EXPECTATION IN ECONOMICS

Incertainty that once gave scope to dream
Of laughing enterprise and glory untold.

ROBERT BRIDGES, *Later Poems*

EXPECTATION IN ECONOMICS

BY

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PREFACE

The economist has gradually provided himself with an outfit of most powerful, elegant, and ingenious tools, with which he can now attack with assurance a great variety of important problems. But there is one purpose for which he is extremely ill-equipped. The frequency-ratio concept of probability is suitable and essential for the purposes of mathematical statistics. But as a means of analysing those originative acts of mind, involving degrees of doubt and belief assigned to the products of imagination, which are what I mean by *expectation*, it is essentially and radically inappropriate. I have, then, two tasks. One is to justify in by logic and example my rejection of the frequency-table as a description of *uncertainty*; the other is to provide a new and proper instrument in its place. Although it might be thought that clearance of the site should precede construction, I have preferred to begin with my positive suggestions and to leave to the last chapter my criticism of the orthodox approach. The problem and the germ of the solution here offered were already in my mind in the summer of 1937. Since then a series of published articles has traced the stages of advance towards what I hope is now a presentable theory; but since those articles were written, development, refinement, and clarification have gone far, and those articles can now be considered as no more than sketches for the finished picture. Only three out of the twelve main divisions (chapters and appendices) of the book are based upon the articles; the rest has never been published in any form; and one of these three (Chapter II) has been radically transformed from the embryonic theory of the articles. These articles were as follows:

1. 'Expectations and Employment' (*Economic Journal*, September 1939).
2. 'The Nature of the Inducement to Invest' and 'A Reply to Professor Hart' (*Review of Economic Studies*, October 1940).
3. 'A Means of Promoting Investment' (*Economic Journal*, June–September 1941).
4. 'A Theory of Investment Decisions' (*Oxford Economic Papers*, No. 6, April 1942).
5. 'The Expectational Dynamics of the Individual' (*Economica*, May 1943).
6. 'An Analysis of Speculative Choice' (*Economica*, February 1945).

Except for Chapter VI, which was written in January 1948, my book had been brought to its present form by August 1946.

In the last of the articles listed above I used an indifference-map with positively-sloped indifference-curves. Some months after finishing the article, but before it was published, I found a formally similar map in the admirable article by Evsey D. Domar and Richard Musgrave called 'Proportional Income Taxation and Risk-Taking' (*Quarterly Journal of Economics*, May 1944). The variables measured on their axes, however, are derived from a frequency-table, and in a footnote to that article they reject my conception in favour of the orthodox approach, believing (mistakenly, as I think) that the latter gives results which are in some sense more precise or determinate than mine.

One thing I am most anxious to make clear. A reader of my manuscript has told me that in places it gives the impression of dogmatic assertion. Nothing could be farther from my mind. In those sentences where I have said, for example, that the enterpriser in given circumstances 'will do' this or that, the statement refers to what he will do *on my assumptions* and *in my model*; it is not a sweeping generalisation about the real world.

It remains to acknowledge the debt I owe to those who have encouraged me in this line of thought. Professor Phelps Brown was the first to whom I broached these ideas, and later, though still, I think, unconvinced, he was willing, at a time of extreme pressure of work, to read the manuscript and send me some most penetrating criticism. To Mr C. J. Hitch, Fellow of Queen's College, Oxford, who as Editor of Oxford Economic Papers accepted for publication the fourth of the articles listed above, I owe thanks for brief but greatly valued words. Mr W. E. Armstrong, of University College, Southampton, an ideal critic of a work lying on the borders between logic, philosophy, and economics, has read the greater part of my manuscript, and I have benefited greatly by a comparison of views with him. I have left till the last two names that I wish most specially to mention. Mr Roy Harrod has been generous in the extreme in his willingness to read, criticise, and encourage my work. Mr Austin Robinson is the god-parent of this book; the gratitude I feel to him I cannot well express, and I shall never forget his kindness.

My contacts with the staff of the Cambridge University Press have been the source of much happiness.

G. L. S. SHACKLE

26 *August* 1948

CHAPTER I

INTRODUCTION

By *expectation* I mean the act of creating imaginary situations, of associating them with named future dates, and of assigning to each of the hypotheses thus formed a place on a scale measuring the degree of our belief that a specified course of action on our own part will make this hypothesis come true. Such a hypothesis I shall call an expectation. A treatment of expectation should, I think, try to answer the following questions:

1. What purpose is served by the forming of expectations? What is the form of the questions which they answer, and the answers which they give?

2. What materials are used in forming expectations, and how is an expectation, or a system of expectations, constructed from these materials?

3. What determines the degree of belief accorded to an expectation?

4. How can such degrees of belief be measured and compared?

5. What determines the influence upon our decisions for personal action (including our decisions on what actions to suggest to other people) which we allow a given imagined future course of events to exert?

6. What is the consequence of disagreement between expected and realised situations?

7. Can any general types of development of situations one out of another through time (organic sequences of situations) be distinguished and ascribed to the essential nature of the expectation-forming process and its part in determining our actions?

Of these questions, numbers 1, 4, 5 and 6 have led me to ideas which are, I believe, possible constituents of a *general* theory of expectation, applicable to no matter what concrete subject-matter. Numbers 2 and 3, however, have yielded nothing which can serve as part of a general apparatus of analysis. On the problem of how expectations are derived from the news and the train of recent situations, and on the nature of any 'spectral set of selective operators' by which we could suppose a man to distribute the hypotheses he has formed, in answer to some question about the future, along his scale of belief, I have only incidental suggestions to offer. The answer 'Yes' to question 7 requires for its proper elaboration a theory dealing with questions 2 and 3. Economists, indeed, have been driven to construct rudimentary answers of their own to these two questions for the very purpose of explaining such phenomena as the business cycle. While, therefore, trains of situations, engendered by and engendering expectations, ought to be our ultimate object of study, a systematic treatment will be beyond the scope of this book. But perhaps this is not altogether a disadvantage. The ideas which I have to offer constitute, I think, a distinct and integral conception in themselves, and its novelty will call for a very careful, exact, and complete presentation, such as will get more attention, and be more conveniently judged by the reader, if it has the book to itself. The plan of the latter is accordingly as follows:

The ideas which I wish to propose on the nature and role of expectation and the character of our response to irreducible uncertainty, together with an interconnected group of new tools of analysis derived from them, are described in Chapter II and its Appendices. In Chapters III, IV, V and VI these tools are applied, by way of illustration of their capabilities rather than in direct pursuit of results (though I hope some of these will be found suggestive by economists), to analyse a number of economic processes which take their form and motive from expectation. Finally, in Chapter VII I have tried to set out those grounds of dissatisfaction with the orthodox treatment of uncertainty (or rather, with the settled and traditional evasion by which writers refuse it entry to their theories altogether and cover its absence by an irrelevant discussion of actuarial principles*) which led to this attempt at a radically different attack.

The various arguments of Chapter II being each presented at some length might not at a first reading be readily grasped as a single whole. I shall therefore try to give the reader at once some idea of my line of approach.

Pure curiosity and the search for philosophical truth play their part in human affairs. In these disinterested forms of uncertainty we merely wish to know the answer to some question; we do not mind what this answer is. But sometimes the question is a practical one: What would be the respective outcomes of a set of rival courses of action? And here we are by no means indifferent between the several hypotheses concerning the outcome of any one course. We have to decide on one course out of all that seem open; we have

* Which are a means of *reducing or eliminating* uncertainty in circumstances where it can be so eliminated, not of *digesting, enjoying, and turning to psychic profit* the uncertainty in those many situations where it *cannot* be removed.

therefore to make some particular *assumption*, fairly clear-cut and definite, about what the outcome of each course would be; and in the task of selecting or creating this assumption, we shall be influenced by desire and imagination as well as by rational belief or unbiased intuitions. There will be so-called 'wishful thinking'; but we shall see that there is a form of wishful thinking that is by no means stupid or even illogical, but is, indeed, the natural and reasonable response of human nature to intractable uncertainty; the main attractiveness of a given course may spring paradoxically from a hypothetical outcome which is regarded by the individual himself as less likely than some others. Let us see more precisely how this may be.

In some situations we prefer a course of action whose relevant consequences cannot be known in advance, and can therefore be imagined as high success or as catastrophe, to a course whose relevant consequences are known, and are dull and unexciting, or, while making it certain that we shall not suffer a general crippling of our powers of action for the further future, also make it *certain* that some particular purpose will not be attained. In short, we are sometimes willing to forgo the certainty that some specific bad thing will not happen, for the sake of escaping from the certainty that some specific good thing will not happen. For example, a military commander may have to choose between keeping his forces intact by attempting nothing, and throwing them into an enterprise whose outcome, for all he can tell, may be an historic victory or, for all he can tell, may be a disaster. A more clear-cut example is that of a man proposing to make a bet, who has to decide whether he prefers to know that his pound will remain a pound, and only a pound, or to be in ignorance whether it will become ten pounds or nothing. And still another example, which can serve for this first glance at our line of approach, is that of an enterpriser proposing to lay out money on constructing concrete equipment.

Let us exclude by assumption the possibility of lending money at fixed interest, so that an enterpriser having at his disposal a given sum of cash has the choice between (*a*) retaining his cash, and thus experiencing in his mind the certainty that his wealth in terms of cash will remain unchanged, and (*b*) creating equipment, and thus dissolving the only barrier of *certain knowledge* which would otherwise both prevent him from enjoying in imagination a very high level of success, and at the same time defend him from suffering in imagination the loss of the means for further ventures. Let us suppose first of all that there is only a single type of investment-opportunity, and that this has only two possible outcomes: the enterpriser knows that the sum C which he lays out will at the end of some given period either have become A, greater than C, or have become B, less than C. And let us suppose that there is nothing which inclines him to feel that one of these outcomes would surprise him more than the other, if the assemblage of indications and his own frame of mind were to remain unchanged until the result will be known. When he is deciding whether or not to invest, he will place himself in imagination in the situation of having actually done so; if the gain $A-C$ is big enough, the enjoyment of it in anticipation will outweigh the contemplation of the other possibility, the making of the loss $C-B$; then the venture will seem preferable to retaining his wealth in cash, and he will decide to invest.

This appears at first sight to be a simplified case; investment-opportunities do not in reality offer only two possible outcomes, but a continuous range, from large gains to the loss of the whole sum invested. And not all outcomes will seem to the enterpriser to have equal power, or lack of power, to surprise him, supposing he remains in his existing state of mind. Some would certainly cause great surprise, and others some surprise, if one of them were to occur without his having altered his existing conception of the current situation and the ones that will grow out of it. Nevertheless, we shall suggest that the above example is in essentials typical of reality.

In forming our expectations we feel it easy to exclude some outcomes as 'impossible'. The actual occurrence of any one of these would cause us such extreme surprise, if we were to retain until then our existing assessment of the situation, that they are simply ignored. With other outcomes we associate a rather less degree of potential surprise, with others less still, and so on, until we are left with a range of outcomes any one of which could occur without surprising us at all. In general, there are two possible reasons for the individual to pay more attention to one of a pair of outcomes than the other: first, there may be a difference in the respective degrees of potential surprise he associates with them, and secondly, when both outcomes are desirable or when both are undesirable, one outcome may be more desirable or less desirable than the other. But among all the outcomes for which his potential surprise is *nil*, there will be only one reason for him to concentrate his attention on some rather than others, namely, that some are more to be desired or the opposite. I suggest, therefore, that when he contemplates this inner range of outcomes, each of which carries no potential surprise, the enterpriser does, in fact, concentrate his attention exclusively on the *best* and the *worst* hypotheses in this range.

If the motive for investing is to give free play to the investor's hopes, and allow him to enjoy in imagination a very high level of success, why should he confine himself to the best outcome which could occur without surprising him? It seems certain that, if some still better outcomes would only surprise him a little, these will outbid for his attention even the best of the 'inner' range. But some critical point there will be, beyond which the increase in the degree of potential surprise will outweigh the associated increase in the amount of gain. This critical point is one of the two on which the investor's attention will be focused. The other may, correspondingly, be the loss of a rather larger proportion of the sum initially invested than the biggest proportion which carries no potential surprise.

To summarise this conception, we suggest that an enterpriser who is deciding whether to invest or not will place himself in imagination in the position of having actually laid out a cash sum on constructing concrete equipment, and will then weigh against each other the two elements of the immediate mental experience which this position would afford him: the enjoyment by anticipation of the greatest gain whose attractiveness is not undermined by association with too high a degree of potential surprise, and the suffering, by anticipation, of the greatest loss whose unpleasantness is not weakened by being associated with too high a degree of potential surprise. It is these two *extremes* which will focus the enterpriser's attention.

The value which a person sets on a speculative asset is essentially and logically (and not merely as a matter of mathematical convenience) the sum of two components of opposite sign: there is the positive component deriving from the hopes and the negative component springing from the fears which the possession of a speculative asset engenders. The two variables of which the value is thus a function, the hope of gain and the fear of loss, are in the general case mutually independent. Thus it is valuable to have a device which can display the separate movements of these variables, and not merely take account of their resultant or net effect. By this means, for example, we can classify the kinds of events which move one and not the other, which move them in opposite directions or in the same direction, and in similar or in different degrees; and this may enable us to disentangle many complexities of the movements of the market value of speculative assets. The line of thought we have briefly traced above provides us with such a device, and this and other ideas arising from it will be a main tool of analysis in what follows. In the next chapter we shall try to give these ideas a greater precision and an adequate logical foundation.

In what relationship does such a *theory of expectation* stand, the reader may ask, to the traditional theories of *probability*? Our detailed answer will be attempted in Chapter VII. Meanwhile, the reader is entitled to some brief indication before he embarks.

The theory of probability, in the form which has been given to it by mathematicians and actuaries, is adapted to discovering the tendencies of a *given* system under *indefinitely repeated* trials or experiments. In any set of such trials, each trial is, for the purpose of discovering such a tendency, given equal weight with all the others. No individual trial is considered to have any importance in itself for its own sake, and any tendency which may be inductively discovered, or predicted *a priori*, for the system, tells us NOTHING about any *single* individual trial which we may propose to make in future. It follows that in forming expectations, actuarial general principles and particular facts will only help us when the following conditions are satisfied:

1. We are sure that the system, whose future behaviour we wish to know, will remain a *given* system and not undergo changes during the interval of future time in question.

2. We are interested only in the total result of a 'large number' of trials, all of which count equally or virtually so in building up this total.

3. *We feel sure that we shall, in fact, have the opportunity* to carry out a sufficient number of trials, and not, for example, find that our plans depended, in fact, on the successful issue of a few early trials whose failure has deprived us of the means of continuing.

In many of the most important affairs of life these conditions are not fulfilled. The conditions which have led to a given frequency of success for a given type of business enterprise in the past cannot, perhaps, be counted on to continue in the future; if they do not, the 'system' will have changed. Perhaps a more important matter is that great acts of risk-taking, in which an enterpriser ventures an important proportion of his resources on a block of equipment which will succeed or fail as a whole, are not performed by each enterpriser every month or every year; they can be engaged in only at long intervals. But the enterpriser cannot, even if he has the 'immortality' of a joint-stock company, attain large numbers for the purpose of offsetting the possible failure of one such gamble against the success of another, by proposing to himself a series of such acts stretching over a century or more; for in order to allow for time-discounting at compound interest, which would otherwise reduce the more remote members of the series to negligible 'present value', the scale envisaged for these distantly future members would have to be gigantic.

Even more fundamental, and more theoretically interesting, is our third consideration: What 'large numbers' do is to turn ignorance into knowledge, or doubt and fear into assurance. But this assurance that there will be no disaster provided our gamble is many times repeated can give no comfort unless there is assurance that it will be many times repeated. If shilling bets are the smallest which can be made, and I have only a shilling in the world, I cannot apply actuarial knowledge to my *first* bet; for suppose I should lose...?

Finally, there are some kinds of decision, of the highest importance for the happiness of individuals or the well-being of whole peoples, which are, actually or virtually, by their nature *unique*; each man in choosing a career or a wife, a general in choosing his plan of campaign, may well consider that his decision is 'once and once only for ever'.

Our last preliminary task is to explain our handling of certain topics which are of the most general importance for our whole subject, and for which, none the less because they constantly arise in the loose language of every day, it is essential to have a set of exact ideas and terms.

By a *calendar date* I mean a named point of time, specified in principle by its year, month, day and hour; and by a calendar interval the interval separating two calendar dates, for example, the month of February in some specified year. Our subject in this book is the analysis of some of the thoughts and decisions of an individual. By his *viewpoint* we mean the point of time at which those thoughts and decisions, with which the argument is for the time being concerned, are taking place in his mind. Thus we think of the calendar as an axis graduated into intervals by named calendar dates, and of the individual's viewpoint as a variable point of time for which different locations on this axis can be successively selected. Amongst the essential characteristics of any thought which can be classified as an expectation there is, besides the *viewpoint* or instant when this thought is generated, the *image-date* or point of time with which it is concerned. This image-date must also be looked on as a variable point of time, and the viewpoint and image-date as variable independently of each other, since the same named calendar date can be the subject of a man's thoughts at successive dates through which his viewpoint advances towards it, and since, on the other hand, in any named short interval (brief enough for its length to be neglected) his thoughts can travel over a succession of different image-dates.

The expression *change of expectations* carries two meanings between which it is vital to distinguish. We have defined an expectation as an association of a particular hypothetical state of affairs, a particular future date and a particular degree of belief. The future date in question is what we have just called the image-date, and, as we have pointed out, a given image-date can remain the object of attention while the viewpoint advances towards it through a succession of different calendar locations. If this advance of the viewpoint is accompanied by a modification of the state of affairs associated by the individual with a *given* image-date, this is one, and the primary, meaning of change of expectation. This, however, is a crude way of expressing the matter. When we say that the individual modifies the state of affairs 'associated with' a given image-date, we really mean that he changes the degree of belief accorded to different associations of states of affairs with that date, perhaps promoting one to a position of complete ascendancy, so that we can say he feels certain of this event; while others formerly dominating his mind are reduced to the level of possibilities which he excludes from his working assumptions. This, then, is one meaning. The advance of the viewpoint need not, of course, be accompanied by any such change; the situations expected, with given degrees of belief, or, as we shall prefer to say, with given degrees of potential surprise, to exist on given calendar dates, may remain unchanged until those dates are actually reached. But in another sense the individual's forward vista must inevitably change as the viewpoint advances; those dates which are reached and passed are of course eliminated from it, while all those calendar dates which still lie ahead are brought nearer. Thus if, instead of considering named, fixed calendar dates, we consider dates or intervals at fixed distances ahead of the moving viewpoint, then each shift of the viewpoint from one calendar date to another brings a fresh set of situations or events into the foreground or the middle distance, as it were, of the forward vista of time. There is perhaps a third sense of changes of expectation which we ought to distinguish. The relative degrees of belief accorded to different contingencies may change while leaving all contingencies in the same order in respect of the belief they command; those contingencies to whose vividness in the individual's mind the contribution made by belief is highest may be the same ones now as before, but this contribution may be even higher for them. Or the range of different contingencies carrying this high degree of belief may have contracted, so that its most extreme members at either end are now less dissimilar from each other. Both these phenomena we shall comprise under the name of clarifying of expectations; but we shall be able to give a much more clear-cut definition of this term when, in the next chapter, we have developed the concept of potential surprise.

The desire to use the word *value* for each of two quite distinct purposes has occasioned me some trouble. It is indispensable in its algebraic sense in any general and abstract quantitative discussion. In algebra a variable quantity is a set of particular quantities, each constant and unique, to any one of which we can point. When we point to any one particular member of the set, we thereby specify a *value* of the variable. On the other hand, in the field of economics the word value has a more special meaning. When the algebraic sense of the word is intended, I shall use it without qualification. Whenever it is to be understood as a term of economics, it will be accompanied in what follows by some qualifying word, whatever seems most appropriate to the context, so that we shall speak of 'subjective value', 'money value', or 'economic value', and so forth.

CHAPTER II

THE NATURE OF EXPECTATION

We decide on one particular course of action out of a number of rival courses because this one gives us, as an immediately present experience, the most enjoyment *by anticipation* of its outcome. Future situations and events cannot be experienced and therefore their degree of desirableness cannot be compared; but situations and events can be *imagined*, and the desirableness of these experiences which happen in the imagination can be compared. What gives imagined things a claim to be treated as the equivalents of future things? It is some degree of belief that the imagined things will take actual shape at the dates we assign to them. This implies that the valid comparison is not between imagined situations or sequences of situations considered without regard to their possibility of becoming true, but between these when each has been assigned some particular degree of belief. Thus the entity which gives us enjoyment by anticipation (or distress by anticipation) or, as I shall say indifferently, by imagination, has two sets of characteristics. The first set specifies or describes the situation or sequence of situations, saying what it would be like if it were to happen (without saying anything as to whether it *will* happen). And it can do this in concrete, and even in quantitative terms: the picture can consist, for example, in a meeting with an actual and living friend, or the conferment on us of some honour, or success in some attempt; the degree of this success may even be capable of statement in numbers, as, for example, the number of marks gained in an examination. The other set (of one member only) consists merely in our degree of belief that this picture will come true. But how is this degree of belief presented to our minds, and put on the same footing of concreteness, of capacity to influence our decisions, as the feelings aroused by the content of the picture? What *is* belief in terms of feeling, of actual mental experience?

The state of mind which accompanies a feeling of certainty or a high degree of belief is one of *repose*. A man who is making plans on a basis of working assumptions which he feels to be very doubtful is always, as it were, looking over his shoulder at these assumptions, on the watch for events which would compel him to abandon them; he is on the alert, and the occurrence of such events would not shock him to the same degree as if he had fully accepted his working assumptions. It is only a man who feels very sure of a given outcome who can be greatly *surprised* by its non-occurrence. A degree of belief is not in itself a sensation or an emotion; but a high degree of belief is a condition of our being able to feel a high degree of surprise. The concrete mental experience which corresponds to any given degree of belief in some particular hypothesis is, I think, the degree of surprise to which this belief exposes us (supposing it to remain unchanged until the truth will be known) and will subject us in case the hypothesis proves false. Accordingly, we can use the degree of surprise which we judge would be caused to us by the non-occurrence of a given outcome, supposing there had been meantime no change in our relevant knowledge, as an indicator of our degree of belief in this outcome. The range of possible intensities of surprise lies between zero and that intensity which would arise from the occurrence of an event believed impossible, or held to be *certain* not to occur. Within this range each one of us will find in his own past experience particular occurrences or types of occurrence each of which has caused him some degree of surprise, the memory of which remains with him vividly enough to serve, in conjunction with the memory of the event which caused it, as one of a series of fixed levels, not necessarily evenly spaced, providing together a scale of potential surprise. We shall discuss in the Appendix to this chapter the question whether a unit can be defined such as to make possible comparisons between degrees of surprise experienced by different individuals. For the main part of the analyses attempted in this book, however, we are concerned only with the feelings of one person at a time, and it is sufficient if we can claim that the existence in his mind of such a series of fixed levels, with the possibility of interpolations between them, entitles us to treat surprise as a continuous variable defined in a certain range, and subject to manipulation by the methods of the differential calculus. This I propose to claim. Thus we shall say that a person can compare his own respective degrees of belief in two different outcomes of some course of action or two different answers to a question by taking each of these outcomes or answers in turn and asking himself what intensity of shock or surprise he would feel if, without there having been meantime any change in the knowledge available to him on which he based his belief in it, he were to learn that this belief is mistaken. The measure so obtained is what we may call the *potential surprise* associated, by a particular person at a particular date, with the falsity of the answer or the non-occurrence of the outcome. This formula, however, is not quite satisfactory for our purpose. In answer to any question about the future, there will typically be in the mind of any one person a number of rival hypotheses, and amongst these there will be a subset of which each member is superior, as regards his degree of belief in it, to any hypothesis outside the subset, but of which no one member is superior to any other member. In this case he cannot attach any degree of surprise greater than zero to the falsity or non-fulfilment of any one particular member of this subset; for to do so would *ipso facto* mark this member off as claiming a higher degree of his belief than any of the others. The most he can do is to attach nil potential surprise to the *fulfilment* of any member of the subset. But he *can* attach some positive degree of potential surprise to every hypothesis *outside* the subset, and by doing so he will express its inferiority, in the matter of the degree of his belief in it, to every member of the subset. And further, he can attach different degrees of potential surprise to different hypotheses outside the subset. It will be convenient, therefore, to invert our formula, and say that by assigning different degrees of potential surprise to the occurrence, rather than the non-occurrence, of different hypothetical outcomes, he assigns to these outcomes positions on a scale of belief.

The subset referred to above we shall call the 'inner subset', or in case the total set of rival answers to any question about the future consists in the values of a continuous variable, we shall call that range within which every value carries nil potential surprise the 'inner range'. Contexts or cases may arise in which there will be two or several such ranges, separated from each other by ranges over which the potential surprise is greater than zero. But in many instances it is the small or moderate numerical values of the variable, both positive and negative in some neighbourhood of zero, which will be assigned nil potential surprise, while all others will carry it in some positive degree increasing with their numerical size. For our purpose in this chapter of describing our main tools of analysis we need only study cases where there is a single 'inner' range. It is likely that the degree y of potential surprise associated with a continuous variable x will itself be a continuous function of the variable, and we shall call the function $y = y(x)$ the potential surprise function. In all cases, at least one value of x must carry nil potential surprise. Typically, however, in such a field as the economic, the difference between the highest and the lowest values of x for which $y = 0$ will be considerable in relation, for example, to the actual value of the same variable at the moment from which the individual is looking forward. As we consider values of x increasingly remote from the inner range, either above or below it, the degree of potential surprise considered as a function of this remoteness will ordinarily increase monotonically, so that above the upper extreme of the inner range y will increase with increase of x, while below the lower extreme y will decrease with increase of x. And since the range of values carrying nil potential surprise will ordinarily merge imperceptibly into those carrying some positive degree, there is likely to be some range of values immediately above the upper extreme of the inner range, and another immediately below the lower extreme, over which the rate of increase of y with increasing remoteness from the inner range will itself increase. This curvature will be reversed, however, over two ranges extending inwards from those two values of x for which our

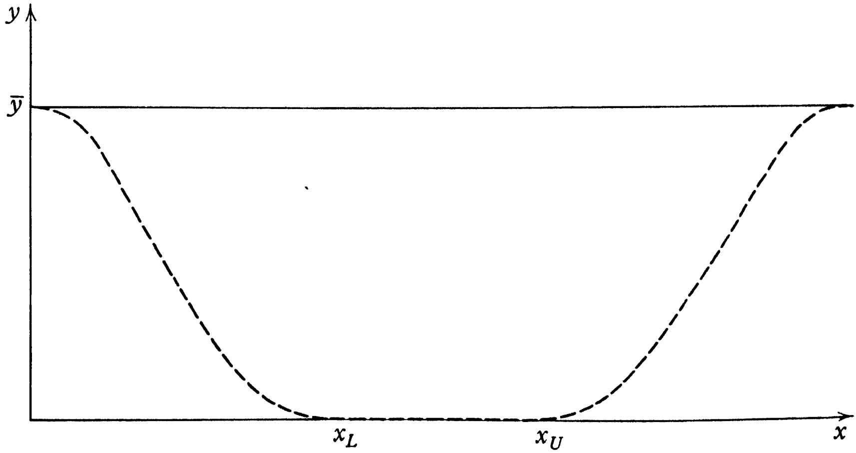

FIG. II 1. x_L, lower extreme of inner range; x_U, upper extreme of inner range.

potential surprise is the highest we can conceive ourselves to feel, corresponding to absolute rejection of the possibility of the hypothesis in question. For this absolute rejection is likely at the last to be gently and hesitantly approached, through a fair range of values which are still just admitted to be barely conceivable. Thus the increase of potential surprise as we pass outwards through those values of x which are regarded as 'next door to impossible' will be decreasingly rapid, and $y=y(x)$ (with y measured on the vertical axis) will tend to become horizontal. Thus $y=y(x)$ will resemble Fig. II 1. For convenience we shall hereafter refer to potential surprise curves of this general shape as bell-shaped curves. The degree of potential surprise which corresponds to absolute rejection of a hypothesis, absolute disbelief in the possibility of an outcome, we shall call the *absolute* maximum of potential surprise. This is the highest level that y can ever attain. We shall write it $y=\bar{y}$, and represent it in Fig. II 1 and subsequent diagrams by a straight line thus labelled lying parallel to the x-axis. We shall find it convenient to define the upper branch of the curve as all that part of it which corresponds to *higher* values of x than the *lower* extreme of the inner range, and correspondingly we define the lower branch as all that part of the curve which corresponds to *lower* values of x than the *upper* extreme of the inner range. The reason for this definition, in which the inner range belongs to both the upper and lower branches, is to allow us to refer briefly to any change of form of $y(x)$ which assigns higher values of y to all values of x above some particular x, even when this particular x lies in the former inner range.*

The entire set of advantages and disadvantages which we attribute to our adoption of one course of action rather than another can sometimes be reduced to the value of a single variable. Such is the case, for example, when we are faced with the choice between a number of different investment-opportunities or the speculative purchase of different assets. When the question is whether or not to buy at a given price a specified outfit of productive equipment, we are simply concerned, as a rule, with the difference between this price and the total discounted net returns to be expected from the use of the equipment. It is mainly in such senses as this that we shall use the word outcome in subsequent chapters, and in discussing the

* I have had no opportunities for systematic psychological experiments, and for some years after I first put forward the concept of potential surprise, in my article 'Expectations and Employment' (*Economic Journal*, September 1939), it rested so far as I was aware purely on my own intuition and logic, and I did not know that any similar idea had ever occurred to anyone else. In March 1942, however, I was excited to find in Professor Sir Cyril Burt's superb book, *The Factors of the Mind* (p. 121, footnote), the following reference to N. F. Campbell's *Measurement and Calculation*. Professor Burt says:

'In an earlier chapter still he explicitly recognises "degrees of knowledge"; and suggests that the "degree of knowledge is measured by the subjective mental discomfort we should suffer if we found it was not true" (p. 160). When endeavouring to show the objective or impersonal character of judgments of beauty, I carried out parallel experiments on judgments of truth. In both cases, according to the introspections, "degree of mental discomfort" was a common criterion.'

The term 'degree of knowledge' is an excellent one, and I might have felt tempted to adopt it, had I not in evolving my own concept invented the other term; and I now think that the latter is superior, for it suggests implications, developed in the subsequent chapters of this book, which the other term does not. Nevertheless, the two concepts are basically the same, and I feel that my analysis gains greatly in authority for having the reality and truth of one of its underlying ideas thus attested by experiment and vouched for by so eminent a professional psychologist.

potential surprise function $y=y(x)$ we may think of x as standing for some such measure. We need not, however, invoke this simplification just yet, and for the moment we shall mean by outcome the specific course of events in all those respects, recognised by the individual as affecting him favourably or unfavourably, in which the sequel to one choice of action on his part differs from that of another choice. Experience teaches us that except in the most trivial sorts of case a person faced with a number of rival courses of action amongst which he must choose will virtually never be able to specify for each of these courses a unique outcome which he can feel completely certain will turn out to be the true one. For any one course of action he will have in mind many different hypothetical outcomes, and out of these there will be a large number, any one of which would cause him some degree of surprise if it were to turn out to be the actual one without there having been in the meantime any change in the knowledge on which he is now basing his expectations. The degree of this potential surprise will differ between different hypotheses. Some will seem to involve assumptions which conflict with elements in his viewpoint situation, and therefore to involve the further assumption that special factors will arise, of which there is no evidence at present, capable of cancelling these adverse elements. The potential surprise attached to these hypotheses may be extreme. Others will seem less potentially surprising, and others less still, in descending steps which will lead ultimately to the subset of hypotheses for each of which the potential surprise is nil. Now it is clear that in all ordinary cases this 'inner' subset must comprise at least one member; but the knowledge which any person can possess of the present intentions and means of action of other people, and of what will be their reactions in the further future to each other's more immediately future acts, is so extremely slight and insecure that, in reality, the inner subset will always consist of a large number of hypotheses whose mutually most dissimilar members will differ from each other very widely. This conclusion brings us to our central problem: How will an individual choose between different courses of action when the outcome of each cannot be known in advance, but is only represented in his mind by a whole set of differing hypotheses?

In order to assess the merits of any given course of action, a man must find some way of reducing the great array of hypotheses about the relevant consequences of this course, which will usually be present in his mind, to some compact and vivid statement. The real incentive for embarking on some given venture, whose objective results will not develop and their character become known until some date in the future, is the *immediate* mental experience which the decision to embark on this course will give us, namely, the *enjoyment by anticipation* of a high level of success. For any specified hypothetical outcome associated with some named future date, there will be in any person's mind a set of actions on his own part, distributed, in general, over a number of dates, whose performance at the due dates he regards as necessary (though not usually sufficient) conditions for the realisation of this outcome. At any time when he has performed such of those actions as have already fallen due, and is resolved and believes himself able duly to perform the rest, he will be able to enjoy by anticipation the outcome in question, even if his degree of belief in its realisation is less than certainty. I do not believe that the impossibility of feeling *certain* that a particular unique result will be attained by the contemplated course of action implies that the individual will not desire a unique focus for his imagination, that is to say, that he will not centre his hopes on *one particular* level of success. He will, it seems to me, desire a single clear-cut mental image concerning the outcome to provide the *content* of his hope, and this will need to be given, as it were, an extra dimension, by association with a sufficiently low degree of potential surprise, to render it a motive force entering into his decision. There is a phrase of common speech which seems to throw much light on this matter; such phrases, moulded unconsciously by the actual modes of working of our minds, should surely be a safe and fruitful source of instruction: we speak of having 'a *lively* expectation' of something. This must surely mean a *life-like, vivid and active* expectation,* one which carries conviction by the realism and insistent presence, as it were 'in the flesh', of the imagined thing. A lively expectation, something which seems a living projection of the future into the present, is what is needed to induce us to take action now to attain the reality. This 'liveliness', this realism, cannot conceivably be possessed by a bundle of divergent hypotheses or conceptions; it must require uniqueness in the mental image on which we finally focus our attention, even if there is no such uniqueness in the contingencies we pass in review in our minds. Again, the enjoyment by anticipation of a favourable result will not be experienced unalloyed, except in the rare case where the outcome is felt to be uniquely certain. In most cases there will be some positively hurtful outcomes which will be not less insistently

* We also say 'This will *quicken* expectations (of something)', 'a *quickened* expectation', the verb 'to quicken' meaning here, of course, 'to bring (vividly) to life', 'to kindle'. This phrase brings out even more strongly the idea of an expectation as a lively and active element in our minds, insistently claiming attention.

present to our minds than the favourable ones. Here again the individual will tend, I think, to concentrate his fears at some specific point, selecting *one particular* degree of misfortune to represent what he 'stands to lose'. The question that we have to answer, if this view be accepted, is how will the particular level of success upon which he will settle his hopes, and that other level which will epitomise his fears, be determined from amongst the whole range of hypothetical outcomes which in varying degree he entertains?

The answer to this question follows directly from two postulates:

First, that hypotheses representing different levels of success will differ from each other in their power to afford the individual enjoyment by imagination, to *stimulate* him agreeably; and that hypotheses representing different levels of misfortune or disaster will differ from each other in their power to cause him distress by imagination, to stimulate him disagreeably.

Secondly, that the power of *mutually exclusive* hypotheses of success to afford enjoyment by imagination is *not additive*, and that therefore the power of the entire set of hypotheses of success associated with any course of action to afford enjoyment by imagination is simply that of one alone amongst these hypotheses, whichever has this power in higher degree than any of the others; and similarly, that the power of the entire set of hypotheses of misfortune associated with this course of action to cause distress by imagination is simply that of the most powerful amongst these hypotheses.

For it is natural to say, I think, that if amongst all the hypotheses of success which the individual could entertain in regard to any venture, one alone is accountable in full for the enjoyment which he derives from the thought of this venture, and by itself determines the intensity of this enjoyment (when he dwells only on the possibilities of success); and if amongst all the hypotheses of misfortune which he could associate with the venture, one alone is responsible for the full intensity of distress which he feels, when he dwells on the possibilities of misfortune; then his attention will be specially and even exclusively *focused* upon these two hypotheses. The second of these postulates makes the same kind of assertion as the familiar propositions: 'the strength of a chain is the strength of its weakest link'; 'the pace of an army is the pace of its slowest man'. It is discussed further in Appendix C to this chapter, which seeks to throw into higher relief the matters here under discussion, and to which the reader's attention, if he feels any difficulty in accepting the argument of this chapter, is very specially directed. Let us turn now to the first of the two postulates.

