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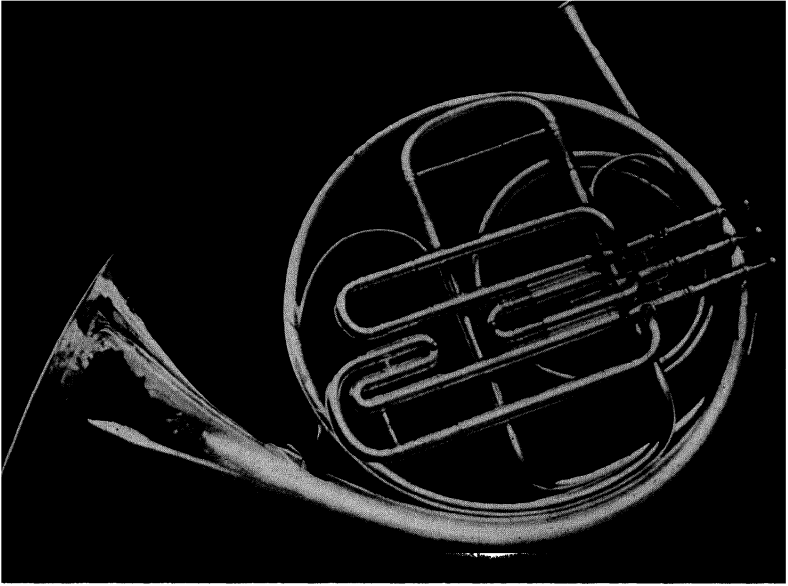
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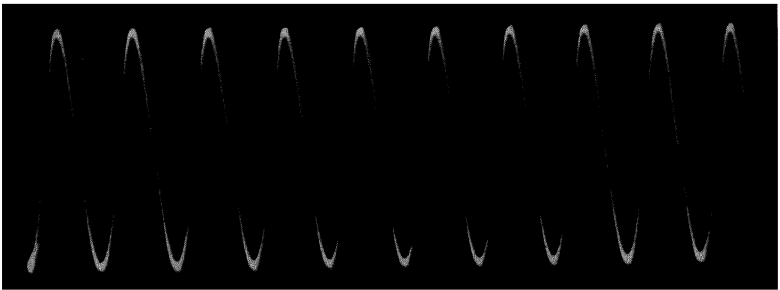
Title Sound

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PLATE I



THE ORCHESTRAL HORN



OSCILLOGRAM

The curve shows the way in which the pressure in the surrounding air varies with time while a note is sounded on the horn. The note is f' of frequency 350 cycles per second, the eighth harmonic of the twelve-foot air column in the instrument.

SOUND

By

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PREFACE

OF the branches of Physics studied in schools, Sound has always been somewhat of a "Cinderella". Much of it does not lend itself to test by simple questions, with the result that it has rarely been given a fair representation in examination papers. It is only natural that teachers, pressed for time, have concentrated the attention of their pupils upon the "uglier sisters".

This neglect has been a pity, for many of the phenomena of sound are of everyday occurrence and general interest. This is more so than ever to-day, with the spread of electrical apparatus for sound reproduction. The sisters are not nearly so independent as the fairy tale would lead us to believe: indeed electricity and sound are very closely linked.

In the present volume an attempt has been made to deal with all matters of general interest in Sound, and its connections with other branches of Physics. The mathematical basis of the subject has been included for the benefit of those who are sufficiently equipped to make use of it, though it is fully realised that not every student will be in this position. The book contains sufficient matter for Advanced Level and Scholarship candidates: Degree candidates also will in many cases find their syllabus covered. It is further hoped that students of Music will find something useful, particularly in the sections on Temperament and Musical Instruments. Most of the more difficult sections of the book are printed in smaller type, in order to facilitate selection.

In preparing the book I have naturally had recourse to many standard text-books on this subject, especially those by E. H. Barton, E. G. Richardson, and Alexander Wood. I have also had the assistance of many friends, whose help has made the task a real pleasure. Dr. H. Martyn Cundy, of Sherborne, and Mr. M. Nelkon, M.Sc., of the William Ellis School, have read the work in MS. and in proof, and made many valuable suggestions. Mr. H. H. Brown, of Sherborne, was kind enough to give his photographic skill to the production of the plates, both of oscillograms and of musical instruments. Miss M. Child, of Sherborne, produced from her oboe the sounds, steady in pitch and amplitude, that are depicted in the oscillograms of Plate IV. To these, and others who have helped, my best thanks are due. Finally I should like to thank the publishers, who, through Mr. Alan J. W. Hill, have co-operated fully with me in trying to make the book both complete and attractive.

F. G. MEE.

SHERBORNE, 1950.

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CHAPTER I

INTRODUCTORY

1. Sound and Hearing

WHEN studying the radiation produced by hot bodies, we learn that it is possible to produce electromagnetic waves having an immense range of wavelengths, of which a very narrow band is capable of stimulating the eye. This narrow band we call *light*, and a good deal of our knowledge of the outside world is received through the sensation it produces. Our sensation of hearing is equally useful, for the production and hearing of sounds provides the main channel through which we communicate with one another. We shall shortly learn that fluctuations of air pressure, of the correct extent and rapidity, are responsible for the sensation of sound..

The ear is a remarkable organ, for it is capable of dealing with sounds of a very wide range of intensity, and has great discriminating powers. Not only can we identify different sources by the sounds they make, but we can pick out one sound when several are going on together, which is almost always the case. And when it comes to detail, we can recognise the voices of different persons although they may say the same words or sing the same notes.

Some sounds are more or less continuous in character—that is, they last for an appreciable time without change, during which time they have a definite pitch. They are scientifically known as musical notes. Other sounds, which change their character in a minute time and thus have few or no regular features, are usually termed noises. In everyday life we commonly mean by a noise “sound that is not desired by the recipient”—and we vaguely assume that a “musical” note ought to be pleasant to listen to. This is by no means always the case with the more general scientific musical note. Such notes are easier to study than the more complex noises, and we find no difficulty in recognising some as deep or low in pitch, while others are shrill or high in pitch. It is a matter of common observation that sounds are produced by vibrating bodies, and that the pitch is connected with the rapidity or frequency of the vibrations of the source.

The loudness of a sound, like the brightness of a light, has a connection with the energy of the source and will also depend on the fraction of it that reaches the ear. But again as in the case of light, other factors are involved also : not all sounds of the same energy are equally

loud. The ear is not equally sensitive at all pitches, just as the eye is not equally sensitive to all colours of the rainbow.

Normally, the sound is carried from the source to the ear through the air, though other materials are equally, if not more, effective. For example, if we strike a tuning fork and place the shank of it on the teeth or the temple the sound is clearly perceived, having been carried by the bones of the head.

2. The Action of a Vibrating Body on the Surrounding Medium

We now have to enquire into the process by which the vibration of the source is carried to the ear. A convenient example is the loud-speaker¹ forming part of a radio set or radiogramophone. This

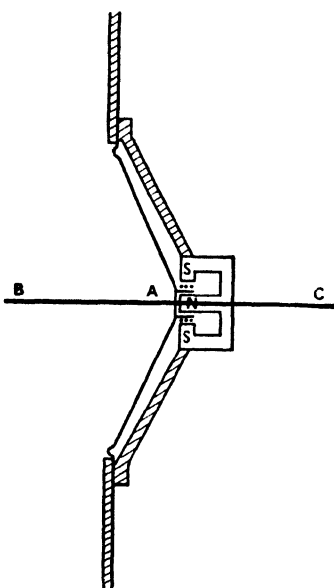


FIG. 1.

frequently consists of a conical diaphragm of stiff paper, flexibly mounted around the circumference, so as to permit motion along the axis of the cone. The diaphragm carries a coil which lies in a strong magnetic field, so that a force acts on the coil when a current passes through it. The diaphragm is usually placed behind a hole in a large board, called a baffle-board (Fig. 1).

Suppose the diaphragm A is caused to make a forward movement—along AB. Because the air in front of it has *inertia*, due to its mass, it cannot instantaneously move away with the velocity of A, but will become to some extent compressed. As air has *elasticity*, this layer of compressed air will subsequently re-expand, compressing the next layer, and so on. Thus, as the result of a single movement of A in the forward direction, a *pulse of compression* travels away from it, to some

extent in all directions, though with the greatest intensity along the line AB.

At the same instant that the compression is produced in front of A, the air behind the diaphragm is subjected to a reduction of pressure. This results in the transmission of a pulse of reduced pressure, or *rarefaction*. This will be most intense along AC, if there is no obstruction, but will be present to some extent in all directions.

If the compression from the front of A and the rarefaction from the back of A were to reach the same point at the same moment with the

¹ This is dealt with more fully in § 75.

same intensity, they would obviously cut one another out. It is the purpose of the baffle-board to prevent this, partly by influencing the intensity distribution of both, and partly by providing a time delay due to the difference of the distances that the two pulses have to travel.

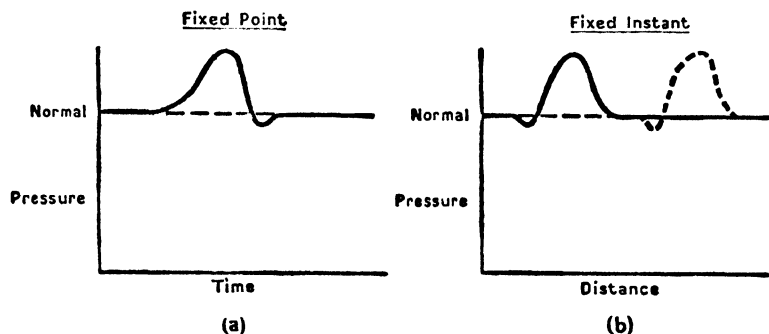


FIG. 2.

The single movement of the diaphragm towards the front therefore causes the pressure at any point B in front of it to vary with time, as shown in Fig. 2a. The elevation of pressure above normal occurs later at greater distances from A, so that the pressure at a fixed instant varies with distance as shown in Fig. 2b. At a later instant the whole curve will have moved on to take the dotted position. Points along AC experience similar changes, but these are diminutions of pressure instead of elevations.

The disturbance created by A—an increase or decrease of pressure—is thus eventually reproduced at distant points. The actual air near to and compressed by A only executes quite a small movement from its original position: but the *state of being compressed* may occur at great distances. Hence the circumstance is described as “the propagation of a condition in a medium without propagation of the medium bearing it.” This type of propagation is known as a *wave*.

If the diaphragm executes a regular vibratory motion, as it will do if the coil carries an alternating current, the air on each side of it will be alternately compressed and rarefied. The curves for pressure variation will then become as in Fig. 3. At a *fixed point* (Fig. 3a), the pressure goes through a complete cycle of its changes in one complete cycle of the motion of the source, this time interval being called the *period*. The reciprocal of this, the number of cycles per second, is called the *frequency*. The actual shape of the curve naturally depends on the equation of motion of the source. At a *fixed instant* (Fig. 3b), after the start, the pressure will vary along a line through the source, since the compressions take time to travel. Thus while one compression is at the point X, that born one period later is at Y, the next at Z, and so on.

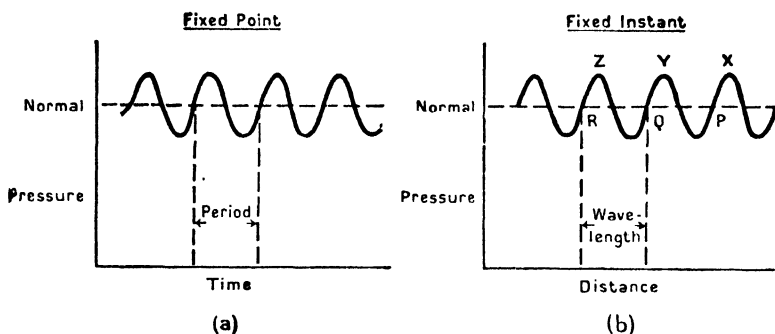


FIG. 3.

These points, like the others, P, Q and R, are points where the disturbance has the same *phase* at the fixed instant. They are equidistant, and the distance XY, YZ, PQ or QR is called the *wavelength*. For audible sound waves in air this distance is from a few inches to a few feet: it is obviously the distance travelled by the disturbance in the time of one period.

The maximum extent of the disturbance of pressure above or below normal is called the *pressure amplitude*. This becomes smaller with increasing distance, partly because the compression is spread in a larger volume of medium, and partly because of imperfect elasticity. Not all the energy used in compression is restored in the subsequent expansion; this occurs particularly in fluids, on account of viscosity or other frictional effects. The dying out of the wave which results is called *attenuation*.

3. Reception of Sound from a Tuning Fork

A tuning fork consists of a U-shaped bar of metal with arms of equal length and mass, with a shank or handle at the middle point. It is commonly encountered in the laboratory and elsewhere as a standard of pitch, for the frequency of the note it gives is subject to very little variation. The prongs are pinched together, or one of them is struck on a reasonably resilient surface. This sets the fork into a transverse vibration, the two prongs moving inwards together, then outwards together, and so on. The disturbance of the air is slightly more complicated than that produced by the loudspeaker diaphragm described in § 2, since there are two prongs in motion.

When both prongs, D, D (Fig. 4), are moving inwards, the air between them is "squeezed out," so that pulses of compression travel outwards with the greatest intensity along the two lines FG. At the same time rarefactions travel with their greatest intensity along the two lines DE. The result is that if the ear is placed at a point on any of these lines the sound will be heard considerably, but that the intensity will fall

to zero at points along the lines HFH and KFK, for the compression and rarefaction will be equal in intensity there. The path difference for the compression and the rarefaction is too small to be of any significance, as the fork is small compared with the wavelength of the sound in air. Thus if the vibrating fork is rotated by the side of the ear, the sound goes through four minima and corresponding maxima for each revolution of the fork.

Normally, when we wish to hear the sound of the fork we bring the shank into contact with a table or other large surface. The vibration is conveyed down the shank to the table, and although the vibration of this is minute it disturbs a large quantity of air and makes a correspondingly louder sound.

It will be realised from the case of the tuning fork that the disturbance of the air by a source of sound such

as a musical instrument or voice is often very complicated. When we add to this the fact that the sound in a room can reach the ear by a great variety of paths, owing to reflection at the walls and other objects, we must expect phenomena of which a detailed theoretical account would be extremely involved.

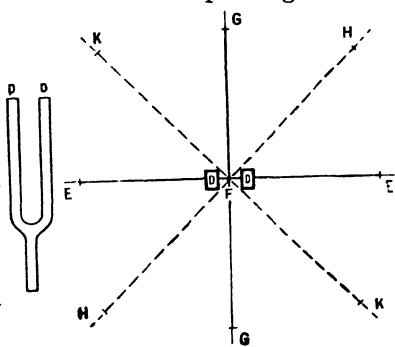


FIG. 4.

4. Types of Wave Disturbance

The action of a vibrator on the surrounding medium is to produce a train of compressional waves. Any portion of the medium moves to and fro *along* the line of propagation, causing the pressure to be alternately increased above normal and decreased below it. Such a wave may therefore be described as one of compression, or of longitudinal displacement. For sounds of ordinary intensity the displacement amplitude is always quite small—from about 10^{-10} to about 10^{-3} cm. In order to be perceived as a sound by the human ear, the frequency of the disturbance must lie between certain limits. These limits vary with the individual and with the intensity of the disturbance: roughly speaking, the lower limit for a pure tone is about 15 cycles per second (c.p.s.), while the upper limit is from 10,000 to 30,000 c.p.s. This matter will be taken up more fully in Chapter IX, when the variation of sensitiveness of the ear is discussed. Compressional waves of frequencies much above the audible range can be produced and are of considerable practical importance: they are said to be supersonic, or ultrasonic, disturbances.

Many kinds of disturbance other than compression can be propagated as waves. In waves in deep water, for example, any particular element of the water moves in a vertical circle. Thus (Fig. 5) elements of water whose undisplaced positions are at P and Q are, at the instant

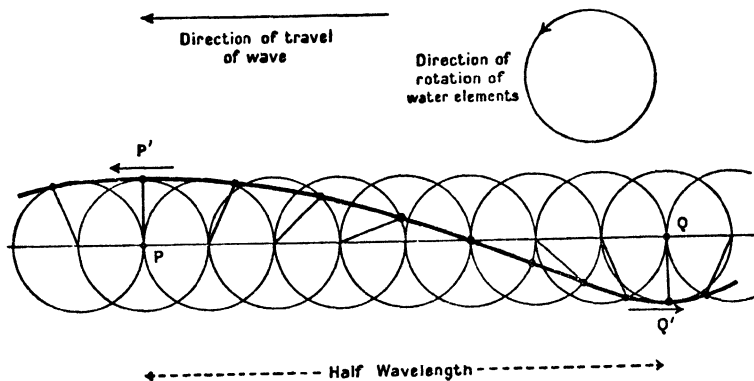


FIG. 5.

represented, actually at P' and Q'. The water at the crest of the wave is, in fact, moving at that moment in the direction of propagation, while that at the trough is moving backwards.

If a stretched cord or wire is displaced to one side and released, the condition of being displaced travels along it, being in due course reflected at the supports. A *transverse displacement* wave is set up: this may produce sound waves in the air, mostly by its disturbance of the supports. A *longitudinal displacement* wave can be produced on the same wire by rubbing it in the direction of its length with a cloth sprinkled with resin. This is a true sound wave *in the wire*, and its velocity and frequency are quite different from those of the transverse wave. It, too, will communicate a sound wave to the air through the supports, the pitch of the sound being quite different from that of the sound set up by the transverse vibration.

Light, and electromagnetic radiation generally, are wave disturbances in which the condition propagated is an electric and magnetic field, both lying in a plane at right angles to the direction of propagation. In this case a very wide range of wavelengths exists—from about 10^{-11} cm. for γ -rays to thousands of metres for long wireless waves. The narrow band of wavelengths from about 4×10^{-5} cm. to about 8×10^{-5} cm. is capable of stimulating the eye, and is called light.

5. Simple Harmonic Motion

If we hang a body on the end of a rubber cord, or helical spring, and pull it down and then release it, the body moves up and down in

a vertical line. Each oscillation takes a certain time, which does not depend upon the size of the oscillation provided the spring obeys Hooke's Law that the extension is proportional to the stretching force. The time of an oscillation depends upon the mass of the body and the force required to produce unit extension in the spring. In practice, the motion dies down owing to loss of energy in air resistance and the imperfect elasticity of the spring: if these were absent the oscillation would continue for ever. This type of motion is an example of Simple Harmonic Motion, which is important because it is the simplest brick out of which all periodic motions can be built.

Definition

If a particle moves in a straight line in such a way that its acceleration is always directed towards a fixed point in that line and proportional to its distance from that point, it is said to move in Simple Harmonic Motion (S.H.M.).

The motion of the body on the spring is easily seen to be a case of Simple Harmonic Motion.

Suppose that the body attached to the spring has mass M , so that the force of gravity acting on it is Mg .

Let this force produce a steady extension l in the spring (Fig. 6b). When the mass is disturbed, let it be at some instant at a position x below the steady equilibrium position, so that the extension of the spring is $l + x$ at this instant (Fig. 6c). Assuming the spring to obey Hooke's Law, the tension T in it in this position is given by

$$T = Mg \left(\frac{l + x}{l} \right).$$

Neglecting air resistance, the resultant force on the mass = $Mg - T$. Neglecting the mass of the spring, the whole of this force is used in giving the mass the acceleration $\ddot{x}$.

$$\begin{aligned} \therefore M\ddot{x} &= Mg - T \\ &= Mg - Mg \left(\frac{l + x}{l} \right) \end{aligned}$$

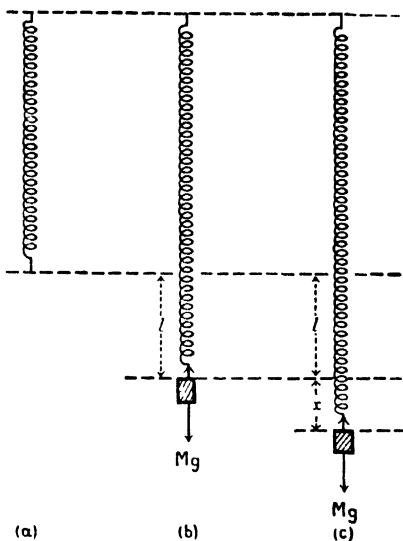


FIG. 6.

$$= -Mg\frac{x}{l}$$

$$\therefore \ddot{x} = -\frac{gx}{l}.$$

Thus the acceleration is directed towards the steady rest position of extension l , and proportional to the distance x from this position. The mass therefore moves in Simple Harmonic Motion.

Solution of the equation.

For simplicity, write $\ddot{x} = -\omega^2 x$ (1)
so that $\omega^2 = \frac{g}{l}$ in the case of the mass on the spring.

Multiplying by $2\dot{x}$, $2\dot{x}\ddot{x} = -2\omega^2 x\dot{x}$

Integrating with respect to t , $\dot{x}^2 = -\omega^2 x^2 + c$.

If $\dot{x} = 0$ where $x = a$, i.e., if a represents the greatest displacement,

$$\text{then } 0 = -\omega^2 a^2 + c$$

$$\therefore \dot{x}^2 = \omega^2(a^2 - x^2) \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$\therefore \frac{\dot{x}}{(a^2 - x^2)^{\frac{1}{2}}} = \omega.$$

Integrating again, $\sin^{-1} \frac{x}{a} = \omega t + \delta$

$$\frac{x}{a} = \sin(\omega t + \delta).$$

$$x = a \sin(\omega t + \delta) \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Thus the motion is *periodic*, x varying with time t according to a sine curve. The *amplitude* a does not decay, but each oscillation is as large as those preceding it. The *phase* of the motion depends on δ . If $\delta = 0$, then $x = 0$ when $t = 0$.

The velocity $\dot{x} = a\omega \cos(\omega t + \delta) \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$

From (3) and (4), the displacement x and the velocity $\dot{x}$ repeat their former values when the "angle" increases by 2π . This happens in time $\frac{2\pi}{\omega}$, which is therefore the *period* T , of the motion. The *frequency*,

or number of oscillations in unit time, is $n = \frac{\omega}{2\pi}$.

From (2), kinetic energy, when $x = 0$,

$$= \frac{1}{2}M\dot{x}^2 = \frac{1}{2}M\omega^2 a^2 = 2\pi^2 Mn^2 a^2.$$

As the body moves the energy changes from being all Kinetic in this position to all potential in the momentary rest positions. Thus this maximum value of kinetic energy represents the TOTAL energy, and is

seen to be proportional to the square of the amplitude a , and the square of the frequency n . Equation (2) is, in fact, the energy equation for the motion, $\frac{1}{2}M\dot{x}^2$ being the Kinetic Energy and $\frac{1}{2}M\omega^2x^2$ the Potential Energy, when the displacement is x .

For the special case of the mass on the spring, for which $\omega^2 = \frac{g}{l}$, the period $\frac{2\pi}{\omega}$ becomes $2\pi\sqrt{\frac{l}{g}}$. This is the same as that of a simple pendulum of length l , this being the steady extension produced by the weight of the mass attached to the spring.

In actual practice, a mass suspended by a helical spring would not perform a true S.H.M., owing to air resistance. This gradually takes energy away from the vibration, so that the amplitude dies away. There is no practical case of a natural vibration which is exactly simple harmonic, as energy is always dissipated in one form or another. Fortunately the effect of decay of amplitude upon the frequency is very small indeed, so that damping can usually be neglected when calculating this quantity. For a full discussion see § 22. Failure of Hooke's Law, and of proportionality generally in other cases, will also cause departure from the perfect sine motion. This happens, for example, with a pendulum, and a magnet oscillating in a magnetic field, if the amplitude of the motion is more than a few degrees.

6. Periodic Motion Generally

In a Simple Harmonic Motion the displacement x varies with the time according to a sine curve (Fig. 7a). For various reasons, of which failure of the simple proportionality between displacement and restoring force is one, vibratory motion is seldom Simple Harmonic, even if energy is supplied to maintain the motion. Thus, for example, a point on a violin string performs a motion of the kind shown in Fig. 7b, when the string is being bowed continuously.

This motion is *periodic*, in so far as it repeats itself exactly at time intervals represented by AB.

It was shown by Fourier that any such motion having a period T , and frequency $n \left(= \frac{1}{T} \right)$, could be made up of a S.H.M. of frequency n together with other S.H.M.'s whose frequencies are multiples of this, if the correct choices of amplitude and phase of the individual harmonic motions are made. Any periodic curve whatever can be matched by choosing these variables correctly. Fig. 7c shows the result of adding S.H.M.'s having frequencies in the ratios 1 : 2 : 3 : 4, and respective amplitudes in the ratios 1 : $\frac{1}{2}$: $\frac{1}{3}$: $\frac{1}{4}$, the phases being such that the zeros occur together. It will be seen that this curve is not unlike

that for the motion of the point on the bowed string, shown in Fig. 7b. To obtain a more exact match it would be necessary to include higher multiple frequency terms.

The various constituents of a periodic motion are called "partials." One of them has the same period and frequency as the motion itself, and is known as the *fundamental*, while the others are commonly

referred to collectively as "harmonics," "upper partials," or "overtones." A certain amount of confusion exists in the nomenclature in this matter: in acoustics the first *multiple* frequency actually present has often been referred to as the "first harmonic" or "first overtone." In alternating current practice it is customary to refer to the fundamental, of frequency n , as the first harmonic term, and to any other term of frequency pn as the p th harmonic. In view of the intimate connection now in existence between alternating currents and sound production and reproduction, the alternating current

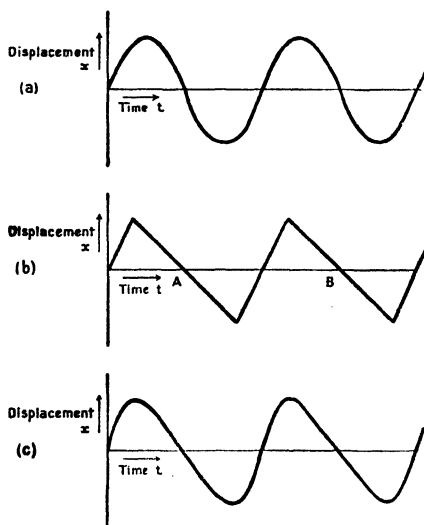


FIG. 7.

custom will be adopted in this book. For example, a note of fundamental frequency 196 c.p.s. produced by a clarinet (§ 62) contains, in addition to the fundamental, a powerful constituent of frequency 588 c.p.s. This will be referred to as the *third harmonic*, though it might also be called the *first overtone*, since the term of frequency 392 c.p.s. is very weak or absent. The curve representing the wave-form of this actual note sounded on a clarinet is shown in Plate V.

7. Characteristics of Sounds

Musical sounds—that is, those which consist of regularly sustained notes, are produced by bodies performing some periodic type of motion. These sounds have three distinguishing characteristics—quality, pitch and loudness.

Quality

What is generally known as "the same note" has a different sound when produced by different musical instruments. It may be shown by experiment that we regard two notes as "the same" if the time

taken for a complete repetition of the wave-form is the same: the quality differs because of difference in the wave-form itself. In other words, notes are classed as "the same" if their *fundamental* periodicity is the same. We can say that differences in quality are due to differences in the form of the curve representing the vibration; or we can say that the number and intensity of the multiple frequency terms which accompany the fundamental are different. It is often possible to pick out these "upper partials" by ear, as they are frequently more powerful than the fundamental. Generally speaking, however, we recognise an instrument or a voice by the total effect of all rather than by the singling out of individual powerful harmonics.

The question of quality and wave-form will be taken up more fully in Chapter VIII, and in the descriptions of musical instruments in Chapters VI and VII.

Pitch

When we state the pitch of a note in cycles per second we give only the fundamental periodicity. There is no doubt that differences of quality cause, subjectively, apparent variations of pitch. If a scale is played on a musical instrument or on a rank of organ pipes, a certain note or pipe may sound "out of tune" if its quality is substantially different from that of the others in the series, even though its fundamental frequency is quite correct.

If notes of suitable pitches are sounded in succession it is possible to recognise definite relationships, called *intervals*, between them. This may be investigated by a simple experiment, in which a disc, perforated with a number of rings of holes, is used (Fig. 8). The rings may contain 12, 15, 16, 18, 20 and 24 holes, equidistant in any one ring. The disc is rotated at a uniform speed by a quiet motor, and air is blown through a jet that can be directed at any of the rings at will. When this is done sounds are produced, each ring giving a fundamental frequency equal to the number of holes passing by in unit time.

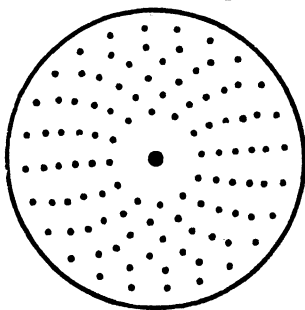


FIG. 8.

If the jet is directed in turn at the 12 and 24 rings, the notes are at once perceived to be an octave apart. This is true whatever may be the speed of the motor. Other pairs of rings sounded in succession also give recognisable musical intervals, again unaffected by the speed of the motor. Thus it is frequency *ratio*, and not numerical difference, that settles the interval between two notes. This may very easily be verified by playing a gramophone record at widely different speeds:

the "tune" sounds the same in each case, though the "key" is altered.

In a sense, then, pitch discrimination is logarithmic, for two notes stand in a fixed relation to each other if the logarithms of their frequencies differ by a fixed amount. Thus the lowest note A_1 on the pianoforte has a fundamental frequency of approximately $27\frac{1}{2}$ c.p.s., while A_2 , an octave above it, has frequency 55 c.p.s. : the top a'''' has frequency 3,520 c.p.s., and a''' , an octave below it, 1,760 c.p.s. But there is just the same "space" in pitch between the one pair and the other, and just the same number of notes on the ordinary scale between them.

Loudness

The loudness of a sound at a given place depends upon the energy of the disturbance there, which is proportional to the square of the amplitude and the square of the frequency. The sensitivity of the ear is, however, quite different at different frequencies, so that sounds of equal energy may be quite different in loudness if they differ in pitch or harmonic distribution.

Loudness discrimination, like that of pitch, is logarithmic in character. If one sound is to be recognised as perceptibly louder than another of the same pitch and quality there must be a larger increase of energy if the original sound is loud than if it is soft. The proportionate energy increase necessary is fairly constant. This matter is considered more fully in Chapter IX.

CHAPTER II

WAVE MOTION

8. The Equation of Propagation of Plane Compressional Waves in a Fluid

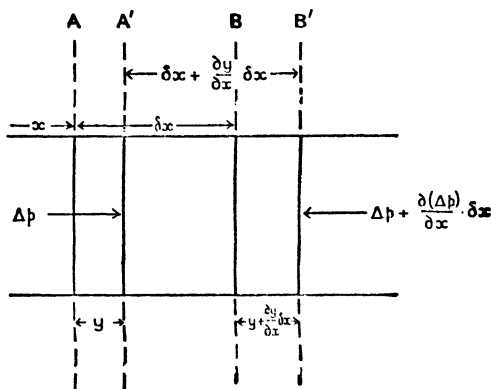
IN § 2, the spreading of a pulse of compression was considered in general terms. The discussion may be given mathematical precision, and the velocity of propagation shown to be $\sqrt{\frac{E}{\rho}}$, where ρ is the density and E the Modulus of Elasticity of the medium. The following is a method of proving this.

Any Modulus of Elasticity is the ratio of a stress to the strain it produces. In the case of a fluid, in which a pressure acts equally in all directions, the only practical Modulus is the Bulk Modulus, defined by

$$\text{Bulk Modulus, } E = \frac{\text{Stress}}{\text{Strain}} = \frac{\text{Increase of Pressure}}{\text{Decrease of volume per unit volume}}.$$

Consider a tube of fluid of unit cross section, and two right plane sections A, B, of the tube (Fig. 9). Let the undisturbed positions of these planes, measured along the tube from any arbitrary zero, be x and $x + \delta x$.

At some later instant, when the medium is disturbed, let the plane A be displaced along the tube by an amount y to A', while B is correspondingly displaced by $y + \left(\frac{\partial y}{\partial x}\right) \delta x$ to B'.



B. (y is a function both of x and t .)

The disturbance has resulted in an alteration of *volume* of the sample lying between the planes, and a variation of *pressure* from point to point along the tube.

Original volume of sample between planes = δx

New volume of sample between planes = $\delta x + \left(\frac{\partial y}{\partial x}\right) \delta x$

$\therefore$ Increase of volume = $\left(\frac{\partial y}{\partial x}\right) \delta x$

$$\therefore \text{Decrease of volume per unit volume} = -\frac{\partial y}{\partial x} \quad . \quad . \quad . \quad . \quad (1)$$

$$\text{Also, Mass of sample} = \rho \delta x$$

ρ being the density.

Let the pressure be Δp above normal for the section initially at A (now at A'), and consequently $\Delta p + \frac{\partial(\Delta p)}{\partial x} \delta x$ for the section initially at B (now at B').

These two pressures are equivalent to

(a) Equal and opposite pressure excesses Δp , which provide the stress on the sample and are linked with its volume change.

(b) A resultant force $\frac{\partial(\Delta p)}{\partial x} \delta x$ in the direction from B' to A', causing the sample to accelerate.

$$\text{But} \quad \text{Stress} = \text{Strain} \times \text{Modulus}$$

Inserting the value of the strain given in (1)

$$\Delta p = -\frac{\partial y}{\partial x} \cdot E \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Also, Resultant force from B' to A' = Mass $\times$ Acceleration from B' to A'.

$$\frac{\partial(\Delta p)}{\partial x} \delta x = \rho \delta x \left(-\frac{\partial^2 y}{\partial t^2} \right) \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$\therefore \frac{\partial(\Delta p)}{\partial x} = -\rho \frac{\partial^2 y}{\partial t^2}$$

Substituting for Δp the value $-E \frac{\partial y}{\partial x}$ given in (2)

$$-E \frac{\partial^2 y}{\partial x^2} = -\rho \frac{\partial^2 y}{\partial t^2}$$

or

$$\frac{\partial^2 y}{\partial t^2} = \frac{E}{\rho} \frac{\partial^2 y}{\partial x^2}$$

For convenience, we write c^2 for $\frac{E}{\rho}$, so that

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

This differential equation is satisfied if

$$y = f(ct - x) \quad . \quad . \quad . \quad . \quad . \quad (5)$$

since $\frac{\partial y}{\partial x} = -f'(ct - x)$, so that $\frac{\partial^2 y}{\partial x^2} = +f''(ct - x)$;

also $\frac{\partial y}{\partial t} = cf'(ct - x)$, so that $\frac{\partial^2 y}{\partial t^2} = c^2 f''(ct - x)$.

f is any function.

Similarly, another solution of the equation (4) is $y = f(ct + x)$. . . (6)

These two expressions, (5) and (6), are known as particular solutions of the equation: the general solution is

$$y = f_1(ct - x) + f_2(ct + x) \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where f_1 and f_2 are any functions, alike or different.

Expression (7) connects the longitudinal displacement y with the position x and time t . Using equation (2), the excess pressure Δp is seen to be given by

$$\Delta p = E f_1'(ct - x) - E f_2'(ct + x) \quad . \quad . \quad . \quad (8)$$

This expression shows how the excess pressure varies with position and time.

Examining (5), we find that the value of y obtaining at any place and time is repeated exactly if t increases by Δt and x by $c\Delta t$, i.e., the disturbance is travelling with velocity c in the positive direction of x .

Similarly, equation (8) represents a disturbance travelling with this same speed in the negative direction of x .

We have therefore the propagation of the condition y (or Δp) without propagation of the medium bearing it, that is, a *wave motion*, and

$$\text{velocity with which the waves travel} = c = \sqrt{\frac{E}{\rho}}$$

The function f_1 or f_2 is settled by the form of the wave: it may be a single pulse, or a periodic function.

9. Simple Harmonic Waves. Relation between Velocity, Frequency and Wavelength

With many practical sources of sound the function f_1 or f_2 is oscillatory in character, having an approximately constant period for several oscillations at least. These are the sounds that have a definite recognisable pitch, classed earlier as "musical notes." An example of a sound wave for which the function is not periodic would be the large pulse of "blast" produced by an explosion, though even this might acquire something of an oscillatory character as the result of successive reflections.

As the simplest type of periodic function is the simple harmonic variation, the simplest type of wave is one in which the function is the sine or cosine.

Consider the expression

$$y = A \sin \frac{2\pi}{\lambda} (ct - x)$$

where y represents the displacement of a particle of the medium at distance x along the line of propagation at time t , c and λ being constants.

- (i) At a *fixed point* (x constant), y varies harmonically with the time, and goes through one complete cycle in time λ/c . (Fig. 3a, p. 3).

$$\therefore \text{Period} = \lambda/c$$

$$\therefore \text{Frequency } n = c/\lambda \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

- (ii) At a *fixed instant* (t constant), y varies harmonically with the distance, and if x changes by λ , y is restored to the same value and the same rate of change. The distance apart of such points,

where the disturbance is in the same phase, is, by definition, the *wavelength*. (Fig. 3b, p. 3.)

$$\therefore \text{Wavelength} = \lambda \quad (2)$$

(iii) If t increases by Δt and x by $c\Delta t$, y is restored to the same value and the same rate of change: i.e., a disturbance at one place is repeated Δt later at a position $c\Delta t$ further from the source.

$$\therefore \text{Velocity with which the waves travel} = c \quad (3)$$

The expression
$$y = A \sin \frac{2\pi}{\lambda} (ct - x)$$

therefore represents a progressive wave for which the variations of displacement with time and distance are simple harmonic.

Substituting (3) and (2) in (1), we obtain

$$\text{frequency} = \frac{\text{velocity}}{\text{wavelength}}$$

$$\text{velocity} = \text{frequency} \times \text{wavelength} \quad (4)$$

The similar expression $y' = A \sin \frac{2\pi}{\lambda} (ct + x)$ represents a wave travelling with velocity $-c$, i.e. travelling in the direction of decrease of x .

The relation (4) applies to waves of all kinds, e.g.:

- (a) A sound wave travels in air with a velocity approximately 330 metres per sec. Thus a sound of frequency 440 c.p.s. ("A" in Orchestral Pitch) has a wavelength given by $330 \div 440$, i.e., 0.75 metre.

The same sound in water, in which the velocity is approximately 1,500 metres per sec., has a wavelength given by $1,500 \div 440$, i.e., 3.41 metres.

- (b) Wireless waves travel with velocity 3×10^{10} cm. per sec. For a wavelength of 300 metres, the frequency is given by $3 \times 10^{10} \div 3 \times 10^4$, i.e., 10^6 c.p.s. or 1,000 kilocycles.

The equation
$$y = A \sin \frac{2\pi}{\lambda} (ct - x)$$

represents only a simple harmonic *plane wave* without attenuation—i.e., one in which the amplitude does not become less as the distance from the source increases. If the wave is diverging, even if there is no loss of energy, the amplitude will fall off as the distance increases, since the wave is covering a spherical surface of ever-increasing area. As this area is proportional to the square of the radius, the energy per unit area in the wave must be inversely proportional to this, which means that the *amplitude* must be inversely proportional to the *first power*

of the distance from the source. A spherical wave without energy loss could be represented by

$$y = \frac{a}{r} \sin \frac{2\pi}{\lambda} (ct - r)$$

r being the distance from the source.

The effect of attenuation is to cause the amplitude to decay exponentially with the distance, so that the equation becomes

$$y = \frac{a}{r} e^{-kr} \sin \frac{2\pi}{\lambda} (ct - r).$$

As Simple Harmonic Motion itself is of very rare occurrence, Simple Harmonic compressional waves are correspondingly seldom encountered. If the manner of excitation causes the source to vibrate in S.H.M., the waves will similarly have this character: but if the motion of the source is as depicted in Fig. 7*b* or *c*, the displacement of the air and eventually the pressure changes will follow a similar course. However, since any periodic variation can be built up from a series of Simple Harmonic variations (§ 6), we can carry out all our calculations on sound waves by using the Simple Harmonic expressions. This is a great convenience.

10. Longitudinal Displacement and Compression in Simple Harmonic Waves

The curves of Fig. 3, p. 3, represented variations of *pressure*: we will now consider how these are related to the displacements along the line of propagation. Such displacements, at a fixed value of t , are shown for a series of values of x by the ordinates of the curve of Fig. 10.

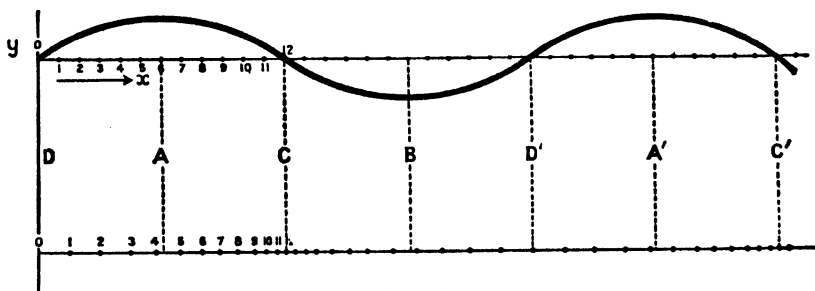


FIG. 10.

It must be borne in mind that this is only a representation: the displacement y shown in the curve at right angles to x is, in fact, *along* the direction of x in the waves we are studying. Thus a particle of the medium, such as that whose undisplaced position is at A, is

displaced at this moment along x by a distance equal to the ordinate of the curve at A.

Applying this to the particles shown by the dots, we find their displaced positions to be those shown on the lower line in the figure, assuming that a positive y represents a displacement to the right. For convenience of recognition the particles in the first half wave are numbered.

At positions such as A, A' or B, where the displacement was at its greatest value, the *spacing* of the displaced particles is the same as if there were no disturbance.¹ At points such as C, C', the particles are much closer than normal, while at D, D' they are correspondingly further apart. Thus the greatest compression is at C or C', and the greatest rarefaction at D or D', in all of which places the displacement is in fact zero. Displacement and compression therefore differ in phase so far as their variation with x is concerned. If the wave is travelling in the positive direction of x , the maximum positive displacement shown at A arrives later at C and B. The maximum compression, already at C, is therefore one quarter of a wavelength in advance of the maximum positive displacement. Similarly, if the wave is travelling in the negative direction of x (displacements being reckoned positive in the same direction as before), the maximum positive displacement precedes the greatest compression by a quarter of a wavelength or quarter of a period.

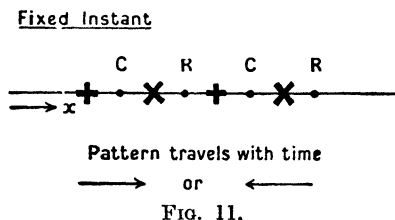


FIG. 11.

The pattern, represented for a fixed instant, travels with time, either to left or right according to the direction of travel of the wave.

These results may be deduced analytically, making use of the relation

$$\Delta p = -E \frac{\partial y}{\partial x} \quad . \quad . \quad . \quad . \quad . \quad . \quad (\text{Eqn. 2, p. 14}).$$

If the displacement y is given by

$$y = A \sin \frac{2\pi}{\lambda} (ct - x). \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

¹ The large amplitude necessary to make the figure clear causes the points at which the spacing is normal to be somewhat displaced from A, A', B. In practice, of course, the amplitude is minute compared with the wavelength, so that the positions of normal pressure are exactly at the points stated.

$$\begin{aligned}\frac{\partial y}{\partial x} &= -\frac{2\pi A}{\lambda} \cos \frac{2\pi}{\lambda} (ct - x) \\ \therefore \Delta p &= +\frac{2\pi EA}{\lambda} \cos \frac{2\pi}{\lambda} (ct - x) \\ &= \frac{2\pi EA}{\lambda} \sin \frac{2\pi}{\lambda} (ct - x + \lambda/4) \dots (2)\end{aligned}$$

Thus the x in this expression (2) must be $\lambda/4$ more than the x in (1) in order that the angle may take the same value in the two cases: i.e., Δp is a quarter of a wavelength or a quarter of a period in advance of y .

