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DESCRIPTIVE GEOMETRY

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Preface

THE specific solutions of problems encountered by the engineer are based on empirical data which, in general, are graphical in nature. A scale, whether it be on a thermometer, on an electric meter, or on a transit, is a graphical device. The conversion from graphical data to numerical data is made directly from the scale and is limited by the accuracy of the scale. Thus the engineering problem at the outset is limited by graphical accuracy.

Algebraic and functional mathematics are, in general, theoretically capable of absolute accuracy. This fact tends to obscure the fact that no solution can be more accurate than the data upon which it is based.

Graphical solutions are not capable of absolute accuracy. However, most engineering problems are capable of being solved graphically, and such solutions are, in general, considerably quicker and simpler than algebraic solutions.

The point of importance, which needs heavy emphasis, is that the complete graphical solution is absolutely as accurate as the algebraic solution so long as the graphical accuracy is within the accuracy of the empirical data upon which the solution is based.

It is the conviction of the authors that in a substantial percentage of cases the graphical method is the preferable method. It is as accurate as the algebraic method; it requires less time in effecting the solution; it is more readily understandable; and it is more fruitful in yielding hidden relationships.

Certainly the engineering student should be equally well acquainted with both approaches so that he may choose the better one to suit the specific problem confronting him.

The present volume is meant to cover the basic material of elementary graphical theory applicable to engineering problems and to develop the ability of the student to solve such problems by the simplest graphical methods. The great bulk of such theory lies in the descriptive geometry which consequently constitutes the greater part of the volume.

The authors do not believe in limiting the teaching of descriptive geometry to one method of attack, as is done when either the "direct" or the "Mongean" method is presented alone. They do believe that all methods are a part of the subject and that the student does not develop adequate power unless he is capable of choosing the simplest of the

possible solutions of any given problem. Consequently, this book attempts to point out the various possibilities of attack on the basic group of problems underlying the subject.

Since the graphical solution of an engineering problem does not always fall in the field of descriptive geometry, chapters have been added in the Appendix covering "Precision in Drawing" and the "Properties of Plane Figures." The latter chapter assumes that the student is acquainted with the elementary plane geometry constructions. It, therefore, contains only those constructions useful in engineering drawing and certain special constructions, chiefly with regard to conics, which are of great importance in engineering work.

A ɔter on stereoscopic drawing has also been placed in the Appendix, si this subject is not contained in any other text and should be availa

Th ɛview figures at the ends of some of the chapters are one-eye views en directly from a set of stereoscopic projection slides used by the au rs, which may be obtained from the Society for Visual Education, , 100 East Ohio Street, Chicago 11, Illinois. There are fifty slides he set, which are available in vectograph form either as prints or for ɔjection by means of a standard slide projector. The slides presen ıe diagrams in their full three-dimensional spatial relationships. The n ɔers on the cuts correspond to those on the slides.

Th ıthors are indebted to Professor A. L. Goodrich of the Massachuset nstitute of Technology for his general assistance and advice.

E. F. W.
J. T. R.

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Notation

POINTS of space are represented by capital letters: A, B, C, . . . A line determined by two points A, B, is termed the line AB.

Unlimited lines of space are represented by small letters: a, b, k, . . .

Unlimited planes of space are represented by capital letters: H, V, P, Q, R, . . .

A plane determined by three points A, B, C, is called the plane ABC.

A plane determined by two intersecting lines a and b is called plane ab.

In general, given elements are denoted by early letters of the alphabet, required elements by late letters; given A_H, A_V, A_P, required P_H, P_V.

The projection of a point on a plane is represented by a letter for the point and a subscript for the plane. Thus, A_H is the projection of point A on the horizontal plane. The projection of a line is represented in the same manner. Thus, k_V is the vertical projection or front view of the unlimited line k. A_HB_H is the top view of the line segment AB.

The line of intersection of two planes is represented by the letters of the two determining planes. Thus, the intersection of plane Q with the plane H is denoted as HQ.

CHAPTER I

Principles of Orthographic Projection

1. Descriptive geometry defined

Descriptive geometry is essentially the technique of accurately representing objects by means of drawings and of solving graphically all problems related to their form and position. Descriptive geometry provides the theoretical basis for technical drawing.

In general, an object or structure may be considered as a combination of elementary geometrical forms. These forms are commonly prisms and cylinders, but may also include pyramids, cones, surfaces of revolution, and warped or twisted surfaces. These various space figures can be analyzed into points, lines, and surfaces as the basic geometrical elements. Descriptive geometry deals specifically with the graphical representation on a plane (the drawing surface) of the basic geometrical elements and the solution of space problems connected with their representation.

2. Orthographic projection

The concept of *projection* provides a useful medium for explaining the process of drawing. If a straight line is passed through any point

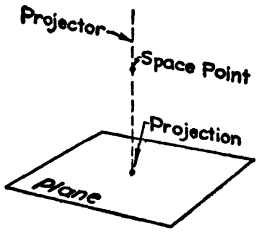


Fig. 1

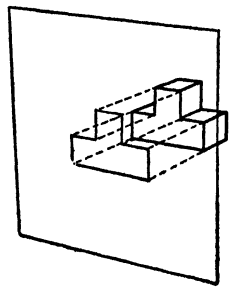


Fig. 2

of space and perpendicular to a plane, the point of space is said to be *projected orthogonally* on the plane at the point where the perpendicular intersects the plane (Figure 1). The perpendicular is called the *projector*. The plane is the *projection plane* and is represented by the drawing paper. The projection of a straight line is determined by the projections of two of its points. The projection of an object on a plane is simply the projection of a sufficient number of lines to define the object (Figure 2). A

projection is often called a *view* in practice, since it represents the object as it would appear to the eye if sight lines were assumed parallel. The term *orthographic projection* usually refers to the general process of representing an object by means of its projections or views as described here.

3. Correspondence

The elementary principles of orthographic projection can best be understood by considering the point as the unit of graphical representation.

It is evident that a single projection of a point on a plane does not completely represent the position of the point with respect to the plane. Every point lying in the projector has the same projection. In order that a projection system be useful, it is necessary that a given point of

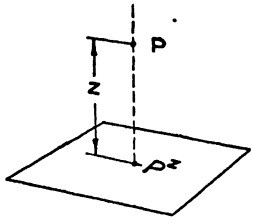


Fig. 3

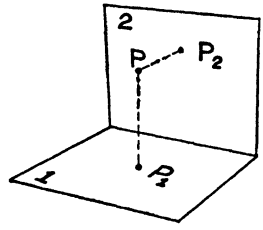


Fig. 4

space have a unique graphical representation, and, conversely, that the graphical representation of a point correspond to a single point of space.

One method for accomplishing this correspondence is to give the distance between the space point and its orthographic projection by means of an index number attached to the projection (Figure 3). This method is called *horizontal projection* and has its application in mapping.

For example, the summit of a hill is located on a map by its geographical coordinates, and its elevation above mean sea level is given as a number.

A second method for obtaining a correspondence between a space point and its graphical representation employs a second projection plane taken perpendicular to the first plane (Figure 4). The orthographic projection of the point on the second plane measures the distance between the space point and the first plane. The representation of an object by means of its projections on two mutually perpendicular planes is the system in which we are interested and will be developed in the material immediately following.

It may be noted here as a general proposition that the two projection planes can be assumed in any orientation in space so long as they are mutually perpendicular. In practice, they are taken in some definite position with respect to the object.

It remains to be shown how the projection system of Figure 4 is represented on the drawing surface.

4. To represent on a plane (the drawing surface) the projections of a point on two mutually perpendicular planes of space

The graphical representation of a space figure is based on the method for graphically representing a point of space. Two mutually perpendicular planes are assumed as projection planes (Figure 5). The line in which these two planes intersect is called the *reference line* (*RL*).

The point P of space is projected perpendicularly on planes 1 and 2, giving the projections P_1 and P_2. These projections, considered as marks on the planes, are the graphical representations of point P. In general,

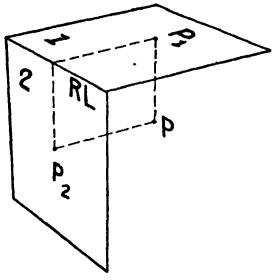


Fig. 5

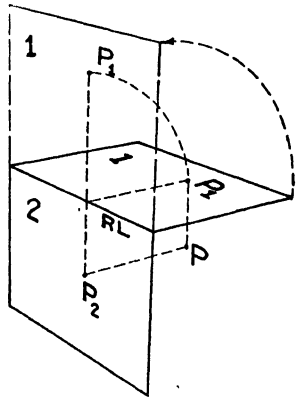


Fig. 6

each point of space has a unique pair of projections, and, conversely, any pair of projections corresponds to a single point of space.

The important feature of projection drawing is the method for representing on one plane (the drawing surface) the two projections which lie on two mutually perpendicular planes. Projection planes 1 and 2 are represented on the drawing surface by assuming them revolved about the reference line *RL* as an axis so that they lie in a single plane (Figure 6). The projections P_1 and P_2 now lie on a single line which is perpendicular to *RL*. This is a consequence of the fact that the point P together with its projections P_1 and P_2 lie in a plane which is perpendicular to *RL*. Figure 7 shows the two projection planes together with the projections of point P as they are actually represented on the drawing paper. Since the planes are considered unlimited in extent, boundary lines are not drawn. Thus, the presence of a reference line on a drawing indicates that there is a projection plane on each side of this line and that they were at right angles to each other before revolving. Also, the projections

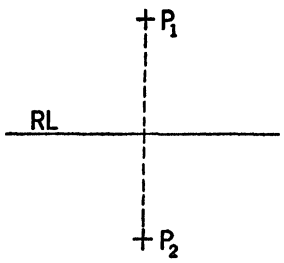


Fig. 7

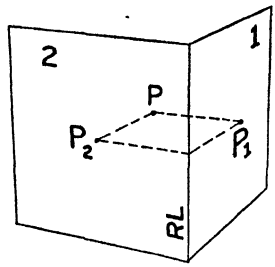


Fig. 8

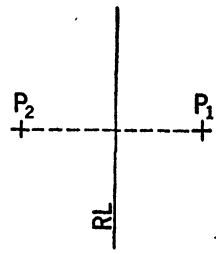


Fig. 9

of a point on the two planes must lie on a line perpendicular to the reference line.

Since two mutually perpendicular planes can have an indefinite number of positions in space, the set of planes shown in Figures 8 and 9 illustrate the principles of representation as well as the set assumed in Figures 5 and 7.

5. Position of the observer

The position of a point of space is interpreted from the conventional representation of Figure 7 by considering each projection plane separately. The projection plane here is assumed to be between the observer and the object. When viewing plane 1, the direction of sight is perpendicular to the plane, and point P appears as projection P_1 (Figure 10).

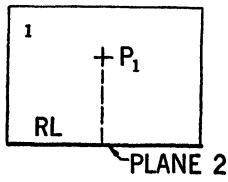


Fig. 10

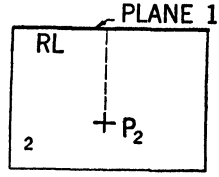


Fig. 11

Since plane 2 in space is at right angles to plane 1, it appears edgewise and is represented by RL. The distance between point P and plane 2 is measured by the perpendicular from P_1 to RL.

When viewing plane 2, the direction of sight is perpendicular to the plane, and point P appears as P_2 (Figure 11). Plane 1 appears edgewise and is represented by RL. The distance between point P and plane 1 is measured by the perpendicular from P_2 to RL.

6. Objects

The principal axes of the majority of objects are three mutually perpendicular lines. The word *object* as used here means a geometrical solid or a space figure susceptible to geometrical analysis. The axes of

an object are usually related to the earth by making one vertical and the other two horizontal. The principal dimensions of an object are measured parallel to the principal axes and are generally called the *width*, *depth*, and *height*. These are familiar terms commonly associated with a reference system oriented with respect to space and will be assigned as in Figure 12.

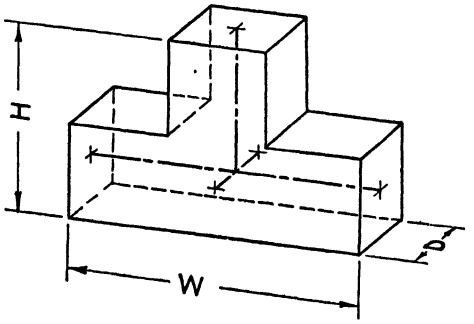


Fig. 12

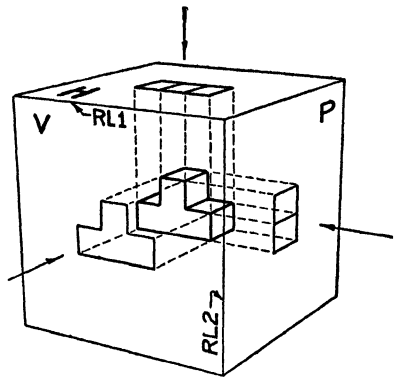


Fig. 13

7. Principal projections

The principal projections of an object are those that show its principal dimensions. It is evident that but two of these dimensions can appear in true size in a single orthographic projection. Thus, the width and depth appear on a horizontal projection plane (Figure 13). This projection is variously called the *horizontal projection*, *top view*, or *plan*. The width and height appear on a vertical projection plane. This projection is called the *front view* or *front elevation*. When the object is projected on a second vertical plane to show its depth and height, the projection plane is called a *profile plane*. The projection is the *profile* or *side view*.

8. Arrangement of views

The top and front views of the object of Figure 13 are represented on the drawing paper by applying the principles of Article 4. The top view is placed above the front view, and the width shows as a dimension common to both views (Figure 14). This object can also be represented by its front and side views (Figure 15), the height being the common dimension.

When an object consists of a combination of geometrical shapes, more than two views may be required for its representation. The top, front, and side views of the object of Figure 16 are necessary to describe its form clearly.

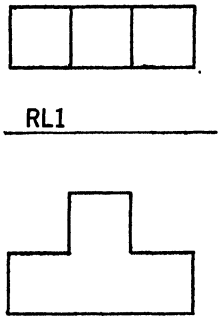


Fig. 14

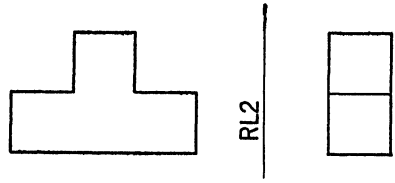


Fig. 15

9. Visualization

Reading a drawing is the mental process of visualizing the object from its projections. A view should be regarded as the object itself. The observer should assume that his direction of sight is vertical when looking at a top view, and horizontal when looking at a front view. A single view shows two dimensions of an object. The third dimension is in the direction of sight and appears in the adjacent view.

Thus, visualizing a top view requires that one build up a mental picture of the object by estimating the relative height of each point and line of the drawing. A front view is visualized by estimating the relative depth of each part of the drawing.

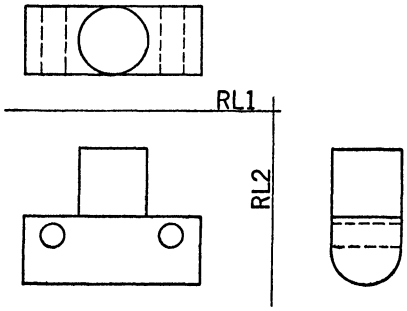


Fig. 16

While the foregoing description of drawing as a process of projection is convenient for the purposes of descriptive geometry, it should be noted that the method of representing an object by its top and front views is a natural one. In setting down on paper a concept of an object, one instinctively draws views which show its various faces in true proportion. The ancient Egyptians and Greeks made plans and elevations of buildings without recourse to any theory of projection, so far as we know.

10. Projections of simple objects

A few examples of objects represented in orthographic projection will serve to illustrate the principles developed in the preceding paragraphs. The horizontal, vertical, and profile projection planes will be referred to as H, V, and P respectively.

Example 1 (Figure 17). *A group of points.* If a point of space A is chosen as an origin, any second point B can be described with respect to it by stating the distance that B lies to the right or left, above or below, and in front or in back of point A. This is a natural system of rectangular coordinates in which directions are related to the observer.

Point A, given by projections A_H and A_V, is one vertex of a triangle ABC.

Point B is 4 units to the right of, 3 units in back of, and 2 units below A.

Point C is 7 units to the right of, 2 units in front of, and 2 units above point A.

It will be seen that distance forward or to the rear of a given point is measured in the top view. Distance above or below a given point is measured in the front view.

The use of rectangular coordinates will be treated more extensively in Chapter IX.

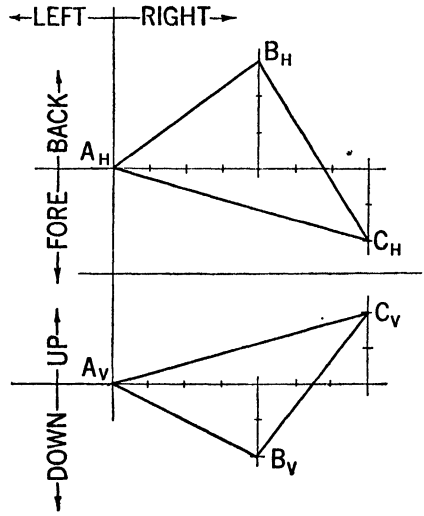


Fig. 17

Example 2. *Straight lines.* If a line is perpendicular to a plane, its projection on the plane is a point. A line segment parallel to a plane projects in its true length on the plane.

A *vertical* line projects as a point on the H-plane (Figure 18). A *horizontal* line segment appears in true length in the top view (Figure 19). The angle between the horizontal projection, C_HD_H, and RL is the true angle between line CD and the vertical plane. A *frontal* line is defined as a line parallel to the V-plane (Figure 20). The front view E_VF_V is a *true-length* projection and also shows the true angle between line EF and the H-plane. A *profile* line is parallel to the profile plane (Figure 21). The side view G_PK_P is a *true-length* view. The angle between G_PK_P and RL is the true angle between line GK and the V-plane.

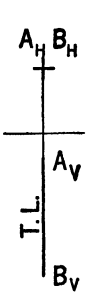
It should be noted that when a line segment is parallel to the reference line in one view, the adjacent view is a *true-length* view.

Example 3. *Plane figures.* The projection of a plane figure on a plane parallel to itself represents the figure in its true shape. Such a projection is generally called a *normal view*, since the observer's direction of sight is perpendicular to the plane of the figure. The projection of a

plane figure on a plane perpendicular to itself is a straight line. This projection is called an *edge view* or *line view* of the plane.

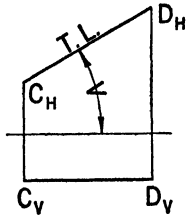
In Figure 22, the triangle P is horizontal. The top view is the normal view. The front view is a line view.

In Figure 23, the triangle Q is parallel to the V-plane. This position is called *frontal*. The front view is the normal view.



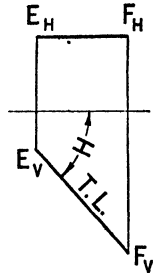
VERTICAL

Fig. 18



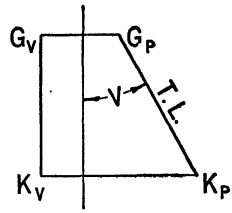
HORIZONTAL

Fig. 19



FRONTAL

Fig. 20

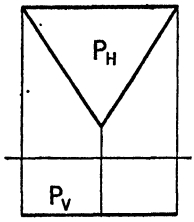


PROFILE

Fig. 21

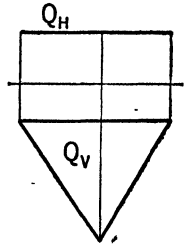
In Figure 24, the circle R is parallel to the profile plane. The side view is the normal view.

Example 4. *A square right prism* (*Figure 25*). One base is in the H-plane. The lateral edges of the prism are vertical. The top view is a normal view of the base and shows the true angle between one edge of the base and the V-plane to be 30°. The front view shows the true altitude of the prism. The bases project edgewise in this view.



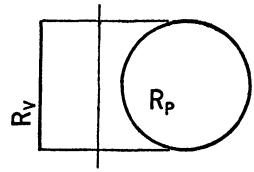
HORIZONTAL

Fig. 22



FRONTAL

Fig. 23



PROFILE

Fig. 24

Example 5. *A hexagonal right pyramid* (*Figure 26*). The base lies in the V-plane. The front view is a normal view of the base and shows the angle between one edge of the base and the H-plane to be 45°. The true length of the altitude appears in the top view. Since the apex is in back of the base, the lateral edges are hidden in the front view.

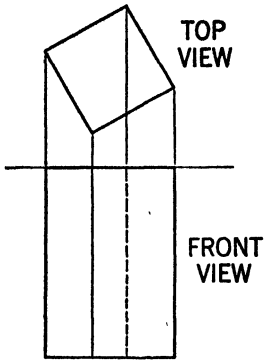


Fig. 25

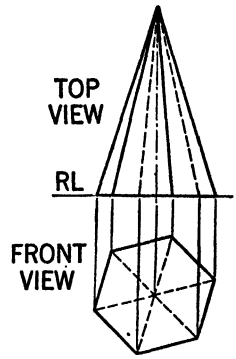


Fig. 26

11. Visibility of edges of a solid

Since only a portion of the surface of a solid can be seen from a given position, the projection of a solid usually contains both visible and hidden lines. The matter of correctly representing these visible and hidden edges is an essential part of the drawing of any solid. In any view of a solid, the boundary lines of the view are always visible lines. Also, the nearest edge to the observer is always visible in a view on any projection

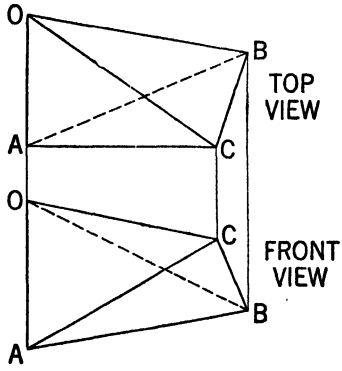


Fig. 27

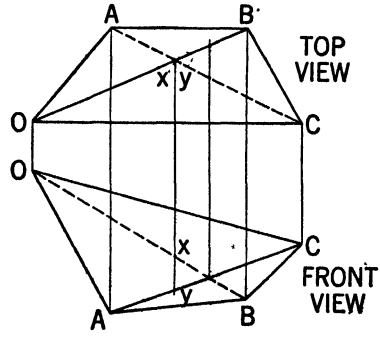


Fig. 28

plane. Thus, the highest edge of a solid is visible in the top view, and the foremost edge is visible in the front view.

The general method for determining visibility is illustrated by the tetrahedron (Figure 27). In the top view the edges OA, OB, BC, and AC are boundary lines of the view and are therefore visible. Edge OC is the highest edge (as can be seen in the front view) and is visible in the top view. Edge AC is the foremost edge (as can be seen in the top view) and is visible in the front view. The boundary lines of the front view are visible.

In the event that the visibility of certain lines of a view cannot be readily determined by inspection, the following procedure is useful. In the top view (Figure 28), assume points x and y on lines AC and OB at their crossing. Find the front view of these points. The higher point x is on the line OB, which is therefore visible in the top view. The visibility of the front view is determined in a similar fashion.

12. Quadrants

In the course of solving a problem, it may be necessary to extend a space figure through one or both of the projection planes. In this event

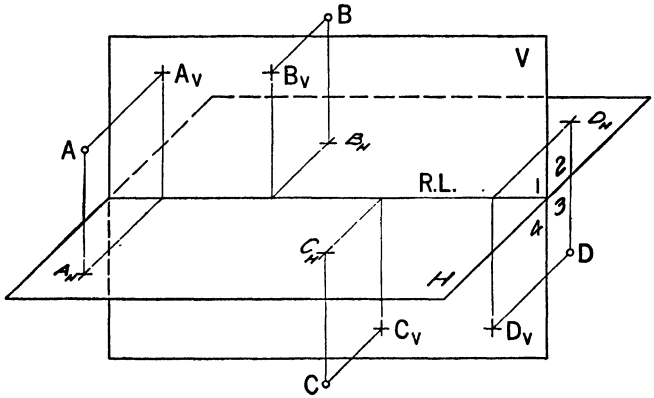


Fig. 29

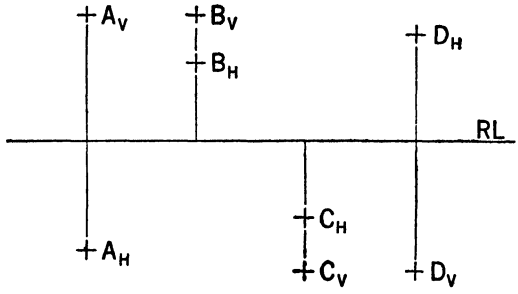


Fig. 30

the projection planes must be extended to both sides of the reference line. Hence, for general purposes, the H- and V-projection planes are considered to be indefinite in extent (Figure 29), thus dividing all space into four angles or quadrants, numbered as shown. Points A, B, C, and D lie in the first, second, fourth, and third quadrants respectively. Every point of space has, in general, two projections: one on each plane.

Both planes and the projections of the points are represented on a

single plane (the drawing surface) by assuming that the H-plane is rotated about RL as an axis until it coincides with the V-plane. The direction of rotation is such as will open the first and third quadrants and close the second and fourth.

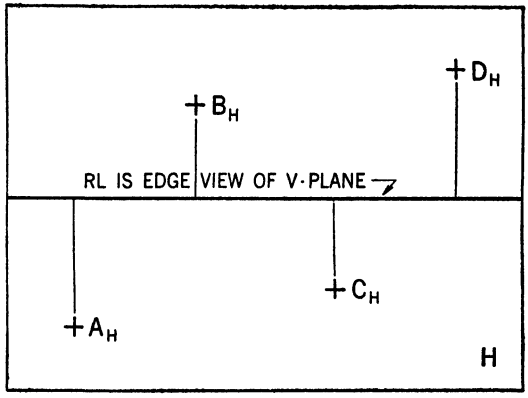


Fig. 31

The points of space $ABCD$ are now described by their pairs of projections A_HA_V, B_HB_V, C_HC_V, and D_HD_V (Figure 30). Each pair of projections lies on a projecting line perpendicular to the reference axis.

When viewing any horizontal projection, the observer is assumed

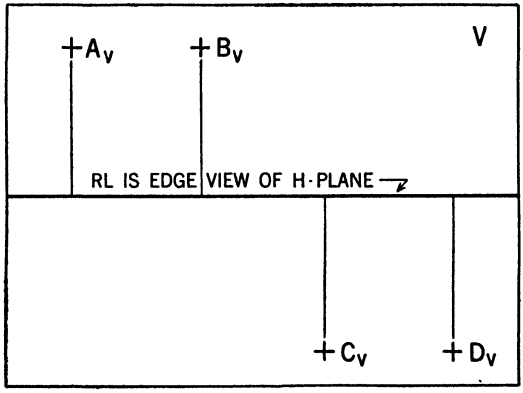


Fig. 32

above H and looking down (Figure 31). The distance that a point lies in front or in back of V appears in this view. When viewing any V-projection, the observer is assumed to be in front of V. The distance that a point lies above or below H appears in this view (Figure 32).

13. Arrangement of views

The arrangement on the drawing of the top and front views of an object depends on the position of the object with respect to the projection planes. American practice in engineering drawing places the object in the third quadrant (Figure 33). Third-quadrant projection places the

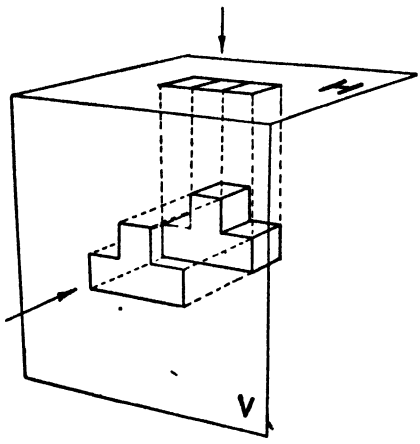


Fig. 33

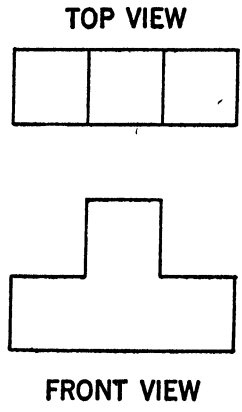


Fig. 34

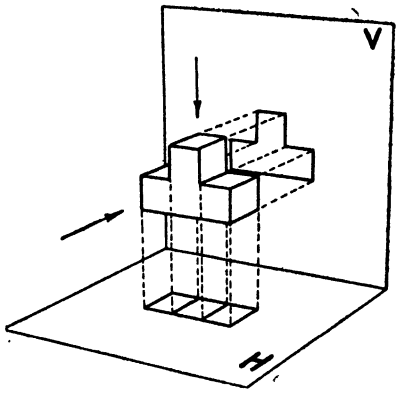


Fig. 35

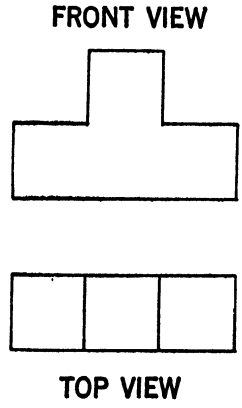


Fig. 36

top view above the front view on the drawing (Figure 34). In third-quadrant projection, the projection plane is always between the observer and the object. In most foreign countries, and to some extent in architectural and structural drawing in the United States, the object is placed in the first quadrant (Figure 35). This results in placing the front view above the top view on the drawing (Figure 36). In first-quadrant projection, the object is always between the observer and the projection

plane. It is to be noted that changing the quadrant of projection does not alter the views themselves but merely their arrangement on the drawing sheet.

14. Additional views

Although two projections completely represent any object determined by points, additional projections are often desirable for clarity or to obtain normal or true-size views of inclined figures. Article 15 establishes a method for constructing a third projection of an object if two projections are given.

The principle of Article 4 that any pair of adjacent projection planes shall be at right angles in space is always followed. Thus, any third view which is deduced from two given views must be a projection on a plane taken perpendicular to one of the given planes.

15. Given the projections of a point on two planes at right angles to each other; to determine a third projection on any third plane taken perpendicular to one of the given planes

I. *Special Case: To determine a profile projection (side view) of a point when the horizontal and frontal projections are given.* The profile plane is perpendicular to both the horizontal and frontal projection planes (Figure 37).

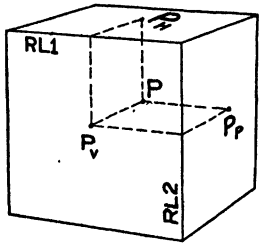


Fig. 37

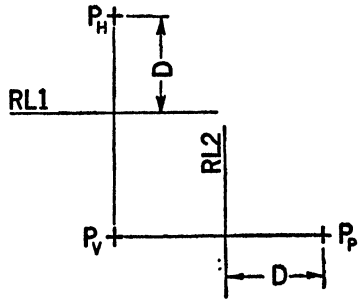


Fig. 38

Given: Projections P_H and P_V (Figure 38).

Take plane P perpendicular to H and V by drawing $RL2$ perpendicular to $RL1$. Project point P on plane P by drawing a projector through P_V perpendicular to $RL2$. Then lay off on this projector the distance from $RL2$ to P_P equal to the distance from $RL1$ to P_H.

When constructing the profile view of an object, it is convenient to place $RL1$ through the front corner of the figure in order to eliminate one measurement from the profile-view construction (Figure 39).

II. *General Case.*

Given: Projections P_1 and P_2 on planes 1 and 2 (Figures 40 and 41). Take any third plane perpendicular to either 1 or 2, such as plane 3 which is taken perpendicular to plane 2. Draw $RL2$ (Figure 41). Draw the projector from P_2 perpendicular to $RL2$. Now both planes 1 and 3 are perpendicular to plane 2 and parallel to the line of projection PP_2, which may be considered the distance of the point P from plane 2. Furthermore, since PP_1 and PP_3 are parallel to plane 2, P_1 and P_3 are the same distance from plane 2 as P. Thus, the distances of P_1 from RL_1 and of P_3 from RL_2 are equal. Thus, we may lay off this distance from RL_2 on the projector we have drawn to locate P_3.

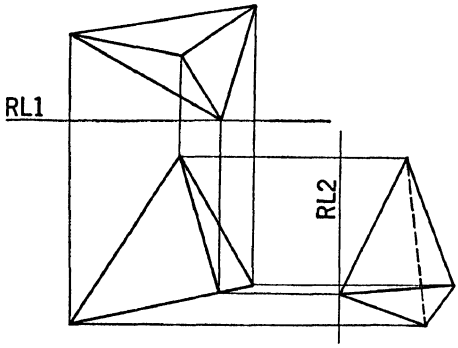


Fig. 39

Consider any group of three consecutive projection planes. Since adjacent planes are at right angles in space, each of the two extreme planes is perpendicular to the middle plane. Therefore, the projections of any given point on the two extreme planes appear equally distant from the respective reference lines.

This process is of fundamental importance in obtaining views. It

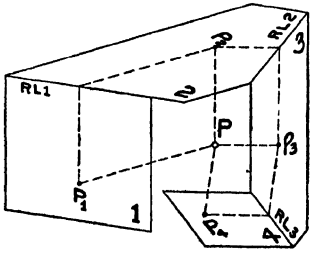


Fig. 40

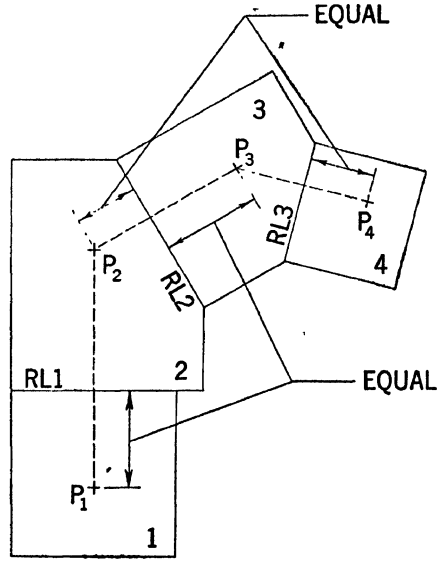


Fig. 41

should be thoroughly understood by the student. It may be generally stated as follows: *In any three successive views, the distances of the two outside views of any point from the respective reference lines are equal and represent the distance of this point from the central plane.*

16. Series of views

This principle may be extended to locate any number of additional projections. Thus, in Figures 40 and 41 assume that P_2 and P_3 are given projections. Take plane 4 perpendicular to plane 3 by drawing $RL3$. Project point P on plane 4 by drawing a projector through P_3 perpendicular to $RL3$. Lay off on this projector the distance from $RL3$ to P_4 equal to the distance from $RL2$ to P_2.

Figure 42 shows five successive views of a tetrahedron. Since the above principle applies to any three successive views and since planes 3

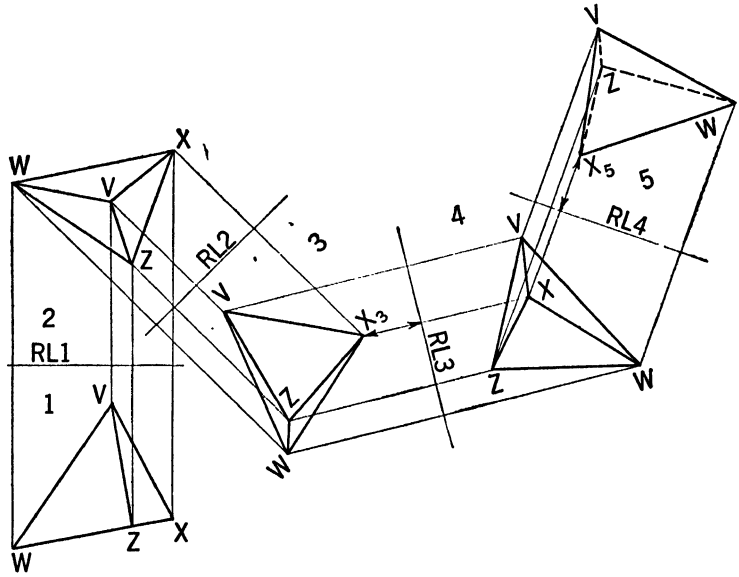


Fig. 42

and 5 are each at right angles to plane 4, the distance from X_3 to $RL3$ is equal to the distance from X_5 to $RL4$.

17. Auxiliary view of an object

Example 1 (Figure 43). *To draw an auxiliary elevation of the tetrahedron as seen in the direction DE.* The desired view is the projection of the figure on a plane taken perpendicular to the given direction of sight DE. This plane is perpendicular to the *horizontal plane.*

Draw $RL2$ perpendicular to DE, allowing a convenient space between $RL2$ and the top view. Draw $RL1$ between the top and front views, and at right angles to the projectors. Draw projectors from V_H, A_H, B_H, C_H perpendicular to $RL2$. Locate V_3, A_3, B_3, C_3 on the projectors from $RL2$ at distances equal to the distances of V_V, A_V, B_V, C_V from $RL1$ respectively.

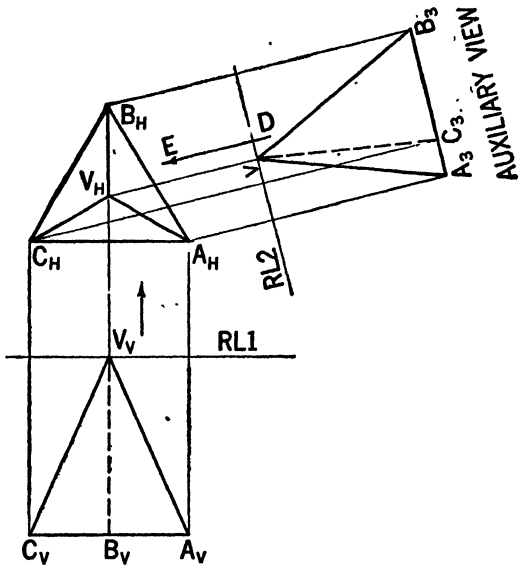


Fig. 43

The speed and accuracy with which an auxiliary view can be drawn is determined to some extent by the location of the reference line. It can be seen in Figure 43 that placing *RL*1 through V_V, instead of above it, eliminates one measurement and allows the auxiliary view to be constructed by using three equal distances. It can further be seen that if *RL*1 had been taken through $C_V A_V$, *RL*2 placed at the distance of $A_3 B_3$, and the measurement to *V* laid off toward the central view, only one measurement need have been made. Note that measurements may be made either toward or away from the central view, so long as they are in the same sense in both outside views. In general, it is convenient to place the reference lines through at least one point of a figure.

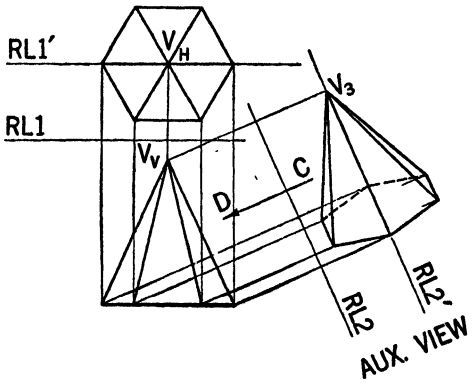


Fig. 44

Example 2 (Figure 44). *To draw an auxiliary view of the pyramid as seen in the direction CD.* The desired view is the projection of the object on a plane at right angles to *CD*. This plane is perpendicular to the *vertical plane.*

Draw *RL*2 perpendicular to *CD*. Draw *RL*1 between the top and front views. Project the points locating the view on the third plane. Three different distances are transferred from the top view to the auxiliary view.

Since the top view is symmetrical about an axis parallel to *RL*1, the auxiliary view can be constructed by using a single distance if *RL*1 is moved to the position *RL*1′ and if *RL*2 is moved to *RL*2′. Where such an axis of symmetry exists, it is generally more convenient to make the *RL* coincident with it.

18. Visualizing auxiliary views

The visibility rules of Article 11 are followed. The boundary lines of the view must be visible; also, every edge or face of the object which is adjacent to the auxiliary plane is visible.

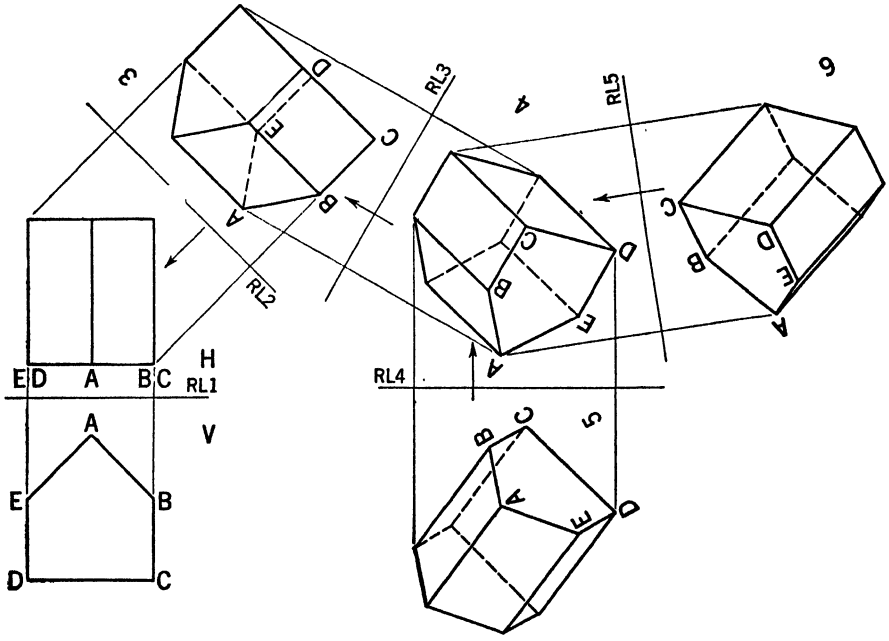


Fig. 45

The auxiliary projection is visualized as a view of the object by turning the drawing so as to bring any vertical lines of the object into a vertical position with respect to the observer. If the object contains no vertical lines, a vertical axis may be drawn in the given views and projected to the auxiliary view for reference purposes.

The correct orientation for a view can be indicated by the use of subtitles lettered at right angles to a vertical line of a view.

The subtitle "Auxiliary View" of Figure 43 is lettered at right angles to the vertical axis of the pyramid. When the drawing is turned to bring the title into a horizontal position, the view is correctly oriented.

The subtitle "Auxiliary View" of Figure 44 is lettered at right angles to the vertical axis.

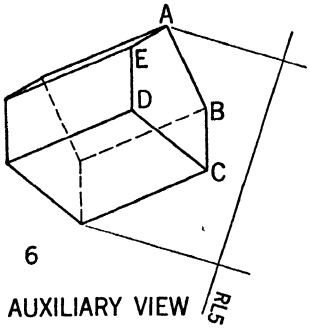


Fig. 46

The series of views of Figure 45 represent an object surrounded by a set of six projection planes. The edges *BC* and *DE* are vertical, as indicated by the top and front views.

Any particular view of the series of views is properly oriented by turning the drawing until the lines *BC* and *DE* appear vertical.

For example, if the projection planes are assumed folded about their reference lines into their true positions in space, an observer looking perpendicular to plane 6 would see the object as in Figure 46. He is below the object and is looking upward.

Problems

The problem sheet. The problems are designed for a working space 8 inches wide and 10 inches high. While any good quality $8\frac{1}{2}$- × 11-inch paper is suitable, quadrille paper ruled in $\frac{1}{4}$-inch squares is more convenient. A specially printed problem sheet (Figure A) is also available.*

Notation. Points are denoted by capital letters; lines of indefinite length by lower-case letters; planes, in general, by capitals.

Locating points. The top and front views of a given point are plotted on the sheet by means of three dimensions which are always given in the same order. The first is the distance from the left border line to the line joining the top and front views; the second is the distance from the lower border line to the front view; the third is the distance from the lower border line to the top view.

These dimensions are in inches and follow the letter which names the point. Thus, the point P:237 (Figure B) is 2 inches to the right of the left border. The front view is 3 inches above the lower border; the top view is 7 inches above the lower border.

A distance not needed in the solution, or which must be found in the course of the solution, is designated by "X." Thus, A:2,X,6 locates only the top view of point A.

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* This form is printed by the Technology Store, Cambridge, Mass.

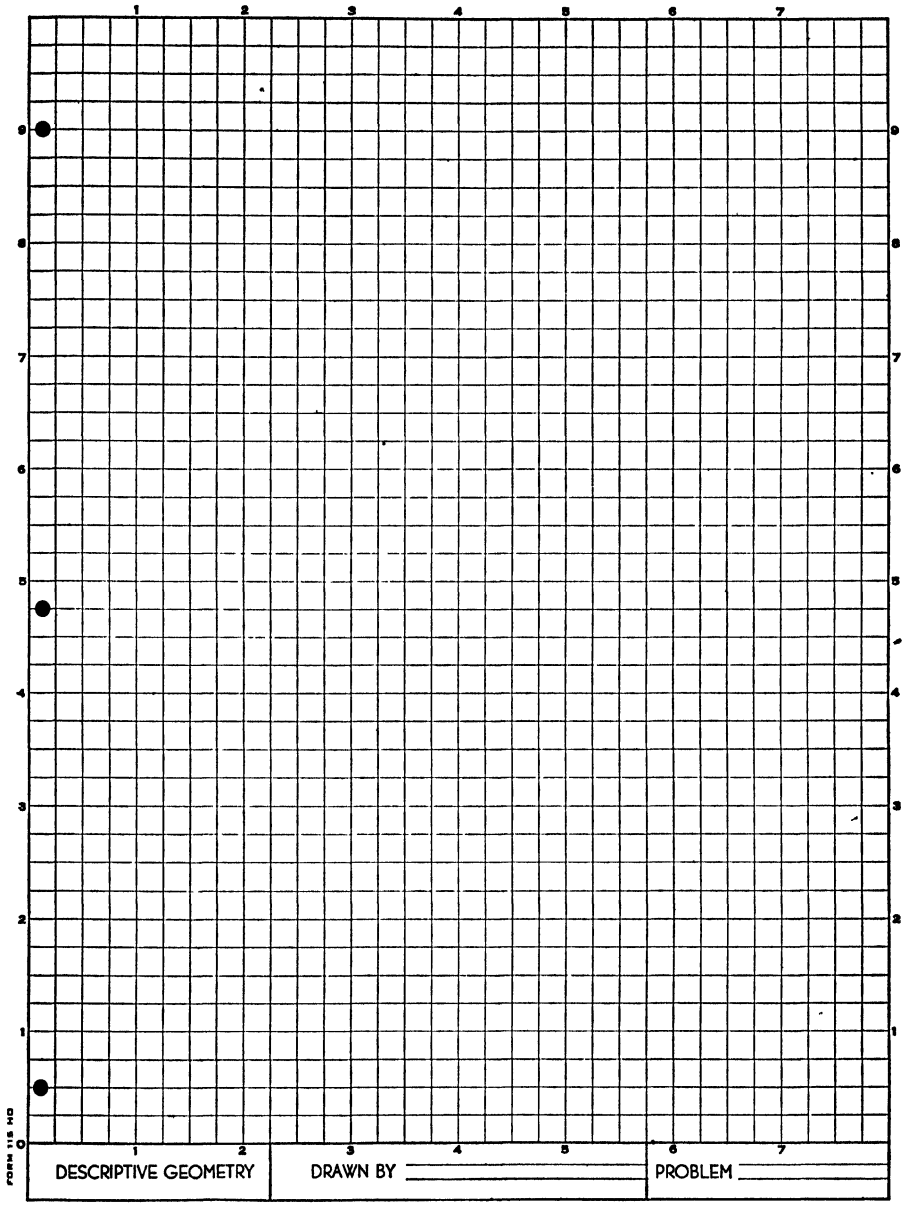

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Fig. A

Plotting lines. A line segment is located by the projections of its end points. Thus, a line AB may be described as A:124 B:337. A line k, of indefinite length, is described by two unnamed points thus: k:528;746 (Figure B).

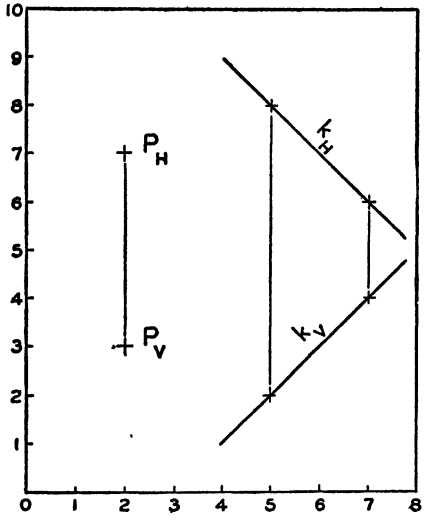


Fig. B

PROBLEMS ON CHAPTER I

GROUP A

1-A1. Draw the top, front, and side views of the building (Figure C) estimating the proportions.

1-A2. Draw the principal views of the building (Figure D).

1-A3. Draw the principal views of the casting (Figure E).

1-A4. Draw the principal views of the casting (Figure F).

1-A5. Draw the principal views of the casting (Figure G).

1-A6. Draw the principal views of the casting (Figure H).

1-A7. Draw the principal views of the casting (Figure K).

GROUP B. PRINCIPAL VIEWS OF DESCRIBED OBJECTS

1-B1. The shape of a cast-iron base is described as follows: From a block 5 inches wide, 2½ inches deep, and 1 inch high, remove the left and right quarters of the upper half. In the center of the top face of the block, cut a 1½-inch square hole to a depth of ½ inch. The sides of the square are parallel to the outside edges of the block. The four outside corners of the block, as seen in the top view, are rounded with a ½-inch

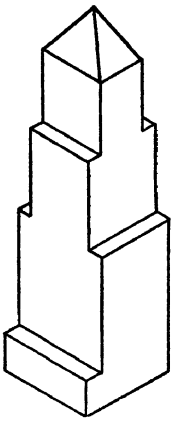


Fig. C

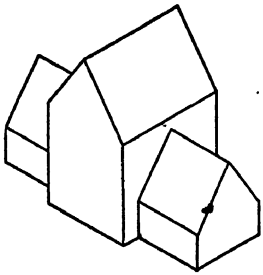


Fig. D

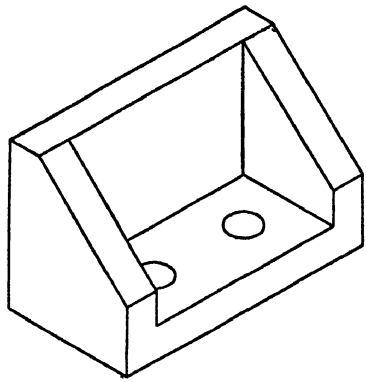


Fig. E

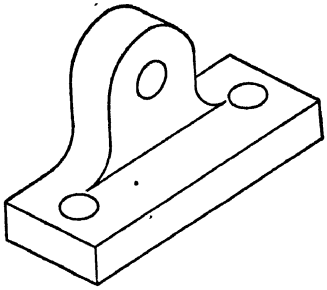


Fig. F

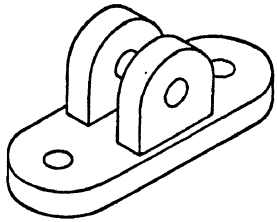


Fig. G

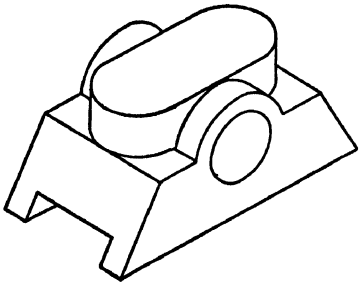


Fig. H

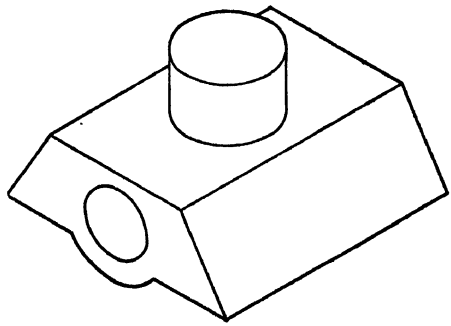


Fig. K

radius. On these same centers, four $\frac{1}{2}$-inch diameter holes are drilled. Draw the top and front views.

1-B2. A cylinder $3\frac{1}{2}$ inches in diameter and $\frac{1}{2}$ inch long has its axis horizontal. A second cylinder $1\frac{1}{2}$ inches in diameter and $1\frac{1}{4}$ inches long is centered on the front face. A circular recess 2 inches in diameter and $\frac{1}{8}$ inch deep is centered in the back of the $3\frac{1}{2}$-inch cylinder. A $\frac{3}{4}$-inch diameter hole is drilled axially through both cylinders. Four $\frac{3}{8}$-inch diameter holes are drilled through the $3\frac{1}{2}$-inch cylinder. These holes are equally spaced on a $2\frac{3}{4}$-inch circle which is concentric with the $3\frac{1}{2}$-inch circle. Draw the top and front views.

1-B3. A cylinder $2\frac{1}{2}$ inches in diameter and 3 inches high has the upper two-thirds of the right and left sides cut away to produce two flat surfaces which are parallel and $1\frac{1}{2}$ inches apart. A 1-inch diameter hole passes through the centers of the flattened surfaces. Draw top, front, and side views.

1-B4. A cast angle appears L-shaped in the front view. Each leg is $3\frac{1}{4}$ inches long, $1\frac{1}{2}$ inches deep, and $\frac{1}{2}$ inch thick. The inside corner is rounded with a $\frac{1}{2}$-inch radius. The upper end of the vertical leg as seen in the side view is semicircular. A $\frac{1}{2}$-inch hole is centered $\frac{3}{4}$ inch from the upper end of this leg.

The corners of the horizontal leg, as seen in the top view, are rounded with a $\frac{1}{4}$-inch radius. A slot $\frac{1}{2} \times 1\frac{1}{2}$ inches is centered in the left end of this leg. The right end of the slot is semicircular.

1-B5. From a block 4 inches wide, 2 inches high, and $1\frac{1}{2}$ inches deep, the center three-fourths of the upper three-fourths is removed, leaving a piece that appears U-shaped in the front view. The top of the left and right arms of this piece, as seen in the side view, are made semicircular. A $\frac{1}{2}$-inch hole is centered $\frac{3}{4}$ inch from the top of each arm. A cylindrical lug 1 inch in diameter and $1\frac{1}{4}$ inches high is centered on the bottom of the block.

GROUP C. PRINCIPAL VIEWS OF GEOMETRICAL ELEMENTS

1-C1. Draw the top, front, and right side views of the tetrahedron A:036 B:$2\frac{1}{4}$,1,8 C:$4\frac{1}{2}$,2,7 D:$3\frac{1}{2}$,5,8.

1-C2. Point A:017 is one vertex of the tetrahedron $ABCD$. Point B is 1 inch to the right of, 2 inches in back of, and 3 inches above A. Point C is 3 inches to the right of, 4 inches above, and $1\frac{1}{2}$ inches in back of A. Point D is $3\frac{3}{4}$ inches to the right of, 1 inch in front of, and 1 inch above A. Draw the top, front, and right side views.

1-C3. Point A:249 is one vertex of the tetrahedron $ABCD$. Point B is 2 inches to the right of, 1 inch in front of, and $1\frac{3}{4}$ inches below A.

Point C is $\frac{1}{2}$ inch to the right of, $1\frac{1}{4}$ inches below, and 3 inches in front of A. Point D is 2 inches to the left of, 2 inches in front of, and $2\frac{1}{2}$ inches below A. Draw the top, front, and left side views.

1-C4. Divide the working space into four equal rectangles, and place one example in each rectangle. Draw the top and front views of a line AB, 2 inches long and placed as follows: (*a*) The line is horizontal and makes an angle of 30° with the V-plane. (*b*) The line is frontal and makes an angle of 45° with the H-plane. (*c*) The line is profile and makes an angle of 60° with the H-plane. (*d*) The line is vertical.

1-C5. Divide the working space into four equal rectangles, and place one example in each rectangle. Draw the top and front views of a $1\frac{1}{4}$-inch square placed as follows: (*a*) The square is horizontal with two edges frontal. (*b*) The square is frontal with one diagonal vertical. (*c*) The square is profile with two edges making an angle of 30° with the H-plane. (*d*) The square is perpendicular to the V-plane and makes an angle of 30° with the H-plane. Two edges are frontal.

1-C6. Divide the working space into four equal rectangles, and place one example in each rectangle. Draw the top and front views of the following plane figures: (*a*) A horizontal equilateral triangle. Each side is 2 inches long, and the left edge is perpendicular to the V-plane. (*b*) A frontal square. The upper edge is $1\frac{1}{2}$ inches long and makes an angle of 15° with the H-plane. (*c*) A profile triangle. The upper front edge is 2 inches long and makes an angle of 45° with the H-plane. The remaining edges are $1\frac{1}{2}$ inches long. (*d*) A horizontal regular hexagon. Each edge is 1 inch long, and two edges are frontal.

1-C7. Divide the working space into four equal rectangles, and place one example in each rectangle. Draw the top and front views of the following solids: (*a*) Square right prism. Each base is a horizontal square $1\frac{1}{2}$ inches on each side. The right front edges of the bases make a 30° angle with the V-plane. The altitude is 1 inch. (*b*) Square right pyramid. The base is a $1\frac{1}{2}$-inch horizontal square, the left front edge making a 15° angle with the V-plane. The vertex is $1\frac{1}{2}$ inches below the base. (*c*) Rectangular right prism. The bases are frontal rectangles, $\frac{3}{4} \times 1\frac{1}{2}$ inches. The prism is 2 inches long. The $1\frac{1}{2}$- $\times$ 2-inch faces make an angle of 30° with the H-plane. (*d*) Hexagonal right pyramid. The base is a frontal hexagon, $\frac{3}{4}$ inch on each side. One diagonal of the base makes a 15° angle with H. The vertex is $1\frac{3}{4}$ inches in front of the base.

Group D. The Complete Reference System

1-D1. *Four quadrants.* (*a*) In the upper half of the working space draw a pictorial view of the H- and V-projection planes similar to Figure

29, Chapter I, making the V-plane about 3×4 inches. Take the left end of RL as an origin O, and represent the following points, indicating both the points in space and their orthographic projections on H and V:

Point A is $\frac{1}{2}$ inch to right of O, $1\frac{1}{4}$ inches above H, and 1 inch in front of V.
Point B is $1\frac{1}{2}$ inches to right of O, $1\frac{1}{4}$ inches above H, and $\frac{3}{4}$ inch behind V.
Point C is $2\frac{1}{2}$ inches to right of O, $1\frac{1}{4}$ inches below H, and 1 inch in front of V.
Point D is $3\frac{1}{2}$ inches to right of O, 1 inch below H, and $1\frac{1}{4}$ inches behind V.

(*b*) In the lower half of the working space, construct the H- and V-projections of points A, B, C, and D, assuming the projection planes revolved into the plane of the drawing paper as in Figure 30. Indicate the quadrant in which each point lies.

1-D2. *First quadrant projection.* Divide the working space into four equal rectangles, and place one example in each rectangle. Draw the top and front views of the following solids using first quadrant arrangement: (*a*) Triangular right prism. The bases are horizontal equilateral triangles $1\frac{1}{2}$ inches on each side. The rear lateral face of the prism makes a 15° angle with V. The altitude is 1 inch. (*b*) Rectangular right prism. The bases are frontal rectangles 1×2 inches. A lower lateral face of the prism makes a 30° angle with H. The prism is $\frac{3}{4}$ inch long. (*c*) Square right pyramid. The base is a $1\frac{1}{2}$-inch square lying in the H-plane. One edge of the base makes 15° with the V-plane. The vertex is 2 inches above the base. (*d*) Hexagonal right pyramid. The base is a frontal hexagon 1 inch on each side. Two edges of the base make a 45° angle with H. The vertex is $1\frac{1}{2}$ inches in back of the base.

GROUP E. AUXILIARY VIEWS

1-E1. Draw the top and front views of the tetrahedron A:$4\frac{1}{2}$,4,$6\frac{1}{2}$ B:$4\frac{1}{2}$,$1\frac{1}{2}$,$6\frac{1}{2}$ C:$2\frac{1}{2}$,3,$8\frac{1}{2}$ D:$5\frac{1}{2}$,4,$9\frac{1}{2}$. Draw an auxiliary view of $ABCD$ taking the direction of sight at right angles to face ABC. Draw an auxiliary view of $ABCD$ on a projection plane taken parallel to face ABD.

1-E2. Line V:636 O:639 is the axis of a right pyramid. The base is a frontal pentagon centered on point O. Point A:$7\frac{1}{2}$,3,9 is one vertex of the base. The solid is cut by a frontal plane taken at the mid-point of the axis. Draw the top and front views of the frustum and an auxiliary view with the direction of sight at right angles to the left lateral face.

1-E3. Line A:226 D:229 is a diagonal of the horizontal hexagonal base of a right pyramid. The vertex is 3 inches above the base. The pyramid is truncated by a plane which passes through points P:1,$4\frac{1}{2}$,X and Q:4,$1\frac{1}{2}$,X, and projects edgewise in the front view. Draw the top and front views of the truncated pyramid and an auxiliary view taken in a direction to show the sloping base in true shape.

1-E4. Line A:236 B:239 is a diagonal of a profile hexagon which is the common base of two right pyramids. The axis of each pyramid is 2 inches long. Draw the front view, right side view, and an auxiliary view for which the direction of sight is perpendicular to the upper right-hand face of the solid. Omit the top view.

1-E5. Points A:7¾,4,10 B:7¾,4,9 C:7¾,2,8 D:7¾,1,10 determine the right-hand base of a right prism. The prism is truncated by a plane which passes through points R:3¾,1,X and S:6¾,4,X and is edgewise in the front view. Draw the top, front, and left end views and an auxiliary view taken in a direction perpendicular to the inclined base.

1-E6. Line A:5½,3,6 B:836 is the front edge of a horizontal rectangle 1½ inches deep. This rectangle is the common base of two right pyramids, each of 2 inches altitude. Draw the top, front, and left side views. Also, draw an auxiliary view taking the direction of sight at right angles to the rear face of the upper pyramid.

1-E7. Line A:226 B:229 is the axis of a thin cylindrical tube 2 inches in diameter. The end at A lies in a vertical plane and makes an angle of 30° with AB. The end at B is perpendicular to the axis. Draw the top and front views and an auxiliary view taking the direction of sight at right angles to the base through A.

1-E8. Triangle A:½,2,6 B:½,3½,8½ C:½,2,8½ is the left base of a right prism 3½ inches long. A vertical hole 1½ inches in diameter and centered on the lower face is cut through the prism. Draw the top, front, and side views and an auxiliary view which shows the sloping face in true shape.

1-E9. Points A:147 B:447 C:5¾,4,8¾ locate the upper edges of two vertical rectangles 2 inches high. These rectangles are the front faces of the ½ inch thick arms of an angle bracket. Two ½-inch holes 1½ inches apart are centered on the front face of the left arm. A hexagonal hole, ⅝ inch on a side is centered on the front face of the right arm. One diagonal of the hexagon is vertical. Draw the top and front views and an auxiliary view which shows the right arm in true shape.

1-E10. Line A:2¼,4,7 B:5¾,4,7 is a diagonal of a frontal hexagonal which forms the front face of a right prism 1½ inches thick. A ½-inch diameter hole ¾ inch deep is centered on the top face of the prism. Two ¼-inch diameter holes ¾ inch deep and ¾ inch apart are centered on the diagonal of the lower left inclined face. The upper hole is nearer the front face. Two holes of the same size are similarly located on the lower right-hand face of the prism. Draw the top and front views of the prism and auxiliary views which show the true shape of the lower left and lower right inclined faces. Show visible lines only in the top view.

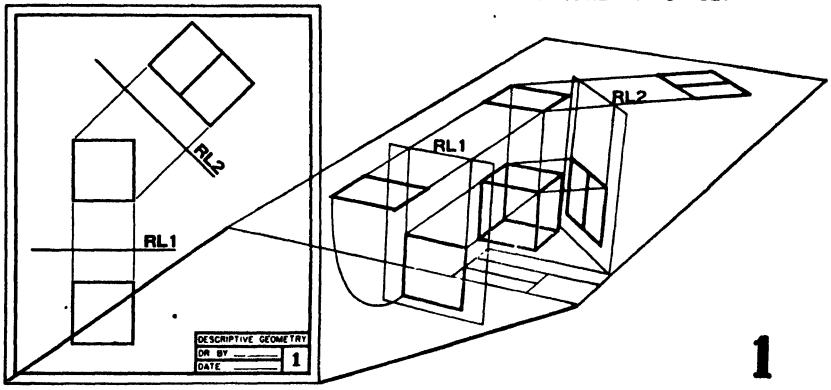
1-E11. Point A:4,1½,7½ is the center of the upper face of a prism 4½ inches wide, 2½ inches deep, and ½ inch high. The corners of the prism as seen in the top view are rounded to a ½-inch radius. Four ⅜-inch holes are drilled through the prism on the same centers as the corner radii. A frontal line AB, 1½ inches long and making an angle of 60° with the horizontal, is the axis of a cylindrical hub 1½ inches in diameter. The end at B is perpendicular to the axis. A ¾-inch hole is drilled along AB. Draw the top and front views and the necessary auxiliary view. Show visible lines only in the top view.

Descriptive Geometry

THE GRAPHICAL REPRESENTATION & SOLUTION OF SPACE PROBLEMS ON A TWO DIMENSIONAL SURFACE – YOUR DRAWING PAPER.

WE OBTAIN
THE SOLUTION OF THE PROBLEM ON YOUR PAPER.

FROM
THE VISUALIZATION OF THE PROBLEM IN SPACE.

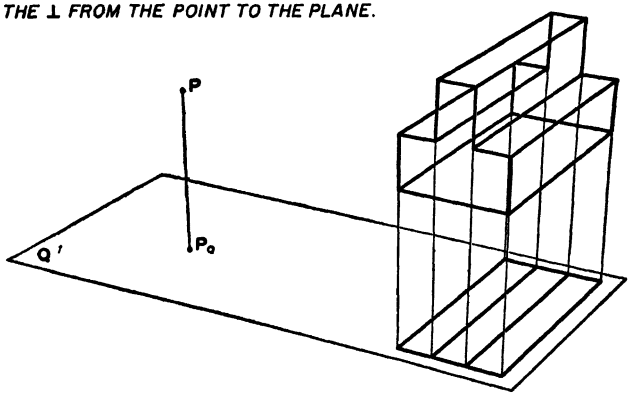


1

THE SPACE MODEL UNFOLDS INTO YOUR PAPER USING THE INTERSECTIONS OF THE PROJECTION PLANES AS FOLDING LINES, CALLED REFERENCE LINES.

Orthographic Projection

THE ORTHOGRAPHIC PROJECTION OF A POINT ONTO A PLANE IS THE INTERSECTION WITH THE PLANE OF THE ⊥ FROM THE POINT TO THE PLANE.

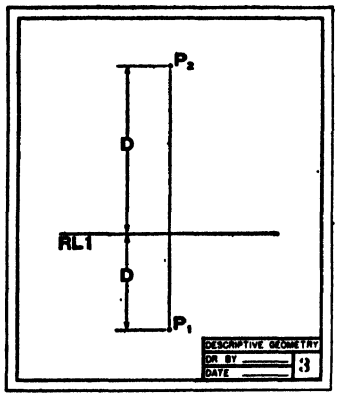


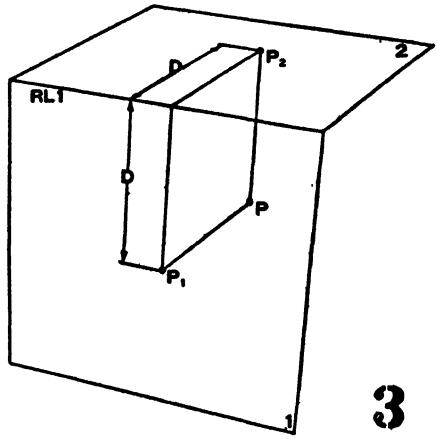
THE ORTHOGRAPHIC PROJECTION OF AN OBJECT ONTO A PLANE IS THE SUM OF THE PROJECTIONS OF ALL THE POINTS ON THE OBJECT

2

Representation of a Point

A POINT IN SPACE MAY BE REPRESENTED BY ITS PROJECTIONS ON 2 MUTUALLY ⊥ PROJECTION PLANES.



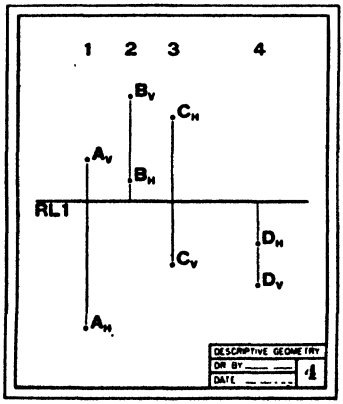


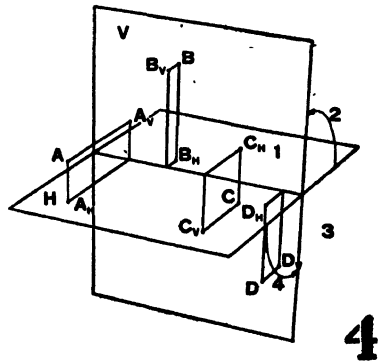
3

NOTE THAT THE PROJECTION LINE MUST BE ⊥ TO THE REFERENCE LINE.

Reference Quadrants

A POINT MAY LIE IN ANY OF 4 QUADRANTS FORMED BY 2 PROJECTION PLANES, GENERALLY ASSUMED TO BE HORIZONTAL & VERTICAL

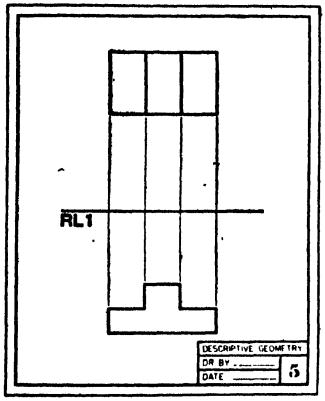


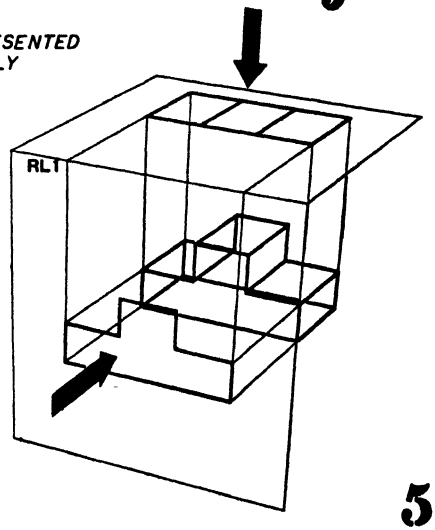


4

Representation of an Object

AN OBJECT IN SPACE MAY BE REPRESENTED BY ITS PROJECTIONS ON 2 MUTUALLY ⊥ PROJECTION PLANES.



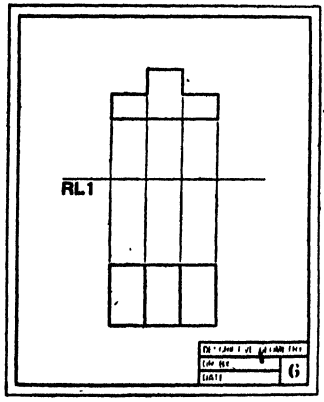


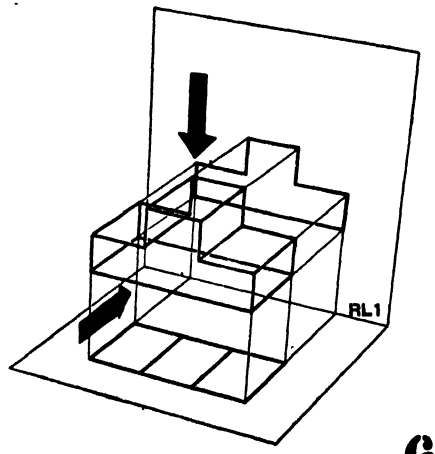
5

THIS IS 3RD ANGLE PROJECTION - THE AMERICAN WAY. VIEWS ARE GENERALLY CALLED FRONT & TOP OR V & H.

Representation of an Object

AN OBJECT MAY BE REPRESENTED BY ITS PROJECTIONS ON 2 MUTUALLY ⊥ PROJECTION PLANES.



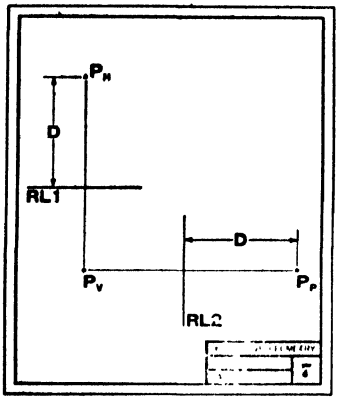


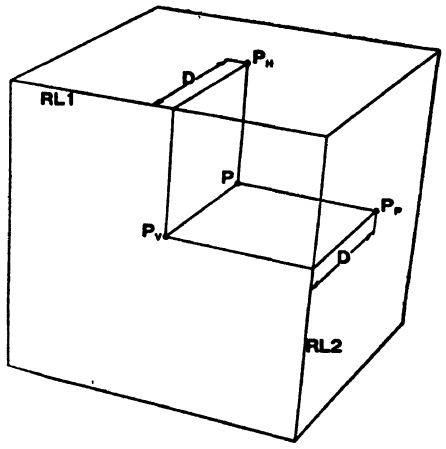
6

THIS IS 1ST ANGLE PROJECTION - THE EUROPEAN WAY.

Standard Views

THE 2 STANDARD PROJECTION PLANES ARE HORIZONTAL & VERTICAL. A 3RD PLANE ⊥ TO EACH (PROFILE PLANE) IS FREQUENTLY CONVENIENT.



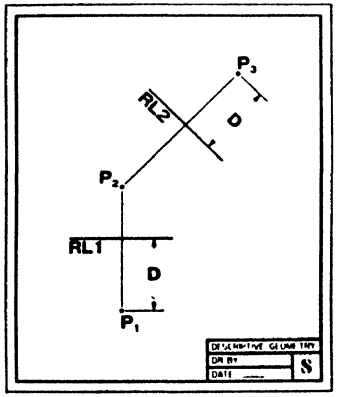


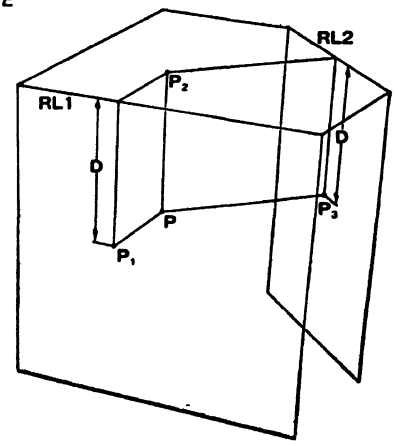
NOTE THAT THE DISTANCE, D, AWAY FROM THE CENTRAL VIEW IS EQUAL IN THE 2 OUTSIDE VIEWS.

7

Auxiliary View of a Point

IF 2 VIEWS OF A POINT ARE GIVEN, A 3RD VIEW CAN BE OBTAINED ON ANY PLANE ⊥ TO EITHER OF THE GIVEN 2

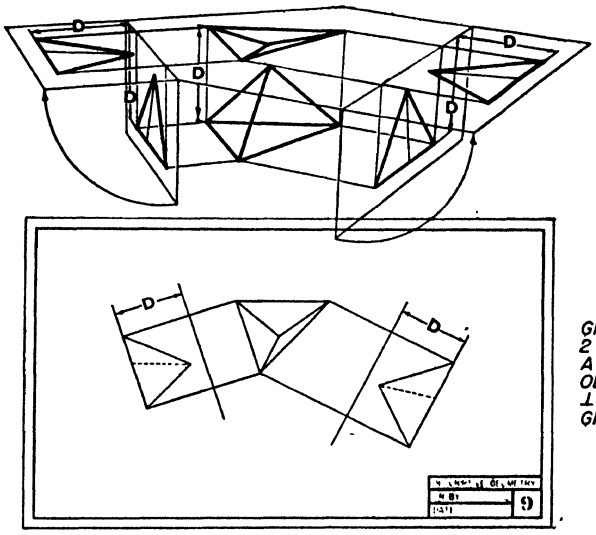




NOTE THAT THE DISTANCE AWAY FROM THE CENTRAL VIEW IS EQUAL IN THE 2 OUTSIDE VIEWS REPRESENTING THE DISTANCE OF THE POINT FROM THE CENTRAL PLANE.

8

The General 3 View System



GIVEN ANY 2 VIEWS ON 2 MUTUALLY ⊥ PLANES A 3RD VIEW MAY BE OBTAINED ON ANY PLANE ⊥ TO EITHER OF THE GIVEN 2

9

CHAPTER II

Fundamental Problems Relating to Lines and Planes

In addition to representing an object by means of its projections on two or more projection planes, the engineer is concerned with the solution of space problems which involve distances, angles, intersections of surfaces, developments of surfaces, and the mutual positions of objects. Although the number of such problems is indefinite, every solution is obtained by use of certain basic geometrical operations. These basic operations are few in number and their graphical representations are regarded as the fundamental problems of descriptive geometry. The application of the fundamental problems to the solution of general space problems is discussed in Chapter III.

19. The fundamental problems

1. To assume a point on a line.
2. To draw a line through two points.
3. To draw a line through a point and parallel to a given line.
4. To draw a true-length view and an end view of a given line.
5. To pass a plane through two intersecting or parallel lines.
6. To draw an edge view and a true-shape view of a plane figure.
7. To find the intersection of a line and plane.
8. To find the intersection of two planes.
9. To draw a perpendicular to a plane.
10. To pass a plane perpendicular to a line.

Generally there are two parts to the solution of a problem: (1) *the analysis*, or method of solution, in which the geometrical operations in space which lead to the solution are described; and (2) *the construction*, or drawing, in which the operations described in the analysis are carried out by the methods of orthographic projection.

20. Problem 1: To assume a point on a line

Analysis: If a point lies on a line, a pair of projections of the point will lie on a line which is perpendicular to the reference line.

If a space line is divided in a given ratio, its projection is divided into the same ratio. For example, the mid-point of a line projects into the mid-point of the projection of the line.

Construction A (*Figure 47*): Let P_H be assumed on c_H. P_V is found by drawing the projector through P_H to cut c_V.

Construction B (*Figure 48*): P_V is assumed on the front view of the profile line AB. Construct the side view of AB. Project P on side view of AB. P_H is found by laying off the distance y_H equal to y_P.

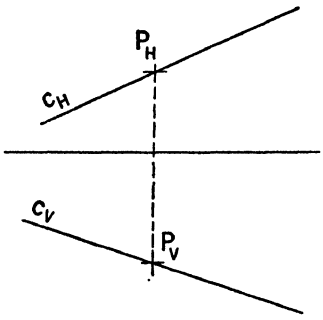


Fig. 47

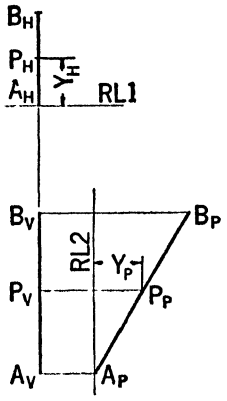


Fig. 48

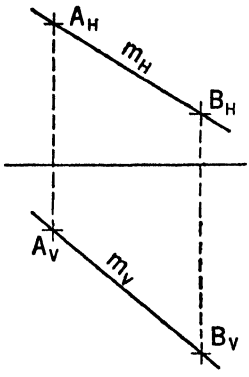


Fig. 49

21. Problem 2: To draw a line through two points

Analysis: The line passing through the projections of the points on a given plane will be a projection of the required line.

Construction: Let two points A and B be given by A_H, A_V; and B_H, B_V (Figure 49). Draw m_H through A_H and B_H, and m_V through A_V and B_V. The required space line is represented by the two projections m_H and m_V.

22. Problem 3: To draw a line through a point and parallel to a given line

Analysis: If two lines are parallel in space, their projections on a plane are parallel. In general, if the H- and V-projections of two lines are parallel, the lines are parallel in space. Profile lines are an exception and are parallel only when the profile projections of the lines are parallel.

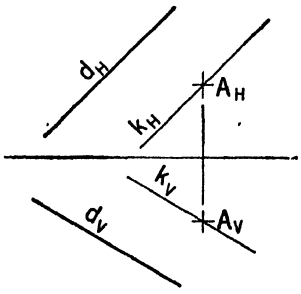


Fig. 50

Construction (*Figure 50*): Let the given line and point be d and A, respectively. Draw the required line k through point A, making k_H parallel to d_H and k_V parallel to d_V.

23. Problem 4: To determine the normal view and end view of an oblique line

A line is in its simplest position with respect to the H- and V-planes when it is perpendicular to one of them. In this position, one view is a point or *end* view and the other a normal or *true-length* view. If a given line is inclined to the H- and V-planes, the end and true-length views can be obtained by proper selection of auxiliary projection planes.

Given: Projections A_VB_V and A_HB_H (Figures 51 and 52).

(*a*) To obtain the normal or true-length view of AB.

Principle: A line projects in its true length on a plane parallel to the line. Project AB on an auxiliary plane taken parallel to AB.

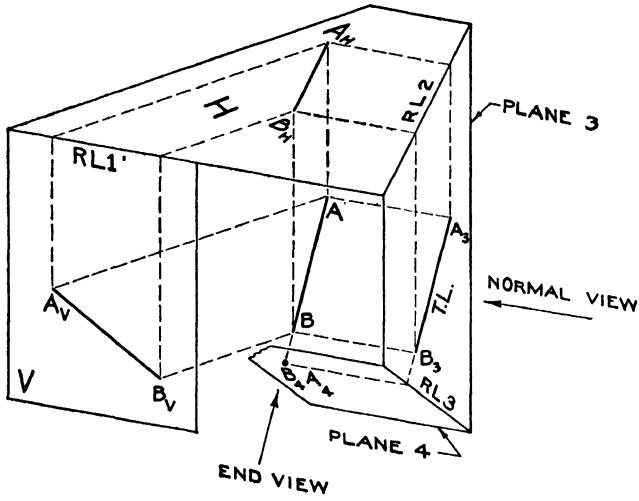


Fig. 51

Construction: Take plane 3 parallel to AB by drawing $RL2$ parallel to A_HB_H. Project AB on plane 3. Then A_3B_3 is the true length of AB. Also the angle H between A_3B_3 and $RL2$ is the true size of the angle between line AB and the H-plane.

The angle between a line and plane appears in true size in a view in which the line projects in true length and the plane projects edgewise.

Note: The normal view of AB can also be obtained by taking $RL3$ parallel to A_VB_V.

(*b*) To determine the end view of AB.

Principle: A line projects as a point on a plane perpendicular to the line. The line and plane will appear perpendicular in the view in which the line projects in true length.

Construction: Take plane 4 perpendicular to AB by drawing $RL3$ at

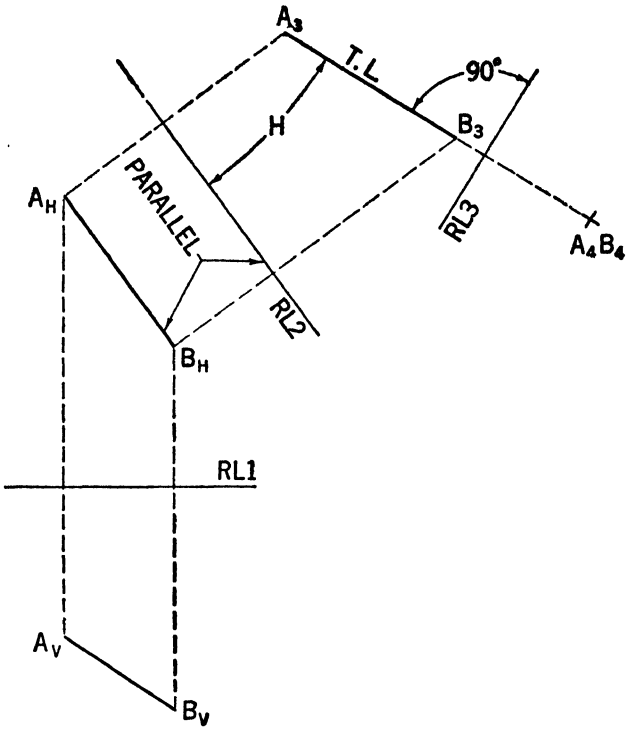


Fig. 52

right angles to A_3B_3 (the true-length view of AB). Project AB on plane 4. Projections A_4 and B_4 coincide, giving the end view of AB.

24. Oblique view of an object

An oblique view is a view taken in a direction inclined to the horizontal and vertical planes.

Let it be required to draw a view of the object (Figure 53), as seen in the direction AB. The required view is one in which the line AB appears as a point. The normal view and end view of AB is obtained by the method of Article 23.

Since the solution of this problem requires the construction of two additional views, it is convenient to enclose the object in a rectangular prism as a reference frame and to draw the auxiliary and oblique views of the prism and direction line only. The oblique view of the given object is then obtained by constructing it in the reference prism by proportional division, referring to the top and front views for the dimensions.

This procedure makes the projection of a large number of points unnecessary and results in a more accurate drawing.

Construction: (*a*) Draw a view of AB and the prism to show AB in true length. (*b*) Draw a view of AB and the prism in which AB appears as a point. (*c*) Construct the oblique view of the given object by proportional division.

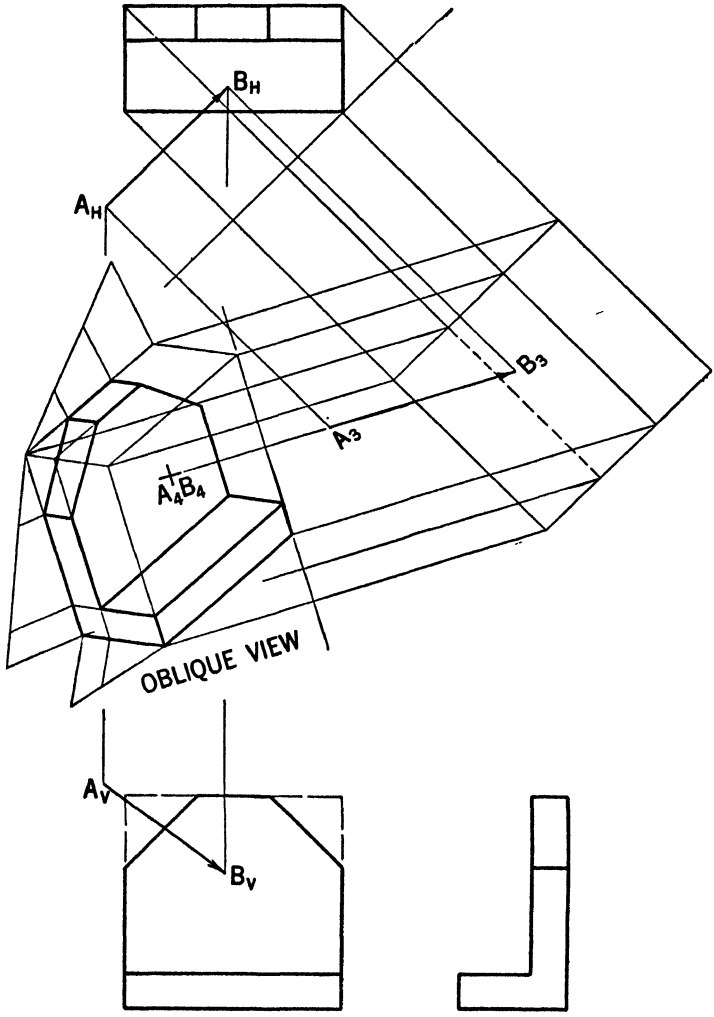


Fig. 53

The oblique view may be correctly oriented with respect to the observer by turning the drawing so as to bring the vertical edges of the object into a vertical position with respect to the observer. The correct position for reading the view may also be indicated by lettering the name

of the view on a horizontal line. A horizontal line in an oblique view is located by drawing it at right angles to a known vertical line of the view.

25. The plane

A plane is generated by a straight line moving so as to touch constantly two intersecting straight lines. A plane usually appears as the face of an object and, as such, is represented on a drawing by the projection of its boundary lines. A plane of unlimited extent can be represented on a drawing by the projections of three points not in a straight line. In practice, these points are usually joined to form two intersecting lines or a triangle, the latter being a convenient form for graphical purposes. A plane can be uniquely represented by its intersections with the coordinate planes (Article 42).

26. Intersecting lines

If two lines intersect, the projections of the common point lie on the same projector (Figure 54). If the common point of two intersecting

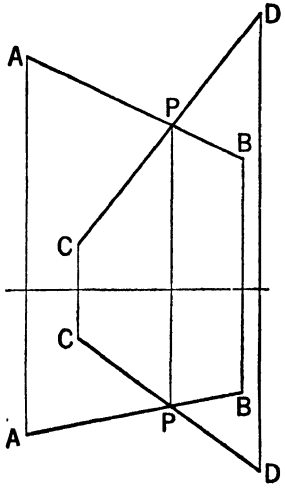


Fig. 54

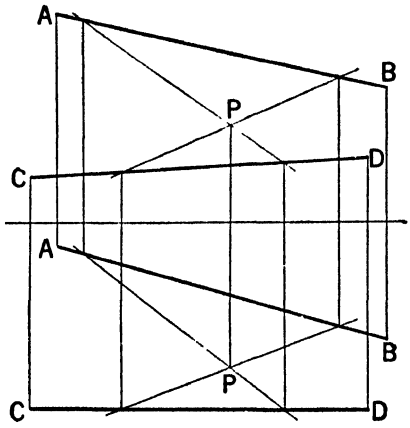


Fig. 55

lines is beyond the limits of the drawing, the condition for intersection is that a pair of lines drawn to intersect the given lines have their common point on the same projector (Figure 55).

27. Problem 5: To pass a plane through two intersecting (or parallel) lines

The space operation of passing a plane through two intersecting lines

requires no construction beyond the representation of the two determining lines. In Figure 56, plane Q is represented by lines b and c. Plane R is represented by lines d and e.

