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COPERNICUS

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COPERNICUS

The Founder of Modern Astronomy

by

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PREFACE

IT is my aim in the following pages to present a short account of the astronomer Copernicus, and of the historic book in which he laid the foundations of the heliocentric theory of the planetary motions. I hope that the work may appeal, not only to students of the history of astronomy, but also to the wider class of readers who are interested in Copernicus as one of the makers of modern thought.

The plan of the book is as follows. By way of introduction to the work of Copernicus, it has been necessary to summarize the development of planetary theories from Babylonian times down to his day. This is done in Chapter I. Chapter II contains a concise biography of Copernicus, with some account of his instruments and of the results of the critical examination of his manuscripts. With Chapter III we begin the exposition of Copernicus' epoch-making work, the *De Revolutionibus.* This occupies the rest of the book, except for a short Epilogue on the establishment of the heliocentric theory, and several supplementary Notes. The *De Revolutionibus* has been studied comparatively with Ptolemy's *Almagest,* and I have inserted numerous references to the latter *in* order to bring out the close connection between the two works.

I owe a deep debt of gratitude to my former teachers. Professor A. Wolf, and the late Professor L. N. G. Filon, for their advice and encouragement, and for the pains they have taken in reading critically through the typescript.

My thanks are also due to Professor H. E. Butler, of University College, for his expert advice on the translation of certain Latin passages. The responsibility for any uncorrected errors or omissions is, however, solely mine.

A. A.

June 1938

CONTENTS

NOTE

IN the present work numerous references will have to be made to various passages in Copernicus' book *De Revolutionihus*, and in Ptolemy's *Almagest*. Such references to a given Book and Chapter of the *De Revolutionihus* will, in general, simply be indicated thus (III, 15); references to a given Book and Chapter of the *Almagest*, thus (*Aim.*, XI, 7); and cross-references to a given Chapter and Article of this book thus (Chapter I, § 2 *supra*), or (Chapter IV, § 3, *infra*).

CHAPTER I

PLANETARY THEORIES BEFORE COPERNICUS

OF all the age-long problems which have prompted human inquiry, there is none whose progressive elucidation has contributed more to the enlargement of scientific insight than that of tracing regularity in the motions of the heavenly bodies. In this immemorial quest,ior the secrets of cosmic order, the name of Copernicus stands for a crisis and a revaluation, but not for a discontinuity. For the problem upon which he expended half a lifetime of intellectual labour was, even in his day, already of ancient standing; moreover, the technique which he employed, and much of the data upon which he relied, in his attempt to devise a new solution for it, and many of the preconceptions with which he approached his task, were an inheritance from antiquity. Unless, then, we know something of the history of astronomy in the ages before Copernicus, we shall scarcely follow his arguments and calculations, nor judge his achievements aright. In this opening chapter, therefore, we shall briefly review such antecedent developments of the science, in Antiquity, in Islam, and in mediaeval Christendom, as are relevant to our purpose, so as to provide the reader with the historical and technical information necessary for the understanding of what follows.

§i. ANTIQUITY

The recorded history of astronomy down to the seventeenth centuary is occuiped, not so much with discoveries of previously unknown phenomena, as with a series of attempts to systematize and to interpret certain facts whose earliest recognition lies altogether beyond the horizon of history. The ancient and mediaeval systems of astronomy were concerned with celestial phenpmena which had been conspicuous to unaided Human perception from of old. For ages the stars had been observed to form permanent groups, or Constellations, wheeling in hourly and seasonal sequences across the vault of the night sky. The Moon had been watched, waxing and waning on her monthly circuit through the central belt of the constellations, and the Sun, varying his daily course, and rising and setting with different stars, according to the season of the year. The periods associated with these bodies formed the basis of calendars for regulating the repetition of the agricultural operations and ritual acts of primitive communities. Less conspicuous than Sun or Moon, but still distinguished from the stars since prehistoric times, were the planets, circulating slowly and erratically through the constellations. Add to these such occasional spectacles as eclipses, and the visitations of comets, and we have before us practically all the phenomena with which astronomers were concerned throughout the whole period covered by this chapter. Several of the earliest historic civilizations possessed a considerably developed knowledge of these basic celestial phenomena, and of the periodicities connected with them. But the highest standard of refinement in this respect seems to have been attained by the Babylonians, whose achievements most directly and appreciably influenced the Greeks in their task of establishing astronomy as a science.

In Babylonia the priests were wont to observe the heavens assiduously, for the purposes of regulating their lunar calendar, and of drawing omens from all striking celestial or atmospheric phenomena. The temples were the observatories and the archives where recorded observations were preserved; and, through the stormy centuries of Babylonian history, the temple schools were able to develop elaborate systems of star-lore, since invaders generally spared the national shrines of the old gods of the land. During the past century, a growing light has been shed on the astronomical achievements of the Babylonians, by the decipherment of clay tablets upon which their observations and calculations were recorded. Babylonian astronomy seems to have passed through three main stages in its development, though these phases overlapped one another, and the dates at which they were reached are still partly matters of controversy. In the earliest stage, observations were, for the most part, indiscriminate, and lacked numerical precision, the phenomena being treated merely as portents of impending national crises, etc.; the next stage (well established by the eighth century B.C.) was characterized by dated records, with numerical specifications, and estimates of the periods of recurrent celestial phenomena, leading, in the two centuries preceding the Christian era, to the highest level of Babylonian astronomical attainment, represented by the construction of ephemerides serving to predict celestial phenomena for years in advance.

The Babylonians attached especial significance to the movements of the seven "planets," namely, the Sun, the Moon, and the bodies to which (following the Romans) we give the names of Mercury, Venus, Mars, Jupiter, and Saturn, and which we class as planets in the modern sense of the term. They associated, or identified, their chief gods with these bodies. The planetary phenomena to which they paid particular attention were (i) the *heliacal risings and settings* of a planet, when it was observed to rise before sunrise for the first time, or to set after sunset for the last time (occasions which marked the limits of its period of extinction in the Sun's rays); (ii) the *stationary points* where a planet's course among the constellations was arrested and reversed; (iii) *oppositions,* when the planet was in the opposite quarter of the sky to the Sun; (iv) *conjunctions,* when the planets appeared to pass close to one another, or to bright stars, and (v) eclipses of the Sun or Moon. The phenomenon of *stationary points* calls for some further explanation, because of the complications which it necessitated in planetary theories until the time of Copernicus. The Sun and Moon, in their periodic circuits round the heavens, travel continuously through the constellations from west by south to east; but the apparent motions of the five planets are more complicated. For example, if the planet Mars be observed in the southern sky night after night for some weeks, it will, in general, be found to be slowly moving from west to east in relation to the background of stars. (Such motion is, of course, to be distinguished from the planet's apparent diurnal revolution about the Earth which it shares with the stars.) At fairly regular intervals, however (about 780 days for Mars), the planet's eastward motion is arrested (its apparent path showing a *stationary point)* and reversed, and Mars moves from east to west through an *arc of retrogression* of about 15^0 before resuming its normal eastward motion. The same is true of the planets Jupiter and Saturn, though their arcs of retrogression are less considerable; while the planets Mercury and Venus have the additional peculiarity that the angular distance of each from the Sun never exceeds a moderate limiting value (about 25^0 for Mercury and 45^0 for Venus).

From at least the beginning of the fourth century BX. the Babylonians were in the habit of defining the apparent positions of the planets in the sky by reference to a series of bright stars distributed fairly regularly round that belt of the heavens in which all the planets move, and which (following the Greeks) we term the *Zodiac.* The place of a planet was defined by specifying its angular separation from the standard star nearest to it. A scale of angujar measurement was afforded by the division of the Zodiac into twelve equal parts, (the *Signs* of the Zodiac), with further subdivisions, and the system was gradually developed to admit of such measurements being made both along the Zodiac and at right angles to it. This system eventually led to the conception of the *ecliptic*—the great circle of the celestial sphere traversed by the Sun in his annual circuit through the constellations. Thus, by the second century B.C., the *celestial longitudes* and *latitudes* of the planets, and of other celestial bodies, were being defined by reference to the circle of the ecliptic, much as we define the geographical longitudes and latitudes of places on the Earth by reference to the terrestrial equator.

By diligent observations, continued over several centuries, the Babylonians were able to arrive at remarkably accurate estimates of the principal *time-periods* associated with the heavenly bodies—the year, the several kinds of months, the *sidereal period* of each planet (in which, on the average, the planet performs a complete circuit of the heavens relatively to the background of stars), and its *synodic period* (the time for a complete circuit relatively to the Sun). By the second century B.C. the Babylonians had arrived at an estimate of the (sidereal) year, 365 days, 6 hours, 13 minutes, 43-4 seconds, representing an error of only about four and a half minutes in excess of the modern estimate for that age (Kugler: *Sternkunde und Sterndienst in Babel*, Vol. I I , p. 8). And already about the fourth century B.C. they had discovered that lunar eclipses form sequences which recur periodically at intervals of about eighteen years. In their intricate ephemerides of the Sun, Moon, and planets, the Babylonians made allowance for the principal periodic non-uniformities observable in the apparent motions of these bodies through the constellations. It is difficult, in a few words, to give any idea of the refinement and complexity of their methods. Tables for showing the dates of successive new moons (second century B.C.) included columns of corrections for the yearly inequality in the Sun's motion, and for the monthly inequality in the Moon's motion; the amounts of the corrections for successive months oscillated in a regular manner between maximum and minimum values. As regards the remaining five planets, the Babylonians took account, not only of the element of inequality in the motion of each which depends upon its position in relation to the Sun (and which we now know to be due to the Earth's motion), but also of the further element, complicating the first, which depends upon the planet's position in the Zodiac (and which results from the planet's elliptic motion). They represented this latter inequality by assigning to the planet, as it moved round the Zodiac, a succession of different rates of angular motion, which varied between maximum and minimum values and recurred in the appropriate period.

It is unlikely that the Babylonians ever arrived at any clear-cut, objective conception of the constitution of the Universe as a whole, such as we encounter in Greek philosophy. It is possible, however, to piece together a composite picture of the cosmology which must have formed the background of Babylonian thought, at least throughout the historic period. The Earth was conceived to be roughly circular in contour, rising towards the centre to form a huge mountain, and resting upon a great ocean which girdled the land with a moat of sea; beyond this rose a circular mountain wall, forming the boundary of the world, and supporting the hemispherical vault of heaven, or firmament. The heavenly bodies seem to have been regarded as moving freely through space.

We find a system of cosmology closely resembling that of the Babylonians in the earliest extant Greek literature—the poems of Homer and Hesiod. And, throughout the period of their intellectual development, the Greeks continued to draw upon the astronomical knowledge of the older civilizations around them to an extent which is receiving ever increasing recognition nowadays. But they transformed this knowledge into a science capable of progressive development, by virtue of their conception of Nature as an orderly system whose constitution and phenomena were not to be attributed to supernatural agencies, but Were to be rationally deduced as consequences of the inherent properties of the one or more primary substances of which the entire Universe was held to be composed. This transformation was initiated by certain schools of philosophers which flourished in the sixth and fifth centuries before Christ, among the outposts of early Greek civilization in Asia Minor and southern Italy. On the one hand, the Ionian natural philosophers, discarding mythology, elaborated a succession of naturalistic, if somewhat crude and speculative, cosmological systems, in which the Earth generally figured as a disc, or a vortical condensation, floating at the centre of the Universe. On the other hand, Pythagoras and his followers, in their search for simple mathematical principles underlying the apparent complexity of Nature, enriched astronomy with conceptions destined to regulate its whole subsequent development.

Besides establishing a naturalistic attitude towards the problems of the Universe, the pre-Socratic Greek philosophers made several special contributions to the stock of astronomical facts and ideas. Thus, Anaximenes' conception of the stars as fixed to a rotating crystal sphere, persisted from the sixth century B.C. down to the age of Galilei. To Pythagoras himself (sixth century B.C.) is credibly attributed the earliest declaration that the Earth is a sphere resting at the centre of a spherical universe, and not a floating disc as the Ionians taught. Pythagoras seems also to have been aware (probably following the Babylonians in this) that the complicated apparent motion of the Sun in the course of the year is, to all appearance, made up of two simple motions—(i) the motion common to all the heavenly bodies, whereby they appear to revolve about an axis through the Earth once in a day, and (ii) a motion peculiar to the Sun, in a contrary direction to the first motion, and taking place about a different axis, in the space of one year. The first of these motions would account for the daily rising and setting of the Sun; the second would account for the Sun's annual circuit among the constellations and for the seasonal fluctuations in his rising and setting points and in his meridian altitude. This analysis-seems also to have been extended by Pythagoras (though with less eligibility) to the apparent motions of the Moon and planets; it gave rise to the idea that the complicated movements of the heavenly bodies could all be resolved into uniform circular motions. This doctrine was established by the authority of Plato, Aristotle, and Ptolemy; as we shall see, it still dominated astronomy in the time of Copernicus two thousand years after Pythagoras, and it was first formally abandoned by Kepler at the beginning of the seventeenth century. By the end of the fifth century B.C., the Pythagorean school had evolved the remarkable system of cosmology associated with the name of Philolaus. To this hypothesis Copernicus directed especial attention, for it was the earliest historic system to displace the Earth from the centre of the Universe, and to set it in revolution about the centre like any other planet. According to Philolaus, the finite sphere within which the Universe was contained had fire at its centre and fire at its circumference. It was divided by concentric spheres into three layers. The outermost of these contained the stars. The intermediate layer contained the five planets, the Sun, and the Moon, in order of approach to the Central Fire, about which these bodies all revolved in circles in the several periods of their circuits round the heavens. Lastly, within the sphere which formed the core of the Universe, was the Earth, which itself revolved daily about the Central Fire, turning towards it the hemisphere opposite to that inhabited by mankind who, in consequence, could never behold the Fire. By making the Earth revolve, as described, in a plane inclined to that in which the other planets moved, Philolaus was able to account, not only for the risings and settings of the heavenly bodies, but also for all seasonal phenomena now attributed to the inclination of the Earth's equator to the ecliptic.

The somewhat fanciful system of Philolaus never established itself; but the primitive Pythagorean cosmology lived on in the natural philosophy of Plato (427-347 B.C.), who conceived the Universe as a rotating sphere in the midst of a boundless void, and the Earth as a stationary sphere in the midst of the Universe. Under picturesque allegories in the *Timaeus* and the *Republic*, it is easy to recognize Plato's expressions of the principle that the motion of each of the seven planets is compounded of two uniform revolutions about the Earth which take place about different axes and in opposite senses, and one of which is common to all the heavenly bodies. It was obvious, however, that this simple Pythagorean conception of a planet's motion took no account of the recurrent *retrogressions* of the planets, to which we have already alluded, or of their departures from the ecliptic. It was probably with a view to remedying this defect in the current theory, that, according to Simplicius, Plato set the astronomers of his time the general problem of adequately representing the observed movements of the heavenly bodies by combinations of uniform circular motions having a common centre in the Earth.

This problem was taken up, early in the fourth century B.C., by Eudoxus of Cnidos, a pupil of Plato, and one of the greatest mathematicians of antiquity. Eudoxus regarded each planet as attached to the equator of an ideal sphere, which rotated uniformly about two opposite poles, with the Earth at its centre. The poles of t|?is sphere were embedded in the surface of a second sphere, external to the first, but concentric with it, and itself in uniform rotation about an axis inclined at a constant angle to that of the first. This second sphere was similarly related to a third one, and so on. Eudoxus* problem was then to choose, for each planet, a particular combination of these spheres, having such axes and periods of rotation that the superposition of their motions would make a point on the equator of the innermost sphere move about the common centre with a motion similar to that with which the planet in question was observed to move about the Earth. The system of each planet had one sphere whose function it was to impart the diurnal motion about the polar axis of the heavens common to all heavenly bodies. In the system of the Moon, there was a second sphere whose rotation corresponded to the monthly eastward revolution of the Moon in the plane of its orbit, and a third sphere which provided for the slow westward regression of the *line of nodes* in which that orbit intersects the ecliptic. (That the Moon's path was inclined to the ecliptic, intersecting it at two *nodes*, was proved by **the** fact that eclipses do not take place at every new and full moon; and these nodes, near which all eclipses occur, were found to revolve round the ecliptic from east to west.) An analogous group of three spheres was postulated for the Sun, which was mistakenly believed to deviate, like the Moon, from the ecliptic. In the application of his theory to the motions of the five planets, Eudoxus was able to give a fair representation of the characteristic motion of a planet among the constellations. In addition to two spheres respectively conferring upon the planet its diurnal revolution, and a uniform eastward revolution in its own sidereal period (as in the simple Pythagorean theory), Eudoxus introduced two further spheres—the first of these rotating about an axis lying in the plane of the ecliptic, and the second, about an axis inclined at a constant angle to that of the first, in the same period (the synodic period of the planet, see p. 18), but in the opposite sense of rotation. The effect of imparting these two motions to the planet would be to make it describe about its mean position a curve like a figure eight lying on its side. This motion, when superimposed upon the uniform eastward motion already possessed by the planet, would give rise to such periodic retrogressions as are actually observed, provided the inclination of the axes of the two last-mentioned spheres were suitably chosen. The departures of the planets from the plane of the ecliptic involved in Eudoxus* hypothesis, however, had no relation to the motions in latitude which are actually observed; and the hypothesis was not applicable, in its original form, to the planet Mars, whose arcs of retrogression are more considerable than those of the more distant planets. Nor did the system of Eudoxus take account of the effects now known to arise from the eccentricity of planetary orbits, which, in the case of the Sun's apparent motion, produces the inequality of the four seasons. Some of these defects were removed by Eudoxus' immediate successors; but one fundamental objection against the whole theory remained: it took no account of variations in the distances of the heavenly bodies from the Earth, such as are suggested by variations in the brightnesses of the planets, and by the occurrence, sometimes, ef total, and sometimes of annular, solar eclipses, which proves that the relative distances from us of the Sun and Moon are liable to vary. The spheres of Eudoxus, however, determined the pattern of Aristotle's cosmological system, whose main outlines and regulative physical principles retained, down to the age of Copernicus, an authority altogether disproportionate to their value.

Aristotle (384-322 B.c.) conceived the physical Universe as a finite sphere, embracing all that exists. At its centre was the Earth, a stationary sphere, around which the rest of the Universe was built up symmetrically in concentric spherical shells. About the central mass of earth lay successive layers predominantly occupied by water, air, and fire respectively, which completely filled the region comprised within the sphere carrying the Moon. The remainder of the Universe, to the outermost sphere carrying the fixed stars, was occupied by the successive systems of planetary spheres, which Aristotle regarded as physically real and in mutual contact, each sphere transmitting its motion to the one next within it. The Moon's sphere was supposed to separate two fundamentally different regions of the Universe. Within this sphere, all things were composed of the four elements, earth, water, air, and fire, which were constantly undergoing transformation one into another, in virtue of their common substratum of formless primary matter, so that the sub-lunary region was characterized by incessant generation, change, and decay. Beyond the Moon's sphere, however, the celestial bodies and their carrying-spheres were all composed of an incorruptible fifth element, or *aether*, capable of undergoing only change of *place*. This doctrine of the immutability of the superlunary realm was still current in the sixteenth century. It was reluctantly abandoned in the seventeenth century, when comets and "new stars" had been definitely admitted to be celestial phenomena, while the distinction between terrestrial and celestial matter was only conclusively abolished after the development of spectrum analysis in the nineteenth century. Aristotle's dichotomy between elemental and celestial bodies involved a fundamental difference in their natural modes of motion. It was upon his theory of motion that Aristotle based his arguments for retaining the Earth as a stationary mass at the centre of the Universe; but we shall deal with these arguments later in connection with Copernicus' attempts to refute them (see Chapter III, § 3 *infra)*.

Among the pupils of Plato and Aristotle in the fourth century B.C., Heraclides of Pontus must be mentioned here. For he taught that the apparent diurnal revolution of the heavens was actually due to an axial rotation of the Earth in the opposite direction. This hypothesis is also attributed by certain classical writers to Ecphantus and Hicetas, both

Pythagoreans of Syracuse (dates unknown, but probably anterior to Aristotle), to whom Copernicus alludes in his book. The relation of these men to Heraclides is obscure. It is suggested by Heath that Ecphantus may have been introduced by Heraclides as a typical Pythagorean into one of his lost dialogues, to serve as a mouthpiece for the writer's own opinions. The ascription of the hypothesis to Hicetas may be due to a similar confusion. Heraclides also correctly explained the curious property of Mercury and Venus, whereby each appears alternately east and west of the Sun, and never recedes far from it. He suggested that these planets revolved in circles about the Sun as centre, while the Sun revolved in a larger circle about the Earth. This limited heliocentric system (wrongly attributed by the Latin commentator Macrobius to the Egyptians, and hence sometimes known as the "Egyptian System") was probably extended (by whom is unknown), during the century following Heraclides, to include Mars, Jupiter, and Saturn, so that all the five planets would thus be supposed to revolve about the Sun while the Sun revolved about the Earth. (In this system, the superior planets would be nearest to the Earth when in *opposition* to the Sun, which would explain why, in fact, they appear brightest in that situation.) This would have marked an important step towards the Copernican scheme; and, early in the third century B.C., Aristarchus of Samos actually anticipated the full Copernican system in its broad outlines. That is to say, he put forward the hypotheses that the sphere of stars was motionless, so that its apparent daily revolution was due to a diurnal rotation of the Earth; that the Sun was at rest at the centre of the sphere of fixed stars; that the Earth and planets described circles about the Sun as centre, and that the radius of the sphere of stars was so incomparably greater than that of the Earth's orbit, that no corresponding apparent motion was produced in the stars. The hypotheses of Aristarchus, however, were generally rejected as impious and contrary to sound physical principles. Moreover, such undeveloped speculations had soon to compete with the carefully articulated systems of the Alexandrian astronomers, where inequalities in the apparent motions of the Sun and planets were represented with steadily increasing numerical accuracy. These systems were conceived on the geocentric hypothesis, and their success did much to establish it.

The development of orthodox planetary theories for a century or so after Heraclides, is obscure. We have already mentioned the possibility that his hypothesis was extended from the inferior to the superior planets during the third century B.C. In such a system, the distinction between these two classes of planets would be that the circle described about the Sun by an inferior planet would be smaller than the circle described by the Sun about the Earth, while that described about the Sun by a superior planet would be larger than the Sun's circle, and would embrace the Earth. It was probably by generalizing these two alternative kinds of motion of a planet about the Earth, that the Greeks arrived at two geometrical devices, which were to be extensively employed, for representing such motion, in all planetary theories down to the time of Kepler. As we shall repeatedly encounter these devices in our study of Copernicus we must now explain their nature once for all (see Fig. i:

no account is taken, in this explanation, of the diurnal motion common to all the heavenly bodies).

For the sake of completeness, we begin with the ideally simplified planetary system of Pythagoras and Plato, in

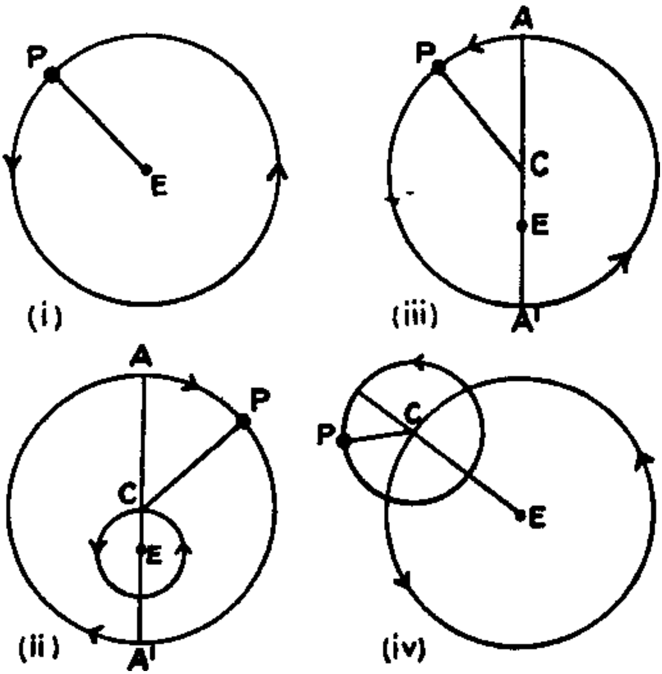


FIG. I

which a planet P described a circle at a uniform **rate about** the Earth E at the centre (Fig. 1, i). This **was developed, as** we have seen, into **the spheres of Eudoxus, which were** merely combinations **of** such **geocentric circular motions, and in which no account was taken of variations in the distances of the planets from the Earth. From the third** century B.C. onwards, however, such variations in the distance, as well as in the rate of apparent motion, of a planet, were represented in either of two ways. In the first of these (Fig. i, ii) the planet P was conceived as uniformly describing a circle (called a *movable eccentric)* about a centre C, which itself meanwhile described a circle about the Earth E. This system was most obviously suited to represent the motions of the superior planets, the point C (which might be identified with the Sun) revolving about the Earth from west to east in one year, relatively to the stars. The planet P performed a complete circuit round the Earth from west to east (relatively to the stars) in its sidereal period. But the Greeks regarded the planet's circle APA' as being carried round E by the motion of C, as if it were a material circle attached to the moving line ACA'; and they reckoned the motion of the planet on this circle, not in relation to the stars, but from the moving point A *(apogee)* where it was farthest from the Earth. Hence, in order just to complete a circuit about the Earth from west to east in its *sidereal* period, the planet had to be supposed to travel in a retrograde direction (from east to west) on its circle, completing a revolution, relatively to A, in its *synodic* period. A particular case of (ii) was occasionally employed in which C was stationary, and the line AA' preserved a fixed direction relatively to the stars, while the planet P revolved uniformly about C from west to east in its sidereal period (Fig. i, iii). Lastly, the planet P could be supposed uniformly to describe a small circle (called an *epicycle)* about a centre C which itself meanwhile described a larger circle (the *deferent)* about the Earth E (Fig. i, iv). By suitably proportioning the radii of such combinations of circles, and assigning the appropriate periods to them, it was possible to give a fairly correct geometrical representation of the retrogressions and other characteristic phenomena of the planetary motions.

According to Ptolemy *(Alm.,* XII, 1), Apollonius of Perga *(fl. c.* 230 B.C.), the "great geometer" of Antiquity, was acquainted with the systems both of movable eccentrics, and of epicycles, and he understood their mathematical properties. Now, on Heraclides' hypothesis, Mercury and Venus described *epicycles* about tkte Sun (which described a *deferent* about the Earth), while, in the supposed extension of this hypothesis, Mars, Jupiter, and Saturn described *movable eccentrics* about the Sun, which occupied the point C in Fig. 1, ii (though by the end of the third century B.c., these systems had been generalized, so as to make the planets, and the Sun itself, revolve about imaginary points). This gave a heliocentric system, so far as the planets were concerned, and it might soon have been developed into the Copernican system (it probably was the means by which Aristarchus was enabled to take that step), especially as the year figured in each separate planetary theory, and was evidently a fundamentally important period. But the planets appeared to fall into two classes, according as they described *eccentrics* or *epicycles;* and it was known to Apollonius that any eccentric system can be transformed into an epicyclic system, and *vice versa,* by interchanging the radii of the two circles, and transforming the respective periods according to definite rules (see Note 1). Consequently, during the centuries between Apollonius and Ptolemy, the eccentrics of **the** superior planets gave place to epicycles **the centres** of whose deferents revolved about the Earth. The reason for this preference was presumably the apparent gain in simplicity involved in the application of the same device (the *epicycle)* to both superior and inferior planets. But the Sun and the year no longer figured in all the planetary theories (for in accordance with the above-mentioned convention, the period of revolution in the epicycle was the *synodic* period of the planet), and the chance of progressing to a heliocentric system was lost for the time being.

The only astronomer known to have made important contributions to the development of planetary theories during the four centuries from Apollonius to Ptolemy, was Hipparchus of Rhodes *(fl. c.* 150 B.C.). All the important works of Hipparchus are lost, and his achievements, chiefly known to us through the *Almagest,* cannot always be distinguished with certainty from those of the Babylonians (of whose recorded observations he made considerable use), or even from those of Ptolemy himself.

