415. Huygens' Construction for Reflection and Refraction, 416. The Decisive Experiment, 417. Geometrical and Physical Optics, 417. Wave-length; The Complete Spectrum, 417. Longitudinal and Transverse Waves, 419. Double Refraction, 420. Nicol Prism, 421. Natural and Polarised Light, 421. Other Methods of producing Plane Polarised Light, 422. Rotation of the Plane of Polarisation; Polarimeter, 422. The Quantum and the Einstein Star Shift, 423. Examples, 425.

CHAPTER X

INTERFERENCE AND DIFFRACTION

Interference of Two Point Sources, 427. Proof of Formula, 428. Fresnel's Biprism, 428. Fresnel's Mirrors, 430. Fundamental Condition for Interference, 431. Colours of Thin Films, 431. Newton's Rings, 433. Diffraction Grating, 434. Rectilinear Propagation of Light, 436. Diffraction, 437. Examples, 437.

PART V

ELECTRICITY AND MAGNETISM

CHAPTER I

THE FUNDAMENTAL FACTS OF MAGNETISM

Artificial Magnets, 441. Poles, 441. Magnetic Induction. Permanent and Temporary Magnetism, 442. Inverse Square Law. Unit Quantity of Magnetism, 444. Magnetic Field, 445. Example, 446. Lines of Force, 446. Mapping

a Magnetic Field by the Compass Needle, 446. Mapping a Magnetic Field by Iron Filings, 448. Physical Reality of Lines of Force, 449. Molecular Magnetism, 450. Examples, 452.

CHAPTER II

MAGNETIC MEASUREMENTS

Moment of a Magnet. Couple on a Magnet in the Earth's Field, 453. Examples, 454. Field Strength due to a Bar Magnet in the Two Standard Positions. End-on Position, 455. Side-on Position, 455. Tangential Positions of Gauss. "A Position," 455. "B Position," 456. Magnetometer, 457. Comparison of Moments of Bar Magnets. Null Method, 458. Proof of Inverse Square Law, 459. Period of Vibration of a Magnet, 459. Comparison of Magnetic Moments and Field Strengths by the Vibration Method, 460. Determination of Earth's Magnetic Field and of Magnetic Moment of a Magnet in Absolute Measure, 460. Comparison of Magnetic Moments when Moments of Inertia are not known, 460. To determine the Distribution of Magnetism along a Bar, 461. Astatic Needle, 461. Hibbert's Magnetic Balance, 462. Examples, 462.

CHAPTER III

TERRESTRIAL MAGNETISM

Declination, 464. Dip, 464. Magnetic Elements, 466. Measurement of Declination, 467. Dip Circle, 467. Variation in Earth's Magnetic Field, 468. Magnetic Poles, 469. Magnetic Compass, 470. Examples, 471.

CHAPTER IV

ELECTROSTATICS. FUNDAMENTAL IDEAS

Fundamental Experiment, 472. Two Kinds of Electrification, 472. Conductors and Insulators, 473. Gold-Leaf Electroscope, 474. Test for Presence of an Electric Charge, 476. Charges of Opposite Sign are always produced in Equal Amounts, 476. Electrophorus, 476. Distribution of Electrical Charge upon Conductors, 477. Discharging Effect of Points, 478. Faraday's Ice-Pail Experiment, 479. Law of Force between Electrical Charges, 480. Torsion Balance, 480. Example, 481. Unit Charge, 481. Example, 481. Intensity of Electric Field, 482. Lines of Force, 482. Illustrations of the Action of Lines of Force, 483. Nature of Electricity, 484. Examples, 485.

CHAPTER V

POTENTIAL AND CAPACITY

Potential, 486. Zero of Potential, 487. Measurement of Potential, 488. Example, 488. Equipotential Surfaces, 488. Potential due to a Charged Spherical Conductor, 488. Capacity, 490. Condensers, 490. Leyden Jar. Variable Air Condenser, 491. Capacity of a Spherical Condenser, 492.

Capacity of a Plate Condenser, 493. Specific Inductive Capacity, 493. Examples, 495. Condensers in Parallel and in Series, 495. Energy of a Charged Conductor, 496. Example, 497. Frictional Machine, 497. Experiments with a Leyden Jar, 498. Lightning, 499. Wimshurst Electrical Machine, 500. Quadrant Electrometer, 502. Examples, 503.

CHAPTER VI

THE ELECTRIC CURRENT

Leclanché Cell, 504. Electric Current, 504. The Swimming Man Rule, 505. Magnetic Field of Electric Current, 506. Unit Current, 507. Unit Electromotive Force, 507. Connection between Frictional and Current Electricity, 508. Theory of Tangent Galvanometer, 509. Tangent Galvanometer, 510. Sine Galvanometer, 511. Thomson's Mirror Galvanometer, 511. Astatic Galvanometer, 513. Solenoid, 513. Electromagnet, 514. Examples, 515.

CHAPTER VII

OHM'S LAW

Resistance of a Conductor, 516. Ohm's Law, 516. Unit Resistance, 517. Illustration of Ohm's Law, 517. Resistance of Three Wires in Series, 518. Resistance of Three Wires in Parallel, 518. Shunts, 519. Ohm's Law applied to a Complete Circuit, 519. Cells in Series and in Parallel, 520. Arrangement of Cells to produce Maximum Current, 521. Kirchhoff's Laws, 521. Example on Kirchhoff's Laws, 522. Work done by a Current, 522. Example, 523. Joule's Law, 523. Experimental Verification of Joule's Law, 524. Examples, 524.

CHAPTER VIII

MEASUREMENT OF RESISTANCE AND ELECTROMOTIVE FORCE

Resistance Box, 527. Keys, Commutators, 528. Measurement of Resistance by Substitution, 529. Wheatstone's Bridge, 530. Wire Bridge, 530. Post-Office Box, 531. Resistance of Galvanometer. Kelvin's Method, 532. Platinum Thermometer, 532. Graphical Method of Determining the Constant Resistance in a Circuit, 533. Force on Straight Conductor in a Uniform Magnetic Field, 533. Verification of Rule, 534. Couple on Coil in Magnetic Field, 534. Moving Coil Galvanometer, 535. Voltmeter, 536. Action of Currents on Currents, 536. Ballistic Galvanometer and Comparison of Capacities, 537. Measurement of Battery Resistance, 538. Potentiometer, 539. Examples, 540.

CHAPTER IX

ELECTROLYSIS. VOLTAIC CELLS

Two Kinds of Conduction, 541. Facts of Electrolysis, 541. Faraday's Laws of Electrolysis, 542. Mass of the Hydrogen Atom, 543. Voltameters, 543. Electrolytic Dissociation, 544. Applications of Electrolysis, 545. Simple Voltaic Cell, 545. Leclanché Cell, 546. Daniell Cell, 546. Bichromate

Cell, 547. Dry Cell, 547. Cadmium Cell, 548. Storage Cell, 548. Comparison of Primary, Secondary, and Electrolytic Cells, 550. Thermoelectricity, 550. Thermopile, 551. Peltier and Thomson Effects, 551. Examples, 551.

CHAPTER X

INDUCED CURRENTS

Faraday's Experiments, 553. Lenz's Law, 554. Magnitude of the Induced E.M.F., 555. Quantity of Electricity which passes, 555. Earth Inductor, 556. Alternating Current, 556. Faraday's Ring, 557. Foucault or Eddy Currents, 557. Examples, 558. Induction Coil, 558. Magnetic Induction, 560. Examples, 563.

CHAPTER XI

TECHNICAL APPLICATIONS OF ELECTRICITY

Incandescent Lamp, 564. Example, 565. Arc Lamp, 565. Electric Furnace, 565. Electric Bell, 566. Electric Telegraph, 566. Telegraphic Relay, 567. Telephone, 567. Microphone, 568. Dynamo, 568. Electric Motor, 570. Barlow's Wheel. Faraday's Disc, 571. Back E.M.F. in Motor, 572. Application of the Energy Principle, 572. Efficiency of Motor, 573. Magneto, 573. Transformer, 573. Examples, 574.

CHAPTER XII

CATHODE RAYS, X-RAYS, AND RADIOACTIVITY

Electric Discharge in High Vacua, 575. Cathode Rays, 576. Nature of Cathode Rays, 577. Calculation of the Results, 577. Example, 579. Theory of Conduction, 579. Positive Rays, 580. X-rays, 581. Nature of X-rays, 582. Discovery of X-rays, 582. Discovery of Radioactivity, 583. Nature of the Rays, 583. Radioactive Disintegration, 584. Atomic Energy, 585. Examples, 585.

CHAPTER XIII

ELECTRICAL OSCILLATIONS AND WAVES

Electromagnetic Inertia, 586. Analogy of Pendulum, 587. Oscillatory Discharge of a Condenser, 587. Verification of the Formula, 588. Example, 589. Electric Resonance, 589. Electromagnetic Waves, 590. Generation of Electric Waves, 590. Crystal Detector, 591. Test of a Crystal, 592. Receiving Aerial, 592. Example, 593. Examples, 594.

EXAMINATION QUESTIONS	595
ANSWERS	619
INDEX	625
CONSTANTS	637

PART I
DYNAMICS AND PROPERTIES OF
MATTER

INTERMEDIATE PHYSICS

CHAPTER I

UNITS AND MEASUREMENT

Units of Length and Mass.—In dynamics and physics generally we deal with the relations between the measurements of different quantities. Now, before we measure any quantity we must have a unit in which to measure it. We must therefore begin our study of the subject by describing the units employed in it. They are of two kinds, fundamental units and derived units. The fundamental units are those of length, mass, and time; as examples of derived units we may take the unit of volume which can be expressed in terms of the unit of length, or we may take the unit of density, which can be expressed in terms of the unit of mass and the unit of length.

There are two systems of fundamental units established by law in Great Britain, the foot-pound-second system, sometimes referred to as the British system, and the centimetre-gram-second system or metric system, which is usually referred to, for short, as the c.g.s. system.

The standard unit of length on the ft.-lb.-sec. system is the yard, which is defined by Act of Parliament as follows: "The straight line or distance between the centres of the transverse lines in the two gold plugs in the bronze bar deposited in the Office of the Exchequer shall be the genuine standard at 62° F., and if lost it shall be replaced by means of its copies." The foot is, of course, $\frac{1}{3}$ of the yard. The standard unit of mass on the ft.-lb.-sec. system is the pound; this is defined as the mass of a certain piece of platinum, marked "P.S., 1844, 1 lb.," which is kept at the same place as the standard yard.

The yard and pound are arbitrary units, that is, they are not intended to be the length or mass of any object existing in nature; they are the descendants of units employed in ancient times. The c.g.s. system, on the contrary, was a new system devised by

a scientific committee appointed in France after the great revolution. The standard unit of length on this system, the metre, was based on the earth's dimensions, and was intended to be one ten-millionth part of the distance from the North Pole to the Equator measured along the meridian passing through Paris. A rod of platinum was made by Borda which should fulfil this condition at 0°C. , and as it would be inconvenient to alter the standard of length slightly every time a more accurate determination of the length of the earth's meridian was made, the metre is defined, not as the ten-millionth part of the quadrant of the meridian, but as the distance between the ends of Borda's rod at 0°C. The kilogram was originally intended by Borda to be the mass of one cubic decimetre of water at 4°C. , the decimetre being $\frac{1}{10}$ of the metre. It does not fulfil this condition exactly, and is defined as the mass of a certain lump of platinum, preserved at Paris and called the "Kilogramme des Archives."

The advantage of the metric system over the ft.-lb.-sec. system is that all the multiples and submultiples of its standard units are powers of 10; thus the centimetre is $\frac{1}{100}$ of the metre, the millimetre $\frac{1}{1000}$ of the metre, and the kilometre, the unit on the metric system corresponding to the mile, is 1000 metres, while the kilogram is 1000 grams. Length and mass can be easily changed from the one system to the other if we remember two relations, namely, that

$$1 \text{ foot} = 30.48 \text{ centimetres}$$

$$1 \text{ lb.} = 453.6 \text{ grams.}$$

The unit of volume on the c.g.s. system is the litre, which is one cubic decimetre or 1000 cubic centimetres. Owing to the ease with which calculations can be made by the metric system, it has been adopted by most civilized countries, and is used almost exclusively for scientific work.

Unit of Time.—As a consequence of the earth's rotation on its axis the sun appears to move across the sky. When it is at its highest altitude any day, it is said to be in the meridian. The interval of time between the passage of the sun across the meridian on any one day and its corresponding passage on the next succeeding day is an **apparent solar day**.

Owing to different causes the length of the apparent solar day varies throughout the year, and it is thus not suited for a standard of time. But if we take its average value, *i.e.* if we add the lengths of all the days in the year together and divide by the number of days, we obtain an interval of time which is constant, and which is known as the **mean solar day**. The unit of time on both the c.g.s. and ft.-lb.-sec. systems is the mean solar second, which is defined as one 86,400th part of the mean

solar day. The time given us by our clocks, mean solar time, is the time given by a fictitious sun which moves at a uniform speed across the sky and is during one part of the year in front of, and during another part of the year behind the real sun. The difference between the times at which the real sun and the fictitious sun cross the meridian on any day, measured in mean solar time, is called the equation of time for that day, and may be as much as 16 minutes.

The stars rise, travel across the sky, and set in the same way as the sun does. But the interval between two successive transits of any particular star across the meridian is always the same, no matter at what season of the year it is measured. This interval constitutes a **sidereal day**; the sidereal day is four minutes shorter than the solar day.

Dimensions of a Unit.—The area of a rectangle 2 centimetres broad and 3 centimetres long is $2 \text{ cm.} \times 3 \text{ cm.} = 6 \text{ cm.}^2$ or 6 sq. cm. The volume of a rectangular block 2 cm. broad, 3 cm. long, and $1\frac{1}{2}$ cm. high is $2 \text{ cm.} \times 3 \text{ cm.} \times 1\frac{1}{2} \text{ cm.} = 9 \text{ cm.}^3$ or 9 cub. cm. The unit of length consequently occurs to the second power in area and to the third power in volume. If it be denoted by L, we express this by saying that the dimensions of area are L^2 and of volume are L^3 . The density of a substance is defined as its mass divided by its volume; thus the density of lead is about 11.4 gm. per cm.^3 , or, as it is often written, 11.4 gm./ cm.^3 . Hence if we denote the unit of mass by M, the dimensions of density are ML^{-3} . In the same way, as will be seen from the next chapter, the dimensions of velocity are LT^{-1} , and the dimensions of all other derived units can be expressed as a product of powers of L, M, and T.

The dimensions of all the terms in an equation should be the same. This is sometimes a means of detecting errors. Also, as will be seen from the following example, the dimensions of a quantity must be carefully borne in mind when changing from one set of units to another.

EXAMPLE.—Change a density of 50 lb./ft.³ into lb./inch³ and gm./c.c.

$$\begin{array}{l} 50 \text{ lb.} \\ \text{ft.}^3 \end{array} = \frac{50 \text{ lb.}}{(12 \text{ inch})^3} = \frac{50 \text{ lb.}}{1728 \text{ inch}^3} = 0.02894 \text{ lb./inch}^3$$

$$\frac{50 \text{ lb.}}{\text{ft.}^3} = \frac{50 \times 453.6 \text{ gm.}}{(30.48 \text{ cm.})^3} = \frac{50 \times 453.6 \text{ gm.}}{80.48^3 \text{ cm.}^3} = 0.8011 \text{ gm./c.c.}$$

Vernier.—We shall now describe certain measuring instruments, namely, the vernier, vernier callipers, micrometer screw gauge, and spherometer.

If an object is being measured by a scale divided into centimetres and millimetres, and its end comes between two divisions,

it is possible to estimate the length to about a quarter of a millimetre. The eye does this by mentally subdividing the millimetre into quarters and estimating by how many of these imaginary quarters the end of the object exceeds the nearest whole division. If we wish, however, to have the result correct to $\frac{1}{20}$ of a millimetre, it is difficult to obtain this degree of accuracy by judgment and the eye alone, and it is usual to employ an auxiliary scale, which slides along the edge of the scale and is called a vernier, after the name of its inventor, a seventeenth-century French mathematician.

The principle of the vernier will be made clear by consideration of Fig. 1, which represents a portion of a centimetre scale

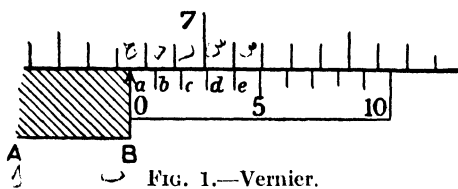


FIG. 1.—Vernier.

considerably magnified. AB is the object to be measured, and we see that its end B lies between 6.7 and 6.8 cm. Let v = length of a vernier division and s = length of the smallest scale division. Then it will be found by inspection that $9s = 10v$ or $v = \frac{9}{10}s$. That is $v = s - \frac{1}{10}s$, and the difference in length of a vernier division and scale division, or "least count" as it is called, is equal to $\frac{1}{10}$ scale division or $\frac{1}{10}$ mm.

Now look along the vernier to see which division on it is most nearly opposite to a division on the scale. It is the fourth, the one marked e in the diagram. If e coincides with a scale division, d is consequently $\frac{1}{10}$ mm. to the right of a scale division, c $\frac{2}{10}$ mm. to the right of a scale division, b $\frac{3}{10}$ mm. to the right of the corresponding scale division, and a $\frac{4}{10}$ mm. to the right of the corresponding scale division. Thus B exceeds the 7 mm. mark by $\frac{4}{10}$ mm., and the correct reading of the scale is 6.74 cm. The rule, therefore, in using this vernier is simply to look for the point where coincidence occurs; if it is at the fourth division on the vernier, the fraction is $\frac{4}{10}$ mm. If it is at the seventh division on the vernier, the fraction is $\frac{7}{10}$ mm.

The vernier represented in Fig. 1 is a particularly simple one. More generally if n vernier divisions equal $n-1$ scale divisions

$$nv = (n-1)s \quad \text{and} \quad v = s - \frac{1}{n}s$$

Thus the least count is s/n , and if coincidence occurs at the m th division the fraction is $(ms)/n$. The method of reading a strange vernier may always be made out by inspecting it, and the first few readings should always be checked by estimating the fraction by the eye.

Sometimes, but very rarely, $nv=(n+1)s$ and the vernier division is larger than the scale division. This is the form which was given to the vernier by its inventor. It has the advantage of a "more open scale," *i.e.* the divisions are further apart than in the other form and consequently easier to read, but this is outweighed by the disadvantage that the numbers on the vernier run the opposite way to the numbers on the scale itself.

Vernier Callipers.—Fig. 2 represents one form of sliding vernier callipers, consisting of a thin steel rod with a fixed jaw at

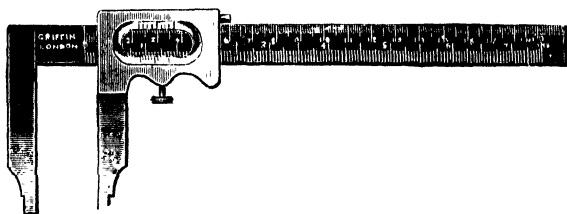


FIG. 2.—Vernier callipers.

one end and a movable jaw, which can be clamped by a screw, and the position of which can be read either in centimetres or inches according to the vernier used. The object to be measured is placed between the jaws, and the movable jaw adjusted until it *just* touches it. Then its dimensions are given directly on the scale, for when the faces of the jaws are in contact, the zero division of the vernier coincides with the zero division of the fixed scale.

Micrometer Screw Gauge.—The micrometer screw gauge, which is shown in Fig. 3, is an instrument used for measuring

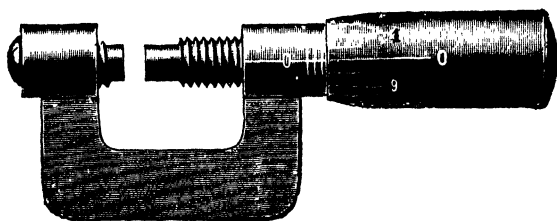


FIG. 3.—Micrometer screw gauge.

small thicknesses, for example, the thickness of a metal plate. The object to be measured is inserted in the gap at the end of the screw. The latter is moved in and out by turning its head. As the screw moves out, it uncovers a scale. When the end of the screw is pressed against the piece opposite to it, the reading on the scale is zero. In the form of screw gauge most commonly used in schools the pitch of the screw is $\frac{1}{2}$ mm., and the head

of the screw is divided into 50 equal parts, enabling fractions of a rotation to be read. Thus for every complete turn of the screw the end moves out one half-millimetre. As each division on the "drumhead" corresponds to a movement of the end of the screw through a distance of $\frac{1}{50} \times \frac{1}{2}$ mm., the instrument obviously reads to $\frac{1}{100}$ mm.

Spherometer.—The spherometer is founded on the same principle as the screw gauge. The usual construction of the instrument is shown in Fig. 4. It rests on three fixed legs, the feet of which are at the same distance from one another. These three legs support a framework in which a micrometer screw turns. The micrometer screw is equidistant from the three fixed legs. The pitch of the screw is usually $\frac{1}{2}$ mm. The number of half-millimetres through which the screw is raised is given by the vertical scale and the fraction of a half-millimetre is given by the circular scale.

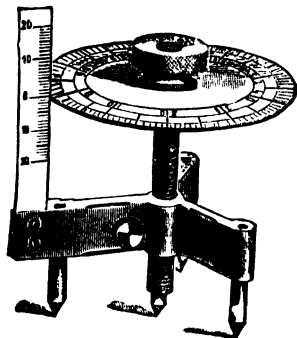


FIG. 4.—Spherometer.

The spherometer is used for determining the radius of curvature of curved surfaces such as the surfaces of lenses. Let us suppose that the surface to be measured is a concave one. The instrument is first placed upon a plane surface, usually the surface of a glass plate, and the micrometer point screwed down until it just touches the plane. The four feet of the instrument are then all in the same plane. The reading of the circle and of the scale is then taken. The instrument is next placed upon the curved surface, and the micrometer point screwed down until it makes contact again. The reading of the circle and of the scale is then taken again. Let l be the distance between two of the fixed feet, and let d be the vertical distance traversed by the point of the screw. Then the radius of curvature is given by

$$r = \frac{l^2}{6d} + \frac{d}{2}$$

It is sometimes difficult to decide when the micrometer screw is just making contact. If it goes too far, of course, the instrument rocks, but this is a clumsy test. If the surface is a polished one, as contact is approaching, an image of the point in the surface can be seen approaching the point itself, and at contact point and image touch. If the screw is turned very steadily and carefully indeed, there is a slight jar at contact, and the whole

instrument may swing round on the screw as a pivot. This is probably the best test.

Proof of Formula.—In order to prove the formula consider Fig. 5. Let A, B, and C be the points of contact of the fixed feet, and D the point of contact of the screw on the curved surface. Let G be its position when it is in the same plane as the fixed feet, and let O be the

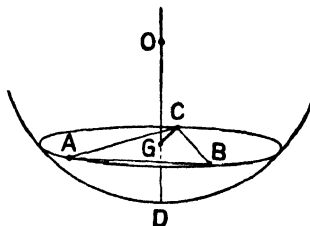


FIG. 5.

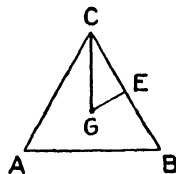


FIG. 6.

centre of curvature of the spherical surface. Then $GD = d$, $AB = BC = CA = l$, and $OD = r$.

Join CG. Then

$$r^2 = OC^2 = OG^2 + GC^2 = (r - d)^2 + GC^2$$

Draw GE perpendicular to CB. Then it is clear from Fig. 6 that

$$CG = \frac{CE}{\cos 30^\circ} = \frac{l}{\sqrt{3}}$$

$$\text{Hence } r^2 = (r - d)^2 + \frac{l^2}{3} = r^2 - 2rd + d^2 + \frac{l^2}{3}$$

$$\text{i.e. } 2rd = \frac{l^2}{3} + d^2 \text{ or } r = \frac{l^2}{6d} + \frac{d}{2}$$

the required formula. The second term in the formula may usually be neglected in comparison with the first one.

EXAMPLES I

1. Calculate the number of cubic centimetres in one cubic inch and in one cubic foot.

2. Find the number of litres in a cubic foot. Hence find the mass of 1 cub. ft. of water in kg. and in lb.

3. Change a density of 11.4 gm./c.c. into lb./ft.³ and lb./inch.³

CHAPTER II

KINEMATICS

Divisions of the Subject.—**Dynamics** is that part of physics which deals with the behaviour of matter under the action of force. It is usually subdivided into **statics** which includes the cases in which the forces do not produce any change in the motion of the bodies to which they are applied, and **kinetics** which includes the cases in which change of motion does occur as a result of the application of force. **Kinematics** deals with the geometry of motion without reference to the application of force, and may also be regarded as a subdivision of dynamics. **Hydrostatics** deals with the equilibrium of liquids under the action of forces.

Sometimes, however, the word dynamics is used as opposed to statics to denote the portion of the subject to which the name kinetics has been given above, and the name **mechanics** is then applied to the whole subject.

Speed.—Let us suppose that a motor car covers a distance of 20 miles in one hour. Then its average speed is 20 miles per hour. Either of two cases may occur. It may cover $29\frac{1}{2}$ feet in every second of the motion. In this case the speed is said to be **uniform**. This case is, of course, extremely improbable. What is more likely is that the car slows down to 10 miles per hour in passing through villages and increases its speed to 30 miles per hour in the open. In this case the speed is said to be **variable**.

If the speed is variable, in order to determine its magnitude at a particular point on the road, it is necessary to time the car over a short distance. If it is timed over 500 yards and takes 50 seconds, the average speed over this distance is 30 feet per second. If the distance is made smaller, say 100 yards, and the time taken is 9 seconds, the average speed over this distance is $33\frac{1}{3}$ feet per second. If it were possible to time the car accurately over a smaller distance still, the average speed would have another value.

The speed at the point is the value which the average speed takes, when the interval is made indefinitely small.

Speed Time Diagram.—Let us suppose that a train starts from rest at a station, and that after 5, 10, 15, 20, and 25 seconds, its speed is respectively 14, 25, 34, 41, and 47 feet per second. Then if these values are entered as ordinates in the graph in Fig. 7, and the ends joined by a curve, the ordinates at intermediate times give the speeds at these times, and the curve is known as the speed time diagram of the motion. The speed, of course, increases continuously from the beginning of the motion.

Let us suppose now, that instead of the speed varying continuously, the train moves during each five seconds with the speed that it actually possesses at the end of the five seconds, and that at the end of this period the speed changes suddenly to the value it possesses at the end of the next five seconds. The speed would then be represented by the stepped line in Fig. 8.

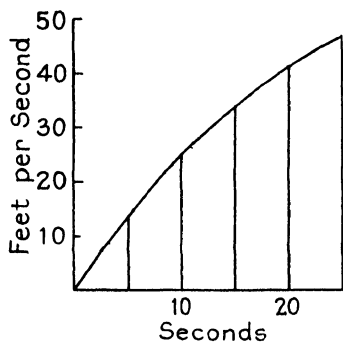


FIG. 7.

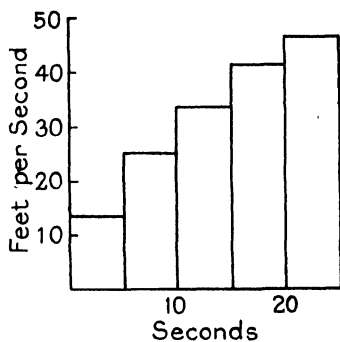


FIG. 8.

Since the speed during the first five seconds is now 14 feet per second, the distance travelled during this time is $14 \times 5 = 70$ feet. This is numerically equal to the area of the first rectangle in Fig. 8, since the length of the base is 5 and the height is 14. Similarly the distance travelled during the second five seconds, $25 \times 5 = 125$ feet, is numerically equal to the second rectangle, and the whole distance travelled during the first 25 seconds of the motion is represented by the sum of the areas of the five rectangles.

The distance travelled in the motion represented in Fig. 8 is greater than the distance travelled in the actual motion. If, however, the train travels during each five seconds with the speed it has at the beginning of the period, the distance travelled is less than the distance travelled in the actual motion. It would be represented in this case by the rectangles in Fig. 9. The difference between the distances travelled in the two hypothetical cases is represented by the shaded areas in Fig. 9.

Let us now take intervals of two and a half seconds and suppose that the speed remains constant during each such interval, and then changes abruptly. The difference in the distances traversed in the two hypothetical cases is now represented by the shaded areas in Fig. 10, and is only about a quarter as large as in the previous case.

Thus by diminishing the size of the intervals we can make the difference between the two distances as small as we please, while the true distance traversed is always intermediate between them. Hence we infer, that if the number of the intervals is made infinitely large, and the stepped lines both coincide with the actual speed time curve, the distance traversed is represented by the area of the curve itself.

The distance traversed by a moving body during any time is numerically equal to the area included between the curve repre-

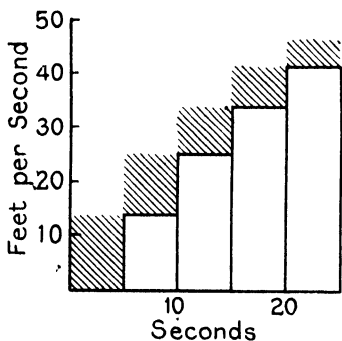


FIG. 9.

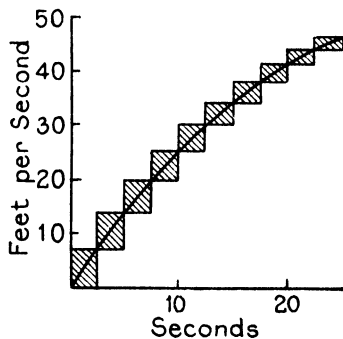


FIG. 10.

senting its motion on a speed time diagram, the axis of time, and the ordinates drawn through the points corresponding to the beginning and end of the interval considered.

Composition and Resolution of Velocities.—The word velocity is used in physics in a different sense from the word speed. When we use the word speed, we refer only to the space covered; when we use the word velocity, we consider at the same time the direction in which the motion takes place. Thus we speak of a speed of 10 miles per hour, and of a velocity of 10 miles per hour due north.

The velocity of a body at any given instant can be represented by a straight line drawn in the direction of the motion, with its length numerically equal to the magnitude of the velocity. The direction in which the motion takes place is usually specified by an arrow head placed on the line. Quantities such as velocity which can be represented by a straight line are known as **vectors** ;

quantities such as density and mass which have only magnitude and no direction are called **scalars**.

A body may have two velocities at the same time. Suppose, for example, a steamer is moving with a velocity of 20 feet per second. Then (Fig. 11) if a man is standing on its deck at O, after the lapse of one second he has been carried to A. If, however, he is running across the deck with a velocity of 12 feet per second, and would have

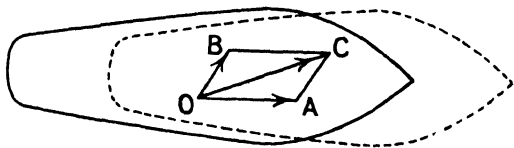


FIG. 11.—Illustrating the parallelogram of velocities.

arrived in one second at B, had the ship been at rest, owing to the motion of the ship he is carried to C. By considering intervals smaller than one second it can be shown that he is carried to the final position C along the line OC.

Now OA, since it is 20 units long, may be taken to represent the velocity of the ship, and OB, since it is 12 units long, to represent the velocity with which the man runs. These are referred to as his **component velocities**. If he had moved with the single velocity represented by OC, he would have arrived at the same place in the same time; this velocity is called his **resultant velocity**.

If a body possesses simultaneously two velocities represented in magnitude and direction by two sides of a parallelogram, drawn from one point, they are equivalent to the single velocity represented in magnitude and direction by the diagonal of the parallelogram passing through the point.

The above theorem is known as the **parallelogram of velocities**. It is proved by an appeal to experience such as in the case of the ship described above; no satisfactory formal proof of it can be given.

In the case of the man on the deck of the ship we compounded his velocity relative to the ship, OB, with the velocity of the ship, OA, in order to find his resultant velocity, OC. We might have been given the converse problem, that is, we might have been given the velocities of the man, OC, and of the ship, OA (Fig. 12),

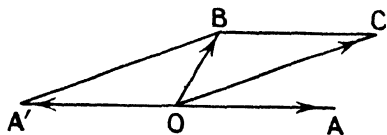


FIG. 12.

and have been asked to find the velocity of the man relative to the ship. In this case we simply superpose on both man and ship a velocity OA' equal and opposite to OA . This reduces the ship to rest, but does not alter the velocity of the man relative to

the ship. Consequently the latter is equal to his new resultant velocity, and is found by completing the parallelogram A'OC and drawing the diagonal OB. To state the rule generally :

In order to find the relative velocity of one point P with respect to another point Q, we compound with the velocity of P a velocity equal and opposite to the velocity of Q.

Any quantities that can be represented by straight lines, that is, all vectors, can be compounded and resolved by the parallelogram construction. The parallelogram construction is, in fact, simply the method of adding vectors.

EXAMPLE.—A boat is being rowed at 3 miles per hour relatively to a river which is flowing at 4 miles per hour. It is being rowed in a direction making an angle of 30° with the direction of the river. Find its resultant velocity.

We have $v^2 = v_1^2 + v_2^2 + 2v_1v_2 \cos \theta = 3^2 + 4^2 + 2 \times 3 \times 4 \cos 30^\circ$

Hence $v = 6.76$ miles per hour.

Let α be the angle which the direction of the resultant velocity makes with the direction of the river.

$$\begin{aligned} \text{Then} \quad \frac{\sin \alpha}{3} &= \frac{\sin 30^\circ}{6.76} \\ \text{hence} \quad \alpha &= 12^\circ 49' \end{aligned}$$

EXAMPLE.—A plane is inclined at an angle of 25° to the horizontal and a wooden block is sliding down it with a velocity of 30 cm./sec. Find the horizontal component of its velocity.

When we are asked to find one component, it is usually assumed that the other is at right angles to it. Thus in this problem the parallelogram of velocities becomes a rectangle with a horizontal and a vertical side and the diagonal making an angle of 25° with the horizontal. The required component is consequently

$$30 \cos 25^\circ = 27.19 \text{ cm./sec.}$$

Acceleration.—The acceleration of a body is its **rate of change of velocity**. Now velocity possesses both magnitude and direction; both may change and the one may change without the other. The simplest case is that in which only the magnitude changes, and the motion takes place in a straight line. We shall consider this case first.

Let us suppose that a body is moving in a straight line, that its velocity at a certain time is 10 ft. per sec., and that at 1, 2, 3, . . . seconds after this time its velocity is, say, 13, 16, 19, . . . ft. per sec. That is, in each second the velocity increases by the same amount, namely 3 ft. per sec. In this case the acceleration is said to be **uniform**. If, however, after 1, 2, 3, . . . seconds, the velocity had been 13, 17, 22 . . . ft. per sec., the increases in successive seconds would not have been the same. In this case the acceleration is said to be **variable**.

Let us suppose that at a certain time the velocity is 15 ft. per

sec., and that after five seconds it is 35 ft. per sec. Then the **average** acceleration during this period is

$$\frac{\text{change of velocity}}{\text{time}} = \frac{35 - 15 \text{ ft. per sec.}}{5 \text{ sec.}} = \frac{20 \text{ ft./sec.}}{5 \text{ sec.}} = \frac{4 \text{ ft.}}{(\text{sec.})^2}$$

and is written 4 ft. per sec. per sec. or 4 ft./sec.² The second "per. sec." is at first difficult to understand, but a little consideration shows the need for it. Velocity, which is rate of change of distance, is measured in units of length per second; acceleration, which is rate of change of velocity, is measured in units of velocity per second, i.e. in (ft. per sec.) per sec. The dimensions of acceleration are thus LT⁻².

In the example cited above the average acceleration over 5 sec. is 4 ft./sec.². The average acceleration over 1 sec. would probably have a different value, and if the interval of time were made indefinitely small, the average acceleration would have a still different value. The acceleration at a given instant in the case of variable acceleration is the value which the average acceleration takes when the interval becomes indefinitely small.

Uniformly Accelerated Rectilinear Motion.—Let a body be moving in a straight line with uniform acceleration, let u be its velocity at a certain instant of time, and let α be the acceleration. Let v be the velocity after a time t has elapsed.

The change of velocity in the time is $v - u$. Hence by definition

$$\alpha = \frac{v - u}{t}$$

which gives

$$v = u + \alpha t \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Let s be the space traversed during t . In order to find s consider the speed time diagram of the motion, Fig. 13. If AB and CD each represents one second, BC represents the change of velocity during the first second and DE the change of velocity during the next second. BC = DE. Hence the triangles are equal, and AE is a straight line. By proceeding in this way it may be shown that AF is a straight line. Hence

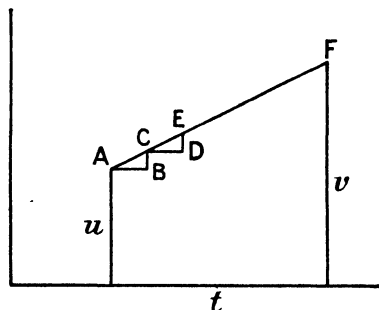


FIG. 13.

$$\begin{aligned} s &= \text{area of figure} \\ &= \frac{1}{2}(u + v)t \\ &= \frac{1}{2}(u + \{u + \alpha t\})t \\ &= ut + \frac{1}{2}\alpha t^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2) \end{aligned}$$

If formula (1) be squared, we have

$$\begin{aligned} v^2 &= u^2 + 2u\alpha t + \alpha^2 t^2 \\ &= u^2 + 2\alpha(ut + \tfrac{1}{2}\alpha t^2) \\ &= u^2 + 2\alpha s \text{ from formula (2)} \end{aligned}$$

Hence $v^2 - u^2 = 2\alpha s$ (3)

This formula does not contain the time. The above three formulæ are of great importance for solving problems in uniformly accelerated rectilinear motion. But they should not be used mechanically; it is often better not to use them and to work out the problem from first principles.

Special Cases.—If the body starts from rest, $u=0$, and the formulæ become $v=\alpha t$, $s=\tfrac{1}{2}\alpha t^2$, and $v^2=2\alpha s$. If the motion is retarded instead of accelerated, then v is less than u , and the formulæ become

$$\begin{aligned} v &= u - \alpha t \\ s &= ut - \tfrac{1}{2}\alpha t^2 \end{aligned}$$

and $v^2 - u^2 = -2\alpha s$.

EXAMPLE.—A body is moving with uniform acceleration in a straight line. Its velocity changes from 4 ft./sec. to 14 ft./sec. in 6 sec. Find the acceleration and the space passed over during the interval.

$$\begin{aligned} \alpha &= \frac{14-4}{6} = \frac{5}{3} \text{ ft./sec.}^2 \\ s &= \tfrac{1}{2}(4+14)6 = 54 \text{ ft.} \end{aligned}$$

Falling Bodies.—The importance of uniformly accelerated motion lies in the fact that it includes the case of falling bodies. In air bodies fall with unequal velocities; a piece of lead falls more rapidly than a feather or a piece of paper. It was formerly thought that this difference was due to the materials themselves, and Aristotle taught that bodies fall with "velocities proportional to their weights." But Galileo showed by dropping weights and other articles from the leaning tower of Pisa about 1590, that the difference is due to the resistance of the air which increases with the extent of surface of the body. This can also be shown by crushing a piece of paper up into a ball and seeing how much faster it falls, or by a well-known experiment performed by Newton, called the guinea and feather experiment. In this experiment a guinea and feather are placed in a long straight glass tube about four feet long, from which the air can be exhausted. When the tube is inverted, the coin and feather fall to the end of the tube in exactly the same time. But when air is introduced, the feather is retarded, and the retardation increases with the quantity of air introduced.

If the air exerted no resistance, all bodies would fall with the same acceleration. This acceleration varies slightly from place to place on the earth's surface, being least at the equator and greatest at the poles. It is always denoted by the letter g . Its average value for Great Britain is about 981 cm./sec.² or 32.2 ft./sec.²

In calculations on falling bodies the resistance of the air can be neglected, unless the surface is large in comparison with the weight, as in the case of a feather or piece of paper. On the ft.-lb.-sec. system g is usually taken as 32 ft./sec.² unless special accuracy is required.

Galileo made his celebrated experiment on falling bodies when he was 26 years of age. He was professor of mathematics at Pisa on a very small salary, and in the presence of the assembled university he ascended the famous leaning tower, balanced a 100-lb. and a 1-lb. shot on the edge, and dropped them together. They struck the ground together, while according to Aristotle the one should have taken 100 times as long as the other. The tower is 188 feet high. Aristotle's teachings were regarded as inspired, the university was not convinced, and Galileo, as a consequence of his youthful energy and imprudence, had to leave Pisa.

The formulæ for falling bodies and projectiles under gravity are due to Galileo, who is the founder of experimental physics; before Galileo it was the custom to settle a question of fact, not by appeal to experiment, but by quoting chapter and verse from Aristotle.

EXAMPLE.—A body thrown upwards against gravity reaches a greatest height of 200 ft. Find (i) the velocity with which it is projected, (ii) the time taken to reach the top, (iii) the velocity which it has after ascending 60 feet, (iv) the velocity it has two seconds after the start of the motion, and (v) the height ascended in the first two seconds of the motion.

$$(i) \quad v^2 = 2gs = 2 \times 32 \times 200 \\ v = 113 \text{ ft./sec.}$$

$$(ii) \quad s = \frac{1}{2}gt^2 \quad 200 = \frac{1}{2} \times 32 \times t^2 \\ t = 3.53 \text{ sec.}$$

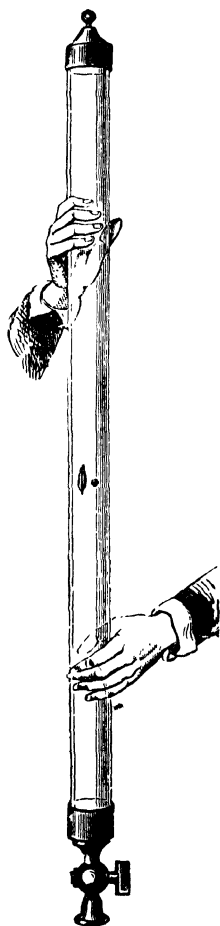


FIG. 14.—Guinea and feather experiment.

$$(iii) \quad v^2 - u^2 = -2gs \quad v^2 = 113^2 - 2 \times 32 \times 60 \\ v = 94.6 \text{ ft./sec.}$$

$$(iv) \quad v = u - gt = 113 - 32 \times 2 \\ = 49 \text{ ft./sec.}$$

$$(v) \quad s = ut - \frac{1}{2}gt^2 = 113 \times 2 - \frac{1}{2} \times 32 \times 2^2 \\ = 162 \text{ ft.}$$

The body has the same velocity on passing each point in the downward motion as it had on the upward motion, but in the opposite direction.

EXAMPLE.—A stone falls over a cliff, and travels 80 ft. in the last second before it strikes the ground at the foot of the cliff. How high is the cliff?

Let h = height of cliff and t = time of fall.

Then

$$h = \frac{1}{2} \times 32t^2 \\ h - 80 = \frac{1}{2} \times 32(t-1)^2$$

Hence

$$80 = \frac{1}{2} \times 32(t^2 - (t-1)^2) \\ t = 3 \text{ sec. and } h = 144 \text{ ft.}$$

Definition of Acceleration.—We shall now consider the definition of acceleration in the general case, when the motion is not in a straight line. Let OA (Fig. 15) represent the velocity

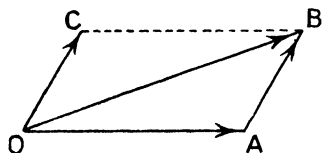


FIG. 15.

of a body at a given time, and OB its velocity after an interval of time t . Draw OC parallel and equal to AB. Then it is obvious from the parallelogram OACB, that OC represents the change of velocity during the interval t ; hence OC/t represents the average acceleration

during the interval. Since AB is parallel and equal to OC, we might equally well choose AB to represent the change in velocity, and we would in this case be saved the trouble of completing the parallelogram. We shall therefore represent the average acceleration by AB/t .

Now make t indefinitely small. The value which AB/t then takes, and the direction which AB then takes, are the magnitude and direction of the acceleration at the time in question. Thus the direction of the acceleration in the general case is not the same as the direction of the velocity.

Acceleration is a vector and can be compounded and resolved by the parallelogram construction.

Uniform Motion in a Circle.—Let us suppose that a particle is moving in a circle with uniform speed, and that it makes a complete revolution in time T . In this time it describes the angle 2π about the centre of the circle. The **angular velocity** or rate of description of angle about this point is consequently

$$\frac{2\pi}{T}$$

It is usually denoted by ω . If v is the speed in the path and r is the radius of the circle, we have

$$v = \frac{2\pi r}{T}$$

Hence

$$v = \omega r.$$

Suppose that at a certain instant the particle is at A (Fig. 16), and that after an interval t it is at B. When it is at A, it has a velocity along the tangent at A equal to v . Represent this velocity by the straight line Oa . When the particle is at B, it has a velocity along the tangent at B. Represent this velocity by the straight line Ob . $Ob = Oa$. Then the change in the velocity during the time is represented by ab .

Let $\angle aOb = \theta$. Then, if ab is bisected at c and c joined to O ,

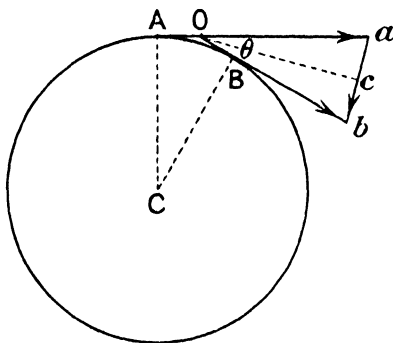


FIG. 16.

$$ab = 2ac = 2Oa \sin \frac{\theta}{2} = 2v \sin \frac{\theta}{2}$$

The average acceleration during t is

$$\frac{2v \sin \frac{\theta}{2}}{t}$$

When θ is made very small, the point B moves up to the point A, $\sin \frac{\theta}{2}$ becomes equal to $\frac{\theta}{2}$, and the average acceleration becomes equal to the acceleration at A. It then takes the value,

$$\frac{v\theta}{t}$$

But $\theta = \angle ACB = \omega t$. Hence

$$\frac{v\theta}{t} = v\omega = \frac{v^2}{r} \text{ or } \omega^2 r$$

When B moves up to A, ab becomes parallel to AC.

Hence, when a particle is moving with uniform speed in a circle,

it has an acceleration equal to v^2/r or $\omega^2 r$ towards the centre of the circle.

EXAMPLE.—A train is moving at a speed of 60 miles per hour round a curve of one-quarter mile radius. Find the acceleration towards the centre of the curve.

$$v = 60 \text{ miles/hour} = 88 \text{ ft./sec.}$$

Hence
$$\alpha = \frac{v^2}{r} = \frac{88^2 \times 4}{5280} = 5.87 \text{ ft./sec.}^2$$

Simple Harmonic Motion.—Let a point Q travel with uniform speed v round a circle of radius r and centre C, and from its instantaneous position let a perpendicular QP be drawn to AB, any fixed diameter of the circle. Then, as the point Q travels round the circle, the auxiliary circle as it is called, the perpendicular QP moves parallel to itself, and the point P travels back and forwards along the diameter AB. The point P is then said to describe a simple harmonic motion or S.H.M., as it is written for short. Simple harmonic

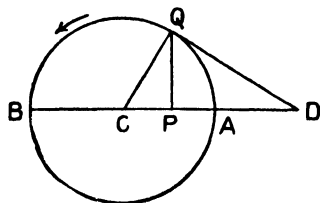


FIG. 17.

motion is thus the *projection upon a straight line of uniform motion in a circle*. It must be noted that both the point Q and the auxiliary circle are mathematical fictions used for defining the S.H.M. When S.H.M. occurs in nature there is only the one real motion, that of P in the straight line.

Simple harmonic motion occurs frequently in nature. It is described, for example, by the bob of the pendulum of a clock and by the prongs of a vibrating tuning fork. The angle QCP is called the **phase** of the motion. The whole distance that P traverses, *i.e.* AB, is called the **range** of the S.H.M., and half the range, *i.e.* the radius of the auxiliary circle, is called the **amplitude** of the S.H.M. The time taken by P to go from one end of the range to the other end and back, or the time taken by Q to go once round the auxiliary circle, is called the **period** of the S.H.M. Let it be denoted by T. Then

$$T = \frac{2\pi r}{v}$$

Join QC and let $\angle QCP = \theta$. Then the displacement of P from its mean position is given by $CP = CQ \cos \theta = r \cos \theta$.

Draw QD, the tangent at Q, and produce BA to meet it at D. Then the velocity of Q is equal to v , and is in the direction of DQ. The velocity of P is the component of the velocity of Q along DB, that is, it is $v \cos QDP = v \sin \theta$.

The acceleration of Q is v^2/r , and is in the direction QC. The acceleration of P is the component of the acceleration of Q in the direction AB, that is, it is

$$\frac{v^2}{r} \cos QCP = \frac{v^2}{r} \cos \theta$$

We have therefore

$$\frac{\text{Acceleration of P}}{\text{Displacement of P}} = \frac{\frac{v^2}{r} \cos \theta}{\frac{v^2}{r} \cos \theta} = \frac{v^2}{r^2} = \frac{4\pi^2}{T^2}$$

Thus, when a particle is describing a S.H.M., the ratio of its acceleration to its displacement is constant and equal to $4\pi^2$ divided by the square of the period.

Conversely, when a particle is moving in a straight line under an acceleration towards a fixed point in the line proportional to its distance from that point, it is describing a S.H.M. This statement, which is often taken as the definition of S.H.M., may be seen to be in harmony with the previous definition, if the foregoing proof is gone through in the reverse direction.

The number of periods per second or $\frac{1}{T}$ is called the **frequency** of the S.H.M.

EXAMPLE.—If the frequency of a S.H.M. is 250 per second and the amplitude is 1 millimetre, determine the velocity and the acceleration when the displacement has half its maximum value.

$v = 250 \times 2\pi \times 0.1 = 50\pi$ cm./sec. In the case in question $\cos \theta = \frac{1}{2}$ and $\theta = 60^\circ$. Hence

$$\text{velocity} = v \sin \theta = 50\pi \sin 60 = 136 \text{ cm./sec.}$$

$$\text{acceleration} = \frac{v^2}{r} \cos \theta = \frac{(50\pi)^2}{0.1} \cos 60 = 1.234 \times 10^5 \text{ cm./sec.}^2$$

Parabolic Motion.—Let us suppose, that a body starts off from O (Fig. 18) with a horizontal velocity v , and that it has an acceleration g downwards. This is the motion of a stone thrown out horizontally from the top of a cliff. The horizontal and vertical motions take place independently of one another, so if we denote distance outwards from the cliff by x and downwards by y

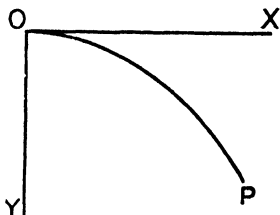


FIG. 18.

$$x = vt, \text{ and } y = \frac{1}{2}gt^2$$

Hence, eliminating t ,

$$y = \frac{g}{2v^2} x^2$$

This equation is one that is often graphed in schools. It represents a parabola with its axis vertical and its vertex in the point of projection.

Let us suppose now that when the stone reaches P, its velocity is reversed in direction, the magnitude being kept unaltered. It will retrace its path, pass through O horizontally, and descend the other side of the same parabola. The path of a body projected at any point under gravity is consequently a parabola.

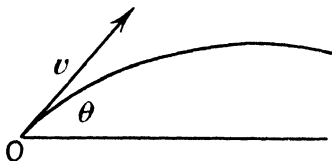


FIG. 19.

Range and Time of Flight of a Projectile.—Let a body be projected from the point O (Fig. 19) with a velocity v in a direction making an angle θ with the horizontal. The horizontal and vertical components of the

the motion are considered independently. The vertical component of the initial velocity is

$$v \sin \theta$$

After time t it is

$$v \sin \theta - gt$$

Consequently it is zero, and the projectile has reached its highest point when

$$t = \frac{v \sin \theta}{g}$$

The projectile takes as long to descend to the initial level as to ascend from it. Hence the whole time of flight is

$$\frac{2v \sin \theta}{g}$$

During the whole flight the horizontal component of the velocity has its initial value

$$v \cos \theta$$

Hence the range on a horizontal plane is given by

$$v \cos \theta \times \frac{2v \sin \theta}{g} = \frac{2v^2 \sin \theta \cos \theta}{g} = \frac{v^2 \sin 2\theta}{g}$$

EXAMPLE.—An aeroplane travelling horizontally at 60 miles per hour drops a bomb at a gasometer 1000 feet beneath. What angle must the line joining bomb and gasometer make with the vertical, when the bomb is let go?

Let t sec. = time to fall.

Then $1000 = \frac{1}{2} \times 32t^2$, $t = 7.907$ sec.

Horizontal distance travelled in this time $= 88 \times 7.907 = 695.9$ ft.

Let θ be the required angle

$$\tan \theta = \frac{695.9}{1000} \text{ and } \theta = 34^\circ 50'$$

EXAMPLES II

1. A train starts from rest and is uniformly accelerated during the first $\frac{1}{4}$ minute of its run, runs $\frac{1}{2}$ minute at a constant speed, and is then brought to rest in $\frac{1}{8}$ minute, the retardation being uniform. Draw the speed time diagram of the motion. If the whole distance traversed is $\frac{1}{4}$ mile, find the maximum speed.

2. A ship making 5 miles per hour in a northerly direction is drifting westward with the tide at the rate of 3 miles per hour. What is its resultant velocity?

3. A train travels at a speed of 8 miles per hour round a hair-pin bend consisting of a semicircle connecting two parallel tracks. The train is longer than the curved portion of the track. Find the velocity of the engine relative to the guard's van, (i) when the engine has turned through 45° , (ii) when it has turned through 90° .

4. A train moving at 30 ml./hr. is brought to rest in 100 sec., the retardation being uniform. Find how far it travels in this time. What is the retardation in ft./sec.²?

5. A body is thrown vertically upwards with a velocity of 128 ft./sec. Find the height to which it rises and the time it takes to reach the highest point.

6. A body is moving in a straight line with a velocity of 15 ft./sec. and an acceleration of 10 ft./sec.² After what time will its velocity be tripled, and what space will it describe in this time?

7. With what velocity must a body be thrown vertically upwards in order that it may rise to a height of 30 ft.? What time does it take to reach this height?

8. How far must a body fall to acquire a velocity of 10 ft./sec.?

9. A stone is falling with a velocity of 20 ft./sec. How much further must it fall for the velocity to increase to 30 ft./sec.?

10. A man on a motor cycle dashes into a wall at a speed of 40 ml./hr. How far would he have to fall to hit the ground with the same speed?

11. A train moving with uniform acceleration takes 4 min. to travel the first mile. What velocity has it acquired at the end of that time? What is its acceleration in ft./sec.²?

12. Two trains are running in the same direction at the speed of 48 ml./hr., the first train leading by 4 ml. If the second train is brought to rest in $\frac{1}{3}$ of a mile, kept at rest for 1 min., and then brought to its former speed in 1 mile, find the extra lead gained by the first train.

13. A cyclist turns a corner at 10 ml./hr. in a curve of 15 ft. radius. What is his acceleration towards the centre of curvature?

14. Taking the earth's equatorial radius as 3962 miles and the sidereal day, *i.e.* the time taken by the earth to make a complete revolution about its axis, as 86,164 sec., find the acceleration of a point on the equator in ft./sec.²

15. What is the value of the ratio of acceleration to displacement for one of the prongs of a tuning fork, which makes 256 complete vibrations per second? If its range is $\frac{1}{16}$ mm., find its maximum velocity.

16. A tuning fork makes 512 complete vibrations per second. The range of the end of one of the prongs is $\frac{1}{32}$ mm. Find its maximum velocity and maximum acceleration.

CHAPTER III

STATICS

Parallelogram of Forces.—Fig. 20 represents a drawing board which is supported vertically by a wooden frame. Round wooden pegs are attached to the drawing board at D and E. The upper ends of two spiral springs B and C are attached to

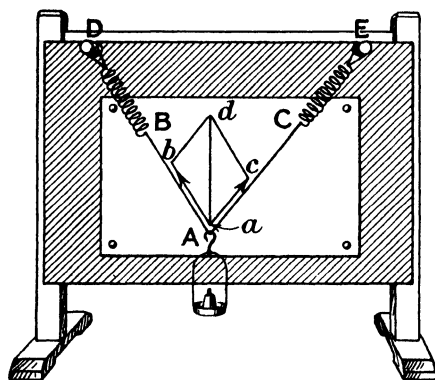


FIG. 20.—Apparatus for proving parallelogram of forces.

these pegs by means of loops of thread, and to the lower ends of these springs are tied two pieces of thread which have each a loop at their lower ends at A. Both these loops are passed over a hook which carries a pan for holding weights. When weights are placed in the pan both the springs are pulled out. Now at the point A three forces meet, one due to the weights in the pan and the weight of the pan itself; this force acts vertically downwards

through A. The other two forces act upwards, one along the thread AB and the other along the thread AC. A piece of drawing paper is pinned on the board behind the threads. The purpose of the apparatus is to determine the relations between these three forces, when the threads AB and AC are inclined at different angles to one another and to the vertical. The springs, threads, and scale pan hang clear of the drawing board.¹

¹ It should be noted that the springs B and C in Fig. 20 cannot be replaced by spring balances; owing to friction the latter do not give satisfactory readings when inclined to the vertical. Also it is not satisfactory to dispense with springs and pass the threads AB and AC over pulleys, attaching scale pans to their other ends, because owing to friction in the pulley the force in a thread is never the same on both sides of it.

The use of the apparatus will be understood best from the description of a particular case. The weight of the pan was 9.2 gm.; in it was placed a 200-gm. weight, and thus the total downward force was 209.2 gm. The springs were originally each about 7.85 cm. long. When the weight was placed in the pan, they were elongated and their lengths were measured very carefully with a centimetre rule. It was found then that the length of the left-hand one had become 12.7 cm., and the length of the right hand one 11.75 cm. Pencil marks were made on the paper behind the two threads so as to get their line, and a thread carrying a weight was held up in front of the paper, and pencil marks made from it giving the line of the vertical. The scale pan was then unhooked from the thread AC, so that it hung on the thread AB vertically below D, and weights were placed in it until the same extension was produced as was obtained during the experiment. The necessary weight, including the pan itself, was 129.2 gm. Similarly the weight necessary to produce the same extension in the spring CE, as was obtained in the experiment, was found to be 109.2 gm.

Now a force is completely defined when its magnitude, its direction, and the point at which it acts are given. Forces are consequently vectors and can be represented by straight lines. The paper was unpinned from the drawing board, and the traces *ab*, *ac* of the threads AB and AC drawn. For the sake of clearness *ab* and *ac* are shown a little out of position in the figure. The force along AB was 129.2 gm.; *ab* was made exactly 129.2 mm. long. The force along AC was 109.2 gm.; *ac* was made 109.2 mm. long. Thus, by taking one millimetre to represent one gram, *ab* and *ac* represented the forces applied by the springs in magnitude, direction, and point of application.

The parallelogram was then completed and the diagonal *ad* joined. It was found to coincide very nearly with the trace of the vertical through *a*, the difference in the directions of the two lines being about half a degree. Its length was measured and found to be 208.8 mm. Now the downward force exerted by the weights in the pan was 209.2 gm., and 209.2 coincides with 208.8 within the error of experiment. Thus the diagonal of the parallelogram represents the force exerted by the weights and pan at A in magnitude and direction. If the experiment is repeated with other weights in the pan and with different inclinations of the threads to the vertical, the result is always found to be the same; if the sides of the parallelogram represent the forces exerted by the springs, the diagonal represents the force exerted by the weights.

The downward force at A is called the **equilibrant** of the two upward forces because it neutralises them. The line *ad* does

not represent the equilibrant in direction ; it goes the wrong way. It represents a single force which produces the same effect as the two upward forces, and is called their **resultant**. We thus arrive at the proposition known as the parallelogram of forces.

If two forces act at a point, and if lines be drawn from that point representing the forces in magnitude and direction, and a parallelogram be constructed on these lines as sides, then the resultant of the two forces will be represented in magnitude and direction by the diagonal which passes through the point.

The forces represented by the sides are called the **components** of the force represented by the diagonal.

EXAMPLE.—An upward force of 100 lb. is exerted on a weight by means of a rope inclined at 20° to the vertical. Find the vertical component of this force.

When one component is asked for, it is assumed, unless otherwise specified, that the other component is at right angles to it. Hence the parallelogram is in this case a rectangle, and the result is

$$100 \cos 20^\circ = 93.97 \text{ lb.}$$

Triangle of Forces.—Let AB and AC (Fig. 21) represent two forces. Then AD represents their resultant, and DA their equilibrant. Now AC and BD are equal and parallel, since they are the opposite sides of a parallelogram. Hence the force represented by AC may be also represented by BD in magnitude and direction but not in position, and the three forces represented by AB, BD, and DA are in equilibrium. Thus :

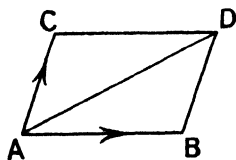


FIG. 21.

If three forces represented in magnitude and direction by the sides of a triangle taken in order act on a particle, then the forces are in equilibrium.

This proposition is known as the triangle of forces.

Polygon of Forces.—Let AB, BC, CD, and DE (Fig. 22) represent in magnitude and direction four forces which act at a point. Then, if we join AC, it is obvious by the triangle of forces that CA is the equilibrant and AC the resultant of AB and BC. Similarly, if we join AD, we find that AD is the resultant of AC and CD, and if we join AE, AE is the resultant of AD and DE, that is, it is the resultant of the four forces AB, BC, CD, and DE, and EA is their equilibrant. Proceeding in this way we

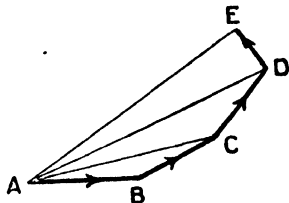


FIG. 22.

arrive at the proposition known as the polygon of forces, which is stated as follows:

If any number of forces acting on a particle are represented in magnitude and direction by the sides of a closed polygon taken in order, then the forces are in equilibrium. If the lines representing the forces do not form a closed polygon, the resultant will be represented by the line joining the ends of the first and last-drawn sides of the incomplete polygon.

EXAMPLE.—Forces of 4, 3, 1, 5, and 6 lb. act on a particle in one plane in directions making angles 45° , 90° , 120° , 150° , and 180° with a given straight line. Find the resultant in magnitude and direction.

The polygon may be drawn and the result obtained graphically, but it is better to proceed as follows: Resolve each of the forces into two components, one parallel and the other perpendicular to the given straight line. The sum of the parallel components is

$$4 \cos 45^\circ - \cos 60^\circ - 5 \cos 30^\circ - 6$$

$$= -8.001$$

The sum of the perpendicular components is

$$4 \sin 45^\circ + 3 + \sin 60^\circ + 5 \sin 30^\circ$$

$$= 9.194$$

Hence the resultant $= \sqrt{8.001^2 + 9.194^2} = 12.19$ lb. in a direction making an angle of $131^\circ 2'$ with the initial line.

Law of Moments.—Let AB represent a force of P units in magnitude, direction, and position. From any point P draw PD perpendicular to AB, and let $PD=p$. Then the product Ep is called the **moment of the force** about the point.

A metre stick was balanced on the sharp edge of a wedge-shaped piece of wood, and was found to come to rest when the edge was below 50.25 cm. This was due to the stick not having quite the same thickness at both ends; otherwise it would have balanced at the middle. When the stick was balanced in this way it was in equilibrium under the action of two forces—

- (1) Its weight acting vertically downwards.
- (2) The upward force exerted by the wedge.

When two forces are in equilibrium, they must be equal, and act in opposite directions in the same straight line. Hence the weight of the stick must act vertically downwards through 50.25 cm., and the upward force exerted by the wedge must be equal to the weight of the stick.

A loop of thread was tied to a 200-gm. weight and the latter hung over the stick at 40.25 cm. Loops of thread were tied to another 200-gm. weight, a 100-gm. weight, and a 50-gm. weight,

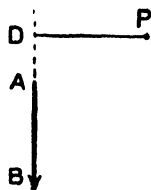


FIG. 23.

and these were in succession one at a time hung on the stick on the other side of the wedge (Fig. 24). It was found, when the



FIG. 24.

wedge was at 50.25, that in order to produce equilibrium the second 200-gm. weight had to be placed at 60.20 cm., the 100-gm.

weight at 70.15 cm., and the 50-gm. weight at 90.40 cm.

The moment of the first 200-gm. weight about the edge of the wedge was

$$200 \times (50.25 - 40.25) = 2000 \text{ units}$$

The moments of the other three weights about the same point were in succession

$$200 \times (60.20 - 50.25) = 1990 \text{ units}$$

$$100 \times (70.15 - 50.25) = 1990 \text{ units}$$

and $50 \times (90.40 - 50.25) = 2007 \text{ units}$

Now the numbers 2000, 1990, and 2007 agree with one another within the limits of experimental error. Hence, when the stick is balanced, the moment of the weight suspended from the one side is equal to the moment of the weight suspended from the other.

The weight on the right of the wedge tends to make that side fall, *i.e.* it tends to make the stick rotate in the direction of the hands of a clock, or in the clockwise direction as it is called. The weight on the left of the wedge tends to make the stick turn the other way, in the direction opposed to the hands of a clock, or in the counter-clockwise direction. The stick does not turn at all, when the turning power of the force on each side is the same. *Thus the moment of a force about a point measures the turning power of the force about the point.*

Instead of hanging a weight on the other side the stick may be balanced by passing a loop of thread over the hook of a spring balance, and supporting it in the manner shown (Fig. 25). In an actual experiment made it was found, that when the loop of thread was at 28.7 cm., the spring balance read 94 gm. The spring balance in this case obviously exerted a clockwise moment of

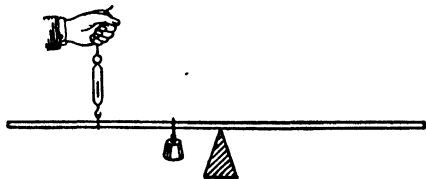


FIG. 25.

$$94 \times (50.25 - 28.7) = 2026 \text{ units}$$

about the edge. This result agrees with the former ones, the

greater error in this respect being due to the difficulty of reading the spring balance accurately.

If we have several weights suspended on each side of the edge, and one or more spring balances acting on it, we find the stick balances when the sum of the moments tending to turn it in the clockwise direction is equal to the sum of the moments tending to turn it in the counter-clockwise direction.

We thus, as a result of experiment, arrive at the general law known as the **law of moments**, which may be stated as follows :

When a body which is free to turn about an axis is in equilibrium, the sum of the moments of the forces tending to turn it in the clockwise direction is equal to the sum of the moments tending to turn it in the counter-clockwise direction.

EXAMPLE.—A metre stick weighing 88 gm. carries a 50-gm. weight at 10 cm., a 60 gm. weight at 20 cm., and a 70 gm. weight at 80 cm. At what point will it balance ?

Let it balance at x cm. Assume that the weight of the stick acts downwards through its midpoint. Then the clockwise moments are

$$88(50-x) \text{ and } 70(80-x)$$

The counter-clockwise moments are

$$50(x-10) \text{ and } 60(x-20)$$

On equating

$$88(50-x) + 70(80-x) = 50(x-10) + 60(x-20)$$

whence

$$x = 43.66 \text{ cm.}$$

The Lever.—A lever is a rigid bar capable of rotation about a point and employed for changing one force into another, which is more suitable for the purpose in view. The first force is called the **power** or effort, the second the **weight** or resistance, and the point about which rotation takes place is called the **fulcrum**.

Fig. 26 depicts a lever which is being used to roll a large stone over. The point C where the end of the lever rests on the ground is the fulcrum. The force P represented as applied by the workman at right angles to the bar at A is the power, and the resistance of the stone represented as acting at right angles to the bar at B is the weight. The distances CA and CB, which we shall denote by a and b , are called the arms of the lever.

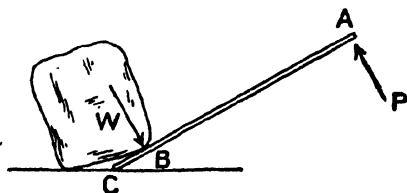


FIG. 26.

Let us suppose that the stone is poised halfway, that it is lifted a certain distance but that the workman is just holding his

own, neither gaining nor losing. Then, if the weight of the lever itself is disregarded, the moments of the two forces about C are equal, that is

$$Pa = Wb$$

or
$$\frac{W}{P} = \frac{a}{b}$$

Now a is many times greater than b , consequently W is many times greater than P , and the workman is enabled to apply a much greater force at B than if he were to use the direct force of his hands. The ratio W/P is called the **mechanical advantage** of the lever.

P and W are represented as acting perpendicularly to the lever in Fig. 26. This is not necessary. The equations hold whatever the directions of P and W , provided we regard a and b not as the lengths of CA and CB, but as the perpendicular distances from C to the lines of action of P and W .

In the example cited above the weight is between the fulcrum and the power. This arrangement is usually referred to as a lever of the second class. A lever of the first class is one in which the fulcrum is between the power and weight, as, for example, in a pair of scissors, which may be regarded as two levers crossing one another. An oar when used for splashing is another example of a lever of the same kind; the rowlock is then the fulcrum, the jerk exerted by the oarsman the force, and the resistance of the water to the blade the weight. But when used for rowing it becomes a lever of the second class. A lever of the third class is one in which the power is between the weight and the fulcrum, as in a pair of tongs or in the treadle of a lathe. In this case the mechanical advantage is less than 1, but, as will be shown later on, there is a gain in speed.

The law of moments and the principle of the lever were known to Archimedes (287–212 B.C.). It was with reference to the latter that he uttered the famous saying, "Give me where I may stand and I will move the world." The parallelogram of forces was first fully explained by Galileo, though special applications of it had been discovered much earlier.

Equilibrium of Parallel Forces.—The law of moments may be regarded from a somewhat different standpoint. Let Fig. 27 represent the same metre stick as was used above. It is now balanced at the point C, a weight P is hung on at A, and B is the point at 50.25 cm. where it balanced before. It is now in equilibrium under the action of three forces—

- (1) P acting downwards through A.

(2) The weight of the whole stick itself, which we shall denote by Q , acting downwards through B .

(3) The upward force through C exerted by the wedge which we shall denote by R .

It is clear that the weight of the stick acts through B , because the stick would balance there if the weight P were removed. Also, since the stick does not move, the whole downward force must equal the upward force, *i.e.*

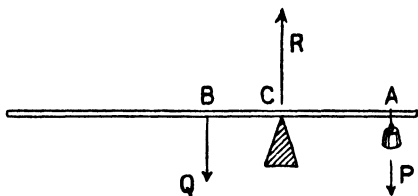


FIG. 27.

$$P + Q = R$$

This can also be shown experimentally by suspending the stick at C in a loop of thread hung from a spring balance, instead of resting it on the wedge. If, for example, the stick weighs 95 gm., a likely value, and P is 50 gm., it will then be found that the balance is pulled out to $95 + 50$, that is, to 145 gm.

By the law of moments

$$P \times AC = Q \times BC$$

or

$$\frac{P}{BC} = \frac{Q}{AC} = \frac{P+Q}{BC+AC}$$

by a well-known algebraical result.

But $P + Q = R$ and $BC + AC = BA$.

Therefore we have the following symmetrical result, which is very easily remembered—

$$\frac{P}{BC} = \frac{Q}{CA} = \frac{R}{AB}$$

The meaning of this result will be more evident from Fig. 28, which represents the forces of Fig. 27. themselves, with the metre stick and wedge removed. We had a system of three parallel forces. We have arrived at the result, that for these three forces to be in equilibrium the middle one must have the opposite direction to the two outside ones, and each of the three must be proportional to the perpendicular distance

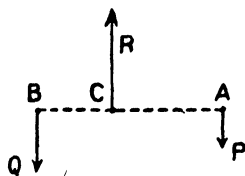


FIG. 28.

between the other two.

If the direction of the force R is reversed, it gives the resultant, not the equilibrant, of the forces P and Q .

EXAMPLE. — A weight of 120 lb. is slung from a pole which is carried on the shoulders of two men, A and B. The distance between the points where the pole presses on their shoulders is 6 ft., and the point where the weight is slung is 4 ft. from the point where the pole presses on A's shoulder. Find the weight borne by each, the weight of the pole being neglected.

Let P and Q be the weights carried by A and B respectively. Then by the rule above

$$\frac{P}{2} = \frac{Q}{4} = \frac{120}{6}$$

Thus A carries 40 lb. and B 80 lb.

EXAMPLES III

1. A weight of 30 lb. is suspended by two strings inclined to the vertical at 30° and 60° . Find the stretching force in each string.

2. Two forces of 6 and 8 gm. act upon a body in lines which meet in a point and are at right angles. Find the magnitude of their resultant.

3. A 100-gm. weight is suspended by a thread. A second thread is then tied to the weight, and pulled in a horizontal direction until the first thread makes an angle of 45° with the vertical. Find the stretching forces in both threads.

4. The resultant of two forces at right angles is 26 lb., and one of the forces is 10 lb. Find the other.

5. Forces 1, 3, 4, 2, and 7 gm. act on a particle in one plane in directions making angles 30° , 60° , 90° , 135° , and 180° respectively with a given straight line. Find their resultant in magnitude and direction.

6. Are three forces in equilibrium when they are represented in magnitude, direction, and position by the sides of a triangle taken in order?

7. Two men, A and B, carry a weight slung from a pole resting on their shoulders. If the maximum weight A can carry is 120 lb. and the maximum weight B can carry is 90 lb., what is the maximum load they can carry between them, and where must it be slung on the pole? The weight of the pole is to be neglected.

8. The arms of a balance are of slightly unequal length. When a body is put in the right-hand pan and the weights in the left-hand pan, its weight is found to be w_1 ; when the body is placed in the left-hand pan and the weights in the right-hand pan, its weight is found to be w_2 . Show that the true weight is $\sqrt{w_1 w_2}$.

9. A metre stick weighs 92.0 gm., and balances on a knife-edge at the 50.2 cm. mark. A 50-gm. weight is slung on it by a piece of thread at the 10.0 cm. mark. Where will the stick now balance?

10. The same metre stick rests on a fulcrum or knife-edge at 10 cm., and carries a 50-gm. weight at 60 cm. It is supported at 90 cm. by a loop of thread, which is passed over the hook of a spring balance? What is the force exerted by the balance?

11. Weights of 7, 5, 3, and 1 lb. are hung at 1, 2, 3, and 4 ft. from one end of a rod of length 5 ft. and of negligible weight, the rod being hinged at this end. What force must act upwards at the other end to sustain them? What is the upward force exerted by the hinge?

CHAPTER IV

THE LAWS OF MOTION

IN Chap. II we have considered the motion of bodies apart from the action of force and in Chap. III we have considered problems in which forces acted without producing motion. We have now to consider how force causes motion. This part of the subject is more difficult than the preceding chapters, and will not be understood unless they are very carefully mastered.

In Chap. III we used force without defining it. The easiest way of becoming acquainted with what it means is simply to do the experiments, or, if this is not possible study the descriptions of the experiments recorded in that chapter. In these experiments force was measured in grams weight. Grams weight and lbs. weight or their multiples are the most natural units to take and are called the **gravitational units of force**. The ton weight is much used by engineers. But, as will be explained later, the gravitational units have the disadvantage that their values vary slightly over the earth's surface.

Momentum.—In physics we attach a more precise meaning to the word "momentum" than in daily life.

The momentum of a body is a property which it possesses in virtue of its mass and velocity, and which is measured by the product of its mass and its velocity.

The word will have to be taken more or less on trust at first, but it may be noted at present, that the greater the momentum of a body the greater is the effort required to stop it. A steam road roller is difficult to stop on account of its mass and a bullet on account of its velocity, and it may be shown by allowing weights to fall into the hand from different heights, that the violence of the impact increases roughly with the product of mass and velocity.

The mass of a body remains constant. Hence its momentum can change only on account of the velocity changing. Now the rate of change of velocity is acceleration. Hence for every body

rate of change of momentum = mass $\times$ acceleration.

Newton's Laws of Motion.—The whole of dynamics is based on the following three laws which were first stated clearly

and accurately by Newton and are hence called Newton's laws, although they are founded on Galileo's work. They are not capable of direct proof, but they are known to be true, because calculations based on them are always in agreement with facts.

I. Every body continues in its state of rest or of uniform motion in a straight line, except in so far as it is compelled by impressed forces to change that state.

II. The rate of change of momentum of a body is proportional to the impressed force, and takes place in the direction in which the force acts.

III. To every action there is an equal and opposite reaction.

The First Law of Motion.—The first law contains two statements. The first of these, namely, that a body at rest continues in a state of rest unless it is compelled by a force acting on it to change that state, appears self-evident. A book resting on the table remains at rest unless it is lifted or pushed. The second statement, namely, that a body already in motion continues in a state of uniform motion in a straight line, unless it is compelled by impressed forces to change that state, does not appear at first sight so evident, because, for example, a curling stone sliding over the ice, when once started, does not go on for ever, but gradually comes to a stop. A little consideration, however, shows that it is the impressed force due to the surface of the ice rubbing on it, that brings it to rest. Also the smoother the ice, the farther it is possible to make the stone go. Hence we come to the conclusion, that if the ice were ideally smooth, that if there were no rubbing at all between it and the stone, then the latter would go on with undiminished velocity until it came to the edge of the pond.

Various illustrations may be given of the first law. If a half-penny is placed on a card which rests on the top of a wine-glass, and the card is flicked away smartly with the finger, the half-penny does not go with it, but remains in a state of rest until the card is gone from beneath it, and then drops into the glass with a ringing sound.

When a passenger leaves a tramcar in motion, all his body has the same forward motion as the car has. But on touching the ground his feet are brought to rest, and, unless he runs forward or leans back, he will fall. When a passenger is standing in a tramcar, and the brakes are applied suddenly, the floor of the car and the passenger's feet are brought to rest, but the upper part of his body continues in its state of uniform motion, and he is consequently thrown forward in the direction of the motion.

Again, when a circus rider jumps through a hoop and alights on the same place on the horse's back, he requires only to jump

up, not forwards. He remains in the same state of forward motion in the air as when on the horse's back. So when he descends again, the horse is still beneath him.

The Second Law of Motion.—Let a force of F units act on a body of mass m and produce an acceleration α . Then the rate of change of momentum is $m\alpha$, and the second law of motion states that

$$F = k m \alpha$$

where k is a constant of proportionality.

Hitherto we have measured force in grams weight or lb. weight, the gravitational units of force. We shall now define two other units called **absolute units of force**, which are chosen so as to make F not merely proportional to, but equal to, $m\alpha$.

The dyne is that force which acting upon a mass of one gram produces an acceleration of one centimetre per second per second in the direction in which it acts.

The poundal is that force which acting upon a mass of one pound produces an acceleration of one foot per second per second in the direction in which it acts.

Now the weight of one gram acting on one gram produces a downward acceleration equal to g or 981 cm./sec.², i.e. a value of $m\alpha = 981$. The gram weight is therefore equal to 981 dynes. The weight of one lb. acting on one lb. produces a downward acceleration of 32.2 ft./sec.² Hence the lb. weight is equal to 32.2 poundals.

When absolute units are used, the second law of motion takes the simple form

$$F = m\alpha$$

It is better always to use the law in this form, and, if the force happens to be given in gravitational units, to *change it into absolute units before inserting it in the formula.*

It is implied in the second law, that a given force produces the same change of momentum in unit time, whether the body is at rest or in motion when the force begins to act, also that, if several forces are acting simultaneously on the body, each produces the same change of momentum as would be produced, if it acted alone.

EXAMPLE.—A force of 10 dynes acts on a mass of 5 gm. for 4 sec. Initially the mass is at rest. Calculate the acceleration and the velocity produced.

$$10 = 5\alpha, \quad \text{hence } \alpha = 2 \text{ cm./sec.}^2$$

The velocity produced is $2 \times 4 = 8$ cm./sec.

EXAMPLE.—A force of 4 lb. weight acts on a mass of 8 lb. Find the acceleration produced.

$$4 \text{ lb. wt.} = 4 \times 32 \text{ pdl., taking the approximate value of } g.$$

$$\text{Hence } 4 \times 32 = 8\alpha \quad \text{and} \quad \alpha = 16 \text{ ft./sec.}^2$$

EXAMPLE.—Express a force of 1 lb. wt. in dynes.

$$1 \text{ lb.} = 453.6 \text{ gm.} = 453.6 \times 981 \text{ dynes} = 4.45 \times 10^5 \text{ dynes.}$$

Mass and Weight.—The mass of a body is the quantity of matter in it. It is always the same no matter where the body is situated. The weight of a body is the force with which the body is attracted downwards by the earth.

Let us suppose that we have two bodies of masses m_1 and m_2 at the same place. Then since they fall with the same acceleration g , their weights are m_1g and m_2g . Consequently their weights are proportional to their masses.

If a body is carried over the earth's surface, its weight varies slightly from place to place, being least at the equator and greatest at the poles. If we took a spring balance reading to 250 gm. and hung an object weighing 200 gm. on it, the spring would not be pulled out to the same extent at different places. If the reading on the scale were 200 gm. at London, it would be $\frac{6}{10}$ gm. less than this at the equator and $\frac{4}{10}$ gm. greater than this at the poles. Such differences would not be perceptible on an ordinary spring balance—the Salter's letter balance for 0 to 250 gm. is only divided for every 5 gm., and reads to about 1 gm. Hence the gravitational units are used for practical purposes, as their use often avoids multiplication by g and simplifies calculation; thus engineers are accustomed to measure forces in tons weight and kilograms weight. But for scientific work it is extremely important that the unit should be constant, and hence in scientific work the dyne is always used as the unit of force. The poundal and its multiples, one of which, *e.g.* the tondal, has a special name, are used only in text-books and in examination questions.

It should be noted, that if an object were weighed in an ordinary balance against a 200-gm. brass weight, the result would be the same both at the equator and the poles. For bodies which have equal weights at one place have equal weights at any other place. But the force with which the earth attracts both the object and the brass weight is less at the equator than at the poles. The weight of a pound of matter is therefore not a definite standard of force, but the pound of matter itself is a perfectly definite standard of mass.

The distinction between the words "mass" and "weight" is not carefully adhered to, and the word "weight" is often used when the word "mass" is meant.

The Third Law of Motion.—The third law of motion states that forces always occur in pairs, each pair consisting of two equal and opposite forces. If the hand presses down on the table, the table presses up on the hand with an equal and opposite force. If a weight is hung on the end of a spring, it pulls the

end down, and the spring pulls it up with an equal and opposite force. If a magnet attracts a piece of iron, the iron at the same time attracts the magnet with an equal and opposite force. Each of the pairs of forces is said to constitute a stress. A force is simply one aspect of a stress; a force in one direction cannot exist by itself.

No difficulty is experienced in accepting the truth of the law in the case of objects which do not move, as, for example, the hand pressed on the table. But in the case of, for example, a horse pulling a canal boat by means of a tow rope, it is sometimes argued by beginners, that the forward force exerted by the horse along the rope on the boat must be greater than the backward force exerted by the boat along the rope on the horse, otherwise the boat would not move forward at all. This is quite wrong. The two forces in the rope are exerted on different things, one on the boat and the other on the horse, and they are exactly equal. Only if they were both exerted on the same thing, would the one neutralise the other.

Conservation of Momentum.—Let us suppose that two bodies strike one another and rebound. Then they are in contact for a very short interval of time during which the first exerts a force on the second, and the second exerts a force on the first. By the third law of motion these forces are equal and in opposite directions. By the second law of motion they each produce equal changes of momentum in opposite directions. Hence the total momentum of the two bodies is unaltered by the collision. The result holds true if the bodies act on each other without touching, and if there are more than two bodies. Thus we arrive at the principle of the conservation of momentum which is stated as follows:

The total momentum in any direction of a system of bodies which act and react on each other is constant, and is not affected by the actions between the parts.

EXAMPLE.—The mass of a gun is 2 tons and that of the shot 14 lb. The shot leaves the gun with a velocity of 3000 ft./sec. What is the initial velocity of the recoil?

Let v be the initial velocity of the gun. Then, as a consequence of Newton's third law, after the explosion the backward momentum of the gun is equal to the forward momentum of the shot. Thus

$$2 \times 2240v = 14 \times 3000$$

$$\text{or } v = \frac{14 \times 3000}{2 \times 2240} = 9.375 \text{ ft./sec.}$$

Body on Inclined Plane.—Let us suppose that a body of mass m is placed on a smooth plane inclined at an angle θ to the horizontal. Let its weight mg acting downwards be represented

by BE. Draw EC at right angles to AB and complete the rectangle ECBD; $\angle CEB = \theta$. Then the weight mg may be resolved into two components, one $mg \cos \theta$ represented by BD acting at right angles to the plane and the other $mg \sin \theta$ represented by BC acting down the plane.

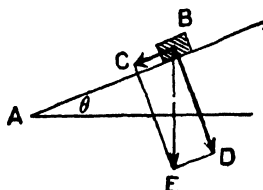


FIG. 29.

The first of these is neutralised by the upward reaction or normal force exerted by the plane. The magnitude of the latter is consequently $mg \cos \theta$.

The second causes the body to move down the plane with an acceleration $g \sin \theta$. The smaller the inclination of the plane, the smaller $\sin \theta$, and the smaller the acceleration.

If the body were kept in equilibrium by attaching a string to it and applying an upward force to it parallel to the surface of the plane, then the magnitude of this force would be $mg \sin \theta$.

The Simple Pendulum.—If a body is suspended, so that it is free to oscillate about a horizontal axis, it is called a pendulum or more correctly a compound pendulum, whatever its form, and Fig. 30 represents one of a type used in clocks. All pendulums have the property, that the period is always the same provided that the arc of swing is small, not more than 2° or 3° .

The simple pendulum consists of a "heavy particle attached to one end of an inextensible, perfectly flexible, weightless thread," the other end of which is fixed. This description is realised with sufficient accuracy by a small lead sphere suspended by a thin thread. The distance between the point of support and the centre of the lead sphere is called the length of the pendulum, and will be denoted by l . Let m be the mass of the bob of the pendulum.

When the pendulum is at rest, the weight of the bob acting vertically downwards is balanced by the stretching force in the string acting vertically upwards. But suppose the pendulum is in the position shown in Fig. 31, making $\angle \theta$ with the vertical. Then the weight mg , acting vertically downwards and represented by BE, can be resolved into two components, one $mg \cos \theta$ represented by BD, and the other $mg \sin \theta$ represented by BC.

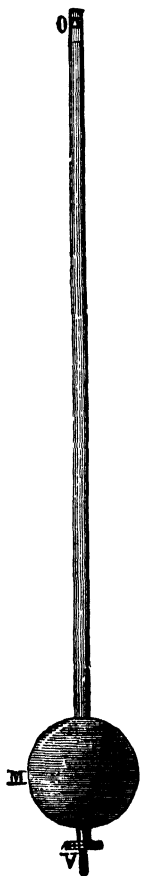


FIG. 30.—Pendulum.

The first of these components is neutralised by the stretching force in the string. The second, acting on the mass m of the bob, gives it an acceleration $g \sin \theta$ in the direction BA.

Now, if θ is small, we may write $\sin \theta = \theta$, and we may regard the arc AB as straight and $= l\theta$. The bob of the pendulum is therefore moving in a straight line towards A, and its acceleration divided by its displacement is equal to $g\theta/(l\theta)$, or g/l , *i.e.* the ratio is constant. The motion is consequently a S.H.M., and the period according to the equation on p. 21 is given by

$$\frac{\text{Acceleration}}{\text{Displacement}} = \frac{g}{l} = \frac{4\pi^2}{T^2}$$

or

$$T = 2\pi\sqrt{\frac{l}{g}}$$

It is thus independent both of the arc of swing, provided the latter is small, and of the mass of the bob. This formula forms our most accurate means of determining g .

Half the period is sometimes referred to as the time of vibration of the pendulum. The seconds pendulum is that one which has the time of vibration equal to one second. We have for its length

$$l = \frac{gT^2}{4\pi^2}$$

If we substitute for g the value for London, namely 981.19 cm./sec.², this gives

$$l = \frac{981.19 \times 2^2}{4\pi^2} = 99.41 \text{ cm.}$$

that is, it is just a little short of the metre.

The fact that the period of the pendulum remains constant as the amplitude diminishes in extent, or the law of the isochronism of the pendulum, as it is called, was discovered by Galileo in 1582, when he was a medical student at Pisa. One day in the cathedral his attention was caught by a great lamp hanging by a chain, which the verger had left swinging to and fro after lighting it. He timed the oscillations by his pulse, the only watch he possessed. He noticed that the time of swing remained constant, as nearly as he could tell, in spite of the fact that the swings were getting smaller and smaller. Afterwards he verified the law by more accurate experiments.

EXAMPLE.—A pendulum, which beats seconds at a place where g is 32.20 ft./sec.² is taken to a place where g is 32.28 ft./sec.² How many seconds does it gain or lose in a day?

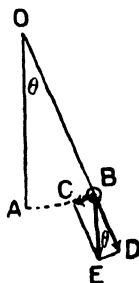


FIG. 31.

Let T be the original period of the pendulum and T_1 the new period. A pendulum which "beats seconds" is one which has a period of 2 sec., but this fact is not required.

$$T = 2\pi\sqrt{\frac{l}{32 \cdot 20}}, \quad T_1 = 2\pi\sqrt{\frac{l}{32 \cdot 28}}$$

Hence
$$T_1 = T\sqrt{\frac{32 \cdot 20}{32 \cdot 28}} = T\left(1 - \frac{8}{3228}\right)^{\frac{1}{2}}$$

Now when α is very small, $\sqrt{1-\alpha} = 1 - \frac{1}{2}\alpha$

Hence
$$T_1 = T\left(1 - \frac{4}{3228}\right)$$

Since $T_1 < T$, the pendulum will go too fast. Let n = number of seconds gained per day. The number of seconds in a day is 86,400.

$$86,400T = (86,400 + n)T_1$$

$$86,400 = (86,400 + n)\left(1 - \frac{4}{3228}\right)$$

or
$$n = \frac{86,400 \times 4}{3228} = 108$$

since the other term is too small to be taken into account.

Centrifugal Force.—It has been shown (p. 19), that when a particle is moving in a circle of radius r with velocity v , it has an acceleration towards the centre of the circle equal to v^2/r . To produce this acceleration a force is required towards the centre of the circle, given by

$$F = \frac{mv^2}{r}$$

This force overcomes the inertia of the particle which would otherwise cause it to move in a straight path. If a stone is swung round at the end of a string, we feel it tugging at the string, and if the string breaks or is let go, the stone flies off at a tangent.

If we consider any point in the string, by Newton's third law there are two forces acting through it, the force exerted by the hand on the stone and the force exerted by the stone on the hand. The latter is called the centrifugal force. It is, of course, given by the expression for F above.

Centrifugal Force on Vehicles.—Fig. 32a represents a cyclist turning a corner. It is necessary for him to lean to the inside of the curve in order to get round. If we regard him and the cycle as forming one body, they are acted on by a resultant force equal to mv^2/r in the direction shown through their centre of gravity—this point will be defined later. Their weight mg acts downwards. The only other force acting on them is the

reaction of the ground. This acts through the track of the wheels on the ground and through the centre of gravity of man and cycle, and hence must be parallel to the frame of the cycle.

Consequently if we draw the triangle of forces (Fig. 32b), where AC represents the resultant and AB the weight, the other component which represents the reaction must be parallel to the

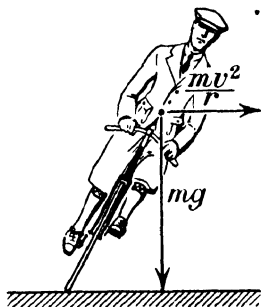


FIG. 32a.

Illustrating centrifugal force.

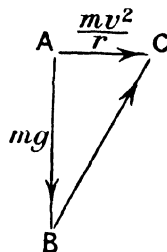


FIG. 32b.

frame of the cycle. Let θ be the angle which the latter makes with the vertical. Then

$$\tan \theta = \frac{AC}{AB} = \frac{mv^2/r}{mg} = \frac{v^2}{gr}$$

Thus the greater the speed and the sharper the curve, the further must the cyclist lean over. But if he leans over beyond a certain angle, a side slip takes place.

When a motor car takes a curve on a racing track or an engine goes round a curve on a railway, the reaction of the ground makes an angle θ with the vertical which is given by the same formula. In order to prevent side slip the racing track is banked as shown in Fig. 33, and in order to prevent lateral pressure on the outside rail in the case of the railway, its level is raised above the level of the inside one. This adjustment can be made only for one particular speed.

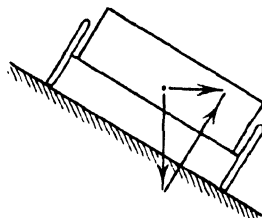


FIG. 33.—Car on banked racing track.

EXAMPLE.—A locomotive has a speed of 30 miles per hour when going round a curve of radius of curvature one-fifth of a mile. The distance between the rails being four feet, find the difference of level of the rails, so that the engine wheels may have no outward pressure on the outside rail.

$$\begin{aligned} 30 \text{ miles per hour} &= 44 \text{ ft./sec.} \\ \frac{v^2}{gr} &= \frac{44^2 \times 5}{32 \times 5280} = \frac{11}{192} = \tan \theta \end{aligned}$$

If we assume that 4 feet is the distance between the rails measured horizontally, the difference in level is

$$48 \tan \theta = 48 \times \frac{1}{10} = 2\frac{1}{5} \text{ inches.}$$

Elastic Impact.—If a man allows an ivory ball to fall upon a marble floor, it rebounds to a considerable height. If the same ball is allowed to fall upon a wooden floor, it rebounds through a smaller distance. If instead of the ivory ball a lead ball is dropped upon the same floors from the same height, the distances through which it rebounds are much smaller than in the former cases. The velocities before and after impact can be calculated from the distance fallen and the height to which the ball rebounds, and as a result of experiment Newton established a general law of which the following is a particular case :

The velocity after impact bears a constant ratio to the velocity before impact.

This ratio depends on the material of the ball and the floor, and is independent of the mass of the ball. It is referred to as the **Coefficient of Elastic Restitution** and is usually denoted by e . The value of e has widely different values for different materials; in the ideal cases for which it is zero and unity, the impact is said to be inelastic and perfectly elastic respectively.

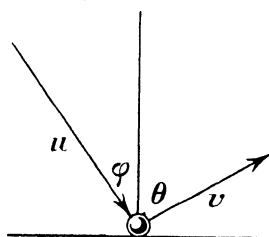


FIG. 34.—Elastic impact.

Let us suppose (Fig. 34) that a sphere is incident on a fixed plane with velocity u in a direction making angle ϕ with the normal to the surface at the point of incidence, and that it has no spin or motion of rotation. After impact let the velocity be v in a direction making angle θ with the normal. It is in the same plane as the normal and the incident velocity. Resolve the incident velocity into components $u \sin \phi$ and $u \cos \phi$ parallel and perpendicular to the plane respectively. The parallel component is unaltered by the impact, no force acting on the sphere in this direction. Hence

$$v \sin \theta = u \sin \phi$$

The perpendicular component is altered according to Newton's law. Hence

$$v \cos \theta = eu \cos \phi$$

On dividing the second equation by the first

$$\frac{v \cos \theta}{v \sin \theta} = \frac{eu \cos \phi}{u \sin \phi}$$

whence

$$\cot \theta = e \cot \phi$$

If the impact is perfectly elastic, $\cot \theta = \cot \phi$, and the angle of reflection is equal to the angle of incidence. This result is of importance for the kinetic theory of gases, and it was used in the corpuscular theory of light.

Let m be the mass of the sphere. It suffers no change of momentum parallel to the plane. Before impact it had a momentum $mu \cos \phi$

downwards and after impact a momentum $mv \cos \varphi$ upwards. As a consequence of the impact it suffers a change of momentum $m(u \cos \varphi + v \cos \theta)$. If a succession of similar spheres were incident with the same velocity at the same point in the same direction at the rate of n per second, they would experience a rate of change of momentum in an upward direction equal to

$$nm(u \cos \varphi + v \cos \theta)$$

Consequently the plane exerts an upward force on them of this magnitude, and by Newton's law of the equality of action and reaction they exert an equal downward force on the plane.

EXAMPLES IV

1. A force of 500 dynes acting on a certain mass for one second gives it a velocity of 20 cm./sec. Find the mass in grams.
2. A force of 80 pdl. acts on a mass of 20 lb. for 2 sec. Find the velocity generated.
3. Change a force of 1 ton weight into dynes.
4. A force of 6 lb. wt. acts on a mass of 20 lb. for 1 sec. Find the velocity generated.
5. A body is moving in a straight line with a velocity of 10 cm./sec. In 3 sec. the velocity increases to 19 cm./sec., and the motion is uniformly accelerated. Find the force acting on the body, (i) in dynes, (ii) in gm. wt., given that the mass of the body is 4 grams.
6. A force of 8 lb. wt. acting on a certain mass for 3 sec. gives it a velocity of 6 ft./sec. Find the mass in lbs.
7. What horizontal force in tons wt. is required to stop a train of 200 tons mass running at 30 ml./hr., (i) in one minute, (ii) in 400 yards?
8. The last waggon on a train weighs together with its load 10 tons and is being accelerated at the rate of 3 ft./sec.² What is the force in the coupling in tons wt.? Frictional resistances are to be neglected.
9. A man weighing 12 stone stands on a hoist, which has a constant acceleration of 4 ft./sec.² Find the force on the hoist due to his weight, (i) when it is ascending, (ii) when it is descending?
10. A cage of mass one ton is lowered down a pit with an acceleration of 16 ft./sec.² What is the stretching force in the rope? A mass of 8 lb. lies on the bottom of the cage. With what force does it press on the latter?
11. The mass of a gun together with its carriage is 7 tons. It fires a 100-lb. shot with a velocity of 1000 ft./sec. What is the velocity of recoil of the gun?
12. Two bodies initially at rest attract each other with a force of $\frac{1}{16}$ dyne. If the masses are 10 and 50 gm. respectively, find the velocities acquired in 1 sec.
13. A pendulum which beats seconds at a place where g is 981.4 cm./sec.² is taken to a place where g is 981.0 cm./sec.² How many seconds does it gain or lose in a day?
14. The thread of a simple pendulum is shortened by 1 inch and the period diminishes by $\frac{1}{100}$ of its value. What was the original value of the length?

15. If a simple pendulum which is intended to beat seconds, loses 20 minutes per week, by what percentage of its own length should it be shortened ?

16. A stone weighing 1 lb. is swung round in a horizontal circle at the end of a string 1 yard long, making a complete revolution every $\frac{2}{3}$ second. What is the stretching force in the string ?

17. An iron ball falls on an iron plate from a height of two metres. How high does it rebound, given that the coefficient of elastic restitution is about 0.66 for iron on iron ?

CHAPTER V

FRICTION AND CENTRES OF GRAVITY

Friction.—It has been explained on p. 38 that when a body is placed on a smooth plane inclined at an angle θ to the horizontal, it moves down the plane with an acceleration $g \sin \theta$. If, however, we take a wooden board, place a small block of wood on it and tilt it, we find that for small values of θ the block does not move at all. This is because the wooden board has a certain amount of roughness. The perfectly smooth plane is an ideal not realised in practice. If the mass m is placed on any actual plane, a force F called the force of friction acts up the plane in the direction CB (Fig. 35) and keeps it at rest, just as much friction being called into action as prevents motion. Consequently $F = mg \sin \theta$, and as θ increases, F increases. When, however, θ reaches a certain limiting value, α , which is called the **limiting angle of friction**, the component of gravity down the plane overcomes the resistance of friction, and the body begins to slide. The limiting angle depends on the substances in contact; for an oak block upon an oak plank it is about 23° when the fibres are parallel, and 16° when they are cross. The resistance of friction always acts tangentially to the surfaces of contact.

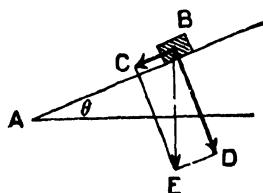


FIG. 35.

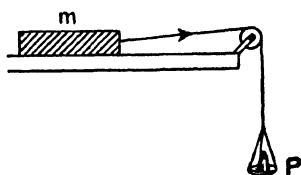


FIG. 36.

In the same way, if a block of wood of mass m is placed on a horizontal plane, and a string attached to it and passed over a pulley, so as to carry a scale pan (Fig. 36), and if weights are placed in the pan, it will be found that at first the body does not move. Only when the weights in the pan together with the weight of the pan itself reach a certain critical value, which we shall call P , is the resistance of friction overcome and the body begins to move.

Laws of Friction.—Friction obeys the following laws :

- (i) *It always opposes motion.*
- (ii) *It depends on the nature of the surfaces in contact, but is independent of the extent of the areas in contact, as long as the reaction or total pressure between the two surfaces is the same.*
- (iii) *The maximum friction between two surfaces is proportional to the total pressure between them.*

These laws are the result of experiment. If a wooden block and a piece of metal of exactly the same weight and with the same extent of rubbing surface are placed on the inclined plane in succession, and the angle with the horizontal is gradually increased, it is found that sliding begins at different angles, showing that the maximum resistance exerted by friction in each case is not the same. Also if the surfaces in contact are oily or greasy, friction is diminished and sliding starts at a smaller angle. It is because oil diminishes friction that it is used in machinery and between axles and their bearings.

If the area in contact in the arrangement represented in Fig. 36 is halved, say by sawing the block in two, sliding will start when P has only half the value previously necessary. If, however, the block is loaded by placing weights on it until the force pressing it down on the horizontal plane is the same as before, it is found that sliding starts when P has its original limiting value.

Coefficient of Friction.—If the weights which the block carries are altered, it is found that sliding always starts when $P/(mg)$ has a constant value. This value, the coefficient of friction or more strictly the coefficient of static friction between the two surfaces in question, is the maximum resistance offered to relative motion divided by the force pressing the surfaces together. It is usually denoted by μ . It can be measured experimentally by the two methods depicted in Figs. 35 and 36. When the block in Fig. 35 is on the point of sliding, the resistance of friction to motion is $mg \sin \alpha$. The normal force is $mg \cos \alpha$. Hence

$$\mu = \frac{mg \sin \alpha}{mg \cos \alpha} = \tan \alpha$$

or the coefficient of friction is simply the tangent of the angle at which sliding begins. In the arrangement of Fig. 36, when mg is the total weight of the block and weights it carries, and P the combined weight of scale pan and weights on it just sufficient to start motion

$$\mu = \frac{P}{mg}$$

Kinetic Friction.—When the two surfaces are moving relatively to one another, the resistance of friction is much less than when motion is on the point of starting, even though the normal force is the same in both cases. The ratio of friction to normal force *after motion is started* is called the coefficient of kinetic friction. It can be determined by either of the arrangements shown in Figs. 35 and 36, if the inclined plane in Fig. 35 is continually tapped as its inclination is increased, or if the horizontal plane in Fig. 36 is continually tapped as the weights are added to the pan. The coefficient of kinetic friction is the limiting value of $\tan \theta$ or of $P/(mg)$ sufficient to keep up motion after it is started. The tapping starts the motion.

Inclined Plane with Friction.—Let us suppose that a body of mass m is placed upon a plane inclined at angle θ to the horizontal, and that the coefficient of kinetic friction between the body and the plane is μ . Then (Fig. 35) the weight mg of the body can be resolved into a component $mg \sin \theta$ down the plane and a force $mg \cos \theta$ at right angles to the plane. Let the inclination be great enough to make the body slide. The resistance due to friction is $\mu mg \cos \theta$ and the resultant force down the plane is

$$mg \sin \theta - \mu mg \cos \theta$$

Hence the acceleration is

$$g \sin \theta - \mu g \cos \theta = g \sin \theta - g \cos \theta \tan \alpha \\ = g \frac{\sin (\theta - \alpha)}{\cos \alpha}$$

If a force F acts on the body parallel to the surface of the plane and causes it to move up the plane, the resultant force is

$$F - mg \sin \theta - \mu mg \cos \theta \\ = F - mg \frac{\sin (\theta + \alpha)}{\cos \alpha}$$

and the acceleration up the plane is

$$\frac{F}{m} - g \frac{\sin (\theta + \alpha)}{\cos \alpha}$$

EXAMPLE.—A mass of 4 lb. hangs over the edge of a smooth table. It is connected by a string, which passes over a pulley at the edge of the table, to a mass of 6 lb. laid on the table. Find the space described and the velocity acquired in a quarter of a second from rest.

$$a = \frac{F}{m} = \frac{4 \times 32}{6 + 4} = \frac{64}{5} \text{ ft./sec.}^2$$

$$v = \frac{64}{5} \times \frac{1}{4} = \frac{16}{5} \text{ ft./sec.}$$

$$s = \frac{1}{2} \times \frac{64}{5} \times \left(\frac{1}{4}\right)^2 = \frac{2}{5} \text{ ft.}$$

Rolling Friction.—The chief cause of sliding friction is the interlocking of minute projections on the two surfaces. When a wheel or any round solid rolls upon a plane surface, sliding friction is done away with. There is, however, a resistance produced at the point of contact called rolling friction which is very much less than in the case of sliding friction for the same pressure between the bodies. Rolling friction is due to the wheel being

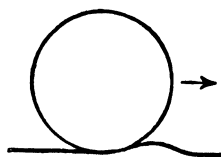


FIG. 37.

slightly flattened at the point of contact and to the surface being rolled up into a little ridge in front of the wheel as shown in Fig. 37. It is, of course, owing to rolling friction being less than sliding friction, that wheeled carriages are used on the roads instead of sledges, and that ball bearings are used in bicycles and elsewhere.

Centre of Gravity.—Let us consider a rigid body, that is one the parts of which cannot be displaced relatively to one another. We can conceive of it as divided up into a great number of particles of weights w_1, w_2, w_3 , etc. These weights constitute a system of parallel forces acting vertically downwards.

Let A, B, and C (Fig. 38) be three of the particles of weights w_1, w_2 , and w_3 respectively. Then by the theorem of p. 31 the parallel forces w_1, w_2 have a resultant equal to $w_1 + w_2$ acting through a point D which divides AB inversely in the ratio of w_1 to w_2 . No matter how the body is turned, the resultant of w_1 and w_2 always passes through D. This point is called the centre of gravity of the two particles A and B. The resultant of w_1 and w_2 would be produced by the two particles collected at their centre of gravity D. Combine this resultant with the weight of the third particle at C. We then obtain a force $w_1 + w_2 + w_3$ acting at some point E in DC, which divides DC inversely in the ratio of $w_1 + w_2$ to w_3 . The weights of the three particles may now be supposed collected at E. By proceeding in this way we can show that the weights of the different particles, into which the body can be divided, all combine into one force equal in magnitude to their sum, which passes through a single fixed point in the body. This point is called the centre of gravity of the whole body.

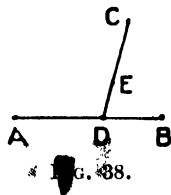


FIG. 38.

The centre of gravity of a body is a point fixed relatively to the body through which the resultant action of gravity on the body always passes, no matter how it is turned.

Particular Cases.—(1) The centre of gravity of a rod which has the same breadth, thickness, and density throughout is obviously at the middle point. For if it is divided into particles,

to every particle on one side of the middle point there corresponds an equally heavy particle on the other side at the same distance from the middle.

(2) If we have a rectangular lamina or thin rectangular plate ABCD of uniform thickness and density (Fig. 39), it can be divided into rods parallel to the ends. Let MN be such a rod. Then the centre of gravity of MN is at its middle point P, which is on the straight line FH drawn from F, the middle point of AD, to H, the middle point of BC. Similarly the centres of gravity of all the other rods are on the same straight line. We can suppose each rod concentrated at its middle point, and thus suppose the rectangle replaced by a line FH uniformly loaded with matter from end to end. And the centre of gravity of such a line is obviously at its middle point G.

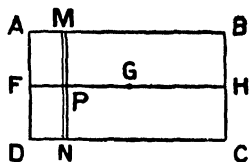


FIG. 39.

(3) Let ABC be a triangular lamina (Fig. 40), and let D be the middle point of BC. Join AD and divide the lamina into strips parallel to BC. Let MN be any one of these strips. Then each side of MN is bisected by AD; hence P, the centre of gravity of MN, lies on the median AD. Similarly the centres of gravity of all the other strips lie on AD. It follows that the centre of gravity of the triangle lies on AD.

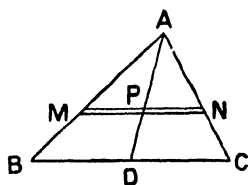


FIG. 40.

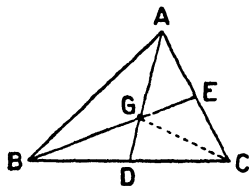


FIG. 41.

of gravity of the triangle lies on BE. Hence it must be G, the point where AD and BE meet.

Join GC. Then since they are on equal bases $\triangle ABD = \triangle ADC$. Similarly $\triangle GBD = \triangle GDC$. Hence by subtraction $\triangle ABG = \triangle AGC$. Similarly it may be shown that $\triangle ABG = \triangle BGC$. Hence, since the three triangles are equal, each is equal to one-third of $\triangle ABC$, that is, $\triangle ABG = \frac{1}{3} \triangle ABD$ or $AG = \frac{2}{3} AD$. We thus arrive at the result, that the centre of gravity of a triangular lamina is situated on any one of the three medians at a distance of $\frac{2}{3}$ of its length from the corner of the triangle. We have also incidentally proved that the medians of a triangle are concurrent.

If the triangle is replaced by three equal particles situated at its corners, the centre of gravity of the system is still at the same point. For the particles at B and C can be supposed collected at D, the point midway between them. Then, since the weight of the particle at D is twice that of the particle at A, their centre of gravity must divide DA in the ratio of 1 to 2.

(4) The centre of gravity of a circular lamina is obviously at its centre. Also the centres of gravity of a solid homogeneous sphere and solid cube are obviously at their centres.

The centre of gravity of a tetrahedron is on the line joining a vertex to the centre of gravity of the opposite face at a point distant $\frac{3}{4}$ of the length of this line from the vertex.

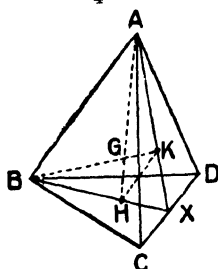


FIG. 42.

For let the tetrahedron ABCD (Fig. 42) be divided into laminae parallel to the base. Then obviously the centre of gravity of each lamina lies in the point where it is intersected by the line joining the vertex to the centre of gravity of the base. Consequently the centre of gravity of the whole tetrahedron lies on this line. Let $CX = XD$, and let H be the centre of gravity of $\triangle BCD$. Then the centre of gravity of the tetrahedron lies on AH. Similarly if K is the centre of gravity of $\triangle ACD$, the centre of gravity of the tetrahedron lies on BK. Consequently it is at G, the point of intersection of AH and BK.

Join HK. Now $BH = 2HX$ and $AK = 2KX$; hence HK is parallel to AB. Then by similar triangles

$$\begin{aligned} AG : GH &= AB : HK \\ &= AX : KX \\ &= 3 : 1 \end{aligned}$$

It follows that the centre of gravity of a pyramid or cone is on the line joining the vertex to the centre of gravity of the base at a point distant $\frac{3}{4}$ of the length of this line from the vertex.

For if we consider the case of the right circular cone shown in Fig. 43, it is clear, by dividing it up into circular laminae parallel to the base, that its centre of gravity lies on the line AO joining the vertex to the centre of gravity of the base. Also by dividing it up into tetrahedra such as MA it is clear that the centre of gravity of each tetrahedron lies on the plane HK drawn parallel to the base and cutting AB and AC in the ratio 3 : 1. Hence the centre

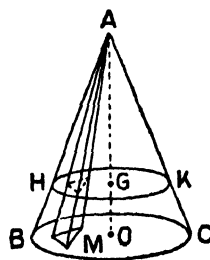


FIG. 43.

of gravity of the whole cone is at G, the intersection of AO with this plane.

Since a triangle or other plane figure is infinitely thin, it has no weight and consequently in strictness no centre of gravity. It is, however, customary to speak of the centres of gravity of areas and lines, the area being identified in thought with a uniform plate and the line with a uniform wire.

EXAMPLE.—A circular hole, 4 inches in diameter, is cut out of a circular disc of wood, 10 inches in diameter, close to the edge. Find the centre of gravity of the remaining portion.

By symmetry the centre of gravity of the remainder is on the straight line joining the centre of the disc and the centre of the part cut out. Let it be a distance x from the centre of the disc, on the side opposite to the part cut out.

The area of the part cut out is 4π and of the remainder 21π . The whole disc may be regarded as composed of the remainder and the part cut out; hence, since the centre of gravity of the whole disc cuts the line joining the centres of gravity of its parts inversely as their areas

$$\frac{x}{3} = \frac{4\pi}{21\pi} \quad \text{or} \quad x = \frac{4}{7} \text{ inch.}$$

Experimental Determination of Centre of Gravity.—If a body is suspended by a string, it comes to rest with its centre of gravity vertically under the point of support. The centre of gravity is thus restricted to one line in the body. By suspending the body from another point we can restrict the centre of gravity to another line. It lies consequently at the intersection of the two lines.

This method can be applied very well to a plate. Holes are bored at two of its corners, it is suspended by passing one of the holes over a nail, and a plumb line is hung on the same nail. When the latter comes to rest, its line is marked in chalk on one face of the plate. The plate is then suspended from the other hole, and the operation repeated. The intersection of the two chalk lines gives the centre of gravity.

Centroid.—The definition of centre of gravity can be approached from another standpoint.

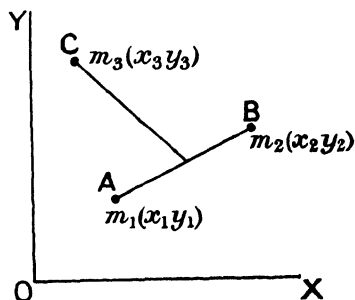


FIG. 44.

Let A, B, C (Fig. 44) be three particles of masses m_1 , m_2 , m_3 situated in a plane at points x_1y_1 , x_2y_2 , and x_3y_3 . The centre of

gravity of A and B divides AB in the ratio m_1 to m_2 . Thus, if its coordinates are $x'y'$,

$$\frac{x' - x_1}{x_2 - x'} = \frac{m_2}{m_1}$$

whence

$$x' = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

The centre of gravity of the three particles divides the line joining C and $x'y'$ in the ratio $m_1 + m_2$ to m_3 . Let its x coordinate be $\bar{x}$. Then

$$\frac{\bar{x} - x_3}{x' - \bar{x}} = \frac{m_1 + m_2}{m_3}$$

This gives after substitution

$$\bar{x} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

By continuing the process we can show that for a great number of particles

$$\bar{x} = \frac{\sum m x}{\sum m}$$

where the summation is extended over all the particles of the system. It is also obvious from the manner of proof, that if the particles are not confined to a plane, the equation still holds together with two similar equations for $\bar{y}$ and $\bar{z}$, provided that x , y , and z denote the distances of the particles from three co-ordinate planes which are mutually at right angles to one another.

In the equations for $\bar{x}$, $\bar{y}$, and $\bar{z}$ only the masses and positions of the particles occur. No use is made of their weights, and the definition would still hold, if the force of gravity were annihilated. When defined in this way the point is referred to as the centroid or centre of mass of the system.

Stability.—For a body to stand on a given base it is necessary for the vertical through its centre of gravity to fall within the base. Thus if we have a table with a heavy weight resting on it (Fig. 45), the table remains erect, provided the vertical through the centre of gravity of table and weight meets the floor at a point G within the triangle formed by joining the feet of the table. If G lies outside this triangle, the table topples over.

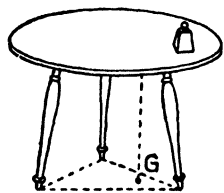


FIG. 45.

A body is said to be in **stable** equilibrium if it returns to its original position when it is slightly disturbed from it. A cube resting on its side or a

cone resting on its base is an example of a body in stable equilibrium.

A body is said to be in **unstable** equilibrium, if, on being slightly disturbed from its original position, it tends to move further from it. A cone balanced on its point or an egg on its end are examples of unstable equilibrium.

A body is said to be in **neutral** equilibrium, when after a slight displacement it neither returns to its original position nor moves further from it. A sphere resting on a horizontal plane is an example of neutral equilibrium.

In stable equilibrium the centre of gravity of the body is in its lowest position and any displacement tends to raise it. In unstable equilibrium the centre of gravity is in its highest position and any slight displacement tends to lower it. In neutral equilibrium the centre of gravity is neither raised nor lowered by a slight displacement.

If the body is free to roll on a supporting surface or is suspended from a point, the centre of gravity always tends to descend just as it would do if all the weight of the body were collected there.

EXAMPLES V

1. A block of wood is placed on an inclined plane, and the inclination of the latter gradually increased. When it makes an angle of 32° with the horizontal, the block begins to slide. What is the coefficient of static friction between it and the plane?

2. A body weighing 10 lb. is placed on a plane inclined at 30° to the horizontal. The coefficient of kinetic friction between the body and the plane is 0.6. Find the force (i) acting up the plane, (ii) acting down the plane, necessary to give the body an acceleration of 5 ft./sec.²

3. A body is placed on a plane inclined at 30° to the horizontal. The coefficient of kinetic friction between the body and the plane is 0.4. If the body is moving down the plane with a velocity of 100 cm./sec., what does this velocity increase to in the next 2 sec., and how far does the body travel in this time?

4. A block of wood, which weighs 400 gm., rests on a table. A string is attached to it, and passes horizontally to a pulley at the edge of the table, carrying a scale pan at its other end. The pan weighs 50 gm. Weights are gradually added to the pan, and, when the total weight in the pan is 182 gm., the block begins to slide. What is the coefficient of static friction between it and the table?

5. If in question 4 a weight of (i) 200, (ii) 300 gm. is placed in the pan, what is the acceleration of the block on the table? Take the coefficient of kinetic friction between the block and the table as 0.4.

6. A train running down an incline of 1 vertical to 150 along the slope has a velocity of 10 ft./sec. at a certain time. What velocity

will it have half a minute later, and how far will it have run in the interval ?

7. A body slides down an incline of 5 vertical to 13 along the slope, the coefficient of friction being $\frac{1}{4}$. After the body has moved a distance d from rest, the coefficient of friction changes to $\frac{1}{3}$. Prove that the body will come to rest after traversing a further distance $2d$.

8. Weights of 1, 2, and 4 gm. are placed at the corners A, B, and C of a triangle. Find the position of their centre of gravity.

9. A square of side a is divided by its diagonals into four triangles and one of these is cut out. Find the position of the centre of gravity of the remainder.

10. A square of side $2a$ is divided into four squares of side a by straight lines through its centre parallel to its sides. One of these squares is removed. Find the centre of gravity of the remainder.

11. A wooden cylinder of radius 4 cm. and density 0.60 gm./c.c. has a cylindrical hole of radius 1 cm. bored in it, parallel to its axis. The axes of the two cylinders are 2 cm. apart. The cylindrical hole is filled with lead of density 11.4 gm./c.c. Find the centre of gravity of the whole figure.

CHAPTER VI

WORK—MACHINES—ENERGY

Work.—In physics the word “work” has a special meaning attached to it. Whenever a force acts upon a body in such a way that motion takes place, work is said to be done; if the motion takes place in the direction of the force, the work is done by the force, but if, owing to the action of other forces, the motion takes place in a direction opposite to that of the force, work is done against the force. More precisely:

If a force F acts on a body and its point of application moves a distance s in a direction making an angle θ with the direction of F , then the force is said to do an amount of work $Fs \cos \theta$ on the body.

Thus if a body of mass m is allowed to fall a height h from a table to the floor, the force acting on it is mg and the distance moved through by the point of application of the force is h , consequently the work done by gravity on it is mgh . If the body is lifted from the floor to the table, the work mgh is done against gravity.

If a body of mass m is allowed to slide a distance s down a smooth plane inclined at an angle φ to the horizontal (Fig. 46), the distance s being measured along the slope, the cosine of the angle between the direction of F and s , *i.e.* $\cos ABC$, is equal to $\sin \varphi$; hence the work done by gravity is $mgs \sin \varphi$.

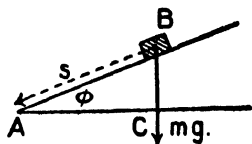


FIG. 46.

If the bob of a simple pendulum of mass m and length l is allowed to swing from a position B (Fig. 47) making an angle φ with the vertical to its lowest position A, and BC be drawn perpendicular to OA, the force is mg and the component of the displacement in the direction of the force is CA. Now $CA = OA - OC = l(1 - \cos \varphi)$. Hence the work done by gravity is $mgl(1 - \cos \varphi)$.

The product $Fs \cos \theta$ may be regarded either as the product of the force and the component of the displacement in the direction of the force, or as the product of the displacement and the component of the force in the direction of the displacement.

No work is done if the point of application of the force does not move, or if it moves only at right angles to the direction of the force. An example of the first case is a book resting on a table; it is acted on at its centre of gravity by its weight vertically downwards, but there is no displacement of the point of application of the force. An example of the second case is that of the stretching force in the string of a simple pendulum. The bob moves always at right angles to the direction of the string; $\cos \theta = \cos 90^\circ = 0$, and the stretching force in the string does no work.

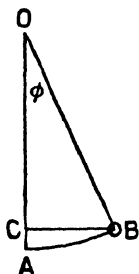


FIG. 47.

Units.—Just as there are four different kinds of units for measuring force, so there are four different kinds of units for measuring work.

The erg is the amount of work done by a force of one dyne acting through one centimetre.

The foot-poundal is the amount of work done by a force of one poundal acting through one foot.

The gram-centimetre is the amount of work done by a force of one gram weight acting through one centimetre.

The ft.-lb. is the amount of work done by a force of one lb. weight acting through one foot.

The first two of these units are absolute units, the erg being the unit used for scientific work. The last two are gravitational units; they or their multiples are used by engineers. The erg has the disadvantage that it is a very small unit; to lift a kilogram weight from the floor to a table one metre above the floor requires $1000 \times 981 \times 100 = 9.81 \times 10^7$ ergs. Hence for many purposes a multiple of it, called the **joule** and equal to 10^7 ergs, is used.

EXAMPLE.—A ladder 20 ft. long leans against a vertical wall, and is inclined at an angle of 30° to it. How much work measured in ft.-lb., does a man who weighs 12 stone do against gravity climbing the ladder.

$$\text{Height climbed} = 20 \cos 30 = 17.32 \text{ ft.}$$

$$\begin{aligned} \text{Work done} &= 12 \times 14 \text{ lb.} \times 17.32 \text{ ft.} \\ &= 2909 \text{ ft.-lb.} \end{aligned}$$

Machines.—Any appliance used for performing work can be called a machine in physics. The term includes appliances as elaborate as the steam locomotive, but here we shall consider only some of the simplest, namely the lever, the screw press, the wheel and axle, and systems of pulleys. These and similar arrangements were described at length in the older books on physics under the name of “mechanical powers.”

The lever has already been described on p. 29, and it was shown there that if P was the "power" and W the "resistance," and CA and CB the perpendicular distances from the fulcrum to their respective lines of action, then

$$P \times CA = W \times CB$$

Let us suppose, for example, that the lever under discussion is one of the second class, and that it is rotated through a small angle about the fulcrum (Fig. 48). Then A traces out the arc AA' and B traces out the arc BB' . The lengths of the arcs are in the ratio of their respective radii, *i.e.*

$$\frac{AA'}{BB'} = \frac{CA}{CB}$$

If the angle ACA' is small, the arcs may be regarded as elements of straight lines. The work done by the power is then $P \times AA'$, and the work done against the resistance $W \times BB'$. But

$$\frac{AA'}{BB'} = \frac{CA}{CB} = \frac{W}{P}$$

or

$$P \times AA' = W \times BB'$$

That is, the work done by the power is equal to the work done against the resistance, or, in other words, you only get as much work out of the lever as you put into it. Although by its means a comparatively small power may be enabled to overcome a great resistance, yet the displacement of the point of application of the power must then be much greater than the displacement of

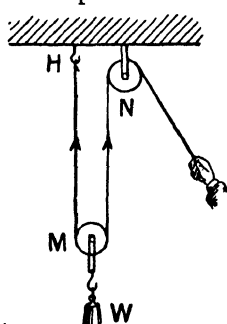


FIG. 49.

the point of application of the resistance. Or, as it is popularly expressed, *what is gained in force is lost in speed*. The ratio of the weight to the power or effort which must be applied to raise it, is known as the **mechanical advantage** of the machine.

Fig. 49 represents the single movable pulley. A hand is shown pulling a cord which passes over the fixed pulley N , then round the movable pulley M to the hook H , to which its end is fastened. The pulley M carries a weight W . The pulley is the name given to the revolving wheel. The pulley and frame in which it is fixed

together form the block.

Let P be the force with which the hand is pulling the cord.

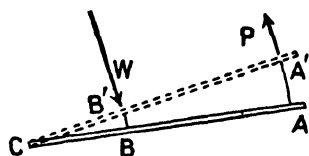


FIG. 48.

Then the stretching force is the same throughout the whole length of the cord to the hook H. Now consider the equilibrium of the block M, neglecting its weight. It is acted on by the weight W vertically downwards and by the stretching force P in the cord applied vertically upwards on both of its sides. The total upward force must balance the downward force. Hence

$$W = 2P$$

Let us suppose now that the weight is pulled up a distance h . Then the hand must move a distance $2h$. The work done by the hand is $P2h$, and the work done against the weight is Wh . By the equation above, these are equal. Hence, although by the arrangement it is possible to lift a weight twice as heavy as could be done by direct lifting, yet no more work is obtained from the arrangement than is put into it. Indeed, if we take into consideration the weight of the block M and the friction in both pulleys, we find that W is less than $2P$, and the useful work obtained from the arrangement is less than the work put into it.

The study of other machines leads us to the same result, which is known as the **principle of work** and can be stated as follows :

In all mechanical devices of any sort whatever, the work accomplished by the machine is never greater than the work expended upon it.

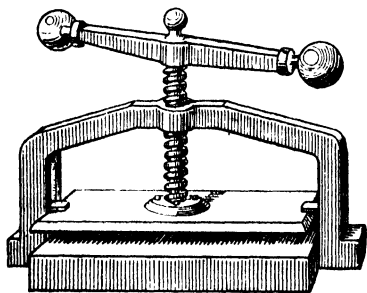


FIG. 50.—Screw press.

We shall now consider the application of the principle to the screw press shown in Fig. 50. When the screw is turned through one complete revolution, its point is lowered through a vertical distance d , equal to the distance between two adjoining threads. This distance is called the pitch of the screw; let r be the radius of the cross arm, and let a force P be applied at each end at right angles to the cross arm. In a complete

revolution the point of application of each force moves round a circle of radius r ; hence the work expended is

$$4\pi rP$$

Let F be the force exerted by the point of the screw. Then the work accomplished in the same time is

$$Fd$$

The principle of work states, that if friction is neglected, $Fd = 4\pi rP$. In the screw press, however, much of the work is lost overcoming friction, and Fd is considerably less than $4\pi rP$.

Efficiency of a Machine.—The efficiency of a machine is defined as the ratio of the useful work done to the work expended upon it in the same time. The **velocity ratio** of a machine is defined as the ratio of the distance moved by the power to the distance moved by the weight in the same time. It can be calculated from the dimensions of the machine.

Let H be the distance moved by the power P , and let h be the distance moved by the weight W in the same time. Then

$$\text{Efficiency} = \frac{Wh}{PH} = \frac{\text{mechanical advantage}}{\text{velocity ratio}}$$

and owing to work lost in overcoming friction this is in general less than 1. If no work is lost, this is equal to 1. Then

$$\text{mechanical advantage} = \text{velocity ratio.}$$

EXAMPLE.—The pitch of the screw in a screw press is 0.5 mm. and the downward force exerted by the point of the screw is 20 kg., when horizontal forces of 100 gm. are applied at right angles to the cross arm at distances of 20 cm. from its centre. Calculate the efficiency of the arrangement.

In a single revolution the work done is

$$20 \times 0.05 = 1 \text{ kg.-cm.}$$

The work put in is

$$0.1 \times 4\pi \times 20 = 8\pi \text{ kg.-cm.}$$

Hence efficiency $= \frac{1}{8\pi} = 0.0398$. Much the greater part of the work is consequently lost overcoming friction.

Wheel and Axle.—In this machine two cylinders A and B are fitted together so as to turn on the same axis. One cylinder B, which has a much larger diameter than the other, is called the wheel, and the other is called the axle. Ropes are wound round the two cylinders in opposite directions, so that when the machine is set in motion, the end of the one rope ascends and the end of the other descends. The power is applied to the end of the rope coiled round the wheel and the weight to the end of the rope coiled round the axle.

Let a be the radius of the axle and let b be the radius of the wheel. Then when they have made one complete revolution, the end of the one rope has descended a distance $2\pi b$ and the end of the other has ascended $2\pi a$. Hence by the principle of work

$$P2\pi b = W2\pi a$$

and the mechanical advantage

$$\frac{W}{P} = \frac{b}{a}$$

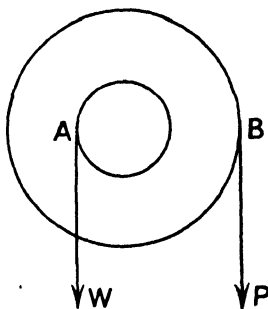


FIG. 51.—Wheel and axle.

If instead of two cylinders, we have a horizontal roller with a handle attached at one end (Fig. 52), the device is called a *windlass*, and is used for raising water out of wells and similar purposes. In the *capstan* used on board ship the rope is coiled round a vertical cylinder, and the power is applied at the end of spokes.

Systems of Pulleys.—In the case of the single movable pulley represented in Fig. 49 the mechanical advantage is only 2. In the systems of pulleys, examples of which are shown in Figs. 53 and 54, the mechanical advantage can be made much greater than this.

In Fig. 53 there are two blocks the upper one of which, AB, is fixed, while the other one, CD, is movable. Each carries two pulleys, and the

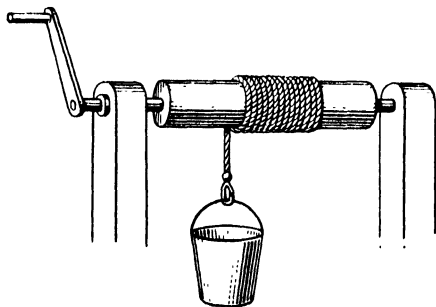


FIG. 52.—Windlass.

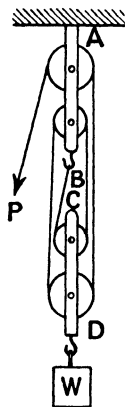


FIG. 53.

one string passes round all the pulleys. The weight W is suspended from the lower end of the movable block, and the effort P is applied to the free end of the string. The lower block is suspended by four strings which are all practically vertical and the stretching force in each of which is P . Hence

$$W = 4P$$

In the general case, if there are n pulleys in each of the blocks,

$$W = 2nP$$

We can take the weight of the lower block and pulleys into the calculation by supposing them included in W .

In Fig. 54 a separate string goes round each pulley. The weight W is hung from the lowest pulley, and the power P is applied by a hand at the free end. The stretching force in the string passing under pulley A is obviously $\frac{1}{2}W$, and we consider this as the weight suspended from B. The stretching force in the string passing under B is consequently $\frac{1}{2}W$. Similarly the

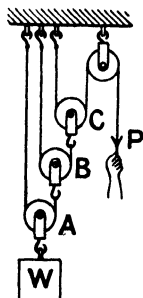


FIG. 54.

stretching force in the string passing under C is $\frac{1}{2}W$, and this is the same as P . Hence

$$\frac{W}{P} = 8 = 2^3$$

In the general case when there are n movable pulleys the theoretical value of the mechanical advantage is 2^n . But in this and the preceding case the actual value obtained is much less than the theoretical value owing to friction in the pulleys and want of flexibility of the string.

Activity.—*The rate at which an agent does work is called its power or activity.*

Activity is measured in any unit of work per unit of time, in ergs per second or foot-lbs. per second. It has, however, three special units of its own. James Watt, the inventor of the steam engine, introduced a unit called the **horse-power** (H.P.), which is equal to 33,000 ft.-lb. of work per minute or 550 ft.-lb. of work per second. His estimate of the horse's activity was intentionally high, as he did not wish to over-advertise his engines; the average horse can work only at about $\frac{3}{4}$ of this rate, and the average man at about $\frac{1}{7}$ of this rate.

The other special units are the **watt**, named after James Watt, which is defined as 1 joule per second, and the **kilowatt**, which is defined as 1000 watts. The kilowatt is very nearly $\frac{1}{3}$ H.P.

EXAMPLE.—A man whose weight is 10 stone climbs a mountain 3800 ft. high in 3 hours. Find in H.P. the average rate at which he works against gravity.

$$\text{Total work done in ft.-lb.} = 10 \times 14 \times 3800$$

$$\text{Activity} = \frac{10 \times 14 \times 3800}{3 \times 60 \times 60} \text{ ft.-lb./sec.}$$

$$= \frac{10 \times 14 \times 3800}{3 \times 60 \times 60 \times 550} \text{ H.P.} = 0.090 \text{ H.P.}$$

Energy.—*The energy of a body is the work that can be obtained from it in virtue either of its motion or position. If it is obtained in virtue of its motion, it is termed **kinetic energy**; if it is obtained in virtue of its position, it is termed **potential energy**. Energy, being work, is measured in the same units as work.*

Kinetic Energy.—Let us suppose that a bullet of mass m is moving with velocity v in a straight line, that it hits a block of wood perpendicularly, and is brought to rest inside it. Let us suppose, also, that the wood offers a constant resistance F to the motion of the bullet, that this resistance produces a negative acceleration α , and that the bullet is brought to rest in a distance s . Then the work it does against the wood is Fs .

Now

$$F = m\alpha$$

and, since the negative acceleration α annuls the velocity v in distance s ,

$$v^2 = 2\alpha s$$

The work obtained from the bullet in consequence of its motion is therefore given by

$$\begin{aligned}Fs &= m\alpha s \\ &= \frac{1}{2}mv^2\end{aligned}$$

The same reasoning can be applied to any body of mass m moving with velocity v which is brought to rest by a constant force F . It can also be extended to include the case when the force F is not constant. Hence the kinetic energy of a body of mass m moving with velocity v is $\frac{1}{2}mv^2$.

EXAMPLE.—A bullet travelling at the speed of 400 metres/sec. is just able to pierce a beam 16 cm. thick. Its mass is 80 gm. Find the resistance of the beam to the bullet in dynes on the assumption that it is constant. What velocity would enable the bullet to pierce a plank of the same wood 3 cm. thick?

Let F dynes be the resistance offered by the wood.

$$\text{Then } Fs = \frac{1}{2}mv^2$$

$$F = \frac{mv^2}{2s} = \frac{80 \times 40,000^2}{2 \times 16} = 4 \times 10^9 \text{ dynes}$$

Let v be the required velocity. Then

$$v = \sqrt{\frac{2Fs}{m}} = \sqrt{\frac{2 \times 4 \times 10^9 \times 3}{80}} \text{ cm./sec.} = 173.2 \text{ metres/sec.}$$

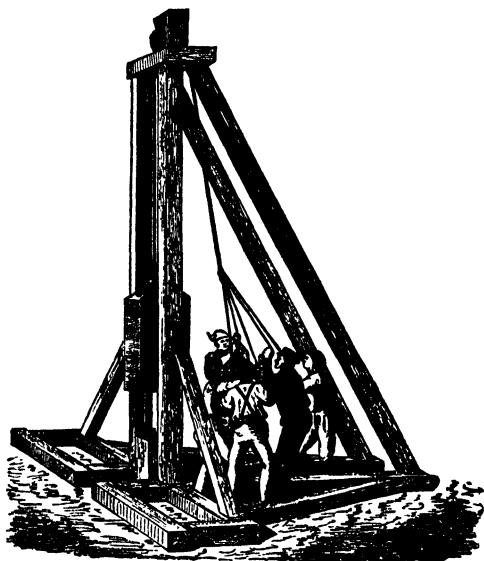


FIG. 55.—Piledriver.

Potential Energy.—

Fig. 55 represents a pile-driver. A massive lump of iron called the ram is raised by means of a rope and pulley several feet above the pile which is to be driven in. It is then let go, and falls with increasing velocity until it hits the pile, and drives it a certain distance into the ground.

Let m be the mass of the ram and v the velocity with which it hits the pile; let h be the height it falls before hitting the pile. Immediately before

hitting the pile it has the kinetic energy $\frac{1}{2}mv^2$, which is suddenly spent in doing work on the pile.

Before starting to fall the ram had only potential energy, energy due to its position, and this was all converted into kinetic energy during the fall. Now $v^2=2gh$. Hence

$$\frac{1}{2}mv^2=\frac{1}{2}m2gh=mgh$$

This last expression depends only on the initial position of the ram, and expresses its potential energy in that position.

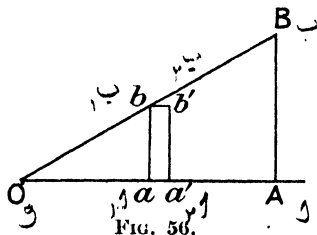
By similar reasoning it can be shown that any mass m raised a distance h above a plane acquires potential energy mgh with reference to that plane.

Potential Energy of a Stretched Spring.—It is a matter of common knowledge that potential energy can be stored up in springs. Thus, when a watch is wound up, its mainspring is bent, and energy is stored up sufficient to keep the watch going for the next day. In the same way in a spring gun it is the potential energy of a compressed spring which is converted into the kinetic energy of the bullet.

The potential energy of a stretched spiral spring is given by the formula "half tension by extension." Suppose that the spring is pulled out 2 cm. and that it then exerts a force of 100,000 dynes. As it is pulled out, its resistance gradually increases from zero to this value. We substitute for the tension 100,000 dynes and for the extension 2 cm. Then the potential energy is

$$\frac{1}{2} \times 100,000 \times 2 = 100,000 \text{ ergs.}$$

The formula is best proved graphically. Let OA (Fig. 56) represent the final extension and AB the final value of the force stretching the spring. Then, since the force is proportional to the extension, ab represents the force corresponding to the extension Oa . If the extension increases to Oa' , where aa' is small, and we neglect the increase in the force, the work done



$$Fs=ab \times aa'=\text{area of rectangle.}$$

If we suppose now, that the spring is extended from O to A in a series of steps, during each of which the force remains constant, the total work done will be represented by the sum of the areas of a series of such rectangles, and if we make the steps infinitely small, this sum will pass over into the area of the triangle OAB, just as on p. 12 in the proof of the analogous formula for the space described in uniformly

accelerated rectilinear motion. The work done in the extension is consequently

$$OAB = \frac{1}{2}AB \times OA = \frac{1}{2} \text{ tension} \times \text{extension}.$$

The Principle of the Conservation of Energy.—Suppose a stone is carried up a cliff h units high. Then the work done on it is mgh , and this is equal to its potential energy when placed on the edge at the top. Let us suppose now, that it is pushed over. When it has fallen a distance x , its velocity is given by $v^2 = 2gx$, and its total energy is

$$\begin{aligned} \frac{1}{2}mv^2 + mg(h-x) &= mgx + mg(h-x) \\ &= mgh \end{aligned}$$

the value which it had at the top. As it falls the energy is gradually changed from potential energy into kinetic, but its total amount remains the same.

This is a particular case of a very general principle, namely the principle of the conservation of energy, which may be stated as follows :

Energy can never be created or destroyed. It may be communicated from one body to another, and may be transformed from the potential to the kinetic form, or into other forms, but the total quantity inside a system not acted on by external forces remains always the same.

The principle appears to be contradicted by the example we have chosen, because when the stone strikes the ground at the foot of the cliff both its kinetic and potential energy disappear. This loss is, however, only apparent. The energy reappears again as heat and noise. The stone is slightly warmer as a result of the blow than it was before it. The full evidence for the truth of the principle cannot be appreciated without a knowledge of heat and electricity, but in the meantime it must be clear that heat is a form of energy, since it is the heat of the coal in the furnace of the engine that makes the train go.

No general proof of the principle is possible, but it has been verified in so many cases that it is now our firm belief, that all the different manifestations of energy, potential, kinetic, heat, energy of chemical constitution, electrical energy, atomic energy, are interchangeable, and that energy is a constituent of the universe equally important with matter, and, like it, indestructible. The possibility of perpetual motion machines, machines which perform work without consuming energy and which there are attempts to invent even at the present day, is absolutely denied by the principle ; the work taken out of a machine must always be less than the energy put in, because some work is always wasted against friction.

EXAMPLE.—Determine the velocity of the bob of a pendulum at its lowest point.

This is an example on the principle of the conservation of energy. Let l be the length of the thread, m the mass of the bob, and φ the angle which it makes with the vertical at the extremity of a swing. Then, as has been shown on p. 55, the work done in raising the bob against gravity from A to B is

$$mgl(1 - \cos \varphi).$$

This is its potential energy at B, when the kinetic energy is zero. If now the pendulum is let go, when the bob reaches A its potential energy is converted wholly into kinetic energy. Let v be the velocity at A. Then

$$\frac{1}{2}mv^2 = mgl(1 - \cos \varphi)$$

or

$$v^2 = 2gl(1 - \cos \varphi) = 4gl \sin^2 \frac{\varphi}{2}$$

and

$$v = 2\sqrt{gl} \sin \frac{\varphi}{2}$$

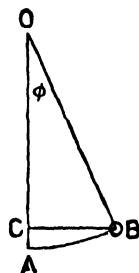


FIG. 57.

EXAMPLES VI

1. How many ergs are there in the ft.-lb. ?
2. Find in foot-tons the work done in drawing a truck weighing 10 tons up an incline a mile long, which rises 1 in 120, the coefficient of friction being $\frac{1}{4}$.
3. Find the work done by
 - (i) a force of 220 dynes acting through 80 cm.,
 - (ii) a force of 16 gm. acting through 2 metres,
 - (iii) a force of 18 lb. acting through 6 inches.
4. The Kinlochleven reservoir is situated 935 ft. above the level of the works. What is the potential energy of the water in ft. lb./cub. ft. ? One cubic foot of water weighs 62.4 lb.
5. Find in ft.-lb. the kinetic energy of a body of mass 20 lb. moving with a velocity of 33 ft./sec.
6. A spiral spring is pulled out by the fingers a distance of 10 cm., and when extended this length exerts a force of 250 gm. What is the potential energy stored in the spring in ergs ?
7. Find in ergs the kinetic energy of a mass of 17 gm. moving with a velocity of 22 cm./sec.
8. The length of a simple pendulum is 80 cm., and the bob weighs 100 gm. Initially it makes an angle of 20° with the vertical. Find the kinetic energy and hence the velocity of the bob, when (i) it is at its lowest point, (ii) it is 10° beyond its lowest point.
9. A 10 H.P. engine is employed to pump water from the bottom of a mine 300 ft. deep. How many cubic feet of water will it raise in an hour ?
10. The mass of a train is 300 tons and its velocity is 30 miles per hour. What H.P. would be necessary to give it this velocity in 1 minute ? The frictional resistances are to be neglected.

CHAPTER VII

ROTATIONAL MOTION—GRAVITATION

Angular Momentum. Kinetic Energy of Rotation.—Let us suppose that a body is rotating about an axis. Then every particle of the body moves in a circle about the axis in a plane at right angles to the axis. Let ω be the angular velocity of the body about the axis measured in radians per second, and let m_1 be the mass of a particle of the body the perpendicular distance of which from the axis is r_1 . Then ωr_1 is the velocity of the particle, and $m_1 \omega r_1$ its momentum, or more strictly its linear momentum. If we multiply this quantity by the radius of the circle in which the particle is moving, we obtain

$$m_1 \omega r_1^2$$

which is called the angular momentum or moment of momentum of the particle about the axis. If we repeat the process for all the particles into which the body can be divided, and take the sum, we obtain

$$m_1 \omega r_1^2 + m_2 \omega r_2^2 + m_3 \omega r_3^2 \dots = \Sigma m \omega r^2 = (\Sigma m r^2) \omega$$

This quantity is called the angular momentum or moment of momentum of the body about the axis.

If we return to the single particle, its kinetic energy is

$$\frac{1}{2} m_1 (\omega r_1)^2$$

By forming similar expressions for the other particles into which the body can be divided and taking the sum we obtain

$$\frac{1}{2} m_1 (\omega r_1)^2 + \frac{1}{2} m_2 (\omega r_2)^2 + \frac{1}{2} m_3 (\omega r_3)^2 \dots = \Sigma \frac{1}{2} m (\omega r)^2 = \frac{1}{2} (\Sigma m r^2) \omega^2$$

for the kinetic energy of rotation of the body about the axis.

Moment of Inertia.—If we multiply each particle of the body by the square of its distance from the axis and add, the sum

$$m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 \dots = \Sigma m r^2$$

is known as the moment of inertia of the body about the axis, and is usually written I . Thus the angular momentum and kinetic energy of the body are $I \omega$ and $\frac{1}{2} I \omega^2$.

Let M be the mass of the body. Let $I = M k^2$. Then the kinetic energy of rotation is

$$\frac{1}{2} I \omega^2 = \frac{1}{2} M k^2 \omega^2 = \frac{1}{2} M (k \omega)^2$$

Thus if the whole mass of the body were concentrated at a point distant k from the axis, its kinetic energy would be unaltered. k is called the **radius of gyration** of the body about the axis. Both I and k vary for a body with the position of the axis.

The following table gives values of k^2 for some simple cases. The bodies are supposed to be of uniform density throughout. The results are obtained by calculations which are beyond the scope of this book.

	Value of k^2
Rod, length $2a$, about an axis through its middle point perpendicular to its length	$\frac{1}{3}a^2$
Rod, length $2a$, about an axis through one end perpendicular to its length	$\frac{1}{3}a^2$
Disc, radius a , about a diameter	$\frac{1}{4}a^2$
Disc, radius a , about an axis through its centre perpendicular to its plane	$\frac{1}{2}a^2$
Solid cylinder, radius a , about its axis	$\frac{1}{2}a^2$
Solid sphere, radius a , about a diameter	$\frac{3}{5}a^2$

When the moment of inertia of a body is known for an axis through the centroid, it can be calculated for any parallel axis. For (Fig. 58) let G denote the centroid, A the point in which the parallel axis intersects the plane of the diagram, and let P be a typical particle. The axes are at right angles to the plane of the diagram. Then

$$\begin{aligned} I_A &= \sum mAP^2 \\ &= \sum m(AG^2 + GP^2 + 2AG \cdot GP \cos \theta) \\ &= AG^2 \sum m + \sum mGP^2 + 2AG \sum mGP \cos \theta \end{aligned}$$

$GP \cos \theta$ is the distance of the particle from the plane through G at right angles to AG , and by the property of the centroid proved on p. 52, $\sum mGP \cos \theta = 0$. Hence substituting M for $\sum m$,

$$I_A = M \cdot AG^2 + I_G$$

i.e. *the moment of inertia about any axis is equal to the moment of inertia about a parallel axis through the centroid plus the product of the mass of the body and the square of the perpendicular distance between the two axes.*

EXAMPLE.—A cylinder of length 20 cm., radius 4 cm., and density 0.6 is rotating about an axis in its surface parallel to its axis of figure with an angular velocity of 10 radians per second. Calculate its kinetic energy.

$$M = 20 \times \pi \times 4^2 \times 0.6 = 603.3 \text{ gm.}$$

$$I = M \frac{a^2}{2} + Ma^2 = M \frac{3a^2}{2} = 603.3 \times \frac{3 \times 4^2}{2} = 1.447 \times 10^4 \text{ gm.-cm.}^2$$

$$\text{Hence kinetic energy} = \frac{1}{2} \times 1.447 \times 10^4 \times 10^2 = 7.235 \times 10^5 \text{ ergs.}$$

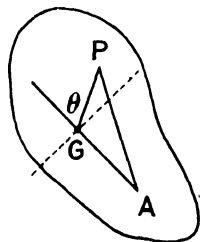


FIG. 58.

Couple.—A couple in dynamics is a system of two equal and opposite parallel forces.

The forces P (Fig. 59) represent the forces of a couple, and AB the perpendicular distance between them. AB is called the **arm** of the couple, and the product of either force P and the arm AB is called the **moment** of the couple. The moment of a couple is usually denoted by the letter L .

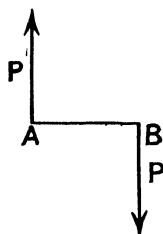


FIG. 59.

If we attempt to find the resultant of the forces of a couple by the method of p. 31, we find that its magnitude is zero, and that the construction for its line of action breaks down. A single force cannot be found which is equivalent to a couple. Now a single force produces a motion of translation. A couple, therefore,

must cause a motion of a different kind, and it can be shown that it always causes a motion of rotation.

Couples are classified into clockwise and counter-clockwise couples, according to the direction of the rotation they produce.

Dynamics of Rotation.—Hitherto we have dealt only with the dynamics of translation. A force acted on a body and moved it in a straight line. We have now to deal with problems in rotation. A couple acts on a body, and it is required to find the result. The simplest way of solving problems of this nature is by analogy with corresponding problems in translation. An analogy exists between the formulæ used for translational and rotational motion; we shall exhibit this analogy in a table, and then use it to solve typical problems.

Translation.	Rotation.
Mass M	Moment of Inertia $I = Mk^2$
Force F	Couple L
Displacement s	Angle θ
Velocity v	Angular velocity ω
Acceleration a	Angular Acceleration $\dot{\omega}$
Work Fs	Work $L\theta$
Momentum Mv	Angular Momentum $I\omega$
Kinetic Energy $\frac{1}{2}Mv^2$	Kinetic Energy $\frac{1}{2}I\omega^2$
$F = Ma$	$L = I\dot{\omega}$

Examples on Rotational Motion.—(1) A wheel has a uniform angular acceleration of 5 rad./sec.^2 . What angular velocity will it acquire, and what angle will it describe in 4 sec. starting from rest?

We have in analogy with $v = at$ and $s = \frac{1}{2}at^2$, $\omega = \dot{\omega}t$ and $\theta = \frac{1}{2}\dot{\omega}t^2$. Hence

$$\omega = 5 \times 4 = 20 \text{ rad./sec.}$$

and

$$\theta = \frac{1}{2} \times 5 \times 4^2 = 40 \text{ rad.}$$

(2) A couple consists of forces of 50 dynes acting in lines 4 cm. apart. Calculate the work it does in rotating a body through 100° .

Here $L = 50 \times 4 = 200$ units

$$\theta = \frac{100 \times \pi}{180} = 1.745 \text{ rad.}$$

Hence work done $= 200 \times 1.745 = 349.0$ ergs.

(3) To a flywheel weighing 4 cwt. whose radius of gyration is 2 ft., a constant couple of 400 ft.-lb. units is applied. Find the angular velocity acquired by the wheel in 10 sec.

The equation is $L = I\omega$. L must be changed to absolute units before insertion, i.e. 400×32 ft.-pd. $I = 4 \times 112 \times 2^2$. Hence

$$\dot{\omega} = \frac{400 \times 32}{4 \times 112 \times 2^2} = \frac{50}{7} \text{ rad./sec}^2.$$

and

$$\omega = \frac{50}{7} \times 10 = 71.4 \text{ rad./sec.}$$

(4) A plane is inclined at an angle θ to the horizontal, and a uniform sphere of radius a is allowed to roll down it a distance l measured along the plane, starting from rest. Find the velocity acquired by the sphere.

Let ω be the angular velocity it acquires. Then, since at each instant the centre is rotating about the instantaneous point of contact, the linear velocity of the centre is ωa , where a denotes the radius of the sphere. Let M be the mass of the sphere. It has the kinetic energy of translation

$$\frac{1}{2} M v^2 = \frac{1}{2} M \omega^2 a^2$$

and in addition the kinetic energy of rotation

$$\frac{1}{2} I \omega^2 = \frac{1}{2} M \frac{2}{5} a^2 \omega^2$$

making a total of

$$\frac{1}{2} M \omega^2 a^2 + \frac{1}{2} M \frac{2}{5} a^2 \omega^2 = \frac{7}{10} M \omega^2 a^2$$

for the kinetic energy gained.

During the motion the centre descends a height $l \sin \theta$. Consequently the potential energy lost is

$$M g l \sin \theta$$

By the conservation of energy

$$\frac{7}{10} M \omega^2 a^2 = M g l \sin \theta$$

hence

$$\omega a = \sqrt{\frac{10 g l \sin \theta}{7}}$$

Compound Pendulum.—Any body which is supported on a horizontal axis not through its centroid, and which is free to rotate about that axis in a vertical plane under the action of gravity is called a compound pendulum. Let Fig. 60 represent a vertical section through the centroid G of a compound pendulum, and let A be the point in which the section intersects the

axis. The axis is at right angles to the plane of the diagram. Let $AG=h$ and let Mk^2 be the moment of inertia of the pendulum about a horizontal axis through G . Then it follows that the moment of inertia about the parallel axis through A is $Mk^2 + Mh^2 = M(h^2 + k^2)$.

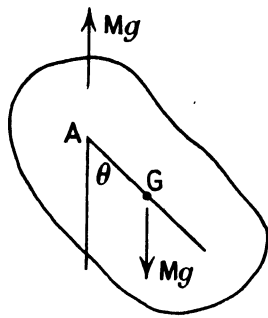


FIG. 60.

Let AG make angle θ with the vertical. The pendulum is acted on by two forces, its weight Mg vertically downwards and the reaction on the axis. For small displacements the latter is vertically upwards and equal to Mg . Thus the two forces constitute a couple of arm $AG \sin \theta = h \sin \theta$ and moment $Mgh \sin \theta$. This couple causes the pendulum to rotate, and the angular

acceleration ω is given by

$$\omega = \frac{L}{I} = \frac{Mgh \sin \theta}{M(h^2 + k^2)}$$

or

$$\frac{Mgh\theta}{M(h^2 + k^2)}$$

when θ is small. The ratio

$$\frac{\text{Angular Acceleration}}{\text{Angular Displacement}} = \frac{\frac{gh\theta}{h^2 + k^2}}{\theta} = \frac{gh}{h^2 + k^2} = \text{constant.}$$

If the ratio angular acceleration to angular displacement is constant for the body, the ratio linear acceleration to linear displacement must also be constant for every point on the body, since the latter are obtained from the former on multiplying them by the distance of the point from the axis. Hence (cf. p. 21) every point in the body is describing a S.H.M., and the period is given by

$$\frac{gh}{h^2 + k^2} = \frac{4\pi^2}{T^2}$$

or

$$T = 2\pi \sqrt{\frac{h^2 + k^2}{gh}}$$

Law of Gravitation.—It has already been mentioned, that bodies are attracted downwards by the force of gravity, as a consequence of which they fall with the uniform acceleration g . This is a particular case of the law of universal gravitation discovered by Newton, which can be stated as follows :

Every particle of matter in the universe attracts every other

particle with a force proportional to the product of their masses and inversely proportional to the square of the distance between them.

Expressing the law in symbols

$$F = \frac{kmm'}{d^2}$$

where F is the force on either particle, m and m' are the masses of the particles, d is the distance between them, and k is a constant of proportionality known as the **gravitational constant**.

The earth attracts bodies on its surface, as if its whole mass were concentrated at its centre; its centre is at a distance of 4000 miles, but its mass is so great that its attraction far outweighs the attraction of bodies on the surface for one another, and the latter cannot be rendered evident, unless special precautions are taken.

Newton arrived by the aid of astronomical calculations first at the result, that the law held between the sun and planets. Then in 1665, when he was a young man of 23, he had to leave Cambridge owing to the plague, and the story goes that, when he was seated in his mother's garden at Woolsthorpe, the thought occurred to him on seeing an apple fall, that perhaps it was the same force which held the planets in their orbits, that also caused bodies to fall to the surface of the earth.

Verification of Law of Gravitation.—The moon's orbit is approximately circular, and its distance is 60 times the earth's radius. Denote the latter by R . The time taken by the moon to go round the earth, *i.e.* the sidereal month, is 27.32 days. The acceleration of the moon towards the earth is consequently given by

$$\begin{aligned} a &= \omega^2 60R \\ &= \left(\frac{2\pi}{27.32 \times 24 \times 60 \times 60} \right)^2 60 \times 4000 \times 5280 \\ &= 0.00897 \text{ ft./sec.}^2 \end{aligned}$$

If the moon is held in its orbit by the force that causes a stone to fall

$$\frac{a}{g} = \frac{1}{60^2}$$

and

$$g = a60^2 = 32.29 \text{ ft./sec.}^2$$

a value which shows that the supposition is right.

The value of g varies over the earth's surface owing to two causes. In the first place, the earth is not a sphere but an oblate spheroid, the poles being nearer the centre than points on the equator. Consequently g is less at the equator. In the second place, owing to the rotation of the earth, bodies on its surface are travelling in circles round its axis. A small part of the earth's attraction on them is consequently utilized in supplying the acceleration $\omega^2 r$ necessary to keep them in these circular paths; this proportion increases in magnitude as we approach

the equator. The remainder is utilized in causing the body to fall. Owing to this cause also g is less at the equator.

The Cavendish Experiment.—The first direct measurement of the force of attraction between two bodies of such a size that we can handle them, was made by Henry Cavendish in 1798.

The instrument employed was a torsion balance consisting of a horizontal rod which carried lead spheres of 2 inches diameter at its ends, and was suspended by a long fine wire. If this rod was acted on by a couple in a horizontal plane, it turned, the wire becoming twisted, and took up a new position of equilibrium. The angle through which it turned was proportional to the couple. Fig. 61 represents a plan of the arrangement. ab is the horizontal

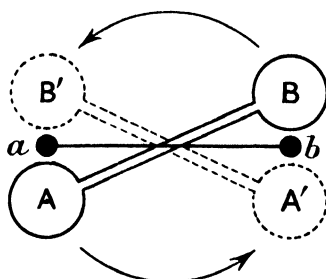


FIG. 61.—Cavendish experiment.

rod. A and B are two large lead spheres of 12 inches diameter which can be rotated from the position AB to the position $A'B'$ as shown by the arrows. In the first position A attracts a and B attracts b , so there is a counter-clockwise couple on the rod. In the second position B' attracts a and A' attracts b , so the couple is clockwise. Thus by rotating the large spheres from the one position to the other, a couple in one direction is changed into an equal couple in the opposite direction, and the rod consequently twists through a small angle from one position of equilibrium to another. This angle is measured, and from it the magnitude of the couple and of the force between a large sphere and the small sphere opposite it is deduced. This force is given by

$$\frac{kMm}{d^2}$$

where M is the mass of a large sphere, m the mass of a small sphere, and d is the distance between their centres. Hence, when it is known, the value of k can be calculated.

In this way Cavendish made the first determination of the value of the gravitational constant. Subsequent measurements with more refined apparatus have given the mean value of 6.66×10^{-8} , when the masses are expressed in grams and the distance in centimetres.

Density of the Earth.—When k is known, the density and mass of the earth can be calculated. Thus Cavendish was said to have “weighed the earth.”

For let M be the mass of the earth and R its radius. Then the force with which a mass of one gram on the earth's surface is attracted downwards is

$$\frac{kM}{R^2}$$

measured in dynes. This force is also g dynes. Hence

$$\frac{kM}{R^2} = g$$

and if we substitute $\frac{4}{3}\pi R^3\rho$ for M , where ρ is the density of the earth,

$$\frac{4}{3}\pi kR\rho = g$$

$$\begin{aligned} \text{Thus } \rho &= \frac{3g}{4\pi kR} = \frac{3 \times 981}{4\pi \times 6.66 \times 10^{-8} \times 4000 \times 5280 \times 30.48} \\ &= 5.46 \text{ gm./c.c.} \end{aligned}$$

EXAMPLES VII

1. A disc is rotating about an axis in its plane tangential to its edge at the rate of 10 revolutions per second. Its mass is 10 gm. and its radius is 5 cm. Calculate its kinetic energy.

2. A wheel has an angular velocity of 5 rad./sec. and an angular acceleration of 2 rad./sec.² What is its angular velocity after two complete revolutions?

3. A flywheel which is uniformly accelerated, makes 4 complete revolutions in 8 sec. starting from rest. What is its acceleration in rad./sec.²?

4. A couple of 20 ft.-lb. units rotates a body through 2 right angles. Calculate the work done in ft.-pdl.

5. A flywheel weighs a ton and has a radius of gyration of 4 ft. Calculate in ft.-lb. the couple required to give it an angular velocity of 50 revolutions per minute in 30 seconds, starting from rest.

6. A uniform rod of length l is suspended from one end and allowed to oscillate as a pendulum. What is its period?

7. A thin rope is wound round the flywheel of an engine of which the diameter is 10 ft., and when the speed of the engine is 200 revolutions per minute, the difference in stretching force at the two ends of the rope is 30 lb. What is the horsepower?

8. Given that the acceleration of gravity is 32.00 ft./sec.² on the earth's surface, what will it be in a balloon 2 miles up?

9. By how much does the rotation of the earth diminish the acceleration of gravity at the equator?

10. Lead spheres of 12 and 2 cm. diameter are suspended with their centres 8 cm. apart. Calculate the force of attraction between them in dynes. The density of lead is 11.4.

CHAPTER VIII

FLUID PRESSURE

States of Matter.—Matter exists in three states, the solid, the liquid, and the gaseous. A solid body has both a definite shape and a definite volume. A liquid has a definite volume, but no definite shape; it takes the form of the vessel into which it is poured. A gas has neither a definite shape nor a definite volume, but tends to expand indefinitely, so as always to fill the vessel which contains it. Both liquids and gases are included under the name of fluids. The distinction between the different states will be gone into in greater detail later; this account is intended to be merely a preliminary one.

The word “gas” was invented by van Helmont, a Belgian chemist who died in 1644. He suggested two names, *gas* and *blas*, connected etymologically with the words “ghost” and “blow,” and the first of these has been adopted.

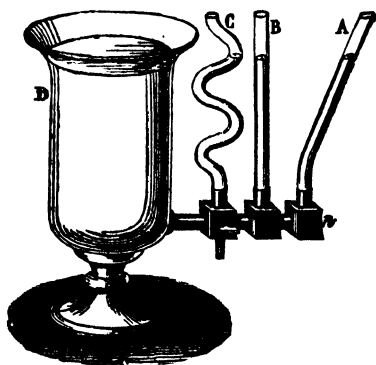


FIG. 62.—Free surface in communicating vessels.

Hydrostatics is the part of physics which deals with the equilibrium of fluids; we are now entering on the study of this part of the subject.

Water Maintains its Level.

—If several different vessels are connected as shown in Fig. 62 and filled with water, then, when the water comes to rest, its surface is at the same level in the different vessels. This is also the case with two reservoirs miles apart connected by a water main, and is popularly expressed by the phrase that “water maintains its level.”

The free surface of a liquid at rest is always horizontal, always at right angles to the plumb line. For, if it were not, and one part of the surface were higher than another, gravity would

cause the liquid to move from the higher part of the surface to the lower part.

This fact is taken advantage of in the construction of the **spirit level**, which consists in its most usual form of a slightly curved glass tube filled with alcohol, except for one small air bubble. Alcohol is used as it is more mobile than water. The tube is fixed in a brass mount.

The air bubble forms the free surface of the liquid, and always takes up a position at the highest part of the tube.

Two points on the tube are

marked, the distance between them being equal to the usual length of the bubble. The instrument is adjusted so that, when it is resting on a horizontal surface, the air bubble comes to rest between these marks.

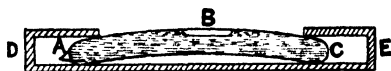


FIG. 63.—Spirit level.

If a vessel or a pipe containing liquid at rest is pierced by a small hole, a thin jet squirts out. Such a jet is sometimes seen in the case of a leaky garden hose. It is always at right angles to the surface of the containing vessel or pipe. Consequently the liquid must press against every part of the surface of the containing vessel with a force at right angles to that part, and the surface must press back with an equal and opposite force.

Pressure at a Point.—Let us suppose that a block of wood rests on a table. The block presses down with a force equal to its weight, known as the thrust on the table, which we shall denote by F ; this force is balanced by an equal and opposite upward thrust exerted by the table on the block. If we divide the thrust F by the area A of the base of the block in contact with the table, the quotient

$$\frac{F}{A}$$

is known as the **average pressure** exerted by the block on the table.

Consider a portion of the base of area a surrounding a point P . Let f be the thrust on this area. Make the area smaller and smaller; f at the same time diminishes in size. The value of f/a when a is made indefinitely small is known as the pressure at the point P .

The ratio of the thrust on a small area surrounding a point to the area itself, when the latter is made indefinitely small, is known as the pressure at the point.

Now consider a liquid of density ρ filling a cylindrical vessel to a depth h , as shown in Fig. 64. Take a small circular area of magnitude a surrounding a point P on the bottom of the vessel,

and consider the liquid contained in the cylinder which has this area as base. Suppose it to become solid without change of volume. Then the forces acting on it are unaltered. It is in equilibrium under (1) the pressure of the surrounding liquid, (2) its own weight, (3) the upward thrust on the base. Since the pressure of the surrounding fluid is perpendicular to the surface, it acts horizontally, and has therefore no vertical component. Hence the two remaining forces must balance, and the thrust on the base is consequently equal to the weight of the column $ahg\rho$. The average pressure on the base is $ahg\rho/a$ or $hg\rho$. This remains unaltered when a is made indefinitely small. Hence the pressure on the base at P is $hg\rho$.

If, instead of taking the base of the cylinder on the bottom of the vessel, it is taken on a plane some distance up, we can

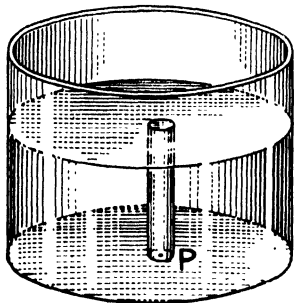


FIG. 64.

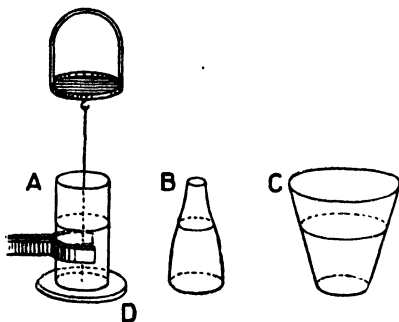


FIG. 65.—Pascal's vases.

show in the same way, that the pressure at a point in this plane is $hg\rho$, where h denotes its depth below the surface.

It will be noticed that pressure has not the dimensions of force but of force/area. It is thus, for example, not measured in dynes, but in dynes/cm.²

Pascal's Vases.—The fact that the pressure depends only on the density of the liquid and the depth below the surface, was verified by an experiment due to Pascal, known as Pascal's vases. In this experiment three vases A, B, and C are taken with equal bases, A cylindrical, B tapering upwards, and C widened upwards (Fig. 65). These vessels have no bottoms. In the figure A is shown held by a clamp. D is a movable disc held against its lower edge by a string attached to one of the scales of a balance. Weights are placed in the other pan to keep the disc against the edge. The vase is filled with water until the pressure is just sufficient to detach the disc from the edge, and the depth of the water noted. The experiment is then repeated

with the other two vases, the weights in the other pan being kept the same. It is found that the disc is detached, when the depth of the water is the same in each case.

The force exerted on the disc must be the same in each case, and the area on which it acts is the same. Hence the pressure on that area is the same, or, in other words, the pressure on the bottom of a vessel is given by the formula $h\rho g$, irrespective of the shape of the vessel.