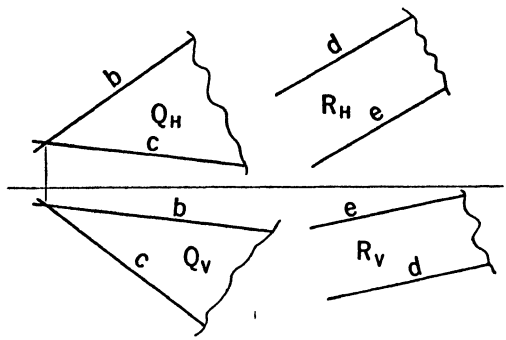


Fig. 56

28. Projections of a line lying in a given plane

If one view of a line which lies in a plane is given or assumed, the other view can be found. Let the top view of line k, lying in the plane determined by lines m and n, be given (Figure 57). Line k_H cuts m_H and n_H in points A_H and B_H respectively. The front view of k is determined by points A_V and B_V.

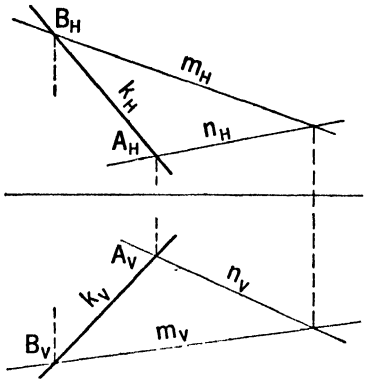


Fig. 57

29. The principal lines of a plane

The lines of a plane which are parallel to the projection planes are the *principal lines* of the plane. The system of lines parallel to the H-plane form the *horizontal* lines of the plane (Figure 58). The system of lines parallel to the V-plane form the *frontal* lines of the plane (Figure 59). In general, through a given point of a plane one horizontal and one frontal line can be drawn. In Figure 60, the line SF is a *horizontal* and the line TF a *frontal* line of the plane DEF.

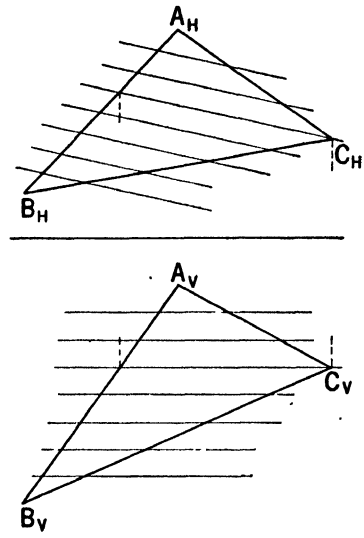


Fig. 58

30. Lines of maximum inclination

The lines of a plane which have the greatest inclination to the H-plane

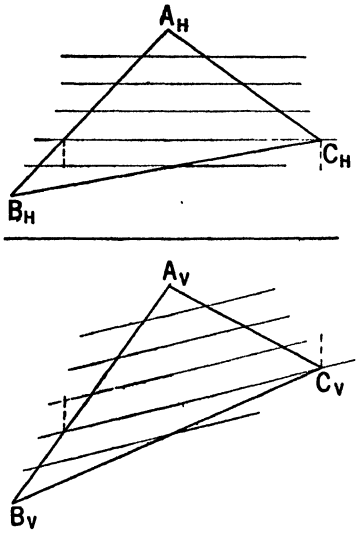


Fig. 59

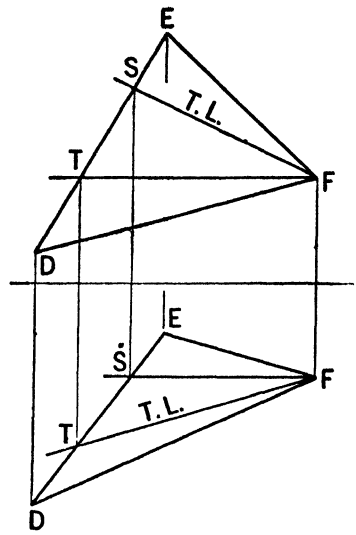


Fig. 60

are perpendicular to the horizontal lines of the plane. In Figure 61, line AB is drawn perpendicular to the horizontal line EF. Its projection, BC, on the H-plane is at right angles to EF. (Refer Article 46.) Let line AD be any other line of plane AEF. Then, $\frac{AC}{AB} > \frac{AC}{AD}$; hence, angle

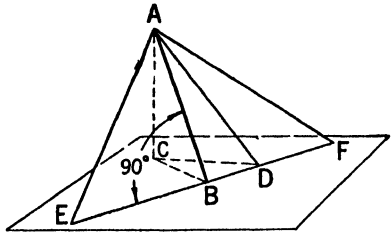


Fig. 61

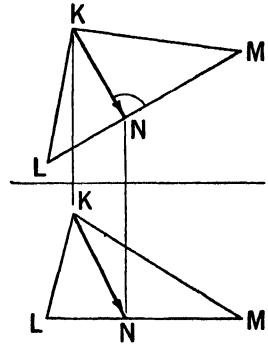


Fig. 62

$ABC >$ angle ADC. Thus, AB is a line of maximum inclination.

A sphere rolling on a plane and acted upon by gravity alone would follow a line of maximum inclination. The direction of the slope of a plane can be described by reference to this line. Plane KLM (Figure 62) slopes down, forward, and to the right as indicated by the maximum slope line KN.

31. To assume a point in a plane

In order that a point lie in a plane, it must lie on a line of the plane.

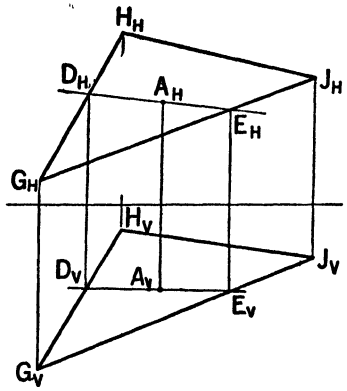


Fig. 63

Let the top view of point A, lying in plane GHJ (Figure 63), be given. Draw the projections of the random line DE so that D_HE_H contains A_H. The front view of A lies on D_VE_V. Obtain D_V E_V. Obtain A_V by projecting from A_H (Article 20).

32. Parallel relations between lines and planes

A line is parallel to a plane if it is parallel to some line in the plane. Conversely, a plane is parallel to a line if the plane contains a line parallel to the given line. Thus, in Figure 64, a plane through line MN and parallel to line KL is represented by drawing the line $K'L'$ parallel to KL and through an assumed point O on line MN. A plane containing point O (Figure 65) and parallel to two given lines PQ and RS, is represented by drawing through point O two lines $P'Q'$ and $R'S'$, parallel to PQ and RS respectively.

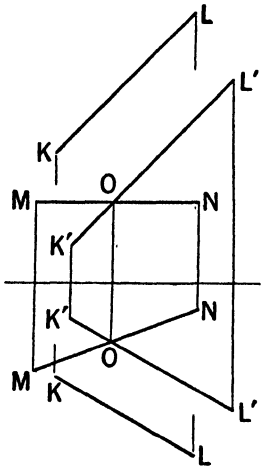


Fig. 64

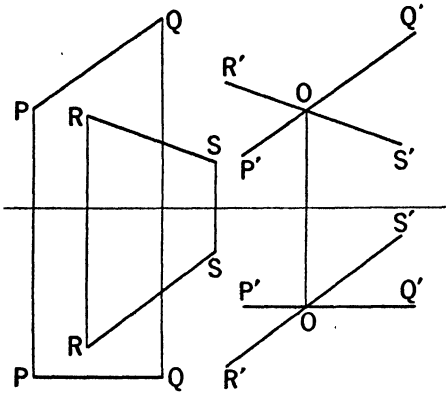


Fig. 65

33. Problem 6: To determine the edge view and the normal view of an oblique plane

A plane is in its simplest position with respect to the H- and V-planes when it is parallel to one plane and therefore perpendicular to the second. In this position, one projection is a normal view and the other an edge view. If the plane is represented as a polygon, the normal view gives

the true shape of the figure. When a given plane is inclined to the H- and V-planes, the edge and normal views can be obtained by selecting appropriate auxiliary planes.

Given: Projections $A_VB_VC_V$ and $A_HB_HC_H$ (Figures 66 and 67).

(*a*) To determine the edge view of ABC.

Analysis: A plane projects as a straight line on a projection plane taken perpendicular to any line in the plane. For convenience, the projection plane is taken perpendicular to a principal line of the plane.

Construction: Draw the auxiliary line CD parallel to the horizontal plane. Then C_HD_H is the true-length view of CD. Take plane 3 per-

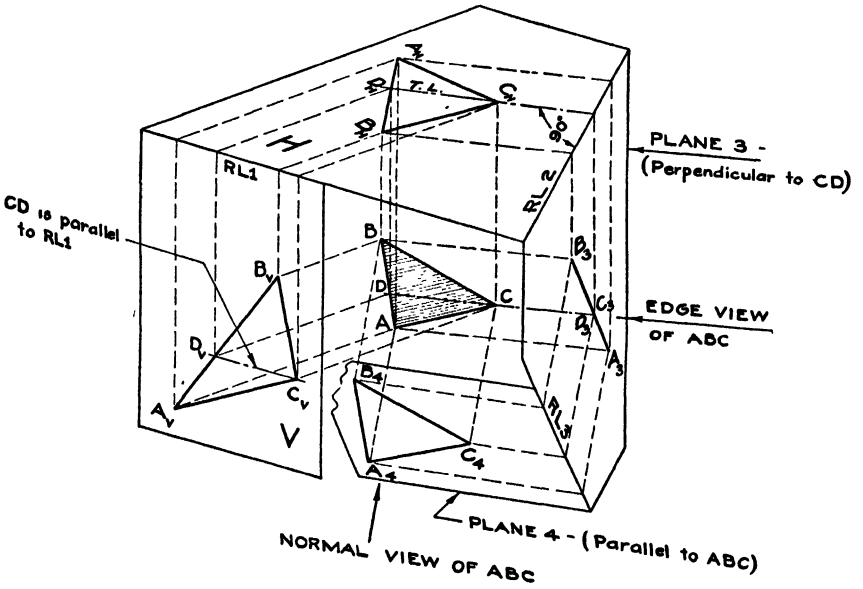


Fig. 66

pendicular to line CD by placing $RL2$ at right angles to C_HD_H. Project ABC on plane 3, determining the edge view $A_3B_3C_3$.

The angle H between $A_3B_3C_3$ and $RL2$ is equal to the angle between plane ABC and the horizontal plane.

Note: The edge view of ABC can also be drawn on a third plane taken perpendicular to a frontal line of ABC.

(*b*) To determine the normal view of ABC.

Analysis: The normal view of a plane will appear on a projection plane taken parallel to the given plane.

Construction: Take plane 4 parallel to ABC by drawing $RL3$ parallel to $A_3B_3C_3$. Project ABC on plane 4.

$A_4B_4C_4$ is the normal view of the plane and the true-size view of triangle ABC.

34. To execute a plane construction on an oblique plane of space

In general, a plane construction is drawn out on the normal view of the given plane.

Example 1 (Figure 68). *To construct a hexagon of diameter BC lying in the plane of ABC.*

Given: The triangle *ABC*.

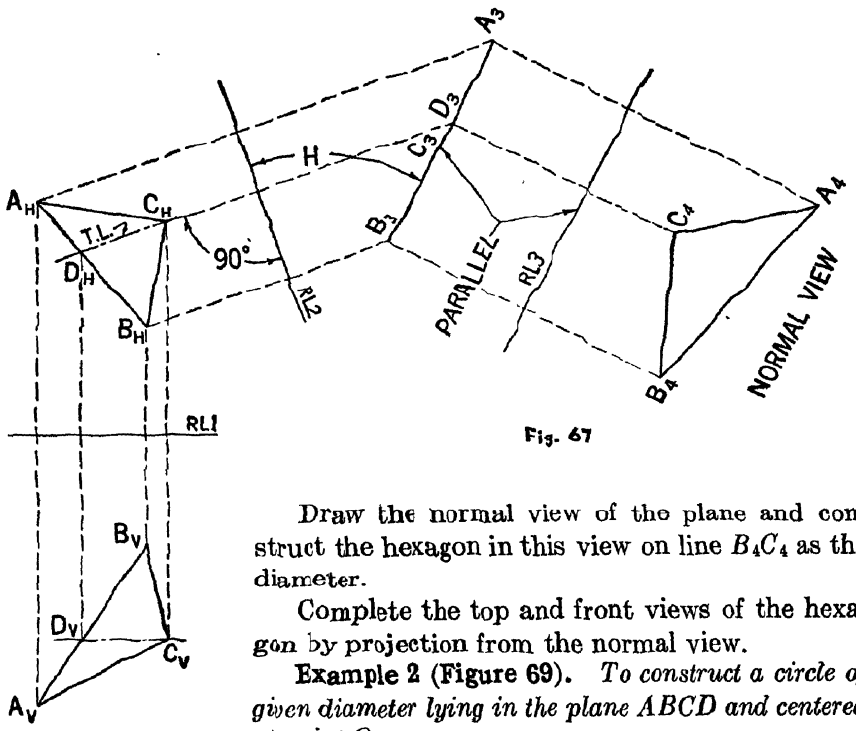


Fig. 67

Draw the normal view of the plane and construct the hexagon in this view on line B_4C_4 as the diameter.

Complete the top and front views of the hexagon by projection from the normal view.

Example 2 (Figure 69). *To construct a circle of given diameter lying in the plane ABCD and centered at point O.*

Given: The parallelogram *ABCD* and point *O* in its plane.

Analysis: A circle having its plane inclined to a projection plane projects as an ellipse on that plane. The diameter of the circle which is parallel to the projection plane projects in true length and becomes the major axis of the ellipse. A second diameter at right angles to the first projects as the minor axis.

Construction: Draw the horizontal line *EF* and lay off the diameter of the circle on E_HF_H. This diameter is the major axis of the top view of the ellipse. The minor axis is drawn through O_H and at right angles to E_HF_H. The minor axis in the top view is a projection of the diameter of the circle constructed on the edge view of *ABCD*. The ellipse is constructed on the axes thus determined by the methods of plane geometry.

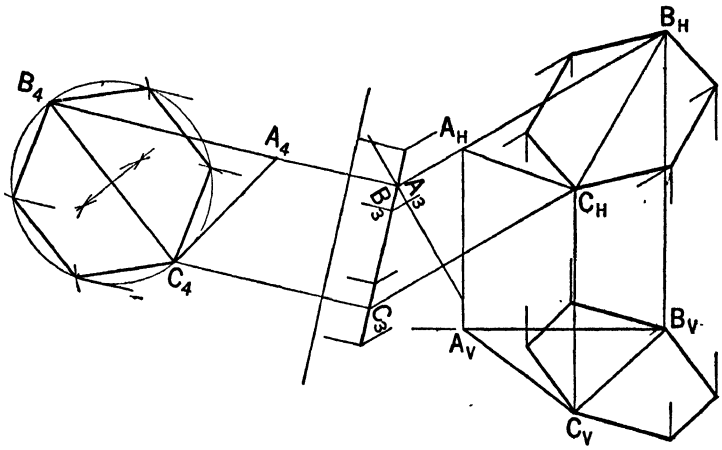

B_H
B_4
A_4
A_H
A_3
B_3
C_H
C_4
C_3
B_V
A_V
C_V

Fig. 68

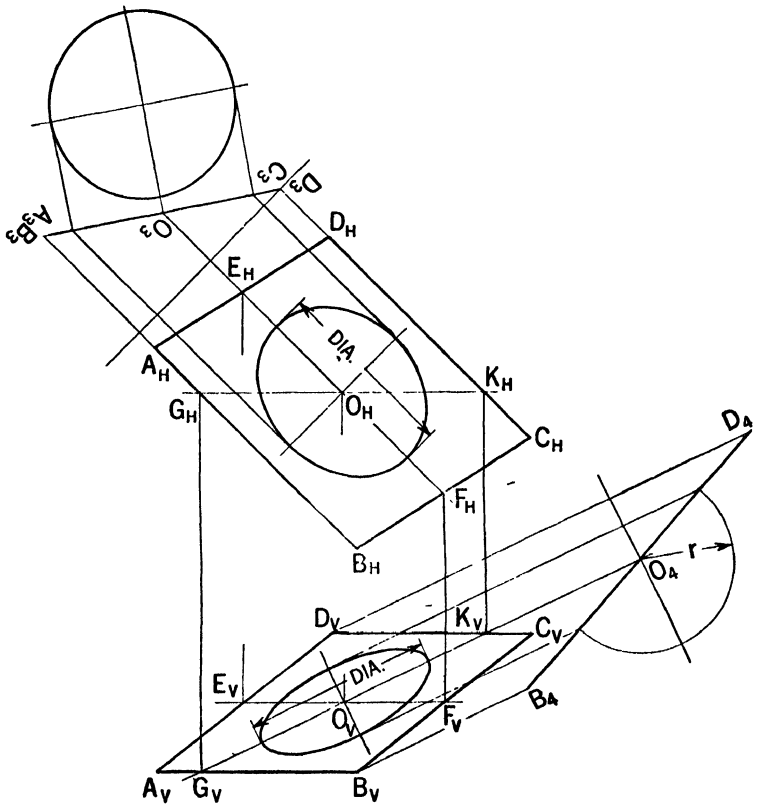

C_3
D_3
A_3B_3
O_3
D_H
E_H
DIA.
A_H
K_H
G_H
O_H
C_H
D_4
F_H
B_H
r
O_4
D_V
K_V
C_V
E_V
DIA.
O_V
F_V
B_4
A_V
G_V
B_V

Fig. 69

The major axis of the ellipse which represents the front view of the oblique circle lies along the frontal line G_VK_V and is equal in length to the diameter of the given circle. The minor axis is the projection of the diameter of the circle constructed on the edge view of $ABCD$.

35. Intersections

The determination of the line in which two surfaces meet is a common problem of practical drawing. The line of intersections of two surfaces is defined as the locus of all points common to the two surfaces. The general principle involved in problems on intersections is that employed in finding the intersection of a line and plane.

36. Intersection of a line and plane

(*Special Case*) The point in which a line pierces a plane that is perpendicular to a projection plane is determined by inspection in the view in

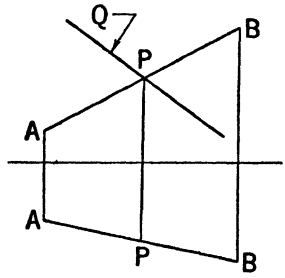


Fig. 70

which the plane appears edgewise. Line AB (Figure 70) intersects the vertical plane Q in point P.

37. Intersection of two planes

(*Special Case*) The line of intersection of two planes is determined by two points which are common to both planes. In this case plane Q, one

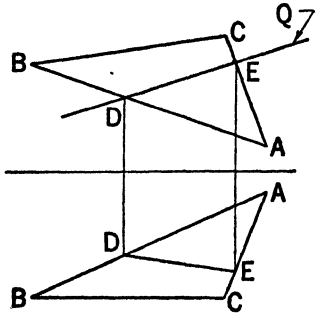


Fig. 71

of the given planes, is perpendicular to a projection plane. Lines AB and AC of plane ABC (Figure 71) pierce plane Q in points D and E respectively. Therefore, planes ABC and Q intersect along line DE.

38. Problem 7: To find the point in which a line intersects an oblique plane

First Analysis (*Figure 72*): Let XY be the given line and ABC the given plane. A vertical plane Q passed through line XY intersects

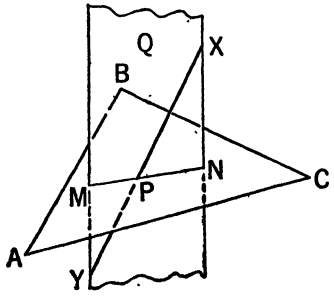


Fig. 72

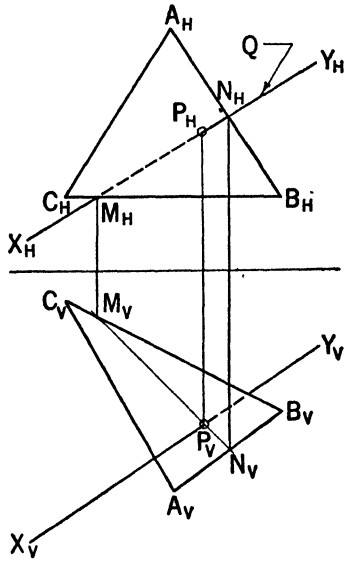


Fig. 73

plane ABC in the line MN. Line MN cuts XY in the point P. Since point P is common to line XY and plane ABC, it is the required point of intersection.

First Construction (*Figure 73*): Given: Line XY and plane ABC. The projection of the vertical plane Q passing through line XY coincides with X_HY_H, cutting plane ABC along M_HN_H. Project points M and N to obtain M_V and N_V. Line M_VN_V cuts X_VY_V in P_V, the front view of the point of intersection. The top view of P is obtained by projecting from P_V.

Second Analysis: Draw an auxiliary view in which the given plane projects edgewise. The point in which the given line cuts the edge view of the plane is the piercing point.

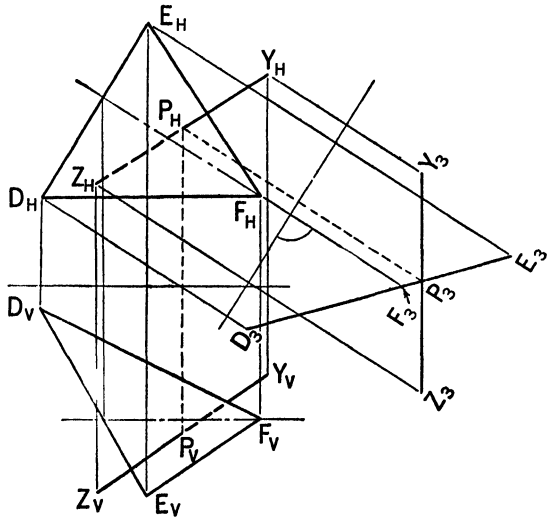


Fig. 74

Second Construction (*Figure 74*): Given: Line ZY and plane DEF. By use of an auxiliary view this general case is reduced to the special case of Article 36. Draw the auxiliary view $D_3E_3F_3$ in which plane DEF appears edgewise. Also draw Y_3Z_3, the projection of YZ on plane 3. The intersection of the line and plane is given by P_3 in the auxiliary view. P_H and P_V are found by projecting from P_3.

39. Traces of a line

The point in which a line intersects a projection plane is the *trace* of the line. The point S (Figure 75) in which line k pierces the H-plane is the *horizontal trace* of k; the point T in which k pierces the V-plane is the *vertical trace*. If a line is parallel to a projection plane, it will have no trace on that plane.

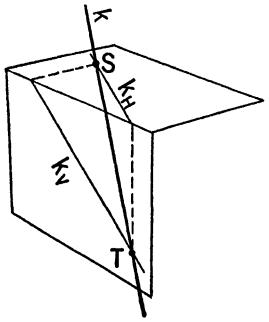


Fig. 75

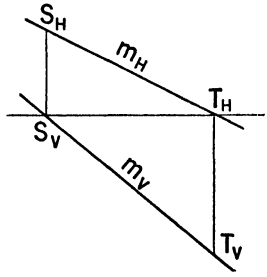


Fig. 76

40. Problem 7.1: To find the traces of a line (Figure 76)

Given: The line m.

The point S_V in which m_V meets the reference line is the vertical projection of the point in which line m pierces the H-plane. The projection S_H is the H-trace of m and lies on the projector through S_V. The point T_H in which m_H meets the reference line is the horizontal projection of

V-trace of m and lies on the projector through T_H. The projections S_V and T_H are always on the reference line.

41. Problem 8: To find the intersection of two planes

First Analysis: The intersection of two planes is a straight line determined by two points which are common to both planes.

A point common to two planes may be obtained by finding the point in which a line of one plane pierces the second plane, using the method of Problem 7.

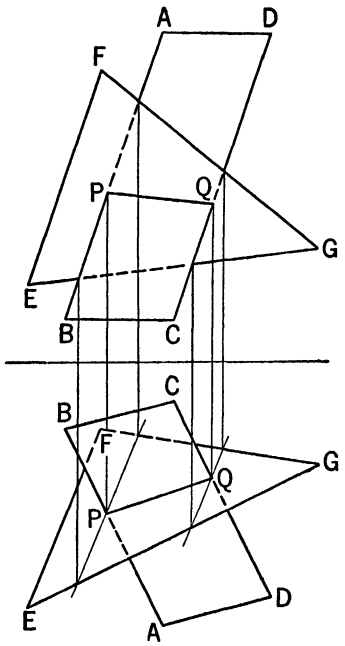


Fig. 77

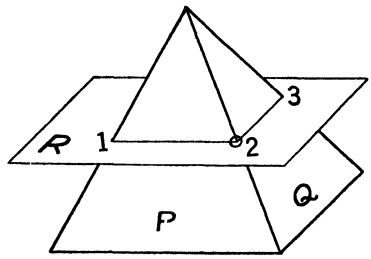


Fig. 78

First Construction (Figure 77): Given: Planes $ABCD$ and EFG. Find the intersection P of line AB with plane EFG. Find the intersection Q of line DC with plane EFG. The line through points P and Q is common to planes $ABCD$ and EFG.

Second Analysis: A third plane which intersects two given planes cuts a line from each of the given planes. The point in which the two lines intersect is common to the given planes.

For example, in Figure 78, the plane R cuts the faces P and Q of the pyramid in lines 1-2 and 2-3. These lines meet at point 2 which lies on the line of intersection of planes P and Q.

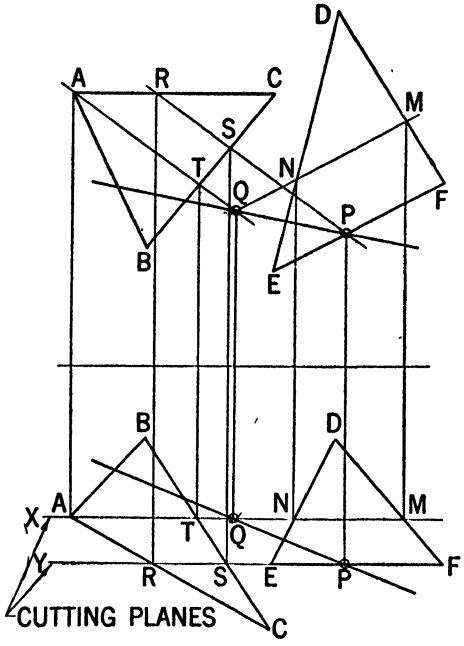


Fig. 79

Second Construction (Figure 79): Let planes ABC and DEF be given. Assume the cutting plane X perpendicular to the V-plane. Plane X cuts planes ABC and DEF in lines AT and NM respectively. These two lines intersect in point Q, a point common to planes ABC and DEF. To determine a second common point, repeat this operation assuming a second cutting plane Y. Plane Y cuts ABC in line RS and plane DEF in line EF.

Lines RS and EF intersect in point P. The line determined by points P and Q is common to planes ABC and DEF.

42. Traces of a plane

The intersection of one plane with another is called its *trace* on that plane. The intersections of a plane with the H- and V-projection planes are its *horizontal* and *vertical traces* respectively (Figure 80). The traces of a plane are particular principal lines of the plane. The H- and V-

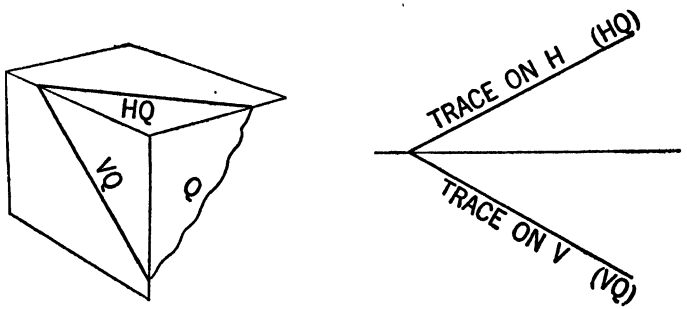
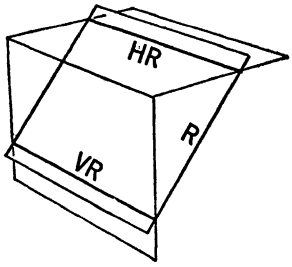


Fig. 80

traces of a given plane are simply the principal lines which lie in the H- and V-projection planes. In general, the two traces of a plane meet at a point on the reference axis. If the plane is parallel to the reference axis, the traces are parallel (Figure 81). A plane parallel to the H-plane will have no H-trace; the V-trace will be an edge view of the plane. Simi-

larly, a plane parallel to the V-plane will have no V-trace; the H-trace will be an edge view of the plane. The traces of a plane provide a unique representation of a given plane and are useful in certain applications.



TRACE ON H (HR)

TRACE ON V (VR)

Fig. 81

Traces of planes are commonly denoted by the names of the two determining planes. Thus, a plane Q may have traces HQ, VQ, and PQ on the H-, V-, and P-planes respectively.

43. Problem 8.1: To find the traces of a plane

Construction (Figures 82 and 83): Let the plane Q be passed through points A, B, and C. The intersection of Q and the H-plane is determined by the points 1, 2, 3 in which the lines AB, BC, and AC of plane Q intersect the H-plane. Similarly, the intersection of plane Q and the V-plane is determined by points 4, 5, and 6 where these same lines pierce the V-plane. Of course, but two points on each trace are required for its location.

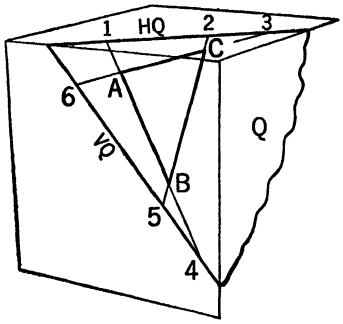


Fig. 82

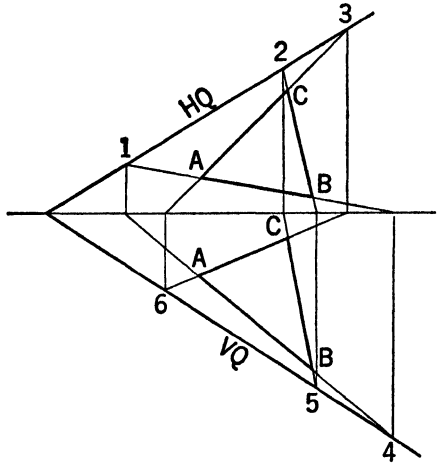


Fig. 83

Since points 1 and 4 are the H- and V-traces of the line AB, it is evident that when a line lies in a plane, a trace of the line must lie on a trace of the plane.

44. Problem 8.2: To find the intersection of two planes (Planes given by their traces)

Analysis: If in any plane of projection, as H, V, or P, the traces of two planes intersect, then the planes must intersect. The intersection of the two traces is a point common to both planes and, therefore, on the line of intersection of the planes. The line of intersection is, in general,

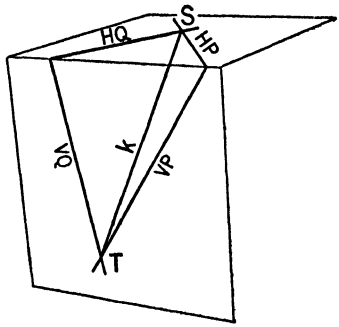


Fig. 84

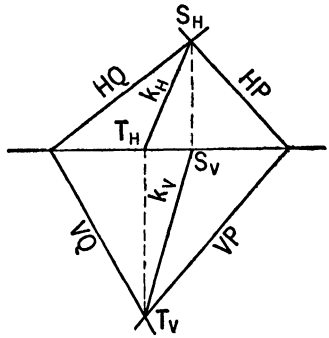


Fig. 85

determined by the two points in which two pairs of traces intersect.

Construction: Let P and Q (Figures 84 and 85) be the given planes. The horizontal traces HP and HQ intersect in point S. The vertical traces VP and VQ intersect in point T. The points S and T are the traces of the required intersection k. k_H is determined by S_HT_H. k_V is determined by S_VT_V.

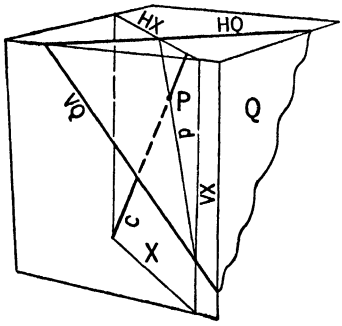


Fig. 86

45. Problem 7.2: To find intersection of a line and plane (Plane given by its traces)

Analysis: Let the line c and the plane Q be given (Figure 86). Pass an auxiliary plane X through line c. The line d in which plane X intersects plane Q contains every point common to the two planes. Therefore, it contains the point common to the given line and plane. The point P in which line d cuts line c is the point where line c pierces plane Q.

Construction (Figure 87): Pass the vertical plane X through line c by drawing HX to coincide with c_H. VX is at right angles to RL. In the front view planes X and Q intersect in line d_V, which is determined by S_V and T_V. The point P_V in which d_V cuts c_V is the front view of the required piercing point. P_H lies on c_H.

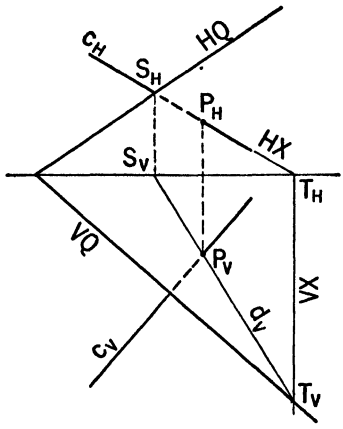


Fig. 87

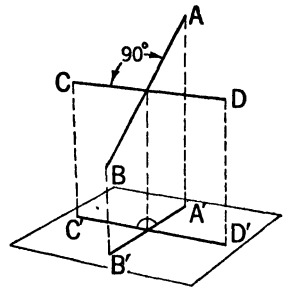


Fig. 88

46. Definitions of perpendicularity

(*a*) Two planes are perpendicular to each other if either plane contains a line perpendicular to the other plane.

(*b*) Two lines are perpendicular to each other if either line lies in a plane perpendicular to the other.

(*c*) A line perpendicular to a plane is perpendicular to every line of the plane.

The solution of any problem involving perpendicular lines is based on the following theorem:

Theorem I. If two lines of space are perpendicular, their orthographic projections on a plane parallel to one of the lines are perpendicular, and conversely (Figure 88).

Therefore, two perpendicular lines will appear at right angles in a view which is a true-length view of one (or both) of the lines.

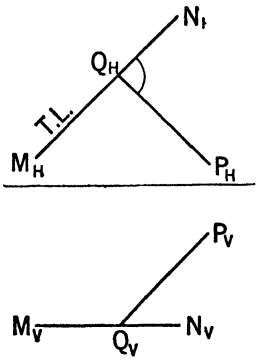


Fig. 89

Example 1 (Figure 89). *To draw a line from point P which will be perpendicular to the horizontal line MN.* Construct P_HQ_H at right angles to M_HN_H, the true-length view of MN. Project point Q to Q_V and draw P_VQ_V.

Example 2 (Figure 90). *To draw a horizontal line BC which will be perpendicular to the oblique line AB.* Draw B_HC_H at right angles to A_HB_H, B_HC_H being the true-length view of line BC. The front view B_VC_V will be parallel to RL.

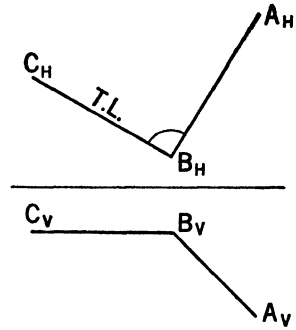


Fig. 90

47. Problem 9: To draw a line perpendicular to a plane

Analysis: Since a normal to a plane is perpendicular to every line of the plane, it is perpendicular in particular to every horizontal and frontal line of the plane. Consequently, from Theorem I, in any top view, a normal to a plane is perpendicular to any horizontal line of the plane. In any front view, a normal to a plane is perpendicular to any frontal line of the plane. If a plane projects edgewise as a straight line, a normal to the plane is at right angles to the edge view. Thus, in Figure 91, line n is perpendicular to plane Q when n_H is at right angles to A_HB_H, and n_V is at right angles to A_VC_V.

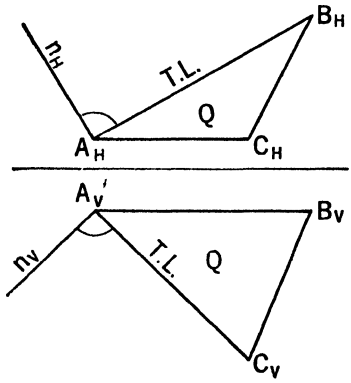


Fig. 91

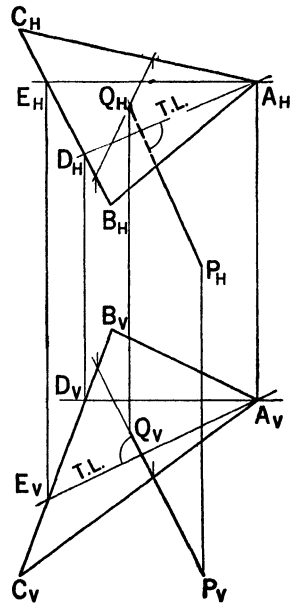


Fig. 92

First Construction (Figure 92): To draw the normal from point P to plane ABC. In plane ABC, draw a horizontal line AD and a frontal line AE. Construct a line through P_H, perpendicular to A_HD_H; and a line through P_V, perpendicular to A_VE_V. These two lines are the projections of the required normal to plane ABC. Point Q is the foot of the

perpendicular. It is the point in which the normal intersects plane ABC, and is found by the method of Problem 7.

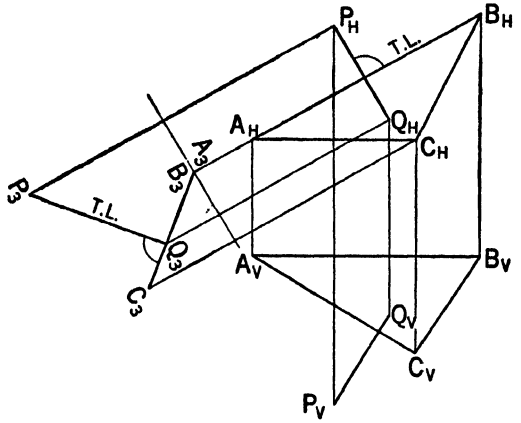


Fig. 93

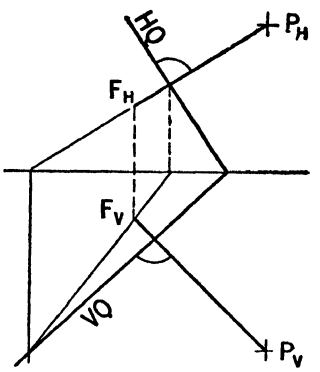


Fig. 94

Second Construction (Figure 93): To draw the normal from point P to plane ABC. Choose auxiliary plane 3 so that the plane ABC projects edgewise as $A_3B_3C_3$. Draw the normal P_3Q_3 at right angles to $A_3B_3C_3$. Since P_3Q_3 is perpendicular to $A_3B_3C_3$, it is parallel (in space) to plane 3. Therefore, P_HQ_H is parallel to $RL2$.

Third Construction (Figure 94): To draw a line through point P and perpendicular to plane Q. Since the traces of a plane are two particular principal lines of the plane, the H- and V-projections of a normal to plane Q are perpendicular to the corresponding traces of the plane. The foot of the perpendicular is point F, and is found by the method of Problem 7.2.

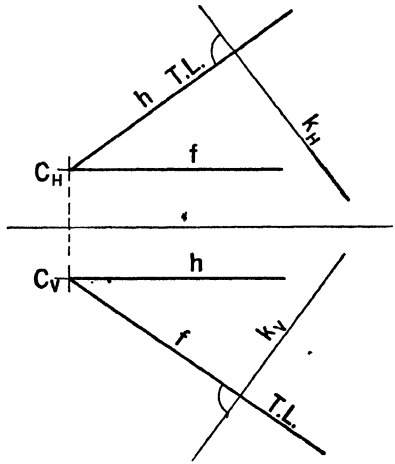


Fig. 95

48. Problem 10: To pass a plane through a point and perpendicular to a line

Analysis: Through the given point draw two lines each of which is perpendicular to the given line. (Theorem I.)

Pass a plane through the two lines drawn. (Problem 5.)

Construction (Figure 95): To pass a plane through point C and perpendicular to line k. Draw the horizontal line h perpendicular to line k by principle of Theorem I. Draw the frontal line f perpendicular to k. Lines h and f determine the required plane.

If the traces of the plane through lines h and f are drawn, it will be found that they will be perpendicular to the corresponding projections of line k. This follows from the fact that the H- and V-traces of a plane are particular horizontal and frontal lines of the plane.

PROBLEMS ON CHAPTER II

Group A. Fundamental Problems 1, 2, and 3

2-A1. (*a*) Given point A: $\frac{1}{2}$,$6\frac{3}{4}$,$7\frac{3}{4}$. Point B is 1 inch to the right of, 1 inch below, and $1\frac{1}{2}$ inches to the rear of point A. Draw the three principal views of line AB and describe the position of its mid-point M with respect to point A. (*b*) Given line C: 5,$7\frac{1}{4}$,$9\frac{1}{4}$ D: 5,$5\frac{1}{2}$,8. Find the projections of point E which lies in CD and is 1 inch below point C. (*c*) Assuming equal weights at points E: $\frac{1}{2}$,$1\frac{1}{4}$,$3\frac{1}{4}$ F: $1\frac{1}{2}$,$\frac{3}{4}$,$4\frac{1}{4}$ G: $3\frac{1}{4}$,$2\frac{1}{2}$,$3\frac{3}{4}$, find the position of the center of gravity of the system with respect to point G. (*d*) Do lines J: $4\frac{1}{4}$,$1\frac{1}{2}$,$4\frac{1}{2}$ K: $5\frac{3}{4}$,$\frac{3}{4}$,3 and L: 5,$\frac{1}{2}$,$4\frac{1}{4}$ M: 523 intersect?

2-A2. Draw the top and front views of the tetrahedron A: 119 B: 236 C: 409 D: 756. Show that the four medians of the solid intersect in a common point. This point is the center of gravity of the tetrahedron.

The medians of a triangular face intersect in a point called the centroid of the triangle. The line joining the centroid of a face and the opposite vertex of the solid is a median of the tetrahedron.

2-A3. Three intersecting edges of a parallelepiped are given by the lines AB, AD, and AE. A: $1\frac{3}{4}$,5,$5\frac{1}{2}$ B: $\frac{1}{2}$,$3\frac{1}{4}$,$5\frac{1}{2}$ D: $1\frac{3}{4}$,$3\frac{1}{2}$,$7\frac{1}{4}$ E: $3\frac{1}{4}$,$3\frac{1}{2}$, $6\frac{3}{4}$. Complete the top and front views, lettering the opposite parallel faces $ABCD$ and $EFGH$. Construct the right-side view.

Group B. Fundamental Problem 4

2-B1. Draw the top and front views of a rectangular right prism having a horizontal base 2×3 inches and a 4-inch altitude. Find the true length of a body diagonal and the true angle that this diagonal makes with the plane of the base. Check the true length by calculation, referring to Article 146.

2-B2. Find the true-length view and the point view of the line A: 335 B: 547. Measure the true angles between line AB and the H- and V-projection planes.

2-B3. Line A: $\frac{1}{2}$,1,4 B: 214 is the front edge of a horizontal rectangle 2 inches deep. This rectangle is the base of a right pyramid 2 inches high. Complete the top and front views of the pyramid, and draw an auxiliary view in the direction E: $2\frac{1}{2}$,$1\frac{3}{4}$,$4\frac{1}{4}$ V: $1\frac{1}{4}$,3,5. Letter the subtitle "Auxiliary View" under each auxiliary view drawn in the correct orientation.

2-B4. The pentagon A:$\frac{1}{2}$,2,4 B:$\frac{1}{2}$,1,4 C:214 D:224 E:1$\frac{1}{4}$,2$\frac{3}{4}$,4 is the front face of a right prism 2 inches long. Draw the top and front views and an auxiliary view in the direction F:2$\frac{1}{4}$,3$\frac{1}{4}$,7 G:1$\frac{1}{4}$,2$\frac{3}{4}$,6. Letter the subtitle "Auxiliary View" as in Problem 2-B3.

2-B5. Point A:018 is the lower left corner of the front face of a rectangular block 3 inches wide, 2 inches high, and 1$\frac{1}{2}$ inches deep. The left and right thirds of the upper half of the block are removed. Draw an auxiliary view of the tee-shaped prism as seen in the direction P:1,4,7$\frac{1}{2}$ Q:1$\frac{1}{2}$,2,9. Obtain the required view by proportional division of the enclosing "box" as described in Article 24.

2-B6. Lines A:136 B:236 and E:217 F:317 are respectively the front edges of two horizontal squares which form the ends of an oblique prism. Complete the top and front views of the prism, and draw an auxiliary view in which the lateral edges project as points.

2-B7. Draw a view of the tetrahedron V:534 A:606 B:707 C:825 in which the edge VB projects as a point.

GROUP C. PLANES. FUNDAMENTAL PROBLEMS 5 AND 6

2-C1. Is the figure A:046 B:129 C:519 D:457 a plane or a skew quadrilateral? Draw the top, front, and side views. Is the figure formed by joining the mid-points of the adjacent sides a plane figure?

2-C2. A plane is determined by points A:118 B:5,5,10 C:716. Points K:4$\frac{1}{4}$,2,X L:7$\frac{1}{2}$,X,7$\frac{1}{2}$ and M:X,3,8$\frac{3}{4}$ lie in this plane. Find the missing projections.

2-C3. Show that lines A:135 B:7,3,7$\frac{1}{2}$ and C:118 D:5$\frac{1}{2}$,1$\frac{3}{4}$,8$\frac{3}{4}$ lie in a plane. Points K:3,X,8 and L:7$\frac{1}{2}$,4,X lie in this plane. Find the missing projections of K and L, and check the fact that line KL lies in plane $ABCD$.

2-C4. Show that lines d:0,2,8$\frac{1}{4}$; 8,2,8$\frac{3}{4}$ and f:$\frac{1}{2}$,4,4$\frac{1}{4}$; 7$\frac{3}{4}$,3$\frac{1}{4}$,6$\frac{1}{4}$ intersect. Through point E:2$\frac{1}{4}$,$\frac{3}{4}$,7 draw a line which will pass through the point in which lines d and f meet.

2-C5. Draw the top, front, and side views of the triangle A:017 B:439 C:356. Through point K:2,2$\frac{3}{4}$,X draw in plane ABC, lines which are parallel to the horizontal, vertical, and profile projection planes. Mark the true-length views (TL).

2-C6. (a) Through point A draw the line of maximum inclination of plane A:056 B:129 C:327. Find the true angle that this line makes with the horizontal plane.

(b) Through point E draw the line of maximum inclination of plane E:649 F:825 G:516, and find the true angle it makes with the horizontal plane.

2-C7. Draw the edge view and the true-size view of the triangle A:$\frac{1}{4}$,4$\frac{3}{4}$,6$\frac{1}{4}$ B:2,3,5$\frac{1}{4}$ C:3$\frac{1}{4}$,3$\frac{3}{4}$,6. Determine also the true angles between plane ABC and the H- and V-projection planes, the true length of each side of the triangle, and the true size of each angle of the triangle.

2-C8. Given the triangle A:4$\frac{3}{4}$,4$\frac{3}{4}$,7$\frac{1}{2}$ B:4$\frac{3}{4}$,2$\frac{1}{2}$,5$\frac{1}{2}$ C:6$\frac{1}{2}$,3$\frac{1}{2}$,6$\frac{1}{4}$. Solve for same data required in the preceding problem.

2-C9. In the plane K:$\frac{1}{2}$,3,8 L:346 M:337 construct a square having the line KM as a diagonal. Draw the top, front, and normal views of this square.

2-C10. Line PQ is a diagonal of a regular hexagon which lies in the plane determined by points P:634 Q:716 R:824. Draw the top, front, and true-size views of the hexagon.

2-C11. Line AB is a diameter of a circle which lies in the plane determined by points A:1$\frac{1}{4}$,3,8 B:437 C:4,4,6$\frac{1}{2}$. Draw the top, front, and true-size views of the circle. Show the major and minor axes of the elliptical views.

GROUP D. INTERSECTIONS. FUNDAMENTAL PROBLEMS 7 AND 8

2-D1. Find the point P in which line E:046 F:418 intersects triangle A:017 B:239 C:426. Solve by two different methods. Show correct visibility for line EF.

2-D2. Find the point Q common to line D:446 E:758 and the plane passing through points A:336 B:625 C:519.

2-D3. Find point E in which line J:2,3$\frac{3}{4}$,7 K:6,$\frac{3}{4}$,7 pierces the parallelogram A:345 B:3,1$\frac{1}{2}$,8 C:509 D:5,2$\frac{1}{2}$,6. Show correct visibility.

2-D4. Find the line common to the triangles A:$\frac{3}{4}$,4,7$\frac{1}{2}$ B:4$\frac{1}{2}$,4,9 C:6$\frac{1}{2}$,1,5$\frac{1}{4}$ and D:1$\frac{1}{4}$,5,5$\frac{3}{4}$ E:4$\frac{1}{4}$,1$\frac{1}{2}$,8$\frac{1}{2}$ F:7$\frac{1}{4}$,2,7. Show correct visibility for the triangles.

2-D5. Find the line of intersection of the triangle A:1$\frac{1}{4}$,2,5 B:3$\frac{1}{2}$,$\frac{1}{2}$,9$\frac{1}{2}$ C:6$\frac{1}{2}$,4,6$\frac{3}{4}$ and the parallelogram D:118 E:7,2$\frac{1}{4}$,7$\frac{1}{4}$ F:7,4$\frac{1}{2}$,5$\frac{3}{4}$ G:1,3$\frac{1}{4}$,6$\frac{1}{2}$. Show correct visibility.

2-D6. Find the line of intersection of the planes Q and R. Plane Q passes through points A:$\frac{1}{2}$,2$\frac{1}{4}$,9 B:2$\frac{1}{4}$,$\frac{1}{4}$,6$\frac{1}{4}$ C:2$\frac{1}{2}$,3,8$\frac{1}{2}$. Plane R passes through points D:5,3$\frac{1}{4}$,9 E:5$\frac{3}{4}$,1$\frac{1}{2}$,6$\frac{1}{2}$ F:6$\frac{3}{4}$,2$\frac{3}{4}$,9$\frac{1}{4}$.

2-D7. Find the line of intersection of planes S and T. Plane S is determined by points A:1,2,6$\frac{3}{4}$ B:2$\frac{1}{4}$,$\frac{1}{2}$,9$\frac{1}{2}$ C:419; plane T by D:4,5,7$\frac{1}{4}$ E:5$\frac{1}{2}$,1$\frac{3}{4}$,9$\frac{1}{2}$ F:6$\frac{1}{4}$,3$\frac{3}{4}$,6$\frac{3}{4}$.

2-D8. Find the line in which the plane passing through triangle A:046 B:308 C:435 cuts triangle D:424 E:508 F:845.

GROUP E. INTERSECTIONS OF LINES AND PLANES WITH THE PROJECTION PLANES

The point common to the horizontal, vertical, and profile projection planes is called the *origin*. The reference lines between the *H*- and *V*-planes, and between the *V*- and *P*-planes intersect in this point.

2-E1. (*a*) Find the points *S* and *T* in which the line which passes through points A:1¼,7½,8½ and B:2½,6¼,7¾ pierces the *H*- and *V*-projection planes. Origin: 0,7½,7½. To the right of the orthographic views draw a pictorial view showing the *H*- and *V*-planes, a portion of line *AB*, and the piercing points *S* and *T*.

(*b*) Find the points *S* and *T* in which the line which passes through points C:2½,2,3½ and D:3¾,¾,4 pierces the *H*- and *V*-planes. Origin: 0,2½,2½. To the right of the orthographic views, draw a pictorial view as in (*a*).

2-E2. Find the traces *S* and *T* on the *H*- and *V*-planes for the lines which contain the following pairs of points:

(*a*)	A:½,5½,9	B:1¾,6¼,7¾	Origin: 0,7½,7½
(*b*)	C:5¼,5½,8½	D:6¼,6½,8	Origin: 8,7½,7½
(*c*)	E:1½,2,3½	F:3,1,3½	Origin: 0,2½,2½
(*d*)	G:524	K:5,¾,3	Origin: 6,2½,2½

2-E3. Represent the following planes by their traces on the *H*- and *V*-projection planes.

(*a*) Plane *Q* is horizontal and contains point A:279. Origin: 0,7½,7½.
(*b*) Plane *R* is frontal and contains point B:668. Origin: 8,7½,7½.
(*c*) Plane *S* is vertical, contains point C:114, and makes 45° with *V*, sloping forward to the right. Origin: 0,2½,2½.
(*d*) Plane *T* is perpendicular to *V*, contains point D:614, and makes 60° with *H*, sloping downward to the left.

2-E4. The plane of triangle A:2½,4¼,5¾ B:3¾,4½,7½ C:5¼,2,5¼ is extended to cut the *H*- and *V*-planes. Find the traces. Origin: 055.

2-E5. Find the *H*- and *V*-traces of the plane which contains the points A:4¼,3½,7¼ B:6½,1¼,9 C:6½,2½,5¾. Origin: 055.

2-E6. Plane *Q* is determined by points A:½,5,5 B:3¼,3,5 C:2¼,5,7¾. Plane *R* is determined by points D:7½,5,5 E:5,¾,5 F:4¼,5,7¼. Represent planes *Q* and *R* by their *H*- and *V*-traces, and find the line of intersection of *Q* and *R*. Origin: 055.

2-E7. Plane *X* is determined by points A:¼,5,5 B:2½,1,5 C:3½,5,7½. Plane *Y* is determined by points D:355 E:6½,1,5 F:4¾,5,9. Represent

planes X and Y by their H- and V-traces, and find the line of intersection of the two planes. Origin: 055.

2-E8. Plane W is determined by points A:$\frac{1}{2}$,5,5 B:6,$\frac{1}{4}$,5 C:6,5,8$\frac{1}{4}$. Represent W by its H- and V-traces. Origin: 055. Through point E:2$\frac{3}{4}$,X,5$\frac{1}{2}$ of plane W, draw the principal lines of the plane. Through point F:5,X,6$\frac{1}{4}$ of plane W, draw the line of maximum inclination to the horizontal.

2-E9. Plane Z is determined by points J:5$\frac{1}{4}$,5,5 K:715 L:1$\frac{1}{4}$,5,10. Represent Z by its traces. Origin: 055.

Point C:4,2$\frac{1}{2}$,X is the center of a plane figure which lies in plane Z. In the front view, the figure projects as a regular hexagon with sides 1$\frac{1}{4}$ inches long and one diagonal horizontal. Draw the top view.

2-E10. Through line A:3$\frac{3}{4}$,2,5$\frac{3}{4}$ B:5$\frac{3}{4}$,4,7 pass the following planes, representing each plane by its H- and V-traces. Origin: 055.

(*a*) Plane P perpendicular to the H-plane.
(*b*) Plane Q perpendicular to the V-plane. Note that planes P and Q are the H- and V-projecting planes of line AB.
(*c*) Plane R parallel to the reference axis between H and V.
(*d*) Any plane X which slopes down, forward, right.
(*e*) Any plane Y which slopes down, backward, left.

2-E11. Plane R is determined by points A:$\frac{1}{2}$,5,5 B:4$\frac{1}{2}$,2$\frac{1}{2}$,5 C:4$\frac{1}{2}$,5,7$\frac{3}{4}$. Represent plane R by its H- and V-traces. Origin: 055. Find point K in which line D:3,2$\frac{1}{2}$,8 E:6,4$\frac{1}{2}$,6 intersects plane R.

2-E12. Plane Q is determined by points A:455 B:125 C:659. Represent Q by its H- and V-traces. Origin: 055. Find point P in which line E:2$\frac{1}{2}$,4,9$\frac{1}{4}$ F:7$\frac{1}{2}$,1$\frac{1}{2}$,5 pierces plane Q.

GROUP F. PERPENDICULAR LINE AND PLANE. FUNDAMENTAL PROBLEMS 9 AND 10

2-F1. Perpendicular lines. (*a*) Through point C:2$\frac{1}{2}$,7,7$\frac{1}{2}$ construct a line perpendicular to and intersecting line A:168 B:369. (*b*) Through point E:5,6,10 construct a line intersecting and perpendicular to line C:589 D:769. (*c*) Through the mid-point of line F:104 G:323 draw a horizontal line perpendicular to FG. (*d*) Through the mid-point of the line K:502 L:714 draw a frontal line which shall be at right angles to KL.

2-F2. Point P:4,X,8 lies in plane J:208 K:549 L:726. At point P construct a line PQ perpendicular to plane JKL. Point Q is 2 inches to the left of point P.

2-F3. Through point P:$5\frac{3}{4},\frac{1}{4},6\frac{1}{4}$ draw a line perpendicular to triangle A:$1\frac{1}{2},\frac{1}{4},8\frac{1}{4}$ B:$3\frac{1}{2},3\frac{1}{2},6\frac{1}{4}$ C:7,5,$8\frac{3}{4}$. Find the foot of the perpendicular.

2-F4. Through point P:638 draw a perpendicular to plane A:157 B:319 C:435. Find the foot of the perpendicular.

2-F5. Through point P:638 pass plane R perpendicular to line A:319 B:556.

2-F6. Construct a 2-inch square perpendicular to line K:$5\frac{1}{2}$,2,$7\frac{1}{2}$ L:746. The square is centered on point K, and two of its sides are horizontal.

2-F7. The triangle A:436 B:648 C:827 is the upper base of a triangular prism $1\frac{1}{2}$ inches thick. Complete the projections of the prism.

2-F8. The line A:2,5,$6\frac{1}{2}$ B:3,4,$8\frac{1}{2}$ is the upper edge of a vertical rectangle $ABCD$. The edges of the rectangle perpendicular to AB are 3 inches long. This rectangle is the right face of a right prism $\frac{1}{2}$ inch thick. A rectangular hole, perpendicular to face $ABCD$, is cut through the solid, leaving walls $\frac{1}{2}$ inch thick on each side. Draw an auxiliary view which shows the true shape of face $ABCD$, and complete the top and front views.

2-F9. Line V:739 O:437 is the axis of a right pyramid. The base, centered on point O, is a 2-inch square having one diagonal horizontal.

2-F10. Line K:125 L:427 is the axis of a square right prism. One corner of the base centered on L is located by point A:$3\frac{1}{2}$,$2\frac{3}{4}$,X.

2-F11. Line A:$4\frac{1}{2}$,$3\frac{1}{2}$,$8\frac{1}{4}$ B:6,2,$8\frac{1}{4}$ is the axis of a right hexagonal prism. Line C:$3\frac{1}{2}$,$2\frac{1}{2}$,$8\frac{3}{4}$ F:$5\frac{1}{2}$,$4\frac{1}{2}$,$7\frac{3}{4}$ is a diagonal of one base. Draw an auxiliary view showing the true shape of the bases, and complete the top and front views of the prism.

2-F12. The line A:2,2,$8\frac{1}{2}$ B:427 is the axis of a $3\frac{1}{2}''$ circle centered on B. Draw the views of the circle.

Note: A circle which makes an acute angle with a projection plane projects as an ellipse on that plane. The diameter of the circle parallel to the plane projects as the major axis of the ellipse. A second diameter at right angles to the first projects as the minor axis. When the principal axes of an ellipse are easily found, points on the curve can readily be located by means of a *trammel.*

2-F13. The line A:5,3,$7\frac{1}{2}$ B:638 is the axis of a 4-inch diameter right cylinder. Draw the following views of the cylinder: the top view, front view, and auxiliary view to show true size.

2-F14. Point V:5,$4\frac{1}{2}$,$6\frac{1}{2}$ is the vertex and C:4,3,$7\frac{1}{2}$ is the center of the base of a right cone. The base is a $2\frac{1}{2}$-inch circle. Since the axis is

oblique, the base will project as an ellipse in the principal views. Find the major and minor axes for each base, and trammel the ellipses. Complete the projections of the cone.

2-F15. Plane Q is determined by points A:155 B:515 C:659. Represent the plane by its H- and V-traces. Origin: 055. Through point P:2,1$\frac{1}{4}$,9$\frac{1}{4}$ draw a line perpendicular to plane Q. Find the foot of the perpendicular.

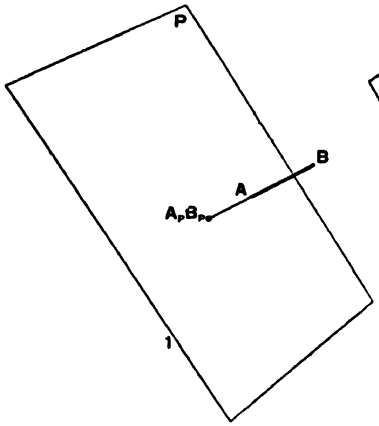
2-F16. Draw the H- and V-traces of plane R which is determined by points A:4$\frac{3}{4}$,5,5 B:$\frac{3}{4}$,2,5 C:7$\frac{1}{2}$,5,9. Origin: 055. Through point P:2$\frac{1}{4}$,4$\frac{1}{2}$,8$\frac{3}{4}$ construct a line perpendicular to plane R, and locate the foot of the perpendicular.

2-F17. Pass a plane Q at right angles to line A:1,4$\frac{3}{4}$,5$\frac{1}{2}$ B:7,1,9$\frac{3}{4}$ and through its mid-point. Find the traces of plane Q. Origin: 055.

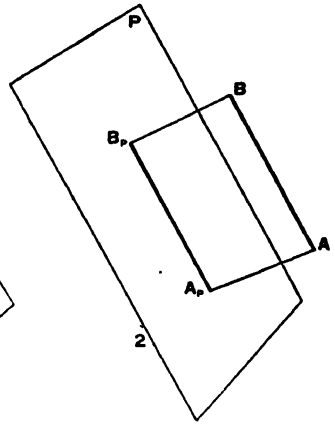
2-F18. Through point P:3,1,8$\frac{3}{4}$ pass a plane R perpendicular to line A:1$\frac{1}{2}$,1$\frac{1}{4}$,5$\frac{1}{4}$ B:6$\frac{1}{2}$,4$\frac{1}{4}$,9$\frac{3}{4}$. Find the traces of plane R. Origin: 055.

Basic Principles

1. A LINE ⊥ TO A PLANE PROJECTS ON THE PLANE AS A POINT.
2. A LINE PARALLEL TO A PLANE PROJECTS ON THE PLANE ITS TRUE LENGTH.



NOTE THAT THE ⊥S FROM A & B MUST COINCIDE.

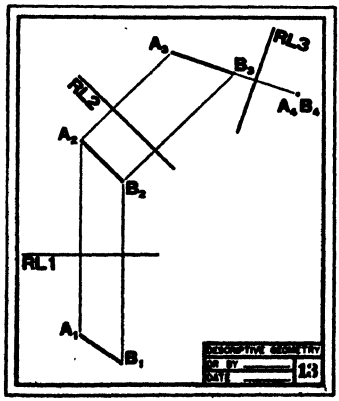


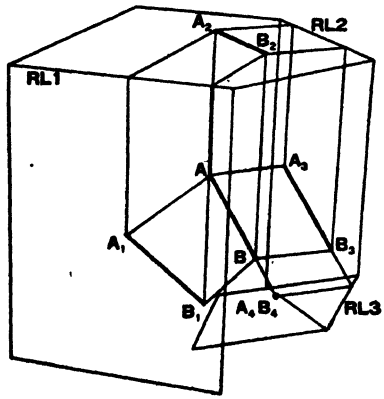
12

NOTE THAT AB & A₁B₁ ARE OPPOSITE SIDES OF A RECTANGLE & MUST BE EQUAL.

True Length and End View of a Line

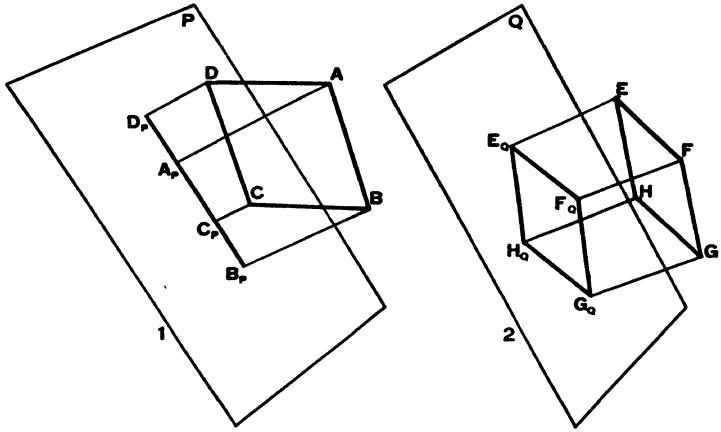
GIVEN 2 VIEWS OF A LINE TO OBTAIN TRUE LENGTH & END VIEWS.





NOTE THAT PLANE 3 IS TAKEN PARALLEL TO AB & PLANE 4 ⊥ TO AB. 13

Basic Principles

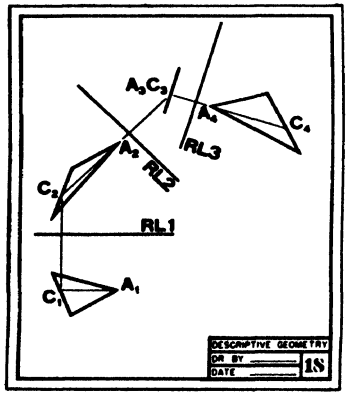


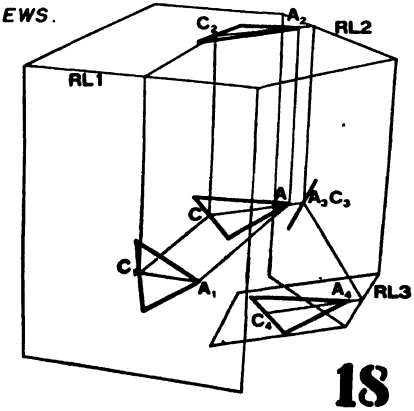
1. A PLANE ⊥ TO A PROJECTION PLANE PROJECTS AS A STRAIGHT LINE - CALLED AN EDGE VIEW.
2. A PLANE PARALLEL TO A PROJECTION PLANE PROJECTS ITS TRUE SIZE - CALLED A NORMAL VIEW.

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Edge and Normal View of a Plane

GIVEN 2 VIEWS OF A PLANE TO OBTAIN THE EDGE & NORMAL VIEWS.

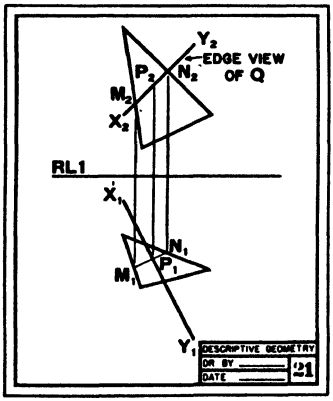


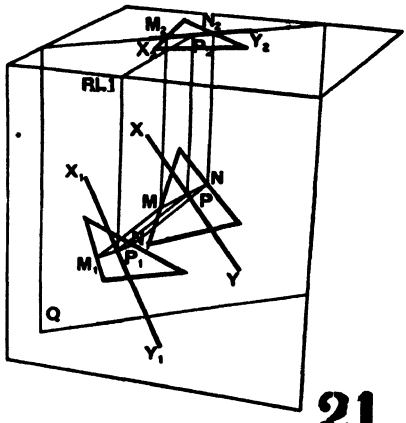


18

NOTE THAT PLANE 3 IS TAKEN ⊥ TO A PRINCIPAL LINE IN THE GIVEN PLANE & PLANE 4 PARALLEL TO THE EDGE VIEW. ANY PLANE GEOMETRIC CONSTRUCTION MAY BE PERFORMED IN A NORMAL VIEW.

Intersection of a Line & a Plane



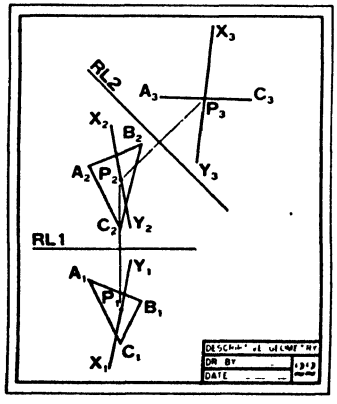


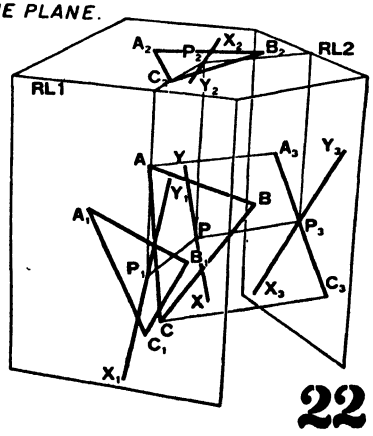
USE A CUTTING PLANE CONTAINING THE LINE & ON EDGE IN ONE VIEW.

NOTE THAT MN IS THE INTERSECTION OF THE GIVEN PLANE & CUTTING PLANE, Q. SINCE XY LIES IN Q, MN MUST INTERSECT XY AT P.

Intersection of a Line & a Plane (Alternate Method)

TO FIND THE INTERSECTION OF A LINE & A PLANE OBTAIN AN EDGE VIEW OF THE PLANE.

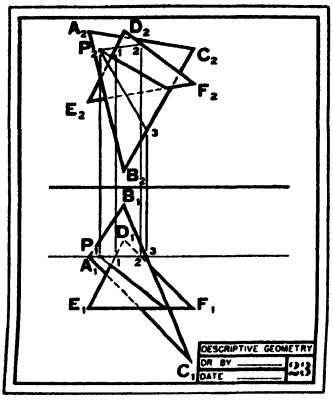


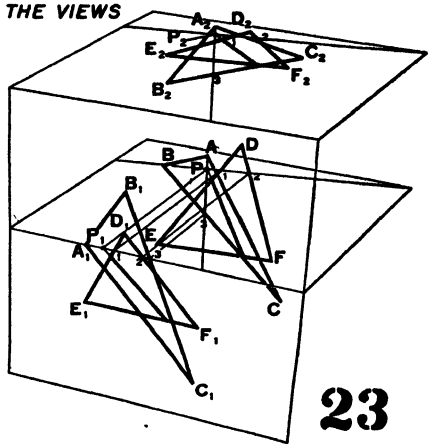


NOTE THAT IN THE EDGE VIEW THE POINT OF INTERSECTION IS APPARENT.

Intersection of 2 Planes

TO FIND THE INTERSECTION OF 2 PLANES USE 2 CUTTING PLANES ON EDGE IN ONE OF THE VIEWS



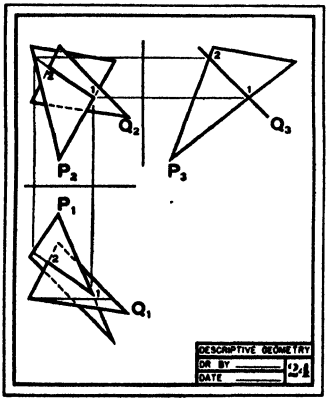


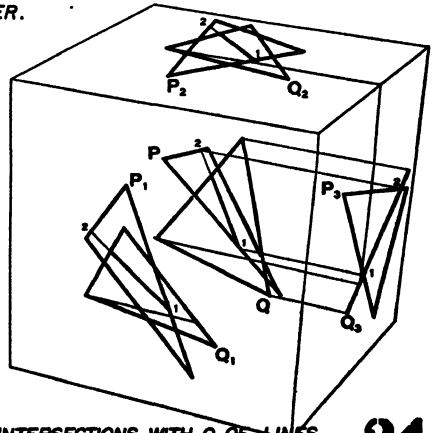
23

NOTE THAT THE INTERSECTIONS OF THE GIVEN PLANES WITH THE CUTTING PLANE INTERSECT IN A POINT ON THE DESIRED LINE. A 2ND CUTTING PLANE WILL DETERMINE A 2ND POINT & THUS THE LINE

Intersection of 2 Planes (Alternate Method)

TO DETERMINE THE INTERSECTION OF 2 PLANES OBTAIN AN EDGE VIEW OF EITHER.



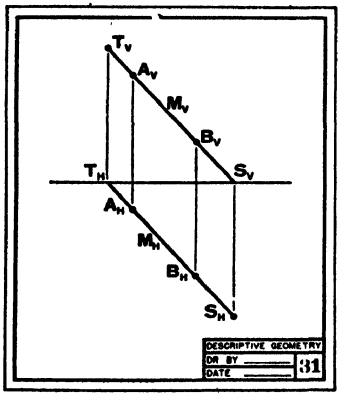


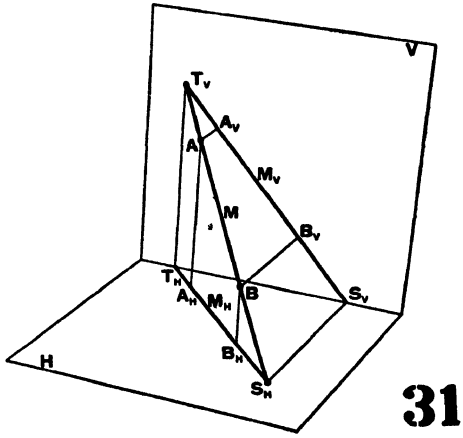
NOTE THAT IN THE EDGE VIEW THE INTERSECTIONS WITH Q OF LINES IN P ARE APPARENT, YIELDING POINTS ON THE DESIRED LINE.

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Traces of a Line

A LINE MAY BE REPRESENTED BY ITS POINTS OF INTERSECTION WITH THE PROJECTION PLANES THESE POINTS ARE THE TRACES OF THE LINE.

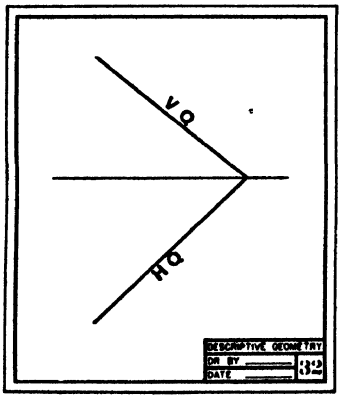


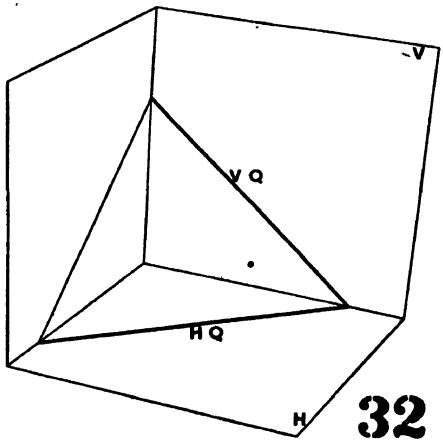


NOTE THAT ONE VIEW OF EACH TRACE ALWAYS LIES ON THE REFERENCE LINE

Traces of a Plane

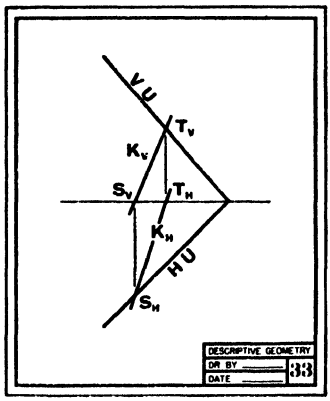
A PLANE MAY BE REPRESENTED BY ITS LINES OF INTERSECTION WITH THE PROJECTION PLANES. THESE LINES ARE THE TRACES OF THE PLANE.

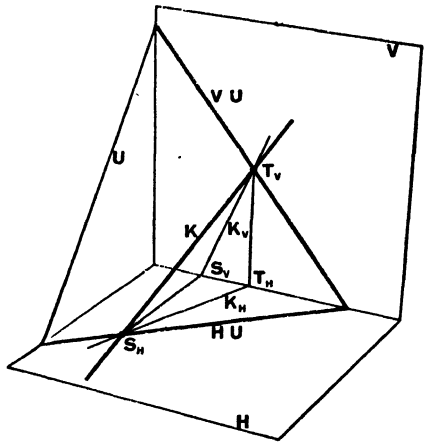




NOTE THAT THE TRACES ALWAYS INTERSECT ON THE REFERENCE LINE.

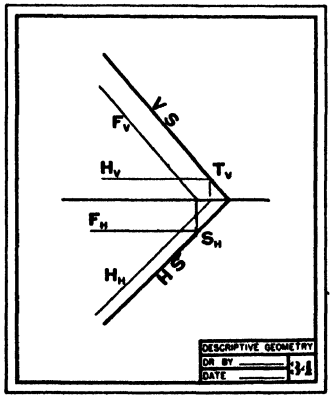
Traces of a Line in a Plane

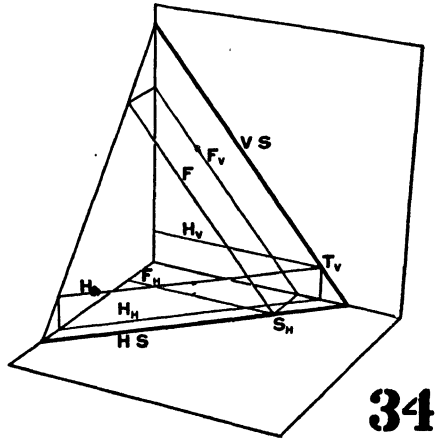




IF A LINE LIES IN A PLANE, THE TRACES OF THE LINE LIE IN THE CORRESPONDING TRACES OF THE PLANE. **33**

Traces & Principal Lines

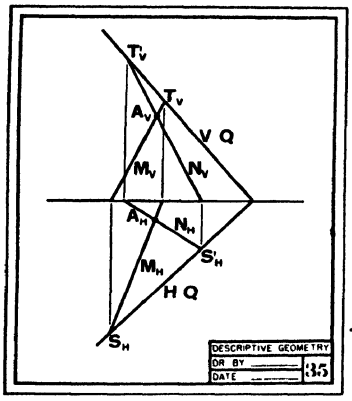


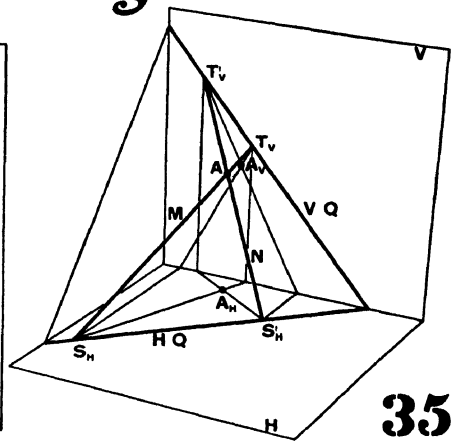


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THE PRINCIPAL LINES IN A PLANE ARE PARALLEL TO ITS TRACES.

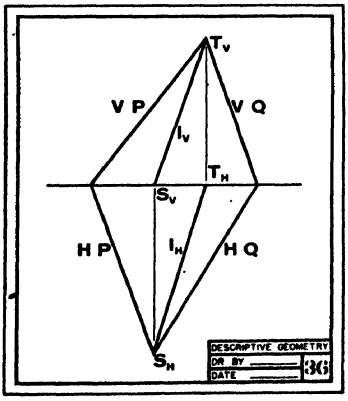
Plane Determined By 2 Intersecting Lines

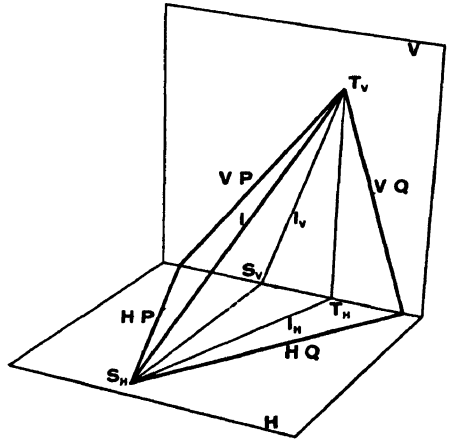




THE PLANE THROUGH 2 INTERSECTING LINES MAY BE DETERMINED BY ITS TRACES WHICH PASS THROUGH THE TRACES OF THE LINES.

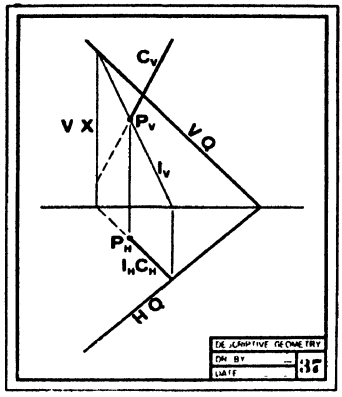
Intersection of 2 Planes By Traces

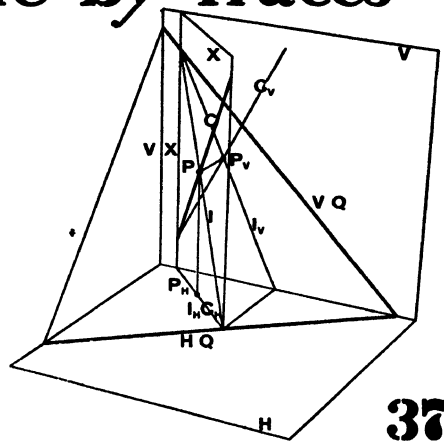




THE INTERSECTION OF 2 PLANES MAY BE DETERMINED BY THE 2 POINTS AT WHICH THEIR TRACES INTERSECT.

Intersection of a Line & a Plane by Traces



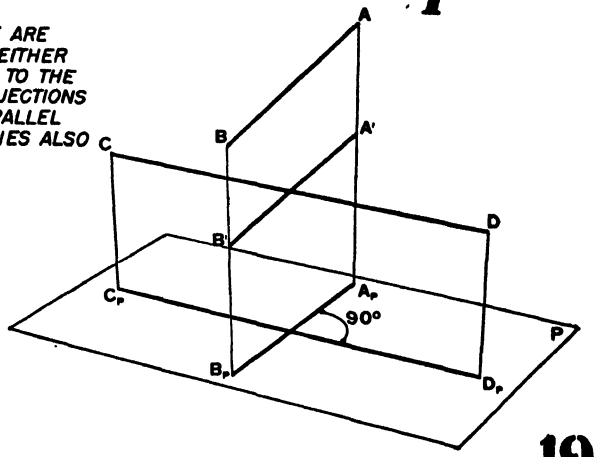


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TO OBTAIN THE INTERSECTION OF A LINE & A PLANE USE A CUTTING PLANE THROUGH THE LINE ⊥ TO EITHER PROJECTION PLANE.

Perpendicular Relationship of Lines in Space

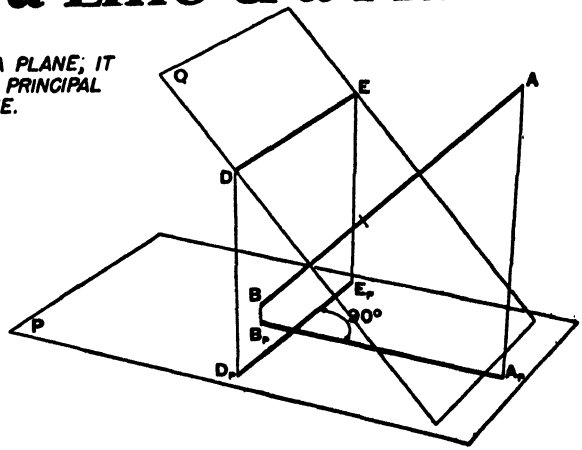
2 LINES IN SPACE ARE CONSIDERED ⊥ IF EITHER LIES IN A PLANE ⊥ TO THE OTHER. THEIR PROJECTIONS ON ANY PLANE PARALLEL TO ONE OF THE LINES ALSO WILL BE ⊥.



GIVEN AB OR A'B' ⊥ CD & P PARALLEL TO CD THEN A_PB_P WILL BE ⊥ C_PD_P. **19**

Perpendicular Relationship of a Line & a Plane

IF A LINE IS ⊥ TO A PLANE; IT WILL PROJECT ⊥ TO PRINCIPAL LINES IN THE PLANE.



GIVEN AB ⊥ Q, ED IN Q PARALLEL TO P. THEN A_PB_P IS ⊥ TO E_PD_P. **20**

CHAPTER III

On the Systematic Solution of Problems

49. Analysis

A geometrical problem requires that, from certain given geometrical elements, one find other elements that satisfy specified conditions. The systematic solution of a problem consists of two parts—the *analysis* and the *construction*. The *analysis* is a statement of the geometrical operations which are required to effect the solution. The *construction* is the graphical representation, by the method of orthographic projection, of the space operations described in the analysis.

The graphical solutions of certain basic problems of space geometry are considered the fundamental problems of descriptive geometry. These problems involve the postulates of construction on which all graphical methods rest. Any space problem capable of solution can be solved by analyzing it in terms of the fundamental problems and carrying out the constructions indicated.

The analysis of a problem may be likened to building a model of the problem, the construction elements used being those described in the fundamental problems. In the analysis, one is concerned with the operations in *space* which lead to the solution rather than with the orthographic projections. This systematic attack on a problem is valuable in that it employs and develops the type of spatial visualization that is useful in the invention and design of mechanical arrangements.

Also, it may be noted that the process of analyzing a problem in terms of fundamental concepts and proceeding logically therefrom illustrates a pattern of thinking applicable to problems in any field.

Many problems of descriptive geometry are variations of one of the fundamental problems. The method of solution for these cases is clear once the fundamental problem involved is recognized.

Very often, however, problems are of a more complex nature, and the method of application of the fundamental problems may be far from obvious. In this type of problem, the discovery of an analysis becomes the first and most difficult part of the solution.

Since problems vary greatly in their nature, the method of making an analysis cannot be set forth in exact terms. There are certain procedures, however, that one can follow which generally will yield a result.

First, imagine the problem solved, and make a sketch showing the given and required elements. Then, study the relations between the parts of the problem with the help of the fundamental problems to find the key to the solution.

If this attempt is not successful, auxiliary geometrical elements may be introduced in order to establish new relations which may suggest the solution.

In this study, the possibility of using loci or new simplifying views must be considered.

50. Loci

Geometrical loci are often useful in devising a solution for a problem. A *locus* is a geometrical element which contains every point that satisfies a given condition and no other points. Thus, the locus of a point equally distant from two given points is a plane which passes through the mid-point of the line segment joining the given points and is also perpendicular to the line.

When the element required in a problem is a point or line, it may be determined by the intersection of loci; for example: To find a point equidistant from four given points A, B, C, and D, which do not lie in a plane.

The locus of a point equidistant from A and B is a plane P perpendicular to line AB at its mid-point. The locus of a point equidistant from B and C is a plane Q perpendicular to line BC at its mid-point. The locus of a point equidistant from C and D is a plane R perpendicular to line CD at its mid-point. The intersection of the three loci P, Q, and R determines the point O which is equidistant from points A, B, C, and D.

51. Change of projection planes

The introduction of auxiliary projection planes provides another method of attack on a problem. A general case may often be reduced to a special case having a simple solution by choosing suitable auxiliary views. The auxiliary-view method of solution is especially useful when the given data lies in an unfavorable position with respect to the projection planes. For example, the true distance between two parallel straight lines which are inclined to the H- and V-projection planes can be measured readily in the auxiliary view in which the two lines project as points. This view is obtained by the method of Problem 4. Solutions based on the use of auxiliary projection planes will employ either Problem 4 or Problem 6 of the fundamental problems.

A problem may be capable of several different analyses. For practical purposes, the analysis that yields the simplest construction is usually

the best solution since greater precision can be expected from a simple construction than from an elaborate one.

52. Examples

A few examples will serve to illustrate the principles described above.

Example 1. *To find a point on a line k which is equidistant from two given points A and B, not on k.*

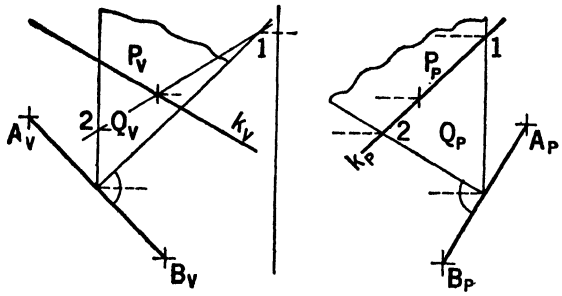


Fig. 96

Analysis: The locus of a point equidistant from points A and B is a plane Q which is perpendicular to the line AB at its mid-point. The point P in which line k cuts this plane is the required point.

1. Draw line AB. (Problem 2.)
2. Locate point M, the mid-point of AB. (Problem 1.)

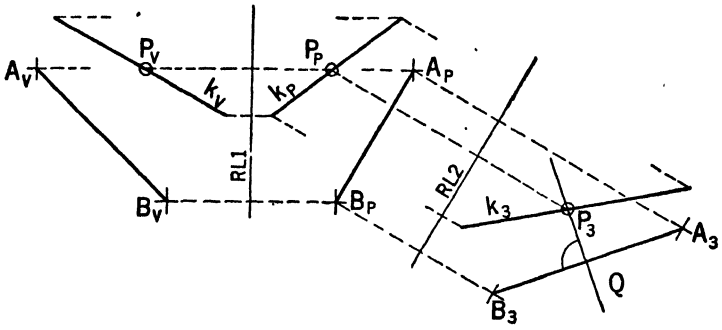


Fig. 97

3. Pass plane Q through point M and perpendicular to AB. (Problem 10.)
4. Find point P in which line k pierces plane Q. (Problem 7.)

First Construction (*Figure 96*).

Second Construction (*Figure 97*): The auxiliary projection plane 3 is taken parallel to line AB. In this view, line AB projects in true length,

and the plane Q perpendicular to AB at the mid-point M, is edgewise. Line k intersects plane Q at P_3, a projection of the required point.

Example 2. *To find the shortest distance between a straight line and a circle.*

Let the line k and the circle C be given.

Analysis: Construct a view of the line and circle in which the line projects as a point. (Problem 4.)

The shortest line R between the point view of the line and the curve is the required distance.