In order to account for the apparent motion of the Sun in the Zodiac, with the annually recurring fluctuation in its rate, Hipparchus supposed the Sun S to revolve on a fixed eccentric circle uniformly about the stationary centre C, which was displaced some distance from the stationary Earth E (Fig. 2; cp. Fig. 1, iii). From a knowledge of the approximate durations of Spring and Summer, and of the length of the year, he was able to calculate the constants of the Sun's orbit, viz. the *eccentricity* (the ratio CE : CA), and the direction, in relation to the equinoctial and solstitial points, of the *apse-line* AA' joining the Sun's *perigee* A and his *apogee* A'. Ptolemy adopted this representation of **the**

Sun's motion, and Copernicus retained it as a means **of** accounting for the principal solar inequality. Hipparchus sought, less successfully, to represent the inequality in the Moon's motion by assuming the satellite to describe an epicycle which was carried round upon a deferent intersecting the ecliptic in regressing nodes (cp. p. 23); so far as the five planets were concerned, Hipparchus merely classified and supplemented existing observations of these bodies, thus preparing the way for Ptolemy three centuries later. Hipparchus' reputed discovery of the precession of the equinoxes, his star-catalogue, and his attempt to determine the distances of the Sun and Moon, will be referred to later, at the point where they become relevant to our account of Copernicus.

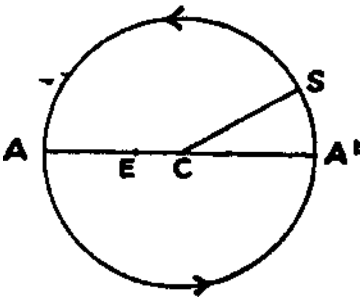


FIG. 2

The Babylonians seem to have made allowance (as already noted) not only for the inequality in a planet's motion which gives rise to the retrogressions, and which recurs in the planet's *synodic* period, but also for the inequality which depends upon the planet's position in the Zodiac, and which recurs in very nearly its *sidereal* period. Hipparchus was apparently aware of this **twofold** inequality ***(Aim.,* IX, *2).*** **and** he supposed **that, in** order **to represent** it, *combinations* **of** eccentrics **and** epicycles **would be required. A solution of this problem which survived, in its broad outlines, for fourteen centuries, was the essential contribution** to planetary theory made by Ptolemy of Alexandria *(fl. c.* A.D. 150).

Ptolemy's mode of representing the motions of Venus and of the three superior planets was, essentially, as follows (Fig. 3): The planet P described from west to east an epicycle whose centre C described in the same sense, a deferent whose stationary centre, O, was eccentric to the Earth, E. The point C was conceived to move with uniform angular velocity, not about the centre O (as in previous planetary theories), nor even about the Earth E, but about a point 0' lying in EO produced, and such that EO' = 2. EO. The motion attributed to Mercury was considerably more complicated ; and there were elaborate schemes for representing the motions of the planets in latitude.

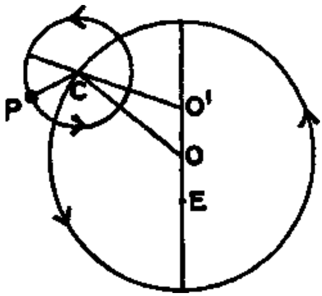


FIG. 3

Ptolemy's lunar theory may be summarized as follows (Fig. 4): The Moon was supposed to move on an epicycle MN, whose centre A moved from west to east on an eccentric deferent, whose centre F, in turn, revolved from east to west about the Earth E, the whole lying in the plane ABCD of the Moon's apparent motion. To an observer at E, the opposite motions of A and F were equal relatively to the line AC joining Earth and Sun. Thus the epicycle was at apogee on the eccentric at the times of new and of full moon, and at perigee at the time of half-moon. This theory (to which some further refinements were added) made allowance for a periodic fluctuation in the eccentricity of the Moon's orbit whose effects seem to have been first clearly distinguished by Ptolemy, and which is now known as the *evection* (see Chapter V, § 1 *infra)*.

Ptolemy's lunar and planetary theory completed the achievement of Greek and Alexandrian astronomy, whose roots lay deep in the ancient civilizations of the East. He wrought the whole into the comprehensive, logical system of astronomy set forth in a treatise, the *MAOHMATIKH ΣΥΝΤΑΞΙΣ* now commonly known as the *Almagest.*[1] The authority of this work (which was completed about A.D. 145) was solidly on the side of the geocentric theory, and it dominated all the developments of astronomy with which we are here concerned, down to the sixteenth century, when the book served as a quarry from which Copernicus extracted many of the data and geometrical methods which he employed in his effort to subvert that authority.

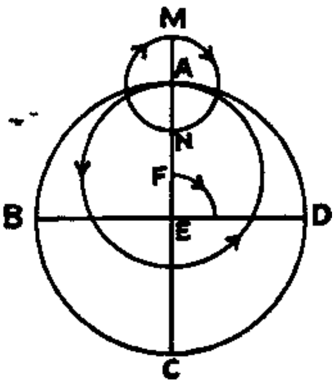


FIG. 4

The developments in cosmology and planetary theory during the fourteen centuries from Ptolemy to Copernicus

1 So called after the Arabic title, Al-majisti, which was formed either by prefixing the article *al* to the Greek adjective μεγίστη (greatest) applied to the *Syntaxis, or,* more probably, according to Sarton *(History of Science,* I, p. 562) from an artificial contraction of the words μεγάλη συνταξις *(great Collection)* by which the work was known to the commentators.

were relatively unimportant when compared with those of the seven centuries between Pythagoras and Ptolemy, and they may be dealt with more briefly. A succession of commentators and compilers kept alive the tradition of Alexandrian astronomy for several centuries after the death of Ptolemy. But even such activities ceased almost entirely in Christendom about the beginning of the sixth century, when the Athenian schools of philosophy were closed by the Emperor Justinian.

§2. THE MIDDLE AGES

A remarkable revival of the astronomical knowledge and activities of classical Antiquity occurred during the Middle Ages in the countries under Islam. On their conquering march westward through Egypt, and eastward to the Indus, the Arabs came into contact both with what remained of Alexandrian culture, and with Hellenic ideas originally implanted in the Middle East by the conquests of Alexander the Great. As the wave of Muslim expansion reached its limits, the Arabs fell under these influences, and scientific pursuits flourished at their courts and academies from the eighth century to about the thirteenth century, at Bagdad, in Egypt, and in Moorish Spain. Scholarly Arab potentates had Greek and Hindu scientific works translated into Arabic (including Ptolemy's *Syntaxis* about 820); and they founded observatories, where important astronomical constants were redetermined, star-catalogues were compiled, and tables of planetary motions were computed.

As to how the motions of the planets should be represented, the judgment of Muslim philosophers was divided between the two traditions respectively associated with the names of Aristotle and of Ptolemy. The complicated motions attributed by Ptolemy to the planets were clearly inconsistent with the physical principles of planetary motion laid down by Aristotle. Whether the acceptance of Ptolemaic astronomy necessarily involved the rejection of Aristotelian physics, however, depended upon what kind of status was to be assigned to planetary hypotheses such as those in the *Almagest.* Upon this question Muslim philosophers showed a difference of opinion* which had already arisen among the late Alexandrian writers, and which (as we shall see) was to reappear in the schools of Christendom. Some of them considered, with Averroes, that a planetary hypothesis should conform to the physical laws of motion of celestial bodies; they adhered to the teachings of Aristotle, and they rejected the Ptolemaic system altogether. Among these was the Spanish astronomer al-Bitrugi (twelfth century), who proposed a system of homocentric planetary spheres in which some allowance was made for the precession of the equinoxes; it proved a serious rival to the Ptolemaic system, and later gained considerable acceptance in northern Europe. Other thinkers regarded planetary hypotheses as mere artifices for systematizing the observed motions of the planets, and for computing tables to predict their future motions; they recognized the inadequacy of the conception of homocentric spheres for this purpose, and they accordingly adopted the Ptolemaic system. Others, again, sought to resolve the conflict by showing that Aristotle's words would bear interpretation in a sense broad enough to comprehend all that Ptolemy had postulated concerning the planetary motions. Several of the Arab disciples of Ptolemy tried to give a physical significance to the details of his system by substituting hypothetical, material mechanisms for the purely geometrical schemes of the *Almagest,* as, indeed, Ptolemy himself had sought to do in one of his later works, Ὑποθέσεις τῶν πλανωμένων *(Planetary Hypotheses).* They supposed each planet to turn upon an epicyclic sphere in the interspace between the concentric spherical surfaces of two aethereal solids which themselves slid freely over other solids. The interspace was eccentric to the Earth, and the solids were endowed with such rotations as to provide for the various phenomena of the planets' motions. One of the greatest of the Arab astronomers, al-Battani (ninth and tenth centuries), upon redetermining the elements of the Sun's apparent orbit, noticed that the longitude of its apogee differed appreciably from that given by Ptolemy, so that he is generally regarded as the discoverer of the progressive motion of the Sun's apses upon the ecliptic.

Despite centuries of concentration upon the details of the Ptolemaic planetary system, however, the Muslims made no significant improvements in principle upon it. They lacked the mathematical genius and the critical faculty of the Greeks; and, under the influence of Neo-Platonist writings (some of them falsely ascribed to Aristotle), their 'natural philosophers became involved in fantastic speculations concerning the hierarchy of Intelligences which were supposed to animate the successive planetary spheres. However, they left many recorded observations which were utilized by succeeding astronomers, including Copernicus.

They transmitted to the West our system of numerals, and the conceptions of the *sine* and of other trigonometrical functions. Above all, it was the Muslims who kept alive the tradition of ancient science and philosophy, and who preserved many of the texts in which it was enshrined, during the period of intellectual stagnation in Europe.

About the eleventh century this tradition began to be communicated to western Christendom, where it produced a considerable widening of the intellectual horizon. The conversion of the Roman Empire to nominal Christianity in the fourth century had been followed by the irruption of the Barbarians; the tradition of ancient science was submerged, and the civilization which arose on the ruins of the Western Empire mostly derived its ideas about the Universe from the teachings of the Church. The earlier ecclesiastical writers had sought to harmonize the cosmology of the Schools with the words of Scripture (allegorically interpreted where necessary). But, with their rise to power, the leaders of the Church showed a growing intolerance of the doctrines of classical science, so that, from about the fourth to the sixth centuries, even the spherical form of the Earth was frequently denied, and the structure of the Universe was sometimes conceived as analogous to that of the Tabernacle of Moses. In the eighth century, however, Bede gave an elementary account of the planetary motions, based upon the particulars in Pliny's *Natural History*. The scientific knowledge available in the West at this period was, in fact, practically limited to what could be gleaned from the writings of the Fathers of the Church, and from Pliny's book; and it was embodied in a succession of encyclopaedias and commentaries. By the end of the ninth century, Christian scholars had become acquainted, through the Latin writings of certain Neo-Platonist commentators, with the cosmological ideas of Plato's *Timaeus* (whose influence upon[4] mediaeval Christian thought remained almost unchallenged down to the twelfth century), and with the planetary hypothesis of Heraclides of Pontus (§ I *supra)*, allusions to which occur throughout mediaeval literature down to the time of Copernicus.

From about A.D. 1000 the influence of Muslim science began to be felt in western Christendom. During the twelfth century, Latin translations were made from the Arabic versions of some of the chief scientific and philosophical works of Antiquity. These classics included Ptolemy's *Almagest* (translated from the Arabic by Gerard of Cremona in 1175), and the physical and astronomical works of Aristotle. (A direct translation of the *Almagest* from the Greek had been made about 1160, but it never became widely known.) Following the discovery of the astronomical systems of Aristotle, of Ptolemy, and of al-Bitrugi, the old controversy over the status of planetary hypotheses and the rival claims of these systems to represent physical reality, was renewed in the schools of western Christendom. It figures largely in the writings of the great scholastic Doctors of the thirteenth century,—Albertus Magnus, who favoured the Ptolemaic theory, and Roger Bacon and St. Thomas Aquinas, who admitted that the system of eccentrics and epicycles agreed best with the facts of observation, while conscious that it was incompatible with the physical principles which alone satisfied the reason, and who awaited some solution of the problem which should be acceptable in all respects. These philosophers, however, dealt with astronomical topics from a purely formal standpoint, without introducing any new facts or principles of scientific value. By the end of the thirteenth century, the homocentric systems of astronomy had been almost entirely superseded by the Ptolemaic theory (usually conceived, at this period, in terms of material mechanisms constraining the motions of the planets); this was especially so in the University of Paris, where a group of astronomers revived the practice of observation as a means of testing the available planetary tables,—the *Toleian Tables,* edited by al-Zarqali *(c.* 1080), which at this period were yielding place to the *Alfonsine Tables* (1272), constructed under the patronage of King Alfonso X of Castile. (In Italy, however, a last attempt was made, by Fracastoro, less than ten years before the publication of Copernicus' book, to represent the planetary motions by means of an elaborate combination of spheres.) The natural philosophy of Aristotle was also criticized, from the religious standpoint, as teaching, or implying, doctrines which conflicted with Jewish, Muslim, and Christian orthodoxy alike. Its study was even prohibited, though ineffectively, in Paris at the beginning of the thirteenth century. St. Thomas Aquinas *(d.* 1274), however, harmonized as much as possible of the doctrines of Aristotle and his Muslim commentators, with Christian theology, to form a type of philosophy against whose conservative authority Copernicus, and the other pioneers of modern science, had later to contend.

The disorders of the Hundred Years' War and the Papal Schism brought about the eclipse of the Parisian School of astronomers. In the fifteenth century, however, there occurred a remarkable revival of astronomical studies, chiefly associated with the Italian and German Universities, which was part of the wider movement of the Renaissance. It was stimulated by the recovery and widespread study of the classics of ancient science and philosophy in their original Greek, instead of in corrupt translations; by the invention of printing, which made possible the rapid multiplication of copies of these works, and the easy diffusion of new ideas; and by the demands of ocean navigators, in an age of competitive exploration and trade, for the nautical instruments and tables which they required on their voyages. This revival culminated in the achievements of Copernicus, and with it we reach the end of the period covered by this introductory survey.

CHAPTER II

THE LIFE-STORY OF COPERNICUS

BEFORE proceeding to discuss the contents of the historic book which was to bring the age-long supremacy of the Ptolemaic system to an end, we shall endeavour to give some account of the life-history of its author. The generations immediately following Copernicus allowed many precious memorials of him to perish, and the reconstruction of the great astronomer's career and intellectual development effected by the laborious researches of the past century still lacks completeness. For the substance of the following biographical sketch, we shall rely principally upon the elaborate studies of Prowe and Birkenmajer.

§ i. BIRTH AND PARENTAGE

The astronomer whom we know as Nicolaus Copernicus devised that appellation for himself, according to the scholarly custom of his day, by latinizing both his Christian name of Niklas, and the family surname, which was variously spelt in contemporary records, but which seems to occur most frequently in the form Koppernigk. He employed the Latin form only in his learned publications, and occasionally in letters and inscriptions; for ordinary official purposes he wrote his name Coppernic. He usually doubled the *p*, and this may perhaps be regarded as the more correct spelling. But in the last few years of his life, he seems to have preferred to spell his Latin name with a single *p*; this is the spelling found in the *De Revolutionibus*, and preserved through all the editions of that book; it is, moreover, that which has usually been adopted by English writers, and it will be retained in the present work.

Copernicus was born on February 19, 1473, at Thorn, on the Vistula, in a house still identified by tradition. The town, which is now in Poland, had been founded in the thirteenth century by the Teutonic Knights to form an outpost of the independent state which they had carved out for themselves by their conquest of the heathen Prussians. During the fourteenth century Thorn had flourished as a port of the Hanseatic League, and an *entrepdt* for the trade of western Europe with Poland; but, by the end of the fifteenth century, the town had lost much of its former prosperity through the rivalry of Danzig, and the frequent hostilities between Poland and the Teutonic Knights. Shortly before the birth of Copernicus, the rebellious subjects of the now decadent Order made common cause with the Poles; the Knights lost their independence and much of their territory, and Thorn passed, with West Prussia, under the suzerainty of the King of Poland.

The astronomer's father, who also bore the name Niklas Koppernigk, was a well-to-do merchant of Cracow (then the chief town of Poland), who migrated, not later than 1458, to Thorn, where he prospered and became a magistrate. Little else is known about him, and his nationality has been a matter of keen controversy between German and Polish writers, all anxious to claim his illustrious son as a fellow-countryman. The fact that Cracow, as well as Thorn, was founded by Germans, and that in the fifteenth century the populations (and especially the wealthier classes) of both towns were preponderantly German, makes it inherently probable that Copernicus was of German blood. Prowe has shown that, while the family name occurs both at Cracow and at Thorn in legal records dating back to the end of the fourteenth century, there is some documentary support for the view that the astronomer's ancestors originally came from Silesia, where the name may once have betokened some connection with the copper-mining industry. The prevalent spelling of the name with a double a also has some bearing upon the controversy, since there is normally no doubling of consonants in Polish.

Shortly after settling at Thorn, Niklas Koppernigk married Barbara Watzelrode, the daughter of a wealthy Thorn merchant belonging to an established German family from which the town had drawn many of its councillors and magistrates. There were four children by the marriage, of whom Niklas, the future astronomer, was the youngest. He had a brother, Andreas, later the companion of his foreign travels and studies, and two sisters—Katharina, who married a merchant of Cracow, and Barbara, who became a nun.

§2. YOUTHFUL STUDIES AND TRAVELS

When Niklas was ten years of age his father died, and the children were adopted by their maternal uncle, Lucas Watzelrode (1447-1512), a man of strong character, who, after a distinguished academic career in Italy had entered the Church, and was now well on his way to a bishopric. Lucas sent his nephew to school at Thorn, and thence, in 1491, to the University of Cracow, which, at that period, enjoyed a reputation second to none among the northern Universities. A brilliant school of mathematics and astronomy had been built up at Cracow by Albert Brudzewski, with whom Copernicus was almost certainly brought into personal touch during his years of residence there. Although adhering to the Ptolemaic system in his published works, Brudzewski was a man of liberal sympathies. He was on friendly terms with leaders of the new humanistic movement, which, emanating from Italy, had already begun at Cracow to challenge the traditional scholastic discipline, and to produce such cleavages in the University as occasionally led to street fighting between rival student factions. It was probably at the Polish University that Copernicus became accustomed to the use of astronomical instruments, and to the practice of observing the heavens.

After spending probably three years at Cracow, Copernicus returned home. His uncle had been, since 1489, the Bishop of Ermland (or Varmia), one of the four dioceses into which Prussia had been divided. It formed a little principality enjoying a large measure of independence, the Bishop's palace being situated at Heilsberg, and the Cathedral at Frauenburg on the coast. Bishop Lucas was anxious to provide for his nephew's future by securing his election to a canonry of Frauenburg Cathedral. The first attempt proved unsuccessful, and the young astronomer was given permission to resume his University studies, this time in Italy. Setting out in 1496, he made his way across the Alps to Bologna, and for the next four years he studied there in the school of law for which the city had long been famous. In 1498 he was joined by his brother Andreas, who had been with him at Cracow. They were both enrolled in the *Natio Germanorum*—the most considerable of all the "nations" into which foreign students were organized at Bologna. The two young men seem to have joined freely in the student life of the city, and upon at least one occasion they were obliged to appeal urgently for funds to an emissary from Ermland, who happened to be within reach. While at Bologna, Copernicus came into close personal touch with Domenico Maria da Novara (1454-1504), the Professor of Astronomy there; it is possible that he even lodged in Novara's house. Novara was a brilliant teacher, and a critically minded observer. He believed (mistakenly) that he had discovered a systematic increase, since the time of Ptolemy, in the latitudes of several places in southern Europe, which he attributed to a progressive displacement of the pole of the heavens. He was also one of those who detected the diminution that had occurred in the obliquity of the ecliptic since ancient times. These considerations disposed Novara to sit rather lightly to the accepted system of astronomy. Moreover, he was one of the leaders in a revival of Platonism which was just then sweeping over southern Europe. In the spirit of this movement he would strive to conceive the constitution of the Universe in terms of simple mathematical relations; and unfettered converse with such a man must have encouraged Copernicus in any plans which he might already have framed for the reform of astronomy along similar lines. Novara and Copernicus observed the heavens together while they had opportunity, the younger man being, according to Rheticus, "not so much a pupil as a helper and witness of the observations" *(Narratio Prima)*.

The earliest observation of his own which Copernicus explicitly utilized in his book (an occultation of Aldebaran by the Moon, *De Rev.*, IV, 27) belongs to this Bologna period. Copernicus did not graduate at Bologna. In the spring of 1500 he went to Rome to be present at the Easter celebrations of that great Jubilee Year, and he remained for a whole twelvemonth in the city, teaching mathematics privately; to this period, also, belongs an observation of a lunar eclipse which he subsequently employed in constructing his lunar theory (*De Rev.*, IV, 14).

About 1497 Copernicus had been elected a Canon of Frauenburg in his absence; his brother had secured similar preferment in 1499. Like many ecclesiastics of his day, Copernicus appears to have entered the service of the Church from temporal motives, and never to have proceeded beyond the vows necessary for his admission to the Chapter. In the summer of 1501 the two young men returned to Ermland to crave further leave of absence, so that they might continue their studies in Italy. This was granted, and Copernicus set out for Padua, where he completed his legal studies. It was at about this period, and probably at Padua, that Copernicus learned Greek; he thereby gained direct access to the works of Plato, and to the other Greek writings from which he later claimed to have derived inspiration. Copernicus did not graduate at Padua, but went on to Ferrara to take his doctorate in Canon Law in 1503. Returning to Padua, he embarked on the study of medicine, which was mostly taught in those days out of the works of authorities such as Galen and Avicenna, with occasional dissections to illustrate these. It was not unusual, at that period, for a churchman to learn something of the healing art, though he was expected to eschew surgery, Copernicus probably did not graduate in medicine, and, by the beginning of 1506, his years of study abroad were ended, and he was back in Ermland.

Just as Copernicus was at last settling down to his official duties at Frauenburg, however, he was bidden to take up his residence at the Bishop's palace, some forty miles away, to act as medical adviser to his uncle, whose health was uncertain. His next six years, until Lucas Watzelrode's death in 1512, were accordingly ,spent in the stately castle of Heilsberg. It was most probably here that Copernicus began to give a definite literary form to the new system of cosmology whose elaboration was to preoccupy him during the remaining thirty years of his life. The first fruit of this activity was the short synopsis of the new planetary system, in its earlier form, given by Copernicus in his manuscript *Commentariolus* (see Note III), if we follow Birkenmajer in assigning its composition to 1512 or earlier, as against the much later date previously adopted by Curtze. It was during these years at Heilsberg, too, that Copernicus became acquainted at first hand with the intricacies of Ermland politics. The little principality was hard put to it to maintain its independence against two overbearing and mutually hostile neighbours—Poland, and the Teutonic Knights (whose territory surrounded it on three sides). To the Bishop fell also the delicate task of mediating between the German population of West Prussia, which aspired to complete national freedom, and the Poles, who were bent upon incorporating the province in their **kingdom.** Copernicus accompanied, or represented, his **uncle on a number of diplo-**

matic missions. While staying at Cracow in 1509, he took the opportunity to secure the publication of his own Latin translation of the Greek Epistles of the Byzantine poet Theophylactus Simocatta. This version, which Copernicus dedicated to his uncle, was the only book he ever published apart from his great work on astronomy. It was on the return journey from another visit of the Bishop and his nephew to Cracow, three years later, that Lucas Watzelrode was overtaken by mortal illness; he was carried to Thorn, his birthplace, and there he died, at the end of March 1512. With the death of his protector, Copernicus' attendance at Heilsberg came to an end, and he returned to Frauenburg. In later years, however, he was not infrequently summoned to the palace to give medical advice and treatment to the elderly and ailing Bishops who successively occupied the see; there is also a credible tradition that the poor of the district were allowed to benefit by his skill. Two years before his death, Copernicus was hastily summoned to Konigsberg by the Duke of Prussia to attend one of his counsellors who had fallen dangerously ill. He returned to Frauenburg after about a month's absence, leaving the patient out of danger. A number of Copernicus' medical books have been preserved, together with recipes written in his own hand on margins and fly-leaves; they give the impression that, in matters of physic, Copernicus followed the accepted authorities, and applied the customary remedies, of his day.

§3. THE CANON OF FRAUENBURG

Frauenburg Cathedral, about which the life of Copernicus was henceforward to revolve, stands on a low hill overlooking the Frisches Haff, a fresh-water lagoon opening into the Gulf of Danzig. It was fortified against the perils of those disordered times by a surrounding wall; and a long-standing tradition claims the three-storeyed tower which forms the north-west corner of the enclosure, as the astronomer's old abode. Several of his new companions were natives of Thorn, who would be personally known to him. He was tragically deprived, however, of the society of his brother Andreas, who, shortly after his return from Italy, developed an incurable disease, probably leprosy, and was forced to retire from his cathedral duties; after vainly seeking relief abroad, he died, not later than 1519.

It was at Frauenburg that Copernicus made nearly all those observations of his own which he utilized in the numerical determination of the elements of his planetary theory. His book contains twenty-seven of his own recorded observations; but if to these we add those written on the fly-leaves and margins of books, etc., the total amounts to more than double that number. Of his activities and resources as an observer, relatively little is known. His instrumental equipment at Frauenburg seems to have been of a modest order. The instruments whose construction and use he describes in his book are those of Ptolemy and his successors. There is, in the first place, an account of a device for determining the meridian altitude of the Sim *{De Rev.,* I I , 2; cp. *Aim.,* I, 10). It consisted of **a** square slab of stone or metal, one of whose faces was made exactly plane. Upon this face, with one of **the upper** comers as centre, **there** was **engraved** a quadrant **of a circle,** which **was divided into degrees and minutes of arc. A short cylindrical style was** attached at the geometrical centre of the quadrant, and at right angles to its plane. The quadrant was set up in the plane of the meridian, and the shadow of the style cast at noon upon the graduated arc indicated the Sun's meridian altitude. This instrument was suited to determine the inclination of the ecliptic to the celestial equator—an angle given by half the difference between the meridian altitudes of the Sun at the summer and winter solstices respectively. Copernicus seems to have possessed such an instrument, and to have employed it to determine this angle, which he found to have suffered a diminution since Ptolemy's day *(De Rev., loc. cit.)*. Copernicus also explains the construction and use of the Ptolemaic astrolabe *(De Rev.,* II, 14; cp. *Aim.,* V, 1); this was a combination of concentric metal rings which were made respectively to coincide with the planes of the principal circles of the celestial sphere. The direction of a celestial body could be referred to graduations on these rings with the aid of movable indices, and its celestial co-ordinates (e.g. longitude and latitude) could thus be determined. The instrument chiefly employed by Copernicus in his own observations, however, was of the type known as a *triquetrum*, which he called, after Ptolemy, an *instrumentutn parallacticum (De Rev.,* IV, 15; cp. *Aim.,* V, 12), and which he employed to measure altitudes of the heavenly bodies. This instrument, which, according to Gassendi, he constructed with his own hands, consisted of three graduated pine-wood rulers (Fig. 5); one of these, AB, was fixed in a vertical position, and it had, at its upper end, a pin A about which the second ruler, AC, was free to turn in a vertical plane. This second ruler carried two sights, and it could be directed accurately towards any celestial body whose angular distance from the zenith (or elevation above the horizon) was required. On these two rulers, and at equal distances from the pin A, were two other pins, B and C. About B the third graduated ruler,

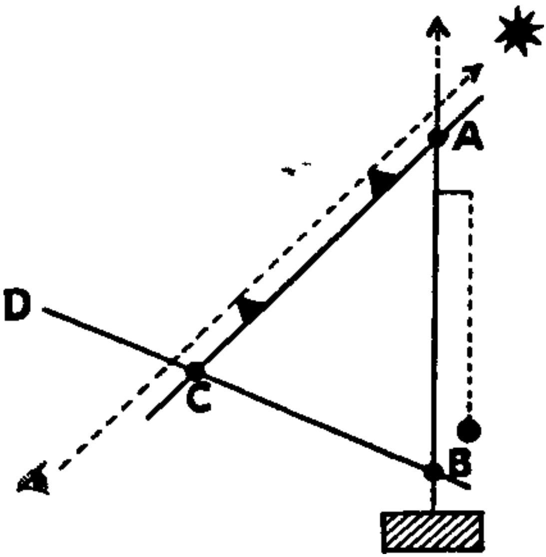


FIG. 5

BD, was free to turn; it was longer than the other two, and at each division it had a perforation into which the pin C could be inserted. It thus served as a cross-piece to hold the two rulers AB, AC at any required angle with each other. The length of this cross-piece intercepted between B and C was the chord of the angle BAC which the line of sight CA made with the vertical; this angle could be deduced from the length of BC with the aid of a Table of Chords (see

Note II); it gave the zenith-distance of the object towards which the line of sights was directed. To facilitate the calculation, the lengths AB, AC, were each divided into 1,000 units, and the cross-piece was made long enough to contain at least 1,414 (= 1,000 *z)* of these units, so that all values of the angle BAC up to 90^0 could be measured, to a certain degree of approximation. The triquetrum of Copernicus was preserved at Frauenburg for some forty years after the astronomer's death; it was then given to Tycho Brahe, who valued it highly; but its subsequent fate is unknown (see J. L. E. Dreyer: *Tycho Brahe,* Edinburgh, 1890, pp. 103,125). Copernicus never excelled as an observer, and he does not seem to have claimed for his results any high degree of refinement. "If only I can be correct to ten minutes of arc," he once said to his disciple Rheticus, "I shall be no less elated than Pythagoras is said to have been when he discovered the law of the right-angled triangle" (Rheticus: *Ephemerides Novae,* p. 6).