The quantity $\frac{2\pi EA}{\lambda}$ in (2) is the *pressure amplitude*, while A in (1) is correctly called the *displacement amplitude*.

It is interesting to note that, using the frequency to which the ear is most sensitive (about 3,500 c.p.s.), the minimum disturbance necessary to cause the sensation of sound has a pressure amplitude of about 10^{-4} dyne/cm.²—that is about 1 part in 10^{10} of the normal atmospheric pressure. This corresponds to a displacement amplitude of about 10^{-10} cm., and an energy flow of about 10^{-17} watt per sq. cm. For very loud sounds the displacement amplitude might be as much as 10^{-2} cm.

11. Wave-front and Ray. Huygens' Principle

Imagine a point source of sound at P (Fig. 12). A compression that left it t seconds previously will have spread to a sphere S of radius ct , where c is its velocity. All points on the sphere will therefore be in the same phase of their motion. Such a surface, which is the continuous locus of points in the same phase of their motion, is called a *wave-front*. The disturbance is being propagated along the radii of the sphere, known as *rays*.

If the source is very distant the curvature of the sphere is very slight, and we have an approximation to the "plane waves" dealt with in § 8.

The condition of the medium all around the surface of S is the same as if we could instantaneously produce all over it a multitude of point sources each emitting a compression at that moment. In time Δt each of these compressions would travel the distance $c\Delta t$ in all directions. They

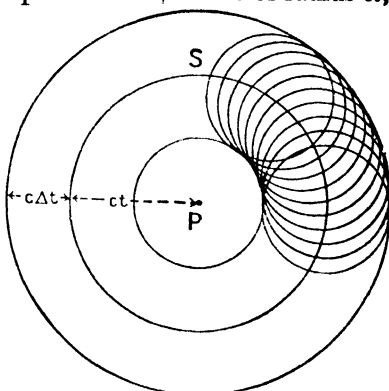


FIG. 12.

would reinforce one another at the new position of the wave-front, which is thus everywhere at this distance from the old, i.e., it is a sphere of radius $c(t + \Delta t)$ or $c(t - \Delta t)$. This is expressed in Huygens' Principle, that "the new position of the wave-front is the envelope of spherical wavelets of radius $c\Delta t$ with their centres on the old wave-front." Which of the two envelopes is chosen depends on the direction of travel of the waves. With the diverging wave of the example, the new wave-front is the sphere of radius $c(t + \Delta t)$.

12. Reflection and Refraction of Waves

When waves strike a boundary between two media in which they have different velocities, some of the energy is reflected back into the original medium. In general, there is also a change in the direction of travel of that part of the energy which enters the new medium, giving the effect known as refraction.

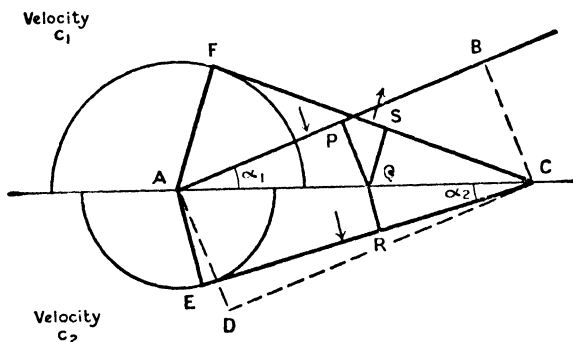


FIG. 13.

Suppose a plane wave-front represented by AB (Fig. 13), travelling in a medium in which its velocity is c_1 , strikes the boundary AC of a medium in which its velocity is c_2 .

Had there been no boundary, the front would have advanced equally at all parts of its length—i.e., while the wavelet from B reached C, that from A would have reached D.

On account of the difference of velocity, however, the wavelet from A acquires a radius $\frac{c_2}{c_1} \cdot BC$ in the new medium, while any portion of it still in the old medium still has the radius equal to BC. If we draw CE and CF tangential to the semicircles representing these wavelets we know that C and E are two points on the wave-front, if any, in the new medium, while C and F are two points on the wave-front, if any, in the old medium.

Let P be any point on AB, and Q a point on AC such that PQ is perpendicular to AB. Let QR be perpendicular to CE and QS to CF.

$$\text{Time taken for a wavelet to travel from B to C} = \frac{BC}{c_1}$$

$$\begin{aligned} \text{Time taken for a wavelet to travel from P to S} &= \frac{PQ}{c_1} + \frac{QS}{c_1} \\ &= \frac{AQ}{AC} \cdot \frac{BC}{c_1} + \frac{QC}{AC} \frac{BC}{c_1} \\ &= \frac{BC}{c_1} \end{aligned}$$

$$\begin{aligned} \text{Time taken for a wavelet to travel from P to R} &= \frac{PQ}{c_1} + \frac{QR}{c_2} \\ &= \frac{AQ}{AC} \frac{BC}{c_1} + \frac{QC}{AC} \frac{AE}{c_2} \\ &= \frac{AQ}{AC} \frac{BC}{c_1} + \frac{QC}{AC} \frac{c_2}{c_1} \cdot \frac{BC}{c_2} \\ &= \frac{BC}{c_1} \end{aligned}$$

Hence the wavelets arrive at C, F, E, S and R at the same instant, irrespective of the position of P.

∴ CF and CE are the new wave-fronts.

Obviously $\widehat{BAC} = \widehat{FCA}$, which is the ordinary *Law of Reflection*.

$$\text{Also } \sin BAC = \frac{BC}{AC} \text{ and } \sin ACE = \frac{AE}{AC}$$

$$\text{so that } \frac{\sin BAC}{\sin ACE} = \frac{BC}{AE} = \frac{c_1}{c_2},$$

$$\text{or } c_2 \sin \alpha_1 = c_1 \sin \alpha_2,$$

which is the ordinary *Law of Refraction*.

The above discussion does not, of course, prove that there will necessarily be any reflected or refracted ray: it gives the directions of the waves, but not their relative intensities. These are settled by the conditions that must be satisfied at the boundary AC.

The definite plane wave-fronts CF and CE depend for their production upon the freedom of the boundary AC from irregularities. If there are in AC irregularities *comparable with the wavelength*, the distances PQR and PQS will become modified by amounts of that order, so that CE and CF will no longer be loci of points that are in the same phase. There is then no simple wave-front, but a general scattering effect.

As sound waves are of comparatively long wavelength, quite rough surfaces—brick walls for example—are capable of giving regular reflection for sound waves. Refraction is much less commonly encountered except that of a gradual kind occurring in the atmosphere, not unlike the optical “mirage” (see § 31).

Unless an obstacle is large compared with the wavelength it will not cast effective shadows, and the waves appear not to obey the law of rectilinear propagation. This effect, known as *diffraction*, is very common with sound waves, again because of their considerable length. In the case of light, for which the wavelength is very short, noticeable diffraction is the exception rather than the rule, and the effect has to be looked for carefully. In sound, the position is quite the reverse: only very high-pitched sounds, of short wavelength, cast well-defined shadows.

13. Superposition or Interference

If two or more wave disturbances arrive at the same point, it is generally true to say that the instantaneous value of the resultant disturbance at any moment is the vectorial sum of the instantaneous values of the separate disturbances. Thus, if two sound waves of exactly the same frequency having displacement amplitudes A and B along the same line arrive at a certain point exactly in phase, they will produce there a disturbance of amplitude $(A + B)$. At points such that the phases are exactly the opposite of each other, the resultant would have amplitude $(A \sim B)$, which is zero for the case in which $A = B$. It is thus possible for two waves apparently to destroy each other at certain places—hence the name *interference*.

Of course, all possible amplitudes between $(A + B)$ and $(A \sim B)$ are represented at appropriate points, according to the phase difference. If the disturbance at a point due to one wave is represented by

$$y_1 = A \sin \theta$$

where θ varies with time, and that due to the other by

$$y_2 = B \sin (\theta + \alpha)$$

where α is the phase difference, constant at a fixed point, the resultant is given by

$$\begin{aligned} y_1 + y_2 &= A \sin \theta + B (\sin \theta \cos \alpha + \cos \theta \sin \alpha) \\ &= \sin \theta (A + B \cos \alpha) + \cos \theta \cdot B \sin \alpha \\ &= (A^2 + B^2 + 2AB \cos \alpha)^{\frac{1}{2}} \{ \sin \theta \cos \phi + \cos \theta \sin \phi \} \end{aligned}$$

$$\text{where } \tan \phi = \frac{B \sin \alpha}{A + B \cos \alpha}$$

$$\therefore y_1 + y_2 = (A^2 + B^2 + 2AB \cos \alpha)^{\frac{1}{2}} \sin (\theta + \phi).$$

Thus the resultant amplitude is $(A^2 + B^2 + 2AB \cos \alpha)^{\frac{1}{2}}$.

This may be illustrated graphically as in Fig. 14. If PQ represents the amplitude A , and QR the amplitude B , the angle α being as marked. PR represents the amplitude of the resultant. If we imagine α varying, PR goes through all values between $(A + B)$ and $(A - B)$ and back again in a complete cycle of α .

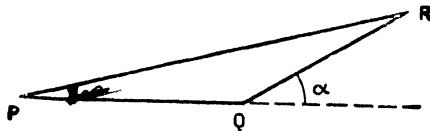


FIG. 14.

There is no contravention of the Principle of the Conservation of Energy involved in interference. The energy in a disturbance is proportional to the square of the amplitude. The mean value of this over all values of α is given by

$$\frac{\int_0^{2\pi} (A^2 + B^2 + 2AB \cos \alpha) d\alpha}{\int_0^{2\pi} d\alpha} = A^2 + B^2$$

The average energy is therefore equivalent to the sum of the separate energies of the waves. As a simple example, if $A = B$, the energy is zero at some points, but four times that of a single wave at points where the disturbances are exactly in phase. The average over all the possible values is twice that of the single wave.

For a distribution of amplitude to remain *permanently* there must be no change of relative phase with time, i.e., the matching of frequency must be absolutely exact. Such wave trains are said to be *coherent*. In the case of light waves, we are faced with the fact that a light "source" consists of millions of emitting atoms, so that the only practical way of producing interference effects with light is to take the light from one source to the point of observation by two paths differing in length. In the case of sound waves, the sources producing them are sometimes (but rarely) "single," and they can be matched in frequency more or less exactly, so that interference between waves originating from separate sources is observable. The common cases, however, are those in which there is one source and a path difference, such as that between direct and reflected waves. Alternatively the sources can be linked in some way—e.g., two loudspeakers driven by the same alternating current.

14. Stationary Waves

This is perhaps the simplest example of interference, occurring when *two wave trains of equal frequency travel along the same straight line in opposite directions*. Suppose first that they are equal in amplitude, so that they may be represented by

$$y_1 = A \sin \frac{2\pi}{\lambda} (ct - x)$$

$$y_2 = A \sin \frac{2\pi}{\lambda} (ct + x)$$

Then
$$y_1 + y_2 = 2A \cos \frac{2\pi}{\lambda} x \sin \frac{2\pi}{\lambda} ct.$$

It will be seen that the variations of y with t and with x have become separated, so that if t is increased by Δt it is no longer possible to change x by an amount (independent of t) so as to preserve $y_1 + y_2$ at the same value as before. In fact, the disturbance no longer travels.

We may regard $2A \cos \frac{2\pi x}{\lambda}$ as an *amplitude*, as it does not vary with time. At certain points, where $x = \lambda/4, 3\lambda/4$, etc., this amplitude is permanently zero, so that there is never any disturbance there. Such points are called *nodes* (P, R, T in Fig. 15). At points midway between these, such as Q, S, where $x = 0, \lambda/2, \lambda$, etc., the amplitude has the maximum value $2A$. These points are called *antinodes*.

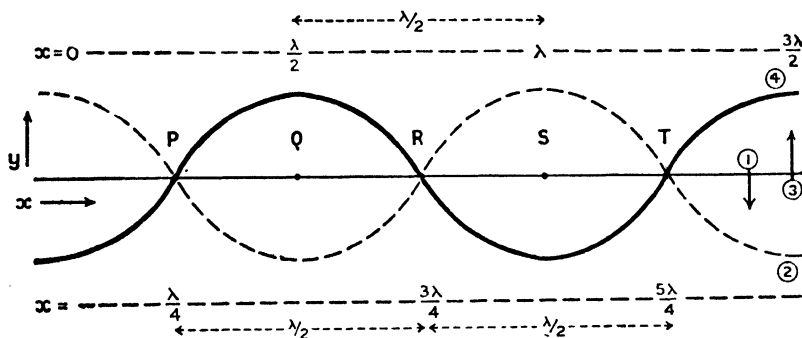


FIG. 15.

In Fig. 15, the combined disturbance, $y_1 + y_2$ is represented as an ordinate at right angles to x .

At $t = 0$ the whole curve lies along the axis of x , and there is no disturbance anywhere. Position (1).

At $t = \frac{\lambda}{4c}$ the curve is as shown at position (2), $y_1 + y_2$ having reached its full value at every point.

At $t = \frac{2\lambda}{4c}$ the curve again lies along the axis of x , position (3), though the motion of all points of it is in the opposite direction to that in position (1).

Therefore the points where there is permanently no pressure change, i.e. the pressure nodes, are those where $\sin \frac{2\pi}{\lambda}x = 0$, i.e. $x = 0, \lambda/2, \lambda$, etc.

They therefore coincide with the displacement antinodes.

15. Stationary Sound Waves by Reflection

In the commonest cases of stationary waves, the wave travelling in the reverse direction occurs as the result of a reflection, caused by the original wave striking normally upon a boundary of some kind.

Let the displacement y_1 in a sound wave moving to the right along the negative x -axis and incident upon a boundary at $x = 0$ be given by

$$y_1 = A \sin \frac{2\pi}{\lambda} (ct - x),$$

and the displacement in the reflected wave by

$$y_2 = mA \sin \frac{2\pi}{\lambda} (ct + x)$$

At $x = 0$ the combined displacement

$$y_1 + y_2 = (1 + m)A \sin \frac{2\pi}{\lambda} ct \quad . \quad . \quad . \quad . \quad (1)$$

The pressure increment in the incident wave, given by $\Delta p = -E \frac{\partial y}{\partial x}$ (p. 14),

is
$$\Delta p_1 = \frac{2\pi EA}{\lambda} \cos \frac{2\pi}{\lambda} (ct - x).$$

In the reflected wave, $\Delta p_2 = -\frac{2\pi EAm}{\lambda} \cos \frac{2\pi}{\lambda} (ct + x).$

Thus, where $x = 0$, $\Delta p_1 + \Delta p_2 = \frac{2\pi EA}{\lambda} (1 - m) \cos \frac{2\pi}{\lambda} ct \quad . \quad . \quad . \quad (2)$

If the boundary is *perfectly rigid*, $y_1 + y_2 = 0$ for all values of t , so that, from (1), $m = -1$.

If the boundary is *perfectly free*, $\Delta p_1 + \Delta p_2 = 0$ for all values of t , so that, from (2), $m = +1$. This case corresponds to the analysis used in § 14.

Thus reflection at a perfectly rigid boundary takes place with a change of phase of the displacement but not of pressure—the boundary becoming a displacement node and pressure antinode. The reflection at a “free end,” such as the open end of a tube, takes place with a change of phase in the pressure but not in the displacement, becoming a pressure node and displacement antinode.

All cases of reflection lie between these extremes. If the reflecting wall is not perfectly rigid, m , though still negative, is less than unity, the reflected wave having a smaller amplitude and energy than the incident wave. The result with a wave incident along the normal is a system of stationary waves with imperfect nodes, since the interference can never be complete. If the incident wave is of amplitude

A , and the reflected of amplitude mA , the antinodes will show an amplitude $(1 - m)A$ and the nodes $(1 + m)A$. (N.B. m is negative.)

These results are of importance in that they enable a measurement to be made of the coefficient of reflection, m , and therefore of the absorption, for different types of surface. As the energy is proportional to the square of the amplitude, that in the reflected wave is m^2 times that in the incident wave, so that the absorption coefficient is $1 - m^2$. A knowledge of this quantity is important in connection with the acoustics of buildings, though it is usually considered more reliable to find absorption coefficients for this purpose by the direct method of the reverberation chamber (§ 90).

The stationary wave method has, however, been used by Taylor and Paris, who employed a slab of the material under test to close the end of a long tube in which standing waves are formed by reflection at the slab. The intensities at nodes and antinodes were explored, using a Rayleigh disc or hot-wire microphone (§§ 78, 67). In the latter case the source was an enclosed loudspeaker.

The method suffers from the disadvantage that the backing of the specimen is unlikely to be comparable with that which exists when it forms part of an extended surface such as a wall: also that the absorption coefficient obtained is only that for normal incidence.

16. Reflection at Oblique Incidence

A practical case of interference in sound is encountered when an aeroplane passes over an observer, near a good reflecting surface, such as a road. As the machine passes there is a noticeable alteration in "quality," apart from the gradual lowering of apparent frequency known as the Doppler effect (§ 19). The change of quality is not unlike a fall in pitch while the machine is approaching, followed by a rise after it has passed. If the observer bends down, so that his ear is nearer the reflector, a change of quality comparable with a rise in pitch is produced.

The effect is caused by the fact that the sound reaches the observer by two paths, one direct, and the other involving

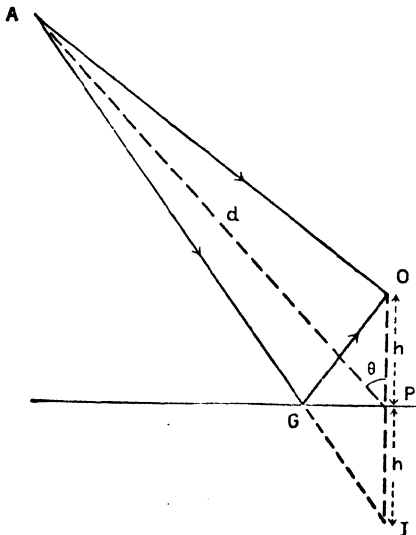


FIG. 16.

reflection at the ground. These differ in length, causing certain frequencies to be reinforced, and others practically eliminated—an effect comparable with the “colours of thin films” produced by interference in light.

Suppose that the source is at A (Fig. 16) and the observer at O, the sound arriving by the direct path AO and by the path AGO involving reflection at the ground G. If I is the “image” of the observer in the ground, the path AGO is equal in length to AI. If d , h and θ have the significances shown in the diagram,

$$\begin{aligned} AI^2 &= d^2 + h^2 + 2dh \cos \theta, \\ AO^2 &= d^2 + h^2 - 2dh \cos \theta. \\ \therefore AI^2 - AO^2 &= 4dh \cos \theta \\ \text{But } AI + AO &\simeq 2d \quad (d \gg h). \\ \therefore AI - AO &\simeq 2h \cos \theta. \end{aligned}$$

The path difference is therefore greatest when $\theta = 0$, i.e., when the machine is directly overhead.

The sound made by the machine consists of a great many vibrations of different frequencies—so many, in fact, that the effect is like a continuous spectrum. Assuming that there is no phase reversal on reflection, any frequency for which the wavelength λ satisfies the relation

$$n\lambda = 2h \cos \theta,$$

where n is an integer, will be reinforced by the interference, while any for which

$$(2n - 1)\frac{\lambda}{2} = 2h \cos \theta$$

will be greatly weakened. This is the cause of the change in quality as θ varies.

As pointed out in § 15, there is no phase reversal so far as pressure increment is concerned, on reflection at a rigid boundary. Provided the eventual detector of the sound responds to changes of pressure (rather than displacement or velocity), the reinforcement will therefore take place as stated above. This is the case with the ear as detector. E.g., for an observer for whom $h = 5$ ft., the first order of interference ($n = 1$) will give reinforcement when

$$\lambda = 10 \cos \theta,$$

and cancellation when

$$\lambda' = 20 \cos \theta.$$

Assuming the velocity of sound to be 1,100 ft. per sec., the first expression means that a tone of frequency 110 c.p.s. (including all its harmonics) will be reinforced when the machine is overhead, and that the frequencies reinforced will be higher at all other values of θ .

A somewhat similar case occurs in the detection of submarine sounds by a hydrophone, i.e., an under-water microphone. The sound made at

a point P (Fig. 17) can reach the hydrophone H by the direct path PH, and by the path PSH involving reflection at the surface S. The path PSH is equal in length to PH', so that the path difference, as before, is $2h \cos \theta$.

In the present instance, however, there is a phase reversal, so far as pressure variations are concerned, on reflection at the water-air boundary (see § 15) so that the first *maximum* of pressure amplitude occurs where

$$\lambda/2 = 2h \cos \theta,$$

$$\text{or } h = \frac{\lambda}{4 \cos \theta},$$

and the first *minimum* where $h = 0$, for all values of λ .

A hydrophone responding to pressure may therefore entirely fail to pick up sounds if it is used too near the surface, especially if $\cos \theta$ is small. It must be borne in mind that λ in water is nearly five times that for the same sound in air.

The corresponding difficulty has been encountered in the detection of low-flying aircraft by radar methods.

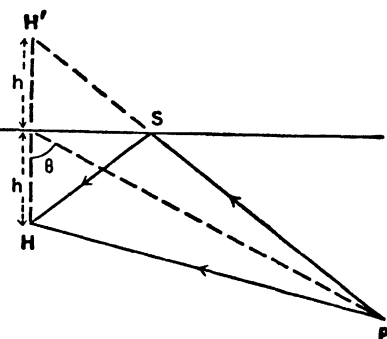


FIG. 17.

17. Beats

The foregoing cases of interference have all been concerned with the superposition of wave disturbances identical in frequency. When the two frequencies differ slightly, however, the effect is that of a gradual change of relative phase. At any one moment the two waves may be in phase at a particular point: subsequently one will have gained half a period upon the other so that reinforcement will give place to weakening, and so on. In the case of sound waves the effect perceived by the ear is a regular increase and decrease of intensity, known as *beats*. Naturally, the sound is not at its maximum at all points at the same moment: the beats constitute a slowly travelling interference pattern (see § 19, p. 40).

Suppose that the separate disturbances have amplitudes A_1 and A_2 , and frequencies n_1 and n_2 ($n_1 > n_2$). At a given point they will arrive in phase at a certain instant, and constitute at that instant a disturbance of amplitude $A_1 + A_2$. This situation will recur after a time interval t in which one source gains one complete cycle on the other, so that

$$n_1 t = n_2 t + 1,$$

or

$$t = \frac{1}{n_1 - n_2}.$$

The successive maxima of amplitude are therefore separated by the time interval $1/(n_1 - n_2)$, so that there are $n_1 - n_2$ maxima in unit time. In fact, the frequency of the beats is the difference of the

frequencies of the notes. Between the maxima the amplitude falls off to the value $A_1 \sim A_2$. The simple relation between the frequency of the beats and the frequencies of the notes renders them most useful for telling when notes are nearly "in tune." When the tuning is exact the beats disappear.

The resultant disturbance of which the intensity is varying has, in general, no exactly constant frequency, the time interval between successive zeros or successive turning values being different at different parts of the cycle. These factors, and the precise form of the variation of displacement with time, depend upon the relative amplitudes and phases of the separate disturbances. In the special case in which the separate amplitudes are equal, the resultant displacement becomes

$$\begin{aligned} y_1 + y_2 &= A \sin(2\pi n_1 t + \delta) + A \sin 2\pi n_2 t \\ &= 2A \cos\left\{2\pi\left(\frac{n_1 - n_2}{2}\right)t + \delta/2\right\} \cdot \sin\left\{2\pi\left(\frac{n_1 + n_2}{2}\right)t + \delta/2\right\} \end{aligned}$$

Successive zeros of displacement occur whenever

$$2\pi\left(\frac{n_1 + n_2}{2}\right)t + \delta/2 = k\pi \text{ (from the sine term)}$$

where k is an integer, that is,

$$n_1 + n_2 \text{ times per second ;}$$

and also whenever

$$2\pi\left(\frac{n_1 - n_2}{2}\right)t + \delta/2 = \frac{(2k - 1)\pi}{2} \text{ (from the cosine term)}$$

that is, $n_1 - n_2$ times per second.

There are therefore likely to be $2n_1$ zeros per second altogether, unless there are any coincidences. ($n_1 > n_2$.)

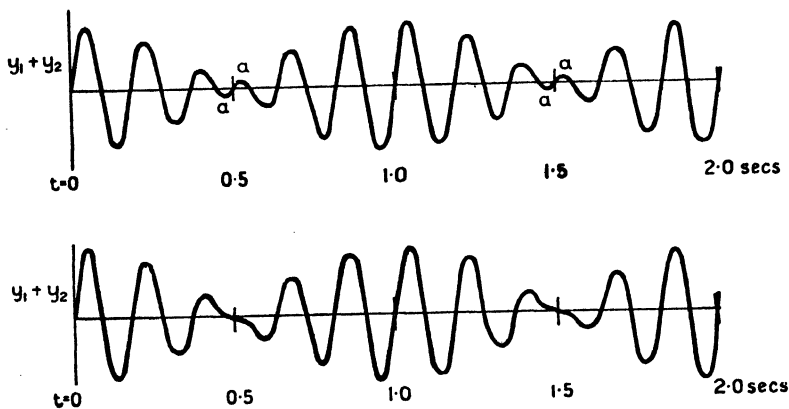


FIG. 18 : a (above) ; b (below).

This is illustrated in Fig. 18*a*, which is drawn for component waves of equal amplitudes, with $n_1 = 6$ c.p.s., and $n_2 = 5$ c.p.s., with zero phase difference at $t = 0$. For the bulk of the time the zeros occur at time intervals of $1/11$ second, corresponding to a frequency of $(n_1 + n_2)/2$. At this rate there would be time for eleven half waves in the period of one second between $t = 0.5$ sec. and $t = 1.5$ sec. Actually, however, there are twelve half waves in this period, the additional half wave being squeezed in at the minimum of intensity.

If the amplitudes of the individual disturbances are adjusted so as to be in the inverse ratio of their frequencies, the resultant curve becomes as in Fig. 18*b*. The small waves *aa* of Fig. 18*a* have disap-

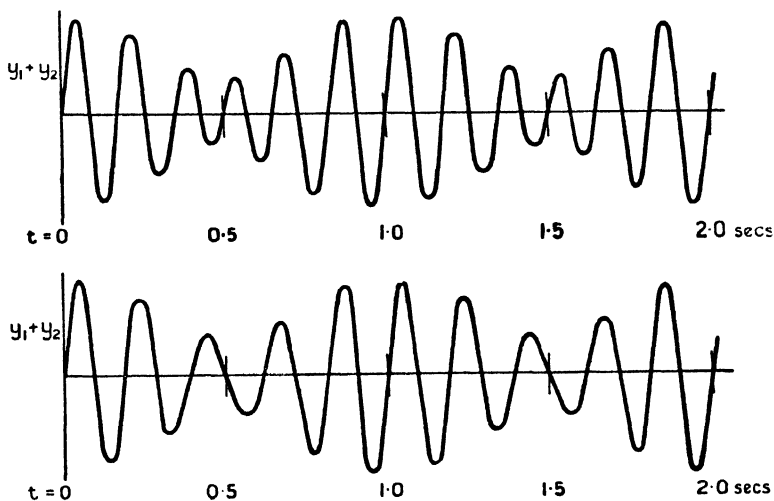


FIG. 19 : *a* (above) ; *b* (below).

peared. The zeros are no longer equidistant at $1/(n_1 + n_2)$ apart, though the turning values are, and there are only five complete waves between $t = 0.5$ sec. and $t = 1.5$ sec.

Figs. 19*a* and 19*b* are drawn for the cases in which individual waves are of frequencies 6 c.p.s. and 5 c.p.s., with the amplitude ratio of 2 : 1 and 1 : 2 respectively. It will be observed that the loss or gain of the half wave is spread more evenly over a greater part of each cycle. Initial phase differences also cause slight modifications in the wave curve: also, when $n_1 + n_2$ is not an integral multiple of $n_1 - n_2$ the curve will not be exactly repetitive, corresponding to a gradually changing phase relation. The vibration curve for two beating tuning forks, obtained by the oscillograph method (§ 71) is shown in Plate IV. The two forks had frequencies of 440 c.p.s. and 391 c.p.s. (ratio 9 : 8), and were

adjusted so that their separate amplitudes were equal. The photograph includes three beat periods of approximately $\frac{1}{80}$ th second each.

It must be emphasised that beats do not in themselves constitute a new low-frequency note. The only frequencies of wave disturbance actually present are the original ones: if, for example, these were supersonic, nothing should be heard. Two powerful tones do, in fact, produce new frequencies equal to the sum and the difference of the individual frequencies, though this effect is caused by the *failure* of the simple principle of superposition used to account for beats.

18. Combination Tones

When two notes are sounded together, a third tone can often be distinctly heard having a frequency equal to the difference of the frequencies of the generating tones. This is commonly known as a "difference tone," and the violinist Tartini appears to be the first who noticed it, in 1714. He found that it was produced in double-stopping—i.e., when producing notes together from two strings on the same violin. A summational tone, having a frequency equal to the sum of the frequencies of the generators, also occurs, but is usually much weaker than the differential.

The difference tone was very noticeable in the sound given by the air-raised sirens used in Great Britain during the second World War. The steady note for the "All-Clear" contained frequencies approximately 500 and 600 c.p.s., and a low note of frequency 100 c.p.s. was plainly audible, especially where the sound was intense. The ordinary police whistle, consisting of two short barrels giving high notes comparatively close together in frequency, emits a penetrating low-pitched sound of characteristic quality. The same principle is used for the production of extremely low notes on organs without incurring the expense of enormous pipes. An open pipe 16 ft. long, having a fundamental frequency of about 34 c.p.s., may be sounded with a shorter pipe giving a frequency of 51 c.p.s. The two together give a certain amount of sound of frequency 17 c.p.s., especially if they are blown from the same wind duct. A single pipe giving this fundamental frequency would have to be 32 ft. in length. The 16-ft. pipe with the fundamental of 34 c.p.s. would have harmonics of frequencies 68, 102, etc., while the other pipe having a fundamental of 51 c.p.s. would have harmonics of frequencies 102, 153, etc. Thus the two together do in fact produce by sum and difference all the harmonics of the 32-ft. pipe, although the third (51 c.p.s.) is certainly in a disproportionate strength. The stop which causes the appropriate pairs of pipes to sound is called the "Acoustic Bass." The same principle was used on old organs much higher up in the register, making the whole instrument very much more compact.

Much research has been carried out with a view to finding out whether combination tones are entirely subjective—i.e., manufactured in the ear of the hearer—or whether they have an objective reality. If the simple principle of superposition held good they would not be produced at all. That they have objective reality has now been established with certainty, by using a resonator adjusted to respond to the expected tone. It is found that reinforcement by resonance does in fact occur, proving the existence of the note outside the ear. This is especially so when the two generators violently agitate the same mass of air or the same vibrating system, such as a soundboard. In any case the effect is added to by the ear itself (see § 85).

A delicate method of detecting the resonance was employed by Rucker and Edser, who used a heavily mounted tuning fork, one prong of which carried a plate, while the other carried one of the mirrors of a Michelson interferometer. Sound was concentrated on to the plate by means of a horn. The sound was made by a double siren, the speed of which could be adjusted to bring either the differential or summational tone to the same frequency as that of the fork. It was found that at these speeds the interference pattern was disturbed, proving the presence of this frequency in the sound. On the other hand, however, no effect was obtained when the sound was made by two separate tuning forks instead of the siren.

König considered beats and difference tones to be identical—the distinct note occurring when the beats follow in sufficiently rapid succession. In this case they would be subjective only—which is now known not to be the case. In addition, this theory would offer no explanation of summational tones, or of the fact that combination tones are much more prominent with loud tones, whereas beats occur equally plainly with faint ones.

Helmholtz suggested that there was a form of “distortion”—i.e., that the simple superposition rule did not apply. The receiver, whether ear drum or other membrane, and the vibrators agitated in the original source do not respond to the double forcing in a way equal to the vector sum of the response to separate frequencies. This could happen if the restoring forces, instead of being proportional to the displacements, varied in a more complicated way, such as

$$F = ax + bx^3$$

where F is the restoring force and x the displacement. Helmholtz showed that the presence of the term involving the square of x in a system forced with frequencies p and q caused the frequencies $2p$, $2q$, $(p - q)$, $(p + q)$ to be present in the response, in addition to p and q . The amplitude of the difference tone is shown to be proportional to the product of the primary amplitudes. This is rather a weak point, for the difference tone generally seems too loud to be due to a small asymmetric term that is negligible in a first approximation. In addition, the summational tone is in practice much weaker than the differential, whereas on the Helmholtz theory they are of equal amplitude.

Waetzmann has to some extent met these objections in his theory of “general asymmetry,” in distinction to the square-law asymmetry proposed by Helmholtz. The general idea is that the response of the system is “one-sided,” an arrangement that he was able to imitate experimentally by loading a membrane with a central mass on one side only.

The free vibrations showed greater amplitude on one side of the undisplaced position. He recorded the motion of the membrane under double forcing and performed the Fourier analysis of the resulting curve. The analysis gave the original primary tones of frequencies p and q , the difference tone ($p - q$) with an amplitude several times that of either primary, a weak tone of frequency ($2p - q$) and occasionally the summational tone ($p + q$).

An example of the use of a combination tone other than in sound is the *supersonic-heterodyne* (*super-het*) type of radio receiver. A radio transmitter sends out electromagnetic waves usually having a frequency of a million or more, which produce a corresponding alternating current in the receiving aerial. For transmission of telephony, this "carrier wave" is modulated—i.e., varied in amplitude—according to the wave form of the sound to be transmitted. At the receiving end the minute voltage obtained from the aerial is magnified at "radio frequency" and selected from unwanted frequencies, and then rectified. It then becomes a small voltage varying at speech frequency and is amplified again to operate a loudspeaker.

The operation of rectification—"detection" as it is called—is inefficient if the radio frequency voltage is small, so that for weak stations two or more stages of amplification before detection are necessary. This causes great difficulties when the carrier frequency is more than about two million c.p.s. (equivalent to 150 metres wavelength), as capacitative feed-back through valves and wiring is apt to cause instability. In the supersonic-heterodyne receiver the original radio frequency voltage, usually after one stage of amplification, is "mixed" with another radio frequency voltage from an oscillator in the receiving set. The two are fed together into a valve with a non-linear response, and, among others, a difference tone ("heterodyne") is produced. The frequency of the local oscillator is arranged to make this tone supersonic—often 465,000 c.p.s., and it is selected from all the others present by a suitably tuned circuit. When the original radio-frequency carrier wave is modulated at speech-frequency, the "difference tone" will be similarly modulated, since its amplitude depends on the amplitudes of both generating oscillations. The tone is subjected to several stages of amplification (usually termed "intermediate-frequency"), which is an easy matter, as the frequency is not sufficiently high to cause instability. It is then rectified and fed to a speech-frequency amplifier in the usual way.

19. The Doppler Effect

It is a common experience that the apparent frequency of a note changes if there is a change in the relative motion of the source and the observer. If we are passed by a motor-car sounding its horn, a railway engine sounding its whistle, or a noisy motor-cycle, a drop in pitch

is that in which the stationary observer O is a little way away from the line of motion SP of the source (Fig. 22), OP being perpendicular to the line of motion. In the position when $\angle SOP = \theta$, the relative velocity of approach along $SO = v_s \sin \theta$, so that

$$n' = n \frac{V}{(V - v_s \sin \theta)} \quad (\text{stationary medium}).$$

When the source is very distant, $\sin \theta = 1$: when it is "broadside on" $\sin \theta = 0$, and when it has completely receded $\sin \theta = -1$. The limits of n' are therefore $\frac{nV}{V - v_s}$ and $\frac{nV}{V + v_s}$, and the rapidity of the change depends on the rapidity of the variation of θ . This is in turn dependent upon the distance OP in comparison with the velocity v_s .

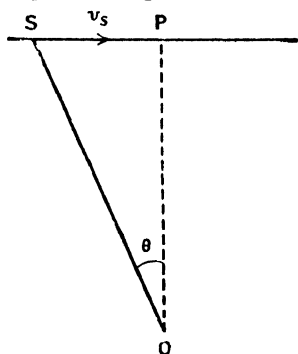


FIG. 22.

An interesting connection exists between the Doppler effect, stationary waves and beats. Consider the motion of an observer along the line joining two sources emitting notes of the

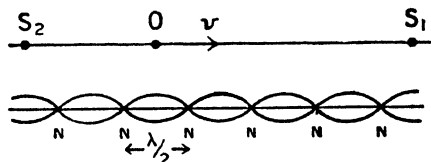


FIG. 23.

same frequency n . If the observer approaches S_1 (Fig. 23) with velocity v ,

$$\text{Frequency received from } S_1 = n \left(\frac{V + v}{V} \right)$$

$$\text{Frequency received from } S_2 = n \left(\frac{V - v}{V} \right)$$

$$\begin{aligned} \therefore \text{Beats are heard, of frequency} &= \frac{2nv}{V} \\ &= \frac{2v}{\lambda} \end{aligned}$$

where λ is the wavelength of the sound, since $V = n\lambda$.

In fact, the observer walks along the system of stationary waves. As the distance between successive nodes is $\lambda/2$, the sound he hears will go through $2v/\lambda$ minima per second, which agrees with the result obtained from the Doppler formula.

Show with reference to the transverse vibrations of a stretched string how reflection of a wave results in the formation of a system of stationary waves. (O.)

6. Give an account of the formation of beats and deduce an expression for the frequency of the beats in terms of the frequencies of the sources. Two trains approach each other on the same line, each moving with velocity 10 ft. per second. A whistle of frequency 250 c.p.s. is sounded on each train. Find the frequency of the beats between the two sources as heard by an observer on either train. (Velocity of sound = 1,100 ft./sec.) (C.S.)

7. Give an account of the Doppler effect in sound and light. A man directs a beam of sound of frequency 12,000 cycles per second at an approaching motor car. He listens to the beats between the reflected sound and that emitted. What is the frequency of the beats if the car is travelling at 30 m.p.h. (44 ft./sec.) ? (Velocity of sound, 1,100 ft./sec.) (C.S.)

8. An observer on a railway station platform observed that as a train passed through the station at 60 m.p.h. the frequency of the whistle appeared to drop by 400 vibrations per second. Find the frequency of the whistle. (Velocity of sound = 1,100 ft./sec.; 60 m.p.h. = 88 ft./sec.) (C.S. Part question.)

9. A source of sound gives out waves of frequency n . Discuss the effect of motion (a) of the source, (b) of the observer, (c) of both source and observer, on the frequency of the sound as heard by the observer. The motion, in each case, may be assumed to be along the line joining source and observer.

A train is moving with uniform velocity v on a straight track between two bridges A and B over the track, the motion being towards A. An observer on the train hears the echo of the train's whistle reflected from each of the bridges. If the velocity of sound is V find the ratio of the wave-lengths of the waves reflected from A and B and the ratio of the frequencies of the echoes heard by the observer. (C.S.)

10. A man is standing beside a railway line observing the whistle of a passing train. The whistle, which has a frequency of 1,000 cycles per second, suffers an apparent change of 100 cycles per second. What is the speed of the train ? (Velocity of sound, 1,100 ft./sec.) (C.S. Part question.)

11. Discuss the motion of a medium which is transmitting two equal trains of simple harmonic waves in opposite directions. (C.S.)

12. A vibrating tuning fork is moving steadily with a velocity of 150 cm./sec. normally towards a wall from which the sound waves are reflected. If the frequency of the fork is 512 sec.^{-1} , what will be the frequency of the beats heard by a stationary observer who has just been passed by the fork ? (Velocity of sound = 330 m./sec.) (C.S.)

13. Give a general account of the Doppler effect, with examples in sound. Consider the bearing which the velocity of the wind may have on the effect.

A plane is travelling horizontally in a straight line in still air with a velocity of 400 ft. per sec. Find the percentage difference between the

frequency of the sound of the 'plane heard by an observer on the ground at the instant when he sees the 'plane vertically overhead and the "true" frequency. Assume the velocity of sound in air to be 1,120 ft. per sec. (*N.U.J.M.B.S.*)

14. Derive expressions for the change of frequency of a sound wave produced by (a) movement of the source, and (b) reflection by a moving mirror on which the sound is incident normally.

Devise an experimental arrangement by which *one* of these relations could be tested. (*O.*)

15. Define the terms amplitude, period, phase as applied to a wave motion. Show how to find the resultant of a number of simple harmonic motions executed in the same straight line and having the same period but differing in amplitude and phase. Prove that it is also a simple harmonic motion.

When an aeroplane is flying overhead a rise in the general pitch of its noise is noticeable if the observer stoops so as to bring his head nearer the ground. Suggest an explanation for this phenomenon. (*Trip.*)

16. Discuss the effect on the frequency of the note heard by an observer when the source of sound and the observer are in motion. What would be the effect, if any, of the presence of a steady wind?

A tuning fork of frequency 1,100 is moved outwards from a wall with a uniform velocity of 1 ft./sec. towards an observer. Explain why beats are heard and calculate their frequency given that the velocity of sound is 1,100 ft./sec. (*Trip.*)

CHAPTER III

FORCED VIBRATION AND RESONANCE

20. Forced Vibration

WHEN we strike a tuning fork and stand the shank upon the table, the table top vibrates and we hear the sound. The vibration has the frequency of the fork, not that of the table, so that the latter is being made to vibrate at a frequency that is not one of its own natural frequencies. It is said to perform a *forced vibration*. The amplitude of this vibration depends greatly upon how nearly the frequency of the fork is matched to one of the natural frequencies of the table, and upon the damping associated with this natural frequency. It is a common experience that when a piano is being played certain objects in the room, such as vases, will boom out when particular notes are sounded. They are said to *resound*, or to act as *resonators*, the phenomenon occurring when the natural and forcing frequencies are suitably matched. Poor quality loudspeakers sometimes give an enhanced response in certain portions of the musical scale as the result of this *resonance*: indeed this is sometimes made use of to give an artificial increase to the bass response. The result is usually dull and boomy reproduction.

An ordinary motor car provides many examples of forced vibration and resonance. Various parts, such as the flexibly mounted engine itself, brake rods, gear lever, etc., each have their own natural frequencies. The engine communicates to them all a forcing frequency proportional to the speed: as this speed alters it matches the natural oscillations of different members, so that each performs its maximum vibration and rattles vigorously at the appropriate engine speed.

The operation of "tuning" an electrical circuit, as in a radio set, makes use of the same principle. A circuit of low resistance, like a mechanical system of low damping, has a natural frequency, dependent in this instance on the inductance and capacity. By altering one or both of these the frequency may be matched to that of the carrier wave of a station that it is desired to receive. Stations radiating other frequencies, to which the receiver is not tuned, will only cause a comparatively small "forced vibration," which may be rendered quite negligible if two or three resonating circuits of low damping are employed.

As other examples of resonance, see Rucker and Edser's experiment (§ 18), Barton's Pendulums (§ 28), the resonance tube (§ 35), the sonometer (§ 53), Melde's experiment (§ 54) and the Helmholtz resonator (§ 66).

21. Simple Harmonic Motion

In § 5 the oscillations of a body suspended by a light helical spring were discussed, air resistance being neglected. The equation of motion was of the form

$$\ddot{x} + \omega^2 x = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1),$$

which embodies the definition of Simple Harmonic Motion—motion of a particle in a straight line such that the acceleration of the particle is always directed towards a fixed point in the line and proportional to its displacement from that point.

This equation will now be solved by a different method from that previously used: a method that can easily be extended to cover a case in which there is air resistance. For a solution to (1), try

$$x = Ae^{mt}$$

so that

$$\dot{x} = mAe^{mt}$$

and

$$\ddot{x} = m^2 Ae^{mt}$$

The value satisfies the equation if

$$m^2 e^{mt} + \omega^2 e^{mt} = 0.$$

$$\therefore e^{mt}(m^2 + \omega^2) = 0,$$

or

$$m^2 + \omega^2 = 0.$$

$$\therefore m = \pm i\omega \text{ where } i = \sqrt{-1}.$$

The equation is satisfied by $x = e^{i\omega t}$ and by $x = e^{-i\omega t}$.

