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Author

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2. *If a character representing a smaller number stands to the left of one representing a larger number, it is subtracted.*

That is, IV = 5 − 1 = 4, and XL = 50 − 10 = 40.

3. *If a character representing a smaller number stands to the right of one representing a larger number, and does not at the same time stand to the left of one representing a larger number, it is added.*

Thus, VI = 5 + 1, and CXII = 100 + 10 + 2. But XIX = 19.

These principles are further illustrated by the following.

VIII = 8	XIV = 14	LXIV = 64	DCCLXII = 762
IX = 9	XXIII = 23	CCXLIII = 243	MCMXVI = 1916

On timepieces IIII is sometimes used instead of IV.

The principle of place value (§ 3), which is characteristic of the Arabic notation, is entirely absent from the Roman notation.

Thus, V represents five *units* no matter in what position it stands, though its position indicates whether it is to be added or subtracted.

EXERCISES (A)

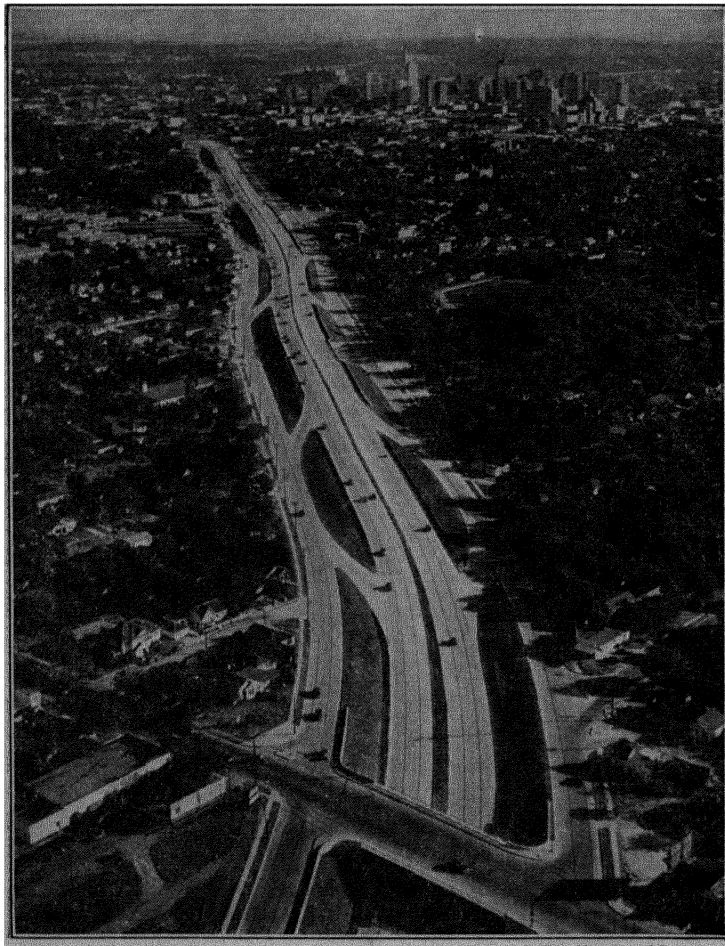
Read the following Roman numerals.

1. CXIV 114
2. CCXIX
3. CCXL
4. CD
5. DC 600
6. DCCCLXVI 866
7. MD 1500
8. MCCXL 1260
9. CXC 210 2165
10. MCMXLV
11. MMXIX 2019
12. MCMXXV 2125
13. $\overline{\text{V}}$IDCCC 6800
14. $\overline{\text{XXV}}$IXV
15. $\overline{\text{XLIII}}$D
16. $\overline{\text{MDCCC}}$ 1800000

Write the following numbers in Roman numerals.

17. 391 CCCXCI
18. 299
19. 504 DIV
20. 2702 MMDCCII
21. 1914 MCMXIV
22. 2000 MM
23. 5426 $\overline{\text{V}}$CCCCXXVI
24. 10,900 XMCM
25. 25,250 $\overline{\text{XXV}}$CL
26. 205,710 $\overline{\text{CC}}$$\overline{\text{V}}$DCCX
27. 316,005 $\overline{\text{CCCXVI}}$V
28. 756 DCCLVI
29. 793,614 $\overline{\text{VII}}$
30. 81,926
31. 23,876
32. 41,760
33. 87,204
34. 59,356
35. 16,734
36. 27,343

8. Origin of the Arabic Numerals. — The Arabians learned the system of numerals which now bears their name from the Hindus and brought it with them to Europe when they invaded Spain about



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ECONOMIC MATHEMATICS

BUSINESS ARITHMETIC FOR THE CONSUMER

152804

BY

N. J. LENNES, PH D.

AND

C. W. SUTTON

1955

ALLYN AND BACON

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FOREWORD

ECONOMIC MATHEMATICS, 1950, is designed to equip high school students to deal accurately, speedily, and confidently with the everyday figures of Business Arithmetic. It reviews and makes certain that control of the four fundamental operations of basic arithmetic is positive and quick. It trains for proficiency in mental calculations and use of short cuts. It aims to improve ability to estimate results and to interpret word problems clearly. It provides a solid foundation for the mathematics of bookkeeping and accounting. It is, in short, a course in the arithmetic of everyday business that covers the needs of both producer and consumer.

ECONOMIC MATHEMATICS makes its work realistic by dealing with daily problems that are up-to-date in data and matter-of-fact in content. Current prices, postal rates, vital statistics, and various other practical applications of arithmetic in daily living reflect the up-to-dateness of this new edition. A glance at the table of contents reveals its comprehensive coverage, and the useful index indicates the careful detail with which the subject matter is treated.

Special features appear in such chapters as Installment Buying, Graphic Representation, Individual Income Tax, Property Insurance, Life Insurance, and United States Customs.

Since the first and chief requirement of a course of this kind is unremitting drill on fundamental operations with integers, fractions, mixed numbers, and decimals, the authors have included not only quick drills in addition and division at the ends of many chapters, but have prepared a mass of new material for drill, testing, and special work, adapted to pupils of different learning levels. In order not to interfere with minimum requirements, this supplementary work has been placed at the end of the book as *Cumulative Reviews* and *Standardized Tests*.

The *Cumulative Reviews* are based on specific sections, but may come any time following study of the corresponding topics. Their purpose is to give practice on those topics which are fundamental, and there are enough of them so that one may be assigned for nearly every day of the school year.

The psychological purpose of these reviews is to make provision for systematic relearnings. The subject matter of the text has been so analyzed, and the exercises in the reviews are so spaced, that each topic recurs with the proper spacing and frequency to conform to the *curve of forgetting and relearning.*

The *Standardized Tests* should be transcribed on the blackboard or mimeographed. Answers written out in the book should be a cause for rejection of all credit. When results disclose deficiencies, remedy should be sought by use of the drill exercises at the ends of various chapters. The score charts should be used first as a competing-with-self device, but they also can be made to yield dividends by way of group and intra-class competition.

The book is exceptionally practical in subject matter and in mode of presentation:

1. It confines itself to the natural and legitimate work of a course in business arithmetic.
2. It includes a store of drill material which results in time-saving for the teacher.
3. It proceeds in natural, logical fashion from topic to topic.
4. Its examples of solutions are taken from the approved daily practice of modern business.

ECONOMIC MATHEMATICS applies the seven cardinal features of a good textbook in mathematics: interest, simplicity, attractiveness, thoroughness, logical sequence, careful gradation, and practical applications.

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ECONOMIC MATHEMATICS

ECONOMIC MATHEMATICS

CHAPTER I

COUNTING. NUMBERS

At a very early age children begin to count small groups of things, and as they proceed counting, a series of words is used. These words are called number-words, or simply *numbers*.

☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
one	two	three	four	five	six	seven	eight	nine	ten

But the groups of things to be counted are often so large that without special arrangement or meaning the series of number-words would be so long and complicated as to be impractical. How can we count and record the number of people in a city, in a state, or even the number of pupils in a school? The way we have found to do this is one of the very great inventions of all time. This invention we shall now study.

1. The Decimal Number System. — The number-words shown above, which we use when we count from one to ten, are represented by the symbols 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. The first nine of these symbols together with the symbol 0 (meaning "nothing to be counted") are called the *Arabic numerals*, or simply Arabic figures.

To count a group of things we begin with 1, 2, and so on up to 10. Then we count ten and one, written 11; ten and two, written 12; and so on up to ten and ten, which we write 20 (two tens). Then we count by saying twenty and one, written 21, up to twenty and

ten, which we write 30 (three tens), and so on. That is, we count successively groups of ten and arrange as just suggested. This system, which we shall describe more fully, is called the *decimal system* because of the use of the number-word ten which comes from the Latin word *decem* meaning *ten*. Because of this special use of ten, this number (10) is called the *base* of the Arabic system.

2. The Arabic Notation. — Using the Arabic numerals and the principle of place value (see § 3), which we are now about to study, we can continue counting as far as we please and recording the results. Thus in counting from ten to twenty we write

$$10, 11, 12, 13, \ldots, 20,$$

which are one ten, no ones; one ten, one one; one ten, two ones; one ten, three ones; . . . ; two tens, no one. In counting the next group of ten we write

$$21, 22, 23, 24, 25, 26, 27, 28, 29, 30.$$

That is, in each of these two-figure numbers the figure to the right represents so many ones, the figure in the next place to the left represents so many tens. Thus 68 means 6 tens and 8 ones.

3. The Principle of Place Value. — The following more general examples will enable us to understand this more fully. In the number 777,777 the numeral 7 to the right represents *seven ones* or *units*. The next numeral represents *seven tens*, the next *seven hundreds*, and so on. That is, any one of the numerals 7 used in writing this number represents ten times as much as the next one to the right of it.

The Arabic notation is exhibited in the following.

billions	hundreds of millions	tens of millions	millions	hundreds of thousands	tens of thousands	thousands	hundreds	tens	ones or units
9,	4	3	1,	5	8	6,	4	7	3

By using the principle of place value, any integral number whatever can be expressed by means of the ten characters:

1, 2, 3, 4, 5, 6, 7, 8, 9, 0.

4. Practical Value of Arabic Notation. — It is difficult to appreciate fully the value to the human race of this apparently simple device for counting and writing numbers. On pages 4 and 5 (§ 7) are examples of very clumsy, slow and cumbersome methods for performing operations which to us are very simple. With the Arithmetic in vogue in Europe 1000 years ago, our present civilization would be impossible. Arithmetical computation would be so laborious that we could not find enough people to do all of it that is necessary for us to do now.

For the sake of convenience in reading large numbers, groups of three, called *periods*, are pointed off by means of commas or by extra spaces. The first period to the right represents ones, the second period represents thousands, the third represents millions, the fourth represents billions, and so on.

5. The Order of Figures. — In a whole number the figure in the first place to the right is said to be of the *first order*, the figure in the second place to be of the *second order*, and so on.

Thus, in the number 975 836, 6 is of the first order, 3 is of the second, 8 of the third, 5 of the fourth, and so on.

6. Reading Numbers. — Numbers are read as simply as is consistent with clearness. In reading whole numbers the word *and* is never used between figures of different order.

Thus we say *three hundred fifty-one*, not *three hundred and fifty-one*. In reading a telephone number, such as 3758, we may say *three, seven, five, eight*. In reading a number like 374, we often say *three, seventy-four*. A number like 24,680 may be read *twenty-four, six eighty*. The number 196,483 may be read *one ninety-six, four eighty-three*. In dictating long tabulations such methods of reading are usually used.

Notice that the comma or the space is not used to separate groups in numbers containing only four figures.

EXERCISES (A)

Dictate the following numbers while someone else writes them down. Then check by having them dictated back.

1. 49,041	**8.** 934,507	**15.** 7,849,643
2. 140,916	**9.** 271,450	**16.** 2,040,928
3. 31,047	**10.** 1,603,912	**17.** 35,649,912
4. 7841	**11.** 25,716,241	**18.** 390,187,243
5. 22,716	**12.** 195,303,216	**19.** 171,275,307
6. 20,225	**13.** 591,684,206	**20.** 319,122,480
7. 785,619	**14.** 37,194,640	**21.** 672,819,193

7. Roman Numerals. — The numerals used by the Romans are still used in a few rare instances. The chapters of a book, the years of laying cornerstones of buildings, the value of paper money, the hours on the faces of many clocks are represented by Roman numerals. The characters used in the Roman notation are the following.

I	V	X	L	C	D	M
1	5	10	50	100	500	1000

A stroke above any one of these, or above any group, multiplies its value by 1000. M, of course, takes the place of $\overline{\text{I}}$. Thus:

$\overline{\text{V}}$	$\overline{\text{X}}$	$\overline{\text{L}}$	$\overline{\text{C}}$	$\overline{\text{D}}$	$\overline{\text{M}}$
5000	10,000	50,000	100,000	500,000	1,000,000

The principles according to which these characters are combined to represent other numbers are illustrated in the following.

$\text{XX} = 2 \cdot 10 = 20$ $\quad \text{IV} = 5 - 1 = 4$ $\quad \text{VI} = 5 + 1 = 6$

$\text{XCII} = 100 - 10 + 2 = 92$ $\quad \text{CXII} = 100 + 10 + 2 = 112$

1. *If a character is repeated, the number represented by the character is multiplied by the number of times it is used.*

That is, $\text{XX} = 2 \cdot 10 = 20$ and $\text{CCC} = 3 \cdot 100 = 300$.

2. *If a character representing a smaller number stands to the left of one representing a larger number, it is subtracted.*

That is, IV = 5 − 1 = 4, and XL = 50 − 10 = 40.

3. *If a character representing a smaller number stands to the right of one representing a larger number, and does not at the same time stand to the left of one representing a larger number, it is added.*

Thus, VI = 5 + 1, and CXII = 100 + 10 + 2. But XIX = 19.

These principles are further illustrated by the following.

VIII = 8	XIV = 14	LXIV = 64	DCCLXII = 762
IX = 9	XXIII = 23	CCXLIII = 243	MCMXVI = 1916

On timepieces IIII is sometimes used instead of IV.

The principle of place value (§ 3), which is characteristic of the Arabic notation, is entirely absent from the Roman notation.

Thus, V represents five *units* no matter in what position it stands, though its position indicates whether it is to be added or subtracted.

EXERCISES (A)

Read the following Roman numerals.

1. CXIV
2. CCXIX
3. CCXL
4. CD
5. DC
6. DCCCLXVI
7. MD
8. MCCXL
9. CXC
10. MCMXLV
11. MMXIX
12. MCMXXV
13. $\overline{\text{VIDCCC}}$
14. $\overline{\text{XXVIXV}}$
15. $\overline{\text{XLIII}}$
16. $\overline{\text{MDCCC}}$

Write the following numbers in Roman numerals.

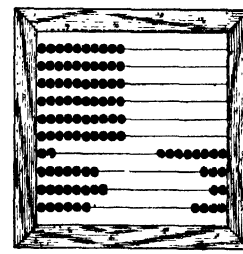
17. 391
18. 299
19. 504
20. 2702
21. 1914
22. 2000
23. 5426
24. 10,900
25. 25,250
26. 205,710
27. 316,005
28. 756
29. 793,614
30. 81,926
31. 23,876
32. 41,760
33. 87,204
34. 59,350
35. 16,734
36. 27,343

8. Origin of the Arabic Numerals. — The Arabians learned the system of numerals which now bears their name from the Hindus and brought it with them to Europe when they invaded Spain about

1100 years ago. After that the system spread very slowly, and it was not until 400 years later that it became known even to the scholars of Europe outside of Spain. But by the time Columbus discovered America, the Arabic System had almost entirely displaced the old Roman System for commercial purposes in southern Europe. In England it gained a firm hold somewhat later.

9. Comparison of the Arabic and Roman Systems. — To appreciate the superiority of the Arabic over the Roman numerals one may write two large numbers in Roman numerals and try to multiply them. The operations of addition and subtraction were not even performed by using numerals, but by means of an instrument called the *abacus*.

10. The Abacus. — The abacus consists of a frame with either horizontal or vertical parallel rods, on each of which are ten beads. Each bead on the lowest or right-hand rod represents one. Each bead on the next rod represents ten, each bead on the third rod represents one hundred, and so on. Numbers are expressed by pushing beads to the right or top of the frame. The arrangement of the beads in the figure represents the number 8324. By means of the abacus numbers can be added and subtracted very rapidly. It is said that by using it an expert Chinese computer can add numbers as rapidly as they can be read to him.



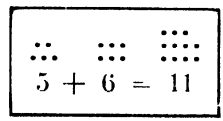
11. Fundamental Operations. — In making practical use of numbers we have to add, subtract, multiply, and divide them. These are called the *fundamental operations of arithmetic*. In the chapters that follow we shall study these operations on whole numbers. In using numbers to solve problems, we always need to use one or more of these operations.

CHAPTER II

ADDITION

12. Use of Addition in Business. — In practical business, addition is the most generally used of the fundamental processes, and considerable skill in adding is essential for one who is to make practical use of numbers. This includes practically every citizen in ordinary civilized life.

13. Definition of Addition. — *Addition is the process of taking two or more numbers together to form one number.*



14. Addends, Sum, Sign of Addition. — The numbers to be added are called *addends*, and the result is called the *sum*.

The sign of addition, +, is called *plus*. Numbers to be added are usually arranged in a column.

Accuracy in addition depends partly upon attention to the proper writing of the figures. They should be made of uniform size, uniformly spaced, and legible. Figures of the same order should stand in a straight column.

Example. Write 317, 52 304, 67, 8 730, 181 462 in a column ready for adding. Note that spaces are used to separate the first and second period.

```
   317
52 304
    67
 8 730
181 462
-------
```

EXERCISES (A)

From dictation write the following sets of numbers in columns ready for adding.

1. 47 607, 304 783, 2 364 189, 402 683, 50 763 457, 37 126 872.

2. 269 478, 9 251 260, 849, 200 458, 752 645, 9 654, 212, 78 769.

3. 75 254, 9 287 484, 20 953 787, 548 175, 25 658 214, 174 195, 25 310, 378 061, 19 756 480, 900 465 628.

4. 122 759, 984 561, 67 458, 200, 045, 17 479, 452 890, 17 284.

ORAL EXERCISES (A)

Read the sums rapidly.

2	7	2	2	5	6	1	4	4	3	6	1	8	2	2	6
1	9	8	9	7	6	4	7	4	6	7	2	8	5	2	8
9	8	5	7	8	9	6	7	1	4	1	2	2	8	3	5
9	2	5	3	3	7	9	5	7	1	3	7	4	9	9	4
3	9	4	9	1	7	7	3	5	4	9	3	8	1	1	9
1	8	6	3	5	8	7	4	6	5	2	3	7	6	8	6
6	6	4	5	4	7	3	1	3	2	3	2	8	8	9	7
1	5	9	1	8	1	8	9	7	3	5	6	1	6	1	6
5	3	5	4	9	5	8	6	7	9	4	8	5	7	6	6
9	2	8	2	5	2	5	2	2	4	3	4	3	4	3	4

ORAL EXERCISES (A)

Give the sums below very rapidly. Note that in $7 + 8$ and $57 + 8$ we have the same combination of figures of the first order.

7	17	27	57	87	4	24	54	7	47	8	38
8	8	8	8	8	9	9	9	6	6	5	5
9	39	79	5	35	75	3	43	73	83	7	37
3	3	3	7	7	7	8	8	8	8	9	9
63	45	37	57	54	38	48	38	78	32	42	52
7	8	9	5	9	7	7	7	7	9	9	9
45	75	35	57	43	53	58	27	47	57	77	87
6	6	6	9	8	8	9	6	8	8	8	8
29	49	59	79	69	39	89	21	34	44	54	64
8	8	8	8	8	8	8	7	7	7	7	7

Beginning with 4 count to 95 by 7's, that is, say 4, 11, 18, . . .

Count from 8 to 98 by 9's, also from 6 to 94 by 8's.

Count from 5 to 97 by 4's, also from 6 to 94 by 11's.

ORAL EXERCISES (A)

Add the columns below as rapidly as possible. Add each column both downward and upward to make sure that the results are correct. A wrong result is worthless.

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
4	5	2	5	9	8	6	5	3	4	3	4
3	4	9	4	8	2	2	6	7	6	4	2
7	1	8	2	3	8	3	3	6	8	2	3
8	2	3	9	9	1	3	5	4	2	6	7
2	7	7	0	7	5	2	2	3	9	8	1
4	8	4	8	4	8	1	3	4	3	2	9
9	9	5	3	3	1	4	2	7	7	9	8

13.	14.	15.	16.	17.	18.	19.	20.	21.	22.	23.	24.
7	8	6	7	4	3	3	8	9	5	1	2
8	2	3	6	2	5	0	6	3	6	3	4
8	3	4	4	8	5	5	2	8	3	2	3
6	6	2	2	6	1	9	0	6	4	5	6
4	7	9	1	5	2	7	5	7	9	7	4
6	8	8	5	7	4	6	2	0	1	6	1
4	7	3	8	5	6	3	8	3	5	5	6
1	5	5	4	3	0	5	9	1	5	7	5

25.	26.	27.	28.	29.	30.	31.	32.	33.	34.	35.	36.
5	4	7	4	7	6	0	2	2	4	3	4
2	2	9	0	1	3	5	3	0	7	8	2
1	1	2	8	9	2	4	7	2	8	9	7
3	0	4	2	6	5	1	5	9	2	1	2
4	7	6	7	4	0	2	9	1	3	0	1
2	0	8	6	2	7	9	0	7	4	8	9
7	8	3	5	8	9	7	7	4	5	7	3
9	5	5	1	0	4	6	6	2	6	3	4
8	9	7	3	5	5	0	0	0	7	4	5
4	2	8	5	9	8	8	8	9	1	5	6

Examples in Addition. The general process of addition is illustrated by the following example.

First add the right column, getting 25 as the sum. This is 5 ones and 2 tens. The 5 ones is written in the ones' place below the line. The 2 tens are added to the tens in the second column. We say we *carry* the 2. A small figure 2 may be written above the 7 to indicate the number carried. This is particularly useful in case the computer is liable to interruptions. Adding the second column, including the 2 carried, the sum is 26. The 6 is written in the tens' place below the line. The 2 is carried to the next column.

322
3974
894
8709
6688
20265

This indicates the nature of the process of addition in general. To make sure that the results are correct, each column should be added both downward and upward.

DRILL EXERCISES (A)

Add:

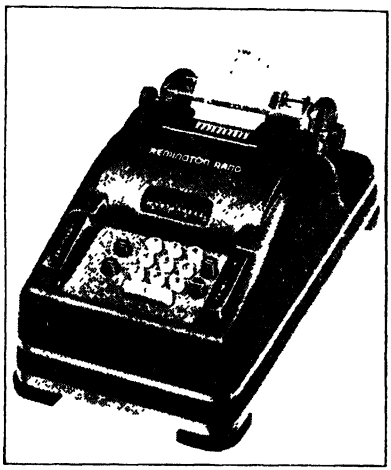
1. 46842	**2.** 86534	**3.** 11111	**4.** 45754	**5.** 65432
9203	34389	28763	29684	53279
24869	29284	91374	92437	45200
3427	26764	22222	50171	43178
59653	86275	51629	67994	25720
6. 7751	**7.** 27839	**8.** 72128	**9.** 3717	**10.** 43204
26812	964510	33333	2608	62132
19627	44444	58967	21479	16204
5274	97586	65555	10265	79141
61269	37642	46798	45824	20560
31558	56948	256921	276540	42715
7392	28475	376458	93761	31200
11. 29237	**12.** 12347	**13.** 999999	**14.** 294084	**15.** 74820
43079	74732	103057	176379	14206
5763	61671	778845	245187	62800
74522	34490	864258	4994	21043
8841	99815	666666	76591	61060
21285	23238	379562	487258	31212
2607	10626	633505	17947	19432

15. Necessity of Checking. — It should never be forgotten that to a business man a result is worthless and probably a great deal worse than worthless unless it is accurate. But it is not sufficient that the result should be accurate; the computer must *know* that it is correct. Hence the necessity of checking every result.

16. Checking by Repeating Work. — The most usual check in addition, and in practice the most useful, is to add each column of figures twice and in the reverse order.

It is practically useless to add the same column twice in the same direction, since the tendency then is to repeat the mistakes, if any. If different results are obtained by adding in opposite directions, add again, omitting the first figure of the column, or grouping differently. This will make it unlikely that the same mistake will be repeated.

2 pkgs. prunes		82
2 pkgs. crackers		53
1 loaf bread		20
10 lbs. potatoes		49
2 lbs. bananas		40
2 doz. eggs	1	10
1 cabbage		21
1 pkg. peas		28
	4	06



AN ELECTRIC ADDING MACHINE

17. Checking Addition by Casting out 9's. — Another method of checking results in addition is by casting out 9's. The process is best understood from an illustration.

437	$4 + 3 + 7 - 9 =$	5
579	$5 + 7 + 9 - 18 =$	3
238	$2 + 3 + 8 - 9 =$	4
464	$4 + 6 + 4 - 9 =$	5
353	$3 + 5 + 3 - 9 =$	2
841	$8 + 4 + 1 - 9 =$	4
2912	$2 + 9 + 1 + 2 - 9 = 5$	$23 - 18 = 5$

To test the addition in this example, add $4 + 3 + 7 = 14$ and subtract 9, leaving 5. Add $5 + 7 + 9 = 21$ and subtract $2 \times 9 = 18$ (because 21 is larger than two 9's), leaving 3. Add $2 + 3 + 8 = 13$ and subtract 9, leaving 4

Add $4 + 6 + 4 = 14$ and subtract 9, leaving 5; and so on for all the numbers. The numbers 5, 3, 4, 5, 2, 4 are called the excesses after all multiples of 9 have been rejected. These are added and from the sum 23, $2 \times 9 = 18$ is subtracted. This leaves a final excess of 5.

Now the numbers 2, 9, 1, 2 of the sum are added and from the sum 14, 9 is subtracted, leaving an excess of 5. Hence the excess in the sum is the same as the excess in the sum of the excesses.

18. Accuracy of Check by Casting out 9's. — The casting out of 9's is not an absolute test, since the result may be wrong in just such ways as to leave the excess unchanged. Thus 2912 and 2921 have the same excess. On the other hand, if the excess of all the numbers and of the sum is not the same, the result cannot be correct.

Use the example at the right for a deliberate study of casting out 9's.

(*a*) How is 3 obtained by casting out 9 from 4836?

(*b*) How is 1 obtained by casting out 9 from 748?

(*c*) How are 5, 4, 8 obtained by casting out 9's in the next three lines?

(*d*) How is 3 obtained?

(*e*) How do we get the check?

4836	→ 3
748	→ 1
38543	→ 5
4918	→ 4
124568	→ 8
173613	→ 3

EXERCISES (A)

Add and test the following by casting out 9's. Write down results only.

1.	2.	3.	4.	5.
74591	73847	2468	2244	7590
43280	91462	3470	6698	2423
56482	87654	5735	7788	6573
27919	90123	7524	5543	1930
38254	35791	6435	2233	8640
39472	74837	8646	4422	1957
19018	26419	2743	8966	3242
4678	45673	5375	8877	3756

19. Horizontal Addition. — In order to save laborious copying it is sometimes necessary to add numbers horizontally. Thus $347 + 591 + 260 + 495 + 398 = 2091$.

Horizontal addition is just like vertical addition, except that it is necessary to take great care that numbers of the same order are added.

ORAL EXERCISES (A)

Add the following horizontally; write the sums but do not copy the figures to be added.

1. $145 + 169 + 847$
2. $856 + 391 + 670$
3. $319 + 1810 + 2780$
4. $540 + 2900 + 8694$
5. $674 + 592 + 547$
6. $492 + 276 + 542$
7. $192 + 1370 + 430$
8. $187 + 2476 + 896$

PROBLEM (B)

The following are the numbers of men, women, and children admitted to a series of baseball games.

Group	Wednesday	Thursday	Friday	Saturday	Sums
Men	2,685	2,437	3,937	10,680	19,739
Women . . .	1,439	1,763	2,184	8,638	14,024
Children . . .	376	485	765	2,306	3,932
Sums . . .	4,500	4,685	6,886	21,624	G. T. 37,695

a. Draw a blank form suitable for the above and on this form copy all of the figures except the sums or totals.

b. Find the total number of men by horizontal addition, also the total number of women and the total number of children. Then add these totals to get the grand total for the series.

c. Find the total attendance each day and add these horizontally to find the total for the series.

In the above example the total is found in two ways. Such problems are said to be self-checking.

PROBLEMS (B)

1. Following is a week's record of the sales of six clerks working in the same department in a large store. Make a blank, copy, and find totals as above.

Clerk	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.	Totals
J. F. Anderson	178 45	217 75	184 30	241 50	310 10	480 60	
R. D. Jenkins	214 20	206 15	152 60	192 60	212 60	394 75	
Ray O. Pattersen	247 35	131 45	207 55	135 45	228 38	192 37	
N. J. Doyle	391 72	124 76	164 82	221 73	286 35	372 90	
Sam V. Tripp	137 15	286 34	173 61	207 04	302 09	408 31	
R. T. Clapp	198 17	193 84	185 45	227 97	273 89	384 56	
Totals							G. T.

2. In a large city high school the number of boys below normal age, of normal age, and above normal age, in a recent year, was as follows.

Year	1st	2d	3d	4th	Totals
Below Normal Age	137	184	33	31	
Normal Age	498	510	316	221	
Above Normal Age	156	161	100	82	
Totals					Grand Total

Make blank, copy, and find the totals

3. The number of girls in the same school and under the same conditions was as follows.

Year	1st	2d	3d	4th	Totals
Below Normal Age	196	184	50	47	
Normal	654	661	390	304	
Above Normal Age	132	111	70	62	
Totals					Grand Total

Make blank, copy, and find totals.

20. Horizontal Addition of Small Numbers. — The method used in adding two or more columns at a time is useful in horizontal addition, since it avoids the danger of adding numbers of different order.

That is, $28 + 45 + 61 + 35 + 78 + 16 = 263$. We say, 28, 68, 73, 133, 134, 164, 169, 239, 247, 257, 263.

Or, 28, 73, 134, 169, 247, 263.

ORAL EXERCISES (B)

In this manner add the following.

1. $73 + 21 + 67 + 82 + 35 + 61$.
2. $14 + 18 + 16 + 23 + 7 + 27$.
3. $15 + 17 + 12 + 16 + 14 + 19$.
4. $24 + 18 + 9 + 17 + 16 + 15$.
5. $25 + 37 + 91 + 8 + 24 + 15$.
6. $14 + 7 + 21 + 42 + 63 + 91$.
7. $22 + 44 + 88 + 17 + 18 + 26$.
8. $24 + 96 + 8 + 127 + 225$.
9. $64 + 125 + 216 + 343 + 512 + 1728$.

Add the following as above. Write down results only.

10. $41 + 64 + 31 + 14 + 71 + 24 + 62$.
11. $19 + 20 + 46 + 39 + 78 + 22 + 61$.
12. $26 + 35 + 19 + 17 + 66 + 42 + 18$.
13. $47 + 39 + 62 + 45 + 20 + 76 + 33$.
14. $18 + 76 + 23 + 48 + 18 + 81 + 46$.
15. $42 + 91 + 27 + 15 + 56 + 73 + 38$.
16. $65 + 87 + 23 + 39 + 22 + 38 + 17$.
17. $91 + 15 + 26 + 14 + 17 + 83 + 67$.
18. $76 + 23 + 10 + 23 + 93 + 77 + 29$.
19. $28 + 55 + 26 + 95 + 49 + 26 + 35$.

Add the following as on page 13. Write down the results only.

A	B	C	D	E	Sums
81476	3491	64509	6459	87160	
12495	7629	9802	18703	4561	
427	36782	54809	6871	98429	
2004	7519	7687	792	4829	
7647	1357	22240	1987	1880	
Sums					Total

ORAL EXERCISES (A)

Add the following. Do not copy the examples. Write down results only.

1.	2.	3.	4.	5.	6.	7.	8.	9.
47	61	51	91	18	38	63	58	35
91	49	38	67	52	55	39	91	83
42	53	19	48	35	92	27	34	74
19	28	42	72	83	74	54	85	56
67	45	75	29	76	19	72	62	49
24	19	96	35	42	23	29	27	62

ORAL PROBLEMS (A)

1. In the number at the right what does the 3 represent? What does the 5 represent? What do each of the numerals 6, 7, 4, 2 represent?

2 4 7, 6 5 3

2. What is meant by the *principle of place value* in Arabic notation?

3. What is meant by *the first order, the second order,* and so on in Arabic notation?

4. Describe the system of dividing large numbers into periods. What is the name of the first period at the right? the second period? the third period? the fourth period?

ORAL PROBLEMS (B)

Not all of the following are correct. Find correct sums for those that are now wrong.

1. $21 + 76 + 382 + 507 + 91 + 1560 + 45 + 87 = 2769$
2. $389 + 704 + 51 + 357 + 42 + 8 + 49 + 987 = 2587$
3. $670 + 54 + 3 + 11 + 8501 + 12 + 18 + 64 = 9333$
4. $8 + 395 + 97 + 714 + 21 + 536 + 207 + 20 = 1998$
5. $394 + 818 + 284 + 696 + 7 + 84 + 5 + 798 = 3186$
6. $73 + 217 + 21 + 375 + 98 + 7 + 49 + 72 = 922$
7. $589 + 552 + 76 + 80 + 87 + 2 + 852 + 887 = 2625$
8. $209 + 767 + 42 + 251 + 91 + 891 + 46 + 54 = 2351$

TOPICS FOR DISCUSSION (A)

2358
373
820
9547
8678
459
378
659
816

1. Which is the easier method for checking work in addition, adding again in the opposite direction or casting out 9's? To try to find a definite answer to this question, add the column at the right, then check your work both ways. Also add some of the examples given earlier in this chapter and check your work both ways. You will be interested to know that casting out 9's is not used as much in checking addition as in checking multiplication.

2. Discuss the importance in practical business of having all answers correct. If a clerk or salesman has only 90% of his answers right, what will happen to him?

3. Discuss the amount of trouble of checking answers that are already correct and also the amount of trouble in checking and getting the correct and checked answer if the first answer is wrong.

4. Discuss the importance of working accurately so that your first answer will be correct.

5. Study the way the number words are repeated and combined in reading numbers.

(*a*) The eleven numerals 0, 1, 2, . . . 9, 10 have what different names?

(*b*) 11, 12, 13, . . . 19 may be regarded as "10 and 1," "10 and 2," "10 and 3," . . . "10 and 9."

(*c*) 20, 21, 22, . . . are clearly "2 tens," "2 tens and 1," "2 tens and 2," . . . .

(*d*) All the number words up to a *hundred* are combinations of the eleven words up to *ten*. Then we have no new ones up to a *thousand*, then no new ones up to a *million*, then no new ones up to a *billion*, and then no new ones up to a *trillion*.

So you see that all numbers up to 999,999,999,999 are read by using combinations of how many number names?

6. The Romans actually counted to 10 in this way: One, two, three, five less one, five, five and one, five and two, five and three,

ten less one, ten. Compare this counting with the Roman numerals to 10.

7. Think an X before each of the following Roman numerals.

I II III IV V VI VII VIII IX

8. Read the following Roman numerals.

X XX XXX XL L LX LXX LXXX XC

Think a C before each of these and read the resulting Roman numerals.

9. Add:

347
86
224
740
194
402

10. Write the numbers in the columns to the left in Roman numerals and then add these numbers. Write the numbers in Arabic numerals.

11. Multiply LXVII, XXVII, XXXIV by two (2). Write each product in Roman numerals.

CHAPTER III

SUBTRACTION

21. Definition of Subtraction. — *Subtraction is the process of taking one number away from another.*

Thus: How much is left when 30 is taken away from 70? | $70 - 30 = 40$ |

22. Minuend, Subtrahend, Remainder. — The number from which a number is taken away in subtraction is called the *minuend*, the number that is taken away is the *subtrahend*, the number that is left is the *remainder*.

Thus in $70 - 30 = 40$, 70 is the minuend, 30 is the subtrahend, and 40 is the remainder.

A short dash ($-$) indicates subtraction and is read *minus* or *less*. The number before this sign is the minuend, and the number after it is the subtrahend. The expression "$70 - 30 = 40$" is read "70 minus 30 equals 40."

23. Other Definitions of Subtraction. — We also say that "Subtraction is the process of finding the difference between two numbers," or that "it consists in finding how much must be added to a given number to make the sum another given number."

All these definitions of subtraction amount to the same thing. Thus we may ask, "What is the difference between 30 and 70?" or "How much must be added to 30 to make 70?" or "How much is left when 30 is taken from 70?"

In this book subtraction will be regarded as the process of finding how much is left when a given number is taken away from another given number.

24. Uses of Subtraction. — Subtraction is used constantly in business. "Making change" is one of its simplest uses. Finding balances of accounts, gains or losses, and so on, all involve subtraction.

ORAL EXERCISES (A)

Subtract the following.

Read the first example $14 - 8 = 6$ or 14, 8, **6**. After a little practice the remainder should be seen at a glance.

14	16	15	15	12	13	14	17	17
8	7	8	7	7	6	5	9	8
13	12	14	11	16	15	11	13	15
7	5	6	7	8	9	4	5	6
11	13	16	11	13	11	11	11	14
7	8	9	3	4	5	6	8	9
11	13	11	12	12	12	12	18	12
9	9	2	3	4	9	8	9	6

ORAL EXERCISES (A)

Read rapidly, giving the difference between the numbers in each of the following pairs. Read the first example $11 - 3 = 8$ or simply 11, 3, **8**. After a little practice the differences should be seen at a glance and only the differences given.

3, 11; 8, 14; 16, 7; 8, 15; 7, 15; 12, 7; 13, 6; 5, 14; 9, 17; 17, 8; 7, 13; 5, 12; 14, 6; 14, 7; 8, 16; 15, 9; 4, 11; 13, 5; 15, 6; 7, 11; 8, 13; 16, 9; 3, 11; 13, 4; 5, 11; 6, 11; 11, 8.

Subtract:

8	18	38	58	17	37	57	77	87
3	3	3	3	9	9	9	9	9
15	35	25	75	55	13	43	23	73
7	7	7	7	7	7	7	7	7
14	44	24	74	64	54	94	13	53
9	9	9	9	9	9	9	8	8
73	83	43	63	16	66	46	76	96
8	8	8	8	9	9	9	9	9

25. Subtrahend Written above the Minuend. — Sometimes the smaller number (the subtrahend) is written above the larger (the minuend).

It is best to get some practice in finding the difference between numbers written this way, since in many cases that will save the work of copying.

ORAL EXERCISES (A)

In the following give the difference between the numbers above the line. Read the first example: 17 − 9 = **8** or simply 17, 9, **8**, or better still, read the difference at a mere glance.

9	9	9	9	9	8	8	8	8
17	27	37	57	67	17	47	87	67
9	9	9	9	9	7	7	7	7
16	36	66	76	96	13	33	53	73
6	6	6	6	6	5	5	5	7
14	24	44	64	84	13	33	53	54
8	7	5	6	8	4	9	9	8
24	26	24	32	31	32	13	42	53

ORAL EXERCISES (A)

In the following sequences of numbers read the difference between each number and the one which follows it. Thus in 4, 9, 15, 23, 29, 37, 45, 53, 68, 81, read 5, 6, 8, 6, 8, 8, 8, 15, 13.

1. 6, 8, 12, 19, 21, 29, 40, 51, 59, 67, 73, 82, 95, 100.

2. 14, 19, 31, 39, 47, 53, 64, 71, 83, 84, 96, 100.

3. 7, 19, 27, 41, 53, 62, 73, 85, 100.

4. 9, 14, 24, 32, 43, 53, 62, 70, 79, 89, 100.

5. 15, 27, 34, 45, 54, 63, 75, 82, 93, 100.

6. 8, 19, 31, 43, 52, 65, 74, 81, 93, 100.

7. 14, 21, 34, 47, 55, 64, 72, 84, 92, 100.

8. 13, 25, 37, 48, 56, 67, 73, 82, 94, 100.

9. 19, 24, 31, 39, 52, 63, 78, 84, 93, 99.

26. Subtraction without Carrying. — Example. From 8647 subtract 3235.

Write the numbers so that figures of the same order are in the same column. The subtrahend is usually written below the minuend.

We say, $7 - 5 = 2$, $4 - 3 = 1$, $6 - 2 = 4$, $8 - 3 = 5$.

With sufficient practice one sees the differences by a mere glance at the figures.

8647
3235
5412

The "without carrying" in the title of this paragraph is significant (compare § 28). In the examples given on this page every figure in the minuend is equal to or greater than the corresponding figure in the subtrahend. This avoids problems containing the greatest difficulty in subtraction.

WRITTEN EXERCISE (A)

Copy and write the difference in the following.

1. 14572	**3.** 55678	**5.** 5497	**7.** 8395
3260	34567	2377	3284
2. 9635	**4.** 7980	**6.** 78899	**8.** 3284
8422	2460	64376	2172

From such examples as $70 - 30 = 40$, $80 - 40 = 40$, $90 - 50 = 40$, we see that when the subtrahend and minuend are both enlarged or both diminished by the same amount, the remainder is unchanged.

Thus $15 - 7 = 8$, and $25 - 17 = 8$. Likewise $17 - 8 = 9$, $37 - 28 = 9$.

These examples illustrate an important principle which we state.

27. *Increasing or diminishing the subtrahend and minuend by the same amount does not change the difference.*

ORAL EXERCISES (A)

Read the differences in the following rapidly.

15	25	35	55	57	67	87	85	37
7	17	27	47	65	75	95	77	45
13	33	48	78	88	18	43	58	73
8	28	53	83	93	23	38	63	68

28. Subtraction with Carrying.

Example 1. From 3924 subtract 2718.

$$\begin{array}{r} {\scriptstyle 1} \\ 3924 \\ 2718 \\ \hline 1206 \end{array}$$

Since we cannot take 8 from 4, we think of 10 as being added to both minuend and subtrahend. We then write 1 just before and a little above the 4 in the minuend. This changes 4 to 14, thus adding 10 to the minuend. To make up for this we think 2 instead of 1 in the subtrahend. That is, we add one ten to the subtrahend. We say we *carry* 1. According to § 27 this does not change the remainder. Then we say: $14 - 8 = 6$, $2 - 2 = 0$, $9 - 7 = 2$, $3 - 2 = 1$.

Example 2. From 2946 subtract 1487.

$$\begin{array}{r} 2946 \\ 1487 \\ \hline 1459 \end{array}$$

We say $16 - 7 = 9$, carry 1; $14 - 9 = 5$, carry 1; $9 - 5 = 4$, $2 - 1 = 1$.

In each of these we apply the principle of § 27. If the minuend is made 10 larger, we add 10 to the subtrahend by increasing the figure in tens' place by 1. If the minuend is made 100 larger, we increase the subtrahend by 100 by adding 1 to the figure in hundreds' place, and so on. We describe this process by means of the word "carrying" to bring out the similarity to the analogous process in addition.

However, in practice we simply add one to the next number in the subtrahend without using the word "carry" at all.

Example 3. From 34,001 subtract 17,268.

$$\begin{array}{r} 34001 \\ 17268 \\ \hline 16733 \end{array}$$

We say $11 - 8 = 3$, $10 - 7 = 3$, $10 - 3 = 7$, $14 - 8 = 6$, $3 - 2 = 1$.

WRITTEN EXERCISES (A)

Copy the following and subtract.

1. 47200 − 26898	**2.** 81410 − 27384	**3.** 18142 − 13786	**4.** 38120 − 26987
5. 12450 − 7893	**6.** 453207 − 176308	**7.** 18300 − 14764	**8.** 15600 − 12840
9. 723004 − 654192	**10.** 46930 − 27380	**11.** 2560 − 1780	**12.** 15427 − 3942

29. Checking Subtraction. — The fundamental idea in subtraction is that the sum of the subtrahend and the difference equals the minuend.

Thus, $17 - 9 = 8$ and $17 - 8 = 9$ *because* $8 + 9 = 17$. This affords a convenient check for subtractions.

Example. From 492,472 subtract 368,987.

We say, $12 - 7 = 5$, $17 - 9 = 8$, etc. To test it we say $5 + 7 = 12$, 1 to carry, $9 + 8 = 17$, etc. It will be noticed that the test amounts simply to adding in the opposite direction. That is, instead of $12 = 7 + 5$, we say $5 + 7 = 12$, etc. This is practically the same check as the one used in addition.

492472
368987
123485

We should never lose sight of the importance of checks. Results are worse than useless unless we *know* they are correct.

WRITTEN EXERCISES (A)

Subtract and check each of the following.

1.	274910 132623	**9.**	579246 264597	**17.**	100952 92347	**25.**	5004692 3942783
2.	3649872 1892965	**10.**	207691 38587	**18.**	154975 29386	**26.**	209074 192458
3.	10011010 9620937	**11.**	56340 29482	**19.**	14235 7941	**27.**	392456 196587
4.	205467 119785	**12.**	750491 684378	**20.**	409381 308593	**28.**	20574 19358
5.	19428 17659	**13.**	17405 13676	**21.**	74241 56475	**29.**	317625 178296
6.	200504 19768	**14.**	293072 176498	**22.**	192342 176451	**30.**	500046 390578
7.	192458 76974	**15.**	30507 15629	**23.**	30492 15786	**31.**	30875791 29484976
8.	84628 30490	**16.**	19408 2693	**24.**	80201 23764	**32.**	53021 24264

PRACTICAL APPLICATIONS OF SUBTRACTION

30. Finding Net Weight. — In weighing articles contained in barrels, bags, or any container whose weight is not to be included, the total weight is called the *gross weight* and the amount to be deducted is called *tare*. The difference between these is the *net weight*.

Thus a load of coal with the wagon weighed 9840 pounds, while the wagon weighed 2470 pounds. In this case the gross weight was 9840 pounds, and the tare 2470 pounds. The net weight in pounds was 9840 − 2470 = 7370.

Following is the record of twelve loads of coal hauled from a certain yard.

(G_1)	(T_1)	(N_1)	(G_2)	(T_2)	(N_2)
Gross Weight	Tares	Net Weight	Gross Weight	Tares	Net Weight
9840	2470	7370	9520	2470	
9280	2530		8760	2440	
8760	2340		8950	2420	
9370	2440		8890	2430	
9180	2460		9020	2430	
9760	2530		9170	2510	

In making out bills gross and tare are written horizontally with a sign, −, between. Thus the above would be written 9840 − 2470, 9280 − 2530, etc.

SIGHT WORK (A)

1. In the above $S(G_1) + S(G_2)$ = the sum of the gross weights.

2. In the above $S(T_1) + S(T_2)$ = the sum of the tares.

3. Discuss $S(G_1) + S(G_2) - S(T_1) - S(T_2) = S(N_1) + S(N_2)$.

4. Is it possible that in some cases $(G_1) + (G_2)$ should be greater than $(T_1) + (T_2)$?

5. Is it possible that the items in G_1 and G_2 should differ greatly? Is it possible that these should be equal?

6. Ask these questions about the items in (T_1) and (T_2).

31. Finding Bank Balances. — In keeping record of the daily balances of customers, banks sometimes use the following form:

R. H. JESSE, JR.

		(A)		(B)		(C)	
DATE		CHECKS		DEPOSITS		BALANCE	
Nov.	10					1280	67
		34	10	171	90		
		6	20				
		8	40			—	
Nov.	11	108	00				
		36	50			—	
Nov.	12	9	15	276	40	—	
Nov.	14	200		124	75		
		76					
		31	40			—	
Nov.	17	4	76	375	20		
		38	90			—	
Totals							

SIGHT WORK (A)

1. Discuss items in column (A).

2. Discuss items in columns (B) and (C).

3. What is the effect upon your balance in the bank if you deposit a certain amount in your account? What is the effect if you draw a check on your bank?

WRITTEN EXERCISES

Design suitable blanks and copy the data in §§ 30, 31. Find the net weights in § 30. In § 31 fill in the daily balances where the marks, —, are. To find each balance add the deposits of the day to the balance of the preceding day and subtract the amounts of the checks. To check, find the difference between the amount of the total deposits and amount of the total checks. This should equal the difference between the first and the last balance.

32. Finding Net Proceeds of Collected Bills. — Notes and bills due in distant cities are frequently sent to banks for collection. The net

proceeds remitted to the owner of the bill or note may be less than the face of the paper for several reasons.

(*a*) The bill or note may be paid before it is due and hence a discount may be deducted.

(*b*) The bank may make a charge by way of commission for collecting.

(*c*) There may be a deduction for exchange.

WRITTEN EXERCISES (A)

Make a blank like the one below, copy the data given, and fill in the "net proceeds" column. To check, add "face of paper" column and deduct from the sum the sums of the "discount" and the "commission and exchange" columns. The result should be equal to the sum of the "net proceeds" column.

Name	Face of Paper	Discount		Commission and Exchange		Net Proceeds	
A	1200	3	80	2	90		
B	840	4	15	1	60		
C	195			1	80		
D	350	1	75	1	50		
E	9650	24	60	8	50		
F	280	2	10	1	00		
G	340		85	1	60		
H	670			2	50		
I	2000	15	45	2	00		
Sums							

GENERAL DRILL IN SUBTRACTION (A)

Subtract and check each of the following.

1. 439078 273949	**2.** 547521 278619	**3.** 378409 293576	**4.** 340765 152696
5. 634769 285943	**6.** 109205 63729	**7.** 252783 171948	**8.** 780456 492387
9. 380528 192717	**10.** 862091 753485	**11.** 863482 792675	**12.** 379208 197524

13. On a certain day the collection department of a bank showed the following items: In each case deduct "Com. & Exch."

A's paper, face	$3600;	discount	$41.60;	Com. & Exch.	$1.50
B's paper, face	800;	discount	none;	Com. & Exch.	2.50
C's paper, face	120;	discount	1.20;	Com. & Exch.	none
D's paper, face	8600;	discount	54.20;	Com. & Exch.	15.00
E's paper, face	420;	discount	6.30;	Com. & Exch.	1.20
F's paper, face	1920;	discount	4.60;	Com. & Exch.	2.50
G's paper, face	3250;	discount	27.40;	Com. & Exch.	5.60
H's paper, face	930;	discount	5.85;	Com. & Exch.	2.75

Enter and fill in the Net Proceeds column as on page 27. Check.

In each of the following find how much must be added to the sum of the numbers below the first number to make the sum equal to the first number.

14.	**15.**	**16.**	**17.**	**18.**
74286	53786	80432	90285	952400
12727	15072	12908	39078	86140
8943	9476	30492	14903	182764
15643	20543	7645	6506	214375
948	4028	1987	7908	89680

In each of the following, which column has the greater sum and how much?

19.		**20.**		**21.**	
4390	348	1294	767	8028	942
846	5927	803	1422	352	3832
246	9060	5378	6349	4946	81
8400	745	901	7285	72	7596
578	2193	8722	298	9621	845
2600	3912	1056	2653	259	9302

22.		23.		24.	
2081	923	3592	5225	11296	9529
5978	8765	7648	369	8537	4772
304	843	291	4214	4285	8205
6875	5926	83	9527	792	928
98	75	5096	3857	3081	2587
6232	4041	3948	918	134	1795

Copy the following from dictation. Find the differences by horizontal subtraction.

		Difference			Difference
25.	4926	3170	**26.**	3918	1296
	2371	1297		4568	3792
	8710	6245		9308	4872
	10470	2480		4371	198
	8426	927		8610	2478
	491	274		517	194
	628	342		895	246
	880	370		1740	1487
	1970	445		3726	2437
	2408	1928		327	164
	7200	3849		6204	3190
	4370	1246		4790	2846
	5400	2890		2192	1467
	8591	3460		9170	2910

CHAPTER IV

MULTIPLICATION

33. Definition of Multiplication. — *Multiplication is the process of taking one number, called the multiplicand, as many times as the number one is contained in another number, called the multiplier.*

The result of this process is called the *product.*

Example. Multiply 3897 by 4.

3897
3897
3897
3897
15588

At the right we have shown a direct application of this definition. If we are to use this process to any extent we shall find it entirely impractical. How long would it take to find the product of 3897 and 378 by using this method?

34. Uses of Multiplication. — Even in the simpler business multiplication in very frequent use we must master thoroughly a very simple process by which products can be found. We shall therefore proceed to develop the necessary steps.

35. Multiplication Table. — All practical work in multiplying is based on the following table, which should be memorized perfectly.

0	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

36. Product, Factors. — The result obtained by multiplication is called the *product*. The multiplier and multiplicand are called the *factors* of the product. Thus 6×8 is read "6 times 8" and indicates that the product of 6 and 8 is to be found.

The sign $\times$ placed between two numbers indicates that one of the numbers is to be multiplied by the other

37. Multiplying by 10, 100, etc. — It follows directly from the principles of the decimal notation that annexing a zero at the right of a number multiplies it by 10, since this moves each figure into the place of the next higher order. Similarly, annexing two zeros at the right multiplies the number by 100, annexing three zeros multiplies it by 1000, and so on.

38. Multiplying by a Product. — The following example illustrates an important principle in multiplication.

To multiply 8 by 15 we may multiply 8 by 3 and the product by 5, obtaining $3 \times 8 = 24$ and $5 \times 24 = 120$. Or we may multiply 8 by 5 and the product by 3, obtaining $5 \times 8 = 40$ and $3 \times 40 = 120$. Hence we have the

Rule: *To multiply by the product of two numbers we may multiply by one of the factors and the product so found by the other factor.*

This principle is used when multiplying by such numbers as 60, 700, 8000, 340, and so on.

Thus to multiply by 60 we first multiply by 6 and then multiply the product by 10 by annexing a zero to the right. To multiply by 700 we first multiply by 7 and then multiply the product by 100 by annexing 2 zeros to the right. Similarly to multiply by 8000 we first multiply by 8 and then multiply by 1000 by annexing three zeros to the right.

39. Multiplying a Sum. — The following example illustrates another principle in multiplication.

To multiply the sum of two numbers as 6 and 8 by 4 we may multiply 6 and 8 by 4 separately, and then add the products.

Hence we have the

Rule: *To multiply the sum of two numbers we may multiply each number separately and then add the products.*

This principle is used when multiplying a number having two or more figures.

40. General Process of Multiplication. — The general process of multiplication is a direct result of the two principles in §§ 38, 39.

Example. Multiply 3894 by 378.

	3894	
	378	
First partial product	31152	$= 8 \times 3894$
Second partial product	27258	$= 70 \times 3894$
Third partial product	11682	$= 300 \times 3894$
Total product	1471932	$= 378 \times 3894$

In the above example how do we multiply by 70? How do we multiply by 300? What zeros are omitted in writing down the work?

WRITTEN EXERCISES (A)

Multiply each of the following. Check by repeating work. Remember that a result is useless until you *know* it is correct.

1.	4374 291	**9.**	8341 628	**17.**	5749 193	**25.**	176294 792
2.	7542 215	**10.**	45973 822	**18.**	87592 193	**26.**	459264 518
3.	345276 324	**11.**	29512 328	**19.**	279358 793	**27.**	12793458 3847
4.	158318 1728	**12.**	192467 2348	**20.**	59371 246	**28.**	37421 639
5.	298542 8729	**13.**	998245 3276	**21.**	29174 423	**29.**	9476 648
6.	787914 2375	**14.**	49865 567	**22.**	738 284	**30.**	39107 328
7.	47241 289	**15.**	2953 173	**23.**	72482 179	**31.**	157928 754
8.	3568 492	**16.**	12945 316	**24.**	241921 249	**32.**	283726 798

41. Multiplication of Numbers Containing Zeros.

Example 1. Multiply 374 by 208.

374
208
2992
748
77792

To multiply by 208 we multiply by 8 and by 200. Explain how the multiplication by 200 is effected.

Example 2. Multiply 4700 by 830.

4700
830
141
376
3901000

To multiply 4700 by 830 multiply 47 by 83 and then multiply the product by 1000 by annexing three zeros. We notice that multiplying either factor by a number multiplies the product by that number. That is, annexing one zero to one of the factors requires us to annex a zero to the product. Hence, omit all zeros to the right and multiply the remaining numbers. Annex to the result as many zeros as were omitted in both factors.

Remark. In finding the product of two numbers use that one as a multiplier which has the smaller number of figures other than zero.

Thus, in 4007×241 use 4007 as the multiplier

WRITTEN EXERCISES (A)

Multiply.

1. 4271 6109	**7.** 3761 207	**13.** 6340 504	**19.** 3700 308
2. 4900 600	**8.** 5400 700	**14.** 9700 3500	**20.** 1040 70085
3. 7040 350	**9.** 90909 2300	**15.** 2500 120	**21.** 76504 1090
4. 2070 460	**10.** 90054 2050	**16.** 7904 308	**22.** 3050 170
5. 94080 3056	**11.** 120408 90050	**17.** 52054 1008	**23.** 275040 6020
6. 9040 270	**12.** 870450 2008	**18.** 91430 290	**24.** 76040 1908

42. Checks by Interchanging Factors or by Dividing the Product. — Multiplication may be checked by interchanging multiplier and multiplicand.

Thus:

496	372
372	496
992	2232
3472	3348
1488	1488
184512	184512

The most certain check, however, is to divide the product by one of the factors. The quotient must be the other factor.

43. Casting Out 9's.

Example. Multiply 493 by 78.

To check by casting out 9's (see page 12), first find the excesses left after casting out 9's in 493 and 78, which are 7 and 6. Then find the excess after casting out 9's in the product of 7 and 6, which is 6. The excess after casting out 9's in the product 38,454 is also 6. Hence the multiplication checks. As in addition, the casting out of 9's is not an absolute test.

$493 \rightarrow 7$
$78 \rightarrow 6$
$3944 \quad 42 \rightarrow 6$
3451
$38454 \rightarrow 6$

Rule: *The excess of 9's in the product is equal to the excess of 9's in the product of the excesses.*

WRITTEN EXERCISES

Multiply each of the following and check both ways.

1. 1397 × 685	**5.** 7229 × 304	**9.** 2356 × 234	**13.** 9231 × 164
2. 9528 × 129	**6.** 9234 × 273	**10.** 1437 × 152	**14.** 4983 × 693
3. 2345 × 216	**7.** 5894 × 792	**11.** 2315 × 203	**15.** 9617 × 139
4. 3046 × 107	**8.** 9213 × 175	**12.** 6235 × 691	**16.** 3057 × 275

44. Denomination of the Product. — The multiplier must always be an abstract number. That is, we do not multiply by such numbers as 6 days or 45 cents, but simply by 6 or 45. If the multiplicand is a denominate number, the product is of the same denomination as the multiplicand.

That is, 8×6 days $= 48$ days, and 9×45 cents $= 405$ cents. In practice, however, it is usually best to regard all numbers as abstract for the purpose of performing arithmetic operations on them. We can easily decide from the nature of the problem what kind of thing is represented by the result.

Thus we know that if the length and breadth of a rectangle are given in feet, the product of the numbers representing these dimensions gives the area in *square feet*. Again, if a man earns \$4 per day, we know that if we multiply the number of days he works by 4, the product is the total number of *dollars* he earns, and so on.

In practice the smaller factor is usually used as the multiplier. But see Remark in § 41.

Thus, to find how many dollars a man earns in 147 days at \$9 a day, multiply 147 by 9, and not 9 by 147.

ORAL EXERCISES

State what number should be used as the multiplier in solving each of the following problems.

Also state what kind of thing is represented by each product.

1. A train runs 6 hours at the rate of 43 miles per hour. How far does it go?

2. A freight steamer sails at an average rate of 11 miles per hour for 287 hours. How far does it go?

3. What is the total wealth of a country containing 49,500,000 inhabitants if the average *per capita* wealth is \$1600?

4. At 38 cents per square foot what is the cost of paving a street containing 38,750 square feet?

5. At \$16.75 a ton what is the value of a hay crop of 280 tons?

6. If one square mile contains 640 acres, how many acres are there in a state containing 56,480 square miles?

45. One-figure Multiplier. — In filling out many little business forms, such as sales slips, it is convenient to be able to find the products of numbers without doing any written work. A special method is useful when one of the numbers to be multiplied is a one-figure number. In such cases use the one-figure number as the multiplier. Then multiply from the left.

1. In the example 8×59, think $8 \times 50 = 400$; remember the 400. Think $8 \times 9 = 72$, $400 + 72 = 472$. Then write the whole product. $8 \times 59 = 472$

2. To multiply 6×927. Think $6 \times 900 = 5400$ (say mentally, "54 hundred"); remember 5400. Think $6 \times 20 = 120$. Think $5400 + 120 = 5520$. Think $6 \times 7 = 42$, $5520 + 42 = 5562$. $6 \times 927 = 5562$

Notice that it is easier to remember "54 hundred 20" than "5 thousand 4 hundred 20."

This method of multiplying is especially convenient in deciding on the correct quotient figures in long division.

ORAL EXERCISES (A)

Using the method of § 45, find the products in the following. Make a definite effort to get all answers correct. Write down the products in order. After you have found all the products, check your work to see whether they are correct.

1. 3×87	**13.** 3×87	**25.** 5×867	**37.** 2×743
2. 8×96	**14.** 5×38	**26.** 2×182	**38.** 4×847
3. 7×38	**15.** 2×78	**27.** 6×639	**39.** 9×728
4. 4×65	**16.** 8×46	**28.** 7×659	**40.** 7×163
5. 9×128	**17.** 4×94	**29.** 9×784	**41.** 4×420
6. 6×307	**18.** 9×87	**30.** 8×708	**42.** 9×981
7. 3×398	**19.** 8×67	**31.** 4×392	**43.** 8×542
8. 5×436	**20.** 7×82	**32.** 9×456	**44.** 9×378
9. 8×670	**21.** 6×78	**33.** 7×853	**45.** 5×907
10. 7×87	**22.** 3×74	**34.** 5×928	**46.** 2×587
11. 9×67	**23.** 4×67	**35.** 3×637	**47.** 3×86
12. 6×46	**24.** 3×87	**36.** 4×873	**48.** 6×89

PROBLEMS (A)

Find the answers to the following in the shortest way.

1. How far will a train go in 9 hours if it goes an average of 47 miles per hour?

2. At $1.46 a bushel, what is the value of 1465 bushels of corn?

3. At $175 a head, what is the cost of 14 horses?

4. At 65 cents a square foot, what is the value of a lot containing 8670 square feet?

5. If 47 men work 268 days, how many days' work of one man is this equal to?

6. How many days' work is done by 24 men working 146 days each?

7. At an average yield of 74 bushels of corn to the acre, how many bushels are there in a field containing 134 acres?

8. At an average yield of 187 bushels of potatoes to the acre, how many bushels are there in a field containing 14 acres?

9. If it requires 352 rails to lay one mile of single track railway, how many rails are required to lay 1785 miles of track?

10. If there are an average of 2640 ties to each mile of track, how many ties will be required for 79 miles of track?

11. At $125 an acre what is the value of a farm containing 286 acres?

12. At $13 per head what is the value of a flock of sheep containing 876 head?

13. At 65 cents a pound what is the value of 48,760 pounds of coffee?

14. What is a man's yearly salary* if he gets $275 a month?

15. At $46 a week how much does a girl earn in a year of 52 weeks?*

16. In a book of 468 pages each page contains an average of 34 lines. How many lines are there in the book?

17. A book of 250 pages contains an average of 395 words on a page. How many words are there in the book?

* (without deductions)

SHORT CUTS

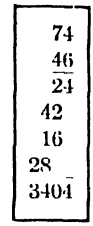
46. Means for Finding Short Cuts. — For multiplying numbers many clever little short cuts have been invented that are very easy when they are once understood. The trouble is that in practical work any one of them is used but seldom, and is very likely to be forgotten. There are some short cuts, however, that are so obvious that once used are not easily forgotten. When the problem presents itself, the little trick also presents itself. We shall now present some such examples together with appropriate short cuts.

47. Cross Multiplication. — By a little study of the multiplication of two numbers like 74 by 46 we can find a method which will enable us to multiply mentally two such numbers, using pencil and paper only to put down the product.

Example. Multiply 74 by 46.

6 ones × 4 ones = 24 ones. 6 ones × 7 tens = 42 tens, and 4 tens × 4 ones = 16 tens. 4 tens × 7 tens = 28 hundreds. Adding the result is 3404.

This work can all be done mentally. We simply say 6 × 4 = 24, 2 to carry. 6 × 7 = 42 and 4 × 4 = 16, which with the 2 carried gives 60. Carry 6. 4 × 7 = 28. To this add the 6 carried, obtaining 34.



48. Product of a One-figure and a Two-figure Number. — It is very important in practical work in multiplication to be able to give almost instantly the product of a two-figure and a one-figure number.

Thus 7 × 59 = 413. We know at once that 7 × 50 = 350 to which we add 7 × 9 = 63.

ORAL EXERCISES (A)

In this manner find the product of each of the following without using pencil and paper.

1. 8 × 43	**3.** 8 × 91	**5.** 7 × 37	**7.** 5 × 78
2. 4 × 83	**4.** 7 × 24	**6.** 4 × 28	**8.** 9 × 84

49. Multiplying by a Two-figure Number. — If we can give at sight the product of a two-figure and a one-figure number, then we

can write down at once the product of any number multiplied by a two-figure number.

Example. Multiply 28943 by 64.

28943
64
1852352

We use the larger number as the multiplier. $3 \times 64 = 192$, carry 19. $4 \times 64 = 256$. $256 + 19 = 275$, carry 27. $9 \times 64 = 576$, $576 + 27 = 603$, carry 60. $8 \times 64 = 512$, $512 + 60 = 572$, carry 57. $2 \times 64 = 128$. $128 + 57 = 185$.

The chief advantage in this kind of multiplication is that it enables us t multiply horizontally. This is convenient in extending bills and in many othe kinds of work.

EXERCISES (A)

Multiply each of the following horizontally.

1. 14×59	**11.** 56×19	**21.** 64×31
2. 73×59	**12.** 74×184	**22.** 27×46
3. 35×17	**13.** 37×192	**23.** 49×12
4. 54×224	**14.** 61×304	**24.** 81×71
5. 29×657	**15.** 78×600	**25.** 67×91
6. 98×311	**16.** 96×594	**26.** 74×39
7. 63×198	**17.** 19×78	**27.** 23×84
8. 42×147	**18.** 39×174	**28.** 68×39
9. 17×61	**19.** 72×21	**29.** 72×46
10. 98×1901	**20.** 67×257	**30.** 84×19

50. Multiplying by 11.

Example 1. Multiply 3946 by 11.

3946 × 11
3946
43406

By arranging the work as shown here, we save writing one line of figures. The work can be shortened still further if we notice that the first figure at the right in the multiplicand is the first figure in the product and that the remaining figures are obtained as follows: $6 + 4 = 10$, 1 to carry. $4 + 9 + 1 = 14$, 1 to carr $9 + 3 + 1 = 13$, 1 to carry. $1 + 3 = 4$.

Example 2. Multiply 1947204 by 11.

1947204 × 11
21419244

In the manner just described the product is written down at once.

51. Multiplying by 22, 33, etc.

Example. Multiply 2947 by 33.

2947 × 33
97251

We say $3 \times 7 = 21$. Write **1** and carry 2. $4 + 7 = 11$, $3 \times 11 = 33$, $33 + 2 = 35$. Write **5** and carry 3. $4 + 9 = 13$, $3 \times 13 = 39$. $3 + 39 = 42$. Write **2** and carry 4. $9 + 2 = 11$, $3 \times 11 = 33$, $4 + 33 = 37$. Write **7** and carry 3. $3 \times 2 = 6$, $3 + 6 = 9$.

With a little practice products of this sort can be written down very rapidly.

WRITTEN EXERCISES

Write down the product of each of the following.

1. 9317×22	**6.** 7804×11	**11.** 2191×33
2. 3451×44	**7.** 493×66	**12.** 8075×77
3. 9276×99	**8.** 5837×44	**13.** 4857×88
4. 9826×55	**9.** 3605×88	**14.** 7984×99
5. 8357×66	**10.** 4308×77	**15.** 14809×66

52. Multiplying by 25, 50, 75.

Example 1. Multiply 4938 by 25.

Since 25 is one fourth of a hundred, we may multiply a number by 25 by multiplying it by 100 and then dividing by 4.

That is, $4938 \times 25 = \frac{493800}{4} = 123450$.

The result can be written down at once by annexing the zeros mentally and then dividing by 4.

That is, $4938 \times 25 = 123450$.

Example 2. Multiply 59264 by 50.

Solution. $59264 \times 50 = \frac{5926400}{4} = 2963200$.

In this case also the result may be written at once.

That is, $59264 \times 50 = 2963200$.

Example 3. Multiply 37202 by 75.

Solution. $37202 \times 75 = 37202 \times \frac{300}{4} = \frac{11160600}{4} = 2790150$.

In this case very little time is saved.

The accompanying form may be used.

37202
300
4)11160600
2790150

53. Multiplying by Numbers like 104, 501, etc.

Example 1. Multiply 2047 by 104.

To multiply by 104, multiply by 100 and by 4 and add the products.

$$\begin{array}{r} 2047 \times 104 \\ 8188 \\ \hline 212888 \end{array}$$

Notice how this is accomplished by putting the figure 8 in the second place to the right of 7. This has the effect of putting 7 into hundreds' place, the 4 in thousands' place, and so on, thus multiplying the whole multiplicand by 100.

Example 2. Multiply 19274 by 301.

$$\begin{array}{r} 19274 \times 301 \\ 57822 \\ \hline 5801474 \end{array}$$

Explain how 19274 is multiplied by 300 in this example.

Example 3. Multiply 837 by 299.

$$\begin{array}{r} 837 \times 299 \\ 251100 \\ \hline 250263 \end{array}$$

299 = 300 − 1. Hence multiply by 300 and then subtract 1 × 837. To save writing we place 300 × 837 = 251100 below 837 and then subtract the upper from the lower number.

Example 4. Multiply 1267 by 494.

$$\begin{array}{r} 1267 \times 494 \\ 633500 \\ 7602 \\ \hline 625898 \end{array}$$

494 = 500 − 6.

54. Multiplying by a Product.

Example. Multiply 3748 by 56.

$$\begin{array}{r} 3748 \times 8 \\ 29984 \times 7 \\ \hline 209888 \end{array}$$

First multiply by 8 and then the product by 7. This saves writing one line of figures and adding two numbers.

WRITTEN EXERCISES

Multiply, using an appropriate short cut in each case.

1. 22×1974	**8.** 55×8503	**15.** 25×1938
2. 50×3058	**9.** 107×4379	**16.** 710×8391
3. 401×725	**10.** 33×1842	**17.** 98×1252
4. 125×2284	**11.** 94×3172	**18.** 77×2640
5. 99×5645	**12.** 999×7821	**19.** 150×6124
6. 503×1951	**13.** 22×6348	**20.** 104×7963
7. 55×2164	**14.** 101×2255	**21.** 402×1729

TOPICS FOR STUDY AND DISCUSSION

Addition	*Multiplication*
2749	2749
2749	× 6
2749	16494
2749	
2749	
2749	
16494	

Subtraction	*Division*
1837	7 Quo.
241	241)1837
1596	1687
241	150 Rem.
1355	
241	
1114	
241	
873	
241	
632	
241	
391	
241	
150	

Addition

$$\tfrac{3}{16} + \tfrac{3}{16} + \tfrac{3}{16} + \tfrac{3}{16} + \tfrac{3}{16} + \tfrac{3}{16} + \tfrac{3}{16} = \tfrac{21}{16} = 1\tfrac{5}{16}$$

Multiplication

$$7 \times \tfrac{3}{16} = \tfrac{21}{16} = 1\tfrac{5}{16}.$$

From the above pairs of examples, addition and multiplication, subtraction and division, the relation in each pair is clear.

CHAPTER V

DIVISION

55. Uses of Division. — In business division is used less frequently than addition or multiplication. Nevertheless, it occurs so often that a candidate for a business position should be able to divide with reasonable speed and accuracy.

56. Definition of Division. — *Division is the process of finding one of two numbers when their product and the other number are given.*

57. Dividend, Divisor, Quotient. — The given product is called the *dividend*, the other given number is the *divisor*, and the number to be found is called the *quotient*. The sign ÷ means that the number preceding it is to be divided by the number following it.

Thus $72 \div 8$ is read "72 divided by 8." To find the missing number in $6 \times ? = 48$ is a problem in division. 48 is the dividend, 6 is the divisor, and the number to be found is the quotient.

58. Other Definitions of Division. — Division is also defined as the process of finding how many times one number is *contained* in another.

From this point of view division may be regarded as a short cut for subtracting a number a certain number of times.

Thus 782 may be divided by 146 by repeated subtraction in the manner shown in the margin. When the remainder becomes less than 146, the process ends. We thus find that 146 is contained 5 times in 782 with 52 as a remainder.

782
146
636
146
490
146
344
146
198
146
52

This method of dividing is required occasionally in certain Civil Service examinations. As late as the year 1000 A.D. this was the only method of division in general use.

We shall now learn a method that we can actually use in practice.

59. Short Division. — The following example illustrates the practical process of short division.

Example. Divide 394708 by 7.

We say $39 \div 7 = 5$ and 4 over, $44 \div 7 = 6$ and 2 over, $27 \div 7 = 3$ and 6 over, $60 \div 7 = 8$ and 4 over, $48 \div 7 = 6$ and 6 over. Hence 56386 is the quotient and 6 is the remainder.

$$7\overline{)394708}$$
$$56386 - 6$$

EXERCISES (A)

In this manner find the quotients and the remainders in the following.

1. $147902 \div 6$	**6.** $39743 \div 7$	**11.** $53827 \div 9$
2. $49840 \div 8$	**7.** $57849 \div 4$	**12.** $9746 \div 8$
3. $98445 \div 7$	**8.** $19487 \div 9$	**13.** $3764 \div 7$
4. $44892 \div 5$	**9.** $37462 \div 8$	**14.** $6347 \div 8$
5. $54976 \div 9$	**10.** $49287 \div 5$	**15.** $5787 \div 6$

60. The First Step in Long Division. In dividing a number like 483750 by a number such as 739 we must first decide what is the least number of figures which must be taken from the left in the dividend in order that 739 shall be contained at least once in that number. We see at once that 739 is contained at least once in 4837 but not in 483. Hence 4837 are the figures which must be taken.

ORAL EXERCISES (A)

In each of the following, state the smallest number of figures which must be taken in the dividend so the divisor shall be contained at least once. The answers to these questions are very easy and should be given rapidly.

1. $3904 \div 94$	**6.** $364291 \div 394$	**11.** $84276 \div 784$
2. $78200 \div 38$	**7.** $234700 \div 486$	**12.** $29814 \div 621$
3. $5242 \div 98$	**8.** $379000 \div 810$	**13.** $21941 \div 176$
4. $7400 \div 78$	**9.** $37608 \div 392$	**14.** $19760 \div 874$
5. $19246 \div 83$	**10.** $79672 \div 895$	**15.** $54862 \div 474$

61. The Second Step in Long Division. — The most difficult step in division is to estimate accurately each figure of the quotient.

Example 1. Find the quotient in $3864 \div 574$.

We see at once that the quotient is at least 6, since $6 \times 600 = 3600$, which is less than 3864; but it is not so easy to decide whether or not the quotient is 7. To decide this we multiply 574 by 6 and subtract from 3864, obtaining 420 as the remainder. Since this is less than the divisor, 6 is the quotient.

$$\begin{array}{r} 6 \\ 574\overline{)3864} \\ \underline{3444} \\ 420 \end{array}$$

Example 2. Find the quotient in $8327 \div 1216$.

We see at a glance that the quotient is 6, since $6 \times 12 = 72$, while $7 \times 12 = 84$. The remaining part of the divisor is so small that we do not need to make much allowance for it.

Example 3. Find the quotient in $5187 \div 713$.

7 is contained 7 times in 51. Since the remaining part of the divisor is small, we do not need to make much allowance for it. Hence the quotient is 7.

Example 4. Find the quotient in $7894 \div 895$.

In this case the divisor is nearly 900. 9 is contained 8 times in 78. Since 78 lacks considerable of being $9 \times 9 = 81$, we conclude that 8 is the quotient.

To be certain that we have the correct quotient it may be necessary to multiply the divisor by the quotient. Indeed, that is just what is done in carrying out long division. It is important, however, that the first estimate should be correct, since that saves the trouble of writing the numbers again and repeating the multiplication.

EXERCISES (B)

Write down the quotient in each of the examples below. Do the work very rapidly, and at the same time be careful to get the right quotient. After you have written down all the quotients multiply to see whether they are correct.

1. $3749 \div 514$	**8.** $7295 \div 938$	**15.** $1765 \div 426$
2. $4293 \div 629$	**9.** $1295 \div 185$	**16.** $2145 \div 586$
3. $7490 \div 934$	**10.** $7149 \div 927$	**17.** $2840 \div 437$
4. $947 \div 421$	**11.** $4565 \div 819$	**18.** $4286 \div 693$
5. $973 \div 159$	**12.** $3427 \div 678$	**19.** $5428 \div 739$
6. $5300 \div 790$	**13.** $2594 \div 456$	**20.** $6210 \div 841$
7. $5294 \div 1814$	**14.** $2419 \div 629$	**21.** $5974 \div 962$

62. Complete Process of Long Division.

Example. Divide 438400 by 748.

First divide 4384 by 748, obtaining 5 as the quotient. Since in the dividend the 4384 is really that many hundreds, the quotient is 500 and the 5 is put in hundreds' place. Then subtract $500 \times 748 = 374000$ from the dividend, leaving 64400 as a remainder. Now subtract successively 80×748 and 6×748, leaving 72 as the final remainder. In practice the canceled zeros are omitted.

	586	(Quotient)
(Divisor)	748)438400	(Dividend)
$500 \times 748 =$	374000	
	64400	
$80 \times 748 =$	59840	
	4560	
$6 \times 748 =$	4488	
	72	(Remainder)

EXERCISES (B)

Find the quotient and the remainder in each of the following.