In 1514 Copernicus was invited by the Lateran Council to assist in a proposed attempt to reform the calendar, which had become deranged, with the lapse of centuries, partly through the over-estimate of the mean length of the civil year in the Julian reform, and partly through the inaccuracy of the relation assumed to hold between the lengths of the lunar month and the tropical year. Copernicus pointed out, however, that any attempt at reform would be nugatory unless the motions of the Sun and Moon were first precisely ascertained; but he promised to keep the problem in mind; he refers to it at the end of the Preface of his book of 1543 as partly justifying his efforts to refashion

astronomy, and, indeed, the improved tables based upon the Copernican theory paved the way for Gregory's reform later in the sixteenth century.

During the early years at Frauenburg, his part in the ordinary duties of the little community left Copernicus ample leisure for pursuits of a more philosophical character. But his wide experience of the world, and the knowledge of affairs which he had acquired while in attendance upon his uncle, marked the young canon out for duties of especial responsibility. Accordingly, in November 1516, he was appointed to administer the temporal and spiritual affairs of some outlying estates belonging to the Chapter. He held this commission for three years, and, subsequently, for a further half-year (1520-21). During this period he lived at Allenstein Castle, only occasionally visiting Frauenburg to see his old friends and to make isolated celestial observations. His term of office fell at a time of growing difficulty and danger for the whole of Ermland. The war-clouds were gathering between Poland and East Prussia; they broke, at the end of 1519, in a campaign of pillage in which the little principality suffered severely from the plundering bands of the Teutonic Knights. Heilsberg was bombarded; Frauenburg had to repel a raid; and Copernicus, at Allenstein, was, for a time, threatened by the forces of the Order. The hour of crisis revealed remarkable qualities of leadership and resourcefulness in Copernicus. He **had** to undertake exceptional responsibilities during the dispersal of the Chapter to various places **of refuge; when the armistice of 1521 had brought large-scale hostilities to an end, he was active in resettling the deserted Allenstein estates; and it fell to him**

to draw up a memorial on the wrongs suffered by Ermland in the war, **and** to present it at the peace conference.

About this period the debasement of the Prussian coinage, which had been aggravated by the war, was giving rise to much inconvenience and hardship. Copernicus gave this matter very earnest attention. His analysis of the causes and evil consequences of such debasement, and his recommendations for remedying it, were set forth in 1522 in a memorandum, written in German, which he subsequently revised, and drew up in Latin, for presentation to the Prussian *Landtag* of 1528. He urged that the minting of coins should be a State monopoly, instead of each city or district having its own currency; that the quantity of money in circulation should be controlled; that in each denomination the coins should contain not less than a certain weight of precious metal apiece (though this might be alloyed to give the coins bulk and hardness), and that the old coinage should be withdrawn when the new was issued, in order to prevent the good new coins from being bought up and melted down. Transitional hardships attending the fulfilment of contracts based on the old currency, would have to be sympathetically considered. These recommendations of Copernicus were made the basis of legislation. Under more favourable conditions he might have rendered his country services as distinguished as those of Newton at the Mint; but the obstructive tactics of interested powers prevented any effective action for the time being.

About the same **period Copernicus circulated copies of an** open **letter to** his **friend Bernhard** Wapowski, **severely criticizing the speculations of a certain Niirnberg astronomer,** Johann Werner, who had sought to revive the old hypothesis that the equinoctial points oscillate slowly about their mean positions (see Chapter IV, § 2 *infra)*.

In 1523 Copernicus was appointed Administrator-General of the diocese during a six months' interregnum between two Bishops. Thereafter, however, his more arduous duties and responsibilities gradually passed into the hands of younger men. By now the political background had become less alarming, the Grand Master of the Teutonic Knights having agreed to become a hereditary secular duke under the suzerainty of the King of Poland. On the other hand, the astronomer's later years were somewhat troubled by the advent of an unsympathetic Bishop, by the domestic dissensions of the Chapter and its disputes with the King of Poland over the right to elect Bishops, and especially by the unrest and cleavage accompanying the spread of Lutheran doctrines from Germany, and the attempts to repress these. Judging by the tone of a polemical book published in 1525, at Copernicus' instigation, by his old friend Tiedemann Giese, a fellow-member of the Chapter, Copernicus seems to have been orthodox in his opposition to Luther, but anxious to resolve the conflict in a conciliatory spirit, and to avoid the disruption of the Church.

In the spring of 1539 Copernicus received an unexpected visit from a young German scholar, Georg Joachim von Lauchen (1514-76), who had adopted the appellation Rheticus after the old Roman name *Rhaetia* of the district of his birth. Rheticus was a *protege* of Melanchthon, and, although only twenty-five years old, he was already a Professor of Mathematics at the Protestant University of

Wittenberg. He explained that his interest had been awakened by what he had heard of the doctrines of Copernicus, and that a keen desire to know more about them had brought him all the way to Frauenburg. Rheticus was cordially received by the astronomer, who gave him every assistance in mastering the intricacies of the new system of cosmology, both by permitting him to consult the manuscript in which it was set forth, and by way of personal explanation. Rheticus had promised to send details of the Copernican system to his old teacher, Johann Schoner, the Nurnberg astronomer, should his mission to Frauenburg succeed. After some ten weeks of study and discussion of Copernicus' manuscript, Rheticus drew up an account of its contents and addressed it to Schoner; it was published, with the approval of Copernicus, at Danzig, in 1540, under the title *Narratio Prima de libris revolutionum.* This was the earliest explicit account of the Copernican system to be published (for a short summary of its contents, see Note I I I). Rheticus spent over two years staying at Frauenburg, or travelling about the district. He made many acquaintances, among them Copernicus' old and trusted friend Tiedemann Giese, now Bishop of Kulm. The prolonged visit of Rheticus to Copernicus was not without peril to both of them. For Rheticus had come from a stronghold of Protestantism into the jurisdiction of a Bishop notorious for his stern measures against heretics, and he had been welcomed by a man whose opinions were at that time more obnoxious to the Wittenberg authorities than to orthodox Catholics. Nevertheless, Rheticus lingered in Prussia until the autumn of 1541, when he left to resume his duties at Wittenberg. He took back

with him a transcript of the two chapters of the *De Revolutionibus* (1,13 and 14) which deal with the elements of plane and spherical trigonometry (mostly collected and generalized from the *Almagest),* and he published them (separately from the main work) at Wittenberg in 1542, under the title *De lateribus et angulis triangulorum turn planorum rectilineorum, turn sphaericorum libellus,* etc. Rheticus further sought to express his gratitude to his host and instructor by the gift of a number of recently printed books on mathematics and astronomy; these included the-first Greek edition of Ptolemy's *Almagest* (Basle, 1538). They formed a notable addition to the little private library which Copernicus bequeathed to the Cathedral at his death; these books were later removed to Sweden by Gustavus Adolphus, and many of them are now preserved at Upsala.

In the Preface to his great work of 1543, Copernicus alludes to the urgent exhortations of Tiedemann Giese and other friends that he should publish the manuscript which he had kept under periodical revision for some thirty years *("in quartum novennium").* The gist of his teachings had become generally known among scholars through the circulation of the *Commentariolus.* This little tract probably served to provide the material for a lecture on the Copernican system which was delivered in 1533 to Pope Clement VII and his court, by the papal secretary, Johann Widmanstad, in the gardens of the Vatican. This lecture prompted Cardinal Schonberg, who, as *nuncio* in Poland and Prussia, had met Copernicus years before, to write to the astronomer from Rome in 1536, strongly urging him to make the full details known to the learned world, and, in any case, to send particulars to the Cardinal himself, at the latter's expense. Copernicus had reason to be well pleased with the laudatory tone of this letter, coming from such a quarter; and it appears in the place of honour in his book. (Fame of a different sort came to him through a satirical play staged at Elbing, a few miles from Frauenburg, which poked fun at the star-gazing ecclesiastic and his extraordinary opinions.) Moved by the persistent entreaties of his friends, by the youthful enthusiasm of Rheticus (whose *Narratio* had fulfilled a useful preparatory purpose), and, perhaps by the sense that his own life was drawing towards its close, Copernicus at length consented to the publication of his book. He entrusted the manuscript to Giese, who sent it to Rheticus, probably in accordance with a provisional arrangement already made during the young Protestant's visit to Prussia. The work was printed at Nurnberg, and was published early in 1543.

During the winter, however, Copernicus had been overtaken by serious illness. A paralytic stroke supervened, leaving no hope of his recovery. For weeks he lay awaiting the final summons. At length, on May 24, 1543, an advance copy of the newly published book was brought to him. He saw and handled his completed work, and some hours later passed away.

§4. THE COMPOSITION AND PUBLICATION OF THE *De Revolutionibus*

While the external facts concerning the life-history of Copernicus can be traced in some detail, we are almost entirely without information as to the motives and influences which may have prompted him to undertake the reformation of Astronomy, and as to any transitional stages through which his planetary theory may have passed before it reached the form in which it is presented in the *De Revolutionibus.* The elaborate investigations of Dr. L. A. Birkenmajer (see Bibliography) have, however, thrown much light, both upon the sources of information which Copernicus utilized, and upon the gradual development of his *technique.* Birkenmajer has made a critical examination of the original manuscript of the *De Revolutionibur,* and of the notes written in Copernicus' own hand which have been found in some of the books formerly in his possession, as well as of other relevant material. From the results of his researches it appears that Copernicus acquired his copies of several fundamental works on mathematics and astronomy while he was still a student at Cracow (c. 1491-94); these included the *Alfonsine Tables,* and the *Tabulae Directionum* of Regiomontanus. The manuscript notes of Copernicus in the former book appear to date back to the Cracow period of his career, and they include calculations apparently relating to the heliocentric planetary scheme presented in the *Commentariolus.* This would seem to suggest that Copernicus took the first steps in the construction of his system some time before his departure for Italy. The numerical data for the *Commentariolus* seem all to have been derived from the *Alfonsine Tables.* The chief sources of information employed in the composition of the *De Revolutionibus,* however, were the *Epitome in Almagestum* (1496) of Purbach and Regiomontanus, and the Latin translation of the *Almagest* by Gerard of Cremona (Chapter I, §2 *supra),* published at

Venice in 1515. The Greek edition of the *Almagest* (Basel, 1538), which Copernicus received as a present from Rheticus shortly before his death, came too late to be of any use to him in the elaboration of his system.

The original manuscript of the *De Revolutionibus* affords evidence that the heliocentric theory underwent a gradual development in the mind of Copernicus, as regards its precise form; moreover, it warrants inferences as to what the stages of this development were, and when they were reached. During the three centuries following the publication of the book, the manuscript passed from hand to hand; it ceased to be recognized for what it was, and all trace of it had been lost when, in the middle of the nineteenth century, it was fortunately discovered in a nobleman's library at Prague. The manuscript is not simply a fair copy written out after Copernicus had put the last touches to his masterpiece; it is a heterogeneous document, embodying numerous and extensive alterations, insertions, and cancellations, evidently made at various dates. It has been the aim of critics to analyse the final text, and the suppressed passages, into successive layers, as it were, and to assign to each of these an approximate period of composition, based upon the dated observations embodied therein, or upon other, more indirect, evidence. The manuscript was examined by Curtze for the purpose of establishing the text of the 1873 edition; later it was scrutinized more minutely by L. A. Birkenmajer and A. Czuczynski, who took account of the variations in the characteristics of the handwriting, and in the quality of the paper and ink employed. Curtze formed the opinion that the manuscript had undergone **two** successive recensions. The second of these was presumably not completed before 1529, as the final version contained particulars of an observation made in that year (see Introduction to edition of 1873). Birkenmajer assigned the two revisions of the manuscript to the periods 1515-19 and 1523-32 respectively; and he thought that probably some finishing touches were applied about 1540, during Rheticus' visit. Among the cancelled passages of the manuscript he found traces of a geometrical theory of the planetary motions in longitude, differing slightly from that set forth in the published work. It evidently corresponded to an early stage in the elaboration of the Copernican system; and it is of interest to note that it is this earlier form of the theory that is outlined in the *Comtnentariolus* (see Note III). Birkenmajer was convinced that the editors of 1873 had misread the manuscript in a number of places, and, further, that some of the textual alterations previously attributed to Copernicus had in fact been made by another hand, in time, however, for them to be followed by the printers of the 1543 edition. He also compared the manuscript of the *De Revolutionibus* with that of the unpublished and long-lost commentary on Copernicus by Erasmus Reinhold, which he discovered at Berlin, and he came to the conclusion that Reinhold must have seen Copernicus' manuscript before it went to press, and that it was probably he who interfered with the text. There are also remarkable discrepancies between the manuscript and the printed edition of 1543. The manuscript doubtless called for some editorial attention before it could be sent to the press. For it nowhere bears the name **of** its **author, and it** is **untitled; internal evidence** suggests **that**

Copernicus intended to entitle it *De Revolutionibus* simply. The divergences of the printed text from the manuscript are, however, so considerable and arbitrary as to suggest to the editors of 1873 that the Niirnberg printers did not work from the manuscript, but from a transcript made by someone acquainted with astronomy, who sought to improve the style, and did not scruple to make unauthorized additions and omissions. It may be that Reinhold was the scribe. There were, however, other exceptional circumstances attending the first appearance of the *De Revolutionibus,* as we must now relate.

The task of printing and publishing Copernicus' book was entrusted by Rheticus to his friend Johann Petrejus of Nurnberg. Rheticus seems to have intended to see the book through the press himself, but, before the work had progressed very far, he was obliged to leave Nurnberg for Leipzig. He handed over the task of supervision to Andreas Osiander, a local Lutheran theologian and mathematician of some note. Osiander had previously had some correspondence with Copernicus, portions of which were subsequently published by Kepler. It seems that Copernicus inquired (July 1, 1540) whether it would be possible for him to publish his theory of the Earth's motion without exciting hostile criticism. Osiander replied (April 20, 1541): 'Tor my part, I have always felt about hypotheses that they are not articles of faith, but bases of calculation, so that, even if they be false, it matters not so long as they exactly represent the phenomena of the [celestial] motions. . . . It would therefore seem an excellent thing for you to touch a little on this point in the Preface. For you would thus render more complacent the Aristotelians and theologians whose contradiction you fear" (Kepler's *Apologia Tychonis contra Ursum).* Copernicus, who had a very different notion of an astronomical theory (see Chapter III, § 5 *infra),* ignored this advice. But, as we have seen, a train of circumstances placed Osiander in control of the final stages of the publication of the book; and he took advantage of the situation to insert, in the most prominent position, a brief Preface such as he had counselled Copernicus to prepare. Scholars will doubtless be shocked (writes Osiande4*by the unsettling hypothesis of the Earth's motion set forth in this book. They should remember, however, that the astronomer is not concerned with the true causes of celestial motions. He is therefore free to adopt any hypothesis which may enable him to give a geometrical representation of the motions that have been observed in the past, and to predict the motions that will occur in the future. Such an hypothesis need not be true or even probable; it is sufficient that it should lead to results in agreement with the facts of observation, and be the simplest hypothesis capable of so doing. Osiander's *Praefatiuncula,* as Gassendi called it, probably represented a well-meaning effort to disarm criticism, and to ensure a favourable reception for the book; and it seems to have succeeded in its object. It is anonymous, but it can scarcely be called a forgery, since it does not purport to be by Copernicus, to whom it refers in the third person, and in laudatory terms. Its authorship early became known to one or two, but was first revealed to the learned world in general by Kepler *(Apologia Tychonis contra Ursum, cap.* I; *Ed.* Frisch, **Vol. I, p. 245. Also *Astronomia* Nova: *Aucior Ramo,* on the** *verso* of the title-page; *Ed.* Frisch, Vol. I l l , p. 136). To Osiander also would seem to be due the "blurb" on the title-page of the *De Revolutionibus,* bidding the public "buy, read, and enjoy" the book.

Of the later editions of the *De Revolutionibus,* those of 1566, 1617, and 1854 (see Bibliography) followed the printed text of 1543. The *Sacular-Ausgabe* of 1873, however, which was published in celebration of the fourth centenary of the birth of Copernicus, was based for the first time upon a critical study of the original manuscript.

CHAPTER III

THE MOBILITY OF THE EARTH

THE ancient Eastern peoples, whose culture formed the matrix of Greek intellectual development, conceived the Earth as a stationary platform having a central or otherwise privileged position in the Universe. Similarly, in the archaic Greek cosmologies the Earth figured as a central, motionless body having the form of a disc, cylindrical frustum or sphere, which floated on the primeval ocean or hovered in the abyss. And, in fact, it was the conception of the Earth as a motionless sphere, poised symmetrically at the centre of sphere-shaped space, which the ancients finally embraced, and which they imposed upon mediaeval thought. True, there were not wanting in Hellas bold thinkers who deliberately allowed for the possibility of the Earth's being neither stationary nor central in space, and who partly realized how the construction of planetary hypotheses might thereby be simplified. Speculative progress along these lines might have led, and indeed, with Aristarchus did momentarily attain, to a completely heliocentric system. But it was prematurely arrested by a combination of factors, among which must be included (i) the reluctance of naive common sense to believe in a motion of the Earth not directly perceptible; (ii) the influence of religious conservatism, anxious to claim a unique and privileged status for man's abode, and successively manifested in Greek, Muslim, **and** Christian circles; (iii) **the growing authority of Aristotle, whose philosophical** arguments were solidly in support of the geocentric theory; and (iv) the relative excellence of the planetary tables constructed by Ptolemy and his successors from the standpoint of that theory. Thus it was that the conception of the Earth as motionless and centrally situated prevailed throughout the Middle Ages. The gradual supersession of the geocentric theory in the sixteenth and seventeenth centuries dates from the appearance of Copernicus' great work of 1543. It will be our task, in this and the three following chapters, to explain what is of most significance in the contents of that remarkable book.

§ 1. THE SCOPE AND PLAN OF THE "DE REVOLUTIONIBUS"

The whole contents and arrangement of Copernicus' book *De Revolutionibus* had better be indicated briefly at the outset. The body of the work falls into six Books, each subdivided into a number of chapters. In Book I, Copernicus sets forth his general arguments for believing in the mobility of the Earth, and for substituting the heliocentric for the geocentric point of view; he sketches the heliocentric arrangement of the solar system in broad outlines, and he gives the modern explanation of the seasons. The Book concludes with some elementary plane and spherical trigonometry, including a Table of Chords, or, more precisely, of sines (see Note II). Book II deals with spherical astronomy (definitions of the circles of the celestial sphere, transformation of co-ordinates, etc.), and treats of problems connected with the rising and setting of the Sun and of other heavenly bodies. As the treatment of such diurnal phenomena is independent of rival physical theories as to their causation, no part of this Book will again concern us except the star-catalogue with which it concludes. Coming down now to detailed geometrical schemes of the motions in the solar system, Copernicus treats of the Earth's several motions, and the elements of its orbit, in Book III. In Book IV he deals with the theory of the Moon's motions, and with the determination of the distances of the Sun and Moon. In Book V, the longest and the most vital of the six, he investigates the motions in longitude of the five planets, and the sizes of their orbits in relation to that of the Earth. In Book VI the motions of the planets in latitude are considered.

We shall proceed now to a more detailed study of the significant portions of Copernicus' book. In the present chapter we shall examine his grounds for questioning, and his arguments for finally rejecting, the accepted verdict of authority on the matter of the Earth's status in the Universe.

§2. THE APOLOGIA OF COPERNICUS

When Copernicus had at last consented to the publication of his manuscript, he took the bold course of dedicating the work to the reigning Pope, the scholarly Paul III. In the Dedicatory Preface he claims for his speculations the Pope's interest and protection, giving some account of how they first took shape. He admits that he has long hesitated to publish his book for fear of the censure which its doctrine of the Earth's motion might incur; but his friends have at last prevailed upon him to commit it to the press. The inadequacy of the planetary theories so far proposed, and the great diversity existing among them, must be his chief excuses for adding yet another to their number. The theories which traced their descent from the homocentric spheres of Eudoxus might, he thought, be in accordance with sound physical principles; but there seemed no prospect of their ever affording a precise representation of the planetary phenomena. The rival theories, which had developed from the eccentrics and epicycles of the Alexandrian astronomers, yielded tables of practical value; but they admitted much that was contrary to sound physics, and the Universe, as conceived on such lines, was a monstrosity. Copernicus relates how, in his disappointment at such a condition of things, he turned to the ancient philosophers to see what alternative theories they might have proposed. He found that Cicero attributed a belief in the motion of the Earth to one Hicetas, and that similar statements were made by Plutarch about Philolaus, Ecphantus, and Heraclides of Pontus. We have already noted the opinions ascribed to these men in antiquity, and we shall consider later the passages *to* which Copernicus here alludes, and the part that they may have played in the development of his own ideas (see § 6 *infra).* Whether Copernicus really derived his inspiration from these classic passages, or whether he merely quoted them for the sake of the impression which they would produce upon his readers, we cannot be certain. At all events he makes them the point of departure for his own exploration of the problem:

"Taking occasion thence," he writes, "I too began to reflect upon the Earth's capacity for motion. And though the idea appeared absurd, yet I knew that others before me had been allowed freedom to imagine what circles they pleased in order to represent the phenomena of the heavenly bodies. I therefore deemed that it would readily be granted to me also to try whether, by assuming the Earth to have a certain motion, representations more valid than those of others could be found for the revolution of the heavenly spheres.

"And so, having assumed those motions which I attribute to the Earth farther on in the book, I found at length, by much long-continued application, that, if the motions of the remaining planets be referred to the revolution of the Earth, and be calculated according to the period of each planet, then not only would the planetary phenomena follow as a consequence, but the order of succession and the dimensions of the planets, and of all the spheres, and the heaven itself, would be so bound together that in no part could anything be transposed without the disordering of the other parts and of the entire Universe."

So Copernicus commends his book to the Pope, whose authority and scholarly fame will surely protect it from the bite of calumny. As for the detraction of those unskilled in mathematics, he will disregard it: *Mathemata Mathematicis scribuntur* (Mathematics are written for mathematicians).

§3. THE NEW ASTRONOMY AND THE OLD PHYSICS

Copernicus follows the practice of Ptolemy, and of other old writers on astronomy, by beginning his treatise with a number of general physical propositions relating to the Earth and to the Universe as a whole. In the first four chapters of the *De Revolutionisms* he keeps close to the corresponding introductory portions of the *Almagest.* Thus we read that the Universe is spherical, for the form of the sphere is the most perfect and capacious, and so forth (I, i; cp. *Aim.*, I, 2). The Earth also is a sphere, if we neglect surface irregularities (I, 2). The reasons here adduced are the sound ones of *Almagest*, I, 3: anyone travelling northward sees a proportionate increase in the elevation of the north pole of the heavens above the horizon; a given eclipse appears later in the day to dwellers in the East than to those in the West, and departing ships seem to sink gradually below the horizon. The ocean fills up the depressions in the Earth's surface; land and water possess the same centre of gravity, so as to cast upon the eclipsed Moon an invariably circular shadow (I, 3; cp. Aristotle: *De Caelo*, II, 14). Chapter 4 brings Copernicus to a cardinal doctrine of classical astronomy: that the motions of the heavenly bodies are uniform, eternal, and circular or compounded of circular motions. This doctrine, which was probably of Pythagorean origin, was supported by the authority of Aristotle *{De Caelo*, I, 2, 3, and II, 6), and Ptolemy gave it his formal assent *[Aim.*, III, 3), thoiigh Copernicus considered him to have departed from it in practice in admitting that the centre of an epicycle might move non-uniformly about the centre of its deferent (e.g. in *Aim.*, IX, 5). Copernicus regards circular motion of the planets as alone compatible with the regular recurrences which we observe in their phenomena. "For it is the circle alone which can bring back again what has already taken place" (I, 4). True, a planet does not appear to move uniformly, and its distance from us seems to vary. A heavenly body, however, cannot move in a single circle at a variable rate, for this would argue variableness in the motive force, or else in the body moved, and the mind recoils alike from either explanation. Hence we must attribute the planetary inequalities either to the multiplicity of the component motions affecting the planets, or else to the Earth's being displaced from the common centre of those motions.

How, then, is the Earth related, in respect of its position and possible motion, to the rest of the Universe? Previous writers, says Copernicus, have generally assumed, almost as a foregone conclusion, that the Earth is at rest at the centre of the Universe; but the matter is not beyond dispute. "For every apparent change of position is due, either to a motion of the object observed, or to a motion of the observer, or to unequal changes in the positions of both. . . . If, then, a certain motion be assigned to the Earth, it will appear as a similar but oppositely directed motion affecting all things exterior to the Earth, as if we were passing them by" (I, 5). Now the daily rotation of the heavens is a motion affecting everything exterior to the Earth, and "if you will allow that the heavens have no part in this motion, but that the Earth turns from west to east, then, so far as pertains to the apparent rising and setting of the Sun, Moon, and stars, you will find, if you think carefully, that these things occur in this way" (I, 5). Next, concerning the position of the Earth in the Universe, Copernicus finds it almost universally held that the Earth is at the centre. But "if someone states that the Earth does not occupy the centre of the Universe, but nevertheless does not admit that its displacement is so great as to be comparable with the sphere of fixed stars, though appreciable and obvious in comparison with the spheres of the Sun and of the other planets; if then he supposes that the motion of those planets will therefore appear non-uniform, being referred to a centre other than the centre of the Earth, he will perchance be able to offer a not unfitting explanation of this non-uniform apparent motion" (I, 5). That is, the planetary inequalities might reasonably be explained by supposing the Earth to be displaced from the centre of the planetary motions by an amount comparable with the distances of the planets, but incomparably less than the distance of the fixed stars. Such a displacement must indeed be negligible compared with the dimensions of the Universe, for the horizon divides the Zodiac and the celestial sphere into two equal parts (I, 6; cp. *Aim.*, I, 5). It does not follow, however, that the Earth must be *at rest* at the centre; rather it appears incredible that a Universe so immense should revolve in twenty-four hours, while its least part, the Earth, remains at rest.

Copernicus next addresses himself (I, 7) to the arguments based on mechanical grounds which the ancients had directed against all theories involving the motion of the Earth, or its displacement from the centre of the Universe. To understand the objections of this type which Copernicus had to overcome it is necessary to revert for a moment to the cosmological ideas of Aristotle (Chapter I, § 1 *supra).*

In the Aristotelian system the elementary bodies constituting the Earth and filling the whole region within the Moon's sphere differed from the aethereal bodies forming the surrounding heavens, not only in their substance, but also in their natural modes of motion. Thus it was supposed that, while elementary bodies moved naturally in straight lines, outwards from the centre of the Universe, or inwards towards the centre, celestial bodies revolved eternally in circles round the centre. These celestial motions were supposed to be maintained by virtue of an incorporeal "unmoved mover," or by a plurality of such movers, inspiring the spheres to an activity represented by their uniform rotation. The rectilinear motions of the terrestrial elements, on the other hand, were attributed to a sifting agency of space itself, whereby these elements were relegated by "natural motions" to their "natural places," i.e. to the layers in which they were respectively supposed to congregate. Such laws of motion were not derived from intelligent experimentation, but were suggested by mere appearances, support for them being sought in plausible deductions from very general statements about the supposed nature of things (many of them little more than popular maxims), or sometimes even from the etymology of the terms employed. Thus, in this matter of natural motions, Aristotle lays it down *(De Caelo,* I, 2) that every motion must be either rectilinear or circular, or compounded of the two, and that the most excellent motion is that which can go on unaltered for ever. Now rectilinear motion cannot be indefinitely maintained in a finite universe without sooner or later being stopped at the boundary of that universe; it is hence inferior to circular motion, which can be so maintained. But the natural motion of each terrestrial element is manifestly rectilinear—fire and air move straight upwards, earth and water straight downwards. Hence there must be some superior element to which circular motion is natural, and this is readily identified with the aether composing the heavens, and the heavenly bodies.