As the equation is linear—i.e., it involves x and its derivatives to the first power only—any linear combination of these two solutions will also satisfy it. Another solution is therefore

$$x = A_1 e^{i\omega t} + A_2 e^{-i\omega t}.$$

As this contains two arbitrary constants A_1 and A_2 , it is the most general solution. A_1 and A_2 may be complex quantities.

This solution may be rewritten

$$\begin{aligned} x &= A_1(\cos \omega t + i \sin \omega t) + A_2(\cos \omega t - i \sin \omega t) \\ &= (A_1 + A_2) \cos \omega t + i(A_1 - A_2) \sin \omega t \\ &= A \cos \omega t + B \sin \omega t \\ &= C \sin(\omega t + \delta) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2), \end{aligned}$$

where $C = \sqrt{A^2 + B^2}$ and $\delta = \tan^{-1} \frac{A}{B}$

The values of C and δ depend upon the initial conditions, but if x has a real value at any real value of t , C and δ are also real, though A_1 and A_2 are complex. E.g., if $x = 0$ when $t = 0$, $\delta = 0$, and C is the turning value of x , occurring first when $t = \pi/2\omega$.

Thus the period of the motion is $2\pi/\omega$, and the frequency $\omega/2\pi$, while the amplitude is constant and equal to C (Fig 24a).

22. Damped Harmonic Motion

In all practical cases the motion is resisted by various frictional effects. To a first approximation, when the resistance is that due to motion through a fluid, such as air, the resisting force is proportional to the velocity.

Taking its value to be $2k$ per unit mass per unit velocity the equation of motion becomes

$$\ddot{x} + 2k\dot{x} + \omega^2x = 0 \quad (3)$$

Trying, as before,

$x = Ae^{mt}$ as a solution, we have

$$\dot{x} = mAe^{mt},$$

and

$$\ddot{x} = m^2Ae^{mt},$$

so that the value satisfies if

$$m^2 + 2km + \omega^2 = 0.$$

i.e., if

$$m = -k \pm \sqrt{k^2 - \omega^2}.$$

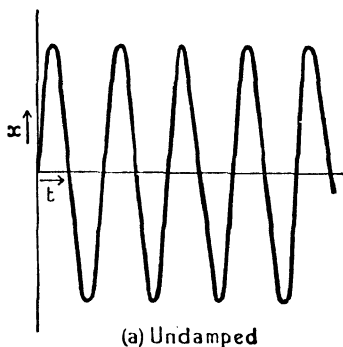
Thus the general solution becomes

$$x = A_1e^{(-k + \sqrt{k^2 - \omega^2})t} + A_2e^{(-k - \sqrt{k^2 - \omega^2})t}$$

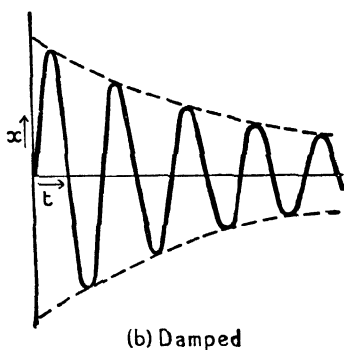
The actual form of this solution depends upon the relative magnitudes of k and ω .

(i) If $k > \omega$, $\sqrt{k^2 - \omega^2}$ is real (but less than k).

Hence x consists in this case of two terms, both dying off exponentially, and there is no oscillation. It is represented in practice by such examples



(a) Undamped



(b) Damped

FIG. 24.

as a "pendulum" moving in thick oil, or an overdamped moving-coil galvanometer. It is of little interest in sound.

(ii) If $k = \omega$, the above method of solution fails. The solution is of the form $x = e^{-kt}(A_1t + A_2)$. The case is known as that of *critical damping*, the decay of x being most rapid when k has this value.

(iii) If $k < \omega$, $\sqrt{k^2 - \omega^2}$ is imaginary, and may be written $\sqrt{k^2 - \omega^2} = ip$, or
so that the solution becomes

$$\begin{aligned} x &= A_1e^{(-k + ip)t} + A_2e^{(-k - ip)t} \\ &= e^{-kt} \{A_1e^{ipt} + A_2e^{-ipt}\} \end{aligned}$$

By the same process as that used in §21, this may be put in the form

$$x = Ce^{-kt} \sin(pt + \delta) \quad (4)$$

Comparing this with the solution (2), it is seen that the introduction of k has two effects :—

(a) The amplitude is no longer constant at C , but dies away according to the factor e^{-kt} (Fig. 24b).

(b) The frequency, formerly $\omega/2\pi$, is now $p/2\pi$, where $p = \sqrt{\omega^2 - k^2}$.

This difference of frequency is usually extremely small if the damping is sufficiently small that a reasonable number of vibrations is performed before the motion becomes negligible.

E.g., suppose that the frequency $p/2\pi = 200$ c.p.s., and that the oscillation dies away to $1/e$ of its former amplitude in 1 second. Then $k = 1$, $p = 400\pi$.

$$\therefore p^2 = 160,000\pi^2 \simeq 1,600,000.$$

$$\therefore \omega^2 = 1,600,001.$$

I.e., ω^2 only exceeds p^2 by 1 part in 1,600,000, so that ω exceeds p by 1 part in 3,200,000. Thus the reduction of the frequency brought about by the damping is exceedingly small.

23. Analytical Treatment of Forced Vibration

We may extend the treatment of the last article to the case when the oscillations of the system are not just allowed to decay away, but an external periodic force $F \sin qt$ per unit mass is applied. The equation of motion then becomes

$$\ddot{x} + 2k\dot{x} + \omega^2x = F \sin qt \quad (5)$$

As a particular solution of this equation try

$$x = A \sin (qt - \epsilon),$$

so that

$$\dot{x} = Aq \cos (qt - \epsilon)$$

and

$$\ddot{x} = -Aq^2 \sin (qt - \epsilon).$$

Writing $F \sin qt$ as $F \sin \{(qt - \epsilon) + \epsilon\}$

$$= F\{\sin (qt - \epsilon) \cos \epsilon + \cos (qt - \epsilon) \sin \epsilon\},$$

it is seen that the chosen expression satisfies the equation (5) if

$$-Aq^2 \sin (qt - \epsilon) + 2kAq \cos (qt - \epsilon) + \omega^2A \sin (qt - \epsilon)$$

$$= F\{\sin (qt - \epsilon) \cos \epsilon + \cos (qt - \epsilon) \sin \epsilon\}.$$

As the solution is to hold for all values of t , the coefficients of $\sin (qt - \epsilon)$ and $\cos (qt - \epsilon)$ on the two sides must be separately equal.

$$\text{Coefficients of } \sin (qt - \epsilon): -Aq^2 + A\omega^2 = F \cos \epsilon.$$

$$\text{Coefficients of } \cos (qt - \epsilon): 2kAq = F \sin \epsilon.$$

$$\text{Squaring and adding, we obtain } F^2 = A^2 \{(\omega^2 - q^2)^2 + 4k^2q^2\},$$

$$\text{or} \quad A = \frac{F}{\{(\omega^2 - q^2)^2 + 4k^2q^2\}^{\frac{1}{2}}}$$

$$\text{Dividing, we obtain } \tan \epsilon = \frac{2kq}{\omega^2 - q^2},$$

the quadrant being such that $\sin \epsilon$ is positive.

Thus the expression $x = A \sin (qt - \epsilon)$ satisfies the equation (5) if A and ϵ have these values. It represents a motion having the *frequency of the driving oscillation*, $q/2\pi$, and lagging behind it in phase by ϵ (between 0 and π since its sine is positive).

The equation of motion will still be satisfied if to x as above derived we

add any term which of itself would make the left-hand side of equation (5) zero. This term is any solution of equation (3), p. 45, that is

$$x = Ce^{-kt} \sin(pt + \delta).$$

The complete solution to (5) is thus

$$x = Ce^{-kt} \sin(pt + \delta) + A \sin(qt - \epsilon) \quad \dots \quad (6)$$

where

$$p = (\omega^2 - k^2)^{\frac{1}{2}},$$

$$A = \frac{F}{\{(\omega^2 - q^2)^2 + 4k^2q^2\}^{\frac{1}{2}}}$$

and

$$\tan \epsilon = \frac{2kq}{\omega^2 - q^2} \text{ (sine positive).}$$

The motion may therefore contain an oscillation of frequency $p/2\pi$, the natural damped oscillation of the system, dying out and eventually becoming negligible, in addition to one of the forcing frequency $q/2\pi$, having constant amplitude. If these frequencies are nearly equal, they may give beats for a time while the natural oscillation is still comparable in amplitude with the forced.

24. Phase of Forced Vibration

It will be observed that the forced vibration $A \sin(qt - \epsilon)$ lags behind the driving force $F \sin qt$ by the angle ϵ , which lies between 0 and π since its sine is positive.

Also, since $F \cos \epsilon = A(\omega^2 - q^2)$, if $q < \omega$, $\cos \epsilon$ is positive, so that ϵ is between 0 and $\pi/2$. If $q > \omega$, $\cos \epsilon$ is negative, so that ϵ is between $\pi/2$ and π .

Thus the phase of the forced oscillation varies with the frequency of the driver as shown in Fig. 25. The rate of change of phase is more rapid the less k becomes, since $\tan \epsilon$ is proportional to k .

When $q = \omega$, so that the frequency of the forcing is identical with the natural frequency that the system would have if there were no damping, the phase lag is $\pi/2$.

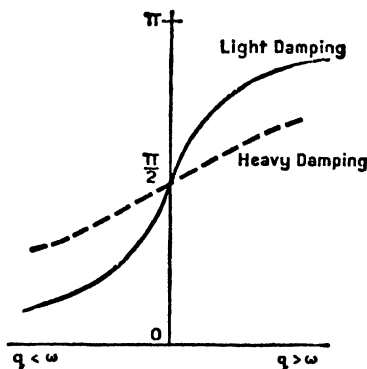


FIG. 25.

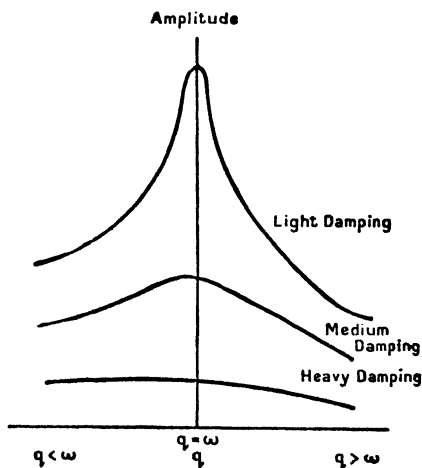


FIG. 26.

25. Amplitude Resonance

The amplitude A of the forced oscillation varies with the frequency of the forcing, dependent upon q . In the solution (6),

$$A = \frac{F}{\{(\omega^2 - q^2)^2 + 4k^2q^2\}^{\frac{1}{2}}} \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (7)$$

A is greatest when the denominator is least, i.e., when

$$\frac{d}{dq} \{(\omega^2 - q^2)^2 + 4k^2q^2\} = 0.$$

$$\therefore -4(\omega^2 - q^2)q + 8k^2q = 0.$$

$$\therefore q^2 = \omega^2 - 2k^2.$$

Thus the amplitude is a maximum when the forcing frequency is

$$\frac{(\omega^2 - 2k^2)^{\frac{1}{2}}}{2\pi}$$

This is not the natural frequency of the system, $(\omega^2 - k^2)^{\frac{1}{2}}/2\pi$, nor the value this would have in the absence of damping, $\omega/2\pi$, but a frequency slightly lower than either.

Substituting $q = (\omega^2 - 2k^2)^{\frac{1}{2}}$ in equation (7), we obtain

$$A = \frac{F}{2k(\omega^2 - k^2)^{\frac{1}{2}}}$$

or

$$A = \frac{F}{2k(q^2 + k^2)^{\frac{1}{2}}}.$$

Hence the maximum amplitude is greater the lower the value of the damping constant k . The *sharpness* of the resonance—that is, the reduction in amplitude accompanying a change of q from the resonance value—is also greater if k is small, as the effect of the term $4k^2q^2$ in the denominator of (7) is smaller compared with the variations of $(\omega^2 - q^2)^2$ (Fig. 26).

26. Velocity Resonance and Energy Intake

At some considerable time after the start, when the natural oscillation has died away, the solution to equation (5) becomes

$$x = A \sin (qt - \epsilon)$$

so that

$$\dot{x} = Aq \cos (qt - \epsilon)$$

Thus $\dot{x}$ is a maximum when $\cos (qt - \epsilon) = 1$, i.e. when $x = 0$, and this maximum value is Aq , which is known as the *velocity amplitude*.

$$\text{Using the relation (7), } A = \frac{F}{\{(\omega^2 - q^2)^2 + 4k^2q^2\}^{\frac{1}{2}}}$$

$$\begin{aligned} \text{the velocity amplitude } Aq &= \frac{Fq}{\{(\omega^2 - q^2)^2 + 4k^2q^2\}^{\frac{1}{2}}} \\ &= \frac{F}{\left\{\left(\frac{\omega^2 - q^2}{q}\right)^2 + 4k^2\right\}^{\frac{1}{2}}} \end{aligned}$$

This is greatest when the denominator is least, that is, when $\omega^2 = q^2$, and its value is then $\frac{F}{2k}$.

Hence the velocity amplitude is greatest when $q = \omega$, that is, when the frequency of the forcing is equal to the natural frequency that the system would have in the absence of damping. The oscillation so produced is sometimes spoken of as a *maintained oscillation*, to distinguish it from a free or forced oscillation.

The energy of the system is proportional to the square of the velocity in the position of zero displacement. Thus, in the present instance, the maximum energy per unit mass is $\frac{1}{2} \cdot \left(\frac{F}{2k}\right)^2$, i.e., $\frac{F^2}{8k^2}$. The maximum energy is therefore inversely proportional to the square of the damping constant.

In each cycle, work is done against the damping, equivalent per unit mass to $\int 2k\dot{x}dx$, the integral being taken over a cycle.

$$\begin{aligned}\text{As} \quad \dot{x} &= Aq \cos (qt - \epsilon) \\ dx &= Aq \cos (qt - \epsilon) dt.\end{aligned}$$

Energy dissipated *per cycle* per unit mass

$$\begin{aligned}&= 2kA^2q^2 \int_0^{2\pi} \cos^2 (qt - \epsilon) dt \\&= 2kA^2q^2 \frac{\pi}{q} \\&= 2\pi A^2qk.\end{aligned}$$

$\therefore$ Energy dissipated *per unit time* per unit mass = A^2q^2k .

Substituting for A, this becomes $\frac{F^2q^2k}{(\omega^2 - q^2)^2 + 4k^2q^2}$.

This is the energy that is taken from the driving source in unit time : it is greatest when $q = \omega$, and then becomes $\frac{F^2}{4k}$ per unit mass per unit time.

27. Summary of Analytical Results concerned with Forced Vibration and Resonance

(1) A system without damping, in which the restoring acceleration at unit displacement is ω^2 , has a natural frequency $\omega/2\pi$.

(2) If the above system is subject to a damping force $2k$ per unit mass, the amplitude dies away in proportion to e^{-kt} and the frequency becomes $p/2\pi = (\omega^2 - k^2)^{1/2}/2\pi$.

(3) When a forcing frequency $q/2\pi$ is applied to the system of (2), vibration of this frequency is present, the amplitude showing no variation with time. In addition the decaying natural oscillation of frequency $p/2\pi$ will also be present.

(4) The amplitude of the forced oscillation of frequency is greatest when $q^2 = p^2 - k^2 = \omega^2 - 2k^2$.

(5) The velocity in the mean position of the forced oscillation is greatest when $q = \omega$, and the energy taken in to maintain this oscillation is then a maximum. This is the normal condition of *resonance*.

(6) The phase of the forced vibration lags behind the driving force, the lag being $\pi/2$ in the condition of (5).

(7) The maximum velocity at resonance and the energy intake per unit time are inversely proportional to the damping constant k .

(8) The sharpness of resonance is greater—that is, the effect of mistuning is more pronounced—when the damping is small.

28. Barton's Pendulums

The principal results of the analysis of forced vibrations can be illustrated by an arrangement of pendulums due to Barton.

A horizontal cord AB several feet in length is stretched between rigid supports, and from it are suspended several pendulums. One of these, CD, has a heavy iron or lead bob, and is the "driver." The others consist of small paper cones, and in this condition their motion is heavily damped (Fig. 27).

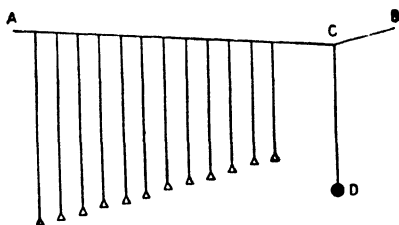


FIG. 27.

The heavy pendulum is set swinging and it disturbs all the others by pulling the cord from side to side at C. The natural part of the vibration of each soon dies away, and after a time each vibrates with a fairly constant amplitude. Being of different lengths, and different natural frequencies, they show different amplitudes.

If split curtain rings are slipped on to the paper cones their mass is considerably increased while the resisting force is unaltered, so that the damping constant k is reduced. This time the initial beats are more pronounced and take longer to die away, but when the amplitudes have settled to a steady figure the resonance is much sharper.

It will be found that the pendulum showing the greatest vibration is a quarter of a period behind the driver. When the latter is passing with its maximum velocity through the middle of a swing, the resonant pendulum is just about to begin a swing in the same direction. The

shortest pendulum, much mistuned with $\omega > q$, is almost in phase with the driver, while the longest, much mistuned with $\omega < q$, is almost half an oscillation behind it.

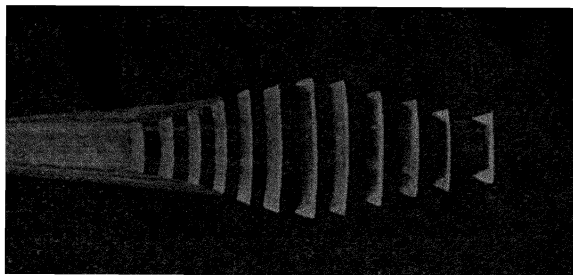


FIG. 28a.

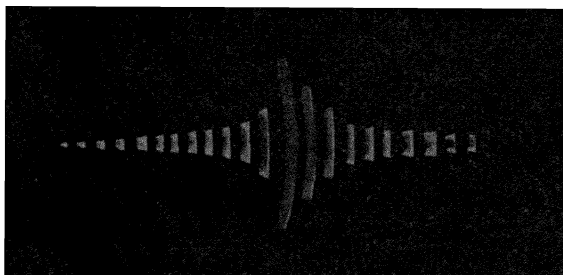


FIG. 28b.

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A photograph can be taken from above or below if a horizontal beam of light is arranged to illuminate the paper cones. If a long exposure is given, the travel of each cone is recorded as a white strip. Figs. 28a and 28b were obtained in this way. Fig. 28a shows the effect with the unloaded cones, and Fig. 28b the sharper resonance obtained by reducing the damping constant.

QUESTIONS

1. Distinguish between *free*, *forced* and *resonant* vibrations, giving one example of each.

A student, performing an experiment with a tuning fork and a resonance tube closed at one end obtained resonance when the length of the air

column was 49.5 cm. and again when it was 82.9 cm. Comment on these results and calculate the frequency of the tuning fork if the temperature of the air was 15°C . and the velocity of sound in air at 0°C . is 33,150 cm. per sec. (*N.U.J.M.B.*) [See also p. 61.]

2. Explain the terms *free* and *forced* vibration, giving examples taken from acoustics.

A particle of mass m moves along a line Ox , being attracted towards O by a force equal to μ times the distance, where μ is an adjustable modulus. It is also acted on by a periodic force $F \sin pt$ directed along Ox , and it experiences a resistive force proportional to the instantaneous value of the linear velocity. Find expressions for, in the steady state,

- (a) the value of μ for which the amplitude is a maximum ;
- (b) the value of the maximum amplitude and the power absorbed at resonance from the source of oscillation ;
- (c) the values of μ for which the difference in phase between the particle velocity and the applied force is $\frac{1}{4}\pi$. (*O.*)

3. Consider the effect of damping on the period of a simple harmonic vibration, and show that it is generally likely to be negligible in the case of a tuning fork vibrating in air. (*Lond. B.Sc.*)

4. Write a short account of the phenomenon of resonance, illustrating your answer with examples. (*Trip.*)

CHAPTER IV

THE VELOCITY OF SOUND

29. Theoretical Value for the Velocity of Sound in a Gas

It was shown in § 8 that the velocity of a plane compressional wave is given by

$$c = \sqrt{\frac{E}{\rho}}$$

where E = the modulus of elasticity of the medium
and ρ = the density of the medium.

In the case of a fluid the relevant modulus is the bulk modulus, since pressures act equally in all directions. An increase of pressure (stress) Δp causes a decrease in volume $-\Delta v$ and therefore a strain $-\frac{\Delta v}{v}$. Thus, the bulk modulus $E = -v \frac{\Delta p}{\Delta v}$.

If the strains are very small, this will differ very little from its limiting value $-v \frac{\partial p}{\partial v}$.

For a perfect gas at constant temperature

$$pv = k \text{ (constant)}$$

$$\text{or } p = k/v$$

∴ for isothermal changes of pressure and volume

$$\frac{\partial p}{\partial v} = -\frac{k}{v^2}$$

so that

$$-v \frac{\partial p}{\partial v} = \frac{k}{v} = p.$$

Hence, if the pressure variations in a sound wave are isothermal and minute, and the gas is perfect, the velocity of the wave is given by

$$c = \sqrt{\frac{p}{\rho}}$$

This formula was arrived at by Newton, who substituted the relevant numerical data and obtained the figure 979 ft. per sec. for the velocity of sound in air. The best experiments at the time gave the value as 1,142 ft. per sec., a discrepancy that Newton endeavoured to account for by making suppositions as to the size of molecules present in the air, and the percentage of "vapours."

The true explanation of the difference was given by Laplace, but not until 140 years later : in 1816 he pointed out that, owing to their rapidity, the pressure changes in a sound wave are likely to develop heat that does not leak away during the compression. In other words, the changes are adiabatic rather than isothermal.

For such changes in a perfect gas

$$pv^\gamma = k \text{ (constant)}$$

where γ is the ratio of the specific heat at constant pressure to that at constant volume.

If

$$\begin{aligned} pv^\gamma &= k, \\ p &= kv^{-\gamma}. \end{aligned}$$

Under these conditions,

$$\frac{\partial p}{\partial v} = -k\gamma v^{-\gamma-1} = -\frac{\gamma p}{v}$$

$$\therefore -v \frac{\partial p}{\partial v} = \gamma p$$

$$\therefore \text{Velocity } c = \sqrt{\frac{\gamma p}{\rho}}.$$

This means that the velocity is $\sqrt{\gamma}$ times that calculated for isothermal compressions : taking γ as 1.414, there is then good agreement between the calculated and experimental figures.

It has been shown by Stokes, using a thermodynamical argument, that the pressure changes must be either very nearly isothermal or very nearly adiabatic, for otherwise attenuation would be extremely heavy. The fact that sound waves do travel large distances before dying out therefore precludes a type of change midway between adiabatic and isothermal, or indeed, appreciably different from either.

Measurements of the velocity of sound provide us with one of the most convenient methods for determining the ratio of the specific heats of gases (see § 36), a knowledge of which is valuable for finding atomicities.

It will be observed that the elasticity $-v \frac{\Delta p}{\Delta v}$ is not constant at its

limiting value $-v \frac{\partial p}{\partial v} = \gamma p$, since p is one of the varying quantities.

The result is that no sound wave is actually propagated in a gas without a change of form, though the effect is only considerable when very large intensities are concerned. For very intense sounds anomalous values for the "velocity" are observed experimentally (§ 32).

30. Influence of Atmospheric Conditions on the Velocity of Sound in Air

Perhaps the simplest method of measuring the velocity of sound in air is to find the time taken for a sound to travel a known distance of some miles in the open air (§ 32). If such results are to be useful it is necessary to know how the various atmospheric factors—temperature, pressure, etc., are likely to affect them. This can be done simply by consideration of the theoretical equation

$$c = \sqrt{\frac{\gamma p}{\rho}} \quad (1)$$

For a gram-molecule of perfect gas

$$pv = RT$$

$$\text{so that for such a gas } \rho = \frac{M}{v} = \frac{Mp}{RT}$$

where M is the molecular weight, and R the universal gas constant.

$$\therefore c = \sqrt{\frac{\gamma RT}{M}} \quad (2)$$

(i) *Pressure.* The expression (2) shows that the velocity is unaffected by changes of pressure. This may also be seen quite easily from (1), if we realise that density is proportional to pressure for a perfect gas at constant temperature.

(ii) *Temperature.* From (2) it is seen that the velocity is proportional to the square root of the absolute temperature. Apart from its effect in large-scale open air measurements of the velocity, this fact is of importance in connection with the pitches of notes emitted by wind instruments (§ 65).

(iii) *Humidity.* If part of the pressure of the air is caused by unsaturated water vapour the density of that air will be less than the density of dry air at the same total pressure. (Molecular weights : $N_2 = 28$, $O_2 = 32$, $H_2O = 18$). On this account the velocity would be slightly greater in moist air than in dry air. The rise is to some extent offset by the fact that γ for the triatomic unsaturated water vapour is about 1.3, as against 1.4 for oxygen and nitrogen.

(iv) *Wind.* A wind, being a drift of the medium bearing the compressions and rarefactions, simply adds its own velocity (vectorially) to that of the waves in it. The velocity of sound in a wind is therefore different in all directions, the maximum being the numerical *sum* of the separate velocities of wind and waves, which will occur *with* the wind, while the minimum will be the numerical *difference*, occurring against it.

31. Abnormal Audibility due to Atmospheric Conditions

As the temperature in the atmosphere is different at different levels, the velocity of sound will be different at different heights above the ground. This causes a refractive effect, the sound "rays" becoming curved. In the case of waves travelling nearly horizontally, there is a simple relation between the curvature of the path and the temperature gradient.

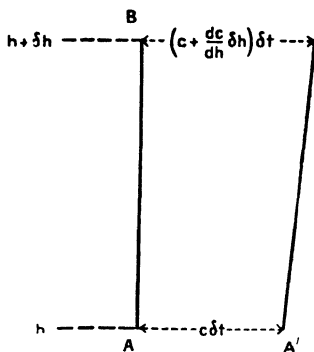


FIG. 29a.

Let AB (Fig. 29a) represent a portion of a plane wave front, initially vertical, A being at height h above the ground, and B at height $h + \delta h$. Let the velocity of sound at height h be c , so that at height

$h + \delta h$ it takes the value $c + \frac{dc}{dh} \delta h$.

In time δt a wavelet from A travels a distance $c\delta t$ to A', while that from B travels $(c + \frac{dc}{dh} \delta h) \delta t$ to B'.

$$\therefore \text{Angle turned through by wave front} = \frac{\frac{dc}{dh} \delta h \delta t}{\delta h} = \frac{dc}{dh} \delta t.$$

$$\therefore \text{Radius of curvature of path, } R = \frac{c \delta t}{\frac{dc}{dh} \delta t} = \frac{c}{\frac{dc}{dh}}$$

If the differences of velocity are due solely to temperature changes,

$$c = k\sqrt{T},$$

where T is the absolute temperature and k is a constant.

Then

$$\frac{dc}{dh} = \frac{k}{2\sqrt{T}} \frac{dT}{dh}$$

$$\therefore R = \frac{2T}{\frac{dT}{dh}}$$

A ray will have this radius of curvature so long as it is travelling nearly horizontally.

The usual effect of this is to cause the rays to be curved as in Fig. 29b, since dT/dh is usually negative. It causes there to be an "acoustic horizon"—a sound cannot be heard at ground level beyond a certain distance from the source, this distance depending upon the height of the source above the ground.

A similar effect is produced by wind. The velocity of a wind is generally different at different heights: in this case, however, since the

velocity of sound relative to the earth is the vectorial sum of its velocity in still air and that of the wind, its velocity gradient will have opposite signs on opposite sides of the source. Thus, if the wind velocity is greater at higher levels, which is usually the case, dc/dh is positive on the leeward side and negative on the windward side of the source.

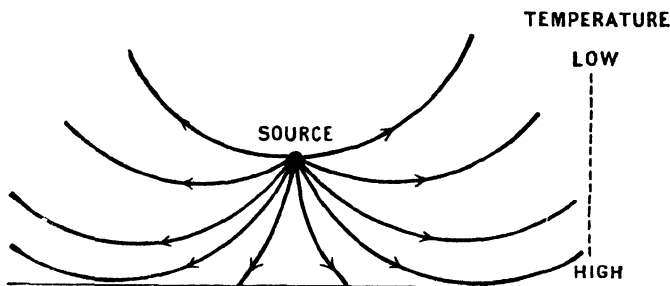


FIG. 29b.

The result is that the "rays" are curved upwards on the windward side, but downwards on the leeward side (Fig. 29c). An observer at ground level is therefore likely to receive a small share of the emitted energy if he is situated on the windward side, and a correspondingly large share on the leeward side, causing abnormally good audibility there.

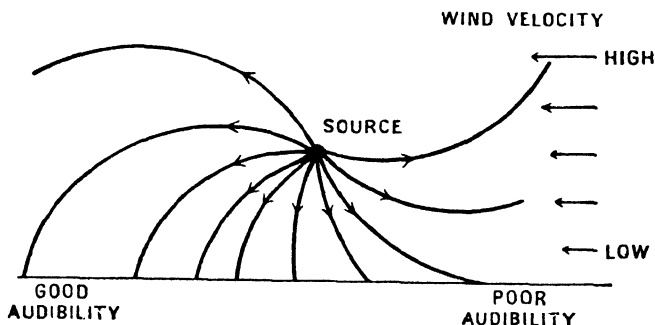


FIG. 29c.

The gradients of temperature or wind that are responsible for curving the sound rays upwards may become reversed in the upper parts of the atmosphere, causing the sound to be deflected down again. The sound may then be audible at a great distance from the source, though it is inaudible at intermediate points at ground level. Such "zones of silence" are of frequent occurrence in the distribution of sound from gunfire or other explosions.

32. Experimental Measurement of the Velocity in Free Air

The general basis of this method is to fire a gun, and to measure the time interval between the arrivals of the flash and the sound at a distant point. Early determinations were inaccurate owing to the "personal error"—that is, the time lag between receiving a stimulus such as a sound, and the response to it, such as pressing the lever of a stopwatch. This lag is not necessarily constant for a given type of stimulus, nor is it likely to be the same when the stimulus is a light flash as it is for a sound. Nevertheless, the experiments carried out by a Commission of the French Academy in 1738 were sufficiently accurate to establish the following conclusions :—

- (1) The velocity is independent of the pressure of the air.
- (2) It increases with the increase of temperature of the air.
- (3) It does not vary with the distance from the source of sound.
- (4) With the wind, the sound is propagated more quickly than against the wind, the speeds being in the first case the sum, and in the second the difference of those of sound and of the wind.
- (5) Reduced to 0° C. the velocity in still dry air came to 332 metres per second.

The distances used were up to about 28 kilometres.

In 1864 Regnault used a method in which the instant of firing caused the rupture of a wire forming part of an electric circuit, while the arrival of the sound at a distant point moved a membrane which broke a second electric circuit. The breaking of the two circuits was recorded on the same revolving drum. Regnault attempted to evaluate and make a correction for the time lag of the apparatus. He was able to show that the intense sound wave near a source such as a cannon did in fact travel with a slightly greater velocity than a wave of more normal intensity. The difference is to be expected owing to the failure of the air to have a constant elasticity and to obey the gas laws for large pressure variations (§ 29).

The method of recording the arrival of sounds has been improved latterly by the use of the hot-wire microphone (§ 67), which was developed during the 1914–18 War for the location of guns. It consists of a container with a fine resistance wire across its aperture, the wire being heated to redness by an electric current. The wire forms one arm of a balanced Wheatstone's network, and for the present purpose an Einthoven string galvanometer is used to record the balance. When a sound wave passes across the aperture of the container the wire is slightly cooled, upsetting the balance momentarily and causing a kick on the galvanometer. This is recorded photographically on a moving film, from which the time interval between the arrival of the sound at various microphones can be found. This

receiver is sufficiently sensitive to be practically instantaneous in action, but the other difficulties of large scale free air methods remain.

Method of Coincidences. This method, originally proposed by Bosscha in 1853, eliminates the "personal error" of an individual or electrical recording apparatus. Two small electromagnetic hammers are arranged so that the current which excites them causes them to produce simultaneously a series of loud taps at equal time intervals. They are actuated by a fork interrupter, arranged so that the taps are made at intervals of a tenth of a second. One of the tappers is stationary, while the observer takes the other with him and moves away from the stationary one. On account of the time taken by the sound to cross the space the sound from the stationary tapper lags behind that from the one carried by the observer. At a suitable distance the sounds are again in step: this distance is about 34 metres, so that the sound travels this distance in $1/10$ second.

The method can be modified using a single hammer and a reflecting wall, but the intensities of direct and reflected sound are very unequal, and it is then less easy to state exactly when the coincidence occurs.

33. The Velocity of Sound in Air in Pipes

Uncertainty is caused in "open air" measurements by variation of wind, humidity, and temperature. In addition, in the older methods at any rate, a very intense source is necessary for an open air method, particularly if its arrival at a distant point is to be recorded electrically. For propagation in a pipe a much less intense source is adequate. For these reasons Regnault took advantage in 1862 of new pipes laid for water and gas in Paris, the sources used being a pistol, explosions and musical instruments. Pipes from 11 cm. to 110 cm. in diameter were used. Violle and Vautier carried out further experiments later in a double conduit which was arranged to form a U-shaped pipe 12 kilometres long and 70 cm. in diameter. The results of the experiments may be summarized thus:—

- (1) Though the intensity affects the velocity near the source, the wave soon "settles down" and is then propagated without change. Its velocity in this condition does not depend upon the original intensity.
- (2) The velocity is independent of the pressure.
- (3) For different gases, the velocity varies inversely as the square root of the density.
- (4) Within the limits customary for musical sounds the velocity is independent of the intensity.
- (5) Differences of pitch are without effect upon the velocity of musical sounds—that is, there is no effect analogous to dispersion in light.
- (6) The velocity in a pipe is less than that in the open air.

The last is the weak point of all pipe or tube methods. It is probably caused by two things : (a) friction between the air and the walls, which would presumably depend upon the roughness of the walls ; and (b) the thermal conduction of the walls, which would tend to equalize temperatures and thus reduce the elasticity of the gas to the isothermal value. A correction formula was calculated by Helmholtz and Kirchhoff, and is in good agreement with the best modern experimental results. According to this formula, the decrease of velocity is inversely proportional to the diameter of the tube, and to the square root of the frequency of the sound, so that it becomes considerable for cases in which the wavelength is large compared with the diameter. As a numerical example, for a tube 2 cm. in diameter and a sound of wavelength about 3 metres (frequency approximately 110 c.p.s.), the velocity is less than that in the open air by about one part in 60.

34. Hebb's Telephone Method (1904)

This method is used in free air, so that " pipe difficulties " do not arise, and the scale of the apparatus is sufficiently small that temperature and humidity factors can be accurately known.

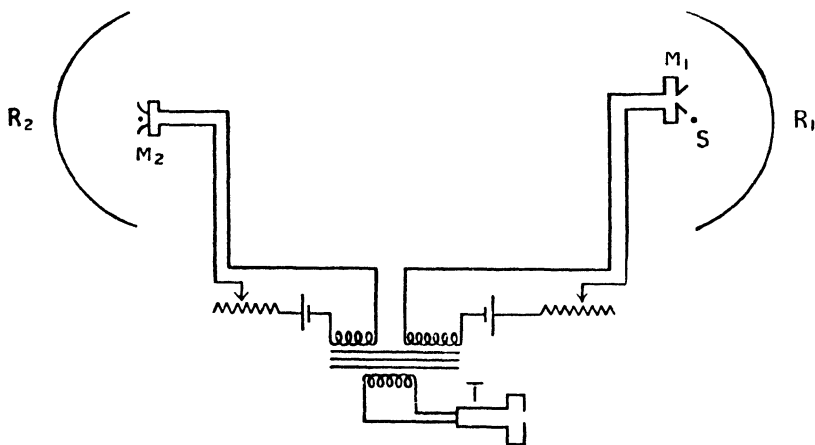


FIG. 30.

A source of sound, S , is placed at the focus of the fixed paraboloidal reflector R_1 (Fig. 30). The source must be of a high frequency in order to render the wavelength short : in some experiments a whistle was used, and in others a tuning fork of frequency 1,280 c.p.s. was substituted. The reflector is made of plaster of Paris, and is some 6 feet in diameter, its function being to produce a parallel beam of sound. This beam falls upon the second similar reflector R_2 , which can be moved along the line R_1R_2 , carrying with it the microphone M_2 which is fixed at its focus.

This microphone is joined with its battery into one primary of a transformer, while the other microphone, M_1 , which receives sound direct from S , is joined with its battery to the other primary winding. A telephone receiver T is connected to the secondary winding of the transformer.

The sound in T will be loud if the flux changes in the transformer core caused by the primary currents are in phase, but weak if they are opposite in phase. R_2 (and M_2 with it) is moved until the sound is at its weakest, and the intensity of the sound entering M_1 , or the numbers of turns used in the primary circuits, or the resistance of these circuits, adjusted until the sound in T is reduced to zero, or almost so. R_2 can then be moved along the line R_1R_2 and its successive positions found which give this smallest intensity. Each of these positions differs from the next by *one wavelength*, since the wave received by M_2 is in the same phase in each case, being opposite to that actuating M_1 . Over 200 successive settings were made, covering a distance of 100 ft. The pitch of the whistle (if used) was compared with that of a tuning fork, itself compared with a standard clock. The velocity was then calculated as the product of frequency and wavelength. The figure obtained was 331.4 metres per second for the velocity in dry air at 0°C .

35. Resonance Tube Method

This is a simple method, now used as a students' exercise. As in Hebb's method, the velocity is obtained as the product of frequency and wavelength.

It will be seen in § 59 that the vibrations natural to an air column in a cylindrical tube are those that produce a displacement node at a closed end and a corresponding antinode just outside an open end. Such a tube will "resound" to a tuning fork held over an open end if its length is adjusted so that one of its natural frequencies matches that of the fork.

The experiment is commonly conducted using a glass tube 2–3 cm. in diameter and 60–80 cm. long, with one end immersed in water in a tall jar (Fig. 31). This provides a convenient method of altering the vibrating length of the air column. The vibrating fork is held above the open end of the tube, and the length l of the tube above the water surface is altered until the sound is most strongly reinforced by resonance.

For the fundamental resonance

$$\lambda/4 = l + x \dots \dots (1)$$

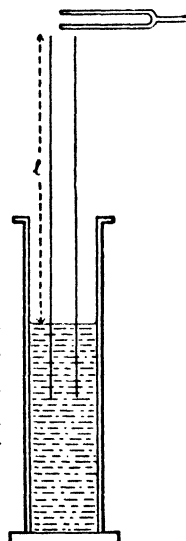


FIG. 31

λ being the wavelength, and x the distance from the end of the tube to the antinode, called the "end-correction."

As $V = n\lambda$, (1) may be rewritten

$$\frac{V}{4n} = l + x,$$

V being the velocity of sound.

The value of l is found for several forks of different frequencies n , so that the end-correction may be eliminated by plotting l against $1/n$. A straight line of slope $V/4$ will be obtained.

The tube will resound again at a length l' , where

$$3\lambda/4 = l' + x \quad . \quad . \quad . \quad . \quad . \quad (2)$$

since a displacement node and antinode are once more in the correct positions. Hence the end-correction could be eliminated by the alternative method of finding the difference of successive resonating lengths with a single fork, this difference being $\lambda/2$. In any case, the method is not an accurate one, partly because the resonance is not very sharp, and partly because the velocity obtained is that in a narrow tube.

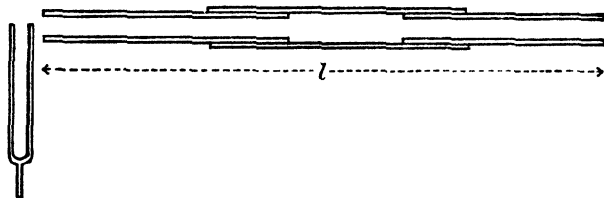


FIG. 32.

A similar but slightly less convenient method uses a tube open at both ends, the length variation being accomplished by a telescopic arrangement (Fig. 32). In this case

$$\lambda/2 = \frac{V}{2n} = l + x_1 + x_2,$$

where x_1 and x_2 are the end corrections. Plotting l against $1/n$ would give a straight line of slope $V/2$.

The resonances with the open tube are generally less sharp than those of the closed tube, owing to the doubled damping.

36. Kundt's Tube

This also makes use of stationary waves in a tube, but the method differs from the resonance tube in the manner of measuring the wavelength. The apparatus has existed in many forms, one of the older designs being illustrated in Fig. 33.

THE VELOCITY OF SOUND

being $\lambda/2$ apart. In both designs of apparatus elaborate methods (not shown) were employed to render the apparatus gas-tight, also to heat the gas and measure its temperature.

17. The Velocity of Sound in Water

An early determination was that made by Colladon and Sturm (1827) on the Lake of Geneva. Two boats were moored at a distance of some 14 kilometres apart. At one of them a bell was struck under water at the same instant that some gunpowder was ignited. The observer in the other boat measured the time that elapsed between the arrival of the flash and that of the sound, which was received through a horn immersed in the water. The time taken for the sound to travel the 14 km. was found to be 9.4 seconds, giving the figure 1,440 metres per second for the velocity. The sound was heard again, more faintly, after a further interval, presumably by reflection from the bottom—the first suggestion of the practicability of echo sounding.

Many observations have since been made to determine the velocity in water, particularly sea-water, accurately. This quantity is of great importance as the figures obtained in echo sounding, or in the acoustic method of determining the positions of ships, depend upon it. The general principle of the methods is to fire a small depth charge and to record the arrival of the sound at a series of microphones along a line through the source. The results show that the velocity shows a slight increase with temperature—about 0.2 per cent. per Centigrade degree, a slight increase with increased salinity—about 0.2 per cent. for 0.2 per cent. increase in salinity, and a small increase with depth—about 0.2 per cent. per 100 fathoms.

38. Echo-sounding

In a method that has been used by the British Admiralty the sound is made, under water, by striking a diaphragm at regular time intervals with an electromagnetic hammer. The hammer is held away from the diaphragm by an electromagnet except when the circuit of the latter is broken: this occurs when an insulating bar on a rotating metal drum passes under the brushes which press upon it. The diaphragm then emits a damped train of waves, which are reflected from the sea bottom, and picked up by a hydrophone (an under-water microphone) on the other side of the vessel. This hydrophone is joined to a telephone: this is, however, short-circuited by brushes bearing upon a second rotating metal drum, except for one instant in each revolution when an insulating bar passes under them, as in the transmitter. The two drums revolve on the same shaft, and the receiving brushes can be "swung" through any angle in relation to those concerned in transmission. If they are swung through the correct angle so that the telephone is

"unshorted" just at the instant at which the sound is received by the hydrophone, the sound will be heard in the telephone: otherwise there will be silence. The time lag of the reflected sound can be deduced simply from the angle through which the brushes must be moved and the speed of rotation of the shaft. In practice the hand-wheel rotating the brushes is calibrated directly in fathoms or feet.

39. Velocity of Sound in Liquids in Tubes

The effect of the containing tube, and more particularly its yielding, is considerably greater in the case of liquids than in the case of gases. The apparatus also tends to be rather bulky unless high frequencies are used, on account of the fact that the velocities are so much higher. Super-sonics—i.e., frequencies above the audible range, provide convenient practical methods, for the influence of the walls can be made negligible, and the experiment can be carried out with small amounts of material. The chief object of such experiments is the determination of the adiabatic compressibility, which has now been found for a great many liquids.