1.	$14790 \div 789$	**19.**	$640901 \div 365$
2.	$25487 \div 206$	**20.**	$1089548 \div 1644$
3.	$37853 \div 93$	**21.**	$987646 \div 671$
4.	$54365 \div 648$	**22.**	$38948 \div 209$
5.	$135429 \div 254$	**23.**	$376954 \div 1721$
6.	$76948 \div 650$	**24.**	$158375 \div 604$
7.	$593871 \div 104$	**25.**	$905729 \div 8463$
8.	$63824 \div 575$	**26.**	$195381 \div 3076$
9.	$12843 \div 294$	**27.**	$78251 \div 948$
10.	$95474 \div 903$	**28.**	$984732 \div 1793$
11.	$242639 \div 824$	**29.**	$72409 \div 659$
12.	$69346 \div 791$	**30.**	$34796 \div 826$
13.	$308704 \div 1241$	**31.**	$87508 \div 1429$
14.	$758609 \div 2548$	**32.**	$1862057 \div 2386$
15.	$295763 \div 806$	**33.**	$34871 \div 9017$
16.	$96299 \div 373$	**34.**	$279408 \div 1294$
17.	$26938 \div 245$	**35.**	$173046 \div 701$
18.	$119376 \div 307$	**36.**	$354762 \div 9231$

63. Checking Division. — By the definition of division $72 \div 8 = 9$ *because* $8 \times 9 = 72$. Hence we see that the product of the quotient and the divisor equals the dividend provided there is no remainder.

Again, $99 \div 12 = 8$ with a remainder 3. To prove this, multiply 12 by 8 and add 3 to the product, obtaining 99. We therefore obtain the following rule for checking results in division:

Multiply the divisor by the quotient and add the remainder. If the result equals the dividend, the division is proved correct.

Thus to check the example in § 62 multiply 748 by 586 and add 72.

EXERCISES (C)

Divide and check as above.

1. $90480 \div 1764$	**5.** $98056 \div 749$	**9.** $49768 \div 364$
2. $125684 \div 2305$	**6.** $31821 \div 245$	**10.** $29800 \div 479$
3. $398409 \div 1876$	**7.** $80467 \div 309$	**11.** $98240 \div 719$
4. $89761 \div 1876$	**8** $75408 \div 1281$	**12.** $82490 \div 917$

64. Casting Out 9's. — To check by casting out 9's, first subtract the remainder from the dividend. Thus in the example on page 46, $438400 - 72 = 438328$. It then remains to test whether $748 \times 586 = 438328$. This is done exactly as in § 43.

EXERCISES (C)

Find the quotient and remainder in each of the following. Check each by casting out 9's.

1. $217011 \div 497$	**7.** $317020 \div 376$	**13.** $680321 \div 8350$
2. $750045 \div 1720$	**8.** $296309 \div 9240$	**14.** $854300 \div 3721$
3. $76348 \div 7645$	**9.** $790308 \div 1796$	**15.** $254700 \div 4751$
4. $964200 \div 408$	**10.** $269872 \div 8934$	**16.** $134674 \div 1763$
5. $316240 \div 3764$	**11.** $389208 \div 1785$	**17.** $754180 \div 9472$
6 $908376 \div 2941$	**12.** $425604 \div 387$	**18.** $987646 \div 3562$

65. Dividing by 10, 100, etc. — To divide a number like 74,000 by 10, strike out the first zero to the right. To divide it by 100 strike out two zeros, and to divide it by 1000 strike out three zeros.

ORAL EXERCISES (A)

Read the quotients in the following:

1. $62000 \div 10$	**4.** $93400 \div 10$	**7.** $80000 \div 1000$
2. $49800 \div 100$	**5.** $8000 \div 10$	**8.** $3200 \div 10$
3. $58000 \div 1000$	**6.** $7000 \div 100$	**9.** $49000 \div 1000$

66. Dividing by Numbers like 200, 5000. — To divide a number by 200 we may first divide by 100 and then by 2. To divide by 5000 we may first divide by 1000 and then by 5.

ORAL EXERCISES (A)

1. State how to divide by 700, by 3000, by 80, by 4000, by 400.

In the following write down the results or read them orally.

2. $630 \div 30$	**5.** $547200 \div 900$	**8.** $384000 \div 400$
3. $3570 \div 30$	**6.** $735000 \div 3000$	**9.** $49700 \div 700$
4. $12800 \div 800$	**7.** $54000 \div 500$	**10.** $7848000 \div 80$

67. Dividing by a Product. — In general, to divide a number by the product of two numbers, we may divide by each of the factors in succession. Thus to divide a number by 54 we may divide by 9 and then by 6, or by 6 and then by 9.

Example 1. Divide 1728 by 72.

Dividing by 9 and then by 8 we find 24 as the result.

$9\overline{)1728}$
$8\overline{)192}$
24

Example 2. Divide 47940 by 192.

Since $192 = 4 \times 6 \times 8$, we divide by these successively:

To get the proper remainder, multiply 2 by 6×8, 4 by 8, and add the products to 4. Thus, $2 \times 6 \times 8 + 4 \times 8 + 4 = 96 + 32 + 4 = 132$ = remainder.

$8\overline{)47940}$
$6\overline{)5992}$ − 4, 1st Remainder
$4\overline{)998}$ − 4, 2d Remainder
249 − 2, 3d Remainder

WRITTEN EXERCISES (A)

1. State how we may divide by 48, by 56, by 72, by 81.

In this manner divide each of the following.

2. $3780 \div 35$	**4.** $116154 \div 81$	**6.** $249300 \div 15$
3. $4380 \div 45$	**5.** $37952 \div 64$	**7.** $7481 \div 72$

In estimating the value of short cuts account should be taken not only of the saving of time but of the compactness of the form in which the work may be written out. This is especially important when the operation is a part of a larger piece of work. Long division in the usual form is particularly to be avoided on this account.

68. Dividing by 25, 50, 125. — Since $25 = \frac{100}{4}$, or one fourth of 100, it follows that a number may be divided by 25 by dividing it by 100 and then multiplying by 4. Similarly to divide by 50, divide by 100 and multiply by 2.

Thus, $7900 \div 25 = 4 \times 79 = 316$ and $19400 \div 50 = 2 \times 194 = 388$.

Since 125 is one eighth of 1000, a number may be divided by 125 by dividing by 1000 and then multiplying by 8.

For further short cuts in multiplication and division, see Chapter XIV after common and decimal fractions have been studied.

WRITTEN EXERCISES (A)

Find the quotient and remainder in each of the following.

1.	$19470 \div 380$	**11.**	$24800 \div 75$	**21.**	$4130 \div 50$
2.	$43600 \div 500$	**12.**	$92400 \div 150$	**22.**	$59200 \div 760$
3.	$393400 \div 700$	**13.**	$452400 \div 50$	**23.**	$432700 \div 490$
4.	$53820 \div 430$	**14.**	$763400 \div 125$	**24.**	$830060 \div 6700$
5.	$754500 \div 3600$	**15.**	$85600 \div 250$	**25.**	$382700 \div 1900$
6.	$472460 \div 720$	**16.**	$924000 \div 125$	**26.**	$27380 \div 500$
7.	$182600 \div 24100$	**17.**	$4525600 \div 1200$	**27.**	$5340 \div 890$
8.	$3924700 \div 600$	**18.**	$12724000 \div 375$	**28.**	$437000 \div 9700$
9.	$49700 \div 25$	**19.**	$68900 \div 125$	**29.**	$659800 \div 1900$
10.	$164000 \div 125$	**20.**	$74980 \div 25$	**30.**	$719000 \div 1420$

69. Quotients to the Nearest Integer. — In many problems it is required to get the quotient to the nearest integer, neglecting the remainder.

Example 1. Divide 845921 by 389, finding the quotient to its nearest integer.

On dividing we find a quotient of 2174 and a remainder of 235.

Since the remainder is more than one half the divisor, 1 is added to the last figure in the quotient and the remainder is neglected. Hence, 2175 is the required quotient.

Example 2. Divide 562190 by 874, finding the quotient to the nearest integer.

On dividing we find a quotient of 644 and a remainder of 334.

Since the remainder is less than one half the divisor, it is neglected and 644 is the required quotient.

When the remainder is exactly equal to one half the divisor, it is customary to add 1 to the quotient.

70. Finding Averages. — It is frequently required to find the average of several numbers. Thus, the average attendance in a school is found, for a week, a month, or even a year.

Rule. *To find the average of several numbers, divide their sum by the number of numbers.*

Example. In a certain school the attendance for the five school days of a week was 897, 882, 904, 906, 898. Find the average for the week.

Adding and dividing by 5, the quotient is 897 and the remainder 2. If it is required to get the average attendance to the nearest integer, 897 is the required result.

$$\begin{array}{r} 897 \\ 882 \\ 904 \\ 906 \\ 898 \\ \hline 5)4487 \\ 897,\ -2 \end{array}$$

ORAL EXERCISES (B)

1. What is meant by the average of a given set of numbers?
2. How would you find the average weight of the boys in a class?

WRITTEN EXERCISES (B)

In the following find the quotients to the nearest integer.

1. $8493 \div 76$ **4.** $1548 \div 64$ **7.** $54780 \div 319$

2. $30094 \div 137$ **5.** $39870 \div 185$ **8.** $47808 \div 139$

3. $4900 \div 271$ **6.** $93870 \div 581$ **9.** $80874 \div 931$

Find the average to the nearest integer of each of the following sets of numbers.

10. 891, 736, 1078, 459, 1280, 936, 1006, 998.

11. 1980, 376, 2490, 549, 2180, 396, 1470, 456.

12. 98, 396, 475, 3891, 1280, 4370, 1520, 274.

13. 670, 94, 52, 74, 31, 17, 21, 391, 84, 3, 40.

DRILL IN DIVISION

Divide and test by casting out 9's. See § 64.

14. $49741 \div 987$ **20.** $178395 \div 401$ **26.** $646789 \div 176$

15. $78452 \div 602$ **21.** $42836 \div 674$ **27.** $844582 \div 902$

16. $35873 \div 39$ **22.** $34821 \div 492$ **28.** $459673 \div 1271$

17. $56345 \div 846$ **23.** $47159 \div 309$ **29.** $573851 \div 406$

18. $924531 \div 452$ **24.** $109046 \div 563$ **30.** $927509 \div 3648$

19. $84967 \div 556$ **25.** $8459801 \div 4461$ **31.** $183591 \div 6703$

In the following write down the quotients and remainders.

32. $3680 \div 400$ **35.** $63000 \div 700$ **38.** $189200 \div 200$

33. $4550 \div 500$ **36.** $5005 \div 600$ **39.** $729000 \div 300$

34. $8190 \div 900$ **37.** $362000 \div 6000$ **40.** $2700 \div 900$

Find the quotients in the following as in § 67.

41. $89640 \div 72$ **43.** $12852 \div 63$ **45.** $52608 \div 96$

42. $5355 \div 45$ **44.** $19712 \div 77$ **46.** $41202 \div 54$

Find the following quotients as in § 68.

47. $348700 \div 25$ **49.** $247500 \div 250$ **51.** $494000 \div 125$

48. $678400 \div 50$ **50.** $14600 \div 25$ **52.** $8927000 \div 125$

CHAPTER VI

DRILL IN FUNDAMENTAL OPERATIONS

71. Necessity for Continued Drill. — The practical business man must be ready at all times to perform the operations of addition, subtraction, multiplication, and division with considerable speed and practically absolute accuracy. To acquire such skill it is necessary to practice on these operations at short intervals extended over a long period of time. For this reason drills on fundamental operations will be given at the ends of the chapters throughout this book.

DRILL IN ADDITION (A)

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.
3	4	7	1	5	3	8	6	3	8	1	5	6	8	1
9	6	9	3	2	9	2	5	8	8	5	3	5	2	9
8	3	3	9	4	2	7	4	9	6	9	9	7	4	6
6	2	4	7	6	4	4	3	2	4	3	7	4	3	4

16.	17.	18.	19.	20.	21.	22.	23.	24.	25.	26.	27.	28.	29.	30.
4	3	9	6	3	6	8	9	7	7	9	9	7	7	5
5	7	8	7	3	8	4	2	6	6	8	1	2	6	9
3	8	6	9	4	8	7	5	5	6	5	4	3	5	6
4	1	4	7	9	7	6	3	2	8	6	2	4	9	2

31.	32.	33.	34.	35.	36.	37.	38.	39.	40.	41.	42.	43.	44.	45.
8	4	5	2	8	9	3	8	5	8	7	5	3	3	8
4	2	7	3	1	4	7	6	7	5	9	7	8	5	5
7	9	9	5	6	2	6	4	4	5	4	3	2	8	3
3	1	8	7	4	7	4	2	6	4	7	4	1	9	1
9	6	4	9	3	2	5	1	2	7	9	2	9	1	7
2	3	2	2	9	5	9	7	4	6	1	5	2	2	6

DRILL IN ADDITION (B)

Add and check each of the following.

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.
3	5	7	2	2	1	6	7	6	8	2	3	1	9	2
9	9	8	5	6	3	3	6	5	2	5	8	5	7	7
4	6	4	7	4	9	9	9	6	3	4	4	1	5	8
8	2	3	9	9	8	2	7	4	5	7	6	7	3	2
2	3	2	8	2	9	2	3	5	4	6	2	8	1	3
3	6	2	3	7	4	7	8	7	6	4	5	1	2	9
7	5	4	4	4	9	7	5	7	7	2	1	3	4	4
6	3	3	7	2	7	8	2	8	9	3	9	4	6	5
5	9	2	6	6	6	3	4	3	2	3	7	5	8	6
9	6	6	0	5	6	2	1	9	8	7	8	2	0	7
3	4	5	1	9	2	4	3	2	5	6	2	6	7	2
1	2	3	3	3	5	5	7	4	0	4	9	7	9	8
0	8	2	5	0	0	8	9	1	4	5	5	8	2	3
2	7	6	2	2	4	4	6	2	2	8	3	5	4	5
4	3	9	9	6	7	5	4	3	1	9	2	7	3	4
3	6	7	7	7	6	6	5	3	3	3	4	3	5	7
6	7	8	6	8	9	2	5	7	7	7	0	8	6	2
5	2	3	8	4	7	3	3	4	9	2	9	2	8	4
8	3	7	3	3	8	7	7	5	6	8	7	6	7	6
7	4	5	1	9	2	4	4	6	8	4	6	4	9	7

Add the following horizontally and check each result.

16. $47 + 93 + 45 + 78 + 63 + 31 + 24.$

17. $193 + 264 + 782 + 831 + 963 + 713 + 645.$

18. $619 + 348 + 943 + 678 + 397 + 842 + 680.$

19. $3192 + 2176 + 3410 + 2617 + 1793 + 459 + 571.$

20. $1782 + 2548 + 5690 + 7064 + 2904 + 410 + 945.$

21. $939 + 729 + 540 + 1729 + 2172 + 469 + 369.$

22. $4500 + 6402 + 3009 + 745 + 4081 + 765 + 1008.$

23. $7931 + 7675 + 218 + 125 + 276 + 701.$

PRACTICAL DRILL (B)

1. Add the following, both horizontally and vertically.

					Sums
	7461	1946	3750	2741	
	9378	1273	6782	4129	
	5917	1764	3125	2714	
	398	2917	2191	2918	
	764	1548	3715	4517	
	591	2704	9876	5787	
	1376	7654	2468	6127	
	945	8924	3579	6526	
	2047	1742	2028	9887	
	3192	4372	5372	3474	
Sums					Total

2. The cost of instruction in the high schools of a certain city, for a period of three years, was as follows.

School	1st Year	2d Year	3d Year	Sums
Academic High Schools	$288,103	$298,150	$283,212	
Technical	71,569	87,601	110,154	
Commercial	37,687	10,136	54,755	
Sums				Total

Find the total cost of high school instruction for each of the three years, the cost of each kind of school for the three-year period, and the total amount paid during the three years.

3. For one week a butcher's sales in pounds were as follows.

Day	Pork	Beef	Lamb	Chicken	Fish	Sums
Monday . . .	112	202	76	68	45	
Tuesday . . .	187	199	96	75	28	
Wednesday . .	214	247	117	91	49	
Thursday. . .	206	175	128	101	61	
Friday. . . .	129	192	171	183	127	
Saturday . . .	78	287	156	228	56	
Sums . .						Total

Find the total sale for each day, the total sale of each kind of meat for the week, and the total of all kinds for the week.

72. Taking Inventory. – At certain stated times a merchant counts the number of articles of each kind which he has in stock and thus finds approximately the value of the goods in the store. This process is called *taking an inventory.*

PROBLEMS (B)

Following is part of the inventory of a store. Copy the data on a proper blank, extend each item, and add to find the total value.

1. *Farm Implements*

No. of Articles	Articles	Value		Total
11	plows	18	50	
12	plows	16		
7	plows	8	50	
34	plowshares	1	50	
3	harrows	5	25	
2	harrows	15	50	
4	disk harrows	65	00	
5	cultivators	32	50	
3	cultivators	30	00	
2	mowers	70	00	
1	feed cutter	40	50	
3	feed cutters	20	75	
7	sprayers	6	50	
9	sprayers	7	25	
6	sprayers	6	75	
3	sprayers	15	50	
3	sprayers	21	50	
285 ft.	spray hose		10	
27	spray nozzles		38	
14	spray nozzles		67	
17	milk cans	3	85	
11	milk cans	4	15	
$2\frac{1}{2}$ gross	milk bottles	8	50	
13	churns	3	45	
4	separators	42	50	
5	wagons	62		
3	wagons	55	50	
4	wagons	95	00	
12	harness	22	50	
7	harness	35	00	
8	harness	42	50	
4	harness	65	00	
	Total Value			

For each of the following partial inventories, copy data on a suitable blank, extend, and find the total values.

2. *Hardware*

No. of Articles	Kind of Article	Value per Unit	Total Value
14	lawn mowers	14 40	
12	lawn mowers	8 80	
1260 ft.	rubber hose	12	
590 ft.	rubber hose	13	
28	nozzles	50	
43	nozzles	45	
56	sprinklers	45	
17	sprinklers	95	
27	wheelbarrows	2 75	
13	wheelbarrows	3 85	
72	hoes	60	
168	railroad picks	75	
41	weeding hoes	45	
46	grub hoes	85	
24	pruning shears	55	
37	pruning shears	85	
29	pruning shears	75	
17	pruning saws	85	
9	hedge shears	2 15	
61	hay forks	2 30	
11	hay forks	2 40	
19	hay forks	1 45	
34	corn knives	60	
14	bench drills	1 90	
9	bench drills	4 50	
17	vises	2 30	
12	vises	4 50	
27	vises	1 25	
86	hammers	1 50	
126	hammers	1 35	
43 bx.	carriage bolts	65	
33	wrenches	1 45	
38	wrenches	2 55	
19	pipe wrenches	2 25	
8	screw plates	4 50	
13	screw plates	2 75	
9	child's wagons	3 60	
15	child's wagons	6 50	
57	screw drivers	85	
19	screw drivers	75	

3. *Furniture*

No. of Articles	Kind of Article	Value per Unit	Total Value
9	parlor sets	84 50	
12	library tables	35 50	
21	rockers	9 75	
17	rockers	12 00	
8	rockers	18 50	
14	rockers	9 75	
34	rockers	5 25	
15	chairs	12 50	
18	couches	29 45	
7	couches	23 50	
6	davenports	48 00	
4	davenports	62 50	
19	bookcases	11 00	
13	bookcases	20 50	
16	tables	9 00	
26	tables	2 50	
43	beds	7 50	
31	beds	15 00	
22	beds	18 50	
11	beds	20 50	
84	pillows	1 55	
114	pillows	2 95	
56	pillows	3 25	
39	springs	8 15	
53	springs	6 50	
42	springs	14 75	
36	mattresses	22 75	
27	mattresses	21 50	
14	mattresses	14 25	
19	dressers	32 50	
31	dressers	36 25	
31	dressers	38 50	
4	dressers	45 00	
3	dressers	55 00	
17	porch swings	19 50	
37	mirrors	6 75	
16	mirrors	8 60	
18	clocks	6 90	
23	clocks	7 20	
47	kitchen chairs	3 40	

73. Rounded Numbers. — Many large numbers cannot be given exactly. Such numbers may be given only to the nearest thousand, and are then said to be *rounded* to the nearest thousand. Thus the number of bushels of corn produced in Illinois in one year is given as 398,149,140. But it is known that this cannot be exactly correct. It is a fact that 398,150,000 or 398,000,000 is equally correct.

The following table gives the number of acres in corn and the total yield for the states named as given by a recent census.

State	Acres in Corn	Bushels Yield
Illinois	10,266,355	398,149,140
Kansas	8,266,018	229,937,430
Nebraska	7,335,187	210,974,740
Indiana	4,499,249	178,967,020

PROBLEMS (B)

1. Read the number of acres and bushels in Illinois to the nearest thousand.

2. Read the same numbers to the nearest millions for Kansas, Nebraska, and Indiana.

TOPICS FOR DISCUSSION

1. How can the total acreage in corn in a state for a certain year be found? Does each farmer know exactly how many acres he has in corn? How is the number of acres found for a certain county? Is there any element of estimating in finding this number? Are we justified in saying that the actual number of acres for a state may not be known within a tenth of 1% (one acre in a thousand)? May it possibly not be known within 1%?

2. Discuss the closeness of approximation within which the total yield in a state may be known.

3. Is the exact population of the United States ever known? Is the population known within a hundred thousand?

CHAPTER VII

DIVISIBILITY OF NUMBERS, FACTORS, MULTIPLES

This chapter serves as an introduction to the chapters on fractions, since dealing with fractions requires a knowledge of the divisibility of numbers, and of the factors and multiples of numbers.

74. Integers. — The numbers obtained by ordinary counting, beginning with 1, are called *integers*. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 are all the integers from 1 to 10 inclusive.

75. Odd and Even Integers. — Every second integer, beginning with 2, is called an *even* integer. Zero is also regarded as an even integer. Thus, 0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20 are all the even integers from 0 to 20 inclusive.

All integers that are not even are called *odd* integers.

We frequently say odd or even *numbers*, instead of odd and even *integers*.

76. Factors. — If a given integer is the product of two or more integers, these are said to be *factors* of the given integer.

Thus, 2 and 3 are factors of 6, as are also 1 and 6.

77. Prime and Composite Numbers. — A number (integer) which has no factors except itself and 1 is called a *prime* number. All integers that are not prime are called *composite* numbers.

Thus, 1, 2, 3, 5, 7, 11 are prime numbers, while 4, 6, 8, 9, 10 are composite.

A prime number which is a factor of another number is called a prime factor of that number.

Thus, 1, 2, 3 are prime factors of 6, and also of 12.

ORAL EXERCISES

1. Give all prime numbers from 1 to 50.
2. Give all prime factors of the numbers 6, 8, 10, 15, 18, 20.
3. Give all composite factors of 12, 16, 24, 36.

78. Divisibility. – A number is said to be *divisible* by any one of its factors.

79. Tests of Divisibility. A number is divisible by 2 if its last digit is divisible by 2. Zero is here regarded as divisible by 2.

Thus, 364, 748, 196, 48780 are divisible by 2.

A number is divisible by 4 if the last two of its digits represent a number that is divisible by 4.

Thus, 324, 25632, 70976, 134568 are all divisible by 4.

A number is divisible by 8 if the last three of its digits represent a number that is divisible by 8.

Thus, 7144, 245632, 19248, 3749672 are all divisible by 8.

All numbers that end in 5 or 0 are divisible by 5. All numbers that end in 0 are divisible by 10.

A number is divisible by 3 if the sum of its digits is divisible by 3.

Thus, 432423 is divisible by 3 because $4 + 3 + 2 + 4 + 2 + 3 = 18$ is divisible by 3.

A number is divisible by 9 if the sum of its digits is divisible by 9.

A number is divisible by 6 if it is even and is divisible by 3.

Thus, 48732 is divisible by 6, while neither 3442 nor 753 is divisible by 6.

A number is divisible by 12 if it is divisible by both 3 and 4.

80. Divisors. – If a given number is divisible by a number, then this second number is called a *divisor* of the given number.

ORAL EXERCISES (A)

State which of the numbers 1, 2, 3, 4, 5, 6, 8, 10, 12, if any, are divisors of each of the following numbers.

1. 3789	**7.** 7542	**13.** 4293	**19.** 59482
2. 1090	**8.** 27891	**14.** 1275	**20.** 987648
3. 2454	**9.** 142365	**15.** 3920	**21.** 700542
4. 3795	**10.** 378900	**16.** 8728	**22.** 1256931
5. 1592	**11.** 15592	**17.** 6309	**23.** 2781450
6. 3763	**12.** 124672	**18.** 1111	**24.** 72951

81. Factoring. — Factoring a number consists in finding all the prime factors of the number. The product of all the prime factors of a number is the number itself.

Thus, the prime factors of 5 are 1 and 5, and $1 \times 5 = 5$. The prime factors of 6 are 1, 2, and 3, and $1 \times 2 \times 3 = 6$.

Sometimes one number is used as a factor several times.

Thus, 2 is used as a factor 3 times in 8, because $2 \times 2 \times 2 = 8$.

82. Common Factors. — If a number is a factor of each of two or more numbers, it is said to be a *common factor* of these numbers.

Thus, 2 is a common factor of 6, 8, and 10. 3 is a common factor of 9, 18, 21; and 7 is a common factor of 28 and 42.

83. Greatest Common Divisor. — The greatest number which is a common factor of two or more numbers is called the *Greatest Common Divisor* (G. C. D.) of these numbers.

The G. C. D. of two or more numbers is the product of all the prime common factors of these numbers.

Example. Find the G. C. D. of 32, 64, 96, 128.

The work is conveniently arranged thus,—

2)32,	64,	96,	128
2)16,	32,	48,	64
2)8,	16,	24,	32
2)4,	8,	12,	16
2)2,	4,	6,	8
1,	2,	3,	4

Divide each number by the common factor 2, obtaining 16, 32, 48, 64 as the quotients. 2 is a common factor of the quotients. Continuing in this manner, we finally get 1, 2, 3, 4 as quotients, and these have no common factor except 1. Hence, the G. C. D. $= 2 \times 2 \times 2 \times 2 \times 2 = 32$.

EXERCISES (A)

1. Find all prime factors of 6, 8, 10, 12, 16, 18, 21, 24, 32, 36, 42.

2. Find all prime common factors of 18, 24, 36, 42.

Find the G. C. D. of each of the following sets of numbers.

3. 30, 42, 64, 72

4. 16, 96, 108, 124

5. 14, 21, 49, 63

6. 256, 768, 508, 1164

7. 196, 408, 1752, 504

8. 45, 90, 360, 1440

9. 17, 51, 85, 102

10. 9, 33, 117, 141

11. 57, 95, 152, 190

12. 195, 270, 450

13. 75, 125, 340

14. 39, 52, 91, 104

84. A General Principle on Common Factors. — If a number is a common factor of two numbers, it is a factor of their sum and also of their difference.

Thus, 8 is a common factor of 24 and 56 and therefore of $56 + 24 = 80$ and of $56 - 24 = 32$.

This principle may be used in finding common factors.

Example. Find the G. C. D. of 1292 and 1615.

If these numbers have a common factor, it is a factor of their difference, that is, of 323. By dividing we find 323 to be a factor of 1292 and of 1615. Hence 323 is the required G. C. D.

The general application of this principle is too complicated for this book and is very seldom if ever required in business practice.

Solution

1615	4
1292	323)1292
323	1292

85. Multiples. — The product obtained by multiplying a given number by an integer is called a *multiple* of the given number.

Thus, 6, 12, 18, 24 are multiples of 6. These numbers are also multiples of 2 and 3. Every integer is a multiple of 1.

86. Rule on Multiples. — *If one of two numbers is a multiple of the other number, then every multiple of the first number is a multiple of the second.*

That is, every multiple of 16 is a multiple of 2, 4, and 8; and every multiple of 60 is a multiple of 2, 3, 5, 10, 20, and 30.

87. Common Multiples. — A number which is a multiple of each of two or more numbers is called a *common multiple* of these numbers.

That is, 30 is a common multiple of 5, 6, 10, 2, 3.

ORAL EXERCISES

Find two common multiples of each of the following sets of numbers.

1. 2, 4, 6	**5.** 3, 5, 6	**9.** 9, 2, 6
2. 3, 6, 9	**6.** 2, 1, 7	**10.** 12, 10, 8
3. 5, 10, 15	**7.** 4, 5, 6	**11.** 16, 8, 4
4. 2, 6, 8	**8.** 3, 6, 8	**12.** 15, 10, 6

88. Least Common Multiples. — The smallest number which is a common multiple of two or more numbers is called the *Least Common Multiple* of these numbers. The Least Common Multiple is usually denoted by L. C. M.

Thus the L. C. M. of 10, 15, 20 is 60, and the L. C. M. of 8, 16, 24 is 48.

Finding the L. C. M. of numbers is of importance in dealing with fractions.

Example. Find the L. C. M. of 18, 32, 48.

We factor each number into its prime factors, as shown.

$18 = 2 \times 3 \times 3$
$32 = 2 \times 2 \times 2 \times 2 \times 2$
$48 = 2 \times 2 \times 2 \times 2 \times 3$

The L. C. M. of 18, 32, 48 must be a multiple of 48. That is, it must contain the factors 2, 2, 2, 2, 3.

Since the L. C. M. is a multiple of 32 it must contain an additional factor 2, and since it is a multiple of 18, it must contain an additional factor 3.

Hence the L. C. M. of 18, 32, 48 is $2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 288$.

This problem may also be solved as follows.

First divide each number by 2. Then divide 9 and 24 by 3, bringing down the 16. Finally divide 16 and 8 by 8, bringing down the 3.

2)18, 32, 48
3)9, 16, 24
8)3, 16, 8
3, 2, 1

The L. C. M. is $2 \times 3 \times 8 \times 3 \times 2 = 288$.

The process of division ends when no two numbers below the last line contain a common factor. The L. C. M. is the product of the numbers below the last line and the factors to the left.

At each step divide by the largest number possible.

In finding the L. C. M. of the numbers 18, 24, 36, 48, we need only to find the L. C. M. of 36 and 48, since 18 and 24 are contained in these. For the purpose of finding the L. C. M. of a set of numbers any number which is contained in one of the others may be rejected.

EXERCISES (B)

Find the L. C. M. of each of the following sets of numbers. In each case, state what numbers may be rejected.

1. 15, 24, 30
2. 9, 27, 81
3. 16, 48, 64
4. 17, 51, 108
5. 16, 24, 64, 96
6. 3, 6, 9, 21
7. 5, 25, 40, 80
8. 7, 28, 49, 63
9. 8, 12, 16, 18

MISCELLANEOUS WORK (B)

1. Loads of grain were hauled to an elevator. Find the net weight in pounds if the gross weights and tares were as follows.

Gross Weight	Tares	Net Weight	Gross Weight	Tares	Net Weight
6860	2680		7380	2570	
7530	2590		8430	2340	
6780	2660		7480	2560	
8630	2580		9540	2580	
8980	2670		9720	2610	
7190	2140		10140	2600	

In each of the following find by how much the first number exceeds the sum of the numbers below it.

2.	**3.**	**4.**	**5.**	**6.**
23768	67827	76249	89470	182714
1541	4976	4982	2164	21712
8077	18029	7245	13970	41608
2962	3763	10866	2612	28739
7603	1522	3079	3970	11479
3245	9248	45034	12891	14731
210	10500	1910	16100	11000

In each of the following which column has the greater sum and how much?

7.		**8.**		**9.**	
3942	4190	7642	5430	15921	7106
8176	1297	1420	2780	7203	2432
9140	2112	1926	7947	14170	9219
2028	2040	3223	3928	2614	9874
1347	1172	3456	8724	9125	7654
2421	3192	1987	9210	14620	2123
3921	2178	2134	7824	1910	8860
7100	5160	5110	6186	2080	3100
1520	1520	2130	2180	1760	4170
1310	2120	2240	1114	3100	2140



MARKDOWNS — A FAMILIAR USE OF FRACTIONS

CHAPTER VIII

FRACTIONS, CANCELLATION

89. Use of Fractions in Business. — Simple fractions occur frequently in business, and no one is competent to do the figuring required in everyday transactions unless he understands fractions.

90. Fractions. — A number in the form $\frac{2}{3}$ is called a *fraction*. There are several definitions of fractions in general use.

Thus $\frac{2}{3}$ may be regarded as one of the 3 equal parts of 2, or as 2 of the 3 equal parts of 1.

In more advanced works on arithmetic, it is usual to define $\frac{2}{3}$ as a number such that $3 \times \frac{2}{3} = 2$.

91. Numerator, Denominator. — A fraction is always written by means of two numbers (usually integers), one above a horizontal line, and the other below it. The number above the line is called the *numerator*, and the one below it, the *denominator*.

Every fraction is, therefore, of the form: $\frac{numerator}{denominator}$.

The denominator states into how many equal parts a unit has been divided; and the numerator states how many of these parts are represented by the fraction.

Thus, in the fraction $\frac{4}{5}$ the denominator 5 states that a unit has been divided into five equal parts and the numerator 4 states that four of these parts are represented by the fraction.

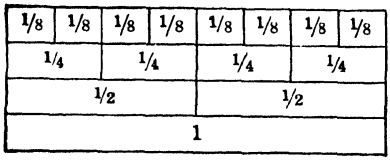
The numerator and denominator of a fraction are called the *terms* of the fraction.

A fraction may also be regarded as an indicated quotient, the numerator being the dividend, and the denominator, the divisor.

Thus the fraction $\frac{4}{5}$ may be regarded as the quotient when 4 is divided by 5. A problem in division is often represented in the form of a fraction.

To deal effectively with fractions it is now necessary to study them more in detail.

92. A General Property of Fractions. — From the figure it is clear that $\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$.



These are instances of a general property of fractions:

Multiplying both terms of a fraction by the same number does not change its value.

Construct a figure similar to the above to show that

$$\frac{6}{9} = \frac{2}{3},\ \frac{3}{9} = \frac{1}{3},\ \frac{4}{8} = \frac{2}{4} = \frac{1}{2}.$$

The above property may then be stated:

Dividing both terms of a fraction by the same number does not change its value.

These two rules may be stated in one as follows:

Multiplying or dividing both terms of a fraction by the same number does not change its value.

93. Fractions in Lowest Terms. — When the numerator and denominator of a fraction have no common factor except 1, the fraction is said to be in its *lowest terms*.

94. Proper Fractions. — A fraction whose numerator is less than its denominator is called a *proper fraction*.

95. Improper Fractions. — A fraction whose numerator is not less than its denominator is called an *improper fraction*.

Thus, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{7}{9}$ are proper fractions, while $\frac{3}{3}$, $\frac{3}{2}$, $\frac{7}{6}$ are improper fractions.

96. Mixed Numbers. — A number consisting of an integer and a fraction is called a mixed number.

97. Reduction to Lowest Terms. — A fraction not in its lowest terms may be reduced to its lowest terms by dividing both numerator and denominator by their greatest common factor.

Thus, $\frac{4}{12} = \frac{1}{3}$, $\frac{3}{15} = \frac{1}{5}$.

98. Reduction to Improper Fractions. — An integer may be reduced to an improper fraction.

Thus, $1 = \frac{2}{2} = \frac{3}{3} = \frac{4}{4} = \frac{5}{5} = \frac{6}{6} = \ldots$ and, $2 = \frac{4}{2} = \frac{6}{3} = \frac{8}{4} = \frac{10}{5} = \ldots$

In a similar manner a mixed number may be reduced to an improper fraction.

Thus, $1\frac{1}{2} = \frac{2}{2} + \frac{1}{2} = \frac{3}{2}$, $2\frac{1}{3} = \frac{6}{3} + \frac{1}{3} = \frac{7}{3}$.

99. Reduction to Integers or Mixed Numbers. — An improper fraction may be reduced to an integer or to a mixed number.

Thus, $\frac{3}{3} = 1$, $\frac{6}{3} = 2$, $\frac{8}{2} = 4$. Also, $\frac{3}{2} = \frac{2}{2} + \frac{1}{2} = 1\frac{1}{2}$, $\frac{7}{2} = \frac{6}{2} + \frac{1}{2} = 3\frac{1}{2}$

To reduce an improper fraction to an integer, or to a mixed number, divide the numerator by the denominator. The quotient is the integral part of the mixed number, and the remainder is the numerator of the fractional part, while the divisor is its denominator.

100. Numbers in the Simplest Form. — A number in which every fraction is a proper fraction reduced to its lowest terms is said to be in its *simplest form*.

Numbers representing *final results* should always be put into the simplest form.

EXERCISES (A)

Reduce the following fractions to their lowest terms.

1.	$\frac{1}{12}$	**3.**	$\frac{124}{218}$	**5.**	$\frac{91}{117}$	**7.**	$\frac{148}{396}$	**9.**	$\frac{57}{114}$	**11.**	$\frac{150}{725}$
2.	$\frac{116}{314}$	**4.**	$\frac{25412}{50841}$	**6.**	$\frac{316}{1580}$	**8.**	$\frac{225}{1000}$	**10.**	$\frac{111}{259}$	**12.**	$\frac{204}{872}$

Reduce the following to improper fractions.

13.	$1\frac{3}{4}$	**15.**	$2\frac{7}{8}$	**17.**	$9\frac{5}{12}$	**19.**	$8\frac{2}{3}$	**21.**	$17\frac{4}{5}$	**23.**	$27\frac{5}{12}$
14.	$91\frac{2}{15}$	**16.**	$128\frac{1}{7}$	**18.**	$250\frac{10}{17}$	**20.**	$185\frac{1}{11}$	**22.**	$283\frac{5}{16}$	**24.**	$9040\frac{9}{13}$

Reduce each of the following to integers or to mixed numbers.

25.	$\frac{7}{3}$	**27.**	$\frac{174}{3}$	**29.**	$\frac{81}{1}$	**31.**	$\frac{54}{3}$	**33.**	$\frac{125}{6}$	**35.**	$\frac{1729}{14}$
26.	$\frac{2589}{121}$	**28.**	$\frac{8193}{27}$	**30.**	$\frac{15051}{37}$	**32.**	$\frac{712147}{214}$	**34.**	$\frac{911452}{33}$	**36.**	$\frac{620451}{45}$

37. If the numerator of a fraction is a multiple of the denominator, to what kind of number may the fraction be reduced?

38. Change $\frac{3}{4}$, $\frac{2}{7}$, and $\frac{3}{9}$ to other fractions.

101. Cancellation. — The property of a fraction by which its numerator and denominator may both be multiplied or divided by the

same number without changing its value may be used in simplifying many examples in division.

Example. Divide $34 \times 45 \times 64$ by $96 \times 84 \times 16$.

Solution. $(34 \times 45 \times 64) \div (96 \times 84 \times 16) = \dfrac{34 \times 45 \times 64}{96 \times 84 \times 16}$.

First divide 64 and 16 by 16, 45 and 84 by 3, and 34 and 96 by 2. Then divide 4 and 48 by 4 and finally 15 and 12 by 3. In the numerator the factors 17 and 5 are left and in the denominator the factors 4 and 28 are left.

$$\frac{\overset{17}{\cancel{34}} \times \overset{5}{\cancel{45}} \times \overset{4}{\cancel{64}}}{\underset{48}{\underset{12}{\underset{4}{\cancel{96}}}} \times \underset{28}{\cancel{84}} \times \cancel{16}} = \frac{17 \times 5}{4 \times 28} = \frac{85}{112}$$

The method used in simplifying this fraction is called *cancellation*.

EXERCISES (A)

By means of cancellation simplify each of the following as much as possible and then reduce the result to the simplest form.

1. $36 \times 34 \times 72 \times 12 \div 48 \times 27 \times 14 \times 18$.
2. $108 \times 90 \times 84 \times 64 \div 32 \times 26 \times 45$.
3. $16 \times 18 \times 32 \times 42 \times 54 \div 24 \times 14 \times 28$.
4. $42 \times 19 \times 9 \times 12 \times 16 \div 35 \times 24 \times 12$.
5. $76 \times 43 \times 36 \times 98 \div 22 \times 18 \times 12$.
6. $64 \times 36 \times 94 \times 18 \div 12 \times 34 \times 46$.
7. $34 \times 97 \times 105 \times 16 \div 12 \times 28 \times 46$.
8. $82 \times 68 \times 72 \times 96 \div 48 \times 16 \times 27$.
9. $94 \times 36 \times 48 \times 56 \div 18 \times 24 \times 14$.
10. $12 \times 15 \times 25 \times 36 \div 8 \times 18 \times 46$.
11. $72 \times 81 \times 90 \times 32 \div 54 \times 27 \times 12$.
12. $154 \times 124 \times 360 \div 105 \times 18 \times 6$.
13. $15 \times 24 \times 32 \div 30 \times 18 \times 12$.
14. $48 \times 36 \times 27 \div 32 \times 16 \times 42$.
15. $14 \times 17 \times 21 \times 6 \div 9 \times 24 \times 42$.

CHAPTER IX

ADDITION AND SUBTRACTION OF FRACTIONS

102. Addition and Subtraction of Fractions in Business. — The addition and subtraction of simple fractions is of very common occurrence in business. Sometimes even more complicated fractions occur.

103. Reducing Fractions to a Common Denominator. — Even such simple fractions as $\frac{1}{2}$ and $\frac{1}{1}$ cannot be added until they are changed into fractions having a common denominator.

This, however, is easily done, since $\frac{1}{2} = \frac{2}{4}$.

Then $\frac{1}{2} + \frac{1}{4} = \frac{2}{4} + \frac{1}{4} = \frac{3}{4}$.

The first step in adding fractions not having a common denominator is to change them into equal fractions having a common denominator.

A fraction can always be changed into other fractions whose denominators are multiples of the given fraction.

Thus, $\frac{3}{4}$ can be changed into fractions whose denominators are multiples of 4, and to no other fractions.

That is, $\frac{3}{4}$ can be changed into $\frac{6}{8}$, $\frac{9}{12}$, $\frac{12}{16}$, but not into 6ths, or 9ths, or 10ths.

If fractions such as $\frac{1}{3}$ and $\frac{1}{4}$ are to be changed into fractions having a common denominator, the new denominator must be a common multiple of 3 and 4. Hence the first step in changing fractions into fractions having a common denominator is to find a common multiple of the denominators of the given fractions.

That is, to change $\frac{1}{2}$, $\frac{1}{3}$, $\frac{3}{4}$ into fractions having a common denominator we must first find a common multiple of 2, 3, 4. In practice we select the L. C. M.

We see at once that the L. C. M. of 2, 3, 4 is 12.

The process is then as follows.

$\frac{1}{2} = \frac{6}{12}$
$\frac{1}{3} = \frac{4}{12}$
$\frac{3}{4} = \frac{9}{12}$

Since both terms of each fraction must be multiplied by the same number, it follows that both terms of $\frac{1}{2}$ must be multiplied by 6, both terms of $\frac{1}{3}$ by 4, and both terms of $\frac{3}{4}$ by 3.

WRITTEN EXERCISES (A)

Find the L. C. M. of each of the following sets of numbers. (See § 88.)

1. 3, 4, 6, 8
2. 2, 4, 10, 16
3. 4, 12, 16, 32
4. 5, 25, 125
5. 10, 20, 40
6. 7, 14, 21, 42
7. 9, 15, 27, 63
8. 11, 44, 77
9. 6, 12, 9, 36
10. 8, 32, 64
11. 9, 18, 21, 24
12. 16, 24, 42, 36
13. 14, 3, 6, 11
14. 8, 9, 10, 12
15. 10, 11, 12, 15, 22
16. 8, 6, 4, 12
17. 7, 9, 12, 14, 18
18. 8, 12, 16, 20
19. 16, 20, 24, 32
20. 15, 18, 21, 27

ADDITION OF FRACTIONS (A)

Example 1. Add $\frac{1}{2} + \frac{1}{4} + \frac{1}{6}$.

We see at once that the L. C. M. of the denominators is 12.

Then $\frac{1}{2} = \frac{6}{12}$

$\frac{1}{4} = \frac{3}{12}$ Hence the sum is $\frac{11}{12}$.

$\frac{1}{6} = \frac{2}{12}$

Example 2. Add $\frac{1}{4} + \frac{1}{8} + \frac{1}{12}$.

$\frac{1}{4}$ 6

$\frac{1}{8}$ 3

$\frac{1}{12}$ 2

$\frac{11}{24}$

In practice we omit writing the denominator. The numerators then appear in a column and are readily added.

WRITTEN EXERCISES (A)

In this manner add the following.

1. $\frac{2}{3} + \frac{3}{4} + \frac{1}{8}$
2. $\frac{5}{6} + \frac{3}{8} + \frac{5}{12}$
3. $\frac{5}{8} + \frac{3}{10} + \frac{2}{5}$
4. $\frac{2}{7} + \frac{1}{3} + \frac{3}{4}$
5. $\frac{7}{9} + \frac{2}{15} + \frac{4}{5}$
6. $\frac{3}{4} + \frac{5}{12} + \frac{7}{8}$
7. $\frac{2}{9} + \frac{4}{15} + \frac{3}{5}$
8. $\frac{1}{4} + \frac{3}{8} + \frac{4}{16}$
9. $\frac{1}{3} + \frac{5}{9} + \frac{7}{18}$
10. $\frac{4}{5} + \frac{9}{10} + \frac{11}{15}$
11. $\frac{2}{3} + \frac{5}{6} + \frac{7}{8}$
12. $\frac{1}{2} + \frac{3}{4} + \frac{5}{8}$

104. Adding Mixed Numbers. — To add mixed numbers, add the integers separately, and the fractions separately.

Thus, $3\frac{1}{2} + 4\frac{3}{8} + 5\frac{1}{12} = 12 + \frac{12}{24} + \frac{9}{24} + \frac{2}{24} = 12\frac{23}{24}$.

WRITTEN EXERCISES (A)

In this manner add the following.

1. $1\frac{1}{5} + 3\frac{3}{10} + 6\frac{1}{15}$
2. $8\frac{2}{3} + 7\frac{5}{8} + 9\frac{7}{12}$
3. $9\frac{4}{5} + 3\frac{3}{8} + 2\frac{2}{3}$
4. $1\frac{7}{8} + 4\frac{3}{16} + 7\frac{7}{32}$
5. $38\frac{8}{9} + 4\frac{2}{3} + 6\frac{4}{15}$
6. $12\frac{3}{7} + 8\frac{1}{9} + 12\frac{4}{21}$
7. $8\frac{4}{9} + 7\frac{3}{8} + 2\frac{1}{3} + 3\frac{3}{4}$
8. $45\frac{3}{5} + 19\frac{2}{3} + 1\frac{1}{4} + 2\frac{1}{10}$
9. $31\frac{3}{8} + 42\frac{7}{11} + 1\frac{1}{24}$
10. $180\frac{1}{2} + 216\frac{3}{4} + 3\frac{5}{8} + 1\frac{3}{16}$
11. $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8}$
12. $1\frac{3}{4} + 2\frac{4}{5} + 3\frac{3}{20} + 1\frac{3}{8}$
13. $2\frac{1}{4} + 1\frac{1}{3} + 4\frac{7}{12} + 3\frac{5}{8}$
14. $3\frac{9}{10} + \frac{1}{5} + \frac{4}{15} + \frac{3}{20}$

105. Short Methods for Adding Certain Fractions.

Example 1. Add $\frac{1}{3} + \frac{1}{7}$.

The common denominator is 3×7 and the numerators are 3 and 7. Hence the sum is $\frac{3+7}{21} = \frac{10}{21}$.

The common denominator is the product of the denominators and the numerator of the sum is the sum of the denominators.

Example 2. $\frac{2}{5} + \frac{2}{7} = \frac{2 \times 7 + 2 \times 5}{5 \times 7} = \frac{2(7+5)}{35} = \frac{24}{35}$.

Again the denominator of the sum is the product of the denominators and the numerator is twice the sum of the numerators.

In the same manner, $\frac{3}{7} + \frac{3}{11} = \frac{3(7+11)}{7 \times 11} = \frac{54}{77}$.

ORAL EXERCISES (B)

In this manner add the following.

1. $\frac{1}{5} + \frac{1}{8}$
2. $\frac{1}{3} + \frac{1}{10}$
3. $\frac{1}{7} + \frac{1}{12}$
4. $\frac{1}{5} + \frac{1}{9}$
5. $\frac{2}{3} + \frac{2}{5}$
6. $\frac{2}{7} + \frac{2}{9}$
7. $\frac{2}{5} + \frac{2}{7}$
8. $\frac{2}{9} + \frac{2}{11}$
9. $\frac{3}{5} + \frac{3}{7}$
10. $\frac{3}{7} + \frac{3}{8}$
11. $\frac{3}{5} + \frac{3}{10}$
12. $\frac{3}{7} + \frac{3}{11}$
13. $\frac{4}{5} + \frac{4}{7}$
14. $\frac{4}{7} + \frac{4}{9}$
15. $\frac{4}{9} + \frac{4}{11}$
16. $\frac{4}{11} + \frac{4}{3}$

In actual business practice, most of the fractions to be added are quite simple and the work can be done orally.

ORAL EXERCISES (A)

Add each of the following sets of fractions and reduce each result to simplest form.

1. $\frac{1}{2} + \frac{1}{4}$
2. $\frac{1}{2} + \frac{3}{4}$
3. $\frac{1}{2} + \frac{1}{8}$
4. $\frac{1}{2} + \frac{3}{8}$
5. $\frac{1}{2} + \frac{5}{8}$
6. $\frac{1}{2} + \frac{7}{8}$
7. $\frac{1}{2} + \frac{1}{3}$
8. $\frac{1}{2} + \frac{2}{3}$
9. $\frac{1}{2} + \frac{1}{5}$
10. $\frac{1}{2} + \frac{2}{5}$
11. $\frac{1}{2} + \frac{3}{5}$
12. $\frac{1}{2} + \frac{4}{5}$
13. $\frac{1}{3} + \frac{1}{4}$
14. $\frac{2}{3} + \frac{1}{4}$
15. $\frac{1}{3} + \frac{3}{4}$
16. $\frac{2}{3} + \frac{3}{4}$
17. $\frac{1}{2} + \frac{1}{6}$
18. $\frac{1}{2} + \frac{5}{6}$
19. $\frac{1}{2} + \frac{1}{10}$
20. $\frac{1}{2} + \frac{3}{10}$
21. $\frac{1}{2} + \frac{7}{10}$
22. $\frac{1}{2} + \frac{9}{10}$
23. $\frac{1}{2} + \frac{1}{12}$
24. $\frac{1}{2} + \frac{5}{12}$
25. $\frac{1}{2} + \frac{7}{12}$
26. $\frac{1}{2} + \frac{11}{12}$
27. $\frac{1}{2} + \frac{1}{16}$
28. $\frac{1}{2} + \frac{3}{16}$
29. $\frac{1}{2} + \frac{5}{16}$
30. $\frac{1}{2} + \frac{7}{16}$
31. $\frac{1}{2} + \frac{9}{16}$
32. $\frac{1}{2} + \frac{11}{16}$
33. $\frac{1}{2} + \frac{13}{16}$
34. $\frac{1}{2} + \frac{15}{16}$
35. $\frac{1}{3} + \frac{1}{5}$
36. $\frac{1}{3} + \frac{2}{5}$
37. $\frac{1}{3} + \frac{3}{5}$
38. $\frac{1}{3} + \frac{4}{5}$
39. $\frac{2}{3} + \frac{1}{5}$
40. $\frac{2}{3} + \frac{2}{5}$
41. $\frac{2}{3} + \frac{3}{5}$
42. $\frac{2}{3} + \frac{4}{5}$
43. $\frac{1}{3} + \frac{1}{8}$
44. $\frac{1}{3} + \frac{3}{8}$
45. $\frac{1}{3} + \frac{5}{8}$
46. $\frac{1}{3} + \frac{7}{8}$
47. $\frac{2}{3} + \frac{1}{8}$
48. $\frac{2}{3} + \frac{3}{8}$
49. $\frac{2}{3} + \frac{5}{8}$
50. $\frac{2}{3} + \frac{7}{8}$
51. $\frac{1}{4} + \frac{1}{5}$
52. $\frac{1}{4} + \frac{2}{5}$
53. $\frac{1}{4} + \frac{3}{5}$
54. $\frac{1}{4} + \frac{4}{5}$
55. $\frac{3}{4} + \frac{1}{5}$
56. $\frac{3}{4} + \frac{2}{5}$
57. $\frac{3}{4} + \frac{3}{5}$
58. $\frac{3}{4} + \frac{4}{5}$
59. $\frac{1}{4} + \frac{1}{8}$
60. $\frac{1}{4} + \frac{3}{8}$
61. $\frac{1}{4} + \frac{5}{8}$
62. $\frac{1}{4} + \frac{7}{8}$
63. $\frac{3}{4} + \frac{1}{8}$
64. $\frac{3}{4} + \frac{3}{8}$
65. $\frac{3}{4} + \frac{5}{8}$
66. $\frac{3}{4} + \frac{7}{8}$
67. $\frac{1}{4} + \frac{1}{10}$
68. $\frac{1}{4} + \frac{3}{10}$
69. $\frac{1}{4} + \frac{7}{10}$
70. $\frac{1}{4} + \frac{9}{10}$
71. $\frac{3}{4} + \frac{3}{10}$
72. $\frac{3}{4} + \frac{7}{10}$
73. $\frac{1}{4} + \frac{1}{16}$
74. $\frac{3}{4} + \frac{3}{16}$
75. $\frac{3}{4} + \frac{5}{16}$
76. $\frac{1}{3} + \frac{1}{7}$
77. $\frac{1}{4} + \frac{7}{16}$
78. $\frac{3}{8} + \frac{3}{19}$
79. $\frac{1}{4} + \frac{1}{7}$
80. $\frac{1}{4} + \frac{1}{9}$
81. $\frac{1}{4} + \frac{1}{11}$
82. $\frac{7}{8} + \frac{7}{16}$
83. $\frac{1}{5} + \frac{1}{6}$
84. $\frac{1}{5} + \frac{1}{7}$
85. $\frac{1}{5} + \frac{1}{8}$
86. $\frac{1}{5} + \frac{1}{9}$
87. $\frac{1}{5} + \frac{1}{11}$
88. $\frac{1}{5} + \frac{1}{12}$
89. $\frac{1}{5} + \frac{1}{13}$
90. $\frac{1}{6} + \frac{1}{7}$
91. $\frac{1}{6} + \frac{1}{11}$
92. $\frac{1}{6} + \frac{1}{13}$
93. $\frac{1}{7} + \frac{1}{8}$
94. $\frac{1}{7} + \frac{1}{9}$
95. $\frac{1}{7} + \frac{1}{10}$
96. $\frac{1}{7} + \frac{1}{11}$
97. $\frac{1}{7} + \frac{1}{2}$
98. $\frac{1}{8} + \frac{1}{11}$
99. $\frac{2}{5} + \frac{2}{7}$
100. $\frac{2}{5} + \frac{2}{9}$
101. $\frac{2}{5} + \frac{2}{11}$
102. $\frac{3}{5} + \frac{3}{8}$
103. $\frac{3}{5} + \frac{3}{11}$
104. $\frac{4}{7} + \frac{4}{9}$
105. $\frac{4}{7} + \frac{4}{11}$
106. $\frac{4}{7} + \frac{4}{13}$
107. $\frac{5}{9} + \frac{5}{12}$
108. $\frac{5}{8} + \frac{5}{16}$

106. Subtraction of Fractions. — Fractions having a common denominator are subtracted by subtracting the numerators. Fractions not having a common denominator must be reduced to a common denominator before any subtraction can be done.

ORAL EXERCISES (A)

1–108. Find the difference between each pair of fractions given on page 72.

Thus, Ex. 1 is $\frac{1}{2} - \frac{1}{4} = ?$

107. Subtracting Fractions from Integers. — To subtract a proper fraction from an integer, one unit of the integer must be changed to an improper fraction having the same denominator as the subtrahend.

Thus, $7 - \frac{5}{6} = 6\frac{6}{6} - \frac{5}{6} = 6\frac{1}{6}$.

ORAL EXERCISES (B)

Find the remainder in each of the following.

1. $8 - \frac{3}{5}$	**5.** $11 - \frac{7}{8}$	**9.** $17 - \frac{1}{2}$
2. $7 - \frac{5}{9}$	**6.** $13 - \frac{1}{9}$	**10.** $21 - \frac{1}{10}$
3. $6 - \frac{2}{3}$	**7.** $16 - \frac{2}{7}$	**11.** $19 - \frac{2}{11}$
4. $5 - \frac{1}{4}$	**8.** $4 - \frac{1}{5}$	**12.** $49 - \frac{7}{12}$

108. Subtraction of Mixed Numbers. — To subtract a fraction or a mixed number, first subtract the fractional part of the subtrahend, and then the integral part. In case the fraction in the subtrahend is greater than the fraction in the minuend, the latter must be changed into an improper fraction by changing one unit of the integral part of the minuend into a fraction.

Thus, $12\frac{1}{3} - 7\frac{1}{1} = 5\frac{1}{12}$, $12\frac{1}{3} - 7\frac{1}{3} = 11\frac{15}{12} - 7\frac{4}{12} = 4\frac{11}{12}$, $34 - 16\frac{4}{7} = 33\frac{7}{7} - 16\frac{4}{7} = 17\frac{3}{7}$.

ORAL EXERCISES (B)

Find the remainder in each of the following.

1. $7\frac{3}{4} - 2\frac{3}{5}$	**4.** $4\frac{5}{6} - 1\frac{2}{3}$	**7.** $7\frac{1}{3} - 6\frac{2}{3}$	**10.** $3\frac{2}{5} - 1\frac{3}{10}$
2. $9\frac{3}{5} - 2\frac{1}{3}$	**5.** $6\frac{7}{12} - 4\frac{1}{2}$	**8.** $9\frac{1}{5} - 5\frac{1}{6}$	**11.** $12\frac{14}{15} - 7\frac{1}{3}$
3. $8\frac{1}{6} - 4\frac{1}{7}$	**6.** $10\frac{7}{8} - 4\frac{1}{6}$	**9.** $11\frac{1}{2} - 4\frac{1}{7}$	**12.** $8\frac{7}{8} - 2\frac{2}{5}$

CHAPTER X

MULTIPLICATION AND DIVISION OF FRACTIONS

109. Multiplication and Division of Fractions in Business. — Multiplication of fractions occurs frequently in business. Thus, in extending bills one may find the numbers of things bought, and also the prices, expressed as fractions. Division of fractions occurs less frequently, but still often enough to make a knowledge of it necessary.

110. Product of a Fraction and an Integer. — We have already defined multiplication as a process of taking a number called the multiplicand as often as there are units in the multiplier. See § 33. The application of this definition when the multiplier is an integer is obvious, even when the multiplicand is a fraction or a mixed number.

Thus, $2 \times \frac{2}{3} = \frac{2}{3} + \frac{2}{3}$ and $3 \times \frac{4}{5} = \frac{4}{5} + \frac{4}{5} + \frac{4}{5}$.

Hence we see that $2 \times \frac{2}{3} = \frac{2 \times 2}{3}$, $3 \times \frac{4}{5} = \frac{3 \times 4}{5}$, and so on.

That is, *multiplying the numerator of a fraction by the integer multiplies the fraction.*

In case the numerator and denominator of the resulting fraction have a common factor it is best to cancel before multiplying.

Thus, in $8 \times \frac{7}{12} = \frac{56}{12} = \frac{14}{3} = 4\frac{2}{3}$ we cancel thus, $\overset{2}{\cancel{8}} \times \frac{7}{\cancel{12}_{3}} = \frac{14}{3} = 4\frac{2}{3}$.

ORAL EXERCISES

In this manner find the products of the following. Before multiplying cancel common factors if any.

1. $4 \times \frac{3}{5}$	**6.** $6 \times \frac{5}{9}$	**11.** $8 \times \frac{4}{5}$	**16.** $7 \times \frac{5}{12}$
2. $2 \times \frac{5}{6}$	**7.** $5 \times \frac{7}{8}$	**12.** $7 \times \frac{3}{5}$	**17.** $8 \times \frac{7}{12}$
3. $3 \times \frac{4}{9}$	**8.** $8 \times \frac{3}{4}$	**13.** $6 \times \frac{7}{12}$	**18.** $9 \times 1\frac{1}{2}$
4. $5 \times \frac{3}{7}$	**9.** $7 \times \frac{3}{5}$	**14.** $8 \times \frac{1}{7}$	**19.** $12 \times \frac{3}{4}$
5. $6 \times \frac{5}{8}$	**10.** $6 \times \frac{2}{3}$	**15.** $9 \times \frac{3}{5}$	**20.** $12 \times \frac{5}{8}$

111. Interchanging Multiplier and Multiplicand. — It is a general rule that the product of two numbers is the same no matter which is the multiplier and which is the multiplicand.

Thus, $6 \times 8 = 8 \times 6$, $7 \times 9 = 9 \times 7$, and so on.

Applying this rule, we see that $\frac{3}{5} \times 7 = 7 \times \frac{3}{5}$ and $\frac{2}{7} \times 8 = 8 \times \frac{2}{7}$. Hence $\frac{3}{5} \times 7 = 7 \times \frac{3}{5} = \frac{21}{5} = 4\frac{1}{5}$, and $\frac{2}{7} \times 8 = \frac{16}{7} = 2\frac{2}{7}$.

ORAL EXERCISES (A)

In this manner find the products of the following. Cancel all common factors before multiplying.

1. $\frac{3}{5} \times 6$	4. $\frac{2}{7} \times 3$	7. $\frac{4}{7} \times 2$	10. $\frac{3}{5} \times 9$
2. $\frac{2}{3} \times 7$	5. $\frac{1}{3} \times 9$	8. $\frac{8}{9} \times 4$	11. $\frac{1}{5} \times 19$
3. $\frac{5}{6} \times 4$	6. $\frac{3}{4} \times 12$	9. $\frac{9}{10} \times 3$	12. $\frac{3}{16} \times 5$

112. Product of Two Fractions. — From Figure 1 we see that $\frac{1}{2}$ of $\frac{1}{3}$ is $\frac{1}{6}$, and from Figure 2 that $\frac{2}{3}$ of $\frac{1}{2} = \frac{2}{6} = \frac{1}{3}$.

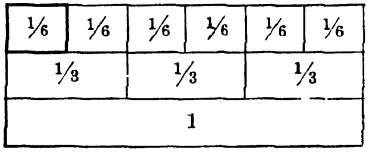


Figure 1.

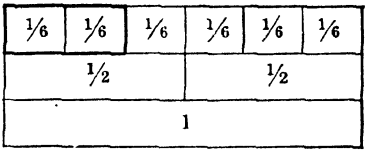


Figure 2.

Draw a diagram similar to Figure 1 to show that $\frac{1}{3}$ of $\frac{1}{2}$ is $\frac{1}{6}$.

Draw diagrams to show that $\frac{1}{2}$ of $\frac{1}{4} = \frac{1}{8}$, $\frac{1}{3}$ of $\frac{1}{4} = \frac{1}{12}$, $\frac{1}{2}$ of $\frac{1}{5} = \frac{1}{10}$.

Draw a diagram similar to Figure 2 to show that $\frac{1}{2}$ of $\frac{2}{3} = \frac{2}{6}$.

Draw diagrams to show that $\frac{3}{4}$ of $\frac{1}{2} = \frac{3}{8}$, $\frac{2}{3}$ of $\frac{1}{4} = \frac{2}{12}$, $\frac{3}{4}$ of $\frac{1}{3} = \frac{3}{12}$.

That is, $\frac{2}{3} \times \frac{1}{2} = \frac{2}{6}$, $\frac{1}{2} \times \frac{2}{3} = \frac{2}{6}$, $\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$, $\frac{2}{3} \times \frac{1}{4} = \frac{2}{12}$, $\frac{3}{4} \times \frac{1}{3} = \frac{3}{12}$.

These are instances of the general rule for multiplying one fraction by another:

To find the product of two fractions, multiply the numerators together and the denominators together.

That is, $\frac{4}{7} \times \frac{3}{5} = \frac{12}{35}$, and $\frac{5}{9} \times \frac{3}{10} = \frac{15}{90} = \frac{1}{6}$.

All possible cancellation should be performed before multiplying.

Thus, $\frac{\cancel{2}}{\cancel{3}} \times \frac{\cancel{3}}{\cancel{10}} = \frac{1}{6}$.
$\quad 3 \quad 2$

WRITTEN EXERCISES (A)

Find the product of each of the following, cancelling all factors common to the numerators and the denominators before multiplying.

1. $\frac{4}{5} \times \frac{3}{8}$
2. $\frac{3}{4} \times \frac{7}{9}$
3. $\frac{2}{3} \times \frac{5}{6}$
4. $\frac{1}{7} \times \frac{7}{10}$
5. $\frac{8}{9} \times \frac{5}{16}$
6. $\frac{3}{7} \times \frac{2}{5}$
7. $\frac{5}{12} \times \frac{2}{5}$
8. $\frac{9}{16} \times \frac{2}{3}$
9. $\frac{2}{3} \times \frac{2}{7}$
10. $\frac{3}{4} \times \frac{3}{7}$
11. $\frac{3}{4} \times \frac{5}{7}$
12. $\frac{5}{6} \times \frac{5}{9}$
13. $\frac{7}{8} \times \frac{2}{3}$
14. $\frac{3}{5} \times \frac{2}{7}$
15. $\frac{5}{16} \times \frac{8}{15}$
16. $\frac{9}{14} \times \frac{7}{27}$
17. $\frac{1}{6} \times \frac{3}{8}$
18. $\frac{17}{32} \times \frac{64}{85}$
19. $\frac{15}{39} \times \frac{78}{95}$
20. $\frac{9}{28} \times \frac{14}{45}$
21. $\frac{19}{47} \times \frac{94}{133}$
22. $\frac{8}{27} \times \frac{9}{32}$
23. $\frac{11}{17} \times \frac{85}{121}$
24. $\frac{15}{16} \times \frac{7}{90}$

113. Product of a Mixed Number and an Integer. — *To multiply a mixed number by an integer, multiply each part separately and then add the products.*

Thus, $3 \times 1\frac{4}{5} = 3 \times 1 + 3 \times \frac{4}{5} = 3 + \frac{12}{5} = 3 + 2\frac{2}{5} = 5\frac{2}{5}$.

ORAL EXERCISES (A)

Find the product of each of the following.

1. $4 \times 2\frac{1}{3}$
2. $3 \times 4\frac{1}{5}$
3. $2 \times 6\frac{2}{3}$
4. $5 \times 3\frac{2}{5}$
5. $8 \times 4\frac{3}{7}$
6. $9 \times 2\frac{3}{4}$
7. $7 \times 5\frac{1}{2}$
8. $5 \times 2\frac{5}{6}$
9. $4 \times 3\frac{7}{12}$
10. $6 \times 7\frac{2}{5}$
11. $9 \times 3\frac{2}{7}$
12. $10 \times 2\frac{1}{3}$
13. $12 \times 3\frac{5}{8}$
14. $21 \times 8\frac{2}{3}$
15. $36 \times 2\frac{4}{5}$
16. $16 \times 3\frac{7}{8}$

114. Product of a Mixed Number and a Fraction. — *To multiply a mixed number by a fraction, multiply each part separately and then add the products.*

Thus, $\frac{3}{4} \times 4\frac{2}{5} = \frac{3}{4} \times 4 + \frac{3}{\cancel{4}_{2}} \times \frac{\cancel{2}}{5} = 3 + \frac{3}{10} = 3\frac{3}{10}$.

WRITTEN EXERCISES (A)

Find the product of each of the following.

1. $\frac{1}{2} \times 5\frac{1}{3}$
2. $\frac{2}{3} \times 6\frac{2}{5}$
3. $\frac{1}{5} \times 4\frac{1}{2}$
4. $\frac{2}{7} \times 3\frac{1}{4}$
5. $\frac{9}{10} \times 7\frac{1}{16}$
6. $\frac{3}{4} \times 5\frac{1}{2}$
7. $\frac{7}{8} \times 9\frac{2}{3}$
8. $\frac{5}{6} \times 3\frac{7}{8}$
9. $\frac{2}{9} \times 9\frac{1}{3}$
10. $\frac{7}{12} \times 6\frac{1}{7}$
11. $\frac{1}{4} \times 2\frac{1}{6}$
12. $\frac{3}{10} \times 5\frac{3}{4}$
13. $3\frac{4}{5} \times \frac{7}{8}$
14. $\frac{5}{9} \times 6\frac{2}{3}$
15. $12\frac{1}{3} \times \frac{3}{4}$
16. $\frac{7}{12} \times 4\frac{7}{16}$

115. Product of Two Mixed Numbers.

Problem. Multiply $6\frac{3}{4}$ by $8\frac{2}{3}$.

Solution: $6\frac{3}{4} \times 8\frac{2}{3} = \frac{27}{4} \times \frac{26}{3} = \frac{117}{2} = 58\frac{1}{2}$.

Four-step process:

Step 1. $6 \times \frac{2}{3} = 4$.
Step 2. $8 \times \frac{3}{4} = 6$.
Step 3. $\frac{3}{4} \times \frac{2}{3} = \frac{1}{2}$.
Step 4. $6 \times 8 = 48$.

$$\begin{array}{r} 6\frac{3}{4} \\ 8\frac{2}{3} \\ \hline 4 \\ 6 \\ \frac{1}{2} \\ 48 \\ \hline 58\frac{1}{2} \end{array}$$

WRITTEN EXERCISES (A)

Find the product of each of the following.

1. $2\frac{1}{2} \times 3\frac{2}{3}$	**12.** $11\frac{2}{5} \times 2\frac{1}{2}$	**23.** $9\frac{1}{4} \times 4\frac{1}{6}$
2. $5\frac{1}{3} \times 4\frac{1}{2}$	**13.** $10\frac{1}{4} \times 4\frac{1}{3}$	**24.** $4\frac{1}{2} \times 2\frac{3}{8}$
3. $6\frac{3}{4} \times 7\frac{1}{6}$	**14.** $5\frac{1}{2} \times 7\frac{1}{8}$	**25.** $10\frac{3}{8} \times 5\frac{2}{5}$
4. $3\frac{7}{8} \times 2\frac{1}{5}$	**15.** $4\frac{1}{7} \times 9\frac{2}{3}$	**26.** $20\frac{1}{4} \times 4\frac{5}{9}$
5. $4\frac{1}{3} \times 6\frac{2}{9}$	**16.** $6\frac{3}{8} \times 12\frac{7}{9}$	**27.** $21\frac{4}{5} \times 3\frac{3}{8}$
6. $3\frac{1}{3} \times 4\frac{2}{3}$	**17.** $3\frac{5}{6} \times 10\frac{3}{8}$	**28.** $14\frac{1}{8} \times 9\frac{2}{5}$
7. $2\frac{1}{2} \times 5\frac{1}{5}$	**18.** $2\frac{9}{10} \times 11\frac{1}{3}$	**29.** $6\frac{1}{6} \times 5\frac{1}{4}$
8. $3\frac{1}{3} \times 4\frac{1}{4}$	**19.** $1\frac{4}{5} \times 9\frac{5}{6}$	**30.** $3\frac{2}{3} \times 4\frac{2}{9}$
9. $7\frac{1}{6} \times 3\frac{1}{2}$	**20.** $7\frac{2}{5} \times 8\frac{1}{3}$	**31.** $14\frac{1}{3} \times 3\frac{1}{4}$
10. $6\frac{2}{9} \times 8\frac{3}{4}$	**21.** $15\frac{2}{3} \times 5\frac{2}{5}$	**32.** $21\frac{1}{2} \times 12\frac{1}{3}$
11. $12\frac{1}{3} \times 1\frac{2}{3}$	**22.** $12\frac{2}{5} \times 7\frac{2}{9}$	**33.** $4\frac{1}{7} \times 3\frac{1}{6}$

PROBLEMS (A)

1. If one pound of sugar costs $10\frac{1}{4}¢$, what will 25 pounds cost?

2. If a boy earns $62\frac{1}{2}¢$ per day, what will he earn in $8\frac{1}{3}$ days?

3. Of a flock of 320 sheep, $\frac{1}{4}$ are valued at \$12 each, $\frac{1}{6}$ of the remainder at \$10 each. The rest are valued at \$8 each. What is the average value of the sheep?

4. If $7\frac{1}{2}$ tons of feed are worth \$245, what will $11\frac{2}{3}$ tons be worth?

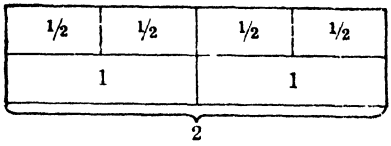
5. At the rate of 24 cents a square inch, what is the cost of making an engraving $2\frac{1}{2}'' \times 3\frac{3}{4}''$?

116. Fractions Divided by an Integer. — To divide a fraction by an integer, either (1) *Multiply* the denominator by the integer, the result becoming the new denominator; or (2) *Divide* the numerator by the integer, the result being the new numerator. This latter method is used only when the integer is an exact divisor of the numerator.

Thus, $\frac{1}{3} \div 2 = \frac{1}{6}$, $\frac{5}{7} \div 3 = \frac{5}{21}$, and $\frac{7}{8} \div 4 = \frac{7}{32}$.
$\frac{2}{3} \div 2 = \frac{1}{3}$, $\frac{3}{7} \div 3 = \frac{1}{7}$, and $\frac{4}{9} \div 2 = \frac{2}{9}$.

In this manner find the quotients in the following.

1. $\frac{5}{8} \div 3$	5. $\frac{1}{2} \div 4$	9. $\frac{5}{7} \div 2$	13. $\frac{7}{8} \div 5$
2. $\frac{3}{7} \div 4$	6. $\frac{3}{8} \div 5$	10. $\frac{8}{9} \div 4$	14. $\frac{3}{5} \div 8$
3. $\frac{4}{5} \div 7$	7. $\frac{9}{10} \div 6$	11. $\frac{3}{5} \div 5$	15. $\frac{7}{12} \div 14$
4. $\frac{1}{9} \div 5$	8. $\frac{3}{7} \div 3$	12. $\frac{6}{11} \div 3$	16. $\frac{3}{16} \div 9$



From the diagram we see that $\frac{1}{2}$ is contained 4 times in 2.

That is. $2 \div \frac{1}{2} = 4$.

Draw diagrams to show that $2 \div \frac{1}{3} = 6$, $3 \div \frac{1}{3} = 9$.

It is not so easy, however, to illustrate in this manner the division of 2 by $\frac{3}{4}$. To obtain a general rule for division of fractions, we make use of the definition of division:

117. Definition of Division. — *Division is the process of finding one of two factors when their product and one of the factors are given.*

118. Dividing an Integer by a Fraction. — The known product is the *dividend* and the known factor is the *divisor*.

Hence, to divide 2 by $\frac{3}{4}$ is the same as finding the missing number in $\frac{3}{4} \times ? = 2$.

If $\frac{3}{4}$ of the missing number is 2, then $\frac{1}{4}$ of it is $\frac{1}{3}$ of 2, or $\frac{2}{3}$.
Hence, the missing number is $4 \times \frac{2}{3} = \frac{8}{3}$.
We test the work by multiplying the quotient by the divisor. The result should be the dividend.

That is, $\frac{3}{4} \times \frac{2}{\frac{8}{3}} = 2$.

ORAL EXERCISES (A)

In the manner shown on page 78 find the missing number in each of the following. Also state each example by using the division sign. Thus, instead of $\frac{4}{5} \times ? = 2$, write $2 \div \frac{4}{5}$.

1. $\frac{4}{5} \times ? = 2$
2. $\frac{7}{8} \times ? = 3$
3. $\frac{5}{6} \times ? = 5$
4. $\frac{3}{7} \times ? = 7$
5. $\frac{3}{8} \times ? = 8$
6. $\frac{2}{3} \times ? = 3$
7. $\frac{5}{8} \times ? = 6$
8. $\frac{3}{10} \times ? = 10$

119. Dividing by a Fraction. — From the examples above we may derive the following rule:

To divide by a fraction, invert the terms of the divisor and multiply.

Thus, to divide by $\frac{3}{5}$, multiply by $\frac{5}{3}$.

ORAL EXERCISES (A)

Find the quotient in each of the following.

1. $\frac{1}{2} \div \frac{2}{3}$
2. $\frac{2}{3} \div \frac{1}{4}$
3. $\frac{3}{4} \div \frac{5}{6}$
4. $\frac{4}{7} \div \frac{3}{7}$
5. $\frac{2}{5} \div \frac{4}{9}$
6. $\frac{3}{8} \div \frac{5}{6}$
7. $\frac{9}{11} \div \frac{4}{11}$
8. $\frac{11}{15} \div \frac{3}{8}$
9. $\frac{6}{7} \div \frac{3}{4}$
10. $\frac{9}{11} \div \frac{9}{10}$
11. $\frac{2}{3} \div \frac{7}{9}$
12. $\frac{4}{5} \div \frac{8}{11}$

120. Division of Mixed Numbers. — If the divisor is not an integer, and if either the dividend or the divisor is a mixed number, multiply the divisor and dividend by the common denominator of the fractions. Then proceed as in § 59.

Thus, $2\frac{1}{4} \div \frac{2}{3} = \frac{2\frac{1}{4} \times 12}{\frac{2}{3} \times 12} = \frac{27}{8} = 3\frac{3}{8}$, and $5 \div 1\frac{4}{7} = \frac{5 \times 7}{1\frac{4}{7} \times 7} = \frac{35}{11} = 3\frac{2}{11}$.

WRITTEN EXERCISES (B)

Find the quotient in each of the following.