The intensity of enjoyment of a given hypothetical outcome by imagining it in advance is no doubt a function of several variables, but two of these are, I think, likely to be dominant; this intensity will plainly be an *increasing* function of the *desirability* of the outcome in question, and a *decreasing* function of the degree of potential surprise associated with it. If the outcome in question, instead of being desirable, is positively hurtful or disadvantageous, the experience can be called 'distress by anticipation', and this again will have an intensity increasing with the degree of hurtfulness or disadvantage and decreasing with the degree of potential surprise. Thus there are two possible reasons for the individual to find one of a pair of hypotheses more keenly stimulating to his imagination, either agreeably or disagreeably, than the other: first, there may be a difference in the respective degrees of potential surprise he associates with them, and secondly, when both outcomes are advantageous one may be more advantageous than the other, and when both are hurtful one may be more hurtful than the other. Among all the outcomes for which his potential surprise is nil, there will be only one reason for him to concentrate his attention on some rather than others, namely, that some are more desirable or more undesirable in their *content*, and it seems clear, therefore, that when he contemplates this inner subset of outcomes, each of which carries no potential surprise, the individual will concentrate his attention on the *best* and the *worst* hypotheses in this range. But evidently he need not confine himself to considering only the inner subset; outside it there may be outcomes even more desirable or hurtful than any of those inside it. But it will usually be true that outside the inner subset the greater the desirability or hurtfulness of a given outcome the higher the degree of potential surprise it will carry; and since, for some outcomes, the potential surprise is such as to imply absolute disbelief in the possibility of these outcomes, and since in such a case there can be no enjoyment of these outcomes by anticipation, there will be a point beyond which no outcome offers a sufficient extra advantage (or extra detriment) over the next most desirable (or hurtful) to compensate for the extra potential surprise which it carries. At such a point the total differential of the degree of enjoyment by anticipation, or distress by anticipation, will be zero, and the degree of enjoyment or distress a maximum. At these two points will be found that particular pair of hypotheses which will mainly capture the individual's attention, will represent for him what he 'stands to gain' and what he 'stands to lose' by committing himself to the course of action in question, and will epitomise for him its attractiveness in comparison with others. Such pairs of hypotheses cannot themselves, however, serve directly as the means of comparison between rival courses of action. For any hypothesis so determined carries some positive degree of potential surprise, and these degrees will in general be different for all the different hypotheses occurring in any set of such pairs. Before it is legitimate to compare, for example, the 'favourable' member* of any such pair with the favourable member of another pair, both must be expressed in such terms that the potential surprise is equal for both. For the purpose of our formal analysis, therefore, we shall suppose the individual to ask himself, in regard to each hypothesis of the pair, what hypothesis carrying nil potential surprise would have exactly equal power to stimulate his interest, and would afford him the same degree of enjoyment (or distress) by anticipation. His method of comparing amongst themselves the resulting pairs of hypotheses, one pair for each of the rival courses of action, so as to pick from amongst all these courses the one most preferred, we shall suggest below. By the mental process we have described, the individual reduces any uncertainty-situation to the simplicity of an ordinary bet, in which only two possible outcomes are considered, one of which is a definite amount of gain and the other a definite amount of loss. It remains to show formally, for the case where the hypotheses are values of a continuous variable x, that there will in fact ordinarily be two determinate values of x, for one of which the enjoyment, and for the other the distress, by anticipation is a maximum.

Let us write $\phi = \phi(x, y)$ for the intensity of the experience, whether of enjoyment or distress, due to the anticipation of any given value of x with its associated potential surprise. We shall also speak of ϕ as measuring (or rather, as *ranking* or assigning *relative* levels to) the degree of (agreeable or disagreeable) *stimulus* imparted to the individual's mind by the thought of any particular pair of values (x, y), or as ranking the 'interestingness' of such a combination. By all these expressions we shall mean the same thing, and shall be asserting that there is a function of x and y whose values, by showing two maxima, determine which two particular hypotheses *matter* to the individual, according to his own view. Since $y = y(x)$, we can write $\phi = \phi\{x, y(x)\}$. Now let us suppose that at some value within the inner range x is such that the thought of it causes neither enjoyment nor distress; and for simplicity let us assume that this neutral value of x happens to be $x = 0$. Over the interval of x between zero and the

* I.e. the one representing success.

upper extreme of the inner range $dy/dx=0$ and therefore $\frac{\partial\phi}{\partial y}\frac{dy}{dx}=0$ while $\partial\phi/\partial x$ is positive. Over this interval therefore ϕ will increase as x increases. Beyond the upper extreme of the inner range, dy/dx is positive, and therefore, since $\partial\phi/\partial y$ is everywhere negative, $\frac{\partial\phi}{\partial y}\frac{dy}{dx}$ is here negative, while $\partial\phi/\partial x$ is, of course, still positive. Now for values of x which the individual regards as impossible, ϕ will be zero. But in most cases there will be finite values of x which he thus rejects. Between the least of such values and the upper extreme of the inner range, there will, therefore, be some value at which

$$\frac{\partial\phi}{\partial x}+\frac{\partial\phi}{\partial y}\frac{dy}{dx}=0,$$

and here ϕ will be a maximum. Turning to the negative values of x let us put $z=-x$, so that increasing values of z stand for increasing degrees of hurtfulness or disadvantage in the outcome. Then an argument similar to the one above shows that there will be a point where $\frac{\partial\phi}{\partial z}+\frac{\partial\phi}{\partial y}\frac{dy}{dz}=0$, and here ϕ will again have a maximum. The two values of x for which ϕ is a maximum we shall call the 'primary focus-values' of x, or the 'primary focus-outcomes' of the course of action under consideration. When these outcomes are a gain or loss expressible in money, we shall call these the 'primary focus-gain' and the 'primary focus-loss'.

The function $\phi=\phi\{x, y(x)\}$ will resemble in shape the one shown in Fig. II 2. This figure ought, of course, to be a three-dimensional model with, for example, the xy-plane horizontal and the ϕx-plane vertical. In Fig. II 2, however, the ϕx-plane is considered to have been rotated about the x-axis through one right angle, so that both planes lie flat on the page. Thus in the upper half of the diagram we see the projection of the ϕ-curve on the ϕx-plane, while in the lower half we see the y-curve placed so that each point on it lies in a position on the page exactly beneath that point on the ϕ-curve which corresponds to it. In the true three-dimensional model, $\phi=\phi\{x, y(x)\}$ would, of course, be a twisted curve whose projection on the xy-plane would coincide exactly with the y-curve. We shall call $\phi=\phi\{x, y(x)\}$ the *stimulation function*.

It has been pointed out to me by Mr R. F. Harrod that the notion of focus-outcomes as the principal criteria of an enterpriser's decisions involves some element of paradox. For it assumes the individual to guide his conduct by reference to outcomes which he expects not to happen, either of which, that is to say, will surprise him if it actually occurs. But I do not find this mode of decision to be out of keeping with what we observe every day. A man may insure his life, although the idea of an early death is not real or vivid to him; he may insure

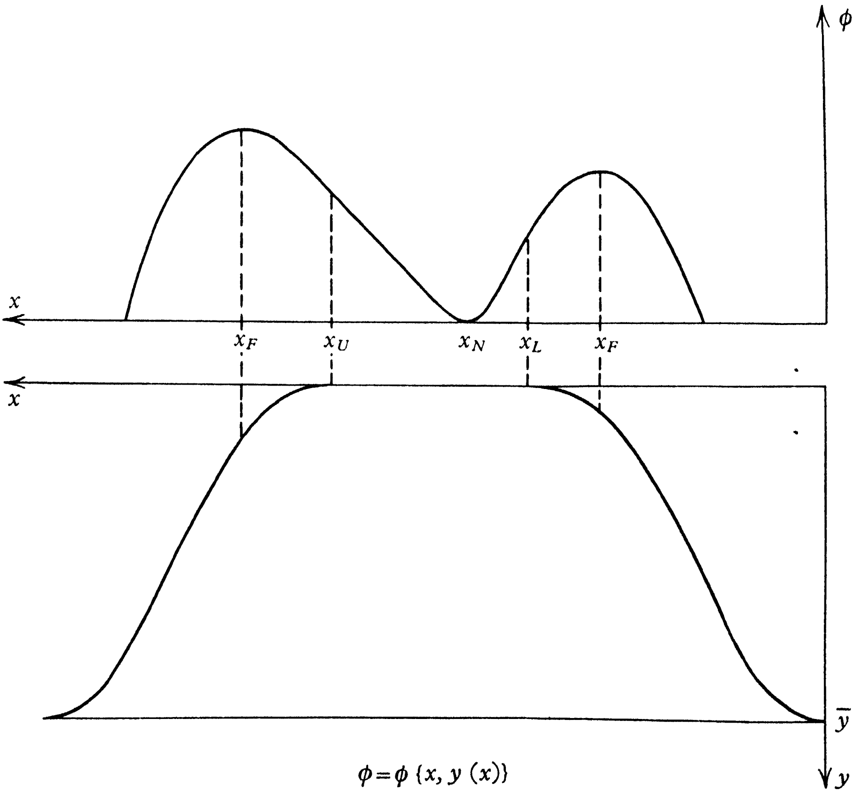

FIG. II 2. x_N, neutral value of x; x_U, upper extreme of inner range; x_L, lower extreme of inner range; x_F, focus-values of x.

his house against fire, although he does not really think that a fire will ever happen. I have a lightning conductor on my chimney, although I should consider it remarkable if my house were actually struck by lightning. I apply for posts to which I have only a remote hope of being appointed. Many a man buys a sweepstake ticket who would be astonished if he won. Thus, although formally my theory seems paradoxical, I think that in this it does no more than reflect human nature itself.

Our argument so far has yielded two results: first, the suggestion that the individual will assess the attractiveness of any course of action by reference to only two hypotheses concerning the outcome of this course, out of all those which present themselves to him as possible; secondly, a suggestion as to his manner of determining or selecting these two hypotheses. The means of this selection was a function ϕ, into whose character we must now further inquire.

Hitherto we have mainly considered the twisted curve $\phi = \phi\{x, y(x)\}$. Let us now turn to the surface $\phi = \phi(x, y)$, in which all such curves, constructed by the individual at a given date (and hence in given circumstances) for the purpose of comparing different courses of action, must lie. We can perhaps best visualise such a surface by viewing it 'in plan' by means of contour-lines. We consider ourselves to be looking down perpendicularly upon the xy-plane and to see traced upon it curves such that the value of $\phi = \phi(x, y)$ is the same for all points of any one curve. It seems natural and reasonable to assume, as we have suggested, that when we compare two hypotheses about the outcome of any one course of action, which are both desirable rather than hurtful, and which both carry equal potential surprise, the more desirable of the two will more powerfully stimulate the enterpriser's imagination and afford him keener enjoyment, that is, will correspond to a higher value of ϕ. This means that $\phi = \phi(x, y)$ has $\partial\phi/\partial x$ everywhere positive for all $x > 0$, where $x = 0$ stands for a 'neutral' outcome neither advantageous nor hurtful, and for all $y < \bar{y}$, where $\bar{y}$ stands for the 'absolute maximum' of potential surprise. Similarly it has $\partial\phi/\partial x$ everywhere negative for $x < 0$ and $y < \bar{y}$. Where $y = \bar{y}$, that is, for all hypotheses which are rejected as impossible, there can be no enjoyment or distress by anticipation. Along the line $y = \bar{y} =$ constant, therefore, we shall have ϕ everywhere zero and hence $\partial\phi/\partial x$ everywhere zero. This is a clue to the shape of the curves projected on the xy-plane by the contour-lines $\phi =$ constant; these projected curves, lying in the xy-plane, must evidently spring from the x-axis at some angle greater than zero, since as we pass along this axis towards successively higher values of x we attain successively higher values of ϕ, that is to say, we climb past successively higher contours. But none of these contour-lines, except those representing $\phi = 0$, can ever actually reach the line (parallel to the x-axis) $y = \bar{y}$. Hence these contour-lines must bend round, lying concave to the x-axis, so as to approach $y = \bar{y}$ asymptotically. As for the value of $\phi(x, y)$ at $x = 0$, $y = 0$, this is not a matter of practical importance for our argument, and we are free to choose whatever seems the most convenient convention. We are free, so far as the internal logical consistency of our argument is concerned, to *define* ϕ as the *excess* of the degree of interest felt by the individual in any given hypothesis, carrying a given degree of potential surprise, over that which he feels in the hypothesis of neither gain nor loss, when this hypothesis is coupled with zero potential surprise. In this case the two contour-lines $\phi = 0$ will together form a cusp with its tip resting on the x-axis at $x = 0$ (the neutral value of x) as in Fig. II 3. To picture the resulting configuration of the whole three-dimensional model which would then represent our conception, we can think of a mountain valley with a level and flat floor formed by the xy-plane, that is, the plane $\phi = 0$. Below this floor nothing is of any concern to us. This valley

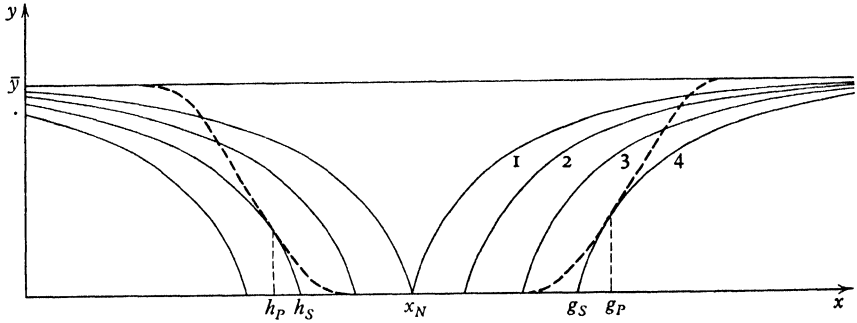

FIG. II 3. $\bar{y}$, degree of potential surprise representing absolute disbelief; x_N, neutral outcome; h_P, primary focus-loss; h_S, standardised focus-loss; g_S, standardised focus-gain; g_P, primary focus-gain. Curves numbered 1, 2, 3, 4 exemplify the contour-lines.

has a broad mouth, across which stretches the line $y = \bar{y}$ (the absolute maximum of potential surprise, a constant), and narrows down funnel-like towards lower values of y until it ends at the point $x = 0$, $y = 0$. Rising on either side from the floor of the valley are the two sheets of the sloping ϕ-surface. It would, as we have said, make no difference to our argument if we supposed the ϕ-surface to be translated up the ϕ-axis, that is, to be lifted bodily without deformation, so that, for example, $\phi(0, 0) > 0$. In this case the two contour-lines $\phi = 0$ would not reach the x-axis at any point, the tip of the cusp formed by them being at some value of y greater than zero; while there would be two other contour-lines which, when projected on the xy-plane, would give a cusp lying with its tip at the point $x = 0$, $y = 0$. We shall, however, adopt the definition making $\phi(0, 0) = 0$.

Upon the same xy-plane on which we have drawn the contour-map (the family of projected contour-lines) of some individual enterpriser's ϕ-surface, we now draw the potential surprise curve $y=y(x)$ which describes his view of the possibilities of some particular venture whose primary focus-points he wishes to determine. These primary focus-points are the two maxima of the twisted curve $\phi=\phi\{x, y(x)\}$, that is, the curve in which a wall, erected along the y-curve perpendicularly to the xy-plane, would meet the ϕ-surface itself. As the y-curve begins, above the upper or below the lower extreme of the inner range, to diverge from the x-axis, it will at first continue, as it did along the inner range itself, to cross successively higher contour lines as it proceeds to larger numerical values of x. As the y-curve swings away towards high values of y, however, the contour-lines begin to swing round in the contrary direction, so that the y-curve and the contour-lines in its neighbourhood become more and more nearly parallel. There will be a point where the y-curve ceases to cross successive contour-lines and instead lies *tangent* to some contour-line; and beyond this point it will begin to cross successively *lower* contour-lines as it proceeds towards still higher numerical values of x and of y. That contour-line with which it has a point of tangency is evidently the *highest* with which it anywhere comes in contact, and the point of tangency itself represents the highest level which the twisted curve attains. There will of course be two such points of tangency, one on the upper and one on the lower branch of the y-curve. There is no reason in general why they should be at the same level of ϕ, and thus two different contour-lines will ordinarily be involved. These two points of tangency are the two primary focus-points assigned by the particular enterpriser, in his circumstances of the moment, to the particular venture in question.

What we have called a *primary focus-outcome* is best looked on not merely as a value of x but as some combination of a particular value of x and a particular value of y, that is, the combination of the idea of some particular size of gain with some particular degree of potential surprise. The primary focus-gains of two ventures which rival each other for the enterpriser's choice may differ both in their values of x and also in their values of y. Suppose the enterpriser should wish to compare these ventures in regard to their desirable aspects only, that is to say, their respective primary focus-gains, leaving out of account for the moment their potentialities of loss. These primary focus-gains cannot be immediately compared with each other by a direct comparison of their respective values of x; each must be replaced by an expression *equivalent* to it and such that both equivalents carry *equal potential surprise*. What must we mean here by an *equivalent* combination of gain and potential surprise? Plainly it is one having equal power to afford the enterpriser enjoyment by imagination. But the power of hypotheses to do this is measured by ϕ. Hence we shall say that two points (x_1, y_1) and (x_2, y_2), that is, two combinations of gain and potential surprise, are equivalent to each other if they are such that, for the individual concerned, in his circumstances of a given date, $\phi(x_1, y_1) = \phi(x_2, y_2)$. That is to say, that two points in the xy-plane will be equivalent if they lie on the same projected contour-line. In order to compare two primary focus-gains we must choose for them equivalents whose respective values of y are equal to each other; and although we are free to select for this purpose any value of $y < \bar{y}$, it will be natural and convenient to reduce all primary focus-outcomes invariably to *zero* potential surprise. What we obtain in this way may be looked on if we like as a virtual extension of the inner range, and the equivalents carrying nil potential surprise would be the extremes of this extended inner range. But this is no more than a possibly helpful metaphor. The principle we have reached is that before any further step can be taken in comparing the merits of two rival ventures, for each of which the primary focus-outcomes have been determined, these primary focus-outcomes must all be replaced by equivalents carrying nil potential surprise. Each of these equivalents will be a value of x, say x_S, such that if the primary focus-outcome is (x_1, y_1), then $\phi(x_S, 0) = \phi(x_1, y_1)$. We shall call such equivalents *standardised* focus-outcomes, and each of them will evidently be represented on the contour-line diagram by the point where the projected contour-line, which contains the primary focus-outcome in question, meets the x-axis. These standardised focus-outcomes are in effect simply values of x, and as such they are all legitimately comparable amongst themselves. Each will lie between the relevant extreme of the inner range (upper or lower as the case may be) and the value of x of the primary focus-outcome in question. And, finally, there will be a one-one correspondence between standardised focus-outcomes and values of ϕ; if we turn our attention to a higher standardised focus-outcome (that is one consisting in a higher value of x), the corresponding value of ϕ will always be higher also. We shall be mainly concerned in what follows with standardised focus-outcomes rather than primary focus-outcomes, and we shall therefore omit the word 'standardised' except where there is danger of confusion. When we speak of focus-outcomes without any qualifying word, we shall invariably mean standardised focus-outcomes.

We can see, from the study we have just made of the shapes of the contour-lines, that there is little likelihood of the point of tangency of such a curve with any y-curve degenerating into a range over which the two curves coincide. Such ranges would, of course, leave the primary focus-values of x (those values which give maxima of $\phi = \phi\{x, y(x)\}$) indeterminate within each such range. But since the potential surprise function $y = y(x)$ will in its relevant segment (that near the x-axis) lie convex to the x-axis, while as we have just seen the curves projected on the xy-plane by the contour-lines ϕ = constant will lie concave to the x-axis, no such coincidence of the curves, except at a single point of tangency, can in general arise. It is interesting to notice, however, that even if such ranges of coincidence do occur, they are of no practical importance; for though the *primary* focus-outcome concerned would thus be rendered indeterminate, yet, whether the y-curve's contact with a contour-line occurred at a single point or over a range of values of x, only one contour-line would in either case be involved, and so the *standardised* focus-outcome would still be unique and unambiguously determined. Moreover, in the far more likely case where, through the curvature both of the y-curve and of the contour-lines being rather slight in some neighbourhood of the primary focus-value, the latter without being actually indeterminate is extremely sensitive to slight changes in the slope of the y-curve, there will be no comparable sensitiveness of the standardised focus value; for large movements of the point of tangency in directions *nearly parallel* to the projected contour-lines, though implying considerable movements of the *primary* focus-value concerned, will only alter the standardised value by a very small proportion.

This description of the shape of the surface $\phi = \phi(x, y)$, its relation to any potential surprise curve $y = y(x)$, and the manner in which surface and curve between them give the twisted curve $\phi = \phi\{x, y(x)\}$ and determine its maxima, so locating the primary focus-outcomes which, when the contour-lines on which they lie are traced back to the x-axis, yield in turn the standardised focus-outcomes, is illustrated in Fig. II 3. Here x_N is the neutral outcome, the thought of which gives neither enjoyment nor distress, and since we are taking money profit, expected to be yielded by an investment or speculation, as our concrete example of a kind of outcome which can be expressed in terms of a single variable or dimension, we consider x_N to be $x = 0$. Positive values of x then stand for gains and negative values for losses. The contour-lines $\phi(x, y)$ = constant are numbered in ascending order, number 1 being the contour-line $\phi = 0$. The y-curve is shown by a dotted line whose point of tangency with contour-line number 4 gives us the primary focus-gain, the value of x at which this is located being marked g_P. The point where contour-line number 4 meets the x-axis gives us finally the standardised focus-gain g_S, lying between the x-value of the primary focus-gain and the upper extreme x_U of the inner range. An exactly similar description, *mutatis mutandis*, applies to the negative range of x, where we have the primary focus-loss located at $x = h_P$ and the standardised focus-loss at $x = h_S$. The twisted curve $\phi = \phi\{x, y(x)\}$ cannot, of course, be shown on this diagram otherwise than in projection on the xy-plane, in which its projection is coincident with the y-curve.

The need to consider standardised rather than primary focus-outcomes is vividly apparent from such an example as that of Fig. II 4, where the primary focus-gain of a venture A has a higher

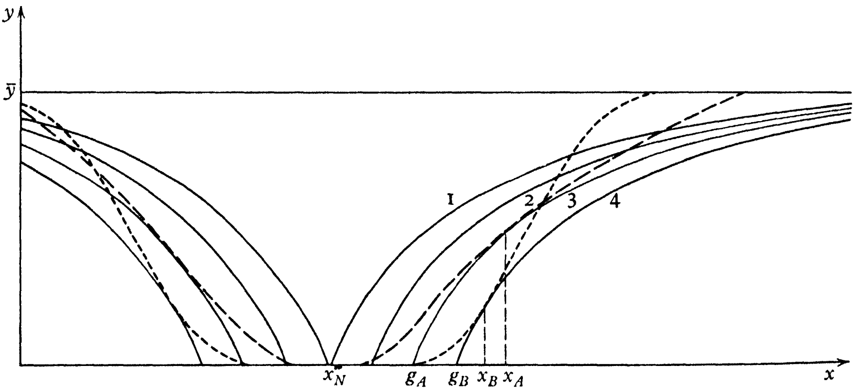

FIG. II 4. ——— y-curve of venture A; ------ y-curve of venture B; x_N, neutral value of x; x_A, the x-co-ordinate of the primary focus-gain of venture A; g_A, the standardised focus-gain of venture A; x_B, the x-co-ordinate of the primary focus-gain of venture B; g_B, the standardised focus-gain of venture B.

value of x, but a lower value of ϕ, and therefore a lower standardised focus-gain g_A, than venture B. In this figure the standardised focus-gains of the two ventures A and B are marked respectively g_A and g_B, while the x-values of their primary focus-gains are marked x_A and x_B.

One further result, important for our later argument, may be stated here. If a change occurs in the form of the y-curve, such that some values of x carry higher, but none carry lower, potential surprise than before, and amongst the values of x affected is the former primary focus-gain (or loss), the corresponding standardised focus-gain (or loss) will move to a numerically smaller value of x. The truth of this statement will at once be clear if we remember that $\partial\phi/\partial y < 0$ everywhere (except along the line $y = \bar{y}$). Hence it is impossible for the y-curve, by changes in its form involving only *increases* in y for given x, to attain a higher value of ϕ (a higher contour-line) than before. At most, the movement of the primary focus-outcome could carry it along the path of the former y-curve in one direction or the other away from its former position. But any such movement would, of course, carry it on to a lower contour-line. And a lower value of ϕ for the primary focus-outcome implies a lower value of x for the standardised focus-outcome. Hence our statement is proved. Now future changes in the form of the y-curve for a given venture (when the latter is strictly assigned to a fixed calendar date) which are anticipated or looked on as possible by the individual concerned, are logically confined to those which fulfil the condition stated at the beginning of this paragraph, namely, that they involve no *decreases*, but only increases, in the values of y associated with given values of x. A man cannot logically at one and the same time say to himself: 'I shall be surprised if x turns out to have a higher value than so and so; but I shall not be surprised if this judgement turns out to be wrong.' In Chapter III we shall be much concerned with *expected* changes in the form of the y-curve; but these expected changes will naturally be of the kind which are logically possible, namely, those where y increases or remains unchanged for given x. Hence future movements of standardised focus-outcomes, of a kind which the individual can expect, or look on as possible without involving himself in logical contradiction, must always be movements towards numerically smaller values of x, that is, movements inwards towards the neutral value of x.

What is the real meaning and final outcome of the mental process, of which we have given a formal representation or mathematical model? It is simply this: that the enterpriser epitomises and represents to himself any venture's promise of gain by means of a single, particular quantity; that size of gain, namely, which, if it carried *nil* potential surprise, would stimulate his interest and imagination just as powerfully as the most interesting of all the combinations of hypothesis of gain and associated potential surprise which he actually entertains; this latter, the true 'most interesting' hypothesis, being actually one which carries some greater degree than zero of potential surprise; and that he does the same in regard to the venture's threat of loss. Or again we may express the matter thus: the enterpriser does not take, as the two magnitudes representing the venture's potentialities of advantage or misfortune, the two extremes of the actual inner range; for there are other hypotheses, outside this inner range, which are even more delightful or distressing to imagine in spite of their carrying some positive degree of potential surprise; but he telescopes, as it were, these outer hypotheses into a new *virtual* inner range, whose extremes are hypotheses which have sacrificed something in size in order to shed all their potential surprise.

And here let me enter a caution against a possible misunderstanding, into which the reader may have been led by shortcomings in the foregoing exposition of my theory. We have said that

$$\phi = \phi\{x, y(x)\}$$

will have a maximum at each of two values of x; and also that the enterpriser's *attention will be focused* on these two values of x. The former of these propositions is a stage in our argument leading up to the latter; they are not both saying the same thing. ϕ indicates the degree of enjoyment due to, or of agreeable stimulus imparted by, some combination of hypothesis of gain and potential surprise; or the 'interestingness' of this combination, its *capacity* to impart such stimulus; all these are equivalent to each other, and could be measured by the same unit; ϕ, that is to say, is something assigned to a pair of values (x, y) in this pair's own right, independently of other pairs, and varying in degree from point to point of a y-curve. We conceive the enterpriser to pass these points in review, and in this process to pause upon one only out of the hypotheses of gain and there to fix his *effective* attention; and to do the same amongst the hypotheses of loss. A man can, as it were, find various girls attractive in varying degrees, but he can only marry one of them.

The concept of focus-outcomes leads us at once to that of an indifference-map of uncertainties, from which, given the standardised focus-gains of any two investment-opportunities or speculative outlays of money, and their standardised focus-losses, we can read off which of these two opportunities the individual whose map it is will prefer. Such a map will summarise comprehensively everything about the attitude to uncertainty of one particular person, in his circumstances of a given moment, that is relevant for determining his actions. On one of a pair of cartesian axes (the horizontal in Fig. II 5) we measure focus-losses and on the other focus-gains. Any point on the plane will stand for the combination of some particular standardised* focus-gain and some particular standardised focus-loss, and will be one of a set of such points all the members of which are mutually equal in attractiveness to the given person in his given circumstances. It is evidently likely that all the points of such a set will lie on a con-

* We shall henceforth usually omit the word 'standardised'; but whenever the terms 'focus-outcome', 'focus-gain', 'focus-loss', are used without qualification, the 'standardised' concept will be meant.

tinuous and smooth curve sloping upwards to the right. The positive slope indeed is certain, for if the passage from one pair of focus-outcomes to another equally attractive involves a small increase of focus-loss, then it must also involve, as compensation for this, a small increase of focus-gain. Such curves we shall call 'gambler indifference-curves'. They will be infinitely many, and every point for which both co-ordinates are positive will lie on one or other of them. (Points in other quadrants plainly have no relevant meaning for us.) The shapes, in general and rough terms, of all these curves are likely to

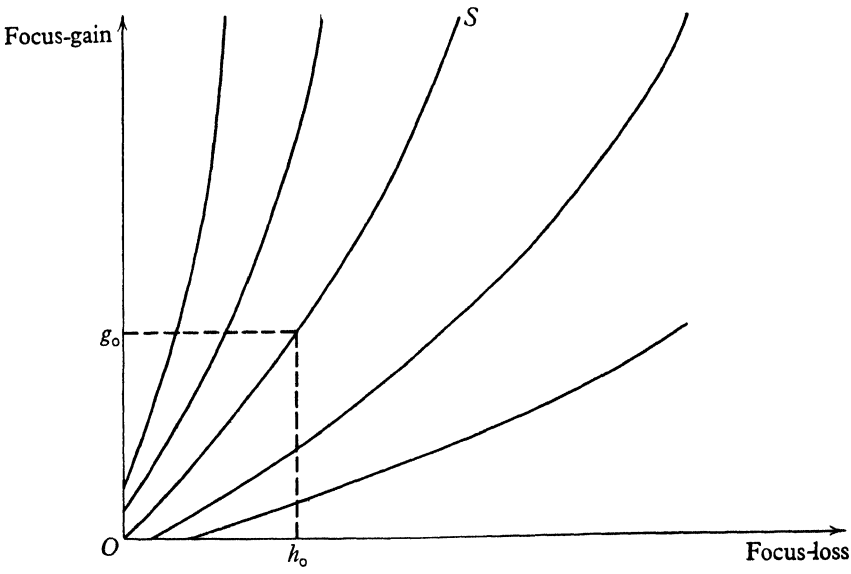

FIG. II 5. A gambler indifference-map. *OS* is the origin indifference-curve. The point (g_0, h_0) is considered to be a variable point on the origin indifference-curve. Then $h_0/g_0(h_0)$ is the schedule of gambler-preference.

be broadly similar. One of them will pass through the origin and will consist of all those points whose attractiveness is equal to that of the situation represented by the origin, namely, a combination of focus-gain and focus-loss which are both zero. This curve we shall call the *origin indifference-curve.* Once the individual has settled in his mind what are the respective focus-gains and focus-losses of any two investment- or speculation-opportunities which he is comparing, his mode of deciding which of the two he prefers is illustrated by their relative positions on his gambler indifference-map; out of these two he will prefer that one which lies on a curve above and to the left of the curve containing the other. By thus comparing in turn every pair of opportunities which can be formed from the entire list of rival opportunities which he has in mind, he can decide which is his most preferred course of action.

The concept of the origin indifference-curve leads us in its turn to a new and very simple definition of an individual's risk-preference, and enables us to measure the latter. A given individual in given circumstances will be able to name for any specific focus-loss a focus-gain such that if he is faced with this pair of focus-outcomes his situation seems to him neither more nor less desirable than if he had the assurance of experiencing neither gain nor loss. We shall say that this focus-gain *compensates* the focus-loss. The ratio of the focus-loss to its compensating focus-gain will in general be different when the focus-loss is different. The set of all such ratios obtained by varying the focus-loss, other circumstances remaining unchanged, is what we shall mean by the *schedule of gambler-preference* of the given individual in these circumstances. It is evident that this is simply the set of ratios of abscissa to ordinate of all points on the origin indifference-curve, when these ratios are placed in one-one correspondence with the respective focus-losses concerned.

Appendix A to Chapter II

ON GAMBLER-PREFERENCE

In order that the concept of focus-outcomes may have meaning and be used in analysis, it is sufficient to assume that each individual can identify for his own mental purposes those respective amounts of gain and loss which, on a balance between doubtfulness and size increasing together, seem to him to epitomise the merits of a given venture. We need not suppose that he is able to measure in units, and compare numerically, the different degrees of doubtfulness or potential surprise. But when we come to the concept of gambler-preference, and inquire just what temperamental qualities this coefficient reflects, we shall find it desirable to ask not only whether such measurement is possible for each individual in isolation, but further whether the units can be so defined that comparisons between persons are possible. Now for each person, considered alone and without reference to others, there are two fixed points on the scale of belief which have an unambiguous meaning: these are, respectively, nil potential surprise, and that degree of potential surprise which represents absolute disbelief. Between these two extremes lies the whole range of different intensities of surprise which he can conceive himself to feel. Referring to his memory of actual occurrences, no matter whether trivial or important in themselves, which have surprised him in the past, he will, I think, be able to class some of these experiences as lying nearer to one extreme and some as lying nearer to the other. There will perhaps be some which he cannot place definitely in either class; these will mark a first division of the whole range into two intervals which are in some sense 'equal'. Taking each of these intervals in turn, he may similarly be able to find some experience that divides this interval again into two equal parts, and thus eventually divide up the whole range into a number, perhaps rather small, of 'equal' intervals. When this has been done by each person for himself, it may turn out that the number of intervals attained is not the same for all persons; but since each person's number must necessarily be a power of two, we can take the smallest of these numbers, and everyone will find himself in possession of a scale having this number of divisions. Now if we can assert that nil surprise means the same thing for all persons, and that maximum possible surprise (corresponding to absolute disbelief) means the same thing for all persons, and finally that in selecting an intensity of surprise 'equally near to' each of two other intensities, each person's mind will work in much the same way, and apply similar standards of 'nearness', they will all arrive, by the process we have described, at scales of measurement which are mutually equivalent, and such that an experience which one person places at, say, one-quarter of the distance from nil to maximum surprise can be regarded as having involved the same intensity of surprise, in some objective sense, as another experience which another person places in the same position on *his* scale. Thus we shall have a scale of potential surprise, anchored and defined in each person's mind by actual experiences of his own, on which any person to whom a particular hypothesis is suggested as the answer to a carefully stated question or the outcome of a properly specified course of action in given circumstances, can assign a place to this hypothesis. It is true that the degree of fineness of subdivision of the whole range of potential surprise which can be attained by means of actual experiences may in itself allow only a poor approximation to continuity; but interpolation between the points actually established by concrete instances will be possible and may reasonably be assumed not to spoil the mutual equivalence of the scales obtained by different individuals or the comparability of the potential surprise functions which they set up on them. Thus two persons, to whom a profit-seeking venture is described, will each be able to draw his own potential surprise function for gain or loss from this venture, and it will be legitimate to compare these functions and meaningful, in case they are the same, to speak of these two persons as having identical potential-surprise functions for this particular venture. We shall make use of this idea in a moment.* Let us now consider again our definition of gambler-preference.

The *schedule of gambler-preference* of a given individual in given circumstances we define as follows: For any specific focus-loss there will be some specific focus-gain such that if he is faced with this pair of focus-outcomes his situation seems to him neither more nor less desirable than if he had the assurance of experiencing neither gain nor loss. We shall say that this focus-gain *compensates* the focus-loss. The ratio of the focus-loss to its compensating focus-gain will in general be different when the focus-loss is different. The set of all such ratios obtained by varying the focus-loss, other circumstances remaining unchanged, is what we mean by the schedule of gambler-preference of the given individual in these circumstances. This definition may seem open to criticism on the ground that it takes account only of those differences in temperament between persons which are revealed by the shapes of their respective gambler indifference-maps, and neglects those which are expressed in the differences of form of their respective ϕ-surfaces, and which, before the gambler indifference-map comes into play, have already influenced the locations of the focus-outcomes.

It might be thought that before we could draw any inferences depending on differences of shape between the respective ϕ-surfaces of two individuals, it would be necessary to define some unit or scale by which ϕ could be measured for both these persons at the same time. But this is not so. The differences in shape which are involved are differences in the relation of one part of one individual's ϕ-surface to the other parts of the same surface. Nothing is implied, in such a comparison as we require to make of the ϕ-surfaces of two persons, regarding the *absolute* values of ϕ attained by either surface at given points (x, y). All that we require is, first, that both surfaces shall use the same unit of x and that both shall use the same unit of y, and for this we have provided; and secondly, that each surface considered by itself shall be so shaped as to give maxima of any twisted curve $\phi = \phi\{x, y(x)\}$ at the correct points (x, y), i.e. so as to represent correctly the individual's temperament in this respect.