The apparatus, known as an acoustic interferometer, follows the same general lines as Kaye and Sherratt's modification of Kundt's tube (§ 36). The source of vibration is a valve-maintained quartz crystal, and the positions of the reflecting piston for resonance are found with great certainty by observing the anode current of the oscillator. The general level of accuracy obtained is in the region of 0.05 per cent.

It is found that, with saturated hydrocarbons, the velocity increases with increasing carbon content, due mostly to a decrease in compressibility. A similar variation occurs with the monohydric alcohols. In the case of isomers such as ortho, meta and para substitution compounds the ortho compound shows the smallest compressibility. The velocity is apparently lowered by the presence of a double bond. Solutions of electrolytes give a linear relation between the change of velocity and the molecular concentration, whereas non-electrolytes do not.

An alteration of velocity with frequency—that is, dispersion—has never been detected with certainty.

QUESTIONS

1. Sound consists of longitudinal waves of alternate compression and rarefaction transmitted through the air with a finite velocity. Discuss the experimental evidence for this statement.

Assuming that the velocity of sound in a medium is given by $\left(\frac{E}{\rho}\right)^{\frac{1}{2}}$, where E is the elasticity of the medium and ρ is its density, show that the velocity of sound in a gas is independent of the pressure, and proportional to the square root of the absolute temperature. ($O. \& C.$)

2. Deduce the relation between the frequency, wave-length, and velocity of a wave motion.

Describe a method of finding the velocity of sound in a gas other than air. ($O. \& C.$)

3. How would you find by experiment the velocity of sound in air?

Calculate the velocity of sound in air in cm. per second at $100^{\circ} C.$ if the density of air at S.T.P. is 0.001293 gm. per c.c., the density of mercury at

0° C. 13.60 gm. per c.c., the specific heat of air at constant pressure 0.2417, and the specific heat of air at constant volume 0.1715. (*Lond. H.S.C.*)

4. Explain in general how stationary undulation is produced and state the features of this kind of wave motion.

A resonance tube has a jagged end and it is used to find the velocity of sound in air. A tuning fork of frequency 250 causes it to resound when it is filled with water to a mark 28 cm. below a reference mark near the open jagged end. A fork of frequency 500 causes resonance when the water reaches a mark $11\frac{1}{2}$ cm. below the reference mark. Both these resounding lengths are the shortest possible with these forks. Find the velocity of sound from these results and also the position of the antinode at the upper end in relation to the mark. (*Lond. H.S.C.*)

5. How would you compare the velocity of sound in wood or brass with that of sound in air? Describe the apparatus, method of procedure, and calculation. (*Lond. H.S.C.*)

6. Give an account of the action of a resonance tube and how you would determine the velocity of sound at 0° C. using a resonance tube. (*Lond. Inter.*)

7. Give an account of Kundt's tube and explain how it may be used to compare (a) the velocity of sound in gases, (b) the velocity of sound in solids. (*Lond. Inter.*)

8. Distinguish between progressive and stationary waves.

Describe an experiment based on the production of stationary waves, which shows that sound travels more slowly in carbon dioxide than in air at the same temperature. Show how to calculate the ratio of the velocities of sound in these two gases from the experimental measurements. (*N.U.J.M.B.*)

9. Describe an experiment to find the velocity of sound in air at room temperature. A ship at sea sends out simultaneously a wireless signal above the water and a sound signal through the water, the temperature of the water being 4° C. These signals are received by two stations, A and B, 25 miles apart, the intervals between the arrival of the two signals being $16\frac{1}{2}$ sec. at A and 22 sec. at B. Find the bearing from A of the ship relative to AB. The velocity of sound in water at t° C. = $4,756 + 11t$ ft. per sec. (*N.U.J.M.B.*)

10. How would you find the velocity of sound along a metal rod, assuming that the velocity of sound in air at 0° C. is known?

A weight of 5 kg. suspended from the lower end of a uniform wire of length 1 metre and diameter 0.36 mm. produces an extension of 2.5 mm. Find the frequency of the note emitted when the stretched wire is stroked by a resined cloth. The density of the material of the wire is 7.8 gm. per c.c. (*N.U.J.M.B.*)

11. Describe and explain how, being provided with a tuning fork of known frequency, you would determine the velocity of sound in air at 0° C.

An expression for the velocity of sound in a gas was first obtained by Newton. Later on his result was modified by Laplace. How was Newton's expression defective, and how did Laplace correct it? (*N.U.J.M.B.*)

12. Explain the terms frequency, wave-length amplitude, phase, as applied to wave propagation.

A long uniform rod of circular section is clamped at its centre and is caused to vibrate longitudinally by stroking with a resined cloth. Describe in detail how you would determine the frequency of the note emitted by the rod assuming that the velocity of sound in air at 0°C. is known. (N.U.J.M.B.)

13. Assuming that the velocity of sound in a gas is equal to $\sqrt{\frac{\text{Adiabatic elasticity}}{\text{Density}}}$, find an expression showing how it depends on the temperature and molecular weight of the gas.

The planet Jupiter has an atmosphere composed principally of ammonia (NH_3) and methane (CH_4) at a temperature of -130°C. Estimate the velocity of sound on this planet assuming the ratio of the principal specific heats of these gases to be 1.3.

(Gas constant $R = 8.3$ joules per deg. per gm. mol.)

Molecular weight of ammonia = 17
 " " methane = 16.) (O.P.)

14. What is wave motion? Explain the terms longitudinal and transverse waves.

Describe an accurate experiment to measure the velocity of sound in air. (O.P.)

15. Explain how sound is propagated. How would you expect the velocity of sound to vary (a) with temperature, (b) with pressure, (c) with type of gas? (O.P.)

16. When the air column in a tube of adjustable length is set into vibration by means of a tuning-fork, a loud sound is heard for certain lengths of the column. Give an account of this effect and explain how it arises.

How can such an apparatus be used to measure the velocity of sound in the tube? (O.P.)

17. Describe a method of measuring the velocity of sound in air.

Give an account of the factors which determine the "pitch", "intensity," and "quality" of a musical note. (O.P.)

18. How has the velocity of sound in water been measured? Of what practical importance is a knowledge of this quantity? (O.P.)

19. Describe how you would measure the velocity of sound in carbon dioxide. Explain what information about the physical properties of a gas can be obtained from a measurement of the velocity of sound. (O.P.)

20. Describe a method of measuring the velocity of sound in a gas like hydrogen of which only limited quantities are available. Explain carefully what length and what time ($V = L/T$) are measured in your method. Discuss on what properties of a gas the velocity of sound depends and draw a rough curve showing how you would expect the velocity of sound in a mixture of oxygen and nitrogen to depend upon the composition of the mixture. (O.S.)

21. Give an account of the behaviour of sound waves of audible frequency in air, illustrating your answer by everyday phenomena. (C.S.)

22. Give a brief account of the evidence in support of the view that sound is propagated as a wave motion through the air. What are the physical factors that determine the velocity of propagation of such waves?

Indicate the chief sources of error in measuring the velocity of sound in the open air and describe a good method of finding this velocity. (*N.U.J.M.B.S.*)

23. Give a brief account of any important and characteristic wave phenomena which occur in sound. Why are sound waves in air regarded as longitudinal and not transverse ?

An observer looking due north sees the flash of a gun 4 seconds before he records the arrival of the sound. If the temperature is 20°C . and the wind is blowing from east to west with a velocity of 30 miles per hour, calculate the distance between the observer and the gun. The velocity of sound in air at 0°C . is 1,100 ft. per sec. Why does the velocity of sound in air depend upon the temperature but not upon the pressure ? (*N.U.J.M.B.S.*)

24. An observer standing close beside an anti-aircraft gun notices that a shell explodes 5 seconds after it has been fired. The sound of the explosion reaches him 9 seconds later. If the angle of elevation of the gun is 45° , calculate to within the nearest hundred feet the height at which the shell explodes. (Velocity of sound in air = 1,110 ft. per sec. : $g = 32$ ft. per sec.²) (*C.S.*)

25. Give a short account of the factors which affect the audibility of distant sounds. (*C.S.*)

26. A ship travelling due North at 1 m./s. in a thick fog fires a detonator in the sea alongside and receives an echo from a buoy on the port side 1.2 s. later. Fifteen minutes later a repetition of the experiment yields the same result. What is the bearing and distance of the buoy ?

Describe the type of apparatus you would use to make these measurements. (Velocity of sound in sea water = 1,500 m./s.) (*O.S.*)

27. Describe how the variation in the velocity of sound in a gas with temperature could be experimentally investigated. Show how the velocity of sound in a perfect gas at temperature T may be calculated from a knowledge of the molecular weight and specific heat at constant volume. (*O.*)

28. Describe the method of propagation of sound. Give an expression for the velocity of sound in a gas, and describe how the velocity of sound in hydrogen may be measured.

How do you account for the "zones of silence" which occur when a big explosion takes place, although the explosion is heard at more distant places outside these zones ? (*O.*)

29. Starting from the fact that the velocity of sound in a gas is given by

$$\text{Velocity} = \sqrt{\frac{\text{adiabatic elasticity}}{\text{density}}},$$

derive an expression to show how it varies with (a) temperature, (b) molecular weight of the gas. What could you deduce about a gas of molecular weight 40 which was found to transmit sound with a velocity of 3.24×10^4 cm. per sec. at a temperature of 27°C . ? Explain your deductions. ($R = 8.4 \times 10^7$ ergs per degree C. per gm. mol.) (*Trip.*)

30. Describe briefly the methods which may be used for an accurate determination of the velocity of propagation of sound. How do you explain the different values obtained for the velocity of sound in free air and in air contained in long pipes of circular section ? (*Trip.*)

31. Describe the effect of a temperature gradient on the propagation of sound in the atmosphere. Assuming the sound rays to be approximately circular and neglecting the effect of diffraction calculate approximately the maximum horizontal distance to the position at which an aeroplane flying at 10,000 ft. would be audible on the ground. (Velocity of sound = 1,100 ft./sec. ; temperature gradient = $-2^{\circ}\text{C}/1,000\text{ ft.}$) (*Trip.*)

32. Write a short essay on the factors affecting the transmission of sound through the atmosphere. (*Lond. B.Sc.*)

33. Show that, in general, the velocity of sound waves in a perfect gas is given by $\sqrt{\frac{\gamma p}{d}}$, where p is the pressure, d the density, and γ the ratio of the specific heats at constant pressure and constant volume.

What are the explanations of

(i) the abnormally high wave-velocities (more than three times the value given by the above expression) which have been measured close to explosive sources of sound, (ii) the "skip-distance" phenomenon associated with the sound from explosions ? (*O.F.*)

CHAPTER V

PITCH, INTERVAL AND TEMPERAMENT

40. Comparison of Pitch by Beats

WE have seen (§ 7) that the pitch of a note depends upon the number of cycles per second executed by its source and the air disturbed by it. If we have a standard source of nearly the same frequency, such as a standard tuning fork, beats will be heard when the two are sounded together, if they are reasonably nearly equal in intensity. By counting these beats we have the difference of the frequencies (§ 17): which of the two frequencies is the greater may be ascertained by making a small alteration in one of them and finding the effect on the beat rate.

For example, suppose that a tuning fork of unknown frequency, sounded at the same time as a standard fork of 200 c.p.s., gives 11 beats in 4 seconds. The frequency of the unknown may be $200 + 2.75$ or $200 - 2.75$. If the prongs of the fork under test are loaded with small pieces of wax, the beat rate may be found to become 5 per second. The addition of the wax lowers the frequency of the fork, and as it has quickened the beat rate, its original frequency must have been less than that of the standard—i.e., the correct value is 197.25 c.p.s. It is conceivably possible, if the pieces of wax added were rather large, that they could depress the frequency all the way from 202.75 to 195; to guard against possible error on this account, the amount of wax can be doubled, and the beats again listened for. If the original frequency were 197.25, the doubled wax would bring it down to about 193, giving 7 beats per second: if 202.75 were the original frequency, the doubled wax would bring the frequency down to about 187. This would give 13 beats per second—too many to count, but a note considerably flatter than the standard.

Scheibler's Tonometer consisted of a set of tuning forks of accurately known frequencies. Other frequencies could be found by the beat method. If the forks in the set are sufficiently close in frequency, the wax method is not necessary, as beats can be counted with one fork above and another below the unknown.

41. The Siren

The siren, as used for this purpose, consists essentially of a disc pierced with a ring of holes, which rotates above a similar ring of holes

in a wind chest (Fig. 36). The disc is provided with revolution counters: the two rings of holes have opposite obliquities along the circumferences, so that the pressure of the air as it escapes through them causes the disc to rotate.

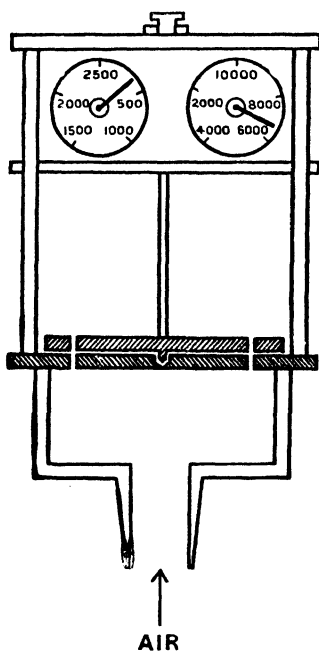


FIG. 36.

The air blast must be derived from a large reservoir in which the pressure is maintained constant by a suitable pump. The supply to the siren is regulated by a screw clip, and adjusted until a slow beat rate is observed (about 4 per second) when the source of unknown frequency is also sounded. Holding this beat rate as constant as possible, the time taken by the disc to perform some known number of revolutions is determined. If there are n holes in the ring, the speed of revolution is r per second, and there are b beats per second, the required frequency is

$$nr \pm b.$$

The correct sign can easily be decided by making a slight reduction in the speed of the siren and finding whether this increases or decreases the beat rate. If only an approximate answer is required it is sufficient to find the revolution speed for what is judged to be unison.

In any case the accuracy depends upon the watch or clock used.

42. The Dropping Plate

In this case, instead of relying on a watch or clock, we make use of the accurately known value of the acceleration of gravity, g .

A "hog's bristle" (obtainable from a disused hairbrush) is attached to one prong of a tuning fork by an adhesive, and the fork mounted so that the bristle just touches a vertical smoked-glass plate (Fig. 37). The smoking may conveniently be done by holding the plate over burning camphor. The fork is pinched vigorously, and at the moment at which it is released the thread supporting the plate is cut. After a few trials the experimenter may succeed in obtaining a trace on the plate of the form shown in Fig. 38.

Very frequently the oscillations near the start are not easy to count, though the actual point of start is fairly definite so far as distance

measurements are concerned. The distances from the start to such points as K, L, M, etc., where the oscillations are clear enough to be counted, are measured with a travelling microscope.

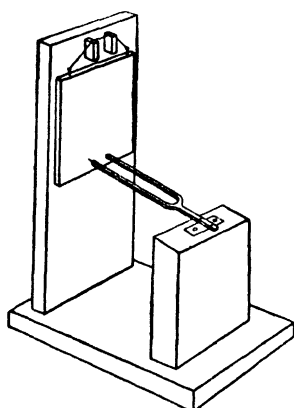


FIG. 37.

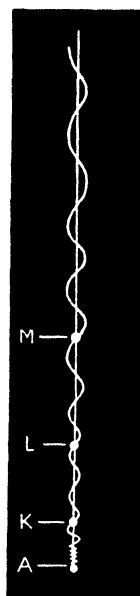


FIG. 38.

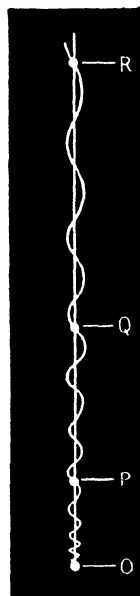


FIG. 39.

If x_0 oscillations elapse between the start A and the point K, and a further x oscillations between K and a point such as L distant s from A

$$s = \frac{1}{2}g \left(\frac{x + x_0}{n} \right)^2$$

where n is the frequency of the fork.

$$\therefore \sqrt{s} = \sqrt{\frac{g}{2}} \left(\frac{x + x_0}{n} \right).$$

Hence, if $\sqrt{s}$ is plotted against x , a straight line of slope $\frac{1}{n}\sqrt{\frac{g}{2}}$ is obtained, from which n is calculable, without knowing x_0 .

An alternative but less reliable method is to select three points, P, Q and R, on the trace, such that PQ and QR contain the same number m of complete oscillations (Fig. 39). PQ and QR then represent the distances travelled in successive time intervals m/n in length. It is then true that

$$QR - PQ = g \frac{m^2}{n^2}$$

To prove this expression, let the time elapsing from the start **O** to the point **P** be t_o .

$$\text{Then } OP = \frac{1}{2}gt_o^2$$

$$OQ = \frac{1}{2}g \left(t_o + \frac{m}{n} \right)^2 = \frac{1}{2}g \left(t_o^2 + \frac{m^2}{n^2} + 2t_o \frac{m}{n} \right)$$

$$OR = \frac{1}{2}g \left(t_o + \frac{2m}{n} \right)^2 = \frac{1}{2}g \left(t_o^2 + \frac{4m^2}{n^2} + 4t_o \frac{m}{n} \right)$$

$$\therefore PQ = OQ - OP = \frac{1}{2}g \left(\frac{m^2}{n^2} + 2t_o \frac{m}{n} \right)$$

$$\text{and } QR = OR - OQ = \frac{1}{2}g \left(\frac{3m^2}{n^2} + 2t_o \frac{m}{n} \right)$$

$$\therefore QR - PQ = \frac{1}{2}g \cdot \frac{2m^2}{n^2} = g \frac{m^2}{n^2}.$$

Thus n can be calculated if QR and PQ are measured, and m and g known.

The mass of the "hog's bristle," and the friction of it on the plate, make a slight difference to the frequency of the fork (and to the rate of fall of the plate). It is not desirable to find the frequency of an unknown directly in this way, but better to use a subsidiary fork and compare the unknown with it by beats.

The experiment may also be used to prove that a freely falling body moves with a constant acceleration, which is not easy to show directly in other ways.

43. Electrically Maintained Tuning-Forks

A tuning-fork is one of the best sources of a sound of constant frequency, and comparison or measurement is greatly facilitated if the

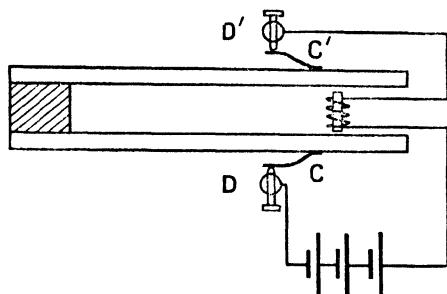


FIG. 40.

fork can be sounded continuously instead of requiring repeated bowing or striking. The vibration can be maintained electrically, either by an electromagnetic method similar in action to the electric bell, or by using a thermionic valve.

In the older method, a small iron-cored solenoid is fixed between the prongs, which carry the contacts C , C' (Fig. 40). When the current passes the prongs are attracted inwards, drawing C and C' away from

the fixed members D and D' and so cutting off the current. The iron core then ceases to be a magnet, so that the prongs open out again, contact is re-established and the cycle repeated. The frequency of vibration is affected to a small extent by the amplitude, and in the Tinsley fork a suitable stop device is used to keep this constant. The supports of the fork must be massive, or they may have a slight effect on the frequency.

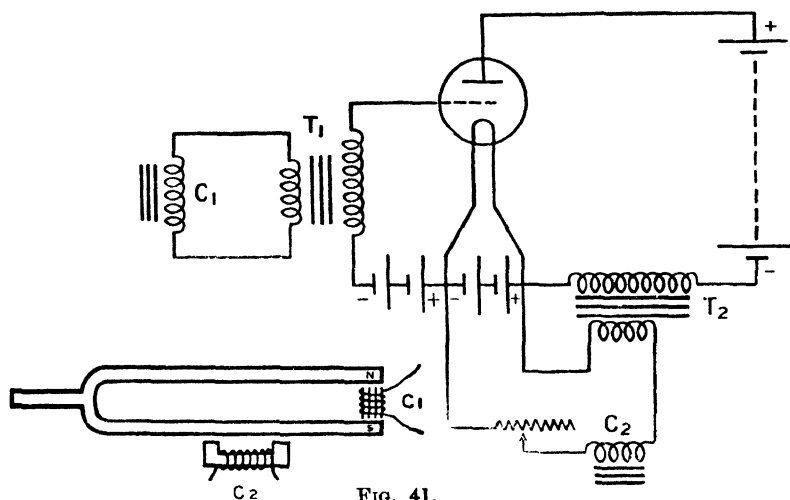


FIG. 41.

The arrangement of a simple valve-maintained fork is shown in Fig. 41. The fork is magnetised, and any vibration of its prongs changes the magnetic flux through the small iron-cored coil C_1 , inducing an E.M.F. in it. After a suitable step-up with the transformer T_1 this is communicated to the grid of a triode valve. The current in the anode circuit of the valve passes through the transformer T_2 , so that variations in it, caused by the variations of the grid potential, cause an induced E.M.F. in the secondary of T_2 . The secondary current from this, polarised with D.C., passes through the coil C_2 of the small electromagnet acting on the side of the fork. If the amplifying power of the valve is sufficient this larger current generated from the initial current in C_1 keeps the fork in vibration. More elaborate circuits have feed-back arrangements for limiting the vibration to a fixed amplitude.

44. The Stroboscopic Method

In its original form, this method enables the frequency of a tuning fork, with certain attachments, to be found. Suppose that each prong of the fork bears a metal plate with a slit in it. The slits are positioned so that they are exactly opposite one another in the undisplaced position

of the fork (Fig. 42a). They then form a "window," through which light can pass when the fork is in this position, but which is closed at other times. If the fork is vibrating, the window will be "open" twice in each complete oscillation.

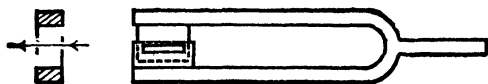


FIG. 42a.

A drum or disc is made having a known number of equidistant stripes drawn on it in a colour with a good contrast to the remainder of the drum (e.g. black on white or vice versa). The drum is mounted on an axle, provided with a revolution counter, that can be driven at a range of constant speeds—e.g. by means of a D.C. electric motor with a field controlling resistance.

The rotating drum is examined (Fig. 42b) through the window of the vibrating fork, which is, for preference, electrically maintained. If the drum is moving at such a speed that one stripe exactly succeeds the former one in the time interval between openings of the window, the stripes appear stationary. This is the case because the retinal images are all in the same position, and are identical in other respects. The

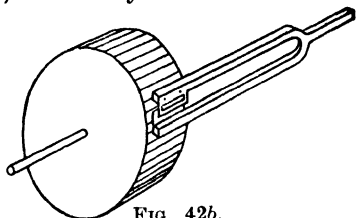


FIG. 42b.

same effect will occur if the drum is rotating at twice—or any exact number of times—this "fundamental" speed. If it is rotating at one half of the fundamental speed, alternate openings of the window will reveal stripes in the same position, but between successive openings the drum will only have moved one half of the space between successive stripes. The result, if the stripes are sufficiently fine, is that a stationary pattern with stripes at half the natural spacing is seen. A similar increase may occur at other fractional speeds.

If the speed of rotation of the drum is just short of that necessary to produce a stationary pattern, successive retinal images will not fall in quite the same place, so that the pattern appears to rotate slowly in the direction *opposite* to that of the drum. If the speed is gradually increased, the pattern will become stationary, and then move on in the direction of rotation of the drum.

To determine the frequency of the fork, the drum is started from rest, and the appearance through the window examined. As the speed increases, the appearance passes through the multiple patterns, and reaches the first stationary pattern having the natural spacing of the drum. The speed is held at this value for some considerable time, t seconds, during which the drum makes r revolutions (obtained from the counter). If s is the number of stripes on the drum, sr stripes pass

by in t seconds, or sr/t per second. The frequency of the fork is one half of this, since the window opens twice in each oscillation. Once the frequency of this fork is known, the frequencies of other sources can be found by comparing them with it by the beat method.

The stroboscopic method can be applied more conveniently by using a flashing light instead of the window formed by a vibrating fork. The sound of which the frequency is required is picked up by a microphone (§ 69), the current from which is amplified (§ 70), and taken to a discharge lamp, such as a Neon lamp. This requires a minimum voltage to strike it, so that it goes completely out at certain parts of the cycle, even if the frequency is quite high. A circular disc bearing rings of radial stripes is rotated beneath the light: a stationary appearance is obtained for any ring for which the stripes pass by at the same rate as the flashing of the light. If the lamp is polarised with a D.C. voltage just below the striking voltage, it will give only one flash per cycle of the current, so that the flash rate is equal to the frequency of the sound. By measuring the speed of rotation when the stationary appearance is achieved the frequency can therefore be calculated as in the original method. The disc may bear several rings in which the numbers of stripes are in the proportions of the frequencies of the notes of a musical scale (§ 49). If the speed of rotation is adjusted so as to give a stationary pattern on the correct ring when one note is sounded it will be possible to check whether other notes bear the correct frequency relation to it, for they too will give stationary patterns on the appropriate rings. This is the basis of the "chromatic stroboscope," which greatly facilitates checking the accuracy of tuning of musical instruments.

The stroboscopic principle is often used in the reverse manner—for determining speed of rotation in terms of the frequency of a fork or other source. For this purpose it has the advantage that it gives the speed at an instant of observation (or almost so), not the average over a long period. Small variations of speed are also readily detectable by means of it. A simple application is the speed indicator used on some gramophones, which consists of a disc bearing 77 equally spaced radial black stripes on a white background. When used in the neighbourhood of an electric lamp actuated by 50 cycle A.C. mains, the pattern will appear stationary if the speed of revolution is correct. An electric lamp, even of the filament type, carrying 50 cycle current is appreciably brighter at the peaks of the cycle than at the zeros, so that it has this extra brightness 100 times per second, or 6,000 times per minute. If one stripe on the disc succeeds the former one at this same rate the speed of revolution must be $6,000/77$, = 77.92 r.p.m., the standard correct rate for gramophone records being 78 r.p.m. It must be borne in mind that the ordinary electric mains seldom have the exact frequency of 50 c.p.s.: when used for electric clocks it is only necessary that this should be the *average* frequency maintained. The departures, however, are not great, and the accuracy of the 77 stripe stroboscopic indicator is ample so far as the correct reproduction of gramophone records is concerned.

45. The Phonic Wheel Method

The phonic wheel, devised originally by Lord Rayleigh, is similar in principle to the synchronous motor used in the mains-driven electric clock of to-day.

An iron rotor (Fig. 43) having equidistant teeth can revolve in the field of an electromagnet, the poles of which have similar serrations. If a current is passed through the coil, the position of least potential energy will be that shown in the diagram—that is, with the teeth of the rotor directly opposite those of the stator, giving minimum air gap. If a regularly interrupted current is passed the rotor would normally remain stationary in this position: but if it is given a sufficient spin (in either direction) it may continue to rotate at such a speed that it moves on one tooth for each pulse of the current. It will then always be in the position of least P.E. when the current is at its

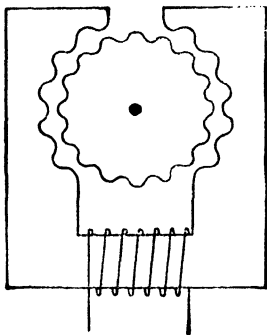


FIG. 43.

maximum. If the current is alternating at 50 cycles per second, there are 100 maxima per second, so that a rotor with 100 teeth would continue to make one revolution per second if spun to about this speed, provided the energy intake is sufficient to overcome frictional losses. Such a motor, suitably geared down, forms the ordinary electric clock for use on mains of controlled frequency. In fact, it counts the cycles made by the current: for every $50 \times 3,600$, or 180,000 cycles, the minute hand makes a complete revolution. By comparing the indications of such a clock with a standard clock, the *average* frequency of the mains can be measured with a high degree of accuracy.

The current in the circuit of an electrically maintained tuning-fork of the electromagnetic type (§ 43) is an interrupted direct current, and if this is passed through the coil of the phonic motor, the rotating iron wheel will go forward one tooth for each complete cycle of the vibration of the fork. In Rayleigh's determination a fork having a frequency of about 32 c.p.s. was used, and the phonic wheel had four poles, so that it made approximately eight revolutions per second. This speed was determined accurately by comparison with the pendulum of a standard clock, and the frequency could then be calculated.

The same current, of frequency now accurately known, was used to produce forced oscillations in another fork having a natural frequency of about 128 c.p.s.: this forced oscillation has a frequency *exactly* four times that of the driver. Any other source of frequency in the neighbourhood of 128 c.p.s. could then be compared with this driven fork by the beat method.

46. Interval

If one note is sounded after another the hearer perceives a certain musical relationship between them. It is an experimental fact that this relationship depends upon the *ratio* of the frequencies, and not their numerical difference (§ 7). Thus one note is one octave above another if the frequency ratio is 2 : 1, whatever the actual frequencies are.

Scientifically, we can state the interval between two notes by simply giving the ratio of their frequencies, or we can set up a scale by dividing the octave logarithmically. The logarithms (base 10) of the frequencies of any two notes that are one octave apart differ by 0.30103 : consequently any two notes are said to be n centioctaves apart if the logarithms of their frequencies differ by $0.30103n/100$. The scales of *cents* and *savarts* are similar : two notes differ by c cents if the logarithms of their frequencies differ by $0.30103c/1200$, and to be s savarts apart if the logarithms differ by $s/1,000$. A whole octave is thus equivalent to 100 centioctaves, 1,200 cents, or 301 savarts.

Musically, intervals are labelled by giving the number of "notes" that the one is distant from the other on the ordinary Doh-Ray-Me-Fah-Soh-Lah-Tc-Doh scale. The two notes themselves are counted in the number. Thus Doh—Doh is an octave, or Eighth : Doh to Soh or Ray—Lah, is a fifth, and so on. If we make a scientific test we find that all the fifths (with one exception) are substantially the same in frequency ratio, namely 3 : 2 ; the third Doh—Me is a ratio 5 : 4, while that for Me-Soh is only 6 : 5. The former is called a Major Third and the latter a Minor Third.

47. Musical Pitches

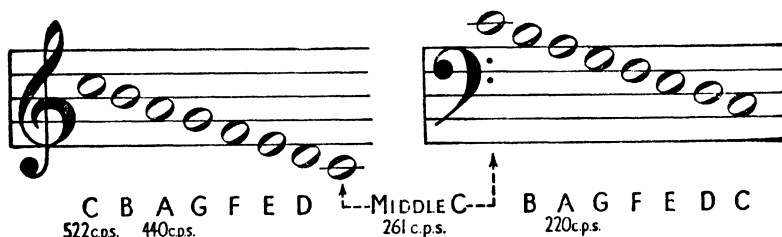
Scientifically we speak of a certain note as having a frequency of so many cycles per second, which determines its pitch. Musically we label certain notes as A, B, C, etc. The frequencies of the notes for which these symbols stand are by no means definite, and have varied greatly in different times and places. Even when one of them is fixed, the others are slightly indefinite (see temperament, § 50). The note commonly chosen as the "fixed point" is the "A"—the note to which various members of an orchestra tune their instruments before (and during) a performance.

Early in the present century there were three main pitches in general use : the "Old Philharmonic," known as High Pitch, with A* at 452 c.p.s. : the "New Philharmonic," or Low Pitch, with A at 439 c.p.s., and the French "Diapason Normal" at 435 c.p.s. The possible use of three different pitches causes a great deal of confusion, particularly because most wind instruments cannot be altered in pitch at will by these large amounts. Hence a flautist or other woodwind

* In "Helmholtz" notation — a' .

player must possess two different instruments, one slightly longer than the other, if he is to be able to play both in the High and the Low Pitch. Recently (1939), however, an International Committee has recommended that $A = 440$ c.p.s. shall be adopted as the standard pitch, and the principal orchestras do in fact attempt to adhere to this standard. Of course, such things as variations in temperature of a concert hall during a performance will inevitably cause slight departures from the standard.

With the A at 440 c.p.s., the C above it takes the value 522 c.p.s. This is the C in the third space of the treble clef.



In the past, the scientist has had his own pitch, based on 256 c.p.s. for Middle C.

48. Harmony

Notes that are pleasant when sounded together or in succession are found to have simple frequency ratios. Thus in the common chord the notes have the relative frequencies 4, 5, 6 and 8. If we add the notes shown in and below the bass stave, whose frequencies on the same scale are 1, 2 and 3, we have really a long series of harmonics of the common fundamental.

If one or more of the notes is put slightly out of tune, so that the simple relations no longer exist, the resulting sound is much rougher.

When the frequency ratios are simple and exact, there will be no beats between the fundamentals or harmonics of the notes, or between any summational or difference tones they produce. For example, if we sound the notes of frequencies 200, 250, 300 and 400 together, they will all probably have several harmonics, so that the following frequencies will in fact be present :—

From 200 . . .	200			400		600		800	
From 250 . . .		250			500		750		
From 300 . . .			300			600			900
From 400 . . .				400				800	

It is seen that the various harmonics fall exactly in tune, and, therefore, produce no beats, and that there are several possible occurrences of the same difference tone of 50 c.p.s. Indeed, the whole combination is a set of harmonics belonging to this as fundamental frequency. If, however, the frequency of one of the notes is altered by a few vibrations per second, for example, 200 replaced by 204, its harmonics at 408 and 612 will give appreciable beats with the 400 and 600 provided by the other notes: the difference tone with 250 will be of frequency 46 c.p.s., no longer a "common factor" of all the frequencies present. The beats produced, and the fact that the combination is no longer a single rich sound, are the cause of the roughness and discordant effect obtained.

Unfortunately, harmony is rarely as perfect as it might be, as it is not usually possible to arrange that the frequencies are absolutely exactly in the simple ratios (§ 50), but it is important to keep the departures as small as possible.

49. Sharps and Flats

If we consider the diatonic scale—Doh, Ray, Me, Fah, Soh, Lah, Te, Doh—the ideal frequency relations between the notes are found to be the following:—

C	Doh	1			
D	Ray	$9/8$	$\rangle$	$9/8$	
E	Me	$5/4$	$\rangle$	$10/9$	
F	Fah	$4/3$	$\rangle$	$16/15$	Semitone.
G	Soh	$3/2$	$\rangle$	$9/8$	
A	Lah	$5/3$	$\rangle$	$10/9$	
B	Te	$15/8$	$\rangle$	$9/8$	
C'	Doh	2	$\rangle$	$16/15$	Semitone.

The first column of figures shows the relation of the frequency of the note to Doh, the first of the scale: the bracketed figures in the next column show the ratio of the frequency of each note to that of the one next below it. These ratios are the "ideal" ones because the notes would have no dissonant harmonics, if suitable combinations are sounded together. A set of notes that might conform to this scale would be represented musically by the letters C to C' in the first column. In the next octave the frequency of each note would be doubled, and so on.

On inspection of the bracketed ratios, which represent the intervals between successive notes, it will be seen that there are three kinds of interval present: the ratios $9/8$, $10/9$, and $16/15$. Of these, the $9/8$ and $10/9$ are *nearly* but not quite equal, and are called *whole tones*, while the much smaller $16/15$ is called a *semitone*. The semitones in this scale lie between the third and fourth, and the seventh and eighth notes. If the

frequency ratios have the exact values given, the succession of notes is said to form a scale of C major in *just intonation*.

If we start a scale on another note, say G, and sink for the moment the difference between $10/9$ and $9/8$, we find that we have two whole tones G-A and A-B, followed by the semitone B-C, which is in the correct place as the third interval of the series. Continuing from there (the ratios will be the same in any octave), we have C-D and D-E whole tones, E-F a semitone, and lastly F-G a whole tone. The last two intervals are in the wrong order: the semitone is wanted last and the whole tone just before it. It is therefore necessary to raise the pitch of the note F, making a new note called F sharp, written F#. This will be used instead of the original F when playing a scale starting on G, or a piece of music "in the key of G." If we choose the frequency of the F# correctly, we should have the following frequency ratios present in the scale of G, assuming that all the other notes have exactly the same frequencies as before:—

G	$> 10/9$
A	$> 9/8$
B	$> 16/15$
C	$> 9/8$
D	$> 10/9$
E	$> 9/8$
F#	$> 16/15$
G'	

It will be observed that the frequency ratios, or intervals, are exactly the same as those of the original scale of C except for the first two. In the original scale of C the first interval is $9/8$ and the second $10/9$: in the scale of G this position is inverted if we use the same frequencies. The note next above the keynote, or tonic, called the *supertonic*, will have to be adjusted slightly in frequency (in the ratio $81/80$) if the scale of G is to have exactly the intervals of just intonation. The interval corresponding to the ratio $81/80$ is known as the *Comma of Didymus*.

If we start a scale on any other note except F, we find that two or more of the notes have to be raised substantially in pitch in order to secure that the semitones are in the right position in the scale. Thus, if we start on D we need F# and C#: on A F#, C#, and G#: on E F#, C#, G# and D#: on B F#, C#, G#, D# and A#. In addition to these large changes small adjustments to the extent of the comma mentioned above will also be required for exactness.

If we start a scale on F we have at first two correct whole tones F-G and G-A, but B is a whole tone above A, whereas an interval of a semitone is required in this position. The B must therefore be replaced by a note substantially lower in frequency, called B flat, and written Bb. This is nearly the same as the A# required when playing in the key of B—but not quite.

If, therefore, we add to the original notes six new ones, that is, five sharps and one flat, we should be able to play a major scale starting on any of the original notes A to G without having an inaccuracy anywhere greater than the comma of Didymus. But if we start scales on these new notes, we naturally find that more new notes are wanted. For example, the keynote of a scale of E major is at $5/4$ of the frequency of C. The third note of this scale, G#, will bear to C the relation $25/16$. If we start a scale on this note, we need for the third note of it B#, at a frequency $125/64$ above the original

C. This is almost an octave—128/64 would be exactly so—but not quite. The interval by which C differs from B# is equivalent to the ratio 128/125, and is known as the *enharmonic diesis*.

Similarly, if we start a scale on the note B flat that occurred in the scale of F, we need for its fourth note a note somewhat lower than E, called E flat. This note is not the same as the D sharp required for the seventh note of a scale starting on E: the two differ again by the enharmonic diesis. If we start a scale on A#, we need the note C double sharp, written C×, also F× and G×, the sequence being A#, B#, C×, D#, E#, F×, G×, A#. C× is very nearly D, and F× and G× are very nearly G and A, but not quite.

Of course there is no end to this: the more scales we construct the more new notes we require, and this gives us more notes upon which to start further scales. Fortunately, however, musical practice does not go beyond the double sharp and the double flat. In actual fact, a great deal of simplification by approximation has to be made.

50. Temperament

It will be appreciated from the previous account that musical instruments would require an enormous number of notes to the octave if they are to play absolutely accurately in all keys. Not only are there all the "extra" notes discussed but even the original ones do not enjoy the same meaning on all occasions. With the human voice, stringed instruments of the violin family, and the slide trombone, there is no difficulty, for there are no "fixed" notes (except open strings), and any frequency can be obtained. This enables the expert to play with accuracy at all times, but puts some considerable strain upon the novice. With wind instruments generally it is always possible to make a slight adjustment of the pitch of a given note by manipulation of the lips, or the wind pressure: as a result of this it is sufficient to provide twelve notes to the octave. These are the main notes C to B of the diatonic scale, together with five additional notes, each one approximately in the middle of each whole tone. Just as in the case of stringed instruments or the voice, the player's ear is the guide which tells him when he is playing each note with correct intonation.

With the keyboard instruments—piano and organ—the position is different. A string with fixed length and tension or a given organ pipe can only emit a certain set of frequencies. Thus if we sound the note D, for example, we shall have to put up with the same frequency, no matter of what scale it may be a member. If we simply tune the strings or pipes so as to be correct for the scale of C, they are very considerably out in other keys—more so in some than in others. The process of spreading these errors, so that no scale is noticeably worse than another, is known as *temperament*, and such a scale is said to be *tempered*.

For a great many years the system of *mean tone temperament* was employed. As its name suggests, this aims at equalising the tones. Thus

the two tones between C and E, instead of being ratios $9/8$ and $10/9$, are made to have the ratio $\sqrt{5}/2$ each. The other notes are arrived at by making all the major thirds perfect at the ratio $5/4$, that is, equal to two of the tempered whole tones. This necessitates a slight reduction of the interval known as the fifth from the ratio $3/2$: for the interval from C to E'', two octaves and a major third, is four fifths. If the third is perfect the ratio of E'' to C is 5 or $80/16$, whereas $(3/2)^4 = 81/16$. This difference of one comma is spread by making each fifth less by one quarter of a comma. The ratios in the scale of C thus become

Frequency Ratio to C					Frequency Ratio to note next below
C	1	.	.	.	$> \sqrt{5}/2$.
D	$\sqrt{5}/2$	.	.	.	$> \sqrt{5}/2$.
E	$5/4$	.	.	.	$> 16/15 + \frac{1}{4}$ comma.
F	$4/3$ sharpened $\frac{1}{4}$ comma	.	.	.	$> 9/8 - \frac{1}{4}$ comma = $\sqrt{5}/2$
G	$3/2$ flattened $\frac{1}{4}$ comma	.	.	.	$> 10/9 + \frac{1}{4}$ comma = $\sqrt{5}/2$
A	$F \times 5/4 = 5/3$ sharpened $\frac{1}{4}$ comma	.	.	.	$> 9/8 - \frac{1}{4}$ comma = $\sqrt{5}/2$
B	$G \times 5/4 = 15/8$ flattened $\frac{1}{4}$ comma.	.	.	.	$> 16/15 + \frac{1}{4}$ comma.
C	2	.	.	.	

To these are added the five notes

					Frequency
F#	.	.	.	.	$5/4$ times D
C#	.	.	.	.	$5/4$ „ A
G#	.	.	.	.	$5/4$ „ E
Bb	.	.	.	.	$4/5$ „ D
Eb	.	.	.	.	$4/5$ „ G

It is then possible to play in the keys of C, G, D, A, F and Bb with only very slight inaccuracies, using twelve notes to the octave in all. It will be observed that there is one additional note somewhere inside each of the whole tones. There is no D# or Ab: if an attempt is made to play in a key for which these notes are necessary, and Eb or G# is used instead, the result is very inaccurate.

The system employed to-day is known as that of *equal temperament*. Regarding each whole tone as the equivalent of two semitones, we have in all twelve semitones to the octave. Hence if each semitone is made to have a frequency ratio of $\sqrt[12]{2}$ they will all be exactly equal, and the inevitable inaccuracies will be equally spread in all scales. A single fixed note represents each of C, D, E, F, G, A, B, on all occasions and another single fixed note will represent each pair C# and Db, D# and Eb, F# and Gb, G# and Ab, A# and Bb. These are the black notes of the keyboard, while the former seven are the white notes. For B# we use C, for F# we use G, and so on.

The comparison between the set of notes in just intonation and the two systems of temperament is easiest to make by expressing the various

frequencies as the number of cents (§ 46) they are above the lowest note of the octave. This is done in the following table :—

Just Intonation			Mean Tone Temperament		Equal Temperament	
Note	Ratio to C	Cents above C	Note	Cents above C	Note	Cents above C
C	1.000	0	C	0	C	0
			C#	76	C# or D $\flat$	100
D	1.125	204	D	193	D	200
			E $\flat$	310	D# or E $\flat$	300
E	1.250	386	E	386	E	400
F	1.333	498	F	503	F	500
			F#	577	F# or G $\flat$	600
G	1.500	702	G	697	G	700
			G#	773	G# or A $\flat$	800
A	1.667	884	A	890	A	900
			B $\flat$	1,007	A# or B $\flat$	1,000
B	1.875	1,088	B	1,083	B	1,100
C'	2.000	1,200	C	1,200	C	1,200

It will be seen that each of the semitones on the original scale of just intonation is equivalent to 112 cents, against 100 only on the equally tempered scale. A whole tone of ratio 10/9 is 182 cents, and one of ratio 9/8 204 cents, against 200 for all in the equally tempered system. The fourth C–F, and fifth C to G or F to C are very nearly correct on either temperament system. The major third, instead of being 386 cents, comes out at 400 on the system of equal temperament. The minor third is 316 cents on the exact scale, and 300 on that of equal temperament, again quite a considerable difference. These differences are the price that must be paid for being able to play in all keys with only twelve fixed notes to the octave.