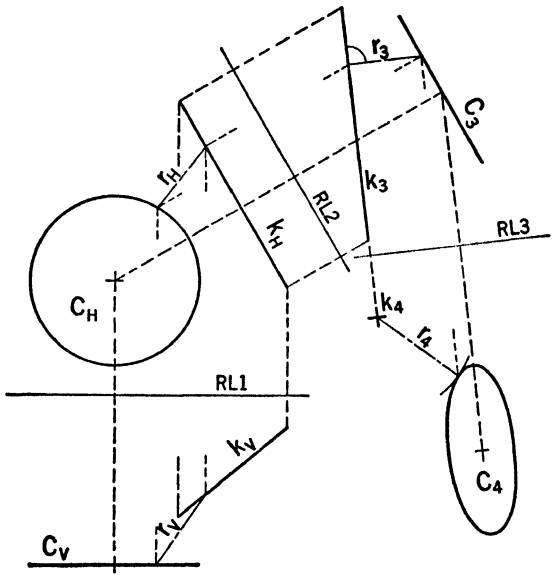


Fig. 98

Construction (*Figure 98*): The radius of the arc centered on k_4 and tangent to the curve C_4 is the shortest distance between line k and circle C.

Example 3. Pass a plane through line t and perpendicular to plane ABC.

Analysis:

1. Assume any point P in line t. (Problem 1.)
2. Draw line m through P and perpendicular to plane ABC. (Problem 9.)
3. Pass a plane through lines t and m. (Problem 5.)

Construction (*Figure 99*).

Example 4. *To draw a line through point P and intersecting the skew lines b and c.*

Analysis: The locus of every line passing through P and intersecting line b is a plane. The point in which this plane cuts line c is the second point which determines the required line.

1. Assume point D on line b.
2. Pass plane Q through lines PD and b.
3. Find point E in which plane Q cuts line c.
4. Draw line PE. PE is the required line.

Construction (*Figure 100*).

Note: If P is a point of a given straight line not intersecting lines b and c, the above construction provides a method for drawing a straight line to intersect three given straight lines.

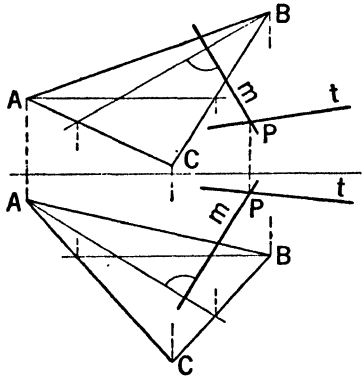


Fig. 99

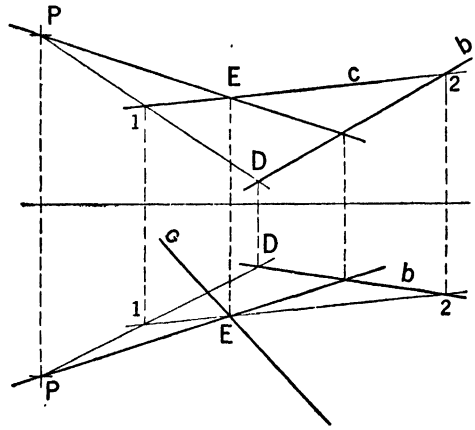


Fig. 100

Example 5. *To draw a line parallel to plane ab and intersecting lines d and e.*

Analysis: The required line lies in a plane parallel to plane *ab* and is determined by the points in which this plane cuts lines d and e. It is evident that an infinite number of lines can be drawn which will satisfy the conditions. Therefore, any point may be assumed on d and a line drawn through it parallel to plane *ab* and cutting line e.

1. Assume point P on line d.
2. Through point P draw lines m and n parallel to lines a and b respectively.
3. Pass plane Q through lines m and n.
4. Find point R in which plane Q cuts line e.
5. Draw line PR.

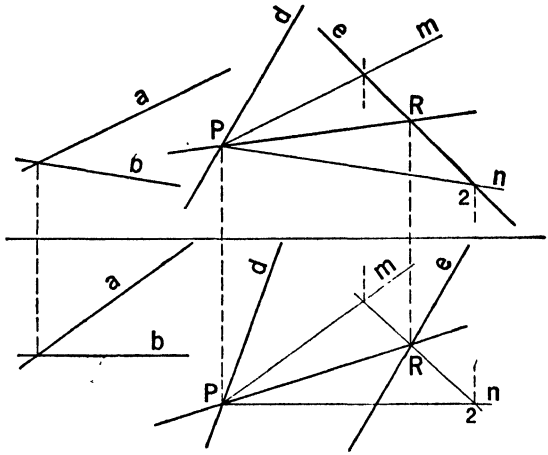


Fig. 101

The aggregate of all lines intersecting d and e and parallel to plane ab is a curved surface called the *hyperbolic paraboloid.*

Construction (*Figure 101*).

Example 6. *To construct a parallelepiped having been given the directions of three concurrent edges and the body diagonal passing through the given corner.*

Let the lines r, s, and t, intersecting in point A, be given (Figure 102). The body diagonal is line AK.

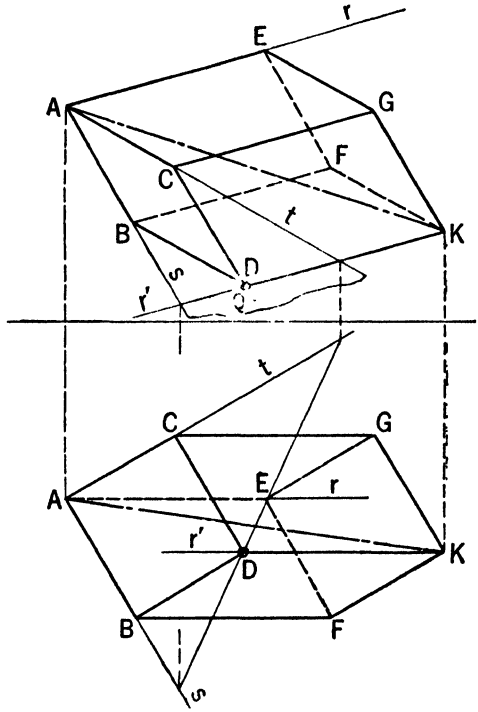


Fig. 102

Analysis:

1. Pass a plane Q through lines s and t.
2. Through point K draw line r' parallel to line r.
3. Find point D in which r' intersects plane Q.
4. Draw lines from D parallel to s and t, establishing face $ABCD$.
5. Draw from C parallel to r and from K parallel to s to establish point G.

6. Draw from B parallel to r and from G and K parallel to t, establishing F and E.
7. Draw EF completing the parallelepiped.

Construction (*Figure 102*).

PROBLEMS ON CHAPTER III

GROUP A. WRITE ANALYSES AND CARRY OUT CONSTRUCTIONS FOR THE FOLLOWING PROBLEMS

3-A1. Through a given point P draw line k which intersects the skew lines c and d. Point P:$\frac{1}{2}$,1,7$\frac{1}{4}$. Line c:2$\frac{1}{2}$,$\frac{1}{2}$,6; 6,4$\frac{3}{4}$,8. Line d:4$\frac{3}{4}$,$\frac{1}{2}$,8$\frac{1}{2}$; 7$\frac{3}{4}$,3,5.

3-A2. Construct line q parallel to line m and intersecting the skew lines s and t. Line m:$\frac{3}{4}$,1,9; 4$\frac{1}{2}$,$\frac{1}{4}$,9$\frac{3}{4}$. Line s:$\frac{1}{2}$,1$\frac{1}{2}$,5; 3,3$\frac{3}{4}$,7$\frac{1}{4}$. Line t:6,$\frac{1}{2}$,7$\frac{1}{2}$; 7$\frac{1}{2}$,4$\frac{1}{4}$,5.

3-A3. Through the given skew lines f and g pass two parallel planes, Q and R. Line f:1$\frac{1}{2}$,1,5$\frac{1}{4}$; 3,4$\frac{1}{4}$,6$\frac{3}{4}$. Line g:4$\frac{1}{4}$,4$\frac{1}{2}$,7; 5$\frac{1}{4}$,3,5$\frac{1}{4}$. Find the lines in which planes Q and R cut the H- and V-projection planes. Origin: 055.

3-A4. Pass plane Q through point E and parallel to plane W. Point E:4$\frac{1}{2}$,3$\frac{1}{4}$,7. Plane W:$\frac{1}{4}$,2,7$\frac{1}{4}$; 1$\frac{1}{4}$,4$\frac{1}{2}$,5$\frac{3}{4}$; 2$\frac{1}{2}$,3,9. Find the lines in which plane Q cuts the H- and V-projection planes. Origin: 055.

3-A5. Through point P draw line k which intersects line d and is parallel to plane Q. Point P:718. Line d:329; 645. Plane Q:047; 219; 337.

3-A6. Find the locus of a point which is equidistant from points A:619 B:857. Find the intersections of the locus with H- and V-projection planes. Origin: 055.

3-A7. Find the locus of a point which is equidistant from points A:146 B:329 C:768.

3-A8. Find four equidistant parallel planes which contain the points A, B, C, and D in that order. A:2,4$\frac{1}{4}$,7 B:4$\frac{3}{4}$,2$\frac{1}{2}$,7$\frac{1}{2}$ C:5$\frac{3}{4}$,3$\frac{1}{4}$,5$\frac{1}{4}$ D:6$\frac{1}{2}$,3$\frac{1}{2}$,5$\frac{1}{4}$.

3-A9. Three non-intersecting edges of a parallelepiped lie along the given lines q, r, and s. Draw the top and front views of the parallelepiped. Line q:247; 538. Line r:457; 625. Line s:318; 736.

3-A10. Project the line segment AB on the triangle PQR. A:245 B:456 P:217 Q:549 R:626.

3-A11. Through the given line m pass a plane P perpendicular to the given plane Q. Line m:4$\frac{1}{2}$,3$\frac{3}{4}$,7$\frac{1}{4}$; 629. Plane Q:5$\frac{1}{2}$,5,5; 425; 7$\frac{1}{2}$,5,7$\frac{1}{2}$.

3-A12. Through point C draw line k perpendicular to and intersecting line t. Point C: 1,2¾,6. Line t: 4,4½,9½; 7,2,5½.

3-A13. Pass a sphere through the given points A, B, C, and D. A: 2,2½,6¼ B: 3½,3½,5½ C: 4½,2½,7¾ D: 5,1,6¼.

3-A14. Pass a sphere through points A: ¼,1½,5½ B: 1,2¼,6¼ C: 2,1,6¼ D: 2½,2,5.

GROUP B. WRITE ANALYSES OF THE FOLLOWING PROBLEMS. CONSTRUCTIONS MAY BE CARRIED OUT BY ASSUMING SUITABLE DATA

3-B1. Pass a plane through a given point and equally distant from three given points.

3-B2. Determine a plane which is equally distant from four given points.

3-B3. Draw a line cutting three given skew lines in such a manner that the points of intersection determine two equal segments.

3-B4. Draw a line which is symmetrical to a given line, the symmetry being with respect to a given plane.

3-B5. On a given line find a point equidistant from two given points.

3-B6. Given a dihedral angle and a line lying in one face. Pass a plane through the line which cuts the dihedral angle in two lines which are perpendicular.

3-B7. Given four points A, B, C, D, not lying in the same plane. Find the point on the circle through A, B, and C which is nearest point D.

3-B8. Find an auxiliary projection plane on which two given skew lines will project parallel.

CHAPTER IV

Graphical Computation

53. The tetrahedron

Problems which concern the determination of linear and angular measurements are known as *metrical problems.*

In plane geometry, the measurement of a plane figure is reduced by a system of triangulation to the solution of a triangle, a rigid figure. There are three problems on the triangle: (1) to find the length of a line; (2) to find the angle between two lines; (3) to find the distance from a point to a line.

In space geometry, the tetrahedron, a rigid figure, is analogous to the triangle of plane geometry, and the measurement of a space figure is reduced to the measurement of a tetrahedron. There are seven metrical problems connected with the tetrahedron: (1) to find the length of a line; (2) to find the angle between two lines; (3) to find the angle between two planes; (4) to find the angle between a line and a plane; (5) to find the distance from a point to a plane (an altitude of the tetrahedron); (6) to find the distance from a point to a line (slant height of a face); (7) to find the distance between two skew lines (two non-intersecting edges of the tetrahedron).

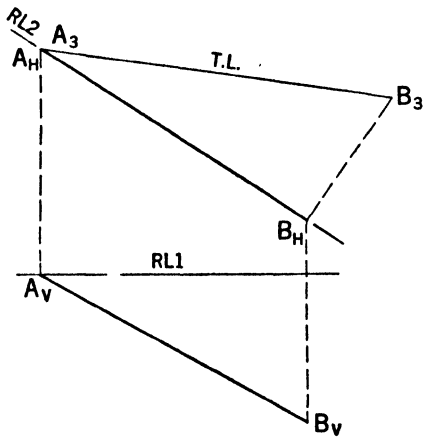


Fig. 103

The graphical solutions of these problems are obtained by application of the fundamental problems developed in Chapter II.

54. To find the length of a line

First Analysis: Let AB be the given line segment. Construct the normal view of AB by the method of Fundamental Problem 4 (Article 23).

First Construction (Figure 103): The construction is simplified by taking $RL1$ through A_V and $RL2$ coinciding with A_HB_H. The normal view A_3B_3 is the true length of line AB.

55. To find the angle between two lines

First Analysis: Let the given lines be g and k. Construct the normal view of the plane figure gk by the method of Fundamental Problem 6 (Article 27).

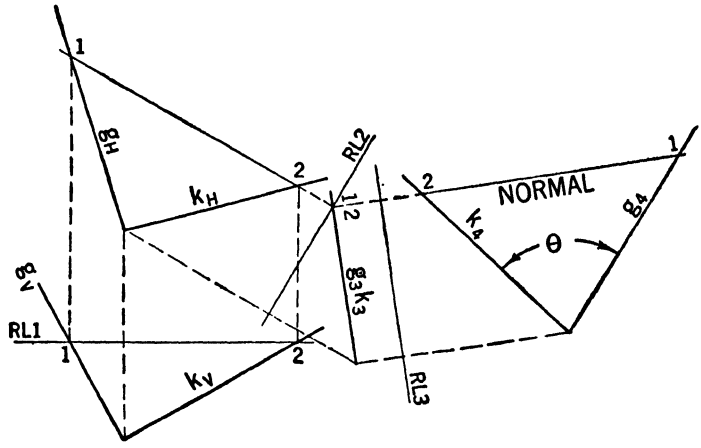


Fig. 104

First Construction (Figure 104): The true angle between lines g and k, is measured between g_4 and k_4.

56. Rotation

An alternate method for determining normal views of oblique lines or planes is to rotate the line or plane into parallelism with one of the projection planes. The method of rotation provides a simple, direct solution for the two preceding problems.

57. The principle of rotation

The fundamental construction of the method of rotation consists in rotating a space point into a projection plane about an axis which lies in that plane. A point rotated about an axis moves at a constant distance from the axis. The path is a circle whose plane is perpendicular to the axis.

58. The rotation of a point about a vertical axis (Figure 105)

Point P, rotated about axis AB, moves in a horizontal circle of radius PC. Points P' and P'' represent the two possible projections of P when it is rotated into the frontal plane containing AB.

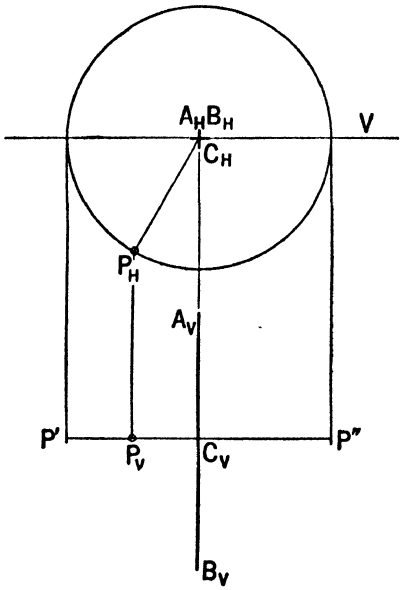


Fig. 105

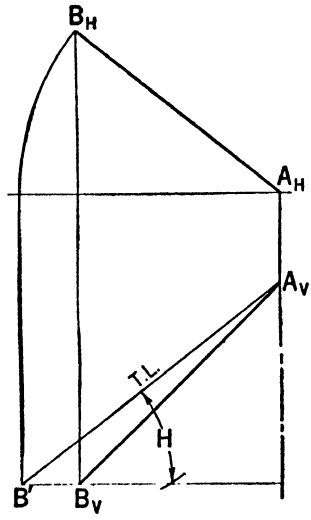


Fig. 106

59. To find the true length of a line

Second Analysis: An oblique line projects in its true length when rotated into parallelism with the vertical or horizontal plane about an axis through one point of the line.

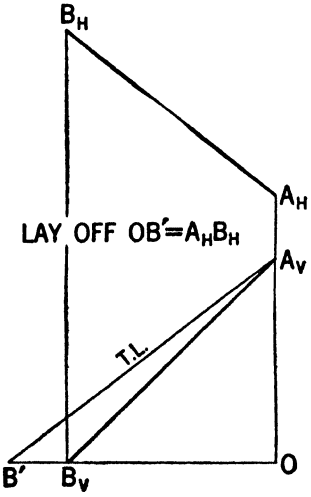


Fig. 107

Second Construction:

Example 1 (Figure 106). *To find the true length of line AB.* Rotate point *B* about a vertical axis through *A* until *AB* becomes parallel to the vertical plane. The revolved front view (A_VB') is the true-length view. The angle *H* is the true angle between *AB* and the horizontal plane.

This construction for true length is usually simplified by omitting the revolved top view (Figure 107) and setting off the distance OB' equal to A_HB_H.

Example 2 (Figure 108). The true length of line *AB* may also be obtained by assuming the axis through point *A* and perpendicular to the vertical plane. Line *AB* is rotated parallel to the horizontal plane and projects in true length in

the top view. The angle V is the true angle between AB and the vertical plane.

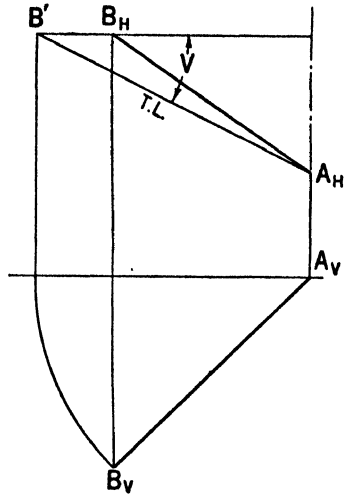


Fig. 108

60. To find the true shape of a plane figure by triangulation

To find the true shape of triangle ABC (*Figure 109*).

Find the true lengths of lines AB, BC, and AC by method of Problem 4. Construct the triangle ABC (Figure 110) using the lengths found for the three sides. This triangle is congruent to the space triangle.

The true shape of any polygon can be found by dividing the figure into triangles and then laying off these triangles in true size. The process of triangulation has important applications in the layout and development of surfaces.

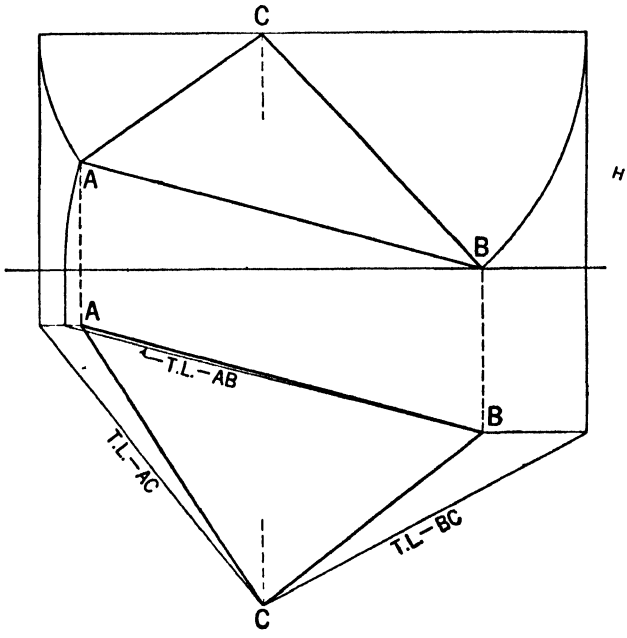


Fig. 109

61. Rotation of a point about a horizontal axis (Figure 111)

Given: Point P and the horizontal line AB. To rotate P about AB as an axis until it lies in the horizontal plane which contains AB.

The path of rotation is a circle. In the top view, this circle projects as a straight line perpendicular to A_HB_H, with its center at C_H. The circle projects in true shape in an auxiliary view in which the axis AB projects as a point. Take $RL1$ through A_VB_V and $RL2$ at right angles

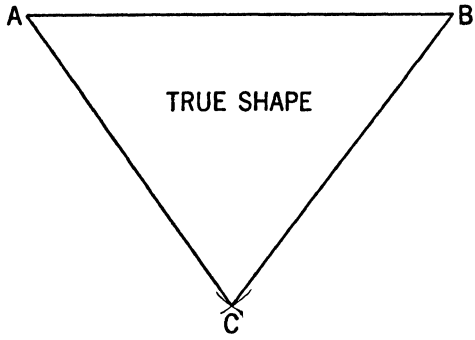


Fig. 110

to A_HB_H. Project P and AB to the auxiliary view obtaining P_3 and A_3B_3. In this view, point P rotating about AB describes a circle of radius P_3C_3. Point P lies in the horizontal plane containing AB at the

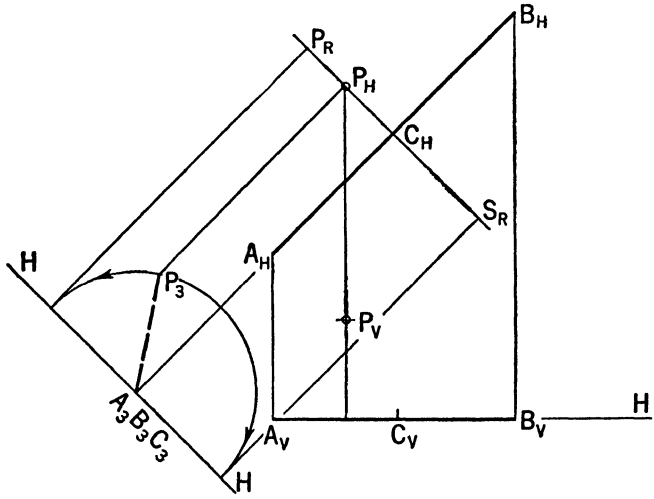


Fig. 111

points in which $RL2$ cuts the circle. The top views of these two points are P_R and S_R.

The construction of Figure 111 can be made more compact by taking $RL2$ perpendicular to A_HB_H and through P_H (Figure 112). The auxiliary view is constructed as before.

A point can be rotated about a frontal line as an axis in a similar fashion. In this case, the circle which represents the path of rotation will project as a straight line in the front view.

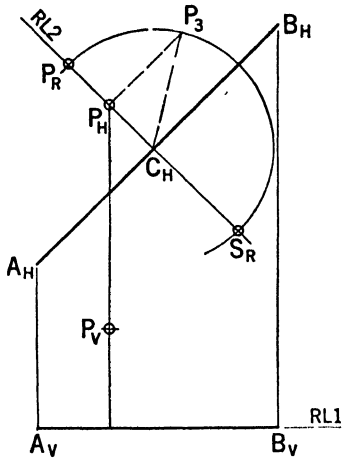


Fig. 112

62. To find the true shape of a plane figure by the method of rotation

Analysis: A plane figure appears in its true shape when rotated into or parallel to a projection plane. The axis about which the figure is rotated must be contained in the plane and be either in the projection plane or parallel to it.

First Construction (Figure 113): To find the true shape of the triangle ABC. Draw the projections of the horizontal line AD. Take $RL1$ through A_VD_V, and $RL2$ perpendicular to A_HD_H. The auxiliary view $A_3B_3C_3$ is an edge view of the triangle. Rotate points B and C about AD as an axis by the method of Figure 111, obtaining B_R and C_R. The rotated view $A_HB_RC_R$ is the true shape of ABC.

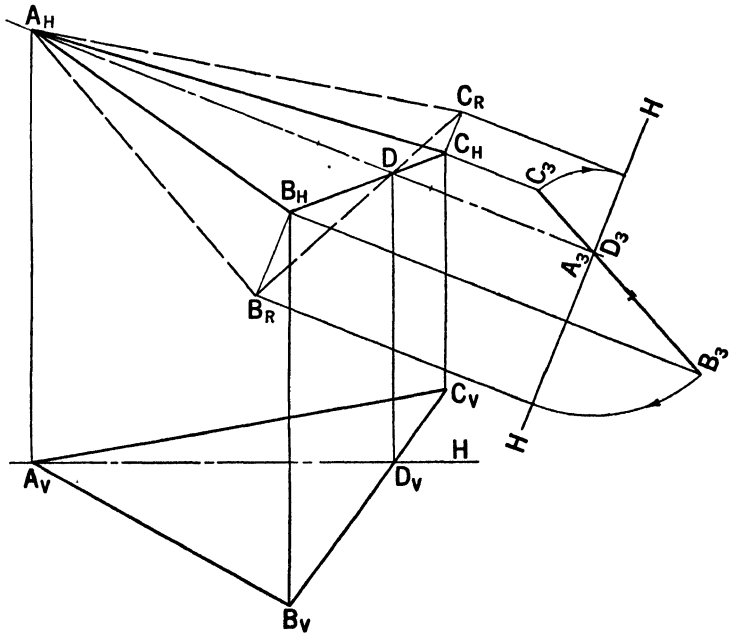


Fig. 113

Second Construction (Figure 114): To find the true shape of triangle ABC. The triangle is to be rotated into the V-plane about its trace on the V-plane. Take RL through B_H. Produce line CA to intersect the V-plane at point T. The line drawn through B_V and T_V represents the intersection of triangle ABC and the V-plane. Rotate point C about B_VT_V by the method of Figure 112, obtaining C_R. Draw B_VC_R and T_VC_R. Find A_R by the line which represents its path of rotation perpendicular to B_VT_V. The rotated figure $A_RB_VC_R$ is the true shape of ABC. This construction avoids the overlapping views of the preceding example.

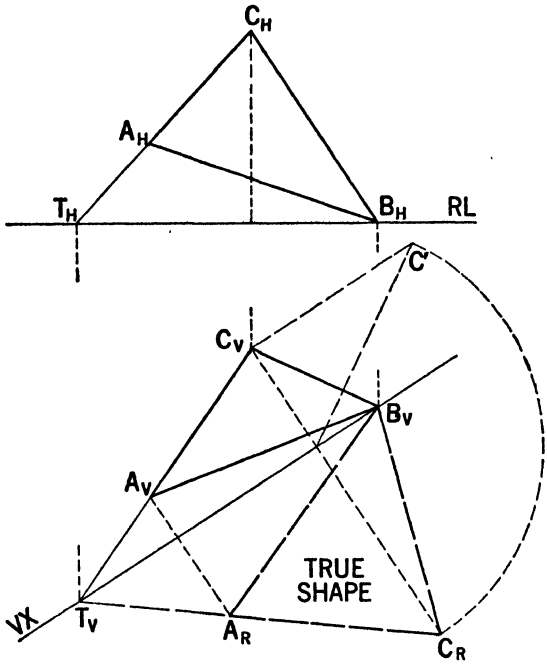


Fig. 114

63. To find the angle between two lines (rotation method)

Analysis: To find the true angle between lines AB and BC (Figure 115). Draw a horizontal line AD in the plane ABC and rotate the given lines into a horizontal plane about AD as an axis. The true size of the angle ABC is measured in the rotated view.

Construction (Figure 115): Point B is rotated about AD to B_R by the construction of Figure 112. The segments AB and BD rotate to A_HB_R and D_HB_R respectively. The true angle is θ.

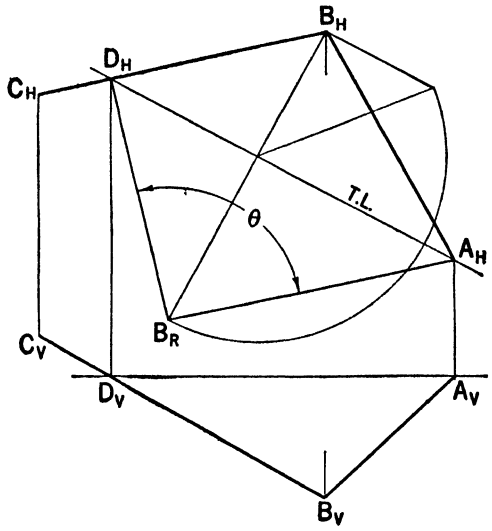


Fig. 115

64. To find the angle between two planes

Let it be required to find the angle between planes VAB and VAC.

First Analysis: The true size of the dihedral angle between two planes appears in a view in which the line common to the two planes projects as a point. Construct the view of $VABC$ in which line VA projects as a point.

First Construction (Figure 116): The true angle between planes VAB and VAC is equal to $B_3A_3C_3$.

Second Analysis: The dihedral angle between two planes is measured by the plane angle cut from the two planes by a third plane passed perpendicular to their line of intersection.

Let it be required to find the angle between planes VAB and VAC (Figure 117).

1. Assume point P on line VA.

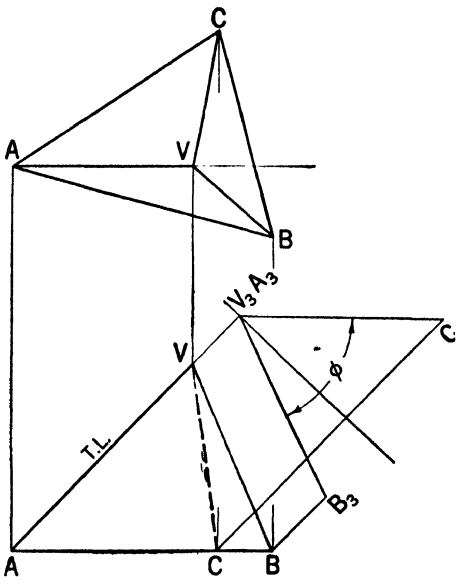


Fig. 116

2. Pass plane X through P and perpendicular to VA.

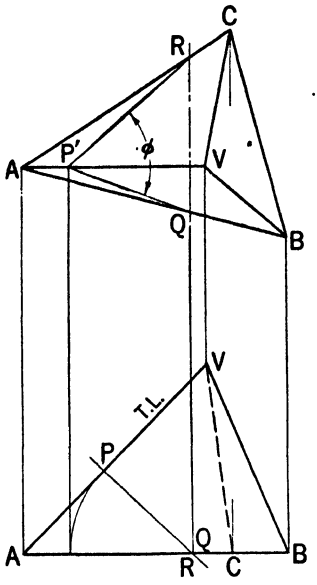


Fig. 117

3. Draw lines PQ and PR in which plane X cuts planes VAB and VAC respectively.

4. Find true size of QPR.

Second Construction (Figure 117): The plane angle QPR is rotated about the horizontal line QR so as to bring point P into the horizontal plane ABC. The required dihedral angle is measured by $QP'R$.

65. Solution of a general case using both first and second analyses

Construction (Figure 118): To find the true size of the dihedral angle between two adjacent sloping faces of the frustum of a square pyramid.

Pass plane Z perpendicular to the line of intersection of two adjacent faces. Plane Z appears edgewise in the view in which the intersection projects in its true length and cuts the plane angle WSX from the given faces. The true size of this angle can be found by rotating it into a horizontal plane $W_{H}S_{R}X_{H}$ or by constructing the normal view $W_4S_4X_4$ by auxiliary projection.

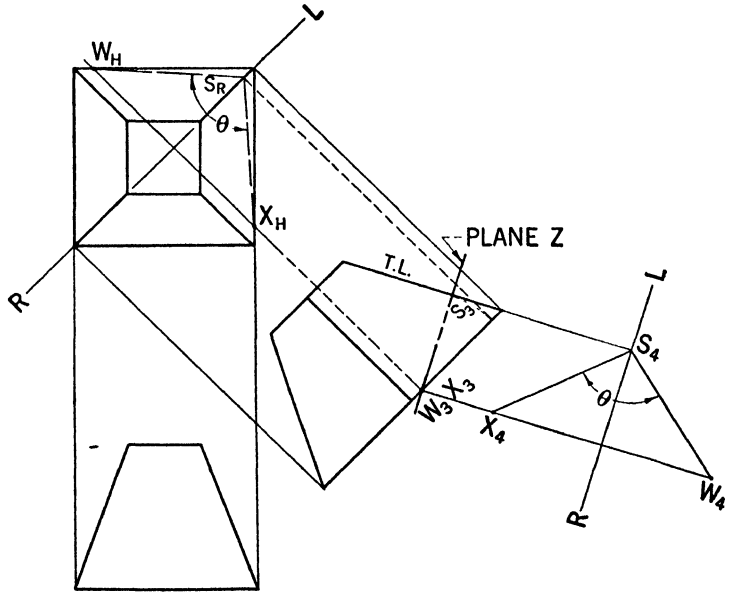


Fig. 118

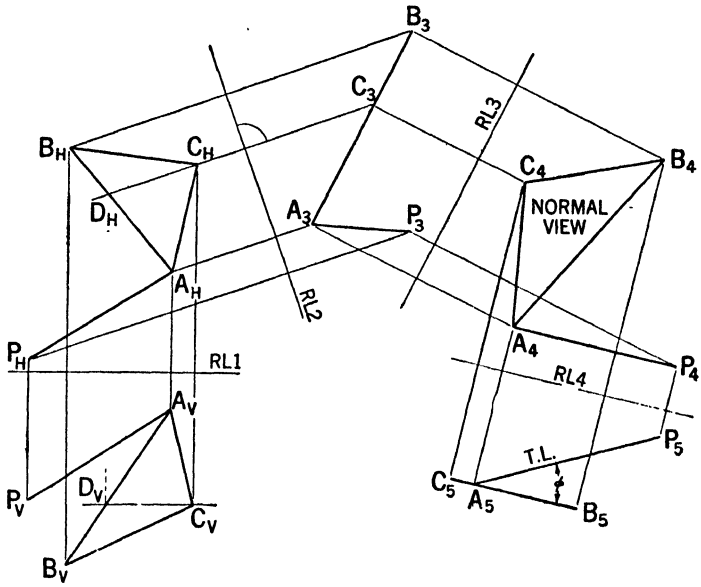


Fig. 119

66. To find the angle between a line and plane

Analysis: The angle between a line and a plane is defined as the angle between the line and its projection on the plane. The true angle between a line and plane appears in the view in which the plane projects edgewise and the line projects in true length.

Construction (Figure 119): Auxiliary View Method. Let it be required to find the angle between plane ABC and line AP.

Draw the horizontal CD in plane ABC. $RL2$ is taken perpendicular to C_HD_H. View 3 shows ABC edgewise and line AP obliquely. $RL3$ is taken parallel to $A_3B_3C_3$. Plane ABC projects in true shape in View 4. $RL4$ is taken parallel to A_4P_4. View 5 shows ABC edgewise and line AP in true length. The angle ϕ is the true angle between AP and plane ABC.

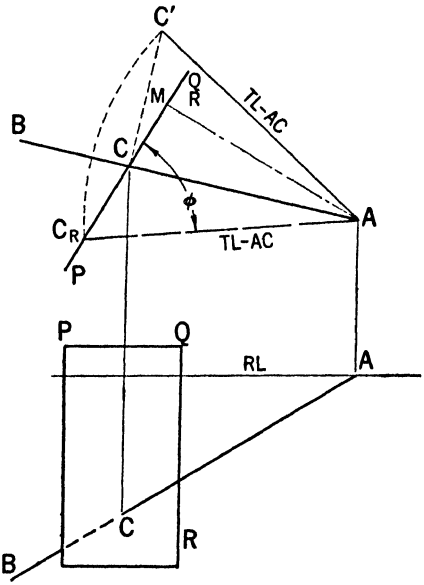


Fig. 120

Construction (Figure 120): Rotation Method. To find the angle between line AB and plane PQR.

Find point C in which line AB pierces plane PQR. Draw the line AM from point A and perpendicular to plane PQR. Rotate the triangle AMC about AM as an axis until AC appears in true length. This operation is readily accomplished by finding the TL of AC $(=AC')$ and laying off $AC_R = AC'$. The required angle is AC_RM.

67. To find the angle between an oblique plane and either reference plane

Let it be required to find the angle between plane Q and the H- and V-planes (Figures 121 and 122).

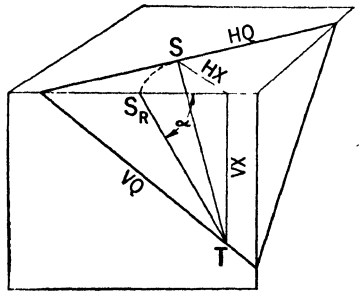


Fig. 121

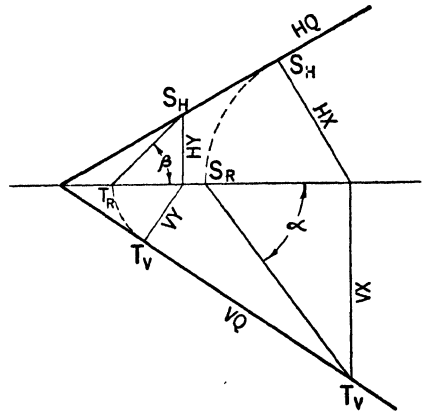


Fig. 122

Analysis: The angle between any two planes is measured in a plane perpendicular to their line of intersection. Planes H and Q intersect in line HQ. Assume any auxiliary plane X, perpendicular to HQ. The portion of plane X included between planes H, V, and Q is a right triangle having the sides HX, VX, and ST. The angle between lines HX and ST is the angle between planes H and Q. The true size of this angle is α and is found by rotating the triangle into the V-plane about VX as an axis.

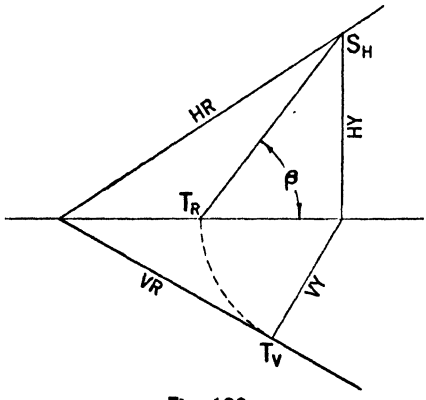


Fig. 123

The angle between planes Q and V is measured on a plane Y, perpendicular to VQ. The portion of plane Y included between planes H, V, and Q forms a right triangle, having the line ST as its hypotenuse. The angle between planes V and Q is β and appears in true size when the triangle is rotated into the H-plane about HX as an axis.

The above construction can be reversed to solve the following prob-

lem: Given one trace of a plane and the angle that the plane makes with either projection plane; to find the other trace.

Let the trace VR (Figure 123) and the angle β between planes R and V be given. Pass plane Y perpendicular to VR. Rotate plane Y into the H-plane about HY as an axis. Point T, the known vertex of the triangle to be formed by planes R, H, and V, rotates to T_R on the reference line. Through T_R draw line $T_R S_H$, making the angle β with the reference line. Draw the trace HR to pass through point S_H.

68. To find the angle between two planes (Planes given by their traces)

Analysis: The dihedral angle between two planes is equal to the angle between their respective normals. Through any assumed point of space draw a perpendicular to each plane. Determine the angle between these lines.

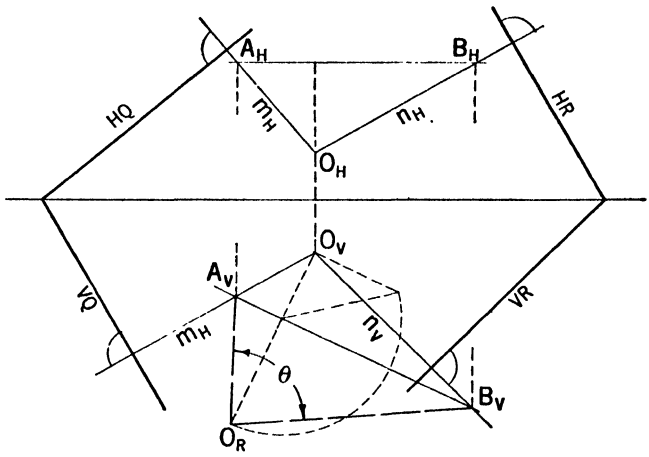


Fig. 124

Example (Figure 124). *To find the angle between planes Q and R.* Assume the projection of a point of space O. Through point O draw line m perpendicular to plane Q, and line n perpendicular to plane R (Problem 9). Draw a frontal line AB of the plane mn. Find the true size of triangle AOB. Angle θ of the rotated triangle $A_V O_R B_V$ is equal to the angle between planes Q and R.

69. To determine the angle between a line and a plane (Plane given by its traces)

Analysis: The angle between a line and a plane S is defined as the angle between the line and its projection on the plane. It is there-

fore the complement of the angle between the line and a normal to the plane.

Example (Figure 125). *To find the angle between plane S and line b.* From any point O on line b draw a normal m to plane S. Draw the frontal line AB to form the triangle AOB. Construct $A_VO_RB_V$, the true shape of triangle AOB. The complement of AOB is found graphically. The angle between line b and plane S is ϕ.

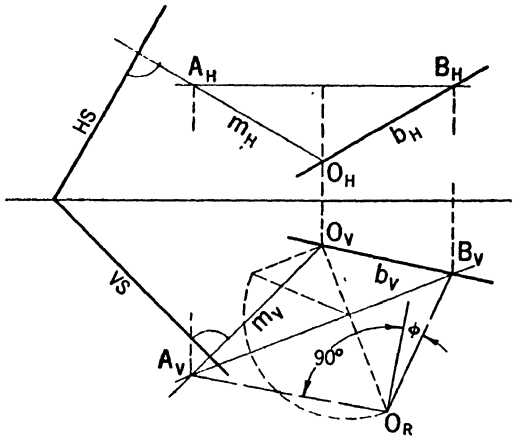


Fig. 125

70. To find the distance from a point to a line

The distance from a point to a line is the length of the perpendicular from the point to the line.

First Analysis: Draw a view of the line and point in which the line projects in its true length. Construct the perpendicular from the point to the line in this view by the method of Theorem I. Find the true length of the perpendicular.

First Construction (Figure 126): Let P be the given point and k the given line. Draw k_3, the normal view of line k, (Article 23). Project P on the auxiliary plane 3. Draw O_3P_3 perpendicular to k_3. Complete the projections of line OP and find its true length. O_VP_R is the required distance.

Second Analysis: Let the given point and line be P and k respectively.

1. Pass a plane Z through point P and perpendicular to line k.
2. Find the point Q in which line k cuts plane Z.
3. Draw line PQ.
4. Find true length of PQ.

Second Construction (Figure 127).

71. To find the shortest distance between two skew lines

First Analysis: Using the principle of Theorem I, the common perpendicular can be drawn in the view in which one of the given lines projects as a point (Figure 128). The distance between the lines is equal to the true length of UV. A general case, in which the given lines are oblique, can be reduced to this simple case by the method of Problem 4.

Second Analysis: The distance between two skew lines a and b is equal

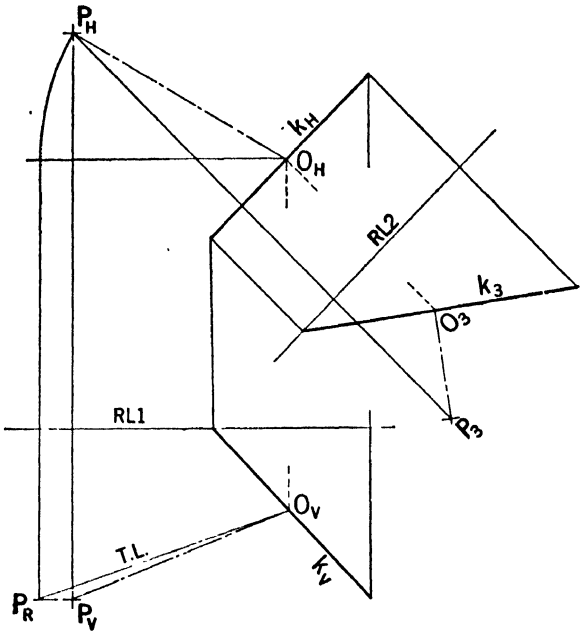


Fig. 126

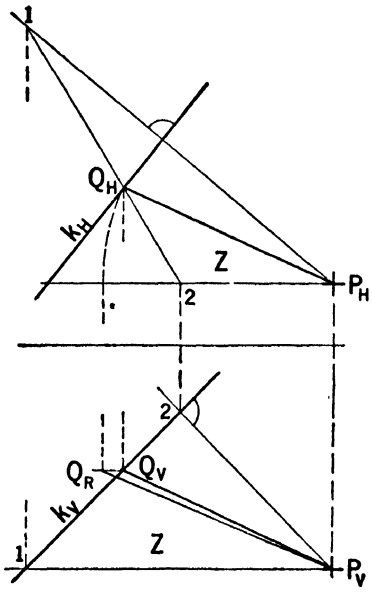


Fig. 127

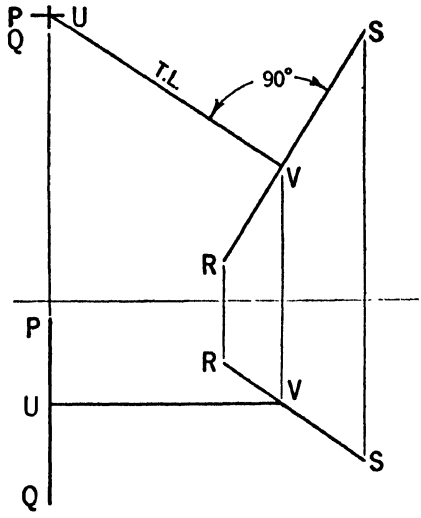


Fig. 128

to the distance between two parallel planes, each of which contains one of the given lines.

1. Assume a point P on line a.
2. Through point P draw line x parallel to line b.
3. Pass a plane through lines a and x.
4. Construct a view in which plane ax projects edgewise.

The shortest distance between lines a and b appears in this view.

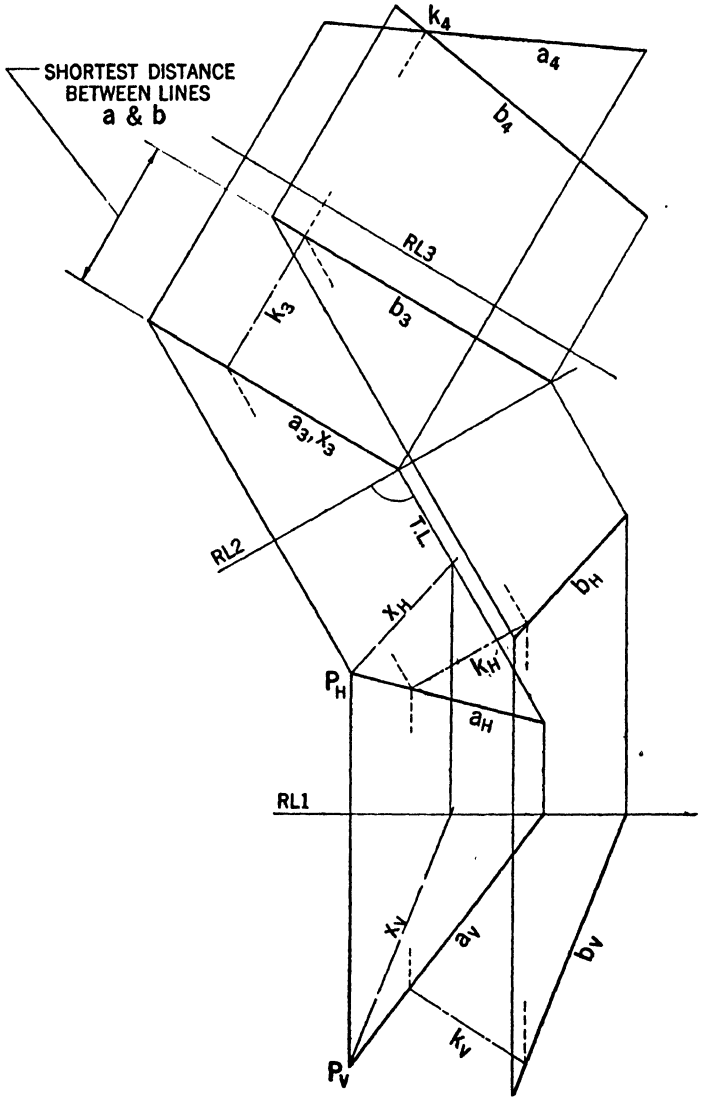


Fig. 129

Second Construction (*Figure 129*): Let a and b represent two skew lines. Through any point of line a draw line x parallel to line b. Pass a plane through lines a and x. Construct a_3x_3, the edge view of plane ax. Project line b on the auxiliary plane 3, determining b_3. In view 3 a plane through line b and parallel to line a appears edgewise, and is also represented by b_3. The distance between the parallel projections a_3 and b_3 is the shortest distance between lines a and b.

The projection of the common perpendicular to lines a and b will project as a point on a projection plane taken parallel to a_3 and b_3. Take $RL3$ parallel to a_3, b_3 and construct a_4, b_4. Lines a and b project in their true lengths in this view. The projection k_4 in which they cross is the end view of the common perpendicular.

The following constructions describe relationships between a line or plane and the projection planes. These constructions are special cases of certain of the preceding metrical problems.

72. To draw the projections of a line making specified angles with the projection planes

To find the line through point O which makes the angle θ with the horizontal plane, and the angle ϕ with the vertical plane.

Draw the horizontal and vertical projections of a right circular cone whose elements make the angle θ with the horizontal plane, assuming any convenient length OA for elements (Figure 130). The problem now is to select the particular element, or elements, of this cone that makes the angle ϕ with the vertical plane. It is evident that the length of the vertical projection of the required element must equal $OA \cos \phi$. This is found graphically by constructing the right triangle AOX, having the angle $AOX = \phi$. Distance OX is the length of the vertical projection of the required element. This length is transferred to the cone by describing arc XB, determining the projection O_VB_V. Then the required line is OB. It will be noted there are four possible solutions to this problem when $(\theta + \phi)$ is less than 90°. If $(\theta + \phi) = 90°$, there are two solutions. There is no solution if $(\theta + \phi)$ is greater than 90°.

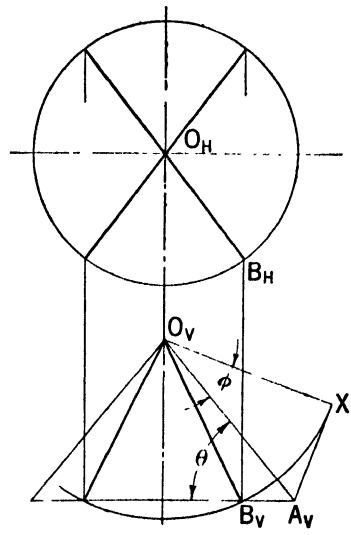


Fig. 130

73. Through a given point to pass a plane which makes specified angles with the principal projection planes

Let it be required to pass a plane through point O which makes the angle θ with a horizontal plane, and the angle ϕ with a vertical plane (Figure 131). The normal to this plane makes an angle of $(90° - \theta)$

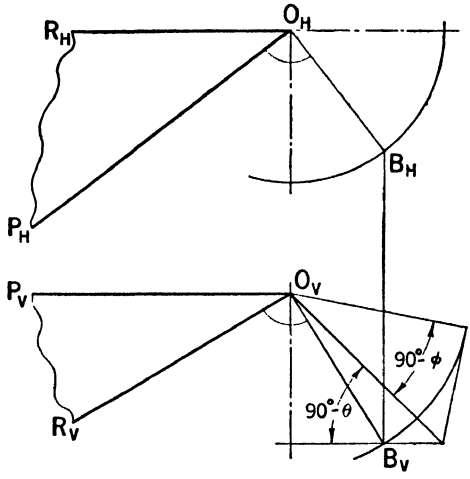


Fig. 131

with the horizontal plane and $(90° - \phi)$ with the vertical plane. Construct this normal OB through point O by the method of Article 47. Pass the plane POR through point O and perpendicular to line OB by method of Problem 10. Plane POR is the required plane. It is evident that values of $(\theta + \phi)$ must lie between 90° and 180°.

74. Bearing and slope of a line

The position of a line is often given in engineering data by its bearing and slope or grade. The bearing of a line is the direction of its horizontal projection with respect to the meridian. It is expressed in degrees measured east or west of the meridian. The slope or grade of a line is its inclination to the horizontal plane. It may be expressed as an angle, a ratio, or a per cent. When given as a ratio, it is the ratio of a horizontal distance to the rise or drop of the line in that distance. The slope in per cent is the rise or fall (in feet) in a horizontal distance of 100 feet (Figure 132).

75. Strike and dip of a plane

Strike and dip are terms used in mining and geology to describe the direction and slope of a plane. The *strike* of a plane is the direction of a

horizontal line of the plane, and is generally given by its bearing. The *dip* of a plane is the angle the plane makes with a horizontal plane and is measured downward (Figure 133). A plane is determined if one of its points and its strike and dip are given.

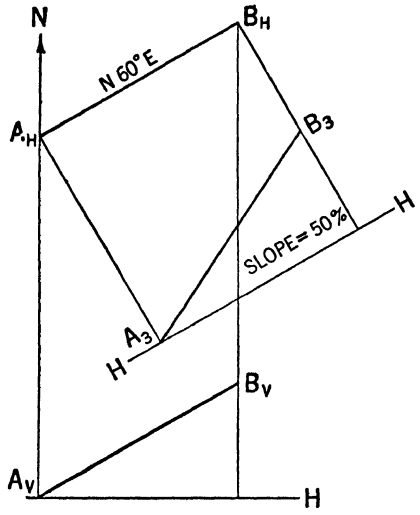


Fig. 132

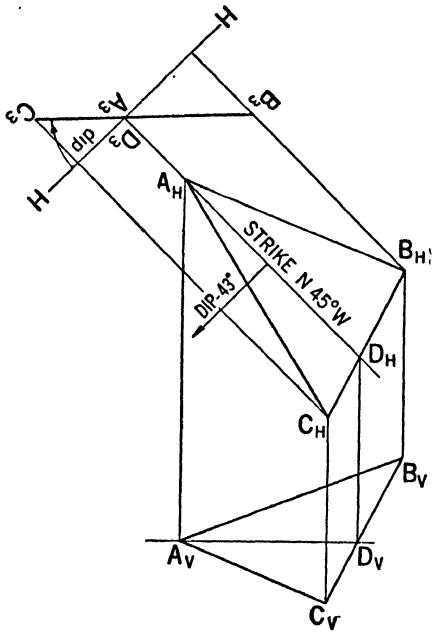


Fig. 133

PROBLEMS ON CHAPTER IV

GROUP A. NORMAL VIEW OF A LINE OR PLANE FIGURE BY ROTATION

4-A1. (*a*) Find the true length of line A:078 B:3,5,10 and the true angle with a horizontal plane through point A. (*b*) Find the true length of line C:678 D:8,5,10 and the true angle it makes with a horizontal plane through point C. (*c*) Find the true length of line E:023 F:304 and the true angle it makes with a frontal plane through point E. (*d*) Find the true length of line G:635 K:803 and the true angle it makes with a frontal plane through point G.

4-A2. Find the true shape of the triangle A:126 B:218 C:434 by the method of triangulation.

4-A3. To execute a plane construction on an oblique plane of space. (*a*) Determine the true shape of the plane quadrilateral C:046 D:1,2,8½ E:4½,4,6¾ F:2,4½,5¾ by the method of triangulation. Lay off the side DE along the right-hand border. (*b*) Construct a 1¼-inch square in

the plane *CDEF*. Two sides of the square are to be parallel to *DE*. One corner of the square is $\frac{1}{2}$ inch from line *DE* and 1 inch from *CD*.

Method: Lay out square in the true-shape view. Produce sides of square to cut sides of the quadrilateral. Locate these intersection points on the top and front views of the quadrilateral by proportional division.

4-A4. Construct a normal view of triangle *P*:416 *Q*:548 *R*:839, using the method of rotation.

4-A5. Rotate triangle *A*:236 *B*:3,1$\frac{1}{2}$,4$\frac{1}{2}$ *C*:545 into the horizontal plane passing through point *C*.

4-A6. Rotate triangle *A*:256 *B*:658 *C*:4,4,5$\frac{1}{2}$ into the frontal plane passing through point *C*. Draw bisectors of interior angles in the normal view, and rotate them back into the front view. Check solution by rotating triangle into the horizontal plane containing line *AB*.

4-A7. Construct a 2$\frac{3}{4}$-inch diameter circle tangent to lines *A*:1$\frac{1}{2}$,3,4$\frac{1}{4}$ *B*:3$\frac{1}{2}$,3,6$\frac{1}{4}$ and *B-C*:6$\frac{1}{4}$,$\frac{3}{4}$,6$\frac{1}{4}$. Find the points of tangency and the major and minor axes of the elliptical views.

GROUP B. ANGLE BETWEEN TWO LINES

4-B1. Find the true size of the angle *A*:1,2$\frac{1}{2}$,5 *B*:4,4$\frac{1}{2}$,7$\frac{1}{2}$ *C*:6,3$\frac{1}{2}$,5.

4-B2. Find the true size of the angle *B*:345 *A*:127 *C*:616.

4-B3. Find the true size of the angle *A*:1$\frac{3}{4}$,2$\frac{1}{4}$,8$\frac{1}{4}$ *B*:3,4,5$\frac{1}{2}$ *C*:6$\frac{1}{4}$,1$\frac{1}{4}$,7$\frac{1}{2}$.

4-B4. Find the true size of the angle *J*:3$\frac{1}{2}$,2$\frac{3}{4}$,6 *K*:$\frac{3}{4}$,$\frac{1}{2}$,6 *L*:4$\frac{3}{4}$,$\frac{1}{2}$,8$\frac{1}{4}$.

4-B5. Find the true size of the angle *P*:4$\frac{1}{2}$,$\frac{1}{2}$,9$\frac{1}{4}$ *Q*:2$\frac{3}{4}$,2$\frac{1}{2}$,8 *R*:5$\frac{3}{4}$,2,6$\frac{1}{4}$.

GROUP C. ANGLE BETWEEN TWO PLANES

4-C1. Find the true size of the dihedral angle between the faces *ACD* and *BCD* of the tetrahedron *A*:036 *B*:215 *C*:238 *D*:346.

4-C2. The inlet to a symmetrical hopper is a horizontal rectangle 3 inches wide and 2 inches deep. One rear corner is located at point *A*:0,3$\frac{1}{2}$,9. The outlet is a 1$\frac{1}{2}$ × $\frac{1}{2}$-inch horizontal rectangle 2 inches below the inlet. The 1$\frac{1}{2}$-inch sides are frontal. Draw the top and front views, and find the true size of the dihedral angle between two adjacent faces.

4-C3. The plate of a hip roof is represented by the trapezoid *A*:016 *B*:416 *C*:519 *D*:019. The roof planes make an angle of 45° with the horizontal. Draw the top, front, and side views of the roof. Find the true size of the dihedral angles between the roof planes.

4-C4. Find the true size of the dihedral angle between planes *A*:556 *B*:619 *C*:825 and *D*:437 *E*:749 *F*:816.

4-C5. Find the true size of the dihedral angle between planes A:4¼,3, 8¼ B:6,4¾,9¼ C:7½,3¾,7¼ and D:4,3¾,7½ E:7¾,4¼,9¾ F:6¾,1,6¼.

4-C6. Find the true size of the dihedral angle between the planes G:7¼,1,5¾ K:7¼,4¾,5¾ L:5,4¾,7¼ and M:6¾,½,8¾ N:5,2,6¼ O:8,5,6¼.

4-C7. Plane Q is determined by points A:½,5,5 B:615 C:7,5,7½. Represent plane Q by its H- and V-traces. Origin: 055. Find the true size of the dihedral angle α between Q and the H-plane, and the dihedral angle β between Q and the V-plane.

4-C8. Find the angles α and β as described in the preceding problem for the plane determined by points D:4½,5,5 E:1,3¼,5 F:7,5,7½.

GROUP D. ANGLE BETWEEN A LINE AND AN OBLIQUE PLANE

4-D1. Find the true size of the angle between the plane of the rectangle A:517 B:747 C:749 D:519 and the line E:337 F:6,2½,8.

4-D2. Find the size of the angle between the line A:156 B:4,2,7½ and the plane of the triangle P:129 Q:716 R:756.

4-D3. Find the true size of the angle between the edge CD and the face ABC of the tetrahedron A:1¼,3¼,5¼ B:2¾,1¼,9¼ C:4½,4¾,7½ D:4½,2½,6.

4-D4. Find the angle between triangle R:0,1½,5½ S:2,4½,8½ T:3,½,6½ and line Y:3¼,4¼,6¼ Z:6¼,1¾,9¼.

4-D5. Find the true size of the angle between the plane D:½,1¾,8¾ E:3¼,3¼,9¾ F:2,5,6¼ and the line A:4¾,1¼,8½ B:8,1¼,7½.

4-D6. Points A:1¼,3,7 B:3¾,3,9½ and C:4¾,1,7 lie in a plane mirror. A light ray passes through points L:6½,4,8¼ and M:4¾,3,8. Find the angle of incidence and the path of the reflected ray.

GROUP E. OBLIQUE LINES AND PLANES

4-E1. Through point A:437 draw a line 3 inches long, making an angle of 45° with the H-plane, an angle of 30° with the V-plane, and sloping downward. (Four solutions.)

4-E2. Point B:346 is the upper end of a line 3 inches long which slopes downward, backward, and to the right. The line makes 30° with the H-plane and 45° with the V-plane.

4-E3. Through point C:437 pass plane Q which slopes downward, forward, and to the right. Plane Q makes an angle of 67° with H and 38° with V. Represent plane Q by its H- and V-traces. Origin: 055.

4-E4. Through point D:4,2¾,7¾ pass plane R which slopes downward, backward, and to the left. Plane R makes 62° with H and 42° with V. Represent plane R by its H- and V-traces. Origin: 055.

GROUP F. PERPENDICULAR DISTANCES

4-F1. Draw the views of the line PQ which represents the shortest distance between point P:014 and line A:136 B:3,1,4½. Mark the true-length view of PQ.

4-F2. Find the shortest distance between point C:037 and line A:247 B:225.

4-F3. Find the shortest distance between point C:2¾,3½,6¾ and line A:¼,4¼,6½ B:2½,2,8¼. Solve by drawing a normal view of the triangle ABC.

4-F4. Find the shortest distance between point P:226 and line A:404 B:735. Solve by rotating the figure PAB into the horizontal plane taken through point B.

4-F5. Find the shortest distance between the skew lines A:048 B:219 and C:237 D:359. Solve by drawing a view in which one of the given lines projects as a point.

4-F6. Find the shortest distance between the skew lines L:409 M:637 and R:729 S:816.

4-F7. Find the shortest distance between the skew lines A:¼,3½,6 B:1¾,1½,5½ and C:1¼,3¼,7¾ D:¼,½,5½. Draw the top and front views of the perpendicular common to AB and CD. Solve by drawing a view in which the projections of AB and CD appear parallel.

4-F8. Lines J:019 K:349 and L:158 M:116 represent respectively the positions of a guy wire and a lead wire of an antenna system. Determine the minimum clearance between these wires. Draw the line which represents this shortest distance in each view. Scale ¼ inch = 1 foot.

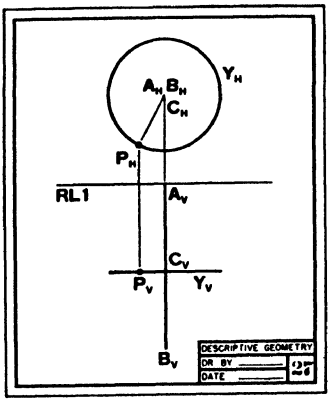
4-F9. Compute the volume of the tetrahedron V:156 A:329 B:515 C:647.

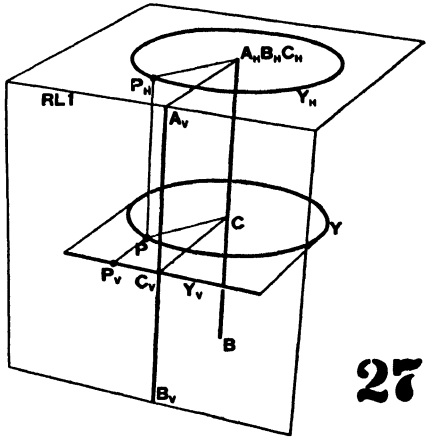
4-F10. Find the common perpendicular PQ between lines A:5,½,7½ B:5,2¾,9½ and C:1,1¾,7¾ D:3¼,4½,5½. Base the construction on the following analysis:

1. Through point B draw line k parallel to CD.
2. Pass plane W through lines AB and k.
3. Through point C draw line m perpendicular to plane W.
4. Through point D draw line n perpendicular to plane W.
5. Find points R and S in which lines m and n intersect plane W.
6. Draw line RS.
7. At point Q, where line RS cuts line AB, draw line t parallel to m and meeting line CD in point P.

Basic Principle of Rotation

ANY POINT ROTATED ABOUT ANY AXIS DESCRIBES A CIRCLE IN THE PLANE THROUGH THE POINT ⊥ TO THE AXIS, THE RADIUS BEING THE DISTANCE FROM THE POINT TO THE AXIS.

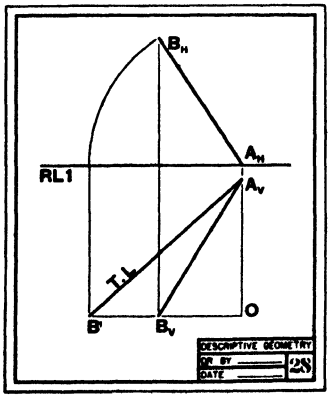


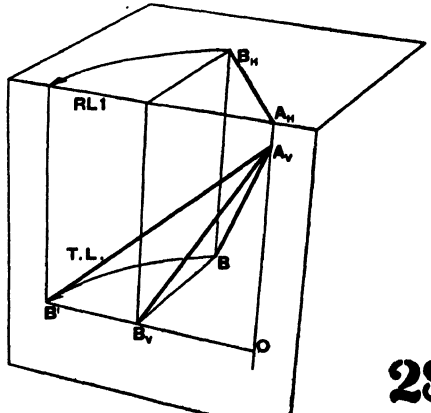


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Rotation of a Line into a Projection Plane

ROTATE THE LINE ABOUT AN INTERSECTING AXIS ⊥ TO ONE PROJECTION PLANE & PARALLEL TO OR CONTAINED IN THE OTHER.



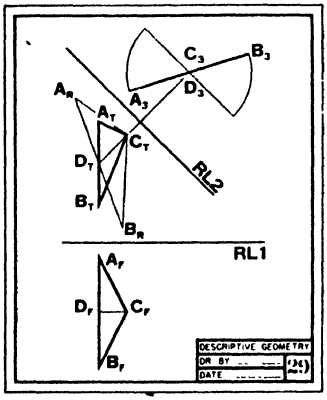


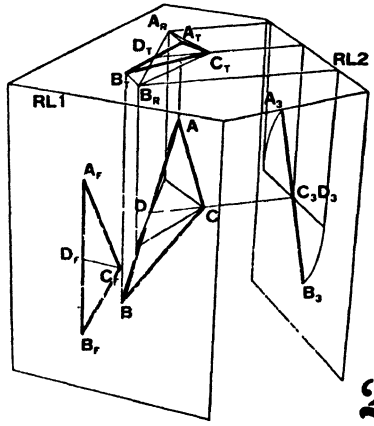
28

NOTE THAT WHEN AB BECOMES PARALLEL TO THE V PLANE IT PROJECTS T.L.

Rotation of a Plane

A PLANE MAY BE ROTATED TO BE PARALLEL TO A PROJECTION PLANE BY USING A PRINCIPAL LINE IN THE PLANE AS AN AXIS.



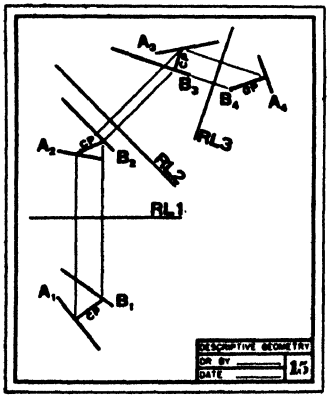


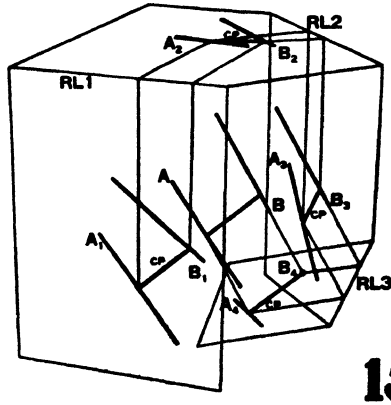
29

NOTE THAT THE PLANE IS TRUE SIZE IN THE ROTATED VIEW. GEOMETRIC RELATIONSHIPS IN THE PLANE SUCH AS THE ANGLE BETWEEN 2 LINES CAN BE DIRECTLY MEASURED IN THIS VIEW.

Perpendicular Common to 2 Skew Lines

TO DRAW THE COMMON ⊥ BETWEEN 2 SKEW LINES IN SPACE OBTAIN AN END VIEW OF EITHER LINE.





15

NOTE THAT THE COMMON ⊥ APPEARS ⊥ TO B IN VIEW 3 & TO A IN VIEW 4.

CHAPTER V

Polyhedrons

76. Regular polyhedrons

Solids which are bounded by planes are called *polyhedrons.* Precisely, it is the surface of the solid that is the polyhedron. The term is commonly used, however, to describe either the surface or the solid. The boundary planes intersect to form the *edges* of the solid. A plane polygon formed by a set of edges is a *face.* The point in which a set of faces intersects is a *vertex.* When all the faces of a solid are congruent, the figure is a regular polyhedron. There are five regular polyhedrons; the tetrahedron, cube, octahedron, dodecahedron, and icosahedron. All other polyhedrons are *irregular.*

77. Irregular polyhedrons

When the faces of a solid are not congruent, the figure is irregular. The prism, pyramid, and prismoid are common examples of this class.

The prism has two polygons called *ends* or *bases* which are congruent and parallel, making all the other surfaces parallelograms which are called *faces.* The solid is a right prism when each face is a rectangle; otherwise, it is oblique. A line connecting the centers of the bases is an *axis.* The axis is parallel to one set of edges. The description of a prism includes the shape of a base. Thus, when the bases and faces are rectangles, the solid is a rectangular right prism. When the bases and faces are parallelograms, the solid is a parallelepiped.

The pyramid can have any polygon for one face called the base; all other faces are triangles. The triangular faces meet in a common point called the *apex.* A line connecting the apex and the center of the base is the *axis.* The solid is a right pyramid when the axis is perpendicular to the base; otherwise, it is oblique.

The prismoid has two parallel ends or bases which are dissimilar polygons having the same number of sides. The faces are plane quadrilaterals. The frustum of a pyramid is a limiting case of a prismoid.

Many polyhedral forms occur in nature as crystals.

78. To find the intersection of a plane and a polyhedron

Analysis: The line of intersection is determined by the points in which the edges of the solid pierce the given plane. The basic problem is,

therefore, the one of finding the point in which a line pierces a plane. There are two general methods for solving this problem:

1. Auxiliary View Method (Article 38, Second Analysis).
2. Cutting Plane Method (Article 38, First Analysis).

First Construction: Let the pyramid *O-ABC* (Figure 134) be intersected by the plane *ef*. Draw the Auxiliary View 3 in which the plane projects edgewise. The points 1, 2, 3 in which the edges *OA*, *OB*, *OC* pierce the plane *ef* are determined in this view. The top and front views of the line of intersection are found by projection from the auxiliary

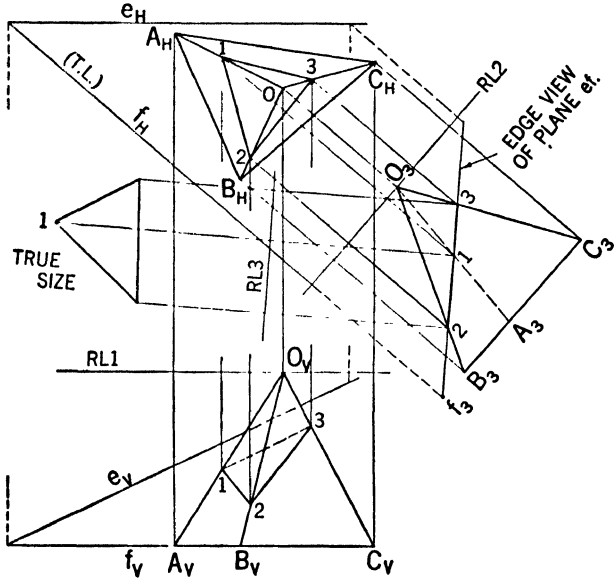


Fig. 134

view. The true shape of the section is found by projecting it on a plane taken parallel to the section.

Second Construction: Let the pyramid (Figure 135) be intersected by the plane *QRST*. The points in which the lateral edges of the pyramid pierce the plane are found by the method of Problem 7. Pass the vertical plane *X*, represented by its trace *HX*, through line *OC*. Plane *X* intersects plane *QRST* in line 5-9. The point 1_V in which lines 5_V-9_V and O_VC_V intersect is a point on the intersection. Points 2, 3, and 4 are determined similarly by planes *Y*, *Z*, and *W*.

79. Given the plane ABC and a hexagonal prism

Analysis: The points in which the edges of the prism intersect the plane are easily found in this example without the use of an auxiliary

view. Find the point in which each edge of the prism pierces the plane by the method of Fundamental Problem 7. The polygon having these points as vertices represents the section cut from the prism by plane ABC.

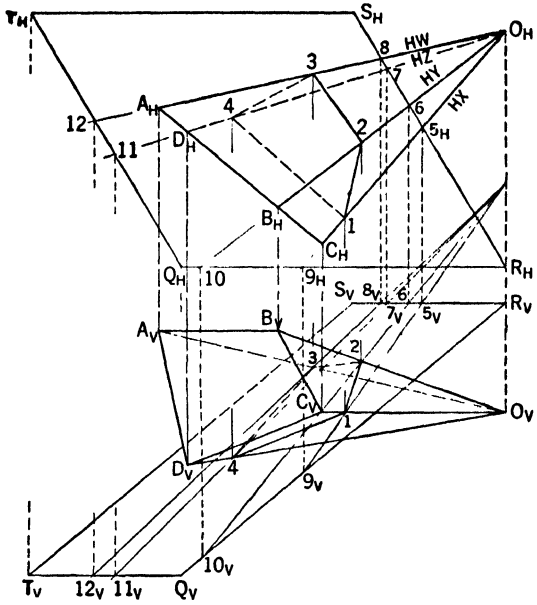


Fig. 135

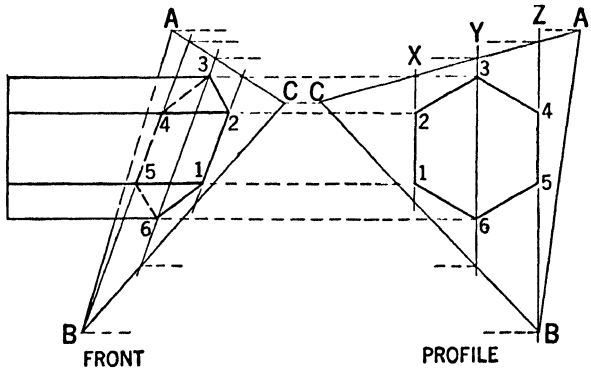


Fig. 136

Construction (*Figure 136*): The cutting plane X, edgewise in the profile view, determines the points 1 and 2 in which the two front edges of the prism intersect plane ABC. Similarly, planes Y and Z determine points 3, 6 and 4, 5 respectively. The hexagon 1-2-3-4-5-6 is the required intersection.

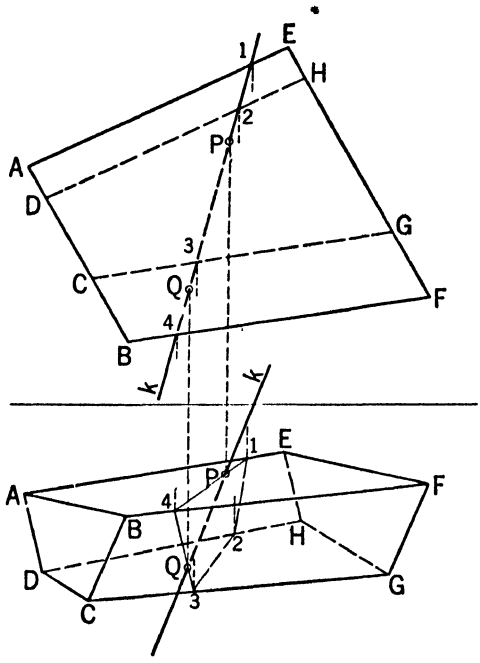


Fig. 137

80. To find the points in which a line intersects a polyhedron

Given: The line k and the prismoid $ABCD$-$EFGH$.

Analysis: Pass a vertical plane through line k. Find the section cut from the solid by this plane. The points P and Q in which line k crosses the edges of the section are the points in which line k intersects the solid.

Construction (*Figure 137*): A vertical plane through line k cuts the prismoid in the section 1-2-3-4. Points P and Q lie on the edges of this section.

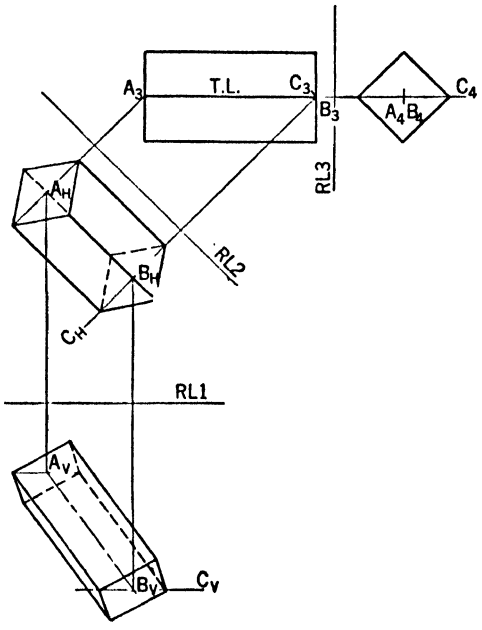


Fig. 138

81. To draw the views of a prism when the right section and axis of the surface are given

Analysis: Draw the normal and end views of the axis. The true size of the right section appears in the view in which the axis projects as a point. The right section appears edgewise in the view in which the axis projects in its true length. Therefore, construct the right section on the end view of the given axis, and complete the required views by projecting from this view.

Construction (*Figure 138*): A square right prism is constructed on a given line AB as an axis. The square bases

are centered on points A and B, and one diagonal of each base is horizontal.

Draw the horizontal line c through point B and perpendicular to line AB. One diagonal of the base centered on B will lie along line c. Take $RL2$ parallel to A_HB_H and draw the normal view of AB. Line c projects as a point in this view, and the bases appear perpendicular to A_3B_3. Take $RL3$ perpendicular to A_3B_3 and draw the point projection of AB. Line c projects to c_4. Construct the true-shape view of the base or right section with one diagonal coinciding with line c_4. Complete the views by use of the existing relationships.

82. To find the right section of a solid

Analysis: A right section of a solid is a section cut by a plane perpendicular to the axis or center line of the solid. It is commonly called

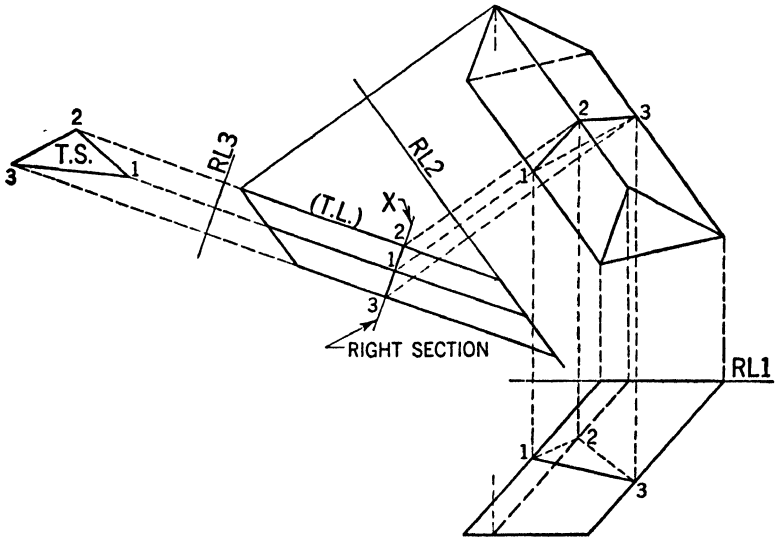


Fig. 139

a *cross section.* The right-section plane appears edgewise and perpendicular to the axis in a view showing the axis in true length. Points in which edges of the solid pierce the section plane are found in this view. The true size of a right section of a solid appears in the view in which the axis of the solid projects as a point.

Construction (Figure 139): To find the projections and true shape of a right section of a triangular oblique prism. Take $RL2$ parallel to the lateral edges of the prism in the top view. Draw the first auxiliary view showing the lateral edges of the prism in true length. Any plane X passed perpendicular to these edges projects edgewise in the first

auxiliary view and cuts the edges in points 1, 2, and 3. The top and front views of the right section are found by projecting points 1, 2, and 3 from the auxiliary view.

The second auxiliary view is located by taking $RL3$ parallel to the edge view of the right section. The second auxiliary view represents the true shape of section 1-2-3.

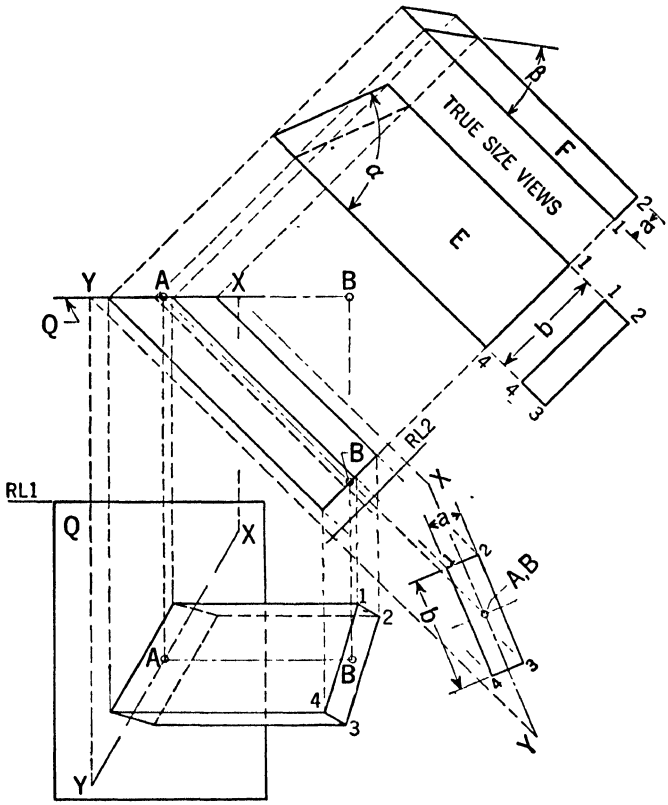


Fig. 140

Example 2 (Figure 140). Line AB is the axis of a rectangular prism having face widths equal to a and b. The end at point A is cut to fit against the plane Q, the long edges of the base being parallel to line XY. The base at point B is a right section.

Take $RL2$ perpendicular to the top view of AB. Draw the auxiliary view of AB (a point) and line XY. Construct the end view of the prism, using the given dimensions and making the longer sides parallel to line XY.

Complete the top and front views by projection. The base at B will appear at right angles to the top view of AB.

The views of the prism in which the lateral faces appear in true size are laid out by a combination of projection and rotation.

Project the edges of the bases at right angles to the top view of the axis AB (this being the true-length view). Draw view E, constructing the lateral edges parallel to the top view of AB and making the width of the face equal to b. The true angle of the bevel cut on the wide face is equal to α.

The lateral edges of view F are parallel to the top view of AB and the width of the face is made equal to a. The true angle of the bevel cut on the narrow face is equal to β.

83. Developments

A development of a polyhedron is a drawing which shows the true size and relative position of each face of the solid. It represents the surface cut open along certain edges and folded out into a single plane. When a development is cut from sheet material and properly bent, it reproduces the surface of the solid very closely. In practical work, the development usually shows the inside of the surface, since the working dimensions of sheet-metal structures are often the inside dimensions. Also, certain allowances must be made for seams and "crowding" due to the thickness of the material. In the following examples, only the purely geometrical aspects of development will be treated.

84. Development by triangulation

Right Pyramid (*Figure 141*). The development of the lateral faces consists of four triangles. The surface is opened along the edge 0-1, and the faces are laid out in sequence. The lateral edges are equal in length. Assume the point O' as the vertex of the development. Using the true length of O-4 as a radius and point O' as the center, strike an arc. Starting at point 1′, set off the true lengths of the basal edges 1′-2′, 2′-3′, 3′-4′, 4′-1′ along this arc. Connect these points to form the faces. The rectangular base may be attached to any basal edge of the development.

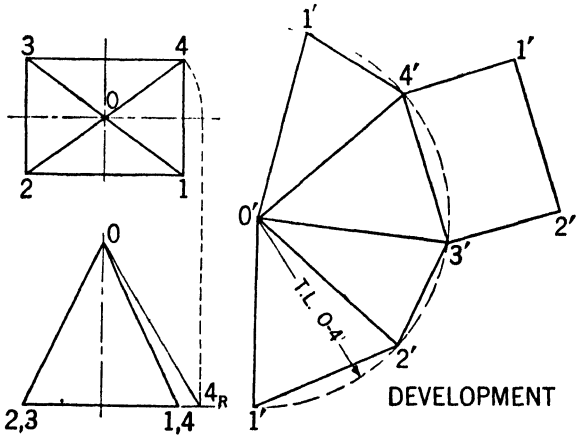


Fig. 141

Truncated Pyramid (*Figure 142*). The upper base of the pyramid is neglected at the outset, and the surface of the oblique pyramid with vertex at O is developed. For convenience, the true lengths of the lateral edges are assembled in a "true-length diagram." It will be seen that the true length of an edge is equal to the hypotenuse of a right triangle having the horizontal projection of the line for its base, and the difference in elevation between the ends of the line as its altitude. Thus, the true length of edge O-1 is found by drawing $O'P'$ equal to

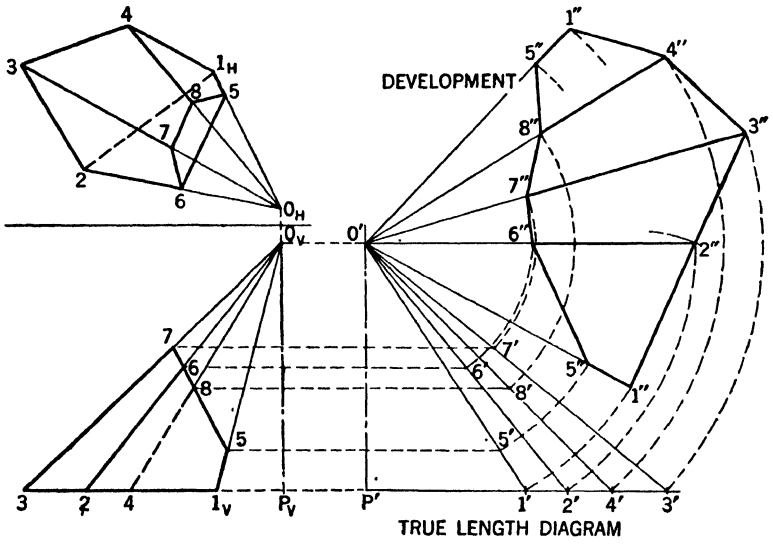


Fig. 142

O_VP_V and setting off P'-1′ equal to O_H-1_H. The true length is O'-1′. The true lengths of the basal edges appear in the top view.

Using the true lengths now available, construct the lateral faces of the pyramid by triangulation. The surface is cut open along its shortest edge O-1. It is convenient to have the vertex of the development coincide with point O' of the true-length diagram.

The points of the upper base are located on the development by laying off the true distance from the vertex of the pyramid to each point. For example, the true length of the segment O-5 is equal to O'-5′. This distance is transferred to edge O'-1″ on the development locating point 5″.

Frustum of a Pyramid (*Figure 143*). When the vertex is inaccessible, the frustum of a pyramid is developed by dividing each trapezoidal face into two triangles. The development is made by laying off each triangle in true size and in its proper sequence. One half of the lateral surface is shown in this development.

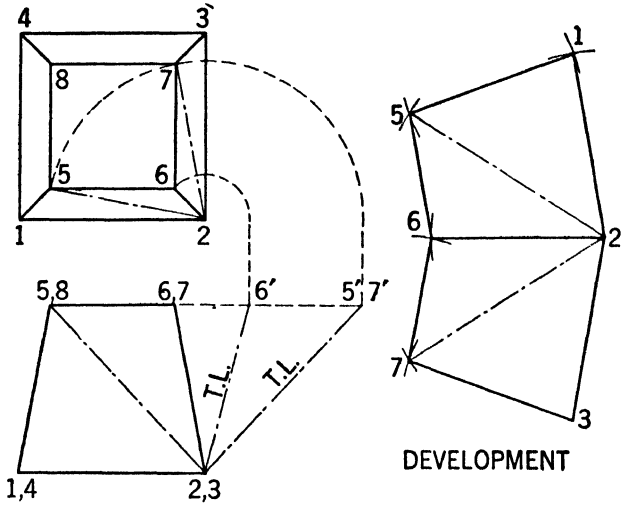


Fig. 143

85. Parallel line developments

The prism (*Figure 144*). Each lateral face of the prism unfolds in true size and in sequence. The parallel edges of the prism produce parallel lines in the development. Since lengths and angles on the faces are preserved in the development, a right section of the prism develops into a straight line which is perpendicular to the parallel edges. This straight line is called the *girth line*, and its length is equal to the perimeter of the right section. In Figure 144, the right section is taken at any convenient point on the axis of the prism. The perimeter of the right section is given in the top view. Placing the development opposite the front view, as shown, allows the true lengths of the lateral edges which appear in the front view to be carried across directly to the developments of these edges.