This result is at first puzzling, because in the experiment the vase B holds less water than A, and the vase C more water than A, and one might expect that for the same depth the pressure on the base would be less in B than in A, and greater in C than in A. But part of the weight of the water in C is supported by the side, and in B the side exerts a downward force against the base.

EXAMPLE.—A cubical vessel of 5 cm. edge is filled with mercury to a depth of 2 cm., and then to the top with water. Find (i) the pressure on the bottom, (ii) the total downward thrust on the bottom.

$$\text{Pressure on bottom} = 2 \times 13.6 + 3 = 30.2 \text{ gm./cm.}^2$$

$$\text{Thrust on bottom} = 25 \times 30.2 = 755 \text{ gm.}$$

Pressure inside a Liquid the same in all Directions.—If a glass tube with an end ground flat against which a thin plate presses, is lowered vertically into a liquid, it can be shown experimentally by blowing air into it, that the pressure at a depth h below the surface is $h\rho g$, because it is only when the pressure exceeds this limit that the plate can be blown away from the end of the tube. If the tube is inclined instead of vertical (Fig. 66), with its end at the same place, the pressure required to force the plate from the end is still the same. Hence the pressure at a point inside a liquid is the same in all directions. This result can be proved mathematically as follows :

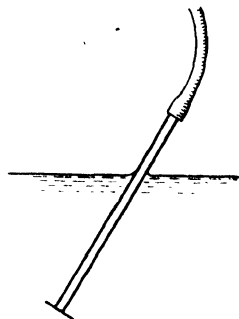


FIG. 66.

Isolate in imagination a small quantity of the fluid bounded (Fig. 67) by a horizontal plane EBCF, three vertical planes, namely FED, FCAD, and CBA, and a plane ABED inclined at any angle to the horizontal, and suppose that the point for which the result is to be proved is inside this figure. It is clear that, if the liquid were solidified without change of volume, its equilibrium would not be disturbed. Let us suppose it solidified. It is then in equilibrium under the pressure of the liquid on its five faces and its weight acting vertically downwards.

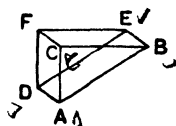


FIG. 67.

It is clear from symmetry that the thrusts on the faces FED and CBA balance one another, and do not require to be considered. Let p denote the pressure on the horizontal face, let p_1 denote the pressure on the inclined face, and let p_2 denote the pressure on the vertical end. The thrust on the upper face is $pFC \times CB$. The weight is $g\rho \frac{1}{2}FC \times CB \times CA$. The ratio of the weight to the thrust is, therefore

$$\frac{g\rho \frac{1}{2}FC \times CB \times CA}{pFC \times CB} = \frac{g\rho}{p} \frac{1}{2}CA$$

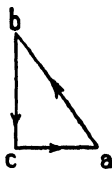


FIG. 68.

Let us suppose that the figure is very small; then CA becomes very small, and the weight can be neglected in comparison with the thrust on the upper face.

The figure is now in equilibrium under the action of three thrusts, one $p_1DA \times AB$ perpendicular to AB, one $pFC \times BC$ perpendicular to CB, and one $p_2FC \times CA$ perpendicular to CA. These three thrusts must obey the triangle of forces. Draw ab , bc , and ca (Fig. 68) to represent these three thrusts respectively. Then

$$ab : bc : ca = p_1DA \times AB : pFC \times BC : p_2FC \times CA \\ = p_1AB : pBC : p_2CA$$

But $\triangle abc$ is similar to $\triangle ABC$. Therefore

$$ab : bc : ca = AB : BC : CA$$

Thus $AB : BC : CA = p_1AB : pBC : p_2CA$

which is only possible if $p = p_1 = p_2$.

Thus the pressure on the plane ABED is equal to p . But the plane ABED was inclined in any direction. When the figure is made indefinitely small, the plane passes through the point in question. Thus the pressure at any point is the same in all directions.

U-Tube.—Let us suppose that two liquids which do not mix occupy the arms of a U-shaped tube as shown in Fig. 69. For the sake of clearness let the liquid DC be oil and the liquid AC be water. Draw a horizontal plane through C to cut the other branch in B. Then the pressure is the same at the two points B and C in the same horizontal. Consequently the pressure due to the column AB of the heavier liquid is equal to the pressure due to the column CD of the lighter liquid. Let h and h' be the heights of the columns of the two liquids respectively and ρ and ρ' their densities; then

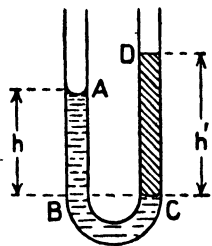


FIG. 69.

$$h\rho = h'\rho' \quad \text{or} \quad \frac{\rho}{\rho'} = \frac{h'}{h}$$

that is, the densities of the liquids are inversely as the heights of the columns.

Thrust on an Area.—If a plate of area A is immersed horizontally in a liquid of density ρ at a depth h below the surface, the pressure on it, $h\rho g$, is everywhere the same, and the thrust on it is $Ah\rho g$. Sometimes the words pressure or “whole pressure” are used for this quantity, but it is better to call it the thrust or “pressure-thrust.”

Let us suppose now that the plate is inclined to the surface as in Fig. 70, and that its outline is of irregular form. The pressure is everywhere at right angles to the plate and increases as we go downwards. Divide the plate into strips parallel to the surface; let the areas of these strips be s_1, s_2, s_3 , etc., and let the centres of gravity or centroids of the strips be situated at depths z_1, z_2, z_3 , etc., below the surface. Then, since the strips are narrow, the pressure on the first may be regarded as uniform all over it and equal to the pressure at its centroid, $z_1\rho g$.

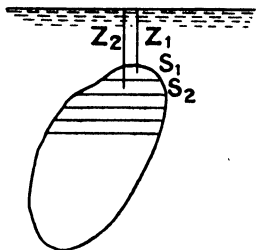


FIG. 70.

The thrust on the first strip is consequently $s_1 z_1 \rho g$. Similarly the thrusts on the second and third are $s_2 z_2 \rho g, s_3 z_3 \rho g$, and the resultant thrust on the plate is

$$s_1 z_1 \rho g + s_2 z_2 \rho g + s_3 z_3 \rho g \dots = \rho g \Sigma s z$$

The centroid of the plate is given (cf. p. 52) by the formula

$$\bar{z} = \frac{\Sigma m z}{\Sigma m}$$

where m is the mass of one of the particles into which the body is divided. In this case the strips take the place of the particles and their areas are proportional to their masses. The centroid of the plate is consequently given by

$$\bar{z} = \frac{\Sigma s z}{\Sigma s} \quad \text{and} \quad \Sigma s z = \bar{z} \Sigma s = \bar{z} S$$

where S is the area of the whole plate. Hence the resultant thrust on the plate is given by

$$\rho g \Sigma s z = S \rho g \bar{z}$$

Thus the resultant thrust on a plate immersed in a liquid is equal to the area of the plate multiplied by the pressure at its centroid.

EXAMPLE.—A triangular plate is immersed in a liquid of density ρ with a side in the surface. The sides are equal and of length a , and the surface of the plate is inclined at 45° to the vertical. Calculate the thrust exerted on it by the liquid.

The area of the plate is $\frac{a^2 \sqrt{3}}{4}$. The length of the median is $\frac{a \sqrt{3}}{2}$.

The centroid being one-third along the median is distant $\frac{1}{3} \frac{a\sqrt{3}}{2}$ from the surface along the median, and is

$$\frac{1}{\sqrt{2}} \frac{1}{3} \frac{a\sqrt{3}}{2} = \frac{a}{2\sqrt{6}}$$

vertically below the surface. The resultant thrust is consequently

$$\frac{a^2\sqrt{3}}{4} g\rho \frac{a}{2\sqrt{6}} = a^3 g\rho \frac{\sqrt{2}}{16}$$

Centre of Pressure.—If a surface is immersed in a liquid, the point at which the resultant thrust on the surface acts, is called the centre of pressure of the surface.

The centre of pressure of the surface does not coincide with its centre of gravity, and in general the determination of its position requires the aid of the calculus. One or two cases can, however, be treated by elementary methods.

Let AB in Fig. 71 represent a rectangle immersed vertically. Divide it into equal strips parallel to the surface, and let the arrows represent

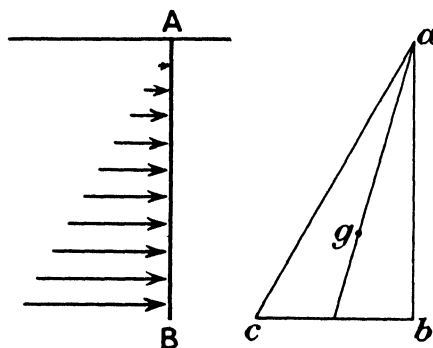


FIG. 71.

the thrusts on these strips in magnitude and direction. Each acts at the centre of its own strip, and its length is proportional to its distance from the surface. It is required to find the line of action of the resultant thrust.

The problem is best solved by analogy. Suppose we are required to find the centroid of triangle *abc*. If this triangle is divided into strips parallel to the base, the weight of each strip is proportional to its area, and

hence can be represented by a horizontal arrow the length of which is proportional to its distance from *a*. The resultant of this system of forces is the weight of the whole triangle, which acts through its centroid at *g*. This centroid is two-thirds of the distance down the median. Similarly the resultant of the thrusts on AB acts at a point P in AB, such that AP = $\frac{2}{3}$ AB.

The centre of pressure is thus on the middle vertical, two-thirds down.

EXAMPLE.—A rectangular board 3 ft. long and 2 ft. broad is immersed vertically in water with its length parallel to the surface and its upper edge 2 ft. below the surface. Find the magnitude and line of action of the resultant thrust on it.

Suppose the board completed to the surface. Then the thrust on the whole board is

$$3 \times 4 \times 2 \times 62.4 = 1497.6 \text{ lb.}$$

since the cu. ft. of water weighs 62.4 lb. This acts in a line $\frac{2}{3} \times 4 = 2.67$ ft. below the surface. The thrust on the upper part is

$$3 \times 2 \times 62.4 = 374.4 \text{ lb.}$$

This acts in a line $\frac{2}{3} \times 2 = 1.33$ ft. below the surface. The thrust on the lower part is consequently $1497.6 - 374.4 = 1123.2$ lb. Let its line of action be x ft. below the surface. Then by taking moments

$$1497.6 \times 2.67 = 374.4 \times 1.33 + 1123.2x$$

whence

$$x = 3.11 \text{ ft.}$$

Transmission of Fluid Pressure.—Any pressure communicated to any part of a fluid is communicated without diminution to every other part of the fluid. This law was first stated by Pascal.

Let us suppose we have a vessel of the shape represented in Fig. 72; terminating above in a tube and filled to the level A with water. Then the pressure at a point P situated h cm. below A is $h\rho g$, or, since $\rho = 1$, in gravitational units simply h gm. per cm.² If now water is poured into the vessel so as to raise the level 1 cm. from A to B, then the pressure at P becomes $h + 1$ gm. per cm.² Thus at every point inside the vessel the pressure increases by 1 gm. per cm.² The increase of pressure might

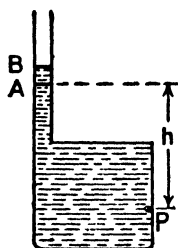


FIG. 72.

equally well have been produced by pressing a closely fitting piston down the tube instead of pouring water into it. The result is the same in each case, and it is also obviously independent of the shape of the vessel. We see, therefore, that Pascal's law for the transmission of fluid pressure is an inevitable consequence of the formula for the pressure.

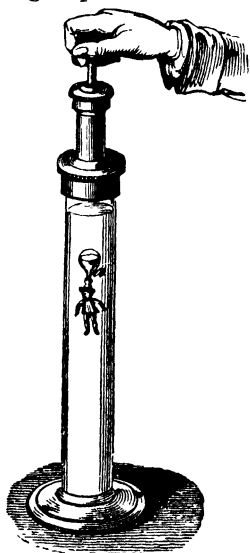


FIG. 73.

Cartesian divers.



FIG. 74.

The Cartesian Diver.—The transmission of fluid pressure is illustrated by the toy called the Cartesian diver (Fig. 73). This consists of a glass cylinder nearly full of water and closed at the top, through which works a piston. By lowering the piston the pressure of the air and consequently of the water inside the cylinder is increased. Inside the water is a little figure attached to a glass ball a which contains

water and air; in the lower part of this ball there is a small hole by which water can enter or escape. When the pressure in the water is increased, more water is forced into the ball, and it becomes heavier and sinks; when the pressure is diminished, the air expands, forcing the water out, and the ball rises. Thus by taking pressure off and putting pressure on to the piston the diver can be made to move rapidly up and down the cylinder. The apparatus can be simplified by using instead of the cap and piston an ordinary jar with a piece of india-rubber tied over its mouth (Fig. 74). The pressure of the hand on the india-rubber is then sufficient to make the divers sink to the bottom of the jar.

The submarine is simply a large Cartesian diver which is made to sink and rise by taking in and expelling water.

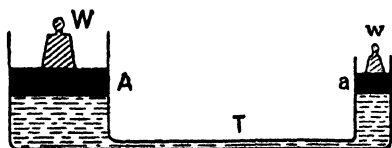


FIG. 75.—Hydrostatic paradox.

The Hydrostatic Paradox.

—Let us suppose we have two cylinders (Fig. 75), one of large cross-sectional area A and one of small cross-sectional area a , and that these are fitted with water-tight pistons and filled with water.

The pistons are in equilibrium, each resting on the top of the water.

Let us suppose now that a weight w is placed on the smaller piston; then the increase of pressure produced is given by $p=w/a$. The increase of pressure at once exerts an upward force pA on the other piston, and to prevent it rising we have to place on it a weight $W=pA$. Then the weights W and w balance one another. Now

$$\frac{W}{w} = \frac{pA}{pa} = \frac{A}{a}$$

and W is many times greater than w . This result has often appeared paradoxical. For example, it seems astonishing, that the thin column of water on the right in Fig. 76 should balance the man standing on the bellows on the left. The bellows are simply a collapsible drum filled with water, which has a wooden top and bottom.



FIG. 76.—Hydrostatic bellows.

The result is, however, quite in accordance with the principle of work. For let the small piston

be lowered through a distance d , and let the large piston be consequently forced up a distance D . Then the volume of water expelled from the small cylinder is ad , and the volume forced into the large cylinder is AD . These must be equal; therefore $ad=AD$. The work done by the weight w in falling a distance d is $wd=pad$; the work done against the weight W in being raised through D is $WD=pAD$. But $pad=pAD$. Hence the work done by the small weight is exactly equal to the work done against the large weight. The contrivance is, in fact, a "machine" similar to the lever.

Hydraulic Press.—This machine embodies the principle of the hydrostatic paradox, and was invented by Bramah in 1795. It is used for compressing paper and cotton bales, making dies,

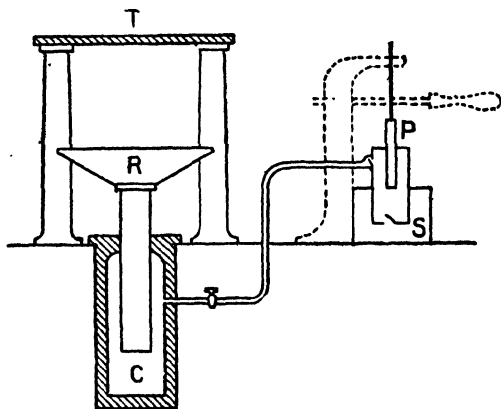


FIG. 77.—Hydraulic press.

etc. The object to be compressed is placed between the ram R and the crosspiece T (Fig. 77). The ram is connected to a plunger or solid cylinder, which works in the large metal cylinder C ; the latter has very strong sides. The pressure on the sides of the plunger has no tendency to drive it in or out, so it may be regarded as a piston with the same area as its end. The large cylinder C is connected by a metal pipe with the small cylinder, in which works the plunger P actuated by a lever handle. The small cylinder is provided with two valves, and acts as a force pump (p. 104) taking water from the reservoir S and pumping it into the large cylinder.

Let F be the force exerted in pushing down the handle, let L be the distance of its line of action from the fulcrum, and let l be the distance of the line of action of the plunger P from the fulcrum. Then the force exerted by the plunger P is LF/l .

Let a and A be the cross-sectional areas of the plungers P and R respectively. The pressure in the two cylinders is consequently $LF/(la)$ and the force exerted on the ram

$$\frac{LFA}{la}$$

The total mechanical advantage of the press is consequently $LA/(la)$, or the product of the mechanical advantage of the lever handle by the mechanical advantage due to the inequality of the areas of the plungers.

Hydraulic power is used in cities to operate lifts, cranes, and other machines. Special pipes of a diameter about 7 in. laid under the streets supply water from a central station at a pressure of 700 or 1000 lb./sq. in. to the machine in question. Hydraulic power can be stored by using it to raise rams which carry loads of 100 tons or so ; when the power is required, the potential energy of these loads is released.

EXAMPLES VIII

1. What is the pressure in kg./cm.² at a depth of 1 kilometre below the surface of the sea, the density of sea water being 1.025 gm./c.c. ? Neglect the pressure of the atmosphere.

2. A uniform U-tube is about half filled with water. How many c.c. of oil of density 0.85 gm./c.c. must be poured into one limb to make the surface of the water rise 6 cm. in the other ? The diameter of the tube is 1 cm.

3. A glass tube open at the top and containing water is gradually tilted, none of the water being spilt, until the pressure at the foot of the tube due to the water is diminished by half. What is then the inclination of the tube to the vertical ?

4. Find the total pressure in grams weight on a triangle of area 8 sq. cm. whose corners are at depths 4, 5, and 6 cm. in water.

5. A cistern is full of water. What is the ratio of the thrust on the upper half of one side to the thrust on its lower half ?

6. A canal lock gate is 14 ft. wide and 9 ft. deep. Find the magnitude and line of action of the thrust on one side of it, given that the water is level with the top and that the density of water is 62.4 lb. per cu. ft. Express your answer in tons weight.

7. The cross-sectional area of the small plunger in a Bramah press is 1 sq. inch, and that of the large plunger 100 sq. inches. The lever handle gives a mechanical advantage of 5. What is the force exerted by the ram, when a force of 1 cwt. is applied to the handle ?

8. The two plungers of a hydraulic press have diameters of an inch and a foot respectively. The downward force on the smaller plunger is 56 lb. What is the force exerted by the ram ? If the stroke of the smaller piston is $1\frac{1}{4}$ inches, through what distance has the ram moved after 10 strokes ?

CHAPTER IX

ATMOSPHERIC PRESSURE—BOYLE'S LAW

Torricelli's Experiment.—In 1643 Torricelli, a pupil of Galileo, took a glass tube closed at one end, about a yard in length and a third of an inch in diameter, filled it with mercury, stopped the open end with his finger, and inverted the tube with the open end below the surface in a vessel containing mercury. He then removed his finger, and found that the mercury descended in the tube, and after a few oscillations remained stationary with its surface at about a height of 30 inches above the surface of the mercury in the dish (Fig. 78). If the glass tube was lowered into the mercury, or inclined, the difference in level between the two surfaces remained the same (Fig. 79). Obviously the space between the closed end of the tube and the upper surface of the mercury column was a vacuum. It is referred to as the **Torricellian vacuum**. If the tube is

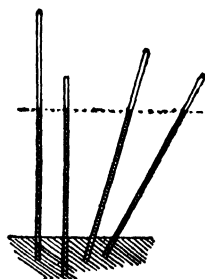


FIG. 79.

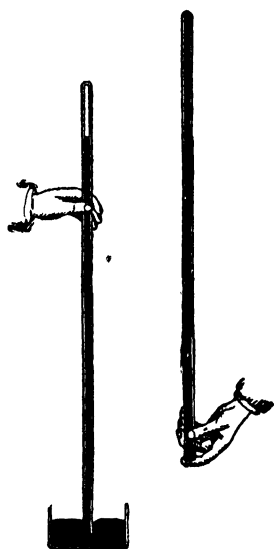


FIG. 78.—Torricelli's experiment.

lowered or inclined, so that the head of the mercury column hits the closed end of the tube, it does so with a characteristic click; there is no air between to act as a cushion. Torricelli concluded from the experiment, that the mercury column was supported in the tube by a pressure due to the atmosphere acting upon the surface of the mercury in the dish.

Pascal's Experiments.—According to Torricelli's experiment the atmospheric pressure is given by hgp , where h is the height of the mercury column

and ρ is the density of mercury, 13.59 gm./c.c. If the mercury were replaced by water, the column should be 13.59 times as high, or about 34 ft. in height. Reasoning in this way Pascal had a glass tube longer than 34 ft. filled with water and set up vertically. He found his conclusion verified.

Also, since the pressure becomes less in a liquid as we ascend in it, it occurred to Pascal that the atmospheric pressure should be less on the top of a mountain. He accordingly got his brother-in-law Perrier, who lived near Puy-de-Dôme, a mountain near the centre of France, to carry a barometer up this mountain. The latter found that the height of the column diminished 8 cm. for a rise of 1000 metres. This decisive experiment was performed in 1648.

A simple way of demonstrating the pressure of the atmosphere is to fill a glass of water to the brim, and then carefully slide a sheet of cardboard over it. If the glass is inverted, the water does not fall out, for the upward thrust of the atmosphere on the lower side of the cardboard balances the weight of the water.

The Mercury Barometer.—The pressure of the atmosphere is not constant, but varies from place to place, and at any one place varies from day to day. In all its variations it seldom departs more than five per cent. from its average value. The mercury barometer is an instrument founded on Torricelli's experiment devised to measure these variations. It is made in two forms, the siphon barometer and the Fortin barometer.



FIG. 80.—
Siphon
barometer.

The **siphon** barometer (Fig. 80) consists simply of a U-shaped tube, the longer limb of which is closed and is about 90 cm. long, and the shorter limb of which is open to the atmosphere. The atmospheric pressure is given by the difference in level of the mercury surfaces in the two limbs, and is expressed usually as a pressure of so many inches or centimetres of mercury. The positions of the mercury surfaces are given by two scales. When the mercury rises in the one limb it falls in the other, and if the diameter of both limbs of the tube is the same, the increase in pressure is equal to twice the rise of the mercury in the closed limb.

Sometimes the open limb ends in a kind of bulb, and the free surface of the mercury has consequently a much greater diameter than the diameter of the closed tube. In this case the variations of level in the open limb are very small and are neglected.

In the **Fortin** barometer the tube is straight, and dips into a

cistern containing mercury. The upper part of the cistern is of glass (Fig. 81), and through the glass the surface of the mercury can be seen. The lower part of the cistern is a leather sack, B, which carries a wooden button C. This wooden button rests on a screw, and by turning the screw the bottom of the sack can be raised or lowered, and consequently the level of the surface of the mercury in the cistern altered. A small ivory pin D is fixed to the top of the cistern. Before taking a reading the surface of the mercury in the cistern is adjusted, until it just touches the point of this pin. The pin is usually mirrored in the surface of the mercury, and when the level is right, the pin and its image just touch. The vertical scale, which is usually engraved on a metal tube surrounding the glass tube of the barometer, has its zero at the point of the pin.

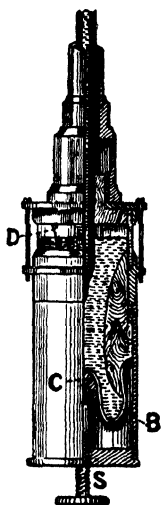


FIG. 81.—Fortin barometer.

EXAMPLE.—The tube of a siphon barometer is 7 mm. in diameter, and the open limb terminates in a bulb, so that the free surface of the mercury has a diameter of 20 mm. In taking readings of the daily variations of the atmospheric pressure the change of level of the open surface is ignored. What is the percentage of the error involved?

Let x cm. = change in level of open limb when level in closed limb changes 1 cm. Then

$$7^2 = x20^2, \text{ i.e. } x = 0.122$$

The correct change in pressure is therefore 1.122 cm. and the percentage error involved is

$$\frac{0.122}{1.122} \times 100 = 10.9 \text{ per cent.}$$

The Aneroid Barometer.—The mercury barometer, on account of its size and the risk of spilling the mercury, cannot easily be carried about from place to place. So, when a portable barometer is required, another type of instrument, called the aneroid barometer from the Greek words for “without liquid,” is used instead of it.

The aneroid barometer consists of a shallow cylindrical box, the upper surface of which is corrugated. It is exhausted of air. The atmosphere presses on the corrugated surface, and the latter rises and falls as the atmospheric pressure alters from day to day. This motion is magnified by a system of levers, and communicated to a pointer which moves round a dial. The dial is calibrated by comparison with a mercury barometer, and so the pressure can

be read on it directly in centimetres or inches of mercury. Fig. 82 shows the mechanism of an aneroid barometer. The rise and fall of the top of the vacuum chamber is communicated directly to the spring B, and thence by the levers C and D to the pointer F.

Aneroid barometers are often made as small as watches, and this type is used for determining the heights of mountains. For this purpose the dial is furnished with another scale, which reads heights directly in feet (Fig. 83). This scale rotates round the scale of pressures, and is clamped in position at the foot of the mountain with its zero opposite the pointer. Then, if the atmospheric pressure at the foot remains constant, as the mountain is

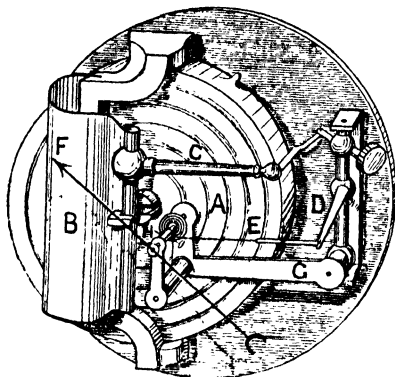


FIG. 82.—Aneroid barometer.

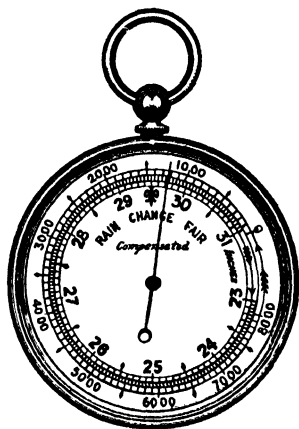


FIG. 83.—Pocket aneroid with altitude scale.

ascended the pointer always registers the height climbed. The rough rule is that for every 900 ft. the barometer falls an inch.

The **barograph** is an instrument for recording the fluctuations of the barometer automatically on a chart. It consists of an aneroid barometer and a cylindrical drum which carries the chart and is rotated by clockwork; the levers of the barometer actuate a pen the point of which presses against the drum.

Barometers for domestic use have usually the words

	Fair,	Change,	Rain,	Much Rain,
at	30·5,	29·5,	28·5,	and 27·5 inches.

These are not to be regarded as absolute predictions, but must be considered together with other factors such as direction of the wind, temperature, and the meteorological peculiarities of the place. But as changes in the wind usually start in the upper regions of the atmosphere,

and spread downwards, the barometer being affected by the weight of the whole air above it gives early warning of these changes. And moist air is lighter and generally warmer than dry air and causes the barometer to fall. In Great Britain the barometer usually falls for rain and south-west wind, and rises for dry weather and north-east wind.

The rise and fall of the barometer was discovered by Torricelli and also by his contemporary Otto von Guericke; the latter learned to foretell storms by it, and so acquired the reputation of being in league with the Evil One.

Standard Atmospheric Pressure.—By the words “standard atmospheric pressure” is understood a pressure equal to that given by a column of mercury 76 cm. high at a temperature of 0°C . on latitude 45° at the sea-level. To make the pressure definite it is, of course, necessary to state the temperature, since in the expression $h\rho g$ the density ρ varies with the temperature, and it is necessary to state the place, since the value of g varies from place to place. At the place specified the value of g is 980.6 cm./sec.^2 . As has already been mentioned, the atmospheric pressure varies about its standard value. Standard atmospheric pressure $=h\rho g=76\times 980.6\times 13.596=1,013,250$ dynes per sq. cm. or approximately one million dynes per sq. cm. On the ft.-lb.-sec. system this is equal to 14.7 lb. per sq. inch. In meteorology a pressure of one million dynes per sq. cm. is called a *bar*; hence standard atmospheric pressure is approximately equal to 1013 *millibars*.

On the English system of units standard atmospheric pressure is sometimes taken as 30 inches. This equals 76.2 cm. and differs slightly from the other definition.

The Magdeburg Hemispheres.—If the atmosphere is pressing down on us with a pressure of 14.7 lb. per sq. inch, the question arises as to why hollow bodies are not crushed by such a pressure. Why is the lid of a cardboard box, for example, not crushed in by this pressure? The answer is, because the air inside the box has the same pressure as the air outside it. But for this the box would collapse at once. In a celebrated experiment made at Magdeburg by Otto von Guericke, the inventor of the air pump, two metal hemispheres of 22 inches diameter with flanged edges were fitted together (Fig. 84), and the air pumped out. The force thrusting them together due to external atmospheric pressure was then so great, that it is said that teams of horses were required to pull them apart.

In finding the pressure on the base of the cylindrical vessel in



FIG. 84.—
Magdeburg
hemispheres.

Fig. 64 the atmospheric pressure was neglected. This is correct, because the latter acts on both sides of the base. The pressure

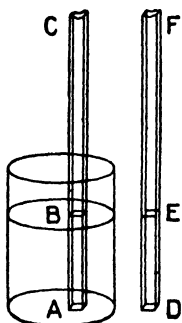


FIG. 85.

inside the vessel at A (Fig. 85) is that due to the column of liquid AB plus that due to the air column BC extending from B to the upper limit of the atmosphere. The pressure outside the vessel at A is equal to that at D, which is on the same level as A, that is, it is equal to the pressure given by the air column DF extending to the upper limit of the atmosphere. Since the weight of the air in the portion DE can be neglected, the pressures due to the two air columns are the same. Hence the excess of pressure or resultant downward pressure inside the vessel at A is that due to the column of liquid AB.

In the same way the thrust on the side of a rectangular box containing liquid and the position of the centre of pressure of the side are unaffected by the pressure of the atmosphere, for the latter is the same in both directions.

Watt's Hydrometer.—Fig. 86 represents Watt's hydrometer, which consists of an inverted U-tube, the arms of which stand in beakers containing different liquids. At the top of the bend a piece of rubber tubing is attached, by which air can be sucked out of the tube. When this is done, the liquids rise above their levels in the beakers. Let the heights reached above these levels be h in AB and h' in CD, and let the densities of the liquids in B and C be ρ and ρ' respectively. Let P be the atmospheric pressure. Then, since the pressure is the same inside the tube at A as inside the tube at D, we have

$$P - h\rho g = P - h'\rho'g$$

which gives

$$h\rho g = h'\rho'g$$

or

$$\frac{\rho}{\rho'} = \frac{h'}{h}$$

the same result as was obtained with the U-tube on p. 78, namely, that the densities of the liquids are inversely as the heights of the columns.

If a U-tube is set up in an erect position and half filled with mercury, the liquid comes to the same level in both arms of the tube. If the end of one arm has a piece of rubber tubing attached and we blow into it, the mercury sinks in this arm, the difference of level in the two arms in cm. of mercury being equal to the

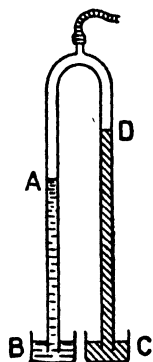


FIG. 86.—Watt's hydrometer.

excess of the pressure of the lungs over the atmospheric pressure. Also, if the rubber tubing is connected to the gas supply, the difference in level gives the excess of the pressure of the gas above atmospheric pressure.

Boyle's Law.—This law is stated as follows :

The pressure of a given quantity of gas varies inversely as its volume, provided that the temperature remains constant.

If p denotes the pressure and v the volume, the law may be stated as p varies as $1/v$, or

$$pv=c$$

where c is a constant. Let m denote the mass of the gas and ρ its density. Then $\rho=m/v$, $v=m/\rho$, and

$$p \frac{m}{\rho} = c \quad \text{or} \quad p = \frac{c}{m} \rho$$

Hence we may state the law in the alternative form :

The pressure of a gas varies as its density, provided that the temperature remains constant.

Boyle published a work proving the law in 1662. Fig. 87 represents one of the arrangements he used. A long glass U-tube was set up vertically, one of the limbs of which was much shorter than the other; the end of the shorter limb was closed and the other end was open. Mercury was poured in at the open end, so as to enclose a column of air AB. The pressure is the same at the two points B and C on the same level; hence the pressure of the enclosed air is that due to the atmosphere plus that due to the column CD. Various quantities of mercury were poured into the tube, and the corresponding positions of B and D noted. The following table gives the results of one of the experiments :

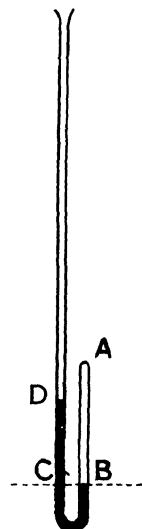


FIG. 87.—
Boyle's tube.

AB	CD	Pressure.		AB	CD	Pressure.	
		Experi- mental.	Theoretical.			Experi- mental.	Theoretical.
12	0	29 $\frac{2}{16}$	29 $\frac{2}{16}$	5 $\frac{1}{2}$	32 $\frac{1}{16}$	61 $\frac{5}{16}$	60 $\frac{1}{8}$
11 $\frac{1}{2}$	1 $\frac{7}{16}$	30 $\frac{9}{16}$	30 $\frac{9}{16}$	5 $\frac{1}{2}$	34 $\frac{1}{16}$	64 $\frac{1}{16}$	63 $\frac{1}{8}$
11	2 $\frac{3}{16}$	31 $\frac{1}{16}$	31 $\frac{1}{16}$	5 $\frac{1}{2}$	37 $\frac{1}{8}$	67 $\frac{1}{8}$	66 $\frac{1}{4}$
10 $\frac{1}{2}$	4 $\frac{1}{16}$	33 $\frac{1}{16}$	33 $\frac{1}{16}$	5	41 $\frac{9}{16}$	70 $\frac{1}{16}$	70
10	6 $\frac{3}{16}$	35 $\frac{1}{16}$	35	4 $\frac{1}{2}$	45	74 $\frac{1}{8}$	73 $\frac{1}{4}$
9 $\frac{1}{2}$	7 $\frac{1}{8}$	37 $\frac{1}{8}$	36 $\frac{1}{8}$	4 $\frac{1}{2}$	48 $\frac{1}{8}$	77 $\frac{1}{8}$	77 $\frac{1}{8}$
9	10 $\frac{2}{16}$	39 $\frac{1}{8}$	38 $\frac{1}{8}$	4 $\frac{1}{2}$	53 $\frac{1}{16}$	82 $\frac{1}{16}$	82 $\frac{1}{8}$
8 $\frac{1}{2}$	12 $\frac{5}{16}$	41 $\frac{5}{16}$	41 $\frac{5}{16}$	4	58 $\frac{5}{16}$	87 $\frac{5}{16}$	87 $\frac{1}{4}$
8	15 $\frac{1}{8}$	44 $\frac{1}{8}$	43 $\frac{1}{8}$	3 $\frac{1}{2}$	63 $\frac{1}{8}$	93 $\frac{1}{8}$	93 $\frac{1}{8}$
7 $\frac{1}{2}$	17 $\frac{1}{8}$	47 $\frac{1}{8}$	46 $\frac{1}{8}$	3 $\frac{1}{2}$	71 $\frac{1}{8}$	100 $\frac{1}{8}$	99 $\frac{1}{8}$
7	21 $\frac{3}{16}$	50 $\frac{3}{16}$	50	3 $\frac{1}{2}$	78 $\frac{1}{16}$	107 $\frac{1}{16}$	107 $\frac{1}{16}$
6 $\frac{1}{2}$	25 $\frac{1}{16}$	54 $\frac{1}{16}$	53 $\frac{1}{8}$	3	88 $\frac{7}{16}$	117 $\frac{9}{16}$	116 $\frac{3}{8}$
6	29 $\frac{1}{16}$	58 $\frac{1}{16}$	58 $\frac{1}{8}$				

The first column gives AB, the length of the air column, measured in inches. This was proportional to the volume of the air. The second column gives CD, and the third was obtained by adding $29\frac{2}{10}$, the height of the barometer in inches, to the second; it gave the pressure of the air in inches of mercury. The fourth column is what the pressure should have been, if the law were strictly true. It will be observed from the maximum value of CD in the table, that the tube must have been about eight feet long. The agreement is not good according to modern standards, but the results are, of course, of historic interest.



FIG. 88.—
Boyle's law
apparatus.

Fig. 88 represents a form of apparatus much used in schools for verifying Boyle's Law. The two glass tubes can be fixed in different positions on opposite sides of a vertical wooden scale. Their lower ends are connected by a piece of rubber tubing. The verification is carried out in the

same manner as with the U-tube already described, but this apparatus has the advantage over the U-tube, that the volume is altered by altering the relative position of the two tubes. It is not necessary to pour in more mercury. Also it can be used for pressures below as well as above atmospheric without changing the quantity of mercury in the apparatus.

Fig. 89 represents an arrangement known as Mariotte's apparatus, which is used for verifying Boyle's Law for pressures less than atmospheric. It consists of a tall glass cylinder containing mercury into which dips a straight glass tube BC closed at its upper end and about 36 inches long. The tube BC is first filled about two-thirds full of mercury; then the end is closed with the finger and it is inverted with the end below the surface of the mercury at B. The pressure of the enclosed air is

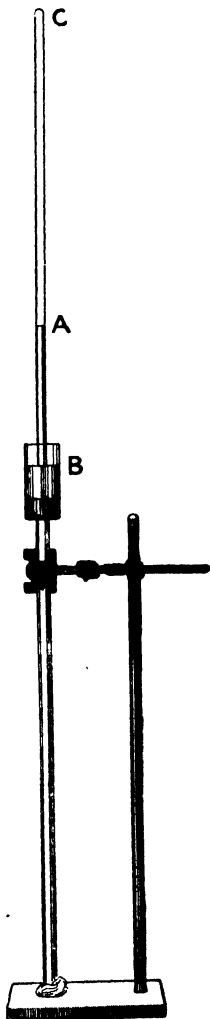


FIG. 89.—Mariotte's
apparatus.

then less than atmospheric by the height of the column AB. The tube is lowered into the cylinder, and corresponding lengths of AC and AB measured; the length AC is found to vary inversely as the pressure.

Boyle's Law is called Mariotte's Law on the Continent, because Mariotte announced the discovery of the law fourteen years after Boyle, without, it is said, being aware of Boyle's work; it is, however, not certain that his discovery was an independent one.

Accuracy of the Law.—Since Boyle's time the law has been tested for different gases over a wide range of pressures. The most extensive range was that employed by Amagat, who worked up to 3000 atmospheres. To produce the pressure he used a steel tube which extended up the shaft of a coal mine to a height of 330 metres. A pump was employed to force mercury in at the lower end of the tube, until it began to run out at one of a series of stopcocks which were inserted at measured intervals up the tube.

The result of these investigations has been to show that the law is not absolutely true. Gases are not compressible enough at very high pressures, and different gases behave differently as regards their deviations from the law. The permanent gases obey it best. As an example of the magnitude of the deviations from it the case of hydrogen may be cited; when the pressure of this gas is increased from 1 to 1000 atmospheres, the product pv instead of remaining constant increases continuously to one and three-quarter times its initial value.

"Short Mercury Column."—This is the name given to a simple experiment which is often performed as an illustration of Boyle's Law.

A thin tube of soft glass of about $2\frac{1}{2}$ mm. internal diameter is taken, and a thread of mercury drawn into it. One end of the tube is then sealed off in the bunsen flame. The tube is next held erect in the two positions shown in Fig. 90, first with the closed end A up, then with it down. The tube is narrow enough to prevent the column from breaking, but wide enough to permit of the easy motion of the column as a whole up and down it. Let P be the atmospheric pressure. When the closed end is up, the pressure on the enclosed air is equal to P less the pressure due to a column of mercury BC high, and the enclosed air consequently expands. When the closed end is down, the pressure on the enclosed air is equal to P increased by the pressure due to a column of mercury EF high, and the enclosed air contracts. Its volume is proportional to the length of the air column. Hence, applying Boyle's Law,

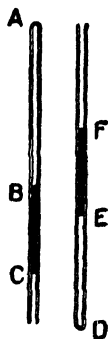


FIG. 90.

$$(P - BC)AB = (P + EF)ED$$

which gives

$$P = \frac{BC \times AB + EF \times ED}{AB - ED}$$

$$= \frac{BC(AB + ED)}{AB - ED}$$

since $EF = BC$. If the lengths AB , BC , and ED are measured, P can be calculated to about $\frac{1}{2}$ cm. of mercury.

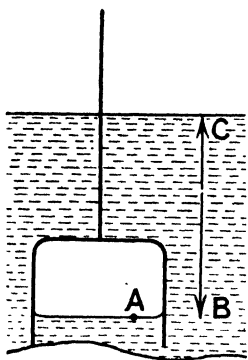


FIG. 91.—Diving bell.

Diving Bell.—The diving bell is a heavy iron vessel open at the bottom, which is lowered into water and used for laying the foundations of piers and similar work. In former practice it took down with it only the air which it contained when above the surface, and when it reached the bottom, the water had risen inside it, as shown in Fig. 91. The pressure is the same at the two points A and B on the same level, and the pressure of the air inside is greater than atmospheric pressure by that due to the column BC . Hence, if H is the height of the water barometer, it follows from Boyle's Law, that

$$\frac{\text{volume of air}}{\text{volume of bell}} = \frac{H}{H + BC}$$

and if BC is 17 ft., the water will have risen one-third of the way up the bell.

Nowadays air is pumped into the bell from the surface through a tube. Besides giving the workmen fresh air this arrangement forces down the level of the water to the bottom of the bell; then the air escapes in a stream of bubbles to the surface.

Height of the "Homogeneous Atmosphere."—Since the pressure at a point in the atmosphere is due to the weight of the superincumbent air, it decreases as we ascend. On the top of Mont Blanc (15,781 ft.) it is only 38 cm. According to Boyle's Law the density is proportional to the pressure, if the temperature is constant. Hence, if we neglect the difference of temperature, the density of the air at the top of Mont Blanc is only half as great as the density of the air at the foot. Thus the density diminishes as we ascend, and there is no definite limit to the atmosphere corresponding to the surface of a sheet of water. A balloon carrying instruments only has gone up as high as 19 miles, and there is sufficient air at a height of 45 miles to reflect the rays of the sun at sunset.

Let us, however, assume that the atmosphere has the same density the whole way up and then ceases abruptly, and apply the same formula for the pressure at the foot as holds for a column of water, namely $p = h\rho g$. If we substitute in this formula, $p = 76$ cm. and $\rho = 0.001293$ gm./c.c., its value for air at 0° C. and a pressure of 76 cm., we obtain

$$h = \frac{p}{\rho g} = \frac{76 \times 13.6 \times 981.0}{981.0 \times 0.001293} \text{ cm.}$$

$$= \text{approximately 8 km. or 5 miles.}$$

This result is known as the height of the homogeneous atmosphere. It is the height of a hypothetical atmosphere of constant density, which would give the same pressure and density on the earth's surface, as the real atmosphere does. The tops of the Himalayas would project above it.

EXAMPLE.—The tube of a barometer has a cross-sectional area of 1 cm.², and when the column stands at 74 cm., 2 c.c. of air at atmospheric pressure are passed up the tube. How far is the surface of the mercury depressed, if initially it was 10 cm. below the closed end of the tube? The tube stands in a dish containing mercury the level of which is not affected appreciably when the air enters.

Let x cm. = depression of surface. Then x represents the pressure of the enclosed air. Its volume is $10 + x$. Hence, applying Boyle's Law,

$$2 \times 74 = (10 + x)x$$

$$x^2 + 10x - 148 = 0$$

which gives $x = 8.15$ cm., the other root being inadmissible.

Force exerted by a Jet.—Hitherto we have considered only the pressure due to stationary columns of liquid or gas. We shall now consider the pressure due to a moving column.

Let us suppose that a horizontal jet of liquid of density ρ impinges against a metal plate with velocity v (Fig. 92), that the

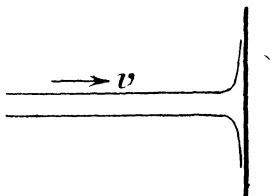


FIG. 92.

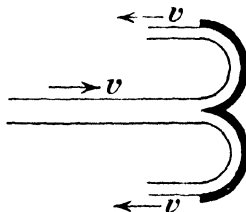


FIG. 93.

cross-sectional area of the jet is a , and that after impact the jet flows off in a direction parallel to the surface of the plate. Then in one second a column of liquid of length v reaches the plate. Consequently the mass arriving per second is ρav . Its momentum is ρav^2 . This momentum is destroyed by the plate. Thus the rate of destruction of momentum per second is ρav^2 , and this is the force exerted by the jet on the plate.

If the surface of the plate is curved, as in Fig. 93, so that the liquid slides round it and leaves with a velocity v in the direction opposite to that in which it arrived, in addition to the momentum destroyed momentum is created in the reverse direction at the rate ρav^2 per second. Hence the force on the plate is in this case $2\rho av^2$. In the Pelton wheel, which has replaced the old-fashioned

water-wheel for so many purposes, the blades consist of cup-shaped pockets which give the water a velocity in the direction opposite to that in which it arrives.

Let us suppose now that instead of a jet of water we have a horizontal stream of air directed against a plane the surface of

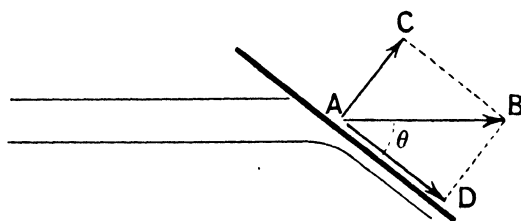


FIG. 94.

which is inclined at an angle θ to the horizontal (Fig. 94). In this case the air after impact flows along the surface of the plane as shown in the figure, so that if AB represents the momentum

incident per second, it is only the component AC which is destroyed. AC consequently represents the force exerted on the plane in magnitude and direction. Its magnitude is $\rho av^2 \sin \theta$. It has a horizontal component $\rho av^2 \sin^2 \theta$ called the drift and a vertical component $\rho av^2 \sin \theta \cos \theta$ called the lift. The drift tends to push the plate backwards in a horizontal direction, the lift to raise it vertically.

Fig. 94 gives the action of the wind on a kite. The force exerted by the string, not shown in the figure, has two components, a horizontal and a vertical one. The horizontal one neutralises the drift; the vertical one and the weight of the kite together neutralise the lift.

Fig. 95 is a plan of a lug-sail boat sailing across the wind, the direction of the latter being represented by the line WA.

We shall suppose that after reaching the sail, it slips along the front of it towards the stern; thus if AB represents the rate at which momentum is arriving, it is the component CB which represents

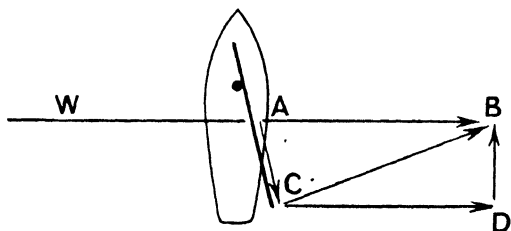


FIG. 95.—Action of wind on a sail.

the rate at which it is destroyed and consequently the force on the sail. Resolve this force into components CD and DB perpendicular and parallel to the keel of the boat. Then the component DB makes the boat go, while the component CD causes leeway.

If in Fig. 94 we superimpose on the diagram a velocity v

towards the left, we get the case of the **aeroplane**. The air in front of the plane then receives momentum at rates represented by AD and BA, *i.e.* at the resultant rate BD. Consequently DB represents the force exerted on the plane by the air. This resolves as before into the lift and the drift, the lift balancing the weight and the drift being neutralised by the thrust of the propellers. As the lift is represented approximately by $\rho av^2 \sin \theta \cos \theta$ or $\frac{1}{2} \rho av^2 \sin 2\theta$, it follows that the greater v , *i.e.* the faster the plane goes, the smaller the angle which its surface must make with the horizontal.

Torricelli's Theorem.—If a hole be made in the side of a tank containing liquid, as shown in Fig. 96, the velocity of efflux of the jet is given by

$$v^2 = 2gh$$

where h is the depth of the hole below the surface. The result is known as Torricelli's theorem.

It may be proved very simply from the principle of energy. For suppose that the level of the surface is maintained by allowing liquid to flow in at the top; the liquid consequently goes from A to B. Consider a mass m . At A it is moving slowly and its kinetic energy can be neglected. At B it is $\frac{1}{2}mv^2$. Now the loss of potential energy in going from A to B is mgh . Hence equating potential energy lost to kinetic energy gained,

$$mgh = \frac{1}{2}mv^2$$

whence

$$v^2 = 2gh$$

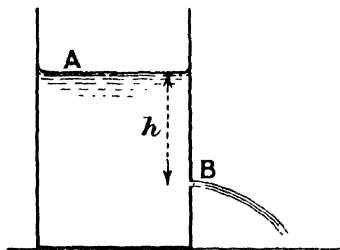


FIG. 96.

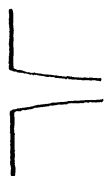


FIG. 97.—
Vena contracta.

The jet when it issues is not cylindrical in form. The stream lines converge inside the vessel, and this convergence continues until a point is reached where the cross-sectional area of the jet is a minimum (Fig. 97). This part of the jet is called the *vena contracta*. At the vena contracta the jet is approximately cylindrical. The area of the vena contracta depends on the nature of the hole and on whether it is fitted with a mouthpiece or not; in the case of a simple circular hole in a thin wall the area of the vena contracta is found by experiment to be about 0.62 times the area of the hole. It is at the vena contracta that the velocity is given by Torricelli's theorem.

EXAMPLES IX

1. The height of the mercury barometer is 752 mm., and the density of mercury is 13.6 gm./c.c. What is the height of an oil barometer in which oil of density 0.845 gm./c.c. is used?

2. A vertical tube closed at the top is full of water and its lower end stands in water, the surface of the water being 10 ft. below the closed end of the tube. What is the pressure of the water against the top of the tube, given that the water barometer stands at 33 ft. ?

3. A man takes a pocket aneroid up a mountain. At the foot it registers 29.0 inches, at the top 26.8 inches, but on descending to the foot again he finds it gives 28.7 inches. He takes 3 hours to go up and $1\frac{1}{2}$ to come down. What is the height of the mountain ? —

4. A "sucker" consisting of a disc of leather 3 inches in diameter with a string attached to its middle is moistened and stuck on the pavement. With how many lbs. wt. does the atmosphere press down on it ?

5. A U-tube is half filled with mercury and a piece of rubber tubing attached to one end of it. A man blows into the tubing, and creates a difference of level in the two limbs of the tube of 4 cm. What pressure is he exerting in dynes/cm.² ?

6. Rewrite the first and second columns of the table on p. 91 in decimals, and graph the reciprocal of the first against the second. Hence determine the atmospheric pressure.

7. The volume of a quantity of gas is 159 c.c. when the barometer stands at 72 cm. What is it at standard pressure ? At what pressure would it have a volume of 200 c.c. ?

8. A piston is situated in the middle of a cylinder 12 inches long, and there are equal quantities of air on each side of it. The piston is pushed down until it is an inch from one end. What is then the ratio of the pressures on the two sides ?

9. A tube 150 cm. long, closed at one end, is half filled with mercury and is then inverted with its open end just dipping into a mercury trough. No air escapes from the tube, and the barometer stands at 75 cm. What is the height of the mercury inside the tube ?

10. A barometer has a little air in the space at the top of the mercury column, and as a result reads 28 inches when it ought to read 29 inches, and reads 29 inches when it ought to read 30.2 inches. What is the correct value of the atmospheric pressure when it reads 29.4 inches ?

11. A column of mercury 8 cm. long is introduced into a thin tube, and one of the ends of the tube sealed off in a bunsen flame enclosing a quantity of air between this end and the mercury. After the tube has cooled to the temperature of the room, it is suspended with its closed end up, and the length of the air column is found to be 20.0 cm. ; when it is suspended with the closed end down, the length of the air column is 16.1 cm. Find the atmospheric pressure.

12. If in the previous question the atmospheric pressure had been 73 cm., the length of the mercury column 12 cm., and the length of the air column 15 cm. when the closed end was down, what would it have been when the closed end was up ?

13. A cylindrical diving bell 8 ft. in height is lowered until the top of the bell is 12 ft. below the surface. The height of the mercury barometer is 30 inches. How high will the water rise inside the bell ?

14. A hole is made in the side of a tank containing water 40 cm. below the surface. Calculate the velocity of efflux.

CHAPTER X

PUMPS

Air Pump.—The air pump is an instrument used for increasing or decreasing the pressure of gas in a vessel. It was invented by

Otto von Guericke in 1650. It is represented in its simplest form in Fig. 98. R is the receiver or glass vessel to be exhausted. It consists of a bell-jar with a carefully ground edge, which is smeared with lard, and rests on a flat metal plate, making an air-tight contact. T is the tube connecting the receiver to the pump; S is a stopcock. The pump itself consists of a cylinder

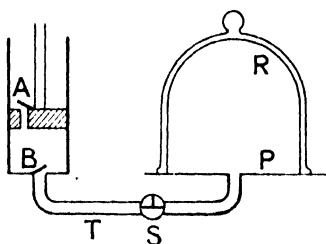


FIG. 98.—Simple air pump.

in which a piston fits air-tight. There is a hole in the piston, closed by the valve A, which opens upward. The end of the tube from the receiver is closed by a valve B, which also opens upward. When the piston descends, the valve A opens, the valve B closes, and all the air in the cylinder is forced out into the atmosphere. On the upstroke the valve A closes, the valve B opens, and some air is sucked out of the receiver through the tube into the barrel of the pump. On the next downstroke

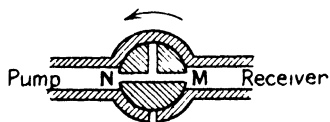


FIG. 99.

this air is forced out into the atmosphere. And so on; every double stroke of the piston removes some more air, until finally the pressure of the air left in the receiver is not sufficient to lift the valve B.

The stopcock S is shown on a large scale in Fig. 99. As shown in the diagram it is set for exhausting the receiver. When the vacuum is as good as can be obtained, the stopcock is turned through 90° in the direction of the arrow. This closes the receiver and prevents air leaking into it through the pump; at the same time it puts the barrel of the pump into communication with the atmosphere. If the cock is turned

to operate. It can reduce the pressure in the course of time to the vapour pressure of mercury, about 0.0007 mm. of mercury at room temperature.

“Hyvac” Pump.—This is one of the best modern pumps, and it is used in the manufacture of wireless valves, X-ray tubes, and incandescent lamps, as well as for experimental purposes. A is a rotor mounted eccentrically on a shaft which passes through the centre of the cylindrical casing (Fig. 102). C is a vane which moves in a vertical

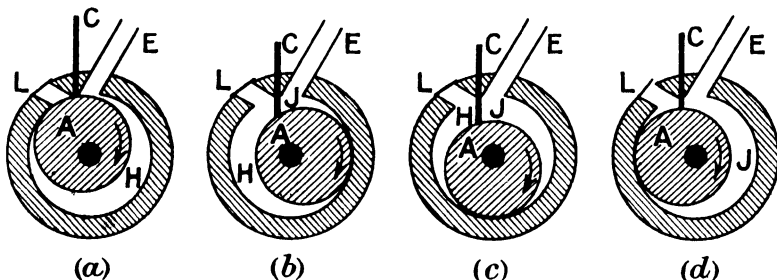


FIG. 102.—Principle of Hyvac pump.

direction, and is kept pressed against the surface of the rotor by a spring which is not shown in the diagram. The vessel to be exhausted is connected to the inlet tube E. L is the exhaust valve, shown diagrammatically.

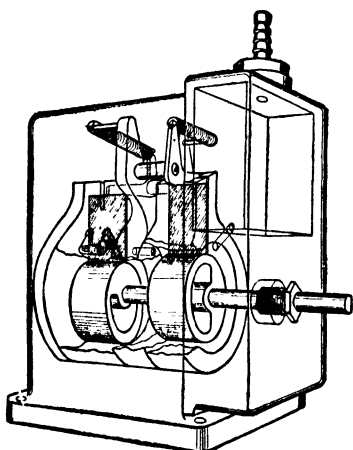


FIG. 103.—Two-stage Hyvac pump.

When the rotor is in position *a*, the whole space H is in communication with the vessel to be exhausted. When position *b* is reached, space H has diminished in volume, and space J has opened up behind it, and is in communication with the vessel to be exhausted. H is further contracted in position *c*, and in position *d* the air in H is compressed so much, that it lifts the valve and is pushed out. The same cycle is then gone through with the air in J.

Usually two cylinders are connected in series, the first raising the pressure to an intermediate value and the second to atmospheric pressure. Fig. 103 shows in outline the arrangement of such a two-stage pump. The two rotors are on the same shaft. They are immersed in a cast-iron box filled with oil to prevent air leakage into the high vacuum, and are rotated by an electric motor. Such a pump produces an X-ray vacuum from atmospheric pressure in a few minutes.

Filter Pump.—Fig. 104 represents another type of pump, the filter pump or water pump. It is usually made of glass. The tube A is connected by a piece of rubber tubing to a water pipe, and the side tube B by a piece of rubber tubing to the vessel to be exhausted. The water issues from the glass nozzle C at a very high speed, and escapes by the tube D carrying out some of the air with it. Consequently air is sucked in through the opening B. The filter pump is never employed when a good vacuum is required, but it is convenient and economical, working without any attention at all.

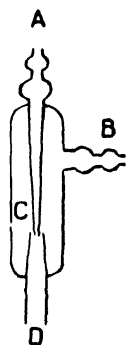


FIG. 104.—
Filter pump.

Compression Pumps.—So far we have described only exhaustion pumps. The mechanical pump represented in Fig. 98 acts as a compression pump, if the valves A and B are arranged so as to work the other way, so that A closes on the downstroke and B closes on the upstroke.

The commonest example of a compression pump is the bicycle pump. In this pump the piston consists of a cup-shaped piece of leather clamped to a piece of metal of slightly smaller diameter than the barrel. When the piston forces the air into the tyre, the leather is pressed up against the side of the barrel, and so the air does not pass it. When the piston is drawn back, the leather leaves the side of the barrel, allowing the air to pass. The arrangement is called a cup valve. There is no other valve in the pump, the other valve necessary to the working of the arrangement being on the tyre. The pump is not an efficient one; in addition to leakage in the valve and the connecting tube there is the loss due to "untraversed space," that is, due to the piston not sweeping out the air in the connecting tube.



FIG. 105.

Fig. 105 shows a section of the valve of a bicycle tyre. The air entering at A forces up the rubber tubing at B and passes into the tyre at C. But the air in the tyre presses the rubber tubing against the opening at B, and so cannot get out. In some pumps a piece of oiled silk is stretched across an opening, and is raised by the air in much the same way as the rubber is raised in the bicycle valve. Sometimes a valve consists of a small steel ball resting in a conical cup with a hole in its bottom.

Common Suction Pump or Lift Pump.—Fig. 106 represents the lift pump which has been used from ancient times for raising water out of wells, etc. A represents the level of the water in the well. From A a vertical pipe AB leads to the barrel of the pump.

At B there is a valve opening upwards ; at C there is a valve in the piston also opening upwards.

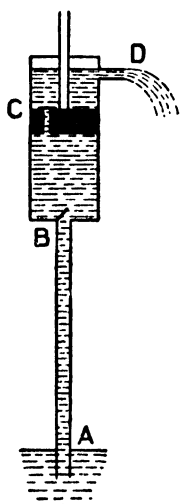


FIG. 106.—Lift pump.

Initially the water stands at the same level at A both inside and outside the pipe. During the first strokes, therefore, the pump acts as an air pump, removing the air from the pipe AB and sucking up the water, until it reaches the barrel of the pump. Then on the downstroke the valve B closes, and C opens ; consequently the water passes from below to above the piston. On the upstroke the valve B opens and the barrel fills again with water, while the valve C closes, and the water already above the piston is lifted up and escapes by the outlet at D.

Since the pump acts at first by creating a vacuum in the barrel and so letting the atmospheric pressure force the water up the pipe AB, the pump should not work if AB is greater than 34 ft., the height of the water barometer. Owing to mechanical imperfections the limit is less than this, namely about 28 ft. Pouring in a little water at the top makes the valve C and the contact between the piston and the side of the barrel more air-tight, and assists greatly in starting the pump.

Force Pump.—Fig. 107 depicts the force pump. It differs from the suction pump in having no valve in the piston, in having the outlet at the foot of the barrel, and also in having a valve D opening outwards at the outlet.

Initially the pump works as an air pump, until the level of the water reaches the barrel. Consequently it is subject to the same limitation as the suction pump, *i.e.* AB must be less than the water barometer. Then, when the water reaches the barrel, on the downstroke the valve B closes and D opens, and the piston forces the water up the tube DF ; on the upstroke D closes, B opens, and the barrel fills with water.

The force pump is used when the water has to be delivered at a point higher than the position of the pump itself, as, for example, in the case of the fire engine. The latter consists of a double-barrelled force pump, the downstroke in the one barrel corresponding to the upstroke in the other. It is capable of forcing a jet of water to the top of a high building.

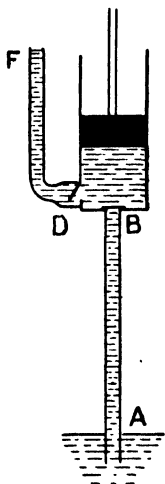


FIG. 107.—Force pump.

EXAMPLE.—The radius of the barrel of a lift pump is 3 inches. The piston is 15 ft. above the surface of the water in the well, and the height of the column of water above the piston is 2 ft. Find in lb. weight the force necessary to sustain the piston in its position.

Let x be the height of the water barometer. The pressure on the under side of the piston is equal to a head of $x-15$ ft. of water. The pressure on the upper side is equal to a head of $x+2$ ft. of water. The difference of pressure is $(x+2)-(x-15)=17$ ft. of water. The force is $17 \times 62.4 \times \pi(\frac{1}{2})^2 = 208.3$ lb.

Siphon.—The siphon is an arrangement for draining off the upper layers of liquid in a vessel without disturbing the lower layers, or for emptying a vessel that cannot be overturned. It consists of a bent tube, one arm of which is longer than the other.

Let us suppose that it is to be used for removing water from the vessel in Fig. 108. It is first filled with water, the ends being closed with the fingers, and placed in the position shown in the figure. The ends are then opened, and a steady stream of water flows through the tube from A to D.

In order to understand the action of the siphon consider two points B and C in the tube at the top of the bend. Let h be the height of B above the level of the surface at A, let h' be the height of C above the level of the surface at D, let H be the height of the water barometer, and suppose the liquid in the tube for an instant at rest. Then the pressure at B calculated from the end A is $H-h$ cm. of water, while the pressure at C calculated from the end D is $H-h'$ in cm. of water.

If we consider the element of liquid between B and C, the pressure on the end B exceeds the pressure on the end C by $(H-h)-(H-h')$ or $h'-h$. Hence the liquid will flow from B to C and down to D, while the atmospheric pressure forces more liquid into the tube at A. Thus a steady stream is maintained.

The siphon works as long as h' is greater than h . Also it is not necessary for the end D to dip below a surface of water; the water may issue from D as a jet. In that case h' is measured from the end of the tube. The siphon would not work if h were greater than H , for in that case a vacuum would form at the top.

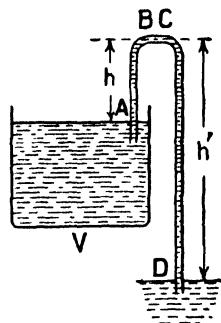


FIG. 108.—Siphon.

M'Leod Gauge.—The M'Leod gauge is a pressure gauge used for measuring low pressures, such as exist in an electric light bulb. It consists of a glass bulb (Fig. 109) to the top of which is joined a narrow bore tube B sealed at the end. The tube CD leads to the vessel the pressure in which is to be measured. At the lower end of the tube CE is attached a rubber tube which communicates with the mercury

reservoir F. The volume V of the bulb and narrow tube from C to the closed end is known from a previous calibration, as are also the volumes of the narrow tube between the graduations on the scale.

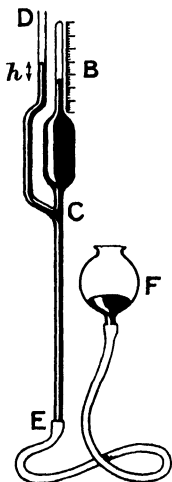


FIG. 109.—
M'Leod gauge.

Let us suppose that the reservoir is raised slowly, until CB is just sealed off. The product of pressure and volume for the gas in CB is then pV , where p is the pressure to be measured. The reservoir is then raised further until the mercury occupies some such position as that shown in the Fig. The volume v occupied by the enclosed gas is now very much smaller. Let h be the difference in level of the mercury in the two tubes. The pressure of the gas in the tube D is practically unaltered by the change, since the additional volume occupied by the mercury in that tube is small compared with the volume of the vessel sealed on at its end. Hence the pressure of the gas imprisoned in the narrow tube is now $p+h$, and the product of its pressure and volume is $(p+h)v$. Consequently by Boyle's law

$$pV = (p+h)v$$

whence

$$p = \frac{hv}{V-v}$$

EXAMPLES X

1. To what fraction of its original value has the pressure fallen in a theoretically perfect air pump after 10 strokes, if the volume of the receiver is four times that of the barrel?

2. After five complete strokes the density of the air in the receiver of an air pump is $\frac{2}{3}$ of its original value. What is the ratio of the volume of the receiver to the volume of the barrel?

3. The receiver of an air pump has a volume of 1400 c.c., and it contains air of density 0.001293 gm./c.c. at normal pressure. The pump is worked until the density of the air is reduced 30 per cent. Find the mass of the air removed.

4. Air is forced into a vessel by a compression pump, the barrel of which has $\frac{1}{10}$ the volume of the vessel. Calculate the density of the air in the vessel after 15 complete strokes.

5. The tube of a lift pump is 6 yards high. The cross-sectional area of the barrel is 12 times that of the tube, and the length of stroke of the piston 18 inches. To what height will the water be raised in the tube by the first upstroke? The air inside the pump is originally at a pressure of 30 inches of mercury, and the piston is originally at the foot of the barrel.

6. Would a siphon flow in a vacuum?

7. In a M'Leod pressure gauge $V=50$ c.c.; in measuring the pressure in an exhausted vessel h was 6 mm. and v was 0.3 c.c. Find the pressure.

CHAPTER XI

PRINCIPLE OF ARCHIMEDES—SPECIFIC GRAVITY

The Principle of Archimedes.—Let us suppose that a liquid is at rest and that a certain portion of it, bounded by the closed surface *P* indicated in Fig. 110, suddenly becomes rigid without change of density, *i.e.* suddenly becomes a solid. Then this solid is still in equilibrium with the rest of the liquid. Now it is acted on by (i) its own weight vertically downwards; (ii) all the pressure-forces exerted by the rest of the fluid at right angles to its surface and indicated by the arrows in the diagram. Since the solid is in equilibrium, these pressure-forces must have a resultant equal and opposite to the weight.

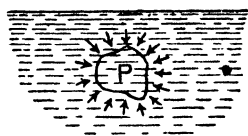


FIG. 110.

Let us suppose now that the solidified liquid is removed, and that in its place we have a solid body of exactly the same shape. Then the pressure-forces on its surface are the same as on the surface of the solidified liquid. Consequently the liquid exerts an upward force on it equal to the weight of the liquid it displaces. If it is heavier than the liquid it displaces, it will sink, and if it is lighter than the liquid it displaces it will float.

Let us suppose now that the portion of liquid which becomes solidified is bounded partly by the surface of the liquid (Fig. 111).

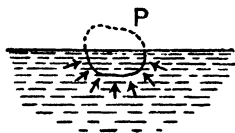


FIG. 111.

The resultant of all the pressure-forces exerted by the rest of the liquid on the solidified part is still equal and opposite to its weight. If we suppose the solidified part replaced by a solid body, which has exactly the same shape below the surface of the liquid as the solidified part has, then the upward force exerted by the

liquid on the solid is equal to the weight of the solidified part, *i.e.* to the weight of the liquid it displaces.

We arrive therefore at the following result :

If any body is wholly or partially immersed in a fluid, it is acted on by an upward force equal to the weight of the fluid it displaces.

This result is known as the principle of Archimedes. It is a consequence of the laws of fluid pressure.

It is said that Hiero, tyrant of Syracuse, ordered a gold crown, but suspected that the goldsmith had used silver as well as gold in its composition. He asked Archimedes to discover if this were the case without, of course, destroying the crown. Noticing the loss of weight of his own body in his bath, when pondering on the subject, the thought suddenly occurred to Archimedes that all bodies would lose weight in this way. Hence the name of the principle. Having grasped the principle he had no difficulty in understanding the nature of specific gravity and in determining its value for the crown.

Experimental Illustration.—The principle is illustrated by a well-known class experiment in which two cylinders A and B are used (Fig. 112); B is of solid brass, and A is hollow and of such a size that B can just fit inside it. B is attached by a hook to the bottom of A, and A is suspended from one of the arms of a balance. Weights are then placed in the other pan until the balance is in equilibrium. A vessel containing water is then raised from below, so as to immerse B completely, and the other side of the balance goes down at once. When water is poured into A so as to fill it to the top, equilibrium is once more restored.

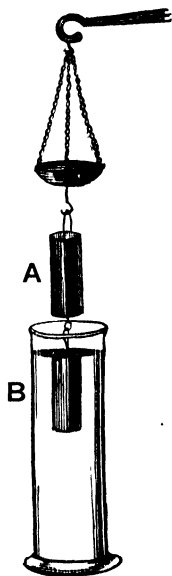


FIG. 112.—Apparatus for proving the principle of Archimedes.

Thus the loss of weight experienced by B when immersed in water is equal to the weight of water contained by A, *i.e.* to the weight of the water displaced by B.

The principle of Archimedes applies to gases as well as to liquids. Thus all bodies immersed in the atmosphere experience an upthrust equal to the weight of air they displace. As the density of air is small, being only 1.293 gm. per litre when dry, at atmospheric pressure and at 0° C., this upthrust can usually be neglected in comparison with the weight of the body. The weight of large volumes of air is, however, by no means negligible. Thus a room 14 ft. long, 9 ft. broad, and 10 ft. high contains under standard conditions about 102 lb. of air. It is the upthrust due to the weight of the displaced air that causes balloons and airships to float.

EXAMPLE.—An electric light bulb has a weight of 50.02 gm. The tip is broken off with a pair of pliers, and the weight of the lamp and fragment is found then to be 51.15 gm. The lamp is next filled with

water and its weight found to be 244.4 gm. Calculate the density of air at the temperature of the experiment.

The volume of the air contained is $244.4 - 51.15 = 193.3$ c.c.

Its weight $= 51.15 - 50.92 = 0.23$ gm.

Hence its density $= 0.00119$ gm./c.c.

Buoyancy Correction.—In very accurate weighings it is necessary to take account of the weight of the air displaced by the body. The weighing is then said to be reduced to vacuo. Let W be the true weight of the body, *i.e.* its weight in vacuo, and let w be the true weight of the counterpoising weights. Let D be the density of the body, d the density of the weights, and σ the density of the air. Then the volume of the body is W/D , the volume of the weights is w/d , the weight of air displaced by the body is $\sigma W/D$, and the weight of air displaced by the weights is $\sigma w/d$. For equilibrium

$$W - \frac{\sigma W}{D} = w - \frac{\sigma w}{d}$$

$$W = \frac{w \left(1 - \frac{\sigma}{d} \right)}{1 - \frac{\sigma}{D}}$$

$$= w + w \sigma \left(\frac{1}{D} - \frac{1}{d} \right)$$

since σ is small compared with D and d .

If, for example, a wooden cube of side 3 cm. is weighed in air and found to be 20.50 gm., and the weights are of brass of density 8.4, then $w = 20.50$ gm., $d = 8.4$ gm. per c.c., $D = 20.5/27$ nearly, *i.e.* 0.759 gm. per c.c., and $\sigma = 0.001293$ gm. per c.c. Thus

$$w \sigma \left(\frac{1}{D} - \frac{1}{d} \right) = 20.50 \times 0.001293 \left(\frac{1}{0.759} - \frac{1}{8.4} \right)$$

$$= 0.0265(1.32 - 0.12)$$

$$= 0.0265 \times 1.20$$

$$= 0.032 \text{ gm.}$$

and the true weight of the body is 0.032 gm. greater than the uncorrected result. The density of the air at the time of weighing would probably differ somewhat from the value taken above, but the difference would not affect the result appreciably.

Specific Gravity.—The density of a substance has already been defined as the ratio of its mass to its volume. Thus if M is the mass and V is the volume, the density ρ is given by

$$\rho = \frac{M}{V}$$

On the c.g.s. system it is measured in gm./c.c., on the ft.-lb.-sec. system in lb./cub. ft., and its numerical value is different on the two systems.

If, however, we compare the masses of equal volumes of two substances, the ratio is obviously independent of the system of

units used. The quantity of mercury which fills a bottle is 13.6 times as heavy as the quantity of water which fills it, no matter what units are used, the ratio depending only on the nature of the two substances. It is known as the **specific gravity** or **relative density** of mercury.

The specific gravity or relative density of a substance is the ratio of the mass of any volume of it to the mass of an equal volume of water.

Let V be the volume, M and M_0 the masses, and ρ and ρ_0 the densities of the substance and water respectively. Then

$$\begin{aligned} \text{specific gravity} &= \frac{M}{M_0} = \frac{\frac{M}{V}}{\frac{M_0}{V}} = \frac{\rho}{\rho_0} \\ &= \frac{\text{density of substance}}{\text{density of water}} \end{aligned}$$

Since the density of water is unity on the c.g.s. system, the specific gravity of a substance is numerically equal to its density on the c.g.s. system. But the one quantity is a ratio with zero dimensions and the other has the dimensions mass over volume. On the ft.-lb.-sec. system the density of water is 62.43 lb./cub. ft. Hence applying the equation to the case of mercury

$$13.6 = \frac{\text{density of mercury}}{62.43}$$

and the density of mercury on the ft.-lb.-sec. system is $13.6 \times 62.43 = 849.0$ lb./cub. ft. It is sometimes useful to remember that 62.43 lb. = 998.9, almost 1000, ounces.

In stating the specific gravity of gases it is usual to take hydrogen or air as the standard substance.

Hydrostatic Balance.—The most obvious way of determining the specific gravity of a solid is by means of the hydrostatic balance. This is an ordinary balance, over one of the pans of which is placed a small wooden bridge or stool carrying a beaker filled with water. The stool and beaker are arranged so that the balance swings without touching either of them.

If the solid sinks in water, the beaker is removed, and the solid is suspended by a thread from the hook that carries the scale pan, so that it hangs above the stool, and its weight in air, a , is determined. The beaker is then inserted so that the solid is suspended in the water, clearing the side of the beaker, and the weight of the solid in water, w , determined. The weight of the water displaced is consequently $a - w$, and the specific gravity of the solid is

$$\frac{a}{a - w}$$

If the solid floats in water, it is first weighed in air. Let the result be a . It is then tied to a piece of metal sufficient to sink it, and they are weighed together in water. Let the result be W . The sinker is next weighed alone in water. Let the result be s ; then s is greater than W . It is not necessary to know the weight of the sinker in air. The solid alone has an apparent negative weight in water equal to $W-s$. Thus the weight of the water it displaces is $a+s-W$, and its specific gravity is

$$\frac{a}{a+s-W}$$

If the solid dissolves in water, it can be weighed in another liquid of known specific gravity, and from its loss of weight in this liquid, its loss of weight in water can be calculated.

The hydrostatic balance can also be used to determine the specific gravity of a liquid. In order to do this we find the weight of a solid first in air (a), then in water (w), and finally in the liquid (l). The weight of water displaced is $a-w$, and the weight of the liquid displaced $a-l$. The specific gravity of the liquid is thus

$$\frac{a-l}{a-w}$$

The solid employed should not have a high density, otherwise a , w , and l do not differ much, and consequently the differences cannot be obtained with such accuracy. The methods of the hydrostatic balance can, of course, be used with the spring balance. It is faster, but not nearly so accurate as the ordinary arrangement.

Specific Gravity Bottle, or Pyknometer.—The specific gravity bottle, which is shown in Fig. 113, consists of a small bottle with a carefully ground-in stopper which has a fine hole passing through it. It is used for determining the specific gravity of a liquid, or of a solid which is in the form of small pieces or a powder, and so cannot be used with the hydrostatic balance.

In determining the specific gravity of a liquid the bottle is first dried and weighed empty, then filled with the liquid, and the excess which escapes through the hole in the stopper wiped off; it is then weighed again. The difference of the two weighings gives l , the weight of the liquid contained. The bottle is then emptied, dried, filled with water and weighed. The difference of this weighing and the first one gives w , the weight of water contained. The specific gravity of the liquid is then simply l/w .



FIG. 113.—
Specific gravity
bottle.

In determining the specific gravity of a powder the bottle is first weighed empty, then full of water, and the difference gives w , the weight of the water contained. It is next emptied, dried, filled about three-quarters full of the powder and weighed again. The difference of this weighing and the first one gives a , the weight of the solid in air. The bottle is then filled up with water, care being taken to get rid of the air bubbles clinging to the powder, and weighed again. The difference of this weighing and the second one gives e , the excess of the weight of the powder over the weight of the same volume of water. Hence $a - e$ gives the weight of the water displaced by the solid, and consequently

$$\frac{a}{a - e}$$

the specific gravity of the solid.

Variable Immersion Hydrometer.—The variable immersion hydrometer or common hydrometer is used in commerce for measuring the specific gravity of alcohol, milk, acids, and sugar and other solutions. One is shown in Fig. 114. It consists of a glass tube with a hollow bulb B in the middle and is loaded at the foot A with mercury or shot, so as to float erect. The stem is graduated. The greater the specific gravity of the liquid, the higher the hydrometer floats out of the liquid. The specific gravity is given by the reading on the stem at the point where the surface cuts it.

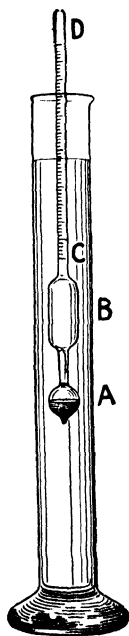


FIG. 114.—Hydrometer.

Instruments of this type are graduated by immersion in liquids of known specific gravity. To obtain great accuracy the stem would have to be inconveniently long. Hence they are usually supplied in sets; for example, a set might consist of seven instruments, one for each of the ranges 0.700–0.850, 0.850–1.000, 1.000–1.200, 1.200–1.400, 1.400–1.600, 1.600–1.800, 1.800–2.000. Also, since specific gravity is determined usually for some special purpose, *e.g.* for finding the percentage of alcohol in spirit, to save calculation the hydrometer may be graduated in percentage of alcohol, and not in specific gravity.

Let V be the volume of the hydrometer below C, let a be the cross-sectional area of the stem, and let h and h' be the lengths of stem immersed when the instrument is floated in liquids of density ρ and ρ' respectively. Then by the principle of Archimedes the upthrusts are equal in the two cases, and

$$\rho(V + ha) = \rho'(V + h'a)$$

This gives
$$\frac{1}{\rho'} - \frac{1}{\rho} = \frac{1}{V} \left(\frac{h'}{\rho} - \frac{h}{\rho} \right)$$

or
$$\frac{1}{\rho'} - \frac{1}{\rho} = \frac{a(h' - h)}{\rho(V + ha)}$$

on substituting for ρ' . Hence the difference between the reciprocals of the densities is proportional to the difference between the lengths of stem immersed.

Nicholson's Hydrometer. — This instrument is not very accurate and is used mostly for instructional purposes. It consists of a hollow metal cylinder B (Fig. 115) with conical ends, terminated at its upper end by a thin rod carrying a scale pan A and carrying at its lower end another pan C loaded with lead. Hence it floats vertically. On the upper rod there is a mark, and when in use the scale pans are always loaded, so that this mark is in the surface.

Let h be the weight of the hydrometer. Then, in using it to determine the specific gravity of a liquid, it is floated in the liquid and weights l are added to the upper pan to sink it to the mark. It is then floated in water and weights w added to the upper pan to sink it to the mark. The weight of the liquid displaced is $h + l$, and the weight of water displaced $h + w$. The volume is the same in each case. Hence the specific gravity of the liquid is

$$\frac{h + l}{h + w}$$



FIG. 115.—
Nicholson's
hydrometer.

In order to determine the specific gravity of a solid, the hydrometer is floated in water and weights w added to the upper pan to sink it to the mark. The weights are then removed, the solid placed in the upper pan, and weights a added to sink the instrument to the mark again. The difference $w - a$ is obviously the weight of the solid in air. The upper pan is again emptied, and the solid next placed in the lower. If it floats, it is tied down. Weights b are then placed in the upper pan to sink the instrument to the mark again. The difference $w - b$ consequently gives the weight of the body in water, $(w - a) - (w - b)$, *i.e.* $b - a$ the weight of the liquid displaced by the solid, and

$$\frac{w - a}{b - a}$$

its specific gravity.

The various formulæ for the determination of specific gravity should never be memorised, but, when a numerical result is wanted, it should be worked out from first principles.

Stability of Ships.—Let Fig. 116 represent a section of a ship floating on an even keel. Then it is acted on by an upthrust due to the weight of the water displaced. Let the magnitude of this be W . It follows from the reasoning of p. 107, that this upthrust acts through H , the centre of gravity of the water displaced or **centre of buoyancy**, as it is called. When the ship is on an even keel, its weight acting downwards through its centre of gravity G balances the upthrust of the displaced water acting through H .

Let us suppose now that the ship is inclined, as shown in Fig. 117. The upthrust of the water has still the same value W , but

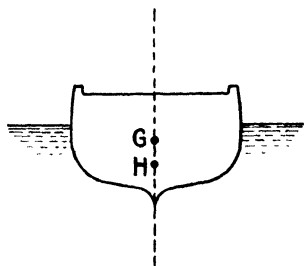


FIG. 116.

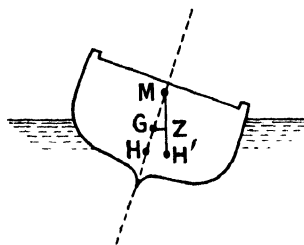


FIG. 117.

the centre of buoyancy has shifted to the point H' , so that the line of action of the upthrust intersects the original vertical in M . The point M is called the **metacentre** of the ship. The weight of the ship still acts downwards through G , so we have two parallel forces W , one downwards through G and the other upwards through M . These constitute a couple. Draw GZ perpendicular to $H'M$. Let θ be the inclination of the ship to the vertical. Then the moment of the couple is

$$W \times GZ = W \cdot MG \sin \theta = W \cdot MG \theta$$

if θ is small. The distance MG is called the **metacentric height**, and the couple in this case tends to diminish θ and is a restoring one.

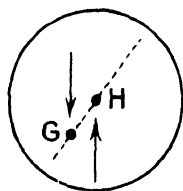


FIG. 118.

If, however, the vertical $H'M$ intersects HG on the other side of G , the couple tends to increase θ , and is a capsizing one. The position of the metacentre is consequently of great importance for the stability of the ship. If it is above the centre of gravity, equilibrium is stable; if it is below the centre of gravity, equilibrium is unstable. The object of putting ballast in a ship is to keep the centre of gravity below the metacentre.

In the case of a submarine wholly immersed (Fig. 118) the centre of buoyancy does not shift its position when the ship is inclined, and obviously coincides with the metacentre. For stable equilibrium it must be above the centre of gravity.

EXAMPLES XI

1. A wooden sphere of diameter 4 cm. is weighed in air and the result found to be 22.57 gm. The weights are of brass of density 8.4 gm./c.c. Find the weight of the sphere in vacuo.

2. According to the newspapers the Zeppelin L33 was 680 ft. long, her envelope was filled with 2,000,000 cub. ft. of gas, and she weighed, together with her equipment and crew, 50 tons. What is the lifting capacity in tons weight of this volume of (i) hydrogen, (ii) coal gas? The density of hydrogen is 0.0899 gm./litre and the density of coal gas 0.52 gm./litre.

3. Calculate in kg. wt. the upward force on a spherical balloon of oiled silk which is filled with impure hydrogen of density 0.100 gm./litre. The total weight of the silk envelope is 55.3 kg., and it weighs 200 gm./metre.²

4. A piece of metal weighs 31.57 gm. in air and 28.8 gm. in water. What is its specific gravity?

5. The weight of a solid in air is 29.05 gm., in water 18.72 gm., and in a salt solution 18.01 gm. What is the specific gravity of the salt solution?

6. A piece of wax weighs 18.03 gm. in air. A piece of metal is found to weigh 17.03 gm. in water. It is tied to the wax, and both together weigh 15.23 gm. in water. What is the specific gravity of the wax?

7. When 48.84 gm. are placed in the upper pan of a Nicholson's hydrometer, it sinks to the mark. When a piece of metal and 34.40 gm. are placed in the upper pan, it sinks to the mark. When the metal is placed in the lower pan, and 36.01 gm. are placed in the upper pan, it sinks again to the mark. Find the specific gravity of the metal.

8. A piece of wood is placed in the upper pan of the same hydrometer, and 36.29 gm. have to be placed in the upper pan to sink the instrument to the mark. When the wood is tied to the lower pan, 55.5 gm. must be placed in the upper pan to sink the instrument to the mark. What is the specific gravity of the wood?

9. A Nicholson's hydrometer weighs 226.60 gm. To sink it to the mark in water a load of 67.36 gm. is necessary in the upper pan; to sink it to the mark in a salt solution a load of 83.72 gm. is necessary. Find the specific gravity of this solution.

10. A specific-gravity bottle weighs 21.57 gm. empty, 69.90 gm. when full of water, and 74.17 gm. when full of a certain solution. What is the specific gravity of this solution?

11. A specific-gravity bottle weighed 21.57 gm. empty. Some lead shot was introduced and the bottle weighed again. The weight was found to be 226.35 gm. The bottle was then filled up with water and weighed again. The result was this time found to be 256.64 gm. The weight of the bottle when full of water was 69.90 gm. Find the specific gravity of the shot.

12. The specific gravity of ice is 0.918 and of sea water 1.03. An iceberg floats with 1000 cubic yards of its volume above the surface. What is its total volume?

13. A ship displaces 10,000 tons of water and its metacentric height is 2 ft. It is heeled over through 5°. Calculate the restoring couple in foot-ton units.

CHAPTER XII

PROPERTIES OF MATTER

Hooke's Law.—Let us suppose that a copper wire, AB, 5 or 6 metres long, is suspended from a support on the wall, and that a scale pan is attached to its lower end (Fig. 119). Then if a weight is placed in the pan, the wire increases in length, the increase in length being proportional to the weight. If the weight is removed, the wire returns to its original length. The weight per unit area of the cross-section is in this case referred to as the stress, and the increase in length per unit length as the strain.

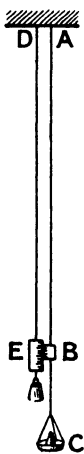


FIG. 119.—
Apparatus for
determining
Young's
modulus.

If the room becomes warmer, this rise in temperature also causes an increase in length, and if the scale E is fixed to the wall, part of the increase observed may be due to the rise of temperature. It is customary, therefore, in making the measurement to hang the scale on a wire ED of the same length and same material as AB, which carries a weight to keep it tight. Then if a rise of temperature takes place, both vernier B and scale E suffer the same displacement, and any relative motion must be due to the weights in the pan C.

Again, if one end of a stick is clamped down on a table and a force applied to the other end (Fig. 120), the latter is displaced downwards through a distance proportional to the magnitude of the force.

If the force is removed, the stick returns to its original shape. In this case the force is called the stress, and the displacement is called the strain.

Again, if a cube of india-rubber is subjected to a uniform pressure on all its faces, it suffers a decrease in volume proportional to the pressure applied, and returns

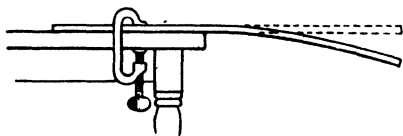


FIG. 120.

to its original volume when the

pressure is removed. The experiment might be carried out by immersing the cube in water and subjecting the water to pressure. In this case the pressure applied is called the stress, and the change in volume per unit volume is called the strain.

It is the same in other cases when forces produce a displacement of the various particles of a body relatively to one another.

The force producing the relative displacement of the different parts of the body is called a stress, and the change produced is called a strain.

And we always have the same law connecting stress and strain, which was first given by Hooke in 1676, and runs as follows :

The strain is proportional to the stress producing it.

Hooke published the law as an anagram : *ceiinossttuu*. Two years later he gave the key to the anagram, *ut tensio, sic vis*, and described various ways of proving the law.

Elasticity.—In physics a meaning is attached to the word “elasticity” quite different from its use in ordinary life.

Elasticity is that property of matter in virtue of which it resists change of shape or change of volume with a force depending on the amount of change produced.

There are thus two kinds of elasticity, elasticity of shape and elasticity of volume. These are synonymous in physics with rigidity and incompressibility respectively, and the degree in which a body possesses them is measured by two coefficients, the rigidity modulus and the bulk modulus of elasticity. In general, when a body is strained, they are both called into play, but it is possible for the one kind of elasticity to be involved without the other, and we shall proceed to consider these two cases.

Shearing Strain.—Let us suppose that the unit cube ABCD is acted on by equal forces P on four of its sides, as shown in Fig. 121. Then its shape is changed in the manner indicated. Every plane in it parallel to the base is displaced relatively to the plane below it, but the height and consequently the volume is unaltered. An idea of the

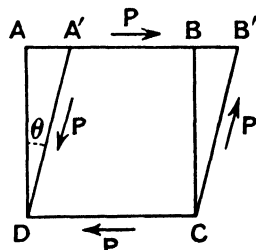


FIG. 121.—Shearing strain.

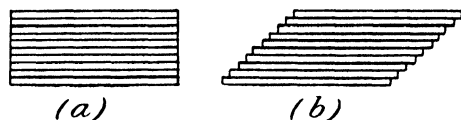


FIG. 122.—To illustrate shearing strain.

nature of the change may be obtained from Fig. 122, which represents a pack of cards first piled in a rectangular block and then displaced

with each card in the pack the same distance to the right of the one below it. But once the pack of cards is displaced, it remains displaced, whereas the cube in Fig. 121 springs back to its original shape when the forces P are removed.

The strain represented in Fig. 121 is a pure change of shape. It is called a shearing strain or simply a shear, and it is measured by the angle θ . The **rigidity modulus**, which measures the resistance of the body to change of shape, and is usually denoted by n , is defined by the equation

$$n = \frac{\text{shearing stress}}{\text{shearing strain}} = \frac{P}{\theta}$$

If the body is very rigid, the value of θ produced is small, and consequently n is large: the greater the rigidity, the greater the rigidity modulus.

Pure Change of Volume.—Let us suppose that a cube of volume V is subjected to a uniform pressure p per unit area on all its six faces. Then all its sides contract to the same extent and there is consequently no change of shape. Let v be the decrease in volume. The **bulk modulus** of elasticity of the substance, which is usually denoted by k , is defined by the equation

$$k = \frac{\text{pressure}}{\text{change of volume per unit volume}} = \frac{p}{v/V} = \frac{pV}{v}$$

The greater the resistance to compression, the smaller v , and consequently the greater k : the greater the incompressibility, the greater the bulk modulus.

EXAMPLE.—Water diminishes $\frac{1}{21,000}$ of its volume, when the pressure on it increases one atmosphere. Find its density at the bottom of a freshwater lake 200 metres deep.

One atmosphere corresponds to 76×13.6 cm. of water = 10.33 metres. Hence the pressure at the bottom is $\frac{200}{10.33}$ atmospheres, and the diminution in volume per unit volume

$$\frac{200}{10.33} \times \frac{1}{21,000} = 0.000922$$

The density is thus $\frac{1}{1 - 0.000922}$ or 1.00092 gm./c.c.

Young's Modulus.—The elastic change easiest to investigate is the increase in length produced in a wire by attaching a weight to its end, and an experiment on this subject forms part of most physics laboratory courses. The apparatus used is that represented in Fig. 119. Before starting the experiment sufficient weight is placed in the pan to take slack out of the wire and make it hang straight without kinks. Then the weight is increased by equal steps to a maximum and decreased by steps of the same magnitude to the initial value, and the scale reading at E taken

for each weight. The readings for the way down should be the same as for the way up, and when they are graphed against the weight, the result should be a straight line.

By experimenting with wires of the same material but of different lengths and different cross-sections, we can show that the increase in length l is

- (1) proportional to the force applied, F ,
- (2) proportional to the total length of the wire, L ,
- (3) and inversely proportional to the cross-sectional area of the wire, A . Stating this result in the form of an equation

$$l = \frac{FL}{YA}$$

where Y is a constant depending on the material of the wire. This can be rewritten

$$Y = \frac{\frac{F}{A}}{\frac{l}{L}} = \frac{\text{force per unit area}}{\text{increase in length per unit length}}$$

The quantity on the right is the stress divided by the strain, and the constant Y is known as Young's modulus of elasticity for the material. It measures the resistance of the wire to stretching; the greater Young's modulus for the material, the smaller is the increase in length produced by a given force.

The increase in length is accompanied by a decrease in the radius, and there is a change in the volume of the wire as well as a change in its shape. Thus both kinds of elasticity are called into play to resist the change, and it can be shown by reasoning beyond the scope of this book, that Young's modulus is connected with the rigidity modulus and the bulk modulus of the material by the equation

$$Y = \frac{9kn}{3k+n}$$

So far we have regarded the stress as the applied force causing the strain, in this case the force per unit area applied at the lower end of the wire. It is more properly the equal and opposite force called into play to resist the strain, which acts throughout the whole volume of the wire.

If the weights in the pan are increased beyond a certain limit, after their removal the wire no longer returns to its original length. It is then said to be overstrained, and the stress beyond which recovery is no longer complete is called its **elastic limit**. If the stress is increased beyond the elastic limit, the **yield point** is reached and the wire breaks. The quotient of the

maximum weight which the wire can carry by its cross-sectional area is called the **breaking stress** of the specimen.

EXAMPLE.—Young's modulus for copper is 1.2×10^{12} dynes/cm.². Calculate the increase in length of a copper wire 5 metres long and 1 mm. in diameter, when stretched by a weight of 5 kg.

$$l = \frac{FL}{YA} = \frac{5 \times 10^3 \times 981 \times 500}{1.2 \times 10^{12} \times \pi (0.05)^2} \text{ cm.}$$

$$= 2.6 \text{ mm.}$$

Viscosity.—If a jug of water is emptied upon the floor, it flows away at once, but if a jug of treacle were emptied upon the floor, it would take a long time to spread out. Liquids which behave like treacle are said to be viscous, and liquids which flow away rapidly are said to be mobile. Viscosity is a property of fluids in virtue of which they resist change of shape, but the resistance in this case is quite independent of the amount of change effected, and when the deforming force is removed, the fluid does not show any tendency to return to its original shape.

Fig. 123 represents liquid flowing along a channel. If we divide it into horizontal layers, we find that these layers are moving with different velocities. The layer in contact with the

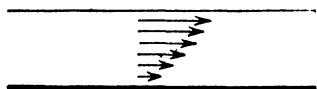


FIG. 123.

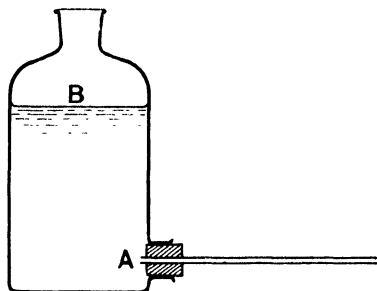


FIG. 124.—Apparatus for measuring viscosity of a liquid.

bottom is at rest, the layer immediately above it is moving slowly, the third layer from the foot is moving faster, and so on until we come to the topmost layer moving with velocity v . The velocities of the different layers are represented by arrows in the diagram, which is drawn for a very viscous liquid. In the case of water the change from zero to the value v is completed within a very short distance from the bottom. Thereafter the velocity has the uniform value v the whole way up.

Each of the layers rubs against the layer above it, keeping it back. Thus viscosity is due to friction between the particles of the liquid. Viscosity is shown by gases as well as liquids.

Fig. 124 shows the arrangement usually adopted for measuring the viscosity of a liquid. A capillary glass tube is stuck through a cork

and fixed horizontally in a hole in the side of a vessel filled with the liquid. The volume of liquid which flows through the tube in unit time is then measured. Let it be V : let l be the length of the tube and r its radius, and let p be the difference of pressure between the two ends, *i.e.* due to the head AB , measured in dynes/cm.². Then it may be shown by reasoning beyond the scope of this book, that

$$V = \frac{\pi p r^4}{8 l \eta}$$

where η is a constant depending on the liquid, which is known as its coefficient of viscosity. *viscosity*

Distinction between a Solid and a Liquid.—It is now possible to go into the difference between the solid and the liquid state with greater precision. A solid has been defined on p. 74 as a body with a definite shape and a liquid as having a definite volume but no definite shape. While this definition is satisfactory with regard to ordinary solids and liquids, there are certain substances which are difficult to place.

One of these is pitch. A piece of hard pitch breaks with a glassy fracture and feels solid to the touch, but if given sufficient time—months or even years—it flows, and takes up the shape of the vessel in which it is placed. A very striking experiment can be performed with it. If pieces of wood are placed on the bottom of a shallow glass dish, then a slab of pitch placed on the top of the wood, and finally pieces of lead on the top of the pitch, it is found that in the course of time the wood rises into the pitch and the lead sinks into it, and after months the lead rests on the glass at the bottom while the wood has come to the surface. Pitch is therefore a very viscous fluid: its apparent solidity is due to viscosity and not to elasticity of shape.

Thus the distinction between the solid and the liquid state is that a solid has both elasticity of shape and volume, whereas a liquid has only elasticity of volume.

Molecules and Atoms.—There was much discussion in ancient times as to whether or not matter is infinitely divisible. Suppose, for example, a lump of copper is taken and divided into parts, and each part divided into parts, can the process be repeated an infinite number of times without the parts losing the properties which the substance possesses in bulk, or shall we come finally to a stage beyond which it is impossible to go? This question has been decided by modern investigation. We know now that matter is composed of very small particles called molecules, much too small to be seen with the most powerful microscope, and if we were to divide far enough, we would get down to the molecules. Spaces exist between the molecules and they are in constant motion. In gases they are relatively far apart and travelling in all directions with high velocities.

In a cubic centimetre of air under standard conditions there are about 27×10^{18} of them. They strike and rebound off on another, and the pressure which a gas exerts on the vessel containing it is due to the bombardment of the side of the vessel by the molecules. In solids and liquids the molecules are much closer together, and thus the greater density of solids and liquids is explained. In solids the molecules oscillate about a mean position, while in liquids they move through the body of the liquid. In solids and liquids the molecules knock against one another, but a force of attraction, molecular attraction, operates between them, and prevents them separating as far from one another as in the case of gases.

The evidence for the existence of molecules is partly chemical and partly physical. In order to explain chemical combination it is necessary to assume that substances are composed of molecules, and that molecules are made of atoms, the molecule being the smallest part of a substance that can exist in a free state and the atom being the smallest part of a substance that can take part in a chemical change. Thus the molecule of common salt is made up of one atom of sodium and one atom of chlorine. The physical evidence for the existence of molecules is taken principally from the behaviour of gases; we do not demonstrate their existence by any one experiment, but if we assume they exist and work out the consequences, we obtain a natural explanation of all the phenomena connected with gases.

The fact that there must be spaces inside a liquid can be shown by a very simple experiment. When 50 c.c. of water and 50 c.c. of alcohol are mixed together, the resultant mixture has a volume of only 97 c.c.

Molecular Forces.—As has been mentioned, molecules attract one another. This attraction is quite distinct from gravitational attraction; it is more powerful than the latter at small distances, and exists in addition to it. The law according to which it acts is not known, but molecular attraction is only “sensible at insensible distances”; we are accustomed to think of it as constant throughout a space surrounding the molecule, known as its sphere of action, and to ignore it elsewhere.

The attraction which binds like molecules together is called **cohesion**, and that which binds molecules of different kinds together **adhesion**. The forces holding a drop of water to a glass rod or nickel plating to iron are examples of adhesion. Many of the properties in which solid bodies differ from one another—their elasticity, tenacity, ductility, malleability, hardness, brittleness, etc., are due to differences in the cohesion between the molecules.

The **tenacity** of a solid is measured by the weight necessary to break it, when in the form of a wire. The relative **ductility** of two solids is tested by seeing which can be drawn into the thinner wire, the **malleability** by seeing which can be hammered into the thinner sheet, the **hardness** by seeing which will scratch the other, and the **brittleness** by seeing which will break most readily under a blow from a hammer.

Surface Tension.—Let MN be a water surface, and let A be a molecule situated some distance below the surface. With A as centre describe a sphere with the range of molecular attraction as radius. We can neglect the action on A of the molecules outside this sphere. The molecules inside the sphere are distributed equally all round it, and there are

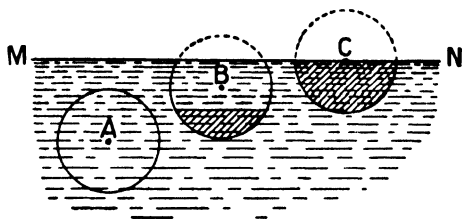


FIG. 125.

as many pulling one way as another. Consequently the resultant molecular attraction on A is zero.

Consider now a molecule situated near the surface at B, and with B as centre and the range of molecular attraction as radius describe a sphere. The density of the air molecules is so small that their attraction can be neglected; hence the force due to the shaded part of the sphere is unbalanced, and there is a resultant downward force on B. Consider now a molecule situated on the surface at C, and describe a sphere with the same radius round C as centre. The downward attraction of the whole lower hemisphere is unbalanced, and hence there is in this case a greater resultant downward force on C.

There are similar forces on other molecules on or near the surface. Hence any body of liquid which is free to take its natural shape, must contract until it has the smallest surface compatible with its volume, for the forces on the surface molecules tend to pull them into the body of the liquid. The sphere is the solid which has the smallest area for a given volume. Thus if a body of liquid is freed from the action of gravity, it should take the spherical form.

Owing to the tendency of the surface to contract every body of liquid behaves as if it were surrounded by an elastic skin in which there is a definite stretching force. This stretching force is known as the surface tension of the liquid. Its presence is very obvious in blowing a soap-bubble, for the process is exactly the same as in blowing up a rubber balloon.

It is owing to the action of gravity, that liquids do not more frequently take the spherical form. If by means of a pipette a quantity of oil is introduced below the surface of a mixture of alcohol and water, which has been arranged to have the same density as the oil, it takes the form of a sphere. The oil should be coloured to make the sphere easily visible, and the experiment should be carried out in a vessel with flat sides, otherwise the sphere will appear distorted. In this case the effect of gravity is eliminated.

If the body of liquid is small, the action of gravity may be neglected in comparison with the surface forces. Hence rain-drops are spherical, as are also the small drops which form when mercury is spilled on a table.

Capillarity.—It was mentioned on p. 74 that a liquid stands in a number of communicating vessels at the same level.



FIG. 126.—Rise of water in capillary tubes.

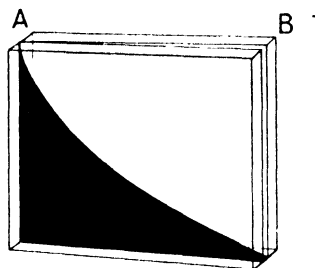
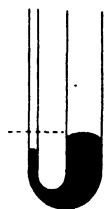


FIG. 127.—Rise of water between plates.

This is, however, not the case when capillary tubes, or tubes of small diameter, are used. For if a series of such tubes with gradually diminishing diameter are arranged with their ends dipping in water, as shown in Fig. 126, the level of the water is higher



in the tubes than outside, and the narrower the bore of the tube the greater the height to which the water rises. Also if two glass plates are arranged dipping into water, pressed tight together at the end A and with a separation of a millimetre or so at the end B, as shown in Fig. 127, the water rises between the plates, the rise being greatest where the separation is least. In these two experiments the water is usually coloured black to render the effect visible. If mercury is used instead of water, the surface is depressed instead of raised, but to make the depression visible of the proper magnitude must employ a U-tube of the shape shown in their elasticity, Fig. 128. Also when a liquid rises the surface inside the tube is concave upwards, and when it is depressed, the surface is convex upwards.

To the phenomenon just described the name capillarity has been given. It is caused by surface tension and it plays a large part in everyday life. The rise of oil in a lamp wick, the absorption of water by a lump of sugar or of ink by blotting paper, the wetting of a whole towel by allowing one corner to dip into water, are all examples of the process illustrated by Figs. 126 and 127.

By experimenting on capillary tubes we find the following results :

(1) *Liquids rise in the tubes, if they wet them, but are depressed in tubes they do not wet.*

(2) *The elevation or depression is inversely proportional to the diameter of the tube.*

Measurement of Surface Tension.—The elevation or depression of the surface in a capillary tube is easily explained by means of the surface film, and gives us the simplest method of measuring the numerical value of the tension existing in that film.

If water is contained in a glass vessel, the surface of the water does not meet the side at right angles, but curves up and meets it tangentially as in Fig. 129. This is because the glass attracts the molecules at C more than the water does, or, in other words, because the adhesion of water for glass is greater than the cohesion of water for water. But if the vessel contains mercury instead of water, the surface bends down and meets the glass at an angle of about 52° , as in Fig. 130, because the molecules of mercury on the surface at C are attracted more by the mercury than by the glass. This angle is known as the angle of contact between glass and mercury. The fact that the molecular attraction of glass for water is greater than that of water for water, and that the molecular attraction of glass for mercury is less

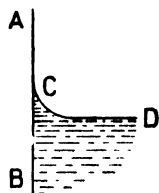


FIG. 129.

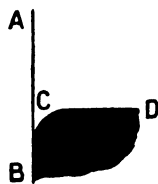


FIG. 130.

than that of mercury for mercury, can be shown by dipping glass rods into these two liquids and then withdrawing them; for the rod withdrawn from the water is wet, and the rod withdrawn from the mercury is dry.

Fig. 131 represents a section of a glass tube with a narrow bore dipping into water. Owing to the surface rising to meet the glass it is concave upwards inside the tube and the surface tension pulls the water up the tube, so that the level is higher inside than outside. Fig. 132 represents a section of a similar tube dipping

into mercury ; here the surface inside the tube is convex upwards, and surface tension depresses the level of the mercury inside the tube.

Let r be the radius of the tube, h the elevation of the liquid in the tube, and T its surface tension, and let us suppose that, as in the case of water, the angle of contact is zero. Then the cylindrical column of height h inside the tube is in equilibrium under the action of its weight and the surface tension in the film at the top.

The weight is equal to $\pi r^2 h g \rho$. The surface tension acts vertically upwards at the top round the inside of the tube, where

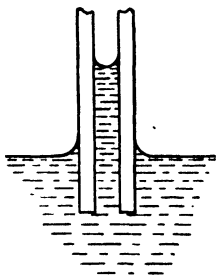


FIG. 131.

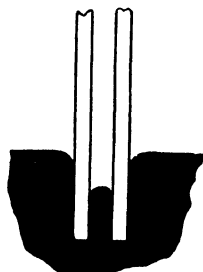


FIG. 132.

the film joins on to the glass, a force T being applied to each cm. of the circumference. It gives a resultant force $2\pi rT$. Equating,

$$2\pi rT = \pi r^2 h g \rho$$

whence

$$T = \frac{r h g \rho}{2}$$

If, as in the case of mercury, there is a depression h and an angle of contact θ , the component of surface tension which acts downwards is $T \cos \theta$, and the resultant downward force given by it is $2\pi rT \cos \theta$. This is equal to the weight of a mercury column of height h , whence

$$T = \frac{r h g \rho}{2 \cos \theta}$$

It will be noticed that according to the above formulæ the elevation or depression is inversely proportional to the diameter of the tube.

In determining the surface tension of water by the above formula, the tube is cleaned, and its bore obtained by drawing a thread of mercury into it, measuring the length of the thread, and weighing it. The tube is then fixed in a vertical position with its end dipping into water, and the level inside the tube measured by a vernier microscope.

To find the level of the surface outside the tube a pointer is fixed in a vertical position with its end just touching the surface. The beaker containing the water is then removed, and the level of the end of the pointer measured. It is not possible to focus directly on the surface owing to the film at its edge and the distorting effect of the glass.

EXAMPLE.—The surface tension of water is 74 dynes/cm. How high should water rise in a capillary tube of 1 mm. diameter?

We have
$$h = \frac{2T}{r\rho g} = \frac{2 \times 74}{0.05 \times 981} = 3.0 \text{ cm.}$$

Further Examples of Surface Tension.—If a needle is placed very carefully on a water surface it floats, although its density is nearly eight times that of water. If it is carefully observed, it is seen to rest in a depression on the surface. The surface does not wet the needle, so where the film joins on to the needle, the tension has an upward component. This upward component supports the weight of the needle. The ability of certain insects to walk on the surface of water is explained in the same way.

If a camel-hair brush is dipped into water the single hairs project

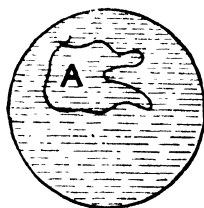


FIG. 133.

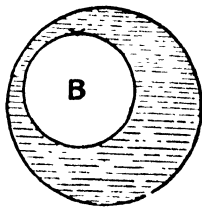


FIG. 134.

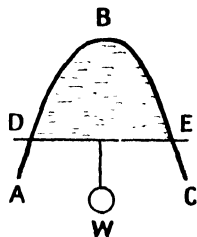


FIG. 135.

in all directions. But when it is removed, they are held together, as if they were surrounded by a skin.

If a metal ring is dipped into a soap solution, so as to form a film across it, and if a loop of fine thread which has been previously wet with the solution is laid on the film, it can be made to take up any form such as A (Fig. 133). If, however, a hot wire is thrust through the loop, the latter springs into the circular form B (Fig. 134), because the surface tension now acts only on the outside of the loop, and the area of the surface outside the loop contracts as much as possible. This makes the area inside the loop as large as possible, and the circle has the largest area for a given perimeter.

Again, if a bent wire ABC (Fig. 135) is taken with a straight wire DE resting against it and a film is formed in the enclosed space, it is found that the film is able to support the wire and also a small weight W. If T denotes the surface tension, the upward force exerted by each side of the film is $T \times DE$. Hence, if W is the total weight supported,

$$2T \times DE = W$$

If a thin rectangular plate of glass such as a microscope slide is

hung from one arm of a balance with its plane vertical and its long edge touching the surface of water but not displacing any water, the surface tension of the water acts down on it with a total force of $2Tl$ dynes, where l is the length of the edge. Hence the weight on the other arm of the balance necessary to produce equilibrium must be increased by this amount. This affords a method of measuring T .

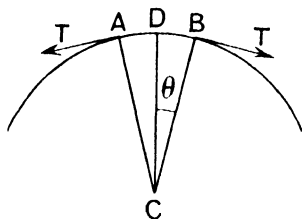


FIG. 136.

Excess of Pressure inside a Soap Bubble.—Let us suppose we have an elastic band of breadth 1 cm. stretched round a cylinder of radius r (Fig. 136). Consider the equilibrium of the portion AB subtending a small angle 2θ at the centre of the section. Let p be the pressure it exerts on the surface of the cylinder, and let T be the stretching force in the band. The area of the band is $2\theta r$: consequently, since θ is small, the resultant upward force on AB

due to the reaction of the cylinder is $2\theta rp$. This acts along CD. It is balanced by the sum of the components of the forces at the ends in the same direction. This is $2T \sin \theta$, or $2T\theta$, since θ is small. Hence

$$2\theta rp = 2T\theta$$

whence

$$p = \frac{T}{r}$$

Let us now suppose that the surface is a spherical one, and that an elastic sheet is stretched round it. If we consider the equilibrium of an area 1 cm. square, there are stretching forces on all four edges. Consequently in place of the above result we obtain

$$p = \frac{2T}{r}$$

This would also hold for a football bladder, where T is the tension in the rubber. The latter acts throughout the body of the rubber.

If we pass to the case of the soap-bubble, there is a tension T on both the inside and outside of the film. Hence the excess of pressure inside is given by

$$p = \frac{4T}{r}$$

Diffusion of Gases.—Fig. 137 represents a well-known lecture experiment. A is a cylindrical porous pot of unglazed earthenware, which is supported in an inverted position with a bent glass tube BC fitted into a cork in its mouth. The pot contains air; the glass tube contains coloured water, and initially the level of the two surfaces B and C is the same. D is a glass jar held over the pot.

If a rubber tube is connected to the gas pipe and gas introduced below the jar, the gas, being lighter than air, ascends and displaces the air. At the same time the level at B sinks and the level

at C rises, showing that the pressure inside the pot has increased. If the glass jar is lifted, so as to allow the gas to escape, the pressure inside the pot decreases to its original value.

The explanation of the experiment is as follows : The pressure of a gas on a surface is due to the impact of the molecules on the surface. Let us suppose that we have two gases on different sides of a surface, both at the same temperature and pressure, and that the molecules of the one gas are lighter than the molecules of the other. According to Avogadro's law the two gases have the same number of molecules per unit volume. Consequently to produce the same pressure the molecules of the lighter gas must move faster than the molecules of the other. This ensures that they strike the surface both oftener and harder. In the experiment in question the holes in the earthenware take the place of the surface. We have air molecules entering them from the inside and gas molecules from the outside. The latter move faster and arrive oftener. Hence the number of molecules and consequently the pressure inside the pot increases, until the excess of pressure makes the outward flow equal to the inward one. The experiment affords convincing evidence of the motion of the molecules of a gas.

To the above process the name of diffusion has been given. It takes place always when we have two gases meeting, and removes inequalities of temperature and composition. A room may be ventilated by the inward diffusion of the outside air in the entire absence of air currents.

Diffusion of Liquids.—Let a tall glass cylinder such as is represented in Fig. 138 be half filled with water. Then by means of a thistle funnel let a strong solution of copper sulphate be introduced into the bottom of the jar. This solution is denser than water, and if the operation is done carefully, it takes up a position below the water with a sharp surface of separation. Above the surface the water is perfectly clear; below it we see the bright blue of the solution. If, however, the jar is left to itself, the surface becomes ill defined, and in the course of days the blue passes up into the water. The copper sulphate has diffused upwards into the water in opposition to gravity. We have here a proof of the motion of the molecules of a liquid.

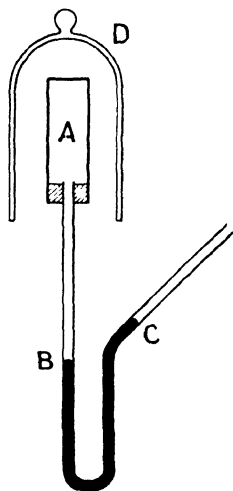
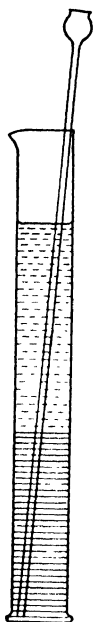


FIG. 137.—Apparatus to show diffusion of gases.

There is a great difference in the rapidity with which substances diffuse. The mineral acids and their salts diffuse comparatively rapidly in solution and are called **crystalloids**; substances such as sugar, starch, gum, albumen, etc., diffuse very slowly, and are called **colloids**.

If a sheet of parchment or the bladder of an animal is used as a partition in a vessel, with pure water on one side and a solution of a crystalloid or a colloid on the other, the crystalloid diffuses through the parchment, but the colloid does not. Such membranes are called **semi-permeable membranes**, because they are permeable to the one class of substance but not to the other. They can be used as a means of separating crystalloids and colloids from a mixed solution. This method was introduced by Graham and is called **dialysis**. The separation produced by dialysis is due to the size of the colloid particles which are each aggregates of molecules too large to pass through the holes in the membrane.

FIG. 138.—
Experiment
to show
diffusion of
liquids.



After a time water makes its way in through the parchment, and the level A stands higher inside the funnel. No sugar passes from the inside to the outside. If, instead of being immersed in water, the funnel is dipped in a sugar solution and filled to the level B with a solution of the same strength, no change in level takes place.

The experiment may be accounted for by assuming that the pores in the membrane are of such a size, that they allow water to pass through them, but stop the passage of the large sugar molecules. The water molecules pass through the pores both ways, but a smaller number strikes the inner side, since some of the molecules inside are sugar. Hence on the whole there is a passage of water to the inside and an accumulation of water there.

If a piston is fitted to the inside of the funnel, the inward diffusion of the water can be prevented. The pressure which

Osmotic Pressure.—Let a thistle funnel be closed at its lower end with a piece of parchment, and then immersed in water as shown in Fig. 139. Then pour a strong solution of sugar into it, so that the level is the same both inside and outside the funnel. After a time water makes its way in through the parchment, and the level A stands higher inside the

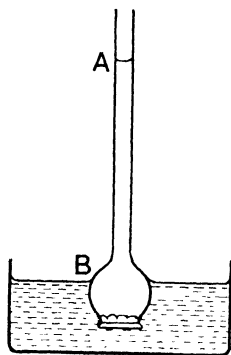


FIG. 139.—Apparatus
to show osmosis.

must be applied to do this is called the **osmotic pressure** of the solution, and the phenomenon itself is called **osmosis**. As many of the membranes existing in animals and plants are semi-permeable, osmosis plays a large part in physiology. Parchment and bladder allow both crystalloids and water to pass, but there are certain membranes, such as the gelatinous precipitate of copper ferrocyanide, which allow water to pass and stop certain crystalloids.

Absorption of Gases by Liquids and Solids.—If a beaker of water is warmed, small bubbles collect upon its walls and rise to the surface. These are air, as is shown by the fact that they do not condense, when they ascend through the colder part of the liquid. Thus ordinary water contains air dissolved in it.

Soda-water is water which has been made to absorb carbon dioxide gas. The absorption is produced by bringing the water together with the gas at a high pressure. Whenever the pressure is taken off, the gas passes rapidly out of solution, and it is to this that the effervescence is due. Generally speaking the amount of gas absorbed by a liquid is proportional to the pressure of the gas, and decreases with the temperature.

Some solids also absorb gases, particularly porous substances such as charcoal. The absorption is selective, being greater in the case of some gases than others, and it seems to be a surface phenomenon, depending on the amount of surface the body possesses.

EXAMPLES XII

1. Calculate Young's modulus for a steel wire from the following data : Length of wire 497 cm. ; diameter of wire 0.075 cm. ; elongation of 0.35 cm. produced by a weight of 6300 gm.

2. If Young's modulus for steel is 2×10^{12} dynes/cm.², what weight must be suspended from a steel wire 4 metres long and 1 mm. in diameter to stretch it 1 mm. ?

3. A capillary tube has a diameter of 1.20 mm. It is dipped vertically in water, and the surface rises to a height of 2.37 cm. inside the tube. What value does this give for the surface tension of water ?

4. A glass plate is suspended from a balance with its plane vertical and an edge 15 cm. long and 3 mm. wide touching the surface of water. Given that the surface tension of water is 74 dynes/cm. find the apparent increase in weight of the plate.

5. A soap bubble of 3 cm. radius is formed from a solution the surface tension of which is 50 dynes/cm. Calculate the excess of pressure inside in cm. of mercury.

6. A glass tube 6 mm. in internal diameter is pushed vertically into mercury. Taking the surface tension of mercury as 547 dynes/cm., and the angle of contact as 52° , calculate the difference in level of the mercury in the tube and outside the tube.

7. Will mercury ascend a lamp wick as oil and water do ?

8. How would you show that different substances diffuse at different rates through the same liquid ? How would you explain the difference and the fact that the rate of diffusion depends on the temperature ?

PART II

HEAT

CHAPTER I

TEMPERATURE AND THERMOMETERS

Temperature.—If a pot of cold water is placed on the fire and a finger is dipped into it from time to time, we feel the water gradually becoming hotter. We describe this by saying that the temperature of the water is increasing; the temperature of a body is thus its **degree of hotness**.

We regard the change in temperature of the water as due to something called heat passing from the fire to the water. The reasons for this assumption will be made clear later. It is to this flow of heat to or from our body that our sensations of hot and cold are due. If we touch a piece of iron and a piece of wood on a cold day, heat flows from the hand to them and they feel cold. More heat flows to the iron than to the wood, because the iron conducts away the heat better than the wood, and the iron consequently feels colder than the wood, even though they are at the same temperature. Thus we cannot compare temperatures correctly by the sense of touch when we are dealing with different substances.

If water feels cold to the hand, it is at a lower temperature than the hand, and heat flows to it from the hand. Thus heat flows from the body at the higher to the body at the lower temperature. It is on this fact that the accurate definition of temperature is based.

One body is said to have a higher temperature than another if, when they are placed in contact, heat flows from that body to the other.

If we have two tanks containing water connected by a pipe below the surfaces, water flows through the pipe from the tank in which the surface is higher to the tank in which it is lower. Thus if the heat in a body is compared with water in a tank, the temperature of the body corresponds to the level of the surface of the water.

Thermometers.—As has already been mentioned, the hand is an unreliable means of estimating temperature. Of two equally cold bodies, that one feels colder which conducts away the heat better; also if the hand has been immersed in hot water, tepid water feels cold to it, but if it has been immersed first in cold water, tepid water feels warm to it. We consequently employ instruments for measuring temperature. Such instruments are called thermometers.

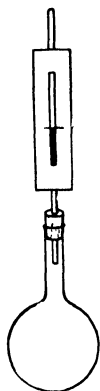


FIG. 140.—Principle of thermometer.

Bodies generally expand when heated, and the thermometers in common use measure temperature by the expansion of a quantity of liquid. Fig. 140 represents the principle of the arrangement. A flask fitted with a cork and capillary tube is filled with water up to a point in the capillary tube. A piece of paper is placed behind the tube and the water is coloured, so that the level of its surface is clearly seen. The flask is then immersed in warm water; the glass expands and so does the coloured water inside it. The latter expands more than the glass does, so its level rises in the capillary tube, and by the position of the level we can measure the temperature of the water in which the flask is placed. Whenever the water returns to the same temperature, the level returns to the same place.

One disadvantage of this arrangement is the size of the flask, which would absorb too much heat from the substance the temperature of which is to be measured, and which would take too long to acquire the temperature of that substance. Another disadvantage is the fact that the upper end of the tube is open, so that liquid escapes by evaporation. Consequently we employ a small spherical or cylindrical bulb of thin glass blown on the end of a capillary tube the other end of which is closed; the cylindrical form is preferable because, owing to its having a larger surface for the same volume, it enables the **thermometric fluid**, or liquid contained, to acquire the temperature of the surrounding liquid more rapidly. It can also be passed more easily through corks.

Two thermometric fluids are employed in practice, mercury and alcohol, mercury to much the greater extent. It has the advantage of a high boiling point and low freezing point (357° C. and -39° C.), it does not wet glass and is easily visible, and it has a low specific heat and conducts heat well, thus responding rapidly to the temperature to be measured. Also a fine mercury column is less liable to break than columns of other liquids. Alcohol has the advantage of a very low freezing

point (-130°C.), and so can be used to measure temperatures in Arctic regions at which mercury is solid. It is also cheaper than mercury. But it has the disadvantage of a low boiling point (78°C.), and it wets the side of the tube when the column is falling. As it is colourless it has to be rendered visible by the addition of some dye. It expands about five times as much as mercury.

Construction of a Thermometer.—Suppose that a simple mercury-in-glass thermometer is to be constructed. Then the following operations have to be gone through :

1. A capillary glass tube is cleaned, a short column of mercury introduced into it, and the length of this column measured by a microscope at different parts of the tube. If the length of the column is always the same, the bore is uniform, and the tube is suitable for the purpose.

2. A bulb is then blown on one end of the tube, and a funnel attached by a piece of rubber tubing to the other end.

3. Mercury is placed in the funnel ; owing to the narrowness of the tube it does not run down into the bulb. The air in the bulb is then warmed and expands, bubbling up through the mercury. It is next allowed to cool and contracts, sucking mercury down into the bulb. The process is then repeated until the bulb is full.

4. The funnel is removed and the mercury boiled, so that its vapour expels all the air from the tube. The latter is then sealed off with the blowpipe.

5. The tube now contains a definite volume of liquid with an air-free space above it. It is put away for some days, until it recovers from the violent heatings undergone in its construction. Then we determine the positions of the column when the instrument is at the temperature of melting ice, and when it is at the temperature at which water boils under standard atmospheric pressure. Marks are put on the stem at these points ; they are known respectively as the lower and upper **fixed points** of the thermometer.

Thermometric Scales.—There are three different thermometric scales in use. On the **Centigrade** scale, which is the scale used for scientific purposes, the interval between the fixed points is divided into 100 degrees, the lower point being taken as 0° and the upper one as 100° . To put the matter more precisely :

Let v be the volume of the part of the tube between the two fixed points, let v be divided into 100 equal parts, and let a mark be put on the stem at the termination of each of these parts, the temperature of the tube being kept constant ; then if at a subsequent time the mercury column occupies t of these divisions, the temperature is said to be t degrees on the mercury-in-glass

Centigrade thermometer, and is written $t^{\circ}\text{C}$. The temperatures as defined in this way and read on mercury-in-glass thermometers made of different kinds of glass do not agree exactly, the maximum divergence on the range 0° to 100° being about $\frac{1}{10}^{\circ}\text{C}$.

On the **Fahrenheit** scale, the common household scale of this country and America, the interval between the fixed points is divided into 180 degrees, the lower one being marked 32° and

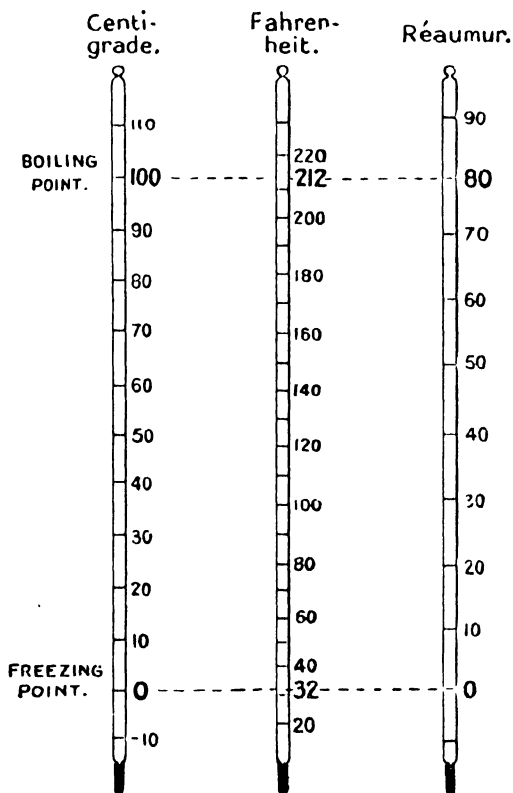


FIG. 141.—The three thermometric scales.

the upper one 212° , and on the **Réaumur** scale which is used in Germany and some other parts of the Continent, the interval between the fixed points is divided into 80 degrees, the lower one being 0° and the upper one 80° . The relation between the three scales is made evident by Fig. 141. On each scale the temperatures below zero have the minus sign prefixed.

In converting temperatures from one scale to another the

important thing to remember is *how many degrees the temperature is above or below the freezing point*. Also a range of 100 Centigrade degrees = a range of 180 Fahrenheit degrees = a range of 80 Réaumur degrees, hence 5 Centigrade degrees = 9 Fahrenheit degrees = 4 Réaumur degrees. Thus to convert a temperature from the Fahrenheit to the Centigrade scale, subtract 32 and multiply by $\frac{5}{9}$. To convert a temperature from the Centigrade to the Fahrenheit scale, multiply by $\frac{9}{5}$ and add 32. To convert a temperature from the Centigrade scale to the Réaumur scale, multiply by $\frac{4}{5}$; to convert a temperature from the Réaumur scale to the Centigrade, multiply by $\frac{5}{4}$. These rules may be combined in the formula:

$$\frac{C}{100} = \frac{F-32}{180} = \frac{R}{80}$$

EXAMPLE.—Change 88° F. to the Centigrade scale.

88° F. is 88–32 or 56° above the freezing point. 56 Fahrenheit degrees = $\frac{5}{9} \times 56 = 31.1$ Centigrade degrees. Hence, since the freezing point is at 0° C., 88° F. = 31.1° C.

EXAMPLE.—Change 50° C. to the Fahrenheit scale.

50° C. is 50° above the freezing point. 50 Centigrade degrees = $\frac{9}{5} \times 50 = 90$ Fahrenheit degrees. Hence, since the freezing point is at 32° F., 50° C. = 90 + 32 = 122° F.

Historical.—The Centigrade scale is sometimes called the Celsius scale, because it was first devised by Celsius of Upsala in 1742. The mercury thermometer itself was invented by Fahrenheit in 1714. Previous to that other liquids had been used; Newton, for example, experimented with oils, but they smeared the glass. To standardise his thermometers Fahrenheit used three fixed points, the temperature of the human body which he took as 96°, the freezing point of water, and the temperature of a mixture of ice, salt, and water, which he took as 0°. This coincided accidentally with the lowest temperature recorded at Dantzic in the winter of 1709, and he thought apparently that it was the lowest temperature possible to obtain anywhere. He found that the boiling point of water was 212° on his scale, and that it varied with the atmospheric pressure, but he did not use it as a fixed point.

The temperature of the body is taken by putting the thermometer in the armpit or the mouth. Its correct value for a person in health varies only slightly from 98.4°; Fahrenheit's measurements were not very accurate.

Verification of Thermometer Fixed Points.—The method of determining or verifying the fixed points of a thermometer will now be described in detail. The latter procedure is more important from the point of view of the student, because the fixed points are usually slightly out, and by correcting them the accuracy of the instrument can be increased. They may also change; if,

when it is first constructed, a mercury thermometer reads correctly, it will read too high, perhaps about $\frac{3}{10}^{\circ}\text{C}$. after half a year. This is due to the gradual recovery of the glass from the effects of the heating it was subjected to, when the thermometer was made; this recovery is rapid at first but slower afterwards.

If a thermometer has been heated and then is cooled, the glass does not take up its proper volume immediately, but "lags" above it; consequently there is a temporary depression in the readings which may be as much as $\frac{1}{10}^{\circ}\text{C}$. The lag can be neglected at high temperatures. For this reason the freezing

point of a thermometer should always be verified before its boiling point.

Sometimes when the thread of mercury is descending rapidly it breaks, leaving an isolated portion near the top of the tube. This can be joined on again by heating the bulb to make the mercury ascend again to touch it, or by jerking the instrument so as to drive the detached portion downwards.

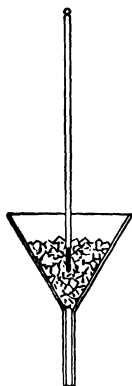


FIG. 142.

To verify the freezing point of a Centigrade thermometer it is held in a vertical position with its bulb surrounded up to the zero mark by pure melting snow or small pieces of ice, and the reading taken as soon as the temperature becomes stationary. The ice is best held in a funnel as shown in Fig. 142, with a clip to hold the thermometer in position and a dish below to catch the water which drains away as the ice melts. The ice must be free from impurities such as salt, otherwise the temperature will be too low. If a thermometer is surrounded in this way with pure melting ice, the mercury stops at the same point under whatever circumstances the experiment is performed, and remains stationary there until all the ice is melted. Variations of atmospheric pressure have no appreciable effect on the freezing point.

To determine the boiling point of a thermometer it is placed in the arrangement shown in Fig. 143, which is called a hypsometer from the Greek words for height and measure, because it can also be used for determining the heights of mountains. The temperature is then read as soon as it becomes stationary. Steam

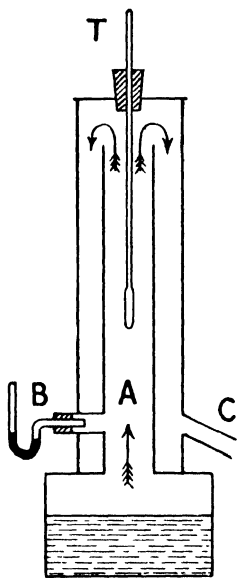


FIG. 143.—Hypsometer.

from boiling water passes up an inner cylinder A round the stem of the thermometer, down between the inner cylinder and outer cylinder, and then escapes by the outlet at C. B is a pressure gauge which measures any excess of pressure in the space round the thermometer over the outside atmospheric pressure. If the outlet C is large enough, the difference in levels at the boiling point is inappreciable. The boiling point of the thermometer should just show above the cork.

The thermometer is placed in steam and not in boiling water, because slight impurities in the form of dissolved salts affect the temperature of the water considerably, but not the temperature of the steam given off by the water. Also water contained in a clean vessel sometimes rises above the correct boiling point and then there is a violent burst of ebullition, after which the temperature falls to the usual value.

The temperature of the steam depends on the atmospheric pressure under which the water boils according to the following table : —

Pressure.	Temperature.
70 cm.	97·71° C.
72	98·40°
74	99·25°
76	100·00°
78	100·73°
80	101·44°

Thus, after the boiling point has been determined, the barometer must be read, and the correct value of the boiling point calculated according to the table for the pressure in question.

When the error of the thermometer has been determined for the two fixed points, a straight-line graph may be drawn giving it for all intermediate values. Thus if, for example, in the case of a Centigrade thermometer the reading at the freezing point is 0·3° too low, and at the boiling point 0·8° too high, Fig. 144 would give the correction to be added at all intermediate values. Thus no correction is required at 27°. The ordinates 0·3° and -0·8° should be plotted against -0·3° and 100·8°, but no appreciable error is involved in taking them at 0° and 100°.

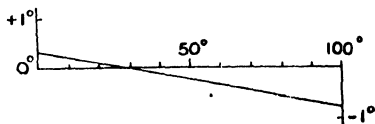


FIG. 144.—Graph for correcting a thermometer.

Stem Correction.—For a thermometer to read correctly it

must be at the same temperature up to the division in question. Thus in determining the boiling point the stem should be immersed in steam up to 100°C . If only the bulb is immersed in a hot liquid and the stem is cold, the mercury in the stem does not expand, and the reading is consequently too low. This error may be allowed for by adding on what is known as the stem correction, which may be as much as 1.5°C . if the bulb is at 100°C . and the stem at 0°C . throughout. But stem corrections are always unsatisfactory owing to the impossibility of knowing definitely what the temperature of the stem is.

EXAMPLE.—Water boils at 100°C . at the foot of a mountain and at 97.71°C . at its top. What is the height of the mountain?

The barometric pressure at the foot is 76 cm., and at the top (*cf.* table on p. 141) 70 cm. The difference = 6 cm. = 2.36 in. Using the rule 1 inch for 900 feet, this gives 2124 feet.

Thermometers for Special Purposes.—The portion of the tube not occupied by the mercury column is usually a vacuum. But in the case of a thermometer to be used at high temperatures, 250°C . to 500°C ., in order to prevent the mercury distilling and condensing on the upper part of the tube, the space above the mercury is filled with carbon dioxide or nitrogen at a pressure which may be as high as 20 atmospheres.

In instruments intended for accurate work the scale is engraved on the outside of the stem. In some instruments the stem is made very thin, and the scale is engraved on a piece of opal glass, or merely printed on a piece of paper, and fixed to the back of the stem, the whole being encased in a wide glass tube to prevent relative displacement of scale and stem. This type is known as the sleeve thermometer in contra-distinction to the solid-stem thermometer, and has the advantage of greater legibility.

Fig. 145 represents a **clinical thermometer**, the thermometer specially adapted for taking the temperature of the human body. As has

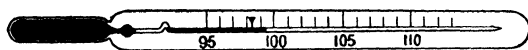


FIG. 145.—Clinical thermometer.

already been mentioned, this is done by placing the thermometer in the mouth or under the armpit of the patient and withdrawing it to be read. There is a constriction near the bulb, and the mercury in expanding flows through the constriction. But when the mercury cools, the column breaks at the constriction leaving its end at the maximum position. After the reading is taken, the mercury is got back through the constriction by holding the instrument vertically in the hand and jerking the arm in somewhat the same manner as when cracking a whip. As the temperature to be measured varies only through a range of from 95°F . to 110°F ., the scale requires to be graduated only for this interval.

Maximum and minimum thermometers record respectively the highest and lowest temperatures reached between two settings of the instrument. One form of maximum thermometer, known as Rutherford's maximum thermometer, is simply a mercury thermometer with a horizontal stem, which contains in the vacuum a small steel index shaped like a dumb bell. When the mercury expands, it pushes the index along in front of it; when it contracts, it leaves the index behind. Thus the end of the index nearer the mercury records the highest temperature reached. The instrument is re-set by tilting the stem and letting the index slide back to the mercury surface. Another form of maximum thermometer works on the same principle as the clinical thermometer.

Rutherford's minimum thermometer is an alcohol thermometer with a similar index inside the liquid column. When the column contracts, the meniscus pushes the index back in front of it. When the column expands, the liquid flows past the index. Thus the end of the index nearer the meniscus gives the lowest temperature reached. The index is re-set by tilting the tube, so that it slides back to the surface.

Fig. 146 represents Six's combined maximum and minimum thermometer. This instrument is not an accurate one, but is much used by gardeners. The bulb A in the middle is filled with alcohol which extends as far as C. From C to D there is a column of mercury and above D more alcohol, the upper half of F being occupied by alcohol vapour. B and E are two dumb-bell-shaped iron indexes supported against the glass by weak springs. The alcohol in the bulb A is the thermometric fluid. When it expands, it pushes the mercury and the index E in front of it, and the lower end of E registers the maximum temperature; when it contracts, the mercury pushes B up and the lower end of B registers the minimum. The indexes are re-set by means of a small magnet applied to the outside of the tube. The small figure at the side shows an index in detail.

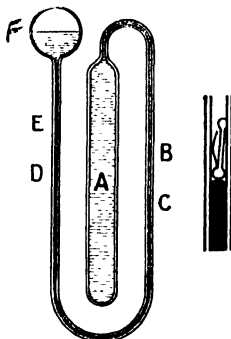


FIG. 146.—Six's maximum and minimum thermometer.

EXAMPLES I

1. Change blood heat, 98° , summer heat, 76° , and temperate, 56° , all from the Fahrenheit to the Centigrade scale.
2. Convert -4° , 22° , 88° , and 112° C. to (i) the Fahrenheit scale, (ii) the Réaumur scale.
3. Convert -22° , 6° , 47° , 112° F. to the Centigrade scale.
4. At what temperature has the Fahrenheit scale the same numerical value as (i) the Centigrade scale, (ii) the Réaumur scale?
5. At what temperature does water boil when the barometric pressure is (i) 71 cm., (ii) 72.8 cm., (iii) 78.5 cm.?

6. About what temperature will water boil on the top of (i) Ben Nevis, (ii) Mont Blanc ? Ben Nevis is 4406 ft. and Mont Blanc 15,781 ft. high. Water boils at 75.9°C . when the pressure is 30 cm., and at 93.5°C . when it is 60 cm.

7. When the fixed points of a Centigrade thermometer are verified, it reads 0.5° at the freezing point and 99.2° at the boiling point. The barometric pressure is 75 cm. What is the correct reading when it reads 15° , and at what temperature is it exactly right ?

8. Two thermometers are made of the same glass ; the bulbs are spherical, the internal diameter of the first being 7.5 mm. and of the second 6.2 mm. The diameter of the tube of the first is 2.5 mm. and of the second 1.8 mm. What is the ratio of the length of a degree of the first thermometer to a degree of the second ?

CHAPTER II

EXPANSION OF SOLIDS AND LIQUIDS

Expansion of Solids.—Solid bodies in general expand when heated. This can be demonstrated by a well-known apparatus known as Gravesande's ring. This apparatus consists of a metal ring mounted on a stand, and when a metal sphere is lowered by a chain it can just pass through the ring. If, however, the sphere is heated by holding it over a flame, it will no longer pass through, as it has expanded and become too large. But if the ring is also heated so as to make it expand also, then the sphere can again pass through.

Or if we have a vertical wire AB (Fig. 147), the upper end of which is attached to a horizontal lever FC, the end C of which passes over a vertical scale, and if the wire is heated by bringing a bunsen flame near it, then the point C travels up the scale, showing that the wire has increased in length. The motion of C can be increased by increasing the ratio of the arms FC and FB of the lever.

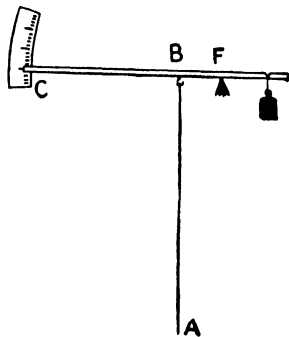


FIG. 147.

Coefficient of Linear Expansion.—It is found that when a bar expands, the increase in length is

- (i) proportional to the original length,
- (ii) proportional to the change of temperature,
- (iii) and varies with the nature of the material.

Thus, if l_0 is the length at 0°C. and l the length at $t^\circ \text{C.}$ the increase in length

$$l - l_0 = \beta l_0 t$$

$$l = l_0(1 + \beta t)$$

or

where β is a constant for the material in question known as its coefficient of linear expansion. To put the definition in words :

The coefficient of linear expansion of a solid is the increase in

length of a bar of unit length when its temperature is raised one degree.

In scientific work the Centigrade scale is always used. If the Fahrenheit scale were employed, the coefficient of linear expansion would be only $\frac{5}{9}$ as large. If the range through which the solid is heated is a very great one, the coefficient of linear expansion has not quite the same value throughout it, but this variation can be neglected unless in exceptional cases. The value of the coefficient of linear expansion is always very small; for example, in the case of copper it is 0.000017.

Let us suppose that a copper rod is 10.000 cm. long at 10° C., and that its length is required at 18° C. Then from the above equation we have

$$l_{10} = l_0(1 + 10\beta)$$

$$l_{18} = l_0(1 + 18\beta)$$

$$l_{18} = l_{10} \frac{1 + 18\beta}{1 + 10\beta} = l_{10}(1 + 8\beta - 80\beta^2 \dots)$$

Now for copper $8\beta = 0.000136$ and $80\beta^2 = 0.0000000231$; hence $80\beta^2$ can be neglected in comparison with 8β . It is the same with all other metals. We can thus write simply

$$l_{18} = l_{10}(1 + 8\beta)$$

Thus, if l is the length at t° C. and the rod is heated to t_1° C., the increase of length is $\beta l(t_1 - t)$ within experimental error. It is not necessary to substitute l_0 for l .

EXAMPLE.—A copper bar is 37.20 cm. long. By how much does it increase in length when it is heated from 15.0° C. to 23.6° C.?

$$\text{Answer} = 0.000017 \times 37.20 \times 8.6 = 0.0054 \text{ cm.}$$

EXAMPLE.—A metal metre scale was carefully measured at 5° C. and its length found to be 99.977 cm. At 35° C. its length was found to be 100.011 cm.; calculate the coefficient of expansion of the metal, and find the temperature at which the scale is exactly 1 metre long.

$$\begin{aligned} \text{Coefficient of expansion} &= \frac{100.011 - 99.977}{99.977(35 - 5)} \\ &= 0.0000113 \end{aligned}$$

$$\begin{aligned} \text{To become exactly right scale has to expand } 100 - 99.977 \\ = 0.023 \text{ cm.} \end{aligned}$$

The required rise of temperature is

$$\begin{aligned} \frac{0.023}{0.0000113 \times 99.977} &= 20.4^\circ \text{ C.} \end{aligned}$$

The required temperature is consequently 25.4° C.

Measurement of Linear Expansion.—The measurement of the expansion of a solid consists essentially of the measurement of a small increase in length, and two types of apparatus are employed for the purpose, a type in which the increase is magnified by a lever as in Fig. 147 and a type in which it is measured directly with great accuracy. The first type suffers from the disadvantage that it is difficult to measure the magnification produced by the lever. Fig. 148 shows an elementary apparatus of the second type much used in colleges.

S is a heating jacket consisting of two concentric cylinders between which cold water or steam can be passed. The rod to be experimented on stands upright inside the jacket. Before it is inserted in position, its length is measured at the temperature of the room with a metre-stick. The upper end of the rod is flat and a screw M can be made to touch it. This screw has usually a pitch of $\frac{1}{2}$ mm. and a large circular head divided into 100 divisions, so that, if it is turned through one division, its point advances through $\frac{1}{200}$ mm.

Water is passed through the jacket, and the temperature taken by lowering a thermometer inside the jacket. As soon as it is steady, the screw is brought into contact with the end of the rod and the reading taken. It is then screwed upwards to allow space for the rod to expand, and steam passed through the jacket until the temperature becomes constant again. The reading for contact is then taken, and the difference between the two readings gives the increase in length of the rod.

Instead of using a screw, a microscope with a scale in its field of view may be focussed on the end of the rod, and the number of divisions the end moves through counted.

EXAMPLE.—A metal rod, 40.0 cm. long, is heated from 15.0° C. to 100.0° C., and it is found that in order to maintain contact, the screw must be turned through 82 divisions on the circular head. Each division corresponds to $\frac{1}{200}$ mm. Find the coefficient of expansion of the rod.

$$\begin{aligned}\beta &= \frac{82 \times \frac{1}{200}}{40 \times 85} \\ &= 0.000121\end{aligned}$$

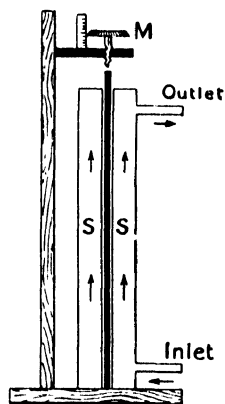


FIG. 148.—Apparatus for measuring expansion of a rod.

Roy and Ramsden's Method.—The apparatus shown in Fig. 148 does not give very accurate results, for part of the rod is exposed to the

outside air, and does not attain the proper temperature. Also the wooden frame and screw themselves expand. Better results are obtained when the rod is rested horizontally on rollers in a bath of water. The first determination by this method was that of Roy and Ramsden.

General Roy made a survey of Hounslow Heath in 1784 to get a base line for the first Ordnance Survey of England, and he had an apparatus designed by Ramsden with which he investigated the variation in length of his measuring rods with temperature. This is shown in Fig. 140; it consists of three troughs which contain the bar to be experimented on and two similar bars of iron. The bar CD to be experimented on is in the middle, and carries the object glasses of two

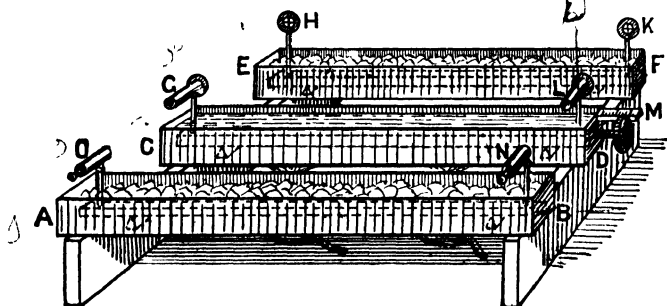


FIG. 140.—Roy and Ramsden's apparatus.

telescopes at G and L. The eyepieces of the same telescopes are carried by the bar AB in the first trough at O and N, and the telescopes are focussed on cross-wires carried by the bar EF in the third trough at H and K. The bar in the middle trough presses against a spring at C and against the point of a micrometer screw M. The upright G can also be screwed along the bar.

The three troughs are filled with melting ice, and the screw M and position of the upright G adjusted, until the images of the cross-wires at H and K coincide with the cross-wires in the eyepieces. Then the middle trough is heated. The ice melts and the bar expands towards C, since the point of the screw at M does not permit it to expand in the other direction. When the temperature is steady, M is turned until coincidence of the cross-wires is obtained at the right-hand end. It is then turned until coincidence is obtained at the left-hand end. The difference of the two readings gives the expansion of the middle bar between G and L.

It will be observed that with this apparatus the constancy of the distances between the measuring instruments can be relied upon, as the bars carrying them are maintained at the temperature of melting ice.

Applications.—In railways a small space is left between the ends of the rails in order to allow for the **expansion of the rails** in the hot days of summer. The force with which metals expand is very great indeed, and if there were no air space, the rails would

curve and twist from their positions. Similar provision has to be made for the expansion and contraction which takes place in metal structures, such as large bridges, etc. In fitting the iron tyre to a cart wheel it is made very hot, and consequently expands ; it is then placed in position. When it cools it embraces the wheel with great force, keeping itself in position and pressing home the spokes into their joints.

The period of a pendulum depends on its length. In the case of a clock which is regulated by a pendulum, if the latter expands, the clock goes slower. It is thus a problem of practical importance to design a pendulum, the length of which is the same at all temperatures. One solution, the so-called **gridiron pendulum**, is represented in Fig. 150 ; A is the point of attachment of the pendulum which is supported by steel and brass rods. The steel rods are shown white, and the brass rods black, in the diagram. It will be observed from the diagram that the effect of an expansion of the steel rods is to lower the bob, and the effect of an expansion of the brass rods is to raise the bob. Let l be the total length of the steel rods, l' the total length of the brass rods, let β and β' be their coefficients of expansion and let t be the rise in temperature. Then, if the bob remains at the same distance below the point of suspension,

$$\beta l t = \beta' l' t \quad \text{and} \quad \beta l = \beta' l'.$$

The coefficients of linear expansion of steel and brass are respectively 0.000011 and 0.0000187. Hence, if the total lengths of steel and brass rod are inversely in this ratio, the period is unaltered.

A nickel steel called **invar** containing 36 per cent. of nickel has an extremely small coefficient of expansion, and is now much used for the construction of pendulum rods and standards of length. It is sufficient that the pendulum rod should be constructed of invar, except when extreme accuracy is required.

When the temperature rises, the **balance-wheel** of a watch expands, and at the same time the elasticity of the hair-spring decreases. Both these effects make the watch go slower. If, however, the rim of the balance wheel is made of strips of two different metals as shown in Fig. 151, the outer strip having the larger coefficient of expansion, when the temperature rises the

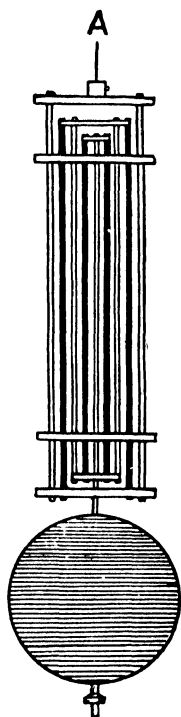


FIG. 150.—Grid-iron pendulum.

outer strip expands more than the inner strip, and the semi-circles become more curved. The mass of the wheel consequently

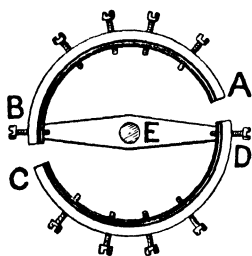


FIG. 151.—Balance-wheel of a watch.

moves in towards E, the axis of rotation, and the moment of inertia of the wheel diminishes instead of increasing. Hence by choosing the dimensions of the wheel suitably and adjusting the small screws the change in the elasticity of the hair-spring may be compensated for, and the period kept constant.

The **glass flasks** and beakers used in a laboratory for heating liquids must have thin walls. This is because glass is a bad conductor of heat, and if thick walls are used, the outside surface expands while the inside is still cold; the glass consequently becomes strained and cracks. Nowadays vessels can be made of fused quartz instead of glass; such vessels are not unlike glass in appearance, but they are considerably more expensive. They can be plunged into a hot bath without fear of injury, even if comparatively thick, because, although fused quartz is a bad conductor of heat, it has a much smaller coefficient of expansion than glass.

In experiments on the passage of electricity through gases it is often necessary to have wires passing through the sides of glass vessels and making an **air-tight joint**. Since the wire is sealed in when the glass is hot, it is necessary for its material to have the same coefficient of expansion as the glass has, if the joint is to remain air tight as it cools down. Platinum fulfils this condition, and was formerly used widely for the purpose. But since the war it has been replaced in electric lamps by platinite, a nickel iron alloy, with a great saving in the cost of manufacture.

Stress produced by Change of Temperature.—In order to get an idea of the magnitude of the forces which bodies are capable of exerting when their temperature is increased, we shall calculate the value for one simple case.

Let a wire of length L and cross-sectional area A hang vertically under gravity, and have its temperature increased by t° C. Its length is consequently increased by $\beta L t$. Then suppose that the temperature falls to its original value, but that the length of the wire is maintained at the increased value by the application of a force F to its lower end. The increase in length produced by the force must consequently be equal to the decrease produced by the fall in temperature. Thus, if E denotes Young's modulus for the material

$$E = \frac{\text{force per unit area}}{\text{increase in length per unit length}} = \frac{F/A}{\beta L t/L} = \frac{F}{A \beta t}$$

Hence

$$F = EA\beta t$$

Let us suppose now that the wire is a copper one, that A is 1 sq. mm., and that t is 1°C . For copper $E = 12.6 \times 10^{11}$ dynes per sq. cm., and $\beta = 0.000017$. Hence

$$\begin{aligned} F &= 12.6 \times 10^{11} \times 10^{-2} \times 0.000017 \\ &= 214,200 \text{ dynes, or about 218 gm. wt.} \end{aligned}$$

A weight of about 20 kg. attached to the end of this wire would be sufficient to break it.

Cubical Expansion.—In the case of crystals such as quartz or Iceland spar the coefficient of linear expansion is different according as the direction of expansion is parallel or perpendicular to the axis of the crystal. In the case of woods it is different along and across the grain. Such substances are called *anotropic*.

Let us suppose that a rectangular block, the lengths of the sides of which are a , b , and c , is heated through a range of temperature $t^\circ \text{C}$., and that the coefficients of linear expansion in the directions of the three sides are β_1 , β_2 , and β_3 . Then the lengths of the sides become $a(1+\beta_1 t)$, $b(1+\beta_2 t)$, and $c(1+\beta_3 t)$, and the volume of the block becomes

$$\begin{aligned} abc(1+\beta_1 t)(1+\beta_2 t)(1+\beta_3 t) \\ = v\{1 + (\beta_1 + \beta_2 + \beta_3)t + (\beta_1\beta_2 + \beta_2\beta_3 + \beta_3\beta_1)t^2 + \beta_1\beta_2\beta_3 t^3\} \end{aligned}$$

where v is its original volume. Since β_1 , β_2 , and β_3 are very small, only their first powers require to be retained, and the volume becomes

$$v\{1 + (\beta_1 + \beta_2 + \beta_3)t\}$$

Suppose now that the block is isotropic, *i.e.* the same in all directions, then $\beta_1 = \beta_2 = \beta_3 = \beta$, and the volume becomes

$$v(1 + 3\beta t)$$

3β is defined to be the **coefficient of cubical expansion** of the block. *The coefficient of cubical expansion is consequently three times the coefficient of linear expansion.* Also the above expression gives the volume of the solid after heating, whether the original form is rectangular or not, because an irregular form can be divided up into a great number of little rectangular blocks.

To put the definition in words:

The coefficient of cubical expansion of a substance is the increase in volume of unit volume when its temperature is raised one degree.

Since a liquid has a definite volume but no definite shape, when we refer to its coefficient of expansion it is always the coefficient of cubical expansion that is meant.

In the same way the coefficient of areal or superficial expansion is twice the coefficient of linear expansion.

Effect of Temperature on Density.—Let α denote the coefficient of cubical expansion of a substance. Then if v_0 denote its volume at 0° C. and v denote its volume at t° C.,

$$v = v_0(1 + \alpha t)$$

Let ρ_0 denote the density at 0° C. and ρ the density at t° C. Then, since the mass is constant,

$$v\rho = v_0\rho_0$$

Hence, substituting in the former equation

$$\rho = \frac{\rho_0}{1 + \alpha t}$$

Thus the density of the substance decreases with rise of temperature. The result, of course, applies to liquids as well as solids.

Dilatometer.—Fig. 152 represents a dilatometer, which consists of a glass vessel with a long straight graduated neck. Let γ be the coefficient of cubical expansion of the glass. Let the dilatometer contain liquid up to the level A at a temperature of 0° C., let the coefficient of cubical expansion of the liquid be α , and let the volume of the liquid be v . Let the vessel and liquid be heated through a range of t° C. Consider their expansions separately.

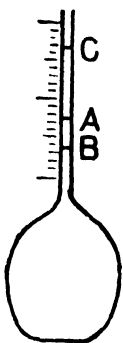


FIG. 152.—
Dilatometer.

Suppose the vessel were filled with glass up to the level A instead of with liquid. Then it would expand as a solid mass. The volume of glass enclosed would be v . Consequently the volume of the vessel up to A at the higher temperature would be $v(1 + \gamma t)$. If the liquid did not expand, its level would sink to B, the volume of AB at the higher temperature being $v\gamma t$. But owing to the expansion of the liquid its level rises to C and its volume becomes $v(1 + \alpha t)$, the volume of BC at the higher temperature being $v\alpha t$.

If we had not allowed for the expansion of the glass, we would have taken AC for the expansion of the liquid. This volume is called its apparent expansion. *The coefficient of apparent expansion of a liquid is defined as the apparent increase in volume per unit volume divided by the increase in temperature.* Let it be denoted by α' . We have then

$$\alpha' = \frac{AC}{vt} = \frac{BC - BA}{vt} = \frac{v\alpha t - v\gamma t}{vt} = \alpha - \gamma$$

or putting the result in words: *The coefficient of true expansion of a liquid is the sum of the coefficient of apparent expansion and*

the coefficient of cubical expansion of the vessel. The coefficient of true expansion is often referred to as the coefficient of absolute expansion.

In order to make a determination of the coefficient of apparent expansion by the dilatometer it is necessary to know (i) its volume to the foot of the scale, and (ii) the volume of a division on the scale. This can be obtained by weighing the vessel empty, full of mercury to the foot of the scale, and full of mercury to the top of the scale, all at the same temperature. The volumes can then be obtained from the density of mercury.

EXAMPLE.—A thermometer is sunk to the 20° mark in a liquid and reads 80° C. The mean temperature of the rest of the stem is 30° C. Find the true temperature of the liquid, given that the coefficients of cubical expansion of mercury and glass are 0.000182 and 0.000024.

The mercury thermometer behaves like a dilatometer, and we can calculate the expansion of the mercury column relative to the glass simply by ignoring the expansion of the glass and using the coefficient of apparent expansion of mercury. The latter is given in this case by

$$\alpha' = \alpha - \gamma = 0.000182 - 0.000024 = 0.000158$$

The length of the mercury column between the 20° and 80° marks is 60 divisions. It should be at 80° C., but is only at 30° C. If it were at its proper temperature, its length would be

$$60(1 + 50 \times 0.000158) = 60.47 \text{ div.}$$

The true temperature is therefore 80.47° C.

Weight Thermometer.—Another method of measuring the coefficient of apparent expansion of a liquid is by means of the weight thermometer, a little glass vessel with a piece of bent capillary tube as neck, as shown in Fig. 153.

It is weighed first empty, then filled to the end of the capillary with the liquid in question by alternate heating and cooling with the mouth below the surface of the liquid, and weighed again. The difference of these two weighings gives w , the weight of the liquid contained. It is next transferred to a water-bath and heated through a range of temperature t° C.; the liquid expands more than the glass does, and some is expelled from the capillary. The vessel is then weighed a third time, and w' , the weight of the liquid left in it, determined. The weight expelled is $w - w'$. The apparent increase in volume is to the original volume as $w - w'$ to w' . Hence the coefficient of apparent cubical expansion is



FIG. 153.—
Weight thermometer.

$$\frac{w - w'}{w' t}$$

The coefficient of true expansion can then be obtained by adding the coefficient of cubical expansion of the glass to the result. As the coefficient of cubical expansion of glass varies with the nature of the glass, it is best determined experimentally for any particular weight thermometer by using the latter first to determine the coefficient of apparent expansion of mercury, the absolute expansion of which is known very accurately. The value for the glass can then be obtained by subtraction. Mercury is very suitable for this purpose because its coefficient of expansion is small, and the expansion of the vessel has consequently a large influence on the result.

Fused quartz has, as has already been mentioned, a very small coefficient of cubical expansion, only about one-hundredth of the coefficient of cubical expansion of mercury. Hence if the weight thermometer is made of fused quartz, the expansion of the vessel can for most purposes be neglected.

A small specific gravity bottle may be used instead of a weight thermometer; in this case the liquid which is expelled through the hole in the stopper must be wiped off.

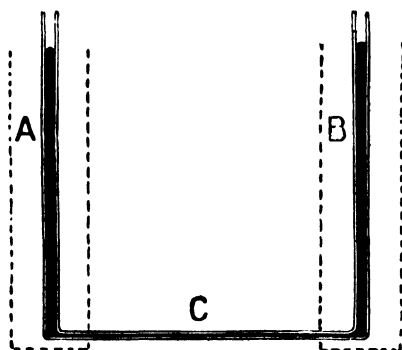


FIG. 154.—Principle of Dulong and Petit's apparatus.

Absolute Expansion of Mercury.—A direct method of determining the coefficient of absolute expansion of mercury was devised and carried out by Dulong and Petit about 1816. This method is based on the principle that, if two columns produce equal pressures, their heights are inversely as their densities.

It is represented in principle in Fig. 154. A and B are two vertical glass tubes connected by a horizontal capillary C. Each tube is surrounded by a metal case indicated by dotted lines; the case surrounding A is filled with ice, and that surrounding B contains oil heated to temperature $t^{\circ}\text{C}$.

Let h_0 , ρ_0 be the height and density of column A, and h , ρ the height and density of column B. Then, since the pressure at the foot of each column is the same,

$$\begin{aligned} h_0 g \rho_0 &= h g \rho \\ h_0 \rho_0 &= h \rho \end{aligned}$$

and

But we have already shown that $\rho = \frac{\rho_0}{1 + \alpha t}$

Hence

$$1 + \alpha t = \frac{h}{h_0}$$

and

$$\alpha = \frac{h - h_0}{h_0 t}$$

If the difference in height of the columns is measured, α can be calculated. The method was improved by Regnault in 1842 and another determination made, and recently it has been further improved by Callendar.

Regnault's Apparatus for Absolute Expansion of Mercury.—

In Dulong and Petit's apparatus the top of the hot column of mercury had to project above the cover of the oil bath to be visible, and hence did not attain the proper temperature. The two mercury surfaces also had different curvatures, because the surface tension of mercury is much less when it is hot than when it is cold, and this difference in curvature is difficult to allow for. Regnault consequently used a modified form of the apparatus.

Two upright iron tubes AA' and BB' (Fig. 155) about 15 cm. long and of 1 cm. internal diameter are joined at the top by a horizontal tube AB of about 3 mm. diameter and at the foot by glass tubes DG and HF. These are connected by a T-joint to a reservoir containing compressed air. The cross tube AB has a small hole in it at C, so that the upper surfaces of the two columns are at the level of C. The column AA' was surrounded by an oil bath the temperature of which was read by an air thermometer, and the column BB' was surrounded by cold water the temperature of which was read by mercury thermometers. The pressure of the air in the reservoir was adjusted, so that the level of the mercury at G was near the bottom of the tube.

The pressure is the same on both sides at the level of C, but the density of AA' is less than the density of BB'. Hence the pressure at A' is less than the pressure at B'. This difference in pressure is measured by the difference in level of the mercury at G and H. Denote this by l , let the difference in level of C and A' or B' be L , let the densities of the cold and hot mercury be respectively ρ_0 and ρ , and let t be the difference in temperature.

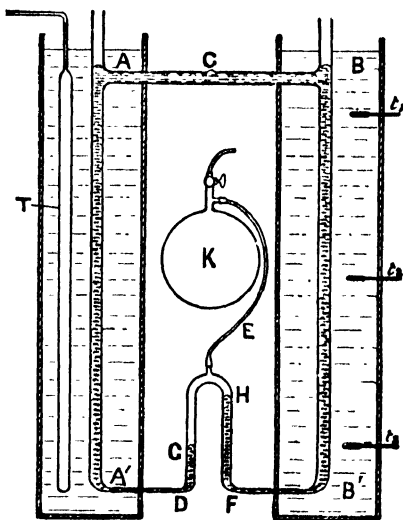


FIG. 155.—Regnault's apparatus for the absolute expansion of mercury.

Then
$$Lg\rho = Lg\rho_0 - lg\rho_0$$

Thus
$$\rho = \rho_0 \frac{L-l}{L}$$

But
$$\rho_0 = \rho(1 + \alpha l)$$

Hence
$$\alpha = \frac{l}{l(L-l)}$$

Hydrostatic Method of determining Coefficient of Expansion.—Let a solid be weighed in air, in liquid at $t_1^\circ \text{C.}$, and in the same liquid at $t_2^\circ \text{C.}$, t_2 being greater than t_1 . Then we know m_1 and m_2 , the masses of liquid displaced at $t_1^\circ \text{C.}$ and $t_2^\circ \text{C.}$ respectively. Let ρ_1 and ρ_2 be the densities of the liquid at the same two temperatures, let α be the coefficient of cubical expansion of the liquid, and let γ be the coefficient of cubical expansion of the solid.

The volumes of the solid at the two temperatures are m_1/ρ_1 and m_2/ρ_2 .

Hence
$$\frac{m_2}{\rho_2} = \frac{m_1}{\rho_1} \{1 + \gamma(t_2 - t_1)\}$$

But by the result proved on p. 152,

$$\rho_2 = \frac{\rho_1}{1 + \alpha(t_2 - t_1)}$$

Hence, eliminating ρ_1 and ρ_2 ,

$$\frac{m_2}{m_1} = \frac{1 + \gamma(t_2 - t_1)}{1 + \alpha(t_2 - t_1)}$$

Thus, if either γ or α is known, the other can be calculated as a result of the experiment.

Maximum Density of Water.

The expansion of water is anomalous; when it is heated from 0°C. to 100°C. , it contracts from 0°C. to 4°C. , then expands to 100°C. , the coefficient of expansion increasing with the temperature. At 4°C. its density is a maximum; as is well known, the unit of mass is defined on the c.g.s. system, so as to make the density of water unity at this temperature.

The temperature of maximum density can be determined by Hope's apparatus, which is shown in Fig. 156, and consists of a glass jar surrounded at its middle by an ice jacket and carrying two thermometers, one above and one below the ice. If the temperature of the water is initially 15°C. , as the water is cooled

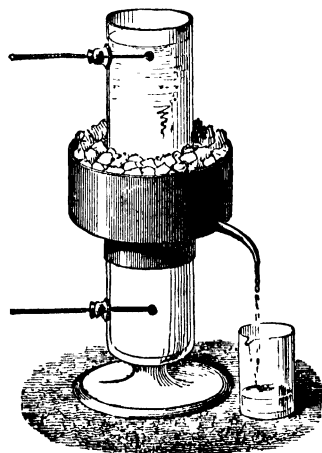


FIG. 156.—Hope's apparatus.

by the ice it sinks, and so the lower thermometer gives a lower temperature than the upper one until 4°C. is reached, when it remains stationary at this temperature; then the colder water begins to ascend, and as a consequence the upper thermometer descends to 4°C. and passes below it.

The fact that, when the cold air causes the temperature of a lake to fall below 4°C. , the colder water remains on the surface, is of great importance in nature, for it prevents convection currents downwards, and the water below remains at 4°C. even in the most severe winter, thus preventing plants and animals from being frozen.

Owing to its variation with temperature, when the coefficient of expansion of water is determined by the weight thermometer, the result obtained, which is of course an average, varies greatly with the range experimented on. A table of the density of water at different temperatures is given at the end of the book.