For comparison it may be noted that the Comma of Didymus corresponds to 21.5 cents, and the enharmonic diesis to 41 cents. Looking at the figures for mean tone temperament it will be seen that the greatest discrepancy is on the supertonic D, where it amounts to 11 cents. The major third is everywhere the correct 386, while the minor third is 311, a discrepancy of 5. The two semitones are rather large, at 117 cents, as against 112, but are much nearer than the 100 of equal temperament. The rise of pitch due to a sharp or the fall due to a flat is only 76 cents : much less than half the tone of 193 cents. That is how it comes about that the "same note" will not do for G# and A $\flat$ on this system, the difference being the 41 cents of the enharmonic diesis.

When a singer or instrumentalist is playing with accompaniment of piano or organ he must conform largely to the temperament in which that instrument is tuned, being guided by his ear. In a string quartet the players frequently play in just intonation—by ear, of course, not by making conscious differences between the same notes in different keys. In an orchestra there is frequently considerable variation and it is impossible to say exactly what frequency ratios exist. The matter

cannot really be reduced simply to numerical terms, though the difficulties can be well illustrated in that way.

QUESTIONS

1. Describe a method of measuring the frequency of vibration of a tuning-fork.

How may the wavelength in air of the note emitted by the fork be determined ? (*O.P.*)

2. Explain what is meant by wave motion, and derive the relation between velocity, wavelength and frequency of a wave.

Describe how you would determine the frequency and wavelength of the sound emitted by a tuning-fork. (*O.P.*)

3. Describe how you would measure the frequency of vibration of the fundamental note of an organ pipe.

How can you deduce the velocity of sound in air from this measurement and the length of the organ pipe ? (*O.P.*)

4. On what physical characteristics do the pitch, loudness and quality of a musical note depend ? Explain what is meant by the terms "Fundamental," "Harmonic" and show how they are related in the note from a violin string. How would you measure the frequency of the fundamental of the string ? (*O.P.*)

5. Describe and explain the determination of frequency by (i) the Kundt's tube method, (ii) the dropping plate method, and give an indication of the frequencies for which each method is suitable.

In what respects do musical tones differ from one another, and what is the physical explanation of such differences ? (*N.U.J.M.B.*)

CHAPTER VI

THE VIBRATIONS OF STRINGS, BARS AND PLATES

51. Transverse Displacement Waves on Strings

If a point on a stretched string is displaced to one side and set free, the "hump" travels along the string, forming a transverse displacement wave. In due course the wave is reflected at the supports of the string, so that a stationary wave system is set up.

To calculate the velocity with which the "hump" travels, suppose the wire or string to be perfectly flexible, and the displacements sufficiently small not to affect the tension T .

Let the mass per unit length be m .

Suppose a displaced element AB of length δl (Fig. 44) to form a circular arc of radius R , so that it subtends an angle at the centre equal to $\delta l/R$.

$$\text{Thus} \quad 2\phi = \frac{\delta l}{R}.$$

The two equal tensions each have a component towards this centre equal to $T \sin \phi$, which approaches $T\phi$ as ϕ tends to zero.

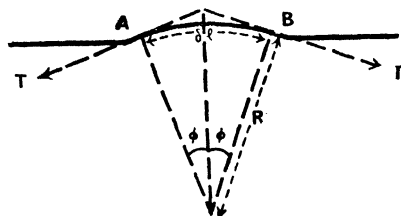


FIG. 44.

$$\therefore \text{Total force towards centre} = 2T\phi = 2\frac{\delta l}{R}.$$

If the hump travels along the wire with velocity V , it can be brought to rest in space by moving the whole wire in the reverse direction with this same speed. At AB we shall then have a portion of wire of mass $m\delta l$ moving with velocity V in the arc of a circle of radius R . To do this a force

$$\frac{m\delta l V^2}{R}$$

must be acting on the element towards the centre of the circle.

$$\therefore \frac{m\delta l V^2}{R} = \frac{T\delta l}{R}$$

$$V^2 = \frac{T}{m}$$

$$V = \sqrt{\frac{T}{m}}$$

This expression gives the velocity of a transverse displacement wave on a string.

52. Frequency of Transverse Vibrations of Strings

Normally, the string is fixed more or less rigidly at its ends. No displacement can therefore occur there, so that reflection with a phase change takes place. The result is a stationary wave system, of the same frequency as the travelling waves, with nodes at the two fixed ends. As there may be any number of nodes in between, various wavelengths and frequencies are possible.

If l is the length of the string, the longest possible wavelength λ_1 is such that $\lambda_1/2 = l$, or $\lambda_1 = 2l$. This is shown in Fig. 45 (1).

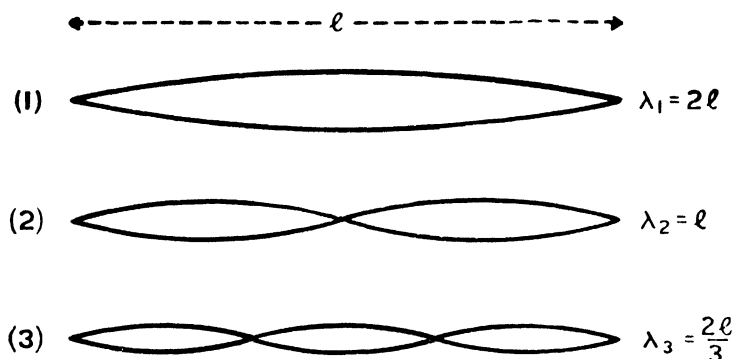


FIG. 45.

In the case (2) in the figure, when the wavelength is λ_2 , we have

$$\frac{2\lambda_2}{2} = l, \text{ or } \lambda_2 = l.$$

Similarly in (3), $\frac{3\lambda_2}{2} = l$, or $\lambda_3 = \frac{2l}{3}$; and so on.

The frequencies corresponding to these modes of vibration are deduced from the relation

$$\text{frequency} \times \text{wavelength} = \text{velocity},$$

the velocity being $\sqrt{\frac{T}{m}}$.

Thus, for (1), $n_1 = \frac{1}{2l} \sqrt{\frac{T}{m}}$ (fundamental vibration);

for (2), $n_2 = \frac{2}{2l} \sqrt{\frac{T}{m}};$

for (3),
$$n_3 = \frac{3}{2l} \sqrt{\frac{T}{m}};$$

and so on.

The string is therefore capable of emitting a full harmonic series—that is, a fundamental and all its multiples. The actual harmonics present depend upon the way in which the vibration is excited and sustained. What kind of a sound the string makes depends on the way in which it is allowed to influence the surrounding air. A string by itself disturbs very little air, and scarcely causes any sound waves: these are usually the result of the vibration of the end supports of the string, magnified by the resonances of the board or box on which they are mounted.

The expression for the fundamental frequency of vibration

$$n_1 = \frac{1}{2l} \sqrt{\frac{T}{m}}$$

shows that this frequency

- (a) is inversely proportional to the length for a given wire under fixed tension,
- (b) is directly proportional to the square root of the tension for a given wire of fixed length,
- (c) is inversely proportional to the diameter for wires of the same material of the same length and under the same tension. (The mass per unit length is proportional to the square of the diameter.)

These statements are sometimes known as Mersenne's Laws, having been published by him in 1636.

53. Experiments with the Sonometer

The sonometer, or monochord, consists of a long resonance box upon which one or more wires can be stretched, usually by means of weights (Fig. 46). The wires pass over fixed bridges B_1 and B_2 , while a third movable bridge B_3 can be used to adjust the vibrating length. The wire can be made to vibrate by plucking it with finger and thumb, or by bowing with a violin-bow. The sonometer is sometimes used almost vertically, an arrangement which lessens the friction at the bridge B_2 and therefore makes the tension in the vibrating portion B_1B_3 known with greater certainty.

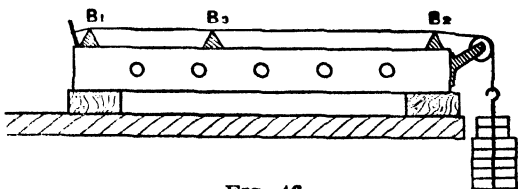


FIG. 46.

To demonstrate the possible modes of vibration of a wire the movable bridge is taken away and the wire placed under considerable tension. It is then bowed at from one-quarter to one-third of its length from one end, and the note observed. A feather, or the tip of the little finger is then held at the exact centre of the wire, and the bowing repeated. The sound will be found to have risen an octave, this being the interval produced by doubling the frequency of the fundamental. If the feather is placed at exactly one-third of the way along the wire the note rises a further fifth, and is of three times the original frequency. It makes no difference which portion of the wire is bowed, the best results being obtained if the bowing is done somewhere near a point that will be an antinode for the desired vibration. The process may be carried on up to the sixteenth or even higher harmonics, the notes forming a natural scale like those of the bugle or valveless horn, but perfectly exact in intonation. Violinists make use of this process when they make their highest notes, which they call "harmonics": touching a point one-quarter of the way along the string with the little finger puts the note up two octaves. An ancient musical instrument, called the Tromba Marina ("Marine Trumpet") used just this principle. It had a single heavy string like that of a Double Bass, with the usual resonating box. The string was caused to emit high harmonics selected by the process given above, and the instrument is stated to have been used by nuns to accompany hymn-singing. Hence its other name—the "Nonnengeige." Its fundamental note—never used, of course—has been compared (unfavourably) with the bray of an ass.

The other experiment commonly carried out with the sonometer is the test of Mersenne's Laws (§ 52).

(a) With a given wire under constant tension, the vibrating lengths required for the emission of various known frequencies are found. This is done by comparing the note given with that emitted by a tuning-fork of known frequency, and adjusting the position of the bridge B_3 until a match is secured. This is repeated for several forks.

An experimenter with a "good ear" can do this tuning accurately without any difficulty: a person with less musical training can obtain the same results by listening for beats between the two sounds, and making adjustments until they are too slow to be distinguishable. The best way of carrying this out is to hold the vibrating fork at such a distance from the ear that the intensity it gives is comparable with that produced by the string. Even a deaf experimenter can obtain good results by making use of the phenomenon of resonance. A small paper rider is placed upon the wire near the centre of the part that is being tuned. The tuning-fork is struck fairly heavily and the shank stood on the box of the sonometer. If the string is in tune or nearly in tune it will vibrate considerably under this stimulus, causing the rider

to tremble, or perhaps, to be "thrown" completely. The length adjustment is made until it is thrown with the maximum vigour. In any of these ways a series of values of the length l with corresponding values of the frequency n is obtained: the accuracy of the first law can be tested by multiplying them together and examining the constancy of the product nl . Alternatively, the logarithms of n and l can be plotted and the inverse first power relation deduced. With the value of nl known, the string can be used for rough measurements of frequency.

(b) The second law is that the frequency given by a wire of fixed length is proportional to the square root of the tension. It is practically inconvenient to test this just as it stands: instead, it is usual to show that for a fixed frequency the length must be proportional to the square root of the tension, which amounts to the same thing if the first law is correct. The procedure is as before, a single fork being used, and length adjustments made until the wire matches it under each tension T in turn. The constancy of $\sqrt{T}/l$ can be examined, or the logarithms of T and l can be plotted.

(c) To complete the test of the laws it is necessary, strictly speaking, to use a number of wires of different diameters. The value of $nl/\sqrt{T}$ could be found for each by the method of experiment (a), and shown to be proportional to $1/\sqrt{m}$, m being the mass per unit length—or to the reciprocal of the diameter if the wires are all of the same material. Usually, however, only one wire is used, and the experiment is completed by calculating $T/4n^2l^2$, which should equal the value of m obtained by direct weighing of a sample of measured length.

$$\text{If} \quad n = \frac{1}{2l} \sqrt{\frac{T}{m}},$$

$$n^2 = \frac{T}{4l^2m},$$

$$\text{and} \quad m = \frac{T}{4n^2l^2}.$$

54. Melde's Experiment

This can be used for demonstration of the various modes of vibration of a string, or for testing the formula $n = \frac{1}{2l} \sqrt{\frac{T}{m}}$. In its original form it made use of a heavy tuning-fork, to one prong of which was attached a thread or fine wire, passing over a pulley to a scale-pan containing weights (Fig. 47).

The fork may be arranged to disturb the string transversely, in which case the frequency of the wave set up in the string is the same as that of

the fork : or the fork may disturb the string longitudinally, as in the diagram. In this case the maximum displacement of the string, up or down, takes place when the prongs of the fork are at their widest

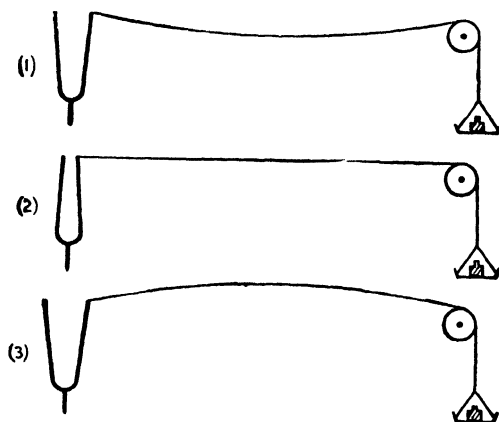


FIG. 47.

(1 and 3, Fig. 47), so that while the fork performs one oscillation each point on the string has only executed half a cycle. Thus in this case the frequency of the forced oscillation set up in the string is one-half of the frequency of the fork.

The velocity of waves upon the string may be varied by adjusting the tension in it, so that any single mode of vibration of the string can be matched to the forcing frequency. When the tension is correct and the fork is vigorously bowed, the string will vibrate in the appropriate number of segments, demonstrating the nodes and antinodes very well (Fig. 48).

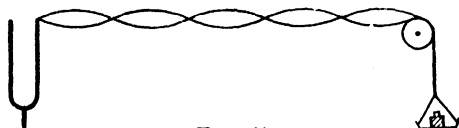


FIG. 48.

A convenient method for carrying out the experiment to-day is to use a vibrator actuated by the A.C. mains, instead of the tuning-fork. This can be made from an electric bell, without the interrupter, and should be fed with a polarising direct current in addition to the A.C. (Fig. 49). The current then varies with time as shown in Fig. 49, and the vibrating blade (preferably a springy piece of steel), performs an oscillation of the same frequency as the mains. If the polarising current is not employed its frequency is double that of the mains.

The string can be made to vibrate in various modes by adjusting the tension : if a numerical calculation, such as the measurement of mains frequency, is being made, it is desirable to measure the exact distances

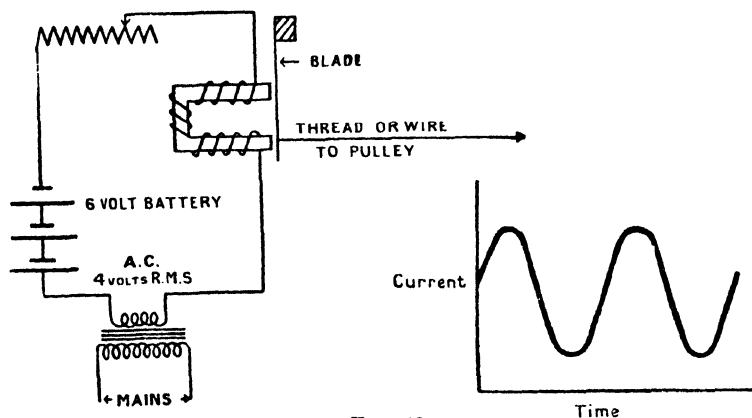


FIG. 49.

between the nodes by placing pins at these points. This is not usually an exact sub-multiple of the length of the string, as there is an "end-correction" at the vibrator end, where the node is imperfect.

Beating between a decaying natural oscillation and that of the forcing frequency (§ 23) is often noticeable, particularly just after making a change of the load in the pan.

55. Means of producing Vibration in Strings

The string may be "plucked"—that is, pulled to one side and let go, as in the violin played "pizzicato", or mechanically in the harpsichord ; or it may be struck with a suitable hammer, as in the piano-forte. Both these methods only initiate a vibration : they do not sustain it, and it dies away gradually on account of damping. To produce a sustained note the bow is used. This consists of a large number of hairs from a horse's tail, half of them being laid in each direction, under tension in a suitable holder. The bow is rubbed with resin to increase the grip on the string. When it is drawn across the string the latter is first drawn to one side with it : when the restoring force due to the tension in the string reaches the value of the limiting friction, slipping begins. The frictional force is then much less, and the string moves back to or beyond the normal equilibrium position. Contrary to what is often stated in very elementary text-books of mechanics, the frictional force does depend very considerably on the relative velocity between the slipping surfaces, in this case the string

and the bow : it is much less when the string is moving against the bow than when it is moving in the same direction. If this were not so there would be no energy intake and the process could not sustain the vibration.

56. Stringed Instruments

The violin family. The members of this family now in common use are the violin, viola, violoncello and double bass. In each case the instrument has four strings, of wire or catgut,¹ mounted upon a resonating box shaped so as to allow freedom of motion of the bow. The strings pass over a bridge B (Fig. 50), which communicates the vibrations to the wood of the box. The tensions in the strings can be adjusted by turning the pegs P, this being the means by which the instrument is tuned, so that the open strings emit notes a perfect fifth apart (except the double-bass, which is tuned in fourths). Thus, in the case of the violin, the open strings are tuned to the notes g, d', a' and e''.

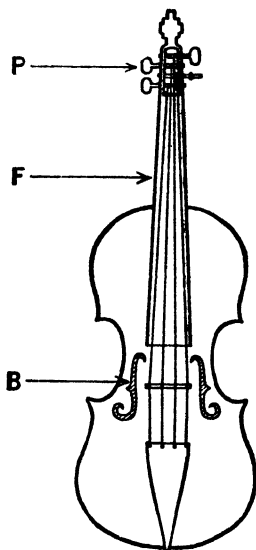


FIG. 50.

Notes other than those given by open strings are "made" by pressing the string on to the fingerboard F at the correct point, thus altering the vibrating length. There are no frets



or other guides to tell the player at what point to put his finger, and there is thus no restriction at all upon the frequencies that can be obtained. To the novice this is an additional difficulty : but it permits the expert to play any scale he chooses and therefore to be unfettered by any system of temperament (§ 50).

As explained in § 52, a string can vibrate in ways that produce any or all of the multiples of the fundamental frequency. The relative intensities of the different harmonics in the final sound, however, depend upon the degree of reinforcement of these by the resonator and other vibrating parts of the instrument. The analysis of notes from violins shows a long series of harmonics to be present, but there is little general agreement between different workers. Undoubtedly the harmonic retinues of different instruments, of different strings on the same instrument, and even of different notes on the same string, show considerable variations. A particular instrument reinforces chiefly

¹ The dried and twisted intestines of the horse, sheep, or ass (not cat).

frequencies in a band or bands known as its "formant": in the case of a first-class violin this appears to extend from about 3,200 to 5,200 c.p.s. in fairly uniform strength. In a poorer instrument the resonant frequencies are lower and less evenly spread. The wood of which the instrument is made appears to be the most important factor, not the varnish upon it, as was once held to be the case.

Keyboard instruments

In the harpsichord the strings are "plucked" by means of a mechanism actuated from the keyboard: in the pianoforte the pressing of a key causes a string to be struck with a hammer covered with soft felt. A second felt, called the damper, comes into contact with the string and stops its vibration when the finger is removed from the key. By pressing a pedal the dampers can be put out of action, allowing the vibration to die away at its own rate. There are usually three strings for each of the higher notes: if the "soft pedal" is pressed only one or two of the three is used in each case. The strings are struck at about one seventh of the way along, weakening the seventh harmonic: this is desirable as the seventh multiple is apt to cause dissonance with other notes in a chord. The actual harmonic retinue depends not only upon the string and the position of striking, but upon the reinforcement produced by other parts of the instrument.

The effect of a rise of temperature upon the frequencies emitted by an instrument such as the pianoforte is generally to cause a slight flattening. The string takes up the air temperature more quickly than the massive supporting frame, the resulting "relative expansion" causing a slight slackening of tension. The slight reduction in frequency resulting from this is, of course, the opposite kind of change to that occurring with wind instruments (§ 65).

57. The Tuning Fork

In discussing the original form of Kundt's tube (§ 36) it was stated that the sounding rod was made to perform *longitudinal* vibrations, with displacement antinodes at the free ends and the corresponding node at the centre. The fundamental frequency is such that the wavelength in the rod is twice the length of the rod, so that the frequency has the value

$$\frac{1}{2l}\sqrt{\frac{E}{\rho}}$$

where E is Young's Modulus for the material and ρ its density, and l the length of the rod.

(The use of Young's Modulus in this expression is, in fact, an approximation, though it is sufficiently near for practical purposes.)

A bar can be made to perform a *transverse* oscillation, the commonest

practical case being the U-shaped bar used as a tuning-fork. Although the elasticity, in the form of Young's Modulus, the length, and the density still affect the frequency, the area and shape of the cross-section are now of importance. The frequencies of the partial tones are no longer multiples of a common fundamental. The derivation and solution of the relevant differential equations are to be found in more advanced text-books.¹

The tuning-fork owes its importance to the fact that it provides a simple source of almost constant frequency. Further, it gives almost a "pure note," the wave-form being almost a perfect sine curve (see Plate IV). Its upper partials are very much higher than the fundamental, and soon die away on account of heavy damping. Changes of temperature have a small effect on the frequency emitted; with a steel fork a rise of 1 Centigrade degree lessens the frequency by about 1 part in 10,000, due principally to a drop in the value of the elasticity. Modern standard forks are made of "elinvar," which reduces the effect to about one-tenth of that occurring with steel. An electrically maintained fork (§ 43) with arrangements for ensuring a standard amplitude is thus a very reliable source of constant frequency. Such forks have been used as the basic control for the frequency of the electromagnetic waves sent out by radio stations, the frequency being "multiplied" many times with a system of distorting amplifiers.

58. The Vibrations of Plates. Chladni's Figures

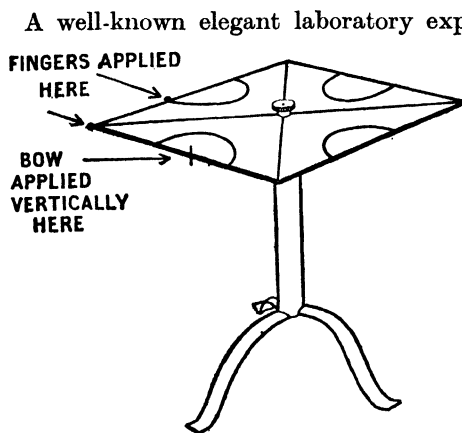


FIG. 51.

A well-known elegant laboratory experiment is the study of the vibrations of a brass plate supported at its centre (Fig. 51). The plate can be made to vibrate in various ways by drawing a violin bow across its edge. Sand is sprinkled upon the plate, and collects upon the nodal lines, forming a symmetrical pattern. The plate may be square or circular, while the place at which the bow is applied, and the position at which a finger is placed to restrain the vibration, provide the means

of selecting the various possible modes of vibration. Chladni, to whom this experiment is due, obtained 52 different figures with a square plate, some of which are easily explicable theoretically.

¹ Such as *Acoustics* by Alexander Wood.

THE VIBRATIONS OF STRINGS, BARS AND PLATES 97

Plates are of importance as the diaphragms of microphones and loudspeakers. To prevent their resonances from destroying fidelity of reproduction it is desirable to make their principal natural frequencies either very high or very low. They can be made very high by subjecting the thin diaphragm to a considerable radial tension, as in the condenser microphone. On the other hand, the ordinary moving-coil loudspeaker uses a paper cone diaphragm mounted almost "freely" at its edges so that its fundamental natural frequency is very low: higher partials are feeble and heavily damped owing to the poor elastic properties of the material.

QUESTIONS

1. Two identical wires stretched on a sonometer under the same tension of 5 kilograms weight emit notes in unison (frequency 300 vibrations per sec.) when plucked. One wire has its tension increased by 100 grams weight, and when the wires are plucked beats are heard. Explain the formation of the beats and calculate the number of beats per second which would be heard. (*Lond. H.S.C.*)

2. A tuning-fork is maintained electrically, the prongs oscillating vertically. One end of a horizontal string is attached to one of the prongs and the string passes over a pulley near the other end, to which a weight is attached. Describe what is observed as the vibrating length of the string is gradually altered.

Explain the state of motion in the string. How can a similar condition be generated by sound waves in the air, and how can the wavelength of such waves be measured? (*Lond. H.S.C.*)

3. Explain the meaning of the term "resonance," giving in illustration two methods of obtaining resonance between the stretched string of a sonometer and a tuning fork of fixed frequency.

A sonometer wire of length 76 cm. is maintained under a tension of 4 kgm. wt. and an alternating current is passed through the wire. A horseshoe magnet is placed with its poles above and below the wire at its mid-point, and the resulting forces set the wire in resonant vibration. If the density of the material of the wire is 8.8 gm. per c.c. and the diameter of the wire is 1 mm., what is the frequency of the alternating current? (*Lond. H.S.C.*)

4. What data would be required in order to predict the frequency of the note emitted by a stretched wire (a) when it is plucked, (b) when it is stroked along its length?

A weight is hung on the wire of a vertical sonometer. When the vibrating length of the wire is adjusted to 80 cm. the note it emits when plucked is in tune with a standard fork. On adding a further weight of 100 gm. the vibrating length has to be altered by 1 cm. in order to restore the tuning. What is the initial weight on the wire? (*Lond. H.S.C.*)

5. Theory shows that the fundamental frequency n of transverse vibration of a stretched string of length l , radius r , tension T and density ρ is given by

$$n = \frac{A}{lr} \sqrt{\frac{T}{\rho}}$$

where A is a constant. Describe how you would use a sonometer and tuning forks of known frequency to verify this relation.

A brass wire and a steel wire, of the same length, diameter and tension, give 5 beats per second when mounted on a sonometer and made to give their fundamental tones simultaneously. Calculate the frequencies of vibration of the two tones, given that the densities of brass and steel are 8.4 and 7.8 gm. per c.c. respectively. (*O. & C.*)

6. Distinguish between progressive and stationary waves.

Describe an experiment to illustrate the formation of stationary waves (with more than one loop) on a string or wire. If a copper wire 0.193 mm. in diameter is used, what must be the tension to form loops 30 cm. long when the frequency of vibration of the wire is 64 per second? Give the result in grams weight. (The density of copper is 8.93 gm. per c.c.)
(*N.U.J.M.B.*)

7. It is found that, when the length l and the tension T of a string are varied so as to keep the frequency of transverse vibration of the string constant, Tl^p is constant, p being a numerical index. Describe how you would verify this result, and determine the value of p .

Two wires, A and B, made of the same metal, of equal lengths and with diameters in the ratio 7 : 4, when set into transverse vibration give notes whose frequencies are in the ratio 4 : 5. These notes are brought into unison when the tension in B is diminished by 1.5 kgm. wt. Find the tension in A and the original tension in B. (*N.U.J.M.B.*)

8. How would you (a) demonstrate the presence of overtones in the sound emitted by a stretched string when it is plucked at its centre; (b) verify the relation between the frequency of the fundamental note emitted by a stretched wire, vibrating transversely, and its tension?

Two wires vibrate transversely in unison. The tension in one wire is increased by 1 per cent. and now, when they vibrate simultaneously, three beats are heard in 2 sec. What was the original frequency of vibration of the two wires? (*N.U.J.M.B.*)

9. Obtain an expression for the velocity of transverse waves along a stretched string.

Describe Melde's experiment for demonstrating the vibrations of a stretched string. (*Lond. Inter.*)

10. A stretched steel wire 0.5 mm. in diameter is supported at two points 30 cm. apart. What tension must be applied to make its lowest characteristic frequency of transverse vibration 1,000 cycles/second? What approximations do you make? (Density of steel = 7.8 gm./c.c.) (*C.S. Part Question.*)

11. Prove that the velocity of transverse waves in a stretched string is (Tension/mass per unit length)^{1/2}.

A wire is stretched between two fixed points so that its natural length is increased by 1 per cent. at room temperature. The coefficient of linear expansion of the wire is 2.3×10^{-3} per cent. per deg. C. and the temperature coefficient of its Young's Modulus is 0.012 per cent. per deg. C. Calculate the percentage change in the frequency of transverse vibrations when the temperature of the wire rises by 10° C. (*O.S.*)

12. Obtain an expression for the velocity of propagation of a transverse wave along a stretched string and deduce from it the laws of transverse vibration of strings. (*Trip.*)

13. Show that the velocity of waves along a stretched string is given by $\sqrt{\frac{T}{m}}$, where m is the mass per unit length and T is the tension. If the length of the string is l , what are the frequencies of its possible oscillations? Write down an expression for the motion of a point of the string in the general case. (*Trip.*)

14. Deduce an expression for the velocity of a transverse wave on a string of mass m per unit length subjected to a tension T . If the string is of finite length l fixed rigidly at each end, find for what frequencies standing waves can be set up on it.

Waves of wavelength λ are travelling along the string. If the maximum transverse displacement is a , find the energy of a length of the string equal to λ . (*O.F.*)

CHAPTER VII

THE VIBRATIONS OF AIR COLUMNS

59. Air Columns in Cylindrical Pipes

IN a cylindrical pipe, plane sound waves may be produced. They will be reflected at the ends, with reversal of the displacement phase at a closed end, and without it at an open end (§ 15). Hence a stationary wave system can be set up, with a displacement node at a closed end, and a displacement antinode at or near an open end. In fact, the displacement antinode is usually a short distance outside the end of the pipe, this distance being known as the *end-correction*. For a

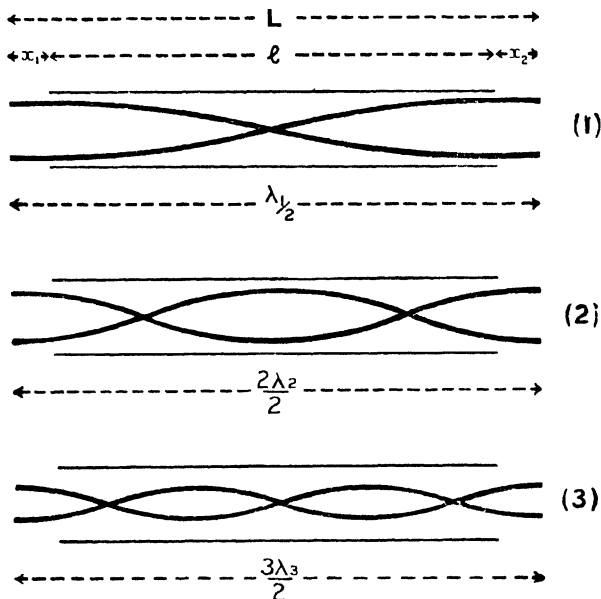


FIG. 52.

cylindrical tube of radius R this end-correction is about $0.6 R$, and it varies only slightly with frequency. If the end is flared no such simple formula holds good.

The wave in the tube, which can be maintained in various ways is, of course, a *stationary sound wave*, and the displacements occurring in it are along the lines of propagation of the travelling waves com-

posing it—that is, along the length of the tube. For convenience, however, we illustrate (Figs. 52 and 53) the limits of the displacement as if this took place at right angles to the length, like the transverse wave on a stretched string. These curves illustrate how the displace-

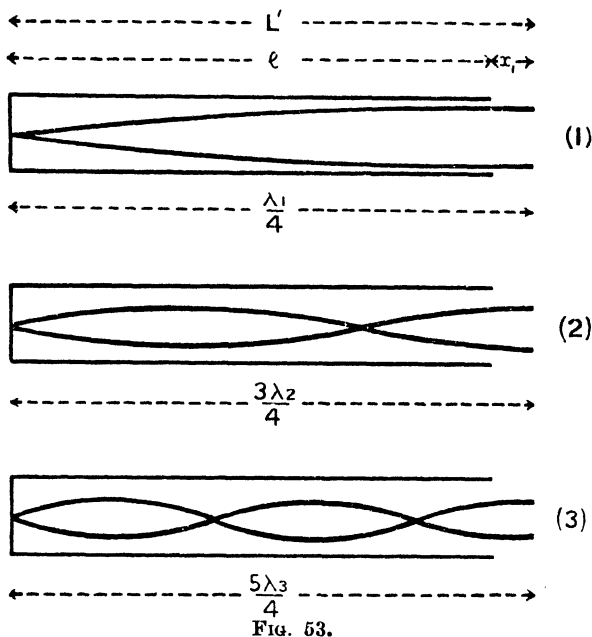


FIG. 53.

ment amplitude varies along the length of the tube, for various possible modes of vibration.

For the open tube (Fig. 52) there will be an antinode just outside each end. The vibrating length L is therefore given by

$$L = l + x_1 + x_2$$

where l is the actual length and x_1 and x_2 are the end-corrections.

For the fundamental vibration as in (1) the wavelength λ_1 is given by

$$\frac{\lambda_1}{2} = L$$

Similarly, the other modes of vibration are such that

$$\frac{2\lambda_2}{2} = L$$

$$\frac{3\lambda_3}{2} = L, \text{ etc.}$$

If V is the velocity of sound in the pipe, the frequencies to which these wavelengths correspond are

$$n_1 = \frac{V}{\lambda_1} = \frac{V}{2L}, \quad n_2 = \frac{2V}{2L}, \quad n_3 = \frac{3V}{2L}.$$

Hence the pipe is capable of emitting notes which form a full harmonic series.

For the pipe closed at one end (Fig. 53), there is only one end-correction, so that the vibrating length L' is given by

$$L' = l + x_1.$$

The fundamental vibration will be such that there is a displacement node at the closed end, and the adjacent antinode near the open end, so that the wavelength λ_1' is given by

$$\frac{\lambda_1'}{4} = L'.$$

For the next possible vibration $\frac{3\lambda_2'}{4} = L'$,

for the next $\frac{5\lambda_3'}{4} = L'$,

and so on.

Thus the frequencies that this pipe can emit are

$$n_1' = \frac{V}{\lambda_1'} = \frac{V}{4L'}, \quad n_2' = \frac{3V}{4L'}, \quad n_3' = \frac{5V}{4L'}$$

forming a series in which only the odd multiple harmonics are present. Further, the fundamental note of the closed tube has only one-half of the frequency of that given by an open tube of the same vibrating length.

60. Maintenance of Vibration of Air Columns. Organ Pipes

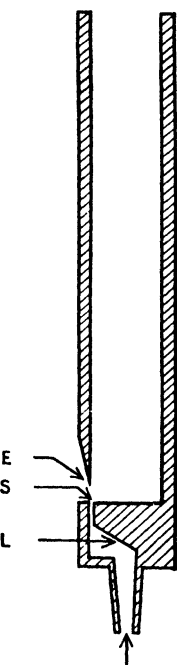
The vibration may be sustained merely by blowing over the end of the tube. This is developed in the flute, in which the player directs a jet of air across a hole in the side of the tube, and in organ pipes of the "flute" or "flue" variety.

In such a pipe a blade-like stream of air, directed by the languet L (Fig. 54), issues from the slit or flue S , and impinges upon the edge E , or passes very near to it. It sets up an "edge-tone"—like the wind whistling through a keyhole or through a crack in a door. If this tone contains suitable frequencies it initiates and maintains the vibration in the pipe.

There is a large end-correction at the blown end of the pipe—two or three times the radius—so that the vibrating length is substantially

more than the geometric length. As this end-correction is not entirely independent of frequency, the natural vibrations of the pipe are not quite exact multiples of the fundamental frequency. When the air in the tube is executing a vibration as the result of its "coupling" with the air producing the edge-tone, it reinforces the partials present in that tone to extents depending upon their nearness to resonance with the natural vibrations of the tube. When the tube is narrow and the end-correction small the partials will be all more or less in resonance together, so that the sound from a narrow tube is likely to be rich in harmonics. With a wide tube, on the other hand, with a large end-correction, the higher frequencies natural to the column are less nearly in resonance with the exact multiples present in the forcing tone. This is part of the cause of the relatively feeble harmonic retinue in the sound given by a wide pipe. The ratio of diameter to length is known as the "scale" of the pipe; a pipe of small scale is richer in harmonics than one of larger scale. The material of the pipe, and its rigidity and uniformity, also have a marked effect on the quality of the sound emitted.

If the pressure of the air with which the pipe is blown is gradually increased, the sound eventually jumps an approximate octave (in the case of an open



AIR
FIG. 54.

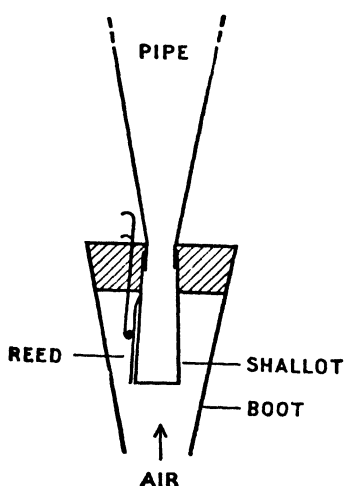


FIG. 55.

pipe), when the fundamental vibration of the column is no longer present. This effect, known as "overblowing," occurs more easily if the scale is small, and is caused by the variation of frequency of the edge-tone. A similar effect results if the pipe is "underblown"—that is, if too slow an air stream is used: this is somewhat of a nuisance in organ construction, making it desirable to admit the air blast suddenly and not gradually.

The basic stop of the organ—the Diapason—consists of open flue pipes, now usually cylindrical and made of metal. The Bourdon is a stopped pipe, that is, it has the top end closed, and it is usually made of wood and square in section. Its fundamental note is the same

as that of an open pipe of approximately twice the length, which makes it a favourite pedal stop in small organs. It is feeble in tone compared with open pipes.

The vibration of the air column may also be sustained by means of a reed. In the case of an organ pipe this is usually a metal tongue covering an opening in the side of a small tube called a shallot (Fig. 55). The tip of the pipe itself, which is usually conical or otherwise of variable diameter, fits into the top of the shallot, which is fed with air in a chamber called the boot. The reed is tuned by moving a wire spring up or down upon it. The quality of tone depends principally upon the shape of the pipe and shallot, the design of the reed, and the clearances.

61. Experimental Investigation of the Motion of Air in Organ Pipes

The diagrams of Figs. 52 and 53 show the theoretical variation of displacement amplitude along a pipe: the pressure amplitude curves would be similar but with nodes replaced by antinodes, and vice versa (§ 10). König investigated the variation of pressure amplitude along a pipe using the "manometric capsule."

This is a small hollow metal chamber, with a gas inlet and a jet, one face of the chamber consisting of a rubber membrane R (Fig. 56). If the rubber is subjected to pressure variations the gas alight at the jet flickers correspondingly. To use this with an organ pipe, several holes are bored in the side of the pipe, and a manometric capsule fitted in each. The jets are lit, and the pipe sounded. The oscillations of the flames are not visible unless their images are spread on the retina, which is done by viewing the images of the flames formed in a rotating mirror. Each flame gives a bright band with a serrated edge: the greater the pressure amplitude the greater the serration. With the capsule half way along an open pipe a serrated edge is produced when the pipe is sounded normally, but a smooth edge if it is overblown so as to elicit the second harmonic.

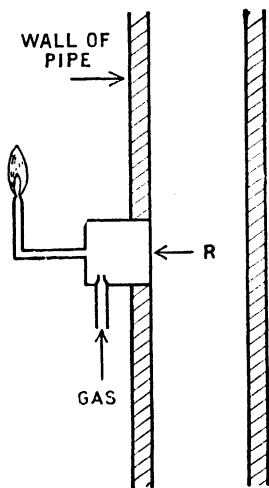


FIG. 56

The optical interference method of Töpler by which pressure amplitude can be measured will be described in § 77: this would provide another method of finding the variation of amplitude along a sounding pipe. Small microphones of various kinds could also be used instead of the manometric capsule.

An adaptation of the Rubens' tube, described by Lawson,¹ though

¹ *Nature*, 160, 4065 (1947).

it is not applicable to an organ pipe, illustrates the pressure conditions in a resonating tube. It consists of a long horizontal thin-walled brass tube, about $2\frac{1}{2}$ inches in diameter, closed at one end and excited at the other by an attached loudspeaker, operated by a valve oscillator of variable frequency. The upper edge of the tube is pierced with a large number of very small equidistant holes, and the tube is supplied with coal gas, which is burned at the holes as jets. In the quiescent state these flames burn uniformly to a height of about $\frac{1}{4}$ inch, but on exciting the tube at any resonant frequency the flames vibrate, and their length shows an undulatory variation along the tube, being greatest at the pressure antinodes.

The dust of Kundt's tube, and the variation in level of the surface of a liquid spread along the bottom of such a tube (§ 96) are also means of rendering the vibrations of an air column visible.

62. The Flute and Clarinet

The *flute* consists of an almost cylindrical tube closed at one end and open at the other, with a hole in the side near the closed end. Between this hole and the open end are a number of other holes, closed or opened by the fingers acting either directly or working upon keys. The sound is made by directing a jet of air across the hole near the closed end, the flute being held transversely. If all the finger holes are closed the vibrating column has the full length of the tube (open at both ends, and with the usual end corrections) as in an open organ pipe: this length is reduced, and the frequency therefore raised, by opening the various finger holes in turn.

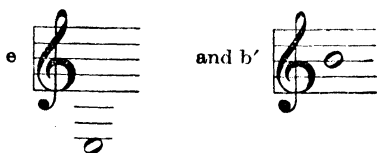
By "overblowing" the tube can be made to speak an octave higher: this may be facilitated by partially uncovering a hole about half-way along the tube, which prevents the formation of a pressure antinode there. Thus all the odd modes of vibration in Fig. 52 are eliminated, while the even ones remain. The scale is extended further by a cross-fingering encouraging a vibration of type (3) as fundamental.

Flutes are made of wood, silver, sometimes of gold, or frequently of an alloy known as "New Metal." The material of the tube, as well as the exact dimensions of the head (the joint containing the hole where the sound is excited) have a considerable influence upon the tone and the ease with which the sound is produced. Harmonic analysis shows that the lower notes played softly are almost "pure tones," while the energy in high harmonics is small under all circumstances.

The *clarinet* (Plate II) consists of a cylindrical tube of wood or ebonite, flared at one end, and almost closed by a single flat reed of cane (*arundo donax*) at the other (Plate III). The reed rests upon the lower lip of the player, the upper lip being curved over the instrument so as to make an air-tight joint. The tip of the tongue is brought to bear upon

the tip of the reed : when the sound is required the tongue is suddenly withdrawn and air forced down the instrument. With its fine clearance from the aperture which it covers, the reed acts practically as a closed end, a pressure antinode being developed near here. The modes of vibration are thus those of a closed tube, the vibrating length being one-quarter, or three-quarters, or five-quarters, etc., of a wavelength (Fig. 53). Only the odd harmonics are thus emitted, and the fundamental is an octave lower than that given by an open tube of similar length. Quite low notes are therefore obtainable from an instrument of reasonable dimensions.