* The idea that it is possible to select or imagine an experience whose intensity is 'equally distant from' the intensities of two other experiences was suggested to me by a memory of Mr W. E. Armstrong's brilliant and stimulating article, 'The determinateness of the utility function' (*Economic Journal*, September 1939).

Suppose that the function associating degrees of potential surprise with amounts of gain or loss to be had from the purchase of a given asset is identically the same in form for each of two people, that each of them feels that his focus-gain is just sufficient to compensate his focus-loss, so that he is indifferent between purchase of this asset and retention of his cash, and that the focus-*loss* is the same for both persons. There is nothing in the foregoing assumptions to imply that the focus-*gain* is the same for both; for although at any point (x, y) our assumptions make dy/dx the same for both men, the ratio $\frac{\partial \phi}{\partial x} \Big/ \frac{\partial \phi}{\partial y}$ between $\partial\phi/\partial x$, the temptation to entertain larger notions of gain, and $\partial\phi/\partial y$, the reluctance to accept higher potential surprise, need not be the same for both of them, and hence a point (x, y) which satisfies the condition $\frac{\partial \phi}{\partial x} + \frac{\partial \phi}{\partial y}\frac{dy}{dx} = 0$ for one man need not satisfy it for the other. Now when any one of the contour-lines $\phi = \text{constant}$ is written so that y appears as an explicit function of x, the derivative of this function at any point (x, y) is numerically equal and of opposite sign to the ratio $\frac{\partial \phi}{\partial x} \Big/ \frac{\partial \phi}{\partial y}$ at that point.* This ratio, whose psychological meaning we have expressed above as the relation between the strengths of opposing 'temptation' and 'reluctance', may, for example, be numerically larger for one of the two men, say B, than for the other, A, at low or moderate values of y, and numerically smaller for B than for A at high values of y. Then plainly the contour-lines of B's ϕ-surface will be more concave to the x-axis than A's, running out from the x-axis at a larger angle at first and then bending more rapidly so as to avoid contact with the line $y = \bar{y} = \text{constant}$, and, as we can see in Fig. II App. A 1, this difference of shape makes it possible for B's standardised focus-gain g_B to be higher than A's, g_A. According to our definition, B's gambler-preference will in this case be *lower* than A's, since the focus-loss is the same for both men and is exactly compensated by their respective focus-gains. But is this a just interpretation of the facts we have assumed? B's focus-gain is higher than A's because his desire for gain is sufficient to overcome a higher degree of potential surprise.

* For if the contour-line is written so that y is given as an explicit function $y = f(x)$ of x, then the values of ϕ along this curve can be written

$$\phi = \phi\{x, f(x)\} = \text{constant}.$$

Differentiating we have $\frac{d\phi}{dx} = \frac{\partial \phi}{\partial x} + \frac{\partial \phi}{\partial f}\frac{df}{dx} = 0$, so that $\frac{df}{dx} = -\frac{\partial \phi}{\partial x} \Big/ \frac{\partial \phi}{\partial y}$.

But to be strongly swayed by hypotheses of gain in spite of their being associated with high degrees of potential surprise may be held to indicate a temperament of somewhat the same quality as that which is strongly attracted by hypotheses of gain in spite of their being associated with a large focus-loss. If this be accepted, can we appropriately say that *B* has a lower gambler-preference than *A*?

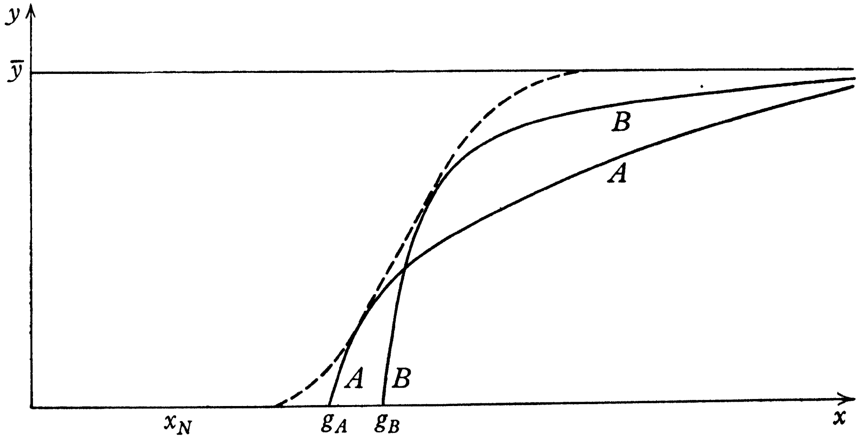

FIG. II App. A 1. *A*, a contour-line of *A*'s ϕ-surface; *B*, a contour-line of *B*'s ϕ-surface. g_A, *A*'s standardised focus-gain; g_B, *B*'s standardised focus-gain; x_N, neutral value of x.

The answer to this criticism is that, in so far as the shape of the gambler indifference-curves, and the form of the ϕ-surface, of a given individual are influenced by, or express, the same traits of his character, these two formal statements of certain aspects of that character are, of course, not independent of each other; if, by inspection of his ϕ-surface, we should learn something about his audacity or caution, or about his taste for or dislike of uncertainty, then we shall learn the same thing, or a broadly similar thing, from inspection of his gambler indifference-map. The schedule of gambler-preference may not, as it were, give true readings in an absolute sense, but it will still serve very well for comparisons of the frame of mind of the same individual at different times, and of different individuals.

Appendix B to Chapter II

SOME SPECIAL CASES

The purpose of this book is to offer new conceptual tools for the analysis of expectation and uncertainty. In order that the reader may, if he so desires, concern himself only with those features of these tools which are essential to the understanding and use of them, we have relegated from Chapter II to this Appendix a discussion of some special forms of the potential surprise function; to Appendix C a section on the radical difference of meaning between the basic ideas of our theory and those of probability theory; and to Appendix D an analysis of changes in his state of expectation which the individual can himself expect.

An interesting advantage of 'standardised' focus-values arises from the possible indeterminacy of primary focus-values. It is conceivable that the y-curve may be of such a form that over some range of x it coincides with one of the projected contour-lines $\phi = \text{constant}$, that is, that its changes, as we move along it from smaller to larger numerical values of the outcome x, exactly compensate those of x itself, so that the twisted curve $\phi = \phi\{x, y(x)\}$, instead of showing a true maximum at a single value of x, shows constant values of ϕ over some range of x, within which range the primary focus-value of x is therefore indeterminate. It will be immediately clear that such indeterminacy of the primary focus-value leaves no indeterminacy of the standardised focus-value; the fact that the y-curve coincides over part of its length with a particular contour-line, instead of being tangent to this line at a single point, still leaves us concerned only with this one contour-line, and therefore with only one value of x as the standardised focus-value. If in some region the contour-lines have a relatively gentle concavity to the x-axis, and the y-curve, having within this region a point of tangency with one of the contour-lines, and being, let us suppose, approximately straight in some neighbourhood of this point, undergoes a small change of slope, this point of tangency may be displaced by a large amount along the y-curve, and if the latter lies at not too great an angle to the x-axis, the primary focus-value of x may also be considerably changed. Such a change arising in this way, however, would have little effect on the standardised focus-value; for the conditions to which the sensitivity of the primary focus-value in such a case is due would cause its movement to occur in a direction nearly parallel to the contour-lines, so that the contour-line on which the new point of tangency would lie would be a close neighbour of the old one, and give a very similar value to the standardised focus-value.

The y-curve represented by the *origin* of the gambler indifference-map can only consist of a cusp with its tip resting on the x-axis at the neutral value of x. For the origin stands for a venture whose standardised focus-outcomes are both zero, and this requires that the y-curve shall not encounter any contour-line higher than those two which meet the x-axis at $x=0$. But these two themselves form a cusp with its tip on the x-axis at $x=0$, and since the y-curve must not pass outside the region defined by these two contour-lines, yet must of course reach the x-axis at some one point at least, the y-curve is necessarily itself either cusped, that is, V-shaped, or else in a more extreme form Y-shaped or T-shaped.

It is, of course, perfectly possible for the y-curve assigned to any venture to show two or many separate ranges over which $y=0$. This will not imply that there will be more than two focus-outcomes; the focus-gain will lie somewhat above the upper extreme* of the highest of these ranges, the focus-loss somewhat below the lower extreme of the lowest of the ranges.

A discontinuous function arises (for example) in an ordinary bet. There are (so far as the bettor is concerned) only two possible outcomes, a gain of one specific amount or the loss of another specific amount, all other outcomes carrying the absolute maximum of potential surprise, that is, the degree corresponding to absolute rejection of their possibility. In such cases the two values of x which alone carry less than the absolute maximum of potential surprise are themselves the primary focus-values.

Appendix C to Chapter II

ON THE NATURE AND MEANING OF FOCUS-VALUES

In this Appendix I wish to make a very special effort to convey the fundamental and absolute difference of meaning between the potential surprise curve and focus-values, on the one hand, and on the other, the 'sham' frequency-distribution and the approximative specifications of it, such as the mathematical expectation, which are the traditional tools for analysing uncertainty. The moments of a frequency-table are mere approximations to the content of the complete table, or rough descriptions of it, which we substitute for the full

* Or if at this extreme of the highest range the curve is discontinuous or has a discontinuous first derivative, the extreme may itself be the focus-gain; and, similarly, *mutatis mutandis*, with the focus-loss.

description. But focus-values are not approximations or briefer substitutes for something more precise, they are in themselves the significant psychic measures or indicators that we seek.

Let us refer to a combination of some hypothesis of gain (or of loss) and some degree of potential surprise as an *element*. We shall speak of gain-elements and loss-elements. To every such element there will correspond some determinate degree ϕ of stimulus, agreeable or disagreeable, which it can impart to the enterpriser's mind. We shall speak of one gain-element as *more powerful* than another (or of one loss-element as more powerful than another) if it gives rise to a higher ϕ. Then the basic proposition, which underlies the whole logic of focus-values, is this:

The degree of stimulus (if we like, of enjoyment by imagination) derived from the entire set of gain-elements of any venture is that derived from the most powerful of these elements, and is not increased beyond this by the co-existence of the other, less powerful, gain-elements. And similarly, the degree of stimulus (if we like, of distress by imagination) due to the entire set of loss-elements is that caused by the most powerful of these loss-elements.

The effect of gain-elements, that is to say, is *not additive*. To make clear my grounds for this assertion, I shall proceed by stages. Let us first consider a *y*-curve of the form shown in Fig. II App. C 1, such that potential surprise is *nil* over an inner range extending on either side of the neutral outcome and is the absolute maximum for all other hypothetical outcomes. In this case it will be agreed, I think, that the enterpriser is concerned solely with the extremes of the inner range. What does it matter to him that he could enjoy, undimmed by any potential surprise, the thought of a gain of 10 %, if the thought of 40 % is *equally* open to him, that is, is equally unclouded by any potential surprise greater than zero? No man will give a thought to the possibility of a loss of 20%, if a loss of 50% seems equally possible, that is, if both these hypotheses carry *nil* potential surprise. Hopes which are mutually exclusive are not additive; fears which are mutually exclusive are not additive. In each case the greatest prevails, and alone determines the power of the attractive or of the deterrent component of the venture's 'dual personality'. In this last sentence, the word 'greatest' is insufficiently precise. It might be asked: Do we mean by 'the greatest' of a number of mutually exclusive fears, the fear of the worst outcome amongst them? Or do we mean the fear of that outcome which has the 'greatest likelihood'? What we mean is the *most powerful element* amongst them; the relative power of different elements depending both on their respective badness-in-themselves and also on the respective degrees to which disagreeableness is subdued by association with high potential surprise. Thus when we pass from the 'rectangular' potential surprise function to consider a bell-shaped function, we shall be justified in accepting the suggestion

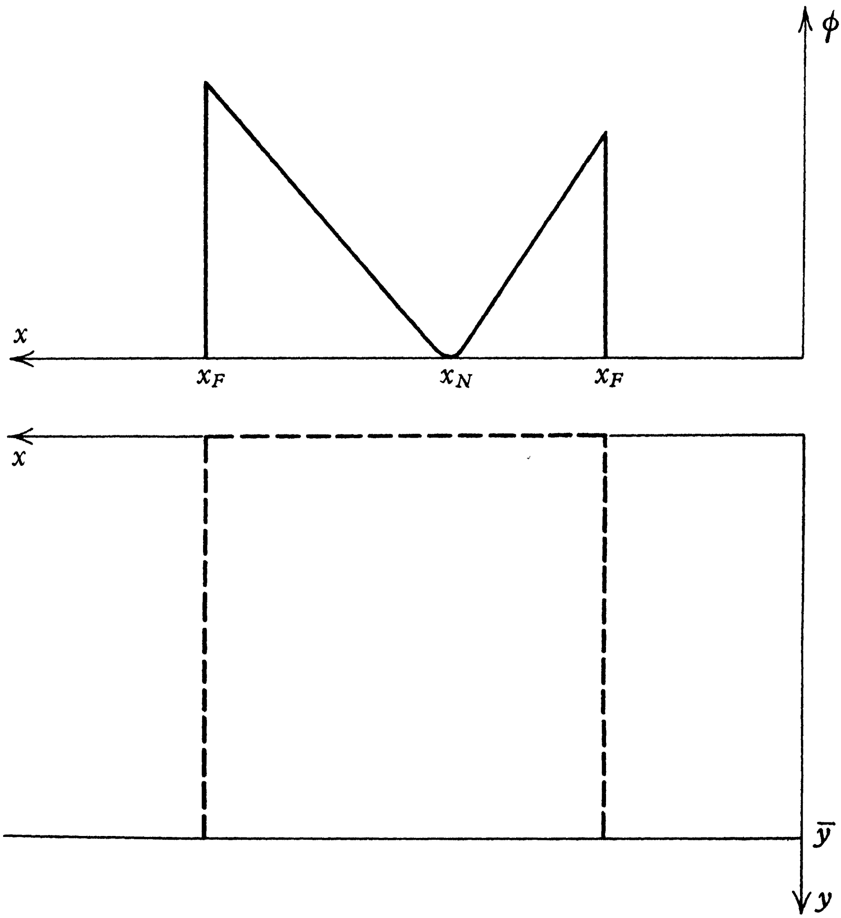

FIG. II App. C 1. x_N, neutral value of x; x_F, extremes of inner range and focus-values.

offered by analogy, namely, that here again only two hypotheses of gain and loss matter. Amongst all the gain-elements, when all have as it were been passed in review by the individual, one alone will have the ascendancy over his mind, and will by itself be accountable for the whole effect upon him of this entire set of gain-elements. And similarly, amongst the loss-elements, one alone will be dominant and will by itself be accountable for the full intensity of disagreeableness of the disagreeable aspect of his expectations concerning this venture. And this is because the gain-elements of any venture are mutually exclusive and therefore not additive; and the loss-elements of a venture are all mutually exclusive and therefore not additive.

The meaning of a potential surprise function is entirely and radically different from that of a frequency-distribution. It is difficult to say this with sufficient emphasis. Whereas a frequency-table distributes cases into additive classes, and it is meaningful, for example, to multiply values of the variable by their respective frequencies and add or integrate the results so as to obtain the mathematical expectation, no similar or even broadly analogous operation with potential surprise would make any sense; and this, *not* because of the difficulty of obtaining a numerical measure of potential surprise. The role of potential surprise is to permit or deny to the imagination effective access to some particular idea. A man's judgement compels him to attach certain degrees of potential surprise to given hypotheses of gain. These degrees determine what will be the most powerful amongst all the gain-elements which the venture in question provides; determine, that is to say, what is the most powerfully stimulating idea (amongst those connected with gain) to which the venture can give rise; increasing potential surprise forms a barrier denying access to still more powerful and exciting ideas. The degree of favourable stimulus obtained from the most powerful element is the degree of favourable stimulus obtained from the whole idea of the venture. And similarly, the degree of unfavourable or deterrent stimulus obtained from the idea of the venture is that obtained from the most powerful amongst the loss-elements. It would be a simplification of our apparatus of analysis if, instead of an indifference-map whose axes measure the amounts of hypothetical *gain or loss* belonging, respectively, to the most powerful gain-element and the most powerful loss-element,* we could use one whose axes directly measured the *power* of these elements themselves, that is, the degree of enjoyment derived from the thought of the venture's potentialities of gain and the degree of distress derived from the thought of its potentialities of loss. But though this direct mode of comparison may well be what the individual himself in the first instance does, our own purpose (namely, to create a working model which reproduces† the relevant

* Or rather, the 'standardised' equivalents of this gain and loss.

† That is to say, a model which, in a sense analogous to the mathematical meaning of the term, is 'isomorphic' with the real mental and extramental processes connected with expectation.

features of the expectation-forming and expectation-using process) is better served by not attempting to invent a unit for measuring ϕ itself, but by using instead the standardised focus-values, which, as we have seen (when we consider the 'gain' side alone, or the 'loss' side alone), are in one-one correspondence with values of $\phi\{x, y(x)\}$ at its maxima in different ventures. For these standardised focus-values are simply numerical amounts of (hypothetical) gain or loss, and are thus ideas of concrete and familiar content, and, moreover, can in some contexts be made to serve for interpersonal comparisons.

I think that the reader may experience some considerable difficulty in fully assimilating, and adopting as part of his apparatus of thought, the idea that gain-elements are non-additive, that the whole power of the attractive aspect (as distinct from the deterrent aspect, in regard to which the same argument applies to loss-elements) of a venture resides, in effect, in the most powerful gain-element and is not added to by the others, whether their values of x are smaller *or larger* than that of the focus-element. 'For', he may say, 'I can agree that elements which lie *inside* the inner range are of no concern to the individual; if a relatively large gain seems "perfectly possible" he will not be more strongly attracted to the venture merely because a relatively small gain also seems "perfectly possible". But will he not derive some additional satisfaction from the thought that the venture offers some small chance of very much larger gains than the focus-gain?' It is the word 'chance' in this sentence which suggests the source of the difficulty; one is constantly tempted to think in terms of a frequency-table; and a frequency-table *is* additive, in a sense which we can illustrate by contrasting it with a potential surprise function in the following way. If the shape of the frequency-diagram were changed by a slight reduction in the probability assigned to a large loss, and an equal slight increase of the probability of a large gain, the mathematical expectation would be increased; while if, for example, a small reduction of the values of y attached to some gains far outside the primary focus-gain were judged by the enterpriser (upon his acquisition of some new knowledge) to be appropriate, this would make no difference to the focus-gain or the net attractiveness of the venture.* This contrast seems a veritable paradox. Yet the explanation is simple. The potential surprise

* We do not, of course, have to suppose that any values of x carry *higher* values of y than before, in order to suppose that some values of x carry *lower* values of y than before; but our argument above would be quite unaffected if, in addition to supposing that y is reduced for some gains, we also suppose it to be increased at the same time for some losses.

function is talking about the outcome of ONE particular venture, the frequency-table is talking about the total outcome of an experiment consisting of indefinitely many repetitions of ventures which are in some sense 'like' this one. The frequency-table tells us that, if only we make enough repetitions, we can look forward, with as near *certainty* as we like, to the total outcome being so and so; the mathematical expectation is a statement about this *total* outcome. In order to get this total outcome of all the repetitions, we must of course *add together* the results of the various separate trials. That is why the frequency-table or function is additive and why its arithmetical mean, the mathematical expectation, is changed by any change, anywhere, in its shape. The potential surprise function, on the other hand, is analogous to a diagram showing the temperatures of various parts of a room or a country. Such a diagram answers the questions 'How warm can we get?' and 'Where can we get warmest?' The degree of warmth we can enjoy in a given country depends only on how warm is *the warmest* part of that country.

One further question which may arise in the reader's mind must be answered here. The mental experiences derived respectively from thoughts of gain and from thoughts of loss are different in *quality* or tone, so that although we say that of two rival (i.e. mutually exclusive) hypotheses of gain (or of loss) the weaker will be dismissed and attention confined to the more powerful, we cannot suppose that thoughts of a venture's possibilities of gain will enable the enterpriser to dismiss from his mind thoughts of its possibilities of loss, or vice versa. Hope and fear can co-exist in a man's mind (or can at least be successively the objects of oscillating attention) even when both arise from a single issue and therefore cannot both be (objectively) well grounded. Yet we have drawn diagrams in which we measure ϕ, whether derived from gain-elements or from loss-elements, upon the same axis and in the same sense of this axis. The reason for this is no more than the convenience of being able to show both branches of the y-curve on the same diagram. We might equally well (perhaps in strict logic it would be better) draw two separate diagrams, one for the 'gain' branch of the y-curve and one for its 'loss' branch, and use different symbols, say ϕ and ψ, for the stimulus derived from gain-elements and from loss-elements respectively. But the danger of any confusion which this precaution would be designed to eliminate seems hardly sufficient to warrant the extra complication.

Appendix D to Chapter II

CHANGES IN HIS STATE OF EXPECTATION WHICH THE INDIVIDUAL CAN HIMSELF EXPECT

In this Appendix we pass to a feature of the focus-values conception which is important not only for the applications we shall make of it in Chapter III (which chapter is wholly based upon it), but also for the light it throws on the essential character of the conception itself. The topic we now wish to discuss arises from the possibility that the individual may expect that some one or more of those unanswered questions, whose presence in his mind is the reason, or at any rate the expression, of his uncertainty about the outcome of the venture, may be answered at some future calendar date intermediate between his viewpoint and that date when the outcome itself will become known. For example, uncertainties of taxation will be disposed of for a time at the date of the next Budget. In looking forward to such a date the enterpriser will foresee that he will then be able to assign to the venture a new shape of potential surprise function, in which some values of the outcome x will be associated with respective degrees of potential surprise different from those given them in the function he can assign now. The implications of this fact are the subject-matter of this Appendix.

The potential surprise function, which at any moment expresses an individual's state of expectation concerning the outcome of some venture, has the particular shape which it has because the individual has in mind a particular set of data, that is, a particular set of relevant questions to which he knows the answers, and another set of questions, also bearing on the issue, to which the answers are still unknown. If the answer to one of these latter questions becomes known to him, he will give the y-curve a different shape. Such a question may be a 'Yes or No' question, or some other question having only two possible answers; in that case there will be two possible new forms, one or other of which the y-curve will adopt if the answer to this particular question becomes known to the individual without there being any other change in his relevant knowledge. Or this question, at the other extreme, may be of such a kind as 'How much...?', to which the possible answers, even if limited to a finite range of the variable in question, can be infinitely many. In that case there will correspondingly be in general infinitely many possible new forms for the y-curve, a family of curves forming what we might loosely call a continuous 'spectrum' with definite characteristics of shape and bounded by definite extreme forms if the answers to the question are so bounded.* If, therefore, the individual looks forward from his viewpoint, located at a calendar date [1], to a date [m] lying intermediately between the viewpoint and that date at which the outcome x itself will be known, and expects that at date [m] some one or more of the unanswered questions, which bear on his state of expectation concerning x, will be answered, he will necessarily expect to have to adopt, at date [m], a new form for $y(x)$ differing from that which he gives to this function now, at date [1]. But he cannot, of course, *specify* now the form which he will adopt at date [m]; to be able to do so would be to know the answer to the question and to adopt the new form of $y(x)$ at once without waiting for date [m]. What he *can* do is to list in his mind the range of possibilities from which, as it now seems to him, the new form of $y(x)$ must eventually be selected, each possibility in this list corresponding to one of the possible distinct answers to the question. Corresponding to each of these answers there will also be some particular degree of potential surprise which, at his viewpoint, the individual attaches to the idea that this answer will, at date [m], prove to be the right one, and that the corresponding form of $y(x)$ will then be adopted. We have now, on the one hand, a schedule which assigns to each hypothesis, concerning the form which the individual will give, at date [m], to $y(x)$, some degree of potential surprise; and on the other hand, we have the form which he gives to $y(x)$ now, at the viewpoint. Between these two there must evidently be some special kind of relationship or harmony; plainly they cannot be independent of each other. It is this relationship which we have now to study.

Let us begin with the simplest case, where the individual has in mind only two rival hypotheses regarding the form which at date [m] he will adopt for $y=y(x)$, and where each of these two carries *zero* potential surprise. Let us refer to Fig. II App. D 1. The upper part of this diagram shows two potential surprise functions, both concerned with the outcome x of the same venture. The knowledge which we assume to be available to the individual, and to underlie these curves, is the same for both of them except in one respect: in constructing one of them he assumes the truth of answer A out of the two possible answers A and B to some particular question, in con-

* In the simplest case, what we have in mind might be obtained merely by rigid transformations, mere translations along the x-axis of the curve corresponding to one particular answer to obtain those corresponding to other answers. If the answers are values of a continuous variable, there can of course be infinitely many curves corresponding to any finite range of this variable.

structing the other he assumes the truth of answer *B*. If, in fact, he does not know the answer to this question, and assigns nil potential surprise to both of the conceivable answers, the *y*-curve which he must actually adopt will be compounded from these two forms by taking the upper branch of that one which shows the lowest values of *y* for given high values of *x* and the lower branch of that one which shows the lowest values of *y* for given low values of *x*; that is to say, he will take the outermost of the two upper branches and the outermost of the two lower branches to form the curve shown in the

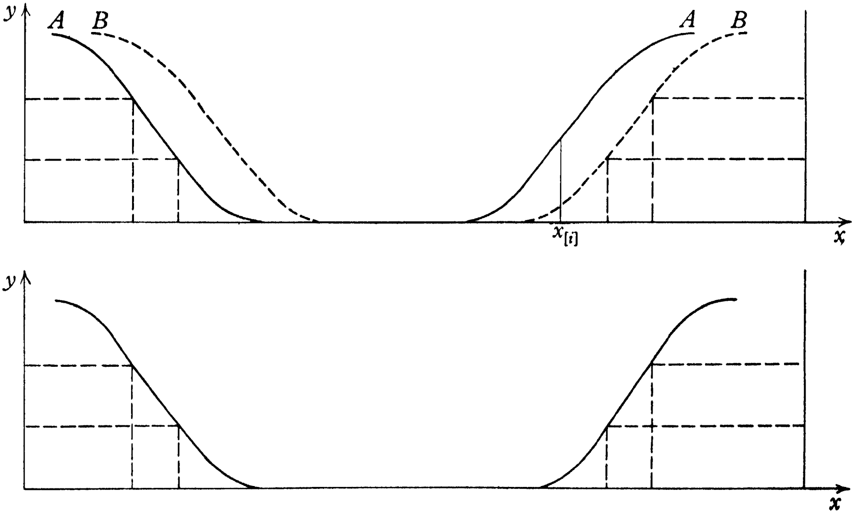

FIG. II App. D 1. In the upper diagram, the curve drawn with full line corresponds to answer *A*, the curve drawn with broken line to answer *B*. The curve in the lower diagram, composed of the lower branch of *A* and the upper branch of *B*, is the *y*-curve which the individual will actually adopt while the answer is still unknown.

lower part of the figure. This statement must rely for its acceptance on the reader's intuition. It exemplifies a general proposition, formally stated on p. 49 below, whose relevancy or 'truth' is suggested to me by my own introspection or intuition, and whose logical status in my system is that of a postulate or axiom. Logical rigour apart, however, I think this statement will be easily seen to be in accord with the description I have given of the concept of potential surprise. Now let [*m*] again stand for a date intermediate between the viewpoint and the date when *x* will be known, and let us suppose that at his viewpoint the individual attaches to the idea that the question will not be answered at date [*m*] a degree of potential surprise greater than zero. Then we can say in ordinary language that, in the situation we have described, the individual at his viewpoint expects a change in his state of expectation concerning x to occur at date $[m]$. All we can say about the nature of the expected change is that the individual thinks that at date $[m]$ he will substitute, for the form he now gives to $y=y(x)$, a new form which will be *either* that appropriate to answer A *or* that appropriate to answer B. This brings us to a result which is one of the main upshots of this part of our argument, viz. that, *whichever* of the two things does happen, the effect will be to give to some values of x a *higher*, but to none a *lower*, degree of potential surprise than before. Such an effect, which, as it were, concentrates the potential surprise curve, shortening the inner range or folding inwards the arms, or both, we shall call a *clarifying* of expectations. Such a clarifying, if the values of x which receive a higher value of y include one of the original primary focus-outcomes, implies a *reduction* or movement *inwards*, towards the neutral value of x, of the corresponding standardised focus-outcome. For if the change in the form of $y=y(x)$ leaves unchanged the x co-ordinate of this primary focus-outcome, it will give it a higher y co-ordinate; and since $\partial\phi/\partial y<0$ everywhere where $\phi>0$, it follows that the new primary focus-outcome will stand on a lower contour-line than did the old. If, instead, the change in the form of $y=y(x)$ moves the primary focus-outcome to a value of x whose associated value of y has not changed, so that the new primary focus-outcome is a point of the old y-curve, this point must evidently stand on a lower contour-line than the old primary focus-outcome, since otherwise it would itself have been the former primary focus-outcome. If, lastly, both of the co-ordinates of the new primary focus-outcome are different from those of the old one, then both of the above reasons will be relevant and *a fortiori* the new primary focus-outcome will stand on a lower contour-line than the old primary focus-outcome. If the curves respectively appropriate to answers A and B are those shown in Fig. II App. D 1, then if the true answer turns out to be A, the standardised focus-*gain* will be reduced, while if the true answer turns out to be B, the standardised focus-*loss* will be reduced. In *neither* case will either of the standardised focus-outcomes be *increased*. The result we have reached is that whenever, at the viewpoint, all the rival hypotheses, concerning the form which will be adopted at date $[m]$ for $y=y(x)$, carry either zero or else the absolute maximum $y=\bar{y}$ of potential surprise, then the only kind of shift of the standardised focus-outcomes which the individual at his viewpoint can entertain, i.e. to which he can attach less than the absolute maximum of potential surprise, is an *inward* shift, a *reduction* of the standardised focus-gain or the standardised focus-loss to smaller numerical values. To the idea that an *inward* shift can occur he can attach zero potential surprise. A specially interesting type of case (as we shall see in Chapter III) is one where the hypothesis, that the question will not be answered at date [*m*], carries some degree greater than zero of potential surprise, and where the alternative answers are such that, for example:

Answer A implies *no change* in the form of $y = y(x)$.
Answer B implies a decrease of the standardised focus-*loss*.

Now suppose that at date [1], instead of regarding both answers as equally possible, the individual attaches some degree greater than zero of potential surprise, instead of nil, to, say, answer B; will he in this case at date [1] adopt curve A as it is, or will he modify it on account of his judgement that answer B is not impossible, and that he cannot assign it the absolute maximum of potential surprise? It seems plain that as regards those values of x which lie within the inner range of curve B but outside that of curve A, he will assign them either the degree of potential surprise accorded to them by curve A, or the degree of potential surprise accorded to the truth of answer B, whichever is the lower. Regarding those values of x that lie a little outside the inner range of B (and also outside that of A), it is harder to justify any specific rule; but if the potential surprise attached to the hypothesis that B is true is very low, then it seems clear that values of x (for example, x_t in the diagram) which are assigned very low potential surprise by B, but very high potential surprise by A, will in the curve actually adopted at date [1] be given a rather lower potential surprise than that given them by A. In the more general and rigorous analysis of expected clarifying of expectations, to which we now proceed, we shall attain a more precise result for cases such as this.

We begin our attempt at a more general and rigorous development by formally setting down the two postulates on which (in addition to those already involved in our description of the concept of potential surprise) this analysis will be based.

Let Q be a set of questions which, if the answers to all were known, would determine the value which will be taken at some named future date by a variable x. And let K be a set of suggested answers to some subset of these questions, one suggested answer for each member of the subset. When the subset is a proper subset, let K be insufficient to determine x. Let $\overset{1}{K}, \overset{2}{K}, \dots, \overset{n}{K}$ each be a set such as K, and let them differ from each other by offering answers to different subsets of questions or by offering a different answer to a given question, or in any combination of such ways. Let $Q_1, Q_2, \dots, Q_N$ be sets of questions, each set sufficient when fully answered to determine the value which will be taken by one and the same variable x at some named future date, and let this list of such sets include all those and only those that the individual has in mind at some viewpoint. Let our list $\overset{1}{K}, \overset{2}{K}, \dots, \overset{n}{K}$ be extended to include all sets, which the individual has in mind, of answers to any subsets of any of the sets Q. Let $\eta = \eta(\overset{i}{K}, x)$ be the function according to which the individual, when his relevant knowledge is $\overset{i}{K}$, assigns to each value of x some degree η of potential surprise.* Let $S = S(\overset{i}{K})$ be the degree of potential surprise assigned by the individual at the viewpoint to the hypothesis that at some named future date, say [m], earlier than that at which the value of x will become known, he will assume $\overset{i}{K}$ to be true and to subsume all that he knows bearing on x (i.e. to be his 'relevant knowledge'). Let G be the totality of assumptions, bearing on the value to be assigned to each of the $S(\overset{i}{K})$ or on the form to be given to each of the functions $\eta = \eta(\overset{i}{K}, x)$, which the individual makes at the viewpoint; that is to say, let G be his relevant knowledge at the viewpoint; and let $y = y(G, x)$ be the function according to which at the viewpoint he assigns degrees of potential surprise to values of x. Let $U_{i,j}$ be the degree of potential surprise which, at the viewpoint, the individual assigns to the *joint* truth of the hypotheses, first, that at date [m] he will adopt $\overset{i}{K}$, and secondly, that when the value of x shall have become known it will turn out

* With *given* relevant knowledge, say $\overset{1}{K}$ or $\overset{4}{K}$, η will be a function of the one variable x only, and the values it can then take can be represented by an ordinary curve. The function $\eta = \eta(\overset{i}{K}, x)$ will consist of a number of such curves, one for each $\overset{i}{K}$, and each having its own distinct form. In order to keep in view this dual plurality of the η-curves on the one hand and of the distinct 'dossiers' of relevant knowledge $\overset{i}{K}$ on the other, and the one-one correspondence between these two sets, we shall include the parameter $\overset{i}{K}$ in every instance, even when we are considering only one curve at a time.

to be x_j. Then we postulate* that $U_{i,j}$ will be the *greater* of the pair $S=S(\overset{i}{K})$ and $\eta=\eta(\overset{i}{K}, x)$. Let us lastly suppose that the individual excludes (assigns the absolute maximum of potential surprise to) the idea that his relevant knowledge might change at some date other than [m]. Then we postulate* that $y=y(G, x)$ will be the *least* degree of potential surprise to be found amongst the entire set of $U_{i,j}$.

Let us illustrate what these postulates imply by means of Fig. II App. D 2. Here we suppose that the individual excludes the idea that there will be any change in his relevant knowledge except at

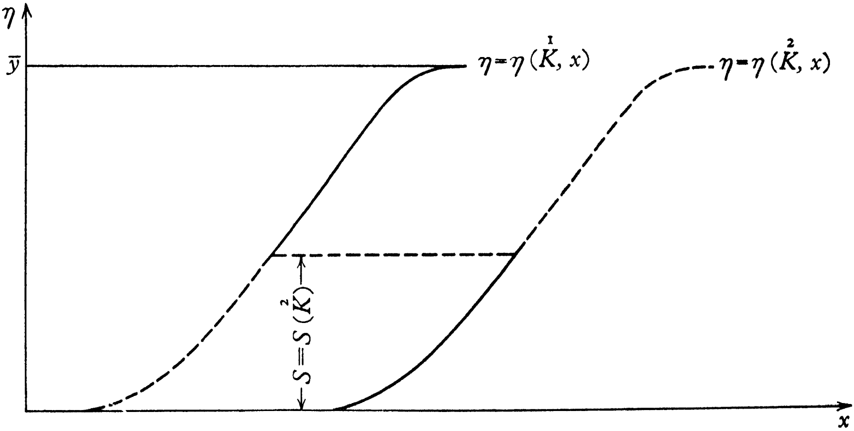

FIG. II App. D 2.

date [m]. We suppose that he has in mind only two rival hypotheses about what his relevant knowledge will become at date [m], viz. that it will become $\overset{1}{K}$, to which hypothesis he assigns potential surprise $S(\overset{1}{K})=0$, or that it will become $\overset{2}{K}$, to which hypothesis he assigns potential surprise $S(\overset{2}{K})$ such that $\bar{y} > S(\overset{2}{K}) > 0$. In order to determine the shape of the function $y=y(G, x)$ according to the principle arising from our two postulates, we can proceed by first finding for each value of x what degree U_1 of potential surprise is equal to the greater of $S(\overset{1}{K})$ and $\eta(\overset{1}{K}, x)$, and what degree U_2 is equal to the greater of $S(\overset{2}{K})$ and $\eta(\overset{2}{K}, x)$, and then selecting the lesser of the two values

* These are intuitions which seem to me to form an integral and harmonious part of the concept of potential surprise developed in the foregoing parts of this book.

of U thus found as the degree $y = y(G, x)$ of potential surprise which the individual at the viewpoint will actually assign to x. The most usual case will be one where the two curves $\eta = \eta(\overset{1}{K}, x)$ and $\eta = \eta(\overset{2}{K}, x)$ have no points in common except in the coincident parts of their respective inner ranges. Our ultimate purpose is to determine the form of a function by which the individual assigns at his viewpoint degrees of potential surprise to various amounts of shift, occurring at dates such as [m] lying intermediately between his viewpoint and the date when x will be known, of the standardised focus-gain. For this purpose, as will appear, we need only consider those cases where the potential surprise assigned to any x by $\eta = \eta(\overset{2}{K}, x)$ is *less* than that assigned to this x by $\eta = \eta(\overset{1}{K}, x)$; cases, that is to say, where the curve $\eta = \eta(\overset{2}{K}, x)$, whose adoption at date [m] is a hypothesis carrying potential surprise greater than zero, lies to the right of the curve $\eta = \eta(\overset{1}{K}, x)$, whose adoption at date [m] is a hypothesis carrying zero potential surprise.