Thermostat.—The expansion of a liquid is used to maintain a water-bath or other apparatus at a constant temperature. Fig. 157 shows one form of thermostat, in this case a gas regulator. It is placed in the bath, and the gas passes through it before reaching the burners which do the heating. The bulb A contains toluene or some other highly expansible liquid; the lower part of A and the narrow tube contain mercury. When the temperature of the bath rises, the toluene expands, forces the mercury up the tube, and shuts off the supply of gas at B. A small bye-pass at C prevents the flame from being wholly extinguished. When the temperature falls, the toluene contracts, and the pressure of the gas increases. The temperature to be maintained is regulated by raising or lowering the outlet at D.

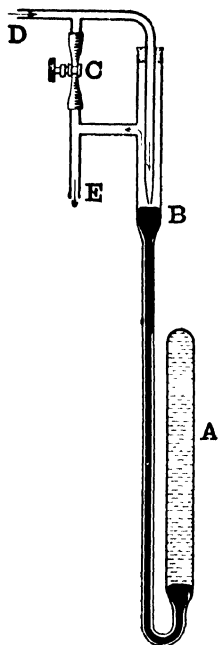


FIG. 157.—Gas regulator.

From Willow's *Physics*
(Edward Arnold & Co.).

EXAMPLES II

1. A brass rod and a steel rod are both measured at 0°C. and their lengths found to be 150.00 and 150.20 cm. respectively. At what temperature are their lengths equal, and what is their common length at this temperature? The coefficients of expansion of brass and steel are 0.000187 and 0.00011 respectively.

2. Find the least distance that must be distributed over 20 miles of railway, which is laid at 37°F. , in order to allow for expansion to

120° F. in summer. The coefficient of expansion of wrought iron is 0·0000122 per degree Centigrade.

3. A pendulum which is made wholly of brass beats seconds when the temperature is 15° C. Find how many seconds it will gain or lose in a day, if the temperature rises to 20° C. The coefficient of linear expansion for brass is 0·0000189. (You may assume for purposes of calculation, that the pendulum consists of an infinitely light brass rod with a weight at its end; the form of the pendulum does not affect the influence of temperature on its period.)

4. The height of the barometer is found to be 78·25 cm. at 22° C. Find at what height the column would stand, if its temperature were 0° C. Coefficient of absolute expansion of mercury = 0·000182.

5. The area of a brass plate is 123·02 sq. cm. at 5° C. What will it be at 122° C., given that the coefficient of linear expansion of brass is 0·0000189?

6. By how many inches is the circumference of an iron wheel increased, when its temperature is raised 400° C.? The diameter of the wheel is 3 ft., and the coefficient of expansion of iron is 0·0000122.

7. The coefficient of linear expansion of Iceland spar is 0·0000251 parallel to the axis and -0·0000056 at right angles to the axis. The crystal contracts at right angles to the axis, when it is heated. The volume of a piece of Iceland spar is 30·20 c.c. at 15° C. By how much has it increased at 40° C.?

8. The coefficient of linear expansion of pine is about 0·000005 along the grain and 0·000034 across the grain. Find how much a sphere of 6 cm. diameter made of pine will expand on being heated from 10° to 40° C.

9. The density of mercury is 13·5955 gm. per c.c. at 0° C. Its coefficient of cubical expansion is 0·000182. What is its density at 60° C.?

10. The density of water is 1·0000 at 4° C. and 0·9718 at 80° C. A cube of glass of 3 cm. edge is weighed in water at both temperatures. Find the difference of the two results, given that the coefficient of linear expansion of the glass is 0·000009.

11. A thermometer is sunk to the 25° mark in a liquid and reads 85° C. The mean temperature of the rest of the stem is 30° C. Find the true temperature of the liquid, the coefficient of expansion of mercury-in-glass being 0·00015.

12. An accurate Centigrade thermometer is immersed to its -10° mark in steam at 100° C., *i.e.* its bulb is wholly immersed, but none of the stem is, and the stem is exposed to a cold draught, so that its mean temperature is 15° C. What does the instrument read, given that the coefficient of apparent expansion of mercury-in-glass is 0·00015?

13. The bulb of a thermometer, together with as much of the stem as is below 0° C., contains at that temperature 2·83 gm. of mercury. Each degree on the stem is 1 mm. long. Calculate the diameter of the stem on the assumption that the apparent expansion of mercury-in-glass is 0·000155.

14. A specific gravity bottle weighs 21·57 gm. empty and 69·90 gm. when full of water at 20° C. When full of water at 90° C. it weighs 68·39 gm. The coefficient of cubical expansion of the bottle is 0·000025.

What is the average coefficient of cubical expansion of water over the range 20°C. to 90°C. ?

15. A weight thermometer, which contains 100 gm. of mercury at 10°C. , is placed in an oil-bath, and the mercury expelled is found to weigh 1.89 gm. Find the temperature of the bath, the coefficient of apparent expansion of mercury in glass being 0.000155.

16. A weight thermometer contains 122.70 gm. of mercury at 15°C. It is heated to 82°C. What weight of mercury is expelled, given that the coefficient of apparent expansion of mercury relative to glass is 0.000155 ?

CHAPTER III

EXPANSION OF GASES

Expansion of Gases.—When gases are heated they expand much more than solids or liquids. This can be shown by the arrangement depicted in Fig. 158, which was used by Galileo as a thermometer, and is often used for demonstration purposes. It consists of a glass bulb containing air, with a long, straight neck dipping into water usually coloured blue or black to make it visible. The level of the water is made to stand halfway down the stem. When the temperature of the bulb is increased, for example, by clapping the fingers round it, the air in it expands, and the top of the water column travels down the stem.

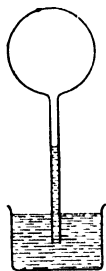


FIG. 158.

When measuring the coefficient of expansion of a liquid or solid it is not necessary for the atmospheric pressure on it to be constant throughout the operation. Any change in its volume produced by an alteration of pressure is so small in comparison with the change produced by the increase in temperature, that it can be ignored. But it is quite otherwise in the case of gases; they are so compressible that we must specify the pressure conditions before we can investigate the effect of heat upon them. There are two cases of special importance :

(i) The pressure is kept constant, and the alteration in volume is observed when the temperature is increased. Let v_0 be the volume of a gas at 0°C. in this case, and v its volume under the same pressure at $t^\circ \text{C.}$, measured on the mercury-in-glass thermometer. Then it is found that

$$v = v_0(1 + \alpha_p t)$$

where α_p is a constant for the gas called its **coefficient of expansion at constant pressure**. Putting the definition in words :

The coefficient of expansion of a gas at constant pressure is the ratio of the increase in volume for a rise of one degree in temperature to its volume at 0°C. , the pressure of the gas remaining constant.

(ii) The volume of the gas is kept constant by shutting it up in a closed vessel and the temperature altered. It is found in this case that the pressure rises as the temperature rises. Let p_0 be its value at 0°C. and p its value at $t^\circ \text{C.}$ Then

$$p = p_0(1 + \alpha_v t)$$

where α_v is a constant for the gas called its **coefficient of increase of pressure at constant volume**. It is also often called the coefficient of expansion at constant volume, in spite of this expression being a contradiction in terms. Putting the definition in words :

The coefficient of increase in pressure of a gas at constant volume is the ratio of the increase in pressure for a rise of one degree in temperature to its pressure at 0° C., the volume of the gas remaining constant.

It should be noted, that if v is the volume of a gas at t° C. and v_1 its volume at t_1° C. under the same pressure, we can **not** use the approximation made on p. 146, in the case of solids, and write

$$v_1 = v\{1 + \alpha_p(t_1 - t)\}$$

α_p and α_v are too large in the case of gases for their squares to be neglected.

We have

$$v = v_0(1 + \alpha_p t)$$

and

$$v_1 = v_0(1 + \alpha_p t_1)$$

Hence

$$v_1 = \frac{v(1 + \alpha_p t_1)}{1 + \alpha_p t}$$

and this rigorous equation must be used when obtaining one volume in terms of another. A similar equation must be used when calculating one pressure from another.

Equality of the Coefficients.—It may be shown theoretically, that if the gas obeys Boyle's law, $\alpha_p = \alpha_v$. For let the volume of the gas be v_0 at 0° C. and pressure p_0 , and let the temperature be raised to t° C., keeping the pressure constant. Then the volume is given by

$$v = v_0(1 + \alpha_p t)$$

If, on the contrary, the temperature is raised keeping the volume constant, the pressure is given by

$$p = p_0(1 + \alpha_v t)$$

But by Boyle's law

$$p_0 v = p v_0$$

that is

$$p_0 v_0(1 + \alpha_p t) = p_0 v_0(1 + \alpha_v t)$$

whence

$$\alpha_p = \alpha_v$$

As the gases we deal with obey Boyle's law, we shall drop the suffix and write $\alpha_p = \alpha_v = \alpha$.

Charles's Law.—The following law is known as Charles's Law :

The common gases when kept at constant pressure expand by a definite fraction of their volume at 0° C. for each degree rise in temperature. This fraction is the same for them all.

We have already stated the first part of the law by the formula for v on p. 160. The first thorough proof of the law was given by Gay Lussac in a paper published in 1802. He showed then

that the volumes of the common gases at 0° C. were to their volumes at 100° C. in the ratio 100 to 137. At a slightly earlier date Dalton in England had obtained the ratio 100 to 139 for air. Later Regnault made a very accurate determination of α , and showed that it was independent of the pressure. His result for air was 0.00366 or $\frac{1}{273}$. Charles's law holds with almost the same degree of exactitude as Boyle's law does, α being practically constant for the permanent gases for all ranges of temperature recorded by a mercury-in-glass thermometer.

Charles held a minor post in the Civil Service, but lost it during an economy campaign. He thereafter became a lecturer on popular science, and was very successful in drawing electricity from thunder-clouds—it was the epoch of Franklin's work, and science was fashionable at Paris. When the brothers Montgolfier introduced their hot-air balloons in 1783, he invented and made the first hydrogen balloon, and ascended in it to a height of 7000 ft.

Determination of α at Constant Pressure.—Fig. 159 represents a simple arrangement for determining the coefficient of expansion of a gas at constant pressure. It consists of a piece of wide

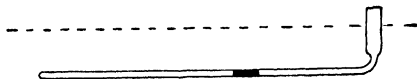


FIG. 159.

capillary tube, of uniform bore, and about 50 cm. long. One end is sealed and a wider tube is joined at right angles to the other end. A thread of mercury is introduced into the capillary so as to rest one-third of the way along it at ordinary temperatures. The capillary is kept horizontal. Thus a definite mass of gas is confined between the sealed end and the mercury thread at atmospheric pressure. The tube is immersed in a water-bath, the surface of the water being represented by the broken line, and the temperature of the water varied, first by putting ice in it and then by heating it up to boiling point. The length of the column of gas enclosed is measured at the different temperatures, and the volume is assumed to be proportional to the length.

Another method is to take a dry flask which is closed by a rubber stopper through which passes a short capillary tube, and immerse it neck upwards in a vessel containing water at 60° or 70° C. It is held down with the end of the capillary just projecting above the surface of the water. Then when the gas in the flask has attained the temperature of the water, the mouth of the tube is closed with a finger, and the flask removed and immersed completely, as quickly as possible, with its neck downwards in a large vessel of ice-cold water. The finger is then removed.

Owing to the fall of temperature, the gas contracts and water runs into the flask. When the flask has attained the temperature of the water, it is raised until the surface of the water is at the same level inside and outside it. The gas inside is then once more at atmospheric pressure. The mouth of the tube is then closed, and the flask removed and inverted. Then, when its outside surface has been dried, it is weighed. It is next completely filled with water and weighed again. Before starting the experiment it had been weighed empty. Thus we know the weight of the water that ran in at the lower temperature and the weight that would be required to fill the flask, and consequently the volume v occupied by the water that ran in and the volume V of the whole flask. The volume at 0°C. is consequently $V - v$ and the increase in volume v . Hence

$$\alpha = \frac{v}{(V - v)t}$$

where t is the difference of temperature.

Instead of weighing the flask the volumes may be obtained by a measuring cylinder.

In both the above methods the expansion of the glass is neglected, but the methods are not accurate enough to make it worth while taking the expansion of the glass into consideration.

Determination of α at Constant Pressure. Regnault's Method.—Fig. 160 represents an accurate method for determining α at constant pressure, the method which Regnault found most satisfactory for the purpose. A is a glass bulb connected by a capillary tube through the three-way stop-cock at E with the graduated tube B. The lower part of B is filled with mercury; D is a tap by which the mercury can be drawn off, and the tube C is open to the atmosphere. The tubes B and C are immersed in water in order to keep their temperature constant.

The gas is admitted to the bulb A through the stop-cock E after passing through drying tubes, and the process of exhaustion and refilling repeated several times, the bulb being gently heated, so as to remove all traces of the moisture which is always present on a glass surface. The cock is then turned so that A and B are in communication, and the tube by which the gas entered closed. The bulb is next immersed in melting ice, and the level of the mercury

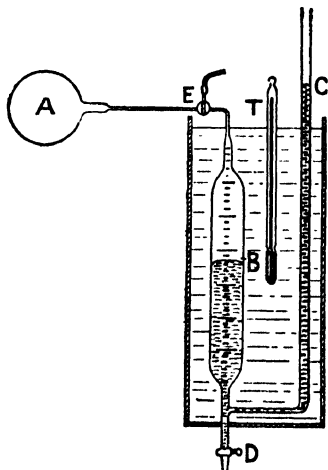


FIG. 160.—Regnault's apparatus for determining α at constant pressure.

at B and C noted ; the pressure of the gas is equal to the atmospheric pressure increased by the pressure due to the difference in level of C and B. The bulb A is then heated to a known temperature. The air expands, pushing the surface down at B and up at C, but mercury is withdrawn at D until the difference in level, and consequently the pressure on the gas, is the same as before. The increase in volume is then obtained from the graduations on B.

The result has to be corrected for the expansion of the bulb and for the fact that the gas in the graduated tube is not at the same temperature as the gas in the bulb. The volumes of the bulb and of the different portions of the graduated tube are obtained by weighing them empty and full of mercury.

Miller's Air Thermometer.—Miller's air thermometer (Fig. 161) is a piece of apparatus for determining α at constant pressure, very suitable for use in schools. The air or other gas is introduced through the stopcock at K. S is a siphon connected by the rubber tube GF with the long cylindrical measuring glass.

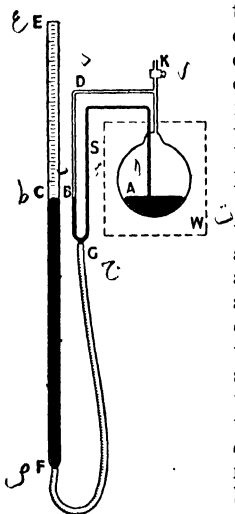


FIG. 161.—Miller's air thermometer.

The latter can be raised and lowered in a vertical direction. The tube B indicates the level of the mercury in the bulb; the latter cannot be seen for the copper heater W, which is shown in outline.

In performing the experiment the heater is filled with ice-cold water, and the measuring cylinder adjusted until the level of the mercury is the same at B and C. The pressure in the bulb is then atmospheric. The water is next heated; the air expands, driving mercury out of the bulb through the siphon and lowering the level at A and B simultaneously. The cylinder EF is then lowered until the level at C is the same as at B. The volume of mercury which has run into the cylinder during the operation is then read off; it measures the increase in volume of the air in the bulb during the change of temperature. And the pressure is, of course, again atmospheric.

In order to determine the initial volume of the air the temperature is brought back to 0° C. and the pressure to atmospheric. The stopcock at K is then opened, and mercury poured into the cylinder, until the bulb is filled with mercury. The difference of the volume of the mercury poured in and the increase in volume of the mercury in the cylinder then gives the volume of the bulb. Sometimes the apparatus is fitted with a tap at F for withdrawing mercury.

Determination of α at Constant Volume.—Fig. 162 represents an apparatus for determining α at constant volume much used in students' courses. The gas to be experimented on is contained in the glass bulb A; this is made in one piece with the tube BC, the whole being kept fixed in position by supports not shown in

the figure. The glass tube BC is connected by rubber tubing with the vertical glass tube DFG; the latter can be moved up and down and clamped in different positions on the scale. The lower parts of both tubes and the rubber tubing contain mercury. The bulb A is surrounded by a can W containing water, the position of which is indicated by the dotted lines. A mark is engraved on the left-hand tube at C. The position of the right-hand tube is adjusted, until the surface of the mercury in the left-hand tube just rises to this mark, then the difference of level of the mercury in the two tubes and the temperature t of the water in the can are noted. The water is then heated to temperature t_1 , the right-hand tube raised up the vertical scale until the surface of the mercury in the left-hand tube is brought back to the mark at C, and the difference of level in the two tubes noted. The volume of the gas enclosed by the mercury is thus the same in each case. The pressure is obtained by adding the barometric pressure to the difference of level of the mercury in the two tubes; let p be its value in the first case, and p_1 its value in the second case. Then

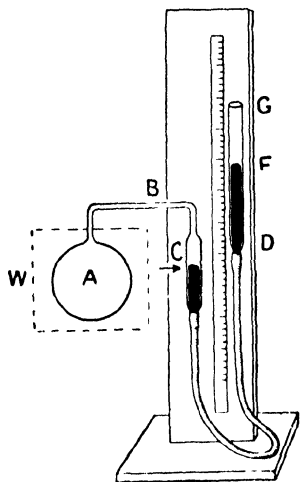


FIG. 162. Apparatus for determining α at constant volume.

$$p = p_0(1 + \alpha t) \quad \text{and} \quad p_1 = p_0(1 + \alpha t_1)$$

whence

$$\frac{p}{p_1} = \frac{1 + \alpha t}{1 + \alpha t_1}$$

and

$$\alpha = \frac{p_1 - p}{p t_1 - p_1 t}$$

The volume of gas in the part of the tube protruding out of the water should be small in comparison with the volume of the bulb. If the right-hand tube is left in too high a position, when the gas is cooled the mercury sometimes runs over into the bulb, and there is difficulty in getting it out again.

Air Thermometer.—So far we have measured temperature on the mercury-in-glass scale. The facts that the different gases have all the same coefficient of expansion, that a gas can be used as thermometric substance a long way below the freezing point and above the boiling point of mercury, and that the expansion of gases is so much greater than the expansion of their containing

vessels that the latter is comparatively unimportant, are all strong reasons for using a gas as thermometric substance instead of mercury.

Accordingly the International Committee of Weights and Measures (1887) has taken as its standard thermometric scale that of the constant volume hydrogen thermometer. The latter instrument is in principle like the apparatus shown in Fig. 162, only it has the top of the right-hand tube closed and the space FG a vacuum, so the pressure on the gas is obtained directly from the difference in level of the two mercury surfaces without the use of a barometer; this makes the scale independent of the fluctuations of the pressure of the atmosphere. The bulb is filled with hydrogen. The surface of the mercury is always brought back to the mark C. To calibrate the instrument the bulb is immersed in melting ice and the level of F' marked on the scale; it is then immersed in steam given off by water boiling under standard pressure and the level of F' again marked. These two marks are taken as the 0° and 100° C. graduations, and the vertical scale is divided into 100 equal parts between the marks, and into parts of the same length above the 100° C. mark and below the 0° C. mark.

The hydrogen thermometer on account of its size is seldom used directly, but forms a standard of reference for correcting mercury thermometers by. The maximum divergence between the mercury thermometer and the hydrogen thermometer is less than 0.2° C. for the range 0° to 100° C. Mercury thermometers made of different kinds of glass differ, of course, slightly from one another. By using a porcelain vessel to contain the gas temperatures as high as 1100° C. can be measured by the hydrogen thermometer.

Absolute Temperature.—The pressure of the hydrogen in the constant volume hydrogen thermometer is given by the formula

$$p = p_0(1 + \alpha t)$$

t being the temperature on the instrument's own scale. Suppose t is negative; then as the temperature decreases p decreases, and finally when $t = -\frac{1}{\alpha}$, or -273° C., the pressure becomes zero.

As it is impossible to think of the pressure of a gas being less than zero, this temperature is considered to be the lowest temperature possible and to denote entire absence of heat; it is termed the "absolute zero." When temperature is measured in degrees Centigrade from the absolute zero, it is referred to as "absolute temperature," and is said to be measured in degrees absolute; thus 100° C. is 373° abs., -10° C. is 263° abs.

At present the name degrees Kelvin is coming into use instead of degrees absolute, especially in America; thus $100^{\circ}\text{C.} = 373^{\circ}\text{K.}$ Kelvin's name has been associated with this scale because he first put it upon a rigorous basis.

The lowest temperature that has ever been reached is that of the liquefaction of helium, namely -271.3°C. Helium was liquefied by Kamerlingh Onnes in 1911.

The Gas Equation.—Let us suppose that at temperature t the pressure and volume of a mass of gas are given by p and v . Keep the pressure constant, but decrease the temperature to 0°C. and suppose the volume decreases to v_1 . Then

$$pv = pv_1(1 + \alpha t)$$

Now change the pressure p to standard atmospheric pressure, denoting the latter by p_0 and at the same time let v_1 become v_0 , the temperature remaining constant. Then by Boyle's law

$$pv_1 = p_0v_0$$

$$pv = p_0v_0(1 + \alpha t)$$

Hence

Let T denote the absolute temperature of the gas. Then

$$T = t + \frac{1}{\alpha} = \frac{1 + \alpha t}{\alpha}$$

Consequently, on substituting in the previous equation,

$$\frac{pv}{T} = \alpha p_0v_0 = \text{const.} = R$$

The equation

$$\frac{pv}{T} = R$$

is referred to as the gas equation, equation of state, or characteristic equation of a gas, and is of great use in solving numerical examples. As it is derived from Boyle's law and Charles's law, it holds naturally only for gases for which these laws hold.

It follows from the gas equation, that if the volume is constant, the pressure varies as the absolute temperature, and that if the pressure is constant, the volume varies as the absolute temperature.

The Gas Constant.—Let m be the mass of the molecule of a gas and N the number of molecules per unit volume under standard conditions, that is, under atmospheric pressure p_0 and at 0°C. Then Nm is its density and $1/(Nm)$ the volume of unit mass. The molecular weight of a gas is the ratio of the mass of one molecule to the mass of one atom of hydrogen, and the number of grams of a substance equal to the number which represents its molecular weight, is spoken of as its **gram-molecule**. The gram-molecule of the gas is m/m_H gm., where

m_H is the mass of an atom of hydrogen. The volume of a gram-molecule of the gas under standard conditions is consequently

$$\frac{m}{m_H} \times \frac{1}{Nm} = \frac{1}{Nm_H}$$

The gas equation becomes, for standard conditions,

$$\frac{p_0 v_0}{T_0} = R$$

If the mass of the gas is one gram-molecule, this becomes

$$\frac{p_0}{Nm_H T_0} = R$$

But by Avogadro's hypothesis N is the same for all gases. Hence if we assume that the gas equation refers to a mass of one gram-molecule, R has the same value for all gases.

EXAMPLE.—Given that the volume of a gas is 2300 c.c. at 0°C . and a pressure of 76 cm., what is it at 15°C . and a pressure of 78 cm. ?

$$\begin{aligned} \text{The required volume is } 2300 \frac{273+15}{273} \cdot \frac{76}{78} \\ = 2363 \text{ c.c.} \end{aligned}$$

EXAMPLES III

1. Gas is contained in a vessel of 2000 litres capacity at 0°C . and under a pressure of 74 cm. What does the pressure become when the temperature rises to 24°C . ?

2. Compare the densities of the air at the top and bottom of a mine shaft. The temperature and pressure are 10°C . and 30 inches at the top and 14°C . and 30.5 inches at the foot.

3. At what temperature will a litre of air weigh a gram when under a pressure of 74 cm. ? A litre of air weighs 1.293 gm. at 0°C . and 76 cm.

4. An empty flask with a capacity of 1 litre is immersed in water at 80°C . with only the mouth above the surface of the water, until the air inside it takes the temperature of the water. Its mouth is then closed with the thumb, it is suddenly inverted, and completely immersed in water at 14°C ., and the thumb is removed. What weight of water enters the flask ?

5. A balloon at its utmost capacity can hold 1000 cubic ft. of gas. 900 cubic ft. are let into it at a pressure of 29 inches and a temperature of 40°F . The valve then chokes, and the balloon rises into a region where the pressure is 27 inches and the sun's rays heat it to 70°F . Will it burst ?

6. Ten cubic feet of dry air measured at 32°F ., when the barometric height is 28 inches, are pumped into a globe whose diameter is 8 inches. The globe is then heated to 200°F . Find the pressure.

CHAPTER IV

CALORIMETRY

Fundamental Experiment.—When a pot of water is heated on the fire, we regard the increase in temperature of the water as due to something called heat passing to the water from the fire. It is now necessary to get more definite ideas on the subject. It is a well-known fact, that when the pot is full, it takes longer to boil than when it is only one quarter full. Thus, when the heat is distributed over a larger quantity of water, the temperature rises more slowly. But a pot of water on a fire is not suitable for accurate experimental work.

Better results can be obtained with an immersion electric heater. This gives out heat at a constant rate, and nearly all this heat goes to warm the water. It is found, that if the heater is immersed in a vessel containing water and the latter kept well stirred, the temperature read by a thermometer dipping into the water rises at a constant rate. If the mass of water is doubled, the temperature rises at half the former rate. If the mass is halved, the temperature rises at twice its original rate. If the water is replaced by an equal mass of alcohol or of some other liquid, the rate at which the temperature rises is changed. If an additional heater similar to the first one is placed in the water, the rate at which the temperature rises is doubled. Thus we see that the rise of temperature in a given time is directly proportional to the heat supplied in the time and inversely proportional to the mass of the liquid ; it depends also on the nature of the liquid. Putting the results in the form of an equation, we have

$$t_2 - t_1 = \frac{Q}{ms}$$

where t_1 and t_2 are respectively the initial and final temperatures, Q is the quantity of heat, m the mass of the substance, and s a constant of proportionality. The equation can be rewritten

$$Q = ms(t_2 - t_1)$$

Thus the quantity of heat supplied to a substance is proportional to its mass and to the rise in temperature produced.

Definitions.—We shall now define **unit quantity of heat**.

Unit quantity of heat is the amount of heat necessary to raise one gram of water 1° C.

Water is chosen as the standard substance simply as a matter of convenience. The unit as thus defined is called the **calorie** and sometimes the gram-degree. The quantity of heat necessary to raise the temperature of one gram of water 1° C. is not rigorously the same at all temperatures, but decreases from 0° C. to a minimum at 37.5° C., and then increases to 100° C., the extreme variation over the range 0° to 100° C. being less than 1 per cent. Hence in very accurate work it is necessary to specify for what particular 1° interval the calorie is defined; the interval most generally chosen is from 15° to 16° C.

Since there are different units of mass and more than one scale of temperature, the unit of heat might have been defined otherwise. The calorie is the unit invariably used in scientific work. If the unit of mass is a kilogram, the unit of heat is called a large calorie. The heat required to raise 1 lb. of water 1° F. is used as a unit in engineering, and is called the **British thermal unit**; it is denoted by the letters B.Th.U. The heat required to raise 1 lb. of water 1° C. is also used as a unit in Great Britain, being known as the Centigrade unit (lb. deg. Cent.).

The gas companies have a special unit of their own for measuring heat, called the **therm**. It is the quantity of heat required to raise 1000 lbs. of water through 100° F., and thus equals 100,000 British thermal units. This is approximately 29 times as great as the kilowatt-hour, the unit used by the electricity companies in charging for heat and energy.

*The **specific heat** of a substance is the quantity of heat required to raise the temperature of one gram of the substance 1° C.*

If in the equation

$$Q = ms(t_2 - t_1)$$

we write $m = 1$ gm. and $t_2 - t_1 = 1^{\circ}$ C., Q becomes $= s$. Consequently in this equation s denotes the specific heat of the substance.

*The **thermal capacity** of a body is the quantity of heat required to raise its temperature 1° C.*

Thus the specific heat is the thermal capacity of unit mass. The specific heat of copper, for example, is 0.093; the thermal capacity of a lump of copper weighing 12.2 gm. is $12.2 \times 0.093 = 1.135$ calories, and the quantity of heat necessary to raise its temperature from 10.2° C. to 15.3° C. is $12.2 \times 0.093 \times 5.1 = 5.79$ calories.

It follows from the definition, that the specific heat of water is unity. The thermal capacity of a given quantity of water is consequently equal to its mass. The thermal capacity of a body

is often referred to as its **water equivalent**, because it gives the mass of water which would absorb the same quantity of heat as the body under the same circumstances.

Law of Mixtures.—The science of the measurement of quantities of heat is called calorimetry, and the piece of apparatus used for the purpose is called a **calorimeter**. The most common form of calorimeter consists simply of a cylindrical copper vessel which is provided with a copper stirrer, and is suspended on three threads inside another cylindrical vessel. The outer surface of the inner cylinder and the inner surface of the outer cylinder are both highly polished. If the inner vessel is filled with warm water, heat cannot escape from it except by conduction along the threads or by radiation and convection from the one polished surface to the other. Now the radiation loss from a polished surface is small, and since the outer cylinder shields the inner one from draughts, the convection loss is also small. Thus the whole loss of heat from the inner vessel is small.

It is found as a result of experiment, when solids and liquids are mixed together in such a calorimeter, that there is no heat gained or lost by the calorimeter and its contents as a whole, provided that chemical action does not take place between the different substances. This result is known as the law of mixtures. It can be stated in the following very convenient form :

The heat lost by the hot bodies is equal to the heat gained by the cold ones.

We shall now proceed to make some applications of this law.

Determination of the Water Equivalent of a Calorimeter.—In applying the law of mixtures the calorimeter itself must be taken as one of the bodies involved, and it becomes important to determine its water equivalent or thermal capacity. This can be done experimentally in the following manner :

The inner vessel of the calorimeter is first weighed empty, then rather more than half full of cold water, and so the mass m of the water is determined. It is then replaced in the outer vessel, and the temperature of the water t noted. Next some water at a higher temperature T is poured in, sufficient to raise the temperature of the water in the calorimeter about 3° ; the mixture is stirred well, and θ , the exact value of the highest temperature reached, noted. The inner vessel is then weighed again, and a determination made of M , the mass of the hot water poured into it.

When the mixture is stirred, the hot water loses heat, and the cold water and the metal of the calorimeter gain heat. The heat lost by the hot water is

$$M(T - \theta)$$

The heat gained by the cold water is

$$m(\theta - t)$$

Let w be the water equivalent of the calorimeter. Then the heat gained by the calorimeter is

$$w(\theta - t)$$

Equating heat lost and heat gained, we obtain

$$M(T - \theta) = m(\theta - t) + w(\theta - t)$$

whence
$$w = \frac{M(T - \theta)}{\theta - t} - m$$

The method does not give accurate results, since the two quantities on the right of the above equation are nearly equal in size, and experimental error in determining the first of them has thus a large influence on the result. The latter may be found more accurately by multiplying the mass of the stirrer and of the inner vessel of the calorimeter by the specific heat of the metal of which they are composed.

EXAMPLE.—A calorimeter contains 80.2 gm. of water at 14.9° C. To this 42.9 gm. at 30.2° C. are added, and, when the mixture is stirred, its temperature is found to be 20.1° C. Find the water equivalent of the calorimeter.

Let w = water equivalent.

Then Heat gained = $(80.2 + w)(20.1 - 14.9) = 417.0 + 5.2w$

Heat lost = $42.9(30.2 - 20.1) = 433.0$

Hence $5.2w = 433.0 - 417.0 = 16.0$

and $w = 3.1$ gm.

Determination of Specific Heat.—In determining the specific heat of a solid the inner vessel of the calorimeter is first weighed empty, then rather more than half full of cold water, and m , the mass of the cold water, determined. It is next replaced in its outer vessel and t , the temperature of the water, is taken. Then the specimen, which has meantime been heated to a high temperature T , is dropped into the water, the water is stirred and θ , the highest temperature reached, is noted. The inner vessel is next weighed and M , the mass of the specimen, determined. Let w be, as before, the water equivalent of the calorimeter, and let s be the specific heat of the specimen.

The heat lost by the specimen is

$$Ms(T - \theta)$$

The heat gained by the water and calorimeter is

$$(m + w)(\theta - t)$$

On equating we obtain

$$Ms(T-\theta) = (m+w)(\theta-t)$$

whence

$$s = \frac{(m+w)(\theta-t)}{M(T-\theta)}$$

In performing the experiment it is very important that the temperature of the specimen should be accurately determined, and that it should be dropped rapidly into the water without having time to lose heat to the air. For this purpose special pieces of apparatus have been devised, and Fig. 163 represents a section of a simple and very efficient type. It consists of a fixed metal tube AB bent at an obtuse angle, and surrounded by a steam or water jacket. Telescoping into the vertical part of the tube is another movable one, which is closed at the top by a cork carrying a thermometer C. The cork is removed and the specimen dropped into this tube. When the temperature indicated by the thermometer has been steady for five minutes, the upper tube is raised, and the specimen falls out of the heater in the direction of the arrow into the calorimeter.

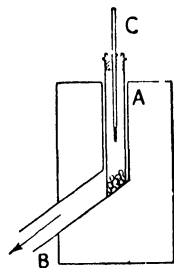


FIG. 163.—Heater for calorimetry.

The specific heat of a liquid can be determined by half filling the calorimeter with it, instead of with water, and dropping in a hot solid of known specific heat. The reasoning given above then still holds, if m be taken to denote the thermal capacity of the liquid.

A small tin without any outer vessel, such as a $\frac{1}{4}$ -lb. cocoa tin, may be used instead of a calorimeter. But it has the disadvantage of losing heat through its bottom to the table on which it rests, and so does not give such accurate results.

In solving problems on calorimetry it is better not to memorise formulae but to work out each case from first principles, equating heat gained to heat lost.

EXAMPLE.—An aluminium calorimeter of mass 26.2 gm. contains 83.8 gm. of water at 14.6° C. A piece of aluminium is heated to 100.1° C. and dropped into the water. What is its mass, given that the final temperature of the water is 19.7° C. and the specific heat of aluminium is 0.21?

Let M be the mass.

$$\begin{aligned} \text{Then Heat gained} &= (83.8 + 26.2 \times 0.21)(19.7 - 14.6) \\ &= 89.3 \times 5.1 = 455.4 \end{aligned}$$

$$\begin{aligned} \text{Heat lost} &= 0.21M(100.1 - 19.7) \\ &= 0.21 \times M \times 80.4 = 16.88M \end{aligned}$$

$$\text{Hence } 16.88M = 455.4 \text{ and } M = 26.93 \text{ gm.}$$

Law of Dulong and Petit.—The product of the atomic weight of an element and its specific heat is constant, and equal to about 6.4. This law was given by Dulong and Petit in 1818 ; it can be expressed in the equation

$$w_1 s_1 = w_2 s_2$$

where w_1 and w_2 are the atomic weights, and s_1 and s_2 the specific heats of any two elements. Since the number of atoms of the first element in w_1 grams is equal to the number of atoms of the second element in w_2 grams, there is the same number of atoms on both sides of the equation, and the law amounts to stating that the thermal capacity of every atom is the same. It is not strictly true, boron, carbon, and silicon being exceptions at ordinary temperatures. But these elements obey the law at high temperatures, and it is now assumed that the specific heats of all the elements decrease to zero at the absolute zero of temperature.

Latent Heat.—When a vessel containing a mixture of ice and water is warmed, it is found that the temperature remains at 0° C. as long as any ice remains unmelted, provided that uniformity of temperature is maintained throughout the vessel by stirring. Since heat is being supplied, and the temperature does not rise, it follows that the heat must be used for another purpose, namely, for melting the ice. In the same way if a large piece of ice is taken with a cup-shaped depression on its upper surface, and a red-hot lump of copper placed in the cup, ice is melted, and both the lump of copper and the water produced take the temperature of melting ice. Here the copper has lost its heat, and there is nowhere the heat could go except into the water. Consequently, to use an expression introduced by Joseph Black, who first discovered what happened, the heat is latent in the water.

Other substances behave in the same way as water in this respect. To convert any of them from the solid to the liquid state without a change of temperature a definite quantity of heat is required. This is known as the **latent heat of fusion**. Conversely, when the substance passes from the liquid to the solid state, the same quantity of heat is given out again. To put the definition in a sentence :

The latent heat of fusion of a substance is the quantity of heat required to convert unit mass of the substance from the solid to the liquid state without change of temperature.

When water starts to boil, its temperature remains constant no matter how much it is heated. The heat supplied, since it is not used for raising the temperature, must be used for changing the state. It consequently becomes latent in the steam, and is called the latent heat of steam or the **latent heat of vaporisation of water**. It is given out again when the steam condenses.

Other liquids behave in the same way as water in this respect. We have consequently the definition :

The latent heat of vaporisation of a substance is the quantity of heat required to convert unit mass of the substance from the state of liquid to the state of vapour without change of temperature.

The latent heat of steam varies with the temperature at which vaporisation takes place. If Q , the total heat of steam, is defined as the quantity of heat necessary to raise one gram of water from 0° C. to a temperature of t° C. and evaporate it at that temperature, then, according to Regnault, measured in calories,

$$Q = 606.5 + 0.305t$$

This equation, which is not very accurate at low temperatures, was proved for a range of from 63° to 194° C. If we put $t = 100$,

$$Q = 606.5 + 30.5 = 637$$

Hence if we subtract 100, the quantity of heat necessary to raise one gram of water from 0° to 100° C., we find that the latent heat of steam at 100° C. is 537 calories.

Determination of Latent Heat of Water.—The latent heat of water or latent heat of fusion of ice can be determined with an ordinary calorimeter as follows :

Fill the calorimeter more than half full with m grams of warm water, and let w be its water equivalent. Let t be the temperature of the water. Dry two or three lumps of ice about the size of walnuts on blotting-paper, drop them into the calorimeter, stir, note the lowest temperature reached (θ), and then weigh the inner vessel so as to find M , the mass of the ice dissolved. Then, if L denotes the latent heat of water, the amount of heat gained by the ice in melting is ML , and in being raised to temperature θ is $M\theta$. The amount of heat lost by the water in being cooled from t to θ is $m(t - \theta)$, and the amount of heat lost by the calorimeter in being cooled through the same range is $w(t - \theta)$. Equating heat gained to heat lost, we obtain

$$M(L + \theta) = (m + w)(t - \theta)$$

which gives

$$L = \frac{(m + w)(t - \theta)}{M} - \theta$$

Thus L can be calculated. The correct value is 80 calories.

Determination of Latent Heat of Steam.—Fig. 164 shows a simple arrangement sometimes adopted for determining the latent heat of steam at atmospheric pressure. As calorimeter a rectangular copper box is used. This is filled to the level indicated with water, and contains a copper condenser with entrance and exit tubes A and B showing above the surface of the water. In

the neck of the flask there is a cork into which a short piece of glass tube fits; to this glass tube is attached a short piece of rubber tubing. The experiment is performed by waiting until

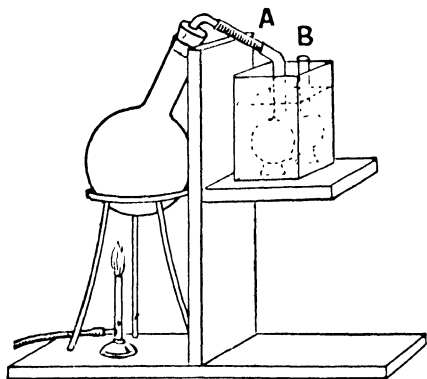


FIG. 164.—Apparatus for determining latent heat of steam.

the water in the flask is boiling vigorously, then inserting the rubber tube in the entrance tube of the condenser for a minute or two. The steam which does not condense escapes by the exit tube. After the rubber tube is disconnected, the water in the calorimeter is stirred, and the highest temperature reached is noted.

Let L be the latent heat of steam, w the water equivalent of the calorimeter and condenser, m the original mass of water, M

the mass of steam condensed, t the initial temperature of the water, and θ its final temperature. The heat gained by the calorimeter and water in being raised from t° to θ° is

$$(m+w)(\theta-t)$$

The heat lost by the steam in condensing is

$$ML$$

and in being cooled from 100° to θ° C. is

$$M(100-\theta)$$

Equating heat gained to heat lost,

$$(m+w)(\theta-t)=ML+M(100-\theta)$$

whence

$$L=\frac{(m+w)(\theta-t)}{M}+\theta-100$$

Convection Correction.—It has hitherto been assumed, that no heat escapes from or to the calorimeter, during the course of the experiment. This is not strictly true, and for accurate work the radiation loss or convection loss, as it ought to be called, for convection plays a larger part in it than radiation, must be corrected for.

Fig. 165 shows how to do this in the case of a determination of the latent heat of steam. The crosses represent temperature readings of the water in the calorimeter plotted as a function of the time. The temperature of the room was 15.0° C. At 0 min. the temperature of the water was 16.0° C. At 1 min. it was still 16.0° C., and the steam

was connected on. At 2 min. it was 25.0°C. , and the steam was disconnected. At 3 min. it was 29.0°C. At $3\frac{1}{2}$ min. the temperature reached its highest value, namely, 29.5° ; thereafter the temperature began to fall. Now the rise of temperature took $2\frac{1}{2}$ min., from the vertical A to the dotted vertical B. In the equal time from B to C the fall of temperature was 0.7°C. The fall of temperature per min. is roughly proportional to the temperature difference of the calorimeter and the room. The average difference of temperature in the interval AB is 9.9° , and the average difference of temperature in the interval BC is 14.2° . Hence we are justified in supposing that there was a loss of $\frac{9.9}{14.2} \times 0.7^{\circ}$ during the interval AB, and in adding this as a correction to the highest temperature reached, in the expression for the heat gained by the water and calorimeter; θ in this expression should consequently be $29.5 + 0.5 = 30^{\circ}\text{C.}$ But in the expression for the heat lost by the steam θ should be unaffected, for the temperature of the condensed steam falls to 29.5°C. , not 30°C.

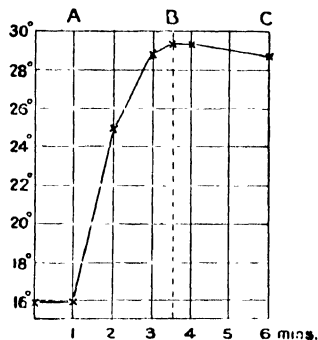


FIG. 165.

The average difference of temperature in the above calculation was obtained graphically by measuring the area between the curve and the axis of abscissæ.

A less accurate way of applying the correction consists in measuring the rates of cooling before passing in the steam and after the maximum temperature has been attained, and assuming that the rate of cooling during the experiment is the mean of these two values. It is multiplied by the time the rise of temperature takes and then added as a correction to the latter.

Other cases may be treated similarly. Convection and radiation corrections should be used with extreme caution, and the extent to which they affect a result should always be clearly indicated.

In determining the latent heat of water it is sometimes advised that the initial temperature of the water should be as high above the temperature of the room as the final temperature is below it, the idea being that the convection loss to the room during the first part of the experiment should be equal to the gain from the room during the second part. But the temperature falls most rapidly at first, and the convection loss cannot be eliminated in this way.

Bunsen's Ice Calorimeter.—When water freezes, its volume increases, 1 c.c. of water forming 1.09 c.c. of ice. This change of volume has been utilised by Bunsen in the construction of a calorimeter which is more sensitive and accurate than the simple calorimeter already described.

Bunsen's ice calorimeter is represented in Fig. 166. A thin-

walled test tube A is sealed into a wider glass vessel B which terminates in a tube CD bent thrice at right angles. The tube and lower part of the glass vessel are filled with mercury. The upper part of the glass vessel is filled with distilled water which has been boiled to free it from dissolved air. The upper portion of CD is a capillary of uniform bore, and the position of the end of the mercury thread can be read on a scale.

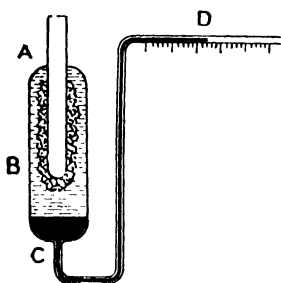


FIG. 166.—Bunsen's ice calorimeter.

Before using the instrument it is surrounded with a bath of melting ice. A freezing mixture is then placed in A until the water round about it is frozen. When a sufficient coating of ice has been formed, the freezing mixture is

removed, the end of the mercury thread at D is read, and the hot body placed in the test tube. The heat given out melts some of the ice and diminishes its volume. The surface of the mercury in B rises, and the end of the thread at D moves towards the left. When it becomes steady, its position is noted, and from the difference in the two readings the amount of heat given out by the hot body can be calculated.

The scale is usually calibrated by pouring a quantity of hot water into A and measuring the displacement produced by the known quantity of heat it contains. The value of its divisions may also be obtained by calculation, if the bore of the capillary is known. For let Q be the quantity of heat given out, let L be the latent heat of water, and let a be the cross-sectional area of the capillary. Then the mass of ice melted is Q/L , the diminution in volume is $0.09Q/L$ c.c., and the displacement is

$$\frac{0.09Q}{aL}$$

If a is 1 sq. mm. and the displacement is 1 mm., Q works out at 0.89 calorie. The instrument can thus be used for measuring very small quantities of heat. It has, of course, the advantages that no thermometer is required, there is no water equivalent to take into consideration, and there is no convection loss.

Joly's Steam Calorimeter.—Fig. 167 represents one form of Joly's steam calorimeter, arranged for determining the specific heat of gases at constant volume. The gas is contained

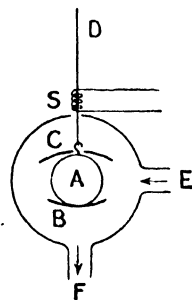


FIG. 167.—Joly's steam calorimeter.

in a copper sphere A which is suspended by a wire D from the pan of a balance. The sphere and the gas it contains are allowed to take up the temperature of the room. Steam is then allowed to enter through the channel E and to pass out through the channel F. It condenses on the sphere until the temperature of the latter is raised to 100°C. ; the mass condensed multiplied by the latent heat of steam gives the amount of heat necessary to raise the copper sphere and gas contained to this temperature. The mass condensed collects on the pan B, and is weighed by the balance. C is a shield which prevents condensed steam dropping on the sphere from above, and in order to prevent the condensation of steam on the wire D, the latter is heated electrically by the spiral S through which it passes without touching.

British Thermal Unit.—Specific heat was defined on p. 170, as the thermal capacity of unit mass. It might have been defined as the ratio of the thermal capacity of unit mass to the thermal capacity of unit mass of water, as is done in some books, and the numerical value would be the same, since the thermal capacity of water is 1 calorie. When defined in this way it is simply a ratio, and independent of the units employed. Now the thermal capacity of unit mass of water is also 1 when measured in British thermal units. It follows that specific heat, when defined as is done in this book, has the same value in B.Th.U. as in calories.

It is otherwise, however, with the latent heats. The heat required to melt 1 lb. of ice raises 1 lb. water through 80°C. which is equivalent to $\frac{9}{5} \times 80$, *i.e.* 144°F. ; hence the latent heat of water is 144 B.Th.U. The latent heat of steam is similarly $\frac{9}{5} \times 537 = 967$ B.Th.U. But the latent heats have the same values in lb. deg. Cent. units as in calories.

Calorific Value of a Fuel.—The calorific value of a fuel is the amount of heat given out when a definite weight, for example 1 lb., is burned. It is measured by the bomb calorimeter. In making the measurement a small quantity of the fuel is powdered and dried, and placed inside a strong steel shell or bomb. This bomb has a gas-tight cover and is filled with oxygen at high pressure. It is then immersed in the calorimeter, and when it has taken up the same temperature as the water, the fuel is fired; this is done by passing an electric current through a wire in contact with the fuel, so as to make it red hot. The fuel burns up completely in the excess of oxygen. The water in the calorimeter is stirred, and the highest temperature reached is noted. In calculating the result the water equivalent of the bomb must be reckoned in with the water equivalent of the calorimeter.

The heat produced by the complete combustion of one gram of good coal is about 8000 calories.

EXAMPLES IV

1. A calorimeter contains water at 40°C. , the equivalent mass of water being 100 gm. In order to cool the water (a) 30 gm. of water are withdrawn and replaced by 30 gm. of water at 10°C. ; or (b) 15 gm.

of water are withdrawn and replaced by 15 gm. of water at 10°C. , and, after stirring, this process is repeated. Assuming there is no cooling due to other causes, determine the final temperature in the two cases.

2. In determining the latent heat of water the mass of the calorimeter (inner vessel) was 39.31 gm., the mass of calorimeter and warm water 99.90 gm., the initial and final temperatures 36.0°C. , and 16.2°C. , and the mass of calorimeter, water, and ice melted 113.4 gm. Work out the result on the assumption that the water equivalent is $\frac{1}{10}$ the mass of the inner vessel of the calorimeter.

3. In a second attempt with the same calorimeter the mass of the calorimeter and warm water was 102.02 gm., the initial and final temperatures were 34.6° and 18.0°C. , the mass of calorimeter, water, and ice melted 113.00 gm. Work out the result.

4. In determining the specific heat of lead the mass of calorimeter and stirrer is 46.50 gm., the mass of calorimeter, stirrer, and cold water 103.15 gm., the lead is heated to a temperature of 93.3°C. , the initial and final temperatures of the water are 15.7° and 28.1°C. , and the mass of the lead is 359.0 gm. Work out the result. Assume that the water equivalent of the calorimeter is $\frac{1}{10}$ the mass of the inner vessel and stirrer.

5. The following data were obtained in a determination of the latent heat of steam: Mass of calorimeter and condenser 304.95 gm., mass of cold water 305.25 gm., initial temperature 21.0°C. , final temperature 32.0°C. , mass of steam condensed 6.3 gm. The water equivalent of the calorimeter and condenser can be taken as $\frac{1}{10}$ of their mass. Work out the result.

6. The following data were obtained in a determination of the specific heat of a metal: Mass of calorimeter 49.22 gm., of calorimeter and water 117.92 gm., temperature of heater 97.5°C. , initial temperature of water 15.5°C. , final temperature of water 19.4°C. , mass of calorimeter, water, and metal 172.9 gm. Work out the result on the usual assumption as to the water equivalent of the calorimeter.

7. A calorimeter contains 129.4 gm. of water at 16.2°C. To this 38.7 gm. are added at 37.3°C. , and, after stirring, the temperature of the mixture is found to be 21.0°C. What is the water equivalent of the calorimeter?

8. What is the temperature of a piece of iron weighing 5.2 lb., which, when immersed in 11.0 lb. of water, raises its temperature from 14.0° to 26.4°C. ? The specific heat of iron is 0.119.

9. A piece of silver weighing 12.6 gm. was heated to 101.4°C. and dropped into 68.2 gm. of turpentine at 11.2°C. The final temperature was 13.0°C. The water equivalent of the calorimeter was 3.1 gm. The specific heat of silver is 0.056. Find the specific heat of turpentine.

10. A glass bottle of weight 3 oz. contains 1 lb. of water, the glass and water being at 20°C. ; 4 lb. of mercury at 45°C. are added. What is the common temperature when the mercury and water have been shaken together? The specific heats of mercury and glass are 0.033 and 0.2 respectively.

11. The following figures were obtained with a latent heat of steam apparatus: Water equivalent of calorimeter and condenser 30.2 gm.

weight of cold water 288.1 gm., initial temperature 16.3°C . Steam was then passed into the condenser, the increase in weight being 7.2 gm., but 0.5 gm. of this condensed in the neck of the flask, and was blown into the condenser as water. What ought the final temperature of the water in the calorimeter to be?

12. Equal volumes of alcohol at 5°C . and turpentine at 60°C . are shaken together; find the temperature of the mixture. The specific gravities of alcohol and turpentine are 0.80 and 0.87, and their specific heats are 0.60 and 0.42.

13. The specific heat of ice is 0.5. What quantity of heat will be required to change 10.2 gm. of ice at -7.6°C . to water at 14.0°C .?

14. What is the result of mixing 1 lb. of ice at -10.0°C . with 2 lb. of water at 25.0°C .?

15. At Quito (2808 metres above sea-level) water boils at 90.1°C ., and at Madrid (610 metres above sea-level) water boils at 97.8°C . What is the value of the latent heat of steam at these two places?

16. How many calories are there in the British thermal unit?

17. A mass of 0.80 gm. of a substance was heated to 100°C . and dropped into a Bunsen's ice calorimeter. The thread of mercury was displaced through a distance of 6.9 mm. in the capillary tube of 1 sq. mm. section. If 1 gm. of water expands by 0.091 c.c. on freezing and evolves 80 calories, calculate the specific heat of the substance.

18. In a test with a bomb calorimeter 0.740 gm. of petroleum was burned. The calorimeter contained 2010 gm. water. The rise in temperature was 3.0°C . Find the calorific value of the petroleum in calories per gram, given that the water equivalent of the calorimeter and bomb was 710 gm.

19. The price of gas is 10*d.* per therm and of electricity 1*d.* per kilowatt-hour. What is the relative cost of obtaining the same quantity of heat from gas and electricity?

CHAPTER V

CHANGE OF STATE

Cooling Curve.—If ice and salt are mixed together in the proportion of 33 parts by weight of salt to 100 parts by weight of ice, a temperature as low as -20°C. can be obtained. Let us suppose that some water at room temperature is placed in a test tube with a thermometer and stirrer dipping into it, and that the whole is immersed in such a freezing mixture. Then the temperature of the water will fall. Let us suppose that it is read every minute as long as it continues to fall, and that a graph or cooling curve is drawn showing how the temperature varies with the time.

It is found that the temperature falls, until 0°C. is reached. Then ice begins to form, and the temperature remains stationary until all the water is frozen. During this time the test tube is losing heat to the freezing mixture, but the water is giving out latent heat on freezing, and this fills the place of the heat lost. Only when all the water is frozen does the temperature begin to fall once more.

Similar curves are obtained for other substances, and are used as a means of determining melting points. Fig. 168, for example, is such a curve for phenol. It is seen from the curve, that after the start of the experiment the temperature drops for six minutes until 42°C. is reached. It then remains stationary for about twenty minutes before commencing to fall once more. Thus 42°C. is the melting point of phenol.

A curve such as Fig. 168 which possesses one well-defined "step" is characteristic only of a pure substance. In the case of paraffin wax, which is a mixture, solidification takes place over a range of temperature, and there is no definite melting point; the slope of the curve changes on entering this range but does not become horizontal. Glass and sealing wax are like paraffin wax in this respect; there is an intermediate state between solid and liquid in which they are plastic. Some cooling curves again have several horizontal steps corresponding to the melting points of different constituents.

If water is freed from dissolved air, and cooled slowly and

carefully, it is possible to take it to a temperature well below 0°C . without freezing setting in. It is then said to be **super-cooled**. But if it is shaken, or if a small piece of ice is dropped in, it at once freezes, and the temperature rises to 0°C . All liquids with definite melting points exhibit the phenomenon of supercooling. It takes place only in the one direction; it is not possible to heat a solid above its melting point without its melting.

The freezing point of a solution is always lower than that of the pure solvent. The exact temperature depends on the strength

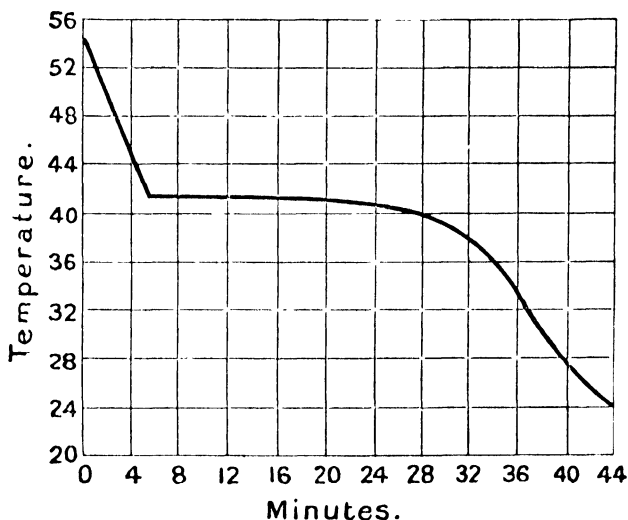


FIG. 168.—Cooling curve for phenol.

of the solution, the depression being proportional to the concentration for dilute solutions.

Determination of Melting Point.—When only a small quantity of the substance is available, the melting point can be determined as follows:

A thin-walled capillary glass tube is sealed at one end. Some of the substance is put in the tube, and the latter is fastened to a thermometer by rubber bands, with its sealed end close to the bulb. The arrangement is then supported in a beaker of water with the end of the capillary projecting above the surface of the water. The water is slowly heated and kept well stirred, and as soon as the substance melts, the temperature is noted; when the substance melts, it becomes more transparent. The water is then allowed to cool, and the temperature at which the substance freezes is noted. The two readings should not differ much

from one another, and their mean is taken as the melting point.

Change of Volume on Solidification.—When water freezes its volume increases, 1 c.c. of water forming 1.09 c.c. of ice. Since ice floats on water, it is, of course, obvious that its density must be less than that of water. The change from the liquid to the solid state is always accompanied by a change of volume, but in most cases the change is in the opposite direction, and the volume diminishes on solidification. Thus paraffin wax contracts on solidifying, and solid paraffin wax sinks when thrown into its own liquid.

When water freezes, the expansion takes place with very great force. This may be demonstrated by filling a cast-iron bottle fitted with a screw stopper with water, screwing the stopper tight, and immersing the bottle in a freezing mixture. After a time it bursts with a loud report. The experiment should be carried out under a strong cover, as fragments of the bottle may be driven to a distance.

It is the expansion of water on freezing which bursts the domestic water-pipes in winter, though the leak is of course not apparent until the thaw comes. It is also responsible for the breaking up of soil and the weathering of rocks. Water percolates into the soil, expanding when it freezes, and the soil is loosened when the thaw comes; in the same way when the water in the cracks of rocks freezes, it expands, and splits off pieces.

Sublimation.—Solids can pass directly to the state of vapour without passing through the intermediate liquid state. They are then said to sublime. Iodine crystals form a good example of sublimation, passing into a dark vapour when gently warmed. Snow and ice sublime slowly; in Arctic regions this is the only method by which they can pass into the state of vapour.

Experiment to determine the Density of Ice.—Drop a small piece of ice into methylated spirits, and add water until it just sinks. Then stir; the ice gradually melts, increasing the density of the mixture. As soon as it floats, remove it as rapidly as possible. Its density is then equal to that of the mixture of methylated spirits and water, which may be determined by the ordinary methods.

Change of Melting Point with Pressure. Regelation.—Since water expands on freezing, we would expect that increase of pressure, which renders expansion more difficult, would make a greater degree of cold necessary to produce freezing, and hence lower the freezing point. On the other hand, in the case of substances like paraffin wax which contract on freezing, we would expect increase of pressure to raise the melting point. This is actually what takes place. But the effect is a small one, being in the case of water only 0.0075° C. for an increase of pressure of one atmosphere. We can consequently neglect the

variations in the barometer, when the lower fixed point of a thermometer is being determined.

If two blocks of ice are pressed together, they become frozen into a single block. Owing to the pressure on the surfaces of contact, the freezing point is lowered below 0°C. , and some ice melts there. But on releasing the pressure the freezing point goes up again and the water freezes, binding the blocks together. This process—melting produced by pressure followed by freezing when the pressure is removed—is known as regelation. It was first noticed by Faraday, and can be demonstrated by the following striking experiment due to Tyndall :

A large block of ice is placed on two stools in a room where the temperature is 0°C. , and weights are hung over it by a copper wire in the manner shown in Fig. 169. Owing to the pressure under the wire the ice melts, and the wire sinks into it ; the water flowing round the wire is released from the pressure and freezes again. This goes on, until in a couple of hours or so the wire works its way through the block and the weights fall to the floor. But the ice is left in one block, a few bubbles marking the plane in which the wire cut through.

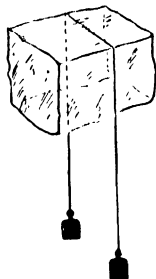


FIG. 169.—
Regelation.

We have also an example of regelation when snow is pressed together to form a snowball. The pressure melts some of the snow ; when the pressure is released, the water freezes and binds the snow together. When the edge of a skate presses on ice, it lowers the freezing point and causes the ice to melt ; hence the blade sinks in, and is enabled to “bite.”

Boiling.—If a flask of water containing a thermometer is heated over a burner, the temperature rises steadily. After a time little bubbles begin to form on the bottom of the flask, ascend a short distance, then collapse. These bubbles require something to start from, for example, a roughness on the side of the vessel. Finally, large numbers of bubbles form throughout the body of the liquid, ascend to the surface, and are given off as steam. The temperature at the same time becomes stationary. The water is then said to boil. The steam as it passes through the upper part of the flask is an invisible colourless vapour. When it comes into contact with cold air, it condenses into a visible cloud of small particles of water. This cloud is often referred to incorrectly as steam. It is small water particles ; steam itself is invisible.

The change of state from liquid to vapour is in many respects similar to the change from solid to liquid.

(1) There is no rise in temperature until the change is completed ; then, if the vapour is enclosed in a boiler and heated, the temperature begins to rise again, just as the temperature of water rises after the ice is all melted.

(2) Latent heat is absorbed during the change.

(3) The change is accompanied by an increase in volume, 1 c.c. of water forming about 1700 c.c. of steam under standard pressure.

(4) Just as liquids may be cooled below their freezing points without freezing, so also they may be heated above their boiling points without boiling. This is very apt to occur if the liquid contains no dissolved air. If a bubble is then formed, it grows with extreme rapidity, and the steam is given off with such violence that it may shake the vessel. This effect is known as "bumping" or delayed boiling. It can be prevented by placing pieces of broken brick or glass or fragments of wire in the vessel, so as to provide nuclei for the bubbles to form on; this should always be done when the boiling point of a solution is to be determined.

(5) The boiling point of a solution is higher than that of the pure solvent, the difference depending on the strength of the solution. It has been mentioned that the freezing point of a solution is lower than that of the solvent, so the effect here is in the opposite direction. A saturated solution of common salt boils at about 108°C . and freezes at about -22°C .

Determination of Boiling Point.—When a solution is boiled, the temperature of the vapour is that of the boiling point of the pure solvent. Hence in determining the boiling point of a pure liquid, the thermometer should be placed in the vapour as in the case of the hypsometer described on p. 140. This eliminates error due to delayed boiling and traces of dissolved substances. On the other hand, in determining the boiling point of a solution the thermometer should be in the liquid, and precautions should be taken, as described above, to prevent delayed boiling.

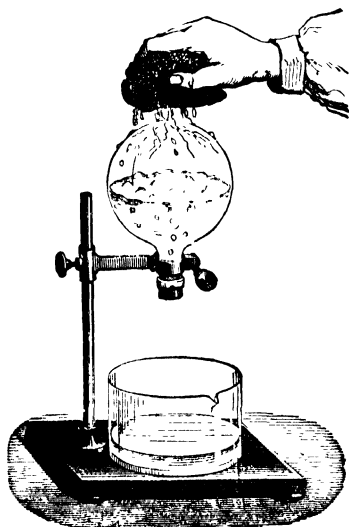


FIG. 170.—Franklin's experiment.

Effect of Pressure on the Boiling Point. Franklin's Experiment.—As has been mentioned on p. 141, increasing the pressure to which a liquid is exposed raises its boiling point, and diminishing the pressure lowers its boiling point. The effect is a much larger one than in the case of the melting point. Fig. 170 represents an

experiment due to Franklin which is often used for demonstrating it to a class.

A round-bottomed flask is filled half full of water, and boiled until all the air in the upper part of the flask is expelled by the steam. It is then corked, inverted, and supported on a retort stand, and after a time ebullition ceases. If the flask is then cooled by cold water from a sponge, the water begins to boil again, because the cold water condenses some of the vapour and lowers the pressure; hence boiling continues until the pressure is restored. The addition of more water causes renewed boiling, and the process can be repeated many times. Each time, of course, boiling takes place at a lower temperature.

Papin's Digester.—This apparatus was exhibited in 1679 to the Royal Society by Papin, one of the discoverers of the steam engine. It consists of an iron or copper boiler the lid of which can be screwed down air-tight. The boiler is fitted with a pressure gauge and also a safety valve which is operated by a lever and a sliding weight, and is of interest as being the first boiler ever fitted with a safety valve. The maximum pressure inside is greater than atmospheric pressure by an amount due to the downward force acting on the valve. When the boiler is heated, the pressure inside increases to this maximum value. The valve then “blows off,” and the water boils. Owing to the increased pressure the boiling point is, of course, above its usual value. The force on the valve and consequently the boiling point can be regulated by sliding the weight along the lever.

The barometric pressure is less at great heights, and the temperature of water boiling in open vessels at great heights is not sufficient to cook food properly; consequently Papin's digester is used in such circumstances for cooking with. It is also used for softening bones and extracting the gelatine they contain.

Evaporation and Boiling.—If a kettle is left on the fire too long, it boils dry, all the water passing away as steam. If the water in a pool on the road is exposed to the action of the wind it evaporates away, although its temperature may not be as much as 15°C . It is interesting to consider these phenomena from the molecular standpoint.

The molecules in a liquid are all in a state of motion in different directions and with different velocities, some moving much faster than others. If a molecule reaches the surface it is acted on in a downward direction by molecular attraction, and consequently it usually flies back into the liquid again. If, however, its velocity is very great, the molecular attraction is not sufficient to stop it, and it passes out into the atmosphere above. The number of molecules with a velocity greater than the average is continually being reinforced as a result of the impacts and jostlings of the molecules with one another. Consequently, there is a steady

stream of molecules through the surface. This is the mechanism by which evaporation goes on. As a result of the abstraction of the molecules with high velocities, the average kinetic energy of the molecules diminishes. But it will be shown later that the heat of a body is simply the kinetic energy of the motion of its molecules. Consequently, as a result of evaporation, the liquid cools.

If a kettle is half full of water, the space above the water contains a mixture of water vapour and air. The resultant pressure exerted by this mixture is the sum of the pressures exerted by the vapour and air separately. Let us suppose that the lid of the kettle is off. Then the sum of the pressures of the air and vapour is equal to the atmospheric pressure.

Suppose, now, that the temperature of the water is gradually raised. The rate of evaporation from its surface increases owing to the velocities of the water molecules increasing, and the mixture of air and water vapour in the upper part of the kettle contains more vapour and less air. When the boiling point is reached, the vapour expels the air, and takes exclusive possession. Its pressure alone is then equal to the pressure of the external air. The boiling point is consequently the temperature at which the pressure of the aqueous vapour is equal to the pressure of the atmosphere.

The essential difference between evaporation and boiling lies in the fact, that molecules evaporate only at the free surface of a liquid, but, when boiling takes place, they also pass into the vaporious condition as bubbles inside the body of the liquid. Consequently, at boiling the rate of formation of vapour molecules is very greatly increased.

Cooling Effect of Evaporation. The Cryophorus.—

Whenever liquid evaporates, latent heat is absorbed, and there is a fall in the temperature of the surface from which the evaporation takes place. The reason why hot tea produces a cooling effect in very hot weather is simply because it starts perspiration, and the evaporation of the latter carries heat away from the body. The danger of standing around in wet clothes is due to the cold produced by the evaporation of the moisture, which is more to be feared than a low temperature. And dentists use an ether spray to produce a low temperature and local anæsthesia.

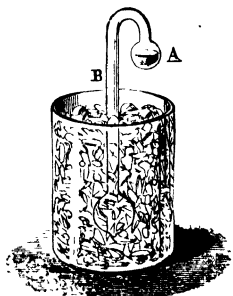


FIG. 171.—The cryophorus.

The cooling effect of evaporation can be demonstrated in a striking manner by an instrument called the cryophorus. This consists of a bent glass tube with a bulb at each end (Fig. 171),

containing water and water vapour, but no air. When the water falls down the tube, it hits the wall at the foot with a metallic click owing to the absence of an air cushion. The water is introduced into the bulb A, and the other bulb immersed in a freezing mixture. This causes the vapour in the tube to condense, and produces rapid evaporation of the water in A. After a short time the temperature of the water in A is so much reduced that it freezes.

EXAMPLES V

1. Describe how you would determine the freezing point and boiling point of an aqueous solution.
2. Give an account of the difference between evaporation and boiling.

CHAPTER VI

VAPOUR PRESSURE. HYGROMETRY

Vapour Pressure.—Let us suppose we have two barometric tubes (Fig. 172), and that by means of a curved tube a few drops of water are introduced into the right-hand one. They at once pass up

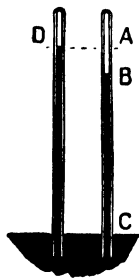


FIG. 172.

the mercury column into the vacuum and vaporise there. Consequently, the level of the surface is depressed to B, the height AB being a measure of the pressure exerted by the vapour. If the temperature of the room is 15°C. , this depression, AB, increases to a maximum of 13 mm. It may be less than this if insufficient water has been introduced into the vacuum, but it refuses to increase beyond this. If more water is introduced after the maximum value of the depression has been attained, it does not vaporise, but lies simply as a liquid on the top of the mercury. If the barometer tube AC is then raised slightly, more water vaporises, but the difference in level of the two columns

always remains the same. If the tube AC is pushed down into the mercury, the vapour condenses on the surface at B, the difference in level of D and B again remaining constant. If the tube AC is pushed down until its end reaches B, all the vapour condenses. Similar results are obtained when other liquids are used instead of water.

To sum up the results of these experiments :

*When vapour is in contact with its liquid in a closed space it has a definite pressure. It is then said to be **saturated**, and the value of its pressure is referred to as the **saturation pressure**.*

If now the barometer tube were surrounded with a water jacket, so that its temperature could be increased, it would be found, that as the temperature was gradually increased, the saturation pressure would increase also, and that as the temperature approached 100°C. , the saturation pressure would approach the pressure of the atmosphere, when the mercury would stand at the same level inside and outside the tube. This holds for other liquids as well as water.

Thus the boiling point is the temperature at which the saturation pressure is equal to the pressure of the atmosphere.

If the saturation pressure is less than the pressure of the atmosphere, only evaporation can take place.

The experiment of Fig. 172 is not very striking in the case of water, as its vapour pressure is so small at 15°C . If, however, ethyl ether is used instead of water, a depression of 35.6 cm. is obtained at 15°C .

Determination of Boiling Point of Alcohol.—Fig. 173 represents an interesting method of determining the boiling point of alcohol and other liquids which boil at a lower temperature than water. A U-tube, closed at the end of the shorter limb, is filled with mercury, and

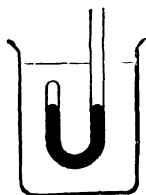


FIG. 173.

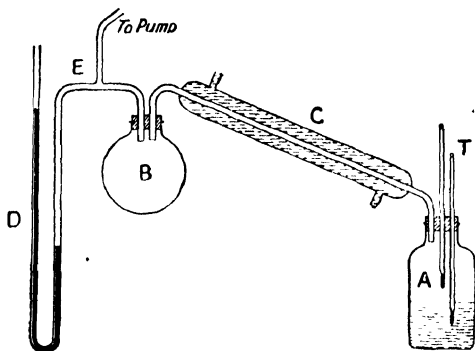


FIG. 174.—Regnault's method of determining vapour pressure at high temperatures.

a small quantity of the liquid passed round the bend to the top of the closed limb. The tube is immersed in water as shown; then if the water is gradually heated, the mercury stands at the same level in both limbs, when the liquid boils.

Measurement of Vapour Pressure.—A barometric tube surrounded by a water jacket is not a very satisfactory way of determining the saturation pressure for different temperatures, since it is difficult to read the level of the mercury through the water jacket, and, in any case, it does not go above pressures of one atmosphere. Fig. 174 shows an arrangement devised and used by Regnault, which is suitable for all pressures.

A is an air-tight vessel in which the liquid is boiled. The temperature of the liquid and the vapour above it are given by thermometers T; when great pressures are used, these dip into iron tubes containing mercury. The vapour rises out of A into the condenser C, a long inclined tube surrounded by a jacket in

which a stream of cold water is flowing. It is all condensed before it reaches the end of the tube, and flows back into the boiler. The pressure under which the liquid boils is altered by a compressing or exhausting pump connected on at E, and is read by the manometer at D. B is a large globe containing air, which minimises fluctuations in the pressure.

In making measurements with this apparatus the pressure in B is first adjusted by the pump to a definite value. The liquid is next boiled, and the temperature of the vapour read as soon as it becomes constant; the vapour is then condensed as rapidly as it is produced. If heat is supplied more rapidly, the vapour is produced and condensed in greater quantities, but once the boiling point is attained there is no further rise of temperature. When the liquid boils, the pressure of the air in the globe is equal to the saturation pressure of the vapour for the temperature in question.

The pressure is then altered to a new value, and after the temperature has become steady again, its value for the new pressure is determined. By this means Regnault was able to determine the saturation pressure of water up to 230°C ., the value for this temperature being $27\frac{1}{2}$ atmospheres.

For low temperatures the arrangement shown in Fig. 175 can be used. An ordinary barometer tube is bent over, so that its end can be immersed in a freezing mixture. The liquid to be experimented on is contained in the end of the tube. The other tube measures the atmospheric pressure, and the difference in level gives the pressure due to the vapour. Ice and several other solid substances have appreciable vapour pressures, and Regnault measured the vapour pressure of ice by an apparatus of this nature.

Fig. 176 represents the vapour pressure of water for the range from 0°C . to 100°C .

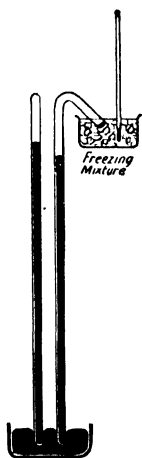


FIG. 175.—Method of determining vapour pressure at low temperatures.

Marcet's Boiler.—The pressure of saturated aqueous vapour at high temperatures may be determined by Marcet's boiler (Fig. 177). This consists of two iron hemispheres about four inches in diameter bolted together, containing mercury, on the surface of which rests a layer of water. The mercury column M acts as pressure gauge, S is a stopcock, and a thermometer is fitted

into the side at T to read the temperature.

In performing an experiment a suitable amount of water is introduced by S. The boiler is then heated with S open, until all the air is expelled. The stopcock is then closed, and the heating continued. The tempera-

ture and pressure rise together, the pressure corresponding to each temperature being the saturation pressure for that temperature. In reading the pressure corresponding to a particular temperature, care must always be taken to see that the latter is steady.

Distillation.—Distillation consists of boiling a liquid and condensing the vapour produced. It is used for separating a liquid from the solid matter dissolved in it, for example, in obtaining fresh from sea water, and also for separating the more volatile constituents of a mixture from the less volatile. The apparatus used for the purpose is called a still; if the condenser in Fig. 174 were sloped the other way, so that the condensed liquid ran down to the left into a vessel placed to collect it there, and not back into the boiler, then the arrangement would form a still.

Fresh water is obtained on board ship by distilling sea water. When sea water is evaporated, the salt remains behind and the vapour given off condenses as pure water. If it is to be used for drinking, it is made to absorb air, and some mineral salts added to improve the taste. It is advisable not to carry the process of distillation too far in this case, but to throw away the liquid remaining behind when diminished to about a quarter of its original volume, because otherwise there is a risk of some of the solid matter being carried over with the vapour.

Alcohol boils at 78°C ., hence in distilling spirits if the fermented liquor is heated up to 78°C ., the alcohol passes off as vapour with some of the water. They are condensed together in the *worm*, as the condenser is called, owing to its spiral form. A second and third distillation may be necessary to raise the spirit to its proper strength.

Distinction between a Vapour and a Gas.—

Let us suppose that in Fig. 172 the tube on the right is a very long one dipping deep into the mercury. Then if it is raised higher than is necessary for all the enclosed liquid to be evaporated, the vapour is said to become **unsaturated**. If the tube is raised further, so that the pressure falls considerably below the value required to produce

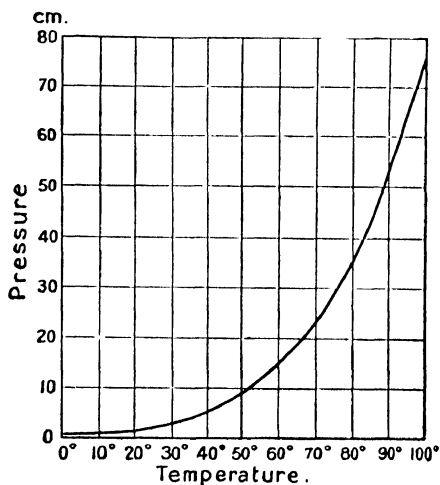


FIG. 176.—Vapour pressure of water.

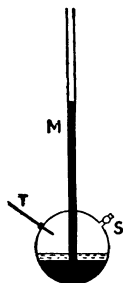


FIG. 177.—
Marceet's
boiler.

saturation, the vapour obeys Boyle's law and Charles's law. We are accustomed to think of water vapour, alcohol vapour, and ammonia vapour as vapours, because we are familiar with these substances in the liquid form, and we are accustomed to think of oxygen, hydrogen, air, etc., as something different, *i.e.* gases. But under suitable conditions a gas can become a vapour and a vapour can become a gas.

Up to a temperature of 365°C . water vapour, for example, can be condensed into liquid by the application of pressure, the pressure necessary at this temperature being 194.6 atmospheres. Above 365°C . it cannot be liquefied by the application of pressure. It is the same with the other vapours; there is a temperature for each below which it can be liquefied by the application of

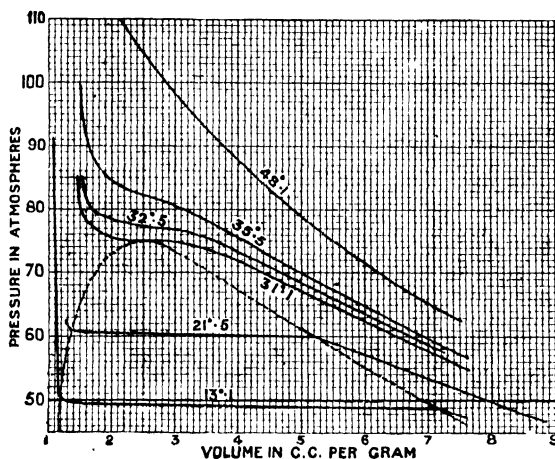


FIG. 178.—Isotherms for carbon dioxide.

pressure, and above which it cannot be liquefied by the application of pressure. This temperature is known as its **critical temperature**. The pressure necessary to liquefy a gas at its critical temperature is known as its critical pressure.

A gas is simply a vapour above its critical temperature.

The critical temperatures of the gases formerly known as permanent gases are very low, the values for hydrogen, oxygen, and nitrogen being -234.5°C ., -118°C ., and -146°C . respectively.

Isotherms for Carbon Dioxide.—Fig. 178 shows the results of a celebrated series of experiments performed on carbon dioxide by Andrews in 1863. They represent the connection between the pressure and volume of carbon dioxide at different temperatures. The first

curve is for 48.1°C. ; it is a hyperbola, so for this temperature Boyle's law holds and the substance is a gas. The next three curves all have indentations in them, but contain no parts parallel to the axis of abscissæ. The curve for 21.5° shows that if we start at this temperature with a large volume and small pressure, and gradually increase the pressure, the volume diminishes continuously until we reach 60 atmospheres, then it suddenly falls without further increase of pressure from about 5.2 c.c. per gram to about 0.4 c.c. per gram. This pressure has liquefied it. The curve for 13.1° shows by its vertical portion the effect of further increase of pressure on the substance, after it has become liquid.

If curves are taken for other temperatures, the parts lying within the dotted curve are parallel to the axis of abscissæ. Hence the temperature corresponding to the curve which just touches the dotted curve, is the critical temperature of carbon dioxide. It is obvious from the diagram that it is near 31.1°C. ; careful experiment showed that it was 30.92°C.

Dalton's Law.—If some water is left inside an air-tight vessel, a portion evaporates, and fills the space above the surface. The quantity of vapour formed depends only on the temperature of the enclosed space, and practically not at all upon the pressure of the air inside the enclosed space. This may be shown by an experiment with a barometer tube. Some air is introduced into the space above the mercury, and the depression noted so as to calculate its pressure. Some liquid is next introduced, so as to saturate the enclosed space and leave a little liquid on the surface of the mercury, and the further depression noted. As a result of the further depression, the pressure of the air falls according to Boyle's law. If allowance is made for this, it is found that the same pressure is produced by the saturated vapour whether the air is there or not.

But the presence or absence of air has a great influence on the speed at which the vapour is formed; the whole evaporation takes place at once in a vacuum, but may require several hours if the space is already full of air.

The result that just as much liquid evaporates into a space which is full of air as into a vacuum, is a particular case of a law given by Dalton, which is stated as follows:

The total pressure exerted in an enclosed space by a mixture of gases and vapours which do not react on one another chemically, is equal to the sum of the pressures which each would produce if it occupied the space alone.

The law is not true for high pressures.

Relative Humidity.—Evaporation is constantly taking place from the surface of ponds and rivers, and water returns to the earth in the form of rain. Consequently there is always a certain amount of water vapour present in the atmosphere. This vapour

is, of course, invisible. If, however, the air cools, a temperature is reached for which the saturation pressure of aqueous vapour is equal to the actual pressure of the vapour present in the atmosphere. If the temperature falls below this, some of the vapour condenses in small drops forming a **cloud** or **mist**, according as the condensation takes place in the upper atmosphere or near the surface of the earth.

*The ratio of the quantity of aqueous vapour present in a given volume of air to the quantity of aqueous vapour it would contain if saturated, the temperature remaining constant, measured in per cent., is called the **relative humidity** or **hygrometric state** or **fraction of saturation** of the air.*

The science of the measurement of the hygrometric state of the atmosphere is termed hygrometry and the instruments used for the purpose are called hygrometers.

Aqueous vapour obeys Boyle's law quite well at moderate temperatures close up to the saturation pressure. Its density is consequently proportional to its pressure. Thus the

$$\text{Relative humidity} = \frac{\text{actual density}}{\text{saturation density}} = \frac{\text{actual pressure}}{\text{saturation pressure}}$$

It is the relative humidity on which our feeling of the dryness of the air depends, not the actual density of aqueous vapour present. If the relative humidity is low, evaporation takes place readily into the air. On a hot day in summer the air may feel drier than on a cold day in winter, because the relative humidity is less, although on the hot day there is much more aqueous vapour present in the atmosphere.

Relative humidity determinations are made in hot-houses to see that the plants receive their proper supply of moisture. They are also made regularly at meteorological stations, and they afford information as to the likelihood of rain or frost.

Formation of Dew.—If a cold surface is presented to the atmosphere, the air in contact with it may be cooled so far, that the vapour present in the air is sufficient to saturate it, and moisture is deposited on the surface. This moisture is known as dew. The air in this case is cooled only in the neighbourhood of the surface, and its pressure remains unaltered during the process.

*The temperature to which the air must be cooled for the aqueous vapour present in a given volume to saturate it is known as the **dew-point**.*

The actual vapour pressure in the air is therefore equal to the saturation vapour pressure at the dewpoint.

On a clear night after a warm day heat is lost by radiation from the earth, and its surface becomes very cold. The air in contact with it cools down, especially if, owing to the absence

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VAPOUR PRESSURE. HYGROMETRY

piece of muslin, which is constantly kept moist by being connected with a reservoir of water. Unless the air is saturated with moisture, the right-hand thermometer indicates a lower temperature than the left-hand one; this is due to the cooling effect of the evaporation from the muslin. The dewpoint is obtained by multiplying the difference of temperature of the wet and dry bulbs by a factor known as Glaisher's factor, which is obtained empirically by comparison with another instrument, and subtracting the result from the temperature of the dry bulb. Glaisher's factor varies with the temperature of the dry bulb, and is given for one type of instrument by the following table :

Dry Bulb Temp.	Factor.	Dry Bulb Temp.	Factor.	Dry Bulb Temp.	Factor.
1° C.	2.81	10° C.	2.06	18° C.	1.83
2	2.54	11	2.02	19	1.81
3	2.39	12	1.99	20	1.79
4	2.31	13	1.95	21	1.77
5	2.26	14	1.92	22	1.75
6	2.21	15	1.89	23	1.74
7	2.17	16	1.87	24	1.72
8	2.13	17	1.85	25	1.70
9	2.10				

As an illustration of the table let us suppose that the dry bulb reads 14.9° C. and the wet bulb

11.7° C. Then the factor is 1.89 and the dewpoint

$$14.9 - 1.89(14.9 - 11.7) =$$

$$14.9 - 6.0 = 8.9^{\circ} \text{ C.}$$

When once the dewpoint is obtained, the relative humidity can be calculated in the same way as Regnault's hygrometer.

The wet and dry bulb hygrometer is not an absolute instrument, since it has to be calibrated by reference to another hygrometer, and its accuracy is affected by draughts. But it is simple and easy to use, and is consequently the instrument generally employed at meteorological stations.

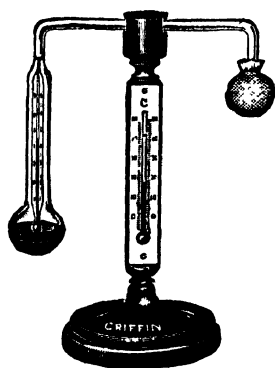


FIG. 181.—Daniell's hygrometer.

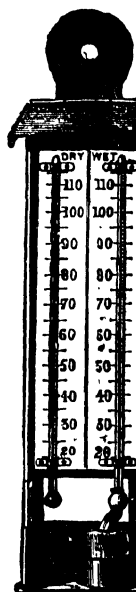


FIG. 180.—Wet and dry bulb hygrometer.

Other Hygrometers.—Fig. 181 represents Daniell's hygrometer, the oldest form of hygrometer. This instrument consists of two bulbs connected by a glass tube, from which the

has been exhausted ; the bulb on the left contains some ether and the rest of the space is filled with ether vapour. A thermometer is enclosed in this bulb, and a piece of muslin is tied round the other one.

To use the instrument ether is poured on the muslin. Its evaporation cools the bulb and condenses the vapour inside it. Consequently ether evaporates in the other bulb and lowers its temperature. As soon as moisture condenses on this bulb, its temperature is noted. The temperature of the apparatus is then allowed to rise, and the reading for which the moisture disappears, is noted. The mean of the two readings is the dewpoint. The instrument does not give very satisfactory results. It will be observed that it employs the same principle as the cryophorus.

Fig. 182 represents **Dines' hygrometer**. A is a reservoir which contains water cooled by ice. B is a metal box closed at the top by a thin plate of black glass.

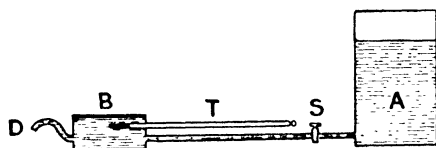


FIG. 182.—Dines' hygrometer.

The thermometer T has its bulb just below the glass plate. When the stopcock S is opened, cold water flows through B, lowering the temperature of the glass plate, and escapes by the outlet at D. When dew deposits on the glass, the flow is stopped and the temperature is noted. The temperature is noted again when the dew disappears as a consequence of the plate becoming warmer, and the mean of the two readings gives the dewpoint.

In the **chemical hygrometer** a known volume of air is drawn through drying tubes which contain pumice moistened with strong sulphuric acid. These are weighed before and after the experiment, and the increase in weight gives the amount of water in the air.

Substances which absorb moisture from the air are said to be **hygroscopic**. A hair when freed from its film of oil changes in length according to the humidity of the atmosphere. In the **hair hygroscope** this change is magnified by a system of levers which operate a pointer moving over a dial. The scale is calibrated so as to give relative humidity directly.

Rain, Hail, Snow, Hoar Frost.—As has been mentioned, clouds are formed by cooling a mass of vapour in the atmosphere below its dewpoint. Condensation takes place at first in the form of minute drops, but as the process continues, the particles coalesce and form larger drops. As the drops grow, their weight increases more rapidly than the area of their surface on which the frictional resistance which supports them in the air depends ; thus they fall to the ground as rain. The cooling which produces the condensation is usually due to the expansion of the air with which the vapour is mixed, and the condensation takes place on dust particles floating in the air. If the falling rain passes through cold regions it may freeze into hail, but many points

connected with the formation of rain and hail are still obscure. If the temperature at which the condensation takes place in the upper atmosphere is below the freezing point, the aqueous vapour condenses into snow. If the temperature of the earth's surface sinks below the freezing point before saturation occurs, the aqueous vapour in the atmosphere is deposited on it as hoar frost; thus hoar frost is not frozen dew, but is formed directly from the state of vapour.

EXAMPLES VI

1. Explain how the sun causes a morning mist to disappear, and why fogs are formed round icebergs.

2. The temperature of the air is 16.3°C . and the dewpoint is 9.2°C . Find the relative humidity and the quantity of aqueous vapour present in the atmosphere in grams per litre.

3. The temperatures of the wet and dry bulb thermometers stand respectively at 13.5°C . and 15.4°C . Find the dewpoint and the quantity of aqueous vapour present in the atmosphere in grams per litre.

4. A mass of air is at a temperature of 100°C . Its dewpoint is 48°C . If the saturation pressure of aqueous vapour at 48°C . is 8.35 cm., find the relative humidity of the air.

5. A cubic metre of air at 20°C . is found to contain 10 gm. of aqueous vapour. What is its relative humidity?

6. The quantity of water given out by the lungs and skin may be taken as 30 ounces in 24 hours. How many cubic feet of half-saturated air at 15°C . and 78 cm. will this quantity of water saturate?

7. The temperature of the air at sunset is 10°C . and the temperature of the ground is 4°C . No dew falls. What is the greatest value which the relative humidity of the air can have?

8. It is sometimes stated, as a practical rule, that when the saturation pressure of aqueous vapour is x mm. of mercury, the weight of water present in a cubic metre is x gm. What is the percentage error in this statement for a pressure of 8 mm.?

9. A mass of air extending over an area of 60,000 square metres to a height of 300 metres has the dewpoint at 15°C ., its temperature being 20°C . How much rain will fall if the temperature sinks to 10°C .? Express your answer in mm. The pressure of water vapour is 9.2, 12.8, and 17.5 mm. at 10° , 15° , and 20°C . respectively. The density of aqueous vapour is $\frac{8}{10}$ the density of dry air under the same conditions of temperature and pressure.

10. A mass of oxygen standing over water measures 200 c.c., the temperature being 20°C . and the pressure 740 mm. What is the volume of the oxygen dry and when measured at normal temperature and pressure?

CHAPTER VII

THE CONNECTION BETWEEN HEAT AND WORK

The Caloric Theory.—According to the view prevalent in the eighteenth century heat was an invisible imponderable fluid called *caloric*, which filled the spaces between the particles of matter. The pressure of this fluid was greatest where the temperature was highest; hence it flowed from a body at a higher to a body at a lower temperature. Some bodies had a greater capacity for storing the fluid than others; hence the difference in their specific heats. If a gas was suddenly compressed or a body was hammered, caloric was squeezed out of it like water from a sponge, and thus the production of heat in this case was explained. The heat caused by the friction of two bodies on one another was explained by supposing that the particles rubbed off had not the same capacity for storing caloric as the original solids had; thus when the particles were rubbed off, heat was liberated.

Caloric could neither be created nor destroyed, hence according to the caloric theory the total quantity of heat in any system must remain constant. If it increases in one part, there must be a corresponding loss in another part.

Rumford's Experiments.—Count Rumford, an American who had to leave his country owing to the War of Independence, and who took service under the Bavarian Government, was impressed by the enormous amount of heat generated during the boring of cannon at Munich. In one experiment made in 1798 he noticed that a cylinder of gun-metal was raised from 60° F. to 130° F. by the friction of a blunt steel borer, although only $\frac{1}{550}$ of the whole mass of the cylinder was rubbed off. In another experiment in order to see whether the exclusion of the atmosphere had any effect on the result he surrounded the metal with water, and found that after 2½ hours the water boiled.

It seemed impossible that this great quantity of heat could come from a change in the thermal capacity of the metal chips. Besides, he showed experimentally that the specific heat of the chips was the same as that of the solid metal. Consequently as

a result of the experiments he came to the conclusion, that no material substance could be produced in this unlimited manner, and that heat must consist in motion.

Davy's Experiment.—Sir Humphry Davy rubbed together two pieces of ice by clockwork, under the receiver of an air pump, and found that the ice melted where the surfaces rubbed, but not elsewhere. There is no doubt that heat was used up in melting the ice, also that the thermal capacity of the ice was greater after melting. Hence the heat could not have come out of the ice, and it is difficult to see where it could have come from unless from the motion. Still in spite of Rumford and Davy's results the caloric theory maintained its position until about 1842, when, as a result of the experiments of Joule and others, the mechanical theory of heat gradually replaced it.

Joule's Experiments.—The apparatus for Joule's most celebrated experiment is shown in Fig. 183. Resting on a wooden

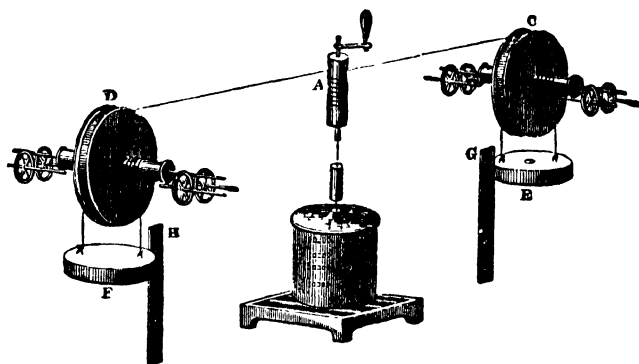


FIG. 183.—Joule's apparatus.

grating is a copper calorimeter, which is provided with a brass paddle wheel. The paddle wheel is attached to the roller A, and so rotates about a vertical axis. The calorimeter is fitted with vanes, so that, when the paddle wheel rotates, the water does not rotate as a whole, but there is a maximum of friction and irregular motion. Cords are wound on the roller A, pass over the pulleys C and D, and carry the weights E and F. The pulleys are mounted on friction wheels. The experiment was performed by allowing the weights to fall on the floor. The distance they fell through was given by the scales G and H.

As the weights fell, the cords unwound from the roller A, the paddle wheel turned, and the temperature of the water in the calorimeter rose. When the weights reached the ground, the paddle shaft was disconnected from the roller, and the weights

wound up. The shaft was then connected again and the weights allowed to fall once more. And so on; a number of descents was made.

If m is the mass of each weight, h the distance fallen through, and n the number of descents, the potential energy lost is

$$2nmgh$$

If W denotes the mass of water together with the water equivalent of the calorimeter, and t is the whole rise in temperature, the amount of heat produced is

$$Wt$$

Some of the energy of the falling weights was destroyed by the impact against the floor, but when this was allowed for, it was found, that, no matter how n and m were varied, $2nmgh$ was always proportional to Wt , i.e. the potential energy destroyed was proportional to the heat produced. This result has been confirmed by other experiments. It was also shown by Joule, that when electrical energy disappears, heat is always produced in the circuit proportional to the electrical energy lost. It is therefore natural to conclude that

Heat is a form of energy.

This view as to the nature of heat is referred to as the mechanical theory of heat, and is now accepted everywhere.

*The ratio of the work done to the heat produced, when work is transformed into heat, is known as the **Mechanical Equivalent of Heat or Joule's Equivalent**. It is usually denoted by **J**.*

According to the best modern determinations J is 4.185×10^7 ergs per calorie. The value 4.2×10^7 is usually taken, except when great accuracy is required. The unit of work used in electricity, the joule, is 10^7 ergs. Hence 1 calorie = 4.2 joules.

In Joule's original experiment each weight was 28.5 lb., the distance fallen through was 5.9 ft., the water equivalent of the calorimeter was 0.99 lb., and the mass of the water 13.9 lb. Consequently the temperature rose only about 0.030° F. when the weights dropped once. The weights were usually dropped about 20 times; thus the total rise of temperature operated with was about 0.60° F. There was a relatively large radiation correction. The temperatures were measured by a very sensitive mercury thermometer. The result obtained was 772 ft.-lb. per B.Th.U.

EXAMPLE.—Change the value for J given above, 4.185×10^7 ergs per calorie, to ft.-lb. per B.Th.U.

$$1 \text{ ft.-lb.} = 30.48 \times 453.6 \times 981 \text{ ergs}$$

$$1 \text{ B.Th.U.} = 453.6 \times \frac{1}{2} \text{ cal.}$$

$$\begin{aligned} \text{Hence } J &= \frac{4.185 \times 10^7 \times 453.6 \times 5}{30.48 \times 453.6 \times 981 \times 9} \\ &= 778 \text{ ft.-lb. per B.Th.U.} \end{aligned}$$

EXAMPLE.—Given that the melting point of lead is 327°C. , that its latent heat of fusion is 5.4 cal. , and that its specific heat is 0.032 , find with what velocity a lead bullet at 20°C. must strike an iron target, in order that all the heat produced should be just sufficient to melt it.

The mass is immaterial, so assume that it is 1 gm. Then heat required

$$\begin{aligned} &= 0.032 \times 307 + 5.4 \\ &= 15.2\text{ cal.} = 6.38 \times 10^8\text{ ergs.} \end{aligned}$$

This must equal the kinetic energy, $\frac{1}{2}v^2$. Hence

$$v = 358\text{ metres per second.}$$

Searle's Apparatus for Determining J.—In a later determination of J , Joule suspended the calorimeter by means of a fine wire, so that it was free to rotate about a vertical axis. When the paddle wheel was rotated, owing to the viscosity of the water the calorimeter tended to rotate in the same direction as the paddle wheel, but was prevented by a couple; the work done was calculated from the magnitude of this couple and the angle through which the paddle wheel was turned.

Searle has designed a simplified form of this apparatus which is in wide use in laboratory courses. In this apparatus the mechanical energy supplied is converted into heat by the friction between two well-fitting brass cones, the inner of which is stationary while the outer rotates. A section of the cones is shown in Fig. 184; the inner cone contains water the temperature

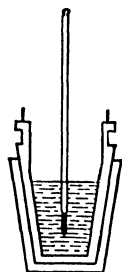


FIG. 184.—Section of the cones.

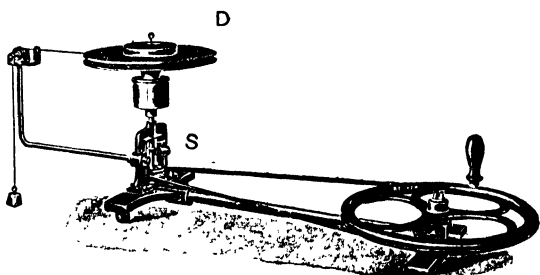


FIG. 185.—Searle's apparatus for determining J .

of which is read by a thermometer. The whole apparatus is shown in Fig. 185. The spindle is rotated by the hand wheel and leather band, and the number of revolutions is read on the revolution counter at S . The outer cone is fixed to the spindle. The inner cone is furnished with two steel pins which fit in two holes in the wooden disc D . A string is fastened round the grooved edge of the disc and then passes over the pulley P , carrying a weight at its end.

The magnitude of the weight and the speed at which the handle is turned are adjusted, so that the weight hangs stationary clear of the floor, neither ascending nor descending. The inner

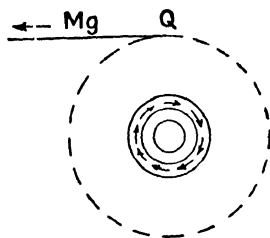


FIG. 186.

cone is then in equilibrium under two forces. If Fig. 186 represents a plan of the cones, the stretching force in the string Mg applies a moment at Q of magnitude Mgr , where r is the radius of the disc. This is balanced by the frictional forces which act on the inner cone in the manner shown. The couple exerted by the frictional forces is consequently Mgr , and this is also the couple exerted on the spindle by the leather band. The work converted

into heat is consequently

$$Mgr2\pi n$$

where n is the number of revolutions. The heat produced is

$$Wt$$

where W is the water equivalent of both cones together with the mass of the water and t is the rise in temperature.

Hence

$$J = \frac{Mgr2\pi n}{Wt}$$

Nature of Heat.—Joule's experiments showed that heat is a form of energy. We shall now go more fully into the connection between heat and energy.

Let us suppose that a certain quantity of gas is contained in a closed vessel. Then the molecules of the gas travel in all directions, and are reflected by the sides. It is the impact of the molecules on the side that causes the pressure on the side; this pressure is equal to the number of molecules impinging on unit area per second multiplied by the average change in momentum suffered by the molecules in the impact. If the velocity of the molecules is increased in a certain ratio, the molecules travel across the vessel in a shorter time, and the number of impacts per second is increased in the same ratio. The molecules also hit the side harder; for the change in momentum which is proportional to the velocity, is also increased in the same ratio. Hence the pressure is increased in the square of the ratio, *i.e.* it is proportional to the mean square of the velocity.

But if the temperature of the gas is increased, the pressure increases in proportion to the temperature, and thus to the heat contained by the gas. The heat contained by the gas is therefore

proportional to the mean square of the velocity of the molecules. But since the number of molecules in the vessel is constant, their kinetic energy is proportional to the mean square of their velocity. It is therefore natural to conclude that

The heat contained in a substance is the kinetic energy of its molecules.

This result is borne out by exact calculation, and applies to solids and liquids as well as to gases. In solids and liquids the molecules oscillate about their mean positions; the amplitude of the oscillations increases when the temperature increases.

It will be instructive to consider the changes in a gram of water, when it is taken from the state of ice below the melting point to the state of vapour at the boiling point. Initially the ice molecules oscillate about their mean positions. As the temperature increases, the oscillations increase in amplitude, and the increase in kinetic energy is equal to the heat supplied. When the melting point is reached, the oscillations become so large, that the elastic forces on the molecules are no longer able to bring them back to their mean positions, and they break loose. The solid then becomes a liquid. The latent heat absorbed increases the potential energy of the molecules.

The water molecules move about the liquid, but knock incessantly against their neighbours. Their motion is consequently an oscillatory one about a position which is always changing. If a molecule reaches the surface, it is prevented from passing through it by the molecular attraction of the surface molecules, unless its velocity is very great, when it is able to overcome this attraction. At any one temperature the velocities of the water molecules vary about a mean, and, as the temperature rises, there is an increase in the number of molecules with velocities high enough to enable them to escape through the surface. Hence the explanation of evaporation.

Any heat given to the water increases the kinetic energy of the molecules until the boiling point is reached. Then latent heat is absorbed in doing work against their mutual attraction, and causes a further increase in their potential energy.

Work done by an Expanding Gas.—Let a mass of gas be contained inside a cylinder of cross-sectional area A , and let it be subjected to a pressure p by means of a piston (Fig. 187). Let v_1 be its volume.

Suppose now that the gas is heated, and that the piston is displaced upwards through a small distance h . Since the change is a small one, we can assume that it does not affect the pressure appreciably. The force acting on the piston during the displacement is then pA , and the external work done by the gas during the displacement

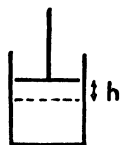


FIG. 187.

is $p\Delta h$. Let v_2 be the final volume. Then $\Delta h = v_2 - v_1$, and the external work done is

$$p(v_2 - v_1)$$

The result holds, no matter what the shape of the envelope containing the gas is. For the surface of the latter can always be divided into plane elements, and, when the volume changes, each element is displaced outwards in the direction of its normal. We have thus the general rule:

The work done by a gas in a small expansion is equal to the product of the pressure and the increase in volume.

Difference of the Specific Heats of a Gas.—The specific heat of a substance was defined on p. 170 as the quantity of heat required to raise the temperature of one gram of the substance 1°C . In the case of a solid or a liquid, if the pressure changes during the determination, the change of volume produced is so slight, that it can be disregarded. Thus the pressure does not require to be specified in the definition. It is, however, otherwise in the case of a gas, and here we distinguish two special cases:

The specific heat of a gas at constant pressure, C_p , is the quantity of heat required to raise the temperature of one gram of the gas 1°C ., keeping its pressure constant.

The specific heat of a gas at constant volume, C_v , is the quantity of heat required to raise the temperature of one gram of the gas 1°C ., keeping its volume constant.

Let us assume that we have unit mass of a gas in the cylinder represented in Fig. 187, and that its temperature is raised through a small interval from T_1 to $T_2^\circ \text{K}$, while p , the pressure, is kept constant. The volume consequently increases from an initial value v_1 to a final value v_2 , and the piston is pushed up. The quantity of heat supplied is, by definition,

$$C_p(T_2 - T_1)$$

and the external work done is

$$p(v_2 - v_1)$$

The latter can only have been done at the expense of the heat supplied. Hence, if we change the calories to ergs, the difference

$$JC_p(T_2 - T_1) - p(v_2 - v_1)$$

is the gain of energy by the gas.

If the volume had been kept constant, while the temperature was raised, the gain of energy, in ergs, would have been

$$JC_v(T_2 - T_1)$$

The question arises, is this expression equal to the difference above? The initial pressure and temperature are the same in both cases, the only factor that distinguishes the first from the second being

that the final volume is greater. Now Joule and Kelvin showed that the internal energy of a gas depends only very slightly on its volume, *i.e.* very little work has to be done in separating the molecules from one another. Thus we are justified in equating the two expressions, and

$$JC_p(T_2 - T_1) - p(v_2 - v_1) = JC_v(T_2 - T_1)$$

or
$$C_p - C_v = \frac{p(v_2 - v_1)}{J(T_2 - T_1)}$$

But from the gas equation
$$\frac{pv_2}{T_2} = \frac{pv_1}{T_1} = R$$

This ratio is equal to
$$\frac{pv_2 - pv_1}{T_2 - T_1} = \frac{p(v_2 - v_1)}{T_2 - T_1}$$

Hence substituting above, we obtain the final expression

$$C_p - C_v = \frac{R}{J}$$

for the difference of the specific heats.

It is interesting to note that Mayer used this expression for the case of air as a means of calculating the value of J , before Joule determined it experimentally. But the result was not good, owing to the data available at that time being inaccurate.

To Calculate the Difference of the Specific Heats of Air.—
We have

$$\begin{aligned} C_p - C_v &= \frac{R}{J} = \frac{p_0 v_0}{273J} \\ &= \frac{76 \times 13.6 \times 981}{0.001293 \times 273 \times 4.2 \times 10^7} \\ &= 0.068 \end{aligned}$$

Proof that the Kinetic Energy of the Molecules of a Gas varies as the Absolute Temperature.—Let us assume that we have a gas contained in a vessel with a plane wall, that the mass of each molecule is m , that the molecules are equally spaced, and that there are N of them per unit volume. We shall also assume that they have all the same velocity V , that one-third of them is moving at right angles to the plane wall, and that the other two-thirds are moving parallel to the wall, equally divided between two directions at right angles to one another. Only half those moving at right angles to the wall are moving towards the wall, *i.e.* $\frac{1}{6}N$ per unit volume. All those moving towards the wall that are within a cylinder of length V and base 1 sq. cm. reach it in one second. Hence the number of molecules hitting the wall per sq. cm. per sec. is $\frac{1}{6}NV$. Each suffers a change of momentum $2mV$ on impact, since the impact is perfectly elastic. Hence the pressure on the wall is given by

$$p = \frac{1}{6}NV \times 2mV = \frac{1}{3}NmV^2 = \frac{1}{3}\rho V^2$$

where ρ is the density of the gas.

$$\text{Now} \quad \frac{p}{\rho} = pv = RT$$

where v is the volume of unit mass. Hence

$$V^2 = 3RT$$

and it follows that the kinetic energy also varies as T .

We have, of course, made certain simplifying assumptions. If these are dropped, the result still holds if $\bar{V}^2$, the mean square of the velocity of the molecules, is substituted for the square of the velocity V^2 . It will be noticed that the equation $p = \frac{1}{3}\rho\bar{V}^2$ affords a means of calculating $\bar{V}^2$.

If we have two gases at the same temperature and pressure,

$$p = \frac{1}{3}N_1m_1\bar{V}_1^2 = \frac{1}{3}N_2m_2\bar{V}_2^2$$

where the suffix 1 refers to the first gas and the suffix 2 to the second. The mean kinetic energy of the molecules of the two gases must be the same, for otherwise, when they are mixed and collisions take place between them, the one gas would gain energy and consequently heat at the expense of the other. Thus

$$\frac{1}{2}m_1\bar{V}^2 = \frac{1}{2}m_2\bar{V}^2$$

Hence $N_1 = N_2$, or under the same conditions of temperature and pressure equal volumes of different gases contain an equal number of molecules. This result is known as Avogadro's law; it was discovered by chemical reasoning.

Conservation of Energy.—Joule showed, that when mechanical energy or electrical energy disappeared, it reappeared as heat. When heat is supplied to a gas in a cylinder, and work is done by the gas in raising a piston, we have the reverse change, heat converted into mechanical energy. When a stone falls from the edge of a cliff, its potential energy is converted into kinetic energy; when it strikes the ground, the kinetic energy disappears, but reappears as heat. When coal burns, its chemical energy is converted into heat. Thus energy is never lost, but only changed from one form to another. This doctrine is known as the *principle of the conservation of energy*, and is one of the chief foundations of modern science. It is generally expressed by the statement, that the sum total of the energy in the universe is constant. Joule's experiments helped very considerably to establish the principle.

EXAMPLE.—An engine consumes 56 lb. of coal per hour. The calorific value of coal is 3.6×10^6 calories per lb., and 1 ft.-lb. = 13.56×10^6 ergs. Given that the efficiency of the engine, *i.e.* the ratio between the useful work performed and the work represented by the heat of combustion of the fuel, is 5 per cent., find the horse-power that the engine develops.

Calorific value of coal consumed per minute

$$\begin{aligned}
 &= \frac{56 \times 3.6 \times 10^6}{60} \text{ cal.} \\
 &= 56 \times 6 \times 10^4 \text{ cal.} \\
 &= \frac{56 \times 6 \times 10^4 \times 42 \times 10^6}{13.56 \times 10^6} \text{ ft.-lb.} = 1.041 \times 10^7 \text{ ft.-lb.}
 \end{aligned}$$

Useful work done per min. $= 5.205 \times 10^5 \text{ ft.-lb.}$

$$\text{H.P.} = \frac{5.205 \times 10^5}{33,000} = 15.8$$

Dissipation of Energy.—In steam and other heat engines work is obtained by supplying heat to a vapour or gas which thereupon pushes a piston up a cylinder. Of the heat supplied part is lost by conduction through the sides of the containing vessels, part is carried away by the vapour into the air, and only the remainder is converted into work. In conduction and radiation heat passes directly from a warmer to a colder body without yielding work at all. In all machines there is a certain amount of work lost in overcoming friction. It is easy to burn coal and warm the air around it, but it is impossible to collect this diffused heat into the chemical energy of the coal again. Thus, although the sum total of the energy in the universe remains constant, it is running to waste as regards convertibility or availability. The universe is consequently like a clock which is running down. The process by which all energy is slowly but surely passing into a state of uniformly diffused heat was first pointed out by Lord Kelvin, and is known as the *diffusion, degradation, or dissipation of energy*.

We derive our energy from the sun's rays, the earth's rotation, and gravitational attraction. The coal and oils which drive our engines owe their existence to the action of the sun at a distant epoch of the earth's history. The vegetation which sustains life is ripened by the sun. The waterfalls which work turbines owe their energy to gravitation, to the sun's rays which caused the water to rise into rain clouds, and to the earth's rotation which helped to make the winds that carried the clouds.

But the sun will become cold, the earth will cease to rotate, and all the bodies which can fall, will eventually fall together.

Adiabatic Change. Adiabatic Equation of a Gas.—When a gas is compressed, the mechanical work done in the compression reappears as heat in the gas, raising its temperature. The compression must be done suddenly, as in the case of a bicycle pump, for the change to be noticeable. If it is done slowly, the heat produced has time to diffuse away out of the gas, before the rise becomes appreciable. In this case the change is said to be an **isothermal** one. If a substance

undergoes a change, and no heat is gained or lost by the substance during the change, the latter is said to be an **adiabatic** one.

Let unit mass of a gas be contained inside a cylinder of cross-sectional area A under a pressure p_1 . Let v_1 be its volume and T_1 its absolute temperature. Let it be compressed to a volume v_2 , where v_2 is only slightly different from v_1 , and let the temperature become T_2 . The pressure at the same time becomes p_2 , but as p_2 is only slightly different from p_1 , we may write $p_1(v_2 - v_1)$ for the work done by the gas in the compression. The heat received during the compression measured in units of work is

$$JC_v(T_2 - T_1) + p_1(v_2 - v_1)$$

Let us assume that the change is an adiabatic one. Then the expression becomes zero, and

$$p_1(v_1 - v_2) = JC_v(T_2 - T_1)$$

On substituting for T_1 and T_2 this becomes

$$\begin{aligned} p_1(v_1 - v_2) &= \frac{JC_v}{R}(p_2 v_2 - p_1 v_1) \\ &= -\frac{JC_v}{R}(p_2 - p_1)v_2 - \frac{JC_v}{R}(v_1 - v_2)p_1 \end{aligned}$$

or
$$\frac{JC_p}{R}(p_2 - p_1)v_2 = \frac{JC_p}{R}(v_1 - v_2)p_1$$

since
$$C_p = C_v + \frac{R}{J}$$

Denote the ratio of the specific heats, C_p/C_v , by γ and divide by $p_1 v_2$. Then

$$\frac{p_2 - p_1}{p_1} = \frac{\gamma(v_1 - v_2)}{v_2}$$

Add 1 to both sides. Then

$$1 + \frac{p_2 - p_1}{p_1} = 1 + \frac{\gamma(v_1 - v_2)}{v_2}$$

or
$$\frac{p_2}{p_1} = \left(1 + \frac{(v_1 - v_2)}{v_2}\right)^\gamma$$

since the R.H. side may be regarded as the first two terms in a binomial expansion. This simplifies to

$$p_1 v_1^\gamma = p_2 v_2^\gamma$$

We have assumed that the change is a small one, but any actual change may be divided into a number of small ones for each of which the above relation holds. Hence for any adiabatic transformation

$$pv^\gamma = \text{constant.}$$

This is referred to as the adiabatic equation for a gas. For air, oxygen, nitrogen, and hydrogen γ has the value 1.41.

EXAMPLE.—A quantity of air at 80 cm. pressure is suddenly compressed to half its volume. Calculate the new pressure.

We have

$$p_2/p_1 = (v_1/v_2)^{\gamma} = 2^{1.41}$$

$$\log p_2 = \log 80 + 1.41 \log 2$$

whence

$$p_2 = 212.5 \text{ cm.}$$

If the change had been an isothermal one, the new pressure would have been 160 cm. Hence the resistance to an adiabatic compression is greater than to an isothermal one.

Liquid Air Machine.—When a gas expands adiabatically and does work against the pressure of the atmosphere, it cools, because the work is done at the expense of its own internal energy. When air expands, work is also done against the mutual attraction of the molecules. It was mentioned on p. 209, that Joule and Kelvin found that this work was small, and it was neglected in proving the formula for the difference of the specific heats of a gas. But when the pressure of the air is great, it becomes appreciable, and gives rise to a cooling effect; it is this cooling effect due to the work done against intermolecular forces that forms the basis of Linde's apparatus for liquefying air.

Fig. 188 shows a form of the apparatus. Air is compressed by the pump A to a pressure of 200 atmospheres, and forced into the spiral B. The heat produced by the compression is carried away by the running water which circulates round the spiral. The air then passes to the cock D through a narrow spiral inside the wide spiral as shown in the figure. When

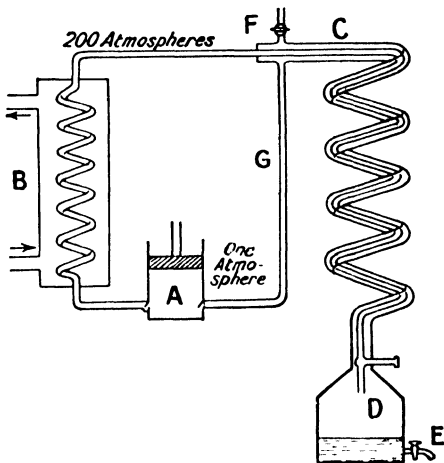


FIG. 188.—Liquid air machine.

D is opened, the air expands to a pressure of 1 atmosphere, and the temperature falls. The cooled air then returns by the wide spiral to C and by the tube G to the pump again. On its upward passage, it cools the air which is coming down to D through the inner spiral. Thus the temperature of the air issuing from D falls, until the temperature of liquefaction is reached. Liquid air then collects in the receiver, and can be drawn off through the stopcock at E. In order to replace the quantity liquefied, air is allowed to enter from the atmosphere at F.

In factories for the manufacture of ice the low temperature required is usually produced by the rapid evaporation of ammonia. Ammonia gas is liquefied at ordinary temperatures by the application of pressure

alone. It is next passed through coils surrounded by running water in order to remove the heat of compression. The pressure is then reduced, whereupon it evaporates; this evaporation is used to cool brine, which in turn is used to make the ice.

EXAMPLES VII

1. Find the equivalent in ergs of the amount of heat required to raise 1 lb. of water through 1°F .

2. Mercury falls from a height of 15 metres upon a perfectly non-conducting surface. How much is its temperature raised? The specific heat of mercury is 0.033.

3. A waterfall is 80 ft. high. How much warmer should the water be at the foot than at the top of the fall, if all the energy acquired is changed into irregular motion in a pool at the foot, and then into heat, none of the heat escaping to the bed of the river?

4. From what height must a raindrop fall to the ground in order that its temperature may be raised 1°C .? (The result will be only of theoretical interest; in practice the resistance of the air cannot be neglected.)

5. A lead bullet of specific heat 0.032 strikes an iron target with a velocity of 350 metres per second. If the bullet falls dead, and half the heat produced is lost to the target, find its rise in temperature.

6. In determining Joule's equivalent by Searle's apparatus radius of the disc was 12.5 cm., the number of revolutions 597, weight of the pan and contained weights 177.2 gm., the water equivalent of the cones 18.5 gm., and the weight of water contained 20 g. In 4 min. the temperature of the water rose 4.5°C . from room temperature. The rotation was then stopped, and the temperature fell 0.2°C . in the next 4 min. Calculate the result.

7. A cylindrical copper tube closed at the ends, and of 1 sq. cm. cross-sectional area, has an internal length of 40 cm., weighs 20 gm., contains 40 gm. of lead shot, and is filled up with water. It is held in a vertical position, and then rapidly reversed end for end 300 times. Find the rise in the temperature of the water, neglecting the thermal capacity of the copper and lead and the space occupied by the lead.

8. One litre of air is contained in a cylinder under a piston at 0°C . and a pressure of 76 cm. The temperature is raised 1°C . Calculate in ergs the work done.

9. Calculate the difference between the two specific heats of hydrogen, being given that a litre of hydrogen at a temperature of 0°C . and under a pressure of 76 cm. of mercury weighs 0.08987 gm.

10. Calculate $\bar{V}$ for the hydrogen molecule at 0°C . One litre of hydrogen weighs 0.08987 gm. at 0°C . and 76 cm. pressure.

11. What quantity of coal would produce the same quantity of heat as the work of one horse for one hour? The calorific value of coal is 3.6×10^8 calories per lb., and 1 ft.-lb. = 13.56×10^6 ergs.

12. A quantity of air at 76 cm. pressure is compressed adiabatically to two-thirds of its volume. Calculate the new pressure.

13. A gas at 0°C . is suddenly compressed to half its volume. What does its temperature become?

CHAPTER VIII

CONDUCTION AND CONVECTION

The Transference of Heat.—Heat is transferred from one body to another by three different processes :

(1) If one end of an iron rod is held in a bunsen flame, the other end being held in the fingers, after a little while the end in the fingers feels warm. The sense of touch shows us that the rod becomes hotter, the nearer we approach the hot end. Heat travels along the rod, but there is no bodily motion of the particles from the one end to the other end of the rod. The heat is said to be transmitted by **conduction**.

(2) If a flask rather more than half full of water, is heated by a bunsen as shown in Fig. 189, the water directly above the bunsen becomes lighter as its temperature rises. It consequently ascends, its place being taken by the colder water which comes from the sides. The currents can be made evident by dropping in a few grains of an aniline dye. They carry heat from the hotter to the colder parts of the liquid by transferring the molecules themselves bodily together with their heat. In this case the heat is said to be transmitted by **convection**, and the currents are referred to as convection currents.

(3) The earth derives its heat and light from the sun. But the earth's atmosphere becomes thinner and thinner as we ascend upwards from its surface, ceasing altogether at a height estimated between 100 and 500 miles. Above this is merely empty space. Now we cannot have conduction or convection where there is no matter, so that the process by which the heat travels from the sun must be of an essentially different nature. The process by which heat passes from one place to another without the action of any material medium is called **radiation**. As will be shown later, the radiation of heat obeys the same laws as the radiation of light.

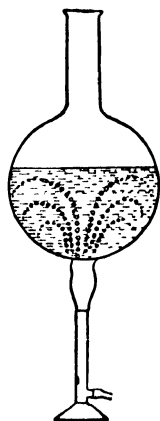


FIG. 189.—Convection currents.

Flow of Heat along a Bar.—The process of conduction can be made very clear by the arrangement shown in Fig. 190. AB is a copper bar one end of which dips into a crucible containing molten lead. Small holes equally spaced are drilled in the bar and thermometers are inserted in these holes, mercury being placed

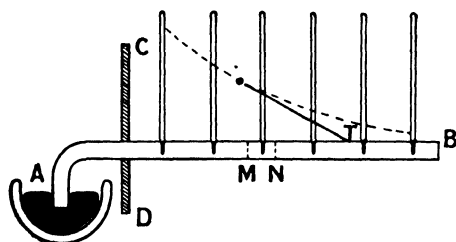


FIG. 190.—Conduction of heat along a bar.

in each hole in order to ensure good contact. The progress of the heat along the bar is shown by the rise of the thermometer readings. CD is a screen for protecting the rod from the flame which heats the lead.

We may distinguish two stages in the experiment; (i) the variable state during which all parts of the bar are rising in temperature; and (ii) the permanent state, or state of steady flow, which sets in a few minutes after the experiment has started. If we consider a small portion of the bar, MN, heat flows into this portion from the left. Part of this heat is used to raise the temperature of MN; the amount used for this purpose is equal to the thermal capacity of MN multiplied by the rise of temperature. The remainder either flows across the section N into the bar to the right or is lost by convection and radiation through the surface of MN to the air. After a time the temperature of MN ceases to rise. All the heat which flows in through M then passes out through N, or is lost through the surface. The permanent state is then reached. The variable state can thus be defined as the state of the bar during the time it is *filling up with heat*.

The readings of the thermometers in the permanent state are shown connected by a dotted curve in the diagram. The slope of this curve is referred to as the **temperature gradient**; thus the tangent of the angle PTN is the temperature gradient at P.

The heat that enters the bar from the lead at A, leaves it through the surface and the end B. The quantity passing through any cross-section per second diminishes as we approach B. But the greater the conductivity of the material, the farther along the bar the heat travels before escaping to the air, once the permanent state is reached; and consequently the higher the temperatures attained.

When the end A is dipped into the lead, the molecules at that end of the bar are thrown into violent to-and-fro motion. They collide with the neighbouring molecules, and throw them in

turn into motion; these in turn collide with molecules farther along the bar, and presently all the molecules in the bar have their kinetic energy increased. And heat, as has already been mentioned, is simply the kinetic energy of the molecules. The molecules themselves do not travel along the rod; the mean position of each molecule remains unaltered. But the kinetic energy is handed on from one molecule to another.

Fundamental Law of the Conduction of Heat.—Fig. 191 represents another arrangement for studying the conduction of heat. A rectangular metal block has two holes drilled in it and thermometers inserted in these holes with their bulbs a distance d apart. On the one face is a steam chamber B surrounded by an outer steam chamber C, and on the other face an ice chamber D surrounded by an outer ice chamber E. Thus the one face is kept at the temperature of steam and the other at the temperature of melting ice, and heat flows across the block from right to left. In the middle the lines of flow are parallel to one another and at right angles to the faces of the block, but above and below they are curved owing to heat escaping to the surroundings. We consider only the portion of the block between the inner chambers B and D, where the lines of flow are straight. These inner chambers are, of course, exactly opposite to each other, and their ends have the same area A . The outer chambers are merely a device, referred to as a “guard-ring,” for ensuring that the lines of flow are straight in the inner portion of the block.

The heat which flows across, melts some of the ice in the inner chamber D. The melted ice drains off as water, and can be collected and weighed. The product of the weight and the latent heat of water gives the quantity of heat which flows across. This quantity might also be obtained from the weight of the steam that condenses in B.

It is found as a result of experiment that the quantity of heat, Q , is

- (i) directly proportional to the area A of the face;
- (ii) directly proportional to the time t ;
- (iii) directly proportional to the difference in temperature, $T_2 - T_1$, recorded by the two thermometers;
- (iv) inversely proportional to d , the distance between the thermometer bulbs.

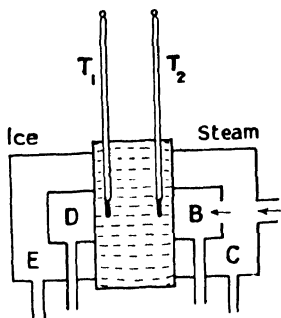


FIG. 191.—Conduction of heat through a block.

This result is known as the fundamental law of the conduction of heat. It can be put in the form of the equation

$$Q = \frac{kA\ell(T_2 - T_1)}{d}$$

where k is a constant of proportionality, called the **conductivity** of the substance of the block.

To put the definition in words :

The conductivity of a substance for heat is the quantity of heat which passes in unit time between the opposite faces of a cube of unit side, when the faces are kept at one degree difference of temperature.

In the equation $(T_2 - T_1)/d$ measures the temperature gradient across the block.

EXAMPLE.—How much heat is conducted per second through an iron plate 3 mm. thick and of 1 sq. metre area, when its two faces are kept at temperatures differing by 2.0°C . ? The conductivity of the plate is 0.140 c.g.s. unit.

$$Q = \frac{0.140 \times 10^4 \times 2}{0.3} = 9933 \text{ cal.}$$

Ingenhousz's Experiment.—The thermal conductivities of different substances can be compared by Ingenhousz's experiment,

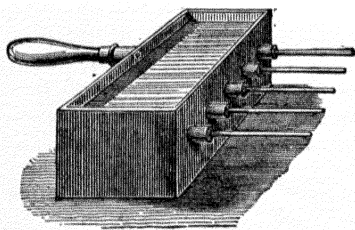


FIG. 192.—Ingenhousz's experiment.

the apparatus for which is shown in Fig. 192. It consists of an iron trough, into the sides of which are fixed rods of the different substances. These rods are all of the same shape and size, and extend the same distance into the trough. They are coated with paraffin wax which melts at about 52°C ., and the trough is filled with boiling water. Heat from the water is conducted along the rods, and the wax melts to a certain distance along each rod, and drops off. The wax melts farthest along the rod which conducts best, and it may be shown by reasoning beyond the scope of this book, that if the bars are so long that their ends do not rise appreciably above the temperature of the room, then the conductivity of the substance is proportional to the square of the distance to which the wax melts.

It should be noted, that the quickness with which the wax starts to fall off is no guide to the conductivity, but depends as well on the thermal capacity of the rod. As has already been mentioned, the heat has to fill up the rod as it travels along it,

and may at first travel faster along a rod with a low thermal capacity. Only when the wax ceases to fall off can a conclusion be drawn as to the conductivity.

Measurement of the Conductivity of a Good Conductor.—The straightforward way of measuring the conductivity of a good conductor such as a metal is to use the method of Fig. 191. A modification of this method is in considerable use in laboratory courses under the name of Searle's apparatus (Fig. 193). In this apparatus the guard-ring is dispensed with, but the bar under experiment is surrounded by felt, so as to prevent loss of heat from the side. Steam plays on one end, and the other end has a $\frac{1}{8}$ " of thin copper tubing soldered to it, through which a steady

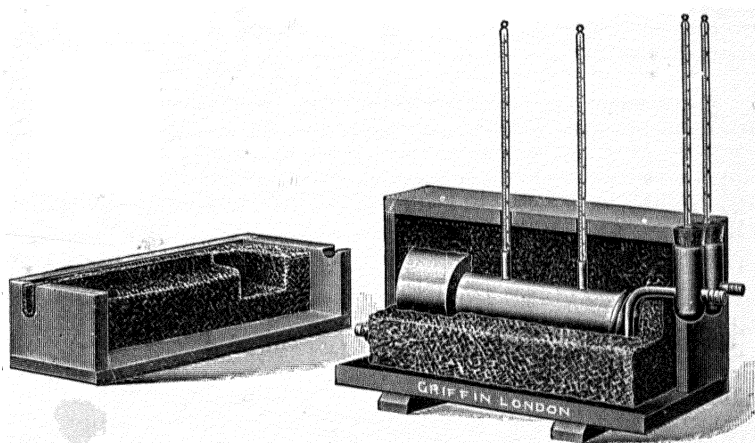


FIG. 193.—Searle's apparatus for measuring the conductivity of a bar.

flow of water passes. The amount of heat received by the water is determined by finding the weight of the water which flows through, and multiplying it by the difference in temperature before and after passing through. This amount of heat is the heat conducted through the bar. The temperature gradient is obtained from the readings of two thermometers which are placed in copper tubes brazed into the bar a known distance apart.

Forbes determined the conductivity of metal bars by using an arrangement similar to that shown in Fig. 190. The temperature gradient was determined directly from the slope of the curve at P. The area of the cross-section was known. The only factor wanting to complete the determination was consequently the quantity of heat Q , which crossed the section of the bar

directly below P per second. This quantity is equal to the quantity lost per second by the surface of the bar to the right of the section, and depends only on the temperature of that surface, which, of course, is known. It was determined by an auxiliary experiment. A short bar of the same material and same cross-section as the original bar was heated to a uniform temperature slightly higher than the temperature of the bar at the section, and then left to cool in air, the temperature being read at short intervals by a thermometer the bulb of which was placed in a hole in the bar. Thus the rate of loss of heat from the short bar to air was measured for different temperatures, and from the results the quantity lost per second by each unit of length of the bar in the original experiment to the right of P to the air could be calculated. The sum of these quantities gives Q .

Measurement of the Conductivity of a Bad Conductor.—The arrangements just described are unsuitable for determining the conductivity of a bad conductor such as glass, because the heat would travel round a glass block or glass bar instead of through it. We have consequently in this case to adopt the arrangement shown in Fig. 194.

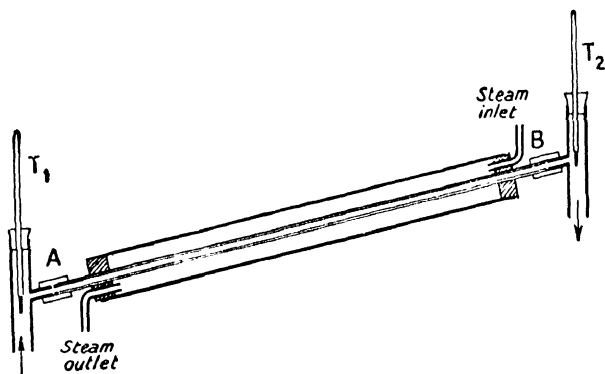


FIG. 194.—Apparatus for measuring the conductivity of glass.

The substance to be experimented on, glass or whatever it is, is used in the form of a tube, AB , through which a steady stream of water flows, entering at A and leaving at B . This tube is surrounded by a steam jacket, and is joined at its ends by short pieces of rubber tubing to brass tubes containing thermometers, T_1 and T_2 . The thermometers are held in position by rubber stoppers.

Let l be the length of the tube exposed to the steam, and let r_1 and r_2 be its internal and external radii. The mean radius is $(r_1 + r_2)/2$, and, since it is thin, the wall of the tube through which the heat has to pass is equivalent to a plate of area $\pi l(r_1 + r_2)$ and thickness $r_2 - r_1$.

Let t_1 be the temperature of the water before entering, and let t_2 be its temperature after leaving the tube, when the steady state is established. These temperatures will not differ much from the temperature of the room, and their mean value $(t_1+t_2)/2$, may be taken as the temperature of the inner surface of the tube. Its outer surface is at 100° C., the temperature of steam. The quantity of heat flowing through the glass per second is consequently

$$\frac{k\pi l(r_1+r_2)[100-\frac{1}{2}(t_1+t_2)]}{r_2-r_1}$$

where k is the conductivity of the glass. Let m be the mass of water which flows through per second, when the steady state is established; this can be obtained by receiving the outflow in a measuring cylinder. Then the quantity of heat received by the water per second is $m(t_2-t_1)$, and on equating this expression to the one above we obtain

$$k = \frac{m(t_2-t_1)(r_2-r_1)}{\pi l(r_1+r_2)[100-\frac{1}{2}(t_1+t_2)]}$$

In order to have the water well mixed a piece of thick twisted wire or similar obstruction is often placed along the axis of the tube.

All metals conduct heat well, the best conductor being silver, then copper and gold following in the order named. It is found that for metals and alloys the ratio of the thermal to the electrical conductivity has approximately the same value at any given temperature. This result is known as the **Wiedemann-Franz law**. Wood does not conduct nearly so well as the metals. If a thin piece of paper is wound round a pencil and held in a flame, it is scorched at once. But if the pencil is replaced by a copper rod, the paper may be held in the flame a few seconds without being scorched, for the metal conducts away the heat.

Conductivity of Liquids and Gases.—Liquids, with the exception of mercury, are bad conductors of heat. If they are heated from below, the heat travels upwards mainly by convection. In a conductivity determination, therefore, the heat must be made to travel downwards. The heated liquid is less dense than the other liquid and remains floating above it; convection is consequently eliminated.

The low conductivity of water is illustrated by a well-known experiment. A piece of ice is wrapped in gauze so as to make it sink, and allowed to fall to the bottom of a test tube filled with water. The test tube is then held in an inclined position, and its upper part heated by a bunsen burner. It is found that the water at the top can be made to boil for a considerable time without melting the ice.

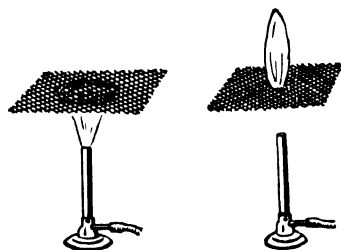
The conductivity of gases is still less than that of liquids, the value for air being about 0.00005 c.g.s. unit. The conductivity of gases is independent of the pressure except at very low pressures. The poor conductivity of feathers, fur, felt, and woollen

materials is due in large measure to the fact, that they are full of numerous small air spaces. The conductivity of woollen materials is greatly increased, if the enclosed air is pressed out of them.

Conductivity and Sensation.—It is well known, that in a room where there is no fire, the iron fender feels colder to the touch than a rug beside it, although, as may be shown by a thermometer, they are both at the same temperature. This is simply because the iron is a better conductor of heat than the rug, and conducts away heat from the hand more rapidly than the rug does. But if they are both heated by the fire to the same temperature above the temperature of the hand, the fender feels hotter to the touch than the rug does, because it conducts heat more rapidly to the hand.

The Davy Safety Lamp.—For a gas to burn in air it must be raised to a definite temperature known as the ignition temperature. If the temperature of the inflammable mixture is cooled below

this value, the flame goes out. This fact forms the basis of the Davy lamp used by miners, and can also be demonstrated very neatly by the experiments shown in Figs. 195 and 196.



FIGS. 195 AND 196.—The principle of the safety lamp.

In the experiment represented in Fig. 195 a piece of copper gauze is placed over a bunsen burner. Owing to the gauze conducting away the heat, the flame below the gauze cannot heat the gas above the gauze to the ignition

point. Consequently the flame cannot pass through the gauze, and, if the gauze is pressed down upon the bunsen, the flame may go out.

In the experiment represented in Fig. 196 the gauze is placed upon the bunsen. The mixture of gas and air is then allowed to pass through it, and lit above the gauze. The latter is then raised. As it moves upwards, it conducts away the heat and lowers the temperature below ignition point; consequently below the gauze the flame goes out.

The atmosphere of some mines contains an inflammable gas, called methane or fire damp, and if this comes into contact with a naked light, an explosion follows. A society which had been formed to discover some way of preventing these explosions, asked Sir Humphry Davy for his assistance, and the discovery of the safety lamp was the result. This is an oil lamp the flame of which is surrounded by a cylinder of wire gauze. The air necessary for the combustion of the oil passes in through the gauze.

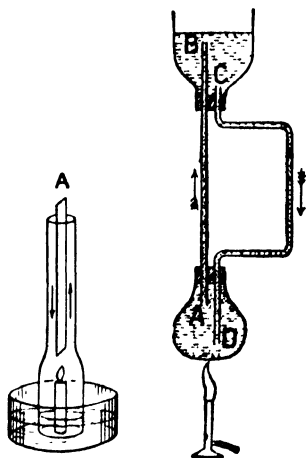
If fire damp is present it enters into the lamp, and burns inside it with a bluish flame. But owing to the conduction loss through the gauze to the base of the lamp, the temperature of the fire damp immediately outside the gauze never rises to the ignition point. Thus the flame cannot travel outside the lamp, and explosions are prevented.

Scale in Boilers.—In the furnace of a boiler heat passes from a hot gas through a metal plate to water. Owing to the hardness of the water a deposit called scale is sometimes deposited on the one side of the plate, and soot may be deposited on the other side. These diminish the conductivity of the plate very considerably, and owing to being deposited unequally set up temperature differences and consequently inequalities in expansion, that may weaken the boiler. Again, if a layer or film of gas after parting with its heat remains in contact with the plate, the heat from the remainder of the gas has to pass through this badly conducting film. The rate at which the heat flows from the furnace to the boiler depends more on not allowing such films to form than on the conductivity of the metal itself.

Experiments on Convection.—Figs. 197 and 198 show two lecture experiments on the convection of heat. Fig. 197 shows a candle which has been lighted and mounted in a shallow vessel containing water. When a glass chimney is placed over the candle, so that the water prevents air entering from below, the flame dies down and goes out. But if a metal partition A is inserted in the chimney as shown in the figure, convection currents establish themselves in the direction of the arrows, as may be demonstrated by the smoke of a match. These provide air for the flame, and the candle remains lighted.

The water heated at the bottom of the lower vessel in Fig. 198 expands and becomes lighter than the surrounding water. It consequently ascends and passes up the tube AB, its place being taken by the colder water in the upper vessel which travels down the tube CD. The motion can be shown very clearly by dropping some dye into the upper vessel and watching it travel down the tube CD.

Convection currents are due to the fact, that substances change their density when heated.



FIGS. 197 AND 198.

Hot Water Supply.—Fig. 198 illustrates the principle of the domestic hot water supply. The actual arrangement is shown in Fig. 199. B is the boiler, placed behind the kitchen fire, and T the hot water tank. The water heated in B ascends by the pipe M and enters at the top of T; then, as it cools, it falls to the bottom of T, and descends by the pipe N to the bottom of the boiler. Water is drawn off at W to the hot water taps, and the

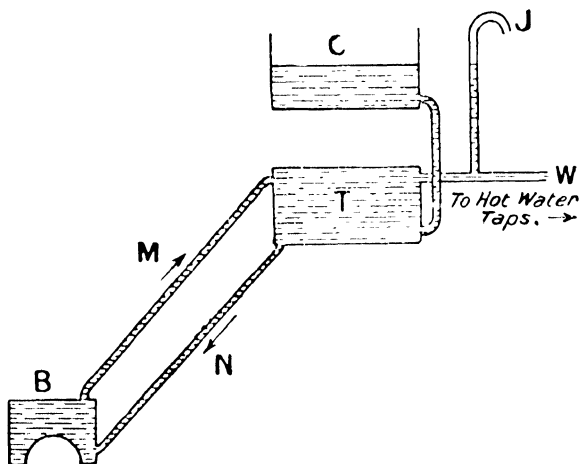


FIG. 199.—Hot water supply.

place of this water is taken by the water that descends from the cold water cistern, C. The open pipe at J is for the escape of steam, should the water reach the boiling point.

In the same way in a hot water heating system a pipe goes from the top of the boiler to the highest point of the system. From there it is led through the rooms to be warmed and then back to the bottom of the boiler.

Example on Chimney.—A chimney 80 feet high and 8 square feet in internal sectional area at the top, has an air-tight cover put over it. The air outside has a temperature of 15°C . and pressure of 76 cm., and the air inside has an average temperature of 60°C . Find the force necessary to keep the cover in position.

Let h be the height of the chimney, ρ the density of the air outside, ρ' the density of the air inside, and let P be the pressure of the atmosphere on the top of the cover. Then

$P + hg\rho$ = pressure at the base of the chimney
 and $P + hgp - hg\rho'$ = pressure on lower surface of the cover.
 Hence $Ahg(\rho - \rho')$ = upward thrust exerted by the air inside.

Under standard conditions $\rho = 1.293$ gm. per litre
 $= 0.08071$ lb. per cub. ft.
 Hence required force $= 8 \times 80 \times 0.08071 (31.3 - 31.1)$
 $= 6.6$ lb. wt.

The result is only approximate, since the gases in the chimney will include carbon dioxide, which is heavier than air at the same temperature. But it shows how the draught increases with the height of the chimney, and explains why sometimes a newly kindled fire will not "draw," until the chimney is warmed.

Land and Sea Breezes.—The land and sea breezes which are observed near the coast, especially in hot countries, are due to convection. During the day the heat of the sun raises the temperature of the land considerably higher than that of the sea, because the sea has a greater specific heat than the land has. During the night the land cools more rapidly than the sea does, because it has not the same store of heat to radiate away. Thus during the day the layers of air near the earth expand and rise, causing a cool breeze to blow in from the sea to fill their place. This usually reaches its maximum in the late afternoon. But at night the air over the sea is the warmer and rises, causing a breeze to blow out from the land.

Trade Winds.—The trade winds are examples of the same process on a larger scale. Heated air rises near the equator, its place being supplied by an inflow of cooler air from the north and south. If the earth were not in rotation, this inflow would take the form of a north wind in the northern hemisphere and a south wind in the southern hemisphere. But owing to the rotation of the earth points on its surface near the equator are moving from west to east with a velocity greater than points in higher latitudes. Hence a wind which starts south towards the equator with the component eastward velocity belonging to the region of its origin, finds as it moves southwards, that the surface is moving eastwards below it; thus with reference to the surface its direction is from north-east to south-west. In the same way the trade winds in the southern hemisphere blow from south-east to north-west. There are belts of calm north and south of the trade winds, where the air descends, which rose at the equator, and has now become cool.

EXAMPLES VIII

1. What would be the effect of wrapping up ice in blankets?
2. How much heat is conducted per second through a slab of copper 3 cm. thick and of 1 sq. metre area, when its two faces are kept at temperatures differing by 1° C.? The conductivity of copper is 0.018 c.g.s. unit.

3. A room is kept at a uniform temperature of 15°C. , while the outside temperature is 5°C. Find the quantity of heat lost in an hour through a wall 3 metres high, 4 metres broad, and 40 cm. thick. The conductivity of the wall may be taken as 0.005 c.g.s. unit.

4. In making a determination of the conductivity of a glass tube by the method described on p. 220, the length of tube exposed to the steam was 54.4 cm., its internal and external radii were 0.8675 and 0.9775 cm., the temperature of the water was 13.8°C. before, and 28.4°C. after passing through the tube, and the quantity of water that passed through per second was 23.8 gm. Calculate the result.

5. A boiler is made of iron, 0.5 cm. thick and of conductivity 0.2 c.g.s. unit. On its inside surface a crust forms, 0.4 cm. thick and of conductivity 0.001 c.g.s. unit. The temperature outside is 300°C. and inside is 120°C. How much heat is conducted through each square centimetre per second?

CHAPTER IX

RADIATION

Apparatus.—As a means of detecting and measuring heat rays we may use either an ordinary thermometer having its bulb blackened with soot or covered with black paper, a differential air thermometer, or a thermopile. The differential air thermometer is represented in Fig. 200. The two bulbs have their surfaces blackened and one is placed to receive the rays; the heat causes the air in it to expand and force the coloured liquid down the tube, and the displacement of the end of the column measures the intensity of the heat rays received. The bulbs are connected by pieces of pressure tubing, and so can be turned up or down.

If the ends of two wires of different metals are soldered together and heat rays are allowed to fall on one of the junctions, an electric current flows round the circuit proportional in intensity to the difference in temperature of the two junctions. In the thermopile a number of pieces of wire of two metals—for example, bismuth and silver—are joined together to form a circuit, a wire of the one metal always being followed by a wire of the other, and the alternate junctions are brought together to the one point, and exposed to the rays. By this means the current is magnified, and the arrangement rendered more sensitive.

Which of the instruments, thermometer, differential air thermometer, or thermopile, should be used, depends on the purpose in question. The ordinary thermometer will not render the results visible to a large audience; the differential air thermometer is more suited for this purpose, and the thermopile is best of all, because the reading of the galvanometer can be made visible by the motion of a spot of light on a scale. The size of the bulb of the differential air thermometer makes it unsuitable

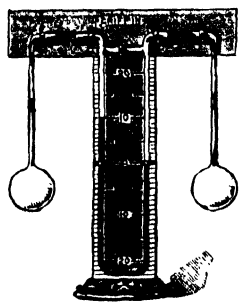
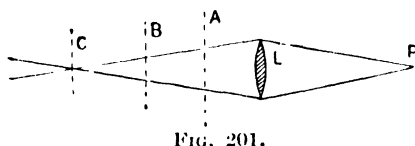


FIG. 200.—Differential thermometer.

for work with narrow pencils of rays. The thermopile has the disadvantage of requiring a very sensitive galvanometer; the most sensitive thermopile on the market at present, when connected on to the Weston student galvanometer and placed as close as possible to an 8-candlepower lamp, gives a deflection of only 4 divisions, corresponding to 0.000067 amp.

As sources of heat rays we can use a metal ball heated to red heat, a tin filled with boiling water, or the arc lamp. One particular pattern of arc lamp which takes less than five amperes and has carbons as thin as pencils, is very suitable for experiments on heat. In this lamp the positive carbon is horizontal, and the arc is surrounded by a glass cover which restricts the supply of air and prolongs the life of the carbons.

Demonstration of Heat Rays.—In Fig. 201 P represents the crater of an arc lamp. The rays from this lamp fall upon the



lens L and are brought to a focus at C. After passing through the focus they diverge again. The convergence and divergence of the light rays may be made evident by receiving them

on a piece of paper and moving this paper backwards and forwards.

The bulb of a thermometer was covered with black paper, placed in succession at several points in the beam along the line marked A, and a reading taken of the rise of temperature produced

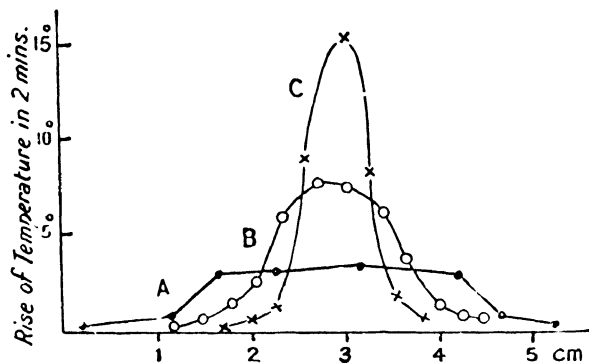


FIG. 202.—Distribution of energy across beam.

in two minutes. After taking one reading the thermometer was allowed to cool to room temperature before taking the next. These readings are represented by the curve A in Fig. 202; the

abscissæ denote the distance in cm. moved by the thermometer across the beam. The same process was then repeated for the sections B and C, and the readings for these sections represented by the curves B and C. It should be stated that the image of the arc at C is three or four millimetres wide, and the bulb of the thermometer has a definite size; hence the breadth of the curve C.

The curves show, that as the beam becomes narrower the rise of temperature produced increases, or in other words *the heat rays converge to a focus* in the same way as the light rays do. If a piece of Ilford infra-red gelatine filter is placed in the path of the rays close up to the lens, it stops the light, and there is no visible image on a paper screen placed at C. But if the thermometer is moved along the sections A, B, and C, as before, curves are obtained similar to the curves A, B, and C in Fig. 202, except that this time the ordinates are only 64 per cent. as high. Consequently, although all the light is stopped by the filter, heat rays get through, and these dark heat rays converge to a focus in the same way as light rays do.

Thus we have two different kinds of rays converging to a focus at C, light rays which produce a visible image on a paper screen and at the same time heat the bulb of a thermometer, and dark heat rays which produce no visible effect on a screen, but heat the bulb of a thermometer. Since the action of a lens depends on its power to refract light, it follows that dark heat rays are refracted in the same way as light rays. By refraction is meant the deviation of a ray which occurs when it passes from air into a medium such as glass, or from glass into air.

Reflection and Absorption of Heat Rays.—Fig. 203 represents an arrangement for demonstrating the reflection of heat rays.

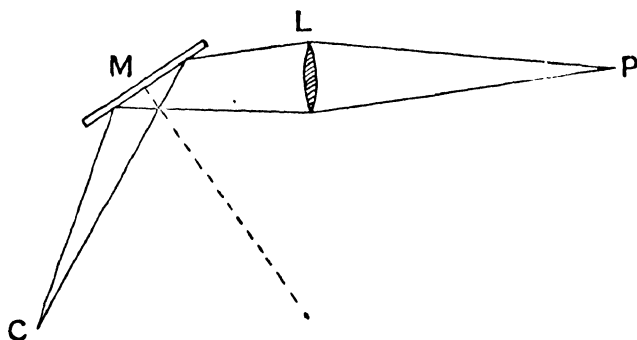


FIG. 203.

The radiation from an arc lamp P falls on a lens L, and after reflection by a glass plate or metal mirror at M is brought to a

focus at C. By taking readings with a thermometer at different points on a section at C it may be shown that the focus of the heat rays coincides with the focus of the light rays, no matter what the inclination of the mirror is, both when the infra-red filter is interposed in the path of the rays, and when it is removed. Thus heat rays obey the same laws of reflection as light rays, *i.e.* the incident and reflected beams make equal angles with the normal to the mirror at M.

If an obstacle is set up to cast a shadow on a screen, and the edge of the shadow is explored with a thermometer or thermopile, it is found that the heat rays are cut off by the obstacle in the same way as the light rays are. Thus as regards propagation, reflection, and refraction, heat rays behave exactly as light rays, and it may be shown by reasoning which lies outside the scope of this chapter, that both light and heat are propagated by a wave motion through space. This wave motion consists of waves of different wave-length; they have all the property of raising the temperature of the bodies on which they fall, but only certain wave-lengths have the property of exciting the sensation of light. The rays from the arc lamp which have the property of passing through the Ilford infra-red filter, have a longer wave-length than the rays of light have.

Substances possess the property of transmitting heat rays to different degrees. Those which allow radiant heat to pass through them are said to be **diathermanous**, and those which absorb the radiant heat falling on them and prevent its transmission are said to be **adiathermanous**; these names are, however, falling out of use. A vacuum is perfectly diathermanous, but the atmosphere, especially the aqueous vapour in it, absorbs an appreciable proportion of the radiant heat from the sun.

The proportion of the radiant heat from the arc lamp transmitted by various substances may be determined by holding them up between the source and the blackened bulb of a thermometer, noting the rise of temperature produced, and comparing it with the rise when there is nothing in the path. The most accurate results are obtained by waiting until the temperature ceases to rise, but for rough work the rise in two minutes may be taken instead. When measured in this way, the Ilford infra-red filter is found to transmit 64 per cent., a glass plate 78 per cent., and a glass cell containing a thickness of one centimetre of water 32 per cent. of the incident radiation, when that radiation has already had its character changed by passing through a lens.

The statement is to be found in many books, that a solution of alum is particularly efficient as an absorber of heat rays. This is not the case; a strong solution of alum absorbs appreciably less than the same volume of water.

Different Kinds of Heat Rays.—The fact that there are different kinds of heat rays each characterised by its own wave-length, should be borne constantly in mind. A body may be transparent to one kind and opaque to another. When a piece of metal is heated, it radiates at first heat of a long wave-length; then as the temperature rises, the wave-lengths become shorter and shorter, until finally they are short enough to excite the sensation of light.

Glass is transparent to light and the heat rays which have a wave-length not much different from the wave-length of light. These rays pass through the glass roof of a hot-house and warm up the interior. The latter then radiates heat of a longer wave-length to which glass is opaque. Thus the hot-house is a kind of light trap; the radiation gets in, but cannot get out.

Rock salt is transparent to the longer wave-lengths which glass absorbs, and hence lenses and prisms of rock salt are widely used for experiments with heat rays.

All the different kinds of heat rays from the sun pass through interstellar space without absorption and therefore without warming it, and are not absorbed much by air, but the aqueous vapour in the atmosphere absorbs the longer heat rays very strongly. Consequently the temperature never falls so rapidly at night as when there is a clear dry atmosphere.

Experiment with Conjugate Mirrors.—Fig. 204 represents an alternative arrangement for experimenting with heat rays. Two

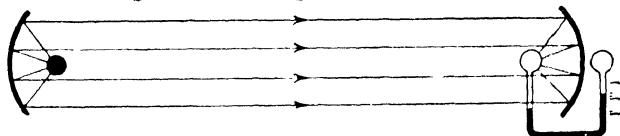


FIG. 204.—Reflection of heat rays by concave mirrors.

concave metal mirrors of about six inches diameter are set up facing each other about two yards apart. The bulb of a differential air thermometer is placed at the focus of the one mirror, and a metal sphere is heated red-hot and placed at the focus of the other. As soon as it is in position, the column of liquid moves up the scale, showing that the heat rays after reflection by the first mirror travel in a parallel beam to the second, and are reflected towards its focus. If the bulb of the thermometer is removed from the focus, there is no effect. The fraction of the radiation transmitted by a glass plate or a cell containing water can be measured by placing them in the path of the beam, and the law of reflection can be further verified by allowing the parallel beam to fall on a plane mirror and then moving the second mirror about, until the direction of the reflected beam is found again.

Heat Rays in History.—Archimedes, the most celebrated mathematician of ancient times, is said to have set the Roman fleet on fire

and destroyed it at the siege of Syracuse in 212 B.C. by concentrating the heat rays of the sun upon it with concave mirrors. The fleet was then one bowshot distant from the walls. But the story is either a pure invention, or arose from a misunderstanding. It is not mentioned by Polybius, who was almost a contemporary of Archimedes, and who has given a detailed account of the siege. Only in the second century after Christ do we first hear that Archimedes destroyed the Roman fleet by fire, and then mirrors are not mentioned; that part of the story dates only from the fourth century of our era.

The most celebrated burning mirrors in history were quite moderate affairs. They were made by Tschirnhausen about 1687. With the aid of a large copper mirror he brought the sun's rays to a focus and used them to kindle wood, boil water, melt tin which was three inches thick, and make a hole in a coin. He also made large burning glasses, and amused himself by boiling fish and crabs in water with them. One of his burning glasses was used at a later date in a well-known experiment at Florence to burn up diamonds; the fact that diamonds are combustible was then demonstrated for the first time.

Large lenses and concave mirrors should not be left where the sun's rays can fall on them; they may set the laboratory on fire.

Experiments with Leslie's Cube.—So far we have been concerned with the properties of the heat rays themselves. We shall now consider the question of their emission and their absorption.

Fig. 205 represents a very instructive experiment. A cubical tin box, known as Leslie's cube, of about six inches side, has one of its sides covered with lamp black, the opposite side brightly polished, and the other two treated in other ways. The cube is filled with boiling water and placed midway between the bulbs of a differential thermometer, the blackened side opposite the one bulb and the polished side opposite the other. Both bulbs consequently receive heat by radiation. It is found that the column is depressed on the right, *showing that a blackened surface radiates more heat than a polished one.*

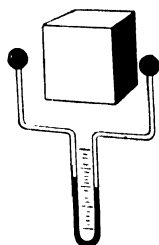


FIG. 205.—
Leslie's cube.

Let us suppose now that one of the glass bulbs of the differential thermometer is blackened and the other is silvered, and that two opposite surfaces of the cube are blackened. Then, if the cube is filled with boiling water as before, and placed midway between the bulbs, with the blackened faces opposite them, it is found that the blackened bulb becomes hotter than the other one. But the blackened surfaces of the cube are radiating heat at the same rate. *Hence a blackened surface absorbs more heat than a polished one.*

By treating the surfaces in other ways it may be shown

generally, that surfaces which radiate heat well, also absorb it well. This may also be proved by other experiments. For example, if letters are written with ink on a piece of bright platinum foil and the latter then heated in the bunsen flame, the writing stands out brighter than the surrounding foil. It absorbs more and consequently emits more.

Ritchie's Apparatus.—Fig. 206 shows an apparatus devised by Ritchie for showing that the emissive and absorptive properties of a surface increase in proportion with one another. It consists of two cylindrical metal vessels C and D, which are filled with air and connected by a glass tube in which a column of coloured liquid has been placed. Between them is a third and larger vessel AB which can be filled with hot water. The surfaces D and B are coated with lamp black, and the surfaces A and C are nickel plated and polished. When boiling water is poured into the middle vessel, the column of liquid does not move. B emits more than A, but D absorbs more than C, so the one vessel gains exactly as much heat as the other.

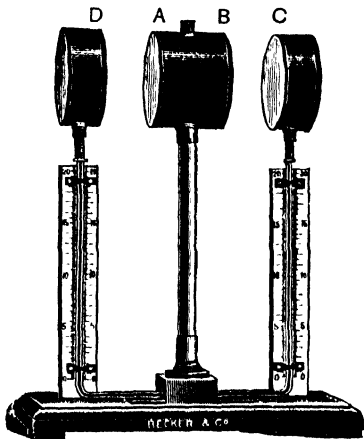


FIG. 206.—Ritchie's apparatus.

Prevost's Theory of Exchanges.—If a hot body is suspended in an evacuated enclosure in such a manner that there is no conduction loss to the enclosure, it loses heat solely by radiation. If the temperature of the body is read from time to time, it is found to decrease until it becomes the same as the temperature of the enclosure. There is then no further change, and it might be thought, that radiation from the body has ceased.

If, however, the body is removed to another enclosure at a lower temperature, for example, a large evacuated tin surrounded by ice-cold water, radiation apparently starts again, and continues until the temperature of the second enclosure is reached. Now there is no material connection between the body and the second enclosure which could make the radiation start again. We are therefore forced to the conclusion, that the body has been radiating all the time.

During the first part of the experiment the body was emitting radiation to the enclosure and receiving radiation from it. The amount received per second was constant, but the amount

emitted per second fell off as the temperature diminished, until finally when equilibrium of temperature was established, the two quantities balanced. When the body was removed to the second enclosure, its temperature and consequently its rate of radiation started to fall again. But when the final equilibrium of temperature was attained, the radiation loss from the body balanced the gain from the enclosure.

Arguing in this way Prevost propounded his theory of exchanges: *All bodies are radiating all the time, and the temperature of a body becomes stationary when it sends as much radiation to the surrounding objects as it receives from them.*

If in the experiment with the conjugate mirrors described on p. 231, a lump of ice is placed in the focus of the one mirror, the temperature of the bulb at the focus of the other mirror falls. This is not because the ice is radiating cold, but because the bulb is radiating more heat to the ice than the ice is radiating to the bulb.

Newton's Law of Cooling.—Let us suppose that a small calorimeter, which can be closed by a lid containing holes for a thermometer and a stirrer, is suspended inside a large vessel which

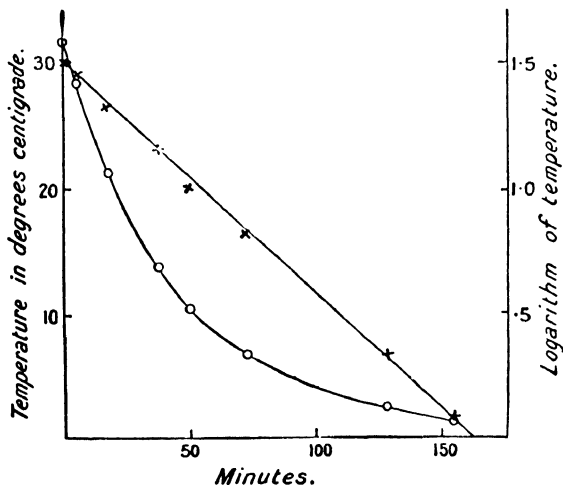


FIG. 207.—Cooling curve.

is itself surrounded by water. Then, if the calorimeter is filled with a hot liquid and left to itself, it cools, mainly by convection and partly by radiation. If the temperature is read from time to time, and plotted against the time, we obtain a cooling curve such as is represented by the circles in Fig. 207. The ordinates

represent difference of temperature between the calorimeter and the surrounding enclosure, the latter being in this case at 13.0°C . It will be noticed that the temperature falls rapidly at first, and then more slowly, and that after the lapse of two and a half hours the calorimeter is still more than one degree above the temperature of the enclosure.

If instead of plotting temperature difference we plot the logarithm of the temperature difference against the time, we get a curve which is practically a straight line shown by the crosses. Consequently, if θ_1 and θ_2 denote the temperature differences at times t_1 and t_2 ,

$$\log \theta_1 - \log \theta_2 = k(t_2 - t_1)$$

where k is a constant. This gives

$$\frac{\theta_2}{\theta_1} = 10^{-k(t_2 - t_1)}$$

If we subtract both sides from 1

$$\frac{\theta_1 - \theta_2}{\theta_1} = 1 - 10^{-k(t_2 - t_1)}$$

Now let us suppose that $t_2 - t_1 = 1$ min. Then the right-hand side, and consequently the left-hand side, is constant. Thus the fall in temperature per minute is proportional to θ_1 . This result was first observed by Newton, but under very different conditions from those described here. Stating it formally:

The rate at which a given body loses heat is proportional to the difference of temperature between the body and its surroundings.

This result has already been made use of on p. 176 in connection with the convection correction in calorimetry.

Specific Heat by the Method of Cooling.—The arrangement described in the preceding section can be used to determine the specific heat of a liquid. For suppose that when the calorimeter is filled with water, it takes x minutes to cool from θ to θ_n measured from room temperature. Divide the fall of temperature into n steps, θ to θ_1 , θ_1 to θ_2 , . . . θ_{n-1} to θ_n , and let the times taken for these separate steps be $\tau_1, \tau_2, \dots, \tau_n$. Then if m_1 is the mass of the water and w the water equivalent of the calorimeter, the rate of loss of heat during the first step

$$(m_1 + w) \frac{\theta - \theta_1}{\tau_1} = c\theta$$

by Newton's law of cooling, c being a constant. This may be rewritten

$$\frac{\tau_1}{m_1 + w} = \frac{\theta - \theta_1}{c\theta}$$

Similarly for the second interval

$$\frac{\tau_2}{m_1 + w} = \frac{\theta_1 - \theta}{c\theta_1}$$

and summing for all the intervals

$$\frac{\tau_1 + \tau_2 + \dots + \tau_n}{m_1 + w} = \frac{x}{m_1 + w} = \frac{\theta - \theta_1}{c\theta} + \frac{\theta_{n-1} - \theta_n}{c\theta_{n-1}}$$

Let us now replace the water in the calorimeter by an equal volume of the liquid whose specific heat is to be measured. Let m_2 be its mass, let s denote its specific heat, and let y minutes be the time it takes to fall from θ to θ_n . Then by reasoning exactly similar to that given above

$$\frac{y}{m_2 s + w} = \frac{\theta - \theta_1}{c\theta} + \frac{\theta_{n-1} - \theta_n}{c\theta_{n-1}}$$

Hence

$$\frac{x}{m_1 + w} = \frac{y}{m_2 s + w}$$

whence

$$s = \left(\frac{m_1 + w}{m_2} \right) \frac{y}{x} - \frac{w}{m_2}$$

If covers of different pieces of cloth are fitted over the calorimeter used in the above experiment and cooling curves taken, the "warmths" of the cloths may be compared.

Emissivity of a Surface.—The emissivity of a surface for heat is defined to be the quantity of heat emitted from one square centimetre of the surface in one second for each degree difference of temperature between the surface and its surroundings.

It follows from the experiments already described, that the emissivity of a surface covered with lamp black is greater than the emissivity of a polished metal surface, and that in general dark surfaces have higher emissivities than bright ones.

The **thermos flask** or vacuum flask is a double-walled glass vessel, the space between the walls being a vacuum. The inner surface of the outer wall and the outer surface of the inner wall are silvered. Thus the convection and radiation losses through the side of the flask are a minimum, and it keeps a liquid hot or cold, as is required, for a very long time. The flask was originally designed by Dewar for containing liquefied gases.

EXAMPLE.—Find the rate of cooling of a blackened copper ball of 2 cm. diameter with its temperature 30° C. higher than the surroundings. The density of copper is 8.9, its specific heat is 0.095, and the emissivity of the surface is $\frac{1}{1000}$ c.g.s. unit.

$$\text{Quantity of heat lost per second} = 4\pi \times \frac{1}{1000} \times 30 = \frac{3}{100}\pi$$

$$\text{Thermal capacity of sphere} = \frac{4}{3}\pi \times 8.9 \times 0.095 = 1.127\pi$$

$$\text{Hence rate of cooling} = \frac{3}{100 \times 1.127} = 0.027^\circ \text{ C. per sec.}$$

Stefan's Law.—Newton's law of cooling does not deal with the total energy radiated by the hot body, but with the excess

of the energy lost by radiation and convection over the energy gained by the same processes. It does not hold for great differences of temperature. In 1879 Stefan put forward the suggestion, that the total radiation from a body is proportional to the fourth power of its absolute temperature. He found that this law agreed with the observations made by Dulong and Petit many years previously. It was shown theoretically by Boltzmann, that the law holds only for a "black body," a body the surface of which absorbs all the radiation incident on it, and it is consequently sometimes referred to as the Stefan-Boltzmann law. The surface of the carbon arc or of the carbon filament of a glow lamp is very nearly "black" in this special sense.

Let us suppose that a black body is radiating in an enclosure, and that it is in equilibrium with the enclosure at temperature $T_0^\circ \text{K}$. Then the rate at which energy is being radiated by the black body and the rate at which it is being received by the black body may each be represented by cT_0^4 , where c is a constant. Let us now suppose that the black body is raised to temperature T . It consequently then radiates at the rate cT^4 and loses energy at the rate

$$c(T^4 - T_0^4)$$

If $T=1000^\circ \text{K}$. and $T_0=288^\circ \text{K}$., *i.e.* about room temperature, T^4 is 145 times greater than T_0^4 , and the second term may be neglected in comparison with the first.

Optical Pyrometry.—Stefan's law has been applied to the measurement of high temperatures. The best known instrument for this purpose, Fèry's radiation pyrometer, consists of a short tube open at one end, which is pointed towards the furnace the temperature of which is to be measured. At the other end is a concave mirror which concentrates the radiation on a thermocouple, *i.e.* the junction of two wires of different metals. The focussing of the radiation on the thermocouple is done by an eyepiece which looks through a hole in the mirror. The thermocouple is connected to a galvanometer, and the current is proportional to the fourth power of the absolute temperature of the source.

There are other instruments in which the temperature is measured by matching radiation of one particular colour against a standard lamp. Optical pyrometry, as the method of determining temperature from the properties of the radiation is called, is the only method suitable for temperatures over 1500°C . The numerical values it gives hold in the first instance only on the assumption that the source is a "black body." If this is not the case, the results will be too low. But the method is very useful in industrial processes, where it is not necessary to measure

a high temperature in absolute units, but only to reproduce it with accuracy.

Optical pyrometry has been applied to the sun and the crater of the carbon arc, and results have been obtained somewhat below 6000°C. for the former and between 3500°C. and 4000°C. for the latter.

EXAMPLES IX

1. A piece of ice is placed in front of a thermopile. Describe exactly what happens.

2. A glass fire screen seems to cut off very little of the light of a bright fire. Of what use is it? How would you prove your statement?

3. A copper calorimeter weighing 10 gm. is filled first with water, then with another liquid, and the times taken in the two cases to cool from 40°C. to 35°C. are 85 and 75 seconds respectively. The weight of the water is 7.3 gm. and that of the liquid 8.7 gm., and the specific heat of copper is 0.095. Calculate the specific heat of the liquid.