As in the flute, the vibrating length is altered by uncovering a series of holes, some normally covered directly with the fingers, and some through the medium of keys. "Overblowing" results in a note, not an octave higher, but of three times the original frequency—that is, higher by a twelfth, or octave and a fifth. In this way the full length of the tube gives the notes

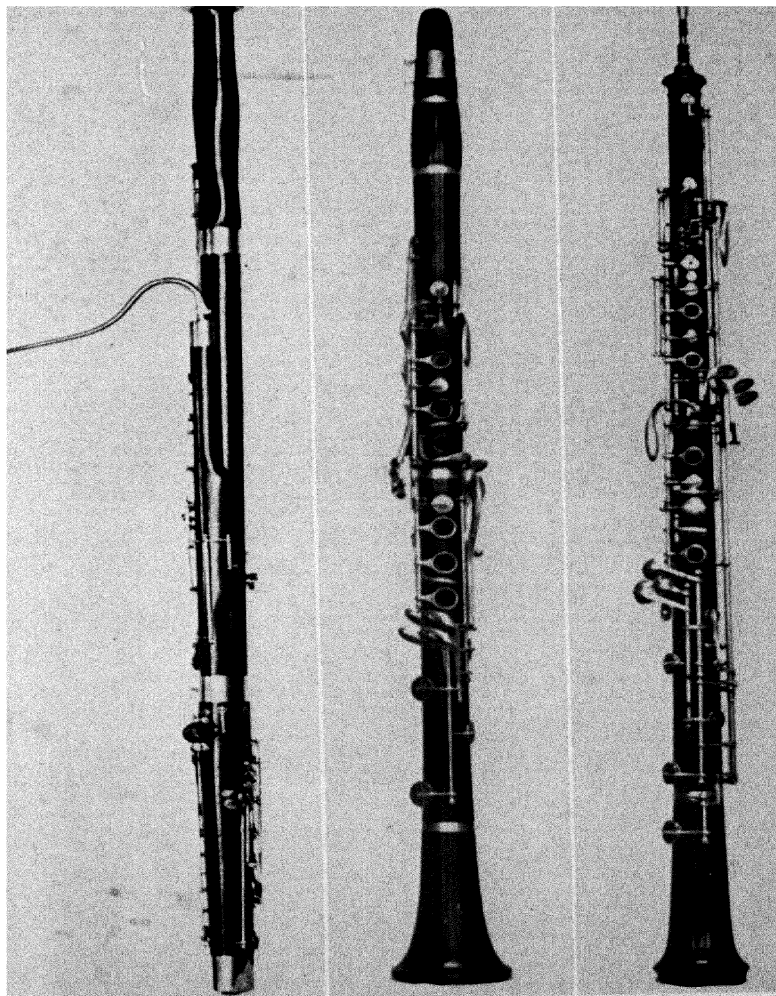


as the two successive harmonics. The overblowing is facilitated by opening a small hole about one-third of the way along the instrument, called the "speaker." The long gap between the two successive notes obtainable on the same tube length (amounting to 19 notes of the chromatic scale) makes the fingering system somewhat more complicated than that of the flute. After all the notes given by the vibration in the form of the third harmonic of the tube have been exhausted, a cross-fingering is used to elicit the fifth harmonic, resulting finally in an immense range of scale. In the early instruments it was difficult to play in a key with several sharps or flats in the signature, with the result that clarinets differing slightly in length came to be used, the music being transposed to suit them. The orchestral player of to-day commonly uses the "Clarinet in B_♭" or the "Clarinet in A," according to the key in which the particular piece of music is written.

The analysis of tones from the clarinet agrees amply with the simple theory outlined above. Indeed, the third and fifth harmonics are quite easy to recognise directly by ear in the lower notes. The two wave-form curves of Plate V illustrate this. In the lower photograph, there are six main peaks in the curve. Between each of these main peaks are two subsidiary ones, caused by the prominent third harmonic. The additional "shoulders" on the up-and-down slopes are caused by the fifth harmonic. In the upper photograph, which was

PLATE II

WOODWIND INSTRUMENTS

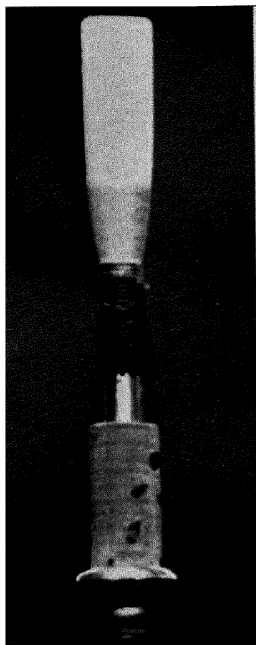


BASSOON

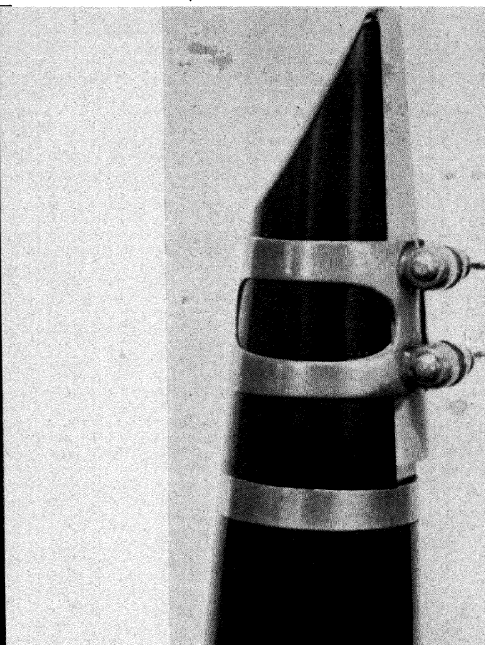
CLARINET

OBOE

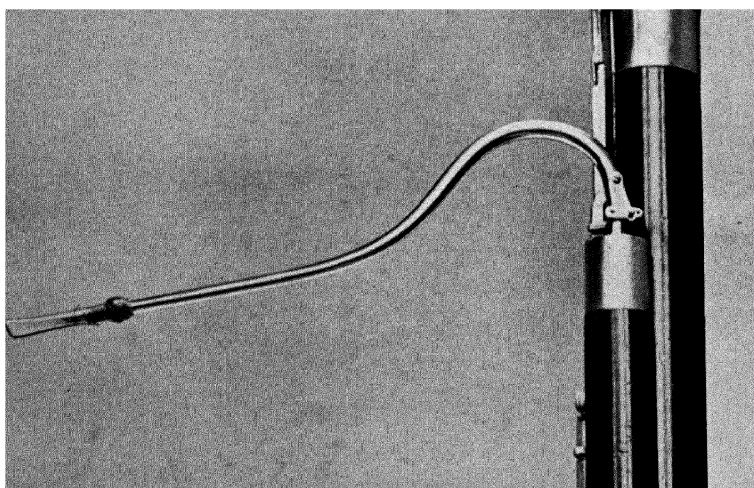
PLATE III
MOUTHPIECES OF WOODWIND INSTRUMENTS



OBOE REED



**CLARINET MOUTHPIECE
AND REED**



BASSOON CROOK, SHOWING THE REED

taken for a slightly higher note on the same instrument¹, the general form is much the same, but the "shoulders" have become distinct peaks, the fifth harmonic being more prominent than before. More accurate investigation shows that higher harmonics such as the ninth and eleventh are present: but there are great differences between different instruments, different reeds, and different notes. More advanced theoretical treatment, based upon the assumption that the reed never quite closes the end of the tube, has been used to account for the fact that the various frequencies obtainable from a given tube length are not quite exact multiples of a common fundamental. The existence of feeble partials of even order is accounted for by their presence in the motion of the reed, causing the air column to emit them as forced oscillations far from resonance.

63. Conical Pipes. The Oboe and Bassoon

A case of practical importance is that of a sound wave in a conical tube with the wide end open, the narrow end being at the apex of the cone or very near it. It is then immaterial whether this end is open or closed, and it may be shown that the system of spherical waves gives a fundamental tone of the same frequency as that given by a doubly open cylindrical pipe of the same length. The possible harmonics are also the same—the full series. Conical pipes are used in the *oboe*, *bassoon* and *saxophone*, and also in certain *brass instruments*.

The *oboe* (French, *hautbois*) consists of a conical tube of wood or ebonite about 18 inches long, the bore at the thin end being only about $\frac{1}{8}$ inch, increasing to about $\frac{1}{2}$ inch and eventually opening out to a small bell (Plate II). The vibration is sustained by means of a reed, made of two almost flat blades of cane mounted on a fine metal tube (Plate III). This tube is inserted, using a cork joint, into the narrow end of the instrument. The instrument is so narrow that viscosity effects are certainly not negligible, and the velocity of sound waves in it is abnormally low, especially near the reed. The result is that the frequency obtained is somewhat lower than would be expected with a tube of this length. The effect also causes the nodes and antinodes to be distributed unequally in distance along the tube. In addition, as it is more marked for low frequencies than for high ones (§ 33) it causes the higher frequencies obtainable from a given tube length by overblowing to be not quite exact multiples of the fundamental. Overblowing produces an approximate octave with most of the notes, as would be expected, the process being facilitated by opening tiny pinholes at the end of the instrument near the reed. The overblown notes tend to be sharp, for the reason given above: the player brings them back into tune by adjusting the pressure of the lips on the reed.

¹ The instrument used was a clarinet in A.

Considerable variation is possible in this way, and the player's ear is the guide, just as in the case of stringed instruments.

Analysis of the sound of the oboe reveals the presence of a very extended series of partials, of which the third, fourth and fifth are often stronger than the fundamental. Indeed, the fundamental may be regarded as the difference tone between two powerful notes. The oscillograms of Plate IV, obtained by the method of § 71, show the wave forms of two notes sounded on an oboe. The note of the top curve is an octave above that of the lower curve. In the note depicted by this lower curve, the third and fourth harmonics are of considerably greater amplitude than the fundamental, causing the curve to cross the axis several times in each complete vibration.

The *bassoon* (Plate II) is effectively a conical tube of wood or ebonite about 100 inches in length, having a bore of about $\frac{1}{8}$ inch at the narrow end and $1\frac{1}{2}$ inches at the wide end. For convenience, the tube is doubled on itself, the lowest joint having two bores connected at the bottom by a U-shaped metal tube. The narrow end is brought out to the side by a curved metal tube called the crook, the free end of which carries the reed (Plate III). This consists of two almost flat pieces of cane, like the oboe reed but considerably larger. Holes and keys for altering the vibrating length are provided, the "doubled" construction permitting each hand to control some on the "down" part of the tube, and some on the "up." As with the oboe, the tube is narrow compared with the wavelength of the sound, so that effects due to reduced sound velocity are prominent—in fact even more so than in the oboe. For the lowest notes the effect of this and of end-corrections is equivalent to adding about 15 inches to the length of the tube, this amount diminishing with increasing frequency.

As in the oboe, a long series of partials is present, but the quality of different notes is often markedly different. The curve of Plate V shows the extremely complicated wave-form of a note sounded on a bassoon. The curve, taken for a note near the bottom of the compass of the instrument, shows powerful harmonics up to and beyond the fifteenth. Although the fundamental frequency is only 78 c.p.s., harmonics having frequencies of the order of 1,000 c.p.s. are very prominent. On account of the reduced velocity mentioned above, the fingering does not obey any very simple rules: in some parts of the scale overblowing produces an approximate octave, but more generally a cross-fingering has to be employed. There are many alternative fingerings for certain notes, and each single instrument has its own peculiarities of tone and tuning. The player's ear is again the ultimate guide in pitching the notes correctly, and these often have to be humoured by the alteration of lip or wind pressure. As an instrument the bassoon is very imperfect, though in the hands of an expert

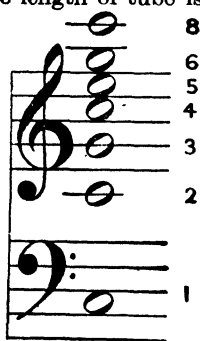
the intonation can be all that is desired. It is a good example of a case in which practice has long outstripped theory. Attempts have been made to improve the instrument, with some success : doubtless real progress could be made if its theoretical complexities were unravelled.

In the *saxophone*, the conical tube is of metal, and is very much wider than that of the oboe or bassoon. The reed is a single piece of cane, mounted upon the mouthpiece exactly as in the case of the clarinet (Plate III). Saxophones of several different sizes are used, the *soprano* being short and straight, while *alto*, *tenor* and *bass* are curved with an upturned bell. This chorus finds its application principally in dance bands and military bands : the instruments are only occasionally prescribed in orchestral scores, mostly of the modern French School.

64. Brass Instruments

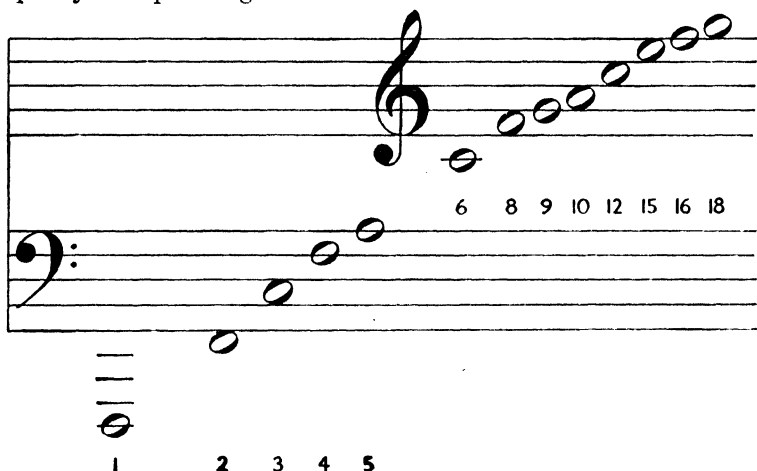
In brass instruments the air column is contained in a metal tube of gradually increasing cross-section, terminating in a bell. The form of the increase may be that of the cone or hyperboloid, depending on the instrument, and the tube is in most cases coiled up. The mouthpiece consists of a hollow cup into which the lips of the player are pressed : the shape and size of this cup are also quite different in different instruments. With the lips pressed in the cup and adjusted to a suitable tension, the forcing of air through them causes them to behave like the double reed of the oboe or bassoon, and a vibration is sustained in the tube.

In the simplest brass instrument, the bugle, only one length of tube is used, and it is possible by adjustment of lip tension and wind pressure to select different possible modes of vibration. The process may be compared with that in which the string of a sonometer (§ 53) is made to vibrate in different numbers of segments by applying a feather in the correct positions. Normally the fundamental vibration cannot be supported in the case of the bugle, and the third, fourth, fifth, sixth and eighth harmonics of the tube constitute the notes normally used. The seventh harmonic is not infrequently produced by accident, with unpleasant effect. If we represent the fundamental as the C in the second space of the bass stave, the harmonics would be the notes shown. With such an instrument there is no way of bridging the gaps between these notes in order to obtain a complete scale.



In the French Horn of the original valveless type the bridging was partially accomplished by making the tube very much longer, so that the harmonics used were higher multiples of the frequency of the fundamental, and therefore lay closer together. Using a tube some 12 ft. long in all

the fundamental wavelength would be about 24 ft., corresponding to a frequency of about 45 c.p.s.—roughly the note F, well below the bottom of the bass stave. The harmonics of this include the following notes, the frequency multiple being stated beneath each.



The multiples 11, 13, 14 and 17 do not correspond exactly to notes of the normal scale, but they can be altered slightly in pitch by placing the hand in the bell or by doing some "forcing" with the lips. They then serve as further useful notes. The lowest notes are far apart and difficult to produce, but are useful for special effects. Great skill is required in playing the horn in order to select the desired harmonic precisely without "wobbling" from one to another.

Although the notes obtained in this way are much closer together than those of the bugle, they still fall far short of a complete chromatic scale. This difficulty was traditionally circumvented by using a different length of tubing for music in different keys. Additional pieces of tubing, known as crooks, were added to the horn to change the open note to the keynote of the piece, so that it became a "Horn in F," or "Horn in D," etc., as the case might be. Temporary key changes, evidenced by "accidentals" in the music, could sometimes be accommodated and sometimes not. Instruments were therefore produced in which the effective length of tubing was controlled by side holes covered with the fingers or with keys; this was not a very successful arrangement, and such instruments have died out, the "Ophicleide" being the one of most recent memory. Two other ways of changing the tube length have survived and are universally employed in appropriate instruments—the use of "valves" and of the "slide."

The modern orchestral horn (Plate I) has valves with pistons, or rotary valves, which bring into "circuit" additional lengths of tubing when they are depressed. When the piston is not depressed the additional section is by-passed by a boring through the piston itself. Three valves are used: one which adds about a fifteenth to the original tube length, another which adds about an eighth, while the third adds about a fifth. Depressing the piston of the first valve will lower all the harmonics by a semitone, the

second by a whole tone, and the remaining valve by a minor third (three semitones). This enables all the gaps to be bridged, and the complete chromatic scale to be obtained.

The notes of the horn, when played softly, have a remarkably simple character. The oscillogram on Plate I (frontispiece) shows the wave-form of the note $f' = 350$ c.p.s., the eighth multiple of the lowest possible vibration of the tube. It is almost a sine curve—that is, the upper partials in it are weak. The third curve of Plate V, shows the wave-form of a somewhat louder note on the horn. The departure from the sine curve is greater than before, but is still very small compared with that of the oboe, clarinet and bassoon.

Other brass instruments, such as the trumpet, cornet, euphonium and bass tuba use valves in just this same way: but the harmonics used are not such high multiples as in the horn, making the instrument easier to play but without the same tone. In the slide trombone, the gaps are bridged, not by valves, but by pulling out the slide, thereby increasing the length of the air column. Some four or five notes are obtainable from every length, just as in the bugle, so that a few different positions of the slide are sufficient to enable the chromatic scale to be produced.

65. Pitch of Wind Instruments

In any wind instrument or organ pipe it is the *wavelength* of the sound that is settled by the dimensions of the tube. The frequency emitted has the value given by the relationship

$$\text{frequency} \times \text{wavelength} = \text{velocity},$$

so that any variation of velocity will affect the frequency.

It was seen (§ 30) that the velocity of sound in a gas is independent of the pressure of the gas, but proportional to the square root of its absolute temperature. The frequency emitted by a wind instrument will therefore change in this same proportion, since the expansion of the material of the tube is quite negligible by comparison.

In the case of an instrument like the organ the temperature of the air in the pipes is presumably that of the air used for blowing them. A rise of 1 Centigrade degree in this temperature will therefore produce a rise of about 1 part in 600 in the velocity and the emitted frequency. In the case of instruments blown with air from the mouth, the temperature eventually reached inside them will be between that of the player's breath and that of the outside air. In the first few minutes of playing the average temperature inside the instrument is low, so that all the notes tend to be flat, becoming sharper as the instrument "warms up." In the case of a small instrument like the flute or oboe, this effect of warming up is quite considerable, but the temperature eventually reached is regulated mostly by the player's breath, and is only comparatively slightly affected by the temperature of the room. With the larger instruments such as the bassoon or bass tuba the effect of "warming up" is not so marked, but the atmospheric temperature has a greater effect on the pitch of the notes emitted. For such

instruments the rise in frequency is about 1 part in 800 for each Centigrade degree rise in the room temperature, while for the flute and oboe when properly warmed up the difference is as little as 1 part in 2,000 for the same temperature change.

These changes look very small, but they are by no means insignificant, especially when we bear in mind that the temperature of a concert hall may rise by 5 Centigrade degrees or more during a performance. For this, and other reasons, it is necessary to be able to "tune" wind instruments—that is, to make small adjustments in the pitch of the notes emitted, which is normally done by making small adjustments in the length.

In the case of a brass instrument such as the horn or trumpet, the slight changes necessary are made by moving the portion known as the tuning slide. This will make up to a few inches difference in the tube length. If the change to be made is at all large, corresponding alterations should be made in the additional lengths of tubing brought into action by the valves. In woodwind instruments, tuning is accomplished by pulling the instrument out slightly at one or more joints. This is a highly unsatisfactory procedure, as a slight increase of length obviously makes a greater proportionate frequency reduction for notes that are made with a short length of tube than for those which use the full length of the instrument. In addition, there is no way of shortening the instrument, so that it will not be possible to reach a pitch slightly *higher* than normal, unless the instrument is deliberately built sharp. As a result of these facts a woodwind instrument can only be brought into correct intonation over quite a narrow range of pitches, the total frequency range being up to about 2 per cent. of the normal value. For this reason, if for no other, it is desirable that standards of pitch should be adhered to universally (see § 47).

QUESTIONS

1. Explain with the aid of diagrams how (1) the pressure, and (2) the amplitude of vibration of the air, vary from point to point of an open organ pipe sounding its fundamental tone. Describe the experiments you would make to verify your account. (*Lond. H.S.C.*)

2. Describe, with the aid of diagrams,

- (a) the stationary waves existing in a sounding organ pipe,
- (b) the progressive waves by which the sound reaches the ear.

Show that a stationary wave system may be regarded as the resultant of two progressive waves.

Use your diagrams to show how the particle velocity and pressure excess are distributed in stationary waves. (*Lond. H.S.C.*)

3. Distinguish between progressive and stationary waves. Give two examples of stationary waves and show how you would show the presence of nodes and antinodes. Explain the difference in quality of the notes

emitted by a "closed" and an "open" pipe of the same fundamental frequency. (*C.S.*)

4. What are the resonance frequencies of the air in a narrow pipe one metre long open at both ends, at 20° C. and 740 mm. of mercury? (Ratio of specific heats of air = 1.4; density of air at N.T.P. = .0013 gm./c.c.; density of mercury = 13.6 gm./c.c.; $g = 981$ cm./sec.²) (*C.S. Part question.*)

5. Distinguish between progressive and stationary waves. Describe the motion of the air in a closed sounding organ pipe adjusted to resonance, and the changes of pressure with time at various points along the pipe. Describe some experimental evidence in support of your statements. (*C.S.*)

6. Explain the possible modes of vibration of a gas in open and closed pipes.

What is the effect of a change of temperature on the note from a metal organ-pipe?

If the frequency of the note is to be independent of the temperature, find the ratio of the cubical expansions of the gas and the metal. (*O.*)

CHAPTER VIII

ANALYSIS, RECORDING, AND REPRODUCTION OF SOUND

66. Analysis by Ear and with Resonators

It was stated in §§ 6, 7 that the differences of quality between the sounds produced by different musical instruments are caused by differences in wave-form, or, in other words, that the intensities of the various partials present are different. By "analysis" of a sound we mean the determination of the frequencies actually present, preferably with an estimate of their intensities.

It is frequently possible to detect some of the partials present in a musical note by ear alone, especially if the observer knows what to listen for. For example, if the note c'' is sounded on a clarinet, followed by f , the c'' can be heard clearly continuing with the f . It is the third harmonic of f , and is very powerful in the clarinet. When the frequency separation is less, or the harmonic less intense, it is more difficult to distinguish the partials with certainty with the unaided ear.

The ear may be helped by means of a resonator, which will "magnify" a certain frequency selectively. The common type of resonator is that designed by Helmholtz (Fig. 57), consisting of a hollow bulb of metal or glass with a narrow aperture A closed by insertion into the ear, and a small open neck B . The air in the neck is capable of performing vibrations similar to those of the body attached to the helical spring considered in § 5, the air in the bulb taking the place of the spring. The natural frequency of oscillation depends upon the volume of the bulb and the length and area of cross-section of the neck. In fact this frequency n is given by

$$n = \frac{c}{2\pi} \sqrt{\frac{S}{lV}}$$

where c = velocity of sound in air

V = volume of bulb

S = area of cross-section of the neck

l = effective length of the neck, which is somewhat greater than its actual length

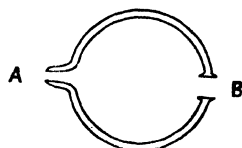


FIG. 57.

This expression may be obtained as follows :-

Let ρ be the density of air, and E its bulk modulus of elasticity.

Mass of air in the neck = ρSl .

Suppose that this air is displaced outwards by a distance Δx , which will involve an increase in volume ΔV of the air in the bulb given by $\Delta V = S\Delta x$.

This will cause a pressure decrease - Δp given by the expression

$$E = - V \frac{\Delta p}{\Delta V}; \text{ so that } - \Delta p = \frac{ES\Delta x}{V}.$$

The "plug" of air in the neck is thus acted upon by a resultant force inwards equal to $S\Delta p = \frac{ES^2\Delta x}{V}$.

Neglecting viscosity effects, this will give it an acceleration inwards equal to $\frac{ES^2 \cdot \Delta x}{\rho SlV}$.

Hence the equation of motion is $\ddot{x} + \frac{ESx}{\rho lV} = 0$.

The motion of the air in the neck is therefore Simple Harmonic, the period being $2\pi\sqrt{\frac{\rho lV}{ES}}$, and the frequency $n = \frac{1}{2\pi}\sqrt{\frac{ES}{\rho lV}}$.

Since the velocity c of sound in the air in the bulb is $\sqrt{\frac{E}{\rho}}$, this expression may be rewritten as

$$n = \frac{c}{2\pi}\sqrt{\frac{S}{lV}}.$$

If a set of such resonators is available, it is possible to pick out the partials present in a sound and to make a rough estimate of their intensities.

An example of the use of a tuning-fork as a resonator for analysis purposes has already been given in connection with the detection of combination tones (§ 18).

67. The Hot-wire Microphone

The estimate of the intensity of a partial can be improved so as to become a measurement by using this instrument, which is based upon the following principle.

A fine platinum wire having a steady electric current passing through it will reach a steady temperature. If a sound wave passes over it the "alternating draught" will cool the wire slightly, reducing its electrical resistance and causing an out-of-balance current in a suitable Wheatstone network. By mounting the microphone in a resonator tuned to a particular frequency, the intensities of notes of that frequency can be compared. If the frequency is that of one of the partials of a sounding note, the intensity of that partial can be measured. The apparatus is

also used in the location of guns by time delay, and in measurements of the velocity of sound (§ 32).

The platinum wire has to be exceedingly thin. It is known as Wollaston wire, made in the first instance of a platinum core with a silver sheath. This is drawn out to the correct fineness, and the silver sheath is later dissolved away, leaving a platinum wire so fine that it is maintained just below red-heat when a current of 30 milliamperes passes through it. The length used has a resistance of about 300 ohms. It is mounted on a glass rod (Fig. 58), which bridges a circular hole in a disc of mica. The ends of the wire are soldered to rings of silver foil, one on each side of the mica.

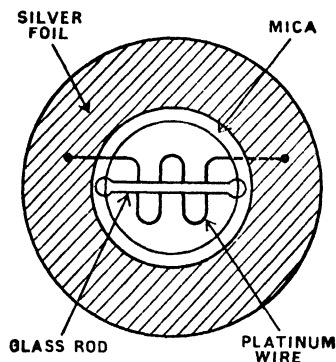


FIG. 58.

The wire in its mounting is placed in the neck of the resonator, which may be similar to the Helmholtz type except that there is no aperture A (Fig. 57). The whole may be arranged to form the stopped end of a tube of the correct

length, giving an additional degree of selectivity. ✓

68. The Phonodeik

The phonodeik of D. C. Miller was one of the first successful instruments which aim at depicting the wave-form of a given sound, i.e. the way the pressure varies with time at a given point. If the curve representing this variation is obtained by experiment, the intensities of the various components present in it can be found by the process of Fourier analysis.

In the phonodeik (Fig. 59) the horn A collects the sound and concentrates it upon the very fine glass diaphragm *d*. A fine wire is fixed to the centre of this diaphragm, and passes round a vertical steel spindle bearing a tiny mirror *m*, the wire being held taut by the spring *s*. Any minute movement of the diaphragm, caused by the sound, pulls or releases the wire, giving rise to rotation of the spindle and mirror. This reflects a beam of light, with the result that the spot falling on a distant screen performs a horizontal oscillation when

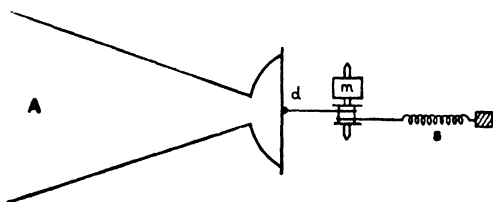


FIG. 59.

the diaphragm is disturbed. In Miller's arrangement the spot fell on a film strip moving vertically, so that the film gave a magnified record of the displacement-time curve of the diaphragm. With this apparatus a very large number of curves produced by the sounds of musical instruments, and of the voice, have been obtained.

It is not difficult, even if somewhat laborious, to analyse such curves so as to find the intensities of the different harmonic components present, making use of Fourier's Theorem.

Any function of period 2π can be expressed as a sum of terms $\Sigma a_k \sin k\theta$ for integral values of k , plus $\Sigma b_k \cos k\theta$ for $k = 0, 1, 2 \dots$

These various terms are the harmonic partials in the vibration, and the coefficients a_1, a_2 , etc., and b_1, b_2 , etc., are related to the amplitudes of these partials. The coefficients can be found as follows:—

Let the periodic function

$$f(\theta) = \sum_1^{\infty} a_k \sin k\theta + \sum_0^{\infty} b_k \cos k\theta.$$

Then
$$\int_0^{2\pi} f(\theta) \sin m\theta d\theta = \pi a_m,$$

since
$$\int_0^{2\pi} \sin k\theta \sin m\theta d\theta = 0 \quad \dots \dots \dots (1)$$

unless $k = m$, when it equals π ,

and
$$\int_0^{2\pi} \cos k\theta \sin m\theta d\theta = 0 \text{ always.}$$

Similarly
$$\int_0^{2\pi} f(\theta) \cos m\theta d\theta = \pi b_m \quad (m \neq 0),$$

since
$$\int_0^{2\pi} \cos k\theta \cos m\theta d\theta = 0 \quad \dots \dots \dots (2)$$

unless $k = m$, when it equals π ,

and
$$\int_0^{2\pi} \sin k\theta \cos m\theta d\theta = 0 \text{ always.}$$

Also,
$$\int_0^{2\pi} f(\theta) d\theta = 2\pi b_0.$$

Thus, if we multiply the ordinates of a periodic curve of frequency n by the corresponding ordinates of a sine curve of the same frequency and integrate over a cycle, the result is a number equal to πa_1 . If we repeat this with the ordinates of a cosine curve of frequency n instead of the sine, the process gives us a number equal to πb_1 . Then, taking $\sqrt{a_1^2 + b_1^2}$, we have a number proportional to the amplitude of the partial of frequency n (that is, the fundamental) actually present in the vibration, while the relative values of a_1 and b_1 tell us the phase of this partial. Any other harmonics of frequencies $2n, 3n$, etc., will not contribute to the number so obtained because the integrals of their products with the sine and cosine ordinates are all zero, as stated in (1) and (2) above.

Repetition of the process, multiplying the ordinates of the original curve by those of a sine and cosine curve of frequency $2n$ and integrating over the cycle in each case (time $1/n$), will give a_2 and b_2 , and hence the relative amplitude and phase of the second harmonic present. In this way the amplitudes and phases of all the partials can be found: the whole process is carried out mechanically with an integrating machine known as a "harmonic analyser," for otherwise the arithmetical labour would be prodigious.

The real difficulty with the phonodeik and similar instruments is that the curves obtained are those of the response of the system, which involves the resonances of the horn, diaphragm, and other parts. Miller used several different horns, and diaphragms of different sizes and thicknesses, and also "calibrated" his apparatus with a special set of organ pipes. The work on loudness (§ 80, 81) had not been done at that time (1916), and Miller's method was to have a set of pipes voiced by an expert until they were judged to be of equal loudness. The amplitudes which they registered on the phonodeik were measured: this revealed the resonances in the apparatus, and enabled the estimates of amplitudes of different harmonics to be corrected to this loudness scale.

69. Microphones

A microphone (other than the hot-wire type, § 67) is a device for producing an electric current which varies with time in the same way as the pressure or displacement in the sound wave falling on it. Apart from its importance for reproduction of sound, as in telephones and broadcasting, and in recording, the microphone facilitates analysis, as the wave-form of the current obtained can be determined by various methods. Like the horn and diaphragm of the phonodeik, however, the moving and electrical parts of the microphone will impress their own characteristics on the resulting current, so that the instrument needs to be calibrated.

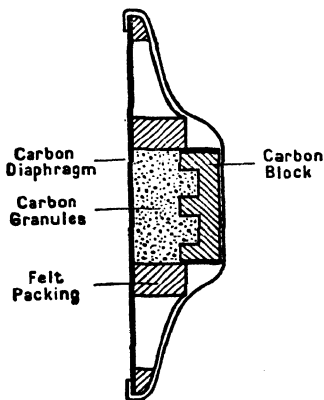


FIG. 60.

Carbon microphone. In the microphone used for ordinary telephone systems, carbon granules are confined between a carbon diaphragm and a carbon block (Fig. 60). When a sound wave falls on the diaphragm so that the latter vibrates, a varying pressure is exerted on the granules, which causes the electrical resistance between diaphragm and block to vary. It is a kind of "faulty contact" effect. The microphone is

Basically the amplifier depends upon the triode valve. If this is connected in the circuit of Fig. 64*a*, so that various potentials can be applied to a perforated electrode (the "grid"), the variations of current in the anode circuit can be observed upon the millimeter MA. The form of the variation of anode current with grid potential is shown in the curve of Fig. 64*b*, in which the central portion is reasonably straight though the ends are curved.

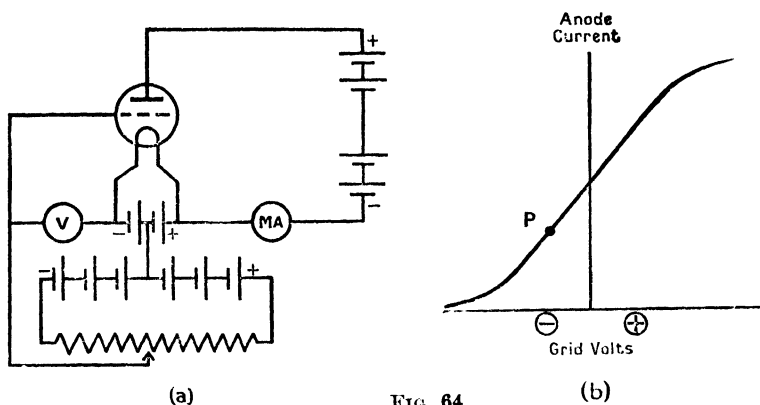


FIG. 64.

If small alternating voltages are applied to the grid of such a valve, about the steady potential represented by the point P, the anode current will have a corresponding alternating component superimposed upon the steady current also indicated by P. P must, of course, be upon the straight part of the line, for otherwise the wave-form will be distorted: in addition P must be at a negative value of grid potential,

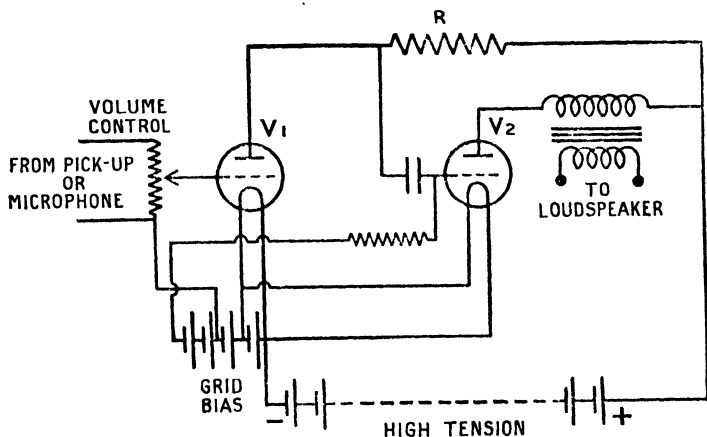


FIG. 65.

for otherwise currents may flow in the grid circuit for part of the cycle, which is likely to distort the wave-form at the input end.

A simple two-stage battery-operated amplifier is shown in Fig. 65. Provided that the anode current-grid volts characteristic of the valve V1 is straight over the region used, the varying part of the potential difference developed across R will be proportional to that originally supplied by the microphone. Similarly, if the working characteristic of V2 is straight it will not distort the wave-form.

As the final output valve has to deal with considerable amounts of power a fair length of its characteristic will be used : a certain amount of curvature of the line may be involved, which will cause the production in the final current of harmonics and combination tones that were not present in the original. This is avoided to some extent in the system known as "push-pull," in which two matched output valves are employed. The current from the penultimate stage passes into a transformer T1 of which the secondary has a tapping at the centre. The potential difference across one half of this secondary is applied between grid and filament of one of the pair of valves, and that across the other half to the other valve (Fig. 66). Thus, when the grid of V1

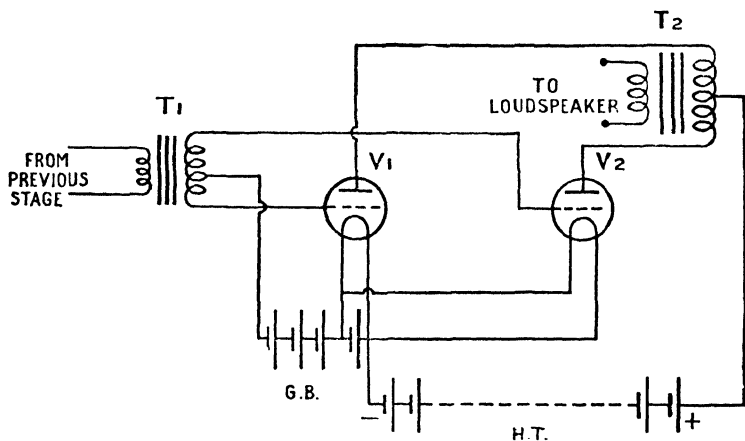


FIG. 66.

falls in potential, that of V2 rises by the corresponding amount, and vice versa. The changes of potential may cause quite unequal changes of anode current in the two valves if the characteristics are curved : but the flux change in the core of the transformer T2 is the result of their *summational* effect. On the reverse half of the cycle, the grid of V1 rises in potential, and that of V2 falls, so that the summational effect is the same as before if the valves are matched. The result, therefore, is that the wave-form of the flux change in T2 is symmetrical,

if the input is symmetrical. Such a symmetrical form is devoid of even harmonics. What it comes to is that the even harmonics that would be produced if V1 and V2 were used separately become subtracted from each other (and therefore eliminated entirely if the valves are matched), while the odd harmonics (including the fundamental as principal) become added together. If the individual characteristics are parabolic over the range used each valve alone would produce only a second harmonic in addition to the fundamental, so that the pair then reproduce without distortion at all. In triode valves the characteristic often does approximate to the parabolic form over a wide range, so that the push-pull system is certainly worth applying in such a case. Pentode output valves, on the other hand, usually add a considerable proportion of harmonics higher than the second, leaving a percentage of odd multiples even in the push-pull arrangement.

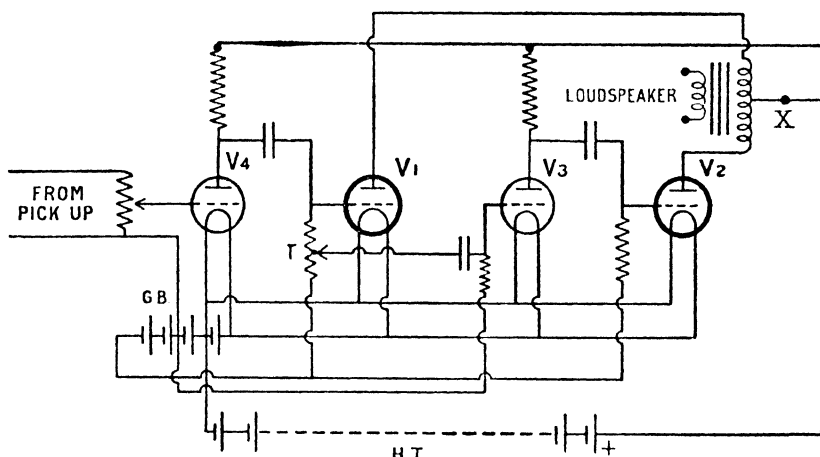


FIG. 67.

The transformer T1 may cause an uneven frequency response and its use is avoided in the "paraphase" amplifier shown in Fig. 67. The valves V1 and V2 are the matched output valves as before. V4 is a previous stage, and V3 is a triode which performs a phase reversal. The output from V4 actuates V1 in the usual way, and part of the p.d. existing between the grid and filament of V1 is led to the grid of V3 and amplified by it, and then handed on to V2. A momentary *increase* of potential at T causes a similar rise on the grid of V3, a rise in its anode current and a fall in its anode potential. This in turn gives a *decrease* in the potential of the grid of the valve V2. By adjusting the tapping point T in relation to the amplification produced by V3 the negative change on the grid of V2 may be made equal to the positive change on the grid of V1, which is the condition for push-pull working. The correct tapping point may be found experimentally by inserting a telephone in the common anode lead at X, while a varying voltage, preferably corresponding to a steady pure tone, is fed

into the commencement of the amplifier. When T is in the correct position there will be a minimum of sound in the telephone since the rises in anode current of one valve will almost equal the falls in the other. It is interesting to note that the *difference* of the changes of anode currents, reproduced by the telephone at X, contains the difference of the odd harmonics given by the two valves and the sum of the even harmonics: whereas the effective *sum* of the changes, reflected as flux changes in the output transformer, contains exactly the reverse. The arrangement is also an illustration of a method by which the frequency of an alternating current can be doubled.

The circuit diagrams given show battery valves with directly heated filaments: in practice valves with cathodes indirectly heated by alternating current would probably be used, while the potential differences necessary for anode and grid circuits would also normally be obtained from A.C. mains by rectification and smoothing, with decoupling arrangements to prevent the output current from influencing earlier stages.

71. Analysis by Electrical Methods

One instrument suitable for depicting the way in which a potential difference varies with time is the *cathode-ray oscillograph*. In this tube a narrow beam of electrons, emitted by a heated filament or cathode F (Fig. 68) and accelerated by a high potential applied to the anodes A_1 and A_2 , passes through the spaces between two pairs of plates XX, YY,

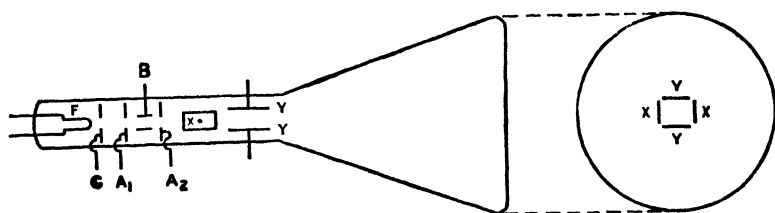


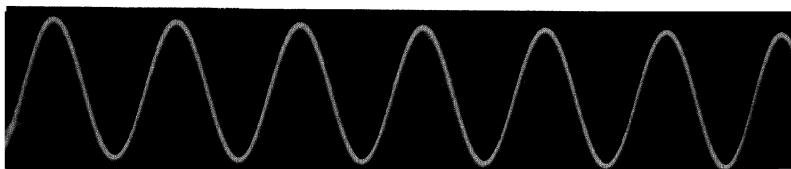
FIG. 68.

YY. The beam strikes a sensitive screen at the end of the tube, producing a bright spot. The spot may be focussed and its brightness controlled by applying suitable potentials to electrodes B and C between the filament and anodes, or by using a magnetic field.