The function $y = y(G, x)$ will coincide with the function $U_1 = U_1(x)$ where this latter lies *below* $U_2 = U_2(x)$, and with $U_2 = U_2(x)$ where this lies below $U_1 = U_1(x)$. Now since $S(\overset{1}{K}) = 0$ we have everywhere $U_1 = \eta(\overset{1}{K}, x)$, which over its own inner range and at some values of x above the upper extreme of that inner range is less than $S(\overset{2}{K})$ and therefore less than U_2. For these values of x therefore, the curve $y = y(G, x)$, marked in Fig. II App. D 2 by a broken line, coincides with $\eta = \eta(\overset{1}{K}, x)$. The curve $\eta = \eta(\overset{2}{K}, x)$ over its own inner range and some values of x above the upper extreme of this inner range is less than $S(\overset{2}{K})$. For these values of x, U_2 is equal to $S(\overset{2}{K})$, and is thus a horizontal straight-line segment. At that value of x where $\eta(\overset{1}{K}, x)$ becomes greater than $S(\overset{2}{K})$, the curve $y = y(G, x)$ ceases to coincide with $\eta = \eta(\overset{1}{K}, x)$ and follows instead the horizontal straight line $S = S(\overset{2}{K})$. But for values of x where $\eta = \eta(\overset{2}{K}, x)$ is greater than $S(\overset{2}{K})$, the curve $U_2 = U_2(x)$ departs from the horizontal straight line $S = S(\overset{2}{K})$ and coincides instead with $\eta = \eta(\overset{2}{K}, x)$, which, however, lies everywhere (except for part of its inner range) below $\eta = \eta(\overset{1}{K}, x)$. Thus $y = y(G, x)$ has the form shown by the broken line in Fig. II App. D 2.

This curve $y = y(G, x)$ subsumes everything in the individual's mind which influences him in assigning, at his viewpoint, degrees of potential surprise to values of x, and it is therefore this curve which, when applied to his ϕ-surface, will determine the standardised focus-gain. Let us name as follows the three parts of a curve $y = y(G, x)$ of the general shape exemplified in Fig. II App. D 2:

The lower bell-shaped segment.
The horizontal segment.
The upper bell-shaped segment.

With a curve of this general shape the primary focus-gain can occur in any one of three ways:

1. As a point of tangency between a contour-line and the lower bell-shaped segment.

2. As a point of tangency between a contour-line and the upper bell-shaped segment.

3. As a point of *contact* of a contour-line and the tip of the cusp formed by the horizontal segment and the upper bell-shaped segment.

Let us refer to these as cases 1, 2 and 3. We cannot discuss each of the endless variety of situations which could arise with contour-lines differing widely from each other in shape or with η-curves differing widely from each other in shape, or with both these features at once. Our discussion is intended only to illustrate general principles, and this purpose will be served sufficiently if we consider only the situation where each contour-line can be obtained from another, and each η-curve can be obtained from another, by translation parallel to the x-axis. Let us take each of the above three cases in turn and consider what degree of potential surprise the individual at his viewpoint will attach to the ideas that at date $[m]$ the standardised focus-gain will increase, that it will remain unchanged, and that it will decrease. For ease of reference, let us call $\eta = \eta(\overset{1}{K}, x)$ the first η-curve, and $\eta = \eta(\overset{2}{K}, x)$ the second η-curve. Let us refer to Fig. II App. D 3.

In case 1, if $S(\overset{2}{K})$, whose actual value is $S_2 > 0$, were to become zero, the viewpoint potential surprise curve $y = y(G, x)$ would alter its shape so as to coincide everywhere with the second η-curve, and the standardised focus-gain would shift from g_{S_1}, the x-intercept of the contour-line tangent to the first η-curve, to g_{S_2}, the x-intercept of the contour-line tangent to the second η-curve. To the idea of such an *increase* of the standardised focus-gain the individual will at his viewpoint evidently assign potential surprise equal to $S(\overset{2}{K}) = S_2 > 0$.

In case 1 again, to the idea that the standardised focus-gain will remain unchanged he will evidently attach zero potential surprise, since this is the degree of potential surprise which at his viewpoint he assigns to the η-curve (number 1) on which the primary focus-gain is actually located. And in case 1, lastly, he will attach the absolute maximum of potential surprise to the idea that the standardised focus-gain could at date [m] *decrease*; for there is no other η-curve, carrying potential surprise $S < \bar{y}$, to which the primary focus-gain could shift.

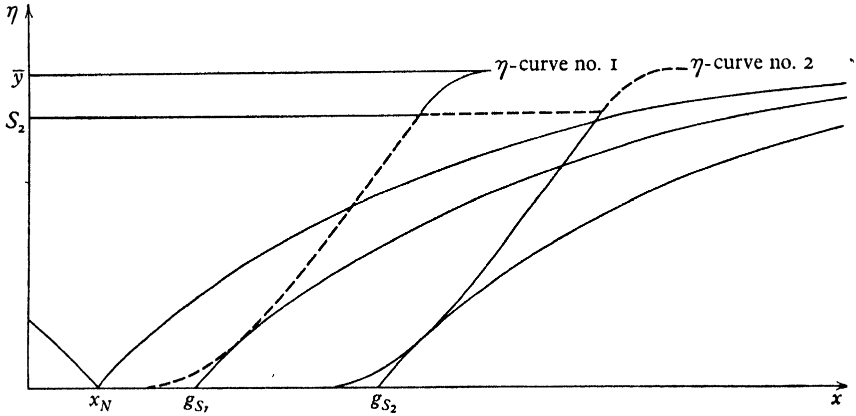

FIG. II App. D 3.

For case 2 let us turn to Fig. II App. D 4. Here the situation of case 1 is reversed. The individual will attach potential surprise equal to $S(\overset{2}{K}) = S_2 > 0$ to the idea that the standardised focus-outcome will *remain unchanged* at its initial value of g_{S_1}, and *zero* potential surprise to the idea that it will *decrease* to g_{S_2}, viz. the x-intercept of the contour-line tangent to the first η-curve. He will attach the absolute maximum of potential surprise to the idea that it will *increase*.

In case 3 we have a seeming paradox. Since there are only two η-curves for which $S(\overset{i}{K}) < \bar{y}$, any imagined shift of the primary focus-gain must be to the point of tangency of some contour-line with one or other of them. There is no η-curve whose adoption at date [m] would cause the primary focus-outcome or the standardised focus-outcome to remain at their viewpoint positions. Consequently, if the individual at his viewpoint attaches a degree of potential surprise of say Y to the *non-answering* of the question at date [m], then he must attach this same degree Y to the idea that at date $[m]$ the standardised focus-gain will remain unchanged; for according to his relevant knowledge at the viewpoint, it is only the non-answering of the question which could leave the standardised focus-gain unchanged. Referring to Fig. II App. D 5, we see that an increase of the standardised focus-gain from g_{S_1}, the x-intercept of the contour-line which passes through the cusp, to g_{S_2}, the x-intercept of the contour-line tangent to η-curve number 2, would carry potential surprise of $S(\overset{2}{K}) > 0$, while a decrease to g_{S_3}, the x-intercept of the contour-line tangent to η-curve number 1, would carry zero potential surprise.

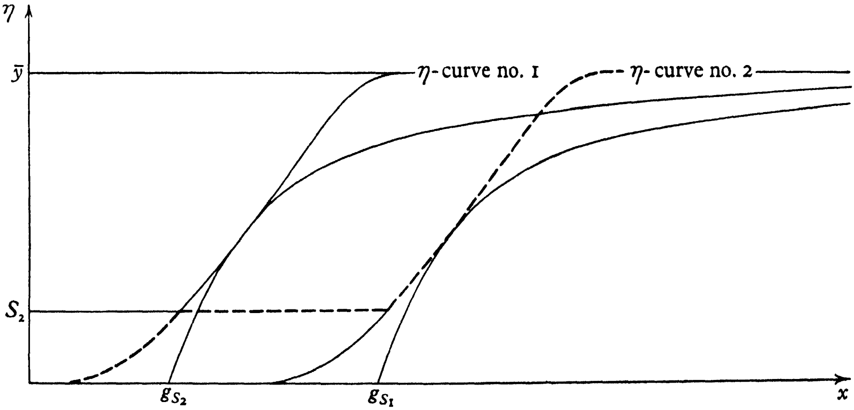

FIG. II App. D 4.

In all three cases, it will be seen, some degree of potential surprise greater than zero will be attached by the individual at his viewpoint to the idea that at date $[m]$, intermediate between the viewpoint and the date when x will be known, the standardised focus-gain will *increase*. We can go further, and say that in all three cases even a *small* increase of the standardised focus-gain will be associated with a rather high degree of potential surprise, and a larger increase will carry a very high degree. In case 2 we have the extreme example of this, since every increase no matter how small carries the absolute maximum $\bar{y}$ of potential surprise. Case 1, since it requires that every contour-line, even $\phi = 0$, should pass *below* the cusp, can only occur if the horizontal segment is either very short or else if it stands for a very high level of potential surprise $S = S(\overset{\angle}{K})$. If the horizontal segment is very short, the maximum possible amount of shift of the standardised focus-gain is correspondingly small; if $S = S(\overset{\angle}{K})$ is very high, a longer horizontal segment, and consequently a larger shift, are possible than with a low value of $S(\overset{2}{K})$; but then the associated degree of potential surprise is this very high value of $S(\overset{2}{K})$. Lastly, in case 3, in so far as, in the neighbourhood of the cusp, the slope of the contour-line is little less than that of the η-curve, the shift of the standardised focus-gain from g_{S_1} to g_{S_2} in Fig. II App. D 5 will be small, even when associated with fairly high potential surprise, and will be smaller, the lower this potential surprise.

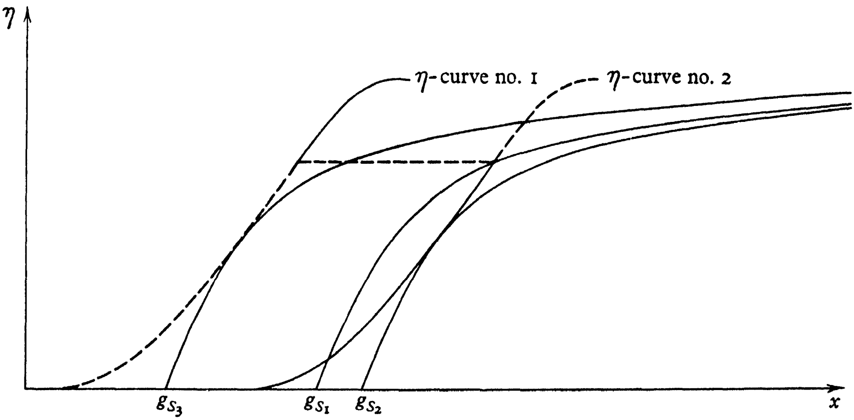

FIG. II App. D 5.

Let us now turn to what we may call the continuous case. We suppose that instead of having in mind only two hypotheses $\overset{1}{K}$ and $\overset{2}{K}$, the individual has infinitely many, each carrying a different degree $S=S(\overset{i}{K})$ of potential surprise (these degrees ranging from zero up to $\bar{y}$), and each giving rise to a distinct member of a continuous 'spectrum' or family of curves such as $\eta=\eta(\overset{i}{K}, x)$. As the number of hypotheses $\overset{i}{K}$ and of curves $\eta=\eta(\overset{i}{K}, x)$ becomes larger and larger, the length of each horizontal segment by which the curve $y=y(G, x)$ transfers itself from one η-curve to another, and also the lengths of the segments in which $y=y(G, x)$ coincides with one or other of the η-curves, will become shorter and shorter, until $y=y(G, x)$ eventually appears as a smooth instead of a kinked curve. This curve must, by its nature and meaning, intersect each successive η-curve at that level of potential surprise which the individual at his viewpoint attaches to the hypothesis that, at date [m], he will adopt this particular η-curve as his sole and effective potential surprise curve concerning the outcome x of the venture. It is plain that the slope of this smooth curve will be everywhere *less* than that of the η-curves which it is crossing. The three ways in which we saw that the primary focus-gain could occur when there were only two η-curves will in the continuous case merge into one another, and in order to find the primary focus-gain we have only to seek the point of tangency of the smooth curve $y=y(G, x)$ with one of the projected contour-lines of the ϕ-surface. The standardised focus-gain will be the x-intercept of this contour-line.

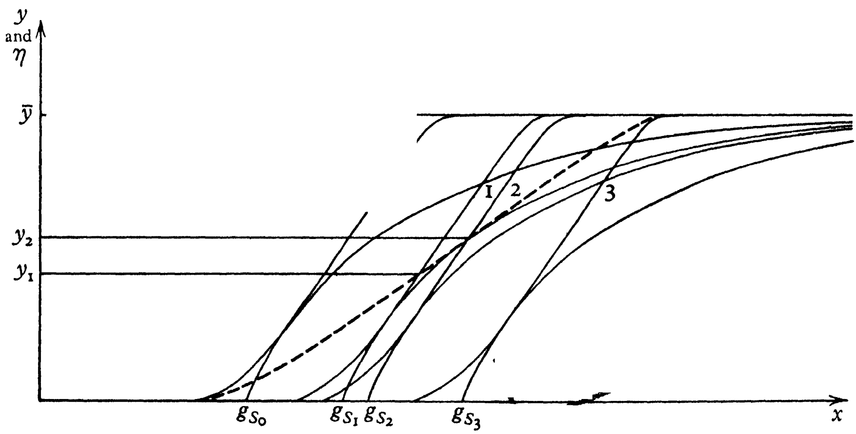

FIG. II App. D 6.

The situation is illustrated in Fig. II App. D 6. Here the curve $y=y(G, x)$ (shown as before by a broken line) takes naturally the same general bell-shaped form as each of the η-curves, but with a lesser average slope. This curve $y=y(G, x)$ is, as we have pointed out above, the effective y-curve from which the primary and standardised focus-gains are determined. It is what we have hitherto referred to simply as 'the y-curve' or 'the potential surprise curve'. It is tangent to a contour-line of the ϕ-surface at a point through which there also passes that member, which in the figure we have labelled 2, of the family of curves $\eta=\eta(\overset{i}{K}, x)$. This member (or rather the hypothesis that at date [m] it will be adopted by the individual as his effective y-curve) carries potential surprise $S(K)$ equal to the y-co-ordinate of its point of intersection with the curve

$y = y(G, x)$, and represented in the figure by a horizontal line labelled y_2. If, then, the individual were to alter the degree of potential surprise he attaches to the η-curve labelled 2 from y_2 to zero, and at the same time increase the degrees of potential surprise he attaches to every other η-curve up to the absolute maximum, $\bar{y}$ (that is, if he were to adopt η-curve number 2 as his sole and effective potential surprise curve), the standardised focus-gain would cease to be the x-intercept (marked g_{S_1}) of the contour-line which is tangent to $y = y(G, x)$ and would shift instead to the x-intercept, marked g_{S_2}, of that contour-line which is tangent to curve 2 of the family of η-curves. Thus at his viewpoint the individual attaches to the idea that the standardised focus-gain might, at date [*m*], increase from g_{S_1} to g_{S_2}, a degree of potential surprise $y = y_2$. From Fig. II App. D 6 we can read off the degree of potential surprise which, at his viewpoint, he attaches to any other amount of shift of the standardised focus-gain from the position it has at the viewpoint. For example, a shift to any value greater than g_{S_3} carries the absolute maximum of potential surprise. Our discussion thus far has led us to the conclusion that any *increase* of the standardised focus-gain, imagined by the individual to occur at some intermediately future date such as [*m*], will carry some degree greater than zero, and increasing rapidly with the size of the imagined increase of the standardised focus-gain, of potential surprise. What, then, is the individual's attitude to the idea that the standardised focus-gain will *remain unchanged* at its viewpoint level through and beyond date [*m*]? This hypothesis is evidently equivalent to the hypothesis that the particular member, labelled 1 in the figure, of the family of η-curves might at date [*m*] be adopted by the individual as his new effective potential surprise function; for this is the particular η-curve which is tangent to the same contour-line as the viewpoint potential surprise curve $y = y(G, x)$. This hypothesis carries potential surprise $S(\underset{i}{K})$ equal to the y-co-ordinate of the point of intersection of the η-curve number 1 with the curve $y = y(G, x)$. This degree is shown on the diagram by a horizontal straight-line segment marked y_1. This result, that the individual attaches a degree of potential surprise *greater than zero* to the idea that the standardised focus-gain may *remain unchanged* by the answering of the question at date [*m*], may at first sight seem rather paradoxical; but not, I think, when we remember that the standardised focus-gain is itself, in any case, a value of x which carries some degree greater than zero of potential surprise. What of a future *decrease* of the standardised focus-gain? In Fig. II App. D 6 we suppose that the member labelled 0 of the family of curves $\eta = \eta(\overset{i}{K}, x)$ carries zero potential surprise, i.e. that $S = S(\overset{i}{K})$ is zero for this curve. Then it is plain that the hypothesis that at date [*m*] the standardised focus-gain will *decrease from* g_{S_1} *to* g_{S_0} carries, at the viewpoint, zero potential surprise. A *smaller* decrease than this will carry some degree greater than zero of potential surprise. If, to the *left* of the member labelled 0, there are other members of the family of η-curves, and these other members also carry zero potential surprise, i.e. $S(\overset{i}{K})$ is zero for each of these also, then hypotheses that the standardised focus-gain will decrease by *larger* amounts than to g_{S_0} correspondingly carry, at the viewpoint, *zero* potential surprise.

We spoke on p. 50 above of constructing the function according to which, at his viewpoint, the individual assigns degrees of potential surprise to various amounts of shift, conceived to occur at date [*m*], of the standardised focus-gain. To construct such a function in the *continuous* case, all we need do is to plot against each other the y-co-ordinate of the point of intersection of the viewpoint potential surprise curve, $y = y(G, x)$, with each η-curve, and the x-intercept of the contour-line tangent to this η-curve. The result will evidently be a curve rather steeper than the viewpoint potential surprise curve itself, but having the same general bell-shaped form.

From our discussion the following proposition, important for some of our subsequent analysis, has emerged: To the idea of any *increase* of the standardised focus-gain, imagined to occur at some date intermediate between the viewpoint and that future date when the value of x will be known, the individual at the viewpoint must of logical necessity (because of the nature of potential surprise curves) attach some degree, greater than zero, of potential surprise, and to any considerable amount of increase he must attach very high potential surprise. To the idea of a *decrease* of the standardised focus-gain, on the other hand, no matter how large (provided in the 'continuous' case that it be larger than a certain minimum), he can, so far as the general inherent logic of potential surprise curves is concerned, attach *zero* potential surprise. Correspondingly, by a parallel argument to that developed above, he must attach some degree greater than zero of potential surprise to the idea of any *increase* of the standardised focus-*loss*, and again he can attach zero potential surprise to the idea of a *decrease*, however large (provided in the continuous case it be larger than a certain minimum), of the standardised focus-loss. In sum, the idea of any movement imagined to occur at some intermediate future date of one or both of the standardised focus-outcomes, *inwards*, towards *smaller* numerical values, can carry zero potential surprise; and there is nothing in the inherent logic of potential surprise curves which restricts the size of inward movements to which zero potential surprise can be assigned; but the idea of any movement, however small, *outwards* must carry potential surprise greater than zero, and a degree of potential surprise increasing rapidly with increase of the size of the movement.

From the argument of this Appendix there follows a consequence of prime importance for our later analysis:

We have seen in Chapter II that the relative attractiveness to an individual of different courses of action amongst which he must choose can be expressed by their relative positions on his gambler indifference-map. Any one such course will be rejected by him if he has also in mind some other course located on a higher gambler indifference-curve. Our discussion in this Appendix has shown it to be possible for him to reject a course of action on this ground, and yet to have in mind the possibility that at some future date the attractiveness of the temporarily rejected venture may improve; that is to say, that it may shift to a higher gambler indifference-curve. But there is only one mode of occurrence of such a hypothetical or contingent future improvement, to which he can attach zero potential surprise, viz. the hypothesis that the standardised focus-*loss* may *decrease*. We have seen that he must always, by the nature and meaning of potential surprise curves, attach some degree greater than zero of potential surprise to the idea that the standardised focus-*gain* could *increase*. From this proposition we shall later draw an inference helping to explain a feature of the business cycle.

CHAPTER III

EXPECTED CLARIFYING OF EXPECTATIONS AND ITS EFFECT ON THE PACE OF INVESTMENT

We shall be concerned in this chapter with some of the bases of an individual's decision whether or not to lay out within a short interval of time lying in his immediate future an amount of money, large in proportion to his total resources, on radically altering the character or scale of his business: in particular, on buying or having constructed to his order civil engineering works, buildings, industrial plant, or machines; productive equipment of a durable kind in the most general sense. All such equipment embraced within a single decision we shall call 'a plant'. The value of a plant to an enterpriser is the excess of its total gross earnings, including its scrap value, or its market value if sold before it ceases to be usable, over his total outlays on constructing and operating it, due allowance being made by discounting for the fact that these outlays and earnings will occur at a number of different dates and must therefore be reduced to terms of their equivalents in spot cash at some one date. An exactly equivalent value for the plant is obtained if we distinguish the outlays on constructing the plant from those which will be necessary after its completion to pay for those things, such as labour, fuel or electric power, and materials to provide the substance of the product, needed to operate the plant; and if we correspondingly subtract, from the assumed gross sale-proceeds of the product in each future interval, the amount which the enterpriser assumes will be spent in that interval on these latter co-operating means of production. The value of a plant can be most compactly expressed as an integral. Let μ be the distance of a variable point of time from some earlier date $\mu = 0$, and let ΔW be the enterpriser's outlay during $\Delta\mu$ on labour, etc., to operate his plant. Then $\theta(\mu) = \lim_{\Delta\mu \to 0} \frac{\Delta W}{\Delta\mu}$ is his instantaneous time-rate of outlay on running the plant at date μ. Let $\Delta\Omega$ be the proceeds during $\Delta\mu$ from the sales of the product or of the plant itself if disposed of for scrap or as a going concern to another enterpriser, and let $\omega(\mu) = \lim_{\Delta\mu \to 0} \frac{\Delta\Omega}{\Delta\mu}$. Let $f(\mu) = \omega(\mu) - \theta(\mu)$. Let t be the distance of the enterpriser's viewpoint from the same earlier date $\mu = 0$ from which μ is measured, so that $\mu - t > 0$ stands for a stretch of future time* measured forward from the viewpoint. Then if ρ is the yield on consols and L the distance from $\mu = 0$ of the date when the plant will be sold or abandoned, we define the enterpriser's *viewpoint subjective value* for the plant as

$$v = \int_t^L f(\mu)\, e^{-\rho(\mu - t)}\, d\mu.$$

This formula is applicable whether the plant already exists and has been in use, or whether it is merely conceived in an individual's mind. For in either case all the value it possesses, potentially or actually, at the viewpoint must be derived from the powers attributed to it to yield valuable services *in the future*; anything that has happened in the past is at most merely a guide or indication of what this attribution of powers, this *expectation*, ought to be. All the variables and functions which occur in this expression, except the location of the viewpoint itself, must therefore have values or forms *assumed* for them; this is true of the functions $\omega(\mu)$ and $\theta(\mu)$, of the rate of interest ρ (which must be looked on as a function of μ) and of the remaining length of time during which the enterpriser will possess or use the plant, $L - t$. An expression precisely similar in structure to that written above for v, with $f(\mu)$ replaced by $q(\mu)$ the enterpriser's instantaneous time-rate of outlay on constructing the plant, will give us the *viewpoint construction-cost*. The difference between the viewpoint subjective value and the viewpoint construction-cost is what we mean by the gain or loss from constructing such a plant. This gain or loss is a specially clear-cut example of an outcome to which the conception of focus-values described above is appropriate. Even the construction-cost, though it refers to proposed operations in the immediate future, cannot always be exactly known in advance. The *value* of a new plant to an owner who intends to retain possession of it during the whole of its working life clearly depends on the circumstances of a long series of years stretching into the future. If instead he hopes to sell the plant, once he has brought it to concert pitch as a going concern, then its value to him depends on what he thinks will be the opinion concerning those future years held by the most sanguine potential purchaser; or on what that purchaser himself thinks will be the opinion of some subsequent purchaser; and so on indefinitely.† Clearly in either case, whether the potential con-

* Namely, the distance of the image-date ahead of the viewpoint.

† The dependence, for example, of Stock Exchange values on what people think other people are going to think is going to happen, and so forth, was first pointed out by Lord Keynes in *The General Theory of Employment, Interest and Money*, Chapter 12.

structor intends to hold or to sell the plant, the present value to be assigned to it, while it still exists only in his mind, is open to extremely divergent conjectures. Thus the outcome of constructing the plant will present itself as a wide range of hypotheses varying from a gain of several times the amount laid out on construction to the loss of the whole of it, and a considerable inner range of these hypotheses will carry *nil* potential surprise. The choice which faces the holder of a large sum of cash, between putting into execution one or other of a large number of projects for constructing equipment of widely differing characters, or retaining the cash or lending it at fixed interest, will be resolved according to our theory by comparing, in the manner suggested in the next paragraph, the pairs of focus-values assigned to the different courses of action.

Let us use the term 'blueprint' to mean a project, existing only in a person's mind or on paper, for constructing a block of equipment of a specific character. One of the focus-values which the individual assigns to the outcome of each blueprint will represent a gain and the other a loss, and we shall call these the 'focus-gain' and the 'focus-loss' of the blueprint. If we write v for the value and c for the construction cost of a plant, and put $x=v-c$, then positive values of x will represent a gain and negative values a loss, and we could speak of a *numerical* increase of the loss $c-v$ as a decrease of x. But for simplicity of statement we shall say that the loss increases when we mean that it increases *numerically*, passing to an algebraically smaller value of x. We shall sometimes speak of the focus-values of x as the 'focus-outcomes' of a given course of action. Suppose that for a blueprint A the standardised focus-gain is g_A and the standardised focus-loss is h_A, while for a blueprint B the standardised focus-outcomes are g_B and h_B. If on the gambler indifference-map representing the individual's attitude, in his current circumstances, to uncertainty, the point (g_B, h_B) is on a gambler indifference-curve which lies above and to the left of that containing (g_A, h_A), then the investor will decide, so far as the choice between A and B is concerned, in favour of B. All other pairs of uses for his cash can be compared in the same way and his course thus decided on.

A blueprint has amongst its attributes not only the nature of its product and its own technical design and its location, but also the specific period of future time in which it is intended to be operated. An investor who has in mind at a particular date a given type of plant will assign to it different pairs of focus-outcomes according as he considers constructing it now or at various distances in future time. For not only will he expect the circumstances of two different specific periods to differ, but the more distant the period considered, the less light his present knowledge will seem to throw on it, and the wider will be the inner range of values of each important variable* for dates within the period. Thus if we designate different physical types of plant by letters A, B, etc., we must also attach to each letter a subscript indicating the particular calendar date at which we are supposing that the plant will be completed and begin its operating life.

Let each of the symbols $A_{[1]}$, $A_{[2]}$, ..., $A_{[r]}$, ... represent the idea of constructing a plant of given technical design on some fixed, specified date, denoted by the subscript, the design of the plant being the same for all the dates. When his viewpoint is located at date [1] the investor will have in mind for each of these blueprints its own schedule assigning to each hypothetical outcome of this blueprint its particular degree of potential surprise. That is to say, at date [1] he will have in mind for any blueprint $A_{[r]}$ a unique specific form of the potential surprise function, such functions differing from each other in general for technically identical blueprints of different construction dates. But this form which he can specify *now* is not the only form† of the potential surprise function for $A_{[r]}$ which he must take into account. He can foresee that when the present moment will have arrived at date [2], and again at date [3], and so on, up to date [r] itself, he will construct a fresh potential surprise function for $A_{[r]}$ which, since at each of those dates he will have in mind knowledge and ideas which will have come to him in the interval, can differ from the form he assigns now. He cannot, of course, specify uniquely *now* the form of function that he will assign to $A_{[r]}$ *at date* [r]; but we shall see that he can sometimes form some judgements about it which are as important, for deciding on his present action, as the form which he assigns now. For convenient reference we must have a notation distinguishing from each other the ideas, present in his mind at date [1], of constructing potential surprise functions for $A_{[r]}$ at different future dates. For this purpose let us attach to the letter y, standing for some form of potential

* Such as prices, total value of backlogs of orders, size of inventories in various hands, rates of taxation, relative numerical strength of parties in Congress or Parliament, etc.

† A potential surprise function $y(x)$ will be considered to have assumed a different *form* if the degree of potential surprise it assigns to *any* value of x has changed. Thus, of course, any translation, parallel to the x-axis, of the curve as a whole without deformation (any *rigid* transformation parallel to the x-axis) will be spoken of as a change of form, equally with all transformations (e.g. linear or affine transformations) which do deform the curve.

surprise function, two subscripts, of which the first will specify the viewpoint or moment at which the investor is actually doing his thinking, and the second will specify a date, which may be the same as the viewpoint or may be still in the future, at which he will be assigning a particular form to $y(x)$. Thus $y_{[1][3]}, A_{[3]}$ is the identity disk, as it were, which the investor attaches, at date [1], to that particular form of potential surprise function which he will assign to $A_{[3]}$ at date [3]. By its nature $y_{[1][3]}, A_{[3]}$, if it is other than identical with $y_{[1][1]}, A_{[3]}$, clearly cannot be a unique form, but may consist, for example, in two or more hypotheses, each carrying zero potential surprise, one of them being a form of $y(x)$ having its *upper branch** identical with that of $y_{[1][1]}, A_{[3]}$, while another is a form having its *lower* branch identical with that of $y_{[1][1]}, A_{[3]}$, so that the combined presence in his mind of these *two* hypotheses, choice between which will not be possible till date [3], accounts for the form which he assigns *now* to $y(x)$. When the investor's viewpoint or present moment arrives at date [3] then $y_{[3][3]}, A_{[3]}$ will come into existence and will be a unique form, and any differences between this form and $y_{[1][1]}, A_{[3]}$ will have been due to fresh news or inferences which came into the investor's mind between dates [1] and [3]. Thus so far as he can make, at date [1], any judgements as to what questions will be answered by the fresh news or knowledge which will come to him before date [3], he can give to the symbol† $y_{[1][3]}, A_{[3]}$ a content consisting of a number of hypotheses, each of which will name a different specific form of potential surprise function as the one to be adopted at date [3]; each of these hypotheses will correspond to one particular set of answers to the questions; and to each of these hypotheses he can attach its particular degree of potential surprise. If the particular form of potential surprise function, named by one of these hypotheses, assigns to some value of x a lower degree of potential surprise than that assigned to this value of x by $y_{[1][1]}, A_{[3]}$, the enterpriser must at date [1] attach to this hypothesis some degree greater than zero of potential surprise; for he cannot at one and the same time regard a given value of x as *unlikely*, and also regard the possibility of his changing his mind in this respect as *likely* (i.e. as deserving zero potential surprise), without involving himself in a logical contradiction. But there is

* See Chapter II, p. 14.

† For vividness of presentation, we will continue for a time to use an actual numeral [3], to typify a future date. It will be understood that the use of the particular numeral [3] for this purpose has no significance; any other numeral would have done. In later sections we shall represent future dates by [*m*] or [*n*].

no logical hindrance to his attaching zero potential surprise to any of the hypotheses constituting $y_{[1][3]}$, $A_{[3]}$ which assigns to $y(x)$ a *higher* value than is assigned to the latter, for the same value of x, by $y_{[1][1]}$, $A_{[3]}$. Now we have seen (Chapter II, p. 27, and Appendix D to Chapter II, p. 46) that if, when we pass from a y-curve of some given shape to one of a different shape, some values of x become associated with higher, but none with lower, values of y, and if one of the values of x thus affected was, with the former curve, one of the primary focus-outcomes, then the standardised focus-outcome obtained from the new curve will be a numerically smaller value of x than the standardised focus-outcome obtained from the old curve. (The standardised focus-outcome in question, if a gain, will be a smaller gain, or if a loss, will be a smaller loss, with the new curve than with the old.) Thus it is in the nature of (standardised) focus-outcomes that the investor will attach some degree greater than zero of potential surprise to the idea that the focus-gain or the focus-loss of a blueprint of distant construction date might increase as the present moment advances towards that date. But he can, so far as logical consistency is concerned, and in some circumstances will, attach zero potential surprise to the idea that either of them may decrease.*

A change in the size of the standardised focus-gain of a blueprint, or in its standardised focus-loss (and in general also a change in both at once), will shift the blueprint to a different gambler indifference curve. We have seen (briefly in this chapter, and in more detail in Appendix D to Chapter II) that there are four possibilities to which it is logically possible, in view of the nature and meaning of potential surprise curves, for the individual to attach zero potential surprise.

1. That between the viewpoint and the date when the outcome of the blueprint will be known, no change will occur in either of the standardised focus-outcomes.

2. That at some date $[m]$, or at more than one such date, intermediate between the viewpoint and the date when x will be known, the standardised focus-*gain* will *decrease.*

* In this paragraph we have touched only briefly on the question what kinds of imagined change in his own expectations (conceived as occurring at some future date intermediate between the viewpoint and that date when the outcome itself of the venture will finally be known) the individual can take account of at his viewpoint; and what degrees of potential surprise he will, in various circumstances, assign to different kinds and degrees of such change. This topic is analysed at length in Appendix D, to Chapter II.

3. That at [m] the standardised focus-*loss* will *decrease.*

4. That at [m] *both* of the standardised focus-outcomes will *decrease.*

Of these possibilities, number 3 implies that the individual at his viewpoint can attach zero potential surprise to the idea that at date [m] the attractiveness of the blueprint will greatly increase. Number 2 implies that he can attach zero potential surprise to the idea that at date [m] its attractiveness will greatly decrease.

Sometimes the individual will be able, not merely to attach zero potential surprise to the occurrence, at some intermediate date, of an improvement or diminution of the attractiveness of a given

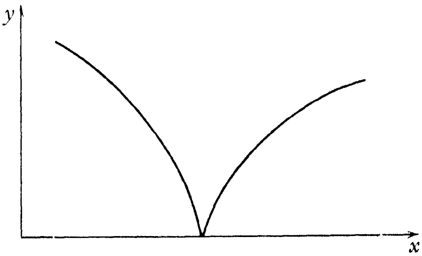

FIG. III 1.

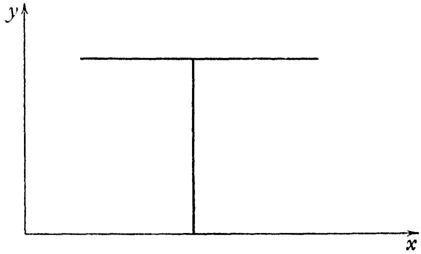

FIG. III 2.

blueprint, but even to attach some degree greater than zero of potential surprise to the *non*-occurrence of such a change. Let us briefly consider this. When some given result depends on the fulfilment of several conditions, then the non-attainment of the result can carry potential surprise greater than zero only if the non-fulfilment of *each separate* condition carries potential surprise greater than zero.* In order that the investor may be able to attach potential surprise greater than zero to the *failure* of, say, the standardised focus-loss of a blueprint to decrease at some date [m], he must attach potential surprise greater than zero to the failure of each of the hypotheses (1) that a suitable question bearing on the prospects of the blueprint will be answered at date [m] and (2) that the answer will be such as

* It may be worth while to digress on this point for a moment. In general, it will be seen, if we attach potential surprise greater than zero to the *non*-occurrence of some particular result, this implies, of course, that the *occurrence* of any and every alternative result, each considered by itself, carries potential surprise greater than zero; in the case of a continuous variable x, for example, $y(x)$ might have the form shown in Figs. III 1 or III 2, amongst an infinite variety of others. The degree of potential surprise attached to the non-occurrence of any given result is evidently identical with the least degree of potential surprise attached to the occurrence of any alternative result.

to effect this amount of decrease in the standardised focus-loss. Now it will often be possible for the enterpriser to feel quite sure, or in other cases fairly sure, that a suitable question will be answered on, or before, some particular calendar date. The question may be, for example, what will be the result of some election, the provisions of some new statue, the character of a Budget, or the outcome of a mineralogical survey: And it will sometimes occur, as we have seen in Appendix D to Chapter II, that some particular amount of decrease of a standardised focus-outcome is the only hypothesis, regarding date $[m]$, to which the individual attaches zero potential surprise. Thus our two requirements can be fulfilled.