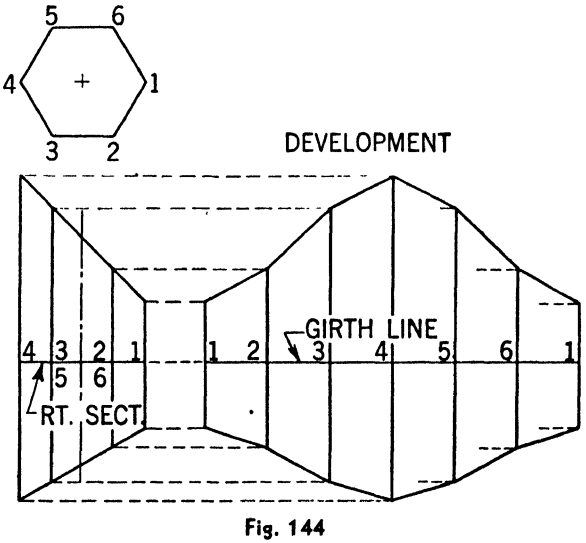


Fig. 144

86. Intersection of solids

When one solid cuts into or penetrates another, the lines formed by the intersection of their surfaces is the intersection of the solids. The angular points of the intersection are determined by the points in which the edges of each solid pierce the faces of the other. Intersecting polyhedrons may be regarded as hollow surfaces, or one may be represented as a complete solid and the second cut away to fit against it.

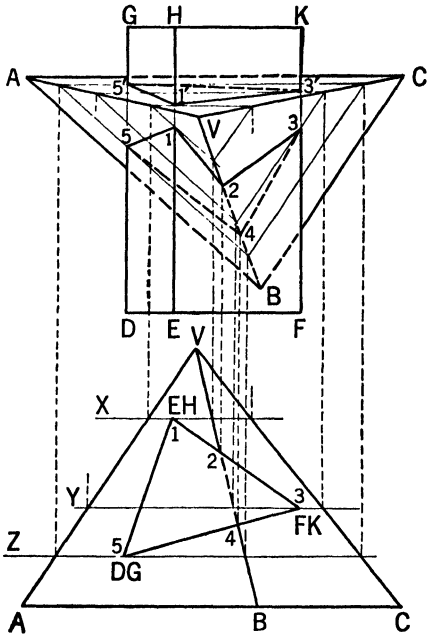


Fig. 145

87. To find the intersection of a pyramid and prism

Analysis: The intersection consists of one or two polygons. Each is determined by two groups of points: the points in which the edges of the prism pierce the faces of the pyramid, and the points in which the edges of the pyramid pierce the faces of the prism. The basic problem involved is that of finding the intersection of a line and polyhedron (Article 80).

Example 1 (Figure 145). A horizontal cutting plane X passed through line EH of the prism cuts a triangle from the surface of the pyramid. The points 1 and 1′ in which edge EH meets this triangle are points on the intersection of the solids. The piercing points for edges DG and FK are found in a similar manner by use of the cutting planes Y and Z.

The points 2 and 4, in which edge VB pierces the surface of the prism, are found in the front view.

Example 2 (Figure 146). A horizontal cutting plane X passed through edge AD of the prism cuts a square from the surface of the pyramid. The points 1 and 7 in which edge AD meets the square are points on the intersection. The piercing points for edges BE and CF are found in a similar manner.

The points 10 and 11 in which edge VL of the pyramid pierces the surface of the prism are found by use of the vertical cutting plane Q. Cutting plane Q also locates the points 4 and 5 in which edge VH pierces the prism.

If a vertical cutting plane is passed through edges VG and VK, it will be found that these edges do not intersect the prism.

The polygons 1-2-4-5-6 and 7-8-10-12-11 represent the intersection

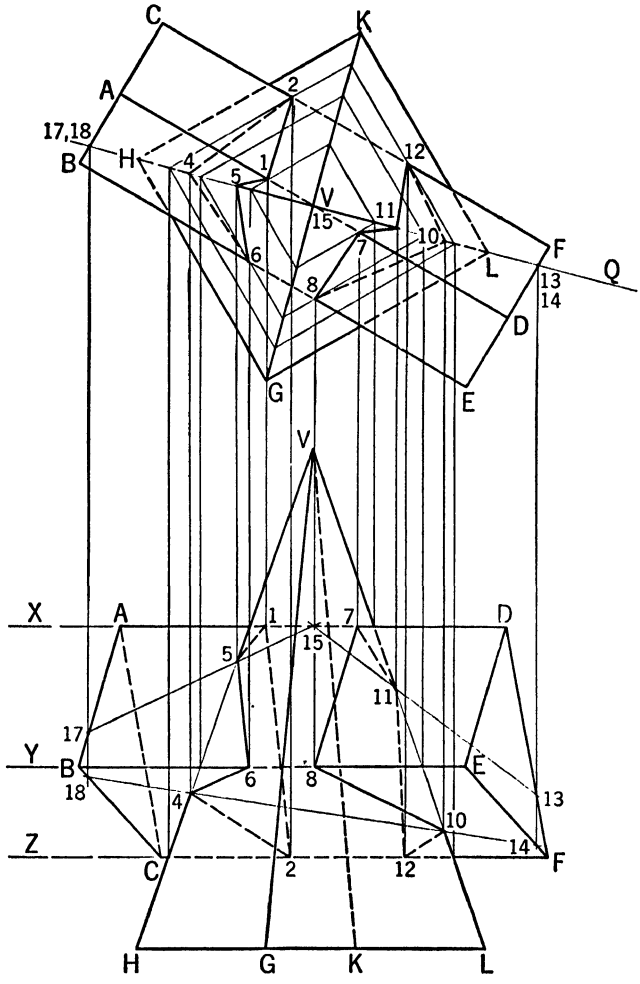


Fig. 146

of the solids. The prism is complete; the pyramid is cut away to receive the prism.

PROBLEMS ON CHAPTER V

GROUP A. INTERSECTIONS

5-A1. Find the points in which line A:$1\frac{1}{4},3\frac{1}{2},6\frac{3}{4}$ B:$6\frac{1}{4},3,7\frac{1}{2}$ cuts the surface of the tetrahedron P:138 Q:$3\frac{3}{4},1,5$ R:$4\frac{1}{2},4\frac{1}{2},7$ S:$6\frac{1}{2},2\frac{1}{4},8\frac{3}{4}$. Show correct visibility.

5-A2. The parallelogram A:017 B:216 C:417 D:218 is the base of a right pyramid 4 inches high. Draw the top and front views and the true-shape view of the section cut from the pyramid by the plane which passes through the points P:129 Q:327 R:146.

5-A3. Draw the top and front views and the true-shape view of the section cut from the tetrahedron V:347 A:426 B:229 C:017 by the plane passing through points P:517 R:119 S:246.

5-A4. Line A:116 B:318 is the diagonal of one square base of a right prism 3 inches high. Draw the views of the portion of the prism which is below the plane determined by points J:147 K:246 L:4,$\frac{1}{4}$,7. Find the true shape of the oblique section.

5-A5. Line A:239 B:3$\frac{1}{4}$,3,9 is the upper edge of a vertical cross section of an I-Beam which is 2 inches high. The flanges and web are $\frac{1}{4}$ inch thick. The beam is terminated by a plane passing through line P:1,3,4$\frac{1}{2}$ Q:3$\frac{1}{2}$,3,7 and having a slope of 60° downward and forward. Draw the top view and true-shape view of the oblique section.

5-A6. Line A:217 B:319 is a left side of a horizontal rectangle which forms the base of a right pyramid 3$\frac{1}{2}$ inches high. The long edges of the base are 3 inches. Line E:415 F:4,3$\frac{1}{2}$,5 is the diagonal of a frontal square. The square is the right section of a prism which intersects the pyramid. Assuming that the prism is cut to fit against the pyramid, find the lines of intersection.

5-A7. Line A:1$\frac{3}{4}$,$\frac{1}{2}$,7$\frac{3}{4}$ B:4$\frac{1}{4}$,$\frac{1}{2}$,7$\frac{3}{4}$ is a diagonal of one horizontal square base of a right prism 5 inches high. Line C:$\frac{3}{4}$,2,8 D:5$\frac{1}{4}$,3$\frac{3}{4}$,8 is the axis of a right hexagonal prism. The edges of the base are 1$\frac{1}{4}$ inches long. The front and rear lateral faces are vertical. Find the lines of intersection of the two prisms.

GROUP B. SOLIDS IN OBLIQUE POSITIONS

5-B1. Point V:4,3$\frac{1}{2}$,7 is the vertex and C:628 is the center of the base of a hexagonal right pyramid. A diagonal of the base is horizontal and 2 inches long. Complete the top and front views.

5-B2. Line A:136 B:2$\frac{1}{2}$,1,8 is the axis of a square right prism. Point C:$\frac{1}{2}$,X,6 locates one corner of the front base. Complete the top and front views.

5-B3. Line A:5$\frac{1}{2}$,3,8 B:6$\frac{1}{4}$,2,6$\frac{3}{4}$ is the axis of a hexagonal right prism. One diagonal of each base is frontal and 1$\frac{1}{2}$ inches long. Complete the top and front views.

5-B4. Line C:538 D:726 is the axis of a square right pyramid. The base is centered on point C, one corner being located at E:4,2$\frac{1}{2}$,X. Complete the top and front views.

5-B5. Line J:1½,2,7½ K:2½,3,8½ is an axis of a cube. One corner of the cube rests on a horizontal plane located 1 inch below point J. Complete the top and front views of the cube.

GROUP C. DEVELOPMENT

5-C1. Develop the surface of the tetrahedron of problem 1-C1. Place point B in the center of the sheet.

5-C2. Develop the parallelepiped of problem 2-A3 by triangulation. Locate point C at ½,5,5, and draw edge CG parallel to the left edge of sheet. Lay out faces in the following order: *CGHD, HDAE, AEFB, FBCG, ABCD, EFGH.*

5-C3. The line A:017 B:317 is the diagonal of the horizontal regular hexagonal base of an oblique pyramid having its vertex at V:347. The pyramid is truncated by a plane passing through points R:237 and S:327 and perpendicular to a frontal plane. Develop the lateral surface which lies between the base and plane RS. Make the joint along the shortest edge.

5-C4. Line A:127 D:2½,2,7 is a diagonal of a horizontal hexagon which forms a right section of a truncated regular hexagonal prism. The upper base contains points P:157 and Q:2½,2½,7; the lower base contains points R:1,½,7 and S:2½,1½,7. Each base appears edgewise in the front view. Develop the surface of the prism, making the joint along the shortest edge.

5-C5. The lines A:037 B:137 and E:119 F:219 are the lower edges of two frontal rectangles 1½ inches high. These rectangles are the open ends of a pipe connection. Complete the top and front views. Determine the true shape of a right section, and develop the surface of the pipe.

5-C6. (*a*) Line A:½,4¼,6½ B:2½,2,8½ is the axis of a prism. The base centered on point A is a frontal rectangle 1 × 1½ inches with the 1½-inch sides vertical. The lower base lies in a horizontal plane through point B. Complete the top and front views of the prism. (*b*) Find the projections and true shape of the right section of the prism determined by the plane perpendicular to the axis AB at its mid-point. (*c*) Develop the lateral surface of the prism.

CHAPTER VI

Curved Surfaces

88. Geometrical surfaces

A *surface* may be the locus of a line moving according to some law. There are as many kinds of such surfaces as there are ways of controlling the movement of a line. The infinity of lines which represent the various positions of the moving line are called *elements* of the surface.

Surfaces are divided into two general classes, depending on the form of the generating line. A surface which can be generated by a straight line is a *ruled surface.* Planes, cones, and cylinders are representative of this class. A surface which can be generated only by a curved line is a *double curved surface.* The *surface of revolution* is the most common double curved surface. It is generated by the rotation of a plane curve about a straight line as an axis. Right sections of a surface of revolution are circles. The sphere is an example of this class. Also, objects turned on a lathe are bounded by surfaces of revolution.

Two ruled surfaces, the circular right cone and cylinder, can be regarded as surfaces of revolution since they can be generated by revolving a straight line about an axis.

An *axis* of a surface is a line so placed that the section cut from the surface by any plane containing the axis will be symmetrical with respect to it. Any given surface may have more than one axis, or it may have none.

Solutions of problems on surfaces are obtained by means of the fundamental problems on the point, line, and plane together with the use of the special geometric properties of the surfaces under consideration. The majority of problems on curved surfaces can be illustrated by examples involving the cone, cylinder, and sphere, elementary surfaces with which almost everyone is familiar. The cone and cylinder typify the ruled surface; the sphere is the most common double curved surface.

89. Representation of surfaces

Surfaces existing within definite limitations are generally represented by the projections of the limiting lines. Surfaces of indefinite extent are represented by the projections of a sufficient number of elements or

sections of the surface to reveal the nature of some desired portion of the surface.

90. The cone

The cone is a ruled surface generated by a straight line which passes through a fixed point and moves so that it always intersects a curved line. The fixed point is the *vertex*, and the curved line is the *directrix* of the cone. The surface may be open or closed; that is the directrix may be continuous or have two ends. When the generating element is of indefinite length, two equivalent surfaces are generated simultaneously on opposite sides of the vertex. These are the *nappes* of the cone. Ordinarily, but one nappe is considered, and the cone is represented by the projections of the vertex, directrix, and the limiting elements of each view.

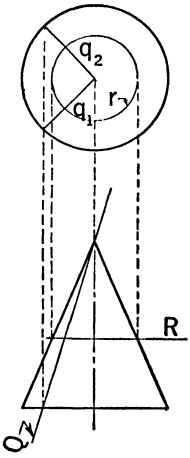


Fig. 147

The intersection of a conical surface by a plane, which cuts every element, is a *section* or *base* of the cone. A plane perpendicular to the axis of the surface cuts it in a right section. Conical surfaces are classified according to the character of their right sections, as circular, elliptical, and so forth.

A *cone of revolution* is formed when either of two intersecting lines is revolved about the other as an axis. A plane Q, containing the vertex of a conical surface and intersecting the surface, cuts the surface in straight lines q_1 and q_2 (Figure 147). A plane R, perpendicular to the axis of a cone of revolution, cuts the surface in a circle.

91. The cylinder

The cylinder is a ruled surface generated by a moving straight line which remains parallel to its initial position and always intersects a curved line. A section of the surface by a plane which intersects every element is taken as a base of the cylinder. A plane perpendicular to the elements cuts the surface in a right section. A cylindrical surface may be open or closed and is classified as circular, parabolic, and so forth, depending upon the shape of its right section. The cylinder is represented by the projections of one of its bases and the limiting lines of each view.

A *cylinder of revolution* is generated when either of two parallel lines is revolved about the other as an axis. A plane S, parallel to the elements of a cylinder and intersecting the surface, will cut the surface in straight lines s_1 and s_2 (Figure 148).

It is often convenient to regard the cylinder as a limiting case of a

cone having its vertex at infinity. It is evident that the cone and cylinder degenerate into a plane when the directrix becomes a straight line or when the vertex lies in a plane with the directrix.

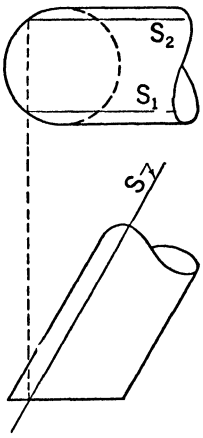


Fig. 148

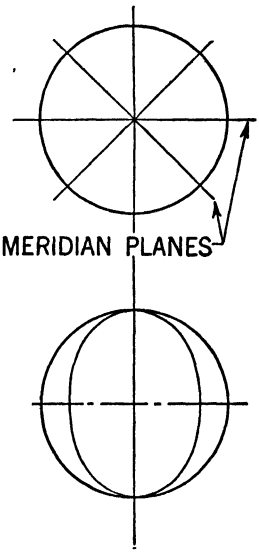


Fig. 149

92. The sphere

The sphere is generated by revolving a circle about a diameter as a axis. It is both a double curved surface and a surface of revolution The limiting line of any given view is the projection of the great circle which is parallel to the projection plane. Every section of a sphere is a circle. A plane containing the vertical axis of a sphere is a *meridian plane*, and its intersection with the sphere is a *meridian circle* (Figure 149).

Strictly speaking, a surface is a two-dimensional magnitude. It has area but not thickness. A solid which is bounded by a surface bears the same name as the surface. Thus, the term *sphere* may describe either a surface or a solid.

93. To assume a point in a surface

Analysis: One projection of a point in a surface can be assumed at random. Through this assumed projection, draw the corresponding projection of an element of the surface. In the case of a ruled surface, this element will generally be a straight line; in a double curved surface, it may be a circle or other curved line. Draw the projection of this element in the other view. The second projection of the assumed point lies on this element.

Construction:

Example 1 (Figures 150 and 151). Given: P_V on the surface of a cone. To find P_H. Draw element $O_V P_V$, cutting base in point 1 or 2. This element has two possible positions since P may lie on either the front

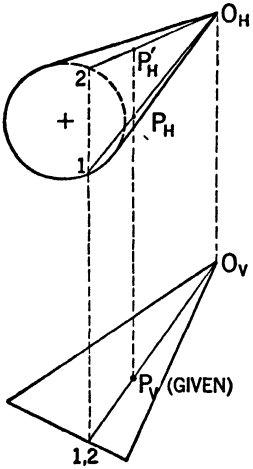


Fig. 150

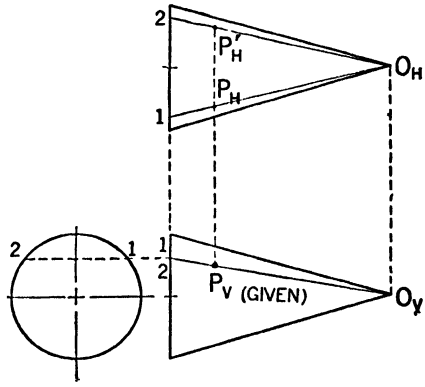


Fig. 151

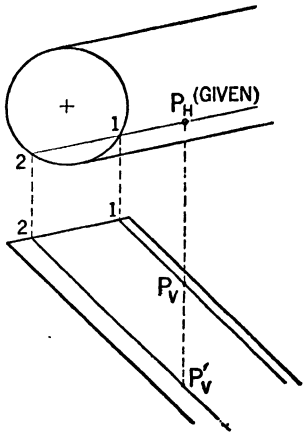


Fig. 152

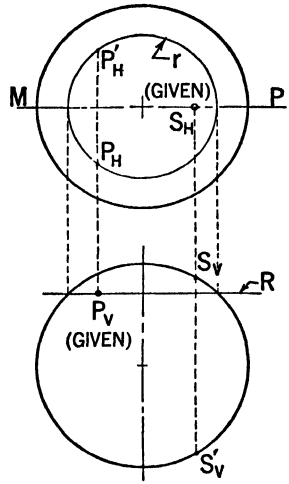


Fig. 153

or rear side of the cone. Draw the projection O_H-1 and O_H-2. P_H may lie on either line.

Example 2 (Figure 152). Given: P_H on the surface of a cylinder. To find P_V. Since P may lie either on the upper or lower side of the

surface, P_V has two possible locations. The construction is similar to that of Example 1.

Example 3 (Figure 153). Given: P_V on the surface of a sphere. To find P_H. A horizontal plane R through P_V cuts the surface of the sphere in the circle r. The projections of point P lie on the projections of the circle r. The point S lies on the frontal great circle of the sphere.

94. Tangent lines and tangent planes

Definitions:

(*a*) If a secant line cuts a curve in points A and B, and point B is moved along the curve until it coincides with A, point A remaining fixed, the secant becomes *tangent* to the curve at point A.

(*b*) A tangent to a plane curve lies in the plane of the curve.

(*c*) A plane is tangent to a curved surface if it has at least one point in common with the surface, through which, if any section plane be passed, the straight line cut from the plane will be tangent to the curved line cut from the surface.

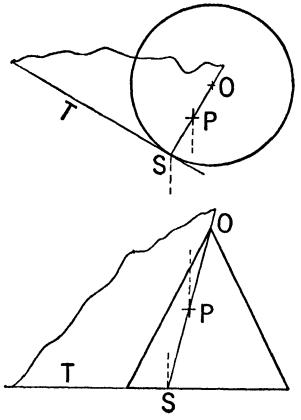


Fig. 154

(*d*) The plane tangent at any point to a ruled surface will contain the rectilinear element (or elements) of the surface which passes through the point.

95. To pass a plane tangent to a cone (or cylinder) at a given point in the surface

Analysis: The tangent plane will contain the element of the surface through the given point and a line tangent to the base at the point in which the element intersects the base.

Construction (*Figure 154*): Let P be the point at which the plane T is to be tangent to the cone. The required plane is determined by the element OS and the line T, which is tangent to the base at point S.

96. To pass a plane tangent to a cone (or cylinder) through a given point without the surface

Analysis: Since the required plane must contain the vertex of the cone, it will contain the line passing through the vertex and the given point P. The second line determining the plane is a tangent to the base, drawn through the point Q where the line joining the given point and the vertex pierces the plane of the base.

If the given surface is a cylinder, the auxiliary line PQ is drawn parallel to the axis of the cylinder.

Construction (Figure 155): Draw line OP, intersecting the plane of the base of the cone in point Q. Draw lines QA and QB tangent to the base of the cone. Planes OQA and OQB are each tangent to the cone. The elements of tangency are OA and OB respectively.

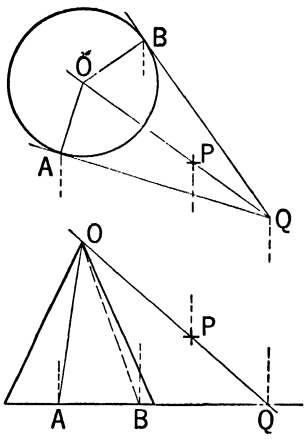


Fig. 155

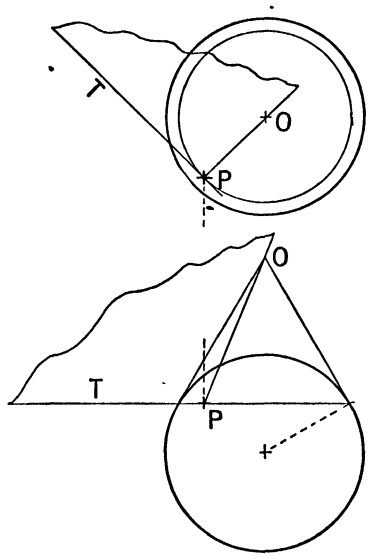


Fig. 156

97. To pass a plane tangent to a sphere (or surface of revolution) at a given point in the surface

Analysis: Construct a cone of revolution tangent to the sphere and containing the given point. Pass a plane tangent to the cone at the given point, using the construction of Article 95.

Construction (Figure 156): Let point P be given in the surface of the sphere. Draw the horizontal circle of the sphere passing through point P. Using this circle as the base, construct the cone which is tangent to the sphere. The axis of the cone is vertical with vertex at point O. The plane tangent to the cone is determined by the element OP and the tangent T to the base circle.

98. To find the intersection of a line and surface

Analysis: A plane passed through the given line will intersect the given surface in a line. The points in which the given line cuts this line of intersection are the points in which the line pierces the surface. The plane should be located so as to cut straight lines or a circle when

possible. This problem is a general case of Problem 7 (Intersection of a line and plane).

Construction:

Example 1 (Figure 157). *Line and Cone. To find the points in which line k intersects the surface of the cone.* Draw any line through the vertex O, intersecting line k at A. Pass a plane X through lines k and OA. Find the intersection HX of this plane with the plane of the base of the cone. The points B and C, in which HX cuts the base of

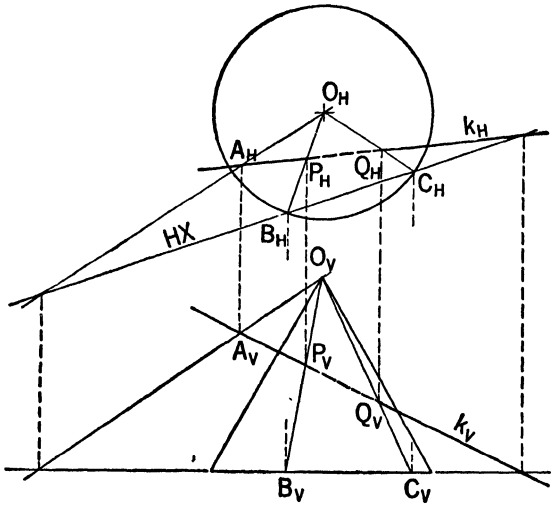


Fig. 157

the cone, locate the elements OB and OC, which are cut from the cone by plane X. The points P and Q, in which line k intersects OB and OC, are the points where line k pierces the surface of the cone.

99. To find the intersection of a plane with a ruled surface

Analysis: Points common to the plane and the given surface can be found by the use of auxiliary cutting planes. A cutting plane passing through an element of the surface cuts the given plane in a straight line. The point where this line intersects the element is a point of the required intersection.

The process of finding points on the line of intersection can also be described as follows: Represent the given surface by a system of elements. Find the points in which the elements pierce the given plane by the method of Article 38. A smooth curve drawn through these points represents the line of intersection.

Example 2. *The Intersection of a Plane and Cylinder. Method 1.* Let the cylinder (Figure 158) be intersected by the plane *KM*. A vertical plane *Y*, parallel to the axis of the cylinder, cuts the surface in elements 2 and 4. Plane *Y* also cuts plane *KM* in line *t*. Line *t* cuts elements 2 and 4 in points *B* and *D* on the intersection. Additional points are located by planes *X*, *Z*, and so forth. Since the cutting planes are parallel, the auxiliary lines *t*, *u*, and so forth are parallel.

Example 3. *Intersection of a Plane and Circular Cylinder. Method 2.* Any plane which makes an angle greater than 0° and less than 90° with the axis of a circular cylinder cuts an ellipse from the surface.

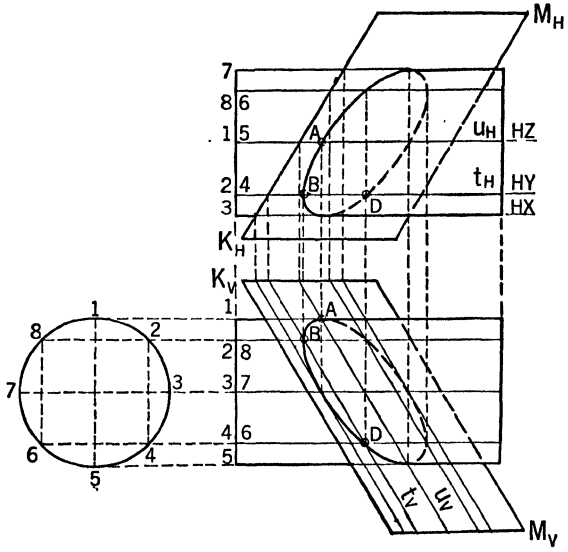


Fig. 158

The length of the minor axis of the ellipse is equal to the diameter (*D*) of the cylinder. The length of the major axis is $D/\sin\theta$, where θ is the angle between the plane and the axis. In Figure 159, the true angle between the cylinder and plane appears in the front view. Elements of the cylinder are first located in the end view of the cylinder and then projected to the front and top views.

The point in which each element pierces the plane is determined in the front view. The true shape of the ellipse determined by these points appears in the top view. Since the major and minor axes of the ellipse are given by lines 1-5 and 3-7 respectively, the curve could be constructed by a plane geometry construction such as the trammel method.

Example 4. *General Case* (*Figure 160*). Let the oblique plane *ABCD* intersect the circular cylinder which has for its axis the oblique

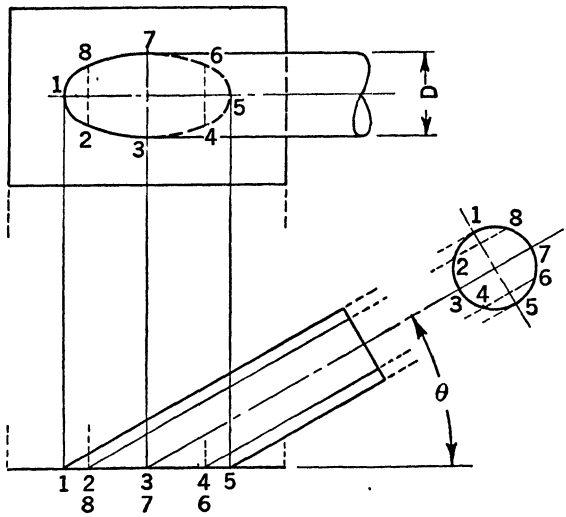


Fig. 159

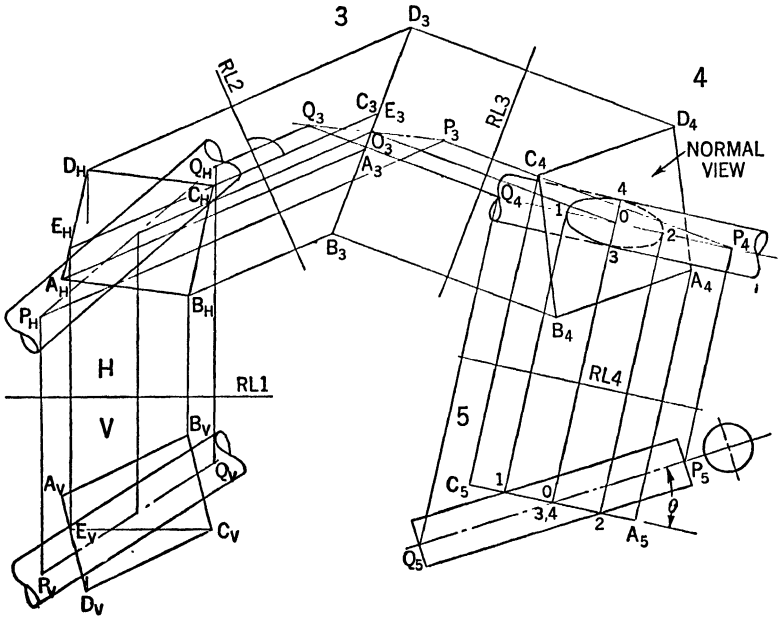


Fig. 160

line *PQ*. This general case can be reduced to the special case of Figure 159 by obtaining a view in which the plane projects edgewise and the line projects its true length.

Having given the top and front views, take *RL2* perpendicular to the horizontal line C_HE_H. Plane *ABCD* projects edgewise in view **3.**

Line PQ projects obliquely. Take $RL3$ parallel to B_3D_3. Plane $ABCD$ projects in true shape in view 4. Draw $RL4$ parallel to P_4Q_4. In view 5, plane $ABCD$ projects edgewise and line PQ projects in its true length. The true angle θ between the axis PQ and plane $ABCD$ is given by this view.

The solution developed for Figure 159 can now be applied to views 4 and 5. The major and minor axes of the ellipse are lines 1-2 and 3-4 respectively.

100. To find the intersection of a plane and a surface of revolution

Analysis: A series of cutting planes is passed perpendicular to the axis of the surface. Each cutting plane intersects the surface in a circle and the given plane in a straight line. The points in which the circle and line cut by the same plane intersect are points on the required section.

Construction:

Example 1 (Figure 161). Let the cone of revolution be intersected by the plane AB which appears edgewise in the front view. The horizontal plane P cuts the cone in the circle T and the plane in the line

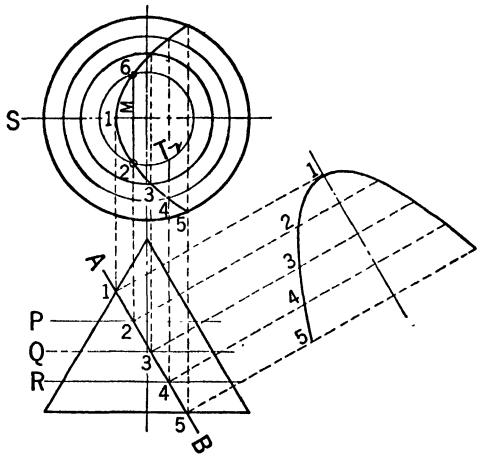


Fig. 161

M. Line M cuts the circle T in points 2 and 6 on the line of intersection. Additional points on the intersection are determined by planes Q and R. Point 1 is the highest point on the curve and lies on the contour element. Since the angle between plane AB and the axis of the cone is equal to that between the elements and the axis, the intersection is a parabola. The top view of the parabola is symmetrical about the center line S.

The true shape of the parabola is obtained by drawing an auxiliary view on a projection plane taken parallel to AB.

Example 2 (Figure 162). The cone of revolution is cut by the oblique plane ABC. This problem is reduced to the preceding basic case by drawing an auxiliary view in which the plane ABC projects edgewise. The construction of Example 1 is carried out in the auxiliary and top views. The front view of the intersection is projected from the

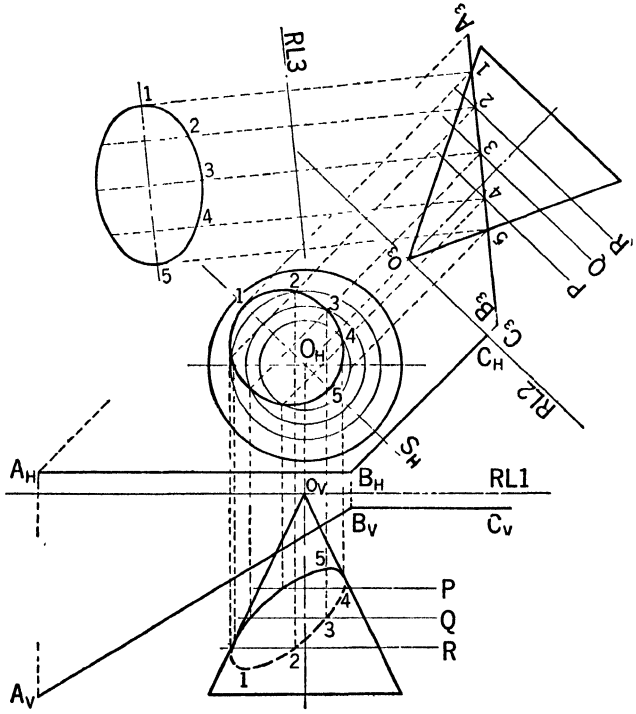


Fig. 162

top view. Since the plane ABC cuts every element of the cone, the intersection is an ellipse. The top view of the ellipse is symmetrical about line S which is drawn parallel to $RL2$.

Example 3 (Figure 163). *Plane and Torus.* The *torus* is a surface generated by revolving a circle about a line outside the circle as an axis. The torus is intersected by a vertical plane S. Frontal cutting planes cut circles from the torus. The intersection is a single curve. Certain positions of the plane produce an intersection consisting of two curves.

Example 4 (Figure 164). *Plane and Turned Solid.* The surface is intersected by planes R and Q. Frontal cutting planes cut circles from the turned surface. Each plane cuts sections composed of two parts.

101. Intersecting curved surfaces

The line of intersection of two curved surfaces is in general a space curve. The line is determined by the points which are common to both surfaces. Points common to two surfaces are found by use of an auxiliary surface selected to cut lines from the given surfaces. The points in

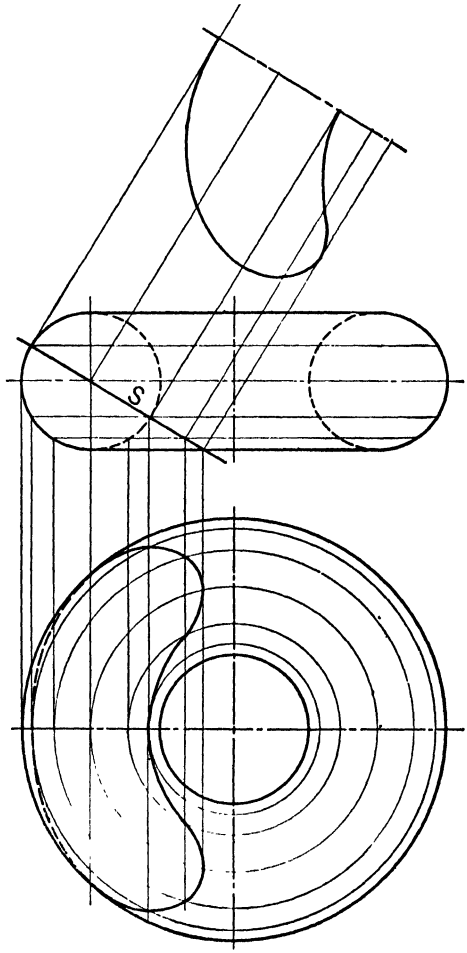


Fig. 163

which the lines cut from one surface intersect those cut from the other are on the line of intersection. A single auxiliary surface will determine one set of points.

The auxiliary cutting surface may be a plane, cylinder, or sphere, the plane being used most frequently. It should be located so as to cut the given surfaces in simple lines if possible; namely, straight lines or

circles. The location of cutting planes for three common cases is illustrated in Figures 165, 166, and 167.

Two Cylinders (*Figure 165*). A plane which is parallel to the elements of each cylinder cuts straight lines from the surfaces. Plane Q, edgewise in the front view, cuts elements AA', BB' and CC', DD' from the given

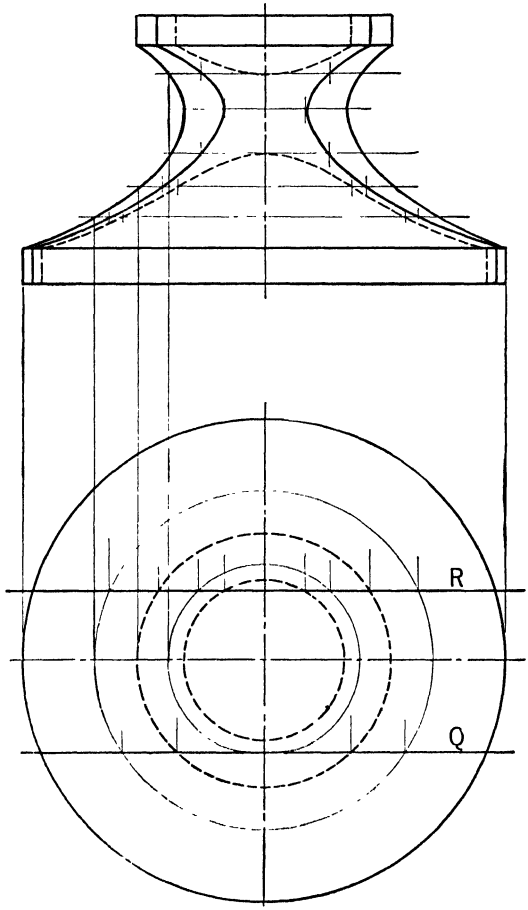


Fig. 164

surfaces. These elements intersect to determine four points common to both cylinders.

Circular Cone and Cylinder (*Figure 166*). A plane which contains the vertex of the cone and is parallel to the elements of the cylinder cuts straight-line elements from each surface. Plane Q cuts the cylinder in elements CC' and DD'. The cone is cut in elements OA and OB. This set of elements intersect each other in four points which are common to the cone and cylinder.

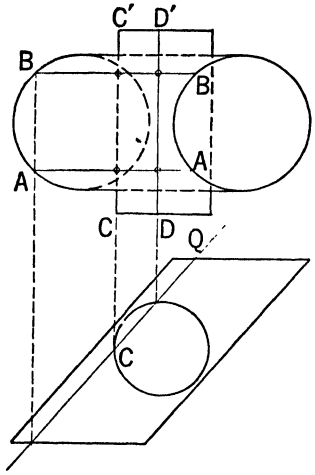


Fig. 165

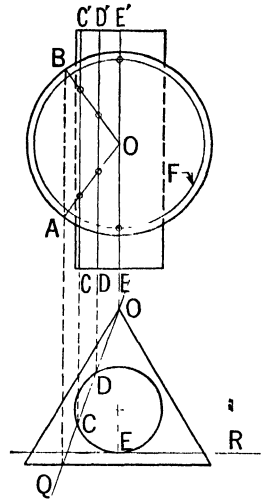


Fig. 166

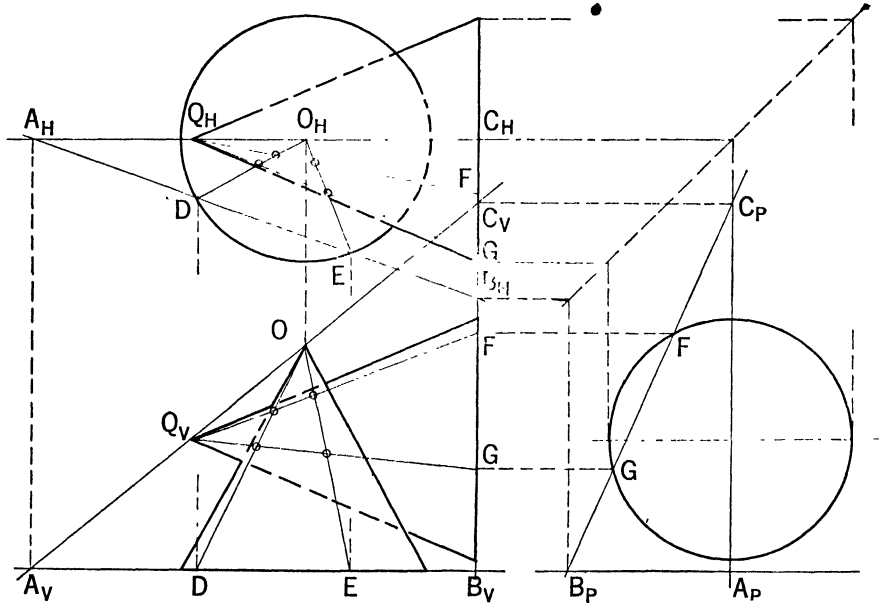


Fig. 167

A cutting plane R, perpendicular to the axis of the cone, intersects the surface in the circle F. Plane R is also tangent to the cylinder, containing the element EE'. The two points in which the element cuts the circle are on the line of intersection of the surfaces.

Two Cones (*Figure 167*). A plane which contains the vertex of each cone cuts each surface in straight-line elements.

Draw line OQ, which intersects the plane of the base of cone O in point A; the plane of the base of cone Q, in point C. Draw a line AB, cutting the base of cone O in points D and E and plane of the base of cone Q in point B. A plane passed through lines AC and AB cuts cone O in elements OD and OE.

Construct the profile view of the base of cone Q. Plane CAB cuts this base curve along line BC in the points F and G, and, therefore, in the elements QF and QG. Four points common to both cones are determined by the two sets of elements.

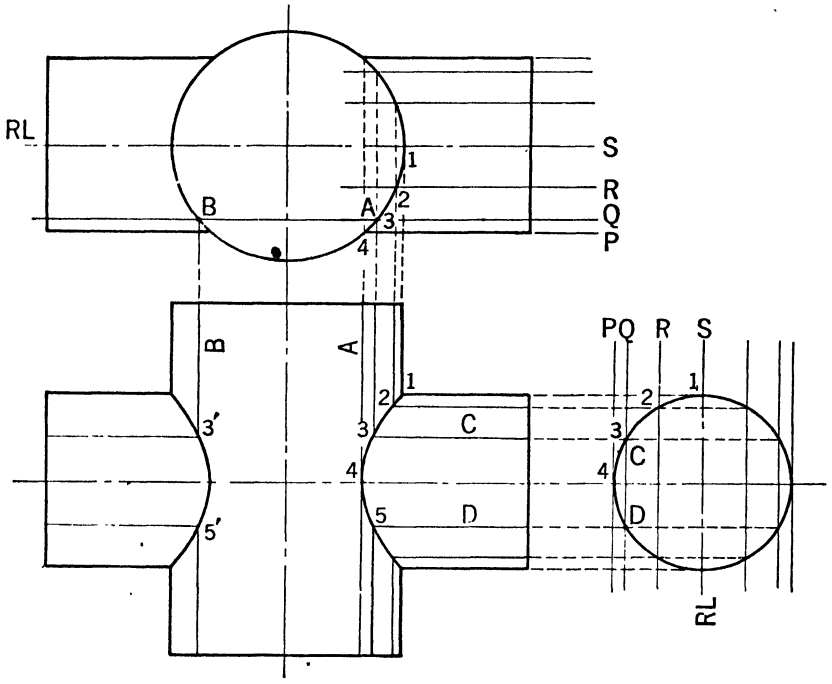


Fig. 168

A series of auxiliary cutting surfaces is necessary to determine a sufficient number of points to define the entire intersection between two curved surfaces.

Example 1. *Two Cylinders* (*Figure 168*). As in Figure 165, cutting planes are taken parallel to the axes of the two intersecting cylinders.

The axes of the given cylinders are parallel to the V-plane. Therefore, the cutting planes P, Q, R, and S, taken parallel to V, project edgewise in the top and side views. Plane Q cuts the vertical cylinder in elements A and B; Q cuts the horizontal cylinder in elements C and D. These elements intersect in points 3, 3′, 5, and 5′ respectively. Additional points are located by planes S, R, and P.

Example 2. *Cylinder and Sphere* (*Figure 169*). Auxiliary planes, as Q, Q', taken perpendicular to the axis of the cylinder, cut circular sections from both surfaces. These circles intersect in the points 1,2 and 1′,2′ on the line of intersection. The highest and lowest points, 3 and 3′ on the intersection, are determined by the planes R and R',

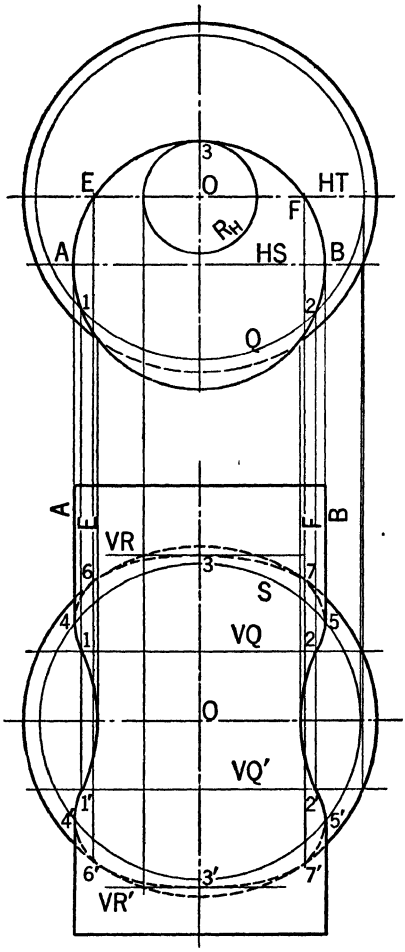


Fig. 169

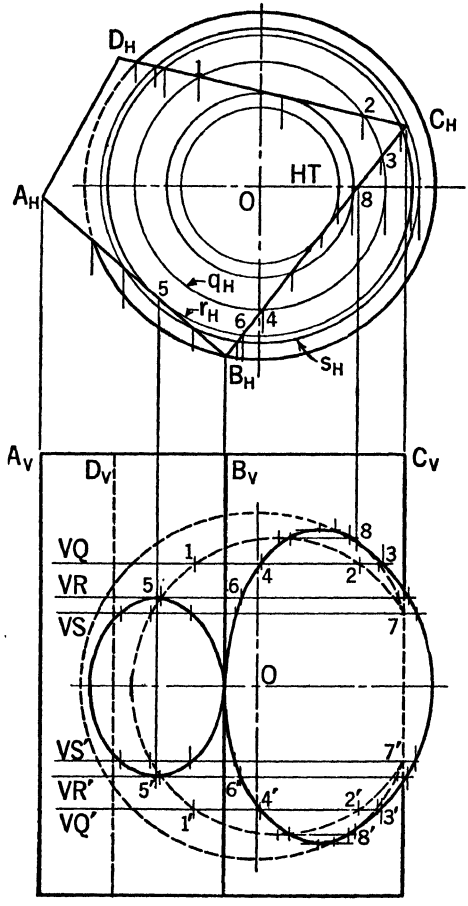


Fig. 170

located from the assumed circular section R_H, tangent to the circle view of the cylinder. The points 4,5 and 4′,5′, where the contour elements of the cylinder pierce the sphere, are determined by the plane S, taken parallel to V through the axis of the cylinder. The plane T, through the center of the sphere and parallel to the V-plane, cuts the cylinder in the elements E and F, locating points 6,6′ and 7,7′, at which the line of

intersection is tangent to the contour of the sphere. The visible points of the line of intersection lie on the front half of the cylinder.

Example 3. *Sphere and Prism* (*Figure 170*). Any auxiliary cutting plane parallel to the H-plane intersects the sphere in a circle and the prism in a quadrilateral. The intersections of the boundary lines of these sections are points common to both solids and, therefore, lie in their line of intersection.

The plane Q cuts a circle from the sphere, projected at q_H, and quadrilateral from the prism which, in the top view, coincides with the base $A_H B_H C_H D_H$. These sections intersect in the top view in points 1, 2, 3, and 4. The front views of these points lie on the edge view of plane Q.

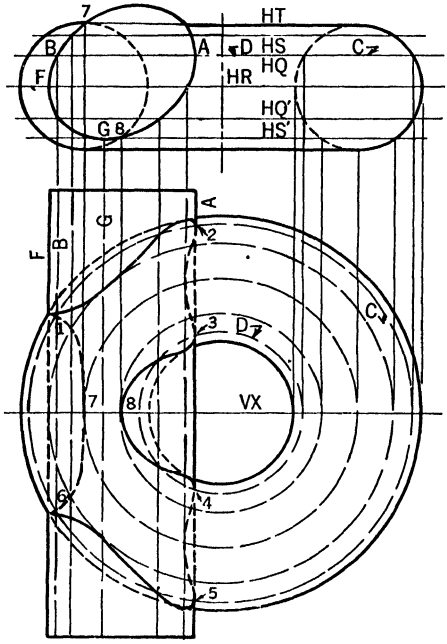


Fig. 171

To find the highest and lowest points on the line of intersection. The highest point on face AB of the prism is found by drawing the circle r_H tangent to $A_H B_H$. This circle lies on the sphere, and determines a plane R which is found in the front view. The point of tangency 5 is projected to plane R. This is the highest point in face AB. Other limiting points are found in a similar manner.

The points 8 and 8′ where the line of intersection is tangent to the contour of the sphere are found by using the meridian plane T of the sphere.

Example 4. *Torus and Elliptical Cylinder* (*Figure 171*). Auxiliary planes are taken parallel to the axis of the cylinder and perpendicular to the axis of the torus. A plane, as Q, cuts the cylinder in the elements A and B, and the torus in circles C and D, giving six points, 1-2-3-4-5-6, on the line of intersection. The chosen secant planes include the following: planes R and Q, containing the contour elements F and A of the cylinder; the limiting planes S and T, parallel to V and tangent to the cylinder and torus respectively; the plane X, through the axis of the torus and parallel to H, locating points 7 and 8.

Example 5. *Two Surfaces of Revolution with Intersecting Axes* (*Figure 172*). The solution is determined in the view in which the axes of both surfaces are parallel to the projection plane. Therefore, only this view

is shown in Figure 172. The auxiliary cutting surfaces used in this case are spheres. A sphere having its center in the axis of a surface of revolution and cutting the surface intersects it in two circles.

With the intersection of the axes as a center, describe a series of spheres which cut the given surfaces, and let the sphere R be taken as typical. It will be seen that R intersects the cone A in the circles U and V; the cone B, in the circles S and T. These circles appear edgewise. Circles S and U intersect in point 1; circles T and V, in point 2; circles S and V, in point 3, and so forth. Other points are found by the use of additional spheres. The limiting points 4 and 5 are determined by the sphere Q which is tangent to cone B. Any sphere smaller than Q will not cut cone B.

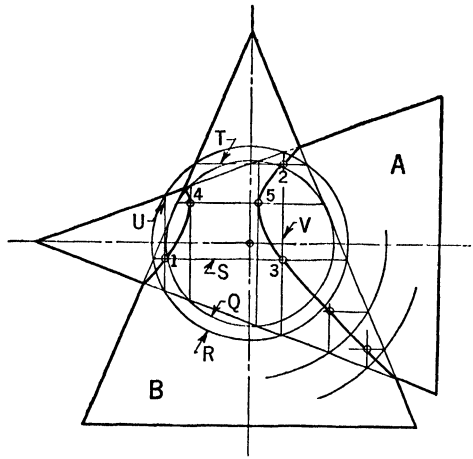


Fig. 172

The intersection consists of two separate curves. The contour elements on the cones determine limiting points on the curves.

If the given surfaces are of such a size that they are tangent to the

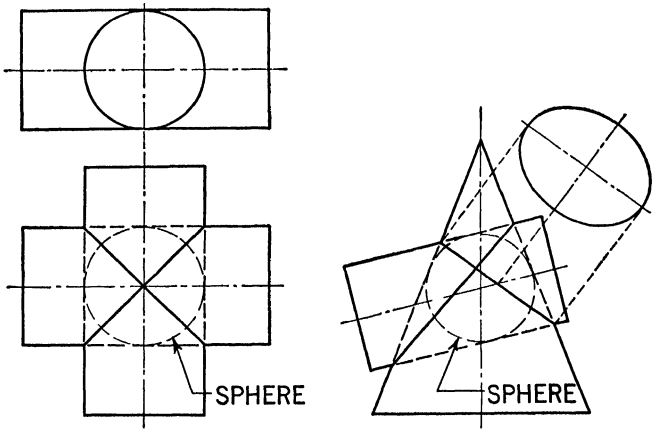


Fig. 173

same sphere, the two lines of intersection join as shown in the following example.

Example 6. *Intersection of Surfaces, Special Case (Figure 173).* Cones and cylinders enveloping the same sphere intersect in plane curves

which are conic sections. The lines of intersection are straight lines in the view in which the axes of the given surfaces appear in true length. The intersections of the surfaces illustrated are ellipses which appear edgewise in the front views.

Sheet-metal elbows and breechings are generally designed to utilize this principle. Fabrication of such structures is simplified when cross sections are circular and joints are plane curves.

102. Development of curved surfaces

The only curved surfaces capable of exact development are those about which a sheet of flexible material can be wrapped without stretching or other distortion. Exact development is thus limited to the surfaces of single curvature; namely, the cone, cylinder, and a surface known as the *convolute* which is described in the following chapter. Approximate developments of other curved surfaces can be made. These are sufficiently precise for many practical purposes.

103. To develop a cone (Figure 174)

The method of development used for the pyramid (Article 84) can be applied to the cone.

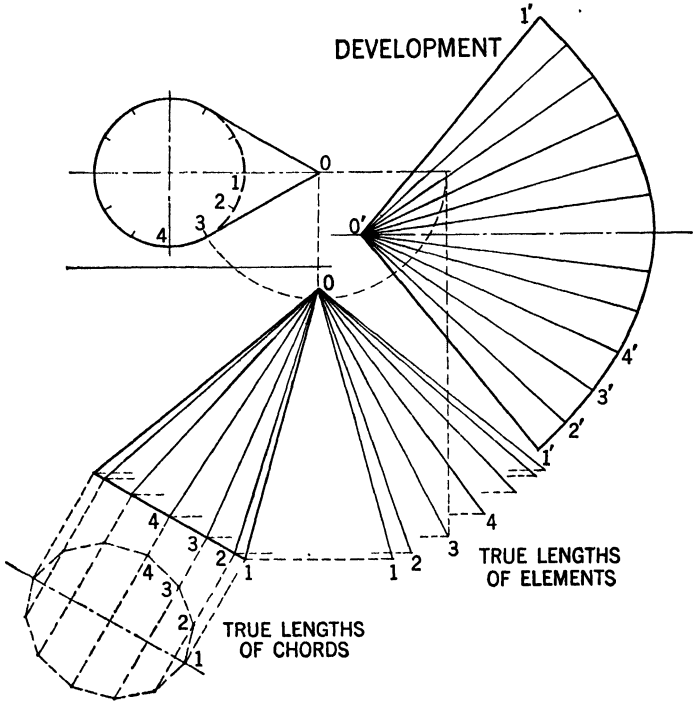


Fig. 174

The base of the cone is divided into a number of parts, and elements of the surface are drawn through the division points. The surface is then developed as a many-sided pyramid. The chord between two consecutive points of the base is used to measure the distance between the ends of two consecutive elements on the development. A smooth curve drawn through ends of the developed elements represents the base line of the cone expanded to the development of the surface. The length of the expanded base line is equal to the perimeter of the base of the cone. There will be an error in this length due to the use of basal chords in laying

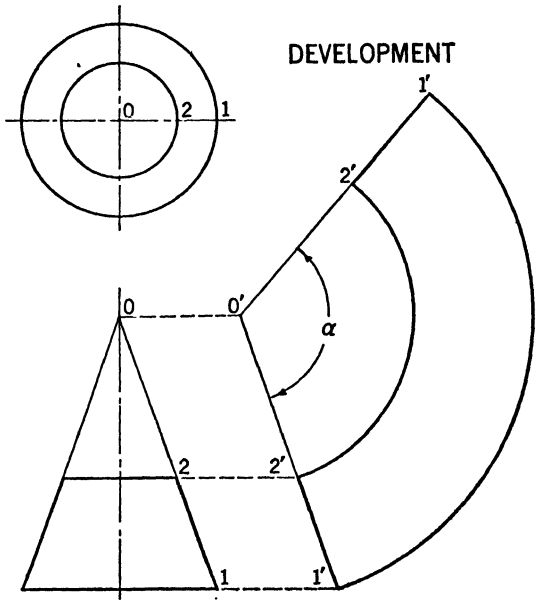


Fig. 175

off the elements. This error can be kept small, however, by using short chords.

The right cone (Figure 175) can be developed without approximation since the base expands into a circular arc having the slant height of the cone as its radius. The angle at the vertex of this development (the sector angle) is equal to

$$360° \times \frac{\text{radius of base}}{\text{slant height}}.$$

104. To develop a cylinder (Figure 176)

The method of development used for the prism (Article 85) can be applied to the cylinder. The true-size view of the right section of the cylinder is divided into a number of equal parts (twelve in this case),

and elements of the surface are drawn through the division points. The surface is then developed as a many-sided prism.

The right section expands into a straight line on the development. This is the girth line, and the elements are perpendicular to it. The

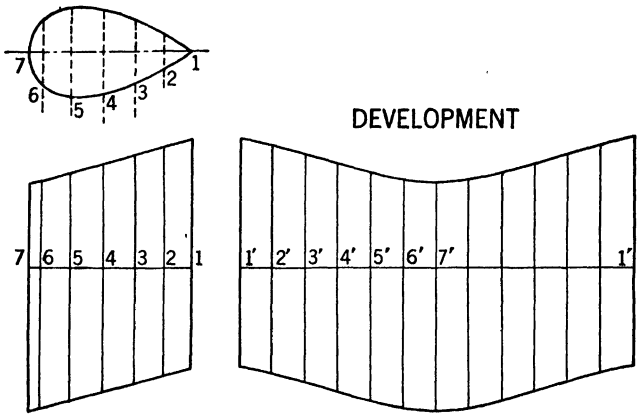


Fig. 176

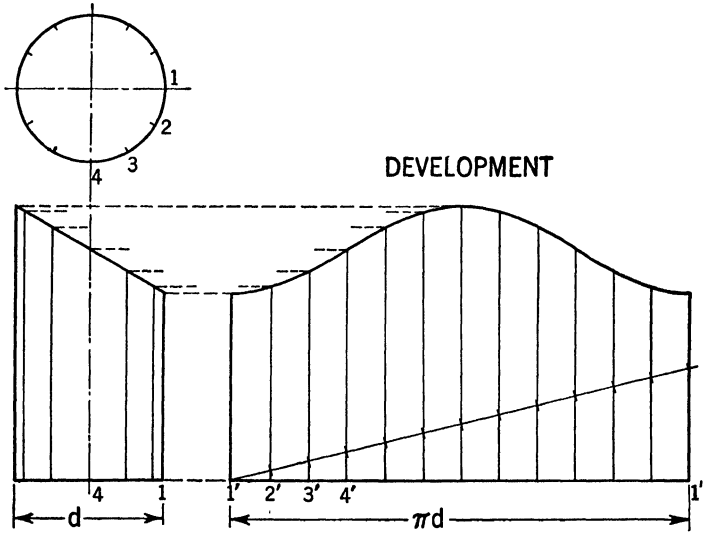


Fig. 177

spacing of the elements on the girth line is taken from the true-size view of the right section.

The chord between two consecutive points of the base is used to space the elements on the girth line. Points should be taken sufficiently close on the right section to keep the error in the length of the girth line small.

The development of a circular cylinder can be made accurate by calculating the length of the girth line. In Figure 177, the length of the girth line is πD. It is divided into twelve equal parts to correspond to the twelve divisions of the right section.

105. To develop the frustum of a cone (Figure 178)

It is assumed that the vertex of the cone is inaccessible. The upper and lower bases of the frustum are divided into a number of parts starting with the same element. The surface between two consecutive elements

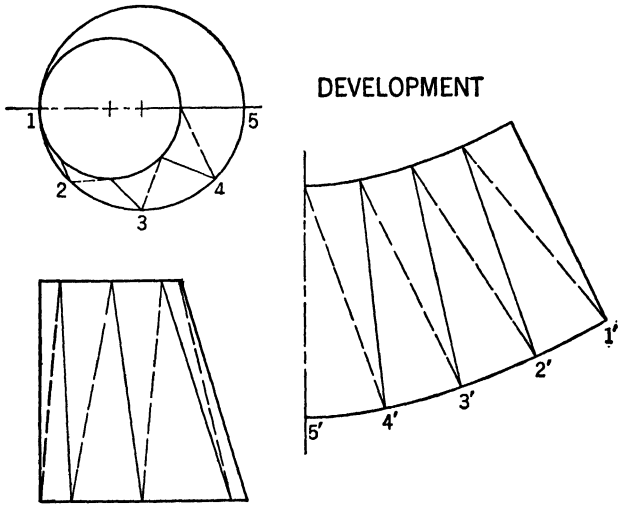


Fig. 178

is approximated by the trapezoid formed by the two elements and the two basal chords joining their ends. This trapezoid is divided into two triangles and the surface developed approximately as a many-sided pyramidal frustum. A symmetrical half of the surface is developed.

106. Approximate developments

Surfaces that are theoretically non-developable may be developed approximately by considering that the surface is made up of small portions of developable surfaces. Double curved surfaces may be approximated by small portions of cones or cylinders. A sphere, ellipsoid, or torus can be divided into narrow gores by a series of meridian planes, and each gore developed as a cylindrical surface. Also, these surfaces can be cut by a series of parallel planes into zones and each zone developed as a conical frustum.

The gore shown in Figure 179 is tangent to the surface of the sphere

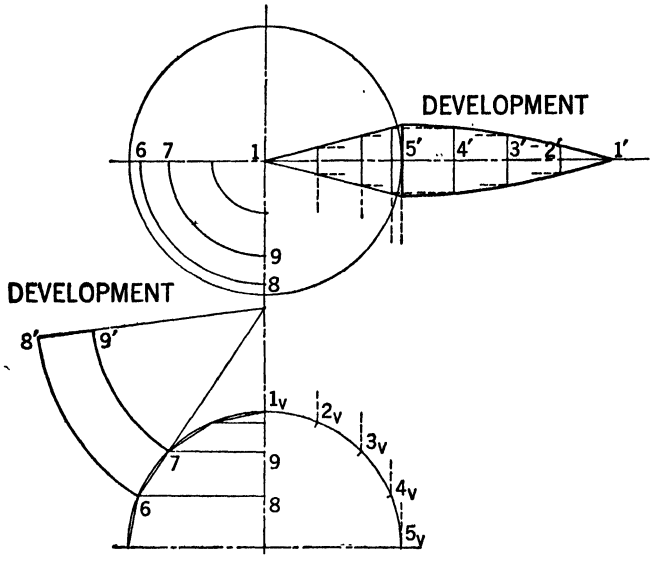


Fig. 179

and represents one-twelfth of the hemisphere. The quarter circle 1_V-5_V is the right section and 1′-5′ the stretch-out of the cylindrical surface.

The conical surface 6-7-8-9 approximates one-fourth of the surface of an intermediate zone of the sphere.

PROBLEMS ON CHAPTER VI

Group A. Tangent Planes

6-A1. A horizontal circle, $2\frac{1}{2}$ inches in diameter and centered on point C:417, is the base of a right cone 3 inches high. (*a*) Through point A:$3\frac{1}{2}$,X,$6\frac{1}{2}$, which lies in the surface of the cone, pass a plane Q tangent to the cone. (*b*) Through point B:$5\frac{3}{4}$,$1\frac{3}{4}$,$7\frac{3}{4}$ pass plane R tangent to the front surface of the cone. (*c*) Find the intersections of planes Q and R with the H- and V-projection planes. Origin: 055.

6-A2. Point C:4,$2\frac{1}{2}$,$7\frac{1}{4}$ is the center of the 3-inch base and A:1,$2\frac{1}{2}$,$7\frac{1}{4}$ the apex of a right circular cone. Through point B:$3\frac{1}{4}$,3,X, which lies in the surface of the cone, pass a plane Y tangent to the cone. Represent the plane by its H- and V-traces. Origin: 455.

6-A3. Point C:$3\frac{1}{2}$,$2\frac{1}{2}$,$6\frac{1}{2}$ is the center of the 3-inch base and point A:1,$2\frac{1}{2}$,8 is the apex of a right circular cone. Through point P:3,2,$8\frac{1}{4}$ pass two planes, each tangent to the cone. Represent each plane by its element of tangency and the tangent to the base curve.

6-A4. Point C:$3\frac{1}{2}$,1,$8\frac{1}{4}$ is the center of the 2-inch base and A:$1\frac{1}{4}$,3,$6\frac{1}{2}$ is the apex of a right circular cone. Through point P:3,$2\frac{3}{4}$,$8\frac{1}{2}$ pass two

planes, each tangent to the cone. Represent each plane by the element of tangency and the tangent to the base curve.

6-A5. Point P:5½,X,6¼ lies in the upper portion of the surface of a torus generated by the rotation of the 1½-inch frontal circle centered on C:2¾,2,7½ about line J:4,4½,7½ K:4,½,7½ as an axis. Pass a plane tangent to the torus at point P. Represent the plane by (*a*) its H- and V-principal lines through P, or (*b*) its H- and V-traces. Origin: 055.

6-A6. Point P:3¼,X,6 lies in the surface of a 3-inch diameter sphere centered at point O:427. Find the two possible positions of P. Pass a plane tangent to the sphere at each position of P. Represent each plane by its H- and V-principal lines drawn through point P.

6-A7. Line C:557 D:318 is the axis of a cylinder. The base centered on point C is a horizontal 2-inch circle. Through point P:3,X,7 in the upper side of the surface of the cylinder, construct a tangent plane W. Represent plane W (*a*) by its H- and V-principal lines through point P, (*b*) by its H- and V-traces. Origin: 055.

GROUP B. THE CONE. DEVELOPMENT AND PLANE SECTIONS

6-B1. Point C:218 is the center of the 3-inch horizontal circular base of a right cone 4 inches high. Develop the surface of the cone omitting the base. Points A:1,X,7 and B:2¾,X,8 lie in the surface of the cone. Find the shortest distance between A and B measured on the surface of the cone. Calculate the sector angle.

6-B2. Point C:1½,¾,8 is the center of the horizontal 2½-inch circular base of an oblique cone. The vertex is at A:2¾,4¼,5¾. The cone is truncated by a plane which passes through point E:1¾,3½,X, slopes downward to the right, makes an angle of 60° with the base and projects as a line in the front view. Develop the surface of the cone which lies between the base and the sloping plane.

6-B3. Line A:258 B:218 is the axis of a reducing connection which has the form of a frustum of a right cone. The upper and lower bases are 3½-inch and 4-inch circles respectively. Develop one quarter of the surface by triangulation. Divide the base into sixteen parts.

6-B4. Point O:2,1,7½ is the center of the horizontal 3¼-inch circular base of an oblique cone. Point V:8,5,9⅛ is the vertex. Find the line of intersection between the cone and the plane determined by line A:½,1½,9¼ B:5¾,1¾,9¾ and point C:3,3¾,6. Draw twelve elements of the surface equally spaced on the base.

6-B5. Point C:5¼,¾,7¾ is the center of the horizontal 3-inch circular base of an oblique cone having its vertex at O:7¾,3¾,6¼. Find the line

of intersection between the cone and the plane determined by points D:6,$\frac{1}{4}$,5$\frac{3}{4}$ E:4$\frac{1}{2}$,3$\frac{3}{4}$,7$\frac{3}{4}$ F:8,3$\frac{3}{4}$,9$\frac{3}{4}$. Find the true shape of the section.

6-B6. Point C:2$\frac{3}{4}$,$\frac{1}{4}$,9$\frac{3}{4}$ is the center of the 5-inch horizontal circular base of a right cone 6 inches high. Through point E:2$\frac{3}{4}$,3$\frac{1}{2}$,X pass a plane which makes a 45° angle with the axis of the cone, slopes downward to the right, and appears edgewise in the front view. Find the true shape of the elliptical section. Locate the foci and directrices.

6-B7. Point C:2$\frac{1}{4}$,$\frac{3}{4}$,7$\frac{1}{2}$ is the center of the 4-inch horizontal circular base of a right cone 3$\frac{7}{8}$ inches high. Through point E:3,$\frac{3}{4}$,X pass a receding plane which slopes upward to the left and makes the same angle with the base as the elements. Complete the plan, showing the line of intersection; and draw the true-shape view of the section, showing the focus and directrix of the curve. (See Article 211).

6-B8. Line A:5$\frac{3}{4}$,$\frac{1}{4}$,9$\frac{3}{4}$ B:5$\frac{3}{4}$,7$\frac{1}{4}$,9$\frac{3}{4}$ is the axis of a cone of revolution of two nappes. Point C:5$\frac{3}{4}$,3$\frac{3}{4}$,9$\frac{3}{4}$ is the vertex. The two bases are each 4$\frac{1}{2}$ inches in diameter. The two nappes of the cone are cut by a profile plane taken $\frac{3}{4}$ inch to the left of the axis. Draw a view showing the true shape of the section and the foci and directrices of the curve.

6-B9. Construct, by use of plane geometry definitions, the conics found in problems 6-B4, 6-B5, and 6-B6 by actual sectioning of a cone.

(*a*) *Ellipse.* Locate center at 1$\frac{3}{4}$,7$\frac{1}{4}$ with major axis parallel to left edge of sheet. Take the major axis and foci from the front view of problem 6-B6. (See Article 211).

(*b*) *Parabola.* Locate the focus at 5,7$\frac{1}{4}$. Draw the axis parallel to top edge of sheet. Take the focus and directrix from the front view of problem 6-B7.

(*c*) *Hyperbola.* Locate the center at 4,2$\frac{1}{2}$. Draw the transverse axis parallel to lower edge of the sheet. Take the foci and vertices from the front view of problem 6-B8.

GROUP C. THE CYLINDER. DEVELOPMENT AND PLANE SECTIONS

6-C1. Line A:125 B:128 is the axis of a cylinder 1$\frac{1}{2}$ inches in diameter. The front base lies in a vertical plane which passes through point A, makes an angle of 45° with the axis, and slopes backward to the right. The rear base lies in a vertical plane which passes through point B, makes an angle of 30° with the axis, and slopes forward to the right. Develop the lateral surface of the cylinder. Calculate the length of the girth line. Draw 12 elements of the surface.

6-C2. Line A:1,4$\frac{1}{4}$,8 B:4$\frac{1}{2}$,6$\frac{1}{4}$,8 is the axis of an elliptical cylinder. The base at A is a 1$\frac{3}{4}$-inch horizontal circle. The base at B lies in a profile plane. Develop the surface omitting the bases. Use 12 elements equally spaced on the right section. Make joint on shortest element.

6-C3. Find the line of intersection between the plane A:047 B:519 C:745 and a right circular cylinder 2 inches in diameter having the line D:417 E:457 for its axis.

6-C4. The lower base of a cylinder is a $2\frac{1}{2}$-inch horizontal circle centered on point C:2,1,$7\frac{3}{4}$. Point D:5,$3\frac{1}{2}$,6 lies on the axis of the cylinder. The upper base lies in the plane which passes through points P:1,$2\frac{1}{2}$,$5\frac{1}{2}$ Q:4,1,$4\frac{1}{2}$ R:5,$4\frac{1}{2}$,8. Complete the top and front views of the cylinder.

6-C5. Line C:$4\frac{1}{4}$,6,$8\frac{1}{2}$ E:$6\frac{3}{4}$,4,$8\frac{1}{2}$ is the axis of a cylinder. The upper base is a 2-inch horizontal circle centered on point C. The lower base lies in the plane passing through points Q:$3\frac{3}{4}$,4,7 R:$5\frac{3}{4}$,6,7 S:$7\frac{3}{4}$,6,9. Complete the top and front views. Develop the surface of the cylinder.

GROUP D. THE SPHERE. DEVELOPMENT AND PLANE SECTIONS

6-D1. Point C:$2\frac{1}{4}$,$2\frac{1}{2}$,7 is the center of a 3-inch sphere. Point D:2,$1\frac{1}{2}$,X is on the back and point E:$3\frac{1}{2}$,$2\frac{1}{2}$,X on the front of the surface. Draw the top and front views of the sphere and the great circle passing through points D and E.

6-D2. Point O:437 is the center of a $3\frac{3}{4}$-inch sphere. Find the line of intersection between the sphere and the plane containing points A:$\frac{3}{4}$,$1\frac{1}{4}$,7 B:$2\frac{1}{2}$,$4\frac{3}{4}$,5 C:$6\frac{3}{4}$,5,$7\frac{1}{2}$.

6-D3. Approximate development of sphere. Point C:419 is the center of a hemisphere 8 inches in diameter. Draw front view only. Divide the surface into eight zones of equal width, and develop the front half of each of the four upper zones. Calculate the sector angles for each cone.

6-D4. Approximate development. Point C:419 is the center of a hemispherical dome 6 inches in diameter. Divide the front half of the surface in twelve equal meridian sections, or gores. Develop the surface of one gore. Approximate the spherical surface with a cylindrical surface *tangent* to the sphere along the center line of one gore.

GROUP E. INTERSECTIONS

6-E1. Cutting Planes. Point C:217 is the center of the $2\frac{1}{2}$-inch horizontal circular base of a right cone 3 inches high. Line A:$\frac{1}{2}$,$2\frac{1}{4}$,$7\frac{1}{4}$ B:$3\frac{1}{2}$,$2\frac{1}{4}$,$7\frac{1}{4}$ is axis of a circular right cylinder 2 inches in diameter. Draw the top, front, and side views of the cone and cylinder. Pass a cutting plane through the vertex of the cone and parallel to the axis of the cylinder intersecting the base of the cone 1 inch in front of point C. Considering the cone and cylinder as solids, section-line the portion of each solid which is cut by the plane. Encircle the points which are common to both surfaces.

6-E2. Cutting Planes. Draw the top, front, and side views of the cone and cylinder of problem 6-E1. Pass a cutting plane perpendicular to the axis of the cone and $\frac{3}{4}$ inch above the base. Section-line the portion of each solid which is cut by the plane. Encircle the points which are common to both surfaces.

6-E3. Cone and Cylinder. Point C:219 is the center of the 4-inch horizontal circular base of a right cone 4 inches high. Line D:2,2$\frac{1}{4}$,6 E:2,2$\frac{1}{4}$,9 is the axis of a 2-inch right circular cylinder. Determine the line of intersection of the two surfaces. Draw the top, front, and side views representing the cylinder and the front half of the cone.

6-E4. Line A:5$\frac{3}{4}$,5,7$\frac{1}{4}$ B:5$\frac{3}{4}$,1,7$\frac{1}{4}$ is the axis of a right cylinder 3$\frac{1}{2}$ inches in diameter. Line C:3$\frac{1}{2}$,3,6$\frac{3}{4}$ D:8,3,6$\frac{3}{4}$ is the axis of a right cylinder 3 inches in diameter. Find the line of intersection between the two cylinders.

6-E5. Line A:7,4,7$\frac{1}{2}$ B:4$\frac{1}{2}$,$\frac{3}{4}$,7$\frac{1}{2}$ is the axis of an oblique cylinder. The horizontal bases are 3$\frac{1}{2}$ inches in diameter. Line C:3,4,7$\frac{1}{4}$ D:4$\frac{1}{2}$,1,7$\frac{1}{4}$ is the axis of an oblique cylinder. The upper base is horizontal and 2$\frac{1}{2}$ inches in diameter. Draw the left half of the surface of cylinder AB, and find the line in which this surface cuts cylinder CD.

6-E6. Two Cones. Line A:3,4$\frac{1}{2}$,7 B:317 is the axis of the frustum of a right cone. The diameters of the upper and lower bases are 4 and 2 inches respectively. Line C:417 D:6$\frac{1}{2}$,4$\frac{1}{2}$,7 is the axis of the frustum of an oblique cone. The upper base is a 1$\frac{1}{2}$-inch horizontal circle. The lower base is a 4-inch horizontal circle. Find the line of intersection of the two surfaces.

6-E7. Line A:5,2$\frac{1}{4}$,5 B:5,2$\frac{1}{4}$,9 is the axis of a right cylinder 2 inches in diameter. The 3$\frac{1}{2}$-inch horizontal circle centered at C:317 is the base of an oblique cone having its vertex at O:7,5,8$\frac{3}{4}$. Find the line of intersection of the surfaces.

6-E8. Point A:3$\frac{1}{2}$,1,7$\frac{1}{2}$ is the center of the 3$\frac{1}{2}$-inch horizontal base of a cone of revolution. The altitude is 4 inches. Line B:5$\frac{1}{4}$,2$\frac{1}{4}$,6 C:1$\frac{3}{4}$,2$\frac{1}{4}$,8 is the axis of a right cylinder 1$\frac{3}{4}$ inches in diameter. Find the line of intersection between the two surfaces.

6-E9. Point C:4,1,6$\frac{3}{4}$ is the center of the 5-inch horizontal base of a cone of revolution. The altitude is 3 inches. Line D:316 E:3,4$\frac{1}{4}$,6 is the axis of a right cylinder. The radius of each base is equal to CD. Find the line of intersection between the two surfaces.

6-E10. Line A:3$\frac{1}{2}$,4,8 B:3$\frac{1}{2}$,$\frac{1}{2}$,8 is the axis of a right cylinder 3 inches in diameter. Line C:1$\frac{3}{4}$,1$\frac{3}{4}$,7$\frac{1}{2}$ D:5$\frac{1}{4}$,3$\frac{3}{4}$,7$\frac{1}{2}$ is the axis of a right cylinder 2 inches in diameter. Find the line of intersection of the solids.

6-E11. Line B:$3\frac{1}{2},\frac{1}{2},7\frac{1}{2}$ C:$3\frac{1}{2},5,7\frac{1}{2}$ is the axis of a right cylinder $2\frac{3}{4}$ inches in diameter. Point A:$3\frac{3}{4},2\frac{3}{4},8$ is the center of a 4-inch sphere. Find the line of intersection between the two surfaces.

6-E12. Line B:$3\frac{1}{2},\frac{1}{2},7\frac{1}{2}$ C:$3\frac{1}{2},5,7\frac{1}{2}$ is the axis of a right cylinder 3 inches in diameter. Point A:$3\frac{3}{4},2\frac{3}{4},8$ is the center of a 4-inch sphere. Find the line of intersection between the two surfaces.

6-E13. Line B:$3\frac{3}{4},\frac{1}{4},7\frac{3}{4}$ C:$3\frac{3}{4},5,7\frac{3}{4}$ is the axis of a right cone. The base is a 4-inch circle centered at B. Point A:$4,2,7\frac{1}{4}$ is the center of a $3\frac{1}{4}$-inch sphere. Find the intersection of the surfaces.

6-E14. Point A:$3,3\frac{1}{4},7\frac{3}{4}$ is the center of a 4-inch sphere. Point B:$4\frac{1}{2},2\frac{1}{2},7\frac{1}{4}$ is the center of a $4\frac{1}{2}$-inch sphere. Find the intersection of the two surfaces.

6-E15. Line B:$4,4\frac{1}{2},7\frac{1}{4}$ C:$4,\frac{1}{4},7\frac{1}{4}$ is the axis of a right cone having a circular base 5 inches in diameter centered at C. Line D:$1\frac{1}{4},2\frac{1}{4},7\frac{1}{4}$ E:$6\frac{3}{4},2\frac{1}{4},7\frac{1}{4}$ is the axis of a right cone having a 4-inch diameter base centered at D. Find the intersection of the surfaces.

6-E16. Cone and Cylinder Enveloping Same Sphere. Point C:459 is the center of a 2-inch sphere. On line A:149 B:769 as an axis, draw a cylinder tangent to this sphere. With point V:489 as a vertex and VD:429 as an axis, draw a cone tangent to the sphere. Draw the lines of intersection of the cone and the cylinder. Draw the normal view of the axes of one ellipse. Omit the top view in this problem.

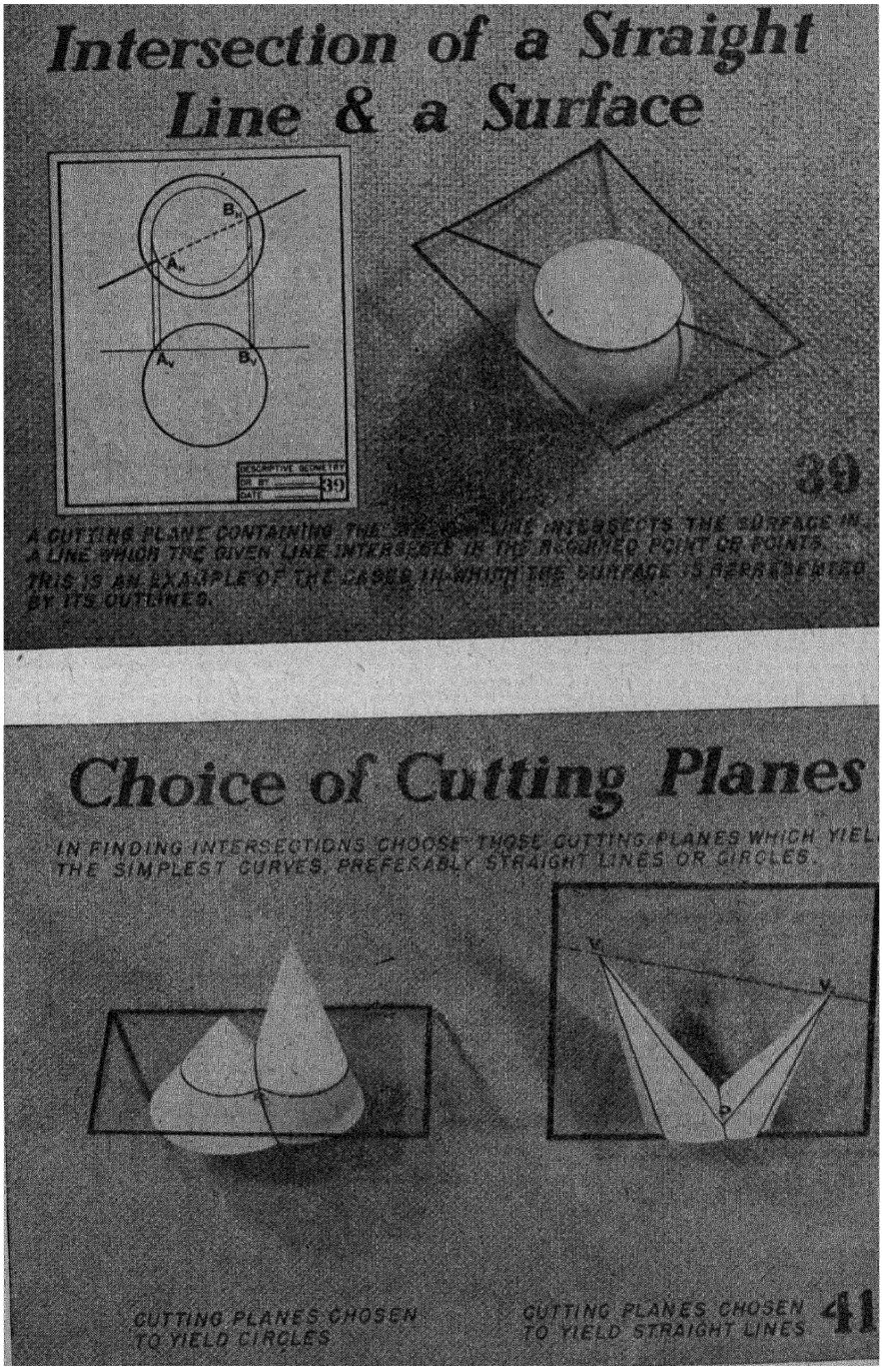
6-E17. A torus is generated by revolving a 2-inch frontal circle centered at point A:$2\frac{1}{2},2,6$ about a vertical axis through point B:426. Draw the top and front views of the torus and find the line of intersection between this surface and the receding plane J:$1\frac{1}{2},1,X$ K:$5,3\frac{1}{2},X$.

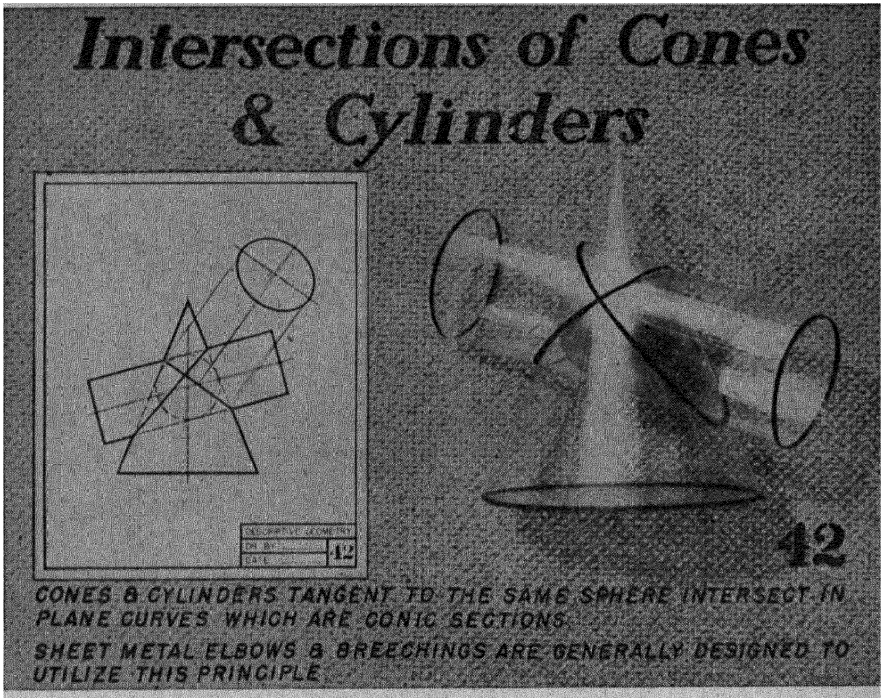
6-E18. The $2\frac{1}{2}$-inch horizontal circle, centered at A:$1\frac{3}{4},1,7\frac{1}{2}$, is revolved about the axis B:416 C:419 to form a 180° pipe bend. The $2\frac{1}{2}$-inch horizontal circle, center D:$5,5\frac{1}{2},7\frac{1}{2}$, is the open end of a vertical pipe which intersects the bend. Complete the views showing the line of intersection.

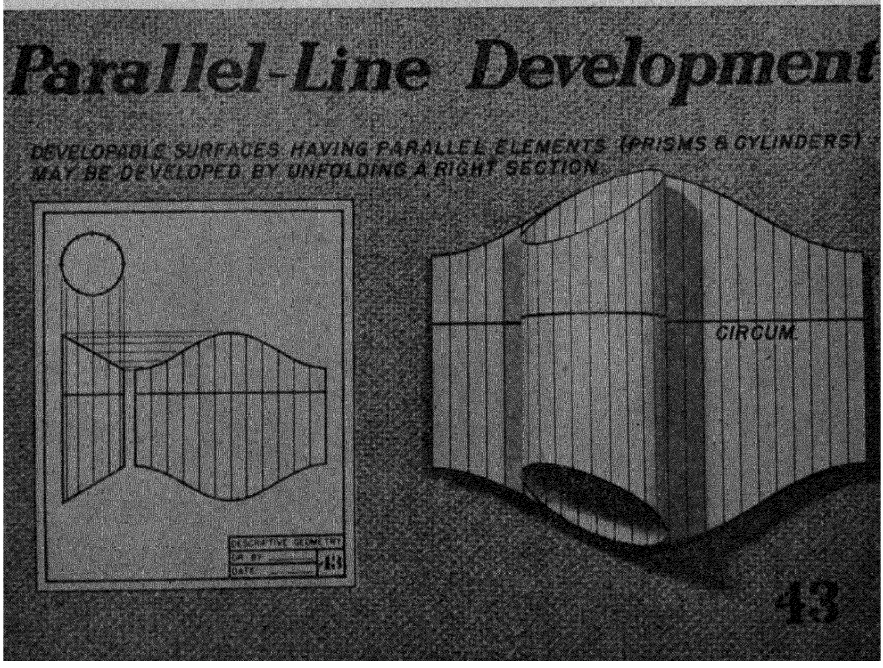
6-E19. Point D:417 is the center of the 4-inch horizontal circular base of a right cone $3\frac{1}{2}$ inches high. The triangle A:$4,4\frac{3}{4},5$ B:$2\frac{1}{2},4\frac{3}{4},7\frac{3}{4}$ C:$5\frac{1}{2},4\frac{3}{4},7\frac{3}{4}$ is the base of a right prism which is cut to fit the surface of the cone. Draw the top and front views showing the line of intersection.