Now Copernicus so far belonged to his age as not to find any fault with mechanical principles of this sort, which indeed were not effectively challenged until about a century after his death. His concern was only to rebut Aristotle's and Ptolemy's application of such principles to prove that the Earth must be at rest. For his own part, he employs closely similar, and to a modern mind equally artificial and worthless, mechanical arguments, to prove that the Earth is more probably in motion. We may summarize as follows the typical mechanical arguments with which Copernicus particularly sets himself to deal.

(i) A simple substance possesses a single natural motion, directed towards, away from, or round, the centre of the Universe. Earth and water have rectilinear downward motions; air and fire have rectilinear upward motions. If the Earth performed a daily rotation, this principle of simple natural motions would be violated (cp. Aristotle: *De Caelo,* I, 2).

(ii) Heavy bodies tend to move in straight lines towards the centre of the Earth (as the centre of the Universe), and to come to rest there. They show no natural tendency to move in any other direction. Hence the whole Earth, which is simply a collection of such heavy bodies, can have no natural tendency to move in any other direction; and no motion of the Earth which was not natural could be eternal (cp. *De Caelo,* II, 14; *Aim.,* I, 6).

(iii) If the Earth were in motion, clouds and other bodies floating in the air would appear to be always travelling in the direction opposed to that motion (cp. ***Aim.,*** I, 6).

(iv) If the Earth performed a daily rotation, the rapidity of its motion would need to be enormous, and anything rotating is more likely to throw bodies off than to draw them to itself. The Earth would long since have been scattered abroad and life destroyed.

Let us look now at the way in which Copernicus meets these several objections (I, 8).

(i) He lays down the principle that every body, terrestrial as well as celestial, possesses a natural *circular* motion. So long as the body remains in its "natural place," this is the only motion that it does possess. Rectilinear motion, however, is superadded to this in any body which is out of its natural place. Thus rising and falling bodies appear to move perpendicularly upwards or downwards because, being parts of the Earth, they partake of the Earth's circular motion; when they are at rest on its surface, they possess this motion *only*.

(ii) Copernicus does not explicitly distinguish this objection from (i), but it clearly vanishes if one is at liberty to add any (natural) motion of the Earth on to the apparent rectilinear motion of a terrestrial body. Moreover, in Chapter 9 Copernicus takes a view of gravity which has an important bearing on this point.

"I am of opinion," he writes, "that gravity is nothing but a certain natural tendency to draw together, which is implanted in parts by the divine providence of the Maker of all things, that they may collect themselves into unity and completeness, being assembled into the form of a sphere. It is to be supposed that this influence resides also in the **Sun, Moon, and other planets in order that, by its agency,** they may remain in that globular shape in which they appear. Nevertheless, these bodies perform their divers circuits" (I, 9).

Now if gravity be thus attributed to bodies demonstrably in motion, terrestrial gravity cannot be admitted as evidence that the Earth is stationary.

(iii) As for the behaviour of clouds, etc., we must suppose that a considerable proportion of the atmosphere, together with the bodies floating in it, is carried round with the Earth in its motion, just as, on the traditional view, the upper portion of the atmosphere partook of the diurnal motion of the heavens, and carried round comets in its course.

(iv) Copernicus denies that the Earth would be disrupted by a diurnal rotation. For it is implied that such motion would be *natural,* producing effects contrary to those produced by *violence.* Why not rather fear the disruption of the Universe, whose parts, on the traditional view, must move with a celerity incomparably greater than that involved in the rotation which it is now proposed to attribute to the Earth?

Summing up, Copernicus concludes that it appears more probable that the Earth is in motion than that it is at rest.

Can the Earth, then, be regarded as a planet? If it possesses any motions, corresponding motions must appear in many external objects (I, 9), in accordance with the principle of the reciprocity of apparent motions already laid down. Copernicus proceeds to apply this principle to the case of the Sun's annual circuit.

"If [this circuit] be transposed from being a solar to being a terrestrial [phenomenon], and it be granted that the Sun is at rest, then the risings and settings of the signs and the fixed stars, whereby they become morning and evening stars, will appear after the same manner [as before]. The stations, retrogressions, and progressions of the planets will also be seen to be a motion of the Earth, not belonging to the planets themselves, but borrowed by them in their apparent behaviour. Lastly, the Sun himself will be deemed to occupy the centre of the Universe, all of which we are taught by the order in which the planets succeed one another, and by the harmony of the entire Universe, if we will but look at the matter with both eyes, as they say" (I, 9).

Thus, in the first nine chapters of the *De Revoluiionibus* Copernicus clears away what his contemporaries would consider the chief objections to regarding the Earth as one of the planets. He has next to establish the relation in which the planet Earth stands to the other bodies of the Universe.

§4. THE COPERNICAN UNIVERSE

Copernicus approaches his new geometrical scheme of the solar system through a discussion of the order in which the planets succeed one another with increasing distance from the centre of the Universe (I, 10; cp. *Aim.*, IX, 1). Ever since the time of Anaximenes (sixth century B.c.) the stars had been recognized, by common consent, as the most distant of the visible celestial bodies. They were thought **to** be situated upon, or within **the** substance of, **a crystal** sphere, which formed, in **the Aristotelian** system, **the boundary of** space. **Its** characteristic **motion was the diurnal rotation about the Earth in which all the seven planets participated. In order to account for Hipparchus' discovery** of the precession of the equinoxes (Chapter IV, § 2 *infra)*, Ptolemy thought it necessary to attribute to the stellar sphere a slow rotation about the axis of the ecliptic, in addition to the diurnal one about the axis of the equator. Now, in the ancient systems of planetary spheres, each component sphere, as we have seen, possessed only a single rotation of its own. In order that the stellar sphere should conform to this rule, Ptolemy (in his *Planetary Hypotheses)* postulated a sphere exterior to that of the stars, and possessing the diurnal rotation only; this motion it transmitted to the stellar sphere, whose own characteristic motion was now simply that required to account for the precession. During the Middle Ages, however, the idea established itself that the sphere of stars, besides showing a continuous rotation about the axis of the ecliptic, was subject also to a small, periodic disturbance which caused the equinoctial points to oscillate about their mean positions (see Chapter IV, § 2 *infra)*. In the time of Copernicus it was customary to attribute this oscillatory motion to the sphere of stars (which was called the *eighth* sphere because additional to the systems of the seven planets), and to postulate two further spheres, called the ninth and the tenth spheres respectively, exterior to the eighth. Of these, the ninth imparted the precessional motion, and the tenth (or *primum mobile)* transmitted the diurnal motion, to all the systems lying within. Outside of all these spheres there was the motionless Empyrean—the abode of God and of the saints. This system is depicted in Plate I, Fig. I, which is reproduced from the 1533 edition of the *Cosmographicus Liber* of Peter Apian, a contemporary of Copernicus.

PLATE I

FIG. I—THE UNIVERSE ACCORDING TO PETER APIAN (1533)

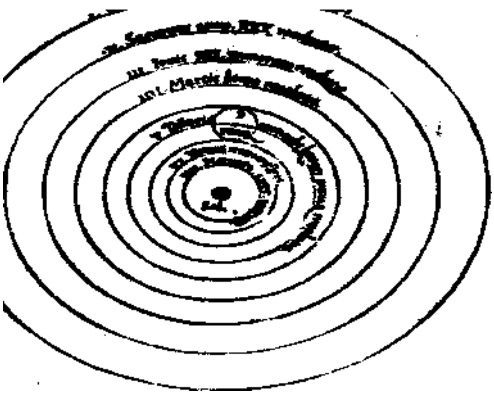


FIG. II—THE UNIVERSE ACCORDING TO COPERNICUS **(1543)**

Copernicus retains the traditional conception of a finite space bounded by the sphere of stars; only now he places the Sun instead of the Earth at its centre. The ancients supposed that the distances of the Moon, the Sun, Mars, Jupiter, and Saturn, from the centre of the Universe, increased in the order in which these bodies are here mentioned—presumably because the periods in which they complete their respective circuits of the Zodiac increase in that order, from the Moon's period of one month up to Saturn's thirty years. As to the positions of Venus and Mercury in relation to the Sun, however, there was much uncertainty. These planets, unlike Mars, Jupiter, and Saturn, always keep within moderate angular distances of the Sun. If, as Ptolemy supposed, their paths lay between the Earth and the Sun, it was argued that they should occasionally be seen against the solar disc, and that, if they were dark bodies, they might even show phases like the Moon, when their illuminated hemispheres were partly turned away from us. If, on the other hand, they were more distant than the Sun, as Plato taught in the *Timaeus,* there must be a great empty space between the Sun and the Moon, contrary to the prevailing belief that the sphere of one planet was contiguous to that of the succeeding one (see VI, § 2 *infra).*

Copernicus declares for the hypothesis of Heraclides, which he associates with the name of Martianus Capella (see § 6 *infra),* that Venus and Mercury describe circles with the Sun as centre; and he extends this hypothesis to the superior planets also. "**If** anyone **takes** occasion from this **to** refer Saturn, Jupiter, **and Mars also to the same centre, he will not err, provided he understands the sixe of their** spheres to be such as to contain and surround the Earth lying within" (I, 10). This assumption that Saturn, Jupiter, and Mars revolve about the Sun in orbits which embrace the Earth will explain why these planets are nearest to us when they rise in the evening, and farthest from us when they set in the evening. "But, indeed, if all these planets depend upon one centre, a space must be left between the outside of the sphere of Venus and the inside of the sphere of Mars, and an orb or sphere be separated off, concentric with the other spheres in respect of each of its boundaries, and capable of receiving the Earth with her handmaiden the Moon, and everything contained within the lunar sphere. . . . Wherefore we are not ashamed to confess that everything the Moon goes round, with the centre of the Earth, travels through that great orb among the other planets, with an annual revolution round the Sun. Further, that the centre of the Universe is in the Sun, which remains unmoved, so that whatever motion it appears to possess should more truly be ascribed to the mobility of the Earth. Also that the magnitude of the Universe is such that, while this displacement of the Earth from the Sun is of appreciable magnitude, compared with any of the other planetary spheres, it is inappreciable in comparison with the sphere of the fixed stars" (I, 10). Copernicus thus arrives at a scheme of things best illustrated, in its broad outlines, by his well-known diagram (Plate I, Fig. II).

"In the midst of all dwells the Sun. For who could set this luminary in another or better place in this most glorious temple, than whence he could at one and the same time lighten the whole? . , . And so, as if seated upon a royal throne, the Sun rules the family of the planets as they circle round him. . . . That nothing of these things appears in the fixed stars proves their immense distance above us, which is sufficient to cause even the annual orbit, or its appearance [in the stars], to vanish from our eyes" (I, 10). This last sentence calls for some further explanation. It is a matter of common experience that an observer in motion (e.g. a passenger in a moving railway train) is aware of progressive changes in the apparent directions and grouping of stationary objects in his vicinity. Such apparent displacement of an object due to a change in the position of the observer is known as *parallax.* It had been recognized from antiquity that an annual revolution of the Earth about the Sun should cause each star to appear to revolve about its mean position with an annual periodicity. For many centuries astronomers sought in vain for any traces of such parallax; and their failure constituted one of the most serious objections to all planetary hypotheses involving a translatory motion of the Earth. Aristarchus of Samos and Copernicus were both forced to meet this objection to the heliocentric system by postulating that the distance of the stars from us must be incomparably greater than the diameter of the Earth's orbit, so that our annual motion could have no sensible effect upon their apparent positions. The annual stellar parallax was first detected, and measured with reasonable accuracy, a century ago. The earliest authentic claim put forward was that of the great German astronomer F. W. Bessel of Konigsberg *(Astronomische Nachrichten,* Nos. 365, 366, December 13, 1838; and Nos. 401, 402, 1840), who established that the star 61 *Cygni* appears to oscillate annually about its mean position with an amplitude of about o"-3. Bessel was anticipated in the discovery of stellar parallax by a Scotsman, Thomas Henderson, whose results, however, were first made public some weeks after those of the German astronomer.

§5. THE STATUS OF THE COPERNICAN THEORY

In the discussion of the philosophical first principles of Astronomy which was carried on intermittently from the time of Plato to the sixteenth century, a fundamental distinction was drawn between two types of planetary hypotheses (cp. Chapter I, § 2 *supra).* Both of these aimed at "saving the phenomena" (accurately representing the facts of observation) by postulating for each planet a set of simple and regularly recurrent motions which, when compounded together, should produce the complicated motion exhibited by the body in question. But, on the one hand, there were hypotheses which, besides being adjusted to fit the facts of observation, were further restricted to conform to those principles of celestial physics to which the planets, as physical bodies, were thought to be subject. On the other hand, there was a more restricted type of hypothesis whose sole justification was that it conformed to, and systematized, the motions of a planet observed in the past, and that it enabled the future motions to be successfully predicted. It made no attempt to explain, by reference to physical principles, *why* the planet should possess the component motions attributed to it; and if two or more alternative sets of such motions represented the planet's behaviour equally well, it afforded no criterion (apart irom mere convenience) for preferring any particular one of them. A typical hypothesis of the former type was the planetary system of Aristotle; on the other hand, that of Ptolemy, with its accumulated empirical refinements, was a complex of hypotheses of the latter type.

In consequence of the exceptional circumstances under which the *De Revolutionibus* was published, some doubt was at first raised in the minds of many readers, who were not personally acquainted with Copernicus, as to the relation in which he considered his theory to stand to physical facts. Did he believe that the Earth actually possessed the motions which he assigned to it? Or did he regard his theory as a mere computing device, intended to facilitate the construction of improved planetary tables? This question would, perhaps, scarcely have arisen had it not been for Osiander's Preface (see Chapter II, § 4 *supra),* whose tendency was so obviously at variance with that of the rest of the book. It is soon answered if we turn to the authentic portions of the *De Revolutionibus,* where we note the following points.

(i) Copernicus designates his doctrine of the Earth's mobility by such words as *opinio, doctrina, cogitatio.*

(ii) He refers to himself as the man "into whose mind it came to dare to imagine some motion of the Earth, contrary to the received opinion of the mathematicians and well-nigh contrary to common sense" *(Dedicatory Preface).* He would scarcely have written in such terms about a computing device, since such an artifice is exempt from all physical restrictions.

(iii) Foreseeing that some may condemn his opinion as unscriptural, Copernicus likens such an attitude to that of

Lactantius, a Latin Father of the Church, who poured scorn on the idea that the Earth was a sphere *(Divinae Institutions,* **III,** 24). "It ought not to surprise scholars," writes Copernicus, "if they pour such scorn on us also" *{Dedicatory Preface).* Copernicus here implies that his doctrine is comparable to that of the *sphericity* of the Earth; and this point is made at the risk of offending the Pope by the reference to the ignorance of Lactantius. We may further recall the space devoted, and the important place assigned, in the *De Revolutionibus* to proofs of the physical admissibility of the Earth's motion.

(iv) On the other hand, in referring to the detailed geometrical schemes which represent the terrestrial and planetary motions, he ordinarily uses the word *hypothesis* (e.g. I, 11; IV, 3; V, 4).

(v) When, on several occasions, he describes or suggests alternative eccentric and epicyclic systems, his choice between them is in each case governed by reference to tradition or convenience (e.g. Ill, 15; III, 20; IV, 3; V, 4): "which of the two [systems] exists in the heavens," he writes in one such case, "it is not easy to distinguish" **(iii, 15).**

Thus the study of the *De Revolutionibus* leaves little room for doubt

(i) that Copernicus intended his hypothesis of the Earth's motion to be understood as a statement of physical fact, and not merely as a methodological device;

(ii) that, in regard to the technical and **numerical details** of his system, he adopted **the empirical Ptolemaic attitude *(Aim.,* XIII, 2), contenting himself "with the simplest geo-**

metrical scheme that would "save the phenomena" in terms of uniform circular motions.

§6. PRECURSORS OF COPERNICUS

In reviewing the contents of the *De Revolutionibus,* we shall have occasion to make numerous references to the *Almagest* of Ptolemy. From its pages Copernicus derived many of his observational data and geometrical devices, with or without acknowledgment 'The ideas which constitute Copernicus' essential contribution to astronomy, however, were certainly not derived from Ptolemy; if we are to trace them to any source, it must be sought in the speculations of a few classical and mediaeval thinkers who stood apart from the main current of opinion. Certain of the writings, or recorded teachings, of these men were known to Copernicus; how far they anticipated his doctrines may best be gathered from a study of several critical passages—in particular of those to which explicit reference is made in the *De Revolutionibus.*

We begin, then, with the extract from the *De Placitis Philosophorum* of pseudo-Plutarch, quoted in Greek in the *Dedicatory Preface* to Paul III:

"It is commonly maintained that the Earth is at rest. But Philolaus the Pythagorean held that it revolves round the Fire in an oblique circle in like manner to the Sun and Moon. Heraclides of Pontus and Ecphantus the Pythagorean suppose the Earth to move, not with a motion of translation, but after the manner of a wheel turning upon an axle about its own centre, from west to east" *(De Plac. Philosophy III.* 13).

In the same work, and only a few pages back from the foregoing passage, we read:

"Aristarchus places the Sun among the fixed stars, and holds that the Earth moves round the Sun's circle" *(ibid.,* I I , 24). Copernicus does not quote this latter passage; he mentions Aristarchus only three times (I I I , 2, 6, and 13), and in no case refers to his heliocentric theory. There is, however, in the original manuscript (at the close of I, 11) a long passage which Copernicus has scored out, but which contains the words: "Although we admit that the motion of the Sun and Moon might also be demonstrated on the assumption that the Earth is immovable, this agrees less with the other planets. It is probable that it was on these and similar grounds that Philolaus judged that the Earth moves, and some say that Aristarchus of Samos was of the same opinion" *(De Rev.,* 1873 ed., p. 34).

In the *Dedicatory Preface,* and again in I, 5, Copernicus alludes to Cicero's account of Hicetas (see Chapter I, § 1 *supra)*; he evidently has the following passage in mind:

"Hicetas of Syracuse, as Theophrastus relates, is of the opinion that the heavens, the Sun, the Moon, the stars, and, in short, all the heavenly bodies are at rest, and that nothing in the Universe moves except the Earth; and as the Earth turns and twists itself with extreme rapidity about its axis, all the same appearances are produced as if the heavens were in motion and the Earth stood still" (Cicero: *Academica,* I I , 39).

Lastly, in constructing his scheme of the solar system (I, 10), Copernicus refers to the account of the "Egyptian" (restricted heliocentric) system (see Chapter I, § 1 *supra)* given by Martianus Capella:

"For although Venus and Mercury show daily risings and settings, yet their orbits by no means surround the Earth, but encircle the Sun with a wider circuit. In short, they make the Sun the centre of their orbits" *(De Nuptiis Philologiae et Mercurii,* Liber VIII).

Copernicus appears to acknowledge no inspiration from later writers than Martianus, but some of his ideas seem to have been anticipated by at least two mediaeval scholars with whose works he was probably acquainted.

There is first a passage, pointed out by Duhem, in one of the works of Johannes Scotus Erigena, the ninth-century theologian, in which the heliocentric system described by Martianus is extended to embrace two of the superior planets:

"The planets, indeed, which revolve round [the Sun] change their hues according to the qualities of the regions in which they are travelling: I speak of Jupiter, Mars, Venus, and Mercury, which always describe their circles round the Sun" *(De Divisione Naturae,* III, 27).

The name of Copernicus is more often linked with that of Nicolaus de Cusa, or Cusanus, a fifteenth-century ecclesiastic. There are passages in his *De Docta Ignorantia* (written about 1440, and first printed in 1514) which are entirely in **the** spirit of Book I of the *De Revolutionibus.* Thus, having laid it down that the Universe is infinite, **and** can **therefore have** no centre, Cusanus continues:

"Since, then, **the Earth cannot be the centre [of the Universe],** it **cannot be entirely devoid of motion. . . . And since we cannot perceive motion except by reference to something fixed, such** as **poles or centres which we pre suppose in measuring motions . . . we find ourselves to be** at fault in all things, and we are surprised when we do not find the stars agreeing in their positions with the rules of the ancients, because we imagine that these held right views about centres and poles and measurements" *(op. cit.,* I I , II). Again:

"Now it is clear to us that the Earth is really in motion, though this may not be apparent to us, since we do not perceive motion except by a certain comparison with something fixed" *(ibid.,* I I , 12).

To sum up, it seems clear from such passages as these:

(i) that the possibility of explaining the apparent diurnal rotation of the celestial sphere by assuming an equal and opposite rotation of the Earth was fairly widely recognized in Antiquity;

(ii) that certain of the ancients had a theory that Venus and Mercury describe circles about the Sun in the centre;

(iii) that certain Pythagoreans suggested that the Earth might possess a motion of translation, though their system was in no sense a heliocentric one;

(iv) that Aristarchus anticipated the Copernican system in an undeveloped hypothesis;

(v) that Copernicus was acquainted with all these surmises when he developed his own system.

It appears, then, that the reformative ideas which we associate with Copernicus are not to be regarded as original products of his genius. His great contribution to Astronomy lay rather in his development of those ideas into a systematic planetary theory, capable of furnishing tables of an accuracy not before attained, and embodying a principle whose adoption was to make possible the triumphs of Kepler and Newton in the following century.

CHAPTER IV

THE COPERNICAN SYSTEM:

THEORY OF THE EARTH'S MOTION

HAVING thus assigned to the Earth a place among the planets, Copernicus has next to specify what motions must be attributed to it in order to account for the diurnal and seasonal phenomena, and for certain slow, secular changes in the Earth's axis of rotation and in the elements of its orbit.

§ I. DIURNAL AND ANNUAL MOTIONS OF THE EARTH

The motions which Copernicus attributes to the Earth (I, n) are essentially as follows (we shall consider the refinements of the theory in the succeeding two articles of this chapter):

(i) a diurnal rotation from west to east about the polar axis;

(ii) an annual revolution about the Sun from west to east in the plane of the ecliptic;

(iii) a variation in the inclination of the Earth's axis to the line joining Sun and Earth, of nearly the same period as (ii), but of opposite tendency.

The motion (i) accounts **for the daily apparent rotation of** the celestial sphere **about the poles of the heavens. The motion (ii) is the cause of the Sun's yearly apparent circuit through the zodiacal constellations. The motion** (iii), introduced as part of the explanation of the phenomena of the seasons, requires a little further explanation.

It was obvious to Copernicus that seasonal changes result from variations in the attitude of the Earth's axis to the Sun; and he saw that it would be possible to account for such variations by supposing the Earth's axis to preserve an approximately invariable direction relatively to the fixed stars. For let ABCD be the Earth's circular orbit in the plane of the ecliptic, with the Sun at its centre E (Fig. 6). Let EA, EB, EC, ED be directed towards the first points of the zodiacal Signs of Cancer, Libra, Capricomus, and Aries respectively. Let FGHI be the Earth's equator, intersecting the plane of the ecliptic in the diameter GAI. Draw FAH perpendicular to GAI in the plane of the equator, F being to the south and H to the north of the ecliptic. Let the Earth now travel round its orbit while GI and FH retain unaltered directions in relation to the fixed stars. When the Earth is at A, it will be the winter solstice, and the Sun will appear south of the equator, at the first point of Capricomus. If now the Earth travels to B, the Sun will appear to lie on the equator in the direction GI, i.e. at the first point of Aries, and it will be the spring equinox on the Earth. When the Earth reaches C, the Sun will appear at the summer solstitial point; and when it reaches D, the Sun will once again be on the equator, and we shall have the autumn equinox. We have here the explanation of the seasons which is given in modem text-books; the condition for its validity is that the Earth's axis of rotation should, to all appearance, continue to point in the same direction, while the Earth's centre describes a circle about the Sun. The mechanical ideas of a later age conduced to the instinctive expectation of such behaviour in an unconstrained rotating body. Copernicus, however, adopted the attitude of the classical writers on astronomy, who regarded a planet, epicycle, etc.,

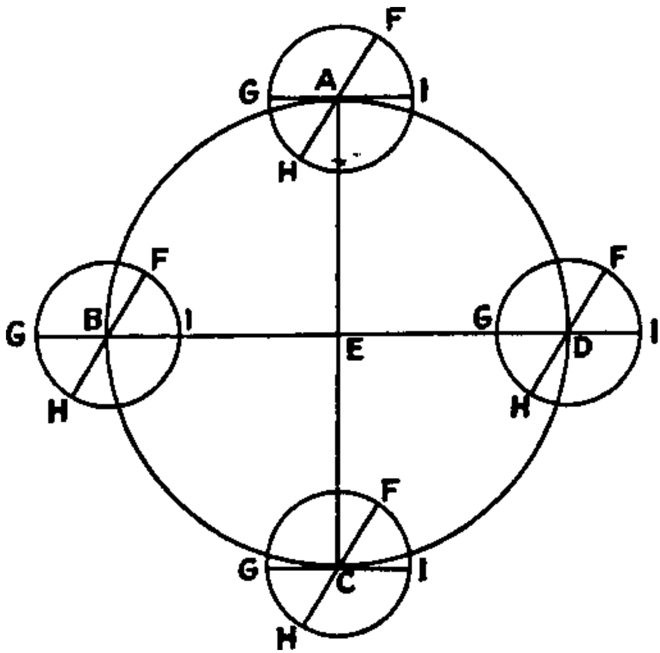


FIG. 6

as being carried round the centre of its revolution like an object attached to the rim of a wheel (cp. Chapter I, § 1 *supra)*. Such an object must present always the same aspect to the hub of the wheel, unless affected by extra motions of its own. Thus, for example, the ancients regarded the Moon as having no rotation of its own *because* it presents always the same face to the Earth (Aristotle, *De Caelo,* I I , 8).

And similarly Copernicus must have thought that the motion (ii), of itself, would cause the Earth's axis of rotation to remain inclined at a constant angle to the line joining Earth and Sun, thus describing a cone in space, and admitting of no seasonal fluctuations in the angular distance of the Sun from the equator. He therefore introduced the motion (iii), whose function it was to keep the direction of the Earth's axis invariable throughout the year, by causing it to describe a cone in space which should just neutralize the conical motion supposed to be produced by the revolution (ii). Kepler, however, writing about fifty years after Copernicus, recognized the superfluity of this motion (iii): 'The said motion," he wrote, "is, in truth, not motion at all, but rather rest" (see *Mysterium Cosmographicum, cap.* I, Author's Notes *g* and *p;* Frisch's edition, Vol. I, pp. 119 and 121). This principle, thus admitted by Kepler in 1596, received more explicit formulation from Newton a century later: "Suppose an uniform and exactly sphaerical globe [rotating about an axis]. . . . Because this globe is perfectly indifferent to all the axes that pass through its centre, nor has a greater propensity to one axis or to one situation of the axis than to any other, it is manifest that by its own force it will never change its axis, or the inclination of it" *(Principia,* Book I, Prop. LXVI, Cor. 22; Motte's Translation).

Besides the periodic diurnal and seasonal phenomena, however, there are certain slow changes in the relation of the Earth to the sphere of stars which constitutes the ultimate system of reference; and Copernicus makes use of the motion (iii) in his explanation of these changes. His treatment of this problem occupies the first twelve chapters of Book III of the *De Revolutionibus,* to which we shall now turn.