1. $3\frac{1}{2} \div \frac{4}{5}$
2. $5 \div 2\frac{3}{4}$
3. $4\frac{1}{3} \div 3\frac{1}{2}$
4. $5\frac{1}{2} \div 1\frac{7}{8}$
5. $9 \div 5\frac{1}{2}$
6. $8\frac{3}{4} \div 3\frac{2}{3}$
7. $5\frac{1}{2} \div 9\frac{2}{3}$
8. $6\frac{2}{3} \div \frac{1}{4}$
9. $4\frac{3}{8} \div \frac{2}{5}$
10. $1\frac{4}{5} \div 3\frac{1}{7}$
11. $8 \div 4\frac{1}{2}$
12. $4\frac{1}{5} \div 3\frac{1}{6}$
13. $7\frac{1}{2} \div 4$
14. $9\frac{1}{4} \div 2\frac{1}{3}$
15. $4 \div 4\frac{1}{7}$
16. $2\frac{2}{3} \div 8\frac{1}{2}$
17. $5\frac{6}{7} \div 2\frac{3}{5}$
18. $6\frac{9}{10} \div 3\frac{3}{10}$
19. $4\frac{1}{2} \div 6\frac{1}{3}$
20. $10\frac{3}{5} \div 4\frac{1}{5}$
21. $11\frac{4}{7} \div 2$
22. $2\frac{1}{8} \div 8\frac{1}{3}$
23. $12\frac{3}{5} \div 9$
24. $6\frac{3}{7} \div 4\frac{3}{5}$
25. $12 \div 4\frac{1}{2}$
26. $9\frac{3}{8} \div 2\frac{1}{3}$
27. $6\frac{1}{7} \div 8\frac{1}{3}$
28. $4\frac{2}{9} \div 3\frac{1}{7}$

121. Dividing a Mixed Number by an Integer. — If the dividend is a mixed number, and the divisor is an integer, then the division should be carried out as follows.

$4\frac{2}{5} \div 3 = 4 \div 3 + \frac{2}{5} \div 3 = 1\frac{1}{3} + \frac{2}{15} = 1\frac{7}{15}$.

Again $169\frac{2}{3} \div 5 = 169 \div 5 + \frac{2}{3} \div 5 = 33\frac{4}{5} + \frac{2}{15} = 33\frac{14}{15}$.

In these cases, reducing the dividend to an improper fraction would be more laborious.

WRITTEN EXERCISES (B)

Find the quotient in each of the following.

1. $43\frac{5}{6} \div 7$	**7.** $228\frac{7}{8} \div 19$	**13.** $381\frac{5}{8} \div 8$
2. $341\frac{2}{3} \div 16$	**8.** $792\frac{11}{13} \div 14$	**14.** $673\frac{4}{9} \div 13$
3. $75\frac{1}{2} \div 6$	**9.** $181\frac{2}{7} \div 13$	**15.** $152\frac{7}{10} \div 17$
4. $119\frac{1}{4} \div 8$	**10.** $344\frac{11}{18} \div 9$	**16.** $139\frac{1}{3} \div 5$
5. $223\frac{5}{9} \div 9$	**11.** $1724\frac{1}{2} \div 14$	**17.** $242\frac{1}{4} \div 3$
6. $183\frac{5}{16} \div 15$	**12.** $259\frac{3}{16} \div 12$	**18.** $619\frac{1}{2} \div 6$

PROBLEMS (C)

1. Turning a certain screw around once drives it $\frac{5}{16}$ of an inch into the wood. How many times must it be turned around to enter the wood $1\frac{7}{8}$ inches?

2. If a train travels 30 miles in $\frac{3}{4}$ of an hour, how far will it travel in $9\frac{5}{6}$ hours?

3. A man gave to one son $\frac{1}{3}$ of his property, to another $\frac{1}{6}$, to another $\frac{1}{9}$, and the balance to a fourth who received \$1100. What was his estate, and what did each receive?

4. If a wheel travels over $10\frac{2}{3}$ feet at each revolution, how many times will it turn around in going a mile?

5. If $\frac{15}{16}$ of a farm of 150 acres cost \$18,000, what was the price per acre?

6. Two men hired an automobile for \$200: the first man used it 21 days; the second used it 29 days. What should each pay?

PROBLEMS (B)

1. A man owned $\frac{4}{9}$ of a shop and sold $\frac{2}{5}$ of his share. What fraction of the shop did he sell?

2. If a cow eats $1\frac{1}{4}$ tons of hay in one month, how much hay will she eat in $3\frac{1}{2}$ months?

3. A farmer finds that his wheat field yields on an average $17\frac{1}{4}$ bu. per acre. At this rate how much should he expect from a field containing $23\frac{1}{2}$ acres?

4. A room is $16\frac{1}{2}$ feet long and $14\frac{1}{4}$ feet wide. How many square feet of lumber should be bought to put a hard wood floor in this room if $\frac{1}{6}$ of the lumber is wasted in matching and fitting?

5. A man sold $\frac{3}{8}$ of his farm land at one time and $\frac{3}{16}$ of what remained at another time. What fraction of the whole farm did he sell the second time?

6. At $3\frac{3}{4}$ dollars per square yard, what is the cost of laying a sidewalk $17\frac{2}{3}$ yards long and $1\frac{2}{3}$ yard wide?

7. At $62\frac{1}{2}$ cents a yard, what is the cost of $7\frac{3}{4}$ yards of cloth?

8. A certain automobile averages $13\frac{1}{4}$ miles to the gallon of gasoline. At this rate how far will it go on a tank filled with $17\frac{1}{2}$ gallons? At 27 cents a gallon how much will this gasoline cost?

9. If $9\frac{3}{4}$ tons of hay are worth \$150, what will $12\frac{5}{6}$ tons cost?

10. If $\frac{3}{4}$ of an article is worth \$150, find the value of $\frac{1}{1}\frac{5}{6}$ of it.

11. A boy catches 18 trout weighing all together 13 pounds. At this rate how much would 54 such trout weigh?

12. A farmer raises 376 bushels of wheat on $19\frac{3}{4}$ acres. What is the average yield per acre?

13. A man runs his automobile 53 miles on $3\frac{1}{4}$ gallons of gasoline. At this rate how many gallons will he need to go 270 miles?

14. A herd of 27 cows consumes $58\frac{1}{2}$ tons of hay in one year. What is the average consumption per cow? At this rate how many tons would 67 cows consume in one year?

ORAL EXERCISES (A)

Find the products of the following.

1. $7 \times \frac{3}{8}$
2. $5 \times \frac{8}{15}$
3. $12 \times \frac{2}{9}$
4. $8 \times \frac{3}{16}$
5. $6 \times \frac{3}{7}$
6. $15 \times \frac{8}{17}$
7. $21 \times \frac{9}{16}$
8. $24 \times \frac{3}{17}$
9. $15 \times \frac{2}{9}$
10. $20 \times \frac{5}{11}$
11. $12 \times \frac{7}{8}$
12. $8 \times \frac{5}{12}$
13. $2 \times 1\frac{1}{3}$
14. $7 \times 2\frac{3}{4}$
15. $3 \times 3\frac{1}{2}$
16. $15 \times \frac{1}{11}$
17. $19 \times 3\frac{1}{7}$
18. $15 \times 4\frac{2}{3}$
19. $18 \times 7\frac{1}{3}$
20. $16 \times \frac{4}{19}$
21. $7 \times \frac{3}{14}$
22. $\frac{1}{3} \times 6\frac{1}{2}$
23. $\frac{2}{3} \times 4\frac{2}{3}$
24. $\frac{3}{6} \times 2\frac{1}{3}$

Find the quotients of the following.

25. $\frac{1}{2} \div 3$
26. $\frac{2}{3} \div 4$
27. $\frac{7}{8} \div 6$
28. $\frac{5}{6} \div 7$
29. $\frac{9}{10} \div 8$
30. $1\frac{1}{2} \div 9$
31. $1\frac{4}{15} \div 8$
32. $\frac{9}{16} \div 12$
33. $\frac{4}{5} \div 5$
34. $\frac{1}{3} \div 4$
35. $\frac{5}{9} \div 9$
36. $1\frac{3}{4} \div 8$
37. $8\frac{2}{3} \div 12$
38. $\frac{5}{16} \div 16$
39. $\frac{9}{17} \div 3$
40. $3\frac{5}{6} \div 8$
41. $1\frac{5}{19} \div 5$
42. $8\frac{1}{2} \div 4$
43. $7\frac{1}{3} \div 11$
44. $5\frac{1}{4} \div 6$
45. $8\frac{1}{5} \div 14$
46. $1\frac{5}{23} \div 6$
47. $1\frac{6}{27} \div 9$
48. $1\frac{5}{23} \div 5$
49. $\frac{3}{4} \div 7$
50. $\frac{9}{16} \div 6$
51. $\frac{5}{9} \div 4$
52. $1\frac{3}{4} \div 5$
53. $5\frac{2}{3} \div 4$
54. $4\frac{3}{5} \div 6$
55. $5\frac{1}{4} \div 7$
56. $7\frac{3}{5} \div 8$

WRITTEN EXERCISES (B)

For special arrangement, see § 115.

1. $1\frac{1}{3} \times 2\frac{1}{2}$
2. $2\frac{1}{2} \times 3\frac{2}{3}$
3. $1\frac{1}{3} \times 4\frac{1}{5}$
4. $4\frac{1}{2} \times 2\frac{1}{2}$
5. $9\frac{1}{3} \times 9\frac{1}{4}$
6. $8\frac{1}{2} \times 3\frac{1}{2}$
7. $11\frac{1}{5} \times 4\frac{1}{6}$
8. $9\frac{1}{6} \times 7\frac{1}{8}$
9. $7\frac{8}{9} \times 5\frac{1}{3}$
10. $6\frac{5}{6} \times 1\frac{1}{3}$
11. $5\frac{1}{3} \times 4\frac{1}{8}$
12. $9\frac{1}{2} \times 1\frac{1}{19}$
13. $4\frac{1}{2} \times 7\frac{1}{3}$
14. $3\frac{1}{3} \times 4\frac{1}{4}$
15. $1\frac{1}{4} \times 3\frac{2}{5}$
16. $6\frac{1}{8} \times 1\frac{1}{7}$
17. $7\frac{1}{5} \times 4\frac{1}{5}$
18. $10\frac{1}{4} \times 9\frac{1}{3}$
19. $15\frac{1}{6} \times 11\frac{1}{13}$
20. $8\frac{2}{7} \times 2\frac{1}{8}$
21. $8\frac{2}{3} \times 5\frac{2}{3}$
22. $1\frac{3}{1} \times 7\frac{5}{8}$
23. $15\frac{3}{8} \times 16\frac{1}{4}$
24. $19\frac{1}{2} \times 14\frac{1}{3}$
25. $5\frac{1}{2} \times 1\frac{1}{11}$
26. $19\frac{1}{3} \times 7\frac{1}{5}$
27. $52\frac{1}{2} \times 2\frac{1}{35}$

ORAL EXERCISES (A)

1. $\frac{4}{5} \div \frac{7}{8}$
2. $\frac{3}{7} \div \frac{2}{3}$
3. $\frac{1}{4} \div \frac{3}{4}$
4. $\frac{1}{5} \div \frac{4}{5}$
5. $\frac{1}{6} \div \frac{3}{7}$
6. $\frac{2}{5} \div \frac{1}{2}$
7. $\frac{4}{7} \div \frac{3}{14}$
8. $\frac{9}{10} \div \frac{3}{21}$
9. $\frac{8}{15} \div \frac{5}{6}$
10. $\frac{17}{19} \div \frac{7}{38}$
11. $\frac{11}{19} \div \frac{5}{38}$
12. $\frac{2}{9} \div \frac{1}{3}$
13. $\frac{3}{8} \div \frac{4}{5}$
14. $\frac{11}{13} \div \frac{19}{26}$
15. $\frac{1}{21} \div \frac{5}{9}$
16. $\frac{5}{6} \div \frac{4}{5}$
17. $\frac{1}{4} \div \frac{7}{8}$
18. $\frac{7}{8} \div \frac{5}{9}$
19. $\frac{11}{13} \div \frac{1}{21}$
20. $\frac{14}{15} \div \frac{5}{21}$
21. $\frac{3}{5} \div \frac{5}{9}$
22. $\frac{7}{8} \div \frac{4}{7}$
23. $\frac{9}{10} \div \frac{8}{11}$
24. $\frac{11}{12} \div \frac{12}{17}$
25. $\frac{15}{16} \div \frac{8}{11}$
26. $\frac{17}{18} \div \frac{5}{6}$
27. $\frac{4}{9} \div \frac{3}{4}$
28. $\frac{7}{18} \div \frac{7}{10}$
29. $\frac{4}{9} \div \frac{3}{8}$
30. $\frac{6}{11} \div \frac{11}{12}$
31. $\frac{5}{9} \div \frac{5}{6}$
32. $\frac{9}{16} \div \frac{7}{8}$

WRITTEN EXERCISES (B)

1. $3\frac{9}{10} \div 2$
2. $7\frac{4}{5} \div 5$
3. $2\frac{2}{3} \div 3$
4. $11\frac{1}{2} \div 4$
5. $6\frac{1}{8} \div 6$
6. $9\frac{1}{5} \div 9$
7. $4\frac{7}{8} \div 11$
8. $11\frac{5}{16} \div 2$
9. $8\frac{2}{3} \div 7$
10. $9\frac{7}{16} \div 6$
11. $24\frac{2}{3} \div 8$
12. $19\frac{7}{16} \div 12$
13. $8\frac{5}{8} \div 11$
14. $9\frac{2}{7} \div 3$
15. $18\frac{7}{9} \div 8$
16. $15\frac{2}{3} \div 6$
17. $11\frac{5}{8} \div 4$
18. $19\frac{1}{5} \div 9$
19. $21\frac{7}{9} \div 11$
20. $50\frac{1}{2} \div 15$
21. $1\frac{3}{8} \div 5$
22. $5\frac{1}{3} \div 7$
23. $7\frac{2}{3} \div 9$
24. $11\frac{5}{6} \div 3$
25. $19\frac{1}{3} \div 11$
26. $14\frac{1}{8} \div 18$
27. $17\frac{2}{3} \div 14$
28. $16\frac{1}{4} \div 7$
29. $15\frac{1}{8} \div 8$
30. $19\frac{1}{2} \div 3$
31. $7\frac{3}{7} \div 9$
32. $41\frac{5}{8} \div 12$

WRITTEN EXERCISES (C)

1. $1\frac{3}{4} \div 1\frac{1}{2}$
2. $4\frac{2}{5} \div 1\frac{3}{4}$
3. $3\frac{1}{2} \div 3\frac{1}{4}$
4. $5\frac{2}{3} \div 3\frac{1}{6}$
5. $4\frac{1}{7} \div 2\frac{1}{21}$
6. $9\frac{2}{3} \div 5\frac{2}{3}$
7. $16\frac{1}{2} \div 5\frac{1}{2}$
8. $3\frac{2}{3} \div 4\frac{1}{5}$
9. $6\frac{7}{8} \div 9\frac{2}{3}$
10. $9\frac{1}{2} \div 7\frac{1}{8}$
11. $5\frac{1}{4} \div 8\frac{2}{3}$
12. $16\frac{2}{3} \div 4\frac{1}{6}$
13. $11\frac{1}{2} \div 7\frac{2}{3}$
14. $19\frac{1}{4} \div 5\frac{1}{2}$
15. $7\frac{1}{2} \div 9\frac{1}{3}$
16. $9\frac{1}{3} \div 7\frac{1}{6}$
17. $10\frac{2}{3} \div 2\frac{1}{3}$
18. $16\frac{3}{8} \div 3\frac{5}{8}$
19. $7\frac{1}{8} \div 4\frac{5}{7}$
20. $9\frac{2}{3} \div 9\frac{1}{6}$
21. $23\frac{1}{3} \div \frac{1}{8}$
22. $11\frac{1}{2} \div 1\frac{1}{12}$
23. $9\frac{2}{3} \div 2\frac{2}{3}$
24. $50\frac{1}{3} \div 9\frac{1}{6}$
25. $61\frac{11}{12} \div 7\frac{2}{11}$
26. $9\frac{1}{8} \div 3\frac{1}{4}$
27. $11\frac{1}{6} \div 5\frac{2}{3}$
28. $17\frac{2}{3} \div 9\frac{1}{6}$
29. $21\frac{1}{5} \div 1\frac{1}{10}$
30. $14\frac{1}{8} \div 12\frac{1}{2}$

MISCELLANEOUS ORAL PROBLEMS (A)

1. The rainfall in May was $3\frac{1}{4}$ inches and in June it was $2\frac{3}{8}$ inches. What was the rainfall in the two months?

2. John is $68\frac{1}{2}$ inches tall and Tom is $66\frac{1}{2}$ inches tall. What is their average height?

3. Mae is $64\frac{5}{8}$ inches tall and Jane is $63\frac{1}{4}$ inches tall. How much must Jane grow before she is as tall as Mae?

4. A carpenter puts a screw $2\frac{3}{4}$ inches long into a built-up plank $3\frac{1}{8}$ inches thick. By how much does the screw fail to reach through the plank?

5. Mrs. Ward buys $7\frac{3}{8}$ yards of goods for two dresses. She uses $3\frac{3}{4}$ yards for one dress; how many yards does she have left for the other dress?

6. Clair did $\frac{1}{2}$ of her practicing in the morning and $\frac{1}{3}$ of it in the afternoon. What fraction of her practicing does she still have left to do?

7. Mrs. Stanton ordered $4\frac{1}{2}$ pounds of meat and the market sent her $5\frac{3}{8}$ pounds. How much more than she ordered did the market send?

8. Pete is 15 years and 8 months ($15\frac{2}{3}$ years) old, and Sam is 14 years and 9 months ($14\frac{3}{4}$ years) old. What is the sum of their ages? What is the average of their ages? What is the difference of their ages?

9. Two bricks, each $2\frac{1}{8}$ inches thick, are placed on edge side by side with a layer of mortar $\frac{1}{4}$ inch thick between them. What is the total thickness of the bricks and the mortar?

10. A book $7\frac{3}{4}$ inches high is placed on a book shelf $8\frac{1}{2}$ inches high. What is the space between the top of the book and the shelf above it?

11. A book shelf is $8\frac{1}{2}$ inches deep. A strip of wood $1\frac{1}{8}$ inches thick is put at the back of the shelf to keep the books from going too far in. How much space is left for the books?

ORAL PROBLEMS (B)

In the following, answer all questions and supply all missing words and numbers.

1. To take $\frac{1}{6}$ of a number is the same as to divide the number by ____.

2. To take $\frac{5}{6}$ of a number take 5 times ____ of it.

3. To divide a fraction by 4 divide its ____ by 4 or multiply its ____ by 4.

4. If you multiply a number by its reciprocal, the product is ____. (Multiply 6 by $\frac{1}{6}$, and also $\frac{3}{4}$ by $\frac{1}{3}$.)

5. If you know the distance a man travels and the time it takes, how do you find his speed?

6. A wheel goes $8\frac{3}{8}$ feet in one revolution. How far does it go in 8 revolutions? in 80 revolutions? in 800 revolutions?

7. An airplane goes 120 miles in 45 minutes ($\frac{3}{4}$ of an hour). What is the speed in miles per hour?

8. I have done $\frac{7}{8}$ of my work and today I expect to do $\frac{1}{2}$ of what is left. How much will I have left to do then?

9. If a farmer feeds $\frac{5}{8}$ ton of hay to each cow in one month, how much will he feed to 16 cows in 4 months?

10. One farmer feeds 28 tons of hay to his 12 cows. How much does he average per cow? Reduce the answer to the simplest form.

11. At $66\frac{2}{3}$ cents per yard what is the cost of 4 yards of cloth?

12. If 8 yards of cloth are sold for \$6, what is the cost per yard?

13. A clerk sold $2\frac{1}{2}$ yards of cloth, then $3\frac{1}{4}$ yards and $1\frac{7}{8}$ yards, all from the same piece. How many yards did he sell?

14. If 27 inches are cut off from 1 yard, what fraction of the yard remains?

15. If cork weighs twenty-four one hundredths (.24) times as much as water, how many times as much as cork does water weigh?

CHAPTER XI

REVIEW DRILL IN FUNDAMENTAL OPERATIONS

DRILL IN ADDITION (A)

In Examples 1–5 add both vertically and horizontally.

1.

48	+ 56	+ 91	=
74	+ 37	+ 45	=
19	+ 63	+ 38	=
28	+ 10	+ 75	=
14	+ 81	+ 35	=
	+	+	=

2.

18	+ 29	+ 16	=
42	+ 92	+ 81	=
76	+ 28	+ 55	=
40	+ 96	+ 21	=
64	+ 84	+ 16	=
	+	+	=

3.

34	+ 11	+ 27	=
15	+ 51	+ 72	=
23	+ 57	+ 82	=
59	+ 31	+ 48	=
52	+ 31	+ 42	=
	+	+	=

4.

343	+ 112	+ 249	=
142	+ 247	+ 197	=
372	+ 400	+ 432	=
171	+ 111	+ 100	=
241	+ 683	+ 261	=
	+	+	=

5.

4874	+ 8780	+ 9098	=
6329	+ 7463	+ 5382	=
2637	+ 3055	+ 6668	=
3763	+ 4684	+ 8274	=
5731	+ 8171	+ 4820	=
	+	+	=

6. VALUE IN MILLIONS OF DOLLARS OF EXPORTS OF THE UNITED STATES

YEARLY AVERAGE	CRUDE MATERIALS	CRUDE FOODSTUFFS	MANUFACTURED FOODSTUFFS	SEMIMANUFACTURES	FINISHED MANUFACTURES	TOTAL VALUE
1876–1880	214	159	162	30	99	
1881–1885	262	163	197	37	116	
1886–1890	277	109	182	40	119	
1891–1895	295	151	239	55	136	
1896–1900	297	215	273	110	242	
1901–1905	432	174	316	161	344	
1906–1910	555	156	317	249	474	
1911–1915	717	206	334	359	716	
1916–1920	1169	588	1133	987	2540	
1921–1925	1187	420	601	537	1566	
1926–1930	1144	300	456	663	2126	

Find the total for each period. Do not copy. Add horizontally.

DRILL IN ADDITION AND SUBTRACTION (B)

1. In three successive years, the United States imported from and exported to the countries named below as follows:

Country	Exports in Millions of Dollars			Imports in Millions of Dollars		
	1st year	2d year	3d year	1st year	2d year	3d year
Bermuda . . .	3.8	3.5	4.0	1.1	0.9	0.8
British Honduras	2.1	1.8	1.9	3.7	2.9	3.3
Canada . . .	836.5	914.7	948.5	475.0	489.3	504.3
Newfoundland and Labrador .	8.5	9.2	12.5	9.1	10.1	10.4

Copy these data on a suitable blank.

Find the total imports from each country for the three years; the total imports from all countries for each year, for the three years. Find the same for the exports. Find the balance of trade for or against each country for each year, for the three years; for all the countries for each year, for the three years.

Add horizontally.

(The balance of trade in favor of a country is the amount by which the exports of that country exceed the imports. The balance of trade against a country is the amount by which imports exceed exports.)

2. In three successive years the imports and exports of the United States to the countries named below were as follows. Find results similar to those required under Example 1.

Copy the data on a suitable blank.

Country	Exports in Millions of Dollars			Imports in Millions of Dollars		
	1st year	2d year	3d year	1st year	2d year	3d year
Europe	2313.8	2374.9	2340.7	1276.5	1248.7	1332.7
North America .	845.3	924.2	961.5	484.5	500.0	515 2
Latin America .	845.9	878.0	973.0	1019.2	1030.2	1106.0
Asia	659.6	654.5	643.4	1256.8	1169.0	1280.3
Oceania . . .	193.7	180.0	192.0	54.5	53.5	56.6
Africa	107.1	116.7	130.5	93.3	90.2	108.6

DRILL IN MULTIPLICATION (A)

In the manner explained in § 48, find the products of the following.

1. 8×46	**5.** 7×58	**9.** 5×78
2. 9×36	**6.** 6×94	**10.** 7×44
3. 6×89	**7.** 9×83	**11.** 3×97
4. 8×74	**8.** 4×49	**12.** 4×76

In the manner explained in § 49, find the products of the following.

13. 84×6498	**17.** 59×8140	**21.** 93×4217
14. 35×4806	**18.** 68×1930	**22.** 78×1860
15. 47×3904	**19.** 74×3940	**23.** 39×4821
16. 63×1486	**20.** 94×6230	**24.** 62×3108

In the manner explained in § 47, find the products of the following.

25. 67×49	**30.** 59×121	**35.** 91×780
26. 83×24	**31.** 19×762	**36.** 45×225
27. 79×144	**32.** 31×112	**37.** 55×362
28. 99×326	**33.** 75×240	**38.** 41×118
29. 125×272	**34.** 250×104	**39.** 27×96

Multiply. Check by casting out 9's. See § 43.

40. 8491 846	**45.** 7548 119	**50.** 12524 361	**55.** 91764 237
41. 5604 391	**46.** 8327 209	**51.** 78591 2945	**56.** 84679 875
42. 7892 4104	**47.** 4954 3045	**52.** 39486 1057	**57.** 40276 894
43. 9194 726	**48.** 8759 4526	**53.** 79355 20507	**58.** 96270 798
44. 3247 2649	**49.** 7863 2008	**54.** 82451 71248	**59.** 18141 456

DRILL IN DIVISION (B)

Divide. Check by casting out 9's. See § 64.

1.	$8491 \div 846$	**11.**	$7548 \div 119$	**21.**	$12524 \div 261$
2.	$5604 \div 391$	**12.**	$8327 \div 209$	**22.**	$14263 \div 521$
3.	$7892 \div 401$	**13.**	$4954 \div 345$	**23.**	$785912 \div 945$
4.	$9194 \div 726$	**14.**	$87594 \div 526$	**24.**	$519492 \div 587$
5.	$74320 \div 2649$	**15.**	$78632 \div 849$	**25.**	$394867 \div 501$
6.	$97446 \div 232$	**16.**	$83600 \div 872$	**26.**	$139405 \div 867$
7.	$6492 \div 719$	**17.**	$69527 \div 548$	**27.**	$793557 \div 2050$
8.	$4209 \div 814$	**18.**	$59450 \div 943$	**28.**	$729035 \div 5271$
9.	$14093 \div 265$	**19.**	$97202 \div 328$	**29.**	$154287 \div 8412$
10.	$61942 \div 848$	**20.**	$11984 \div 265$	**30.**	$872142 \div 5418$

Find the results in the following as suggested in §§ 65, 66.

31.	$3940 \div 100$	**36.**	$760000 \div 1000$	**41.**	$5937000 \div 800$
32.	$27450 \div 1000$	**37.**	$875000 \div 4000$	**42.**	$92750000 \div 5000$
33.	$39840 \div 200$	**38.**	$1952000 \div 300$	**43.**	$75280000 \div 200$
34.	$17854300 \div 1000$	**39.**	$2460000 \div 5000$	**44.**	$8624400 \div 200$
35.	$825750 \div 500$	**40.**	$3780000 \div 600$	**45.**	$751945600 \div 2000$

Find the results in the following by the method of § 67.

46.	$1686 \div 24$	**51.**	$19292 \div 81$	**56.**	$146400 \div 75$
47.	$2468 \div 36$	**52.**	$13728 \div 88$	**57.**	$107576 \div 56$
48.	$5580 \div 45$	**53.**	$18396 \div 21$	**58.**	$12312 \div 54$
49.	$14208 \div 96$	**54.**	$63504 \div 42$	**59.**	$22356 \div 81$
50.	$11968 \div 64$	**55.**	$58464 \div 63$	**60.**	$180288 \div 72$

Find the results in the following by the method of § 68.

61.	$3500 \div 25$	**66.**	$1945000 \div 250$	**71.**	$72000 \div 750$
62.	$6500 \div 50$	**67.**	$3876500 \div 500$	**72.**	$128000 \div 250$
63.	$75400 \div 100$	**68.**	$4593800 \div 50$	**73.**	$976300 \div 50$
64.	$8750 \div 25$	**69.**	$72000 \div 500$	**74.**	$3872000 \div 125$
65.	$964500 \div 125$	**70.**	$36000 \div 250$	**75.**	$42718500 \div 50$

DRILL ON FRACTIONS (A)

Give orally as many as possible of the following.

1. $\frac{1}{4} + \frac{1}{8}$
2. $\frac{5}{6} + \frac{2}{3}$
3. $\frac{1}{3} + \frac{4}{5}$
4. $\frac{7}{8} + \frac{5}{6}$
5. $\frac{9}{10} + \frac{9}{8}$
6. $\frac{15}{16} + \frac{9}{20}$
7. $\frac{19}{21} + \frac{7}{24}$
8. $\frac{3}{5} + \frac{3}{11}$
9. $\frac{1}{2} + \frac{5}{14}$
10. $\frac{17}{33} + \frac{7}{18}$
11. $\frac{2}{3} + \frac{3}{4} + \frac{5}{6}$
12. $\frac{5}{12} + \frac{11}{15} + \frac{9}{20}$
13. $\frac{1}{2} + \frac{2}{5} + \frac{3}{11} + \frac{7}{12}$
14. $\frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}$
15. $\frac{2}{7} + \frac{3}{8} + \frac{4}{9}$
16. $3\frac{1}{2} + 7\frac{5}{12}$
17. $4\frac{1}{6} + \frac{15}{16} + 3\frac{7}{8}$
18. $2\frac{1}{9} + 3\frac{1}{4} + \frac{5}{16}$
19. $30\frac{7}{10} + 40\frac{1}{5} + \frac{9}{20}$
20. $10\frac{1}{16} + 11\frac{5}{24} + \frac{11}{30}$
21. $9\frac{1}{2} + \frac{35}{7} + \frac{43}{14}$
22. $6\frac{7}{8} + \frac{21}{4} + \frac{15}{8}$
23. $\frac{9}{4} + \frac{11}{16} + \frac{17}{24}$
24. $1\frac{5}{6} + \frac{27}{12} + \frac{3}{8}$
25. $\frac{4}{5} - \frac{2}{3}$
26. $\frac{7}{8} + \frac{3}{4}$
27. $\frac{7}{15} - \frac{3}{24}$
28. $\frac{9}{14} - \frac{14}{63}$
29. $\frac{13}{15} - \frac{5}{21}$
30. $\frac{5}{6} - \frac{3}{8}$
31. $\frac{9}{13} - \frac{14}{39}$
32. $18\frac{2}{9} - 4\frac{5}{6}$
33. $25 - 16\frac{2}{3}$
34. $96\frac{8}{9} - 17\frac{4}{7}$
35. $20\frac{1}{4} - 14\frac{5}{6}$
36. $18\frac{1}{2} - \frac{7}{38}$
37. $\frac{2}{3} \times \frac{1}{5}$
38. $\frac{1}{2} \times \frac{5}{6}$
39. $\frac{3}{7} \times \frac{2}{15}$
40. $\frac{3}{8} \times \frac{16}{21}$
41. $\frac{4}{5} \times \frac{7}{8}$
42. $\frac{9}{10} \times \frac{15}{23}$

Before solving the following, study again the example solved on page 77.

43. $16 \times 3\frac{7}{8}$
44. $27 \times 2\frac{5}{11}$
45. $\frac{7}{16} \times 5$
46. $\frac{8}{27} \times 3\frac{3}{8}$
47. $\frac{19}{21} \times 6$
48. $\frac{17}{18} \times 4\frac{1}{5}$
49. $1\frac{1}{2} \times 1\frac{1}{2}$
50. $2\frac{1}{3} \times 2\frac{3}{4}$
51. $3\frac{5}{9} \times 2\frac{6}{11}$
52. $4\frac{7}{16} \times 12\frac{3}{5}$
53. $\frac{1}{4} \div \frac{1}{3}$
54. $\frac{3}{5} \div \frac{2}{7}$
55. $\frac{5}{16} \div \frac{3}{8}$
56. $\frac{7}{11} \div \frac{5}{22}$
57. $\frac{13}{14} \div \frac{11}{28}$
58. $1\frac{1}{2} \div 2$
59. $2\frac{2}{3} \div 4$
60. $5\frac{1}{6} \div 7$
61. $9\frac{3}{8} \div 3$
62. $8\frac{7}{12} \div 6$
63. $4\frac{7}{11} \div 5$
64. $1\frac{1}{2} \div 2\frac{2}{3}$
65. $3\frac{7}{8} \div 1\frac{4}{5}$
66. $2\frac{3}{8} \div 1\frac{5}{6}$
67. $3\frac{1}{3} \div 1\frac{7}{10}$
68. $14\frac{1}{2} \div 7\frac{1}{3}$
69. $9\frac{2}{11} \div 4\frac{5}{11}$
70. $6\frac{1}{4} \div 5\frac{1}{2}$
71. $9\frac{9}{10} \div 4\frac{3}{10}$
72. $7\frac{7}{16} \div 9\frac{5}{8}$
73. $8\frac{1}{3} \times 71\frac{4}{5}$
74. $384\frac{4}{7} \times 26\frac{2}{3}$
75. $81\frac{1}{2} \div 5\frac{2}{3}$
76. $272\frac{1}{3} \times 61\frac{4}{7}$
77. $93\frac{3}{8} \times 64\frac{2}{5}$
78. $398\frac{8}{9} \times 46\frac{5}{7}$

MISCELLANEOUS WORK (B)

1. A picture $12\frac{1}{2}$ inches long and $8\frac{3}{4}$ inches wide is put in a frame $2\frac{3}{4}$ inches wide outside the picture. What are the dimensions of the frame?

2. 4 is what part of 20? $\frac{3}{7}$ is what part of $4\frac{2}{3}$?

3. $\frac{3}{4}$ is what part of $\frac{2}{3}$? $\frac{3}{5}$ is $\frac{2}{7}$ of what number?

4. If a wheel goes $9\frac{3}{4}$ feet at each revolution, how many times will it turn around in going 150 miles?

5. A farm containing $74\frac{5}{8}$ acres was sold for \$10,000. What was the price per acre?

6. An airplane went $34\frac{3}{4}$ miles in $21\frac{1}{2}$ minutes. What was its speed in miles per hour?

7. A field containing $48\frac{1}{4}$ acres yields 924 bushels of wheat. At this rate what would be the yield of a field containing $267\frac{1}{2}$ acres?

8. In one department of a store the sales of eight clerks for one week were: \$347.20, \$412.60, \$298.40, \$481.25, \$397.50, \$407.65, \$424.85, \$445.65. What is the average of these sales?

9. August 1, W. K. Warren had a balance in his bank of \$4795.80. During the next week he made deposits of \$465.40, \$276.80, \$194.70, \$562.67, \$845.92, \$374.50. Then he withdrew by checks \$100.00, \$87.20, \$25.00, \$2850.00, \$32.50, \$35.00, \$65.80, \$294.65, \$1720.35. What was his balance at the end of the week? Arrange the work as in § 31.

10. One day the collection department of a certain bank showed the following: *A*. Face of paper, \$2160; discount, \$12.80; commission and exchange, \$5.60; *B*. Face of paper, \$3500; discount, \$41.60; commission and exchange, \$12.50; *C*. Face of paper, \$340; discount, none; commission and exchange, \$2.50; *D*. Face of paper, \$950; discount, none; commission and exchange, none; *E*. Face of paper, \$5265.50; discount, \$94.75; commission and exchange, \$25.00; *F*. Face of paper, \$720; discount, none; commission and exchange, \$7.20. Arrange the work and show totals as on page 27.

CHAPTER XII

DECIMALS

122. Decimals in Business. — A very large part of the fractions encountered in business are in the form of decimals. In fact, decimals occur nearly as frequently as ordinary integers. Hence, the prospective business man should not only understand the theory of decimals, but he should acquire effectiveness in handling them.

123. Decimal Fractions. — Fractions whose denominators are 10, 100, 1000, etc., are called *decimal fractions* and are written in a special way. Thus $\frac{1}{10}$ is written .1, $\frac{1}{100}$ is written .01, $\frac{17}{100}$ is written .17, and $\frac{38}{1000}$ is written .038.

The most common use of decimal fractions in the United States is in representing amounts of money.

Thus, 10 cents, or $\frac{1}{10}$ of a dollar, is written \$.10 and 15 cents, or $\frac{15}{100}$ of a dollar, is written \$.15.

124. The Principle of Place Value in Decimals. — A period, called the *decimal point,* is used to separate the integers from the decimal fractions. The first place to the left of the decimal point is *one's* place; the second place to the left, ten's place; and so on, as in the case of purely integral numbers. A digit in the first place to the right of the decimal point represents tenths; a digit in the second place to the right of the decimal point represents hundredths; and so on.

Thus, in the number 1234.5678, the 1 represents one thousand; the 2 represents hundreds; the 3, tens, and the 4, ones. The 5 represents tenths; the 6, hundredths; the 7, thousandths, and the 8, ten-thousandths.

Thus, we see that in a decimal number representing an integer or a fraction, or both, any digit represents ten times as much as the same digit in the next place to the right of it. This enables us to add, subtract, multiply, and divide decimal fractions, or mixed integers and decimal fractions, exactly as we do integral numbers. The only difficulty is in placing the decimal point in the result. Hence, placing the decimal point is the only new element of arithmetic in performing the fundamental operations on decimals.

125. Reading Decimals. — In the number .178, there are 1 tenth, 7 hundredths, 8 thousandths. This makes 178 thousandths and is read 178 thousandths. The number 14.78 is read 14 *and* 78 hundredths.

ORAL EXERCISES (A)

Read each of the following numbers.

1. 8.94, 2.748, 1.904, .37, .07, .004, .0008, .042, .0741, .1901.

Write the following numbers, using figures.

2. One hundred forty-six and thirteen hundredths.

3. Four-hundred ninety-six thousandths.

4. Three million forty thousand and ten-millionths.

5. Ninety-nine thousand and nineteen-billionths.

6. One trillionth.

7. Ten thousand one hundred and three hundred sixty-one hundred-thousandths.

126. Addition of Decimals. — To add decimals they should be so placed that the decimal points stand in a column.

Thus, to add 12., 1.49, .078, 640.2, 4.904, .007, write the numbers as follows:

12.
1.49
.078
640.2
4.904
.007
658.679

The first column to the right represents thousandths. Adding, we get 19 thousandths. Write 9 in thousandths' place in the sum, and carry 10 thousandths, or 1 hundredth. The next column represents hundredths, and its sum including the 1 carried is 17 hundredths. Write 7 in hundredths' place in the sum, and carry 1. Proceed in this manner until all the columns have been added. The decimal point in the sum is placed directly under the decimal points in the addends.

Thus we see that the process of addition is exactly the same when decimal fractions are involved as it is in the case of ordinary integers. Care must be taken, as in the addition of integers, that digits of the same order are added.

That is, we must add thousandths to thousandths, hundredths to hundredths, and tenths to tenths. For this reason the decimal points are placed in a straight column.

12.000
1.490
.078
640.200
4.904
.007
658.679

Sometimes zeros are annexed to the right to make the number of decimal places the same in all the numbers.

Thus, the numbers added above on this page may be written as at the right.

WRITTEN EXERCISES (A)

Write the following sets of numbers in columns and add them.

1. 1.94 + 42.64 + .049 + 365. + 912.7 + 12.841 + 1226.70.
2. 2.75 + 315.96 + 1.0582 + 1.72 + 84.059 + 101.0861 + 1.0587.
3. 13.41 + 1.905 + 87.9123 + 121.089 + 1056. + 987.306 + 1.0632.
4. 9056.0038 + 2067.921 + 10056.9862 + .000567 + 98246.5.
5. 105.692 + 203.7845 + .02597 + 204.567 + 981.506 + 824.961.
6. 245.065 + 5093.81 + 9.24385 + .059365 + 12875.049 + 328.4595.
7. 902.456 + .005693 + 1728.045 + 921.056 + 803.4578 + 205.983.
8. 71.248 + 1.045792 + 386.0791 + 2845.0759 + .053761 + 903.047.
9. 1924.05 + 2345.921 + 1726.902 + 305.076 + 1921.087 + 5925.086.
10. 38762.904 + 208.347 + 92.45461 + 87.059 + 80.904 + 70.9829.
11. 105626.959 + 4.009 + .007386 + 2905.0876 + 9.24876 + 209.045.
12. 3789245.935 + 37875.42 + 494.63871 + 2.08523 + 91.062308.

127. Horizontal Addition of Decimals. — This is in every respect similar to the horizontal addition of integers. In both cases it is necessary to take care that digits of the same order are added.

ORAL EXERCISES (B)

Find the sums of the following by horizontal addition. Do not copy the figures. Write down the sums only. Work rapidly.

1. 3.48 + 6.40 + 14.35 + 29.75 + 51.94 + 31.90.
2. 27.09 + 36.05 + 7.21 + 19.08 + 45.27 + 83.42.
3. 6.40 + .0571 + 26.45 + 7.96 + 10.01 + 46.02.
4. 18.96 + 15.28 + 91.06 + 10.15 + 8.76 + 19.08.
5. 27.05 + 4.08 + 16.24 + 9.18 + 17.01 + 86.61.
6. 904.25 + 87.63 + 901.40 + 76.25 + 4.80 + 92.71.
7. 1845.06 + 2406.45 + 307.96 + 281.26 + 1907.08 + 8704.09.
8. 308.19 + 156.19 + 1729.01 + 63.87 + 207.45 + 46.87.
9. 87.20 + 19.04 + 27.98 + 41.81 + 15.47 + 28.10.
10. 1.005 + 10.05 + 1.005 + .0005 + 100.5 + 1000.5.

PROBLEMS (A)

Copy data in Examples 1 and 2 on suitable blanks.

1. In the following add the columns to find the total sales in each department for the whole week. Add horizontally to find the total sales for the whole store for each day. Check by adding totals for the days and totals for the departments.

SALES IN A SMALL DEPARTMENT STORE

DAY	CLOTHING	GROCERIES	HARDWARE	FURNITURE	NOTIONS	TOTALS
Monday . . .	$174.65	$249.34	$ 84.37	$465.50	$213.70	
Tuesday . .	209.28	206.15	51.28	112.76	200.05	
Wednesday .	197.21	210.08	62.49	228.91	199.11	
Thursday . .	246.40	238.27	87.25	305.10	211.19	
Friday . . .	300.00	261.04	93.08	256.91	247.87	
Saturday . .	326.25	293.97	112.15	311.10	263.49	
Totals						

2. The monthly sales of a hardware store were as follows.

MONTH	HOUSE FURNISHINGS	BUILDING MATERIAL	PAINTS, OILS	AUTO SUPPLIES	MISCELLANEOUS	TOTALS
Jan.	$10,215.09	$12,009.10	$ 7,591.05	$1,256.14	$6,503.76	
Feb.	9,204.87	10,234.06	5,092.76	931.51	5,979.05	
March	12,810.12	11,007.91	7,809.47	1,175.29	6,204.92	
April	13,762.71	17,216.07	9,376.08	1,829.57	7,693.09	
May	14,095.38	16,378.96	10,247.22	2,315.68	6,904.79	
June	15,093.59	14,962.53	8,234.97	4,592.76	8,793.90	
July	12,386.01	17,005.68	9,172.84	5,487.92	7,238.07	
August	7,892.83	18,059.64	5,276.03	4,286.44	5,204.62	
Sept.	8,310.75	15,075.09	6 315.88	3,716.15	3,748.96	
Oct.	10,287.12	12,071.32	7,692 07	2,245.79	6,004.72	
Nov.	7,506.48	7,863.76	6,004.07	1,954.07	5,909.50	
Dec.	6,921.73	5,297.08	5,296.42	1,249.82	6,004.22	
Totals						

Find the total sales for each month, the total sales of each item, and the total sales for the year.

3. A family expense account for one month is given on page 96. Find each day's expenditure by horizontal addition. Fill in all spaces for totals. Check as on page 13.

	RENT	LIGHT AND FUEL	TELEPHONE	CLOTHING	LAUNDRY AND CLEANING	MEAT AND FISH	MILK	BREAD	VEGETABLES	MISCELLANEOUS FOODS	AMUSEMENTS	BOOKS AND SUBSCRIPTIONS	CARFARE AND TRAVEL	NEWSPAPERS AND PERIODICALS	DOCTOR AND ILLNESS	FURNITURE	MISCELLANEOUS EXPENSES	TOTAL
1	$55	$5.50				$ .60				$ .30	$ .60		$ .20	$ .05			$2.00	
2									$1.15	.30							.50	
3					$1.50	.90											6.00	
4						1.20		$ .36						.15			.10	
5						.85			.45	.20	1.10							
6				$2.00		.65	$1.20		.30	1.45		$ .50					.45	
7									.25	1.35				.05			.35	
8						1.35			.05	3.30							.25	
9								.36			.90			.05			1.55	
10				.50		.70				.20								
11						3.20			1.10	1.20				.05			1.15	
12						.90							.20				1.50	
13																	.50	
14								.36		.20	1.20					$35.50		
15				3.50								1.50						
16				1.00		1.20	2.31		.10	.60				.15				
17					4.35					.15			.10					
18				.25		.45			.30	.50	4.40						.10	
19						.40		.36		.60		1.50						
20																		
21				5.25		.85			.50	1.35	2.55			.25				
22						.75												
23						3.20		.36					.20					
24				.05					.35	.85								
25										.15								
26				5.00		.55			.45	.80				.15		45 00	.85	
27								.18										
28							2.52											
29			$3.75	2.50	2.49	.95			.40				.20	.15				
30	Totals																	

128. Subtraction of Decimals. — This is in all respects similar to the subtraction of integers. In both cases it is necessary to take care that digits of the same order are subtracted. In case the minuend and subtrahend are written in a column, the decimal points must be in a column.

Thus, to subtract 3.49 from 420.45, we write

$$\begin{array}{r} 420.45 \\ 3.49 \\ \hline 416.96 \end{array}$$

To subtract 13.074 from 47.21, we write

$$\begin{array}{r} 47.21 \\ 13.074 \\ \hline \end{array}$$

In the last case a zero may be annexed at the right in the minuend.

WRITTEN EXERCISES (A)

Find the difference in each of the following.

1. 301.6 − 24.8

2. 42.71 − 24.17

3. 234.9 − 178.65

4. 109.27 − 87.42

5. 226.1 − 195.01

6. 328.07 − 190.707

7. 1595.286 − 1287.9082

8. 2984.063 − 1905.9216

In each of the following write the difference without first copying the numbers.

9. 124.74 − 35.86

10. 108.26 − 47.97

11. 325.04 − 187.21

12. 789.21 − 395.47

13. 620.09 − 429.42

14. 209.46 − 98.79

15. 128.06 − 96.29

16. 76.20 − 45.81

17. 209.86 − 143.79

18. 87.25 − 49.38

In each of the following columns subtract from the first number the sum of those below it.

19.	**20.**	**21.**	**22.**
6249.700	3892.628	9485.004	10746.093
319.820	45.091	629.912	396.725
56.421	308.706	703.075	1287.304
386.494	115.291	2521.903	443.826
41.920	86.005	223.008	3091.065
580.640	245.975	114.293	906.224

Example. A man who has \$1746.80 in the bank draws checks as follows: \$240.00, \$157.65, \$44.80, \$174.35, \$115.00. How much has he left?

Write this down as shown here. Subtract the sum of the checks from \$1746.80.

\$1746.80
240.00
157.65
44.80
174.35
115.00

WRITTEN EXERCISES (A)

1. Mr. Buckley deposits \$2460 in the bank and draws checks as follows: \$35, \$27.50, \$47.65, \$19.80, \$270.50, \$54.60, \$105. How much has he in the bank?

Find the balance in the bank indicated on each of the following check stubs.

2.

Bal. brought forward	\$349.60
Checks	46.70
	28.46
	39.53
	17.65
Balance	

3.

Bal. brought forward	\$894.35
Checks	147.40
	259.60
	54.00
	27.50
Balance	

4.

Bal. brought forward	\$1247.45
Checks	540.00
	231.90
	27.50
	37.64
Balance	

5.

Bal. brought forward	\$12,680.37
Checks	1,700.00
	247.60
	24.97
	193.00
Balance	

6.

Bal. brought forward	\$1982.70
Checks	287.60
	300.00
	84.56
	12.60
	19.80
	490.60
Balance	

7.

Bal. brought forward	\$4970.60
Checks	1280.00
	360.00
	45.80
	120.00
	67.40
	197.40
Balance	

PROBLEMS (B)

Find the remainder in each of the following by horizontal addition and subtraction. Do not copy the figures, write down the results only.

1. A man had $869.80 in the bank, and drew out $140.50, $65.00, $245.20, and $54.00. How much had he left?

2. From $1260 subtract the sum of $496, $250.60, $90.40, $12.65.

3. From $736.00 subtract the sum of $35.80, $130.00, $89.30.

4. From $934.10 subtract the sum of $105.21, $99.16, $510.12, $17.86.

5. From $2785.27 subtract the sum of $1591.08, $529.47, $202.76.

6. From $9670.83 subtract the sum of $2540.80, $946.52, $899.42, $1058.10.

7. From $276.05 subtract the sum of $10.84, $108.40, $1.84, $96.29.

8. From $1010.95 subtract the sum of $425.87, $93.07, $147.87.

Find the balance in each of the following accounts.

	Dr.	Cr.		Dr.	Cr.		Dr.	Cr.
9.	$48.60	$50	10.	$117.28	$ 95.27	11.	$ 11.27	$ 45.67
	35.40	30		940.15	103.04		148.76	229.51
	26.75	80		87.29	16 29		95.08	376.95
	34.25	25		345.90	179.81		87.29	147.82
	89.70			104.87	73.69		58.16	89.28
	14.50			296.45	205.10		95.12	679.53

12. During one week the daily bank deposits and withdrawals of a storekeeper were as follows. Which was the greater, the total deposits or the total withdrawals, and how much?

Days	Deposits	Withdrawals
Monday	$736.80	$ 191.40
Tuesday	670.50	380.50
Wednesday	589.76	260.00
Thursday	632.50	105.15
Friday	593.60	880.47
Saturday	890.00	1650.80

129. Multiplying by 10, 100, 1000, etc. — It follows as a direct consequence of the place-value in the decimal notation that a number may be multiplied by 10 by moving the decimal point one place to the right. Hence moving the decimal point two places to the right multiplies by 100; moving it three places to the right multiplies by 1000; etc. Annex zeros if necessary.

ORAL EXERCISES (A)

Read the products in the following.

1. 10×74.89	**3.** 100×26.8	**5.** 1000×99.864
2. 100×3.987	**4.** 100×364	**6.** 1000×319.32

130. Placing the Decimal Point in the Product.

From $\frac{3}{10} \times \frac{4}{10} = \frac{12}{100}$, we see that $.3 \times .4 = .12$.

Similarly, $7 \times \frac{5}{100} = \frac{35}{100}$, and hence $7 \times .05 = .35$. $1\frac{4}{10} \times 3\frac{1}{10} = \frac{14}{10} \times \frac{31}{10} = \frac{434}{100}$. Hence $1.4 \times 3.1 = 4.34$.

Rule. — *The number of decimal places in a product is equal to the sum of the numbers of decimal places in the factors.*

ORAL EXERCISES

How many decimal places are there in each of the following products?

1. $.07 \times 1.35$	**3.** 2.07×35	**5.** $.0042 \times 1.02$
2. 4.57×26	**4.** $.009 \times 2.06$	**6.** 9.078×3.04

131. Rule for Multiplying Decimals. — *To find the product of two numbers containing decimal fractions, multiply exactly as if they were integers. Locate the decimal point so that the number of decimal places in the product is equal to the sum of the decimal places in the factors.*

ORAL EXERCISES

Find the products of the following.

1. $.03 \times .9$	**4.** $.04 \times .025$	**7.** $.009 \times .6$	**10.** $.07 \times .09$
2. $.09 \times .08$	**5.** $.15 \times .04$	**8.** $.020 \times .9$	**11.** $.9 \times .25$
3. $.02 \times .6$	**6.** $.45 \times .05$	**9.** $.60 \times .4$	**12.** $.08 \times .94$

WRITTEN EXERCISES (A)

Find the products of the following.

1. 4.35	**4.** 45.08	**7.** 8.27	**10.** 200.87	**13.** 7.693
27.4	1.29	.093	10.26	1.45
2. 1.26	**5.** 53.04	**8.** 1.0051	**11.** 35.29	**14.** 2.903
.048	.068	30.4	.0076	.405
3. 80.24	**6.** 942.4	**9.** 75.026	**12.** 26.907	**15.** .004592
9.36	3.27	27.94	150.4	3.605

132. Horizontal Multiplication of Decimals. — Decimal fractions are multiplied horizontally in the same manner as integers, taking proper care in placing the decimal point. See § 47.

Thus, $38 \times .64 = 24.32$ and $.083 \times .94 = .07802$.

WRITTEN EXERCISES (A)

Write down the products without copying the numbers.

1. $5.6 \times .04$	**4.** 75×4.04	**7.** 8.1×7.5
2. 3.4×2.8	**5.** 2.5×12.60	**8.** $.15 \times 2.6$
3. 1.4×48	**6.** $.99 \times 3.26$	**9.** $.025 \times 1.07$

133. Division by 10, 100, 1000, etc. — It follows from the place value in decimals that a number is divided by 10 by moving the decimal point one place to the left. Hence moving the decimal point two places to the left divides by 100; moving it three places to the left divides by 1000; etc. If necessary prefix zeros.

Thus, $8.94 \div 100 = .0894$.

ORAL EXERCISES

Read the quotients in the following.

1. $794.8 \div 10$	**5.** $.078 \div 10$	**9.** $.084 \div 100$
2. $43.9 \div 10$	**6.** $54.89 \div 100$	**10.** $587.4 \div 1000$
3. $7.91 \div 10$	**7.** $8.39 \div 100$	**11.** $31.97 \div 1000$
4. $.891 \div 10$	**8.** $.978 \div 100$	**12.** $8.974 \div 1000$

134. Division of Decimals. — Division of numbers containing decimal fractions is exactly like division of integral numbers. But care must be taken to put the decimal point in the right place.

Example 1. Divide 3947.49 by 264.

Solution. We first divide 394 tens by 264, obtaining 1 (ten) as the quotient. The 1 is therefore written in tens' place in the quotient. We then proceed with the division as in the case of whole numbers. The decimal point in the quotient is placed directly above the decimal point in the dividend. If we wish to carry out the quotient to a larger number of decimal places, we annex zero to the remainders. Show why 14.953, and not 14.952 is the nearest approximation in thousandths.

```
      14.953
264)3947.49
    264
    1307
    1056
     2514
     2376
      1389
      1320
       690
```

WRITTEN EXERCISES (A)

In the following find the quotient to the nearest thousandth.

1. $174\overline{)281.927}$
2. $814\overline{)5391.4}$
3. $528\overline{)37.394}$
4. $592\overline{)15.292}$
5. $493\overline{).0026}$
6. $879\overline{)4.5}$
7. $350\overline{)7}$
8. $482\overline{).1}$
9. $1470\overline{)892.5}$

Example 2. Divide 13.47 by 1.84.

Solution. First multiply both dividend and divisor by 100. This is done to make the divisor a whole number. The example then is: Divide 1347 by 184. The quotient is 7.320 with a remainder 120. Since the remainder is more than half of 184, the nearest approximation in thousandths is 7.321.

```
      7.321
184.)1347.
    1288
      590
      552
       380
       368
        120
```

Example 3. Divide 1.4704 by .081.

Show that this is equivalent to dividing 1470.4 by 81.

EXERCISES

Find the quotients in the following, carrying results to 3 decimals.

1. $1.98\overline{)43.67}$
2. $36.3\overline{)674}$
3. $14.25\overline{)5280}$
4. $75.43\overline{)250.06}$
5. $1.43\overline{)478.26}$
6. $3.25\overline{)259.63}$
7. $5737\overline{)4273}$
8. $35.45\overline{)9.825}$
9. $32.46\overline{)18.79253}$

135. Dividing by 200, 500, etc. — To divide by 200 we may divide by 100 and then by 2. See § 66. To divide by 500 we may divide by 100 and then by 5, or divide by 1000 and multiply by 2.

Example. Divide 34780 by 300.

We first strike out two zeros in the divisor and move the decimal point two places to the left in the dividend.

$$300\overline{)347.80}\quad 115.93$$

WRITTEN EXERCISES (A)

Find the quotients in the following, carrying results to 3 decimals.

1. $3547 \div 400$	**7.** $73.80 \div 800$	**13.** $69.4 \div 400$
2. $7468 \div 600$	**8.** $5460 \div 300$	**14.** $473.2 \div 500$
3. $8900 \div 7000$	**9.** $104.7 \div 200$	**15.** $327.4 \div 600$
4. $3970 \div 700$	**10.** $49.07 \div 700$	**16.** $273.4 \div 700$
5. $534.2 \div 300$	**11.** $159.47 \div 600$	**17.** $582.6 \div 800$
6. $54.06 \div 900$	**12.** $2894.30 \div 900$	**18.** $856.2 \div 900$

136. Dividing by 25, 50, 125, etc. — To divide by 25, divide by 100 and multiply by 4. See § 68. To divide by 50, divide by 100 and multiply by 2, etc.

Example 1. $6947 \div 25 = 69.47 \times 4 = 277.88$.
Example 2. $74.04 \div 50 = .7404 \times 2 = 1.4808$.
Example 3. $459.75 \div 125 = .45975 \times 8 = 3.67800$.
Example 4. $1987.25 \div 250 = 1.98725 \times 4 = 7.94900$.

In practice the quotients are written down at once.

Thus, $798.42 \div 125 = 6.38736$.

WRITTEN EXERCISES (A)

1. $1984.2 \div 25$	**7.** $1891 \div 250$	**13.** $4.681 \div 25$
2. $64.95 \div 25$	**8.** $5.97 \div 250$	**14.** $64.18 \div 50$
3. $1.84 \div 50$	**9.** $1.864 \div 25$	**15.** $81.46 \div 125$
4. $.609 \div 50$	**10.** $.937 \div 50$	**16.** $73.92 \div 250$
5. $39.27 \div 125$	**11.** $.0364 \div 125$	**17.** $23.97 \div 125$
6. $403.8 \div 125$	**12.** $.8468 \div 250$	**18.** $726.8 \div 50$

DRILL EXERCISES IN THE MULTIPLICATION OF DECIMALS (A)

It is of great importance to the practical accountant to be able to perform multiplication horizontally when one of the factors contains one, two, or even three digits. See § 49.

Write down the products of the following without copying the examples.

1. 3×19.25
2. $4.7 \times .92$
3. $7.8 \times .9$
4. 87.4×2
5. 3×6.72
6. 9.4×7
7. 12.5×6
8. $49.8 \times .4$
9. $11.2 \times .6$
10. $4.5 \times .75$
11. $93.4 \times .7$
12. 8.16×3
13. $4.59 \times .3$
14. $2.4 \times .16$
15. 1.31×16.5
16. $3.87 \times .003$
17. $.9 \times 1.5008$
18. $.00062 \times .04$
19. $45.75 \times .001\frac{1}{2}$
20. $.048 \times .0002$
21. $.059 \times .03$
22. $.072 \times .025$
23. $2.0245 \times .05$
24. $3.875 \times .001$
25. $.02934 \times .03$
26. $16.205 \times .07$
27. $.400 \times .004$
28. $1.001 \times .005$
29. $16.48 \times .7$
30. $17.28 \times .02$
31. 6×678.4
32. 4×125.2
33. $78 \times .6$
34. $0.4 \times .016$
35. $1.111 \times .11$
36. $34.26 \times .025$
37. $8.5 \times .042$
38. $.002 \times .075$
39. $.6 \times 60$
40. $.06 \times .6$
41. $.0006 \times 600$
42. $.60 \times .006$
43. $.0006 \times .10000$
44. $.6 \times .0600$
45. $.4 \times .225$
46. $12.50 \times .125$
47. $.25 \times 2.05$
48. 3.08×19
49. $.074 \times .9$
50. $59.80 \times .8$
51. $.375 \times .2$
52. $.891 \times 0.5$
53. $60.25 \times .0050$
54. $76.84 \times .4$
55. $37.8 \times .5$
56. $876.5 \times .8$
57. 1.934×7
58. $.8914 \times .03$
59. $1.872 \times .06$
60. $.0758 \times .02$
61. 4.2×16.7
62. $.968 \times 40$
63. $.6 \times .03036$
64. $.06 \times .0128$
65. $.4888 \times .02$
66. $42.3 \times .25$
67. 9.3×48
68. $4.59 \times .03$
69. 106.8×1.5
70. $96.7 \times .8$
71. $39.2 \times .6$
72. 7.6×1.05
73. $5.2 \times .163$
74. $.543 \times .04$
75. $.038 \times .06$

MISCELLANEOUS WORK

1. (A) During December, January, February, and March a family paid bills as follows.

Item	Dec	Jan.	Feb.	Mar.	Totals
Fuel	$15.90	$20.10	$14.25	$ 9.80	
Groceries	25.95	30.28	21.98	32.71	
Meat	14.71	15.60	12.79	16.92	
Milk and Butter .	10.50	12.75	9.35	13.69	
Totals					

Find the total expenses for all these items for each month and also find the total expenses by horizontal addition for the four months.

2. (B) Following is a partial inventory. Copy on a suitable blank, extend, and foot the total values.

No. of Articles	Kind of Article	Value	Total Value	No. of Articles	Kind of Article	Value	Total Value
$13\frac{1}{2}$ doz.	pairs gloves	$ 9.50		6 doz.	ties	$ 6.00	
9 doz.	pairs gloves	11.00		42 doz.	collars	1.00	
$3\frac{1}{4}$ doz.	pairs gloves	12.50		32 doz.	collars	1.25	
17	hats	7.50		12 doz.	collars	1.40	
27	hats	1.80		18	suits	20.00	
31	hats	1.25		14	suits	18.00	
42	hats	2.80		21	suits	16.00	
37	caps	1.50		35	suits	14.00	
85	caps	.80		60	overcoats	12.00	
$4\frac{3}{4}$ doz.	shirts	12.00		42	overcoats	15.00	
$2\frac{1}{2}$ doz.	shirts	14.50		32	overcoats	20.00	
5 doz.	shirts	17.00		17	raincoats	10.00	
14 doz.	shirts	16.00		12	raincoats	13.00	
17 doz.	ties	2.00		15	white trousers	6.25	

3. (B) A man sold a house for $8400, thereby gaining $\frac{1}{5}$ of the purchase price. What was the purchase price?

4. (C) The net income from a certain business for two years was $18,500. Find the income for each year if that of the second year was $\frac{1}{4}$ less than that of the first year.

5. (P) Multiply the sum of $3\frac{1}{4}$ and $7\frac{5}{8}$ by the difference between $18\frac{3}{5}$ and $7\frac{2}{3}$.

CHAPTER XIII

REDUCING FRACTIONS TO DECIMALS

137. Common Fractions and Decimals Used Interchangeably. — It is frequently necessary to change common fractions into decimals. Several examples of this are given in this chapter, and the student can easily find many more.

138. Examples of Reducing Common Fractions to Decimals. — Since a fraction may be regarded as an indicated division, it follows that it may be reduced to a decimal by carrying out the division.

Thus, the decimal value of $\frac{3}{8}$ is found by dividing 3 by 8.
Hence $\frac{3}{8} = .375$.

$$8\overline{)3.000}\quad .375$$

Most fractions do not have exact decimal equivalents. Thus, $\frac{1}{3}$ can be written approximately as .33, or .333, or .3333, etc., but there is no decimal which equals exactly $\frac{1}{3}$. However, for any given fraction, it is possible to find a decimal which differs from it by as little as we please. Thus, .33 differs from $\frac{1}{3}$ by $\frac{1}{300}$; .333 differs from $\frac{1}{3}$ by $\frac{1}{3000}$; and .3333 differs from $\frac{1}{3}$ by $\frac{1}{30000}$.

Example. Reduce $\frac{3}{7}$ to a decimal.

Carrying the division to four places of decimals, we get .4285 and a remainder 5. (The remainder is really .0005.) Since 5 is more than one half of 7, it follows that .4286 is the nearest approximation to four places of decimals. This decimal differs from $\frac{3}{7}$ by less than $\frac{1}{10000}$.

$$7\overline{)3.0000}\quad .4286$$

139. Combination of Decimal and Common Fractions. — Sometimes numbers are expressed as combinations of common and decimal fractions.

Thus $\frac{1}{3} = .33\frac{1}{3}$ or $.333\frac{1}{3}$. The first is read "thirty-three and one third hundredths" and the second, "three hundred thirty-three and one third thousandths."
Similarly $\frac{1}{6} = .16\frac{2}{3}$, or $.166\frac{2}{3}$, or $.1666\frac{2}{3}$.

140. Important Decimal Equivalents of Common Fractions. — There are certain common fractions whose decimal equivalents should be memorized perfectly. Such are —

$\frac{1}{2} = .5$	$\frac{1}{5} = .2$	$\frac{1}{10} = .1$	$\frac{1}{16} = .06\frac{1}{4}$	$\frac{1}{30} = .03\frac{1}{3}$
$\frac{1}{3} = .33\frac{1}{3}$	$\frac{1}{6} = .16\frac{2}{3}$	$\frac{1}{12} = .08\frac{1}{3}$	$\frac{1}{20} = .05$	$\frac{1}{40} = .02\frac{1}{2}$
$\frac{1}{4} = .25$	$\frac{1}{8} = .12\frac{1}{2}$	$\frac{1}{15} = .06\frac{2}{3}$	$\frac{1}{25} = .04$	$\frac{1}{50} = .02$

Compare these with aliquot parts of 100. See § 148.

Based on these the decimal equivalents of many other fractions may be obtained.

Thus $\frac{2}{3} = 2 \times .33\frac{1}{3} = .66\frac{2}{3}$, $\frac{5}{8} = 5 \times .12\frac{1}{2} = .62\frac{1}{2}$, etc.

ORAL EXERCISES (A)

Give the decimal equivalents of the fractions below. Use two decimal places and give the remainders, if any, as common fractions, as in the table above.

1. $\frac{2}{5}$	7. $\frac{7}{8}$	13. $\frac{7}{16}$	19. $\frac{7}{20}$	25. $\frac{7}{30}$
2. $\frac{3}{5}$	8. $\frac{5}{12}$	14. $\frac{9}{16}$	20. $\frac{9}{20}$	26. $\frac{1}{32}$
3. $\frac{4}{5}$	9. $\frac{7}{12}$	15. $\frac{11}{16}$	21. $\frac{11}{20}$	27. $\frac{3}{32}$
4. $\frac{5}{6}$	10. $\frac{11}{12}$	16. $\frac{13}{16}$	22. $\frac{13}{20}$	28. $\frac{5}{32}$
5. $\frac{3}{8}$	11. $\frac{3}{16}$	17. $\frac{15}{16}$	23. $\frac{17}{20}$	29. $\frac{7}{32}$
6. $\frac{5}{8}$	12. $\frac{5}{16}$	18. $\frac{3}{20}$	24. $\frac{19}{20}$	30. $\frac{9}{32}$

WRITTEN EXERCISES

Reduce each of the following common fractions to a decimal. Give each result correct to four places of decimals. Use long division only when necessary.

1. $\frac{3}{14}$	7. $\frac{7}{16}$	13. $\frac{11}{17}$	19. $\frac{1}{64}$	25. $\frac{5}{48}$
2. $\frac{5}{13}$	8. $\frac{9}{13}$	14. $\frac{16}{29}$	20. $\frac{3}{64}$	26. $\frac{11}{48}$
3. $\frac{5}{16}$	9. $\frac{4}{21}$	15. $\frac{4}{33}$	21. $\frac{5}{64}$	27. $\frac{7}{45}$
4. $\frac{13}{16}$	10. $\frac{16}{23}$	16. $\frac{7}{36}$	22. $\frac{7}{64}$	28. $\frac{9}{42}$
5. $\frac{3}{13}$	11. $\frac{7}{11}$	17. $\frac{11}{36}$	23. $\frac{9}{64}$	29. $\frac{11}{39}$
6. $\frac{4}{19}$	12. $\frac{14}{15}$	18. $\frac{13}{36}$	24. $\frac{11}{64}$	30. $\frac{13}{35}$

141. Reasons for Reducing Common Fractions to Decimals. — Fractions are reduced to decimals for several reasons. If fractions like $\frac{3}{11}$, $\frac{7}{15}$, $\frac{5}{9}$, $\frac{8}{17}$, are to be added, the result may be found more readily by adding their decimal approximations.

This sum is certain to contain the nearest approximation to four places of decimals. That is, 1.7655 may be taken as the nearest approximation to four places. And 1.76554 is obviously the nearest approximation to five places. By the exercise of a little common sense, we can readily decide how far the decimals need to be carried to obtain a given degree of accuracy in the result.

Thus, $\frac{3}{11} = .272727$
$\frac{7}{15} = .466667$
$\frac{5}{9} = .555556$
$\frac{8}{17} = .470588$
1.765538

142. Comparing Magnitudes of Fractions. — Another reason why fractions are reduced to decimals is that the magnitudes of numbers can be compared more readily in that form.

Example. One ball team has played 37 games and won 21, and another has played 59 games and won 32. Which has won the greater fraction of the games played?

The first team has won $\frac{21}{37}$ of its games and the second $\frac{32}{59}$ of its games. It is not at all obvious at a mere glance which is greater, $\frac{21}{37}$ or $\frac{32}{59}$. On reducing these fractions to decimals we find $\frac{21}{37} = .568$ and $\frac{32}{59} = .542$.

In one year the number of persons in the State of Wisconsin between the ages of 5 and 21 years inclusive was 972,500. Of these, 493,932 attended elementary schools; 35,914, secondary schools; and 11,827, higher schools. In the same year, the number of persons between those ages in the State of Montana was 142,800, while the numbers attending elementary, secondary, and higher schools respectively were 66,651, 3655, and 545.

These numbers do not give any idea as to the comparative school attendance in these states. However, when we know that in Wisconsin .508, .037, and .012 are the fractions of the total number which attend each kind of school, while in Montana these fractions are .467, .026, and .0038, we can make comparisons at once.

WRITTEN EXERCISE (A)

At a certain stage in the pennant race in the American League the number of games played and won respectively by the various teams were: Boston 73, 46; Chicago 74, 48; Cleveland 78, 41; Detroit 72, 36; New York 70, 36; Philadelphia 70, 25; St. Louis 76, 30; Washington 71, 30. Reduce the fraction of games won by each team to a three-place decimal and arrange the teams in the proper order.

143. Fractions Which Can Be Reduced to Exact Decimal Equivalents. — Fractions whose denominators have no prime factors except 2 and 5 can be reduced to exact decimal equivalents. Thus fractions whose denominators are 2, 4, 5, 8, 10, 16, . . . can be so reduced, while fractions whose denominators are 3, 6, 7, 9, 11, 12, 13, 14, 15, 17, . . . cannot be so reduced.

ORAL EXERCISES (A)

Which of the following fractions can be reduced exactly to decimals: $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{5}{6}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$, $\frac{5}{9}$, $\frac{5}{12}$, $\frac{7}{16}$, $\frac{1}{15}$?

144. Fractions in Common Use. — The following fractions represent the diameters in inches of standard sizes of wire manufactured in the United States: $\frac{1}{2}$, $\frac{15}{32}$, $\frac{7}{16}$, $\frac{13}{32}$, $\frac{3}{8}$, $\frac{11}{32}$, $\frac{5}{16}$, $\frac{9}{32}$, $\frac{17}{64}$, $\frac{1}{4}$, $\frac{15}{64}$, $\frac{7}{32}$, $\frac{13}{64}$, $\frac{3}{16}$, $\frac{11}{64}$, $\frac{5}{32}$, $\frac{9}{64}$, $\frac{1}{8}$, $\frac{7}{64}$, $\frac{3}{32}$, $\frac{5}{64}$, $\frac{9}{128}$, $\frac{1}{16}$, $\frac{9}{160}$, $\frac{1}{20}$, $\frac{7}{160}$, $\frac{3}{80}$, $\frac{11}{320}$, $\frac{1}{32}$, $\frac{9}{320}$, $\frac{1}{40}$, $\frac{7}{320}$, $\frac{3}{160}$, $\frac{11}{640}$, $\frac{1}{64}$, $\frac{9}{640}$, $\frac{1}{80}$, $\frac{7}{640}$, $\frac{13}{1280}$, $\frac{3}{320}$, $\frac{11}{1280}$, $\frac{5}{640}$, $\frac{9}{1280}$, $\frac{17}{2560}$, $\frac{1}{160}$.

The following fractions represent the thickness in inches of commercial iron and steel sheets: $\frac{1}{500}$, $\frac{1}{250}$, $\frac{3}{500}$, $\frac{1}{125}$, $\frac{1}{100}$, $\frac{3}{250}$, $\frac{7}{500}$, $\frac{1}{64}$, $\frac{9}{500}$, $\frac{1}{50}$, $\frac{11}{500}$, $\frac{1}{10}$, $\frac{7}{250}$, $\frac{1}{32}$, $\frac{9}{250}$, $\frac{1}{25}$, $\frac{9}{200}$, $\frac{1}{20}$, $\frac{11}{200}$, $\frac{1}{16}$, $\frac{13}{200}$, $\frac{7}{100}$, $\frac{3}{40}$, $\frac{2}{25}$, $\frac{17}{200}$, $\frac{9}{100}$, $\frac{19}{200}$, $\frac{1}{10}$, $\frac{11}{100}$, $\frac{1}{8}$, $\frac{27}{200}$, $\frac{3}{20}$, $\frac{32}{200}$, $\frac{9}{50}$, $\frac{1}{5}$, $\frac{11}{50}$, $\frac{6}{25}$, $\frac{1}{4}$.

WRITTEN EXERCISES (B)

1. Make a list of the fractions in the first group above, giving the decimal equivalents and arranging them in the order of their magnitude. If there are any fractions in this list which do not have exact decimal equivalents, give their nearest approximations to five places of decimals.

2. Make a similar list of the fractions in the second group.

PROBLEMS INVOLVING DECIMALS (B)

1. American gold coin weighs 25.8 grains per dollar and American silver coin weighs 412.5 grains per dollar. Both coins contain .9 precious metal and .1 alloy. How much pure gold is there in a five-dollar gold piece? How much pure silver is there in a silver dollar?

2. A nautical mile is 6080 feet. Express this as a decimal of the statute mile (5280 ft.). By how many feet does a nautical mile differ from 1.151 statute miles?

3. In her fastest trial trip a cruiser covered the course at 34.92 knots (nautical miles per hour). How many miles per hour did she make, counting 1.151 miles per knot?

4. An ocean liner made the trip across the Atlantic Ocean at the rate of 32.10 knots. How many miles per hour was this, counting 1.151 miles per knot?

5. An American busnel contains 2150.42 cubic inches. By how many cubic inches does this differ from the English imperial bushel which contains 2218.19 cubic inches?

6. Multiply the number of cubic inches in an American bushel (2150.42) by .804. By how much does the product differ from 1728 cubic inches (one cubic foot)?

7. A bin is $6\frac{1}{2}$ (6.5) feet wide, $8\frac{3}{4}$ (8.75) feet long, and $5\frac{1}{4}$ (5.25) feet deep. How many bushels of grain will it hold if one cubic foot holds .804 of one bushel?

8. Fresh water weighs about 62.5 pounds per cubic foot. Ice weighs .92 times as much as water. Find the weight of a cubic foot of ice.

9. Using the result obtained in problem 8, find by how many pounds 35 cubic feet of ice differs from one ton (2000 lb.).

10. Cork weighs .24 times as much as water. How many cubic feet of cork will weigh one ton?

11. The water of the Dead Sea weighs 1.24 times as much as fresh water (see problem 8). How much would one cubic foot of water from the Dead Sea weigh?

12. If milk weighs 8.6 lb. per gallon, how many gallons are there in 100 lb. of milk?

13. An empty milk can weighs 19.3 pounds, and when filled with milk it weighs 93.8 pounds. How many gallons of milk does it hold? (See problem 12.)

14. A farmer hauled milk to the creamery as follows: Sunday, 474 lb.; Monday, 464 lb.; Tuesday, 507 lb.; Wednesday, 493 lb.; Thursday, 501 lb.; Friday, 497 lb.; and Saturday, 437 lb. At $37\frac{1}{2}$ cents a gallon what was the value of this milk? (See problem 12.)

15. At 85¢ per thousand cubic feet for gas what is the bill of a family consuming 7840 cubic feet of gas?

16. At 95 cents per thousand cubic feet how many cubic feet does a family consume whose gas bill is $6.45?

17. A lot in New York City is sold for $127 per square foot. What is the price of the lot if it is 27.4 feet wide and 97.3 feet deep?

18. A lot in Boston is sold for $114 per square foot. What is the area of the lot if it sells for $1,248,000?

19. A school baseball team plays 19 games and wins 11 of them. Reduce the standing of the team to a three-place decimal.

20. It has been found by experimentation that on an average 5.6 pounds of corn fed to hogs cause them to gain 1 pound in weight. A farmer feeds 7806 lb. of corn to his hogs. How much should they gain in weight?

21. In a recent year the population of the state of New York was 9,373,000. This same year 145,654 deaths occurred within the state. Find to the nearest tenth the number of deaths per thousand inhabitants. (This is the usual mode of expressing death rates.)

22. During a recent year the populations and the number of deaths in the states and cities named below were as follows:

State or City	Population	Deaths
Minnesota	2,686,000	24,688
Indiana	3,150,000	37,677
New Hampshire	455,000	6,329
Chicago	3,048,000	34,852
New York (City)	5,970,800	70,456
Philadelphia	2,035,900	24,838

Find the death rate for each of these.

ORAL PROBLEMS (A)

Answer all of the following questions. Supply the missing words and numbers.

1. To multiply a decimal by 10, move the decimal point ____ ____ to the ____. To multiply by 100, move the decimal point ____ ____ ____ ____ ____.

2. To divide a decimal by 10, move the decimal point ____ ____ to the ____. To divide a decimal by 100, move the decimal point ____ ____ to the left.

3. Explain how the rules stated under problems 1 and 2 can be used to multiply or divide a whole number by 10, 100, or 1000.

4. Under what conditions may it be necessary to annex or prefix 0's in applying the above rules?

5. To multiply decimals, multiply like whole numbers and then point off as many decimals in the product as there are in the ____ and ____ combined.