We have now reached an important stage in the argument of this chapter, viz. the conclusion that in appropriate circumstances the investor can without logical contradiction feel that a great increase or a great diminution in the attractiveness which a given blueprint derives from his expectations could occur without surprising him; and that he may sometimes even feel that he will be very surprised, at some named future date, unless one or other of these things does occur. In the following paragraphs we shall see two ways in which this feeling may induce him to refrain from immediate construction of a blueprint which he would otherwise find attractive. If for either reason he does so refrain, the aggregate national investment-flow will be lower, over some period of the immediate future, than it would otherwise have been.

Let us suppose that the investor has in mind a blueprint for immediate construction $A_{[1]}$, which he would embark on, rather than retain his cash or lend it at fixed interest, if he had no other blueprint in mind. And let us suppose that he has in mind a blueprint for deferred construction $B_{[n]}$ to which $y_{[1][1]}, B_{[n]}$ assigns an extremely high focus-gain, but also assigns a focus-loss so large that, if the blueprint's attractiveness depended on this pair of focus-outcomes alone, the investor would dismiss it from consideration. And further, let us suppose that $y_{[1][n]}, B_{[n]}$ comprises a number of different hypotheses about the form of the potential surprise function which the investor will assign to $B_{[n]}$ when date $[n]$ is reached, and that amongst these there is one, carrying *nil* potential surprise, which would imply the same focus-gain as $y_{[1][1]}, B_{[n]}$, but a focus-loss so small as to render $B_{[n]}$, in view of its high focus-gain, an extremely attractive investment-opportunity. If the opportunity which would thus be created were actually and immediately open to him, the investor would construct $B_{[1]}$ in preference to $A_{[1]}$. But the creation of this opportunity is in fact only a contingency, and both the occurrence and the non-occurrence of this contingency carry *nil* potential surprise. Accordingly, he must consider what alternative investment-opportunity would be open to him at date [n] should the focus-loss of $B_{[n]}$ fail to decrease. If this alternative is not less attractive than $A_{[1]}$, then a decision to retain his cash until date [n], in order to be assured of the power to venture on $B_{[n]}$ should he then wish to, will imply at worst a postponement and not the losing altogether of an investment-opportunity of the quality of $A_{[1]}$. In such a case, provided the date [n] is not too remote from the present, it seems clear that the investor will decide to retain his cash. But this will imply, of course, that the national aggregate investment-flow will be lower, over some period of the immediate future, than it would have been in the absence of any blueprint such as $B_{[n]}$.

Even if it seems to the investor that the best alternative to $B_{[n]}$ which will be open to him at date [n], should the answers to the special questions turn out unfavourable to $B_{[n]}$, will be less attractive than $A_{[1]}$ now is, he may yet decide to retain his cash; for the contingency of a deterioration in his position, such that he will be offered at date [n] a worse instead of a better opportunity than $A_{[1]}$, may be outweighed in his mind by the contingency of a great improvement. On the other hand, it is possible that even if the best alternative to $B_{[n]}$ is actually better than $A_{[1]}$, he may yet decide to construct $A_{[1]}$; for the length of the period from date [1] to date [n] is also, as we shall see, a factor in his decision. If this period is short in comparison with the useful lives of plants such as A and B, and if, for example, as will frequently be the case, he expects $A_{[n]}$ to be not less attractive than $A_{[1]}$, then the existence in his mind of a blueprint for deferred construction which is *contingently* very much more attractive than $A_{[1]}$ will cause him to retain his cash.

Why should a decision to invest now in $A_{[1]}$ preclude the investor from investing a little later on in $B_{[n]}$, if by then he has come to prefer the latter, even supposing his cash resources, owned and borrowed together, are only sufficient to pay for one or other of the blueprints at a time? Could he not sell plant A when it is complete, or obtain a loan secured on it? This is the heart of the problem of *liquidity*, which consists essentially in the fact that a man can hold with perfect sincerity (and often, as it turns out afterwards, with justice), expectations which imply a far higher value for the plant than he can persuade others to believe in. The focus-gain which he himself assigns to A may be larger, and the focus-loss smaller, than the corresponding ones in the mind of any potential buyer of plant A. If in such a case he yet insists on selling A, he will make, according to his own expectations and corresponding valuation of A, a loss, and this may well offset the superior attractiveness which $B_{[n]}$ may by then have acquired. Thus, if having constructed A he finds when date $[n]$ is reached that $B_{[n]}$ has become a very attractive proposition, it may seem better, none the less, to hold on to A: but he will regret not having retained his cash. If at date [1] he foresees that this position may arise at date $[n]$, he will decide to retain his cash.

The mere existence in the investor's mind of a blueprint for deferred construction whose focus-gain and loss are both very high in relation to its construction-cost will not by itself constitute an inducement to him to retain his cash. For in regard to many such blueprints he will attach high potential surprise to the occurrence of any considerable decrease of the focus-loss at or before the construction date. He may often feel sure that no extra light will be thrown on the prospects of a given blueprint except in the actual course of using the plant after it has been constructed. He may have hypothetical future events in mind which could occur without surprising him, and which would reduce the focus-loss, but by an amount insufficient to render the blueprint attractive. In the nature of things, events must be rare which can seem so comprehensively to block up all paths by which misfortune might strike the enterprise in the course of its career that they reduce the focus-loss to nothing. But there are certain classes of 'experiment', such as elections, parliamentary voting on particular pieces of legislation, lawsuits, and harvests, the date of whose occurrence can be known in advance, and whose outcome can, if it is of the right kind, defend the enterprise against some of the hazards whose impact would be most disastrous and would otherwise carry no potential surprise. If the question, the answer to which is expected to throw light on the prospects of a blueprint, is one whose answer will be some value of a continuous variable, then $y_{[1][n]}, B_{[n]}$ may contain not merely a finite set of different hypotheses about the form of potential surprise function which will be assigned to $B_{[n]}$ at date $[n]$, but an infinite set, each member of which will, of course, imply a different focus-loss or gain, and in this case the *amount of the decrease* of either of these latter which can be imagined to occur at date $[n]$ will itself have to be looked on as a continuous variable associated with a varying degree of potential surprise. The idea of a large decrease of the focus-loss will, of course, be more interesting to the investor than that of a small decrease; but beyond a certain size each larger hypothetical decrease will carry a higher degree of potential surprise. One particular amount of decrease will have more power than any other to hold the investor's attention: a slightly larger one will lose more of this power through its higher potential surprise than it gains through its extra size; a slightly smaller one will lose more of this power through its smaller size than it gains through its lower potential surprise. Thus in such cases we can identify and speak of the *focus-decrease of the focus-loss*. When the investor is deciding whether the prospect of a decrease of the focus-loss of a particular blueprint for deferred construction is a sufficient inducement to him to retain his cash, the particular amount of decrease which concerns him is this focus-decrease.* We shall refer to that one of the hypotheses contained in $y_{[1][n]}, B_{[n]}$ which corresponds to the focus-decrease of the focus-loss as the favourable† focus-hypothesis of $y_{[1][n]}, B_{[1]}$.

The strength of the inducement to retain cash, arising in the way we have described in this section, will depend on four sets of factors:

(α) The focus-gain and loss of $A_{[1]}$; the focus-gain and loss of $B_{[n]}$ according to the favourable focus-hypothesis of $y_{[1][n]}, B_{[n]}$; and the focus-gain and loss of that blueprint say $C_{[n]}$, which would become the most attractive one in the investor's mind for construction at date $[n]$ if the questions should be answered in a sense *unfavourable* to $B_{[n]}$.

(β) The length of the interval between date [1] and date $[n]$.

(γ) The degree of potential surprise attached to the hypothesis that the questions will be answered not later than date $[n]$.

(δ) The amount by which the subjective value he attributes to $A_{[1]}$ if he assumes that its focus-gain will be realised exceeds the price for which plant A could be readily sold at date $[n]$.

Regarding (α), the investor may have in mind as an alternative to $B_{[n]}$, in case the latter's focus-loss fails to decrease at date $[n]$,

* For full precision and rigour, we must call this the primary focus-decrease of the standardised focus-loss. Is there any need for the investor to *standardise* this primary focus-decrease by asking himself what amount of decrease carrying *zero* potential surprise would have equal power to arrest his attention? He may have in mind, for comparison with a blueprint for immediate construction, not merely one but several blueprints for deferred construction, each having its own primary focus-decrease of its focus-loss, and these several primary focus-decreases may carry differing degrees of potential surprise. Each of them must, in that case, be standardised by the determination of an equivalent amount of decrease carrying zero potential surprise. This will be finally the standardised focus-decrease of the standardised focus-loss. We introduce this notion here for the sake of logical rigour and completeness. However, in the actual use we shall make of the notion of contingent decrease of the focus-loss of a blueprint for deferred construction, the distinction between primary and standardised focus-decrease has no part to play.

† There will also be a focus-decrease of the focus-gain; this we shall call the unfavourable focus-hypothesis.

a blueprint $C_{[n]}$ whose focus-loss would be greatly reduced, and $C_{[n]}$ thus rendered extremely attractive, by that same set of answers to the special questions which would reduce the focus-*gain*, instead of the focus-loss, of $B_{[n]}$. This circumstance would, of course, tend greatly to strengthen the inducement to retain cash. Next it is clear that the *length of time* he must wait in order to know the answers to the special questions is an influence on the inducement to wait. For, by assumption, there is something he would prefer to do with his cash rather than lend it at fixed interest. To wait, therefore, involves an item on the debit side, a cost, and the longer the prospective time which it will be necessary to wait, the stronger the temptation to choose the other alternative, the immediate construction of $A_{[1]}$. Regarding (γ), it is plain that if the investor feels some doubt as to whether the questions will be answered at date [n], then the stronger this doubt is, the weaker, *ceteris paribus*, will be the inducement to retain cash exerted by any given hypothesis regarding the character of the answers.

Clearly we cannot explore all the possible combinations of circumstance which can face the investor in making a choice between constructing a certain blueprint now and retaining his cash in view of the contingency that some other much more attractive opportunity may present itself a short time hence. What we have done in this section is to isolate the essentials of such a situation. It will be understood that throughout this discussion the word 'attractive' is used in a purely subjective sense. An investment-opportunity, or blueprint, is something which exists in the investor's mind. Its characteristics and qualities are thoughts, and not something which can be observed without reference to the individual.

We have now to consider how a particular blueprint for immediate construction, say $A_{[1]}$, whose focus-outcomes according to $y_{[1][1]}, A_{[1]}$ are in themselves highly attractive, may be rendered unattractive by the fact that one of the hypotheses contained in $y_{[1][3]}, A_{[1]}$, and carrying *nil* potential surprise, implies a much lower focus-gain than $y_{[1][1]}, A_{[1]}$. To explain this proposition we must consider the inherent structure of enjoyment by anticipation.

If we attach low potential surprise to the future occurrence of an event which, when it occurs, will give us pleasure, we can take pleasure in imagining this event in advance. To do this is to get enjoyment by anticipation. Since to enjoy by anticipation is itself a pleasurable act, and can itself be imagined in advance, it can give rise to a secondary enjoyment by anticipation, in which we enjoy the prospect of enjoying the prospect of a pleasure-giving event.

And there can evidently be a tertiary, etc., enjoyment by anticipation. To make matters precise, let us consider an arbitrary short period called a 'day', and let us call the day on which the event itself is due to occur day n, let the preceding day be day $n-1$, and so forth. Then on day $n-1$ the individual will enjoy the direct anticipation of the event itself. On day $n-2$ he will enjoy, first, a direct anticipation of the event itself, though at a greater distance in the future, and therefore less vivid and intense, than that which will be felt on day $n-1$; and secondly, he will enjoy looking forward to the pleasurable feelings which he is going to experience on day $n-1$. On day $n-3$ this twofold enjoyment which is going to be experienced on day $n-2$ can itself be looked forward to, as well as the feelings which are going to be experienced on day $n-1$, and the event itself; and so on. Each day added to the intervening period has two distinct and opposite effects on the total degree of enjoyment by anticipation felt by the individual at the moment from which he is looking forward. First,* it will remove the event itself, and also day $n-1$, day $n-2$, etc., one day further off from the present moment, and hence the intensity of enjoyment from looking forward to each of these days will be reduced. Secondly, it will add an extra day to those future days on which pleasurable feelings are going to be experienced, and which can therefore themselves be looked forward to with pleasure. This second effect has not, so far as I know, been anywhere discussed heretofore; but the first effect is a familiar idea in economic theory since it has no doubt been in the minds of all those who have used the concept of 'impatience' in discussing the rate of interest, and this first effect will be generally admitted to be often strong and important. Hence if we can show that the second effect is sometimes powerful enough to overcome the first effect, the importance for theory of the second effect will have been established. Now it may be that the first of these effects is always stronger than the second, so that *any* lengthening of the intervening period, however short that period was, causes a net reduction of the intensity of present enjoyment by anticipation. But there seems to be strong evidence that this is not so. Introspection and the testimony of others indicate that a prospective enjoyable event can be *too near* in time for

* We shall refer to these as the first and second effects. These 'effects' merely describe the comparison of what a person would feel when looking forward to a given event n days ahead with what he would feel when looking forward to the same event $n+1$ days ahead. The 'cause' of these 'effects' is merely the shift of our attention from one idea to another, and the 'effects' are therefore, of course, simultaneous.

the intensity of enjoyment by anticipation due to it to be a maximum. Suppose we know that a holiday is due to us at some unspecified time in the coming twelve months. Then if we are suddenly informed that it is to begin the day after to-morrow, we shall feel that we have lost something by the shortness of the notice, even supposing that all our plans and preparations have been made. In greater matters I believe this factor is correspondingly more important. For example, a man who might be prepared to venture a quarter of his fortune on an expedition to prospect for gold, whose outcome will not be known, perhaps, for years, might decline to risk it on the turn of a card, though the focus-gain and loss were the same in both cases. However, the question whether the first of the two effects is or is not stronger for all lengths, however short, of the period which separates the present moment from the date of the anticipated event, is irrelevant for our present purpose. What we can say, on the basis of pure logic, is that an investor will feel that a blueprint for immediate construction, of given focus-gain and focus-loss, whose outcome will be known at a specific future date, say T, will be rendered less attractive by the presence in his mind of some question, due to be answered at a much nearer date than T, which question if answered in one particular way would greatly reduce the focus-gain of the blueprint while leaving its focus-loss unchanged. For the possibility, carrying low potential surprise, that the question might be answered in that way, will cause him to fear the loss of part of the period during which he could otherwise hope to enjoy by anticipation a favourable outcome of the venture. When he begins construction of the blueprint, he looks forward to being in 'enjoyable doubt' (less enjoyable, of course, than certainty of a gain equal to the focus-gain, but more enjoyable than certainty of a very small gain) about its outcome during the whole of a period of years. It will be a serious impairment of the attractiveness of the blueprint if he feels that he would not be surprised to learn something one month hence which will greatly reduce the focus-gain; for this contingency would transform his

* I find evidence for the reality of the second effect in such remarks as the one I have italicised in the following extract from the City column of the *Evening Standard*, 8 February 1946:

'The bottom dropped out of the Mexican Eagle Oil share market this morning on the news of the agreement with the Mexican Government....

'The bulls of the shares are chiefly disappointed at the lack of an immediate cash settlement. Strong rumours have been put out in the past few days that the settlement would provide 20*s*. to 25*s*. a share instead of which holders will probably have to wait a year before they know what they are going to get.

'Against the immediate disappointment the speculators *will have a longer run for their money*.'

prospective state of mind, for the rest of the period up to date T, into one of 'disagreeable doubt'.

If the argument of the preceding sections is accepted, it is clear that the approach of some event, such as a Presidential election or an international conference, some conceivable outcomes of which are widely different from each other and would have extremely different effects on the valuation of a specific investment blueprint, will tend to make the aggregate investment-flow less than it would otherwise have been. There will be a *general* tendency amongst investors to wait and see. At a date when such an event is still remote, most of the investors who are then making such comparisons as that between $A_{[1]}$ and $B_{[n]}$, which we described above, will feel that the waiting time would be too long, and will each decide to construct at once the plant representing his $A_{[1]}$. But at a date nearer to the event, most of them will feel it worth while to wait and see. Thus the strength of the tendency for investment to be discouraged will continually increase as the event draws nearer, and then jump discontinuously to zero at the moment when the event has just occurred.

It follows that such an event when it actually occurs may release a large number of investment-decisions, even though it has not lessened the focus-loss nor raised the focus-gain of any blueprint in the mind of any investor. Blueprints for immediate construction will no longer have to compete in the investor's mind with the contingency that opportunities of still higher promise may arise at a near future date; and their attractiveness will not be decreased by the fear that a near future date may bring an abrupt reduction of their focus-gain. Thus many blueprints, such as the one which on p. 62 we labelled $A_{[1]}$, may begin to be constructed.

An actual occurrence which causes surprise proves that the individual's structure of expectations* either contained a misjudgement or was incomplete. Either the event was included amongst his hypotheses but excluded from the inner subset, where clearly it ought to have been, or else it formed no part of any hypothesis.† If his

* I.e. the whole assemblage of sets of variants in his mind.

† I believe the distinction may be important between these two types of surprising events, which I propose to call *counter-expected events* and *unexpected events*:

Counter-expected event:	an hypothesis which has been considered and to which as a consequence of this examination a high degree of potential surprise has been assigned.
Unexpected event:	a contingency which has entirely escaped attention, which has never entered the individual's mind, and has formed no part of any hypothesis.

A person's structure of expectations may be more completely demolished by an unexpected event than by a counter-expected event; the former reveals not merely exclusion of this event from the inner subset was wrong, so may be his exclusion, based on similar assumptions and reasoning, of other hypotheses still to be tested, and he must consider again his other judgements of this kind. If the event was something entirely unthought of he must consider a mass of new ideas which it will generate about the paths which the course of events may follow. Thus an important surprising event will require him more or less to create afresh his structure of expectations. Such an event will cause his whole existing set of judgements, by which he has assigned to each hypothesis its particular degree of potential surprise, to become a dead letter.* He cannot instantly substitute for this old set of judgements a new set emerging from a thorough examination of the surprising event in all its bearings and implications, for this examination will take time. But in the nature of things it must be possible for him at any moment to answer the question whether the actual realisation of such and such an hypothesis would surprise him. It follows that he will have in mind, during the interregnum between the abandonment of one set of fully considered judgements and the establishment of a new one, some set of provisional judgements. This must, in the nature of the case, consist in a wide allowance for events, and values of variables, which formerly he would have labelled as potentially surprising, but which he cannot now condemn offhand. Every y-curve in his mind is likely to be expanded, as it were, by extending the inner range at both ends and perhaps by lessening the slope of the increasing and decreasing segments; in fact, by decreasing for every value of x, except those already within the inner range, the value of y assigned to it. Thus the impact effect in his mind of a major surprising event† will include a great precautionary increase in the focus-gain and focus-loss of each one of the blueprints he has in mind. But at the moment when he makes these increases, he will be able to look forward to a moment, that moment, namely, when he will have completed the examination of the significance of the surprising event, when many or all of these focus-gains and focus-losses will have again been reduced; not, of course, in general back

a misjudgement, but the fact that the individual is not only unable to know some essential features of the situation but has been ignorant of the existence and extent of his ignorance.

* These existing judgements will not necessarily be altered as a consequence of his studying the implications of the surprising event, but until this examination has shown whether or not any change is called for, their authority will be suspended.

† By a major surprising event I mean one which is both very surprising to the individual and highly relevant to his judgements; that is, a very surprising change in some very important circumstance.

to their original positions, but to new levels higher or lower than their original levels but lower than their provisional levels. He will, at least, attach very high potential surprise to the supposition that most of them will *not* by then have been thus reduced.* Thus, if at the moment when a surprising event has just occurred, he looks ahead to the moment when his examination of this event will have been completed, he is looking ahead to a future date at which the answers to special questions, in the sense we have given to this phrase above, will become known to him. Thus the occurrence of a surprising event will give him the same incentive to 'wait and see' as the approach of a date when the outcome of special 'experiments', such as elections or harvests, will be known. He will be tempted to hold off from embarking immediately on the construction of any blueprint $A_{[1]}$ for two reasons: (1) in case some other blueprint may emerge from his process of examination with a more attractive pair of focus-outcomes than those assigned to $A_{[1]}$ by his provisional judgements; and (2) in case his new set of fully considered judgements assigns to $A_{[1]}$ a much less attractive pair of focus-outcomes than those assigned to it by his provisional judgements. The former possibility will be made more insistent to his mind by the fact that the precautionary increases of focus-gains and focus-losses will, by their nature, have been such as to *level up* the attractiveness of different blueprints. These precautionary increases will have been generous in all cases, and more or less determined by a factor common to all the blueprints, namely, the need to make sure that these increases shall be large enough. The individual will tend to push them all up to some common high level.

Our main conclusion from the argument of this section is as follows: One effect of an event which causes surprise will be to heighten at first the attractiveness of liquidity, that is, of deferment of choice of a specific blueprint, and discourage the immediate construction of equipment. If a large number of investors are thus affected by the same event, the aggregate investment-flow in some period closely† following this event will be lower than it would otherwise have been.

The time occupied by the process of examination may be considerable. The sorting out and assembling of fresh impressions, the gradual evolution of new ideas, the tracing out of all the bearings

* Since *both* its focus-outcomes will be reduced, he cannot tell whether any blueprint will be rendered more attractive or less attractive by the process of examination; either can happen.

† Closely, rather than immediately, since plans for the immediate future can be altered only at high cost.

of the event on what was in his mind before, and the canvassing or waiting for signs of other people's reactions to the same event, will make an arduous process not compassed in a day or two. We shall call this process the assimilation of the event into the structure of expectations. To this process may be added a *testing* of the new set of judgements by waiting to see if more events will come along such that they would have been surprising had they occurred before the event which occasioned the process of examination, but are not so now that this event has been assimilated.

If an event, after its meaning has been assessed, *increases* a focus-gain or a focus-loss, then by the definition of these latter it must have been surprising. A hitherto unthought-of event, even though it turns out, after assimilation into the structure of expectations, to *decrease* a focus-gain or a focus-loss, will also at its occurrence have been surprising. And, as we have seen on p. 52, it is possible for a hypothetical decrease of a focus-gain or loss, when this decrease exceeds a certain size, to carry a degree greater than zero of potential surprise, and such a decrease if it occurred could therefore be the outcome of a counter-expected event. But many of the events which *decrease* a focus-gain or loss will be non-surprising; they will be some of those very events which the investor had in mind in assigning *nil* potential surprise to those profit-outcomes of the venture falling within the inner range; such events will be assimilated easily and quickly into the individual's structure of expectations, and he will be in as good a position to make a decision immediately after such an event as before it. Thus we find that there is an important asymmetry between investment-stimulating and investment-depressing events. Investment can be stimulated, after the assimilation of the event, by either an increase of focus-gains or a decrease of focus-losses. But only in the latter case will the assimilation and therefore the effect be immediate. In the former case the *impact effect* will be depressive. Investment can be depressed by either an increase of focus-losses or a decrease of focus-gains, and the effect in *both* these cases will seem to be immediate, though in fact the downward movement following an event which is going to increase focus-losses will be due at first to the mere 'standstill' effect of the surprising event. This asymmetry seems at least partly to explain why the downturn of investment and employment after a boom is usually more abrupt and rapid than their upturn after a slump.

CHAPTER IV

THE CHOICE OF ASSETS TO BE HELD FOR SPECULATIVE GAIN

A man who feels sure that he knows the time-rates at which the prices of all kinds of assets are going to change during some short interval measured forward from his viewpoint will decide, if all assets can at his viewpoint be exchanged at given prices, to hold the whole value of his existing set of assets through this interval in the form of that one asset whose money price is going to increase most. But if instead of looking on a unique time-rate of price change for each kind of asset as certain, he entertains several hypotheses about this time-rate, we can no longer speak of him simply as expecting one price to increase faster than any other; the degree of belief he accords to each hypothesis has now also to be considered. On what principle will he then make his choice of assets, and what actually observed market phenomena can be accounted for by supposing that all or many individuals make use of this principle? This problem seems to invite attack by means of the tools introduced in Chapter II. The gambler indifference-map in particular will enable us to draw some conclusions regarding the sensitiveness of speculative markets to events small in themselves, minute straws in the wind which seem to have disproportionate effects. Finally, our justification in substituting the concept of potential surprise for that of numerical probability, in such problems as the present, is sought in an actual market phenomenon of the highest interest, namely, the nature of 'floating value' of land mentioned in the Uthwatt *Report*.*

We suppose our enterpriser to possess at date $[n-1]$ a given collection of assets, and to be faced with a set of given prices at which at that date he can exchange any quantities of these assets for others. We suppose him to look forward to a date $[n]$ which is so near to date $[n-1]$ that it will surprise him if new price-determining elements or features of the situation, of which there is no sign at date $[n-1]$, begin to influence prices powerfully before date $[n]$; and having chosen an interval short enough to justify him in neglecting (in order to meet the practical necessity for some kind of choice) the possibility of discontinuities arising from unthought-of new elements, we

* *Expert Committee on Compensation and Betterment, Final Report* (Cmd. 6386), paragraphs 23, 24.

suppose him to apply a further criterion and choose his interval so short that those changes which he does expect can be regarded as approximately linear functions of time, so that he need only concern himself with net price differences between the beginning and end of the interval, and not with changes occurring in subintervals within this interval. The interval thus defined, beginning at a named date $[n-1]$ and ending at a named date $[n]$, we shall refer to throughout as interval n. The date $[n-1]$ can be thought of as his viewpoint or as a date still in the future. Prices and other variable quantities will be assigned to particular dates by writing $[n-1]$ and $[n]$ as subscripts to the symbols concerned. Let B, C, ..., M be the entire list of different kinds of assets or goods with which the enterpriser concerns himself, and let a_B, a_C, ..., a_M be the ratios in which, according to some hypothesis in his mind, the money-prices of these goods will respectively change in interval n. That is to say, if $W_{B,[n-1]}$ is the price of good B at date $[n-1]$, and $W_{B,[n]}$ is the price it will have at date $[n]$, then $a_B = \frac{W_{B,[n]}}{W_{B,[n-1]}}$. Then if each of the ratios a_B, a_C, ..., a_M has in his mind a unique value which he looks on as *certain*, he will decide to exchange on date $[n-1]$ all the assets he then possesses for goods of that one kind, say B, whose ratio a_B is highest; where there is no subjective uncertainty, his preference will be for a *single* good. (If the ratios of expected price-change are equal for two or more goods, he will be indifferent between these goods or any combinations of them.)

In reality, however, the ratios a_B, a_C, ..., a_M in which the prices of the different goods will change in any interval n are not known for certain to anyone. For each such ratio the enterpriser will have in mind not a unique value but a function associating with each of a number of hypotheses concerning this ratio a particular degree p of potential surprise. Let us suppose at first that the enterpriser concerns himself with only two goods B and C, and that for those two goods the respective potential surprise functions are $p_B = p_B(a_B)$ and $p_C = p_C(a_C)$, and let us also assume, for the purpose of our later argument, that these goods are not subject to any natural physical change. The condition $p_B = p_C > 0$, for the increasing branch of each curve, will determine a one-one correspondence between values of a_B and values of a_C, each particular pair of values being associated with a particular degree of potential surprise; and similarly for the decreasing branches. We shall call such a pair of values a pair of linked values.* For any specific set of quantities of the two goods, any

* More generally we can speak of a set (of two or more) linked values. Such a set is illustrated in Fig. IV 3.

particular pair of linked values a_B and a_C, each greater than unity, will correspond to some specific amount of gain to be had from holding this set of quantities of goods through interval n; and any particular pair each less than unity will correspond to some specific amount of loss. If x is a variable hypothesis about the amount of this gain or loss, and for each value of x $p=p(x)$ is the degree of potential surprise associated with that particular pair of linked ratios a_B and a_C which gives this value of x, there will be some pair of mutually corresponding values of x and p which, amongst all such pairs where x is positive, has a greater power $\phi=\phi\{x, p(x)\}$ to concentrate the enterpriser's attention upon itself than any other pair. This we shall call the *primary focus-gain* from holding the *given set of quantities* of the two goods B and C through interval n. In exactly parallel fashion we define the *primary focus-loss* from holding this set of quantities through this interval. Now each different set of quantities of the two goods, amongst all those sets which are accessible to the enterpriser through his being able, at date $[n-1]$, to exchange any part or the whole of his holding of C for some quantity of B, or vice versa, at some fixed ratio of exchange, will have its own pair of primary focus-outcomes, to each of which there will correspond a standardised focus-outcome. The enterpriser's problem is to select that set of quantities of B and C whose pair of standardised focus-outcomes lies on a gambler indifference-curve above and to the left of all those containing other such pairs. We wish ultimately, therefore, to discover something about the forms of two functions, which we may write $g_S(R)$ and $h_S(R)$, which show respectively how the standardised focus-gain g_S and the standardised focus-loss h_S will vary with the number R of units of one good, say C, which the enterpriser considers exchanging, out of his initial holding of C, for extra units of B to add to his initial holding of B. (If we take as our unit of each good the quantity which, at the given market prices of date $[n-1]$, is worth one unit of money, R stands for the total value of the units given up and likewise, of course, of those received.)

By the enterpriser's initial holdings of the two goods we mean the quantities of them which are in his possession at the moment when he starts to consider his problem, before he has decided what quantities he will hold through interval n. Let β be the value of his initial holding of good B, let γ be the value of his initial holding of good C, and let R, thought of as a variable hypothesis, be the value of those units of C which he proposes, under different tentative plans, to exchange for units of B. R, that is to say, is the amount by which he proposes to decrease the market value, at date $[n-1]$, of his holding of C and increase that of B. Now let us write the ratios

a_B and a_C in any pair of linked ratios as functions $a_B = a_B(p)$ and $a_C = a_C(p)$ of the degree of potential surprise associated with this pair of ratios. Then to *any given* degree $p = p_B = p_C$ of potential surprise there will correspond a hypothesis of gain x given by

$$x = a_B(p)(\beta + R) + a_C(p)(\gamma - R),$$

so that

$$\frac{\partial x}{\partial p} = \frac{da_B}{dp}\beta + \frac{da_C}{dp}\gamma + \left(\frac{da_B}{dp} - \frac{da_C}{dp}\right)R.$$

Let us choose for p that value which, at some one size of R, makes $x = g_P$ the primary focus-gain. Now it can perfectly easily happen (as in the case illustrated in Fig. IV 1) that at this, or any, value of p

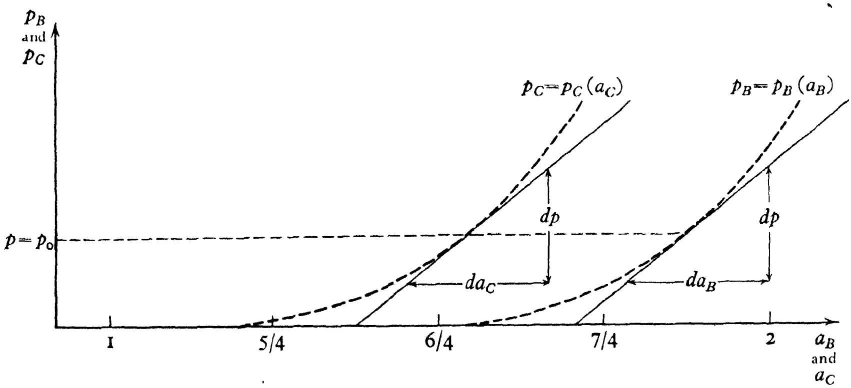

FIG. IV 1. The graphs of the functions $p_B = p_B(a_B)$ and $p_C = p_C(a_C)$ can also, of course, be looked on as graphs of $a_B = a_B(p)$ and $a_C = a_C(p)$. The graphs are here drawn so that at $p = p_0$ we have $da_B/dp = da_C/dp$.

we have $\frac{da_B}{dp} = \frac{da_C}{dp}$. If this equality holds then $\partial x/\partial p$ is independent of R, and a change of the latter will leave unaffected the value of p which gives $x = g_P$, and thus will leave unaffected the values of a_B and a_C which are to be used in computing g_P by means of the expression

$$g_P = a_B(p)(\beta + R) + a_C(p)(\gamma - R).$$

Then further we have

$$\frac{\partial x}{\partial R} = a_B(p) - a_C(p),$$

which, if a_B and a_C remain constant under all changes of R, is itself a constant over the whole relevant range of R (that is, the range from $R = \gamma$ to $R = -\beta$). Hence if, at that value of p which, for some one value of R, gives $x = g_P$, we assume that $\frac{da_B}{dp} = \frac{da_C}{dp}$, we have g_P as a *linear* function of R. If $a_B > a_C$ then g_P will evidently be an increasing function of R, if $a_B < a_C$ a decreasing function, the definition of R being the amount of money-value transferred *from* the form of good C *to* that of good B. Finally, if the shapes of the contour-lines $\phi = \text{constant}$ are such (e.g. if, as is perfectly possible and as we happen to have actually drawn them in Fig. II 3, they are all identical) that when, as is true on our assumptions, p is the same for all the values which g_P can assume under changes of R, $g_S = g_P - \epsilon$, where ϵ is a constant, it follows that g_S, the *standardised* focus-gain, will be a linear function of R. Thus we have found that under certain simple conditions, by no means unlikely to be *approximately* true, the function $g_S(R)$, one of the two which we set out to study, will be *linear*. By a parallel argument the other function, $h_S(R)$, can also quite easily be approximately linear. To show that it is indeed quite easy to imagine $g_S(R)$ and $h_S(R)$ being linear, let us insert into the expression

$$\begin{aligned} x &= a_B(p)\ (\beta + R) + a_C(p)\ (\gamma - R) \\ &= a_B(p)\beta + a_C(p)\gamma + \{a_B(p) - a_C(p)\}R, \end{aligned}$$

those values of a_B and a_C which are greater than unity and carry potential surprise barely exceeding zero. The value we shall thus get for x will, of course, be the upper extreme of the inner range, and it is evident that, a_B and a_C being now constants, this upper extreme is a linear function of R. So also will be the lower extreme. But the standardised focus-gain is plainly likely in most cases to behave in a very similar manner, under changes of R, to the upper extreme of the inner range; and the standardised focus-loss is likely to behave rather like the lower extreme of the inner range. Hence in what follows we shall assume that what is *strictly* true of the extremes of the inner range, namely, their linearity under changes of R, and is likely to be *approximately* true of the standardised focus-outcomes, is strictly true of these latter also.