§2. THE PRECESSION OF THE EQUINOXES

The Earth's motions of diurnal rotation, and of annual revolution round the Sun (§ 1 *supra),* define two great circles on the celestial sphere: (i) the *celestial equator,* in which the plane of the Earth's equator cuts the celestial sphere, and which has as its poles the two points about which the sphere appears daily to rotate, and (ii) the *ecliptic,* in which the plane of the Earth's annual orbit cuts the celestial sphere. If the Earth's axis of rotation retained a constant direction in relation to the sphere of stars, and if the plane of its orbit were likewise invariable, then the two *equinoctial points,* in which these two great circles intersect, would remain fixed in relation to the stars. In actual fact, however, the equinoctial points are not stationary, but they move slowly round the ecliptic, relatively to the stars, in the opposite direction to that in which the Sun performs his yearly circuit. In consequence, a progressive increase is observed in the *longitudes* (Chapter I, § 1 *supra)* of all the stars, which are measured eastward from the vernal equinoctial point (where the Sim crosses the equator at the spring equinox). This steady increase in the stellar longitudes was recognized, in the second century B.C., by Hipparchus of Rhodes *(Aim.,* VII, 2), and Ptolemy interpreted it as due to a slow eastward rotation of the sphere of the stars about the poles of the ecliptic. In consequence of this motion, the period in which the Sim completes a circuit of the heavens relatively to the stars—the *sidereal year,* exceeds (by about 20 minutes) the period in which he completes a revolution relatively to the equinoctial points—the *tropical year.* The consequent recurrence of the spring equinox, and the beginning of a fresh seasonal cycle, *before* the expiry of a complete sidereal year from the last spring equinox, has caused the whole phenomenon to be termed the *precession of the equinoxes* (Copernicus uses the terms *praecessio, anticipatio, praeventio).*

Copernicus begins his third Book with a brief historical introduction to the problem of precession (III, 1). He then proceeds to explain the phenomenon as due, not to a motion of the stars, but to a continuous alteration in the plane of the Earth's equator, whereby the Earth's axis of rotation describes a cone about the axis of the ecliptic. In order to account for such a circulation of the Earth's axis, Copernicus merely postulates a slight inequality between the periods in which the two motions designated as (ii) and (iii) in the last article, are respectively completed. That is, he makes the period of the variation in the inclination of the Earth's axis to the line joining Earth and Sun, slightly less than the period of revolution of the Earth round the Sun, the *obliquity* (inclination of the equator to the ecliptic) meanwhile retaining a fairly constant value. "For . . . the two revolutions, I mean the annual one affecting the inclination, and that of the Earth's centre, are not exactly equal, the restoration of the inclination to its original value[x] occurring a little before the centre completes its circuit. Whence it necessarily follows that the equinoxes and solstices appear to fall early, not because the sphere of fixed stars moves eastward, but rather because the equator moves westward, remaining inclined to the ecliptic according to the measure of the inclination of the axis of the terrestrial globe" (III, 1). This conception of precession, as due to a motion of the Earth's axis, eventually estabhshed itself, especially after it had enabled Newton to give a dynamical explanation of the whole phenomenon); though nowadays we have further to take account of minute alterations in the plane of the *ecliptic*, unsuspected by Copernicus.

The diversity in the estimates of the rate of precession (i.e. of the rate of regression of the equinoctial points on the ecliptic) made throughout the centuries, had given rise to the idea that this rate was subject to considerable variations, and even that the equinoctial points, instead of moving continuously round the ecliptic, relatively to the stars, oscillated about their mean positions, with a period and amplitude which were variously estimated. This idea of a *trepidation* of the equinoctial points seems to have originated during the Alexandrian period; it prevailed among Indian writers on astronomy from about the fourth century A.D.; from them it spread to the Arabs, though the great al-Battani remained unconvinced of its truth. In the Latin version of the *Alfonsine Tables*, which became known to astronomers in northern Europe early in the fourteenth century, it was assumed that the sphere of stars was subject to a continuous precessional motion *plus* an oscillation about its mean position.

Copernicus, while rejecting the idea of the "trepidation" of the equinoctials, seeks to establish that the rate of precession is non-uniform, and to ascertain its periodicity and mean value, by reference to a series of recorded determinations of the longitudes of the bright stars *Spica, Regulus,* and ß *Scorpii,* made by the ancient astronomers Timocharis (third century B.C.), Hipparchus, Menelaus, and Ptolemy, by the Arab al-Battani, and by Copernicus himself (III, 2). Upon these data he bases a geometrical theory intended to account, both for the inequality in the rate of precession, and for a slow diminution in the values of the obliquity of the ecliptic recorded through the ages. Unfortunately, Copernicus never made any allowances for the possibility of serious errors in the actual observations of his predecessors (even supposing these to be genuine), or of textual corruptions in the manuscripts by which the results had been handed down. As Delambre remarks: "Apres avoir renverse le systeme des anciens, il n'ose meme suspecter leurs observations." This uncritical acceptance of recorded observations led Copernicus, in this instance, to construct a laborious theory of precession, having but little relation to the facts as at present understood, though perhaps of sufficient interest to justify a brief study.

Copernicus believed that the fluctuations in the rate of precession, and in the value of the obliquity, were related phenomena. He sought to account for them by attributing two independent oscillatory motions to the Earth's polar axis (III, 3). Each of these oscillations is to be performed "after the manner of suspended bodies," the motion taking place "over the same course, between two limits, most rapidly in the midst . . . and slowest at the extremities." (They are, in fact, what we should now call simple harmonic motions, as is clear from I I I , 4.) The motions are applied to the polar axis in the following manner:

Let ABCD be the ecliptic (Fig. 7), E its north pole, A, C the solstices, and B, D the equinoxes. Draw the solstitial colure AEC, and let F, G be the positions of maximum and minimum displacement of the pole of the equator from E, respectively corresponding to the maximum and the minimum obliquity of the ecliptic. Let BHD be the mean equator (cutting the ecliptic in the mean equinoctials B, D), and let J be its pole. Let J and the mean equator turn slowly and uniformly about E from east to west (clockwise in the Figure). This uniform motion constitutes the mean precession, and is accounted for, as we have seen, by a slight disparity between the rates of the annual motion of the Earth and of the compensatory motion of the Earth's axis.

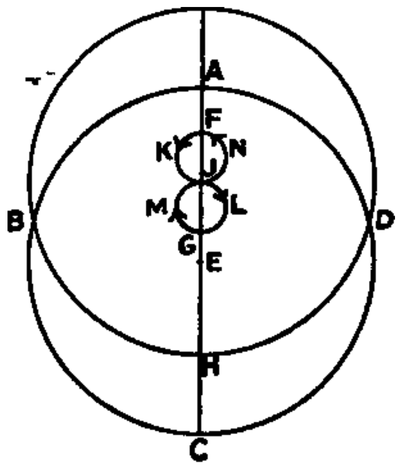


FIG. 7

Now suppose the pole of the equator to be affected by two simultaneous oscillations of the kind described above:

(i) an oscillation between F and G;

(ii) an oscillation perpendicular to FG, completed in half the period of (i). These two motions are to account for changes in the obliquity, and in the rate of precession, respectively, and their joint effect is to cause the pole to describe the figure-eight FKJLGMJNF on the celestial sphere (the opposite pole, of course, meanwhile traces out a corresponding figure in the reverse sense).

Copernicus has next to satisfy the demands of traditional physics by showing that rectilinear oscillations such as those above described can be produced by compounding certain uniform circular motions; he proceeds as follows (III, 4):

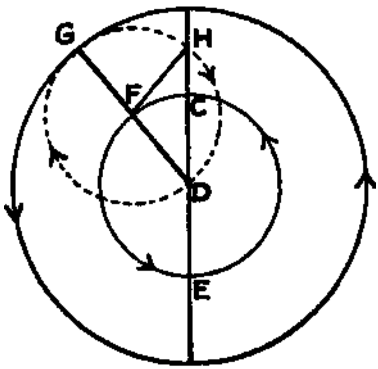


FIG. 8

Let the straight line AB (Fig. 8) be divided into four equal parts in C, D, E. With centre D and radii DA, DC, draw two circles. Let F be any point on the inner circle, and, with centre F and radius FD, draw a circle cutting AB in H, D. Let F describe the circle CFE, and let H describe the circle GHD with twice the velocity of F in its circle, and in the opposite direction. Then

$$Z.GFH - 2.GDH,$$

and it easily follows that H must lie constantly on the straight line AB, and must oscillate between the limits A and B. (Actually DH = DG.cos Z.GDH, and the motion is simple harmonic.)

The next step is to show that, in consequence of the oscillations of the pole of the equator, the equinoctial points vary about their mean positions, and the obliquity fluctuates about its mean value, with oscillations of the same kind as those postulated for the pole (III, 5). To show that this is true of the oscillations of the equinoctials, let the mean equator AEB cut the ecliptic CED at the mean equinoctial point E (Fig. 9), and let the pole P of the equator suffer a small' displacement to P' at right angles to the solstitial colure APB. Then the equator is displaced into the position shown by the dotted line AE'B (where P'E" = PE = 90°), cutting the ecliptic at the new equinoctial point E', where EE' is approximately PP' multiplied by a constant (the cosecant of the obliquity). Hence as P' oscillates about P, the equinoctial E' must oscillate about its mean position E, in a similar manner; this oscillation is, of course, superimposed upon the steady retrograde motion of the equinoctial accounted for in (III, 3), and it operates alternately to increase and to decrease the rate of precession, which is thus subject to a periodically recurring cycle of changes.

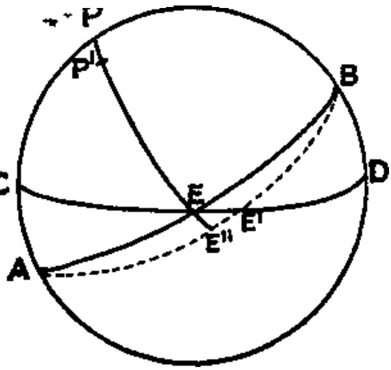


FIG. 9

Copernicus typifies this cycle by means of the circle ABCD (Fig. 10), the rate being least at A, increasing through its mean value at B, being greatest at C, and decreasing through its mean value at D. When any quantity shows periodic changes regulated according to such a cycle, it should be possible to recognize the occasions of its passing through the stages represented by the points A, B, C, D, respectively. Copernicus accordingly seeks to deduce, from an examination of the extant observations, the dates at which the rate of precession passed through these four points in its cycle, and hence, the total duration of the cycle (III, 6). He finds this period to be about 1,717 years, the minimum value of the rate of precession appearing to fall roughly midway between the dates of Timocharis and Ptolemy. This is, accordingly, the period in which, in Copernicus' scheme, the pole performs a complete transverse oscillation about its mean position, the period of oscillation assigned to the obliquity being twice as long (3,434 years).

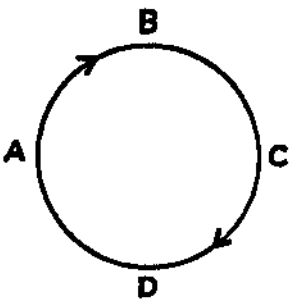


FIG. 10

The mean rate of precession is deduced from the total precession in 1,717 years (for which period the inequality cancels out), and it is found to be 1° 23' 40' per century (modern value 1° 23' 46"), the equinox performing a complete circuit of the ecliptic in 25,809 years.

Copernicus' mode of representing periodic variations such as those affecting the rate of precession, and of determining the values of quantities subject to them, must now be explained; it was essentially as follows:

Consider a circle ADBC (Fig. 11), with centre O, and two perpendicular diameters AB and CD. Take any point P on the circle, and draw PM perpendicular to AB. Now let P move round the circle so that the .COP increases uniformly with the time. Then the oscillation of M about 0 represents the type of fluctuation of a point about its mean position, or of a quantity about its mean value, now being considered. The ZCOP, which increases steadily with the time, and which governs the value of the variable quantity in the manner indicated, represents what Copernicus calls the *argument of the anomaly,* or, more shortly, the *anomaly.* (For example, the anomaly of the precession, in Copernicus' scheme just described, increases through 360^0 in 1,717 years.)

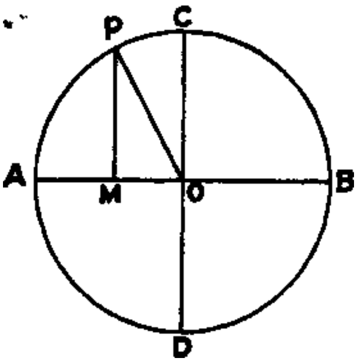
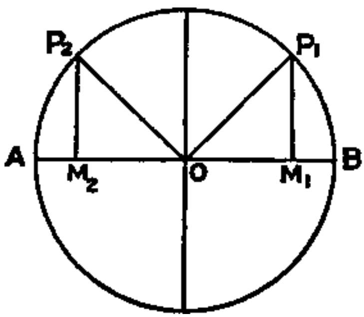


FIG. 11

Copernicus tabulates the mean precessional motion, and the increase in its anomaly, for daily and yearly intervals (III, 6). A knowledge of the anomaly is, in fact, necessary in the calculation, for any given date, of the *equation of the equinoxes* (the angular distance between the true and the mean equinoctials). Before it can be so employed, however, the maximum possible value of this equation has to be determined. This problem corresponds to that of finding the amplitude of a simple harmonic motion when the phase, and the displacement from the mean position, are given; and Copernicus proceeds somewhat as follows (III, 7): He

finds that in the 432 years that elapsed between two measurements of stellar longitudes, made by Timocharis and by Ptolemy respectively, the equinoctial point suffered an apparent displacement of 4^{0} 20', which differs from the *mean* precessional motion for 432 years (viz. 6° o') by i° 40'. His data further lead him to assume that, roughly midway between the observations of Timocharis and Ptolemy, the rate of precession must have passed through a minimum, when the mean and apparent equinoctials would coincide and the argument of the anomaly would be zero. Hence the anomaly must have had equal and opposite values at the beginning and end of the period of 432 years; and since, during this interval, the anomaly must have increased through $(360 \times \frac{432}{1717})^{\circ}$ or 90^{0} 35', its initial and final values may be represented in a diagram (Fig. 12) constructed on the principle of Fig. n by the points *P_v* P_2 where $\angle P_1OC = \angle COP_2 = \frac{1}{2}(90^{\circ}\ 35') = 45^{\circ}\ 17\frac{1}{2}'$. Draw PiMp P_2M_2 perpendicular to AB. In this Figure, AOB may be regarded as an arc of the ecliptic (treated as a straight line), having the mean equinoctial O at.its centre, and M_{1f} M_2 as the true equinoctials at the beginning and end of the 432 years. Then the arc M_1M_2 represents the difference between the mean and the apparent precessional



motions of that period, which (as already stated) was 1° 40'. Hence

$$OM_1 = OM_2 = 50', \text{ and}$$

$$OA = \left(50 \times \frac{\text{radius of circle}}{\text{half-chord of } 90^\circ\, 35'}\right)'$$

$$= (50 \times \tfrac{10000}{7107})' \text{ (from the Table of Chords)}$$

$= 1^\circ\, 10'$, which is the required maximum departure of the true from the mean equinoctial.

The equation of the equinoxes can now be computed for any given date (III, 8): given the amplitude and phase of a simple harmonic motion, it is possible to find the corresponding displacement of the moving point. Copernicus tabulates the equation for each 3° of the anomaly from 0° to 360°; and he provides *radices,* specifying the place of the mean equinox and the value of the anomaly, at certain standard epochs. These radices, in conjunction with the tables, can be used to determine the place of the apparent equinox for any given date (III, 11, 12).

Considering next the obliquity of the ecliptic, Copernicus shows that the recorded estimates of the obliquity are consonant with the theory that this quantity oscillates between the values 23° 28' and 23° 52', in a complete period of 3,434 years (III, 10). The above-mentioned table of the equation of the equinoxes includes a column of corrections to be added to 23° 28' to obtain the true obliquity corresponding to each 3° of anomaly.

The erratic motion of the vernal equinoctial point upon the ecliptic makes it, in Copernicus' opinion, no fit origin from which to measure the longitudes of the fixed stars. He therefore reckons his longitudes from the bright star "at the head of Aries" *(y Arietis);* but his star-catalogue (II, 14) is otherwise a mere *richauffe* of Ptolemy's catalogue *(Aim.,* Books VII and VIII). The star-catalogue in the *Almagest* is the earliest extant synopsis of its kind. It has long been disputed whether Ptolemy himself made the observations upon which it is based, or whether he merely brought up to date the catalogue known to have been constructed nearly three centuries earlier by Hipparchus, which has not survived. The view that Ptolemy himself independently determined at least a substantial proportion of his star-places, has gained ground following Vogt's partial restoration of Hipparchus' original catalogue from data in his surviving *Commentary on the Phenomena of Eudoxus and Aratus (Astr. Nacht.,* 1925, Nos. 5354-55)-

§3. THE EARTH'S ECCENTRIC

The theory of precession leads Copernicus, by a natural transition, to the problem of giving an exact representation of the Earth's annual revolution round the Sun. For if the equinoctial points traverse the ecliptic at a variable rate, it must follow that successive tropical years cannot be all of the same duration. And when evidence of this irregularity was sought among determinations of the length of the tropical year made throughout the ages, the question arose whether the discrepancies brought to light might not be due in part to certain inequalities affecting the *Sun's* apparent motion relative to the stars.

Ptolemy recognized but one solar inequality, viz. the annual cycle of changes in the Sun's rate of motion in longitude whereby equal arcs of the ecliptic are, in general, described in unequal times (*Alm.*, III, 4); we have seen how Hipparchus represented this inequality by assuming the Sun uniformly to describe a circle eccentric to the Earth, and had determined the magnitude and direction of the eccentricity (Chapter I, § 1, *supra)*. Copernicus, as we shall see, distinguishes further causes of non-uniformity in the Sun's apparent motion, but he retains the eccentric to represent this fundamental inequality, subject to an interchange of the parts played by Earth and Sun. He reproduces the calculation of the elements of the eccentric given in the *Almagest*, and then, having noted that several mediaeval observers disagreed with Ptolemy's estimates, he re-determines these quantities from observations of his own, made in 1515, the procedure being, briefly, as follows (III, 16):

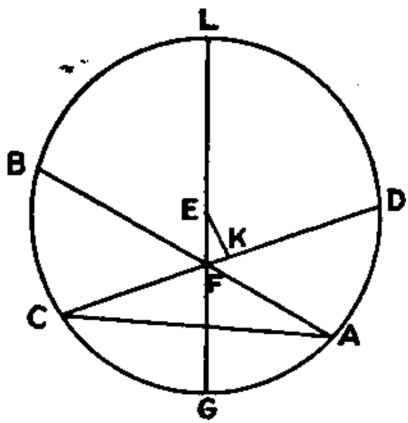


FIG. 13

Let ABC be the Earth's orbit with centre E (Fig. 13), G, L being the apses, at which the Earth is respectively nearest to, and farthest from, the Sun F; and let A, B, C be the positions of the Earth viewed from which the Sun appears at the spring equinox, at the autumn equinox, and at the mid-point of the Sign of Scorpio, respectively. Then the eccentricity (EF : EG), and the direction of the line LG, have to be determined. The points A, F, B form a straight line; produce CF to meet the circle again in D, and draw EK perpendicular to CD.

Copernicus observed the time-intervals required for the Earth to travel from B to C, and from B to A. From tables of the mean rate of motion of the Sun in longitude (III, 14), he calculates the arcs BC, BA, uniformly traversed by the Earth during these intervals. From the arc BC, thus known, the $\angle BAC$ (= $\frac{1}{2}$ $\angle BEC$) is obtained; also $\angle BFC$ is known (= 45°); hence $\angle ACD$ (= $\angle BFC - \angle BAC$) is found, and the arc AD. Hence, and from the known arcs BC and BA, the arc CAD is obtained. From the Table of Chords, Copernicus finds the ratio of chords (CD : CA), and also, in the triangle ACF whose angles are all known, he finds (CF : CA), and hence (CF : CD), (CF : FD), and, eventually, FK (which is half of the difference of FD and CF) in terms of the diameter of the circle ABC. The perpendicular EK is found in the same units, from $\angle EDK$, which is half of ($180° - \angle CED$); hence the right-angled triangle FKE can be solved. The eccentricity (EF : EG) of the Earth's orbit is found to be about $\frac{1}{31}$, and the Sun's apogee is found to lie 6° 40' east of the summer solstitial point.

It is now possible to calculate, for any given day in the year, the Sun's *equation of centre*, i.e. the difference between the mean and the apparent places of the Sun in the ecliptic, which arises from the eccentricity of the Earth's orbit (III, 17). For let AEB (Fig. 14) be the Earth's eccentric, centre C, with the Sun at S. To an observer on the Earth, say at E, the apparent direction of the Sun (ES) differs from its mean direction (EC, which appears to revolve uniformly about E) by the angle CES. This angle can be readily calculated from a knowledge of the Earth's elongation (ACE) from the farther apse A, and of the eccentricity (CS : CE). The equation of centre Z.CES can be shown to be a maximum when ES is perpendicular to AB (III, 15; cp. *Alm.* III, 3). This maximum, as calculated by Ptolemy, was 2° 23'; with Copernicus' elements, it amounts to i° 51' (modern maximum value of the equation of centre: i° 55').

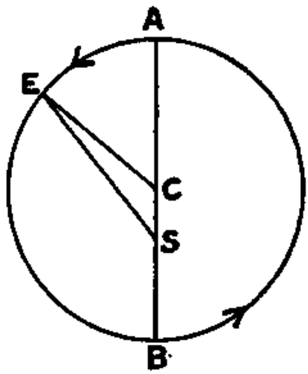


FIG. 14

Having found reason to suppose that the direction of the apse-line of the Earth's eccentric, and the amount of the eccentricity, are alike subject to change with lapse of time, Copernicus proceeds to investigate the law governing these variations (III, 20). His stock of observational data was small, and his theory is carefully adjusted to harmonize with Ptolemy's assertion that the eccentricity was the same in his time as in the days of Hipparchus. Copernicus supposes that the apse-line swings round non-uniformly from west to east, and he assumes somewhat arbitrarily that the variations in its rate of motion, and in the eccentricity, occur in a cycle of roughly the same period as that governing the variations in the obliquity (§ 2, *supra).* He proposes two alternative, equivalent methods of representing these variations; we shall here describe the simpler of the two.

The centre C of the Earth's circular orbit KLM (Fig. 15) describes a small circle EF from east to west about a point G, which is fixed relatively to the Sun S. As C revolves, the apse-line LSCK of the Earth's orbit evidently oscillates about its mean direction BSGA (which itself turns uniformly

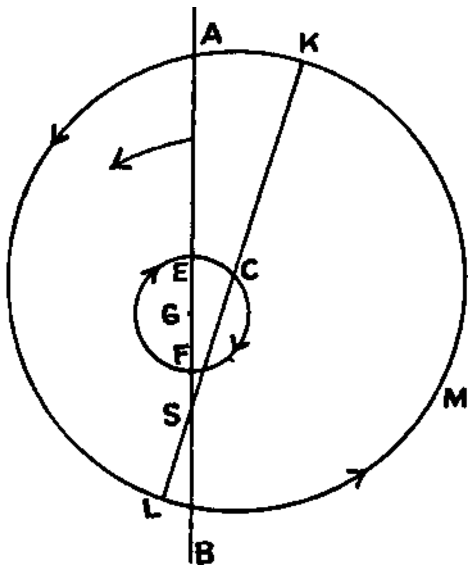


FIG. 15

about S from west to east), while the eccentricity SC oscillates between its maximum and minimum values, SE and SF.

Second and third solar inequalities arise from these variations in the eccentricity, and in the motion of the apse-line, respectively; and Copernicus, as we have already noted, regards these as governed by the same anomaly as the obliquity. This assumption enables him readily to determine the mean apse-line, and the correct elements for computing the equation of centre, at any given epoch.

The anomaly goes through its cycle of 360^0 in 3,434 years (§ 2 *supra)*, which is therefore the period in which the centre of the Earth's eccentric revolves round its mean position. Taking the diameter of the Earth's orbit as 10,000 parts, the eccentricity was 414 between the times of Ptolemy and Hipparchus (in which period the anomaly passed

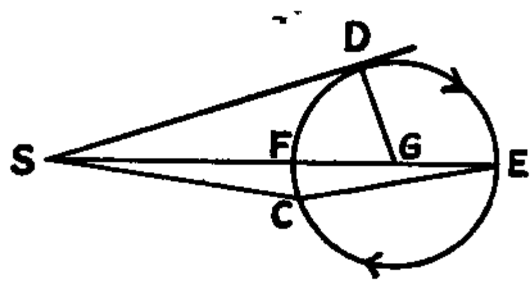


FIG. 16

through its zero value, see § 2 *supra)* and 323 at the date (1515) of Copernicus' own observation (when the anomaly would be 165^0 39'). From these data, the required elements are obtained as follows (III, 21):

Let EFD be the small circle described about G by the centre of the Earth's orbit (Fig. 16, cp. Fig. 15), and S the Sun; then SGE is the direction of the mean apse-line, and SE, SF, measure, on a certain scale, the maximum and minimum eccentricities, respectively. From E, set off the arc EC subtending 165° 39′ at G. Then Copernicus takes the direction of SC as that of the true apse-line in 1515, and its length as measuring the eccentricity in that year. To calculate EF, SG, and $\angle$ESC, we have

$$SE = 414 = \text{max. eccentricity};\ SC = 323,$$

and

$$\angle CES = 7^\circ\ 10',$$

whence, by calculation, $EC = 95$, $EF = 96$,

whence $SF = (414 - 96) = 318 = \text{min. eccentricity}$,

$$SG = 366, \text{ and } \angle ESC = 2^\circ\ 7':$$

this is the *equation of the apogee* (i.e. the difference between the longitudes of the mean and apparent apogees arising from the non-uniformity of motion of the apse-line) in 1515.

Now draw SD to touch the circle at D, and join DG. Then

$$SG = 366, \text{ and } DG = 48,$$

whence $\angle DSG$, the *maximum* equation of the apogee, is found to be *f* 28'.

The mean progressive motion of the apse-line is deduced (III, 22) from a comparison of the approximate place of apogee in 64 B.C. (when the anomaly was supposed to have vanished, and the mean and apparent apogees to have coincided) with its calculated mean place in A.D. 1515. The motion is found to be about 24* *per annum* from west to east, relative to the fixed stars (modern value: 11*-25 *per annum).*

A correction has thus to be applied (III, 23) to the Sun's mean rate of motion relative to the stars, so as to give his motion as measured from mean apogee; and a further correction is required, in calculating the equation of centre, for the departure of the apparent from the mean apogee. These corrections are provided for in the tables by which Copernicus completes his solar theory (III, 24). These tables show

(1) the equation of the apogee corresponding to each 3^0 of the argument of anomaly;
(2) the Sim's equation of centre for each 3^0 of his elongation from the apparent apogee, when the eccentricity has its minimum value;
(3) the amounts to be added to the equations of centre (2) when the eccentricity is a maximum;
(4) the proportions of the corrections (3) to be applied for each 3^0 of the anomaly.

Our study of the Copernican theory of the Earth's motions, then, reveals the following innovations of importance to the progress of astronomy:

(1) The phenomena of the day and year are referred to a combined rotation and revolution of the Earth;
(2) Precession is interpreted, for the first time, as due to a slow conical motion of the Earth's polar axis about the axis of the ecliptic;
(3) The progressive motion of the apse-line of the Earth's orbit, detected by al-Battani (Chapter I, § 2 *supra),* is confirmed, and a reasonable estimate of its magnitude offered.

For the rest, Copernicus' detailed theories of the supposed fluctuations in the precession, the obliquity, the eccentricity, and the progression of the apse-line, are worthless for the reasons already indicated. Fluctuations in these quantities do indeed occur, and our methods of representing them bear, in some instances, a formal resemblance to the devices of Copernicus; but any comparison of our numerical coefficients with his would be meaningless.

CHAPTER V

THE COPERNICAN SYSTEM *(continued)*

THEORY OF THE MOON'S MOTION

IN the Copernican system the Moon retained its former relation to the Earth, exchanging its status as one of the seven planets for that of a satellite, whose daily rising and setting was now to be attributed to the Earth's axial rotation. Hence the substitution of the heliocentric for the geocentric point of view did not fundamentally affect the problem of giving a geometrical representation of the Moon's apparent motion; and the fourth Book of the *De Revolutionibus,* which is devoted to the lunar theory and related problems, is among the less original portions of the work. Nevertheless, while adopting much of Ptolemy's observational data and employing his characteristic geometrical methods, Copernicus was able to make considerable improvements upon the lunar theory *set* out in the fourth and fifth Books of the *Almagest*

§i. THE LUNAR INEQUALITIES

Copernicus begins his study of the Moon's motion by examining Ptolemy's lunar theory, whose essential features we have already outlined (IV, i; cp. *Aim,,* V, 2, and Chap. I, § 1 *supra).* He regards this theory as objectionable and inadequate (IV, 2). For it was supposed that the centre of the epicycle appeared to move uniformly with regard to the

Earth's centre, and this would mean that it must actually move non-uniformly about the centre of its own deferent, which would conflict with the fundamental axiom of celestial motion (I, 4). For the same reason Copernicus rejects the convention whereby the Moon was supposed to move in its epicycle with a uniform angular velocity relatively to a line joining the centre of the epicycle to an arbitrary point within the deferent. He further declares that the classical lunar

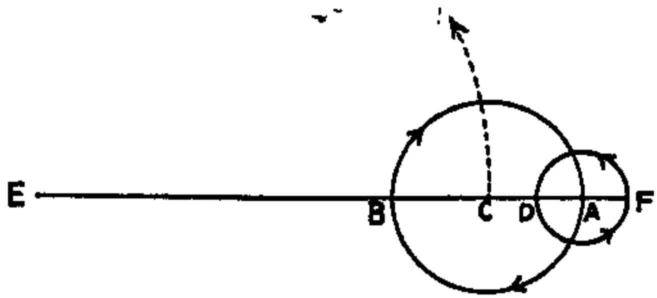


FIG. 17

theory indicates fluctuations in the Moon's distance and angular diameter which are out of all proportion to those actually observed.