6. (B) Tell how you would multiply a decimal number by 25, by 50, by 75.

7. (B) Tell how you would multiply a decimal by 125, by 250, by 750.

8. (B) Tell how you would divide a decimal by 25, by 50, by 75, by 125, by 250, by 500.

9. Our basketball team has won 5 of the 9 games it has played. What fraction of the games played has the team won? Reduce this fraction to a three-place decimal. Make sure that the last figure is correct. Then find what decimal fraction of the games that were played the team has lost.

10. Which of the fractions $\frac{7}{9}$ or $\frac{5}{7}$ is the greater? Reduce these two fractions to three-place decimals and compare. Which can be compared more easily, decimals or common fractions?

11. Which fraction is greater, $\frac{7}{11}$ or $\frac{9}{11}$?

PROBLEMS (B)

1. One cubic foot contains 1728 cubic inches and one gallon contains 231 cubic inches. One cubic foot contains how many gallons? Find answer correct to two decimal places.

2. A tank 8 feet long, 3 feet wide, and 2 feet deep will hold how many gallons?

3. If one barrel contains $31\frac{1}{2}$ gallons, what decimal fraction of a barrel does one cubic foot hold? Use one cubic foot equivalent to $7\frac{1}{2}$ gallons. Thus $7\frac{1}{2} \div 31\frac{1}{2}$ should give the answer. Reduce the result to a decimal.

4. A cistern holds 790 cubic feet. How many barrels will it hold?

5. If one cubic foot contains .4 of a bushel of corn on the ear, how many such bushels will a crib hold that is 25 feet long and 6 feet wide, if the corn averages 5 feet in depth?

6. A farmer delivered 496 pounds of milk gross weight. The cans weighed 87.4 pounds. At 8.6 pounds to the gallon how many gallons of milk did he deliver?

7. In a bin 20 feet long and 8 feet wide there is coal to an average depth of 6.5 feet. At 39 cubic feet per ton, how many tons of coal are there in this bin? Find answer to the nearest tenth of a ton.

TOPICS FOR DISCUSSION

1. If you are to arrange in order of magnitude the fractions $\frac{2}{3}$, $\frac{3}{5}$, $\frac{5}{8}$, how would you proceed? Which is easier, to reduce these fractions to a common denominator, or to reduce them to two-place decimals?

2. If our unit of weight were much smaller than it is, as, for instance, one hundredth of a pound, would we need very often to use fractional parts of the unit of weight?

3. Would very small units of measure be convenient for practical use? Notice that we have *principal* units, as *pound, yard, hour.* Why, do you think, do we use several different units of length?

CHAPTER XIV

ALIQUOT PARTS

145. Use of Aliquot Parts in Business. — The uses of aliquot parts are illustrated in § 140 and in the exercises of this chapter. They are of frequent occurrence, and much drill is necessary in order to handle them effectively. Those who wish to enter business as a profession should not lose sight of the fact that their work is that of specialists and that hence they require extensive training which is not needed by people in other walks of life.

146. An *aliquot part* of a number is contained an integral number of times in the given number.

Thus: 2, $2\frac{1}{2}$, $3\frac{1}{3}$, 5 are aliquot parts of 10, since $10 \div 2 = 5$, $10 \div 2\frac{1}{2} = 4$, $10 \div 3\frac{1}{3} = 3$ and $10 \div 5 = 2$.

The number 1 is an aliquot part of any integer and hence is not enumerated among the aliquot parts of numbers.

147. Prices of Goods Aliquot Parts of a Dollar. — Prices of goods are frequently fixed as aliquot parts of one dollar, since that is equivalent to a certain number of units per dollar.

Thus $12\frac{1}{2}$ is $\frac{1}{8}$ of 100 and hence $12\frac{1}{2}$ cents apiece is the same as 8 for one dollar or 2 for a quarter.

148. Aliquot Parts of 100. — The following are the more important aliquot parts of 100.

ALIQUOT PARTS OF 100

$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{1}{10}$	$\frac{1}{12}$	$\frac{1}{15}$	$\frac{1}{16}$	$\frac{1}{20}$	$\frac{1}{25}$	$\frac{1}{30}$	$\frac{1}{40}$	$\frac{1}{50}$
50	$33\frac{1}{3}$	25	20	$16\frac{2}{3}$	$12\frac{1}{2}$	10	$8\frac{1}{3}$	$6\frac{2}{3}$	$6\frac{1}{4}$	5	4	$3\frac{1}{3}$	$2\frac{1}{2}$	2

There are other aliquot parts of 100. Thus $14\frac{2}{7} = \frac{1}{7}$ of 100; $11\frac{1}{9} = \frac{1}{9}$ of 100; $9\frac{1}{11} = \frac{1}{11}$ of 100; $7\frac{9}{13} = \frac{1}{13}$ of 100, etc., but these occur less frequently in business.

149. Multiplication by Aliquot Parts of 100. — Since goods are frequently priced at aliquot parts of a dollar, multiplication by aliquot parts of 100 is important.

Since $33\frac{1}{3}$ is $\frac{1}{3}$ of 100, multiplying by 100 and dividing by 3 is the same as multiplying by $33\frac{1}{3}$.

Similarly, to multiply by $16\frac{2}{3}$, multiply by 100 and divide by 6. Give rules for multiplying by $12\frac{1}{2}$, $8\frac{1}{3}$, $6\frac{2}{3}$, $6\frac{1}{4}$.

Example 1. Multiply 394 by $33\frac{1}{3}$.

Solution 1. $394 \times 33\frac{1}{3} = 394 \times \frac{100}{3} = 39400 \div 3 = 13133\frac{1}{3}$.

Example 2. $280 \times 16\frac{2}{3} = 28000 \div 6 = 4666\frac{2}{3}$.

Example 3. $1260 \times 12\frac{1}{2} = 126000 \div 8 = 15750$.

Example 4. $2900 \times 8\frac{1}{3} = 290000 \div 12 = 24166\frac{2}{3}$.

Example 5. $790 \times 6\frac{2}{3} = 79000 \div 15 = 5266\frac{2}{3}$.

Example 6. $39800 \times 6\frac{1}{4} = 3980000 \div 16 = 248750$.

WRITTEN EXERCISES (A)

Write down the results of the following.

1. $54 \times 33\frac{1}{3}$	**17.** $49 \times 6\frac{2}{3}$	**33.** $960 \times 6\frac{1}{4}$
2. $47 \times 16\frac{2}{3}$	**18.** $74 \times 6\frac{1}{4}$	**34.** $597 \times 33\frac{1}{3}$
3. $156 \times 8\frac{1}{3}$	**19.** $196 \times 8\frac{1}{3}$	**35.** $340 \times 6\frac{2}{3}$
4. $240 \times 12\frac{1}{2}$	**20.** $38 \times 12\frac{1}{2}$	**36.** $570 \times 16\frac{2}{3}$
5. $1470 \times 6\frac{2}{3}$	**21.** $48 \times 33\frac{1}{3}$	**37.** $840 \times 33\frac{1}{3}$
6. $374 \times 6\frac{1}{4}$	**22.** $64 \times 6\frac{2}{3}$	**38.** $320 \times 16\frac{2}{3}$
7. $126 \times 6\frac{1}{4}$	**23.** $85 \times 6\frac{1}{4}$	**39.** $480 \times 33\frac{1}{3}$
8. $324 \times 8\frac{1}{3}$	**24.** $140 \times 12\frac{1}{2}$	**40.** $740 \times 12\frac{1}{2}$
9. $780 \times 6\frac{2}{3}$	**25.** $24 \times 33\frac{1}{3}$	**41.** $590 \times 8\frac{1}{3}$
10. $360 \times 33\frac{1}{3}$	**26.** $37 \times 12\frac{1}{2}$	**42.** $376 \times 6\frac{1}{4}$
11. $980 \times 16\frac{2}{3}$	**27.** $49 \times 8\frac{1}{3}$	**43.** $428 \times 6\frac{2}{3}$
12. $1546 \times 12\frac{1}{2}$	**28.** $128 \times 6\frac{1}{4}$	**44.** $536 \times 8\frac{1}{3}$
13. $128 \times 6\frac{2}{3}$	**29.** $70 \times 8\frac{1}{3}$	**45.** $784 \times 6\frac{2}{3}$
14. $94 \times 8\frac{1}{3}$	**30.** $544 \times 6\frac{2}{3}$	**46.** $684 \times 6\frac{1}{4}$
15. $276 \times 12\frac{1}{2}$	**31.** $270 \times 12\frac{1}{2}$	**47.** $486 \times 6\frac{2}{3}$
16. $5780 \times 16\frac{2}{3}$	**32.** $380 \times 8\frac{1}{3}$	**48.** $937 \times 16\frac{2}{3}$

WRITTEN EXERCISES (A)

Find the total cost of each of the following. As far as possible perform all multiplications mentally.

1.	2.	3.
18 yd. at $33\frac{1}{3}$ ct.	144 lb. at $33\frac{1}{3}$ ct.	580 yd. at 10 ct.
28 yd. at $16\frac{2}{3}$ ct.	86 lb. at 25 ct.	650 yd. at 20 ct.
54 yd. at $8\frac{1}{3}$ ct.	350 lb. at 50 ct.	540 yd. at 25 ct.
76 yd. at $12\frac{1}{2}$ ct.	230 lb. at $16\frac{2}{3}$ ct.	410 yd. at 50 ct.
64 yd. at $6\frac{1}{4}$ ct.	480 lb. at $8\frac{1}{3}$ ct.	1290 yd. at $8\frac{1}{3}$ ct.
50 yd. at $6\frac{2}{3}$ ct.	290 lb. at $6\frac{2}{3}$ ct.	360 yd. at $12\frac{1}{2}$ ct.

4.	5.	6.
340 yd. at 50 ct.	2500 yd. at 20 ct.	8900 lb. at $6\frac{1}{4}$ ct.
480 yd. at $33\frac{1}{3}$ ct.	5200 yd. at $12\frac{1}{2}$ ct.	4300 lb. at 25 ct.
620 yd. at 25 ct.	340 yd. at $16\frac{2}{3}$ ct.	64 lb. at 50 ct.
700 yd. at $12\frac{1}{2}$ ct.	1260 yd. at $33\frac{1}{3}$ ct.	860 lb. at $16\frac{2}{3}$ ct.
450 yd. at $6\frac{2}{3}$ ct.	530 yd. at $6\frac{1}{4}$ ct.	1200 lb. at $12\frac{1}{2}$ ct.
2700 yd. at $6\frac{1}{4}$ ct.	850 yd. at $8\frac{1}{3}$ ct.	420 lb. at $33\frac{1}{3}$ ct.

The special method for multiplying by aliquot parts of 100 leads to a similar method for multiplying by certain other numbers.

Thus, to find the cost of 48 articles sold at \$12.50 apiece notice that $12.50 = \frac{1}{8}$ of \$100. Hence $48 \times \$12.50 = \$48 \times \frac{100}{8} = \$600$. Notice that $\$6.66\frac{2}{3}$ is $\frac{1}{15}$ of \$100, $\$8.33\frac{1}{3} = \frac{1}{12}$ of \$100, $\$16.66\frac{2}{3} = \frac{1}{6}$ of \$100, $\$3.33\frac{1}{3} = \frac{1}{30}$ of \$10, etc.

In this manner find the cost of each of the following, performing all multiplications mentally.

1.	2.
144 articles at \$12.50	36 articles at $\$3.33\frac{1}{3}$
68 articles at \$ 2.50	48 articles at $\$6.66\frac{2}{3}$
60 articles at $\$6.66\frac{2}{3}$	260 articles at $\$8.33\frac{1}{3}$
232 articles at $\$8.33\frac{1}{3}$	380 articles at \$ 2.50
648 articles at $\$16.66\frac{2}{3}$	156 articles at \$12.50
342 articles at $\$33.33\frac{1}{3}$	84 articles at $\$16.66\frac{2}{3}$
126 articles at \$ 2.50	96 articles at $\$33.33\frac{1}{3}$
242 articles at $\$3.33\frac{1}{3}$	72 articles at \$25.00

150. Multiplication by Multiples of Aliquot Parts of 100. — Since $\frac{1}{15}$ of $100 = 6\frac{2}{3}$, $\frac{2}{15}$ of $100 = 13\frac{1}{3}$, $\frac{4}{15} = 26\frac{2}{3}$, etc. Hence we may multiply by $13\frac{1}{3}$ by multiplying by 200 and taking $\frac{1}{15}$ of the result.

The following list contains the more important numbers of this kind.

$\frac{2}{3}$ of $100 = 66\frac{2}{3}$	$\frac{7}{8}$ of $100 = 87\frac{1}{2}$	$\frac{3}{16}$ of $100 = 18\frac{3}{4}$
$\frac{2}{5}$ of $100 = 40$	$\frac{5}{12}$ of $100 = 41\frac{2}{3}$	$\frac{5}{16}$ of $100 = 31\frac{1}{4}$
$\frac{3}{5}$ of $100 = 60$	$\frac{7}{12}$ of $100 = 58\frac{1}{3}$	$\frac{7}{16}$ of $100 = 43\frac{3}{4}$
$\frac{4}{5}$ of $100 = 80$	$\frac{11}{12}$ of $100 = 91\frac{2}{3}$	$\frac{9}{16}$ of $100 = 56\frac{1}{4}$
$\frac{5}{6}$ of $100 = 83\frac{1}{3}$	$\frac{2}{15}$ of $100 = 13\frac{1}{3}$	$\frac{11}{16}$ of $100 = 68\frac{3}{4}$
$\frac{3}{8}$ of $100 = 37\frac{1}{2}$	$\frac{4}{15}$ of $100 = 26\frac{2}{3}$	$\frac{13}{16}$ of $100 = 81\frac{1}{4}$
$\frac{5}{8}$ of $100 = 62\frac{1}{2}$	$\frac{7}{15}$ of $100 = 46\frac{2}{3}$	$\frac{15}{16}$ of $100 = 93\frac{3}{4}$

Example 1. Find $453 \times 66\frac{2}{3}$.

Solution. Multiply by 200 and take $\frac{1}{3}$ of the product.

```
  453
  200
3)90600
 30200
```

Example 2. Find $2460 \times 83\frac{1}{3}$.

Solution. $2460 \times 500 = 1230000$; $1230000 \div 6 = 205000$. The work is shown to the right:

```
 2460
  500
6)1230000
 205000
```

Example 3. Find $1740 \times 56\frac{1}{4}$.

Solution.

```
  1740
   900
16)1566000
 97875
```

Example 4. Find $286 \times 37\frac{1}{2}$.

Solution.

```
  286
  300
8)85800
 10725
```

WRITTEN EXERCISES (B)

Find the product of each of the following and, in case the result is not an exact decimal, find it correct to two decimal places.

1. $490 \times 46\frac{2}{3}$	**5.** $795 \times 56\frac{1}{4}$	**9.** $1940 \times 62\frac{1}{2}$
2. $794 \times 18\frac{3}{4}$	**6.** $1260 \times 58\frac{1}{3}$	**10.** $4910 \times 37\frac{1}{2}$
3. $1640 \times 31\frac{1}{4}$	**7.** $560 \times 41\frac{2}{3}$	**11.** $4310 \times 83\frac{1}{3}$
4. $390 \times 43\frac{3}{4}$	**8.** $1430 \times 87\frac{1}{2}$	**12.** $650 \times 93\frac{3}{4}$

WRITTEN EXERCISES (B)

Find the total in each of the following.

1.	2.	3.
48 yd. at $66\frac{2}{3}$ ct.	64 lb. at $93\frac{3}{4}$ ct.	460 lb. at $87\frac{1}{2}$ ct.
190 yd. at $83\frac{1}{3}$ ct.	75 lb. at 80 ct.	320 lb. at $58\frac{1}{3}$ ct.
420 yd. at $37\frac{1}{2}$ ct.	160 lb. at $68\frac{3}{4}$ ct.	2300 lb. at 40 ct.
360 yd. at $62\frac{1}{2}$ ct.	610 lb. at $56\frac{1}{4}$ ct.	1250 lb. at 60 ct.
480 yd. at $87\frac{1}{2}$ ct.	310 lb. at $31\frac{1}{4}$ ct.	2500 lb. at 80 ct.
840 yd. at $41\frac{2}{3}$ ct.	1300 lb. at $43\frac{3}{4}$ ct.	830 lb. at $37\frac{1}{2}$ ct.
350 yd. at $58\frac{1}{3}$ ct.	2600 lb. at $68\frac{3}{4}$ ct.	530 lb. at $87\frac{1}{2}$ ct.
1240 yd. at $37\frac{1}{2}$ ct.	1450 lb. at $56\frac{1}{4}$ ct.	96 lb. at $66\frac{2}{3}$ ct.
630 yd. at 40 ct.	780 lb. at $31\frac{1}{4}$ ct.	35 lb. at $83\frac{1}{3}$ ct.

4.	5.	6.
4830 sq. ft. at $37\frac{1}{2}$ ct.	600 yd. at 80 ct.	1280 lb. at $81\frac{1}{4}$ ct.
3950 sq. ft. at $58\frac{1}{3}$ ct.	350 yd. at 50 ct.	1600 lb. at $68\frac{3}{4}$ ct.
4300 sq. ft. at 40 ct.	280 yd. at 60 ct.	3200 lb. at $18\frac{3}{4}$ ct.
8700 sq. ft. at 50 ct.	460 yd. at 40 ct.	2400 lb. at $46\frac{2}{3}$ ct.
6400 sq. ft. at $62\frac{1}{2}$ ct.	1240 yd. at $83\frac{1}{3}$ ct.	1820 lb. at $13\frac{1}{3}$ ct.
3200 sq. ft. at $87\frac{1}{2}$ ct.	980 yd. at $66\frac{2}{3}$ ct.	1460 lb. at $26\frac{2}{3}$ ct.
4500 sq. ft. at $41\frac{2}{3}$ ct.	650 yd. at $18\frac{3}{4}$ ct.	940 lb. at $58\frac{1}{3}$ ct.
5300 sq. ft. at $58\frac{1}{3}$ ct.	860 yd. at $46\frac{2}{3}$ ct.	870 lb. at $41\frac{2}{3}$ ct.
1250 sq. ft. at $18\frac{3}{4}$ ct.	1540 yd. at $26\frac{2}{3}$ ct.	380 lb. at $66\frac{2}{3}$ ct.

7.	8.	9.
350 yd. at $18\frac{3}{4}$ ct.	6800 lb. at $87\frac{1}{2}$ ct.	9860 lb. at $58\frac{1}{3}$ ct.
500 yd. at $58\frac{1}{3}$ ct.	4300 lb. at $91\frac{2}{3}$ ct.	2670 lb. at $83\frac{1}{3}$ ct.
380 yd. at $41\frac{2}{3}$ ct.	3800 lb. at $26\frac{2}{3}$ ct.	380 lb. at $66\frac{2}{3}$ ct.
1694 yd. at $87\frac{1}{2}$ ct.	1400 lb. at $46\frac{2}{3}$ ct.	94 lb. at $87\frac{1}{2}$ ct.
86 yd. at $13\frac{1}{3}$ ct.	2600 lb. at $18\frac{3}{4}$ ct.	86 lb. at $37\frac{1}{2}$ ct.
1245 yd. at $62\frac{1}{2}$ ct.	1900 lb. at $66\frac{2}{3}$ ct.	320 lb. at $58\frac{1}{3}$ ct.
2400 yd. at 50 ct.	3400 lb. at $87\frac{1}{2}$ ct.	464 lb. at $91\frac{2}{3}$ ct.
860 yd. at 40 ct.	2600 lb. at $41\frac{2}{3}$ ct.	1850 lb. at $31\frac{1}{4}$ ct.
640 yd. at $58\frac{1}{3}$ ct.	4600 lb. at $58\frac{1}{3}$ ct.	980 lb. at $56\frac{1}{4}$ ct.
3280 yd. at $37\frac{1}{2}$ ct.	3800 lb. at $46\frac{2}{3}$ ct.	330 lb. at $68\frac{3}{4}$ ct.

MISCELLANEOUS WORK (A)

1. Using the form on page 26 enter the following data, fill in the daily balances, and find the totals: August 20, balance: $319.80; deposits: $130, $60, $25; checks: $2.80, $4.56, $10.60. August 21, checks: $5.25, $7.40. August 22, checks: $1.50, $12.50. August 23, deposits: $15.00; checks: $3.40, $5.00, $7.20. August 24, checks: $25.00, $2.50, $45.00. August 25, checks: $1.50, $7.50, $18.60.

2. Following is the record of 16 loads of corn hauled to the elevator. Find the net weight of each load and the total net weight.

Gross Weight	Tare	Net Weight	Gross Weight	Tare	Net Weight
4960	1840		5130	1860	
3940	1810		5070	1850	
4760	1850		4980	1850	
4530	1860		4730	1830	
5160	1830		4850	1840	
4860	1830		4750	1870	
4335	1830		4640	1890	
4455	1860		4330	1920	

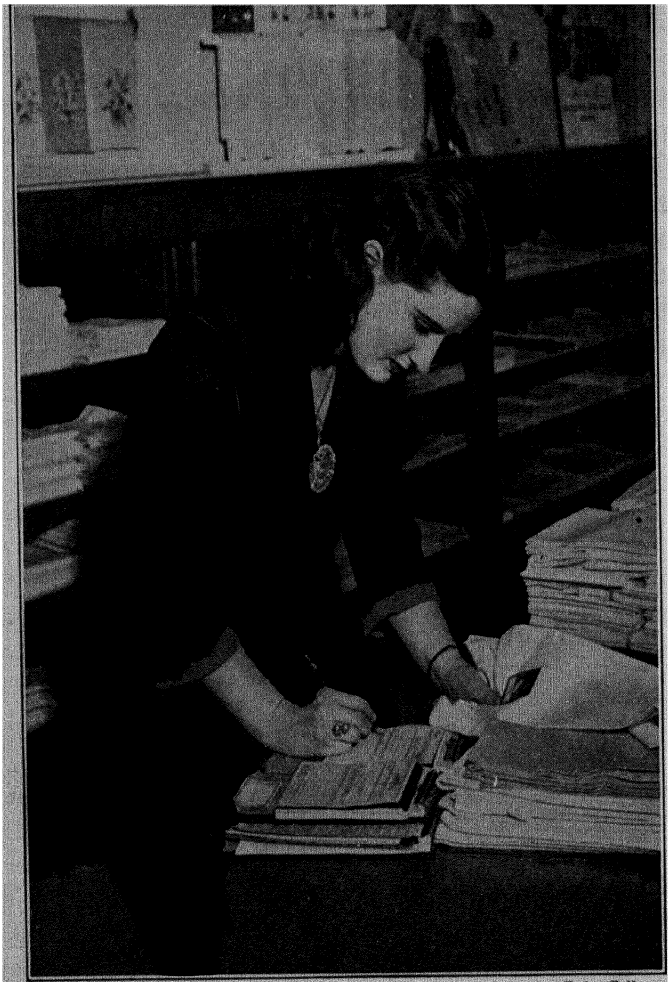
3. An elevator contains 840,000 bushels of wheat. Twelve train loads of wheat are removed containing numbers of bushels as follows: 27 840, 31 690, 29 360, 24 980, 27 360, 31 240, 30 380, 29 080, 28 900, 23 430, 32 670, 31 240. Find the number of bushels left in the elevator.

4. In an elevator amounts of wheat were received as follows in one day: 2860 bushels, 1430 bushels, 4930 bushels, 830 bushels, 150 bushels, 3980 bushels, 1740 bushels, 2360 bushels, 940 bushels, 380 bushels. The same day carloads of wheat were removed containing numbers of bushels as follows: 1570, 1380, 1445, 1260, 1480, 1435, 1395, 1540, 1510, 1490. Did the elevator gain or lose wheat this day and how much?

Find the sums of the following by horizontal addition.

5. 389, 468, 590, 773, 857, 745, 1370, 2914, 3180.

6. 541, 792, 298, 347, 1592, 438, 964, 1874, 4912, 2672.



Ewing Galloway

MAKING OUT A SALES SLIP FOR LATER BILLING

CHAPTER XV

BILLS AND ACCOUNTS

151. Importance of Bills and Accounts. — In practically every kind of business, accounts must be kept and bills must be made out. Everyone engaged in business should be able to make out a bill in proper form. The accountant should be able to design blank bills to meet the requirements of the business in which he is employed. A bill for any kind of merchandise is also called an *invoice*.

152. Items of a Bill. — A bill is a statement rendered by a seller to the purchaser of goods. A bill should contain the following items.

1. Name and location of seller.
2. Name and location of buyer.
3. Date of sale of each item.
4. Quantity, nature, and price per unit of each item.
5. Total price of each item and total of the whole bill.
6. Extra marks on the goods, if any.
7. Terms of sale such as cash, credit for 30 days, 60 days, etc.
8. Extra charges, if any, such as cartage, freight, boxing, etc. Like express and parcel post, freight costs are figured according to classification, weight, and distance. Those who use freight only occasionally usually consult the local freight agent for rates.
9. Any deductions, such as credits, discounts, part payments, etc.

153. Extending and Footing Bills. — Multiplying the price by the number of units to find the cost of each item is called *extending* the bill. Totaling the cost of the items is called *footing* the bill.

In extending bills one of the numbers to be multiplied is usually small enough so that the multiplication can be done at sight.

In case it is necessary to extend some items by ordinary long multiplication this work should be done on separate paper and the product only should be entered on the bill.

154. Examples of Bills. — The following bills show a variety of forms used in different kinds of business.

Cleveland, Ohio, Apr. 24, 1950.

MR. JAMES LYSANDER,
1040 Belleview, City

Bought of JOHN J. STONE & CO.

Terms. Cash the first of each month.

12 bu. Potatoes	$1.45	17	40		
3 bbl. Flour	8.80	26	40		
144 doz. Oranges	.45	64	80		
110 lb. Tea	.52	57	20		
96 lb. Coffee	.47	45	12		
112 Canned Goods	.38	42	56		
75 bu. Apples	1.01	75	75		
				329	23

EXERCISES

Extend the amounts in the following invoices and find the total of each one. Copy each on an appropriate blank.

1. Wholesale Groceries

(Note the pre-war prices of some bill.)

Chicago, May 4, 1937.

SMITH & PERKINS,
Milwaukee

Bought of SPRAGUE, WARNER & CO.

Terms: Cash 30 days.

42 chests of Green Tea	at	$27.50				
27 chests of Black Tea	at	20.19				
14 chests of Uncolored Japan Tea	at	47.22				
15 sacks of C. & R. Coffee	at	20.25				
25 bbl. of Granulated Sugar	at	36.58				
12 bbl. of Coffee Sugar	at	26.50				
40 boxes of Lemons	at	4.56				
56 boxes of Oranges	at	$3.87\frac{1}{2}$				
22 boxes of Grapefruit	at	$4.62\frac{1}{2}$				
29 boxes of Raisins	at	3.10				
Cartage		24.80				
Freight (prepaid)		15.30				

2. **Furniture**

New York City, Feb. 15, 1950.

GRAND RAPIDS FURNITURE CO.

Sold to HORACE MANN SCHOOL,
New York.

Terms: Cash.

264	4 Chairs	at	$12.40	
893	2 H. B. W. & Co. Chairs 119A	at	9.95	
1260	2 H. B. W. & Co. Chairs 119B	at	14.10	
293	4 Book Cases	at	35.25	
741	2 Library Tables	at	34.75	
626	100 Chairs	at	12.25	
547	1 Principal's Desk	at	75.50	
827	12 Office Chairs	at	22.50	

The numbers entered in the left margin are identification numbers used by the store.

3. **Wholesale Fruit**

Cleveland, Ohio, Apr. 8, 1937.

MR. A. C. BLAINE,
1024 Broadway, City.

Bought of LAKESIDE FRUIT CO.

Terms: 30 days

120 boxes of Oranges	at	$ 3.00	
125 doz. Pineapples	at	.75	
25 bbl. Greenings	at	2.25	
56 bbl. Baldwins	at	3.60	
12 bbl. Kings	at	4.25	
32 bunches Bananas	at	1.02	
16 crates Strawberries	at	2.25	
45 bbl. Sweet Potatoes	at	3.63	
18 bbl. Potatoes	at	3.00	
14 crates Lemons	at	2.10	
15 crates Grapefruit	at	2.80	
11 bbl. Northern Spy	at	4.60	
17 boxes Floridas	at	3.12	
190 lb. Dates	at	.08	
Cartage		15.50	

4. Drawing Instruments

Cleveland, June 4, 1937.

SOLOMENDEN OPT. CO.

Sold to EAST TECH. HIGH SCHOOL,
East 55th St.

20 Federal Ruling pens $5\frac{1}{2}''$	at	$ .45
20 Federal Bow pencils	at	.64
15 Federal Bow pens	at	.64
25 Compass pens for federal compass	at	.60
10 Federal compasses complete	at	1.48
36 Federal drawing boards	at	.55

5. Shop Supplies

Cleveland, June 5, 1937.

M. SHERWOOD CO.

Sold to EAST TECH. HIGH SCHOOL,
East 55th St.

50 ft. Leather Belting $\frac{1}{5}''$ wide	$ .048 per foot
400 ft. Belt Lacings $\frac{1}{4}''$ cut lacing	.80 (per 100 feet)
200 ft. Belt Lacings $\frac{3}{8}''$ cut lacing	1.13 (per 100 feet)
100 ft. Leather Belting $\frac{7}{8}''$ single ply	.057 per foot
100 ft. 2" wide XLCR Leather Belting	.255 per foot
36 ft. Leather Belting $9\frac{1}{2}''$ single ply	.460 per foot
36 ft. Leather Belting 5" double ply	.64 per foot
3 Bales White cotton waste XXX2434 (100 lb. to the bale)	.07$\frac{7}{8}$ per lb.
To be delivered on or before Sept. 14, 1937	

155. Credit Memorandum. — Sometimes goods which are ordered do not prove satisfactory, and are returned, and a rebate or deduction is made to the purchaser for the value of the goods returned.

In such a case, a memorandum of credits showing the credit is made out by the seller and sent to the purchaser.

Credit Memorandum

CLEVELAND, O., Feb. 1, 1950

MR. I. W. SMITH,
Buffalo, N. Y.

In account with HENRY & CO.
E. Third and Williams St.

	500 bd. ft. of quarter oak @ $120.00 per M		$60	00

This statement resembles an invoice in form, except that it shows a credit and not a debit. For this reason the printing on the credit memorandum is usually in red or some other color than black.

When the bill is rendered the amount credited should be deducted from the bill. In case such deduction is not made the credit memorandum may be used as part payment.

Find the amount due on the following bill.

Wholesale Dry Goods

ST. LOUIS, Feb. 15, 1947.

MORGAN CO.

Sold to JOHNSON & WILLIAMS
Columbus, Ohio.

326	1150 yd.	Gingham	@ $ .11½	per yd.
745	1250 "	Cashmere	@ .90	" "
819	960 "	Silk	@ 2.10	" "
317	2550 "	Crash Toweling	@ .28	" "
269	765 "	India Linen	@ .21½	" "
	Credit 960 yd. of Silk @ $2.08			

The silk proved to be unsatisfactory, and was returned. Write a credit memorandum properly signed for the amount of the credit.

156. Receipting a Bill. — When a bill is paid it is receipted by the concern receiving the payment. This is done by writing "Paid"

across the face of the bill together with the date of payment and the signature of the recipient. When a clerk receipts a bill for a firm, he should write the name of the firm and then his own initials.

Thus: Paid Aug. 9, 1949.
United Stove Co.
per E. S.

157. Accounts. — Sometimes only part of the amount of a bill is paid. Such payments must be shown as credits on the bill and their amounts deducted from the total of the bill. A statement showing purchases and payments and the balance due is called a *statement of account* or simply an *account*.

The United Stove Co.

CLEVELAND, O., Feb. 1, 1947.

Sold to EARL AND BROWN,
Warren, Ohio.

Terms: Cash 30 days

Jan.	6	10 Oakland Gas Ranges	@ $93.25	$932	50		
	14	12 New Federal Electric Ranges	@ 158.50	1902	00		
	25	8 Climax Wood-coal Ranges	@ 60.50	484	00	$3318	50
		By Cash		$1300	00		
				1200	00	2500	00
		Balance				$818	50

PROBLEMS (A)

Arrange each of the following in the form of a bill and find the amount due on each.

1. Edwards Co., Springfield, Mass., sold to Mrs. Frank Morse, 10 pounds of sugar at $12\frac{1}{2}$ ct. a pound; 1 sack of flour, at $1.50 a sack; $1\frac{1}{4}$ bushels of potatoes at $1.25 a bushel; 2 bags of salt at 6 ct. a bag; 2 pounds of lettuce at 25 ct. a pound; 2 loaves of bread at 18 ct. a loaf; $2\frac{1}{4}$ dozen eggs at 72 ct. a dozen; 2 pounds of Elgin butter at 70 ct. a pound; $1\frac{3}{4}$ dozen oranges at 55 ct. a dozen.

2. A merchant bought of the Rudd Co., Cleveland, the following: 1400 bushels of apples at 92 ct. a bushel; 580 boxes of oranges at \$4.14 a box; 15 bunches of bananas at \$5.98 a bunch; 160 boxes of lemons at \$2.50 a box.

3. In the month of March, John Wilson, a farmer, residing at Norwalk, Ohio, bought of Martin & Co., Columbus, lumber and mill products as follows: March 1, 10,000 feet of pine lumber at \$62 per M (thousand); 5000 shingles at \$7.75 per M; March 8, 50 sacks of cement at 57 ct. a sack; 5000 ft. of hemlock at \$58 per M; March 15, 3000 ft. of lumber consisting of $2'' \times 4''$ and $2'' \times 6''$ at \$58 per M.

March 15 a payment of \$400 was made and on March 25 another payment of \$200 was made. Render the statement to Wilson, April 1, showing the amount due; and receipt the bill as having been paid in full April 6.

4. Johnson Co. of Pittsburg sold William Earle, Avon, Ohio, on January 17:

20 bushels of clover seed at \$16.25 per bushel; 18 bushels of alfalfa seed at \$16.75 per bushel; 8 bushels of Timothy seed at \$9.30 per bushel; 24 bushels of small clover seed at \$16.60 per bushel.

Receipt the bill as having been paid February 8.

F. O. B. (Free on board) Avon. (The shipper paid the freight.)

Also find the amount of this bill if the price per bushel of each is advanced 50 ct.

5. J. W. Brown of Detroit, Mich., buys a quantity of groceries from the Edwards Co., Chicago. From a chest of tea, 25 pounds are spilled out and Mr. Brown asks for a memorandum of credit covering the amount of leakage. Make out a credit memorandum like the one which the Edwards Co. sends him, the tea being worth 50 ct. a pound.

6. Messrs. Aeter & Smith, Dayton, Ohio, sold to the Mechanical Engineering Co., Chicago, 20 gasoline engines at \$250.00 each to be sent by freight. Make out the invoice.

Terms of Sale: Cash 30 days.

158. Pay Rolls.

PAY ROLL FOR THE WEEK ENDING Feb. 27

No.	NAME	NO. OF HOURS WORKED EACH DAY						T. NO. HR.	WAGES P. HR.	WAGES P. WK.
		M.	T.	W.	Th.	F.	S.			
1.	Edward Dutton	$6\frac{1}{2}$	7	$7\frac{1}{2}$	$7\frac{3}{4}$	8	$3\frac{1}{4}$	40	$1.25	
2.	Ray Edwards	7	6	$7\frac{1}{2}$	8	$6\frac{1}{2}$	4	39	.95	
3.	James E. Johns	7	6	6	6	7	4	36	1.10	
4.	Harold Johns	$6\frac{1}{2}$	7	$6\frac{1}{2}$	8	$5\frac{1}{2}$	$3\frac{1}{2}$	37	.95	
5.	John Richards	$6\frac{3}{4}$	$7\frac{1}{4}$	6	6	5	3	34	1.00	
6.	Edmund Smith	$7\frac{1}{2}$	7	$6\frac{1}{2}$	7	6	4	38	1.05	
7.	Reg. Williams	7	7	$7\frac{1}{4}$	7	$6\frac{3}{4}$	4	39	1.15	
8.	Geo. B. Wilson	8	7	5	$7\frac{1}{4}$	7	$3\frac{3}{4}$	38	1.50	

Find the wages earned this week by each man. Find the total pay roll.

Many firms pay their workmen in cash, not in checks. Hence it is necessary to determine the amount of money or change required and the amount of each kind of coin or bill.

CITIZENS' SAVINGS BANK
TOLEDO, OHIO.

Pay Roll Memorandum
EARLE & WILLIAMS
require the following:

	NO.	AMOUNT
Pennies		
Nickels		
Dimes		
Quarters		
Halves		
Dollars		
Twos		
Fives		
Tens		
Twenties		
Total Amount		

The pay roll is prepared from time slips like the following.

TIME SLIP, Oct. 21

No. of Worker	In	Out	In	Out
1.	8.15	12.25	1.35	4.41
2.	8.17	11.33	2.29	5.29
3.	8.20	12.05	1.35	5.30
4.	8.00	12.30	1.40	4.40
5.	8.15	12.15	1.30	5.30
6.	8.15	12.20	1.45	4.25
7.	8.05	12.25	1.20	4.30
8.	8.30	12.30	1.30	5.30
9.	8.05	12.45	1.45	4.20
10.	8.00	1.00	2.05	4.40
11.	8.30	1.10	2.15	5.35
12.	8.15	12.50	1.55	5.15
13.	8.10	12.10	1.20	5.00
14.	8.20	12.30	1.40	4.30

From this time slip prepare part of a pay roll like the one given on page 128. Find the time for each man to the nearest $\frac{1}{4}$ hour.

From the data given below prepare a pay roll, and as part of it prepare a pay roll memorandum. (One line is filled in completely.) By adding each of the columns to the right find how many ten dollar bills, how many fives, etc. are needed.

No.	Name	No. Hr. Each Day						Tot. Hr.	Hr. Wage	Total Wage	Money of Various Kinds							
		M	T.	W.	Th.	F.	S.				$10	$5	$1	$ 50	Q	D	N	P
3	James Wilson	7	8	7²	6	5³	4		$0.98		3	1	2	0	1	2	0	1
1	W. B. Hart	7	7²	8	7	6	3¹		1.10									
5	R. T. Wallace	8	8	7	6²	7	3¹		.95									
7	A. C. Jones	7²	7	6	5²	7	3²		1.25									
8	Art. Higgins	8	7	7²	6²	5²	4		1.15									
9	Joe Callahan	7²	7	8	7	6	4		1.00									
10	Buck Smead	8	7	7	7	6³	3²		1.05									
11	Frank Powers	7	7²	6	7	5³	3²		1.08									
12	E. B. Billings	7	7	6	6²	6	4		1.50									

NOTE. The small numbers (¹, ², and ³) indicate quarter hours.

MISCELLANEOUS WORK (A)

Add and check by casting out 9's.

1.	2.	3.	4.	5.	6.	7.
3784	1357	2197	7204	9140	3767	8587
1126	5209	6318	7408	1582	7692	7743
1778	7913	7829	7439	4593	5663	6647
6462	8642	3421	8764	5427	5443	5326
7890	2046	2324	3914	6512	5991	7298
4321	1357	7353	3914	5324	2454	2248
5678	4567	5754	1678	9105	4347	3367
2109	2098	2686	9432	6480	1465	4455
3456	5920	3431	7678	3590	3998	9988
2987	3642	2132	2472	7670	1648	5310
1023	7829	4150	9381	1918	2782	3670
4104	1416	2147	1114	9162	3765	7243

Find the difference in the sums in the following pairs of columns.

8.		9.		10.	
14987	8476	46300	5170	2100	3617
9026	9320	2780	19780	1807	19240
8400	7371	9146	4460	4304	5330
9273	2340	3390	56214	5245	27462
7886	2570	2800	17690	3904	39500
30080	22600	5600	84800	8380	40800
1905	10800	18380	37700	6140	32480
465	3980	22600	59850	6934	3190
876	4760	39300	62380	3700	1914
8470	5490	24400	77240	2540	217

Find the products of the following, using the method given in § 49.

11. 95×8762	**17.** 42×1340	**23.** 56×47
12. 81×3040	**18.** 37×6874	**24.** 59×385
13. 73×945	**19.** 29×5392	**25.** 62×296
14. 65×1392	**20.** 58×4785	**26.** 73×138
15. 47×2847	**21.** 67×2967	**27.** 79×592
16. 53×5563	**22.** 74×3762	**28.** 82×7874

CHAPTER XVI

DENOMINATE NUMBERS

159. Uses of Measurement. — Practically everything we buy and sell is measured. We buy meat by the pound, cloth by the yard, land by the acre, and milk by the quart. We describe our houses, our machinery, and even our horses by giving their dimensions.

Buildings and machinery are designed and the dimensions of the various parts are specified. These parts are "made to measure," often in distant places, but when brought together and assembled they fit properly into their places because the measurements were correctly made and faithfully followed out.

One of the most important differences between uncivilized and civilized peoples is that the latter make extensive use of measuring, while the former have no definite means of finding the exact magnitude of a quantity. The Indian would say "much," or "heap much," and that is about as near as he could come to a definite expression of a magnitude. Civilized peoples designate quantities definitely as so many times certain units.

160. Definition of Measurement. — Measurement is the process of finding the magnitude of a quantity by comparing it with a certain magnitude of the same kind, called a unit of measure.

161. Denominate Numbers. — Denominate numbers are expressed in terms of units of measurement, which have been established by law or by custom.

Thus, 8 feet, 6 quarts, 18 pounds, 4 hours are denominate numbers.

162. Simple Denominate Numbers. — A simple denominate number is expressed in terms of one unit only.

Thus, 5 yards, 15 acres, 18 cubic feet are simple denominate numbers.

163. Compound Denominate Numbers. — A compound denominate number is expressed in terms of two or more units.

Thus, 5 yards 2 feet, 2 pounds 3 ounces are compound denominate numbers.

164. Kinds of Measurement. — The principal different kinds of measurement are: measurement of distance, of area, of volume or capacity, of weight, of time, of angles, and of value.

165. Units Used. — In each kind of measurement there are several different units adapted to different magnitudes to be measured.

Thus, the units of distance are the inch, foot, yard, rod, or mile. The unit used depends upon the magnitude of the distance to be measured.

166. A Single Unit Usually Used. — To measure any one thing, a unit is selected, and the magnitude is then usually given in terms of this unit and fractions of the unit.

Thus, we speak of $2\frac{1}{4}$ pounds of meat rather than of 2 pounds 4 ounces, etc. In some cases, however, two or more units may be used. Thus, we speak of two weeks and three days, one hour and twenty minutes.

167. Relations among Various Units. — It is necessary, however, to know the relation between the different units.

Thus, the length of a room is given in feet, while we buy carpet for it by the yard. Vegetables are bought by the bushel, and sold by the quart or pound.

MEASUREMENT OF LENGTH

168. Principal Units of Linear Measure. — The principal units of linear measure are the inch, foot, yard, rod, and mile. In the tables, the units which are used most frequently are printed in heavy type.

TABLE OF LINEAR MEASURE

Measures in Common Use		Surveyors' Measure	
12 **inches** (in.)	= 1 **foot** (ft.)	7.92 inches	= 1 link (l.)
3 feet	= 1 **yard** (yd.)	100 links	= 1 chain (ch.)
$5\frac{1}{2}$ yards	= 1 **rod** (rd.)	25 links	= 1 rod
320 rods	= 1 **mile** (mi.)	4 rods	= 1 chain
5280 feet	= 1 mile	80 chains	= 1 mile

Special Measures		Nautical Measures	
8 furlongs	= 1 mile	6 feet	= 1 fathom
3 feet	= 1 pace	120 fathoms	= 1 cable length
4 inches	= 1 hand	6080 feet	= 1 nautical mile (knot)
$\frac{1}{3}$ inch	= 1 size	3 knots	= 1 league

The regular mile is sometimes called a statute mile. The knot or nautical mile is also called the geographical mile. The yard is the fundamental unit of length from which the others are derived according to the above table. The length of the English yard was fixed by law in 1866 as the length of a certain bronze bar at 62° F. The yard as adopted by the U. S. Coast Survey is very slightly longer, and is nearly .91 meter. The meter is really regarded as the fundamental unit of length the world over. Its length was first based on one ten-millionth of the distance from the equator to the poles along a meridian. The yard of the U. S. Coast Survey is such that one meter equals 39.37 inches.

169. Uses of Special Units of Length. — The *hand* is used in measuring the height of horses. The *furlong* is used in measuring the length of race courses. The *size* is used in measuring shoes. *Rods* and *yards*, or *rods* and *feet*, are not frequently used together. However, it is not uncommon to divide a forty-acre piece of land (measured in rods) into streets and building lots which are measured in feet.

The *link* and *chain* are used by surveyors in measuring large tracts of land. Thus, in the U. S. forestry service, these are used almost exclusively in measuring large areas.

The name chain may have been derived from the instrument used in measuring. Now, however, a steel tape is usually used, but the units of measure are retained. The link and chain are convenient because of the decimal relation between them (100 links = 1 chain), and for other reasons which will appear later (see § 176).

170. Reduction of Denominate Numbers. — Reduction of a denominate number consists in changing the units in which it is expressed, without changing the value.

Thus, 40 in. = 3 ft. 4 in.	14 ft. = 4 yd. 2 ft.
68 in. = 5 ft. 8 in.	25 ft. = 8 yd. 1 ft.

Reduction *descending* consists in reducing a denominate number to units of lower denomination.

Thus, in 2 yd. 1 ft. = 84 in., and 4 ft. 8 in. = 56 in., the changes are reductions descending.

Reduction *ascending* consists in reducing a denominate number to units of higher denomination.

Thus, in 42 in. = 3 ft. 6 in., and 38 ft. = 12 yd. 2 ft., the changes are reductions ascending.

ORAL EXERCISES (A)

1. How many feet and inches are there in each of the following: 36 in., 45 in., 65 in., 80 in., 94 in., 110 in., 120 in., 135 in.?

2. How many yards and feet are there in each of the following: 16 ft., 27 ft., 40 ft., 70 ft., 85 ft., 92 ft., 120 ft.?

3. How many feet are there in 2 rd.? in 3 rd.? in 4 rd.? in 7 rd.?

4. How many rods are there in $\frac{1}{2}$ mile? in $\frac{1}{4}$ mile? in $\frac{3}{4}$ mile?

5. How many rods are there in one furlong? in 3 furlongs?

6. How many chains are there in a half mile? in a quarter mile? in $\frac{1}{8}$ of a mile? in $\frac{3}{4}$ mile?

7. If a horse is 16 hands high, how many feet high is he?

8. How many yards are there in 3 rd.? in 8 rd.? in 12 rd.?

9. In the above exercises, which are reductions ascending, and which are reductions descending?

Example. Reduce 14 yd. 2 ft. 8 in. to inches.

Solution

14 yd. $= 3 \times 14$ ft. $= 42$ ft.
42 ft. $+ 2$ ft. $= 44$ ft.
44×12 in. $= 528$ in.
528 in. $+ 8$ in. $= 536$ in.

WRITTEN EXERCISES (A)

Reduce each of the following to the lowest denomination named in the example.

1. 45 yd. 1 ft. 9 in.
2. 40 rd. 5 yd. 1 ft.
3. 3 mi. 80 rd.
4. 26 ch. 71 l.
5. 7 mi. 24 ch. 50 l.
6. 3 mi. 2640 ft.

Example. Reduce 486 in. to inches, feet, and yards.

Dividing 486 by 12, we obtain a quotient of 40 and a remainder of 6.

That is, 486 in. $= 40$ ft. 6 in.

Dividing 40 by 3, we obtain a quotient of 13 and a remainder of 1.

That is, 40 feet $= 13$ yd. 1 ft.

Hence, 486 in. $= 13$ yd. 1 ft. 6 in.

Solution.

12) 486
3)40 − 6
13 − 1

EXERCISES

1. Reduce 1246 inches to yards, feet, and inches.
2. Reduce 786 feet to yards and feet.
3. Reduce 12,698 feet to miles and feet.
4. Reduce 4,796,850 links to miles, chains, and links.
5. Reduce 8947 links to chains, rods, and links.
6. Reduce 25,790 feet to nautical miles and feet.

ADDITION OF DENOMINATE NUMBERS

171. *Example.* Add 15 yd. 2 ft. 8 in.; 6 yd. 1 ft. 3 in.; and 5 yd. 2 ft. 7 in.

Solution. Write the numbers so that numbers of the same denomination are in a column.

Thus,

15 yd.	2 ft.	8 in.
6	1	3
5	2	7
28 yd.	0 ft.	6 in.
	2 yd.	1 ft.

The sum of the right column is 18 inches, which is 1 foot 6 inches. Write the 6 inches in inches' column under the line, and carry the 1 foot to the next column. The sum of the next column, including the 1 foot carried, is 6 feet, which equals 2 yards. The sum of the next column including the 2 yards carried is 28 yards. Hence the sum is 28 yards and 6 inches.

EXERCISES

Add the following.

1.

1 mi.	40 rd.	1 ft.	10 in.
3	190	5	6
5	260	10	7

2.

40 yd.	2 ft.	10 in.
10	1	6
5		8

3.

45 rd.	3 yd.	2 ft.	6 in.
10	2	1	8
23	4	2	9
18	1	1	7

4.

30 mi.	3200 ft.	5 in.
150	1650	4
26	1490	8

5.

50 rd.	9 ft.	1 in.
20	12	9
62	14	11

6.

8 yd.	2 ft.	3 in.
7	1	9
10	0	2
12	1	11

SUBTRACTION OF DENOMINATE NUMBERS

172. *Example.* From 15 miles 160 rods subtract 3 miles 245 rods.

Since we cannot subtract 245 rods from 160 rods, we change 1 mile (taken from the 15) to rods. Then 480 − 245 = 235.

Solution.

	320
15 mi.	160 rd.
3	245
11 mi.	235 rd.

WRITTEN EXERCISES

Subtract each of the following:

1.

20 rd.	12 ft.	10 in.
10	9	11

2.

40 mi.	210 rd.	8 ft.	5 in.
32	260	3	7

3.

14 rd.	3 ft.	8 in.
7	9	10

4.

6 mi.	240 rd.
5	300

5.

8 mi.	4000 ft.
5	4500

6.

2 mi.	160 rd.	10 ft.
1	200	5

7.

12 rd.	10 ft.	8 in.
8	8	10

8.

5 mi.	120 rd.	8 yd.	27 in.
3	210	4	30

9.

8 mi.	1760 ft.
5	2000

10.

10 mi.	200 rd.	8 ft.
	400	10 in.

MULTIPLICATION OF DENOMINATE NUMBERS

173. *Example.* Multiply 3 yards 2 feet 10 inches by 4.

4 × 10 in. = 40 in. = 3 ft. 4 in. Write 4 in inches' column, and carry the 3 ft. 4 × 2 ft. = 8 ft. 8 ft. + 3 ft. = 11 ft. 11 ft. = 3 yd. 2 ft. Write 2 in feet's column, and carry the 3 yd. 4 × 3 yd. = 12 yd. 3 yd. + 12 yd. = 15 yd. Hence the product is 15 yd. 2 ft. 4 in.

Solution.

3 yd.	2 ft.	10 in.
		4
15 yd.	2 ft.	4 in.

EXERCISES

1. Multiply 2 miles 45 rods by 12.
2. Multiply 20 rods 8 feet 10 inches by 8.
3. Multiply 13 miles 400 feet 8 inches by 9.
4. Multiply 42 miles 80 rods 3 yards by 24.

DIVISION OF DENOMINATE NUMBERS

174. *Example 1.* Divide 26 yards 2 feet 7 inches by 6.

26 yards ÷ 6 = 4 yards with a remainder of 2. Reduce the 2 yards to feet, and add to the 2 feet. 8 feet ÷ 6 = 1 foot with a remainder of 2. Reduce the 2 feet to inches, and add to the 7 inches. 31 ÷ 6 = $5\frac{1}{6}$.

Solution.

	6	24
6)26 yd.	2 ft.	7 in.
4 yd.	1 ft.	$5\frac{1}{6}$ in.

Example 2. Divide 9 yards 1 foot 10 inches by 8.

Since 8 is not contained once in 4, we reduce the 4 feet to inches. The result is 1 yd. $7\frac{1}{4}$ in. (no feet).

Solution.

	3	48
8)9 yd.	1 ft.	10 in.
1 yd.	0 ft.	$7\frac{1}{4}$ in.

Sometimes the number of highest denomination must be reduced to the next lower denomination before dividing.

Example 3. Divide 3 yards 2 feet 4 inches by 5.

First reduce 3 yards to feet and then proceed as above.

Solution.

	9	12
5)3 yd.	2 ft.	4 in.
	2 ft.	$3\frac{3}{5}$ in.

WRITTEN EXERCISES

1. Divide 18 rd. 2 ft. 3 in. by 7.
2. Divide 125 mi. 40 rd. 12 ft. 6 in. by 4.
3. Divide 116 mi. 14 ft. 8 in. by 5.
4. Divide 40 mi. 180 rd. by 16.
5. Divide 110 yd. 2 ft. by 14.

Subtract in Examples 6 and 7.

6. 17 mi. 265 rd. 19 ft.
   8 mi. 125 rd. 12 ft.

7. 12 rd. 7 ft. 8 in.
   6 rd. 8 ft. 11 in.

8. Multiply 8 rd. 2 ft. 7 in. by 12.
9. Multiply 5 mi. 164 rd. 2 yd. by 8.
10. Multiply 13 yd. 2 ft. 8 in. by 67.
11. Divide 18 rd. 6 ft. 4 in. by 4.
12. Divide 8 mi. 145 rd. 3 yd. by 8.
13. Divide 54 yd. 1 ft. 7 in. by 21.

MEASURES OF AREAS

175. Units of Area. — The units of area are naturally squares whose sides are units of length. Thus, we use the square inch, the square foot, the square yard, the square rod, and the square mile.

TABLE OF SQUARE MEASURE

144 **square inches** = 1 **square foot** (sq.ft.)
9 square feet = 1 **square yard** (sq. yd.)
$30\frac{1}{4}$ square yards = 1 **square rod** (sq. rod)
160 square rods = 1 **acre** (A.)
640 acres = 1 **square mile**
16 square rods = 1 square chain
10 square chains = 1 acre 100 sq. ft. = 1 square
43,560 square feet = 1 acre 36 sq. mi. = 1 township

176. Uses of the Units of Area. — Large sections of land, such as counties or states, are measured in *square miles*. Farms are measured in *acres;* land in great cities is measured in *square feet*. In surveying tracts of land in the U. S. forest reserves, the chain is used as a unit of length, and hence the area is found in *square chains*. To find the area in acres, the number of square chains is divided by 10. *Square inches* are used in measuring the areas of small surfaces. Carpets, areas of walls, and ceilings are measured in *square yards*. The *square* is used in measuring roofing.

ORAL EXERCISES

1. What part of an acre are 40 square rods? 80?
2. Reduce 3200 square rods to acres.
3. Reduce 12,800 square rods to square chains.
4. Reduce 2880 square inches to square feet.
5. Reduce 1600 square rods to acres.
6. Reduce 5 square miles to square rods.
7. Reduce 1440 square inches to square feet.
8. Reduce 121 square yards to square rods.
9. Reduce 180 square feet to square yards.

WRITTEN EXERCISES

1. Reduce 124,382 square inches to square inches, square feet, and square yards. (See example, page 134.)

2. Reduce 25,760 square rods to acres.

3. Reduce 12,800 square rods to square inches.

4. Reduce 1526 square inches to square feet and square inches.

5. Reduce 22,560 square yards to acres, square rods, and square yards.

6. Reduce 10 square yards 3 square feet 40 square inches to square inches.

Add:

(Explain Example 7 fully. See page 135.)

7.
10 A. 120 sq. rd.
5 A. 80 sq. rd.

8.
8 sq. ft. 100 sq. in.
60 sq. ft. 60 sq. in.

9.
2 sq. mi. 320 A.
5 sq. mi. 420 A.

10.
30 sq. yd. 8 sq. ft. 100 sq. in.
5 sq. yd. 6 sq. ft. 72 sq. in.

Subtract:

(Explain Example 12 fully. See page 136.)

11.
$\frac{2}{3}$ acre
100 sq. rd.

12.
$\frac{2}{3}$ acre
80 sq. rd. 30 sq. yd. 6 sq. ft. 100 sq. in.

13.
$\frac{5}{6}$ sq. mi.
20 A. 100 sq. rd.

14.
$\frac{2}{3}$ sq. rd.
$\frac{1}{2}$ sq. yd.

Multiply:

(Explain Example 15 fully. See page 136.)

15. 7 square yards 4 square feet by 8.

16. 10 square rods 6 square feet by 16.

17. 16 square rods 10 square yards 6 square feet 14 square inches by 20.

18. Reduce $\frac{5}{8}$ of a mile to lower denominations.

19. Divide 10 square yards 5 square feet 120 square inches by 8.

20. Divide 12 acres 140 square rods 20 square yards by 9.

177. Reduction of Denominate Numbers to Decimal Form. — *Example 1.* Reduce 7 square feet to a decimal fraction of a square yard.

Solution. 7 square feet = $\frac{7}{9}$ square yard = .778 square yard, which is correct to three decimals.

Example 2. Reduce 14.84 square yards to square yards, square feet, and square inches.

.84 sq. yd. = 9 × .84 = 7.56 sq. ft.
.56 sq. ft. = 144 × .56 = 80.64 sq. in.
Hence, 14.84 sq. yd. = 14 sq. yd. 7 sq. ft. 80.64 sq. in.

Solution.

.84	.56
9	144
7.56	224
	224
	56
	80.64

Example 3. Reduce 1,849,824 square feet to acres correct to three decimal places.

Such examples are of frequent occurrence in actual practice. If a piece of land is measured by a 100-foot tape the area is obtained in square feet. The reduction to acres is then made by dividing by 43,560 as in the example.

Solution.

42.466 (Acres)
43560)1849824
174240
107424
87120
203040
174240
288000
261360
266400

ORAL EXERCISES (A)

1. If the area of a field is given in square feet state a rule for reducing it to acres.

2. If a distance is measured in chains and links state a rule for reducing to rods and feet.

3. The distance which a ship travels is measured in knots. State a rule for finding the equivalent number of miles.

4. If land is bought by the acre state a rule for finding the cost per square foot.

5. If the area of a wall is given in square feet state a rule for finding the number of square yards.

PROBLEMS

1. A field is 880 feet square. Reduce the area to acres expressed as a decimal.

2. Reduce 445,678 square feet to acres.

3. In 76,314 square yards how many acres are there?

4. In a city certain land is worth 30¢ a square foot. At this rate, how much will one acre cost?

5. A farm sold for $180 an acre. At this rate, how much will one square foot cost?

6. A new addition to a city consists of 40 acres. Three fifths of this is laid out in building lots, and the rest in streets and alleys. If the average sales price of the lots is 3¢ a square foot how much is realized out of the 40-acre tract?

7. A surveyor measures a distance of 1640 ft. How many chains and links is this?

8. How many 160-acre farms in one township?

9. A railway replaces rails on $163\frac{1}{2}$ miles of single track road. At 32 pounds a foot, how many tons of rails are required? (1 ton = 2000 lb.)

10. If a steamer makes 25.4 knots an hour how many statute miles per hour is this?

11. A battle cruiser made 29 knots an hour chasing the enemy. How many statute miles per hour was this?

12. In measuring a field a surveyor finds the length of it to be 1765.6 feet. Reduce this to rods correct to two decimal places.

13. A freight train averages 31 miles per hour. How many knots is this?

14. A horse race is run over a distance of 6 furlongs. How many feet is this?

15. A ceiling is 340 square feet. Reduce to square yards.

16. Reduce 76,480 square rods to square miles.

17. Reduce 704,840 square feet to acres.

18. Reduce 38 miles to knots.

MEASURES OF VOLUME, OR CAPACITY

178. Units of Volume. — Some units of measure of volume or capacity are cubes, the length of whose edges are units of length. Thus, we use the cubic foot and the cubic yard as units of volume.

TABLE OF CUBIC MEASURE

1728 **cubic inches** (cu. in.) = 1 **cubic foot** (cu. ft.)
27 cubic feet = 1 **cubic yard** (cu. yd.)
$24\frac{3}{4}$ cu. ft. = 1 perch (P.)
128 cu. ft. = 1 cord (cd.)
1 load of earth = 1 cubic yard

179. Units of Capacity. — The principal units of capacity are the pint, quart, gallon, peck, and bushel.

TABLE OF MEASURE OF CAPACITY

Liquid Measure		Dry Measure	
4 gills (gi.)	= 1 **pint** (pt.)	2 **pints** (pt.)	= 1 **quart** (qt.)
2 pints	= 1 **quart** (qt.)	8 quarts	= 1 **peck** (pk.)
4 quarts	= 1 **gallon** (gal.)	4 pecks	= 1 **bushel** (bu.)
1 gallon = 4 qt.	= 8 pt. = 32 gi.	1 bu. = 4 pk.	= 32 qt. = 64 pt.
$31\frac{1}{2}$ gallons	= 1 **barrel** (bbl.)		
63 gallons	= 1 hogshead		

In practice, however, hogsheads, pipes, etc. differ much in size.

The liquid gallon, technically known as the *wine gallon*, contains 231 cubic inches. A gallon of water weighs nearly 8.5 pounds.

The imperial gallon of England contains 277.274 cubic inches. It contains 10 pounds of water. The Winchester bushel contains 2150.42 cubic inches. The heaped bushel contains 2747.71 cubic inches. The imperial bushel of England contains 2218. 192 cubic inches. It is 8 times the imperial gallon.

180. Comparison of Liquid and Dry Measure.

	Gallon	Quart	Pint
Liquid	231 cu. in.	$57\frac{3}{4}$ cu. in.	$28\frac{7}{8}$ cu. in.
Dry	$268\frac{4}{5}$ cu. in.	$67\frac{1}{5}$ cu. in.	$33\frac{3}{5}$ cu. in.

181. Weight of a Bushel of Various Commodities. — The weight of a bushel of certain grains and seed has been fixed by statute in many

of the states, and these must be used in buying and selling unless specific agreements are made to the contrary.

Commodities	Wt. in Avoir.	Exceptions
Barley	48	Ala., Ga., Ky., Pa. 47; Ariz. 45; Cal. 50
Buckwheat	48	Cal. 40; Ida., N. Dak., Okla., Ore., S. Dak., Tex., Wash. 42; Ind., Can., Minn., N. J., N. C., O., Tenn., Wis. 50
Corn (in ear)	70	Miss. 72; Ohio 68
Corn (shelled)	56	Mass. 50
Oats	32	N. J., Va. 30
Potatoes	60	Me., Pa. 56
Rye	56	Cal. 54; Me. 50
Timothy Seed	45	Ark. 60; Okla., S. Dak. 42
Wheat	60	

Apothecaries' Measure

Apothecaries' fluid measure is used by druggists for fluid medicines. The gallon of this measure is the same as the wine gallon.

TABLE OF APOTHECARIES' FLUID MEASURE

60 minims (m) = 1 fluid drachm (f3)
8 fluid drachms = 1 fluid ounce (f3)
16 fluid ounces = 1 pint (O.)
8 pints = 1 gallon (Cong.)

182. Approximate Measures. — In practice the following approximate measures are used.

One bushel = 1.25 cubic feet. 1 cubic foot = .8 bushel.
One heaped bushel = 1.6 cubic feet.
One cubic foot = .625 heaped bushel.
One ton of stove coal (hard) = 35 cubic feet.
One ton of timothy hay in well-settled mow = 450 cubic feet.
One ton of clover hay in well-settled mow = 550 cubic feet.

183. Some Uses of Units of Volume and Capacity. — Cubical contents of rooms, household gas, etc., are measured in *cubic feet;* excavations, in *cubic yards;* masonry, by the *perch,* and wood, by the *cord.*

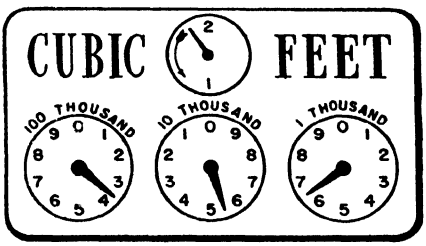
Milk is measured by the *pint* and *quart*, and sometimes by the *gallon;* kerosene and gasoline are measured by the *gallon.*

Vegetables, such as peas and beans, are measured by the *dry quart;* potatoes, apples, and turnips, by the *peck* and *bushel.*

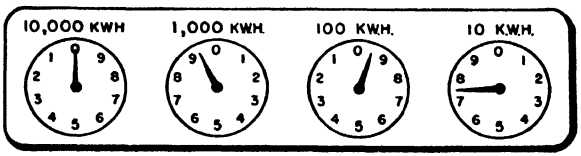
Grains of all sorts are measured by the *Winchester bushel.*

The *heaped bushel* is used for potatoes, apples, vegetables, corn on the ear, etc. Apples are frequently sold by the *barrel.*

READING GAS AND ELECTRIC METERS



All gas meters are much alike. The dial at the top is simply a test dial; those at the bottom measure the gas used. Note that two are read clockwise, and one counterclockwise. Read the dials from *left to right.* A hand between two numbers is read as the preceding number. The figures denote tenths of the numbers over the dials and so two ciphers must be added to the reading of the right-hand dial. Thus these dials read: 35600. The difference between any two readings measures the amount of gas used for that period.



Electricity is measured in kilowatt hours. The dials are read as above save that the figures represent whole units instead of tenths and so no ciphers are added at the end. Thus these dials read: 9970.

1. Secure as many receipted gas and electric bills as you can and construct dial diagrams to represent the different amounts.

PROBLEMS

1. A bin holds 1250 cubic feet. How many exact Winchester bushels does it hold? How many bushels does it hold containing 1.25 cubic feet to the bushel?

2. How many cubic feet in a carload of wheat containing 1650 bu. (*a*) when a Winchester bushel is used? (*b*) When approximate measure is used?

3. A potato bin contains 2400 cubic feet. How many bushels does it contain, (*a*) using approximate measure? (*b*) Using exact measure? Use heaped bushel.

4. A haymow contains 78,600 cubic feet. How many tons of timothy hay will it hold? How many tons of clover will it hold?

5. A wagon box contains 100 cubic feet. How many tons of coal will it hold when full but not heaped?

6. Reduce 20 cubic yards 12 cubic feet to cubic inches.

7. Reduce 24,400 cubic inches to cubic yards.

8. Add: $\frac{2}{5}$ of a cubic yard, 5 cubic yards, and 1500 cubic inches.

9. Reduce 2624 pints to bushels; 1728 quarts to bushels.

10. Multiply 2 barrels 16 gallons 4 quarts 2 gills by 24.

11. Multiply 8 bushels 3 pecks 2 quarts 1 pint by 16.

12. Divide 15 gallons 2 quarts 1 pint 3 gills by 6.

13. Divide 180 bushels 3 pecks 6 quarts 2 pints by 16.

14. A herd of cows eat 2 bushels 3 pecks 6 quarts 1 pint of grain each day. How much will they eat in a month of 30 days?

15. A gardener sold at 14¢ a quart his berries amounting to 28 bushels 3 pecks 6 quarts. Find the selling price.

Subtract:

16.

bu.	qt.	pt.
8	3	0
6	3	1

17.

gal.	qt.	pt.	gi.
8	3	0	1
2	2	1	3

MEASURES OF WEIGHT

184. Units of Weight. — There are four kinds of measure of weights in the United States; namely, Avoirdupois, Troy, Apothecaries', and Metric weights.

Table of Avoirdupois Weight

16 **ounces** (oz.)	= 1 **pound** (lb.)
100 pounds	= 1 hundredweight (cwt.)
2000 pounds	= 20 hundredweight = 1 **ton** (T.)

185. Uses of Avoirdupois Weight. — The avoirdupois pound is the most common unit of weight. The hundredweight is used sometimes in weighing cattle, grain, etc. The ton is used in weighing hay, freight, etc.

186. The Long Ton. — The long ton is used in the U. S. custom houses, and in weighing coal and ores at the mine.

Table of Long Ton

28 pounds	= 1 quarter (qr.)
4 quarters	= 1 long hundredweight (cwt.)
20 long hundredweight	= 1 long ton (L. T.)

Table of Troy Weight

24 **grains** (gr.)	= 1 pennyweight (pwt.)
20 pennyweight	= 1 **ounce** (oz.)
12 ounces	= 1 **pound** (lb.)

187. Uses of Troy Weight. — Troy weight is used in weighing gold, silver, and other precious metals, and in weighing coins at the United States mint. It is also used in the manufacture and sale of jewelry.

188. The Carat. — The carat is used in weighing precious gems.

$$1 \text{ carat} = 3.168 \text{ grains}$$

The word carat is also used in another sense to describe the fineness of gold. Gold 18 carats fine contains $\frac{18}{24}$ gold and $\frac{6}{24}$ alloy. In this sense 1 "carat" means $\frac{1}{24}$.

Table of Apothecaries' Weight

20 **grains** = 1 scruple (sc. or ℈)
3 scruples = 1 dram (dr. or ʒ)
8 drams = 1 **ounce** (oz. or ℥)
12 ounces = 1 **pound** (lb. or ℔)

189. Uses of Apothecaries' Weight. — Apothecaries' weight is used by doctors in writing prescriptions, and by druggists in filling them. It is also used in the wholesale drug trade, and in the manufacture of drugs.

Comparison of Weight

	Pound	Ounce	Grain
Avoirdupois	7000 gr.	437½ gr.	1 gr.
Troy	5760 gr.	480 gr.	1 gr.
Apothecaries'	5760 gr.	480 gr.	1 gr.

The grain is the same in all these weights.

In the table given below, the avoirdupois pound is used.

1 bbl. flour = 196 lb.
1 bbl. salt = 280 lb.
1 firkin of butter = 56 lb.
1 bbl. pork = 200 lb.
1 cu. ft. fresh water = 62½ lb.
1 keg of nails = 100 lb.

ORAL EXERCISES

1. State a rule for reducing ounces (avoirdupois) to hundredweight and tons.

2. If the weight of a bag of coins is given in grains, state how to find the weight in pounds (avoirdupois).

3. A coal dealer buys 100 tons (long) of coal and sells it by the ton (avoirdupois). State how he may find the number of tons that he may expect to sell if the weight shrinks 50 pounds per long ton in handling.

4. Give rules for changing avoirdupois pounds to troy pounds, and apothecaries' pounds to avoirdupois pounds.

PROBLEMS

1. Reduce 21,464 ounces to higher denominations.

2. Reduce 2 T. 7 hundredweight 24 pounds to ounces.

3. Find the total cost of

3240 lb. of coal at $9.75 per ton.
3750 lb. of coal at $8.50 per ton.
3860 lb. of coal at $9.50 per ton.
4160 lb. of coal at $8.75 per ton.
3940 lb. of coal at $12.50 per ton.
4080 lb. of coal at $10.75 per ton.

4. Find the cost of 8 pwt. 4 gr. of gold at $1.50 per pennyweight.

5. A farmer in Ohio sold 5250 lb. of wheat at $1.75 per bushel; 6848 lb. of oats at 50 cents per bushel; 3456 lb. of barley at $1.60 per bushel; and in part payment he accepted 210 lb. of clover seed at $14.25 per bushel; and 250 lb. of timothy seed at $7.25 per bushel. Make a statement showing the transaction and the amount due the farmer.

6. If I buy 10 tons of coal by the long ton, and sell it by the short ton, how much is my margin * if the price in each case is $10.00 a ton?

7. Reduce 4 oz. 6 pwt. 10 gr. to a decimal fraction of a pound.

8. Change 12 lb. avoirdupois to troy pounds.

9. If a druggist buys medicine at $4.80 a pound avoirdupois, and sells it at 18¢ per dram, how much more or less than cost does he receive for one pound avoirdupois?

10. Reduce 16,875 grains to higher denominations (troy).

11. Reduce 24,382 grains to higher denominations (apoth.).

12. Reduce 2 lb. 10 oz. 16 pwt. 20 gr. to grains.

13. Reduce 4 lb. 6 oz. 2 dr. 1 sc. 10 gr. to grains.

14. A farmer took three loads of hay to market. The first weighed, including the wagon, 4380 lb.; the second 4464 lb.; the third 4852 lb. If the wagon in each case weighed 2160 lb., what did he receive for his hay at $32 per ton? See § 30.

* Margin is the difference between the selling price of an article and what this article costs the merchant who sells it.

UNITED STATES MONEY

190. Units of United States Money. — The principal units of United States money are the cent and the dollar.

Table

10 mills (m.)	= 1 **cent** (¢ or ct.)	
5 cents	= 1 **nickel**	
10 cents	= 1 **dime**	
25 cents	= 1 **quarter**	
50 cents	= 1 **half dollar**	
100 cents	= 1 **dollar** ($)	
5 dollars	= 1 half eagle (gold coin)	No longer minted
10 dollars	= 1 eagle (gold coin)	
20 dollars	= 1 double eagle	

The United States silver dollar weighs 412.5 grains, of which $\frac{9}{10}$, or 371.25 grains, is pure silver.

The 10-dollar gold piece weighs 258 grains, of which $\frac{9}{10}$, or 232.2 grains, is pure gold. All gold coin has been withdrawn from circulation by the government.

The half dollar, quarter, and dime are silver coins somewhat lighter in proportion than the silver dollar. That is, a dime does not weigh one tenth as much as a silver dollar.

There is no *coin* whose value is less than 1 cent, but the mill is used in such matters as levying taxes. (See page 415.) The monetary units of Canada are the same as those of the United States.

The denominations of United States money are arranged on the decimal scale the same as our number system. This makes reduction from one denomination to another particularly simple.

ORAL EXERCISES

1. Reduce 648 cents to dollars and cents.

2. Reduce 8 dollars and 14 cents to cents.

3. How many cents in a double eagle ($20)? in a half eagle ($5.00)? in 3 dollars and 84 cents?

4. How many dollars, cents, and mills are there in 79,464 mills?

ENGLISH MONEY

191. Units of English Money. — The principal units of English money are the penny, the shilling, and the pound.

Table

4 farthings (far.)	= 1 penny (d.)
12 pence (pennies)	= 1 shilling (s.)
20 shillings	= 1 pound sterling (£)

Par value of the pound is usually \$4.8665, but this value fluctuates. In the fall of 1949 the value of the pound fell as low as \$2.28.

FRENCH MONEY

192. Units of French Money. — The principal units of French money are the centime and the franc.

Table

10 millimes (m.)	= 1 centime (c.)
10 centimes	= 1 decime (d.)
10 decimes	= 1 franc (F.)

The franc contains 4.4803 grains of pure gold, and was formerly equal practically to \$.039, which was par of exchange on France.

GERMAN MONEY

193. Units of German Money. — The principal units of German money are the pfennig and the mark.

Table

100 pfennig (pf.) = 1 mark (M.)

The mark contains 5.5513 grains of pure gold, and was formerly equal to \$.238, which was par of exchange on Germany.

The value of the money of one country in terms of the money of other countries varies so much that problems of commuting the value of one of them in terms of others have been omitted in this edition. In case of immediate need, the current quotations must be looked up.

MEASURE OF TIME

194. Units of Time. — In the measure of time all units in the table are of practically equal importance. Time is measured more frequently than any other magnitude.

Table

60 **seconds** (sec.)	= 1 **minute** (min.)	7 days	= 1 **week** (wk.)
60 minutes	= 1 **hour** (hr.)	365 days	= 1 **year** (yr.)
24 hours	= 1 **day** (da.)	366 days	= 1 leap year.

195. Length of the Day and the Year. — One year is the time it requires the earth to make one complete journey around the sun. One day is the time from noon to noon.

The year is 365 da. 5 hr. 48 min. 49.7 sec., or very nearly $365\frac{1}{4}$ days. Because of this extra $\frac{1}{4}$ day in each year, every fourth year is made 366 days long. The extra day of the leap year is added to the month of February.

The years whose numbers are divisible by 4 are leap years.

E.g. 1944, 1948 are leap years.

Since the year is not quite $365\frac{1}{4}$ da., the years marking the completion of a century, such as 1700, 1800, 1900, are not leap years, except that all years whose numbers are divisible by 400 are leap years.

Days of the Months

January (Jan.)	= 31 days	July	= 31 days
February (Feb.)	= 28 days	August (Aug.)	= 31 days
March (Mar.)	= 31 days	September (Sept.)	= 30 days
April (Apr.)	= 30 days	October (Oct.)	= 31 days
May	= 31 days	November (Nov.)	= 30 days
June	= 30 days	December (Dec.)	= 31 days

The measurement of time is one of the few cases in which several units are used for the same measurement. Thus, the time of a foot race is reported in minutes, seconds, and fractions of a second. The time of a cross-country run is given in hours, minutes, and seconds. The time of a ship's journey across the ocean is given in days, hours, minutes, and seconds.

MEASURE OF ANGLES AND ARCS

196. Units of Circular Measure. — The principal unit of circular measure is the degree, which equals $\frac{1}{360}$ of a whole circumference.

Table

60 **seconds** (″)	= 1 **minute** (′)
60 minutes	= 1 **degree** (°)
360 degrees	= 1 circumference (cir.)

In geography, one degree is $\frac{1}{360}$ of the distance around the earth. One degree on the equator equals 60 knots, or geographical miles.

The units of measure of angles and arcs are the same.

MEASURE OF TEMPERATURE

197. The Fahrenheit Thermometer. — Temperature is measured by means of the thermometer, which consists essentially of a glass tube containing mercury. The thermometer in common use is called the Fahrenheit thermometer. On this thermometer the freezing point is marked 32 degrees (32°) above zero, and the boiling point is marked 212° above zero. The distance between the freezing point and the boiling point is divided into 180 equal parts or degrees.

198. The Centigrade Thermometer. — The so-called centigrade thermometer is used in scientific work. On it zero is placed at the freezing point, while the boiling point is marked 100°. The intervening space is divided into 100 equal parts.