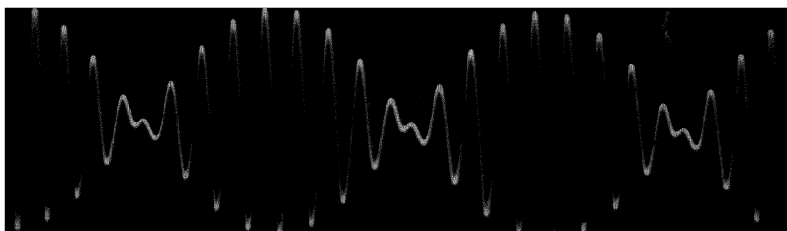
If a potential difference is applied between the X plates the electrostatic field so produced causes the beam to deflect to right or left, while a similar application to the Y-plates will cause the beam and the spot to move up or down. If a sinusoidal p.d. is applied to the Y-plates, and a steady p.d. to the X plates, the spot will describe a vertical line. If, however, the p.d. on the X-plates is made to decrease uniformly with time, this will cause the spot to travel across the field, so that it will describe a sine curve. Further, if the X p.d. can be suddenly

PLATE IV

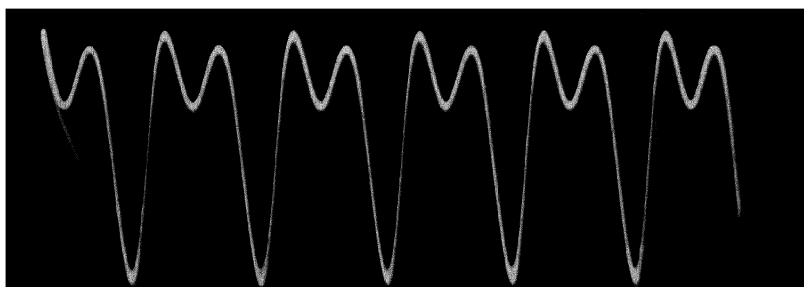
OSCILLOGRAMS



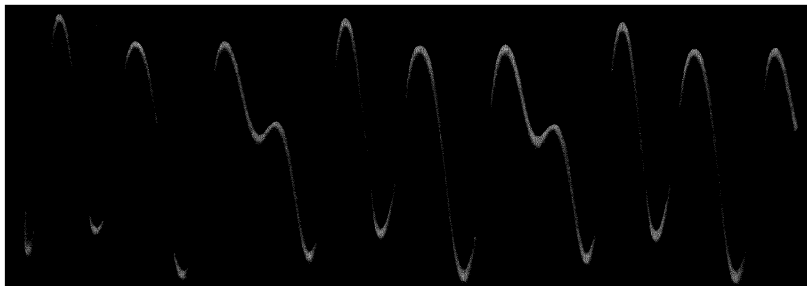
TUNING FORK. g' 391 cycles per second.



TWO BEATING TUNING FORKS. a' , 440 c.p.s., and g' , 391 c.p.s.
(Frequency ratio $9/8$)

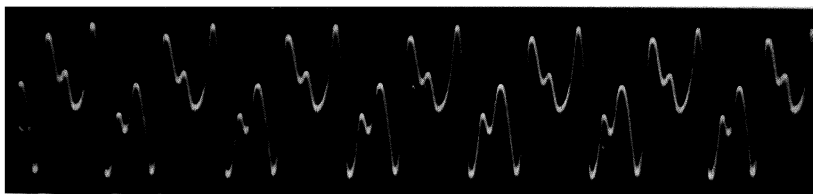


OBOE. Six vibrations of the note d'' 587 c.p.s.

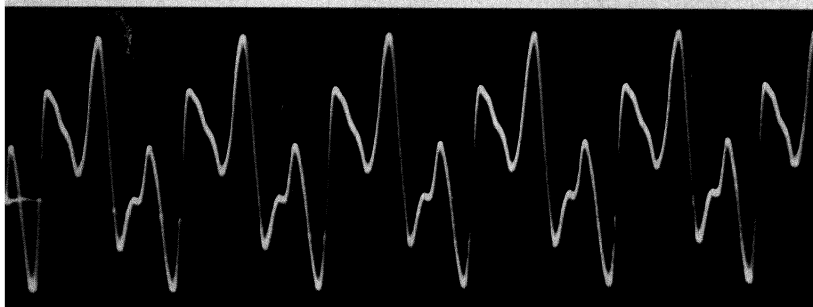


OBOE. Three vibrations of the note d' , 293.5 c.p.s.

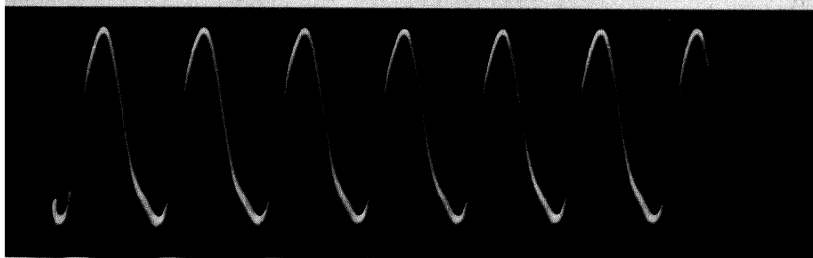
OSCILLOGRAMS



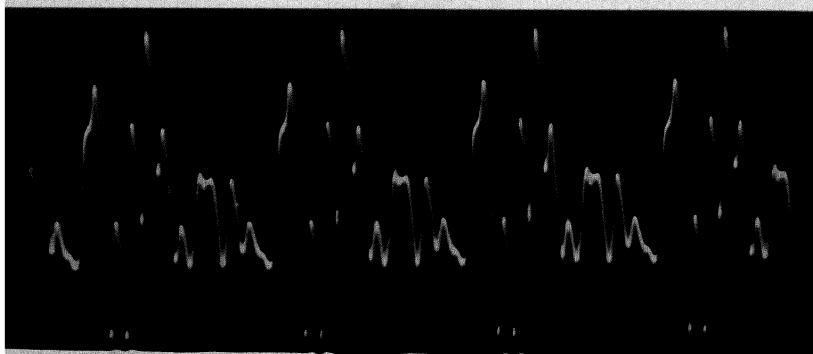
CLARINET. Seven vibrations of the note g, 196 c.p.s.



CLARINET. Six vibrations of the note e, 165 c.p.s.



HORN. Seven vibrations of the note c', 262 c.p.s.



BASSOON. Four vibrations of the note E flat, 78 c.p.s.

jerked back to the initial value after it has fallen sufficiently to take the spot across the screen, and then allowed to fall again, the curve will be traced again. If there is synchronism between the repetition of the X p.d. and an exact number of cycles of the sine p.d. applied to the Y-plates, the successive curves will fall in the same place on the screen, giving a stationary appearance. This can be photographed if desired.

Whether the p.d. applied to the Y-plates is sinusoidal or not, its actual wave-form will be revealed by the spot on the screen: a photograph can be subjected to the usual process of harmonic analysis. The p.d. for the Y-plates, for this purpose, may be obtained with any high quality microphone (§ 69) and amplifier (§ 70). The p.d. for the X-plates, known as a "time-base" voltage, may be obtained by charging a condenser through a pentode valve, and discharging it automatically at suitable time intervals. The wave-form curves, usually called *oscillograms*, shown in Plates IV and V, were obtained by photographing the traces obtained on the screen of a cathode-ray tube by this method. The same apparatus is obviously applicable for testing the fidelity of an amplifier by examining the wave-form of the potential difference at different stages.

An alternative method of analysing the partials present in an electric current is to inject it with another current into a suitable "mixing" valve, which will form combination tones with the various frequencies present. The frequencies necessary for the injected tone in order to give a summational or differential tone of known frequency are measured. The intensity of the combination tone, measurable by using a tuned amplifier and valve voltmeter, is proportional to the intensity of the partial concerned in its production.

72. Disc Recording and Reproduction

The recording of a sound consists, broadly, in "storing" the wave-form for subsequent reproduction. There are various methods for doing this, one of them involving the use of a fine groove in a cylinder or disc of wax. The wave-form may be represented by the variation in the depth of the groove, or its undulation from side to side, which is the basis of gramophone recording. The first recordings were made upon wax cylinders, the groove being of variable depth ("hill and dale" recording). This was soon replaced by the flat disc and spiral groove of constant depth, with the wave-form represented as a transverse undulation ("lateral cut"). This is the system commonly used in the commercial gramophone record of to-day.

In making a record, a disc of specially prepared soft wax is rotated at constant speed, while a sharp cutting stylus is impressed upon it, the stylus being slowly moved radially inwards by means of a system of gears. The stylus is made to vibrate transversely in relation to

the general direction of the groove, with the result that the latter has similar displacements. The amplitude of these displacements is always very small—certainly never more than a hundredth of an inch. In the old “acoustic recording,” used until 1924, the sound to be recorded was collected by a horn, and caused a diaphragm to vibrate, this motion being communicated in due course to the stylus. Such an arrangement had many defects: horn and diaphragm resonances were prominent, quite apart from the fact that it was not always easy to collect enough sound energy to cause adequate displacements of the cutting point. With the coming of high-quality microphones and amplifiers this method has been superseded by “electrical recording.” The sound is collected by a microphone and thus converted into a similar alternating current: after amplification this actuates a moving-coil or other reproducer bearing the stylus, which is usually a sapphire point lapped to a chisel edge. The electrical circuits make it possible to “even out” many of the resonances that do not yield to direct treatment: the balance can be adjusted as regards high or low notes, or soloist and accompaniment: the general intensity, reflected as the amplitude of the groove, can be adjusted to the optimum value. Several waxes can be cut at the same time, so that one can be used for an immediate “play-back”: if this is satisfactory the others are processed.

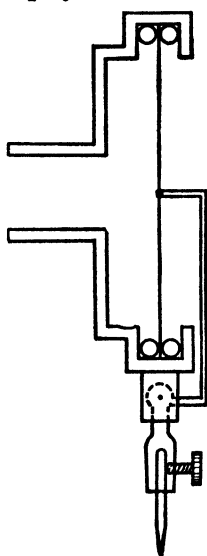


FIG. 69.

Processing consists first in dusting the surface of the recorded wax with very fine bronze powder, followed by polishing. Copper is then deposited electrolytically upon this surface, until the grooves and face are covered to a depth of a few thousandths of an inch. This “negative” is peeled off, and a further deposit of copper made on the side of it formerly in contact with the wax: this is again peeled off, giving a working “positive.” The same process repeated gives another “negative,” which, after suitable mounting, is used for making the impression upon the shellac disc that forms the final “record.” This last process is performed under hydraulic pressure with the record material hot: on cooling we have a hard, very smooth disc with a groove of the same form as the original cut wax.

Prior to 1925 the sound was reproduced from the record using a “sound-box” (Fig. 69). This consists of a metal or mica diaphragm, closing the aperture of a cylindrical box which communicates with a horn. Attached to the centre of the diaphragm is a lever pivoted at the bottom of the sound-box, and clamped in the shorter arm of this lever is a renewable needle. The sound-box

is mounted upon a "tone-arm," which can revolve about a point at some distance from the centre of the revolving record, so that the needle can follow the groove with little side thrust.

When the record is rotating with the needle in the groove, the point is forced to move from side to side according to the wave-form. The centre of the diaphragm is compelled to do likewise, its edges remaining almost still, as the sound-box as a whole has too much inertia for it to perform any considerable vibration. Like the acoustic recorder, this system will impress resonances of diaphragm and horn upon the wave-form supplied to it: in addition, the energy it can deliver and therefore the loudness of the sound are limited by the amplitude of the wave on the record. A short, thick needle will produce a louder sound than a longer or thinner one since the diaphragm movement will be greater on account of the length ratio of the lever, and because the thick needle will bend less under the stresses involved. The thick needle, however, subjects the walls of the groove to greater forces and is therefore likely to increase wear. The purpose of the horn is chiefly to enable the diaphragm to disturb a larger mass of air, which "loads" it more efficiently.

In 1925 the electrical method was applied to the reproducing system, replacing the sound-box by a "pick-up," which is effectively a microphone actuated mechanically by the movement of the needle instead of by the varying pressure of sound waves. The common types of pick-up are the magnetic and the crystal types, though the moving-coil principle has also been applied. In the ordinary magnetic pick-up a small horse-shoe magnet with four pole-pieces N1, N2, S1, S2 is used, with a fixed coil CC placed as shown in Fig. 70. An iron armature (which may be the needle itself), is pivoted and suitably damped by rubber pads. With the needle in the position shown, lines of induction pass through the armature from N1 to S2, threading the coil in the process. When the needle has moved over to the right the armature "leads" the lines of induction from N2 to S1, so that they pass through the coil in the opposite direction. As the needle moves from side to side with the groove there are therefore flux changes through the coil and an E.M.F. induced in it. This E.M.F. given by the pick-up is amplified with a valve system, and eventually reproduced by the loudspeaker.

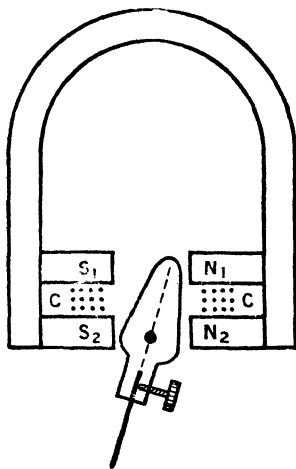


FIG. 70.

The crystal pick-up, like the crystal microphone, uses the piezo-electric effect (§ 95) in Rochelle salt. The crystal is strained, usually torsionally, by the movement of the needle, and the corresponding E.M.F. obtained. As a rule the crystal pick-up is light in weight and effective in output.

Owing to the small space between adjacent grooves of a record the amplitude which they are given cannot be large. This means, in fact, that bass notes cannot be recorded in their full intensity, since the amplitude required for such sounds is far greater than that for sounds of higher pitches to which the ear is more sensitive (§ 80). This weakening, however, can be compensated to some extent in the pick-up, amplifier, and loudspeaker, which is an advantage of electrical reproduction over the older acoustic method. The actual volume is also under control, since the amplification can be varied: this makes it possible to use needles that are naturally "soft," in order to reduce record wear, without the disadvantage of faintness of sound.

73. Steel-tape Recording

In this system the wave-form is represented by the variation in magnetisation of steel. The procedure is, first, to magnetise a steel tape by drawing it along the pole of an electro-magnet, known as a wiping head. The tape is then drawn at a speed of about 90 metres per minute past a "recording head," consisting of an electro-magnet with knife-edge poles, actuated by the varying current from an amplifier. This varies the magnetisation of the tape in accordance with the output of the amplifier, which corresponds to the output of the microphone affected by the sound wave.

The reproducing head consists of a single pole piece with a coil wound round it, the tape being drawn past it at the same speed used in recording. The variations in magnetic flux give an induced E.M.F. which will correspond in wave-form to the original sound, if all stages of the process are free from distortion. This E.M.F. is amplified in the usual way.

The method has the advantage that long programmes can be recorded without interruption, and the record can be replayed at once without any processing. The tape can be "wiped clean" and used for other recordings if the first is not required permanently. More recently a plastic tape impregnated with a magnetic oxide of iron has been used, in the German "Magnetophon."

74. Film Recording and Reproduction

In this method of recording, the stored wave-form is represented by the variation in transparency of the film.

In a sound film of the standard 35 mm. size, the "sound track" is

Allotted a space about 2.5 mm. wide. In the "variable density" method of recording the full width of this space is in use all the time, but with different degrees of density of image: the track might be said to consist of a series of transverse parallel "lines" of different opacities. These lines are in fact images of a narrow slit, illuminated by a source which is varied in intensity by an amplified microphone current.

In a former method of recording this amplified current was applied directly to an electric lamp. If this is of the gas discharge type it will respond to the changes of the current with changes in brightness, even if the current varies with a frequency of some hundreds of cycles per second. The lamp illuminates a slit, the image of which is cast on the moving film. On development, the film will be most opaque in places representing the instants when the lamp was brightest, and its actuating current greatest. This situation will be reversed when a "positive" is made from the film for reproduction purposes. Thus, if light is shone through the sound track as it passes by a narrow slit, the light transmitted will follow the same time variation as the light emitted by the recording lamp.

In more modern methods the amplified microphone current is used to vary the width of a narrow slit, using an arrangement not unlike the moving-coil loudspeaker. Light from a powerful steady source passes through this slit, and through another slit of which an image is cast on the moving film. The varying slit width modulates the brightness of the transmitted light beam, and hence the opacity of each particular area of track.

In the "variable width" method of recording some part of the width of the sound track at any place is left transparent, while the rest is quite opaque. The extent of the transparent part (opaque in the original negative) is proportional to the intensity of the amplified microphone current. The record is produced by illuminating a variable length of the slit of which the record is an image. This may be done by attaching a mirror to a wire loop carrying the current and situated in a magnetic field, and reflecting light from it on to the slit. The mirror rotates under the effect of the currents in the wire loop and thus illuminates a variable length of slit.

For either method of recording the reproduction method follows the same lines. Light from an "exciter" lamp passes through a narrow slit, of which an image is produced in the plane of the sound track by the lenses L1 (Fig. 71). The light passing through the track is collected by L2 and falls upon the sensitive surface of a photoelectric cell, causing the emission of electrons and a small circuit current (Fig. 72). A potential difference is developed across the high resistance R, proportional to the light received by the cell, and this is led to the first valve of an amplifier. As the light passed by the film follows the wave-form

of the original speech currents, the final output of the amplifier has the same time-variation. Failure of proportionality at any stage of the process will cause distortion of the wave-form and therefore give rise to harmonics and combination tones not present in the original sound.

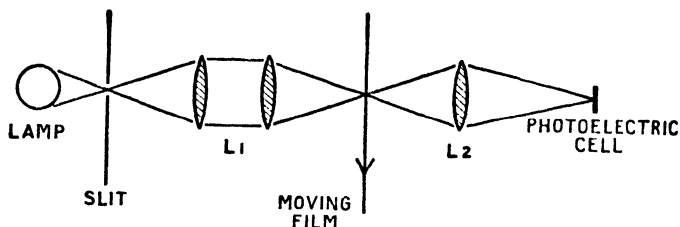


FIG. 71.

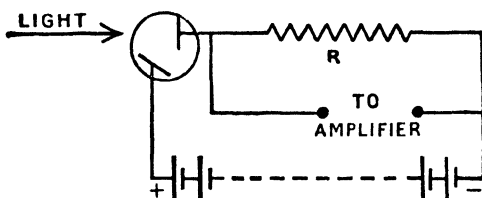


FIG. 72.

The sound track has to pass by the image of the slit in the reproducer at a uniform speed, whereas the "vision" is produced by the film moving through the projector in a series of jerks. There must therefore be a suitable loop between the projector and the sound gate, and this must be allowed for in the original recording so that sound and vision are eventually properly synchronised.

75. Loudspeakers

The last link in the chain of reproduction by an electrical method lies in the conversion of the varying currents into corresponding mechanical movements and eventually into sound waves.

In the *telephone* reproducer or ear-piece (Fig. 73), this is accomplished by using a small permanent magnet with a thin soft-iron (stalloy) diaphragm mounted at a small clearance from its poles. The magnet has a coil wound round it, and the varying or alternating current is passed through this coil. If the current, at one instant, flows in such a direction that the effect of the coil is to strengthen the magnet, the diaphragm is attracted to the poles more forcibly than without the current: if the current is in the reverse direction the attraction will be lessened and the diaphragm less "bent in" than normal. Thus, if an alternating current is passed, the diaphragm moves in and out in a way

depending on the frequency and wave-form of the current. A varying direct current (in the correct direction) would be equally effective. It should be noted that, with A.C. at least, it is necessary for NS to be a permanent magnet upon which the variations of strength are superposed: a soft-iron core would give attractions on both halves of the cycle and therefore reproduce all notes an octave too high.

Though adequate for reproduction of the voice when used close to the ear, the telephone is not suitable if the amplitude required is at all large: as the diaphragm gets closer to the poles the attractive forces are increased, so that the response becomes non-linear. The resonant frequencies of the diaphragm also render the response very uneven.

For *loudspeakers*, therefore, a means is used whereby larger amplitudes can be produced without distortion. This is especially necessary for reproduction of bass notes. The method used is the moving-coil principle, alluded to in § 1 and in § 69. A magnet, either permanent or excited by a current in a coil, is made in the shape shown (Fig. 63, p. 120), with a central pole N, and the opposing pole S upon a surrounding ring. The lines of force radiate in the air gap, in which lies the circular coil. In the usual form the coil is attached to the apex of a large cone of stiff paper, flexibly mounted around the edges (Fig. 1, p. 2). The passage of a current through the coil causes it to move at right angles to the lines of force—that is, in or out along the axis of the cone. An alternating current therefore causes the diaphragm to vibrate in a manner similar to the wave-form of the current. To render this device at all efficient, the diaphragm must be large, and a rigid “baffle-board” must be provided to prevent the waves from the back of the diaphragm cancelling out with those from the front.

Alternatively, a small relatively rigid diaphragm may be used and made to agitate a larger mass of air by mounting it at the narrow end of a horn.

Loudspeakers based upon the piezo-electric effect (§ 95) have also been used, principally for the reproduction of very high frequencies.

76. Electronic Organs

The valve amplifier and loudspeaker make possible the production of sounds of any desired pitch, quality and intensity if small alternating voltages of the correct frequency and wave-form are available. A number of musical instruments using this principle have been made, the Hammond organ and the Compton Electrone being among them.

In the Hammond the basic alternating voltages are obtained by electro-magnetic induction, using a series of 91 tone generators driven by a constant

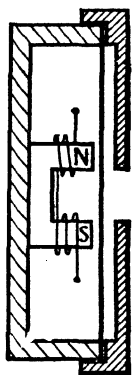


FIG. 73.

speed motor. Each generator consists of a "tone-wheel"—a plate in the shape of a regular polygon with the corners rounded off—which rotates in front of the pole of a magnet upon which a coil is wound (Fig. 74). The changes of magnetic flux induce in the coil an E.M.F. which is of the pure

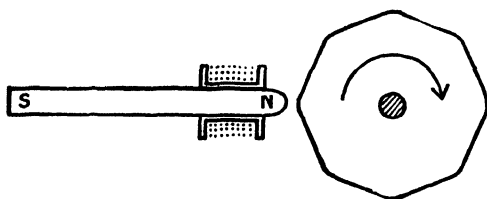


Fig. 74.

sine form. The number of sides on the polygons, and their speeds of rotation, are selected so that the 91 give all the frequencies necessary either as fundamentals or as upper partials for building up various qualities of tone.

The console from which the instrument is played has two manuals and a

pedal board. Each of the manuals is provided with twelve additional keys (these are replaced by pistons in some models), which serve the purpose of stops in that they settle the tone quality obtained from that manual. Nine of these keys produce pre-set mixtures of harmonics, one is a cancelling key, while the other two produce mixtures that can be selected by the player at will. The selection is carried out by adjusting draw-bars above the manual: the draw-bars give for each playing key the following nine frequencies: sub-fundamental, sub-third harmonic, fundamental, second harmonic, third harmonic, fourth harmonic, fifth harmonic, sixth harmonic, and eighth harmonic, with eight possible degrees of intensity for each. The use of the sub-fundamental and sub-third harmonic corresponds to the use of a 16-foot stop on a pipe organ. The combined voltage selected either by a pre-set key or the draw-bars is fed to an amplifier, there being a general volume control actuated by a swell pedal. The amplified current is rendered as a sound by one or more loudspeakers. In the Hammond the "harmonics" used are also the fundamentals of appropriate notes of the scale: as these are "tempered" (§ 50), except in the case of octaves, the "harmonics" are not quite exact multiples of the fundamental frequencies with which they are employed.

By using the adjustable draw-bars many interesting tone qualities can be synthesised. The makers suggest various imitative combinations by giving the intensities for the harmonics in the form of a code number—e.g., a concert flute is given as 00,6300,000 and a clarinet as 00,6070,330. The numbers refer to the intensities of the harmonics in the order given in the preceding paragraph, starting with the sub-fundamental and finishing with the eighth harmonic. It is interesting to note the small harmonic retinue in the flute imitation and the powerful third harmonic in that of the clarinet, which are well-known characteristics of these instruments.

Obviously the possibilities of an instrument such as the Hammond for creating new tone colours are boundless, but it does not give a completely satisfactory imitation of orchestral instruments, or indeed, of standard organ tone. The fact is that the harmonic texture in any orthodox instrument is different at different parts of the scale, depending upon the resonances or "formant" (§ 56) of the instrument. In the Hammond the harmonic texture selected is constant for all notes on that manual: an elaborate arrangement of filters would be necessary to imitate the resonances of instruments. A further criticism commonly made is that the sound is "too smooth" to be confused with the classical organ. This is

partly due to the fact that the "harmonics" used are actual tempered scale notes "borrowed" for the purpose: thus beating between "harmonics" of one note and the fundamental or octaves of another will not occur when a chord is sounded. An additional fact is that in a pipe organ many pipes are concerned in speaking each note when anything like the full organ is used: the phases of the sounds they give are unrelated, neither do the differences remain constant, as tuning is never exact. We may liken the source in the classical organ to a large string orchestra, while the sound of the Hammond would correspond to that of an amplified string quartet.

In the Compton Electrone the original alternating voltages are produced by electrostatic means—by varying the capacitance of a condenser. In a recent model formants are used instead of "all sine-wave" generators. There are twelve generators, each of which gives voltages corresponding to one note of the lowest chromatic scale and a large number of its exact frequency multiples, the twelve being driven by one belt passing over pulleys having diameters inversely proportional to the equal tempered ratios. Harmonics up to the thirtieth are used at present, but experiments are being made to go much higher in order to obtain greater diversity of registration and effects.

QUESTIONS

1. The note which is emitted when the cork is rapidly withdrawn from an empty bottle is due to the air in the neck acting as a piston to the air in the bottle itself, which contracts and expands adiabatically as a whole.

Find the note at N.T.P. emitted from a Winchester quart bottle ($2\frac{1}{2}$ litres), the length and diameter of whose neck are each 2 cm. (Density of air at N.T.P. is 1.293 gm./litre: 1 atmosphere = 10^6 dynes/cm.²: $\gamma = 1.41$.) (O.S.)

2. What is meant by the wavelength and frequency of a sound? Describe how you would measure them.

Why is it possible to distinguish between two musical instruments sounding notes of the same pitch? (O.P.)

3. Explain how you would analyse a note from a violin so as to find the amplitudes and frequencies of all its component pure tones. (C.S.)

4. Notes of the same frequency produced by different musical instruments are said to be of different quality. Give a physical description of this difference and say how it could be investigated experimentally. (O.S.)

5. What is meant by "progressive" and "standing" waves? Give examples from mechanics and from sound of the production of standing waves.

Derive a relation between the frequency of standing waves in a stretched string, the velocity of propagation, and the length of the string.

Suggest how the tone of a violin may be reproduced on a cinema organ. (O.S.)

6. Write a short essay on the physics of musical sounds. (Trip.)

CHAPTER IX

INTENSITY, LOUDNESS AND HEARING

77. Measurement of Intensity

THE intensity of a sound wave is defined as the average rate of flow of energy per unit area normal to the direction of propagation. For plane waves this is equal to the product of the mean energy density, and the velocity of sound. The mean energy density depends upon the amplitude and the frequency,¹ so that in order to measure intensity we have to measure amplitude in some form : displacement amplitude, pressure amplitude, or velocity amplitude.

Measurement of these quantities is difficult for two main reasons. Firstly, the presence of a receiver of some kind alters the quantity that is being measured, unless the receiver is small compared with the wavelength. Secondly, the energy involved is very small, varying from about 10^{-16} watts per sq. cm. for the feeblest audible sounds to about 10^{-6} watts per sq. cm. for a loud sound.

A method which surmounts the first difficulty neatly is the optical one originally used by Töpler and Boltzmann in 1870 to determine the pressure amplitude in a sounding organ pipe. The variations in air pressure in the stationary sound waves cause variations of refractive index, which are made to cause variations in an interference pattern. One of the interfering light beams passes transversely through the pipe, which has transparent walls; the other passes outside the pipe. Fringes are produced by recombining them suitably, and on sounding the pipe the fringes perform an oscillatory movement. The movement can be slowed down by using stroboscopic observation. Unfortunately only very powerful vibrations cause a sufficient change of refractive index to produce an observable effect.

A direct measurement of displacement amplitude in a powerful sound wave such as that existing in a Kundt's tube (§ 36) has been made by using smoke particles. With loud sounds the displacement amplitude may be as much as a tenth of a millimetre, so that the lines executed by the particles are easily visible with a microscope. No difference of amplitude is observed between large and small particles, so that it is likely that all take part fully in the motion of the air.

The main instrument used for absolute measurement is the Rayleigh disc; microphones of various kinds can be calibrated with its aid or in other ways, and used under circumstances in which the direct use of the disc would be impracticable.

¹ In § 5, the energy of a body of mass M moving in S.H.M. with amplitude a and frequency n was shown to be $2\pi^2 Mn^2 a^2$. This expression gives the energy density in a S.H. wave, M being the mass of unit volume of medium.

78. The Rayleigh Disc

A light disc, suspended in a stream of air, tends to set itself at right angles to the stream. The direction of flow is irrelevant, so that the alternating stream existing in a sound wave has the same effect as a steady stream of the same mean square velocity.

For a disc of radius r , in a gas of density ρ , moving with velocity V , the couple G is given by

$$G = \frac{4}{3} \rho r^3 V^2 \sin 2\theta,$$

where θ is the angle between the normal to the disc and the direction of flow.

In practice the disc is of mica, 1 cm. in radius, suspended by a quartz fibre 10 cm. long, and is arranged so that $\theta = 45^\circ$, thereby securing the maximum couple. The deflection of the disc from the 45° position is proportional to G , the constant of proportionality of the fibre being determined by previous experiments. The instrument is usually employed to measure the root-mean-square (R.M.S.) velocity at a suitable point in a tube arranged to be in resonance with a source of constant frequency.

79. Calibrated Microphones

A hot-wire microphone (§ 67) or condenser microphone (§ 69) can be exposed to sounds of which the R.M.S. velocities have been measured by the Rayleigh disc method. If this is done for various frequencies the microphone can be calibrated and used to measure sound intensity directly.

Calibration can also be carried out by means of the thermophone and the pistonphone. In the former, a thin platinum strip is used, and an alternating current, either alone or superimposed upon D.C., is passed through it. The strip, being of low thermal capacity, oscillates in temperature, causing calculable variations of the pressure of the surrounding air in a cavity. These variations are allowed to affect a microphone of which the voltage output is measured. In the pistonphone a piston moves in a small cylinder forming part of a cavity closed by the microphone diaphragm. The piston is moved mechanically or electrically with known amplitude, causing calculable pressure variations in the cavity.

80. Sensitiveness of the Ear to Intensity

It is well known that notes having a frequency greater than a certain figure—from 10,000 to 30,000 c.p.s., depending upon the person concerned—are not audible, neither are pure tones having less than a certain frequency. These limits of audibility are gradual in their

onset. The sensitivity of the ear, that is, the least audible intensity, varies greatly with frequency: at the other end of the scale, very large intensities cause a painful sensation, the intensity required again varying with frequency.

The results of experiments on this subject are expressed in the two curves of Fig. 75, the "threshold of audibility" (1) and the "threshold of

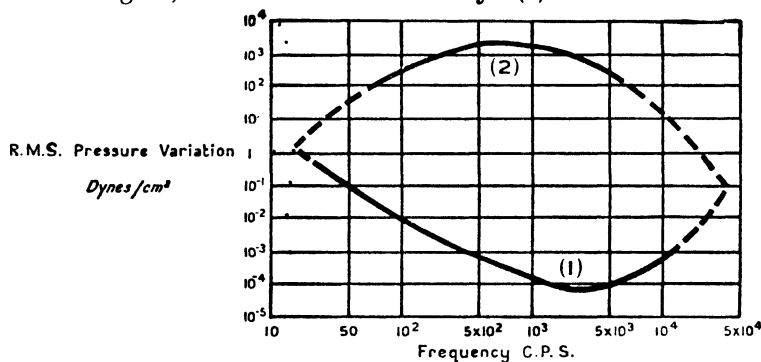


FIG. 75.

feeling" (2). To obtain the first curve by experiment, a sound of known frequency is made and adjusted in strength so as to be just audible, an average for several observers being taken. The intensity in the free field is then measured with a calibrated microphone. This is repeated for many frequencies. To obtain the second curve, the same procedure is followed, except that the intensity of the sound is increased until the observer reports a painful sensation.

The greatest sensitiveness of the ear occurs at a frequency of about 3,500 c.p.s. (nearly four octaves above middle C), when the root-mean-square pressure variation necessary just to cause the sensation of sound is approximately 8×10^{-5} dyne/cm.². Using the expression derived in § 10 this is found to correspond to a displacement amplitude of 1.25×10^{-10} cm., which is considerably less than the diameter of a hydrogen molecule.

At 1,000 c.p.s. the threshold of audibility occurs at an R.M.S. pressure variation of approximately .0002 dyne/cm.², corresponding to an intensity of 10^{-16} watts/cm.². This latter figure is an important one, being accepted as the standard "threshold" or "zero" from which other measurements are reckoned.

It will be seen from the curve that the sensitivity is much less at low frequencies—being about 1 dyne/cm.², equivalent to 2.5×10^{-9} watts/cm.², at 20 c.p.s. It is also seen that the threshold intensity rises again at very high frequencies, though not to the same extent as at low ones.

The intensity required for producing a painful sensation is greatest at about 800 c.p.s., where the threshold of audibility is still fairly low. At 1,000 c.p.s. the ratio of the two pressure amplitudes is about $10^7 : 1$, so that the ratio of the intensities is $10^{14} : 1$, a considerable range for the same receiving system to accommodate.

As the two curves approach at their ends they might eventually meet, as shown by the dotted lines. They would then enclose an area which would define the limits of audibility. Cases of deafness show a restriction of this area in different regions: detailed measurements of this kind are valuable for diagnosis and for the prescription of suitable hearing aids.

81. Intensity Level, Sensation Level, and Equivalent Loudness

As stated in § 77, intensity is measured in watts per square centimetre. Relative intensities often have more practical significance than the absolute value, and the *Intensity Level* of a sound is a way of stating the ratio of its intensity to a standard intensity.

The standard chosen is 10^{-16} watts/cm.², previously alluded to as the threshold of audibility for a note of frequency 1,000 c.p.s. A sound having 10 times this energy is said to have an intensity level of 1 *bel* or 10 *decibels*¹: one of 100 times the energy to be of 2 *bels* or 20 *decibels*. The scale is, in fact, logarithmic, so that

$$\text{Intensity level in decibels, (dB)} = 10 \log_{10} \frac{I}{I_0}$$

where I = intensity of the sound

I_0 = the standard intensity of 10^{-16} watts/cm.².

This definition applies no matter what the frequency of the sound may be.

The reason for using the logarithmic scale is twofold. Firstly, it is practically convenient on account of the immense range of intensities encountered in practice. The feeblest and loudest sounds differ by the immense factor of 10^{14} , though this is now represented by the modest figure of 140 dB. Further, the response of the ear, or "sensation" produced by the sound, roughly obeys a logarithmic law.

The connection between a physical stimulus and the corresponding response was first investigated by Weber. In experimenting with weights he came to the conclusion that the minimum increase ΔW which gave a just perceptible increase in the sensation of pressure was proportional to the weight W already in use. Many sensations, loudness and brightness among them, are roughly in accordance with this rule. For sounds of moderate intensity it is true to say that ΔI

¹ An alternative unit is the *neper*.

Intensity level in nepers = $\log_e (I/I_0)$ where e is the base of natural (Napierian) logarithms. Thus 1 neper = $\cdot 4343$ bel = 4.343 dB.

for a just perceptible increase in loudness lies between 5 and 15 per cent. of I , the figure depending on the frequency but being practically constant for any one frequency. If we adopt the logarithmic scale one sound will just be distinguishable in loudness from another of the same pitch if they differ by a fixed number of units on the scale—between 0.2 and 0.6 dB in fact, which correspond to increases of 5 and 15 per cent. respectively.

The statement that the sensation produced is proportional to the logarithm of the intensity is known as the Weber-Fechner law.

The threshold of audibility curve of Fig. 75, in which the R.M.S. pressure variation is plotted against the frequency, can be re-scaled to show how the *threshold intensity level* varies with the frequency as in

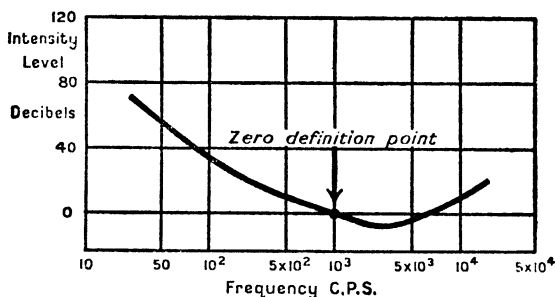


FIG. 76.

Fig. 76. The threshold level has the following values at selected frequencies :

Frequency c.p.s.	Threshold Intensity Level
50	54 dB
100	38 dB
1,000	0 dB (definition)
5,000	-7 dB

The *Sensation Level* of a sound of any frequency is measured by the ratio of its intensity to the threshold intensity at that particular frequency. Using the same logarithmic scale,

$$\text{Sensation Level in Decibels} = 10 \log_{10} \frac{{}_nI}{{}_nI_o}$$

where ${}_nI$ is the intensity of the sound of frequency n and ${}_nI_o$ is the threshold intensity at frequency n .

The sensation level of a sound is therefore obtained by subtracting

from its intensity level in dB the threshold intensity in dB for that particular frequency. Changing from intensity level to sensation level is a kind of "adjustment of the zero."

A sound of intensity level 100 dB would have the following sensation levels at selected frequencies.

Frequency c.p.s.	Threshold Intensity Level db	Sensation Level db
50	54	$100 - 54 = 46$
100	38	62
1,000	0	100
5,000	-7	107

It might perhaps be hoped that sounds of the same sensation level would sound equally loud, whatever their pitch. This, however, is not so, for not only is the "zero" of the scale different at different pitches, but the size of the "unit" is different also. In obtaining the sensation level scale we have adjusted the zero, but done nothing about the unit. In fact, loudness seems to be regulated by a quantity usually lying in between sensation level and intensity level.

For measuring loudness, the following method is used. The sound concerned (of any pitch whatever), and another sound which is a pure tone of frequency 1,000 c.p.s., are listened to alternately. The pure tone is adjusted in strength until the observer judges that the sounds are equally loud. The intensity level of this reference tone, which is the same as its sensation level because its frequency is 1,000 c.p.s., is

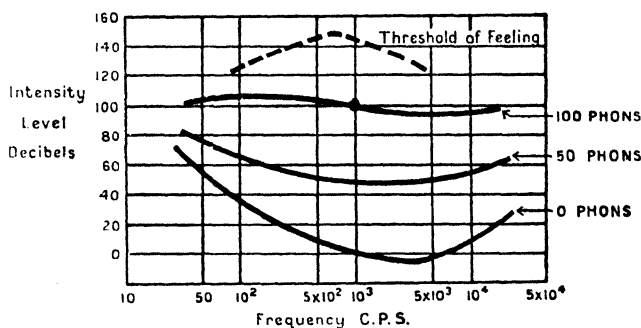


FIG. 77.

then measured. If this quantity is n decibels, the other sound is said to have an *Equivalent Loudness* of n phons.

The intensity levels required for sounds of various loudnesses are shown in Fig. 77. The lowest curve, 0 phons, is simply the threshold

curve of Fig. 76. The following table shows the Intensity Level and Sensation Level for sounds of 20, 50 and 100 phons at selected frequencies.

Frequency c.p.s.	Equivalent Loudness Phons	Intensity Level dB	Sensation Level dB
50	20	65	11
100	20	53	15
1,000	20	20	20
10,000	20	20	27
50	50	76	22
100	50	68	30
1,000	50	50	50
10,000	50	52	59
50	100	102	58
100	100	103	65
1,000	100	100	100
10,000	100	94	101

One point of interest that is illustrated by the table is that for a loud sound of 100 phons the intensity levels over the wide range of frequencies selected show little variation—only 9 dB; at 50 phons the difference has risen to 24 dB and at 20 phons to 45 dB. If, therefore, we take a composite sound and magnify the intensities of all frequencies equally, the balance will alter, becoming much more favourable to low notes as the general intensity increases. This effect is quite noticeable with an ordinary radio receiver: an increase of volume produces a considerable increase in the relative bass in the sound. This is particularly important in reproduction of the speaking voice, which is so often too loud to be natural, carrying with it an excess of low frequencies for the above (and other) reasons. Reducing the volume often produces a marked improvement.

The phon scale is commonly used in the measurement of noise.

82. Sensitiveness of the Ear to Pitch

If two pure tones of the same loudness but differing slightly in frequency are sounded in succession, a certain minimum difference in frequency is necessary for a perceptible change to be recognisable. If this difference is Δf , the frequencies both being approximately equal to f , it is found that $\Delta f/f$ is not constant with f , but varies with this and also with the loudness of the sound. At the frequency of 2,000 c.p.s. and sensation level 70 dB, $\Delta f/f = .0017$ —that is, a

frequency difference of 1 part in 600 can be detected. At frequencies of about 60 c.p.s. and low sensation levels the minimum changes necessary may be as much as 1 part in 20. Pitch discrimination in the case of low notes is always easier if the notes are not pure, as the harmonics will lie in a region where a smaller relative difference is recognisable.

83. The Ear

For convenience we consider the structure of the ear in three distinct sections, (1) the *outer ear*, which comprises the visible part and the passage leading to the ear drum, (2) the *middle ear* which consists of a small chamber containing a chain of levers, and (3) the labyrinth or *inner ear* in which the movements are communicated to the appropriate nerve endings.

The visible part of the ear—the *pinna*—acts as a sound collector, and leading from it is a passage about $2\frac{1}{2}$ centimetres long. This passage ends in the ear drum, or *tympanum* (Fig. 78) which is a delicate

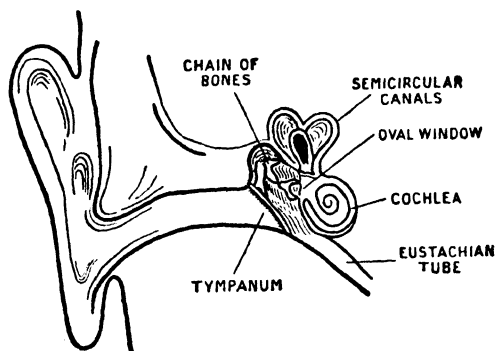


FIG. 78.

membrane with a muscle for keeping it taut. It vibrates when sound waves enter the ear.

The middle ear contains the chain of bones—the *malleus* or hammer (attached to the tympanum), the *incus* or anvil, and the *stapes* or stirrup. The latter closes the aperture to the inner ear, called the *oval window*. The chain of levers reduces the displacement amplitude, increasing the force available correspondingly. The Eustachian tube leads into the middle ear from the back of the throat, and thus equalises the general pressure (but not the variations caused by the sound wave) on the two sides of the tympanum.

The inner ear is a bony cavity containing the semicircular canals (which play no part in hearing but are concerned with balance), and the *cochlea*, which resembles a snail shell. It is filled with fluid, and is

partially divided by a bony shelf, the division being completed by two membranes—the *membrane of Reissner* and the *basilar membrane*. If taken out and uncoiled, the latter is about 3 centimetres long, 0.16 millimetre wide at one end and 0.5 millimetre wide at the other. It is strengthened by transverse fibres, and there is evidence that it is more tightly stretched transversely at its narrower end. It carries the *organ of Corti*, which contains the nerve endings in small hair cells, and the tectorial membrane. When the vibrations are passed into the cochlea through the oval window, the membrane of Reissner, basilar membrane and tectorial membrane vibrate, producing relative motion between the last and the hair cells. This gives rise to the sensation of sound.

84. Theories of Hearing

The resonance theory, originally put forward by Cotugno (1736–1822), was supported by Helmholtz, and is still under consideration. The basic supposition is that the fibres of the basilar membrane are like the strings of a harp, and behave as a series of heavily damped resonators. A sound of given pitch would cause a stimulation in one area of the membrane, with the maximum at the resonance point. This localised stimulation of nerve endings is assumed to give rise to the sensation of pitch.

There is no conclusive evidence for the existence of such a set of resonators, but the lengths and tensions in the fibres of the membrane are such that it would not be impossible for them to have the required frequency characteristics. There is strong evidence that the perception of frequency is distributed along the basilar membrane. Animals that have been subjected to excessively loud sounds of various pitches have afterwards been found to show damage of the membrane in different places. Mechanical damage to parts of the membrane is also known to cause selective deafness.

The alternative theory is that the whole membrane responds to all loud sounds, and that the impulses conveyed to the brain by a nerve or nerve bundle have the same frequency as the stimulus and are recognised accordingly. On the whole, this seems unlikely, as there is no evidence that the auditory nerve exhibits variations of potential at frequencies in excess of about 2,000 c.p.s.