With a view to removing these objections, Copernicus proposes to represent the Moon's motion as follows (IV, 3):

He makes the Moon describe a small epicycle DF (Fig. 17), so that starting from F it moves from west to east; the centre A of this circle describes, in the opposite sense, a larger epicycle AB, whose centre C is carried from west to east on a deferent whose centre is that of the Earth E. When EC is directed towards the mean Sun, the Moon is to be at D on its epicycle; when EC has turned through 90^0 from this position, the Moon is to be at F. It thus traverses the circle DF twice in a synodic month (mean interval between two successive new moons) relatively to the line DC. Similarly A describes the larger epicycle in an anomalistic month (interval between two successive passages of the Moon through its perigee), while C describes its deferent (relatively to the mean Sun) in one synodic month.

Copernicus provides Tables (IV, 4; cp. *Aim.*, IV, 3) showing the accumulated angular motion of the Moon from day to day and from year to year, relative (i) to the Sun (synodic motion), (ii) to the apses of the Moon's orbit (anomalistic motion), and (iii) to its nodes on the ecliptic (draconitic motion).

The next few chapters (V-XII) are devoted to an investigation of the two principal lunar *inequalities,* or periodic fluctuations in the Moon's rate of motion about the Earth; they are represented by the two epicycles of Fig. 17, and respectively correspond to the *equation of centre* and the *evection* in modern lunar theory. Both inequalities were known to Ptolemy *(Aim.,* IV, 4); and no more were discovered until after the time of Copernicus. The First Inequality is an oscillation of the Moon about its mean position in its orbit, vanishing at apogee and at perigee, and recurring in the Moon's anomalistic period. It arises in consequence of the eccentricity of the Moon's orbit, and allowance was made for it by the Babylonian astronomers and by Hipparchus. The Second Inequality is here treated as a variation in the first one, disappearing about new and full moon, and becoming most pronounced at quadrature; it arises from fluctuations in the eccentricity of the Moon's orbit, and seems to have been recognized first by Ptolemy *(Aim.,* V, 2).

The problem before Copernicus was that of deducing from observations the ratios (CD : EC) and (CF : EC) in Fig. 17, so as to determine the elements of his lunar theory. He calculates the former ratio from a group of three lunar eclipse observations, in imitation of Ptolemy's procedure for determining the First Inequality *(Aim.,* IV, 5). He applies the method to two independent groups of three eclipses each. We shall pass over the discussion of the first triad of observations (those which Ptolemy had utilized), and shall sketch Copernicus' application of the method to three eclipse observations which he had made for himself (IV, 5). On each occasion he had recorded the Sun's longitude, and the time of the central phase of the eclipse to within a fraction of an hour, with the following results (we shall give merely the dates of the eclipses and the true positions of the Sun in the Zodiac):

DATE	TRUE SUN
(1) 1511, Oct. 7	22^0 25' of Libra
(2) 1522, Sept. 6	22^0 12' of Virgo
(3) 1523, Aug. 26	11° 21' of Virgo.

In the mean interval of 10 yrs., 337 d., 48 mins., between the eclipses (1) and (2), the Moon evidently moved through 329^0 47', over and above complete circuits, since at each eclipse it was diametrically opposite to the Sun. In the mean interval of 354 d., 3 hrs., 9 mins., between (2) and (3), the Moon's motion was 349^0 9'. For the former interval of time, the mean synodic motion of the Moon relative to the Sun, rejecting multiples of 360^0, was given by the tables as 334^0 47', and the motion of the Moon in anomaly (motion in the circle AB of Fig. 17, relative to the apogee) as 250^0 36'. For the latter interval of time, the tabular synodic motion was 346^0 10', and the anomalistic motion, 306^0 43'. Hence, in the former interval, a motion of 250^0 36' in anomaly had the effect of taking $(334° \; 47' - 329° \; 47')$, or 5°, off the Moon's mean motion; while in the latter interval a motion of 306^0 43' in anomaly added 2° 59' on to the Moon's mean motion.

Now let E be the centre of the Earth (Fig. 18), and ABC

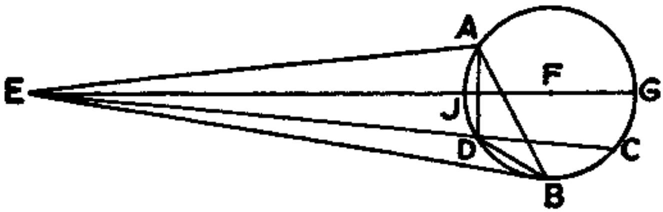


FIG. 18

the circle upon which the Moon lies when new or full, the centre F of this circle being also the centre of the Moon's larger epicycle. Let A, B, and C represent the Moon's positions on this circle at the first, second, and third eclipses respectively. Then, from the data given above,

$$\text{arc } ACB = 250° \; 36',$$
$$\text{arc } BAC = 306° \; 43', \text{ whence}$$
$$\text{arc } AGC = 197° \; 19', \text{ and}$$
$$\text{arc } CB = 53° \; 17' = \angle CFB;$$
$$\angle AEB = 5°, \text{ and } \angle BEC = 2° \; 59'$$

Since the arc AGC exceeds 180^0, and C is behind A in longitude, evidently the apogee must lie on AGC. Let G, J be the apogee and perigee respectively. Let EC intersect the circle ABC in D. Then $\angle CDB = \frac{1}{2} \angle CFB = 26° \ 38'$, and $\angle BED = 2° \ 59'$; therefore $\angle DBE = 23° \ 39'$, whence, in the triangle BED, the Table of Chords gives (BD : ED) $= \frac{1042}{8024}$.

Likewise $\angle ADC = 98° \ 40'$, and $\angle AEC = 5° - 2° \ 59' = 2° \ 1'$, whence EAD $= 96° \ 39'$, and, in the triangle ADE, (AD : ED) $= \frac{702}{19865}$. But (BD : ED) $= \frac{1042}{8024}$ (proved above), whence (AD : BD) $= \frac{283}{1042}$. Also $\angle ADB = \frac{1}{2}$ (re-entrant) $\angle AFB = 125° \ 18'$, whence, solving the triangle ADB, (AB : BD) $= \frac{1227}{1042}$.

Expressed in terms of the radius of the circle ABC (FG = 10,000 parts),

$$AB = 16{,}323;\ BD = 13{,}853;\ ED = 106{,}751;$$

whence the arc DB subtends 87° 41′ at the centre. Add the arc BC, and the arc DC will subtend 140° 58′ at the centre. On the scale, FG = 10,000, we have (from the Table of Chords) DC = 18,851, and EC = ED + DC = 125,602. Now

$$EJ \cdot EG = EF^2 - JF^2 = ED \cdot EC \quad (\textit{Euc.}, \text{III}, 36);$$

whence EF = 116,226, where FG = 10,000,
or FG = 860, where EF = 10,000.

This is found to be in fair agreement with the value of FG (870) which Copernicus had deduced from Ptolemy's observations.

Copernicus proceeds to calculate the angles FEA, FEB, FEC, representing the differences between the apparent place of the Moon, and its mean place (F), at each of the three eclipses. Knowing what the apparent places were on these occasions, he is thus able to calculate the corresponding mean places. A comparison (IV, 6) of one of these mean places with one similarly deduced from Ptolemy's triad of observations enables him to verify the accuracy of the Moon's rates of synodic and anomalistic motion assumed in the tables (IV, 4).

Copernicus has next to investigate the second lunar inequality. The problem presents itself in this way: It was found that when the Moon was near quadrature (90^0 from

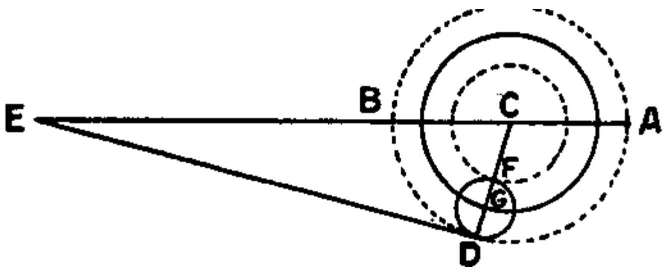


FIG. 19

the Sun), the difference between its mean and its apparent positions might amount to as much as 7^0 40', although, if the Moon were restricted to move on the circle ABC of Fig. 18, this difference should not exceed about 5^0 (IV, 8; cp. *Aim.*, V, 3). Hence the necessity for the second smaller epicycle in addition to the first.

About C, the mean position of the Moon in its orbit (Fig. 19), draw the circle ADB upon which the Moon lies at the times of its greatest departure from its mean position. Through E, the centre of the Earth, draw the line EBCA, and the tangent ED meeting the circle ADB at D. Join CD. The ∠CED being 7° and ∠CDE being 90^0, the ratio (CD : CE) is found to be $\frac{1334}{10000}$. But the Moon's distance from the centre C at full moon was found above to be about 860 parts (where CE = 10,000). Now draw a circle with centre C and radius CF = 860; this is the circle upon which the Moon lies at new and at full moon. Bisect FD at G, and draw a circle with centre G and radius GF. This circle is the Moon's second epicycle; its radius GF is $\frac{1}{2}$ (1,334 − 860) or 237 parts, the radius CG of the first, larger epicycle being

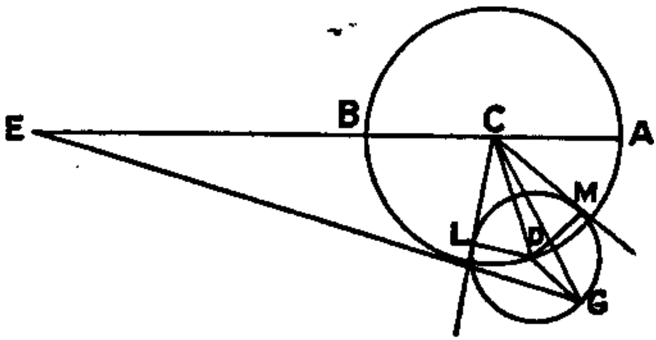


FIG. 20

1,097 parts. The angles subtended by these radii at the Earth's centre E ($\angle CEG = 6° 18'$, and $\angle GED = 1° 22'$) correspond respectively to the amplitudes of the equation of centre and of the evection (6° 17′, and 1° 16′)* in our terminology.

This second inequality itself introduces a non-uniformity into the rate at which the Moon revolves about the centre C of the larger epicycle AB, relatively to the apogee A, i.e. the rate at which its anomaly increases (IV, 9). For let E be the centre of the Earth, and D that of the smaller epicycle, and draw the tangents CL, CM, to the latter (Fig. 20). It has been shown above that $(DL : DC) = \frac{237}{1,097}$

$= \frac{2190}{10000}$, whence $\angle DCL = \angle DCM = 12° 28'$, and this is the greatest difference that can occur between the Moon's mean and true anomalies. It will occur when EC is inclined at an angle of \ (yy° 32') or 38° 46' to the line joining the centre of the Earth and Sun.

Having now arrived at a numerically determinate lunar theory, and having partly verified it (IV, 10) by reference to an observation of the Moon ascribed to Hipparchus (*Aim.*, V, 5), Copernicus next proceeds to draw up a table by means of which the Moon's apparent place can be deduced from its mean place at any given date (IV, 11, 12). The construction of the table depends upon the following principle: Let G be the Moon (Fig. 20). Then in the triangle CDG the sides CD, DG, are constant, and $\angle CDG$ is determined at any time by the Moon's mean elongation from the Sun. Hence CG and $\angle DCG$ may be found. Add $\angle DCG$ to

/ECD]

the Moon's mean elongation from perigee (__~7~), and the $\angle ECG$ is obtained; this, with EC and CG, enables the triangle ECG to be solved, and $\angle CEG$, the difference between the Moon's mean and apparent places, to be obtained.

§2. THE MOON'S MOTION IN LATITUDE

The cycle of changes in the Moon's celestial latitude recurs in the period required for the Moon to perform a complete revolution relatively to the nodes in which its orbit cuts the ecliptic. This is the *draconitic month*, to the evaluation of which Copernicus next addresses himself **(IV, 13).**

The ideal method of determining this period would, he

holds, be to consider two partial lunar eclipses, occurring near the same node, showing shadows equal in extent and similar in situation, and falling on dates which, though widely separated in time, correspond to equal elongations of the Moon from its apogee (cp. *Aim.,* IV, 2). The interval between two such eclipses would be that required for a whole number of circuits relative to the node, the particular whole number being given by a previous approximate knowledge of the required period. Having failed to find such a convenient pair of eclipses, Copernicus resorts to another method, and considers two eclipses occurring at nearly diametrically opposite points of the Moon's orbit (relatively to the line of nodes), the shadows encroaching from opposite points of the Moon's limb and differing in extent by a known amount. The dates, which fell in 174 B.c. and A.D. 1509 respectively, corresponded to approximately equal distances of the Moon from the Earth. Copernicus finds the *apparent* motion of the Moon, relative to a node, in the interval, by allowing for the difference in the extents of the eclipses (a difference of ^r of the Moon's diameter in the extent of the shadow being taken to correspond to |° difference of angular distance from the node); he converts this into *mean* motion, and utilizes the result to verify his tables of the Moon's motion in latitude (given in IV, 4).

§3. THE DISTANCES OF THE SUN AND MOON FROM THE EARTH

The earliest known attempt to determine the distances of the Sun and Moon from the Earth by rigorous geometrical reasoning from the results of actual observations was that made by Aristarchus of Samos (*fl. c.* 280 B.c.). Aristarchus considered the triangle having the Sun (S), the Moon (M), and the Earth (E) at its three angles. When the Moon appeared half illuminated, he estimated the Z.SEM to be 87^0, the angle SME being then 90^0; thus the three angles of the triangle, under these circumstances, were known, and Aristarchus was able to approximate to the ratio (ES : EM) which gave the relative distances of the Sun and Moon from the Earth. (That he found this ratio to lie between 18 and 20, while actually it is about 380, was due, not to errors of reasoning, but to the fact that, at half moon, SEM is about 89^0 50', and not 87^0.) From further observational estimates of the angular diameters of the Sun and Moon, and of the cross-section of the Earth's shadow-cone at the Moon's distance, Aristarchus was able to express the distances and sizes of the Sun and Moon in terms of the length of the Earth's radius (though far from accurately). The Earth's radius was itself determined in standard units of length by the ancients on more than one occasion (e.g. by Eratosthenes of Alexandria in the third century B.C.). They employed a method of measuring the size of the Earth which has persisted, with increasing refinement, down to modern times. The procedure was to find the difference of latitude between two stations lying on the same meridian and a known distance apart; this distance stood to the entire circumference of the Earth in the same proportion as the observed difference of latitude stood to 360^0,

Aristarchus had proceeded on the assumption that the size of the Earth was negligible in comparison with the Moon's distance from the Earth. In actual fact, however, the distance of the Moon is only about sixty times the Earth's radius, and, in consequence, the direction in which a terrestrial observer sees the Moon's centre at some given instant varies appreciably according to his position on the Earth. The *parallax,* or difference in apparent direction, may amount to as much as about i°, which was an angle easily measurable by the observers of the ancient world; and it was by the measurement of such parallax that the Moon's distance was eventually (determined. The nearer the Moon is to the observer's horizon, the more its apparent position among the surrounding stars differs from its position as seen, at the same instant, from a place where it is vertically overhead. But, given the Moon's altitude above the horizon, the amount of the parallax depends simply upon the proportion of the Earth's radius to the Moon's distance. Hence Ptolemy was able to determine this proportion by comparing observations of the Moon made when it was near the zenith with others made when it was near the horizon *(Aim.,* V, 13). Ptolemy gives the apogee distance of the Moon as $64^1/_6$ times the Earth's radius, and he calculates the corrections to be applied for parallax, so as to convert the observed position of the Moon among the stars into what its position would appear to be if it were vertically overhead, for various positions of the Moon in its orbit.

Copernicus rejects Ptolemy's estimate of the Moon's distance, and his system of corrections for parallax, as resting upon a lunar theory which has been superseded by his own (IV, 16). He bases his own estimate upon two observations which he made, in 1522 and 1524, of the Moon's meridian zenith distance **(angular** distance **from** the **zenith** when crossing the meridian). The differences between the observed and the calculated zenith distances on these two occasions represent the respective parallaxes, and they can be used to determine the corresponding distances of the Moon by the following method (IV, 17):

Let ABA' be a section through the centre C of the Earth (Fig. 21). Let AD be the vertical of the observer A, and E the Moon's centre. From the observed zenith distance, /.DAE, of the Moon, as seen from A, and the parallax, Z.AEC, by which it differs from the calculated zenith distance, the angles of the triangle AEC are known, and (CE : AC) can be calculated, giving the Moon's distance in terms of the Earth's radius at the time of the observation.

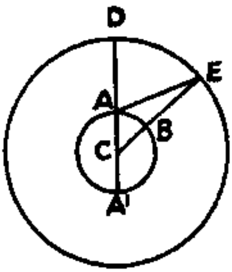


FIG. 21

To arrive at the greatest and least possible distances of the Moon from the Earth, let ABC Fig. 22) be the Moon's larger epicycle with centre D, FGK the smaller epicycle with the Moon at K, and E the Earth's centre.

The Moon's true anomaly, /.ADK (re-entrant), and hence Z.EDK, are obtained, for the date of observation, from the tables, and also the difference ZDEK between the mean and apparent places of the Moon in its orbit. Hence the angles of the triangle KDE are known, and Copernicus finds the ratio (DE : EK). But he has already found the Moon's distance EK in terms of the Earth's radius; and the elements of his lunar theory (IV, 5, 8) give DF and DG in terms of DE. Hence all these lengths can be expressed in terms of the Earth's radius (and its sexagesimal fractions), the results being

$$EK = 56\tfrac{42}{60}, \qquad DF = 5\tfrac{11}{60},$$
$$DE = 60\tfrac{18}{60}, \qquad DG = 8\tfrac{2}{60};$$

whence, the Moon's greatest distance from the Earth $= 68\tfrac{1}{3}$, and the Moon's least distance from the Earth $= 52\tfrac{4}{15}$, in the same units.

In his attempt to calculate the distance and size of the

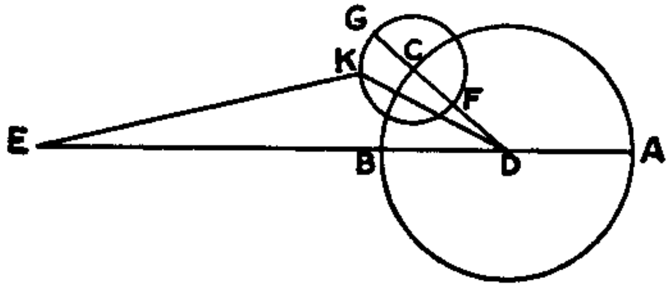


FIG. 22

Sun and the extent of the Earth's shadow-cone (IV, 18-20), Copernicus follows the method described by Ptolemy *(Alm.*, V, 15), and attributed by him to Hipparchus. This method may be briefly indicated as follows:

In Fig. 23, let S, M, O, be the centres of the Sun, the Moon (both at apogee), and the Earth respectively, when they are in the same straight line, and let AGV, BHV, be common tangents to the Sun and Earth in the plane of section, intersecting in V, the vertex of the cone of shadow cast by the Earth. The method presupposes that we know the following quantities: (i) the approximate distance OM of the Moon from the Earth in terms of the Earth's radius

OH; (ii) the (equal) angular diameters, $\angle AOB$, $\angle DOE$, of the Sun and Moon, as observed from the Earth; (iii) the ratio in which the diameter JK of the Earth's shadow at the Moon's distance stands to the diameter DE of the Moon (Ptolemy takes this ratio to be $\frac{13}{5}$; Copernicus makes it $\frac{403}{150}$). Knowing the Moon's distance OM in terms of OH, and its angular diameter $\angle DOE$, we have its linear diameter DE. Knowing DE and (JK : DE), we have JK. Also since JK, CF, are equidistant from O, JK, GH, and CF are in

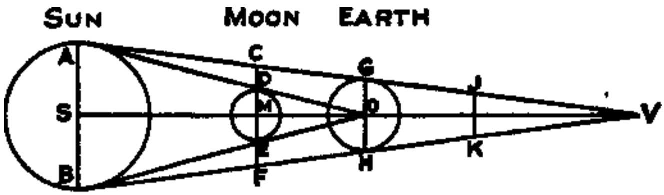


FIG. 23

arithmetical progression, so that CF = 2 GH — JK, and is known in terms of OH. Since EF = ½ (CF — DE), the ratio (EF : OH) is known. But

$$\frac{EF}{OH}=\frac{MS}{OS}, \text{ and } 1-\frac{EF}{OH}=1-\frac{MS}{OS}=\frac{OM}{OS}=\frac{\text{Moon's distance}}{\text{Sun's distance}}$$

The Sun's distance is thus known in terms of that of the Moon, and hence in terms of the Earth's radius. Ptolemy gave the ratio (Sun's distance) : (Earth's radius) as 1,210. Copernicus, reworking Ptolemy's calculation with slightly different data, found for it a mean value of 1,142; but his solar theory indicated that it should vary between 1,105 and 1,179. (The correct value would be about 23,000; the method is liable to give very inaccurate results, since the slightest error in estimating the acute and nearly equal vertical angles of the shadow-cones AOB, AVB, greatly affects the resulting estimate of the distance of the Sun to which both these cones are tangential.)

In the next few chapters (IV, 21-23) Copernicus deduces the limiting values of the horizontal parallaxes and angular diameters of the Sun and Moon from the previously determined constants of their orbits; and he investigates the effect, upon the cross-section of the Earth's shadow at the Moon's distance, of variations in the distances of the Sun and Moon from the Earth. His next problem is that of tabulating the corrections for the parallax in altitude of the Sun and Moon (IV, 24, 25). Here again the work is modelled on the *Almagest (Attn.,* V, 17), but with such alterations as the new lunar theory requires. The remaining chapters of Book IV (26-32), which deal descriptively with eclipse problems, etc., are not of such interest or originality as to call for discussion here.

The lunar theory of Copernicus represents no advance in fundamental principles on that of Ptolemy, for it recognizes no further inequalities in the Moon's motion beyond those already known. It had the merit, however, of representing the apparent motion of the Moon without grossly exaggerating the observed variations in its angular diameter, as Ptolemy's theory had done. The reform which Copernicus effected in this respect may be judged from the following table showing the liniits within which the angular diameter varies according to the respective *theories* of Ptolemy and

Copernicus, with the limits given by modern observation, for purposes of comparison:

	MOON'S ANGULAR DIAMETER	
	max.	min.
Ptolemy	1° 0′ 26″	31′ 36″
Copernicus	37′ 34″	28′ 45″
Modern values	33′ 30″	29′ 26″

CHAPTER VI

THE COPERNICAN SYSTEM *(continued)*

THEORY OF THE PLANETARY MOTIONS

WE have seen how Copernicus claims, in the early pages of the *De Revolutionibus,* that his new theory can be harmonized with the facts of planetary motion, and that it affords a simple explanation of certain well-known planetary phenomena. In the concluding two Books of his work he seeks to justify this claim by constructing numerically determinate geometrical theories to represent the motions of the five planets, Mercury, Venus, Mars, Jupiter, and Saturn.

§i. SIDEREAL AND SYNODIC MOTIONS OF THE PLANETS

A distinction is drawn at the outset (V, i) between the two components in the apparent motion of a planet in longitude. On the one hand, there is the orbital motion which the planet would be seen to possess by an observer stationed at the Sun, whereby it completes a circuit of the heavens in its sidereal period. On the other hand, this motion is complicated for the terrestrial observer by his own motion on the moving Earth, which gives rise to an apparent or parallactic motion *(motus commutation^)* of the planet, superimposing upon its orbital motion an inequality which recurs in the planet's synodic period. In consequence of this second component, a superior planet's apparent motion through the constellations is arrested and temporarily reversed at regular intervals. We have already referred to this phenomenon (pp. 16, 22), and have explained how ancient astronomers sought to account for it by postulating special spheres (Eudoxus) or epicycles (Ptolemy) as essential constituents of their planetary hypotheses. This had to be done separately for each planet; and it was found impossible to explain, on physical grounds, why the planetary motions should be complicated in this way. Copernicus, however, was able (V, 3) to account for this phenomenon in all the planets, as a class, by showing it to be a direct consequence of the annual revolution of the Earth round the Sun—a single antecedent factor of whose actuality he had already adduced independent physical evidence. This simple explanation of the peculiar behaviour of the planets marked an important advance upon the *ad hoc* hypotheses of Eudoxus and Ptolemy. It constituted the chief argument for the scientific truth of the Copernican theory, and its best claim to acceptance, when it was first put forward.

In order to understand, in a general way, how the retrogressions arise in consequence of the annual motion of the Earth, suppose the respective orbits of the Earth, E, and of a superior planet, P, to be coplanar circles having a common centre at the Sun, S (Fig. 24). Draw the tangents PE_1, PE_2 from the planet to the Earth's orbit. If the planet remained at rest at P, and the Earth uniformly described its orbit, then, while the Earth traversed the arc E_1E_2, the planet would appear, to a terrestrial observer, to be moving against the background of stars in the retrograde direction (from east to west) through the $\angle E_1PE_2$; but while the Earth described the remainder of its orbit, from E_2 to E_1 the planet would appear to move progressively (from west to east) through the same E_2PE_1 That is, the planet would appear to oscillate about the direction SP with an amplitude /.SPE_1 either way. But actually the planet P is not at rest, but travels round its orbit in the same direction as the Earth (though with less angular velocity), and it carries the tangents PE_1 PE_2 with it. Thus the oscillations are superimposed upon a general eastward motion of the planet, and they give rise to the characteristic fluctuations in its apparent motion. The corresponding phenomena in the apparent motion of an inferior planet may be explained on analogous lines, the positions of the Earth and planet in Fig. 24 being interchanged. The period of recurrence of this inequality in a planet's motion is that in which the Earth gains a complete revolution upon the planet, or *vice versa,* that is to say, it is the synodic period. Copernicus gives tables (based on Ptolemy's Tables) showing how the synodic motion of each planet (its angular motion reckoned from the moving line SE) accumulates from day to day and from year to year (V, 1; cp. *Aim.,* I X , 4).

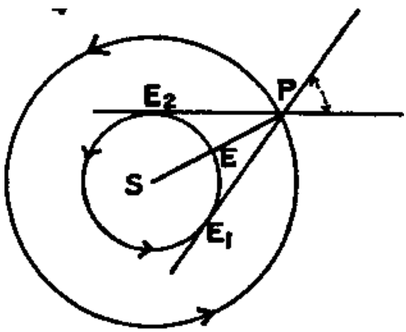


FIG. 24

A numerically precise theory of this phenomenon subsequently given by Copernicus (V, 35, 36) is adapted from the investigation of the stationary points on an epicycle given by Ptolemy *(Aim.,* XII, i) and attributed by him to ApoUonius. The procedure is equivalent to determining the condition that the speeds of the Earth and of a planet in the direction perpendicular to the line joining them should be instantaneously equal; the planet will then have no angular velocity about the Earth, and will appear stationary to a terrestrial observer.

§2. MOTIONS OF THE PLANETS IN LONGITUDE

Before embarking on the development of his own planetary theory, Copernicus gives a brief account (V, 2) of the hypothesis by which Ptolemy sought to represent the motions of Saturn, Jupiter, Mars, and Venus *(Aim.,* IX, 5, 6; see Chapter I, § 1 *supra).* The essential feature of Ptolemy's treatment was the epicycle carried round on a deferent eccentric to the Earth. Copernicus stresses the fact that the centre of this epicycle was supposed to move uniformly, not about the centre of the deferent, nor about the Earth's centre, but about a third, arbitrarily chosen point, while the motion of the planet on its epicycle was measured from a radius passing through the same arbitrary point. He claims that his theory renders unnecessary such a travesty of the established principles of Physics. Having shown (§ 1 *supra)* how the Earth's annual revolution accounted, at one stroke, for the most conspicuous inequality in the apparent motion of each planet, he was free to concentrate on the residual effects arising from the fact that the planets do not uniformly describe circles concentric and coplanar with that traversed by the Earth. The planets did not all yield to the same mode of treatment, three different types of hypotheses being required in order to represent the motions in longitude of (A) the superior planets, (B) Venus, and (C) Mercury, respectively. We shall now examine these several hypotheses, and shall indicate how the constants which they involved were numerically determined from suitably chosen observations.