Now the forms of the potential surprise functions $p_B = p_B(a_B)$ and $p_C = p_C(a_C)$ may be such that in every pair of linked ratios the same member, say a_B, is greater than the other. In this case the standardised focus-gain will be greatest, and the standardised focus-loss smallest, when the set of quantities held consists exclusively of good B, with none of good C. But more often one of the ratios, say a_B, will be greater than the other when both are greater than unity, but less than the other when both are less than unity.* In this case, by holding less of good B and more of good C, the enterpriser can have a smaller focus-loss at the cost of having also a smaller focus-gain. Measuring standardised focus-losses along the horizontal and

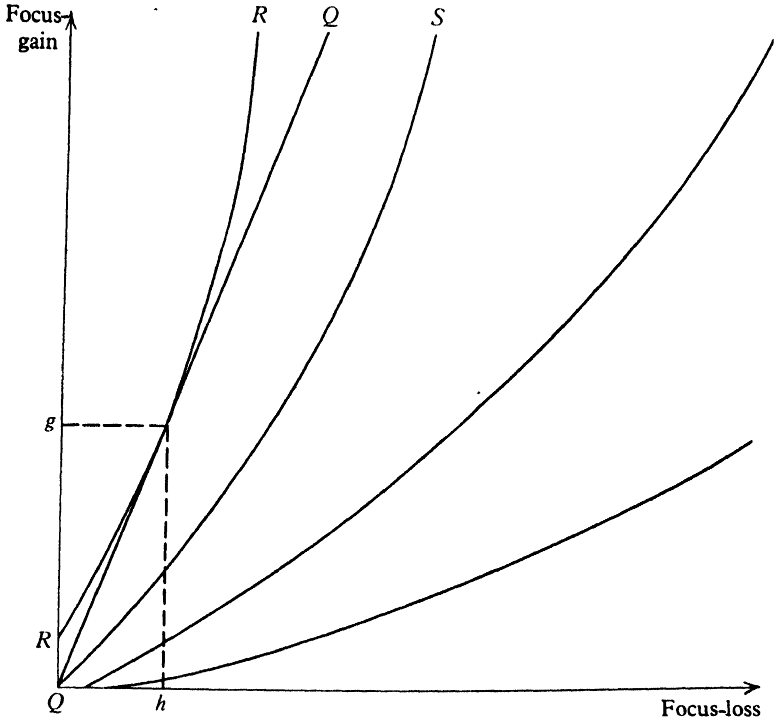

FIG. IV 2. This figure is drawn on the assumption that one of the two goods is money. The gambler opportunity-curve QQ therefore starts from the origin. Amongst the gambler indifference-curves which it encounters (cuts, meets, or is tangent to) the one representing the most desired situations is the curve RR, with which it has a point of tangency (g, h) representing a focus-gain g combined with a focus-loss h. The enterpriser will choose those determinate quantities of the two goods B and C, which correspond to the point (g, h) on the opportunity-curve. The curve QS is the origin indifference-curve.

standardised focus-gains along the vertical axis of Fig. IV 2, we can display the range of choice open to the enterpriser as a curve QQ such that the two focus-outcomes increase together. If one of the two goods in question is money, this curve will start from the origin, otherwise it will in general start some distance from the origin. For

* The potential surprise function for a_C will then lie wholly 'inside' that for a_B in the manner illustrated for three functions in Fig. IV 3.

by gain and loss we mean gain and loss of money-value, and by holding all his capital in the form of money, the enterpriser can make sure of having neither gain nor loss, and the situation in which he has this assurance is represented by the origin. Now since we are assuming that both of the standardised focus-outcomes are linear functions of R, it follows that when they both change, if this change is due solely to a change of R, they will be linear functions of each other, and the curve QQ, which we shall call the gambler opportunity-curve, will be a straight line. Its relevant part will be a segment of finite length bounded at one end by a point corresponding to the holding of the whole of the enterpriser's capital in the form of one

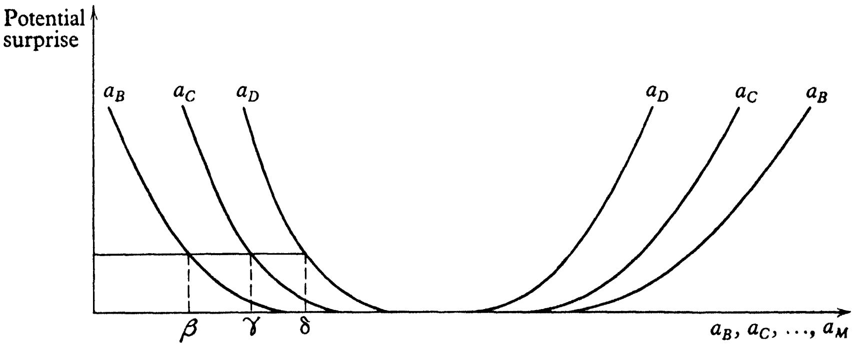

FIG. IV 3. β, γ and δ are a set of 'linked values' of a_B, a_C and a_D respectively. The curves of the functions $p_B = p_B(a_B)$, etc., are labelled a_B, etc.

good only, say B, with none of C, and at the other by a point similarly corresponding to the holding of C only with none of B; any extension of the curve beyond these points has no meaning.

Every point with both co-ordinates positive on the plane of Fig. IV 2 stands for a distinct pair of standardised focus-outcomes; the combination of some specific focus-gain and some specific focus-loss. Starting with any one such point it will be possible to find others which, to a specified person with a specified total value of stocks of goods, will be neither more nor less attractive than this point. An entire set of points mutually equal in their attractiveness to a given person at a given point of time, lying within some relevant range of focus-losses (say from zero up to the total value of the particular individual's possessions) we may call a 'gambler indifference-curve'. For any one person there will be a family of these curves such that any point on the plane will lie on one or other of them. Every member of this family of curves, or indifference-map, will be altered in shape by any alteration in the total value of the enterpriser's possessions, for the significance to him of a given hypothetical gain or loss per unit of time depends on its ratio to his total capital (as well as to his habitual level of consumption and some other standards). Thus a gambler indifference-map describes the tastes, in regard to hopes and uncertainty, of a given individual in given circumstances.

The gambler indifference-curve which passes through the origin will indicate all those combinations of focus-gain and focus-loss which to the particular person concerned, in his circumstances of a particular instant, are no more nor less attractive to him than a high degree of confidence that he will experience neither gain nor loss. For the origin of the gambler indifference-map represents the combination of a standardised focus-gain and a standardised focus-loss both equal to zero, and we have seen in Appendix B to Chapter II that the y-curve which it represents must therefore have a cusp with its tip meeting the x-axis at the zero outcome, so that the amount of gain (and likewise the amount of loss) vanishes to a higher order than the associated potential surprise. Thus we are justified in referring to the origin of the gambler indifference-map as representing 'a high degree of confidence' that there will be neither gain nor loss; for this outcome is the *only* one carrying nil potential surprise, all others carrying some positive degree of it. It seems reasonable to suppose that the gambler indifference-curves will all resemble each other in the main features of their shape, and it follows that if we can infer from a general knowledge of human nature and circumstances the general shape of that particular indifference-curve which passes through the origin, which we will call the 'origin indifference-curve', we shall have a clue to the general shape of the indifference-map as a whole. Now this origin indifference-curve is likely to slope upwards to the right with increasing steepness; for a focus-loss equal to the whole of one's capital would in the minds of most people need a focus-gain many times as large as itself to compensate it (i.e. to render the combination equivalent to an assurance of constancy of one's capital in terms of money), while a focus-loss which is a hardly noticeable proportion of one's capital can perhaps be offset by a focus-gain equal to itself; and as we consider successively larger focus-losses in the range between these extremes, the focus-gain just sufficient to compensate each size of loss is likely to bear a larger and larger ratio to the latter. Thus in some neighbourhood of the origin we may expect the origin indifference-curve to have fairly typically a constant slope of unity (half right angle) and thence to rise with increasing steepness, perhaps in some cases eventually becoming asymptotic to a finite focus-loss. The typical 'gambler indifference-map' will therefore resemble the one indicated by means of a few sample curves in Fig. IV 2. In an indifference-map where the axes have the meanings here assigned to them, the combination of focus-outcomes represented by any point on an indifference-curve is preferred, by the individual whose map it is, to every combination represented by a point on any curve lying below and to the right of the former curve.

We have now to superpose the gambler opportunity-curve on the gambler indifference-map. Let us begin by assuming the shapes of the two functions $p_B = p_B(a_B)$ and $p_C = p_C(a_C)$ to be such that, whether the enterpriser elects to hold all his capital in the form of good *B*, or all in the form of good *C*, or in any proportions of both goods, the standardised focus-gain and the standardised focus-loss will be equal. If one of the goods, say *C*, is money, and the whole capital is held in this form, the two focus-outcomes will necessarily be equal, since both will be zero; and no argument is needed to show that equality in the case of a good other than money is a perfectly ordinary case. On this assumption the opportunity-curve will be a straight line bisecting the angle between the axes; and we have seen that there will be an indifference-curve which in some neighbourhood of the origin also bisects this angle. If the opportunity-curve extends near enough to the origin, it will coincide over some part of its length with this indifference-curve, and elsewhere will lie below and to the right of it. Thus the latter will be the highest (i.e. most desirable) indifference-curve which the opportunity-curve encounters. If the opportunity-curve and the origin indifference-curve should chance to have only one point in common (which would necessarily be the extreme end of the opportunity-curve nearest the origin), the enterpriser, choosing that combination of quantities of *B* and *C* which corresponds to this point on the opportunity-curve, will prefer to hold his whole capital in the form of the less exciting, the more narrowly predictable in value (according to his own judgement) of the two goods. And this will still be approximately true if the two curves have a range of points, instead of a single point, in common, since this range is not likely to be large in relation to the length of the opportunity-curve as a whole.

Now suppose that the form of the function $p_B = p_B(a_B)$ changes so that, if the whole capital is held in the form of good *B*, the standardised focus-gain g_S will be greater than the standardised focus-loss h_S, while the form of the function $p_C = p_C(a_C)$ remains such that, if the whole capital is held in the form of good C, the two standardised focus-outcomes will be equal. In this case that end of the opportunity-curve which is nearest the origin will represent two equal focus-outcomes, while the other end will represent a focus-gain larger than the corresponding focus-loss. An opportunity-curve tilted in this fashion will be common enough, since again equality of the focus-outcomes at one end of it will always occur if one of the two goods is money, so that both of the outcomes are zero. The origin indifference-curve will now no longer be the highest indifference-curve touched by the opportunity-curve; the latter, if it is long enough, will evidently be tangent at some point to one of the indifference-curves lying above and to the left of the origin indifference-curve, and this point of tangency will no longer be the extreme end of the opportunity-curve nearest the origin, but will be some way along it, and will represent a combination of goods no longer consisting exclusively of C but comprising also some B. If the increase in the slope of the indifference-curves is very gentle, so that they approximate somewhat to straight lines, a relatively small pivoting shift of the opportunity-curve, in such a way as to increase its slope, will carry the point of tangency a long way up the opportunity-curve to a point which implies a combination of goods consisting mainly of B. It is possible, indeed, as a by no means exceptional case, that this combination will consist *exclusively* of B; for what we have called the opportunity-curve is in fact a finite segment only (since the gain or loss from holding a finite quantity of any goods through a specific interval cannot be infinite), and hence although if produced far enough it would eventually have a point of tangency such as we have described, it may in fact stop short of this point, and have its extreme furthest from the origin resting on an indifference-curve which it meets at an angle greater than zero.

We have in the foregoing an explanation of how some event which attracts little direct notice in its own right can initiate a boom of activity in the market, first inducing for each of many enterprisers some slight change in the form of one of the p-curves, say a small, bodily upward shift of the upper branch of $p_B = p_B(a_B)$, thus causing a slight tilting of the opportunity-curve and thus a *great* change in the relative quantities of two goods (one of which may be money) which individuals wish to hold *at the initial prices*, thus inducing large offers to buy one good and causing a rise of its price, a rise which will seem to justify, or probably more than justify, the small bodily upward shift of the upper branch of $p_B = p_B(a_B)$ which expressed each individual's interpretation of the originating event itself, and lead, in this latter case, to a further revision of expectations and a new cycle of the same sequence of events. The validity of this explanation, it will be remembered, depends on certain assumed characteristics of the gambler indifference-map which we suppose to be typical of the indifference-maps of a substantial proportion of the enterprisers; these characteristics are, first, an origin indifference-curve which in some neighbourhood of the origin approximately bisects the angle between the axes, and, secondly, only a gentle curvature in the indifference-curves, so that a small pivoting shift of the opportunity-curve causes the point of tangency to sweep more or less from one end to the other of the opportunity-curve. The shape of the indifference-curves is, of course, a mathematical or graphic description of the temperament of the enterpriser concerned. The particular shape which we assume in the above argument represents what we may express in rough and ready language as a low rate of increase of the reluctance to gamble as the amount at stake, considered as a proportion of the individual's total capital, increases. Common sense approves the conclusion that, if such temperaments abound, small events will be the more readily able to generate booms of market activity.

Let us turn now to the case where more than two goods are involved. We have seen that if, say, $a_B > a_C$ in *every* pair of linked values, both above and below unity, the good C can be neglected. Thus when we consider more than two goods, we need only concern ourselves with cases where, say, $a_B > a_C > a_D > \dots > a_M$ in every *linked set* in which all these ratios are greater than unity, and where $a_B < a_C < a_D < \dots < a_M$ in every linked set in which all these ratios are less than unity. The potential surprise functions in such a case are illustrated in Fig. IV 3. Now if, taking each of the goods $B, C, \dots, M$ in turn, we suppose the enterpriser to hold the whole value of his capital in the form of the equivalent quantity of this good alone, we shall have on the plane of Fig. IV 2 a number of points, each of which we may label with the name of the good concerned. Each of these points will represent a higher focus-gain than any point which shows a smaller focus-loss. For the value, at the beginning of interval [n], of the stock of goods then held by the enterpriser is the same no matter what its form. Its value at the end of the interval [n] will have risen (or fallen) in a ratio a_B if it is wholly in the form of good B, in a ratio a_C if it is a pure stock of good C, and so on. If, then, the potential surprise function of a_C lies wholly 'inside' that of a_B, and the function for a_D inside that of a_C, and so on, it is plain that a lower focus-gain implies a smaller focus-loss. Thus, the points representing the paired focus-outcomes attached to a pure stock of *B*, a pure stock of *C*, etc., will form a chain of straight-line segments all sloping upwards to the right. But it is possible that even some of these points can be neglected; for suppose that one of them, representing, say, good *C*, lies below and to the right of a straight-line

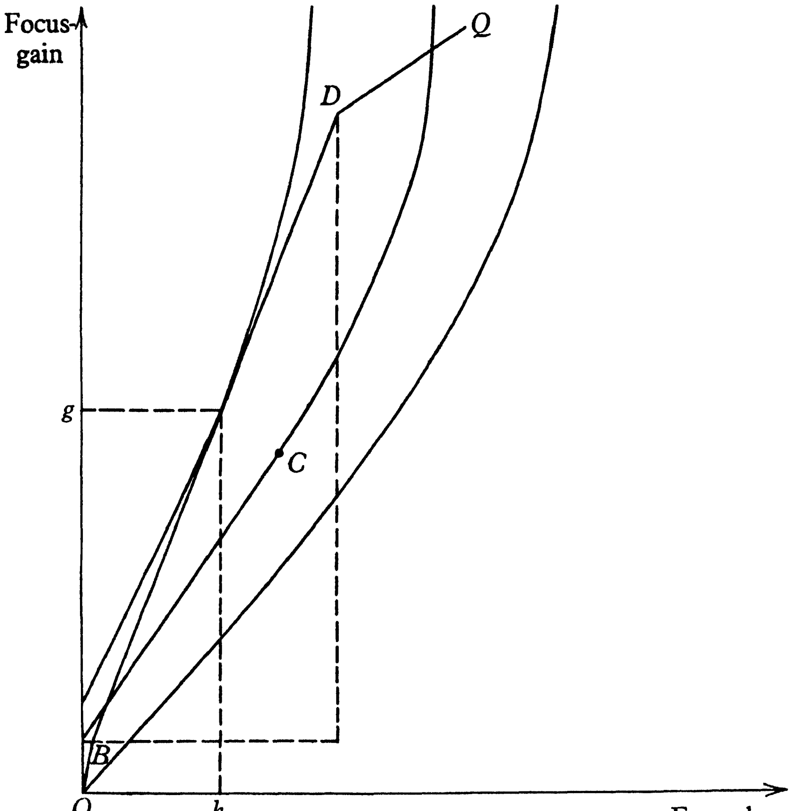

FIG. IV 4.

segment joining two others of the points, say *B* and *D*, and within a triangle bounded by this line segment and by a straight line parallel to the loss axis and another parallel to the gain axis, as shown in Fig. IV 4. Then it will be possible to find a point on the line joining *B* and *D* which stands for the same gain but a smaller loss than *C*, and another which stands for the same loss but a larger gain than *C*, and between these two latter points there will be a subsegment every point of which will represent both a larger focus-gain and a smaller focus-loss than *C*. Plainly the enterpriser will prefer a combination of *B* and *D*, corresponding to some point of this subsegment, to the holding of a pure stock of *C*, or any combination of *B* or *D* with *C*. When all points such as *C* have been eliminated, the remainder will determine a chain of straight-line segments lying convex to the gain axis. This chain will constitute the curve of most favourable opportunities, and the enterpriser will choose that combination of goods which corresponds to the point of tangency, or other contact, of this chain with the highest indifference-curve which it meets. What combination of goods will such a point represent? Evidently it will be the combination, in determinate proportions, of that *pair* of goods which corresponds to the adjacent 'corners' of the opportunity-chain. We are at once led to ask: 'Will it never seem to the enterpriser advantageous to hold, with a view to speculative gain, a set of more than two goods?' We know that it sometimes does seem so in real life, and we must therefore inquire in what respects the analysis we have just performed is inappropriate to particular cases or neglects relevant features of real situations. One assumption, which is implicit in our procedure, can certainly not be supposed to hold in all cases, namely, that the degree of potential surprise associated with a given change of value of a collection of goods as a whole will always be that degree which, if applied to the functions such as p_B, p_C, ... for price change of individual goods each considered by itself, would give such price-changes of the individual goods as would yield the required weighted sum.

So long as the potential surprise function for the ratio of price-change of each good remains valid in unchanging form no matter whether this good is considered singly or as contributing to the total value-change of a collection of goods, there will always be, for any combination of more than two goods, some combination of not more than two goods which is superior (or, at least, not inferior) to it. In order to see this, let us first see how we can plot on the plane of Fig. IV 4 a point representing the focus-gain and loss from holding some specific combination of three goods, say *B*, *C* and *D*. We first plot the three points corresponding to pure holdings of these goods. Then, as we have seen, the opportunity-curve for combinations of any two of them, say *B* and *D*, will on plausible assumptions lie approximately along the straight-line segment joining the two points *B* and *D* corresponding to pure holdings of these goods. Now any point on this straight-line segment can be looked on as representing the focus-outcomes from holding a specific quantity of a composite good made up of *B* and *D* in fixed proportions. Let us call this new composite good *E*. We can then join this fixed point, labelled *E*, to the one remaining point *C* of the three original points and thus obtain a new straight-line segment each of whose points will represent the focus-outcome from holding some specific combination of *E* and *C*, that is to say, some combination of specific quantities of *B*, *D* and *C*. By a similar procedure we can determine a point representing the pair of focus-outcomes corresponding to *any* combination of three goods, and, analogously, for any combination of any number of goods. We shall thus get a 'swarm' of points, which will evidently cover entirely a simply-connected region of the plane, leaving no 'holes' or interstices between the points within its outer boundary. But this outer boundary, above and to the left of the swarm, will be simply the gambler opportunity-chain of Fig. IV 4, and we know that any point lying below and to the right of this chain will be inferior to some point of the chain itself, that is, some point on one of the straight-line segments composing the chain. Thus (except in the case where the points for three goods all lie on the same straight 'link' of the chain) any combination of more than two goods will be inferior to some combination of not more than two goods.

If, then, combinations of more than two goods are ever chosen, one or other of two things must in such cases be true: either some qualities of the combination besides its focus-gain and loss are being taken into account; or else the assumption that the focus-gain and loss of such a combination can be obtained from the potential surprise functions for the ratios of price-change of the individual goods in the direct way we have described does not always hold. Let us consider this latter possibility first. It is not easy to think of any simple (mental) mechanism by which a combination of many goods could be assigned a *higher* focus-gain than a pure stock, equivalent in initial market value, of the most promising of the constituent goods of the combination. But the idea that a collection of assets can be assigned a *lower focus-loss* than would result from adding up the focus-losses of the assets when each is looked on as an isolated individual is familiar from the practice of insurance, of which, from the policy-holder's point of view, it is the basis. 'Spreading of risks' is, indeed, so familiar a principle that, under the aspect of the risk of *loss*, we need not discuss it further here. But it is not only the 'risk of loss' which is reduced by holding many and varied assets, it is also the 'risk of gain', and there is nothing to ensure that these two reductions, effected by one and the same diversification of assets, will be equally valued by any given individual. He may rate very much higher the glimpse of great gains afforded him by the holding of a single asset than the dull security afforded by holding many. If, then, a number of poor men each hold one asset, the total of their individual valuations of this asset may be many times the value which a rich man, if he held all the assets, would assign to the whole collection, even if the poor men and the rich man all have identical temperaments, and if the marginal utility of wealth is in some sense equal for them all in spite of the diversity of their circumstances. Let us translate this into terms of our own concepts and apply it to solve an actual problem. There has recently come to light an exceedingly interesting phenomenon of an actual market, namely, the market for land adjacent to towns, and this phenomenon, and our own immediate problem, seem to illuminate each other. Evidence taken by the Uthwatt Committee* shows that, where the belt of land encircling a town is parcelled up amongst a large number of separate ownerships, the market value of each piece is such that, when the separate values are aggregated, the total is several times as great as would be warranted by any reasonable estimate of *aggregate* future building development round the town as a whole. It is as though each actual and potential owner of a plot of land near the town were convinced that, out of a far more than adequate total supply of similarly situated land, the particular plot in question was almost *certain* to be selected as part of the site for such new houses as will be required during, say, the next twenty years. We can, I think, go a long way towards explaining this curious phenomenon by supposing that, instead of assigning to his own plot the value it would have if development of it within a moderate time were *certain*, multiplied by the *probability* of this particular plot being chosen out of, say, fifty equally good plots each sufficient by itself for all likely development within a period short enough to influence current values, each owner in effect considers what I have called the focus-outcomes from holding such a plot. He believes that there will certainly be some extension of the built-up area of the town; he sees no reason, other than pure accident, why his plot should *not* be the one chosen, no positive disability tending to exclude it; consequently he attaches nil or very low *potential surprise* to the hypothesis that his plot will be required; and though he must also attach nil potential surprise to the hypothesis that it will *not* be required, yet the price he assigns to the plot must be well above that which would emerge from a calculation of numerical probabilities, in order that the focus-gain may not be out

* *Expert Committee on Compensation and Betterment, Final Report* (Cmd. 6386), paragraphs 23, 24.

of all proportion to the focus-loss; for if the market valuation were as low as that based on probability, the plot would offer any potential buyer a very high focus-gain, even perhaps of the same order of magnitude as the difference between the value of the plot if development were certain and its value if non-development were certain. Now let us suppose that the relevant belt of land surrounding a town is divided into ten equal plots each divided into ten equal subplots, and that the amount of land likely to be required for building development within a period short enough to influence current values appreciably is equal to one plot. Let us further assume it to be publicly known that the land required will not be taken from a number of different plots but will consist of one or other of the individual plots as a whole. And let us suppose that our enterpriser, intending to buy some of the land in the hope that the pieces he acquires, or some of them, will be selected for development, finds his capital sufficient, at the current prices, for the equivalent of one plot. He has the choice, then, of buying one whole plot or, for example, one subplot in each of ten plots. If he chooses this second course, his focus-gain may, for some temperaments, be only *one-tenth* of what it would be if he bought the whole of one plot. This assertion will raise immediate objection from those accustomed to think only in terms of numerical probability; for they will say that, though admittedly the largest gain which the enterpriser can hope for is in the second case only one-tenth that of the first case, yet in the second case he is *certain* to make this gain, while in the first case his chances are only one in ten. This brings out, albeit in a rather exaggerated form, the essence of my contention regarding the difference between numerical probability and potential surprise. In order to enjoy with a high intensity, or even to the full, the anticipation of some gratifying outcome, a man requires only that there should be no solid, identifiable reason to disbelieve in the possibility of this outcome; he does *not* require solid grounds for feeling sure that it will be the true outcome. If we look upon the ten subplots, each from a different plot, as ten distinct commodities, it is plain that this is another case where it is by no means legitimate to derive the focus-outcomes of a combination of the commodities from the potential surprise functions of price-change of the individual commodities.

It is possible, then, for the focus-loss which a speculator assigns to a collection of goods to be smaller than the weighted sum obtained by treating each of the individual goods composing the collection as though it were the only asset possessed by the speculator in question, and taking the focus-loss he would assign to it in those circumstances; and similarly, as we have just seen, it is possible for the focus-gain he assigns to the collection to be smaller than the corresponding weighted sum of individual focus-gains.

We will not here consider the other class of reasons why a combination of many goods might be preferred to a combination of two only, namely, that qualities of other kinds besides its profitability as a store of value are taken into account; amongst such other qualities *liquidity* or marketability will no doubt be prominent.

CHAPTER V

DESIGN OF TAXATION TO PRESERVE THE INCENTIVE TO ENTERPRISE

By a venture we shall mean in this chapter any purchase of physical equipment with a view to deriving from it returns net of operating costs, whose discounted total will be greater than the cost of the asset; we shall suppose that at the moment of such purchase (in all cases where the cost is more than some specified amount) the enterpriser concerned is required by law to name a future date at which the books shall be closed, the remaining market value of any part of the equipment not already used up or sold determined, and the results of the venture computed; and that if the outcome thus arrived at is a gain, some determinate proportion of it will then have to be paid as tax, but if it is a loss, no offsetting of this loss will be allowed, for the purpose of tax assessment, against gains accruing to the enterpriser from other sources; and we shall assume that no practical complication, such as the need to pay dividends at intervals *during* the time while the equipment is earning them, will make it impracticable to have a tax whose *rate* depends on the final outcome as a whole, as computed at the closing of the books. We wish to consider the influence which different forms of tax will exert on the inducement to employ funds in different sorts of venture, or to retain them as cash.

In order to make his choice between different ventures the enterpriser, we shall suppose as before, will in effect plot the position of each venture on his gambler indifference-map. But the standardised focus-gain and focus-loss used for this plotting must now be derived from a potential surprise curve which associates degrees of potential surprise with hypothetical gains *net of tax*. Instead of applying to his ϕ-surface the curve $y=y(x)$, where x stands for untaxed gain or loss, he must apply a curve $p=p(u)$, where p stands for degrees of potential surprise and $u=x\{1-T(x)\}$ stands for a gain x diminished by a tax at a rate $T(x)$. The degree p of potential surprise associated with any given value of u will of course be simply the degree of potential surprise which the enterpriser attaches to the *untaxed* gain x from which the given value of u is obtained by subtraction of an amount $xT(x)$ of tax.

Since if the enterpriser elects to retain his funds in cash he can feel certain of incurring no gain, no loss, and no taxation, it is plain that no idea of investing in equipment will attract him unless, after taking tax into account, it lies on a gambler indifference-curve which is above and to the left of the origin indifference-curve. Now on our assumptions no tax will make any difference to the focus-*loss* of any venture. But we shall easily see that many types of tax will reduce the standardised focus-*gain* of every venture, and thus its attractiveness in comparison with the retention of cash. If, for example, $T(x) > 0$ for all $x > 0$, we have $u = x\{1 - T(x)\} < x$, while $p(u) = y(x)$. Thus a replacement of the curve $y = y(x)$ by the curve $p = p(u)$ can be thought of as a transformation of every point (x, y) of the former into a point (u, p) of the latter such that $p = y$, $u < x$. Since we have $\partial\phi/\partial x > 0$ everywhere except along the line $y = \bar{y}$, this transformation will carry every point (x, y) (except points $(x, \bar{y})$) into a point (u, p) to which there corresponds a *lower* value of ϕ. This will be true of the primary focus-point of untaxed gains, namely, (g_P, y), that is, the primary focus-gain obtained from the curve $y = y(x)$ and lying on it. But this point (g_P, y) gives a higher value of ϕ than any other point of the curve $y = y(x)$, and all these other points are by the transformation moved so as to give still lower values of ϕ than they did before. It follows that whatever point on the curve $p = p(u)$ represents the primary focus-point of gains *net of tax*, it must necessarily give a lower value of ϕ, and thus a lower standardised focus-gain j_S, than did the former primary focus-point (g_P, y). Many of the ventures, which enterprisers have in mind at any time, whose standardised focus-gains are thus reduced, will in this way be moved from a gambler indifference-curve lying above and to the left of the origin indifference-curve to one lying below and to the right of it. Thus the imposition of any tax whose rate is greater than zero for all gains greater than zero is likely to reduce the aggregate investment-flow, that is, the quantity of money which enterprisers spend in a unit of time with the purpose of effecting a net improvement or enlargement of their capital equipment. Even if gains which, when reduced to terms of a percentage per annum on the sum invested in the venture, lie below some specified level, are exempt from tax, the latter will evidently still tend to discourage the adoption of any of those ventures whose primary focus-point of untaxed gains lies above the exemption limit.

Is it possible to devise a form of tax by which the majority of actual ventures will be caused to yield some revenue, but which will leave the incentive to enterprise, that is, the *ex ante* attractiveness of every venture, entirely unaffected? Can such a tax be so fashioned that actual gains realised *ex post* are taxed at lower rates when they accrue to those who have been able rather than merely lucky?

Luck should be taxed, but imagination and knowledge should be allowed to keep the high rewards which they can win in combination. But how are we to determine when a high rate of profit is due to luck and when to instructed imaginative enterprise? Our earlier analysis suggests a means by which we can accept the *ex ante* judgement of the enterpriser himself. A rate of profit which lies far above his primary focus-gain is for him a counter-expected outcome;* he cannot claim it as the gain in hope of which he was led to venture his stakes. If it occurs, it is due to luck. But a high rate of profit which lies near his primary focus-gain he *can* claim as the vindication of his hopes, in view of which he risked his capital. What of a level of profit which is positive but far *below* his primary focus-gain? This lies within his inner range, and therefore does not discredit his judgement. But it is a level of gain which, if our concept of focus-outcomes correctly interprets human nature, did not *ex ante* have any influence, favourable or discouraging, on his decision to make the venture. Thus if our taxation reduces still further a profit which is small in relation to the primary focus-gain, the prospect of such reduction will itself have no influence on decisions whether to invest or not. Thus hypothetical rates of profit far above and those far below the enterpriser's primary focus-gain for a given venture are alike in having *ex ante* no influence on his decisions whether to make this venture or not. Moreover, the former if they actually occur can be ascribed to luck, though the more modest rates of profit lying within his inner range must be accounted, should any one of them turn out to be the realised rate, as confirming his judgement. If these propositions be accepted, our principle of taxation will be to tax at the lowest rate those levels of profit which lie nearest to the primary focus-value of untaxed gains, and to tax at progressively higher rates gains increasingly remote from the primary focus value of untaxed gains, whether above or below it. Thus if z is the numerical difference, whether positive or negative, between any gain x and the primary focus-value g_P of *untaxed* gains, the rate $T = T(z)$ of our tax will be a function of z such that $dT/dz > 0$ for all z and that $T(0) = 0$, the form of the function $T(z)$ perhaps being different according as z is positive or negative. Such a tax can be looked on as a transformation carrying every point (x, y) of the curve $y = y(x)$ into a corresponding point (u, p) of the curve $p = p(u)$ such that $u \leqq x, p = y$.

* See Chapter III, p. 73 footnote.

The primary focus-point* of untaxed gains, namely, (g_P, y_P), will be common to both curves. For when $x = g_P$ we have $z = 0$ and $u = x = g_P$, while for all pairs of mutually corresponding points, one on each curve, we have $p = y$. Except at (g_P, y_P), however, the pairs of mutually corresponding points (u, p) and (x, y) will be such that $p = y$, $u < x$. Then since $\partial\phi/\partial x > 0$ everywhere (except on the line $y = \bar{y}$), it is clear that, except at (g_P, y_P) every point (u, p) will give a lower value of ϕ than its corresponding point (x, y), and the latter,

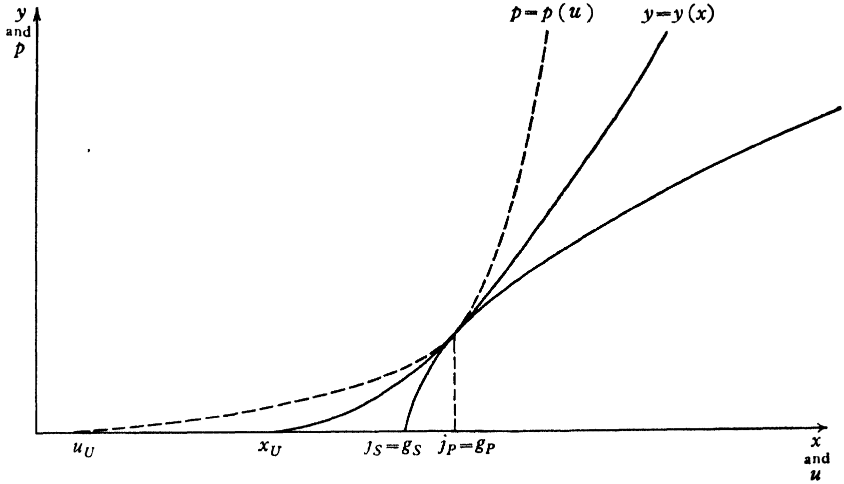

FIG. V 1. u_U, upper extreme of inner range of u; x_U, upper extreme of inner range of x; j_P, primary focus-value of taxed gains; g_P, primary focus-value of untaxed gains; j_S, standardised focus-value of taxed gains; g_S, standardised focus-value of untaxed gains.

of course, gives a lower value of ϕ than that given by (g_P, y_P). Thus every point on the curve $p = p(u)$, except (g_P, y_P), will give a lower value of ϕ than that given by (g_P, y_P), so that the latter will be the primary focus-point of *taxed* gains as well as of untaxed gains. That is to say, the standardised focus-gain will be the same for both curves and will be unaffected by the imposition of the tax. This result will be immediately clear from Fig. V 1, where g_S stands for the standardised focus-value of untaxed gains and j_S for that of taxed gains, the two values being one and the same. But any divergence

* I use the expression 'focus-*point*' to mean a *pair of magnitudes in combination*, namely, those associated values of x and y which together give a maximum of the twisted curve $\phi = \phi\{x, y(x)\}$. The phrase is meant to direct attention to the fact that two magnitudes are involved. When we are concerned only with that value of x which is one member of this pair of magnitudes, we speak of the 'focus-*value*'.

between the actual, *ex post*, result of the venture and the primary focus-value of untaxed gains will cause some revenue to be yielded to the Exchequer. Such a method of taxation is illustrated in Fig. V 1. The question then arises: How is the taxing authority to identify the primary focus-value of untaxed gains of any venture? The enterpriser himself must declare this in advance. Will there be a temptation for him to declare a level of his primary focus-gain different from the genuine one? No. For in order to make the venture as attractive as possible when tax is taken into account (and

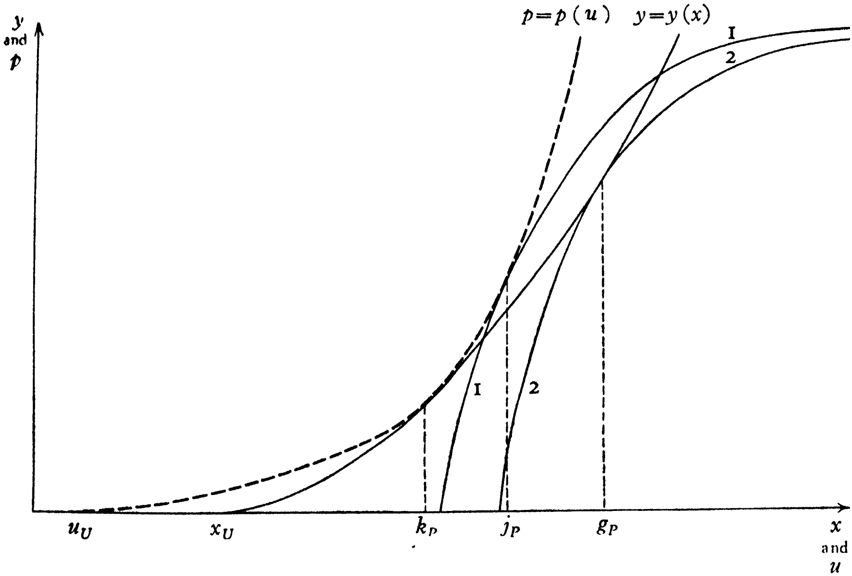

FIG. V 2. u_U, upper extreme of inner range of u; x_U, upper extreme of inner range of x; k_P, enterpriser's declared primary focus-value of untaxed gains; j_P, primary focus-value of taxed gains; g_P, enterpriser's genuine primary focus-value of untaxed gains. Curves numbered 1, 2 are contour-lines.

this will naturally be his aim) he must put k_P, his *declared* primary focus-value of untaxed gains, at that value of x which will give the highest standardised focus-value of taxed gains. But, as in the case illustrated in Fig. V 2, to put k_P elsewhere than at that value g_P of x which is his *genuine* primary focus-value of untaxed gains will at best merely put j_P the primary focus-value of taxed gains at some point of the curve $y=y(x)$ other than (g_P, y_P) (and will almost always move it to an even worse position, as in Fig. V 2); but of all points of this curve, it is (g_P, y_P) which gives the highest value of ϕ and will therefore give the highest standardised focus-value of taxed gains. To put k_P equal to g_P, therefore, is the condition for maximising j_S, the standardised focus-value of taxed gains.