Degrees above zero are marked +, and degrees below are marked −.

Temperature measured by the Fahrenheit is marked F., and when measured by the centigrade is marked C. We see at once that 32° F. = 0° C., and 212° F. = 100° C. The difference between boiling and freezing temperatures is 100° C. and 180° F. respectively for the two thermometers. Hence a change of 1° C. = a change of 1.8° F. From these facts, we get:

199. Rule for Interchanging Fahrenheit and Centigrade Readings. — *To change degrees F. to degrees C., subtract 32° from the number of degrees Fahrenheit, and then divide by 1.8.*

To change degrees C. to degrees F., multiply the number of degrees by 1.8, and add 32.

WRITTEN EXERCISES (B)

1. Add:

12 yr.	6 mo.	11 da.	40 min.
8	5	20	50
4	7	45	45
7	9	18	16

2. Subtract:

10 mo.	12 da.	14 hr.	40 sec.
7	16	15	50

3. George Washington was born Feb. 22, 1732. How many years, months, and days ago was that?

4. Find the exact number of days between July 24, 1948, and Feb. 12, 1949.

5. Find the exact number of days between Sept. 19, 1948, and Jan. 30, 1949.

6. If a note is discounted Feb. 15, 1949, and is due May 14, 1949, how many months and days has it to run?

7. If a note is dated July 16, 1949, and has 4 months and 20 days to run, when is it due?

8. Reduce 45° 21′ 24″ to seconds.

9. Reduce 45,915″ to higher denominations.

10. The latitude of New York is 40° 42′ 43″ North; the latitude of New Orleans is 30° 57′ 46″ North. Find the difference in latitude.

11. Again the latitude of New York is 40° 42′ 43″ North; the latitude of Rio Janeiro is 22° 56′ South. Find their difference in latitude.

12. Change .375 of a minute to the decimal fraction of a degree.

13. If a thermometer reads 40° (F.) above zero, change the reading to centigrade.

14. Change +28° C. to Fahrenheit reading.

15. Change 58° F. to centigrade reading.

16. Reduce .725 of a day (24 hours) to hours, minutes, and seconds.

17. The longitude of a certain place is 75° 28′ 36″ E., of another 60° 40′ 28″ W. What is the difference in longitude?

200. Miscellaneous Measures.

Paper Table		Counting Table	
24 sheets	= 1 quire (qr.)	12 units	= 1 **dozen** (doz.)
20 quires	= 1 ream (rm.)	12 dozen	= 1 **gross** (gro.)
2 reams	= 1 bundle (bdl.)	12 gross	= 1 great gross (gr. gro.)
5 bundles	= 1 bale (bl.)	20 units	= 1 **score**

MISCELLANEOUS PROBLEMS (B)

1. Reduce 1 ream 16 quires 20 sheets to a fraction of a bale.

2. If a man is 3 score and 10 years, how old is he?

3. Reduce 8 gross 9 doz. to units.

4. Reduce $\frac{5}{6}$ of a quire to a fraction of a bundle.

5. Find the gain on a carload of coal weighing 25 long tons bought at \$5.25 per long ton, and sold at \$6.12 per short ton.

6. Reduce $\frac{4}{5}$ of a day to hours, minutes, and seconds.

7. I bought a city lot containing $2\frac{1}{4}$ acres for \$6500 an acre, and sold it for \$.25 per square foot. Did I gain or lose, and how much?

8. I bought 12 bu. of cherries at \$2.50 a bu., and sold them at 12¢ a quart. How much did I gain or lose?

9. Find the cost of 3856 laths at \$3.25 per M.

10. Find the cost of 4380 pounds of hay at \$16 a ton.

11. An American bought in Bermuda 12 yards of cloth at 16 shillings a yard, 2 suits of clothes at £6 and 10 shillings each, and an overcoat for £7. What was his bill in U. S. money at the par value of the pound? At today's value?

12. An American lady bought in Paris 8 pairs of gloves at 8 francs a pair. What was her bill in U. S. money?

13. If a piece of city property 240 feet deep by 350 feet front costs \$10,000 and is sold in lots of 50 feet frontage at \$2000 each, what is the gain?

14. How many times will a wheel whose circumference is 15 feet revolve in going 10 miles?

15. Reduce 125 yards 2 feet 6.5 inches to a decimal fraction of a mile.

THE METRIC SYSTEM OF WEIGHTS AND MEASURES

201. The Decimal Relation of Units of Metric System. — The metric system of weights and measures is a decimal system. That is, each unit is a decimal multiple of the next lower unit. In the measures of length, weight, and capacity, each unit is 10 times the next lower unit. In the measure of area, each unit is 100 times, and in the measure of volume, 1000 times the next lower unit.

202. Metric System, Where Used. — Nearly all civilized countries, except England and the United States, now use the metric system of weights and measures. In 1866 its use was permitted by law in the United States, but it has never been generally adopted except for scientific use. The United States requires the use of the metric system in the medical work of the army and navy.

203. Use of Metric System in Great Britain. — In Great Britain the use of the metric system has been adopted for the pharmacopœia, which contains the official description of drugs. This substitutes the metric system for apothecaries' weights and measures.

204. Fundamental Unit of Metric System. — The fundamental unit of this system is the meter, since all the other units are derived from it.

MEASURE OF LENGTH

Table

10 **millimeters** (mm.)	= 1 **centimeter** (cm.)
10 centimeters	= 1 decimeter (dm.)
10 decimeters	= 1 **meter** (m.)
10 meters	= 1 decameter (Dm.)
10 decameters	= 1 hectometer (Hm.)
10 hectometers	= 1 **kilometer** (Km.)
10 kilometers	= 1 myriameter (Mm.)

The length of the meter was first determined by taking one ten-millionth the distance from the pole to the equator. Later it was discovered, however, that this had not been done accurately. The meter equals nearly 39.37 inches in length (see § 168).

MEASURES OF AREA

205. Units of Area. — The units of measure of area are the squares of the units of measure of length.

Table

[$(cm.^2)$

100 square millimeters (sq. mm.), $(mm.^2) = 1$ sq. centimeter (sq. cm.),
100 square centimeters = 1 sq. decimeter (sq. dm.), $(dm.^2)$
100 square decimeters = 1 **square meter** (sq. m.), $(m.^2)$
100 square meters = 1 square decameter (sq. Dm.), $(Dm.^2)$
100 square decameters = 1 square hectometer (sq. Hm.), $(Hm.^2)$
100 square hectometers = 1 **square kilometer** (sq. Km.), $(Km.^2)$

The **Hectare** = 100 ares = 10,000 sq. m. is used in measuring land where we would use the acre as the unit.

The square meter is also called a centare. The square kilometer is used where we would use the square mile.

CUBIC MEASURE

Table

1000 **cubic millimeters** $(mm.^3)$ = 1 **cubic centimeter** $(cm.^3)$ or (cc.)
1000 cubic centimeters = 1 cubic decimeter $(dm.^3)$
1000 cubic decimeters = 1 **cubic meter** $(m.^3)$

The cubic centimeter is much used in scientific work.

MEASURES OF WEIGHT

Table

10 milligrams (mg.) = 1 centigram (cg.)
10 centigrams = 1 decigram (dg.)
10 decigrams = 1 **gram** (g.)
10 grams = 1 decagram (Dg.)
10 decagrams = 1 hectogram (Hg.)
10 hectograms = 1 **kilogram** (Kg.)
10 kilograms = 1 myriagram (Mg.)
10 myriagrams = 1 **quintal** (q.)
10 quintals = 1 **tonneau** (t.)

206. Uses of Various Units of Weight. — The gram is used in cases where we would use the grain. The kilogram is used in weighing all kinds of merchandise where we would use the pound.

The quintal and tonneau are used where we would use the hundredweight and ton respectively.

MEASURES OF CAPACITY

Table

10 milliliters (ml.)	=	1 centiliter (cl.)
10 centiliters	=	1 deciliter (dl.)
10 deciliters	=	1 **liter** (l.)
10 liters	=	1 decaliter (Dl.)
10 decaliters	=	1 **hectoliter** (Hl.)
10 hectoliters	=	1 kiloliter (Kl.)

207. Use of Units of Capacity. — The liter is the principal unit and is used for measuring all kinds of liquid and dry substances where we use the pint, the quart (liquid or dry), the gallon, etc.

The hectoliter is used for measuring grains, vegetables, etc.

208. Equivalents of Metric Units in U. S. Units.

1 meter = 39.37 inches
1 square meter = 1.196 sq. yd.
1 hectare = 2.471 acres
1 liter = .908 dry quart = 1.0567 liquid quarts
1 hectoliter = 2.8377 bushels
1 gram = 15.432 grains

1 yard = .9144 meter
1 square yard = .8361 sq. m.
1 acre = 40.47 ares = .4047 hectare
1 bushel = .3524 hectoliter
1 grain = .0648 gram
1 pound (avoir.) = .4536 kilogram

1 kilogram = 2.2046 pounds (avoir.)

The metric units of weight and capacity are derived from the meter as follows:

1 cc. of water weighs 1 gram.
1 dm.^{3} of water weighs 1 kilogram.
1 dm.^{3} = 1 liter.
1 m.^{3} of water = 1 tonneau = 2204.6 lb.

ORAL EXERCISES

1. Read 18,460 millimeters as meters, decimeters, and centimeters.

2. Read 648,094 cubic centimeters as cubic decimeters and cubic centimeters.

3. Read 86,476 grams in terms of higher denominations.

4. Reduce 47 kilograms, 486 grams to grams.

5. Reduce 746 hectoliters, 59 liters to liters.

6. Reduce 34,678 liters to hectoliters and liters.

7. Reduce 137,894 grams to kilograms and grams.

8. By how much does one tonneau differ from a long ton?

EXERCISES

1. Reduce to higher denominations: 44,864 centigrams; 25,432 deciliters; 62,456 millimeters.

2. Reduce 16 kilograms to pounds.

3. Reduce 30 pounds to kilograms.

4. Change 37 yards 2 feet 6 in. to meters.

5. Change 300 meters to yards, feet, etc.

6. Change 20 gallons to liters.

7. Change 16 liters to gallons.

8. A farmer sold his wheat in the U. S. at \$1.50 per bu. A French farmer sold his for \$3.50 a hectoliter. If 100 bu. were sold in each case, which farmer received the more money, and how much?

9. Add .625 m.; 840 cm.; 2540 mm.; .175 Km.

10. If a farm contains 250 hectares 10 ares of land, what is it worth at \$75 an acre?

11. If a kilogram of flour is worth 8¢, what will 2 barrels be worth?

12. The Empire State express runs 60 miles an hour, and a European train runs 80 kilometers an hour. Which is the faster, and how much? Express the difference in miles per hour.

13. A dime weighs 2.5 grams. What is its weight in grains?

CHAPTER XVII

RATIO AND PROPORTION

209. Uses of Ratio and Proportion. — While ratio and proportion are not of such constant use in business as are the fundamental operations, denominate numbers, or percentage, it is useful in many of the problems of commerce. The problems on page 163 are instances in point. No one can have a complete knowledge of the arithmetic of business who does not understand the elements of ratio and proportion.

210. Ratio. — The relative magnitudes of two numbers may be expressed by saying that one is a certain number of times as great as the other, or a certain fraction of the other.

Thus, 8 is twice as great as 4, 12 is 3 times as great as 4. Again 8 is $\frac{1}{3}$ of 24. 7 is $\frac{1}{6}$ as great as 42.

In each of these the result is obtained by dividing the first number by the second.

The ratio of one number to another is the indicated quotient in which the first number is the dividend, and the second the divisor.

Since any division may be indicated as a fraction, every ratio may be expressed as a fraction.

Thus, the ratio of 8 to 4 is $8 \div 4 = \frac{8}{4}$, and the ratio of 7 to 21 is $7 \div 21 = \frac{7}{21}$.

A fraction expressing a ratio may be reduced to its simplest form.

Hence every ratio may be expressed as an integer or as a fraction reduced to its lowest terms.

Thus, the ratio 8 to 4 is $\frac{8}{4} = 2$; the ratio 4 to 8 is $\frac{4}{8} = \frac{1}{2}$, and the ratio 12 to 8 is $\frac{12}{8} = \frac{3}{2}$.

If, however, a ratio is expressed by an improper fraction not equal to an integer, it should not be reduced to a mixed number.

Thus we speak of a ratio $\frac{8}{5}$ and not $1\frac{3}{5}$.

211. Antecedents and Consequents. — In speaking of the ratio of two numbers, it is important to note the order in which they are given.

Thus, the ratio of 2 to 8 is $\frac{2}{8} = \frac{1}{4}$, while the ratio of 8 to 2 is $\frac{8}{2} = 4$.

The first number of a ratio is called the *antecedent*, and the second number the *consequent*.

The special sign of ratio is the colon. Thus, the ratio 2 to 8 is written 2:8. Hence we have:

$$\text{Antecedent:consequent} = \frac{\text{antecedent}}{\text{consequent}} = \frac{\text{numerator}}{\text{denominator}}.$$

In some respects, the ratio of two numbers is their most important relation. If we know that two numbers differ by 10, this may mean much or little, according to the magnitude of the numbers. Thus, if two streets differ in length by 10 miles, we think of one street as much longer than the other, while two railways which differ by 10 miles may be regarded as of the same length for some practical purposes. On the other hand, if two distances are known to be in the ratio $\frac{1}{2}$ we know a significant relation between them whether they are large or small.

212. Numbers of the Same Kind Compared. — Since only magnitudes of the same kind can be compared, it follows that if either the antecedent or the consequent is a concrete number, the other must be a concrete number of the same kind. But a ratio is always abstract.

Thus, we speak of the ratio of two distances *both* expressed in miles, or *both* in yards, or *both* in feet, or *both* in inches.

We speak of the ratio of the areas of two fields both expressed in acres; of two quantities of grain, both expressed in bushels, etc.

In practice, however, it is best to regard all numbers as abstract for the purpose of computation. See next page.

To find the ratio of one number to another, divide the first by the second.

ORAL EXERCISES (A)

Express each of the following ratios in its simplest form.

1. 3:9	**5.** 7:2	**9.** 8:24	**13.** 24:9
2. 5:15	**6.** 11:4	**10.** 9:21	**14.** 15:12
3. 7:3	**7.** 18:6	**11.** 12:15	**15.** 21:9
4. 8:5	**8.** 16:5	**12.** 9:24	**16.** 36:8

EXERCISES (A)

Express each of the following ratios in its simplest form.

17. 27:105	**24.** 36:96	**31.** $\frac{1}{2}:\frac{1}{3}$	**38.** $2\frac{3}{4}:\frac{4}{5}$
18. 32:128	**25.** 45:81	**32.** $1\frac{1}{2}:3$	**39.** $4\frac{1}{2}:11\frac{1}{3}$
19. 45:75	**26.** $2:\frac{1}{2}$	**33.** $2\frac{1}{4}:9$	**40.** $6\frac{2}{3}:9\frac{1}{3}$
20. 64:12	**27.** $3:\frac{1}{2}$	**34.** $2\frac{3}{4}:11$	**41.** $5\frac{1}{2}:24$
21. 56:8	**28.** $\frac{1}{2}:2$	**35.** $\frac{1}{2}:\frac{1}{5}$	**42.** $7\frac{1}{4}:45\frac{1}{2}$
22. 108:32	**29.** $\frac{2}{3}:2$	**36.** $\frac{2}{3}:\frac{3}{4}$	**43.** $\frac{5}{6}:8\frac{2}{3}$
23. 96:16	**30.** $\frac{3}{4}:3$	**37.** $1\frac{1}{2}:2\frac{1}{2}$	**44.** $\frac{3}{5}:1\frac{1}{4}$

213. Numbers of Different Kinds Compared. — It is frequently convenient to consider the ratio of two numbers which would not be of the same kind if they were considered as concrete numbers. This offers no difficulty, however, since the numbers may always be considered as abstract.

Thus, the average yield per acre of a wheat field may be regarded as the ratio of the number of bushels to the number of acres.

Again, the per capita expenditure of the government of a certain city may be regarded as the ratio of the number of dollars expended to the number of people.

214. Proportion. — The equality of two ratios constitutes a proportion.

Thus, the ratios 3:7 and 6:14 form a proportion if written $3:7 = 6:14$, or $\frac{3}{7} = \frac{6}{14}$.

215. Means and Extremes. — In a proportion such as $3:7 = 6:14$ the first and the last numbers are called the *extremes*, and the two remaining numbers are the *means*.

When three of the four numbers in a proportion are given, the fourth number may always be found. It is this fact which makes proportion useful in solving problems.

In the following example, the numbers to be found are represented by x.

Example. Find x in $\frac{x}{7} = \frac{3}{8}$.

Solution. If one seventh of x, or $\frac{x}{7}$, $= \frac{3}{8}$, then $x = 7 \times \frac{3}{8} = \frac{21}{8} = 2\frac{5}{8}$.

ORAL EXERCISES (A)

Find the value of x in each of the following:

1. $\frac{x}{2} = \frac{2}{3}$	**5.** $\frac{x}{4} = \frac{4}{5}$	**9.** $\frac{x}{7} = \frac{2}{21}$	**13.** $\frac{x}{3} = \frac{7}{15}$
2. $\frac{x}{3} = \frac{4}{5}$	**6.** $\frac{x}{5} = \frac{3}{4}$	**10.** $\frac{x}{5} = \frac{4}{15}$	**14.** $\frac{x}{9} = \frac{5}{17}$
3. $\frac{x}{5} = \frac{3}{7}$	**7.** $\frac{x}{6} = \frac{3}{4}$	**11.** $\frac{x}{8} = \frac{5}{16}$	**15.** $\frac{x}{11} = \frac{3}{22}$
4. $\frac{x}{4} = \frac{2}{3}$	**8.** $\frac{x}{5} = \frac{7}{10}$	**12.** $\frac{x}{9} = \frac{2}{15}$	**16.** $\frac{x}{14} = \frac{5}{21}$

216. Product of Extremes Equals Product of Means. — In solving a problem by proportion, the proportion may always be written so that the unknown number is the antecedent of the first ratio. The finding of the unknown number is then a problem like those just solved.

Sometimes, however, a proportion may be given, in which the missing number is not the antecedent of the first ratio.

Thus, we may be required to find the value of x in such proportions as $\frac{3}{x} = \frac{4}{5}$, $\frac{4}{5} = \frac{x}{3}$, $\frac{5}{7} = \frac{8}{x}$. We shall now study a method for finding the value of x in any proportion.

The proportion $\frac{2}{5} = \frac{4}{10}$ or $2:5 = 4:10$ may be written $2:5 = 2:5$ by reducing the second ratio to simplest form. We then notice that the means and the extremes are the same. This is true of any proportion in which the ratios are reduced to simplest form. Hence,

In a proportion the product of the extremes equals the product of the means.

This statement holds for any proportion whatever, whether the ratios are reduced to simplest form or not.

ORAL EXERCISES (A)

Verify the above rule for each of the following proportions.

1. $\frac{3}{4} = \frac{6}{8}$	**3.** $\frac{1}{5} = \frac{3}{15}$	**5.** $\frac{6}{11} = \frac{18}{33}$
2. $\frac{7}{4} = \frac{21}{12}$	**4.** $\frac{2}{3} = \frac{8}{12}$	**6.** $\frac{2}{5} = \frac{8}{20}$

217. Rule for Finding the Unknown Number in Proportion. — The following is a rule for finding the value of x in any proportion.

One extreme equals the product of the means divided by the other extreme.

One mean equals the product of the extremes divided by the other mean.

Example. Find x in $\frac{7}{x} = \frac{5}{9}$.

Solution. By the rule $x = \frac{7 \times 9}{5} = \frac{63}{5} = 12\frac{3}{5}$.

WRITTEN EXERCISES (A)

Find the value of x in each of the following.

1. $\frac{3}{8} = \frac{x}{4}$
2. $\frac{1}{x} = \frac{7}{8}$
3. $\frac{3}{x} = \frac{7}{9}$
4. $\frac{2}{5} = \frac{3}{x}$
5. $\frac{2}{7} = \frac{x}{10}$
6. $\frac{6}{x} = \frac{8}{11}$
7. $\frac{5}{x} = \frac{9}{13}$
8. $\frac{7}{8} = \frac{x}{12}$

218. Problems Solved by Proportion. — Proportion may be used in solving a variety of problems, as is illustrated in the following.

Example 1. A field containing 17 acres yields 328 bu. of wheat. What should be the yield of a field containing 45 acres?

Solution. The ratio of the areas of the fields equals the ratio of the yields.
Let x = number of bushels from the second field.
Then $\frac{x}{328} = \frac{45}{17}$, and $x = \frac{328 \times 45}{17} = 868\frac{4}{17}$ (bushels).

Example 2. If it costs \$18,630 to build $6\frac{1}{2}$ miles of stone road, what is the cost of building $\frac{3}{4}$ mile of such road?

Solution. $x : 18630 = \frac{3}{4} : 6\frac{1}{2}$

and $x = \frac{18630 \times \frac{3}{4}}{6\frac{1}{2}} = \frac{18630 \times 3}{26} = 2149.62$ (dollars).

Example 3. In feeding 14 cows for 17 weeks, 23 tons of hay is used. How much will be needed to feed 25 cows for 19 weeks? (Read § 101 at this point.)

Solution. 14×17 = number of cows fed one week in the first lot
and 25×19 = number of cows fed one week in the second lot.

$\frac{x}{23} = \frac{25 \times 19}{14 \times 17}$, and $x = \frac{23 \times 25 \times 19}{14 \times 17} = 45.90$ (tons).

219. Cause and Effect. — In solving problems by proportion the idea of cause and effect is of value. In order that a problem shall give rise to a proportion there must be present what may be regarded as two causes and their corresponding effects.

The ratio of the causes will be equal to the ratio of the effects. This is illustrated in the examples on page 163.

Thus, in Example 1, the areas of the fields may be regarded as the causes and the yields as the effects. In Example 2, the lengths of the roads are the causes and their costs are the effects. In Example 3, the numbers of cows fed one week are the causes and the amounts of hay consumed are the effects.

These examples are excellent illustrations of the reason why we regard the numbers as abstract. We cannot multiply 328 *bushels* by 45 *acres*.

PROBLEMS

1. If it takes a man 4 days to plow 9 acres, how long will it take him to plow 42 acres?

2. An automobile runs 57 miles on 3 gallons of gasoline. How far will it run on 20 gallons?

3. If 40 bushels of wheat cost \$60, what will 250 bushels cost?

4. If 36 cords of wood cost \$114, what will 125 cords cost?

5. If 8 men can dig 6000 cubic feet of dirt from a ditch in 10 days of 8 hours each, how many cubic feet can they dig in 12 days of 9 hours each?

6. If 36 men build 1080 cubic feet of stone wall in a certain time, how many men will it require to build a wall containing 5760 cubic feet in the same time?

7. If a party of campers consisting of 60 people eat 1500 pounds of food in 14 days, how many pounds will it take to last 100 people 21 days?

8. If a pile of wood containing 252 cubic feet is worth \$64, what is a pile worth which contains 3024 cubic feet?

9. If an automobile runs 118 miles on $6\frac{1}{4}$ gallons of gasoline, how much gasoline will it use in going 374 miles?

220. Partitive Proportion. — It is sometimes required to separate a number into two or more numbers whose ratios shall be equal to the ratios of two or more given numbers.

Example 1. Divide 48 into two parts in the ratio $\frac{3}{7}$.

Solution. Let x represent one of the parts. Then

$$\frac{x}{48} = \frac{3}{10}, \text{ or } x = 48 \times \frac{3}{10} = 14.4.$$

Hence the other part is $48 - 14.4 = 33.6$.

If 48 is divided into 10 equal parts and 3 of these parts taken, this will clearly give one of the required numbers.

Example 2. Divide 165 into three parts in the ratios 2:3:4.

Solution. Let x = the smallest of the numbers.

Then $$\frac{x}{165} = \frac{2}{9} \text{ or } x = \frac{2 \times 165}{9} = \frac{2 \times 55}{3} = \frac{110}{3} = 36\tfrac{2}{3}.$$

If y is the next larger number, then

$$\frac{y}{165} = \frac{3}{9} = \frac{1}{3} \text{ and } y = \frac{165}{3} = 55.$$

Hence the third number is $165 - 36\frac{2}{3} - 55 = 73\frac{1}{3}$.

This type of problem is of rather frequent occurrence in business. See pages 430, 431.

WRITTEN EXERCISES

1. Divide 360 into two numbers in the ratio of 5:8.
2. Divide 740 into three numbers in the ratios 3:7:10.
3. Divide 1260 into three numbers in the ratios 3:4:5.
4. Divide 1800 into two numbers in the ratio 2:9.
5. Divide 2750 into three numbers in the ratios 1:4:7.
6. Divide 3170 into three numbers in the ratios 2:1:5.
7. Divide 18.36 into three numbers in the ratios 2:1:9.
8. Divide 48.32 into four numbers in the ratios 1:2:3:4.
9. Divide 465.4 into four numbers in the ratios 2:1:6:5.
10. Divide 987.8 into four numbers in the ratios 3:5:6:8.
11. Divide 845 into three numbers in the ratios 2:3:4.
12. Divide 100 into two numbers in the ratio $\frac{1}{2}:\frac{1}{3}$.
13. A man divided his estate of \$18,000 among his three sons in the ratios of 2:3:4. How much did each receive?

DRILL IN FUNDAMENTALS

Copy from sight, add, and check.

1.	2.	3.	4.	5.	6.
21784	67890	12345	67890	11121	14156
18642	29876	54321	42975	21274	12123
31579	24680	13579	80642	97531	24680
72900	16745	62958	46240	29876	54321
81420	62490	37890	62700	27843	39560
91417	76152	59372	90809	87065	42732
19147	67251	27395	90809	56078	23724
64422	49876	54321	29876	54321	71925
10020	26000	45333	75607	56789	23976
90760	12344	56789	72620	41743	92476
49700	56200	57300	27300	27920	65400
76542	29922	34272	62729	98326	19798
16171	81920	21222	32425	26272	82930
54443	42414	40938	37363	53433	32313
46474	84950	51525	35456	57585	96061
67574	73727	17079	68676	61564	63626
77787	98081	82838	48586	87888	99091
30120	11010	20098	97969	59493	92929
49504	51525	35455	56575	85960	61626
26949	59473	72717	27968	67666	56263
13746	77926	64297	82890	24466	79824

Copy from sight, divide, and check. Find quotients and exact remainders.

1. $4781\overline{)9376400}$	**2.** $642\overline{)81927000}$	**3.** $1745\overline{)34102700}$
4. $934\overline{)5764000}$	**5.** $498\overline{)19672000}$	**6.** $3782\overline{)94280600}$
7. $1394\overline{)10649000}$	**8.** $1084\overline{)70456000}$	**9.** $749\overline{)4764000}$
10. $1426\overline{)87492000}$	**11.** $358\overline{)61892000}$	**12.** $8417\overline{)38976000}$
13. $781\overline{)53642000}$	**14.** $4920\overline{)37896000}$	**15.** $3764\overline{)15497000}$
16. $914\overline{)8764900}$	**17.** $794\overline{)7146200}$	**18.** $492\overline{)7476000}$
19. $1264\overline{)9424000}$	**20.** $3642\overline{)1467200}$	**21.** $2946\overline{)4647000}$

CHAPTER XVIII

SQUARE ROOTS

221. Uses of Square Root. — Many problems require the length of the hypothenuse of a right triangle when the sides are known, or the diagonal of a rectangle when its length and width are known. Such problems require the finding of the square roots of numbers.

222. Square of a Number. — The result obtained by multiplying a number by itself is called the *square* of that number.

Thus, 4 is the square of 2, 9 is the square of 3, and $\frac{25}{64}$ is the square of $\frac{5}{8}$.

The square of a number is indicated by writing a small figure 2 a little above and to the right of it.

Thus, $2^2 = 4$, $3^2 = 9$, and $(\frac{5}{8})^2 = \frac{25}{64}$.

223. Perfect Square. — A number which is the square of an integer or of a fraction is called a *perfect square*.

Thus, 4, 9, $\frac{25}{64}$ are perfect squares.

224. Square Root. — One of the two equal factors of a number is called the square root of the number.

225. The Radical Sign. — The square root of a number is indicated by placing over it the symbol $\sqrt{}$, called the *radical sign*.

Thus, $\sqrt{4} = 2$, $\sqrt{9} = 3$, $\sqrt{\frac{25}{64}} = \frac{5}{8}$.

226. Approximating $\sqrt{2}$. — If a number is not a perfect square, we cannot find its square root exactly. But we may approximate such square root as nearly as we please.

Thus, $(1.4)^2 = 1.96$, and $(1.5)^2 = 2.25$. Hence $\sqrt{2}$ lies between 1.4 and 1.5.

Again, $(1.41)^2 = 1.981$, and $(1.42)^2 = 2.0164$. Hence $\sqrt{2}$ lies between 1.41 and 1.42.

$(1.414)^2 = 1.999396$, and $(1.415)^2 = 2.002225$. Hence $\sqrt{2}$ lies between 1.414 and 1.415.

1.414 is the nearest approximation to $\sqrt{2}$ to three places of decimals.

227. Approximating Square Root. — The first step in approximating a square root is to estimate a first approximation.

Example. Approximate the square root of 79.

Solution. $8^2 = 64$ is less than 79, and $9^2 = 81$ is greater than 79. Hence, $\sqrt{79}$ lies between 8 and 9. Moreover, it is considerably nearer 9 than 8. Hence we take 8.8 as a first approximation.

Dividing 79 by 8.8, the quotient is 8.98 (nearly). Hence the root lies between 8.80 and 8.98. We take half the sum of these, or 8.89 as a second approximation.

Dividing 79 by 8.89, the quotient is 8.886 (nearly). Hence the root lies between 8.890 and 8.886. Half the sum of these is 8.888, which is correct to three decimal places.

```
        8.98
8.8)790
    704
    860
    792
    680

        8.886
8.89)79000
     7112
     7880
     7112
     7680
     7112
      5680
      5334
```

228. Rule for Finding Square Root. — The process of approximating a square root is described in the following rule:

(1) *Estimate a first approximation, and divide the number by it, obtaining a first quotient.*

(2) *Use half the sum of the first approximation and first quotient as a second approximation, and divide the number by it.*

(3) *Use half the sum of the second approximation and the second quotient as a third approximation, and divide the number by it.*

(4) *Continue this process until the required closeness of approximation has been obtained.*

The root always lies between the last divisor and the last quotient.

WRITTEN EXERCISES

Find each of the following accurate to two places of decimals.

1. $\sqrt{89}$	**4.** $\sqrt{93}$	**7.** $\sqrt{60}$	**10.** $\sqrt{87}$
2. $\sqrt{67}$	**5.** $\sqrt{39}$	**8.** $\sqrt{73}$	**11.** $\sqrt{61}$
3. $\sqrt{58}$	**6.** $\sqrt{45}$	**9.** $\sqrt{84}$	**12.** $\sqrt{42}$

229. Finding First Approximation of Root. — The most difficult step in approximating a square root is to make a close first estimate of the root.

Example 1. Estimate the square root of 385.

Solution. The root lies between 19 and 20, because 385 is greater than $19^2 = 361$, and less than $20^2 = 400$. Further, 385 is somewhat nearer 400 than 361. Hence we use 19.6 as a first approximation.

Example 2. Estimate the square root of 741.

Solution. The root lies between 27 and 28 because 741 is greater than $27^2 = 729$, and less than $28^2 = 784$. Further, 741 is considerably nearer 729 than it is to 784. Hence we use 27.2 as a first approximation.

It makes little difference whether we use 27.2 or 27.3 as the first approximation.

Divide 741 by 27.2. The quotient is 27.24. This gives 27.22 as a second approximation. If we use 27.3 as a first approximation, we get a quotient 27.14 and this likewise gives 27.22 as a second approximation.

```
      27.24
27.2)741
     544
     1970
     1904
       660
       544
       1160
```

This method for approximating square roots has the following advantages over the usual method.

1. It can be understood in a very few minutes.
2. It forces on the mind the essential nature of an approximated root.
3. It gives constant drill in estimating roots.
4. The operation itself is equally as short as the other.

WRITTEN EXERCISES

Find the square root of each of the following correct to two places of decimals.

1. $\sqrt{342}$	6. $\sqrt{620}$	11. $\sqrt{531}$	16. $\sqrt{914}$	21. $\sqrt{467}$
2. $\sqrt{291}$	7. $\sqrt{213}$	12. $\sqrt{461}$	17. $\sqrt{886}$	22. $\sqrt{284}$
3. $\sqrt{820}$	8. $\sqrt{191}$	13. $\sqrt{791}$	18. $\sqrt{694}$	23. $\sqrt{455}$
4. $\sqrt{847}$	9. $\sqrt{397}$	14. $\sqrt{825}$	19. $\sqrt{419}$	24. $\sqrt{627}$
5. $\sqrt{297}$	10. $\sqrt{496}$	15. $\sqrt{673}$	20. $\sqrt{688}$	25. $\sqrt{942}$

230. Rule for Finding First Approximation. — To estimate a first approximation to the square root of a larger number, we make use of the following:

$10^2 = 100$, and $100^2 = 10{,}000$. Hence the square root of any number between 100 and 10,000 lies between 10 and 100.

Again, $1000^2 = 1{,}000{,}000$. Hence the square root of any number between 10,000 and 1,000,000 lies between 100 and 1000.

Following is a rule for estimating a first approximation to the square root of a number.

(1) *Divide the number into groups of two digits in each, beginning at the right side of the number or at the decimal point if there is one.*

(2) *Estimate the root of the last group to the left, and annex enough zeros to make one digit for each group.*

Example. Find the square root of 578,946, correct to two decimals.

Solution. We estimate the root of 57, the left group in 57,89,46 to be 7.6. Annex one zero and remove the decimal point, obtaining 760. $578946 \div 760 = 761.77$ Hence 760.88 is the second approximation. $578946 \div 760.88 = 760.890$. Hence 760.8850 is the third approximation. This is certainly accurate to three places of decimals.

To approximate the square root of a decimal, divide it into groups of two digits, beginning at the decimal point, and going toward the right. Estimate the root of the first group containing figures other than zero, and then prefix one zero for each group preceding this one, placing a decimal point to the left of the last one.

Thus, to estimate the square root of .000746, we write .00,07,46, and estimate the root of 7 as 2.7. Then we prefix one zero, obtaining .027 as a first approximation.

WRITTEN EXERCISES

Approximate each of the following to two decimals.

1. $\sqrt{89164}$	3. $\sqrt{1941.62}$	5. $\sqrt{287.39}$	7. $\sqrt{598.30}$
2. $\sqrt{35.86}$	4. $\sqrt{28739}$	6. $\sqrt{28.739}$	8. $\sqrt{8953.4}$

9. Find the length in rods of a side of a square piece of land containing one acre (160 square rods).

CHAPTER XIX

MENSURATION

231. Uses of Mensuration. — Mensuration (indirect measurement, see § 234) is of constant use in many trades, professions, various kinds of business, and in the household. The manifold and frequent uses of mensuration will appear throughout this chapter. It is one of the most important parts of arithmetic.

232. There are two kinds of measurement, namely *direct* measurement and *indirect* measurement.

233. Direct Measurement. — Direct measurement consists in applying a unit to a magnitude of the same kind to see how many times the unit is contained in it.

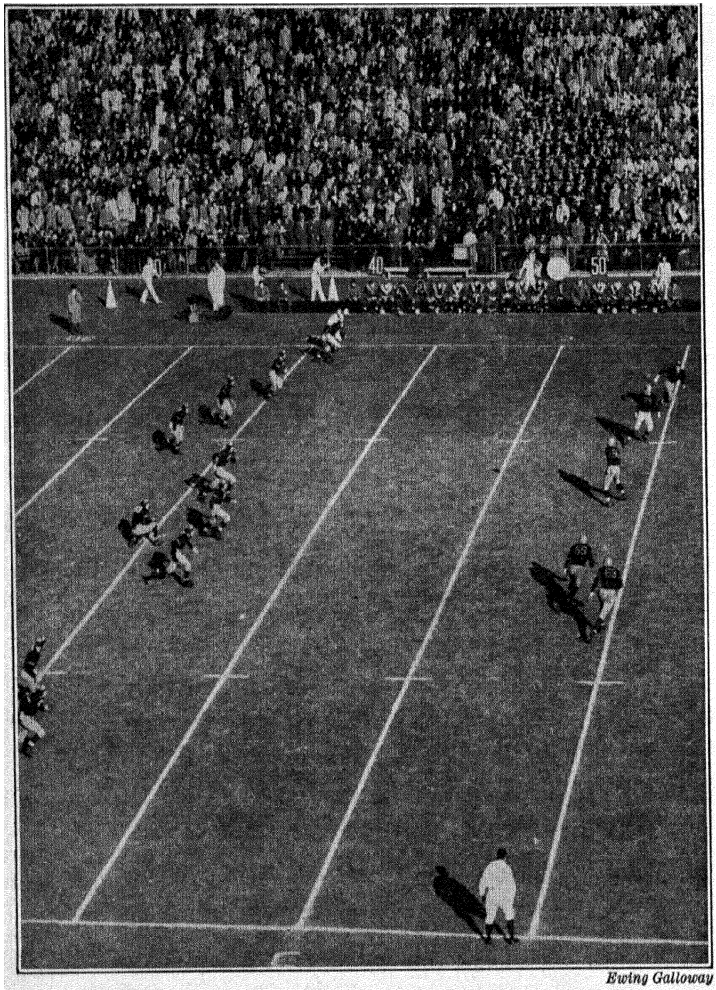
Thus, we may measure the *length* of an object by applying a foot measure or a yard measure directly to the object to see how many times the unit is contained in its length. Similarly, we may measure the contents of a milk can by finding how many times a quart measure may be filled from it.

234. Indirect Measurement. — Indirect measurement consists in measuring a quantity by any means other than the direct application of the unit of measure to the quantity to be measured.

Thus, we measure the area of a rectangle by taking the product of its length and width. The unit of measure (the square foot or the square rod) is not applied at all to the quantity to be measured. Similarly, the volumes of bins, cisterns, and tanks are obtained indirectly by measuring the dimensions (length, width, depth) by means of a linear unit, and then computing the volume.

235. Definition of Mensuration. — *Mensuration is the art of indirect measurement.*

236. Measurement of Lengths. — Indirect measurement of length is, in general, more difficult than indirect measurement of area or volume. In this book, the only method given for measuring length indirectly is the one given in §§ 242, 243.



Ewing Galloway

WHERE LINES AND MEASUREMENT ARE VERY IMPORTANT

237. Angles. — Two lines having a common end point are said to form an *angle* (Figure 1). If two lines cut each other so that all four angles formed are equal, then each of these angles is a *right angle* (Figure 2). A four-sided figure which has four right angles is a *rectangle* (Figure 3). If a rectangle has all its sides equal, it is a *square*.

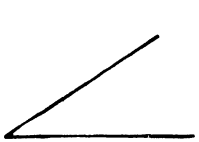


Figure 1.

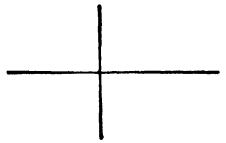


Figure 2.

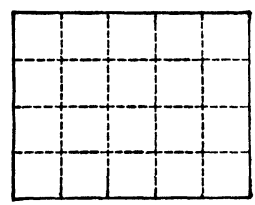


Figure 3.

238. Equations and Formulas. — An *equation* is the statement that two quantities are equal. The sign = is used to express this relationship. The expressions on either side of the sign are called *members* of the equation. One of the members sometimes contains an unknown quantity whose value is found by solving the equation.

A *formula* is a general rule expressed in symbols, usually in the form of an equation, stating relations between quantities. The quantities are usually represented by letters, as A for area, l for length, and so on.

239. Area of a Rectangle. — The number of unit squares contained in a rectangle is equal to the product of its length and width.



Thus, if the rectangle is five inches long and four inches wide, it contains 4×5 square inches, since there are 4 rows with 5 square inches in each.

The general rule may be stated as follows

$$Area\ of\ rectangle = length \times width, \text{ or } A = lw.$$

From this rule it follows that, $Area \div width = length$, or $\frac{A}{w} = l$, and $Area \div length = width$, or $\frac{A}{l} = w$.

To use this rule, the length and the width must be measured in the same unit. If the unit is feet, the area will be given in square feet; if it is rods, the area will be given in square rods; etc. If the units are different, they must be reduced to the same unit to find the area.

ORAL EXERCISES (A)

1. How many square feet are there in a blackboard 3 feet wide and 14 feet long?

2. How many square feet are there in a blackboard $2\frac{1}{2}$ feet wide and 24 feet long?

3. How many square yards are there in a wall 15 feet long and 9 feet high?

4. How many square yards are there in a sidewalk 6 ft. wide and 60 ft. long?

5. How many square yards of cement flooring are required for a basement 27 feet wide and 42 feet long?

PROBLEMS (B)

1. How many acres are there in a field 60 rods wide and 110 rods long? See page 138.

2. A surveyor finds that a rectangular field is 1650 feet wide and 2395 feet long. How many acres does it contain? See page 138. Find the result accurate to two places of decimals.

3. Find the ratio of the area of a rectangle 17 in. long and 15 in. wide, to the area of a rectangle 14 feet long and 10 feet wide.

4. How many bricks 4 inches by 6 inches will it take to pave a rectangular court 18 feet by 24 feet?

5. The perimeter of a rectangular plot is 96 rods, and its length is twice its breadth. What is the area? (The perimeter of a figure is the distance around it.)

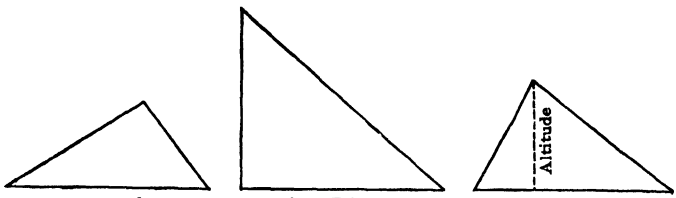
6. The base of a rectangle is 58 feet and 4 inches. Its altitude is 21 feet. Find the side of an equivalent square.

7. The base of a rectangle is $3\frac{1}{2}$ yards. Its area is 27,516 square inches. What is the perimeter?

8. Find the dimensions of a rectangle whose perimeter is 48, if the length is twice the width.

240. Triangles. — A *triangle* is a three-sided figure.

Any side of a triangle may be regarded as its *base*. The line drawn from the vertex opposite the base and perpendicular to the base is called the *altitude* on that base.



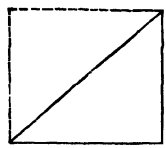
Obviously, every triangle has three bases and three altitudes.

The words *base* and *altitude* are usually used to mean the *lengths* of these lines.

A triangle is *right-angled* if one of its angles is a right angle.

In a right triangle, either side about the right angle may be regarded as its base, and the other side as the altitude.

A right triangle is just half a rectangle having the same base and altitude. (See Figure 1.) Hence, the area of a right triangle equals *half* the product of the base and altitude.


Figure 1.

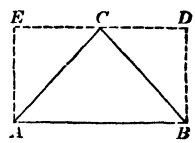


Figure 2.

It is easily seen from Figure 2 that the area of the triangle ABC is half the area of the rectangle $ABDE$.

Hence, for any triangle,

$$\textit{Area of a triangle} = \frac{\textit{base} \times \textit{altitude}}{2}, \text{ or } A = \tfrac{1}{2}\, bh.$$

From this rule it follows directly that,

$$\textit{Area of triangle} \div \textit{base} = \frac{\textit{altitude}}{2}, \text{ or } \frac{A}{b} = \frac{h}{2}.$$

$$\textit{Area of triangle} \div \textit{altitude} = \frac{\textit{base}}{2}, \text{ or } \frac{A}{h} = \frac{b}{2}.$$

ORAL EXERCISES (A)

1. The base and altitude of a triangle are 8 inches and 6 inches respectively. What is its area?

2. The area of a triangle is 60 square feet and its base is 12 feet. What is its altitude?

3. The area of a triangle is 80 square yards and its altitude is 8 yards. What is its base?

4. If the area of a rectangle and also its length are known how may its width be found?

5. If the area and also the width of a rectangle are known how may its length be found?

6. If the area of a triangle and also its base are known how may its altitude be found?

7. If the area and also the altitude of a triangle are known how may its base be found?

EXERCISES (A)

1. The base of a triangle is 17 feet and its altitude on that base 14 feet. Find its area.

2. The area of a triangle is 36 square feet. If the base is 9 feet, what is the altitude?

3. Find the area of a triangle whose base is 40 inches, and whose altitude is 3 feet.

4. The base and the altitude of a right triangle are 18 feet and 16 feet respectively. What is the area?

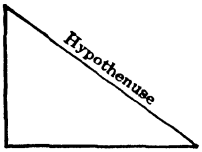
5. Find the area in acres of a field which is 140 rods long and 80 rods wide. (160 square rods equal one acre.) At $125 an acre what is the value of this field?

6. A rectangular lot of ground is 150 feet deep. How wide must it be to contain 20,000 square feet?

7. A field is 120 rods wide. A farmer wishes to cut off a pasture containing 40 acres from one end of this field. Find the dimensions of the pasture.

THE THEOREM OF PYTHAGORAS

241. The Right Triangle. — The side of a right triangle opposite the right angle is called the hypothenuse.

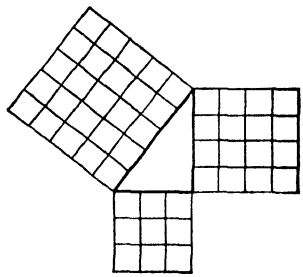


If the sides of a right triangle are 3 and 4, the hypothenuse is found to be 5. If the sides are 8 and 6, the hypothenuse is 10.

242. The Theorem of Pythagoras. — In all right triangles, the following relation holds: *The sum of the squares of the two sides of a right triangle equals the square of the hypothenuse.*

Hence, to find the hypothenuse of a right triangle whose sides are given, find the squares of the two sides, add them and find the square root of the sum.

Thus, in the triangle shown in the figure, $3^2 + 4^2 = 9 + 16 = 25 = 5^2$.



Example. Find the length of the hypothenuse of a right triangle whose sides are 12 and 15.

Solution. We know that the square of the hypothenuse equals $12^2 + 15^2 = 144 + 225 = 369$.

Hence, the hypothenuse equals $\sqrt{369} = 19.21$.

If we let x = length of hypothenuse, then the solution is written $x^2 = 12^2 + 15^2 = 369$, and $x = \sqrt{369} = 19.21$.

243. Rule for Finding One Side when the Hypothenuse and the Other Side Are Given. — *To find one side of a right triangle when the hypothenuse and the other side are given, subtract the square of the given side from the square of the hypothenuse and find square root of the result.*

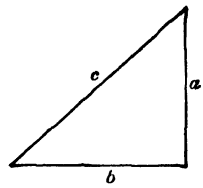
Example. The hypothenuse and one side of a right triangle are 12 and 8 respectively. Find the remaining side.

Solution. Square the hypothenuse and subtract from it the square of the given side, obtaining $12^2 - 8^2 = 144 - 64 = 80$. The required side $= \sqrt{80} = 8.94$.

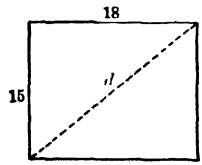
Sections 242, 243 give methods for indirect measurement of lengths or distances.

WRITTEN EXERCISES (A)

In the figure, the lengths of the sides are indicated by a, b, c.



1. If $a = 10$, $b = 9$, find c.
2. If $a = 18$, $b = 12$, find c.
3. If $c = 21$, $b = 16$, find a.
4. If $c = 34$, $b = 20$, find a.
5. If $c = 17$, $a = 10$, find b.
6. If $c = 51.6$, $a = 40.7$, find b.



7. Find the diagonal of a rectangle whose sides are 15 and 18.

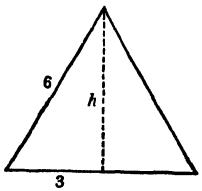
8. Find the diagonal of a room whose dimensions are 16 feet and 12 feet.

9. Find the diagonal of a rectangle whose sides are 12 inches and 14 inches.

10. Find a side of a rectangle if the other side is 18 inches and the diagonal is 32 inches.

11. Find a side of a rectangle if the other side is 20 rods and the diagonal is 30 rods.

12. Find a side of a right triangle if the hypothenuse is 42 inches and the other side is 28 inches.



13. Find the altitude of an equilateral triangle whose sides are 6. (Find h in the figure.)

14. Find the altitude of an equilateral triangle whose sides are 8 feet.

15. Find the altitude of an equilateral triangle whose sides are 10 inches.

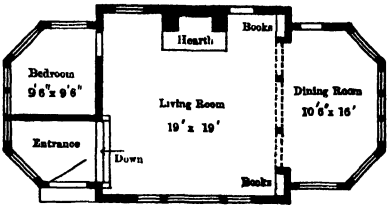
16. Find the area of an equilateral triangle whose sides are 12.

Suggestion. First find the altitude.

17. A carpenter "squares" a corner by means of a triangle whose sides are 6, 8, and 10 feet respectively. Is this method correct?

PLASTERING AND PAINTING

244. Plastering and painting are usually undertaken at so much per square yard. Sometimes allowance is made for doors and windows, and sometimes not. Only rarely is full allowance made for these, because working around them is more difficult than working over a clear surface. The allowance to be made must be specified in the contract. For painting, it is usual not to make any allowance for the openings.



In giving the dimensions of a room, they are usually given in the order, length, width, height.

Thus, $16' \times 14'\ 6'' \times 9'$ means 16 feet long, 14 feet 6 inches wide, and 9 feet high.

The figure shows the form in which the ground plan of a house is prepared for the builder. Some of the rooms are omitted to save space.

ORAL EXERCISES (A)

1. If a room is 15 feet wide, 18 feet long, and 9 feet high, the area of its walls (making no allowance for openings) is equal to that of a wall 22 yards long and 3 yards high. Explain.

2. Find the area in square yards of the walls of a room 12 feet wide, 15 feet long and 9 feet high, allowing 15 square yards for openings.

3. Find the area in square yards of the ceiling of the room in Example 2.

4. Find the area in square yards of a wall 16 feet long and 9 feet high.

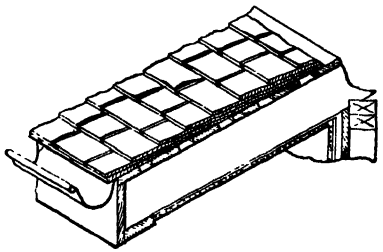
5. At 45 cents a square yard what is the cost of plastering a wall 18 feet long and 10 feet high?

ROOFING AND FLOORING

245. The unit of roofing is usually the *square*, which contains 100 square feet. See §§ 175, 176.

Carpenters and builders have tables which show how many shingles or tiles of a certain size are required for a square of roofing.

Thus, it is usual to allow 850 to 900 standard shingles for one square.



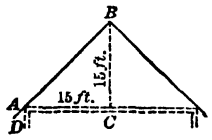
Shingles are estimated as averaging 4″ in width. They are laid 4″ to 6″ to the weather depending on the steepness of the roof.

In calculating the amount of lumber needed for a floor it is customary to find the area of a floor and add $\frac{1}{5}$ of the area to allow for waste.

Thus if the area of a floor is 360 square feet the lumber required is 360 + $\frac{1}{5}$ of 360 = 432 board feet.

WRITTEN EXERCISES (A AND B)

1. The figure shows the gable end of a barn. Find the length of the rafter *BD*, if it extends $1\frac{1}{2}$ feet beyond the wall.



Notice that *ABC* is a right triangle whose sides *AC* and *CB* are each 15 feet.

2. (B) The barn in the preceding example is 45 feet long. How many squares of roofing are required if the roof extends $1\frac{1}{2}$ feet beyond the building at each end?

3. (B) How many bundles of shingles, 250 shingles in each, are required for the roof in the preceding building, allowing 875 shingles to the square?

4. How many square feet of flooring are required for the living-room in the house on the opposite page, adding $\frac{1}{5}$ for waste?

5. The rooms in a house are 16′ by 12′ 6″, 18′ by 14′, 14′ by 12′, 13′ by 11′, 12′ by 12′, 14′ 6″ by 12′, 12′ by 9′, and 10′ by 9′. How many board feet of flooring are required, adding $\frac{1}{5}$ to allow for waste?

246. Estimating Wall Paper. — Wall paper is sold in single rolls, 8 yards long, and in double rolls, 16 yards long. The paper is usually 18 inches wide. One cannot buy less than a single roll.

The steps in estimating the amount of paper for a room are:

First, find the length of a vertical strip (from the border around the ceiling, to the mop board).

Second, find how many vertical strips 18 inches wide will be required to go around the room, omitting the complete width of doors and windows.

It is assumed that the ends left from the rolls after cutting the complete lengths will be sufficient to paper over and under the doors and windows.

Third, find how many rolls are necessary to obtain the required number of strips.

Fourth, notice whether the wastage in cutting will be sufficient to paper above and below the openings.

Thus, if the length of the strips is 9 feet, 5 such strips may be cut from a double roll, leaving 3 feet over. However, if the length of the strips is only 8 feet, there will be nothing left over. Sometimes it may be necessary to allow from 6 inches to 10 inches on each strip for matching the pattern.

Suppose we wish to paper the ceiling of a room 16 feet × 14 feet. From a double roll, we can cut exactly three strips each 16 feet long. Since each strip is $1\frac{1}{2}$ feet wide, 9 strips will cover a width of $13\frac{1}{2}$ feet. For the remaining half foot, we must cut a complete strip, however. Hence 10 strips, each 16 feet long, will be required to paper this ceiling.

This will require three double rolls and one single roll, or four double rolls.

ORAL EXERCISES (A)

1. How many strips 18 inches wide will be required to paper a room 15 feet long and 13 feet wide, allowing 14 feet in width for doors and windows?

2. How many rolls 16 yards long are required for 36 strips $9\frac{1}{2}$ feet long, allowing 6 inches on each strip for matching the pattern? How many feet wastage are there in each roll?

3. How many rolls 16 yards long are required for 72 strips 10 feet long, allowing 6 inches for matching?

4. How many rolls 16 yards long are required to paper a ceiling 12 feet wide and 15 feet long?

MISCELLANEOUS PROBLEMS (A AND B)

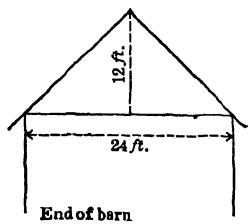
1. The first story in the house on page 179 is 10 feet high. Allowing 1 foot for the mop board and nothing for the openings, how much does it cost to rough finish the living room at 55¢ a square yard?

NOTE. The ceiling must be done as well as the walls.

2. At 20¢ per square yard, how much will it cost to paint the walls of a room 24′ square and 11′ high, allowing 200 square feet for doors and windows?

3. At 45¢ per square yard, what will it cost to plaster the walls and ceiling of a room 16′ 4″ long, 12′ 6″ wide, and 9′ high, allowing for 2 doors 7′ 4″ by 3′ 8″ and 4 windows 6′ × 3′ 6″?

4. At 40¢ a square yard, what will it cost to paint a barn if it is 60 feet long, 24 feet wide, and 18 feet high to the rafters, the height of the ridge above the eaves being 12′?



5. How many shingles averaging 4″ in width and laid 4″ to the weather will be required to cover both sides of the roof of a barn 40′ long, the roof being 18′ from ridge to eaves?

6. If the first course is laid double, how many more shingles will be needed in the preceding example?

7. (B) How many square feet in the roof of a barn 40 feet long, 32 feet wide, if the ridgepole is 12 feet higher than the side walls and the rafters extend 2 feet beyond the wall, and the roof extends 2 feet beyond the building at each end?

8. (B) At 90¢ a double roll for the walls and 45¢ a double roll for the ceiling, find the cost of the paper for a room 18 feet long, 16 feet wide, and 8 feet high, allowing 12 feet in width for doors and windows.

9. At $1.10 a double roll, find the cost of the paper for a room, ceiling included, if the room is 20 feet long, 14 feet wide, 8 feet high, the border costing 45¢ a yard, and no allowance being made for doors and windows.

CARPETING

247. Estimating Carpets. — In estimating the length of carpet needed for a floor, it must be borne in mind that one cannot buy a fractional width of carpet. Any length of carpet may be bought.

Suppose a floor 16 feet by 14 feet is to be carpeted, the width of the carpet being one yard. If the carpet is laid lengthwise of the room, five full widths must be used. In that case 5×16 feet $= 80$ feet, or $26\frac{2}{3}$ yards will be needed. It may be necessary, however, to allow 6 or 8 inches on each strip for matching the pattern. If 6 inches are allowed, the total length will be $82\frac{1}{2}$ feet, or $27\frac{1}{2}$ yards.

248. Paving, Engraving, etc. — There are several other cases in the industries where the cost is computed at so much per square unit. Thus paving is done by the square foot or the square yard, engraving and other forms of reproduction of pictures by the square inch, clearing of land by the acre, etc. In such cases a knowledge of the rules for finding the areas of rectangles and triangles and the exercise of a little common sense will enable one to solve the problems as they arise.

ORAL EXERCISES (A)

1. What would be the length of carpet required for the room described above if the strips were laid crosswise instead of lengthwise of the room?

Notice that 6 strips would be needed. Allow 6 inches on each strip for matching the pattern as before.

2. How many yards of carpet one yard wide will be needed for a room 16 feet wide and 20 feet long, (*a*) if the carpet is laid lengthwise of the room, (*b*) if it is laid crosswise? No allowance is made for matching the pattern.

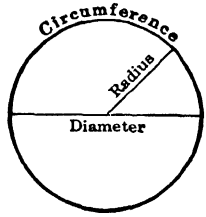
3. Find the length of carpet one yard wide required to carpet a room 13 feet wide and 17 feet long, (*a*) if the carpet is laid lengthwise of the room, (*b*) if it is laid crosswise, 6 inches being allowed on each strip for matching the pattern.

4. Find the length of carpet one yard wide required for the living room in the house on page 179, making no allowance for the hearth.

CIRCUMFERENCE OF THE CIRCLE

249. Definition of a Circle. — The *circle* is a part of a plane bounded by a curve which is everywhere equally distant from a fixed point called the *center*. The curve is called the circumference, while in higher mathematics the curve itself is called the circle.

250. Diameter, Radius, Circumference. — A straight line through the center of a circle and having its end points in the circumference is called a *diameter*. A line connecting the center with the circumference is called a *radius*.



The *distance* around a circle is also called its *circumference*. It may be found by careful measurements, and also in other ways, that the circumference of a circle is about $3\frac{1}{7}$ times the diameter. A closer approximation to the circumference may be obtained by multiplying the diameter by 3.1416.

The ratio of the circumference of a circle to its diameter is frequently represented by π. $3\frac{1}{7}$ and 3.1416 are approximations to π.

Diameter multiplied by π = *circumference, or* $2\pi r = C$.

Hence, *Circumference* $\div$ *diameter* $= \pi$, or $\frac{C}{2r} = \pi$,

and *Circumference* $\div \pi$ = *diameter*, or $\frac{C}{\pi} = 2r$.

ORAL EXERCISES (A)

1. Find the circumference of a circle whose radius is 5 inches.

2. Find the circumference of a circle whose radius is 15 inches.

3. If the diameter of a wagon wheel is 4 feet, how far will it roll in turning around once? (What is its circumference?)

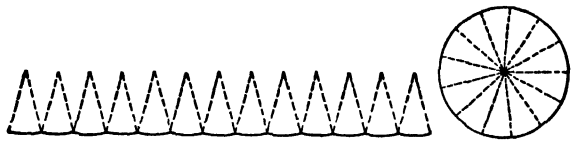
4. The diameter of a cylindrical steam boiler is 6 feet. What is the length of an iron band just long enough to go around it?

5. The diameter of a locomotive drive wheel is 6 feet. How far does it roll in making one revolution?

6. The diameter of an automobile wheel (including the tire) is 42 inches. How far will this wheel roll in making one revolution?

AREA OF THE CIRCLE

251. Area of Circle. — The circle may be cut into pieces, each of which is nearly the shape of a triangle. The narrower the pieces are cut, the more nearly straight is the base of each piece.



Since the area of a triangle equals half the product of the base and altitude, it follows that the area of each of these small triangle-like pieces equals the length of the arc forming its base multiplied by half the altitude of the triangle. But the altitude of each of these triangles is the radius of the circle and the sum of their bases equals the circumference of the circle. Hence we have:

The area of a circle equals the circumference multiplied by half the radius.

But the circumference equals 3.1416 × twice the radius. Hence it follows that the area = 3.1416 × twice the radius × $\frac{1}{2}$ radius = 3.1416 × radius × radius = 3.1416 radius^2.

Denoting 3.1416 by π, and the radius by r, we have

$$Area\ of\ a\ circle = \pi \times r^2, \text{ or } A = \pi r^2.$$

(The expression $\pi \times r^2$ is also written πr^2.)

ORAL EXERCISES (A)

Find the areas of circles whose radii are given below. Use $3\frac{1}{7}$ as the value of π.

Radius	Radius	Radius	Radius	Radius
1. 3	**5.** 2	**9.** 9	**13.** $\frac{1}{2}$	**17.** 300
2. 4	**6.** 7	**10.** 20	**14.** $\frac{1}{3}$	**18.** 400
3. 5	**7.** 8	**11.** 30	**15.** 100	**19.** 500
4. 6	**8.** 10	**12.** 40	**16.** 200	**20.** 1000

MISCELLANEOUS WRITTEN EXERCISES (A)

1. How many yards of carpet $2\frac{1}{4}$ feet wide will be required for a room $18' \times 14'$, if the strips run lengthwise of the room?

2. Find the cost of carpeting a room 24 feet by 20 feet with carpet $\frac{3}{4}$ of a yard wide at \$3.10 per yard, (*a*) if the strips run lengthwise, (*b*) if the strips run crosswise.

3. Find the cost of carpeting a room with carpet $\frac{7}{8}$ of a yard wide at \$3.50 per yard, if the room is $26' \times 16'$, the strips running lengthwise. There is a waste of $\frac{1}{4}$ of a yard on each strip for matching.

4. Find the cost of plastering a room $32' \times 28' \times 12'$ at \$.38 per square yard, allowing 20 square yards for openings.

5. The roof of a house is 38 feet long, and each side is $20\frac{1}{2}$ feet from ridgepole to eaves. What will the roofing cost at $\$6.72\frac{1}{2}$ a square?

6. Find the circumference of a circle whose radius is $4\frac{3}{4}$ feet.

7. The drivers on a freight locomotive are 4 feet 6 inches in diameter. How far do they go in making one revolution? (What is their circumference?)

8. How many times do the drivers in the preceding problem revolve in going one mile?

9. Find the diameter of a wheel whose circumference is 22 feet.

10. Find the diameter of a circular race track whose length is one mile.

11. The circumference of a tree is 4 feet 9 inches. Find its diameter.

12. The circumference of an iron pipe is 17 inches. What is its diameter?

13. Find the area of a circle whose radius is 18 inches.

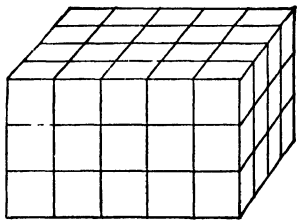
14. The radius of a circle is 23. Find the circumference and area.

15. The circumference of a circle is 96. Find the radius and area.

16. A circular pond whose circumference is 440 feet is surrounded by a path 8 feet wide. Required the area of the pond and also of the path.

VOLUMES

252. Volume of a Rectangular Solid. — A solid figure in the shape of a box is called a rectangular solid. We can easily see from the figure that *the number of cubic units of a rectangular solid equals the product of the length, width, and depth.*



Thus, if a box is 5 feet long, 4 feet wide, and 3 feet deep, its cubical contents is $5 \times 4 \times 3 = 60$ cubic feet.

The product 5×4 gives the number of cubic feet in a layer 5 feet by 4 feet and *one* foot deep.

The three dimensions of the solid must be expressed in the same unit. The volume will then be expressed in terms of a cubic unit whose edge is the unit of length used in measuring the length, width and depth.

That is, if all the dimensions are expressed in inches then the volume will be expressed in cubic inches. Again, if the dimensions are expressed in feet, the volume will be expressed in cubic feet, etc.

ORAL EXERCISES (B)

1. How many cubic feet in a box $2' \times 4' \times 6'$?

2. How many cubic feet in a box $3' \times 5' \times 6'$?

3. How many cubic yards in an excavation 12 yards wide, 20 yards long, and 2 yards deep?

4. How many cubic inches in a box $4'' \times 5'' \times 8''$?

5. If the length and width of an excavation is given in yards and the depth is given in feet, how may the number of cubic yards be found? How may the number of cubic feet be found?

6. If the dimensions of a rectangular solid are given in inches, how may the volume be found in cubic feet?

7. If the volume of a rectagular solid is known and also its length and width, how may its depth be found?

8. If the volume of a rectagular solid is known and also its length and depth, how may its width be found?

BRICKLAYING AND MASONRY

253. Standard and Roman Bricks. — An ordinary standard brick measures about $8\frac{1}{4} \times 4 \times 2$ inches, which is equal to 66 cubic inches, or 26.2 brick to a cubic foot. The average weight of such a brick is $4\frac{1}{2}$ pounds.

Roman bricks, which are used for facing and various ornamental purposes, are $12'' \times 4'' \times 1\frac{1}{4}''$.

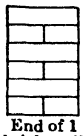
When laid in a wall, a brick will occupy considerably more than 66 cubic inches, since it is bedded in mortar on all sides except the one facing out.

254. Number of Bricks in a Wall. — For various thicknesses of wall the number of bricks required are estimated as follows:

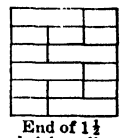
Wall 1 brick in thickness ($8\frac{1}{4}$ inches), 14 bricks per square foot of wall.

Wall $1\frac{1}{2}$ bricks in thickness ($12\frac{3}{4}$ inches) 21 bricks per square foot of wall.

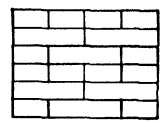
Wall 2 bricks in thickness (17 inches) 28 bricks per square foot of wall.



End of 1 brick wall



End of $1\frac{1}{2}$ brick wall



End of 2 brick wall

It should be understood that there are many possible arrangements of the bricks in these walls besides those shown here.

It will be noticed that the two-brick wall is more than twice as thick as the one-brick wall. This is because there is a layer of mortar between the two bricks when placed end to end; and when the bricks are placed lengthwise, there will be four bricks side by side and hence three layers of mortar instead of one as in the two-brick wall.

Bricks are sold by the thousand (M).

Thus bricks may be quoted at \$25.00 per M. In this case 52,800 bricks will cost $52.8 \times \$25.00 = \$1320$.

Rule: *To find the cost of a given number of bricks point off three decimal places and multiply by the price per M.*

255. Cancellation Used in Solving Problems. — The use of cancellation in solving problems is illustrated in the following examples.

Example 1. How many tons of timothy hay are there in a well-settled mow, $35' \times 24' \times 12'$?

Solution. The number of cubic feet in the mow is $35 \times 24 \times 12$ and the number of cubic feet in one ton is 450.

Hence, $\frac{35 \times 24 \times 12}{450}$ is the number of tons.

$$\frac{\overset{7}{\cancel{35}} \times \overset{8}{\cancel{24}} \times \overset{2}{\cancel{12}}}{\underset{\cancel{450}}{\underset{\cancel{90}}{\underset{\cancel{10}}{\underset{5}{}}}}} = \frac{112}{5} = 22\tfrac{2}{5} \text{ (tons)}.$$

Example 2. What is the capacity in gallons of a cistern $11' \times 8' \times 6'$?

Solution. The number of cubic inches in the cistern is $11 \times 8 \times 6 \times 1728$. Hence the number of gallons is

$$\frac{\overset{2}{\cancel{11}} \times \cancel{8} \times \overset{6}{\cancel{6}} \times 1728}{\underset{\cancel{231}}{\underset{\cancel{77}}{\underset{7}{}}}} = \frac{27648}{7} = 3949\tfrac{5}{7} \text{ (gallons)}.$$

In such examples the expressions $\frac{35 \times 24 \times 12}{450}$ and $\frac{11 \times 8 \times 6 \times 1728}{231}$ should be written down at once.

Example 3. How many tons of stove coal (hard) will a bin hold if it is $12' \times 10' \times 5'$? (See § 182.)

Solution. $\frac{12 \times 10 \times \overset{5}{\cancel{5}}}{\underset{7}{\cancel{35}}} = \frac{120}{7} = 17\tfrac{1}{7}$ (tons).

Example 4. At $1\frac{2}{3}$ cubic yards to the load, how many loads of dirt must be hauled away to make an excavation $86' \times 50' \times 6'$?

Solution. $1\frac{2}{3}$ cubic yards = 45 cubic feet.

Hence the number of loads is $\frac{86 \times \overset{10}{\cancel{50}} \times \overset{2}{\cancel{6}}}{\underset{\cancel{45}}{\underset{\cancel{9}}{\underset{3}{}}}} = \frac{1720}{3} = 573\tfrac{1}{3}$.

Example 5. Find the number of bushels of wheat in an elevator bin $14' \times 16' \times 56'$. (See § 182.)

Solution. $\frac{14 \times 16 \times 56}{1.25} = 10{,}035\frac{1}{5}$ (bushels).

Example 6. A bin is $10' \times 12' \times 5'$. How many bushels of potatoes will it hold? (See § 182.)

Solution. $\frac{10 \times 12 \times 5}{1.6} = \frac{5 \times 3 \times 5}{.2} = \frac{75}{.2} = 375$ (bushels).

PROBLEMS (A)

1. How many loads of earth must a contractor remove to make an excavation $46' \times 38' \times 5'$ if the loads average 30 cu. ft.?

2. Find the capacity in gallons of a cistern $8\frac{1}{2}' \times 6' \times 5'$.

3. How many tons of clover hay are there in a mow $40' \times 28' \times 12'$?

4. How many square feet are there in a surface $27'' \times 42''$?

5. How many cubic yards are there in an excavation $18' \times 24' \times 7'$?

6. How many bu. of wheat does a bin hold which is $14' \times 8' \times 7'$?

7. At 36 cubic feet to the ton how many tons are there in a bin $30' \times 18' \times 5'$?

8. How many acres are there in a field 65 rods by 90 rods?

9. At \$20 per M what is the cost of brick for a $1\frac{1}{2}$-brick wall $50' \times 18'$?

Suggestion. The cost is $\frac{21 \times 50 \times 18 \times 20}{1000}$ (dollars).

10. At \$9 per square yard what is the cost of paving a block $48' \times 360'$?

11. At \$4.75 per square yard what is the cost of laying a hardwood floor in a room $16' \times 18'$?

12. What is the ratio of the areas of two fields, one 72 rods by 108 rods and the others 98 rods by 148 rods?

13. At \$3.00 a square yard what will a sidewalk $5' \times 480'$ cost?

14. At \$2.75 a cubic yard, what is the cost of making an excavation $84' \times 45' \times 8'$?

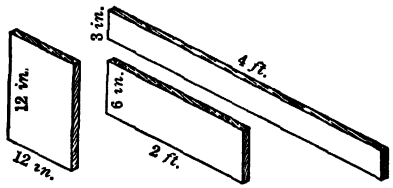
BOARD MEASURE

256. The Board Foot. — A piece of lumber containing 144 cubic inches or one twelfth of one cubic foot is called a board foot.

In measuring ordinary lumber, boards less than one inch in thickness are regarded as one inch thick.

Thus a piece of board one foot square is a board foot as are also pieces 6 inches wide and 2 feet long, 4 inches wide and 3 feet long, 3 inches wide and 4 feet long, and 2 inches wide and 6 feet long, provided they are all one inch thick or less.

An exception to this is made in the case of very costly woods, such as mahogany, which are often sold in thin sheets for veneering purposes.



257. Finding Board Feet. — The obvious method for finding the number of board feet in a piece of lumber is to divide the number of cubic inches by 144, or 12×12.

However, if one of the dimensions is given in feet, we may multiply feet by inches and divide by 12.

If two dimensions are given in feet, and one in inches, we obtain the number of board feet by taking the product of the dimensions expressed in these units.

258. Lumber-Yard Rule. — From the above we have the lumber-yard rule:

Feet by feet by inches = board feet.

In estimating the amount of lumber required for floors, etc., allowance must be made for waste in matching the lumber.

Thus, 3-inch flooring may run only $2\frac{5}{8}$. It is customary to add $\frac{1}{5}$ or $\frac{1}{6}$ for matching, and for waste in cutting.

In putting on siding, the boards overlap, for which allowance must be made. (See the figure on page 192.)

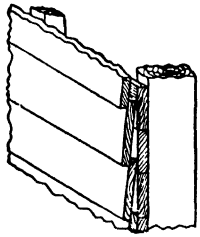
Lumber is sold by the thousand (M) board feet.

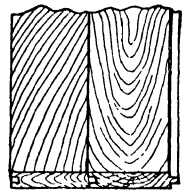
Example. How many board feet are needed for the flooring of a room 15′ by 18′, adding $\frac{1}{5}$ to allow for waste, etc.?

Solution.

$15 \times 18 \times \frac{6}{5} = 324$ (bd. ft.)

The $\frac{1}{5}$ is added by multiplying by $\frac{6}{5}$. The flooring is counted as one inch thick.





PROBLEMS (A)

1. The floor in a store is made of $1\frac{1}{2}$-inch boards. How many board feet are used, if the store is 45 feet long, and 26 feet wide? At \$68 per M, what is the cost of this lumber? (Add $\frac{1}{5}$ for waste.)

2. Find the cost of the following lots of lumber:

860 pieces $14' \times 10'' \times 1''$ at \$44.60 per M.
250 pieces $10' \times 8'' \times 1\frac{1}{2}''$ at \$49.50 per M.
165 pieces $16' \times 9'' \times 2''$ at \$60.00 per M.
52 pieces $12' \times 4'' \times 2\frac{1}{2}''$ at \$62.50 per M.
34 pieces $14' \times 10'' \times 3''$ at \$62.50 per M.
2460 linear feet 3″ wide, one inch thick at \$82.00 per M.

LUMBER YARD PRACTICE

259. Another lumber-yard rule is:

To find the number of board feet in lumber 1 inch thick and:

3 inches wide, take $\frac{1}{4}$ the length.
4 inches wide, take $\frac{1}{3}$ the length.
6 inches wide, take $\frac{1}{2}$ the length.
8 inches wide, deduct $\frac{1}{3}$ the length.
9 inches wide, deduct $\frac{1}{4}$ the length.
10 inches wide, deduct $\frac{1}{6}$ the length.
12 inches wide, take the length.
14 inches wide, add $\frac{1}{6}$ the length.
15 inches wide, add $\frac{1}{4}$ the length.
16 inches wide, add $\frac{1}{3}$ the length.

$2'' \times 4''$, deduct $\frac{1}{3}$ length.
$3'' \times 4''$, take the length.
$2'' \times 6''$, take the length.
$2'' \times 8''$, add $\frac{1}{3}$ the length.
$2'' \times 10''$, add $\frac{2}{3}$ the length.

MISCELLANEOUS PROBLEMS (A)

1. How many cubic feet in a bin $8' \times 7' \times 5\frac{1}{2}'$?

2. How many cubic yards in an excavation 28 yards long, 20 yards wide, and 8 yards deep? At \$2.75 a cubic yard, what is the cost of making this excavation?

3. A ditch is 6 feet deep, $2\frac{1}{2}$ feet wide, and 254 rods long. How many cubic feet must be removed in digging it?

4. In a grain bin 12 feet long, and 9 feet wide, the grain is $5\frac{1}{2}$ feet deep. How many bushels of grain are there in this bin? (See § 182.)

5. How many tons of coal in a bin $10\frac{1}{2}$ feet long and 8 feet wide if the coal averages 4 feet in depth? (See § 182.)

6. How many tons of timothy hay in a mow 25 feet by 22 feet if the hay is 10 feet deep? (See § 182.)

7. How many bushels of potatoes in a bin 12 feet wide by 14 feet long, if they average 5 feet in depth? (Potatoes are measured by the heaped bushel. See § 182.)

8. A farmer wishes to put 25 tons of clover hay into a mow 30 feet by 24 feet. How deep must the hay be after it is well settled?

9. At \$18.50 per M (thousand) what is the cost of brick for a $1\frac{1}{2}$-brick wall 180 feet long and $4\frac{1}{2}$ feet high?

10. If the wall in Example 9 were built of masonry, how many perches of masonry would it contain? (Regard wall as one foot thick.)

11. How many thousand bricks are required to build a two-brick wall 54 feet long and 21 feet high if $\frac{1}{10}$ is deducted for openings?

12. What would be the cost of this wall if the bricks cost \$27.75 per M laid in the wall?

13. A house 42 feet long, 38 feet wide, and 18 feet high has $1\frac{1}{2}$-brick walls. How many thousands of bricks are used if no allowance is made for openings or corners? (It is assumed that the bricks saved by the openings and corners are used for gables, chimneys, etc.)

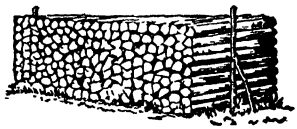
14. What is the weight of 1000 standard bricks?

15. How many standard bricks are there in 1 ton? in $2\frac{1}{2}$ tons?

WOOD MEASURE

260. The Cord. — A pile of wood 8 feet long, 4 feet wide, and 4 feet high is called a cord of wood.

The cord contains 128 cubic feet of piled wood.



Thus, a pile of 2-foot wood must be 16 feet long and 4 feet high to make a cord, and 16-inch wood requires a pile 24 feet long and 4 feet high to make one cord.

A pile of cord wood is about 56% solid wood. The rest is interstitial space.