4. A metal sphere at 40°C. cooling in air at 10°C. loses half a degree of temperature in the first quarter-minute. What is its temperature at the end of 12 minutes?

5. A long round copper bar 1 sq. cm. in cross-sectional area is heated and hung up in a room which is at a uniform temperature. By what fraction of its excess of temperature will it cool per second? The emissivity of the surface is $\frac{1}{4000}$ c.g.s. unit, the specific gravity of copper is 8.9, and its specific heat is 0.095. The emission from the ends is to be neglected.

6. Compare the amount of heat lost per second by a black body at 500°C. and at 100°C. in an evacuated vessel which is kept at 10°C.

PART III

SOUND

CHAPTER I

WAVE MOTION

Water Waves.—We shall begin the study of wave motion by considering the special case of waves formed on the surface of water. When a stone is dropped into a pool, waves diverge from the point where it strikes the surface and travel out in circles to the edge of the pool. They carry energy with them, as may be seen by the manner in which they roll pebbles up and down at the edge of the pool. This energy wasted in friction was part of the energy of the falling stone. But, though the energy moves on, the water particles which transmit it remain in the same place. This may be seen by watching a small piece of cork floating on the surface. Initially it is at rest; during the passage of the wave it oscillates to and fro, and, after the wave has passed, it is back in its original position.

If the profile of the wave is examined at a distance from its origin, it is seen to have the shape represented in Fig. 208, that of a sine curve. It is hence said to be a harmonic wave. This profile moves bodily in the direction of the wave-motion.

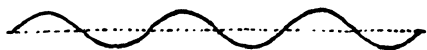


FIG. 208.

*The distance between two successive crests or two successive troughs is called the **wave-length**.*

*The velocity with which any crest or trough moves is called the **velocity** of the wave.*

*The time taken by a crest to travel one wave-length is called the **period** of the wave.*

*The number of complete wave-lengths which pass a given point in one second is called the **frequency** of the wave.*

If we denote the wave-length, velocity, period, and frequency by λ , V , T , and n respectively, according to the definitions

$$T = \frac{\lambda}{V} \quad \text{whence } \lambda = VT$$

and
$$n = \frac{1}{T} \quad \text{whence } V = n\lambda$$

The height of the crests above, or the depth of the troughs below, the level of the undisturbed surface is called the **amplitude** of the wave. During the passage of the wave, water which would otherwise fill a trough is raised up to form a crest. The wave thus possesses potential energy. Also, owing to the motion of the water particles, it possesses kinetic energy. This potential and kinetic energy comes from the energy of the falling stone. The velocity acquired by the pebbles at the edge of the pool is proportional to the amplitude of the wave, and the kinetic energy they acquire is proportional to the square of the amplitude. *The intensity or rate of delivering energy of the wave is consequently proportional to the square of its amplitude.*

A wave shows the characteristic properties of reflection, refraction, interference, and diffraction.

Reflection.—Fig. 209 shows what happens when a train of water waves approaches a wall or breakwater, as seen from above. WW is the wall. The lines are the crests of the waves. As each crest reaches the wall, its direction of motion is altered, until, finally, the whole train is turned round. The direction of pro-

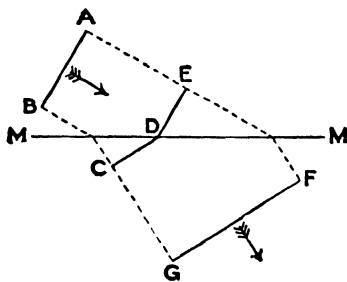
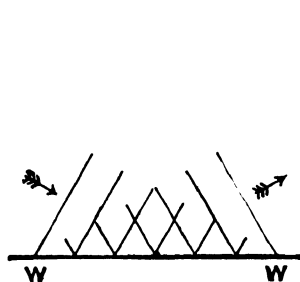


FIG. 209.—Reflection of plane waves. FIG. 210.—Refraction of a plane wave.

pagation of the wave makes equal angles with the wall before and after reflection. In the middle of the diagram the incident and reflected waves occur superimposed.

Refraction.—The velocity of a water wave varies with the depth of the water, being greatest where the latter is greatest. In Fig. 210 let AB be one of a train of waves travelling on the surface of a sheet of water of uniform depth. On the other side of MM let the depth suddenly become shallower, but let its value on this side be uniform also. The velocity of the wave becomes less when it crosses MM, and as a consequence its direction changes as shown by the figure, GF representing the wave after it has crossed, and CDE giving its position at an intermediate stage. This bending of the wave on entering a medium where the velocity is different is known as refraction. It has sometimes been likened

to a company of soldiers marching on to a ploughed field where they cannot go so fast.

Refraction is often visible in the case of steamer waves, when they pass from the dredged-out channel of a river to the shallow water at the sides. It can also be seen whenever we have waves

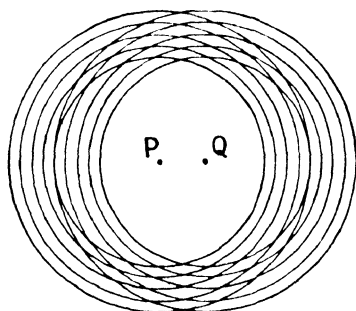


FIG. 211.

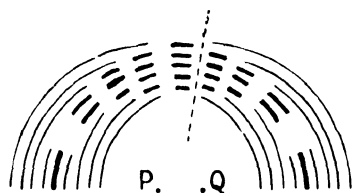


FIG. 212.

breaking on a shelving beach; for, although the waves may be approaching the beach obliquely some distance out, when they reach shallower water they go more slowly, and consequently wheel round, breaking with their crests parallel to the beach. In this case the refraction takes place gradually, not suddenly on crossing a line.

Interference. — Fig. 211 is a plan of a pool of water into which stones have been dropped simultaneously at P and Q, causing the emission of two trains of five waves each. These trains are shown superimposed, the circles representing crests. Now, when a crest is superimposed on a crest, we have a crest twice as high; when a trough is superimposed on a trough, we have a trough twice as deep, and when a crest is superimposed on a trough, they neutralise each other, and the level is the same as if the surface had not been disturbed. This is shown in Fig. 212; the thick lines represent crests twice as high, and



FIG. 213.—Interference of wave trains on a mercury surface.

the spaces at their ends represent undisturbed level. As both halves of the diagram are the same, only the part above PQ is drawn. It is possible to pass through the wave train from the front to the back along the dotted line and have undisturbed level the whole way. The one wave has neutralised or interfered with the other; we have added wave to wave, and got as a result nothing.

Fig. 213 is a photograph taken from above of two overlapping wave-trains on a mercury surface. The regions where they interfere can be distinguished quite clearly. Figs. 214 and 215 show another way of approaching the subject; *a* and *b* are two waves

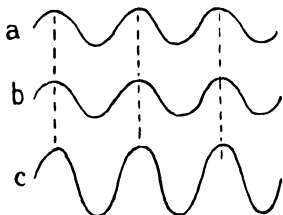


FIG. 214.

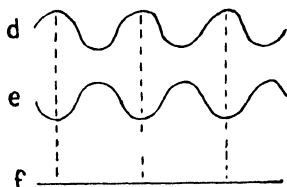


FIG. 215.

which superimpose with crest on crest and trough on trough and give *c*; *d* and *e* are two waves which superimpose with crest on trough and trough on crest and give *f*. The crests fill up the troughs.

Diffraction.—Fig. 216 represents a water wave passing perpendicularly through a hole, AB, in a wall. P is a plan of the wave before it reaches the wall, Q shows it actually at the opening, and R and S give two of its positions on the other side. It will be noticed that after passing through the hole the wave is not restricted to the space between the lines AH and BK, but curves round behind the wall at X and Y. To this bending round of the wave behind an obstacle the name “diffraction” has been given.

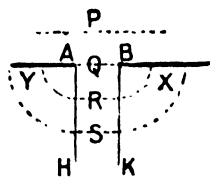


FIG. 216.

Stationary Waves.—The dotted curves in the top diagram in Fig. 217 represent two water waves moving in opposite directions. They have the same amplitude, the same period, and the same velocity. The full curve, which is obtained simply by adding the ordinates of the dotted curves, gives the resultant displacement of the particles on the surface. The second diagram represents conditions a time $\frac{1}{2}T$ later, when the dotted waves have moved respectively $\frac{1}{2}\lambda$ to the right and $\frac{1}{2}\lambda$ to the left; the full curve again gives the resultant displacement. The third diagram represents conditions a time $\frac{1}{4}T$ later, when the dotted curves

have moved respectively $\frac{1}{2}\lambda$ to the right and $\frac{1}{2}\lambda$ to the left; the dotted curves now neutralise one another exactly, and the full curve is a straight line. The fourth and fifth diagrams represent respectively conditions after times $\frac{3}{4}T$ and $\frac{1}{2}T$, the full curves giving the resultant of the dotted curves in each case. If we take later times, $\frac{5}{4}T$, $\frac{3}{2}T$, etc., we simply get the same changes in the reverse order.

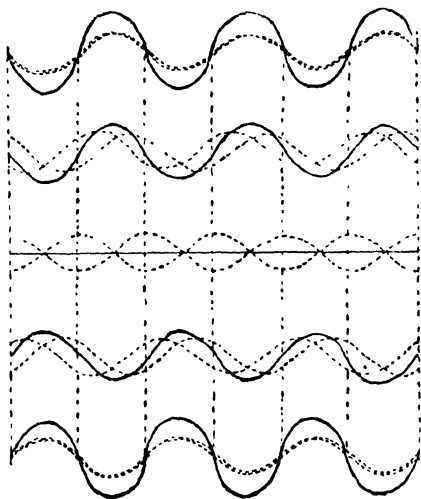


FIG. 217.

The points in which the full curves are intersected by the seven vertical dotted lines are always at rest. These points are called **nodes**. The points midway between the nodes are called **loops** or **anti-nodes**;

at the loops the displacement changes from a maximum on the one side to a maximum on the other. The resultant displacement changes simply from the form given by the full curve in Fig. 218 through the straight line to the dotted



FIG. 218.—Stationary Wave.

curve, and then back again through the straight line to the full curve. There is no forward motion of the wave as a whole, and the wave is said to be a stationary one.

Thus two progressive waves of equal amplitude in opposite directions produce a stationary wave with an amplitude twice as great as that of either of its components.

Motion of the Particles.—The circle in Fig. 219 has twelve points on it, *a, b, c, d, e, f, g, h, i, j, k, l*, each point being distant 30° from the next one, and is rotating in the direction of the arrow with uniform speed. Let T be the time required to make a complete revolution. Then if the topmost diagram represents the position of the circle for zero time, the other diagrams represent its positions for times $\frac{1}{12}T$, $\frac{2}{12}T$, $\frac{3}{12}T$, $\frac{4}{12}T$, $\frac{5}{12}T$, and $\frac{6}{12}T$. Consider the two points *a* and *A*. Let *A* move up and down a vertical line in such a manner that its perpendicular distance from the horizontal line is equal to the distance of the point *a* from the

same line. A thus describes a simple harmonic motion, and a is the auxiliary particle used in the definition of its motion.

In the same way let B, C, D, etc., move up and down, so that the distance of each from the horizontal line is the same as the distance of the particle on the circle with the same letter from the horizontal line. B, C, D, etc., consequently also describe S.H.M.'s of the same period as A, but each in a different phase.

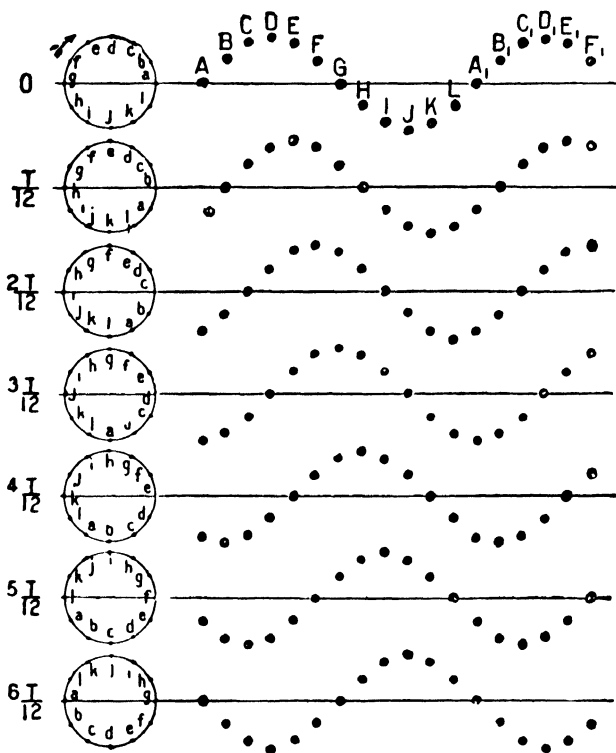


FIG. 219.—Motion of particles in a transverse wave.

Let A_1 , B_1 , C_1 , etc., have the same distances from the horizontal line as A, B, C, etc., have.

If all the particles in each diagram are joined by a curve, it will be noticed that this curve has the form of a sine curve, and that as the motion goes on, it appears to move bodily towards the right. If instead of stopping at F_1 we add other particles, making the displacement of every twelfth particle the same, the diagram represents a harmonic progressive wave moving towards

the right. Since the particles move at right angles to the direction of propagation of the wave, the latter is said to be **transverse**. The period and amplitude of the S.H.M.'s of the individual particles coincide with the period and amplitude of the wave.

If the displacements of the particles of the medium from their equilibrium positions are laid off along the horizontal line, instead of at right angles to it as in Fig. 219, we obtain the result shown in Fig. 220. In the top row the particles are crowded together at G and beyond F₁; at these two points a **condensation** or **compression** is said to take place. At A and at A₁ the particles are farthest apart: here a **rarefaction** is said to take place. And as the individual particles perform their S.H.M.'s these condensations and rarefactions move towards the right, just in the same way as the crests and troughs moved towards the right

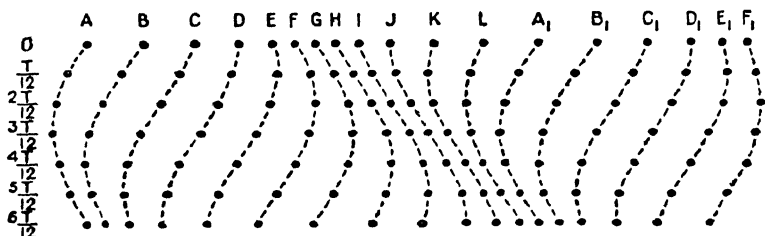


FIG. 220.—Motion of particles in a longitudinal wave.

in Fig. 219. A **longitudinal** wave is then said to pass along the particles from left to right, and the distance between any condensation and the one next to it, or between any rarefaction and the one next to it, is called the **wave-length** of the wave. At condensations and rarefactions the particles are passing through their equilibrium positions; it is evident also, by comparing Figs. 219 and 220, that the crests and troughs of the transverse wave correspond to the points in the longitudinal wave where the density is unaltered.

We have shown that if the particles of the medium are vibrating in the above manner, a wave passes. The converse may also be proved; if a wave is passing through a material medium, the particles are executing either transverse or longitudinal vibrations or, as in the case of water waves, a combination of both.

We can have stationary longitudinal waves in the same way as we have stationary transverse waves. At the loops the displacement and velocity of the particles are greatest; at the nodes the particles are at rest, and the condensations and rarefactions take place there.

Solitary Wave.—It is not necessary that a wave should occur as part of a train: it may consist of a single irregularly shaped

elevation or depression, or pulse as it is called. The passage of such a pulse up a rope can be demonstrated by a well-known lecture experiment. A rope about 25 feet long is suspended vertically, the lower end held with one hand, and the rope struck with the other hand a few inches above this end. The indentation then travels up the rope, is reflected at the top, and can be seen quite clearly coming down the other side.

Resultant of Two S.H.M.'s in the Same Straight Line.—Simple harmonic motion is defined as the projection upon a diameter of uniform motion in a circle.

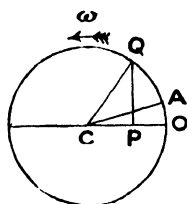


FIG. 221.

Let a point Q (Fig. 221) be moving round a circle of radius a with constant angular velocity ω . When $t=0$, let it be at A. Then $\angle QCA = \omega t$. Let $\angle ACO = a$. Draw QP perpendicular to CO. Then as Q moves round the circle, P executes a S.H.M. Its displacement $CP = CQ \cos QCP = a \cos (\omega t + a)$. a is the phase of the S.H.M., and the time taken by Q to go round the circle, $2\pi/\omega$, is its period.

Let us suppose that we have a particle executing two S.H.M.'s in the same straight line at the same time, both with the same period but with different amplitudes and phases. Then its resultant displacement is equal to the sum of the displacements due to the separate motions, and is given by

$$a \cos (\omega t + a) + b \cos (\omega t + \beta)$$

where b and β are the amplitude and phase of the second S.H.M. This is equal to

$$\begin{aligned} a \cos \omega t \cos a - a \sin \omega t \sin a + b \cos \omega t \cos \beta - b \sin \omega t \sin \beta \\ = (a \cos a + b \cos \beta) \cos \omega t - (a \sin a + b \sin \beta) \sin \omega t \end{aligned}$$

If we write
and
it becomes

$$\begin{aligned} a \cos a + b \cos \beta &= R \cos \delta \\ a \sin a + b \sin \beta &= R \sin \delta \\ R \cos (\omega t + \delta) \end{aligned}$$

Thus the two separate S.H.M.'s compound into a S.H.M. which has the same period as the component S.H.M.'s.

The expression $a \cos (\omega t + a)$ represents the displacement of one particle for different times. The expression $a \cos (\omega x + a)$, when graphed as a function of x , represents the profile of a wave, *i.e.* the displacement of a great number of particles for one time, and it will be found from the graph that the wave-length is given by $2\pi/\omega$.

Consequently $a \cos (\omega x + a) + b \cos (\omega x + \beta)$

represents the superposition of two wave trains with the same wave-length. It follows from the above reasoning, that they compound into a wave represented by

$$R \cos (\omega x + \delta)$$

which has the same wave-length as the component waves.

If the wave-lengths are not the same, the resultant does not take a simple form. It may be found by drawing the separate waves and adding their ordinates graphically. The case when the amplitudes are equal and the two wave-lengths are nearly equal is specially impor-

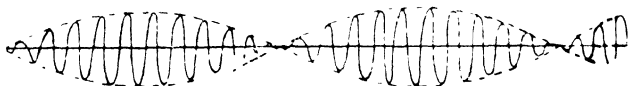


FIG. 222.

tant. It is represented by Fig. 222. In this case the resultant is a harmonic wave the amplitude of which varies periodically.

Resultant of Two S.H.M.'s at Right Angles to each other.—Let us suppose that a point P (Fig. 223) is executing a S.H.M. about O in the line AA' between the points A and A'. Take any straight lines through O at right angles to each other as OX and OY. The components of OP along OX and OY will vary in S.H.M. Hence, conversely, two S.H.M.'s at right angles to one another with the same period and in the same phase compound into a S.H.M.

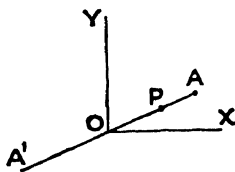


FIG. 223.

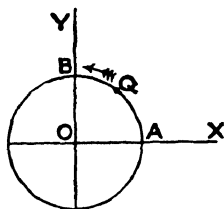


FIG. 224.

Consider next Fig. 224. The point Q is moving round the circle with uniform angular velocity. The projections of its motions on two diameters at right angles to each other vary in S.H.M. When the particle which represents the projection of the motion on OX is at A, the particle which represents the projection of the motion on OY is at O, starting out towards B. It is hence a phase difference $\frac{\pi}{2}$ behind, or $\frac{3\pi}{2}$ in front of the other. If Q had been moving round the circle the other way, this particle would have been a phase difference $\frac{\pi}{2}$ in front of the other. Hence, conversely, when two S.H.M.'s at right angles have the same period and amplitude, but a phase difference of $\frac{\pi}{2}$ or $\frac{3\pi}{2}$, they compound into uniform motion in a circle.

It may be shown by reasoning beyond the scope of this book that two S.H.M.'s at right angles to each other which have the same period,

different amplitudes, and any phase difference, compound into motion in an ellipse.

When the two S.H.M.'s have not the same period, they are best compounded graphically. Let us suppose, for example, that the motion

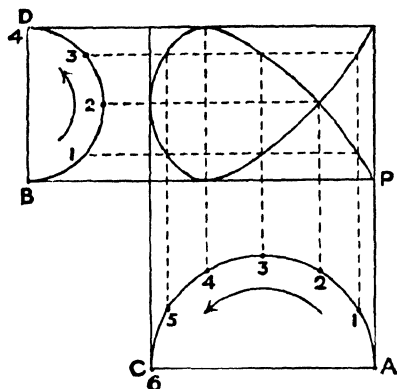


FIG. 225.—Composition of two S.H.M.'s of different periods.

They arrive simultaneously at the points indicated by the same number. The S.H.M.'s to be compounded are the projections of the motions of these particles. Draw lines through the points with the same numbers to meet as shown. Then if a particle start out from P and pass simultaneously through the intersections of these lines, its motion will have the same projections as the particles which move on the semicircles. It is consequently the resultant of the two given S.H.M.'s.

If the components are not in the same phase, the resultant motion is different. Fig. 226 shows it for various cases; (a) is simply Fig. 225 over again; (b), (c), (d), and (e) are for the cases, where the phase of the vertical motion is respectively $\frac{\pi}{4}$, $\frac{\pi}{2}$, $\frac{3\pi}{4}$, and π behind the phase of the horizontal.

If the ratio of the periods of the two S.H.M.'s can be expressed by whole numbers, the motion repeats itself after a time which is the least

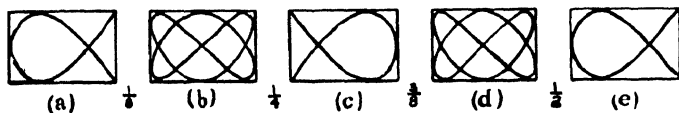


FIG. 226.—Lissajous' figures.

common multiple of the periods. If the periods are not commensurable, the tracing particle never returns to its starting-point.

The case where the ratio of the periods can nearly be represented by two simple numbers is of practical importance. For example, suppose that the vertical period is to the horizontal one in the ratio

101 to 150, and that the components start off in the same phase. Then initially they give a path very like (a) of Fig. 226. The vertical motion then falls behind the horizontal one. When its phase is $\frac{\pi}{4}$ behind, the path is very similar to that of Fig. 226 (b); and so on. The path in succession passes through all the forms of Fig. 226. When it returns approximately to its initial shape, the one motion has gained a period on the other, and if this time is observed, the ratio of the periods can be obtained very accurately. Curves similar to those shown in Fig. 226 were studied very exhaustively by Lissajous about 1873 by optical methods, and are hence known as Lissajous' figures.

Blackburn's Pendulum.—The easiest way of compounding S.H.M.'s of different periods at right angles to one another is by means of Blackburn's pendulum. This piece of apparatus is shown in Fig. 227. A glass funnel containing fine, dry sand is fitted through a lead ring and supported by string. For vibrations at right angles to the plane of the frame the arrangement forms a pendulum of a length equal to the distance from the lead ring to the top bar; for vibrations in the plane of the frame it is equivalent to a pendulum of length equal to the distance from the ring to the clip r . The ratio of the periods can be adjusted by moving this clip. As the pendulum vibrates, the sand runs out, tracing the path on the paper below.

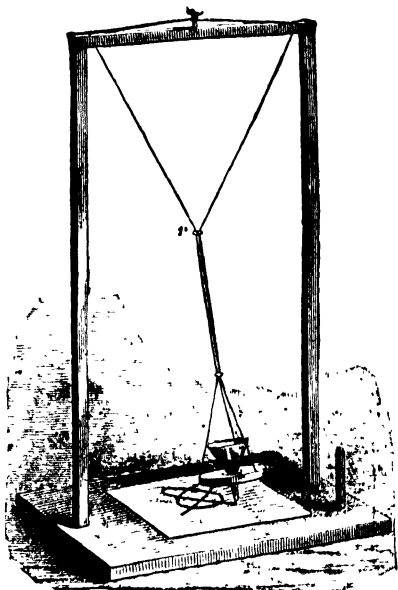


FIG. 227.—Blackburn's pendulum.

EXAMPLES I

1. If the velocity of a wave is 340 metres per sec. and its frequency 256 per sec., find the wave-length.
2. Find graphically the resultant of two simple harmonic motions at right angles to one another. The period of the one is twice the period of the other, the amplitudes are the same, and the phase of the motion with the shorter period is $\frac{\pi}{2}$ ahead of the other.
3. Two S.H.M.'s at right angles to one another give Lissajous' figures. The same figure recurs at intervals of 6 seconds. If the one S.H.M. has a frequency of 100 per second, and the other has a frequency of nearly twice this, find its possible values.

CHAPTER II

PRODUCTION AND PROPAGATION OF SOUND

Source in Vibration.—The source of a sound is always in a state of vibration, and, as the vibration dies down, the sound diminishes in intensity. This can be shown by a variety of experiments. If a bicycle bell is touched with the fingers, the ringing sound stops, because the fingers stop the vibrations. If it is clasped firmly in the hand, the vibrations cannot be started, and the hammer of the bell simply makes a rattling noise.

If a noise is caused by dropping a weight on the floor, it and the part of the floor which it strikes are both set into violent vibration by the impact. If a stretched string or wire is plucked, so as to give a musical note, owing to the vibrations taking place too fast for the eye to follow them the string appears drawn out into a ribbon at the middle. Also small V-shaped pieces of paper placed on the string are thrown off by the vibrations. When a tuning-fork is sounding, its prongs alternately approach and recede from one another. This is visible in the case of large vibrations; also the vibrations can be felt on lightly touching the prongs with the fingers.

Tuning-forks are sometimes mounted on wooden boxes open at one end, called resonance boxes or resonators. This has the effect of making the note much louder. The vibrations of the fork are communicated to the box and to the air inside the latter; this happens very easily, because the air inside the box has the same period as the fork itself. But, though the box makes the sound louder, it causes it to die away faster. What is gained in intensity is lost in duration.

Velocity of Sound.—It is a fact of common observation, that sound takes an appreciable time to travel. Thunder is not heard until some time after the lightning flash, and the flash of a cannon at a distance is seen some time before the report is heard.

A famous determination of the velocity of sound in air was made by Arago and other French observers in 1822. Two stations were chosen 18·6 kilometres apart; at each station a cannon was placed, and observers were stationed with chrono-

meters. The cannon at the two stations were fired alternately, and the time noted which elapsed between the flash and the report. Observations were made in both directions, for if the wind assisted the velocity in the one direction, it would retard it in the other. The temperature was observed at a number of places between the two stations, the mean result being 15.9°C . The value of the velocity found was 340.9 metres per second for this temperature. The velocity of light is so great that the time taken by the light of the flash to go from the one station to the other could be neglected.

The velocity of sound in water was experimentally determined by Colladon and Sturm in 1827 on the Lake of Geneva. The measurements were carried out at night. Two boats were moored at a distance of 13.5 km. apart. The first carried a bell below the surface of the water; the second carried an ear trumpet, the mouth of which dipped below the surface of the water and was closed by an indiarubber membrane. When a hammer struck the bell, some gunpowder was automatically ignited. An observer in the second boat measured the time between the flash of light and the arrival of the sound by the water. In this way it was found that the velocity of sound in water was four times as great as in air.

The fact that sound travels in solid bodies, such as wood and iron, can be shown very prettily by an experiment with a tuning-fork and its resonance box. A rod of wood about 14 feet long and $\frac{1}{4}$ inch thick with sharpened ends is taken, and the one end held lightly against the resonator. If the tuning-fork is sounded and held against the other end, the sound travels along the rod to the box, and is audible to a large audience. But if the tuning-fork is removed from the other end, the sound becomes very much fainter, and is audible only to the person holding the fork.

Variation with Intensity.—The speed of sound is not independent of the intensity. This was shown in 1864 by Regnault at Versailles. Measurements were carried out in the open air by the method of the reciprocal firing of cannon, two distances, 1280 and 2445 metres, being used. The instants of firing and of receiving the sound were recorded electrically, and it was found that for the distances of 1280 and 2445 metres the velocities were 331.37 and 330.7 m. per sec. respectively, when reduced to a temperature of 0°C . Hence nearer the cannon, *i.e.* where the sound is more intense, the velocity is slightly greater.

Medium of Transmission.—If an electric bell is suspended inside the receiver of an air-pump by the two wires carrying the current, and the air pumped out, the sound of the bell will become less and less as the air is removed, but becomes stronger at once if the air is suddenly readmitted. This shows that sound cannot be propagated in a vacuum.

The experiment would not succeed if the bell were allowed simply to rest on the bedplate of the receiver, for the vibrations would then be transmitted directly to the plate and thence to the external air, and the air inside the receiver would not be necessary as an intermediary.

Reflection.—If an observer stands at a distance of about 300 feet from a cliff or large building, and shouts towards it, his words are echoed back to him. The sound is reflected back by the cliff or building. At this distance the sound takes about half a second to go to the cliff and back, and as the observer utters only two syllables in this time, only his last two syllables are echoed back clearly to him. The rest of the echo is not heard distinctly owing to its being superimposed on the sound itself.

The rolling of thunder is due to the reflection of the original sound by clouds at different distances from the observer. The sounding boards placed behind pulpits are for the purpose of reflecting towards the audience the sound that would otherwise travel upwards and backwards, and be lost. A ship in a fog can sometimes discover when it is approaching a line of cliffs or an iceberg by hearing the echo of its horn.

Interference of Sound.—The simplest way of producing interference of sound is by means of a tuning-fork. In Fig. 228, A and B are the prongs of a tuning-fork, viewed from the ends. They vibrate in such a way that they alternately approach and recede from one another. When they approach, they make a condensation in the air between and a rarefaction in the air towards Y and Y'; when they recede from one another, the opposite conditions are produced. An ear situated at X or X' is affected by both prongs in the same way, and if an ear is placed at Y or Y', the one prong screens off the effect of the other. Hence

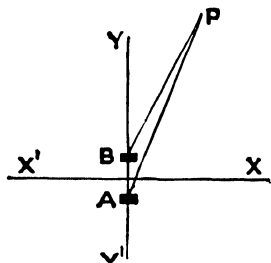


FIG. 228.

in both these directions a loud sound is heard. But to an ear situated at P the one prong is sending out a compression when the other is sending out a rarefaction. Hence in this direction only a faint sound is heard. If the fork is struck and rotated about its handle, the sound waxes and wanes according as the ear comes into the lines X'X and Y'Y or the intermediate directions.

Fig. 229 shows another way of producing interference of sound. The arrangement consists of glass or brass tubes about 3 cm. in diameter, the narrower tube C sliding in and out of the other. A whistle is sounded at A. The sound travels to B

inside the tube by the two paths D and C. If these are equal in length, a loud sound is heard at B. If the tube is then gradually drawn out, so as to increase the length of the path C, the intensity of the sound diminishes at B. If it is drawn out further, the intensity increases again; if the tube is drawn out further still, the intensity again diminishes. This apparatus is known as Kundt's interference tube.

Diffraction of Sound.—It is a well-known fact that sound is diffracted. If a noise is coming along the side of a high building where there are no walls or other buildings to reflect the sound, and an observer at a corner steps round the corner, he still hears it.

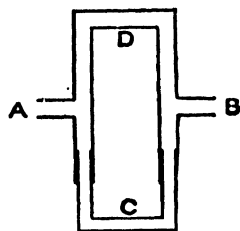


FIG. 220.

Nature of Sound.—We have seen that sound possesses all the characteristics of wave motion. It has a finite velocity, requires a medium to travel in, and is capable of reflection, interference, and diffraction. It is therefore natural to assume that it is propagated by wave motion in air, water, or wood, or whatever the medium is. Since the source is always in a state of vibration, the wave motion must be started by the vibratory motion of some piece of matter, just as water waves might be started by dipping the end of a stick regularly below the surface.

The only other explanation that might be offered of the propagation of sound is that it is carried by small particles shot out from the source in all directions, like bullets from a machine gun. But this explanation is ruled out by the fact that sound cannot cross a vacuum. We can also calculate what the velocity of a longitudinal wave in air ought to be, and the result agrees very well with the value obtained for the velocity of sound in air. So the matter is settled beyond all possible doubt.

Calculation of the Velocity.—Newton showed that the velocity of a longitudinal wave in a gas is given by the formula

$$V = \sqrt{\frac{k}{\rho}}$$

where k is the bulk modulus of elasticity for the gas, and ρ is its density. Now we may calculate k for a gas on either of two assumptions. We may take for granted either (i) that Boyle's law holds, or (ii) that the adiabatic equation holds, owing to the compressions taking place so rapidly, that the heat produced in them has not time to flow away.

Let p and v be the pressure and volume of unit mass of a gas,

T

and when the pressure is increased to p_1 , let the volume become v_1 . Then

$$k = - \frac{p_1 - p}{\frac{v_1 - v}{v}}$$

The minus sign is prefixed in the definition to make k a positive quantity. If Boyle's law holds, $pv = p_1v_1$. Substitute for p . Hence

$$k = - \frac{p_1 - p_1 \frac{v_1}{v}}{\frac{v_1 - v}{v}} = p_1$$

which becomes, since the change is a small one, simply

$$k = p$$

If the adiabatic equation holds, $pv^\gamma = p_1v_1^\gamma$. This gives

$$p = p_1 \left(\frac{v_1}{v} \right)^\gamma = p_1 \left(1 + \frac{v_1 - v}{v} \right)^\gamma = p_1 \left(1 + \gamma \frac{v_1 - v}{v} \right)$$

by the binomial theorem, since $v_1 - v$ is small. Hence

$$p - p_1 = p_1 \gamma \frac{v_1 - v}{v}$$

and on substitution in the definition

$$k = - \frac{p_1 - p}{\frac{v_1 - v}{v}} = \gamma p_1 = \gamma p$$

since p_1 is nearly equal to p . Thus on the two assumptions the velocity of sound is given respectively by

$$V = \sqrt{\frac{p}{\rho}} \quad \text{and} \quad V = \sqrt{\frac{\gamma p}{\rho}}$$

Newton made the first assumption. If we substitute for p and ρ the standard values for 0°C , his formula gives the result $V = 280$ metres/sec., which is much too low. Laplace pointed out the source of the discrepancy in 1816; the compressions and rarefactions take place so rapidly that the heat produced has no time to flow from one part of the gas to another. Hence the second formula must be used. Since $\gamma = 1.40$ for air, it gives $V = \sqrt{1.40} \times 280 = 331.3$ metres/sec., or 1087 feet/sec. This is the mean of the best modern experimental results.

Effect of Temperature.—The variation of the velocity of sound in a gas with temperature follows very simply from Laplace's

formula. Let V_0 =velocity at 0°C. , and V =velocity at $t^\circ \text{C.}$ Then, from the gas equation,

$$\frac{p}{\rho} = \frac{p_0}{\rho_0} \left(1 + \frac{t}{273} \right)$$

Hence

$$V = \sqrt{\gamma p / \rho} = \sqrt{\gamma p_0 / \rho_0} \sqrt{1 + \frac{t}{273}}$$

$$= V_0 \sqrt{1 + \frac{t}{273}} = V_0 \left(1 + \frac{t}{516} \right)$$

very nearly, since t is usually 15°C. or thereabouts. Thus the velocity increases with the temperature, and is uninfluenced by changes in pressure and density, as long as the temperature remains constant.

Refraction of Sound.—The variation of the velocity with temperature sometimes causes refraction of a sound wave. For, in Fig. 230, let EF represent the surface of the earth, and A a wave front proceeding from left to right. Let the air be hotter near the earth's surface. Consequently the lower edge of the wave travels faster, the direction of propagation is gradually altered, and the wave is refracted into the upper regions of the atmosphere.

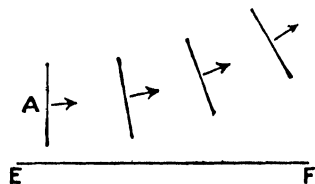


FIG. 230.—Refraction of a sound wave.

The velocity of sound is increased when it is travelling with the wind, and diminished when it is travelling against the wind. If a wave were opposed by a head wind the velocity of which increased with the distance above the earth's surface, it would be refracted in exactly the same manner as is represented in Fig. 230.

Proof of Formula for Velocity.—We shall now prove the formula $V = \sqrt{k/\rho}$. The proof is usually regarded as difficult, and may be omitted without prejudice to what follows.

Let us assume that we have a tube of unit cross-sectional area along which a wave is proceeding from left to right with velocity V (Fig. 231). The condensations are represented by the shading. Let p, ρ, u and p_1, ρ_1, u_1 be the pressure, density, and velocity of the particles on the planes A and B respectively.



FIG. 231.

Imagine two planes moving with velocity V from left to right, which are instantaneously coincident with the fixed planes A and B .

Since these moving planes always preserve the same position relative to the compressions and rarefactions, the mass of gas between them remains constant. When they are in the positions indicated, the rate at which gas is entering through A is

$$\rho(u-V)$$

and at which it is leaving through B is

$$\rho_1(u_1-V)$$

Hence

$$\rho(u-V) = \rho_1(u_1-V)$$

or

$$\rho u - \rho_1 u_1 = V(\rho - \rho_1)$$

On the left-hand side of this equation we may write $\rho_1 = \rho$, since the density does not vary much about its mean value, while u varies from a positive to a negative value. Thus the equation becomes

$$\rho(u - u_1) = V(\rho - \rho_1)$$

Consider now the gas between the two stationary planes A and B. Let the distance between them be l . The time taken by the wave to go from A to B is l/V , and during this time the velocity at B changes from u_1 to u . Hence the acceleration is

$$\frac{(u - u_1)V}{l}$$

and the rate of change of momentum of the gas contained between A and B is

$$\rho l \frac{(u - u_1)V}{l} = \rho(u - u_1)V$$

The resultant force on it is the difference of the pressures on the two faces, $p - p_1$. Thus

$$p - p_1 = \rho(u - u_1)V$$

If we substitute for $\rho(u - u_1)$ from the equation above, this becomes

$$p - p_1 = V^2(\rho - \rho_1) \quad \text{or} \quad \frac{p - p_1}{\rho - \rho_1} = V^2$$

But

$$k = -\frac{\frac{p_1 - p}{1 - 1}}{\frac{1}{\rho}} = \rho \frac{(p - p_1)}{\rho - \rho_1}, \quad \text{whence} \quad \frac{p - p_1}{\rho - \rho_1} = \frac{k}{\rho}$$

Hence

$$V^2 = \frac{k}{\rho} \quad \text{and} \quad V = \sqrt{\frac{k}{\rho}}$$

Sensitive Flame. Galton's Whistle.—The arrangement represented in Fig. 229 can be used to determine the wave-length of the sound wave. Initially there is the same number of wave-lengths in each path, so when the two waves arrive at B, compression is superimposed on compression and rarefaction on rarefaction. When interference is first produced, the one path is half a wave-length longer than the other. When the sound reaches a maximum again, it is

a wave-length longer than the other. Hence, if the lengths of the paths are measured, the wave-length can be determined.

For detecting the effect at B it is better to use a sensitive flame instead of the ear. This is a gas flame produced by a pin-hole burner, the pressure of which has been increased until it is on the point of flaring, as shown in Fig. 232, B. When a sound of suitable pitch is produced, the flame shortens and roars, as shown in Fig. 232, A. The flame will work only with a good gas pressure, $3\frac{1}{2}$ to 5 inches of water, but at this pressure it is extremely sensitive to waves of a high pitch.

In Rayleigh's sensitive flame (Fig. 233) the gas is delivered into the interior of a small box which is closed on one side with tissue paper or a sheet of mica. It then passes up a vertical tube and burns at the top. The pressure is adjusted with the clip, until the flame appears to be detached from the top of the tube; it is in this

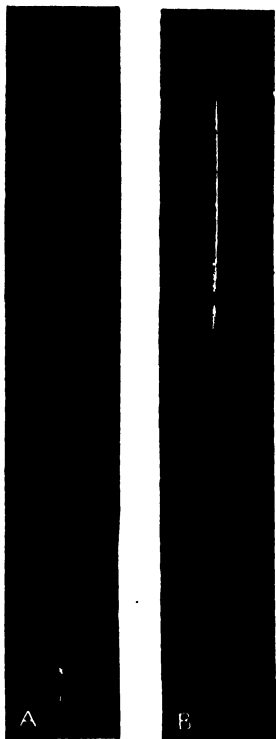


FIG. 232.—Sensitive flame.

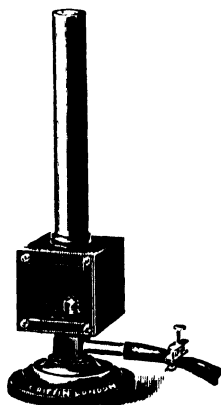


FIG. 233.—Rayleigh's sensitive flame.

condition that the flame is sensitive. This flame works at ordinary gas pressures.

The best source of sound to use with the sensitive flame is Galton's whistle. This produces a shrill note the pitch of which can be raised to the limit of audibility by sliding a piston along a tube.

When a sound wave is reflected by a wall, it is superimposed on the incident wave, and stationary waves are produced as shown in Fig. 217. There can be no motion of the air particles at the wall, as the wall stops the motion. Hence there is a node there. If a sensitive

flame is moved out from the wall, the positions of the loops and other nodes can be determined, and hence the wave-length calculated.

Sound Ranging.—During the Great War the batteries on the west front took up fixed positions which were carefully camouflaged to screen them from aerial observation. "Sound Ranging" was the name given to the method of locating these hidden batteries by noting the time at which the sound was received by three different stations. Fig. 234 shows the principle of the method. A sound starts from the source S, and is received at the three stations A, B, and C. At each station the time is recorded at which the sound wave arrives. Hence the intervals between its reaching A and B and A and C are known, as are also the distances travelled by sound in these intervals. If circles are drawn with B and C as centres, and these distances as radii, the centre of the circle which passes through A and touches these circles is the source S.

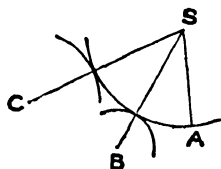


FIG. 234.

The recording stations were situated at distances of from three to nine miles from the source. Each had a microphone connected by cable to a station behind the base, and the variation of the current in each cable was recorded photographically on the same moving film. The time intervals could be measured in this way to $\frac{1}{500}$ second. Wind distorted the wave-front, and this involved elaborate corrections.

Energy of the Sound Wave.—When a sound wave diverges from a point in open space, its energy spreads over the surface of a sphere of ever-widening radius. If E is the total amount of energy emitted per second, the quantity received per sec. per sq. cm. of the surface is

$$\frac{E}{4\pi r^2}$$

The intensity of the sound is consequently inversely proportional to the square of the distance. Speaking tubes confine the energy and increase the distance at which sound is audible.

When a lecturer speaks in a hall, in addition to the direct wave to the hearer, there are all sorts of reflected waves. Sound is reflected from wall to wall, weakened by each reflection; hence the larger the hall, the longer it takes to die down. Curtains, cushions, rugs, etc., absorb sound very much better than plaster, glass, or boards; hanging the walls with heavy curtains may greatly improve the acoustics of a building. The reverberation is also, for the same reason, usually much less when the hall is full than when it is empty. Speakers in large halls go slow, so as to allow the reverberation of one word to die down before uttering the next. A sounding board behind the speaker is often a great help; the reflected wave then follows so closely on the direct wave that there is no confusion.

EXAMPLES II

In order to simplify calculation, take $V = 340$ metres/sec. or 1100 ft./sec. if the temperature is not specified.

1. Given that the velocity of sound is 331.3 metres/sec. at $0^{\circ}\text{C}.$, find its value at $15^{\circ}\text{C}.$ in metres/sec. and ft./sec.

2. A peal of thunder is heard four seconds after the lightning flash is seen. How far distant was the flash?

3. A ship enters a fog and blows its siren. The blast is echoed back in $1\frac{1}{4}$ sec. Assuming that the reflection is from an iceberg right ahead, and that the ship's speed is 12 knots, how long would it take to reach the iceberg? (1 knot = 6080 ft./hr.)

4. It is observed that waves pass a fixed point once in every four seconds, and that the distance from crest to crest is 23 ft. Find their velocity in miles per hour.

5. A bullet fired from a rifle with a velocity of 2000 ft./sec. is heard to strike a target 2.5 sec. later. Find the distance of the target from the marksman.

6. A stone is dropped into a well and heard to strike the water 2 sec. later. How deep is the well?

7. A litre of hydrogen weighs 0.08987 gm. under standard conditions. The ratio of its specific heats, γ , is 1.41. Find the velocity of sound in hydrogen when the temperature is $10^{\circ}\text{C}.$

8. The positions of three stations, A, B, and C, are specified by the rectangular coordinates (4, 0), (0, 0), and (0, 2) on a map, distances being measured in miles. A noise is recorded 13.14 sec. later at B than at A, and 8.89 sec. later at C than at A. Find the coordinates of the source by a geometrical construction.

CHAPTER III

MEASUREMENT OF FREQUENCY. MUSICAL SCALE

Loudness, Pitch, Quality.—Sounds can be divided into two classes, noises and musical notes.

The distinction between a noise and a musical note is that a musical note is propagated by a regular wave with a definite frequency, while a noise has no definite frequency and is propagated by a wave of an irregular form.

A musical note has three characteristics, loudness, pitch, and timbre or quality.

The loudness of a note depends on the intensity of the wave producing it, the latter being proportional to the square of the amplitude of vibration of the air particles. It may be measured at a point by the quantity of energy delivered there by the wave per sq. cm. per sec.

In the language of everyday life we speak of the pitch of a note as being high or low.

The pitch of a note depends only on the frequency of the wave producing it; the greater the frequency, the higher the pitch.

Even when two notes have the same pitch and intensity they may differ from one another. Thus the notes of the piano are quite different from those of the flute. This difference is referred to as difference in quality or timbre. It

is due to the note not being simple, but containing other fainter notes of a higher frequency.

Siren.—The fact that the pitch of a note is determined by its frequency can be shown by an instrument called the siren. In its simplest form this consists of a wooden or cardboard disc (Fig. 235) pierced with a ring of small equally spaced holes. The disc is mounted on an axle perpendicular to its plane, and can be rotated at different speeds. A jet of air from a bellows is directed against the disc as shown; as each hole comes into position in front of the jet a puff is forced through it, starting a pulse of compression on the other side. If the wheel is rotating uniformly, these pulses follow one another at regular intervals and combine to form a note of definite pitch. As the disc is rotated faster, the pitch of the note rises.

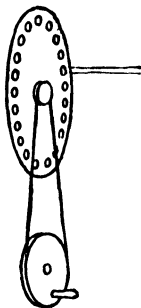


FIG. 235.—Simple siren.

Figs. 236, 237, and 238 show the usual form of the instrument. Fig. 236 shows its external appearance, and Fig. 238 is a vertical section. A is a brass disc with 18 holes in it, which is free to rotate about a vertical spindle. Fig. 237 gives a plan and section of this disc, one of the holes being shown at *n*. Below this movable disc there is a fixed disc B, also with 18 holes in it; these are placed so as to coincide simultaneously with the 18 holes on the upper disc. One of the holes in B is shown at *m*. The holes in A and B are inclined obliquely to one another; consequently when wind from a bellows is forced up the tube O, and escapes through *m*

FIG. 236.

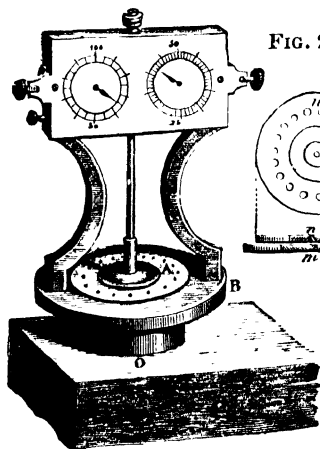


FIG. 237.

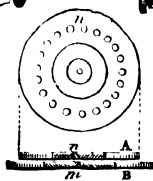
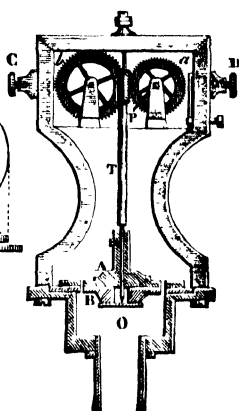


FIG. 238.



Siren.

and *n* and the other holes to the atmosphere, it strikes obliquely against the sides of the holes in A, and imparts a rotary motion to A. As the pressure of the air in the bellows increases, the speed of rotation of the disc A increases. At first when motion starts, the siren gives only a hissing sound; this changes into a note of very low pitch which goes higher and higher as the speed of rotation increases. By regulating the pressure of the air it is possible to keep the speed of rotation and consequently the pitch of the note constant.

The spindle T is connected to a speed counter with two dials, one of which reads single revolutions and the other hundreds of revolutions. The speed counter is started and stopped by the screws C and D.

Let us suppose that the pitch of the note emitted by the siren is kept constant and that *N* revolutions are recorded in an interval of time *t*. In each revolution the holes come opposite one another

18 times. When the holes come opposite one another, 18 simultaneous blasts of air are emitted, and a wave is started. Thus in time t there are $18N$ waves started, and the frequency of the note emitted is consequently $18N/t$.

In using this siren to determine the pitch of a note there is considerable difficulty in keeping the speed of rotation constant at such a rate that the siren is in unison with the note. Hence in the best instruments the disc is rotated by an electric motor. The holes are then bored vertically.

Savart's Wheel.—Savart's wheel has teeth somewhat like a circular saw. It is used with a short strip of steel which is clamped, so that the free end presses against the teeth. When the wheel rotates, the spring vibrates and emits a note the pitch of which rises as the velocity of the wheel is increased.

Tuning-fork.—The tuning-fork is a U-shaped metal rod used as a standard of pitch. It is sometimes screwed into a wooden box,

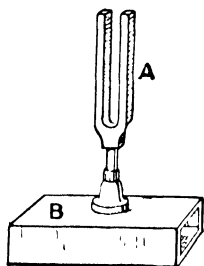


FIG. 239.—Tuning-fork with resonator.

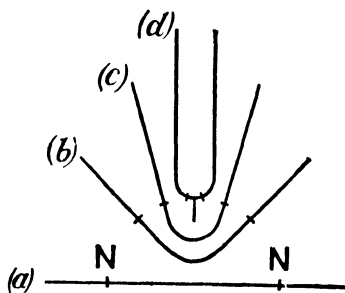


FIG. 240.—Illustrating mode of vibration of tuning-fork.

open at one end, which resounds to the note of the fork, making it louder, but at the same time shortening its duration (Fig. 239).

Let us suppose that a metal rod which is free at both ends is sounding with two nodes at NN , as shown in Fig. 240 (a). The rod is vibrating up and down, and if the wave form is drawn through the nodes, it will be noted that both ends are up at the same time. Now gradually bend the rod through the shapes (b) and (c) to (d). The nodes come closer together, and in (d), which forms a tuning-fork, they practically coincide. Since both ends were up at the same time in (a), it is clear that they will vibrate in (d), so as to approach, and recede from, each other alternately.

The reason why tuning-forks are used so much in sound is because they give a simple tone, and the wave they produce approaches the ideal sine curve more closely than any wave

originated in another manner. Their pitch is also only slightly affected by changes in temperature.

Electrical Tuning-fork.—For some purposes tuning-forks are required to maintain a continuous note for a long time, and in this case it is usual to drive them electrically. Fig. 241 shows one way of doing this. The fork is supported on a heavy stand, and a small electromagnet fixed between the prongs. The current from a battery B enters the fork below, leaves it by the spring contact at S, flows round the coil of the magnet, and then goes back to the battery. This brings the magnet into action. The latter attracts the prongs and breaks the circuit at S. The magnet then demagnetises, the prongs fly back, and the circuit is once more made. And so on, the action being similar to that of an electric bell.

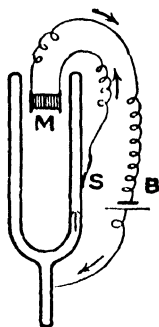


FIG. 241.

Measurement of Pitch of a Tuning-fork.—Since tuning-forks are used as standards of pitch, it is necessary to be able to measure their frequency with accuracy. One way of doing this is to attach a bristle to a prong of the fork and cause this bristle to trace a wavy line on a smoked cylinder or drum which is rotating rapidly. One end of the axle is a screw working in a nut, so as the cylinder rotates its surface moves in the direction of the axle and the tracings made in successive revolutions do not superimpose. At the same time a tracing is made by a point which is periodically raised by a small electromagnet, the current of which is made by the pendulum of a clock. The frequency is then determined by counting the number of vibrations which occur between two of the time marks. The arrangement is called a **chronograph**. An alternative method of making the time marks is to use an electrically driven tuning-fork of known frequency. The cylinder is usually rotated mechanically.

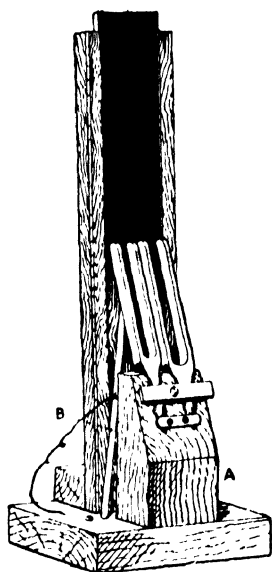


FIG. 242.—Falling plate method of comparing frequencies.

A simpler apparatus much used in students' courses is the **falling plate**. This is shown in Fig. 242, arranged for comparing the frequencies of two forks. These are clamped on the block A, and have fine bristles attached to their inside prongs. The

surface of a brass plate is smoked, and the latter is then supported in the slide. The ends of the bristles should just touch the smoked surface. The forks are bowed until they both sound loudly. The plate is then made to fall by pulling away the prop B, or, better still, by releasing a spring clamp. The bristles trace wavy lines on the plate, and by counting the number of wave-lengths on equivalent lengths of the two traces the ratio of the frequencies can be determined.



FIG. 243.

Formula for Falling Plate Method.—The frequency of a single fork can be calculated from a trace made by this method as follows: Let PQR be the trace (Fig. 243), suppose that there are exactly m waves in PQ and in QR, and let the lengths of PQ and QR be respectively l_1 and l_2 . The time of fall is the same between P and Q and between Q and R; denote it by t . Let u be the speed of P as it passes the bristle. Then

$$l_1 = ut + \frac{1}{2}gt^2 \quad \text{and} \quad l_2 = (u + gt)t + \frac{1}{2}gt^2$$

Hence

$$l_2 - l_1 = gt^2 \quad \text{and} \quad t = \sqrt{\frac{l_2 - l_1}{g}}$$

Thus the frequency of the fork is given by

$$\frac{m}{t} = m\sqrt{\frac{g}{l_2 - l_1}}$$

In order to obtain good results contact with the bristle should be light, and there should be no friction between plate and slide.

Stroboscopic Disc Method of Finding Frequency of a Fork.—In this method two light pieces of card or metal foil, C, D, are attached to the

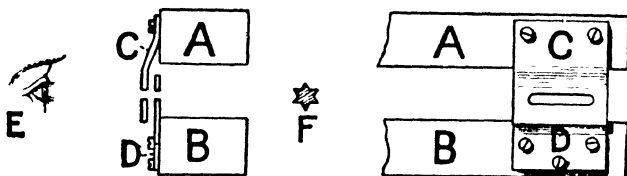


FIG. 244.

ends of the prongs, A and B, as shown in Fig. 244. Each has a slit in it, and these slits are situated so that they are exactly opposite each other when the prongs are in their equilibrium position. When the fork is sounding, it is possible for the eye at E to see the object F through the slits only when the prongs are passing through this position, that is, twice every period.

A white circular disc on which rings of equally spaced black dots have been painted (Fig. 245) is placed behind the fork and kept in rotation by an electric motor, the number of revolutions being read by

a speed counter. If it is observed through the slits, it is, of course, only seen intermittently, but the fork vibrates so fast that the different impressions blend, and in general the dots appear to be in continuous motion. The speed is, however, adjusted, so that while vision is cut off, each dot moves into the position previously occupied by its neighbour; they then appear to be at rest. Let the disc in this case make N revolutions per second, let n be the frequency of the fork, and let there be m dots in the ring. The number of dots which pass any point in a second is Nm . Hence the disc must be seen Nm times per second. But it is seen $2n$ times per second. Consequently

$$n = \frac{Nm}{2}$$

In cinematograph pictures of carriage wheels the spokes sometimes appear to revolve slowly the wrong way. If the pictures were taken at the rate of 20 per second, and during the twentieth of a second each spoke has moved into the position occupied by the previous one, they appear to be at rest. If they have not moved quite so far, the eye naturally fuses the two pictures; hence the phenomenon in question.

Determination of Frequency by Lissajous' Figures. It has already been stated that when two S.H.M.'s, the periods of which are in the

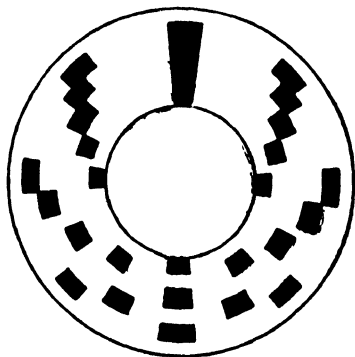


FIG. 245.—Stroboscopic disc.

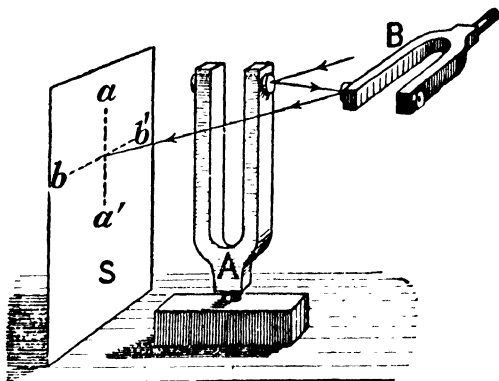


FIG. 246.—Arrangement for combining S.H.M.'s at right angles.

ratio or nearly in the ratio of two simple whole numbers to one another, are combined at right angles, they give characteristic curves. If a mirror is fixed to the prong of a tuning-fork, when the fork sounds,

the mirror vibrates, and if a ray of light is reflected by the mirror and received on a screen, instead of giving a spot as image, it gives a line. Owing to the vibration of the mirror, the image executes an S.H.M. up and down a line, but as the movement is too fast for the eye to follow it, the S.H.M. is seen as a line of light.

Let us suppose now we have two forks at right angles to each other, as shown in Fig. 246, each with a mirror fixed to one of its prongs, and that a ray of light is reflected by both forks. If A alone were sounding, the image would execute an S.H.M. along aa' and the line aa' would be produced on the screen. If B alone were sounding, the image would execute an S.H.M. along bb' , and the line bb' would appear on the screen. If the two forks sound together, these motions combine into a curve. If the periods are commensurable, the form of the curve remains stationary, but if they are not quite commensurable, it passes through a series of changes and then back to its original form. As has been mentioned on p. 251, by measuring the time required for the curve to pass through the cycle of changes we can determine the ratio of the periods with great accuracy. Hence, if the one period is known, the other can be calculated.

What makes the method of special value is the fact that the periods of the forks of a series should be to one another in the ratio of simple whole numbers.

Beats.—If two tuning-forks of nearly the same frequency are sounded together, we do not get unison, but throbbing or pulsating bursts of sound with intervals of comparative silence between them. The bursts of sound are called beats, and the two notes are said to beat or give beats. The number of beats per second can be counted with a stop-watch, if not more than four per second.

The phenomenon may be thus explained. Let m and n be the frequencies of the two forks, n being greater than m . The wave trains superimpose at the ear of the observer. Starting from the instant when a condensation in the one train coincides with a condensation in the other, the one train gradually falls behind the other, until there is a difference of half a period, the waves oppose one another, and the sound is a minimum; then the difference increases to a period, two condensations coincide again, and we have a maximum of sound. Since the one train gains $n - m$ periods on the other per second, the condensations obviously coincide $n - m$ times per second. Each coincidence gives a beat. There are thus $n - m$ beats per second. The process is shown graphically in Fig. 222.

Of a pair of forks intended to give the same frequency one may be far enough out to give beats with the other. If their frequencies are too close to give beats, one may be made to vibrate more slowly by fixing a rider to one of its prongs. The frequency of the beats may then be altered by adjusting the position of the

rider. If two forks are already giving beats, and we weight the prongs of the fork which has the smaller frequency with small pieces of wax, the frequency of the beats is increased. When two forks are giving beats, this affords a means of finding which has the lower frequency.

Interval between Two Notes. Musical Scale.—When two notes have the same pitch, they are said to be in unison. When they have not the same pitch, *the ratio of their frequencies is called the interval between them.* Thus, if the frequencies of two notes are 256 and 192, their interval is $\frac{256}{192} = \frac{4}{3}$, and if the frequencies are 512 and 384, the interval is $\frac{512}{384} = \frac{4}{3}$. The ear is able to estimate the interval between two notes. Certain intervals have names; thus 2 is the octave, $\frac{3}{2}$ the fifth, $\frac{4}{3}$ the fourth, $\frac{5}{4}$ the major third, $\frac{6}{5}$ the minor third.

Now it is found that when the interval between two notes is represented by a simple ratio such as 2 to 1, 3 to 2, etc., the notes produce a pleasant sensation when sounded, either one after the other, or together. Consequently the earliest musical instruments were tuned so that their intervals were simple ratios. The ancient Greeks had several scales, one of which appears to have developed into our modern scales during the Middle Ages, as a result of the requirements of harmony.

<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>	<i>a</i>	<i>b</i>	<i>c'</i>
do	re	mi	fa	so	la	si	do
24	27	30	32	36	40	45	48

The notes of the major diatonic scale are shown above, first in musical notation in the treble clef for the octave starting on middle *c*, then their names are given, and finally their names on the sol-fa notation. The row of numbers beneath should be memorised, as it gives the easiest way of finding the interval between two notes. Thus the interval between *g* and *c* is $\frac{36}{24} = \frac{3}{2}$, a fifth, and between *c'* and *g* is $\frac{48}{36} = \frac{4}{3}$, a fourth. The intervals between consecutive notes of the scale are $\frac{9}{8}$, $\frac{10}{9}$, $\frac{16}{15}$, $\frac{9}{8}$, $\frac{10}{9}$, $\frac{9}{8}$, $\frac{16}{15}$; of these $\frac{9}{8}$ is called a major tone, $\frac{10}{9}$ a minor tone, and $\frac{16}{15}$ a diatonic semitone. The note from which the scale starts is called the tonic.

The combination of three notes with the frequency ratios, 4, 5, 6, is called the *major chord*, and produces a very pleasing effect when sounded together. The major diatonic scale contains three major chords: *c, e, g*; *f, a, c'*; *g, b, d'*. This fact doubtless played an important rôle in fixing the scale.

Standard middle *c* forks used in physical laboratories all have the frequency 256. The octaves above the one represented on the preceding page are denoted in order by the indices ⁱ, ⁱⁱ, ⁱⁱⁱ, ^{iv}, and the octaves below by capital letters and capital letters with suffixes _i, _{ii}. Thus we have the series C_{ii}, C_i, C, *c*, *c*ⁱ, *c*ⁱⁱ, *c*ⁱⁱⁱ, *c*^{iv}, for the notes with the frequencies 32, 64, 128, 256, 512, 1024, 2048, 4096.

Equally Tempered Scale.—In the piano and other instruments with fixed notes, if the frequencies of the notes were proportional to the numbers on p. 269, it would be possible to play music only in one key. For example, if *g* were taken as *do*, the interval between it and the next note above it is $\frac{40}{36} = \frac{10}{9}$, whereas the interval between *do* and *re* should be $\frac{27}{27} = \frac{9}{8}$. Hence such instruments are tuned, not according to the natural scale, but according to the equally tempered scale, which represents a compromise between all possible keynotes. On this scale the frequencies of the notes are in the following ratio :

<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>	<i>a</i>	<i>b</i>	<i>c'</i>
1	$2^{\frac{1}{12}}$	$2^{\frac{2}{12}}$	$2^{\frac{3}{12}}$	$2^{\frac{4}{12}}$	$2^{\frac{5}{12}}$	$2^{\frac{6}{12}}$	2

Thus on the equally tempered scale a major tone is $2^{\frac{1}{12}}$ and a semitone $2^{\frac{1}{24}}$. If the frequency of *c* is 256, the frequencies of the notes on the two scales are as follows :

	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>	<i>a</i>	<i>b</i>	<i>c'</i>
Natural Scale	256	288	320	$341\frac{1}{3}$	384	$426\frac{2}{3}$	480	512
Tempered Scale	256	287.4	322.5	341.7	383.6	430.5	483.3	512

EXAMPLE.—If the frequency of *c* is 274 (concert pitch), what are the frequencies of *f*, *f*[#], *b*, and *c*_b on the equally tempered scale? *f*[#] reads *f* sharp and is a semitone higher than *f*, *c*_b reads *c* flat and is a semitone lower than *c*.

$$\begin{aligned} f &= 274 \times 2^{\frac{3}{12}} = 365.8 \\ f^{\#} &= 274 \times 2^{\frac{4}{12}} = 387.6 \\ b &= 274 \times 2^{\frac{10}{12}} = 517.2 \\ c_b &= 274 \times 2^{\frac{1}{12}} = 325.8 \end{aligned}$$

Doppler Effect.—If an observer is standing by the side of a railway line, and an engine passes at a high speed sounding its whistle, the pitch of the whistle appears to fall suddenly at the moment of passing. This apparent change of pitch is referred to as the Doppler effect.

It can be explained very simply from the accompanying diagram (Fig. 247). Let *V* be the velocity of the sound, *v* the

velocity of the engine, n the frequency of the whistle, and let the circles give positions of successive sound waves all drawn for the same time. The circles are not concentric; their centres are displaced in the direction of motion of the engine. In one second n waves are emitted, but these are crowded together into a distance $V - v$ in front of the engine instead of being spread over a distance V . Consequently the wave-length is diminished, and to any one in front of the engine the pitch appears to be

$$\frac{nV}{V-v}$$

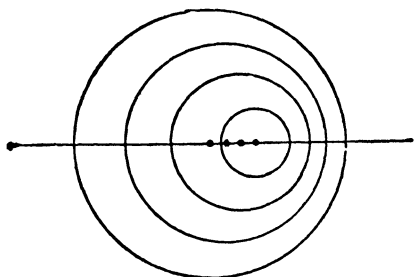


FIG. 247.

Similarly in the space behind the engine we have n waves spread over a distance $V + v$ instead of over a distance V . The wave-length is consequently increased, and the pitch appears to be

$$\frac{nV}{V+v}$$

Hence the change of pitch, as the whistle passes, corresponds to an interval

$$\frac{V+v}{V-v}$$

If the whistle is stationary, and the observer approaches it with velocity v , in one second he meets the waves which occupy a distance $V + v$. Consequently the apparent pitch is

$$\frac{n(V+v)}{V}$$

After he passes the whistle, he is overtaken in one second by the waves which occupy a distance $V - v$. Consequently the apparent pitch is

$$\frac{n(V-v)}{V}$$

EXAMPLES III

1. A siren is tuned to unison with a fork of frequency 256, and kept in unison for 7.2 seconds. If the disc has 18 holes, what number of revolutions should the speed counter record for this time?

2. A disc which is provided with 36 holes arranged at equal distances from one another on a circle concentric with its edge is rotated rapidly

about an axis which passes perpendicularly through its centre. A jet of air from a bellows is directed against the holes, and the note produced is found to be in unison with a fork of frequency 320. How many revolutions does the disc make per second?

3. A bristle is attached to one of the prongs of a tuning-fork, and the fork is sounded. A smoked glass plate is held vertically so that the bristle rests against it, and then allowed to fall under the action of gravity. The bristle traces a wavy line upon it. Show that if N is the number of waves marked in a distance d , the frequency of the fork is $N/\sqrt{2d/g}$.

4. Two tuning-forks make 4 beats per second. The frequency of one is 320. When a prong of the other is weighted with a small piece of wax which reduces its frequency, the number of beats per second is increased. What was its original frequency?

5. There are seven octaves on an ordinary piano, the lowest note being A_{41} and the highest a^{41} . Calculate their frequencies, given that the frequency of c is 261.

6. What are the wave-lengths of these two notes? Take the velocity of sound as 340 metres/sec.

7. Given that the frequency of c is 261 (international pitch), calculate the frequencies of g , b , c' , both on the natural and equally tempered scales.

8. An engine is standing in a siding blowing a whistle, which has a frequency of 600. A train passes at 60 miles per hour. What does the frequency of the whistle appear to be to the passengers in the train, before and after passing?

9. The frequency of the whistle of an engine is 500. It passes an observer at the side of the railway at 60 miles per hour. What is the apparent frequency of the whistle before and after passing?

10. An observer is standing by the side of a railway when a train passes with the engine sounding its whistle. As the engine passes, the pitch of the whistle appears to drop a minor third ($\frac{2}{3}$). What is the speed of the train in miles per hour?

CHAPTER IV

VIBRATIONS OF STRINGS AND COLUMNS OF AIR

Vibrating Strings.—If a violin string is bowed or a piano wire struck by the hammer, it is set into vibration, and a musical note is heard. Such a refined arrangement, however, is not necessary; even an elastic band stretched round the legs of a stool gives a note when plucked.

Fig. 248 represents a piece of apparatus known as the **mono-**

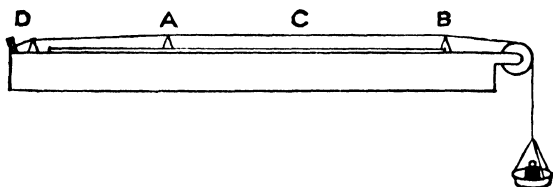


FIG. 248.—Monochord.

chord or **sonometer**, used for investigating the notes emitted by a stretched string or wire. The wire, usually a piano wire, is attached to a peg at D, passes over the fixed bridge at D, the movable bridge at A, the fixed bridge at B, and then over the pulley. Its end carries a pan of weights to give it the necessary tension. The bridges are simply wedge-shaped pieces of wood. They are mounted on a wooden box which acts as resonator, and increases the intensity of the note. In some forms of the apparatus, instead of using a pan and weights the wire is attached to a spring balance which is stretched by a winch. Sometimes the box is mounted vertically, in which case the pulley is unnecessary. It is the part AB of the wire that is experimented on; there is a wooden scale below it for measuring its length.

Let us suppose, first, that the stretching force is kept constant, and that a series of tuning-forks is taken and the length of AB adjusted, so that when it is plucked at C it is in unison in succession with each of the forks. It is found in this case that the frequency of the fork, n , is:

- (i) *Inversely proportional to the length of the wire, l .*

Let us suppose next that l , the length of AB, is kept constant, and that T , the stretching force in the wire measured in dynes, is varied, so that when the wire is plucked it is again in unison with each of the forks. It is found in this case that n is :

(ii) *Directly proportional to the square root of the stretching force, T .*

Again, keep the length of AB constant and the stretching force constant, but change the wire. It is found in this case that n is independent of the material of the wire, but

(iii) *Inversely proportional to the square root of m , the mass of the wire per unit length.*

These results may be combined in the statement that n varies as $\sqrt{T/l} \sqrt{m}$. It may be shown experimentally that the constant of proportionality is $\frac{1}{2}$. Hence

$$n = \frac{1}{2l} \sqrt{\frac{T}{m}}$$

It was found by Pythagoras (572-492 B.C.) that when a stretched string was plucked to give a certain note, and when the length of the vibrating portion was diminished to one-half and then the string was plucked again, the stretching force being kept constant, the string gave out a note an octave higher. If the length of the string was diminished to two-thirds, it gave out a note one-fifth higher, and if it was diminished to three-quarters of the original length, it gave out a note one-fourth higher; and so on. When the stretching force is constant the lengths corresponding to the different notes of the octave bear simple relations to one another.

This is one of the oldest discoveries in physics; it inspired a great quantity of semi-mystical thinking from the time of the music of the spheres down to Kepler, the contemporary of Galileo.

Harmonics of a Stretched String.—Let us consider the stretched string from another standpoint. It may be shown that the velocity of a transverse wave in such a string is given by

$$V = \sqrt{\frac{T}{m}}$$

Two transverse waves in opposite directions produce stationary waves, the distance between successive nodes being $\frac{1}{2}\lambda$. When stationary waves are produced in a stretched string, the ends must be nodes, since they cannot move up or down. Hence the string must vibrate, as shown in Fig. 249, between the positions indicated by the full and dotted lines.

Since the curvature is small, the length l along the curve may be assumed to be the same as the length along the chord. Obviously in the case of (a) the wave-length $\lambda = 2l$, in the case of (b) it is $\frac{3}{2}l$, in the case of (c) it is $\frac{4}{3}l$, and it is clear that by inserting 3, 4, 5, etc., nodes between the ends of the string we would obtain

the wave-lengths $\frac{2}{4}l$, $\frac{2}{5}l$, $\frac{2}{6}l$, etc. Now $V = n\lambda = \sqrt{T/m}$. Consequently the frequencies of the note emitted by the string should be given by

$$n = \frac{1}{2l} \sqrt{\frac{T}{m}}, \quad \frac{2}{2l} \sqrt{\frac{T}{m}}, \quad \frac{3}{2l} \sqrt{\frac{T}{m}}, \text{ etc.}$$

The first of these notes (Fig. 249 (a)) is that obtained by plucking the string at its midpoint. It is called the **fundamental**. If, however, the wire is plucked with one hand at a quarter of its length from the end, and then touched for an instant lightly and almost simultaneously at its midpoint by a finger of the other hand, the note an octave higher than the fundamental, given by the second formula, is distinctly heard. This is called the **first harmonic**. Touching the wire at its midpoint has produced the node at G (Fig. 249 (b)). In the same way, if the wire is plucked

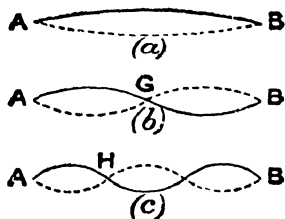


FIG. 249.—Harmonics of a stretched string.

at a point $\frac{1}{3}$ of its length from one end, and then touched at a point H (Fig. 249 (c)), $\frac{1}{3}$ of its length from the end, it emits a note of the frequency given by the third formula above, i.e. an octave plus a fifth above the fundamental. This is called the **second harmonic**. In the same way by plucking the wire at a loop and damping it at a node we can produce the harmonics the frequencies of which are given by the other members of the series.

The harmonics of a string are sometimes referred to as its **overtones** or **upper partials**. The emission of overtones is not peculiar to strings, but is shared by organ pipes, bells, etc. But in the case of bells the frequencies of the overtones are not exact multiples of the frequency of the fundamental. It is only when the frequencies of the overtones are exact multiples of the frequency of the fundamental that they are referred to as harmonics of the latter.

If the wire is plucked at a point one-quarter of its length from one end, without being touched at the middle, the note most distinctly heard is the fundamental; if now the wire is touched for an instant at the middle, the intensity is diminished, and the note most distinctly heard is the first harmonic. The latter had been present from the beginning, but was not audible until the fundamental had been damped down. This and similar experiments lead to the following conclusion :

In general when a wire is plucked, the note emitted consists both of fundamental and overtones, the degree in which each is present depending on the manner in which the wire is plucked.

Timbre.—The musical quality or timbre of a note depends on the degree in which the different overtones are present. The note of a tuning-fork is pure, giving only the fundamental, especially if fitted on a resonance box. If the harmonics of a wire are few and weak, the note is soft and mellow; if they are strong up to the fifth, the note is richer; the presence of harmonics above the sixth produces a sharp metallic quality. A piano string is struck at a point one-seventh of its length from one end, in order to eliminate all harmonics above the sixth. A trained ear can detect the presence of harmonics in a note.

EXAMPLE.—What is the frequency of the fundamental note of a wire 50 cm. in length, which is stretched by a force of 15 kilograms? A metre of the wire weighs 2.69 gm.

$$V = \sqrt{T/m} = \sqrt{15,000 \times 981/0.0269}$$

$$= 233.9 \text{ metres/sec.}$$

$$\text{Frequency} = \frac{23,390}{100} = 233.9$$

Proof of Formula for the Velocity.—We shall now prove the formula $V = \sqrt{T/m}$. The proof is somewhat difficult, and may be omitted at a first reading.

Let us suppose (Fig. 250) that T is the stretching force and m the

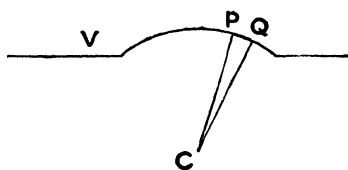


FIG. 250.

mass per unit length of the string, and let a solitary wave be travelling from left to right with velocity V . Consider the motion of the element PQ ; let its length be s . Let θ_1 and θ_2 be the angles which the tangents at P and Q make with the undisturbed direction of the wire. The upward component of the stretching force at P is $T \sin \theta_1$ and the

downward component at Q is $T \sin \theta_2$. The resultant downward force is consequently

$$T \sin \theta_2 - T \sin \theta_1 = T(\theta_2 - \theta_1)$$

since the angles are small.

Let a be the downward acceleration of the element. Then its rate of change of momentum is msa . Let the radius of curvature CP of the wave at P be denoted by r . Then $s = r(\theta_2 - \theta_1)$. Now

$$msa = T(\theta_2 - \theta_1)$$

Substitute for s and this becomes

$$mra = T$$

In order to find a suppose the whole diagram to move from right to left with uniform velocity V . The acceleration is unaltered, since the velocity is uniform. The wave is now stationary, but the element is moving round the curve with velocity V . Its downward acceleration

is V^2/r . This must be equal to a , and on substituting it for a we obtain the desired result, $V = \sqrt{T/m}$.

Melde's Experiment.—This is an interesting experiment on the vibrations of strings. One end of a thread about a yard long is fastened to a prong of a tuning-fork, and the other passes over a glass rod, and carries a scale pan. The weights in the pan are adjusted so that the frequency of the fork is twice the frequency of the fundamental note of the thread.

If the fork is held as in Fig. 251, when the prong moves to the left the thread sags, and when it moves to the right the thread tightens. When the prong moves towards the left again the inertia of the string carries it into the upper position. The stretching force has been adjusted to make it arrive there just when the prong is turning. Hence the thread vibrates in its fundamental mode as shown, the frequency of the fork being twice the frequency of the thread.

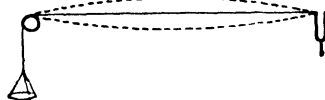


FIG. 251.

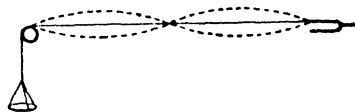


FIG. 252.

If the fork is held as in Fig. 252, when the prong moves up, the thread moves up, and when the prong moves down, the thread moves down. Hence in this case the frequency of the fork is equal to the frequency of the thread, there is a node at the midpoint of the thread, and the latter vibrates in its first harmonic.

Resonance.—Let us suppose we have a simple pendulum suspended by a thread about a yard long, and that the bob is at rest. Take a piece of notepaper folded into a strip 6 inches long by 1 inch wide; hold the paper by the end, and try to deflect the bob of the pendulum by bringing the side of the strip against it and pushing steadily. Owing to the bending of the paper the utmost deflection that can be obtained is probably about a quarter of an inch. If, however, the pendulum is tapped repeatedly in one direction with the strip of paper, the taps being delivered when the bob is at the middle of its swing going in that direction, then the oscillations may become quite large, 3 or 4 inches on each side of the vertical.

The force applied is no greater than before. The paper still bends. But the experiment shows that by timing a weak force so as to make it synchronise with the natural period of a system, large vibrations may be obtained. If the taps are not quite in time with the swings, oscillations are produced, but these oscillations are not so large as when the synchronisation is perfect.

The phenomenon is referred to as resonance, and it plays a large rôle in physics. We have had an example of it in Melde's

experiment just described. In the first case the fundamental, and in the second case the first harmonic, is in resonance with the fork. Soldiers are commanded to break step when crossing a suspension bridge, otherwise the bridge might oscillate dangerously in resonance with the rhythm of their feet. Resonance may be demonstrated very simply by the following experiment :

Let two mounted tuning-forks of the same frequency be placed together with the open ends of their resonators facing each other. Strike the one fork, and then stop its vibrations with the hand. The other fork will be heard sounding. But if the frequency of the second fork is altered by attaching a rider to the end of one of its prongs, when the first fork is damped, the second fork is dumb.

Vibrating Air Columns.—We shall now consider the formation of stationary sound waves in tubes or pipes of constant cross-section. Such waves are formed by the superposition of progressive waves in opposite directions, and it was shown on p. 247

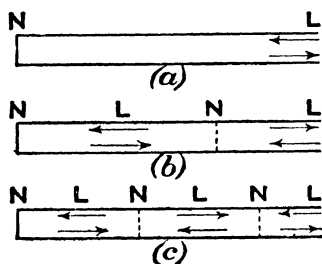


FIG. 253.—Harmonics of a closed pipe.

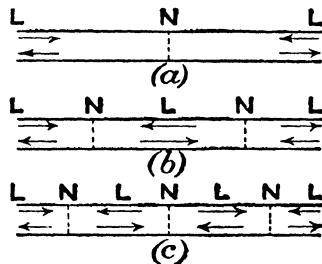


FIG. 254.—Harmonics of an open pipe.

that at the nodes the air particles are at rest, but there is a maximum change of density and pressure there, while at the loops the displacement of the air particles is a maximum, first in the one direction and then in the other, but there is neither change of pressure nor of density. If the end of the tube is closed, since the air particles cannot move into the closed end, the latter is a node. If the end of the tube is open, there can be no change of density there, because any excess of density equalises itself over the outside atmosphere ; hence in this case the end is a loop.

Fig. 253 represents the case of a tube closed at one end. Let l be its length. Then it is clear that in (a) where the tube is sounding its fundamental, $\lambda = 4l$, and that in (b) and (c) where it is sounding harmonics, $\lambda = \frac{4}{3}l$ and $\frac{4}{5}l$. The other possible wavelengths are obviously given by $\frac{4}{7}l$, $\frac{4}{9}l$, etc.

Fig. 254 represents the case of a tube open at both ends.

Let l be the length as before. Then it is clear that in the fundamental mode (a) $\lambda = 2l$, and that in the harmonics (b) and (c) $\lambda = l$ and $\frac{2}{3}l$. The other possible wave-lengths are given by $\frac{4}{5}l$, $\frac{3}{4}l$, etc. In this case all the multiples of the fundamental frequency occur, but in the former case only the odd multiples.

If the tube is closed at both ends, the wave-lengths are the same as when it is open at both ends.

Velocity of Sound by Resonance.—Fig. 255 represents a simple method of determining the velocity of sound based on the properties of vibrating air columns. A cylindrical glass tube is held in a vertical position with its lower end dipping into a vessel containing water, and a tuning-fork of frequency n is caused to sound above the upper end of the tube with its plane vertical. The tube is then lowered into the water, so as gradually to diminish the length AB of the enclosed air column. For one position the air column resounds loudly to the note of the fork; if the tube is long enough, it may resound faintly to the fork in other positions. These positions are determined as accurately as possible, and the length l of the air column measured for each. In each of these positions the air column is set into vibration by the regularly recurring blows which the fork delivers against the air above it.

Let us suppose that at a certain instant the lower prong of the fork is moving downwards. This causes a wave of compression to move down the tube. The wave is reflected by the water at the foot and travels up again, the interval of time for the passage down and up the tube being $2l/V$. If this interval is exactly equal to an odd number of half periods, the reflected wave superimposes on the wave of compression produced by the upward motion of the lower prong, the motion of fork and air column are in agreement, and the blows from the prong are coming at the right intervals. The condition for resonance is therefore

$$\frac{2l}{V} = \frac{1}{2}T, \frac{3}{2}T, \frac{5}{2}T, \text{ etc.}$$

or

$$V = 4nl, V = \frac{4}{3}nl, V = \frac{4}{5}nl, \text{ etc.}$$

where T is the period of the fork. Thus when l is measured, V can be calculated.

The first equation holds for the loudest resonance; in this case the column is vibrating in its fundamental mode, and the length of the column is one-quarter of the wave-length. The other values of l are 3, 5, etc., times as long; in these cases the column is vibrating in one of its harmonics. Under the usual

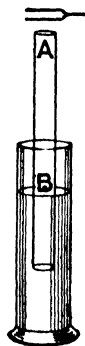


FIG. 255.

conditions of the experiment the tube is rarely long enough to give more than two positions of resonance. It is found in practice that the loop is situated a little above the upper end of the tube, and the equation $V = 4n(l + 0.4d)$ gives better values for the fundamental mode, d being the diameter of the tube.

A tube open at both ends can be made to give resonance with a fork. In this case the length of tube which resounds best is half the wave-length. The experiment can be carried out by sliding one piece of thin tube inside another.

The resonance box of a middle C fork has an internal length of 31 cm. The wave-length of the note given by this fork at 15° C. is 132.9 cm. A quarter of this is 33.2, so allowing for the disturbing effect of the end, the fundamental of the air column enclosed by the box agrees with the note of the fork.

If an unmounted fork is struck, and its stem held against a table or sounding board, the note becomes louder. In this case the surface of the table executes what are called forced oscillations in contradistinction to the free oscillations of the air in the resonance box. The oscillations are forced because they do not coincide with a natural period of the table, and, for the same reason, they are not so loud.

EXAMPLE.—In determining the velocity of sound by resonance tuning-forks of frequencies 256, 320, 384, and 512 were used, and resonating lengths of 31.3, 23.3, 19.7 and 14.3 cm. were obtained. The diameter of the tube was 6.7 cm. Assume the correct value for the velocity of sound at 0° C., namely 331.3 metres/sec., and calculate the temperature at which the experiment was performed.

We have

n	l	$\lambda = 4(l + 0.4d)$	V
256	31.3	136.0	34,810
320	23.3	104.0	33,280
384	19.7	89.6	34,410
512	14.3	68.0	34,820
Mean value of V			34,330 cm./sec.

$$\text{We have therefore } 34,330 = 33,130 \left(1 + \frac{t}{5.46} \right)$$

$$\text{i.e. } \frac{120}{3313} = \frac{t}{5.46} \quad \text{or } t = 20^\circ \text{ C.}$$

Organ Pipes.—Fig. 256 represents an organ pipe which is open at the top. It can be sounded either by blowing it with the mouth or by a bellows. Fig. 257 shows a bellows of a type often used in experiments in sound. P is a pedal worked by the foot, S the bellows themselves, and D a leather reservoir into which the air is pumped; the air escapes from the reservoir through the pipe E to the wind-chest C. In the top of this wind chest there are a number of holes closed by keys into which a siren or organ pipes can be fitted; when the key is pressed the siren or organ pipe

is sounded. Increased pressure can be obtained by forcing down T.

Fig. 258 represents a section of the pipe shown in Fig. 256. The wind is supplied through *i*, and escapes through a narrow slot, opposite which there is a lip *b*. The air-jet impinging against the lip sets the air column inside the pipe vibrating. The pipe is fitted into the wind-chest by means of the foot P. The pipe behaves as if its lower end were open. Thus if *l* is the length of

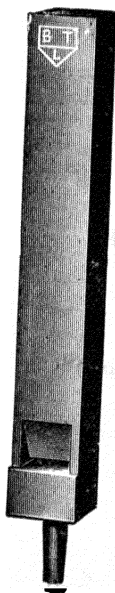


FIG. 256.

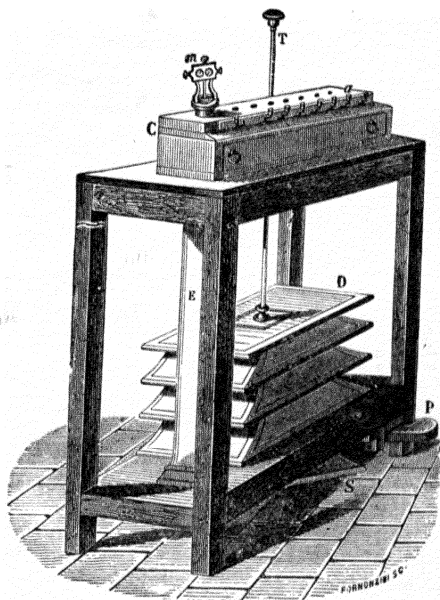


FIG. 257.—Bellows for experiments on sound.

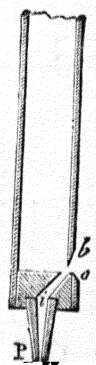


FIG. 258.

the air column, and *V* the velocity of sound, the wave-lengths of the fundamental and overtones are respectively $2l$, $\frac{3}{2}l$, $\frac{5}{3}l$, etc., and their frequencies $\frac{1}{2}V/l$, V/l , $\frac{3}{2}V/l$, etc. In general the note emitted by the pipe is a complex one with the fundamental predominating. But if the strength of the air blast is increased, the first overtone predominates.

The pitch of the pipe can be slightly lowered by partially closing the upper end, for example, by sliding a plate across it. This is an easy way of demonstrating beats, when we have two pipes of the same pitch.

If the upper end of the organ pipe is closed, it becomes a node,

for the particles are prevented from vibrating there. The wave-lengths of the fundamental and overtones are then $4l$, $\frac{4l}{3}$, $\frac{4l}{5}$, etc.

Experiments with smoke show that the jet of air passes alternately to the left and right of the lip b in Fig. 258, thus maintaining the oscillations. Instead of the lip b the pipe may be furnished with a thin plate of metal called a reed. The reed by its vibrations alternately opens and closes an aperture through which the wind passes, and the impulses derived from this intermittent supply of air set the column of air in the pipe into vibration. The pitch of the pipe is slightly modified by the action of the reed. The child's trumpet is an example of a reed pipe.

The pipe open at both ends is approximately realised in the case of the flute. In this instrument a cylindrical air column is set into vibration by the performer blowing against the side of the mouth-hole. The air column extends from the mouth-hole to the hole that is open highest up the flute. This hole and the mouth-hole both act as loops.

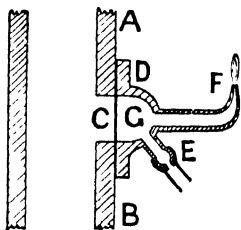


FIG. 259.—Manometric flame.

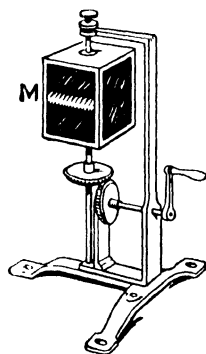


FIG. 260.—Rotating mirror.

Opening a hole is equivalent to cutting off the tube at that hole. For the lowest and middle octave the fingering is the same, but the shape of the lips and direction of the air blasts are different, the fundamental being obtained in the one case and the first overtone in the other. The principle of the penny whistle is the same as that of the flute.

Manometric Flame. Rotating Mirror.—The positions of the nodes and loops in vibrating air columns can be determined by means of an arrangement due to Koenig, called the manometric flame.

A hole is made in the side of the pipe AB (Fig. 259), and a thin sheet of indiarubber C stretched across it. A small wooden box encloses the rubber membrane. Ordinary coal gas enters at E and burns in a fine jet at F . If there is a node inside the tube at C , the pressure fluctuates rapidly. This forces the membrane in and out, and causes a variation in the pressure of the gas. When the pressure of the gas increases, the size of the jet increases, and when the pressure of the

gas diminishes, the size of the jet decreases. The variation in the size of the jet takes place too rapidly to be viewed directly, and must be observed in a rotating mirror.

The arrangement is shown in Fig. 260. Mirrors are mounted on the sides of a box which is rotated rapidly about a vertical axis by means of a handle and toothed wheels. Owing to the motion of the mirror the position of the image of the flame is always changing; consequently when its size alters, the new image appears in a different place. Thus the change can be detected, and we see something like Fig. 261. If we have two notes sounding together, the fundamental and its octave, the flame appears as in Fig. 262. Complex notes can be analysed into their constituent tones from the appearance of the flame.

At a loop there are no fluctuations, and as the position of the exploratory hole is moved down the tube from a loop to a node, the fluctuations increase in magnitude.

Singing Flame.—If a long glass tube is drawn out to a fine jet, connected to the gas pipe and lighted, and then pushed slowly up the middle of a glass tube two or three feet long which has been mounted vertically, it is found that in a certain position a loud harsh note is given out. When the flame is examined in the rotating mirror, it is then seen to be fluctuating violently, and it has been shown that the sound is due, in the first place, to oscillations arising in the tube supplying the jet: these set up oscillations in the air column.

Kundt's Tube.—A rod AB (Fig. 263) about a metre long is clamped at its middle, and carries at one end a light piston which fits into a glass tube, DC, of four or five centimetres diameter. The tube is closed at the end D and a small quantity of lycopodium powder is scattered along it. If the right-hand half of the rod is grasped and pulled to-

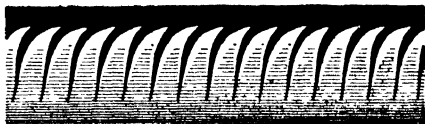


FIG. 261.

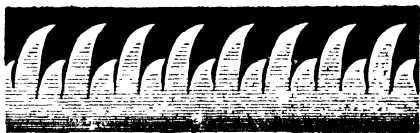


FIG. 262.

Images of flame in rotating mirror.

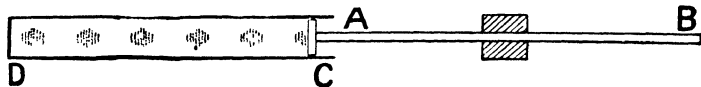


FIG. 263.

wards B with a cloth which has been rubbed with resin, it is set into longitudinal vibrations with a node at one end and a loop at the other. If the frequency of the rod coincides with one of the frequencies of the air in the glass tube, stationary waves are set up in the latter, and the lycopodium powder collects into a periodic arrangement of

heaps ; the figure shows these heaps from above. The wave-length inside the tube is consequently known, as it is twice the distance from one heap to the next. The explanation of why the powder collects in this way is difficult and beyond the scope of this book. The experiment will not work unless the tube and powder are clean and dry.

The arrangement can be used for comparing the velocity of sound in different gases. The tube is filled with the one gas after the other, and the positions of the heaps, and hence the wave-length, determined in each case. Then, since the frequency is unaltered, the velocity is simply proportional to the wave-length.

Vibrations of Rods and Plates.—Rods are capable of executing both transverse vibrations and longitudinal vibrations. An example of longitudinal vibrations is given by the rod used with Kundt's tube. In this case there is a node at the clamp and loops at the free ends, and the wave-length is twice the length of the rod. In the case of transverse vibrations a clamped end is a node and a free end a loop, but there is no simple relation between the dimensions and the wave-length. The tuning-fork is, of course, an example of the transverse vibrations of rods.

Plates also vibrate transversely. As in this case the waves travel in two dimensions, the number of overtones is much greater than in the

case of a string or vibrating air column, and it is difficult to calculate the values of the frequencies. But the positions of the nodes themselves can be obtained easily by experiment. The plate is supported in a horizontal plane, being screwed to a vertical pillar, and a violin bow is drawn across its edge to make it sound. Fine sand is scattered on the surface : when the plate

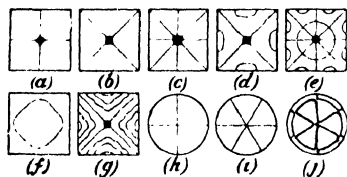


FIG. 264. — Chladni's figures.

vibrates, the sand gathers along the nodes, and we get figures such as are shown in Fig. 264. This diagram is for the case when the plate is screwed at its centre to the pillar.

These figures are called Chladni's figures after a German experimenter who studied them in detail and demonstrated them with great skill towards the end of the eighteenth century. To make each figure appear the plate must be bowed at the appropriate loop and damped at the appropriate node. When this is done well, the experiment is a very striking one.

Audition.—The external ear leads to an air passage closed at its end by a membrane called the drum. The air waves travel down this passage and set the drum into vibration. The auditory nerve terminates in a large number of fibres called Corti's fibres. The manner in which sound travels from the drum to the nerve and thence to the brain belongs to physiology rather than to physics.

If the frequency of a note lies below a certain limit, it is no longer heard as a continuous note, but only as separate impulses.

Helmholtz determined this limit to be about 30 per second. The measurement is difficult, because there is always the risk of an overtone being mistaken for the note itself. If the frequency of a note lies above 20,000 per second or thereabouts, the ear cannot hear it. It is difficult to assign these limits with accuracy, since they vary with age and the individual.

The direction from which a sound comes can be estimated by the ear. The estimate is apparently based on the difference in intensity with which the sound is heard by the two ears and also on the difference in phase with which it arrives at the two ears—precisely the method of sound ranging.

The human voice is produced by two membranes called the vocal chords, stretched across the trachea. When air is forced between them, they are set into vibration. The pitch of the note is determined by the tension and thickness of the chords. The throat, mouth, and the cavities of the nose act as a resonating chamber.

Phonograph. Gramophone.—In the phonograph devised by Edison in 1876 for the purpose of recording and reproducing sounds, a wax cylinder is rotated by clockwork, and the words to be reproduced are spoken into a horn. The sound waves which constitute these words cause a thin glass disc at the end of the horn to vibrate, and the vibrations of the disc cause a chisel-shaped sapphire point to move up and down, cutting a groove of varying depth in the wax. To reproduce the sound a slightly different arrangement is fitted, consisting of a smooth sapphire point which passes along the indentation produced by the recording point. It sets a thin glass disc at the end of the horn into vibration, and the vibration of the glass disc sets up the sound waves in the air.

In gramophone records the grooves have varying width, not varying depth. Thus in the gramophone the needle moves from side to side, not up and down like the sapphire point in the phonograph.

Fig. 265 shows the construction of the gramophone reproducer.

P, the needle point, is held by a screw S in the needle carrier N, which is attached at D to the centre of the glass diaphragm. The latter is held by the cover A against a rubber ring. The tube B is connected to the horn, which presses against the rubber ring C to make an air-tight joint.

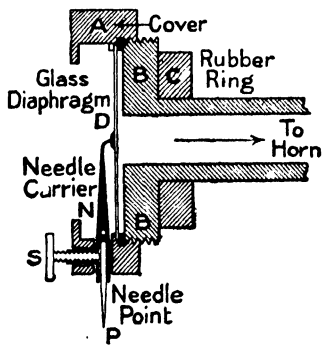


FIG. 265.—Gramophone reproducer.

EXAMPLES IV

1. A vibrating string gives the note *c* when stretched by a weight of 16 lb. What weight must be used to give the note *g*? The length of the string is kept unaltered.

2. What is the stretching force in the *c* string of a violin which vibrates 640 times per second, if the vibrating portion has a mass of $\frac{1}{16}$ gm. and a length of 33 cm.?

3. The internal length of the resonance box of a tuning-fork of frequency 320 is 24.7 cm. What is the frequency of the fundamental note of the box at 0° C.?

4. What length should a glass tube open at both ends have, in order that it may produce resonance when a tuning-fork of frequency 320 is sounded near one end?

5. What percentage variation will take place in the frequency of a pipe organ if the temperature changes from 40° to 55° F.?

6. An organ pipe gives a certain note at 0° C. How much must the temperature rise for the pitch to go up one semitone?

7. In determining the velocity of sound by resonance tuning-forks of frequency 256, 320, 384, and 512 were used, and resonating lengths of 31.05, 24.32, 20.0, and 14.20 cm. were obtained. The diameter of the tube was 6.5 cm., and the temperature at which the experiment was performed 17° C. Calculate the velocity of sound at 0° C. from the above data.

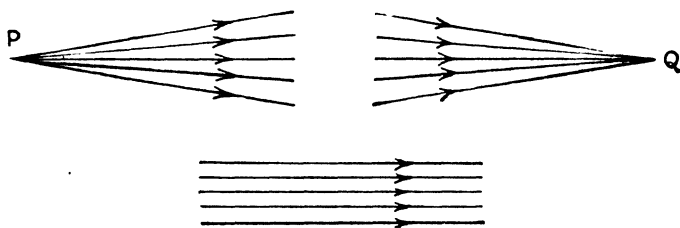
PART IV

LIGHT

CHAPTER I

SHADOWS. PHOTOMETRY

Rectilinear Propagation.—Light travels in straight lines. This is obvious for many different reasons. We always assume that a body exists in the direction of the rays of light which enter our eye from it. Also if the shadow of a stick is cast on a wall by a candle flame, the flame, the top of the stick, and the shadow of the top of the stick are all in one straight line. Sometimes the sun's rays are observed passing down through a hole in a cloud; they are then seen to be straight. This is also the case with the rays entering a darkened room through a hole in the shutter.



FIGS. 266, 267, 268.

The path of these last rays is rendered visible only by floating particles of dust. If chalk dust is shaken into the air, the visibility of the rays is increased, and if the air were wholly free from dust, the rays would not be seen at all.

A collection of adjacent rays is called a pencil. Fig. 266 represents a pencil diverging from the point P, Fig. 267 a pencil converging to the point Q, and Fig. 268 a parallel pencil. The direction in which the light travels is indicated in each case by the arrow heads.

Pinhole Camera.—If rays of light pass into a dark box through a small hole in the side, they form an inverted image of external

objects on the opposite side, as is shown in Fig. 269, where the front of the box is removed for the purpose of the diagram. a photographic plate is placed to receive the image, a picture can be taken. The hole must be small, literally a pinhole, otherwise the image is indistinct. Consequently the image is very fair, and unless the object to be photographed is a bright one, like a

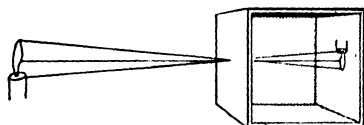


FIG. 269.—Pinhole camera.

candle flame, a very long exposure is necessary. The pinhole camera has, however, the advantage that no focusing is necessary.

If a second hole is made near the first, another image of the flame is seen; if the two overlap, the resultant picture will be blurred. A large hole may be regarded as a number of small holes close together, and this is the reason that it does not produce a clear picture. Suppose, for example, that the hole is triangular, then the images formed at a distance by the different parts of it coincide so nearly, that there is no loss of clearness, and the shape of the hole does not affect the picture. But if the rays are received by a screen close up to the hole, the pictures produced by the different parts of it are very small and do not overlap. So the patch of light on the screen has the same shape as the hole itself. That is, in the case in question, it is triangular.

EXPERIMENT. The easiest way to demonstrate the pinhole camera is to take a small wooden or cardboard box, remove one end, and replace it by a piece of ground glass. Suitable pieces of ground glass in various sizes can be obtained at any large photographic dealer's. Then a small hole is bored in the middle of the opposite end. If the camera is pointed towards a window or an electric lamp, an image of the window frame or of the filament will be seen on the glass. The image will, of course, appear brighter if a dark cloth is held over the head to exclude stray light.

EXAMPLE. — The lens of a camera has a diameter of half an inch, and is situated at a distance of 4 in. from the plate. It transmits 84 per cent. of the light incident on it, and when the full aperture of the lens is used, the correct exposure under certain circumstances is $\frac{1}{80}$ sec. The lens is replaced by a pinhole of $\frac{1}{60}$ in. diameter. Find the correct exposure under the same circumstances.

The amount of light received varies as the area of the surface. The ratio of the areas is the ratio of the squares of the diameters:

$$\left(\frac{1}{2}\right)^2 : \left(\frac{1}{60}\right)^2 = 900$$

Hence, if the lens transmitted all the light that fell on it, the exposure

the second case would be $900 \times \frac{1}{50} = 18$ sec. The correct exposure, therefore, $18 \times 0.84 = 15.12$ sec.

Shadows.—If an opaque body, for example a sphere (Fig. 270), is placed to intercept some of the rays emitted by a luminous point S, the cone of light incident on the sphere is stopped, and the space beyond the sphere enclosed by the continuation of the

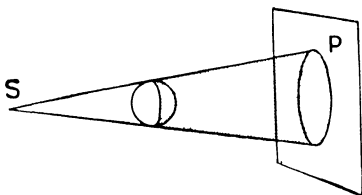


FIG. 270.

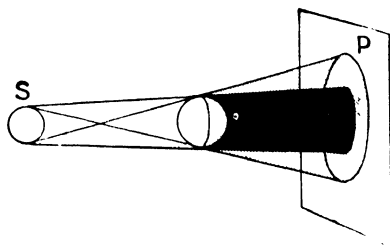


FIG. 271.— Umbra and penumbra.

cone is screened from the source. This space is called the shadow cone, and its intersection by the white screen at P gives the shadow cast on this screen.

Suppose that instead of a point the source of light is a luminous sphere (Fig. 271). Then the shadow is divided into two parts. First of all there is the total shadow or **umbra**, represented in black, which receives no light whatever from the source and from all points in which no part of the source is visible. Second there is the partial shadow or **penumbra**, which is shaded in the diagram; this receives light only from part of the source and from all points in it only part of the source is visible, the remainder being obscured by the opaque object. On the screen at P we have a uniform disc, the section of the umbra, surrounded by a zone, the section of the penumbra; the latter is, however, not one uniform shade as shown in the diagram, but gradually diminishes in intensity from its inner to its outer edge. While the boundaries of umbra and penumbra are perfectly definite geometrically, they are not so optically. The relative sizes of umbra and penumbra depend on the sizes of source and object and on the relative position of source, object, and screen. The umbra may not reach the screen (*cf.* Fig. 272).

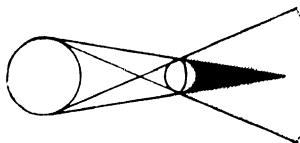


FIG. 272.

Eclipses.—Eclipses of the sun are caused by the moon moving between the sun and the earth. They consequently take place when

the moon is new. The moon is not self-luminous, so it acts the part of the opaque sphere in Fig. 272. If a point of the earth's surface is within the umbra, then for this point the eclipse of the sun is total; if it is within the penumbra, the eclipse of the sun is partial, *i.e.* the moon obscures part of the sun's disc. The distances of sun and moon from the earth are such that the earth traverses the shadow near the end of the umbra. Consequently as the moon moves on, the umbra either traces out a narrow band of total darkness on the earth's surface, or does not reach the earth at all.

Eclipses of the moon are caused by the earth getting between the sun and moon. They consequently occur at full moon. The earth's shadow falls on the surface of the moon, and as the latter is visible only by reflected sunlight, those portions of it within the umbra are obscured; during the eclipse we see the edge of the umbra moving across the surface.

Eclipses of the sun and moon can be illustrated very easily by using an electric lamp with a diffusing globe to represent the sun, a 100-watt "Fullolite," for example, and white cardboard discs to represent the earth and moon.

The Standard Candle.—The unit of intensity of a light source in this country is the standard candle, a spermaceti candle $\frac{7}{8}$ in. in diameter, weighing six to the pound, and burning at the rate of 120 grains to the hour.

The unit of light or lumen is the quantity that falls per second on a screen 1 sq. cm. in area held perpendicular to the rays coming from a point source of unit intensity, placed at a distance of 1 cm. from the screen; or more accurately, the quantity of light that falls per second on unit area of a sphere of 1 cm. radius drawn with such a source as centre.

The illumination of a surface is the quantity of light received by it per sq. cm. per sec. It has nothing to do with the colour or reflecting power of the surface, just as rainfall is independent of the nature of the soil. The illumination of a surface is usually measured in metre-candles. If, for example, the illumination is 5 metre-candles, the surface is as bright as if it were illuminated by 5 candles at a distance of 1 metre. The metre-candle is also called the lux. The names "lux" and "lumen," however, are not widely used.

The Inverse Square Law.—Suppose a pyramidal wire framework, constructed as shown in Fig. 273, with equal edges AD, AE, AF, and AG and a square base DEFG of 3 cm. side. Divide AD into three equal parts by the points B and C, and through B and C suppose planes drawn parallel to the base. By similar triangles these planes will be squares of 1 and 2 cm. side respectively, and the areas of the three planes will be 1, 4, and 9 sq. cm. Suppose that the vertex of the pyramid is placed at the flame, and that the three planes are transparent. Then the same quantity of

light that passes through the plane at B passes also through the planes at C and D. Let this quantity of light be Q ; then the

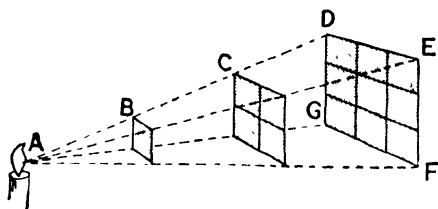


FIG. 273.—Illustrating the inverse square law.

illumination of a piece of white paper placed to coincide in succession with the planes at B, C, and D will be

$$\frac{Q}{1}, \frac{Q}{4}, \text{ and } \frac{Q}{9}$$

light units per sq. cm. per sec.

Thus the illumination produced by a source of light varies inversely as the square of the distance from it.

Inverse Square Law. Otherwise.—Suppose that a source of light is placed at S at the centre of a sphere of radius r_1 , and that it emits Q units of light per second. Let the illumination of the inside surface of the sphere be denoted by I_1 . Then

$$I_1 = \frac{Q}{4\pi r_1^2}$$

Suppose now that this sphere is removed, and the light falls on a concentric sphere of radius r_2 . Let the illumination of the inner surface of this sphere be denoted by I_2 .

Then

$$I_2 = \frac{Q}{4\pi r_2^2}$$

Hence we have

$$I_1 = \frac{Q}{4\pi r_1^2} \cdot \frac{4\pi r_2^2}{r_2^2} = \frac{r_2^2}{r_1^2} I_2$$

i.e. the intensity of illumination at any point of a surface is inversely proportional to the square of the distance of that point from the source of light.

The inverse square law enables us to compare the intensities of different sources and to measure the illumination of surfaces; the instruments used for this purpose are known as photometers,

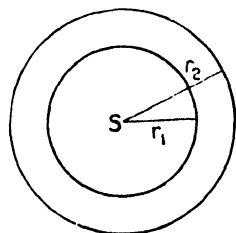


FIG. 274.

and the part of the subject dealing with it as photometry. The word "photometer" was first applied to the instrument by Rumford in a letter to the Royal Society in 1793.

The Rumford or Shadow Photometer.—This consists of a rod of the shape of a lead pencil, which is mounted vertically before a screen. Fig. 275 represents a plan of the arrangement. R is the rod, AC the screen, and L_1 and L_2 the two sources, the intensities of which are to be compared. BC is the shadow of the rod formed by L_1 and AB the shadow of the rod formed by L_2 on the screen. The distances of L_1 and L_2 from the screen are adjusted so as to make these shadows equally dark, and at the same time R is placed so that the shadows just touch at B. If

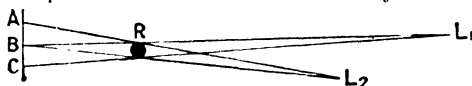


FIG. 275.

they overlap, or if there is a bright space between them, the comparison of their intensities cannot be made so accurately.

All parts of the screen outside the regions AB and BC are illuminated by both sources. The regions AB being in the shadow of L_2 is illuminated solely by L_1 , and the region BC being in the shadow of L_1 is illuminated solely by L_2 . Consequently, if the shadows are equally intense, the illuminations produced by the two sources at B are equally great, and the intensities of the two sources are simply as the squares of their distances from B. Hence, if the intensity of the one source is known in terms of the standard candle, and the distances BL_1 and BL_2 are measured with a metre stick, the intensity of the other source can be determined.

The screen should have a rough unglazed surface. One great advantage of the shadow photometer is that the accuracy of its results is not affected much by the presence of another source of light in the room. For this third source produces a shadow of its own on the screen, which can easily be recognised, and it is only necessary to take care that the edge of this third shadow does not fall on B.

The Bunsen or Grease Spot Photometer.—This consists of a piece of white unglazed paper, in the middle of which a small portion has been rendered translucent by a drop of grease. It is placed between the two sources and in the same straight line with them. It possesses the advantage of simplicity, but it is difficult to make a grease spot with a very sharp boundary, and, of course, it is impossible to detect slight inequalities in the illumination when the boundary is not sharp. As a substitute for the grease spot a disc of tissue paper may be used. The latter is made by placing

two sheets of paper on the top of one another and with a sharp knife cutting a star-shaped opening in both of them at the same time. The tissue paper is placed between them, and they are then stuck together with the stars accurately superimposed.

The grease spot is translucent, and the surrounding paper is opaque. Consequently, if the distance of the photometer is adjusted so that the grease spot appears of the same brightness as the surrounding paper, the intensities of the two sources should be as the squares of their distances from the photometer.

In practice, however, it is found that the grease spot seldom disappears for the one position, when viewed in succession from both sides of the screen. This is due to the light reflected by the white paper being scattered more than the light transmitted by the grease spot. Thus the more obliquely the surface is viewed, the darker appears the grease spot. Consequently, in working with the grease spot we set it so as to obtain the same relative intensity of the two areas when viewed from both sides; or in other words, we adjust, not for disappearance, but for the same contrast effect.

The Wedge Photometer.—The use of this photometer, which is sometimes called the Ritchie wedge, is shown in the adjoining Fig. 276. S_1 and S_2 are the sources. The wedge is arranged so that its faces make equal angles with the line joining S_1S_2 . The faces are rough and reflect the incident light diffusely. The observer looks at the edge of the wedge from the side at E, and adjusts the distances of the sources so that the two faces appear equally bright. The intensities of the sources are then as the squares of their distances from the edge.

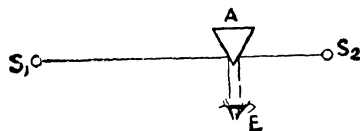


FIG. 276.

A scraped surface of plaster of Paris forms the best diffusing surface; but instead of this a wooden wedge may be used, and a sharply folded piece of unglazed paper placed over it and fastened at the back A with a drawing pin. The sharper the fold, the more accurately can the setting be made. I have also found it an advantage to have a sighting arrangement attached to the wedge, so as to make sure that the edge is viewed at right angles to the axis of the bench.

The Trotter or Perforated Screen Photometer.—This instrument is probably the most accurate of all the simple photometers. It is represented in sketch (Fig. 277) and in plan (Fig. 278). Light from the two sources to be compared falls on two screens in the directions indicated by the lines A and B, making an angle of 35° with the normals to the screens. White Bristol board is an excellent material for the screens.

The glaze must be removed with a damp cloth. A star-shaped opening is cut in the front screen with a sharp knife; this opening should not be more than $\frac{3}{8}$ in. over all. The eye at E sees the back screen through the hole in the front one, and if the latter is neatly cut, it is possible to arrange the distances of the lamps so that it disappears altogether.

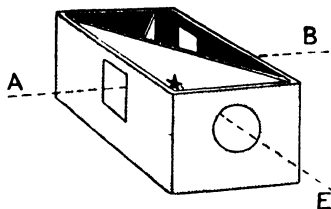


FIG. 277.

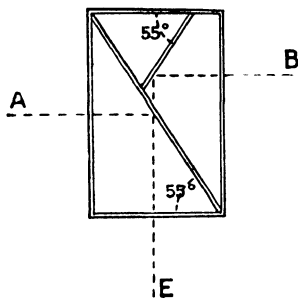


FIG. 278.

The Trotter photometer.

Stray light affects the accuracy of this and the two previous photometers, making the source on the side from which it falls appear too intense. This error can be eliminated by interchanging the positions of the two sources and taking the mean of the results.

Photometry.—Any of the photometers described in the preceding sections may be placed together with the light sources on the top of a table and the distances between them simply measured with metre sticks. But the results obtained in this way would not be accurate. The photometer and light sources are usually mounted on pieces which slide along a bar about 3 metres long. This bar carries a scale on which the distances between the pieces can be read. The bar and the sliding pieces should be blackened to prevent their reflecting light to the photometer. The ceiling of the rooms and the walls, particularly the parts behind the light sources, should also be blackened. If this is not possible, the error due to stray light can be eliminated by mounting diaphragms with dead-black surfaces on the photometer bar in such positions that they transmit all the direct rays from the source to the photometer, but stop light reflected from the walls of the room.

The inverse square law holds rigorously only for point sources, hence care must be taken that the distance between source and photometer is always large in comparison with the linear dimensions of the source.

Generally speaking, in photometry an accuracy of 1 per cent. in the final result is very good, though under the most favourable

circumstances $\frac{1}{10}$ per cent. can be attained. But in fitting up a photometer and taking readings for the first time, the accuracy attained is always very much less than this. Some of the readings diverge widely from the mean, and the temptation to throw these away and use only the "good readings" is a very strong one. This should never be done. If a setting is made carelessly the numbers should not be read, but if they are read and noted down on paper, they should contribute their share to the final result; the "bad readings" keep the average straight.

EXAMPLE.—A 30 candle-power and an 8 candle-power lamp are at the same height 100 cm. apart. At what point on their line of centres do they give the same illumination?

Let the point be x cm. from the 8 candle-power lamp. Then

$$\frac{8}{x^2} = \frac{30}{(100-x)^2}$$

This gives $22x^2 + 1600x - 80,000 = 0$

the roots of which are 34.0 and -106.8. There are consequently two points fulfilling the required conditions, one 34.0 cm. from the 8 candle-power lamp towards the other lamp, and the other 106.8 cm. from the 8 candle-power lamp away from the other lamp.

Fraction of Light transmitted by a Glass Plate.—The fraction of light transmitted by a glass plate can be measured by any photometer. For let there be two lamps of A and B candle-power, and let balance be obtained when these are situated a and b cm. from the photometer respectively. Then

$$\frac{A}{a^2} = \frac{B}{b^2}$$

Let the glass plate be placed between the photometer and the lamp of candle-power A, and let f denote the fraction of the incident light it transmits. To restore balance the other lamp must be moved further away from the photometer. Let its new distance from the latter be c . Then

$$\frac{fA}{a^2} = \frac{B}{c^2}$$

Dividing this equation by the preceding one,

$$\frac{fA}{a^2} \bigg/ \frac{A}{a^2} = \frac{B}{c^2} \bigg/ \frac{B}{b^2}$$

Hence

$$f = \frac{b^2}{c^2}$$

EXAMPLE.—Two lamps are balanced at distances of 50 and 70 cm. from a shadow photometer. A plate of glass is placed between the

stronger lamp and the screen, and it is found that the other lamp must be moved 2.5 cm. away from the screen in order to restore balance. How much light is transmitted by the glass plate ?

By formula
$$f = \frac{b^2}{c^2} = \frac{50^2}{52.5^2} = 0.907$$

Variation of the Illumination of a Surface with the Angle of Incidence of the Light.—The angle of incidence is the angle made by the incident rays with the normal to the surface. Let BC be a small rectangular screen on which a parallel pencil of rays is falling. Let PN be the normal to the mid-point of the screen, and let $\angle SPN = \theta$. Let Q be the quantity of light received by the area BC per second. Then the illumination of the surface is given by

$$I = \frac{Q}{\text{area BC}}$$

Let us suppose now that the screen is rotated about its mid-point P into a position at right angles to the rays. The quantity of light Q now falls upon the area B'C' and the illumination of the surface, which we shall now denote by I_0 , is given by

$$I_0 = \frac{Q}{\text{area B'C'}}$$

Hence

$$I = \frac{Q}{\text{area BC}}$$

or

$$I = I_0 \cos \theta$$

since $\text{area B'C'} = \text{area BC} \cos \angle SPN = \text{area BC} \cos \theta$.

Experiments in Photometry.—It is an easy experiment to compare the intensities of two lamps or to determine the fraction of light transmitted through a glass plate. An optical laboratory is not necessary; the measurements can be made in any dwelling-room at night. In determining the fraction of light transmitted by a plate, it

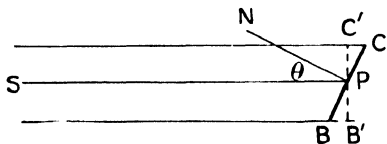


FIG. 279.

is advisable to set up, if possible, three pieces together. The formula then becomes

$$f = \frac{b^2}{c^2}$$

and, since the effect to be measured is greater, the result obtained is more accurate.

The candle-power of carbon filament lamps is usually marked on the bulb; thus 250 16 means 250 volts 16 candle-power. In the case

of metal filament lamps it is generally the rate of consumption of energy that is given; thus 250 40 means 250 volts 40 watts. But with such lamps the candle-power is about $\frac{1}{5}$ of the watts, *i.e.* a 40-watt lamp gives 32 candle-power and a 20-watt lamp 16 candle-power. If a batch of new lamps which are described by the makers as having the same candle-power is taken and tested, it will be found that there is considerable variation between the individual lamps; also the candle-power changes with the age of the lamp. But for rough work a lamp which has the average intensity of the group may be taken as a standard.

The ordinary penny household candle, which has a length of about $7\frac{1}{2}$ in. and a diameter of about $2\frac{1}{2}$ in. and which burns about 8.6 grams per hour, gives, I have found, about 1.1 or 1.2 candle-power. It may also be used as a standard for rough work.

The following are given as examples of experiments that may be performed in photometry:—

To measure the Candle-power of a Lamp in Position.—By means of a piece of string measure the distance from the lamp to a point on the wall of the room. Pin a piece of white paper on the wall at this point. Turn out the other lights, if any, and holding a candle in one hand and a pencil in the other, adjust the distances so that the two shadows touch, and are equally dark. The light from the candle must fall on the paper at the same angle as the light from the lamp. Get a friend to measure the distance of the candle in the critical position. Then the intensities of lamp and candle are inversely as the squares of their distances from the paper.

To measure the Illumination of the Surface of a Desk. We shall suppose at first that there is only one lamp in the room, that it has 32 candle-power, and that it is at a distance of 2 metres from the surface of the desk in a direction making an angle of 45° with the normal to the surface of the latter. Then the result may be obtained from the formula $I_0 \cos \theta$. Here $I_0 = 8$ metre-candles, and θ is 45° . Hence the result is 5.7 metre-candles. If there is more than one lamp, their effects should be calculated separately and the results added.

If there is only one lamp, the determination may be made experimentally with a candle flame and a pencil by arranging that the two shadows touch and are equally dark. Some dexterity is required, as the candle must be tilted for an instant so that the flame may be unobstructed, and if it is tilted for too long, its intensity will alter.

From 10 to 40 metre-candles is supposed to be sufficient for office desks, schoolrooms, etc., though I think a desk lamp should give as much as 150 metre-candles on the desk. The illumination of an object in ordinary daylight is, roughly, 10,000 metre-candles.

Test of a Pocket Flash Lamp.—These lamps have usually a bulb that takes 3.5 volts, and is run off three dry cells mounted in series which give, when fresh, about 4.5 volts. The cells, however, rapidly polarise, and their voltage falls off; consequently the light given by the lamp diminishes.

A pocket flash lamp fitted with a lens was directed towards a screen at a distance of 160 cm., which was placed to receive the rays at right angles, and a stick was set up to cast a shadow on the screen at the

point where the beam was brightest. A candle was arranged to cast an adjacent shadow, and its distance adjusted at intervals of 12 min. to make the shadows equally dark. The lamp was, of course, kept running the whole time of the test. The following table gives the readings and results; the first row gives the time, the second the distance of the candle from the screen, and the third the intensity of the lamp in terms of the candle employed:

0	12	24	36	48	60	72 min.
70	84	100	116	130	140	170 cm.
5.22	3.63	2.56	1.90	1.51	1.31	0.89 candles

Thus in 72 min. the candle-power of the lamp fell almost to one-sixth of its initial value.

Illumination of Rooms.—The illumination given by a window can sometimes be measured by a wedge photometer and a candle at the opposite side of the room. The candle has to be held close up to the wedge on the opposite side of it from the window. The difficulty here is that the wall behind the candle reflects light from the window on to the same face of the wedge as the candle does. It may be necessary to hang a black cloth or piece of black paper on the wall to eliminate this error. In any case the results are never very accurate. But the illumination of a room varies so enormously from time to time that even rough results may have great value.

EXAMPLES I

1. In a pinhole camera the distance from the hole to the screen is 6 inches. The camera is pointed towards a tree 20 ft. high at a distance of 50 ft. and is on the same level as a point halfway up the tree. Find the height of the image.

2. A circular uniform source of light, 10 cm. in diameter, is placed 50 cm. in front of an opaque sphere 5 cm. in diameter. Find the distance from the centre of the latter to the end of the umbra; also find the diameter of the penumbra cast on a screen placed at this distance.

3. If the light of the sun is admitted through a small hole, an image of the sun is formed on a screen placed to receive it, but if the aperture is a large one, we obtain an image of the aperture. Explain this.

4. A stick is placed in front of a screen, and an electric glow-lamp and standard candle throw equally dark shadows of the stick on the screen. The candle and lamp are respectively 34 and 134 cm. from the screen. Find the candle-power of the lamp.

5. How many candles are required to produce at a distance of 420 cm. the same illumination as a single candle at a distance of 60 cm.?

6. A 30 candle-power and a 16 candle-power electric glow-lamp are at the same height, 120 cm. apart. At what points on their line of centres do they give the same illumination?

7. The intensities of two glow-lamps are compared by the grease-spot photometer, and balance is obtained when the photometer is 83 cm. from the one lamp and 53 cm. from the other. The lamps are then interchanged in their sockets, and balance is obtained when the distances are 50 and 77 cm. What is the ratio of the intensities of the lamps?

8. It is found that the light of the full moon falling perpendicularly on a screen gives the same illumination as a standard candle at a distance of 4 ft. Find the candle-power of the moon, given that its distance is 60 times the earth's radius. The earth's radius is 4000 miles.

9. Two lamps are balanced at distances of 57 and 51 cm. from a wedge photometer. A large plate of clear glass, which transmits 90 per cent. of the light incident on it, is then placed midway between the photometer and the stronger lamp. How much nearer must the latter be moved to restore balance?

10. A candle is just visible to the unaided eye on a clear dark night at a distance of 3.1 km. in the open. The diameter of the pupil is then $\frac{1}{4}$ inch. At what distance should the candle be visible, when field glasses are used, the object glasses of which are $\frac{7}{8}$ in. in diameter? Assume that $\frac{1}{4}$ of the light incident on the object glasses enters the eye, the remainder being lost inside the instrument.

11. A small screen is held 4 ft. from a candle flame, in such a position that the light is incident on it normally. It is then removed to a distance of 8 ft. and turned, so that the light is incident on its surface at an angle of 45° . To what fraction of its original value does the illumination of the screen diminish?

12. The shadow which causes an eclipse is, geometrically speaking, a solid cone with its axis in the line joining the centre of the sun and the shadow-casting body (cf. Fig. 272). If the sun is 93,000,000 miles from the earth, and subtends an angle of $32'$ at the earth, find how far the earth's shadow extends from the centre of the earth. Find also the diameter of the earth's shadow at the point where the moon crosses it. The distance of the moon is 60 times the earth's radius.

13. The angle subtended by the moon's diameter at the earth is $31'$. The time taken by the moon to go round the earth is 29 days 13 hours. Hence calculate its hourly motion in minutes of arc, and find, if an eclipse of the moon is central, how long it is total, and also what time elapses between the first and last contacts of the shadow on the moon's disc. The penumbra is to be neglected altogether; it does not affect the brightness of the moon's surface much.

CHAPTER II

REFLECTION

Laws of Reflection.—When light falls on a piece of white paper, it is reflected in all directions. Such reflection is called **diffuse** reflection, and it takes place at all rough surfaces. The rays enter some distance into the paper before being reflected, and if the latter is coloured, are themselves coloured after reflection. A scraped surface of plaster of Paris forms an ideal diffuse reflector.

When a narrow pencil of light is reflected from the polished surface of a transparent medium such as glass, diffuse reflection takes place to only a very small extent, and the reflected light comes off in one particular direction. It is then said to undergo **regular** reflection. The point where the incident ray of light strikes the surface is called the point of incidence, and if a normal is drawn to the surface at this point, the angle which the incident ray makes with it is called the angle of incidence, and the angle which the reflected ray makes with it is called the angle of reflection.

The laws of reflection are as follows :—

The incident ray, the normal to the reflecting surface at the point of incidence, and the reflected ray are all in the same plane.

The angle of reflection is equal to the angle of incidence.

Thus, in Fig. 280, if P is the point of incidence, MK the trace of the reflecting surface, NP the normal, AP the incident ray, and

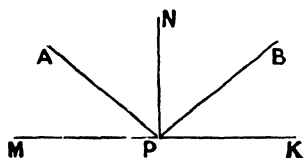


FIG. 280.

PB the reflected ray, AP, PN, and PB are in the same plane, and $\angle NPB = \angle NPA$. When the angle of incidence is zero, the ray is reflected back on its path.

The distinction between diffusely reflecting surfaces and regularly reflecting surfaces cannot be drawn very sharply; both kinds of reflection may be present at the same time to a considerable extent. For example, in the case of a glazed tile light is reflected in all directions, but there is a marked preponderance in the direction of regular reflection. It is diffusely reflected light that renders bodies

visible. A perfect mirror would be invisible; we would see, not the surface, but the objects mirrored in the surface.

Reversibility of the Ray.—If, in Fig. 280, BP is regarded as the incident ray, PA is the reflected ray. Thus Fig. 280 offers an example of what is known as the *reversibility of the ray*, a principle which may be stated as follows: If by any means light is able to travel from a source at A to a point B, then, if the source is placed at B, light will travel back to the point A by the same path.

Image of a Point. Real Image. Virtual Image.—Let MN be the section of a plane mirror perpendicular to the plane of the paper, and let P be a point source of light. Let PA be any ray from P to the mirror; then, after reflection, it has the direction AC. Draw EA, the normal at A, draw PN perpendicular to MN, and produce CA and PN to meet at Q.

In $\triangle s$ APN and AQN we have AN common, $\angle ANP = \angle ANQ$ both being right, and $\angle PAN = \angle QAN$, since $\angle EAN$ is right, and EA bisects $\angle CAP$. Hence the triangles are equal and $PN = NQ$.

The point Q is fixed and quite independent of the direction of PA. If we draw another ray PB, BD its direction after reflection must also pass through Q. All rays diverging from P and striking the mirror appear, therefore, to an eye at CD to diverge from Q. The point Q is thus said to be the image of P.

Images may be either real or virtual. They are defined as follows:—

(1) When a pencil of rays diverging from a point is made to converge to a second point, that second point is called a **real image** of the first point.

(2) When a pencil of rays diverging from a point is made to appear to diverge from a second point, that second point is called a **virtual image** of the first point.

In Fig. 281 the point Q is a virtual image. The essential difference between a real image and a virtual image is that a real image can be received upon a screen and made visible; a virtual image cannot. The images formed by a pinhole camera are real.

EXAMPLE.—An observer stands 20 ft. from the shore of a lake, and sees the top of a mountain 2000 ft. high mirrored in the water at the lake's edge. If his eyes are 7 ft. above the level of the water, what is the distance of the mountain?

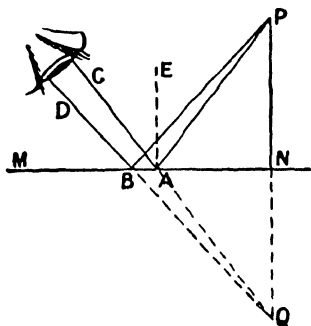


FIG. 281. Image of a point.

Let x ft. be the distance from the edge. Then by similar triangles

$$\frac{x}{2000} = \frac{20}{7}, \quad x = 571\frac{1}{4} \text{ ft.}$$

Image of a Line.—If instead of a luminous point we have a line object PR , an image is formed of every point of it as above, and these point images combine to form the line image QS as shown in Fig. 282. It will be noticed that the object is reversed by reflection, that right and left are interchanged. Consequently, if a piece of writing is held up to a mirror, its image is illegible; if, however, it is blotted, and the blotting paper held up to the mirror, the writing, being reversed twice, becomes legible again.

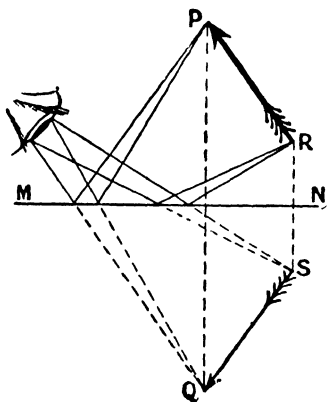


FIG. 282.—Image of a line.

For an image to be formed by reflection it is not necessary for the normals from the object or the eye to meet the reflecting surface itself. It is necessary only for the object and eye to be in front of the plane of the reflecting surface and for the rays to meet the latter.

Experiments on Reflection.—The best proof of the laws of reflection lies in the fact that their truth is assumed in measurements made with the spectrometer (p. 380) and sextant (p. 371), and that the assumption has never led to inaccurate results. They can, however, be proved very simply with a drawing-board, pins, and a rectangular glass slab.

One face of the glass slab is used as reflecting surface. This face should be polished, should be about three-quarters of an inch high, and should stand at right angles to the surface of the drawing-board. The opposite face of the slab should be covered with black paint to obscure the image formed by internal reflection, which may be a source of error.

Instead of the slab, we may employ a piece of thin mirror glass mounted on a wooden block. In this case the image is formed by reflection at the silvered surface on the back of the mirror, and the thickness of the mirror causes error; this may be disregarded if the mirror is a thin one. The most suitable mirror to employ from the theoretical point of view would be one silvered on the front, but such mirrors tarnish, and are not made unless to a special order.

Until about 1840 glass mirrors were backed with an amalgam of mercury and tin. This process has, however, been almost entirely superseded by the silver process. Silver is deposited on the glass from a solution; when the surface is dry, it is brushed over with a dilute solution of mercury cyanide, and then coated with red lead.

(1) Fix a piece of paper on a drawing-board and draw any straight line MPK on it. Place the mirror in position so that it rests along MK , and stick two pins in front of it in positions such as Q and R . Look into the mirror from the right, using one eye, and R and Q will be seen by reflection in the direction PR' ; then place two pins, T and S , in line with the images of R and Q . Remove the mirror and join QR and ST ; they should meet on the surface of the mirror at P . Draw the normal PN , and show with a pair of compasses in the manner indicated in the diagram, that $\angle QPN = \angle NPT$.

When the heads of the pins are in line, it can be shown that they lie in a plane perpendicular to the surface of the mirror. Thus the first law of reflection is proved.

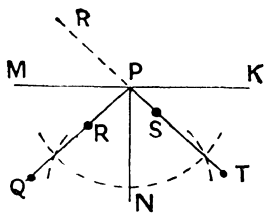


FIG. 283.

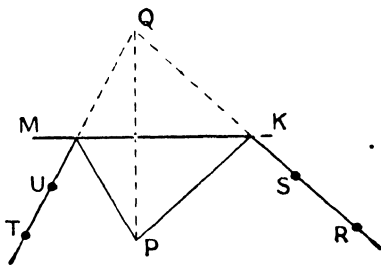


FIG. 284.

(2) Draw a straight line MK , and set the mirror along this line. Place a pin, P , in front of the mirror and, looking into the mirror from the right and using one eye, place two pins, R and S , in line with the image of P . In the same way look into the mirror from the left, and place the pins T and U in line with the image of P . Remove the mirror, join TU and RS , and produce them to meet in Q ; Q is the image of P . Join PQ and show by trial that PQ is bisected by MK and is at right angles to MK .

(3) The experiments represented in Figs. 283 and 284 may be performed in a slightly different manner. Instead of sticking pins in the board at Q and R , as shown in Fig. 283, a line QR may be drawn on the paper. The edge of the ruler is then moved about the paper until it appears in line with the reflection of QR in the glass, and a pencil drawn along the edge of the ruler; thus the line ST is determined. Similarly, instead of sticking pins in the board at R , S , T , and U , as shown in Fig. 284, the position of the image at Q may be obtained by getting the edge of the ruler in line with it in two different positions.

(4) Place a pin P in front of the mirror, and place a longer pin, Q , behind the mirror in line with P and the image of P . Then the upper part of the pin Q will be seen over the top of the mirror. Move the head from side to side; if the long pin is not exactly at the image of P , there will be parallax,

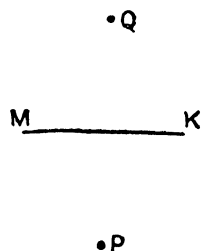
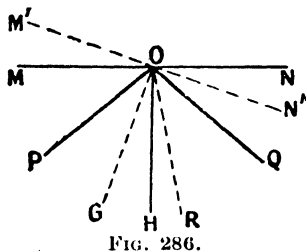


FIG. 285.

i.e. relative motion, between it and the image of *P*. The amount of parallax varies with the distance of *Q* from the position of the image, and by moving *Q* to or from the mirror, so that parallax disappears, the position of the image of *P* may be found.

Rotation of a Plane Mirror.—If a ray of light falls on a mirror and the mirror is rotated through an angle, the reflected ray is turned through twice the angle turned through by the mirror. Let *MN* be the intersection by the paper of a plane mirror perpendicular to the plane of the paper. Let *PO* be the incident ray, *OQ* the reflected ray, and *OII* the normal at the point of incidence. Let the mirror be rotated through an angle *M'OM* about an axis through *O* perpendicular to the plane of the paper, and let *OR* and *OG* be the positions of the reflected ray and normal after the rotation. Then



$$\begin{aligned}\angle M'OM &= \angle GOH = \angle POH - \angle POG = \frac{1}{2} \angle POQ \\ &\quad - \frac{1}{2} \angle POR = \frac{1}{2} (\angle POQ - \angle POR) = \frac{1}{2} \angle QOR\end{aligned}$$

But *QOR* is the angle turned through by the reflected ray; this proves the theorem. It can easily be verified by pins on a drawing-board.

Applications.—The foregoing theorem is the principle of the sextant (p. 371); it is also employed in the measurement of small angles. Suppose, for example, it is desired to measure the rotation of a small magnetic needle. We might attach a long pointer to the needle, and arrange that the end of this pointer moves over a graduated scale. But the pointer would add to the weight of the needle, increase its resistance to motion, and so diminish the size of the angle turned through; and if it were thin and light, it might bend. If, however, a ray of light is used as pointer, we get over these difficulties, for it has no weight and it does not bend.

The ray of light may be used in either of two different methods. We may attach a mirror to the magnet and by its means reflect a pencil of light from a stationary lamp on to a translucent scale. When the mirror rotates, the ray rotates, and the light moves along the scale. This method is carried out in a darkened room.

Or we may direct a telescope towards the mirror and focus it on the scale, as is shown in plan in Fig. 287. In this figure *T* is the telescope, *SS* the scale, and *M* the mirror; the latter is in a deflected position. Since the light rays from *P* are reflected

by the mirror to enter the telescope, the eye looking into the latter sees the scale reading at P in the centre of the field. If the mirror slowly rotates into a position at right angles to the axis of the telescope, the different numbers on the scale between P and T cross the centre of the field in succession, and from the scale reading at the centre of the field at any time the deflection at that time can be calculated. The above method is used in daylight.

The distance TP measured along the scale in Fig. 287 is proportional to $\tan \angle TMP$, since TM is a fixed distance; also $\angle TMP$ is twice the angle of deflection of the mirror. Hence TP is proportional to the tangent of twice the angle of deflection of the mirror. But the angles worked with are usually so small that the tangent can be put equal to the angle itself in circular measure; hence it is customary to take displacement on the scale as simply proportional to change of angle itself.

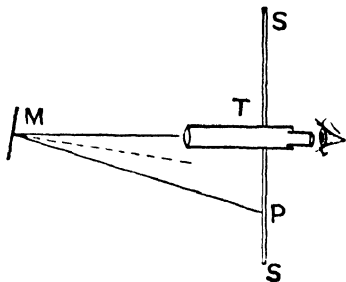


FIG. 287.

EXAMPLE.—In Fig. 287 the distance of the scale from the mirror is 1 metre, and P is initially 10 cm. from the point on the scale opposite the centre of the mirror. During a deflection it moves 20 cm. further away from this point. Find in radian measure the angle turned through by the mirror M: (i) approximately, (ii) accurately.

- (i) $\frac{1}{2} \frac{20}{100} = 0.1000$ rad.
 (ii) $\frac{1}{2} (\tan^{-1} \frac{10}{100} - \tan^{-1} \frac{10}{100}) = \frac{1}{2} (16^\circ 42' - 5^\circ 43') = 0.0958$ rad.

Pepper's Ghost.—At night, if the blinds are not pulled down, objects inside a room can be seen reflected in the window pane. The objects outside are then too dark to send rays through the pane to the eye. If the light is had outside, we see outside objects with some of the brighter objects inside the room superimposed on them; for example, the fire in the grate may be seen burning outside in the street. But in broad daylight the outside objects are so much brighter, and the rays they send to the eye so much more powerful, that the rays reflected by the pane from objects inside the room can be neglected in comparison with them, and only outside objects are seen. Thus the relative visibility of the objects reflected and the objects transmitted by a plate of glass is purely a question of lighting.

It was by the use of a glass plate and alteration of the lighting, that the old illusion known as Pepper's ghost was produced. Professor Pepper was an illusionist who travelled about the country exhibiting the ghost. The same arrangement is sometimes used on the stage

for producing ghosts at the present time. Fig. 288 is a plan showing how it is done in this case.

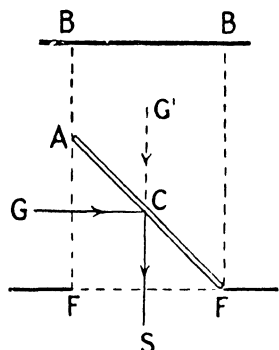


FIG. 288.—Explaining Pepper's ghost.

FF are the footlights, BB the back of the stage, and AF is a large plane sheet of glass which is placed vertical and at an angle of 45° with the line of vision of the spectators. S is a solitary spectator seated in the middle of the stalls. G is the actor who is to fill the rôle of ghost. He is so far to the side that S cannot see him directly, but only by reflection in the glass; the rays of light from G to S traverse the path GCS, but appear to traverse the path G'CS. The background is dim, and the actor G is at first kept dark, but when his time comes, the arc light is turned on him, and he duly "appears" at G'. His image is, of course, transparent; bright parts of the background can be seen through it. The disadvantage of the arrangement is that it requires a deep

stage and a very large glass plate.

Parallel Mirrors.—Let us suppose that a point source is placed between two mirrors A and B which have their faces turned towards one another, so that its distance from A is a and its distance from B is b . Then by reflection in A an image is formed at A_1 . This image is in front of the mirror B, so acts itself as object, and by reflection in B an image is formed at A_2 . By reflection again in A this in turn gives rise to an image at A_3 , and so on. Similarly by considering the reflection of P in B we

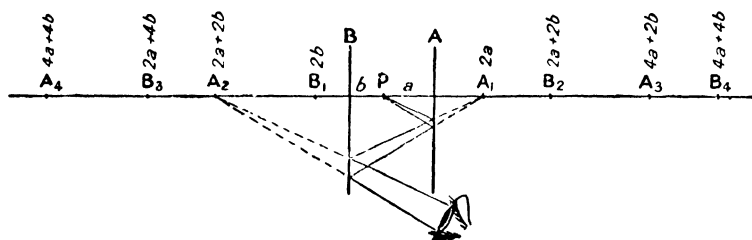


FIG. 289.—Images formed by parallel mirrors.

get another infinite series of images $B_1, B_2, B_3, \dots$. There are thus two infinite series of images, but, of course, owing to the light getting fainter by successive reflection, it is only the first few of each series that are seen. Above each image in the diagram is given its distance from P. The diagram also shows the path taken to the eye by the pencil of rays which forms one of the images, A_2 .

Inclined Mirrors.—Let us consider first Fig. 290. P is a point source of light and OA is a mirror which has its reflecting surface turned towards P . In order to find the position of the image of

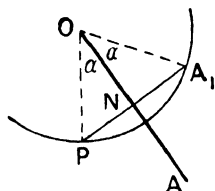


FIG. 290.

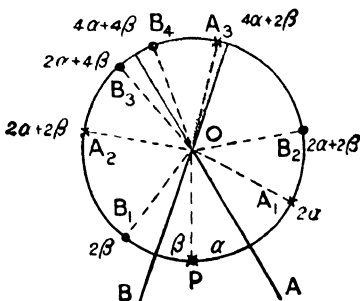


FIG. 291.—Images formed by inclined mirrors.

P draw a normal PN to the mirror, produce it behind the mirror, and mark off $NA_1 = NP$; A_1 is the image. It may be shown that triangles PNO and A_1NO are equal. Consequently, if the point O is taken as centre, and an arc described with OP as radius, it passes through A_1 , and if the angle PON is denoted by α , the point and its image are each distant the same angle α from the surface of the mirror measured around the circle.

In Fig. 291 AO and BO represent two mirrors with their reflecting surfaces turned towards one another, and P is an object placed between them. Draw a circle with O as centre and OP as

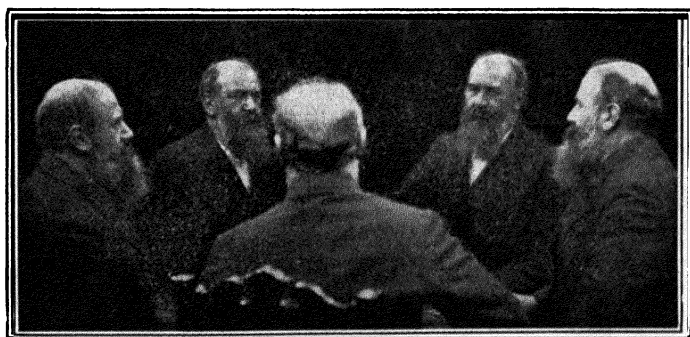


FIG. 292.—Images formed by two inclined mirrors.

radius, and let the angles AOP , BOP be respectively α and β in radian measure. According to the construction of Fig. 290, P forms an image A_1 by reflection in OA , which is situated on the

circle at an angular distance 2α from P. Since A_1 is in front of OB, it forms an image A_2 in it. This image is also on the circle and is at an angular distance of $2\alpha + 2\beta$ from P. A_2 in turn forms an image A_3 in OA distant angle $4\alpha + 2\beta$ from P. A_3 is behind both mirrors, and consequently forms no more images.

If we go back to P it forms an image B_1 in OB, which in its turn forms an image B_2 in OA. The latter forms an image B_3 in OB, which forms an image B_4 in OA; this image is behind both mirrors and consequently no further images are formed. There are seven images in all. Their angular distances are marked on the figure, and it will be seen that the differences are alternately equal to 2α and 2β . A_1, B_2, A_3 , and B_4 ,

it should be noted, are measured one way, B_1, A_2 , and B_3 the other way round the circle.

Fig. 292 is an example of the effects that can be obtained with inclined mirrors. Fig. 293 shows how the rays reach the eye for the case of one of the four images, namely the second on the left.

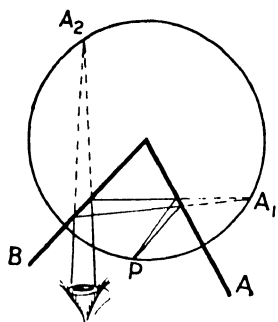


FIG. 293.

EXAMPLE.—An object is placed between two mirrors OA and OB as in Fig. 291, the angles which OP makes with OA and OB being respectively denoted by α and β . Find the number of images formed.

Constructing a figure similar to 291, but with angle $\alpha + \beta$ much smaller, we find that the angles $PA_1, PA_2, PA_3, \dots$ have the values $2\alpha, 2\alpha + 2\beta, 4\alpha + 2\beta, \dots$, and consequently that PA_{2n} is $2n(\alpha + \beta)$, where n is any integer. The number of images that can be formed in this way is limited, the last number of each series being that which is formed within the vertically opposite angle to AOB.

If A_{2n} is the first image to fall within this angle, $PA_{2n} > \pi - \alpha$. $2n(\alpha + \beta) > \pi - \alpha$, and

$$2n > \frac{\pi - \alpha}{\alpha + \beta}$$

Similarly, if A_{2n+1} is the first image to fall within the angle,

$$2n(\alpha + \beta) + 2\alpha > \pi - \beta, 2n(\alpha + \beta) + \alpha + \beta > \pi - \alpha, (2n + 1)(\alpha + \beta) > \pi - \alpha$$

and

$$2n + 1 > \frac{\pi - \alpha}{\alpha + \beta}$$

Hence in both cases the number of images in the series $A_1, A_2, A_3, \dots$ is given by the integer next greater than

$$\frac{\pi - \alpha}{\alpha + \beta}$$

Similarly, the number in the series $B_1, B_2, B_3, \dots$ is given by the integer next greater than

$$\frac{\pi - \beta}{\alpha + \beta}$$

If $\alpha + \beta$ is an exact sub-multiple of π , $\pi/(\alpha + \beta)$ is an integer, and is the integer next greater than either of the two expressions above; hence it gives the number of images in each series. In this case the positions of the last image of each series coincide. For, suppose there are $2n$ images in each series, then

$$PA_{2n} + PB_{2n} = 2n(\alpha + \beta) + 2n(\alpha + \beta) = 4n(\alpha + \beta)$$

But $2n = \frac{\pi}{\alpha + \beta}$, therefore $2n(\alpha + \beta) = \pi$

Hence $PA_{2n} + PB_{2n} = 2\pi$, and A_{2n} and B_{2n} coincide.

Again, if there are $2n + 1$ images in each series,

$$PA_{2n+1} + PB_{2n+1} = 2n(\alpha + \beta) + 2\alpha + 2n(\alpha + \beta) + 2\beta = 2(2n + 1)(\alpha + \beta)$$

But $2n + 1 = \frac{\pi}{\alpha + \beta}$; therefore $(2n + 1)(\alpha + \beta) = \pi$.

Hence $PA_{2n+1} + PB_{2n+1} = 2\pi$, and we have coincidence again.

EXPERIMENT.—The positions of the images should be determined by the parallax method on the drawing-board, both for parallel mirrors and different positions of the inclined mirrors, and the results compared with those obtained by theory.

EXAMPLES II

1. On a moonlight night when the surface of the sea is covered with small ripples, instead of an image of the moon a band of light is seen on the surface extending in the direction of the moon. Explain in a diagram how this happens.

2. If a man walks towards a mirror at 3 miles an hour, with what velocity does he approach his image?

3. The sun is 25° above the horizon. Its image is observed in a pool of water. What is the angle of incidence and reflection in this case?

4. An observer wishes to see the side of his head with the aid of two mirrors. Make a plan showing the course of the rays.

5. A man who is 6 ft. high stands in front of a plane mirror which is hung vertically. The top of the mirror is 6 ft. from the ground. Determine its shortest length in order that the man may see his full-length image in it.

6. Two plane mirrors are placed vertically upon a table. A ray of light is reflected by both mirrors in succession. Show that the angle through which it is deviated by the double reflection is equal to twice the angle between the mirrors.

7. Light from a 16 candle-power lamp falls on a silvered mirror and is reflected thence to a shadow photometer, all direct light from the lamp to the photometer being carefully screened off. The mirror reflects 90 per cent. of the incident light, and the length of the path of the rays from the lamp to the screen via the mirror is 120 cm. Where must an 8 candle-power lamp be placed to produce an equally dark shadow?

8. Two parallel mirrors are 10 ft. apart, and a candle is placed between them 7 ft. from one mirror. Find the distance between the third and fourth images seen in each mirror.

9. Two mirrors are inclined to each other at 90° , and an object placed between them. Draw a diagram showing the positions of the images formed.

10. Two mirrors are inclined to one another at an angle of 60° , and an object is placed between them. Draw a diagram showing the positions of all the images formed. Show how the rays reach the eye from the image formed in the angle vertically opposite that contained by the mirror.

11. A ray of light starts from a point A, is reflected by a plane mirror at a point P, and arrives at a point B. Prove that APB is the shortest possible path from A to B by way of the mirror.

CHAPTER III

REFRACTION

Laws of Refraction.—Let us suppose that a trough with parallel sides of glass is filled with water, and that a beam of parallel light AB from a projection lantern in a darkened room is incident on the surface of the water at B. By dusting chalk in the air the rays may be rendered visible, and it will be noticed that the surface of the water acts as a mirror, and that a reflected beam comes off in the direction BC, as required by the laws of reflection. At the same time a beam BD will be seen below the surface of the water. This beam is called the **refracted beam**, because on entering the water the rays are bent or refracted out of the direction AB into the direction BD; it is rendered visible by the small particles

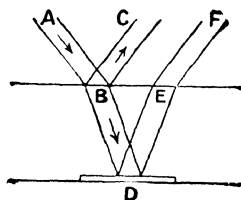


FIG. 294.

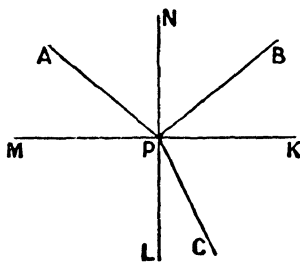


FIG. 295.

floating in the water which scatter light out through the glass side of the trough to the eye, each particle acting as a separate source. If a piece of mirror glass is placed at the bottom of the trough at D, the refracted beam is reflected, and proceeds upwards to the surface of the water where it is refracted into the air. The beams EF and BC are parallel, and EF is brighter than BC.

We obtain similar results with other liquids and with a glass slab. Whenever a narrow pencil of light (Fig. 295) falls on the surface of a transparent medium, it gives rise to two pencils, a reflected one and a refracted one. If the normal is produced

downwards into the transparent medium, the angle which it makes with the refracted ray is called the angle of refraction.

The laws of refraction are as follows :—

The refracted ray lies in the same plane as the normal and the incident ray, and is on the opposite side of the normal from the incident ray.

The sine of the angle of incidence bears a constant ratio to the sine of the angle of refraction for all angles of incidence, the value of the ratio depending on the nature of the media in contact at the surface at which refraction takes place.

Thus, in Fig. 295, if P is the point of incidence, MK the trace of the refracting surface, NP the normal and AP the incident ray, the refracted ray PC is in the same plane as AP and PN and $\sin \text{NPA} / \sin \text{LPC}$ is constant, no matter what the value of NPA is.

Write

$$\frac{\sin \text{NPA}}{\sin \text{LPC}} = \mu$$

Then μ is said to be the **index of refraction** or **refractive index** of the lower medium with reference to the upper, or simply the index of refraction of the lower medium. For all transparent solids and liquids μ is greater than 1. Its value for water is 1.333 and for crown glass is about 1.52, but varies with the glass; in approximate calculations these values are usually taken as $\frac{4}{3}$ and $\frac{3}{2}$.

When the angle of incidence is zero, AP and PB both coincide with NP, and PC coincides with PL. The ray is then undeviated by refraction.

EXPERIMENT.—The best proof of the laws of refraction lies in the fact that they are assumed every time an index of refraction is determined by the spectrometer (p. 380), and there never has been any reason to doubt their accuracy. They can, however, be proved very simply with a slab of glass on the drawing-board.

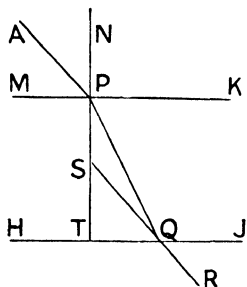


FIG. 296.

Draw a straight line AP (Fig. 296), and place the slab in position, so that MK and HJ are the traces of two of its parallel faces. Draw MK and HJ. Then, looking into the face HJ of the slab, set the edge of a ruler in line with AP as seen through the glass, and draw part of the line QR. Remove the slab, draw the normal NPS, produce RQ to meet the normal at S, and join PQ. The ray which was incident along AP emerges along QR, and consequently follows the path PQ inside the slab. Thus $\angle \text{SPQ}$ is the angle of refraction.

The ratio of the sines can now be determined in different ways, but perhaps the neatest is as follows : It is found from the drawing that SR is parallel to AP. Consequently $\angle APN = \angle TSQ$. Hence

$$\mu = \frac{\sin APN}{\sin TPQ} = \frac{\sin TSQ}{\sin TPQ} = \frac{TQ/SQ}{TQ/PQ} = \frac{PQ}{SQ}$$

No matter how $\angle APN$ varies, the ratio PQ/SQ is always constant.

Instead of drawing a line AP and sighting on it with the edge of a ruler, two pins may be set up in AP, and two pins set in line with them on the other side of the glass.

Twilight.—In the first experiment described in this chapter, a beam of light was rendered visible by reflection from chalk particles in the air or from particles floating in water. Twilight is caused in exactly the same way by the reflection of sunlight from minute solid particles in the air, or from the air itself—authorities differ—in the upper regions of the earth's atmosphere. After the sun has set, its rays continue to shine through the atmosphere above the observer's head, and are reflected down to the earth's surface. Twilight is considered to end when this scattered light becomes imperceptible.

The Coin and Basin.—The law of reflection must have been known from very early times. It is proved in the first proposition of Euclid's *Catoptrics*, a work composed at Alexandria about 300 B.C. But the subject of refraction was not understood until much later.

Fig. 297 represents an experiment which attracted a considerable amount of attention in ancient times. C is a coin resting at the bottom of an empty basin, E an eye which cannot see the coin

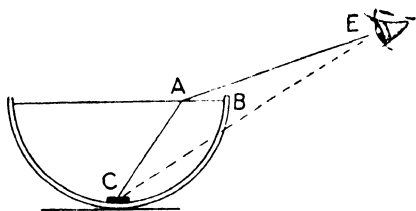


FIG. 297.—Coin and basin.

owing to the edge of the basin at B coming between. But when the basin is filled to the top with water, the rays from the coin to the eye are refracted, and follow the path CAE; hence the coin becomes visible. The experiment was first successfully explained about A.D. 100 by Ptolemy, the Alexandrian astronomer, in his book on optics.

Ptolemy's Data on Refraction.—Ptolemy published a table giving the angles of refraction corresponding to different angles of incidence for water, for glass, and for the case when the one medium was water and the other medium was glass.

In order to measure the refraction from air to water, Ptolemy employed a graduated circle divided into 360° , which was immersed vertically in water up to the horizontal diameter. There were three objects for sighting on, one fixed at the centre of the circle, a second fitted to one of the upper quadrants, and a third fitted to the lower part

of the circle. This last was pushed with a rod, until an eye placed at the object in air saw all three in a straight line. In measuring the refraction from air to glass Ptolemy used a semi-cylinder of glass, and adjusted its plane surface to coincide with a diameter of the graduated circle which was held against its end. In measuring the refraction from water to glass the semi-cylinder was placed with its plane surface on the surface of the water, and the circle was held with its horizontal diameter in the plane of separation of the two media. The following table gives the results; they are of interest as being the earliest physical measurements on record :—

Air to water.		Air to glass.		Water to glass.	
Angle of incidence.	Angle of refraction.	Angle of incidence.	Angle of refraction.	Angle of incidence	Angle of refraction.
0°	0°	0°	0°	0°	0°
10	8	10	7	10	9½
20	15½	20	13½	20	18½
30	22½	30	20½	30	27
40	28	40	25	40	35
50	35	50	30	50	42½
60	40½	60	34½	60	49½
70	45	70	38½	70	56
80	50	80	42	80	62

Ptolemy did not discover the law of refraction. The latter was first stated in its present form by Descartes in his *Dioptrics* in 1637, but it is generally assumed to have been obtained by Snell about 1621, although he never published his discovery.

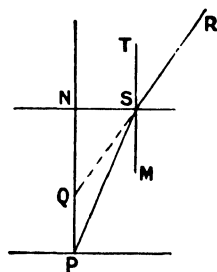


FIG. 298.

Image of a Point formed by Refraction at a Plane Surface.

Let us suppose that a pencil of rays is diverging from a point P on the under-surface of a parallel-sided slab of some refracting material, for example, glass. The ray PN, which meets the upper surface of the slab at right angles, does not change its direction in passing into the air, but any other ray, PS, is refracted at the surface of separation. Let SR be its direction after refraction,

and let μ be the index of refraction of the glass. Let RS meet PN in Q. Then

$$\mu = \frac{\sin TSR}{\sin PSM} = \frac{\sin NQS}{\sin NPS} = \frac{NS/SQ}{NS/SP} = \frac{SP}{SQ}$$

If the angle NPS is small, $SP/SQ = NP/NQ$; consequently, $\mu NQ = NP$, and the point Q is fixed. Thus, if a thin pencil

from P be incident at right angles on the upper surface of the glass, it appears after refraction to come from Q, *i.e.* a virtual image of P is formed at Q.

Fig. 299 shows the same thing in another manner. E is an eye looking down at a point P at the bottom of a pool of water. The point P is apparently raised to Q, and the pool seems shallower than it really is. The real depth is NP and the apparent depth NQ. These are connected by the relation $NP = \mu NQ$. Thus

$$\mu = \frac{\text{real depth}}{\text{apparent depth}}$$

For water μ has the value $\frac{4}{3}$. Hence, if the depth of the pool is one foot, it appears only nine inches. Most boys find sooner or later, when they step off a stone into the water, that it is always deeper than it seems. For a sharp image to be formed by refraction, as in Fig. 299, it is necessary that the pencil should be a narrow one, whereas sharp images are formed by reflection at mirrors, no matter how wide the pencil is. This does not make any real difference in practice, because, owing to the small size of the eye pupil, the pencil of light entering it is necessarily a narrow one.

If a stick is placed obliquely in the water and looked at from above, it appears bent where it enters the water, the part of the stick in the water being raised. If the eye is directly above the image, the end of the stick will be at three-quarters of its actual depth, as in Fig. 300, but if the eye is a considerable distance to the side, the image is higher and not so sharp.

Determination of Index of Refraction by the Microscopic Method.

—A method of measuring the index of refraction of a glass slab is founded on the principle of the foregoing section. A microscope is used, the tube of which can be raised vertically by a rack-and-pinion motion. This microscope is first focussed on a mark on the stage, then the slab of glass is placed on the mark and the microscope focussed on the virtual image of the mark formed by the slab. The distance through which the microscope has to be raised gives PQ (Fig. 299). This is read on a scale. The

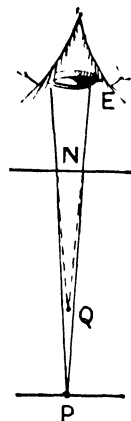


FIG. 299. Image formed by refraction.

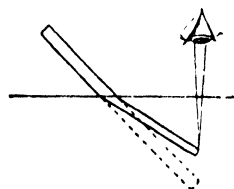


FIG. 300.

microscope is then focussed on a scratch on the upper surface of the slab, and the height through which it has to be raised this time gives QN. Then, when PQ and QN are known, the index of refraction can be calculated.

The method can also be applied to liquids. The liquid is contained in a glass trough. The microscope is first focussed on a mark in the inside of the empty trough. Then the liquid is poured in, and the microscope focussed on the apparent position of the same mark. Finally the microscope is focussed on particles floating on the surface of the water, and the calculation made in the same way as for a solid.

The microscope employed is generally a low-power one, and, since there is a wide region within which the focus does not vary much, the settings are not accurate. If, however, a high-power object glass is used, the focus is sharply defined, and the method is capable of great accuracy.

Experiments on Image formed by Refraction at a Plane Surface. - The refractive indices of glass and water can be determined by the "microscopic method" without the use of a microscope.

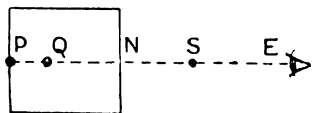


FIG. 301.

(1) A rectangular glass slab is placed on the drawing-board (Fig. 301). It has a vertical scratch on one face at P, and the opposite side is polished. A straight line PNS is drawn perpendicular to the polished face, and a pin set up in this line in such a position S, that its image formed by reflection in the surface of the slab coincides with the image of P formed by refraction at the same surface. The eye should be moved from side to side at E and the position of S adjusted until there is no parallax between the two images. Then

$$\mu = \frac{PN}{QN} = \frac{PN}{NS}$$

(2) Fill a glass tumbler or a milk jug full of water and place a penny inside on the bottom. Measure the depth of the water with a ruler. Then raise a penny above the level of the table outside the jug by putting books, matchboxes or other pennies below it, until both pennies appear the same distance from the eye, when viewed simultaneously from above. The depth of the second penny below the surface of the water is then measured, and the one depth divided by the other gives the refractive index of the water.

(3) Determine the index of refraction of a glass slab by the microscopic method as described on p. 317. The results should be tabulated as follows :

Reading on mark.	Reading on mark through glass.	Reading on top of glass.	Thickness.	Apparent thickness	Refractive index.
3.74	4.39	5.63	1.89	1.24	1.524

One determination is entered in the table. The thickness is the difference of the first and third columns, the apparent thickness the difference of the second and third, and the refractive index the fourth divided by the fifth. To obtain the full accuracy of the method, about six independent determinations should be made, and the mean of the results taken.

(4) Mount a glass slab on two wooden blocks on the stage of a low-power microscope (Fig. 302), which is provided with a vertical scale for measuring the distance through which it is raised, and focus through the slab on a mark P' on the stage. Then remove the glass slab and focus again on the same mark. The difference in readings gives the apparent upward displacement of P' . Is this displacement the same as when the

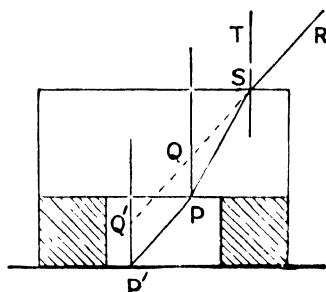


FIG. 302.

slab rests directly on the stage, or does the air space below make a difference? The student should be able to determine theoretically from the diagram whether there is a difference between the two cases or not.

EXAMPLE. A slab of glass 1 in. thick is placed with its lower face parallel to and 1 in. above the surface of a table, and on its upper face rests a layer of water 2 in. thick. The refractive indices of the glass and water are 1.52 and 1.33 respectively. How far does the table appear raised when viewed from above through the glass and water?

Each slab produces an apparent upward displacement of

$$\frac{(\mu - 1)t}{\mu}$$

where μ is its index of refraction and t is its thickness. The effect of the glass is

$$\frac{0.52}{1.52} = 0.342$$

and of the water

$$\frac{0.33}{1.33} \times 2 = 0.496$$

Hence the result is 0.838 inch.

Multiple Images formed by Reflection at a Glass Plate.—Let us suppose (Fig. 303) that a ray of light AP_1 is incident on the surface of a glass plate at P_1 . It gives rise to a reflected ray P_1R_1 and a refracted ray P_1C_1 . On meeting the second surface of the plate the ray P_1C_1 gives rise to a reflected ray C_1P_2 and a refracted ray C_1T_1 . On meeting the first surface of the plate the ray C_1P_2 in turn gives rise to a reflected ray P_2C_2 and a refracted ray P_2R_2 ; and so on. The single incident ray AP_1 gives rise to an infinite number of reflected rays P_1R_1, P_2R_2, P_3R_3 , etc., and an infinite number of transmitted rays C_1T_1, C_2T_2, C_3T_3 , etc. The first two reflected rays are about equally bright, and the others get rapidly fainter, because each reflection diminishes the intensity of the ray very much. The transmitted rays decrease rapidly in brightness, C_1T_1 being much brighter than C_2T_2 , and C_2T_2 about as bright as the first two reflected rays. If the second surface of the plate is silvered, there are, of course, no transmitted rays, but Fig. 303

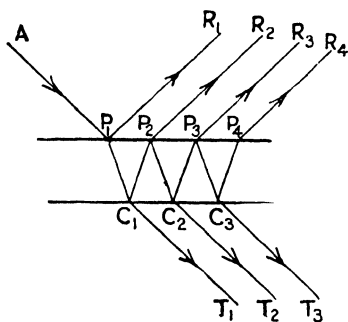


FIG. 303.

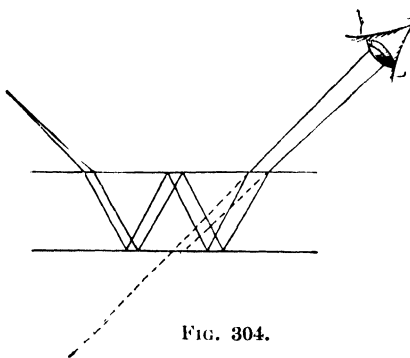


FIG. 304.

otherwise holds good. Only in this case the second reflected ray is much the brightest, the first and third are about equally bright, and the others diminish rapidly in intensity.