85. Aural Harmonics and Combination Tones

In what has been said about the quality of the notes from different instruments, and on analysis generally, it has been assumed that a sound wave of simple harmonic form would be recognised as a simple tone with no multiple frequencies—or, in other words, the ear does not distort the form of a wave incident upon it.

There is, however, definite evidence that with sufficiently strong pure tones as stimuli, harmonics even up to the tenth are in fact audible. The same distortion accounts for the fact that the ear adds considerably to summational and differential tones (§ 18).

To detect and measure the intensity of aural harmonics, the method of beat analysis is used. Suppose the pure tone used to stimulate the ear has frequency n . To investigate the second harmonic, an exploring tone of frequency slightly different from $2n$ and of variable intensity is listened

to at the same time. If there is an aural harmonic of frequency $2n$ of intensity comparable with that of the exploring tone beats will be heard. They will be most distinct if the intensity of the exploring tone is adjusted to equal that of the harmonic, which can thereby be found.

With sufficiently loud tones the harmonics may be very powerful: e.g., for a tone of frequency 100 c.p.s. such that the equivalent loudness of the fundamental is 100 phons, the second, third, and fourth harmonics produced are each in excess of 60 phons, and harmonics up to the tenth are measurable. The production of aural harmonics makes any determination of the lower limit of audible frequency difficult and unreliable.

The existence and intensity of aural combination tones have been established and measured by the same exploring tone method used for measurement of harmonics. Their existence plays an important part in cutting out the deficiencies of sound-reproducing apparatus, particularly so far as bass response is concerned. If a set of notes which are harmonics of a common fundamental—e.g., 100, 200, 300, 400, 500, 600, 700, 800 c.p.s. are sounded together, it is found that the complete suppression of the fundamental (100 c.p.s.) makes little or no difference to the quality. The fact is that each pair of successive harmonics forms this fundamental *in the ear* as a difference tone. Thus it comes about that sound reproducers such as an early gramophone or midget radio set produce *in the air* no bass notes at all, but these are to some extent manufactured *in the ear* and the final result is tolerable.

QUESTIONS

1. Discuss what is meant by the loudness of a sound. How can it be measured in practice? (*Lond. B.Sc.*)

2. Discuss what is meant by a noise level and give an account of some method used for its measurement, explaining the units involved. (*Lond. B.Sc.*)

3. Discuss the measurement of the absolute intensity of the sound from a maintained source, and describe any way in which the least energy required to produce an audible sound can be determined. How does this energy vary with frequency? (*Lond. B.Sc.*)

4. Discuss the difficulties of measuring in absolute units the intensity of a sound wave.

Describe one modern method of making such an absolute determination, and specify its merits and limitations. (*O.F.*)

CHAPTER X

THE ACOUSTICS OF BUILDINGS

86. Reverberation

EVERYONE is familiar with the fact that a lamp will give a brighter illumination of surfaces in a room if the walls and ceiling are sufficiently light in colour to reflect well than it will if they are dark. A similar effect occurs with sound, a given source producing a greater average intensity when the absorption is low.

Looking at the matter from the reception end, it is obvious that when the source starts emitting, the sound it gives will reach any given point first by the direct straight line path. Subsequently it will arrive there by paths of greater length, involving one or more reflections. The result is that several sets of waves, usually progressively diminishing in intensity, arrive at the point. Neglecting the effects of interference which is equivalent to taking an average (§ 13), these will give a disturbance having an intensity equal to the sum of the separate intensities. So far, this increase in loudness is all to the good, assuming that we wish to hear the sound. Owing, however, to the low velocity of sound, an appreciable time elapses between the arrival of the first waves and the last which are of sufficient intensity to matter: some of the waves may have travelled thousands of feet by a hundred or so successive reflections, so that the delay in approaching the maximum of sound may be several seconds. ~~What is perhaps worse~~, if the sound is cut off at the source it will not stop instantaneously at the receiver, which will continue to pick up the successive reflections until they are too weak to stimulate it. The louder the original sound, the longer will this process take. This gradual dying out is known as *reverberation*, and it is the most important single factor in settling whether a room is "good for sound" or not.

Unfortunately the two desirable characteristics, loudness and clarity, are produced by opposite conditions of reflecting power. The better the reflecting power of the walls and objects present in the room, the larger will be the intensity reached by the use of a given source. From the point of view of a singer or instrumentalist such good reflections produce a stimulating effect: a powerful sound is made with little effort, and the inevitable discontinuities in starting and stopping the sound are to some extent rounded off. From the point of view of the audience, presumably interested in clarity as well as sonority, the reverberation must be strictly limited: in a piece of music notes

following one another in rapid succession will become confused if the sound takes too long to rise and fall, while speech may be rendered unintelligible.

87. The Experiments of W. C. Sabine

Prior to about 1900 there was no scientific basis for the design of halls for good acoustic properties. Not infrequently halls were built and found to be unsatisfactory for the purposes for which they were intended. [The Fogg Art Museum of Harvard University provided an example of a hall in which a speaker could hardly make his words intelligible to the audience, and Wallace C. Sabine was consulted with a view to finding suitable remedial treatment.] Here really began the whole of the study of planned acoustics of halls.

A rough test showed that in the hall in question, when empty, a sound persisted for about $5\frac{1}{2}$ seconds after the source had ceased to make it, and that although the time was less when the hall was full, it was still much too long to be satisfactory. Sabine experimented by using an organ pipe of frequency 512 c.p.s., the time of reverberation (i.e. the time for which the sound remained audible after the air supply to the pipe had been cut off) being measured with a stopwatch. By taking a sufficient number of observations, which were in fact remarkably consistent, a reliable average was secured.

In the earliest experiments the absorption was increased gradually by bringing into the room various lengths of cushion from a neighbouring lecture room, the reverberation time being found under these different conditions. On plotting the reciprocal of this time against the length of cushion introduced, a straight line graph was obtained, showing that a relation of the form $AT = K$ held good, T being the reverberation time, A the total absorption present, and K a constant for the room. The graph did not pass through the origin, owing to the absorption present in the empty room: from the intercept on the A -axis this was deduced to be equivalent to 146 metres of cushion. Experiments in different rooms showed that K was proportional to the volume of the room, so that the general relation became

$$AT = kV$$

or

$$T = \frac{kV}{A}.$$

In this empirical relation A is still measured as a length of cushion. To obtain a less arbitrary unit Sabine found experimentally the length of cushion equivalent in absorption to a known area of open window, which he regarded as a complete absorber. With A now measured in

"open window units" (i.e. the equivalent area of open window) his formula became

$$T = \frac{.171V}{A}$$

if V is in cubic metres and A is in square metres ;

or
$$T = \frac{.05V}{A}$$

if V is in cubic feet and A is in square feet.

The reverberation time depends upon the intensity of the sound before the cut-off. The decay is, in fact, exponential, and Sabine converted his actual times to those that would have been observed if the intensity at cut-off were one million times the minimum of audibility. In fact, the "standard reverberation time," as this is now called, is the time taken for a decay in intensity of 60 decibels.

Sabine realised that the absorbing powers of materials were different at different frequencies, and that this would have a considerable influence on the quality of the sound made by musical instruments. He also conducted experiments to discover whether musical taste was consistent in the judgment of optimum reverberation time, finding that it was so to a remarkable degree of accuracy. His accounts of the various experiments, given in his "Collected Papers on Acoustics,"¹ make amusing and profitable reading.

88. Theoretical Treatment of Reverberation

This analysis is based on the assumption that the sound energy becomes distributed sufficiently uniformly to make valid consideration in terms of an average energy density. In practice the energy density is much the same at all parts of a sufficiently large room, excepting near the source, provided that the absorption is low.

The first step is to calculate, in terms of this average energy density E , the rate at which energy falls upon the walls and other surfaces present,

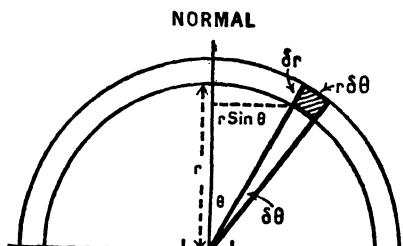


FIG. 79.

forming part of a plane wall.

Imagine two circles drawn with δS as centre, in a plane containing the

and hence the rate at which it is absorbed. An expression can then be obtained for the final steady value of E , in terms of the rate of emission or power P of the source. The third step is to formulate and solve a differential equation on the principle that the rate of change of E at any moment is settled by the excess of P over the total absorption at that moment.

Consider the reception of sound energy by an element of area δS ,

¹ Harvard University Press, 1922.

normal to δS . Let the radii of these circles be r and $r + \delta r$. Consider the portion of the area between the circles also lying between two radii at angles θ and $\theta + \delta\theta$ with the normal (Fig. 79).

Area considered $= r \delta\theta \cdot \delta r$.

If the whole figure is rotated about the normal through an angle $\delta\phi$, this element of area travels through a circumferential distance $r \sin \theta \delta\phi$, tracing out an element of volume $r^2 \sin \theta \delta\theta \delta r \delta\phi$; so that the differential of the volume is $r^2 \sin \theta d\theta dr d\phi$.

The sound energy in this element at any moment

$$= E r^2 \sin \theta d\theta dr d\phi,$$

E being the energy density. This energy is travelling through the element equally in all directions, so that an amount

$$\frac{E r^2 \sin \theta d\theta dr d\phi}{4\pi}$$

leaves it in unit solid angle.

The solid angle subtended by δS at the element

$$= \frac{\delta S \cos \theta}{r^2}$$

$\therefore$ The energy in the element of volume that is travelling towards δS

$$= \frac{\delta S \cos \theta}{4\pi r^2} E r^2 \sin \theta d\theta dr d\phi$$

$$= \frac{E \delta S}{4\pi} \cdot dr \cdot \cos \theta \sin \theta d\theta \cdot d\phi$$

To find the energy received by δS in unit time, we must integrate this for the whole of the volume lying within a distance c of δS , where c is the velocity of sound. Integrating first with respect to ϕ ,

$$\text{energy received} = \frac{E \delta S}{4\pi} dr \cos \theta \sin \theta d\theta \int_0^{2\pi} d\phi$$

$$= \frac{E \delta S}{2} dr \cos \theta \sin \theta d\theta.$$

Integrating with respect to θ ,

$$\text{energy received} = \frac{E \delta S}{2} dr \int_0^{\pi/2} \cos \theta \sin \theta d\theta$$

$$= \frac{E \delta S}{4} dr.$$

Integrating with respect to r ,

$$\text{energy received per unit time} = \frac{E \delta S}{4} \int_0^c dr$$

$$= \frac{1}{4} E c \delta S.$$

(This expression may be compared with the value $E c \delta S$ for plane waves travelling normally towards δS .)

If a is the absorption coefficient of the element δS ,

energy absorbed by it in unit time $= \frac{1}{4} E c a \delta S$,

$\therefore$ total rate of absorption at any time when the energy density is E

$$= \frac{1}{4} E c \Sigma a \delta S.$$

$\Sigma a \delta S$ is the total absorption of all the surfaces upon which the sound can fall, hereafter denoted by A .

If P is the power emitted by the source, the final value of the energy density $E_{\max.}$, is given by

$$P = \frac{1}{4} E_{\max.} c A,$$

or
$$E_{\max.} = \frac{4P}{cA}.$$

At any time after the start, E grows at a rate proportional to the excess of P over the absorption at that moment.

Total sound energy present at any moment = EV , where V is the volume of the room.

$\therefore$ Rate of growth of this energy $= V \frac{\partial E}{\partial t}.$

Rate of absorption of energy at this time $= \frac{1}{4} EcA.$

$$\begin{array}{ccccc} \text{Rate of growth of} & & \text{Rate of absorption} & & \text{Rate of supply} \\ \text{energy in space} & + & \text{by walls} & = & \text{by source} \end{array}$$

$$V \frac{\partial E}{\partial t} + \frac{1}{4} EcA = P,$$

or
$$\frac{\partial E}{\partial t} + \frac{cA}{4V} E = \frac{P}{V}.$$

For convenience, write α for $\frac{cA}{4V}$, so that the equation may be rewritten

$$\frac{\partial E}{\partial t} + \alpha E = \frac{4P}{cA} \cdot \alpha.$$

Multiply by $e^{\alpha t}$,

$$\left(\frac{\partial E}{\partial t} + \alpha E \right) e^{\alpha t} = \frac{4P}{cA} \alpha e^{\alpha t}.$$

$$\therefore \frac{\partial}{\partial t} (E e^{\alpha t}) = \frac{4P\alpha}{cA} e^{\alpha t}.$$

Integrating, we get

$$E e^{\alpha t} = \frac{4P}{cA} e^{\alpha t} + k,$$

where k is a constant of integration.

(1) If $t = 0$ when $E = 0$, $k = -\frac{4P}{cA}.$

$$\therefore E = \frac{4P}{cA} (1 - e^{-\alpha t}), \text{ where } \alpha = \frac{cA}{4V}.$$

This expression shows the *growth* of E with time after the start of the sound.

(2) If the source is cut off when E has reached the maximum, so that

$$P = 0, \text{ and } t = 0 \text{ when } E = E_{\max.} = \frac{4P}{cA},$$

then

$$k = E_{\max.},$$

and

$$E = E_{\max.} e^{-\alpha t}, \text{ where } \alpha = \frac{cA}{4V}.$$

This expression shows the *decay* of E with time, after the source has been cut off.

The growth (1) and decay (2) are illustrated in Fig. 80. The case may be compared with the variation of p.d. across a condenser during charge and discharge through a resistance.

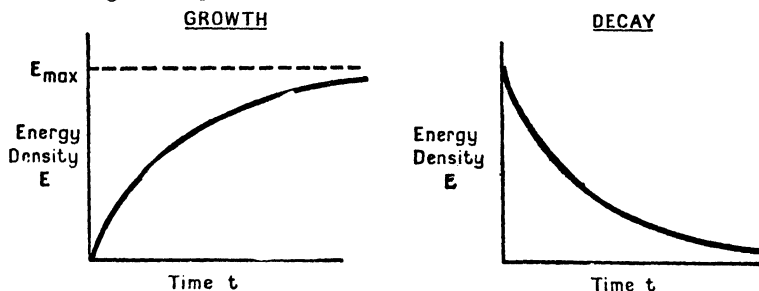


FIG. 80.

The standard reverberation time T , being the time for the energy density and therefore the intensity to decay to one-millionth of its value before the cut-off, is given by the equation of (2), for

$$e^{-\alpha T} = 10^{-6}$$

$$\alpha T = \frac{cAT}{4V} = \log_e 10^{+6} = 2.303 \times 6.$$

Using the value 340 metres per second for c ,

$$T = \frac{4 \cdot 2.303 \cdot 6 V}{340 A}$$

$$= 0.16 \frac{V}{A},$$

or similarly, when the dimensions are in feet, $T = \frac{.05V}{A}$.

The factors affecting T , and the actual numerical constant are thus in good agreement with experiment (§ 87). This is indeed remarkable considering the assumptions in the calculation, though it suggests that they cannot be wildly inaccurate. Briefly, they are

- (a) Energy distribution uniform.
- (b) Interference effects negligible.
- (c) Absorption coefficients independent of intensity.
- (d) Power emitted by the source independent of the energy density reached.
- (e) Loss of energy in the air negligible.

It is interesting to note that the actual reverberation time for a source giving an intensity somewhat less than the million times minimum audibility is not very much less than the standard value, owing to the exponential form of the decay. Thus, if the intensity reached is only 10^4 times the minimum the reverberation time will be two-thirds of the standard value.

The above method of analysis was originally given by Franklin in 1903:

more recently a quite different treatment has been given by Eyring, who obtains the equation

$$T = \frac{kV}{-S \log_e (1 - \bar{a})},$$

where S is the total area of the absorbing walls, and $\bar{a}$ is the average absorption coefficient, defined by

$$\bar{a} = \frac{\sum a \delta S}{S} = \frac{A}{S}.$$

This formula differs considerably from $T = \frac{kV}{A}$ when $\bar{a}$ is large, but not when it is small. In the latter case

$$\log_e (1 - \bar{a}) \simeq -\bar{a},$$

so that the values are approximately the same. In practice the average absorption coefficient is usually sufficiently low to render the simpler formula of Franklin as accurate as the data warrant.

89. Optimum Reverberation Time

Assuming that the absorption coefficients of the various constructional materials are known, it is possible to calculate the reverberation time of any proposed hall in advance of the actual construction. By making the correct choice of materials for the various surfaces, we can arrange for the reverberation time to have almost any desired value. It is therefore necessary to know what value of this time is likely to be most satisfactory for the purpose to which the hall is to be put.

Experience with the best Concert Halls in different countries shows that there is a range of reverberation times which give good results in a hall of given size: this is just as well, for the absorption caused by the audience (mostly their clothes, not their ears) makes a considerable difference. For volumes of about 50,000 cubic feet the best times seem to lie between 1 and 1.5 seconds: for a large hall of 400,000 cubic feet the values are from 1.5 to 2 seconds. These figures refer to a sound of frequency 512 c.p.s. Very much more experimental work has been done in connection with studios for broadcasting and gramophone recording: it is found that acoustic defects due to the wrong reverberation time are accentuated in these cases, possibly on account of the different directional response of a microphone in comparison with a person with two ears.

As regards the intelligibility of speech, it must be borne in mind that this will be affected by loudness. If the absorption in a room is large, the reverberation time will be short so that there will be little confusion of the syllables, but the intensity will be weak. If the speaker's voice is amplified, the shorter the reverberation time the better: cinemas are usually designed with this in mind. On the other hand, if the natural voice is to be relied upon, some reflection is desirable to give the

necessary loudness: it is found that a reverberation time of from 1 to 1.5 seconds gives the best intelligibility in a 400,000 cubic foot hall. It seems that good intelligibility can be secured by a careful speaker with times up to 2.5 seconds.

90. Measurement of Absorption Coefficients

When we state that the absorption coefficient of the material of a surface is a , we mean that a fraction $(1 - a)$ of the sound energy incident upon the surface is returned to the side from which it came. For use in connection with reverberation times, the absorption coefficient must be an average for all directions of the impinging waves.

A method of finding the coefficient by the use of stationary waves has been referred to in § 15: the great weakness here is that the measurement is possible at normal incidence only. The more common method is the use of a reverberation chamber—that is, a specially designed room into which known areas of the materials under test can be introduced and their effects on the reverberation found. This room must be reasonably large, while the absorption due to its walls must be small: extraneous noise must be excluded in order to make it possible to measure the decay time with fair accuracy.

The method generally employed is to find the reverberation times for two sources of emitting powers P and P' . P and P' need not be known absolutely, it being sufficient to know their ratio. For example, in some of the experiments of P. E. Sabine P was the power of one organ pipe, while P' was that of up to four similar pipes. More recently loudspeakers fed with measured audio-frequency currents have been used.

From the equations of § 88, the steady energy densities maintained by the two sources will be

$$\frac{4P}{cA} \text{ and } \frac{4P'}{cA}$$

During the decay they would reach the fixed value E_0 in times T and T' given by

$$E_0 = \frac{4P}{cA} e^{-\alpha T}$$

and

$$E_0 = \frac{4P'}{cA} e^{-\alpha T'}, \text{ where } \alpha = \frac{cA}{4V}.$$

$$\therefore \frac{P'}{P} = e^{\alpha(T - T')},$$

or

$$\alpha = \frac{\log_e P' - \log_e P}{T' - T}$$

and

$$A = \frac{4V \log_e(P'/P)}{c(T' - T)}.$$

The value of A is found for the empty chamber, and its new value with various areas of the material under test.

The reverberation times can be assessed by ear, or the decay studied with

an oscillograph. In the latter case it must be remembered that the intensity is likely to vary from place to place, superposition effects being prominent.

It is, of course, possible to determine absorption by measuring the value of $E_{\max.}$, the steady energy density reached with a source of given power. A source of constant power P will maintain an energy density $E_{\max.}$ when the absorption is A , given by $E_{\max.} = \frac{4P}{cA}$.

If the absorption is increased to A' , the energy density becomes $E'_{\max}$ such that

$$E'_{\max.} = \frac{4P}{cA'}$$

so that

$$\frac{A'}{A} = \frac{E_{\max.}}{E'_{\max.}}$$

The value $\frac{E_{\max.}}{E'_{\max.}}$ is equal to the measured ratio of the intensities maintained

by the same source under the two conditions. It is, however, never an easy matter to determine this experimentally, on account of the variations of intensity due to the complicated superposition patterns that are produced. (Any person who has moved about in a room in which a radio set is reproducing the B.B.C.'s tuning note will be aware of these variations.) If the frequency of the source is slowly varied, and the source itself rotated, observations made over a period at several different points enable reliable averages to be deduced.

91. Calculation of Reverberation Time

The absorption coefficients of all the common constructional and decorative materials have now been determined for a range of frequencies, some typical values being given in the following table :

Material	Frequency, c.p.s.		
	250	500	1,000
Unpainted brick	0.03	0.03	0.05
Hard plaster	0.01-0.02	0.01-0.02	0.02-0.03
Lime plaster	0.02-0.03	0.03-0.04	0.03
Wood blocks	0.03	0.06	0.10
Glass		0.03	
Carpet, $\frac{1}{4}$ -in. Axminster on $\frac{1}{4}$ -in. felt	0.15	0.40	0.65
Acoustic plaster	0.15	0.25	0.30
Fibre boards $\frac{1}{2}$ -in., on battens	0.30-0.40	0.30-0.35	0.25-0.35
Cabot quilt, 3-ply, 2 layers.	0.40	0.70	0.70

The last three materials in the table are special absorbents, used when it is necessary to increase the absorption in order to keep the reverberation time down.

time only by means of an electric spark. This spark followed the spark that initiated the sound wave, after a delay that could be adjusted electrically. In this way series of photographs showing the progress of waves were obtained.

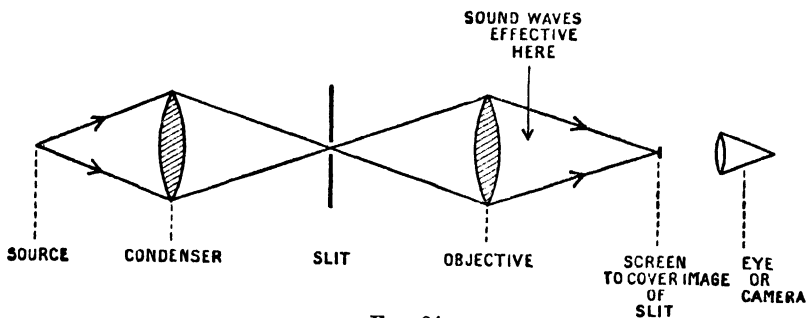


Fig. 84.

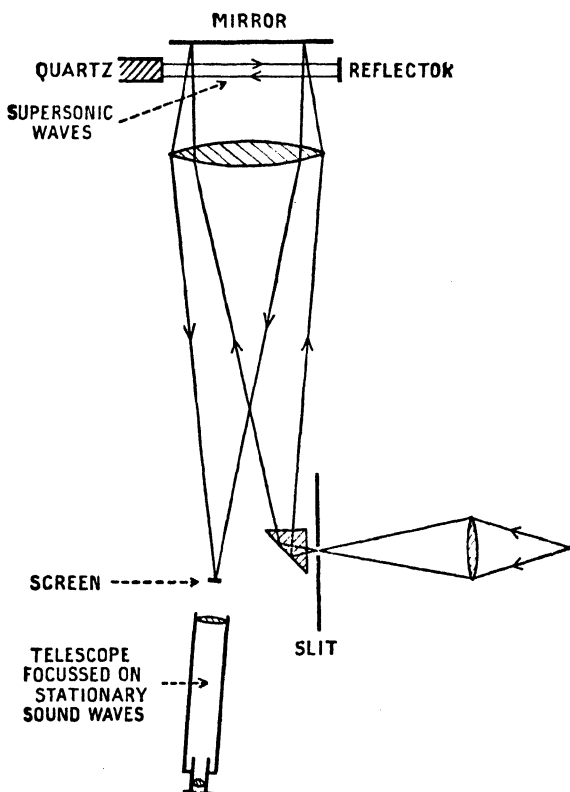


Fig. 85.

With the advent of powerful sources of supersonic waves the arrangement can be used to make visible the pressure nodes and antinodes in a stationary wave system. Thus, if stationary waves are produced across the light beam to the right of the objective, using a quartz oscillator and a reflector, the pressure nodes remain dark while the loops are bright. The arrangement of Pohlmann (Fig. 85) is an improvement on the other for this purpose, as the light passes twice through the sound wave.

Travelling waves can be observed with the same apparatus if stroboscopic illumination is used. This may be produced by means of a Kerr cell actuated by the same alternating potential as the quartz.

Powerful stationary sound waves in a horizontal tube can also be made visible by using a layer of liquid on the bottom of the tube. The excess pressure at the displacement nodes causes the liquid surface to show an undulatory contour: if sufficient intensity is used the layer is completely broken up into sections. This effect has been made use of for wavelength measurement: the curved sections of liquid behave as cylindrical lenses, casting images of the straight filament of a distant lamp.

97. Diffraction of Light by Sound Waves

When a sound wave is present in a medium the pressure varies with the distance from the source according to a periodic curve—a sine curve in the simplest case—the distance between points in the same phase being the wavelength. The density and refractive index of the medium will therefore show a periodic variation with distance along the direction of propagation of the wave. The effect is much more marked with liquids than with gases, and if light is passed through the liquid at right angles to the sound waves the liquid behaves as a diffraction grating. The grating space is, of course, the wavelength of the sound: if the wave is travelling, the grating is in fact moving with the velocity of sound, which gives rise to a Doppler effect, changing the effective frequency of the light.

If stationary waves are used the effect is that of two equal waves travelling in opposite directions, giving opposite Doppler effects in a given order of diffraction. In the k th order the superposition of these “modulates” the light with frequency $2kn$, where n is the frequency of the sound. As in the case of sound, these “beats” are really a travelling interference pattern, which can in this case be rendered stationary by using stroboscopic illumination at the start.

98. Supersonic Echo-sounding

The method of echo-sounding described in § 38 is not capable of giving detailed results, such as those necessary for the location of wrecks, on account of the long wavelengths of the sounds employed. The wavelength in water of a sound of frequency 1,000 c.p.s. is about 1.5 metres; a narrow beam cannot be produced unless the source has dimensions large compared with this, which is not practicable, so that detail is not observable. By using very high frequencies, however, a narrow directional beam and its reflection from a limited area can be studied.

The frequency used is about 40,000 c.p.s., the waves being produced by a specially loaded quartz plate. This is not made to oscillate continuously, but to emit a series of damped wave-trains at intervals controlled by a motor. The reflected waves are received on the same quartz, causing it to vibrate and to generate a small E.M.F. The delay of this E.M.F. on that initiating the waves is registered with a suitable oscillograph, from which

a direct reading of depth can be obtained. Alternatively, the variation of depth can be recorded as a trace on a revolving drum, while the ship is in motion.

QUESTIONS

1. Discuss the generation and use of supersonic waves. (*Lond. B.Sc.*)
2. Describe how you would produce (a) a note of frequency approximately 1,000 cycles per second, (b) an ultra-sonic vibration of frequency approximately 50,000 cycles per second. Explain how you would measure the wavelengths of these disturbances. (*C.S.*)
3. Prove that the velocity of propagation of longitudinal waves in a rod is $\sqrt{\frac{Y}{d}}$, where Y is Young's Modulus and d the density of the rod. What are the frequencies of the notes which can be emitted by such a rod of length l clamped at one end?
Describe concisely a method of maintaining longitudinal oscillations in a nickel rod. (*O.F.*)
4. Write a short account of the generation and properties of supersonic waves. (*O.F.*)

MISCELLANEOUS QUESTIONS

1. Explain the production of the beats heard when two notes of nearly the same pitch are sounded together, and show how the frequency of the beats is related to the frequencies of the separate notes.
The first overtone of an open pipe and the fundamental tone of a pipe closed at one end give 5 beats per second when sounded together. If the length of the pipe closed at one end is 25 cm., what are the possible lengths of the open pipe? (Neglect end corrections, and take the velocity of sound in air to be 340 metres per second.) (*O. & C.*)
2. Distinguish between progressive and stationary sound waves.
Describe and give the theory of some method of comparing the velocity of sound in air with that in argon. (*O. & C.*)
3. A vibrating tuning-fork is held over the open end of a vertical tube closed at the other end. Water is poured into the open end and it is found that at certain levels of the water the intensity of the sound reaches a maximum. Explain this, and describe what readings would have to be made to calculate the velocity of sound from the experiment.
Explain also where the nodes of pressure and gas motion occur in the tube, and how these positions might be verified experimentally. (*O. & C.*)
4. Explain the effect of the motion of the source and of the observer on the apparent pitch of a note.
An express train blowing its whistle travels through a station, and an observer standing on the platform notices that as the engine passes him the drop in pitch of the whistle corresponds to an interval of $6/5$. What is the speed of the train? (Speed of sound in air = 1,100 ft. per second.) (*O. & C.*)

5. Explain the origin of the beats heard when two tuning-forks of slightly different frequency are sounded together. Deduce the relation between the frequency of the beats and the difference in frequency of the forks. How would you determine which fork had the higher frequency?

A simple pendulum set up to swing in front of a "seconds" pendulum ($T = 2$ sec.) of a clock is seen to gain so that the two swing in phase at intervals of 21 secs. What is the time of swing of the simple pendulum? (*Lond. H.S.C.*)

6. What are the distinguishing characteristics of stationary and progressive waves? Show that stationary waves are produced by the superposition of two similar wave trains travelling in opposite directions.

Describe a method of determining the velocity of sound in a gas other than air. (*O. & C.*)

7. State briefly how you would show by experiment that the characteristics of the transmission of sound are such that (a) a finite time is necessary for transmission, (b) a material medium is necessary for propagation, (c) the disturbance may be reflected and refracted.

The wavelength of the note emitted by a tuning-fork, frequency 512 vibrations per sec., in air at 17°C. is 66.5 cm. If the density of air at S.T.P. is 1.293 gm. per litre, calculate the ratio of the two principal specific heats of air. Assume that the density of mercury is 13.6 gm. per c.c.

(*N.U.J.M.B.*)

8. Describe in detail how to determine the frequency of a tuning-fork by the falling plate method.

Describe how the frequency of an alternating current, such as that from the A.C. mains, may be determined with a sonometer. (*N.U.J.M.B.*)

9. Explain the formation of echoes in terms of the behaviour of waves.

Describe how the velocity of sound in either (i) air, or (ii) water, may be determined by the use of echoes. An observer A fires a pistol in the angle between two high walls at right angles to each other and the sounds produced are heard by another observer B. A is 110 yards from one wall and 220 yards from the other; B is 440 yards from each wall. Draw a diagram to scale (110 yards = 1 inch) showing the path of "rays" of sound from A to B, and calculate the time intervals between the sounds heard by B.

[Velocity of sound in air = 1,100 ft. per sec.] (*N.U.J.M.B.*)

10. Explain what is meant by resonance, illustrating your answer with one mechanical and one acoustical example.

How may the frequency of a tuning-fork be found by a resonance tube method when the velocity of sound in air at 0°C. is known? Discuss briefly how the observations and the final result would be affected by a rise in the temperature. (*N.U.J.M.B.*)

11. What factors determine the frequency of transverse vibration of a stretched wire? How may the relation between transverse frequency and tension be established experimentally?

A stone hangs in air from a wire which is stretched over a sonometer. The bridges of the sonometer are 40 cm. apart when the wire is in unison with a certain fork. When the stone hangs immersed in water the length between the bridges must be altered to 32 cm. to re-establish unison with the fork. Calculate the specific gravity of the stone. (*N.U.J.M.B.*)

12. Write down an expression for the velocity of sound in a gas and hence show that this quantity is (a) dependent on the temperature, (b) independent of the pressure of the gas.

Describe carefully how you would use a resonance tube and a tuning-fork of known frequency to determine the velocity of sound in air, showing how to calculate the velocity at 0°C. from your observations. (*N.U.J.M.B.*)

13. Explain why the note emitted by a closed organ pipe differs in quality from that of a note of the same pitch and loudness emitted by (a) an open pipe, (b) a violin.

The fundamental note emitted by a stretched wire vibrating transversely between two bridges 50 cm. apart is in unison with that of an organ pipe when the temperature is 12°C. Find approximately the change in length between the bridges necessary to restore unison when the temperature of the air in the pipe rises to 20°C. , the tension in the wire remaining unaltered. (*N.U.J.M.B.*)

14. Describe the phenomenon of beats, and show that the beat frequency is the difference of the frequencies of the tones producing the beats. Describe *one* quantitative use of the phenomenon.

A certain fork is found to give two beats per second when sounded in conjunction with a stretched string vibrating transversely under a tension of either 10.2 or 9.9 kgm. wt. Calculate the frequency of the fork. (*N.U.J.M.B.*)

15. Explain what is meant by a "progressive wave," and a "stationary wave," and give examples of both. Show how stationary waves can be produced by the reflection of progressive waves.

Describe a method of measuring the wavelength of sound waves in air. (*O.P.*)

16. How does the velocity of sound in a gas depend on temperature and pressure?

The observer in an aeroplane flying horizontally at 240 m.p.h. releases a bomb and hears the sound of the explosion 20 seconds afterwards. Find the height of the aircraft, neglecting air resistance. (Velocity of sound in air = 1,100 ft./sec.) (*O.P.*)

17. Explain what is meant by (a) longitudinal wave motion, (b) transverse wave motion, and give examples of each.

Describe how you would find the fundamental frequency of vibration of a stretched string. (*O.P.*)

18. Explain the difference between stationary and progressive waves. Give *two* illustrations of each type of wave. How would you show that the velocity of waves along a stretched uniform string is independent of the frequency of the waves? (*O.P.*)

19. Explain how sound waves are propagated in solids, liquids, and gases. Describe briefly *three* methods of measuring the velocity of sound, *one* for each type of medium. (*N.U.J.M.B.S.*)

20. Describe and explain the mode of action of an electrically driven tuning-fork or vibrator. How may such a fork or vibrator be used to verify the relation between tension and wavelength of transverse waves on a stretched string or wire?

A fine wire 500 cm. long is fixed at both ends and is under tension, the fundamental frequency of transverse vibrations being 50 cycles per sec. At what distance from its centre must a bridge be placed in order that 4 beats per sec. may be heard when both sections of the wire are made to vibrate transversely ? (*N.U.J.M.B.S.*)

21. Write down expressions for the velocity of propagation along a stretched wire of (a) transverse waves, (b) longitudinal waves. Deduce one of the expressions.

Find the ratio of the fundamental frequencies of transverse and longitudinal vibrations for a steel wire, diameter 1 mm., mounted on a sonometer and stretched by a force of 10 kilograms weight.

[Young's modulus for steel = 20×10^{11} dynes per sq. cm.].

Describe and explain a method of finding the velocity of longitudinal waves in a long steel rod. (*N.U.J.M.B.S.*)

22. Describe the phenomena of (a) stationary waves, and (b) beats, and explain under what conditions each phenomenon is produced. A uniform wire is stretched over two bridges so that each unit of its unstretched length is increased by 0.1 per cent. Compare the frequencies of the fundamental notes of the stretched wire when vibrating longitudinally and transversely. (*N.U.J.M.B.S.*)

23. (a) Explain Huygens' construction for the propagation of waves in a medium and use it to show that a bullet travelling through the air with a uniform velocity u , greater than that of sound, v , gives rise to a conical wave having the bullet at its apex and a semi-vertical angle $\sin^{-1} \frac{v}{u}$.

(b) Give a brief account and explanation of the audibility of sounds over great distances. (*N.U.J.M.B.S.*)

24. How would you demonstrate experimentally that light and sound are each transmitted by a wave mechanism, and that the waves in the former case are transverse and in the latter case longitudinal ? (*C.S.*)

25. Discuss the evidence from the production, transmission and reception of sound which shows that it consists of longitudinal waves. (*C.S.*)

26. Show that the total energy of a particle performing simple harmonic motion is independent of time and that it is proportional to the square of the amplitude and to the square of the frequency of the motion.

The tension in the wire of a sonometer is equal to the weight of 100 gm. If the wire is 100 cm. long, and vibrates in its fundamental mode with a maximum amplitude of 0.1 mm. calculate the energy of the vibration.

(The frequency n of vibration of the wire is related to the tension T , the mass per unit length m , and the length l by the formula $n = \frac{1}{2l} \sqrt{\frac{T}{m}}$. (*C.S.*)

27. Describe and account for the Doppler effect, in sound. A jet-engined aircraft flies so as to pass directly overhead in level flight at a height of 300 metres at a speed of 250 metres/sec., emitting a whine which can be taken to have a frequency of 10,000 cycles/sec. Draw a graph showing how the frequency heard varies with the horizontal distance of the aircraft. (Velocity of sound in air = 330 metres/sec.) (*O.S.*)

28. What do you understand by the Doppler effect ?

A whistle of 1,000 cycles/second pitch is attached to one end of a light tube 2 ft. long so that it can be sounded while the tube is rotating freely in a vertical plane about a horizontal axis through the other end. If the velocity of the whistle when the tube is horizontal is 16 ft./sec., find the upper and lower limits of the pitch of the sound heard by an observer on the ground viewing the motion end-on. (Velocity of sound = 1,100 ft./sec.) (O.S.)

29. Explain the terms pitch, velocity, amplitude, intensity, harmonic content (overtones) when applied to a sound wave and explain how you would measure one of these quantities for the sound given out by a tuning-fork. (O.S.)

30. What explanation can you give of the following :

The singing of telephone wires in the wind.

Sound is heard more easily down-wind than up-wind.

(O.S. Part question.)

31. Calculate the approximate frequency of the lowest note given by the longitudinal vibrations of a steel bar of length 25 cm., density 8 gm. per c.c., and Young's Modulus 2×10^{12} dynes per cm². (C.S. Part question.)

32. How has (a) the compressibility of liquids, (b) the velocity of sound in liquids been measured ? Show how one of these measurements may be used as a check on the other. (Cambridge Mays.)

33. Derive an expression for the velocity of plane sound waves through a gas. Discuss the effect of (a) pressure, (b) temperature, on the velocity.

Neglecting end-corrections, calculate the change in frequency of the fundamental of a 10-ft. open-ended organ pipe when the air temperature changes from 5° C. to 25° C. (O.)

34. Write an essay on the physical principles involved in musical instruments, explaining carefully why the sounds of different instruments are distinguishable. (O.)

35. Find the velocity of propagation of a longitudinal wave in a rod. Show that in the case of a wire under tension the ratio of this velocity to that of the transverse wave is $\frac{1}{\sqrt{\rho}}$ where ρ is the extension of the wire per

unit length. (The velocity of the transverse wave is given by $\sqrt{\frac{T}{m}}$, where T is the tension and m the mass per unit length.) (O.)

36. Find an expression for the velocity of a longitudinal wave along a solid rod of density ρ , and Young's Modulus E , and show that the rate of flow of energy per unit cross-section is $2\pi^2 a^2 \omega^2 \sqrt{E\rho}$, where a is the amplitude and ω is the frequency.

Two rods, with cross-sections, densities and Young's moduli A_1, ρ_1, E_1 and A_2, ρ_2, E_2 respectively, are joined end to end. Show that a longitudinal wave will pass from one to the other without reflection provided

$$A_1 E_1^{\frac{1}{2}} \rho_1^{\frac{1}{2}} = A_2 E_2^{\frac{1}{2}} \rho_2^{\frac{1}{2}}.$$

(O.)

37. Discuss the different acoustical methods by which Young's modulus can be obtained for a metal available in the form of rods. (Lond. B.Sc.)

38. Describe how you would maintain a tuning-fork in continuous vibration.

Show how a cathode-ray oscillograph could be used to determine the frequency of the sound emitted by a source, and as an indicator in the determination of the wavelength. (*O.F.*)

39. Describe an accurate method of measuring the frequency of a tuning-fork. (*Trip.*)

40. Derive the differential equation for the propagation of transverse waves along a perfectly flexible stretched string, and use it to discuss the motion of such a string. In particular determine the frequencies of the possible modes of vibration and indicate roughly how these will be affected by lack of perfect flexibility. (*Lond. B.Sc.*)

41. Deduce an expression for the velocity of sound in a perfect gas.

Show that if the lapse rate (rate of fall of temperature with height) in the atmosphere is constant, a sound wave moving approximately horizontally is refracted in a circular arc of radius $2T / \left(\frac{dT}{dh} \right)$ where T is the absolute temperature and $\frac{dT}{dh}$ is the lapse rate. Hence show that if the lapse rate is $6^\circ \text{C. per kilometre}$, an aeroplane flying at 1,000 metres cannot be heard by an observer on the ground at a horizontal distance greater than 13.5 kilometres. (The curvature of the earth should be neglected.) (*O.F.*)

42. Discuss the relation between progressive and stationary waves, giving examples from various branches of physics. Describe briefly the characteristics of the stationary wave system either (*a*) in a Kundt tube as used in measuring the velocity of sound, or (*b*) in a violin string. (*Trip.*)

ANSWERS

CHAPTER II (p. 40).

1. 107.2 and 32.6 c.p.s.
2. (a) 70 ft. per sec. approaching. (b) 74.7 ft. per sec. receding.
3. 270 c.p.s.
4. 1,375 c.p.s.
6. 4.59 beats per second.
7. 1,000 beats per second.
8. 2,484 c.p.s.
9. $\frac{\lambda_A}{\lambda_B} = \frac{V - v}{V + v} \cdot \frac{n_A}{n_B} = \left(\frac{V + v}{V - v} \right)^2$,
10. 54.8 ft. per sec.
12. 4.67 beats per second.
13. 15% increase.
16. 2 beats per second, approximately.

CHAPTER III (p. 51).

1. 510 c.p.s.

CHAPTER IV (p. 66).

3. 388.6 metres per sec.
4. 330 metres per sec. : 5 cm. above mark.
9. $53^\circ 8'$.
10. 2,490 c.p.s.
13. 301-310 metres per sec.
23. 4,554 ft.
24. 6,900 ft.
26. From second position, 900 metres, 240° .
29. Monatomic : Argon.
31. 14 miles.

CHAPTER VI (p. 97).

1. 3 beats per sec.
3. 49.6 c.p.s.
4. 4 kg. wt.
5. 132.5 and 137.5 c.p.s.
6. 39.3 gm. wt.
7. A, $8\frac{1}{2}$ kg. wt. ; B, $4\frac{1}{2}$ kg. wt.
8. 300 c.p.s.
10. 56 kg. wt.
11. 1.21 per cent. decrease, assuming that Young's Modulus decreases with temperature rise.
14. $\frac{2\pi^2 a^2 T}{\lambda}$ for a single travelling wave of amplitude a .
 $\frac{\pi^2 a^2 T}{\lambda}$ for stationary waves of maximum amplitude a .

CHAPTER VII (p. 112).

4. 171, 342 c.p.s., etc.
6. Gas/Metal = $2/3$.

CHAPTER VIII (p. 133).

1. 131.7 c.p.s.

CHAPTER XI (p. 161).

3. $\frac{(2k+1)}{4l} \sqrt{\frac{Y}{d}}$ where $k = 0, 1, 2, \text{ etc.}$

MISCELLANEOUS (p. 161).

1. 98.5 and 101.5 cm.
4. 100 ft. per sec. = 68.2 m.p.h.
5. 1.83 sec.
7. 1.39.
9. 0.53, 0.40, 0.33 sec.
11. 2.78.
13. 7 mm. decrease.
14. 268 c.p.s.
16. 4,120 ft.
20. 5 cm.
21. Transverse/Longitudinal = 1/40.
22. Longitudinal/Transverse = 31.6/1.
26. 0.242 erg.
28. 1018.1 and 989.8 c.p.s. or 1010.4 and 982.5 c.p.s.
31. 5,000 c.p.s. (one end clamped, the other free).
33. 2 c.p.s.

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