EXERCISES

1. How many cords of wood are there in a pile of 4-foot wood 40 feet long and 5 feet high?

2. How many cords are there in a pile of 2-foot wood 125 feet long and 4 feet high?

3. How many cords are there in 5 piles of 16-inch wood each 25 feet long and 7 feet high?

4. How many board feet of solid wood are there in one cord of wood?

5. If soft coal sells for $4.50 a ton, how much must each of the kinds of wood given in the table below sell for in order to make it equally expensive to burn coal and wood? (Refer to § 261.)

261. Heating Value of Wood. — Weight of one cord of wood and heating value as compared with soft coal are as follows.

Hickory or Hard Maple	4500 lb. equals 1800 lb. coal
White Oak	3850 lb. equals 1540 lb. coal
Beech, Red, and Black Oak	3250 lb. equals 1300 lb. coal
Poplar, Chestnut, Elm	2350 lb. equals 940 lb. coal
The average Pine	2000 lb. equals 800 lb. coal

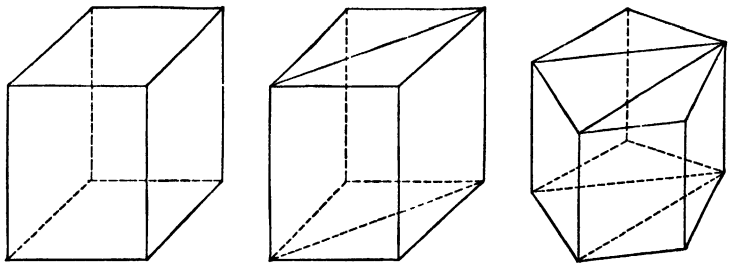
It is important that the wood be dry when burned, as every 10% of moisture detracts 12% from the heating value of the wood. It requires considerable heat to evaporate the water in green wood.

PRISMS AND CYLINDERS

262. Prisms. — The figures below represent solids in each of which the top and the bottom, called *bases*, are parallel planes. The other faces are called *lateral faces*. These solids are called *prisms*.

Boxes, bins, etc., are prismatic in shape. They are represented by the figures to the left, and are called rectangular prisms. We have already found the volume of such prisms by multiplying the length by the width by the height.

The height of a prism is called its *altitude*.



263. Volume of a Prism. — Multiplying the length by the width gives the area of the base if the base is a rectangle.

Hence: *The volume of a rectangular prism equals the product of the base and altitude.*

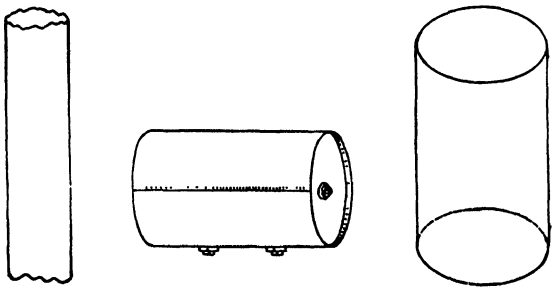
In the middle figure, the rectangular prism has been cut into two triangular prisms, each equal to half the original prism. Hence, in the case of a triangular prism, the volume equals the product of the base and altitude.

The prism to the right has been divided into three triangular prisms. Since the rule just stated holds for each of these triangular prisms, it must hold for the whole prism. Hence, we conclude that for any right prism whatever, we have,

The volume of a prism equals the product of the base and altitude.

It is understood, of course, that what is really meant by this rule is that the number of square units of area of the base is multiplied by the number of linear units in the altitude, and that the product represents the number of cubic units of volume.

264. Cylinders. — Objects such as steam pipes, round smoke stacks, stove pipes, etc. are said to be *cylindrical* in form.



A cylindrical object with a flat top and bottom, such as a gasoline tank, is called a *cylinder.*

The flat ends are called the *bases* of the cylinder. The curved surface is called the *lateral* surface of the cylinder.

When the bases are at right angles to the lateral surface the cylinder is a *right cylinder.* The distance between the bases is the *altitude.*

If the outer layer of a cylindrical roll of paper is taken off, and laid flat, it will be found to be a rectangle, one side of which is the circumference of the cylinder; and the other, the altitude of the cylinder.

265. Lateral Area of a Cylinder. — It is evident from the above that *the lateral area of a right cylinder equals the circumference of the base multiplied by the altitude.*

PROBLEMS (A)

1. Find the lateral area of a cylinder whose circumference is 30 inches and altitude 20 inches.

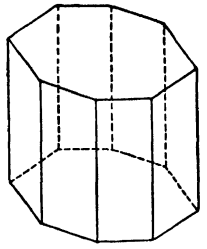
2. Find the total surface of a cylindrical tank 36 inches high and 16 inches in diameter.

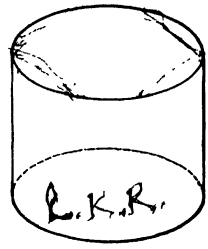
Suggestion. Add the areas of the bases to the lateral area.

3. A cylindrical tank 42 inches high and 18 inches in diameter is made of sheet steel weighing $3\frac{1}{2}$ pounds per square foot. Find the weight of the empty tank, allowing 3 square feet for joints.

266. Volume of a Cylinder. — It is easily seen by comparing the cylinder and prism in the cut that:

The volume of a cylinder equals the area of the base multiplied by the altitude.





PROBLEMS (A)

1. Find the volume of a prism the area of whose base is 10 square inches and whose altitude is 8 inches.

2. Find the volume of a cylinder the area of whose base is 240 square inches and whose altitude is 20 inches.

3. State fully how to find the volume of a cylinder when the altitude and the radius of the base are given.

4. Find the volume of a cylinder whose radius is 10 and whose altitude is 5.

5. A power plant has a cylindrical reservoir 30′ deep and 22′ in diameter. If a cubic foot of water weighs $62\frac{1}{2}$ lb., how many tons of water will it hold?

6. Find the number of gallons in each of the following rectangular cisterns and reservoirs.

(*a*) $8' \times 7' \times 10'$. (*c*) $9' \times 9' \times 9'$. (*e*) $112' \times 78' \times 96'$.
(*b*) $6' \times 6' \times 12'$. (*d*) $50' \times 50' \times 24'$. (*f*) $11' \times 10' \times 8'$.

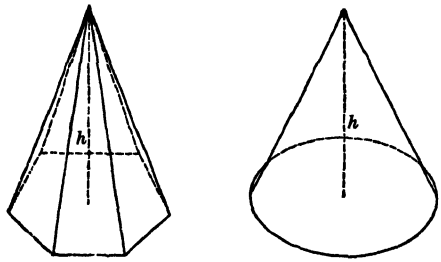
7. Find the number of gallons in each of the following cylindrical cisterns and reservoirs.

(*a*) Diameter 5′, depth 7′. (*d*) Diameter 50′, depth 60′.
(*b*) Diameter 6′, depth 10′. (*e*) Diameter 30′, depth 50′.
(*c*) Diameter 8′, depth 12′. (*f*) Diameter 12′, depth 18′.

267. Volume of Pyramid and Cone. — The volume of a pyramid is equal to one third the volume of a prism of the same base and altitude.

The volume of a cone is equal to one third the volume of a cylinder of the same base and altitude.

That is: *The volume of a pyramid or a cone equals one third the product of the base and altitude.*



PROBLEMS (A)

1. Find the volume of a pyramid with altitude 9 and area of base 16.

2. The area of the base of a cone is 32 and its altitude is 12. Find the volume.

3. The base of a pyramid is a square whose sides are 6 and its altitude is 8. Find its volume.

4. The base of a cone is a circle whose radius is 10 and its altitude is 9. Find the volume.

5. The altitude of a pyramid is 9 and the area of its base is 400. Find the volume.

6. The volume of a pyramid is 4000, and its altitude is 20. What is the area of the base?

7. The area of the base of a cone is 24 and the altitude is 16. Find the volume.

8. The diameter of the base of a circular cone is 12 and the altitude is also 12. Find the volume.

9. The volume of a right circular cone is 400 and the altitude is 16. What is the area of the base?

268. Surface of a Sphere. — Consider a sphere placed in a cylindrical box which is just large enough to receive it. The diameter of the cylindrical box and of the sphere will be the same, and the height of the box will equal the diameter of the sphere. It is a curious fact that the total surface of this sphere will be exactly equal to the lateral area of the box.

The lateral area of the box equals *circumference* $\times$ *altitude*. If r = radius of box then the circumference equals $2\pi \times r$. Since the altitude = twice the radius we have, Lateral area $= 2\pi \times r \times 2r = 4\pi r^2$. This formula also gives the surface of a sphere.

269. Volume of a Sphere. — A sphere may be regarded as made up of a large number of small pyramids, all having their bases in the surface of the sphere and their vertices at its center. The sum of the bases of these pyramids is the surface of the sphere and their altitude is the radius of the sphere.

Hence the volume of the sphere = surface $\times \frac{\text{radius}}{3}$,

or $$\text{Volume} = 4\pi r^2 \times \frac{r}{3} = \frac{4}{3}\pi r^3.$$

Denoting the surface of the sphere by S and its volume by V, we have,

$$S = 4\pi r^2, \qquad V = \tfrac{4}{3}\pi r^3.$$

PROBLEMS (A)

1. Find the surface of a sphere whose radius is 10 inches.

2. At \$2.40 a square foot, find the cost of gilding the hemispherical dome of a cathedral if the diameter of the dome is 80 feet.

Suggestion. Find half the surface of a sphere whose radius is 40 feet.

3. The diameter of the earth is approximately 8000 miles. Assuming it to be spherical in shape, find its total area.

4. Find the volume of a sphere whose radius is 5 inches.

5. If cast iron weighs 450 lb. a cubic foot, find the weight of a solid spherical cast iron ball whose radius is 6 inches.

6. Find the volume of the earth in cubic miles. (See problem 3.)

UNITED STATES LAND SURVEYS

270. In 1802 Colonel Mansfield, the surveyor of the Northwest Territory, inaugurated the plan for surveying the public lands which is still in use. The general features of the plan were as follows: The entire public domain was first divided into parts called *land districts*. In each district a meridian line, called the *principal meridian*, was run through the entire district from North to South, and from some point on this meridian an East and West line was run which was called a *base line*. Parallel to the principal meridian, and to the base line, lines were run six miles apart, dividing the land into squares six miles on the side. These squares are called *townships*. A row of townships extending north and south is called a *range*.

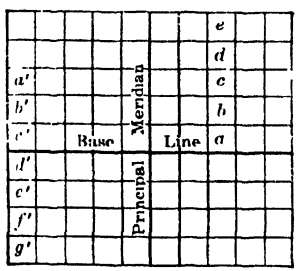


Figure 1.

6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

Figure 2.

The ranges are designated as *east* or *west*. Thus, in Figure 1 the range of townships marked a, b, c, d, e, is range 3, East. The range containing townships a', b', c', d', e', f', g', is range 5, West. The township marked c is township 3 North, range 3 East. The township marked c' is township 2 South, range 5 West.

Each township is divided into 36 square mile plots, called *sections*. They are numbered as shown in Figure 2, the section numbered 1 being the north-east corner of the township.

Section 14 in the township marked b' would be described as section 14, Township 2 North, range 5 West. Added to this, there would be a designation giving the land district in which the section is located.

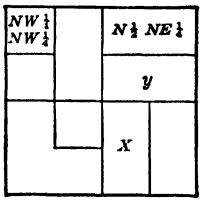
The land surveyed in this manner consists of nearly all the land now in the United States which did not belong to the thirteen States at the time of the Revolution, together with the lands which were later ceded to the United States by these original States.

N.W ¼ (North West Quarter)	N.E ¼ (North East Quarter)
N½ of S.W ¼	
	SE ¼ of SE ¼

A section of land is divided into smaller parts, as shown in the figure. Thus the piece marked S.E. $\frac{1}{4}$ of S.E. $\frac{1}{4}$ is called the south-east quarter of the south-east quarter.

PROBLEMS (A)

1. Make a figure representing a township, and on it mark the following: the S.E. $\frac{1}{4}$ of section 18; the West $\frac{1}{2}$ of section 27; the S.W. $\frac{1}{4}$ of the N.E. $\frac{1}{4}$ of section 9.



2. Describe the piece of land marked X in this figure. Also describe the piece marked y.

3. At \$75 an acre, what is the value of the South $\frac{1}{2}$ of the N.E. $\frac{1}{4}$ of section 1?

4. At 80¢ a rod what is the cost of fencing in a section of land? A quarter section?

5. A piece of land 100 rods wide and 240 rods long is divided into four equal pieces by two fences running through it. At 80¢ a rod how much will it cost to fence the land, including the partition fences and the fences around it?

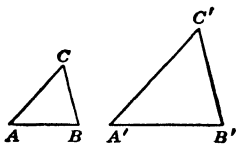
6. What fractional part of a section is the N.W. $\frac{1}{4}$ of the N.W. $\frac{1}{4}$? If the whole section is worth \$20,000, what is this part worth? At 65¢ a rod how much will it cost to fence in this part of the section?

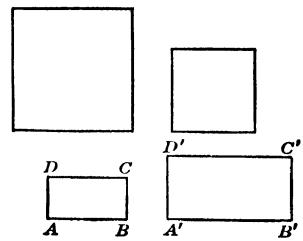
7. If I own the S.W. $\frac{1}{4}$ of section 4, how many acres do I own? Make a diagram representing the section and showing the piece just described. Divide the rest of the section into one half section and eighths of a section.

8. Find the number of acres in each of the pieces shown on the figures in Example 1.

SIMILAR FIGURES

271. Definition of Similar Figures. — Figures having the same form or shape are said to be *similar*.





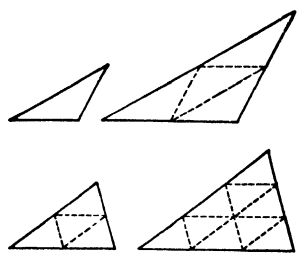
272. Sides of Similar Figures Form Proportion. — Above are shown three pairs of similar figures. One of the essential properties of two similar figures is *that corresponding sides form a proportion.*

That is, in the triangles $\frac{AB}{A'B'} = \frac{BC}{B'C'} = \frac{CA}{C'A'}$.

Similarly, in the rectangles $\frac{AB}{A'B'} = \frac{BC}{B'C'}$.

In the two similar triangles shown here the sides of one are twice the sides of the other. The area of the larger triangle is four times that of the smaller. That is, the ratio of the areas equals the square of the ratio of the sides.

The ratio of the corresponding sides of the triangles in the next figure is $\frac{3}{2}$, and the ratio of their areas is $\frac{9}{4}$. Again, the ratio of the areas equals the square of the ratio of the sides.



These are special cases of a very important property of all similar surfaces.

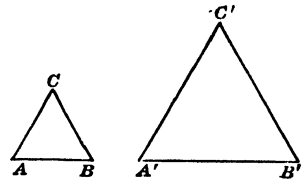
273. Areas of Similar Figures. — *The ratio of the areas of similar surfaces equals the square of the ratio of their corresponding lines.*

Thus, the ratio of the areas of two circles equals the ratio of the squares of their radii, or of their diameters.

ORAL EXERCISES (A)

1. If triangles ABC and $A'B'C'$ are similar and if

$$\frac{AB}{A'B'} = \frac{3}{5}, \text{ find } \frac{BC}{B'C'} \text{ and } \frac{CA}{C'A'}.$$



In Examples 2-7 let $AB:A'B' = 3:5$.

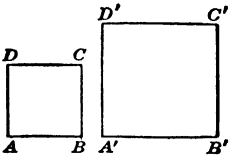
2. Find the ratio of the areas in the triangles ABC and $A'B'C'$.

3. If the area of triangle ABC is 9, find the area of triangle $A'B'C'$.

4. If in triangle ABC, $AB = 6$, find $A'B'$.

5. If $BC = 9$, find $B'C'$.

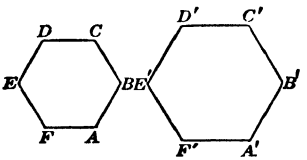
6. If $CA = 12$, find $C'A'$.



7. If the area of triangle ABC is 36, find the area of $A'B'C'$.

8. If $ABCD$ and $A'B'C'D'$ are squares and if $\frac{AB}{A'B'} = \frac{4}{7}$, find the ratio of their areas.

9. If $ABCDEF$ and $A'B'C'D'E'F'$ are similar and if $\frac{AB}{A'B'} = \frac{3}{7}$, find $\frac{BC}{B'C'}$, $\frac{CD}{C'D'}$, $\frac{DE}{D'E'}$, $\frac{EF}{E'F'}$, and $\frac{FA}{F'A'}$.



10. If the area of $ABCDEF$ is 15, find the area of $A'B'C'D'E'F'$.

PROBLEMS (B)

1. Corresponding sides of two similar triangles are $3\frac{1}{2}$ and 11. Find the ratio of their areas.

2. A circular metal disk 9 inches in diameter weighs $3\frac{1}{2}$ pounds. What is the weight of a disk cut from the same plate if its diameter is 15 inches?

3. Two gasoline tanks made of the same thickness of metal are similar in shape, that is the ratio of the diameters is the same as the ratio of lengths. One is 30 inches long; and the other, 42 inches long. Find the ratio of their weights when empty.

SIMILAR SOLIDS

274. Volumes of Similar Solids. — The volume of a one-inch cube is one cubic inch. The volume of a two-inch cube is $2 \times 2 \times 2 = 2^3 = 8$ cubic inches, while the volume of a three-inch cube is $3 \times 3 \times 3 = 3^3 = 27$ (cubic inches).

It is easy to see that if all three dimensions of a rectangular box are doubled, the volume of the box is multiplied by 8, and if the dimensions are trebled, the volume is multiplied by 27.

The following is an important property of all similar solids.

The ratio of the volume of similar solids equals the cube of the ratio of their corresponding lines.

Thus, the ratio of the volumes of two spheres equals the cube of the ratio of their radii, or of their diameters. Similarly, the weights of two animals of similar shape are in the same ratio as the cubes of their lengths or heights. Again, the capacities of two ships of the same shape are in the same ratio as the cubes of their lengths.

PROBLEMS (B)

1. If two corresponding lines of similar solids are in the ratio $\frac{3}{1}$, find the ratio of their volumes.

2. If two trees are similar in shape, but one is twice the height of the other, find the ratio of their volumes.

3. Find the ratio of the weights of the projectiles fired by a four-inch and a sixteen-inch gun, if the projectiles are similar in shape.

4. Find the ratio of the weights of two similar blocks of stone, if the ratio of their lengths is $\frac{2}{3}$.

5. If the bullets for two rifles are of the same shape, find the ratio of their weights if the ratio of their diameter is $\frac{5}{6}$.

6. Find the ratio of the weights of the shells fired by 11 and 13.5 inch guns, assuming that the shells are of the same shape.

7. The ratio of the altitudes of two similar cones is 2:3. Find the ratio of their volumes.

8. If the velocity from a pipe 2″ in diameter is 2 feet a second, how many hours will it take to fill a tank $6' \times 3' \times 4'$? If the diameter of the pipe were doubled, how would it affect the time?

275. Composite Units of Measure. — There are some quantities for the measurement of which composite units are used. Thus, the amount of freight carrying done by a railway is measured in ton miles. That is, one ton carried 100 miles is counted as 100 ton miles, as is also 100 tons carried one mile.

The *passenger mile* is used to measure the amount of passenger carrying done by a railway.

The *train mile* is used to measure the amount of train service rendered by a road.

The *foot pound* is used to measure amount of energy expended. One foot pound is just sufficient to lift one pound weight one foot against the force of gravity.

The *horse power* is used to measure the rate at which a machine is capable of supplying energy. One horse power is a force sufficient to lift 33,000 lb. one foot in one minute.

The *acre inch* is used to measure the amount of water used in irrigation.

MISCELLANEOUS PROBLEMS (C)

1. A wagon is 12 feet long. How high must 4-foot wood be piled upon it to make $1\frac{1}{3}$ cords?

2. If 31,000 pounds of plaster cost $146.25, find the cost per ton.

3. A field is 47.5 rods long, and 38.4 rods wide. How many bushels of corn will it produce, allowing 55 bushels to the acre?

4. What part of a day is 5 hours 37 minutes 30 seconds? Reduce to a decimal accurate to three places.

5. How many heaped bushels will a crib contain whose base is 12 feet square, and whose height is 8 feet? (See page 143.)

6. What is the length of a garden 88 feet wide, if it contains $\frac{1}{3}$ of an acre?

7. A's farm is 240 rods wide. If he sells 18 acres off one end, how much shorter is his farm than it was before?

8. The ice on a circular pond is 14 inches thick. If the diameter of the pond is 1000 feet, how many cubic feet of ice are there on it?

LONGITUDE AND TIME

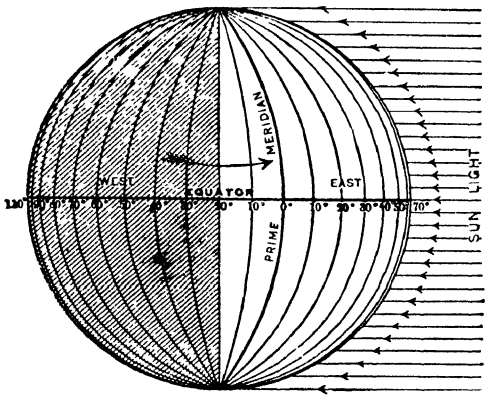
276. Rotation of the Earth. — The earth rotates on its axis from west to east once in twenty-four hours.

The ends of the axis are at two points on the earth's surface called the *poles*.

277. The Equator. — The great circle on the earth halfway between the poles is called the *equator*.

The equator is divided into 360°. (See page 152)

278. The Meridians. — The semicircles extending from pole to pole are called *meridians*.



279. Longitude. — We know from geography that a place on the earth's surface is located by giving its latitude and longitude.

The longitude of the meridian through the observatory at Greenwich (near London, England), is regarded as zero.

The meridian through Greenwich is called the *prime meridian*.

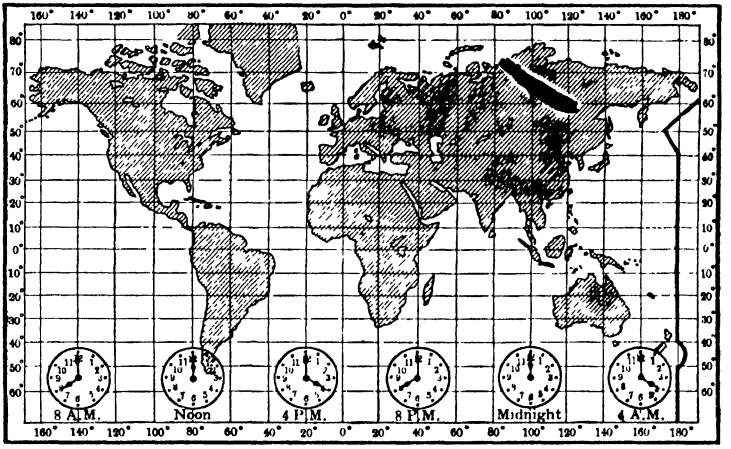
The longitude of any other meridian is so many degrees east or west of Greenwich. Thus, the longitude of New York City is 74° *west* of *Greenwich*, because the meridian through New York meets the equator 74° west from the point at which the meridian through Greenwich meets it. The longitude of Berlin is 13° 24′ *east* of *Greenwich* for a similar reason.

280. Difference in Longitude and Difference in Time. — *The true solar or local time* at any one place is determined by the position of the sun. When the meridian of the place passes directly under the sun, it is approximately noon at that place.

Since the earth makes one complete rotation in 24 hours, it follows that it rotates $\frac{1}{24}$ of 360° or 15° in one hour. Hence of two places whose longitude differs by 15°, one will have noon an hour later than the other. That is, the clocks at these places will differ by one hour. Similarly, if the longitudes differ by 30°, the clocks differ by 2 hours; and if the longitudes differ by 90°, the clocks differ by 6 hours.

The earth rotates toward the east. Hence if a place A is east of a place B, noon will come at A before it comes at B.

Thus the meridian of Greenwich has noon 6 hours earlier than a place that is 90° west.



From the figure it is obvious that if two places are on the same side of the prime meridian, the difference of their longitudes is found by subtracting the longitude of one from that of the other, and if the places are on opposite sides of the prime meridian, the difference of longitude is found by adding their longitudes.

281. First Step in Finding Difference in Time. — The first step in finding the difference in time of two places when their longitudes are given is to find their difference in longitude.

Longitudes of Certain Places.

Alexandria (Egypt)	29° 53′ E.	Liverpool	3° W.
Berlin	13° 24′ E.	Minneapolis	93° 15′ W.
Boston	71° 31′ W.	New York	74° W.
Singapore	103° 51′ E.	Paris	2° 20′ E.
Chicago	87° 36′ W.	Moscow	37° 34′ E.
Cleveland	81° 42′ W.	San Francisco	122° 25′ W.

ORAL EXERCISES (A)

Find the difference in longitude between each of the following pairs of places.

1. New York, Chicago
2. Boston, San Francisco
3. Cleveland, Paris
4. Berlin, Singapore
5. Moscow, Liverpool
6. Minneapolis, Alexandria
7. NewYork, Berlin
8. Paris, Alexandria
9. Chicago, Singapore
10. Berlin, San Francisco
11. Singapore, Moscow
12. Liverpool, Paris

282. Second Step in Finding Difference in Time. — The second step in finding the difference in time at two places when their longitudes are given is to find the time which corresponds to their difference in longitude.

We have seen that the earth turns 15° in one hour.

Hence we get the following:

Difference in Longitude	Difference in Time
15°	1 hr.
1°	4 min.
15′	1 min.
1′	4 sec.

Example. What is the difference in time between two places if their longitudes differ by 49° 43′?

Solution. 45° corresponds to 3 hr., 4° corresponds to 16 min., 30′ corresponds to 2 min., 13′ corresponds to 52 sec. Hence, 49° 43′ in longitude corresponds to 3 hr. 18 min. 52 sec. in time.

ORAL EXERCISES (A)

1. When it is noon at Greenwich, what time is it 15° west of Greenwich? 15° east? 30° west? 45° west? 60° west? 90° west?

2. When it is 8 A.M. 90° west, what time is it 75° west? 60° west? 105° west?

3. When it is 2 P.M. 75° west, what time is it at Greenwich? At 30° east?

Example. The difference in time of two places is 2 hr. 55 min. Find the difference in longitude.

Solution. 2 hours corresponds to 30° of longitude.
52 minutes corresponds to 13° of longitude.
3 minutes corresponds to 45′ of longitude.
Hence, 2 hr. 55 min. corresponds to 43° 45′ longitude.

PROBLEMS (A)

1. On board ocean steamers, the clocks are set daily to true local time. A passenger whose watch keeps New York time (75° west), is 3 hours 43 minutes behind ship's time. In what longitude is the ship?

2. What is the difference in time between Moscow and Chicago?

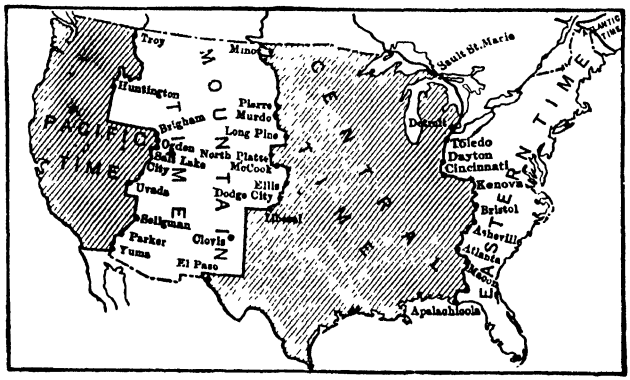
3. When it is 8 A.M. in San Francisco, what time is it in Paris?

4. When it is 2 P.M. in Alexandria, Egypt, what time is it in Boston?

5. A passenger on an ocean steamer whose watch keeps Greenwich time is 4 hr. 12 min. ahead of ship's time. In what longitude is the ship?

6. A passenger sets his watch with the clocks of the ship. Six days later he finds that his watch is 1 hour 47 minutes ahead of ship's time. How many degrees east or west has the ship traveled?

283. Standard Time. — Clearly, no two places have the same local time unless they are on the same meridian. For the sake of convenience, however, the same time is used over large areas. The local time of Greenwich is used all over Britain, and in Holland and Belgium. In Germany, Italy, Switzerland, and Scandinavia, the time of 15° east is used. The United States is divided into four time belts. The eastern part, or New England section, has the time of 75° west. Central time is the time of 90° west. Rocky Mountain time is the time of 105° west, and Pacific time is the time of 120° west. The very eastern extremity of Canada has what is called Atlantic time, which is the time of 60° west.



STANDARD TIME BELTS

This time is called *standard time.*

Since each railway changes its time at the end of a division, the boundaries of the time belts are very jagged. In the case of the mountain time belt the standard meridian is considerably east of the center of the belt.

EXERCISE (A)

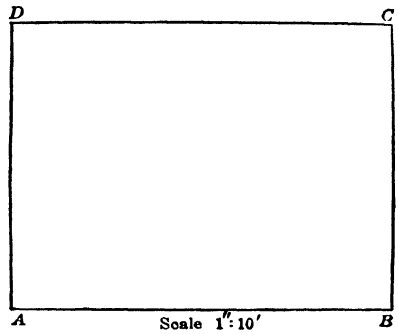
Find the difference between true solar time and standard time at each of the following places:

Berlin, Boston, Chicago, Cleveland, Liverpool, Minneapolis, New York, San Francisco.

DRAWING TO SCALE

284. Definition of Scale. — Drawings are frequently made so that a certain unit of length such as one inch in the drawing will represent a definite distance, such as one foot, or 10 feet, or one mile in the object represented by the drawing. If one inch in the drawing represents 10 feet or 120 inches in the object, then the drawing is said to be to *the scale* 1:120. This is read "scale one to one hundred twenty."

This scale is also written 1″:10′, which is read "scale: 1 inch to 10 feet."



PROBLEMS (A)

1. What are the dimensions of the room represented by this figure?

2. The top of a certain desk is 5 feet long and $3\frac{1}{2}$ feet wide. Make a drawing of the top of this desk to the scale 1:12.

3. Make a drawing of the same desk to the scale 1″:2′. Express this scale without using the words "inches" "feet" in the statement.

4. What is the straight line distance from A to C in the room represented by the figure above?

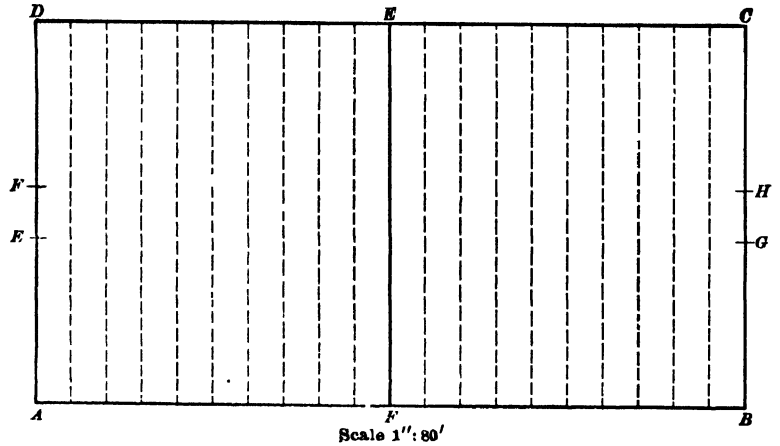
5. Measure a room in the school building with care, and make a drawing to scale representing it. Enter the scale on the drawing.

6. Measure the dimensions of a room in your home, and make a drawing to scale representing it.

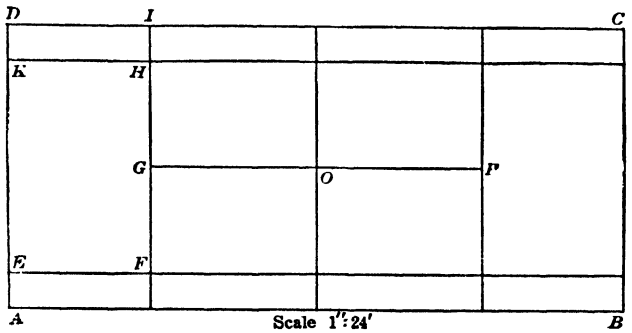
7. Determine the dimensions of a football field by measuring the drawing given on page 212 and then computing them. The points G H and E F mark the goal posts. Determine approximately the distance between each pair of goal posts.

8. Draw a football field to the scale 1″:30′.

9. Express the scale of Example 8 without using the words inches and feet.



10. Determine the dimensions of a tennis court by measuring the drawing given here and then computing them. Give the lengths of the lines *AB*, *BC*, *GO*, *FH*, *HI*.



11. A baseball diamond is a square ninety feet on each side. The pitcher's box is in a straight line between the home plate and second base, 60 feet, 6 inches from the home plate. Draw a baseball diamond to the scale 1″:10′.

MISCELLANEOUS WORK (A)

1. Five loads of corn weigh as follows: 3120, 2840, 3486, 2952, 2662 pounds. The tares are: 1140, 1080, 1250, 1168, 1286 pounds. Find the value of the corn at \$1.08 per shelled bushel.

2. A commission merchant bought 1960 pounds of flour at \$11.60 a barrel; 10 barrels of salt at $\frac{3}{4}¢$ a pound; 1750 pounds of pork at \$20.50 a barrel. What did the whole cost him?

3. Divide 17 tons 875 pounds of coal equally among 9 families.

4. A grocer mixes two kinds of tea as follows: 676 pounds 10 ounces Salada with 590 pounds 12 ounces English breakfast. He sells from the mixture 457 pounds. What is the balance worth at $82¢$ a pound?

5. At \$12.50 a ton find the value of $\frac{9}{16}$ of a ton.

6. What part of a pound avoirdupois is a pound troy? A pound apothecaries'?

7. If one ring weighs 10 grains, how many such rings can be made from a pound of gold?

8. From $\frac{5}{6}$ of a long ton take $\frac{7}{8}$ of a long hundredweight.

9. Find the total value of

3650 lb. of wheat at \$2.15 per bushel.
3780 lb. of wheat at \$2.20 per bushel.
3560 lb. of wheat at \$2.18 per bushel.
3740 lb. of wheat at \$2.25 per bushel.
3820 lb. of wheat at \$2.25 per bushel
3490 lb. of wheat at \$2.28 per bushel.

10. Find the total value of

2160 lb. of hay at \$16.50 per ton.
2230 lb. of hay at \$16.50 per ton.
2060 lb. of hay at \$16.50 per ton.
2140 lb. of hay at \$16.75 per ton.
2310 lb. of hay at \$16.75 per ton.
2420 lb. of hay at \$18.00 per ton.
2360 lb. of hay at \$18.00 per ton.

ORAL PROBLEMS (B)

1. Find the number of board feet in 100 pieces of one-inch timber 16 feet long and 3 inches wide.

2. Find the number of board feet in each of the following:

100 pieces of one-inch lumber, 12 feet long and 4 inches wide.
100 pieces of one-inch lumber, 12 feet long and 6 inches wide.
100 pieces of one-inch lumber, 12 feet long and 8 inches wide.
100 pieces of one-inch lumber, 16 feet long and 9 inches wide.
100 pieces of one-inch lumber, 12 feet long and 10 inches wide.
100 pieces of $2'' \times 4''$ lumber, 12 feet long.
100 pieces of $3'' \times 4''$ lumber, 16 feet long.
100 pieces of $2'' \times 6''$ lumber, 10 feet long.

3. At 28 bricks per square foot (6 two-brick wall), how many M of brick are required for a wall containing 500 square feet?

4. At \$100 per acre what is the cost of a quarter section of land?

5. How many acres are there in a field 60 rods by 80 rods. At $1\frac{1}{2}$ bushels per acre how many bushels of wheat will be required to seed this field?

6. Our long hundredweight contains how many pounds?

7. Our long ton contains how many pounds?

8. By how many pounds do a long and a short ton differ?

9. How many feet are there in one furlong? in 3 furlongs? in 5 furlongs?

10. What is the unit of speed used in measuring the speed of ships? About how many miles per hour is this unit?

11. Why do we have leap year every four years? Is there any exception to this rule for leap years? Why?

TOPIC FOR DISCUSSION

Compare in general and metric system of measures with the system that we use. Try to find out where the metric system was invented, and by what nations it has been adopted. What nations do not use it?

PROBLEMS (B)

1. If a statute mile is 5280 feet and a nautical mile is 6080 feet, by what number must a number of nautical miles be multiplied to convert it into statute miles? By what number must a number of statute miles be multiplied to convert it into nautical miles?

2. From a field 50 rods by 120 rods a farmer harvested 1756 bushels of corn. How many bushels per acre did he get? Find result to the nearest tenth of a bushel.

3. A pile of 4-foot pieces of timber is 12 feet wide, 140 feet long, and $6\frac{1}{2}$ feet high. At \$12.50 per cord what is the value of this pile?

4. At \$1.45 per bushel what is the value of a truck-load of shelled corn that weighs 8460 pounds net.

5. A truck loaded with oats weighs 7490 pounds and the empty truck weighs 3240 pounds. At 72 cents a bushel what is the value of this load?

6. A concrete walk is 5 feet wide and 147 feet long. At \$4.15 a square yard what is the cost of this walk?

7. An excavation 47 feet by 64 feet is $5\frac{1}{2}$ feet deep. At \$2.35 per cubic yard what is the cost?

8. If one rod is $16\frac{1}{2}$ feet, how many square feet are there in a square rod? How many square feet are there in one acre?

9. A lot is 140 feet wide and $252\frac{1}{2}$ feet deep. By how much does this lot differ from one acre?

10. One liquid quart contains $57\frac{3}{4}$ cu. in. and one dry quart contains $67\frac{1}{5}$ cu. in. A given number of liquid quarts must be multiplied by what to change to dry quarts? A given number of dry quarts must be multiplied by what to change to liquid quarts? Find each answer correct to two decimal places.

CHAPTER XX

SPECIFIC GRAVITY

285. Certain objects will float on water while others sink in it. This is because some objects are lighter than water, while others are heavier.

Thus nearly all kinds of wood are lighter than water and hence float on it, while nearly all rocks and metals are heavier than water and hence sink in it.

286. Specific Gravity. — The ratio of the weight of a certain volume of a substance to the weight of the same volume of water is the *specific gravity* of the substance.

Thus, one cu. ft. of cork weighs 15 lb. while one cu. ft. of water weighs 62.5 lb. Hence the specific gravity of cork is $\frac{15}{62.5} = .24$ (approximately).

Again, one cu. ft. of steel weighs 489.6 lb. Hence its specific gravity is $\frac{489.6}{62.5} = 7.834$ (approximately).

WRITTEN EXERCISES

Find the specific gravity of each of the following, the weight of one cubic foot being given.

1.	Aluminum	166.5	**2.**	Sea Water	64.375
3.	Bronze	552.	**4.**	Hemlock	24.
5.	Nickel	548.7	**6.**	Mahogany	51.
7.	Silver	655.1	**8.**	Poplar	30.
9.	Pure Alcohol	49.6	**10.**	Boxwood	70.
11.	Ammonia	55.7	**12.**	Walnut	36.
13.	Milk	64.50	**14.**	Nitric Acid	76.06
15.	Ice	57.50	**16.**	Vinegar	67.5
17.	Salt	133.12	**18.**	Turpentine	54.4

TABLE OF SPECIFIC GRAVITY

Alcohol, Pure	.794	Larch (wood)	.56
Aluminum	2.67	Lard	.947
Asphaltum	1.39	Lead	11.38
Beech (wood)	.73	Limestone	2.90
Birch (wood)	.65	Mahogany (wood)	.81
Box (wood)	1.12	Maple (wood)	.68
Brass	8.40	Marble	2.72
Brick, soft	1.60	Milk	1.032
Brick, common	1.79	Mercury	13.58
Brick, hard	2.	Nitric Acid	1.217
Brick, pressed	2.16	Oak, Live	1.11
Bronze	8.853	Oak, Red	.74
Butter	.942	Oak, White	.77
Cedar (wood)	.62	Oil, Olive	.92
Chestnut (wood)	.56	Oil, Palm	.97
Cement, Portland	3.10	Oil, Petroleum	.88
Copper	8.853	Pine, White	.45
Cork	.24	Pine, Yellow	.61
Diamond	3.55	Platinum	21.5
Ebony (wood)	1.23	Quartz	2.64
Elm (wood)	.61	Salt	2.130
Fir (wood)	.59	Sandstone	2.35
Gold	19.258	Silver	10.505
Granite	2.60	Steel	7.834
Hemlock (wood)	.38	Sugar	1.6
Hickory	.77	Tin	7.35
Honey	1.456	Tungsten (wolfram)	18.6
Iridium	23.	Turpentine	.87
Iron, cast	7.218	Vinegar	1.08
Iron, wrought	7.70	Walnut (wood)	.58
Ice	.920	Willow	.54
Ivory	1.870	Zinc	7.0

Which of these substances will float on water? which substance is the heaviest? the lightest?

PROBLEMS (B)

1. A load of white oak lumber weighs 3640 lb. How many cubic feet of lumber are there in the load? how many board feet, supposing that all pieces are at least an inch thick?

2. A piece of steel is $6' \times 4'' \times 2''$. Find its weight.

3. Find the weight of a block of granite $4' \times 2' \times 1\frac{1}{2}'$.

4. Find the weight of a block of ice $4' \times 2\frac{1}{2}' \times 10''$.

5. The ice chest of a refrigerator is $22'' \times 16'' \times 12''$. How many pounds of ice will it hold, allowing 2 inches in each dimension for irregularities in the ice?

Suggestion. Find the weight of a regular block of ice $20'' \times 14'' \times 10''$.

6. What would be the weight of a cord of white pine if the wood were a solid mass? If a cord of pine weighs 2000 lb., what fraction of the space in the cord is occupied by the wood?

7. What is the weight of a white oak beam $16' \times 8'' \times 6''$?

8. What is the weight per linear foot of a cylindrical wrought iron rod, $1\frac{1}{4}''$ in diameter?

9. What is the weight of one gallon of milk?

10. A milk can weighs 114 lb. when filled with milk and 31 lb. when empty. How many gallons does it hold?

11. A tank contains 24,640 lb. of turpentine. How many barrels, at 31 gallons each, does it contain?

DRILL OF FUNDAMENTALS (A)

Copy from dictation, divide, and check. Find quotients and exact remainders.

1. $1821\overline{)2727800}$	**8.** $273\overline{)19372000}$	**15.** $874\overline{)1974000}$
2. $672\overline{)9148500}$	**9.** $982\overline{)1782900}$	**16.** $415\overline{)3759700}$
3. $727\overline{)7643200}$	**10.** $403\overline{)15450300}$	**17.** $7341\overline{)5370400}$
4. $1276\overline{)7947000}$	**11.** $790\overline{)4526000}$	**18.** $4107\overline{)3415000}$
5. $2346\overline{)8919000}$	**12.** $824\overline{)42192000}$	**19.** $9145\overline{)7243000}$
6. $742\overline{)297600}$	**13.** $852\overline{)3792642}$	**20.** $476\overline{)3175600}$
7. $941\overline{)567200}$	**14.** $278\overline{)893000}$	**21.** $591\overline{)8716490}$

CHAPTER XXI

PERCENTAGE

287. Percentage in Business. — Next to the four fundamental operations, percentage is the most important subject in the arithmetic of business. Its manifold applications to business occupy nearly one half of this book.

288. Meaning of Per Cent. — It has become customary to express a large number of fractions as *hundredths*, or *per cents*.

The words per cent are derived from the Latin words *per* and *centum*, meaning "*in the hundred*," or "*hundredths*."

The symbol for per cent is %.

289. Definition. — *Percentage* is the name used for calculations in which hundredths, or per cents, are used as the basis of comparison.

290. Base, Rate, Percentage. — The principal numbers involved in percentage are the *base*, the *rate*, and the *percentage*.

The *base* is the number of which a certain number of per cents, or hundredths, are taken.

Thus, in "5% of 200 equals 10," 200 is the base.

The *rate* is the number of hundredths or per cents taken.

Thus, in "5% of 200 equals 10," 5 per cent is the rate.

The *percentage* is the result obtained by taking a certain per cent of a number. (Note the two different meanings of "percentage.")

Thus, in "5% of 200 equals 10," 10 is the percentage.

The *amount* is the base plus the percentage.

The *difference* is the base less the percentage.

Per cent has come into such general use that 5% means more to the average individual than the equivalent common fraction $\frac{1}{20}$. We even use fractional per cents. Thus, we say that 3.2% of a certain sample of milk is butter fat, while we would never say that the milk contains $\frac{4}{125}$ butter fat.



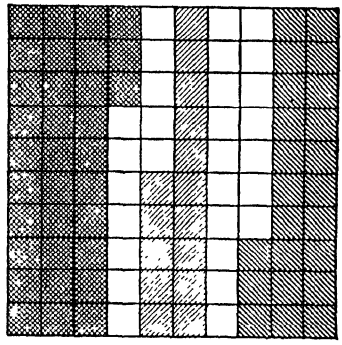
Devaney

WORKING ON COMMISSION

291. Percentage Involves only Decimal Fractions. — Since percentage involves fractions whose denominators are 100, it follows that no new theory of arithmetic is involved. It will be useful, however, to gain a direct acquaintance with magnitudes represented by certain per cents.

ORAL EXERCISES (A)

1. State how many per cent of the whole figure below are shaded thus: ▩.



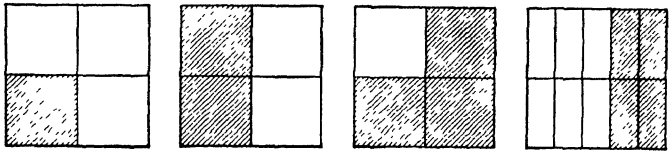
2. How many per cent of the whole figure are shaded thus: ▩?

3. How many per cent are shaded thus: ▨?

4. How many per cent are white?

5. How many per cent of the whole figure are there all together?

6. In the first little square below, how many per cent of the whole are shaded? How many per cent are white? How many per cent are there all together?



7. In the second little square, how many per cent are shaded? How many per cent are white? How many per cent are there all together?

8. In the third little square, how many per cent are shaded? How many per cent are white? How many per cent are there all together?

9. In the fourth little square, how many per cent are shaded? How many per cent are white? How many per cent are there all together?

10. In a class there are 16 boys and 16 girls. How many per cent of the class are girls?

292. Decimals Read as Per Cents. — Since per cent means hundredths, it follows that any decimal may be read as per cent. A number of per cents such as 25% may mean .25, or $\frac{1}{4}$ of some other number, or it may simply represent .25 = $\frac{1}{4}$, just as any other fraction.

Example 1. Express .31 as per cent.

Solution. The number of hundredths or per cent in a number is found by moving the decimal point two places to the right.

Thus, .31 = 31%.

Similarly, .245 = 24.5%.

293. Reduction of Common Fractions to Per Cents. — A common fraction may be reduced to per cents by reducing it to a decimal fraction and then moving the decimal point two places to the right.

Thus, $\frac{1}{4}$ = .25 = 25%,

$\frac{1}{8}$ = .125 = 12.5%,

and $\frac{2}{3}$ = .66$\frac{2}{3}$ = 66$\frac{2}{3}$%.

ORAL EXERCISES

Reduce the following to per cents.

1. .15	**5.** .04	**9.** .416	**13.** .842
2. .72	**6.** .09	**10.** .271	**14.** .261
3. .21	**7.** .7	**11.** .8	**15.** .7
4. .6	**8.** .39	**12.** .09	**16.** .091

Read each of the following as a decimal.

17. 5%	**22.** 24%	**27.** 45%	**32.** 90%
18. 9%	**23.** 30%	**28.** 3%	**33.** 70%
19. 12%	**24.** 76%	**29.** 7%	**34.** 50%
20. 14%	**25.** 75%	**30.** 91%	**35.** 86%
21. 21%	**26.** 80%	**31.** 64%	**36.** 31%

Reduce the following to per cents.

37. $\frac{1}{2}$	**42.** $\frac{2}{3}$	**47.** $\frac{1}{8}$	**52.** $\frac{1}{16}$
38. $\frac{1}{3}$	**43.** $\frac{5}{6}$	**48.** $\frac{1}{7}$	**53.** $\frac{3}{16}$
39. $\frac{1}{4}$	**44.** $\frac{3}{4}$	**49.** $\frac{3}{8}$	**54.** $\frac{5}{16}$
40. $\frac{1}{5}$	**45.** $\frac{2}{5}$	**50.** $\frac{5}{8}$	**55.** $\frac{7}{16}$
41. $\frac{1}{6}$	**46.** $\frac{3}{5}$	**51.** $\frac{7}{8}$	**56.** $\frac{9}{16}$

294. Aliquot Parts of 100. — On page 114 we studied the aliquot parts of one hundred. We will now express these as per cents.

Halves Thirds Fourths Fifths	Per Cents	Sixths Eighths Twelfths	Per Cents	Fifteenths Sixteenths	Per Cents
$\frac{1}{2}$	50%	$\frac{1}{8}$	$16\frac{2}{3}\%$	$\frac{1}{15}$	$6\frac{2}{3}\%$
$\frac{1}{3}$	$33\frac{1}{3}\%$	$\frac{2}{8}$	$83\frac{1}{3}\%$	$\frac{2}{15}$	$13\frac{1}{3}\%$
$\frac{2}{3}$	$66\frac{2}{3}\%$	$\frac{1}{8}$	$12\frac{1}{2}\%$	$\frac{4}{15}$	$26\frac{2}{3}\%$
$\frac{1}{4}$	25%	$\frac{3}{8}$	$37\frac{1}{2}\%$	$\frac{1}{16}$	$6\frac{1}{4}\%$
$\frac{3}{4}$	75%	$\frac{5}{8}$	$62\frac{1}{2}\%$	$\frac{3}{16}$	$18\frac{3}{4}\%$
$\frac{1}{5}$	20%	$\frac{7}{8}$	$87\frac{1}{2}\%$	$\frac{5}{16}$	$31\frac{1}{4}\%$
$\frac{2}{5}$	40%	$\frac{1}{12}$	$8\frac{1}{3}\%$	$\frac{7}{16}$	$43\frac{3}{4}\%$
$\frac{3}{5}$	60%	$\frac{5}{12}$	$41\frac{2}{3}\%$	$\frac{9}{16}$	$56\frac{1}{4}\%$
$\frac{4}{5}$	80%	$\frac{7}{12}$	$58\frac{1}{3}\%$	$\frac{11}{16}$	$68\frac{3}{4}\%$
		$\frac{11}{12}$	$91\frac{2}{3}\%$	$\frac{13}{16}$	$81\frac{1}{4}\%$
				$\frac{15}{16}$	$93\frac{3}{4}\%$

Other common fractions frequently reduced to per cents are $\frac{1}{20} = 5\%$, $\frac{1}{25} = 4\%$, $\frac{1}{40} = 2\frac{1}{2}\%$, $\frac{1}{60} = 1\frac{2}{3}\%$, $\frac{1}{80} = 1\frac{1}{4}\%$, $\frac{1}{9} = 11\frac{1}{9}\%$, $\frac{1}{11} = 9\frac{1}{11}\%$.

The following should be memorized fully.

$\frac{1}{2} = 50\%$, $\frac{1}{3} = 33\frac{1}{3}\%$, $\frac{1}{4} = 25\%$, $\frac{1}{5} = 20\%$, $\frac{1}{6} = 16\frac{2}{3}\%$, $\frac{1}{8} = 12\frac{1}{2}\%$, $\frac{1}{12} = 8\frac{1}{3}\%$, $\frac{1}{16} = 6\frac{1}{4}\%$.

The equivalent in per cent of such fractions as $\frac{5}{12}$ or $\frac{3}{16}$ may be found at once from these.

Thus, $\frac{5}{12} = 5 \times 8\frac{1}{3}\% = 41\frac{2}{3}\%$.

ORAL EXERCISES (A)

1. In this manner find the per cent equivalents of all twelfths.

2. In the same manner find the per cent equivalents of all eighths; of all sixteenths.

3. Find the per cent equivalents of all fifteenths.

4. Find the per cent equivalents of $\frac{3}{20}$, $\frac{7}{20}$, $\frac{9}{20}$.

5. Find the per cent equivalents of $\frac{2}{25}$, $\frac{3}{25}$, $\frac{4}{25}$, $\frac{6}{25}$, $\frac{7}{25}$.

6. Find the per cent equivalents of $\frac{3}{40}$, $\frac{7}{40}$, $\frac{9}{40}$, $\frac{11}{40}$.

7. Find the per cent equivalents of $\frac{7}{60}$, $\frac{11}{60}$, $\frac{13}{60}$.

8. Find the per cent equivalents of $\frac{3}{80}$, $\frac{7}{80}$, $\frac{9}{80}$.

295. A Number Is 100% of Itself. — Since $\frac{100}{100} = 1$, it follows that every number is 100% of itself. That is, 2 = 100% of 2, 37 = 100% of 37, and so on for any number whatever.

Similarly, $\frac{200}{100} = 2$. Hence, 200% of any number is twice the number; 300% of any number is three times the number; and so on.

ORAL EXERCISES (A)

How many per cent of one is each of the following?

1. 2	**4.** 3	**7.** $1\frac{1}{5}$	**10.** $2\frac{3}{4}$
2. 4	**5.** $1\frac{1}{2}$	**8.** $1\frac{2}{5}$	**11.** $3\frac{4}{5}$
3. 6	**6.** $2\frac{1}{4}$	**9.** $1\frac{3}{4}$	**12.** $3\frac{3}{5}$

Read each of the following as per cents.

13. 1.12	**16.** 2.5	**19.** 3.9	**22.** 5.25
14. 1.24	**17.** 2.4	**20.** 2.75	**23.** 4.70
15. 1.91	**18.** 7.21	**21.** 4.50	**24.** 3.67

Read each of the following as decimals.

25. 160%	**28.** 42%	**31.** 5%	**34.** $5\frac{1}{2}$%
26. 240%	**29.** 210%	**32.** 120%	**35.** $2\frac{1}{2}$%
27. 360%	**30.** 760%	**33.** 306%	**36.** $6\frac{3}{4}$%

37. Find 1% of 645, 271, 986, 8470.

Suggestion. Divide by 100 by moving the decimal point two places to the left.

38. Find 2% of 100, of 200, of 700, of 1600.

39. Find 3% of 100, of 300, of 400, of 600, of 1000.

40. Find 4% of 100, of 200, of 500, of 900.

41. Find 5% of 100, of 200, of 700, of 1000.

42. Find 10% of 100, of 1200, of 8940, of 76582.

Suggestion. $10\% = \frac{1}{10}$. Hence to find 10% of a number, move the decimal point one place to the left. Prefix a zero if necessary.

43. Find 10% of 768, of 35.7, of 89.7.

44. Find 10% of 4.98, of 7.67, of 1.246, of 8.93.

45. Find 10% of .72, of .456, of .192, of .216.

46. Find 10% of .041, of .0924, of .0192, of .0217.

296. Decimals and Per Cents. — Per cents are frequently given to one tenth of one per cent and sometimes even to one hundredth of a per cent.

Example 1. Express 45% as a common fraction in its lowest terms.

Solution. $45\% = .45 = \frac{45}{100} = \frac{9}{20}$.

Example 2. Express $6\frac{1}{4}\%$ as a decimal.

Solution. $6\frac{1}{4}\% = 6.25\% = .0625$.

Example 3. How many per cent of one are $\frac{5}{7}$?

Solution. $\frac{5}{7} = .714(3) = 71.4\%$, which is correct to the nearest tenth of one per cent. The fraction is reduced to a four-place decimal, and the decimal point moved two places to the right. To be sure we have the nearest approximation to three places, it is necessary to run the decimal to four places. If the quotient were .7146, then .715 would be the nearest approximation to three places, and 71.5% would be the required answer. See page 50.

ORAL EXERCISES (A)

Express each of the following as a decimal.

1. $2\frac{1}{2}\%$	4. $6\frac{1}{2}\%$	7. $5\frac{3}{4}\%$	10. $1\frac{3}{4}\%$
2. $4\frac{1}{2}\%$	5. $5\frac{1}{4}\%$	8. $6\frac{3}{4}\%$	11. $3\frac{3}{4}\%$
3. $4\frac{1}{4}\%$	6. $12\frac{1}{2}\%$	9. $2\frac{3}{4}\%$	12. $4\frac{3}{4}\%$

WRITTEN EXERCISES (A)

Express each of the following in per cent. Find each result to the nearest tenth of one per cent. Use short division.

1. $\frac{1}{7}$	5. $\frac{5}{7}$	9. $\frac{4}{9}$	13. $\frac{1}{8}$
2. $\frac{3}{7}$	6. $\frac{6}{7}$	10. $\frac{5}{9}$	14. $\frac{3}{8}$
3. $\frac{2}{7}$	7. $\frac{1}{9}$	11. $\frac{7}{9}$	15. $\frac{5}{8}$
4. $\frac{4}{7}$	8. $\frac{2}{9}$	12. $\frac{8}{9}$	16. $\frac{7}{8}$

In some kinds of examples results are carried to hundredths of a per cent, or even to thousandths of a per cent. The extent to which they are carried depends entirely upon the character of the problems involved.

Express each of the following as a fraction in its lowest terms.

17. 25%	21. 72%	25. 12%	29. 75%
18. 42%	22. 65%	26. 8%	30. 40%
19. 50%	23. 84%	27. 4%	31. 80%
20 30%	24. 6%	28. 5%	32. 90%

FINDING THE PERCENTAGE

297. Problem. — Given the base and rate to find the percentage.

Example 1. Find the percentage when the base is 460 and the rate is $5\frac{1}{2}\%$. That is, find $5\frac{1}{2}\%$ of 460.

Solution. $5\frac{1}{2}\% = .055$.
Hence $5\frac{1}{2}\%$ of $460 = .055 \times 460 = 25.3$.
In this case 460 is the base; $5\frac{1}{2}\%$ the rate; and 25.3 the percentage.

460
.055
2300
2300
25.300

Rule. *To find the percentage, express the rate as a decimal, and multiply the base by it.*

For the method to be used when the rate contains more difficult fractions, see examples below.

Example 2. Find $7\frac{1}{8}\%$ of 5260 correct to two decimal places.

Solution. To find $\frac{1}{8}\%$ of 5260 divide 52.60 by 8. The result found is exactly $7\frac{1}{8}\%$ of the given number.

5260
.07
7% = 368.20
$\frac{1}{8}$ of 1% = 6.575
374.775
or 374.78

Since 374.775 is exactly halfway between 374.77 and 374.78, one of these numbers is as near the correct result as is the other. In such cases it is customary to take the larger number. See page 50.

Example 3. Find $9\frac{3}{4}\%$ of 4926 correct to two decimal places.

Solution. First multiply by .09 and then by .03, putting the last product, 147.78, below the first. Divide the last product by 4 and then add result to the first product. After dividing by 4 draw a line through the second product to show that it is not to be added. The required result is 480.29.

4926
$.09\frac{3}{4}$
443.34
~~4)147.78~~
36.945
480.285 or 480.29

Example 4. Find $6\frac{5}{8}\%$ of 79438 correct to two decimal places.

Solution. The solution is arranged as follows:

$$79438 \times .06\tfrac{5}{8}$$
$$= 4766.28 + 496.4875$$
$$= 5262.7675 \text{ or } 5262.77$$

WRITTEN EXERCISES (A)

Find the percentage in each of the following.

	Base	Rate		Base	Rate		Base	Rate
1.	264	8%	8.	1825	$12\frac{1}{2}\%$	15.	7500	$62\frac{1}{2}\%$
2.	790	$5\frac{1}{2}\%$	9.	790	$8\frac{1}{4}\%$	16.	875	250%
3.	850	$4\frac{3}{4}\%$	10.	2450	$30\frac{1}{4}\%$	17.	1920	240%
4.	1600	$10\frac{3}{4}\%$	11.	4650	125%	18.	1450	80%
5.	950	$16\frac{2}{3}\%$	12.	2640	150%	19.	2500	92%
6.	1800	25%	13.	3900	75%	20.	9600	$60\frac{3}{4}\%$
7.	720	24%	14.	8800	50%	21.	2900	$45\frac{1}{2}\%$

298. Finding Percentage when Rate Is an Aliquot Part of 100. — When the rate is a simple aliquot part of 100, the percentage may be found by short division.

Thus, to find $33\frac{1}{3}\%$ divide by 3, to find $12\frac{1}{2}\%$ divide by 8, etc.

Solve the examples below using the following table:

$50\% = \frac{1}{2}$	$20\% = \frac{1}{5}$	$8\frac{1}{3}\% = \frac{1}{12}$
$33\frac{1}{3}\% = \frac{1}{3}$	$16\frac{2}{3}\% = \frac{1}{6}$	$6\frac{2}{3}\% = \frac{1}{15}$
$25\% = \frac{1}{4}$	$12\frac{1}{2}\% = \frac{1}{8}$	$6\frac{1}{4}\% = \frac{1}{16}$

ORAL EXERCISES (A)

Find the percentage in each of the following.

	Base	Rate		Base	Rate		Base	Rate
1.	400	50%	8.	1600	$6\frac{1}{4}\%$	15.	48	$16\frac{2}{3}\%$
2.	650	10%	9.	36	$33\frac{1}{3}\%$	16.	1000	$12\frac{1}{2}\%$
3.	1200	25%	10.	36	25%	17.	48	$12\frac{1}{2}\%$
4.	1200	$33\frac{1}{3}\%$	11.	36	$16\frac{2}{3}\%$	18.	48	$33\frac{1}{3}\%$
5.	1200	$16\frac{2}{3}\%$	12.	36	$8\frac{1}{3}\%$	19.	48	$66\frac{2}{3}\%$
6.	1200	$12\frac{1}{2}\%$	13.	36	50%	20.	48	$37\frac{1}{2}\%$
7.	3000	$6\frac{2}{3}\%$	14.	48	$6\frac{1}{4}\%$	21.	64	$6\frac{2}{3}\%$

FINDING THE RATE

299. Reasons for Finding the Rate. — It is frequently important to know how many per cent one number is of another.

Thus, in a census it is found that a state with a population of 2,649,260 has 78,240 illiterates; while in another state with a population of 678,925 there are 23,284 illiterates. To decide which state has the greater proportion of illiterates, we find how many per cent of each population are illiterate.

300. Rule for Finding the Rate.

Example 1. 16 is how many per cent of 400?

Solution. 1% of 400 = 4, and 4 is contained 4 times in 16. Hence 16 = 4% of 400.

Example 2. 24 is how many per cent of 270?

Analysis: 1% of 270 = 2.70. 2.70 is contained 8.89 times in 24. Hence 24 is 8.89% of 270.

Solution.

$$2.7\overline{)240}\ 8.89$$

216

240

216

240

Rule: *To find the rate per cent, divide the percentage by one per cent of the base.*

ORAL EXERCISES (A)

1. 20 is how many per cent of 100?
2. 40 is how many per cent of 1000?
3. 100 is how many per cent of 500?
4. 80 is how many per cent of 2000?
5. 50 is how many per cent of 3000?

WRITTEN EXERCISES (A)

Find the rate to the nearest tenth of one per cent.

	Base	Percentage		Base	Percentage
1.	860	29	6.	720	21.60
2.	2480	264	7.	890	94
3.	918.4	37.2	8.	7560	4914
4.	2462	21.9	9.	720	288
5.	794	24.5	10.	578	26.01

301. Applications of Finding the Rate. — The table below gives the number of persons in the United States over 10 years of age, and the number of illiterate persons in each class in one year.

Supply the missing per cents in this table. Notice how the per cents enable you to compare the illiteracy of the different classes of people.

POPULATION 10 YEARS OF AGE AND OVER

	TOTAL	ILLITERATE	
		No.	Per cent
Total population	98,723,047	4,283,753	1.3
Male	49,949,798	2,198,293	4.4
Female	48,773,249	2,085,460	4.3
White	87,980,667	2,407,218	
Male	44,554,124	1,214,025	
Female	43,426,543	1,193,193	
Negro	9,292,586	1,513,392	
Male	4,564,690	801,949	
Female	4,727,866	711,943	
Indian	238,981	61,517	
Male	123,469	29,630	
Female	115,512	31,887	
Chinese	63,392	12,912	
Male	53,650	10,735	
Female	9,742	2,177	
Japanese	97,273	8,932	
Male	60,580	4,231	
Female	36,693	4,701	
Native White	74,763,739	1,103,134	
Male	37,175,901	640,048	
Female	37,287,838	463,086	
Native Parentage	53,876,411	986,469	
Male	27,188,095	577,312	
Female	26,688,316	409,157	
Mixed Parentage	20,887,328	116,665	
Male	10,287,806	62,736	
Female	10,599,522	53,929	
Foreign-born White	13,216,928	1,304,084	
Male	7,078,223	573,977	
Female	6,138,705	730,107	

FINDING THE BASE

302. *Example.* If a certain grade of milk contains 4% of butter fat, how many pounds of milk must be delivered to a creamery to furnish 650 pounds of butter fat?

Analysis: Since 4% of the milk equals 650 pounds, $\frac{1}{4}$ of 650 lb., or 162.5 lb., equals 1 per cent of the required amount. Hence $100 \times 162.5 = 16250$ is the number of pounds required.

To simplify the work we divide by 0.4 instead of dividing by 4, and then multiplying by 100.

Solution.
16250.
.04)650.00

Rule: *To find the base, express the rate as a decimal and divide the percentage by it.*

WRITTEN EXERCISES (A)

Find the base accurate to two decimal places in each of the following:

	Rate	Percentage		Rate	Percentage
1.	5%	360	12.	11%	88.8
2.	25%	81	13.	145%	290
3.	15%	45	14.	200%	764
4.	$2\frac{1}{3}$%	71.5	15.	175%	1005
5.	120%	414	16.	$16\frac{1}{2}$%	800
6.	$26\frac{1}{4}$%	$26\frac{1}{4}$	17.	$56\frac{1}{4}$%	196
7.	7%	145	18.	$33\frac{1}{3}$%	750
8.	$14\frac{2}{7}$%	$98\frac{2}{7}$	19.	18%	92
9.	$16\frac{2}{3}$%	400	20.	$12\frac{1}{2}$%	625
10.	81%	985	21.	125%	1125
11.	76%	278			

303. Finding the Base when Rate Is an Aliquot Part of 100. — When the rate is an aliquot part of 100 the base can be found as in the following.

Example 1. 16 is 20% of what number?

Solution. $20\% = \frac{1}{5}$. Hence the base is $5 \times 16 = 80$.

Example 2. 240 is $33\frac{1}{3}$% of what number?

Solution. $33\frac{1}{3}\% = \frac{1}{3}$. Hence the base is $3 \times 240 = 720$.

ORAL EXERCISES (A)

Find the base in each of the following.

	Rate	Percentage		Rate	Percentage
1.	4%	10	13.	$6\frac{1}{4}\%$	800
2.	10%	20	14.	$8\frac{1}{3}\%$	12
3.	5%	40	15.	$12\frac{1}{2}\%$	90
4.	5%	100	16.	$6\frac{2}{3}\%$	20
5.	25%	500	17.	5%	40
6.	50%	50	18.	$12\frac{1}{2}\%$	500
7.	$33\frac{1}{3}\%$	25	19.	$6\frac{1}{4}\%$	1000
8.	20%	600	20.	$16\frac{2}{3}\%$	350
9.	$16\frac{2}{3}\%$	460	21.	25%	260
10.	$12\frac{1}{2}\%$	800	22.	50%	1240
11.	$6\frac{2}{3}\%$	400	23.	$33\frac{1}{3}\%$	6000
12.	$66\frac{2}{3}\%$	50	24.	$8\frac{1}{3}\%$	4000

PROBLEMS (A)

1. A certain grade of fine gunpowder contains 75% niter, 15% charcoal, and 10% sulphur. How much gunpowder can be made by using 15 tons of niter?

2. A certain grade of Tungsten steel contains 9.3% of Tungsten. How many tons of such steel can be made, using one ton of Tungsten?

3. A certain grade of nickel steel contains 27% of nickel. How many tons of such steel can be made, using 45 tons of nickel?

4. A certain grade of Babbitt's metal (used in bearings on railway cars, etc.) contains 88.9% tin, 3.7% copper, and 7.4% antimony. How much copper is required for an order of 25 tons of this metal?

5. How many tons of Babbitt's metal can be made, using 1 ton of copper?

6. How many tons of Babbitt's metal can be made, using 20 tons of tin?

7. How many tons of Babbitt's metal can be made, using 1 ton of antimony?

Problems of the kind shown in the two examples on this page are of frequent occurrence.

Example 1. What number increased by 25% of itself equals 1740?

Analysis: (*a*) The required number equals 100% of itself. Hence 1740 = 125% of the required number and hence the required number is 1740 ÷ 1.25 = 1392.

(*b*) 1740 = 125% of the required number or $\frac{5}{4}$ of it. Hence $\frac{1}{5}$ of 1740 = 348 = $\frac{1}{4}$ of the required number, and 4 × 348 = 1392 = the required number.

Solution.
Written Work

```
                1392
(a) 1.25)1740
      125
      490
      375
      1150
      1125
       250
       250

(b)    5)1740
         348
           4
        1392
```

Example 2. What number decreased by 15% of itself equals 12,760?

Analysis: 12760 = 85% of the required number. Hence the required number = 12760 ÷ .85 = 15011.76, which is the nearest approximation to two decimal places.

Solution.
Written Work
12,760 ÷ .85 = 15011.76

PROBLEMS (A)

1. What number increased by 5% of itself is equal to 378?

2. What number increased by 30% of itself is equal to 1625?

3. Having increased my capital by 12% of itself, I find that I have $380,000. How much had I at first?

4. My crop of oats this year is 15% greater than my crop last year and during the two years I raised 5375 bushels. What was my last year's crop?

5. What number diminished by 5% of itself is equal to 380?

6. What number diminished by 20% of itself is equal to 1008?

7. A merchant received 80% of a consignment of grain; he sold 20% of what he received and then found that he had left 576 bushels. What was the entire consignment?

8. A man having sold $33\frac{1}{3}\%$ of his land had 100 acres left. How many acres had he at first?

9. In one year the population of a certain city was 457,200, which was an increase of 20% for the 10 years preceding. What was the population ten years previous?

10. After paying 75% of the debts of an estate, $625 will discharge the remainder. What was the total indebtedness?

ORAL EXERCISES (A)

Change each of the following to per cents.

1. .404	**4.** 1.09	**7.** .978	**10.** 14.9	**13.** .0004
2. .052	**5.** 2.84	**8.** .002	**11.** 149.	**14.** .0017
3. .008	**6.** 0.49	**9.** 1.48	**12.** 1.006	**15.** .0174

Give the equivalent of each of the following in per cents.

16. $\frac{1}{2}$	**19.** $\frac{1}{3}$	**22.** $\frac{2}{5}$	**25.** $\frac{1}{6}$	**28.** $\frac{3}{8}$	**31.** $\frac{1}{16}$
17. $\frac{1}{4}$	**20.** $\frac{2}{3}$	**23.** $\frac{3}{5}$	**26.** $\frac{5}{6}$	**29.** $\frac{5}{8}$	**32.** $\frac{3}{16}$
18. $\frac{3}{4}$	**21.** $\frac{1}{5}$	**24.** $\frac{4}{5}$	**27.** $\frac{1}{8}$	**30.** $\frac{7}{8}$	**33.** $\frac{5}{16}$

Read each of the following as a decimal.

34. $2\frac{1}{2}\%$	**37.** $3\frac{1}{4}\%$	**40.** $5\frac{1}{1}\%$	**43.** $8\frac{1}{8}\%$
35. $3\frac{1}{2}\%$	**38.** $3\frac{3}{4}\%$	**41.** $5\frac{3}{4}\%$	**44.** $5\frac{3}{8}\%$
36. $1\frac{1}{4}\%$	**39.** $5\frac{1}{2}\%$	**42.** $6\frac{1}{5}\%$	**45.** $6\frac{5}{8}\%$

Find the percentage in each of the following.

	Base	Rate		Base	Rate		Base	Rate
46.	200	8%	**55.**	245	5%	**64.**	320	$6\frac{1}{4}\%$
47.	450	5%	**56.**	75	25%	**65.**	3000	$6\frac{2}{3}\%$
48.	900	30%	**57.**	160	$16\frac{2}{3}\%$	**66.**	4980	$8\frac{1}{1}\%$
49.	1125	10%	**58.**	2025	10%	**67.**	140	10%
50.	540	$33\frac{1}{3}\%$	**59.**	192	$12\frac{1}{2}\%$	**68.**	460	$12\frac{1}{2}\%$
51.	560	$12\frac{1}{2}\%$	**60.**	400	200%	**69.**	520	25%
52.	2500	50%	**61.**	525	3%	**70.**	35	50%
53.	1728	$16\frac{2}{3}\%$	**62.**	750	75%	**71.**	94.8	$33\frac{1}{3}\%$
54.	91	3%	**63.**	2.5	25%	**72.**	35.6	50%

MISCELLANEOUS PROBLEMS (A)

1. From the following data compute the per cent of urban and of rural population in the United States for each of the years given.

Urban and Rural Population	1940	1930	1920
Total population	131,669,275	122,775,046	105,710,620
Urban population	74,423,702	68,954,823	54,304,603
Rural population	57,245,573	53,820,223	51,406,017

2. From the following table compute the rate per cent of increase or decrease of population for 1930–1940 for each of the given cities.

City	1940	1930
New York	7,454,995	6,930,446
Chicago	3,396,808	3,376,438
Philadelphia	1,931,334	1,950,961
Boston	770,816	781,188
St. Louis	816,048	821,960
Cleveland	878,336	900,429
Detroit	1,623,452	1,568,662

3. Find the per cent of the total population which was composed of negroes at each census.

Year	Total	Negroes
1940	131,669,275	13,454,405
1930	122,775,046	11,891,143
1920	105,710,620	10,463,131
1910	91,972,266	9,827,763
1900	75,994,575	8,833,994
1890	62,947,714	7,488,676
1880	50,155,783	6,580,793
1870	39,818,449	5,392,172
1860	31,443,321	4,441,830
1850	23,191,876	3,638,808
1840	17,069,453	2,873,648
1830	12,866,020	2,328,642
1820	9,638,453	1,771,656

304. The Three Problems in Percentage. — We have now solved three problems in percentage, viz. to find any one of the three numbers, base, rate, and percentage, when the other two are given.

These problems are illustrated by the following examples.

1. Find 8% of 396,
   or, find $5\frac{1}{4}\%$ of $3\frac{3}{4}$.
2. 16 is how many per cent of 200,
   or, $\frac{2}{5}$ is how many per cent of $1\frac{3}{4}$?
3. 50 is 10% of what number,
   or, $5\frac{3}{8}$ is 214% of what number?

These are the only types of problem possible in percentage and a careful study of them will make percentage and its applications easy.

305. Direct Method for Finding Rules for Percentage. — We should notice that the rules for solving these three problems can be derived directly from the definition of percentage, viz.

$$Base \times Rate = Percentage$$

If any two of these numbers are given, the third can be obtained. We simply have to find the missing numbers in the following.

$$\text{Base} \times \text{Rate} = \text{————}$$
$$\text{Base} \times \text{——} = \text{Percentage}$$
$$\text{——} \times \text{Rate} = \text{Percentage}$$

WRITTEN EXERCISES (A)

Supply the missing number in each of the following.

	Base	Rate	Percentage
1.	8040	$4\frac{3}{4}\%$	——
2.	5980	——	2627
3.	——	43%	428
4.	1650	25%	——
5.	24.5	——	2.45
6.	——	$33\frac{1}{3}\%$	810
7.	750	——	30
8.	18.56	$10\frac{1}{2}\%$	——

Example. 20 is how many per cent greater than 16?

Solution. 20 is 4 greater than 16. 4 is 25% of 16.
Hence 20 is 25% greater than 16.

ORAL EXERCISES (A)

1. 10 is how many per cent greater than 8?
2. 8 is how many per cent less than 10?
3. 25 is how many per cent greater than 10?
4. 20 is how many per cent less than 25?
5. 12 is 50% of what number?
6. 16 is 25% of what number?
7. 16 is 200% of what number?
8. Find $12\frac{1}{2}\%$ of 1600.
9. Find $8\frac{1}{3}\%$ of 2400.
10. Find $6\frac{1}{4}\%$ of 6400.
11. Find $16\frac{2}{3}\%$ of 5000.
12. Find $33\frac{1}{3}\%$ of 960.
13. Find 25% of 1500.
14. Find 50% of 250.
15. 60 is how many per cent greater than 50?
16. 110 is how many per cent greater then 100?
17. Find 75% of 2700.
18. Find $62\frac{1}{2}\%$ of 800
19. Find $37\frac{1}{2}\%$ of 1600.
20. Find 10% of 1000.
21. Find 20% of 8750.
22. Find $87\frac{1}{2}\%$ of 8750.
23. 25 is 50% of what number?
24. 500 is 250% of what number?
25. 8 is 400% of what number?
26. 50 is what per cent of 500?
27. 200 is what per cent of 1000?
28. 1600 is 40% of what number?

ORAL PROBLEMS (B)

1. In a school containing 540 pupils 45% are boys. How many boys are there?

2. If potatoes shrink 10% in weight from November until March, how many bushels in October will amount to 270 bushels in March?

3. In a town of 1200 inhabitants 51% are females. How many males are there?

4. In 250 bushels of berries 4% are poor. How many bushels are poor?

5. In a distance of 96 miles 25% of the road is paved. How many miles are unpaved?

6. A man sold 10% of his cows and had 9 left. How many had he at first?

7. A man added 20% to his farm and had then 120 acres. How many acres had he at first?

8. I bought 560 fruit trees and lost 80. What per cent did I lose?

9. I sold 40 bushels of apples, which was 25% of my crop. How many bushels did I raise?

10. A paper had 5000 subscribers, and lost 100. How many per cent were lost?

11. A grocer had 72 barrels of flour and sold 18. How many per cent remained?

12. A man owned 250 acres of land, which was 250% of what his neighbor owned. How many acres did his neighbor have?

13. In a class of 250 students 20% failed. How many passed?

14. In a class of 360 students 18 failed. How many per cent failed?

15. At the end of a year I had added 25% to my bank accounts and then had $6250. How much had I at the beginning of the year?