What would be the practical implications for the enterpriser of this mode of taxation? Whereas under a more ordinary scheme he would have to pay some tax whatever the level (above zero) of his profits, and perhaps at a rate increasing with the rate of profit, here he would be exempt from tax at that one rate of profit which he had most vividly in mind when he decided that the venture was sufficiently attractive to warrant his embarking on it; that particular rate of profit which alone, if we are right, is almost the sole source of the positive component of the attractiveness of the venture, though it does not alone determine the net attractiveness; at all other rates of profit (above zero) he would pay some tax, and at some levels more than he would have done under a constant or progressive rate of tax. It may be asked whether this scheme would not increase the uncertainty of uncertain ventures? Fig. V 3 is drawn on the assumption that the enterpriser has put $k_P = g_P$, that is, has declared his primary focus-value of untaxed gains at its true value. This figure shows three potential surprise curves:

$y = y(x)$ for untaxed gains,
$p = p(u)$ for gains taxed according to our own scheme,
$\psi = \psi(s)$ for gains subject to a tax at a constant rate R.

In the third of these $s = x\{1 - Rx\}$, where R is a constant lying between zero and $T(z)$, when z has a value corresponding to the upper extreme of the inner range of x. It will be seen that the potential surprise attached to given rates of tax-diminished profit is increased for gains remote from g_P and decreased for those near g_P by our scheme as compared with a tax at a constant rate R; but, again, if our concept of focus-values is realistic, this change from R to $T(z)$ would increase rather than diminish the attractiveness of the venture; for the *ex ante* rates of profit which interest the enterpriser are those in the neighbourhood of g_P.

The argument underlying our suggested mode of taxation gives one further result of some interest, namely, a simple and pragmatic question, instead of a subtle and abstract one, which we can put to the enterpriser in order to determine the primary focus-gain of any venture he has in mind: we simply ask him to name that rate of profit from which he would choose to have the deviation of the realised rate measured, on condition that the rate of tax on the realised rate of profit should be an increasing function of this deviation, and should be zero if the deviation is zero. If the concept of primary focus-gain, as we have specified it, has reality, its level for any venture conceived by a given individual will be determined if this question is faithfully and competently answered.

The practical application of the principle of taxation I have outlined would no doubt present difficulties, and the reader may feel that I ought to consider what these would be and try to solve

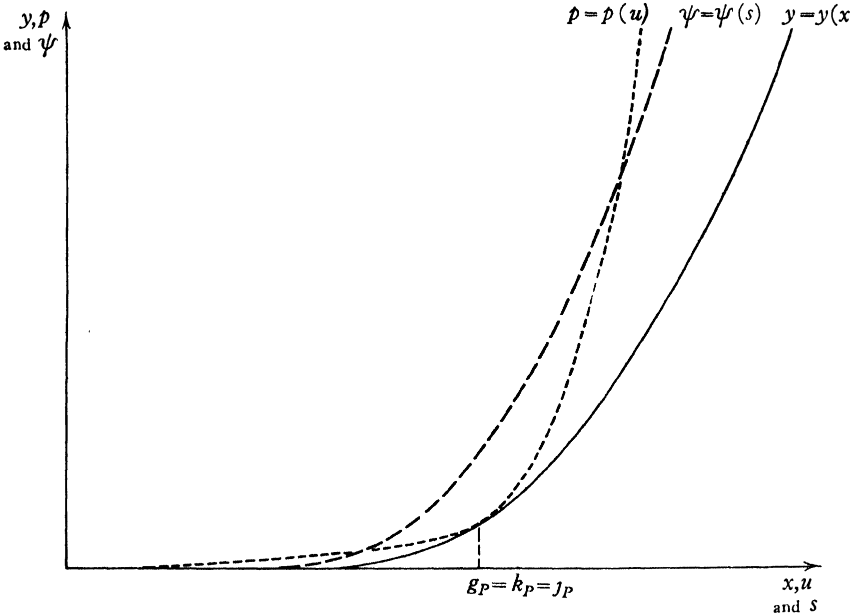

FIG. V 3.

them. But I think that such an attempt would be alien to the purpose of this book, whose concern is with theory and analysis, not with problems of administration. I believe that the principle could be applied, and might contribute an element of some worth to a technique of investment-stimulation. But here it is to the character of the ideas in themselves that I wish to draw the reader's attention, and practical applications concern me only in so far as they can illuminate my central construction. The practical problems, therefore, I must leave to be studied elsewhere.

CHAPTER VI

A THEORY OF THE BARGAINING PROCESS

Suppose there is some unique indivisible object (say, the only house in some particular situation) owned by *B*, who is willing to sell it, and that it is desired at present only by *C*, for whom it has no close substitutes. At what price will the house change hands? Orthodox economic theory says that *B* will have in mind some price which represents his subjective valuation of the house, that is, its value to him as a possession to be retained, so that if he accepted this price he would feel himself neither better nor worse off than before, while *C* will have in mind a price which represents the subjective value of the house to him; if *C*'s subjective valuation is higher than *B*'s, the house will change hands at a price which is *indeterminate* over the range between the two valuations. This theory is plainly incomplete. The actual price must be determined by *something*, and we cannot rid ourselves of the need to explain what that something is by arbitrarily assuming it to be outside the scope of economics. Are we sure that no explanation can be found in terms of accepted economic categories?

This seems to be yet one more problem, like that of the true nature of money and interest as that problem stood before the invention of liquidity preference, where the neglected essential factor is *uncertainty*. In short, determinacy will be attained when we take into account, for each bargainer, the interaction in his own mind, as he settles his bargaining plan, of his greed for profit and his fear, if he overreaches himself, of some form of loss; and if we consider that there can be no such hopes and fears unless there are various and imperfect degrees of belief associated with rival hypotheses about the outcome of this course of action or that.

Our first task is to show that, except by an almost inconceivable coincidence, uncertainty must enter at some stage into the bargaining process. If at the outset each bargainer had in mind a definite hypothesis about what the succession of price-offers (first an offer by one bargainer and then an offer by the other) would be, and if each felt certain that his hypothesis was correct, and if these two hypotheses coincided at every point, then an agreed price would presumably be reached without there having been at any stage any uncertainty in the mind of either bargainer. But if either bargainer attaches nil or low, or indeed anything less than the absolute maximum of potential surprise to more than one hypothesis about what will happen at any stage of the process, uncertainty is of course already present; while if each bargainer has an exact idea, which he feels certain is correct, about what will happen at each stage, and if their respective ideas for any one stage do not coincide, then when that stage is reached one at least of them will be proved wrong and will in general no longer have ground for believing his judgement concerning the subsequent stages to be certainly correct. It is justifiable for us, therefore, either to assume that uncertainty is present from the outset in the mind of a bargainer, or that all those stages where it was absent have already been passed through, and that we take up the developing situation at the point where uncertainty has arisen.

Our procedure in analysing the process of bargaining between *B* and *C* will be:

(i) To picture the state of mind of each bargainer at the outset of the bargaining process, or at the moment when each has conceived afresh the situation which faces him after a disappointment (whether favourable or unfavourable) of his former expectations, i.e. after something has happened to which before its occurrence he attached some degree greater than zero of potential surprise.

(ii) To indicate very briefly the nature of the process which will be generated by the confrontation of the two bargainers' respective plans, in their initial and their successively revised forms.

A negotiation may of course have as its purpose the settlement or determination of any number of different questions or variables: a number of prices and an equal number of quantities of goods to be exchanged; delivery dates; various promises which the parties may wish to exact from each other as to subsidiary conditions to be fulfilled in executing the bargain. And in any negotiation, a bargainer may no doubt have a variety of weapons in his armoury: the style or tone in which his offers are conveyed by writing or by word of mouth; the timing of his successive offers and the length of time elapsing between them; bluff and deception in various forms and degrees; and so forth; and he may even indulge in 'shock tactics', suddenly withdrawing all his previous offers and starting again with a less favourable initial offer. But such possible complications of the purpose and method of a negotiation need not, I think, prevent us from abstracting the essentials, and building a theory of price-determination on certain simplifying assumptions which could afterwards be modified to deal with more complicated cases. Thus, *timing and delay* do not essentially complicate the problem. In making his bargaining plan before opening the negotiation, or in making a fresh plan during its course, a bargainer will plan the timing of his successive offers with a view to getting the greatest effect on the other bargainer's mind from each concession he makes. The intended time-pattern of the sequence of offers may be different for different intended sequences of asked or offered prices. But we shall not consider how the bargainer selects the best time-pattern for any given sequence of prices. The chief variables in a negotiation intended to settle a price are the asked or offered prices themselves, and we shall confine our attention to these. And we shall neglect the possibility of 'shock tactics': we shall suppose that, having once announced a price, the seller will make each of his offers lower than its predecessor, while the buyer will make each of his offers higher than its predecessor, until agreement is reached or the negotiation is broken off.

B's mental process concerns the following prices:

1. His *absolute minimum price* m, to accept which would only just compensate him for losing the direct enjoyment of the house.

2. His *gambit price* g, that is, the price he will announce at the outset.

3. His *effective minimum price* j, the least that some chosen *policy* will allow him to accept.

4. The *agreed price* v, at which, if at all, the house will eventually change hands.

And the following derived quantities:

5. His *gain* x (which in simpler cases will be equal to $v-m$).

6. His *descent* $s=g-v$.

B has three possible policies:

1. He can resolve that, whatever g he announces, he will not let his subsequent offers descend at any one step or in the total of some or all steps by enough to diminish his prestige as a bargainer and so impair his powers in future negotiations. That is to say, he will not allow the descent of his announced price to be steep enough at any stage of the negotiation, or large enough in total, to alter in the mind of the other bargainer *C*, or any other potential bargainer with whom he thinks he may be faced in future, the functions according to which they judge, from *B*'s successive 'asking prices', what is the least price that *B* will accept. Rather than lose face, he will suffer the negotiation to break down. This is the 'possible breakdown' policy.

2. He can resolve that, whatever g he announces, he will subsequently make whatever concessions may be necessary to secure

agreement, short of accepting a price less than *m*. This is the 'possible loss-of-face' policy.

3. He can resolve that, having announced a given *g*, he will set a limit, depending on the particular *g*, to the concessions he will make; if agreement is not secured by these, he will suffer the negotiation to break down. This is the 'combined policy'.

B's task is to choose *simultaneously* a policy and a gambit price (and where the 'combined' policy is concerned, also an effective minimum price). He cannot select them independently of each other. One policy is not by itself better than another, nor one gambit price by itself better than another. Comparison must be made of the entities each consisting of a policy and a gambit price, or of a policy, a gambit price, and an effective minimum price. Such a combination we shall call a bargaining plan. Any such bargaining plan is essentially like an investment blueprint: it offers both hopes and fears; the bargainer must consider what, having accepted it, he will stand to gain and what he will stand to lose. We shall suggest that these opposing factors can be represented in his mind by a *focus-gain* and a *focus-loss*, and all possible plans can thus be compared, and a choice amongst them made, by means of a gambler indifference-map.

What does *B stand to lose* under the 'possible breakdown' policy? The opportunity-cost of adopting this policy is plainly the sacrifice of the best hope he could have entertained under the 'possible loss-of-face' policy. To allow the negotiation to break down is to throw away all the opportunities it represented. But these opportunities are mutually exclusive; only one of them could in fact have been used (that is, only one gambit price could have been tried), so the sacrifice entailed is the hope of that gain which, having regard both to its size and the ease of believing it attainable, is the most attractive possibility held out by the 'possible loss-of-face' approach.

It is to be noticed that, under the 'possible breakdown' policy, what *B* stands to lose is *the same* for every choice he may make of a gambit price.

Against this possibility of breakdown and the sacrifice entailed, what lies in the other scale? For any one level of his gambit price, he will have in mind a set of rival hypotheses about the price he will actually secure. To each of these hypotheses, and to its corresponding hypothetical gain x (viz. the excess of the hypothetical price over his absolute minimum price) he will assign its own degree $y(x)$ of potential surprise, which may be zero for some range of x but will in general begin, at some size of x, to increase with further increase of x. Thus there will be a primary focus-gain with its corresponding standardised value. Now since the *adverse* contingency is the same for all plans under the 'possible breakdown' policy, the most attractive of these plans will simply be that one which has the largest focus-gain. It is not till we come to compare this plan with its rivals under the other policies that we need ask what is the nature and the measure of its focus-loss.

In order to entertain any hope (with however high a degree of potential surprise) of a given gain, he must of course set his gambit price above his absolute minimum price by at least the amount of this gain. Over some range, therefore, a higher gambit price will be associated with a higher focus-gain. But under the 'possible breakdown' policy there will be some level such that any gambit price higher than this level will merely ensure the breakdown; and the potential surprise attached to the hypothesis of *any gain at all* will increase as this level is approached. Thus at some level the sign of the derivative of focus-gain with respect to gambit price will become negative, and it follows that the most attractive plan under the possible breakdown approach will be determinate.

What does *B* stand to lose under the 'possible loss-of-face' policy? 'Loss of face' means simply impairment of future bargaining power, that is, it means loss of *future* gains. The true nature of the two opposed policies is thus that while each of them seeks immediate gain, one does so by risking immediate loss, the other by risking future loss. In order to assess *in terms of spot cash* the loss of face entailed by any given *descent* $g-v$ from any given g, the bargainer must have in mind some notion of the number and scale, and the distribution over future time, of the negotiations in which he expects to engage. For example, if *none*, then loss of face will not matter at all, and its spot-cash equivalent will be zero. Further, it is plain from this view of the nature of loss of bargaining prestige that a 'rate of interest' or 'impatience' comes in and can affect the course of bargaining. The assessment of the loss of face entailed by a given descent $g-v$ from a given g, in terms of spot cash, must plainly be a matter of very rough and ready, and indeed of intuitive, calculation. But some such assessment there must evidently be, if conduct is to be rational.

Under the 'possible loss-of-face' policy, what *B* stands to lose is not invariant under changes of gambit price but depends on the latter. If he sets his gambit price very high, the descent which may ultimately prove necessary in order to secure agreement may be very great and the 'loss of face', measured by its spot-cash equivalent, correspondingly so; the higher the gambit price, the less will be the potential surprise (unless this is already zero) attached to any given degree of 'loss of face', and the larger, therefore, will be the focus-loss of the plan consisting of that gambit price under the possible loss-of-face policy; but again, in order to entertain any hope (with however high a degree of potential surprise) of a large gain, he must evidently set his gambit price above his absolute minimum price by at least the amount of this gain. For each of the possible gambit prices g that he passes in review before deciding amongst them, he will have in mind a set of rival hypotheses about the agreed price v that this gambit price will secure, and to each of these hypotheses he will attach some degree $y=y(g, v)$ of potential surprise. To each such hypothetical value of v there will correspond a determinate *gross* gain $\xi=v-m$ and, g being given, a determinate *descent* $s=g-v$; and to this descent there will further correspond some degree of loss of face expressible, having regard to B's expectations about the number, scale, and time-distribution of his future engagement in negotiations, as an amount z of spot cash. ξ and z will each carry the same degree $y=y(g, v)$ of potential surprise as v. Thus for each gambit price g he will have in mind a set of rival hypotheses about the *net* gain $x=\xi-z$, these hypotheses ranging in general over positive *and negative* values of x; and to each of these hypotheses he will assign some degree $y=y(x)$ of potential surprise. By reasoning which we need not again elaborate there will thus be for *each* g, under the possible loss-of-face policy, a primary and a standardised focus-gain and a primary and a standardised focus-loss. The two standardised focus-outcomes, one pair for each bargaining plan, that is, each g, will simply be amounts of spot cash. By using these two amounts as co-ordinates of a point on his appropriate gambler indifference-map, the bargainer can compare all the bargaining plans under the possible loss-of-face policy and select the one which, for him as an individual with given temperament, offers the most attractive combination of hopes and fears.*

We can now answer the question: What is the nature and measure of the *focus-loss* of all plans under the possible breakdown policy?

* It may be asked: If the bargainer should find in the course of actual bargaining under the 'loss-of-face' policy that he had reached an 'asking price' p where his loss of face was already equal to the difference $p-m$, so that at best he could no longer hope for a positive net gain, would he not then stop bargaining? No, he would continue, because if he stopped, the loss of face already incurred would not be compensated even in part by any difference $v-m$, whereas if he continues there is *at every point* some hope that a further small concession, entailing only a small addition to the loss of face, might avoid the breakdown and secure a relatively big difference $v-m$.

Plainly it is the *focus-gain* of the most attractive plan under the possible loss-of-face policy. And evidently we can compare the best plan under the possible breakdown policy with the plans under the possible loss-of-face policy by plotting it on the same gambler indifference-map.

We have now considered the two 'pure' policies. In one of these the bargainer sets j, his effective minimum price, high enough to avoid all loss of face, in the other he sets it equal to his absolute minimum price. But there is plainly a third policy: he can set his effective minimum price at some intermediate level, and for *each* gambit price there will then in general be a different focus-gain for each different effective minimum price; for what he judges to be the optimal sequence of 'asking' prices, with which to follow up his gambit price, will depend on the depth of the greatest descent he is prepared to make from that gambit price; and the degree of potential surprise assigned to any hypothetical outcome will be different if this sequence is different. Thus when his effective minimum price is relatively close to the gambit price, the steps of his descent towards it can be small and will influence the other bargainer's mind by the air of stiffness and intransigeance which they impart. There will also be, for each given gambit price, a different focus-*loss* for each different effective minimum price, reached in the following way: A particular pair of values of g and j being specified, the bargainer will attach some particular degree $y=y(g,j)$ of potential surprise to the hypothesis that with this g and j the negotiation will break down and thus involve him in the *combined* loss consisting of the loss of face due to the descent $g-j$ plus the sacrifice of the focus-gain of the most attractive plan under the 'possible loss-of-face' policy.

Thus under the 'combined' policy each plan, that is, each pair of values (g,j) of the gambit price and the effective minimum price, will have its standardised focus-gain and its standardised focus-loss, each of these being simply an amount of spot cash. All plans under the combined policy can be compared with each other and with those under the other policies by being plotted on the same gambler indifference-map.

The problem facing C, the potential buyer, is the exact counterpart of B's and can be solved in the same way. To make the description of B's procedure apply to C we must of course replace B's absolute minimum price by C's absolute *maximum* price, B's descent $g-v$ by C's ascent, say $v-G$, and so forth. It makes no difference to our conception of the nature of the bargaining process which of the two bargainers we suppose to be the first to announce a gambit price, though it may well make a difference to the price they ultimately agree on. For it may be necessary for either party to choose a fresh bargaining plan any number of times in the course of the negotiation. Each successive 'asking price' named by *B* is a new piece of information for *C*. It may fit perfectly into his previous conception of what was in *B*'s mind, that is, it may cause him *no surprise*, or at the other extreme it may compel a complete refashioning of his own ideas. On the assumptions we have made, *B*'s 'asking price' and *C*'s 'offered price' will move towards each other. If we knew what effect any given change in *C*'s offered price would have on the focus gain and loss of the various bargaining plans in *B*'s mind, and what effect any given change in *B*'s asking price would have on the focus-gain and loss of *C*'s bargaining plans (in addition to a knowledge of their respective gambler-preference systems) we could conceptually determine at what price agreement or breakdown would be reached. In short, the outcome is *conceptually determinate* without reference to anything except the two bargainers' tastes and *expectations*.

The analysis sketched above leads to two chief conclusions, both of which are new results carrying us beyond what accepted theory has told us, and, I think, actually contradicting it, though they seem agreeable to intuition and experience. That theory, taking as its basic postulate that each bargainer's preference system is given *as regards commodities* and in particular as regards the object to be exchanged, declares, first, that in the general case where monopolist faces monopsonist, exchange *will* take place provided only that the seller's subjective valuation of the object is less than that of the buyer; and, secondly, that the price will lie *indeterminately* somewhere between those valuations. Our analysis leads us to controvert both these findings. For by taking account of the bargainer's attitude to *uncertainty*, that is, by assuming him to have a given preference system for various *gambling situations* as these are assessed in his own mind in terms of focus-values, we find, first, that it is by no means certain that any exchange will take place even when the seller's absolute minimum price is below the buyer's absolute maximum price; and secondly, that if exchange does take place, the price, in an important sense, is determinate: it is conceptually knowable in advance, if we are fully informed about the gambler-preference system of each bargainer, and the functions according to which he will draw inferences from a given sequence of 'asking prices' or 'offered prices' announced by the other bargainer.

CHAPTER VII

A COMPARISON WITH THE ORTHODOX VIEW

The reader now has before him not only a formal description of a new apparatus but also a number of experiments in applying it to particular themes. I was driven to attempt the construction of such an apparatus by a conviction that the orthodox approach involves some serious logical difficulties, is unrealistic, and leaves unexplained a number of important phenomena. Now that the positive aspects of my argument have been presented, it may be well in conclusion to try to explain the basis of this feeling, and show what those difficulties are, which seem to me to be inherent in the traditional view but to be avoided by the one I have advanced.

The orthodox theory of the procedure by which an individual assigns to each of a number of rival courses of action, amongst which he must choose, but whose respective relevant outcomes are not known exactly and for certain, a valuation which allows for and condenses to a compact quantitative statement the whole spectrum of possibilities and the uncertainty inherent in it, which for him represents this outcome, depends on a concept which we may call 'subjective numerical probability'. In this procedure the individual after listing in his mind the whole set of possible outcomes (so far as these are distinct from each other in aspects which involve his advantage or disadvantage) assigns to each of them some proper fraction, the fractions being so chosen that they sum to unity, and that each represents the 'probability' of the outcome in question turning out to be the truth. There is little need to insist on the difficulty of deciding and explaining what we can mean in this connection by 'probability'. All concepts of numerical probability are alike in involving the ideas of uniformity in some specified sense in the conditions of the experiments, and of the number of these experiments being in some sense 'large'. In order to establish empirically a figure for the probability of a given outcome we must have made a 'large' number of trials under conditions which are constant in some specified sense; in order to give meaning to the figure obtained, and to use it, we must have in view the making of a large number of further experiments within the same system of given conditions. Now for many important kinds of decisions which must be taken in human affairs it will be impossible to find a sufficient number of past instances which occurred under appropriately similar conditions; no well-founded figures of probability for different kinds of outcome can be established on the basis of experience. This difficulty, however, is a minor one compared with the fact that, even if by vicarious experience a probability is established, many kinds of decision are for each individual virtually *unique*; the die is to be thrown once and for all, and it will be little comfort to the individual, should the outcome be disappointing, to think that if he had a hundred lives a similar decision in all of them would yield him a known proportion of successes. Few individual enterprisers, for example, even in the course of their whole lives, launch a number of ventures of even broadly similar kinds which is 'large' in any sense required by the theory, or even the practical application, of probability principles. If I am faced with the need to choose a career, or a type of business in which to embark my fortune and perhaps to lose the whole of it, a choice which must in its nature be made once and for all or virtually so, does it help me to know that the proportion of successes in this or that line is such and such? Is 'probability' in this frequency-ratio sense a relevant consideration at all when only one or virtually only one 'throw of the die' is going to be allowed us? It is universally agreed that the probability of a single, isolated event has no meaning. The point that numerical probability (no matter whether reached empirically or deductively) is *irrelevant* when only a single further trial is to be made, is well brought out in the following passage by Charles S. Pierce:†

'If a man had to choose between drawing a card from a pack containing 25 red cards and a black one, or from a pack containing 25 black cards and a red one; and if the drawing of a red card were destined to transport him to eternal felicity and that of a black one to consign him to everlasting woe, it would be foolish to deny that he ought to prefer the pack containing the larger proportion of red cards, although from the nature of the risk, it could not be repeated. It is not easy to reconcile this with our analysis of the conception of chance. But suppose he should choose the red pack and should draw the black card. What consolation would he have? He might say that he had acted in accordance with reason, but that would only show that his reason was absolutely worthless. And if he should choose the red card, how could he regard it as anything but a happy

* Only once in a long time, a large proportion of our life.

† *Chance, Love and Logic*, by Charles S. Pierce, edited by Professor Morris R. Cohen, quoted in *Mathematics and the Imagination*, by Edward Kasner and James Newman (New York, 1940).

accident? He could not say that if he had drawn from the other pack he might have drawn the wrong one, because an hypothetical proposition such as "If *A*, then *B*" means nothing with reference to a single case.'

Now if it be agreed that the 'probability' which the enterpriser assigns to a hypothesis concerning the outcome of a *unique* venture is something different from and unrelated to 'frequency-ratio' probability; that it is subjective, and not arrived at either by rigorous logic or by carefully-controlled induction; and really amounts to the assigning of marks to each hypothesis according to the degree in which it possesses some quality other than intrinsic desirability;* we have to ask: What is this other quality? What we seek, I think, is a quality which we might call 'credibility', 'plausibility', 'possibleness', the quality of being a reposeful basis for imaginations of success, for hopes and dreams; a foundation which obtrudes no visible or insistently disturbing insecurities, no obvious unrealism, no strained holding together of the picture and consequent distraction from enjoyment of its content.

Subjective stability or *reposefulness* is a quality which, from the very nature of the purpose they serve, the individual is bound to seek for his expectations, though all experience would teach us not to look for objective stability. For though in fact his system of expectations will be continually suffering smaller or greater shocks and dislocations and may sometimes be shattered, yet the making and the execution of a plan takes time, and at its inception is expected to take time, and a plan if it is to make sense must be based on one self-consistent scheme of expectations,† that is to say, a system in which, for each of the variables embraced, we have for each hypothesis as to its value at each calendar date one constant degree of belief. To call forth from the individual the mental and nervous energy required to devise and try to execute a plan of any consequence, the incentive arising from the *content* of the outcome he pictures must be insulated from the distractions arising from uncertainty; the picture must be steady and clear while it lasts, not blurred by oscillation between different versions. To attain this subjective stability, the individual will defer repeatedly the crystallisation of any plan, waiting continually for a situation where he can feel that the number of unanswered questions is at a minimum, and

* I.e. the quality of being desirable (in the eyes of the person concerned) regardless of whether or not it will or can come true.

† The point was brilliantly made by Professor F. A. Hayek in his article 'Economics and Knowledge' (*Economica*, February 1937).

is likely rather to increase on balance, through the obsolescence of some of the data now in his possession, than to decrease through the acquisition of fresh knowledge. Only at such a time of *freedom from impending fresh knowledge* will he consent in important matters to decide and act. But this desire for stability has a further consequence; not only must the outcome be represented in his mind by a very small number of dominant hypotheses, which alone vividly hold his attention, rather than by a distribution over which his attention is widely dissipated, but these few special levels of success or misfortune (one of each according to the view put forward in this book) must be determinate maxima of some actual mental experience, not mere mathematical abstractions. Thus we are led to a hypothesis such as that of focus-values put forward in the preceding chapters. The individual's view of the outcome must be a vivid contrast of two pictures, the happy and the unhappy (the most clear-cut arrangement of *content* which still allows for *uncertainty*), not swaying over a wide dispersion or prosaically indexed by an unappealing, unexciting abstraction such as the mathematical expectation.* Such were the ideas which suggested the substitution of potential surprise for numerical probability.

In founding itself upon James Bernoulli's *Principle of Non-Sufficient Reason*, the theory of numerical and mathematical probability seems, it is true, to take a starting-point highly acceptable to an inquirer into the nature and laws of expectation. The principle itself and the service it renders to the conception of numerical probability are thus explained by Lord Keynes: †

'In order that numerical measurement may be possible, we must be given a number of *equally* probable alternatives. The discovery of a rule, by which equiprobability could be established, was, therefore, essential. A rule, adequate to the purpose, introduced by James Bernoulli, who was the real founder of mathematical probability, has been widely accepted, generally under the title of *The Principle of Non-Sufficient Reason*. . . .

'(This principle) asserts that if there is no *known* reason for predicating of our subject one rather than another of several alternatives, then relatively to such knowledge the assertions of each of these alternatives have an *equal* probability. Thus *equal* probabilities must

* If contingencies $x_{[i]}$ have probabilities $z_{[i]}$ and there are n contingencies in all, the mathematical expectation is $\sum_{i=1}^{n} x_{[i]} z_{[i]}$.

† *A Treatise on Probability*, by John Maynard Keynes, Chapter IV, pp. 41, 42.

be assigned to each of several arguments, if there is an absence of positive ground for assigning *unequal* ones.'

The wholly subjective meaning which must evidently be given here to the words 'equally probable', the implied choice of a meaning for probability which makes it refer to the state of mind of some particular individual, and the implication that it is *ignorance* on the part of this individual which alone gives probability a meaning and a part to play, are entirely consonant with our requirements for a concept to be used in the theory of expectation. Yet it does not appear that numerical probability can always perfectly express our state of mind when we are confronted with a number of alternative predicates between which we know of no reason to discriminate. For if there are n such predicates or contingencies we give each of them the probability $1/n$, and if n is large, this procedure implies that we regard each of them as *highly improbable*. But to say that each of them is highly improbable is to say more than we really mean, when what we wish to express is 'I have little relevant knowledge, and so far as I am concerned, any one of the mutually exclusive contingencies A, B, ..., N could happen without seeming incongruous with what I do know of the circumstances. No one of these contingencies seems to me "improbable" in the sense that it calls for any stretch of imagination to conceive it coming true.'* By contrast, the concept of potential surprise can express this frame of mind with exactness. For by assigning nil potential surprise to each of the rival contingencies, we give it a measure of belief or acceptance which will remain unchanged no matter how many other rival (i.e. mutually exclusive) hypotheses are placed alongside it on an equal status. The fundamental question which we are led to ask is this: Does an increase in the number of hypotheses (rivals to a given hypothesis), which we cannot reject as impossible, really reduce the

* As an illustration of my meaning, suppose that there are four candidates for some appointment, each of whom seems, to some detached observer, to be fully qualified for the post, and amongst whom nothing inclines him to believe that one, rather than another, will appeal specially to the preferences of the person making the appointment. Then if this observer should try to represent his opinion concerning the prospects of any one of these candidates, say A, by saying that A's 'chances are $\frac{1}{4}$', he will by implication be asserting that A is *unlikely* to be appointed. But this is in flat contradiction of the observer's actual state of mind; he considers that, so far as his own means of judgement go, A's appointment is a perfectly possible, credible, and 'likely' outcome, entirely congruous with all the circumstances known to him. It will not surprise him in the least if A is in fact appointed, and to say that A's appointment is 'improbable', as the numerical representation would compel him to do, would grossly misrepresent and distort his real feeling.

degree of acceptance we accord to the given hypothesis? Plainly, I think, it does not. The mere recognition of a wider ignorance about what may happen does not alter or reduce the right of the given hypothesis to its place amongst those which our knowledge does *not* enable us to reject. An outcome that we looked on as *perfectly possible* before is not rendered less possible by the fact that we have extended the list of perfectly possible outcomes. For we do not think of the decision between all the rivals as something which will be made in the future by a random selection performed by the Goddess of Chance; often we look upon it rather as a decision which is already implicit in the situation existing at our viewpoint, but which the limitations of our knowledge and powers of reasoning prevent us from working out in advance of the event. Now when all the recognised rivals of a given hypothesis (predicate, contingency) are taken together as a whole, they are equivalent, for the person concerned, to the *contradictory* of the given hypothesis. What we are saying, then, amounts to the assertion that some measure of acceptance or belief is needed which will accord equal status to a *given hypothesis and to its contradictory*. Now it is easily shown* that we cannot indicate this state of mind by assigning a numerical probability $\frac{1}{2}$ to the hypothesis and to its contradictory. For the contradictory is itself composed of mutually exclusive alternatives to each of which, on the same ground, we might have to assign a probability $\frac{1}{2}$. Thus we should be in the position of having *more than two* exclusive alternatives each with the probability $\frac{1}{2}$. Numerical probability, then, cannot represent the frame of mind we are considering, while our own concept is perfectly adapted to do so. This is a particular aspect of a more general ground on which our concept can claim advantage, for the analysis of expectation, over numerical probability, namely, the *mutual independence* of the 'plausibility estimates' which are what we express by means of potential surprise. The measures of degree of belief which our concept assigns to different hypotheses are *independent of each other* and do not have to sum to unity or any other particular total.

Our basic reason for rejecting the numerical probability-distribution as the form of the answer which an enterpriser gives to the question in his mind: What will be the outcome if I adopt this plan? is vividly brought out by the following consideration: Some followers of the

* The paradoxes arising from the belief that the Principle of Non-Sufficient Reason, when applicable, assigns equal probabilities to a hypothesis and its contradictory were for the first time shown in their true character, and successfully resolved, by Lord Keynes (op. cit. Chapter IV).

orthodox approach point out that the establishment by the enterpriser in his own mind of a numerical probability-distribution is not the end of the story; there can be uncertainty in his mind whether this distribution is 'correct'.* Now what can be meant here by 'correct'? What actual, recorded, *ex post* outcome of the venture he has in mind will show the probability-distribution to have been correct or incorrect? The crucial dilemma can be posed as follows: either the probability-distribution is a genuine one having in view a large number of future trials (i.e. business ventures) all arising in the same system or totality of possible phases,† so that there will be eventually an actual *ex post* distribution to be compared with, and to justify or falsify the *ex ante* distribution; or else the alleged probability-distribution refers to a unique or virtually unique experiment, and is therefore entirely bogus and meaningless, since we shall be no better able after than before the outcome is known, whatever this outcome turns out to be, to say whether the 'distribution' was 'correct' or not. The former alternative is not appropriate to the kind of business decisions we have in mind; no business executive has to decide a hundred times in ten years whether or not to spend £1,000,000 on a new factory. The second alternative leads us to say: If the so-called 'distribution' is bogus in this sense, then its real nature is the awarding of marks to different hypotheses and expresses, not *actuarial risk* which is a form of *knowledge*, but *uncertainty*

* As an example of the orthodox approach I quote the following passages from the fine article by Evsey D. Domar and Richard A. Musgrave on 'Proportional Income Taxation and Risk-Taking' (*Quarterly Journal of Economics*, May 1944):

'To handle our problem, quantitative values for the yield and the degree of risk of an investment are needed; and in the absence of a better approach, they are obtained by means of a probability-distribution which the investor will construct for each available investment opportunity....

'Investment decisions are made in spite of uncertainty with respect to the relevant data and their implications. No investor is sure that his estimated probability-distribution is entirely correct, but the degree of uncertainty will vary with different investors and different investments. It will be a factor in the investment decision. Yet it is extremely difficult to express the degree of uncertainty involved in workable terms. For our purposes it is sufficient to say that the prevalence of uncertainty may induce the investor to require a somewhat higher return than would be required otherwise.'

It is not quite clear whether the uncertainty referred to in the last sentence quoted arises from the fact that the distribution *is* a distribution and not a unique value looked on as certain, or from the investor's doubt as to whether the distribution is the right shape, whatever 'right' can mean here. It is such Gordian knots as these that I wish to cut by a radical simplification.

† See, e.g., A. C. Aitken, *Statistical Mathematics*, Chapter I (Edinburgh, 1939).

which is another name for *ignorance.** But what purpose is served by building up this elaborate description of the uncertainty-situation which exists in an individual's mind if, after all, it fails to describe his uncertainty in exact and unequivocal terms and is declared to be itself, not a certainly correct, but a possibly incorrect, description? If it merely describes his state of mind before the event, how will knowledge of the actual event tell him whether that description was correct or not?

Let us ask the same question of our own approach: What actual, *ex post*, outcome of a single venture will show the answer, which the enterpriser gave to himself *ex ante* to the question 'What will the outcome be?' to have been correct, and what will show it to have been in some degree a misjudgement? It is fully justified if the actual outcome falls inside his inner range, and it is wrong in increasing degree according as the potential surprise he attached *ex ante* to what has proved *ex post* to be the truth was higher.

The probability-distribution approach is perhaps more acceptably stated by Myrdal, whose formulation does not involve any such confusing idea as that of the investor being uncertain whether the probability-distribution, which presumably describes his uncertainty regarding the outcome of some one venture, is itself 'correct'. He also, like the authors previously quoted, suggests that uncertainty may be a disutility which will only be accepted if there is some positive compensation (even though purely subjective and anticipatory) for doing so. Myrdal, however, regards this compensation as required by the mere existence of a *dispersion* instead of a unique value looked on as certain, not by any uncertainty about the 'correctness' of the distribution:

'Since there exists a whole series of probabilities for every single element of gross returns and costs, we have to multiply every expectation of incomes or costs by a coefficient before we can discount it. [The word "discount" here refers to pure time-discounting. G.L.S.S.] This coefficient gives the assumed degree of probability. The expression for the net return, calculated in this way, must subsequently be multiplied by a second coefficient which expresses as a valuation the attitude toward risk which is held by the entrepreneurs evaluating

* Since writing this sentence I have found the following in an article on 'Money and the Theory of Assets' by Professor Jakob Marschak (himself an exponent of the probability-distribution approach): 'This may be the rationale of Professor Knight's important distinction between "risk" and "uncertainty": the former is a known parameter of a frequency-distribution, the latter, the lack of knowledge of this (or any other) parameter.'

the probabilities of such future elements of returns and costs.' (*Monetary Equilibrium*, by Gunnar Myrdal (Glasgow, 1939), p. 59.)