(A) THE SUPERIOR PLANETS

The orbit of a superior .planet was constituted as follows (V.4):

The planet's deferent is the circle AGB (Fig. 25), with centre D, and the Earth's orbit is the circle NO, with centre C (the eccentricity being exaggerated in the figure for the sake of clearness). The planet describes the epicycle EF, whose radius is one-third of CD, uniformly from west to east, in the planet's sidereal period (relatively to the moving radius of the deferent), while the centre of the epicycle describes the deferent AGB in the same sense and in the same period. When the epicycle is at the farther apse, or apogee, A, the planet is at F; it is at J when the epicycle is at G (90^0 from A), and at L when the epicycle is at the nearer apse, or perigee, B. (It is worth noting that the curve FJL, described by the planet in a half period, is not an exact semicircle.) If G be the position of the centre of the epicycle at any given time, and J the corresponding position of the planet, then the mean motion of the planet in longitude is measured by the increase in the angle ADG, and its apparent motion, as observed from the centre of the Earth's orbit, by the increase in the angle ACJ.

When the Earth is exactly interposed between the planet and the centre C of the Earth's orbit, the planet's apparent place, as viewed by a terrestrial observer, is unaffected by

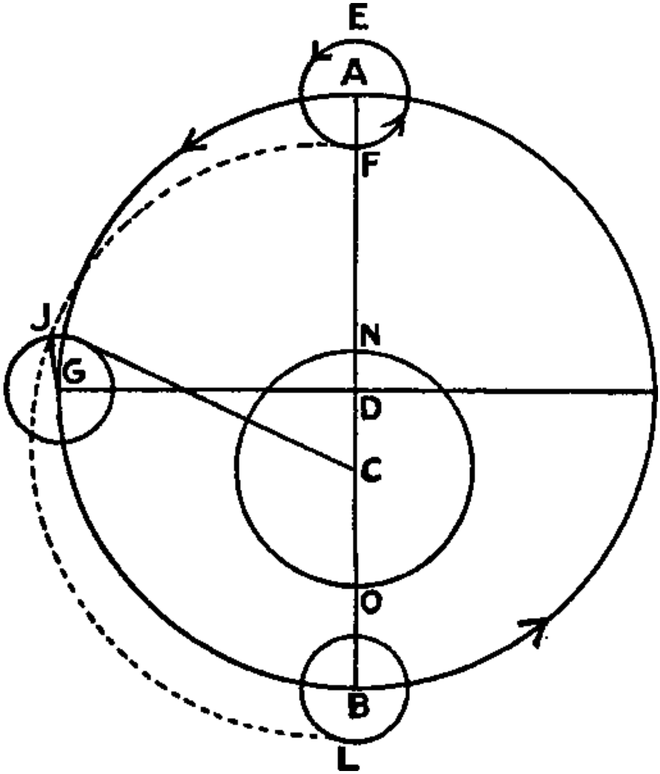


FIG. 25

parallax due to the displacement of the observer from C. Hence, whenever the planet appears in opposition to the centre of the Earth's orbit, the angle ACJ can be obtained free from error. The apse-lines of the deferents of the superior planets were determined by Ptolemy from opposition observations {*Aim*,, X, 6); and Copernicus also determines the constants of his planetary orbits from groups of three opposition observations. He goes through the process twice for each of the three superior planets, first using three of Ptolemy's observations, and then three of his own, and proving that the two sets of results so obtained show a measure of agreement. We can sufficiently illustrate the procedure by considering a single determination of the orbit of a single superior planet, and shall select Copernicus' discussion of his own observations of Saturn (V, 6). The dates of the three oppositions, and the longitudes of the planet on these occasions, were as follows:

Date (A.D.)	Long. of the Planet
(1) 1514, May 5	205° 24′
(2) 1520, July 13	273° 25′
(3) 1527, Oct. 10	0° 7′

The changes in the planet's *apparent* longitude between (1) and (2), and between (2) and (3), are evidently 68° 1' and 86° 42' respectively; and the *mean* motions in longitude corresponding to the intervals of time between these two pairs of observations are found to be 75° 39' and *88*° 29'.

The first step is to determine the eccentricity of the deferent and the orientation of the apse-line (AB in Fig. 25). To obtain a first approximation Copernicus, following Ptolemy (*Aim.*, X, 7), assumes that the planet moves uniformly in a simple eccentric ABC, whose centre is F, that of the Earth's orbit being D (Fig. 26). His procedure is then as follows:

Let A, B, C be the positions of the planet on this eccentric at the times of the first, second, and third oppositions respectively. Join AD, BD, CD, and AB; produce CD to meet the circle in E, and join AE, BE. From the above data, we have ∠BDC = 86° 42′, and ∠BFC = 88° 29′; hence, ∠BDE = 93° 18, and ∠BED = ½. ∠BFC = 44° 14′. Hence, in the triangle BDE the remaining angle DBE is known, and from the Table of Chords the ratio of the sides BE, DE can be determined. Similarly, in the triangle ADE the ratio (AE : DE), and Z.AED can be evaluated, and hence also the ratio (AE : BE). Knowing this ratio, and ∠AEB (= ½. ∠AFB = ½.75° 39′), Copernicus solves the triangle AEB and obtains the ratio (AB : BE). But since ZAFB is known (75^0 39′), AB can be expressed in terms of the radius of the circle ABC, and hence also BE and DE can be obtained in the same units. The value of BE found in this way shows that ∠BFE = 103° 7′, whence the re-entrant ∠CFE amounts to (103° 7′ + 88° 29′) or 191° 36′, which enables the value of CE to be calculated, and hence of CD (= CE − DE), in terms of the radius of the circle ABC. The centre F of the circle must lie in the larger segment EABC. Let GFDH be-the required apse-line

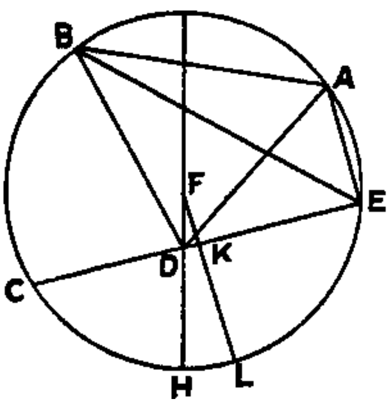


FIG. 26

of the eccentric, and draw FK perpendicular to CE. We have

(rectangle CD.DE) = (rectangle GD.DH)
= (square on GF) − (square on FD).

But CD and DE are known in terms of the radius GF of the eccentric, and hence FD is obtained (FD = 1,200 parts, where GF = 10,000; this is not far from the value of FD, viz. 1,016 parts, which Copernicus had already deduced from Ptolemy's data). In the right-angled triangle DFK, DK is known ($\frac{1}{2}$.CE − CD), and FD, whence ∠DFK, or ∠HFL, can be calculated, and hence ∠CFH (= $\frac{1}{2}$. ∠CFE − ∠HFL) and its supplement ∠CFG are obtained. But ∠CFB is known (88° 29′, as given above); ∠BFG is found by subtraction, and hence ∠GFA (= 35° 36′) is obtained, which fixes the position of the apogee G in relation to the positions of the planet at the three oppositions. (Copernicus noticed that the longitudes of the apses of the three superior planets seemed to have increased appreciably since the time of Ptolemy, the apse-line of Saturn, for example, having swung round through about 14° during the intervening fourteen centuries. Such progressive motions of the apse-lines actually occur, though Copernicus overestimated their amounts.)

The planet's orbit is now constituted as in Fig. 27 (cp. Fig. 25), where, as before, G, H respectively represent the apogee and perigee; F, D are the centres of the orbits of the planet and the Earth respectively; and A, B, C the planet's mean places at the times of the three oppositions discussed above, are now to represent the centre of the planet's epicycle at those times. Of the eccentricity (1,200 parts) just deduced, three-quarters (900 parts) are assigned to the deferent to represent the separation of the centres of the orbits of the planet and the Earth; while the remaining quarter (300 parts) appears as the radius of the epicycle. By distributing the eccentricity in this way, Copernicus was able approximately to reproduce Ptolemy's treatment of the orbital inequality of theplanet's motion while obviating Ptolemy's assumption of a centre of uniform motion differing from the geometrical centre of the deferent. The points N, O, P, on the epicycle, where the planet is situated at the times of the respective oppositions, are defined, as in Fig. 25, by the relations

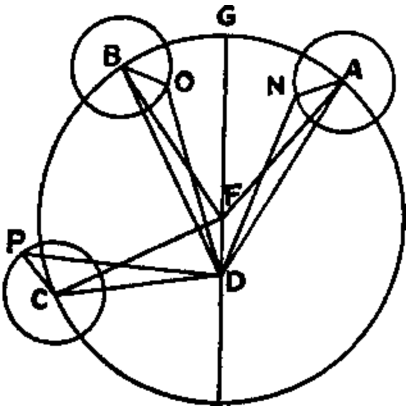


FIG. 27

$$\angle FAN = \angle GFA;$$
$$\angle FBO = \angle GFB;$$
$$\angle FCP = \angle GFC.$$

The angles of *mean* motion, Z.AFB, Z.BFC, are known from the tables; and it is required to calculate the angles Z.NDO, Z.ODP, and to show that they agree with the *observed* differences in the longitudes of the planet at the respective oppositions. Copernicus found that it would be necessary to adjust the calculated eccentricity and apse-line somewhat in order to obtain such agreement, the above determination being only in the nature of an approximation. He adopts the following corrected values:

$$\angle GFA = 38^\circ\ 50';$$
$$FD = 854 \text{ parts};$$
$$\text{radius of epicycle} = 285 \text{ parts},$$
$$\text{where} \quad GF = 10{,}000 \text{ parts}.$$

Agreement between the observations and theory is then established as follows:

In the triangle AFD, AF = 10,000 parts; FD = 854, and ∠AFD = 141° 10′. Solving the triangle, we have AD = 10,679; ∠FAD = 2° 52′, and ∠FDA = 35° 58′.

In the triangle ADN, ∠FAN = ∠GFA = 38° 50′, whence ∠DAN = ∠FAN + ∠FAD = 41° 42′. Also AD and AN are known, whence ∠ADN is found to be 1° 3′. But ∠ADF = 35° 58′, therefore ∠NDF = ∠ADF − ∠ADN = 34° 55′. Similarly, from the second observation, ∠ODF = 33° 5′, whence ∠ODN = 68°, as against the *observed* difference of 68° 1′ between the apparent longitudes of the planet at the first and second oppositions.

In like manner, ∠ODP is found to be 86° 42′, which agrees with the observed difference between the apparent longitudes at the second and third observations.

The longitude of the apogee G is found by subtracting from the observed longitude of the planet at the third opposition (0° 7′) the angle GDP (= ∠ODF + ∠ODP = 33° 5′ + 86° 42′ = 119° 47′); this gives the longitude of G as 240° 20′.

It is thus shown that, so far as this triad of observations is concerned, the hypothesis enables arcs of mean motion to be correctly transformed into arcs of apparent motion. The hypothesis had been equally successful with Ptolemy's triad (V, 5); it is next shown to be applicable to the remaining superior planets (V, 10-19), and Copernicus feels confident in basing his Planetary Tables upon it.

We have seen that Copernicus determined the orbits of the superior planets according to his hypothesis, from observations of these bodies made when they were in mean opposition (i.e. when they appeared at an elongation of 180^0 from the mean Sun at the centre of the Earth's orbit). Under these circumstances they would appear, to a terrestrial observer, to lie in the same direction as if viewed from that centre. But a superior planet may be observed at *any* elongation from the mean Sun, and its direction will, in general, be affected by a parallactic displacement depending upon the relative positions of the Earth and the planet in their orbits: this must be considered in calculating the planet's apparent (geocentric) longitude from its mean longitude, referred to the centre of its own orbit. Moreover, knowledge of the amount of this displacement, under circumstances, enables the dimensions of the planet's to be evaluated in terms of the radius of the Earth's . A single observation of the planet, when not in opposition, suffices for this purpose, assuming the motion to be in accordance with the hypothesis just described; and we shall again illustrate Copernicus' procedure with reference to the planet Saturn (V, 9).

At a certain hour on February 25, 1514, Saturn was observed to be in longitude 209^0. The tables give the Sun's mean longitude at that time (315^0 41'). and Saturn's mean elongation from the mean Sun* (116^0 31'), and, hence, the planet's mean longitude as the difference of these (199^0 10'), the apogee of its deferent being at about 240^0 20', as already stated.

In Fig. 28, let F be the centre, and G the farther apse, of

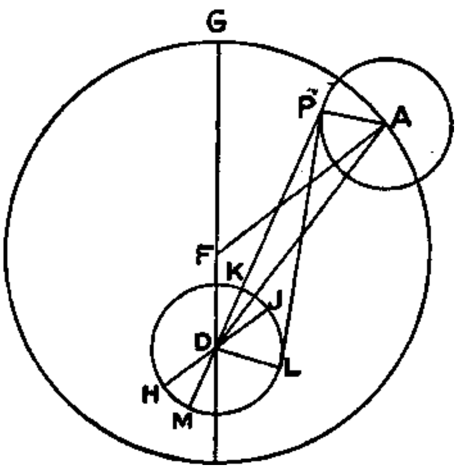


FIG. 28

the planet's deferent; and let D be the centre of the Earth's orbit. Let P be the planet, and A the centre of its epicycle, where $AP = \frac{1}{3}.DF$, and $\angle FAP = \angle AFG = (240° \; 20' - 199° \; 10') = 41° \; 10'$, in accordance with the hypothesis already explained. Let PD produced cut the Earth's orbit in K and M; and draw the diameter HJ parallel to FA. Set off the arc HL, making $\angle HDL = 116° \; 31'$: then L will

* That is to say, the angle between the line drawn from the Earth to the centre of its orbit, and the line drawn from the centre of Saturn's epicycle to the centre of his deferent.

be the position of the Earth in its orbit. In the triangle AFD, the ratio (FD : AF) is known from the theory of Saturn's motion, already reviewed, and Z.AFD = 180^0 — Z.AFG = 138^0 50'. Hence the triangle can be solved for AD, and for Z.DAF (which is 3^0 1'). Next, in the triangle PAD, AP, AD, and ZDAP (= ZDAF + Z.FAP) are known, and the triangle can be solved for DP, and for ADP (which is 1° 5'). The planet's mean place is defined by the direction FA, and its apparent place, referred to D, by the direction DP, and the difference between these is given by (,/ADP + Z.DAF), or 4^0 6'. Hence, if the Earth were at K or M, Saturn's apparent longitude would be (199^0 10' + 4^0 6'), or 203^0 16'. But, the Earth being at L, the planet's longitude was observed, as already stated, to be 209^0. The difference (/.DPL) of 5^0 44' must be due to parallax. Now ZPDL = 180^0 — ZLDM, and ZLDM = (116^0 31' — 4^0 6') = 112^0 25' (since Z.HDM measures the inclination of FA to DP), whence ZPDL = 67^0 35'. Also Z.DPL = 5^0 44'. Hence, in the triangle DPL, the ratio (DP : DL) can be calculated. But (AD : DP) is known from the triangle PAD, and (AF : AD) from the triangle FAD. Hence, eventually, the ratio (AF : DL) is obtained, i.e. the of the radius of the planet's deferent to that of irth's orbit. Thus, if DL be taken as unity, AF its to 9-174. The eccentricity DF, and the radius • '* of the epicycle, can also be evaluated in the same units.

By analogous investigations of the radii of the other planetary deferents, Copernicus arrives at results which may be tabulated as follows, the modern "mean distances" of the several planets being given for general comparison with Copernicus' figures:

PLANET	RAD. OF DEFERENT	MOD. MEAN DISTANCE
Mercury	0-376	0-387
Venus	0-719	0-723
Earth	*1'OOO*	*1'000*
Mars	1-520	1-524
Jupiter	5-219	5-203
Saturn	9-174	9-539

We have here the earliest instancy of an astronomer determining the relative sizes of the orbits of the several planets from actual observations.

The various estimates of the relative distances of the planets from the centre of the Universe, made by astronomers before Copernicus, were all obtained by assuming these distances to be connected by some arbitrary relation or other. Thus the Pythagoreans supposed that the radii of the successive planetary orbits were proportional to the segments of a lyre-string whose vibrations would give notes forming some musical concord. Plato, in the *Timaeus,* seems to have taken the distances of the seven planets from the centre as respectively proportional to the numbers 1, 2, 3, 4, 8, 9, 27—a series formed by combining the first four terms of two geometrical progressions (1, 2, 4, 8, and 1, 3, 9, 27), and whose last term is the sum of all the preceding ones. When it had been recognized that the distance of each planet from the Earth varied within definite limits, certain late Greek and Arab astronomers made the assumption that the maximum distance of any given planet from the Earth, at the centre of the Universe, was equal to the minimum distance of the planet immediately exterior to it, so that no useless empty spaces should exist in the Universe. Now the greatest distance of the Moon from the Earth was known to the Alexandrians with fair accuracy, as we have seen (Chapter V, § 3 *supra)*: this was assumed equal to the least distance of Mercury, the next planet beyond the Moon in the Ptolemaic system. Moreover, knowing a planet's minimum distance from the Earth, it was possible to calculate its maximum distance, given the ratio of the radii of the epicycle and deferent, and the eccentricity of the latter, which were both deducible from observations. Hence, the minimum distance of Mercury being assumed equal to the known maximum distance of the Moon, the maximum distance of Mercury could be calculated; but this was the minimum distance of Venus. The maximum distance of Venus, similarly derived from the constants of its orbit, was equal to the minimum distance of the Sun; and so on, up to the maximum distance of Saturn, which was generally assumed equal to the radius of the sphere of fixed stars. It happened that the distance of the Sun obtained in this manner agreed fairly closely with that given in the *Almagest,* which was obtained by a totally different method. The nrocedure just outlined led to gross underestimates of the xe (and therefore of the magnitude) of the Sun, and . an entirely false idea of the proximity of the stars; „ results, combined with the absence of observable stellar parallax, would further tend to discourage heliocentric hypotheses during the Middle Ages.

We turn next to those chapters of the *De Revolutionibus* which treat of the motions in longitude of the inferior planets.

(B) VENUS

In his attempt to represent the motion of Venus, Copernicus assumes that, to a first approximation, the planet describes a circle about a centre D which is displaced from the centre C of the Earth's orbit AB (Fig. 29). This eccentric has an apse-line ACDB, whose orientation may be determined from observations of maximum elongations of the planet from the mean Sun C, on the principle that, if two successive maximum elongations are observed to be equal, an apse must lie midway between the places of the mean Sun at the respective observations. A comparison of the amounts of different pairs of such equal elongations serves to determine which is the farther and which the nearer apse (V, 20; cp. *Aim.*, IX, 7, and X, 1). Following Ptolemy's data and calculations (*Aim.*, X, 1), Copernicus locates the farther apse in longitude 48^0 20' (that is, C appears from A to be in longitude 48^0 20'), and the nearer apse, in longitude 228^0 20'.

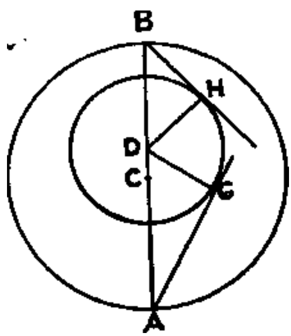


FIG. 29

The radius and eccentricity of the planet's eccentric are found (V, 21) by assuming, again on the authority of Ptolemy, that the maximum elongation of the planet from C is 44^0 48' when it is observed from the Earth at the farther apse, and is 47^0 20' when observed from the nearer apse. Draw the tangents AG, BH, and join DG, DH. In the right-angled triangle ADG, $\angle DAG = 44° \ 48'$, whence (DG : DA) is found to be $\frac{7046}{10000}$. Similarly, in the right-angled triangle BDH, $\angle$DBH = 47° 20′, whence (DH : DB) = $\frac{7346}{10000}$. Taking DG = DH = 7,046, we have DB = 9,582. Hence, AB = AD + DB = 19,582, whence AC = 9,791, and CD = 209. Hence, on the scale AC = 10,000, we have DG = DH = 7,193, and CD = 213. Thus, taking the radius of the Earth's orbit as unity, the radius of Venus' eccentric is 0·719.

Copernicus found, however, that this simple theory broke

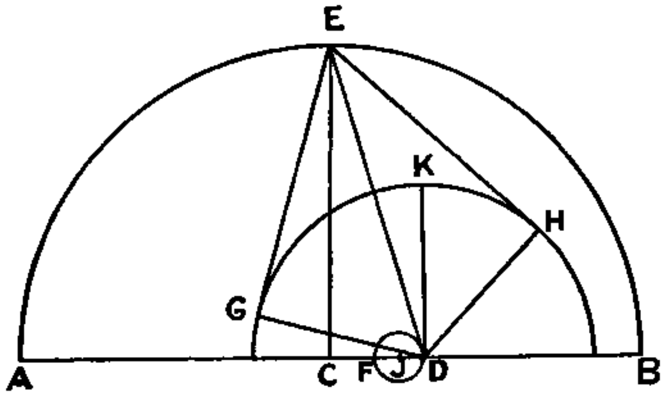


FIG. 30

down when the Earth was not at an apse (V, 22). He takes the following pair of observations (*Alm.*, X, 3) into con-
tion:

DATE	MEAN SUN	ELONGATION OF VENUS
A.D., Feb. 18	318° 50′	43° 35′ (morning star)
0 A.D., Feb. 18	318° 50′	48° 20′ (evening star)

At each observation, the mean Sun C appeared in a direction at right angles to the apse-line ACB of Venus, the Earth being at E (Fig. 30), where ZACE = 90^0. The calculation is then as follows:

Through the centre D of the eccentric of Venus draw DS perpendicular to AB, and construct the tangents EG, EH Then

$$\angle GEC = 43^\circ\ 35';$$
$$\angle HEC = 48^\circ\ 20';$$
$$\therefore\ \angle GEH = 91^\circ\ 55', \text{ and } \angle DEH = \angle DEG = 45^\circ\ 58';$$

$\therefore\ \angle CED = 2^\circ\ 23'$. But $\angle DCE = 90^\circ$; hence the triangle CED can be solved, and (CD : CE) is found to be $\frac{416}{10000}$. But in the former theory, (CD : AC) was approximately $\frac{208}{10000}$. Now bisect CD in F, so that CF = FD = 208. Bisect F*D* in j. Copernicus regards the centre 01 the eccentric of Venus as describing from west to east the circle FD about J, so that when the Earth is at A or B, the centre of the planet's eccentric is at F, but when the Earth is midway between the apses, as at E, the centre is at D.

This hypothesis led to results in agreement with the ancient data; but contemporary observations seemed to indicate that, while FD had remained at 208, CD had diminished from 416 to 350.

(C) MERCURY

The problem of representing the motion of the planet Mercury had taxed the ingenuity of Ptolemy, and the most complicated of all the schemes of planetary motion in the *De Revolutionibus* is the one relating to this member of the solar system (V, 25). The orbit of Mercury, as it was conceived by Copernicus, may be explained by reference to Fig. 31:

The circle ATB is the Earth's orbit, with centre C. About

D, a point on the diameter AB, a circle EF is drawn. With centre F (the point on EF farthest from C) a circle HJ is drawn, which is to be the eccentric of Mercury; and with centre J is drawn the epicycle LK.

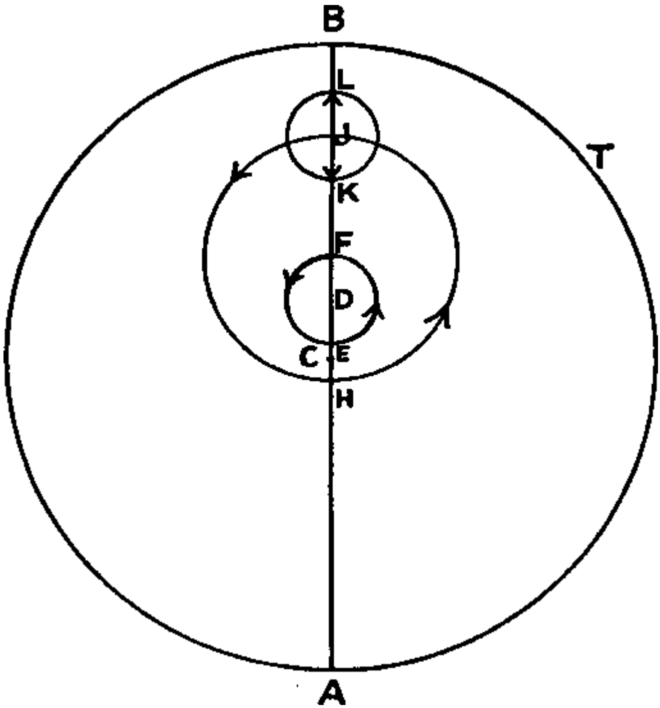


FIG. 31

The centre F of the eccentric HJ is conceived to describe the circle EF from west to east at twice the Earth's rate of revolution round the Sun, and J swings round about the centre F from west to east in the planet's sidereal period of

88 days, relatively to the stars, or in its synodic period of 116 days, relatively to the Earth. The planet meanwhile oscillates between L and K on the diameter LK of the epicycle, with a motion compounded of circular motions, after the manner explained in connection with the theory of precession (Chapter IV, § 2 *supra). It* performs two complete oscillations in a year. When the Earth is at one of the apses, A or B, the centre of the planet's eccentric is at F, and the planet is at K. When the Earth is 90^0 from A and B, the centre of the eccentric^is at E, and the planet is at L.

By the same method as that employed to determine the apse-line of Venus' eccentric, Copernicus fixes the farther apse of Mercury in longitude 183^0 20', and the nearer apse in 3^0 20' (V, 26; cp. *Aim.,* IX, 7).

The numerical elements of Mercury's orbit are next determined (V, 27) from measurements of the maximum elongations of Mercury from the mean Sun, with the Earth (1) at an apse, and (2) 90^0 from an apse:

	LONG, OF MEAN SUN	MAX. ELONGATION OF MERCURY
(1)	182^0 38'	19^0 3' (morning star)
	4^0 28'	23^0 15' (evening star)
(2)	93° 30'	26^0 15' (evening star)
	93^0 39'	20^0 15' (morning star)

(cf. *Aim.,* IX, 8, 9)

The positions of the Earth at the times of the observations (1) were nearly 180^0 apart, and may be taken as coinciding sensibly with the apses. In Fig. 32, let C, D be the centres of the orbits of the Earth and of Mercury respectively, **and** let A, B be the positions of the Earth, at the apses, at the times of the respective observations (i). Draw tangents AG, BH to the planet's eccentric, and join DG, DH. From the observations (i),

$$\angle DAG = 19^\circ\ 3', \text{ and } \angle DBH = 23^\circ\ 15'.$$

Hence the ratios of the sides of the right-angled triangles

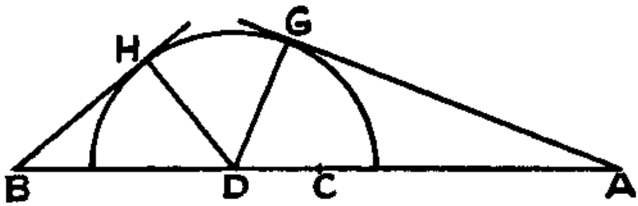


FIG. 32

AGD, BHD can be calculated, and, taking AC = 10,000, we have

$$DG = DH = 3{,}573, \text{ and } CD = 948$$

which gives the radius and the eccentricity of Mercury's eccentric in terms of the radius AC of the Earth's orbit.

idering next the observations (2), we are to assume radius CE joining the Earth E to the centre of its perpendicular to the planet's apse-line AB (Fig. 33), tat the centre of Mercury's eccentric is now at some point F differing from the position D which it occupied (Fig. 32) when the Earth was at an apse. Draw the tangents EG, EH to the planet's eccentric. Then Copernicus' problem is to find the position of F, and to determine DF and FG. From the observations (2),

$\angle CEG = 26° 15'$, and $\angle CEH = 20° 15'$.

$\therefore \angle GEH = 46° 30'$.

$\therefore \angle FEG = \angle FEH = 23° 15'$.

$\therefore \angle CEF = \angle CEG - \angle FEG = 3°$.

Also, $\angle ECF = 90°$.