16. On a certain day 3800 children attended school. This was 95% of the enrollment. What was the enrollment?

17. A man sold 25 head of young cattle for $1200, which was 20% more than they cost. What did they cost per head?

MISCELLANEOUS PROBLEMS (B)

1. If the number of new pupils in a certain city is 3107, of which number 1584 are boys, and 1523 are girls, what per cent of the whole are boys? What per cent are girls? What per cent of the number of boys is the number of girls?

2. The total enrollment for the year in another city was 60,367, of which 278 were in the normal school; 3788 in the high schools; 54,291 in the elementary schools; 2010 in the kindergartens. What per cent of the whole were registered in each kind of school?

3. One year the cost of instruction in the academic high schools of a certain city was $236,015.60; and the next year $276,443.20. What was the per cent increase?

4. The cost of instruction in other schools of the same city for the second year was $320,666.18, which was an increase of 5.31% over the year before. What was the cost for that year?

5. The enrollment of the academic high schools in the above city at the close of the year was 4614, of which 1504 were classical students, and 3110 were scientific. How many per cent of the whole was each of these? The number left or withdrawn during the year was 679. Find the total registration, and the per cent of the total registration which left or withdrew during the year.

6. I owe A $142.80, which is 12% of all my debts. How much do I owe?

7. A man bought 20 cows for $85 per head. He sold 85% of them for $105 per head, and the remainder for $50 a head. Did he receive more or less than the cost of the cows, and how much?

8. A invests 25% of the capital of a firm; B, 40%; and C, the remainder which is $7000. What is the capital of the firm?

9. A man's salary is $4000. He spends 22% for fuel and rent; 12% for clothing; 3% for books; and $1018 for other expenses. What has he left?

10. I have in the bank $2750, which is 25% of what I have invested. What is the total amount of my investment?

CHAPTER XXII

PROFIT AND LOSS; DISCOUNTS

306. Applications of Percentage. — Percentage is of very general application in business. The treatment of profit and loss, discounts, interest, bank discount, brokerage, taxes, customs, stocks and bonds, and many other subjects, is based on percentage.

The element of *time* enters into such subjects as interest and bank discount, and makes them more difficult than subjects such as profit and loss, and trade discount in which time is not an essential element.

307. Profit and Loss. — We must be very clear about the meaning of the words profit and loss. Profit does not mean the difference between the selling price of an article and what this article costs the merchant who sells it. This difference is *margin*. To determine profit, we must deduct from the margin the expense which the merchant incurs in selling the article. The remainder, if any, is profit.

For example, a grocer sells butter for 70¢ a pound. The butter costs him 49¢ a pound. The difference between 70¢ and 49¢ does not represent the grocer's profit, but his margin. To get his profit, we must deduct from the margin the expenses that enter into selling the butter — rent of the store, salaries of employees, delivery, heat, light and water, insurance and taxes, and all other expenses.

When a dealer sells merchandise for less than he paid for it, he loses money. But his loss is more than the difference between the cost and the selling price. To this difference must be added the expenses of selling.

308. What Is Meant by Cost. — Cost of merchandise is the amount a merchant pays for it plus the expense of getting it to his store. Suppose a hardware dealer buys a heating stove for $24, the freight on the stove from the point of manufacture to the merchant's town

is \$4.50, and it costs \$1 to get the stove from the railway freight station to the store. The cost of the stove, then, is \$29.50.

309. The Base on which Profit and Loss Are Computed. — The rate of expense, margin, and profit, or loss, when there is a loss, is computed as a percentage of the selling price. It is sometimes convenient to figure as a percentage of the cost price, the amount more or less than cost, received in a business transaction. These differences are then called increase and decrease, respectively.

310. Finding the Selling Price. — The selling price equals cost plus increase or the cost less decrease.

Thus: A dealer buys a doll carriage for \$2 and sells it at an increase of 50%. The increase is computed on the cost as a base. Hence the increase is 50% of \$2 = \$1 and the selling price is \$2 + \$1 = \$3.

Again, if a dealer pays \$50 for a farm wagon and sells it at an increase of 20%, the increase is 20% of \$50 = \$10, and the selling price is \$50 + \$10 = \$60.

Rule: *To find the selling price, add the increase to the cost, or subtract the decrease from the cost.*

ORAL EXERCISES (A)

Find the selling price of each of the following:

	Buying Price	Rate of Increase or Decrease		Buying Price	Rate of Increase or Decrease
1.	\$10	40% increase	**5.**	\$100	$27\frac{1}{2}$% increase
2.	\$20	5% decrease	**6.**	\$500	10% decrease
3.	\$25	25% increase	**7.**	\$1000	17% increase
4.	\$100	20% increase	**8.**	\$400	$12\frac{1}{2}$% decrease

311. Finding the Rate of Increase or Decrease. — Rule: *To find the rate of increase or decrease, find the increase or decrease, and then find how many per cent of the cost this is.*

Example. A dealer bought a plow for \$16, and sold it for \$20. What was the rate of increase?

Solution. The increase was \$4, which equals 25% of \$16, the cost.

ORAL EXERCISES (A)

Find the rate of the increase or decrease, in each of the following.

	Cost	Selling Price		Cost	Selling Price
1.	\$10	\$8	5.	\$40	\$30
2.	\$.80	\$1	6.	\$15	\$25
3.	\$1	\$1.25	7.	\$20	\$30
4.	\$2.40	\$3	8.	\$1.50	\$3

To find the selling price when the cost and the rate of increase or decrease are given it is often most convenient to add the rate of increase to 1 or subtract the rate of decrease from 1, and then multiply.

Example 1. Find the selling price of an article costing \$45 and sold at an increase of 35%.

Solution. Multiply \$45 by 1.35.

Example 2. Find the selling price of an article costing \$125 and sold at a decrease of 15%.

Solution. Multiply \$125 by .85.

WRITTEN EXERCISES (A)

Find the selling price in each of the following.

	Cost	Rate of Increase or Decrease		Cost	Rate of Increase or Decrease
1.	\$24	25% increase	7.	\$400	10% decrease
2.	\$38	15% increase	8.	\$960	25% increase
3.	\$60	$33\frac{1}{3}$% increase	9.	\$1050	110% increase
4.	\$75	50% decrease	10.	\$4060	22% decrease
5.	\$120	10% decrease	11.	\$7560	$16\frac{2}{3}$% increase
6.	\$600	$12\frac{1}{2}$% decrease	12.	\$4980	$37\frac{1}{2}$% increase

Find the rate of increase or decrease on each of the following.

	Cost	Selling Price		Cost	Selling Price
13.	\$14	\$18	19.	\$25	\$30
14.	\$20	\$24	20.	\$19	\$30
15.	\$36	\$48	21.	\$40	\$45
16.	\$1850	\$1600	22.	\$75	\$80
17.	\$144	\$176	23.	\$1540	\$1600
18.	\$12.50	\$15	24.	\$976	\$800

MARGIN FIGURED AS A RATE OF THE SELLING PRICE

312. Margin Figured as a Rate of the Selling Price. — Although it was formerly common practice to figure margin as a percentage of the cost of the goods, it is now generally recognized that it should be figured as a percentage of the selling price.

Selling price is best represented by the customer's dollar, or 100%, and the cost of the goods, expenses of doing business, margin, and profit, or loss, if there is a loss, are all portions of this 100% and should be so represented.

ORAL EXERCISES (A)

Find the margin per cent on the selling price in the following.

	Cost	Selling Price		Cost	Selling Price
1.	$.50	$1	7.	$.15	$.60
2.	$.40	$1	8.	$.10	$.50
3.	$3	$5	9.	$.25	$.75
4.	$1.40	$6	10.	$.20	$.50
5.	$2	$5	11.	$2.50	$6
6.	$.75	$1.25	12.	$4	$10

WRITTEN EXERCISES (A)

Find the margin per cent, using the selling price as the base.

	Cost	Selling Price		Cost	Selling Price
1.	$3.50	$5	7.	$8.50	$12
2.	$2.40	$6	8.	$11.25	$22
3.	$1.75	$3	9.	$13.50	$25
4.	$4.50	$7	10.	$14	$22
5.	$2.50	$4.70	11.	$7.25	$12
6.	$3.20	$5	12.	$17.50	$40

13. A book agent sold a set of books for $45 and received $20 as his commission. What was his rate on the selling price?

14. Goods sold in a jewelry store average 45% margin on the selling price. What is the margin on sales totaling $8460?

PROBLEMS (B)

Example. Eight pounds of 22¢ coffee is mixed with 12 pounds of 26¢ coffee. At what price per pound must the mixture be sold to make $33\frac{1}{3}\%$ margin?

Solution. 20 lb. of the mixture costs $8 \times 22 + 12 \times 26 = 1.76 + 3.12 = \$4.88$ or \$.244 a pound. Since the margin is to be $33\frac{1}{3}\%$ of the selling price \$.244 per pound equals $66\frac{2}{3}\%$ of the selling price. Dividing \$.244 by $66\frac{2}{3}\%$ equals .00366 or 1% of the selling price. Multiplying by 100 equals \$.366, 100% of the selling price, or 37¢ per lb.

1. A dealer buys eggs for 42¢ a dozen. If $6\frac{1}{4}\%$ of the eggs break, how much per dozen must he charge to get a margin of 15%?

2. From October to January corn shrinks 7%. Which pays better, to sell at 78¢ a bushel in October, or 83¢ a bushel in January?

3. A dealer bought one grade of coffee for 37¢ a pound, and another for 42¢ a pound. He blends these two grades, using one pound of the cheaper grade to two pounds of the more expensive grade, and sells the mixture at 48¢ a pound. What is his margin per cent?

4. A dealer was obliged to sell a stock of damaged goods at $16\frac{2}{3}\%$ below cost, a decrease of \$1825. What was the cost of the goods?

5. A grocer buys 5 barrels of apples each containing $2\frac{1}{2}$ bushels at \$8.25 a barrel. If 10% of the apples decay, at what price per bushel must he sell them to make a margin of 15%?

6. A self-binder cost the dealer \$100, and was sold by him for \$125. What was the margin per cent?

7. A retail dealer buys collars at \$1.10 a dozen, and sells them at 2 for 25 cents What is his margin per cent?

8. A firm in closing its books at the end of the year allows 8% for bad debts. What are its outstanding debts worth if their total is \$12,465?

9. A merchant buys 200 tons of hard coal by the long ton (2240 lb.) for \$3800, and sells it for \$20.50 a short ton (2000 lb.). What rate of margin does he make?

10. How many square feet of flooring must be bought for a room 16 feet by 20 feet, adding $16\frac{2}{3}\%$ to allow for waste?

MISCELLANEOUS PROBLEMS (C)

1. In a certain drug store, increases average 85% on the buying price. What are the increases on goods costing \$6720?

2. Skates are bought at \$9.60 a dozen pairs, and sold at \$1.40 a pair. What is the rate of margin?

3. One dealer makes 15% margin on the selling price, and another makes 18% increase on the buying price. Which makes the greater margin? What is the difference in margins on goods which cost them \$100?

4. A fountain pen is advertised to sell at \$4. At what price per dozen must the company sell it to the dealer to enable him to make a margin of 15%?

5. A set of furniture is advertised to be sold 15% below cost. If the advertised price is \$145, what is the "cost" asserted to be?

6. By careful buying the cost of an article to a merchant is reduced from \$10 to \$9. By how many per cent is the margin increased if the article sells for \$14?

7. An article is manufactured at a cost of \$5.60. The manufacturer makes a margin of 25%, the wholesaler a margin of 25%, and the retailer a margin of 30%. How much does the retailer charge his customer for the article?

8. Find the per cent of increase in the population of the following cities from 1930 to 1940.

City	1940	1930
Atlanta, Ga.	302,288	270,366
Birmingham, Ala.	267,583	259,678
Quincy, Mass.	75,810	71,983
Pittsburgh, Pa.	671,659	669,817
Buffalo, N. Y.	575,901	573,076
Santa Fe, N. M.	20,325	11,176

313. Variation in Price. — Consider a wholesale dealer in some commodity such as building hardware. During the course of say one

year, the market price of hardware may undergo considerable change. Suppose that at the period of highest price he sells a small bill of goods on long time credit to a retail dealer of questionable financial standing. Suppose on the other hand that at the period of lowest price of the year he sells a large bill of goods for spot cash to a valuable customer for whose trade several other wholesale houses are in active competition. It is clear that the prices made in these two bills will differ very considerably. In the first bill, the article will be quoted at a maximum price, and in the second bill at a minimum price.

314. The List Price. — Merchants and especially wholesale dealers issue catalogues describing their goods, and giving prices at which they hold themselves ready to sell to all customers. Such prices are called *list prices*.

The list prices are usually put high to avoid possible loss from unexpected changes.

However, during very unusual times business houses reserve the right to raise prices above the list price without notice.

315. Discounts. — Deductions made from list prices are called *discounts*.

It is evident from the above that wholesale dealers make deductions from their list prices in all except rare instances. That is, nearly every bill from a wholesale house is subject to a discount.

316. Trade Discount. — A trade discount is a discount made on account of the condition of the market, the size of the order, the reliability and desirability of the customer, etc.

317. Time Discount. — A time discount is a discount made because the bill is paid in less time than is allowed by the terms of the sale.

318. Gross and Net Amount of a Bill. — The amount of a bill before any discount is deducted is called the *gross amount*.

The amount left after deducting the discount is called the *net amount*.

Discounts are frequently aliquot parts of 100, thus making computation easy.

319. Single Discount. — A single discount is computed as a certain rate per cent of the list price.

Example. Goods listed at \$9.50 were sold at a discount of $33\frac{1}{3}\%$. Find the selling price.

Analysis: $33\frac{1}{3}\% = \frac{1}{3}$; $\frac{1}{3}$ of \$9.50 = \$3.16$\frac{2}{3}$, which is the discount. \$9.50 − \$3.16$\frac{2}{3}$ = \$6.33$\frac{1}{3}$.

Solution.

List Price	\$9.50
Discount	3.16$\frac{2}{3}$
Selling Price	\$6.33$\frac{1}{3}$

ORAL EXERCISES (A)

Find the selling price in each of the following.

	List Price	Rate Discount		List Price	Rate Discount
1.	\$20	20%	11.	\$40	20%
2.	2.40	25%	12.	50	25%
3.	3.60	10%	13.	25	$6\frac{1}{4}\%$
4.	7.50	12%	14.	45	40%
5.	10.50	8%	15.	10.80	50%
6.	12	30%	16.	12.60	60%
7.	15	40%	17.	17.50	10%
8.	9.20	10%	18.	9.80	4%
9.	11	11%	19.	14.40	5%
10.	8.50	30%	20.	8.75	6%

WRITTEN EXERCISES (A)

Find the selling price.

	List Price	Rate Discount		List Price	Rate Discount
1.	\$162.50	35%	10.	\$780.60	18%
2.	175.50	20%	11.	272.50	27%
3.	225.60	25%	12.	1960.40	30%
4.	1250.40	40%	13.	2540	$62\frac{1}{2}\%$
5.	758.60	18%	14.	1080	$37\frac{1}{2}\%$
6.	90.50	32%	15.	916.52	25%
7.	278.50	40%	16.	1725.41	$17\frac{1}{2}\%$
8.	98.75	50%	17.	862.76	$26\frac{1}{2}\%$
9.	3060.40	11%	18.	1150	40%

320. Wholesaler's Discount. Retailer's Margin. — Many articles are advertised and listed at retail prices and a sufficient discount is made to the retailer to enable him to make a margin. Thus, a certain fountain pen has been listed and sold retail for many years at \$2.50. Such articles are sold to the retail dealer at a certain discount, and the question arises as to the rate of increase on the cost obtained by the retail dealer.

Example. An article listed to sell retail at \$5 was sold to the retail dealer at a discount of 25%. What rate increase did the retail dealer get?

Analysis: 25% of \$5.00 = \$1.25, which is the discount.

\$5.00 − \$1.25 = \$3.75 = buying price.

The margin, \$1.25, is $33\frac{1}{3}$% of \$3.75, or the buying price. Hence, the retail dealer gets an increase of $33\frac{1}{3}$% on his investment.

This is a problem of very frequent occurrence.

Solution.

Retail Price	\$5.00
Wholesale Price	3.75
Margin	\$1.25

$.33\frac{1}{3}$ = rate increase

$$3.75\overline{)1.2500}$$

1 125

1250

WRITTEN EXERCISES (A)

Find the retailer's increase per cent on each of the following if he sells at list price.

	List Retail Price	Wholesaler's Discount		List Retail Price	Wholesaler's Discount
1.	\$1.20	20%	**11.**	\$21.50	25%
2.	3.60	25%	**12.**	9.75	20%
3.	4.50	10%	**13.**	8.60	30%
4.	16	12%	**14.**	14.50	10%
5.	25	6%	**15.**	36	$33\frac{1}{3}$%
6.	3.75	$33\frac{1}{3}$%	**16.**	75	35%
7.	21	10%	**17.**	150	$12\frac{1}{2}$%
8.	32	50%	**18.**	92	23%
9.	17.50	8%	**19.**	76	24%
10.	10.50	$12\frac{1}{2}$%	**20.**	28	14%

21. A book dealer sold text-books at list price, amounting to \$574.80. His expense for freight, etc., was \$12.40. What was his increase per cent if the publishers gave him a discount of 20%?

Example 1. What rate of discount must be given a retailer to enable him to sell at list price and get an increase of 30%?

Analysis: Suppose the list price is \$100. Then 100 = 130% of the buying price, and 100 ÷ 130 = $.76\frac{12}{13}$ = 1 per cent of the buying price. Hence the buying price is $\$76\frac{12}{13}$ and therefore the discount is $23\frac{1}{13}\%$.

Solution.
Written Work.

$$130\overline{)100}\quad 76\tfrac{12}{13}$$
$$\underline{91}$$
$$90$$
$$\underline{78}$$
$$12$$

$$100 \quad 76\tfrac{12}{13}$$
$$23\tfrac{1}{13}$$

Example 2. A retail dealer will not handle an article at a rate of increase less than 40%. Find the lowest discount which may be given him if the only rates of discount given are multiples of 5. What will be his exact increase per cent?

Solution. 100 = 140 per cent of selling price. Hence, as above, the buying price is found to be \$71.43, which represents a discount of 28.57%. Hence 30% is the required discount, and the actual buying price is \$70.

Since 30 is $42\frac{6}{7}$ per cent of 70, it follows that the actual increase per cent will be $42\frac{6}{7}\%$.

WRITTEN EXERCISES (A)

In each of the following, find the exact rate of the discount, the retail dealer selling at list price.

Retail Dealer's Increase	Retail Dealer's Increase	Retail Dealer's Increase
1. 25%	5. 60%	9. 35%
2. 20%	6. 80%	10. 75%
3. 40%	7. 45%	11. 125%
4. 50%	8. 100%	12. 150%

In the following, the rates of discount allowed are multiples of 5.

In each case the minimum rate of increase is given. Find the actual discount on the list price, and the actual rate of increase in each case.

13. 40%	17. 50%	21. $33\frac{1}{3}\%$
14. 60%	18. 100%	22. $66\frac{2}{3}\%$
15. 30%	19. 45%	23. 65%
16. 70%	20. 55%	24. 85%

321. Discounts, How Noted on Bill. — If a bill is rendered with no discount in 90 days, and 3% off if paid in 30 days, or 5% in 3 days, this may be indicated on the bill thus: $n/90, 3/30, 5/3$. This is read, no discount, or "net," in 90 days, 3% in 30 days, and 5% in 3 days.

Example. Find the amount of the following bill for the various discounts quoted on it.

1	Bbl. M S Milk Powder	200 @	14	28 00
10	# Egg Albumen		75	7 50
5	Pails Jelly		1 00	5 00
2	Cans G M Baking Powder	20 @	10	2 00
½	M 8″ Paprus Pie Plates		1 40	70
5	Reams "B" Wax Paper		40	2 00
	15/90, 18/60, 20/10			

Solution.

Total of bill =	45.20
Less 15% in 90 da.	6.78
Due in 90 da.	38.42
Less 18% in 60 da.	8.14
Due in 60 da.	37.06
Less 20% in 10 da.	9.04
Due in 10 da.	36.16

Notice that the $8.14 and $9.04 are subtracted from $45.20 and not from the numbers immediately preceding them.

EXERCISES (A)

Find the various discounts on the following bills.

1.	1	# 261 Steam Cooker Complete			35 00
	1	#246 Boiler Burner			50 00
		5/90, 7/60, 8/30			
2.	1	# 311 B & W Pan Rack			19 00
	5	# Val Almonds		42	2 10
	10	# Almond Paste		33	3 30
	10	# Walnut Pieces		29	2 90
	1	Can O P Malt Extract	56 @	9½	5 32
	1	# Citric Acid			50
	100	Pink Bud Holders			75
		10/90, 12/30			

322. Discount Series. — Frequently several discounts are made on the same bill. Thus, a bill may be subject to one discount because the current market price is below list price, to another because the order is unusually large, and to a third for cash payments. This is called discount series.

323. Reason for Discount Series. — In a discount series the second discount is always based on the amount of the bill after the first discount is deducted, and the third discount is based on the amount after the two preceding discounts are deducted, etc. It is for this reason that several discounts are made instead of one.

There is good reason why each successive discount should be based on the bill reduced by the previous discounts. Thus, if a discount is made for cash, this is naturally based on the amount due at the end of a certain time. Again, if a special discount is made because of a large order, this is naturally based on the amount of the order at the current price, and not on the amount computed at list prices, which may be very much greater.

Example. Find the net proceeds of a bill for \$250, subject to discounts of 25%, 15%, and 5%.

Solution. \$250 less 25% = \$187.50; \$187.50 less 15% = \$159.38; \$159.38 less 5% = \$151.41, which is the net amount of the bill.

WRITTEN EXERCISES (A)

Find the net amount of each of the following bills.

	Gross Amount	Discounts
1.	\$2500	15%, 25%, 35%
2.	\$7600	40%, 25%, 5%
3.	\$12000	50%, 20%, 10%
4.	\$650	$8\frac{1}{2}$%, 8%, 5%
5.	\$500	16%, 10%, 5%
6.	\$4500	30%, 15%, 10%

In each of the following determine which series is more advantageous to the buyer.

7. 15%, 20%, 30% or 40%, 10%, 5%.

8. 12%, 15%, 20% or 25%, 10%, 8%.

9. 3%, 6%, 12% or 15%, 5%, 2%.

324. Order of Discounts. — The order in which successive discounts are taken is immaterial.

Thus, \$800 less 25% = \$600, and \$600 less 20% = \$480, and \$480 less 5% = \$456.

Also, \$800 less 5% = \$760, \$760 less 20% = \$608, and \$608 less 25% = \$456.

A discount of 25% may be deducted by simply taking 75% of the gross amount. In this way a series discount may be computed as follows: $800 \times .75 \times .80 \times .95$.

Since a series of factors may be arranged in any order without changing the product, it is obvious that the order of discounts is immaterial.

Thus, $800 \times .75 \times .80 \times .95 = 800 \times .95 \times .75 \times .80$.

Example 1. A bill for \$1850 is subject to discounts of 40%, 20%, and 10%. Take these discounts in different orders to show that the order is immaterial.

Solution.

1850	1850	1850
.60	.80	.90
1110.0	1480	1665
.80	.90	.80
888	1332	1332.0
.90	.60	.60
799.20	799.2	799.2

In some cases the following method of solution is more convenient.

Example 2. Find the net amount of a bill for \$8640 with discounts of $33\frac{1}{3}$%, 25% and 5%.

Solution.

3)8640
2880 = $33\frac{1}{3}$%
4)5760
1440 = 25%
20)4320
216 = 5%
4104 = net amount.

Example 3. What single discount is equivalent to 40%, 20%, 10%?

Solution. Discounting \$100 40% leaves \$60. Discounting \$60 20% leaves \$48. Discounting \$48 10% leaves \$43.20. Hence the single discount is 56.8%.

This work may all be done mentally.

ORAL EXERCISES (A)

Find the single discount which is equivalent to each of the following discount series.

1. 50%, 20%, 10%
2. 30%, 10%
3. 20%, 10%
4. 25%, 10%
5. 40%, 30%, 5%
6. 20%, 15%, 10%
7. 10%, 5%
8. 20%, 5%
9. 25%, 20%
10. 30%, 20%
11. 40%, 20%, 5%
12. 50%, 40%
13. 50%, 20%, 5%
14. 40%, 10%, 5%
15. 30%, 15%
16. 20%, 10%, 5%
17. 60%, 10%
18. 40%, 25%, 10%
19. 10%, 20%, 30%
20. 20%, 30%, 40%
21. 30%, 10%, 10%
22. 20%, 20%, 10%
23. 20%, 10%, 10%
24. 40%, 30%
25. 30%, 10%, 10%
26. 50%, 20%
27. 50%, 30%
28. 25%, 10%, 5%

WRITTEN EXERCISES (A)

Goods were sold to the retailer at the following discounts from list price. What were his increases if he sold at list price?

Suggestion. Assume \$100 as the list price in each case.

1. 25%, 10%, 5%
2. 30%, 10%, 5%
3. 40%, 10%, 5%
4. 16%, 8%, 2%
5. 20%, 10%, 4%
6. 12%, 10%, 2%
7. 5%, 10%, 20%
8. 2%, 5%, 10%
9. 40%, 15%, 5%
10. 25%, 10%, 5%
11. 30%, 20%, 10%
12. 40%, 20%, 10%

13. One firm offered to sell a certain bill of goods at \$1260, with discounts of 30%, 15%, and 5%. Another firm offered to sell the same bill at \$1500, with discounts of 40%, 20%, and 10%. Which was the better offer, and how much?

325. To Find Marking Price Permitting Given Discounts (B).

Example 1. A wholesaler wishes to sell an article at \$20, but wishes to give his customer a reduction of $33\frac{1}{3}\%$ from the list price. What must he make the list price?

Solution.

Written Work

$$.66\frac{2}{3})\overline{2000}\quad 30$$
$$1980$$
$$20$$

Analysis: (*a*) \$20 equals $66\frac{2}{3}\%$ of the list price. Hence $20 \div .66\frac{2}{3} = 30$ is the list price.

(*b*) $66\frac{2}{3}\% = \frac{2}{3}$. Hence \$20 = $\frac{2}{3}$ of the list price, which is \$30.

Example 2. A wholesale dealer wishes to sell an article for \$12 after giving discounts of 40%, $33\frac{1}{3}\%$ and 25%. What must he make the list price of the goods?

Solution. The discounts 40%, $33\frac{1}{3}\%$, 25% are equivalent to a single discount of 70%. Hence \$12 = 30% of the list price which is \$40.

Example 3. Goods cost a dealer \$15. What must he mark the goods to be able to make a reduction of $16\frac{2}{3}\%$ and still get an increase of $66\frac{2}{3}\%$?

Solution. The increase is $66\frac{2}{3}\%$ or $\frac{2}{3}$ of \$15 = \$10. Hence the selling price is \$25. To permit a discount of $16\frac{2}{3}\%$ the selling price must be $83\frac{1}{3}\%$ or $\frac{5}{6}$ of the marking price, which is \$30.

PROBLEMS (C)

1. A wholesale hatter marked hats at \$42 a dozen. He discounted this price 20% to a retailer, who marked them to get an increase of 25%. Find retail price of the hats.

2. Find the list price of goods that are sold for \$300 after a discount of 25% has been made.

3. Find the list price of goods that sell for \$432 after giving discounts of 40%, 20%, 10%.

4. I sold a car for \$357, which was 16% less than I paid for it. For what sum should I have sold to get an increase of 16%?

5. I bought a watch for \$68, 15% less than its value. What would have been the increase if it had sold at 10% more than its value?

6. I bought land for \$600 an acre. How much must I ask for it that I may make a reduction of 25% and still get an increase of 20%?

MISCELLANEOUS PROBLEMS (C)

1. A piano dealer wishes to sell a piano for \$440 after giving discounts of 50%, 40%, and 20%. What must be his list price?

2. Bought a gun for \$56, which was $16\frac{2}{3}\%$ more than its value. I sold it for $16\frac{2}{3}\%$ less than its value. Find the decrease and the selling price.

3. Find the list price of an article that sells at \$120, after a discount of 40% has been made.

4. A man sold a lot for \$520 after making discounts of 40% and 20%. What was the original price?

5. A dealer reduced the price of cloth $8\frac{1}{2}$ cents a yard, thus cutting his margin from $7\frac{1}{2}\%$ to 5%. What was the selling price?

6. Sold 120 horses for \$18,000, a decrease of 10%. For how much per head should they have been sold to get an increase of 20%?

7. A sold a house to B, at an increase of 25%. B sold it to C for \$2400, a decrease of 20%. What was A's increase?

8. The rent of a house this year is \$507.50, which is $1\frac{1}{2}\%$ more than last year. The rent last year was 5% of the value of the house. What is the value?

9. A man bought a horse for \$72 and sold it for 25% more than it cost and for 10% less than he asked for it. What did he ask for it?

10. A contractor engaged to build a railroad at \$31,200 a mile. What was his increase per cent if the work actually cost \$90 per rod?

11. A man owning $\frac{2}{3}$ of a foundry sold 80% of his share for \$26,320. What was the value of the foundry?

12. A farmer sold $67\frac{1}{2}$ bushels of corn, which was $18\frac{3}{4}\%$ of his crop. What was the remainder worth at \$1.20 a bushel?

13. If I buy a book marked \$4.80 at 25% and 20% discount, and sell it at an increase of 25%, what shall I get as increase?

14. A man's expenses are 80% of his income, and $33\frac{1}{3}\%$ of his income equals 10% of his property, which is valued at \$27,000. What is his income? What are his expenses?

326. Problems Applicable to Any Price (B). — On the preceding pages we have found many examples in which no price was given. It is interesting to note that many important problems can be solved without reference to any particular price. Thus, the answer to the question, which is the more advantageous of the two discount series, 30%, 15%, and 5%, or 40%, and 10%, is the same whatever the amount of the bill may be.

In solving such problems, a price may be assumed for the sake of convenience, but any other price may equally well be assumed. In some cases it may be more convenient to use \$1000, or \$10,000 than \$100, as the price.

Example. A retailer bought an article at 40% discount from list price, and sold it at 10% discount from list price. What was his increase per cent?

Solution. Suppose the list price is \$100. Then at 40% discount, the goods were bought for \$60. The retailer who sold at 10% discount received \$90 for them. Hence the increase was \$30, which is 50% on the \$60 which the goods cost him.

WRITTEN EXERCISES (B)

Find the rate of the retailer's increase.

	Rate of Discount from List Price in Buying	Rate of Discount from List Price in Selling		Rate of Discount from List Price in Buying	Rate of Discount from List Price in Selling
1.	50%	20%	5.	10%	7%
2.	20%	10%	6.	30%	25%
3.	40%	25%	7.	17%	14%
4.	16%	8%	8.	12%	10%

9. I bought 2 houses for \$18,000, paying 25% more for one than for the other. I sold the cheaper house at a 25% increase, and the higher priced house at 20% decrease. Did I receive more or less than the cost of the houses, and how much?

10. The list price of a bill of hardware was \$1200. The purchaser was allowed 25% off the list price, 10% off for cash, and 1% for delay in delivery. What did he pay, and what was the discount?

11. A grocer sold 82 bbl. of apples at 22% increase, an increase of \$45.10. What was the cost per bbl.?

WRITTEN EXERCISES (B)

In each of the following find what the exact selling price would be to make the required increase per cent, and then give a reasonable marking price, which would make a good approximation to this. Sometimes goods are marked at such odd prices as 49¢ instead of 50¢, 98¢ or 99¢ instead of \$1. This is done simply for advertising effect.

	Buying Price per Dozen	Rate of Increase		Buying Price per Dozen	Rate of Increase
1.	\$13	30%	**4.**	\$7	80%
2.	16	35%	**5.**	27.50	40%
3.	10	45%	**6.**	21.75	60%

MISCELLANEOUS PROBLEMS (B)

1. I sold 85 horses for \$9526, 12% less than I paid for them. For how much per head should they have been sold to get an increase of 15%?

2. I loaned A 36% of my money and B 75% of the remainder. If B received \$300 more than A, how much money had I at first?

3. A merchant bought a bill of goods upon which the total discount was \$318, the discount series being 20%, 20%, and 10%. What was the net price of the goods?

Suggestion. Find the total discount on \$100.

4. Goods marked 45% advance of cost are sold at a discount of 20% on the marked price. What did the goods cost if the increase on them was \$25.20?

5. To 350% of 248 add 3.5% of it, and subtract from the result 35% of the sum.

6. How many per cent above cost must a man mark goods so that he may drop 20% from the marked price, and still get an increase of 20%?

7. A board of education purchased 5000 tons of coal. The Lehigh Co. quoted \$12 a ton with discounts of 20% and 10%. The Penn Co. quoted \$15 a ton with discounts of 40% and 5%. Both firms quote the same terms, 2/10, N/20. Which is the better price, and what did the coal cost at the lower price?

SOME REPRESENTATIVE PROBLEMS ON DISCOUNT (C)

1. A manufacturer finds that the public is willing to pay \$6 for an article. At what cost must he aim to make the article to allow an increase of 15% for himself and 35% for the retail dealer?

Analysis: \$6. = 135% of the price made to the retailer. Hence, this price is $\frac{6}{1.35} = \frac{600}{135}$. The wholesale price equals 115% of the cost of manufacturing. Hence $\frac{600}{135}$ = 115% of the manufacturer's cost. Therefore the manufacturer's cost $= \frac{600}{135} \div 1.15 = \frac{60{,}000}{135 \times 115} = 3.86$ (dollars).

Suggestion Cancel 5 in each of the numbers 135 and 115 and cancel 25 in 60,000.

Solution.
Written Work.

800
~~2400~~
~~60000~~
~~135~~ × ~~115~~
~~27~~ × 23
9

3.86 (dollars)
207)800
621
1790
1656
1340
1242

2. A bill for \$8000 is due in 90 days, with 3% discount on any part of it paid within 30 days. If a payment of \$4500 is made within 30 days, what is the balance due at the expiration of the 90 days?

Suggestion. The amount of the bill is reduced by \$1.00 for every 97 cents paid within 30 days.

3. A merchant's stock cost him \$27,800. After selling goods for \$31,260 he took an inventory of the remainder, and found it worth \$9750. What was his average rate of increase on the goods sold?

Suggestion. In taking the inventory the value given to the goods is the price for which the merchant can buy them with a possible reduction because of their being out of season, shopworn, etc. Hence subtract the inventory value from the total cost of the goods given. \$27,800 − \$9750 = \$18,050 is the cost of the goods sold for \$31,260.

4. A merchant bought goods for \$14,500. What should be the inventory value of the goods left after selling goods for \$16,800 at an average rate of increase of 65%?

5. Which of the following series discount is more advantageous to the buyer: 10, 15, 20, 5, or 25, 15, 10?

6. A consignment of goods from England was invoiced at 1460 pounds sterling. The total charges for transportation and insurance were \$165. Part of this consignment consisted of 80 dozen knives and forks billed at 2 pounds sterling per dozen. What part of the charges should be borne by this table silver? At what price per dozen must they be sold to get a 35% increase, the import duty being 25%? One pound sterling is considered as \$4.88.

Suggestion. The silver must bear $\frac{160}{1460}$, or $\frac{8}{73}$ of the charges, — that is, $\frac{8}{73} \times \$165$. The duty is 25% of 160 pounds or 40 pounds.

7. A consignment of goods was invoiced from Paris at 31,500 francs. Total extra charges were 1200 francs. Part of consignment consisted of 260 dozen pairs of gloves billed at 70 francs per dozen. At what price per pair must the gloves be sold to get an increase of 80%, the import duty being \$2.50 per dozen pairs? (Consider the value of the franc as par.)

PROFITS; COST OF DOING BUSINESS

327. Cost of Doing Business. — The margin is by no means an indication of the amount of money made by a business man. Out of the margin he must pay expenses such as rent, wages and salaries, cost of advertising, light, heat, insurance, taxes. There are losses due to bad debts, etc. In short he must pay the total cost of carrying on the business out of the margin. Included in the cost of doing business is a salary for the business man himself.

328. Profit. — The margin less the cost of carrying on the business is *profit*. It is comparatively easy to determine the total profit of a business, but it is not so easy to determine whether or not the sale of any one article yields a profit and how much.

In practical business it is very important, however, to know the profit on a particular line of goods, since that will enable the business man to decide whether or not he should change certain details of his business. A complete treatment of this complicated problem is to be found in advanced works on accounting. We give here only the strictly arithmetical elements of the problem.

329. Apportioning Operating Expenses according to Selling Price of Goods. — In a small store the most practical method is to divide the

cost of carrying on the business among the articles sold in proportion to their selling price. In larger stores each department may be treated separately and special kinds of goods may have costs assessed against them in a special way.

Thus, if in a small store the selling price of the goods bought and sold in one year is \$37,500 and if the total cost of carrying on the business is \$7250, then the cost of doing business is $7250 \div 37{,}500 = .1933 = 19.33\%$.

Hence to find the profit on an article sold in this store 19.33% of the selling price must be added to the purchase price and the sum subtracted from the selling price.

WRITTEN EXERCISES (A)

1. The purchase price of goods sold in a store during one year was \$61,730, the expenses of doing business were \$12,860, and the profit was \$3660. The expenses were what per cent of sales?

In each of the following, the cost of doing business is what per cent of the sales?

	Yearly Sales		Expenses of Doing Business		Expense Per Cent	
2.	\$197,630		\$29,370			
3.	31,934	60	5,930			
4.	84,378		14,630	50		
5.	364,780		63,756	80		

330. Rate of Preceding Years Used in Marking Goods. — For the purpose of marking goods the percentage of expense is estimated from the rate of the preceding year or better still from the average of several preceding years.

331. Gross Cost. — The buying price plus the amount assessed against the article from the cost of doing business is called the *gross cost*.

WRITTEN EXERCISES (A)

In a certain store the expense per cent for several preceding years averaged 18.57%. Find the cost and the profit on each of the articles given below. Enter the results on a suitable blank.

	Buying Price	Gross Cost	Selling Price	Profit
1.	$ 49 50		$ 75 00	
2.	3 75		8 00	
3.	24 00		50 00	
4.	27 50		60 00	
5.	120 00		180 00	
6.	35 00		50 00	
7.	1 20		3 00	
8.	45		1 00	
9.	1 25		2 00	
10.	6 50		15 00	

PROBLEMS (B)

1. During the year a dealer sold for $37,500 merchandise which cost him $28,125. How much margin did he make? This was what per cent of the selling price?

$37,500 minus 28,125 equals $9375; his margin $9375 divided by $37,500 equals 25%, his percentage of margin.

2. His expenses for the year were: salaries, $4500; supplies, postage, $187.50; advertising, $575; heat, light and water, $200; delivery, $425; collections, $275; rent, $900; insurance and taxes, $450; miscellaneous expenses, $500. How much did his expenses total for the year? They were what per cent of the sales?

Adding the various items gives a total expense of $8012.50; $8012.50 divided by $37,500 gives 21.1%, his percentage of expense.

3. How much was his profit for the year? This was what per cent of his sales?

His margin $9375 minus $8012.50, his expenses, leaves $1362.50, his profit; $1362.50 divided by $37,500 gives 3.6+%, his percentage of profit on sales.

4. At the close of the year his stock inventoried $11,000. He had store fixtures valued at $1750, and a truck at $550. Accounts and notes owed him amounted to $4500. He owed wholesalers and manufacturers $3500. How much did he have invested in the business? His year's profit was what per cent of this investment?

Adding

Average stock	$11,000
Fixtures	1,750
Truck	550
Accounts Receivable	4,500
Total	$17,800

Subtracting from this amount the $3500, owed wholesalers and manufacturers, leaves $14,300, the total investment.

His profit $1362.50 divided by $14,300 gives $9.5^{+}\%$, his percentage of profit on investment.

332. Price Yielding the Greatest Profit. — The total sales which can be made of a certain article depend partly upon the price at which it is sold. If the price is made very high, the margin on each article is great, but the total margin may be small because only a small number can be sold. If on the other hand the price is made very low, the volume of sales will be great, but the margin on each article will be so small that the total margin will again be small; in fact, the aggregate margin might conceivably be so small that the expense would be greater. There is likely to be some intermediate price which will result in sales and margin such as to make the profit the greatest. The problem in such a case is to determine the price which will yield the greatest profit.

PROBLEMS (C)

1. A manufacturer finds that by selling a certain article at an increase of 50%, he can sell about 15,000 a year. At an increase of 35% he can sell 25,000 per year. At an increase of 25% he can sell 35,000 per year, and at an increase of 15% he can sell 40,000 per year. Which of these rates of increase will yield him the greatest total margin? At which rate of increase would he make the greatest profit, provided his expense of selling equals 25%, 20%, 15%, and $12\frac{1}{2}\%$ of sales?

Suggestion. Assume that the cost of manufacturing is $10 per article.

There are many other elements entering into such a problem. The costs of manufacturing each article and of advertising may decrease with the growth of the output. Valuable business relations may be established by extending the sales, and so on.

2. Marking suits at \$30, a tailor sold 15 per day. Could he reduce the price to \$25 if the sales increased to 28 a day, the cost being \$17.50, and his expenses remaining 22% of sales? What per cent would his profits be increased or decreased each day?

3. A merchant finds that by selling a certain article at an increase of 40% he can sell about 25,000, at an increase of 30% he can sell 35,000, at an increase of 20% he can sell 45,000, and at an increase of 10%, 60,000. Which of these rates of increase will yield him the greatest total profit if his expenses of operation are 25%, $22\frac{1}{2}$%, 15%, and $12\frac{1}{2}$% of sales, respectively?

ORAL REVIEW ON PROFIT AND LOSS (A)

1. I bought goods for \$600 and sold them at an increase of 15%. Find the selling price.

2. I bought goods for \$500 and sold them for \$600. What was the increase per cent?

3. Goods were sold for \$90 at a decrease of 20%. What was the cost and what was the decrease?

4. A house was sold for \$9000 at a decrease of 10%. What was the cost and what was the decrease?

5. A horse was sold for \$100 at an increase of 10%. What was the cost and what was the increase?

6. A horse was sold for \$90 at an increase of 10%. What was the cost and what was the increase?

7. A horse was sold for \$110 at a decrease of 10%. What was the cost and what was the decrease?

8. I bought a cow for \$100 and sold her at an increase of 15%. How much was the increase? What was the selling price?

9. I sold a cow for \$125, which represented an increase of 25%. What did it cost me?

10. A tailor sold a suit for \$54, a 20% increase. He sold another suit for \$35, a decrease of the same amount as the increase on the first suit. What per cent was the decrease on the last suit?

11. A man paid \$9000 for a farm and sold it for \$7200. What was the per cent of decrease?

12. A man sold a farm for \$7200, which was 20% more than it cost. What did it cost?

13. A merchant bought goods for \$1000 and sold them at a decrease of 8%. What was the decrease and what was the selling price?

14. A boy bought a pony for \$90 and sold it for \$100. What was his increase per cent?

15. A farmer bought a team of horses for \$500 and sold them for \$450. What was his decrease per cent?

16. I bought a house for \$8000 and sold it at an advance of 25%. What was its selling price?

17. I sold a farm for \$11,500, which was an advance of 15% over the cost. What did it cost?

18. A man sold a horse for \$150, which was 25% more than it cost him. If he had sold the horse for \$200, find per cent of increase or decrease.

19. Books were sold for \$150, which was \$50 less than cost. Find the decrease per cent.

20. By selling a house for \$10,000 a real estate dealer made a margin of \$700. What was his per cent of increase?

21. I bought a house for $\frac{3}{5}$ of its value and sold it for 20% more than its value. Find increase per cent.

22. I sold a lot at an increase of 10%, an increase of \$500. Find the selling price.

23. A man sold a house at an increase of 25%, an increase of \$2000. What was the cost?

24. A man sold a house for \$8100, a decrease of 10% on the transaction. What was the decrease?

25. I sold a lot of goods for \$1200, an increase of 20%. What was the cost?

26. A man sold a lot of goods for \$174, a decrease of 13%. What would have been the increase by selling the goods for \$275?

27. I sold a piece of property for \$2500, which was \$250 less than cost. What would have been my per cent of increase by selling it for \$3000?

28. If a merchant buys goods at 10% off the list price, what is his per cent increase if he sells at list price?

29. I sold for \$300 an article that costs me \$250. What was my per cent of increase?

30. By selling a store for \$750 more than the cost, I got an increase of 15%. What did it cost me?

31. Which of the following series of discounts is more advantageous to the buyer, 25, 10, 5, or 20, 15, 10?

32. Which of the following series of discounts is more advantageous to the buyer, 40, 25, 10, or 50, 25, and 5?

MISCELLANEOUS PROBLEMS IN DISCOUNT (B)

1. I have an invoice for \$369.43, subject to trade discount of 5%, and a regular discount of 6/10, 5/30. For how much shall I draw my check if the invoice is paid within the 10 days?

2. I bought goods at 10%, 15%, and 5% off. If the discounts amounted to \$95.64$\frac{3}{4}$, what was paid for the goods?

3. A merchant bought a lot of cloth at 30¢ per yard. What must the goods be marked in order that he may throw off 25% from the marked price, and still get a 25% increase?

4. If I sell an article at an advance of 25% on the cost, and then discount the bill 5% for cash payments, find the cost if my margin is \$63.75.

5. A certain automobile costs the agent \$1875. What must he sell it for to get a commission of 25% on the selling price?

6. I bought 30 flat-top desks at \$34.50 each; 16 typewriter desks (sanitary) at \$43.50 each; and 24 low roll sanitary 55-inch desks at \$64.50 each. If these prices represent a discount of 40% from the

list price, what is the list price of each kind of desk? If I receive a further discount of 3% for cash, what is the cost to me?

7. A merchant's sales for the year amounted to $85,500. He lost 2% of this amount in bad accounts. The cost of the goods sold was $60,800, and the cost of conducting the business was $15,295. What was his margin of profit for the year? What would be his per cent of margin based upon his sales for the year?

8. Chairs cost $65 a dozen less 20% and 15%. At what price should they retail to get an increase of 30%?

9. I bought goods for $400 less 20%, 10%, and 3%. I sold them for $600 less 40% and 10%. Find my increase or decrease per cent.

10. A bill for $10,000 is due in 60 days with 5% discount on any part of it paid within 30 days. If $4000 is paid within 30 days, what is the balance due at the expiration of the 60 days?

11. A merchant invested $15,000 in stock. After selling $12,000 worth he took an inventory and found the stock on hand was worth $9000. What was his average rate of increase on the goods sold?

12. Bought 10 teachers' desks at $37.50; 5 principals' desks at $73.75; 5 filing cases at $58.50; 10 bookcases at $42.75. Discounts: 15%, 10%, 5%. Find the cost of the entire lot.

13. Find the net amount of the following.

12 bu. alfalfa seed at $16
15 bu. potatoes (early) at $2.30
3 cans of Inoculation at $1.15
10 bu. seed corn at $3.50
Discounts 5%, 3%, 2%.

14. A piano listed at $600 is sold subject to discounts of 40%, 15%, 10%. Find the net price.

15. If the fare from Boston to New York by bus one way is $4.75 and the round trip is $8.50, what is the rate of discount on the round trip?

16. Which is the better for the purchaser, and how much per cent, a series of 40%, 25%, and 10%, or a single discount of 60%?

DRILL IN FUNDAMENTALS (A)

Copy from sight, multiply, and check.

1. 47×157	**16.** 71×148	**31.** 23×324
2. 39×89	**17.** 78×916	**32.** 67×766
3. 67×714	**18.** 89×246	**33.** 45×542
4. 53×216	**19.** 83×498	**34.** 56×652
5. 27×492	**20.** 62×134	**35.** 78×876
6. 21×346	**21.** 67×567	**36.** 89×982
7. 32×767	**22.** 79×890	**37.** 15×514
8. 91×394	**23.** 85×123	**38.** 27×726
9. 98×516	**24.** 64×456	**39.** 37×732
10. 87×678	**25.** 59×789	**40.** 49×945
11. 81×347	**26.** 38×346	**41.** 52×257
12. 77×678	**27.** 53×246	**42.** 63×367
13. 73×901	**28.** 32×357	**43.** 74×478
14. 63×234	**29.** 44×148	**44.** 85×586
15. 68×567	**30.** 55×139	**45.** 96×695

MISCELLANEOUS WORK

1. A certain grade of German silver contains 25.8% nickel. How much German silver can be made by using 5680 lb. of nickel?

2. If 84% of butter is butter fat, how much butter can be made from 1240 lb. of butter fat?

3. The dressed weight of a steer is 62% of the live weight. What must be the live weight to make the dressed weight 950 lb.?

4. If a family pay in rent 20% of their total expenditure, what is their total yearly expenditure if they pay $65 a month for rent?

5. In a certain city it was found by the last census that 21.4% of the population was of school age. The present school census gives 128,462 of school age. At this rate what is the present population?

6. A company invested \$35,000 in goods, sold \$45,000 worth and inventoried the balance for \$15,000. Find the average rate of increase on the goods sold.

7. What single discount is equivalent to a discount series of 40%, 25%, 10%, and 5%?

8. Which is the better offer for the seller, and how much, trade discounts of 20%, 10%, and 3%, on a bill of \$750, or a single discount of 30%?

9. A consignment of goods from France was invoiced at 9800 francs. Total charges were 1240 francs. Part of this consignment was 25 dozen gloves invoiced at 80 francs per dozen. What part of the charges should be borne by the gloves?

10. At the beginning of a year a merchant had goods in stock as per inventory, \$14,860. During the year he bought goods for \$87,250, and paid \$8750 freight and cartage. He sold goods for \$108,650, and his inventory at the end of the year showed goods on hand to the amount of \$21,340. What was his average rate of increase?

CHAPTER XXIII

COMMISSION AND BROKERAGE

333. Importance of Commission and Brokerage. — There are very many agents and brokers working on commission. There are real estate agents, insurance agents. There are general agents, or brokers who will sell or buy almost anything on commission. There are financial brokers who buy and sell stocks and bonds and other securities. There are few applications of percentage to business which are of more general use than commission and brokerage.

334. Agent or Broker, Principal. — An *agent, commission merchant,* or *broker* transacts business for the account of another. The business transacted is usually selling or buying. The individual or company for whom the agent buys or sells is called the *principal.*

335. Commission or Brokerage. — The agent receives a compensation for his work, which is usually a certain per cent of the amount involved in the transaction. This is called his *commission* or *brokerage.*

336. Shipment, Consignment. — A lot of goods sent to an agent to be sold is called a *shipment* by the person who sends the goods, and a *consignment* by the agent who receives it.

337. Consignor, Consignee. — The person sending the goods is called the shipper, or *consignor,* and the one receiving them is called the *consignee.*

338. Account Sales. — When goods are sold by an agent, he renders a statement to his principal, which is called an *account sales.* The account sales contains a full account of the gross amount of the sale, the commission charged, and all incidental expenses.

339. Account Purchase. — When goods are bought by an agent, he renders a statement to his principal, which is called an *account purchase.* The account purchase contains a full account of the amount paid, the commission charged, and all incidental expenses.

340. Gross and Net Proceeds. — The amount received by the agent for goods sold is called *gross proceeds.* This amount less the commission and other charges is called *net proceeds.*

341. Gross and Net Price. — The amount which an agent actually pays for goods is called the *net price,* or *net cost;* and the amount after commission and other expenses connected with the purchase are added is called the *gross price* or *gross cost.*

342. Guaranty. — An agent sometimes guarantees the collection of bills for goods sold by him, or he guarantees the quality of goods bought. For this service a certain amount called *guaranty* is charged.

An example will make these definitions clear.

Arthur Ward ships 50 bales of cotton to his agent, R. A. White, of Galveston, Texas, to be sold on commission. White sells the cotton for $3750. He deducts $37.50 as his commission and $18.75 for guaranteeing collection and remits $3693.75 to Mr. Ward.

In this case, Ward is the *principal;* White, the *agent, commission merchant,* or *broker;* $3750 is the *gross proceeds;* $37.50, the *commission* or *brokerage;* $18.75, the *guarantee;* and $3693.75 the *net proceeds.* Ward speaks of the cotton as a *shipment,* and White speaks of it as a *consignment.* Ward is the *shipper* or *consignor,* and White is the *consignee.*

D. L. Thompson of St. Paul requests Martin A. Reyerson Co. of Chicago to purchase for his account certain structural iron. The iron is bought for $9500, brokerage $95.

In this case Thompson is the *principal;* Reyerson, the *agent;* $9500, the *net price;* $95, the *commission;* and $9500 + $95 = $9595, the *gross price.*

343. Commission and Brokerage Compared with Percentage. — Problems on commission and brokerage are problems in percentage.

The sales price, or purchase price, is the *base.*

The rate commission or brokerage is the *rate.*

The commission, or brokerage, is the *percentage.*

The gross cost is the *amount.*

The net proceeds is the *difference.*

344. To Find the Commission. — To find the commission when the rate and the price are given.

Solution. This is the same problem as finding the percentage when the base and the rate are given. See page 226.

ORAL EXERCISES (A)

Find the commission in each of the following.

	Selling or Buying Price	Commission Rate		Selling or Buying Price	Commission Rate
1.	\$ 50	5%	9.	\$ 300	6%
2.	\$150	3%	10.	\$ 500	8%
3.	\$200	5%	11.	\$1200	4%
4.	\$800	10%	12.	\$2000	5%
5.	\$240	4%	13.	\$1500	5%
6.	\$ 75	2%	14.	\$ 900	6%
7.	\$925	3%	15.	\$2500	7%
8.	\$850	8%	16.	\$5000	5%

WRITTEN EXERCISES (A)

Find the commission in each of the following.

	Selling or Buying Price	Commission Rate		Selling or Buying Price	Commission Rate
1.	\$ 580	$1\frac{1}{2}\%$	14.	\$ 240.64	$3\frac{1}{8}\%$
2.	\$ 220	2%	15.	\$ 960.40	$2\frac{7}{8}\%$
3.	\$1250	$3\frac{1}{2}\%$	16.	\$1252.10	$3\frac{1}{2}\%$
4.	\$ 76.50	5%	17.	\$2786.92	$2\frac{1}{6}\%$
5.	\$ 19.38	4%	18.	\$1459.25	$1\frac{7}{8}\%$
6.	\$ 456.50	6%	19.	\$ 726.71	5%
7.	\$ 925.18	$2\frac{1}{2}\%$	20.	\$2945.60	$2\frac{1}{4}\%$
8.	\$ 750.00	$3\frac{1}{8}\%$	21.	\$ 175	$4\frac{1}{2}\%$
9.	\$1820.00	2%	22.	\$ 860	5%
10.	\$1546.70	$3\frac{1}{6}\%$	23.	\$ 900	$4\frac{1}{2}\%$
11.	\$1946.75	$2\frac{1}{4}\%$	24.	\$3500	$3\frac{1}{2}\%$
12.	\$1845.00	$1\frac{1}{2}\%$	25.	\$ 475	7%
13.	\$1729.50	$3\frac{1}{4}\%$	26.	\$ 385	8%

345. To Find Net Proceeds. Problems. — To find the net proceeds when the selling price and the rate of commission are given.

Solution. This is the same problem as finding the difference when the base and the rate are given.

Thus, if the selling price is \$200, and the rate commission is 2%, the net proceeds is \$200 − \$4 = \$196.

ORAL EXERCISES (A)

Find the net proceeds in each of the following.

	Selling Price	Rate Commission		Selling Price	Rate Commission
1.	\$100	3%	8.	\$1500	2%
2.	150	2%	9.	2000	$2\frac{1}{2}$%
3.	175	$2\frac{1}{2}$%	10.	1750	2%
4.	250	$3\frac{1}{2}$%	11.	2500	$2\frac{1}{4}$%
5.	400	$2\frac{1}{4}$%	12.	3000	3%
6.	650	$3\frac{1}{2}$%	13.	700	4%
7.	900	$1\frac{1}{2}$%	14.	350	5%

WRITTEN EXERCISES (A)

Find the net proceeds in each of the following.

	Selling Price	Rate Commission		Selling Price	Rate Commission
1.	\$ 78.50	5%	7.	\$ 764.50	$2\frac{1}{4}$%
2.	1845.50	2%	8.	925.80	$1\frac{1}{2}$%
3.	2640.20	$2\frac{1}{2}$%	9.	1750.20	$3\frac{1}{2}$%
4.	15,459.10	4%	10.	876.40	$4\frac{1}{4}$%
5.	7642.50	$3\frac{1}{8}$%	11.	2640.60	$3\frac{1}{8}$%
6.	8140.60	$4\frac{1}{4}$%	12.	7948.50	$2\frac{1}{2}$%

13. A real estate agent sold a house for \$7500. How much did he remit to his principal if he deducted a commission of $3\frac{1}{2}$%?

14. A cotton broker sold 85 bales of cotton averaging 490 pounds at $14\frac{1}{4}$ cents a pound. How much did he remit to his principal after deducting a commission of $\frac{3}{4}$%?

15. An agent sold a second-hand automobile for \$940. What were the net proceeds from the sale if the agent deducted a commission of $7\frac{1}{2}$%?

346. To Find the Gross Price. — To find the total sales or purchases when rate and commission are given.

Solution. This is the same problem as finding the base when the rate and the percentage are given. See page 230.

ORAL EXERCISES (A)

Find the total sales in each of the following.

	Rate	Commission		Rate	Commission
1.	5%	$10	4.	1%	$2500
2.	2%	8	5.	2%	2000
3.	3%	12	6.	3%	750

EXERCISES (A)

Find the total sales in each of the following.

	Rate	Commission		Rate	Commission
1.	$2\frac{1}{2}\%$	$ 48	4.	$2\frac{1}{8}\%$	$620
2.	2%	56	5.	$2\frac{5}{8}\%$	721
3.	$3\frac{1}{4}\%$	720	6.	$3\frac{1}{4}\%$	214

7. The average rate of commission on sales charged by a real estate agent is $4\frac{1}{2}$ per cent. What must be his total sales in a year to obtain a gross income of $18,000?

347. To Find Net Price in Buying. — To find the net price, the gross price and the rate commission being given.

Solution. This is the same problem as finding the base when the rate and the amount are given. See page 232.

Thus, if $5000 is sent to an agent to invest in grain at a commission of $1\frac{3}{4}\%$, the amount actually invested in grain is $5000 ÷ 1.0175 = $4914.00.

WRITTEN EXERCISES (A)

Find the net price in each of the following.

	Rate Commission	Gross Cost		Rate Commission	Gross Cost
1.	2%	$ 850	5.	$2\frac{1}{6}\%$	$ 980
2.	3%	720	6.	$3\frac{1}{2}\%$	1750
3.	$2\frac{1}{2}\%$	1520	7.	$2\frac{1}{4}\%$	2520
4.	$3\frac{1}{4}\%$	1728	8.	3%	15,426

348. Account Sales. — The following are examples of account sales and account purchases (see §§ 338,339).

Cleveland, O., Feb. 1, 1949.

Sold for the Account of THOMAS COLWIN & Co.

By the SHERWOOD Co.

Terms of Sale: N/60, 3/10.

Jan.	1	1949	25 lb. $\frac{5}{8}$″ 11 semi. fin. Hexagon Nuts est. 150	.85 C		1	28	
			250 lb. $\frac{5}{16}$″ 18 semi. fin. Hexagon Nuts est. 400	.30 C		1	20	
			400 lb. $\frac{1}{4}$″ 20 semi. fin. Hexagon Nuts est. 300	.24 C			72	
			700 $\frac{1}{2}$″ × 3″ sq. hd. mach. bolts	.87			87	
			900 $\frac{1}{2}$″ × 6″ sq. hd. mach. bolts	1.18		1	18	
			Charges Commission 10% Cartage $1.50					

349. Account Purchase.

Boston, Mass., July 1, 1949.

Purchased for the Account of

WEST TECH. HIGH SCHOOL, E. 4th and 6th Ave.

By GEO. STONUS & Co.

			3 doz. 6″ outside firm joint calipers (tempered)	@ $9.68	$29	04		
			3 doz. 6″ outside firm joint calipers (tempered)	@ 9.68	29	04		
			$\frac{1}{2}$ doz. 12″ outside firm joint calipers (tempered)	@ 13.20	6	60		
			$\frac{1}{2}$ doz. 12″ inside firm joint calipers (tempered)	@ 13.20	6	60		
			1 doz. Combination squares and protractors complete with center heads 12″ scales		48	80		
			1 doz. 12″ scales for 12″ combination square stock		17	00		
			1 doz. 18″ scales for 12″ combination square stock		24	40		
			Charges Cartage $5.00 Commission 3%		$161	48		

WRITTEN EXERCISES (A)

1. Prepare an account sales from the following data, using your own name as agent, and any name you wish as principal.

On June 8, you received the following consignment and disposed of it at prices as follows:

Six thousand Millward Needles #8 @ $1.80 per M; 6000 Millward Needles #7 @ $1.80 per M; 12 doz. spools Black O.N.T. Cotton #60 @ $.78 per doz.; 26 doz. spools White O.N.T. Cotton #70 @ 78¢ per doz.; 1 great gross Fish eye pearl buttons $\frac{1}{4}''$ @ 90¢ per gross; 1 great gross De Long Fasteners, black med., 1 great gross white med., each at 35¢ a gross. Your commission is 2%.

2. Prepare an account purchase from the following facts.

William Bros. of Toledo bought for the account of Sampson Bros. of Columbus,

300 light hard Maple pins, $\frac{1}{4}''$ by 36", at 80¢ per C (hundred)
300 light hard Maple pins, $\frac{1}{2}''$ by 36", at 80¢ per C
300 light hard Maple pins, $\frac{3}{8}''$ by 36", at 85¢ per C
200 light hard Maple pins, $\frac{5}{8}''$ by 36", at $1.35 per C;

less 50%. To be delivered on or before September 1, F.O.B. Columbus.

ORAL EXERCISES (A)

1. My agent sells goods to the amount of $5000. What is his commission at 4%?

2. I sent my agent $525 to invest in cloth at 10¢ a yard. How many yards can he buy if his commission is 5%?

3. A man sold real estate on a commission of 5% and sent the owner $9500. What was the selling price of the real estate?

4. An agent sold my house for $6000. What was his commission at $2\frac{1}{2}$%?

5. An agent's commission amounts to $750, at 3%. What was the selling price of the property?

6. An agent receives $204 to buy syrup. At $2 a gallon, how many gallons can he buy if his commission is 2%?

7. A lawyer, in attempting to collect a bill of \$800, compromised for 60%. What was his commission at 3%?

8. A house and lot were sold for \$7500 and the owner received \$7200 as the net proceeds. What was the rate of commission?

9. At a commission of 3% an agent sold cotton and remitted to his principal \$3880. What was the sales price?

10. A wholesale merchant sent his agent in New York \$3090 to buy hats at \$25 a dozen. If the agent charges 3% for buying, what was his commission and how many dozen hats did he buy?

11. My agent received \$126 for collecting a debt of \$2520. What was his rate of commission?

12. Amount of sale, \$6000, rate of commission, 4%. Find commission.

13. Amount of commission, \$500, rate of commission, 2%. Find amount of sale.

14. Amount of sale, \$8000, commission \$200. Find rate of commission.

15. Amount remitted to agent, \$3060, commission 2%. Find commission and amount invested.

16. Amount invested \$8000, commission 5%. Find amount sent to agent.

17. If I sold a farm of 150 acres for \$160 an acre and gave 4% commission, what would be the amount of the commission and the net proceeds?

18. If I sold the same farm for \$150 an acre with 5% commission on the first \$1000 and $2\frac{1}{2}$% on the balance, what would be the proceeds and the commission?

19. Find the net price, when the rate of commission is 3% and the gross cost is \$1236.

20. Find the total sales when the rate of commission is 4% and the commission is \$48.

21. I sold a house for \$7500. Find the net sale, if the rate of commission is 3%.

MISCELLANEOUS PROBLEMS (B)

1. Sent my agent \$2520 to invest in cotton cloth at $12\frac{1}{2}¢$ a yard. How many yards can he buy after deducting his commission of 2%?

2. After paying charges amounting to \$17.20 and retaining a commission of 2%, an agent returned to his principal \$774.15 as the net proceeds of a sale. What was the sales price?

3. A merchant bought through an agent 720 yards of carpeting at \$2.25 a yard, and paid the agent $\frac{3}{4}$% commission. The freight amounted to \$7.37. At what price per yard must he sell the carpet to get an increase of 20%?

4. An agent sold apples for \$984 on a commission of 4%. He invested the net proceeds in flour, charging a commission of $2\frac{1}{2}$% for buying. What was the entire commission?

5. An agent sold a cargo of wheat on a commission of $1\frac{1}{2}$%. He sent the owner \$44,325. What was his commission?

6. Sold lumber for \$1,224 on a commission of 4%. Invested the net proceeds in wheat after retaining my commission of 2% for buying. Find the value of the wheat, and the total commission.

7. A man sent his broker \$6007.50 to invest in U.S. bonds. What amount did the broker invest if he charged $\frac{1}{8}$% commission?

8. A commission merchant received January 1, 14,000 feet of #1 and #2 lumber, kiln dried, from the Southern Mills of Atlanta, Ga The freight was \$140.50; drayage, \$50; insurance $\frac{1}{6}$% of the gross sales. January 10, he sold 4 M @ \$100 per M; Jan. 20, 5 M @ \$102.50; and Jan. 25, 5 M at \$110 per M. Storage charges were \$20.25 and the agent's commission was $2\frac{1}{2}$%. Prepare an account sales, showing the transaction, and the net proceeds.

9. Bought through Brown & Co. 18 quart cans Bridgeport Penetrating Stains @ 56¢; 6 lb. cans prepared wax @ 30¢; 30 lb. cans Bridgeport Standard #10 wood filler @ 18¢; 50 lb. cans Special White wood filler @ 18¢. Brown & Co.'s commission was 2% and drayage \$2. Prepare an account purchase, using the above facts, and any name you may wish as principal.

CHAPTER XXIV

SIMPLE INTEREST

350. Frequency of Problems in Interest. — Of all the applications of percentage, interest is the most general. There are few people who do not need to figure interest at some time in their lives. Next to the four fundamental operations the subject of interest is probably the most important to the business man.

351. Interest. — A man borrows \$100 and at the end of one year he is obliged to pay back \$106. The extra \$6 which he pays is called *interest.*

352. Reasons for Paying Interest. — The custom of paying interest on borrowed money is now universal, and one cannot borrow in a business way without doing so.

On reflection the reasons for paying interest are obvious. If money could be borrowed without interest a man in a city could borrow, say \$1000, to pay for the grading and beautifying of the lot around his house, and for the period between the borrowing of the money and the repayment of the loan, he would enjoy the increased beauty of his home with no extra expense to himself. The person who loaned the money, meanwhile, would be without its use. Again, a farmer might borrow money to pay for clearing land, and up to the time of repaying the loan the increased production from the land would be clear gain. Under such circumstances, everybody would want to borrow and nobody would want to lend.

353. Interest on Investments in General. — The element of interest is by no means confined to cases where money has been loaned at a given rate. Any one engaging in business calculates to make a *profit* which first of all shall pay a reasonable rate of interest on the money invested. Stocks and bonds are bought because they yield dividends, and for no other reason. Every business investment is made on the expectation of a return that may be regarded as interest.

354. Principal, Amount. — The amount on which interest is paid is called the *principal.* The principal plus the interest is called the *amount.*

355. Rate of Interest. — A certain per cent of the amount borrowed is paid each year as interest.

The rate per cent of the principal which is paid each year as interest is called the *rate of interest.*

356. Time. — In computing interest *time* is an essential element. That is, the interest is figured as so many per cent for a *certain unit of time.* The unit of time used in specifying the rate of interest is nearly always one year.

Thus, when we say that the rate of interest is 6%, we mean that 6% of the principal is paid *each year* as interest.

357. Interest Compared with Percentage. — When the time is one year, problems in interest are problems in ordinary percentage.

The principal is the base.

The rate of interest is the rate per cent.

The interest is the percentage.

The amount is the same in both.

ORAL EXERCISES (A)

Find the interest in each of the following, the time in each being one year.

	Principal	Rate of Interest		Principal	Rate of Interest		Principal	Rate of Interest
1.	\$ 100	5%	**10.**	\$1650	5%	**19.**	\$ 600	3%
2.	200	6%	**11.**	240	7%	**20.**	780	10%
3.	300	7%	**12.**	1500	4%	**21.**	900	12%
4.	150	4%	**13.**	675	8%	**22.**	1600	5%
5.	1000	3%	**14.**	900	9%	**23.**	1350	6%
6.	1200	$3\frac{1}{2}$%	**15.**	1400	$6\frac{2}{5}$%	**24.**	750	4%
7.	600	$4\frac{1}{2}$%	**16.**	725	5%	**25.**	2000	6%
8.	800	4%	**17.**	1000	4%	**26.**	2500	7%
9.	2000	10%	**18.**	200	8%	**27.**	3000	8%

WRITTEN EXERCISES (A)

Find the interest in each of the following, the time being one year.

	Principal	Rate		Principal	Rate		Principal	Rate
1.	$ 850	7%	9.	$ 1728	5%	17.	$ 6440	6%
2.	975	6%	10.	956	6%	18.	25000	4%
3.	1610	3%	11.	1728	4%	19.	62450	$4\frac{1}{4}$%
4.	724	10%	12.	54050	7%	20.	18720	$6\frac{1}{2}$%
5.	218	8%	13.	1726	8%	21.	36485	5%
6.	726	9%	14.	3450	9%	22.	7500	$6\frac{2}{3}$%
7.	450	4%	15.	7652	10%	23.	50025	$9\frac{1}{5}$%
8.	12400	5%	16.	2940	12%	24.	6525	$7\frac{2}{5}$%

358. Time Other Than One Year. — In long time loans with simple interest, the interest is payable yearly and the time involved in each computation is therefore one year. Problems arise, however, in which the time is several years. In such cases, the interest is the interest for one year multiplied by the number of years.

Thus, the interest on $800 at 6% for four years is 4 times the interest for one year, or $4 \times \$48 = \$192$.

ORAL EXERCISES (A)

Find the interest on each of the following.

	Principal	Rate	Time		Principal	Rate	Time
1.	$1650	5%	2 yr.	12.	$9142	$6\frac{1}{2}$%	$3\frac{1}{3}$ yr.
2.	1240	4%	3 yr.	13.	7621	4%	2 yr.
3.	1728	3%	4 yr.	14.	8540	5%	$3\frac{1}{4}$ yr.
4.	875	$5\frac{1}{5}$%	6 yr.	15.	1725	8%	$2\frac{1}{2}$ yr.
5.	540	$3\frac{1}{2}$%	2 yr.	16.	950	10%	$2\frac{1}{5}$ yr.
6.	1620	$6\frac{1}{2}$%	3 yr.	17.	450	5%	$3\frac{2}{5}$ yr.
7.	725	7%	4 yr.	18.	2940	$7\frac{1}{2}$%	4 yr.
8.	2160	$4\frac{1}{3}$%	5 yr.	19.	3625	5%	$2\frac{4}{5}$ yr.
9.	2756	5%	4 yr.	20.	4008	6%	4 yr.
10.	3840	6%	$3\frac{1}{2}$ yr.	21.	800	7%	$4\frac{1}{2}$ yr.
11.	2760	3%	$5\frac{1}{2}$ yr.	22.	1600	5%	$3\frac{1}{4}$ yr.

359. Time Less Than One Year. — A very large number of loans are made for less than one year, and such loans give rise to the more difficult problems in interest.

To find the interest for a fraction of a year, the interest for a whole year is multiplied by this fraction.

Thus, to find the interest on \$500 at 6% for $\frac{1}{3}$ of a year, the interest for one year, or \$30, is multiplied by $\frac{1}{3}$, obtaining \$10.

Similarly to find the interest on \$500 at 6% for $\frac{1}{4}$ of a year, \$30 is multiplied by $\frac{1}{4}$, obtaining \$7.50.

In the following exercises count one month as one twelfth of one year.

ORAL EXERCISES (A)

	Principal	Rate	Time		Principal	Rate	Time
1.	\$ 300	6%	6 mo.	10.	\$3000	2%	1 mo.
2.	500	5%	4 mo.	11.	1600	3%	6 mo.
3.	400	8%	9 mo.	12.	1400	6%	5 mo.
4.	600	4%	5 mo.	13.	7000	7%	8 mo.
5.	700	6%	6 mo.	14.	8000	8%	7 mo.
6.	900	10%	8 mo.	15.	270	3%	4 mo.
7.	1000	5%	6 mo.	16.	1900	10%	9 mo.
8.	1200	6%	10 mo.	17.	5000	6%	4 mo.
9.	800	5%	2 mo.	18.	8000	5%	8 mo.

WRITTEN EXERCISES (A)

	Principal	Rate	Time		Principal	Rate	Time
1.	\$4200	7%	9 mo.	10.	\$6750	8%	4 mo.
2.	2600	4%	8 mo.	11.	2486	6%	5 mo.
3.	4500	5%	4 mo.	12.	1950	4%	6 mo.
4.	7600	6%	6 mo.	13.	7822	5%	8 mo.
5.	2100	3%	7 mo.	14.	9160	9%	4 mo.
6.	2150	8%	9 mo.	15.	7480	5%	9 mo.
7.	1675	9%	2 mo.	16.	5400	4%	10 mo.
8.	970	5%	11 mo.	17.	3860	5%	7 mo.
9.	2480	2%	6 mo.	18.	5840	7%	9 mo.

MISCELLANEOUS WORK (B)

1. Find the amount due the owner of a piece of property that sells for \$6000, when 3% is allowed for selling.

2. Find the commission on the sale of 200 dozen eggs at 45¢ a dozen, when 2% is allowed for selling.

3. Find the proceeds on 200 bbl. of apples at \$5.25 a bbl. after deducting 2% for selling, \$40 for freight, and \$25 for carting.

4. What is the rate of income if an investment of \$108.50 yields \$6 per year? (\$6 is how many per cent of \$108.50?)

5. How much must I pay for a permanent source of income of \$7 to yield me 6% on my investment? (\$7 is 6% of what amount?)

6. A merchant bought shirts at \$15 per dozen. At what price per piece must he sell to get an increase of 50%?

7. A retail dealer buys goods listed at \$85 at discounts of 20%, 15%, and 10%. At what price must he sell the goods to get an increase of 35% on the purchase price?

8. A retail dealer bought goods for \$5840, giving a commission of $1\frac{1}{4}$% for buying. He paid \$840 for freight and other expenses. For how much must he sell the goods to get an increase of 30%?

9. A certain retail dealer in furniture sold a bill of goods for \$204.80. The marking price of the articles sold would make them amount to \$296.50. What was the rate per cent of his reduction? If this sale brought the dealer an increase of 17%, what would have been his increase per cent if he had sold at the marking price?

10. A retail dealer wishes to mark his goods so that he may reduce the marking price by 15% to his customers and still get a 25% increase on the sale. How many per cent above his purchase must he mark the goods?

11. One man sells goods at an increase of 35% of the buying price and another sells goods at a margin of 25% of the selling price. Which makes the greater margin? If each of these men sells goods bought for \$1000, what is the difference in their margins?

360. The Commercial Year and Ordinary Interest. — For the purpose of computing interest for short periods the so-called commercial year is usually used. Each month is counted as 30 days and the year as 360 days, or 12 months of 30 days each. Interest computed on this basis is called *ordinary interest* or *bankers' interest.*

361. The Six Per Cent Principle. — *Six per cent for one year is the same rate as one per cent for two months.*

This is a fundamental principle which forms the basis for the most important methods for computing interest when the time is expressed in days or in months and days.

Under this principle the interest at 6% for two months is obtained by pointing off two places in the principal.

362. Bankers' Sixty-day Method. — When the rate of interest is six per cent, the work may be simplified by special devices as is shown in the following examples.

Example 1. Find the interest on \$2460 at 6% for 7 months.

Solution.

24	60	interest for 2 months
73	80	interest for 6 months
12	30	interest for 1 month
86	10	required interest

A vertical line is used to separate dollars and cents. Pointing off two places gives the interest for 2 months. Multiplying by 3 gives the interest for 6 months, etc.

Example 2. Find interest on \$780 at 6% for 35 days.

Solution.

7	80	interest for 2 months
3	90	interest for 30 days ($\frac{1}{2}$ of 2 months)
	65	interest for 5 days ($\frac{1}{6}$ of 30 days)
4	55	interest for 35 days

Example 3. Find interest on \$9300 at 6% for 2 days.

Solution.

93	00	interest for 60 days
3	10	interest for 2 days ($\frac{1}{30}$ of 60 days)

Example 4. Find the interest on \$8460 at 6% for 19 days.

Solution.

84	60	interest for 60 days
28	20	interest for 20 days ($\frac{1}{3}$ of 60 days)
1	41	interest for 1 day ($\frac{1}{20}$ of 20 days)
26	79	interest for 19 days (20 − 1 = 19)

WRITTEN EXERCISES (B)

Using the bankers' sixty-day method, find the interest in each of the following examples, the rate in each case being six per cent.