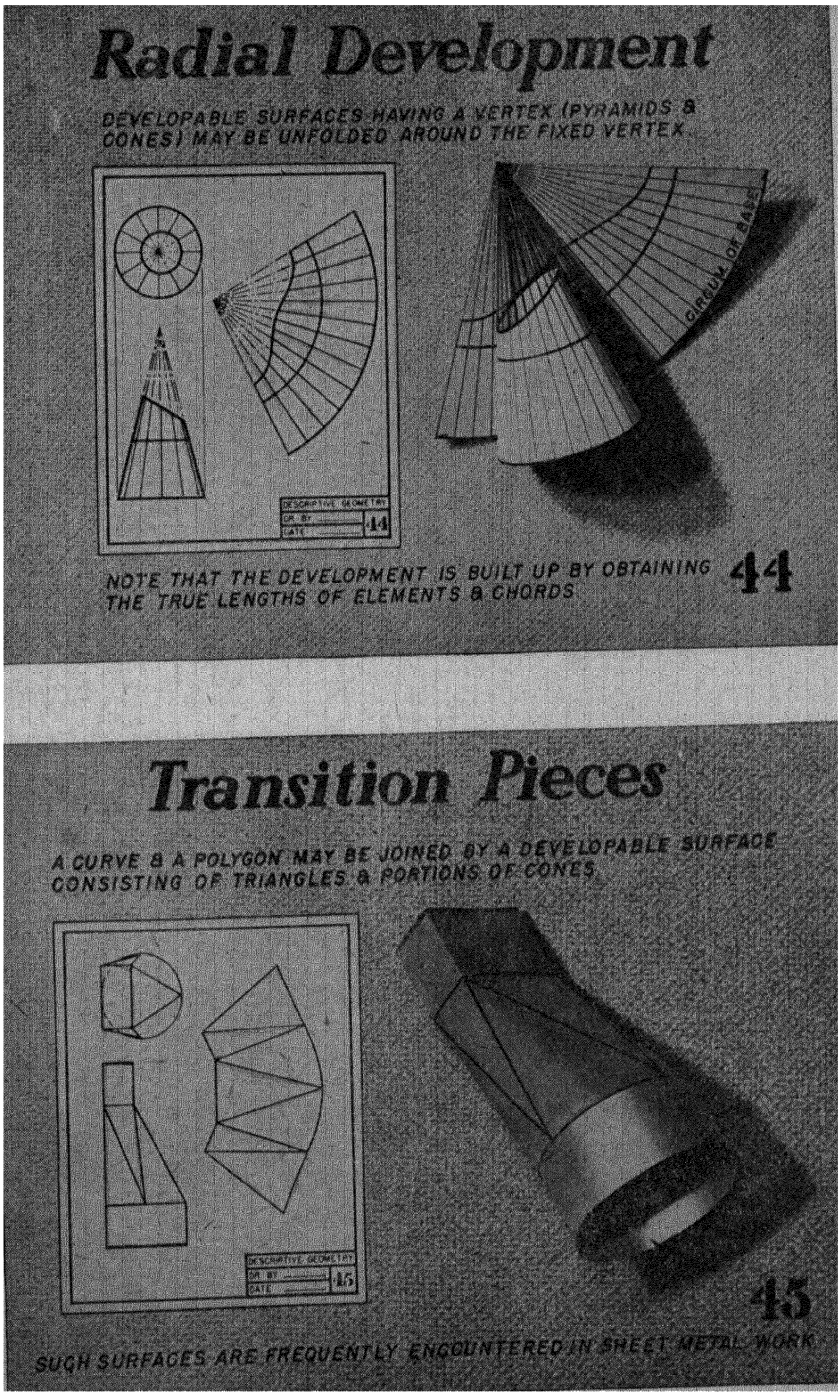
6-E20. Line A:$4,4\frac{1}{2},7$ B:$4,1\frac{1}{2},7$ is the axis of a right prism. The bases are regular hexagons $1\frac{3}{4}$ inches on a side. One diagonal of each base is frontal. Line C:$4,2\frac{1}{4},7$ D:$4,4\frac{1}{4},7$ is the axis of a right cone. The base is a 4-inch circle centered at point C. Find the intersection of the surfaces.

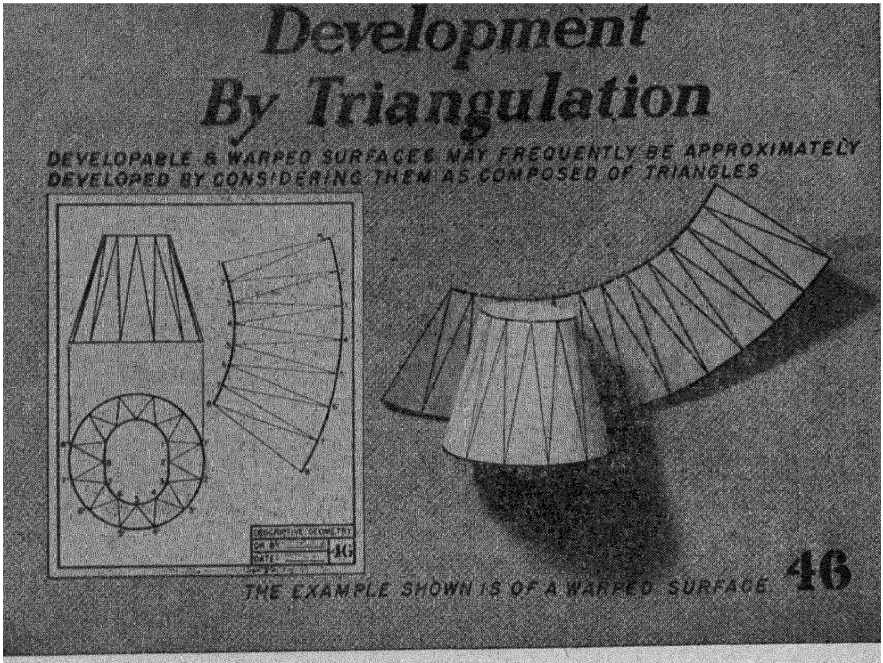
6-E21. Point C:$2\frac{3}{4}$,6,8 is the upper end of the vertical axis of a $1\frac{3}{4}$-inch circular cylinder. This cylinder intersects a square pipe having as its axis the line A:128 B:558. The ends of the square pipe are perpendicular to AB, and one diagonal of each end is horizontal and 2 inches long. Find the line of intersection of the cylinder with the upper surfaces of the prism. Determine the shape of the hole in the square pipe by means of a development.

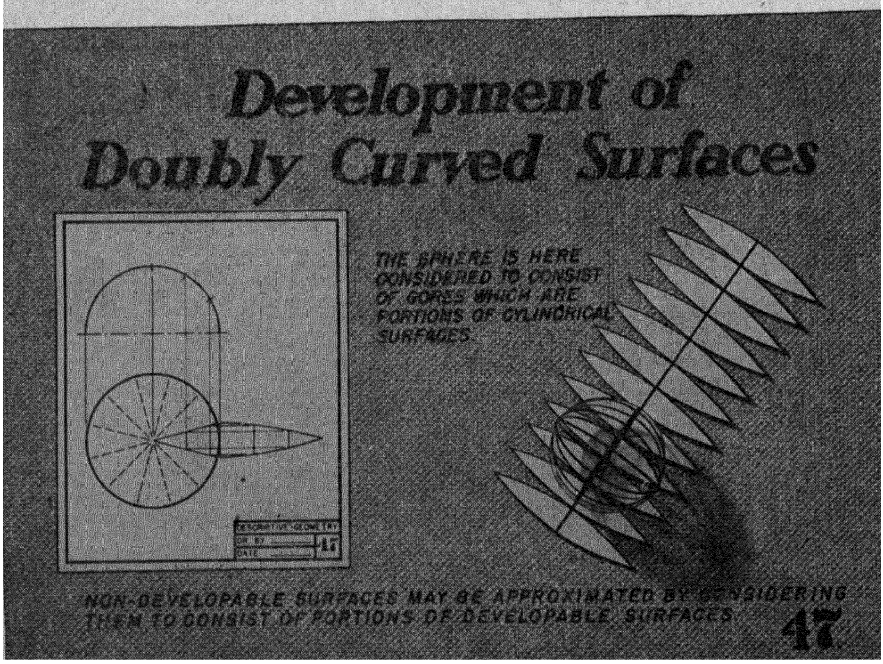

Intersection of a Straight Line & a Surface
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THIS IS AN EXAMPLE OF THE CASE IN WHICH THE SURFACE IS REPRESENTED BY ITS OUTLINES.
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IN FINDING INTERSECTIONS CHOOSE THOSE CUTTING PLANES WHICH YIEL THE SIMPLEST CURVES, PREFERABLY STRAIGHT LINES OR CIRCLES.
CUTTING PLANES CHOSEN TO YIELD CIRCLES
CUTTING PLANES CHOSEN TO YIELD STRAIGHT LINES


Intersections of Cones & Cylinders
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SHEET METAL ELBOWS & BREECHINGS ARE GENERALLY DESIGNED TO UTILIZE THIS PRINCIPLE


Parallel-Line Development
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CHAPTER VII

Surfaces of General Form

107. Warped surfaces

A ruled surface generated by a straight line which moves in such a manner that no two consecutive positions lie in the same plane is called a *warped* or *twisted surface*. This characteristic motion of the generating line makes the accurate development of a warped surface impossible.

The motion of a straight line in space can be completely controlled by requiring that it always intersect three other lines called the *linear directrices* of the surface. Any ruled surface thus may be generated by a straight line moving so as to intersect three given linear directrices. These directrices may be straight lines or curves; they may be alike, or they may differ from each other.

108. Generation of warped surfaces

(*a*) *First Method.* Every warped surface can be generated by a moving straight line which always intersects three linear directrices so selected that no two consecutive positions of the generating line will lie in the same plane.

If the linear directrices are straight lines, only two types of warped surfaces can be generated,—the *unparted hyperboloid* and the *hyperbolic paraboloid*. If one or more of the directrices are curved lines, an unlimited variety of warped surfaces may be generated, only a few of which bear specific names.

(*b*) *Second Method.* Through a given point of space, a set of lines drawn parallel to the successive positions of the generating element of a warped surface form a conical surface. This cone, when substituted for one of the three linear directrices, is called a *cone director*. Thus, a given warped surface may be generated either by a line moving in contact with three linear directrices, or by a line touching two linear directrices and parallel to an element of a cone director.

The motion of the generating line of a warped surface may be such that a set of lines drawn through a fixed point and parallel to its successive positions describe a plane rather than a cone. In this case, the plane becomes the *plane director* of the warped surface and may be substituted for any one of the linear directrices.

109. Classification of warped surfaces

Warped surfaces can be divided into two general groups, *doubly ruled* and *singly ruled.*

A warped surface is *doubly ruled* when two straight-line elements can be drawn through any point in the surface. This is possible when the three directrices of a surface are all straight lines. In this case, it will be seen that, if three positions of the generating element are taken as directrices and any one of the original directrices is used as a generating line, the surface can be regenerated. In a doubly ruled surface, every element of either generation intersects every element of the other generation. Since only the unparted hyperboloid and the hyperbolic paraboloid have straight-line directrices, these are the only doubly ruled surfaces.

A warped surface which cannot be generated by the use of three straight-line directrices is *singly ruled,* and but one straight-line element can be drawn through a given point in the surface.

110. Plane section of a warped surface

The intersection of a plane and a warped surface will be, in general, a smooth curve drawn through the points in which the elements of the surface pierce the plane.

111. Locating a point in a warped surface

One projection of a point in the surface can be assumed at random. If an element of the surface can be readily drawn through this assumed point, the second projection of the point is found on the other projection of the element. Otherwise, the second projection of the point must be found by means of a section of the surface cut by a plane containing the given point.

112. Tangent planes

A plane tangent to a warped surface at a given point will contain the element of the surface which passes through the point. In general, the plane will be tangent to the surface at but one point, and elsewhere it will be secant to the surface. Any plane containing an element of a warped surface will be, in general, tangent to the surface at some point.

In a doubly ruled surface, a plane tangent to the surface at a given point will contain the two straight-line elements of the surface which pass through the given point.

In a singly ruled surface, a plane tangent at a given point may be determined by one straight-line element of the surface drawn through the point and the tangent to any curve lying in the surface and containing the given point.

113. The hyperbolic paraboloid

The hyperbolic paraboloid takes its name from the fact that any plane section of the surface, when not a straight line, is either a parabola or a hyperbola. The surface may be represented by drawing several positions of a generating element which moves parallel to a given plane director and constantly intersects two given straight-line directrices.

A symmetrical portion of the surface is shown in Figure 180. Lines AB and CD are directrices, and the vertical plane X is the plane director. Seven elements of the ruling parallel to X are drawn. Since the surface is doubly ruled, lines AD and BC may be considered as directrices and

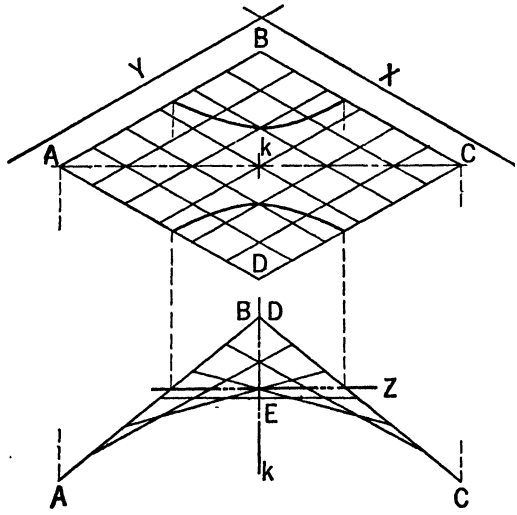


Fig. 180

plane Y, parallel to AB and CD, the plane director. Seven elements of the ruling parallel to Y are shown. The surface has an axis k which is parallel to the line of intersection of the director planes.

The frontal section plane through the axis intersects the surface in the parabola AEC. The horizontal section plane Z cuts the surface in two branches of a hyperbola.

Any series of planes parallel to the plane director will cut the surface in elements and also divide the directrices proportionally. Therefore, if the directrices are lines of definite length, elements of the surface can be found by dividing the directrices proportionally and joining the points of division. The hyperbolic paraboloid of Figure 181 is determined by the directrices AB and CD. Each directrix is divided into five equal parts, and the elements of one ruling are represented. The

second ruling can be drawn similarly by taking AD and BC as directrices and dividing these lines into a number of equal parts.

If the top view of the point P (Figure 181) which lies in the surface is given, its front view is found by means of the section t cut from the surface by the vertical plane X.

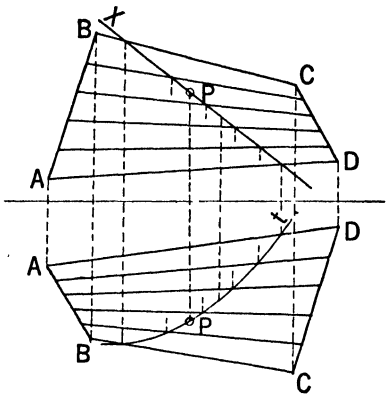


Fig. 181

A plane tangent to the hyperbolic paraboloid, at a given point in the surface, contains the elements of each ruling which pass through the point.

114. The unparted hyperboloid

The unparted hyperboloid is a doubly ruled surface in which the motion of the generating line may be controlled by three straight-line directrices so selected that the surface has no plane director. This surface is generally obtained when three straight-line directrices are taken at random. The method of constructing a line to intersect three given lines is described in Article 52, Example 4. The surface is symmetrical, having one principal axis. Any plane containing the axis cuts the surface in a hyperbola. A plane perpendicular to the axis will, in general, cut the surface in an ellipse. For this reason, the surface is sometimes known as the *elliptic hyperboloid.* It is possible to choose the directrices so that the elliptical cross section becomes a circle. The hyperboloid then becomes a surface of revolution.

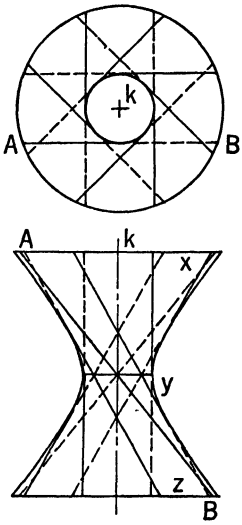


Fig. 182

The hyperboloid of revolution is most easily constructed by rotating one straight line about another not in the same plane.

Let the line AB (Figure 182) rotate about the vertical axis k. Each point of line AB will describe a circle perpendicular to k. Points A and B describe the circles x and z respectively. The point of AB nearest the axis will describe the circle y, which is called the *circle of the gorge.* Eight successive positions of the generating element AB are represented.

Since this surface is symmetrical with respect to the axis k, it is evident that the line CD (Figure 183), which is placed the same as AB except that it has the opposite inclination, can be rotated to generate

the same surface. The hyperboloid is thus doubly ruled. In Figure 183, eight elements of each ruling are drawn. It will be noted that each projection of an element of one system is also a projection of an element of the second system.

The hyperboloid of revolution can be represented by a series of parallel circles (Figure 181). Let k be the axis and AB the generating line. Each point of AB will describe a circle having its plane at right angles to line k. The hyperbola which represents the

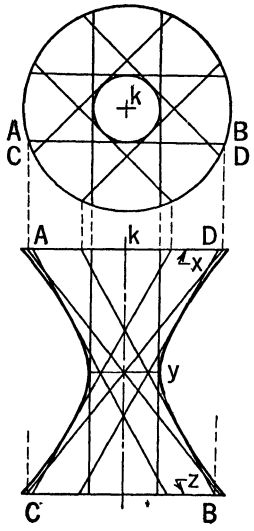


Fig. 183

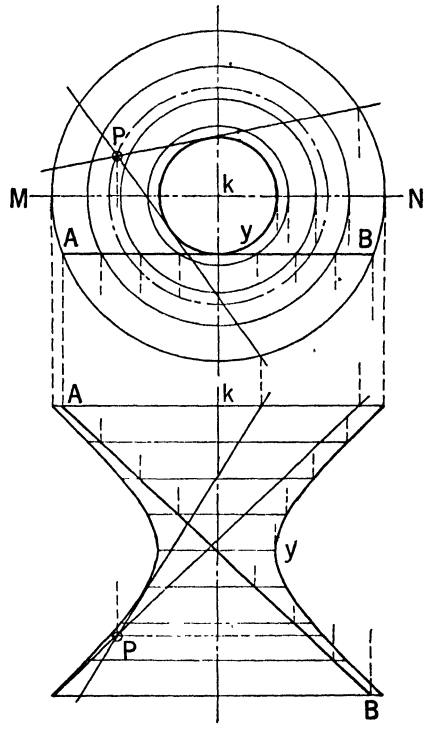


Fig. 184

apparent contour of the front view is determined by the points in which the frontal plane MN, containing the axis k, cuts the parallel circles of the surface.

A plane tangent to the surface at point P contains one element of each generation drawn through the point.

115. The helix

One of the most important warped surfaces has for one of its directrices a space curve called the *helix*. In its usual form, the helix may be defined as the path of a point which remains at a constant distance from a straight line, rotates uniformly around this line as an axis, and at the same time moves uniformly in a direction parallel to the line. Any distance traversed by the generating point in its motion parallel to the

axis is the *axial advance*. The axial advance for one complete revolution of the generating point is the *lead* or *pitch* of the helix.

The projection of the helix on any plane perpendicular to the axis is a circle.

(*a*) *Construction of the helix.* Let the vertical line k (Figure 185) be the axis, L the lead, and P the generating point. Let the circle centered on k_H represent the path of rotation. Since the respective motions of point P around and parallel to k are uniform, the distances traversed by P in these two motions will be proportional. Divide the circle and the lead into the same number of equal parts, as 12, and draw horizontals through the points that divide the lead. Starting from 1_H, at the end of $\frac{1}{12}$ of a complete revolution 2_H will be the H-projection of the generating point. At the same time, point P will have advanced $\frac{1}{12}$ of the lead. Therefore, 2_V will be located by projecting from 2_H to the second horizontal line from the top. The V-projection for the position given by 3_H will lie on the third horizontal from the top. The remaining positions are located in a similar manner. A smooth curve joining the points 1_V to 13_V is the V-projection of the helix.

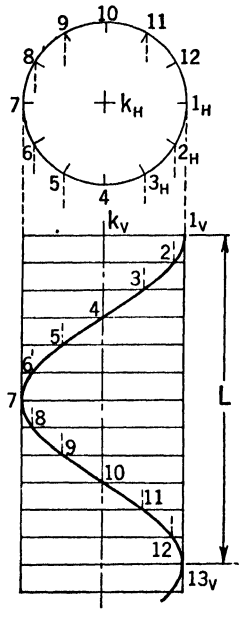


Fig. 185

Since the axial advance of the point P is uniform, every portion of the helix is equally inclined to the H-plane. When the direction of motion of the generating point (in the circular view) is clockwise as it moves away from the observer, the helix is *right-handed.* The opposite direction of rotation gives a *left-handed* curve. The helix of Figure 185 is right-handed.

(*b*) *Development of the helix.* The helix may be regarded as drawn on the surface of a right circular cylinder having the same axis. In Figure 186, the base of this cylinder is the circle lying in H. When the cylinder is developed, this base will develop into the straight line 1, 2, 3 . . . 1. Since every portion of the helix is equally inclined to the plane of the base, this curve will develop into the straight line 1-13 which makes the angle θ with the development of the base. Angle θ is the *helix angle.*

(*c*) *Tangent line to the helix* (*Figure 187*). Since every element of the helix makes the same angle θ with the plane of the base H, every line tangent to the helix will make the angle θ with H. Also, the length of the tangent line between the point of tangency and H will be equal to the length of the helix between this point and H. Let line t be tangent

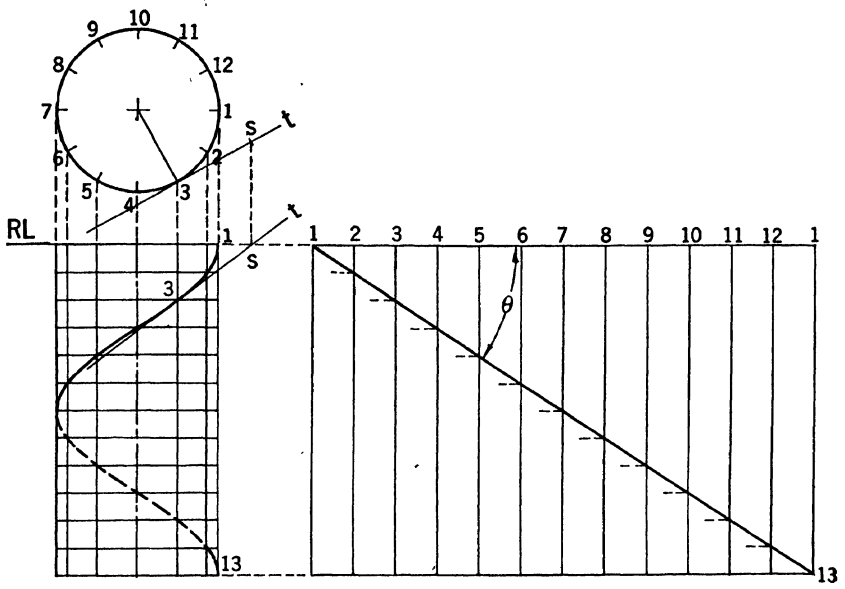


Fig. 186

to the helix at D and pierce H at point S. The length of t is $ABCD$. It will be seen that the length of the H-projection of t, S-D_H, is equal to the arc A-D_H.

(*d*) *To draw a tangent to a helix at a given point.* Let point 3 (Figure 186) be given. The H-projection of the tangent t will be tangent to the circle at point 3_H. Make $3_H S_H$ equal in length to the arc 1_H-3_H. Since point S is in H, S_V is on the RL. Draw t_V through 3_V and S_V. Any plane perpendicular to the axis may be used as the base of the curve.

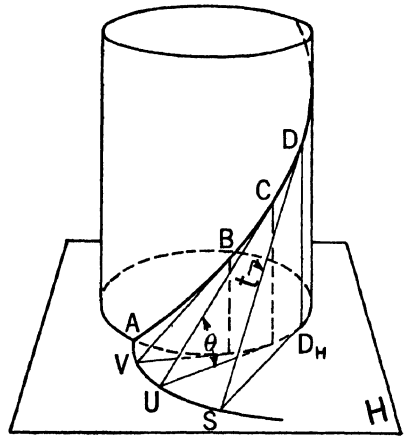


Fig. 187

116. The screw surface or helicoid

In its most common form, the screw surface or *helicoid* is generated by a straight line which slides uniformly along a fixed straight line or axis with which it makes a constant angle and at the same time rotates uniformly around that axis. It is evident that any given point of the generating line describes a helix. The surface thus obtained is a warped surface and is not developable.

A second form of the helicoid is obtained when the generating line does not intersect the axis but remains at a constant distance from it.

When the constant angle between the generating line and the axis is 90°, the surface generated is the *right helicoid.* When the constant angle is other than 90°, the surface is an *oblique helicoid.*

Since the generatrix is of indefinite length, the helicoid is composed of two nappes separated by the axis of the helix. In the following illustrative problem, only one nappe is shown. The surface is usually represented by its axis, helical directrix, and several positions of the generating line.

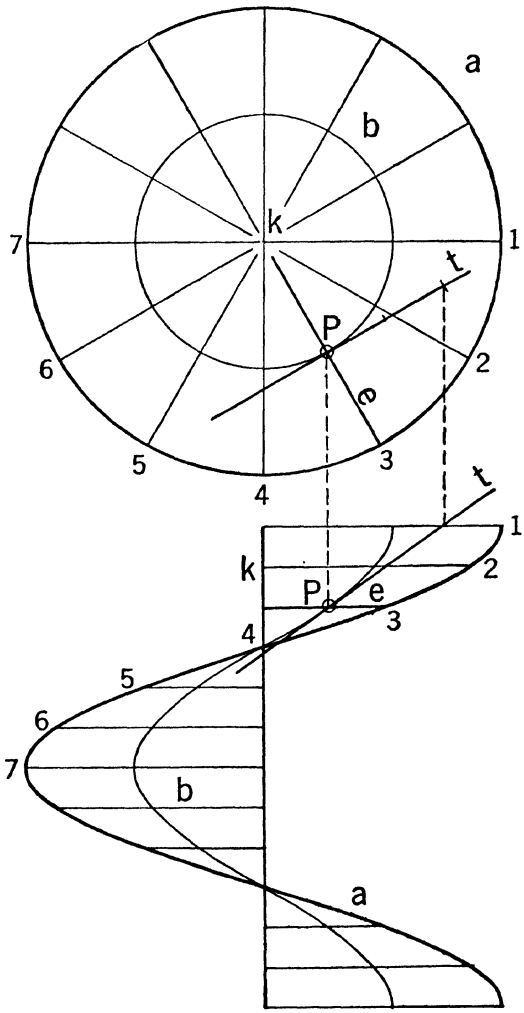


Fig. 188

117. Tangent planes

The principles of Article 112 apply to the helicoid.

Let the helix a (Figure 188) be the directrix and line k the axis of a right helicoid.

A plane tangent at point P is determined by the element e and the line t. The line t is tangent at P to the helix b passing through P. The helix b is determined by the intersection of the cylinder which passes through P and the helicoid.

118. The developable helicoid, or helical convolute

The developable helicoid or *helical convolute* is generated by a moving straight line which remains always tangent to a helix. In the pictorial illustration (Figure 187), $ABCD$ . . . is the helical director; BV, CU, DS positions of the generating line; and $AVUS$ the line in which the surface intersects the plane H taken perpendicular to the axis of

the helix. If the elements are produced indefinitely on either side, two nappes of the surface are formed. The surface is single curved and is therefore capable of development.

(*a*) *Construction of the convolute.* Let the helix *a* (Figure 189) be given. Draw tangents to the helix at points 0, 1, . . . 8, and find their

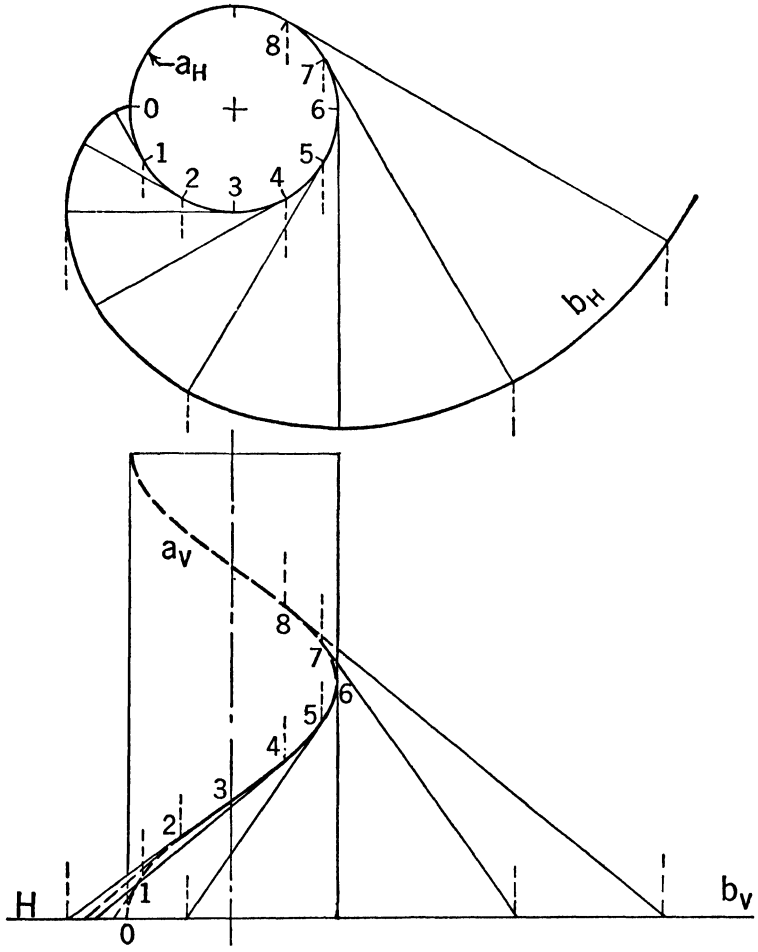


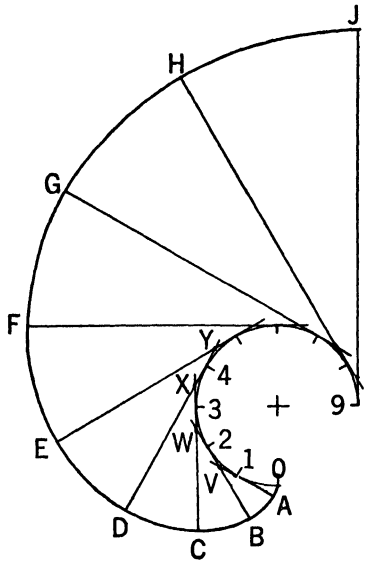
Fig. 189

traces on the *H*-plane. A smooth curve *b* drawn through these traces represents the intersection of the surface with *II*, and may be regarded as the base of the convolute. It will be seen that b_H is the involute of the circle a_H.

Since the convolute is a single curved surface, a plane tangent to the surface at a given point is tangent along the entire length of the

element passing through the point. The tangent plane will also contain a line tangent to the base at the point in which the element intersects the base.

(*b*) *Development of the convolute.* Let it be required to develop a portion of the helical convolute represented in Figure 190. Divide the helix into a number of equal parts and draw the elements $A1$, $B2$, and so forth. Assume that the elements intersect as follows: $B2$ and $C3$ meet in W; $C3$ and $D4$ meet in X; $D4$ and $E5$ meet in Y; and so forth. This assumption is nearly exact, becoming entirely true when the divisions of the helix are infinitely small. The surface can now be developed by triangulation, a portion being shown in Figure 191.



Examining the development (Figure 191), it can be seen that the angles W', X', Y' . . . are equal since they are the true sizes of the angles

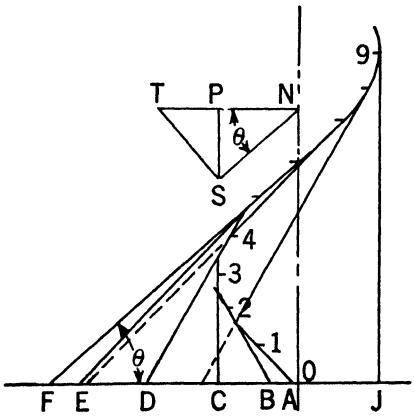


Fig. 190

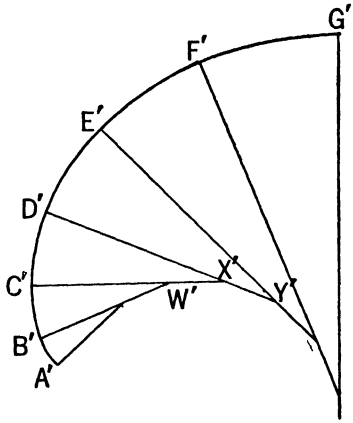


Fig. 191

W_H, X_H, Y_H . . . of the H-projection which are equal. Also, the lengths $W'X'$, $X'Y'$ . . . are equal. Therefore, a circle drawn tangent to these segments will represent the development of a portion of the helix. When the divisions of the helix are infinitely small, the tangent circle becomes the exact development of the helix, and the curve *BCDEFG* becomes its involute. Hence, the true development of a helical convolute

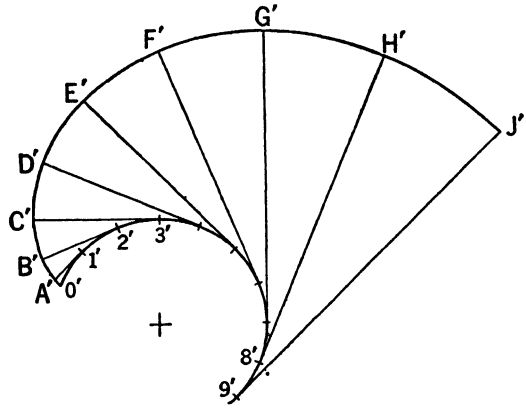
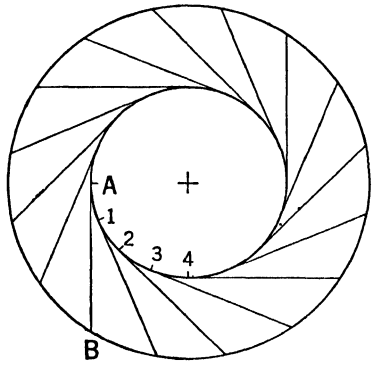


Fig. 192



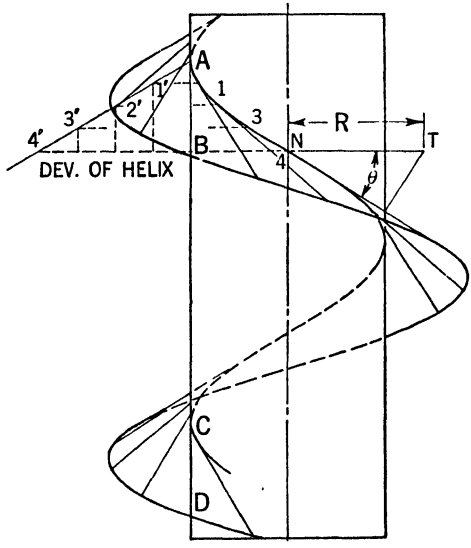


Fig. 193

is bounded by a circle and its involute. The radius of the circle is equal to the radius of curvature of the helix.

(*c*) *To find the radius of the development of the helix.* The radius of curvature of a helix is constant, and is equal to $R \sec^2 \theta$ (by Diff. Calculus), where R is the radius of the cylinder containing the helix, and θ is the helix angle. This radius can be found graphically as follows: (Figure 190) In any convenient position in the V-projection, lay off $NP = R$; draw NS making the angle θ with NP; also draw PS perpendicular to NP. Then $NS = R \sec \theta$. Draw ST perpendicular to NS; then $NT = NS \sec \theta = R \sec^2 \theta$. Therefore, the radius of curvature of the helix is equal to NT.

(*d*) *To construct the exact development of the surface.* With radius NT, draw a circular arc (Figure 192). Make the distances 0′-1′, 1′-2′, . . . 8′-9′ equal to the true lengths of the segments 0-1, 1-2, and so forth, of the helix. These lengths may be taken from the development of the helix. Construct the involute of this arc. The required development is the surface between the circular arc and its involute.

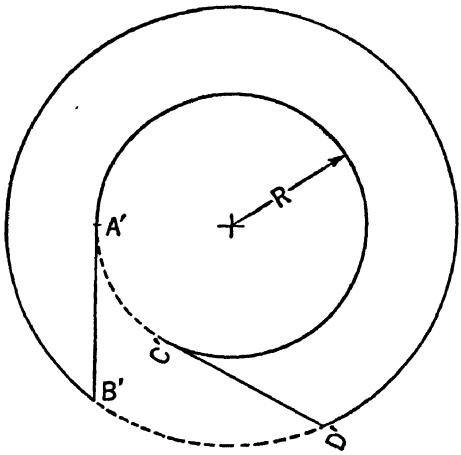


Fig. 194

The portion of a helical convolute lying between the helical directrix and a circular cylinder (Figure 193) having the same axis has been used in practice as the working surface of a screw conveyor. The surface between elements AB and CD represents one turn or *flight* of the convolute.

In order to develop the surface, it is necessary to determine the radius of the development $R = NT$ and the development of a portion of the helix A-4′.

The development (Figure 194) is constructed by striking a circle of radius R and laying off along the arc the developed length of the helix $A'C'$. At point A', the element AB is drawn in true length tangent to the arc, locating point B'. A circular arc passing through point B' represents the developed edge of the convolute. The element $C'D'$ drawn tangent to the arc $A'C'$ at point C' completes the development.

119. The conoid

The *conoid* is a warped surface having a plane director and two linear directrices, one of which is a straight line and the other, a curve. The

surface is called a *right conoid* when the straight-line directrix is perpendicular to the plane director; otherwise, the surface is an *oblique conoid.*

The right conoid of Figure 195 has the straight line AB and the circle c for directrices. The plane director is frontal. A plane tangent to the surface at a given point P will contain the element of the surface through P and a line tangent to some curve of the surface passing through P. A horizontal plane Z passing through P cuts the surface in an ellipse. The plane tangent at P is determined therefore by the element e and line t which is tangent to the ellipse at P.

The conoid can be used in structural work as a transition surface between an arch and a flat ceiling. Transition connections between square and circular openings may have a conoidal form. The conoid is also involved in the mathematical theory of optics.

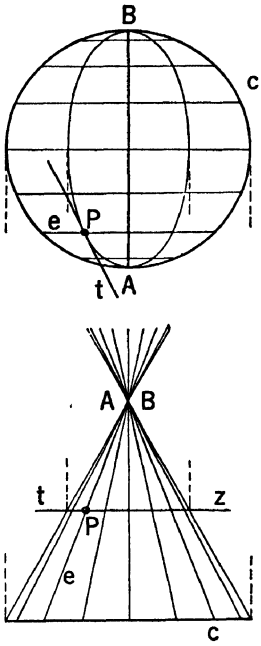


Fig. 195

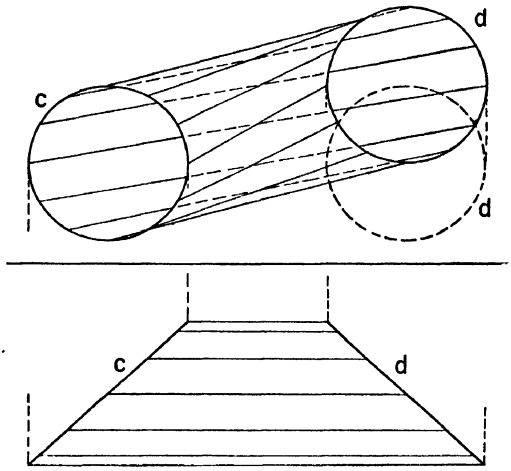


Fig. 196

120. The cylindroid

The *cylindroid* is a warped surface having a plane director and two curved linear directrices. A simple form of the cylindroid is represented in Figure 196. The ellipses c and d are the directrices, and the plane director is horizontal. If the ellipse d were moved forward to the position d', the cylindroid would become a cylinder.

The cylindroid has been used in architectural work as a transition surface between two nonparallel arches.

121. The cow's horn

The *cow's horn* is a warped surface having three linear directrices, two curved and one straight. The curved directrices are circles which

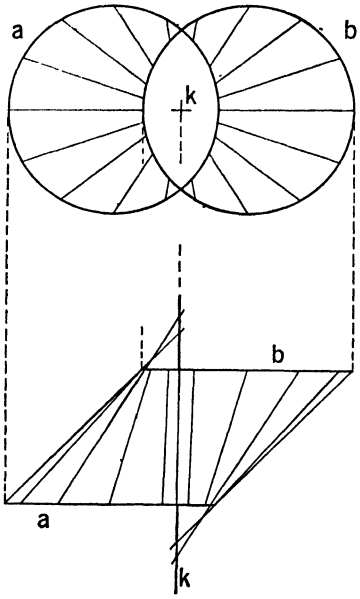


Fig. 197

lie in parallel planes. The straight-line directrix is at right angles to the plane of the circles and intersects the line joining their centers. A simple form of this surface is represented in Figure 197.

122. The warped cone

The *warped cone* has three linear directrices, two circular (or elliptical) and one straight. The circles lie in nonparallel planes; the line joining their centers is the straight-line directrix.

123. The general warped surface

The warped surface having three curved directrices chosen at random will, in general, bear no special name.

(*a*) *To find the element of a general warped surface.* Let the given directrices be the curves *a*, *b*, and *c* (Figure 198). Through any point *P* in directrix *c*, draw the elements *P*1, *P*2, *P*3, and *P*4 determining a conical surface. Pass a vertical cylinder through directrix *a*, cutting the conical surface in the curve *e*. The point *Q*, determined by the intersection of e_v and a_v, locates the element of the surface which passes through point *P*. Other elements are found similarly.

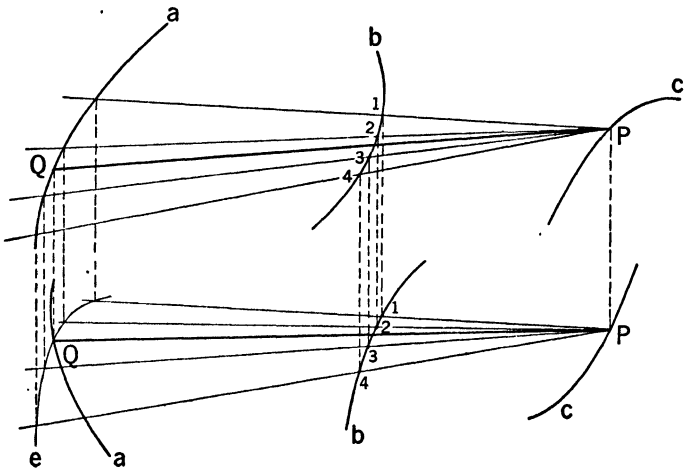


Fig. 198

124. A tangent to a plane curve of unknown properties

Let it be required to construct a line tangent to curve C at point P (Figure 199). Assume several points on curve C, as W, X, Y, Z, lying on either side of P. Draw chords PW, PX, PY, PZ. With P as center, and using any convenient radius, strike the arc k intersecting the chords

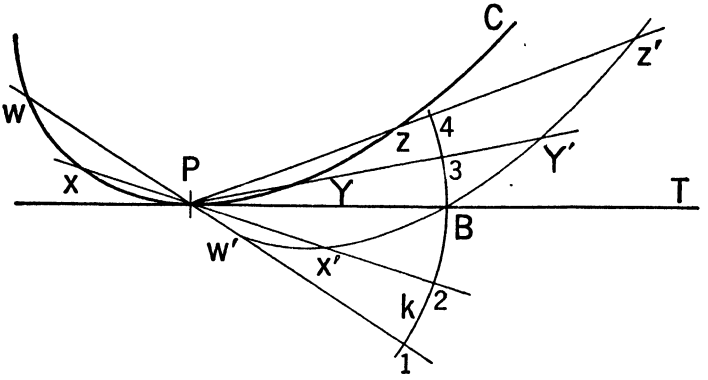


Fig. 199

produced in points 1, 2, 3, and 4. Lay off 1-W' = PW; 2-X' = PX; 3-Y' = PY; 4-Z' = PZ. Pass a smooth curve through points W', X', Y', and Z'. The point B in which curve W'-Z' cuts the arc k lies on the tangent t which passes through point P.

125. Plane tangent to a general warped surface at a given point in the surface

Let the directrices of a warped surface be the curves a, b, and c (Figure 200). Lines d, e, f, and g are four elements of the surface. A point D, lying in the surface, is assumed in the front view. The top view of D lies in the curve n cut from the surface by a plane X, which passes through D_V and is perpendicular to the V-plane. The plane tangent to the surface at D contains the element passing through D and the line tangent to the curve n at point D.

The element k containing point D is found by the method of Article 123. The tangent t to curve n is found by the construction described in the preceding paragraph.

126. Double curved surfaces of general form

Many structures are composed in part of double curved surfaces which do not permit classification into the geometrical forms bearing specific names. Curved surfaces of this general form are used particularly in ship and airplane construction.

A surface of this character is represented by the projections of its limiting contours together with a set of sections of the surface cut by a

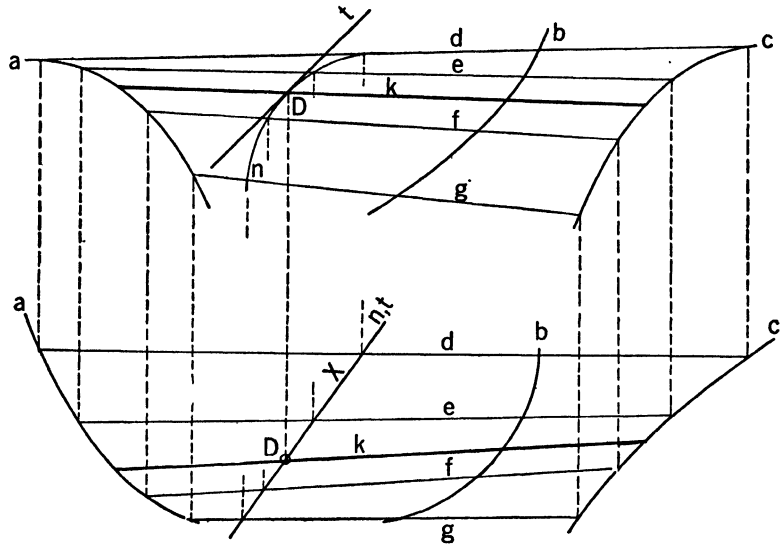


Fig. 200

series of parallel planes. Thus, the streamlined surface of Figure 201 is represented by the limiting contours or boundary curves of the top and profile views and the set of frontal sections which appear in true shape in the front view. To avoid confusion, the front view shows the left half of each forward section and the right half of each rear or after section. In a framed structure, these frontal sections are usually called *frame lines* since the ribs of the structure follow these lines.

In order to test a designed surface for smoothness or "fairness," two additional sets of lines called *water lines* and *buttock lines* are commonly drawn. The *water lines* are sections cut by a set of horizontal planes; the *buttock lines* are sections cut by a set of profile planes.

127. Intersections

Intersection of developable or warped surfaces with surfaces of general form may be reduced to the problem of determining the intersection of a line and a general surface.

Example 1 (Figure 202). *Line and General Surface.* To find the point in which line k cuts the surface represented by the series of contours or sections *A-B-C-D-E-F*. In practice, these sections might represent frames of a structure such as the hull of a ship or the fuselage of an airplane.

Analysis: Pass plane X through line k by drawing VX through k_V. Plane X cuts the contours in points 1, 2, 3, 4, 5, 6. Project these points to the profile view and draw a smooth line X' through them. Line X' cuts line k in point P. This is the required point of intersection.

Construction (Figure 202).

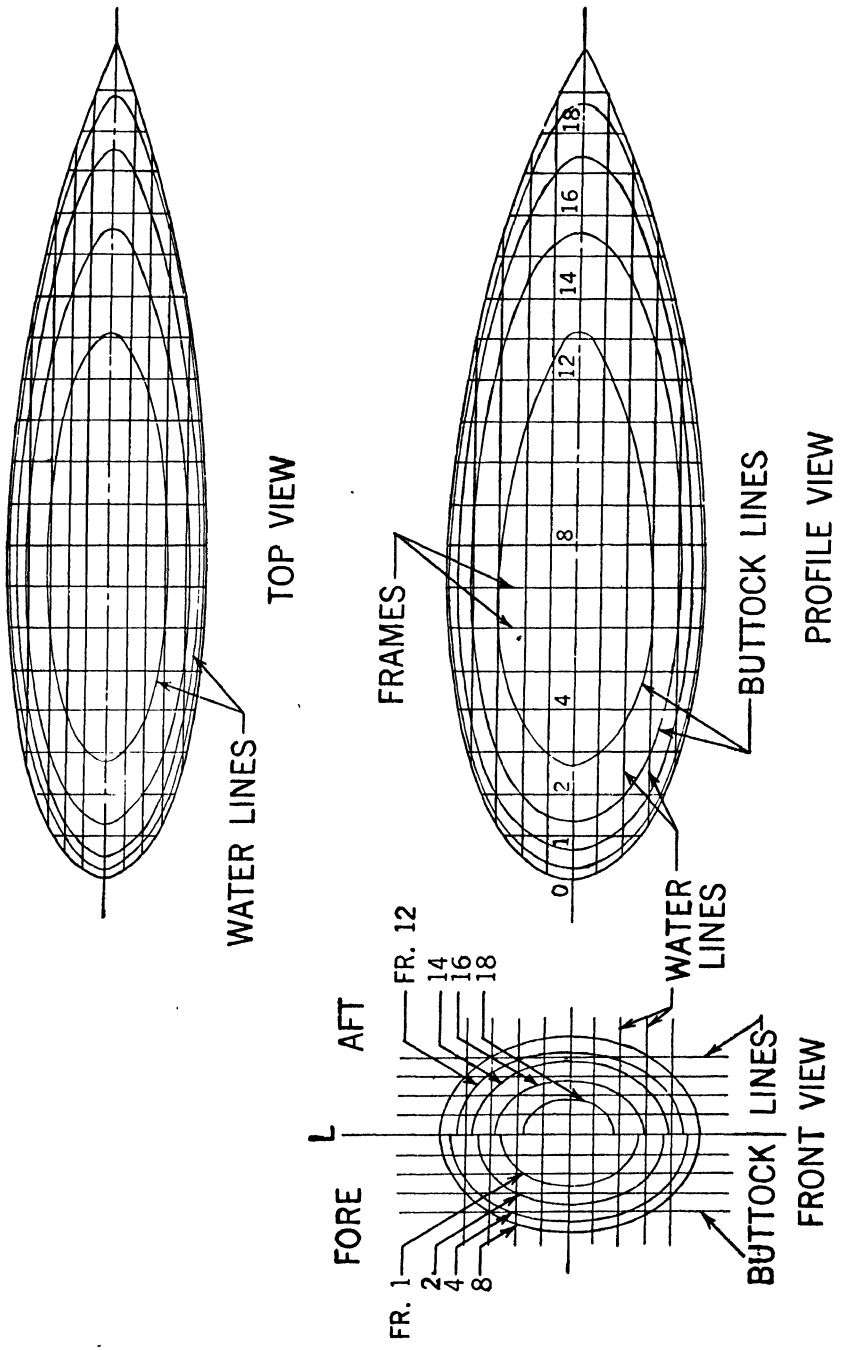

WATER LINES
TOP VIEW
FRAMES
BUTTOCK LINES
PROFILE VIEW
0
1
2
4
8
12
14
16
18
FR. 12
14
16
18
WATER LINES
FORE
AFT
FR. 1
2
4
8
BUTTOCK LINES
FRONT VIEW

Fig. 201

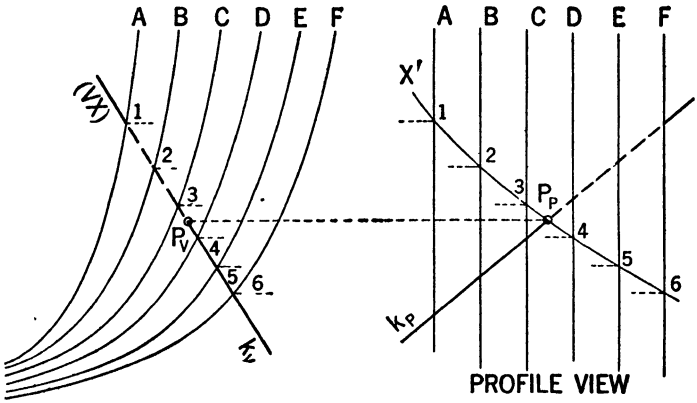


Fig. 202

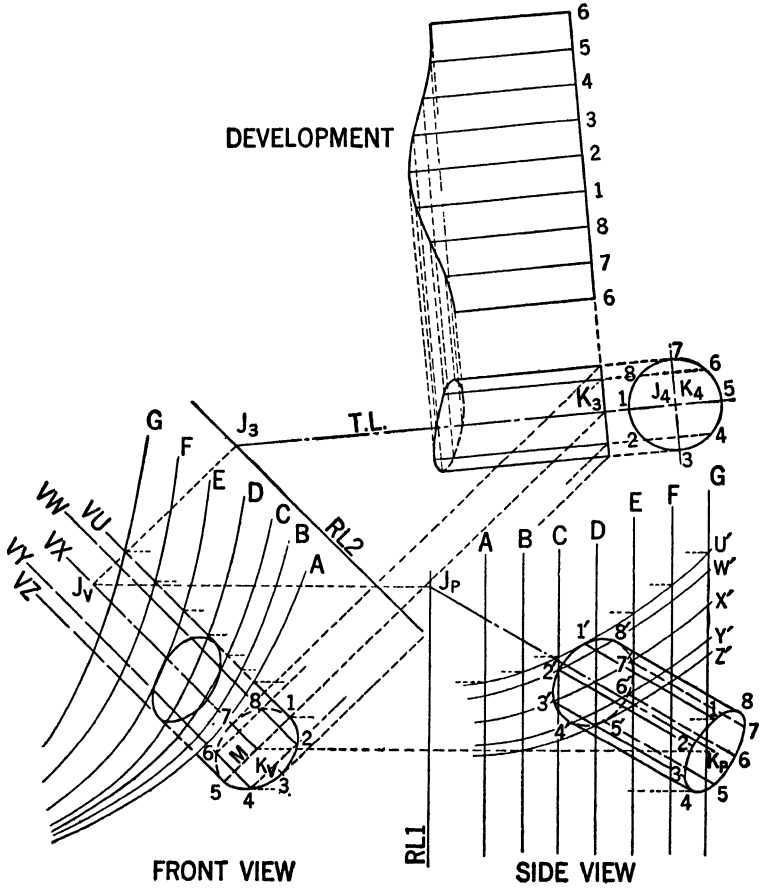


Fig. 203

Example 2 (Figure 203). *Cylinder and Surface of General Type.* A general surface is given by the parallel sections *A-B-C-D-E-F-G*. Let it be required to find the intersection of this surface with a circular cylinder having line *JK* as its axis and diameter of *M*.

Draw the true-length view and the end view of line *JK*, obtaining the projection J_3K_3 and J_4K_4. Draw the edge view of the circular base on K_3 as center and perpendicular to J_3K_3.

Construct the end view of the cylinder on J_4K_4 as center, and *M* as diameter. Locate eight equally spaced elements 1 . . . 8. Project these elements to the front and side views.

The cylinder is now represented in the front and side views by eight elements. The line of intersection is determined by the points in which the elements pierce the surface *A-B-G*. The piercing point for each element is found by the method of Example 1. Thus, in the front view, plane *U* is passed through element 1. Plane *U* cuts the line *U'* (profile view) from the surface *A-G*. The point 1' in which *U'* cuts the profile view of element 1 is a point on the profile view of the intersection. Planes *W*, *X*, *Y*, and *Z* determine the piercing points of the remaining elements.

The front view of the intersection is projected from the profile view. The auxiliary view of the intersection is projected from the front view.

The pattern or development of the surface of the cylinder may be constructed by the method of Article 104.

PROBLEMS ON CHAPTER VII

GROUP A. THE HYPERBOLIC PARABOLOID

7-A1. Lines A:$\frac{1}{4}$,6,8 B:$2\frac{3}{4}$,1,$9\frac{1}{2}$ and C:$2\frac{3}{4}$,1,$6\frac{1}{2}$ D:$5\frac{1}{4}$,6,8 are linear directrices of a hyperbolic paraboloid. The plane director is vertical and parallel to lines *AC* and *BD*. Divide *AB* into ten equal parts, and draw elements of the surface through these points. Show the same number of elements of the second ruling. Draw the top, front, and right side views. Assume two horizontal planes, the first $\frac{1}{8}$ inch above and the second $\frac{1}{8}$ inch below the apex. Draw the top view of each line of intersection.

7-A2. Lines A:$\frac{1}{2}$,$5\frac{3}{4}$,$9\frac{1}{4}$ B:$2\frac{3}{4}$,$1\frac{1}{4}$,7 and C:5,$1\frac{1}{4}$,$9\frac{1}{4}$ D:$7\frac{1}{4}$,$5\frac{3}{4}$,7 are linear directrices of a hyperbolic paraboloid. The plane director is frontal. Represent the surface by ten equally spaced elements. Through point E:3,*X*,$7\frac{1}{2}$ of the surface pass a plane tangent to the surface at this point. Let the plane be represented by the two elements of the surface which contain *E*.

7-A3. Lines A:$3\frac{1}{2}$,5,9 B:$3\frac{1}{2}$,1,6 and C:856 D:819 are the linear directrices of a hyperbolic paraboloid. The plane director is profile. Represent the surface by thirteen equally spaced profile elements. Draw

the top, front, and left side views. Assume a frontal plane $1\frac{1}{4}$ inches in back of point C, and find its intersection with the surface. Through points A and D pass a plane perpendicular to the vertical projection plane, and find the line in which this plane cuts the surface.

7-A4. A surface is generated by a line which moves so as to touch constantly lines A:016 B:259 and C:619 D:856, and at the same time remain parallel to a horizontal plane. Draw the top and front views of the surface, showing at least 9 elements.

Determine the top and front views of the point P in which line R:2,6,$6\frac{1}{2}$ S:519 intersects the surface.

Find the true size of the curve cut by a vertical section plane through line RS. (Use method of rotation.)

7-A5. Lines A:147 B:$2\frac{1}{2},\frac{1}{2}$,9 and C:$5\frac{1}{2}$,3,9 D:756 are linear directrices of a hyperbolic paraboloid. Lines AD and BC are elements of the surface. Complete the top and front views by drawing five additional intermediate elements of the surface. Find the top view of point E:3,$2\frac{1}{2}$,X which lies in the surface. Through point E pass a plane tangent to the surface at this point. Represent the tangent plane by the elements of the surface which pass through point E.

7-A6. Lines O:$1\frac{1}{2}$,1,$9\frac{1}{2}$ P:$1\frac{1}{2}$,4,$9\frac{1}{2}$ and V:$6\frac{1}{2}$,1,$9\frac{1}{2}$ X:$6\frac{1}{2}$,4,$4\frac{1}{2}$ are linear directrices of a rectangular hyperbolic paraboloid. The plane director is horizontal. Represent the surface in the top and front views by thirteen equally spaced elements.

Intersect the surface by frontal planes taken $\frac{1}{4}$, $\frac{1}{2}$, 1 inch, 2, and 3 inches in front of point O, and draw the front views of the lines of intersection.

Group B. The Unparted Hyperboloid

7-B1. Lines j:2,1,10;7,$4\frac{3}{4}$,$8\frac{3}{4}$ k:258;616 m:458;418 are the linear directrices of an unparted hyperboloid. Represent a portion of the surface by a set of elements, and find the section cut by a frontal plane Q taken through point P:7,$4\frac{3}{4}$,$8\frac{3}{4}$.

7-B2. Line A:4,$4\frac{3}{4}$,$7\frac{1}{4}$ B:4,$\frac{3}{4}$,$7\frac{1}{4}$ is the axis of a hyperboloid of revolution. The directing circles centered on points A and B are each 4 inches in diameter. The circle of the gorge is $1\frac{7}{32}$ inches in diameter. Represent the surface between circles A and B by 24 equally spaced elements of one ruling. Omit hidden lines in each view.

7-B3. Find a straight line k which intersects each of the four skew lines a:$\frac{1}{4}$,$2\frac{1}{2}$,$6\frac{1}{4}$;$3\frac{1}{2}$,5,$9\frac{1}{2}$ b:$4\frac{1}{2}$,5,$8\frac{1}{2}$;$4\frac{1}{2}$,$\frac{1}{4}$,$8\frac{1}{2}$ c:$6\frac{1}{4}$,$\frac{1}{4}$,$9\frac{1}{2}$;$7\frac{3}{4}$,$3\frac{3}{4}$,$5\frac{3}{4}$ d:$\frac{1}{2}$,2,$7\frac{1}{2}$; $7\frac{1}{2}$,$4\frac{1}{4}$,8.

Suggestion: Let three of the given lines be directrices of a warped surface. Find the points in which the fourth line intersects this surface. The required line k will contain one of these points. There are, in general, two solutions.

GROUP C. THE HELICOID

7-C1. Line a:417;447 is the axis of a right helicoid of 3-inch pitch. The directing helix is right-handed. Line O:447 C:647 is an element of the surface. Draw one turn of one nappe of the portion of the surface lying between a 4-inch and a $1\frac{1}{2}$-inch cylinder. The cylinders are concentric with the helicoid. Show 16 positions of the generating element.

7-C2. Line b:417;447 is the axis of a right helicoid. The directing helix is left-handed and has a 3-inch pitch. Line O:447 D:247 is one element of the surface. Draw the portion of one turn of one nappe of the surface which lies within a 4-inch cylinder concentric with the helicoid. Construct a plane tangent to the helicoid at point P:3,X,6.

7-C3. Line c:417;447 is the axis of a right helicoid. The directing helix is right-handed, and the pitch is 3 inches. Line A:247 B:647 is an element of the surface. Draw one turn, representing both nappes and showing 16 positions of the generating element. Line P:$2\frac{1}{4}$,1,X Q:$5\frac{1}{4}$,4,X is the edge view of a plane perpendicular to the V-plane. Find the intersection of the plane and the helicoid.

7-C4. Line d:4,$\frac{3}{4}$,7;4,$6\frac{1}{4}$,7 is the axis of an oblique helicoid. The directing helix is right-handed and has a 3-inch pitch. Line A:4,$5\frac{3}{4}$,7 B:$6\frac{1}{2}$,$4\frac{1}{2}$,7 is an element of the surface. Draw the front view and a partial top view of $1\frac{1}{2}$ turns of the portion of the surface which lies between a 5-inch and a 1-inch diameter cylinder. In one turn, show 24 equally spaced elements of one nappe.

7-C5. Line C:545 D:515 is the axis of a left-handed helix having a pitch of 3 inches and a diameter of $1\frac{1}{2}$ inches. The helix is the directrix of a convolute which starts at point E:5,4,$4\frac{1}{4}$. Represent the portion of one nappe of the surface which lies between point E and a horizontal plane through point D. Draw 13 elements, and show the trace of the convolute on the horizontal plane through D.

CHAPTER VIII

Horizontal Projection

128. Horizontal projection

Horizontal projection is a system of representation used principally in map making. A single horizontal projection plane is employed. This plane is called the *datum plane* and is usually taken to represent the mean level of the sea.

A point of space is located by its horizontal projection together with an index number which denotes the position of the point above or below the datum plane in terms of some given unit of distance. The index number is positive when the point is above the plane, zero when in the plane, and negative when below it. A scale of distance must always accompany the drawing. Point P (Figure 204) is five units above the datum plane. Two mutually perpendicular lines are drawn on the H-plane for reference purposes. On a map, these lines would represent a meridian and a parallel of latitude.

129. The straight line

A straight line is represented by the projections of two of its points, including their index numbers (Figure 205). A vertical projection of an

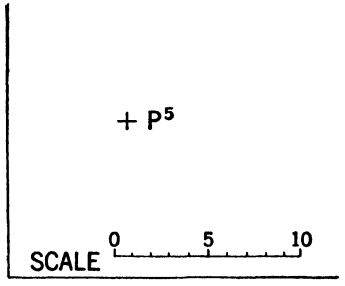


Fig. 204

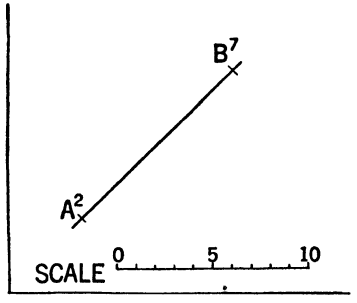


Fig. 205

object can always be constructed from the information contained in the horizontal projection. Thus, the true length of line AB and its angle with the horizontal (α) is found by projecting AB on a vertical plane taken parallel to the given horizontal projection (F gure 206).

A line is *graduated* by marking off points which are integral distances

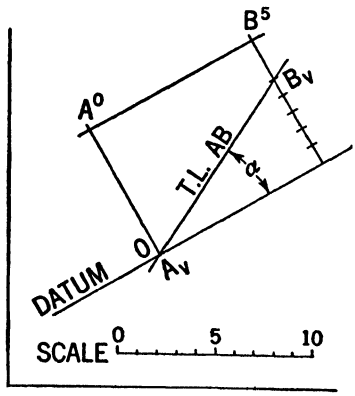


Fig. 206

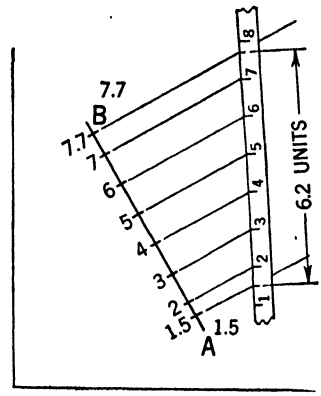


Fig. 207

from the datum plane. The method of proportional division is convenient in determining the subdivisions (Figure 207).

130. The plane

The plane may be represented by the projections of three of its points, two intersecting lines, or two parallel lines.

A plane can be mapped by means of a series of horizontal lines which lie in the plane at integral distances from the datum plane. Let line PQ (Figure 208) be a line of maximum slope of a given plane. The line

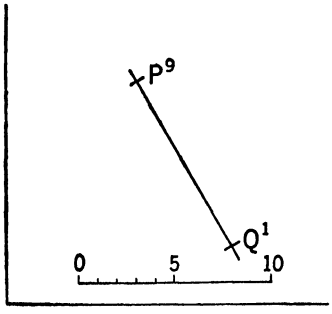


Fig. 208

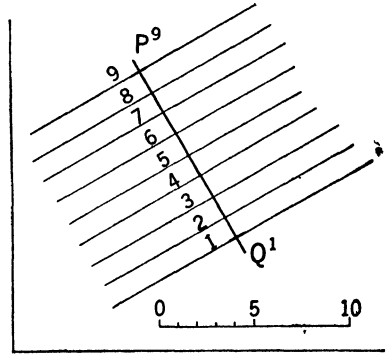


Fig. 209

is graduated as in Figure 209, and lines are drawn through the points of division at right angles to PQ. These lines are horizontals or contours of the plane, and, when indexed according to their distances from the datum plane, provide a graphical representation of the plane.

In horizontal projection, the plane is most conveniently represented by a line of maximum slope (Article 30). Thus, the plane in Figure 210 is

represented by the projection of the line of maximum slope AB. The symbols of Figure 210 are used in geological mapping to represent a plane formed by the face of a layer of rock. The *strike* is a level line through a given point of the plane. The *dip* is the inclination of the plane with the horizontal.

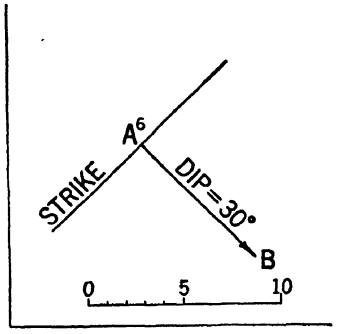


Fig. 210

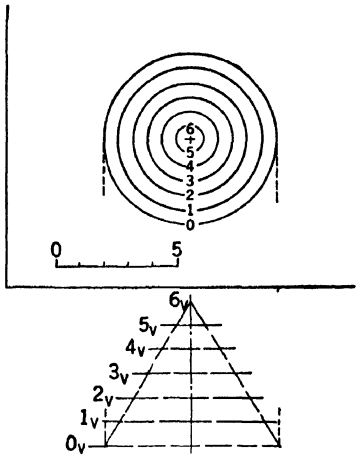


Fig. 211

131. Curved surfaces

A curved surface is represented by a series of sections cut from the surface by planes taken at integral distances from the datum plane. For example, the cone of Figure 211 is represented by the horizontal projections of a set of circles cut from the cone by horizontal planes at unit intervals. The circles are contour lines of the surface. A *contour line* is defined as the locus of the points of a surface which are equally distant from the datum.

132. Surface of the ground

Any general surface can be represented in horizontal projection by the method outlined in the preceding paragraph. The form of the ground surface is represented on a map by the use of contours (Figure 212). Each contour is a line which contains all points of equal height above the mean level of the sea, or more simply, it is a level line on the ground.

133. Applications of horizontal projection

Horizontal projection has its chief application in map making. Also many problems concerning points, lines, and surfaces can be solved as conveniently by horizontal projection as by a two-view drawing. In some solutions (as in determining the true length of a line, Figure 206),

it will be found that the construction is equivalent to drawing a vertical projection. Since the principles involved in solving a problem in horizontal projection are the same as those employed in a two-view drawing, only a few examples will be given.

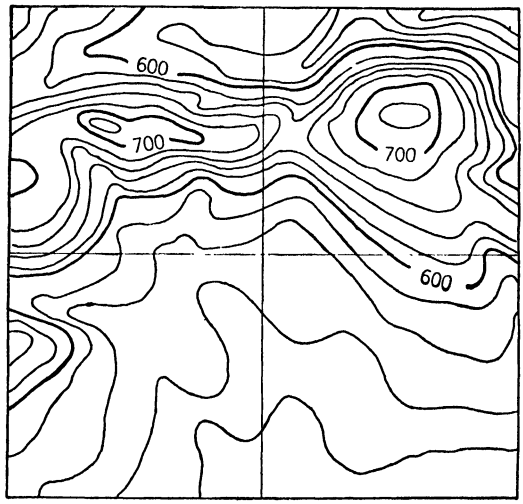


Fig. 212

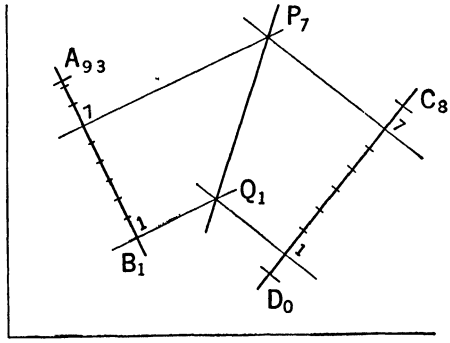


Fig. 213

134. Intersection of two planes

To find the line of intersection between the planes given by their lines of steepest slope AB and CD (Figure 213).

Analysis: Graduate each line. At the same level on each line, draw a horizontal of the plane. The horizontals at the same level cut each other in a point which lies on the line of intersection of the planes.

The horizontals drawn at level 1 intersect in point Q; the horizontals at level 7 intersect in point P. The line of intersection is determined by P and Q.

135. Intersection of plane and curved surface

To find the intersection of the plane given by the line of steepest slope AB and the curved surface represented by its contour lines (Figure 214).

Analysis: The line of steepest slope is graduated to conform to the levels of the given contours of curved surface, and horizontals are drawn through the points of division. At each level, the horizontal of the plane cuts a contour of the surface in two points which lie on the required intersection.

136. Profiles

A vertical section of the ground is called a *profile.* It is drawn as an elevation with the vertical scale exaggerated.

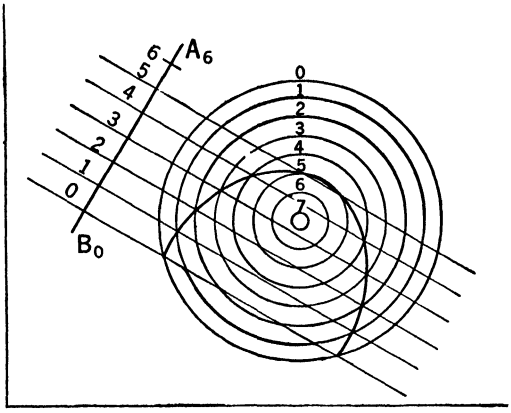


Fig. 214

Use a convenient vertical scale, and draw straight lines to represent the vertical projection of each contour line given in the map (Figure 215). The profile along line *A-A* is determined by the vertical projections of the points in which *A-A* cuts the contour lines of the map. The profile along line *B-B* is found in the same manner.

137. Intersection of inclined plane and ground surface

To find the intersection of the plane ABC and the contoured surface (Figure 216).

Analysis: Draw the projections of a number of horizontal lines of the plane and having the same index numbers as the contour lines. The points in which the lines of the plane meet the contour lines, having the same indices, lie on the required line of intersection.

The horizontal line *BD* of plane *ABC* is found by graduating the line *AC* and joining its 600-foot point *D* with point *B.* Other horizontals are drawn parallel to *BD.*

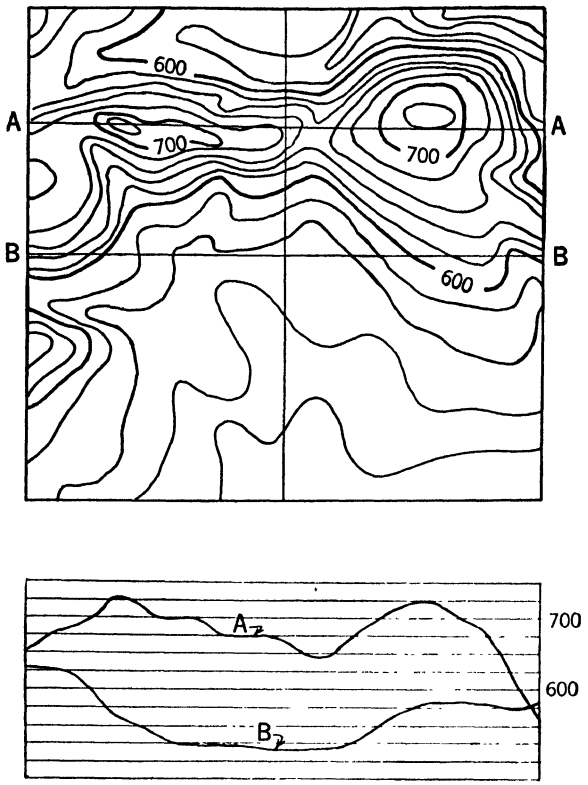


Fig. 215

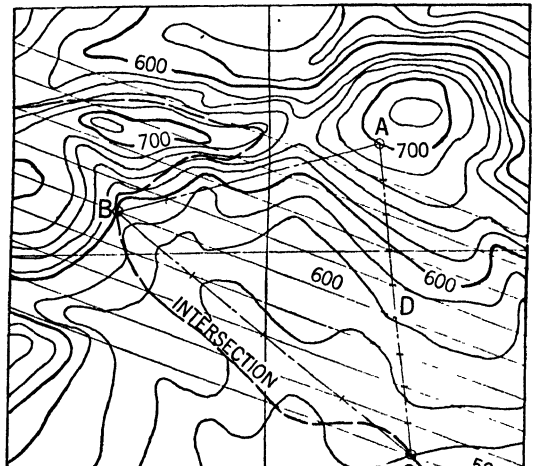


Fig. 216

In practice, points A, B, and C might be points in the outcrop of a thin vein of rock. Assuming that the vein is a plane within a limited area, the line of intersection would represent the line of probable outcrop.

PROBLEMS ON CHAPTER VIII

8-A1. Point A:2,X,3 is 10 feet above the datum plane. Point B:7,X,7 is 7.75 feet above datum. Draw the plan of line AB. Find its true length and its true angle with a horizontal plane. Locate the plan of point C which lies on AB at a distance of 8.50 feet above the datum. Scale: 1 inch = 1 foot.

8-A2. Point A:6,X,2 is 9.75 inches above the datum plane. Point B:3,X,7 is 2.25 inches above the datum. Line AB is a line of maximum slope of the plane Q. Draw a system of contours of Q, taken at integral distances from the datum plane.

8-A3. Point A:2,X,2 is 2.75 inches above the datum plane. Point B:7,X,6 is 1.50 inches below the datum. Line AB is a line of maximum slope of the plane R. Draw a system of contours of R. Find the strike and dip of R, assuming north at top of sheet and meridian parallel to left edge.

8-A4. Point A:3,X,3 is $\frac{3}{4}$ inches below datum. Point B:1,X,6 is $2\frac{1}{2}$ inches above datum. Point C:7,X,7 is $3\frac{3}{4}$ inches above datum. Point D:5,X,4 is $\frac{1}{2}$ inch above datum. Lines BA and CD are lines of maximum slope for planes P and Q respectively. Find the line of intersection of the two planes.

CHAPTER IX

Analytic Solutions

138. Analytical method

Descriptive geometry and space analytic geometry include the same subject matter. In the former, the method of solving problems is graphical and in the latter, analytical. The instrumental precision of graphical methods is sufficient for most practical problems. The tolerances obtainable graphically are generally well within the tolerances of physical data. In cases where analytical results are necessary, the graphical method provides an independent check of the solution. In order that one may pass readily from one form of representation to the other, certain portions of descriptive geometry and analytic geometry will be correlated.

139. Cartesian coordinate planes

Three mutually perpendicular planes constitute the reference planes of space analytic geometry. These planes intersect in a point called the *origin of coordinates*. The planes also intersect one another in the three lines OX, OY, and OZ which are the axes of a system of rectangular

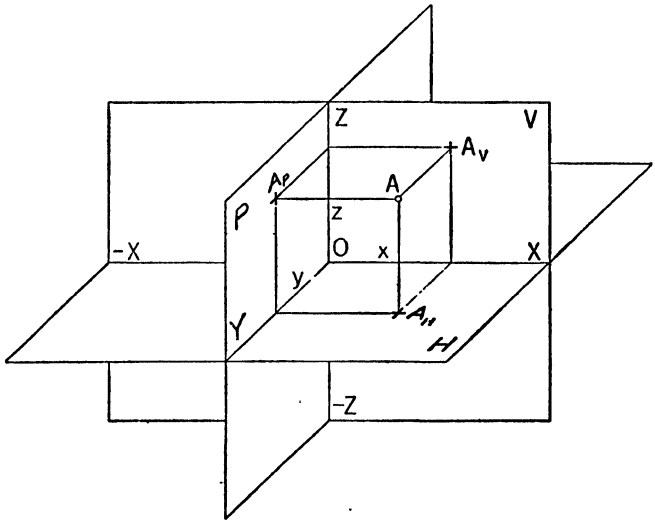


Fig. 217

coordinates (Figure 217). When the cartesian coordinate planes are horizontal, vertical, and profile, they are identical to the principal projection planes of descriptive geometry. Considered as directed lines, the X-, Y-, and Z-axes are taken as positive forward, upward, and to the right of the origin. The distances A-A_P, A-A_V, and A-A_H are

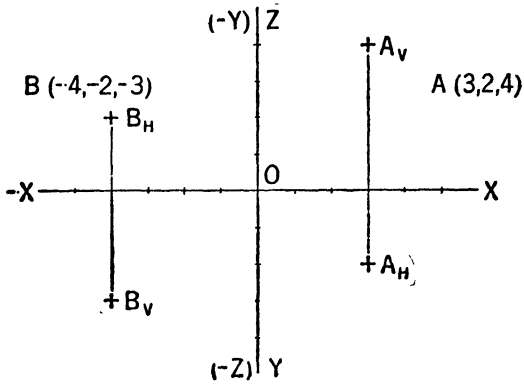


Fig. 218a

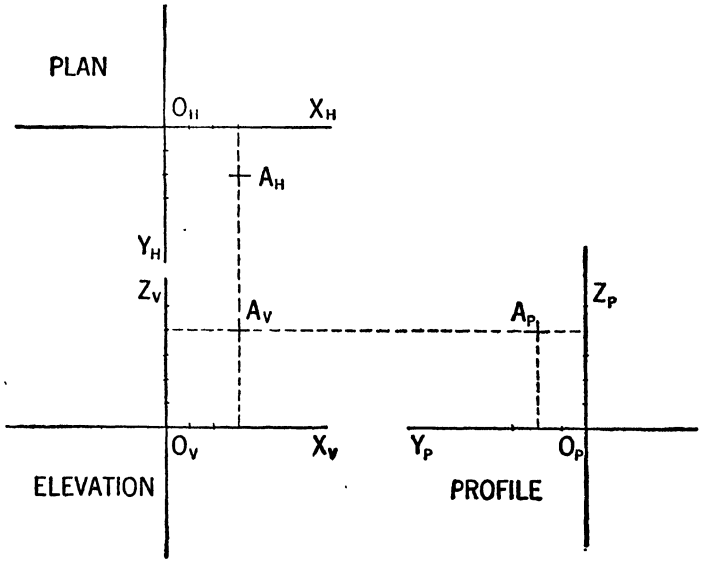


Fig. 218b

respectively the x, y, and z-coordinates of the space point A. It will be noted that only two orthographic projections are needed to fix the location of a point given by three coordinate dimensions.

The coordinate planes are represented on the drawing surface by rotating the H- and V-planes about the axis OX and the P- and V-planes

about the axis OZ (Figure 218*a*). The coordinates of a point of space are given by three measurements in the order x, y, and z. Thus, $A(3,2,4)$ denotes a point in the first quadrant, yielding the projections A_H and A_V. Point $B(-4, -2, -3)$ lies in the third quadrant. The profile projections of A and B are not shown.

An alternative method for representing the reference axes is shown in Figure 218*b*. The top and front views of a point O are assumed to represent the origin. The top views of the three axes are drawn through O_H; the front views through O_V. If a profile view is desired, O_P is assumed, and the profile views of the axes are drawn. This method for representing the axes is useful when the problem extends beyond a single quadrant and leads to overlapping views.

140. Loci

An equation represents a locus if the coordinates of every point of the locus satisfy the equation and every point with coordinates satisfying the equation lies on the locus. A single equation between the coordinates of a point in space generally represents a surface. Thus, the equation $z = 0$ is the equation of the XOY-plane (Figure 217) since the z-coordinate for every point in this plane is equal to zero, and every point whose z-coordinate is zero lies in the XOY-plane. Similarly, the equation of the XOZ-plane is $y = 0$; the YOZ-plane is represented by $x = 0$. The equation $x^2 + y^2 + z^2 = 4$ represents a sphere with center at the origin and radius equal to 2.

Two simultaneous equations generally represent a line. Each equation represents a surface, and the points common to both represent the line of intersection of the two surfaces. The line may be straight or curved, depending on the form of the surfaces.

Three simultaneous equations usually represent a definite number of points. If the equations represent planes, the three planes intersect, in general, in one point. Thus, the planes $x = 0$, $y = 0$, and $z = 0$ intersect in the point (0,0,0), the origin of coordinates.

141. The locus of every first degree equation in rectangular coordinates is a plane

The first degree equation $Ax + By + Cz + D = 0$ represents a surface (Figure 219). Placing $y = 0$, the surface intersects the XZ-plane in the straight line $a = Ax + Cz + D = 0$; $y = 0$. Placing $x = 0$, the surface intersects the YZ-plane in the line $b = By + Cz + D = 0$; $x = 0$. Lines a and b meet in a point on the Z-axis, found by placing both x and y equal to zero. The point is $-\frac{D}{C}$ units from the origin. The plane $X'O'Y'$, taken parallel to and z_1 units from XOY, cuts the

surface in the line $k = Ax + By + Cz_1 + D = 0$; $z = z_1$. Placing $y = 0$ determines the point in which line k intersects XZ-plane. This point also lies in line a. Placing $x = 0$ determines the point in which line k intersects the YZ-plane. This point also lies in line b. Since z_1 can have any value, the surface $Ax + By + Cz + D = 0$ is the locus of a straight line which moves so as to touch constantly two intersecting straight lines. Therefore, $Ax + By + Cz + D = 0$ represents a plane.

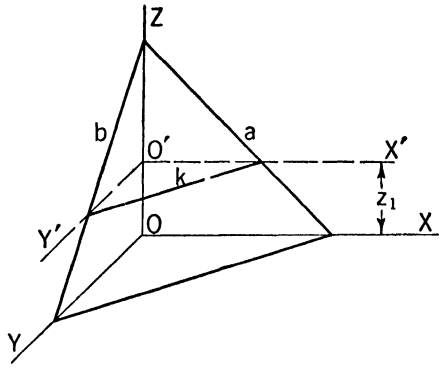


Fig. 219

142. The plane

The plane is represented analytically by a first degree equation of the form $Ax + By + Cz = D$. The H-trace of this plane is found by placing $z = 0$. Similarly, when $y = 0$, the line $Ax + Cz = D$; $y = 0$ represents the V-trace of the plane. The plane cuts the X-axis in a point having y- and z-coordinates equal to zero. Therefore, the X-intercept is equal to $\frac{D}{A}$. Similarly, the Y-intercept is $\frac{D}{B}$, and the Z-intercept is $\frac{D}{C}$. Thus, the plane $Q = 5x + 10y + 6z = 30$ has the line $HQ = x + 2y = 6$; $z = 0$ for its H-trace, and $VQ = 5x + 6z = 30$; $y = 0$ for its V-trace. The intercepts on the X-, Y-, and Z-axes are 6, 3, and 5 respectively (Figure 220). The *intercept* form of the equation of a plane is:

$$\frac{x}{\frac{D}{A}} + \frac{y}{\frac{D}{B}} + \frac{z}{\frac{D}{C}} = 1.$$

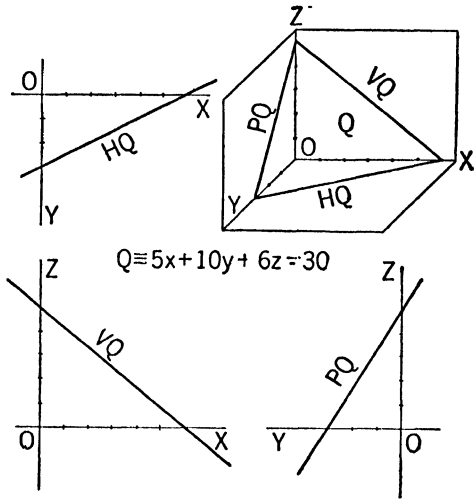


Fig. 220

The traces of the plane are plotted in orthographic projection by assigning some convenient unit of length to the values of x, y, and z. If the constant term D equals zero, the plane passes through the origin. If the coefficient of any variable equals zero, the plane is parallel to the corresponding axis.

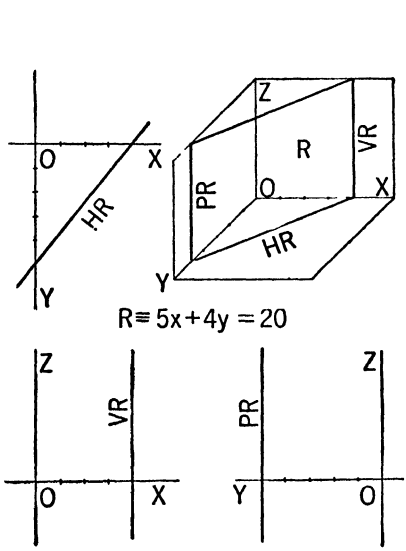


Fig. 221.

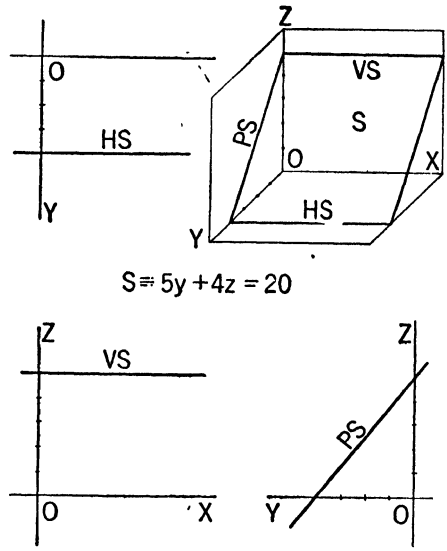


Fig. 222

Thus, the plane $R \equiv 5x + 4y = 20$ has the line $HR \equiv 5x + 4y = 20$; $z = 0$ for its H-trace, and $VR \equiv x = 4; y = 0$ for its V-trace. The intercepts on the X-, Y-, and Z-axes are 4, 5, and infinity respectively (Figure 221).

The plane $S \equiv 5y + 4z = 20$ has the line $HS \equiv y = 4$; $z = 0$ for its H-trace, and $VS \equiv z = 5$; $y = 0$ for its V-trace. The intercepts on the X-, Y-, and Z-axes are infinity, 4, and 5 respectively (Figure 222).

The equations of the H-, V-, and P-projection planes are $z = 0$, $y = 0$, and $x = 0$ respectively.

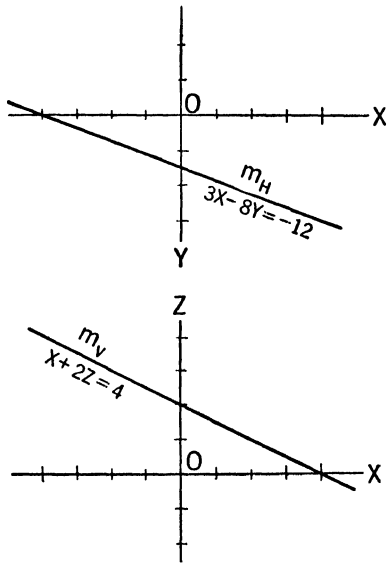


Fig. 223

143. The straight line

In rectangular coordinates, a straight line is determined by the intersection of two planes, and therefore requires two equations for its representation. Thus, a line is determined by two general equations of the form $A_1x + B_1y + C_1z = D_1$, and $A_2x + B_2y + C_2z = D_2$. If the determining planes are the H- and V-projecting planes of the line, the

equations have the form $A_1x + B_1y = D_1$ and $A_2x + C_2z = D_2$ respectively. These equations locate directly the H- and V-projections of the line.