Hence the sides of the triangle ECF are found (in terms of CE = CA = 10,000) to be, CF = 524, and FE = 10,014. But CD was found above to be 948 on this scale. Hence

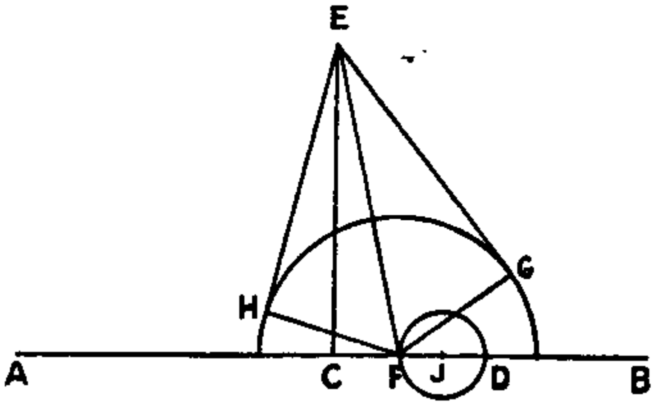


FIG. 33

FD = (948 − 524) = 424, which is the diameter of the small circle FD described by the centre of Mercury's orbit. Hence, if J is the centre of that circle, CJ = 736.

Also, in the right-angled triangle HEF, where $\angle HEF = 23° 15'$, FH is found to be 3,953, where CE = 10,000. But when the Earth was at A or B, it was shown (above) that the planet was at a distance 3,573 from the centre of its eccentric. Hence the amplitude of the planet's oscillation about its mean distance from the centre must be (3,953 — .3573 or 380.

adoption of the heliocentric standpoint was the indispensable pre-condition for the vast subsequent advances in astronomical theory which have made possible the precision of the lunar and planetary tables of to-day.

§4. MOTIONS OF THE PLANETS IN LATITUDE

Copernicus devotes the last and shortest Book of the *De Revolutionibus* to the problem of representing geometrically the observed departures of the planets from the plane of the ecliptic. The superior planets and the inferior planets form two groups, each requiring separate treatment; and we shall consider the two cases separately.

(A) *Case of a Superior Planet*

Copernicus represents the motions in latitude of the superior planets as follows (VI, 1): Each of these planets moves in a plane inclined to the ecliptic; it thus shows a maximum northward displacement from the ecliptic at one point of its orbit, and a maximum southward displacement at the diametrically opposite point. At the points midway between these limits the planet meets the ecliptic as it passes through its nodes. (Copernicus makes the line of nodes 'hrough the centre of the Earth's orbit, not through .) The cycle of changes in the planet's latitude, further depends upon the motion of the terrestrial x, the latitude appearing to vary according to the planet's position in relation to the Earth. This is partly an optical effect, due to variations in the distance between the two bodies; but Copernicus attributes it, in part, to a fluctuation in the inclination of the plane of the planet's orbit to the ecliptic. He likens it to the oscillation of the pole assumed in the theory of precession (Chapter IV, § 2 *supra),* and he supposes it to be regulated according to the following law (VI, 2):

Let ABCD (Fig. 34) be the Earth's orbit with centre E, and FGKL the planet's *mean* orbit inclined to the ecliptic, which it intersects in GEL. Let F be the northernmost and K the southernmost point on FGKL, and G, L the nodes. Let the *true* plane of the planet's orbit (at some given date) be OGPL, intersecting the mean orbit (and the ecliptic) in GEL. Then if the Earth is at A, and the planet at O, in opposition to the Sun, the inclination of its orbit is to have its maximum value (mean value + maximum value of Z.OGF); if the Earth is at B, 90^0 distant in longitude from the planet, the inclination has its mean value (O coincides with F); if the Earth is at C, 180^0 from the planet, the inclination has its minimum value (mean value—maximum value of Z.OGF), while if the Earth is at D, the inclination has its mean value (O again coincides with F). The planet, of course, may lie anywhere on its orbit, and not necessarily at O; but the general rule holds that the inclination is a maximum when the planet is in opposition to the Sun, and a minimum when it is in conjunction, the cycle of changes in the inclination recurring in the planet's synodic period.

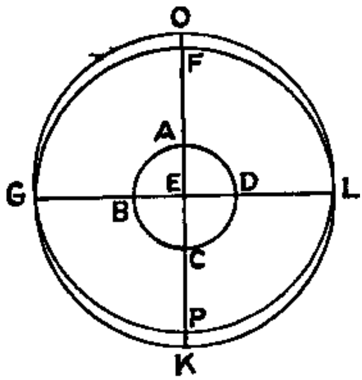


FIG. 34

Copernicus deduces **the mean** inclinations of the orbits of the several superior planets, and the amplitudes of their oscillations, from observations of the planets' maximum angular departures from the ecliptic at opposition and at conjunction (VI, 3; cp. *Altri.,* XIII, 3), and he shows how to calculate the apparent latitude of a superior planet with the Earth at any given point on its orbit (VI, 4).

(B) *Case of an Inferior Planet*

The closing chapters of the *De Revolutionibus* treat of the motions in latitude of Venus and Mercury. In this portion of the work Copernicus can make but little claim to originality; he goes freely to Book XIII of the *Almagest* for observational data and for methods of disentangling the superposed effects of the several independent orbital oscillations postulated in his theory.

The orbit of each of the inferior planets is to be regarded as intersected by the ecliptic in its apse-line; and the inclination of each is affected by two oscillations, whose nature may be explained with the aid of Fig. 35 (VI, 2).
\BCD be the Earth's orbit, and the dotted circle GKFL
n orbit of the planet (whose eccentricity to ABCD
'VT exaggerated in the figure). The two planes inter-
the apse-line FG. When the Earth was at B or D, *90* from an apse, the inclination of the planet's orbit, deduced from observations of its latitude, was found to be less than when it was observed from A or C. This is accounted **for** by assuming an **oscillation** about the apse-line **FG causing the inclination of the planet's true orbit to be a maximum when the Earth is at an apse, and a minimum**

when the Earth is midway between the apses. When, however, the Earth is at A and the planet at G, or the Earth at C and the planet at F, the planet's latitude is not, in general, zero. Accordingly a second oscillation is assumed called the *deviation.* The planet is supposed to travel on a circle RTS intersecting the plane GKFL in RS. The plane of this circle oscillates about RS, alternating between equal and opposite inclinations to GKFL. The planet is at R or S when the Earth is midway between the apses, and at T, 90^0 from R and S, when the Earth is at A, for RS swings round in such a manner that the orbital motion *of* the planet relative to T is the same as that of the Earth relative to A. The *period* of this second oscillation is equal to the period in which the planet traverses the circle RTS relatively to T, and therefore equal also to the period in which the Earth completes the circuit ABCD. The *phase* of the oscillation about RS is so adjusted that the inclination of the circle RTS to the plane GKFL is a maximum when the Planet is at T, which it is when the Earth is at A. Hence, while the planet moves from T to R on the circle RTS, the inclination of this circle to the mean orbit diminishes from its maximum value to zero. Following out the cycle it

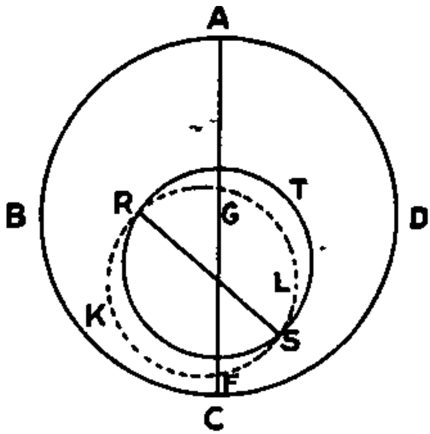


FIG. 35

becomes clear that the planet must lie always on the same side of the plane GKFL, only touching it in passing through R and S. Thus, Venus is always north of GKFL, and Mercury always south. The circle RTS is for Venus concentric, but for Mercury non-concentric, with the orbit GKFL.

Copernicus devotes four chapters (VI, 5-8) to the evaluation of the numerical constants defining the motions of Venus and Mercury in latitude. We shall restrict ourselves to a brief outline of his procedure, suppressing all numerical details.

The first quantity to be examined (VI, 5) is the "declination," i.e. the latitude of the planet when the Earth is midway between the apses of the planet's orbit, and when, therefore, the "deviation," as above defined, vanishes. Ptolemy gives *(Aim.,* XIII, 3) values for the corresponding maximum north and south latitudes of Venus and Mercury when these planets are (i) nearest the Earth, and (ii) farthest from the Earth. From these values, and the known distances of the planets at such times, Copernicus calculates the angles of inclination of the planes of the orbits to the ie at the times of observation, and he shows how to the latitude of each planet when it is at any given its orbit (the Earth being still supposed to be .y between the apses), and also the amount by which tins displacement in latitude alters the longitude of the planet measured along the plane of its orbit (cp. *Aim.,* XIII, 4). The "obliquation" is next considered, i.e. the departure of an inferior planet from the ecliptic, observed when the Earth lies in the planet's apse-line (VI, 6, 7). The "deviation" is involved in this. The inclination of the orbital plane to the ecliptic, under these conditions, *unaffected by deviation,* is obtained from the *mean* of the latitudes of the planet, observed at its greatest elongations on the two sides of the Sun. This value of the inclination is compared with that already found when the Earth was midway between the apses, and the comparison gives both the mean inclination of the orbital plane and the amplitude of its oscillation about the apse-line. The ''deviation*' alone now remains to be determined (VI, 8). It was introduced to account for the fact that the maximum north latitudes *7>l* Venus, observed from the Earth at an apse, exceed the maximum south latitudes, while the corresponding south latitudes of Mercury exceed the north latitudes. The mean values of the north and south latitudes (the "obliquation") observed when the Earth is at an apse, being known (VI, 6) for both planets, the residual northward displacement of Venus, and southward displacement of Mercury, must be attributed to the deviation. This quantity can therefore be determined from a comparison of the mean and extreme values of the latitudes. When the Earth is not at an apse, only a proportion of the maximum deviation has to be applied; and Copernicus shows (VI, 8) how to compute the correction appropriate to any given position of the Earth in relation to the apses. This section of the *De Revolutionibus,* which occupies the last few pages of the work, concludes with tables of planetary latitudes; these are similar in form to those of Ptolemy (VI, 8, 9; cp. *Aim.,* XIII, 5, 6).

The planetary theory of Copernicus represents, not only his most sustained mathematical effort, but also the **most** substantial evidence that he could advance of the scientific truth of his system. The philosophical arguments of Book I could clearly be invoked to prove almost anything. The solar theory of Book I I I merely offered a geometrical equivalent to the traditional point of view. So long as the Sun and Earth were the only bodies considered, there were no grounds for deciding, or even distinguishing, between the rival solutions. Reference to the sphere of stars, while introducing meaning into the controversy, brought also the challenge of a crucial instance (the search for stellar parallax) whose negative character embarrassed the followers of Copernicus down to close upon a century ago. The lunar theory of Book IV could have no bearing upon the matter either way. In his planetary theory, however, Copernicus succeeded in numerically relating to a single cause those principal inequalities of the several planets which had always been regarded as just so many unrelated phenomena to be separately explained by the introduction of an *ad hoc* complication into the economy of each planet. Copernicus, indeed, did not entirely abolish such complications, for we have seen epicycles, etc., playing a prominent part in his « But he initiated the process by which such arbitrary ; have been steadily eliminated from astronomical and cosmic phenomena have been referred to ever laws of ever-increasing generality.

EPILOGUE

WITHIN about a century and a half of its formulation by Copernicus the heliocentric theory had gained almost universal acceptance in scientific circles. The story of its establishment does not come within the scope of this book. We shall, however, in these concluding pages briefly indicate the contributions of a representative group of sixteenth- and seventeenth-century thinkers to the triumph of Copernicanism.

Rheticus, the young mathematician* 'from Wittenberg, whose youthful enthusiasm cheered the last years of Copernicus, was, in all probability, the earliest of his disciples. The first important contribution to the spread of Copernican ideas after 1543, however, was due to Rheticus' colleague, Erasmus Reinhold (1511-53), who held the senior chair of Mathematics at Wittenberg. Reinhold prepared a revised and enlarged edition of the astronomical tables of the *De Revolutionibus,* and published it in 1551 under the title of *Tabulae Prutenicae coelestium motuum.* These tables followed, in the main, the rubrics laid down in Copernicus' book; but they were based upon an independently calculated set of constants; they aimed at greater precision and convenience, and they were free from the arithmetical slips which occasionally marred their Copernican prototypes. Although Reinhold's tables rested upon very inadequate foundations, like all others of their kind down to the seventeentli century, they represented an improvement upon the Alfonsine Tables, and their general adoption helped to make the Copernican theory familiar, if not immediately acceptable, to astronomers.

What seem to be the earliest literary evidences of the spread of Copernican ideas among English writers on astronomy date from 1556. In that year John Field, or Feild (1520(?)-87), a gentleman of Yorkshire, published an ephemeris for the ensuing year *(Ephemeris anni* 1557 *currentts juxta Copernici et Reinhaldi canones supputata,* etc.). In his introduction Field declares that he has followed Copernicus and Reinhold, "whose writings are established and based upon true, sure and authentic demonstrations/' The year 1556 saw also the publication of a cosmological treatise, *The Castle of Knowledge*, by Robert Recorde (1510(?)~58), a figure of some importance in the history of sixteenth-century mathematics. This book contains a passage which, though perhaps intentionally ambiguous, seems to imply some recognition, on the author's part, of the significance of the Copernican theory.

Among the most enthusiastic of these early English disciples of Copernicus was William Gilbert (1540-1603), physician to Queen Elizabeth, and a pioneer in the experimental investigation of magnetism and electricity. In his great book, *De Magnete* (1600), after proving that the magnetic properties of the Earth are roughly analogous to the of a spherical lodestone, Gilbert proceeds to put •guments for the diurnal rotation of the Earth two magnetic poles, some of them of the same those advanced by Copernicus, others more closely upon Gilbert's own peculiar philosophy of magnetism.

The Copernican doctrine found another fearless champion in the Italian philosophical reformer, Giordano Bruno

(1548-1600), who, however, went considerably further than Copernicus in the novelty and boldness of his teachings concerning the constitution of the Universe, and who, for his many heresies, was eventually burned at the stake in Rome. Bruno conceived the Universe as a boundless space occupied by a countless number of heavenly bodies, all composed of the same four elements (earth, air, fire, and water), and all moving freely. The Sun was merely one of the stars, distinguished from the others only by his greater proximity to us. Bruno did his best to demolish the Aristotelian propositions that the Universe has a centre occupied by some privileged body, that the stars and planets are supported by solid spheres, and that the motions of bodies depend in any way upon their positions. His teachings were not entirely acceptable even to many Copernicans. By his written and spoken words, however, Bruno helped, especially in England and Germany, not only to establish the heliocentric theory, but to carry the Copernican revolution a stage further by dethroning the Sun and his train of planets from their distinctive and central position in the Universe.

A series of recorded observations of the planets, more accurate and systematic than any previously made and extending over many years, was the special contribution of Tycho Brahe (1546-1601), a Danish aristocrat, who, next to Copernicus himself, must be accounted the greatest of sixteenth-century astronomers. His labours would, in any case, have led to the great improvement of planetary tables at which he aimed; but, as it proved, they further opened the way to discoveries which laid the foundation of the whole of modern dynamical astronomy, and of much else besides. Though he shared the objections which Copernicus had raised against the Ptolemaic system both on observational and on physical grounds, Tycho Brahe could never bring himself to accept as a substitute a system involving the motion of the Earth; this, he held, "contradicts not only the principles of Physics, but also the authority of the Holy Scriptures, which repeatedly confirm the stability of the Earth/' An argument of greater validity which he directed against the annual motion of the Earth postulated by Copernicus was the seeming absence of any corresponding periodic displacement in the apparent positions of the stars-Tycho Brahe's expressions of his personal opposition to the Copernican theory are, however, far outweighed in significance by the services which he rendered to the new astronomy as one of the greatest observers of all time. He improved upon the design and construction of the types of instruments employed in his day for measuring the positions of the Sun, Moon, and planets in relation to the background of stars and to the standard celestial circles; and the long series of planetary observations which he was thus able to leave to his successors was all the more valuable because accompanied by nu entirely new star-catalogue, which completely super-

Ptolemy and its Copernican derivative.

ore his death at Prague, Tycho Brahe had

little band of assistants a young and com-

pa - unknown German astronomer, Johann Kepler (1571-1630), and it is with his name that the next notable stage in the establishment of the Copernican theory is associated. Kepler had been won over to the new theory as a student at Tubingen University, and thereafter his energies were chiefly directed to the reformation of astronomy. He made his greatest discoveries while attempting to construct schemes of planetary motion agreeing better than those of Copernicus with the results of refined observation, and while searching, in Pythagorean fashion, for the simple mathematical laws by which he believed the elements of the several planetary orbits to be connected. After Tycho Brahe's death Kepler secured access to the Danish astronomer's recorded observations, and for several years he concentrated his efforts on the problem of constructing a geometrical theory of the motions of Mars, conceived, from the Copernican standpoint and based on the data afforded by Brahe's observations. At first Kepler sought to frame his theory in accordance with the long-established geometrical conventions to which the historic planetary systems had conformed. But he failed to secure that measure of agreement between his theory and the observations which he felt that the refinement of the latter justified him in demanding. He was forced to conclude that the orbit of Mars could not be represented in terms of uniform circular motions about fixed points; and he was ultimately led to formulate, for the planet Mars in the first instance, his first two Laws of Planetary Motion:

I. The planet describes an ellipse about the Sun in one focus.

II. The radius vector drawn from the Sun to the planet describes equal areas in equal times.

These laws were announced in Kepler's greatest book, *Astronomia nova* (1609: *cap.* LIX). They were subsequently extended from Mars to the remaining planets, and they were supplemented *ten* years later by the third and last of Kepler's great Laws of Planetary Motion:

III. The squares of the periods of revolution of the several planets are proportional to the cubes of their respective mean distances from the Sun (*Harmonices Mundi,* 1619: V, 3).

Contributions of decisive importance to the establishment of the heliocentric theory were made by the great Italian physicist, Galileo Galilei (1564-1642), during the early part of the seventeenth century. In the first place, Galilei invalidated the typical philosophical arguments against the heliocentric theory by substituting for the Aristotelian scheme upon which they were based a sounder system of Dynamics, and thus placing the problem of the'planetary motions in a new light. Secondly, with the aid of telescopes of his own device and construction, Galilei discovered a number of celestial phenomena, some of which told strongly in favour of the Copernican theory, especially by helping to break down the traditional distinction between the Earth and the heavenly bodies. Lastly, Galilei stood out as a brilliant exponent and an irrepressible champion of the heliocentric theory. His advocacy of Copernicus culminated, in 1632, in the publication of his great *Dialogues on the Two Chief Systems of the Universe,* in which he placed before his countrymen, in their native Italian, the claim of the new cosmology to be consistent with sound physical principles, and marshalled the evidence which his own discoveries had brought to its support.

About the beginning of the seventeenth century the Aristotelian conception of the celestial motions as representing some sort of spontaneous activity on the part of the planets or of their carrying-spheres had begun to give place to the characteristically modern view of the solar system as a mechanism whose parts act upon one another according to universally valid mechanical laws. It was Isaac Newton (1642-1727) who formulated the modern principles of mechanics and applied them to the concrete problems presented by the solar system. He showed that Kepler's Laws could be regarded as expressing certain properties of the orbits of planets moving under attractive forces directed towards the Sun, and varying inversely as the square of the distance from that body. He further accounted for these forces by generalizing terrestrial gravitation into a universal property of bodies. He thus established the heliocentric theory as the accepted cosmology of the modern world by showing how it opened the way to the interpretation of the laws of planetary motion as rational consequences of a few simple dynamical generalizations covering vast fields of observable fact. Occasional voices were, indeed, raised on behalf of the geocentric theory during the eighteenth century, and even later; but such opinions are now to be found only in fanatical circles. It remained for astronomers of the generations following Newton to prove that his mechanical principles sufficed to account for the intricacies in the motions of the celestial bodies, down to all but the last detail. Only an unexplained discrepancy in the rate of progression of the perihelion of Mercury remained outstanding to be the Achilles' heel of the Newtonian theory of gravitation, and to constitute, in due course, the crucial test of a still more comprehensive synthesis.

Unearum); this calls for a word of explanation. In Ptolemy's *Almagest* (I, 9) there is a table giving the lengths of the chord AB of a circle (Fig. 37) corresponding to certain assigned values of the angle AOB which that chord subtends at the centre O. These lengths are expressed in terms of a unit, or "part," which is one-sixtieth of the radius; they are therefore independent of the dimensions of any particular circle. The chords of certain angles are known from elementary geometry (e.g. chord of 90^0 = 60 *2* parts). The calculation of the remainder of the table depends upon a set of theorems analogous to the formulae of elementary trigonometry, and of which Ptolemy gives proofs; he seems, however, to have derived both the table and the underlying propositions from Hipparchus of Rhodes. Such a "Table of Chords" was the Greek equivalent of a sine-table, since

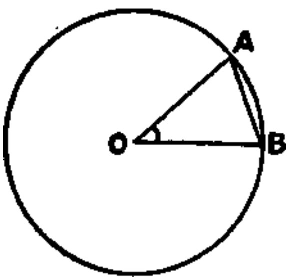


FIG.37

chord of / A O B = 2.AO.sin ½ (Z.AOB)

The conception of the *sine* of an angle, measured by half the chord of double the angle, appears to have been introduced by the Hindus not later than the fifth century A.D.; it was adopted by the Arabs, and a table of sines was included in the *Epitome in Almagestum* of Purbach and Regiomontanus, a copy of which Copernicus acquired soon after its publication in 1496.

Copernicus' Table (I, 12) gives the semichord of twice the angle, for angles increasing by successive increments of 10', from o° to 90°, with a column of mean differences to facilitate interpolation. As Copernicus takes the hundred-thousandth part of the radius of the circle as his unit or "part/' the figures in his table are the same as those in a modern table of sines.

NOTE III

The "Commentariolus" and the "Narratio Prima"

The historic book in which Copernicus elaborated the details of his planetary system was not published until 1543, the year of its author's death. In the meantime, however, the essential doctrines of the book had twice^been put into literary form to satisfy the curiosity which its fame had awakened. Copernicus himself drew up a brief, non-technical account of his system, generally known as the *Commentariolus,* which appears to have been circulated in manuscript in sympathetic quarters. Later the main facts were made known to a wider public through the more detailed *Narratio Prima* which Rheticus published in 1540. We shall now briefly indicate the contents of these two compositions.

(1) Two manuscripts are known to exist of the synopsis entitled *Nicolai Coppernici de hypothesibus motuum coelestium a se constitutis commentariolus.* The first of these was discovered at Vienna by M. Curtze, and edited by him in 1878 *(Mittheilungen des Coppemicus-Vereins zu Thom,* Heft I, pp. 1-17). The second was found shortly afterwards at Stockholm by A. Lindhagen *(Bihang till K. Svenska Vet. Akad. Handlingar,* Bd. 6, 1881). The text was established from these two manuscripts by L. Prowe *(Nicolaus Coppernicus,* Vol. II, pp. 184-202). It has been translated into

English by E. Rosen in *Osiris,* Vol. III, Pt. I, 1937. Curtze assigned the composition of the *Commentariolus* to the period 1533-39. Birkenmajer, however, adduces evidence for supposing that its contents were elaborated before 1509, and that it was composed not later than 1512 *(Mikolaj Kopernik*, 1900). His reasons for assigning so early a date to the *Commentariolus* depend upon his contention (see Chapter II, § 4 *supra)* that Copernicus worked out two successive types of planetary theory, and that the one given in this little tract is the earlier of the two.

The *Commentariolus* begins with a brief allusion to the two typical planetary theories of Antiquity, and indicates their characteristic deficiencies. The system of homocentric spheres failed to account for the undoubted variations in the distances of the planets from the Earth; the rival system of eccentrics and epicycles, as developed by Ptolemy, was free from this objection, but it involved assumptions which conflicted with sound Physics. Copernicus, seeking a way out of this dilemma, at length found that a reasonable and satisfactory explanation of the planetary motions could be given, provided the following postulates were granted:

(i) That the celestial spheres have not all the same centre.

(ii) That the centre of the Earth is not the centre of the Universe, but is only the centre of gravity and of the Moon's sphere.

(iii) That the Sun is at the centre of all the planetary spheres ^nd of the Universe.

(iv) That the ratio of the Sun's distance from the Earth to the height of the firmament is less than that of the Earth's radius to the Sun's distance, and is therefore negligible.

(v) That all apparent motion in the firmament is due to the motion of the Earth, which, with the surrounding elements, daily turns upon poles fixed in the firmament.

(vi) That all apparent motion of the Sun is due to the movement of the Earth, which revolves about the Sun like any other planet.

(vii) That the progressions and retrogressions of the planets are to be attributed to the motion of the Earth, which alone suffices to explain all the apparent non-uniformities in the heavens.

The technical details of the Copernican system are then summarized under seven heads, mathematical "demonstrations "destined for a larger work" being omitted:

(i) The order of succession of the planetary spheres, with their approximate periods of revolution, is laid down in accordance with the heliocentric scheme of Plate II, Fig. II.

(ii) The triple motion of the Earth is indicated, and precession is attributed to a mutability in the direction of the Earth's axis of rotation.

(iii) It is recommended that the length of the year be deduced from observations of the apparent motion of the Sun relative to the fixed stars, and not to the equinoxes.

(iv) A theory of the motion of the Moon is outlined which is precisely that developed in Book IV of the *De Revolutionibus* (see Chapter V, § 1 *supra).*

(v) A theory of the motions in longitude of Saturn, Jupiter, and Mars is here outlined; but it differs from that eventually adopted in Book V of the *De Revolutionibus.* There each of these planets is supposed to describe an epicycle whose centre describes a deferent which is *eccentric* to the Earth's orbit (see Chapter VI, § 2 *supra).* Here, in the *Commentariolus,* each planet's motion is represented by means of two epicycles, one superimposed upon the other, and a deferent which is *concentric* with the Earth's orbit, the whole forming a system closely similar to that assigned to the Moon. (This represents the earlier type of planetary theory which Birkenmajer traced in cancelled passages of the manuscript of the *De Revolutionibus.)*

(vi) The system of Venus is also constituted by a deferent and two epicycles after the manner of the superior planets.

(vii) The system of Mercury, conceived on similar lines, is, however, complicated by a periodic fluctuation in the radius of the deferent. The theories of the motions in latitude of the several planets, outlined under (v), (vi), and (vii), resemble those developed in Book VI of the *De Revolutionibus* (see Chapter VI, § 4 *supra).*

Copernicus finally reckons up the number of separate circular motions involved, explicitly or implicitly, in the foregoing hypotheses; they amount to thirty-four in all.

(2) The *Narratio Prima* is mainly concerned with the contents of Book III of the *De Revolutionibus;* Rheticus had not yet completed his study of the manuscript of Copernicus when he addressed this report to his old teacher, Johann Schoner, and he intended to follow it up with further *Narrationes.* Besides being a competent astronomer, Rheticus was an accomplished classical scholar, and he writes in a somewhat flowery style, with many high-flown, though doubtless sincerely meant, panegyrics on his *"Dominus Doctor/"* Copernicus, whom, however, he does not mention by name. He treats the various topics in a rather

different order from that in which they occur in the manuscript. After briefly indicating the contents of all six Books, he deals, in succession, with the determination of the rate of precession and of the amplitude and period of its anomaly; with the measurement of the tropical and sidereal years; and with the periodic changes in the obliquity of the ecliptic, and in the eccentricity and apse-line of the Earth's orbit (see Chapter IV *supra).* At this point Rheticus turns aside for a moment from his exposition of Copernicus, and seeks to establish a connection between the fortunes of earthly monarchies and the motion of the centre of the Earth's eccentric round the small circle which it describes about its mean position (see Fig. 16, p. in *supra).* He believed that Rome began to decline when this centre started approaching the Sun, and that, when at length it has moved round to the position which it occupied at the Creation, the Second Advent may be expected. (Rheticus* astrological leanings may more probably be attributed to the influence of his mentor Melanchthon than to that of Copernicus, whose writings seem to be free from all trace of the false science.) Returning to his text, Rheticus deals with the remaining heads in the following somewhat arbitrary order: the Copernican lunar theory; the arguments in favour of the mobility of the Earth; the general arrangement of the heliocentric solar system (with the diagram of *De Rev.,* I, n); the threefold motion of the Earth, and the double oscillation of the polar axis; and, lastly, the Copernican schemes of motion of the planets in longitude and latitude, without numerical data or diagrams.

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INDEX

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