When an object is viewed by reflection on a glass plate or by transmission through a glass plate, it follows, in consequence of the reflection of the rays back and forwards inside the glass, that we do not see a single image but a train of images. Fig. 304 shows how the rays reach the eye from the third reflected image. When we stand in front of a plate-glass mirror and view our reflection in it, there is no confusion, because the second image quite overpowers the others, and because the next two brightest, the first and third, are nearly superimposed on the second. But if a candle flame is held close to the mirror and the images looked at very obliquely, they separate, and according to the circumstances from five to ten may be seen, of which, of course, the second is

much the brightest. In the same way, if a candle flame is held close to an unsilvered glass plate, and the images looked at obliquely, from three to six may be seen.

It will be observed, that in Fig. 304 the image is not in the perpendicular from the object to the surfaces of the plate, but displaced to the side. The displacement increases with the obliquity of the rays, and is greater for each image than for the one before it. If, however, the incidence is normal, the images all lie in the perpendicular, each a distance $2t/\mu$ behind the one in front of it, where t is the thickness and μ the index of refraction of the material of the plate. This may be seen by producing the lines R_1P_1 , R_2P_2 , . . . in Fig. 303, and considering the limiting positions of their intersections with the perpendicular through the object, when the angle of incidence becomes small.

Caustic formed by Refraction at a Plane Surface.—In order that the image of P formed at Q in Fig. 298 may be sharp, it is necessary that the rays diverging from P make a small angle with the vertical. Fig. 305 shows what happens when the angle is not small. P is a point source situated in glass of refractive index 1.5, and rays diverging from P meet the plane surface of the glass at A , B , C , . . . etc., and are there refracted into air. In order not

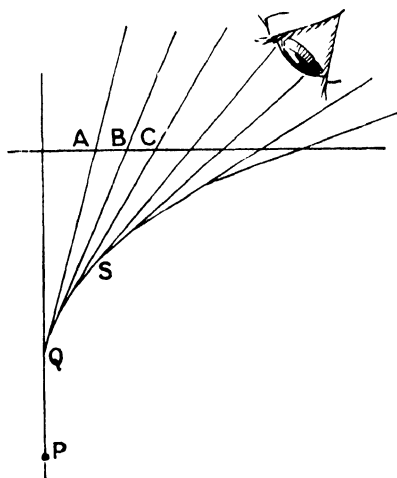


FIG. 305.—Caustic formed by refraction.

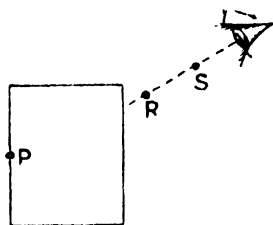


FIG. 306.

to confuse the diagram the paths of the rays inside the glass are not shown; the straight lines give the paths of the rays after their refraction into air, produced backwards into the glass. These rays do not meet in a point at Q , as has already been mentioned, but each two successive rays meet in points which lie on a curve called a *Caustic*. Every ray which diverges from P , after refraction at the surface, is a tangent to the caustic.

If an eye looks down at P obliquely, as shown in Fig. 305, it sees an image of P at S , but the image is not as sharp as when the eye looks vertically down and sees it at Q . S is nearer the surface than Q ; conse-

quently when the eye looks down at the flat bottom of a vessel full of water, the latter appears slightly concave.

EXPERIMENT.—Set up a pin P on a drawing-board close to the face of a glass slab, and then set up two other pins, R and S, in line with the image of P. Draw a straight line through the positions of R and S. Repeat for other positions of R and S, choosing a wide range of directions. Remove the slab, produce the different straight lines backwards, and join the intersections of each successive pair, so as to form the caustic. This experiment requires extremely accurate work.

Total Reflection.—In Fig. 307 APB represents the path of a ray from glass to air, the part AP being in glass and the part PB in air. The angles of incidence and refraction are connected by the relation $\mu \sin \theta = \sin \phi$. Now μ has a value approximately equal to 1.5 for glass. Consequently if θ is gradually increased, it reaches a value given by

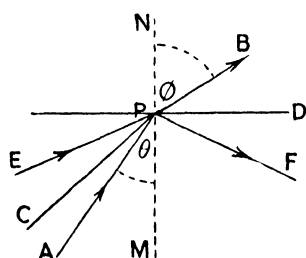


FIG. 307.

for which $\sin \phi = 1$ and the emergent ray goes parallel along the surface. This angle is represented by CPM in the figure, where CP and PD are corresponding incident and refracted rays.

$$\mu \sin \theta = 1 \quad \text{or} \quad \sin \theta = \frac{1}{\mu}$$

Suppose, now, that θ is increased beyond this value and that the ray in the glass has the direction EP. $\mu \sin \theta$ has then a value greater than 1 and $\sin \phi$ an impossible value. There is consequently in this case no refracted ray, only a reflected ray PF, and the light is said to be totally reflected. Since in general the incident beam gives rise to both a refracted and a reflected beam, the intensity of the reflected beam must be less than the intensity of the incident beam. But when the light is totally reflected, there is no refracted beam and no loss of light, and the reflected beam is as bright as the incident beam.

Critical Angle.—If a ray is travelling in any medium in such a direction that the emergent ray just grazes the surface of the medium, the angle which it makes with the normal is called the critical angle or limiting angle of total reflection.

Thus in Fig. 307 $\angle CPM$ is the critical angle. For glass of refractive index 1.52 it has the value $41^\circ 9'$, and for water the value $48^\circ 36'$.

The index of refraction of diamond is 2.417. Consequently its critical angle has a low value of $24^\circ 26'$, and only if the rays make a comparatively small angle with the normal will they be

able to emerge. As a result the number of directions in which rays can emerge are few, and the light may emerge strongly in one of these directions. This gives the diamond its sparkle.

Experiment on Total Reflection.—Hold an empty test-tube obliquely in a glass beaker filled with water. Stand at some distance from the window, so that most of the light enters the beaker horizontally from the side, and view the surface of the test-tube from above through the water. It will appear bright like a mirror or like polished metal; this is because the rays of light incident through the water on the side of the tube are totally reflected. If, however, water is poured into the test-tube, the part filled with water looks dark in comparison with the part above it. Total reflection ceases now, and most of the rays which fall on the side of the tube enter into it.

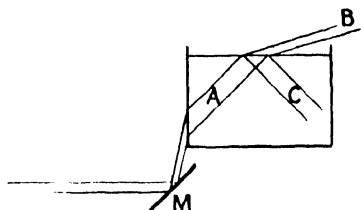


FIG. 308.

Experimental Demonstration of Total Reflection.

Fig. 308 represents a trough with glass sides and ends, which is filled with water. A horizontal beam of light from a projection lantern is received by a mirror M and made to enter at the side of the trough. Its path inside the trough is rendered visible either by reflection at the particles floating in the water or by dissolving a fluorescing substance in the water (*cf.* p. 388). If the angle which the beam A makes with the normal is less than the critical angle, it gives rise both to the reflected beam C and the refracted beam B, which can be rendered visible by dusting chalk in the air. But if the mirror M is inclined, so as to gradually increase the angle of incidence of the beam A inside the water, the beam B becomes horizontal and disappears; simultaneously there is a great increase in brightness of the beam C. The beam A is then totally reflected.

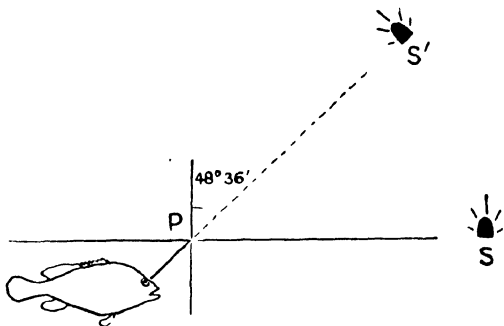


FIG. 309. Fish and setting sun.

The Fish and the Setting Sun.

Fig. 309 gives the answer to the well-known problem of finding in what direction a fish sees the setting sun. The surface of the water is supposed perfectly smooth. The rays from the sun come along the surface at grazing incidence, are refracted

at P, and enter the eye of the fish, making the critical angle with the vertical. The fish consequently sees the sun setting at an angle of $48^{\circ} 36'$ with the vertical. In fact the whole sky for the fish is compressed within a cone of semi-vertical angle $48^{\circ} 36'$. Outside this cone it sees stones, etc., at the bottom of the sea totally reflected at the surface.

Totally Reflecting Prism.—Fig. 310 represents a section of what in optics is simply called a prism, but what would in geometry be called a right triangular glass prism, *i.e.* a solid bounded by plane figures, of which two, the ends, are congruent triangles, and the others, the sides, are rectangles. The section is a principal section, one made by a plane at right angles to the sides, and for the prism in question it is a right-angled isosceles triangle. ABC

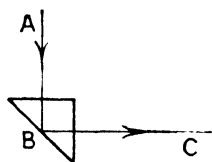


FIG. 310.

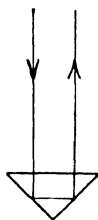


FIG. 311.

represents a ray which is incident perpendicularly on one of the faces, and consequently enters the prism undeviated. It is then reflected internally by the hypotenuse surface, is incident perpendicularly on the third face, and emerges undeviated. The angle of incidence on the hypotenuse surface is 45° . Now the critical angle for crown glass is about $41^{\circ} 9'$, which is less than

the angle of incidence. The rays are consequently totally reflected by the hypotenuse surface, and, except for the comparatively small reflection losses on entering and leaving the glass, emerge from the prism with their intensity undiminished.

Such a prism is called a total reflection prism. It is often used instead of a mirror for reflecting a beam through 90° . If a mirror is silvered on the front it tarnishes, and, if it is silvered on the back, the faint image caused by reflection on the front sometimes causes confusion. By the use of the total reflection prism both these sources of trouble are eliminated.

The prism can also be used to turn the beam through 180° as shown in Fig. 311. In this case it suffers two total reflections.

The Luminous Jet.—If a jet of water falls into a dish from the side of a glass vessel in a darkened room, and a beam of light from a projection lantern is directed into the vessel from the side opposite to the jet and travels into the jet, suitable precautions being taken to screen off stray light, the jet appears dark until it strikes the bottom of the dish. Then a brilliant lighting results. For the light everywhere falls on the surface of the water at an angle greater than the critical angle, and consequently cannot escape from the jet by refraction.

Two Parallel Slabs.—Let Fig. 312 represent the passage of a ray of light from a medium A through two slabs with parallel sides made of media B and C back into A. B and C possess different indices of refraction; in order to have clear ideas we may think of them as water and glass, the medium A being air. The normals to the three surfaces are parallel. The angles of refraction inside the media B and C are respectively β and γ . It is found by experiment that the emergent ray is always parallel to the incident ray; hence it makes the same angle α with the normal as the latter. Let ${}_A\mu_B$, ${}_B\mu_C$, and ${}_C\mu_A$ denote respectively the indices of refraction of B with respect to A, of C with respect to B, and of A with respect to C. Then, from the definition,

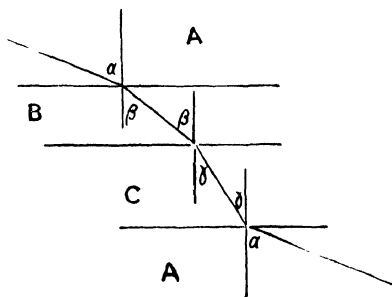


FIG. 312.

$${}_A\mu_B = \frac{\sin \alpha}{\sin \beta}, \quad {}_B\mu_C = \frac{\sin \beta}{\sin \gamma}, \quad \text{and} \quad {}_C\mu_A = \frac{\sin \gamma}{\sin \alpha}$$

whence, multiplying the three equations together,

$${}_A\mu_B \cdot {}_B\mu_C \cdot {}_C\mu_A = \frac{\sin \alpha}{\sin \beta} \cdot \frac{\sin \beta}{\sin \gamma} \cdot \frac{\sin \gamma}{\sin \alpha} = 1$$

This is an important result which enables us to calculate the refractive index of one medium with respect to another, when the refractive indices of both media are known with respect to a third. For example, if B is water which has a refractive index 1.333 with respect to air, and if C is glass of refractive index 1.52, then the refractive index of the glass with respect to water is given by

$${}_B\mu_C = \frac{1}{{}_A\mu_B \cdot {}_C\mu_A}$$

Now we have

$${}_C\mu_A = \frac{1}{{}_A\mu_C}$$

since ${}_C\mu_A = \frac{\sin \gamma}{\sin \alpha}$ and ${}_A\mu_C = \frac{\sin \alpha}{\sin \gamma}$

Hence ${}_B\mu_C = \frac{{}_A\mu_C}{{}_A\mu_B} = \frac{1.52}{1.333} = 1.14$

Absolute Refractive Index.—The refractive index of air under standard conditions with respect to vacuum is 1.0003. Let us suppose that in Fig. 312 the medium A is vacuum, the medium B air, and the medium C glass of refractive index 1.52. Then the refractive index of the glass with respect to vacuum, or its absolute refractive index, as it is called, is given by

$$\mu_{AC} = \frac{1}{\mu_{CA}} = \mu_{AB} \cdot \mu_{BC} = 1.0003 \times 1.52 = 1.5206$$

Thus the absolute refractive index of a substance is always greater than its relative refractive index with respect to air, but the difference is smaller than could be measured with the average school spectrometer.

Several Parallel Slabs.—Let us suppose that a ray incident on the surface of a glass slab in air makes an angle of incidence ϕ

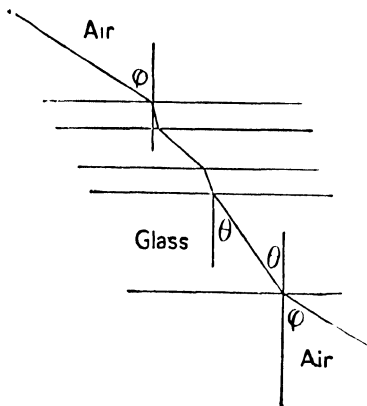


FIG. 313.

and an angle of refraction θ with the normal to the surface. Next let us suppose that the ray is incident at the same angle, but does not enter the glass directly, that a number of different media are inserted between, as shown in Fig. 313; these media have different thicknesses, but are all bounded by planes parallel to the plane surface of the glass. Then the ray makes the same angle θ with the normal in the glass as before; θ depends only on ϕ , and is not affected by the media the ray passes through on its way to the glass. This may be shown by supposing the ray

to enter the air again, for the emergent ray must make the same angle ϕ with the normal at the point of emergence.

EXAMPLE.—Two equal glass plates with parallel sides are cemented together with a thin air film between, thus forming a glass cell containing air. The cell is then immersed in a trough of water with its sides vertical and a horizontal beam of light passed through it. It is found that when the cell is rotated about a vertical axis through an angle θ from the position in which the light is incident on it perpendicularly, the beam no longer passes. Explain the occurrence, and derive an expression for the index of refraction of water.

The two glass plates with the air film may be regarded as forming a series of plates such as is represented in Fig. 313. Under no circumstances will the beam be totally reflected on entering the glass, because the refractive index of glass is greater than the refractive index of

water. Now the angle which the ray makes with the normal in the air film is not affected by the glass plate passed through; we may think of the arrangement as simply a plate of air immersed in water. In going from water to air the beam is entering a medium with a smaller index of refraction. Consequently total reflection takes place when the angle of incidence θ becomes equal to the critical angle. The beam no longer passes, because it is totally reflected by the air film, and the refractive index of water is given by

$$\mu = \frac{1}{\sin \theta}$$

Atmospheric Refraction.—Let us suppose that the atmosphere, instead of diminishing gradually in density as we ascend, keeps its density constant, and then suddenly comes to an end. In other words, we have a uniform slab of air and above that vacuum. If a ray of light, SQ , from a star falls on this slab, it is refracted; consequently an observer situated at P sees the star in the direction PQ . If the air were removed he would see it in the direction PS' , which is parallel to QS , since the star is infinitely distant. Instead of making the angle ϕ with the vertical, the direction of the star makes the slightly smaller angle θ . In other words, it is displaced towards the zenith—the direction vertically overhead—through the small angle $\phi - \theta$. Now

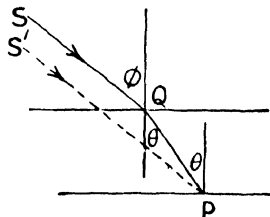


FIG. 314.

$$\begin{aligned}\sin \phi &= \sin \{(\phi - \theta) + \theta\} \\ &= \sin (\phi - \theta) \cos \theta + \cos (\phi - \theta) \sin \theta \\ &= (\phi - \theta) \cos \theta + \sin \theta\end{aligned}$$

because, since $\phi - \theta$ is small, $\sin (\phi - \theta)$ may be written equal to $\phi - \theta$ in radian measure, and $\cos (\phi - \theta)$ may be put equal to 1. Substitute this value of $\sin \phi$ in the law of refraction. Then

$$\begin{aligned}(\phi - \theta) \cos \theta + \sin \theta &= \mu \sin \theta \\ (\phi - \theta) \cos \theta &= (\mu - 1) \sin \theta \\ \phi - \theta &= (\mu - 1) \tan \theta\end{aligned}$$

or

Consequently the displacement equals $\mu - 1$ or 0.0003 times the tangent of the apparent angle which the star makes with the zenith.

The density and refractive index of the atmosphere increase gradually as we approach the earth's surface. Hence the atmosphere must not be represented by a single uniform slab, as in Fig. 314, but by a number of slabs of gradually increasing density, as in Fig. 315. The ray from the star suffers a slight deviation on

entering each of these separate slabs, as shown in Fig. 315. But, as has been proved above, the direction in the final medium is independent of the media passed through. Hence the above formula still holds, provided that $\phi - \theta$ is small. If, however, the

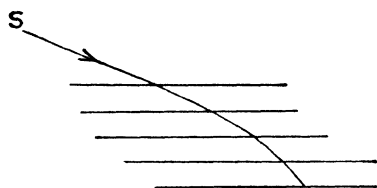


FIG. 315.

star is low in the heavens, $\tan \theta$ is large, the formula would not give a small value of $\phi - \theta$, and hence can no longer be applied.

The upward displacement produced increases very rapidly as the star approaches the horizon, attaining a magnitude of $35'$ when ϕ reaches 90° .

This is greater than the angle subtended by the diameter of the sun or of the moon. Thus, when the lower edge of the sun's disc appears to be touching the horizon, the whole disc is really below the plane of the horizon. Hence, owing to the refraction of the atmosphere, both ends of the day are lengthened at the expense of the night.

Mirage.—It sometimes happens in a desert, that the layer of air immediately above the sand is much hotter than the layers higher up, consequently its density and index of refraction are lower than for the layers higher up. Rays of light from the sky incident on the hot layer at a very large angle are thus totally reflected, and pass upwards again into the colder regions without reaching the sand. If they reach the eye of an observer, he sees a piece of the sky apparently mirrored in

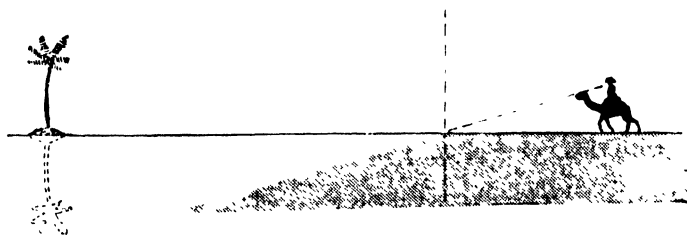


FIG. 316.—Mirage.

the sand, and takes it to be the surface of a lake. To this phenomenon the name of mirage is given. Thus in Fig. 316 the observer sees the tree and sky behind it directly. He also sees them by total reflection on the hot air immediately above the sand, and thinks he sees a lake with the reflection of the tree in it.

EXAMPLES III

1. A ray of light is refracted at the surface of a glass plate of refractive index 1.517, the deviation caused by the refraction being 12° . Calculate the deviation for different angles of incidence, graph the result as a function of the angle of incidence, and determine from your graph the angle of incidence for the case in question.

2. Explain why a transparent substance like glass is opaque when finely powdered.

3. A vessel, 8 in. deep, is filled with methyl alcohol of index of refraction 1.332. Find its apparent depth.

4. The index of refraction of carbon bisulphide is 1.63. Carbon bisulphide is poured into a beaker to a depth of 6.0 cm. What is its apparent depth?

5. A slab of glass, 8 cm. thick, with a refractive index of 1.52 is held with its lower surface horizontal 5 cm. above a piece of paper on which a mark is made. Where does the mark appear to be to an eye looking at it vertically through the slab? Illustrate your answer with a diagram.

6. A small air bubble in a glass slab appears to an eye looking normally at the surface to be 3 cm. from the latter. If 1.52 is the index of refraction of the glass, what is the real distance of the bubble from the surface?

7. Look at the reflected image of an electric light filament in a piece of red glass. Why are there two images, one red and one white?

8. Find the absolute refractive index of water, given that the refractive index of water with respect to air is 1.3330 and the refractive index of air is 1.0003.

9. The absolute indices of refraction of ethyl ether and a certain kind of glass are 1.352 and 1.517 respectively. What is the index of refraction of the glass with respect to the ether?

10. Find the critical angle for a ray passing from glass of index of refraction 1.583 to water. The index of refraction of water is 1.333.

11. What is the greatest zenith distance that a star can have as seen by an eye under water? The zenith distance of a star is the angle which its direction makes with the vertical.

12. The apparent elevation of the centre of the sun's disc above the horizon is 40° . Find its true elevation. The index of refraction from vacuum to air is 1.0003.

13. A glass trough half filled with water is placed beneath a measuring microscope, and the latter focussed (i) on a mark on the table beneath the trough, and (ii) on the upper surface of the water. More water is then poured in, and both readings repeated. It is found, that (i) has been displaced upwards through a distance a , and that (ii) has been displaced upwards through a distance b . What is the index of refraction of water?

14. Kepler believed that the law of refraction was given by

$$\phi = \frac{\mu \theta}{\mu - (\mu - 1) \sec \theta}$$

where ϕ is the angle of incidence, θ the angle of refraction, and μ a constant. By means of this formula calculate ϕ for values of θ increasing in steps of 5° from 5° to 45° inclusive, taking μ equal to 1.333. Then calculate ϕ by the correct formula for the same values of θ and μ , and graph the results of both calculations on the same diagram as functions of θ .

15. A ray of light starts from a point A, is refracted by a plane water surface at a point P, and arrives at a point B inside the water. Show graphically for one particular case, that the value of $AP + \mu BP$ for the actual path is less than for any other possible path from A to B.

CHAPTER IV

SPHERICAL MIRRORS

IF the surface bounding two media is spherical in shape and highly polished, it is said to form a spherical mirror. It is not necessary for it to be silvered; an unsilvered glass surface gives quite as sharp images, but if the surface is silvered, the images are much brighter. The glass must be silvered on the front, if the theory given here is to apply, for if the mirror consists of a thin piece of glass silvered on the back, the light suffers refraction at the front, both before and after reflection at the back.

Spherical mirrors are divided into two classes, concave and convex. In the case of the **concave** spherical mirror, the light falls on the surface from the same side as the centre of curvature or centre of the sphere of which the surface forms part; in the case of the **convex** mirror the light falls on the surface from the opposite side to the centre of curvature.

Spherical mirrors like plane mirrors form images of objects placed in front of them. But whereas the image formed by a plane mirror is always virtual, the same size as the object, and situated at the same distance from the mirror as the object is, the image formed by a spherical mirror may be real, and is usually magnified or diminished in size; its distance from the mirror varies also with the curvature of the mirror and the position of the object. We shall proceed to find the formula which gives the position of the image, restricting ourselves at first to the simplest case, the case of a concave mirror forming a real image. We shall afterwards go on to the more difficult cases.

Proof of Formula for Concave Mirror forming a Real Image.

—Let AB represent a section of a concave spherical mirror, let C be its centre of curvature, and let P be a point source of light. Join PC and produce it to cut the mirror in A.

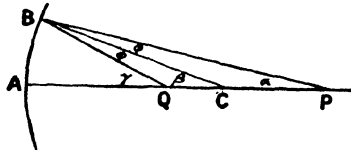


FIG. 317.

Draw any ray making a small angle α with AP to meet the mirror at B. The element of surface at B may be regarded as

plane, and consequently the laws of reflection at plane surfaces may be applied to it. The element of surface coincides with the tangent plane at B, and CB is the normal to it. Let $\angle PBC = \phi$. The ray after reflection at B makes an angle ϕ with the normal and meets AP at Q. Denote $\angle BCQ$ by β and $\angle BQA$ by γ , and let AP, AC, and AQ have respectively the values u , r , and v .

Then, from the figure

$$\beta = \phi + \alpha, \quad \gamma = \phi + \beta,$$

and consequently $\alpha + \gamma = 2\beta$. But since α , β , and γ are small, we can write

$$\alpha = \frac{AB}{u}, \quad \beta = \frac{AB}{r}, \quad \text{and} \quad \gamma = \frac{AB}{v}.$$

On substituting these values in the equation above, and dividing out by AB, we obtain

$$\frac{1}{u} + \frac{1}{v} = \frac{2}{r}$$

i.e. the position of Q is quite independent of the position of B.

Rays diverging from a point P, therefore, and making a small angle with the straight line joining P with the centre of curvature of the mirror, after reflection at the mirror, converge towards another point Q. Or, in other words, the mirror forms an image of the point P at Q.

In Fig. 317 keep the mirror and, consequently, the point C fixed, and rotate the line PC through a small angle about an axis perpendicular to the plane of the figure through C. P and Q then describe arcs which may be taken as straight lines, and the one line will be the image of the other.

Spherical mirrors have usually a circular rim, and the straight line through the centre of curvature perpendicular to the plane of this rim is called the axis of the mirror. We have proved, therefore, for one particular case, that a spherical mirror forms an image of a short line situated on the axis and perpendicular to the axis.

Conjugate Points. Focus.—It is clear that in Fig. 317 the rays may be regarded as starting from Q and going to P. Or in other words, object and image are interchangeable. This is expressed by saying that P and Q are conjugate points.

If the object is at infinity, $1/u$ is zero, and v becomes equal to $r/2$. All the incident rays are then parallel, and after reflection they converge to a point F midway between A and C (Fig. 318). This point F is called the principal focus, or simply the focus of the mirror, and the length AF is called its **focal length**, and is

usually written f . If we substitute f for $r/2$ in the formula for the spherical mirror, the latter becomes

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

If the rays from the sun are focussed by a large mirror in the manner indicated by Fig. 318, the heat produced at F is sufficient to set fire to small pieces of paper.

Graphical Construction.—If the property of the focus, and the fact that the mirror forms images be assumed, the position and size of the image can be found very easily by a graphical construction. Let there be an object at P (Fig. 319), only one half of

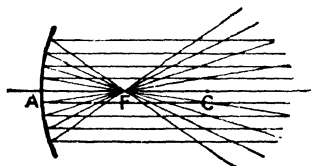


FIG. 318.

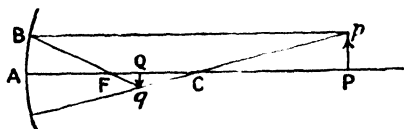


FIG. 319.

which, Pp , is drawn. Draw a ray from p through C ; it falls on the mirror perpendicularly, and consequently after reflection comes back along the same path. Draw another ray, pB , parallel to the axis; after reflection it passes through the focus. The point q where BF cuts pC is therefore the image of p , and the perpendicular qQ to the axis consequently the image of pP .

Alternative Proof of Formula.—The formula for the mirror may be derived from Fig. 319. Since AB is small in comparison with the radius of curvature—it is greatly exaggerated in the figures for the sake of clearness—it may be regarded as straight, and at right angles to AP . Then $AB = Pp$, and consequently $AB/Qq = Pp/Qq$. Triangles ABF and QqF are similar, as are also triangles QqC and PpC . Hence

$$\frac{AB}{Qq} = \frac{AF}{QF} \quad \text{and} \quad \frac{Pp}{Qq} = \frac{PC}{QC}$$

But the left-hand sides of these two equations are already equal; hence

$$\frac{AF}{QF} = \frac{PC}{QC}$$

or

$$\frac{f}{v-f} = \frac{u-2f}{2f-v}$$

which gives

$$f(2f-v) = (u-2f)(v-f)$$

or

$$fu + fv = uv$$

and on dividing out by uvf ,

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} = \frac{2}{r}$$

the equation which has already been found.

Magnification produced by a Mirror.—The relative sizes of the image and object can also be obtained from Fig. 319. For

$$\frac{Qq}{Pp} = \frac{QC}{CP} = \frac{r-v}{u-r} = \frac{v\left(\frac{1}{v} - \frac{1}{r}\right)}{u\left(\frac{1}{r} - \frac{1}{u}\right)}$$

But since
$$\frac{1}{u} + \frac{1}{v} = \frac{2}{r}, \quad \frac{1}{v} - \frac{1}{r} = \frac{1}{r} - \frac{1}{u}$$

Thus
$$\frac{Qq}{Pp} = \frac{v}{u}$$

The ratio Qq/Pp is termed the linear magnification or simply the magnification. If the magnification is a proper fraction, the image is smaller than the object. The above equation can be put into the following useful rule: *the linear dimensions of the image and object are in the ratio of their distances from the mirror.*

EXAMPLE.—A real image produced by a concave mirror is found to be four times the height of the object. If the mirror is 30 cm. from the object, what is its focal length?

We have $v=4u$, $u=30$. Thus $v=120$, and on substitution in

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$f=24$ cm.

Convention of Signs.—Hitherto in dealing with the concave mirror we have restricted ourselves to the case, that the image is on the same side of the mirror as the object. It is then real, and can be received on a screen. Let us suppose that the object lies between the surface of the mirror and the focus; for example, let $r=10$ cm. and $u=3$ cm. Then, by the equation

$$\frac{1}{3} + \frac{1}{v} = \frac{1}{5}$$

which gives $v=-7.5$ cm.

The natural interpretation of this result is that the image is situated 7.5 cm. from the mirror on the opposite side from the object. If we use the graphical construction, it leads to this same position of the image. Now we do not know so far, that we have the right to use either the formula or the graphical construction, when v is negative. We shall, however, make the assumption, that all distances to the right of the mirror are positive, and all

distances to the left negative. That is, u , v , r , and f are to be regarded as the co-ordinates of the object, image, centre of curvature, and focus with respect to the mirror as origin, and we are to use the plus and minus signs in exactly the same manner as in co-ordinate geometry or graph plotting. Then on the basis of this assumption, we shall find formulæ for the position of the virtual image formed by a concave mirror, and for the position of the image formed by a convex mirror.

Concave Mirror. Virtual Image.—Let P (Fig. 320) be the object, C the centre of curvature, and BQ the direction of the reflected ray produced backwards to meet the axis. Let ϕ denote the angle of incidence and reflection at B, and let $\angle APB$, $\angle ACB$, and $\angle AQB$ be equal to α , β , and γ respectively. $AP=u$, $AC=r$, and $AQ=v$. Then, from the figure,

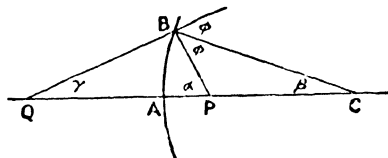


FIG. 320.

Then, from the figure,

$$\alpha = \phi + \beta, \quad 2\phi = \alpha + \gamma$$

which gives $\alpha - \gamma = 2\beta$. Since α , β , and γ are small, we can write

$$\alpha = \frac{AB}{u}, \quad \beta = \frac{AB}{r}, \quad \text{and} \quad \gamma = -\frac{AB}{v}$$

The minus sign is necessary in the expression for γ , since v itself is negative. On substituting these values in the equation above, and dividing out by AB , we obtain

$$\frac{1}{u} + \frac{1}{v} = \frac{2}{r}$$

Convex Mirror.—Let P (Fig. 321) be the object, C the centre of curvature, BQ the direction of the reflected ray produced backwards, and ϕ the angle of incidence and reflection as before. Let α , β , and γ have the values denoted in the figure, and let $AP=u$, $AC=r$, and $AQ=v$. In this case both r and v are negative.

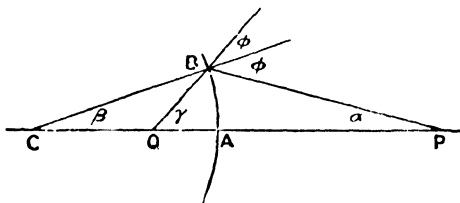


FIG. 321.

Then $\gamma = \beta + \phi$, $2\phi = \alpha + \gamma$ and these give $\gamma - \alpha = 2\beta$. We have

$$\alpha = \frac{AB}{u}, \quad \beta = -\frac{AB}{r}, \quad \text{and} \quad \gamma = -\frac{AB}{v}$$

and on substituting these values and dividing out by AB , we obtain the equation

$$\frac{1}{u} + \frac{1}{v} = \frac{2}{r}$$

as before.

Hence in Figs. 320 and 321 the position of the point Q is independent of the position of B , and the mirror forms an image of the point P at Q . The image is in each case a virtual one; it is not the rays themselves, but only their directions produced backwards that meet. Consequently the image will not show up on a screen.

If in Figs. 320 and 321 the point C is kept fixed, and the line PC is rotated through a small angle about an axis through C perpendicular to the plane of the paper, P and Q describe arcs which may be taken as straight lines, and the one line is the image of the other.

Thus we find that the same formula holds for all cases of image formation by spherical mirrors, provided that it is interpreted algebraically.

Focus of a Convex Mirror.—Let us suppose that a beam of parallel light is incident on a convex mirror, as shown in Fig. 322. Then $1/u$ is zero, and v becomes equal to $r/2$. After reflection at the mirror the rays appear to diverge from a point F midway between A and C , which is called the focus of the mirror; AF is called the focal length of the mirror, and is usually written f . In Fig. 322 f is negative.

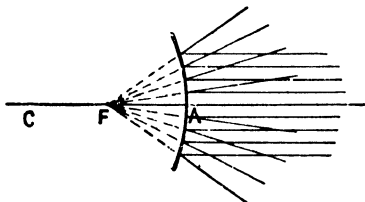


FIG. 322.

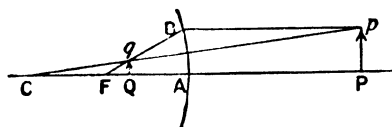


FIG. 323.

Graphical Construction for Convex Mirror.—Fig. 323 shows a determination by the graphical method of the position and size of the image formed by a convex mirror. pP is the upper half of the object. From P a ray is drawn through C ; it falls on the mirror perpendicularly, and consequently after reflection comes back along the same path. Another ray pB is drawn parallel to the axis; after reflection its direction produced backwards passes through the focus F . The point q , where BF cuts pC , is therefore the image of p , and the perpendicular to the axis, qQ , consequently the image of pP .

The formula for the mirror and the law of magnification can be derived from Fig. 323 just as they were derived from Fig. 319.

If regard is paid to the signs of Pp and Qq , reckoning distances up positive and down negative, the law of magnification becomes

$$\frac{Qq}{Pp} = -\frac{v}{u}$$

Then, when Qq/Pp is positive, the image is erect, and when it is negative, the image is inverted.

Rules for solving Problems.—To recapitulate, then, the following facts must be kept in mind in solving numerical problems on mirrors:

(1) The same formula

$$\frac{1}{u} + \frac{1}{v} = \frac{2}{r} = \frac{1}{f}$$

holds for all cases of image formation by a spherical mirror.

(2) The magnification is always equal to the image distance divided by the object distance.

(3) The same graphical construction holds for all cases. And in order to avoid mistakes in sign, which are very difficult to prevent altogether,

(4) Always verify numerical results by the graphical construction.

Some textbooks use a different convention of signs from that adopted here, and always take the direction opposed to the incident light as positive; thus $v=10$ cm. means that the image is 10 cm. from the mirror on the side from which the light is coming, and $v=-7$ cm. means that the image is 7 cm. distant on the other side. The two conventions agree or clash according to the side of the page from which the light comes. As there is no reason why a student should unlearn his co-ordinate geometry when he starts to study light, and as the one convention possesses no advantage over the other, we shall always measure distances to the right of the mirror positive, and distances to the left of the mirror negative.

If a student has not time to learn both the use of the formula and the graphical construction, it is better to concentrate on the graphical construction.

EXAMPLE.—A concave mirror has a radius of curvature of 12 cm. Find the position, nature, and size of the image, when an object 4 mm. high is placed (a) 18 cm. from the mirror; (b) 4 cm. from the mirror.

(a) The formula gives

$$\frac{1}{18} + \frac{1}{v} = \frac{2}{12}$$

Hence $v=9$ cm., and the image is real and inverted. The magnification v/u is $\frac{1}{2}$, and hence the height of the image is 2 mm.

(b) In this case

$$\frac{1}{4} + \frac{1}{v} = \frac{2}{12}$$

Hence $v = -12$ cm., *i.e.* the image is behind the mirror, and is erect and virtual. The magnification is $12/4 = 3$, and the height of the image consequently 12 mm.

EXAMPLE.—A convex mirror has a focal length of 10 cm. Find the position, nature, and size of the image, when an object 5 mm. high is placed on the axis of the mirror at a distance of 15 cm. from it.

In this case the focus is behind the mirror, so that f must be written -10 . Then the formula gives

$$\frac{1}{15} + \frac{1}{v} = -\frac{1}{10}$$

Hence $v = -6$, the image is behind the mirror, erect, and virtual, the magnification is $6/15 = \frac{2}{5}$, and the height of the image is 2 mm. high.

The Optical Bench.—The optical bench is a piece of apparatus used for determining the focal lengths of lenses and mirrors. There are two classes of optical bench, those made with metal bases and fittings, and those made of wood. The former, while more accurate and indispensable for certain special purposes, are too expensive and elaborate for general use, and a wooden bench of the type shown in Fig. 324 will be found sufficient for most purposes.

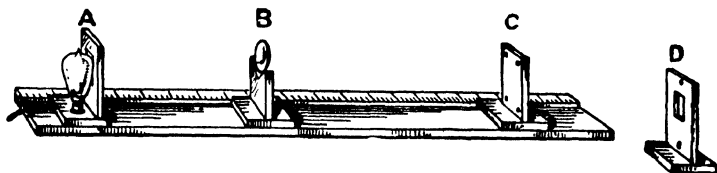


FIG. 324.—Optical bench.

The base of this bench is mahogany. At the side is a scale two metres long for reading the positions of the pieces. The stand A carries an incandescent electric lamp, in front of which there is a wooden upright with a rectangular hole in it, across which are stretched cross-wires. These cross-wires are the object, and it is usually advisable to fasten a piece of tissue paper with drawing-pins between the hole and the lamp to give a more uniform background to the object. Stand B is for carrying the lens or mirror; it has a V-shaped top with a groove in it. C is a screen for receiving images, consisting of an upright with a piece of paper fastened on with drawing-pins, and D a similar screen with a hole in it, also used for receiving images. The positions of the various stands can be read by marks on their bases.

The optical bench is generally used in a darkened room. This is, however, not necessary. If instead of cross-wires, the filament of a 16 candle-power carbon glow lamp is used as object, the image is bright enough to be received on a screen in broad daylight. Also it is not necessary to mount the various stands on a wooden

board as shown in Fig. 324 ; they may be moved along the surface of the table itself, and the distances between them measured with an ordinary metre-stick.

Mirrors suitable for use on an optical bench can be made by taking untrimmed spectacle lenses and painting one face black. The other face is used as reflecting surface. The paint absorbs the light that enters the glass.

Experiments on Spherical Mirrors.—(1) Fix a concave mirror in the stand B (Fig. 324), place it on the optical bench facing stand D, and direct the bench towards a window or distant light. The window is so far away, that the rays from it may be considered parallel ; they pass through the hole in D, are reflected by the mirror, and form an image of the distant object at the focus of the mirror. The stand D is moved towards the mirror, until this image is sharply focussed on it. The distance of the image from the mirror is then the focal length of the latter.

If the axis of the mirror were set parallel to the bench, the image would fall on the hole, and hence be lost. The mirror is consequently tilted a little, so that the image falls on the paper at the side of the hole ; this causes no appreciable error.

(2) Stick a pin in the hole in D with its head in the same plane as the paper which is fastened on this stand, and fix a concave mirror on the stand B so as to face the paper. Place the lamp A on the other side of D with the upright turned away from the latter, so that its rays fall through the hole in D on to the mirror. Focus the mirror until an image of the pin is formed on the paper close beside the pin itself. Then measure the distance of the mirror from the pin.

In this case $v = u$. Hence, substituting in the formula,

$$\frac{1}{u} + \frac{1}{u} = \frac{2}{r}$$

$u = r$, and the radius of curvature of the mirror is equal to the distance of the mirror from the pin.

(3) Let the rays from the cross-wires on stand A pass through the hole in D, fall on a concave mirror mounted on B, and be reflected back to form an image on D. Take different positions of cross-wires and mirror, and measure the corresponding values of u and v . The latter should then be entered in a table as below, and f calculated by the formula

u	v	f

If many values of u and v are taken, they should be graphed against one another ; the curve is a rectangular hyperbola.

(4) Mount a convex mirror on stand B, and let the rays of light from the illuminated cross-wires fall on it. As the image is virtual, it cannot now be received on a screen, and consequently a different procedure must now be adopted, analogous to that of experiment 4 on p. 305. Mount a long pin in a piece of wood, and set it vertical on the axis of the mirror behind the latter. Then placing a hand over the lamp, so as to prevent its direct light entering the eyes, look over the top of the lamp into the mirror. An image of the cross-wires will be visible in the mirror, and at the same time the long pin will be seen over the top of the mirror. Shift the long pin about until it coincides in position with the image of the vertical cross-wire, moving the head from side to side to be sure that there is no parallax between them. The distance of the pin from the mirror gives v , the distance of the cross-wires from the mirror gives u , and f can then be calculated by the formula.

(5) Verify the values of r obtained on the optical bench by means of the spherometer.

EXAMPLE.—The focal length of a concave mirror is obtained by forming the image of a lamp several metres distant. The result is 25.2 cm. How far away must the lamp be for the result to be accurate to 2 mm.?

Let u be the distance of the lamp, when the result is of the degree of accuracy required. Then $v=25.2$ and $f=25.2-0.2=25.0$ cm. Consequently

$$\frac{1}{u} + \frac{1}{25.2} = \frac{1}{25.0}$$

which gives $u=3,150$ cm.

Tracing the Position of the Image.—In the formula for the mirror

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

write $u=f+x$ and $v=f+y$. Then x and y denote respectively the distances of object and image from the focus. We obtain

$$\frac{1}{f+x} + \frac{1}{f+y} = \frac{1}{f}$$

$$f(f+y) + f(f+x) = (f+x)(f+y)$$

and finally

$$xy = f^2$$

This equation may be used for tracing the change in position of the image, when the position of the object is varied. It holds both for the concave and convex mirrors.

Thus in the case of the concave mirror facing towards the right with its focus f in front of it, when $x=\infty$, $y=0$. As the object moves towards the focus, the image moves to the right away from the focus. When $x=f$, $y=f$; so they pass at this point, *i.e.* at the centre of curvature. As x becomes 0, y moves towards infinity.

When the object crosses the focus, x is negative and very small. At the same time y becomes negative and very large; the image is virtual and a long way behind the mirror. As the object passes the focus on its way towards the mirror, the image jumps from positive to negative infinity. Finally, as the object approaches the surface of the mirror, x becomes $-f$, and consequently y becomes $-f$. So object and image meet on the surface of the mirror.

In the case of the convex mirror facing towards the right with its focus distant f behind it, when $x = \infty$, $y = 0$. As x diminishes, y increases; finally, when $x = -f$, $y = -f$, *i.e.* they reach the surface of the mirror together.

The above results are shown below in tabular form. The table at the same time gives the character of the image; this is obtained most easily from the graphical construction.

TABLE OF RESULTS

CONCAVE MIRROR

<i>Position of Object</i>	<i>Position of Image</i>	<i>Character of Image</i>
At infinity.	At focus.	Real.
Between ∞ and C	Between F and C.	Real, inverted, diminished.
At C.	At C.	Real, inverted, same size.
Between C and F.	Between C and ∞ .	Real, inverted, magnified.
At F.	At infinity.	—
Between F and mirror	From an infinite distance behind mirror to mirror.	Virtual, erect, magnified.
At mirror.	At mirror.	Erect, same size.

CONVEX MIRROR

<i>Position of Object</i>	<i>Position of Image</i>	<i>Character of Image</i>
At infinity.	At focus.	Virtual.
Between infinity and mirror.	Between F and mirror.	Virtual, erect, diminished.
At mirror.	At mirror.	Erect, same size.

EXPERIMENT.—Place a candle flame close up in front of a concave mirror, for example, a shaving mirror. On looking into the mirror a magnified erect image of the flame will be seen. Gradually draw the flame away from the mirror; the image grows larger and fills the whole mirror, until finally it ceases to be visible.

If, however, the flame is moved further out, and we look into the mirror along its axis from a distance, an inverted aerial image of the flame will be seen further out than the flame itself. If the flame is again

moved out, this image moves in, until there is no parallax between them, and flame and aerial image are seen side by side, exactly the same size. As the flame moves further out still, the aerial image moves further in, and diminishes in size.

EXPERIMENT.—Form a real image with a concave mirror, and determine corresponding values of u and v , taking as wide a range of values as possible and including the case where v is greater as well as the case when v is smaller than u . Graph the results, and from the graph determine the focal length of the mirror.

The Magic Dagger and the Phantom Bouquet.—Fig. 325 represents an illusion known as the “ Magic Dagger,” in which a concave mirror is employed. A dagger is suspended at A inside a box in an inverted position. The box contains electric lamps not shown in the figure, and hence the dagger is strongly illuminated. The rays issuing from it

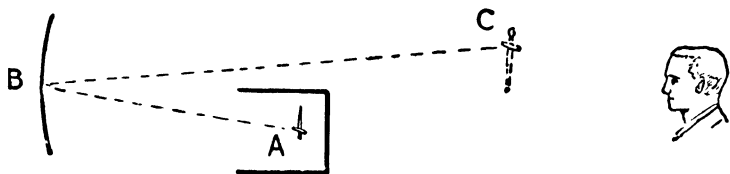


FIG. 325.—Magic dagger.

fall upon the concave mirror at B, are reflected by the latter, and form a real erect image at C. The illusion is shown in a partially darkened room; the spectator sees a luminous dagger before him, which, when he tries to grasp it, melts into thin air. Also if he moves to the side, so that his eyes do not catch the reflected rays from the mirror, the illusion vanishes. The spectator cannot see the real dagger or the lamps which illuminate it, as they are concealed by the box. The illusion is a very effective one, when it is properly staged.

The “ Phantom Bouquet ” is another variant of the same illusion. Instead of a dagger a bouquet of flowers is suspended upside down inside the box, and a vase is placed at C. When the spectator is in line with the vase and the mirror, an erect image of the bouquet appears in the vase, but when he steps to the side, the vase is empty.

✱ **Caustic formed by Reflection at a Concave Mirror.**—We assumed in proving the formula for the mirror (Fig. 317), that the arc AB was small. If it is not small, the approximations made are no longer justified, and the formula does not hold.

Let AB represent a section of a concave spherical mirror, let C be its centre of curvature, and let P be a point object. Then if rays be drawn diverging from P at all angles, by making their angles of reflection equal to their angles of incidence they may be found graphically to occupy after reflection the positions shown in the figure.

Only the rays reflected from the neighbourhood of A pass through the image of P at Q; the others intersect the axis between A and Q. Any two rays reflected from neighbouring points of the mirror intersect each other before reaching the axis, and these points of intersection lie on a caustic. All the reflected rays touch this curve, and it has a

cusps at Q. The form of the caustic alters, of course, if P moves along the axis. Owing to the reflected rays coming closer together at the caustic, a bright curve is formed on a piece of paper or other white surface there. A familiar example of this is nearly horizontal sunlight shining into a teacup filled almost to the top with milk. Here the

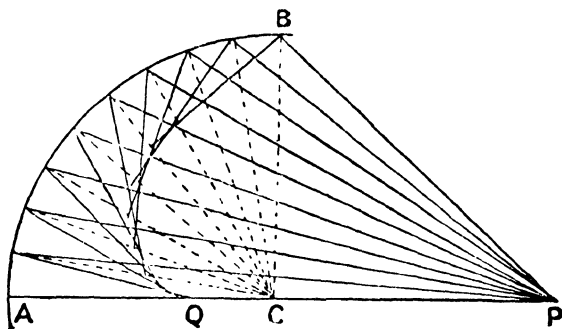


FIG. 326.

inside of the cup acts as mirror. A better way of producing the caustic is by means of a piece of polished steel spring bent circular and placed on a drawing-board.

The shape of the caustic curve can be determined graphically for other positions of P in the same way.

EXAMPLES IV

1. A concave spherical mirror has a focal length of 10 cm. Find the position, size, and nature of the image, when the object is 2 cm. high and distant (i) 24 cm., (ii) 6 cm. from the mirror. Verify by the graphical construction.

2. A concave spherical mirror has a radius of curvature of 24 cm. Find the position, size, and nature of the image, when the object is 1 cm. high and distant (i) 18 cm., (ii) 6 cm. from the surface of the mirror. Verify by the graphical construction.

3. A convex spherical mirror has a focal length of 10 cm. Find the position, nature, and size of the image, when the object is 2 cm. high and distant (i) 24 cm., (ii) 6 cm. from the mirror. Verify by the graphical construction.

4. A convex spherical mirror has a radius of curvature of 24 cm. Find the position, size, and nature of the image, when the object is 1 cm. high and distant (i) 18 cm., (ii) 6 cm. from the surface of the mirror. Verify by the graphical construction.

5. The radius of a curvature of a concave mirror is 20 cm. Find the positions of the object for which a real image three times its height and a virtual image twice its height are formed.

6. A gas flame is placed at a distance of 6 ft. from the wall of a room. Find the radius of curvature of a concave spherical mirror, and where it must be placed, in order that it may produce on the wall an image of the flame magnified four times.

7. An unlit gas burner is placed 3 ft. in front of a concave mirror of radius 2 ft. Where must a flame be placed, so that its image may appear to come out of the unlit burner? Should the flame be erect or inverted?

8. An object is placed at a distance of $\frac{1}{3}r$ in front of a concave mirror of radius r . Show that the image is erect, virtual, and magnified three times.

9. An image produced by a concave mirror is n times the size of the object. What is the distance of the latter from the mirror in terms of the focal length?

10. If the image formed by a concave mirror is 5 in. distant from the object, and the mirror itself is 4 cm. distant from the object in the other direction, find the radius of curvature of the mirror.

11. An object is held in front of a convex mirror, at a distance equal to the focal length of the mirror. Determine the size, nature, and position of the image.

12. A pencil of rays converging to a point 30 cm. behind a mirror, is brought to a focus, by reflection from its surface, at a point 15 cm. in front of the mirror. State whether the mirror is convex or concave, and determine its radius of curvature.

13. A small object on the axis of a concave mirror at a distance of 20 inches from it produces a virtual image which is three times its own size. Find the focal length of the mirror.

14. Describe what you see, when you look at your image in a concave mirror, and move backwards slowly from the mirror.

15. A plane mirror is placed 3 ft. in front of a concave mirror of 1 ft. focal length. Find where an object must be placed between the two mirrors in order that the first two images may coincide.

16. A concave and a convex mirror each 15 cm. in radius are placed opposite to each other and at 40 cm. apart. A point source of light is situated midway between them, and a pencil of rays from it is reflected first at the convex, then at the concave mirror. Find the position of the image formed.

17. A narrow strip of plane mirror is placed between a vertical knitting needle and a convex mirror. The distance between the mirrors is 4 in. When the needle is 7 in. from the strip, it is found that the two virtual images appear to coincide. What is the focal length of the mirror?

CHAPTER V

LENSES

Lenses.—A lens is a portion of a refracting medium bounded by two spherical surfaces or by one spherical surface and a plane surface. The straight line joining the centres of curvature of the surfaces is called the axis of the lens, or, if one of the surfaces is plane, the axis is the straight line normal to it drawn through the centre of curvature of the other. A plane through the axis is said to be a principal section of the lens. A lens forms an image of an object situated on its axis.

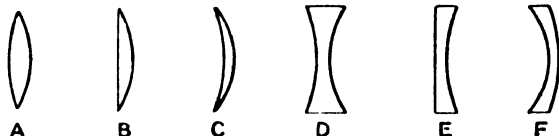


FIG. 327.—Types of lenses.

Lenses are divided into two classes. The first class, **convex** or converging lenses, cause a beam of parallel rays to converge; the second class, **concave** or diverging lenses, cause a beam of parallel rays to diverge. Fig. 327 gives the principal sections of some typical lenses. Of these, A, B, and C are convex; D, E, and F are concave. A is termed a double-convex or bi-convex lens, B a plano-convex lens, and C a convex meniscus, while D is termed a double-concave or bi-concave lens, E a plano-concave lens, and F a concave meniscus. C and F are termed also, somewhat indiscriminately, convexo-concave and concavo-convex lenses.

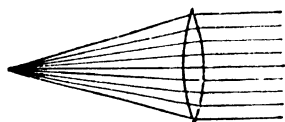


FIG. 328.

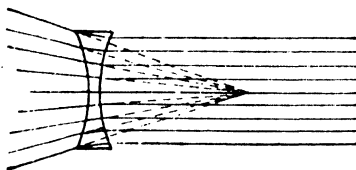


FIG. 329.

Figs. 328 and 329 show beams of parallel rays parallel to the axis incident on a convex and a concave lens respectively. The

rays are in each case going from right to left. After passing through the convex lens the rays meet in a point, the focus of the lens; after passing through the concave lens the directions of the rays produced backwards meet in a focus. The distance from the lens to the focus is in each case called the focal length of the lens, and is usually denoted by f .

Fig. 330 represents a beam of parallel rays incident on a convex

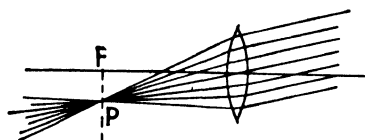


FIG. 330.

lens and making an angle with the axis of the lens. In this case they meet in a point P directly below the focus, F . Or, in other words, P is situated in the plane drawn through F at right angles to the axis of the lens.

The direction of a ray of light which passes through the centre of a lens is unaltered, because to such a ray it acts simply as a parallel plate.

Graphical Determination of Position of Image.—If the properties of the focus and the centre of the lens are assumed, the position of the image can be determined very easily graphically. For let Fig. 331 represent the case of a convex lens forming a real image. The arrow Pp drawn at right angles to the axis represents one half of the object. From p a ray is drawn parallel to the axis to meet the lens in B . After refraction by the lens it passes through the focus F . From p another ray is drawn through C , the centre of the lens. It passes through undeviated. The point

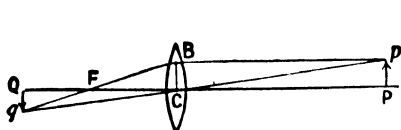


FIG. 331.

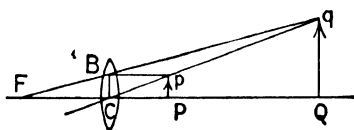


FIG. 332.

where the two rays intersect, that is, q , is the image of p , and the perpendicular to the axis, qQ , is the image of pP .

The rays BF and pC may not meet to the left of the lens, in which case a virtual image is formed. Fig. 332 represents this case. The rays BF and pC are drawn in the same way as before, and their directions produced backwards meet at q ; a virtual image of Pp is formed at Qq .

Fig. 333 represents the case of a concave lens. The ray pB is drawn parallel to the axis as before. After refraction by the lens it appears to diverge from the focus F . It consequently intersects the ray pC at q , and a virtual image of Pp is formed at Qq .

In Figs. 331, 332, and 333 only two representative rays are drawn from the point p . This is all that is necessary in order to determine the position of the image. In order to show more clearly how the lens acts, the paths of all the rays diverging from

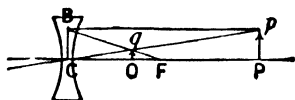


FIG. 333.

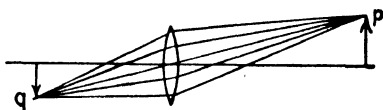


FIG. 334.

p and incident on the lens are represented for the same three cases in Figs. 334, 335, and 336. And similar diagrams might be drawn for the rays diverging from every point in the object.

Formula for Lens.—The positions of the object and image formed by a lens are connected by a formula similar to the one already proved for the spherical mirror. We shall denote the distances of the object, image, and focus from the centre of the lens by the symbols u , v , and f . These symbols are to be used algebraically. If, for example, the object is 40 cm. to the right of the lens, $u = +40$; if it is 40 cm. to the left of the lens, $u = -40$.

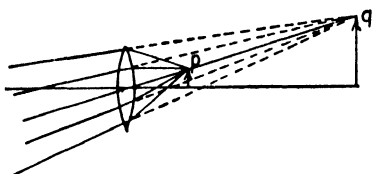


FIG. 335.

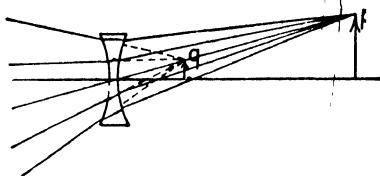


FIG. 336.

The formula can be proved from Figs. 331, 332, and 333. Consider first Fig. 331, the case of a convex lens forming a real image.

$$\begin{aligned} CB &= Pp \\ \text{hence} \quad \frac{CB}{Qq} &= \frac{Pp}{Qq} \end{aligned}$$

Triangles FQq and FCB are similar, as are also triangles CQq and CPp . Hence

$$\frac{CB}{Qq} = \frac{CF}{QF} \quad \text{and} \quad \frac{Pp}{Qq} = \frac{CP}{CQ}$$

But the left-hand sides of these two equations are already equal; hence

$$\frac{CF}{QF} = \frac{CP}{CQ}$$

But $CF=f$, $QF=-FQ=-(CQ-CF)=-v+f$, $CP=u$ and $CQ=v$. Hence, substituting

$$\begin{aligned}\frac{f}{-v+f} &= \frac{u}{v} \\ vf &= -uv + uf \\ uf - vf &= uv\end{aligned}$$

and, on dividing out by uvf ,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f},$$

the required formula.

If Figs. 332 and 333 are considered instead of Fig. 331, it will be found that the above proof holds equally well for these figures also. Hence the formula in its algebraic form is true for all possible cases of image formation by a lens.

Formula for Magnification.—The ratio of the length of the image to the length of the object is called the linear magnification, or simply the magnification. From Figs. 331, 332, and 333 we have

$$\text{Magnification} = \frac{Qq}{Pp} = \frac{CQ}{CP} = \frac{v}{u} = \frac{\text{distance of image from lens}}{\text{distance of object from lens}}$$

When the ratio v/u is positive, the image is erect; when it is negative, the image is inverted. This may be verified by an inspection of the three figures.

Conjugate Points.—It is clear from Fig. 334, that for a convex lens forming a real image the positions of object and image are interchangeable. If a real object is placed at q , a real image is formed at p . Hence p and q are called conjugate points. The definition is extended to include also the cases represented in Figs. 335 and 336 where the image is virtual. In these cases the object placed at q must not be a real one but a virtual one, otherwise there is no image formed at p . The rays must fall on the lens from the left, as if they were going to converge to a point at q ; they are then brought by the lens to a real image at p .

It is the same with the case of a spherical mirror forming a virtual image; if the virtual image is replaced by a virtual object, a real image is formed in the position of the former real object.

EXAMPLE.—A convex lens has a focal length of 24 cm. Find the position, size, and nature of the image formed of an object 1 cm. high, when the latter is situated (i) 60 cm., (ii) 12 cm. from the lens.

In the first case we have $u=60$ and $f=-24$, since it is the focus on the other side of the lens that is used. Therefore

$$\frac{1}{v} - \frac{1}{60} = -\frac{1}{24}$$

which gives $v = -40$ cm., and the magnification $= -40/60$. Hence the image is $\frac{2}{3}$ cm. high and real and inverted.

In the second case

$$\frac{1}{v} - \frac{1}{12} = -\frac{1}{24}$$

which gives $v = 24$. The magnification is $24/12$. The image is consequently on the same side of the lens as the object, 2 cm. high, virtual and erect.

EXAMPLE.—The focal length of a concave lens is 30 cm. Find the position, size, and nature of the image, when the object is distant 40 cm. from the lens, and is 1 cm. high.

We have

$$\frac{1}{v} - \frac{1}{40} = \frac{1}{30}$$

Hence $v = 17.1$ and the magnification is $\frac{12.0}{7} \times \frac{1}{40} = \frac{3}{7}$. The image is consequently $\frac{3}{7}$ cm. high, and virtual and erect.

EXAMPLE.—Rays of light from a luminous object are brought to a focus at a point P. A convex lens of 10 cm. focal length is then placed 12 cm. from P, so as to intercept the rays before they meet at P. If now they meet at Q, find the distance PQ. Where else could the lens be placed, so that, after passing through it, the rays might appear to diverge from Q?

Let us suppose that the rays are coming from the right. Then in the formula $u = -12$, $f = -10$, and

$$\frac{1}{v} + \frac{1}{12} = -\frac{1}{10}$$

which gives $v = -5.45$ cm. PQ is therefore 6.55 cm.

The other position is on the other side of P with Q as the virtual image of P. Let the distance of P from this position be u . Then

$$\frac{1}{u + 6.55} - u = -\frac{1}{10}$$

which gives $u = 5.45$ or -12 cm. The first root gives us the position we are looking for, the second root merely the first position over again. (Cf. Method 4 of p. 351.)

Two Lenses in Contact.—Two thin lenses of focal lengths f_1 and f_2 are placed in contact with their axes coincident.

Their thicknesses are so small in comparison with the other distances involved, that the centre of the combination may be taken anywhere on the axis inside the lenses. It is required to find the focal length of the single lens that is equivalent to the combination.

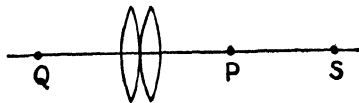


FIG. 337.

Let Fig. 337 represent the combination, let P be the object, and let S be the image formed by refraction through the first lens alone.

S in its turn becomes the object for the second lens ; after passing through both lenses the rays form a final image at Q.

Let the distances of P, S, and Q from the lenses be denoted by u , s , and v . Then, considering the action of the first lens only,

$$\frac{1}{s} - \frac{1}{u} = \frac{1}{f_1}$$

Considering the action of the second lens only,

$$\frac{1}{v} - \frac{1}{s} = \frac{1}{f_2}$$

By adding the above two equations,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2}$$

Now suppose that the two lenses are replaced by a single lens of focal length F which forms an image of P at Q. By applying the formula to this lens,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{F}$$

Hence, from the equation immediately above,

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

This equation gives the required result.

Determination of Focal Length of Convex Lens.—(1) The simplest method of determining the focal length of a convex lens is to form the image of a distant object, for example, a lamp or a window frame, on a screen, and then to measure the distance of the screen from the lens. Care must be taken that the distant object is far enough away for its image to be formed actually at the focus of the lens, otherwise the result will come out too large.

(2) Arrange the optical bench as shown in Fig. 324 and adjust the position of the lens and the screen, so that a sharp image of the cross-wires is formed on the latter. Then measure u and v , the object and image distances. At the same time measure the length Pp of the horizontal cross-wire and the length Qq of the image of the horizontal cross-wire. Repeat for different positions of the lens and enter the results in tabular form as follows :

u	v	Pp	Qq	v/u	Qq/Pp	f

f is calculated from the first two columns by means of the formula, which takes the form

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

for a convex lens producing a real image when used arithmetically, *i.e.* when the numerical values of u and v are substituted directly without prefixing their proper signs. The fifth and sixth columns give the magnification calculated in different ways, and they should agree, if the observations have been made correctly.

The lengths of the cross-wire and its image can be measured simply with callipers or with a rule. In order to obtain greater accuracy a negative should be taken of a white paper scale and from this negative two positives made on lantern plates. These positives should be bound up with pieces of ground glass instead of cover glass, the ground side being next the photographic film. The two scales thus prepared are, of course, identical. They are mounted on the bench, one being used as object instead of the cross-wires and the other being used as image screen. The magnification is then found by getting the image of the one scale directly below the other and finding how many divisions on the one correspond to the whole length of the other.

(3) Place the lens at the middle of the bench and arrange the cross-wires and image screen at equal distances from it. Then in general there will be no image formed. If now the distances of the cross-wires and image screen from the lens are gradually increased or decreased, their values being always kept equal to one another, positions will eventually be reached in which a sharp image is formed on the screen. Then u is numerically equal to v , and by substituting in the formula each is equal to $2f$. If d is the distance between cross-wires and image screen, f is numerically equal to $\frac{d}{4}$.

(4) If d is numerically less than $4f$, no real image is formed. If d is numerically greater than $4f$, for every given position of the cross-wires and image screen two positions can be found for the lens in which it gives a real image. This follows simply from the nature of the formula, for they can be written arithmetically

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}, \quad u + v = d$$

and thus u and v can be interchanged without their form being altered. Thus, if originally $u=10$ cm. and $v=15$ cm., and if the cross-wires are kept fixed and the lens moved out another 5 cm., so that u becomes 15 cm. and v becomes 10 cm., an image again appears on the screen. The magnification in the one case is the reciprocal of the magnification in the other.

If a is the distance between the two positions of the lens, $a = v - u$. But $d = v + u$; hence $2v = d + a$, $2u = d - a$,

$$\text{and} \quad \frac{1}{f} = \frac{1}{v} + \frac{1}{u} = \frac{2}{d+a} + \frac{2}{d-a} = \frac{4d}{d^2 - a^2}$$

$$\text{or} \quad f = \frac{d^2 - a^2}{4d}$$

S in its turn becomes the object for the second lens ; after passing through both lenses the rays form a final image at Q.

Let the distances of P, S, and Q from the lenses be denoted by u , s , and v . Then, considering the action of the first lens only,

$$\frac{1}{s} - \frac{1}{u} = \frac{1}{f_1}$$

Considering the action of the second lens only,

$$\frac{1}{v} - \frac{1}{s} = \frac{1}{f_2}$$

By adding the above two equations,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2}$$

Now suppose that the two lenses are replaced by a single lens of focal length F which forms an image of P at Q. By applying the formula to this lens,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{F}$$

Hence, from the equation immediately above,

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

This equation gives the required result.

Determination of Focal Length of Convex Lens.—(1) The simplest method of determining the focal length of a convex lens is to form the image of a distant object, for example, a lamp or a window frame, on a screen, and then to measure the distance of the screen from the lens. Care must be taken that the distant object is far enough away for its image to be formed actually at the focus of the lens, otherwise the result will come out too large.

(2) Arrange the optical bench as shown in Fig. 324 and adjust the position of the lens and the screen, so that a sharp image of the cross-wires is formed on the latter. Then measure u and v , the object and image distances. At the same time measure the length Pp of the horizontal cross-wire and the length Qq of the image of the horizontal cross-wire. Repeat for different positions of the lens and enter the results in tabular form as follows :

u	v	Pp	Qq	v/u	Qq/Pp	f

f , distance of the cross-wire from the lens is, of course, u , and the focal length can be obtained from the formula.

More Accurate Method of determining Focal Length.—The methods described on pp. 350–352 for determining the focal length of a convex lens are very suitable for spectacle lenses with a focal length of 20 or 30 cm., and give single results in this case which agree to about 1 millimetre. If the focal length of a camera lens or of a magnifying glass or of a microscope objective is to be measured, or if greater accuracy is desired, it will be found advantageous to use the following method :—

A horizontal paper scale is set up in a dark room and illuminated by a lamp. The lens is mounted a great distance in front of the scale, and forms a small aerial image of it. The length of this aerial image is measured by a vernier microscope (*cf.* Fig. 355), and the result divided by the length of the scale gives the magnification m . Let d be the distance of the object from the aerial image. Then, if u and v are used arithmetically,

$$m = \frac{v}{u}, \quad d = u + v = u(1 + m)$$

But

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

hence

$$f = \frac{uv}{u+v} = \frac{mu}{1+m} = \frac{md}{(1+m)^2}$$

m being usually so small, that the denominator may be made unity.

For camera lenses I have taken a 50 cm. scale distant 12 metres. For microscopic objectives the distance should be much less. The vernier microscope should have both a horizontal and a vertical motion.

Provided that d is much greater than f , the method gives accurate results for the focal lengths of thick lenses and systems of lenses, which is not the case with the methods described on pp. 350–352. The theory of thick lenses and systems of lenses is, however, beyond the scope of such a book as the present one.

Combination of Lenses and Mirrors.—(1) Convex lens, plane mirror, inverted image. If a plane mirror is placed behind a convex lens, and an object is placed at the focus of the lens in front of the latter, an inverted image the same size as the object is formed coincident with or alongside of the object. For the rays are parallel after passing through the lens both before and after reflection by the mirror.

(2) Convex lens, plane mirror, erect image. If the object is placed at twice the focal length of the lens on the one side, and the plane mirror at twice the focal length on the other, an erect image will be formed coincident with the object. This arrangement works well, however, only with a small object; better results are given with a concave mirror of radius equal to twice the focal length of the lens.

(3) Concave lens and concave mirror in contact. Let f and f' be respectively the numerical values of the focal lengths of the mirror and the lens. Then it may be shown that the combination acts as a concave mirror, the focal length F of which is given by

$$\frac{1}{F} = \frac{1}{f} - \frac{2}{f'}$$

This is analogous to the formula given on p. 350. The 2 is due to the fact that the light goes through the lens twice. The case of a plane mirror can be included in the above formula by making f infinite. The cases of convex mirror and convex lens can also be included by giving the proper signs to f and f' .

In all problems where the image is made to coincide with the object itself by refraction through a lens and reflection at a mirror, it is clear that rays diverging from a point on the axis must return along the same path. They must therefore travel along a normal to the mirror, and consequently the image formed by the first refraction is situated at the centre of curvature of the mirror.

Tracing the Position of the Image.—In the formula for the lens

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

write $u = -f + x$ and $v = f + y$. Then, if we mark the positions



FIG. 339.

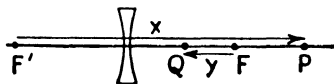


FIG. 340.

of the focus on both sides of the lens, x and y obviously denote the distances of the object and image from these points, as shown in Figs. 339 and 340, where P and Q are object and image respectively. Hence

$$\frac{1}{f+y} - \frac{1}{-f+x} = \frac{1}{f}$$

$$f(-f+x) - f(f+y) = (f+y)(-f+x)$$

$$xy = -f^2$$

and

This equation may be used for tracing the change in position of the image, when the position of the object is varied, as has been done in the corresponding case of the mirror on p. 340. But the results are probably obtained more easily and certainly with

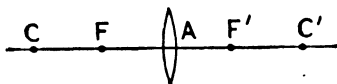


FIG. 341.

less risk of error from the graphical construction. They are shown below in tabular form; the table at the same time gives the character of the image. The references are to Fig. 341; C and C' are twice as far out from A as F and F'.

TABLE OF RESULTS

CONVEX LENS		
<i>Position of Object</i>	<i>Position of Image</i>	<i>Character of Image</i>
At infinity.	At focus.	Real.
Between ∞ and C' .	Between F and C .	Real, inverted, diminished.
At C' .	At C .	Real, inverted, same size.
Between C' and F' .	Between C and $v = -\infty$.	Real, inverted, magnified.
At F' .	At $v = -\infty$.	—
Between F' and lens.	Between $v = +\infty$ and lens.	Virtual, erect, magnified.
CONCAVE LENS		
<i>Position of Object</i>	<i>Position of Image</i>	<i>Character of Image</i>
At infinity.	At focus.	Virtual.
Between ∞ and lens.	Between focus and lens.	Virtual, erect, diminished.

Refraction at a Spherical Surface.—Let us suppose that the common boundary of two transparent media is spherical in shape, and that the rays emitted by a luminous point P in the first medium are refracted at the spherical boundary. Let μ be the index of refraction of the second medium with respect to the first. In order to have clear ideas the first medium may be thought of as air and the second as glass, but the results will be perfectly general. Just as in the case of the spherical mirror, the refracting surface may be either concave or convex with respect to the point P . We shall first take the case that it is concave.

Join P to C , the centre of curvature of the surface, and let PC meet the surface in A . Let PB be any ray making a small angle α with PC and meeting the surface at B . CB is the normal to the surface at B . After refraction the ray will proceed as if it came from Q . Let $\angle CBP = \phi$ and $\angle CBQ = \theta$. Then by the law of refraction $\sin \phi = \mu \sin \theta$. Since the angle α is small, the angles ϕ and θ must also be small. Hence this equation may be written

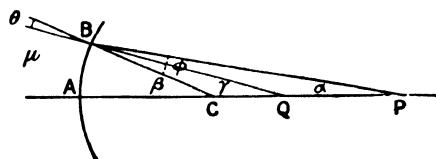


FIG. 342.

Let $\angle AQB = \gamma$ and $\angle ACB = \beta$
 Then $\theta = \beta - \gamma$, $\phi = \beta - \alpha$
 and on substituting in the above equation this gives
 $\beta - \alpha = \mu(\beta - \gamma)$ or $\mu\gamma - \alpha = (\mu - 1)\beta$

Now let $AP=u$, $AQ=v$, and $AC=r$, where u , v , and r are all measured positive to the right of A . We may then write

$$\alpha = \frac{AB}{u}, \quad \gamma = -\frac{AB}{v}, \quad \text{and} \quad \beta = -\frac{AB}{r}$$

and on substituting these results and dividing out the common factor we obtain

$$\frac{\mu}{v} - \frac{1}{u} = \frac{\mu-1}{r}$$

If the point P is on the other side of C , $\phi = \alpha - \beta$ and $\theta = \gamma - \beta$, but the proof is otherwise the same. Thus all rays diverging from

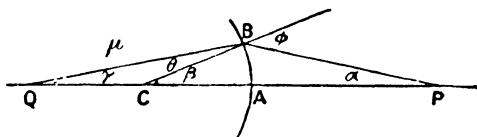


FIG. 343.

P and making a small angle with AP appear to come from Q after refraction, or in other words, a virtual image of the point P is formed at Q .

If the refracting surface is convex (Figs. 343 and 344), there are two cases according to whether BQ meets AP to the left or the right of A .

If we take the former case (Fig. 343), $\phi = \alpha + \beta$, $\theta = \beta - \gamma$, and consequently

$$\mu\gamma + \alpha = (\mu-1)\beta$$

Since $\alpha = \frac{AB}{u}$, $\gamma = -\frac{AB}{v}$, and $\beta = -\frac{AB}{r}$, this reduces to the former result. In this case the image is real.

If we take the latter case (Fig. 344), $\phi = \alpha + \beta$, $\theta = \beta + \gamma$, and therefore

$$\mu\gamma - \alpha = (1-\mu)\beta$$

In this case $\alpha = AB/u$, $\gamma = AB/v$, and $\beta = -AB/r$, so the final equation is once more the same. In this case the image is virtual.

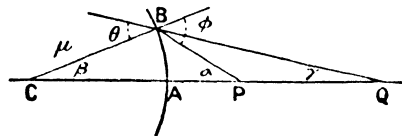


FIG. 344.

If in Figs. 342, 343, and 344 the refracting surface, and consequently C , is kept fixed, and PC rotated through a small angle about an axis through C perpendicular to the plane of the paper, P and Q describe small arcs which

may be regarded as straight lines, and the one straight line is the image of the other.

EXAMPLE.—A small air bubble in a sphere of glass of 4 cm. diameter appears to be 1 cm. from the surface, when looked at along a diameter.

If the refractive index of the glass is 1.52, find the true position of the bubble.

The formula is

$$\frac{1}{v} - \frac{\mu}{u} = \frac{1 - \mu}{r}$$

since the object is inside the glass. Let us suppose that the eye is looking towards the right. Then $v=1$, $r=2$, and

$$\frac{1}{1} - \frac{1.52}{u} = \frac{1 - 1.52}{2}$$

which gives $u=1.21$ cm. Thus the air bubble appears nearer than it really is, the same as in the case of a glass slab.

Refraction through a Lens.—In proving the formula for the lens on pp. 347-48, we assumed both the properties of the focus and the centre of the lens, and also the fact that the lens forms images. We shall now prove the same formula without making these assumptions; the proof will be based on the formula for refraction at a spherical surface, and so will assume nothing but the law of refraction.

Let Fig. 345 represent a lens which is placed in a medium with reference to which the index of refraction

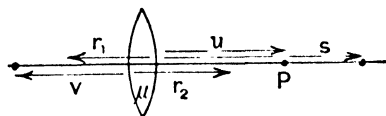


FIG. 345.

of its material is μ . Let a luminous point P be situated on its axis at a distance u from its first surface. Let r_1 and r_2 be the distances of the centres of curvature of the two surfaces each measured from its own surface, and let t be the thickness of the lens measured along its axis.

An image of P is formed by refraction at the first surface. Let its distance from the first surface be s . Then by the formula for refraction at a spherical surface

$$\frac{\mu}{s} - \frac{1}{u} = \frac{\mu - 1}{r_1} \quad \dots \quad (1)$$

The image is distant $s+t$ from the second surface of the lens, and a second image is formed of it by refraction at the latter. Let v be the distance of the second image from the second surface. Then applying the formula a second time, remembering that $1/\mu$ now fills the position formerly occupied by μ ,

$$\frac{1}{v} - \frac{1}{s+t} = \frac{1 - \mu}{r_2}$$

or

$$\frac{1}{v} - \frac{\mu}{s+t} = \frac{1 - \mu}{r_2}$$

Let us now suppose that the lens is thin, and that it may be neglected in comparison with s . The above equation thus becomes

$$\frac{1}{v} - \frac{\mu}{s} = \frac{1-\mu}{r_2} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Add the left-hand and right-hand sides of equations (1) and (2) and we obtain finally

$$\frac{1}{v} - \frac{1}{u} = (\mu - 1) \left(\frac{1}{r_1} - \frac{1}{r_2} \right) \quad . \quad . \quad . \quad . \quad (3)$$

If the object is at an infinite distance from the lens, the incident rays are parallel, $1/u = 0$, and v is given by

$$\frac{1}{(\mu - 1) \left(\frac{1}{r_1} - \frac{1}{r_2} \right)}$$

As has already been mentioned, this quantity is denoted by f , and is called the focal length of the lens. If f is substituted in (3) the fundamental equation of the thin lens becomes

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

the formula we have already obtained. But in proving the formula this time, we have made no other assumption than the law of refraction.

Formula for Focal Length of Lens.—In using the equation for f ,

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

due regard must be paid to the signs of r_1 and r_2 . If the lens is bi-convex or bi-concave, the surfaces are turned different ways and r_1 and r_2 have different signs. Which is positive and which is negative depends upon whether the lens is bi-convex or bi-concave, and also on which side the lens is to the right of the page, but in any case, after the numerical values are substituted, the two terms in the second bracket must have the same sign. If the lens is a meniscus one, r_1 and r_2 have the same sign, and after the numerical values are substituted the two terms in the second bracket have consequently different signs; the two curved surfaces partly neutralise one another. If the lens is plano-convex or plano-concave, either r_1 or r_2 becomes infinite.

EXAMPLE.—The radii of curvature of the two faces of a concave lens are 10 and 20 cm., and the refractive index of the glass of which it is

made is 1.5. Find the numerical value of the focal length (*a*) if the lens is bi-convex, (*b*) if it is convex meniscus.

We have in the first case

$$\frac{1}{f} = (1.5 - 1) \left(\frac{1}{10} + \frac{1}{20} \right), \text{ i.e. } f = 13.3 \text{ cm.}$$

and in the second case

$$\frac{1}{f} = (1.5 - 1) \left(\frac{1}{10} - \frac{1}{20} \right), \text{ i.e. } f = 40 \text{ cm.}$$

Optical Centre of a Lens.—If a ray of light passes through a lens undeviated, that is, if its direction after emergence is parallel to its direction before incidence, the lens must act on it merely as a parallel slab, and the tangent planes at the points of incidence and emergence must be parallel.

Let $E_1G_1G_2E_2$ be such an undeviated ray. Then the radii C_1G_1 and C_2G_2 perpendicular to the tangent planes must be parallel. Let G_1G_2 cut the axis of the lens at C .

Triangles C_1CG_1 and C_2CG_2 are similar, for $\angle G_1C_1C = \angle G_2C_2C$, since C_1G_1 is parallel to C_2G_2 , and $\angle C_1CG_1 = \angle C_2CG_2$. Thus $C_1C : C_2C :: C_1G_1 : C_2G_2$, or, in other words, the distance between the centres is divided in the ratio of the radii and C is a fixed point. All rays, therefore, that are undeviated by the

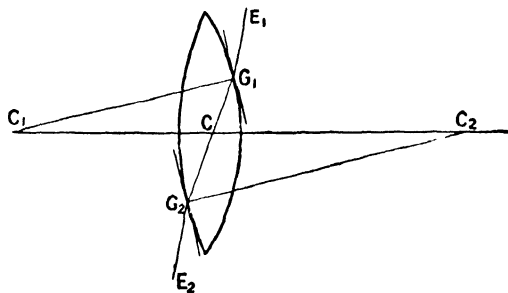


FIG. 346.

lens pass through a fixed point on the axis. This point is called the optical centre of the lens.

Conversely, if any ray passes through the optical centre, it is undeviated, for, if the radii be drawn to the points of incidence and emergence, they can be proved to be parallel.

Fig. 346 is exaggerated for the sake of clearness. It is never necessary to consider rays incident so obliquely as E_1G_1 . When the lens is thin, we may regard its optical centre and the two points in which the axis meets its surface as coincident.

We assumed, on p. 346, that rays which passed through a lens near its centre would be undeviated, or would suffer a very slight

deviation, because in this region the lens has almost the same shape as a plane parallel plate. We have now proved, that for rays passing through a certain definite point the property is not approximately but absolutely true.

Cylindrical Lenses.—Let us suppose that the surfaces of a thin piece of glass are cylinders, the axes of which are parallel. Such a piece of glass is called a cylindrical lens.

If a section is made of it by a plane at right angles to both axes, it has the same shape as the section of a lens with spherical surfaces. Consequently any pencil of rays diverging from a point in this plane is brought to a focus by the lens. On the other hand, the section by any plane parallel to the axes is the same as the section of a thin plate with parallel sides. Consequently a pencil of rays in this plane is unaffected by the lens.

A cylindrical lens thus forms images only of lines parallel to the axes of its surfaces. If such a lens is placed on the optical bench with its axes parallel to one of the cross-wires, it forms a sharp image of that wire, but no image at all of the other wire. As far as the other wire is concerned, the lens acts merely as a plane parallel plate.

With this limitation the focal lengths of cylindrical lenses and mirrors can be determined on the optical bench in the same way as the focal lengths of spherical lenses and mirrors.

EXAMPLES V

1. A convex lens has a focal length of 30 cm. Find the position, size, and nature of the image formed of an object 2 cm. high, when the latter is situated (i) 80 cm., (ii) 10 cm. from the lens.

2. A convex lens has a focal length of 18 cm. Find the position, size, and nature of the image formed of an object 1 cm. high, when the latter is situated (i) 12 cm., (ii) 60 cm. from the lens.

3. A gas flame is at a distance of 8 ft. from a wall. Where must a convex lens of 1 ft. focal length be placed in order to form a sharp image of the flame on the wall?

4. A concave lens has a focal length of 25 cm. Find the position, size, and nature of the image formed of an object 2 cm. high, when the latter is 40 cm. from the lens.

5. A concave lens has a focal length of 18 cm. Find the position, size, and nature of the image formed of an object 1 cm. high, when the latter is situated (i) 54 cm., (ii) 12 cm. from the lens.

6. A beam of light converges to a point P. A convex lens of focal length 20 cm. is then placed 30 cm. from P, so as to intercept the rays before they meet at P. If now they meet at Q, how far is Q from the lens?

What would be the effect of replacing the convex lens by a concave lens of the same focal length in the same position ?

7. An intense point source of light is situated 10 ft. from a wall, and a convex lens of focal length 18 in. is placed directly between the source and the wall at a distance of 3 ft. from the source. What is the diameter of the circle of light thrown by the lens on the wall ? The diameter of the lens is 3 in.

8. A convex lens is mounted on an optical bench, and it is found, that when the object distance is 27.3 cm., the image distance is 42.7 cm., and that when the object distance is 56.0 cm., the image distance is 23.6 cm. Find the mean value of the focal length of the lens.

9. In order to determine the focal length of a convex lens it is used to form an image of a lamp at a distance of about 5 metres. The lamp is supposed to be infinitely far off, and the distance of the image from the lens, which is found to be 24.2 cm., is then taken to be the focal length of the lens. Calculate the magnitude of the error involved.

10. Show that a convex lens cannot form a real image on a screen which is distant less than $4f$ from the object.

11. If an object at a distance of 14.0 cm. from a convex lens has its image magnified three times, what is the focal length of the lens ?

12. At what distance from a convex lens must an object be placed, in order that the image may be (i) real and magnified three times ; (ii) virtual and magnified three times ? Express your result in terms of the focal length.

13. An image of the moon is formed on a piece of white cardboard by means of a lens, and the diameter of the image is measured, and found to be 6 mm. Find the focal length of the lens, given that the moon subtends an angle of $31'$ at the surface of the earth.

14. A camera is provided with a little celluloid scale, on which are marked the positions at which the lens must be placed in order to photograph objects at infinity, 15, 10, 5, 3, and 2 ft. respectively. Construct a plane of this scale, given that the focal length of the lens is 5.2 in.

15. A lantern slide $3\frac{1}{4}$ in. square is to be projected on a screen 8 ft. square at a distance of 30 ft. What is the focal length of the lens required ?

16. A quarter plate, $4\frac{1}{2}$ by $3\frac{1}{4}$ in., is to be enlarged to whole plate size, $8\frac{1}{2}$ by $6\frac{1}{2}$ in., by means of a lens of $5\frac{1}{2}$ in. focal length. How far should the negative and printing paper be placed from the lens ?

17. Light from a luminous object passes through a concave lens and, after reflection from a concave mirror, forms a real inverted image of the object between the lens and the mirror. Make a diagram showing the path of the rays from a corner of the object.

18. The plane side of a plano-convex lens is silvered, and the lens then acts like a concave mirror of 20 cm. focal length. Calculate the radius of curvature of the convex face, given that the refractive index of the glass is 1.52.

19. A small air bubble in a sphere of glass 5 in. in diameter appears to be 2 in. from the surface, when looked at along a diameter. If the refractive index of the glass is 1.50, find its true position.

20. A convex lens throws a real image on a screen. A glass slab 1 in. thick is placed between the lens and screen, and the image becomes indistinct. How far and in what direction must the screen be moved in order to make it sharp again? Illustrate your result with a diagram. The index of refraction of the glass is 1.52.

21. Find the focal length of a plano-convex lens, given that the radius of curvature of its convex surface is 40 cm., and that the refractive index of its material is 1.523.

22. The radii of curvature of the two faces of a concave lens are 12 and 24 cm., and the refractive index of the glass of which it is made is 1.5. Find the numerical value of the focal length: (a) if the lens is bi-concave; (b) if it is concave meniscus.

23. A convex lens of 20 in. focal length forms an image of an arrow which lies along the axis of the lens with its mid-point 30 in. from the lens. The length of the arrow is 2 in. Find the length of the image.

24. A lens of focal length 10 cm. made of glass of refractive index 1.52 is immersed in a long tank filled with water 1 metre from the end of the tank. It forms an image of this end on a screen held under water near the other end of the tank, *i.e.* both object and image are under water. The index of refraction of water is 1.33. How far is the screen from the lens?

25. A convex and a concave lens each of 8 in. focal length are held coaxially at a distance of 4 in. apart. Find the position of the image, if the object is at a distance of 12 in. beyond (a) the convex lens, (b) the concave lens.

CHAPTER VI

OPTICAL INSTRUMENTS

Magnifying Glass.—The apparent size of an object depends on the angle which it subtends at the eye; the nearer it is brought to the eye, the larger it appears. If, however, it is brought nearer than a distance of 10 or 12 in., the eye either cannot focus it, or has to make an exertion to see it distinctly. Hence this distance, which is called the **distance of distinct vision**, is the most favourable position for examining the detail of an object.

Let AB be a convex lens of small focal length placed before the eye at E, let F be its focus, and let Pp be an object within its focal

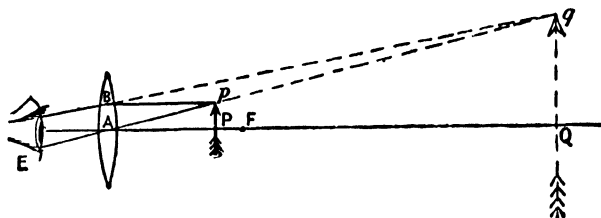


FIG. 347.—Magnifying glass.

length. Then on applying the usual graphical construction, the image is found to be at Qq. All rays proceeding from Pp to the eye, after refraction by the lens, appear to come from Qq.

In using a magnifying glass we hold it so that the image is at the distance of distinct vision. Thus the distance of Q from the eye is equal to D, the distance of distinct vision, or, since the eye is near the lens, we may put $AQ = D$. If the object Pp were to be examined by the eye direct, it would have to be placed at Q. The magnification, that is, the ratio of the apparent lengths with and without the lens, is thus Qq/Pp . But by similar triangles

$$\frac{Qq}{Pp} = \frac{AQ}{AP} = \frac{D}{f}$$

since AP is almost equal to f, the focal length of the lens. *The magnification is consequently equal to D/f.*

A single convex lens used as above is sometimes called a simple microscope. Instead of a single lens a combination of thin lenses may be used cemented together, for example, two concave meniscus lenses of flint glass separated by a double convex lens of crown glass. Such a combination gives a sharper image.

EXPERIMENT.—Rule two lines on a card, and look at them through a magnifying glass held close up to one eye. At the same time look with the other eye at a scale placed at the distance of distinct vision. By getting the image of the lines to superimpose on the scale, and measuring the distance between the lines, the magnification can be determined. Compare it with the theoretical value.

Astronomical Telescope.—A microscope is used for obtaining a magnified image of a near object. Telescopes are employed for

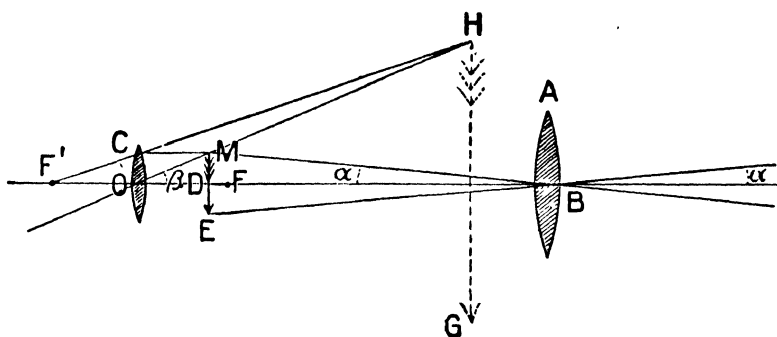


FIG. 348.—Telescope. Diagram for magnification formula.

obtaining distinct vision of distant objects. They are of two kinds, refracting and reflecting, but the same principle underlies them all. An inverted image of a distant object is formed by means of a convex lens—the object glass—or a concave mirror; then this image is examined by an eyepiece.

The most important form of refracting telescope, the astronomical telescope, was invented by Kepler in 1611. The eyepiece of this telescope is usually a compound one, consisting of two lenses an interval apart. It fulfils the same function as the magnifying glass described in the preceding section, and could be replaced by a single convex lens. In order to make the explanation of the instrument simpler, we shall, therefore, suppose the compound eyepiece replaced by a single magnifying lens. The theory of the instrument is then represented in Fig. 348.

AB is the object glass and CO the eyepiece. The object glass forms a real and inverted image ME of the distant object very near its focus. BM and BE are the two undeviated rays from the

extremities of the distant object through the centre of the object glass to the extremities of this image. Let F and F' be the two foci of the eyepiece. Draw from M a ray MC parallel to the axis and a ray MO through the centre of the eyepiece; then, by the graphical construction, we find that a virtual image of ME is formed at HG .

The magnifying power of the telescope is measured by the ratio of the angle subtended at the eye by the image in the telescope to the angle subtended by the distant object itself. Now the length of the telescope is short compared with the distance of the distant object. Hence the angle subtended by the distant object at the eye is 2α (see Fig. 348). The eye is held close up to the eyepiece; hence the angle subtended by the final image HG at the eye is 2β . Consequently the magnifying power is

$$\frac{2\beta}{2\alpha} = \frac{\beta}{\alpha} = \frac{MD/OD}{MD/BD} = \frac{BD}{OD}$$

since the angles are small. But BD and OD are almost equal to the focal lengths of the object glass and eyepiece respectively. Hence the **magnifying power** of the telescope is equal to

$$\frac{\text{Focal length of object glass}}{\text{Focal length of eyepiece}}$$

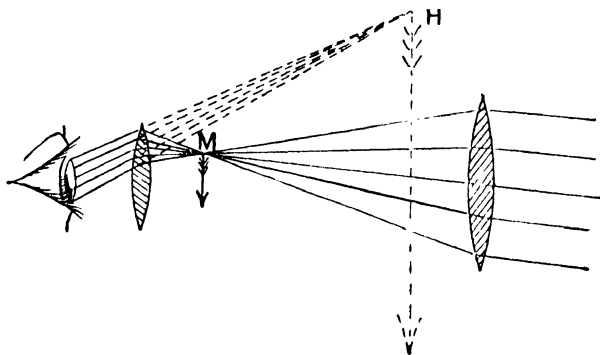


FIG. 349.—Telescope. Diagram for path of rays.

Fig. 348 is drawn for the purpose of proving the magnification formula. Fig. 349 shows the path of a beam of parallel rays through the same telescope to the eye of the observer. The object is so far away that the rays incident on the object glass from any point on it are parallel. A similar diagram might be drawn for every single point on the image. It is clear from the diagram, that the telescope inverts the image, *i.e.* makes objects appear upside down.

The advantage of the compound eyepiece over the magnifying glass is that it gives sharper images. The object glass of the astronomical telescope usually consists of two lenses, cemented together, made of different kinds of glass; the reason for this is given on pp. 392-394. The amount of light received by a telescope is proportional to the area of the object glass; if half the surface of the latter is covered, the image in the field becomes fainter, but it still remains in focus.

Resolving Power of a Telescope.—The magnification of a telescope is given by the ratio of the focal length of the object glass to the focal length of the eyepiece. Hence if we were to increase the focal length of the object glass three times, say, keeping its diameter constant, the magnification would be increased three times. But nothing would be gained by this; the image in the field would become three times larger, but no fresh detail would be revealed. It would be like pulling out an elastic sheet on which a picture is painted.

When a telescope is focussed on a star, the image in the field is not a point, but a small disc surrounded by one or two faint concentric rings. When a telescope is focussed on an extended object, each point of the object forms its own disc, and thus the image has a definite grain.

If two stars are very close together, they are seen as one. If, however, the centre of the disc image of the one has separated a certain distance from the centre of the disc image of the other, the eye recognises them as two. It is found that the angle subtended by the two stars at the eye of the observer, when their images in the telescope can just be separated, is

$$\frac{5.0}{d}$$

measured in seconds of angle, where d is the diameter of the object glass measured in inches. This angle is called the resolving power of the telescope. Thus, if d is two inches, the resolving power of the telescope is $2\frac{1}{2}$ sec. The greater the resolving power, the finer the grain of the image, and the greater the detail that can be recognised in the object.

The resolving power of a telescope depends wholly on the diameter of the object glass. The object glass of the Yerkes telescope, the most powerful one employed hitherto for astronomical observation, has a diameter of 40 in. and a focal length of 65 ft. Its resolving power is consequently about $\frac{1}{8}$ sec. To grind and polish this object glass required great care and took a long time. It is also extremely difficult to obtain pieces of glass this size of the necessary homogeneity, and it is possible that if the object glass were larger it might strain under its own weight.

So that the limit of resolving power in telescopes is probably nearly reached.

The most interesting objects on which to try a telescope are the planets Venus, Jupiter, and Saturn. Their apparent angular diameters vary from 11 to 67 sec., 32 to 50 sec., and 14 to 20 sec. respectively, according to their positions in the heavens. An astronomical telescope of the kind fitted to the larger spectrometers usually shows the phases of Venus and four of Jupiter's moons, but is not powerful enough to bring out Saturn's rings.

To measure the Magnification of a Telescope.—Set up a horizontal white scale at a great distance, and fix a sliding mark on it. Focus the telescope on the scale, and observe the latter simultaneously with one eye through the telescope and with the other direct. The two images superimpose. The mark is then moved along the scale until the image of the part cut off by it, as seen through the telescope, is equal in length to the whole scale as seen direct. The magnification is then obtained by dividing the length of the whole scale by the length of the part cut off.

To measure the Resolving Power of a Telescope.—Get a piece of fine wire gauze about an inch square, and count the number of wires there are per centimetre. Hence calculate the separation between the centres of two adjacent wires. Mount the gauze in front of a lamp in a darkened room, with a piece of ground glass behind it in order to give a uniformly illuminated background, or place it against a window with a north sky as background. Then find the greatest distance at which the wires of the gauze are seen separate through the telescope; when this distance is determined, one step further away from the gauze is sufficient to make the latter appear one uniform blur. The separation between two adjacent wires of the mesh divided by this distance gives the resolving power in radians, from which it can easily be reduced to seconds.

The Galilean Telescope.—The great disadvantage of the astronomical telescope is that it inverts the image. This does not prevent its employment in astronomy or in spectroscopy, where it does not matter whether the image is upside down or not, but it renders it unsuitable for general terrestrial use.

The Galilean telescope, which was made and used by Galileo a year or two previous

to Kepler's discovery of the astronomical telescope, gives an erect image. It differs from the astronomical telescope solely in the eyepiece. This consists of a single concave lens.

In Fig. 350 a convergent pencil of rays is incident on the concave lens from the right. Two rays of the pencil, Bp and qA , are

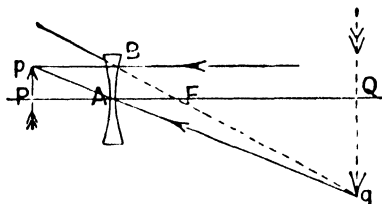


FIG. 350.—Galilean telescope.

shown. If the concave lens were removed, the pencil would form a real image at p ; but as it is, only the directions produced pass through p , and p acts as a virtual object. The ray qA passes through the centre of the lens, and is consequently undeviated, and the ray Bp appears after refraction to come from the focus F . Consequently a virtual image is formed of p at q . Let us suppose that AQ is the distance of distinct vision D , and that AF , the focal length f , is small in comparison with D . Then AP is almost equal to f . Consequently the magnification

$$\frac{Qq}{Pp} = \frac{AQ}{AP} = \frac{D}{f}$$

by similar triangles.

The concave lens therefore acts as a magnifying glass just as the convex lens in Fig. 317. But there are two important differences. The concave lens can be used only with virtual, aerial images; it could not be employed to magnify any real object, for example, a diagram on a piece of paper. Also it inverts the image. But as the object glass of the Galilean telescope already inverts the image, the second inversion produced by the eyepiece brings the image the right way up.

It may be shown by a diagram similar to Fig. 318, that the magnification of a Galilean telescope is given by the ratio of the focal length of its object glass to the focal length of its eyepiece, the same formula as for the astronomical telescope. The Galilean telescope is shorter than the astronomical one, its length being approximately equal to the difference of the focal lengths of the object glass and eyepiece, whereas in the case of the astronomical telescope the length is approximately equal to their sum.

Ordinary field glasses and opera glasses consist of two Galilean telescopes mounted side by side and focussed by the same screw.

Terrestrial Telescope. Prism Glasses.—The principle of the terrestrial telescope is represented in Fig. 351. The object glass O forms

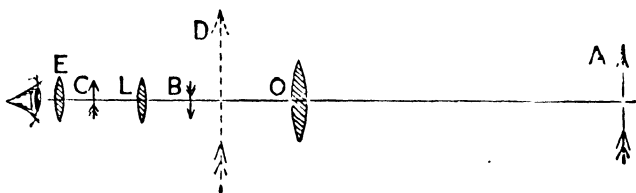


FIG. 351.—Terrestrial telescope.

a real, inverted image of the distant object A at B . The erecting lens L forms a real, inverted image of B at C , and the eyepiece E forms a virtual image of C at D . In practice instead of a single lens L a pair

of lenses is used, and instead of a single eyepiece a compound eyepiece, but their action is the same as that of the single lenses shown in the diagram. The chief disadvantage of the terrestrial telescope is its length.

Fig. 352 represents prism glasses. This instrument was introduced by Messrs. Zeiss in 1895. It consists of two astronomical telescopes mounted side by side, each containing two right-angled glass prisms. The 90° edges of the prisms are at right angles to one another. Each of these prisms turns the rays through 180° , and at the same time reverses the image, as shown in Fig. 353. The one prism reverses the image right and left, the other up and down. The rays traverse the

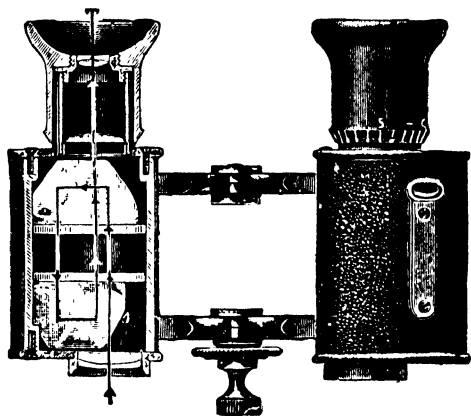


FIG. 352.—Prism glasses.

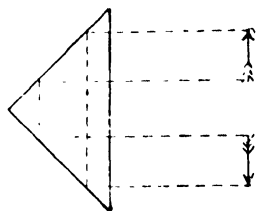


FIG. 353.

body of the instrument three times. Thus prism glasses combine the advantages of the astronomical telescope, large field and bright image, with an erect image and a short length.

Mirror Telescopes.—In reflecting telescopes the object glass is replaced by a concave mirror which forms a real image of the distant object. This is examined by an eyepiece. If the eyepiece were focussed on the image directly, the head of the observer would get in the way of the incident light, and the various types of reflecting telescope differ in the manner in which they overcome this difficulty.

Mirrors for reflecting telescopes were originally made of speculum metal, a somewhat brittle alloy of copper and tin, and when the surface tarnished, it had to be repolished, and thus the most difficult and critical part of its construction repeated. They are now made of glass and silvered, and an old silver film can quite easily be replaced by a new one.

Reflecting telescopes are used only in astronomy. Large reflecting telescopes are considerably less expensive than refracting instruments of the same power, but they are inconstant and require careful attention.

The Microscope.—If the focal length of a simple magnifying glass is diminished in order to obtain increased magnification, the eye has to be placed inconveniently close to the object examined; also lenses of such small focus are difficult to grind accurately, and it is not possible to form such sharp images with a single lens as with a system of lenses. For large magnifications it is therefore necessary to use a compound microscope. The principle of the latter instrument is represented in Fig. 354 in a simplified form.

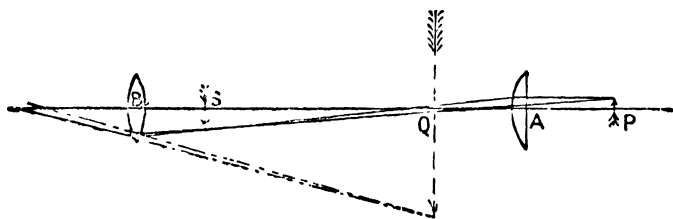


FIG. 354.—Compound microscope.

In the compound microscope the work of forming the image is distributed over two lenses or lens systems, the object glass and the eyepiece. It is the function of the object glass to form an image of the object by as wide angle a pencil as possible, and it is the function of the eyepiece to form a large image of this image by means of thin pencils.

The compound microscope may be regarded as an astronomical telescope adapted to view near objects. Suppose that in the case of Fig. 348 the object is brought nearer and nearer the object glass, but that the focal length of the latter is made shorter and shorter, so that the image ME remains in the same place. Then finally the object is close up in front of the object glass, the focal length of the object glass is smaller than the focal length of the eyepiece, and Fig. 348 has passed over into Fig. 354.

In Fig. 354 P is the object, A the object glass, S the image formed by the object glass, B the eyepiece, and Q the enlarged virtual image of S formed by B. Let the image Q be situated at the distance of distinct vision, D, from the eye, which is the same thing approximately as the distance from the lens B. Let v be the distance of S from the lens A, and let L be the length of the object. Let F be the focal length of the object glass and f the focal length of the eyepiece.

The object is situated just beyond the focus of A, so the length

of S is approximately $(vL)/F$. B acts as a simple magnifying glass, so the length of the image at Q is

$$\frac{vL}{F} \cdot \frac{D}{f}$$

Hence the **magnification** is

$$\frac{vD}{Ff}$$

Owing to the great magnification produced, the object examined by a compound microscope must be brightly illuminated, otherwise the image would be too faint, and most microscopes are fitted with a system of lenses for this purpose, called a **condenser**.

Fig. 355 represents a compound microscope of a type used in physical laboratories. It is designed not for magnifying, but for measuring. The tube, which is shown vertical, can be turned horizontal, and its position can be adjusted in both a vertical and horizontal direction, and read on either of the two scales. The instrument is thus very convenient for making accurate measurements of changes in length or height.

The Sextant.—It has been shown, that when a mirror is rotated, the angle turned through by the reflected ray is twice the angle turned through by the mirror. The sextant, which is founded on this principle, is an instrument used for measuring the angle subtended at the observer by two distant objects. It was invented by Hadley in 1730. It consists of a telescope, T, directed towards a mirror, A, only half of which is silvered and the rest transparent, a mirror, B, which can be rotated, and a pointer attached to B, which reads its position on a graduated circle CD.

Consider an object at such a distance that S_1 and S_2 , the rays from it to the mirrors A and B, are parallel. Let S_1 be the ray from it to the mirror A, and let the mirror B be in such a position that the ray S_2 after reflection at B and A enters the telescope in the direction S_1 . Then the two images of the object will be seen superimposed. The reading of the pointer for this position of B is taken as zero. Suppose that, starting from this zero and

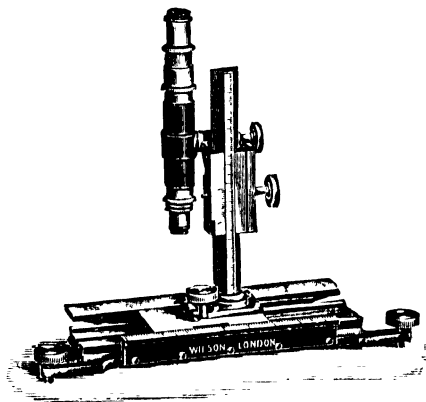


FIG. 355.—Vernier microscope.

keeping the telescope directed on one of the two objects the angle between which is to be measured, it is necessary to rotate B through an angle α to bring the image of the other one into coincidence. Then the angle they subtend at the eye is 2α . The angles on the

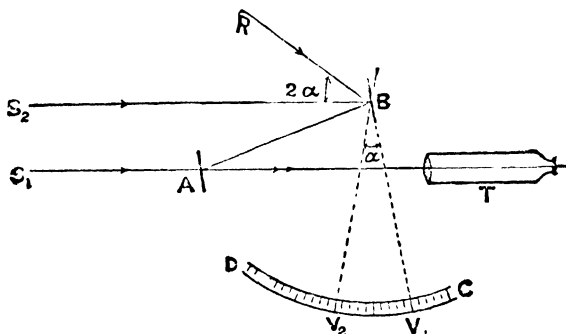


FIG. 356.—Sextant.

scale CD are numbered at twice their proper value. Hence the reading V_1V_2 of the pointer gives the angle subtended directly.

The sextant is used principally for taking the angle of elevation of the sun. The arrangement usually adopted on land is indicated in Fig. 357. The observer sits in front of a mercury trough, resting the elbow of the arm that holds the instrument on the knee. The telescope is pointed in the direction BC towards the image of the sun in the mercury, and the image of the sun formed by reflection on the movable mirror is brought to coincide with it by moving the pointer along the graduated circle. The scale reading then gives 2α (Fig. 357), and since AB and CD are parallel, α is the altitude of the sun. A mercury surface is used instead of a mirror, because it sets itself horizontally under the action of gravity.

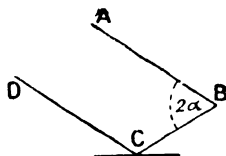


FIG. 357.

At sea the angle between the sun and the horizon is taken.

It is easy to see from a figure, since the position of the sun in the heavens is known, that the latitude can be determined from its altitude at noon. If the time of its highest altitude is noted on the chronometer, which keeps Greenwich time, the time the earth has been turning since the sun was above the meridian at Greenwich is known, and hence the longitude can be determined.

Photographic Camera.—A photographic camera consists essentially of a rectangular box, at one end of which is placed the sensitive plate or film, and at the middle of the opposite end

of which is placed the lens. The inside of the box is painted black, as is usual with optical instruments, in order to absorb stray light. The distance of the lens from the plate can be altered, so as to focus the picture sharply on the latter, and no light must enter the camera except through the lens. By means of a series of stops placed at the lens the aperture of the pencils of light forming the image can be regulated. The diameter of the aperture is always expressed as a fraction of the focal length of the lens; thus, if a lens is working at $f/16$, we mean that the diameter of the stop is $\frac{1}{16}$ of its focal length. A lens working at $f/8$ thus admits about eight times as much light as one working at $f/22$, but, of course, does not give such a sharp picture.

The modern photographic plate consists of an emulsion of silver bromide in gelatine, which has been poured while warm on to a large glass plate and allowed to set. The glass plate is then cut into smaller sizes. The light produces no visible action on the plate until the developer is poured over it. Then the particles of silver bromide on which the light was incident are reduced to black metallic silver. After development the plate is "fixed" by immersion in a solution of hyposulphite of soda, which dissolves away all the sensitive particles on which the light has not acted. It is then washed and dried. As the bright parts of the image, those on which most light falls, come out black on the plate, and the dark parts come out light, the picture is said to be a negative. By placing it in contact with sensitive paper, and allowing light to act through the negative on the paper, an image called a positive is produced on the paper. In this image the light and shade are correctly rendered; it is the picture wanted, the negative being only an intermediate step.

Optical Lantern.—Fig. 358 depicts the optical system of a lantern used for projecting slides. AB represents the slide; it is

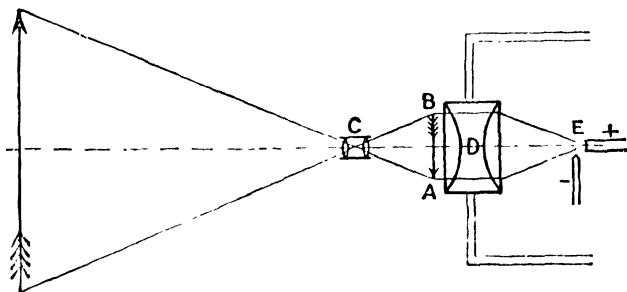


FIG. 358.—Projection lantern.

placed upside down. C is the objective or projecting lens. It forms an erect image of the slide on the screen, and by focussing it

the image can be made sharp for different positions of the screen. The source of light, E, is an electric arc, the positive carbon of which is horizontal and the negative vertical. By this arrangement the full light from the crater falls upon the condenser D. The latter consists usually of two plano-convex lenses with their plane faces outwards, and its function is to make as much light as possible pass through the slide in such a direction that it also passes through the objective. It focusses the crater of the arc on the objective. If the condenser were removed and replaced by a piece of ground glass in order to give a uniform background, the image would be much fainter. The electric arc is enclosed in a box, so as to keep in all light except that which forms the image on the screen. Limelight may be used instead of the arc, but the arc is brighter and more convenient, and is always to be preferred if available.



FIG. 359.—Reversing prism.

The lantern can be employed for projecting pieces of apparatus, if the latter are placed in the position usually occupied by the slide. They cannot be reversed as easily as a slide can, and so would appear on the screen upside down. But when this is undesirable, a reversing prism is used; this is placed close to the projecting lens. It consists simply of a total reflection prism, and acts as shown in Fig. 359.

Cinematograph.—Visual impressions on the retina persist after the removal of the stimulus for about $\frac{1}{16}$ of a second. Thus, if photographs are taken of a moving object at a rate of not less than 16 per sec., and if these photographs are projected on a screen at the same rate, the discontinuous pictures fuse together, and produce an illusion of continuous motion. This is the principle of the cinematograph. The pictures are smaller than lantern slides, the regulation size being about 1 in. by $\frac{3}{4}$ in., and they are projected by a lens of about $2\frac{3}{4}$ in. focal length. They are printed on a film of celluloid or a similar preparation, and the film is uncoiled from one spool, is jerked by a mechanism through the “gate” in the focal plane of the lens, and coiled up on another spool. Each jerk moves the film a distance equal to the height of a picture. Each picture rests for an instant while in the gate and is then projected, and while it is being jerked out and the next one jerked in, a sector passes up in front of the projecting lens and cuts off the light. The films are tough enough to stand the strain of being pulled through, but the celluloid ones are inflammable, and the image of the crater of the arc on one for a short time is sufficient to set it on fire.

EXAMPLES VI

1. An object viewed through a convex lens of 4 in. focal length held close to the eye appears to be 10 in. away. Find its actual position and the magnification.
2. The focal length of the object glass of an astronomical telescope is 13 in. and the focal length of its eyepiece is $\frac{2}{3}$ in. Calculate its magnifying power.
3. If half the object glass of a telescope which is pointed at the moon is covered, how will the appearance of the moon be affected, as seen through the telescope?
4. Show that the magnifying power of the astronomical telescope is $F\left(1 + \frac{f}{D}\right)$, if the final image is formed at the distance of distinct vision D instead of at infinity.
5. What is theoretically the angular distance between the centres of two stars which are just separated by a telescope with a 3-inch object glass?
6. A piece of wire gauze has about 20 wires to the cm. It is looked at with a telescope which has a 1-inch object glass. What is the greatest distance at which the wires can be seen separate?
7. The focal length of the objective of a compound microscope is $\frac{1}{10}$ in., that of the eyepiece $\frac{3}{8}$ in., their distance apart is 11 in., and the distance of distinct vision is 10 in. Find the position of an object which will give a final image at the distance of distinct vision.
8. A snapshot camera is provided with stops marked $f/5.6$, $f/8$, $f/11$, $f/16$, $f/22$, and $f/32$. If the correct exposure for $f/8$ is $\frac{1}{50}$ sec., calculate what it must be for the other stops.
9. A lantern slide $3\frac{1}{4} \times 3\frac{1}{4}$ in. is placed at a distance of 9 in. from a 500 candle-power arc lamp. An image of 8 ft. side is thrown on a distant screen. Find the intensity of illumination of the clear parts of the image, assuming that all the light which passes through the slide falls on the screen and disregarding loss of light in the condenser and the objective.
10. A stick 6 in. long is spun about an axis through its midpoint and perpendicular to its length. How many revolutions must it make per second in order to present the appearance of a disc? Visual impressions persist on the retina for $\frac{1}{16}$ sec.

CHAPTER VII

THE SPECTROMETER AND THE SPECTRUM

Prisms.—If a portion of a medium has two plane faces inclined to each other at an angle, in optics it is called a prism. Any plane at right angles to these faces is called a **principal plane** of the prism, and if the faces meet in an edge, it is called the **refracting edge**. The angle between the two faces is called the **refracting angle** or **angle of the prism**.

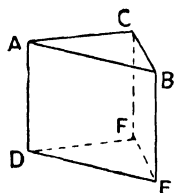


FIG. 360.

Thus in Fig. 360 if ABED and ACFD are the plane faces, AD is the refracting edge. Most prisms in optics have the shape shown in the figure in which the base and top surfaces are both principal sections; in this case $\angle CAB$ is the refracting angle. The prisms used in optics are made of glass or some other transparent material. The faces ABED and ACFD are

polished, the base and top surfaces are matt, and the remaining face BCFE may be either matt or transparent. If it is transparent, BE and FC may also be used as refracting edges of the prism.

Minimum Deviation.—Fig. 361 represents a principal section of a glass prism ABC. PQ is a ray of light incident on the prism at Q. After refraction at Q the ray travels in the direction QR, and after refraction at the second face at R it emerges in the direction

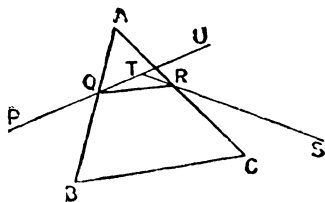


FIG. 361.

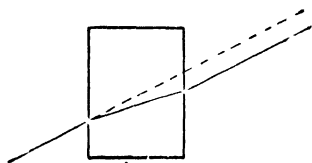


FIG. 362.

RS. Produce PQ to U, and produce SR to meet it in T; then the angle between the directions of the incident and emergent rays is UTS, and the prism produces a deviation in the ray equal to this angle.

It is important to note the difference between a deviation and a displacement. Fig. 362 represents the displacement produced in the path of a ray by its passage through a glass slab with parallel sides; the emergent ray is parallel to the incident ray. But in Fig. 361 it is rotated through an angle from the direction of the incident ray. It is also important to note, that this rotation is away from the refracting edge towards the thicker part of the prism.

If the angle of incidence of the ray PQ on the face AB alters, the deviation alters. If the incident and emergent rays are equally inclined to the prism, *i.e.* if $AQ=AR$, the deviation is a minimum, and the prism is said to be in the position of minimum deviation. This is easily seen. For, let the deviation of any other ray PQRS (Fig. 363) be a minimum; then the deviation of the ray P'Q'R'S' which passes through the prism in the symmetrically opposite direction, so that $AQ=AQ'$ and $AR=AR'$, is the same as the deviation of PQRS. Consequently the latter cannot be a minimum.

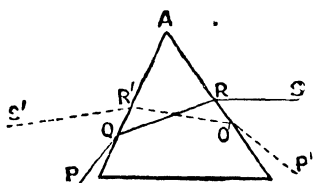


FIG. 363.

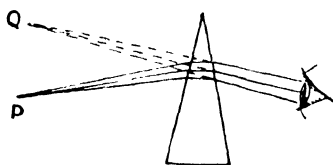


FIG. 364.

Image produced by a Prism.—Let P represent a point source of light. A pencil of rays is incident from this source on a glass prism, and after traversing the prism is received by the eye of an observer, as shown in Fig. 364. If the rays are produced backwards, they come to an approximate focus at Q; hence the point source of light appears to the observer to be at Q.

It may be shown by tracing the paths of the rays on a drawing-board, or better still by calculation, that the image is not sharp except near minimum deviation or when the refracting angle of the prism is small.

Twinkling of Stars.—If objects at a distance are regarded through a stream of hot air, *e.g.* through the hot air issuing from a chimney or rising from a seashore heated by the sun, they have a quivery appearance and wave about and tremble. This is due to the index of refraction of the hot air being less. The patches of hot air act like prisms, deviating the rays and changing the apparent directions from which they come, and the positions and shapes of the patches are always altering.

The scintillation or twinkling of the stars is similarly to be ascribed to changing inequalities in the refractive index of the atmosphere.

Also objects seen through water in which ice, sugar, or acid is being dissolved have a streaky appearance. This is due to the different portions of the solution having different strengths and to the index of refraction varying with the strength; here also portions of liquid with a higher index of refraction act like so many prisms.

EXAMPLE.—Prove that if a lens is held before the eye, and then moved to one side, the objects seen through it appear to move in the same direction as the lens if the latter is concave, but in the opposite direction to the lens if the latter is convex.

Suppose that the object is seen through the centre of a convex glass lens, and that the lens is moved gradually downwards. This is equivalent to introducing a glass prism of gradually increasing angle between the eye and the object. The edge of this prism points upwards; hence the object is apparently displaced upwards, *i.e.* in the opposite direction to that in which the lens is moving. The case of the concave lens can be treated similarly.

Variation of Deviation with Angle of Incidence.—Set a glass prism on a drawing-board and draw pencil marks along the two faces AB, AC (Fig. 361). Place two pins on the line PQ, and looking into the prism from the other side, place other two pins in line with their images, as seen through the glass. Hence draw the line RS. Remove the prism, and measure the angle of incidence of PQ and the angle of deviation by means of a protractor. Repeat for different angles of incidence, graph the deviation against the angle of incidence, and determine the angle of minimum deviation from the graph.

As an example of the kind of results obtained, when the angle of incidence in the case of a 60° glass prism was 40° , 50° , 60° , and 70° , the deviation was 57° , 50° , 50° , and $53\frac{1}{2}^\circ$ respectively. This gives a minimum deviation of about 49° .

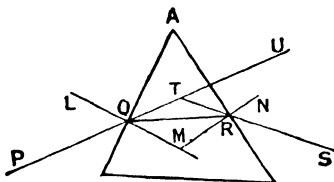


FIG. 365.

Formula for Minimum Deviation.—Suppose now that Fig. 365 represents the position of minimum deviation, and that LM and MN are the normals at the points of incidence of a ray PQRS. Write

$$\angle PQL = \angle SRN = i, \text{ and } \angle RQM = \angle QRM = r$$

and let the angle of minimum deviation be δ . Then

$$\begin{aligned} \delta &= \angle UTS = \angle TQR + \angle TRQ \\ &= \angle TQM - \angle RQM + \angle TRM - \angle QRM \\ &= 2(i - r). \end{aligned}$$

Also

$$\angle AQR + \angle ARQ = \pi - A$$

and

$$\angle AQR + \angle ARQ = \angle AQM - \angle RQM + \angle ARM - \angle QRM = \pi - 2r$$

Hence $A=2r$, *i.e.* $r=\frac{1}{2}A$, and on substitution in the expression above for δ ,

$$\delta=2(i-\frac{1}{2}A) \text{ or } i=\frac{1}{2}(A+\delta)$$

But the index of refraction, μ , is defined by

$$\mu=\frac{\sin i}{\sin r}$$

On substitution for i and r this gives

$$\mu=\frac{\sin \frac{1}{2}(A+\delta)}{\sin \frac{1}{2}A}$$

Hence, if A and δ are known, μ can be calculated. A useful way of remembering this formula is by considering what happens if the prism is made of air instead of glass, *i.e.* δ becomes 0 and μ is equal to 1.

Determination of Index of Refraction of a Glass Prism by Pin Optics Methods : (1) **By Minimum Deviation.**—Draw the trace ABC of the prism (Fig. 366) on the drawing-board, and place a pin at F and one at

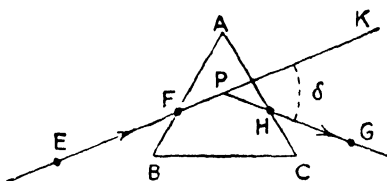


FIG. 366.

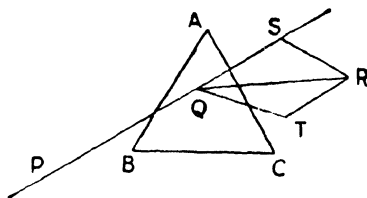


FIG. 367.

H making $AF=AH$ —about two-thirds of side of prism. Observe the image of the pin F by looking into the face AC of the prism, and place a pin at G , so that it is in line with the pin H and the image of the pin F . Similarly by looking into the face AB place a pin at E , so that it is in line with the pin F and the image of the pin H . Note that it is also in line with the image of the pin G .

Remove the prism and draw the lines $EFPK$ and PHG . Read the angles δ and A with a protractor, and calculate μ by the formula of the preceding section.

(2) **Any Position.**—Let ABC (Fig. 367) be the trace of a glass prism and PS any line drawn on the paper. Look into the face AC of the prism, and draw TQ so that it appears to be the continuation of PQ . Remove the prism, and produce TQ to cut PS in Q . Cut off $QS=QT$, and draw TR perpendicular to AC and SR perpendicular to AB . Join QR . Then

$$\mu=\frac{QR}{QS}$$

This method gives extremely accurate results. It does not require the prism to be at minimum deviation.

To prove the formula let ABC (Fig. 368) be the trace of the prism, let PMQ be the incident ray, and let QNT be the emergent ray; join MN , draw QR parallel to MN , and through any point S in PQ produced draw SR perpendicular to AB to meet QR in R . Then angles i and r are equal respectively to the angles of incidence and refraction at M . Consequently

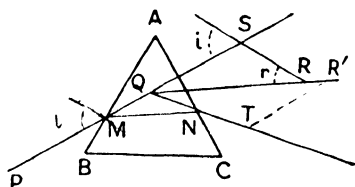


FIG. 368.

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin QSR}{\sin QRS} = \frac{QR}{QS}$$

If QT be cut off $= QS$, and TR' be drawn perpendicular to AC to cut QR in R' , it can similarly be proved that

$$\mu = \frac{QR'}{QS}$$

and hence that R' coincides with R .

Spectrometer.—The spectrometer or goniometer is an instrument used for measuring A and δ , and so determining the indices of refraction of substances. It gives much more accurate results than can be obtained by pin optics methods. Fig. 369 shows a plan of one, arranged for determining δ . It consists essentially of a divided circle ABC about the axis of which a collimator, DE , and a telescope, FG , can rotate. The collimator is a tube at one end of which, E , there is a convex lens, and at the other end of which, D , there is a slit exactly at the focus of this convex lens. The rays of light which enter the slit thus form a parallel pencil after passing through the lens. After deviation by the prism they fall on the object glass of the telescope, which is an astronomical

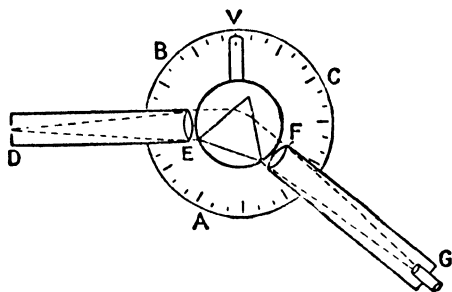


FIG. 369.—Plan of spectrometer.

one, and converge to form a real image, which is observed through the eyepiece. The prism rests on a table which can be rotated about the axis of the divided circle, and the rotation of which can be read by a vernier at V . The position of the telescope on the divided circle can also be read by a vernier.

To measure δ the prism is first removed, and the telescope directed to look into the collimator. An image of the slit is then seen in the field, and the telescope is turned until this image coincides with the cross-wires. The reading is then taken. The

prism is next put into position on the prism table, and the telescope turned so as to see the image formed by the light which passes through the prism. The prism is then rotated, so as to bring it into the position of minimum deviation. The latter is easily recognised, because at it the image turns in the field, and begins to move in the other direction. When it is found, the reading on the circle is again taken. The difference of the two readings gives δ .

To measure A , the angle of the prism, we may proceed in either of two ways. In the first of these (Fig. 370) the prism is placed with its edge A near the centre of the prism table, so that the rays from the collimator are incident on it in the direction PA , some on the face AB and some on the face AC . The rays are reflected in the directions AQ and AR . If the prism is kept fixed and the telescope pointed in the directions QA and RA , images of the slit

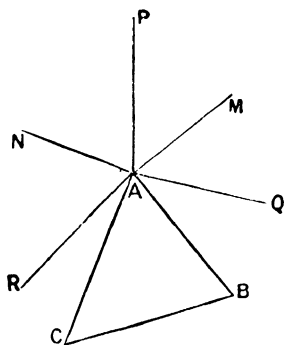


FIG. 370.

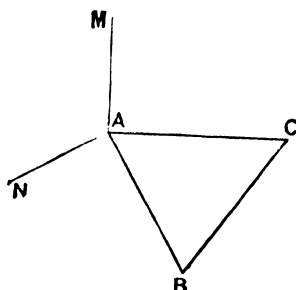


FIG. 371.

will be seen. Hence $\angle QAR$ can be measured. The angle of the prism, A , is half $\angle QAR$. For, draw AM and AN the normals to the two faces. Then

$$\begin{aligned}\angle QAR &= 2\pi - \angle QAP - \angle RAP = 2\pi - 2\angle PAM \\ &\quad - 2\angle PAN = 2(\pi - \angle MAN) = 2A\end{aligned}$$

In the second method the prism is adjusted so that the rays reflected from one of the faces produce an image of the slit coinciding with the cross-wires in the field of view. The telescope is then kept fixed, and the prism table rotated until an image of the slit formed by reflection from one of the other faces is coincident with the cross-wires. The difference of the angle of rotation and two right angles is equal to the angle between the faces from which the light was reflected.

For it is clear (Fig. 371) that in the second position the normal to the one face has the same direction as the normal to the other

in the first position. The angle turned through by the prism is hence equal to $\angle MAN = \pi - A$.

Figs. 372 and 373 represent two types of spectrometer which are in much use. In the second the telescope is counterpoised, so as to prevent strain during rotation.

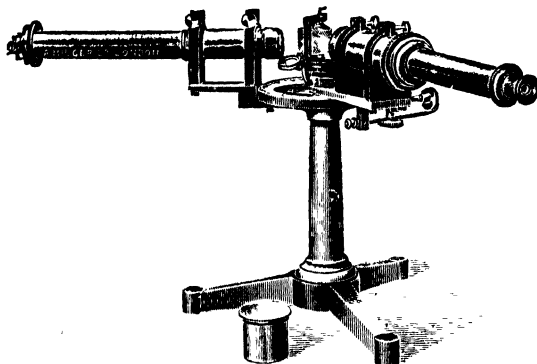


FIG. 372.—Spectrometer.

Experiments with the Spectrometer.—Before using the spectrometer it must be adjusted. The telescope is removed from the instrument and taken to a window. The eyepiece is focussed on the cross-wires; next the window is opened, and the telescope focussed upon a distant

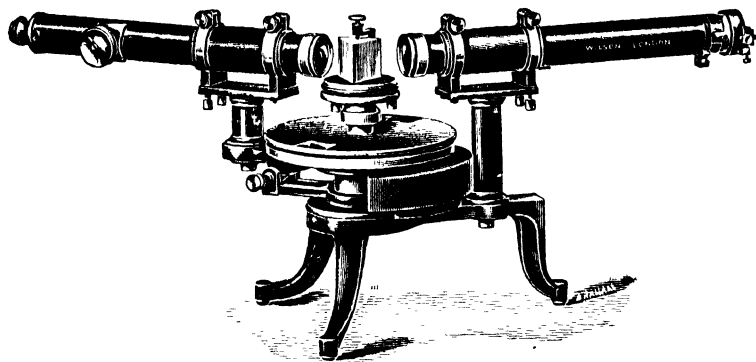


FIG. 373.—Spectrometer with counterpoised telescope.

object, until on moving the eye from side to side the image of the distant object remains at rest relative to the cross-wires. The telescope is then replaced in the instrument, turned to look into the collimator, and the slit of the latter illuminated. An image of the slit is seen in the field,

and the distance of the slit from the lens of the collimator adjusted, until this image is as sharp as possible. Both telescope and collimator are then adjusted for parallel light.

The spectrometer, as has been mentioned above, is used principally for measuring A and δ and so determining μ by the formula

$$\mu = \frac{\sin \frac{1}{2}(A + \delta)}{\sin \frac{1}{2}A}$$

With the instrument shown in Fig. 373 the average first year university student should with a little encouragement be able to get μ correct to the third decimal place; an exceptionally good student is able almost to get the fourth decimal place.

The index of refraction of a liquid can be determined by the spectrometer, if it is put in a hollow prism the sides of which are parallel-sided plates of glass. The fact that the angle turned through by a reflected ray is twice the angle turned through by the mirror (p. 306) can be proved by the spectrometer, and the variation of the deviation with the angle of incidence (p. 378) can be investigated by the spectrometer, with much greater accuracy than by pin optics methods.

Sodium Flame.—In making experiments on refraction by the pin optics method, when the pins and the lines drawn on the paper are looked at through the glass, they appear coloured at the edges. This is, however, not an appreciable source of error, as the degree of accuracy obtained by this method is not high. But in working with the spectrometer, in order to avoid the disturbing effect of colour, it is necessary to illuminate the slit with light of one colour alone, for which purpose a sodium flame is used. The best way of producing the latter is to form a bead of sodium bicarbonate in a loop at the end of a thin platinum wire and hold it in the edge of a bunsen flame, so as to make the wire red hot; this gives the flame an intense yellow colour for a long time.

We are now going to study the colours formed by the refraction of light. The classic experiments in this subject were carried out by Isaac Newton at Cambridge in 1666.

Newton's Fundamental Experiment on the Spectrum.—Newton caused a pencil of direct sunlight to enter a darkened room through a hole in the shutter. It formed an image of the sun on the opposite wall of the room, the room acting as pinhole camera. He then placed a glass prism inside the room close up to the hole with its refracting edge horizontal and pointing downwards, so as to receive the pencil. The latter, as was to be expected, was refracted upwards by the prism. But instead of forming a single image of the sun in its natural colours higher up on the wall, it formed a vertical coloured strip to which Newton gave the name of the spectrum. The colours of the spectrum were, according to Newton, red, orange, yellow, green, blue, indigo, violet, in order of increasing deviation. They pass

imperceptibly into one another through intermediate tints, and are not marked off by sharp boundaries. Fig. 374 represents the formation of the spectrum, the colours being denoted by their initial letters. The fact that the red end is deviated least should be memorised.

As the prism was slowly rotated about a horizontal axis, the spectrum moved down the wall until the position of minimum deviation was reached, after which it moved up again. At minimum deviation the spectrum was shortest.

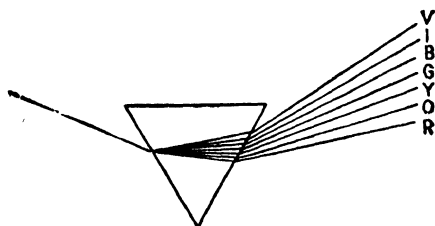


FIG. 374.—Formation of spectrum.

Newton inferred from the experiment, that white light is composite, and that in decomposing it by the prism we are getting at something simpler and more elementary. Also that the

refractive indices of the different colours have slightly different values; hence they are bent up to a different extent on entering the prism, and on leaving it the angles of divergence between each successive pair are further increased. Or, in other words, the white light undergoes *dispersion*, as it is called, in the prism owing to the difference in refrangibility of its constituent rays.

In order to show that the prism did not produce the colours, but that it separated colours that were already existing, Newton performed amongst others the following three experiments :—

Attempt to further decompose a Colour of the Spectrum.—Let a beam of white light fall on a prism P, and form a spectrum VR (Fig. 375) on a piece of cardboard in which a narrow slit is cut. Then certain rays of the spectrum, for example the green, will go through the slit. Now let these rays fall on a second prism Q, and afterwards be received on another piece of cardboard W. It is found that there is no further

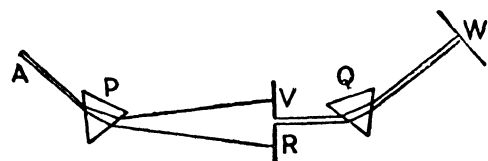


FIG. 375.

decomposition of the light; the green rays remain green after passing through the second prism, and the patch of light on the cardboard W is wholly green. Thus the green rays of the spectrum form something elementary that

cannot be decomposed further. The same holds true of all the other colours of the spectrum, as may be shown by moving the slit in the first piece of cardboard, so that the other colours in succession pass through it.

Light of a definite position in the spectrum and hence of a definite colour is called *monochromatic*. A very satisfactory way of obtaining

monochromatic light is simply to throw a spectrum on a screen with a hole in it, as in Fig. 375, and allow the radiation that is wanted to pass through the hole.

Experiment with the Crossed Prisms.—The arrangement of this experiment is shown in Fig. 376, which is taken from Newton's *Opticks*. S is the sun, F the hole in the shutter. If only the prism ABC was used, the spectrum PT was produced. If the second prism DH was placed with its refracting edge vertical, so as to receive the light immediately it left the first prism, both together produced the inclined spectrum *pt*. Each separate colour of the first spectrum was given a sideways displacement by the second prism, proportional to the vertical displacement it received from the first, and suffered neither further change nor decomposition. This confirms the conclusion of the preceding experiment.

Recomposition of White Light.—Not only can white light be decomposed into the seven colours of the spectrum, but they can be recombined together to form white light. Fig. 377 shows how Newton proved this. The prism forms a spectrum which could be received by a sheet of white cardboard at VR, but falls instead upon a convex lens. This convex lens makes the different colours superimpose again, and they converge to form a white patch on a piece of white cardboard at W. If a pencil or similar obstacle is held in front of the lens, so as to stop

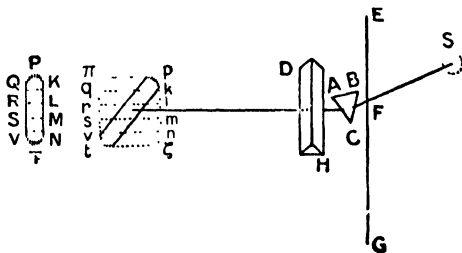


FIG. 376.—Newton's diagram of the crossed prisms.

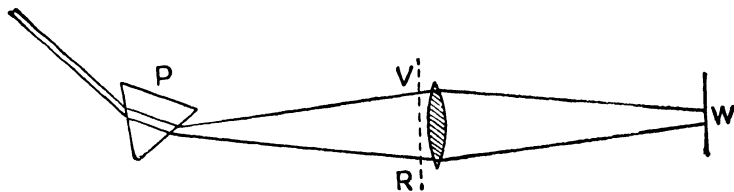


FIG. 377.

some part of the spectrum, then the patch at W appears coloured. The lens must be large enough to receive the whole spectrum at VR, and must focus the first surface of the prism on the screen at W.

Projection of a Spectrum.—Newton worked usually with a circular aperture in a shutter. Thus his spectrum consisted of a series of blurred images of the sun formed by the successive colours somewhat after the manner of the pinhole camera. All these images overlapped considerably, and consequently the spectrum was not pure. Fig. 378 shows the plan of an arrangement which

was occasionally used by Newton, and which is generally employed nowadays to exhibit the spectrum simultaneously to a large number of observers ; this arrangement gives much purer colours.

S is a slit and L a convex lens which forms an image of this slit at Q. When the prism is inserted in the path of the rays at P, a spectrum is formed in focus on the screen RV, which is placed at the same distance from P as Q is. The best source of light to use is the arc lamp ; an image of the crater is thrown on the slit,

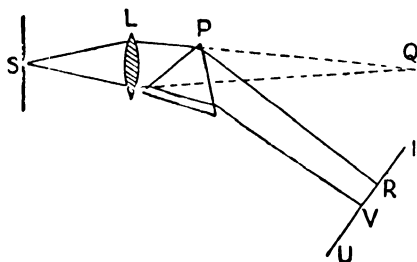


FIG. 378.

somewhat out of the focus, by a lens not shown in the diagram. The spectrum then received on the screen is similar to the first one shown in the frontispiece.

If a piece of red glass is placed anywhere in the path of the rays, it cuts out all the spectrum except the red end. If a piece of blue cobalt glass is placed

in the path of the rays, the spectrum is crossed by two black bands in the red and a broader one in the green which together stop most of the red half of the spectrum. If a skein of red wool is moved along the screen, it appears red when illuminated by the red rays and black when illuminated by the other parts of the spectrum. Similarly with the other colours ; if they are pure, each appears black except when illuminated by light of its own colour, when it appears in that colour. It is, however, difficult to obtain colours of the necessary purity to perform the experiment with success.

Infra-red.—Newton's spectrum was limited by red at the one end and violet at the other. Since his time we have discovered that there is an enormous extent of invisible rays lying beyond the red at the one end and beyond the violet at the other. These rays are termed respectively the infra-red and the ultra-violet.

The infra-red was discovered by Sir William Herschel in 1800. He placed the bulb of a sensitive thermometer on a spectrum, and found that the rays falling on it produced a rise of temperature. To make sure that the rise of temperature was actually due to the rays falling on it, and not to some other cause, two other thermometers were placed in the shade near the spectrum ; it was found that they showed no rise in temperature. The rise in temperature was much greater at the red end of the spectrum than at the violet. When placed beyond the violet (in the region UV, Fig. 378), there was no heating effect, but when the bulb was placed beyond the red (in the region IR, Fig. 378), there was a

marked rise of temperature. The interpretation of the experiment is, that the sun's rays carry energy with them which is converted into heat when it falls upon the bulb of the thermometer, that the violet rays of the solar spectrum have least energy, and that there are rays beyond the red end of the spectrum, invisible heat rays, which carry energy, but do not have any effect upon the eye.

A thermometer can be used for detecting heat rays only in the case of an extremely powerful source like the sun. When small rises of temperature are to be measured, it is customary to employ one of a number of other instruments, the most popular of which is the thermopile. In using the thermopile the rays are allowed to fall upon the junction of two different metals in an electric circuit, and a current is set up proportional to the rise of temperature. In this way rises of temperature as small as one-millionth of a degree Centigrade can be measured.

Experiment on Heat Rays.—Glass transmits the invisible heat rays better than water, quartz transmits them better than glass, and rock salt better than quartz. It is an interesting experiment to allow the rays from a bunsen burner to fall upon a thermopile, and then interpose plates of various substances and determine the fraction transmitted by each. If no thermopile is available, a more powerful source can be used, and its rays focussed on the bulb of a thermometer with a lens.

Ultra-violet.—Modern photographic plates may be divided into three classes as regards their sensitiveness to the colours of the spectrum, namely, ordinary, orthochromatic, and panchromatic. The ordinary are sensitive to the violet and blue, the orthochromatic are sensitive also to the green, and the panchromatic to the whole visible spectrum. But no matter which class of plate is used, if it is exposed to the spectrum, the portion of the plate beyond the violet, UV in the arrangement of Fig. 378, is also acted on, showing that here also there are invisible rays, namely, the ultra-violet. They were discovered first by J. W. Ritter in 1801, who found that they blackened silver chloride. Rays which produce chemical change in this way are referred to as *actinic* rays.

Experiments on Actinic Rays.—The simplest way of demonstrating the chemical action of light is by means of ordinary gelatino-chloride printing paper, or P.O.P. as it is called. If this is held in a very bright spectrum, it is darkened by the action of the light, and the degree of darkening in the different parts may be taken as a measure of the strength of the chemical action in these parts.

Or the paper may be exposed directly to the light of the sun or of a lamp, and the effect of interposing plates of glass investigated. In this case the best way of proceeding is to expose the paper directly to the rays for *a* seconds, then through the glass to the same rays for *b* seconds,

arranging that the degree of blackening is the same in each case. The ratio a/b may be taken as the fraction of actinic rays transmitted by the plate. The results are only approximate. It may be shown in this way, that quartz transmits the actinic rays better than ordinary crown glass, and that crown glass transmits them better than dense flint.

Fluorescence.—If a spectrum is received upon a screen of barium-platinocyanide instead of upon a sheet of paper, it is found that the violet and ultra-violet shine with a green light. The energy of the violet and ultra-violet radiations is absorbed by the screen, and re-emitted as light of another colour. This phenomenon is called fluorescence. It was first explained properly by Stokes in 1852, and it is shown by a great number of substances, for example, fluorspar, from which it derives its name, uranium glass, sulphate of quinine, aqueous solutions of fluorescein and eosin, etc.

The essential characteristic of fluorescence is that light of one colour is absorbed and re-emitted as light of another colour. Ultra-violet radiation may be absorbed and re-emitted as visible light. Stokes came to the conclusion that the absorbed radiation must always be more refrangible, *i.e.* lie further from the red end of the spectrum, than the re-emitted radiation, and this conclusion is known as Stokes' law. Modern investigation has, however, shown that it is certainly not always true.

Fluorescence can be used for detecting the presence of ultra-violet light. It can also be used for rendering visible the track of a beam of light through a liquid, as in the experiment represented in Fig. 294. In this experiment the path of the beam in the water was made visible by floating specks scattering the light. If a little fluorescein is dissolved in the water, each molecule of fluorescein emits a green light when the beam reaches it, and hence the path of the beam in the water shines with a green light.

Phosphorescence.—Certain substances, after being exposed to light, emit light for some time afterwards when placed in a dark room. This phenomenon is known as phosphorescence. It is the violet and ultra-violet rays that are most active in producing it. The duration of the emission after exposure to light varies very widely. Balmain's luminous paint, which is a sulphide of calcium, will shine for hours in the dark after exposure to bright sunshine, while other substances cease to emit in a few seconds.

The phenomenon is quite distinct from the faint greenish-white luminosity which phosphorus shows when exposed to moist air in the dark. The latter is due to the slow combustion of the phosphorus. It is a chemical change which takes place only in the one direction, and is not reversed by exposure to light. In true phosphorescence we have a reversible change; the exciting

light throws into the substance the energy which is radiated away in the dark, and by exposing the substance to the exciting light it can be made to phosphoresce again and again.

The distinction between fluorescence and phosphorescence is that fluorescent substances emit light only as long as the exciting light is acting on them; phosphorescent substances emit light after the exciting light ceases.

The Spectroscope.—The easiest way of examining a spectrum is to set a prism at minimum deviation on the prism table of a spectrometer and let the light fall upon the slit; then a spectrum will be formed in the focal plane of the telescope, and can be examined with the eyepiece. The instruments used for examining spectra are called spectrosopes.

Any spectrometer designed for measuring angles can also be used for examining spectra, but if it is to be used only for the latter purpose its construction may be simplified. First of all, the prism is fixed on the prism table. Then, as the spectrum always appears at the same place on the circle, the latter does not require to be divided the whole way round. It is enough if it is divided through an arc of 60° . Also it is not necessary for the scale to be divided in degrees; any arbitrary unit is sufficient. Finally, the prism is usually covered in to prevent stray light entering the telescope; if the spectrum is at all faint, it will be otherwise swamped by this stray light.

Kirchhoff and Bunsen Spectroscope.—This instrument is represented in Fig. 379. The collimator (to the left) is fixed and

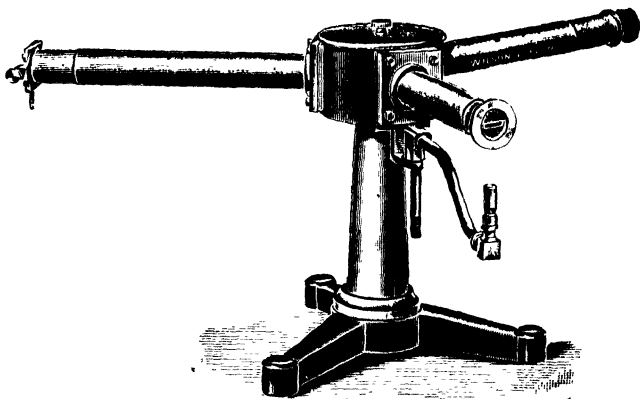


FIG. 379.—Kirchhoff and Bunsen spectroscope

the telescope (behind) moves through a small range. To the right in front there is a second collimator which carries a fine photo-

graphic scale at its end instead of a slit. This scale is illuminated by the gas burner, and is seen through the telescope by reflection on the second face of the prism immediately above the spectrum. Hence the position of any point in the spectrum can be easily read off on the scale.

Spectrum Analysis.—If light from an ordinary arc lamp or an electric glow lamp or a gas or candle flame is allowed to fall upon the slit of a spectroscope, we obtain a continuous spectrum, *i.e.* a spectrum similar to the first one in the frontispiece. If sunlight falls on the slit, we see a continuous spectrum crossed with dark lines, known after their discoverer as the Fraunhofer lines, and denoted by letters of the alphabet. The solar spectrum is shown second in the frontispiece. If sodium light falls upon the slit, we see only two narrow yellow lines; these are known as the D lines, and are so close together that, unless the slit is very narrow, they appear as one. Sodium light is thus practically homogeneous; that is why it is used in determining the index of refraction of a glass prism.

The spectrum of the neon lamp consists of a great number of bright lines. This spectrum and the sodium spectrum are what are known as line spectra. The neon lamp glows with a dull red light, runs off the ordinary lighting circuit, and was introduced as a night light and for advertisement purposes shortly after the war. The blue cone of the bunsen burner gives a spectrum consisting of five bands, called the Swan spectrum after its discoverer; these are broader than lines, and also differ from lines in having each one side sharp and bright and the other side fading imperceptibly into the background. This kind of spectrum is known as a band spectrum. Finally, if we allow light from an electric glow lamp to fall on the slit, but place a sheet of cobalt glass in front of the slit, we obtain an example of an absorption spectrum, the continuous spectrum of the lamp crossed by dark bands due to the cobalt oxide in the glass.

Each chemical element has its own characteristic spectrum, and we can investigate the nature of a substance by heating it or causing it in any other way to emit light, and examining the spectrum of that light. This procedure is known as spectrum analysis.

It will be noticed that the yellow line of the sodium spectrum coincides in position with the dark D line of the solar spectrum (*cf.* frontispiece). It is found that if light, which would otherwise give a very bright continuous spectrum, passes through sodium vapour, the dark D line appears in its spectrum. The sodium vapour has absorbed rays of the same refrangibility as it emits. It is natural to assume, that the body of the sun gives a continuous spectrum, but that it is surrounded by a gas which absorbs in

this way some of the rays of this continuous spectrum. *The Fraunhofer D line thus becomes a proof of the existence of sodium vapour in the sun*; in a similar way the existence in the sun of other terrestrial elements has been proved.

The Rainbow.—It is not necessary for the formation of a spectrum that the prism should be made of glass; a hollow glass prism filled with water does as well. Even a drop of water acts under certain conditions in the same way as Newton's prism. For (Fig. 380) let SA be a ray of white light from the sun incident on a drop of water at A. On entering the water it is broken into a series of coloured rays, of which, for the sake of clearness, only the red and violet are drawn. These are reflected inside the drop at B and enter the air at C, when the angle between them is further increased. If a piece of cardboard is placed at W, a spectrum will be obtained.

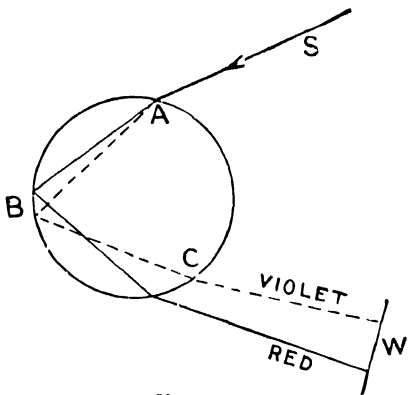


FIG. 380.

Let us calculate the deviation of the red rays caused by their passage through the drop. The radius is, of course, the normal at A. Let i (Fig. 381) be the angle of incidence and r the angle of refraction. Then from the figure the deviation is given by

$$\begin{aligned}\angle SFD &= 2 / SFO = 2(\angle ABO - \angle FAB) \\ &= 2(r - i + r) = 4r - 2i\end{aligned}$$

We have, of course, $\sin i = \mu \sin r$. If the value of the refractive index of water for red light, namely, 1.329, is substituted, a table calculated of corresponding values of i and r , and the deviation graphed as a function of i , the minimum value of the deviation is found to be 42.8° . In the same way the minimum value of the deviation for the violet is 40.8° .

The spectrum produced according to Fig. 380 is very faint, unless the rays go through the drop at minimum deviation. Only then are they sufficiently concentrated in one direction to be easily visible.

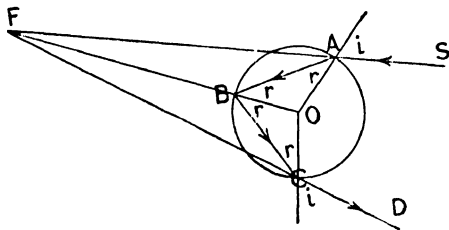


FIG. 381.

Now consider Fig. 382. Let us suppose that an observer is standing at E with his back to the sun, facing the rain drops A, B, C. The rays from the sun may be regarded as all parallel to one another. If we

draw a cone with semi-vertical angle 42.8° , and the direction of the sun's rays as axis, all the red rays which reach the observer, after refraction by rain drops lying in the surface of this cone, will have suffered minimum deviation. Consequently they will be easily visible. In the same way violet will be visible in the direction which makes an angle of 40.8° with SE, and the other colours come in between. Thus the rainbow is formed. It is obvious that the red is to the outside.

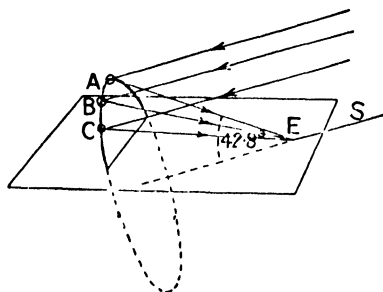


FIG. 382.

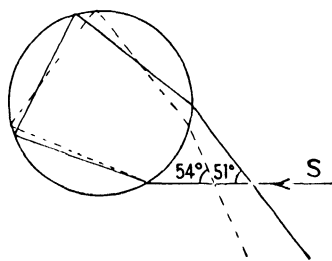


FIG. 383.

A rainbow formed in the above manner is called a **primary** rainbow. If the rays are reflected twice inside the drop, as shown in Fig. 383, for minimum deviation the red rays make an angle of about 51° , and the violet rays an angle of about 54° with the incident light. Consequently a rainbow will be formed with the violet outside. This rainbow is called the **secondary** rainbow, and to see it the observer must also have his back to the sun. The primary and secondary rainbows are sometimes seen together, the secondary one being outside the primary one and fainter and broader than the latter.

Chromatic Aberration.—A lens may be regarded as the limiting form of a prism with a number of faces (Fig. 384), when

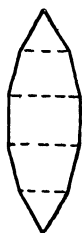


FIG. 384.

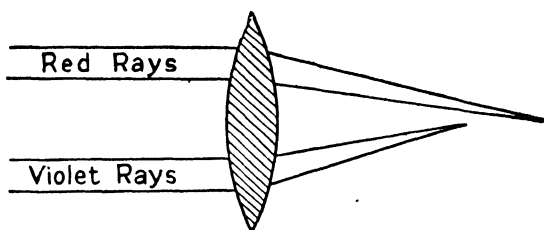


FIG. 385.—Chromatic aberration.

the number is made very great. Now a prism bends the different rays of light out of their path to a different extent; it bends the violet most and the red least. Consequently the same holds true of a lens; if parallel beams of red and violet light fall on it, they

come to a focus at different points, as shown in Fig. 385. Also if a single lens forms an image by means of white light, the images formed by the different constituent colours do not occupy the same place, and hence confuse one another. The blurring of the final image due to this cause is called chromatic aberration.

Chromatic aberration can be greatly diminished by using a compound lens made of two different kinds of glass. Suppose that we take two 60° prisms made of the two principal kinds of glass used in optics, one of hard crown and the other of dense flint, and set them in succession on the prism table of a spectrometer at minimum deviation for sodium light. Then, when the slit is illuminated with white light, the deviations of the middle of the spectrum are about 49° and 54° , and the lengths of the spectra are about $35'$ and $83'$ respectively in the two cases. We may regard the action of the prisms under two heads. First of all, there is the deviation produced which is $\frac{54}{49}$ times as great for the flint prism as for the crown. Second, there is the dispersion of the light into a spectrum which is $\frac{83}{35}$ times as great for the flint prism as for the crown. The one ratio is more than twice the other.



FIG. 386.



FIG. 387.

Let us now consider Figs. 386 and 387. Fig. 386 represents two 60° prisms made of crown glass. A ray of white light P falls on the first, and is decomposed into its constituent rays. These fall on the second, suffer the reverse deviation to what they experienced in the first prism, and emerge at Q parallel to one another. The second prism thus destroys both the deviation and the dispersion produced by the first, and both together act like a slab of glass.

In Fig. 387 the ray is incident on the same crown glass prism as before. But behind the crown glass prism there is now placed a flint glass prism of just sufficient power to neutralise the dispersion produced by the first prism. A much smaller angle than 60° is sufficient for this. Then, since the dispersion of flint glass is relatively so much greater than its deviation, the second prism removes only part of the deviation produced by the first. So the rays of the different colours emerge parallel to one another at Q, but deviated through an angle from the incident ray P.

Fig. 388 shows how the above result is applied in constructing the achromatic lens. The flint glass concave lens acts in the same way as the flint prism, and the crown glass convex lens in the same

way as the crown prism. The flint lens is strong enough to destroy the dispersion, but not strong enough to destroy the deviation that the convex lens is about to produce. Consequently both red and violet rays come to the same focus at F . For the sake of clearness the two lenses are shown separate in Fig. 388, but in actual practice they are cemented together as in Fig. 389.

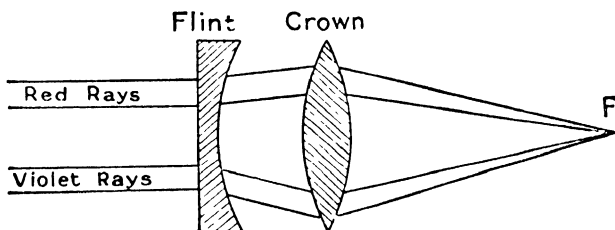


FIG. 388.



FIG. 389.

Experiments on Chromatic Abberation.—The focal length of a lens may be determined for light of different colours on the optical bench. Ordinary red glass gives a pure enough red, but for green and blue it will be necessary to use the tricolour filters described on p. 406. They should be placed between the cross-wires and the lamp.

With the ordinary arrangement of a rectangular aperture covered behind with tissue paper and illuminated with white light, if the image screen be moved out of focus towards the lens, the edge of the image becomes tinged with red; if it be moved out of focus the other way, the edge of the image becomes tinged with blue. For in the first case the red image is more out of focus, each point on the edge forming a red disc, and these discs extend beyond the blue point images, and in the second case these conditions are reversed.

Spherical Aberration.—If two rays, parallel to the axis, are incident on a lens of wide aperture (Fig. 390), one close to the axis and the other close to the margin of the lens, axial and marginal

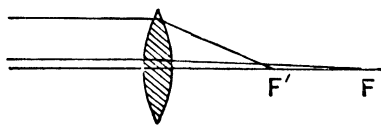


FIG. 390.

rays as they are called, then they do not come to a focus at the same point, even though they are of the same colour. The figure shows the effect in an exaggerated form. The blurring of the image due to this cause is called spherical

aberration. It may be tested for by placing in front of the lens a diaphragm that transmits only the central pencil, replacing this by a disc that blocks out everything except the marginal zone, and determining the position of an image in the two cases.

Direct Vision Spectroscope.—In Fig. 387 the angle of the second prism is chosen so as to give deviation, but no dispersion. It is

possible to design a combination of prisms which will give dispersion, but no deviation in the middle of the spectrum. This is what is used in the direct vision spectroscope.

The direct vision spectroscope (Fig. 391) consists usually of a collimator and compound prism mounted together in a tube about

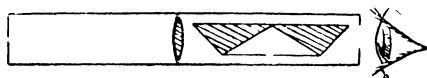


FIG. 391.—Direct vision spectroscope.

3 in. long. The compound prism is composed of a very dense flint prism fitted between two crown prisms. There is no telescope; the eye simply looks in at the aperture. The advantages of the instrument are, that it can be carried in the pocket, and that it is pointed directly at the light to be investigated; there is no troublesome angle to be allowed for.

EXAMPLES VII

1. A prism is to be made of crown glass, the refractive index of which is 1.523, and is required to produce a minimum deviation of 25° . To what angle must it be ground?

2. In determining the index of refraction of a glass prism by the spectrometer, A is found to be $44^\circ 46'$ and δ to be $29^\circ 32'$ for sodium light. What is the value of the index of refraction?

3. If the index of refraction of a glass prism for sodium light is 1.517, and the angle of the prism $59^\circ 57'$, what is the minimum value of the deviation it produces?

4. A glass prism has a refracting angle of 25° . Its index of refraction is 1.563. A ray of light falls perpendicularly on one face of the prism, enters the glass, and is refracted at the other face. What is the value of the deviation produced?

5. What is the greatest allowable angle of a prism, in order that a ray incident on the first face at an angle of 60° may emerge from the second face? The index of refraction of the glass is 1.583.

6. Show that if the angle of a prism is greater than twice the critical angle for the medium of which it is composed, it is impossible for a ray to pass through it.

7. Instead of taking the spectrometer telescope to a window and focussing it on an infinitely distant object, a student turns it towards a lamp in the room at a distance of 12 metres and gets the image of the latter sharp. The cross-wires are then at a distance of 30 cm. from the object glass. How many millimetres must the object glass be racked in to make the focus exactly right?

8. The horizontal bar of a window frame is seen against the sky. If it is viewed through a glass prism held in front of the eye with its refracting edge horizontal and pointing downwards, how will the appearance of the edges of the bar be affected ?

9. The presence of carbon monoxide in the blood is indicated by certain dark bands in the absorption spectrum of the blood. How would you test for the presence of carbon monoxide in a specimen of blood ? Make a diagram showing how you would arrange the apparatus.

10. Show that when light passes through a thin prism, the deviation does not vary with the angle of incidence, provided that the incidence is nearly perpendicular.

11. If a star were observed by a telescope the object glass of which was not corrected for dispersion, what appearances would present themselves on sliding the eyepiece in and out ?

12. White light is incident perpendicularly on one face of a prism of refracting angle 25° . The indices of refraction of the end rays of the visible spectrum are 1.611 and 1.631 for the glass of which the prism is composed. Calculate the angular separation between the end rays of the spectrum produced.

13. Explain how to combine prisms so as to produce deviation without dispersion and dispersion without deviation.

14. Two thin prisms of refracting angles A and A' are made of glass of refractive indices μ and μ' respectively. $(\mu - 1)A = (\mu' - 1)A'$. Show that, when they are turned opposite ways, the one prism neutralises the deviation produced by the other.

CHAPTER VIII

THE EYE AND COLOUR VISION

The Eye.—The human eye consists practically of a spherical chamber with a circular opening in front ; by means of a system of spherical refracting surfaces the light entering this opening forms an image on the back of the chamber.

Fig. 392 represents a section of the eye. It is surrounded outside, except in front, by a hard, horny coating S, called the sclerotic. In front is the cornea C, which bulges outwards, and acts as a transparent window to admit light to the crystalline lens L ; the latter is attached to the walls of the eye by the ciliary muscle M. In front of the lens is a diaphragm, I, called the iris, which is coloured ; it is the colour of the iris that is meant, when the colour of an eye is referred to. In the centre of this diaphragm there is a circular aperture called the pupil. The pupil adapts itself to conditions, contracting when the light is strong and opening when the light is weak, so that as uniform an amount of light as possible may be admitted to the eye. The space A between the lens and cornea, *i.e.* the anterior chamber, is filled with a watery liquid containing a little salt in solution, which is called the aqueous humour. The space V behind the lens, *i.e.* the posterior chamber, is filled with a transparent gelatinous substance termed the vitreous humour.

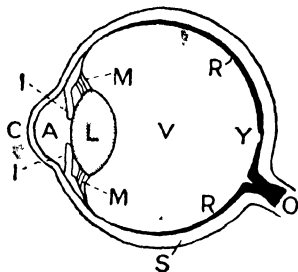


FIG. 392.—Eye.

After passing through the cornea, aqueous humour, lens, and vitreous humour, the rays fall on the retina R. The retina consists principally of a network of nerve fibres connected with the brain by the optic nerve O. In the retina there are two kinds of vision cells, termed rods and cones. In the retina directly behind the pupil is the yellow spot Y, or macula lutea, which has a depression in the centre called the fovea centralis ; here vision is most distinct. There are no rods in the fovea centralis. The point in the retina

at which the optic nerve enters is not sensitive to light, and is called the blind spot.

The index of refraction of the cornea is nearly the same as that of the aqueous humour. Consequently rays entering the eye suffer refraction mainly at three surfaces, the outer surface of the cornea and the two surfaces of the lens.

In focussing the eye the retina cannot be pushed further away from its lens like the focussing screen of a camera. The eye lens is, however, plastic like a piece of india-rubber, and the curvature of its surfaces can be altered by the ciliary muscle. Usually, when the eye is at rest, it is adjusted so that the image of a distant object is in focus on the retina. If, then, it is turned to a nearer object, its pose must be altered, it must be accommodated to the nearer object in order that the latter may be clearly seen. This is done by a forward motion of the lens and an increase in curvature of both its surfaces.

The image formed on the retina is an inverted one, but this causes no confusion, since the impression it produces is always associated mentally with the upright position of things.

The **distance of distinct vision** has already been defined on p. 363, as the distance at which the eye can best examine the detail of an object. For the normal eye it is about 10 in. or 25 cm. When the ciliary muscle is quite relaxed, the eye is focussed on its **far point**. For the normal eye the far point is at infinity. When the ciliary muscle has contracted as much as it can, the eye is focussed on its **near point**; the latter is, of course, nearer to the eye than the distance of distinct vision. The near point can be determined by noting the shortest distance at which very small type is legible.

The focal length of the eye is altered by immersing it in water, just in the same way as the focal length of a lens is altered by immersing it in water. This is noticed by boys when diving; the pebbles at the bottom are not seen distinctly, because it is impossible to make up by increase in curvature what the eye has lost in relative refractive index.

Experiments on the Resolving Power, Chromatic Aberration, and Spherical Aberration of the Eye.—(1) Rule parallel black lines in Indian ink on a piece of cardboard, making the lines and the spaces between them each one millimetre broad. Set up the card in a good light, and determine the distance at which the lines can just be seen separate. The angle subtended by two adjacent lines at this distance is the resolving power of the eye. For a normal eye it should be about one minute.

(2) When the eye is adjusted for parallel light the focus of the violet rays lies 0.43 mm. nearer the lens than the focus of the red rays does. This difference of focus can be shown very well with certain kinds of cobalt glass which let through only the blue and violet and the extreme

red end of the spectrum. If we look through such a piece of glass at the filament of an electric glow lamp, we see two images super-imposed, a red one and a blue-violet one. If the filament is at a great distance, the eye focusses involuntarily on the red image, which consequently appears surrounded by a blue-violet haze. If the filament is so near that the eye cannot focus on the red, the blue-violet image is seen surrounded by a red haze.

(3) Determine the near point. Pierce a piece of cardboard with a pin, hold the pinhole close up to the eye, and look at a page of printing through it. It will be found that the printing can be read easily when closer to the eye than the near point. The focal length is thus less for the axial rays than for the marginal rays, the opposite to what happens in the case of the single convex lens.

To find the Blind Spot.—Make a large and a small disc on a piece of white paper about 3 in. apart, the one an eighth and the other three-eighths of an inch in diameter. Hold the paper so that one eye is exactly opposite to each spot, direct both eyes to the smaller spot, and close the eye opposite to this spot. Then move the paper backwards and forwards, keeping the open eye directed towards the small spot, and it will be found that at a certain distance the large spot completely vanishes, because the rays from it enter the open eye obliquely, and fall upon the blind spot.

Defects of Vision.—There are four defects of the eye of frequent occurrence, which are remedied by the use of spectacles. These are (1) Myopia, or short sight; (2) Hypermetropia, or long sight; (3) Presbyopia; (4) Astigmatism.

In **myopia**, or short sight, the focal length of the eye is too small, and parallel rays are brought to a focus in front of the retina. Consequently distant objects cannot be seen distinctly.

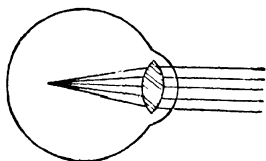


FIG. 393.—Short sight.

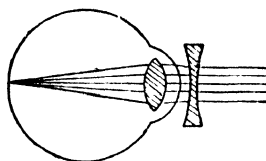


FIG. 394.—Short sight corrected.

Near objects can be seen quite well, and the distance of distinct vision is less than in the case of the normal or emmetropic eye. The defect is corrected by the use of a concave spectacle lens.

In **hypermetropia**, or long sight, the focal length of the eye is too great, and when the eye is relaxed, parallel rays are brought to a focus behind the retina. By an act of accommodation the focus can be brought on to the retina in the case of distant objects, and consequently they can be seen clearly. But even when the lens is curved as much as possible, the images of near objects

still fall behind the retina and appear blurred. In the hypermetropic eye the distance of distinct vision is greater than for the normal eye. Hypermetropia is corrected by the use of a convex spectacle lens.