Professor A. G. Hart in his book *Anticipations, Uncertainty and Dynamic Planning** makes no explicit distinction between those business decisions (such as large-scale extensions of durable plant) which are made only at long intervals† by any one enterpriser or firm, and which must thus be treated as single, isolated events and therefore, according to the definition which he himself gives,‡ involve *uncertainty*, and those (such as the scale of output with *given* durable plant) which are going to be many times repeated during the enterpriser's life and can perhaps therefore be based on principles of actuarial probability. The argument to which he proceeds in Chapter IV of his book seems to me to be logically acceptable provided it is explicitly confined to policy on *constantly recurring* decisions, such as those which a business man must take from week to week or from day to day concerning output. It is *not* appropriate where the decision has to be taken once for all, or once for a long time.† For what he explains to us in this chapter is the policy which, assuming the business man has the kind of knowledge about his contingencies which an insurance company has about *its* contingencies, will maximise his net receipts over a period long enough to embrace many decisions.

The point which Professor Hart is most concerned to make is that in constructing his business plan envisaging production, sales, etc., at a particular calendar§ date it will be advantageous and rational for the enterpriser to secure adaptability of the plan to revisions and improvements of his knowledge (regarding what is going to be the situation at that date) which he must expect to occur between his viewpoint§ and the date which his plan has in view. If decisions on some recurring issue of policy are going to be taken at short intervals, and knowledge of the frequency ratio of different outcomes of various policies has been, or can be, gained, then actuarial principles ought of course to be applied, and it may well turn out that in the long run (i.e. after many applications of the policy) a flexible

* Chicago, 1940.

† Time-discounting at any positive rate of interest entails that remote future decisions cannot be effectively brought into the calculation so as to obtain 'large numbers', since the discounted relative importance of their outcomes will be small in comparison with that of near-future decisions. Thus if the intervals between decisions are sufficiently long, these decisions must be treated as isolated events. Even the 'immortality' of a joint-stock company leaves this principle unaffected.

‡ Op. cit. p. 51.

§ I here adopt my own terminology.

business plan which is not the best for *any one* scale of operation or variant of the product, but can be made reasonably good for many different scales or products, will yield a larger return than would result from the attempt to find the very best plan on the assumption that one particular scale of activity of one sharply defined type would be appropriate, and no other. In this book I have been mainly concerned, not with measurable *risk* and the application, in circumstances which give it meaning, of actuarial probability, but with the totally different problem of what are the human reactions to *uncertainty*, that is, the irreducible core of ignorance concerning the outcome of a virtually isolated act. I assume, of course, that in drawing the blueprint of any venture, and determining his focus-values for it, the enterpriser will take full account of the possibilities of adapting his mode of using any proposed equipment to various rival sets of circumstances which all carry in his mind low potential surprise. By introducing greater flexibility he may be able to reduce the focus-loss; or by some sacrifice of flexibility in favour of more intensive specialisation to a sharply defined purpose, he may be able to increase the focus-gain. Thus my theoretical scheme is fully able to embrace those considerations on which Professor Hart insists.

Professor Jakob Marschak,* also, thinks of each venture or asset as being represented in a person's mind by a distribution in which a numerical † probability is assigned to each hypothesis about the gain or loss which will finally prove to have been reaped when the venture is wound up or the asset disposed of and the books are closed. He distinguishes four influences, or types of influence, which affect the relative market prices of assets:

(i) The 'expected value'‡ of a venture or asset, that is, the mathematical expectation $\Sigma x_t z_t$ (where probabilities z_i are assigned to hypotheses x_i) of x, the total gain or loss which will ultimately have been realised as a consequence of the venture. 'The tendency (he

* I refer in what follows to his article 'Lack of Confidence' (*Social Research*, February 1941) which gives a compact and, I believe, the most recent expression of his views.

† Marschak says (op. cit. p. 52) that his analysis could proceed equally well if probabilities were considered as ranks, not numbers: but he does not elaborate the suggestion.

‡ This term holds the possibility of confusion between the interpretation which, without being quite sure of Professor Marschak's intention, I have here put on it, which in Chapter III we have represented by $x=v-c$, and what we have there represented by v; but it scarcely matters in the present discussion which meaning we adopt.

says)* to put a high estimate on the expected values† of profits (or some other average) we have called "optimism".'

(ii) The dispersion of the probability-distribution 'in whatever way this may be measured (the standard deviation, the difference between the highest and the lowest possible profit or some other measure....Dispersion may be taken as a measure of "risk"; its reciprocal would measure the "safety" of an asset....The tendency to estimate a low dispersion of profits—low risks—may be called "confidence".'

(iii) The 'reliability' of the probability-distribution. 'It must be recognised that an individual may estimate two assets as having the same expected value, yet be aware that the two estimates have different degrees of reliability according to the different types of information at his disposal. (This) calls for the use of additional concepts (reliability) analogous to those developed by the modern theory of statistical inference.'

(iv) The 'audacity' or 'venturesomeness' of the enterprisers or speculators: 'the willingness to sacrifice a given amount of "safety" (the reciprocal of whatever measure is adopted of dispersion) for a given amount of "expected profits"'.

It will be interesting to consider what the relations are between these notions and the most nearly analogous features of our own scheme. In both (i) and (ii) Marschak adopts a measure which is a function of the entire distribution, that is, of the part which consists of losses, with their respective probabilities, as well as of the part which consists of gains with their probabilities. Thus the 'expected value', which he would perhaps render into popular language by the word 'profitability', is an average of *all* the different gains *or losses* which are looked on as not impossible (each one weighted by its probability). And the dispersion of the distribution, whatever measure of it be adopted, is also a characteristic of the distribution as a whole. In contrast to this method, Messrs Domar and Musgrave, in the article cited above, have moved nearer to my own procedure. They split the distribution into its positive and negative parts and take the average, weighted by probabilities, of each of these parts separately, thus obtaining the positive component, which they call the 'yield', and the negative component, which they call the 'risk', of the mathematical expectation or expected value. This plan seems to me greatly superior. For let us at first consider

* Op. cit. p. 53.

† The plural is used because Marschak is thinking of a number of distributions, one for each of a number of assets.

the positive part* of the distribution by itself; the more dispersed this part is, the larger the contribution it will make to the 'risk' in Professor Marschak's sense; but is it appropriate to his, and our, purpose that risk should be so defined that this consequence arises? So long as we exclude all transfer of probabilities† between the positive and negative parts of the distribution, and consider only a shift of probability between different levels of positive profit, then a more dispersed positive part means that low or moderate gains have assumed lower probabilities only in order that higher gains may

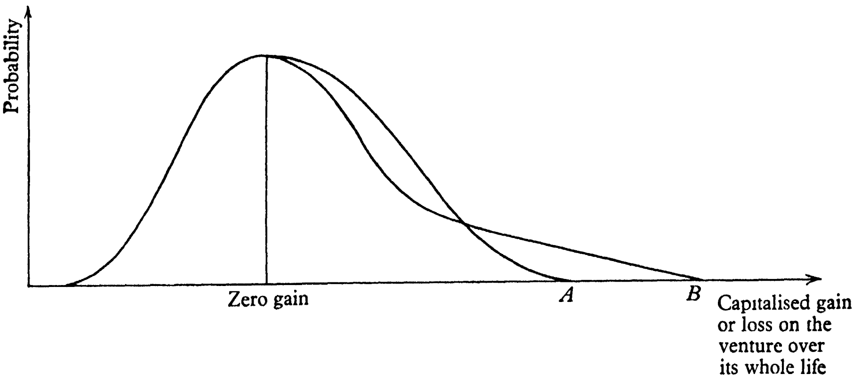

FIG. VII 1. The letters *A* and *B* do not indicate points on the gain-axis, but merely serve to distinguish the curves from each other. The curves coincide for all losses. Curve *B* shows the greater over-all dispersion, and on this account would be regarded by Professor Jakob Marschak as involving greater 'risk'. But plainly it is the more desirable by almost any reasonable criterion.

assume higher probabilities. Thus a greater dispersion *within the positive part* (as in curve *B* of Fig. VII 1) implies nothing detrimental to the venture: on the contrary, the risk (in a more ordinary sense than Professor Marschak's) of the profit being only a low one is *reduced.* It is only a greater dispersion within the *negative* part, or else an increase in the probabilities attributed to some particular levels of loss with a corresponding decrease in those attributed to some particular levels of gain, which really increases the enterpriser's 'danger'. Thus it seems appropriate to consider the two parts of the distribution separately, and the procedure of Messrs Domar and Musgrave, in so far as the probability-distribution approach is admis-

* I.e. the part concerned with gains as distinct from losses.

† By such a transfer of probabilities I mean an increase in the probabilities of, for example, some particular levels of loss and a corresponding decrease in the probabilities of some particular levels of gain.

sible at all, seems the correct one. In our own scheme the two 'components', the two focus-outcomes, are even more completely independent; for each branch of the y-curve can undergo any sort of change, including, for example, those which involve a *general* decrease in y over the positively sloping segment (corresponding loosely to a general increase in the probabilities assigned to gains) without the other branch being in any way affected either in its actual shape and position, or in the meaning to be attributed to this given shape and position; nothing corresponding to a 'transfer of probabilities from the negative to the positive part of the distribution', or vice versa, is ever called for; the focus-gain, for example, can increase by reason of a shift of some segment of the y-curve without affecting the measure of belief assigned to *any* hypothetical outcomes outside this segment, whether gains or losses.

The 'reliability' attributed to a distribution has no counterpart in our scheme. One might at first sight be tempted to paraphrase as follows Marschak's passage quoted above: 'In establishing subjective numerical probability-distributions regarding the outcomes of two different ventures, a man will sometimes regard the available data bearing on one venture as providing in some sense a better basis for conjecture than the available data bearing on the other: as being, for example, either more trustworthy in themselves, or leaving fewer relevant questions unprovided with some sort of answer.' In such a case an additional influence on his choice between the two ventures is introduced: If he looked on both distributions as equally 'well founded' he might choose, say, venture A without hesitation; but if the probability-distribution for venture A is relatively insecure, he may reject it in favour of the more convincingly supported though more modest promises of B. How would this situation be expressed by means of our concepts? We must first point out that untrustworthy data can in general be exchanged for data which are trustworthy but leave us ignorant within wider limits. Likewise data which are *insufficient* to confine our uncertainty within narrow limits can become sufficient to provide us with firm ground if we go far enough in widening the limits. In short, what we have called 'a better basis of conjecture' means a basis which gives a more compact form* to the relevant y-curve, and thus a smaller numerical

* By a more compact form of the y-curve I mean broadly a curve having a narrower inner range and more steeply sloping outer segments. Our statement in the text accords entirely with the following from Professor Marschak's article 'Money and the Theory of Assets' (*Econometrica*, October 1938): 'Greater knowledge means a concentration of the most likely rival hypotheses within a narrow region.'

value to one or other, or both, of the two standardised focus-outcomes. Now if this greater compactness of the y-curve belonging to, say, venture B, as compared with that of venture A, results in B's focus-loss being smaller than A's while their focus-gains are the same, the superiority of B will be adequately expressed by our apparatus. But if it consists in B's focus-*gain* being smaller than A's while their focus-losses are the same, the superiority will lie, in fact, with A, and will be so shown by our scheme. What the latter does, in fact, is to reduce automatically the gain-or-loss possibilities of two ventures to statements in terms of one and the same level of 'subjective reliability' or 'confidence'. (Marschak connects the latter word with 'high reliability' as well as with 'low dispersion').

The reason why no refinement or, as I should say, unrealistic complication, parallel to Marschak's concept of reliability, can or need find a place in our method, can be expressed as follows: A man cannot say 'I attach a degree p_a of potential surprise to this hypothesis; but I also attach a degree $p_b < p_a$ of potential surprise to the idea that my judgement, expressed as p_a, is ill-founded, and ought to be lower.' This roundabout statement would reduce to the simple assignment of some degree of potential surprise $p_k < p_a$ to the truth of the hypothesis. Now in so far as the probability-distributions, referred to in Professor Marschak's passage quoted above, are genuine contingency-tables and have in view a large number of future trials, we can take no exception to the idea that one of them is founded on data more nearly adequate in scope or more respectable in provenance; and that this difference in *ex ante* reliability ought to be taken account of, somehow, in the respective statements of judgements based on the two sets of data; for the assignment of lower reliability to the one estimated distribution *is* then subject to test, and to a verdict of 'disproved' or 'not disproved', by subsequent comparison of each estimated distribution with its own corresponding *ex post*, actually recorded distribution. But if the *ex ante* distributions are bogus, in the sense elaborated earlier in this chapter, and merely describe the individual's state of mind in regard to the outcome of a single isolated trial (and a trial may be properly described as 'isolated' if the individual is not *sure* that he will make further trials, in sufficient numbers, in relevantly similar conditions, in the not too distant future), then this distinction between the 'reliabilities' of the distributions belonging to different ventures is, in my view, inadmissible and meaningless. For what the distributions then describe is a state or degree of ignorance; * and it does not make sense

* This use of what purports to be a frequency-table, but is in fact no more than the awarding of marks for 'plausibility', 'verisimilitude', or the possession of

to say that a man feels uncertain how uncertain, or ignorant, he is;* the chain is no stronger than its weakest link, and the effective degree of doubt in his mind, the only one which matters, is that which combines the effects of *all* sources of doubt bearing on the question at issue.

The fourth of Marschak's influences, the attribute of enterprisers or speculators which he calls 'audacity' or 'venturesomeness', is of course exactly what we seek to measure by our 'schedule of gambler-preference'.

To these four influences which Marschak lists in his *Lack of Confidence* we must add a fifth from his *Money and the Theory of Assets*. There he says '...and (witness football pools) they like "long odds", i.e. high positive skewness of yields'. I should translate this by saying that, in Marschak's view, many people are willing to experience the feeling that they are nearly certain to suffer a small loss, if this is the condition of their having also the feeling that it is just possible they may make a very large gain. A venture affording this type of combination of mental experiences would be represented in terms of our concepts by a y-curve in which the compensation offered for the loss-possibilities represented by a given lower branch was an upper branch where even moderate gains carried high potential surprise, but where some very high gains carried no greater potential surprise than moderate gains. Instead of being concave to the x-axis only at very high values of y much nearer to $y=\bar{y}$ than to $y=0$, such an upper branch would begin its concavity to the x-axis at middle values of y, and might thus have two points of tangency with contour-lines, one such point being on that part of the y-curve where it is convex to the x-axis, the other where it is concave. The latter point might give the higher value of ϕ, especially if the temperament of the person concerned was such that his contour-lines $\phi=$ constant ran out to large values of y before turning, rather sharply, to approach $y=\bar{y}$ asymptotically. Such a case is illustrated in Fig. VII 2.

It may not be out of place in this chapter to mention a reference to my theory by Evsey D. Domar and Richard A. Musgrave in their article on 'Proportional Income Taxation and Risk-Taking' (which I did not see until the first five chapters of this book were already written). They say:

'The theory of investment-behaviour, as developed by G. L. S.

a familiar-seeming structure or sequence of types of situation, by a hidden mental process far removed from an explicit, objective, numerical calculation, is in my view extremely liable to be misleading, and has indeed led many writers on our theme into strange complexities of thought.

* It does make sense to say, as we did in Chapter III, p. 74, that he *is in fact* more ignorant than he realises.

Shackle, divides expectations into those which would and would not cause "surprise" and thus avoids having to attach numerical probabilities to all expected yields. It appears to us that the resulting indeterminacy makes it impossible to derive satisfactory tools for the comparison of relative advantages of different investments and therefore for the analysis of taxation effects.'

But my system is no whit more indeterminate than theirs. Both they and I reduce the enterpriser's conception of the gain and loss possibilities of a venture to a *pair of numbers*, precise numerical magnitudes, one of which represents and summarises the favourable

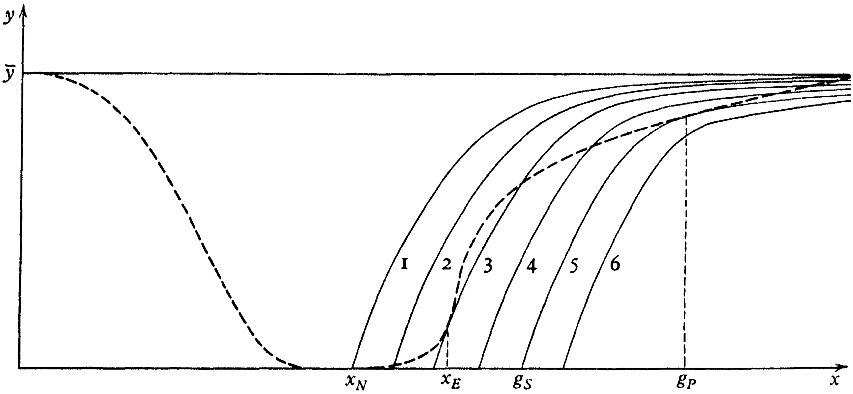

FIG. VII 2. $\bar{y}$, degree of potential surprise representing absolute disbelief; x_N, neutral outcome giving neither enjoyment nor distress by anticipation; g_P, primary focus-gain; g_S, standardised focus-gain; x_E, the inferior of the two maxima of $\phi = \phi\{x, y(x)\}$. The contour-lines $\phi =$ constant are represented by six such lines numbered 1, 2, ..., 6 in order of increasing values of ϕ.

and the other the unfavourable potentialities of the venture, *as judged by the enterpriser.* The difference between the characters of the two resulting expressions is, first, that their figures for 'yield' and 'risk' are mere mathematical abstractions, the answers to a particular kind of sum, while mine are claimed to represent *psychological realities,* the levels of hypothetical outcome which generate the highest intensities of feeling; and, secondly, that their figures and mine are, of course, determined in two entirely different ways. It is, I think, this latter difference which has misled them into claiming for their system a greater determinacy than they concede to mine. And I think their claims are illusory. The essence of the matter is this: in their system, there are numbers (claiming to represent probabilities multiplied by contingent gains or losses) which can be added up to get a weighted average. In my system no such arithmetic is possible; there can be no multiplying of hypotheses by degrees of potential surprise, no adding up of the results, any such procedure is impossible and meaningless. But what are these 'probabilities' which Domar and Musgrave use? Are they objectively obtained frequencies intended to be applied to a series of future trials taken as a whole? No. Are they frequencies expected to arise from the spreading of a given sum over a large number of different lines of investment? No: they are to be used for comparing the merits of one investment-opportunity with those of others. Then they are no more than plausibility-estimates, and have no greater objectivity (but, I think, considerably less realism and appropriateness of form) than my 'degrees of potential surprise'. But if my critics and I both attribute to the enterpriser the power to arrive at precise magnitudes, and both of us suppose him to do it by purely subjective means, what ground have they for claiming superior determinacy?

My last quotation will be from an author who, while adopting the frequency-distribution approach, concedes in the main the chief objections we have brought against it in this chapter. The following passages are from the brief but highly acceptable discussion of this topic given by William Fellner in his article 'Monetary Policies and Hoarding in Periods of Stagnation' (*Journal of Political Economy*, June 1943):

'Statements relating to *the* schedule of the marginal efficiency of capital—and generally speaking to *the* magnitude of expected income—imply that expectations can be reduced to single values (which may be different for different values of some other variable). Expectations, however, are not single valued in reality....

'The main difficulty inherent in the problem of entrepreneur expectations is a consequence of the fact that they are neither single valued nor frequency distributions in the ordinary sense. Consequently, the probability calculus, as defined in statistics, is inapplicable to this problem....

'Individuals actually think and express themselves in concepts of probability when having to make up their minds in conditions to which the notion of probability, in the ordinary sense, cannot be applied. At the same time they are aware of the fact that their forecasts are unreliable to a degree to which true probability judgements are not, and they attempt to allow for this circumstance in one way or another.

'It is important to realise that the lack of reliability of the forecasts under consideration is a consequence of two phenomena which, while being conceptually distinct, are inter-related with each other. These phenomena may best be illustrated with reference to a probability model. Entrepreneurs (or their managers) make, as it were, a forecast concerning the life-time of an individual belonging in an animal species for which there are no satisfactory statistics. The entrepreneur has no experience with the species, although he has experience with related species; aside from this, he has to act on a forecast relating to a single case or, at best, to a small number of cases. In other words, the individual case under consideration does not belong in a homogeneous universe of actual cases of which it would be true that the single cases belonging in the universe are identical, except for properties, the effect of which on the outcome tends to cancel out for aggregates containing increasing numbers of individual cases. Moreover, entrepreneurs are also in no position to disregard the differences between individual cases because a small number of unfavourable outcomes may force them to give up the "game". These two conceptually distinct circumstances are inter-related with each other because, if the single cases belonged in homogeneous universes, insurance would have a much wider applicability, and hence the individual entrepreneur could, in many instances, eliminate the risk arising from the dispersion around mean values.

'The procedure of splitting the forecasts under consideration into a probability judgement and another constituent could then be rationalised as follows: First, the working hypothesis is made that numerous past cases (over which the experience of the forecaster extends) stand in some definite relationship to a hypothetical homogeneous universe consisting of many individuals like the one under consideration; and it is assumed that the forecaster has been able to appraise the effect on observed results of the deviations from homogeneity. Consequently, the entrepreneur has a forecast regarding the frequency distribution which would be established for the species in which the present individual belongs *if the experiment related to many individuals belonging in this species.* In reality the experiment always relates to one individual of a new species: consequently, the behaviour of the entrepreneur depends on two factors in addition to the probability judgement in question. The two additional determinants of his behaviour are (*a*) the degree of confidence he has in his ability to translate his past experience into terms of a homogeneous universe consisting of individuals such as the one now under consideration and (*b*) his attitude towards the risk arising through the dispersion around mean values. Even if he had full confidence in his "probability judgement", he would have to bear the risk of dispersion because his probability judgement is not shared by a sufficient number of other persons to render the risk insurable.'

It will be seen that, if Professor Fellner had felt himself free to reject the frequency-distribution approach, his main grounds for doing so would have been the same two as mine: first, the insufficiency of the facts provided by the record of the past as a means of establishing probabilities regarding such matters as the outcomes of business ventures; secondly (and in my view much the more important and fundamental) the *irrelevance* of estimates of probability (in the sense of relative frequency) to unique or quasi-unique decisions. To this second and finally fatal objection, however, he only concedes by his wording a small fraction of its real force; and in his references to 'rendering the risk insurable' he misses, I think, the point which I have tried to keep conspicuously in view by the choice of such terms as 'gambler-preference', namely, that the enterpriser often *does not wish* to eliminate risk, or rather, uncertainty: on the contrary, it is uncertainty which allows him, and only uncertainty which can allow him, to entertain as possibilities his highest levels of imagined success.

APPENDIX

THE FORMAL PROPERTIES OF THE CONCEPT OF POTENTIAL SURPRISE

The attempt is made in this Appendix to present the notion of potential surprise in the form of a *mathematical system* in the sense in which this term is used by the exponents of mathematical logic; viz. as a collection of propositions of which some are *postulates*, that is, statements whose truth is assumed as a basis for all the rest of the system, and the remainder are *theorems* capable of being deduced from the postulates by rigorous logic. The nature of such a system could not be better set forth, I think, than by Professor W. H. McCrea:*

'Now our goal in geometry is to construct an abstract deductive system, i.e. to exhibit the logical consequences of an explicitly stated set of postulates. The result is a purely mental creation. There can be any number of abstract geometries, and, moreover, equivalent geometries may be derivable from various sets of postulates.

'More precisely, a geometry G is a collection of propositions having the following properties:

'(i) If a finite set I of the propositions are selected as "initial propositions", the remaining propositions can be deduced from them.

'(ii) The propositions I must be logically consistent.

'(iii) The propositions I should be independent, i.e. none can be deduced from the others. (This is a natural, though not essential, requirement.)

'It is desirable that I should be simple in content. They are the postulates mentioned above and there is no question of proving them. Also there is no question of defining the entities or the relations involved in them. We start, in fact, with indefinable elements which are postulated to satisfy certain indefinable relations.

'If I' is any subset of I then I' must certainly satisfy conditions (ii), (iii). All the propositions which can be deduced from I' alone constitute a geometry G', and G' is included in G. (Here a statement that a set of elements *may or may not* satisfy a condition is not regarded

* W. H. McCrea, *Analytical Geometry of Three Dimensions* (Oliver and Boyd, Edinburgh 1942), pp. 139, 140.

as a proposition.) Now the selection of I is arbitrary, subject to conditions (i)–(iii). But it is natural in constructing G to select I in such a manner that it consists of subsets $I_1, I_2, \ldots, I_n$ which render the geometry G_r derived from $I_1+I_2+\ldots+I_r$ $(r=1, \ldots, n)$ an "interesting" geometry. G $(\equiv G_n)$ is then developed through the interesting stages $G_1, G_2, \ldots, G_n$.'

We shall not here try to push the development of such a system very far: the number of theorems offered is quite small. But this method of setting out in formal, abstract, and rigorous fashion some of the ideas which underlie the analysis offered in this book has proved to the author its value in directing attention to *unstated assumptions* and the logical pitfalls arising from them. Appendix D to Chapter II was written, and Chapter III revised, as a consequence of this formal study.*

Although I have sought to make my set of propositions fulfil the requirements of an independent, self-contained, abstract system such as Professor McCrea describes, we are not really unconcerned with the relation of our system to the observable or subjectively experienced world. We wish to make it serve to analyse and interpret that world, and as a consequence the terms we use, though undefined so far as the self-contained system itself is concerned, must be linked up with our notions of reality. The embryonic 'system' which follows is therefore introduced by a passage (falling outside the list of numbered propositions) in which are grouped a number of explanatory statements and definitions. The numbered propositions themselves comprise ten postulates and nine theorems.

By a hypothesis is meant any suggested answer to any question, and by the contradictory of this hypothesis is meant the hypothesis that the suggested answer will turn out to be wrong. By *rival hypotheses* is meant two or more mutually exclusive hypotheses concerning the same question. By an *exhaustive set of rival hypotheses* is meant

* Another possible advantage of such a method is the hope (unrealised in this case) that one may recognise an isomorphism or structural identity between our system, as described by its postulates, and some system already well recognised and highly developed, whose inferences could then be immediately carried over into our own system. An example of such an isomorphism in pure mathematics is the correspondence

$$\begin{bmatrix} x & -y \\ -y & x \end{bmatrix} = x\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + y\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \longleftrightarrow x+yi,$$

where the complex field is represented as a matric algebra over the real field (see C. C. MacDuffee, *Vectors and Matrices*, Carus Mathematical Monographs, no. 7). No such short cut to further results has yet suggested itself to the author.

a set of heads, concerning some one question or experiment, under one or other of which every particular answer or outcome which the individual can think of, which he does not absolutely exclude or reject (in our own language: to which he attaches less than the absolute maximum of potential surprise), can be classified.* Such a set may evidently prove in the event not to have included the true answer or outcome. A set of hypotheses is *equivalent* to another hypothesis when the truth of all the hypotheses in the set implies the truth of the other hypothesis, *or is considered to do so by the individual.* Of the nineteen propositions which follow, numbers 1–10, inclusive, are the *postulates* or *initial propositions*:

1. An individual's degree of belief in a hypothesis can be thought of as consisting in a degree of potential surprise associated with the hypothesis, and in another degree associated with its contradictory.

2. Degrees of potential surprise can be zero or greater than zero. No meaning is assigned to a degree of potential surprise less than zero.

Degrees of potential surprise are bounded above by that degree $\bar{y}$, called the *absolute maximum* of potential surprise, which signifies the absolute rejection of the hypothesis to which it is assigned, absolute disbelief in the truth of the suggested answer to a question or the possibility of the suggested outcome of an 'experiment'.

3. *Equality* between the respective degrees of belief felt by an individual in two hypotheses will then require, for its expression in terms of potential surprise, *two* statements, viz. that some given degree of potential surprise is attached to both hypotheses, and that some given degree is attached to the contradictories of both.

4. The degree of potential surprise associated with any hypothesis will be the least degree amongst all those appropriate to different mutually exclusive sets of hypotheses (each set considered as a whole) to which this hypothesis appears to the individual to be equivalent.

5. The degree of potential surprise that a person associates with the contradictory of a hypothesis cannot be greater than zero unless the degree he associates with the hypothesis is zero; and vice versa.

6. When H is any hypothesis, the degree of potential surprise attached to the contradictory of H is equal to the *smallest* degree attached to any rival of H.

* Any set of rival hypotheses can evidently be made exhaustive by the addition to it of a *residual hypothesis*, defined as covering every particular hypothesis, whether it has been thought of or not, having any character which would exclude it from classification under the hypotheses in the initial set.

7. The degree of potential surprise assigned to the joint (simultaneous) truth of two hypotheses is equal to the *greater* of the respective degrees assigned to the separate hypotheses.

8. Any hypothesis and its contradictory together constitute an exhaustive set of rival hypotheses.

9. All the members of an exhaustive set of rival hypotheses can carry zero potential surprise.

10. At least one member of an exhaustive set of rival hypotheses must carry zero potential surprise. But it is possible for all the rival hypotheses which are in any degree particularised or specified to carry potential surprise greater than zero, only the residual hypothesis carrying zero potential surprise.

The remaining propositions, numbers 11–19, are *theorems*:

11. From (8) and (9) it follows that a hypothesis and its contradictory can each carry zero potential surprise (notwithstanding that the contradictory may itself be capable of being resolved into two or more mutually rival hypotheses).

12. Let H be any hypothesis. Then from (6) it follows that the degree of potential surprise attached to the contradictory of H cannot be greater than zero unless *every hypothesis rival to H* carries some degree of potential surprise greater than zero.

13. From (5) and (12), the contradictories of two rival hypotheses (i.e. two mutually exclusive hypotheses both concerning the same question) cannot both carry degrees of potential surprise greater than zero. For if H_A and H_B are two rival hypotheses, and the contradictory of H_A carries some degree greater than zero of potential surprise, this implies by (12) that H_B carries some degree greater than zero of potential surprise; but (5) implies that the contradictory of H_B cannot then carry any degree of potential surprise greater than zero.

14. From (3) and (13) it follows that if *equal degrees of belief* are felt in two rival hypotheses, the contradictories of both must carry zero potential surprise. The highest degree of *equal belief* which can be reposed in two rival hypotheses consists in assigning to both of them, and to the contradictories of both of them, zero potential surprise.

15. Let $\overset{1}{H}_1, \overset{1}{H}_2, \ldots, \overset{1}{H}_n$ be an exhaustive set of rival hypotheses concerning a first question, and let them carry respective degrees of potential surprise $\overset{1}{y}_1, \overset{1}{y}_2, \ldots, \overset{1}{y}_n$. And let $\overset{2}{y}_t$ be the potential surprise which the individual now thinks that, if $\overset{1}{H}_t$ shall have proved true

he will then assign to a hypothesis $\overset{2}{H}$ suggested in answer to a second question. There are then two cases. First, if $\overset{2}{y}_1 = \overset{2}{y}_2 = \ldots = \overset{2}{y}_n$ we assert that the degree y of potential surprise which the individual will now (in ignorance of the answer to the first question) attach to $\overset{2}{H}$ will be $\overset{2}{y}_i$. For since the set $\overset{1}{H}_1, \overset{1}{H}_2, \ldots, \overset{1}{H}_n$ is by assumption an *exhaustive* set of rival hypotheses, every instance of assigning a degree $\overset{2}{y}$ of potential surprise to any hypothesis $\overset{2}{H}$ must be in fact an instance of assigning some degree, say Y_i, of potential surprise to the *joint* truth of $\overset{1}{H}_i$ and $\overset{2}{H}$. By (7) such a degree will be the *greater* of $\overset{1}{y}_i$ and $\overset{2}{y}_i$. Since the $\overset{2}{y}_i$ are all equal, this degree Y_i cannot, for any pair $\overset{1}{H}_i$ and $\overset{2}{H}$, be less than $\overset{2}{y}_i$. But since the set $\overset{1}{H}_1, \overset{1}{H}_2, \ldots, \overset{1}{H}_n$ is an exhaustive set of rival hypotheses, at least one member of it must, by (10), carry zero potential surprise. Hence one or more Y_i will be equal to $\overset{2}{y}_i$. By (4) the degree y of potential surprise associated with $\overset{2}{H}$ will be the *least* of the Y_i, and therefore y will be equal to $\overset{2}{y}_i$

16. Secondly, if the $\overset{2}{y}_i$ are *not* all equal, it will still be true that, since the $\overset{1}{H}_i$ are an *exhaustive* set of rival hypotheses, every instance of assigning a degree $\overset{2}{y}$ of potential surprise to any hypothesis $\overset{2}{H}$ suggested in answer to the second question must be in fact an instance of assigning some degree Y_i of potential surprise to the *joint* truth of $\overset{1}{H}_i$ and $\overset{2}{H}$. By (7) such a degree will be the *greater* of $\overset{1}{y}_i$ and $\overset{2}{y}_i$. By (4) it follows that the degree y of potential surprise that the individual will now (in ignorance of the answer to the first question) attach to $\overset{2}{H}$ will be the *least* degree which can be found amongst the complete set consisting of the *greater member of each pair* $\overset{1}{y}_i, \overset{2}{y}_i$.

17. Let symbols have the meanings given them in (16). Then the individual will attach a degree *greater than zero* of potential surprise to the hypothesis that, when the answer to the first question shall have become known, he will *reduce* the degree y of potential surprise he attaches to any hypothesis $\overset{2}{H}$ concerning the second question. For to assert the contrary would be to assert that there is some hypothesis $\overset{1}{H}_k$ to which the individual initially assigns potential surprise $\overset{1}{y}_k$ equal to *zero*, and that the degree $\overset{2}{y}_k$ of potential surprise

which this answer to the first question implies for the particular answer $\overset{2}{H}$ to the second question is *less* than the degree y of potential surprise *initially* assigned, in *ignorance* of the answer to the first question, to $\overset{2}{H}$. But this would involve a contradiction. For out of the pair $\overset{1}{y}_k$, $\overset{2}{y}_k$ we have $\overset{1}{y}_k = 0$, and therefore by (2) $\overset{2}{y}_k$ is not less than $\overset{1}{y}_k$. But if $\overset{2}{y}_k$ can stand for the greater of the pair $\overset{1}{y}_k$, $\overset{2}{y}_k$, then it appears in the set amongst which, by (14), the *least* member is y, viz. the degree of potential surprise initially assigned to $\overset{2}{H}$. $\overset{2}{y}_k$ cannot, therefore, be less than y.

18. Let $\overset{1}{c}$ be the potential surprise attached to the *contradictory* of some hypothesis $\overset{1}{H}$ suggested in answer to a first question, let $\overset{2}{c}$ be the potential surprise attached to the *contradictory* of some hypothesis $\overset{2}{H}$ suggested in answer to a second question, and let c be the potential surprise attached to the idea that $\overset{1}{H}$ and $\overset{2}{H}$ are both simultaneously *untrue*. Then by (7) c is equal to the *greater* of $\overset{1}{c}$ and $\overset{2}{c}$.

19. Let symbols have the meanings they had in (18). Suppose that the contradictory of $\overset{1}{H}$ carries some degree $\overset{1}{c}$ greater than zero of potential surprise, and that the individual thinks that if $\overset{1}{H}$ shall have proved true he will then assign some degree $\overset{2}{y}$ greater than zero of potential surprise to $\overset{2}{H}$. Then we assert, concerning the degree y of potential surprise assigned, in ignorance of the answer to the first question, to $\overset{2}{H}$, that $y = \overset{2}{y}$. Similarly, if $\overset{1}{H}$ carries potential surprise $\overset{1}{y}$ greater than zero, but the individual thinks that if $\overset{1}{H}$ shall have proved true he will then assign potential surprise $\overset{2}{c}$ to the contradictory of $\overset{2}{H}$, we assert that $y = \overset{1}{y}$. For by (5), if the contradictory of $\overset{1}{H}$ carries some degree $\overset{1}{c}$ greater than zero of potential surprise, then $\overset{1}{H}$ carries zero potential surprise. By (7), the degree of potential surprise carried by the joint truth of $\overset{1}{H}$ and $\overset{2}{H}$ is equal to the greater of the respective degrees carried by $\overset{1}{H}$ and $\overset{2}{H}$ separately. Since by assumption $\overset{2}{y} > 0$, and since we have shown that $\overset{1}{y} = 0$, we have $\overset{2}{y} > \overset{1}{y}$ and hence $y = \overset{2}{y}$, as asserted. Our second assertion is proved by an exactly parallel argument, *mutatis mutandis*.

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* These terms being so closely associated in meaning, no distinction is made between them in this Index

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