Example. The line $m \equiv 3x - 8y = -12; x + 2z = 4$ is represented graphically by the H-trace of the plane $3x - 8y = -12$ and the V-trace of the plane $x = 2z = 4$ (Figure 223).

144. The point

In general, a point is determined by three planes. The coordinates of a point:

$$P \equiv \begin{cases} A_1x + B_1y + C_1z = D_1, \\ A_2x + B_2y + C_2z = D_2, \\ A_3x + B_3y + C_3z = D_3, \end{cases}$$

are given by the simultaneous solution of the three equations.

145. Direction of a straight line

(*a*) *Direction Components.* Let points $P(x_1y_1z_1)$ and $Q(x_2y_2z_2)$ determine a line (Figure 224). Construct a rectangular prism with PQ as a diagonal and edges a, b, and c parallel to the coordinate axes. The positive direction of each edge is the same as that of the parallel axis. The edges a, b, and c are the *direction components* of the line PQ, and their lengths are written $a = (x_2 - x_1)$, $b = (y_2 - y_1)$, and $c = (z_2 - z_1)$. Hence, we say that line PQ has the direction $a:b:c$.

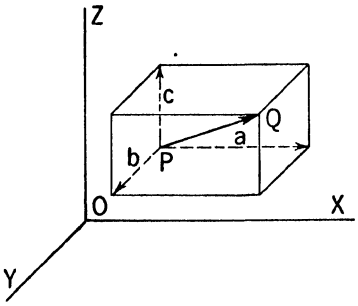


Fig. 224

(*b*) *Direction Cosines.* Let the angles between line PQ (Figure 224) and a, b, and c be given by the letters α, β, and γ respectively. These are the direction angles of the line. The cosines of α, β, and γ are the *direction cosines* of the line.

146. The distance between two points

Referring to Figure 224, the length PQ is equal to $\sqrt{a^2 + b^2 + c^2}$ or $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$.

147. Equations of a line determined by two points

A line passing through points $P(x_1y_1z_1)$ and $Q(x_2y_2z_2)$ has for its direction components $(x_2 - x_1):(y_2 - y_1):(z_2 - z_1)$. Let $R(xyz)$ be any point on this line. Then the direction components of RP are

$(x - x_1):(y - y_1):(z - z_1)$. Since PQ and RP are parts of the same line, the direction components of the two segments are proportional.

Therefore, $$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}. \tag{1}$$

It follows that a line through a point $(x_1y_1z_1)$ and parallel to a line having the direction components $a:b:c$ has for its equations:

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}. \tag{2}$$

148. Problem. To pass a line through two points

Graphic Solution: The line is represented by its projections m_H and m_V which contain the projections of the given points A and B (Figure 225).

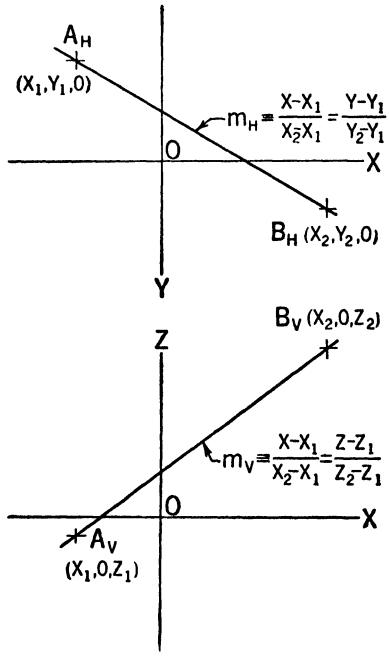


Fig. 225

Analytic Solution: The equations of the line m passing through points $A(x_1y_1z_1)$ and $B(x_2y_2z_2)$ are:

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1}, \quad \text{and} \quad \frac{x - x_1}{x_2 - x_1} = \frac{z - z_1}{z_2 - z_1}.$$

These equations represent the *plane projectors* of line m which are perpendicular to the H- and V-planes respectively.

149. Problem. To find the points in which a line intersects the projection planes

Graphic Solution (*Figure 226*): The point S_V in which m_V meets OX is the vertical projection of the point in which line m pierces the H-plane. The projection S_H is on m_H and the projector through S_V. The point T_H in which m_H meets OX is the horizontal projection of the point in which line m pierces the V-plane. T_V is on m_V. The projections S_V and T_H are always on OX. The projections S_H and T_V are the

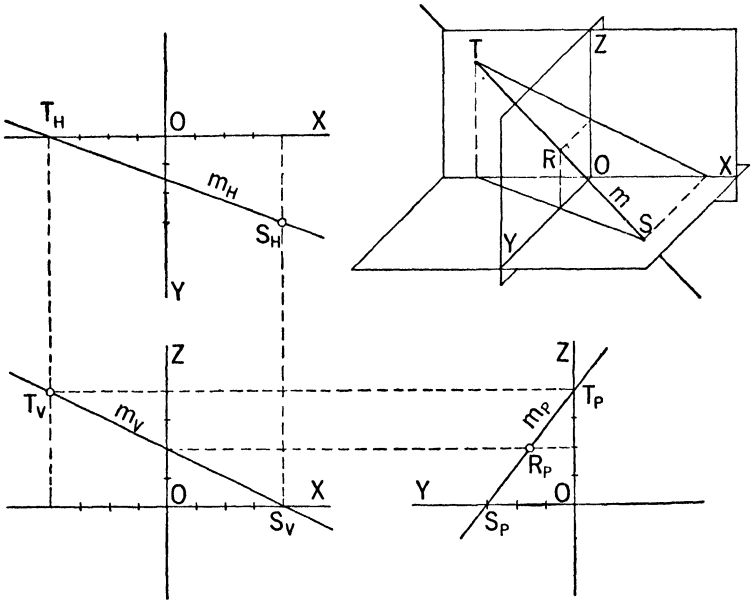


Fig. 226

horizontal and vertical traces of the line. The point R in which line m pierces the profile plane is the profile trace.

Analytic Solution: Let the H- and V-*plane projectors* of the line m be $A_1x + B_1y = D_1$ and $A_2x + C_2z = D_2$. The H-trace (S) is a point common to three planes—the two plane projectors of the line m and the H-plane. The V-trace (T) is the point common to the two plane projectors of m and the V-plane.

$$\text{Therefore, } S \equiv \begin{cases} A_1x + B_1y = D_1 \\ A_2x + C_2z = D_2 \\ z = 0 \end{cases} \text{ and } T \equiv \begin{cases} A_1x + B_1y = D_1 \\ A_2x + C_2z = D_2 \\ y = 0. \end{cases}$$

The simultaneous solution of each set of equations gives the coordinates of points S and T.

Example (Figure 227). Represent the line k which passes through points $A(1,3,6)$ and $B(7,9,10)$. Determine the coordinates of the H-trace (S) and the V-trace (T).

Graphic Solution: Plot points A and B, and draw line k. Line k pierces the H-plane in point $S(-8,-6,0)$ and the V-plane in point $T(-2,0,4)$.

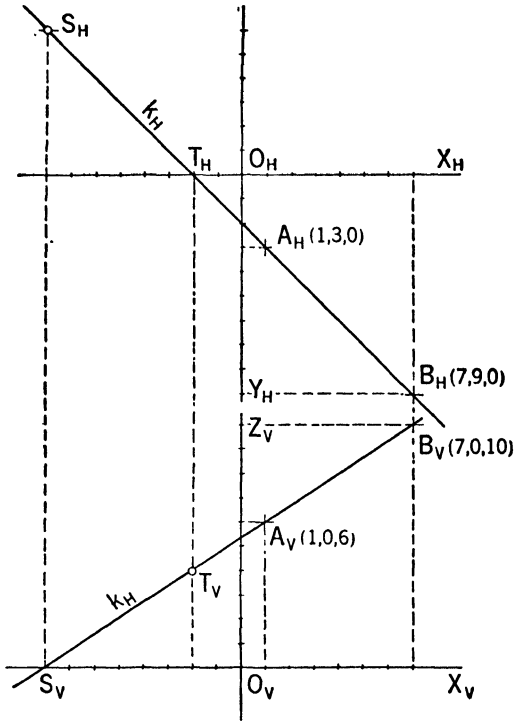


Fig. 227

Analytic Solution: The equations of the line k are:

$$\frac{x-1}{7-1}=\frac{y-3}{9-3}, \quad \text{or} \quad x-y=-2$$

and

$$\frac{x-1}{7-1}=\frac{z-6}{10-6} \quad \text{or} \quad 2x-3z=-16.$$

Therefore,

$$k \equiv \begin{cases} x-y=-2 \\ 2x-3z=-16. \end{cases}$$

To find the traces of line k:

$$\text{Point } S \equiv \begin{cases} x-y=-2 \\ 2x-3z=-16 \\ z=0. \end{cases}$$

A simultaneous solution gives $x=-8$, $y=-6$, and $z=0$.

Therefore, H-trace $\equiv S(-8,-6,0)$.

$$\text{Point } T \equiv \begin{cases} x - y = -2 & \text{A simultaneous solution} \\ 2x - 3z = -16 & \text{gives} \\ y = 0. & x = -2,\ y = 0, \text{ and } z = 4. \end{cases}$$

Therefore, V-trace $\equiv T(-2,0,4)$.

150. Equation of a plane determined by three points

(*a*) *Point Equation of a Plane.* The position of the plane represented by $Ax + By + Cz + D = 0$ depends on the relative values of the coefficients. This equation takes the form $A'x + B'y + C'z + 1 = 0$ (1) when divided by D. The coordinates of any point which lies in this plane satisfy this equation.

To find the equation of the plane determined by points $P(x_1y_1z_1)$, $Q(x_2y_2z_2)$, and $R(x_3y_3z_3)$, substitute these coordinates for x, y, and z in equation (1), and solve the resulting three equations for A', B', and C'. Substitute the values obtained in equation (1).

Example (Figure 228). Represent the plane determined by points $D(-4,2,8)$, $E(-3,1,3)$, and $F(5,7,2)$. Find its intercepts on the X-, Y-, and Z-axes.

Graphic Solution: Represent the plane by two parallel lines. Draw line DF. Through E draw a parallel to DF. The traces of line DF are S_H and T_V. The traces of line E are S'_H and T'_V. Line HQ is determined by S_H and S'_H; line VQ by T_V and T'_V. HQ and VQ are the traces of the required plane. The intercepts on the X-, Y-, and Z-axes are -2.9, 2.3, and -6.4 respectively.

Analytic Solution: The equation of the required plane has the general form $Ax + By + Cz = 1$, where x, y, and z are the coordinates of any point in the plane. Substituting the values of coordinates of points D, E, and F gives the three simultaneous equations

$$\begin{aligned} -4A + 2B + 8C &= 1; \\ -3A + B + 3C &= 1; \\ 5A + 7B + 2C &= 1. \end{aligned}$$

When solved, $A = -\frac{31}{90}$, $B = \frac{39}{90}$, $C = -\frac{14}{90}$.

When these values are substituted in the general expression

$$Ax + By + Cz = 1,$$

and simplified, the equation of plane Q becomes

$$31x - 39y + 14z = -90.$$

The intercepts on the X-, Y-, and Z-axes are -2.90, 2.31, and -6.43 respectively.

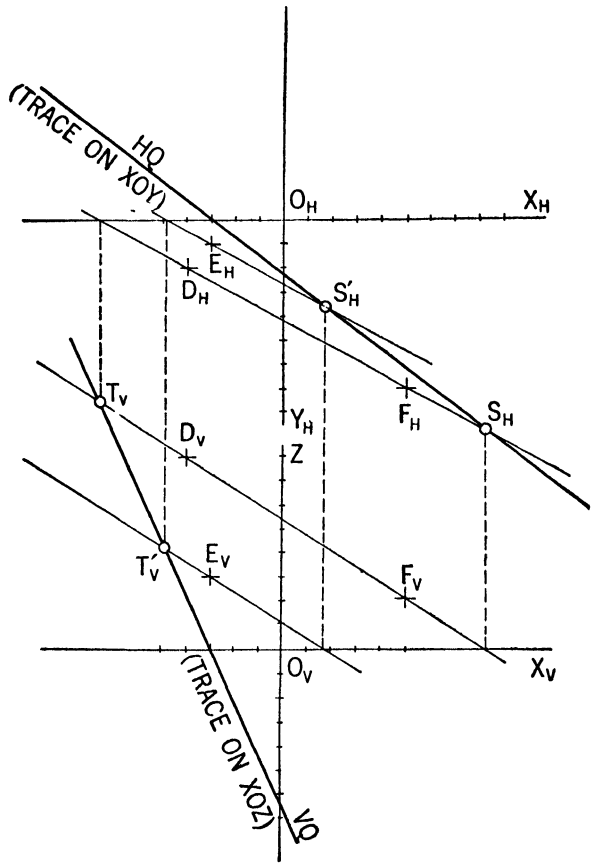


Fig. 228

(*b*) *Normal equation of a plane.* Let the intercepts of a plane on the X-, Y-, and Z-axes be represented by a, b, and c.

The equation of the plane is

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1. \tag{1}$$

Let the length of the normal to the plane through the origin be equal to p. The direction angles of this normal are given by α, β, and γ.

$$\text{Then,} \qquad \frac{p}{a} = \cos\alpha; \qquad \frac{p}{b} = \cos\beta; \qquad \frac{p}{c} = \cos\gamma. \tag{2}$$

When (1) is multiplied by p,

$$\frac{xp}{a} + \frac{yp}{b} + \frac{zp}{c} = p. \tag{3}$$

When the values of (2) are substituted in (3),

$$x \cos \alpha + y \cos \beta + z \cos \gamma = p. \tag{4}$$

This is the *normal* equation of the plane.

151. Intersection of two planes

Two planes $P \equiv A_1x + B_1y + C_1z = D_1$ and $Q \equiv A_2x + B_2y + C_2z = D_2$ intersect in the line

$$k \equiv \begin{cases} A_1x + B_1y + C_1z = D_1 & (1) \\ A_2x + B_2y + C_2z = D_2 & (2) \end{cases}$$

For points in the line of intersection these equations are simultaneous. If z is eliminated from (1) and (2), an equation in x and y results which is the equation of the horizontal projecting plane of the line k. If y is eliminated, the equation of the vertical projecting plane of line k results.

Example. Find the equations of the H- and V-projections of the line of intersection k of planes

$$P \equiv 3x + 5y - 7z = -10, \tag{1}$$
$$Q \equiv 5x - 14y + 3z = 15. \tag{2}$$

Horizontal projecting plane of k: When z is eliminated from (1) and (2),

$$3(1) \quad 9x + 15y - 21z = -30; \tag{3}$$
$$7(2) \quad 35x - 98y + 21z = 105. \tag{4}$$

When (3) and (4) are added, $44x - 83y = 75.$ (5)

Vertical projecting plane of k: When y is eliminated from (1) and (2),

$$14(1) \quad 42x + 70y - 98z = -140, \tag{6}$$
$$5(2) \quad 25x - 70y + 15z = 75. \tag{7}$$

When (6) and (7) are added, $67x - 83z = -65.$ (8)

152. To find the point in which a line intersects an oblique plane

Analytic Solution: The coordinates of point P common to the plane $Q \equiv A_1x + B_1y + C_1z = D_1$ and the line $c \equiv A_2x + B_2y = D_2$; $A_3x + C_3z = D_3$ are given by the simultaneous solution of the three equations.

153. To find the angle between two lines (Figure 229)

Given: Line AB having direction components $a_1:b_1:c_1$ and line AC having direction components $a_2:b_2:c_2$.

The direction components of BC are, therefore,

$$a_2 - a_1 : b_2 - b_1 : c_2 - c_1.$$

Then, if the angle between AB and AC is θ, we have by trigonometry,

$$\cos\theta = \frac{\overline{AB}^2 + \overline{AC}^2 - \overline{BC}^2}{2AB \cdot AC} \tag{1}$$

Since $\overline{AB}^2 = a_1^2 + b_1^2 + c_1^2$, $\overline{AC}^2 = a_2^2 + b_2^2 + c_2^2$,
and $\overline{BC}^2 = (a_2 - a_1)^2 + (b_2 - b_1)^2 + (c_2 - c_1)^2$.

Then, by substitution in (1),

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}. \tag{2}$$

If lines AB and AC are perpendicular, $\cos\theta = 0$, and (2) reduces to

$$(a_1a_2 + b_1b_2 + c_1c_2 = 0). \tag{3}$$

154. The direction of a line perpendicular to a plane

Let $P(x_1y_1z_1)$ and $Q(x_2y_2z_2)$ be any two points in the plane

$$Ax + By + Cz + D = 0. \tag{1}$$

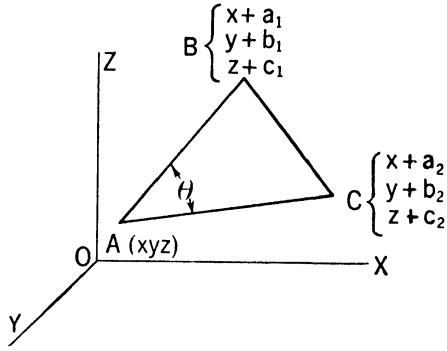


Fig. 229

Substituting the coordinates of P and Q in (1) we have:

$$Ax_1 + By_1 + Cz_1 + D = 0;$$
$$Ax_2 + By_2 + Cz_2 + D = 0.$$

Subtracting the first expression from the second gives:

$$A(x_2 - x_1) + B(y_2 - y_1) + C(z_2 - z_1) = 0.$$

The direction components of PQ are

$$(x_2 - x_1) : (y_2 - y_1) : (z_2 - z_1). \tag{2}$$

Since PQ is any straight line of the given plane, the direction $A:B:C$ is perpendicular to the plane. [Refer to Article 153, (3).]

The equations of a line through a given point $E(x_3y_3z_3)$ and perpendicular to the plane $Ax + By + Cz + D = 0$ are

$$\frac{x - x_3}{A} = \frac{y - y_3}{B} = \frac{z - z_3}{C}. \tag{3}$$

155. Equation of a plane through a given point and perpendicular to a given line

Let the plane pass through the point $P(x_1y_1z_1)$ and be perpendicular to a line having the direction $A:B:C$. If $Q(xyz)$ is any point of the

required plane, the direction components of line QP are $(x - x_1)$: $(y - y_1)$:$(z - z_1)$. Since the given line is perpendicular to the plane, it is perpendicular to line QP which lies in the plane. Therefore, it follows from Article 153, (3), that $A(x - x_1) + B(y - y_1) + C(z - z_1) = 0$ is the equation of the required plane.

156. Angle between two planes

Let θ be the angle between planes:

$$A_1x + B_1y + C_1z + D_1 = 0$$
$$A_2x + B_2y + C_2z + D_2 = 0.$$

The angle between the planes is equal to the angle between the perpendiculars to the planes from any assumed point.

The directions of the perpendiculars to the given planes are $A_1:B_1:C_1$ and $A_2:B_2:C_2$ respectively.

Hence, by Article 153, (2)

$$\cos\theta = \frac{A_1A_2 + B_1B_2 + C_1C_2}{\sqrt{A_1^2 + B_1^2 + C_1^2}\sqrt{A_1^2 + B_1^2 + C_1^2}}. \tag{1}$$

If the two planes are parallel,

$$\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2}. \tag{2}$$

If the two planes are perpendicular, $\cos\theta = 0$.

Hence, $$A_1A_2 + B_1B_2 + C_1C_2 = 0. \tag{3}$$

157. Distance from a point to a plane (Figure 230)

Let $P(x_1y_1z_1)$ be the point and $Ax + By + Cz + D = 0$ the plane. Draw PR parallel to OZ, intersecting the given plane in point R. The coordinates of R are (x_1y_1z).

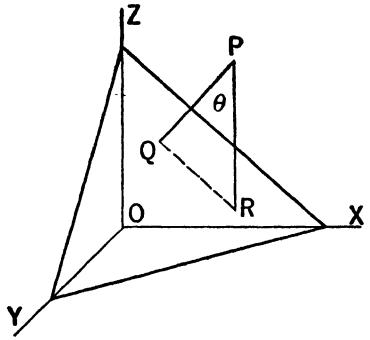


Fig. 230

To find z, substitute x_1 and y_1 in equation of plane.

Whence, $$z = \frac{-Ax_1 - By_1 - D}{C}.$$

Then,

1. Length of $PR = z_1 - z = \dfrac{Ax_1 + By_1 + Cz_1 + D}{C}$.
2. The distance $PQ = PR \cos \theta$.
3. The direction components of PQ are $A:B:C$.
4. The direction components of PR are $0:0:1$.

Hence, $$\cos \theta = \frac{C}{\sqrt{A^2 + B^2 + C^2}}$$ (refer to Article 153)

and

$$PQ = PR \cos \theta = \frac{Ax_1 + By_1 + Cz_1 + D}{\sqrt{A^2 + B^2 + C^2}}. \qquad (1)$$

PROBLEMS ON CHAPTER IX

The problems of this group are to be solved graphically and checked by an analytic solution. The reference planes may be represented as in Figure 218*a* or Figure 218*b* of Article 139. The location on the problem sheet of the origin of coordinates is given in each problem. When O_H and O_V are separated, the method of representation is according to Figure 218*b*. When O_H and O_V coincide, the representation of Figure 218*a* is to be used. Coordinates given in parentheses are cartesian coordinates and are to be measured from the given origin.

Group A. Representation of the Line and Plane

9-A1. Draw the traces of the planes $Q:8x + 16y + 10z = 20$, $R:8x + 16y + 10z = 0$, and $S:8x + 16y + 10z = -20$ on the horizontal and vertical reference planes. Scale: Unit = 1 inch. Origin: $5,2\frac{1}{2},7\frac{1}{2}$ or 555.

9-A2. Draw the traces of the planes $T:x + z = 2$, $U:10x - 15y + 6z = -15$, and $W:3y + 4z = 6$. Scale: Unit = 1 inch. Origin: $4,2\frac{1}{2},7\frac{1}{2}$ or 455.

9-A3. Draw the H- and V-projections of the line $b:3x + 5y = 30$; $2x - 3z = -24$. Find the points S, T, and R in which the line b pierces the H-, V-, and P-reference planes. Scale: Unit = $\frac{1}{4}$ inch. Origin: 419 or 455.

9-A4. Draw the top, front, and side views of the line $d:5x + 8y = 40$; $x + z = 4$. Find the points S, T, and R in which line d pierces the

XOY-, *XOZ*-, and *YOZ*-reference planes. Scale: Unit = $\frac{1}{4}$ inch. Origin: 2,2$\frac{1}{2}$,7$\frac{1}{2}$.

9-A5. Draw the top, front, and side views of the line $k: x - 2y = -4$; $2x - z = 0$. Find the points S, T, and R in which k pierces the H-, V-, and P-reference planes. Scale: Unit = $\frac{1}{4}$ inch. Origin: 2,2$\frac{1}{2}$,7$\frac{1}{2}$.

9-A6. Draw the top and front views of the line which passes through points $A(-12,8,16)$ and $B(8,-8,4)$. Determine the points S and T in which the line pierces the horizontal and vertical reference planes. Scale: Unit = $\frac{1}{4}$ inch. Origin: 4,$\frac{1}{2}$,6$\frac{1}{2}$ or 455.

9-A7. Draw the top, front, and side views of the line which passes through points $C(2,8,-4)$ and $D(828)$. Find the points S, T, and R in which the line pierces the *XOY*-, *XOZ*-, and *YOZ*-planes. Scale: Unit = $\frac{1}{4}$ inch. Origin: $\frac{1}{2}$,3,9.

9-A8. Plane Q passes through points $A(035)$, $B(-6,9,1)$, and $C(8,9,-13)$. Find its traces on planes *XOY* and *XOZ*. Find the coordinates of the intercepts. Scale: Unit = $\frac{1}{4}$ inch. Origin: 4,3$\frac{1}{2}$,9$\frac{1}{2}$ or 455.

9-A9. Find the traces on the horizontal, vertical, and profile coordinate planes for the oblique plane W which is determined by points $A(028)$, $B(064)$, and $C(-6,2,4)$. Find the coordinates of the intercepts. Scale: Unit = $\frac{1}{4}$ inch. Origin: 428.

9-A10. Find the line common to the planes $P: 10x - 15y - 10z = -90$ and $Q: x + y + 2z = 12$. Write the equations of the projecting planes of the line. Scale: Unit = $\frac{1}{4}$ inch. Origin: 429 or 455.

9-A11. Find the line of intersection of the planes $P: 10x + 15y + 6z = 90$ and $Q: 5x - 5y - 6z = -60$. A solid is enclosed by planes P and Q and the H- and V-coordinate planes. Find the section cut from this solid by the plane $W: 6x + 14z = 84$. Section-line the cut surface. Find the coordinates of the point K which is common to planes P, Q, and W. Scale: Unit = $\frac{1}{4}$ inch. Origin: 419 or 455.

9-A12. Find the line common to planes $Q: 2x + 2y + z = 12$ and $R: 3x - y + 4z = -12$. Write the equations of the projecting planes of the line, and find the coordinates of its traces on *XOY* and *XOZ*. Scale: Unit = $\frac{1}{4}$ inch. Origin: 4,3$\frac{1}{4}$,9$\frac{3}{4}$ or 455.

9-A13. Find the coordinates of the point K which is common to the planes $P: 6x - 3y - 4z = -12$ $Q: 2x + y - 2z = -2$ and $R: 9y + 8z = 36$. Draw the projections of the tetrahedron which is enclosed by planes P, Q, R, and *XOY*. Scale: Unit = 1 inch. Origin: 455.

9-A14. Find the point P in which line b: (−2,2,2); (14,10,6) intersects the plane $M: x + y + 2z = 12$. Scale: Unit = $\frac{1}{4}$ inch. Origin: 428

9-A15. Find the point K common to line d:$(-1,3\frac{1}{4},1)$; $(1\frac{1}{2},1\frac{1}{2},3\frac{1}{2})$ and plane W:$3x - 2y + 3z = 3$. Scale: Unit = 1 inch. Origin: 455.

GROUP B. DISTANCES

9-B1. Line A:217 B:$5\frac{1}{2}$,1,7 is the lower front edge of a prism 2 inches deep and 4 inches high. Find the true length of a body diagonal of the prism.

9-B2. Find the perimeter of the triangle A:157 B:319 C:736.

9-B3. Through point $A(-3,-3,-4)$ construct a perpendicular to plane Q:$2x + y + 2z = -2$. Find the foot of the perpendicular and the shortest distance from point A to plane Q. Scale: Unit = 1 inch. Origin: $4,4\frac{1}{2},5\frac{1}{2}$ or 455.

9-B4. Find the shortest distance from the origin to plane R:$3x - 4y - 3z = 12$. Scale: Unit = 1 inch. Origin: $4,4\frac{1}{2},5\frac{1}{2}$ or 455.

9-B5. Through point $P(2,0,0)$ pass a plane N perpendicular to line $A(0,-1,-2)$ $B(3,-4,-4)$. Scale: Unit = 1 inch. Origin: $4,4\frac{1}{2},5\frac{1}{2}$ or 455.

9-B6. Find the volume of the tetrahedron $V(034)$ $A(201)$ $B(440)$ $C(523)$.

Note: Volume of tetrahedron $= \frac{1}{6}\begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_4 & y_4 & z_4 & 1 \end{vmatrix}$.

Scale: Unit = 1 inch. Origin: 119.

GROUP C. ANGLES

9-C1. Find the true size of the angle A:236 B:$3,4\frac{1}{2},7\frac{1}{2}$ C:$5,4,5\frac{1}{2}$.

9-C2. Find the true size of the angle D:226 E:545 F:728.

9-C3. Find the true size of the dihedral angle between planes Q:$15x - 30y - 8z = -120$ and R:$60x + 56y + 105z = 840$. Scale: Unit = $\frac{1}{4}$ inch. Origin: 455 or 419.

9-C4. Find the true size of the dihedral angle between plane R:$2x - 3y - 3z = -30$ and the horizontal reference plane. Scale: Unit = $\frac{1}{4}$ inch. Origin: 419 or 455.

9-C5. Find the true size of the dihedral angle between plane Q:$3x + 5y + 3z = 30$ and the vertical reference plane. Scale: Unit = $\frac{1}{4}$ inch. Origin: 419 or 455.

CHAPTER X

Pictorial Drawing

158. Definition of pictorial drawing

The term *pictorial drawing* is used to describe a projection of an object which permits ease of visualization. The projection itself may be an orthographic projection in which the object has some specified orientation with respect to the projection plane—isometric, dimetric, and trimetric are in this class—or it may be some other form of projection, such as perspective or oblique.

Isometric, dimetric, and trimetric projections presuppose an object whose surfaces lie predominantly in, or parallel to, three mutually perpendicular planes, the intersections of which establish three mutually perpendicular axes. In the sense used here, the orthographic projection of such an object on any oblique projection plane is a pictorial drawing.

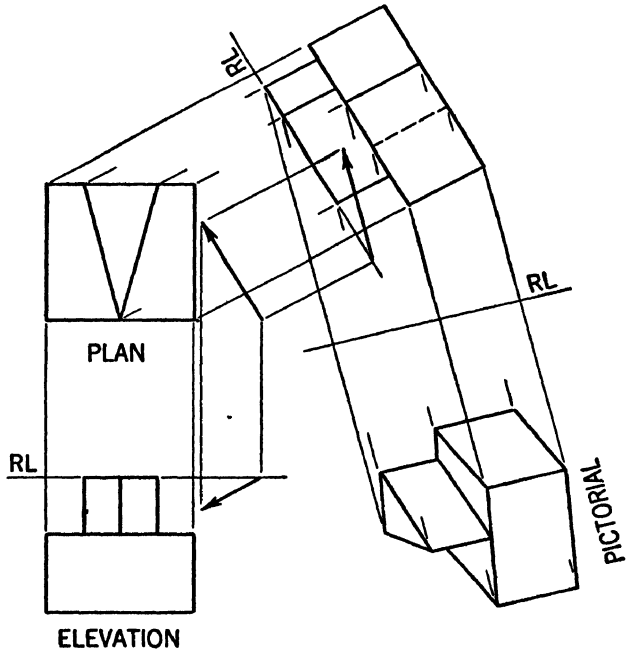


Fig. 231

159. Pictorial view by auxiliary projection

Such a drawing may be obtained consequently from two orthographic views by inserting an arrow in the desired direction of sight and obtaining a view in this direction by auxiliary projection (Figure 231). This is the desired pictorial view.

Such views are classified according to the inclination of the axes to the oblique projection plane. For any given angle of inclination of a line to a plane, the ratio of the length of the projection of any line segment on that plane to its true length is a constant. Consequently, the angle of inclination may be expressed as a scale factor.

160. Pictorial view by direct measurement

Consider a cube whose edges are of unit length (Figure 232). A projection on any oblique plane may be obtained by the auxiliary-view method. The three edges in the oblique view will make angles of α, 90°, and β with the horizontal, and the edges will have scales of $\frac{W}{1}$, $\frac{H}{1}$, and $\frac{D}{1}$

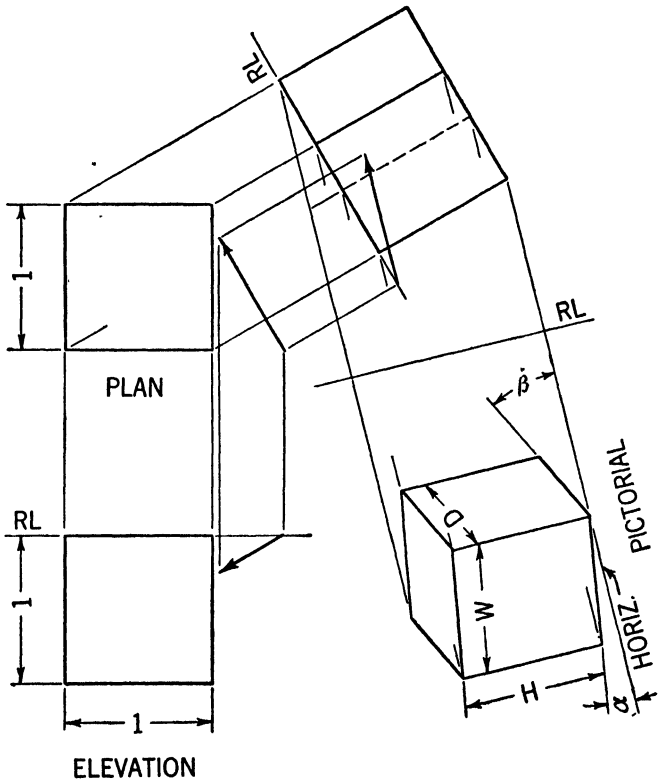


Fig. 232

respectively, where W, H, and D are the projected lengths of the unit edges on the oblique plane.

A drawing of any object may be made on this plane by direct measurement, without reference to orthographic views, by constructing the three axes at the specified angles and using the specified scales, each along its proper axis.

Since any line parallel to an axis projects at the scale of that axis, the entire object may be drawn by measurements along the axes or along lines parallel to them, discrete points being located by their three coordinates from the origin. This process is described more fully later for isometric drawing, the most common form of this class of pictorials.

161. Pictorial view by revolved-view method

Any object may be projected on any oblique plane by projecting directly onto that plane as represented by its traces.

Consider the cube in Figure 233 to be projected on the oblique plane Q. Construct perpendiculars from points on the cube to the oblique plane. Determine the points of intersection of the perpen-

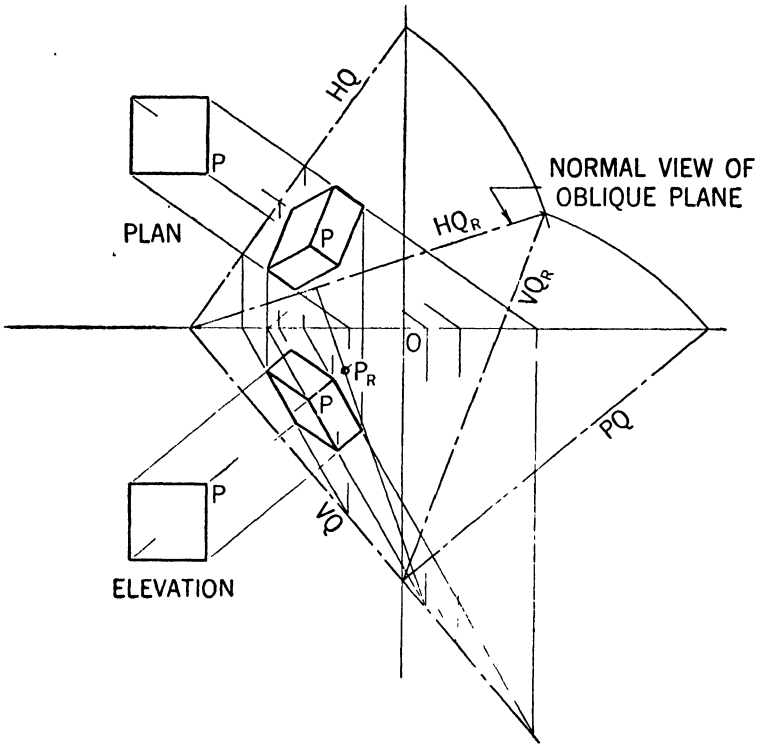


Fig. 233

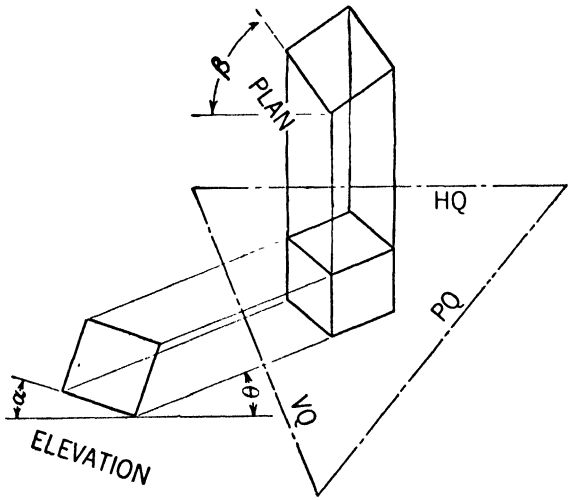


Fig. 234

diculars with the oblique plane. Join the points. The H- and V-planes containing the front and top views may now be revolved into the oblique plane so that all three views lie in a single plane. To do this, a true view of the oblique plane may be obtained by revolving the oblique plane around VQ into the V-plane. In Figure 233, the normal view of the oblique plane is shown by its three traces together with one point (P) of the object.

Since the plan and elevation in Figure 233 are normal views containing HQ and VQ respectively, their relation to these traces is established. Hence, when the three views are revolved into a single normal view, the relation of the plan and elevation to HQ and VQ will be the same as in Figure 233.

We may thus construct the three views lying in a single plane and orient them so that the vertical edges of the cube appear vertical (Figure 234). This establishes the direction of the two horizontal axes.

The directions of the axes may be established also from the projection of the origin on the oblique plane. Since the angles XOY, XOZ, and YOZ are right angles, they will appear as such when rotated into the oblique plane. This may be done by constructing semicircles on XZ, XY, and YZ as diameters and projecting O on the semicircle perpendicular to the diameter. The directions O_VX_V and O_VZ_V (Figure 235) show the directions of the axes in the elevation and O_HX_H and O_HY_H in the plan.

The width parallel to the X-axis in the plan and elevation (Figure 234) will now make angles of α and β with the horizontal. The projectors

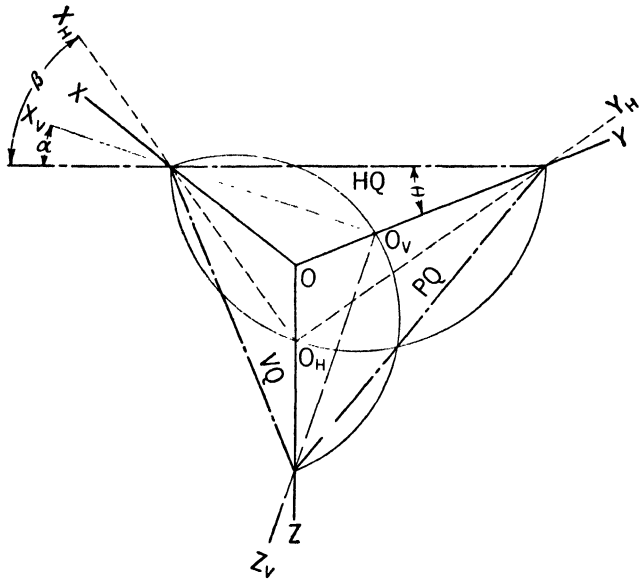


Fig. 235

onto the oblique plane will be perpendicular to the traces and will make angles of 90° and θ with the horizontal. A pictorial view of any object may consequently be made on this oblique plane directly from a plan and elevation by placing the widths in the two views at angles of α and β respectively with the horizontal and projecting at angles of θ and 90° respectively with the horizontal. The traces are immaterial and may be omitted. The two views may be spaced in any convenient manner. Figure 236 shows a pictorial obtained in this manner.

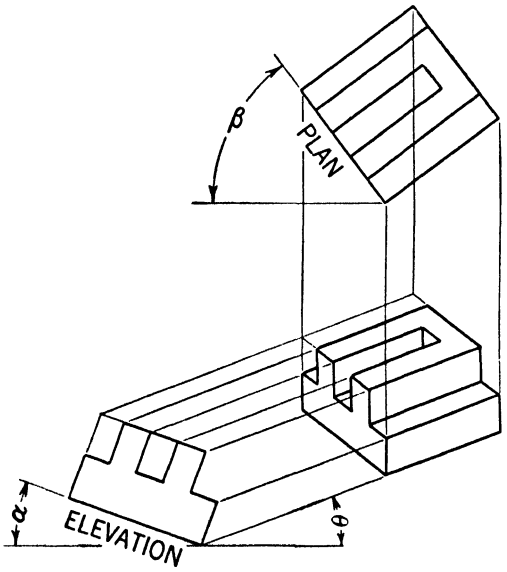


Fig. 236

162. Choice of oblique plane

The image received by the eye from an object in space is a perspective view in which parallel lines in space converge in the view. Since parallel

lines remain parallel in pictorials of the above type, they appear to be visually distorted. This distortion increases rapidly in the direction of the line of sight.

The choice of an oblique plane is made to minimize this distortion. This is best achieved by placing the minimum dimension of the object so that it is parallel to the axis having the maximum inclination to the projection plane.

Any specific orientation of the oblique plane may be chosen as a standard for making a number of pictorial drawings. This is a matter of individual preference. Once a standard is set, scales may be made and triangles cut to the angles α, β, and θ to reduce to a minimum the work of making such drawings.

163. Trimetric projection

The general case discussed above, when the three axes have different inclinations to the oblique projection plane and the three scales are different, yields a *trimetric drawing.*

164. Dimetric projection

A *dimetric drawing* is obtained when two of the axes are equally inclined to the oblique projection plane. Two of the scales are consequently equal.

The necessary condition for this is that the perpendicular to the oblique plane through the origin shall make equal angles with two of the axes. This specifies that in obtaining a dimetric from orthographic views, the projection of the arrow representing the line of sight shall make an angle of 45° with the reference line between the two views in either one of the orthographic views.

165. Isometric projection

Since isometric projection is the most important of this general class of projections, it will be treated at greater length. The processes described for isometric are also applicable to dimetric and trimetric with proper attention to scales and angles.

If three mutually perpendicular lines are equally inclined to a plane, the projections on that plane of equal lengths of each line will be equal, and the angles between these projections will be equal. Thus, since the sum of the angles is 360°, each angle must be 120°.

The projection plane, called the *isometric plane,* may be defined as a plane perpendicular to the diagonal of a cube whose three perpendicular edges project into the isometric axes.

Figure 237 shows a view of this cube projected on the isometric plane. This view is obtained from two standard orthographic views by drawing a view in which the long diagonal appears as a point.

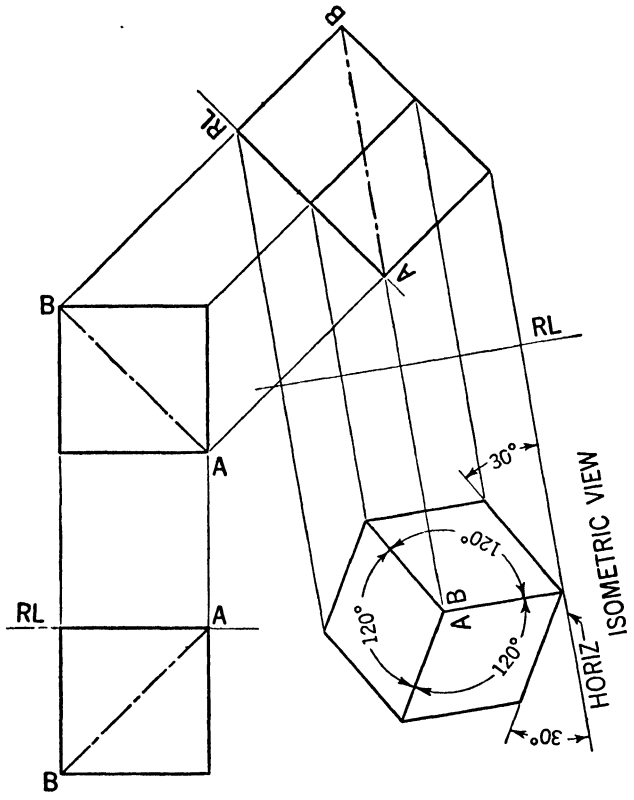


Fig. 237

166. Isometric scale

Since equal distances along the three axes project equally, the scale of all lines parallel to the three axes is a constant in the isometric view. This scale is $\frac{\sqrt{2}}{\sqrt{3}}$, or 81.65 per cent, of true length.

167. Isometric by auxiliary projection

Since an isometric view is an orthographic view on a plane in a specified position, it may always be obtained from the orthographic views by auxiliary-view methods.

The desired direction of sight is, as in Figure 237, that of the diagonal

of a cube whose faces are parallel to the vertical, horizontal, and profile planes. Such a diagonal appears at 45° to the reference line in the front and top views. Thus, to obtain the isometric view of an object, insert an arrow in this direction, and obtain a view in which the arrow appears on end. This will be the isometric view.

168. Isometric by revolved view

An isometric may be obtained by the revolved-view method.

Figure 238 represents the plan and elevation of an object. The object is projected onto the isometric plane represented by its traces HQ and VQ.

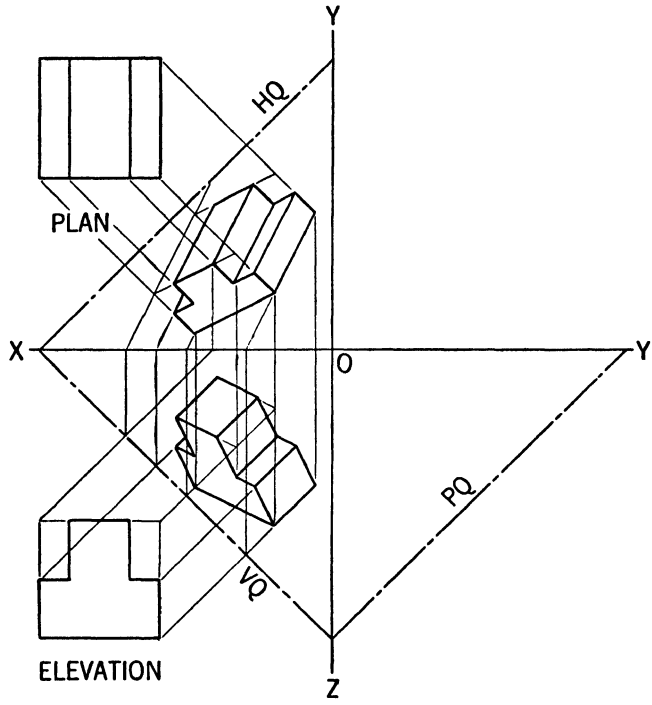


Fig. 238

The three views now may be rotated into a single view and a normal view of this obtained. As in the general case, this can be done best by obtaining a normal view of that portion of the isometric plane bounded by the traces and locating the plan and elevation in the same relation to the traces as in the original views.

Since HQ makes an angle of 60° with VQ and the axis OX makes an angle of 45° with VQ, the latter makes an angle of 15° with HQ or the horizontal.

Figure 239 shows the position of the three views with relation to each other after revolution.

An isometric view may thus be obtained from a front and top view by the following procedure:

Place the front view so that the width makes an angle of 15° with the horizontal, up to the left.

Draw projectors from the points of the front view at an angle of 30° to the horizontal, up to the right.

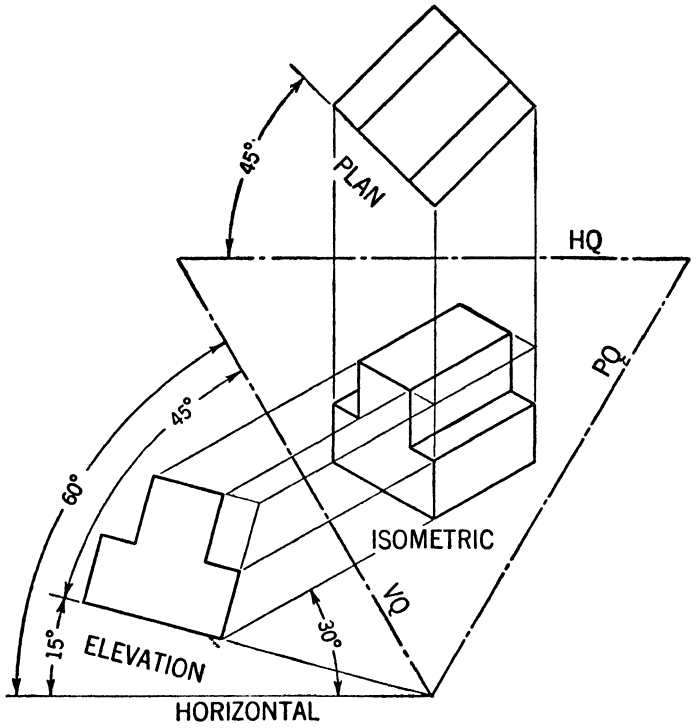


Fig. 239

Place the top view so that the width makes an angle of 45° with the horizontal, up to the left. Draw vertical projectors.

The paired intersections of the projectors determine the isometric view.

The profile or right side view may be used in place of either the front or the top by placing the depth at 15° to the horizontal, up to the right, and projecting at 30° to the horizontal, up to the left.

The above method will be found very useful when two standard orthographic views of the object to be drawn are available. It should be noted that this method yields a view in the isometric scale.

169. Isometric by direct measurement

Since the scales along the three axes are equal, an isometric projection may be constructed by direct measurement from the object. The origin is generally taken at a lower front corner of the object, and measure-

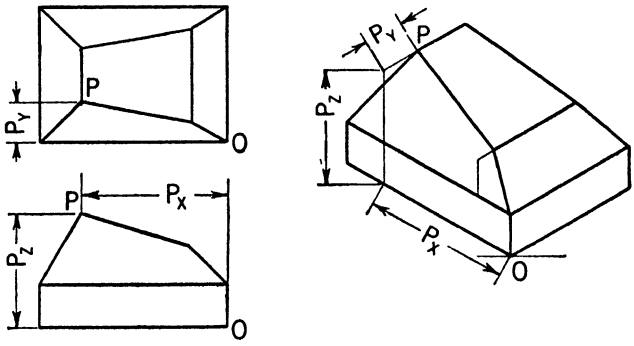


Fig. 240

ments along the axes are made at full scale. The width and depth are scaled at 30° to the horizontal, up to the right and left, and the height is scaled vertically up.

Since only lines parallel to the three axes are at equal scale, any lines not parallel to any axis must be constructed by taking coordinates along the axes to locate points on the line.

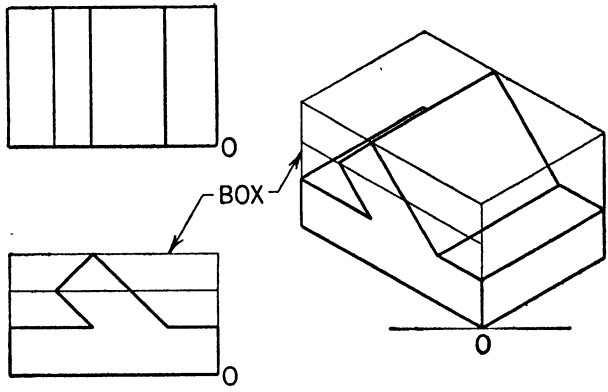


Fig. 241

When the object is irregular, this may be accomplished by offsets from the origin. In Figure 240, the point P is located by its three coordinates from the origin. The isometric is at full scale.

It is frequently convenient with an irregular object to consider it as contained in a rectangular box (Figure 241). The box is drawn

first, and the points where the object touches the box are located on the surfaces of the box. The isometric is at full scale.

170. Inverted isometric

It is occasionally desirable to represent the object in isometric as seen from below rather than from above.

This may be done by the revolved-view method by placing the front view so as to make an angle of 15° with the horizontal, down to the left, and projecting at 30° to the horizontal, down to the right, the top view remaining as before.

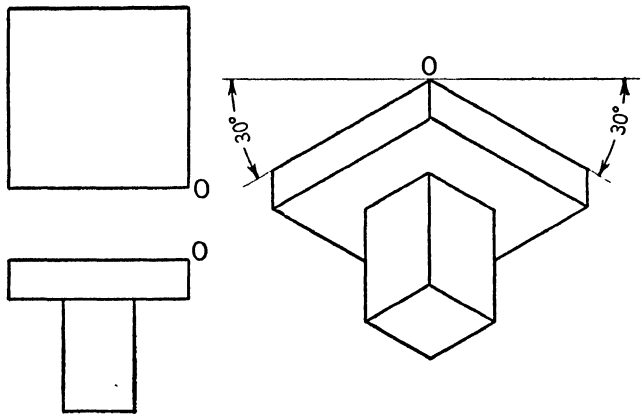


Fig. 242

By the direct-measurement method, the origin is placed at a top front corner and the width and depth axes inclined 30° down from the origin (Figure 242).

171. Circles in isometric

Circles may be constructed in isometric by obtaining a sufficient number of points to construct the resultant ellipse. However, it is customary to use an approximate method when the circle lies parallel to one of the principal planes.

A square whose edges are parallel to the two axes which are parallel to the plane of the circle is circumscribed around the circle. The square is obtained in the isometric view (Figure 243).

Construct perpendiculars from the corners A and B of the short diagonal to the opposite sides. These perpendiculars intersect in two points, E and F. Using A, B, E, and F as centers, draw a circular arc from each. The arcs will be tangent to the edges of the square at the feet of the perpendiculars.

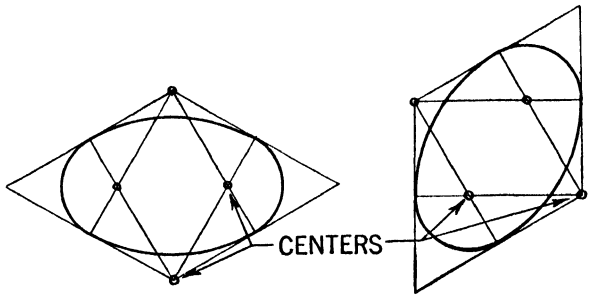


Fig. 243

172. Perspective projection

If a line, or projector, is drawn from a point P to a plane, the intersection of the line with the plane is said to be a projection of the point P on the plane. When a number of points are involved, such as those delineating an object, the projectors from the points may be made to follow any law which will yield a useful result. The law for orthographic projection is that all the projectors shall be perpendicular to the projection plane. The law for perspective projection is that all the projectors shall pass through a single point called the *center of projection* or the *station point.*

The image received by the retina of the eye is substantially a perspective projection, the optical center of the lens being the center of projection. When a perspective projection is viewed by the eye from the center of projection, the bundle of projectors coincides with the bundle that would have existed had the eye viewed the object from the same center. The perspective projection consequently produces substantially the same retinal image as the object. It is, therefore, the most realistic of all forms of pictorial projections.

Figure 244 shows the two standard orthographic views of a perspective system. The object is projected to the oblique plane P using the origin of coordinates O as the center of projection. Lines are drawn from points on the object to O. The intersections of these lines with P are determined and connected properly. This yields the perspective projection on plane P.

173. Vanishing points

Consider any vertical line parallel to the axis OZ such as AB. The intersection of the plane ABO with the projection plane P contains the perspective projection A_PB_P of this line on plane P. Since the plane ABO contains the line AB and the point O, it must contain the line parallel to AB through the point O, that is, the line OZ. Since Z is

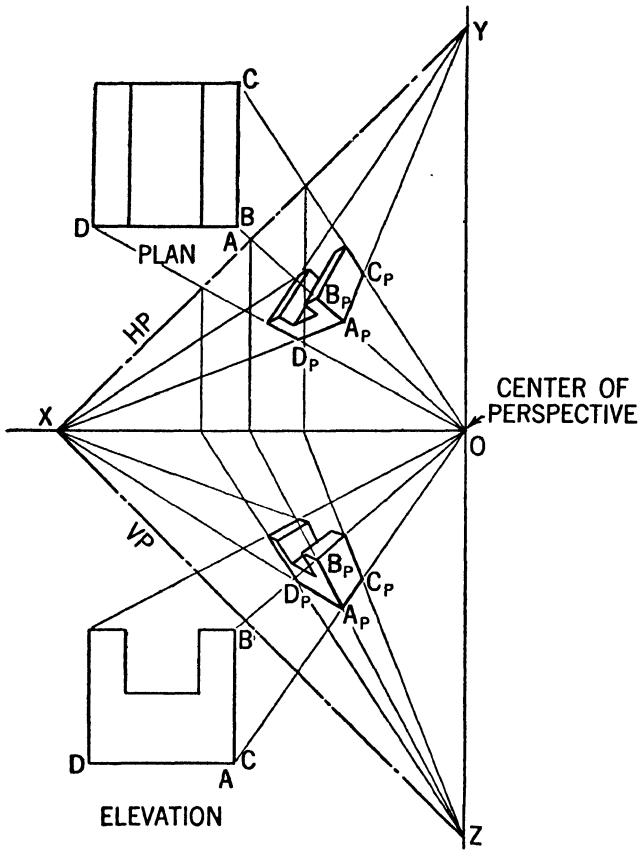


Fig. 244

also in the plane P, it must lie on the intersection of the two planes which contains the perspective projection of AB. Similarly, the perspective projection of any line parallel to OZ must pass through the point Z, that is, the perspective projections of all lines parallel to OZ pass through a common point Z.

Similarly, the projections of all horizontal frontal lines, such as AD, parallel to OX pass through the point X, and the projections of all lines parallel to OY, such as AC, pass through the point Y.

Thus, the three families of lines respectively parallel to each of the three coordinate axes each pass through a common point.

Similarly, the planes established by any random family of parallel lines and the point O must contain the parallel line through O. This line intersects the plane P in one point. The perspective projections on P of all the lines of the family, therefore, must pass through this point. Thus, all parallel lines converge to a point in a perspective projection.

These points are known as *vanishing points.* All rectilinear objects have in general three principal vanishing points, one for each dimension.

Figure 245 shows a normal view of the plane P in Figure 244, showing the three vanishing points X, Y, and Z. The plan and elevation are rotated into the plane P.

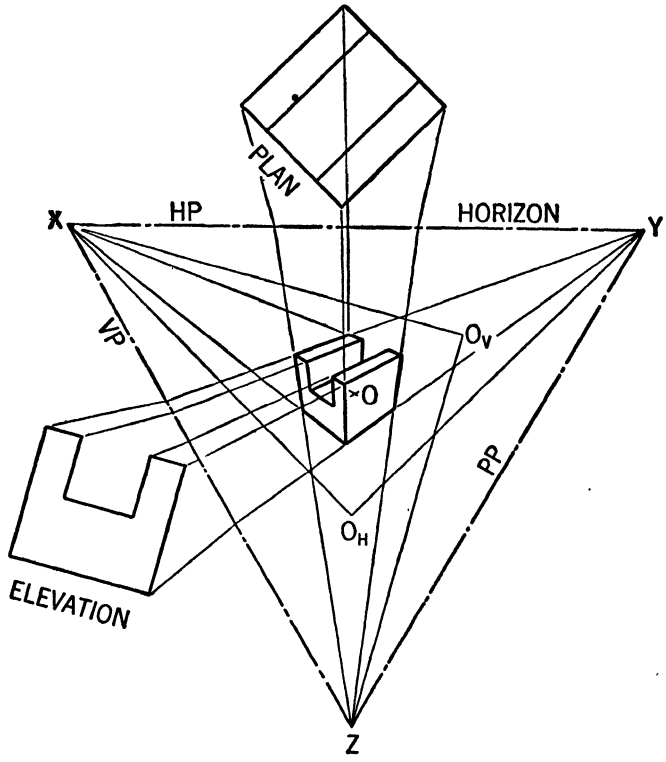


Fig. 245

174. Meaning of a vanishing point

To understand clearly the meaning of a vanishing point, consider a line $ABCD$ (Figure 246) inclined to the projection plane, or picture plane, intersecting this plane at point A and continuing to infinity. Projectors to the center of perspective P from successive points on this line, starting at A and progressing toward infinity, yield the perspective projection of the line $A_PB_PC_P$ . . . . As the point on the line approaches infinity, the projector approaches parallelism with the line. Thus, the projection of the infinite point on the line is the point of intersection VP of the parallel line through P with the projection plane. Furthermore, the projection of the infinite point on any line parallel to ABC will similarly

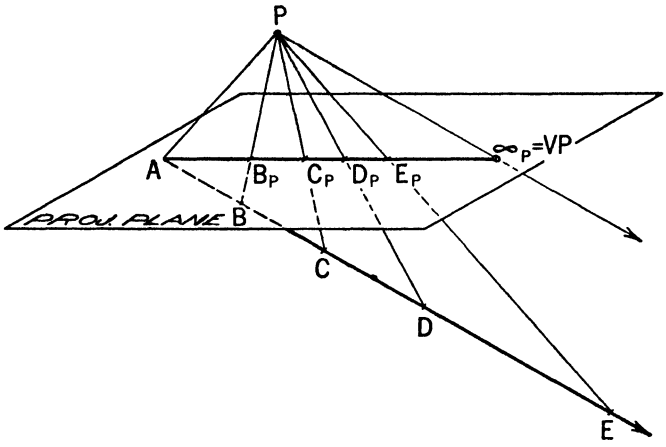


Fig. 246

be the same point *VP*. This point is consequently the vanishing point of the family of lines parallel to *AB*.

175. Three-point perspective

Rectilinear objects having their principal dimensions all inclined to the projection plane have three principal vanishing points. The resultant perspective projection is called a *three-point perspective*.

A perspective projection is meant to be viewed as a picture with the eye at the center of projection. In normal viewing, the eye generally is placed so that its line of vision to the center of the picture is perpendicular to the plane of the picture. Thus, for best results the picture plane should be oriented perpendicular to the projector from the center of the object to the center of perspective. Furthermore, it is desirable in general to look obliquely at the principal surfaces of rectilinear objects so that the maximum total surface will be visible. Such an orientation demands three-point perspective. Thus, this form of perspective usually yields the most natural picture.

176. Two-point perspective

However, it is very convenient to orient the projection plane so that it is parallel to one dimension, usually the vertical. In such a case, the vanishing point of the family of vertical lines is at infinity, and the projections of all vertical lines become vertical. Such a system is called *two-point perspective*. This is probably the most common form of perspective.

In architectural views of houses, for instance, it is common practice to insert the picture plane in a vertical position to obtain a two-point

perspective. When the station point is taken so that the projector from the center of the object is not perpendicular to the picture plane, the eye normally is not placed at the proper viewing point. The distortion introduced by this is, in general, not annoying.

Figure 247 shows a two-point perspective with the picture plane vertical and containing the forward vertical edge of the object. Certain simplifications are possible. The vanishing points of horizontal lines are obtained first. The H-views of the two vanishing points VP_H are obtained by drawing lines through SP_H parallel to the horizontal directions of the edges of the object. These lines meet the picture plane at

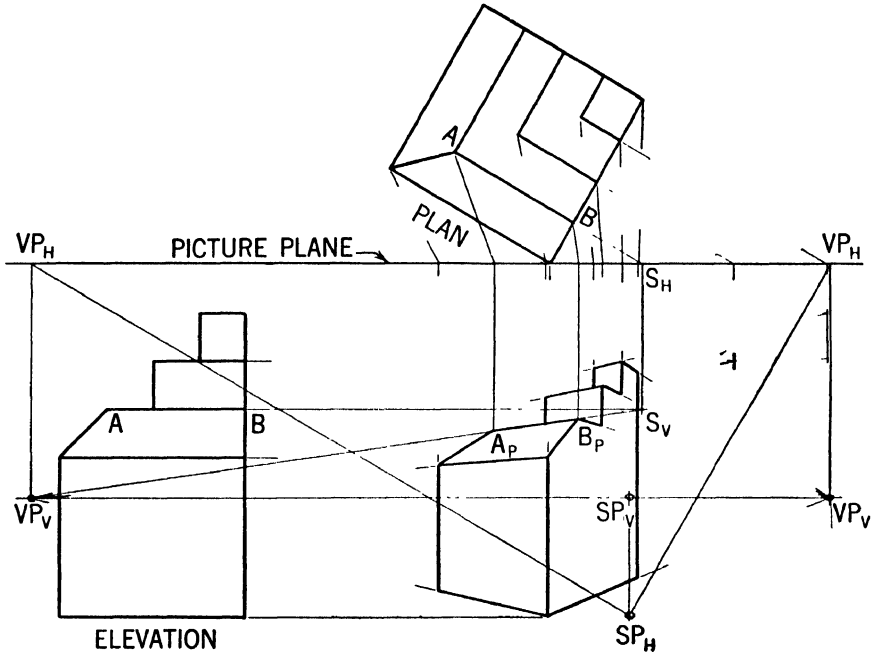


Fig. 247

VP_H. VP_V is then at the elevation of SP_V. Since the picture plane is vertical, any horizontal line (such as AB) of the object, if extended to the picture plane, will pierce this plane in a point S at the same elevation as the line AB on the object.

The elevation of the object is used solely to obtain the heights of such piercing points and thus may be located conveniently anywhere to one side at the proper elevation. Two points are immediately obtainable on the perspective view of every horizontal line—the piercing point S and the vanishing point VP_V. The desired segment of the line then can be obtained by projectors from the plan view. This method of

Vertical lines are projected from the plan view and the desired segment intercepted by horizontal lines through the end points.

Oblique lines may be obtained from their end points or, if a sufficient bundle of parallel lines exists, their vanishing point may be determined.

177. One-point perspective

It is further possible to orient the projection plane parallel to the plane of two principal dimensions, the frontal plane. In such a case, all vertical lines project parallel, and all horizontal frontal lines project

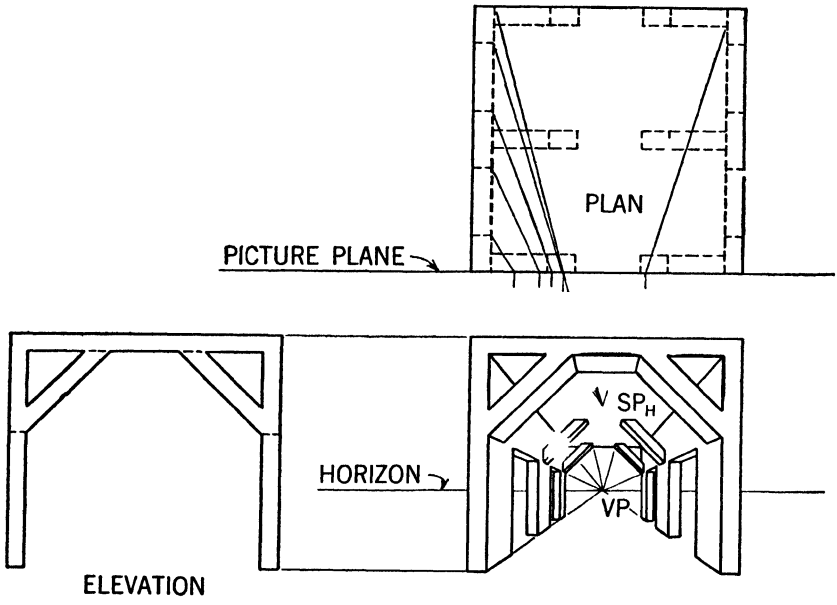


Fig. 248

parallel. Only the profile lines yield a vanishing point. Such a perspective is called *one-point perspective.* Figure 248 shows a one-point perspective.

Perspectives of interiors of rooms, where the eye normally looks perpendicular to the far wall, can be executed most conveniently in one-point perspective.

178. Oblique projections

The law for oblique projection is that all projectors shall be parallel to each other but not perpendicular to the projection plane.

The value of oblique projection lies in the fact that the projection plane may be oriented parallel to one face of the object which will project its true size and shape while the receding dimension becomes visible at some chosen angle with the horizontal and at some chosen ratio

of foreshortening. Objects in which a great majority of the detail and of the irregularities of shape appear in one surface are best suited for oblique projection.

Consider the cube shown in orthographic plan and elevation in Figure 249. Consider a receding plane X through line AB making an angle α with the horizontal. Any projector from B to the vertical plane lying in plane X will intersect the vertical plane along VX. Thus, the

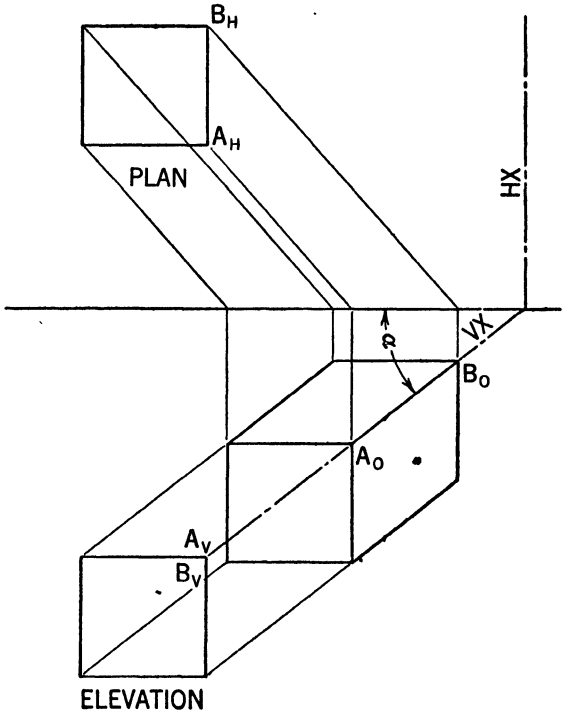


Fig. 249

angle α is established by the choice of the plane X, and the ratio of foreshortening of the line AB is established by the direction of the projector within the plane X.

Consequently, in oblique projection, the principal surface of the object may be drawn in its true size, and the receding dimension may be taken at any angle and scaled at any desired ratio of foreshortening.

179. Oblique projection from plan and elevation

Suppose we are given the plan and elevation of the object in Figure 250 with the reference line through the front surface. We desire to develop the oblique projection so that receding lines will project at an angle of α with the horizontal and at some desired ratio of foreshortening; in this instance, one half their true length. Draw projectors from points on

the elevation making an angle α with the horizontal. On a projector from a point in the front surface, that from A, lay off $A_V B'_V$ equal to $\frac{1}{2}AB$. Find B'_H. Draw $B_H B'_H$. This is the desired direction of projection in the plan view. Draw all projectors parallel to $B_H B'_H$ and $B_V B'_V$ in the plan

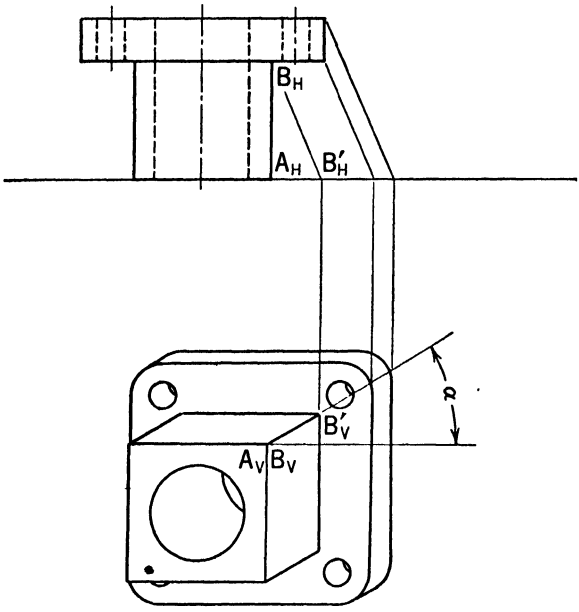


Fig. 250

and elevation respectively to develop the complete oblique projection. Note that the elevation, except for the front surface, actually must be omitted to avoid overlapping the oblique projection.

180. Oblique projection by direct measurement

In practice, it is obvious that the plan and elevation need not appear on the drawing paper as the oblique can be developed from measurements. Draw the front surface only of the object in elevation, draw projectors at the angle α and lay off the lengths of receding lines at the required scale along these projectors. The visible portions of the second frontal surface are drawn as a normal view around any point which may be obtained in this surface.

181. Forms of oblique projection

The angle α with the horizontal is taken usually at 30° or 45°. When the lines in the receding direction project full scale, that is, when the projectors make an angle of 45° with the projection plane, the drawing is called a *cavalier projection.*

When lines in the receding direction appear at one-half scale, the drawing is called a *cabinet drawing.* Cabinet drawings reduce the distortion inherent in oblique projection.

PROBLEMS ON CHAPTER X

GROUP A. ISOMETRIC AND OBLIQUE PROJECTION

10-A1. Make an isometric drawing of the object described in problem 1-B1.

10-A2. Make an isometric drawing of the object described in problem 1-B3.

10-A3. Make an isometric drawing of the object described in problem 1-B4.

10-A4. Make an isometric drawing of the object described in problem 1-B5.

10-A5. Make an oblique drawing of the object described in problem 1-B2.

10-A6. Make an oblique drawing of the object described in problem 1-B2. Remove an upper quarter of the object in order to show the recess in the back.

10-A7. Draw the given elevations of the Bridge Pier (Figure L), and construct its isometric projection using the method of Article 168.

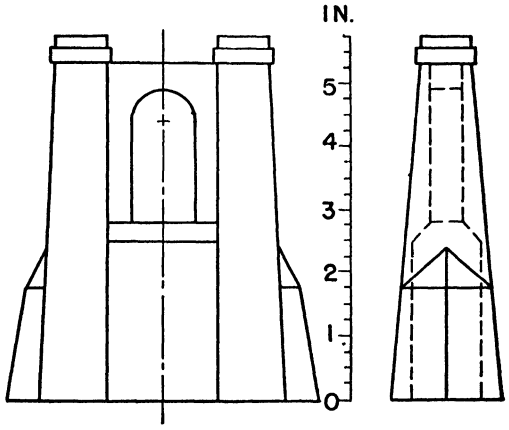


Fig. L

GROUP B. PERSPECTIVE DRAWING

10-B1. Line A:169 B:469 is the upper front edge of a prism 4 inches high and 1 inch deep. Remove the right and left thirds of the lower three-fourths of the prism to form a T-block. With point S:634 as the center of projection, construct the perspective view of the block on a frontal picture plane taken through line AB.

10-B2. Pentagons A:126 B:136 C:1¾,4,6 D:2½,3,6 E:2½,2,6 and F:5½,2,6 G:5½,3,6 H:6¼,4,6 J:736 K:726 represent the front bases of two right prisms, each 3 inches long. With point S:4¼,2½,3 as the center of projection, draw the perspective view of each prism on a picture plane taken through the front faces.

10-B3. The front end of a right cylinder is a frontal circle 2 inches in diameter centered at point C:337. The cylinder is 3 inches long. Line A:2¼,1,7½ B:3¾,1,7½ is the lower front edge of a right prism ¼ inch deep. The upper side of the prism is cut to fit against the cylinder forming a support. A similar support is placed ½ inch from the back end of the cylinder. Using point S:6,1½,3 as the center of projection, draw a perspective view on a frontal plane taken through point C.

10-B4. Point A:2½,1,5 is the lower front corner of a building having a rectangular floor plan 16 feet × 20 feet. The 20-foot side of the building makes an angle of 30° with the vertical plane through point A. The height of the side walls is 10 feet. The gable roof, which has no overhang, has a rise of 8 feet and a span of 16 feet. The point of sight O is 38 feet in front of and 6 feet above point A. Draw the top and side views of the building, omitting the front view. Construct the perspective view by the visual-ray method. Draw the horizon. Scale: ⅛ inch = 1 foot.

10-B5. (*a*) Using point S:4,3½,4 as the center of projection, draw the perspective view of line A:227 B:519 on a frontal plane taken 1 inch in front of point A. (*b*) Consider that line AB is produced indefinitely, and find the perspective of the point which lies at infinity. This is the vanishing point of AB. (*c*) Extend line AB forward to pierce the picture plane in point O. Note that the perspective of O coincides with its front view.

10-B6. Using point S:3,4,½ as the center of projection, draw the perspective view of line A:236 B:539. The picture plane is 1 inch in front of point A. Find the trace and the vanishing point of line AB. Note that, since line AB is horizontal, the vanishing point is on the horizon.

10-B7. Line A:0,¾,5½ B:1¼,¾,5½ is the lower front edge of a right prism 1½ inches high and 1¼ inches deep. Using point O:3½,4,4 as the center of projection, construct a three-point perspective of the prism. Let the H-, V-, and P-projection planes pass through point O, and draw the H-, V-, and P-traces of the picture plane, making the H-trace about 5 inches long. The true-shape view of the picture plane is to be drawn at the top of the problem sheet. Find the distance between the center of projection and the picture plane. This is the viewing distance.

10-B8. Make a three-point perspective drawing of the Bridge Pier (Figure L). Select a suitable center of projection.

10-B9. Make a two-point perspective drawing of Figure M. Sheet size: 11 × 17 inches.

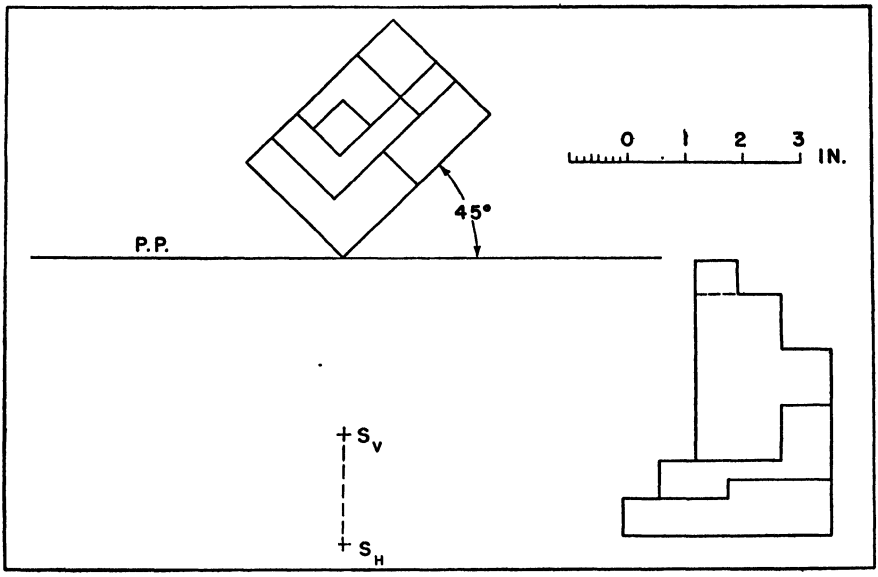


Fig. M

10-B10. Make a two-point perspective drawing of Figure N. Sheet size: 11 × 17 inches.

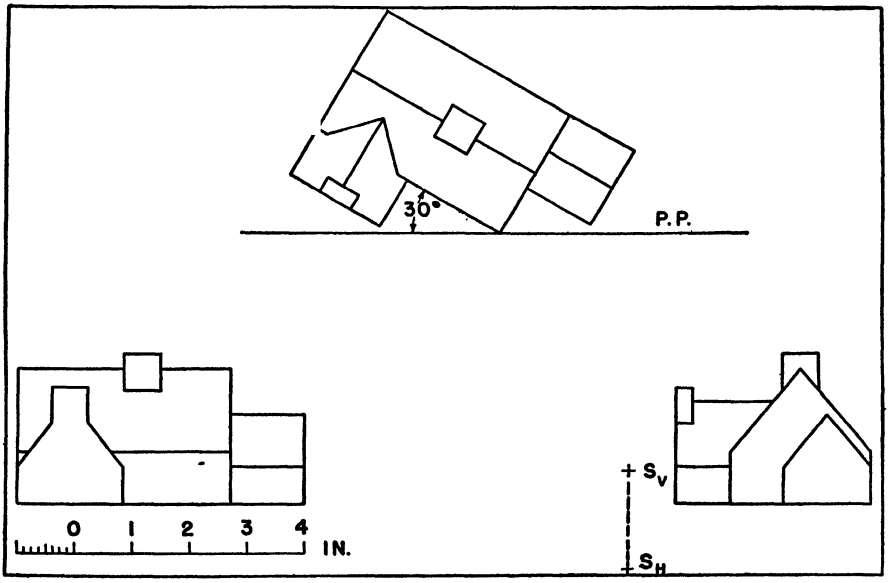


Fig. N

CHAPTER XI

Simple Shadows

182. Shadows

Shadows are added occasionally to a drawing when a more realistic representation of an object is desired. They are used most frequently on certain classes of architectural drawings and may be applied to any orthographic or pictorial view.

The object is assumed to be illuminated by light falling in parallel rays. Although any convenient direction of light may be chosen, it is common to assume light rays which slope downward, backward, and to

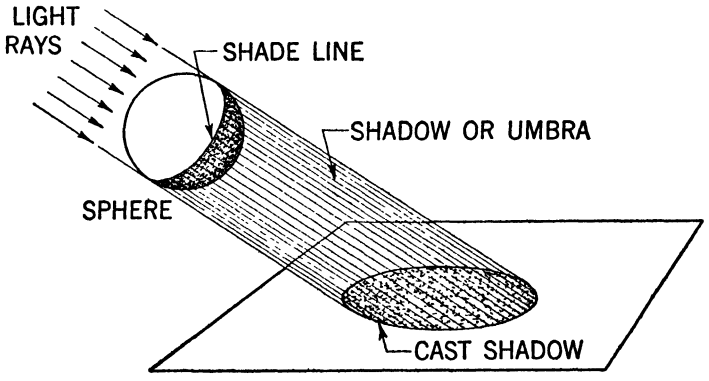


Fig. 251

the right, their projections making an angle of 45° with the reference line in both the top and front views. In some instances, a point source of light might be used, in which case the rays will be divergent.

If an opaque object is placed in the field of a light source, one side of the object is lighted while the other is in shade (Figure 251). The space from which light is cut off is the *shadow* of the object. The shadow is a volume rather than an area and is sometimes called the *umbra.* The line which separates the lighted from the darkened surface of an object is called the *shade line.* It is evident that the *shadow surface* is a ruled surface which has the shade line as its directrix. The intersection of the shadow surface with any other surface is called the *cast shadow*, or, often, simply the *shadow.*

The cast shadow of an object can be considered as its oblique projection on a given plane, the projectors being the light rays. In the case of a point of light, the shadow is a central or perspective projection of the object. The boundary of the shadow is the projection of the shade line.

183. Shadow of a point

The surface on which a shadow is to be cast generally is taken below or in back of the object. Therefore, in the study of shadow projection, it is common to place the object in the first quadrant of the projection

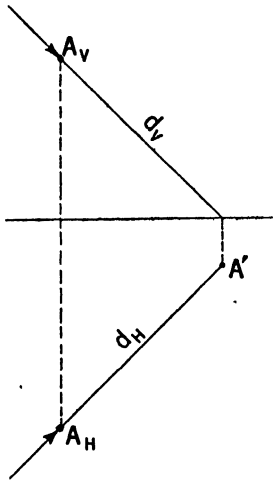


Fig. 252

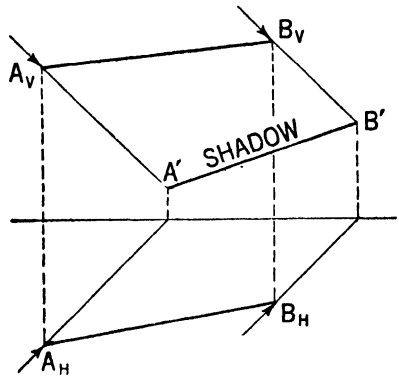


Fig. 253

planes (see Article 12). If a light ray has the direction d (Figure 252), the shadow of point A on the horizontal plane falls at A'. The shadow is the H-trace of the ray.

Taking the conventional direction for light rays, the shadow of line AB (Figure 253) on the V-plane is the line $A'B'$.

184. Shadow of a plane figure

The triangle ABC (Figure 254) intercepts light from the point source P casting the shadow $A'B'C'$ on the horizontal plane. The shadow is determined by the points in which rays through vertices A, B, and C meet the horizontal plane.

When the light source is distant, as in the case of the sun, the rays become parallel. Assuming the conventional direction of light, the shadow of triangle DEF (Figure 255) on the horizontal plane is $D'E'F'$.

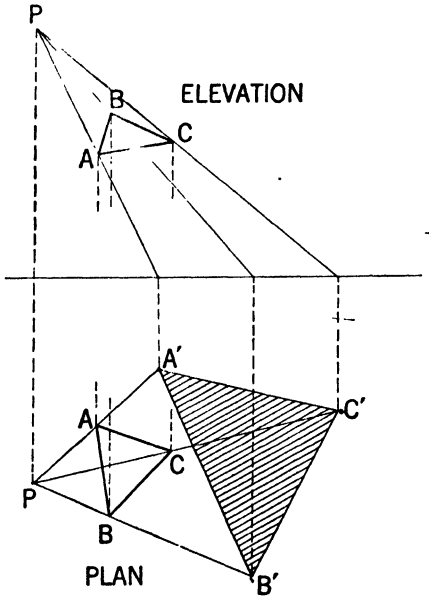


Fig. 254

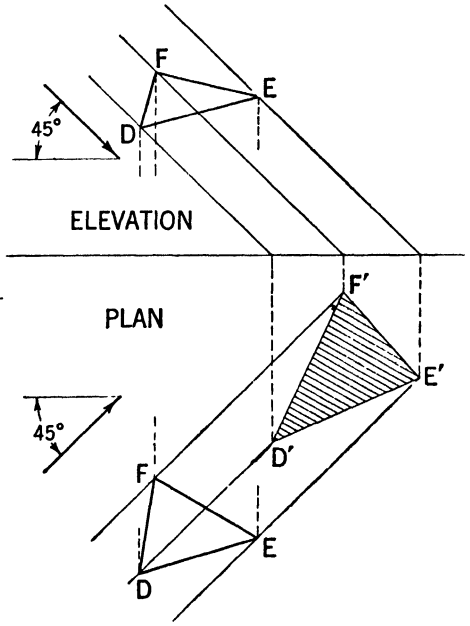


Fig. 255

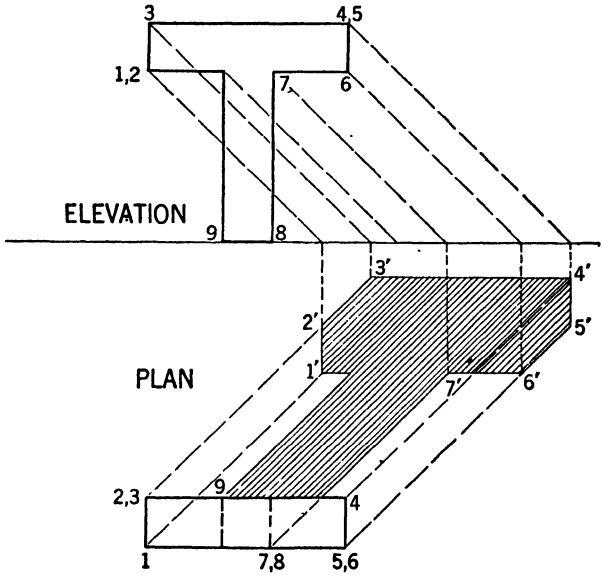


Fig. 256

185. Shadow of a prism

Let it be required to find the shadow of a prism (Figure 256) on the horizontal plane. The conventional direction of light is assumed.

The shade lines of a prism lie along its edges. A point is on a shade line if the light ray passing through the point does not intersect the prism in any other point. Points 1-2-3-4-5-6-7-8-9 determine the shade line of the given prism. The oblique projection of these points on the horizontal plane determines the boundary line of the shadow.

186. Shadow of a cylinder

The shadow of the cylindrical column and cap (Figure 257) is to be cast on a frontal plane through the axis of the cylinder.

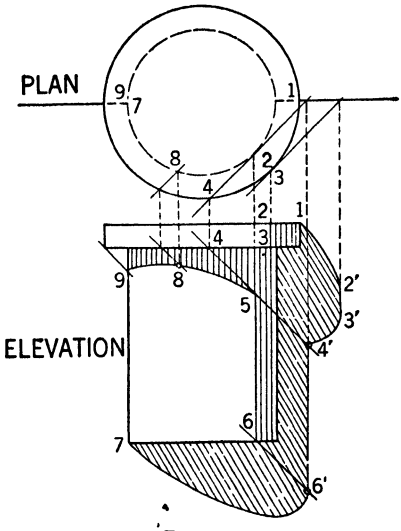


Fig. 257

The shade line of the cap is determined by points 1-2-3-4 which, projected on the frontal plane, give the shadow bounded by line 1-2′-3′-4′. The lower edge of the cap also casts a shadow on the column. The lower boundary of this shadow is a curved line passing through points 5-8-9. The portion of the cap to the right of line 2-3 is in shade.

Points 5, 6, and 7 lie on the shade line of the column. This shade line is projected on the frontal plane completing the shadow. The surface of the cylinder to the right of line 5-6 is in shade.

187. Perspective shadows

Let the perspective view of the vertical line AB be given (Figure 258). Point B lies on the ground plane passing through the horizon. A suitable

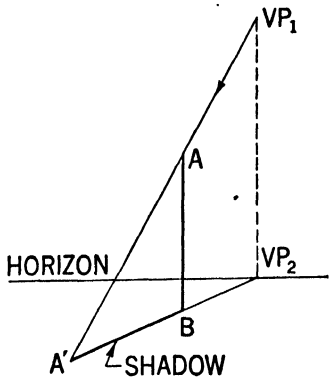


Fig. 258

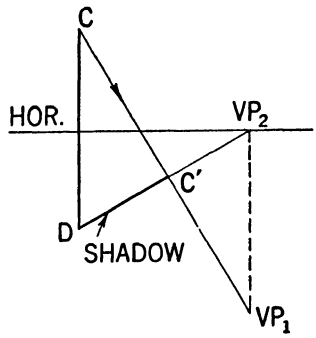


Fig. 259

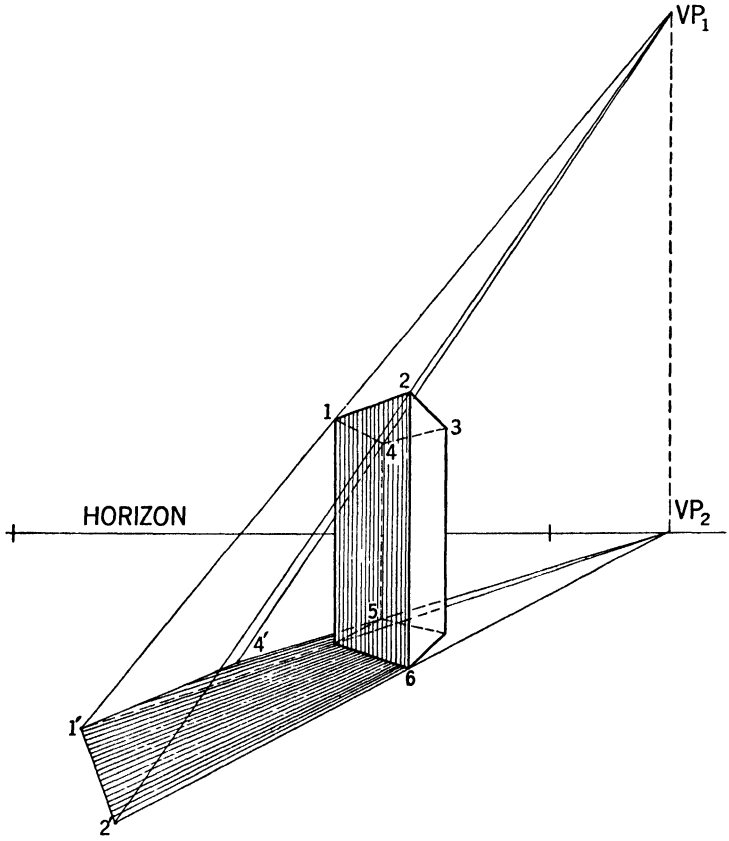


Fig. 260

direction of light is chosen by assuming a vanishing point for the parallel rays. The shadow of point A on the ground plane is the point A' in which the ray through point A meets this plane. Point A' is determined by intersection of the ray VP_1-A and the trace VP_2-B of the vertical plane passing through this ray. $A'B$ represents the shadow of the vertical line AB. Since the light rays are parallel, the traces on the ground plane of vertical planes containing the rays are parallel and vanish in point VP_2. Points VP_1 and VP_2 lie on the same vertical line.