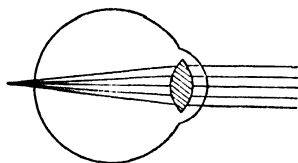


FIG. 395.—Long sight.

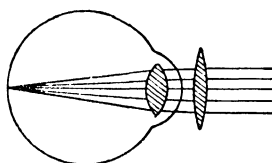


FIG. 396.—Long sight corrected.

Presbyopia is a loss of accommodating power occurring in the case of elderly people owing to the stiffening of the muscle that alters the curvature of the crystalline lens. People afflicted with it can see distant objects clearly, but not near objects. Sometimes the accommodation almost entirely disappears. Convex spectacle lenses, usually powerful ones, must be used for reading and writing, and less powerful ones for getting about with.

In **astigmatism** the focal length of the eye is different in two planes at right angles to one another, that is, if on a card two sets of parallel lines are ruled at right angles to one another, and the card is rotated so that the directions of the lines correspond with the above two planes, only the one set of lines can be seen distinctly at once. If the other set is to be seen distinctly, the accommodation of the eye must be altered, and the first set put out of focus. Astigmatism is due to the surfaces of the lens and the cornea, principally the surface of the cornea, not being symmetrical about their axis. There are few eyes which do not suffer from this defect in a slight degree. It is corrected by the use of cylindrical, sphero-cylindrical, and toric lenses. A sphero-cylindrical lens is one, one surface of which is spherical and the other surface of which is cylindrical. A toric or toroidal, or more strictly a plano-toric lens, has one surface plane and the other with different curvatures in two planes at right angles to one another.

Diopters.—The power of spectacle lenses is always measured in diopters, and is obtained in this unit by dividing 40 by the focal length measured in inches or dividing 100 by the focal length measured in centimetres. If the lens is a convex one, a + is prefixed to the result, and if concave, a —. Thus a lens of power —2 diopters is a concave one with a focal length numerically equal to 20 in.

EXAMPLE.—A long-sighted person can see distinctly only objects which are at a distance of 60 cm. or more. Calculate in diopters the

power of a pair of spectacle lenses which will enable him to see objects distinctly at a distance of 25 cm.

The lenses must be convergent and of such a strength that the rays which diverge from an object at 25 cm. appear after passing through the lens to diverge from a virtual image at 60 cm. Suppose that the eye is facing towards the right. Then

$$\frac{1}{60} - \frac{1}{25} = \frac{1}{f}$$

and $f = -30$ cm. The power of the lens is consequently $+2\frac{1}{3}$ diopeters.

EXAMPLE.—A short-sighted person's far point is at 80 cm., *i.e.* he cannot see objects distinctly which are at greater distances than this. What lens will correct this defect?

The eye is too convergent, and hence a concave lens must be used. Objects which are at an infinite distance must be seen distinctly, *i.e.* after passing through the lens they must appear to diverge from an image at 80 cm. But since the objects are at infinity, the rays which come from them are parallel. Hence the focus of the lens must be at 80 cm. and its power is consequently $-1\frac{1}{4}$ diopeters.

Binocular Vision.—When we look at a near object, the lines of vision of the two eyes have to converge towards it, and an act of muscular accommodation has to be made. The magnitude of the slight exertion involved in this helps to give us an idea of the distance of the object. But our estimation of distance is principally made in another way altogether. The image formed on the fovea of the one eye is not exactly the same as the image formed on the fovea of the other, because the object is regarded by the two eyes from different standpoints. The difference is, of course, easily noticed, when we look through a window, and close first one eye and then the other; the window frame appears to move relatively to the background. This difference in the two images we associate with solidity; if the two images are exactly the same, as is the case when we look at a picture, the images on the retinas of the two eyes fuse together perfectly, and give a sensation of flatness, and the objects in the picture do not stand out in relief.

The Stereoscope.—The stereoscope is an instrument used for looking at photographs with, which gives them a wonderful sense of relief and solidity. The photographs must be taken in pairs, each pair of the same object but from slightly different standpoints. Fig. 397 illustrates the principle of the instrument. ab and $a'b'$ are the two photographs. ab is looked at by the one eye through the half lens M, and $a'b'$ by the other eye

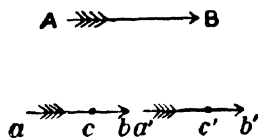


FIG. 397.

through the half lens N, which is of the same power as M. Each of these lenses produces a virtual enlarged image, and also acts as a prism and deviates the rays. The two images thus appear at AB on the top of one another, and, as owing to the photographs being taken from different points the images do not superimpose perfectly, the observer sees one picture which seems to stand out in relief.

Photographs for the stereoscope may be taken with an ordinary camera by placing it first in the one position and then in the other. They are, however, usually taken simultaneously by means of special cameras with two equal lenses mounted a short distance from each other side by side. A special camera is necessary when moving objects are to be photographed.

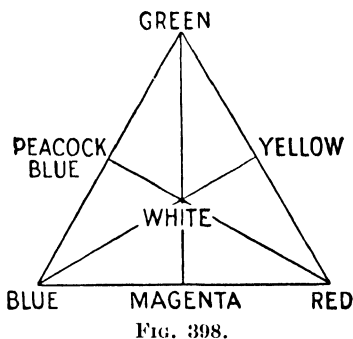
Adding Coloured Lights.—We shall now pass on to the study of the sensation of colour. Let us suppose that by means of three projection lanterns we are able to throw three discs of light upon a screen in a darkened room, also that one of the discs is red, another green, and the third blue. For the success of the experiment it is necessary that the colours should be very pure. Then by tilting the lanterns the discs may be made to move across the screen and superimpose on one another. It will be found that if the red is superimposed on the green we obtain yellow. Strong red imposed on weak green gives orange, weak red on strong green yellowish green. Green superimposed on blue gives peacock blue. Red superimposed on blue gives magenta, and red on a stronger blue gives purple. Red, green, and blue superimposed on one another make white. If white is dimmed, we get grey; if orange is dimmed we get brown. Superimposing white on any colour makes it paler. Thus by means of the three colours, red, green, and blue, we obtain nearly all the colours that occur in nature. They do not give us violet, but pure violet does not occur frequently in nature. So red, green, and blue are termed the **primary colours**. Putting our results in the form of a table we obtain :

$$\begin{array}{l}
 \text{Red} + \text{Green} + \text{Blue} = \text{White} \\
 \text{Red} + \text{Green} = \text{Yellow} \\
 \text{Green} + \text{Blue} = \text{Peacock blue} \\
 \text{Blue} + \text{Red} = \text{Magenta} \\
 \text{Red} + \text{Peacock blue} = \text{White} \\
 \text{Green} + \text{Magenta} = \text{White} \\
 \text{Blue} + \text{Yellow} = \text{White}
 \end{array}
 \left. \vphantom{\begin{array}{l} \text{Red} + \text{Green} + \text{Blue} = \text{White} \\ \text{Red} + \text{Green} = \text{Yellow} \\ \text{Green} + \text{Blue} = \text{Peacock blue} \\ \text{Blue} + \text{Red} = \text{Magenta} \\ \text{Red} + \text{Peacock blue} = \text{White} \\ \text{Green} + \text{Magenta} = \text{White} \\ \text{Blue} + \text{Yellow} = \text{White} \end{array}} \right\}$$

Peacock blue, magenta, and yellow are termed the three **complementaries**, since each of them combined with one of the primaries gives white. Peacock blue is sometimes referred to as

minus-red, since it is the colour obtained by subtracting red from white, and in the same way magenta and yellow are referred to as minus-green and minus-blue.

The Colour Triangle.—The colour triangle, which is represented in Fig. 398 and also in the frontispiece, is a very useful method of exhibiting at a glance the results of adding all possible combinations of coloured lights by the arrangement described in the previous section. The first attempt at a diagram of this nature is given in Newton's *Opticks*, but the method was not developed fully until it was taken up by Helmholtz and Maxwell two centuries later. In the triangle, which is an equilateral one, every point represents a colour. The primary colours are at the corners, the complementaries at the mid-points of the sides, and white at the centre. The spectrum colours, red, orange, yellow, yellowish-green, green, peacock blue, lie along the first and then somewhat

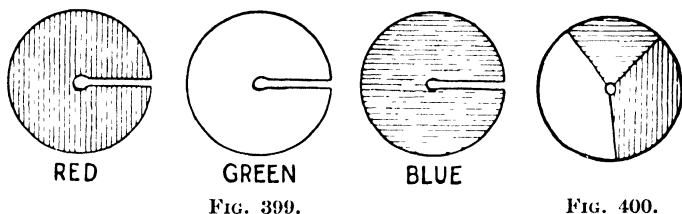


outside the second upper side of the triangle. The violet end of the spectrum is below blue outside the triangle. The colours along the base are, in order, red, rose-pink, magenta, purple, blue. The saturated colours lie along or outside the sides of the triangle; the paler colours inside round about white. For example, if we travel along the median from the red corner, the red becomes paler and paler, until it merges into white; the white then takes on a faint peacock-blue tinge which increases in saturation until the side of the triangle is reached.

It is found that if any two colours in the diagram are superimposed, their intensities being in the ratio of a to b , the result is represented by the point in the line joining their positions which divides this line in the ratio of a to b , and is nearer the colour of which most was taken. Or, in other words, the rule is the same as that for finding the position of the centroid of two unequal masses. Thus, since red and green in equal quantities make yellow, yellow is represented by the point midway between them, and we infer from the diagram, for example, that if yellow and blue are added to make white, the intensity of the yellow must be twice as great as the intensity of the blue. This is sometimes expressed by saying that blue is twice as strong as yellow of the same intensity.

The properties of the colour triangle may be proved also by the following three experimental arrangements.

Maxwell's Colour Disc.—Fig. 399 represents red, green, and blue discs, in which slits are cut. Two of these discs are slipped into the slit of the third, and so combined into a single disc, as shown in Fig. 400.



The arrangement will be understood better, if a model is cut out with a pair of scissors, and fitted together as shown in the figure. The resultant disc has three sectors, all of variable angle, for the angles can be altered by slipping the discs round. Suppose, now, that this disc is spun rapidly round its centre by fixing it on a turntable rotated by a handle and gearing; then the colours of the different sectors fuse owing to the persistence of vision, and the impressions on the retina add. The relative intensities of the different colours can be altered by varying the angles of the sectors, and the arrangement enables us to combine the three primaries in all possible proportions. The disc must be placed in a good light. Instead of the three primaries any two colours may be used. It is found that the results given by this arrangement agree with those obtained by superposing lights upon a screen.

Lambert's Method of Superposing Colours.—Fig. 401 represents another method of mixing colours, Lambert's method. P is a glass plate, a stripped photographic plate, for example. A and B are pieces

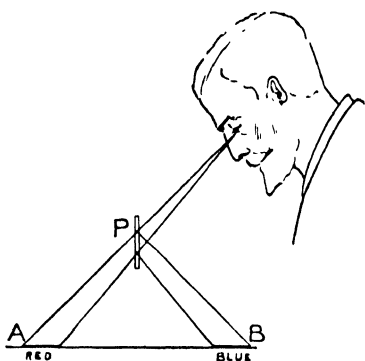


FIG. 401.

of red and blue coloured paper laid out on a piece of black velvet cloth. The eye can see A through the glass, and B by reflection from the surface of the glass; they consequently appear superimposed, and the colours mix. With the arrangement shown in the diagram the red would appear much brighter than the blue. By holding the glass plate so that the reflection takes place at grazing incidence, the blue can be made brighter. This might also be done by illuminating it with a lamp.

Newton's Method of Superposing Colours.—The spectrum colours themselves can be mixed by the arrangement shown in Fig. 402. RV is a spectrum formed by a glass prism. It is received by a lens which focusses it on a screen W, where an image of the prism is formed. If all the colours of the spectrum are

received by the lens, this image is white. But if a sheet of cardboard is placed at VR with two slits in it, one to let through the red and the other to let through the green, then at W we shall have a red and a green image of the prism superimposed, and the result will be yellow. In the same way, by having the slits at other points in the spectrum

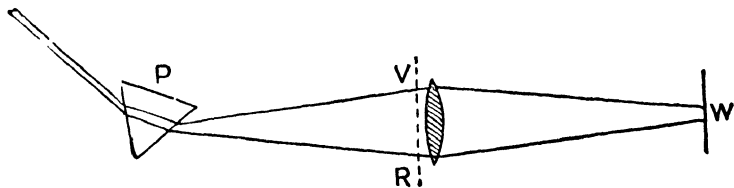


FIG. 402.

the effect of mixing other colours can be tried. This is the manner in which Newton first studied the subject, and laid down its principles two and a half centuries ago. The arrangement was greatly improved by Sir William Abney; his development of it is known as "Abney's Colour Patch Apparatus."

Mixing Coloured Pigments.—According to experiments made by the various methods described above, red, green, and blue are the primary colours. But from our experience with water colour paints in childhood, we have always been accustomed to regard red or rather carmine, yellow, and blue as the primary colours. If we take six large test-tubes, three empty and three filled with a carmine, a yellow, and a blue liquid respectively, for example, solutions of fuchsin, naphthol yellow, and copper sulphate, and if we pour the solutions two and two together into the empty tubes, then the carmine and yellow make red, the yellow and blue make green, and the blue and red purple. This is a good way of exhibiting the mixing of pigments to a large audience. Yellow and blue then make green. According to the colour triangle they should make white. Whence comes the contradiction?

The colours of natural bodies and pigments are due to the fact that most of the white light falling on them goes some distance in, and some of its constituent rays are absorbed before reflection takes place. Thus a yellow pigment appears yellow because, when the constituents of white light fall upon it, the blue and violet are absorbed, and red, yellow, and green reflected. Most yellows occurring in nature are not very pure, and reflect red and green as well as yellow. A blue pigment appears blue because it absorbs red and yellow; most blues occurring in nature are not very pure, and reflect green as well as blue. When the yellow and blue pigments are mixed, the mixture absorbs all the colours absorbed by its components singly, *i.e.* blue, violet, red, and

yellow. Green is the only colour left. It alone is reflected and the mixture appears green. A mixture of pigments gives only the colour which neither absorbs, not the sum of the two colours, as we obtain when adding lights.

The Wratten Filters.—Kodak, Ltd., sell two sets of coloured gelatine filters which illustrate the mixture of coloured pigments excellently, and are very useful for experiments on colour generally. The first set, the standard tricolours, let through each one-third of the spectrum, and give red, green, and blue. The second set, the complementaries, let through each a different two-thirds, and give peacock blue, yellow, and magenta. When the complementaries are combined two and two together, they give the primaries, a perfect match being obtained in each case, and when all three complementaries are combined they give black. Now combining the two complementaries yellow and peacock blue is equivalent to mixing yellow and blue pigments; in each case the resultant colour is that of the third of the spectrum absorbed by neither.

Three-Colour Printing.—Theoretically the best three colours to use in three-colour printing would be the complementaries of the colour triangle; inks of exactly the right colour are, however, difficult to obtain, and the result is generally a compromise. The last diagram of the frontispiece is an example of three-colour printing. The yellow impression is made first, then the magenta or carmine, and finally the blue.

Anaglyphs.—Anaglyphs are pictures printed in two colours which are looked at through pieces of coloured gelatine, and produce a stereoscopic effect. They have recently become well known through being printed in certain illustrated papers.

It was explained on p. 401 that for objects to stand out in relief it was necessary for each eye to see a picture appropriate to its own standpoint. An anaglyph consists of two separate pictures printed on the top of one another, one for each eye, one in red and the other in bluish green. The pieces of gelatine are red and bluish green. The eye which looks through the red gelatine sees the red picture either not at all or very faintly, because to it the white background appears red; it sees the green picture as black, because the rays reflected by green do not go through the red filter. The eye which looks through the green filter does not see the green picture, but sees the picture drawn in red lines as black. Hence each eye sees its own picture, and the two pictures fuse together producing the stereoscopic effect.

Colour Vision.—Nearly all colours can be reproduced by adding the three primaries, red, green, and blue, in varying proportions. This fact is beyond all dispute. But three authorities on the subject, Young, Helmholtz, and Maxwell, made the additional assumption, that there are three primary colour sensations corresponding to these primary colours; this is the fundamental hypothesis of the Young-Helmholtz theory of colour vision, and it has been a subject of debate for years.

Put in its simplest form, the Young-Helmholtz theory states, that in the retina of each eye there are three sets of nerves, one set for the sensation of red, another for the sensation of green, and a third for the sensation of blue. When red light falls on the eye, it stimulates the red nerves. When yellow light falls on the eye, it stimulates both the red and green nerves. When white light falls on the eye, it stimulates all three sets of nerves.

Colour Blindness.—Certain individuals are unable to recognise colours or match them properly; they are referred to as “colour blind.” Some are so bad that they are quite unable to distinguish between a good red and a good green; others make only minor mistakes when seeing is good, but are liable to confuse red, green, and white under difficult circumstances, when, for example, they are tired, and the lights are small and seen at a distance through mist or rain. With a healthy subject the defect occurs much more frequently with men than with women; 5 per cent. of men are usually assumed to have congenital colour blindness, but only two in a thousand women.

It is, of course, very important that the colour blind should not be admitted into occupations in which signal lights have to be distinguished, and there are various tests for colour blindness, of which the Board of Trade lantern test is perhaps the most important. The candidate to be tested sees two apertures in the side of the lantern by reflection in a mirror; they appear to him the same size as signal lights at a distance. Their colours are changed rapidly and in different orders, and he is simply asked to name them.

According to the original form of the Young-Helmholtz theory the colour blind lacked either one or two of the three sets of nerves. All the commonly occurring cases of colour blindness lacked either the red or green set of nerves, and were consequently referred to as red or green blind. It is now universally recognised that conditions are not so simple as this. But the subject is controversial, and it would be out of place to go further into it here.

EXAMPLES VIII

1. Determine the near and far points of your own eyes.
2. A long-sighted person can see distinctly only objects which are at a distance of 50 cm. or more. Find the power of spectacle lenses which will enable him to see distinctly objects at a distance of 25 cm.
3. The far point of a certain short-sighted person is situated at a distance of 30 cm. from the eye. Calculate in diopters the power of a spectacle lens which will enable him to see objects at a distance.

4. A short-sighted person sees printed matter most distinctly when it is at a distance of 12 cm. from his eyes. Find the power of spectacle lenses which will change his distance of distinct vision to 30 cm.

5. A short-sighted person has distinct vision at 4 inches. What kind of spectacle lenses will enable him to read a book at a distance of 15 inches from his eyes ?

6. A person whose distance of distinct vision is 12 cm. uses a magnifying glass of 4 cm. focal length to magnify a small object. What is the distance of the object from the magnifying glass when in focus, and what is the magnification obtained ?

7. What is the apparent colour of a green body, when it is in a room to which only red light is admitted ?

8. Red light is found to penetrate a fog more effectively than light of any other colour. Would the penetrating power of an arc lamp be increased by passing its rays through red glass ?

CHAPTER IX

THE NATURE OF LIGHT

Galileo's Experiment.—Galileo made an attempt to determine the velocity of light by means of two observers furnished with lamps and situated a distance apart. The first observer uncovered his lamp, and the second observer uncovered his as soon as possible after seeing the light from the first observer's lamp. The idea was, that the time which elapsed between the first observer's uncovering his own lamp and his seeing the second lamp would be equal to the time taken by the light to go from him to the second observer and back. The method failed owing to the enormous velocity of light, the time taken by it to travel the distance in question being very much less than the time necessary to uncover one lamp or to see another.

The velocity of light can be determined experimentally by four separate methods. These, taken in order of their discovery, are Römer's method, the aberration method, Fizeau's method, and Foucault's method. Fizeau's method is the easiest of the four to understand.

Römer's Method.—Four of the moons of the planet Jupiter are large enough to be observed easily with a small telescope. Their periods of rotation about Jupiter vary from 42 hours to 16½ days. Their orbits are in approximately the same plane as the orbit of Jupiter about the sun. They are, of course, dark bodies, and are illuminated solely by the reflected light of the sun ; consequently when they enter the shadow cast by Jupiter they are eclipsed or disappear. Now it is natural to assume that they rotate about Jupiter with uniform angular velocity. Consequently the interval of time that elapses between two successive eclipses of any one moon should always be the same.

In 1675, however, the Danish astronomer Römer observed a peculiar variation in the times of occurrence of the eclipses. When the earth was approaching Jupiter they occurred too close together, and, when the distance between the earth and Jupiter was increasing, they occurred too far apart. He explained the difference by means of the time taken by light to pass through space.

In Fig. 403 let S represent the sun, and let the two circles be the orbits of the earth and Jupiter. E and J are the positions of the earth and Jupiter when they are in conjunction, *i.e.* nearest one another, and E' and J' their positions when in opposition, *i.e.* farthest from one another. Jupiter takes 11.86 years to make a revolution round the sun, so that it moves only from J to J' , while the earth moves from E to E' .

We can regard the eclipses of any moon as a uniform series of time signals sent out by Jupiter. Owing to its distance from Jupiter the earth does not receive them until an appreciable time after they are sent out. As the earth moves from E to E' , its distance from Jupiter increases, and the signals are received later and later; as the earth moves from opposition to conjunction again, the distance decreases, the signals are received earlier, and

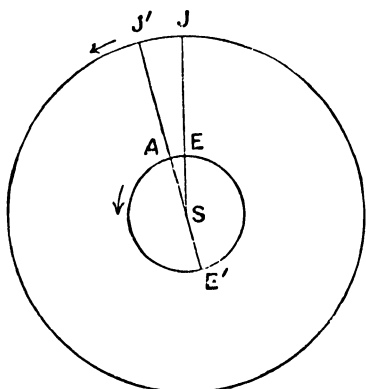


FIG. 403.

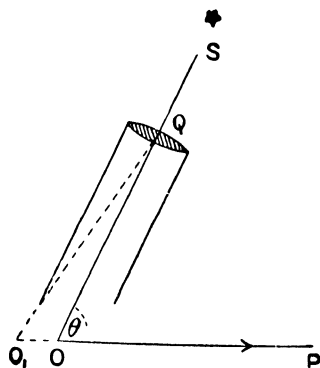


FIG. 404.

when the earth and Jupiter are again in conjunction the lost time is made up. According to Römer, when the earth is in opposition, the signals have fallen 996 sec. behind the uniform rate, and this is the time taken by the light to travel the distance AE' , *i.e.* the diameter of the earth's orbit.

The diameter of the earth's orbit is 1.859×10^8 miles. Hence, using the modern value of 1002 sec. instead of Römer's 996, we find for the velocity of light

$$\frac{1.859 \times 10^8}{1002} = 1.855 \times 10^5 \text{ ml./sec.} \\ = 2.98 \times 10^{10} \text{ cm./sec.}$$

The Aberration Method.—The apparent direction of the light from a star depends on the motion of the telescope. For example, in Fig. 404 let OS be the true direction of a star, let the telescope be pointed in the direction OS , and let the telescope and observer

be moving with velocity v in the direction OP . Then, when the light is passing down the telescope, the latter is moving sideways; consequently the path of the central ray *relative to the telescope* is shown by the dotted line QO_1 , and if the image of the star is to appear in the middle of the field on the cross-wires, the telescope must be pointed in the direction O_1Q . We suppose that the image is a small distance from the centre of the field. Let θ be the true direction of the star, let V be the velocity of light, and let t be the time taken by the light to travel down the telescope. Then $O_1Q = Vt$ and $O_1O = vt$. Also in triangle QOO_1

$$\frac{\sin O_1QO}{O_1O} = \frac{\sin QOO_1}{QO_1}$$

$$\frac{\sin O_1QO}{vt} = \frac{\sin \theta}{Vt}$$

This gives

or, since $\sin O_1QO$ is very small

$$\angle O_1QO = \frac{v}{V} \sin \theta$$

Thus, owing to the motion of the telescope, the star is displaced in the direction of that motion in front of its true position by an angle equal to $(v \sin \theta)/V$.

The earth moves in its orbit about the sun with a velocity of about $18\frac{1}{2}$ ml./sec. If this value be substituted for v , and $\sin \theta$ be put equal to unity, $(v \sin \theta)/V$ takes the value 20 sec. Thus, if the telescope is moving with the velocity of the earth, the stars receive an apparent angular displacement varying from 20 sec. to 0 sec. according to their position in the heavens.

This apparent displacement is known as aberration. It was discovered and measured in 1726 by Bradley, the Astronomer Royal. He gave the correct explanation, and calculated the velocity of light from his observations. Bradley's discovery proved the correctness of Römer's views; until then they had been neglected. Aberration has the effect of making the apparent position of each star execute an annual motion about its true position.

Fizeau's Method.—The first terrestrial determination of the velocity of light was carried out by Fizeau in 1849. His arrangement is shown diagrammatically in Fig. 405. A beam of light from a source S passes through a converging lens, is reflected by a glass plate P , and comes to a focus at F . It is then made parallel by the lens O , traverses a very great distance OL , falls on the lens L , and is brought to a focus on the surface of the concave mirror M . The radius of curvature of this mirror is equal to ML , its distance from the lens; the central ray of any pencil through the lens thus falls on the mirror normally, and is reflected back the way it comes, even though inclined to the axis of the mirror. The lens and

mirror L and M thus direct the beam back through the lens O to form a real image once more at F, and the observer looks at this real image through the eyepiece E and the glass plate P.

W is a toothed wheel, and, as it rotates, its teeth pass one after another through the point F alternately stopping and letting through the light. If the wheel is moving slowly the eye sees a flickering image of S. If the images succeed each other faster than 15 or 20 a second, the flickering ceases owing to the persistence of vision, and the image becomes steady. It is, of course, not as

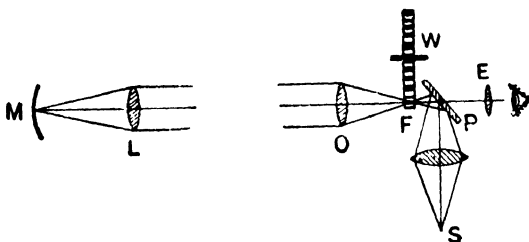


FIG. 405.

bright as it would be if the wheel were away; the teeth in passing stop some of the light.

If the speed of the wheel is still increased, so that the time taken by the light to go from F to M and back is exactly equal to the time required for a tooth to move into the position formerly occupied by an open space, the light is intercepted by a tooth on its return, and the image vanishes. If the speed of the wheel is now doubled, the light passes through the next space, and the image is again visible; if the speed of the wheel is trebled, the light is intercepted by the next tooth, and again vanishes. And so on; as the speed increases, the image alternately appears and vanishes.

In Fizeau's experiments the wheel had 720 teeth, and the widths of the teeth and open spaces were equal. The distance between M and F was 8.6 km. A determination was made of the angular velocity of the wheel for which the image disappeared. Let it be ω radians/sec. for the n th disappearance, and let V be the velocity of light in km./sec. Then (cf. Fig. 406)

$$\frac{2 \times 8.6}{V} = \frac{(2n-1)}{2} \cdot \frac{2\pi}{720} \cdot \frac{1}{\omega}$$

i.e.

$$V = \omega \frac{720 \times 2 \times 8.6}{(2n-1)\pi}$$

whence V can be calculated.

The best determination by Fizeau's method was made by Perrotin at the Nice Observatory in 1900. The distance FM

(Fig. 405) was 40 km., and the result obtained was 2.9986×10^{10} cm./sec.

Foucault's Method.—Fig. 407 shows the details of the method by which Foucault made a determination in 1862. It requires a much shorter distance than Fizeau's method. S is a rectangular aperture illuminated with sunlight, Q a plane parallel plate of glass, L a lens, R a plane mirror, which can be rotated about an axis perpendicular to the plane of the figure, and M a concave mirror. The lens L forms an image of S on M. The centre of curvature of M is at the centre of R, hence no matter what the position of R is, if the light from it falls on M at all, it falls on it normally and is thus reflected back along its path. If R is in the

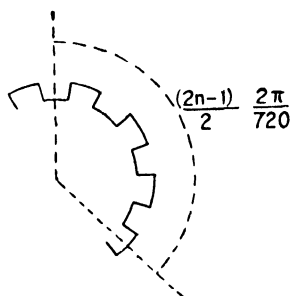


FIG. 406.

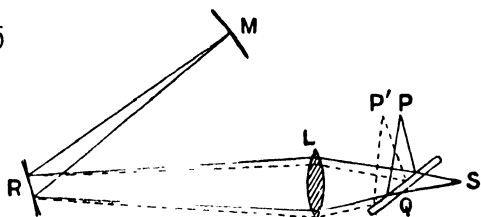


FIG. 407.

same position when the light reaches it again, the rays travel back to Q and are reflected to form an image at the side at P. If R is rotating rapidly, it has moved through an appreciable angle by the time the beam returns from M, and consequently the light is reflected in the direction of the dotted rays to form an image at P'.

In Foucault's experiment the distance RM was 20 metres, and the displacement PP' 0.7 mm. From the displacement the angle turned through by R was calculated, and then the angular velocity of R having been determined, the time taken by the light to go from R to M and back was known.

By placing a tube with water between R and M, Foucault was able to show that the velocity is less in water than in air.

Foucault's method has been used and considerably improved by Michelson and by Newcomb. Michelson's result (1882) was 2.9985×10^{10} cm./sec., and Newcomb's result 2.9986×10^{10} cm./sec., the same value as Perrotin obtained afterwards by the other method. Michelson has recently made a re-determination which has given the result 2.9980×10^{10} cm./sec. with an accuracy of one part in ten thousand. In order to simplify calculation the

value 3×10^{10} cm./sec. is generally used. This is a very great velocity; light could travel round the world $7\frac{1}{2}$ times in a second.

The two terrestrial methods of determining the velocity of light are much more accurate than the astronomical methods.

Emission Theory.—We have seen that light travels with a finite velocity. Light rays carry energy with them; this is shown by the heating effect they exert on bodies on which they fall. Now energy is propagated with a finite velocity in two different ways:

- (1) By the motion of the matter carrying the energy;
- (2) By the transference of the energy alone, as in wave motion, the matter remaining stationary.

An example of the first way is the stream of bullets from a machine gun. There each bullet has an amount of kinetic energy given by $\frac{1}{2}mv^2$, where m is its mass and v its velocity, and if the bullet is stopped by a wall, this energy reappears as heat in the wall. Now we explain things by analogy with other things. Hence it is natural to assume that light rays consist of jets of extremely small particles shot out from the source like a stream of bullets. The particles are so small, that they can be emitted from the source for a long time without causing it to lose weight. This manner of explaining the ray is called the emission theory of light, and it was the theory adopted by Newton.

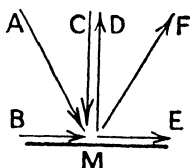


FIG. 408.

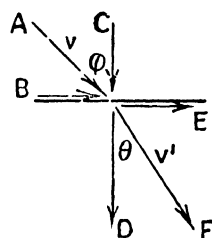


FIG. 409.

The emission theory gives a natural explanation of the rectilinear propagation of light. It also accounts in a simple manner for the laws of reflection and refraction. For, suppose that the particles are perfectly elastic, and rebound from the mirror like rubber balls. Let the arrow A represent the path and also the incident velocity of a particle incident at M (Fig. 408). Resolve this velocity into the two components B and C. The vertical component has its direction reversed by the impact, and so can be represented by D after reflection. The horizontal component is unaltered, and so can be represented by E. D and E compound into F, which makes the same angle with the normal as A does.

To explain refraction let A (Fig. 409) represent the incident velocity as before. Let its magnitude be V , and let ϕ be the angle of incidence. Newton supposed that on approaching the denser medium the particle is attracted by the molecules of the latter. Consequently the normal component of its velocity is increased, and has in the second medium to be represented by an arrow D which is longer than C. Since there is no force on the particle tangential to the surface, the tangential component remains unaltered, and E is equal to B. Thus the resultant velocity in the second medium is represented in magnitude and direction by F. Let V' be its magnitude, and let θ be the angle of refraction. Since

$$B = E, \quad \frac{V \sin \phi}{\sin \theta} = \frac{V' \sin \theta}{\sin \phi}.$$

and

i.e. the law of refraction holds, provided that the velocity in the second medium is constant, also that the velocity is greater in the denser medium.

Hence according to the emission theory the velocity of light must be greater in water and glass than in air.

Wave Theory.—The simplest way of studying wave motion is to drop a stone into a pool, and watch the train of waves propagated out on the surface of the water from the centre of disturbance. When a wave is passing, the water particles oscillate about their positions of equilibrium, and after the wave is past, they return to these positions. But the energy of the initial disturbance is carried on, and transformed into the motion of the stones caused by waves breaking at the edge of the pool.

The analogy between the propagation of light and wave motion is not at first sight very pronounced, but in 1678 Huygens put forward the theory that light consisted of a wave-motion propagated in the ether. The rays were at right angles to the wave-front. The ether was a continuous weightless medium filling all space. It was necessary to postulate the existence of this medium for the light to travel in from the sun, for there is no matter in interstellar space.

Huygens gave the following construction by which, if the

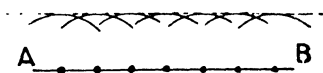


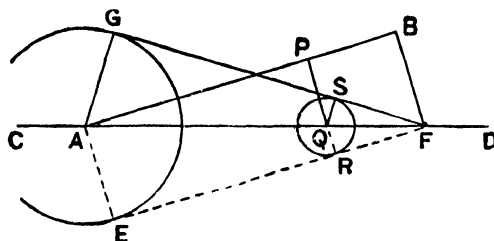
FIG. 410.

position of the wave-front was known, its position could be found after any interval of time. Let AB (Fig. 410) be the wave-front; consider all the points on the wave-front as centres of disturbance, and draw spheres with these points as centres, and the distance the light would travel in this time as radius. Then all these spheres have a common tangent plane. This tangent plane is

the position of the new wave-front after the given interval of time. Thus the wave-front at any point is moving in the direction of the normal to the wave-front at that point. The construction can be applied to curved surfaces as well as to plane ones.

Huygens' Construction for Reflection and Refraction.—To apply Huygens' construction to the reflection of a plane wave, let AB be the trace of a plane wave-front which is perpendicular to the plane of the paper, and let CD be the trace of a reflecting surface, also perpendicular to the plane of the paper. Draw BF perpendicular to AB to meet CD in F ; draw AE parallel and equal to BF , and join EF . Then EF gives a position which the wave-front would have occupied if there had been no mirror. Take P any point on the wave-front, and draw PQR perpendicular to AB .

Let A and Q act as centres of secondary disturbances. With A as centre draw a circle of radius AE , and with Q as centre draw a circle of radius QR .



F.G. 411.—Huygens' construction for reflection.

Then, when the point B has reached F , the light from A and P has reached the circumferences of the two circles. Since FRE is tangent to both circles, it is possible to draw a straight line FSG tangent to both circles.

As P was any point on AB , FG touches the secondary waves derived from all points on AF , and is consequently the trace of the reflected wave-front. It is obvious from the figure, that AB and GF are inclined equally to CD , and hence the angles of incidence and reflection are equal.

To apply Huygens' principle to the refraction of the same wave, let V be the velocity in the upper medium and V' the velocity in the lower medium, let AB be the trace of the incident wave, let ED be the position it would have reached in time t , had there been no refraction, and let CD be the trace of the refracting surface. Then $AE = Vt$. From P , any point on AB , draw PQR perpendicular to AB . With A as centre draw a circle of radius $AG = V't$, and with Q as centre draw a circle of radius $QS = QR V'/V$. Then when the point B has reached D , the light from A and P has reached the circumferences of these two circles. Through D draw DG tangent to the larger circle; it may then be shown to touch the smaller circle and consequently to be the trace of the refracted wave-front.

AG gives the direction of the refracted ray and AE of the

wave-length is called the period of the wave. The number of crests that pass per second is called the frequency of the wave, and is usually denoted by n or ν . Hence

$$T = \frac{\lambda}{V}, \quad \lambda = VT, \quad \text{and} \quad V = n\lambda$$

The maximum displacement of each vibrating particle from its equilibrium position, *i.e.* the perpendicular distance from the top

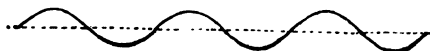


FIG. 413.

of the crest or the bottom of the trough to the dotted line is called the amplitude of the wave.

Let us suppose that a wave of light falls upon the surface of a refracting medium. Then by Huygens' construction the points on the surface of the medium may be regarded as secondary sources. They are vibrating under the influence of the incident wave and sending out waves in the second medium. It consequently follows, that the period T of the wave must be the same in both media. Let V and V' denote the velocities of light and λ and λ' the wave-lengths in the upper and lower media respectively. Then

$$\mu = \frac{V}{V'} = \frac{\lambda/T}{\lambda'/T} = \frac{\lambda}{\lambda'}, \quad \text{or} \quad \lambda' = \frac{\lambda}{\mu}$$

i.e. the wave-length varies inversely as the index of refraction. The length of the wave in water is three-quarters of its length in air, and the length in crown glass is approximately two-thirds of its length in air.

The wave-length of light increases from the violet to the red end of the spectrum, the values for the ends being about 0.00040 cm. and 0.00076 cm. The values for the two components of the sodium line, the D lines, are 0.0005890 cm. and 0.0005896 cm., the mean value being 0.0005893 cm. As it would be inconvenient always to have to write down a decimal point and four nothings every time we wish to express a wave-length, it is customary to use special units for this purpose. These are the **Ångström unit**,* 10^{-8} cm., named after the investigator who introduced it, and commonly denoted by A.U., the micron, 10^{-4} cm., written μ , and the **micromillimetre**, 10^{-7} cm., written $\mu\mu$. Thus the mean wave-length of sodium light is 5893 A.U., 589.3 $\mu\mu$, or 0.5893 μ , according to the unit it is expressed in. The scale above the frontispiece gives the wave-lengths of the different colours in A.U.

* It is becoming customary to refer to this as the angstrom, and to denote it simply by Å.

It will be observed, that the wave-lengths are more crowded together at the red than at the violet end of the spectrum, and that the angular separation between two colours in a prismatic spectrum is not proportional to their difference of wave-length. This fact is sometimes referred to as the *irrationality of the spectrum*.

Heat rays have been measured with a wave-length of 107μ , or one hundred and forty times as long as the wave-length of the red end of the spectrum, and from this limit there is an unbroken series all the way to the end of the visible spectrum. The ultra-



Fig. 414.—The complete spectrum.

violet can be photographed to 1800 A.U. ; it can be traced much further than this, if work is carried on in a vacuum, for the air of the atmosphere exerts a very strong absorption on wave-lengths below this limit. But the infra-red and ultra-violet do not exhaust the spectrum. The electromagnetic waves used in wireless, X-rays, and the γ rays of radium, though differing widely in their properties, are known now to be of essentially the same nature as light. The only real difference between them lies in the value of the wave-length. Electromagnetic waves have a much longer wave-length than the infra-red, and X-rays and γ -rays much shorter wave-lengths than the ultra-violet. Fig. 414 shows their relation to the visible spectrum. The gaps represented in black between X-rays and ultra-violet and between infra-red and wireless have not yet been fully explored.

Longitudinal and Transverse Waves.—PX, PY, and PZ are mutually perpendicular to one another. Let us suppose that a wave is propagated in the direction PX. PX is the normal to the wave-front ; it is one of the rays of which the wave is composed. Consider a particle situated at P. As the wave passes, it executes vibrations. What is the direction of these vibrations ?

They may be in the direction of PX itself. This is what happens in the case of sound waves in air ; the air molecules oscillate backwards and forwards in the direction of propagation of the wave. In this case the wave is said to be longitudinal.

On the other hand, the vibrations may be at right angles to PX, *i.e.* in the plane YPZ. In this case the wave is said to be transverse. It is also possible that the vibrations may be not only in the plane YPZ, but always parallel to a certain direction

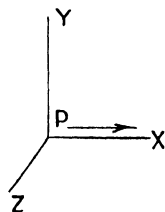


FIG. 415.

in this plane, for example, parallel to PY . In this case the wave is said to be transverse and plane polarised in the plane ZPX .

Let us approach the matter from another standpoint. If a long cord is stretched horizontally and suddenly struck with the hand, a pulse or indentation travels along it, of the shape shown in Fig. 416. It goes too fast to be followed by the eye. If, however, a heavy rope about 20 ft. long is fastened at one end and allowed to hang vertically, and if the loose end is held with the one hand, and the rope struck with the other a little above this end, the pulse can be seen ascending the rope and then descending it after reflection at the top. The pulse is a solitary transverse wave passing along the rope.

Let us suppose now, that we have two boards with slots in them, A and B (Fig. 416), and that pulses in all possible planes

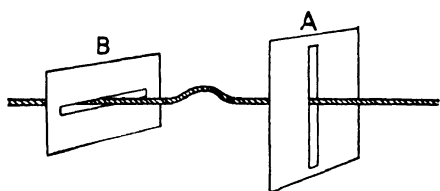


FIG. 416.

travel along the rope from the right towards A. If the plane of the pulse is parallel to the slot, it will get through undiminished. If it is inclined to the slot, only the component parallel to the slot will get through. If it is at right angles to the slot, it will

be stopped altogether. Let us suppose now, that the second slot is at right angles to the first; then the pulses that get through the first will be stopped by the second, and nothing will get through at all.

To recapitulate: if the pulse is plane polarised, it can be stopped by a single slot turned the right way; if we have a series of transverse pulses in different planes, they can be stopped by two consecutive slots at right angles to one another; if the vibration gets through two consecutive slots set at right angles to one another, the wave must be longitudinal.

We shall now apply the above results to the study of light.

Double Refraction.—Calcite or Iceland spar is crystallised calcium carbonate, and was at one time found in great quantities in Iceland in very large crystals of watery clearness. It cleaves very perfectly along three directions forming parallelepipeds, or rhombs as they are called in this connection, with their faces parallel to the planes of cleavage.

Let a pencil of light JK fall on the surface of such a rhomb (Fig. 417). Then in general it forms two refracted rays, one KM a continuation of the incident ray, such as we would get in the case of glass, and another KL which is inclined to the normal at K , even though the angle of incidence is zero. KM is called the

ordinary ray and KL the extraordinary ray. After emerging from the rhomb the two rays are parallel. If the rhomb is rotated round JKM as axis, the extraordinary ray travels round the ordinary ray.

The above phenomenon is called double refraction. It is exhibited by other crystals as well as by Iceland spar, but not in such a marked manner. It was discovered by Erasmus Bartholinus in 1669 for the case of Iceland spar.

Nicol Prism.—The rays (Fig. 417) are too close together to be examined singly. If, however, a rhomb, the edges of the end faces of which are equal and one-third the length of the other edges, is sliced from end to end, as shown in Fig. 418, and if the cut faces are polished and re-united with a film of Canada balsam, then the rhomb transmits only one of the rays. For the index of refraction of the ordinary ray AO is greater than the index of refraction of Canada balsam, the index of refraction of the extraordinary ray AE is less than the index of refraction of Canada balsam, and the angles of incidence of the two rays on the film are such that the extraordinary ray goes through, while the ordinary ray is totally reflected to the side. A rhomb prepared in this way is called a Nicol prism, or simply a nicol, after its inventor.



FIG. 417.



FIG. 418.—Nicol prism.

Natural and Polarised Light.—Let us set up two nicols with their faces parallel. Then a pencil of light which passes through the first also passes through the second. But if the second nicol is rotated through 90° about the ray as axis, the light is stopped completely by the second nicol. The natural explanation of this experiment is, that the nicols play the rôles of the slots in Fig. 416, that the light incident on the first nicol consists of transverse pulses or waves vibrating in all directions at right angles to the direction of propagation, and that after passing through the first nicol these are plane polarised; consequently if the second nicol is set at right angles to the first, or if the nicols are crossed, as it is called, no light gets through. This explanation is borne out by more elaborate experiments. So we arrive at the conclusion, *that light is a transverse wave-motion, that in natural light the vibrations take place in all directions at right angles to the direction*

of propagation, and that after passing through a nicol they are confined to one direction or plane polarised.

If the ordinary and extraordinary rays produced by the passage of a pencil through a rhomb of Iceland spar are examined by a nicol, it is found that they are plane polarised at right angles to one another.

Other Methods of Producing Plane Polarised Light.—In certain kinds of tourmaline, a double refracting crystal which occurs in many different colours, the ordinary ray is so strongly absorbed that plates cut parallel to the axis of the crystal transmit practically no light at all except the extraordinary. Such plates can therefore be used as a means of producing plane polarised light.

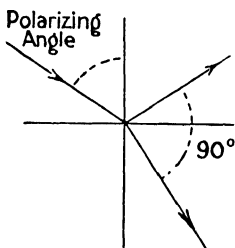


FIG. 419.

In 1808 Malus discovered that when light is reflected at a particular angle from the surface of glass, water, or other transparent substances, it is almost completely plane polarised in the plane of incidence. This particular angle of incidence is called the polarising angle. A few years later Brewster showed that the tangent of the polarising angle was equal to the index of refraction of the medium in question. This fact, Brewster's law as it is called, leads to the result that at the polarising angle the reflected and refracted rays are at right angles to one another, as shown in Fig. 419. For, let ϕ be the angle of incidence and θ the angle of refraction; then

$$\frac{\sin \phi}{\sin \theta} = \mu$$

But by Brewster's law $\mu = \tan \phi$; therefore

$$\frac{\sin \phi}{\sin \theta} = \tan \phi, \quad \sin \theta = \cos \phi$$

and $\phi + \theta = \frac{\pi}{2}$, which gives the required result.

Rotation of the Plane of Polarisation. Polarimeter.—Suppose that two nicols are mounted so as to rotate about the same axis, and that a tube filled with water and with plane glass ends is placed between them, so that the rays from the one nicol pass down it before entering the other. Then when the two nicols are crossed, the field is dark. If, however, some sugar is dissolved in the water, the field becomes light as it goes into solution, but extinction can always be restored by rotating the second nicol through a definite angle. The sugar solution thus rotates the plane of polarisation of the incident light. If the strength of the

solution is constant, the angle of rotation is proportional to the length of the tube. For a given length of tube the rotation is proportional to the strength of the solution measured in grams of dissolved substance per c.c. of solution. The rotation varies with the wave-length and also with the temperature. The rotation of the plane of polarisation is used in commerce and also in medicine as a means of estimating the quantity of sugar in a liquid.

Substances which rotate the plane of polarisation as a sugar solution does, are said to be optically active. Many other liquids besides sugar solution are optically active, as are also plates of quartz and certain other crystals cut perpendicular to the axis of the crystal.

Instruments used for measuring rotation of the plane of polarisation are called polarimeters, polariscopes, and saccharimeters. The simplest form of polarimeter consists merely of two nicols, which rotate about the same axis, and a tube between, for holding the liquid. The first nicol is called the polariser and the second the analyser. Fig. 420 represents a polarimeter of this type suitable for use by elementary students. The sodium flame is placed to the right; a polarising nicol is enclosed in the brass tube at this end of the instrument. At the other end is the divided circle on which the rotation of the analyser is read.

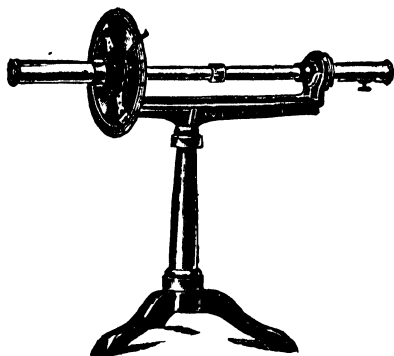


FIG. 420.—Saccharimeter.

The Quantum and the Einstein Star Shift.—It was customary in textbooks written twenty years ago to enlarge on the properties of the ether, how, for example, it offered no resistance to the passage of the planets through it, and yet was rigid enough to vibrate like a bell. At present the least said about the ether the better, for, paradoxical as it may seem, modern opinion is moving towards the view that light is a wave-motion that does not require a medium, because it possesses many of the properties of a substance itself.

Also within the past twenty years two difficulties have arisen in connection with the wave theory which are causing an immense amount of discussion and investigation at present, but can only be touched on very briefly here.

Substances absorb those radiations which they emit. Thus incandescent sodium vapour emits the D lines, and the sodium vapour surrounding the sun absorbs them. In the same way for

a red-hot piece of carbon the emission for every wave-length is proportional to the fraction of the incident radiation absorbed, and the ratio of emission to fraction absorbed is the same for all substances. In a theoretical investigation of this subject Planck found it necessary to assume, to put it crudely, that light was emitted in "quanta," that the source emitted a definite quantity of energy at a time. When a vibrating string is sounding a note, the energy is communicated gradually to the air, and the amplitude of the vibrations dies down gradually. It had always been previously assumed that something similar happened in the case of light. But apparently the store of energy in the source of light diminishes in steps, not continuously. The amount of energy emitted at a time varies with the wave-length; there is a special value of the quantum for every wave-length, which is given by the formula $h\nu$, where h is a constant known as Planck's constant, and equal to 6.55×10^{-27} erg sec., and ν is the frequency of the light wave, *i.e.* the number of crests that pass per second. The result is in ergs. Opinions as to the significance of Planck's result vary all the way from the radical view, that there is something seriously wrong with the wave theory of light, to the conservative view, that the quantum is an amount of energy released suddenly by some change within the atom, which is afterwards emitted continuously. But the general effect of the discussion has been unsettling.

It has always been taken for granted, that when a ray of light from a star passed by the edge of the sun to the eye of an observer at E on the earth's surface, it did so undeflected, as in Fig. 421.

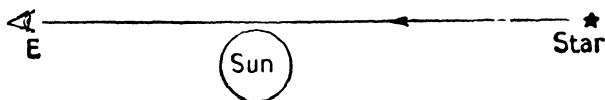


FIG. 421.

Its path was a straight line, and the observer saw the star actually where it was. According to the general theory of relativity put forward by Einstein in 1915, the ray was bent as in Fig. 422, and

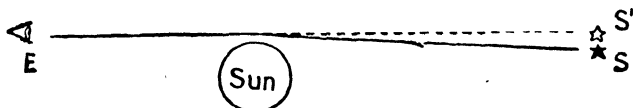


FIG. 422.

the observer saw the star in the direction S' , and not in its true position at S . The ray was deflected by the sun's gravitational attraction. A measurable effect was to be expected only in the

case of the sun's gravitational attraction, which is, of course, much stronger than the earth's ; the acceleration of gravity is 27·6 times as great on the sun as on the earth, and a man who weighs $11\frac{1}{2}$ stone on the earth would weight 2 tons on the sun.

Stars are not visible in the neighbourhood of the sun except at an eclipse, when the light of the latter is hidden. Let a , b , and c be the true directions of three stars, and S the position of the sun as seen at an eclipse ; then, according to Einstein's prediction, the stars should be seen in the directions A , B , and C . They are displaced outwards radially from the sun, and the displacements, aA , bB , cC , are inversely proportional to the distance from the sun's centre. The displacement is extremely small, being only 1·75 sec. at the sun's edge, and during an eclipse we cannot observe close to the sun owing to the light from the corona, so the actual deflections we might expect to observe are considerably less than this. But they are nevertheless within the power of modern instruments. An eclipse of the sun occurred on May 29, 1919, in a part of the sky where there were stars suitable for observing on, and the result of the observations was in agreement with Einstein's prediction.

Another eclipse occurred in 1922. The American expedition sent to Wallal in Western Australia to observe it obtained a mean shift of 1·74 sec., only 0·01 sec. less than the predicted value. This result was based on the measurement of 80 stars. The effect can now be regarded, therefore, as established beyond all doubt. Light has consequently weight, the same as a stone or any other portion of matter, although, of course, its weight is extremely small indeed. It is very difficult to fit this result in with our previous ideas on the subject.

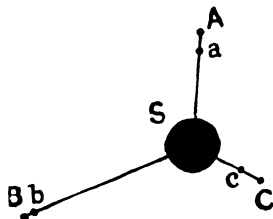


FIG. 423.

EXAMPLES IX

1. Assuming that Jupiter's first moon revolves round the planet in 42 hours, that the velocity of the earth in its orbit is $18\frac{1}{2}$ miles per sec., and that the velocity of light is 186,000 miles per sec., find the apparent interval between successive eclipses : (i) when the earth is approaching Jupiter ; (ii) when the earth is receding from Jupiter.

2. In determining the velocity of light by Fizeau's method the distance between the two stations is 10 km. and the wheel has 720 teeth. Find the angular velocity in radians per second for the third and fourth disappearance of the image.

3. In an experiment for measuring the velocity of light by Foucault's method the fixed mirror was distant 1 km. from the revolving mirror, which made 500 revolutions per sec. Assuming that the velocity of light is 3×10^{10} cm./sec., calculate the angular deviation of the return ray.

4. Show that according to the corpuscular theory of light, when a corpuscle is incident upon an optically dense medium, the increase in the normal component of its velocity is $V \sin (\phi - \theta) / \sin \theta$, where V is the resultant velocity in the initial medium, and ϕ and θ are the angles of incidence and refraction.

5. Calculate the frequency and the value of the quantum for light of wave-length 5893 Å.U.

CHAPTER X

INTERFERENCE AND DIFFRACTION

Interference of Two Point Sources.—If two stones are dropped simultaneously into a pool of water, a train of waves travels out from the point where each strikes the surface. These trains can be followed by the eye as a series of ever-widening circles. The one train superimposes on the other, and it will be noticed, that where a crest superimposes on a crest, we have a crest twice as high; where a trough is superimposed on a trough we obtain a trough twice as deep, and where a crest is superimposed on a trough, they neutralise one another, and the level is the same, as if the surface had not been disturbed.

The effect, to which the name interference is given, is shown in Fig. 424, where M and N are the two sources. The crests are represented by full circles and the troughs by dotted circles. The sources have the same phase, *i.e.* the two series of crests and troughs have the same distances from their respective centres. At the points marked by o's we have crest superimposed on crest or trough superimposed on trough, so that the amplitude is twice that of each component wave. At the points marked by x's we have crest and trough superimposed on one another, the two waves neutralise each other, and the amplitude is zero.

It will be found by drawing the circles for any other time, *i.e.* by increasing the radii of all of them by the same amount, that GH and CD are always lines of zero intensity, and AB, EF, and JK always lines along which the amplitude is doubled and the

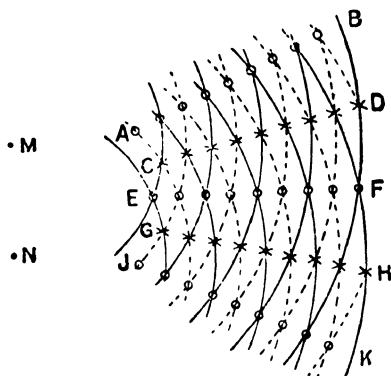


FIG. 424.—Interference of two point sources.

intensity increased four times, for the intensity varies as the square of the amplitude.

Proof of Formula.—Let two particles at M and N (Fig. 425)

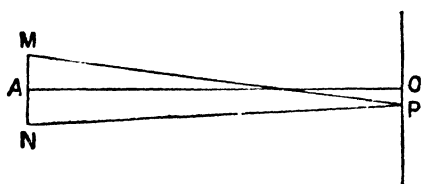


FIG. 425.

vibrate parallel to one another in the same phase and send out waves of light with the same amplitude and same period.

In order to consider the effect at a point P, join MN, and through A, the mid-point of MN, draw AO per-

pendicular to MN. Draw PO perpendicular to AO. Let $MN = 2d$, $AO = D$, and $OP = x$. Then

$$MP^2 = D^2 + (d+x)^2 \quad \text{and} \quad NP^2 = D^2 + (d-x)^2$$

therefore
$$MP^2 - NP^2 = (d+x)^2 - (d-x)^2 = 4dx$$

and
$$MP - NP = \frac{4dx}{MP + NP}$$

Let d and x be small in comparison with D . Then we have approximately $MP + NP = 2D$, and

$$MP - NP = \frac{2dx}{D}$$

If $MP - NP$ is equal to an integral number of wave-lengths, that is, if $2dx/D = n\lambda$ or

$$x = \frac{nD\lambda}{2d} \quad u$$

the two waves reinforce one another, and we have a maximum of intensity at P, while if $MP - NP$ equals an odd number of half wave-lengths, that is, if

$$x = \frac{(n + \frac{1}{2})D\lambda}{2d}$$

the two waves interfere, and we have darkness at P.

Let us suppose now, that there is a screen through OP perpendicular to the plane of the paper, and that the particles are replaced by slits perpendicular to the plane of the paper. It follows from the formulæ just proved, that there is a number of parallel bright and dark bands on the screen, the distance between two consecutive bright or dark bands being

$$\frac{D\lambda}{2d}$$

Such bands are called **interference bands**.

Fresnel's Biprism.—This is the easiest way of getting two sources of light to interfere. The light from the slit S (Fig. 426)

falls on the biprism CEG, which is simply a prism with a very obtuse angle at E. It may be regarded as made up of prisms CEF and GFE placed base to base; hence the name. The edge E of the biprism divides the incident light into two portions, one which passes through the face EC and appears after refraction to come from B, and another which passes through the face EG and appears after refraction to come from A. As the angles GCE and CGE are small and equal, A and B are situated close to S at equal

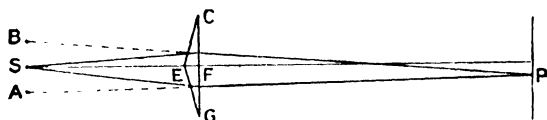


FIG. 426.

distances from it. The effect of the biprism is thus to produce two virtual images of the slit which cause interference bands in the space to the right of CG, for example on a screen at P.

The measurement of the interference bands produced by a biprism is widely used as an exercise for students. A sodium flame is taken as the source of light, and the object of the experiment, as it is generally done, is to determine the wave-length of sodium light. The apparatus is an optical bench, usually of metal, fitted with a vernier microscope, but an elaborate bench is not necessary. The microscope shown in Fig. 355 is sufficient, if the slit and biprism are set up on improvised mounts in front of it. The bands are produced in the focal plane of the microscope instead of on a screen. The slit and edge of the biprism are vertical, and the tube of the microscope is set horizontal and parallel to the line SF (Fig. 426). It is moved horizontally at right angles to SF, so that the cross-wires pass in succession across the different interference bands.

In setting up the apparatus the first requirement is to get the slit, biprism, and microscope in the same straight line. The slit is next made narrow, and then on looking into the microscope a vertical strip of light will be seen. This resolves into a number of equidistant parallel lines, when the biprism is adjusted so that its edge is parallel to the slit, and when the width of the slit is further regulated. It is very important that the slit and the edge of the biprism should be accurately parallel to one another.

The formula for the wave-length is

$$\lambda = \frac{2df}{D}$$

where f is the distance between two successive bands, D is the distance from the virtual images to the focal plane of the micro-

scope, and $2d$ is the distance between the two virtual images. Since the opposite faces of the biprism are almost parallel, the divergence of a thin pencil of rays passing through it is not appreciably altered. Consequently A and B are in the same plane as S, and the distance D can be measured with a metre-stick. The best way to measure f is to read the position of ten or twelve, say ten, consecutive bands, subtract the reading for the first from the reading for the sixth, the reading for the second from the reading for the seventh, and so on, and divide the mean of the results so obtained by five.

To determine $2d$ a convex lens is inserted between the biprism and the microscope and adjusted, so as to form images of A and B in the focal plane of the latter. In order to do this it is usually necessary to move the microscope further away from the biprism, but the positions of the biprism and slit themselves must be kept unaltered. When the images are obtained, the distance between them is measured with the microscope. Let the result be c_1 . The slit, biprism, and microscope are then kept fixed and the lens moved into the other position in which it forms images of A and B

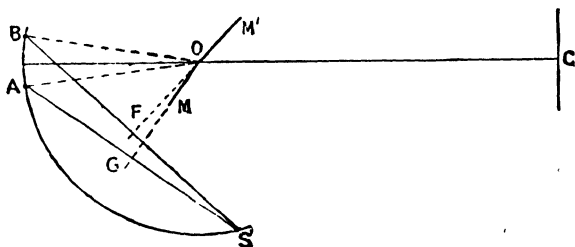


FIG. 427.

in the focal plane of the microscope. Let the distance between the images in this case be c_2 . Then $2d = \sqrt{c_1 c_2}$ since the magnification in the one position is the reciprocal of the magnification in the other.

If white light is used, the bands produced by the different wavelengths fall at different places and confuse one another. So only a few are seen, and these are coloured.

Fresnel's Mirrors.—This is another method of producing interference bands. Two plane mirrors are inclined to each other at an angle of almost 180° , and light falls on them from a slit which is parallel to the line of intersection of their surfaces. Fig. 427 represents a plan of the arrangement; S is the slit and M and M' are the mirrors. To find the positions of the virtual images of S formed by the mirrors, produce M'O to F and OM to G; draw SF perpendicular to M'F and SG perpendicular to MG, and lay off

$FB=FS$ and $GA=GS$. Then the images of S are situated at A and B ; after reflection from the mirrors the light from S proceeds as if it came from A and B , and interference bands are formed on a screen at C .

Let $OS=a$, let $OC=b$, and let the angle FOG be denoted by ω . It is obvious from the construction that A and B are on the circle with centre O and radius OS . Since FS and GS are respectively perpendicular to OF and OG , $\angle FSG = \angle FOG = \omega$. $\angle AOB$ stands on the same arc as $\angle FSG$ and has its vertex at the centre, consequently it equals 2ω . The arc AB thus equals $2a\omega$, and, since ω is small, $2a\omega$ may also be taken as the distance between A and B measured along the chord. The straight line OC is drawn so as to bisect the angle AOB . In the notation of p. 428 the distance between the bands is given by $D\lambda/(2d)$; on substituting $D=a+b$ and $2d=2a\omega$ we obtain in the present case

$$\frac{(a+b)\lambda}{2a\omega}$$

Fresnel was a civil engineer in the employment of the French government. He was suspended from his office for joining the small army which attempted to oppose the return of Napoleon from Elba, and it was during this enforced leisure that he invented the biprism and mirrors.

Fundamental Condition for Interference.—In the two methods for producing interference just described, it will be noticed that the two interfering sources are derived from the same source. Thus A and B are merely images of S in Figs. 426 and 427. This condition has been found indispensable for the successful production of interference bands. As an experimental fact it is impossible to get light from two independent sources to interfere.

This is explained by assuming that every source of light is subject to abrupt changes of phase very many times in a second, possibly owing to the molecule which is sending out the light vibration, coming into collision with other molecules. Now in Fig. 425 if the phase of M falls behind the phase of N , the central band of the system moves above O and the whole system of fringes is displaced. If the relative phase difference of M and N changes very many times in the second the system of fringes is displaced very many times in the second, too frequently for the eye to follow them; the eye sees them superimposed and all trace of the individual bands is lost. But if M and N are derived from the same source, their phases change simultaneously, the system of bands remains stationary, and is consequently always visible.

Colours of Thin Films.—It has already been explained, that when light falls on a glass plate, owing to multiple reflection taking place inside the plate we have several reflected waves.

If the plate is not silvered, the first two are almost equally bright, and much brighter than the others. We shall calculate the path difference of these waves.

AP_1 (Fig. 428) is an incident ray, P_1CP_2 its course after refraction, and P_1R_1 and P_2R_2 the first and second reflected rays. Let ϕ be the angle of incidence, θ the angle of refraction, let e be the thickness of the plate, and let μ be its refractive index.

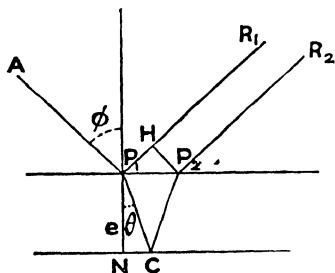


FIG. 428.

Draw P_2H at right angles to P_1R_1 . The first and second reflected wave-fronts superimpose on this line. The number of wave-lengths in P_1H is P_1H/λ ; the number in P_1CP_2 is $2\mu P_1C/\lambda$, since the wave-length in glass is only

λ/μ . Hence in going from P_1 to H P_2 the one wave has gained

$$\frac{2\mu P_1C}{\lambda} - \frac{P_1H}{\lambda}.$$

wave-lengths on the other.

$$\begin{aligned} 2\mu P_1C - P_1H &= 2\mu P_1C - P_1P_2 \sin \phi \\ &= 2\mu P_1C - 2P_1C \sin \phi \sin \theta \\ &= 2\mu P_1C - 2\mu P_1C \sin^2 \theta \\ &= 2\mu P_1C \cos^2 \theta \\ &= 2\mu e \cos \theta \end{aligned}$$

We would consequently expect, that if $2\mu e \cos \theta$ equalled an odd number of half wave-lengths, the one wave would interfere with the other.

As a matter of fact the contrary result is true. We obtain interference when

$$2\mu e \cos \theta = n\lambda$$

n being any integer, and reinforcement of the waves when

$$2\mu e \cos \theta = (n + \frac{1}{2})\lambda$$

because the wave suffers a change of phase of half a wave-length when it is internally reflected at C . This half wave-length must be added to the number of wave-lengths in the path.

If a thin film of oil is spread over the surface of muddy water and viewed in reflected light, brilliant colours are seen. They are due to the interference of light reflected at the upper and lower surfaces of the film, the latter acting as the plate considered above. Owing to the muddy water no light comes from below to dilute the purity of the effect. The same colours can also be obtained

from the skin of a soap bubble or from the thin paper-like pieces of glass that are sometimes the result of an amateur's glass-blowing.

When we look at a small area on the surface of such a thin film, the rays diverging from it to the eye are almost parallel. The rays diverging from the front and back surfaces come originally from the same source, probably a white sky, and hence are in a condition to interfere. If $2\mu e \cos \theta = n\lambda$, the colour corresponding to that wave-length does not appear in the reflected light, and the film appears coloured. Owing to the rays received by the eye being not quite parallel, $\cos \theta$ is not the same for all of them. For one definite part of the spectrum to be blocked out, it is therefore necessary for n to be a small whole number and consequently for the film to be very thin.

Newton's Rings.—If a lens, the surfaces of which have large radii of curvature, is placed on a glass plate and the point of contact viewed in white light, it is seen to be surrounded by coloured rings. These were first observed by Hooke in 1665, they were studied and their radii measured very carefully by Newton, and they were first explained satisfactorily by Young. They are due to interference between the light waves reflected at the upper and lower surfaces of the air film contained between the lens and plate.

In studying the formation of the rings Newton used telescope object glasses, the radii of curvature of which were many feet, and measured the radii of the rings

formed directly with a foot rule. In the experimental arrangement employed in laboratory courses nowadays spectacle lenses are used with a radius of curvature of perhaps one metre, the source of light is a sodium flame, and the diameters of the rings are measured with a vernier microscope. The air film is formed by the lower surface of the lens L and the upper surface of the plate P

(Fig. 429). The microscope is inclined to the vertical at an angle θ , and the light from the flame makes approximately the same angle with the normal to the upper surface of L . Hence, as they are seen obliquely, the rings appear as a system of ellipses.

The air film can be regarded as a plate of slowly varying thickness. If R is the radius of curvature of the under surface of the lens, e the thickness of the air film at P , and $CP = \rho$ (Fig. 430), then

$$\rho^2 = e(2R - e)$$

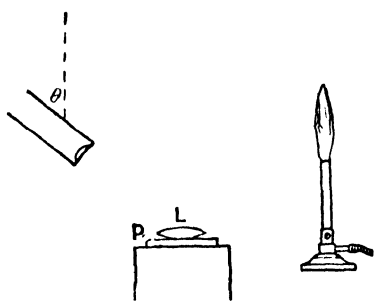


FIG. 429.—Arrangement for observing Newton's rings.

by considering the products of the segments of the horizontal chord through N and of the vertical chord through C. Since e is small this equation may be written :

$$e = \frac{\rho^2}{2R}$$

Now consider the ray incident on the lens in the manner indicated by the dotted line. Let us suppose that the point P is so near C that the sides of the film may be considered parallel. Then, if $2\mu e \cos \theta = n\lambda$, the ray shown in the figure interferes with the ray reflected from the upper surface of the film, and, to an eye situated above, the film appears black at P. On substituting for e , the condition for interference becomes $2\rho^2 \cos \theta / (2R) = n\lambda$, since $\mu = 1$ for air, or

$$\rho^2 = \frac{n\lambda R}{\cos \theta}$$

Hence on the ring with this radius we have blackness, and this ring is one of a series, the radii of the others being obtained by giving n the other integral values. The squares of the first n radii are as the first n natural numbers. Similarly the radii of the bright rings, the loci of the points of greatest brightness, are given by

$$\rho^2 = \frac{(n + \frac{1}{2})\lambda R}{\cos \theta}$$

Diffraction Grating.—The diffraction grating is an instrument used for producing spectra. It consists usually of a number of fine, equidistant, parallel lines ruled with a diamond on a mirror of speculum metal or on a glass plate, or of a celluloid cast made from such a grating. Some of the earliest gratings consisted of fine wires stretched parallel and equidistant from one another between two screws of equal pitch. Gratings ruled with a diamond on a dividing machine are very expensive, hence celluloid casts or replicas are used instead, except for the most important work. Such gratings are often known as Thorp gratings; they are, of course, transparent.

Previous to 1883 all gratings had been plane; they had been ruled on plane surfaces or the wires had been stretched in one plane. They were used with telescope and collimator just as the prism is used in the spectroscope. But in that year Rowland described a method of ruling them on concave metal mirrors, which rendered

the use of collimator and telescope lenses superfluous. Here only plane gratings will be treated.

We shall assume that the grating consists of a glass plate on which there is a great number of equidistant, parallel, opaque strips or lines of equal width. These lines are separated by clear spaces, A_1B_1 , etc. (Fig. 431). Let A_1A_2 , the width of a combined line and clear space, be e .

If a plane wave of wave-length λ is incident perpendicularly on the grating, all the clear spaces act as secondary sources and emit rays in all directions. Let us consider the rays making an angle θ with the normal. Then all the rays from each space can be replaced by a resultant coming from the mid point of that space, and the problem reduces to that of compounding a number of parallel rays, C_1D_1 , C_2D_2 , etc., from a number of equidistant points, C_1 , C_2 , etc.

Draw C_2K perpendicular to C_1D_1 . Then the common difference of these rays is $C_1K = C_1C_2 \sin C_1C_2K = e \sin \theta$, and the rays reinforce in the direction θ if

$$e \sin \theta = n\lambda$$

where n has any integral value, positive or negative. If the telescope is pointed in a direction given by this equation, a bright line is seen on the cross-wires.

Most gratings have about 14,000 lines to the inch, in which case e is $\frac{1}{14,000}$ inch, or about 1.81×10^{-4} cm. If the light used is yellow and of wave-length 5.8×10^{-5} cm., the equation for the maxima becomes

$$\sin \theta = 0.3197n$$

and the values of θ given by $n=0, \pm 1, \pm 2, \pm 3$, are $0^\circ, \pm 18^\circ 39', \pm 39^\circ 45', \pm 73^\circ 33'$ respectively. Larger values of n give impossible values for $\sin \theta$. The image given by $\theta=0$ is called the direct image, the two given by $n=\pm 1$ are said to belong to the first order spectrum, those given by $n=\pm 2$ are said to belong to the second order spectrum, and those given by $n=\pm 3$ to the third order spectrum.

If white light is incident on the grating, each colour into which it can be resolved forms its own images. The direct images all superimpose, hence the direct image is white, but the other images do not superimpose. The first order images form two first order

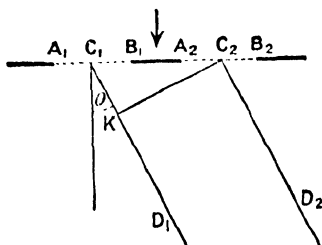


FIG. 431.

spectra, the second order images two second order spectra, and the third order images two third order spectra, so that altogether this grating forms six spectra. These different spectra overlap. For example, if in the equation $e \sin \theta = n\lambda$, we write $e \sin \theta = 1.5 \times 10^{-4}$, a possible value, we find that $n=2$ gives 7.5×10^{-5} cm., and $n=3$ gives 5×10^{-5} cm. for λ , so that the red of the second order is superimposed on the green of the third order spectrum.

The red end of a grating spectrum is not so crowded together as the red end of a prism spectrum, but a spectrum formed by a grating is never so bright as the spectrum formed by a prism.

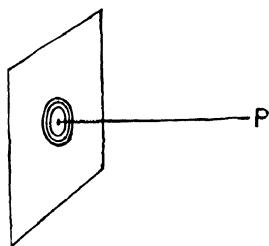


FIG. 432.

Rectilinear Propagation of Light.—

For a long time it was impossible to explain, on the wave theory of light, how shadows are formed. Sound, which was undoubtedly propagated by wave-motion, bent round corners; why was light propagated in straight lines?

The explanation was given by Fresnel. It lies beyond the scope of this book, and so can only be indicated here. Let Fig. 432 represent a plane wave passing through a square aperture.

Then every point on the wave-front can be regarded as sending out waves. Let the distance of P from the wave-front be p . If spheres be described with P as centre and radii $p + \frac{\lambda}{2}$, $p + \lambda$, $p + \frac{3\lambda}{2}$, etc., they cut the wave-front in circles, dividing it into zones. Fresnel considered the effects of these zones separately, and showed that

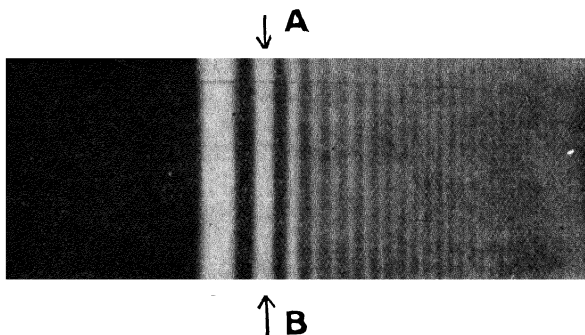


FIG. 433.—Photograph of the diffraction bands produced by a straight edge.

at all points directly in front of the wave, they gave an effect equal to the effect of half the first zone. At points in the shadow they

interfered with one another, and gave nothing. Thus the rectilinear propagation was due to the *destructive interference of the different zones*. It was necessary for the explanation that the wave-length should be small in comparison with p and the dimensions of the wave-front, and it has been found that sound waves of very short wave-length also cast shadows, thus confirming the explanation.

Diffraction.—When a wave of monochromatic light diverges from a very narrow slit and passes an obstacle with a sharp edge parallel to the slit, the edge of the shadow is not sharp. There is a number of dark bands outside the geometrical shadow, and inside it the intensity diminishes gradually to zero. Fig. 433 is a photograph of the effect, highly magnified; the two arrows A and B mark the edge of the geometrical shadow. The bands are more difficult to observe than interference bands.

We have a similar rhythmic variation of brightness at the edge of the shadow, when the wave passes a narrow obstacle or through a narrow slit. It is said to be due to diffraction, and is caused by interference of the rays from different parts of the wave-front. According to the laws of geometrical optics there should be no diffraction, but these laws are only approximately true.

EXAMPLES X

1. Interference bands are produced by a biprism in the focal plane of a reading microscope. The focal plane is distant 80 cm. from the slit. The two images of the slit are 3 mm. apart. If sodium light is used, find the distance between the bands.

2. Newton's rings are formed between a plane surface of glass and a lens. The diameter of the fifth black ring is 8 mm. when sodium light is used, and the light passes through the air film at an angle of 30° with the normal. Find the radius of the under surface of the lens.

PART V

ELECTRICITY AND MAGNETISM

CHAPTER I

THE FUNDAMENTAL FACTS OF MAGNETISM

IN ancient times, in the province of Magnesia in Asia Minor, an iron ore was found, fragments of which had the property of attracting to themselves small pieces of iron. They were hence called magnets. Their chemical constitution is represented by the formula Fe_3O_4 , that is, they consist of equivalent proportions of the two oxides of iron, FeO and Fe_2O_3 . They are also called "lodestones" or "leading stones," because it was found later that they can be used to enable a ship to steer a course.

Lodestones as found in nature have irregular forms. They do not exhibit the characteristic property all over their surface, but if they are dipped into a box containing iron filings, the filings adhere in clusters to corners and edges, leaving other parts clear.

Artificial Magnets.—If a small bar of steel, for example, an ordinary sewing needle, is drawn across a corner of a lodestone several times, always in the same direction, it also acquires magnetic properties which may be even more strongly marked than in the case of the original lodestone. It is then said to be an artificial magnet. All magnets used nowadays are artificial magnets, but they are made, as will be explained on p. 514, by means of an electric current. Artificial magnets made in this way are both stronger and of a more convenient shape than lodestones; the latter, and the method of making magnets by stroking them with lodestones, are now only of historical interest. Consequently, when we speak of magnets, it is always artificial magnets that are referred to.

Poles.—If a magnetised needle or bar magnet is dipped into iron filings, they adhere to it in two tufts, one at each end. Fig. 434 shows the nature of the tufts in the case of a flat bar magnet. The regions at the ends of the magnet, where the power of attracting filings is apparently situated, are called its poles. The straight line joining the poles is called the magnetic axis of the

magnet. If a magnetised needle is suspended in a horizontal position, so that it is free to turn about a vertical axis through its midpoint, and if there is no iron near it, it always comes to rest

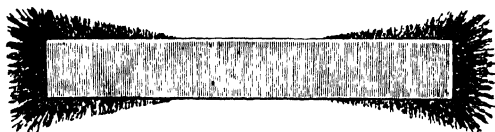


FIG. 434.

in the same direction, which in Great Britain is a few degrees west of north. It is always the same end which points towards the north. If the magnet is displaced from its equilibrium position, it oscillates about it before coming to rest. The pole which points towards the north is called the north pole or north-seeking pole of the magnet; the other one is called the south pole or south-seeking pole.

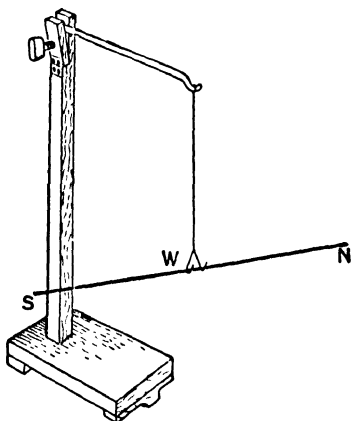


FIG. 435.—Suspended magnetised needle.

Fig. 435 represents a magnetised knitting needle suspended by a cotton thread from a stand. The thread carries a piece of copper wire bent into a double hook W in which the needle rests. The north pole is denoted by N and the south pole by S. If the north pole *n* of another magnet is brought near the north pole N, the latter is repelled. If the south pole *s* of another magnet is brought up to N, the latter is attracted. If the north pole *n* is brought near the south pole S, the latter is attracted. If the

south pole *s* of the other magnet is brought near S, the latter is repelled. These results may be summed up in the following rule:—

Like poles repel, unlike poles attract.

Magnetic Induction. Permanent and Temporary Magnetism.

—When one pole of a strong magnet is held near a small piece of soft iron, the latter jumps up to meet it. This is because the mere presence of the pole is sufficient to convert the piece of soft iron into a magnet with its nearer end a pole of the opposite kind; the two unlike poles then attract one another, and the piece of

soft iron jumps up to adhere to the pole. The soft iron is then said to be **magnetised by induction** or influence, or to possess induced magnetism. This induced magnetism is almost entirely of a temporary nature ; when the soft iron is taken to a distance from the magnet, it loses nearly all its magnetism.

When a magnet is dipped into a box of filings, we have little chains of filings formed. The first filing of each chain sticks to the magnet, the second filing to the end of the first filing, the third filing to the end of the second filing, and so on. Each of the filings becomes a little temporary magnet owing to the inductive action of the magnet upon it, and the north pole of the one magnet sticks to the south pole of the other.

Materials such as pure iron which are readily magnetised and lose their magnetism readily are said to be magnetically **soft**. Magnetically **hard** materials, such as steel that has been cooled very rapidly from a white heat, are difficult to magnetise and difficult to demagnetise. Permanent magnets are susceptible to alterations of heat and cold and to knocks or vibrations of any kind, and lose a part of their magnetism with lapse of time ; the harder the material, the less is this loss.

When a rod of bismuth is suspended by a silk fibre between the poles of a very powerful magnet, it sets itself at right angles to the line joining the poles, not along the line joining the poles as soft iron does. Substances which behave like bismuth are called *diamagnetic*, and substances which behave like iron are called *paramagnetic*. But only a few of these substances, iron, steel, nickel, cobalt, and certain alloys, exhibit magnetic properties to a marked degree. They are called *ferromagnetic* substances, and except in very exceptional circumstances the magnetic properties of other substances may be neglected. When iron, steel, nickel, and cobalt are heated, they lose their magnetism altogether at temperatures which are different for the different substances.

Soft materials, notably an alloy, stalloy, containing $3\frac{1}{2}$ per cent. of silicon, are used extensively in the cores of transformers, and of the armatures of motors and dynamos. Still softer material, permalloy, consisting of 78.5 per cent. of nickel and the remainder iron, is used in the form of a ribbon in the rubber coating of submarine telegraph cables, in order to increase the speed of transmission of messages. Hard materials are required for permanent magnets, as, for example, the binnacle magnets placed near a ship's compass to eliminate the deviation of the compass due to the ship's permanent magnetism, or the control magnet in a moving coil ammeter or voltmeter. The hard materials formerly used were high carbon steels, or steels containing about 6 per cent. of tungsten ; these have given place to still harder materials containing large percentages of the two elements cobalt and chromium.

Inverse Square Law. Unit Quantity of Magnetism.—Magnetic poles always occur in pairs, one at each end of the magnet. If a long, thin magnetised knitting needle is broken into two, new poles appear one on each side of the break, a south pole appearing on the piece which contains the original north pole, and a north pole appearing on the piece which contains the original south pole. Therefore we cannot experiment with one pole by itself.

If, however, we take two long magnetised knitting needles, and bring an end of the one close up to an end of the other, the effects of the distant poles can be neglected, and the force between the magnets is almost wholly due to the two poles close together. By experiments made in this way it is found that the force between two poles is inversely proportional to the square of the distance between them. If the distance between them is halved, the force is increased four times.

Let Fig. 436 represent two magnets with their north poles close together. If there were added to the pole N another exactly like it, the force of repulsion would be doubled; if at the same time two magnets equal in every way to $n\ s$



FIG. 436.

were tied along it with the three north poles together, the repulsion would be increased sixfold. Thus the force between the poles varies as the product of the pole-strengths. Combining this result with the former one we arrive at Coulomb's law:

The force between two poles is proportional to the product of their strengths and inversely proportional to the square of the distance between them.

Expressing the law mathematically

$$F = \frac{1}{\mu} \frac{mm'}{d^2}$$

where m and m' denote the strengths of the poles, d the distance between them, and μ a factor of proportionality depending on the medium. The definition of unit pole is based on this equation:

Two like poles of equal strength have unit quantity of magnetism, when they repel one another at a distance of one centimetre with a force of one dyne, both being in air.

Write F and d each equal to 1 in the equation and suppose that $m=m'$. Then by definition each=1. Consequently $\mu=1$, if the medium is air. The unit pole is thus defined, so as to make the constant of proportionality $\mu=1$ for air, and it is found experimentally that it is also=1 for all other non-magnetic media.

Magnetic Field.—Figs. 437 and 438 represent a magnet mounted on a fine steel point and a compass, both for use in experiments in magnetism. The space round a magnet in which it influences the deflection of a compass needle is called the field of the magnet. The field strength, or the intensity of the magnetic field at a point, is measured by the force in dynes which would act on a unit north pole placed at that point, the presence of the pole being supposed to produce no disturbance in the field. Thus, to take the case of a point distant r from a pole of strength m , if a unit north pole is placed at the point, the force on it is $\frac{m \times 1}{r^2} = \frac{m}{r^2}$

and this is consequently the numerical value of the field strength at the point. Its direction is along the line joining the two poles, outwards from the pole of strength m , if the latter is a north one. If the field strength at a point is H , the field acts on a unit positive pole placed at the point with a force of H dynes, and on a pole of strength m placed at the point with a force of Hm dynes. The unit in which field strength is measured is called the **gauss**; * thus a pole of strength 10 units placed in a field of strength 0.14 gauss experiences a force of 10×0.14 dynes. If the field is everywhere the same in intensity and direction, it is called a uniform field.

Since a compass needle always sets itself in a definite direction when there are no magnets in its neighbourhood, we conclude that there is a magnetic field due to the earth itself. This field is uniform over a small region, if all magnets are removed to a distance.



FIG. 438.—
Small compass.

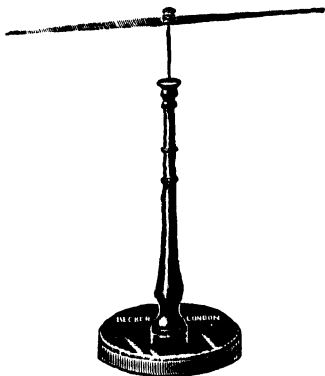


FIG. 437.—Pivoted magnetic
needle.

If a magnet is mounted horizontally on a piece of cork and floated on the surface of water, it turns itself in a definite direction, but does not move as a whole towards the side of the vessel. Consequently the resultant force on it must be zero; if the force on one pole is Hm , the force on the other is $-Hm$. Thus the poles of a magnet are of equal strength; if the strength of the one is m , that of the other is $-m$. It has thus become customary to regard the magnetism of the north pole as positive and that of the south pole as negative.

* The International Electrochemical Commission at Oslo, July, 1930, decided to call this unit the "oersted," and retain the term "gauss" for B (cf. p. 561).

EXAMPLE.—Two thin straight magnets each 12 cm. long are laid in one straight line with their north poles facing one another and 4 cm. apart. Calculate in dynes the repulsion between the two magnets on the assumption that the poles are at the ends of the magnets, and that the pole strengths are 20 and 30 units respectively.

$$\text{Repulsion between N. poles} = \frac{20 \times 30}{4^2} = 37.50 \text{ dynes.}$$

$$\text{Repulsion between S. poles} = \frac{20 \times 30}{28^2} = 0.76 \text{ dyne.}$$

$$\text{Attraction between a N. and a S. pole} = \frac{20 \times 30}{16^2} = 2.34 \text{ dynes.}$$

$$\text{Resultant repulsion} = 37.50 + 0.76 - 2 \times 2.34 = 33.58 \text{ dynes.}$$

Lines of Force.—A curve may be drawn in a magnetic field in such a manner, that the tangent to it at any point is parallel to the direction of the field at that point. Such curves are called lines of force. They are a very useful way of representing the field strength throughout a magnetic field. Since they represent the direction in which a positive pole moves, they start at a positive pole and end at a negative one. They may be made to represent the numerical value of the field strength at a point by drawing them so close together, that the number passing through a plane of unit area which has the point as centre and is at right angles to their direction, is exactly equal to this numerical value; this number is referred to as the density of the lines. If the field is a uniform one, the lines of force are equidistant and parallel.

It has been shown above that the field strength due to a north pole of strength m at a distance r from it is m/r^2 . If we draw a sphere of radius r with the pole as centre, the number of lines of force passing through unit area of this sphere is consequently m/r^2 . The number passing through the whole surface is

$$\frac{m}{r^2} \times 4\pi r^2 = 4\pi m$$

Thus 4π lines of force start from each unit of positive magnetism in the pole, and by similar reasoning 4π lines of force end at each unit of negative magnetism in a south pole.

Mapping a Magnetic Field by the Compass Needle.—A magnetic field can be mapped very easily by a compass needle. The method succeeds best with a bar magnet about 15 cm. long or thereabouts; this is placed on a large sheet of drawing paper which is fastened down on the table. Then a compass needle is placed on the paper near the north pole of the magnet, and, after it comes to rest, marks are placed on the paper opposite the poles of the compass needle. The needle is then shifted so that its first pole comes opposite the second mark, and a third mark drawn

opposite its second pole. It is then shifted a second time, so that its first pole comes opposite the third mark, and a fourth mark drawn opposite the second pole, and so on. By proceeding in this way a series of marks, 1, 2, 3, 4, etc. (cf. Fig. 439), is obtained, and on joining these marks a closed curve is obtained extending from the north to the south pole of the magnet. The curve is, of course, a line of force. Then, when the one curve is completed, a fresh start is made at another point and another curve drawn, and so on, until the paper is filled with curves.

The lines of force represented in the figure do not diverge from points, but from regions situated near the ends of the magnet. This is the usual case. If, however, the magnet consists of a thin cylindrical rod ending in spheres, the lines of force diverge from the centres of the spheres. Such magnets were introduced by Robison at the end of the eighteenth century, and are known as Robison's ball-ended magnets.

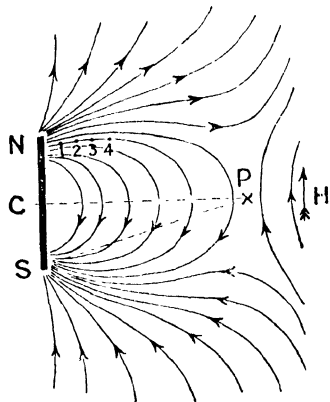


FIG. 439.—Lines of force and neutral point.

The appearance of the resulting figure depends on the position in which the bar magnet was originally placed with reference to the earth's magnetic field. In Fig. 439 the direction of the earth's field is given by the long arrow marked H ; this is the direction in which the compass needle would set itself over the whole paper, if the bar magnet were removed. So Fig. 439 represents the lines of force for the case in which the bar magnet is placed parallel to the earth's field with its north pole pointing north. Of course, only one-half of the field on a horizontal plane is shown; the lines of force have exactly the same shape on the other side of the magnet.

It will be noticed that the lines of force between P and the magnet go the one way, but beyond P they go the other, *i.e.* between P and the magnet the magnet's field is the stronger, but beyond P the earth's field is the stronger. If the compass needle is slowly moved out along the dotted line CP , which is drawn perpendicularly through the centre of the magnet, and its case is tapped gently at the same time, the direction of the needle reverses at P . P is termed the **neutral point**. At P the earth's field balances the field of the magnet.

Let the distance between the poles of the magnet be $2l$, let the strength of its poles be m , and let $CP = d$. As mentioned above the poles are some distance from the ends in the case of a thick magnet. Then, if a unit pole is placed at P , the earth's field will act on it with a force H , and the south pole of the magnet will act on it with a force

m/PS^2 in the direction PS. If we regard $\triangle PCS$ as a triangle of forces, the force along PS can be regarded as equivalent to a force along PC and a force along CS. The latter is equal to

$$\frac{CS}{PS} \cdot \frac{m}{PS^2} = \frac{ml}{(d^2 + l^2)^{\frac{3}{2}}}$$

But the unit pole at P is also acted on by a force m/NP^2 along NP. This also resolves into two components, one of which neutralises the force already existing along PC, and the other of which is equal to the force above.

The resultant force exerted by both poles on the unit positive pole at P is consequently

$$\frac{2ml}{(d^2 + l^2)^{\frac{3}{2}}}$$

and for the neutral point we have

$$H = \frac{2ml}{(d^2 + l^2)^{\frac{3}{2}}}$$

Hence if H is known, and d and l are measured, m can be calculated. Provided the magnet is strong enough, there are always two neutral points, no matter how it is placed with reference to the earth's magnetic field. But the above formula holds only for the one particular case.

Mapping a Magnetic Field by Iron Filings.—When iron filings are placed near a magnet, they become magnetised by

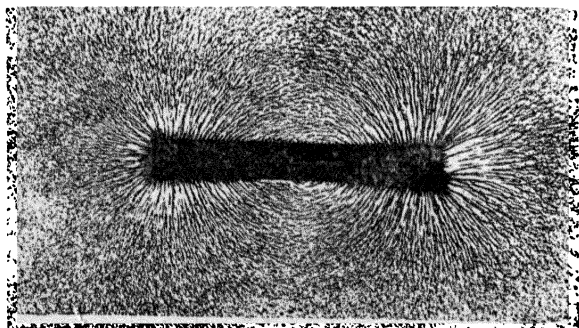


FIG. 440.—Bar magnet.

induction and tend to set themselves parallel to the lines of force. Hence if a glass plate is laid over a magnet and iron filings scattered uniformly over it, and if the plate is lightly tapped so as to assist the filings to set themselves parallel to the lines of force, we can at once obtain the form of the latter.

Figs. 440, 441, 442, 443, 444, have been obtained in this way. Fig. 440 represents a bar magnet in its own field, Fig. 441 two like

poles facing each other, Fig. 442 two unlike poles facing each other, Fig. 443 a cylinder of soft iron placed in a uniform magnetic field the direction of which is shown by the arrow, and Fig. 444 the lines of force of a uniform magnetic field disturbed by a hollow soft iron

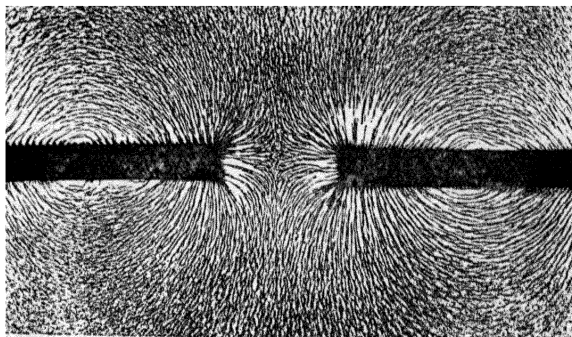


FIG. 441.—Two like poles facing each other.

cylinder. It will be observed from Figs. 443 and 444, that the lines of force prefer passing through the iron to passing through the surrounding air ; in Fig. 443 they enter at one end and leave at the other, and in Fig. 444 they pass from one side of the cylinder to

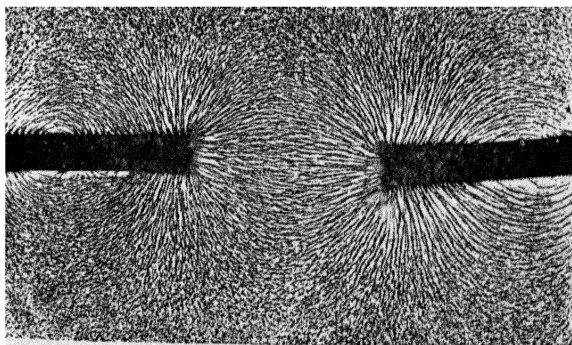


FIG. 442.—Two unlike poles facing each other.

the other through the iron, which thus shields the space within the cylinder from the effects of the magnetic field.

Physical Reality of Lines of Force.—Faraday showed that the lines of force whose presence is made manifest by the iron filings,

are in a state of tension and repel each other sideways. They exist in the field whether the filings are there or not, and it is their

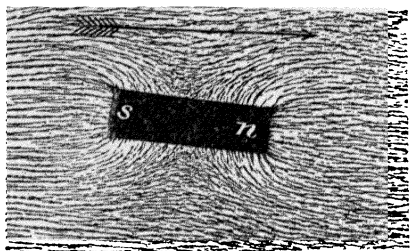


FIG. 443.—Soft iron cylinder in uniform field.

presence which causes the attraction and repulsion of magnetic poles. In Fig. 442 the two unlike poles are pulled together by the tension in the lines of force joining them, and the mutual repulsion of these lines makes them travel from pole to pole in curves spread over a wide space and not by the shortest path. In Fig. 441 it is the sideways repulsion of

the lines that forces the poles apart.

Molecular Magnetism.—If a long thin magnet such as a knitting needle is plunged into iron filings, the tufts adhere almost exclusively to the ends, showing that there is a pole at each end. If the needle is now broken into two pieces, it is found that a new pole appears at each side of the break, a south pole on the part which contains the original north pole and a north pole on the part that contains the original south pole. Each part has become a complete magnet in itself, and when plunged into iron filings exhibits tufts at both ends. If, however, the parts are put together again in the position which they occupied before the magnet was broken, and again tested with filings, it is found that very few filings adhere at the break. This is because the one pole neutralises the other, as shown in Fig. 445; the lines of force go across from N' to S' , and do not diverge out into the space surrounding the magnet as at N or S .

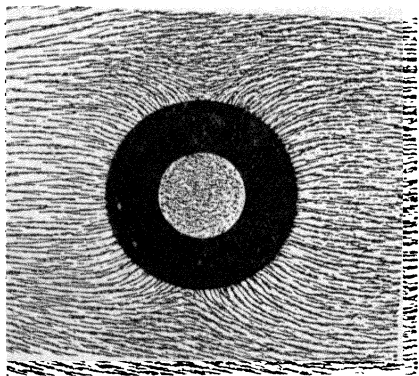


FIG. 444.—Shielding effect of hollow, soft iron cylinder.

If the magnet is broken into a number of parts, it is found in the same way that each part is a complete magnet in itself, but that, when the parts are put together again, the reconstructed magnet shows polarity only at the ends, owing to the other poles

neutralising each other in pairs. This holds for a thick magnet subdivided laterally as in Fig. 443 as well as for a magnetised

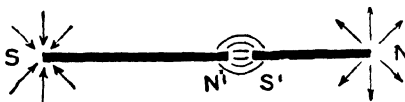


FIG. 445.



FIG. 446.

knitting needle. We thus arrive at the result, that *every small part into which a magnet can be subdivided is itself a magnet*.

Formerly it was supposed that in a ferromagnetic material the molecules were themselves magnets. In the unmagnetised state the axes of these magnets pointed in all possible directions, and the process of magnetisation consisted merely in turning them all into the same direction. Thus Fig. 447 would represent the unmagnetised and Fig. 448 the magnetised state of a soft iron bar. When all the molecular magnets were turned into the same direction, the material was said to be **saturated**. Ferromagnetic materials do possess a saturation value, a limiting value beyond which the magnetism of the material cannot be increased, no matter how great the magnetising field; this saturation value is different for different materials. But in 1922 Ewing showed,



FIG. 447.

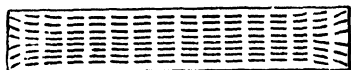


FIG. 448.

To illustrate the molecular theory of magnetism.

that it was not the molecules themselves that turned, but some system inside the atom.

Once the elementary magnets are aligned, the mutual attraction of their poles tends to hold them in position. If, however, the magnet is heated, the molecules are thrown into violent oscillation and the alignment may be destroyed; hammering and vibration have the same effect. The elementary magnets at the ends of the magnet have no neighbours on the one side to hold them in position, and consequently are the most easily displaced. But if the magnet is made in the form of a nearly closed ring with a narrow air gap separating the poles, the elementary magnets on the face of the one pole are held in position by the elementary magnets on the face of the other. Again in the case of a horse-shoe magnet, if the poles are joined by a soft iron "keeper," the latter is magnetised under the influence of the magnet, and there is a chain of elementary magnets from pole to pole. Conse-

quently the positions of the elementary magnets in the poles are rendered more stable.

The difference between soft and hard magnetic materials is that in the former the elementary magnets are easily turned, but at the same time easily fall out of position when the controlling field is removed, whereas in the latter the elementary magnets are difficult to turn, but at the same time difficult to displace from the new positions once these are taken up.

EXAMPLES I

1. With what force will a north pole of strength 5 units (i) attract a south pole of strength 8 units at a distance of 4 cm., (ii) repel another north pole of strength 6 units at a distance of 5 cm. ?

2. The repulsive force between two poles is 20 dynes when they are 5 cm. apart ; what is it when this distance is increased by 1 cm. ?

3. A magnet of pole strength 20 c.g.s. units and magnetic length 20 cm. is placed horizontal and at right angles to the magnetic meridian. Find the resultant horizontal force on a unit pole placed on the axis of the magnet produced and at a distance of 10 cm. from the north pole. $H = 0.17$ c.g.s. unit.

4. A bar magnet was placed in the magnetic meridian with its north pole pointing north, and the neutral points were determined to be 27.5 cm. from the magnet. Calculate the pole strength, given that the length of the magnet is 20 cm., that the poles are distant 1 cm. from the ends of the magnet, and that $H = 0.17$ c.g.s. unit.

5. Two equally strong cylindrical chrome steel magnets each of mass 25 gm. are suspended with their two S. poles together and the N. poles hanging down. The magnets are free to turn about the S. poles, and the N. poles move apart until the distance between them is 3 cm. The magnets are 10 cm. long. Find the pole strength of each magnet. The result will only be approximate, since the magnets have a diameter of 6 mm.

CHAPTER II

MAGNETIC MEASUREMENTS

Moment of a Magnet. Couple on a Magnet in the Earth's Field.

—If m is the pole strength of a magnet and $2l$ the distance between the poles, the product $2ml$, usually denoted by M , is defined to be the magnetic moment of the magnet.

Let NS represent a magnet of pole strength m and length $2l$, pivoted at its middle point C , and free to rotate in a horizontal plane in a uniform magnetic field of strength H . Then on the north pole of the magnet there is a force Hm in the direction EN and on the south pole of the magnet there is a force Hm in the direction DS . These two forces constitute a couple. Let θ be the angle which the axis of the magnet SN makes with the direction of the field. Then DE , the arm of the couple, is equal to $2l \cos ECN = 2l \sin \theta$, and the moment of the couple is $Hm2l \sin \theta = HM \sin \theta$. The couple turns the magnet into a position with its axis parallel to the direction of the field. If the magnet is already in this position, the equilibrium position as it is called, the couple on it is zero. If the magnet is held in the position shown in the diagram and then released, it oscillates about the equilibrium position before coming to rest in it.

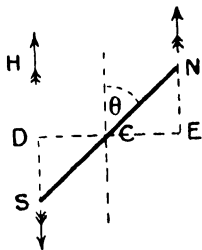


FIG. 449.

So far we have tacitly assumed that the magnet is a thin one with its poles situated at points near the ends. The experiments with iron filings show that generally each pole consists of a number of elementary poles distributed over a certain region. When the magnet is placed in a uniform field, all these elementary poles are acted on by forces parallel to one another. Now, this system of parallel forces reduces to a single force acting at a definite point in the magnet, the centre of the system. Consequently, when the magnet is placed in a uniform field, it behaves as if all the magnetism in the pole were concentrated at this point,

and it has become usual in this case to regard this point as the pole of the magnet. If the magnet represented in Fig. 449 is a thick one, N and S are therefore to be regarded as poles of this nature, points in the magnet at which the resultant attractive and repulsive forces of the field act.

When the magnet in Fig. 449 is held at right angles to the direction of the field, $\theta = \pi/2$, and the moment of the couple is HM , which becomes M if $H=1$. We thus arrive at a new definition of M ; *the moment of a magnet is the moment of the couple required to hold it at right angles to the lines of force in a uniform field of unit intensity*. This definition is more satisfactory than the other one when the position of the poles is not well defined, for it does not assume the definition of a pole.

EXAMPLE.—A magnet suspended by a fine vertical wire sets itself parallel to the earth's field, when the wire is untwisted. On turning the upper end of the wire through 180° the magnet is deflected through 30° from its original position. How far must the upper end of the wire be turned to deflect the magnet through 45° from its original position?

The deflecting couple is proportional to the angle through which the upper end is twisted relatively to the lower. Let θ be the required angle. Then, if c is the constant of proportionality,

$$c150 = HM \sin 30^\circ, \quad c(\theta - 45) = HM \sin 45^\circ,$$

whence
$$\frac{\theta - 45}{150} = \frac{\sin 45^\circ}{\sin 30^\circ}$$

and
$$\theta = 257.1^\circ.$$

EXAMPLE.—A bar magnet whose magnetic moment is 1000 c.g.s. units is turned in a horizontal plane through 60° from the equilibrium position. Find the work done, given that $H = 0.14$ gauss.

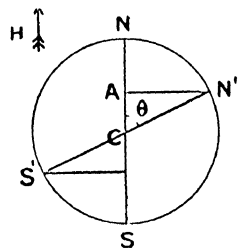


FIG. 450.

Let m be the pole strength and let $2l$ be the length of the magnet. NS is the initial and $N'S'$ the final position of the magnet. The force on the north pole is Hm , and the component of its displacement parallel to the



FIG. 451

direction of the force is $NA = NC - AC = l(1 - \cos \theta)$. Hence the work done on this pole is $Hml(1 - \cos \theta)$. An equal quantity of work is done on the other pole. Thus the whole work done is $2Hml(1 - \cos \theta) = HM(1 - \cos \theta) = .14 \times 1000(1 - \cos 60^\circ) = 70$ ergs.

Field Strength due to a Bar Magnet in the Two Standard Positions. End-on Position.—Let NS (Fig. 451) be a magnet of pole strength m and let $2l$ be the distance between the poles. It is required to find F , the field strength due to the magnet at a point P on its axis distant d from C , the centre of the magnet.

$$\begin{aligned} \text{We have } F &= \frac{m}{NP^2} - \frac{m}{SP^2} = \frac{m}{(d-l)^2} - \frac{m}{(d+l)^2} \\ &= \frac{4mld}{(d^2-l^2)^2} = \frac{2Md}{(d^2-l^2)^2} \quad \dots \dots (1) \end{aligned}$$

where M is the moment of the magnet. If l^2 is small in comparison with d^2 , this becomes

$$F = \frac{2M}{d^3} \quad \dots \dots (2)$$

Generally l^2 cannot be neglected in this way.

Side-on Position.—In this case we are required to find the field strength at a point P situated in a line through C at right angles to the axis of the magnet.

Let $CP=d$. The field strength at P due to N is m/NP^2 along NP . Resolve this into components parallel to NC and CP ; these components are respectively

$$\frac{CN}{NP} \frac{m}{NP^2} \quad \text{and} \quad \frac{CP}{NP} \frac{m}{NP^2}$$

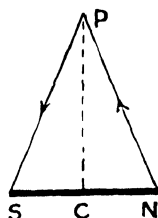


FIG. 452.

The field strength at P due to S is m/PS^2 and resolves into components, one of which is equal to and in the same direction as the first component above, and the other of which is equal and opposite to the second component above. Hence the field strength at P due to the whole magnet is given by

$$F = 2 \frac{CN}{NP} \frac{m}{NP^2} = \frac{2ml}{(d^2+l^2)^{\frac{3}{2}}} = \frac{M}{(d^2+l^2)^{\frac{3}{2}}} \quad \dots \dots (3)$$

since $NP^2 = d^2 + l^2$. If l^2 is small in comparison with d^2 , this becomes

$$F = \frac{M}{d^3} \quad \dots \dots (4)$$

Tangential Positions of Gauss. "A Position."—The two positions in which a stationary bar magnet and a pivoted magnetic needle can be arranged so as to form the letter T before the needle is deflected, are called the tangential positions of Gauss. The position represented in Fig. 453 is called the "A position." SN is at right angles to the earth's magnetic field, M is the magnetic moment of SN , and $CP=d$. Let m' , $2l'$,

and M' be respectively the pole strength, length, and magnetic moment of the needle, and let the axis of the latter make an angle θ with the direction of the earth's magnetic field. The poles of the needle are acted on both by the earth's magnetic field, the



FIG. 453.—“A position” of Gauss.

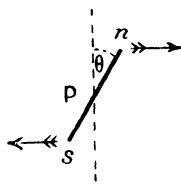


FIG. 454.

strength of which we shall suppose is H , and also by the field of the magnet, the strength of which we shall denote as before by F . The earth's magnetic field exerts a couple on the needle of moment $HM' \sin \theta$, the restoring couple, which tends to diminish θ .

The forces exerted on the poles of the needle by the field of the magnet are shown in Fig. 454 and are each equal to Fm' . The perpendicular distance between their lines of action is $2l' \cos \theta$. Hence they constitute a couple, the deflecting couple, of moment $Fm'2l' \cos \theta$ or $FM' \cos \theta$.

The needle comes to rest in the position in which the deflecting and restoring couples are equal, *i.e.* for which

$$\begin{aligned} FM' \cos \theta &= HM' \sin \theta, \\ F &= H \tan \theta \end{aligned}$$

or substituting the value of F from (1)

$$\frac{M}{H} = \frac{(d^2 - l^2)^2}{2d} \tan \theta \quad . \quad . \quad . \quad (5)$$

If l^2 is small in comparison with d^2 , this becomes

$$\frac{M}{H} = \frac{d^3}{2} \tan \theta \quad . \quad . \quad . \quad (6)$$

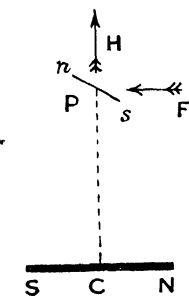


FIG. 455.—“B position” of Gauss.

before by $F = H \tan \theta$. On substituting the value of F from (3) this becomes

$$\frac{M}{H} = (d^2 + l^2)^{\frac{3}{2}} \tan \theta \quad . \quad . \quad . \quad (7)$$

$$\text{or} \quad \frac{M}{H} = d^3 \tan \theta \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

if l^2 is small in comparison with d^2 . In proving these formulæ we assume that the needle is small enough for F to have the same value at both of its poles.

Magnetometer.—The formulæ (5), (6), (7), and (8) may be used for

- (1) Comparing magnetic moments ;
- (2) Comparing magnetic field strengths ;
- (3) Finding the ratio of a magnetic moment to a magnetic field strength.

For these purposes a magnetometer is employed. A simple form of this instrument is shown in Fig. 456. It consists of a short compass needle pivoted at the centre of a graduated circle and carrying a long pointer. The compass box is mounted on a wooden base fitted with a centimetre scale. For more accurate work a smaller needle is used, which is attached to the back of a mirror suspended by a silk fibre, and the deflections of which are

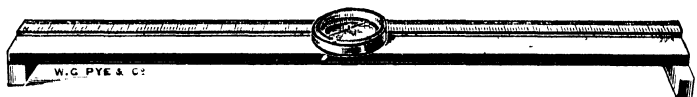


FIG. 456.—Magnetometer.

read by a telescope and scale or, if the room is dark, by lamp and scale.

In using the magnetometer shown in Fig. 456 the compass box is first turned until the diameter joining the zeros is parallel to the centimetre scale. Then, if the A position is to be taken first, the base board of the instrument is set east and west and levelled if necessary ; the east and west direction can be found very accurately by slowly rotating the base-board until the pointer comes to zero. The bar magnet is then placed in position on the board with its middle point a certain distance d from the needle as measured by the centimetre scale, and its length east and west, and the deflection read by both ends of the pointer. The magnet is next reversed end for end, and the deflection again taken at both ends of the pointer. The magnet is now removed to the same distance on the other side of the needle and the deflections observed ; it is then reversed end for end, and the deflections again, observed. We thus obtain altogether eight readings of the deflection, and the mean of these is taken.

If the value of M/H is desired, this mean value is substituted in (5) together with the corresponding values of d and l , and the calculation made. If the moments of two magnets are to be compared, the first magnet is removed, the second one put in its place on the magnetometer

at the same distance d from the needle, and the same operations gone through over again. Then if M_1 and M_2 are the moments, $2l_1$ and $2l_2$ the lengths of the first and second magnets, and θ_1 and θ_2 the deflections they produce respectively, it follows from (5) that

$$\begin{aligned} M_1 &= (d^2 - l_1^2)^2 \tan \theta_1 \\ M_2 &= (d^2 - l_2^2)^2 \tan \theta_2 \end{aligned} \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (9)$$

If the strengths of the earth's field are to be compared at two different places, measurements of the deflection are made for the same magnet and the same value of d with the needle first in the one place and then in the other. If H_1 , H_2 and θ_1 , θ_2 denote the field strengths and deflections produced in the first and second place respectively, it follows from (5) that

$$\frac{H_1}{H_2} = \frac{\tan \theta_2}{\tan \theta_1} \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (10)$$

If the B position is used instead of the A position, the base board of the magnetometer must be set north and south, and the bar magnet must be set at right angles to the centimetre scale. Then instead of (9) we obtain

$$\begin{aligned} M_1 &= (d^2 + l_1^2)^{\frac{3}{2}} \tan \theta_1 \\ M_2 &= (d^2 + l_2^2)^{\frac{3}{2}} \tan \theta_2 \end{aligned} \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (11)$$

(10) holds in this case also. (9) and (11) may be simplified, if l_1^2 , l_2^2 are small in comparison with d^2 .

A word of explanation should perhaps be given, why in taking readings with the magnetometer both ends of the pointer are read and the magnet is reversed. If through a fault in construction the point of support of the needle is not exactly at the centre of the circle, the reading at one end of the pointer is too large and at the other end is too small by the same amount (*cf.* Fig. 457), and by taking the mean these two errors neutralise one another. Again, if the poles of the bar magnet are not the same distance from its centre, the deflection is too large in the one case and too small when the magnet is reversed; hence

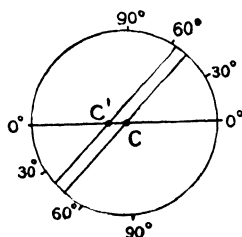


FIG. 457.

the mean gives a more accurate result.

Comparison of Moments of Bar Magnets. Null Method.—Either the A or the B position is used. One magnet is placed on the one side of the needle to produce a deflection; the second magnet is placed on the other side, and its distance adjusted so that it exactly removes the deflection produced by the first. The formula may be derived from (5) or (7).

EXPERIMENT.—To determine the position of the poles of a bar magnet by the magnetometer. Use the B position; then

$$\frac{M}{H} = (d^2 + l^2)^{\frac{3}{2}} \tan \theta$$

$$\text{or} \quad d^2 + l^2 = \left(\frac{M}{H} \right)^{\frac{2}{3}} \frac{1}{\tan^{\frac{2}{3}} \theta} = \left(\frac{M}{H} \right)^{\frac{2}{3}} \cot^{\frac{2}{3}} \theta$$

Observe θ for different values of d , calculate d^2 and $\cot^{\frac{2}{3}} \theta$, and graph d^2 against $\cot^{\frac{2}{3}} \theta$. The result is a straight line which cuts the d^2 axis in a point distant l^2 from the origin. We then obtain l by taking the square root. The method is accurate only for weak magnets.

Proof of Inverse Square Law.—When the magnet is small the deflections in the A and B positions are given by the formulæ

$$\frac{M}{H} = \frac{d^3}{2} \tan \theta_1 \quad \text{and} \quad \frac{M}{H} = d^3 \tan \theta_2$$

respectively; hence if d is the same in both cases, $\tan \theta_1 = 2 \tan \theta_2$. The formulæ are derived on the assumption that the force between two magnetic poles varies inversely as the square of their distance apart. If the law were that of the inverse n th power, it can be shown that we would have $\tan \theta_1 = n \tan \theta_2$. By measuring the ratio of $\tan \theta_1$ to $\tan \theta_2$ Gauss showed that the inverse square law was true; this way of proving the law is a very accurate one.

Period of Vibration of a Magnet.—If a magnet is suspended in a magnetic field, as shown in Fig. 435, and if a second magnet is brought up from the side to it along the horizontal line at right angles to its axis through its centre, and then suddenly withdrawn, it will oscillate in a horizontal plane about its axis of suspension. Now for a motion of rotation such as this

$$\begin{aligned} \text{Angular Acceleration} &= \frac{\text{Applied Couple}}{\text{Moment of Inertia of Rotating System}} \\ &= \frac{HM \sin \theta}{I} \end{aligned}$$

where in this case H is the field strength, M the magnetic moment and I the moment of inertia of the suspended magnet, and θ the angular displacement or the angle through which it is turned from its equilibrium position. If θ is small,

$$\frac{\text{Angular Acceleration}}{\text{Angular Displacement}} = \frac{HM}{I} = \text{constant.}$$

Hence the motion is simple harmonic, and if T is its period

$$\frac{HM}{I} = \frac{4\pi^2}{T^2} \quad \text{or} \quad T = 2\pi \sqrt{\frac{I}{HM}} \quad \dots (12)$$

For a uniform solid cylindrical bar oscillating about an axis through its centroid perpendicular to its length

$$I = m \left(\frac{l^2}{12} + \frac{r^2}{4} \right)$$

where m is the mass, l the length of the cylinder, and r its radius.

For a thin rectangular bar of length l and breadth b oscillating about an axis through its centroid perpendicular to its plane

$$I = m \left(\frac{l^2 + b^2}{12} \right)$$

If l^2 is much greater than r^2 or b^2 , these two formulæ reduce to the same form, namely

$$I = \frac{ml^2}{12}$$

Comparison of Magnetic Moments and Field Strengths by the Vibration Method.—Formula (12) can be used both for comparing the magnetic moments of magnets and for comparing the intensities at different points in a magnetic field. For if two different magnets are caused to oscillate at the same place under the influence of the field we have

$$\frac{HM_1}{I_1} = \frac{4\pi^2}{T_1^2} \quad \text{and} \quad \frac{HM_2}{I_2} = \frac{4\pi^2}{T_2^2}$$

whence
$$\frac{M_1}{M_2} = \frac{I_1 T_2^2}{I_2 T_1^2} \cdot \cdot \cdot \cdot \cdot \cdot (13)$$

Again, if the same magnet is caused to vibrate in succession at different points in a field, we have

$$\frac{H_1 M}{I} = \frac{4\pi^2}{T_1^2} \quad \text{and} \quad \frac{H_2 M}{I} = \frac{4\pi^2}{T_2^2}$$

whence
$$\frac{H_1}{H_2} = \frac{T_2^2}{T_1^2} \cdot \cdot \cdot \cdot \cdot \cdot (14)$$

Determination of Earth's Magnetic Field and of Magnetic Moment of a Magnet in Absolute Measure.—From the magnetometer we obtain an equation for M/H by formula (5) or (7); by determining the period of vibration of the same magnet at the same place we obtain a value for HM by formula (12). We have thus two equations for the two unknowns, and we can determine H in absolute measure for the place in question by eliminating M ; similarly by eliminating H we can determine M , the magnetic moment of the magnet.

Comparison of Magnetic Moments when Moments of Inertia are not known.—Let us suppose that two bar magnets are tied together, centroids together and like poles at the same end, and then vibrated in a horizontal plane about a vertical axis through their common centroid. They are equivalent to a single magnet of magnetic moment $M_1 + M_2$ and moment of inertia $I_1 + I_2$; consequently the period is given by

$$\frac{H(M_1 + M_2)}{I_1 + I_2} = \frac{4\pi^2}{T^2} \cdot \cdot \cdot \cdot \cdot (15)$$

Untie the magnets, reverse one end for end, tie them together, and vibrate them again in a horizontal plane about the vertical axis through their common centroid. They are now equivalent to a single magnet of magnetic moment $M_1 - M_2$ and moment of inertia $I_1 + I_2$; consequently the period is given by

$$\frac{H(M_1 - M_2)}{I_1 + I_2} = \frac{4\pi^2}{T'^2} \quad \dots \quad (16)$$

On dividing (15) by (16) we obtain

$$\frac{M_1 + M_2}{M_1 - M_2} = \frac{T'^2}{T^2}$$

whence

$$\frac{M_1}{M_2} = \frac{T'^2 + T^2}{T'^2 - T^2} \quad \dots \quad (17)$$

This equation enables us to determine the ratio of the magnetic moments of the magnets without determining their moments of inertia. But the method gives only approximate results, because the presence of the one magnet affects the pole strength of the other.

EXPERIMENT.—To determine the distribution of magnetism along a bar. Let T be the period of oscillation of a compass needle in the earth's field H . Place due magnetic south of the compass needle and close to it a long vertical bar magnet which has its positive end at the same height as the needle. Let the additional field strength due to the magnet be F_1 and let the period become T_1 . Then

$$\frac{HM}{I} = \frac{4\pi^2}{T^2} \quad \text{and} \quad \frac{(H + F_1)M}{I} = \frac{4\pi^2}{T_1^2}$$

where M is the magnetic moment and I the moment of inertia of the compass needle. Consequently, on subtracting the first equation from the second, we obtain

$$\frac{F_1 M}{I} = 4\pi^2 \left(\frac{1}{T_1^2} - \frac{1}{T^2} \right) \quad \dots \quad (18)$$

Raise the bar magnet 2 cm. and let the corresponding values of F and T be F_2 and T_2 . Then

$$\frac{F_2 M}{I} = 4\pi^2 \left(\frac{1}{T_2^2} - \frac{1}{T^2} \right) \quad \dots \quad (19)$$

and on dividing equation (18) by equation (19)

$$\frac{F_1}{F_2} = \frac{\frac{1}{T_1^2} - \frac{1}{T^2}}{\frac{1}{T_2^2} - \frac{1}{T^2}}$$

F_1 and F_2 are due principally to the magnetism in the bar at the level of the compass needle, and may be taken as proportional to the latter. The method is not an exact one.

Astatic Needle.—If two magnets of nearly equal magnetic moment are fixed together with their axes parallel but pointing in opposite directions, and suspended in a magnetic field so that they are free to

rotate about a vertical axis through their centroids, the restoring couple acting on them is

$$H(M_1 - M_2) \sin \theta$$

and is consequently very small, since $M_1 - M_2$ is small. Thus the system is easily deflected and takes a long time to return to its equilibrium position. Such a system is called an astatic system or astatic needle.

Hibbert's Magnetic Balance.—The attraction or repulsion exerted by one magnet on another cannot be measured by an ordinary balance, for if a scale-pan is removed and replaced by a magnet, and an attempt made to diminish or increase its weight by placing another magnet below it on the base of the balance, the suspended magnet becomes

unstable. For experiments of this nature Hibbert's magnetic balance is used. This piece of apparatus, which is shown diagrammatically in Fig. 458, consists essentially of a magnet NS joined to a brass bar SD, the whole being pivoted at S and balanced by a sliding weight w at C, together with another magnet N'S' which can be fixed at various positions on a

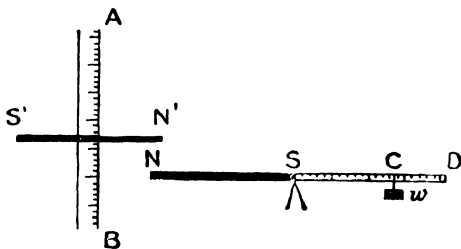


FIG. 458.

vertical scale, so that the poles N' and N are above one another. The force exerted by N' and S' on S can be neglected, since its line of action passes so near the pivot, and the force of S' on N is neglected in comparison with the force of N' on N owing to the greater distance of S' from N. Hence the sliding weight w is balanced solely by the repulsion between N and N', and the latter can be calculated from the position of the weight.

EXAMPLES II

1. A magnet 10 cm. in length lies in a field of intensity $H=0.18$, and the strength of each of its poles is 6 units. Find the moment of the couple required to deflect it (i) through an angle of 30° from the magnetic meridian, (ii) through an angle of 60° from the magnetic meridian.

2. A bar magnet of pole strength m , the distance between the poles of which is $2l$, is placed in the magnetic meridian with its south pole pointing north. Show that the neutral points are situated on the axis of the magnet at a distance d from its midpoint, where d is given by

$$H = \frac{4mdl}{(d^2 - l^2)^2}$$

3. A cylindrical magnet is placed in the magnetic meridian with its north pole pointing south. The neutral points are found to be in the line of the magnet at a distance of 30 cm. from its midpoint. Calculate the pole strength of the magnet, given that $H=0.17$ c.g.s. unit, and that the poles are 18 cm. apart.

4. Two magnets are placed in succession on the magnetometer in the "A" position, and found to give deflections of 26.1° and 23.3° at a distance of 33.5 cm. Calculate their moments, given that $H=0.15$ c.g.s., and that the length of each magnet is 10 cm.

5. Two magnets of the same moment of inertia are made to vibrate in succession under the action of the earth's field. Their periods are 7.0 and 7.55 sec. Calculate the ratio of their magnetic moments.

6. A bar magnet was placed on the magnetometer in the "A" position and found to give a deflection of 21.7° at a distance of 35 cm. When it was suspended at the same place, the time of twenty complete vibrations in the earth's field was 2 min. 38 sec. The length of the magnet was 10.1 cm. and its mass 25.07 gm. Calculate the moment of the magnet and the field strength.

7. A bar magnet was placed on the magnetometer in the "B" position and found to give a deflection of 22.8° at a distance of 27 cm. When it was suspended at the same place, the time of twenty complete vibrations in the earth's field was 2 min. 35 sec. The length of the magnet was 10.1 cm. and its mass 25.27 gm. Calculate the moment of the magnet and the field strength.

8. Two bar magnets, the magnetic moments of which are 100 and 150 respectively, are tied together, like pole to like and midpoints together, and suspended horizontally in the earth's magnetic field. The period of vibration is 8.0 sec. One of the magnets is then reversed end for end. What does the period become?

9. A compass needle is placed inside a hollow vertical cylinder of soft iron, and it is found that the vibrations are slower than they would be, if the cylinder were removed. Give the reason for this.

10. A bar magnet, which is suspended horizontally by a fine vertical wire, lies in the meridian when there is no torsion in the wire, and the upper end of the wire must be twisted through 90° in order to deflect the magnet 15° from the meridian. The magnet is then remagnetised, and it is found that the upper end of the wire must now be turned through 150° instead of 90° in order to produce the same deflection. Calculate the ratio of the magnetic moments in the two cases.

11. A long bar magnet is placed with its N. pole 7 cm. to the south of a small magnetic needle, and the needle is found to make 12 complete vibrations in 6 sec. When vibrating in the earth's field (0.18 c.g.s. unit), the needle makes 6 complete vibrations in 8 sec. Calculate the strength of the N. pole on the assumption that the S. pole is so far away, that its effect may be neglected.

12. A small compass needle makes 10 vibrations per minute under the influence of the earth's magnetic field. A magnet 80 cm. long is placed in a vertical position with its lower end on the same level with the needle and 15 cm. north of it; the needle then makes 12 vibrations per minute. Calculate the pole strength of the magnet, neglecting the influence of the upper pole. $H=0.15$ c.g.s. unit.

13. A magnetic needle makes 10 vibrations in 32 seconds in the earth's field alone. A bar magnet is placed with its S. pole pointing north, in such a position in the meridian, that the needle is due east of its midpoint; the needle then makes 20 vibrations in 50 seconds. Find its period of vibration, if the magnet is reversed pole for pole.

CHAPTER III

TERRESTRIAL MAGNETISM

Declination.—As has already been mentioned, when a compass needle is at a distance from other magnets, it sets in a definite direction. This direction is called magnetic north and south, or the magnetic meridian at the place. If the magnet is displaced from it, it oscillates about it, and finally returns to it. The magnetic north direction does not coincide with the geographical north, which is the direction of the meridian or great circle through the place and the earth's axis of rotation. The angle between magnetic and geographical north or between the magnetic and geographical meridians is called the declination or, by mariners, the variation. The simplest way of determining the geographical meridian is to measure the length of the shadow of a vertical rod at different times near midday; when the shadow is shortest, the sun is in the meridian.

The declination varies from place to place over the earth's surface, and it is usual to represent its distribution by curves drawn on maps joining all the places where its value is the same. Such curves are called **isogonals**; Fig. 459 shows the isogonals for the earth's surface for the year 1922. The full lines indicate westerly declination, *i.e.* places where the compass needle points west of geographical north, while the dotted curves indicate easterly declination. The thick lines which join up the places where the declination is zero, and the compass consequently points towards geographical north, are called the **agonic** lines. One agonic line runs down the east of North and South America; the other runs from Finland south to Arabia, crosses North India, forms a loop over East Asia known as the Siberian oval, and then passes through the Malay Peninsula and the west of Australia.

Dip.—The ordinary compass needle is balanced so as to turn in a horizontal plane. Suppose, however, that an unmagnetised bar of steel is mounted free to rotate about a horizontal axis through its centre of gravity, so that it balances in any position in which we care to set it, and that it is then magnetised; as soon

LINES OF EQUAL DECLINATION FOR THE YEAR 1922

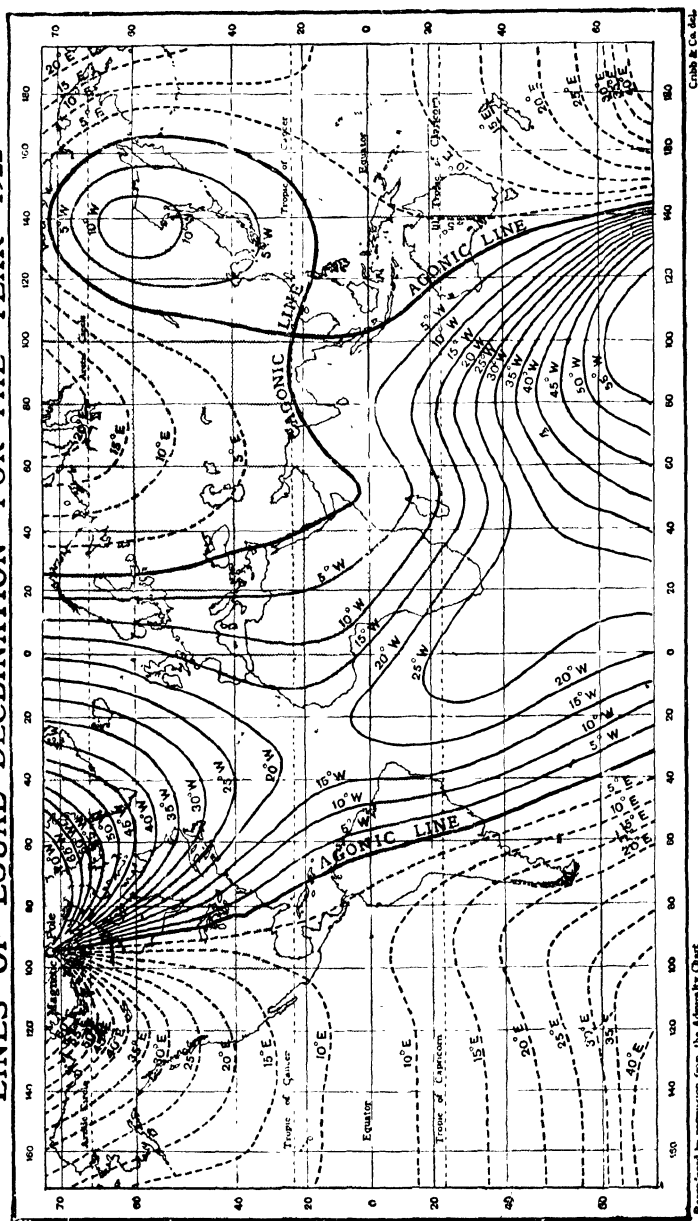


FIG. 459.

Longmans, Green & Co. London, New York, Toronto, Bombay, Calcutta & Madras

Reproduced by permission from the Admiralty Chart

Cribb & Co. del.

as it is magnetised it comes to rest only in one position, which makes a definite angle with the horizontal. The magnitude of this angle depends on the vertical plane in which the needle happens to be. If the needle is in the magnetic meridian, the angle is a minimum, and is then called the magnetic dip or inclination; the needle used for measuring it is called the dip needle.

Like the declination the dip varies from place to place, the value for Greenwich being about 67° . Like the declination it can be represented over the surface of the earth by a map in which places of equal dip are joined by lines known as **isoclinical** lines. These lines are roughly parallel to the geographical lines of latitude, and there is a line of zero dip known as the magnetic equator which coincides roughly with the geographical equator. At all places on this line the dip needle sets itself horizontal. North of the magnetic equator the north pole of the dip needle dips below the horizontal, the angle of dip increasing with the distance from the equator, and south of the equator the south pole of the dip needle dips below the horizontal.

Magnetic Elements.—If an unmagnetised steel bar is suspended by a fine thread through its centre of gravity, and is then magnetised and suspended by a thread through the same point, it comes to rest in the magnetic meridian making the angle of dip with the horizontal. This direction must consequently be the direction of the earth's resultant magnetic field, and the value H which acts on needles pivoted to turn in a horizontal plane must be the horizontal component of the resultant. Let OC (Fig. 460) represent the resultant in magnitude and direction, and let F denote its magnitude. Then

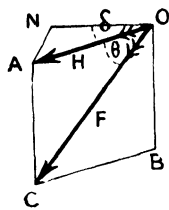


FIG. 460.—Declination and dip.

$$H = F \cos \theta$$

where θ is the dip, and it is clear that the vertical component of the field is given by

$$V = H \tan \theta$$

If ON gives the direction of geographical north, $\angle NOA$ is the declination. Hence if the horizontal component of the earth's field, the declination, and the dip are known, the magnetic intensity at a place is fully determined. These three quantities are called the magnetic elements. Their values at Greenwich for 1927 were as follows: $H = 0.184$ c.g.s. unit; declination $= 12^\circ 45'$ W.; dip $= 66^\circ 51'$.

Lines passing through points for which the horizontal intensity of the earth's magnetic field is the same are called **isodynamic**

lines, and can be drawn on maps in the same way as isogonal and isoclinical lines.

The method of determining H has already been described on p. 460.

Measurement of Declination.—In order to measure the declination we must determine the geographical and magnetic meridians.



FIG. 431.—Determination of magnetic axis.

The former may be obtained by observing the direction of the sun when it is highest in the heavens or by other astronomical means. The latter may be obtained by suspending a needle and observing the direction in which it sets, but there is a source of error to guard against here. The magnetic axis of the needle may not coincide with its geometrical axis. Suppose that AB represents the geometrical axis and CD the magnetic axis of the needle. It will consequently set as indicated in the diagram where NS is the magnetic north-south line, and if the geometrical axis were taken as the direction required, the result would be in error. This is corrected by turning the magnet over and suspending it with the other side up, when it sets itself in the position $A'B'$, which is simply the position AB rotated about the line CD . The mean of the two readings consequently gives the correct result.

Dip Circle.—Fig. 462 represents one type of dip circle, the instrument used for the accurate determination of the dip. It consists essentially of a needle resting on two agate knife edges, which rotates around a vertical circle graduated in degrees. The needle can be raised off the knife edges by the small screw at the right, and is enclosed by two glass plates to shield it from draughts. The instrument is entirely of gun metal, no iron entering into its composition. The horizontal circle carries a level, and the whole is supported by three levelling screws.

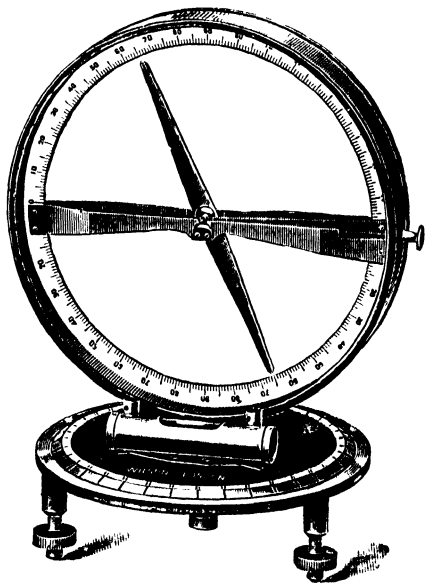


FIG. 462.—Dip circle.

In making a determination the case is first turned about the vertical axis until the needle sets vertically; its axle is then in the magnetic meridian. The case is then turned through 90° to bring the needle into the meridian, and the dip read off the vertical scale at both ends of the needle. This eliminates error due to the axle not being at the centre of the vertical circle (*cf.* Fig. 457). The needle is then reversed in its bearings, back for front, and the readings repeated; this eliminates error due to the magnetic axis not coinciding with the geometrical axis (*cf.* Fig. 461). The case is next rotated through 180° , and the preceding four readings repeated; this removes error due to the zero line of the vertical scale not being truly horizontal, for if the scale is too high up at the one side, it will be too low at the other. Finally the needle is demagnetised, remagnetised the other way, and the preceding eight readings repeated. This removes error due to the centre of gravity of the needle not being in the axis of rotation, for if gravity increases the dip in the one case, it will diminish it in the other.

Variation in Earth's Magnetic Field.—In addition to changing from place to place on the earth's surface, the magnetic

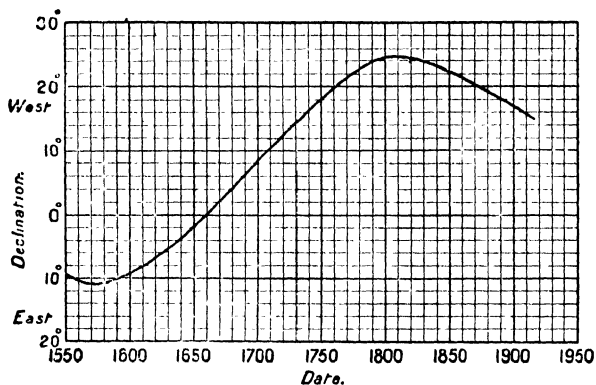


FIG. 463.—Secular change of the declination at London.

elements change with time. A continuous record of these changes is kept in certain observatories, and it is found that the magnetic elements undergo a **diurnal change**, an **annual change**, and also a **secular change**, *i.e.* one that goes on through centuries. For example, at Kew in summer the declination falls 4' or 5' below its mean value every morning, and rises as far above its mean value early every afternoon.

Fig. 463 represents the secular change in the declination at London from 1550. It was originally easterly, but became zero in 1660; in that year at London the compass needle pointed to geographical north. In 1815 it attained a maximum westerly value, and is now decreasing again.

In addition to the regular changes sudden and violent changes of the magnetic elements sometimes take place over a large part of the earth's surface. These disturbances are called **magnetic storms**. Fig. 464 shows the change of declination at Greenwich during a magnetic storm and during an ordinary quiet day. Magnetic storms are connected in some way with sun-spots; the sudden appearance of a large sun-spot is generally accompanied by a magnetic storm and by the **Aurora Borealis**. The latter is seen frequently in northern latitudes and consists of streamers or bands of light which dart across the whole sky. In England the phenomenon occurs rarely, and the bands are of a pale green colour and not bright. By photographing them simultaneously from different standpoints it has been found that they are between 100 and 800 km. above the earth's surface, and that they may be as much as 1000 km. in length. It is natural to

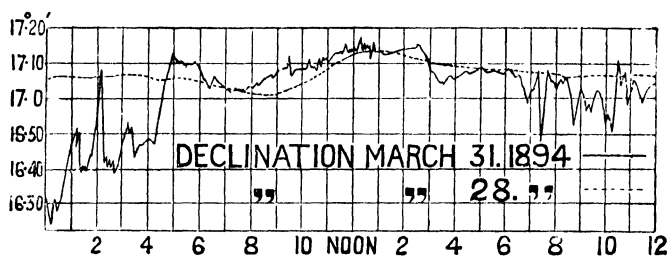


FIG. 464.—A magnetic storm.

suppose that they are electric currents high up in the atmosphere caused by some emission from the sun.

Magnetic Poles.—The compass did not become known to European nations until the eleventh century, though, according to their own annals, the Chinese used it for thirty centuries before. When first employed in Europe it consisted merely of the needle embedded in wood or thrust into a corn stalk and then floated on water. The discovery of the dip was made by an English instrument maker, Robert Norman, about 1576.

It was thought at first that the needle pointed north because of an attraction by the "lode star" or pole star, but Gilbert showed, in a work published in 1600, that the earth itself was a magnet. The needle set in a definite direction because it was in the earth's field, and rods of iron left lying about became magnetised under the influence of this field just as when placed near a large magnet. The earth's magnetic field is at present regarded as due to both internal and external causes, principally the former, and it would be given approximately by a large internal magnet as shown in Fig. 465. The arrows in this figure

represent the directions of the dip needle, and it will be observed that at two points A and B, the north and south magnetic poles, it is vertical. Since the north magnetic pole attracts the north-seeking or north end of the dip needle, it must itself be a south-seeking or south pole. The end of the internal magnet below A is consequently indicated by the letter S.

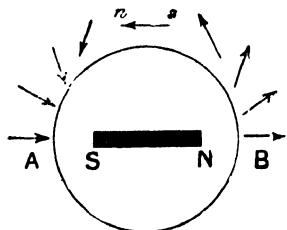


FIG. 465.—The earth as magnet.

The north magnetic pole lies in lat. $70^{\circ} 5' N.$ and long. $96^{\circ} 43' W.$, while the south magnetic pole is at lat. $72^{\circ} 25' S.$ and long. $154^{\circ} E.$ They are consequently not on the same diameter. The north magnetic pole was discovered by Sir James Ross in 1831. The south magnetic pole was discovered by David, Mawson, and Mackay, three members of Shackleton's expedition, in 1908. Three days before the discovery the dip was $89^{\circ} 10'$: the evening before it was $89^{\circ} 45'$, and they estimated then, that if they were to wait where they were, the diurnal variation would bring the pole right beneath them. They decided, however, to go on another 13 miles next day to the mean position. During the last 9 miles of this march the compass needle turned in all directions and was of no use for laying a course; they consequently had to set up marks for finding their way back.

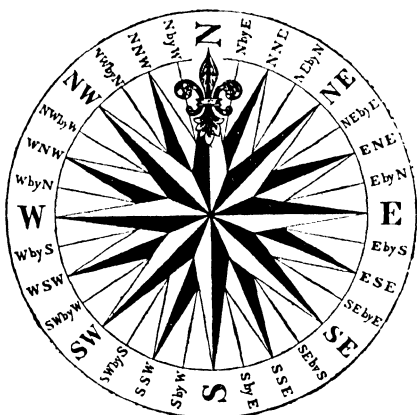


FIG. 466.—Compass card.

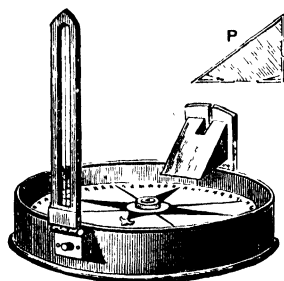


FIG. 467.—Prismatic compass.

Magnetic Compass.—The magnetic compass finds its most important use in the navigation of ships and aeroplanes. The compass card consists of a light disc on which the points of the compass, shown in Fig. 466, are drawn and which carries several

light steel magnets, the whole being suspended upon an agate cup resting on a needle point.

If a steel bar is held in the direction of the earth's magnetic field and struck with a hammer several times, it becomes magnetised. Modern ships are built of steel, and owing principally to the hammering they undergo in construction they become magnetic. This would affect the compass reading, and permanent magnets and soft-iron spheres have to be fixed in such positions round the compass as to destroy exactly the field produced by the ship's magnetism.

Fig. 467 shows a prismatic compass, which is used for surveying and military purposes. Its case is a shallow metal box about $2\frac{1}{2}$ inches in diameter. To use it to take the bearing of a distant object the eye looks through the slit above the prism and gets the distant object in line with the vertical wire in the upright frame; the reading of the card is then seen reflected in the prism directly below the distant object. A section of the prism is shown at P; its under surface is curved so as to magnify the reading of the card. Similar prismatic fittings, called azimuth mirrors, are used for taking compass bearings at sea.

EXAMPLES III

1. The magnetic elements at a place are : Declination 30° W., dip 56° S., and horizontal force 0.20 c.g.s. Calculate the components of the earth's field in the directions geographical north and west, and vertically downwards.

2. Describe the dip circle and explain its action. How does it behave when it is rotated slowly through one complete revolution about a vertical axis?

3. How will a dip needle set itself, when its axis of rotation is in the magnetic meridian at the magnetic equator?

4. A dip needle is 10 cm. long and is placed in the meridian in a locality where the dip is $66^\circ 50'$ and the horizontal component of the earth's field 0.185 c.g.s. unit. A small weight is attached to the upper end of the needle and brings it into the horizontal plane. Calculate the magnitude of the weight on the assumption that the poles of the needle are at its ends and of strength 10 units.

5. A dip circle is placed so that the needle sets vertical. The circle is then rotated through an angle α about a vertical axis, and the dip as measured in this position is found to be ϕ . Find its true value.

6. A dip needle oscillates in the meridian at the rate of 30 oscillations per minute at a place where the dip is $66^\circ 54'$, and the horizontal component of the earth's field 0.185 gauss. At another place where the dip is $31^\circ 1'$, the dip needle makes 28.74 oscillations per minute. Find the value of the horizontal component of the earth's field at the second place.

CHAPTER IV

ELECTROSTATICS. FUNDAMENTAL IDEAS

Fundamental Experiment.—It was discovered by Thales about 600 B.C. that when amber was rubbed it acquired the property of attracting threads and other light objects. It was found by Gilbert, about the year 1600, that this property was shared by many other substances, such as glass, sealing-wax, etc., and he named the effect produced on these substances by rubbing them “electrification,” after *elektron*, the Greek word for amber.

The easiest way of exhibiting the effect is to tear off some little squares of paper about 2 mm. side, and rub the end of a vulcanite fountain pen on your sleeve. It then becomes electrified, and if it is brought near the little bits of paper, the latter jump up and adhere to it.

Two Kinds of Electrification.—If a pith-ball is suspended by a dry silk thread like the bob of a pendulum, and if a glass rod is rubbed with a piece of silk and held towards the pith-ball, the latter is attracted towards the glass rod, touches it, and then is repelled by it. If another pith-ball is suspended by a dry silk thread, and a stick of sealing-wax rubbed with dry flannel and held towards this other pith-ball, the latter is attracted towards the wax, touches it, and then is repelled by it. But if the glass rod is held towards the second pith-ball, it attracts it, and if the sealing-wax is held towards the first pith-ball, it also attracts it. Evidently the pith-ball electrified by the glass rod is in a different state from the pith-ball electrified by the sealing-wax.

These experiments are explained by assuming that there are two kinds of electricity, just as there are two kinds of magnetic poles, and that, as in the case of the magnetic poles, their behaviour can be summed up in the following rule :

Like electrical charges repel ; unlike electrical charges attract each other.

The kind of electricity excited on a glass rod by rubbing it with silk was formerly called **vitreous** electricity, and that excited on the sealing-wax by rubbing it with flannel **resinous** electricity. The pith-ball that touched the glass rod acquired a charge of

vitreous electricity, and hence was repelled by the latter and attracted by the sealing-wax; the pith-ball that touched the sealing-wax acquired a charge of resinous electricity, and hence was repelled by the latter and attracted by the glass. The names "vitreous" and "resinous" are, however, now obsolete. It was found that whenever a charge of the one kind of electricity was produced, an equal charge of the other kind was produced simultaneously; for example, when the glass acquired the vitreous charge, the rubbing silk acquired a resinous charge. And if these two charges were transferred to the same body, the latter showed no signs of electrification at all. The two charges neutralised each other and the final charge was zero. Hence, since the addition of the two kinds of electricity produced zero charge like the addition of numbers with positive and negative signs, it became natural to regard the one kind as positive and the other kind as negative. The vitreous electricity is called **positive** and the resinous **negative**; this choice of terms is perfectly arbitrary, and could quite as well have been made the other way.

All uncharged bodies are assumed to contain large equal charges of positive and negative electricity. If such a body is alone, the charges neutralise each other and it has no electrical action. But if a charged body, for example, a positively charged body, is brought near, it attracts the negative charge within the first body and repels the positive one. Consequently, the end of the first body nearer the second body acquires a negative charge and the other end a positive one, and the negative charge being nearer the second body, its attraction is greater than the repulsion of the positive charge. Thus an uncharged body is attracted towards a charged one. The action is somewhat similar to that of a magnet on a piece of unmagnetised iron. This explains why the pith-balls in the experiments described above were at first attracted by the glass and sealing-wax. Like charges repel, unlike charges attract, and *charges of either sign attract uncharged objects*.

The question arises as to why in the experiment with the fountain pen the pieces of paper stuck to the vulcanite, and were not repelled as soon as they touched, as in the case of the pith-ball. It is because the vulcanite becomes negatively charged, and at first repels all the negative electricity out of the paper into the table, and the negative electricity which flows into the paper from the vulcanite after contact is not enough to make up this deficiency. Consequently, there is an excess of positive in the paper, and it remains sticking to the vulcanite.

Conductors and Insulators.—When a pith-ball suspended by a dry thread of unbleached silk is given a charge of electricity, it retains this charge for a time. But if it were suspended by a

thin copper wire, it would lose the charge at once. The silk thread is called a good insulator or non-conductor, while the copper wire is a good conductor and conducts the electricity away at once. The best non-conductors are ebonite, sulphur, glass, shellac, quartz, paraffin, sealing-wax, and silk. The best conductors are the metals and solutions of inorganic salts in water. There is no sharp division between the classes; for example, dry wood and paper are semi-conductors. When a body is electrified by friction, the nature of the charge produced on it depends on the substance with which it is rubbed; thus glass, which acquires a positive charge when rubbed with silk, acquires a negative charge when rubbed with catskin.

The earth, or at least those portions of it which are moist, may be regarded as one huge conductor. A body which is put into conducting communication with the earth is said to be **earthed**; the best way of doing this is to connect it by a wire to the water-pipe or the gas-pipe, or to carry the wire itself deep below the surface of the earth. Owing to the large size of the earth in comparison with any charged body, the fraction of the original charge which remains on it after earthing is negligible; the charge passes along the conducting path to earth, and the body is said to be discharged.

All the non-conductors can be electrified by rubbing; if a brass rod is held in the hand and rubbed with flannel, it shows no signs of electrification, but if it is fitted with an insulating ebonite handle and then rubbed it becomes strongly electrified. The electricity escapes in the first case through the body of the experimenter to earth.

Absolutely pure water would be a non-conductor, but water usually contains small quantities of dissolved matter which make it a conductor. Thus, if the silk supporting the pith-ball is damp, the latter does not retain its charge, and in the same way, if the glass stems used for insulation purposes in electrical apparatus become covered with a film of moisture, they cease to insulate. Experiments on electric charges succeed best on a dry frosty day, and an open coal fire or a gas fire is the best way of drying the apparatus. If the atmosphere is moist, the charges leak away very rapidly.

Gold-Leaf Electroscope.—Fig. 468 represents the gold-leaf electroscope, which is an instrument used for indicating the presence of an electric charge. It consists of two gold leaves hung from the lower end of a metal rod. This rod passes through an insulating plug in a glass shade, which protects the leaves from mechanical injury and air currents, and terminates above in a metal disc or cap. The gold leaves, rod, and disc may be regarded as one conductor.

If the disc is touched for an instant by a glass rod which has been electrified in the usual way, a positive charge is communicated to the leaves, and they repel one another and diverge. If they diverge too far, they touch strips of tinfoil pasted inside the shade, which conduct away their charge to the earth, and cause them to collapse together again. The electroscope can be discharged at any instant by merely touching the plate with a finger; the charge then escapes through the body of the experimenter to the earth. If a charged electroscope is left to itself, the charge gradually leaks away through the atmosphere.



FIG. 468.—Gold-leaf electroscope.

If the electroscope is discharged, and then a glass rod electrified in the usual manner is brought near the disc, but not allowed to touch it, the leaves diverge. They are then said to be **electrified by induction**. Fig. 469*a* shows what happens. The positive and negative charges on the plate and disc separate out under the influence of the positive charge on the glass, the negative charge being attracted up to the disc and the positive charge being repelled into the leaves; the leaves consequently diverge with positive electricity. If now, while the glass rod is kept in position, the plate is touched for an instant with the finger, as shown in Fig. 469*b*, the positive charge on the leaves escapes through the body of the experimenter to earth and the leaves come together, while the negative charge on the disc is held in position by the attraction of the positive charge on the glass rod. Suppose now

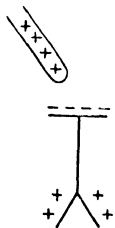


FIG. 469*a*.

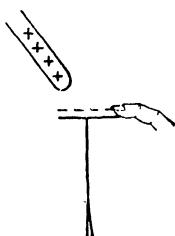


FIG. 469*b*.

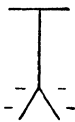


FIG. 469*c*

Charging an electroscope by induction.

that the glass rod is removed; the negative electricity is no longer confined to the plate by the attraction of the positive charge on the glass rod, but spreads over the plate, rod, and leaves, so that the leaves diverge with a charge of negative electricity, as

shown in Fig. 469c. The glass rod has thus been made the means of giving the electroscope a charge of *the opposite nature* to that which it possessed itself. When charged in this way the electroscope is said to be charged by induction. The method of induction is the most convenient way of charging an electroscope, because the charge can be regulated by altering the distance of the charging body.

Test for Presence of an Electric Charge.—If the leaves of the electroscope diverge, and if when a body is brought close up to the disc of the electroscope without touching it, they diverge still further, then the body has an electric charge of the same sign as the leaves, for we have here the induced distribution superimposed on the original one. If, however, the leaves collapse, the body may have a charge of the opposite sign to the leaves, but this is not in itself conclusive, for if an earthed conductor is brought near the disc of a charged electroscope, it causes the leaves to collapse.

Charges of Opposite Sign are always produced in Equal Amounts.—It was mentioned on p. 473 that, whenever a charge of the one kind of electricity was produced, an equal charge of the other kind was produced simultaneously. This can be shown very neatly by making a flannel cap to fit the end of an ebonite rod and fastening a silk cord to it; this cord is wound round the cap and the latter rotated round the ebonite by pulling the cord tight. The friction of the flannel on the ebonite causes electrification. If both rod and cap are then brought near the disc of an electroscope, there is no effect on the leaves, but if the cap is drawn off by the silk cord, and cap and rod tested separately, the rod is found to have a negative and the cap an equal positive charge. In the ordinary method of electrifying the ebonite rod the positive charge of the flannel escapes through the fingers of the experimenter to earth.

Electrophorus.—Fig. 470 represents the electrophorus, a simple apparatus used for producing and collecting larger quantities of electricity than can be obtained by rubbing sealing-wax or glass. Its action depends on electrification by induction.

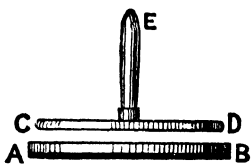


Fig. 470.—Electrophorus.

It consists of a cake of resin or some similar substance about 12 inches in diameter and a half-inch thick, contained in a metal mould AB, and a metal disc or cover CD, of a diameter somewhat less than that of the cake, and provided with an insulating glass handle E. To use the electrophorus the cake is first well dried, and then rubbed with catskin. This gives its surface a charge of negative electricity. The cover is then placed on the top of the cake. Owing to the surface of the latter not being quite plane,

contact is made only at a few points; consequently, since the resin is not a conductor, its electricity does not pass to the cover, but acts inductively on the latter, causing a positive charge to separate out on the under surface of the plate and a negative charge to appear on its upper surface. If the upper surface is then touched with the finger, this negative charge escapes through the body of the experimenter, and if the cover is raised by its insulating handle the positive charge is retained, and diffuses itself over the surface. If a finger is then brought near the edge of the cover, a sharp spark passes, or if the edge is brought near an unlit burner out of which gas is streaming, a spark passes to the burner and lights the gas. The mechanical work done in raising the plate is converted into the electrical energy of the spark.

The cover may be charged a considerable number of times without recharging the resin.



FIG. 471.

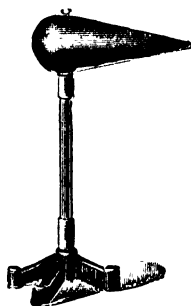


FIG. 472.

Distribution of Electrical Charge upon Conductors.—A **proof plane** consists of a small metal disc, about the same size as a sixpence but thinner, fixed by sealing-wax to the end of a thin glass rod; the glass rod forms an insulating handle. Figs. 471, 472, and 473 all represent metal conductors mounted on insulating glass pillars; in Fig. 471 the conductor is spherical, in Fig. 472 it is pear shaped and drawn out to a point on one side, and in Fig. 473 it is spherical and hollow, and has a little hole at the top.

Now, suppose that the spherical conductor is charged by an electrophorus, and that its surface is touched by a proof plane. Some of the electricity then flows into the proof plane; if the proof plane is then made to touch the disc of a gold-leaf electroscope, the leaves of the latter will diverge. If the electroscope and proof plane are discharged by touching them with the finger, and the proof plane is employed to bring a charge to the electro-

scope from any other point on the surface of the conductor, the gold leaves will diverge again to the same extent as before. Now the extent of the divergence is a measure of the quantity of electricity communicated to the electroscope. Hence the quantity of electricity taken from the two points on the surface of the conductor is the same, *i.e.* the surface density, or amount of charge on unit area is the same over the whole surface.

If, however, the conductor represented in Fig. 472 is charged, and charges are taken from different points on its surface to the electroscope, the divergence is not the same in the different cases, but is greatest when the charge is taken from the point. In the same way, when a flat disc is electrified, much more electricity

can be taken from the edge than from the centre. This is a particular case of a general rule: if the conductor has any sharp points or edges, the electricity collects there in virtue of its own self-repulsion, and if the density is sufficiently great, may escape to the air.

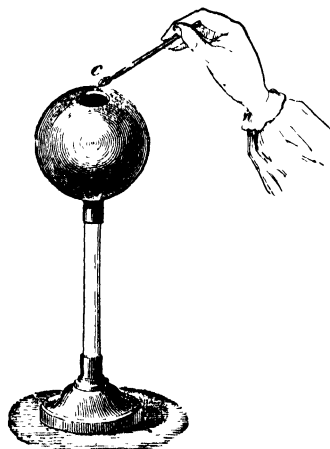


FIG 473.

If the conductor shown in Fig. 473 is charged, and the proof plane is inserted to touch the inside of the conductor, withdrawn very carefully so as to prevent its touching the edge of the hole, and then made to touch the disc of an electroscope, the leaves do not diverge at all. This shows that there is no charge on the inside of the sphere; the electricity resides solely on the outside of a conductor.

Discharging Effect of Points.—The discharging effect of points may be illustrated by a number of experiments. If a needle point is attached to a conductor and the latter charged, the air round the point becomes charged with electricity from the point, and is then repelled by the electricity remaining on the point. This causes a strong current of air, the “electric wind,” and a candle flame held close to the point is deflected by the current away from the point. Instead of the candle flame we may use the “electric whirl,” an arrangement of blades balanced on a pin point which rotates about a vertical axis; the current falls on the blades in succession and causes the arrangement to spin round. Again, if a gold-leaf electroscope is charged and a needle held in the hand close to the disc, the leaves fall together. The needle point becomes electrified by induction, and discharges electricity to the air of sign opposite to that on the electroscope; this passes across to the latter discharging it.

Faraday's Ice-Pail Experiment.—The process of electrification by induction can be investigated very thoroughly by the arrangement shown in Fig. 474, which consists of an insulated, hollow spherical conductor with a small opening in the top, the outer surface of which is in conducting communication with an electroscope. Faraday who performed the experiment first used an ice-pail in place of the spherical conductor; hence the name of the experiment.

The experiment consists of three parts :

(i) A is unelectrified and the leaves hang vertically. The insulated conductor D is given a positive charge and introduced inside A as shown in the figure, moved about, and then withdrawn, care being taken not to touch A during the process. As soon as D is inside A, the leaves diverge, and during the time D is being moved about inside A the divergence of the leaves remains constant; the positive charge on D induces a negative charge on the inside surface of A and a positive charge on the outside surface of A and on the electroscope, and it is the latter which causes the leaves to diverge. When D is withdrawn, they hang vertically once more.

Hence when a charge is almost completely surrounded by a conductor, the magnitude of the induced charge does not vary with the position of the inducing charge inside the conductor.

(ii) A is unelectrified and the leaves hang vertically. D is charged positively and again introduced inside A. The leaves diverge with the induced positive charge. Then D is allowed to touch the inside surface of the conductor. The divergence of the gold leaves remains unaltered. D is then withdrawn, and the divergence of the gold leaves still remains unaltered, but on testing D it is found to be completely discharged. Its charge has exactly neutralised the charge induced on the inside surface of the conductor.

Hence when an inducing charge is almost completely surrounded by a conductor, the charge induced on the inside surface is equal in magnitude to the inducing charge.

(iii) The leaves hang vertically, D is charged positively and introduced as before, and once more the leaves diverge. Then the electroscope is earthed for an instant, and the leaves collapse. D is withdrawn, and immediately the leaves diverge again, to the same extent as before, but it may be shown that this time the sign of their charge is negative. Whenever it is no longer held in

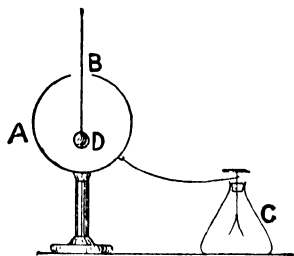


FIG. 474.—Faraday's ice-pail experiment.

position by the attraction of the charge on D, the induced negative charge takes up the position on the outside of the conductor previously occupied by the induced positive charge, and is evidently of the same strength as the latter.

Hence the induced positive and negative charges are exactly equal.

Law of Force between Electrical Charges.—If two charged bodies are small in comparison with the distance between them, we can regard each charge as being situated at the centre of the small body, and it is then found, just as in magnetism, that

The force exerted on one charge by another is proportional to the product of the charges and inversely proportional to the square of the distance between them.

Expressing the law algebraically we have

$$F = \frac{1}{K} \frac{qq'}{d^2}$$

where F denotes the force in dynes, q, q' the magnitudes of the charges, d the distance between them in centimetres, and K a constant of proportionality depending on the medium.

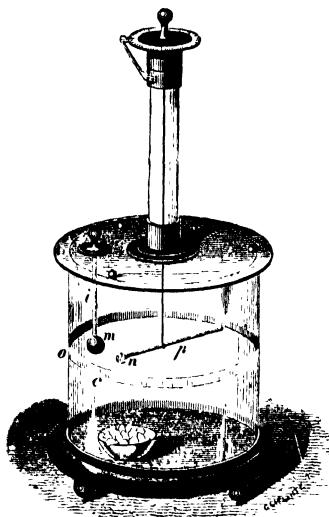


FIG. 475.—Torsion balance.

Torsion Balance.—The law was proved by Coulomb by means of the torsion balance about 1787, and is sometimes referred to as Coulomb's law. Fig. 475 represents the arrangement. There is a glass rod i carrying a gilt pith-ball m , and a fine shellac rod np supported by a thin wire and carrying a disc of copper foil n . oc is a scale fixed round the side of the glass cover, and the air inside the cover is kept dry by a small dish containing calcium chloride. It is arranged by means of the torsion head, that there is no twist on the wire when m and n are in contact. m is then removed, charged, and

placed in position; it touches n , the latter becomes electrified, is repelled, and comes to rest after a few oscillations a certain distance from m under the combined action of the electrical repulsion of m and the torsion of the wire; the couple exerted by the wire is proportional to the angle turned through. By rotating the torsion head more twist is put on the wire and n

brought to rest at half its former distance from m ; then it may be shown by calculation that the force necessary to hold it in equilibrium in this position is four times its former value. If m is touched by another sphere of the same size, its charge is halved, and it is found that half the former amount of twist is necessary to hold n in position.

The law was proved by experiments of the above nature but not accurately, owing to the gradual leaking away of the charges along the insulating supports. Our belief in its accuracy rests on the fact, that it has been made the basis of many theoretical calculations, and that there has never been any reason to doubt the validity of the results obtained.

EXAMPLE.—In a special case Coulomb found that when m touched n , the rod carrying the latter came to rest in a position making an angle of 36° with its initial direction. Through what angle should it be necessary to turn the upper end of the wire to bring n halfway back to its initial position?

The force of repulsion in this position is four times as great, and consequently the one end of the wire should be twisted through $4 \times 36^\circ = 144^\circ$ relatively to the other. But as the lower end turns through 18° in passing to the new position and the wire has already 36° of twist, the upper end of the wire must be turned through an additional $144 - 36 + 18 = 126^\circ$.

Unit Charge.—The definition of unit quantity of electricity is based on Coulomb's law :

Unit quantity of electricity is such, that when placed at a distance of 1 cm. in air from an equal and similar quantity, it repels it with a force of 1 dyne.

In the equation

$$F = \frac{1}{K} \frac{qq'}{d^2}$$

write $q = q' = 1$ and $d = 1$. Then, from the definition, F also $= 1$, and consequently $K = 1$. The unit is thus defined in such a way as to make the constant of proportionality $K = 1$ for air. It has other values for other media.

The unit as defined above is called the **electrostatic unit** in order to distinguish it from another unit of quantity which is used in connection with current electricity.

EXAMPLE.—Two small electrified pith-balls which carry equal charges, are hung by fine silk fibres from a fixed point, and kept apart by their mutual repulsion. The mass of each ball is $\frac{1}{10}$ gm. and the length of each fibre 120 cm. The distance between the balls is 9.9 cm. Find the magnitude of the charges.

Let q = charge on each ball. Then the force of repulsion on each is $q^2/9.9^2$. This is equal to the horizontal component of the stretching

force in each fibre or $\frac{1}{2}g \sin \theta$ dynes, where θ is the angle which the fibre makes with the vertical. We have

$$\sin \theta = \frac{9 \cdot 9}{2 \times 120} \quad \text{Hence} \quad \frac{q^2}{9 \cdot 9^2} = \frac{9 \cdot 9}{40 \times 2 \times 120}$$

which gives $q = 10$ units.

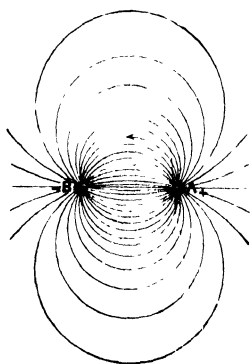
Intensity of Electric Field.—The space round an electric charge or system of charges, throughout which it exerts a force of attraction or repulsion, is called its electric field. The strength of the field is defined in the same way as for a magnetic field :

The electric field at a point is said to have unit intensity or field strength, if it acts on unit positive charge placed at the point with a force of one dyne, the presence of the unit positive charge being supposed to produce no disturbance in the field.

Thus to take the case of a point distant 5 cm. from a charge of 20 units concentrated at a point, the field strength would be $20/5^2 = 0 \cdot 8$ c.g.s. unit. If the field strength is F , a charge of q units placed at the point would be acted on by a force Fq ; it is the product Fq , not F' , which is measured in dynes.

The direction of the field is that in which unit positive charge would move at the point.

Lines of Force.—An electric field can be represented in direction and magnitude by lines of force in the same way as a magnetic



Lines of electric force.

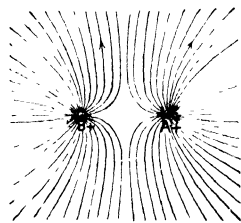


FIG. 477.—Like charges.

FIG. 476.—Unlike charges.

field, but owing to experimental difficulties cannot be mapped so easily as a magnetic field can. Also the convention adopted as to the number of lines drawn to represent the strength of the field is different from in magnetism. In magnetism $4\pi m$ lines of force were assumed to radiate out equally in all directions from a positive pole of strength m ; consequently, if a sphere of radius r were drawn with the pole as centre, the number of lines inter-

secting the surface per unit area would be $4\pi m/4\pi r^2 = m/r^2$, which is equal to the field strength. In electrostatics only q lines of force are supposed to start from a positive charge of strength q , and consequently the number intersecting unit area of the sphere of radius r is $q/4\pi r^2$. To obtain the field strength we must multiply it by 4π . This is the general rule; if N is the number of lines of force per square centimetre, $4\pi N$ is the field strength.

We assume, then, that a line of force starts from each unit positive charge, that a line of force ends at each unit negative charge, and that a line of force cannot end without there being a unit charge at its end. In the case of a charge of strength q , however, the other ends of the lines of force may be so far away, that the effect of the charges situated there may be neglected, and we can treat the charge q as if it existed by itself. Here we have another difference from magnetism, for in magnetism the positive pole is always accompanied by a negative one.

As in magnetism the lines of force tend to shorten themselves like stretched elastic cords, and neighbouring lines repel each other sideways. All the phenomena of attraction and repulsion are explained by means of the properties of the lines of force; thus in Fig. 476 the tension of the lines of force pulls the unlike charges together, and in Fig. 477 the lateral repulsion of the lines of force pushes the like charges apart.

Illustrations of the Action of Lines of Force.—Many electrostatic phenomena are made much clearer by a consideration of the behaviour of the lines of force. Let us take first the earthing of an insulated charged sphere.

Let us suppose that the sphere is earthed by the straight wire AB. Consider the line of force whose position coincided initially with that afterwards occupied by the wire. At first it could not indulge its tendency to contract, for its ends could not leave the sphere and earth; but whenever the latter were joined by the wire, the ends of the line slid along the wire carrying their charges with them, met, and the line of force was annulled.

There was consequently nothing left to support the line CD on the right, and it was pushed by the neighbouring line EF against the wire, and annulled also. And so with the other lines; each in turn pulled a charge along the surface of the sphere on to the wire, where it met and neutralised the charge belonging to the other end.

Let us consider next Faraday's ice-pail experiment. Assume that the sphere which is lowered inside the insulated, uncharged conductor has a positive charge Q . Then Q lines of force radiate out from it equally in all directions. With the exception of a

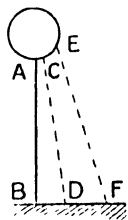


FIG 478.

few which pass out through the hole these meet the inner surface of the conductor. Consequently, the charge on the latter is almost equal to $-Q$, the sum of the charges at the ends of the lines.

Before the charged sphere was lowered into the hollow conductor, the lines of force which started from it terminated on the walls of the room. As the sphere was lowered, each was in turn cut by the edge of the hole, forming two lines, one from the sphere to the inside surface and the other from the outside surface to the walls of the room. The latter started from a unit positive charge on the outer surface of the sphere, and it was the sum of these positive charges which formed the charge induced on the outer surface of the sphere.

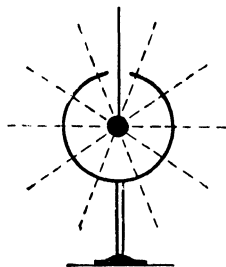


FIG. 479.

Let us consider finally the screening effect of a piece of wire gauze. It is often important to screen an instrument like the electroscope from the effect of charged bodies in its neighbourhood. This can be done by bending wire gauze into a cylinder, earthing it to the gas or water pipe, and standing it round the electroscope with its upper edge about six inches above the surface of the disc. Then the lines of force from charged bodies in the neighbourhood terminate on the gauze and do not reach the disc.

Nature of Electricity.—The **Two-Fluid** theory of electricity maintained that the opposite kinds of electricity were subtle fluids; those of the same kind repelled, and of opposite kind attracted, one another. The union of equal quantities of the two fluids constituted a neutral fluid which was supposed to exist in large quantities in all unelectrified bodies.

The **One-Fluid** theory was originally suggested by Benjamin Franklin, who first used the terms positive and negative to denote the two kinds of electricity. According to this theory, there was only one electric fluid and it was possessed by unelectrified bodies in a normal amount; an excess of the fluid constituted a positive charge and a deficiency a negative charge on the body. The particles of the fluid repelled each other and attracted ordinary matter.

Both these theories could explain all phenomena known up to the end of last century. It was then established experimentally beyond all possible doubt, that atoms contained as constituents small bodies charged with negative electricity; these are known as **electrons**. All electrons have the same mass, about $\frac{1}{1836}$ of the mass of the hydrogen atom, and also the same charge, which is known as the **elementary electrical charge**; according to a very accurate determination made by Millikan, the value of this charge

is 4.774×10^{-10} electrostatic unit. Since the atoms are electrically neutral, it follows that if electrons are withdrawn from an atom, it is left with a positive charge. According to the modern view, when a conductor is electrified by induction, for example, electrons are separated out from the atoms at the one end and move to the other end of the conductor, creating a deficiency of negative electricity at the one end and an excess at the other. The modern view may therefore be regarded as a development of the old one-fluid theory with the rôles of positive and negative electricity interchanged, and the unknown fluid replaced by a cloud of charged particles, about which we have full and accurate information.

But the language of the two-fluid theory possesses a certain convenience for describing phenomena, and hence we still speak of the positive electricity within a conductor being repelled to the one end and the negative being attracted to the other end, though we know that the positive charges remain stationary.

The atom contains a number of electrons varying from one in the case of hydrogen up to 92 in the case of uranium. This number is known as the atomic number. Less is known about the positive electricity than about the electrons. According to the theory of the atom put forward by J. J. Thomson, the positive electricity was distributed uniformly throughout a sphere inside which the electrons moved about under the action of the inverse square law. This theory has now been replaced by Rutherford's view according to which the positive electricity is concentrated in a nucleus round which the electrons circle.

EXAMPLES IV

1. A piece of shellac is rubbed with flannel and used to touch a pith-ball, which is suspended by a silk thread. A stick of sealing-wax which has also been electrified by rubbing it with flannel, is then made to approach the pith-ball, and the latter is repelled. What is the sign of the charge upon the shellac ?

2. A positive charge is given to a gold-leaf electroscope, and the leaves diverge. A charged body is made to approach the disc ; the leaves approach each other and hang down straight, but when the body is brought closer still without touching the disc, they diverge again. State the sign of the charge on the body, and explain the behaviour of the leaves.

3. If you are given a positively charged insulated sphere, how could you charge two other spheres, one positively and the other negatively, without diminishing the charge on the first sphere ?

4. What is the distance between two small spheres which have charges of 16 and 36 units respectively, and repel each other with a force of 4 dynes ?

5. Two small spheres are insulated, charged with 5 and 10 units of positive electricity respectively, and placed 12 cm. apart. Find the force of repulsion between them in grams weight.

CHAPTER V

POTENTIAL AND CAPACITY

Potential.—Fig. 480 represents a pair of discharging tongs; the distance between the two knobs can be altered by changing the inclination of the arms. The handle is an insulating one of glass.

Now, let us suppose that two insulated conductors are charged with positive electricity, and then put into communication by means of the discharging tongs, that is, one knob is made to touch the one conductor and the other knob the other conductor. In general electricity flows through the tongs from the one conductor



FIG 480.—Discharging tongs.

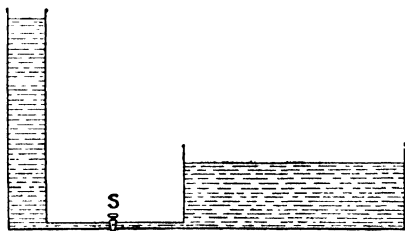


FIG. 481.

to the other, and the question arises as to the direction in which this flow takes place.

This experiment is analogous to two experiments in other branches of physics. Fig. 481 represents a section of two cylindrical vessels containing water connected by a tube, which is closed by a stopcock S. When the stopcock is opened, water flows through the tube from the one vessel to the other. The water flows from the vessel in which the surface has the higher level, even though that vessel may contain less water than the other one.

Again, if a penny is heated red hot and placed on the top of a large mass of copper at room temperature, the latter, owing to its larger thermal capacity, may contain more heat than the penny, but nevertheless the flow of heat is from the penny to the large mass, because the penny is at the higher temperature.

Let us return now to the electrical problem. When the two conductors are put into communication we attribute the direction of flow to a condition of the electricity called its potential, which bears very much the same relation to quantity of electricity as pressure bears to quantity of water, or temperature bears to quantity of heat. The electricity flows from the conductor which is at the higher potential to the conductor which is at the lower potential, even though the latter may contain a larger quantity of electricity. The flow continues until the difference of potential between the two conductors is annulled. It follows that all parts of a conductor in which the electricity is at rest, must be at the same potential.

So far we have assumed that the conductors are charged with positive electricity. The analogy is complicated a little by the fact that we have no negative heat or negative water corresponding to negative electricity. If the conductors are charged with negative electricity, since loss of negative electricity is equivalent to gain of positive, the conductor which loses negative electricity is at the lower potential. If the conductors have charges of different sign, after being connected the sign of the charge on the two will be the same as the sign of the greater charge, while the total charge possessed will be equal to the difference of the original charges. For example, if originally the conductors have charges of $+100$ and -60 units, after being connected they possess a charge of $+40$ units. In this case the conductor with the charge of $+100$ units is originally at the higher potential. To sum up:

Two conductors are said to be at different potentials if, when they are put in conducting communication, the distribution of electricity changes. The body on which the positive electricity decreases is said to be at the higher potential.

Zero of Potential.—Just as measurements of temperature are made with reference to some standard such as the melting point of ice, and as measurements of level are made with reference to sea-level, so measurements of potential are made with reference to the potential of the earth. The earth is at the same potential throughout, at least those parts of it which are large in extent and conduct well, for example, the sea, or the gas and water pipes; its potential is taken as zero. The potential of a positive charge, of glass rubbed by silk, is above the potential of the earth; the potential of a negative charge, of sealing-wax rubbed with flannel, is below the potential of the earth.

Measurement of Potential.—*The difference of potential between two points is measured by the work which must be done to move unit positive charge from the one point to the other.*

If the work in moving unit positive charge from A to B is done against the electrical forces of the field, B is said to be at a higher potential than A ; if it is done by the forces of the field, B is at a lower potential than A.

Let us suppose that the two points A and B are a distance d apart, and that the electrical field is uniform. Let F be the component of its intensity along the line BA. Then the force on unit positive charge is F , and the work done in moving unit positive charge from A to B is Fd . Hence

$$V_B - V_A = Fd$$

and

$$F = \frac{V_B - V_A}{d}$$

where V_A and V_B denote respectively the values of the potential at A and B. The work done in moving q positive units from A to B would be q times as great.

The unit of difference of potential used in current electricity is called the volt. One electrostatic unit of difference of potential is equal to 300 volts.

EXAMPLE.—How much work is done in carrying a charge of 5 electrostatic units from the earth to a point where the potential is one volt or $\frac{1}{300}$ electrostatic unit above the earth ?

$$\text{Answer} = 5 \times \frac{1}{300} = \frac{1}{60} \text{ erg.}$$

Equipotential Surfaces.—An equipotential surface is a surface drawn to pass through all the points in the field which have the same potential. It follows from the definition of potential, that electricity will not flow along an equipotential surface, and conversely a surface on which the electricity is at rest is an equipotential surface. Thus the surface of an insulated conductor is one. Since electricity will not flow along an equipotential surface, the field intensity has no component along the surface. Hence :

The lines of force cut an equipotential surface everywhere at right angles.

Since the lines of force of the external field terminate on the surface of a closed conductor, there is no force inside it, and no work is done in moving a charge about inside it. Thus the whole interior is at the same potential as its surface.

Potential due to a Charged Spherical Conductor.—Let us suppose we have a spherical conductor of radius a , having a charge Q , and at a great distance from all other conductors.

Then the charge Q is distributed uniformly over its surface, and the lines of force radiate symmetrically from it in all directions. The number of lines of force radiating from it is Q . If we now consider a concentric sphere of radius r which is greater than a , the number of lines of force cutting unit area of this sphere is $Q/4\pi r^2$. But, as has been mentioned on p. 483, the field strength $= 4\pi \times \text{number of lines of force per unit area} = 4\pi \times Q/4\pi r^2 = Q/r^2$. Thus the sphere acts at external points, as if its whole charge were concentrated at its centre.

Suppose now that unit positive charge is moved from P_1 to P_2 (Fig. 482), where P_1 and P_2 are very close together. The force of repulsion on it is

$$\frac{Q}{OP_1^2} \text{ at } P_1 \quad \text{and} \quad \frac{Q}{OP_2^2} \text{ at } P_2$$

The average value lies between these two quantities, being greater than the first and less than the second.

If the force changed uniformly, the arithmetic mean would be the correct average to take; the force, however, increases more rapidly at P_2 than at P_1 , and it is found that the average is represented better by the geometric mean,

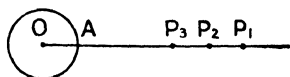


FIG. 482.

$$\frac{Q}{OP_1 OP_2}$$

The work done against the forces of the field in moving unit charge from P_1 to P_2 is consequently

$$\begin{aligned} \frac{Q}{OP_1 OP_2} P_2 P_1 &= \frac{Q(OP_1 - OP_2)}{OP_1 OP_2} \\ &= Q \left(\frac{1}{OP_2} - \frac{1}{OP_1} \right) \end{aligned}$$

In the same way the work done from P_2 to P_3 is

$$Q \left(\frac{1}{OP_3} - \frac{1}{OP_2} \right)$$

and similar results are obtained for all the steps up to A. If all these expressions are added together, the terms cancel out with the exception of the two end ones, and

$$V_A - V_{P_1} = Q \left(\frac{1}{OA} - \frac{1}{OP_1} \right)$$

Now let OP_1 become infinitely large; then $1/OP_1$ becomes zero. Since the lines of force diverge out equally from the sphere in all directions, it must be at an infinite distance from the earth, and

V_P the potential at infinity is equal to the potential of the earth, that is to zero. Hence the potential of the sphere is given by

$$V = \frac{Q}{a}$$

Capacity.—The quantity of heat given to a body is obtained by multiplying its increase of temperature and its thermal capacity. If we have two bodies in contact, and an additional quantity of heat is given to them, it is distributed between them in proportion to their thermal capacities.

Just in the same way all conductors have electrical capacities. The quantity of electricity given to a conductor is equal to the product of its increase of potential and its capacity, potential as before being analogous to temperature. If initially the conductor is uncharged,

$$Q = CV$$

where Q is the quantity of electricity, V the potential to which Q raises it, and C the capacity. Hence

$$C = \frac{Q}{V}$$

The capacity of a conductor is the ratio of the charge on it to the potential to which this charge raises it.

For a spherical conductor at a great distance from other bodies

$$C = \frac{Q}{V} = \frac{Q}{Q/a} = a$$

Thus the capacity of a sphere is equal to its radius. Capacities are consequently measured in cm. on the electrostatic system of units. In dealing with current electricity this unit is too small, and another unit, the **microfarad**, is used. One microfarad $= 9 \times 10^5$ cm. Hence the capacity of a sphere of 10 cm. radius is

$$\frac{10}{9 \times 10^5} = 1.11 \times 10^{-5} \text{ microfarad.}$$

There is one important difference between thermal capacity and electrical capacity; the electrical capacity of a conductor depends not only on its own size and shape, but also on its position with regard to other conductors.

Condensers.—The larger a conductor, the greater its capacity, provided that its shape is unchanged. But there is another method of increasing the capacity of a conductor, better than increasing its size; this method utilises the principle of the condenser.

It is represented in Fig. 483. A is a fixed insulated metal plate, connected to an electroscope, B a similar plate which is

earthed and faces A, and the distance of which from A can be altered.

Let us suppose first, that the plate B is at a great distance from A, that A is given a positive charge, and that the leaves of the electroscope diverge widely. If B is brought up to A, the leaves gradually fall together. The closer the plates are brought together, the closer the leaves approach one another. On withdrawing the earthed plate to its original position the leaves regain their initial divergence, showing that the charge on A has not altered during the experiment.

The positive charge is distributed between the plate A and the electroscope in the ratio of their capacities. When the leaves fall together, the electroscope loses its charge to the plate; consequently, the capacity of the latter has been increased.

When the plate B is brought close up to A, a negative charge is induced on it equal and opposite to the charge on A. The attraction of this negative charge counteracts the self-repulsion of the positive charge on A, keeps it together, and enables A to hold more positive electricity than it otherwise could.

To look at the matter from yet another standpoint, the potential of A is the work done in taking unit positive charge from earth to A. When B is brought close to A, the distance the unit charge is carried is very much less. Thus for the same charge Q on A the work done, and consequently the potential, is less, and hence the capacity of A is increased.

An arrangement by which the capacity of a conductor is increased, as in Fig. 483, by bringing near it another conductor with a charge on it of the opposite sign, is called a condenser. The capacity of the one plate when the other plate is earthed is called the capacity of the condenser.

Leyden Jar. Variable Air Condenser.—Figs. 484 and 485 depict respectively two well-known types of condenser, the Leyden jar and the variable air condenser.

The Leyden jar consists of a glass jar, the inner surface of which is coated with tinfoil halfway up. This tinfoil corresponds to the plate A in Fig. 483. It is in conducting communication with a brass rod surmounted by a knob, which is fixed in the insulating cover of the jar. The outer surface of the glass is also covered with tinfoil halfway up, this coating corresponding to the plate B.

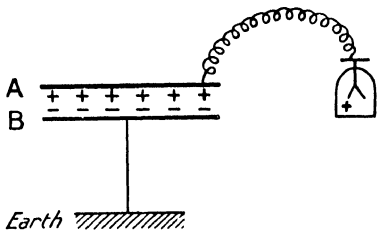


FIG. 483.

The condenser depicted in Fig. 485 consists of a set of fixed, equidistant, semicircular aluminium vanes inside which another set of semicircular vanes moves, the whole being contained between ebonite top and bottom plates and surrounded by a celluloid cover. The one set of vanes corresponds to plate A in Fig. 483, the other set to plate B. The pointer on the top gives a reading proportional to the angle of overlap of the two sets of vanes; the capacity is approximately proportionate to this angle.

The small condensers used in wireless receiving sets consist of sheets of tinfoil separated by sheets of mica or paraffined paper; the latter is cheaper than mica, but is not such a satisfactory insulator as mica.

Capacity of a Spherical Condenser.—The condenser is represented in Fig. 486 and consists of an inner insulated sphere of

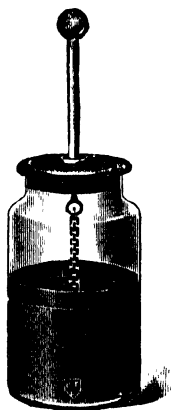


FIG. 484.—Leyden jar

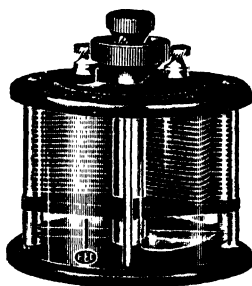


FIG. 485.—Variable air condenser.

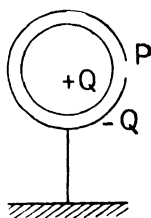


FIG. 486.

radius r_1 and a concentric, earthed, outer sphere of radius r_2 . There is a small hole in the outer sphere at P to allow of the inner sphere being charged and discharged, and the medium between the two spheres is air.

Let Q be the charge on the inner sphere. Except for a slight disturbing effect due to the hole this charge distributes itself uniformly over the surface, and induces a charge $-Q$ on the inner surface of the outer sphere. The line of force from each unit charge on the inner sphere goes radially across to the outer sphere, and terminates there in a unit charge of the opposite sign. If a unit positive charge is brought from infinity to the surface of the inner sphere, the work done against the repulsion of the charge Q on the inner sphere is Q/r_1 . The work done by the charge $-Q$ on the outer sphere in bringing it up to the edge of the

hole is $-Q/r_2$; once it is inside, no further work is done on it by this charge, for the interior of the sphere is an equipotential space, so far as the charge $-Q$ is concerned. The whole work done against the forces of the field is

$$\frac{Q}{r_1} - \frac{Q}{r_2} = Q \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

This is the potential of the inner sphere, and by definition the capacity of the condenser is

$$V = \frac{Q}{Q \left(\frac{1}{r_1} - \frac{1}{r_2} \right)} = \frac{r_1 r_2}{r_2 - r_1}$$

Capacity of a Plate Condenser.—The capacity of the spherical condenser per unit area of the surface of the inner sphere is

$$\frac{r_1 r_2}{r_2 - r_1} \frac{1}{4\pi r_1^2}$$

Write $r_1 = r$, $r_2 = r + d$. Then the expression becomes

$$\frac{r(r+d)}{4\pi r^2 d}$$

Let us suppose now, that r becomes very large in comparison with d , so that the d in the numerator may be neglected. The expression simplifies to

$$\frac{1}{4\pi d}$$

The curvature is now so small that, provided too great an area is not taken, the surfaces may be regarded as plane. Consequently, the capacity of a condenser consisting of two parallel plates of area A situated a distance d apart in air is

$$\frac{A}{4\pi d}$$

Specific Inductive Capacity.—Let us suppose that in the case of the arrangement shown in Fig. 483 the plates are kept at a fixed distance apart, and that the insulated one is charged. Then introduce a thick slab of ebonite, glass, or other insulating material between them. The leaves at once fall together, showing that the capacity of the condenser is increased. The capacity therefore depends on the medium between the plates. This fact is expressed by saying that different dielectrics, as the media between the plates are called, have different specific inductive capacities.

The specific inductive capacity of a substance is the ratio of the

capacity of a condenser when this substance is used as dielectric to its capacity with air as dielectric.

The following table gives the specific inductive capacity, or dielectric constant as it is now more frequently called, of some common substances :

Ebonite	. . .	2.8	Mica.	. . .	7
Glass	. . .	5-10	Benzene	. . .	2.3
Sulphur	. . .	4	Petroleum	. . .	2.1

The capacity of a condenser formed by two plates of area A distant d apart and separated by a medium of specific inductive capacity K is

$$\frac{KA}{4\pi d}$$

In the case of such a condenser the lines of force go straight across from the one plate to the other, as shown in Fig. 487, except in the case of the end lines which are forced out by the lateral pressure of the others. Hence



FIG. 487.

the field intensity between the plates is constant except at the ends, and if Q is the total charge on either plate, the potential difference between the plates

$$\frac{Q}{C} = \frac{4\pi dQ}{KA} = Fd$$

where F is the field strength. Consequently

$$F = \frac{4\pi Q}{KA}$$

thus the force exerted on unit charge in a medium is inversely proportional to the specific inductive capacity of the medium.

Unit charge was defined on p. 481 as one which repelled an equal charge at a distance of 1 cm. with a force of 1 dyne, both being in air. It follows from the above reasoning, that if the medium is one of specific inductive capacity K , the force of repulsion will be only $1/K$ dyne. Thus by measuring the capacity of condensers we can determine the value of the constant K , which entered into the statement of Coulomb's law on p. 480.

The effect of the medium on the force between two charges can be shown by the experiment represented in Fig. 488. Two pith-balls are suspended by silk threads and charged, and come to rest in the positions A.

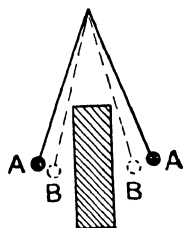


FIG. 488.

If a thick slab of glass is placed between them, they come to rest in the positions B, the force of repulsion having been diminished by the introduction of the glass. When the glass is withdrawn, they return to their original positions.

EXAMPLE.—The diameter of a Leyden jar is 9.3 cm., the height of the cylindrical coating is 23 cm., and the thickness of the glass 0.3 cm. Calculate its capacity on the assumption that the specific inductive capacity of the glass is 6.

Regard the jar as a plate condenser. The area of the plate is the sum of the areas of the side and bottom, *i.e.*

$$9.3\pi \times 23 + \pi \left(\frac{9.3}{2} \right)^2 = 739.8 \text{ cm.}^2$$

Then

$$\begin{aligned} \frac{KA}{4\pi d} &= \frac{6 \times 739.8}{4\pi \times 0.3} = 1178 \text{ cm.} \\ &= 0.00131 \text{ microfarad.} \end{aligned}$$

EXAMPLE.—A variable air condenser consists of 26 fixed semi-circular vanes of 8 cm. diameter and 26 similar movable vanes. The opposite surfaces are 0.09 cm. apart. Calculate the maximum capacity of the condenser.

The system is represented in Fig. 489. The two outside surfaces A and B are uncharged; the electricity spreads itself uniformly over all the other surfaces, so that, with the exception of the outside plates, each plate has a charge on both sides. Let n be the number of plates in each set; then it is clear that the whole system is equal to $2n-1$ separate plate condensers, and that its capacity is given by

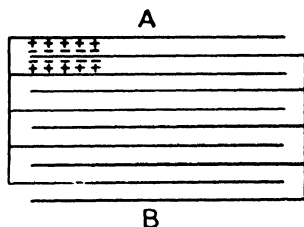


FIG 489.

$$\begin{aligned} \frac{(2n-1)A}{4\pi d} &= \frac{51 \times \pi 4^2}{2 \times 4\pi \times 0.09} = 1133 \text{ cm.} \\ &= 0.00126 \text{ microfarad.} \end{aligned}$$

Condensers in Parallel and in Series.—If the inner coatings of a number of Leyden jars or other condensers are joined together, and the outer coatings are also joined together, the condensers are said to be connected in parallel. In this case the capacity of the resultant condenser is the sum of the capacities of the separate condensers. For if there are three separate condensers, and if the capacities and the charges of each are C_1 , C_2 , C_3 and Q_1 , Q_2 , Q_3 respectively, then

$$V = \frac{Q_1}{C_1} = \frac{Q_2}{C_2} = \frac{Q_3}{C_3} = \frac{Q_1 + Q_2 + Q_3}{C_1 + C_2 + C_3}$$

Hence the resultant capacity $C = C_1 + C_2 + C_3$. The result may be regarded as obvious

If a number of condensers are arranged with the outer coating of one joined to the inner coating of the next, as shown in Fig. 490, they are said to be in series or cascade. In this case the reciprocal of the resultant capacity is equal to the sum of the reciprocals of the separate capacities. For let V_1, V_2, V_3 denote the potentials of the different coatings, as shown in the figure, and let Q be the charge on each coating. Then

$$Q = C_1(V_1 - V_2) = C_2(V_2 - V_3) = C_3V_3$$

and if C denotes the capacity of the resultant condenser,

$$\begin{aligned} \frac{1}{C} &= \frac{V_1}{Q} = \frac{(V_1 - V_2) + (V_2 - V_3) + V_3}{Q} \\ &= \frac{V_1 - V_2}{Q} + \frac{V_2 - V_3}{Q} + \frac{V_3}{Q} \\ &= \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \end{aligned}$$

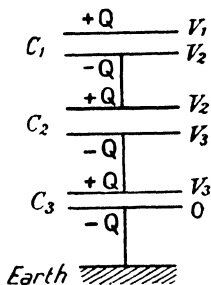


FIG. 490.—Condensers in series.

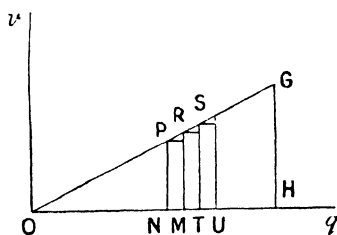


FIG. 491.

The definition of the capacity of a condenser assumed in the above proof is, that it is the charge on either plate divided by the potential difference between the plates. This agrees with the definition given on p. 491, when, as in this case, the charge on the second plate is exactly the same as would be obtained, if that plate were earthed.

Energy of a Charged Conductor.—The potential of a conductor which is at a distance from other bodies is proportional to the charge on it. Let us suppose that a conductor is charged gradually by bringing up small quantities of electricity to it from an infinite distance. Let OH (Fig. 491) represent the final charge Q , and HG the final value of the potential V . Then if ON represents the charge for an intermediate stage, NP represents the potential for the same stage. If a small

additional charge NM is now brought up to the conductor at this stage, the work done is equal to the product of the charge and the potential to which it is raised, *i.e.* to $MN \times NP$ or the area of the rectangle MP. If additional charges MT, TU are brought up, the work done is equal to the areas of the rectangles TR and US. Proceeding in this way it is evident that the whole work done in charging the conductor is equal to the sum of the areas of the rectangles into which $\triangle OGH$ can be divided, *i.e.* in the limit when the steps are made very small, to the area of the triangle itself, or $\frac{1}{2}QV$.

The conductor must have potential energy equal to the work done in charging it. If this potential energy is denoted by E , we have, since $Q=CV$,

$$E = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2} \frac{Q^2}{C}$$

These formulæ also hold for a condenser with one plate earthed, if V is the potential difference between the plates.

The process of charging the conductor in the foregoing proof is analogous to building a brick tower. In the expression for the energy $\frac{1}{2}V$ corresponds to the average gain of potential energy per brick, and Q to the number of bricks used.

EXAMPLE.—Two metal spheres of radii 6 and 12 cm., electrified with 400 and 320 positive units respectively, are placed a long way apart, and then connected by a very fine insulated wire. Find the final charge on each sphere and the energy dissipated as heat in the wire.

$$\text{Final potential} = \frac{Q_1 + Q_2}{C_1 + C_2} = \frac{400 + 320}{6 + 12} = 40.$$

Final charges are respectively 240 and 480 units.

$$\begin{aligned} \text{Heat in wire} &= \frac{1}{2} \frac{Q_1^2}{C_1} + \frac{1}{2} \frac{Q_2^2}{C_2} - \frac{(Q_1 + Q_2)^2}{C_1 + C_2} \\ &= \frac{400^2}{2 \times 6} + \frac{320^2}{2 \times 12} - \frac{720^2}{2 \times 18} \\ &= 3200 \text{ ergs.} \end{aligned}$$

Frictional Machine.—Fig. 492 represents Winter's frictional electrical machine. It consists of a circular glass plate, which is made to revolve rapidly while pressed against a cushion covered with an amalgam of tin and zinc. As a result of the friction against this cushion positive electricity is generated on the surface of the glass. As the electrified part of the glass revolves, it passes between two wooden rings. The inner sides of these rings are each furnished with a row of sharp metal points, which are in conducting communication with the metal sphere supported on

the insulating glass pillar to the left. The metal points come very close to the glass surface; consequently, the positive electricity

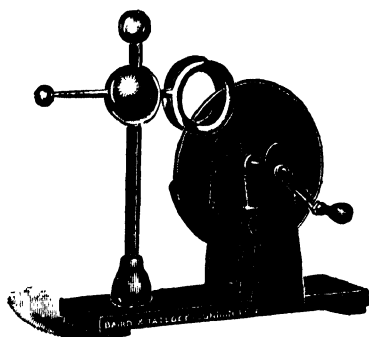


FIG. 492.—Winter's electrical machine.

sparks from the surface of the glass, and collects on the sphere. After the handle has been rotated some time, the charge on the sphere attains a maximum; any gain beyond this is balanced by leakage to the atmosphere and down the glass pillar.

Experiments with a Leyden Jar

Jar.—If a Leyden jar is held with the hand touching its outer coating and the knob close to the spherical collector of the friction machine, or, better still, close to the little

conducting sphere to the left of the latter, sparks of positive electricity pass from the machine to the inner coating of the jar, and at the same time a negative charge is induced from the earth, through the body of the experimenter, to the outer coating of the jar. If the jar is then placed on the table, the outer coating touched with one knob of the discharging tongs, and the other knob of the tongs brought up to the knob of the condenser, when the distance between the two knobs is small, a bright spark passes between them with a sharp report. The charges on the two coatings of the jar then annul one another, and the jar is said to be discharged.

If the jar is charged and placed on a table, and the knob touched by the experimenter, a shock is obtained, which may be a severe one, and possibly dangerous in the case of a large jar, for the inner charge passes to the outer one through the body of the experimenter.

If, however, after charging the jar it is placed on an insulating stool, *i.e.* a little stool with glass legs, or placed on a large block of paraffin wax, which is a very good insulating material, the experimenter may touch the inner coating with impunity. The charge does not move; it is held in position by the attraction of the charge on the outer coating. But if the experimenter touches both coatings at the same time, he receives a shock as before.

When a Leyden jar is discharged, the energy of the jar, that is, part of the work which was done in turning the frictional machine, is dissipated in the heat, light, and sound of the discharge. The question arises as to where this energy was when the jar was charged. The answer to this question is given by an interesting

experiment performed with the particular kind of jar shown in Fig. 493, both the outer and inner coatings of which, C and D, are made of thin metal sheeting, and can be detached from B, the glass itself.

After the jar is charged in the usual way, it is placed on an insulating block. The inner coating is next removed by the hand, or better, by a glass rod; then the glass vessel is removed, and finally the outer coating itself. The two coatings are next put into contact, and little or no spark passes. The jar is then built up again, and its coatings put into communication by means of the discharging tongs, when a spark and report are obtained, almost as strong as if the coatings had not been removed. The energy of the charge is thus apparently stored up in the glass itself.

If a Leyden jar is discharged by momentarily connecting the coatings, a fainter spark may be obtained after a few seconds,

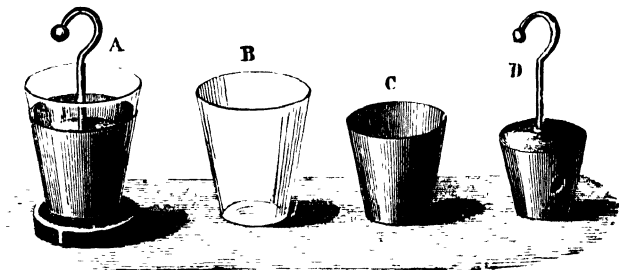


FIG. 493.—Leyden jar with detachable coatings.

showing that the glass has not been able to release all its energy at once.

The Leyden jar was discovered in 1745, and many curious experiments were performed with it about that time. Mussehenbroek, to whom our first knowledge of its properties is principally due, described his chief experiment with it as a terrible one which he would not repeat for the crown of France. The name, Leyden jar, is due to the Abbé Nollet, who some years later formed a ring of Carthusian monks, 1800 yards in length, holding iron wires which connected them in series, and discharged a battery of jars round the ring. The simultaneous jump made by the monks showed that the discharge was instantaneous and of about the same intensity throughout the ring.

Lightning.—Clouds can become charged bodily with electricity. When a disruptive discharge takes place between two clouds or a cloud and the ground, we have lightning. Lightning is thus just a huge electric spark. The reverberation of thunder is the echoing and re-echoing of the original report from the clouds.

Franklin was the first to prove that the clouds in a thunderstorm

are charged with electricity. In 1752 during a thunderstorm near Philadelphia, he flew a kite provided with an iron point which was connected with the hempen string of the kite. The lower end of the string was tied to a key, and the key fastened to a tree by a silk cord in order to insulate the apparatus. On presenting his hand to the key he at first obtained no spark and was beginning to despair of success, when rain fell and the string became a good conductor. Then sparks were drawn in rapid succession. The silk cord was protected from the rain.

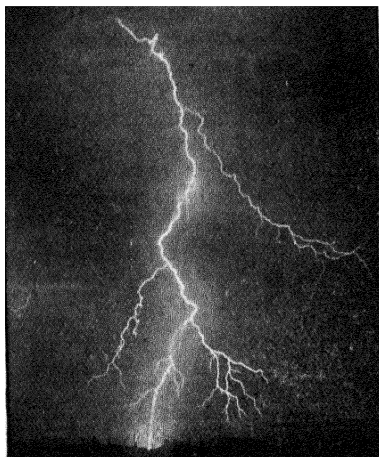


Photo by R. C. Gray

FIG. 404.—A lightning flash.

When a charged cloud is above the building, it induces an opposite charge into the lightning conductor, which escapes silently from the sharp point of the latter and neutralises the cloud; if a discharge does take place, the lightning strikes the rod in preference to the building, and is conducted to the earth without doing any damage.

Fig. 404 is a photograph of a lightning flash. The branches are directed towards the seat of the negative electricity. In taking such a picture the camera must be set for time, and pointed at chance. Ordinary snapshot film will do, but it is better to use backed plates.

Wimshurst Electrical Machine.—Fig. 495 represents the Wimshurst electrical machine, which gives a much more powerful supply of electricity than the frictional machine shown in Fig. 492. It consists of two varnished glass discs on which strips of tinfoil are pasted. The discs are rotated at a high speed in opposite directions by a handle shown at the back of the figure. Two horseshoe conductors provided with rows of points collect positive electricity from the tinfoil strips at the one side, and negative electricity from the tinfoil strips at the other side; the electricity flows through chains to the Leyden jars, where it is stored up. In front of the front plate there is a diametral conductor provided

with brushes at its ends; these brushes graze against the strips of tinfoil. The presence of a similar conductor, which makes contact with the tinfoil on the other plate, is indicated by dotted lines. In front of the instrument are the sparking knobs with

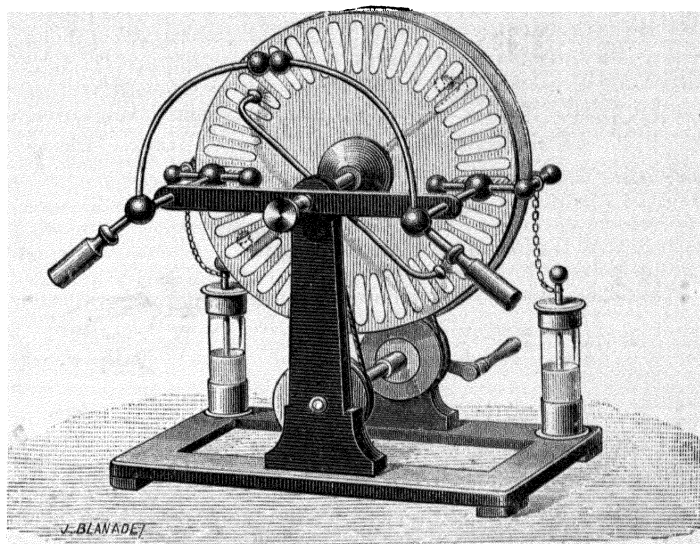


FIG. 495.—Wimshurst electrical machine.

their vulcanite handles for regulating the length of the spark gap. When the machine is working, after the jars are filled, a constant stream of sparks passes between the knobs.

Fig. 496 explains the working of the machine. The front plate is represented by the inner ring, and the back plate by the outer ring. MN and PQ are the two diametral conductors, E and F the combs for collecting the electricity, H and J the Leyden jars, and K the sparking knobs. The directions of rotation of the two discs are shown by the arrows.

Let us suppose that initially the strip A has a negative charge, which may have been induced on it by the neighbourhood of a charged body; then this negative charge induces a

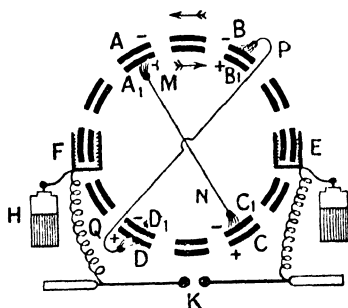


FIG. 496.—Action of Wimshurst machine.

positive charge on A_1 , and a negative charge on C_1 . As the discs rotate the negative charge on A is delivered up at F. When A_1 arrives at B_1 and C_1 at D_1 , they induce a negative charge on B and a positive charge on D; then they themselves give up their charges to the combs at E and F. The positive charge on D is given up to the comb at E. Thus when the machine is working the outer ring receives a negative charge at B and a positive one at D, while the inner ring receives a positive charge at A_1 and a negative one at C_1 , also the machine acts more powerfully as the motion goes on.

Quadrant Electrometer.—The quadrant electrometer is an instrument used for measuring small differences of potential. The original model, which was due to Lord Kelvin, has now been superseded by a simpler form of the instrument due to Dolezalek. The essential features of the instrument are four metal quadrant-shaped boxes mounted on insulating pillars and a light aluminium vane, called the needle, which is

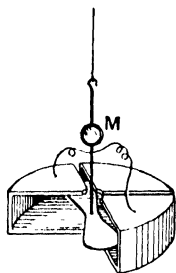


FIG. 497.—Quadrant electrometer with one quadrant removed.

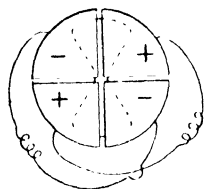


FIG. 498.—Plan of the quadrants.

suspended by a fine wire symmetrically inside the quadrants; they are shown in Fig. 497 with one of the quadrants removed for the sake of clearness. An idea of the shape of the quadrants can be obtained by cutting a pill box into four pieces along two diameters at right angles to one another. Each quadrant is insulated from its neighbours, but connected by a wire to the diagonally opposite one. The needle carries a mirror M, and its rotation is read by the deflection of a beam of light. The whole is enclosed in an earthed metal case in order to screen off external fields.

To use the instrument the needle is charged to a constant high potential, for example, by connecting it to the positive terminal of a battery the other terminal of which is earthed. One pair of quadrants is then connected to one, and the other pair to the other of the sources whose potential difference is to be measured. The needle is repelled by the quadrants at the higher potential and attracted by the quadrants at the lower potential, and rotates until it is brought to rest by the torsion of the wire; the deflection is then proportional to the difference of potential between the quadrants.

EXAMPLES V

1. How much work is done in carrying a charge of 50 units from the earth to a point where the potential is 40 electrostatic units?

2. Calculate the capacity of a parallel plate condenser consisting of two plates each of area 400 cm.^2 situated a distance of 2 mm. apart in air.

3. How many lines of force are there per cm.^2 in the space between the plates of a parallel plate air condenser, given that the area of each plate is 120 cm.^2 , that the charge on each is 500 units, and that they are 2 mm. apart?

4. Three condensers of capacity 0.001, 0.002, and 0.003 microfarad are connected (a) in parallel, (b) in series, (c) with the second and third in series and the first in parallel with them. Calculate the resultant capacity in each case.

5. An insulated metal sphere of radius 20 cm. which is at a distance from other bodies is charged to a potential of 30 units. Calculate the magnitude and the energy of the charge.

6. An air condenser is composed of two circular plates, each 8 cm. in radius, placed 1 mm. apart. The energy required to charge it to a certain potential is 1200 ergs. Assuming the field to be uniform between the plates, calculate the difference of potential and the charge on either plate.

7. The plates of a parallel plate condenser are 2 mm. apart and the potential difference between them is 30 electrostatic units. Without discharging them they are separated to a distance of 3 mm. What does the potential difference now become? How great is the work done in the separation, given that the charge is 3000 units?

8. A Leyden jar of capacity 1000 units is given a charge of 10,000 units, and is allowed to share this charge with an uncharged jar of 500 units capacity by connecting the two in parallel. Calculate the change in the electrical energy of the system, on the assumption that there is no charge lost in the process.

9. What is the capacity of a condenser made of sheet glass 2 mm. thick with tinfoil coatings each 25 cm. square, if the specific inductive capacity of the glass is 7.0?

10. Two small equal spheres 6 cm. apart in air have charges +20 and -10 units. Calculate the force between them. The spheres are connected for an instant by a wire held by an insulating handle. What is then the force between them? If, now, without altering their distance they are both immersed in paraffin oil of specific capacity 2, what does the force between them become?

11. An air condenser with plates 12 cm. square and 2 mm. apart is charged with 1000 units and plunged into oil of specific inductive capacity 2. Find the loss of energy.

CHAPTER VI

THE ELECTRIC CURRENT

Leclanché Cell.—Fig. 499 represents a Leclanché cell. It consists of a glass jar, inside which there is a porous pot containing a plate of carbon, packed round with a mixture of powdered carbon and manganese dioxide. To the top of the carbon plate is fastened a piece of lead in which a brass binding screw is fixed. The carbon plate is called the positive pole, or positive electrode, or positive terminal of the cell. The negative pole consists of the zinc rod to the left. The cell is filled one-third full with a solution of sal ammoniac or ammonium chloride.

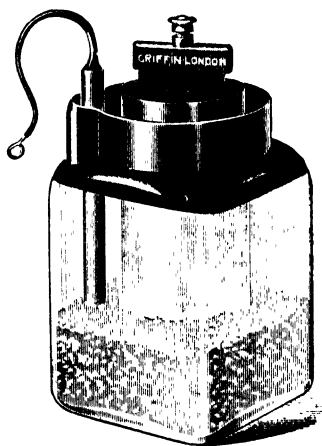


FIG. 499.—The Leclanché cell.

Electric Current.—By means of a Leclanché or other cell the following effects may be produced :

MAGNETIC EFFECT.—Let us suppose that a magnet *ns* (Fig. 500) is pivoted so that it is free to rotate about a vertical axis, and that a straight wire *AB* is set up in a horizontal position in the magnetic meridian close to *ns*. Then if the ends of the wire are connected to the poles of the cell, the magnet is at once deflected, and tends to set

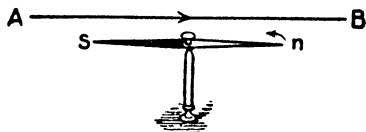


FIG. 500.—Oersted's experiment.

itself at right angles to the wire. This experiment was first performed by Oersted at Copenhagen in 1820. If instead of one cell two cells are taken in series, that is, with the negative pole of the one cell connected by a short piece of wire to the positive pole of the other and with the ends of the wire AB joined to the free poles, then the deflection of the magnetic needle is increased.

HEATING EFFECT.—If the poles of the cell are joined by a short piece of thin iron wire, the latter becomes warm as soon as the poles are joined. The heat may be sufficient in some cases to raise the wire to incandescence, as in the case of the metallic filament lamp.

CHEMICAL EFFECT.—Fig. 501 represents a hydrogen voltmeter. This is a U-shaped tube with stop-cocks at the ends, a thistle funnel at the middle, and short pieces of platinum wire, K and A, sealed through the glass at the corners. It is filled initially to the top of both arms with dilute sulphuric acid. If several cells are arranged in series, and the free positive pole connected to the platinum wire at A, and the free negative pole connected to the platinum wire at K, then as soon as the connections are made, bubbles begin to be given off inside the tube at K and A. These bubbles ascend the arms and collect, the volume of the gas formed in the one branch being exactly twice the volume of gas in the other. The gases are hydrogen and oxygen, the volume of the hydrogen being twice the volume of the oxygen, and the gases are formed by the decomposition of water. When the cells are disconnected, the formation of bubbles stops.

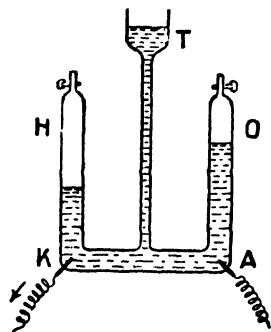


FIG 501.—Hydrogen voltmeter.

Other chemical effects can be produced, if the platinum electrodes are replaced by other metals and the sulphuric acid by other solutions.

To recapitulate : when the wire joins the poles of the battery a magnetic field is produced in its neighbourhood, heat is evolved in it, and if it is cut in two and the ends dipped into acidulated water, chemical change takes place. Thus when the wire is joined to the poles of the battery, it is in a different state from what it was in before connection was made. It is then said to be *conducting an electric current*, and the direction of the current is *from the positive to the negative pole of the battery along the wire*.

The Swimming Man Rule.—The direction of the deflection

of the needle in Oersted's experiment is given by Ampère's rule, which runs as follows :

If a man is swimming in the wire in the direction of the current above the needle, breast stroke, then the north pole is deflected towards his left hand.

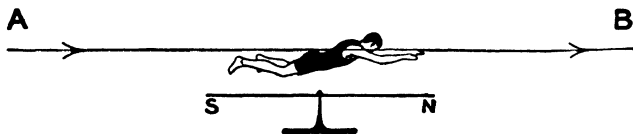


FIG. 502.—Swimming man rule.

If the direction of the current is reversed, the direction of the deflection is reversed. If the wire is stretched below the needle instead of above it, the direction of the deflection is reversed.

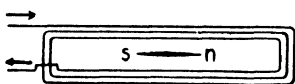


FIG. 503.—Galvanic multiplier.

Hence if the wire is formed into a vertical rectangular coil, as in Fig. 503, and the needle placed at the centre, the effects of the lower sides of the rectangles reinforce the effects of the upper sides. A coil of wire arranged

like this used to be termed a galvanic multiplier or galvanoscope.

Magnetic Field of Electric Current.—Since a wire carrying a current deflects a compass needle just as a magnet does, it has a magnetic field just as a magnet has, and this magnetic field can be plotted in the same way as the magnetic field of a bar magnet, namely, by placing a compass needle on a sheet of paper, making marks opposite its ends, shifting it so that the first end comes opposite the second mark, and so on. The simplest case is when the paper is pinned on a table, and the wire is a long vertical straight one passing through a hole in the table and carrying a very large current. In this case the lines of force are simply a series of concentric circles with their common centre in the point in which the wire intersects the plane of the paper.

The lines of force can also be obtained by spreading iron filings on a plate of glass and tapping the glass, when the filings set themselves along the lines of force under the influence of the current. Fig. 504 represents lines of force obtained in this way; the black mark at the centre of the figure is the wire which is at right angles to the plane of the diagram.

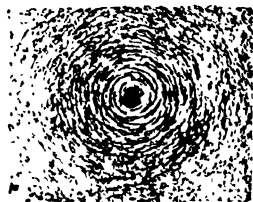


FIG. 504.—Magnetic lines of force due to a straight current.

The strength of the magnetic field at a given point due to a current is proportional to the strength of the current. It also depends on the distance of the point from the current. It can be shown by experiment, that if a wire of length l carrying a constant current is bent into an arc of a circle of radius r , the magnetic force at the centre of the circle is proportional to l and inversely proportional to r^2 . If we denote it by F ,

$$F = \frac{k l C}{r^2}$$

where C is the strength of the current and k is a constant of proportionality. If the wire consists of one complete turn, $l = 2\pi r$, and

$$F = \frac{2\pi k C}{r}$$

Unit Current.—The above result is used to define unit current :

Unit current is such, that when it flows in an arc 1 cm. long of a circle of radius 1 cm., it acts on a unit magnetic pole placed at the centre of the circle with a force of 1 dyne.

According to the definition, if $C=1$, $l=1$, and $r=1$, $F=1$. Hence $k=1$, and the expression for the force becomes

$$F = \frac{lC}{r^2}$$

The unit of current as defined above is known as the theoretical **electromagnetic unit of current** ; it is found to be rather large for practical use, so in practical work a unit one-tenth as large, called the **ampere**, is used instead. Other units of current are the **milli-ampere** and the **micro-ampere**, which are equal to 10^{-3} ampere and 10^{-6} ampere respectively.

Corresponding to the two principal units of current we have two electromagnetic units of quantity, the theoretical unit which is the quantity passing any point in a wire in one second when the theoretical unit of current is flowing in it, and the practical unit, called the **coulomb**, which is the quantity passing any point in a wire in one second, when a current of one ampere is flowing in it.

The coulomb is equal to 3×10^9 electrostatic units of quantity, as defined on p. 481.

Unit Electromotive Force.—*Unit electromotive force is said to exist between two points in a wire, when work equal to one erg is done on unit quantity of electricity in moving it from the one point to the other point in the wire.*

In this definition the unit quantity is the theoretical unit, and the unit of electromotive force thus defined is the theoretical unit. The latter is, however, found to be too small for practical

use, and instead of it a practical unit called the **volt** and equal to 10^8 theoretical units is employed.

It will be observed that electromotive force in current electricity is the same thing as difference of potential in electrostatics. The only difference between the definitions is, that in the one case the charge is moved along a wire and in the other case it is carried on a small body from the one point to the other.

One electrostatic unit of difference of potential = 300 volts.

Connection between Frictional and Current Electricity.—

When a Leyden jar is discharged, electricity flows from the one coating to the other through the discharging tongs. In the same way when a Wimshurst machine is working, if the knobs on both sides of the spark gap are joined by a wire, electricity passes from the one knob to the other along the wire instead of passing in sparks. The question arises as to what constitutes the difference between the passage of electricity in these cases and the current produced by joining the terminals of a Leclanché cell by means of a wire.

Exactly the same thing takes place in each case, only the numerical values of the quantities involved are different. If we represent difference of potential by difference of level, the electric current produced by the cell can be compared to a broad river flowing slowly and steadily with a fall of level of only a few feet per mile, while the current produced by joining the terminals of the Wimshurst machine is like a mountain torrent in which there is not much water, but the gradient of which may be 45° . The current produced by discharging the Leyden jar has the additional difference, that it lasts only for a very short time.

As has been mentioned on p. 484, atoms contain as constituents small bodies charged with negative electricity known as electrons. When a current flows along a wire, the atoms remain stationary, but the electrons move, from the point at the lower to the point at the higher potential. If the electric current is to be compared to a current of water, to be strictly accurate it is the negative fluid that should move; it has, however, become customary to consider the current as passing the other way and as consisting of a motion of positive electricity, and it is not desirable to attempt to change the convention.

The maximum difference of potential of which the Leclanché cell is capable is about 1.5 volts. The smallest difference of potential that will affect an ordinary gold-leaf electroscope is about 100 volts. A rod of glass rubbed by silk may easily be excited up to some thousands of volts. The potential difference necessary to make a spark pass in air between two metal balls 1 cm. in diameter is 4800 volts when the surfaces are 1 mm. apart, and 26,700 volts when the surfaces are 1 cm. apart.

Theory of Tangent Galvanometer.—The tangent galvanometer is theoretically and historically the most important instrument for measuring electric current, although it finds no commercial use nowadays.

Let us suppose that a current C measured in theoretical units is flowing round a wire wound in the form of a circular coil of radius r and n turns, and that the plane of the coil is vertical and in the magnetic meridian. Let us suppose also, that a compass needle or small magnet of length $2l$ and pole strength m is placed at the centre of the coil, and pivoted on an agate point, so that it is free to rotate about a vertical axis. Let Fig. 505 represent a section of the coil by a horizontal plane through its centre; NS is the magnetic needle, TF the direction of the magnetic meridian—*i.e.* of the horizontal component of the earth's magnetic field—and T and F are the points in which the coil intersects the plane of the diagram.

When a current passes round the coil, forces are exerted on each of the poles in agreement with Oersted's experiment. Each of these forces is equal to $2\pi nCm/r$, a part $2\pi Cm/r$ being due to each of the turns of the coil, and they are represented in direction and magnitude by the lines NA and SB.

NA and SB constitute a couple tending to turn the magnet out of the plane of the meridian. Let the angle QON through which the magnet has already been deflected be denoted by θ . Then $QP=2QO=2ON \cos \theta=2l \cos \theta$, and the magnitude of the deflecting couple is

$$\frac{2\pi nCmQP}{r} = \frac{2\pi nCm2l \cos \theta}{r} = \frac{2\pi nCM \cos \theta}{r}$$

where M denotes the magnetic moment of the magnet.

But it has been shown on p. 453 that there is a restoring couple due to the earth's field equal to

$$HM \sin \theta$$

which tends to bring the magnet back into the meridian. Now as θ increases, the restoring couple increases and the deflecting couple diminishes, until, when θ is 90° , it has become zero. Consequently, there is always a position in which they balance one another; this position is given by

$$\frac{2\pi nCM \cos \theta}{r} = HM \sin \theta$$

which becomes

$$C = \frac{r}{2\pi n} H \tan \theta$$

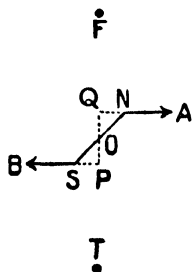


FIG. 505.

Thus the current is proportional to the tangent of the deflection; hence the name of the instrument.

The fraction $2\pi n/r$ depends only on the construction of the galvanometer and is known as the **galvanometer constant**. If it be denoted by G , the formula for the current becomes

$$C = \frac{H}{G} \tan \theta$$

Tangent Galvanometer.—Fig. 506 represents a tangent galvanometer. A long thin pointer is attached at right angles to the compass needle, and its ends move over a scale graduated in degrees. The vertical circle

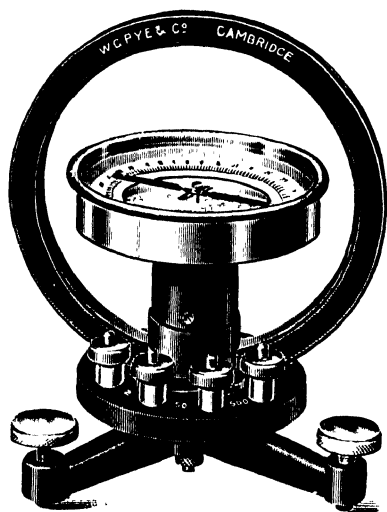


FIG. 506.—Tangent galvanometer.

carries the coils, of which in the case of the instrument depicted there are three, one of 2 turns of thick wire, one of 50 turns of thinner wire, and one of 500 turns of thinner wire still. The terminals of the coils are visible on the front of the instrument. Which coil should be used depends on the strength of the current to be measured; it is obvious from the formula that if C is small, in order to get a large deflection n should be as large as possible.

Before using the galvanometer it must first be levelled. Then the dial must be turned until the 90° marks lie in the plane of the coil. The galvanometer itself must next be rotated until the ends of the pointer are at the 0° marks on the dial; the coil is then in the magnetic meridian. This is an important adjustment, and must be verified by sending a current through the galvanometer and reversing its direction; the magnitude of the deflection should then be the same both ways. If it is not, the adjustment should be repeated until it is.

The fraction by which the tangent of the deflection must be multiplied to give the current in amperes is called the **reduction factor** of the galvanometer. If it is denoted by k we have

$$C \text{ (in amperes)} = k \tan \theta$$

Hence

$$k = \frac{10r}{2\pi n} H$$

Sine Galvanometer.—If the coil of a tangent galvanometer is mounted, so that it can be rotated about a vertical axis, and the angle through which it is rotated can be read off on a scale, the current can be measured in a manner different from that described on the preceding page. The coil is first set in the magnetic meridian and the current passed. The coil is then rotated until the needle lies again in its plane and θ , the angle of rotation, read. The restoring couple is $HM \sin \theta$ as before, but the deflecting forces are now at right angles to the needle, so the deflecting couple is

$$\frac{2\pi nCM}{r}$$

On equating the two couples we obtain

$$C = \frac{r}{2\pi n} H \sin \theta$$

Thus the current is proportional to the sine of the deflection. When used in this manner the instrument is called a sine galvanometer.

Thomson's Mirror Galvanometer.—The tangent galvanometer shown in Fig. 506 is not very sensitive. If we make the pointer longer in order to read smaller angles, it becomes liable to bend and heavy to turn, so nothing is gained by proceeding in this direction. The formula for the current is

$$C = \frac{H}{G} \tan \theta$$

where G is $2\pi n/r$. If the instrument is to be sensitive, that is, give large values of θ and $\tan \theta$ for small values of C , H/G must be small. Now H/G varies with r and is quite independent of the size of the magnet which does not enter into the formula at all. Consequently we shall increase the sensitiveness by making r and the magnet small.

Guided by these considerations Thomson, afterwards Lord Kelvin, introduced a type of tangent galvanometer of which there have been many different forms. The needle of this instrument is very short, is rigidly fixed to the back of a small light concave mirror, and is suspended by a silk fibre at the centre of a vertical coil of very small diameter. A horizontal scale is placed in front of the mirror at about a distance of one metre from it, and behind this scale is placed a lamp which throws a beam of light upon the mirror. The mirror reflects it back to form an image upon the scale. When the magnet is rotated, the reflected beam is rotated and the image moves along the scale, its displacement being proportional to the angle of rotation of the magnet. The beam of light thus forms a long, massless, unbendable pointer.

A control magnet is provided which can be fixed at different heights above the coil, and can be turned in different directions.

When this is low down, its field overpowers the earth's field, and by rotating it the zero of the spot of light can be brought to any desired position on the scale.

The control magnet can also be set in such a position as to almost completely neutralise the earth's field. The restoring couple then becomes very small, a small current produces a large deflection, and the sensitiveness of the instrument becomes very great. When adjusting for maximum sensitiveness in this manner, the observer notes the period of oscillation of the image on the scale; the greater the period, the weaker the field intensity, and the greater the sensitiveness of the instrument.

The control magnet has also another use. One defect of the tangent galvanometer is that the earth's field may vary during the course of the measurements owing perhaps to a current in a lighting circuit below the table or to some similar cause. This would make the readings unreliable. But by bringing the control magnet close down, such accidental variations can be swamped and consistency obtained at, of course, the expense of sensitiveness.

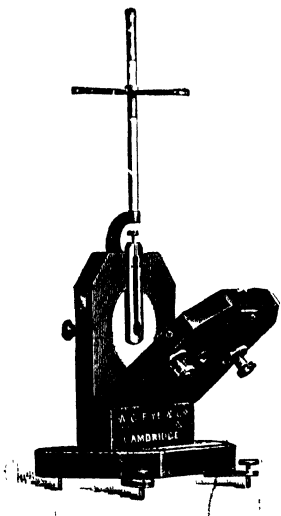


FIG. 507.—Thomson galvanometer.

Fig. 507 represents one type of Thomson mirror galvanometer. There are two coils, one of which can be folded down to give access to the mirror and magnet; the control magnet slides on the vertical rod. The **sensitiveness or sensitivity of this instrument** varies from 7 to 70 cm. per micro-amp. according to the number of turns in the coil, when the scale is at a distance of one metre from the mirror. The scale can be read to $\frac{1}{2}$ mm., so the lower value means that a current of $\frac{1}{20} \times \frac{1}{7}$ micro-amp. or about 7×10^{-9} amp. could just be detected by the instrument. The silk fibre suspension has the advantage that it eliminates friction and torsion, but the disadvantage that it is easily broken; an instrument with a silk fibre suspension is no longer "fool-proof."

While Thomson's mirror galvanometer obeys the tangent law, the deflections used are generally so small that the tangent can be regarded as proportional to the angle itself.

A distinction is made between zero instruments, that is, instruments which are only required to indicate the presence and direction of a current, and instruments used for measuring the

magnitude of currents. The former do not require to be so accurate as the latter.

Astatic Galvanometer.—Fig. 508 represents two needles of nearly equal magnetic moments M_1 and M_2 fixed together with their axes parallel but pointing in opposite directions, and suspended by a fibre PQ, so that they can rotate about a vertical axis. Such a system has been described on p. 461; the restoring couple on it is $H(M_1 - M_2) \sin \theta$. If, however, each magnet is provided with its own coil, and a current runs round the two coils in opposite directions as shown by the arrows, the magnets are similarly placed with regard to the current though differently placed with regard to the earth's field. Consequently, the deflecting couples reinforce one another giving a total of

$$\frac{2\pi nC(M_1 + M_2) \cos \theta}{r}$$

and the system comes to rest in the position given by

$$C = \frac{r}{2\pi n} \frac{M_1 - M_2}{M_1 + M_2} H \tan \theta$$

As compared with an ordinary tangent galvanometer the sensitiveness has been increased in the ratio $M_1 + M_2$ to $M_1 - M_2$, which can be made

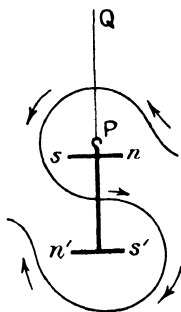


FIG. 508.—Illustrating the astatic galvanometer.

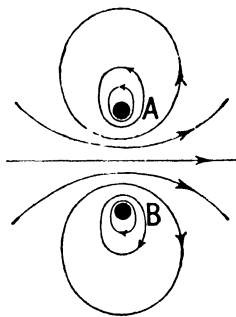


FIG. 509.—Magnetic field due to a circular coil.

very great by making M_2 almost equal to M_1 . Such an arrangement is called an astatic galvanometer.

Solenoid.—Fig. 509 represents the lines of force due to a current flowing in a circular coil, for a plane at right angles to the coil through its centre. A and B are the sections of the coil by the plane of the diagram. It follows from the swimming man rule that the current flows upwards at A and downwards at B. The lines of force are closed curves encircling the current. The diagram represents only the lines of force due to the coil. The field obtained in any actual case will be the resultant of this and the

earth's field, unless the current is so strong as to make the earth's contribution to the resultant negligible.

Fig. 510 represents a piece of tubing upon which insulated wire has been wound, always in the same direction, from the one end to the other. Such a coil of wire is called a solenoid. Fig. 511 represents the lines of force of the solenoid for the case of a plane through its axis. The black discs are sections of the wire by the

plane of the diagram. Each of the turns of the solenoid taken by itself gives a diagram similar to Fig. 509, and these separate diagrams taken together compound into Fig. 511. Each line of force in Fig. 511 encircles every turn of the coil, and inside the solenoid the lines of force are parallel, equidistant, straight lines, showing that the field inside the solenoid is uniform and parallel to the axis. Outside the solenoid the lines of force have the same distribution as for a bar magnet, and if, as viewed from an end of the solenoid, the current goes round the coil in an anti-clockwise direction, that end corresponds to the north pole of the bar magnet. This follows from the swimming man rule.

Electromagnet.—If the solenoid has a large number of turns and carries a large current, the field inside it may be very intense. If a bar of soft iron or of steel is then placed inside the solenoid, it becomes magnetised, just as it would be magnetised by the field of a permanent magnet. When the current is cut off, the soft



FIG. 510.—Solenoid.

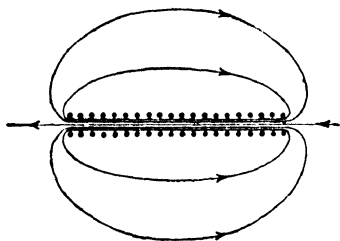


FIG. 511.—Magnetic field due to a solenoid.

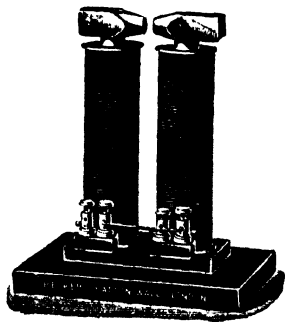


FIG. 512.—Electromagnet.

iron loses its magnetism but the steel retains its magnetism; it is by means of a solenoid that permanent magnets are made.

Fig. 512 represents an electromagnet consisting of cylindrical iron cores bolted to a soft iron base and provided with pole pieces; each core has its own magnetising coil, and the current must, of

course, go round both coils the same way. When the current is cut off, the magnet loses nearly all its magnetism. This may be shown by allowing nails and other pieces of iron to attract themselves to the pole pieces; when the current stops, these all fall down. When the current is reversed, the poles are reversed. An electromagnet gives more intense and uniform fields than an ordinary magnet.

EXAMPLES VI

1. If a penny is lying on the table tails up, and a current is flowing round its edge from Britannia's trident to her helmet, will a north pole placed at the centre of the penny be raised or lowered by the action of the current?

2. Find in amperes the strength of a current which produces a deflection of 45° in a tangent galvanometer with a single turn of wire of 8 cm. radius, given that $H=0.17$ c.g.s. unit.

3. A current of 0.72 amp. flows through a tangent galvanometer consisting of 5 turns of wire each of 30 cm. diameter. What is the field strength at the centre of the coil due to the current, and through what angle is the needle deflected, given that $H=0.17$ c.g.s. unit?

4. Compare the strengths of the fields produced at the centres of a coil of 10 turns and 6 cm. radius and a coil of 20 turns and 9 cm. radius, when the same current is passing through both.

5. What is meant by the constant and the reduction factor of a tangent galvanometer? A tangent galvanometer has a coil of 50 turns each of 10 cm. radius. Calculate the constant and the reduction factor for a place where $H=0.18$ c.g.s. unit.

6. A tangent galvanometer is set up with the plane of its coil at right angles to the meridian, and the period of the needle noted when no current is flowing. A steady current is then passed through the instrument, and the period is increased 40 per cent. How is the period affected when this current is reversed?

CHAPTER VII

OHM'S LAW

Resistance of a Conductor.—If the poles of a cell are connected by a great length of copper wire and then the copper wire is replaced by an equal length of iron wire of the same diameter, it is found that the current in the second case is less than in the first. The same E.M.F. produces a smaller current in the iron wire than in the copper wire. The wires therefore offer a resistance to the current, and the resistance of the iron wire is greater than the resistance of the copper one.

Ohm's Law.—Let us suppose that a cell is connected in series with a long wire and a galvanometer, and that a current is passed. Then replace the cell by two cells in series, each of the same kind as the first. The E.M.F. is consequently doubled, and it is found that the current is doubled. If the cell is replaced by three cells, the current is trebled. The result holds not only for a complete circuit but also for part of a circuit; if the potential difference between two points on a wire is measured by a quadrant electrometer, and if the current through the wire is measured by a galvanometer, it is found that the current and the E.M.F. increase in the same ratio. It is customary to regard the E.M.F. as the cause of the current.

The results of these and similar experiments are summed up in the following law announced by Ohm in 1826 :

For a given conductor the ratio of the potential difference between its ends to the current flowing in it is constant as long as the physical state of the conductor remains the same.

Thus if E is the potential difference between the ends of a conductor and C the current flowing in it, and if there are no changes of state such as change of temperature, then by Ohm's law

$$\frac{E}{C} = \text{a constant, } R.$$

The equation may also be written $E = CR$. R is defined to be the resistance or electrical resistance of the conductor.

Unit Resistance.—A conductor is said to have unit resistance when unit E.M.F. or unit potential difference produces unit current in it. The unit of resistance on the practical system is called the **ohm**; a conductor has a resistance of one ohm when a potential difference of one volt between its ends produces a current of one ampere. There is also a theoretical unit of resistance corresponding to the theoretical units of potential difference and current.

If, for example, the E.M.F. of a cell is 1.45 volts and it produces a current of 0.112 amp., the total resistance of the circuit is $1.45/0.112 = 12.9$ ohms. The E.M.F. is 1.45×10^8 in theoretical units and the current is 0.0112 in theoretical units. Hence the resistance is $1.45 \times 10^8 / 0.0112 = 12.9 \times 10^9$ theoretical units. Consequently, the ohm is equal to 10^9 theoretical electromagnetic units of resistance.

It is found as the result of experiment, that the resistance of a wire varies directly as its length and inversely as the area of its cross-section. This result may be put in the form of an equation

$$R = \frac{kl}{a}$$

where l is the length of the wire, a the area of its cross-section, and k a factor depending on the nature of the material of which the wire is made. k is defined as the **specific resistance** of the material of the wire and is measured in ohm-cm. In the case of metals it increases with the temperature. The specific resistance can also be defined as the resistance of a cube of the substance of 1 cm. edge to a current flowing through it parallel to one of the edges.

The reciprocal of the resistance of a wire is called its **conductance**, and the reciprocal of the specific resistance of a material is called its **specific conductivity**.

Illustration of Ohm's Law.—The flow of electricity along a wire is often compared to the flow of water along a tube. Ohm's law can be illustrated by fitting a piece of capillary tubing into the side of a vessel containing water. The tube is horizontal. The pressure at the end inside the vessel is atmospheric pressure plus the pressure due to the height of the water surface above the level of the tube. The pressure at the other end is atmospheric. Hence the difference of pressure at the two ends of the tube, which corresponds to potential difference, is proportional to the height of the water surface above the tube. The quantity of water which flows through the tube per second corresponds to the current. It will be found, if the experiment is performed, that the pressure difference is proportional to the quantity of water per second, in analogy with Ohm's law, but only for capillary tubes. If the tube is wider, the law does not hold.

Resistance of Three Wires in Series.—When three wires MN, NP, and PQ are connected as shown in Fig. 513, they are said to be arranged in series. Let their resistances be respectively R_1 , R_2 , and R_3 , and let the potential differences between their ends be respectively E_1 , E_2 , and E_3 . The same current must flow in each of them, otherwise there would be an accumulation of electricity at the junctions. Denote it by C . By Ohm's law

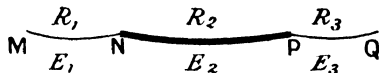


FIG. 513.—Three wires in series.

$$R_1 = \frac{E_1}{C}, \quad R_2 = \frac{E_2}{C}, \quad R_3 = \frac{E_3}{C}$$

Then by definition R , the resistance of the whole system, is given by

$$R = \frac{E_1 + E_2 + E_3}{C} = \frac{E_1}{C} + \frac{E_2}{C} + \frac{E_3}{C} \\ = R_1 + R_2 + R_3$$

The case of any other number of wires can be treated similarly. Thus

The resistance of a number of conductors in series is equal to the sum of their separate resistances.

Resistance of Three Wires in Parallel.—When three wires MPN, MQN, and MSN are connected as shown in Fig. 514, they are said to be arranged in parallel or in multiple arc. Let the resistances of the wires and the currents in them be denoted by R_1 , R_2 , R_3 , and C_1 , C_2 , C_3 , respectively. Let C be the resultant current flowing in at M and out at N; then

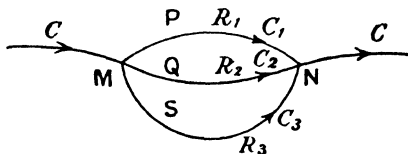


FIG. 514.—Three wires in parallel.

$$C = C_1 + C_2 + C_3$$

Let E be the difference of potential between M and N. Then

$$C_1 = \frac{E}{R_1}, \quad C_2 = \frac{E}{R_2}, \quad C_3 = \frac{E}{R_3}$$

Consequently $C = \frac{E}{R_1} + \frac{E}{R_2} + \frac{E}{R_3}$

Let R be the equivalent resistance of the system, that is, the resistance of the single wire which maintains C and E unaltered when substituted for the three wires. Then by definition

$$C = \frac{E}{R}$$

Thus, substituting above,
$$\frac{E}{R} = \frac{E}{R_1} + \frac{E}{R_2} + \frac{E}{R_3}$$

and
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

The case of any number of wires can be treated similarly. Thus :

The reciprocal of the equivalent resistance of a system of conductors in parallel is equal to the sum of the reciprocals of the separate resistances.

Shunts.—Fig. 515 represents a current which divides at the point A, part flowing through a galvanometer the resistance of which together with its connecting wires is G , and part flowing through a wire of resistance S , to unite again at B. Let E be the potential difference between A and B, let C_G be the part of the current flowing through the galvanometer, let C_S be the other part, and let C be the whole current. By Ohm's law

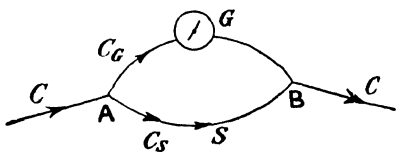


FIG. 515.—A shunted galvanometer

$E = C_G G = C_S S$
Hence
$$\frac{C_S}{C_G} = \frac{G}{S}$$

that is, the currents in the conductors are inversely proportional to their resistances.

$$C = C_G + C_S = C_G + \frac{G}{S} C_G = \frac{G+S}{S} C_G$$

Hence
$$C_G = \frac{S}{G+S} C$$

Let us suppose now that $S = \frac{1}{10} G$. Then $C_G = \frac{1}{11} C$, that is, only one-tenth of the original current passes through the galvanometer. If S had been equal to $\frac{1}{100} G$, only one-hundredth of the original current would have passed through the galvanometer. In these cases the galvanometer is said to be shunted by the resistance S , and S in its turn is said to be a shunt across the terminals of the galvanometer. The effect of shunting a galvanometer is to increase the range of the instrument and make it available for measuring large currents. Suppose, for example, its resistance is 1 ohm and it reads currents up to $\frac{1}{10}$ ampere; by employing shunts of $\frac{1}{9}$ ohm and $\frac{1}{99}$ ohm the range can be increased to 1 ampere and 10 amperes respectively.

Ohm's Law applied to a Complete Circuit.—Fig. 516 represents a circuit composed of a cell of internal resistance B , a coil of resistance R , and a galvanometer of resistance G . Let

E be the electromotive force of the cell. Then by Ohm's law the current in the circuit is given by

$$C = \frac{E}{B + R + G}$$

$R + G$ is referred to as the external resistance in the circuit. By Ohm's law the E.M.F. between the terminals of the cell is

$$(R + G)C = \frac{E(R + G)}{B + R + G}$$

and this is less than E , unless the resistance of the cell can be neglected in comparison with $R + G$.

E is said to be the E.M.F. of the cell "on open circuit," *i.e.* when no current is flowing. It is the difference in potential between its terminals that would be read in this case by a quadrant electrometer or similar instrument. The method of proving Ohm's law by substituting two and three cells for the original cell in the circuit, which was described on p. 516, assumes that

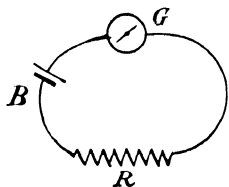


FIG. 516.

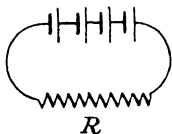


FIG. 517.—Cells in series

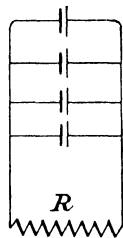


FIG 518.—Cells in parallel

the resistance of the cells is small in comparison with the other resistance in the circuit.

Cells in Series and in Parallel.—Let us suppose that we have n cells of electromotive force E and internal resistance B , and that they are connected in series with an external resistance R . Fig. 517 represents the arrangement; the positive terminal of the one cell is connected to the negative terminal of the next. The E.M.F. of the battery is the sum of the E.M.F.'s of the separate cells, and the resistance of the battery is the sum of the separate resistances. Thus by Ohm's law the current is given by

$$C = \frac{nE}{R + nB}$$

If, on the other hand, the n cells are arranged in parallel as shown in Fig 518, with their positive terminals connected together and their negative terminals connected together, then the E.M.F. of the battery is simply equal to that of one of the single cells, and

the equivalent resistance of the battery is B/n , according to the ordinary formula for the resistance of conductors in parallel. Hence the current in the circuit is given by

$$C = \frac{E}{R + \frac{B}{n}} = \frac{nE}{nR + B}$$

Arrangement of Cells to produce Maximum Current.—Which of the two arrangements, series or parallel, sends the larger current through a given external resistance, obviously depends on which of the denominators of the above expressions is greater. If $R + \frac{B}{n}$ is greater than $nR + B$, *i.e.* if B is greater than R , a larger current will be obtained by connecting the cells in parallel.

Let us suppose that instead of n cells we have n rows each containing m cells, the cells in each row being connected in series, and that all the n rows are connected in parallel, and joined to an external resistance, R . The E.M.F. of a row is mE and its resistance mB . The equivalent resistance of the n rows is mB/n . Hence the current is given by

$$C = \frac{mE}{R + \frac{mB}{n}} = \frac{mnE}{nR + mB}$$

Let us suppose that mn , the number of cells, is given, but that m and n are variable, *i.e.* the manner in which the cells can be arranged in rows is at our disposal. Then the current is largest when the denominator is a minimum. Now

$$nR + mB = (\sqrt{nR} - \sqrt{mB})^2 + 2\sqrt{mnRB}$$

The last term is constant. Consequently, the current is largest when the square is a minimum. But the square cannot be negative. Hence C is a maximum when

$$(\sqrt{nR} - \sqrt{mB})^2 = 0$$

$$\text{i.e. when } nR = mB \quad \text{or} \quad R = \frac{mB}{n}$$

But mB/n is the equivalent internal resistance of the battery. Hence the current in the circuit is a maximum when the cells are arranged in such a way, that their equivalent internal resistance is as nearly equal as possible to the external resistance in the circuit.

Kirchhoff's Laws.—Kirchhoff's laws are two rules for solving problems on branching networks of wires in which currents are flowing. They give no information beyond what is already to be found in this chapter, but they state the fact that electricity does not accumulate at the junctions and enunciate Ohm's law in a form very convenient for application. They are :

(1) In any network of wires the algebraic sum of all the currents which meet at a point is zero, or

$$\Sigma C = 0$$

(2) The electromotive force acting round any closed circuit is found by multiplying the resistance of each portion of the circuit by the current which flows in it, and adding the products thus obtained, or

$$\Sigma E = \Sigma (CR)$$

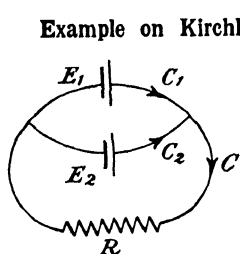


FIG. 519.

Example on Kirchhoff's Laws.—Two batteries of E.M.F.'s E_1 and E_2 and resistances B_1 and B_2 are connected in parallel so as to send currents in the same direction through an external resistance R (Fig. 519). Find the total current in the external resistance.

Let C be the current in the resistance R , C_1 the current in the branch containing E_1 , and C_2 the current in the branch containing E_2 . Then by Kirchhoff's first law

$$C = C_1 + C_2$$

By Kirchhoff's second law

$$E_1 = C_1 B_1 + CR \quad \text{and} \quad E_2 = C_2 B_2 + CR$$

These equations may be written

$$C_1 = \frac{E_1 - CR}{B_1} \quad \text{and} \quad C_2 = \frac{E_2 - CR}{B_2}$$

which give on substitution in the first law

$$C = \frac{E_1 - CR}{B_1} + \frac{E_2 - CR}{B_2}$$

$$C \left(1 + \frac{R}{B_1} + \frac{R}{B_2} \right) = \frac{E_1}{B_1} + \frac{E_2}{B_2}$$

and

$$C = \frac{E_1 B_2 + E_2 B_1}{B_1 B_2 + R(B_1 + B_2)}$$

Work done by a Current.—It follows from the definition of electromotive force, that if Q theoretical electromagnetic units of electricity are raised through a potential difference of E theoretical electromagnetic units, the work done is EQ ergs. Now the coulomb, the practical unit of quantity, is $\frac{1}{10}$ of the theoretical unit and the volt is 10^8 theoretical units. Consequently, the work done in raising one coulomb through one volt or allowing it to fall through one volt is

$$\frac{1}{10} \times 10^8 = 10^7 \text{ ergs.}$$

This quantity is taken as the practical unit of work or energy, and is known as the **joule**.

The joule is the work done when one coulomb is moved through a potential difference of one volt.

Activity or power is the rate of doing work. There is a special unit of activity used in electricity known as the **watt**.

The watt is the rate of working of one joule per second.

The watt as unit is found to be too small for engineering practice, and the **kilowatt** or 1000 watts is used instead. When power is expended in a circuit at the rate of one kilowatt for an hour, the energy consumed is known as the **kilowatt hour** or **Board of Trade unit**. When the price of electricity is stated, for example, as being 6d. per unit for lighting and 1d. per unit for heating, it is the kilowatt hour that is referred to.

EXAMPLE.—A 16-candle-power carbon lamp is run off a 250-volt circuit and takes 0.22 amp. Find how much energy it consumes in an hour.

Rate of working 250×0.22 watts = 0.055 kilowatt.

Energy consumed per hour = 0.055 kilowatt hour.

Joule's Law.—When a battery of electromotive force E maintains a current C in a circuit for time t , a quantity of electricity Ct falls through a potential difference E , and consequently work ECt is done in the circuit. This work can be used for running a motor, if there is one in the circuit, but if the terminals of the battery are simply joined by a wire, it is converted into heat in the wire.

We have

$$ECt = (CR)Ct = C^2Rt$$

Thus the heat produced in the circuit is proportional to the resistance, to the time, and to the square of the current. This result was obtained first by Joule, and is known as Joule's law. When C , R , and t are measured in amperes, ohms, and seconds, the heat evolved is given in joules, the practical unit of work. One joule equals 10^7 ergs and one calorie equals 4.2×10^7 ergs. Hence one calorie equals 4.2 joules, and

$$\text{Heat evolved} = \frac{C^2Rt}{4.2} \text{ calories.}$$

When the terminals of a battery are joined by a wire of small resistance, it may be felt by the fingers to grow warm, and under certain circumstances may become red hot and burn through. It is the function of the fuses included in electric

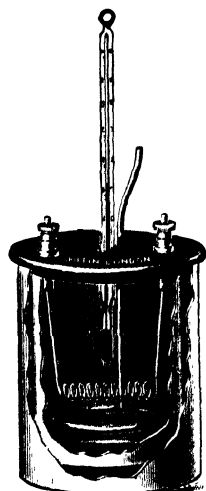


FIG. 520. — Calorimeter for verifying Joule's law.

lighting circuits to burn through and break the circuit, if an excessive current passes. They consist of a wire of low melting point, the diameter and consequently the resistance of which is so chosen, that the wire melts through whenever a particular current is exceeded.

Experimental Verification of Joule's Law.—Fig. 520 represents a calorimeter used for verifying Joule's law. In carrying out the experiment the inner vessel is filled more than half full with water, the temperature noted, and a battery and galvanometer, or other instrument for measuring the current, connected in series with the terminals. After the current has been run for a measured interval of time, it is broken, the temperature again noted, and the amount of heat evolved calculated from the mass of the water, the water equivalent of the calorimeter, and the rise of temperature. It is also calculated by the formula $C^2Rt/4.2$, where C is the current and R is the resistance of the heating coil. The two results should agree.

EXAMPLES VII

1. A 16 candle-power carbon glow lamp takes a current of 0.22 amp. when run off a 250-volt lighting circuit. What is its resistance when in use? (In the case of carbon the resistance decreases as the temperature increases.)

2. Two flash-lamp bulbs are marked 2.5 V 0.3 A and 3.5 V 0.3 A respectively. If the V stands for volts, and the A for amperes, what are the resistances of these two bulbs when lit up?

3. The resistance of a bobbin of wire is measured, and found to be 52.7 ohms; a portion 121 cm. long is cut off, and its resistance is found to be 0.49 ohm. What was originally the total length of wire on the bobbin?

4. One end A of a wire ABC is connected to earth, and the other is maintained at a potential of 1.48 volts by connecting it to the pole of a battery, the other pole of which is earthed. If the resistance of AB is 10 ohms and the resistance of BC 5 ohms, find the current in the wire and the potential of B.

5. The specific resistance of platinum is 11.0×10^{-6} ohm-cm. at 18°C . Calculate the resistance of a platinum wire 1 metre long and $\frac{1}{2}$ mm. diameter at this temperature.

6. The specific resistance of platinum can be represented by the formula

$$R = R_0(1 + \alpha t)$$

where R_0 is the resistance at 0°C ., t the temperature in degrees Centigrade, and $\alpha = 0.0035$. If the resistance of a platinum wire is 100 ohms at 0°C ., what is it at 15°C .?

7. The specific resistance of platinum is 6 times as great as the specific resistance of copper. What is the ratio of the resistance of a platinum wire $\frac{1}{8}$ mm. diameter and 328 cm. long to the resistance of a copper wire 0.42 mm. diameter and 10 metres long?

8. Two wires of resistances 10 and 20 ohms respectively are arranged in parallel. What is their equivalent resistance?

9. Two points are connected by a wire the resistance of which is 10.78 ohms. What resistance must be arranged in parallel with this wire in order to make the resistance of the combination exactly equal to 10 ohms?

10. A galvanometer which has a resistance of 1 ohm is suitable for a range of 0 to $\frac{1}{10}$ amp. What is the resistance of the shunt which must be employed to make it suitable for the range 0 to $\frac{1}{2}$ amp.?

11. Three cells, X, Y, and Z, the E.M.F.'s and resistances of which are as follows:

	X	Y	Z
E.M.F.	1.07	1.50	2.00 volts
Resistance	4	0.4	0.3 ohms

are connected in series through a resistance of 83 ohms. Find the current; find also the current when the cell Z is reversed.

12. If the first two cells described in the preceding example are connected in parallel so as to act in the same direction, find the current which they maintain through the same external resistance.

13. If the three cells described in ex. 11 are connected in parallel so as to act in the same direction, find the current which they maintain through an external resistance of 50 ohms.

14. A current from a battery is flowing through an external circuit which has a resistance 6 times that of the battery. In what ratio does the current increase when the value of the external resistance is halved?

15. A cell, which has an E.M.F. on open circuit of 1.46 volts, and the internal resistance of which is 2 ohms, has its terminals joined by two wires of 4 and 10 ohms placed in parallel. Find the current in each wire.

16. A battery has an E.M.F. of 15 volts and an internal resistance of 5 ohms. Its terminals are connected by a wire, and it is found that, when the circuit is closed, the difference of potential between the ends of the wire is 12 volts. Calculate, in calories per minute, the rate at which heat is being produced in the wire.

17. A wire of 3 ohms resistance is immersed in 60 grams of water in a calorimeter of water equivalent 6 grams, and a current of 4 amps. passed for 5 mins. Find the rise in temperature of the water.

18. A wire of 2.5 ohms resistance is immersed in 55 grams of water in a calorimeter of water equivalent 6 grams, and a current of 0.5 amp. passed for 10 mins. Find the rise in temperature produced.

19. If a Leclanché cell working at 1.46 volts gives a current of 1 amp., what is its rate of expenditure in watts? What is this equivalent to in H.P.? 1 H.P. = 33,000 ft.-lb. per min., 1 ft. = 30.48 cm., and 1 lb. = 453.6 gm.

20. A desk electric fan makes 1400 revolutions per minute, and takes 45 watts. Given that the kilowatt hour costs 1d., what is the expense of running it for one hour?

21. Calculate the resistance of a 250-volt 40-watt lamp.

22. When a current of $\frac{1}{8}$ amp. is passed through a platinum wire its temperature keeps 1° C. above that of surrounding objects. Assuming that the loss of heat is proportional to the difference in temperature, calculate the temperature of the wire when a current of $\frac{1}{5}$ amp. is passed through it. The variation of the resistance of the wire with temperature is to be disregarded.

CHAPTER VIII

MEASUREMENT OF RESISTANCE AND ELECTROMOTIVE FORCE

Resistance Box.—It is sometimes necessary to vary the resistance in an electric circuit. Different types of apparatus are employed for this purpose according to the size of the current used. When the current is small, a resistance box of the type shown in Fig. 521 is employed. The wires are connected to the

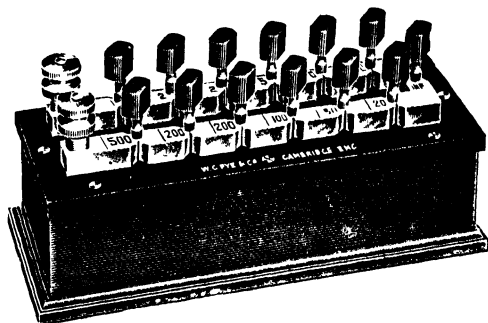


FIG. 521.—A resistance box.

binding screws on the left. The box has a vulcanite top on which are fixed fourteen brass blocks connected by thirteen brass plugs with vulcanite handles. When all the plugs are in position, the current simply flows through the blocks and plugs from the one terminal to the other, and, since the cross-sectional area of the blocks is large, the resistance is practically zero.

But if a plug is out (*cf.* Fig. 522), the current can pass from the one block to the other only through a long insulated wire which is doubled on itself at the middle so as to get rid of induction effects, and is then wound in a coil on a bobbin inside the box. The lengths of the wires are adjusted, so that their resistances are exact numbers of ohms, in the case of

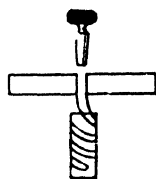


FIG. 522.

the box depicted 1, 2, 2, 5, 10, 20, 20, 50, 100, 200, 200, 500. Thus the box gives any whole number of ohms up to 1110. The end plug gives an infinite resistance, *i.e.* the end blocks are not joined by a coil inside the box.

For larger currents, 1–10 amp., an uncovered wire with a high specific resistance is used, which is wound firmly on a slate cylinder,

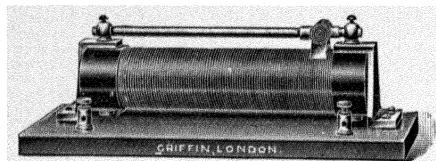


FIG. 523.—Adjustable resistance.

as shown in Fig. 523. The neighbouring turns of the wire do not touch one another. The two terminals shown in front are connected to the ends of the wire. A sliding piece on the top bar makes

contact with the wire on the cylinder. The circuit is connected to the top bar and one of the front terminals; the length of the wire on the cylinder through which the current flows, and consequently the resistance, can then be altered by moving the sliding contact.

For still higher currents the wires are wound on an open frame. A variable resistance for high currents is called a rheostat.

Keys, Commutators.—In carrying out electrical experiments it is often necessary to make or break an electric current, or to reverse the direction of a current through a galvanometer. Special pieces of apparatus are used for this purpose, since it would be inconvenient to be continually connecting and disconnecting the wires.

Figs. 524 and 525 represent respectively a tapping key and a one-way plug switch used for making and breaking a current. In the case of the tapping key the stud is connected below the

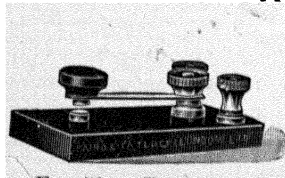


FIG. 524.—Tapping key.

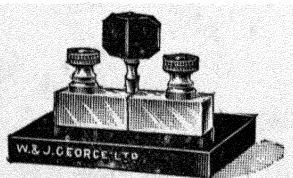


FIG. 525.—One-way plug switch.

wood with the terminal on the right, and contact is made only as long as the key is pressed down; when the finger is withdrawn, the metal strip springs up, and the current is broken. The plug switch is used when the current has to flow for a longer time.

Fig. 526 represents a four-way plug switch which is useful for several purposes. When employed for reversing a current through a galvanometer, the galvanometer is connected to two opposite

terminals and the battery to the other two; then when the plugs are inserted in two opposite holes, the current flows one way, and when they are inserted in the other two holes, it flows the other.

Fig. 527 represents Pohl's commutator, which is used principally for rapidly reversing a current through a circuit. It consists of

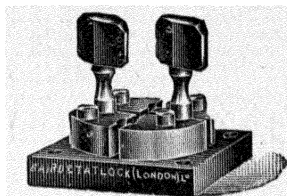


FIG. 526.—Four-way plug switch.

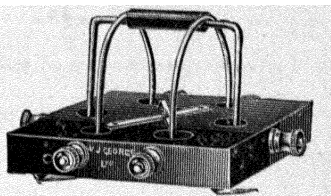


FIG. 527.—Pohl's commutator.

six mercury cups on a vulcanite base, each provided with its own binding screw. One pair is connected by a copper strip, and another pair is also connected by a copper strip which is bent up in order to avoid contact with the first. The wire frame rocks from the position in which the two back legs make contact with the mercury to the position in which the two front legs make contact. The battery is connected to the two front terminals and the circuit to the end terminals; the current is reversed by tipping the rocker over.

Measurement of Resistance by Substitution.—This is the simplest method of measuring an unknown resistance. It is represented in Fig. 528; x is the unknown resistance, R a resistance box, and K a two-way key. The current is first directed through x and the deflection of the galvanometer G noted; it is next directed through R , and the resistance adjusted by taking out plugs until the deflection is again the same. The resistance in the box is then equal to the unknown resistance.

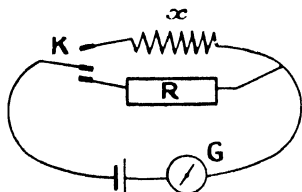


FIG. 528.—Measurement of resistance by substitution

This method requires that the unknown resistance should fall within the range of the box, and that the galvanometer should be an accurate one. Consequently, for general use another method, the method of Wheatstone's bridge, is preferred; this is applicable to a resistance of any magnitude and has also the advantage of being a null method, *i.e.* the apparatus is adjusted, until the deflection of the galvanometer is zero. The galvanometer has merely to indicate the presence of a current, and thus does not require to be so constant in its readings.

Wheatstone's Bridge.—The diamond-shaped system of conductors $AHDF$ shown in Fig. 529, with two opposite corners H and F connected through a galvanometer G and a contact key k , and the other two corners A and D connected through a cell B and a contact key K , is called Wheatstone's bridge.

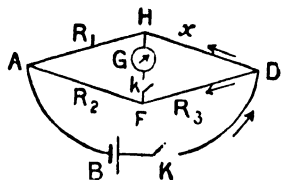


FIG. 529.—Wheatstone's bridge.

Let us suppose that the key K is pressed down. Then a current flows from the battery in the direction indicated by the arrow, and divides at D into two parts, one part of strength C_1 which flows along DH , and the other part of strength C_2 which flows along DF . The two currents C_1 and C_2 unite again at A , and flow back to the battery. Let us suppose that the points H and F are exactly at the same potential; then, if the key k is pressed down, no current will flow from F to H .

Let R_1 , R_2 , R_3 , and x be respectively the resistances of the branches AH , AF , FD , and HD , let E_1 be the potential difference between D and H or D and F , and let E_2 be the potential difference between F and A or H and A . Then, by Ohm's law,

$$E_1 = C_1 x = C_2 R_3$$

and

$$E_2 = C_1 R_1 = C_2 R_2$$

This gives

$$\frac{E_1}{E_2} = \frac{x}{R_1} = \frac{R_3}{R_2}, \quad \text{whence} \quad x = \frac{R_1 R_3}{R_2}$$

Hence the resistance x can be calculated, if either of the adjacent resistances R_1 or R_3 and the ratio of the other two resistances is known.

Wire Bridge.—The wire bridge is a special form of Wheatstone's bridge. One is shown in Fig. 530 and represented in plan

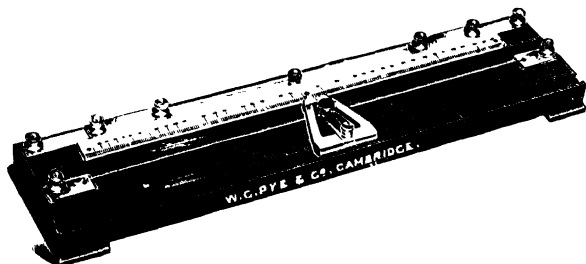


FIG. 530.—The wire bridge.

in Fig. 531. AD is a straight uncovered wire of uniform cross-sectional area and high specific resistance. The thick black lines

are copper strips of negligible resistance; R_1 is a known resistance of constant amount and x an unknown resistance, both attached to the copper strips by binding screws. G is a galvanometer connected to a binding screw at H and to a sliding contact at F . A and D are connected to a battery B and key K .

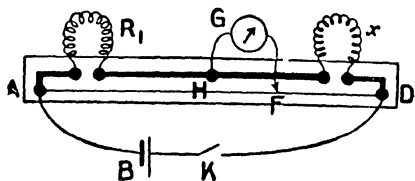


FIG. 531.—Connections for the wire bridge.

Corresponding points in Figs. 529 and 531 have the same letter, so that A , H , D , and F are the corners of the diamond, and the ratio of FD to AF gives the ratio of R_3 to R_2 , since the resistance of the wire is proportional to its length. FD and AF can be read off on the scale. In order to determine the resistance x we have consequently only to press the key K , and move the contact F along the wire until a point is found for which there is no deflection of the galvanometer. Then F and H are at the same potential, and

$$x = R_1 \frac{FD}{AF}$$

Post-Office Box.—The post-office box or P.O. box is another form of Wheatstone's bridge, designed originally for the measurement of the resistance of telegraph wires. In appearance it is like the resistance box shown in Fig. 521 with a third row of resistance coils added. Fig. 532 is a plan of one, lettered in agreement

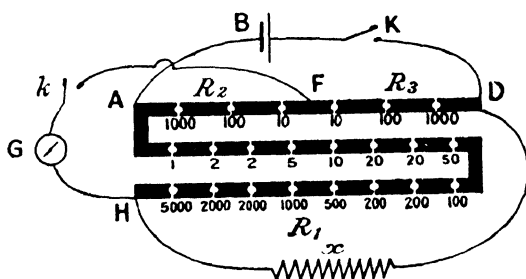


FIG. 532.—The post-office box.

with Fig. 529, on which the positions and values of the resistance coils are indicated. There are terminals at A , F , D , and H for attaching the battery, galvanometer, and unknown resistance. The arms AF and FD are known as the ratio arms; the ratio of the resistances of these arms can be made 1 : 100, 1 : 10, 1 : 1,

10 : 1, and 100 : 1. The remaining arm AH is simply a resistance box giving any whole number of ohms from 1 to 11,110.

Since in the post-office box the resistances are whole numbers, it may not be possible to get an exact balance. For example, 5623 ohms may give a deflection to the left and 5624 ohms to the right. The correct value can then be calculated by the principle of proportional parts.

The key K in the battery branch should be pressed before the key k in the galvanometer branch, because, if k is pressed first, on pressing K a transient current may flow through the galvanometer, and this transient current may be in the opposite direction to the current indicated by the needle when it finally comes to rest.

Resistance of Galvanometer. Kelvin's Method.—In determining the resistance of a galvanometer by this method it is placed instead of the unknown resistance in the arm IID of a Wheatstone's bridge (Fig. 529). The arm HF contains nothing but the key k . The key K is closed and k left open ; then a steady current flows through the galvanometer. If now, when k is closed, there is no change of current through the galvanometer, II and F must be at the same potential. Hence the resistance of the galvanometer is $R_1 R_3 / R_2$, the same expression as was found for the unknown resistance formerly.

Platinum Thermometer.—The mercury thermometer cannot be used much above 300° C., and the air thermometer is awkward to use and really only suited for standardising more handy forms of instrument. Consequently, as the resistance of a wire increases with temperature, and the measurement of a resistance can be carried out with great accuracy, the variation in the resistance of a wire is used as a means of measuring temperature. The wire chosen is a platinum one ; it is wound on a thin mica frame and enclosed for protection in a glass or, for higher temperatures, a porcelain tube, the whole forming what is known as a platinum thermometer. Flexible leads connect the thermometer to a Wheatstone's bridge.

The variation in resistance of a pure metal with temperature is given by the formula

$$R_t = R_0(1 + \alpha t + \beta t^2)$$

where R_t and R_0 are respectively the resistances at t° and 0° C., and α and β are constants. Hence, since the formula contains three unknown quantities, three temperatures are required to calibrate the thermometer. Those usually taken are the melting point of ice, the boiling point of water, and the boiling point of sulphur, 444.6° C. The range of the platinum thermometer extends up to about 1200° C.

For small ranges of temperature the resistance of a wire can be represented quite well by the formula $R_0(1 + \alpha t)$. The resistance of carbon, unlike that of metals, decreases as its temperature rises, the resistance of a carbon filament lamp in use being about half its value when cold. The resistance of the alloy constantan is almost independent of its temperature ; hence constantan is much used for the construction of resistance coils.

Graphical Method of Determining the Constant Resistance in a Circuit.—Let us suppose that a battery of electromotive force E and resistance B , a galvanometer of resistance G , and a resistance box are joined in series. Let R be the resistance unplugged in the box. Then by Ohm's law the current is given by

$$C = \frac{E}{B+G+R}$$

Hence

$$\frac{E}{C} = B+G+R$$

If now the value of C is measured for different values of R and $1/C$ graphed against R (Fig. 533), the observations will lie on a straight line PQ . This straight line intersects the R axis a distance OP beyond the origin. When $1/C = 0$, we have

$$B+G+R=0 \quad \text{or} \quad B+G=-R$$

Hence the numerical value of OP gives $B+G$, the constant resistance in the circuit.

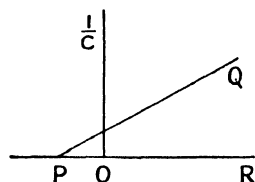


FIG. 533. Diagram for graphical determination of resistance.

It is not necessary that $1/C$ itself should be plotted; any quantity proportional to $1/C$ will do. For example, in the case of a tangent galvanometer it is sufficient to take the cotangent of the deflection. But the method, although a very interesting one, is only useful, when $B+G$ is of the same magnitude as the resistances unplugged in the box.

The only instrument for measuring current described so far has been the tangent galvanometer. Much the greater number of current-measuring instruments at present in use are of another type, the moving coil type, and we shall now proceed to describe the principle on which this type is based.

Force on Straight Conductor in a Uniform Magnetic Field.—Oersted's experiment showed that a wire carrying a

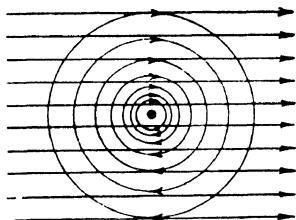


FIG. 534.

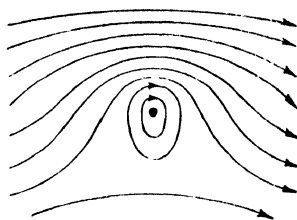


FIG. 535.

Interaction of magnetic fields of current and magnet.

current exerted a force on a magnetic pole in its neighbourhood.

By Newton's third law of motion action and reaction are equal and opposite. Consequently, since the wire exerts a force on the pole, the pole must exert an equal and opposite force on the wire. This force may be ascribed to the joint action of the fields of the current and pole. For example, in Fig. 534 the straight lines represent the lines of force of the magnet and the circles the lines of force of a current flowing downwards into the paper. The resultant of these two fields is shown by the curved lines in Fig. 535. Above the wire the lines of force reinforce one another and their density is great, below the wire they oppose one another and their density is small, and it is clear that their tension and lateral repulsion will force the wire downwards.

The magnitude and direction of the force are given by the following rules :

If a straight wire of length l cm. carries a current of C c.g.s. units and is placed in a uniform magnetic field of strength H c.g.s. units at right angles to the direction of the field, it is acted on by a force of lCH dynes.

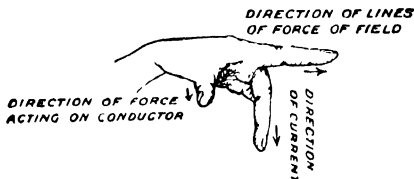


FIG. 536 -- Left-hand rule.

Point the thumb and the first two fingers of the left hand in three directions at right angles to one another ; then, if the first finger gives the direction of the field and the second finger the direction of the current, the thumb gives the direction of the force.

If the current is measured in amperes, the force is $\frac{1}{10}lCH$ dynes.

Verification of Rule.—It was stated in Chapter VI that when a current C flows in a circular coil of one turn and radius r , it exerts a force $2\pi mC/r$ on a magnetic pole of strength m placed at the centre of the coil in a direction at right angles to the plane of the coil. Thus each unit of length of the coil exerts a force mC/r^2 . By Newton's third law this is the reaction of the pole on unit length of the coil, and it will be observed that it agrees with the formula lCH , for in this case H is the field strength at the coil due to the pole, and is consequently equal to m/r^2 .

Couple on Coil in Magnetic Field.—Fig. 537 represents a rectangular coil of wire which is suspended in a uniform magnetic field of strength H , so as to rotate about a vertical axis through its middle point. This vertical axis is shown by a broken line in the figure. A current C flows in the coil in the direction shown by the arrows, and the normal to the plane of the coil makes angle θ with the direction of H . Let l and b be the length and breadth of the coil respectively.

Each of the four sides of the coil is a straight wire carrying a current in a magnetic field, and is hence acted on by a force which can be found by the rules on p. 534. In calculating the force on the top and bottom we take the component of H at right angles to the wire, but the force in this case acts through the midpoint of the wire and has hence no moment about the axis. The force on each of the sides is lCH ; on the side to the left it is forward at right angles to the paper, and on the side to the right backward at right angles to the paper. Thus the forces on the sides constitute a couple tending to rotate the coil about the axis of suspension. The perpendicular distance between their lines of action is $b \sin \theta$, and consequently the moment of the couple is

$$lCHb \sin \theta = ICA \sin \theta$$

where A is the area of the coil. It may be shown that the result holds true for any shape of coil and not merely for a rectangular one.

The couple becomes zero when $\sin \theta = 0$, *i.e.* the equilibrium position of the plane of the coil is at right angles to the direction of the field. The corresponding formula for the couple on a magnet suspended in the field is $HM \sin \theta$. Hence the coil behaves like a magnet of moment CA , which has its axis at right angles to the plane of the coil.

If the solenoid described on p. 514 is suspended in a horizontal position, so that it is free to turn about a vertical axis through its midpoint, it sets itself in the magnetic meridian when a current is passing, for there is a couple on each separate turn of the coil which brings it into a position at right angles to the earth's field. It is also attracted by a magnet, for it sets itself parallel to the field of the latter just as another magnet would. It thus behaves exactly like a suspended bar magnet.

Moving Coil Galvanometer.—In the moving coil galvanometer a small coil of several turns of thin insulated wire is suspended between the poles of a strong horseshoe magnet. One end of the coil is attached to a phosphor bronze strip which acts as a means of suspension, the other end to a flexible spring, and a cylindrical core of soft iron is placed within the coil to concentrate the lines of force as much as possible.

The coil is suspended with its plane parallel to the lines of force of the field. When the current passes, there is a deflecting

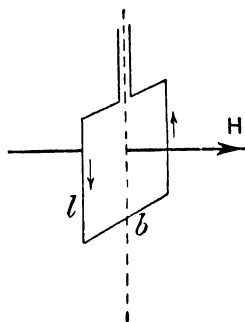


FIG. 537.—Coil in a magnetic field.

couple proportional to the cosine of the angle which the plane of the coil makes with the direction of the field. This is resisted by the torsion of the suspension, which is proportional to the angle turned through, and the coil comes to rest in the position in which these two couples balance. Since the angles used are small, the current is proportional to the deflection itself. The latter is read either by means of a mirror and scale or by the motion of a pointer over a graduated scale.

The advantages of the moving coil instrument over the tangent galvanometer are, that any variation of the earth's field has no disturbing effect, since the coil is already in a very strong field, and that the instrument can be used in any position, and does not require to be set in the meridian. It has the disadvantage that the current cannot be calculated from the dimensions of the instrument as in the case of the tangent galvanometer, and hence the scale has to be calibrated by some other method. But this is a minor consideration, and much the greater number of current-measuring instruments in use are of the moving coil type. If the scale is graduated so as to read directly in amperes, milli-amperes (thousandths of an ampere), or micro-amperes (millionths of an ampere), they are called **ammeters**, **milli-ammeters**, or **micro-ammeters**.

Voltmeter.—By providing a milli-ammeter with a large series resistance it can be used as a voltmeter or instrument for measuring electromotive force, and nearly all the voltmeters in commercial use are of this type. For example, a certain milli-ammeter in wide use has a range of 0–15 milli-amperes and a resistance of 5 ohms. If it is combined with a resistance of 995 ohms, the total potential difference across instrument and resistance when a current of 15 milli-amps. is flowing through it is $0.015 \times (995 + 5) = 15$ volts. Hence it can be used to measure E.M.F.'s up to 15 volts, and a resistance of 995 ohms which fits neatly on to the instrument is supplied for this purpose. In most voltmeters the large resistance is contained inside the instrument.

Of course, when the E.M.F. of a battery is measured in this manner, some current is taken from it, whereas when an electrostatic instrument like the quadrant electrometer is used, no current flows at all. By allowing a current to flow between the terminals whose potential difference is being measured, we diminish the potential difference between these terminals. But, when the resistance of the voltmeter is great, the error introduced in this manner is small, and voltmeters of this type are much more reliable than electrostatic voltmeters.

Action of Currents on Currents.—Since a wire in which a current is flowing possesses a magnetic field, a second wire conveying a current experiences a force when placed in this field.

If the two wires are parallel, the lines of force of the first meet the second at right angles, and it follows from the rules on p. 534, that the force on the second is an attraction towards, or a repulsion away from, the first. If the currents are in the same direction it is an attraction, and if they are in opposite directions it is a repulsion. In the same way the field of the second wire exerts a similar force on the first. So we arrive at the result :

Currents in parallel wires in the same direction attract one another.

Currents in parallel wires in opposite directions repel one another.

The attraction can be shown very neatly by Roget's spiral. This consists of a spiral wire which hangs vertically with its lower end just dipping into a pool of mercury. When a current is sent through it, the adjacent turns attract each other, since they carry parallel currents. The spiral consequently shortens, the lower end rises out of the mercury, and the current is broken. The attractive force being then removed, the spiral lengthens and makes contact again. And so on; the process is constantly repeated.

The attraction or repulsion exerted by a fixed coil on a movable coil in which the same current is flowing is used as a means of measuring current, voltage, and electrical power. The **dynamometer** and **current balance** are instruments of this class.

Ballistic Galvanometer and Comparison of Capacities.—

When current is made, the pointer of a galvanometer does not go directly to its final position. It swings at first too far, and then oscillates about the final position before coming to rest at it, and it is the final position that is observed.

Sometimes a current rises from zero value to a maximum, and then falls again to zero almost instantaneously. In this case it is the total quantity of electricity which passes that is important, and the instrument used for measuring it is called a ballistic galvanometer. Ballistic galvanometers may be either like a tangent galvanometer or of the moving coil type; the essential condition is that they should have a large period, and that the oscillations should not die down rapidly. When a quantity of electricity passes through such a galvanometer, the needle swings from its zero position to one side, and then oscillates about the zero position until it finally comes to rest at it. It is the amplitude of the first throw or swing that is read; this is proportional to the quantity of electricity that passes.

Fig. 538 represents an arrangement by which the ballistic galvanometer may be used to compare the capacities of condensers or the E.M.F.'s of cells. G is the galvanometer, K a spring key, C a condenser of capacity C_1 , and B a battery of electromotive force E. The key initially makes contact on J, so that both

sides of the condenser are connected through the galvanometer. It is then pressed down to make contact on H, and immediately the two sides of the condenser are charged to a potential difference E . The key is then allowed to spring back to its initial position, the condenser discharges, and a quantity of electricity C_1E passes through the galvanometer. The throw of the galvanometer is noted.

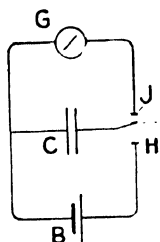


FIG. 538.

If the condenser is replaced by another one of capacity C_2 and the throw noted in this case, the ratio of the throws is equal to the ratio of the capacities, since it is the ratio of C_1E to C_2E . If the battery is replaced by another one of different electromotive force, the same con-

denser being used in each case, the ratio of the throws is equal to the ratio of the E.M.F.'s.

The capacity of a condenser is the ratio of the charge on it to the potential difference between the plates. *When one coulomb of electricity raises the potential difference between the plates to a potential difference of one volt, the capacity is said to be one farad.* This unit is, however, too large for practical purposes, so a unit called the **micro-farad**, which is one-millionth of the farad, is used instead. It has already been referred to on p. 490.

Measurement of Battery Resistance.—Fig. 539 represents an arrangement for determining the resistance of a battery. B is the battery, r its resistance, K a plug key, R the external resistance in the circuit, and V a voltmeter.

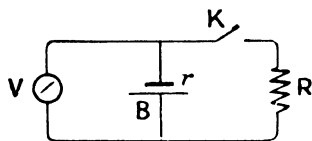


FIG. 539.—Measurement of battery resistance.

The voltmeter is first read with the cell on open circuit, *i.e.* with the key K open. Let the result be E . This is the potential difference between the terminals of the cell when no current is passing. The key is then closed, and we have, by Ohm's law,

$$E = C(R + r)$$

where C is the current in the circuit. The difference of potential between the ends of the resistance R is CR , and this is the quantity now indicated by the voltmeter. Let it be E' . Then

$$E' = CR$$

We have consequently

$$\frac{E}{E'} = \frac{R + r}{R}$$

which gives

$$r = \frac{(E - E')R}{E'}$$

Potentiometer.—The potentiometer is an instrument used for comparing the E.M.F.'s of cells. It is represented in principle in Fig. 540. B is a battery of constant E.M.F. which must be higher than that of any of the cells to be compared; two accumulators in series will be found suitable. DF is a long, straight, uniform wire of high resistance which has its end D connected to the positive pole of the battery. S is one of the cells whose E.M.F. is to be compared. Its positive pole is connected to D and its negative pole through a galvanometer G to a contact key at H. Let its E.M.F. be denoted by E_1 and let the length of DH be l_1 .

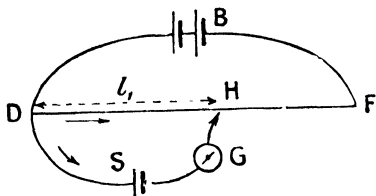


FIG. 540.—Principle of the potentiometer.

Suppose first that there is no contact at H. Then since the wire is uniform, the potential difference between D and H is by Ohm's law proportional to l_1 . If now contact is made at H, the accumulators tend to send a current through the branch circuit DSGH in the direction of the arrow. This is opposed by E_1 , the E.M.F. of the cell S, and if the position of H is such that the potential difference between D and H before contact is exactly equal to E_1 , no current will flow through the galvanometer when the key is depressed. This position of H is found by trial and the length l_1 noted. The cell is then replaced by another one of electromotive force E_2 , and the position of H again found for which there is no current in the galvanometer, when the key is depressed. Let DH now be l_2 . Then

$$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$

If the cell S in the branch circuit has its terminals joined by a resistance and a plug key, and the position of H is found for which there is no current in the galvanometer, both when the plug key is open and closed, the resistance of the cell can be calculated from the results by the equation on the preceding page.

The potentiometer can also be used for measuring a current and calibrating an ammeter. The instrument to be calibrated is placed in the main circuit, for example, between B and D, and a standard cell placed in the branch circuit at S. When balance is obtained, the potential drop between D and H is then known very accurately, and if the resistance of DH is known, the current can be calculated. The method requires a potentiometer in which the portions of the wire, DH, HF, are replaced by resistance boxes, the box which replaces DH being a very accurate one.

EXAMPLES VIII

1. An unknown resistance is measured against a 5-ohm resistance coil by means of a wire bridge. It is found to be greater than the latter, contact having to be made either at the 28.9 or the 70.9 cm. mark on the scale—the wire is 100 cm. long—according to the side of the bridge on which the unknown resistance is placed. What is its value? To what do you attribute the slight difference in the two results?

2. An unknown resistance is measured against a 5-ohm resistance by means of a one-metre bridge. In order to obtain balance contact must be made at 39.7 cm. A 10-ohm resistance is substituted for the 5-ohm resistance, and balance is then obtained at 24.7 cm. What is the value of the unknown resistance?

3. A battery of E.M.F. 1.45 volts and negligible resistance is connected to the opposite corners A and C of a quadrilateral wire frame ABCD. Calculate the potential difference between B and D, given that the resistances of AB, BC, CD, and DA are respectively 40, 30, 80, and 70 ohms.

4. An ammeter is connected in series with a battery and resistance box. When the resistances unplugged in the box are 100, 200, 300, and 400 ohms, the ammeter readings are 99.0, 65.0, 48.3, and 38.6 divisions respectively. Find the other resistance in the circuit as accurately as possible.

5. A current of 3 amp. flows in a vertical wire 30 cm. long. Calculate in dynes the force exerted on the wire by the earth's field, given that the horizontal component of the earth's field is 0.16 c.g.s. unit.

6. A rectangular coil of height 10 cm. and width 8 cm. is suspended in a magnetic field of strength 2.0 units. Calculate the couple acting on it, when its plane makes an angle of 60° with the direction of the field, and a current of 2.0 amp. flows in the wire.

7. A milli-ammeter has a range of 0–15 milli-amp. and a resistance of 5 ohms. What series resistance must be provided to convert it into a voltmeter with a range of (i) 0–75 volts, (ii) 0–150 volts?

8. The E.M.F. of a Daniell's cell is 1.07 volts, and its internal resistance is 4 ohms. Its poles are joined by a wire, the resistance of which is 20 ohms. Calculate the current produced, and the difference of potential between the ends of the wire.

9. A fresh flash-light battery is run for 20 minutes through an external resistance of 8.85 ohms. The current is at first 0.42 amp., but at the end of the test it has fallen to 0.27 amp. The E.M.F., as measured on a voltmeter, was 4.35 volts before, and 3.0 volts after the test. Find the initial and final values of the resistance of the cell.

10. A voltmeter which has a resistance of 2000 ohms is used to measure the E.M.F. of a cell the resistance of which is about one ohm. What is the percentage error caused by neglecting the current through the voltmeter?

CHAPTER IX

ELECTROLYSIS. VOLTAIC CELLS

Two Kinds of Conduction.—When an electric current flows along a wire, there is no motion of molecules or atoms in the direction of the current. If, for example, a copper plate presses against an iron plate, and a current flows across from the copper to the iron, no penetration of the copper molecules into the iron can be detected by the most careful chemical analysis. But when a current passes through a solution of a salt or acid, some of the dissolved substance is carried to the points where the current enters and leaves the solution, and is separated out there. In this case the conduction is said to be **electrolytic** as distinct from metallic, the vessel in which the solution is contained is called an **electrolytic cell**, and the solution itself an **electrolyte**. The decomposition produced by the current is referred to as **electrolysis**. The conductors by which the current enters and leaves the solution are called the **electrodes**, that by which the current enters the **anode** and that by which it leaves the **cathode**. The terminology of the subject is due to Faraday.

All solutions of salts, acids, and bases in water are electrolytes, also fused salts and some solid substances such as glass at a high temperature. Ohm's law and Joule's law hold for electrolytic conduction, provided that the difference of potential is measured between two points inside the liquid. Electrolytic conductors also produce a magnetic field in the same way as metallic conductors do.

Facts of Electrolysis.—Let us study the experiment represented in Fig. 541. Three electrolytic cells are connected in series with a battery. The first one contains a solution of copper sulphate with copper electrodes, the second a solution of silver nitrate with silver electrodes, and the third is the arrangement already described on p. 505, and consists of a dilute solution of sulphuric acid with platinum electrodes. In each cell the

anode is on the left and the cathode on the right, that is, the current flows through the cells from left to right.

It is found that when current passes in the first cell, a certain quantity of copper is deposited on the cathode and the same quantity of copper goes into solution from the anode. When the current passes in the second cell, a certain quantity of silver is

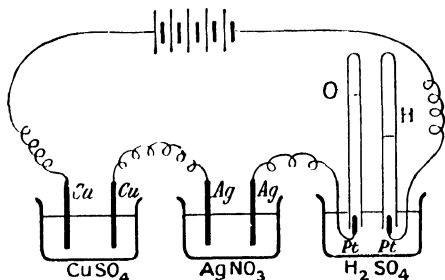


FIG. 541.—Electrolysis.

deposited on the cathode and the same quantity of silver goes into solution from the anode. When the current passes in the third cell, gas is evolved at both electrodes, oxygen at the anode and hydrogen at the cathode, and if the gas is collected in inverted tubes in the usual manner, the volume of the hydrogen is twice

the volume of the oxygen. It is found that the quantities of copper and silver deposited and the quantities of oxygen and hydrogen given off are proportional to the quantity of electricity that passes and independent of the size and shape of the cells and electrodes or of the strength of the solutions. If one coulomb of electricity passes, 0.0003294 gm. of copper, 0.0011183 gm. of silver, 0.00008293 gm. of oxygen, and 0.0001045 gm. of hydrogen are liberated. These quantities are known as the electro-chemical equivalents of the respective substances.

*The mass of a substance deposited by the passage of unit quantity of electricity is called the **electrochemical equivalent** of the substance.* It is usually measured, as above, in grams per coulomb.

Faraday's Laws of Electrolysis.—The facts of electrolysis were summed up by Faraday in the following laws :

1. *The mass of any substance liberated is proportional to the quantity of electricity that passes.*
2. *The mass of any substance liberated by a given quantity of electricity is proportional to the chemical equivalent of the substance.*

It follows from the second law, that the electro-chemical equivalent is proportional to the chemical equivalent. The chemical equivalent of an element is its atomic weight divided by its valency. The atomic weights of copper, silver, oxygen, and hydrogen are 63.57, 107.88, 16.00, and 1.008. Silver and hydrogen are monovalent, oxygen is divalent, and copper is divalent in copper sulphate. Hence the chemical equivalents of copper, silver, oxygen, and hydrogen are respectively 31.78, 107.88, 8.00, and 1.008. Now the electro-chemical equivalent

of these elements have already been stated. Consequently, from the second law we must have

$$\frac{0.0003294}{31.78} = \frac{0.0011183}{107.88} = \frac{0.00008294}{8.00} = \frac{0.00001045}{1.008}$$

and this relation is found to be true.

The electro-chemical equivalent of any substance can be determined from the electro-chemical equivalent of any other substance by an equation of the above type, when their chemical equivalents are known; silver is usually taken as the other substance, as its electro-chemical equivalent has been determined with great accuracy. When an element has two valencies, it has two electro-chemical equivalents, one for each valency. Thus the value for copper given above is for the cupric state; the value for the cuprous state is twice as great.

Faraday's laws of electrolysis can be summed up in the equation $W = zCt$, where W is the mass deposited in grams, z is the electro-chemical equivalent, C the current in amperes, and t the time in seconds.

Mass of the Hydrogen Atom.—Faraday stated the laws of electrolysis in 1833. Writing in the same year he drew the conclusion from the laws that there is a definite electrical charge associated with the atom, since the quantity of electricity passed through the electrolyte may be regarded as possessed by the atoms liberated at the electrodes. In his own words the electro-chemical equivalents of bodies are "simply those quantities of them which contain equal quantities of electricity." We know now that there is an elementary electric charge, the charge on an electron, equal to 4.774×10^{-10} electrostatic unit. Expressed in coulombs this is equal to 1.59×10^{-19} . It follows from the definition of electro-chemical equivalent, that each gram of hydrogen liberated in an electrolytic cell carries a charge of $\frac{1}{0.00001045} = 9.6 \times 10^4$ coulombs.

If each atom carries one elementary electric charge, it follows that there are 6.0×10^{23} atoms in one gram of hydrogen. This is in agreement with determinations of the same quantity made by other methods.

The gram-equivalent of a substance is a quantity equal to its chemical equivalent expressed in grams, *e.g.* the gram-equivalent of silver is 107.9 grams. The mass of any substance liberated by the passage of 95,700 coulombs is equal to its gram-equivalent.

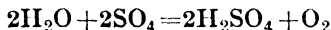
Voltameters.—For the electrolytic cell we have $W = zCt$, where W is the mass of a substance of electro-chemical equivalent z deposited in time t by current C . If W , z , and t are known, C can be calculated. Electrolytic cells which are used in this way for the measurement of current and the standardisation of instruments are called voltameters. The hydrogen voltameter has already been described on p. 505. The silver voltameter is

the most reliable and accurate, though the copper voltameter is much used on account of its cheapness. The silver voltameter consists of a silver rod dipping into a platinum basin which contains silver nitrate. The silver rod acts as anode and silver is deposited on the basin, the quantity deposited being determined by weighing the basin before and after the experiment. The copper voltameter consists of copper electrodes dipping into a solution of copper sulphate.

Electrolytic Dissociation.—Let us now return to the experiment represented in Fig. 541. The most natural explanation of what happens is, that when copper sulphate goes into solution in water, some of the molecules dissociate into two parts called **ions** from the Greek word for wanderer or traveller, a Cu ion consisting of the copper atom with two positive charges, since copper is divalent, and a SO_4 ion consisting of one sulphur and four oxygen atoms, which carries two negative charges. When the circuit is made, the electric field causes the Cu ions to move to the cathode and the SO_4 ions to move towards the anode. Thus the electricity travels by a kind of convection. The ions which move towards the anode are called **anions** and those which move towards the cathode **cathions**. The electric field does not cause the dissociation and make the ions; it merely causes a motion of the ions which are already there. When the Cu ions arrive at the cathode, they give up their charges to it and deposit on it. When the SO_4 ions arrive at the anode, they give up their charges to it; they then attack the anode, forming copper sulphate.

In the same way some of the silver nitrate molecules are dissociated on solution into Ag ions and NO_3 ions. The Ag ions carry each one positive charge, since silver is monovalent, and the NO_3 ions carry each one negative elementary charge. The Ag ions give up their charges to the cathode and are deposited there, while the NO_3 ions give up their charges to the anode and attack it, forming silver nitrate.

In the case of the hydrogen voltameter some of the H_2SO_4 molecules are dissociated into H and SO_4 ions. The former give up their charges to the cathode and then are liberated in the form of a gas; the latter give up their charges to the anode and then attack the water according to the equation



forming sulphuric acid and oxygen, the latter of which is liberated as a gas. This is an example of what is known as a secondary action; such actions often obscure what happens in the cell, for the natural inference on seeing oxygen evolved at the anode would be to regard oxygen as the anion. It should be noted that in the hydrogen voltameter the SO_4 ion is unable to attack the

water until it has given up its charges and ceases to be an ion. The volume of the hydrogen liberated is twice the volume of the oxygen, since it contains twice as many molecules.

In the cases examined above the metals and hydrogen travel with the current. This is a general rule; *the metals and hydrogen always travel with the current*, and are hence called electro-positive, while the acid particles and non-metals travel against the current, and are hence called electro-negative.

Pure water is not an electrolyte and does not conduct electricity. But it is practically impossible to obtain water without traces of some salt or acid dissolved in it.

Applications of Electrolysis.—Electrolysis is used in the arts for electroplating, electrotyping, and the refining of metals.

In electroplating—copper plating, nickel plating, silver plating—the article to be plated is placed in a bath, and forms the negative electrode of an electrolytic cell. The other electrode is of the metal which is to be deposited from the solution; thus in the case of silver plating it is of silver, and the solution consists of 2 parts of silver cyanide and 2 parts of potassium cyanide dissolved in 250 parts of water. When the current passes, silver is deposited on the article to be plated, silver goes into solution from the anode, and the strength of the electrolyte remains constant.

In electrotyping the page is set up in ordinary type, a mould taken in wax, powdered with graphite in order to render it conducting, and then copper plated by electrolysis. The wax is then removed, replaced by a metal backing, and the sheets printed off from the copper-faced plate thus formed, which stands wear much better than ordinary type and is always used for large editions.

In purifying copper, for example, by electrolysis, unrefined copper is used as the anode, and pure copper sulphate used as the electrolyte. As the anode goes into solution, the impurities fall to the bottom of the bath; consequently pure copper is deposited on the cathode. Electrolytic copper is the purest on the market.

Simple Voltaic Cell.—The only cell described hitherto has been the Leclanché cell. We shall now go into the subject of cells in more detail.

Let us suppose that we have a copper plate and a zinc plate dipping into a dilute solution of sulphuric acid. If the difference of potential of the two plates is measured, it is found to be 1.08 volts, the copper plate being at the higher potential. Small bubbles of gas form on the surface of the zinc plate, but these can be eliminated by using chemically pure zinc; no bubbles form on the copper plate.

If, however, the copper plate and zinc plate are joined by a wire, a current flows from the copper plate through the wire to the zinc plate, there is a copious evolution of bubbles at the copper

plate, and the zinc plate wastes away. The arrangement acts as a cell, and can be used for giving current. The H_2SO_4 molecules in solution dissociate into SO_4 and hydrogen ions; the SO_4 attacks the zinc, forming zinc sulphate, and the hydrogen ions are given off as gas on the copper plate. It is the energy given out by the formation of the zinc sulphate which keeps the current going.

If the action is allowed to go on for some time, an accumulation of hydrogen bubbles takes place on the copper plate. This acts in two ways; it increases the resistance of the cell by diminishing the effective area of the plate, and it lowers the E.M.F. of the cell. As a result of these changes the current in the circuit diminishes, and the cell is said to **polarise**. But if the copper plate is removed and dried, and dipped in the solution again, the polarisation is found to have disappeared.

When the cell is giving a current, before polarisation sets in, the potential falls from the copper to the zinc outside the cell,

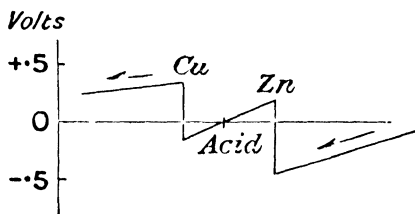


FIG. 512.

makes a sudden rise of 0.62 volt from the zinc to the acid, falls in the acid, and makes a sudden rise of 0.46 volt from the acid to the copper. Fig. 542 represents its distribution in the circuit, the mean potential of the acid being taken as zero.

After polarisation begins, the potential rise from the acid to the copper diminishes. When no current is flowing, the acid is at the same potential throughout, and so are the wires connected to the zinc and copper plates.

If the current is compared to a river, the Zn and Cu plates in Fig. 542 must be regarded as pumping stations at which the water is raised to a higher level.

Leclanché Cell.—The disadvantage of the simple cell described above is, that it polarises too rapidly. All the voltaic cells in use have devices for preventing polarisation. Thus in the Leclanché cell, when the chlorine ion attacks the zinc forming zinc chloride, the NH_4 ion liberated at the carbon plate breaks up into NH_3 which dissolves in the solution, and hydrogen which reacts with the manganese dioxide to form water and manganous oxide. In spite of this the Leclanché polarises rather rapidly, but recovers after a rest, and it possesses the advantage that the solution does not attack the zinc when no current is passing. Consequently, it is much used for electric bells, telephones, and other intermittent work.

Daniell Cell.—The Daniell cell is one in which polarisation has been overcome very successfully. It consequently gives a

very constant E.M.F., but the chemical changes do not stop when the current is broken, and it has to be dismantled when not in use.

In the usual form of the cell the negative plate is a zinc rod dipping into a porous pot which contains a solution of sulphuric acid. This porous pot is placed inside a wider copper vessel which contains a solution of copper sulphate. The porous pot prevents the mixing of the two fluids. The copper vessel itself is the positive plate of the cell. The sulphuric acid is decomposed into hydrogen and SO_4 ions; when a current passes, the SO_4 ions attack the zinc forming zinc sulphate, and the hydrogen ions pass into the copper sulphate forming sulphuric acid and liberating copper ions. These copper ions are deposited on the copper plate, and there is no polarisation, since it is copper and not hydrogen that is deposited.

In another form of the cell, the gravity Daniell, the zinc sulphate solution which is less dense than the copper sulphate solution, simply floats on the top of the latter in a glass vessel, the porous partition being done away with. The zinc electrode is at the top, and the copper electrode at the bottom of the vessel. The liquids do not mix for a considerable time, as the process of diffusion is a slow one.

The E.M.F. of the Daniell cell is 1.08 volts.

Bichromate Cell.—The positive pole of this cell consists of two carbon plates in conducting communication, between which the negative zinc plate is placed. The plates are close together, and hence the resistance of the cell is small. The electrolyte is a mixture of sulphuric acid and potassium bichromate. The sulphuric acid attacks the zinc liberating hydrogen. The hydrogen ions travel to the carbon plates, give up their charge, and are oxidised by the bichromate to form water; the bichromate consequently acts as depolarising agent. The zinc is drawn up out of the liquid when not in use. The E.M.F. of the cell is high, about 2 volts.

Dry Cell.—The disadvantage of a cell containing liquid is that the latter may dry up or be spilled. For this reason cells containing no liquid, called dry cells, are in considerable use; they have different trade names, but in composition are all practically the same, not differing much from the Leclanché cell. Their positive pole is a carbon rod surrounded by a paste, the average composition of which is 10 parts by weight of manganous dioxide, 10 parts of carbon or graphite, 2 parts sal ammoniac, and 1 part zinc chloride. This paste is called the depolarising paste; it is held firmly round the carbon electrode by a sack, and it occupies almost all the volume of the cell. The negative electrode is the zinc box containing the cell. Between the zinc and depolarising

paste is the electrolytic paste, usually white, made of flour, plaster of Paris, sal ammoniac, and zinc chloride; the layer of electrolytic paste is generally very thin. The carbon and depolarising paste must not touch the bottom of the zinc box. There is a small hole in the top of the cell for the escape of ammonia.

The E.M.F. of such a cell when fresh is 1.45 volts. The batteries used in pocket flash lamps consist simply of three such cells in series, and consequently ought to have an E.M.F. of 4.35 volts when fresh. Dry cells deteriorate in the course of time, even though not in use, owing to chemical changes taking place inside them.

Cadmium Cell.—The cells hitherto described are used as sources of current. The cadmium cell has the property that its

E.M.F. is very constant, and it is used solely as a standard of E.M.F. Only a very small current can be taken from it, or it will be damaged, but if it is used with care the E.M.F. remains constant at 1.018 volts for years. The cell consists of various substances contained in an H-shaped glass tube with platinum electrodes sealed through the glass; the details may be made out from Fig. 543.

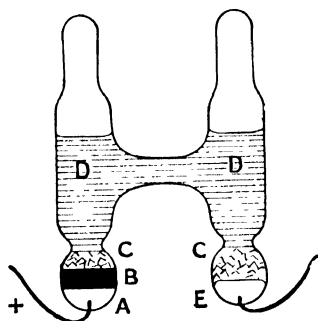


FIG. 543.—Cadmium cell.

A, Mercury. B, Mercurous sulphate paste. C, Cadmium sulphate crystals. D, Cadmium sulphate (saturated solution). E, Cadmium amalgam.

Storage Cell.—If two lead plates are immersed in a solution of one part of sulphuric acid to ten parts of water, and a current passed through the solution, hydrogen bubbles rise from the plate at

which the current leaves the solution, and the other plate becomes brown. This is due to the formation of lead peroxide (PbO_2) by the action of the oxygen, which is liberated by the current at this plate in exactly the same way as oxygen was liberated in the case of the hydrogen voltameter. If the current passing through the liquid is measured by an ammeter, it is found to decrease as the lead peroxide is formed.

If the battery is now removed, and the lead plates joined through the ammeter, it will be found that a current passes through it the other way. The original current virtually changed the two lead plates into a lead plate and a plate of lead peroxide, and thus made the arrangement into a voltaic cell, for whenever we have two dissimilar metals dipping into an acid we have a voltaic cell. It was the E.M.F. of the cell thus formed acting against the charging E.M.F. that made the ammeter reading fall in the first

case. The arrangement acts as a voltaic cell only until the peroxide is used up.

The principal difference between the cell just described and the commercial storage cell or accumulator in common use is, that in the storage cell lead grids are used instead of plates, that the interstices in the grids are filled with lead sulphate before charging, and that after charging the interstices in the positive grid are filled with lead peroxide, and in the negative grid with a porous spongy lead. During charging the SO_4 ions travel to the anode and form lead peroxide according to the equation



The E.M.F. of a freshly charged storage cell is 2.1 volts, and it gives back 75 or 80 per cent. of the energy spent in charging it. The grids disintegrate rapidly and have to be renewed. The test for knowing when to stop charging an accumulator is that, when the cell is fully charged, when all the lead sulphate on the negative grid is converted into lead, hydrogen gas begins to be given off; products of decomposition are then being lost and the cell is no longer working reversibly. The lead grids of an accumulator are shown in Fig. 544.

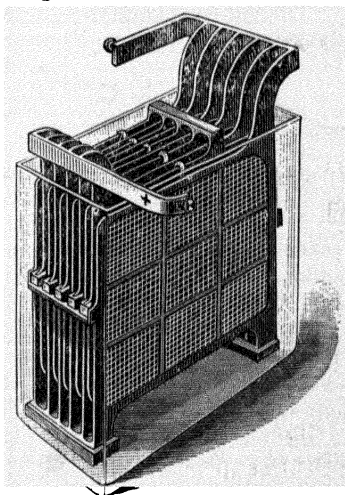


FIG. 544.—Storage cell.

The capacity of a cell is measured in ampere-hours or milliampere-hours, and in the case of a dry cell a value is generally given by the makers. The capacity varies with the current taken from the cell; if the average current is kept small, the capacity is much larger than when the current is large. Increasing the area of the plates in a cell or accumulator increases the maximum current output, and decreases the internal resistance.

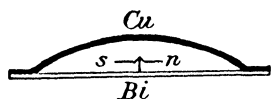
Recently there has been a great development in the manufacture of dry cells. Some have now an E.M.F. of 1.70 volts when new; in 1914 the best value of E.M.F. was 1.48 volts. Large capacity dry cells can now be obtained which have an initial E.M.F. of 1.70 volts, give 112 ampere-hours, a current of one ampere, and have a negligible resistance. They have no appreciable polarisation when used properly, and can be used for purposes for which a storage cell was formerly thought to be indispensable. Also high tension dry batteries can now be obtained with E.M.F.'s up to 150 volts, formed by small cells connected in series.

Comparison of Primary, Secondary, and Electrolytic Cells.—Faraday's laws of electrolysis hold for primary cells and secondary cells as well as for simple electrolytic cells. Thus if a battery of Leclanché cells arranged in series is used to pass a current through a zinc sulphate solution into which two zinc rods dip, the quantity of zinc eaten away from the negative electrode of each of the Leclanché cells is exactly equal to the quantity deposited on the cathode of the electrolytic cell.

The essential difference between the primary voltaic cell, the secondary cell, and the electrolytic cell is, that in the first of these the chemical change goes of itself, in the second the current produces the chemical change during charging, and the chemical change goes of itself when the cell discharges, while the third is neutral as regards the chemical change.

Thermo-electricity.—In 1821 Seebeck soldered the ends of a bent bar of copper to a bar of bismuth, and suspended a magnetic needle horizontally between the bars (Fig. 545). He then heated one of the junctions, and observed that a current flowed round the circuit from the copper to the bismuth through the cold junction. Such currents are called **thermo-electric currents**, and a pair of metals combined in this manner is called a **thermo-element** or **thermo-couple**. Seebeck arranged the metals in a **thermo-electric series**, such that when any pair of them was used to form a thermo-element, the current flowed through the cold junction from the element that was higher up to that which was lower down in the series. The order of the metals in the series, however, depends on the temperatures of the junctions.

FIG. 545.—Seebeck's experiment.



The nature of the effect is best understood by considering a particular case. Fig. 546 gives thermo-electric curves for a copper-iron circuit. The ordinates of the upper curve represent the E.M.F. produced in the circuit, when the cold junction is at 0°C . and the hot junction is at the temperature represented by the abscissa. The current flows from the iron to the copper through the cold junction. The E.M.F. rises with the temperature of the hot junction until it attains a maximum of about $1\frac{1}{2}$ millivolts, the temperature corresponding to this point being called the **neutral temperature**.

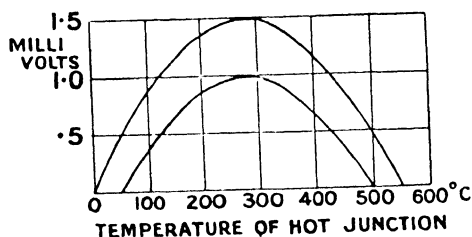


FIG. 546.—E.M.F. of copper-iron thermo-couple.

The E.M.F. rises with the temperature of the hot junction until it attains a maximum of about $1\frac{1}{2}$ millivolts, the temperature corresponding to this point being called the **neutral temperature**.

It then falls until at about 550°C . it has become zero. Beyond this point, which is known as the inversion temperature, the current flows the other way. The lower curve is for the case when the cold junction is at 50°C . Both curves are parabolas, the difference in the heights of the ordinates being everywhere the same. Combinations of other metals give similar results. It will be noticed that the E.M.F.'s produced are small.

Thermopile.—Thermo-elements are used for two purposes :

(i) For measuring radiant heat. For this purpose a number of wires, alternately bismuth and silver, are joined at their ends, and connected to a galvanometer, the junctions being represented by the points AAAA and BBBB in the figure. The combination of bismuth and silver is chosen, because it gives a large E.M.F. The junctions AAAA are exposed to the radiation and become heated, while the junctions BBBB are kept at room temperature. Hence a current flows through the galvanometer which is proportional to the difference of temperature, since the latter is small, and hence to the quantity of heat received. An instrument of this type in which the effect is multiplied by using several elements instead of one, is called a thermopile.

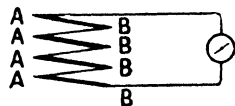


FIG. 547. —Principle of the thermopile.

(ii) For measuring the temperature of furnaces. For this purpose the thermo-couple is protected by a porcelain cover and placed inside the furnace, a combination of platinum with an alloy of platinum and iridium or with an alloy of platinum and rhodium being generally used. The arrangement is referred to as the thermo-electric thermometer.

Peltier and Thomson Effects.—Peltier found in 1834, that when an electric current passes through the junction of two different metals, there is either a liberation or an absorption of heat, according to the direction of the current. The effect is proportional to the current, not to the square of the current as in the Joule effect. In the case of the iron-copper circuit described above, when the two junctions are at different temperatures, and the current flows from iron to copper through the cold junction, there is, according to Peltier, an absorption of heat at the hot junction and a liberation of a smaller quantity of heat at the cold junction. We have here the source of the energy of the current ; it is transformed heat energy, just as in the case of the voltaic cell it was derived from the chemical energy of the constituents of the cell.

Whenever a current is passed along a wire in which the temperature varies from point to point, there is a slight liberation or absorption of heat. This effect, which of course exists in addition to the Joule effect, is known as the Thomson effect after its discoverer, Lord Kelvin.

EXAMPLES IX

1. How long will it take a current of half an ampere to deposit 1 gram of silver from a solution of silver nitrate ?

2. An acidulated water voltameter and an electrolytic cell, which contains copper sulphate and has copper electrodes, are connected in series. 10,000 coulombs are passed. How much hydrogen is liberated, and how much copper is deposited ?

3. A steady current is sent for 20 minutes through (i) a solution of hydrochloric acid (HCl) and (ii) a solution of cuprous chloride (Cu_2Cl_2). The volume of hydrogen liberated in (i) shows that 0.000621 gram of hydrogen was separated from the solution. Find the weight of copper deposited in (ii).

4. A constant current was sent for half an hour through the coil of a tangent galvanometer, producing a deflection of 35° , and also through a copper voltameter containing copper sulphate in which 1.112 grams of copper were deposited. What is the reduction factor of the galvanometer ?

5. A Leclanché cell supplies a current of 1 ampere for 5 minutes. Calculate the diminution in weight of its zinc electrode, given that zinc is divalent, and that its atomic weight is 65, that the electrochemical equivalent of silver is 0.001118, that silver is monovalent, and that the atomic weight of silver is 108.

CHAPTER X

INDUCED CURRENTS

Faraday's Experiments.—If a cylindrical coil of wire is connected with a sensitive galvanometer, and a bar magnet is suddenly thrust into the coil, as shown in Fig. 548, the galvanometer indicates a momentary current through the coil. The current lasts only as long as the magnet is moving, and stops when the magnet comes to rest. If the magnet is suddenly withdrawn, we have again a momentary current, but this time in the

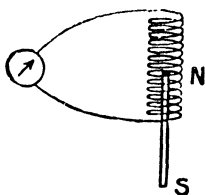


FIG. 548.

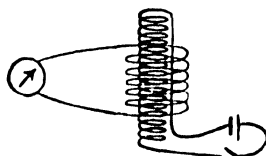


FIG. 549.

opposite direction. Also thrusting in the south pole of the magnet produces a current in the same direction as withdrawing the north pole, and withdrawing the south pole produces a current in the same direction as thrusting in the north pole.

If we have two cylindrical coils (Fig. 549), one called the **secondary** with a sensitive galvanometer in circuit with it, and another called the **primary** inside the first with a battery and key in circuit with it, when the key is pressed down we have an instantaneous current through the galvanometer. When the current is broken we have again an instantaneous current through the galvanometer, this time in the opposite direction. The effect is much increased, if the primary has a soft-iron core.

If the key is pressed down all the time, and the primary is suddenly withdrawn from the secondary, there is an instantaneous current in the secondary in the same direction as when the primary current is broken. This instantaneous current ceases as

soon as the relative motion ceases. If the primary is thrust into the secondary with the key pressed down, an instantaneous current is again produced in the secondary, this time in the opposite direction.

It is not necessary that the circuits should be cylindrical coils as shown in the diagrams. It is not necessary that the circuit containing the galvanometer should be at rest; the instantaneous currents are produced equally well when the coil is slipped over the magnet or when the secondary is slipped over the primary, in fact, whenever there is relative motion of a circuit and a magnetic field.

The above currents are known as **induced currents** and the phenomenon as **electro-magnetic induction**. They were discovered by Faraday in 1831. His results can be summed up in the following law:

Whenever the number of lines of magnetic force threading a conducting circuit is altering, an induced current flows round the circuit which continues only as long as the change is taking place.

All the electric current used nowadays for lighting, heating, and driving machinery is produced by methods depending on the above law.

Lenz's Law.—If the primary and secondary consist of two equal parallel coils situated close to each other, when the current is started in the primary, the induced current in the secondary is in the opposite direction, as shown by the arrow (Fig. 550); it is known as the *inverse* induced

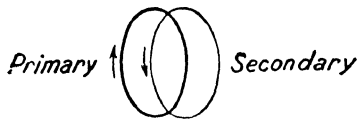


FIG. 550.

current. When the current in the primary is broken, the induced current is in the same direction as the current in the primary and is known as the *direct* induced current.

The field of the inverse current is opposed to the field of the primary current, and the field of the direct induced current is in the same direction as the field of the primary current. Thus, at make the induced current tends to prevent the field growing up and at break it prolongs its existence; in each case it opposes the change taking place. The same result holds true in all cases, and a rule for obtaining the direction of the induced current is based on it. This rule was given by a Russian named Lenz shortly after Faraday's discovery and runs as follows:

The direction of the induced current is such as to oppose the change which produces it.

We shall give two further examples of the application of the rule. Let us suppose there are two parallel straight wires in one of which a current is flowing, and that the distance between the

wires is increased. The current induced in the second wire by the motion must be in the same direction as the original current, because currents in the same direction attract each other, and this attraction will diminish the motion.

Again, if current is flowing in a circular coil in an anti-clockwise direction, it exerts a force on a north pole at the centre of the circle towards the observer. Consequently, if a north pole is moved towards the circle, the induced current is in the anti-clockwise direction, for a current in this direction opposes the motion.

Magnitude of the Induced E.M.F.—If a current is induced in a circuit, and the wire of the circuit is replaced by one of another material, of exactly the same shape, the magnitude of the current changes, but the product current $\times$ resistance is the same in each case. The induced E.M.F. is independent of the resistance of the circuit.

The electromotive force induced in a circuit is equal to the rate at which the number of magnetic lines of force passing through the circuit is altering.

The rule gives the result in theoretical units. It must be divided by 10^8 to bring it to volts.

Quantity of Electricity which passes.—If n is the change in a very short interval of time t of the number of lines of magnetic force passing through a circuit, n/t is the average induced E.M.F. during this time, and the average induced current is consequently given by

$$C = \frac{n}{tR}$$

Hence the quantity of electricity that passes is given by

$$Qt = \frac{n}{R}$$

Suppose that in any actual experiment the total change in the lines of force through the circuit is N . During the experiment the magnitude of the induced current alters greatly, but the time over which the experiment extends may be divided into intervals for each of which the current is approximately constant. Let $n_1, n_2, n_3, \dots$ be the changes in the number of lines of force in these intervals. Then by the above reasoning the total quantity of electricity that passes is given by

$$Q = \frac{n_1}{R} + \frac{n_2}{R} + \dots = \frac{N}{R}$$

The result is independent of the rate at which the change is effected.

Now Q may be measured by the ballistic galvanometer (p. 537). Hence N can be calculated. But if the ballistic galvanometer

is to measure Q accurately, the change must be completed before the needle has time to move appreciably from its zero position.

Earth Inductor.—The earth inductor consists of a circular wooden frame round which is wound a coil of n turns of wire, and which is mounted so that it can be rotated about either a vertical diameter or a horizontal diameter of the circle.

Let A be the area of the coil. Let us suppose that it is standing with its plane vertical and at right angles to the magnetic meridian, and that it is connected to a ballistic galvanometer, the total resistance of coil and galvanometer being R . The number of lines of force through each turn of the coil is AH , where H is the horizontal component of the earth's field, and the number of lines of force through the whole coil is nAH . If now the coil is rotated about its vertical diameter, the number of lines of force passing through it decreases, becomes zero when the coil has turned through 90° , and increases to its former value, but in the opposite direction through the coil, when the latter has turned through 180° . Hence, if the coil is stopped at this point, the number of lines of force passing through the circuit has changed through the rotation from nAH to $-nAH$, which is equal to a total change of $2nAH$. The quantity of electricity which passes through the galvanometer is consequently

$$\frac{2nAH}{R}$$

and if the scale of the galvanometer has been calibrated by the discharge of a known quantity of electricity through it, H can be calculated.

If the plane of the coil is initially not at right angles to the meridian, when it is turned through 180° the quantity of electricity passing through the galvanometer is not so great. Hence the coil can be used as a means of determining the direction of the magnetic meridian.

Let us suppose now that the coil is held with its plane horizontal, and then suddenly rotated through 180° about a horizontal diameter. The number of lines of force through it is initially nAV , where V denotes the vertical component of the earth's field, and finally $-nAV$. Thus, the quantity of electricity which passes through the galvanometer is

$$\frac{2nAV}{R}$$

If we divide this result by the previous one, we obtain $V/H = \tan \theta$, and so determine θ , the dip.

Alternating Current.—Let us suppose that the coil of the earth inductor is kept rotating about a vertical diameter with

uniform angular velocity ω , and that initially the normal to its plane is in the direction of the meridian. Then at time t it makes an angle ωt with the meridian, the projection of its area upon a plane at right angles to the meridian is $A \cos \omega t$, and the number of lines of force passing through it is $nAH \cos \omega t$.

Let a point Q (Fig. 551) describe a circle of radius a with uniform angular speed ω starting from D at $t=0$. Draw QP perpendicular to OD . Then, as Q moves round the circle, P moves backwards and forwards along the diameter DF . The displacement and velocity of P with respect to O are the components of the displacement and velocity of Q with respect to O . We have therefore

displacement of $P = a \cos \omega t$

velocity of $P = \text{rate of change of displacement} = \omega a \sin \omega t$

That is, the rate of change of $a \cos \omega t$ is $\omega a \sin \omega t$. By analogy the rate of change of $nAH \cos \omega t$ is $nAH\omega \sin \omega t$. The current through the circuit is therefore

$$\frac{nAH\omega \sin \omega t}{R}$$

This quantity becomes negative, and the current consequently changes its direction every half revolution of the coil. Such a current is known as an **alternating current**. The period of a revolution is $2\pi/\omega$; hence the frequency of the current is $\omega/2\pi$ **cycles per second**.

For simplicity write the current $C \sin \omega t$, where C is its maximum value. Then its mean value is zero, since $\sin \omega t$ is as often negative as positive. But the mean value of its square

$$C^2 \sin^2 \omega t = \frac{C^2}{2} (1 - \cos 2\omega t) \text{ is } \frac{C^2}{2}$$

The square root of this quantity, $C/\sqrt{2}$, is known as the **root mean square** of the current.

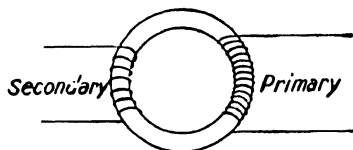


FIG. 552.—Faraday's ring.

Faraday's Ring.—Fig. 552 represents Faraday's ring, the arrangement by which he performed his first successful experiment on induced currents. It is a copy of the first diagram in his *Experimental Researches*. The ring was an iron

one, $\frac{7}{8}$ inch thick and 6 inches in external diameter. The primary had about 315 and the secondary about 262 turns.

Foucault or Eddy Currents.—A changing magnetic field induces

currents, not only in circuits formed of wires, but also in every mass of metal in the field. Such currents are called Foucault or eddy currents.

For example, if a horizontal bar magnet is executing torsional oscillations in the earth's field about a vertical axis through its mid-point, and a large copper plate is placed close below it, the field through the plate changes owing to the motion of the poles, and currents are induced in the plate. These currents by Lenz's law oppose the motion. Consequently, the oscillations of the magnet die down very rapidly.

Again, if a copper disc is rotating with angular velocity ω , with its plane at right angles to a magnetic field of strength H , each of the radii of the disc may be regarded as a wire which cuts through the lines of force of the field. The number of lines of force cut through in one revolution is $\pi r^2 H$, and the number cut through per second is $\frac{1}{2} \omega r^2 H$.

Consequently an E.M.F. equal to $\frac{1}{2} \omega r^2 H$ is induced between the centre and edge of the disc. No current flows, however, because if we consider such a path as AOB (Fig. 553), the fall of potential on OA is equal to the gain of potential on BO, while the arc AB is at the same potential throughout, and there is thus no E.M.F. round the circuit. But if the field is not uniform, the E.M.F. between the ends of one radius is not the same as between the ends of another. Hence, if, while the disc is rotating, it

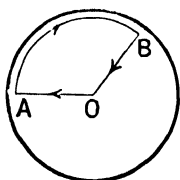


FIG. 553.

is placed with its edge between the poles of a horseshoe magnet, currents are induced, and the action of the external field on these currents brings the disc to rest.

EXAMPLE.—A rectangular wooden frame of area 2000 sq. cm. which carries a coil of 100 turns round its edge, is set up with its plane vertical and at right angles to the horizontal component of the earth's magnetic field. The value of the latter is 0.16 c.g.s. unit. The frame is suddenly rotated about a vertical axis through an angle of 180° . The resistance of the circuit is 40 ohms. Calculate in coulombs the quantity of electricity which flows round it.

$$Q = \frac{2nAH}{R} = \frac{2 \times 100 \times 2000 \times 0.16}{40 \times 10^8}$$

$$= 1.6 \times 10^{-5} \text{ coulomb}$$

$$= 16 \text{ micro-coulombs.}$$

EXAMPLE.—A train is travelling at 60 miles per hour. The component of the earth's magnetic field is 0.44 c.g.s. unit. Calculate the E.M.F. between two points on an axle 4 ft. 6 in. apart.

$$\text{No. of lines cut per sec.} = \frac{9 \times 88 \times 30.48^2 \times 0.44}{2}$$

$$= 1.62 \times 10^5$$

Therefore E.M.F. = 0.00162 volt.

Induction Coil.—The induction coil is a piece of apparatus used for producing a secondary or induced current of very high

voltage from a primary current of low voltage such as is yielded by a few cells. Fig. 554 shows one of a type much used for general laboratory work, and Fig. 555 gives a diagram of its connections.

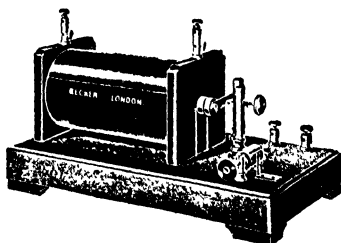


FIG. 554.—Induction coil.

It consists essentially of a bundle of straight iron wires termed the core round which is wound a primary coil of one or two layers of thick copper wire. Outside this is the secondary which consists of a very large number of turns of fine wire insulated with silk and shellac. The core is made of wires instead of solid iron in order to prevent Foucault currents. The current enters and leaves the instrument by the terminals on the right at the back in Fig. 554. On the right at the front of the same figure is a commutator or reversing key for changing the direction of its flow through the primary. In the middle is shown the “break,” or device for stopping and starting the current through the primary, which except in large instruments, always takes the form of the hammer interrupter.

This consists of a spring to one end of which is attached a piece of soft iron; on the side is a platinum surface which rests against a platinum point fixed to a vertical pillar. The current passes up the pillar, through the platinum point to the spring,

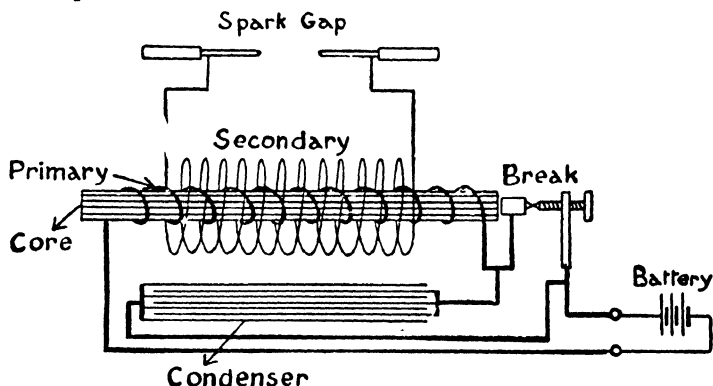


FIG. 555.—Diagram of induction coil.

and then round the primary. This magnetises the core, the core attracts the piece of soft iron, contact between the platinum

point and surface is broken, and the current is interrupted. The core then demagnetises, the spring flies back, contact is made, and the same cycle takes place over again.

Whenever the primary current is made or broken, an E.M.F. is induced in the secondary, and if a wire is attached to one of the terminals of the secondary which are shown at the top of the coil in Fig. 554, and its end brought near the other, sparks pass across the gap in the same way as between the knobs of a Wimshurst machine. The break takes place much more sharply than the make, the induced electromotive force is much greater one way, and the stream of sparks passes mainly in one direction. An induction coil delivers a greater quantity of high voltage electricity than a Wimshurst machine. It is found that if a condenser is connected across the break as shown in Fig. 555, the working of the coil is greatly improved; this condenser is usually contained inside the wooden base of the instrument.

The largest induction coils can be made to give sparks 18 or 24 inches long. The hammer interrupter is not suitable for use with such coils because the current arcs across the platinum contacts and destroys them. When an induction coil is in operation, the spark changes oxygen of the atmosphere into ozone; the latter can easily be recognised by its very characteristic smell.

The induction coil described above is sometimes referred to as the *Ruhmkorff* coil to distinguish it from the *Du Bois Raymond* coil. The latter is used in physiology; the chief difference between the two types is, that in the *Du Bois Raymond* coil the secondary slips off the primary, and that the distance between the two, and consequently the length of the spark, can be regulated. It goes without saying that any one touching the secondary of an induction coil receives a shock, which in the case of a large coil may be dangerous to life.

Magnetic Induction.—Let us suppose that we have a long solenoid with n turns of wire per unit length which carries an electric current. Then, as has been stated on p. 514, there is a magnetic field inside it. It may be shown that the strength of this field is given by

$$H = 4\pi nC$$

where C is the current in theoretical units. The strength of the field does not vary with the distance from the axis, but it falls off at the ends of the solenoid, owing to the lines of force beginning to diverge out.

Let us suppose that a circular coil of N turns is slipped along the outside of the solenoid. Then if A is the area of the cross-section of the solenoid, the number of lines through each turn of this secondary coil is AH , and the number of lines through the whole secondary NAH . If the current in the primary is reversed, the change in the number of lines through the secondary is $2NAH$, and if the secondary coil is con-

nected to a ballistic galvanometer, the whole resistance of the secondary circuit being R , a quantity of electricity

$$\frac{2NAH}{R}$$

flows through the galvanometer. The galvanometer deflection is consequently a measure of the number of lines per cm.^2 through the solenoid. If the secondary coil is slipped along the solenoid, the galvanometer deflection is always the same, except when it approaches an end.

It has already been mentioned (p. 553), that the induced current is much increased when the primary has a soft-iron core. Let us suppose that a uniform solid cylinder of soft iron is introduced inside the solenoid, that it fills it from end to end, and that the galvanometer deflection is increased in the ratio 1 to μ . It will be found by slipping the secondary along the solenoid, that it is increased in the same ratio everywhere except near the ends of the solenoid. The number of lines of force per cm.^2 must consequently be increased by the presence of the iron in the ratio 1 to μ . *The number of lines of force per cm.^2 is called the magnetic induction in the iron* and is always denoted by B . Hence

$$B = \mu H$$

μ is called the **permeability** of the iron. Magnetic induction has not the same dimensions as magnetic field intensity, but it would go beyond the scope of this book to explain the difference fully.

If the iron core is cut in two and the parts replaced in the solenoid with a short air gap between them, the deflection is increased in the same ratio when the secondary coil is placed round the gap, as when it is placed round the iron. This is because poles form on both sides of the gap. *The pole strength per cm.^2 is called the intensity of magnetisation of the iron*, and is always denoted by I . It has been shown, p. 446, that the number of lines of force diverging out from a pole of strength m is $4\pi m$. The number of lines of force leaving one cm.^2 on each side of the gap is consequently $4\pi I$. These go straight across the gap parallel to the axis of the solenoid. Hence

$$B = H + 4\pi I$$

The ratio of the intensity of magnetisation I to the field strength producing it is called the **susceptibility** and is usually denoted by k .

$$I = kH$$

Let us return now to the case of the undivided iron core; it has, of course, poles at each end of strength AI . The lines of magnetisation due to the iron, $4\pi AI$ in number, pass inside the body of the core from the south pole to the north one, and then back through the air, or through the air and the solenoid itself, from the north pole to the south one. They are thus closed curves. This is in contradistinction to electrostatics, where the lines of force always end in a positive or a negative charge. Moreover, in passing back through the solenoid from the north pole to the south one, they oppose H , the original magnetising field of the solenoid, and their action in this respect is

referred to as the demagnetising effect of the poles. If the core is a long one, the demagnetising effect of the poles which are then at a considerable distance, can be neglected at points near its middle.

BH Curve.—Since the working of modern electrical machinery

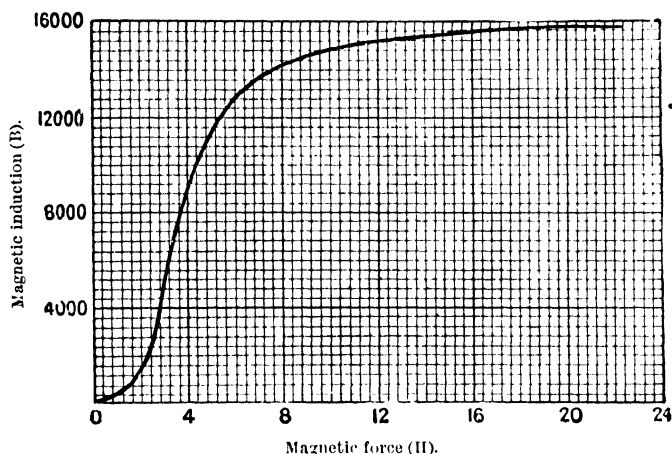


FIG. 556.—BH curve for iron.

depends on induced currents, a knowledge of the permeability of iron is of very great practical importance. It is not constant, but varies

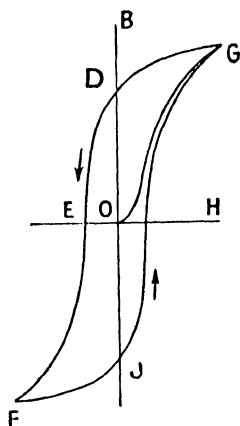


FIG. 557.—Hysteresis curve.

with the magnetising force; it can be measured by two different methods, the ballistic method and the magnetometric method. In the former the iron rod experimented on is placed inside a solenoid with a secondary coil, and the permeability calculated from the quantity of electricity which passes in the secondary when a given field is applied. In the magnetometric method the iron is placed inside a solenoid and magnetised; its poles then deflect a magnetometer needle, the instrument being simply a refinement of that described on p. 457.

Fig. 556 shows the nature of the results obtained for iron, when the magnetising field is gradually increased from zero. The abscissæ represent H , the ordinates B . At first the increase of B with H is slow, then there is a portion of the curve for which it is rapid, then it becomes slow again, and the curve is finally a straight line. At this stage in

the expression $B = H + 4\pi I$, I has become constant, the iron is said to be **saturated**, and the further increase in B is due solely to the increase in H .

If the magnetising field H is increased from zero to a maximum value and then gradually decreased to zero again, the curve representing B is not the same on the way down as on the way up. Fig. 557 represents what happens. OG is the initial portion of the curve, simply a reproduction of part of Fig. 556 on a smaller scale. Then when H is decreased to O , B diminishes to D . If H is reversed and gradually increased, B diminishes to E . When H has attained the value it had at G but with the sign changed, B has diminished to F . If now the value of H is diminished to O , reversed, and increased to its former maximum value, the change in B is represented by the curve FJG . From the curve it is clear that the magnetisation lags behind the magnetising force, that it depends not only on the magnetising force but also on the previous history of the specimen; this lagging effect is known as **magnetic hysteresis**, and the curve $GDEFJG$ is called a hysteresis curve. The value of I corresponding to the point D is known as the residual magnetism, and the value of H corresponding to the point E as the coercive force.

EXAMPLES X

1. A circular coil is suddenly thrust over the N . pole of a magnet which is fixed in a vertical position with the N . pole up. Is the direction of the induced current clockwise or counter-clockwise when viewed from above?

2. A straight horizontal wire one metre long is falling vertically with a velocity of 500 cm. per sec., and its length is at right angles to the magnetic meridian. What is the magnitude in volts of the E.M.F. induced between its ends, given that $H=0.17$ c.g.s. unit?

3. A disc of 10 cm. radius is rotated at a speed of 50 revolutions per second in a vertical plane at right angles to the magnetic meridian. Calculate the E.M.F. between the centre and the edge, given that $H=0.17$ c.g.s. unit.

4. Calculate the magnetic field strength inside a solenoid 120 cm. long which has 3000 turns, when a current of one ampere is flowing in it.

CHAPTER XI

TECHNICAL APPLICATIONS OF ELECTRICITY

Incandescent Lamp.—It has already been explained on p. 523, that when a current is passed through a wire, heat is evolved, and that in certain circumstances the wire may become red hot. This is the principle of the incandescent electric lamp, which consists of a thin filament of carbon or tungsten sealed into a glass bulb from which the air has been evacuated. It is necessary that the filament should have a high melting point, for the hotter it is, the more light it gives for the same consumption of energy. Platinum was tried first, but was not a success, for it had to be raised nearly to its melting point in order to give light economically, and an accidental variation of the current was then sufficient to burn the wire through. Carbon was then used successfully for many years, but has now been supplanted by lamps with tungsten filaments sold under trade names such as Osram, Mazda, etc.; these were first introduced about 1906. The filament is enclosed in a vacuum to prevent it oxidising; the vacuum has also the effect of preventing loss of heat by conduction to the sides of the bulb.

It is customary to rate the performance of an incandescent lamp or glow lamp, as it is sometimes called, in watts per candle. A 16-candle-power carbon lamp, when run off a 250-volt lighting circuit, usually takes about 0.22 amp. Its rate of consumption of energy is $CE = 0.22 \times 250 = 55$ watts, and the number of watts required per candle is consequently $55/16 = 3.4$. The tungsten lamp requires only 1.25 watts per candle, and so gives roughly three times as much light for the same expenditure of energy.

In the ordinary incandescent lamp, as has been mentioned above, the filament is enclosed in a vacuum. It was discovered before the war, that if the bulb of the lamp were filled with an inert gas such as argon or nitrogen, the filament would stand a higher temperature without disintegrating, and more light would be obtained for the same expenditure of energy. Such lamps were consequently put on the market under the name of the

Half-Watt Lamp. As a matter of fact they require six-tenths of a watt per candle.

EXAMPLE.—A room is illuminated by a 40-watt Osram lamp. Calculate the cost of running it per hour, (i) at the lighting rate of 6*d.* per unit, (ii) at the heating rate of 1*d.* per unit.

The unit is the kilowatt-hour. The results are $\frac{1}{100} \times 6 = 0.24d.$ and 0.04*d.* The smallest electric radiators for warming a room take about $\frac{3}{4}$ of a unit, so require about twenty times as much current as a lamp of average size.