When the light source is in back of the object, VP_1 is above the horizon; when in front of the object, VP_1 is below the horizon (Figure 259). The shadow of the vertical line CD is $C'D$. Point D lies in the horizontal plane.

The prism of Figure 260 rests on the ground plane. The light source is in back of the prism, and the parallel rays vanish in point VP_1. The edges 6-2-1-4-5 represent the shade line. The shadow 6-2′-1′-4′-5 is determined by the method just described. The vertical face 1-2-6 is in shade.

CHAPTER XII

Practical Applications

188. Examples

The following examples will serve to illustrate the application of the principles developed in the preceding chapters to particular practical problems. The illustrations are selected to exemplify problems in different classes of engineering and technical work.

1. Wood Structures

189. Hip roof

The hip roof (Figure 261) is a common structure, and is used here to illustrate the general method for obtaining the true shapes of the various members of a structure framed with timbers.

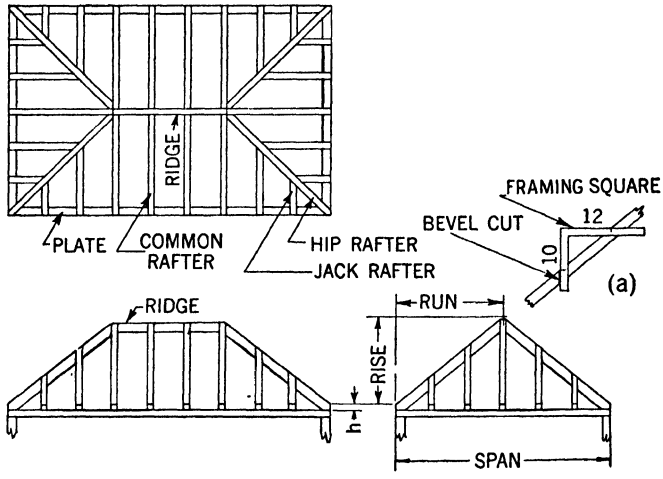


Fig. 261

The roof planes intersect at the ridge and the hip rafters. The inclination of a roof plane is stated generally in terms of its vertical rise in a given horizontal distance called its *run*. The *pitch* of a roof is the rise at its center divided by the span of the roof. To allow space for fastening the rafters to the plate, the lower edge of each roof plane is raised a certain distance (h) above the top of the plate.

Angles are laid off on a timber by means of a framing square having two arms which are divided into inches and fractions. An angle is usually given by its tangent, using a base of 12. Thus, the bevel cut for a rafter having a rise of 10 and a run of 12 is marked by placing the square on the timber as in Figure 261.

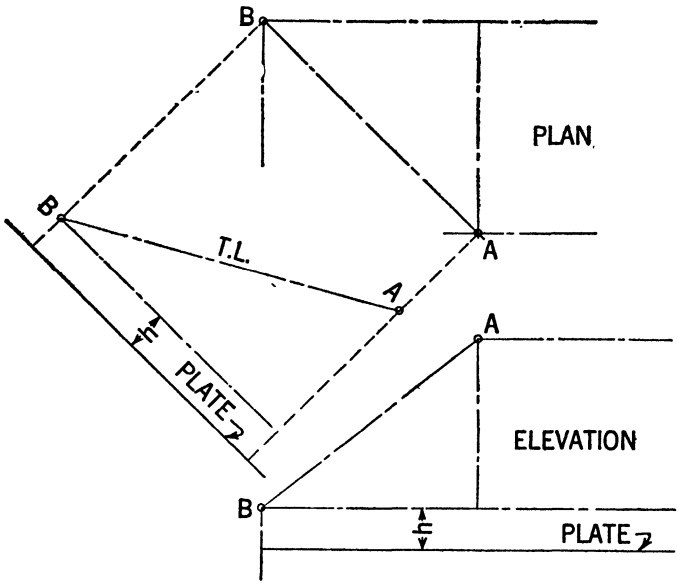


Fig. 262

(a) **True size of hip rafter.** Having been given the rise and run of two adjacent roof planes, a drawing is made showing the plan, elevation, and true-length view of their line of intersection AB (Figure 262). When the roof slopes are equal, line AB makes an angle of 45° with the edge of the plate in the plan view. When the slopes are unequal, the line of intersection is found as in Figure 263. Roof plane R_1 has an angle of slope S_1. Roof plane R_2 has a slope angle of S_2. A horizontal plane Q cuts the roof planes in two lines which meet in the plan view in point X_H. Line A_HX_H is the line of intersection of R_1 and R_2. The top of the plate (Figure 262) is drawn at a given distance h below the edge of the roof.

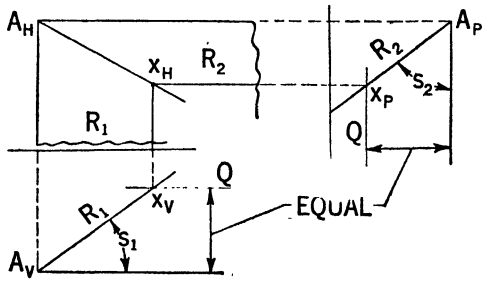


Fig. 263

The top and front views of the ridge, common and hip rafters, are now

drawn to scale (Figure 264). An auxiliary view drawn with the direction of sight perpendicular to the wide face of the hip rafter shows its true length and the angle for the bevel cuts on the wide face.

A second auxiliary view drawn with the direction of sight perpen-

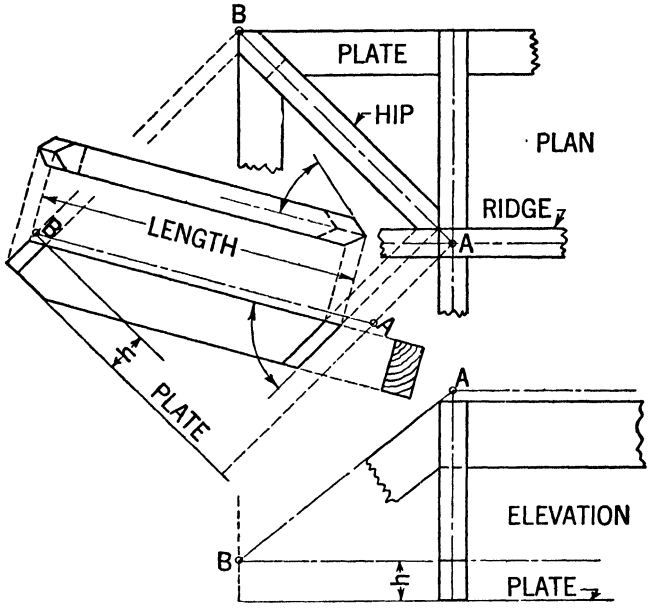


Fig. 264

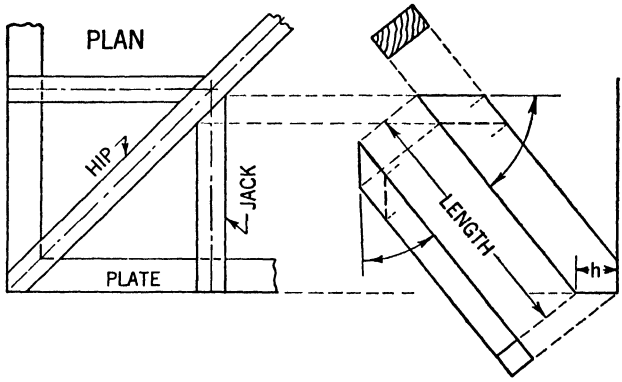


Fig. 265

dicular to the narrow face of the hip rafter shows the angle for the bevel cuts on the narrow face.

It will be noticed that the top of the hip rafter lies below the line AB which represents the intersection of the roof planes. Sheathing laid across the rafters will meet on this line, however.

(b) **True size of jack rafter.** A view taken in a direction perpendicular to the wide face of the jack rafter (Figure 265) shows its true length and the angle for the bevel cuts on the wide face. An auxiliary view taken in a direction perpendicular to the narrow face of the rafter shows the angle for the bevel cut on that face.

2. Steel Frameworks

190. Skew bridge

Structures framed with rolled sections require bent steel plates for connections at the joints. The angles to which these plates are bent are determined from the center line drawings of the framework.

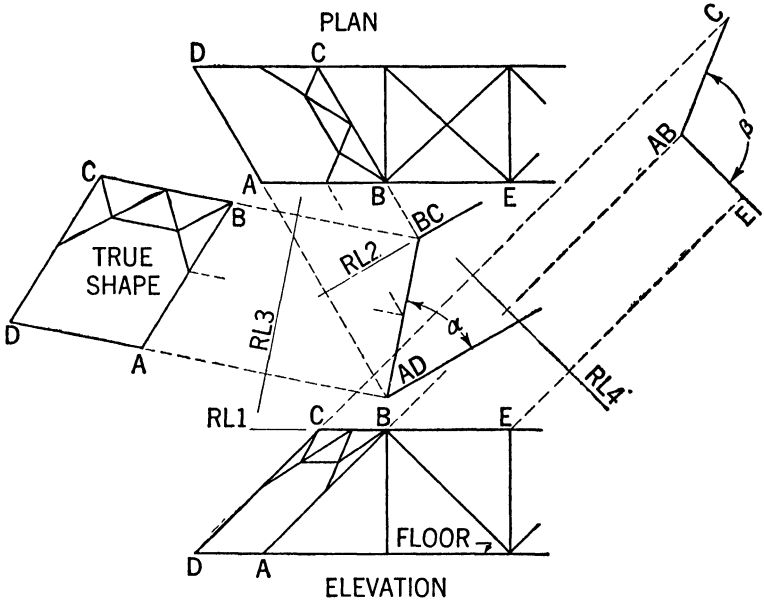


Fig. 266

Let the plan and elevation of one end of a skew truss bridge be given (Figure 266). The following information is required: (*a*) The true shape of the portal framework *ABCD*; (*b*) the dihedral angle between the plane of the portal *ABCD* and the plane of the floor of the bridge; (*c*) the dihedral angle between the plane of the portal *ABCD* and the plane of the truss *ABE*.

(*a*) The true shape of the portal is found by drawing a normal view of *ABCD*. Refer to Article 33.

(*b*) The angle α between *ABCD* and the floor of the bridge appears in true size in the view in which the common line *AD* projects as a point. Refer to Article 64.

(*c*) The angle β between *ABCD* and plane *ABE* appears in true size in the view in which the common line *AB* projects as a point.

It is often desirable to make solutions for angles on a separate drawing. This allows the use of a larger scale and greater precision. The problem stated above can be solved for angles α and β by drawing the traces of the plane of the portal and finding the angle between the portal plane and the *H*- and *V*-reference planes by the method of Article 67.

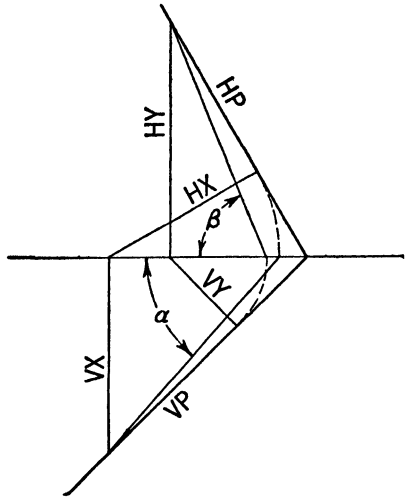


Fig. 267

Since lines *AB* and *BC* of the portal plane are frontal and horizontal respectively, the trace *VP* is parallel to *AB* and trace *HP* is parallel to *BC* (Figure 267).

191. Engine mount

The mounting for a radial airplane engine is a typical tubular structure. The methods used for determining the lengths of various members and the angles at points of connection may be applied to many similar airplane structures.

The engine ring *EFGH* (Figure 268) is connected to the bulkhead *ABCD* by eight tubes represented by their center lines.

(*a*) *To find the true length of tubes BE and BF and their included angle.* Construct the normal view of plane *BEF*. The true lengths are B_4E_4 and B_4F_4; the included angle is θ. Refer to Article 33.

(*b*) *To find the angle between BE and the plane of the engine ring.* The plane of the engine ring is frontal. Rotate the line *BE* into parallelism with the profile plane. The angle α between the rotated line BE_R and the edge view of the engine ring is the required angle.

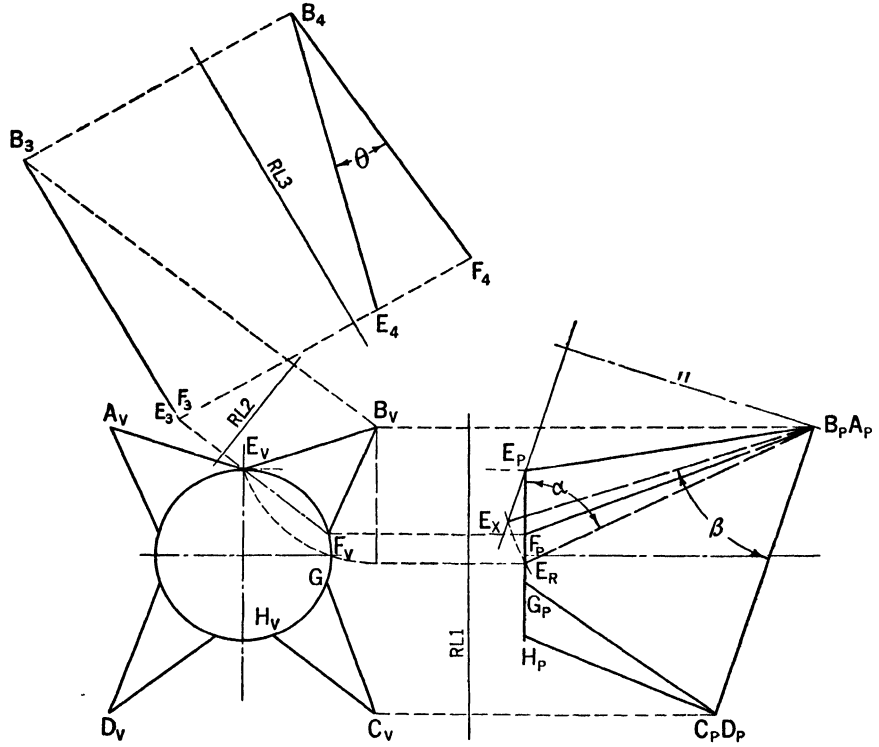


Fig. 268

(*c*) *To find the angle between tube BE and the bulkhead ABCD.* Draw line n perpendicular to plane $ABCD$. Rotate BE about n as an axis, bringing it into the true-length position B_PE_X. The angle between B_PE_X and the edge view $A_PB_PC_PD_P$ is the required angle. Refer to Article 66.

3. Vectors

192. Statics

In space, a force can be resolved in three directions. The magnitudes of the components can be determined by the methods of descriptive geometry.

In mechanics, it can be shown that three components of a given force lie along the edges of a parallelepiped of which the given force, as a vector, is the body diagonal.

Thus, knowing the length of the body diagonal and the directions of three concurrent edges, in order to resolve a given force in three given directions, we have the problem of constructing a parallelepiped. The analysis for this solution is given in Article 52, Example 6.

Let it be required to find the forces acting on the members *a*, *b*, and *c* of the frame which supports the weight *W* (Figure 269).

Draw the front and side views of vector *OR* parallel to *W* and equal in length to 3000 pounds on the force scale. Through point *O* draw lines *a*, *b*, and *c* parallel to frame members *a*, *b*, and *c* respectively. Construct the views of the parallelepiped as outlined in Article 52. The magnitudes of the forces in members *a*, *b*, and *c* are represented by *OQ*, *QS*, and *RS* respectively. The true lengths of these vectors give the magnitudes of the components.

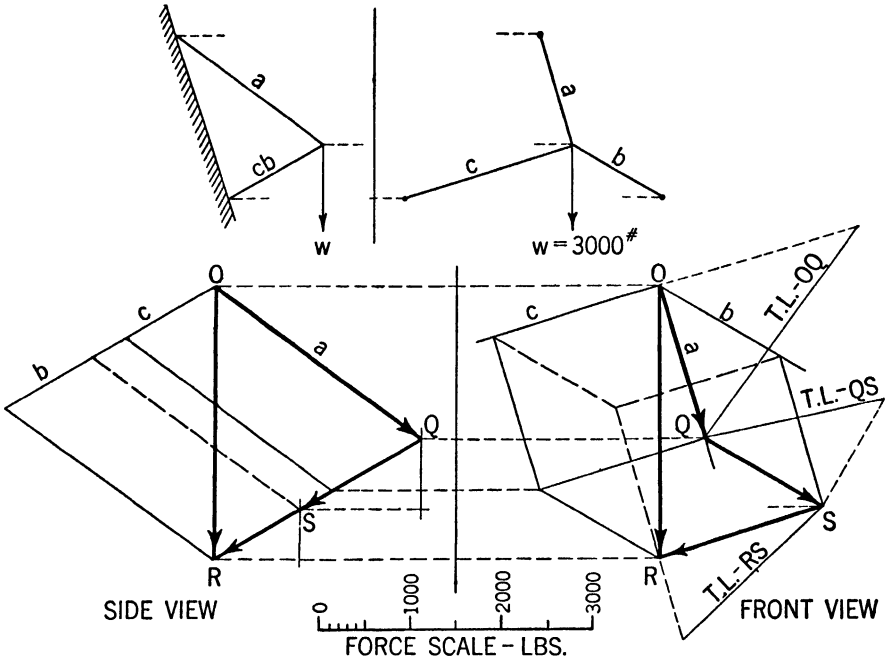


Fig. 269

On examining the completed drawing, it will be seen that the force problem can be solved by drawing a skew quadrilateral having three sides parallel to, and in the same direction as, the three given vectors. The fourth and closing side of the quadrilateral is the balancing force. Such a figure is called a *force polygon*. When the vectors forming a force polygon point in the same direction around the polygon, the forces are in equilibrium. If one vector is drawn in a contrary direction, it represents the resultant of the remaining forces. The arrows shown in Figure 269 represent the components of *OR*.

A second method for the resolution of a force in three directions breaks the resulting force polygon into two force triangles and thus allows its construction without reference to a parallelepiped.

To find the forces in members a, b, and c.

Given: Tripod frame *abc* acted upon by 1000 pounds force with direction w (Figure 270).

The resultant of forces having directions w and a is equal and opposite to the resultant of the forces in the directions b and c. Since this resultant must lie in the plane of w and a, and also the plane of b and c, it lies along the line k which is the intersection of these planes.

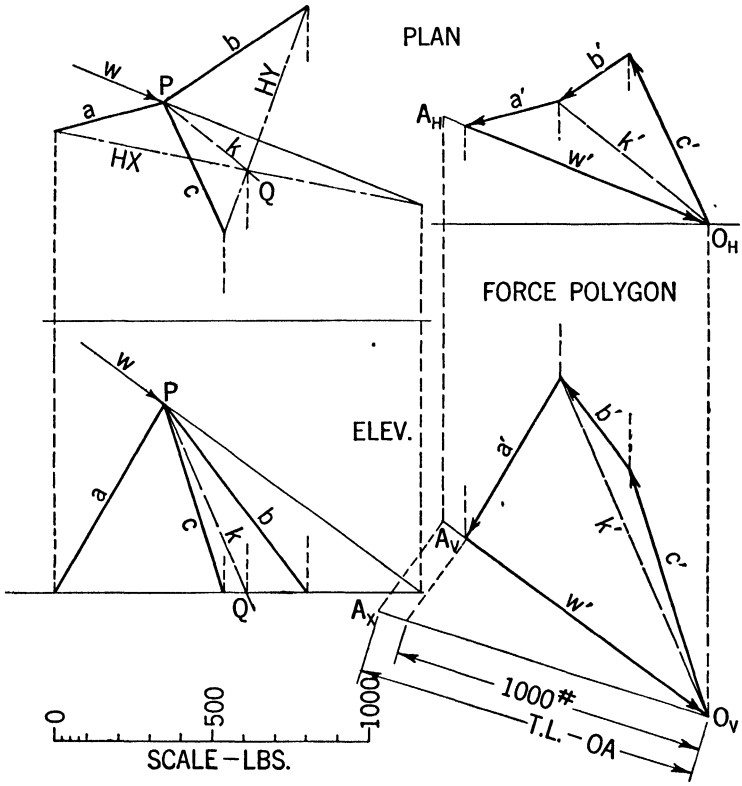


Fig. 270

One point on k is given by the intersection P of a, b, and c. A second point Q is found by passing a horizontal plane to cut the given members. The plane aw cuts this H-plane in HX; the plane bc cuts it in HY. Point Q is determined by the intersection of HX and HY.

Construction of force polygon. Assume the top and front views of a point of space O. Through O draw line OA of any convenient length and parallel to w, making O_VA_V parallel to w_V and O_HA_H parallel to w_H. Lay off on OA a distance representing 1000 pounds, using a suitable force scale. This distance is laid off on the normal view of OA and then

projected to $O_V A_V$ and $O_H A_H$, giving the vector w'. The arrowhead is at O.

Complete the force triangle $w'k'a'$ by drawing vectors k' and a' parallel to frame members a and k respectively.

Complete the force triangle $k'b'c'$ by drawing vectors b' and c' parallel to frame members b and c respectively.

The true lengths of vectors a', b', and c' (not shown here) represent the forces in frame members a, b, and c. Since the system is assumed to be in equilibrium, the arrows have the same sense in each view of the force polygon.

4. Sheet-Metal Structures

193. Steel chute

A chute (Figure 271) enters a wall at point A and passes through the floor at point B. The cross section of the chute is a square of given

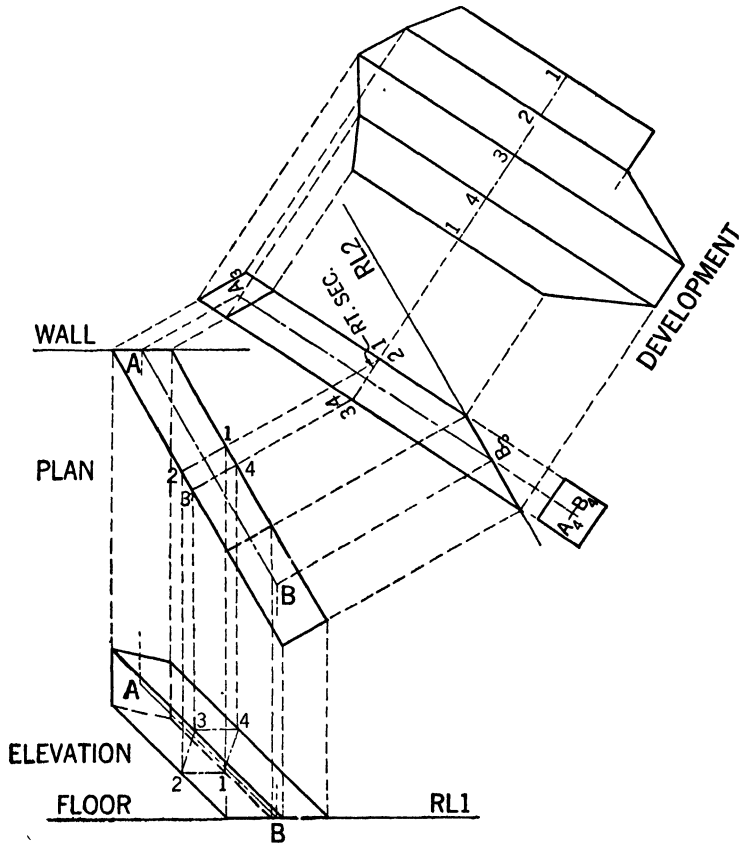


Fig. 271

size having two edges parallel to the floor. The views showing the true shapes of the wall and floor openings and the development of the surface are required.

Draw the true-length view of the center line AB. The floor is represented by $RL2$. A square right section will appear at right angles to the true-length view of AB. At any convenient point on A_3B_3, construct the square of given size with edges 1-2 and 3-4 appearing as points and thus parallel to the floor. The square projects in true size in the view A_4B_4, in which AB appears as a point.

The points of the right section are projected back to the plan and elevation, and these views are completed by drawing the edges of the chute parallel to the center line AB.

The edges of the chute are now projected to the first auxiliary view, where they appear in true length. The perimeter of the right section appears in the second auxiliary view. The development of the surface is constructed using the information contained in the two auxiliary views (see Article 85).

194. Breeching

The circular openings at A, D, and F (Figure 272) are connected by a symmetrical Y of uniform cross section. Each section is cylindrical and equal in diameter. The center lines BC and BE are at right angles. AB, CD, and EF are made equal in length. On points B, C, and E as centers, draw spheres having diameters equal to the diameter of circle A. Construct each cylindrical section of the Y tangent to the spheres drawn at the intersection of the axes. The lines of intersection for adjoining pieces are found as described in Article 101, Example 6.

(a) End section. On point F as a center, draw a semicircle having its diameter equal to the circle F. This semicircle, which represents the front half of circle F rotated through a 90° angle, is divided into six equal parts. Lines drawn through points 1-7 of the semicircle and parallel to center line EF represent equally spaced elements on the surface of the cylinder. Cylinder EF is developed by using circle F as the right section. The length of the girth line for one half of the developed surface is equal to $\frac{\pi D}{2}$. The girth line is divided in six equal parts. The developed elements of the cylinder are drawn through these points.

(b) Intermediate section. Since the joint line between cylinders BE and EF bisects the angle BEF, equally spaced elements of the cylinder BE will meet the elements of the cylinder EF on this line. A right section is cut at a convenient point on BE. For convenience, the girth line 1-7 for the half development is aligned with the right section. This line,

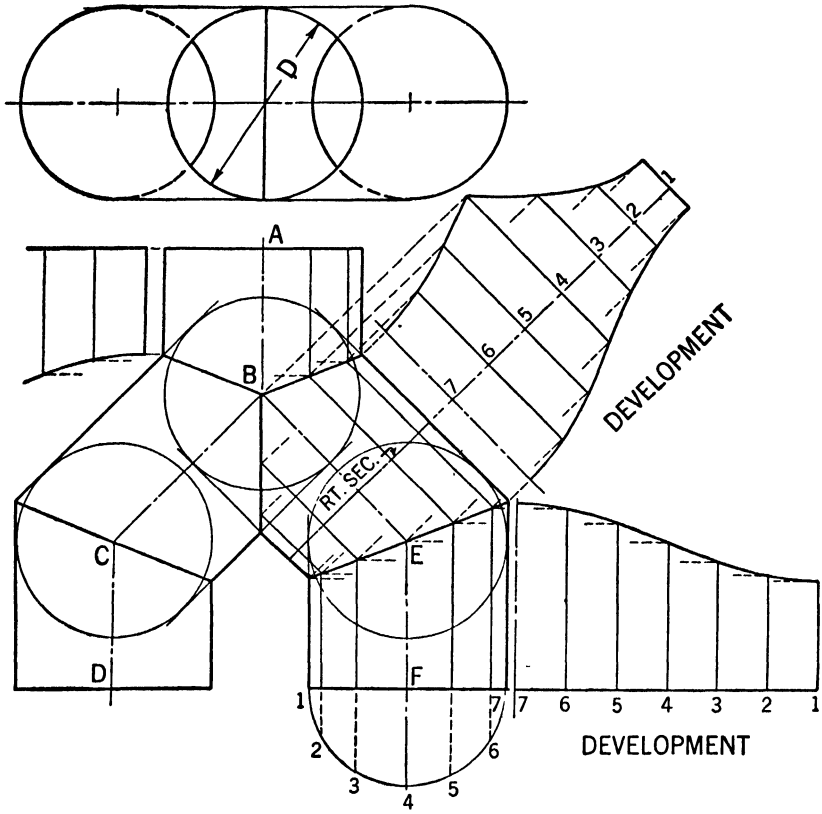


Fig. 272

equal in length to $\frac{\pi D}{2}$, is divided into six equal parts and the developed element of the surface drawn through the points.

The cylinder AB is developed in a similar manner, using circle A as its right section.

195. Uptake and transition piece

Four boilers of a ship are connected to a smokestack by means of ducts called uptakes (Figure 273). The stack is elliptical at the base. The uptakes terminate in a single rectangular opening. A structure called a *transition section* joins this rectangular opening to the elliptical end of the stack. Each piece of this structure is developed from full-size drawings made on a mold loft floor.

(a) Development of transition section. The surface of the transition section is composed of four triangles and four portions of oblique cones. The conical surfaces are divided into small areas and developed by

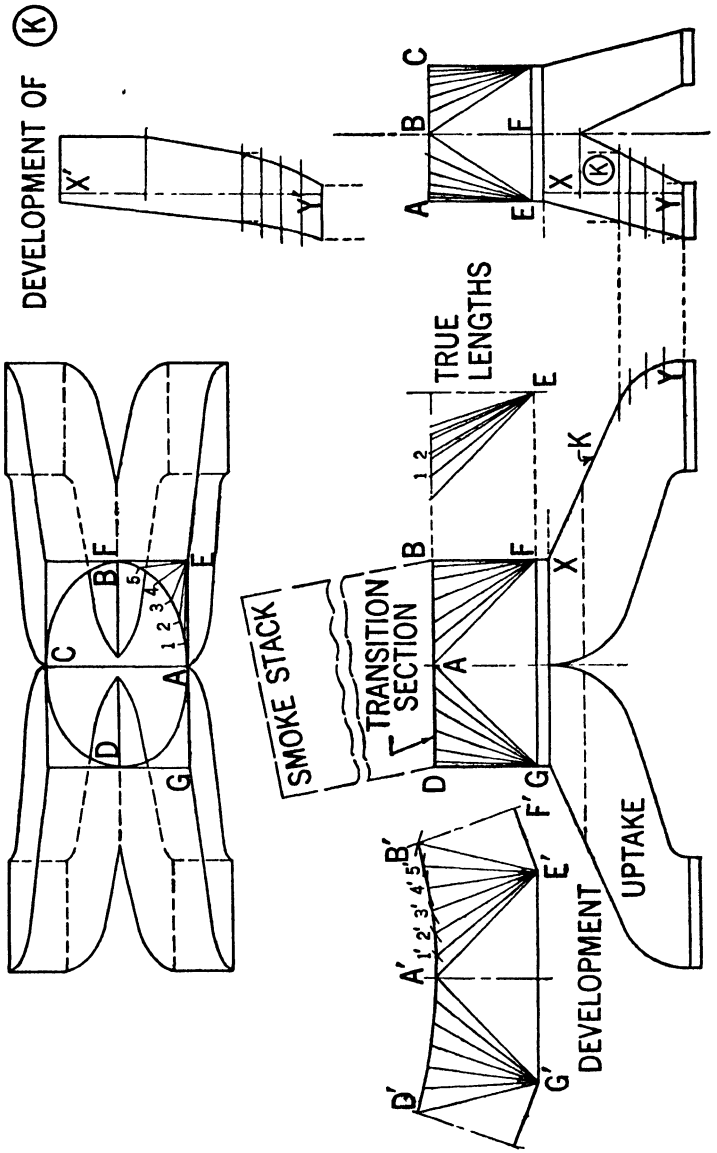

DEVELOPMENT OF (K)
X'
Y'
C
B
A
E
F
X
(K)
Y
TRUE LENGTHS
1 2
K
SMOKE STACK
TRANSITION SECTION
D
G
UPTAKE
DEVELOPMENT
A' B' C' D' E' F' G'
1' 2' 3' 4' 5'
1 2 3 4 5

Fig. 273

triangulation. The cone having its vertex at E is typical of the remaining three. The base curve AB is divided into six equal parts, and the elements of the cone are drawn.

The true length of each element is found by the right-triangle method. These true lengths are drawn to the right of the view in order to avoid confusion. The true length of any given line, as E-1, is the hypotenuse of a right triangle having for its altitude the difference in elevation between the points E and 1 and for its base the length of the plan view of E-1.

The development is started by constructing the triangle $A'E'G'$ equal to AEG. From point E a series of radii, equal to the true lengths of the elements $E1$ to EB, are struck. The lengths of the segments A-1, 1-2, and so forth, of the ellipse, as given in the plan view, are now set off on the development, making A'-$1' = A$-1; $1'$-$2' =$ 1-2, and so forth. In practice, this operation is accomplished by bending a thin batten to fit the plan view of the quarter ellipse and marking the points between A and B. The batten then is laid on the development so that these points lie on the proper radii, and the smooth curve AB is drawn along the edge. Since the transition piece is symmetrical, only one half of the surface is developed.

(b) Uptake development. The plate K of one of the uptakes is composed of a plane and a cylindrical surface. The cylindrical portion is divided into small areas by drawing a series of elements of the surface. The girth line XY is drawn at right angles to the elements. The true length of the girth line $X'Y'$ is taken from the longitudinal elevation by means of a batten. The points at which elements are assumed are marked at the same time. Line $X'Y'$ becomes the base line for the development. The elements are laid off in true length at the proper points along $X'Y'$.

The remaining plates of the uptake can be developed in a similar manner.

5. Metrical Problems on the Sphere

196. Spherical triangles

The portion of a spherical surface bounded by three arcs of great circles is a *spherical triangle.* If each vertex of a spherical triangle ABC is joined to the center O of the sphere by a straight line, a corresponding trihedral angle, O-ABC, is formed (Figure 274). The sides of the spherical triangle opposite the vertices A, B, and C are denoted by the letters a, b, and c respectively. These sides are measured by the face angles of the trihedral angle. The angles of the spherical triangle are measured by the dihedral angles of the trihedral angle. In order that a trihedral angle may be constructed, any one face angle must be less than the sum

of the remaining face angles. The development of the faces is a circular sector, its radius being equal to the radius of the sphere (Figure 275).

The spherical triangle or trihedral angle is composed of six elements: three dihedral angles and three face angles. It can be constructed if any three elements are given. The remaining elements can be deduced from those given. In certain cases, more than one solution may be possible.

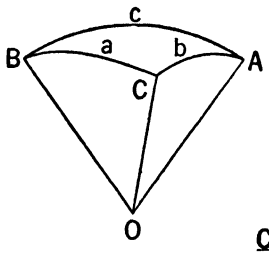


Fig. 274

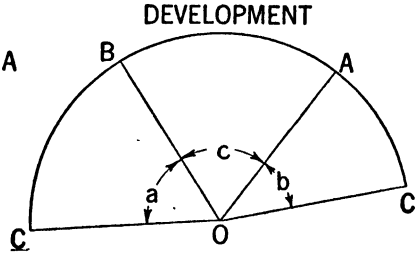


Fig. 275

These considerations lead to six cases in which the given data are as follows:

1. Three face angles.
2. Two face angles and the included dihedral angle.
3. Two face angles and an opposite dihedral angle.
4. One face angle, an adjacent and an opposite dihedral angle.
5. One face angle and the two adjacent dihedral angles.
6. Three dihedral angles.

Solutions for the first three cases only are given since by use of the corresponding polar triangle (described at the end of this section), case 6 reduces to case 1, case 5 to 2, and case 4 to 3.

Case 1. Three face angles.

To find: A, B, and C.

Given: Angles a, b, and c (Figure 276).

Construct the development of the trihedral angle making edge OA vertical. Any convenient radius may be used. Angle $A_VO_VB_V$ is equal to c; $B_VO_VC_Y$ is equal to a; $A_VO_VC_X$ is equal to b. The trihedral is now formed by folding the faces into their correct positions in space.

Allowing face AOB to remain fixed, rotate faces AOC and BOC about edges AO and BO respectively, until the outside edges meet in the edge OC of the solid angle.

A line through C_X and perpendicular to AO represents the path of rotation in the front view for point C_X. This path is a circular arc k in the top view. A line through C_Y and perpendicular to BO represents the path of rotation of point C_Y. The two lines intersect in C_V, one

vertex of the spherical triangle. The other vertices are A and B. The projector through C_V intersects the arc k_H in point C_H. We now have the determining points of the trihedral angle in two views. Then, B_HC_H, B_VC_V, and A_VC_V are added here merely to improve the pictorial effect of the drawing. They are not required in the constructions for the values of the dihedral angles.

Since edge AO is a point in the top view, the dihedral angle A is given by $B_HA_HC_H$. The dihedral angle B is found by drawing the

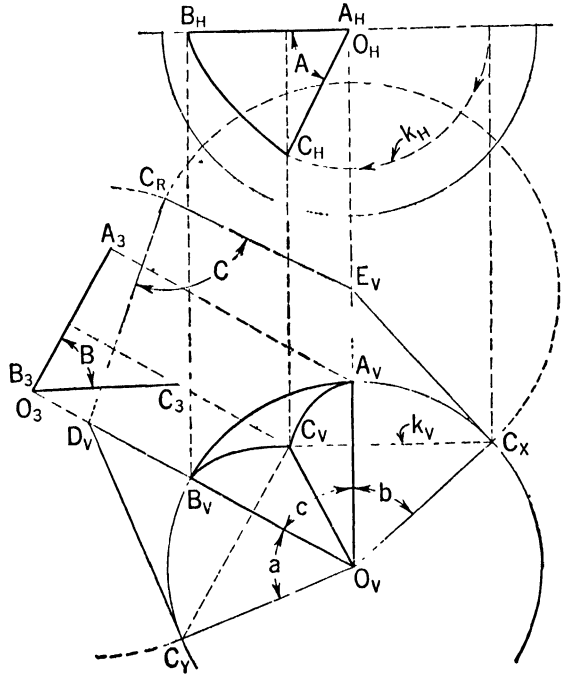


Fig. 276

auxiliary view in which line OB projects as a point, and faces AOB and BOC project edgewise. Angle B is given by $A_3B_3C_3$.

The dihedral angle C can be found in an auxiliary view in which edge OC projects as a point, and faces BOC and AOC project edgewise. The following construction, however, is somewhat shorter and thus more precise. On the development of face BOC, draw line C_YD_Y perpendicular to C_YO_Y. Also, on the development of face AOC, draw line C_XE_Y perpendicular to C_XO_V. If the two faces are now rotated to their true positions in space, lines CD and CE will form an angle DCE which measures the dihedral angle C. The true size of angle DCE is found by rotation to be $D_VC_RE_V$ and is equal to the required angle C. This

rotation is performed easily since D_VC_R and E_VC_R are equal to D_VC_Y and E_VC_X respectively.

A more compact form of the solution for this problem is shown in Figure 277. Unnecessary lines are omitted, and the partial top and auxiliary views of angles A and B are inverted to avoid overlapping.

In this construction, it may be more convenient to regard the views of angles A and B as right sections of the dihedral angles rotated into the vertical projection plane.

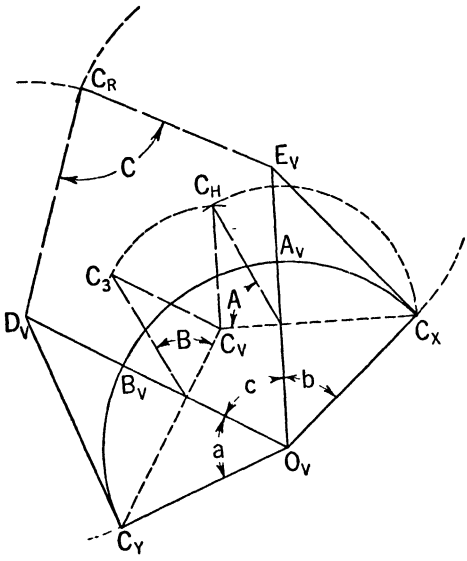


Fig. 277

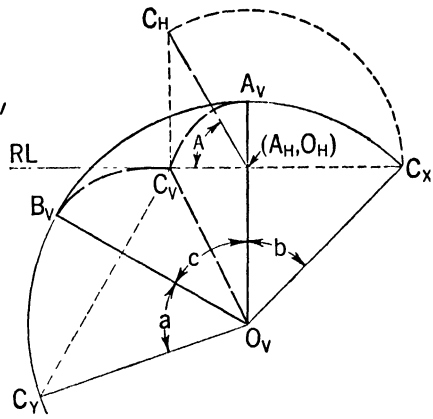


Fig. 278

Case 2. Two face angles and the included dihedral angle.

Given: A, b, and c (Figure 278).

Let AO be a vertical line of any convenient length. On the vertical projection plane draw the development of the two given faces of the trihedral angle making $AOC = b$ and $AOB = c$.

Allowing face AOB to remain fixed on the V-plane, rotate face AOC about AO as an axis until the angle between the two faces is equal to the given dihedral angle A. If a reference line is taken through C_X, the top view of face AOC, after rotation, is $A_HO_HC_H$. In the front view, C_X rotates to C_V, which lies on the projector through C_H.

The face angle a is found by rotating the face BOC about OB as an axis until it lies in the V-plane. Through C_V draw a line perpendicular to O_VB_V and intersecting the circular arc A_VB_V produced at C_Y. Angle $B_VO_VC_Y$ is equal to the required angle a.

Since the three face angles are now known, the remaining angles can be found as in Case 1.

Case 3. Two face angles and an opposite dihedral angle.

Given: B, b, and c (Figure 279). This is an ambiguous case.

Draw the development of the given faces AOB and AOC on the vertical plane, taking edge AO vertical. Assume the reference line RL through point A. Through edge BO, pass a plane Q which makes the given angle B with the vertical projection plane (Article 67). This plane contains the face BOC. Rotate face AOC about AO as an axis until edge OC lies in plane Q.

Produce O_VC_X to cut the RL in K_X. Rotate triangle AOK about AO as an axis until line OK lies in plane Q. This position is determined by drawing the circular arc which represents the top view of the path of

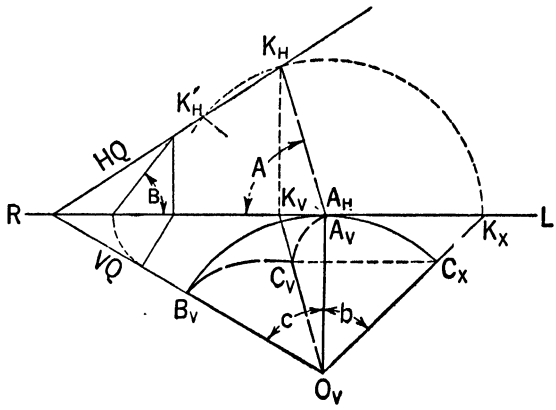


Fig. 279

rotation of K. This arc cuts HQ in K_H and K'_H, indicating two different solutions to this problem. Evidently, there would be one solution if the arc was tangent to HQ; no solution if the arc failed to cut HQ.

To complete one solution, draw the line K_HA_H which represents the top view of plane AOK containing face OAC in its true position. The angle A between faces AOB and AOC appears in the top view.

The front view of edge OC lies along O_VK_V. A line through C_X and perpendicular to AO represents the path of rotation of point C_X. This line cuts O_VK_V in C_V. The remaining angles are found as in the construction for Case 1.

197. The terrestrial triangle

Many problems involving measurements on the earth's surface require the solution of a spherical triangle. For example, a ship following the simplest course steams on a *rhumb line*, a line on the sphere which cuts every meridian at the same angle. Such a course has a constant bearing. The rhumb line or *loxodromic curve* plots as a straight line

on a mercator chart, a form of map in which meridians and parallels are straight lines at right angles to each other (Figure 280). The shortest course between two points is the arc of a great circle. A great circle course cuts the meridians at varying angles (except a course following a meridian or the equator) and plots as a curve on a mercator chart. In practice, the great circle course is approximated by following a series of tangents or chords of the arc plotted on the mercator chart. These short courses are rhumb lines.

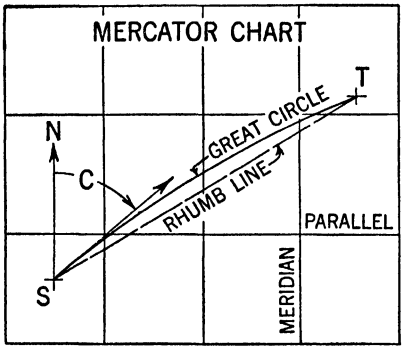


Fig. 280

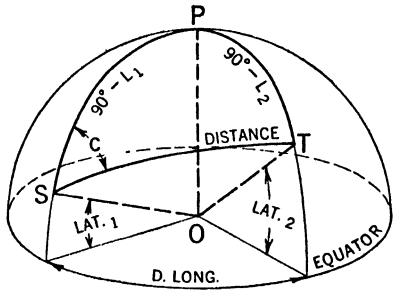


Fig. 281

A great circle course ST is represented on a sphere by plotting points S and T from their known longitude and latitude (Figure 281). The dihedral angle at the pole is equal to the difference in longitude between points S and T. The meridian arcs PS and PT are the co-latitudes of S and T respectively. The arc ST is the length of the course and is measured by the face angle SOT of the trihedral angle O-PST. The angle C of the initial bearing for the course ST is equal to the dihedral angle at S.

The problem to be solved may be stated as follows: In the triangle PST, sides SP and TP and the included dihedral angle P are given. The side ST and the angle S are to be determined.

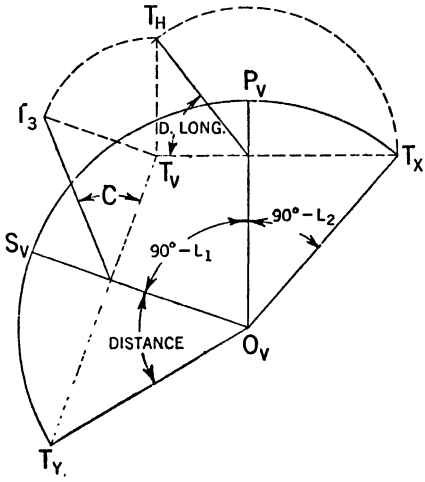


Fig. 282

The solution of this triangle (Figure 282) falls under Case 2 (Article 196). The course angle at S is equal to C. The distance ST, in degrees, is equal to the face angle $S_VO_VT_Y$.

For the purposes of navigation, the earth is regarded as a sphere on which 1 degree of arc on a great circle is equal to 60 nautical miles.

198. The astronomical triangle

The *celestial sphere* is an imaginary sphere of infinite radius concentric with the earth. One hemisphere is shown in Figure 283, the earth being a point at O. The polar axis of the earth, extended, becomes the polar axis of the celestial sphere PO. The plane of the earth's equator extended, cuts the celestial sphere in the celestial equator. The point Z vertically above an observer at O is the *zenith* of the observer. A plane through point O and at right angles to OZ cuts the sphere in the *celestial horizon.* It is assumed that the earth and celestial sphere are fixed in space and that the heavenly bodies move on the surface of the celestial sphere. Point S represents the position of a star on the celestial sphere. The points P, S, and Z form the vertices of a spherical triangle called the *astronomical triangle.*

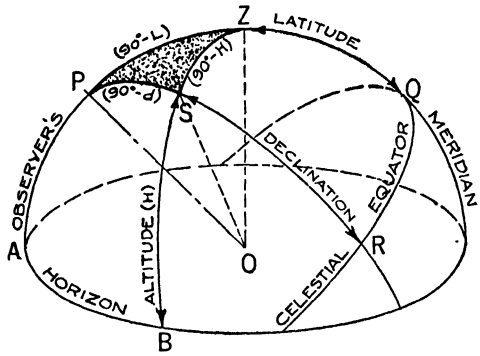


Fig. 283

The arc BS is the altitude of the star or its angle above the horizon. The arc RS is the star's declination or angle above the equator. The arc QZ is equal to the latitude of the observer. The sides of the triangle have the following values:

$$PZ = 90° - \text{Latitude};$$
$$PS = 90° - \text{Declination};$$
$$ZS = 90° - \text{Altitude}.$$

Points P, S, and Z are joined to O to form a trihedral angle.

The dihedral angle at P is measured by the arc QR. This angle is called the *local hour angle* of the star and represents the angle between the observer's meridian and the star's meridian. The dihedral angle at Z is measured by the arc AB and represents the *bearing* or *azimuth* of the star.

(a) The determination of time. The meridian circle PSR is called an *hour circle.* When S represents the sun, the hour circle PSR rotates about the polar axis PO, point R on the celestial equator moving through 360° in 24 hours, or at the rate of 15° per hour.

When the hour circle of the sun moves into coincidence with the observer's meridian, the astronomical triangle reduces to a line, and the

time is noon (or midnight), *local apparent time.* Owing to the slightly irregular motion of the sun, the apparent time at a given instant differs by a small amount from the *civil time* shown on a chronometer. This difference varies from day to day and is called the *equation of time.*

The hour of the day at the observer's meridian is found by subtracting the local hour angle (converted to hours) from 12 o'clock when the observation is made in the morning, or adding it to 12 o'clock when the observation is made in the afternoon. When the *L.H.A* is 0°, the time is 12 o'clock noon.

Example. An observer in latitude 42°00′ N. finds the altitude of the sun to be 44°00′. The declination of the sun at the time of observation

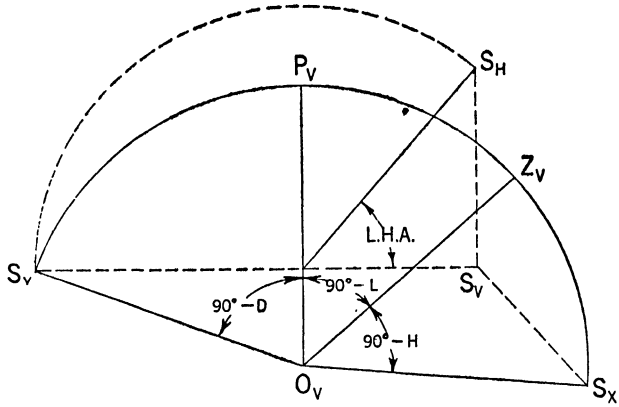


Fig. 284

is 20°00′ N. The observation time is P. M. Find the local hour angle of the sun and the local apparent time at the observer's meridian.

The solution of this problem leads to a trihedral angle *O-PSZ* in which the three face angles are known. The construction of Case 1 (Article 196), therefore, is employed. The development of the trihedral angle is constructed (Figure 284) using the following values:

$$\text{Angle } P_VO_VS_Y = 90° - D = 70°;$$
$$\text{Angle } P_VO_VZ_V = 90° - L = 48°;$$
$$\text{Angle } Z_VO_VS_X = 90° - H = 46°.$$

The construction for the angle at P yields a value of 47°47′ for the *L.H.A.* This angle, converted to hours, gives $3^h11^m08^s$.

Since the observation is after noon, the local apparent time is $15^h11^m08^s$.

(b) The determination of longitude. The determination of longitude requires the use of a chronometer which shows the civil time at the Greenwich meridian, which is 0° longitude. Greenwich civil time is changed to Greenwich apparent time by applying the equation of time.

If the Greenwich apparent time of the observation is noted, the difference between this time and the local apparent time, as found above, serves to determine the longitude of the observer.

If, in the previous example, the G.A.T. is $18^h55^m00^s$, then

G.A.T.	$18^h55^m00^s$
L.A.T.	$15^h11^m08^s$
Long.	$3^h43^m52^s$
	$= 55°58'.0$ West.

The *line of position* method used in modern navigation assumes the known elements of the astronomical triangle to be the co-latitude of the observer, the co-declination of the star, and the local hour angle of the star. This gives two sides and the included angle. The triangle is solved for the co-altitude and the azimuth of the star.

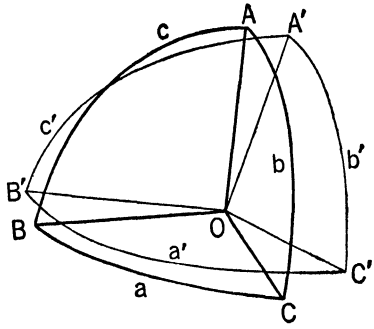


Fig. 285

(c) **Supplementary trihedral angles.** In a given trihedral angle $O\text{-}ABC$ (Figure 285) draw a radius of the sphere OA' perpendicular to the plane OBC and on the same side as point A. Construct radii OB' and OC' in a similar manner, perpendicular to faces OAC and OAB respectively. Points A', B', and C' now define the vertices of the *polar triangle* of ABC. Since OA' and OB' are perpendicular to planes OBC and OAC respectively, the plane angle $A'OB'$ is the supplement of the dihedral angle between these planes. Thus, the sides a', b', and c' of the polar triangle are the supplements of the corresponding sides of the primitive triangle. That is

$$a' = 180° - A; \qquad b' = 180° - B; \qquad c' = 180° - C.$$

Also, it can be shown that ABC is the polar triangle of $A'B'C'$. Therefore

$$a = 180° - A'; \qquad b = 180° - B'; \qquad c = 180° - C'.$$

The trihedral angles $O\text{-}ABC$ and $O\text{-}A'B'C'$ are called *supplementary angles.*

6. Earthwork

199. Roadway

A proposed roadway 40 feet wide is to follow the center line ABC (Figure 286). The road is level and at elevation 45. The earth slopes

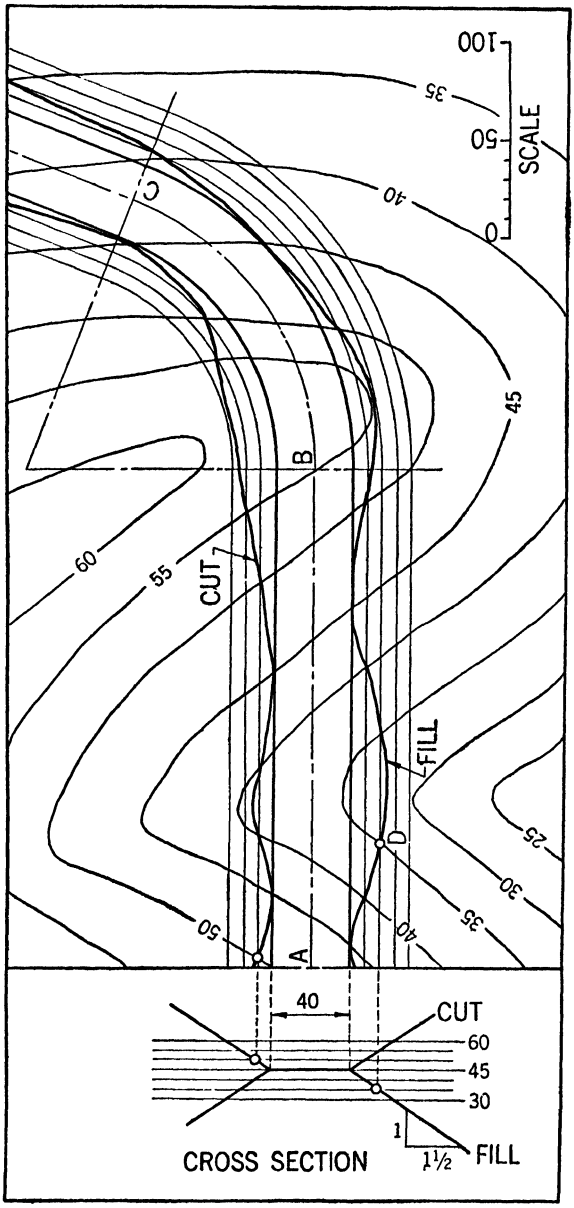

100
50
0
SCALE
35
40
45
60
55
50
40
35
30
25
C
B
A
D
CUT
FILL
40
CUT
60
45
30
1
1½
FILL
CROSS SECTION

Fig. 286

are to be $1\frac{1}{2}$ to 1 in both cut and fill. Determine the top of the slope in the cut and the bottom of the slope in the fill when the earthwork has been completed.

Construct a cross section of the roadway and represent the level planes which correspond to the contours of the ground. The intersection of each level plane with the earthwork is projected from the cross section to the map. The earthwork contours are straight lines between points A and B and circular arcs between points B and C. Points in which contour lines at the same level meet are points in the line of cut or fill. For example, the 35-foot ground contour cuts the 35-foot contour of the fill in point D.

7. Graphical Representation of an Equation of Three Variables

200. PV = KT

The equation $PV = 10{,}000T$ represents the pressure-volume-temperature relationship for a given quantity of helium gas. Pressure is given in centimeters of mercury. Volume is in cubic centimeters. Temperature is in Centigrade units on the absolute scale.

Represent this equation by its orthographic projections, using the following maximum values for the variables:

$$P = 152 \text{ cm}, \qquad V = 50{,}000 \text{ cc}, \qquad T = 760°.$$

The equation $PV = 10{,}000T$, referred to rectangular coordinates, is a surface of the type $xy = kz$, a hyperbolic paraboloid. In order to draw the elements of this surface, it is necessary to determine its plane director and two linear directrices.

Three mutually perpendicular axes OP, OV, and OT are assumed (Figure 287). The following scale values are arbitrarily assigned:

Axis	*Maximum Valve*	*Length*
OV	50 liters	5 units
OP	152 cm (2 Atmos.)	3 units
OT	760 C (Abs.)	5 units

(a) Directrices. Any straight line cut from the surface by a plane can be used for a directrix. In this case, when V is a constant, the equation becomes $kP = 10{,}000T$, a straight line. Let $V = 0$, then

$$P(o) = 10{,}000T.$$

Therefore, one directrix is a vertical line through the origin. Let $V = 50{,}000$, then $5P = T$. Thus, the second directrix is a line in the profile plane POT located at $V = 50{,}000$. It is located by two of its points. When $P = 0$, $T = 0$. When $P = 152$, $T = 760$.

(b) Plane director. When the surface is intersected by horizontal planes of the form $P = k$ (constant), the intersections are straight lines having the equation $kV = 10{,}000T$. These lines can be regarded as elements of the surface. Thus, the plane director is horizontal.

The orthographic views of the surface can now be completed. The two linear directrices are located in the top, front, and side views. Twelve equally spaced horizontal elements are drawn in the front view and projected to the side and top views. According to the given equa-

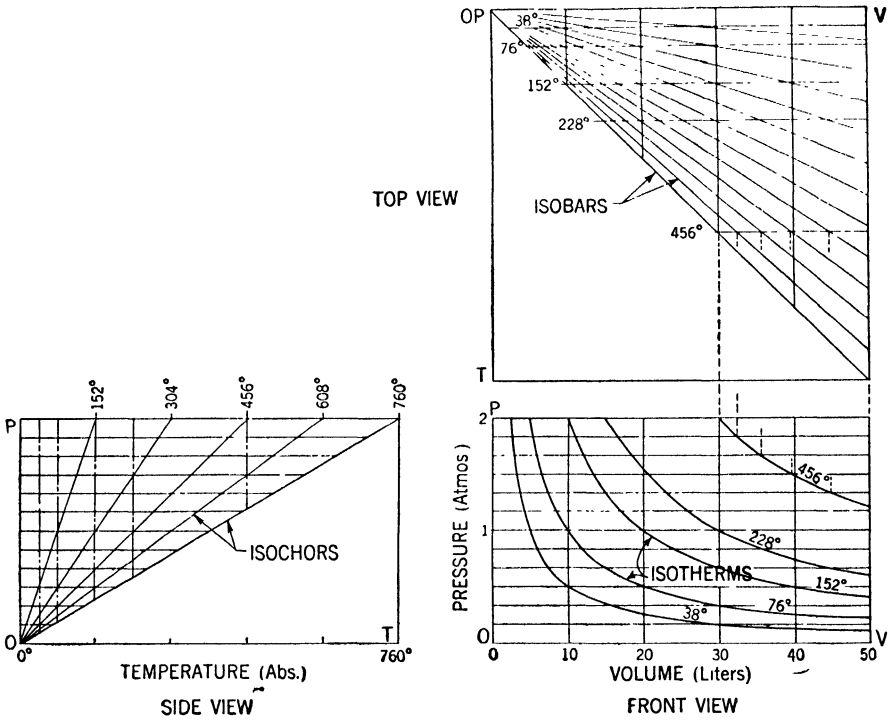


Fig. 287

tion, horizontal lines of the surface represent lines of equal pressure and are called *isobars*.

The horizontal elements passing through points $P = 0$ and $P = 2$ (Atmos.) are divided into five equal parts, and elements of the second ruling are drawn through these points. These are lines of constant volume and are called *isochors*.

Any plane parallel to the vertical projection plane will cut the surface in a hyperbola of the form $PV = constant$. This is an *isothermal line*. Isothermals are plotted for $T = 38°$, $76°$, $152°$, $228°$, and $456°$.

The oblique projection of this surface (Figure 288) is constructed with the cross axis OT taken at an angle of 30°. Dimensions on the cross axis are plotted half size.

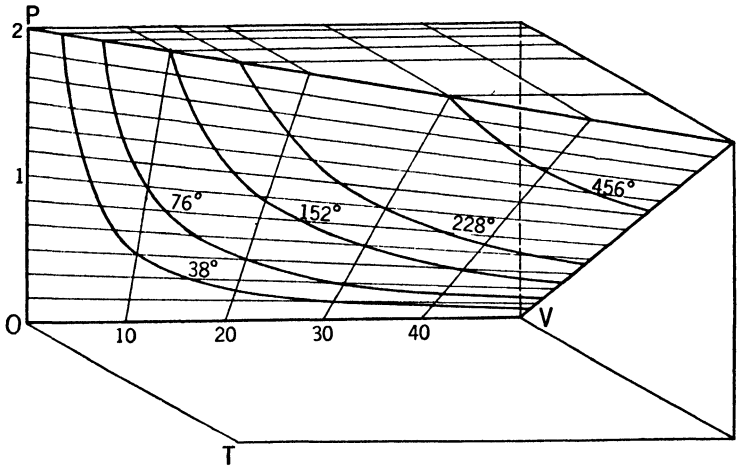


Fig. 288

8. Conic Lofting

201. Streamlined surface

In the three-view drawing of the streamlined surface (Figure 289), the following lines are given:

1. The upper and lower center lines.
2. The top view of the half-breadth line.
3. Five transverse sections taken at stations 0, 11, 25, 48, and 65. Stations 0 and 65 are points.

The problem is to fair the given lines and locate the upper and lower shoulder lines and the profile view of the half-breadth line in such a manner that any transverse section will fair together longitudinally with the given sections.

A conic section is a fair curve. The method of conic lofting consists in finding the conic that contains the critical points of the curve given in the three-view drawing. A conic is determined by five elements. By means of a construction based on Pascal's Theorem, Appendix 2, a conic can be passed through any five points or through one point and tangent to each of two lines at given points.

Fairing the given lines. The transverse sections locate three points on the upper center line. These points together with the end points give five points through which a conic is constructed. The lower center line and the half-breadth lines are faired in the same manner.

In the front view, each transverse section is enclosed in a rectangle formed by four tangents. A plane passing through the axis of the surface and the tangent point T_1 on section 25 fixes the tangent points for

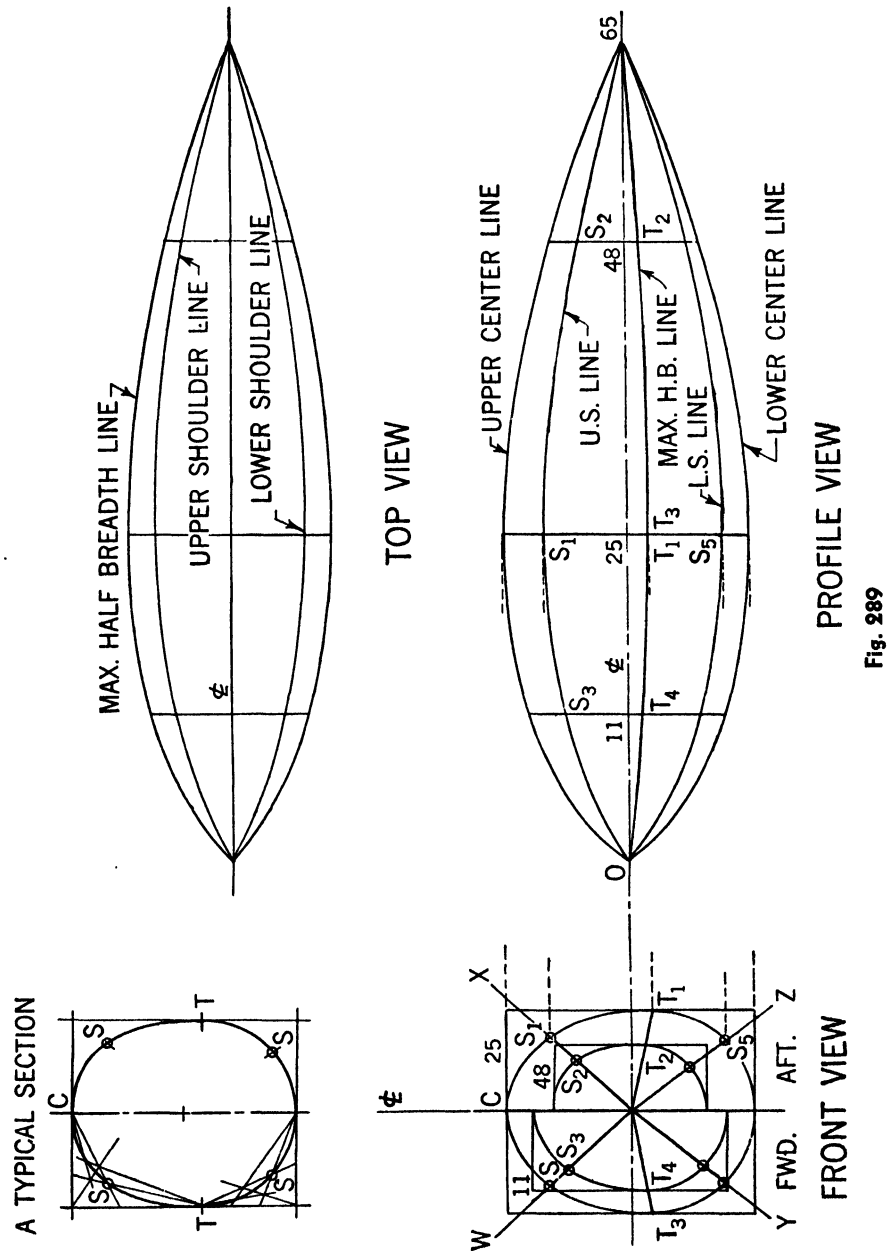


Fig. 289

section 48 at T_2. Similarly, a plane through the axis and T_3 on section 25 locates the tangent T_4 for section 11. Each section is tangent to the upper and lower sides of the rectangle at the center point. The tangent points thus established in the front view are projected to the corresponding sections in the profile view. The tangent points T_1, T_2, and T_3, together with the end points 0 and 65, give five points through which a conic is constructed. This curve is the profile view of the half-breadth line.

A transverse section is faired by selecting a control or shoulder point in each quadrant of the curve. Point S_1 is taken as the upper shoulder point on section 25, and a conic is constructed which passes through S_1 and is tangent to the sides of the rectangle at points T_1 and C.

A plane X passed through point S_1 and the axis of the solid cuts section 48 in its shoulder point S_2. A plane W, symmetrical to X, cuts section 11 in shoulder point S_3. The lower shoulder points are located by assuming the planes Y and Z, which contain the axis of the surface. Sections 11 and 48 now are faired by using the tangent and control points just found. The three upper shoulder points are projected to the profile view. The conic passed through these three points, together with the end points 0 and 65, is the upper shoulder line. The lower shoulder line is found in a similar fashion. The upper and lower center lines also are constructed in the top view.

Any intermediate section now can be constructed. The upper and lower center lines and the half-breadth lines give the height and breadth respectively of the enclosing rectangle. In the front view, the planes of the half-breadth lines locate the tangent points on vertical sides of each rectangle. The control points for the curves are obtained from the planes of the shoulder lines.

PROBLEMS ON CHAPTER XII

Group A. Frameworks

12-A1. Hip roof framing. Points A:$\frac{1}{2}$,1,9$\frac{1}{2}$ B:$\frac{1}{2}$,1,6$\frac{1}{2}$ and C:3$\frac{1}{2}$,1,6$\frac{1}{2}$ locate the plate at a corner of a building. Line AB is equal to one half of the roof span. The rise of each roof plane is 8 inches per 12 inches of run. Draw the top, front, and side views of the ridge pole, two common rafters, one hip rafter, and a pair of jack rafters located at the center of the hip rafter. Construct the true-size views of the hip rafter and a jack rafter. The common and jacks are 3- × 6-inch timbers. The hip rafter and ridge are 3 × 8 inches. Let $h = 4''$. Scale: 1 inch = 1 foot.

12-A2. Skew bridge portal. Points A:1$\frac{1}{2}$,$\frac{1}{2}$,9 B:3$\frac{1}{2}$,$\frac{1}{2}$,6 C:5$\frac{1}{2}$,2$\frac{1}{2}$,6 and D:3$\frac{1}{2}$,2$\frac{1}{2}$,9 determine the corners of a bridge portal. The corners of the portal bracing lie at points C, D, E:329, and F:526. Draw the top

and front views of the portal and a portion of the trusses, using a panel width of 16 feet. Scale: $\frac{1}{8}$ inch = 1 foot. Draw the bracing as shown in Figure 266.

Find: (*a*) The true shape of the portal; (*b*) the true angle between the plane of the portal and the floor of the bridge; and (*c*) the true angle between the plane of the portal and the plane of a truss.

12-A3. The line *A*:239 *B*:3$\frac{1}{4}$,3,9 is the upper edge of a vertical cross section of an I-beam which is 2 inches high. The flanges and web are $\frac{1}{4}$ inch thick. The beam is terminated by a plane passing through the line *P*:1,3,4$\frac{1}{2}$ *Q*:3$\frac{1}{2}$,3,7 and sloping 60° downward and forward. Find the top view and the true-shape view of the oblique section.

GROUP B. VECTORS

12-B1. Three forces acting at point *P*:316 are represented in projection by the vectors *P-A*:204 *P-B*:449 and *P-C*:507. Find the vector which represents the resultant of these forces, and determine its magnitude. Force scale: 1 inch = 100 pounds.

12-B2. Three forces acting at point *P*:457 have the directions *P-A*:138 *P-B*:525 and *P-C*:649. The value of each force is 250 pounds. Draw the top and front views of the vectors which represent the forces, and find the resultant. Force scale: 1 inch = 100 pounds.

12-B3. Resolve the vector *O*:137 *R*:728 into components which lie on the lines *a*:137;2,4,10 *b*:137;435 and *c*:137;417.

12-B4. A vertical force of 4000 pounds acts at point *O*:4,4$\frac{1}{2}$,7 of the tripod frame *O-A*:1,1$\frac{1}{2}$,7 *O-B*:5,1$\frac{1}{2}$,9 and *O-C*:7,1$\frac{1}{2}$,6. Find the force in each leg of the frame. Scale: 1 inch = 1000 pounds.

GROUP C. SHEET-METAL STRUCTURES

12-C1. Circular elbow. Points *A*:049 *B*:1$\frac{1}{2}$,4,9 *C*:3,2$\frac{1}{2}$,9 and *D*:319 locate the center line of a three-piece elbow. Each piece is cylindrical and 2 inches in diameter. Draw the front view only, and develop the center piece of the elbow. Use twelve elements, and make joint on shortest element.

12-C2. Transition connection. The ends of a 2- × 6-inch frontal rectangle are rounded to a 1-inch radius. These semicircles are centered at points *A*:1,5,10 *B*:1,1,10. A 2-inch frontal circle is centered at *C*: 1,3,8$\frac{1}{2}$. The rectangle with rounded ends and the circle form the openings of a sheet-metal transition piece. Draw the top, front, and side views. Show a right section of one of the oblique cylinders which form a portion of the transition surface. Make a pattern for one cylindrical portion and the adjacent plane surface.

12-C3. Transition connection. A horizontal opening is formed by two semicircles of $\frac{7}{8}$-inch radius centered at points A:1,3$\frac{1}{2}$,8 and B:1,3$\frac{1}{2}$,6 which are joined by two tangents. This circular-ended opening is to be connected to a 1$\frac{3}{4}$-inch diameter horizontal circle centered at C:3,$\frac{1}{2}$,7 by a developable transition piece. Draw the top and front views of the connection, and develop one half of the surface.

12-C4. Transition connection. The rectangle A:016 B:416 C:419 D:019 and the 2-inch horizontal circle centered at C:2,4,7$\frac{1}{2}$ are the open ends of two pipes. Draw a developable transition surface to connect the two openings. Develop one half of the surface.

12-C5. Transition connection. Line C:1,1$\frac{1}{2}$,7$\frac{1}{2}$ D:1,$\frac{1}{2}$,7$\frac{1}{2}$ is the axis of a 1$\frac{3}{4}$-inch diameter circular pipe. The plane of the upper end of the pipe makes an angle of 60° with the center line, sloping downward to the right. Line E:2$\frac{3}{4}$,4$\frac{1}{2}$,6 F:3$\frac{3}{4}$,4$\frac{1}{2}$,6 is the front edge of a horizontal rectangular opening 3 inches deep. Draw the top and front views of a transition connected between the two openings, and develop one half of the surface.

12-C6. Breeching. Points A:219 and C:619 are the centers of horizontal 2-inch circles that are the ends of two pipes. Design a Y-connection to join these two pipes to a third pipe having its open end represented by the horizontal 3-inch circle centered at E:479. The design is to use circular cones and cylinders intersecting in plane curves. Draw the front view only.

12-C7. Development of breeching. Develop one half of an intermediate section of the breeching of Problem 12-C6. Redraw this section locating its center line 2 inches from left side of sheet, keeping the vertex of the cone on the sheet.

12-C8. Chute. The line A:1,4,7$\frac{1}{2}$ B:519 is the center line of a chute which passes through a floor at point B and a wall at point A. Represent the floor by a horizontal plane through B and the wall by a profile plane through A. The cross section of the chute is a square which is 3 feet on each side. Two sides of the square are horizontal. Draw the views of the chute, and determine the true shape of the hole which must be cut in the floor and the wall. Scale: $\frac{1}{4}$ inch = 1 foot.

12-C9. Develop the portion of the chute of problem 12-C8 which lies between the floor and the wall.

12-C10. Condenser exhaust connection. Line A:0,2,6$\frac{1}{2}$ B:1$\frac{1}{2}$,2,6$\frac{1}{2}$ is the upper edge of a frontal rectangle 2 inches high. Line C:358 D:3,5,9$\frac{1}{2}$ is the upper edge of a profile rectangle 2 inches high. Join these two openings by a quarter bend having line E:3,0,6$\frac{1}{2}$ F:3,5,6$\frac{1}{2}$ as

its axis. The upper and lower surfaces are to be helicoidal and the sides cylindrical. Develop the sides, and make an approximate development of the upper surface by triangulation.

GROUP D. SPHERICAL TRIANGLES

The following problems may be solved on the $8\frac{1}{2}$- × 11-inch problem sheet using a sphere of 4-inch radius. For greater precision, a sphere of 10-inch radius is suitable. In this case, angles should be measured by their chords.

12-D1. Given: the angles $a = 45°$, $b = 30°$, and $C = 60°$. Solve for angles A, B, and c.

12-D2. Given: $a = 60°$, $b = 30°$, $C = 45°$. Solve for A, B, and c.

12-D3. Given: $a = 45°$, $b = 30°$, $C = 120°$. Solve for A, B, and c.

12-D4. Given: $a = 30°$, $b = 60°$, $c = 45°$. Solve for A, B, and C.

12-D5. Given: $a = 60°$, $b = 30°$, $c = 75°$. Solve for A, B, and C.

12-D6. Given: $A = 45°$, $a = 30°$, $b = 30°$. Solve for B, c, and C.

12-D7. Given: $A = 30°$, $a = 30°$, $b = 45°$. Solve for B, c, and C.

12-D8. Given: $A = 135°$, $B = 120°$, $C = 105°$. Solve for a, b, and c.

GROUP E. TERRESTRIAL AND CELESTIAL SPHERES

12-E1. Determine the length in nautical miles of the great circle course from San Francisco (Lat. 37°48′ N., Long. 122°24′ W.) to Honolulu (Lat. 21°18′ N., Long. 157°55′ W.).

12-E2. Determine the length in nautical miles of the great circle course from New York (Lat. 40°43′ N., Long. 74°00′ W.) to Liverpool (Lat. 53°24′ N., Long. 3°04′ W.). Find the initial and final bearings of the course.

12-E3. An airplane is to travel a great circle course from Washington (Lat. 38°54′ N., Long. 77°03′ W.) to Rio de Janeiro (Lat. 22°54′ S., Long. 43°10′ W.). Find the initial bearing of the course.

12-E4. A navigator in Lat. 42°N. observes the altitude of the sun to be 15° on a morning sight. The declination of the sun at the time of observation is 21°S. Find the local apparent time. If the observation is taken at $13^h56.0^m$ Greenwich apparent time, find the longitude of the ship.

12-E5. A mariner observes the altitude of the sun to be 57°, its declination being 14°N. The latitude of the ship is 35°N. The sight is taken before noon. Find the local apparent time. If the Greenwich apparent time is $18^h05.5^m$, find the longitude of the ship.

12-E6. At what hour (local apparent time) will the sun rise in Boston (Lat. 42°21′) if its declination at sunrise is 18°N.? (At sunrise and sunset the sun's altitude is 0°.)

12-E7. At sea in Lat. 37°N., the star Denebola is west of the meridian at a twilight observation. Its declination is 15°N., and its local hour angle is calculated to be 36° at the time of observation. Find the altitude of the star and its azimuth.

12-E8. The planet Venus is east of the meridian at a morning observation. Its declination is 15°S., and its local hour angle at the time of observation is found to be 30°. Find the altitude and azimuth of the planet.

GROUP F. MISCELLANEOUS PROBLEMS

12-F1. Pipe connection. A straight steam pipe which passes through point P:003 is to be connected into a pipe having A:125 B:3,0,3½ as its center line. The connection is to be made by means of a tee fitting. Determine the distance from point A to the center of the tee and the true distance from point P to the tee. Scale: ¼ inch = 1 foot.

12-F2. Foundation. Scale: 1 inch = 50 feet. Line A:436 B:736 is the front edge of the top of a proposed foundation for a rectangular building 100 × 150 feet. Line AB bears due east and has an elevation of 100.00. A test pit sunk at a point K, 100 feet east of A and 50 feet north of A, struck bed rock at a depth of 55 feet below the top of the foundation wall. An examination of the rock indicated that it dipped 15° in a northwesterly direction, and its strike line ran N60°E. Determine the front view of the foundation assuming it built to bed rock.

12-F3. Culvert. Scale: 1 inch = 20 feet. Point A:526 is on the center line of a level highway 20 feet wide, bearing due north. The slope of the earth fill is 1½ horizontal to 1 vertical. The bed of a stream bearing N45°W. and sloping 15 per cent downward in a northwesterly direction crosses the road at a point 15 feet directly below A. A culvert is to carry this stream under the road. The center line of the bottom of the culvert will follow the center line of the stream bed. Determine the true length of the center line of the bottom of the culvert between the sides of the fill.

12-F4. Mining problem. Scale: 1 inch = 100 feet. A vein of ore is located by the drill holes A, B, and C. Point S:546 is on the surface of the ground. The north-south line is parallel to the left edge of the sheet. Point A is 95 feet north and 293 feet west of point S. Point B is 390 feet north and 214 feet west of point S. Point C is 288 feet north

and 146 feet east of point S. Elevation of S = 3000 feet, A = 2632 feet, B = 2865 feet, C = 2950 feet.

Determine the following information, and tabulate results.

1. Find the strike and dip of the vein.
2. Find the depth of a vertical shaft from S to the vein.
3. Find the shortest incline from S to the vein.
4. Find the shortest crosscut (horizontal) from S to the vein.

Solution I: Determine the above information from plan and elevation.

Solution II: Solve by using an auxiliary projection showing the plane of the vein on edge. Drill hole at point C pierces lower side (foot wall) of vein at elevation 2920. Find the thickness of the vein on the auxiliary view. Use 11- × 17-inch sheet for Solution II.

12-F5. Frame joint. Size of sheet, 11 × 17 inches.

1. Given: Line AB and point C. Locate the front view of point A $2\frac{1}{2}$ inches from the left-hand border line and $2\frac{3}{4}$ inches from the lower border line. The top view of point A is $6\frac{1}{4}$ inches above the lower border line. Locate point B $2\frac{3}{4}$ inches to the right, $1\frac{5}{8}$ inches below, and $2\frac{3}{4}$ inches to the rear of point A. Locate point C $2\frac{3}{8}$ inches to the right, $\frac{9}{16}$ inch above, and $1\frac{1}{4}$ inches to the rear of point A.

2. Draw the projections of a line CD perpendicular to AB. (Locate point A in the auxiliary view $5\frac{1}{4}$ inches from A in the top view.) Draw the second auxiliary view which shows line CD in true length and AB as a point. Draw AB and CD in the form of center lines.

3. Lines AB and CD represent the center lines of two intersecting square tubes. Each tube is $1\frac{1}{2}$ inches square, and the ends of the tubes are cut at right angles to the center lines. One edge of tube AB passes through center line CD. Two opposite faces of tube CD are parallel to center line AB. Draw the top, front, and auxiliary views of the tubes. Note that certain inside edges will be visible in the top and front views. Show visible lines only in the front view, and show complete visibility in the other views. Object lines and reference lines are to be full weight. All projectors are to be very fine sharp lines.

12-F6. Cross connection. Size of sheet, 11 × 17 inches. Draw the projections of the common perpendicular EF between lines A: $1\frac{1}{2}$,$1\frac{3}{8}$,$5\frac{7}{8}$, B: $3\frac{9}{16}$,$4\frac{1}{8}$,$7\frac{3}{16}$ and C: $2\frac{1}{8}$,$2\frac{3}{4}$,$9\frac{3}{4}$ D: $4\frac{3}{4}$,1,$7\frac{1}{2}$. (Project AB and CD on a projection plane parallel to both lines.) Lines AB and CD are the center lines of two pipes $1\frac{1}{8}$ inches square, which are connected by a third pipe with line EF as a center line. The ends of all pipes are cut perpendicular to their center lines. The ends of pipe EF fit against the surfaces of pipes AB and CD, and the sides of EF are in the plane of sides of pipes

AB and CD. The cross section of pipe EF is thus a rhombus. Draw complete projections showing visibility in all views.

12-F7. Clearance problem. Size of sheet, 11 × 17 inches. Line X:2,3½,7 Y:11,3½,7 is the center line of an airplane fuselage. Line A:2½,2 5/16,4 7/16 B:10,1 1/16,4¾ is an aileron control cable. Line C:2½,2,4 13/16 D:10,1¾,3 31/32 is an elevator control cable. The cables are shown in their desired positions. The required clearance between these cables is 2 inches. (Scale of drawing as plotted is 1½ inches = 1 foot.) If the existing clearance is not sufficient, only the end A of the aileron cable can be moved, and only vertical displacement is permitted. If relocation of point A is necessary, show the new position of cable AB.

12-F8. The vector A:4,3½,9½ B:7,1,5½ represents a force which rotates in the plane determined by points A, B, and C:1,1¾,6, and about an axis through point C. Find the moment if the force scale is 1 inch = 100 pounds.

12-F9. A ray of light passing through points M:1,1,5½ and N:3,2,6½ is reflected from a plane mirror A:348 B:426 C:728. Find the path of the reflected ray.

12-F10. Lines O:1½,1,9½ P:1½,4,9½ and V:6½,1,9½ X:6½,4,4½ are linear directrices of a rectangular hyperbolic paraboloid. The plane director is horizontal. Represent the surface by 13 elements taken ¼ inch apart in the elevation. The pressure-volume-temperature relationship for an ideal gas is given by the characteristic equation $\frac{PV}{T} = constant$. This equation is represented in geometrical form by the above surface. Pressure is measured along axis OP, volume along OV, and temperature along an axis through O and perpendicular to plane POV. Planes parallel to POV cut the surface in equilateral hyperbolas and represent pressure-volume relations for a given temperature. Draw the curves cut from the surface by frontal planes taken ¼,½,1 inch, 2, and 3 inches from plane POV.

APPENDIX I

Precision in Drawing

202. Precision in drawing

As stated in the Preface, a graphical solution is not capable of absolute accuracy. The tolerances within which results can be obtained are dependent on a variety of factors, including the scale at which the work is done, the accuracy of the instruments used, the effects of weather on the drawing paper, and the eyesight and care of the draftsman.

Since most engineering data are obtained from instruments which contain scales of one sort or another and which are themselves subject to physical errors, the possible accuracy with which problems using these data can be solved is predetermined before the numerical or the graphical solution is begun. The problem of the draftsman, therefore, becomes one of choosing his scale and his working materials and adjusting his own care to insure that the graphical solution itself is more accurate than the empirical data. If this is done, the graphical solution will be as accurate as a numerical or algebraic solution would be. Therefore, using graphical methods, the draftsman should be acquainted with the tolerances which he may expect from all the factors involved in his drafting. Some of these factors are discussed in the following articles.

When a quantity is to be found by graphical methods, the simplest construction possible should be used. The drawing should be compact, and overlapping views may not be objectionable. Drafting mistakes may be avoided by carefully lettering all points of the construction.

203. Lines

A line, as defined by Euclid, is "that which has length without breadth." Hence, a mathematical line cannot have material existence, and any "line" on a drawing is a rough approximation to a Euclidian line. With reasonable care, however, a pencil line may be made to serve most practical purposes. In geometrical constructions, a line need not exceed 0.005 inch in width. It is possible to reduce the width about half of this amount by use of special materials.

An interesting test is to determine how many separate parallel lines one can draw between two parallel lines a given distance apart. If the lines are drawn as close as possible but clearly separated, it will be found that the distance between the lines will be approximately equal to

the width of the lines. If the widths of the lines and spaces are 0.005 inch, ten lines can be drawn in a space 0.10 inch wide.

The distance between two points can be measured with a probable error as small as the width of the scale mark, which is about 0.005 inch.

The possible error in the position of a point P determined by two intersecting lines depends on the error in the lateral position of one of the lines and the size of the angle of intersection. If E is the amount of lateral error in locating one of the lines and X is the error in the position of the point, then $X = \frac{E}{\sin \theta}$ where θ is the angle between the two lines (Figure 290). When $E = 0.005$ inch and $\theta = 5°$, $X = 0.05$ inch, nearly $\frac{1}{16}$ inch.

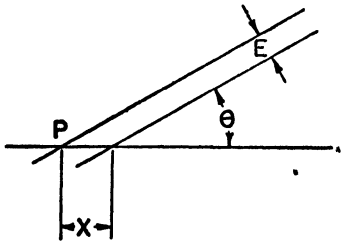


Fig. 290

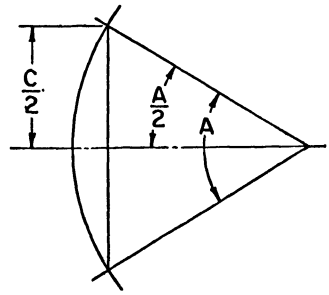


Fig. 291

204. Angles

An angle can be measured or constructed by use of a table of tangents or chords. In both cases, a base of 10 inches is convenient. This results in a probable error of $\frac{1}{10}$ to $\frac{1}{15}$ of one degree. The construction for the chord method is indicated in Figure 291. The chord of an angle is equal to twice the sine of half the angle when the radius is unity. The table of chords (Appendix IV) thus was constructed from a table of sines. Small protractors are unsuitable for precise graphical solutions.

205. Scale

If distances are set off with a needle point, measurements may be read to the nearest 0.01 inch. The needle-point impression must be very small. A measuring scale on which the inch is divided into 50 parts is suitable for this work. Fiftieths can be read without the aid of a magnifying glass and the hundredths estimated very closely.

Measurements should be set off on a line already drawn. This must be a fine line and be longer than the required measurement. Transferring measurements from the scale to the drawing by means of dividers is to be avoided.

Since the accuracy with which a line can be measured is substantially

a fixed quantity, the accuracy of any given solution will be linearly proportional to the scale at which it is drawn. If the scale is doubled, the percentage error will be cut in half. Thus, increase in scale gives increased accuracy up to the limits of the drafting instruments available. The draftsman should choose, in general, a scale which is only slightly greater than the smallest that will yield the desired accuracy and show clearly the method of solution.

206. Shrinkages

All drawing surfaces are subject to change in shape from such factors as change in temperature, humidity, and age. Short solutions which take only a few hours generally are done under uniform conditions and are consequently very little subject to these factors unless the solution is one which may be filed for scaling at some future date. A draftsman working with problems that require high accuracy should be acquainted with the relative percentage of shrinkages of various materials. For very high accuracy, drafting should be done on metal surfaces. Solutions which require considerable time should be worked on under similar atmospheric conditions when this is possible.

APPENDIX II

Properties of Plane Figures

207. Properties of plane figures

Although a study of plane figures is not strictly within the field of descriptive geometry, a knowledge of their properties is desirable since in the solution of a space problem, plane figures are often involved. For example, sections of solids are plane figures, and their correct representation often requires an understanding of their geometric properties.

It may be noted that the projection of any space figure on a plane is a plane figure composed of points and lines between which certain plane relationships exist.

The following discussion is far from exhaustive; in fact, merely a few of the relationships having immediate application in graphical work are presented. It is hoped that an introduction to some of the properties of plane figures, not generally mentioned in courses in elementary geometry, will cause the interested student to investigate this material further in the standard works on projective geometry.

The properties of geometrical figures are of two classes: metric and projective.

208. Metrical properties

Metrical properties are those involving the notions of length, angle, and area; properties of figures that are conserved only under rigid motions of the figure. Thus, the length of a line segment is not altered by changing its position in space. It is changed, however, in its projection on a plane taken at an oblique angle to it. Elementary geometry is, in general, a metrical geometry.

209. Projective properties

Projective properties of a figure are those which remain unchanged in projection. For example, a group of points on a straight line will always project into a group of collinear points. The projections of concurrent lines project into concurrent lines. The properties of collinearity and concurrency are thus preserved in projection.

210. Desargue's theorem

If two triangles lying in a plane are so placed that the straight lines joining the corresponding vertices intersect in a point, then the corresponding sides intersect in three points which lie on a straight line.

Desargue's theorem is illustrated in Figure 292. Point O is the center of projection and line 1-3 the *axis of homology*. The triangles are said to be in *perspective position* when placed as shown. The relationships of Figure 292 can be deduced from a space figure in which the triangles are in different planes (Figure 293). If the triangle ABC of plane Q is projected from point O to plane R, the projection $A'B'C'$ is the perspective of ABC. It is evident that since AB and $A'B'$ are in the same plane,

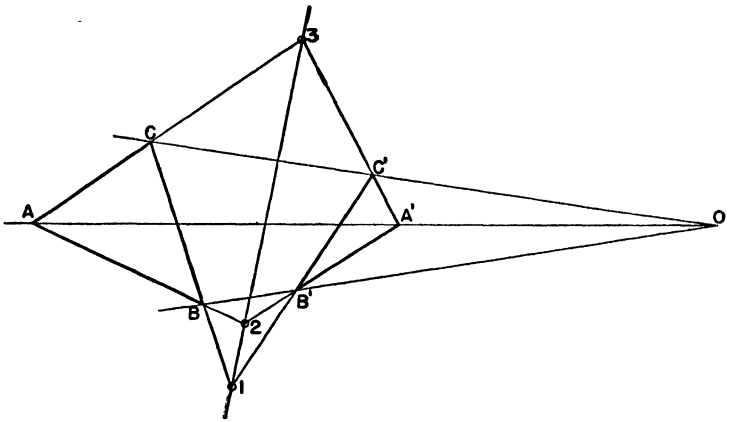


Fig. 292

they will intersect, if produced, in a point 2. Also, since AB lies in plane Q and $A'B'$ in plane R, point 2 will lie on line QR which is the intersection of the two planes which contain the figures ABC and $A'B'C'$. Similarly, the sides AC and $A'C'$, and BC and $B'C'$, will intersect in points 3 and 1 respectively, lying on line QR.

Obviously, the arrangement shown in Figure 293 can be thought of as a pyramid O-ABC intersected by a plane R in the section $A'B'C'$. Line QR is the trace of the section plane on the plane of the base of the pyramid.

If the space configuration of Figure 293 is projected on a plane, the figure which represents Desargue's triangle theorem results, since collinearity and concurrency are preserved in the projection. The vertex of the pyramid becomes the center of projection and the line QR the axis of homology. If the center of projection O (Figure 293) is moved to infinity, the projectors become parallel. The figure ABC-$A'B'C'$, if

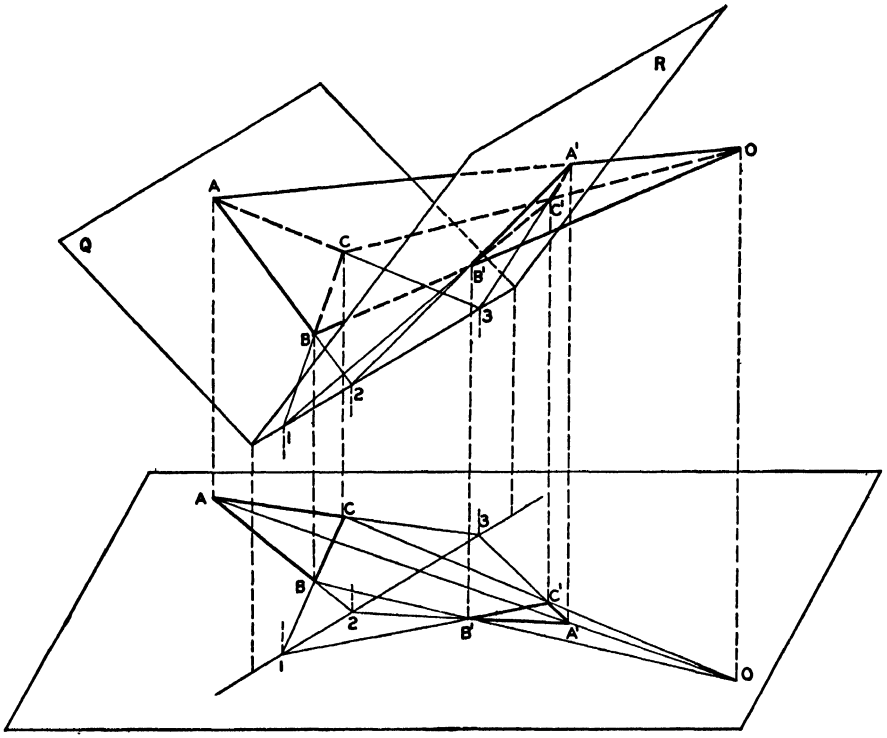


Fig. 293

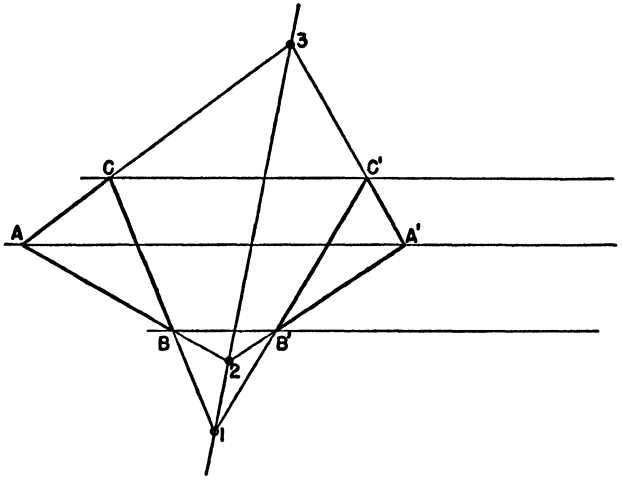


Fig. 294

visualized as a solid, becomes a prism. The triangle theorem now is represented by Figure 294.

Since the H- and V-projections of any triangle ABC (Figure 295) are related projectively, the corresponding sides of each view produced must intersect in three collinear points.

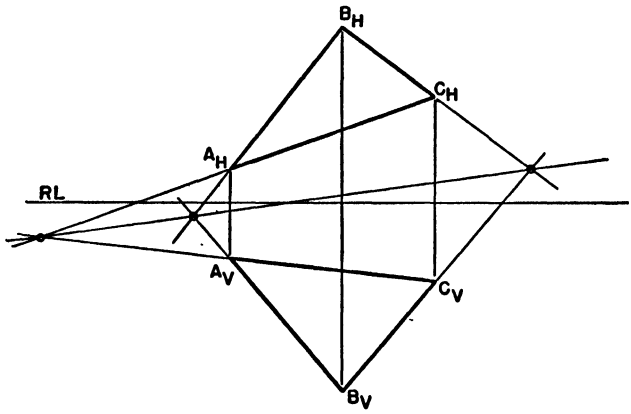


Fig. 295

211. Conics

The most important plane curves are those known as *conics* or *conic sections*. They may be derived from the intersection of a plane with a cone of revolution. The intersection formed by a plane which cuts every element of such a cone is an *ellipse*. In the one situation where the plane is perpendicular to the axis of the cone, the section is a *circle*. The intersection formed by a plane which cuts both nappes of the cone is a *hyperbola*. In the particular case where the plane is parallel to an element of the cone, the section is a *parabola*. Thus, a plane which rotates about a suitably chosen line can cut from a cone one circle, one parabola, and an infinite number of ellipses and hyperbolas.

The conic sections may be defined metrically as follows: The *ellipse* is the locus of points, the sum of whose distances from two fixed points, called the foci, is constant. The *hyperbola* is the locus of points, the difference of whose distances from two fixed points, the foci, is constant. The *parabola* is the locus of points equally distant from a fixed point, the focus, and a fixed line, the directrix.

Theorem. If a cone of revolution is intersected by a plane, and two spheres are inscribed in the cone tangent to the plane, the two points of tangency are the *foci* of the section cut by the plane; the straight lines in which the section plane is intersected by the planes of the circles of contact of the spheres and cone are the *directrices* of the conic section.

The plane Q (Figure 296) cuts an ellipse from the right circular cone. The inscribed spheres are tangent to plane Q at the foci F and F'. The planes R and S are the planes of the circles of contact of the spheres and cone. Planes R and S intersect the section plane Q in the directrices d and d'.

The plane X (Figure 297) intersects the cone in a hyperbola, having foci F and F' and directrices d and d'.

The plane Y (Figure 297) cuts a parabola from the cone. There is but one inscribed sphere tangent to Y and, therefore, but one focus F and one directrix d.

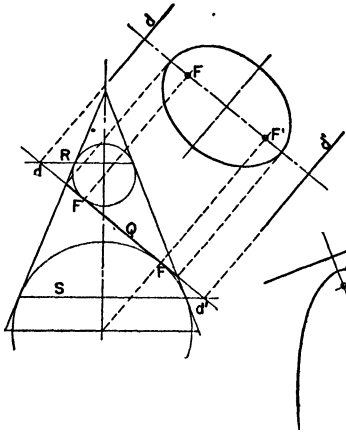


Fig. 296

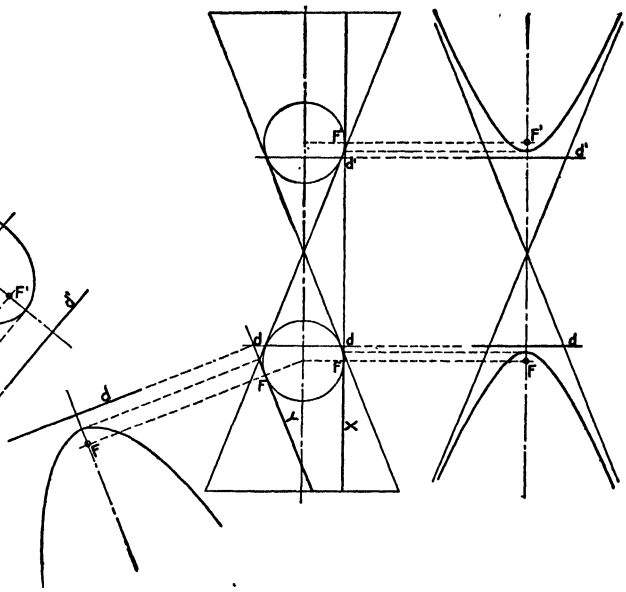


Fig. 297

Degenerate conics. Three limiting or degenerate conics are formed when the cutting plane contains the vertex of the cone.

If in Figure 296 plane Q moves parallel to itself until it passes through the vertex of the cone, the ellipse shrinks to a point.

In Figure 297, if plane X is moved parallel to itself until it passes through the vertex of the cone, the hyperbola degenerates into two intersecting straight lines.

If plane Y (Figure 297) is moved parallel to itself until it contains the vertex of the cone, the parabola degenerates into a straight line.

212. Properties of conics

The following properties of conics are particularly useful in graphical work.

Tangents. A tangent to an ellipse or hyperbola bisects the angle between the two focal radii which intersect in the point of tangency (Figure 298). A tangent to a parabola biscets the angle between the focal radius and diameter which pass through the point of tangency (Figure 299).

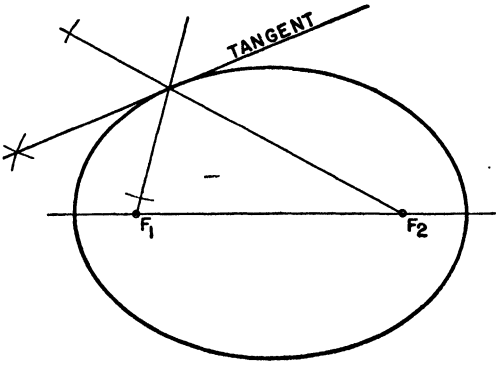


Fig. 298

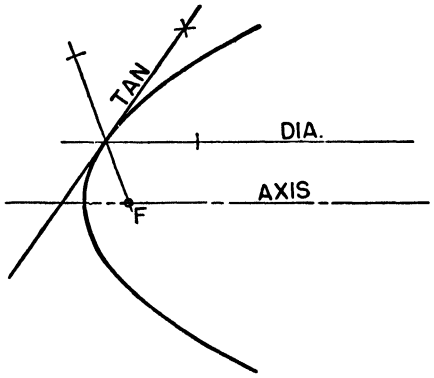


Fig. 299

Diameters. Lines through the center of an ellipse or hyperbola are *diameters*. The diameters of a parabola are at right angles to the directrix. A diameter of a conic bisects a system of parallel chords (Figure 300).

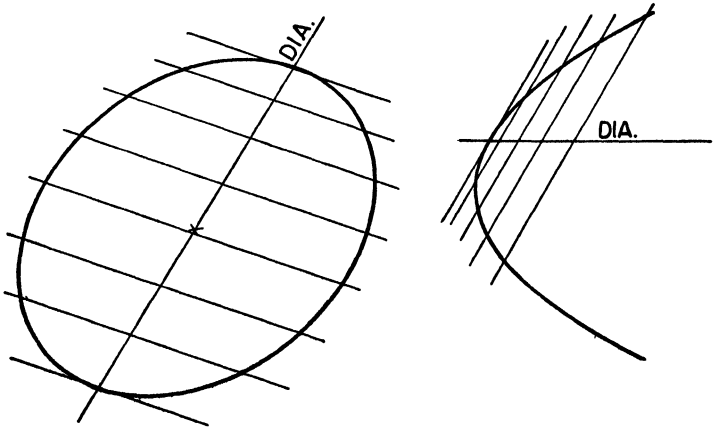


Fig. 300

Conjugate diameters. When each of two diameters of an ellipse is parallel to the tangent at the extremity of the other, the diameters are

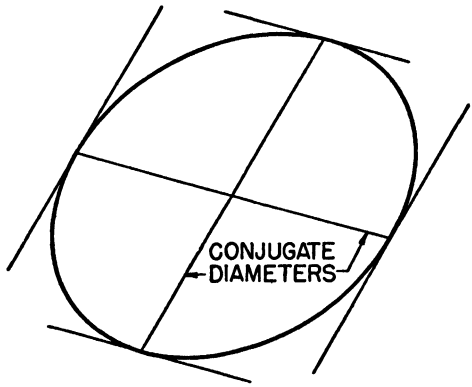


Fig. 301

said to be *conjugate* (Figure 301). Each of a pair of conjugate diameters bisects every chord parallel to the other.

Supplementary chords. Chords from any point of an ellipse to the ends of the transverse axis are called *supplementary*. A pair of supple-

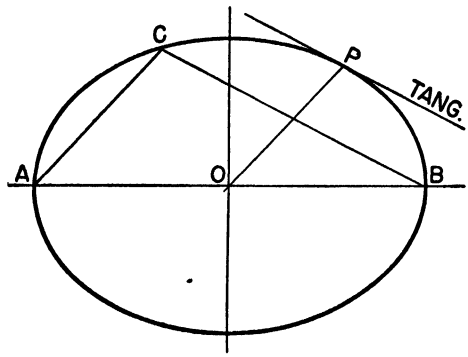


Fig. 302

mentary chords are parallel to a pair of conjugate diameters. This proposition gives a simple construction for a tangent to an ellipse. The tangent at point P is found by drawing the chord AC parallel to PO and

the tangent through P parallel to the supplementary chord BC (Figure 302).

(*a*) *To find axes and foci when the curve only is given.* (*1*) *Ellipse.* Draw a pair of parallel chords. Their bisector is a diameter of the ellipse, and its mid-point is the center of the ellipse (Figure 303). From the center of the ellipse, strike a circle which cuts the ellipse in four points. The axes of the ellipse are drawn through its center and parallel to the sides of the rectangle determined by the four points.

An arc struck off from the end of the minor axis, with radius equal to the semi-major axis, intersects the major axis in the foci.

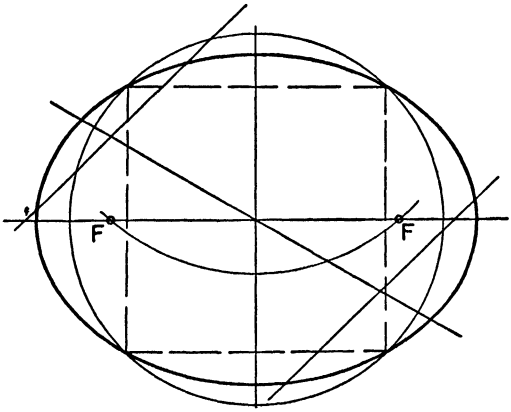


Fig. 303

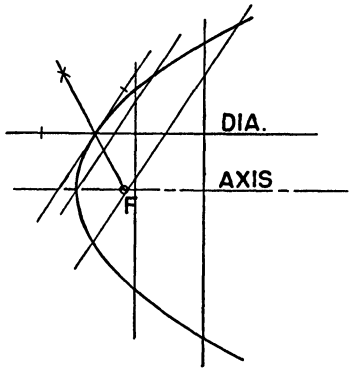


Fig. 304

(*2*) *Parabola* (*Figure 304*). A diameter is obtained by bisecting a pair of parallel chords. The parallel to these chords at the end of the diameter determined by them will be the tangent at this point. The axis is found by bisecting a pair of chords which are perpendicular to the diameter. The focus is determined by the property that the tangent bisects the angle between a diameter and the focal radius through the point of tangency.

(*3*) *Hyperbola.* The axes are found as in the case of the ellipse.

213. Hexagon

A hexagon is a plane figure composed of six points, no four of which are collinear, and the six lines which connect them (Figure 305). Since the six lines which form the sides of the hexagon may be taken in any order, sixty different hexagons can be constructed on six given points.

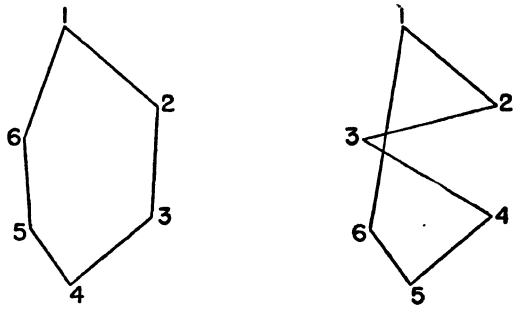


Fig. 305

214. Pascal's theorem

The three pairs of opposite sides of a hexagon inscribed in a conic intersect in three points which lie on a straight line (Figure 306). The straight line is often called *Pascal's Line.*

This construction can readily be checked by the student using a circle as the conic. Any five points determine a particular conic.

Pascal's theorem provides a useful construction for a conic when five points are known. The five given points are taken as five vertices of a

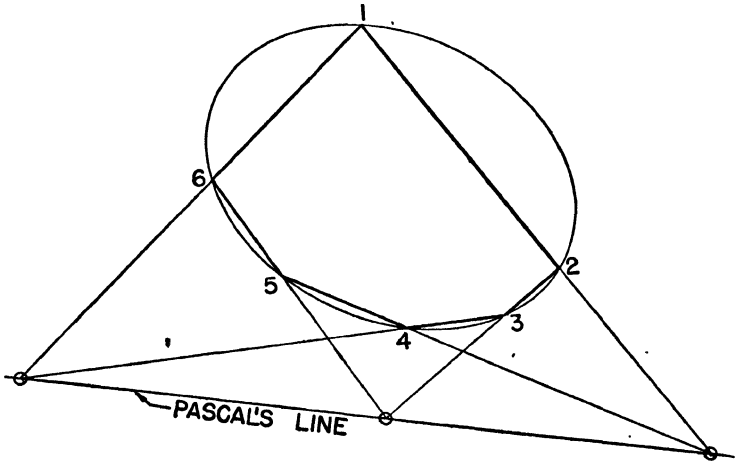


Fig. 306

hexagon inscribed in the conic. The sixth vertex is found by use of the fact that the opposite sides of the hexagon intersect on the Pascal Line. The sixth point lies on the conic.

(*a*) *To construct a conic passing through five given points.*

Given: Points 1-2-3-4-5 (Figure 307).

Draw lines 1-2 and 4-5, producing them to intersect at 0.

Draw lines 2-3 and 3-4 as control axes, labeling them X and Y respectively.

Through point O, draw the random Pascal Line Z, cutting the control axes in points a and b.

Join points 5-a and 1-b. These lines intersect in point 6. Points

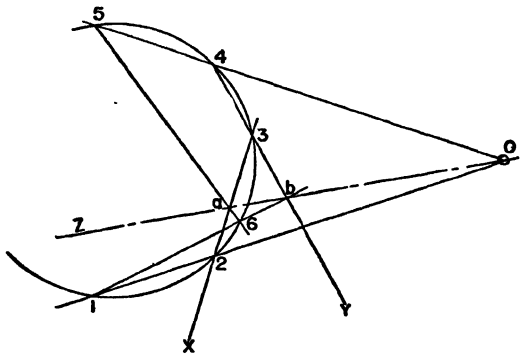


Fig. 307

1-2-3-4-5-6 form an inscribed hexagon, and point 6 is on the conic. This construction is repeated after moving the Pascal Line to a new position.

If point 1 moves along the conic until it coincides with point 2, the chord 1-2 becomes a tangent to the conic at point 2. The conic is now determined by three points and one point-tangent. Similarly, if point 5 moves until it coincides with point 4, chord 4-5 becomes a tangent to the conic at point 4, and the conic is now determined by one point and two point-tangents.

In each case, it is seen that the conic is determined by five elements which may be points or tangents.

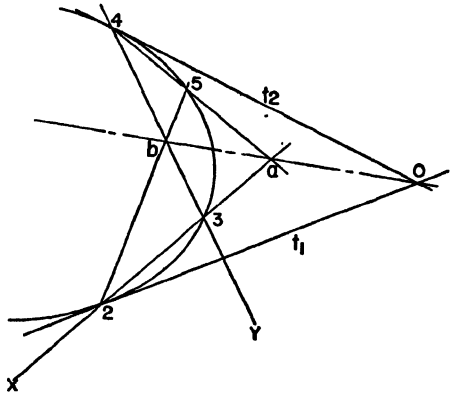


Fig. 308

(*b*) *To construct a conic having given one point and two point-tangents.*

Given: Tangents t_1 and t_2; points of tangency 2 and 4. Point 3 (Figure 308).

Produce tangents t_1 and t_2 to intersect in point O.

Draw control axes X and Y determined by lines 2-3 and 3-4 respectively.

Draw one position of the Pascal Line cutting the control axes in points a and b.

Draw lines 2-*b* and 4-*a* intersecting in point 5, a point on the required conic.

215. Determining the type of conic from a portion of the curve

Draw chord AB of the curve and tangents to the curve at points A and B (Figure 309). The tangents intersect in point O. Determine the mid-point M of AB, and draw OM. If the curve cuts OM in its mid-point D, the curve is a parabola. Any curve cutting OM between M and D is an ellipse. Any curve cutting OM between O and D is a hyperbola.

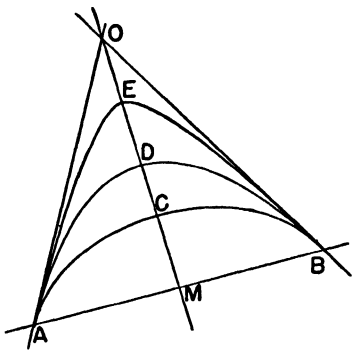


Fig. 309

216. Line conics

A conic determined by the locus of a point is called a *point conic*. A conic also may be considered as the envelope of a continuous set of tangent lines, and when described in this manner, it is known as a *line conic*. A line conic is determined by five tangents, and when these are given, a sixth tangent can be constructed by use of Brianchon's theorem. This theorem on the line conic is analogous to Pascal's theorem on the point conic.

217. Brianchon's theorem

If a hexagon is circumscribed to a conic, the lines joining the opposite pairs of vertices pass through a point P (Figure 310).

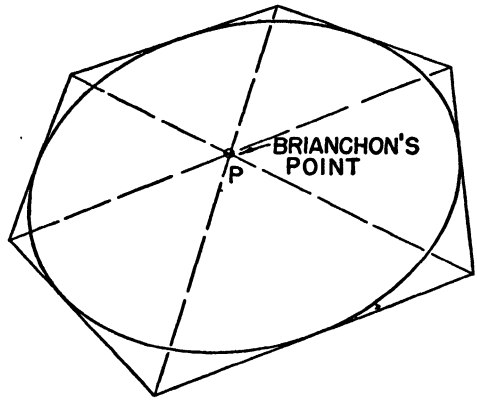


Fig. 310

To construct a conic having given tangents 1-2-3-4-5 *and vertices A-B-C-D (Figure 311).* Join the opposite vertices *A-D*. Through point C draw any line cutting AD in point X and line 1 in point F. A line through B and X cuts tangent 5 in point E. The line determined by points E and F is a sixth tangent to the conic.

Other tangents may be found by altering the position of point X on line AD.

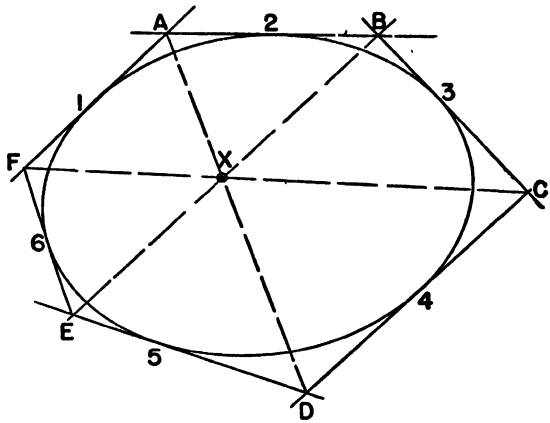


Fig. 311

218. Parabolic envelope

The parabola determined by two point-tangents is constructed easily as a line conic. Let the tangents OA and OB be given together with the points of tangency A and B (Figure 312). Divide OA and OB into the same number of equal parts, numbered as shown. The lines connecting the corresponding numbered points are tangents to the parabola. The distance along tangent 1′-1 to the point of tangency C bears the same

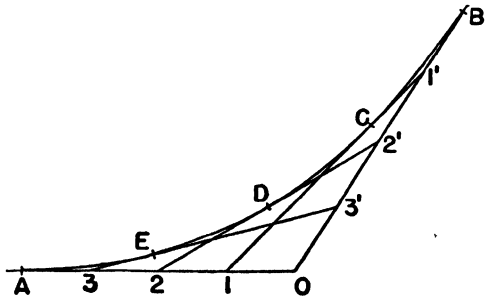


Fig. 312

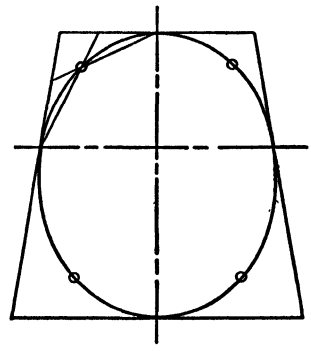


Fig. 313

ratio to 1′-1 as the distance B-1′ bears to B-O. Thus, C-1′ is equal to one fourth of 1′-1. Similarly, the parabola is tangent to line 2-2′ at its mid-point; it is tangent to 3-3′ three fourths the distance from 3′.

The conic constructions which have been described are useful in laying out molds or templets for structures requiring fair or smooth curves. For example, the fuselage cross section of Figure 313 is made up of conics. Four tangents forming a trapezoid enclose the cross section. The conic which forms each quarter is determined by two point-tangents and a control point.

APPENDIX III

Stereoscopic Drawing

219. State of the art

The following discussion of stereoscopic drawing is by no means complete. Many problems both of theory and practice have not yet been solved adequately. Certainly better methods than those developed here are possible. Furthermore, many practical aspects of the subject can be treated only sketchily. However, since no existing textbook contains anything more than a mention of the possibility of making such drawings, it is felt that a chapter setting forth the present state of the art as practiced by the authors should be included in this volume.

220. Basic problem

When a scene is viewed through a window by a single eye, the window may be considered as a picture plane and the eye as a station point. If the scene were replaced by its perspective projection on the window, the projection would produce the same retinal image as the scene produced. If the projection were perfect, the eye would be unable to detect any change. If, however, this eye were closed and the other eye opened, the scene would yield a new projection on the window and a new retinal image which would differ from the first.

In normal binocular vision, the differences between the two retinal images of the space scene are interpreted by the mind in terms of depth, the observer being aware of only one image in three dimensions. Since our two perspective projections on the window produce the same retinal images as the scene, the mind will interpret them as a single three-dimensional image provided that each eye is limited to seeing only the projection intended for it.

The problem of stereoscopic presentation thus becomes that of presenting the two proper perspective projections in such a manner that each eye sees only the projection intended for it. This may be accomplished by a number of means, notably with a stereoscope, by the anaglyphic process using red and green dies and filters, or by the use of polarized light.

221. Stereoscopic drawing defined

Stereoscopic drawing is the process of drawing the two proper perspective projections and orienting them correctly with respect to each other

for viewing by one of the above means. The process differs from single perspective projection in the added factor of maintaining the proper relationship between the two views.

222. Limitations of single perspective

Consider the single perspective projection of a sphere (Figure 314). Any point in this projection, such as P, might represent any one of the infinite number of points P', P'', and so forth on the line EP from the eye through P. The projection as a whole might represent any one of an

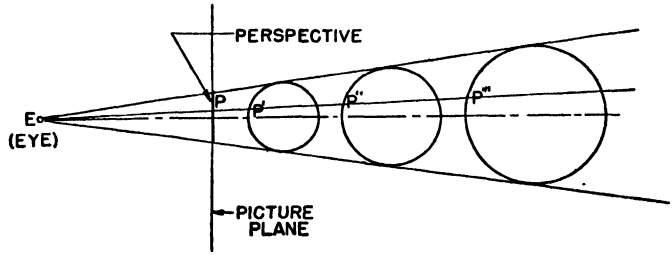


Fig. 314

infinite number of spheres each of which would produce the same projection. In fact, the same projection could be produced by non-spherical shapes, either concave or convex. A single perspective projection thus is fixed in direction only, not in size or shape.

223. Stereoscopic image fixed

A second perspective projection viewed by the other eye might also be any one of an infinite number of spheres. However, since the two

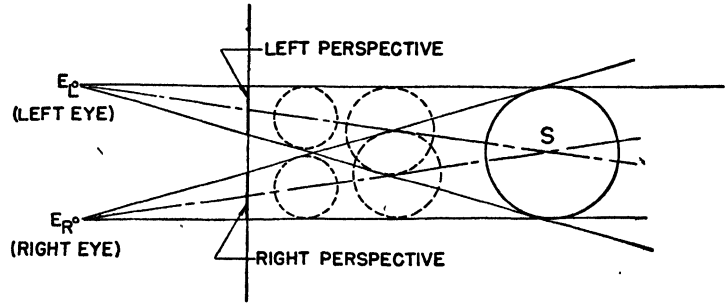


Fig. 315

rays from the two eyes through the two projections of any point can meet only at one point in space, the two projections placed as shown in Figure 315, and each viewed by the proper eye only, can represent together only the one sphere S. No other size, shape, or location is possible. Thus, a stereoscopic image is fixed in size, shape, and location.

224. Terminology

In discussing the process of obtaining a stereoscopic image, the following terminology will be used. An original *object* in space projects to two station points yielding two projections on a picture plane. These projections will be called a *projection pair*. When the *projection pair* is viewed properly by the two eyes, they perceive a stereoscopic *image* in space. Thus, we will speak of an *object* or an *object point* which yields a *projection pair*. Properly viewing the *projection pair* yields an *image* or an *image point*. A *stereograph* is a projection pair obtained from an *object* and yielding an *image* which may or may not be congruent to the object.

225. Functions of projection pairs

Consider an object point P in space (Figure 316), two station points E_L and E_R representing the position of the two eyes, and a picture plane

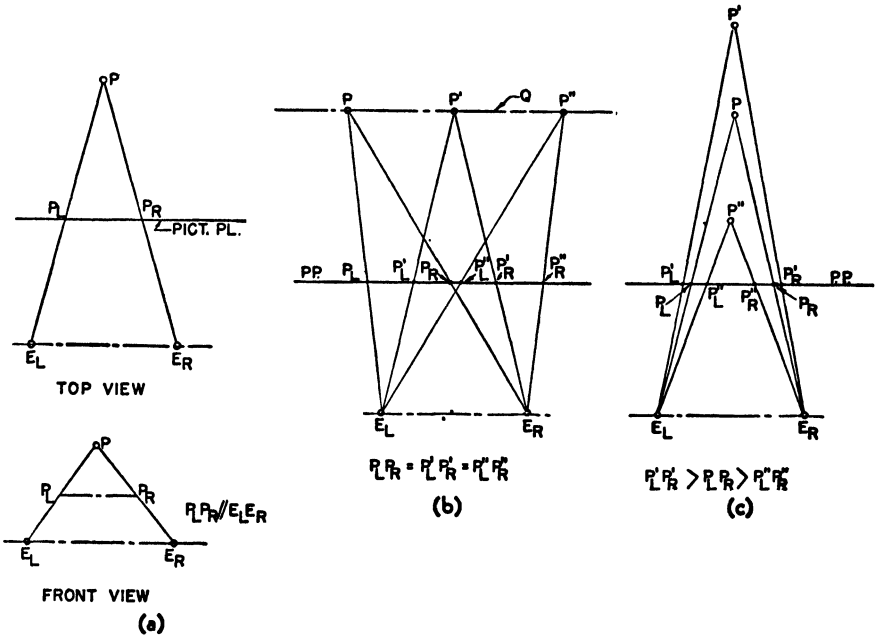


Fig. 316

taken parallel to the line E_LE_R. P yields the projection pair P_L and P_R. Consider the line P_LP_R as P is moved in space. The following facts appear:

1. P_LP_R is always parallel to E_LE_R (Figure 316*a*); that is, the lines joining all projection pairs in a stereograph must be parallel to each other

and to the line between the eyes, which we shall assume always to be horizontal.

2. If we pass a plane Q through P, parallel to the picture plane (Figure 316*b*), every point in this plane will have a projection pair separated by a distance equal to P_LP_R; conversely, all projection pairs of equal separation yield images which lie in a single plane parallel to the picture plane. In particular, the images of all superimposed projection pairs lie in the picture plane.

3. If the point P is moved away from the station points, the distance P_LP_R will increase and yield an image point farther from the eyes and vice versa (Figure 316*c*). Thus, the depth of a point in a stereoscopic image is a function of the separation between its projection pair.

226. Factors affecting image shape

Consider an object line AB perpendicular to the picture plane, and which would pass through E_L if extended (Figure 317). Its projections

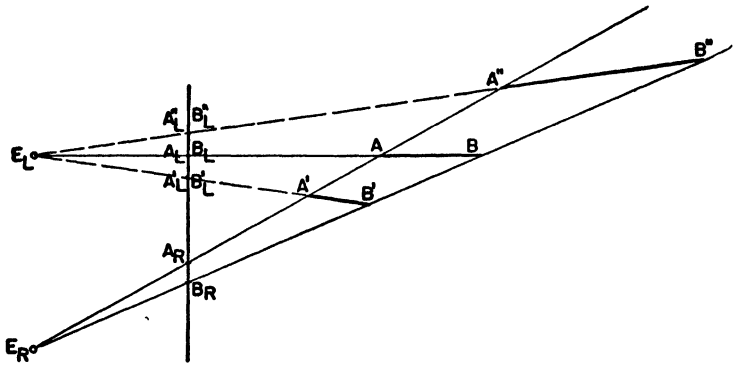


Fig. 317

are the point A_LB_L and the line A_RB_R. When viewed with the eyes at E_L and E_R, the image AB is coincident with the object AB. Now, consider the separation between the two projections to be altered. If the left projection is moved closer to the right $A'_LB'_L$, the stereoscopic image of the line AB will change in direction and also become shorter, $A'B'$; if the left is moved away from the right $A''_LB''_L$, AB will change in direction and become longer, $A''B''$. In both instances, the resultant image changes in direction and in length.

On the other hand, the image of a projection pair of a horizontal line parallel to the picture plane, treated in the same manner, will remain parallel to itself since its end points always have the same separation of projection pairs. It will change in length and location but not in direction. Thus, if we have a projection pair which yields an image

square (Figure 318), and if we alter the separation between the pair, the image changes in size, shape, and location. Though this is a simplified case, it is always true that the shape, size, and location of a stereoscopic image is a function of the separation of the two projections.

It can similarly be shown that size, shape, and location are functions of the viewing distance. If the eyes move closer to the picture plane, the image becomes shallower and nearer; if they move farther away, the image becomes deeper and farther. The same phenomenon occurs if the projections are magnified or reduced (the projection pair of the

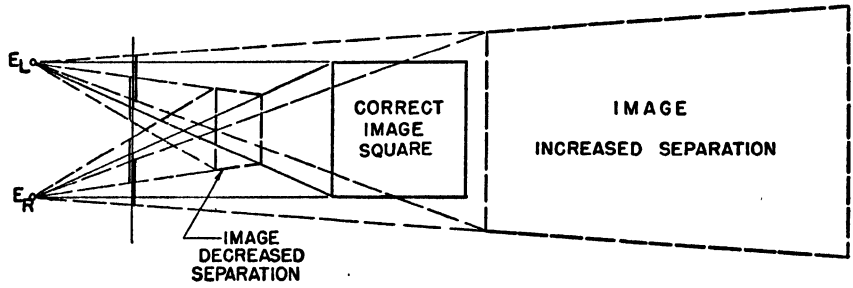


Fig. 318

same point being superimposed), the image increasing or decreasing in its relative depth to width and height as it is magnified or reduced, respectively.

227. Relation of picture plane to projection pair

Assume any object in space and two station points fixed in position with respect to it. If a picture plane is inserted parallel to the line between the station points and moved to remain parallel, the two projections on the picture plane will remain exactly the same shape but will alter in size and separation.

If the eyes replace the station points and view the projections obtained on the picture plane at any position it may have assumed, the resultant image will coincide with the object.

Thus, theoretically, a given projection pair may be shown at any size provided the proper viewing distance and separation for that size are used. Actually, this cannot be done with complete freedom because of problems of eyestrain.

228. Locating the picture plane

In normal vision, the eyes always focus and converge for the same distance. In viewing a stereograph, they focus for the distance to the screen, or to infinity if a lens stereoscope is used, while their convergence

varies over the depth of the image. This is not a natural condition. Unless a lens stereoscope is used, eyestrain increases as the difference in distance between focus and convergence increases. This difference in distance is held to a minimum if we place the picture plane through the center of the object. As we bring the picture plane to the front of the object, the focal distance becomes increasingly shorter than the convergence distance; and as we move the picture plane to the rear of the object, the convergence distance becomes shorter than the focal distance. Both of these lead to eyestrain and, if carried far enough, to inability to fuse the two projections.

The standard practice in stereoscopic photography is to place the front point of the object, and consequently of the image, in the plane of the screen. This leads to the most satisfactory results and should be approached where possible. Thus, in the discussion that follows, we will fix the relation of our image to the screen at the outset by establishing a point in the object, generally the fore point, whose projection pair is to be superimposed in viewing.

229. Image size and viewing distance

Thus, in making a stereoscopic drawing, we choose in advance the size of the final stereograph either in a stereoscope or on the screen upon which it is to be projected and the distance at which we desire to view it, remembering that reasonable changes in magnification still may yield the same image if the separation and viewing distance are altered properly.

In actual practice, considerable latitude is permissible. Drawings for a 6-foot screen viewed from 12 feet can be handled on screens from 4 to 8 feet. If the final stereograph is of a form in which the separation cannot be altered to adjust to the magnification—this is true of the vectograph—the distortions introduced will be acceptable within the above limits and for viewing distances commensurate with any room in which such a screen is generally used. However, a drawing to be made into a 6-inch vectographic print or to be viewed in a stereoscope cannot in general be successfully projected.

In developing the theory of stereoscopic drawing, we shall assume that we are attempting to obtain a properly shaped and located image.

230. Viewing conditions established

Before starting a stereoscopic drawing, we establish:

1. The final size or sizes of the stereograph. By this is meant the size of the actual projection to be viewed. Thus, if projection onto a screen is contemplated, we mean the size on the screen, not on the lantern slide. If more than one size is contemplated, one for a stereoscope and

one for a screen, this can be taken care of by the use of an additional station point without any other change. See Article 242.

2. The plane of the points whose projections are to be superimposed, that is, the plane in the resultant image which we wish to coincide with the screen or the stereograph. All points in front of the plane of superposition will have their projections crossed over and will appear in front of the screen in the final image. All points behind this will have their projections separated positively and will appear behind the screen.

3. The desired viewing distance. In general, this should be two to three times the final width of the stereograph. When a lens stereoscope is used, it should be the focal length of the stereoscope.

231. Drawing size

We are now ready to decide on a size for the original drawing. Problems of accuracy, to be discussed later, dictate that all stereoscopic drawings should be made so that the original projections on the drawing board are from 12 to 24 inches wide. For viewing in small stereoscopes, this size may be reduced as requirements of accuracy are not as great. The ratio of the size of the drawing to the final size of the stereograph is the *scale* of the drawing.

We construct a plan and elevation at the chosen size and with a chosen orientation, placing the picture plane where convenient, usually to coincide with the plane of superposition. Next, we lay out the viewing distance perpendicular to the picture plane on a line through the center of the object. The drawing interocular is then laid out parallel to the picture plane, one-half on each side of the center line.

The distance between the human eyes, called the *interocular*, is fixed by nature and averages about $2\frac{1}{2}$ inches. Consequently the drawing interocular, or distance between station points in our original drawing, must be $2\frac{1}{2}$ inches times the scale so that when the stereograph becomes its final size, the interocular will become $2\frac{1}{2}$ inches.

232. Location of horizon

In viewing stereographs, particularly in stereoscopes, the line of sight to the center of the image is almost invariably substantially perpendicular to the picture plane. Thus, our horizon, as understood in ordinary perspective, always should be located through the vertical center of the object. Because of the increased reality of stereographic images, any departure from this may introduce annoying distortions.

For instance, if we desire a perspective of a house, looking from slightly above, the normal single perspective practice is to raise the horizon above the house so that the resultant perspective should be viewed at an angle to its plane. In making a stereograph, this practice should

be avoided. This, in general, means that our plan and elevation are not the normal ones but, substantially, auxiliary views parallel to our picture plane.

233. Example

To clarify the above discussion, let us consider an example. We have the plan and elevation of a house. We wish to view this from 30° to the right of the front and from 30° above the horizontal. We wish to construct the stereographs for a 6-foot screen viewed from 12 feet. The near point of the house is to be in the plane of the screen. We are going to construct the house having a width of 18 inches on the drawing board.

234. Necessary orthographic views

Construct the normal front and top views (Figure 319), inserting an arrow to indicate the desired direction of sight. Next obtain an auxiliary view showing the arrow in true length. This is the required side view for projection. Insert the picture plane through the front corner perpendicular to the arrow. Obtain a top view looking parallel to the picture plane.

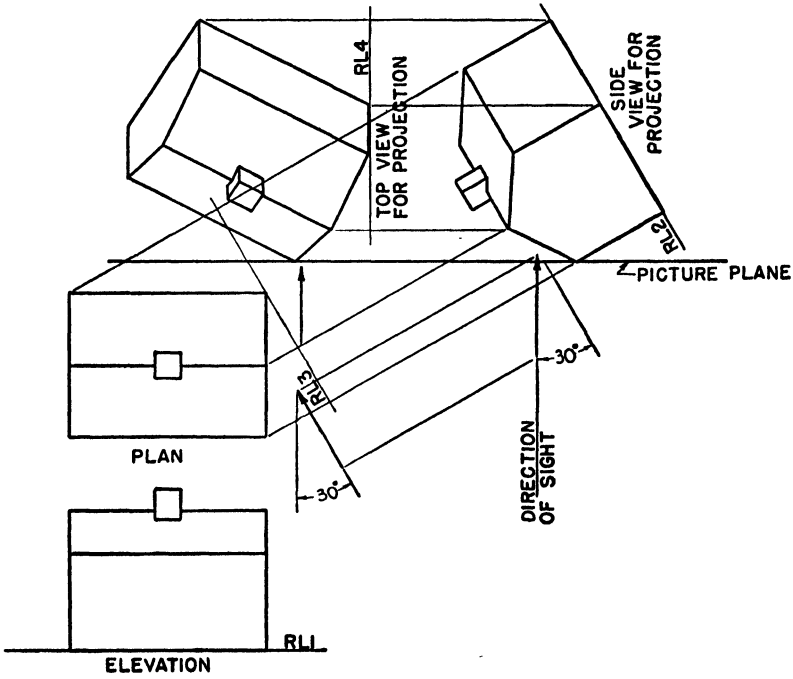


Fig. 319

235. Determining scale

Orient these two views on the drawing board (Figure 320). The width of the house taken parallel to the picture plane is 22 inches. 22 inches is 0.3 of 6 feet, the screen width. Thus, 0.3 is our scale. This establishes a viewing distance of 144 inches $\times$ 0.3 or 43.2 inches and a drawing interocular of 2.5 inches $\times$ 0.3 or $\frac{3}{4}$ inch.

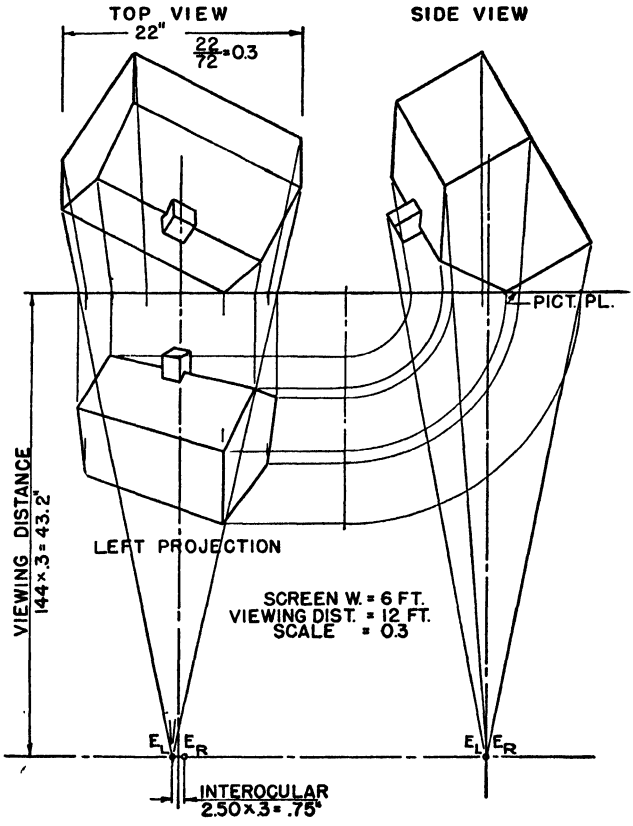


Fig. 320

Draw lines through the center of the object in the two views perpendicular to the picture plane. On these lines, mark points 43.2 inches from the picture plane. This point in the elevation represents both station points. In the plan view, two points $\frac{3}{4}$ inch apart centered on this point are the two station points E_L and E_R.

236. Obtaining first perspective

We are now ready to develop the perspective from one of these station points E_L.

Any method of perspective may be used. However, at the size at which we are working, vanishing points tend to become inaccessible. We shall proceed here without their use. Where easily available, they should be used in obtaining the first view.

Figure 320 illustrates one simple method of obtaining the perspective. Obtain the lateral positions of the points by projection onto the picture plane in the top view and the elevations by projecting in the side view. Rotate the elevations through 90°, and pair the points. It is convenient to label all points and project them on a strip of paper in both views. The two strips thus may be oriented at right angles to each other and the points paired.

237. Method for second perspective

Whatever method is used to obtain the first perspective, it is inadvisable to obtain the second perspective in the same way. It is not possible to draw a line to much closer on the average than 0.005 inch. If an error of any given amount is made in both perspectives in the same direction, the error in the image will not be a depth error and will be of the order of the error in the perspective. However, if an error is made in the separation of the projection pair of any point, a large depth error results. In the example with which we are working, a separation error of 0.005 inch will result in a depth error of about 0.8 inch, a ratio of 160 to 1. If both perspectives are developed independently, many errors considerably larger than this will inevitably occur. If, however, the second perspective is developed from the first by direct measurement of the proper separation of projection pairs, errors in depth will be reduced to a minimum.

238. Curve of separation

This is best accomplished by means of a *curve of separation.* We have seen that every point at the same depth has the same separation of its projection pair. We can determine the proper separation for various depths by projecting points at these depths to both station points.

Figure 321 illustrates the method of obtaining a *curve of separation.* Draw a line XY perpendicular to the picture plane through one station point E_L in the top view, and divide the line over the depth of the object into equal parts Y, 1, 2 . . . X. Project each division point to the second station point yielding Y, 1_P . . . X_P. Scale the separation of each projection pair at the picture plane. For point 1, this distance is $Y1_P$; for point 2, $Y2_P$; and so forth. Set these distances off from their respective points on XY parallel to the picture plane obtaining 1-1′, 2-2′, and so forth. Fair a smooth curve through the points so obtained: $Y1'$, 2′, 3′, and so forth. This is the *curve of separation.* The separation

of the projection pair of any point will be the distance, taken parallel to the picture plane, between XY and the curve of separation at the depth of the point. Except for very deep objects or for objects which extend considerably forward from the plane of superposition, the curve is substantially a straight line. If it is drawn as a straight line, small errors in depth will occur in the image which will in general not be detectable.

In order that the two projections shall not overlap, draw a line ST parallel to XY away from the curve of separation at a distance somewhat greater than the full width of the first perspective. Through

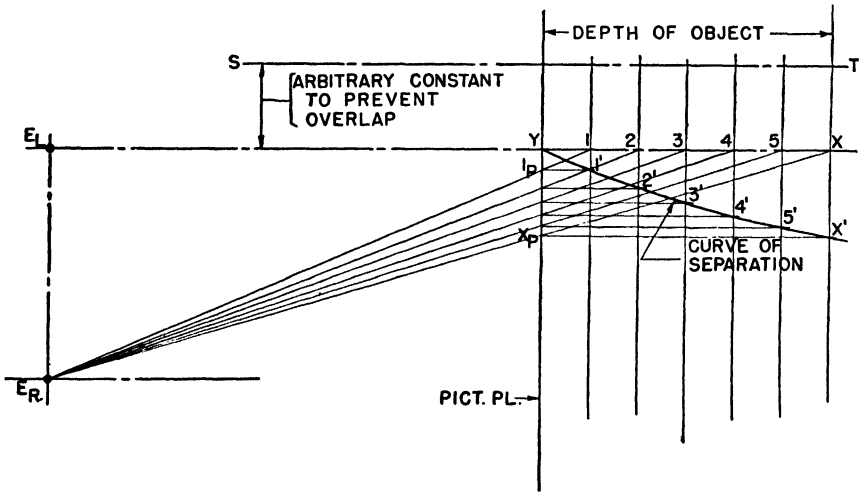


Fig. 321

points on the top view, draw lines parallel to the picture plane. The segment of this line through each point between ST and the curve of separation now becomes the distance between the projection pair of this point.

239. Developing second perspective

Through points on the first perspective, draw lines parallel to the picture plane (Figure 322). Measure very accurately with a beam compass the distance between projection pairs, that is, between ST and the curve of separation. For point C, this is $C'C''$. Set these off along the lines from the corresponding points in the first perspective. This determines the second perspective.

It will be noted that by this method no large depth error, particularly no reversal in depth between two points, can occur so long as the curve of separation is smoothly faired and the measurements are carefully made.

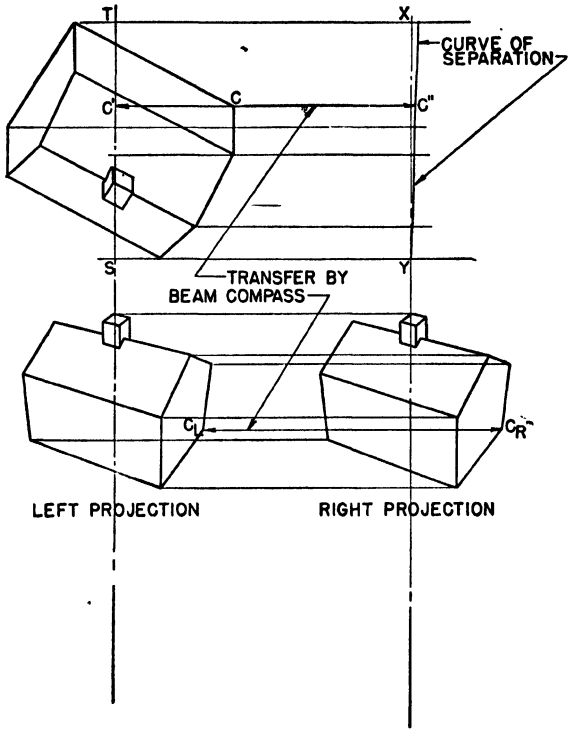


Fig. 322

240. Correcting errors

We have now completed our projection pair except for the correction of any errors which may have occurred. Noticeable errors may be corrected by direct observation of the stereoscopic image. This may be done best under a mirror stereoscope large enough to handle the two original projections. Though the image as a whole will be distorted, the relative depths of points with respect to each other will not be altered. Points that appear too deep may be brought forward by reducing the distance between their projection pairs until they appear correctly. Such corrections involve very small shifts of lines and points, and must be done with extreme care. Considerable practice is necessary to acquire the ability to correct a stereoscopic pair visually.

241. Registration

The projections now may be prepared in the desired form for projection. Careful registry is necessary. The following should be observed:

1. Superposition of the proper point. In our example, the forward upper corner of the house.

2. The pair must be vertically oriented with respect to each other. Errors due to faulty vertical registry are discussed in Article 246.

3. The pair must not be rotated with respect to each other. If the tops are rotated inward and the bottoms outward, the top of the image will come forward, while the bottom will recede. A rotation of 2° or 3° may cause a bad leaning effect.

242. Multiple sizes

If it is desired to have more than one size of stereograph, this can be accomplished by the introduction of one more station point leading to a second curve of separation and a third projection.

The only fixed factor is the human interocular of $2\frac{1}{2}$ inches. In our example, the following conditions existed:

	Drawing	*Final*
Size	22 in.	6 ft.
Viewing Distance	43.2 in.	12 ft.
Interocular	0.75 in.	2.5 in.

Suppose we now introduce a third station point E'_R such that $E_L E'_R$ is $2\frac{1}{4}$ inches. We now have three scales:

$$E_L E_R = \frac{0.75}{2.50} = 0.3;$$

$$E_R E'_R = \frac{1.50}{2.50} = 0.6;$$

$$E_L E'_R = \frac{2.25}{2.50} = 0.9.$$

Since our drawing sizes are already determined, the two new scales determine our new final conditions. We thus have three possible stereoscopic pairs for the following conditions:

Pair	*Final Size*	*Viewing Distance*
$E_L E_R$	6 ft.	12 ft.
$E_R E'_R$	3 ft.	6 ft.
$E_L E'_R$	2 ft.	4 ft.

It can be seen readily that the third station point may be located to yield any desired final size or any desired viewing distance, but the ratio between the two must remain 1 to 2.

Three station points will yield three pairs which will fulfill most requirements. It is usually advisable to choose the three station points to yield one pair for projection and one pair for hand-held vectographs or small stereoscopes.

243. Sources of error

Many sources of error are due to the fact that in a stereoscopic drawing all the shading, color, and minute details of normal vision

are lacking. Therefore, the eye lacks a continuous field of discreet points which automatically are paired properly by the brain so that perfect fusion and interpretation result. The following errors are the most important ones generally encountered.

244. Drafting error

Consider the projection pair of any random frontal line inclined at an angle α with the horizontal (Figure 323). Point A_L should be paired with point A_R. Assume an error of Δ perpendicular to the line in locating one view of the line.

If the lines are perfectly smooth, the eyes now will assume point A_L to be paired with B'_R. The error in horizontal separation is therefore $A_R B'_R$ or Δ cosec α, and the error in depth will be many times this amount.

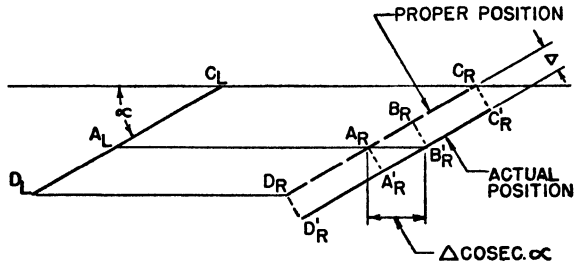


Fig. 323

This error will be present except at the ends of the line, C and D, where the eyes are forced to adjust for the vertical error and to fuse C_L with C'_R. The error is, thus, considerably less at the ends. Since the eyes at any instant can see sharply only a very small segment of the line, they scan the line. As they do this, the line will appear deeper along its length than at its ends. This effect can partly be overcome by putting discrete points on the line which the eyes will be forced to pair properly.

245. Horizontal lines

Note that the depth error for any given drafting error is minimum for vertical lines and approaches infinity as the line approaches the horizontal. In fact, a perfectly smooth horizontal line lies at an indeterminate depth as the two eyes have no information concerning the proper points to pair except at the ends of the line. This is a well-known binocular phenomenon common to everyday vision.

When a horizontal line is present, the tendency of the eyes is to place it at the same depth as any line it may cross. It is, therefore, advisable in orienting objects for stereoscopic drawings to avoid horizontal lines and lines approaching the horizontal as much as possible. Where

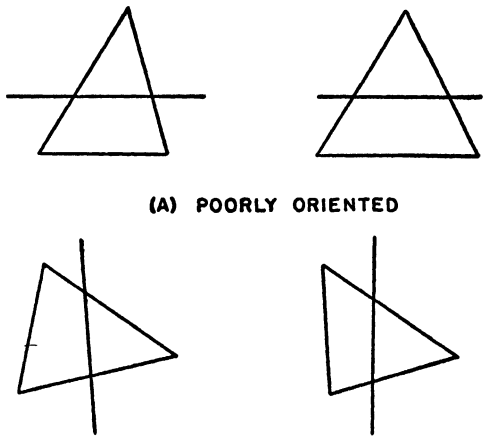


Fig. 324

this is not possible, short lines are preferable to long ones. Discrete points may be added where necessary. The drawing shown in Figure 324 should be oriented as in (*b*) rather than as in (*a*).

246. Vertical errors

Even though a stereoscopic pair is perfectly drawn, a vertical error in the registration of the two projections will introduce errors in image shape.

In the case of inclined lines, this error is exactly similar to that introduced by a drafting error when drawing a line. In Figure 325, a

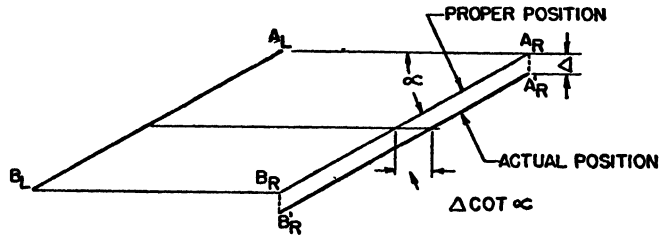


Fig. 325

vertical error of Δ yields a separation error Δ cot α where α is the inclination of the line to the horizontal.

This error becomes particularly annoying when dealing with curved surfaces. The two halves of the projection pair of a circle are ellipses (Figure 326). In the circle shown, A is the near point and B the far point. As the eye scans from A through C or D to B, the separation continuously

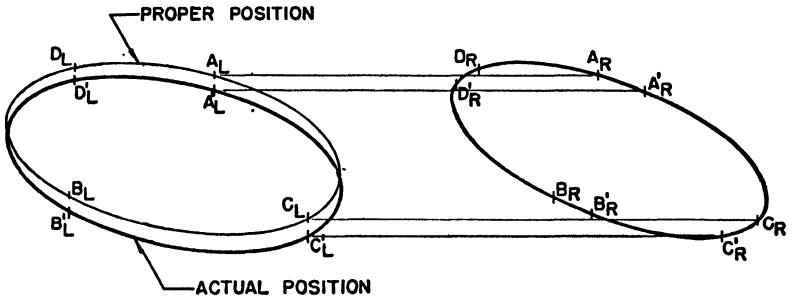


Fig. 326

increases, and the depth of the image, consequently, continuously increases.

Suppose the left projection were slightly lowered. Points A'_L, B'_L, C'_L, and D'_L will now fuse with A'_R, B'_R, C'_R, and D'_R. Note that the separation for C and D has decreased to bring the images of C and D forward while that of A and B has increased putting the images of A and B back. Thus, the entire circle becomes distorted; portions may become even completely reversed in depth. This may be observed quite easily in a stereoscope by drawing such a circle and moving one projection slowly up and down. The phenomenon can be controlled best by accurate registry and by introducing as many discrete points as possible, forcing the eyes to adjust for the vertical error throughout the entire figure. It will be noted that in such a circle, a portion must be inevitably very closely horizontal and, thus, very difficult to control in depth.

247. Edge view planes

No plane in a stereoscopic drawing should intersect the line segment between the station points. If the drawing is of a transparent object,

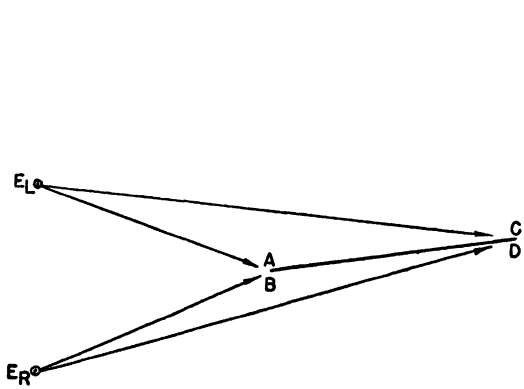


Fig. 327a

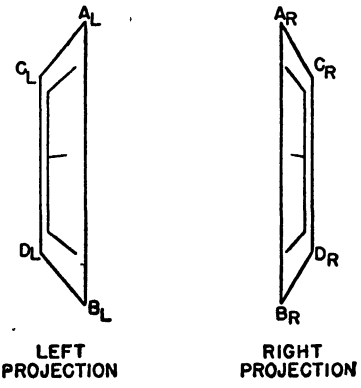


Fig. 327b

one side of the plane will be visible to one eye and the other side to the other (Figure 327*a*). The order of vertical lines in the projection pair (Figure 327*b*), is now reversed. This leads to confusion in interpretation as the eyes, in the absence of the minutia of color and shading, do not know which projections to pair. This is, of course, a condition possible to create in normal binocular vision where it leads to the same confusion.

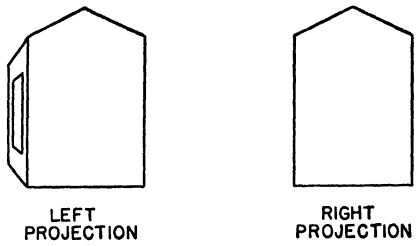


Fig. 328

If the object is opaque, details in the plane may be visible to one eye and not to the other. For instance, Figure 328 shows a projection pair of a house in which the plane of one side intersects the line segment between station points. The window in this plane is visible only to the left eye. It, therefore, has no fixed depth and will float in space. In fact, individuals with a master right eye may fail to see it entirely.

248. Apparent intersections

The tendency for crossing lines to appear as actually intersecting in space is not eliminated fully by the avoidance of horizontal lines. This is particularly true if the drawings are inked drawings in which the lines necessarily must have some width. The fusion of the ink at the point of apparent intersection eliminates the edges of the forward line. Such

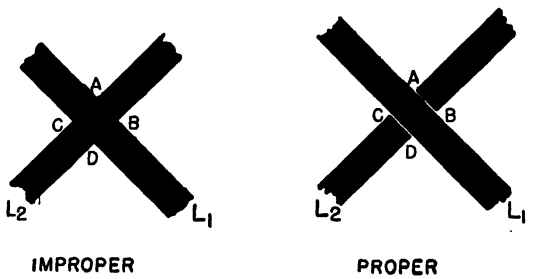


Fig. 329

edges always exist in space vision. Thus, if the eyes concentrate their attention at this point, the true separation of the lines is likely to be lost. Figure 329*a* represents a magnified intersection of lines as inked on a stereograph. Suppose L_1 is the forward line in space. Its edges, AB and CD, would be visible in space, but they do not appear on the stereograph. Small as this fault appears, it causes a very annoying confusion

in viewing the stereograph, particularly if one of the lines approaches the horizontal.

This confusion can be eliminated best by using a razor blade to make breaks in L_2, the rear line where it passes behind L_1, as shown in Figure 329*b*. Every intersection which is not a true intersection in space should be treated in this manner on the finished stereograph.

249. Curved surfaces

In dealing with plane surfaces, if two or more lines are shown in any plane, the psychological tendency is to read the plane into the picture. Thus, in Figure 330, only three lines are shown in each face of the pyramid, yet the mind interprets them as being plane faces, not as merely triangles of wire.

On a curved surface without shading, the mind does not readily

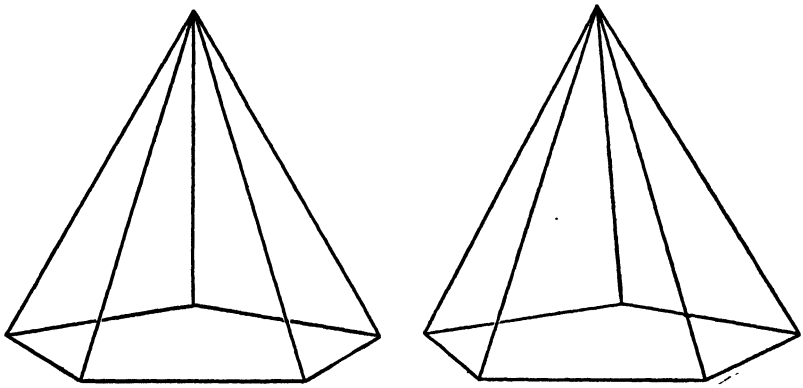


Fig. 330

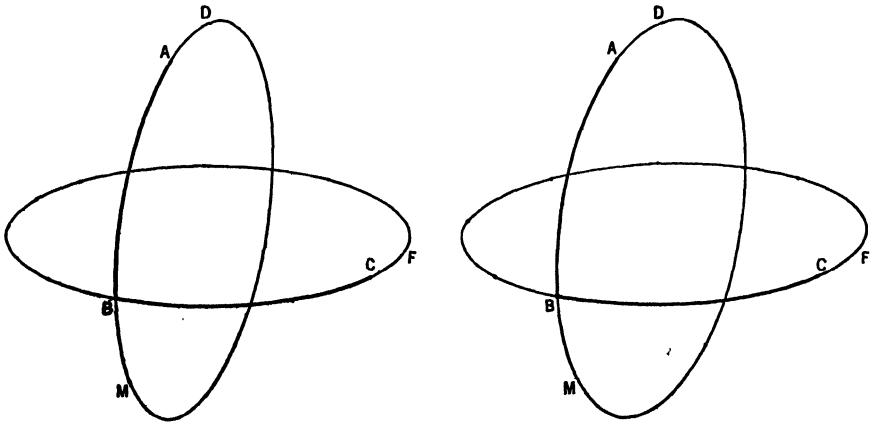


Fig. 331

interpolate the proper surfaces into the empty spaces. Thus, the sphere shown in Figure 331 would not be interpreted as a sphere but as two circular loops of wire with open spaces between. Consequently, in the absence of shading, enough lines must be shown to permit the mind to fill in the intervening surfaces.

Closed surfaces of uniform curvature, either single or double, such as cylinders or spheres, should be drawn with a minimum of four properly chosen loops. Figure 332 contains six loops.

With surfaces of changing curvature, the number of lines required depends upon the complexity of the surface.

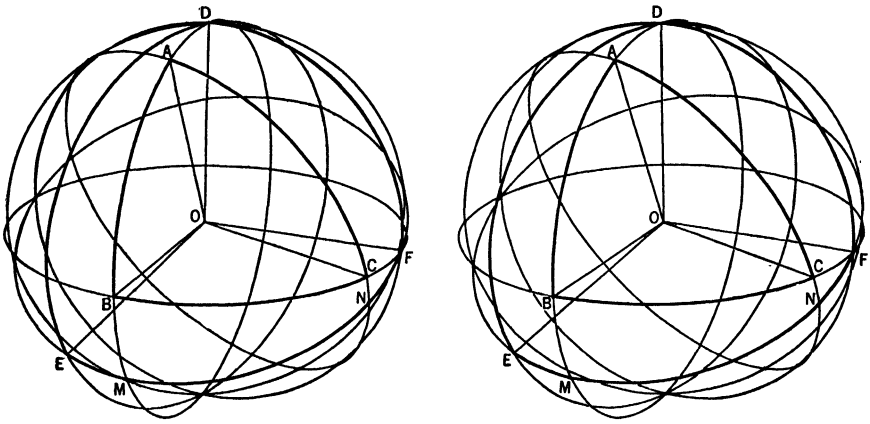


Fig. 332

250. Visibility

One of the great values of a stereoscopic drawing is that simple geometrical figures may be treated as though they were transparent. In such a case the problem of visibility does not arise.

The invisible lines on an opaque object can be shown only by some arbitrary convention. The usual method of using short dashes is not satisfactory because the illusion of reality in a stereoscopic image makes such lines look like a series of unattached wires. Furthermore, such dashes must be shown by projection pairs properly coordinated, a very laborious process. It is possible to show lines in back of surfaces by the use of distinctly different line weights, but this makes the object transparent. It is best either to treat the object as entirely transparent or to treat it as opaque and omit hidden lines entirely.

251. Lettering

The simplest method of lettering a stereoscopic drawing is to make all the lettering frontal. If this is done, the two halves of the projection

pair of each letter are identical, and a lettering machine or printed letters may be used, the pair being given the proper separation to locate the image at the desired depth.

The most convenient and simplest procedure is to make all the lettering in one plane, on a transparent acetate overlay, over one projection. When the pair is photographed, the same overlay may be registered over both views with a separation to place it in any desired plane. Where a letter refers to a point back in the image, arrows may be drawn from the position the letter will occupy in the drawing when the overlay is registered, to the projection pair of the point. The projection pair of the arrow then automatically produces an image from the letter in the frontal plane to the image of the point. This procedure not only greatly reduces the work of lettering but insures better registry.

It is possible, though extremely tedious, to place letters on the surfaces of the object. In this case, the letters become part of the object and must be projected and drawn by the same method and with the same care that the projection pair of the object is drawn.

252. Shrinkage

One of the chief difficulties in obtaining an accurate drawing is due to the shrinkage of the paper on which the drawing is made. Unless both projections are made at the same time under similar atmospheric conditions, a difference in size is very likely to result. This leads to serious depth and vertical errors.

Since a stereoscopic drawing consisting of even a few lines usually takes many hours to draw, the possibility of completing the pair under exactly similar conditions is not great. It is thus advisable to use a material of very low shrinkage or one of uniform shrinkage with the projection pair on the same piece.

Frosted cellulose acetate has been found very satisfactory for this purpose. Ordinary drawing or tracing papers almost invariably cause difficulties.

Since minor corrections under a mirror stereoscope are nearly always necessary for first-rate work, it is also desirable to draw lines in such a way that they may be erased and shifted easily. The best practice for this purpose is to use black scotch tape instead of ink for the lines. Such tape may be cut to any desired width, and it may be laid very rapidly on cellulose acetate. It can be removed instantly without leaving any trace and without effecting the sticking quality of the tape. Thus, a black scotch tape line may be shifted with great ease and rapidity, whereas an ink line requires much time and trouble.

Thus, if the original drawings are made in pencil on acetate and the lines finally covered with tape, they may be corrected with a minimum effort.

APPENDIX IV

Natural Trigonometric Functions to Four Places

	Sin	Cos	Tan	Cot	
0° 00′	.0000	1.0000	.0000		90° 00′
10	029	000	029	343.8	50
20	058	000	058	171.9	40
30	.0087	1.0000	.0087	114.6	30
40	116	.9999	116	85.94	20
50	145	999	145	68.75	10
1° 00′	.0175	.9998	.0175	57.29	89° 00′
10	204	998	204	49.10	50
20	233	997	233	42.96	40
30	.0262	.9997	.0262	38.19	30
40	291	996	291	34.37	20
50	320	995	320	31.24	10
2° 00′	.0349	.9994	.0349	28.64	88° 00′
10	378	993	378	26.43	50
20	407	992	407	24.54	40
30	.0436	.9990	.0437	22.90	30
40	465	989	466	21.47	20
50	494	988	495	20.21	10
3° 00′	.0523	.9986	.0524	19.08	87° 00′
10	552	985	553	18.07	50
20	581	983	582	17.17	40
30	.0610	.9981	.0612	16.35	30
40	640	980	641	15.60	20
50	669	978	670	14.92	10
4° 00′	.0698	.9976	.0699	14.30	86° 00′
10	727	974	729	13.73	50
20	756	971	758	13.20	40
30	.0785	.9969	.0787	12.71	30
40	814	967	816	12.25	20
50	843	964	846	11.83	10
5° 00′	.0872	.9962	.0875	11.43	85° 00′
10	901	959	904	11.06	50
20	929	957	934	10.71	40
30	.0958	.9954	.0963	10.39	30
40	987	951	992	10.08	20
50	.1016	948	.1022	9.788	10
6° 00′	.1045	.9945	.1051	9.514	84° 00′
10	074	942	080	9.255	50
20	103	939	110	9.010	40
30	.1132	.9936	.1139	8.777	30
40	161	932	169	8.556	20
50	190	929	198	8.345	10
7° 00′	.1219	.9925	.1228	8.144	83° 00′
10	248	922	257	7.953	50
20	276	918	287	7.770	40
30	.1305	.9914	.1317	7.596	30
40	334	911	346	7.429	20
50	363	907	376	7.269	10
8° 00′	.1392	.9903	.1405	7.115	82° 00′
10	421	899	435	6.968	50
20	449	894	465	6.827	40
30	.1478	.9890	.1495	6.691	30
40	507	886	524	6.561	20
50	536	881	554	6.435	10
9° 00′	.1564	.9877	.1584	6.314	81° 00′
	Cos	Sin	Cot	Tan	

	Sin	Cos	Tan	Cot	
9° 00′	.1564	.9877	.1584	6.314	81° 00′
10	593	872	614	197	50
20	622	868	644	084	40
30	.1650	.9863	.1673	5.976	30
40	679	858	703	871	20
50	708	853	733	769	10
10° 00′	.1736	.9848	.1763	5.671	80° 00′
10	765	843	793	576	50
20	794	838	823	485	40
30	.1822	.9833	.1853	5.396	30
40	851	827	883	309	20
50	880	822	914	226	10
11° 00′	.1908	.9816	.1944	5.145	79° 00′
10	937	811	974	066	50
20	965	805	.2004	4.989	40
30	.1994	.9799	.2035	4.915	30
40	.2022	793	065	843	20
50	051	787	095	773	10
12° 00′	.2079	.9781	.2126	4.705	78° 00′
10	108	775	156	638	50
20	136	769	186	574	40
30	.2164	.9763	.2217	4.511	30
40	193	757	247	449	20
50	221	750	278	390	10
13° 00′	.2250	.9744	.2309	4.331	77° 00′
10	278	737	339	275	50
20	306	730	370	219	40
30	.2334	.9724	.2401	4.165	30
40	363	717	432	113	20
50	391	710	462	061	10
14° 00′	.2419	.9703	.2493	4.011	76° 00′
10	447	696	524	3.962	50
20	476	689	555	914	40
30	.2504	.9681	.2586	3.867	30
40	532	674	617	821	20
50	560	667	648	776	10
15° 00′	.2588	.9659	.2679	3.732	75° 00′
10	616	652	711	689	50
20	644	644	742	647	40
30	.2672	.9636	.2773	3.606	30
40	700	628	805	566	20
50	728	621	836	526	10
16° 00′	.2756	.9613	.2867	3.487	74° 00′
10	784	605	899	450	50
20	812	596	931	412	40
30	.2840	.9588	.2962	3.376	30
40	868	580	994	340	20
50	896	572	.3026	305	10
17° 00′	.2924	.9563	.3057	3.271	73° 00′
10	952	555	089	237	50
20	979	546	121	204	40
30	.3007	.9537	.3153	3.172	30
40	035	528	185	140	20
50	062	520	217	108	10
18° 00′	.3090	.9511	.3249	3.078	72° 00′
	Cos	Sin	Cot	Tan	

	Sin	Cos	Tan	Cot	
18° 00′	.3090	.9511	.3249	3.078	72° 00′
10	118	502	281	047	50
20	145	492	314	018	40
30	.3173	.9483	.3346	2.989	30
40	201	474	378	960	20
50	228	465	411	932	10
19° 00′	.3256	.9455	.3443	2.904	71° 00′
10	283	446	476	877	50
20	311	436	508	850	40
30	.3338	.9426	.3541	2.824	30
40	365	417	574	798	20
50	393	407	607	773	10
20° 00′	.3420	.9397	.3640	2.747	70° 00′
10	448	387	673	723	50
20	475	377	706	699	40
30	.3502	.9367	.3739	2.675	30
40	529	356	772	651	20
50	557	346	805	628	10
21° 00′	.3584	.9336	.3839	2.605	69° 00′
10	611	325	872	583	50
20	638	315	906	560	40
30	.3665	.9304	.3939	2.539	30
40	692	293	973	517	20
50	719	283	.4006	496	10
22° 00′	.3746	.9272	.4040	2.475	68° 00′
10	773	261	074	455	50
20	800	250	108	434	40
30	.3827	.9239	.4142	2.414	30
40	854	228	176	394	20
50	881	216	210	375	10
23° 00′	.3907	.9205	.4245	2.356	67° 00′
10	934	194	279	337	50
20	961	182	314	318	40
30	.3987	.9171	.4348	2.300	30
40	.4014	159	383	282	20
50	041	147	417	264	10
24° 00′	.4067	.9135	.4452	2.246	66° 00′
10	094	124	487	229	50
20	120	112	522	211	40
30	.4147	.9100	.4557	2.194	30
40	173	088	592	177	20
50	200	075	628	161	10
25° 00′	.4226	.9063	.4663	2.145	65° 00′
10	253	051	699	128	50
20	279	038	734	112	40
30	.4305	.9026	.4770	2.097	30
40	331	013	806	081	20
50	358	001	841	066	10
26° 00′	.4384	.8988	.4877	2.050	64° 00′
10	410	975	913	035	50
20	436	962	950	020	40
30	.4462	.8949	.4986	2.006	30
40	488	936	.5022	1.991	20
50	514	923	059	977	10
27° 00′	.4540	.8910	.5095	1.963	63° 00′
	Cos	Sin	Cot	Tan	

	Sin	Cos	Tan	Cot	
27° 00′	.4540	.8910	.5095	1.963	63° 00′
10	566	897	132	949	50
20	592	884	169	935	40
30	.4617	.8870	.5206	1.921	30
40	643	857	243	907	20
50	669	843	280	894	10
28° 00′	.4695	.8829	.5317	1.881	62° 00′
10	720	816	354	868	50
20	746	802	392	855	40
30	.4772	.8788	.5430	1.842	30
40	797	774	467	829	20
50	823	760	505	816	10
29° 00′	.4848	.8746	.5543	1.804	61° 00′
10	874	732	581	792	50
20	899	718	619	780	40
30	.4924	.8704	.5658	1.767	30
40	950	689	696	756	20
50	975	675	735	744	10
30° 00′	.5000	.8660	.5774	1.732	60° 00′
10	025	646	812	720	50
20	050	631	851	709	40
30	.5075	.8616	.5890	1.698	30
40	100	601	930	686	20
50	125	587	969	675	10
31° 00′	.5150	.8572	.6009	1.664	59° 00′
10	175	557	048	653	50
20	200	542	088	643	40
30	.5225	.8526	.6128	1.632	30
40	250	511	168	621	20
50	275	496	208	611	10
32° 00′	.5299	.8480	.6249	1.600	58° 00′
10	324	465	289	590	50
20	348	450	330	580	40
30	.5373	.8434	.6371	1.570	30
40	398	418	412	560	20
50	422	403	453	550	10
33° 00′	.5446	.8387	.6494	1.540	57° 00′
10	471	371	536	530	50
20	495	355	577	520	40
30	.5519	.8339	.6619	1.511	30
40	544	323	661	501	20
50	568	307	703	1.492	10
34° 00′	.5592	.8290	.6745	1.483	56° 00′
10	616	274	787	473	50
20	640	258	830	464	40
30	.5664	.8241	.6873	1.455	30
40	688	225	916	446	20
50	712	208	959	437	10
35° 00′	.5736	.8192	.7002	1.428	55° 00′
10	760	175	046	419	50
20	783	158	089	411	40
30	.5807	.8141	.7133	1.402	30
40	831	124	177	393	20
50	854	107	221	385	10
36° 00′	.5878	.8090	.7265	1.376	54° 00′
	Cos	Sin	Cot	Tan	

	Sin	Cos	Tan	Cot	
36° 00′	.5878	.8090	.7265	1.376	**54° 00′**
10	901	073	310	368	50
20	925	056	355	360	40
30	.5948	.8039	.7400	1.351	30
40	972	021	445	343	20
50	995	004	490	335	10
37° 00′	.6018	.7986	.7536	1.327	**53° 00′**
10	041	969	581	319	50
20	065	951	627	311	40
30	.6088	.7934	.7673	1.303	30
40	111	916	720	295	20
50	134	898	766	288	10
38° 00′	.6157	.7880	.7813	1.280	**52° 00′**
10	180	862	860	272	50
20	202	844	907	265	40
30	.6225	.7826	.7954	1.257	30
40	248	808	.8002	250	20
50	271	790	050	242	10
39° 00′	.6293	.7771	.8098	1.235	**51° 00′**
10	316	753	146	228	50
20	338	735	195	220	40
30	.6361	.7716	.8243	1.213	30
40	383	698	292	206	20
50	406	679	342	199	10
40° 00′	.6428	.7660	.8391	1.192	**50° 00′**
10	450	642	441	185	50
20	472	623	491	178	40
30	.6494	.7604	.8541	1.171	30
40	517	585	591	164	20
50	539	566	642	157	10
41° 00′	.6561	.7547	.8693	1.150	**49° 00′**
10	583	528	744	144	50
20	604	509	796	137	40
30	.6626	.7490	.8847	1.130	30
40	648	470	899	124	20
50	670	451	952	117	10
42° 00′	.6691	.7431	.9004	1.111	**48° 00′**
10	713	412	057	104	50
20	734	392	110	098	40
30	.6756	.7373	.9163	1.091	30
40	777	353	217	085	20
50	799	333	271	079	10
43° 00′	.6820	.7314	.9325	1.072	**47° 00′**
10	841	294	380	066	50
20	862	274	435	060	40
30	.6884	.7254	.9490	1.054	30
40	905	234	545	048	20
50	926	214	601	042	10
44° 00′	.6947	.7193	.9657	1.036	**46° 00′**
10	967	173	713	030	50
20	988	153	770	024	40
30	.7009	.7133	.9827	1.018	30
40	030	112	884	012	20
50	050	092	942	006	10
45° 00′	.7071	.7071	1.000	1.000	**45° 00′**
	Cos	**Sin**	**Cot**	**Tan**	

APPENDIX V

Table of Chords

The Tabulated Quantities = Twice the Sine of Half the Arc

Deg.	0′	10′	20′	30′	40′	50′
0	.0000	.0029	.0058	.0087	.0116	.0145
1	.0174	.0204	.0233	.0262	.0291	.0320
2	.0349	.0378	.0407	.0436	.0465	.0494
3	.0523	.0553	.0582	.0611	.0640	.0669
4	.0698	.0727	.0756	.0785	.0814	.0843
5	.0872	.0901	.0930	.0959	.0988	.1017
6	.1047	.1076	.1105	.1134	.1163	.1192
7	.1221	.1250	.1279	.1308	.1337	.1366
8	.1395	.1424	.1453	.1482	.1511	.1540
9	.1569	.1598	.1627	.1656	.1685	.1714
10	.1743	.1772	.1801	.1830	.1859	.1888
11	.1917	.1946	.1975	.2004	.2033	.2062
12	.2090	.2119	.2148	.2177	.2206	.2235
13	.2264	.2293	.2322	.2351	.2380	.2409
14	.2437	.2466	.2495	.2524	.2553	.2582
15	.2610	.2639	.2668	.2697	.2726	.2755
16	.2783	.2812	.2841	.2870	.2899	.2927
17	.2956	.2985	.3014	.3042	.3071	.3100
18	.3129	.3157	.3186	.3215	.3243	.3272
19	.3301	.3330	.3358	.3387	.3416	.3444
20	.3473	.3502	.3530	.3559	.3587	.3616
21	.3645	.3673	.3702	.3730	.3759	.3788
22	.3816	.3845	.3873	.3902	.3930	.3959
23	.3987	.4016	.4044	.4073	.4101	.4130
24	.4158	.4187	.4215	.4243	.4272	.4300
25	.4329	.4357	.4385	.4414	.4442	.4471
26	.4499	.4527	.4556	.4584	.4612	.4641
27	.4669	.4697	.4725	.4754	.4782	.4810
28	.4838	.4867	.4895	.4923	.4951	.4979
29	.5008	.5036	.5064	.5092	.5120	.5148
30	.5176	.5204	.5232	.5261	.5289	.5317
31	.5345	.5373	.5401	.5429	.5457	.5485
32	.5513	.5541	.5569	.5596	.5624	.5652
33	.5680	.5708	.5736	.5764	.5792	.5820
34	.5847	.5875	.5903	.5931	.5959	.5986
35	.6014	.6042	.6069	.6097	.6125	.6153
36	.6180	.6208	.6236	.6263	.6291	.6318
37	.6346	.6374	.6401	.6429	.6456	.6484
38	.6511	.6539	.6566	.6594	.6621	.6649
39	.6676	.6703	.6731	.6758	.5786	.6813
40	.6840	.6868	.6895	.6922	.6950	.6977
41	.7004	.7031	.7059	.7086	.7113	.7140

The Tabulated Quantities = Twice the Sine of Half the Arc

Deg.	0′	10′	20′	30′	40′	50′
42	.7167	.7194	.7222	.7249	.7276	.7303
43	.7330	.7357	.7384	.7411	.7438	.7465
44	.7492	.7519	.7546	.7573	.7600	.7627
45	.7654	.7680	.7707	.7734	.7761	.7788
46	.7815	.7841	.7868	.7895	.7921	.7948
47	.7975	.8001	.8028	.8055	.8081	.8108
48	.8135	.8161	.8188	.8214	.8241	.8267
49	.8294	.8320	.8347	.8373	.8400	.8426
50	.8452	.8479	.8505	.8531	.8558	.8584
51	.8610	.8636	.8663	.8689	.8715	.8741
52	.8767	.8793	.8820	.8846	.8872	.8898
53	.8924	.8950	.8976	.9002	.9028	.9054
54	.9080	.9106	.9132	.9157	.9183	.9209
55	.9235	.9261	.9286	.9312	.9338	.9364
56	.9389	.9415	.9441	.9466	.9492	.9518
57	.9543	.9569	.9594	.9620	.9645	.9671
58	.9696	.9722	.9747	.9772	.9798	.9823
59	.9848	.9874	.9899	.9924	.9949	.9975
60	1.0000	1.0025	1.0050	1.0075	1.0100	1.0126
61	1.0151	1.0176	1.0201	1.0226	1.0251	1.0276
62	1.0301	1.0326	1.0350	1.0375	1.0400	1.0425
63	1.0450	1.0475	1.0500	1.0524	1.0550	1.0574
64	1.0598	1.0623	1.0648	1.0672	1.0697	1.0721
65	1.0746	1.0770	1.0795	1.0819	1.0844	1.0868
66	1.0893	1.0917	1.0941	1.0966	1.0990	1.1014
67	1.1039	1.1063	1.1087	1.1111	1.1135	1.1159
68	1.1184	1.1208	1.1232	1.1256	1.1280	1.1304
69	1.1328	1.1352	1.1376	1.1400	1.1424	1.1448
70	1.1471	1.1495	1.1519	1.1543	1.1567	1.1590
71	1.1614	1.1638	1.1661	1.1685	1.1708	1.1732
72	1.1756	1.1780	1.1803	1.1826	1.1850	1.1873
73	1.1896	1.1920	1.1943	1.1966	1.1990	1.2013
74	1.2036	1.2059	1.2083	1.2106	1.2129	1.2152
75	1.2175	1.2198	1.2221	1.2244	1.2267	1.2290
76	1.2313	1.2336	1.2360	1.2382	1.2405	1.2427
77	1.2450	1.2473	1.2496	1.2518	1.2541	1.2564
78	1.2586	1.2609	1.2631	1.2654	1.2677	1.2699
79	1.2721	1.2744	1.2766	1.2789	1.2811	1.2833
80	1.2856	1.2878	1.2900	1.2922	1.2945	1.2967
81	1.2989	1.3011	1.3033	1.3055	1.3077	1.3099
82	1.3121	1.3143	1.3165	1.3187	1.3209	1.3231
83	1.3252	1.3274	1.3296	1.3318	1.3340	1.3361
84	1.3383	1.3404	1.3426	1.3447	1.3469	1.3490
85	1.3512	1.3533	1.3555	1.3576	1.3597	1.3619
86	1.3640	1.3661	1.3682	1.3704	1.3725	1.3746
87	1.3767	1.3788	1.3809	1.3830	1.3851	1.3872
88	1.3893	1.3914	1.3935	1.3956	1.3977	1.3997
89	1.4018	1.4039	1.4060	1.4080	1.4101	1.4121
90	1.4142					

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