Arc Lamp.—If two rods of carbon which are at a potential difference of 70 or 80 volts are brought into contact for an instant and then suddenly pulled apart, an electric discharge characterised by intense light and heat passes between them. This constitutes the electric arc. The operation of bringing the carbons into contact and then withdrawing them is known as “striking the arc.”

After the discharge has passed some time, the end of the positive carbon or anode becomes hollowed out, and the end of the negative carbon or cathode becomes pointed. Both carbons are consumed by the arc, the positive one twice as fast as the negative one. 85 per cent. of the light of the arc comes from the crater or hollow on the positive carbon which has a temperature of about 4000° Abs., the rest from the cathode and the vapour between anode and cathode. If the current is too large, the lamp begins to hiss; if the distance between the carbons becomes too great, the arc goes out. The carbons of the arc lamps used for street lighting strike and feed together automatically, but in the arcs used for projection work this is usually done by hand.

Arcs used for street lighting are enclosed in a tightly fitting glass globe so as to restrict the supply of air and lengthen the life of the carbons. Instead of mounting the carbons coaxially they are inclined to one another with the anode pointing downwards; the crater can thus radiate freely without being obscured by the negative carbon. Also the carbons have cores containing salts of strontium, calcium, barium, and titanium. This has the effect of making the luminous vapour between the electrodes develop into a brilliant flame which then gives the greater portion of the light. The **flame arc**, as this type is called, is the most efficient of all illuminants, requiring only $\frac{1}{4}$ watt per candle.

Electric Furnace.—There are two types of electric furnace. The first, which consists of a fire-clay tube round which is wound a heating coil, is used for temperatures up to 1500° C., the whole being protected against loss of heat. The substance to be heated is placed inside the tube. Electric heaters, radiators, kettles, ovens, all operate on the same principle, the heat being generated in coils of wires.

The second type of furnace is used for higher temperatures, and consists of an arc the carbons of which are embedded in a block of lime or fire-clay. The carbons are horizontal and in the same straight line. The substance to be heated is placed in a cavity between the electrodes, slightly below them, and a very strong current is passed.

Electric Bell.—Fig. 558 shows the construction of an electric bell. G is the gong and H the hammer; the latter is attached to a piece of soft iron, A, called the armature, which carries a light spring S, and is itself held in position by the spring N. The spring S makes contact with the point of a screw. M is an electromagnet. Besides the bell the circuit contains the bell push K, which is simply a contact key, and the battery B, usually two Leclanché cells.

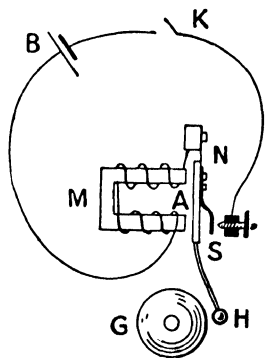


FIG. 558.—Electric bell.

When current is made by the bell push, it flows across the contact at S and round the coils of the electromagnet, magnetising the latter. The electromagnet then attracts the armature, and the hammer strikes the gong. But when the armature is attracted, the circuit breaks at S, the magnet demagnetises, and the armature flies back. Contact is then made again, the armature is attracted, and the hammer strikes the gong. And so on; the bell keeps ringing as long as the bell push is pressed.

Electric Telegraph.—In the following table the Morse code ordinarily employed in telegraphic signalling is given :

A — — —	J — — — —	S — — —
B — — — —	K — — — —	T — — —
C — — — —	L — — — —	U — — — —
D — — — —	M — — — —	V — — — —
E — — — —	N — — — —	W — — — —
F — — — —	O — — — —	X — — — —
G — — — —	P — — — —	Y — — — —
H — — — —	Q — — — —	Z — — — —
I — — — —	R — — — —	

Call — — — — —. One dash=three dots. Space between signals forming same letter=one dot, space between letters=three dots, space between words=five dots.

If a circuit contains a battery, reversing key, and galvanometer, the galvanometer being distant some miles from the battery and key, signals can be sent from the key to the galva-

nometer by deflecting the needle either to the right or to the left. This was the principle of some of the older forms of telegraph; the needle was vertical, a deflection of the upper end to the left being equivalent to a dot, and a deflection of the upper end to the right being equivalent to a dash. Nowadays the Morse sounder, an electromagnet with its armature supported by a spring, is used instead of the galvanometer. Whenever the key at the sending station is pressed, the armature at the receiving station is pulled down against the magnet; when the circuit is broken, it springs back. The armature can be made to carry a pen, below which a tape is drawn at a uniform rate, so that when the current is made, the pen presses down on the paper and writes dots and dashes, but in practice the operators read the messages by ear alone. Whenever the armature strikes the magnet, and whenever it springs up, it makes a click, and the letters can be read by the intervals between the clicks.

In the early days of telegraphy a double circuit was used, but it was found in 1837, that the return wire might be dispensed with,

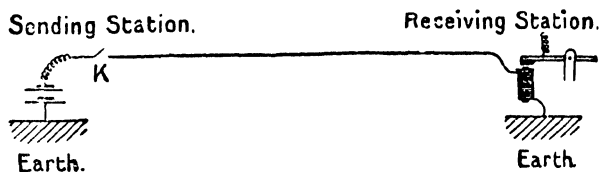


FIG. 559.—Simple telegraph circuit.

and the earth itself used as return conductor. This is the universal practice nowadays; Fig. 559 shows the arrangement, the ends of the wires being attached to copper plates which are buried at some depth in the earth, so as to make good contact with the latter.

Telegraphic Relay.—If the signals are sent a long distance, owing to the resistance of the wire the current may be too weak to work the Morse sounder. In this case it is sent through a small magnet or *relay*, the coil of which has many thousand turns of wire; this magnet attracts a light armature and closes a local circuit at the receiving station, consisting of battery and sounder. When the relay demagnetises, the circuit breaks. The click of the relay itself is too faint to be heard easily. By this arrangement the Morse sounder is worked at the expense of a battery at the receiving station, the function of the signals being simply to turn the current on and off.

Telephone.—The telephone, which was invented by Graham Bell in 1875, consists at each end in its simplest form of a bar magnet round which there is a coil of wire, these coils forming part of a circuit of which the earth is the return wire. In front of each

coil is a thin iron diaphragm, almost touching the end of the magnet. When the diaphragm at the one end is spoken into, it vibrates under the influence of the sound wave, and alters the number of lines of force which pass through the coil. Induced currents are thus set up in the circuit; these alter the strength of the field in which the iron diaphragm at the other end of the line is situated, it executes the same vibrations as the first diaphragm, and consequently reproduces the sounds which cause the latter to vibrate. The distance through which the diaphragms move is extremely small, about one millionth of an inch. The Bell telephone may thus be used both as transmitter and receiver. It is independent of an electric battery, but the voltages generated are so small, that they cannot overcome the resistance of a long circuit.

In modern practice an instrument of this type is still used as receiver, a horseshoe magnet being substituted for the bar magnet, but a return wire is provided, and a transmitter working upon a different principle, sometimes called the **microphone transmitter**, is used. At the back of the diaphragm is a little chamber filled with granulated carbon. As the diaphragm vibrates, the electrical resistance of this carbon alters, and the strength of a primary current which flows through the carbon also alters. This primary current then flows round a soft-iron core, round which the telephone wire is wound as secondary, and its variations induce currents on the telephone line, which actuate the diaphragm at the other end. The primary current is provided by a battery. This type of instrument carries much farther than the original instrument. It was invented in its original form by an English clergyman named Hunnings.

Microphone.—Fig. 560 represents the first microphone, which was invented by Hughes. A piece of gas carbon C, pointed at both ends, rests lightly in notches cut in two pieces of the same kind of carbon A and B, mounted on an insulating support, and the apparatus is connected in circuit with a single cell and a telephone receiver. The resistance of the carbon contacts changes with the pressure to which they are subjected, hence any slight disturbance of the contacts produces a change of the current in the circuit. The sensitiveness of the instrument is very great; the ticking of a watch placed on the base of the instrument is heard in the telephone like the strokes of a hammer. The principle is the same as is applied in the microphone transmitter.

Dynamo.—It has already been explained on p. 557, in connection with the earth inductor, that if a coil of n turns and area A

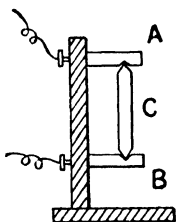


FIG. 560.—
Microphone.

is rotated with angular speed ω in a field of strength H , the direction of which is at right angles to the axis of rotation, an E.M.F. is induced in the coil equal to

$$nHA\omega \sin \omega t$$

The dynamo, the machine by which electricity is generated on a large scale for lighting, heating, traction, etc., is in principle simply an arrangement of this nature in which the rotating coil, or **armature**, is turned by a steam engine or by water-power. The ends of the coil are fastened to two metal rings (Fig. 561) which rotate about the same axis as the coil. The "brushes" of wire gauze or carbon press lightly against these slip rings, and so enable contact to be made with the fixed external circuit.

The current produced in this way is, as has been mentioned on p. 557, alternating current (A.C.) and the machine producing

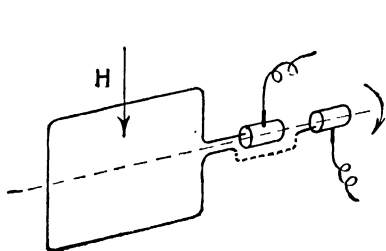


FIG. 561.—Coil producing alternating E.M.F.

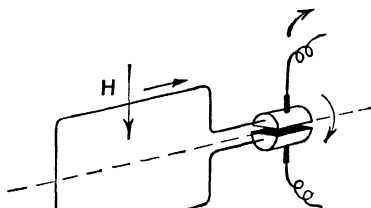


FIG. 562.—Rotating coil with commutator.

it is an **alternating current dynamo**. For lighting and heating alternating current is as useful as direct current (D.C.).

The rotating coil can, however, be converted into a **direct current dynamo** by employing the **commutator** shown in Fig. 562. This consists of a split ring which rotates on the same axis as the coil does, and the two halves of which are insulated from one another. One half is connected to the one end of the coil and the other half to the other end. When the coil is in the position shown in the figure, the rate at which the lines of force are being cut, and consequently the E.M.F., is a maximum, and it can be shown by Lenz's law that for the given direction of rotation the induced current is in the direction indicated by the arrows. When the coil has turned through 90° and the induced E.M.F. is zero, each brush passes from one half of the ring to the other. When the coil has turned through 180° and the E.M.F. is in the other direction round the coil, contact is made on the other halves of the ring. So the current goes round the external circuit the same way. It is represented in magnitude as a function of the time

by the curve shown in Fig. 563, a sine curve with the sign of the negative part reversed.

The current, though now always in the same direction, is not constant in magnitude. In a dynamo for actual use the inequalities must be smoothed out. This is done by using an armature of complicated design, the current from which is represented by the superposition of several curves of the above type,



FIG. 563.

all having the same period but with their maxima arranged at different places, so that the sum of the ordinates is nearly a straight line.

The field in which the armature turns is provided by a large electromagnet, the **field magnet**, which is excited by the current generated by the dynamo itself. There is always sufficient magnetism left in the magnet to start the current, and the latter once started increases the field strength. If the coils of the field magnet are connected in series with the dynamo and external circuit, the dynamo is said to be **series wound**; if the coils of the field magnet are connected in parallel with the external circuit, so that only part of the current produced by the dynamo flows through them, the latter is said to be **shunt wound**; if some of the coils of the field magnet are series wound and some are shunt wound, the dynamo is said to be **compound wound**.

Electric Motor.—It has already been explained on p. 535, that when a current flows round a coil in a magnetic field, the coil turns into a position in which its plane is at right angles to the direction of the field. The coil behaves like a magnet of moment CA , where C is the current and A is the area, and the action of the moving coil galvanometer is based upon this fact.

If an ordinary compass needle is deflected, and then allowed to return to its position of equilibrium, its momentum will carry it beyond this position. Suppose that, when it reaches the position of equilibrium, the control field is reversed in direction; then it swings through another 180° . If the field is again reversed when it is passing through the new equilibrium position, it continues through a second 180° . And so on; by reversing the field, the magnet is kept in continual rotation.

Let us consider once more the coil represented in Fig. 562. If a current is passed through it from a battery, the coil turns into the position in which its plane is at right angles to the direction of the field, and its momentum carries it beyond this position. As it passes this position the commutator reverses its polarity, and a half revolution takes place. Then comes a further reversal and another half revolution, and so on. The case is the same as the preceding one except that here it is the sign of the poles, not

the direction of the field, that is altered. The energy of the battery is converted into energy of rotation of the coil, and can be used for raising a weight or doing other mechanical work. The electric motor consists essentially of a coil rotating in a magnetic field in this manner; it is simply a reversed dynamo.

Barlow's Wheel. Faraday's Disc.—The principle of the dynamo and the motor can be illustrated by certain lecture table experiments:

Let us suppose that a straight wire is hung vertically from a hook about which it is free to turn, with its lower end dipping into a pool of mercury, and that a horseshoe magnet is placed with its poles one on each side of the wire, so that the lines of force intersect the wire at right angles. Then, if a current flows down the wire, a force acts on the wire at right angles to the lines of force and the current. This force causes the wire to swing sideways about its point of support, and its lower end leaves the mercury. The current then breaks, the wire swings back, contact is re-established, the force once more deflects the wire, and so on; the wire keeps swinging in and out of the mercury.

Fig. 564 represents a piece of apparatus known as Barlow's wheel. It consists of a star-shaped copper disc, one point of which dips into a mercury trough. There are two terminals, one of which is connected to the brass pillar, and the other of which is connected to the mercury trough; when the terminals are connected to a battery, a current passes up the pillar to the axle of the disc, and then down a radius of the disc to the mercury. The radius plays the part of the wire in the previous experiment. If a horseshoe magnet is placed with its poles one on each side of the lowest blade of the star, the lines of force meet the disc at right angles, and the star begins to rotate. This causes the point to move out of the mercury and breaks the current. The force consequently ceases, but the momentum acquired is sufficient to bring the next point into the mercury, and the star receives another impulse forward. And so it rotates in a series of jerks, the apparatus forming a simple electric motor.

It is not necessary that the disc should be star-shaped. An ordinary circular disc which makes contact all the time, rotates in the same way when the field is applied, for the radii may be regarded as a series of wires, and when one loses contact, the force acts on the next.

Fig. 565 represents an apparatus due to Faraday. A copper

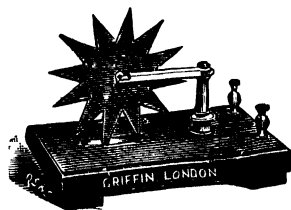


FIG. 564.—Barlow's wheel.

disc is rotated by a handle ; it passes between the poles of a magnet, shown in this case in the diagram, and cuts the lines of force at

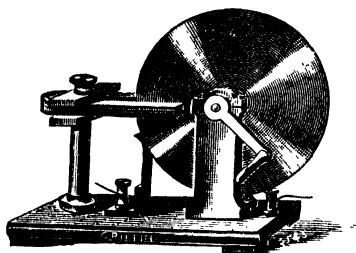


FIG. 565.—Faraday's disc.

right angles. Consequently, an E.M.F. is set up between the centre and rim of the disc. A thin strip of copper makes contact with the rim of the disc, and is connected to the one terminal, and the axle of the disc is connected to the other. Hence if the terminals are connected by a wire, a current passes through it when the disc is rotated, the apparatus forming a simple

dynamo. All the current from the rim to the axle does not pass by the wire ; some, as has been mentioned on p. 557, flows back as Foucault currents through the disc.

Back E.M.F. in Motor.—Let us suppose that a battery of electromotive force E and negligible resistance is connected to a series wound motor the resistance of the armature and magnet of which is R . Then before the armature is set into rotation, the current through the motor is equal to E/R .

When the armature moves, its coils cut the lines of force of the field magnet, and an induced electromotive force, called the back E.M.F., is set up in it, just in the same way as if it were moved by hand. Let us denote this electromotive force by E' . By Lenz's law it tends to stop the motion, that is, it acts against the E.M.F. of the battery which is causing the motion. The resultant E.M.F. in the circuit is consequently $E - E'$, and the current is given by

$$C = \frac{E - E'}{R}$$

When the motor is starting, the back E.M.F. is zero. If the full E.M.F. of the battery were applied to it, the current might overheat the coils and ruin them. It is usual, therefore, to employ a starting resistance in series with the motor. As the speed rises, the back E.M.F. increases, and this resistance is gradually cut out ; the driver of an electric tramcar can be seen throwing out his resistance by means of a switch, whenever he increases speed.

Application of the Energy Principle.—Since for a series motor, one in which the armature and magnet coils are arranged in series,

$$CR = E - E'$$

we obtain, on multiplying by C ,

$$C^2 R = CE - CE'$$

Now the terms in this equation give the rate at which heat is being produced, the rate at which work is being done by the battery, and the rate at which work is being done in turning the armature, and the equation states that the first is equal to the difference of the other two, a result which might have been inferred from the energy principle. It has been derived here without the aid of the energy principle.

The terms C^2R and CE undoubtedly represent the rate of production of heat and the rate at which work is done by the battery. The energy principle requires that there should be a third term in the equation in addition to these two, or, in other words, that there should be a back E.M.F. We can thus infer the existence of the back E.M.F. without using the laws of current induction. In fact, the laws of current induction themselves may be inferred from the energy principle; it was not necessary to discover them experimentally.

Efficiency of Motor.—The electrical efficiency of a series motor, the ratio of power spent in producing rotation to total electrical power supplied, is given by $CE'/(CE)$ or E'/E .

The rate at which power is spent in producing rotation and doing external work, CE' , is equal to

$$\frac{(E - E')E'}{R}$$

It is consequently zero when $E' = 0$ and $E' = E$, and a maximum when $E' = \frac{1}{2}E$. So most work is obtained from the motor when the back E.M.F. is equal to half the applied E.M.F., but, of course, the higher the back E.M.F. is above this value, the more economically the work is being obtained.

Magneto.—The magneto, the spark from which is used to explode the gases in an internal combustion engine, has primary and secondary coils like an induction coil. But instead of using a battery the current in the primary is produced by rotating both primary and secondary coils together in the field of a permanent magnet.

Transformer.—The transformer is in principle somewhat similar to the induction coil, except that the core is usually in the form of a ring, so as to concentrate the lines of force through it as much as possible, and an alternating current is used in the primary instead of a direct current interrupted by a break.

If the current in the primary is $C \cos \omega t$, the number of lines of force through the secondary is proportional to this and also to the number of turns in the secondary. It consequently varies as

$$NC \cos \omega t$$

where N is the number of turns in the secondary. The E.M.F. in the secondary is proportional to the rate of change of this, which has been shown on p. 557 to be

$$NC\omega \sin \omega t$$

The E.M.F. in the secondary is thus an alternating one, of the same frequency as the primary current, and it is proportional to N . It may, therefore, according to the magnitude of N , be either greater or less than the E.M.F. in the primary. In the former case the transformer is said to be a **step-up** transformer and in the latter case a **step-down** transformer.

Since the rate of production of energy by a dynamo is equal to the product of E.M.F. and current generated, it is clear that in order to transmit 20,000 watts to a distance, we may use, for example, either an E.M.F. of 200 volts and a current of 100 amperes or an E.M.F. of 2000 volts and a current of 10 amperes. If the wire is the same in each case, the energy wasted in heat in the wire is 100 times as great in the first case as in the second. Hence in transmitting power to a distance, high voltage currents are employed. These are always alternating, and they are converted by a step-down transformer at the place where they are to be used, to a voltage more suitable for domestic supply.

EXAMPLES XI

1. A room was illuminated by four 16-candle-power carbon lamps which required $3\frac{1}{2}$ watts per candle. These were replaced by two 40-watt metal filament lamps which gave each about 32 candles, and finally by one 60-watt half-watt lamp which gave about 100 candles. Calculate the cost per hour in each case, given that the Board of Trade unit costs 6d.

2. An electric bell is to be installed so that it can be rung from separate rooms from the same battery. Show how to make the connections.

3. Will it take more work to rotate a dynamo armature when the circuit is closed than when it is open?

4. A single dynamo operates 10,000 incandescent lamps at 250 volts. The lamps are arranged in parallel and each takes 0.16 amp. What is the total current furnished by the dynamo, and what is its activity in kilowatts and in horse-power?

5. The same power is to be sent over two lines from a power station to a distant point. The one current is transmitted at 250 volts and the other at 10,000 volts. What must be the ratio of the areas of the cross-sections of the two lines, if the heat loss is to be the same in each case?

CHAPTER XII

CATHODE RAYS, X-RAYS, AND RADIOACTIVITY

Electric Discharge in High Vacua.—The present chapter gives an account of some of the principal results of the modern period of physical research. These deal with the nature of electricity itself and the nature of matter which is closely bound up with it; the investigations leading to them started from a study of the discharge of electricity in high vacua made towards the end of last century.

When an induction coil is working, if the terminals of the secondary are pushed close enough together, sparks pass between them. The **sparking potential** as it is called, or the potential difference at which the resistance of the air just breaks down, is at ordinary temperature and pressure about 20,000 or 30,000 volts when the spark gap has a length of 1 cm.; it varies somewhat with the shape of the terminals, and increases with the length of the gap. If the potential difference between the terminals is less than the sparking potential, the insulation is not quite perfect, but the conductivity in this case is very much less than when sparks pass.

Let us suppose now that a glass tube about 30 cm. long (Fig. 566) is provided with aluminium electrodes A and K attached to platinum wires sealed in through the glass, that it is connected by a side tube to a pump, and that the air or other gas in the tube is gradually exhausted. Let us suppose at the same time that the electrodes A and K are connected to the terminals of an induction coil, A to the positive and K to the negative terminal. As has already been mentioned, the discharge from an induction coil is practically uni-directional.

At first no spark will pass at all. Then as the pressure is decreased, a spark passes in a thin bright line with a crackling noise. As the pumping is continued, this line spreads out until

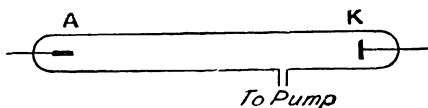


FIG. 566.

its glow fills the whole tube. The discharge becomes at the same time much smoother, and at this pressure, a millimetre or so of mercury, the difference of potential necessary to maintain the discharge is a minimum. The colour of the glow depends on the gas in the tube.

Ordinary **Geissler tubes** are simply tubes of this nature with constricted portions so as to concentrate the discharge and make it brighter, or twisted into fantastic shapes and made of fluorescent glass so as to give pretty colour effects.

Cathode Rays.—If the tube is exhausted to $\frac{1}{1000}$ of a millimetre or so, the character of the discharge is entirely changed.

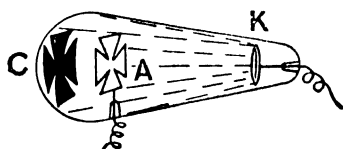


FIG. 567.—Shadow cast by cathode rays.

The residual gas in the tube becomes dark, and a pencil of almost invisible rays, the cathode rays, proceeds out from the cathode towards the anode. These rays have the following properties :

(1) *When they fall on the glass of the tube, they cause it to glow with a greenish fluorescent light.*

(2) *They travel in straight lines.* This may be shown by the tube represented in Fig. 567 which has an aluminium cross for anode. The end of the tube at C glows with a greenish light except where the cross casts a well-defined shadow on it.

(3) *They can pass through thin sheets of aluminium foil or gold leaf without making holes in them.*

(4) *They exert mechanical pressure.* This may be shown by the tube represented in Fig. 568 which contains a small paddle

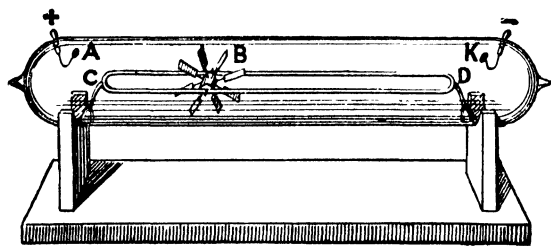


FIG. 568.—Mechanical effect of cathode rays.

wheel with mica vanes. The axle of the tube rolls on glass rails, and the rays from K strike the upper vanes at B, and cause the wheel to roll along the rails.

(5) *They carry a charge of negative electricity.* This can be shown by receiving them in a metal cylinder connected to an electroscope.

(6) *They are deflected both by a magnetic and by an electrostatic field.* In order to show this a narrow pencil is required; the latter is obtained (Fig. 569) by allowing the rays to fall on a brass plate B which has a small hole at its centre. The narrow pencil passing through the hole produces a bright patch of light on the glass at C. If the pole of a magnet is brought near the tube, this patch of light is deflected to some such position as D, the pencil of rays behaving exactly as if it were a flexible wire carrying an electric current towards the cathode. If the tube is fitted with two metal plates like the plates of a parallel plate condenser, one of which is earthed and the other charged to a high potential, so that the lines of force go straight across from the one to the other, and if cathode rays are passed between these plates at right angles to the lines of force, then they are deflected in the same direction as a negative charge would be deflected.

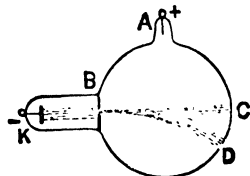


FIG. 569.—Deflection of cathode rays.

Nature of Cathode Rays.—For a long time there was a controversy as to the nature of cathode rays. On the one hand it was thought that they were a kind of invisible light, propagated by wave motion. The property numbered (3) above pointed in this direction. On the other hand it was maintained that they were streams of negatively charged particles shot off with great velocity from the cathode. It was decisively shown by Sir J. J. Thomson and his co-workers in 1898, that the latter view was the correct one, and that the particles involved, now known as **electrons**, were not the atoms or molecules previously familiar to physicists but something new and much smaller, with a mass equal to $\frac{1}{1836}$ that of the hydrogen atom and with a velocity which varies from 10^9 to 10^{10} cm. per sec., *i.e.* from one-thirtieth to one-third of the velocity of light. The ratio of the charge to the mass of the electron was found to be 1.77×10^7 **electromagnetic units per gram**. Since 1898 electrons have been found to play a part in many other phenomena besides those of the cathode ray tube, always with the same charge and the same mass. It is consequently now universally accepted, that they are one of the constituents out of which atoms are built, and that electricity itself has an atomic constitution, all charges of electricity being exact multiples of the charge carried by an electron.

Calculation of the Results.—It has already been stated (p. 534), that when a current C flows in a straight wire at right angles to a magnetic field of strength H , the wire is acted on by a force CH per unit length at right angles to itself and to the field. Let us suppose

that the current consists of a procession of electrons, each of which has a charge e measured in electromagnetic units, and a velocity v , and that there are n of them to the unit of length. All those in a length l will pass a given point in the wire in unit time. Hence the quantity of electricity passing any given point in the wire in unit time is nve , that is, the current $C = nve$, and the force on the wire per unit length can be written $nveH$. The force must be exerted on the moving electrons, since there is no force when there are no moving electrons. Consequently, there is a force on each electron equal to

$$Hev$$

and the electron has an acceleration at right angles to its direction of motion given by

$$a = \frac{Hev}{m}$$

where m is its mass.

Let us suppose that the acceleration is produced by an electric field of strength F acting at right angles to v . Then

$$a = \frac{Fe}{m}$$

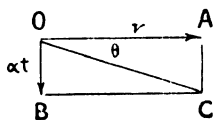


FIG. 570.

If an electron has a velocity v represented by OA (Fig. 570), and in time t acquires a velocity at right angles to v represented by OB , the resultant velocity after time t is represented by OC . The direction of motion has been changed from OA to OC , *i.e.* it has been deflected through an angle given by

$$\tan \theta = \frac{AC}{OA} = \frac{at}{v}$$

Let l be the length which would be traversed in time t if there were no deflection. Then

$$l = vt \quad \text{and} \quad \tan \theta = \frac{al}{v^2}$$

For the electrostatic field $\tan \theta = \frac{Fel}{mv^2}$

which reduces to $\theta = \frac{Fel}{mv^2}$

since θ is usually small.

Let the deflection produced by a magnetic field of strength H acting at right angles to v through the same distance l be denoted by φ . Then in the same way

$$\tan \varphi = \frac{Hevl}{mv^2}$$

or

$$\varphi = \frac{Hel}{mv}$$

If this equation is divided by the previous one

$$v = \frac{\varphi H}{\theta H}$$

hence, since F and H are known and φ and θ can be measured, v can be calculated. When the value of v is known, if it is substituted in the expression for φ above, the value of e/m can be calculated. It was by proceeding on these lines that v and e/m were first determined for a cathode ray particle.

EXAMPLE.—An electron is acted on by an electric field through a potential difference V . Find the velocity it acquires from rest, and calculate its numerical value, if V is 50,000 volts, given that e/m is 1.77×10^7 electromagnetic units per gram.

$$\begin{aligned} \text{Work done by field} &= Ve \\ \text{Kinetic Energy gained} &= \frac{1}{2}mv^2 \\ \text{Hence} &\quad \frac{1}{2}mv^2 = Ve \\ \text{and} &\quad v^2 = 2Ve/m = 2 \times 50,000 \times 10^8 \times 1.77 \times 10^7 \\ \text{whence} &\quad v = 1.33 \times 10^{10} \text{ cm. per sec.} \end{aligned}$$

Theory of Conduction.—In metals a certain number of electrons are so loosely attached to the atoms that they are shaken off and wander about in a free state. If, however, an electric field is applied, they all move in the same direction, and it is this motion that constitutes the electric current.

In insulators the electrons are fixed in the atom and cannot become free of themselves. But by friction, for example the friction of a silk cloth on a glass rod, some can be detached and removed for a time, and so the body acquires a charge. When a conductor is electrified by induction, the electrons in it drift towards one end under the influence of the external body, and that end acquires a negative charge. When an electron leaves an atom, the latter acquires a positive charge; consequently, the other end of the conductor is charged positively.

When a salt, for example sodium chloride, is dissolved in water, the cation sodium loses an electron to the anion chlorine. The former consequently acquires a positive charge and the latter a negative charge, both equal to the charge on the electron. In the case of a divalent ion, for example copper in copper sulphate, the copper ion has lost two electrons to the anion. This shows that each bond of chemical affinity must have a connection with one particular electron.

In the case of a gas under ordinary circumstances, a proportion of the molecules have gained or lost electrons by collision or otherwise. These charged molecules are referred to as **gaseous ions**. They are extremely few in number, and a gas in its normal state is a very good insulator indeed. The number of ions in a gas can be greatly increased by exposing it to X-rays; the latter shake electrons loose from neutral molecules, and these electrons attach themselves to other molecules. The gas then conducts electricity and is said to be **ionised**.

It was discovered by C. T. R. Wilson, that when dust-free air is saturated with water vapour and allowed to expand suddenly, condensation takes place on the negative ions in the air as nuclei, and drops are formed which fall as a cloud to the bottom of the vessel in which the experiment is performed. The total quantity of electricity brought down by the cloud can be measured, and it is possible to calculate the size of a drop from their rate of fall by means of a formula given by Stokes. We thus obtain the number of drops in the cloud, and consequently the charge on each drop. But as each drop condenses on an ion, the charge on a drop is the elementary charge e , the charge on the electron. It was in this way that the latter was first determined.

Positive Rays.—In addition to the cathode rays in a cathode ray tube there are luminous rays which travel towards the cathode and fall upon the surface of the latter. If the cathode is perforated, these rays go through the holes and are visible in the space behind the latter. For this reason they were called by a word which has been translated **canal rays**, though channel rays is a better equivalent. They act upon a photographic plate. They are deflected by a magnetic and an electric field, and it follows from the deflections which are much smaller than in the case of an electron, that they consist of positively charged particles, atoms which have lost electrons. It was shown by Sir J. J. Thomson that the measurement of the deflections afforded an accurate means of calculating the mass of the atoms constituting the rays, *i.e.* all the gaseous atoms within the tube, and he named this method of investigating the nature of a substance **positive ray analysis**.

The method has been improved, and results of the greatest importance have been obtained by Aston. He has shown that many of the atomic weights we are accustomed to deal with in chemistry are merely averages. For example, when the stream of positive rays consists of chlorine atoms, it decomposes under the action of the magnetic and electric fields into two streams which have atomic weights 35 and 37. Now the atomic weight of chlorine is 35.46 when determined by chemical methods. No evidence for the existence of atoms with this weight has been found in the positive ray tube. It has consequently been concluded that there are two kinds of chlorine, chemically indistinguishable, which occur together always mixed in the same proportions, but which are separated out by positive ray analysis. Substances of this nature which have different atomic weights though chemically identical, are called **isotopes**.

Positive ray analysis has shown that the atomic weights of all atoms are whole numbers. Elements like chlorine which give fractional atomic weights by chemical methods occur in chemistry

as mixtures of isotopes ; the atomic weights of these isotopes are whole numbers.

X-Rays.—It was discovered by W. K. Röntgen at Würzburg in 1895, that when cathode rays strike the glass wall of the tube they not only cause it to emit a green fluorescent light, but also to emit invisible rays which have very remarkable properties. He called them X-rays because their nature was unknown ; they are also called Röntgen rays after their discoverer.

X-rays act on a photographic plate. When they fall on a screen of barium platinocyanide they cause it to light up with a green fluorescent light. They travel in straight lines. They pass through a prism without deviation and they are not acted on by lenses. They ionise gases. They are not deflected by a magnetic or an electric field. They have the striking property of penetrating media such as paper, cloth, leather, wood, and aluminium, which are opaque to ordinary light. Flesh is transparent to them, but the bones are opaque, and by their aid we can see the bones through the flesh. They have thus an important application in medicine. They are stopped by the denser metals, especially lead. If a purse with coins in it is held up between the source of the X-rays and the fluorescent screen, in the shadow on the screen the coins are visible through the purse ; if a wooden box containing brass weights is held up, the weights are visible through the box.

Fig. 571 shows an X-ray tube and the method of taking a **radiogram** by it, as a photograph produced by X-rays is called.

The cathode is concave and concentrates the cathode rays upon the anti-cathode or target, which is made of tungsten. The X-rays are radiated out from the anti-cathode, not from the glass of the bulb as in Röntgen's first experiments, and pass through the glass wall of the bulb undeflected. The anode is not essential to the working of the bulb but steadies the discharge : it is in conducting

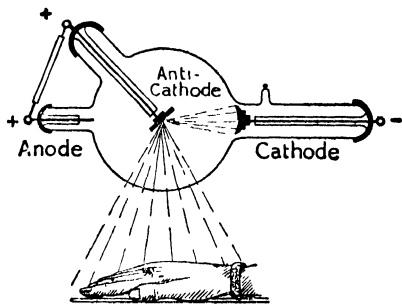


FIG. 571.—X-ray tube.

communication with the anti-cathode. The discharge is also made steadier by having the leading-in wires for the electrodes sheathed in glass tubes as shown in the figure. The cathode and anode are usually made of aluminium. The bulb is, of course, connected to the secondary of a large induction coil.

The rays emitted by the bulb in Fig. 571 are shown falling on

a hand which rests on a photographic plate. The shadow of the bones comes out on the plate when the latter is developed; the picture obtained in this way is a "negative" from which a positive can be obtained in the usual manner. Fig. 572 shows a

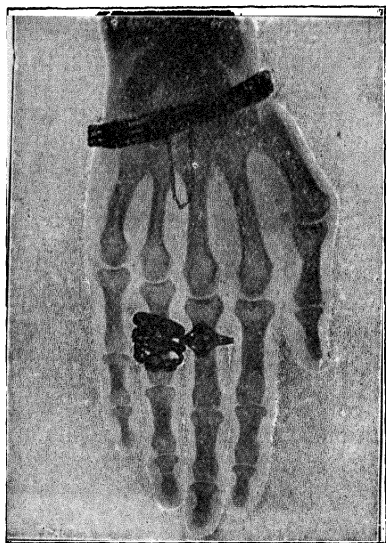


FIG. 572.—Radiogram of hand with bracelet and rings.

positive obtained in this way of a hand with bracelet and rings.

Nature of X-Rays.—For a long time after their discovery the nature of X-rays remained a mystery. They were neither reflected nor refracted, and hence it was thought that they must be essentially different from ordinary light. It was shown, however, by Laue in 1912, and afterwards by W. H. and W. L. Bragg, that they were simply light rays of extremely short wave-length, and that the experiments on reflection and refraction failed to give a positive result merely because of the shortness of the wave-length.

A surface reflects and refracts light only if the inequalities in it are small in comparison with the wave-length of the light. The very process of polishing a mirror consists in removing the inequalities in its surface. Now the wave-length of X-rays is about $\frac{1}{5000}$ of the wave-length of visible light and is of much the same size as the diameters of the molecules themselves; consequently, it is impossible for us by any means whatever to prepare an artificial surface smooth enough to reflect and refract waves of this length.

Discovery of X-Rays.—When Röntgen discovered X-rays he was looking for invisible rays. An electric discharge was passing in a tube which was completely covered with thick black paper. To his surprise a fluorescent screen three yards away shone brightly. There was no apparent cause for this. When objects were placed between the tube and screen, shadows were cast, hence the effect was due to some mysterious radiation proceeding from the tube. The radiation had the property of passing through black paper, a thing that ultra-violet light could not do, hence it could not be ultra-violet light; further investigation revealed the fact that it was radiation of an entirely new type.

Discovery of Radioactivity.—In 1896 Henri Becquerel, in Paris, found that when a small quantity of the mineral uranium was placed above a photographic plate which had been wrapped in black paper, it emitted rays which passed through the paper and affected the plate. If a coin were placed on the top of the paper and the experiment left in a dark room for several days, when the plate was developed there was a shadow picture of the coin upon it, similar to the pictures produced by X-rays. It was also found that the rays ionise air; if a small piece of uranium was placed near an electroscope, the air became conducting and the electroscope lost its charge. The property of emitting rays was shared by all uranium compounds. A substance which emits rays in this way is said to be **radioactive**.

Shortly after Becquerel's discovery Mme. Curie found that thorium and its compounds were also radioactive. During this investigation she noticed that pitchblende, the ore from which uranium is extracted, was about four times as radioactive as uranium itself. She therefore concluded that pitchblende must contain some hitherto unknown element, more radioactive than uranium. By a laborious process of elimination she succeeded in discovering a new element and in separating out a few hundredths of a gram of it from several tons of pitchblende. She named this new element radium. It discharges an electroscope more than a million times as fast as uranium or thorium does.

Nature of the Rays.—Rutherford showed in 1899 at Montreal that the rays emitted by radioactive bodies are a mixture of three types :

(i) Alpha rays. These are absorbed readily by a thin layer of air and have considerable ionising power. They are deflected by a magnetic field and an electric field. From the direction and magnitude of the deflection it has been concluded, that they are positively charged particles with a mass four times the mass of the hydrogen atom, and they have been identified as helium atoms which have each lost two electrons.

(ii) Beta rays. These are found to be identical with cathode rays, *i.e.* they are streams of electrons projected out from the substance with very great velocity. In the case of radium the velocity approaches that of light, and so is greater than the velocity of the electrons in cathode rays. Beta rays have much greater penetrating power but less ionising power than alpha rays.

(iii) Gamma rays. These are much more penetrating than beta rays, and are deflected neither by a magnetic nor an electric field. They are probably X-rays caused by the impact of the beta rays on surrounding matter.

The difference in the behaviour of the three types of rays is illustrated by Fig. 573, which is due to Mme. Curie. A small

portion of radium is placed at the bottom of a hole bored in a block of lead, and a narrow pencil of rays escapes upwards. A strong magnetic field is applied at right angles to, and towards the plane of the figure, and it separates the different types in the manner shown.

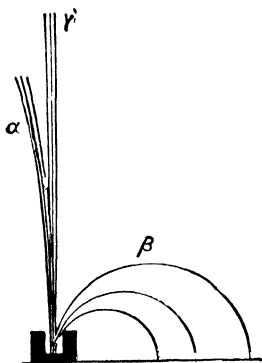


FIG. 573.—Magnetic deviation of rays from radium.

Radioactive Disintegration. — The emission of particles by a radioactive substance is independent of temperature and other physical conditions. It is proportional to the amount of active substance present, and is unaffected by the chemical state of the substance; the latter may be changed, for example, from the nitrate to the chloride without the activity being affected in any way. It is consequently wholly a property of the atom, and Rutherford put forward the view, that the atoms of radioactive

substances are slowly and spontaneously disintegrating into simpler atoms. This view is now universally adopted. Uranium and thorium are the two heaviest atoms. They seem at times to become unstable and eject alpha particles. This is in each case the beginning of a series of changes which continue until a stable form is reached, radium being an intermediate stage in the series which starts from uranium. In transforming from uranium to radium the atom loses three alpha particles; the atomic weight of radium is about 226 which is 12 less than 238.5, the atomic weight of uranium, and the atomic weight of helium is 4.

Thus, at least for some elements, the atom has lost what was once regarded as its characteristic property, indestructibility, and the transmutation of the elements which the alchemists sought for so long is taking place spontaneously before our eyes.

If the atom ejects alpha particles and electrons, it naturally follows that these are constituents of which it is built up. The discovery of isotopes has shown us, that the weights of all atoms are multiples of the weight of the hydrogen atom, consequently, the hydrogen atom may be a constituent. It may be shown by different lines of reasoning that, as has already been stated on p. 485, the atom contains a number of free electrons varying from one in the case of hydrogen up to 92 in the case of uranium. This number is known as the **atomic number**. The chemical properties of the substance are characteristic of its atomic number rather than of its chemical atomic weight. According to Rutherford, the atom consists of a nucleus with a positive charge round which the electrons circle, but this is a matter not yet fully understood.

One fact has, however, been established by experiment beyond all doubt. There is nothing positive corresponding to the electron. Positive electricity appears always as an atom or molecule which has lost one or more electrons, and the mass of the atom or molecule carrying it is very much greater than the mass of an electron.

Atomic Energy.—It was discovered by two Frenchmen, Curie and Labord, in 1903, that radium is continually giving out heat at a rate of about one hundred calories per gram per hour. The measurement was made by a Bunsen's ice calorimeter. As a result the temperature of radium bromide is always about 2° C. higher than its surroundings. Now radium disintegrates as a consequence of its activity. In 2000 years only half of the original quantity is left; this time is known as the transformation period of radium. It is possible from these figures to calculate the amount of heat energy evolved in the disintegration of one gram of radium. The result is 2.5×10^9 calories, three hundred thousand times as much as is evolved in the combustion of one gram of coal. The energy released in radioactive disintegration is energy of constitution of the atom. It is probable that similar huge quantities of energy are locked up in the constitution of all atoms. Thus the atomic energy in coal and motor spirit is enormously greater than the energy of chemical combination used to drive steam and internal combustion engines. But at present there seems no prospect of man being able to tap it and employ it for his purposes, although apparently it plays an important part in the economy of stars.

EXAMPLES XII

1. Cathode rays have a velocity of one-tenth of the velocity of light. A magnetic field of 2 c.g.s. units is applied at right angles to the direction of the rays, and the length of the path in the field is 5 cm. Calculate in radians the magnitude of the deflection produced, given that $e/m = 1.77 \times 10^7$ electromagnetic units per gram.

2. Cathode rays pass between two parallel plates 4 cm. apart. The length of the path between the plates is 7 cm., and it is found that when a battery of E.M.F. 250 volts, connected to the plates, is reversed, the rays are deflected through one-tenth radian. Calculate their velocity, given that 1 volt = 10^8 c.g.s. units, and that $e/m = 1.77 \times 10^7$ electromagnetic units per gram.

3. The atomic weights of the isotopes of chlorine are 35 and 37. If the atomic weight of chlorine is 35.46, find what quantity of each of the isotopes is present in 100 grams of chlorine.

4. If the fraction of a gram of radium which disintegrates in one hour is x , prove that the quantity remaining after n hours is $(1-x)^n$.

CHAPTER XIII

ELECTRICAL OSCILLATIONS AND WAVES

Electromagnetic Inertia.—Every electric current possesses a certain amount of inertia depending on the shape of the circuit. The inertia is especially great in those parts of the circuit which consist of coils or have many turns. This may be demonstrated by the arrangement shown in Fig. 574. When the key K is closed, the current from the battery flows in the directions shown

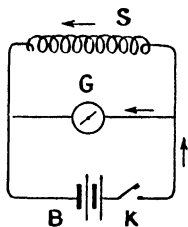


FIG. 574.

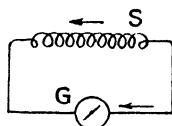


FIG. 575.

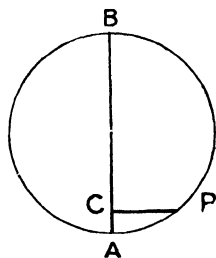


FIG. 576.

by the arrows. When the key is opened, we have only to consider the upper circuit; at the instant in question the current in the two halves is flowing the opposite way round, as shown by the arrows in Fig. 575. But the current in the coil has much the greater inertia and annuls the other current; consequently, the direction of the galvanometer deflection is reversed after the key is opened, before the current dies down.

As a consequence of its inertia the current has energy analogous to kinetic energy. If the strength of the current is denoted by i , its energy is given by the expression

$$\frac{1}{2}Li^2$$

where L is a constant known as the **inductance** or **coefficient of self-induction** of the circuit; this constant depends only on the

shape of the circuit. The practical unit in which L is measured is called the **henry**; if L is measured in henrys and i in amperes, the energy is given in joules.

Analogy of Pendulum.—Let us consider a simple pendulum of mass m and length l vibrating through an arc of length x on each side of the vertical. Let v be its velocity at the lowest point in the path. Then at the lowest point its

$$\text{Kinetic Energy} = \frac{1}{2}mv^2$$

If P is a turning point (Fig. 576)

$$AC = \frac{CP^2}{CB} = \frac{x^2}{2l}$$

since, when x is small, AC can be neglected in comparison with AB , and the distance measured along CP is equal to the distance measured along the arc. Hence at the turning point the

$$\text{Potential Energy} = mgAC = \frac{1}{2} \frac{mg}{l} x^2$$

$$\text{The period} = 2\pi \sqrt{\frac{l}{g}} = 2\pi \sqrt{\frac{\frac{1}{2}m}{\frac{1}{2} \frac{mg}{l}}} = 2\pi \sqrt{\frac{\text{coeff. in expn. for K.E.}}{\text{coeff. in expn. for P.E.}}}$$

As the pendulum moves from the turning point to the lowest point in its range, the energy gradually changes from the potential to the kinetic form.

Oscillatory Discharge of a Condenser.—In Fig. 577 A and B are the two coatings of a Leyden jar and D a spark gap formed by two well-polished knobs. S is a coil of large self-induction. A and B are connected to the terminals of a Wimshurst machine, and the inner coating of the jar is gradually charged positively. It consequently acquires potential energy given by $Q^2/2C$, where Q is the magnitude of the charge and C the capacity of the condenser. Finally, when the sparking potential is reached, the resistance of the air breaks down, and the jar discharges across the gap.

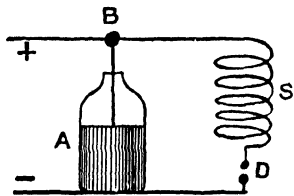


FIG. 577.

When the potentials of the two coatings of the jar are equalised, the energy of the circuit is given by $\frac{1}{2}Li^2$. Now i , the maximum current, stands in the same relation to the charge Q , as v does to x in the case of the pendulum. It seems plausible therefore to assume, that the energy in the circuit oscillates between the potential

and kinetic forms just as in the case of the pendulum, and that the period of these oscillations is given by

$$T = 2\pi \sqrt{\frac{\text{coeff. in expn. for K.E.}}{\text{coeff. in expn. for P.E.}}} = 2\pi \sqrt{\frac{\frac{1}{2}L}{\frac{1}{2C}}} = 2\pi \sqrt{LC}$$

If this is the case, the positive and negative charges do not annul each other, but are carried by their inertia across the gap, so that the outer coating acquires the positive charge and the inner coating the negative one. The jar next discharges through the gap a second time and resumes its initial condition, the positive charge being now on the inner coating and the negative on the outer. These changes are then repeated a number of times, but as the energy of the original charge is gradually dissipated as heat in the circuit, the strength of the current across the gap

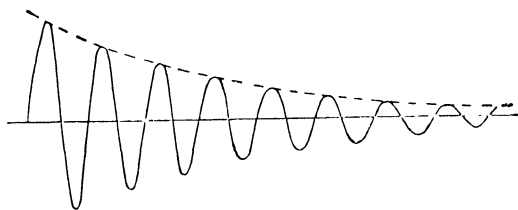


FIG. 578.—Damped oscillation.

diminishes with each repetition, until it is finally insufficient to maintain the gap in a conducting state, and the oscillations stop abruptly. The current across the gap should therefore be given as a function of the time, as

long as it lasts, by the curve shown in Fig. 578; this curve represents a **damped oscillation**.

Verification of the Formula.—The formula $T = 2\pi \sqrt{LC}$ was given first by Lord Kelvin in 1853. It had been noticed by Henry in 1842, that when a needle was magnetised by placing it inside the coil shown in Fig. 577, the north pole was sometimes at the one end and sometimes at the other, even although the condenser was always charged the same way. The current which flowed across the gap last was consequently sometimes in the one direction and sometimes in the other, and this suggested oscillations.

In 1857 Feddersen examined the image of the spark gap formed by reflection in a mirror which was rotating about an axis parallel to the gap, and showed that the discharge was an oscillatory one. Owing to the motion of the mirror the image of the spark was drawn out into a band consisting of bright lines separated by dark spaces, the lines starting first from the one side and then from the other side of the gap. Each line corresponded to a separate discharge. The period of the discharge

could be calculated from the distance between the lines and the speed of rotation of the mirror, and it was found that the result agreed with the formula.

EXAMPLE.—A Leyden jar which has a capacity of 0.0019 microfarad is discharged through a solenoid which has a self-inductance of 15,000 microhenrys. Calculate the period of the resultant oscillations.

L and C must be converted to henrys and farads before they are inserted in the formula. Hence

$$\begin{aligned} T &= 2\pi \sqrt{0.0019 \times 10^{-6} \times 15,000 \times 10^{-6}} \\ &= 3.35 \times 10^{-5} \text{ sec.} \end{aligned}$$

Electric Resonance.—Let us suppose that two tuning-forks of the same pitch, mounted on resonators, are placed near each other with the open ends of the resonators facing each other. Then if the one fork is set into strong vibration, and then damped by grasping the prongs with the fingers, the other fork will be heard sounding. If, however, the pitch of the second fork is altered by attaching a weight to one of its prongs, and the experiment repeated, when the first fork is damped, the second is silent.

The above experiment is an example of resonance, and shows what conditions are necessary for its appearance. The first tuning-fork emits sound waves; pulses of compression and rarefaction travel through the air to the second fork and beat upon it at intervals which correspond exactly to its own period. It is consequently set into vibration in the same way as a heavy pendulum, which receives a slight push every time it passes the middle of the range in a given direction. The effect of each push is slight, but the cumulative effect of a series of pushes produces large results.

Let us suppose now, that the condenser and coil in Fig. 577 are connected to the secondary of an induction coil instead of a Wimshurst machine. Every time the primary breaks, the condenser charges, and then discharges in a series of oscillations across the gap. Let a second coil be connected through a very small spark gap which can be regulated by a micrometer screw, to a variable air condenser of the type shown in Fig. 485, and let them be placed near the first circuit. Then if the capacity in the second circuit is adjusted to make the periods of the two circuits the same, electrical oscillations are set up in the second circuit, and sparks can be seen crossing its gap. But, if the periods of the two circuits are different, there are no sparks. So that we may have resonance between two circuits of this nature, just as we have resonance between tuning forks. The first circuit is usually referred to as the vibrator or oscillator and the second as the receiver or resonator.

Electromagnetic Waves.—The one tuning fork acts on the other by means of sound waves. It follows that the oscillator must also act on the receiver by means of waves which it radiates in all directions, electromagnetic waves they are called. Clerk Maxwell showed mathematically in 1865 that such waves must exist, that their velocity was the same as the velocity of light, and that they were essentially of the same nature as light waves. But it was not until 1887–88 that their existence was demonstrated experimentally. This was done by Hertz. In Feddersen's experiments proving the formula for the period of the oscillator, the periods worked with were about 10^{-5} sec. By reducing the capacity and inductance of the circuit Hertz brought the period down to about 10^{-8} sec., and succeeded in showing that electromagnetic waves differed from light waves only in having a much longer wave-length. Since $\lambda=VT$, the wave-length of the waves used by Hertz was about 3 metres as against the value 5.9×10^{-5} cm. for yellow light.

The waves used in wireless telegraphy and telephony are electromagnetic waves. Oscillations are generated first in a circuit containing capacity and inductance; these induce oscillations in the aerial which is tuned to resonance with it, and which radiates waves out into space much more effectively than the oscillator itself could.

Generation of Electric Waves.—The older way of generating electric waves was to charge a condenser by means of an

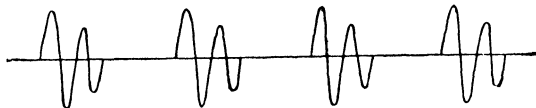


FIG. 579.—Train of damped waves.

induction coil and let it discharge through an inductance and a spark gap. Every time the condenser discharged, a damped wave was radiated which stopped abruptly, and the whole wave train could be represented by some such diagram as Fig. 579. The message was sent by breaking up the train into long and short periods according to the Morse code, and the condenser discharged perhaps one hundred times to the second.

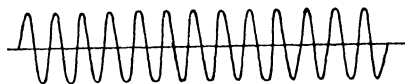


FIG. 580.—High frequency wave.

Nowadays in wireless telephony a continuous wave is used. Suppose a station broadcasts on a wave-length of 400 metres; then the wave may be represented as

in Fig. 580, where the distance between successive crests or successive troughs is 400 metres, and the period is 1.33×10^{-6} sec.

This is known as the **high frequency** wave. If the wave is broadcasting a musical note of frequency 256, the crests vary in height as in Fig. 581, their tops all lying on the dotted curve which represents a wave of frequency 256 and consequently of length 1.17×10^6 metres. This is known as the **low frequency** wave. If ordinary speech is being transmitted, the frequency and amplitude of the low frequency wave vary along the wave train.

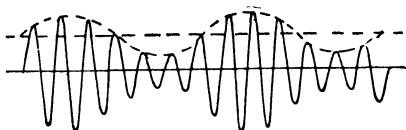


FIG. 581.—High frequency wave with low frequency wave superposed.

It would lead too far to explain how the continuous wave is produced, but the method consists essentially of a circuit with capacity and inductance, the oscillations of which act on a "valve," causing the latter to feed back energy into the circuit from a battery and so maintain them an indefinite time, once they are started. The process is roughly analogous to that by which an escapement feeds the energy of a spring into a pendulum.

Crystal Detector.—The simplest way of detecting the presence of electromagnetic waves would be to observe the sparks crossing the gap of a circuit tuned to the frequency of the wave. This method is, however, very insensitive, and could only be used in the case of extremely strong waves. The telephone is a very sensitive method of detecting variations in the strength of a current, but the high frequency oscillations occur much too rapidly for the diaphragm to follow, and if the telephone is exposed to the full wave, the diaphragm simply stands still and nothing is heard.

If a crystal such as carborundum, zincite, or galena is placed in contact with the end of a wire (the "cat's whisker"), or if two crystals such as zincite and bornite are placed in contact, the electric current passes across the surface of contact much more easily in the one direction than in the other. Such an arrangement is called a rectifier. If, for example, the current in a circuit



FIG. 582.—Rectified wave.

is represented by the full curve in Fig. 582, and a rectifier is inserted, the ordinates on one side of the axis are all diminished in the manner shown, owing to the resistance being greater in that direction. The average value of the ordinate is consequently given by the dotted curve, and varies with the low frequency wave. The latter is slow enough for the diaphragm of the telephone to follow; thus if a telephone is inserted in the

circuit, we hear the sound represented by the low frequency wave.

There are other forms of rectifier besides the crystal detector, but the latter is the simplest of them all.

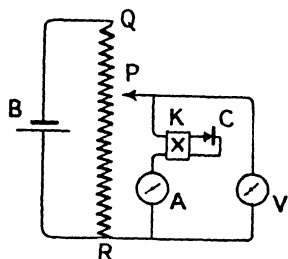


FIG. 583.—Test of a crystal.

crystal. The E.M.F. applied to the crystal is varied by moving P along the resistance. Before starting the test the contact is adjusted, until a "sensitive spot" is found.

Fig. 584 shows the nature of the results obtained for a zincite-bornite combination, which has the advantage that a firm pressure can be applied, so that the setting is not affected by vibration. The abscissæ give volts, the ordinates current in microamperes. If Ohm's law held, the graph would be a straight line through the origin. The inclination, however, changes near the origin showing that the resistance is much less for positive than for negative voltages. For +1 volt the current is 470 microamperes, giving an effective resistance of 2130 ohms; for -1 volt it is 22 microamperes giving a resistance of 45,500 ohms, 21 times as great. The more abrupt the change in the inclination at the origin, the greater the efficiency of the arrangement as a rectifier.

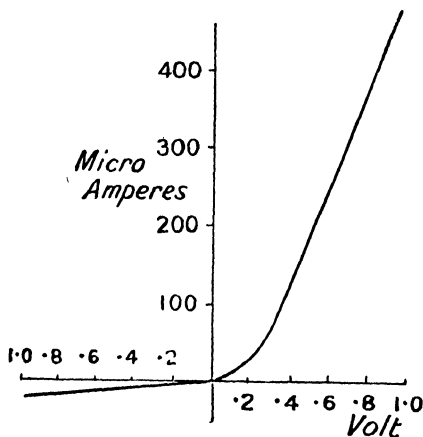


FIG. 584.—Characteristic curve for zincite-bornite.

From a diagram by S. O. Pearson in the *Wireless World*, May 5, 1926.

Receiving Aerial.—Fig. 585 shows a receiving aerial, tuning inductance, and the earth. The aerial is pointed towards the sending station, and may be regarded as forming one plate of a

condenser of which the earth forms the other plate. C is therefore given. The process of tuning consists in altering L until the period of the aerial, $2\pi\sqrt{LC}$, coincides with the period of the waves; then resonance oscillations are set up in the aerial. L is altered either by sliding a contact along a cylindrical coil and so changing the number of turns used, or by rotating a circular coil inside another one of greater radius about a common diameter of both as axis: the second arrangement is known as the variometer; and gives maximum inductance when the coils are in the same plane and the current flows round them in the same direction.

The circuit may also be tuned by altering C instead of L. A variable condenser is arranged in series with the aerial as in Fig. 586 or in parallel with it as in Fig. 587, and the resultant capacity altered by adjusting this condenser; in these figures the aerial



FIG. 585.

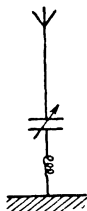


FIG. 586.

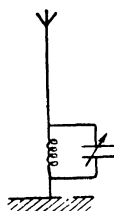


FIG. 587.

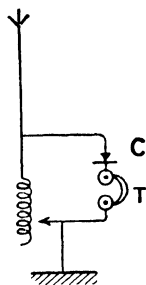


FIG. 588.

and variable condenser are represented in the manner usually employed in diagrams of this nature.

So far we have shown only how the oscillations are set up. In order to detect them a side circuit containing crystal and telephone must be connected on. Fig. 588 shows one way of doing this to the aerial shown in Fig. 585. The resistance of crystal and telephone is so great, that the current drawn off does not disturb the oscillations in the main circuit.

In the diagrams we have distinguished too sharply between the positions of the capacity and inductance in the circuit. There is some inductance in the aerial and some capacity in the coil. A coil alone, with no condenser attached, will oscillate in resonance with an electric wave, provided it has the correct dimensions.

EXAMPLE.—A coil has an inductance of 686 microhenrys and a capacity of 9.5×10^{-6} microfarad. Calculate its natural wave-length in metres. What is its wave-length in metres when shunted by a capacity of 0.0001 microfarad?

$$\text{Natural wave-length} = 2\pi V \sqrt{LC}$$

$$= 2\pi \times 3 \times 10^8 \sqrt{686 \times 10^{-6} \times 9.5 \times 10^{-12}}$$

$$= 152 \text{ metres}$$

$$\text{Wave-length when shunted} = 2\pi \times 3 \times 10^8 \sqrt{686 \times 10^{-6} \times 1.095 \times 10^{-10}}$$

$$= 516 \text{ metres.}$$

EXAMPLES XIII

1. A condenser of capacity 0.001 microfarad is discharged through an inductance of 5 microhenrys. Calculate the period of the oscillations produced in seconds and the wave-length radiated in metres.

2. A rotating mirror is placed at a distance of one metre from the spark gap in the preceding question. How fast must it be rotated, in revolutions per second, if the images of the successive sparks in the same direction are 1 cm. apart?

EXAMINATION QUESTIONS

PART I.—DYNAMICS AND PROPERTIES OF MATTER

OXFORD AND CAMBRIDGE SCHOOLS EXAMINATION BOARD. HIGHER CERTIFICATES¹

PHYSICS

1. Draw the velocity-time diagram for a point describing a straight line with uniform acceleration, and deduce the formula $v^2 = u^2 + 2fs$.

A pit cage descends part of its journey with uniform acceleration f , and the remainder with uniform retardation $2f$. If it starts from rest, and if the shaft is h feet deep, show that the whole time of descent is $\sqrt{3h/f}$. (1922.)

2. Define velocity, uniform velocity, and relative velocity.

Two men can each row in still water at the rate of 5 miles an hour. One rows up stream a distance equal to the width of the river (half a mile) and back again; the other rows straight across and back again, Assuming that the velocity of the stream is 3 miles per hour, find the difference in the time taken by the two men. (1921.)

3. What are the characteristics of simple harmonic motion?

A particle moves in a straight line with simple harmonic motion about a point A. Find an expression for the velocity at any point. The particle starts at a distance of 5 feet from A, and when its distance from A is 3 feet, its velocity is 20 feet per sec. Find its velocity when it passes through A. (1921.)

4. Prove that the motion of a simple pendulum, when of small amplitude, is approximately simple harmonic, and find its period.

A clock pendulum which beats seconds on the earth's surface, where $g=32$, is taken down a mine of depth $\frac{1}{4}$ mile. Assuming g to vary directly as the distance from the earth's centre, and that the earth's radius is 4000 miles, find the number of seconds lost by the clock in a day. (1922.)

5. State the laws of friction, and describe the experiments you would make to illustrate them.

A body resting on a horizontal plane is pulled by a string inclined at an angle of 30° to the horizontal. If the coefficient of friction is $\frac{1}{4}$, find the ratio of the weight of the body to the tension when the body is just slipping. (1923.)

¹ These questions and the questions on pp. 600, 604, 606, and 612 are reprinted here by permission of the Oxford and Cambridge Schools Examination Board.

6. State the principle of the conservation of momentum and describe any experiments you would make to verify it.

A machine-gun is mounted on an aeroplane, and when the latter is flying horizontally with a velocity of 60 miles per hour the gun is fired ahead horizontally for 10 seconds. Find the change in velocity of the aeroplane and the force tending to move the gun, relative to the aeroplane.

The mass of the aeroplane is 1000 lb., the rate of firing of the bullets 10 per second, the mass of a bullet $\frac{1}{2}$ oz., and the muzzle velocity 2000 feet per second. (1923.)

7. State the principle of conservation of momentum in an impact.

A bullet of mass $\frac{1}{2}$ oz. is fired horizontally with velocity 1000 feet per second into a block of wood resting on a smooth horizontal table and weighing 4 lbs. If the bullet remains embedded in the block, find the velocity with which the latter finally moves. (1922.)

8. Prove that in order to keep a body of mass m moving with constant speed v in a circle of radius r , a force $(mv^2)/r$ directed towards the centre must be supplied.

Find the height of the outer rail above the inner rail on a railway track in order that a train may take a curve of radius 500 yards at 30 miles an hour without producing any side-thrust on the rails, the breadth of the track being 4 feet. (1923.)

9. Define work, power, horse-power.

Find the horse-power of an engine which draws a train of mass 200 tons up a slope of 1 in 112 with a steady velocity of 25 miles per hour, the resistance due to friction being 6 lb. wt. per ton. (1923.)

10. Define work, power, horse-power.

A cylinder of an engine is 54 inches in diameter, and the average speed of the piston is 900 feet per minute. If the average pressure of steam on the piston is 32 lb. weight per square inch, find the horse-power indicated by the engine. (1922.)

11. Define kinetic energy, and find an expression for the kinetic energy of a mass m moving with velocity v .

A pile driver weighing 400 lb. falls through 9 feet on to a pile weighing 80 lb., and at each impact the pile is driven in 10 inches. Find the average resistance to be overcome. (1921.)

12. Prove that the pressure at a point in a liquid is the same in all directions.

Find the resultant pressure and the centre of pressure for each of the sides of a cube of which each side is a square metre, and which is filled with water. (1921.)

13. Explain what is meant by the pressure at a point in a liquid and by centre of pressure.

Find the centre of pressure of a square area of side 3 feet, immersed vertically in water with one edge on the surface. (1923.)

14. Find the pressure at a depth x in a liquid of density ρ .

Find also, if the atmospheric pressure is 14 lb. weight per square inch, and if a cubic foot of sea water weighs $62\frac{1}{2}$ lb., the depth in the sea at which a pressure of 20 atmospheres is reached. (1922.)

15. Plot a curve showing the relation between the extension and the stretching force for the given spiral spring.

Find the work done in stretching the spring through a distance of 5 cm., (a) when the initial load is 100 gm., (b) when the initial load is 200 gm. (1921.) (Practical Examination.)

16. Make observations to determine the relation between the weight applied to the given spring and the extension produced.

Represent your results graphically, and from your graph determine the work required to stretch the spring 3 cm. when the initial load is (a) 50 grams, (b) 200 grams. (1923.) (Practical Examination.)

17. Make a suitable cage and place ice in it. When the two are just immersed in water, remove them and weigh the ice. Hence determine the density of the ice. (1922.) (Practical Examination.)

ELEMENTARY SCIENCE

1. State Newton's laws of motion, and describe carefully how you would verify the second law of motion. (1923.)

2. Show that the kinetic energy of a body of mass m , when moving with velocity v , is measured by $\frac{1}{2}mv^2$.

Explain what is meant by the Principle of the Conservation of Energy. How does it apply to (a) a vibrating pendulum, (b) a body sliding down a rough inclined plane? (1923.)

3. Explain what is meant by the pressure at a point in a liquid, and find the value of the pressure at a point at a distance h below the surface of a liquid of density ρ .

A bubble of volume 0.001 cub. in., formed at a depth of 20 ft. below the surface of a lake, rises to the surface. If the height of the barometer is 30 in., and the density of mercury 13.5 grams per c.c., find the volume of the bubble when it reaches the surface. (1923.)

4. Make observations to determine how the square of the time of oscillation of a simple pendulum varies with the length of the pendulum.

Represent your results graphically, and deduce the value of the acceleration due to gravity. (1921.) (Practical Examination.)

5. Suspend three masses at points along the given string which is fixed at its two ends. Given one mass, determine the other two by a graphical method. (1922.) (Practical Examination.)

6. With the given Boyle's Law apparatus, make observations on the relation between the volume (v) of the enclosed gas and the difference (p) between the pressure of the gas and atmospheric pressure. Represent graphically the relation between p and $1/v$, and deduce from your graph a value for the height of the barometer. (1923.) (Practical Examination.)

7. With the given apparatus make experiments to verify the relation between the pressure at a point in a liquid, and the depth of the point below the surface. (1921.) (Practical Examination.)

SUBSIDIARY SUBJECT

1. A body, starting from rest with a uniform acceleration, is observed to travel 20 feet during one second, and 40 feet during the next second. How far had it travelled before the first observation began? (1922.)

2. What is the character of the motion of a body which falls under the action of gravity?

How long will a body take to fall 144 feet from rest, and how far will it go in the last second of its motion? (1921.)

3. Define work and energy, and state what you know of the Principle of Conservation of Energy.

Apply this principle to a pendulum, and find the velocity with which the bob of a simple pendulum must be projected in order to describe a quadrant of a circle. (1923.)

4. Write a short essay on machines, with special reference to levers and systems of pulleys. (1922.)

5. What do you understand by the mechanical advantage and velocity ratio of a machine? Why do they usually differ from one another?

If the mechanical advantage of a machine is 5 and the velocity ratio 8, what force must be exerted, and how much work must be expended, to raise a mass of 200 lb. uniformly through a height of 20 ft.? (1921.)

6. What is meant by (a) the mechanical advantage, (b) the velocity ratio, of a machine? Describe some machine, and find expressions for both of these in the machine you describe.

If a machine has mechanical advantage 6, and velocity ratio 9, find the work done in raising a mass of 1 ton uniformly through 25 ft. (1923.)

7. Describe any form of hydrometer, and give an account of its use in the laboratory. How would you calibrate the hydrometer you describe? (1923.)

8. Define density and specific gravity.

Under what conditions are these represented by the same number?

How would you determine the density of sugar? (1921.)

9. Describe any method of determining the specific gravity of a substance which is soluble in water. (1922.)

PART II.—HEAT

LONDON UNIVERSITY MATRICULATION ¹

1. Describe experiments to justify the use of the boiling point of water as a fixed point in a Thermometric scale, and give a full account of how the 100° mark is actually determined on a centigrade thermometer. (1924.)

2. The absolute coefficient of expansion of mercury per degree Centigrade is 0.000182 and its apparent coefficient measured by a glass

¹ These questions and the questions on pp. 603, 604, 605, 609, 610, and 617 are reprinted here by the permission of the London University Press.

weight thermometer is 0.000155. Find the coefficient of linear expansion of the glass per degree Fahrenheit, giving your reasons for the different steps in your calculations. (1924.)

3. Explain the term *coefficient of expansion*.

A glass bottle at 0°C . has a volume of 50 c.c. What mass of alcohol would it hold at 20°C if the coefficient of linear expansion of glass = 0.000088, the density of alcohol at 0°C . = 0.810 gm. per c.c., and its absolute coefficient of expansion = 0.00105? (1919.)

4. Describe a constant volume air thermometer.

Air at 50°F . is heated at constant volume till the pressure is increased by 10 per cent. Find the new temperature ($^{\circ}\text{F}$.). (1923.)

5. How is *heat quantity* measured?

What mass of steam at 100°C . must be passed into a swimming bath 50 ft. $\times$ 40 ft. $\times$ 6 ft. so that the temperature of the water may rise from 3°C . to 13°C .?

(Latent heat of steam = 540. 1 c. ft. of water weighs 62.5 lb.) (1920.)

6. Taking the latent heat of fusion of lead as 5.0, its specific heat as 0.030, and its melting point as 330°C ., find the temperature to which 5000 grams of water at 12°C . will be raised by pouring into it 200 grams of molten lead at its melting point. (1921.)

7. Give a short account of the chief phenomena occurring in the change of state from solid to liquid. (1920.)

8. What is meant by "saturated vapour"?

How would you find (a) the saturated vapour pressure of water at 30°C ., (b) the temperature at which the water vapour in a room would become saturated? (1920.)

9. Describe how to determine the melting point of a solid by plotting its cooling curve. Explain the theory underlying the experiment. (1921.)

10. Define calorie and British Thermal Unit, and find how many of the former correspond to one of the latter, using such approximate values of the relations between British and Metric Units as you know.

The calorific value of coke is 13,000 B.Th.U. per lb. Find the minimum amount of coke which would have to be burnt in order to heat 30 gallons of water from 60°F . to 100°F . for use in a bath.

(1 gallon of water weighs 10 lbs.) (1924.)

11. State briefly the reasons for the acceptance of the theory that heat is a form of energy in preference to the older theory of a heat fluid (caloric).

On test a steam plant was found to use 2 lb. of coal for every H.P. hour of work supplied by the engines. The calorific power of the coal was 12,000 British Thermal Units per lb.; 1 H.P. is 33,000 foot-lbs. per minute, and Joule found that 770 ft.-lbs. of work are used in generating 1 B.Th.U. of heat. What proportion of the heat obtained from the coal was turned into work? What happens to the rest of the heat? (1923.)

OXFORD AND CAMBRIDGE SCHOOLS EXAMINATION
BOARD. HIGHER CERTIFICATES

PHYSICS

1. Write a short essay on thermometry, including some method of measuring very high temperatures. (1922.)

2. Define the coefficient of linear, and of cubical, expansion of a material when heated, and show that the latter is approximately three times the former.

How would you determine *either* the coefficient of linear expansion of a short rod of quartz, *or* the coefficient of cubical expansion of mercury? (1923.)

3. Distinguish between absolute and apparent expansion.

Describe a method of determining the absolute coefficient of expansion of a liquid. (1921.)

4. Describe experiments which have been made to investigate the relation between the pressure and the volume of a gas at constant temperature. Give an account of the main results. (1921.)

5. Write a short essay on the relation between the pressure and volume of a gas, including some experimental modes of determining it. (1923.)

6. Define temperature (*a*) on the centigrade scale of the mercury-in-glass thermometer, (*b*) on the centigrade scale of a vapour-pressure thermometer.

The table below gives the vapour pressure of a liquid at different temperatures on the centigrade mercury thermometer. Draw a graph showing the relation between the temperatures on the mercury thermometer and the centigrade vapour-pressure thermometer, and find the temperature of the former which corresponds to 60° on the latter.

Temperature on Mercury Thermometer	0	20	40	50	60	70	80	90	100
Vapour Pressure in mm. of Mercury	4.6	17.5	55.1	92.3	149	233	355	526	760

(1923.)

7. Describe the steam calorimeter, and show how it may be used to determine the specific heat of a liquid. (1921.)

8. Give an account of any method of determining the mechanical equivalent of heat, and mention any special precautions against error which are necessary. (1922.)

9. Explain what is meant by the coefficient of thermal conductivity of a substance, and describe briefly some method of measuring it for a metal.

A vessel with an area of 200 sq. cm., and 2 mm. thick, is filled with ice and placed in a vessel kept at 100° C. How many grams of ice will melt per minute when conditions have become steady?

Latent heat of fusion of ice, 80 calories per gram.

Conductivity of the substance, 0.002 C.G.S. units. (1923.)

10. How would you verify the laws of reflection and refraction of thermal radiations? (1921.)

11. Describe the bolometer or any other instrument used for measuring the intensity of thermal radiations. (1922.)

12. Determine the mean coefficient of expansion of turpentine between room temperature and the temperature of boiling water, using a specific gravity bottle as a weight thermometer. (1923.) (Practical Examination.)

13. Make observations on the rate of cooling of a mass of water in the given calorimeter. Draw a curve to represent the variation of temperature with time, and from this curve obtain a second curve, showing how the rate of emission of heat varies with the difference between the temperatures of the calorimeter and the room. (1921.) (Practical Examination.)

ELEMENTARY SCIENCE

1. Distinguish between apparent expansion and absolute expansion. Describe a method of determining the coefficient of absolute expansion of a liquid. (1921.)

2. Define coefficient of increase of volume and coefficient of increase of pressure, with temperature, of a gas, and describe how you would measure the latter. (1922.)

3. What is meant by the latent heat of a substance?

How would you determine the latent heat of an unknown substance which melted at about 50°C .? Describe any precautions you would take. (1922.)

4. What is meant by the vapour pressure of a liquid? How would you measure the vapour pressure of water at temperatures between 70°C . and 105°C .? (1921.)

5. Describe a method of determining the mechanical equivalent of heat.

Mention any sources of error in the method and show how these are eliminated. (1921.)

6. Describe how the specific heat of a substance may be measured by the method of cooling.

500 grams of water and an equal volume of a liquid of mass 400 grams when placed successively in the same calorimeter cool from 55°C . to 50°C . in 280 seconds and 200 seconds respectively. If the mass of the copper calorimeter is 200 grams, and the specific heat of copper 0.1, find the specific heat of the liquid. (1923.)

7. Describe a method of comparing the specific thermal conductivities of two substances.

A vessel 1 cm. thick and with an external area of 400 sq. cm. contains water. The outer surface is kept at a temperature of 150°C . and the water at 100°C . How much water will evaporate in 10 minutes?

The specific thermal conductivity of the material of the vessel may be taken as 0.2 c.g.s. unit, and the latent heat as 536. (1922.)

8. Make experiments to verify the relation between the pressure of a gas and its temperature. Represent your results graphically, and deduce the coefficient of increase of pressure with temperature. (1923.) (Practical Examination.)

9. Determine the latent heat of fusion of ice. (1921.) (Practical Examination.)

10. Make observations to determine the temperature of the given solid three minutes after removal from boiling water. The specific heat of the solid is not given. (1922.) (Practical Examination.)

11. Determine the dewpoint by two methods. For one method a polished calorimeter and ice will be provided; for the other a wet and dry bulb thermometer and suitable tables.

Calculate the relative humidity. (1922.) (Practical Examination.)

SUBSIDIARY SUBJECT

1. Write a short essay on the measurement of temperature. (1921.)

2. Describe some form of weight thermometer, and give an account of its experimental use. (1922.)

3. Show that the coefficient of cubical expansion of a substance is practically three times that of linear expansion.

Give a careful account of any method of finding the coefficient of expansion of a liquid. (1922.)

4. What is meant by the "coefficient of expansion of a substance"?

Describe a method of measuring the coefficient of linear expansion of a metal, pointing out any sources of error in the method.

How is the coefficient of cubical expansion related to that of linear expansion? (1921.)

5. Give an account of any method of determining the coefficient of cubical expansion of a liquid, stating the corrections you would apply to your observations. (1923.)

6. Describe the main sources of error in the determination of specific heats by the calorimetric method, and explain how they may be overcome. (1921.)

7. State the experimental arrangements necessary in the determination of (a) the latent heat of fusion of ice, and (b) the latent heat of steam. Indicate in each case how you would perform the necessary calculations. (1922.)

8. Describe the changes which take place in the volume of a mass of ice initially at -5°C . as its temperature is raised until it is converted into steam at 100°C .

Would a thermometer filled with water be any use in finding temperatures between 2°C . and about 8°C .? (1921.)

9. Show how to determine the specific heat of a liquid by the method of cooling. (1922.)

10. Explain carefully what is meant by vapour pressure, and describe how you would measure the vapour pressure of a liquid at different temperatures. (1923.)

LONDON UNIVERSITY INTERMEDIATE SCIENCE

1. Describe a good method for the determination of the linear coefficient of thermal expansion of a metal bar. (1920.)

2. Explain how the thermal expansion of a metal rod may be measured. (1920.)

3. What is meant by the statement that the coefficient of linear expansion of steel is 0.000012 per degree C. ?

If a clock with an uncompensated pendulum of steel keeps correct time at 15° C., how many seconds would it lose in 24 hours at a temperature of 25° C. ? (1922.)

4. Describe some form of constant volume air thermometer. Explain how you would graduate it and how you would use it to find the boiling point of a salt solution.

In what respects, if any, is an air thermometer superior to a mercury thermometer ? (1923.)

5. Discuss the advantage of a constant volume air thermometer as compared with an air thermometer under constant pressure. (1921.)

6. Define *latent heat* and describe how to determine it for the fusion of ice. (1920.)

7. Describe how you would measure the dewpoint, and explain carefully how the relative humidity of the atmosphere may be ascertained from a knowledge of the dewpoint and of the saturation vapour pressure of water at different temperatures. (1921.)

8. Describe and explain a method of finding the dewpoint. How could you utilise the result to find the weight of water vapour contained in a cubic metre of the air ? (1923.)

9. Explain what is meant by the statement that the mechanical equivalent of heat is 4.2×10^7 ergs per calorie.

An aluminium fly-wheel, radius 7.5 cm. and mass 1000 gm., is driven at the rate of 100 revolutions per minute. Its rim is braked by a silk band, one end of which carries a load of 2 kgm., while the other is attached to a spring balance reading 180 gm. Assuming all the heat generated is spent in raising the temperature of the wheel, calculate its rise of temperature in one minute. (Specific heat of aluminium 0.22.) (1923.)

10. Why is the quantity of heat required to raise the temperature of a given mass of air through 1° C. greater when the air is allowed to expand than when its volume is kept constant ? Explain how this fact has been used to obtain a value for the mechanical equivalent of heat. (1923.)

11. Explain the statement that the thermal conductivity of glass is 0.002 in C.G.S. units.

The inside of a glass window, 0.3 cm. thick, is at a temperature of 15° C. and the outside at 5° C. Estimate the rate at which heat is escaping through the window if its area be 2 sq. metres. (1922.)

PART III.—SOUND

LONDON UNIVERSITY MATRICULATION

1. Draw a diagram showing the passage of transverse waves along a line of particles, and by the help of your diagram indicate the meaning of the words *amplitude*, *phase*, *period*, *wave-length*, *frequency*. (1922.)

2. How does sound travel through gases?

Indicate briefly how, if at all, the velocity of a sound depends (a) on its wave-length, (b) on the density of the gas, (c) on the temperature of the gas. (1921.)

3. During bombing practice in the neighbourhood of a corrugated iron fence an observer noticed that the explosion of each bomb was followed by a shrill echo. Explain this with a rough diagram of the relative positions of the bomb, the fence, and the observer. (1920.)

4. Prove the relation connecting the velocity of propagation of a wave with its frequency and wave-length.

Give an account of the various modes of transverse vibration of a stretched string and explain how you would show that the frequency is inversely proportional to the wave-length, the tension being constant. (1919.)

5. How would you verify with a sonometer the law connecting the frequency of a stretched string with its tension?

If an additional 75 lb. raises the pitch an octave, what was the original tension? (1923.)

6. What is meant by an *overtone*?

Give an account of the first three overtones which may be present in a vibrating air column closed at one end. How could you demonstrate the existence of overtones when a musical note is sounded (*e.g.* when a note is struck on a piano)? (1920.)

OXFORD AND CAMBRIDGE SCHOOLS EXAMINATION
BOARD. HIGHER CERTIFICATES

PHYSICS

1. Describe and compare the characteristics of the wave motion in the following cases:

- (a) a progressive sound-wave in air;
- (b) the motion in an organ pipe adjusted to resonance.
- (c) the vibration of a stretched string. (1923.)

2. Write a short essay on resonance, illustrating your answer by examples from as many branches of physics as possible. (1923.)

3. Describe fully a method of determining the pitch of a tuning-fork. (1922.)

4. Describe a method of producing stationary waves using (a) longitudinal waves, (b) transverse waves.

How would you show the position of the nodes and anti-nodes in each case? (1921.)

5. Adjust the length of the given tube for resonance with tuning-forks of different known frequencies. Represent your results graphically by plotting a curve to illustrate the relation between the reciprocal of the frequency and the corresponding length of the resonance column.

Deduce a value for the "end-correction." (1922.) (Practical Examination.)

LONDON UNIVERSITY INTERMEDIATE SCIENCE

1. Describe a good method to determine the frequency of the note emitted by a tuning-fork. (1923.)

2. A light string carrying a load of 100 grm. is attached to the prong of a tuning-fork and vibrates in three segments when the fork is bowed. Explain this and show how the string may be made to vibrate in five segments by either (a) altering its length, or (b) changing the load. (1923.)

3. Describe the nature of the motion and of the pressure changes within an open organ pipe sounding its fundamental. If the note has the same frequency as that of a siren with a disc containing 12 holes and making 1000 revolutions per min., when the velocity of sound in the air is 1100 feet per second, calculate the length of the organ pipe. How nearly would you expect the measured length to agree with that calculated? (1923.)

PART IV.—LIGHT

LONDON UNIVERSITY MATRICULATION

1. Describe how to compare the effective candle-power of a lamp covered with a glass shade with its candle-power when the shade is removed.

Find the ratio of the distances of the lamp from the photometer if the shade absorbs 9.75 per cent. of the light emitted by the lamp, assuming the distance of the comparison source to be the same in each case. (1921.)

2. Describe an arrangement by which you could investigate the percentage of light reflected at different angles by a piece of plain glass. (1921.)

3. Prove that if a mirror rotates, a ray reflected from it turns through twice the angle through which the mirror turns. How would you verify this experimentally? (1922.)

4. Explain the formation of images by multiple reflections.

Two parallel mirrors, 12 ft. apart, face each other, and an object is placed 3 ft. from one mirror. Represent to scale the first three images seen in each mirror, and draw rays whereby an observer, looking into one of the mirrors, sees an image formed after five reflections in all. (1920.)

5. Draw as accurately as you can the path of three pencils of rays from a point under water (refractive index $\frac{4}{3}$) to three different eyes above the surface, the angles of incidence of the central rays of the pencils in the water being 0° , 30° , 45° . By means of your figure explain the change in the apparent depth of the water as the position of the observer changes. (1923.)

6. A bright point P is 2 in. from the front surface of a plane mirror consisting of a piece of plate glass $\frac{1}{2}$ in. thick silvered at the back. Draw a careful diagram showing the positions of the first two images of P formed by the glass. Does the position of either depend upon your point of view? (1921.)

7. Explain the term *principal focus* of a concave spherical mirror, and find from first principles its approximate position.

Give a geometrical construction for finding the position and nature of the image of a small object formed by a concave mirror, and obtain a formula for the magnification. (1921.)

8. In what circumstances does a convex lens form a real image?

A lens of 12 in. focus is used to throw an image of a lantern slide 3 in. square on a screen 50 ft. away. Where must the lantern slide be placed, and what will be the size of the picture? (1922.)

9. A convex lens of 10 in. focal length forms a virtual image three times the height of the object. Where must the object be placed? Solve the problem first by drawing and then by formula. (1921.)

10. What conditions are necessary for a lens to form a real image smaller than the object? Give an explanatory diagram.

A camera lens of focal length 1 ft. is in focus for distant objects. What change must be made in the distance between the lens and the film in order that an object 10 ft. off may be in focus? What would the corresponding change be for a camera lens of 6 in. focal length? (1923.)

11. ABC is a section of a right-angled glass prism of refractive index 1.5. $\angle ABC = 90^\circ$, $AB = BC = 2$ in. Rays parallel to AC are incident on the face AB at points $\frac{1}{2}$ in. and $1\frac{1}{2}$ in. from A. Trace the subsequent course of the rays. (1921.)

12. Explain why one or more lenses are usually used with a prism to form a spectrum.

A prism is set in the minimum deviation position for yellow light. How must it be turned to bring it into the minimum deviation position for blue light, and what effect will this turning have on the spectrum? (1922.)

OXFORD AND CAMBRIDGE SCHOOLS EXAMINATION BOARD. HIGHER CERTIFICATES

PHYSICS

1. Find an expression for the magnification of a small linear object placed perpendicular to the axis of a concave mirror.

Show that for a small linear object along the axis of the mirror the

magnification is the square of the magnification for the same object placed perpendicular to the axis. (1921.)

2. How would you determine the focal length of (a) a double convex lens, and (b) a double concave lens?

Find an expression for the focal length of a thin lens in terms of the index of refraction of its material, and the radii of curvature of its surfaces. (1923.)

3. A pin, convex lens, and plane mirror are adjusted so that (a) an inverted image, (b) an erect image, is formed coincident with the object: Give diagrams showing how images of points of the object off the axis are produced in the two cases.

How would you determine the focal length of the lens in the two experiments? (1922.)

4. Show how lenses may be combined to form a telescope. Explain what is meant by the magnifying power of a telescope, and find an expression for the magnifying power of a telescope, consisting of two lenses, when used to view distant objects. (1922.)

5. Describe the spectrometer and give an account of its uses. (1921.)

6. Describe the prism spectrometer, illustrating your answer by a careful diagram to show the formation of a spectrum of a source of white light.

Give an account of some of the uses of the spectrometer. (1923.)

7. Write a short essay on wave-motion, with special reference to sound and light waves. (1922.)

8. Describe carefully how the refraction of light is explained on the wave theory, considering in particular the refraction of a plane wave at a plane surface. (1922.)

9. Give an account of the wave theory of light, and show how the reflection of light may be explained on this theory. (1921.)

10. How has it been shown that the velocity of light in air is greater than the velocity of light in an optically denser medium such as glass?

What is the importance of this experimental result? (1921.)

11. Give an account of *either* Fizeau's or Foucault's method of measuring the velocity of light.

In an experiment for the determination of the velocity of light by the revolving mirror method, the distances from the rotating mirror to the source of light and fixed mirror are 40 metres and 500 metres respectively. If the mirror makes 200 revolutions per sec. and the velocity of light is 3×10^{10} cm. per sec., what shift of the image would you expect? (1923.)

12. Write a short essay on colour. (1922.)

13. Measure the distance y between an object and its real image formed by a lens, for different distances x of the object from the lens. Represent your results graphically, and deduce from your graph the focal length of the lens. (1922.) (Practical Examination.)

14. Determine the focal length of the given concave lens by combining it with a concave mirror. A series of observations should be taken. (1923.) (Practical Examination.)

15. Find the deviation produced by the given prism for a series of angles of incidence. Represent your results graphically, and

determine the angle of minimum deviation. Hence find the refractive index of the glass. (1921.) (Practical Examination.)

ELEMENTARY SCIENCE

1. Describe an accurate method of comparing the illuminating powers of two sources of light.

What is the candle-power of a lamp which illuminates a surface at a distance of 15 ft. from it, as much as a 32 candle-power lamp when at a distance of 2 ft. ? (1921.)

2. State the laws of refraction of light.

An object is viewed through a cube of glass. Explain how the refractive index of the glass may be deduced from observations of the apparent position of the object. (1922.)

3. State the laws of refraction of light.

Explain why it is that an object, when viewed through a slab of transparent material with parallel faces, appears nearer to the eye than it really is. Show how this may be applied to determine the refractive index of the material. (1923.)

4. What is meant by total reflection ? Give examples.

What is the relation between the critical angle and the refractive index ? (1921.)

5. What is (a) a real image, (b) a virtual image ?

A concave mirror has a radius of 20 in. A small object is placed on the axis at a distance of 18 in. from the mirror. Find the position of the image.

If a pin 2 in. long is placed on the axis with its point at the centre of curvature, find the length of the image. (1921.)

6. Explain what is meant by (a) a real image, (b) a virtual image.

A thin lens forms a real image of an object four times the size of the object and 50 cm. from the object. If a thin concave lens, of focal length 20 cm., is placed in contact with the first lens, show that a virtual image will now be formed of the same size as the first image. (1923.)

7. What is (a) a real image, (b) a virtual image ?

An image 1 cm. long of an object is formed on a screen by a convex lens. Keeping the object and screen fixed, the lens is moved until a second image is formed on the screen. If this image is 0.75 cm. long, what is the length of the object ? (1922.)

8. How may two lenses be combined to form a microscope ?

A compound microscope contains two convex lenses, an objective of focal length 0.5 in., and an eyepiece of focal length 1 in. An object is placed at a distance of $1\frac{1}{2}$ in. from the objective. What will be the distance between the object-glass and eyepiece if the final image is seen at a distance of 10 in. from the eye ? (1922.)

9. Determine the focal length of the given concave lens by combining it with a convex lens. A series of observations should be made. (1922.) (Practical Examination.)

10. Place a pin at different distances u from the given convex lens, and determine the distance of the image v in each case.

Represent graphically the relation between the reciprocals of u and

v , and deduce the focal length of the lens. (1921.) (Practical Examination.)

11. Place a pin at different distances u from a convex lens and measure the distances v of the corresponding images from the lens. Represent graphically the relation between the reciprocals of u and v , and deduce the value of the focal length of the lens. (1923.) (Practical Examination.)

SUBSIDIARY SUBJECT

1. What do you understand by the focus and focal length of a concave mirror? How far from a concave mirror of 10 in. focal length must an object be placed, in order that an image twice its size may be formed? (1921.)

2. State the laws of reflection and refraction in optics. Give any method of determining the focal length of a convex lens. (1922.)

3. Explain how two lenses may be combined to form (a) a microscope, and (b) a telescope.

Draw careful diagrams illustrating the passage of a pencil of light through the system from a point off the axis. (1923.)

4. Define refractive index. How would you measure the refractive index of (a) a glass plate, and (b) a prism? (1922.)

5. Give an account of the optical system of the eye, and explain any common defects of vision. A long-sighted person cannot see objects any nearer than 25 in. What kind of lens will he require in order to see clearly objects at all distances greater than 10 in., and what must be the focal length of the lens?

6. For what defects of vision are convex and concave lenses, respectively, used in spectacles? Illustrate by diagrams, how they correct these defects. (1921.)

LONDON UNIVERSITY INTERMEDIATE SCIENCE

1. Describe the apparatus with which you would compare the candle-powers of two sources of light. Explain how you would use it to test the statement that a given glass plate transmits only 10 per cent. of the incident light. (1923.)

2. Where must an object be placed with respect to a convex lens of 1 in. focal length in order that (1) a real image, (2) a virtual image, may be formed a foot away from the lens? Give a diagram showing, in each case, how the image is formed. (1923.)

3. A convex lens is placed on a plane mirror and a pin adjusted vertically above until at a height of 15 cm. an image is formed side by side with it. When a convex mirror is substituted for the plane mirror the pin must be raised 15 cm. higher for the image to remain in coincidence. Draw diagrams explaining how these images are formed, and find the focal lengths of the lens and mirror. (1923.)

4. Describe the essential parts of a spectrometer and explain how you would use it to find the refractive index of a substance. (1923.)

5. Being provided with a source of light, a slit, two convex lenses, each of 20 cm. focus, and a prism, describe exactly how you would form

a pure spectrum upon a screen. Draw a careful diagram of the arrangement and use it to show how the spectrum is formed. (1923.)

6. Explain briefly the optical action of a normal eye. How are the defects of long-sight and short-sight overcome?

A person can only see objects distinctly at a distance of 50 cm. from the eye. If he wishes (*a*) to read a book at a distance of 25 cm., (*b*) to see objects clearly at a distance of 5 m., what kind of lenses must he use, and what must be their focal lengths? (1923.)

7. Obtain a formula showing how the focal length of a thin convex lens can be calculated when the radii of its surfaces and the refractive index of the material are known. Describe how you would test the formula experimentally. (1923.) (Intermediate Honours.)

PART V.—ELECTRICITY AND MAGNETISM

LONDON UNIVERSITY MATRICULATION

1. Give a short account of the molecular theory of magnetism.

Describe some simple way of determining quantitatively the distribution of magnetism along a bar magnet, and indicate what results would be obtained with an ordinary bar magnet. (1920.)

2. An iron bar AB is to be magnetised so that A is north-seeking. How may this be done by employing (*a*) a magnet, (*b*) the Earth, (*c*) an electric current? (1921.)

3. An unmagnetised steel needle in this country is pivoted so that it rests in a horizontal position. It is then magnetised and will no longer rest horizontally. How must it be loaded to be level again? Suggest a method of finding the magnetic moment of the needle by measurement of this load. What other quantities will be needed? (1924.)

4. What is meant by the statements (*a*) that the strength of a magnetic pole is *m units*, (*b*) that the strength of a magnetic field is *H units*?

The distance between the poles of a bar magnet is 15 cm., and the strength of each pole is 100 units. Find in magnitude and direction the field due to the magnet at a point distant 15 cm. from each pole. (1920.)

5. "The strength of the Earth's horizontal field in London is 0.18." Explain fully the meaning of this statement.

If a small needle makes 12 oscillations per minute in this field, what addition to the field will be needed for it to make 20 oscillations per minute without change of its mean position? (1923.)

6. Describe the construction of an electroscope and explain carefully how it should be used to determine the sign of the charge on a conductor. (1921.)

7. Describe two experiments to illustrate the discharging effect of a sharp-pointed conductor and two practical applications of the phenomenon. (1921.)

8. Describe the process of charging an electroscope (*a*) positively, (*b*) negatively, using in both operations a negatively electrified ebonite rod. (1922.)

9. Substances may be classed as good and bad insulators, or good and bad conductors. Distinguish between these and give a list of six substances in order covering the whole range. (1923.)

10. An insulated metal plate is connected to an electroscope and given a positive charge. An earth-connected metal plate is brought near the first one. Draw two diagrams showing the distribution of the lines of force and indicating the behaviour of the electroscope. (1922.)

11. How would you show that the potential of a charged conductor is the same all over it? Would you use a small or a large electroscope for this purpose? Give a reason for your choice. (1924.)

12. Define unit charge.

Two small conducting spheres of the same size, and 4 cm. apart in air, have charges of +18 and -2 respectively. What is the force between them? What will it be if they are brought in contact and then replaced as before? (1921.)

13. Define *unit quantity of electricity*.

A pith-ball charged with 30 units attracts another pith-ball of the same capacity 6 cm. distant with a force of 10 dynes. Find the charge on the second ball. If the balls are allowed to touch and again separated at the same distance, find the new force between them. (1923.)

14. How may the inverse square law of electric force be verified by the torsion balance (or otherwise)?

Two small equal spheres carrying charges of +5 and -10 units are placed 20 cm. apart. Find the force they exert on each other (*a*) before, (*b*) after they have been connected for a moment by a conducting wire. (1919.)

15. Why do we believe that the electricity obtained from an electrical machine is the same as that obtained from a Daniell cell? Explain the differences observed in the two cases. (1923.)

16. Explain the principle of the tangent galvanometer. An electric circuit includes a tangent galvanometer which gives a deflection of 45° . When, however, an additional resistance of 5 ohms is put in the circuit this deflection is reduced to 30° . Calculate the total resistance of the circuit in the first instance. (1920.)

17. Given a coil of resistance wire, how would you proceed (*a*) to make a 1-ohm coil from the wire, (*b*) to find the specific resistance of the material? (1924.)

18. State Ohm's law, and show how it provides a definition of electrical resistance.

Two coils in series have a resistance of 18 ohms, and in parallel a resistance of 4 ohms. Find their separate resistances, proving from first principles any formulæ which you employ. (1921.)

19. A resistance coil is in series with an ammeter of negligible resistance and a lamp of 200 ohms resistance, the voltage of the supply being 110. The potential difference, measured by a voltmeter, between the terminals of the coil is 2 volts. What should the ammeter-reading be, and what is the resistance of the coil? Draw a diagram showing

the connections of ammeter and voltmeter, marking + and -. (1922.)

20. A Daniell cell is connected to a voltmeter, the reading being 1.1 volt. The terminals of the cell are now connected to a 2-ohm coil, and the voltmeter reading falls to 0.8 volt. Find the resistance of the cell. What would you expect the voltmeter to read if a 1-ohm coil were substituted for the 2-ohm coil? (1923.)

21. Explain why the original simple cell discovered by Volta is no longer used as a source of current, and describe the method used in some other form of cell to overcome the defects of the original type.

For what purposes are primary batteries used at the present time? (1923.)

22. Describe what happens when an electric current is passed through a solution of sodium sulphate (or other similar salt), platinum electrodes being used.

How long will it take for 100 c.c. of hydrogen to be evolved if the current is $\frac{1}{2}$ ampere?

(96,500 coulombs liberate 1 gm. of hydrogen which occupies $11\frac{1}{2}$ litres.) (1924.)

23. State Faraday's laws of electrolysis, and show how they may be applied to measure the strength of an electric current. (1921.)

24. Give a diagram and brief explanation of an electric bell and of a battery suitable for working it. (1923.)

25. Explain the term *electromagnetic induction*.

Sketch and describe carefully some apparatus or machine whose action depends upon electromagnetic induction. (1919.)

26. Explain the working of an induction coil, pointing out the ways in which the different parts of it contribute to increase the length of the spark. (1921.)

OXFORD AND CAMBRIDGE SCHOOLS EXAMINATION BOARD, HIGHER CERTIFICATES

PHYSICS

1. Describe an experimental method of verifying the law of inverse squares for magnetic poles. (1921.)

2. Describe an experiment by which you could measure the magnetic moment of a bar magnet. (1923.)

3. Explain what is meant by the angle of dip, and describe a method of measuring it accurately.

Observations are made of the apparent angle of dip in two vertical planes at right angles to each other. If α_1 and α_2 are the observed values, show that the true value of the angle of dip α is given by the expression

$$\cot^2 \alpha = \cot^2 \alpha_1 + \cot^2 \alpha_2. \quad (1922.)$$

4. Define electric intensity and electric potential at a point.

Point charges of electricity equal to +100, +100, and -300 respectively are placed at the corners of an equilateral triangle ABC of side 10 cm. Find (a) the force on a point positive charge of 2 units placed at the intersection O of the median lines, and (b) the work done

in taking this charge from the point O to the middle point of AB. (1922.)

5. Explain what is meant by electrical potential, and find an expression for the potential at a point due to a charged spherical conductor.

Show that the capacity of a spherical conductor is numerically equal to its radius. (1923.)

6. Describe some form of electrical induction machine. Why does a sharp point attached to an electrical machine prevent a high potential being obtained? (1921.)

7. Give an account of the magnetic effects due to electric currents.

Calculate the intensity of the magnetic field at the centre of the coil of a galvanometer of 100 turns of wire and 10 cm. mean radius, when a current of 0.2 ampere flows through the coil. (1921.)

8. State Kirchhoff's laws.

In a Wheatstone bridge arrangement, the four resistances AB, BC, CD, and DA are 1 ohm, 2 ohms, 3 ohms, and 2 ohms respectively. The cell connected between A and C has an E.M.F. of 2 volts and internal resistance of 2 ohms. Find the current through the galvanometer if the latter has a resistance of 4 ohms. (1922.)

9. Describe a method of measuring the resistance of a cell.

Six similar cells are connected in series with a resistance of 18 ohms and a key. The terminals of the battery are connected to a high-resistance voltmeter. The readings of the voltmeter with the key open and closed are 12 volts and 9 volts respectively. Find the resistance of one cell. (1922.)

10. State Ohm's law, and describe carefully the experiments you would make in order to verify it.

Does this law hold in the case of an electrolyte such as dilute sulphuric acid? (1922.)

11. Describe a method of comparing a small electromotive force such as that of a thermocouple with that of a Daniell cell. (1921.)

12. Find an expression for the heating effect produced by an electric current in flowing through a resistance.

A current of $\frac{1}{2}$ ampere flows through a resistance of 50 ohms immersed in a calorimeter containing water. If the water equivalent of the calorimeter and contents is 100 grams, find the rise in temperature per second, neglecting losses of heat. (1923.)

13. Give an account of the heating effects of electric currents.

A wire 1 mm. in diameter carries a current of 2 amperes. Assuming that the specific resistance of the wire is 2×10^{-4} ohms per cm.³, and that no heat is lost by convection or conduction, calculate the amount of heat radiated per square centimetre per second when the temperature of the wire has reached a steady state. (1921.)

14. Describe some practical form of ammeter or voltmeter, giving details of the principles which underlie its use. (1923.)

15. Describe some form of suspended coil galvanometer, and explain the advantages of this form of instrument.

The coil of such an instrument has an effective area of 40 sq. cm. and swings in a field of 100 gauss. A couple of 2 dyne-cm. is required to twist the coil through one radian. Find the current which will produce

a deflection of 1 mm. on a scale distant 1 metre from the instrument. (1922.)

16. State the laws of electromagnetic induction.

An earth inductor, which has a coil of 20 turns and an area of 400 sq. cm. per turn, makes 40 revolutions per second in a magnetic field of intensity 0.5 gauss. Assuming that the resistance of the circuit is 40 ohms and that a commutator is used, find the average value of the current. (1921.)

17. State the laws of electromagnetic induction. How have these laws been deduced?

A circular coil of 50 turns of wire revolves 50 times per second in a magnetic field of intensity 20 gauss about a diameter perpendicular to the field. If the area of each turn is 500 sq. cm. and the resistance of the coil is 50 ohms, find the average current in the coil when a commutator is in circuit. (1923.)

18. Make experiments to determine how the magnetic intensity due to a bar magnet varies with the distance from its centre for points on the equatorial plane.

Represent your results graphically, and deduce the distance between the poles of the magnet. (1922.) (Practical Examination.)

19. Make experiments to determine how the magnetic intensity at a point on the axis of a magnet, due to the magnet, varies with the distance of the point from the centre of the magnet.

Represent your results graphically. (1921.) (Practical Examination.)

20. Make experiments with a vibration magnetometer to determine how the intensity of the magnetic field due to a bar magnet at a point on its axis depends on the distance of the point from the centre of the magnet. (1923.) (Practical Examination.)

21. Connect in series a cell of small resistance, a resistance box, and a tangent galvanometer. Make observations of the current for different values of the resistance.

Represent your results graphically, and from the graph deduce the resistance of the galvanometer. (1922.) (Practical Examination.)

22. Connect in series a tangent galvanometer, a resistance box, and a cell of small resistance and known E.M.F. Make observations of the angle of deflection for a series of values of the resistance, and plot a graph showing how the cotangent of the angle varies with the resistance in the box.

Deduce the value of the reduction factor of the galvanometer. (1923.) (Practical Examination.)

23. Use a potentiometer method to measure the difference of potential between the terminals of a Daniell cell (*a*) when the cell is on open circuit, (*b*) when the terminals are joined by a resistance of 2 ohms. Hence determine the resistance of the cell. (1921.) (Practical Examination.)

ELEMENTARY SCIENCE

1. How would you show experimentally that the two poles of a bar magnet are of equal strength?

Explain carefully how you would determine the pole strength of the magnet. (1922.)

2. Define (a) unit magnetic pole, (b) intensity of magnetic field.

A bar magnet of moment 200 units has poles of strength 20 units. Find the ratio between the intensities of the magnetic fields due to the magnet at points 20 cm. from the centre of the magnet (a) on its axis, (b) on a line perpendicular to the magnet through its centre.

How would you verify this experimentally? (1923.)

3. What do you understand by (a) declination, (b) the angle of dip, and how would you measure them? (1921.)

4. Describe carefully how you would show (a) that when electricity is produced by friction, equal and opposite charges are produced; (b) that when electricity is produced by induction, the total induced charge is equal to the inducing charge. (1923.)

5. Describe and explain the method you would use to investigate the distribution of electricity on the surface of charged conductors. Give an account of the distribution in some special cases. (1921.)

6. Explain the action of an electrical condenser. What is meant by the capacity of a condenser?

Three condensers of capacity 3, 2, and 4 units respectively are connected (a) in series, (b) in parallel. Compare the effective capacities in the two cases. (1922.)

7. State Ohm's law and describe the experiments you would make in order to verify it.

The difference of potential between the terminals of a cell in open circuit is 2.2 volts. When the terminals are connected by a wire of resistance 4 ohms, this difference of potential is reduced to 2 volts. What is the internal resistance of the cell? (1921.)

8. Describe an accurate method of measuring a resistance.

Two wires of lengths 100 cm. and 200 cm., areas of cross-section 0.01 sq. mm., and 0.04 sq. cm., and specific resistances 20×10^{-6} ohms per centimetre cube and 4×10^{-6} ohms per centimetre cube, respectively, are connected in parallel. What is their combined resistance? (1923.)

9. State Ohm's law.

The difference of potential between the ends of the filament of an electric lamp through which a current of 0.2 ampere is passing is 200 volts. Find the number of calories of heat developed per minute. $J = 4.2 \times 10^7$ ergs per calorie. (1923.)

10. On what does the heating effect of an electric current depend?

The terminals of a cell are joined by two wires of resistance 2 ohms and 4 ohms respectively. Compare the amounts of heat developed in any time in the two resistances. (1921.)

11. On what does the heating effect of an electric current depend?

A constant current of 1 ampere flows in a platinum wire of resistance 5 ohms, stretched along the axis of a cylindrical tube, through which a steady stream of water passes at the rate of 15 c.c. per minute. The steady difference between the temperature of the water entering and leaving is 4.8°C .

Neglecting losses of heat, calculate a value of the mechanical equivalent of heat. (1922.)

12. Describe a tangent galvanometer, and explain what is meant by (a) the galvanometer constant, (b) the reduction factor.

A constant current is passed for 20 minutes through a tangent galvanometer and silver voltameter in series, and the weight of metal deposited is 0.5 gm. If the deflection of the galvanometer needle is 30° , calculate the reduction factor of the galvanometer.

The electrochemical equivalent of silver may be taken as 0.00112 gm. per coulomb. (1922.)

13. State the laws of electrolysis, and describe how you would verify them experimentally.

Explain what happens when an electric current is passed through solutions of the following substances :

- (a) Sulphuric acid, with platinum electrodes ;
- (b) Copper sulphate, with platinum electrodes ;
- (c) Copper sulphate, with copper electrodes ;
- (d) Sodium sulphate, with platinum electrodes. (1923.)

14. Place the given bar magnet with its axis in the magnetic meridian and its south pole pointing north.

Plot the lines of magnetic force of the resultant field and deduce the pole strength of the magnet. ($H=0.18$ gauss.) (1921.) (Practical Examination.)

15. Place the given bar magnet in the magnetic meridian with its north pole pointing north. Plot the lines of force of the field, and deduce the value of the pole strength of the magnet. (1923.) (Practical Examination.)

16. Compare the moments of the given bar magnets by using them to deflect a magnetometer (a) in the "end on" position, (b) in the "broadside on" position. (1922.) (Practical Examination.)

17. Compare the resistances of the given coils by means of a tangent galvanometer. (1922.) (Practical Examination.)

18. Measure the specific resistance of the material of the given wire. (1921.) (Practical Examination.)

19. Determine the specific resistance of the material of the given wire. (1923.) (Practical Examination.)

SUBSIDIARY SUBJECT

1. What is meant by the moment of a magnet ?

Describe the use of the magnetometer. (1922.)

2. Explain what is meant by (a) unit magnetic pole, and (b) the intensity of a magnetic field.

How would you compare (a) the strengths of two magnetic fields, (b) the pole strengths of two magnets ? (1923.)

3. What do you understand by (a) the pole strength of a magnet, (b) the moment of a magnet ? How would you show that the poles of a magnet are of equal strength ? (1921.)

4. Describe Faraday's ice-pail experiment. (1923.)

5. Explain the action of an electrical condenser.

Describe and explain the effect produced on the spark between the knobs of a Wimshurst machine when they are connected to a Leyden jar. (1921.)

6. Explain the principle involved in Wheatstone's bridge.

How would you use the bridge to measure the resistance of an electrolyte ? (1922.)

7. State Ohm's law.

Find the resistance of a battery which on open circuit gives an E.M.F. of 6 volts, and which, when producing a current of 2 amperes, has a potential difference of 4 volts between its poles. (1923.)

8. Write a short essay on the measurement of electrical potential. (1922.)

9. Upon what does the heat generated in a conductor by the passage of an electric current depend ?

Compare the quantities of heat generated in any time in two wires of 2 and 10 ohms resistance, when a current is passed through them (a) in series, (b) in parallel. (1921.)

10. Write a short essay on *either* (a) the telephone and microphone, or (b) electric batteries. (1922.)

11. Write a short account of the physical theory underlying *either* (a) the telephone, or (b) the dynamo. (1923.)

LONDON UNIVERSITY INTERMEDIATE SCIENCE

1. Sketch the field near three equal bar magnets which lie a little apart in the form of a triangle and having unlike poles together at one corner only. (1921.)

2. Define the magnetic potential at a point. Two equal magnets having poles of unit strength and moments of magnitude 10 lie along the same straight line with their north poles adjacent. Calculate the work done when the distance between the adjacent poles is reduced from 30 to 20 cm. (1920.)

3. Explain the theory of the method you would use to determine the absolute value of the moment of a short magnet. (1920.)

4. Explain how you would make an accurate measurement of the magnetic dip. Describe how this quantity varies at different parts of the earth's surface. (1922.)

5. Write a brief account of the more important properties of the earth's magnetic field. (1920.)

6. Describe and explain the characteristic distinctions between the magnetic properties of iron and steel and state how you would demonstrate them. (1921.)

7. Define the difference of electric potential between two points. In a uniform field of unit intensity a point describes a circle of unit radius of which a diameter is parallel to the direction of the electric intensity. Starting with the point at one extremity of this diameter calculate the changes in the potential at the point for each successive displacement of 30° round the circle. (1922.)

8. Define electric potential, and show how it is related to electric field. How do the electric field and the potential vary from point to point within and without an electrified hollow closed conductor ? (1923.)

9. Two equal spheres some distance apart have equal and opposite

electric charges ; what is the potential at different places along the line joining their centres and how would you prove this experimentally ? (1920.)

10. What is meant by the capacity of a conductor ? Two spheres whose radii are 1 cm. and 2 cm. respectively are each given a charge Q and then placed in contact. Find the potentials and electrical energies of the charged spheres both before and after contact is made. (1920.)

11. Having two equal parallel-plate condensers and a slab of wax that fits between the plates, how would you find the dielectric constant of the wax ? (1920.)

12. Describe some form of electrical influence machine and explain its action. (1921.)

13. What is Ohm's law ?

A circuit consists of a battery whose electromotive force is 1.8 volts in series with a resistance of 10 ohms and an ammeter whose resistance is negligible. The ammeter registers 0.15 ampere. Explain this. (1922.)

14. Explain the principles which govern the distribution of electric currents in branched circuits and deduce the relation between the resistances of the arms of a balanced Wheatstone's bridge. (1923.)

15. What are the laws of development of heat in a homogeneous conductor carrying an electric current ? An electric heater of resistance 10 ohms is connected to a 100-volt circuit. Calculate the time required to raise a kilogram of water from 20° C. to the boiling point, assuming all the heat generated in the heater to be conveyed without loss to the water. (Joule's Equivalent $= 4.18 \times 10^7$ ergs per calorie.) (1923.)

16. Derive the formula for the balanced Wheatstone bridge. How would you use the bridge to measure the resistance of a galvanometer, no other galvanometer being available ? (1923.)

17. Describe some form of very sensitive galvanometer, explaining to what feature it owes this property. (1921.)

18. Describe with diagrams the essential features in the structure of a moving coil ammeter or voltmeter and explain the forces which act on the various parts of the coil when in operation. (1923.)

19. State Faraday's laws of electrolysis.

How much electricity will be required to liberate a cubic metre of dry hydrogen at 15° C. ?

(Assume electrochemical equivalent of copper $= 0.000328$, atomic weights Cu 63.1, H 1.008, density of hydrogen at N.T.P. $= 9 \times 10^{-5}$.) (1923.)

ANSWERS

PART I.—DYNAMICS AND PROPERTIES OF MATTER

CHAPTER I

1. 16.39 c.c. ; 28,310 c.c. 2. 28.31 ; 28.31 kg. ; 62.43 lb. 3. 711.5 lb./ft.³ ; 0.4118 lb./inch.³

CHAPTER II

1. Two-thirds of a mile per minute. 2. 5.83 ml./hr. 30° 55' W. of N.
3. 6.124 ml./hr. making 112° 30' with the initial direction of motion of train ; 11.31 ml./hr. making 135° with the initial direction of motion of train. 4. 2200 ft. ; 0.44 ft./sec.² 5. 256 ft. ; 4 sec. 6. 3 sec. ; 90 ft. 7. 43.8 ft./sec. ; 1.37 sec. 8. 1.56 ft. 9. 7.81 ft. 10. 53.8 ft. 11. 44 ft./sec. ; $\frac{11}{60}$ ft./sec.² 12. $2\frac{2}{15}$ ml. 13. 14.34 ft./sec.² 14. 0.1112 ft./sec.² 15. 2.587×10^6 sec.⁻² ; 8.042 cm./sec. 16. 8.041 cm./sec. ; 2.586×10^4 cm./sec.²

CHAPTER III

1. 25.98 lb. ; 15 lb. 2. 10 gm. 3. 141.4 gm. in first thread ; 100 gm. in second. 4. 24 lb. 5. 10.44 gm. making 125° 24' with the given line. 6. No ; they cause rotation. 7. 210 lb. slung $\frac{3}{7}$ of AB from A. 9. At 36.05 cm. 10. 77.48 gm. 11. 6 lb. ; 10 lb.

CHAPTER IV

1. 25 gm. 2. 8 ft./sec. 3. 9.972×10^8 dynes. 4. 9.6 ft./sec.
5. (i) 12 dynes ; (ii) 0.0122 gm. 6. 128 lb. 7. (i) 4.58 tons ; (ii) 5.04 tons. 8. 0.937 ton. 9. $13\frac{1}{2}$ stones ; $10\frac{1}{2}$ stones. 10. $\frac{1}{2}$ ton ; 4 lb.
11. 6.38 ft./sec. 12. $\frac{1}{1000}$ cm./sec. ; $\frac{1}{8000}$ cm./sec. 13. It loses 18 per day. 14. 40.3 inches. 15. 0.4 per cent. 16. 8.27 lb. 17. 87.1 cm.

CHAPTER V

1. 0.625. 2. 376 pdl. up ; 56 pdl. down. 3. 401 cm./sec. ; 501 cm. 4. 0.58. 5. 135.8 cm./sec.² ; 248.5 cm./sec.² 6. 16.4 ft./sec. ; 396 ft. 8. Take a point D on AB, so that AD=2DB ; 3DG=4GC. 9. In axis of symmetry $\frac{1}{9}a$ from centre. 10. $a\sqrt{2/6}$ from centre along axis of symmetry. 11. 1.06 cm. from centre of large cylinder.

CHAPTER VI

1. 1.357×10^7 ergs. 2. 792 ft.-tons. 3. (i) 17600 ergs ; (ii) 3200 gm.-cm. ; (iii) 9 ft.-lb. 4. 5.843×10^4 ft.-lb./cub. ft. 5. 340 ft.-lb.
6. 1.227×10^8 ergs. 7. 4114 ergs. 8. (i) 4.735×10^5 ergs ; 97.29

cm./sec.; (ii) 3.542×10^5 ergs; 84.14 cm./sec. 9. 1056 cub. ft. 10. 616 H.P.

CHAPTER VII

1. 6.169×10^5 ergs. 2. $\omega = 8.67$ rad./sec. 3. 0.7854 rad./sec.²
4. 2011 ft.-pdl. 5. 195 ft.-lb. 6. $2\pi\sqrt{(2l/3g)}$. 7. 5.71 HP. 8. 31.938 ft./sec.² 9. 0.349 per cent. 10. 0.000513 dyne.

CHAPTER VIII

1. 102.5. 2. 11.09. 3. 60°. 4. 40 gm. 5. 1 to 3. 6. 15.8 tons
6 ft. down. 7. 25 tons. 8. 3.6 tons; 0.087 inch.

CHAPTER IX

1. 12.10 metres. 2. 9.97 lb./sq. in. 3. 1800 ft. 4. 104. 5. 53370.
7. 150.6 cm.; 57.24 cm. 8. 11:1, neglecting thickness of piston.
9. 28.6 cm. 10. 30.7 inches. 11. 71.1 cm. 12. 20.9 cm. 13. 2.70 ft.
14. 280.2 cm./sec.

CHAPTER X

1. 0.107. 2. 4.97:1. 3. 0.543 gm. 4. 2.5 times the original
density, the latter being supposed atmospheric. 5. 10.24 ft. 6. No.
7. 0.0362 mm.

CHAPTER XI

1. 22.610 gm. 2. (i) 67 tons; (ii) 43 tons. 3. 515.8 kg.
4. 11.40. 5. 1.069. 6. 0.909. 7. 8.97. 8. 0.653. 9. 1.056.
10. 1.088. 11. 11.35. 12. 9196 cubic yards. 13. 1743.

CHAPTER XII

1. 2.02×10^9 gm./cm.² 2. 4001 gm. 3. 70 dynes/cm. 4. 2.31 gm.
5. 0.0050 cm. 6. 1.68 mm. 7. No.

PART II.—HEAT

CHAPTER I

1. 36.7° C., 24.4° C., 13.3° C. 2. (i) 24.8°, 71.6°, 190.4°, 233.6° F.,
(ii) -3.2°, 17.6°, 70.4°, 89.6° R. 3. -30.0°, -14.4°, 8.3°, 44.4° C.
4. (i) -40.0° C., (ii) -25.6° R. 5. (i) 98.10° C., (ii) 98.79° C., (iii)
100.91° C. 6. (i) About 95° C., (ii) about 77° C. using the rule 1 inch
for 900 ft.; the rule is, however, not applicable for such great heights,
and the correct value for Mont Blanc is 84° C. 7. 14.6° C.; 54.1° C.
8. 1.000:1.089.

CHAPTER II

1. 174° C.; 150.488 cm. 2. 59.4 ft. 3. Loses 4.1 seconds per day.
4. 77.94 cm. 5. 123.564 sq. cm. 6. 0.552. 7. 0.0105 c.c. 8. 0.248
c.c. 9. 13.449 gm./c.c. 10. 0.708 gm. 11. 85.5° C. 12. 98.7° C.
13. 0.203 mm. 14. 0.000486. 15. 134.3° C. 16. 1.26 gm.

CHAPTER III

1. 80.5 cm. 2. ρ at bottom : ρ at top = 1.0024 : 1.0000. 3. 70.7° C. 4. 187 gm. 5. Volume 1024 cubic feet. Yes. 6. 81 atmospheres.

CHAPTER IV

1. (a) 31.0° C., (b) 31.7° C. 2. 78.4. 3. 82.7. 4. 0.0325. 5. 518. 6. 0.0668. 7. 2.0 gm. 8. 246.9° C. 9. 0.463. 10. 22.8° C. 11. 29.2° C. 12. 28.8° C. 13. 997.6 calories. 14. 9 oz. of ice are melted. 15. 543.9 and 538.5 calories. 16. 252. 17. 0.0758. 18. 11.027 calories per gram. 19. Gas is cheaper, the ratio of the cost being 34 to 100.

CHAPTER VI

2. 62.8 per cent. ; 0.00876 gm./litre. 3. 11.8° C. ; 0.01045 gm./litre. 4. 11.0 per cent. 5. 57.6 per cent. 6. 4661. 7. 60.3 per cent. 8. 3.3 per cent. 9. 0.97 mm. 10. 177.4 c.e.

CHAPTER VII

1. 1.059×10^{10} ergs. 2. 1.06° C. 3. 0.057° C. 4. 428 metres. 5. 228° C. 6. 4.60×10^7 ergs/cal. 7. 0.561° C. 8. 3.714×10^6 ergs. 9. 0.9874. 10. 1.84×10^5 cm./sec. 11. 0.177 lb. 12. 134.6 cm. 13. 89.7° C.

CHAPTER VIII

1. It would keep longer. 2. 612 calories. 3. 5.4×10^5 calories. 4. 0.00154 c.g.s. unit. 5. 0.447 caloric.

CHAPTER IX

2. It cuts off the dark heat rays. 3. 0.728. 4. 23.4° C. 5. 0.00105. 6. The ratio is 27.09 : 1.

PART III.—SOUND

CHAPTER I

1. 132.8 cm. 2. Looks like ∞ ; $a^2y^2 = 4x^2(a^2 - x^2)$. 3. $200\frac{1}{2}$; $199\frac{5}{8}$.

CHAPTER II

1. 340.3 metres/sec. ; 1116 ft./sec. 2. 4400 ft. 3. 33.9 sec. 4. 3.92 mls./hr. 5. 1774 ft. 6. 60.6 ft. 7. 1284 metres/sec. 8. 6, 4.

CHAPTER III

1. 102.4. 2. 8.88. 4. 316. 5. 27.4 ; 3512. 6. 1241 cm. ; 9.68 cm. 7. Natural 391.5, 489.4, 522.0 ; equally tempered 391.1, 492.8, 522. 8. 648 before ; 552 after. 9. 543.5 before ; 463.0 after. 10. 68.2 mls./hr.

CHAPTER IV

1. 36 lb. 2. 5.406×10^6 dynes. 3. 335.3. 4. About 53 cm. 5. 1.5 per cent. 6. 33.4° C. 7. 334.8 metres/sec.

PART IV.—LIGHT

CHAPTER I

1. 2.4 in. 2. 50 cm., 20 cm. approximately. 4. 15.53. 5. 49. 6. 50.6 cm. from the 16 C.P. lamp towards and 32.5 from the 16 C.P. lamp away from the other lamp. 7. 2.41. 8. 1.00×10^{17} . 9. 2.92 cm. 10. 3.49 km. 11. 0.177. 12. 8.68×10^5 ml., 5788 ml. 13. 30.47, 1.70 hr., 3.74 hr.

CHAPTER II

2. 6 ml. per hr. 3. 65° . 5. 3 ft. $2\frac{1}{2}$ in. assuming eyes are 5 in. below top of head. 7. 89.4 cm. from screen. 8. 6 ft. in the one, and 14 ft. in the other.

CHAPTER III

1. $33^\circ 6'$. 3. 6.006 in. 4. 3.68 cm. 5. 2.74 cm. above the paper. 6. 4.56 cm. 8. 1.3334. 9. 1.122. 10. $57^\circ 22'$. 11. $48^\circ 36'$. 12. 51.9" less than the apparent elevation. 13. $b/(b-a)$.

CHAPTER IV

1. (i) 17.1 cm. from mirror, 1.43 cm. high, real, inverted ; (ii) 15 cm. behind mirror, 5 cm. high, virtual, erect. 2. (i) 36 cm. in front, 2 cm. high, real, inverted ; (ii) 12 cm. behind, 2 cm. high, virtual, erect. 3. (i) 7.06 cm. behind, 0.59 cm. high, virtual, erect ; (ii) 3.75 cm. behind, 1.25 cm. high, virtual, erect. 4. (i) 7.2 cm. behind, 0.4 cm. high, virtual, erect ; (ii) 4.0 cm. behind, 0.66 cm. high, virtual, erect. 5. (i) 13.3 cm., (ii) 5 cm. in front. 6. 3.2 ft., at 2 ft. on other side of flame. 7. $1\frac{1}{2}$ ft. from mirror, inverted. 9. $(n+1)f/n$. 10. 6.45 cm. 11. $\frac{1}{3}f$ behind, half size, virtual erect. 12. Concave, 60 cm. 13. 30 in. 15. 1.732 ft. from plane mirror. 16. 8.98 cm. in front of concave. 17. $4\frac{1}{8}$ in.

CHAPTER V

1. (i) 48 cm. behind, 1.2 cm. high, real, inverted ; (ii) 15 cm. in front, 3 cm. high, virtual, erect. 2. (i) 36 cm. in front, 3 cm. high, virtual, erect ; (ii) 25.7 cm. behind, 0.43 cm. high, real, inverted. 3. 1.172 ft. from either the flame or the wall. 4. 15.4 cm. in front, 0.77 cm. high, virtual, erect. 5. (i) 13.5 cm. in front, 0.25 cm. high, virtual, erect ; (ii) 7.2 cm. in front, 0.6 cm. high, virtual, erect. 6. 12 cm. ; rays would diverge from a point 60 cm. from lens on opposite side. 7. 4 in. 8. 16.63 cm. 9. 1.12 cm. too large. 11. 10.5 cm., 21 cm. 12. $\frac{2}{3}f$, $\frac{3}{4}f$. 13. 66.6 cm. 15. 11.79 in., if 30 ft. be measured from lens. 16. $8\frac{1}{2}$, $16\frac{1}{2}$ in. 18. 20.8 cm. 19. 2.14 in. from surface. 20. 0.34 in. away from lens. 21. 76.5 cm. 22. 16 and 48 cm. 23. 8.08 in. 24. 57.2 cm. 25. (a) 9.3 in. from convex lens on same side as object ; (b) 88 in. from convex lens on side opposite to concave.

CHAPTER VI

1. 2.86 in. from lens. 3.5 times. 2. 11.6 times. 3. It will be half as bright. 5. 1.7 sec. 6. 20.6 metres. 7. 0.1010 in. from objective. 8. In hundredths of a second 0.98, 2.0, 3.78, 8.0, 15.12, 32.0. 9. About 1 foot-candle. 10. 16 per sec.

CHAPTER VII

1. $43^\circ 11'$. 2. 1.586. 3. $38^\circ 39'$. 4. $16^\circ 20'$. 5. $72^\circ 21'$. 7. 7.3 mm. 8. Upper edge appears blue, lower edge red. 11. In, it appears blue; out, it appears red. 12. 40.5'.

CHAPTER VIII

2. +2 diopters. 3. -3.33 diopters. 4. -5 diopters. 5. -7.33 diopters. 6. 3 cm., 4 times. 7. Black, if the colours are pure. 8. No, for the quantity of red already in the white would not be increased.

CHAPTER IX

1. Interval is (i) diminished, (ii) increased, by 15.0 sec. 2. 327, 458. 3. $144'$. 5. 5.091×10^{11} per sec., 3.334×10^{-12} erg.

CHAPTER X

1. 0.0157 cm. 2. 470 cm.

PART V.—ELECTRICITY AND MAGNETISM

CHAPTER I

1. (i) 2.5 dynes; (ii) 1.2 dynes. 2. 13.9 dynes. 3. 0.246 dyne in a direction making $46^\circ 17'$ with meridian. 4. 229 units. 5. About 129 c.g.s. units.

CHAPTER II

1. 5.40 c.g.s. units; (ii) 9.35 c.g.s. units. 3. 105 units. 4. 1320, 1161. 5. 1.163 : 1. 6. 1050, 0.128. 7. 1109, 0.127. 8. 17.9 sec. 10. 9 : 5. 11. 53.9. 12. 14.8. 13. 5.32 sec.

CHAPTER III

1. 0.173, 0.100, -0.297 c.g.s. unit. 3. It is in neutral equilibrium in all positions. 4. 0.0088 gm. 5. Angle whose tan is $\tan \phi \sin \alpha$. 6. 0.371 gauss.

CHAPTER IV

1. Negative. 2. Negative. 4. 12 cm. 5. 3.54×10^{-4} gm.

CHAPTER V

1. 2000 ergs. 2. 159.2 cm. 3. 4.17. 4. 0.006, 0.000545, 0.0022 microfarad. 5. 600 units, 9000 ergs. 6. 3.87, 619. 7. 45 electrostatic units, 22,500 ergs. 8. 16,667 ergs. 9. 1741 cm. 10. 5.55, 0.69, 0.347 dyne. 11. 4363 ergs.

CHAPTER VI

1. Raised. 2. 2.16 amp. 3. 0.151 c.g.s., 41.7° . 4. 3 : 4. 5. 31.42, 0.05731. 6. Diminished 18 per cent.

CHAPTER VII

1. 1136 ohms. 2. 8.33 and 11.67 ohms. 3. 130.1 metres. 4. 0.099 amp., 0.99 volt. 5. 0.56 ohm. 6. 105.25 ohms. 7. 1.39 times. 8. 6.67 ohms. 9. 138.2 ohms. 10. 0.25 ohm. 11. 0.0521 amp., 0.0065 amp. 12. 0.0175 amp. 13. 0.035 amp. 14. 4 : 7. 15. 0.215 amp., 0.086 amp. 16. 103. 17. 52° C. 18. 1.46° C. 19. 1.46 watts, 0.00196 IIP. 20. 0.045d. 21. 1562 ohms. 22. 4° C. above its surroundings.

CHAPTER VIII

1. 12.24 ohms. 2. 3.286 ohms. 3. 0.152 volt. 4. 91.48 ohms. 5. 1.44 dynes. 6. 160 dyne-cm. 7. 4995 ohms, 9995 ohms. 8. 0.0446 amp., 0.892 volt. 9. 1.5 and 2.3 ohms. 10. $\frac{1}{20}$ per cent.

CHAPTER IX

1. 29 min., 49 sec. 2. 0.1045 gm. H, 3.294 gm. Cu. 3. 0.0392 gm. 4. 2.68. 5. 0.1009 gm.

CHAPTER X

1. Clockwise. 2. 8.5×10^{-5} volt. 3. 2.67×10^{-5} volt. 4. 31.4 c.g.s. unit.

CHAPTER XI

1. 1.344d., 0.48d., 0.36d. 3. Yes. 4. 1600 amp., 400 kilowatts, 536 HP. 5. 1600 : 1.

CHAPTER XII

1. 0.059 rad. 2. 3.94×10^9 cm. per sec. 3. 76.02 gm. at 35, 23.98 gm. at 37.

CHAPTER XIII

1. 4.42×10^{-7} sec., 133 metres. 2. 1790 revolutions per sec.

INDEX

(The references are to pages.)

- "A" position of Gauss, 455
- Aberration, astronomical, 410
- , chromatic, 392-94
- , spherical, 394
- Absolute expansion of mercury, 154
- refractive index, 326
- temperature, 166
- Absorption of gases by liquids and solids, 131
- spectrum, 390
- Acceleration, 14, 18
- Accumulator, 549
- Acoustics of halls, 260
- Actinic rays, experiments on, 387
- Activity, definition of, 61
- , optical, 423
- Adhesion, 122
- Adiabatic equation of a gas, 211
- Aerial, 593
- Aeroplane, lifting force on, 97
- Agonic lines, 464
- Air columns, vibrating, 278
- pump, 99
- thermometer, 165
- —, Miller's, 164
- Alpha rays, 583
- Alternating current, 556-57
- Amagat, 93
- Ammeter, 536
- Ampere, 507
- Amplitude of wave, 242
- Anaglyphs, 406
- Analysis, spectrum, 390
- Andrews, 194
- Aneroid barometer, 87
- Angle, critical, of total reflection, 322
- , measurement of, by light ray, 306
- of prism, measurement of, 381
- Ångström unit, 408
- Angular momentum, 66
- Anions, 544
- Annual change, 468
- Anti-nodes, 245
- Aperture of photographic lens, 373
- Apparent depth of water, 317
- expansion, coefficient of, 152
- Aqueous humour, 397
- Arago, 252
- Arc lamp, 565
- Archimedes, principle of, 107
- — —, experimental illustration, 108
- Astatic galvanometer, 513
- needle, 461
- Astigmatism, 400
- Aston, 580
- Astronomical telescope, 364
- Atmospheric refraction, 327
- Atomic energy, 585
- number, 485, 584
- Audition, 284
- Aurora Borealis, 469
- "B" position of Gauss, 456
- Back E.M.F. in motor, 572
- Balance-wheel, compensation of, 149
- Ballistic galvanometer, 537
- Balmain's luminous paint, 388
- Band spectrum, 390
- Barlow's wheel, 571
- Barograph, 88
- Barometer, 86-89
- Bartholinus, 421
- Basin, coin and, 315
- Battery resistance, measurement of, 538
- Beats, 268
- Becquerel, Henri, 583
- Bell, electric, 566
- Bell, Graham, 567
- Bellows, for experiments on sound, 281
- , hydrostatic, 82
- Bench, optical 338
- Beta rays, 583
- Bichromate cell, 547
- Bicycle pump, 103
- Binocular vision, 401

- Black, Joseph, 174
 Blackburn's pendulum, 250-51
 Blind spot, 398
 — —, experiment on, 399
 Blindness, colour, 407
 Boiling, 185
 Boiling-point, determination of, 186
 — —, effect of pressure on, 186
 Bomb calorimeter, 179
 Borda, 4
 Bouquet, phantom, 342
 Boyle's law, 91-93
 Bradley, 411
 Bragg, W. H. and W. L., 582
 Bramah press, 83
 Breaking stress, 120
 Brewster, 422
 Brittleness, 123
 Bulk modulus, 118
 Bunsen photometer, 294
 Bunsen's ice calorimeter, 177
 Buoyancy, centre of, 114
 — correction, 109

 Cadmium cell, 548
 Calcite, 420
 Callipers, 7
 Caloric theory, 202
 Calorie, 170
 Calorimeter for verifying Joule's law, 523
 Calorimetry, 169-79
 Camera, photographic, 372
 —, pinhole, 290
 Canal rays, 580
 Candle, standard, 292
 Capacities, comparison of, 537-38
 Capacity, 490
 — of a cell, 549
 Capillarity, 124
 Carboni dioxide, isothermals for, 194
 Cartesian diver, 81
 Cascade, condensers in, 496
 Cathions, 544
 Cathode rays, 576-77
 Caustic by reflection at a concave mirror, 342
 — by refraction at a plane surface, 321
 Cavendish experiment, 72
 Cells, in parallel and series, 521
 —, voltaic, 545-49
 Celsius, 139
 Centigrade scale, 137
 Centre of gravity, 48-53
 — — —, experimental determination of, 51

 Centre, optical, of a lens, 359
 Centrifugal force, 40
 Centroid, 51
 Characteristic equation of a gas, 167
 Charles's law, 161
 Chladni's figures, 284
 Chromatic aberration, 392
 — —, experiments on, 394
 — —, of eye, 398
 Chronograph, 265
 Ciliary muscle, 397
 Cinematograph, 374
 Circle, uniform motion in, 18
 Clinical thermometer, 142
 Coefficient of elastic restitution, 42
 — — expansion at constant pressure, 160
 — — — — —, determination of, 162
 — — — — —, Regnault's method, 163
 — — increase of pressure at constant volume, 161
 — — — — —, determination of, 164
 Cohesion, 122
 Coin and basin, 315
 Colladon and Sturm, 253
 Collimator, 380
 Colloids, 130
 Colour blindness, 407
 — disc, Maxwell's, 404
 — filters, 406
 — triangle, 403
 — vision, 406
 Coloured lights, addition of, 402
 — pigments, mixing, 405
 Combination of lenses and mirrors, 353
 Commutators, 528, 569
 Compass card, 470
 —, prismatic, 470
 Complementary colours, 402
 Compound microscope, 370
 — pendulum, 69
 Concave lens, determination of focal length of, 352
 — mirror, real image, 331
 — —, virtual image, 335
 Concavo-convex lens, 345
 Condensation, 247
 Condenser, capacity of a plate, 493
 —, — — — spherical, 492
 —, lantern, 374
 —, oscillatory discharge of, 587
 —, variable air, 492, 495
 Condensers, 490-93

- Condensers, in parallel, 495
- , in series, 496
- Conductance, 517
- Conduction of heat, 216–23
- — —, fundamental law, 217
- Conductivity of a bad conductor, measurement of, 220
- — liquids and gases, 221
- , thermal, 218
- Cones, rods and, 397
- Conjugate points, 332, 348
- Conservation of energy, 64, 210
- — momentum, 37
- Contact, two lenses in, 349
- Continuous spectrum, 390
- Convection of heat, 223–25
- correction, 176
- Convergent pencil, 289
- Convex lens, determination of focal length of, 350
- mirror, 335
- Convexo-concave lens, 345
- Cooling curve, 182, 234
- Cornea, 397
- Coulomb, 480, 481
- Coulomb's law of magnetism, 444
- Couple, 68
- Critical angle of total reflection, 322
- temperature, 194
- Crossed nicols, 421
- prisms experiment, 385
- Crown and flint glass contrasted, 393
- Cryophorus, 188
- Crystal detector, 591
- —, test of, 592
- Crystalline lens, 397
- Crystalloids, 130
- Cubical expansion, 151
- —, coefficient of, 151
- Curie, Mme., 583
- Current balance, 537
- Cylindrical lenses, 360
- —, used for astigmatism, 400
- D lines, 390
- —, wave-lengths of, 418
- Dagger, magic, 342
- Dalton's law, 195
- Damped oscillation, 588
- Daniell cell, 546
- Daniell's hygrometer, 199
- Davy lamp, 222
- Davy's experiment on nature of heat, 203
- Declination, 464
- , measurement of, 467
- Defects of vision, 399
- Degradation of energy, 211
- Density of earth, 72
- Depth, apparent, of water, 317
- Derived units, 3
- Descartes' law of refraction, 316
- Deviation, minimum, 376
- Dew, formation of, 196
- Dewpoint, 196
- Dialysis, 130
- Diamagnetic substances, 443
- Dielectric constant, 494
- Differential thermometer, 227
- Diffraction, 244, 437
- grating, 434
- of sound, 255
- Diffusion of gases, 128
- — liquids, 129
- Dilatometer, 152
- Dimensions of a unit, 5
- Dine's hygrometer, 200
- Diopters, 400
- Dip, 464
- circle, 467
- Direct vision spectroscope, 394
- Discharging tongs, 486
- Disintegration, radioactive, 584
- Dispersion of light by a prism, 384
- Displacement of a ray, 377
- Dissipation of energy, 211
- Dissociation, electrolytic, 544
- Distance of distinct vision, 363, 393
- Distillation, 193
- Diurnal change, 468
- Divergent pencil, 289
- Diving bell, 94
- Dolezalek, 502
- Doppler effect, 270
- Double refraction, 420
- Dry cell, 547
- Ductility, 123
- Dulong and Petit, law of, 174
- — —, apparatus for expansion of a liquid, 154
- Dynamo, 568–70
- Dynamometer, 537
- Dyne, 35
- Earth inductor, 556
- Earth's field strength, measurement of, 460
- Echo, 254
- Eclipses, 291
- Eddy currents, 557
- Einstein, 424
- Elastic impact, 42
- limit, 119
- Elasticity, 116–20

- Electric field, 482
 Electro-chemical equivalent, 542
 Electrolysis, 541-45
 Electromagnet, 514
 Electrometer, quadrant, 502
 Electron, 484, 577-79
 Electro-negative elements, 545
 Electrophorus, 476
 Electro-positive elements, 545
 Electroscopes, gold-leaf, 474
 Electrotyping, 545
 Emission theory of light, 414
 Emissivity of a surface, 236
 Energy, atomic, 585
 —, kinetic and potential, 61
 — of charged conductor, 496
 Equally tempered scale, 270
 Equilibrant, 25
 Equilibrium, stable, unstable, 52
 Equipotential surfaces, 488
 Erg, 56
 Euclid's *Catoptrics*, 315
 Evaporation, 187
 —, cooling effect of, 188
 Expansion of gases, 160-68
 — — liquids, 152-57
 — — solids, 145-52
 Extraordinary ray, 420
 Eye, 397-401

 Fahrenheit scale, 138
 Falling bodies, 16
 — plate apparatus, 265
 Far point, 398
 Farad, 538
 Faraday, 543
 Faraday's disc, 571
 — experiments on induction of currents, 553-54
 — ice-pail, 479, 483
 — laws of electrolysis, 542
 — ring, 557
 Feddersen, 588
 Ferromagnetic substances, 443
 Filter pump, 103
 Filters, colour, 406
 Fish and setting sun, 323
 Fizeau, 411
 Flash lamp, test of, 299
 Flint and crown glass contrasted, 393
 Fluorescence, 388
 Focal length, accurate method of determining, 353
 Focus of a lens, 346
 — — — mirror, 332
 Foot-poundal, 56
 Force on current in magnetic field, 533
 Force pump, 104
 Fortin barometer, 86
 Foucault, 413, 417
 — currents, 557
 Fovea centralis, 397
 Franklin, Benjamin, 484, 499
 Franklin's experiment on boiling, 186
 Fraunhofer lines, 390
 Frequency, measurement of, 265-68
 — of light, 418
 — of S.H.M., 21
 Fresnel's biprism, 428
 — mirrors, 430
 Friction, 45-48
 —, coefficient of, 46
 —, limiting angle of, 45
 —, rolling, 48
 Frictional machine, 497
 Ft.-lb., 56
 Fundamental units, 3
 Furnace, electric, 565
 Fuse, 523

 Galilean telescope, 367
 Galileo, 16, 17, 39
 —, experiment on velocity of light, 409
 Galileo's thermometer, 160
 Galton's whistle, 258
 Galvanic multiplier, 506
 Galvanometer, astatic, 513
 —, ballistic, 537
 —, mirror, 511
 —, moving coil, 535
 —, shunted, 519
 —, sine, 511
 —, tangent, 509-10
 Galvanoscope, 506
 Gamma rays, 583
 Gas constant, 167
 — equation, 167
 Gauss, "A" and "B" positions of, 455-56
 —, proof of inverse square law, 459
 —, the, 445
 Geissler tubes, 576
 Ghost, Pepper's, 307
 Gilbert, 469
 Glaisher's factor, 199
 Glass plate, light transmitted by, 297
 — sphere, air bubble in, 356
 Gold-leaf electroscope, 474
 Goniometer, 380
 Gram-equivalent, 543
 Gramophone, 285
 Graphical construction for concave mirror, 333

- Graphical construction for convex mirror, 336
 — — — lens, 346
 Grating, diffraction, 434
 Gravesande's ring, 145
 Gravitation, law of, 70
 — — —, verification by motion of moon, 71
 Grease spot photometer, 294
 Gridiron pendulum, 149
 Guerike, Otto von, 88, 99
 Guinea and feather, 17
 Gyration, radius of, 67

 Hadley, 371
 Half-watt lamp, 565
 Hardness, 123
 Harmonics, 274
 Heat, a form of energy, 206
 — rays, 386
 — —, demonstration of, 228-31
 — —, experiment on, 387
 — —, wave-length of, 419
 Heating effect of current, 505, 523-24
 Helmholtz, 403, 406
 Henry, 588
 Henry, the, 587
 Herschel, Sir William, 386
 Hertz, 590
 Hibbert's magnetic balance, 462
 High frequency wave, 591
 "Homogeneous atmosphere," 94
 Hooke's law, 116
 Hope's apparatus, 156
 Horizon, artificial, 372
 Horse-power, 61
 Hughes, 568
 Humidity, relative, 195
 Hunnings, 568
 Huygens, 415
 Huygens' construction, 415-17
 Hydraulic press, 83
 Hydrogen atom, mass of, 543
 — thermometer, 166
 Hydrometer, Nicholson's, 113
 —, variable immersion, 112
 —, Watt's, 90
 Hydrostatic balance, 110
 — method of determining coefficient of expansion, 156
 — paradox, 82
 Hygrometer, chemical, 200
 —, Daniell's, 199
 —, Dines', 200
 —, Regnault's, 197
 —, wet and dry bulb, 198
 Hygrometric state, 196

 Hygrometry, 195-201
 Hypermetropia, 399
 Hypsometer, 140
 Hysteresis, magnetic, 563
 "Hyvac" pump, 102

 Ice calorimeter, Bunsen's, 177
 —, determination of density of, 184
 Iceland spar, 420
 Ice-pail, Faraday's, 479, 483
 Illumination of a surface, 292
 — — — —, measurement of, 299
 — of rooms, 300
 —, variation with angle of incidence, 298
 Image of a line by reflection, 304
 — — — point by reflection, 303
 — — — — refraction, 316
 — produced by a prism, 377
 Images, multiple, 308-11
 Impact, elastic, 42
 Incandescent lamp, 564
 Inclined mirrors, 309-11
 Inclined plane, motion on, 37
 Index of refraction, 314
 Induced currents, 553-58
 Inductance, 586
 Induction coil, 559-60
 —, magnetic, 442
 Inductor, earth, 556
 Inertia, moment of, 66
 Infra-red, 386
 Ingenhousz's experiment, 218
 Interference of light, 427-34
 — — sound, 254
 — — surface waves, 243
 Interval between notes, 269
 Invar, 149
 Inverse square law, 292-93
 Ions, electrolytic, 544
 —, gaseous, 579
 Iris, 397
 Irrationality of the spectrum, 419
 Isoclinal lines, 466
 Isogonals, 464
 Isotopes, 580

 Jet, force exerted by, 95
 —, luminous, 324
 Joly's steam calorimeter, 178
 Joule's experiments on nature of heat, 203
 — equivalent, 204
 — law, 523

 Kelvin, 211, 502, 511, 532, 551, 588
 —, scale of temperature, 167

- Kepler, invention of telescope, 364
 Kepler's law of refraction, 330
 Keys, 528
 Kilowatt hour, 528
 Kirchhoff and Bunsen spectroscope, 389
 Kirchhoff's laws, 521-22
 Kite, 96
 Kundt's interference tube, 255
 — tube, 288

 Lambert, superposition of colours, 404
 Lamp, arc, 565
 —, flame arc, 565
 —, half-watt, 565
 —, incandescent, 564
 Land and sea breezes, 225
 Lantern, projection, 373
 Laplace, velocity of sound, 256
 Latent heat of fusion, 174
 — — — steam, determination of, 175
 — — — vaporisation, 174
 — — — water, determination of, 175
 Laue, 582
 Lechlanché cell, 504, 546
 Left-hand rule, 534
 Lens, formula for, 347
 Lenses, 345-62
 —, two, in contact, 349
 Lenz's law, 554
 Leslie's cube, 232
 Lever, 29
 Leyden jar, 491, 495
 — —, detachable, 499
 — —, discovery of, 499
 Lift pump, 103
 Lightning, 409
 Linear expansion, coefficient of, 145
 — —, measurement of, 147
 Lines of electric force, 482
 — — magnetic force, 446
 Liquid air machine, 213
 Lissajous' figures, 251, 267
 Long sight, 399
 Longitudinal waves, 247, 419
 Loops, 245
 Low frequency wave, 591
 Lumen, 292
 Luminous jet, 324
 Lux, 292

 μ , 418
 $\mu\mu$, 418
 Machines, 56
 M'Leod Gauge, 105
 Macula lutea, 397

 Magdeburg hemispheres, 89
 Magic dagger, 342
 Magnet, making a, 441, 514
 Magnetic balance, Hibbert's, 462
 — elements, 466
 — field, 445
 — —, mapping a, 447-50
 — — of electric current, 506
 — induction, 561
 — storms, 469
 Magnetisation, intensity of, 561
 Magnetism, molecular, 450-51
 Magneto, 573
 Magnetometer, 457, 562
 Magnification of compound microscope, 371
 — of lens, 348
 — of telescope, measurement of, 367
 — produced by spherical mirror, 334
 Magnifying glass, 363
 — —, measurement of magnification, 364
 Malleability, 123
 Malus, 422
 Manometer, 100
 Manometric flame, 282
 Mapping magnetic field, 447-50
 Marcet's boiler, 192
 Mariotte, 93
 Maximum thermometer, 143
 Maxwell, Clerk, 403, 406, 590
 Maxwell's colour disc, 404
 Measurement of resistance by substitution, 529
 Mechanical advantage of a machine, 57
 — equivalent of heat, 204
 Melde's experiment, 277
 Melting-point, change of, with pressure, 184
 Meniscus lens, 345
 Metacentre, 114
 Metallic conduction, theory of, 579
 Metre-candle, 292
 Michelson, 413
 Micro-farad, 490, 538
 Micrometer screw gauge, 7
 Microphone, 568
 — transmitter, 568
 Microscope, 370
 —, simple, 363
 —, vernier, 371
 Microscopic method of determining refractive index, 317
 Miller's air thermometer, 164
 Millikan, 484

- Minimum deviation, 376
 — —, formula for, 378
 — thermometer, 143
 Mirage, 328
 Mirror galvanometer, 511
 —, manufacture of, 304
 —, rotation of, 306
 —, rotating, 282
 — telescopes, 369
 Mirrors, inclined, 309–11
 —, parallel, 308
 —, spherical, 331–44
 Mixtures, law of, 171
 Molecular forces, 122
 — magnetism, 450–51
 Molecules and atoms, 121
 Moment of a magnet, 453
 Moments, law of, 27
 Momentum, 33
 Monochord, 273
 Monochromatic light, 384
 Morse code, 566, 590
 — sounder, 567
 Motor, electric, 570
 — —, back E.M.F. in, 572
 — —, efficiency of, 573
 Moving coil galvanometer, 535
 Multiple images, 308–11, 320–21
 Musical scale, 269
 Myopia, 399

 Natural light, 421
 Near point, 398
 Negative, photographic, 373
 Neon lamp, 390
 Neutral point, 448
 Newcomb, 413
 Newton, 71
 —, combination of colours, 403, 404
 —, discovery of spectrum, 383, 385
 —, velocity of sound, 256
 Newton's law of cooling, 234
 — laws of motion, 33–36
 — rings, 433
 Nicholson's hydrometer, 113
 Nickel plating, 545
 Nicol prism, 421
 Nodes, 245
 Norman, Robert, 469

 Oersted's experiment, 504
 Ohm's law, 516
 One-fluid theory, 484
 Onnes, Kamerlingh, 167
 Optical bench, 339
 — centre of a lens, 358
 — lantern, 373

 Organ pipes, 280–82
 Orthochromatic, 387
 Osmotic pressure, 130
 Overtones, 275

 Panchromatic, 387
 Papin's digester, 187
 Parabolic motion, 21
 Parallax, 305
 Parallel forces, equilibrium of, 30
 — mirrors, 308
 Parallelogram of forces, 24
 — — velocities, 13
 Paramagnetic substances, 443
 Partial, upper, 275
 Pascal's experiments on barometer, 85
 — vases, 76
 Peltier effect, 551
 Pelton wheel, 95
 Pendulum, Blackburn's, 251
 —, compound, 69
 —, gridiron, 149
 —, simple, 38
 Penumbra, 291
 Pepper's ghost, 307
 Perforated screen photometer, 295
 Period of vibration of magnet, 459
 Permeability, magnetic, 561
 Perrotin, 412
 Phantom bouquet, 342
 Phonograph, 285
 Phosphorescence, 388
 Photographic camera, 372
 — plate, 373
 Photometer, Bunsen, 294
 —, grease spot, 294
 —, perforated screen, 295
 —, Rumford, 294
 —, shadow, 294
 —, Trotter, 295
 —, wedge, 295
 Photometry, 296
 —, experiments in, 298
 Pigments, mixing coloured, 405
 Piledriver, 62
 Pinhole camera, 290
 Pin optics experiment on refraction,
 318
 — — experiments, 304–06
 — — methods applied to glass prisms,
 378
 Pipes, organ, 280–82
 Pitch, measurement of, 265–68
 Planck, 424
 Planck's constant, 424
 Plane-polarised light, 422
 — wave, 420

- Platinite, 150
 Platinum thermometer, 532
 Pohl's commutator, 529
 Polarimeter, 423
 Polarisation of cell, 546
 Polariscopes, 423
 Polarising angle, 432
 Polygon of forces, 26
 Positive ray analysis, 580
 — rays, 580
 Post-office box, 531
 Potential, 486-90
 —, due to spherical conductor, 488
 —, sparking, 575
 Potentiometer, 539
 Poundal, 35
 Presbyopia, 400
 Pressure, centre of, 80
 — in a fluid, 75
 — — — — same in all directions, 78
 —, transmission of fluid, 81
 Prevost's theory of exchanges, 233
 Primary colours, 402
 Prism glasses, 369
 —, image produced by, 377
 —, reversing, 132
 —, totally reflecting, 324
 Prismatic compass, 470
 Prisms, 376
 Projectile, range of, 22
 Projection of a spectrum, 385
 Ptolemy's data on refraction, 315-16
 Pulleys, systems of, 60
 Pumps, 99-106
 Pyknometer, 111
 Pyrometry, optical, 237
 Pythagoras, 274
- Quadrant electrometer, 502
 Quantum, 423
- Radiation of heat, 227-38
 Radioactivity, 583-85
 Radiogram of hand, 582
 Radium, discovery of, 583
 —, energy of, 585
 Rainbow, 391
 —, secondary, 392
 Rarefaction, 247
 Ray, reversibility of, 303
 Rayleigh's sensitive flame, 259
 Raymond, Du Bois, 560
 Real image, 303
 Réaumur scale, 138
 Rectilinear propagation of light, 289, 436
- Reduction factor of galvanometer, 510
 Reflecting telescopes, 369
 Reflection, diffuse, 302
 —, experiments on, 304
 —, laws of, 302
 —, of water wave, 242
 —, total, 322
 Refraction at a spherical surface, 355
 —, atmospheric, 327
 —, double, 420
 —, laws of, 313-15
 — of sound, 257
 — of water wave, 242
 —, through a lens, 357
 Refractive index, 314
 — —, absolute, 326
 — — by microscopic method, 317
 Regelation, 184
 Regnault, 253
 Regnault's apparatus for absolute expansion of mercury, 155
 — hygrometer, 197
 Relative density, 110
 — humidity, 195
 Relay, telegraphic, 567
 Relief, perception of, 401
 Resinous electricity, 472
 Resistance box, 527
 Resistances in parallel, 518
 — — series, 518
 Resolving power of eye, 398
 — — — telescope, 366
 — — — —, measurement of, 367
 Resonance, 277
 —, electric, 589
 —, velocity of sound by, 279
 Retina, 397
 Reversibility of ray, 303
 Reversing prism, 374
 Rheostat, 528
 Rigidity modulus, 118
 Ritchie's apparatus, 233
 Ritter, J. W., 387
 Robison, 447
 Rods and cones, 397
 Römer, 409
 Röntgen, W. K., 581
 Roget's spiral, 537
 Rotating mirror, 282
 Rotation of a plane mirror, 306
 — — plane of polarisation, 422
 Rotational motion, 66-70
 Roy and Ramsden's apparatus, 147
 Ruhmkorff coil, 560
 Rumford photometer, 294

- Rumford's experiments on nature of heat, 202
 Rutherford, 485, 583, 584
 S H.M.'s, composition of, 248-51
 Saccharimeter, 423
 Sail, action of wind on, 96
 Saturated vapour, 190
 Saturation pressure, 190
 Savart's wheel, 264
 Scalp's, 13
 Scale in boilers, 223
 Sclerotic, 397
 Screening, electric, 484
 Screw gauge, 7
 — press, 58
 Searle's apparatus for determining J, 205
 — — — measuring conductivity, 219
 Secular change, 468
 Seebeck, 550
 Self-induction, coefficient of, 586
 Sensitive flame, 258
 Setting sun, fish and, 323
 Sextant, 371
 Shadow photometer, 294
 Shadows, 291
 Shearing strain, 117
 Shielding effect of iron, 450
 Ships, stability of, 114
 "Short mercury column," 93
 Short sight, 399
 Shunts, 519
 Sidereal day, 5
 Signs, convention of, 334, 337
 Silver plating, 545
 Simple harmonic motion, 20
 — microscope, 363
 — pendulum, 38
 Sine galvanometer, 511
 Singing flame, 283
 Siphon, 105
 Siren, 262
 Snell, 316
 Soap bubble, excess of pressure inside, 128
 Sodium flame, 383
 — spectrum, 390
 Solenoid, 514
 —, suspended, 535
 Solidification, change of volume on, 184
 Solitary wave, 247
 Sonometer, 273
 Sound ranging, 260
 —, velocity of, 252
 South magnetic pole, discovery of, 470
 Sparking potential, 575
 Specific conductivity, 517
 — gravity, 109-13
 — — bottle, 111
 — heat, 170
 — — by method of cooling, 235
 — —, determination of, 172
 — heats of a gas, difference of, 208
 — inductive capacity, 403, 494
 — resistance, 517
 Spectacles, calculation of, 400
 Spectrometer, 380
 —, experiments with, 382
 Spectroscope, 389
 —, direct vision, 387
 Spectrum analysis, 390
 —, complete, 419
 —, continuous, 390
 —, discovery of, 383
 —, projection of, 385
 Speculum metal, 369
 Speed, 10
 — time diagram, 11
 Spherical aberration, 394
 — — of eye, 398
 — mirrors, 331-44
 — —, experiments on, 339
 — surface, refraction at, 355
 Sphero-cylindrical lens, 400
 Spherometer, 8
 Spirit level, 75
 Spring, potential energy of, 63
 Standard atmospheric pressure, 89
 — candle, 292
 Stars, twinkling of, 377
 Stationary waves, 244
 Steam calorimeter, Joly's, 178
 Stefan's law, 236
 Stem correction of thermometer, 141
 Stereoscope, 401
 Stokes, explanation of fluorescence, 388
 Storage cell, 548
 Stress and strain, 117
 — produced by change of temperature, 150
 Stretched string, 273-75
 Stroboscopic disc, 266
 Sturm and Colladon, 253
 Sublimation, 184
 Sun, temperature of, 238
 —, constitution of, 391
 Supercooled water, 183
 Surface tension, 123-28
 — — —, measurement of, 125

- Susceptibility, magnetic, 501
 Swan spectrum, 390
 Swimming man rule, 506
 Tangent galvanometer, 509-10
 Telegraph, 566
 Telephone, 567
 Telescope, astronomical, 364
 —, formula for magnifying power, 365
 —, Galilean, 367
 —, resolving power of, 366
 —, terrestrial, 368
 Telescopes, mirror, 369
 Temperature and thermometers, 135-144
 — gradient, 216
 Tenacity, 123
 Terrestrial telescope, 368
 Test of a flash lamp, 299
 Thales, 472
 Therm, 170
 Thermal capacity, 170
 — unit, British, 170-79
 Thermo-couple, 550
 Thermo-electric thermometer, 551
 Thermo-electricity, 550
 Thermometer, construction of, 137
 —, differential, 227
 —, platinum, 532
 —, thermo-electric, 551
 —, weight, 153
 Thermometric fixed points, verification of, 139
 — scales, 137
 Thermopile, 227, 387, 551
 Thermos flask, 236
 Thermostat, 157
 Thin films, colours of, 431
 Thomson effect, 551
 Thomson, J. J., 485, 577
 Three-colour printing, 406
 Timbre, 276
 Toepler pump, 101
 Tongs, discharging, 486
 Toric or toroidal lens, 400
 Torricelli's experiment, 85
 — theorem, 97
 Total reflection, 322
 —, experiment on, 323
 Totally reflecting prism, 324
 Tourmaline, 422
 Tracing position of image for lens, 354
 — — — — spherical mirror, 340
 Trade winds, 225
 Transformer, 573
 Transmission of glass plate, 297
 Transmitter, microphone, 568
 Transverse waves, 246, 419
 Triangle of forces, 26
 Trotter photometer, 295
 Tuning-fork, 264
 — —, electrical, 265
 Twilight, 315
 Twinkling of stars, 377
 Two-fluid theory, 484
 U-tube, 78
 Ultra-violet, 387
 Umbra, 291
 Unit charge of electricity, 481
 — current, 507
 — electromotive force, 507
 — quantity of magnetism, 444
 — resistance, 517
 Vacuum, theoretical, of air pump, 100
 Vapour pressure, 190-95
 — —, Regnault's method of determining, 191
 Variation, magnetic, 464
 Variometer, 593
 Vectors, 12
 Velocity of light, aberration method, 410
 — — —, Fizeau's method, 411
 — — —, Foucault's method, 413
 — — —, Römer's determination, 409
 — — —, Galileo's experiment, 409
 — — — sound, 252
 — — — by resonance, 279
 — — —, effect of temperature on, 256
 — — — in air, calculation of, 255
 — — — —, variation with intensity, 253
 — — — —, proof of formula for, 257
 — — — in string, proof of formula for, 276
 Velocity ratio of a machine, 59
 Vena contracta, 97
 Vernier, 5
 Vibrations of rods and plates, 284
 Virtual image, 303
 Viscosity, 120
 Vision, binocular, 401
 —, defects of, 399
 Vitreous electricity, 472
 — humour, 397
 Volt, 488, 508
 Voltaic cell, 545-49
 Voltameter, 505, 543
 Voltmeter, 536

- Water equivalent, 171
— —, determination of, 171
—, maximum density point, 156
— waves, 241
Watt, James, 61
Watt, unit of activity, 61, 523
Watt's hydrometer, 90
Wave theory of light, 415-37
— motion, 241-47
Wave-length, 417-19
Waves, electromagnetic, 590
Wedge photometer, 295
Weight of light, 425
— thermometer, 153
Wet and dry bulb hygrometer, 198
Wheatstone's bridge, 530
Wheel and axle, 59
Whirl, electric, 478
White light, recombination of, 385
Wiedemann-Franz law, 221
Wilson, C. T. R., 580
Wimshurst electrical machine, 500-02
Wind, electric, 478
Winter's frictional machine, 497
Wire bridge, 530
Wireless waves, 419
Work, definition of, 55
— done by expanding gas, 207
—, principle of, 58
Wratten filters, 406

X-Rays, 419, 581-82

Yellow spot, 397
Yerkes telescope, 366
Yield point, 119
Young-Helmholtz theory of colour vision, 406
Young's modulus, 118

Zeiss, 369
Zero, absolute, 166
Zincite-bornite, 592

CONSTANTS

Substance.	Specific gravity.	Coeff. of expansion per degree C.	Specific heat.	Thermal conductivity. cal./cm. sec. deg. C.	Specific resistance, ohm-cm.
Aluminium	2.65	25.5×10^{-6}	0.21	0.50	3×10^{-6}
Brass . . .	8.4 to 8.7	18.7 "	0.090	0.26	6 to 9 "
Copper . . .	8.93	16.7 "	0.093	0.92	1.7 "
Iron . . .	7.86	11 "	0.119	0.16	9 to 15 "
Lead . . .	11.37	27.6 "	0.030	0.083	—
Platinum . .	21.50	8.9 "	0.032	0.17	11.0 "
Steel . . .	7.7 to 7.9	11 "	—	—	—
Tin . . .	7.29	21.4 "	0.055	0.15	—
Glass . . .	About 2.5	8 "	0.12 to 0.2	0.002	10^{11} to 10^{14}
Mercury . .	13.6	182 " (cubical)	0.033	0.018	95×10^{-6}
Methyl alcohol .	0.80	—	0.60	—	—
Turpentine . .	0.87	—	0.42	—	—
Ice . . .	0.916	—	0.50	0.005	—

DENSITY OF WATER AT DIFFERENT TEMPERATURES

Temp.	Density.	Temp.	Density.	Temp.	Density.
0° C.	0.99987	7° C.	0.99993	40° C.	0.9922
1	0.99993	8	0.99988	50	0.9881
2	0.99997	10	0.9997	60	0.9832
3	0.99999	15	0.9991	70	0.9778
4	1.00000	20	0.9982	80	0.9718
5	0.99999	25	0.9971	90	0.9653
6	0.99997	30	0.9957	100	0.9584

INDICES OF REFRACTION FOR SODIUM LIGHT

Crown glass about	1.52	Water	1.333
Flint glass about	1.54 to 1.7	Amyl alcohol	1.41
Diamond	2.417	Ethyl alcohol	1.362
Ice	1.31	Methyl alcohol	1.332
Carbon bisulphide at 0° C. . .	1.644	Ethyl ether	1.352
Carbon bisulphide at 20° C. .	1.628	Glycerine	1.463

The refractive index of air under standard conditions is 1.00029.

A silver mirror reflects about 93 per cent. and a polished glass surface reflects about 4 per cent. of the light incident perpendicularly upon it.

WAVE-LENGTHS IN VISIBLE SPECTRUM

Hydrogen vacuum tube	4102, 4340, 4861, 6563. A. U.
Mercury arc	4046·8, 4078·1, 4358·2, 4916·4, 5460·7, 5780·6 5790·7.
Helium discharge lamp	4389, 4438, 4471, 4713, 4922, 5016, 5048, 5876 6678, 7065.
Sodium flame	5890·0, 5895·9.
Lithium flame	6707·8.
Thallium flame	5350·7.

RATIO OF LIGHT DIFFUSELY REFLECTED TO LIGHT RECEIVED
(Ordinary Daylight)

White cartridge paper	0·80	Emerald green paper	0·18
Ordinary foolscap	0·70	Black cloth	0·015
Yellow wallpaper	0·40	Black velvet	0·005

ELASTICITY MODULI
(In dynes per sq. cm.)

	<i>Young's Modulus.</i>	<i>Rigidity Modulus.</i>	<i>Bulk Modulus.</i>
Brass	$1·0 \times 10^{12}$	$3·5 \times 10^{11}$	10×10^{11}
Copper	1·24 „	4·0 „	14 „
Iron	2·0 „	8·0 „	14 „
Steel	2·0 „	8·0 „	18 „

Surface tension of water 74 dynes per cm.

Surface tension of mercury 547 dynes per cm. Angle of contact between mercury and glass $52^\circ 40'$. These values vary largely with the freshness of the surface.

Gravitational constant, $k = 6·66 \times 10^{-8}$ c.g.s. unit.

g at Equator = 978 cm. per sec.²

g at Greenwich = 981·18 cm. per sec.²

g at North Pole = 983·2 cm. per sec.²

1 Horse-power = 33,000 ft.-lb. per min.

1 Kilowatt = 1·34 horse-power.

1 cubic foot of water weighs 62·43 lb. Specific gravity of paraffin wax 0·91, of cork 0·25, of sea-water 1·01 to 1·05.

1 litre of dry air weighs 1·293 gm. at 0° C., 76 cm.

Velocity of sound in air is 331·3 metres/sec. or 1087 ft./sec. at 0° C.

Velocity of light is $3·00 \times 10^{10}$ cm./sec. = 186,000 miles/sec.

Electrochemical equivalent of silver = 0·001118 gm. per coulomb.

Saturated vapour pressure of aqueous vapour on pp. 141, 193, and 198.

THE END