	Principal	Time		Principal	Time
1.	$ 7825	45 da.	**28.**	$ 1725	87 da.
2.	4240	10 da.	**29.**	2190	95 da.
3.	5781	93 da.	**30.**	650	48 da.
4.	7945	80 da.	**31.**	3761	4 mo.
5.	6256	30 da.	**32.**	2950	3 mo.
6.	8834	21 da.	**33.**	4675	6 mo.
7.	3785	63 da.	**34.**	8762	7 mo.
8.	4920	19 da.	**35.**	19,440	8 mo.
9.	2580	84 da.	**36.**	6787	9 mo.
10.	9786	100 da.	**37.**	9160	11 da.
11.	10,922	27 da.	**38.**	7580	10 da.
12.	6145	75 da.	**39.**	4216	5 mo.
13.	3711	40 da.	**40.**	2852	4 mo.
14.	7500	36 da.	**41.**	1987	75 da.
15.	9862	96 da.	**42.**	4515	18 mo.
16.	8756	17 da.	**43.**	1396	28 da.
17.	4084	15 da.	**44.**	5009	46 da.
18.	2560	82 da.	**45.**	5406	17 da.
19.	1250	76 da.	**46.**	5412	81 da.
20.	4962	42 da.	**47.**	7645	7 mo.
21.	5750	45 da.	**48.**	1954	3 mo.
22.	6900	104 da.	**49.**	8760	4 mo.
23.	7624	78 da.	**50.**	5140	4 mo.
24.	8942	19 da.	**51.**	9287	9 mo.
25.	9540	27 da.	**52.**	1648	11 mo.
26.	11,500	45 da.	**53.**	1721	10 mo.
27.	14,760	76 da.	**54.**	6828	4 mo.

363. Table for Bankers' Sixty-day Method. — In using the bankers' sixty-day method, much depends on the skill with which the interest for a certain number of days is obtained from the interest for 60 days. Thus it is difficult to find the interest direct for 7 days, since this involves multiplying by $\frac{7}{60}$. It is easy, however, to get the interest for 6 days ($\frac{1}{10}$ of 60) and then add to it the interest for one day ($\frac{1}{6}$ of the interest for 6 days).

The interest for any number of days may be found in several ways. Thus 8 days = 6 days + 2 days = 10 days − 2 days.

The following table is merely suggestive. In many cases other combinations may be equally simple.

$1 = \frac{1}{60}$ of 60	$11 = 10 + 1$	$21 = 20 + 1$
$2 = \frac{1}{30}$ of 60	$12 = \frac{1}{5}$ of 60	$22 = 20 + 2$
$3 = \frac{1}{20}$ of 60	$13 = 12 + 1$	$23 = 24 - 1$
$4 = 3 + 1$	$14 = 12 + 2$	$24 = 4 \times 6$
$5 = 6 - 1$	$15 = \frac{1}{4}$ of 60	$25 = 24 + 1$
$6 = \frac{1}{10}$ of 60	$16 = 15 + 1$	$26 = 24 + 2$
$7 = 6 + 1$	$17 = 18 - 1$	$27 = 24 + 3$
$8 = 6 + 2$	$18 = 3 \times 6$	$28 = 30 - 2$
$9 = 10 - 1$	$19 = 18 + 1$	$29 = 30 - 1$
$10 = \frac{1}{6}$ of 60	$20 = \frac{1}{3}$ of 60	$30 = \frac{1}{2}$ of 60

WRITTEN EXERCISES

1–30. Find the interest at 6% on \$189.40 for 1 day, 2 days, etc., up to 30 days.

Suggestion. To divide 189.40 by 60 simply divide by 6 and start writing the numbers one place toward the right.

Thus: $\begin{array}{r|l} 189 & 40 \\ 3 & 157 \end{array}$.

In similar manner divide by 30 and by 20.

31–60. Find the interest on \$91,400 at 6% for 31 days, 32 days, etc., up to 60 days.

Suggestion. First make a table like the one above for the numbers 31 to 60. That is, $31 = 30 + 1$, $32 = 30 + 2$, etc. It is important to notice that 6 is $\frac{1}{10}$ of 60, that $36 = 6 \times 6$, $42 = 7 \times 6$, etc.

364. Checking. — The bankers' six per cent method lends itself readily to checking.

Example. Find the interest on \$3940 at 6% for 37 days.

Solution.

39	40	interest for 2 mo.
19	70	interest for 30 days
3	94	interest for 6 days
	66	interest for 1 day
24	30	interest for 37 days (30 + 6 + 1 = 37)

Check

39	40	interest for 60 days
13	133	interest for 20 days
26	27	interest for 40 days
1	97	interest for 3 days
24	30	interest for 37 days (40 − 3 = 37)

WRITTEN EXERCISES (B)

Find the interest of the following and check each, the interest being 6% in each example.

	PRINCIPAL	TIME		PRINCIPAL	TIME
1.	\$8170	22 da.	17.	\$8561	26 da.
2.	390	17 da.	18.	9250	23 da.
3.	750	21 da.	19.	6280	76 da.
4.	2620	45 da.	20.	3780	43 da.
5.	1780	19 da.	21.	950	16 da.
6.	1950	60 da.	22.	675	12 da.
7.	8720	27 da.	23.	729	75 da.
8.	9640	42 da.	24.	9456	84 da.
9.	8525	87 da.	25.	2375	96 da.
10.	1921	99 da.	26.	12,820	48 da.
11.	1726	102 da.	27.	11,115	42 da.
12.	540	29 da.	28.	775	37 da.
13.	927	40 da.	29.	629	29 da.
14.	856	75 da.	30.	826	31 da.
15.	1450	87 da.	31.	2250	37 da.
16.	1728	40 da.	32.	1940	96 da.

365. Finding Interest at Rates Other than Six Per Cent. — To find interest at other rates than six per cent by the bankers' sixty-day method, first find the interest at six per cent. The following examples illustrate the method.

Example 1. Find the interest on \$2370 at 5% for 52 days.

Solution.

23	70	interest for 60 days at 6%
3	95	interest for 10 days at 6%
19	75	interest for 50 days at 6% ($5 \times 10 = 50$)
	79	interest for 2 days at 6% ($2 = \frac{1}{5}$ of 10)
20	54	interest for 52 days at 6% ($50 + 2 = 52$)
3	42	$\frac{1}{6}$ of the interest at 6%
17	12	interest for 52 days at 5%

The interest at 5% is found by deducting one-sixth of the interest at 6%.

Example 2. Find the interest on \$1270 for 74 days at $4\frac{1}{2}$%.

Solution.

12	70	interest for 60 days at 6%
2	54	interest for 12 days at 6%
	42	interest for 2 days at 6%
15	66	interest for 74 days at 6%
3	91	$\frac{1}{4}$ of interest for 74 days at 6%
11	75	interest for 74 days at $4\frac{1}{2}$%

The interest at $4\frac{1}{2}$% is found by deducting $\frac{1}{4}$ of the interest at 6%.

Example 3. Find the interest on \$7850 for 70 days at $6\frac{1}{2}$%.

Solution.

78	50	interest for 60 days at 6%
13	08	interest for 10 days at 6%
91	58	interest for 70 days at 6%
7	63	$\frac{1}{12}$ of interest for 10 days at 6%
99	21	

In this manner the interest can be found very simply for any of the rates which are generally prevalent.

Thus, to find interest at 4%, find the interest at 6% and deduct $\frac{1}{3}$ of it. Similarly:

For 8% add $\frac{1}{3}$	for 7% add $\frac{1}{6}$
for 9% add $\frac{1}{2}$	for $7\frac{1}{2}$% add $\frac{1}{4}$
for 10% add $\frac{2}{3}$	for $5\frac{1}{2}$% subtract $\frac{1}{12}$

There are, however, many rates of interest for which this method is somewhat complicated. Thus to find interest at $4\frac{3}{4}$% we should have to subtract $\frac{1}{4}$ to get the interest at $4\frac{1}{2}$% and then add $\frac{1}{24}$ of 6% ($\frac{1}{4}$ of 1%) to get the required interest. To find interest at $6\frac{3}{4}$% we should add $\frac{1}{6}$ and then subtract $\frac{1}{24}$.

WRITTEN EXERCISES (C)

Find the interest in each of the following.

	Principal	Rate	Time		Principal	Rate	Time
1.	$1840	5%	53 da.	29.	$6112	$7\frac{3}{4}\%$	78 da.
2.	780	$5\frac{1}{2}\%$	75 da.	30.	76.40	$3\frac{1}{2}\%$	87 da.
3.	1250	3%	27 da.	31.	77.53	$5\frac{1}{4}\%$	46 da.
4.	725	6%	15 da.	32.	80.80	$6\frac{1}{2}\%$	29 da.
5.	841	$6\frac{1}{2}\%$	18 da.	33.	1120	$8\frac{1}{4}\%$	44 da.
6.	784	$4\frac{1}{2}\%$	81 da.	34.	760	$5\frac{1}{4}\%$	54 da.
7.	2176	8%	62 da.	35.	260.50	$4\frac{1}{4}\%$	64 da.
8.	1452	7%	72 da.	36.	2300	$8\frac{1}{2}\%$	29 da.
9.	1948	$8\frac{1}{2}\%$	80 da.	37.	1260	$7\frac{1}{4}\%$	48 da.
10.	962	$7\frac{1}{2}\%$	121 da.	38.	517.60	$6\frac{1}{4}\%$	76 da.
11.	2751	$6\frac{1}{4}\%$	145 da.	39.	527.60	$5\frac{1}{2}\%$	54 da.
12.	2053	$4\frac{1}{4}\%$	92 da.	40.	1250	$5\frac{3}{4}\%$	81 da.
13.	3188	$6\frac{3}{4}\%$	27 da.	41.	1275	$8\frac{1}{4}\%$	76 da.
14.	1308	10%	45 da.	42.	25.62	6%	59 da.
15.	3702	5%	73 da.	43.	7115	4%	61 da.
16.	2338	4%	96 da.	44.	2612	5%	76 da.
17.	5983	3%	81 da.	45.	4012	7%	44 da.
18.	2932	8%	43 da.	46.	3650	8%	87 da.
19.	9520	$6\frac{1}{2}\%$	37 da.	47.	5042	$9\frac{1}{2}\%$	62 da.
20.	2690	9%	29 da.	48.	4242	$5\frac{1}{4}\%$	29 da.
21.	8610	8%	42 da.	49.	7800	$3\frac{3}{4}\%$	47 da.
22.	7452	4%	60 da.	50.	5500	$6\frac{1}{2}\%$	39 da.
23.	2158	5%	76 da.	51.	3800	$5\frac{1}{2}\%$	58 da.
24.	6696	$4\frac{1}{2}\%$	28 da.	52.	9500	$5\frac{1}{4}\%$	87 da.
25.	1275	$6\frac{3}{4}\%$	85 da.	53.	1250	$6\frac{1}{4}\%$	49 da.
26.	31.25	7%	98 da.	54.	3250	6%	89 da.
27.	675.90	$9\frac{1}{4}\%$	26 da.	55.	2500	7%	37 da.
28.	250.60	$8\frac{1}{2}\%$	42 da.	56.	7500	8%	76 da.

366. When the Time Is 6 Days, 60 Days, 600 Days, or 6000 Days. — Inasmuch as finding the interest when the rate is 6% is the basis for finding the interest for any rate, the 6% case deserves further attention. We have already seen, § 361, that when the time is 60 days the interest at 6 per cent per annum is 1% of the principal, and that the interest is therefore found in this case by pointing off two places in the principal.

It is evident that if the time is 6 days ($\frac{1}{10}$ of 60 days) the interest is found by pointing off three places in the principal; if the time is 600, the interest is found by pointing off one place; and if the time is 6000 days the interest equals the principal.

Example 1. Find the interest on \$1240 for 43 days, at 6%.

Solution.

1	240	interest for 6 days
8	680	interest for 42 days
	207	interest for 1 day
8	887	8.89 = interest for 43 days

Example 2. Find the interest on \$9845 for 10 mo. at 6%.

Solution.

984	50	interest for 600 days or 20 months
492	25	interest for 10 months

Example 3. Find the interest on \$2160 for 1 yr. 7 mo. 15 da. at 6%.

Solution.

216	00	interest for 600 days or 20 mo.
10	80	interest for 1 mo.
205	20	interest for 1 yr. 7 mo. (19 mo.)
5	40	interest for 15 days ($\frac{1}{2}$ of mo.)
210	60	interest for 1 yr. 7 mo. 15 days

These examples are solved by means of the principles stated formally in § 367.

367. A General Principle. — 1. *To find the interest when the time is six days and the rate six per cent, point off three places in the principal.*

2. *To find the interest when the time is sixty days and the rate six per cent, point off two places in the principal.*

3. *To find the interest when the time is six hundred days and the rate six per cent, point off one place in the principal.*

Judicious use of this principle will greatly facilitate finding interest when the rate is 6%.

368. When the Time Is an Aliquot Part of 6 Days, 60 Days, or 600 Days. — The method in this case is obvious and is illustrated in the following examples. In the case of 60 days, examples have already been given. See § 363.

Example 1. Find the interest on \$2480 for 2 days at 6%.

Solution.

2	48	interest for 6 days
	83	interest for 2 days ($\frac{1}{3}$ of 6 days)

Example 2. Find the interest on \$3450 for 5 mo. at 6%.

Solution.

345	00	interest for 600 days (20 mo.)
86	25	interest for 5 mo. ($\frac{1}{4}$ of 20 mo.)

369. Combinations of Aliquot Parts of 6, 60, and 600 Days.

Example 1. Find the interest on \$17,650 for 23 days at 6%.

Solution.

176	50	interest for 2 mo.
58	83	interest for 20 days ($\frac{1}{3}$ of 60 days)
8	825	interest for 3 days ($\frac{1}{2}$ of 6 days)
67	655	interest for 23 days

Example 2. Find the interest on \$125,000 for 11 mo. 17 da. at 6%.

Solution.

12,500	00	interest for 20 mo.
6,250	00	interest for 10 mo.
625	00	interest for 1 mo.
312	50	interest for 15 da.
41	67	interest for 2 da. ($\frac{1}{3}$ of 6 da.)
7,229	17	interest for 11 mo. 17 da.

WRITTEN EXERCISES (B)

Find the interest in each of the following, the rate being 6%.

	Principal	Time		Principal	Time		Principal	Time
1.	\$9860	47 da.	**10.**	\$2544	95 da.	**19.**	\$8245	4 mo. 21 da.
2.	2440	50 da.	**11.**	8326	82 da.	**20.**	2978	35 da.
3.	1650	2 mo.	**12.**	7592	27 da.	**21.**	8476	19 da.
4.	1726	3 mo. 10 da.	**13.**	12,450	19 da.	**22.**	9240	56 da.
5.	1945	92 da.	**14.**	7942	29 da.	**23.**	4629	72 da.
6.	8760	26 da.	**15.**	8860	40 da.	**24.**	3480	66 da.
7.	9241	76 da.	**16.**	9242	75 da.	**25.**	4950	21 da.
8.	1600	41 da.	**17.**	7851	4 mo.	**26.**	7822	118 da.
9.	7650	87 da.	**18.**	2940	6 mo. 10 da.	**27.**	7519	220 da.

370. Interest Table. — Bankers and others who need to compute many problems in interest frequently use an *interest table*, part of which is given below.

5%	60 Days	61 Days	62 Days	63 Days	64 Days
1	00833333	00847222	00861111	00875	00888888
2	01666667	01694444	01722222	01750	01777777
3	02500000	02541666	02583333	02625	02666666
4	03333333	03388889	03444444	03500	03555555
5	04166667	04236111	04305555	04375	04444444
6	05000000	05083333	05166666	05250	05333332
7	05833333	05930555	06027777	06125	06222221
8	06666666	06777777	06888888	07000	07111110
9	07500000	07624999	07749999	07875	07999999

5%	65 Days	66 Days	67 Days	68 Days	69 Days
1	00902777	00916666	00930555	00944444	00958333
2	01804555	01833333	01861111	01888888	01916666
3	02708333	02749999	02791666	02833333	02874999
4	03611110	03666666	03722222	03777777	03833333
5	04513888	04583333	04652777	04722222	04791666
6	05416666	05499999	05583333	05666666	05749999
7	06319443	06416666	06513888	06611110	06708333
8	07222221	07333332	07444444	07555555	07666667
9	08124999	08249999	08374999	08499999	08624999

The interest given in the table is ordinary bankers' interest. The method of using it is illustrated in the following example.

Example. Find interest on \$38,940 at 5% for 61 days.

Interest on \$30,000 is	\$254	17
Interest on 8,000 is	67	78
Interest on 900 is	7	62
Interest on 40 is		34
The required interest	\$329	91

In the table no decimal points are given. By a little practice they can be put in without any trouble.

The **5%** in heavy type at the top of the page indicates that on that page the table gives 5% interest. To find the interest on, say, \$700, for 61 days, look under *61 days* and in the line in which stands the

number 7 in the left margin, where we find the number 05930555. We see at a glance that the decimal point must be placed between the 5 and the 9. That is, the required interest is \$5.93. It requires only a little common sense to see that the decimal point could not be placed elsewhere. That is, the interest could be neither \$.59, nor \$59.31. If the principal were \$7000 the interest would be \$59.31 and if the principal were \$70, the interest would be \$.59.

In solving a problem by means of such a table, first turn over the pages until you come to the rate of interest of your problem and then under this rate until you come to the number of days of your problem. The solution is then like that on the opposite page.

WRITTEN EXERCISES (C)

Find the interest at 5% on each of the following.

	Principal	Time		Principal	Time		Principal	Time
1.	\$9400	63 da.	21.	\$7645	66 da.	41.	\$8640	66 da.
2.	2750	65 da.	22.	9316	64 da.	42.	12650	61 da.
3.	4128	61 da.	23.	7850	65 da.	43.	9876	62 da.
4.	9560	68 da.	24.	9240	62 da.	44.	4950	67 da.
5.	7640	63 da.	25.	1726	61 da.	45.	8792	65 da.
6.	12520	69 da.	26.	9040	67 da.	46.	5640	61 da.
7.	14760	61 da.	27.	7516	66 da.	47.	2970	63 da.
8.	3542	65 da.	28.	8348	64 da.	48.	380	65 da.
9.	5376	64 da.	29.	7616	63 da.	49.	9870	67 da.
10.	7892	67 da.	30.	9126	68 da.	50.	2740	69 da.
11.	6500	63 da.	31.	19260	61 da.	51.	2965	68 da.
12.	8760	69 da.	32.	7540	69 da.	52.	3474	66 da.
13.	9750	62 da.	33.	9876	67 da.	53.	6790	64 da.
14.	2825	66 da.	34.	2440	66 da.	54.	8395	62 da.
15.	4346	64 da.	35.	17522	62 da.	55.	6774	60 da.
16.	23450	60 da.	36.	8640	61 da.	56.	890	69 da.
17.	76923	68 da.	37.	9875	63 da.	57.	13940	62 da.
18.	1265	61 da.	38.	2648	66 da.	58.	6530	64 da.
19.	3760	69 da.	39.	9476	68 da.	59.	3425	67 da.
20.	9482	67 da.	40.	7952	69 da.	60.	1750	65 da.

371. Exact or Accurate Interest. — *Exact interest* is obtained when a year is counted as 365 days and the exact number of days between dates is taken.

To find the exact number of days between the date when the loan is made and the day when it is due, use is sometimes made of a table which gives the number of days from January first to any day of the year. Thus to find the number of days from April 8th to July 21st it is noted that April 8th is the 98th day of the year and July 21st the 202d day. Subtracting gives the required number.

372. Table of Time. — The following table may also be used:

Month	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
January	365	31	59	90	120	151	181	212	243	273	304	334
February	334	365	28	59	89	120	150	181	212	242	273	303
March	306	337	365	31	61	92	122	153	184	214	245	275
April	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	31	61	92	123	153	184	214
June	214	245	273	304	334	365	30	61	92	122	153	183
July	184	215	243	274	304	335	365	31	62	92	123	153
August	153	184	212	243	273	304	334	365	31	61	92	122
September	122	153	181	212	242	273	303	334	365	30	61	91
October	92	123	151	182	212	243	273	304	335	365	31	61
November	61	92	120	151	181	212	242	273	304	334	365	30
December	31	62	90	121	151	182	212	243	274	304	335	365

The exact number of days from March 10th to July 18th is obtained from this table by noticing that from March to July is 122 days, and then adding 8 days (the difference between 10 and 18), getting 130 days.

ORAL EXERCISES (B)

Using the above table find the exact number of days from

1. Jan. 7 to April 9.
2. Feb. 3 to Aug. 27.
3. Feb. 24 to July 3.
4. March 3 to Oct. 7.
5. July 6 to Nov. 4.
6. Sept. 3 to Dec. 17.
7. Oct. 13 to Jan. 27.
8. Nov. 24 to March 3.
9. May 6 to Nov. 4.
10. Jan. 7 to July 28.
11. Nov. 8 to Feb. 4.
12. Feb. 3 to Sept. 7.
13. April 4 to Oct. 8.
14. May 2 to Aug. 7.

373. Exact Interest, when Used. — Exact interest is used when very large sums are involved. The United States government always uses exact interest. The state and city governments and banks do so when large sums are involved.

Example. Find the interest on $194,000 at 5% from March 7 to Oct. 28.

Solution. From the table we find that the time is 214 + 21 = 235 (days). Hence the interest is

$$194000 \times \frac{5}{100} \times \frac{\overset{47}{235}}{365_{73}} = \frac{455900}{73} = 6245.21 \text{ (dollars)}$$

WRITTEN EXERCISES (B)

Find exact interest.

	Principal	Time	Rate
1.	$1,860,000	May 1 to Aug. 9	5%
2.	32,280,000	April 27 to July 6	$4\frac{3}{4}$%
3.	98,435,600	May 4 to Oct. 5	$6\frac{1}{2}$%
4.	764,280	July 20 to Dec. 6	4%
5.	9,237,650	Aug. 8 to Nov. 15	$6\frac{1}{4}$%
6.	872,500	Feb. 15 to July 20	7%
7.	9,134,560	Jan. 4 to May 7	$5\frac{1}{2}$%
8.	224,486	Feb. 22 to Sept. 19	5%
9.	1,923,450	Feb. 14 to July 4	$3\frac{1}{2}$%
10.	785,640	Aug. 10 to Dec. 15	9%
11.	964,750	June 29 to Nov. 30	$6\frac{1}{4}$%
12.	452,750	April 5 to Oct. 4	5%
13.	947,682	Jan. 6 to April 8	$4\frac{1}{4}$%
14.	354,560	May 8 to July 15	6%
15.	175,925	March 4 to Aug. 8	9%
16.	85,914	June 16 to Sept. 11	$4\frac{1}{2}$%
17.	326,050	Aug. 4 to Nov. 16	$6\frac{1}{4}$%
18.	769,245	Mar. 10 to June 29	$5\frac{1}{2}$%
19.	879,760	Jan. 5 to May 4	$4\frac{1}{8}$%
20.	243,879	June 4 to Oct. 29	5%

374. Table of Exact Interest.

6%	40 Days	41 Days	42 Days	43 Days	44 Days
1	00657534246	00673972602	00690410958	00706849315	00723287671
2	01315068493	01347945205	01380821917	01413698630	01446575342
3	01972602739	02021917808	02071232876	02120547945	02169863013
4	02630136986	02695890410	02761643835	02827397260	02893150685
5	03287671233	03369863013	03452054794	03534246575	03616438356
6	03945205179	04043835616	04142465742	04241095890	04339726027
7	04602739726	04717808218	04832876711	04947945205	05063013699
8	05260273973	05391780821	05523287670	05654794520	05786301360
9	05917808219	06065753424	06213698629	06361643835	06509589041

6%	45 Days	46 Days	47 Days	48 Days	49 Days
1	00739726027	00756164380	00772602739	00789041095	00805479452
2	01479452054	01512328761	01545205478	01578082191	01610958904
3	02219178081	02268493142	02317808218	02367123287	02416438356
4	02958904119	03024657523	03090410957	03156164383	03221917808
5	03698630136	03780821904	03863013697	03945205479	04027397260
6	04438356163	04536986284	04635616436	04734246574	04832876712
7	05178082191	05293150665	05408219175	05523287670	05638356164
8	05917808218	06049315046	06180821915	06312328766	06443835616
9	06657534245	06805479427	06953424654	07101369862	07249315068

The method of using this table is exactly the same as that given on pages 290–291. This table is often used for very large sums.

ORAL EXERCISES (A)

Using the above table, find the interest at 6% on the following.

	Principal	Time		Principal	Time
1.	$80,000	40 da.	8.	$7,000,000	45 da.
2.	200,000	47 da.	9.	4,000	43 da.
3.	100,000	47 da.	10.	8,000	42 da.
4.	60,000	48 da.	11.	90,000	48 da.
5.	900	49 da.	12.	70,000	45 da.
6.	70	42 da.	13.	300,000	47 da.
7.	40,000	47 da.	14.	400,000	46 da.

Example. Find the interest on $240,800 at 6% from May 3 to June 12.

Solution. From the table, page 292, the time is 40 days. Using the table on the opposite page:

$1315	07	interest on $200.000 for 40 days
263	01	interest on 40,000 for 40 days
5	26	interest on 800 for 40 days
$1583	34	the required interest

WRITTEN EXERCISES (B)

In this manner solve the following, the rate in each case being 6%.

	Principal	Time
1.	$86,400	April 10 to May 24
2.	47,500	June 4 to July 15
3.	7,650	Jan. 15 to Feb. 28
4.	2,940	June 4 to July 22
5.	87,600	Oct. 8 to Nov. 24
6.	4,520	Nov. 15 to Dec. 25
7.	97,750	April 6 to May 25
8.	17,270	May 4 to June 14
9.	93,945	June 29 to Aug. 10
10.	76,400	Aug. 8 to Sept. 19
11.	95,200	Oct. 5 to Nov. 20
12.	66,450	Nov. 14 to Dec. 30
13.	94,870	May 7 to June 23
14.	458,420	July 20 to Sept. 2
15.	87,650	Feb. 15 to April 3
16.	19,470	Jan. 16 to March 1
17.	78,920	June 19 to Aug. 4
18.	46,740	May 17 to July 1
19.	91,290	Oct. 4 to Nov. 16
20.	76,400	March 8 to April 25
21.	50,500	April 10 to May 27
22.	96,700	June 6 to July 24

375. The General Equation of Interest. — By the definition of interest we have

$$Interest = principal \times rate \times time$$

The rate is expressed in terms of per cent and the time in terms of years. The difference between ordinary and exact interest lies in the difference in computing the time.

376. The Cancellation Method of Finding Interest.

Example 1. Find ordinary interest on \$14,600 at 5% from Jan. 16 to Feb. 20th.

Solution. The time is 35 days or $\frac{35}{360}$ of one year.

Hence, $\text{Interest} = \overset{73}{\cancel{14600}} \times \frac{5}{\cancel{100}} \times \frac{35}{\underset{72}{\cancel{360}}} = \frac{2555}{36} = 70.97$ (dollars).
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad 36$

Example 2. Find the exact interest on \$5680 at $5\frac{1}{2}\%$ from May 3 to Aug. 25.

Solution. By the table, § 372, the exact number of days from May 3 to Aug. 25, is 114 days. Hence the time is $\frac{114}{365}$, also $5\frac{1}{2}\% = \frac{11}{200}$.

Hence, $\text{Interest} = \overset{5.68}{\cancel{5680}} \times \frac{11}{\cancel{200}} \times \frac{114}{\underset{73}{\cancel{365}}} = 97.57$ (dollars).
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad 28.40$

This method is called the *cancellation method* and is the most direct and most general method for finding interest. It works equally well so far as the theory is concerned, for all rates, and is equally applicable to ordinary and exact interest.

WRITTEN EXERCISES (A)

By the cancellation method, find the ordinary interest on each of the following.

	Principal	Rate	Time		Principal	Rate	Time
1.	\$4620	6%	48 da.	**7.**	\$7850	9%	108 da.
2.	2675	$4\frac{1}{2}\%$	64 da.	**8.**	12464	$5\frac{1}{4}\%$	120 da.
3.	1940	$6\frac{1}{2}\%$	36 da.	**9.**	9176	$4\frac{3}{4}\%$	56 da.
4.	8756	$3\frac{3}{4}\%$	96 da.	**10.**	4500	6%	200 da.
5.	9122	7%	84 da.	**11.**	2545	5%	50 da.
6.	6340	8%	63 da.	**12.**	2160	$3\frac{1}{2}\%$	90 da.

377. Short Cancellation Method. — The work in computing interest by the cancellation method may be shortened by performing part of the cancellation as the work is first written down.

Example 1. Find the interest on \$9840 at 7% for 93 days.

Solution. The interest is $9840 \times \frac{7}{100} \times \frac{93}{360}$.

We now notice that in all such problems the numbers 100 and 360 are in the denominators. The 100 can always be canceled by pointing off two places in the principal. We point off one more place in the principal to get rid of the 0 in 360.

So we write at once $9.840 \times 7 \times \frac{93}{36}$.

Again, we notice that 9.840 is a multiple of 6. Hence we may write

$$1.64 \times 7 \times \frac{93}{6} = .82 \times 7 \times 31 = 177.94 \text{ (dollars)}$$

If we had been really clever we might have divided 9.840 by 12 at once and 93 by 3, thus writing down $.82 \times 7 \times 31$ immediately

Example 2. Find the interest on \$2730 at 6% for 50 days.

Solution. The interest is $2730 \times \frac{6}{100} \times \frac{50}{360}$.

We can however, write at once $9.1 \times \frac{5}{2} = \frac{45.5}{2} = 22.75$ (dollars)

The 100 is canceled by pointing off in 2730. A zero is stricken off in $\frac{50}{360}$. The 6 is canceled in 6 and 36 and 3 is canceled in 27.30 and 6.

WRITTEN EXERCISES (A)

In each of the following perform as much of the cancellation as you can when you write down the first statement. Find exact interest in each problem.

	Principal	Rate	Time		Principal	Rate	Time
1.	\$790	5%	75 da.	**10.**	\$92,160	6%	75 da.
2.	850	4%	63 da.	**11.**	1850	10%	21 da.
3.	2500	6%	90 da.	**12.**	460	8%	42 da.
4.	1750	8%	112 da.	**13.**	925	6%	84 da.
5.	2640	7%	120 da.	**14.**	650	4%	110 da.
6.	9250	4%	56 da.	**15.**	1040	2%	63 da.
7.	3560	6%	126 da.	**16.**	1460	3%	75 da.
8.	9620	4%	140 da.	**17.**	1780	5%	50 da.
9.	8750	8%	93 da.	**18.**	1990	7%	70 da.

378. Interchanging Principal and Time. — The interest on \$500 for 85 days at 6% is the same as the interest on \$85 for 500 days. This is evident from the fact that the interest in the first case is $500 \times \frac{6}{100} \times \frac{85}{360}$ and in the second case $85 \times \frac{6}{100} \times \frac{500}{360}$.

These products are equal because the factors 500 and 85 may be interchanged.

In every problem in interest the number of days and the number of dollars in the principal may be interchanged. This sometimes simplifies the solution of problems as shown in the following example.

Example. Find the interest on \$1200 for 88 days at 6%.

First Solution.

12	00	interest for 60 days
4	00	interest for 20 days
1	20	interest for 6 days
	40	interest for 2 days
17	60	interest for 88 days

Second Solution. (Find interest on \$88 for 1200 days)

8	80	interest for 600 days
17	60	interest for 1200 days (1200 = 2 × 600)

By this method we can give immediately the interest of \$60 at 6% for any number of days. Thus the interest on \$60 for 134 days at 6% is \$1.31.

That is, pointing off two decimals in the number of days given, we find the interest on \$60. If only one decimal is pointed off, the result is the interest on \$600.

Thus the interest on \$600 for 134 days is \$13.40. If three decimals are pointed off in the given number of days, the result is the interest on \$6. Thus \$.134 is the interest on \$6 for 134 days.

ORAL EXERCISES (A)

Find the interest on the following, the rate in each case being 6%.

	Principal	Time		Principal	Time		Principal	Time
1.	\$ 60	84 da.	**4.**	\$ 60	175 da.	**7.**	\$6000	89 da.
2.	60	43 da.	**5.**	600	142 da.	**8.**	600	72 da.
3.	600	17 da.	**6.**	600	351 da.	**9.**	6000	53 da.

Example. Find the interest on \$120 at 6% for 53 days.

Solution. We know at once that the interest on \$60 is \$.53. Hence the required interest is \$1.06.

In a similar manner the interest on any sum which is an easy multiple of \$60 or \$600 or \$6000 may be found.

ORAL EXERCISES (A)

Find the interest on the following, the rate being 6%.

	Principal	Time		Principal	Time		Principal	Time
1.	$180	31 da.	7.	$300	10 da.	13.	$540	27 da.
2.	360	42 da.	8.	3000	45 da.	14.	3400	32 da.
3.	1200	81 da.	9.	2400	52 da.	15.	720	86 da.
4.	120	24 da.	10.	480	17 da.	16.	7200	110 da.
5.	240	73 da.	11.	4800	95 da.	17.	960	14 da.
6.	1800	64 da.	12.	3600	81 da.	18.	1080	28 da.

Further extension of this method is suggested by the following.

Example. Find the interest on $650 for 97 days at 6%.

Solution.

Interest on $600	$9	70
Interest on $60		97
	10	67
Interest on $10		16
	$10	51 = required interest

WRITTEN EXERCISES (B)

In each of the following the rate is 6%.

	Principal	Time		Principal	Time		Principal	Time
1.	$300	43 da.	14.	$740	42 da.	27.	$840	71 da
2.	350	27 da.	15.	625	76 da.	28.	920	48 da.
3.	400	45 da.	16.	770	24 da.	29.	1920	28 da
4.	450	76 da.	17.	190	54 da.	30.	2640	27 da.
5.	700	81 da.	18.	150	78 da.	31.	2110	76 da.
6.	800	53 da.	19.	100	63 da.	32.	1760	67 da.
7.	950	42 da.	20.	1850	28 da.	33.	2280	22 da.
8.	1000	29 da.	21.	850	66 da.	34.	925	29 da.
9.	1140	31 da.	22.	1300	88 da.	35.	4240	24 da.
10.	1250	97 da.	23.	705	92 da.	36.	240	18 da.
11.	1200	105 da.	24.	1650	40 da.	37.	540	37 da.
12.	910	33 da.	25.	1210	21 da.	38.	840	49 da.
13.	610	91 da.	26.	760	62 da.	39.	6840	53 da.

379. Rate of Interest on Investments. — A real estate agent bought a lot for \$2000 and one year later sold it for \$2400. The taxes and expenses connected with the sale and purchase were \$200. This left a net gain of \$200, which is 10% of the cost of the lot. Hence this investment yielded 10% a year on the amount invested. No interest is here figured on the \$200 paid in taxes and for miscellaneous expenses.

If the sale had been made in six months the investment of \$2000 would have earned \$200 in a half year, which is equivalent to 20% yearly interest.

In problems like the above we must find the rate of interest when the amount invested and the income are known.

WRITTEN EXERCISES (B)

Find the rate of interest yielded by each of the following investments. (See page 228 for method of finding the rate.)

	Amount Invested	Yearly Income		Amount Invested	Yearly Income
1.	\$8640	\$432	16.	\$1680	\$84
2.	2700	135	17.	880	70.40
3.	4500	315	18.	5000	425
4.	9600	432	19.	2440	170.80
5.	12,500	625	20.	9750	585
6.	7840	784	21.	35,640	1425.60
7.	8375	502.50	22.	1970	177.30
8.	1940	155.20	23.	4250	170
9.	7850	314	24.	3490	174.50
10.	9670	773.60	25.	4200	273
11.	3450	147	26.	6600	429
12.	86,540	3461.60	27.	176,500	6177.50
13.	2600	143	28.	7940	317.60
14.	7850	349.50	29.	2870	143.50
15.	16,900	760.50	30.	1450	101.50

Example. A capitalist invested \$47,800 in real estate which yielded a yearly net income of \$3560. What rate of interest did he get on this investment?

Solution.

$$\begin{array}{r} 7.45\% \\ 478)\overline{\$3560} \\ 3346 \\ \hline 2140 \\ 1912 \\ \hline 2280 \end{array}$$

The rate of income is here stated to the nearest hundredth of one per cent.

WRITTEN EXERCISES (B)

In each of the following find the rate of interest to the nearest hundredth of one per cent.

	Amount Invested	Yearly Income		Amount Invested	Yearly Income
1.	\$12,400	\$1150	**7.**	\$1550	\$56.25
2.	15,200	750	**8.**	2860	193.20
3.	18,500	1440	**9.**	9850	832.50
4.	22,540	1350	**10.**	10,560	750.60
5.	25,650	1750	**11.**	14,680	1125.50
6.	29,225	1129	**12.**	17,250	1250

13. A man bought a farm for \$28,500, paid \$760 in taxes and for repair of buildings. He rented the farm one year for \$1200 and then sold it for \$30,000. What per cent interest did the man get on his investment?

Suggestion. The total investment was \$28,500 and the total amount received was \$30,000 + \$1200 − \$760 = \$30,440.

14. A bought a double house for \$7500, rented each side for 12 months at \$30 per month, paid \$220 in taxes, insurance, and water rent. At the end of the year he sold the property for \$8000. What per cent did he make on his investment?

15. Two seamen bought a vessel for \$10,000. They carried 200,000 bushels of grain during the year at a rate which gave them 2¢ a bushel, over and above all expenses; paid \$1000 for insurance, repairs, etc. At the end of the year, the boat was sold for \$9000. What was the gain per cent on the investment?

380. Profits Computed as Interest. — In most business transactions when money is invested, the time of the investment is other than one year. Thus a man may buy and then sell in two months or six months or a year and a half.

The rate of profit on a business investment is often computed as follows: All expenses are first deducted. This involves all outlays for work, rent, salaries of superintendents, etc. Then the investor sets aside a certain sum as compensation for his own work. What remains of the gain is regarded as interest on the money invested. This gives rise to problems like the following:

Example. A man invested \$7800 in a business deal and closed it out in three months, making a gain of \$940 over and above all expenses. What rate of interest did this investment yield?

Solution. Three months is $\frac{1}{4}$ of a year. Hence the gain for one year at this rate would be $4 \times 940 = 3760$. Hence the rate of interest was 3760 divided by 7800 = 48.2%.

WRITTEN EXERCISES (B)

In each of the following find the rate of interest to which the gain is equivalent.

	Amount Invested	Time	Gain		Amount Invested	Time	Gain
1.	\$18,700	4 mo.	\$760	**14.**	\$3150	16 mo.	\$240
2.	2650	6 mo.	66.25	**15.**	5400	10 mo.	300
3.	17,506	7 mo.	1200	**16.**	4200	11 mo.	225
4.	2592	8 mo.	250	**17.**	7200	5 mo.	210
5.	840	9 mo.	63	**18.**	9600	4 mo.	60
6.	765	2 mo.	12.75	**19.**	10,440	8 mo.	840
7.	1560	11 mo.	120	**20.**	8760	9 mo.	328.50
8.	8430	3 mo.	256	**21.**	9320	3 mo.	233
9.	1533	10 mo.	95	**22.**	4860	6 mo.	243
10.	3475	5 mo.	150	**23.**	2150	10 mo.	150
11.	3550	8 mo.	275	**24.**	7500	18 mo.	500
12.	4250	7 mo.	300	**25.**	1980	7 mo.	480
13.	5400	17 mo.	344.25	**26.**	3670	5 mo.	470

381. Quick and Slow Sales. — An article which is sold quickly can be sold on a closer margin than an article which must be kept in stock a long time. Hence there is a disadvantage in keeping a very large stock of goods on hand, that is, a large amount of each kind of goods, for in that case some of the goods are sure to be carried a long time. The disadvantage arises not only because of the item of interest on the money invested in the goods themselves, but also because of cost of insurance, the expenses of storage, etc.

In figuring the profit of a business, we consider the total investment of capital, and the net gain after all expenses have been paid.

PROBLEMS (A)

1. A trader in farm implements has $60,000 invested in his business. His profit for one year is $8000. What is the rate of interest on this investment if the trader's own work is worth $3000 a year?

2. I bought a piece of land for $1500 and gave in payment a note dated September 5, 1945, with interest at 8%. May 20, 1947, I sold the land for $2500. After paying my note, what was my gain, assuming expenses of $400 and income of $500 from rent?

3. Three men engage in business with $62,000 capital. The income for 4 months is $10,000, the expenses of conducting the business are $7500 including salaries. Find what rate of profit was realized on the investment.

4. The income from a farm costing $23,000 was $4000, and the expenses were $2850. What was the gain per cent on the investment?

382. To Find the Time When Principal, Rate, and Interest Are Given. — Since Interest = Principal × rate × time, it follows that the time can be found by dividing the interest by the product of the principal and the rate. But the product of the principal and the rate equals the interest for one year.

Hence we have the

Rule. *To find the time divide the interest by the interest for one year.*

This problem is not of frequent occurrence.

383. To Find the Principal When Rate, Time, and Amount Are Known.

Rule: *Find the amount of* \$1 *for the given time and rate and divide the given amount by it.*

The reason for this rule will be apparent as soon as the statement of the rule itself is really comprehended.

Example. Find the principal if the amount in 5 mo. 18 da. at 5% is \$1600.

```
1.0233)1600.0000(1563.57
       10233
       57670
       51165
        65050
        61398
         36520
         30699
          58210
          51165
           70450
```

Solution.

First find the interest on \$1 at 6%.

03	interest for 6 mo.
002	interest for 12 da.
028	interest for 5 mo. 18 da.
0047	$\frac{1}{6}$ of interest
0233	= interest at 5% for 5 mo. 18 da.

Hence amount of \$1 is \$1.0233 and principal = \$1600 divided by 1.0233 = \$1563.57.

The problem of this section is of comparatively frequent occurrence.

WRITTEN EXERCISES (☰)

Find the principal in each of the following.

	Amount	Rate	Time		Amount	Rate	Time
1.	\$2400	6%	3 mo.	**11.**	\$3840	$5\frac{3}{4}$%	49 da.
2.	1120	6%	2 yr.	**12.**	2013	$4\frac{1}{3}$%	54 da.
3.	1648	6%	6 mo.	**13.**	4260	$3\frac{1}{2}$%	63 da.
4.	1881	6%	9 mo.	**14.**	2767.875	$5\frac{1}{5}$%	45 da.
5.	1230	5%	6 mo.	**15.**	1950	$4\frac{1}{2}$%	30 da.
6.	1515	4%	3 mo.	**16.**	7624	6%	81 da.
7.	1720	6%	15 mo.	**17.**	2940	7%	93 da.
8.	3225	5%	18 mo.	**18.**	8750	$6\frac{1}{2}$%	42 da.
9.	1228	4%	7 mo.	**19.**	2460	$5\frac{1}{4}$%	21 da.
10.	1632	8%	3 mo.	**20.**	7920	5%	36 da.

GENERAL DRILL IN COMPUTING INTEREST (A)

Find the interest on the following and check.

	Principal	Rate	Time
1.	$47,000	$5\frac{3}{4}\%$	2 mo. 6 da.
2.	1240	7%	7 mo. 4 da.
3.	2500	8%	3 mo. 8 da.
4.	1480	$7\frac{1}{2}\%$	6 mo. 15 da.
5.	3980	7%	8 mo. 12 da.
6.	4300	$5\frac{1}{2}\%$	4 mo. 7 da.
7.	860	7%	3 mo. 9 da.
8.	124	6%	7 mo. 18 da.
9.	290	7%	9 mo. 27 da.
10.	7800	8%	3 mo. 2 da.
11.	396	9%	4 mo. 8 da.
12.	2780	7%	7 mo. 9 da.
13.	1380	$6\frac{1}{2}\%$	2 mo. 8 da.
14.	4560	6%	6 mo. 27 da.
15.	7140	$5\frac{1}{2}\%$	7 mo. 24 da.
16.	2300	$6\frac{1}{2}\%$	5 mo. 18 da.
17.	4650	$6\frac{1}{2}\%$	4 mo. 12 da.
18.	2980	7%	5 mo. 15 da.
19.	3500	$5\frac{1}{2}\%$	3 mo. 12 da.
20.	6800	$5\frac{1}{2}\%$	4 mo. 9 da.
21.	4750	6%	3 mo. 9 da.
22.	18,680	5%	7 mo. 8 da.
23.	23,490	5%	6 mo. 19 da.
24.	13,500	$5\frac{1}{2}\%$	9 mo. 26 da.
25.	4860	$6\frac{1}{2}\%$	7 mo. 23 da.
26.	41,600	$5\frac{1}{4}\%$	7 mo. 15 da.
27.	7580	$6\frac{1}{2}\%$	4 mo. 9 da.
28.	18,640	$6\frac{1}{4}\%$	2 mo. 7 da.

384. True Discount and Present Worth. — In large transactions the method shown in the following examples is often used to decide how much an obligation due at a future time is worth at present.

Example 1. A note for \$80,000 bearing no interest is due in 6 months. How much is it worth at present, the rate of interest being regarded as 6%?

Solution.
Amt. of \$1 is 1.03.
\$80,000 ÷ 1.03 = \$77,669.90 = present worth
Analysis: The problem is to find what principal must be invested at 6% to yield an amount of \$80,000 in 6 mo. That is, it is the problem of § 383.

The principal which must be invested to amount to the value of a given sum at a given time is called the present worth of that sum, and the difference between this sum and its *present worth* is called *true discount.*

Example 2. Find the present worth and the true discount of a note for \$1680 bearing interest at 7% and due in 80 days, the value of money being $5\frac{1}{2}$%.

Solution.

16	80	interest for 2 mo. at 6%
5	60	interest for 20 da. at 6%
22	40	interest for 80 da. at 6%
3	73	$\frac{1}{6}$ of interest at 6%
26	13	interest for 80 da. at 7%
\$1706.13		amount of note
.01		interest for 60 da. at 6%
.003$\frac{1}{3}$		interest for 20 da. at 6%
.013$\frac{1}{3}$		interest for 80 da. at 6%
.001$\frac{1}{9}$		$\frac{1}{12}$ of interest at 6%
.012$\frac{2}{9}$		interest at $5\frac{1}{2}$% for 80 da.

Analysis: First, find the amount of \$1680 in 80 days at 7%. This will be the amount actually due on the note at expiration of the 80 days.
Second, find amount of \$1 for 80 days at 1%.
Third, divide amount of note by amount of \$1.

\$1.012$\frac{2}{9}$ = \$1.01222 = amount of \$1 for 80 da.
\$1706.13 ÷ 1.01222 = \$1685.53 = present worth and \$1706.13 − \$1685.53 = \$20.60 is the true discount.

The rate of interest used in discounting the note is called the *rate of discount.* The rate of discount is frequently not the same as the rate of interest of the note.

WRITTEN EXERCISES (C)

Find the present worth and the true discount of each of the following notes.

	Face of Note	Rate of Note	Time	Rate of Discount
1.	$600	6%	120 da.	5%
2.	163.50	6%	1 yr. 6 mo.	6%
3.	9945	6%	1 yr. 9 mo.	6%
4.	300	5%	60 da.	6%
5.	1200	6%	90 da.	5%
6.	1100	4%	30 da.	5%
7.	960	3%	60 da.	6%
8.	725	5%	45 da.	4%
9.	900	3%	30 da.	4%
10.	1500	4%	27 da.	8%
11.	2100	6%	1 yr.	5%
12.	212.50	3%	15 mo.	4%
13.	1728	4%	4 mo.	3%
14.	6000	7%	1 yr. 2 mo.	7%
15.	1250	5%	6 mo.	8%
16.	2760	8%	75 da.	6%
17.	900	4%	90 da.	5%
18.	148.75	6%	18 mo.	4%
19.	2960.40	5%	9 mo.	6%
20.	7645.10	3%	60 da.	5%
21.	8940	6%	30 da.	6%
22.	3150	6%	90 da.	5%
23.	1560	7%	90 da.	6%
24.	680	8%	120 da.	7%
25.	4790	5%	4 mo.	6%
26.	9360	4%	7 mo.	5%
27.	12,400	5%	6 mo.	5%

385. Rates of Interest. — The rate of interest depends upon many factors: the length of time of the loan, the security offered, the locality, and the conditions of the money market. In new sections of the country where capital is scarce, the rates of interest are high.

386. Legal Interest. — Below is a list of the legal interest rates in the various states of the Union. The legal interest is the rate of interest which a debt bears if no *rate* is agreed upon, the agreement merely being that the debt is to bear interest. If no agreement whatever is made about interest then no interest is paid.

387. Legal and Maximum Rates of Interest. — The following table also gives the highest rate of interest permitted in the various states. Legal rates of interest are being changed from time to time.

State	Legal Rate	Highest Allowed	State	Legal Rate	Highest Allowed
Alabama	8	8	Montana	8	10
Alaska	8	12	Nebraska	7	10
Arizona	6	10	Nevada	7	12
Arkansas	6-10	6-10	New Hampshire	6	any
California	7	12	New Jersey	6	6
Colorado	8	12	New Mexico	10	12
Connecticut	6	12	New York	6	6
Delaware	6	6	North Carolina	6	6
District Columbia	6	8	North Dakota	6	10
Florida	8	10	Ohio	6	8
Georgia	7	8	Oklahoma	6	10
Idaho	7	10	Oregon	6	10
Illinois	5	7	Pennsylvania	6	6
Indiana	6	8	Rhode Island	6	any
Iowa	6	8	South Carolina	7	8
Kansas	6	6	South Dakota	7	12
Kentucky	6	6	Tennessee	6	6
Louisiana	5	8	Texas	6	10
Maine	6	any	Utah	8	12
Maryland	6	6	Vermont	6	any
Massachusetts	6	any	Virginia	6	6
Michigan	5	7	Washington	6	12
Minnesota	6	8	West Virginia	6	6
Mississippi	6	8	Wisconsin	6	10
Missouri	6	8	Wyoming	7	10

In the state of New York any rate may be charged by agreement on call loans (see page 340) of $5000 and upwards.

388. Usury. — Charging higher interest than the highest permitted by law is called *usury*. Usury is punishable by penalties ranging from the loss of the extra interest charged to the loss of all interest and the principal.

ORAL EXERCISES (B)

1. Discuss the reasons for the difference in the legal rate of interest in the various states.

2. Give some of the arguments both for and against laws regulating the rate of interest.

3. What would be the effect of a law making the highest rate of interest permitted very low, as, for instance, 2 or 3 per cent?

4. What is the effect of very low rates of interest on the value of a farm or any other income producing property?

5. What would be the effect of a very high rate of interest on the rental value of a house?

6. What is the effect of very low rates of interest on general business? (Remember that nearly all business concerns borrow money at certain times of the year.)

PROBLEMS (B)

1. A merchant borrowed \$85,000 each year for an average of 150 days. Find the interest at the legal rates in Alabama, Illinois, New York, South Dakota. Also find the interest at the highest rates permitted in Michigan, Nebraska, and Washington.

The results obtained in this example give one element in the difference in the cost of doing business in various parts of the United States.

2. A farmer borrowed \$13,000 for $3\frac{1}{2}$ years. Find the interest at the legal rates in Ohio, Illinois, California. At the highest rates in New Mexico, North Dakota.

3. A man borrowed \$2500 for 250 days. Find the interest if he borrowed it at legal interest in Montana, Washington, North Dakota, Michigan.



Courtesy First National Bank of Boston

SAVING FOR SECURITY

CHAPTER XXV

COMPOUND INTEREST

389. Situations Giving Rise to Compound Interest. — Problems in compound interest occur in connection with deposits in savings banks and with many general problems of investment. Thus the life insurance companies in calculating the future value of an investment make use of compound interest. Again in computing the income from stocks and bonds paying dividends, quarterly or semi-annually, and in figuring cost of life insurance, compound interest is used. By all odds the most frequent use of compound interest is in connection with savings deposits.

390. Simple Interest. — In the case of ordinary loans the interest is supposed to be paid when due, but if not paid it becomes an ordinary debt bearing no interest. That is, only the original principal bears interest even if there is interest due and unpaid.

391. Compound Interest. — When money is deposited in a savings bank the interest, when due, is added to the principal, the sum forming a new principal all of which bears interest. The difference between simple and compound interest will be made clear by an example.

Simple Interest

Principal = \$600
Rate 6%, Time 4 yr.

Interest = $600 \times \frac{6}{100} \times 4 = \$144$

Hence the amount due in four years would be \$744.

Compound Interest

1st Principal	\$600
	.06
	36.
	600
2d Principal	636

Continuing in this manner we find that the amount due at end of four years is \$757.49.

The difference between the simple and compound interest is \$13.49.

Compound interest on ordinary loans is prohibited by law in many states, but if offered and accepted it is not regarded as usury.

392. Time of Compounding Interest. — Interest is said to be *compounded* when it is added to the principal. In the problem on page 311 the interest is compounded at the end of each year or yearly. Interest may be compounded *yearly, semiannually,* or *quarterly.*

The actual interest is greater the more frequently the interest is compounded. Thus the interest on \$100 at 4% for one year when compounded annually is \$4, and when compounded quarterly, it is \$4.06.

4% yearly interest when compounded quarterly is computed like 1% interest compounded yearly. Similarly 4% yearly interest compounded semiannually is computed the same as 2% interest compounded annually.

PART OF COMPOUND INTEREST TABLE (B)

393. Amount of \$1 at compound interest compounded annually:

YEARS	2%	3%	4%	4½%	5%
1	1.02000	1.03000	1.04000	1.04500	1.05000
2	1.04040	1.06090	1.08160	1.09203	1.10250
3	1.06121	1.09273	1.12486	1.14117	1.15763
4	1.08243	1.12551	1.16986	1.19252	1.21551
5	1.10408	1.15927	1.21665	1.24618	1.27628
6	1.12616	1.19405	1.26532	1.30226	1.34010
7	1.14869	1.22987	1.31593	1.36086	1.40710
8	1.17166	1.26677	1.36857	1.42210	1.47746
9	1.19509	1.30477	1.42331	1.48610	1.55133
10	1.21899	1.34392	1.48024	1.55297	1.62889
11	1.24337	1.38423	1.53945	1.62285	1.71034
12	1.26824	1.42576	1.60103	1.69588	1.79586
13	1.29361	1.46853	1.66507	1.77220	1.88565
14	1.31948	1.51259	1.73168	1.85194	1.97993
15	1.34587	1.55797	1.80094	1.93528	2.07892
16	1.37279	1.60471	1.87298	2.02237	2.18287
17	1.40024	1.65285	1.94790	2.11338	2.29202
18	1.42825	1.70213	2.02582	2.20848	2.40662
19	1.45681	1.75351	2.10685	2.30786	2.52695
20	1.48595	1.80611	2.19112	2.41171	2.65330
21	1.51567	1.86029	2.27877	2.52024	2.78596
22	1.54598	1.91610	2.36992	2.63365	2.92526
23	1.57690	1.97359	2.46472	2.75217	3.07152
24	1.60844	2.03279	2.56330	2.87601	3.22510
25	1.64061	2.09378	2.66584	3.00543	3.38635

On page 312 is a part of a compound interest table such as is used by bankers.

Example. Find the amount of \$500 at 5% compound interest for 6 years compounded annually.

Solution. From the table the amount of \$1.00 for this time and rate is 1.3401. Hence the amount of \$500 is $500 \times \$1.34010 = \$670.050$.

If the rate is 5% compounded semiannually for 6 years, we take $2\frac{1}{2}\%$ for 12 years. If the rate is 4% compounded quarterly for 5 years, we take 1% compounded annually for 20 years.

WRITTEN EXERCISES (B)

Solve the first four exercises without using the table. For the others use the table.

	Principal	Rate	Compounded	Time
1.	\$1000	4%	Semiannually	3 yr.
2.	2000	6%	Semiannually	2 yr.
3.	5000	3%	Annually	2 yr.
4.	10,000	8%	Semiannually	2 yr. 6 mo.
5.	795	8%	Quarterly	1 yr. 3 mo.
6.	1360	5%	Annually	2 yr. 6 mo.
7.	1580	6%	Semiannually	2 yr. 6 mo.
8.	2250	6%	Semiannually	4 yr.
9.	1670	$4\frac{1}{2}\%$	Annually	5 yr.
10.	2780	4%	Annually	3 yr.
11.	1985	$4\frac{1}{2}\%$	Annually	2 yr. 9 mo.
12.	275.82	6%	Semiannually	3 yr.
13.	860.40	9%	Semiannually	1 yr. 6 mo.
14.	1728.65	8%	Quarterly	5 yr.
15.	917.20	6%	Semiannually	1 yr. 6 mo.
16.	1260.90	4%	Semiannually	3 yr.
17.	2850.16	5%	Annually	4 yr.
18.	1945.22	6%	Semiannually	2 yr. 6 mo.
19.	9176.40	4%	Semiannually	3 yr.
20.	1678.70	5%	Annually	6 yr.

394. Sinking Fund Table. —The table below shows the accumulation at compound interest of an annual investment of one dollar. Such tables are used by all who have much computing of this kind to do and especially by bankers.

Periods	2%	3%	4%	4½%	5%	6%
1	1.02	1.03	1.04	1.045	1.05	1.06
2	2.0604	2.0909	2.1216	2.137025	2.1525	2.1836
3	3.121608	3.183627	3.246464	3.278191	3.310125	3.374616
4	4.204040	4.309136	4.416323	4.470710	4.525631	4.637093
5	5.308121	5.468410	5.632975	5.716892	5.801913	5.975319
6	6.434283	6.662462	6.898294	7.019152	7.142008	7.393838
7	7.582969	7.892336	8.214226	8.380014	8.549109	8.897468
8	8.754628	9.159106	9.582795	9.802114	10.026564	10.491316
9	9.949721	10.463879	11.006107	11.288209	11.577893	12.180795
10	11.168715	11.807796	12.486351	12.841179	13.206787	13.971643
11	12.412090	13.192030	14.025805	14.464032	14.917127	15.869941
12	13.680332	14.617790	15.626838	16.159913	16.712983	17.882138
13	14.973938	16.086324	17.291911	17.932109	18.598632	20.015066
14	16.293417	17.598914	19.023588	19.784054	20.578564	22.275970
15	17.639285	19.156881	20.824531	21.719337	22.657492	24.672528
16	19.012071	20.761588	22.697512	23.741707	24.840366	27.212880
17	20.412312	22.414435	24.645113	25.855084	27.132385	29.905653
18	21.840559	24.116868	26.671229	28.063562	29.539004	32.759992
19	23.297370	25.870374	28.778078	30.371423	32.065954	35.785591
20	24.783317	27.676486	30.969202	32.783137	34.719252	38.992727
21	26.298984	29.536780	33.247970	35.303378	37.505214	42.392290
22	27.844963	31.452884	35.617889	37.937030	40.430475	45.996828
23	29.421862	33.426470	38.082604	40.689196	43.501999	49.815577
24	31.030300	35.459264	40.645908	43.565210	46.727099	53.864512
25	32.670906	37.553042	43.311745	46.570645	50.113454	58.156383

The payment at a future date of an indebtedness such as municipal bonds is frequently provided for by what is called a sinking fund. Thus, to pay $50,000 at the end of twenty years, a certain sum may be invested at compound interest each year. The above table shows that one dollar invested each year at 4% compound interest will amount to $30.969202 in twenty years.

Example 1. What amount must be invested yearly at 4% interest compounded yearly to amount to \$100,000 in 16 years?

Solution. By the table \$1.00 invested each year at 4% amounts to \$22.697512 in 16 years. Hence the required yearly investment is

$$\frac{100{,}000}{22.697512} = 1105.77 \text{ (dollars)}.$$

Example 2. What amount must be invested semiannually at 4% interest compounded semiannually to amount to \$75,000 in 12 years?

Suggestion. The required amount is the same as the annual investment at 2% compounded yearly for 24 years.

WRITTEN EXERCISES (B)

In the following, find the investment required on the dates on which the interest is compounded.

	Total Debt	Rate Interest on Sinking Fund	Compounded	Time
1	\$150,000	4%	Semiannually	5 yr.
2	75,000	4%	Semiannually	2 yr.
3	2,500	6%	Semiannually	3 yr.
4	100,000	4%	Annually	1 yr.
5	26,000	8%	Quarterly	5 yr.
6	79,500	6%	Semiannually	3 yr.
7	16,250	4%	Semiannually	2 yr.
8	202,050	8%	Quarterly	1 yr.
9	8,000	4½%	Annually	6 yr.
10	10,000	6%	Annually	3 yr.
11	53,000	4%	Semiannually	2 yr.
12	99,000	8%	Quarterly	1 yr.
13	133,000	9%	Semiannually	5 yr.
14	76,000	6%	Semiannually	3 yr.
15	98,500	8%	Quarterly	4 yr.
16	105,000	4%	Annually	2 yr.
17	300,000	4%	Semiannually	3 yr.
18	275,000	6%	Semiannually	4 yr.
19	500,000	4%	Semiannually	12 yr
20	450,000	4%	Annually	7 yr.
21	75,000	4½%	Annually	20 yr.
22	125,000	4%	Annually	18 yr.
23	45,000	5%	Annually	19 yr.
24	85,000	3%	Annually	14 yr.

395. Periodic Interest. — It is sometimes agreed that interest on the original principal shall bear interest whenever due and unpaid. This is called *periodic interest.* It differs from compound interest in that the interest on interest due and unpaid cannot bear interest if unpaid. In many states contracts for periodic interest are not legal and cannot be enforced. In such cases the law is evaded by giving interest-bearing notes for the amounts of the interest on the principal, due at the time the interest is due and bearing interest after they are due.

Example 1. Find periodic interest on \$1200 at 6% for four years.

Solution. The simple interest is $4 \times 6 \times \$12 = \$288$. The interest due and unpaid from the first year is \$72 for 3 years, from the second year \$72 for 2 years and from the third year \$72 for 1 year or in all \$72 for 6 years.

The interest on the interest is $6 \times \frac{6}{100} \times \$72 = \$25.92$.

Hence the periodic interest is $\$288 + \$25.92 = \$313.92$.

WRITTEN EXERCISES (C)

Find the periodic interest on each of the following, no interest having been paid.

	Principal	Rate	Time		Principal	Rate	Time
1.	\$800	5%	3 yr.	8.	\$3860	5%	3 yr.
2.	1200	6%	5 yr.	9.	5700	$6\frac{1}{2}\%$	6 yr.
3.	1400	8%	3 yr.	10.	1260	$5\frac{1}{4}\%$	5 yr.
4.	700	7%	4 yr.	11.	3140	7%	4 yr.
5.	500	$6\frac{1}{2}\%$	6 yr.	12.	18,700	$5\frac{3}{4}\%$	5 yr.
6.	2500	5%	7 yr.	13.	4960	$5\frac{1}{2}\%$	3 yr.
7.	2480	4%	7 yr.	14.	7840	$6\frac{1}{2}\%$	7 yr.

PROBLEMS (C)

1. A father invests \$50 at 4% compound interest on the date of his son's birth and a like amount on each succeeding birthday. What will be the amount of these investments when the boy is 21 years old?

2. In 1937, one saving bank offered $4\frac{1}{2}\%$ compounded annually and another, 4% compounded semiannually. Which was the better investment? Find the amount of \$100 invested in each bank for five years.

3. At the beginning of each year for six years a company set aside from the profits of the preceding year $35,000 as a sinking fund. If this sum was invested at 5% compound interest compounded yearly, what did it amount to at the end of the sixth year?

4. A man withdraws the interest on his money at the bank as it falls due. He receives semiannually therefrom $56, interest being at $2\frac{1}{2}\%$ annually. What is the amount of his deposit?

5. A store costing $24,000 has the following expenses charged against it: taxes $252, insurance $148, miscellaneous $350. What should it rent for to realize 6% on the investment?

6. A savings account was opened July 1, with a deposit of $1000, interest being credited each 6 mo. at 4%. What was the amount on deposit July 1, five years later, assuming there were no withdrawals or deposits?

7. In the above problem assuming that an additional $1000 had been deposited on each July 1 during the period, what would have been the amount on deposit at the end of the five-year period?

8. The city of C—— put in an electric lighting plant at an expense of $1,500,000 and sold municipal bonds maturing in 15 years to pay for the plant. What sum should the city set aside each year at $4\frac{1}{2}\%$ to redeem the bonds at maturity?

9. A man borrows $2000 for 5 years at 4% and agrees to pay principal and interest annually in 5 equal payments. What annual payment is required? (The first payment is at the end of the first year.)

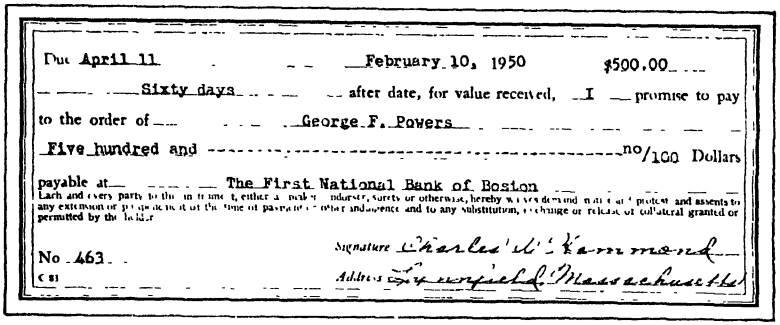
10. John Walters has an account with the Savings and Loan Co. He places on deposit government bonds for $1500 as collateral security for overdrafts. The bank charges him $4\frac{1}{2}\%$ interest on all overdrafts over 2 days. June 29 there was an overdraft of $250 that was settled July 10; Aug. 8 there was an overdraft of $430 that was settled Aug. 25; Sept. 25 there was an overdraft of $220 that was settled Oct. 14. How much interest did Mr. Walters pay?

11. If the government loans money to farmers, to be repaid in 20 equal annual installments, find the annual payment necessary to cancel an indebtedness of $1000, the rate of interest being 5%.

CHAPTER XXVI

PROMISSORY NOTES

396. Reasons for Promissory Notes. — When money is borrowed a promissory note is usually given as evidence of indebtedness. Every business man should know the essential elements of promissory notes and of the various kinds of indorsements of such notes.



Due April 11 February 10, 1950 $500.00

Sixty days after date, for value received, I promise to pay

to the order of George F. Powers

Five hundred and ----------no/100 Dollars

payable at The First National Bank of Boston

Each and every party to this instrument, either as maker, indorser, surety or otherwise, hereby waives demand, notice of protest and assents to any extension or postponement of the time of payment or other indulgence and to any substitution, exchange or release of collateral granted or permitted by the holder.

No. 463

Signature Charles M. Hammond

Address Hopewell, Massachusetts

397. The essential elements of a promissory note are: —

(*a*) *It must specify the sum to be paid.*

A written promise to pay enough to defray the expenses of a trip to Europe may be a valid contract, but it would not be a promissory note.

(*b*) *The note must designate the payee.*

That is, it must read "pay to bearer" or "pay to George F. Powers" (some definite person). It is not sufficient to write "I promise to pay $500.00."

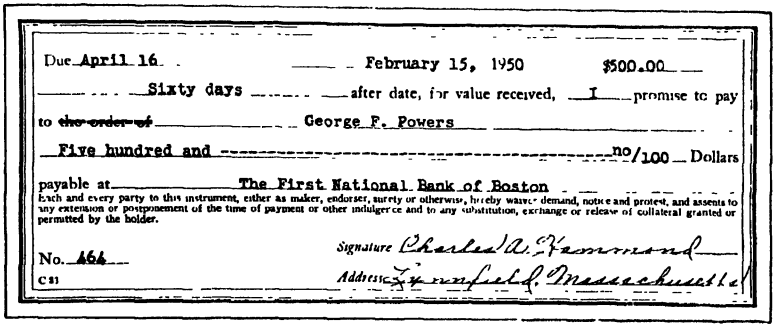
(*c*) *The note must be signed by the maker.*

It is not necessary to insert the place of payment, though this may be convenient. If no place of payment is inserted, the note must be presented for payment to the maker personally or at his usual place of business.

398. Payee. — The person to whom the note is payable is the *payee*. In the notes on this page George F. Powers is the payee.

399. Maker. — The person who promises to make the payment is the *maker* of the note, as Hammond in the notes below.

400. The Date of Maturity is the day on which a note is due.



Due April 16 February 15, 1950 $500.00

Sixty days after date, for value received, I promise to pay

to ~~the order of~~ George F. Powers

Five hundred and no/100 Dollars

payable at The First National Bank of Boston

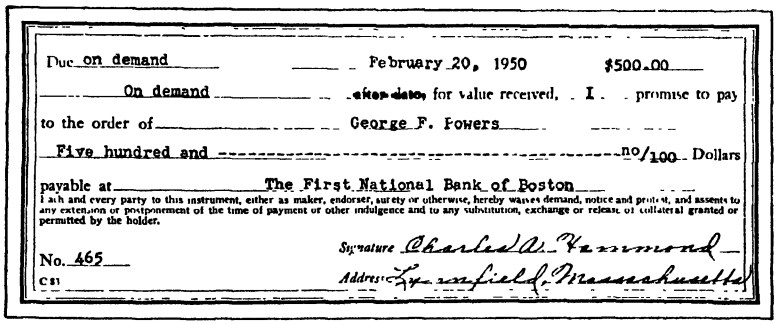
Each and every party to this instrument, either as maker, endorser, surety or otherwise, hereby waives demand, notice and protest, and assents to any extension or postponement of the time of payment or other indulgence and to any substitution, exchange or release of collateral granted or permitted by the holder.

No. 464

Signature Charles A. Hammond

Address Lynnfield, Massachusetts

401. Non-negotiable Note. — The note above is payable to George F. Powers and to no other person. Hence it cannot be transferred to a third person. Such a note is said to be *non-negotiable*.



Due on demand February 20, 1950 $500.00

On demand ~~after date~~, for value received, I promise to pay

to the order of George F. Powers

Five hundred and no/100 Dollars

payable at The First National Bank of Boston

Each and every party to this instrument, either as maker, endorser, surety or otherwise, hereby waives demand, notice and protest, and assents to any extension or postponement of the time of payment or other indulgence and to any substitution, exchange or release of collateral granted or permitted by the holder.

No. 465

Signature Charles A. Hammond

Address Lynnfield, Massachusetts

402. Negotiable Note. — The second note above is payable to the order of George F. Powers, which means that Mr. Powers may order Hammond to pay the note to a third person. That is, the note may be sold. Such a note is called a *negotiable note*.

403. Demand Note. — The second note above is payable on demand. That is, payment may be demanded at any time and the maker holds himself ready to pay as soon as the demand is made.

404. Note with Interest. — Sometimes a note reads "with interest" which means that the legal rate is to be paid. Thus, in Illinois the rate on this note would be 5%. (See page 308.)

405. Note without Interest. — The notes on page 319 bear no interest until due. If it is not paid when due, they bear interest at the legal rate from the date of maturity.

406. Indorsing a Note. — In order to sell the negotiable note George F. Powers must *indorse* it. This he does by signing his name on the back of the note.

407. Different Kinds of Indorsements. — A note may be indorsed in several different ways.

INDORSEMENT IN BLANK

George F. Powers

A note indorsed in blank is payable to bearer and may be sold to any person.

Pay to Arthur Ford, or order George F. Powers

The second indorsement makes the note negotiable, but Arthur Ford must indorse the note if he is to transfer it.

RESTRICTED INDORSEMENT

Pay to Arthur Ford George F. Powers

A restricted indorsement makes the note non-negotiable. That is, Arthur Ford cannot transfer the note so indorsed to another person.

If the maker fails to pay the note when due, each of the indorsements described above makes the indorser liable for payment of the note. If a note is not paid at maturity, the holder should *protest* it. That is, he should send a legal notice to the maker of the note and to all the indorsers that it has not been paid. If this is not done promptly, the indorsers escape liability on the note.

INDORSEMENT WITHOUT RECOURSE

Without Recourse George F. Powers

If George F. Powers wishes to transfer the note without rendering himself liable, he indorses it *without recourse*.

408. Difference between a Promissory Note and an Ordinary Debt. — A promissory note differs from an ordinary indebtedness on account in that it may be sold and collected independently of any indebtedness

which the payee of the note may owe to the maker of the note. In general commercial usage a man who fails to pay a note when it is due is a self-declared bankrupt, while failure to pay an ordinary account when due is only *prima facie* evidence of delinquency and may be explained away by disputing the debt itself, or by offering evidence that there are items which should be used as offsets of the debt. A promissory note can be disputed only by declaring that the note is a forgery, or that it was obtained by fraud. The law in regard to promissory notes should be studied in courses on commercial law.

WRITTEN EXERCISES (A)

1. Make a note payable in a definite time and carrying a specified rate of interest.

2. Make a note not bearing interest and payable to anyone who may present it for payment.

3. Indorse a note in blank.

4. Indorse a note "without recourse" and explain the difference between this indorsement and the usual indorsement.

5. What is a non-negotiable note? May a negotiable note be indorsed so as to make it non-negotiable? How?

6. A note made by Charles A. Hammond in favor of George F. Powers is indorsed as follows.

George F. Powers
Pay to Henry Ward
or order
Carl Woods
Without Recourse
Henry Ward

This indorsement shows that Mr. Powers transferred the note to Carl Woods, Carl Woods to Henry Ward, and Henry Ward to some other person. Explain fully the liability which each indorser assumes.

CHAPTER XXVII

PARTIAL PAYMENTS

409. If a note reads "on or before a certain date I promise to pay," then payments may be made on the note before the date named in it. Such payments are called *partial payments.*

410. United States Rule for Computing Partial Payments.

1. *Find the amount of the note up to the date of the first payment. If this payment is sufficient to cover the interest up to that date, deduct it from the amount. The result is the new principal.*

2. *Using this new principal, find the amount of the note up to the date of the second payment and deduct payment as before.*

3. *Proceed in this manner until the final date is reached.*

4. *If a payment is not sufficient to cover the interest for the period next preceding, no deduction is made, but the payment is added to the next payment.*

This rule has been adopted by the United States Supreme Court and is in very general use.

411. Indorsing a Note. — When payment is made on a note the amount is written on the back of it. This is called *indorsing* the note. On a note dated Jan. 10, 1948, payments are made as follows: — March 20, 1948, $1500; Aug. 10, 1948, $1000; December 1, 1948, $50. These payments are recorded on the back of the note as follows:

Received on this note,

March 20, 1948	$1500
Aug. 10, 1948	1000
Dec. 1, 1948	50

When a partial payment is made on a note the payment *must* be indorsed on the note itself; for otherwise the note might be sold to a third party and the full amount collected in spite of the previous part payment.

The United States Rule for computing partial payments will be most easily understood from an example.

Example. Find the amount due May 1, 1949, on a note for $6000, interest 6%, indorsed as shown in § 111.

Solution. Principal Jan. 10, 1948	$6000
Interest Jan. 10 to March 20	70
Amount March 20	6070
Payment March 20	1500
	4570
Interest Mar. 20 Aug. 10 (1 mo. 20 da.)	106.63
Amount Aug. 10	1676.63
Payment Aug. 10	1000
	3676.63
Interest Aug. 10, 1948, to May 1 1949 (8 mo 21 da.)	159.93
	3836.56
Payment Dec. 1, 1948	50
Amount May 1, 1949	3786.56

We see at a glance that the $50 paid December 1, 1948, is not sufficient to cover the interest from Aug. 10 to that date. Hence the interest is computed from August 10, 1948, to May 1, 1949, and the $50 payment deducted from the amount due on the latter date.

PROBLEMS (C)

1. A note dated May 20, 1948, for $8500 bearing interest at 6% was indorsed as follows: August 1, 1948, $500; Oct. 10, 1948, $600; Dec. 1, 1948, $1000; May 1, 1949, $4000. Find the amount due July 1, 1949.

2. Note for $500, dated May 1, 1947; interest 6%. Payments: Dec. 24, 1947, $75; June 29, 1948, $25; Aug. 22, 1948, $140; June 4, 1949, $250. What was due Jan. 1, 1950?

3. $700 Cleveland, Ohio, August 1, 1947

One year after date I promise to pay James Reynolds, or order, Seven Hundred Dollars with interest at 5%, for value received.

John Simpson.

Payments: Aug. 10, 1948, $250; Jan. 15, 1949, $300; Aug. 15, 1949, $150. What was due Oct. 25, 1949?

412. Compound Interest Avoided in Partial Payments. — The reason that the amount is not computed and the payment deducted in case the payment is not sufficient to cover the interest to the date of payment is that compound interest is to be avoided.

There are certain more complicated questions, however, arising in this connection, the discussion of which is beyond the scope of this book. The student is referred to texts on the mathematics of investment for further information.

413. Merchant's Rule. — The following rule is often used by business men to settle indebtedness.

(*a*) Find the amount of the original principal to the date of final settlement.

(*b*) Find the amount of each payment to the date of final settlement.

(*c*) Add the amounts under (*b*) and subtract from the amount under (*a*). The remainder is the amount due. This is sometimes called the *Merchant's Rule*.

Example. A note for $4800 bearing interest at 7% and dated Oct. 7, 1936 was indorsed as follows:

Dec. 1, 1936, $200; Jan. 10, 1937, $1500; March 15, 1937, $500. Find amount due May 15, 1937, using the Merchant's Rule.

Solution.

Principal Oct. 7, 1936		4800
Interest Oct. 7, 1936 to May 15, 1937		203.47
Amt. of Note May 15, 1937		5003.47
Amt. of the $200 payment	$206.38	
Amt. of the $1500 payment	1536.46	
Amt. of the $500 payment	505.83	
Total Payments		2248.67
Balance Due May 15, 1937		2754.80

ORAL EXERCISES

1. What is meant by partial payment? When partial payment is made on a note, how must this payment be recorded? Why?

2. How is compound interest avoided by computing according to the United States Rule?

PROBLEMS (B)

Use the Merchant's Rule to solve the following.

1. Face of note \$3000, date Aug. 15, 1946, rate of interest 6%. The note is indorsed as follows: Nov. 1, 1946, \$400; Jan. 20, 1947, \$600, May 1, 1947, \$500. Find amount due Aug. 1, 1947.

2. Face of note \$5000, date Dec. 15, 1946, rate of interest 5%. Payments as follows: Jan. 31, 1947, \$2000; June 1, 1947, \$1500; Sept. 15, 1947, \$1000. Find the amount due Jan. 1, 1948.

3. Note of \$1500, dated June 5, 1947, bearing interest at 5%. Payments: Aug. 1, \$500; Oct. 2, \$300; Dec. 10, \$200. Find the amount due Jan. 14, 1948.

4. Note for \$2000 given July 1, 1946, interest 6%. Payments: Oct. 1, \$800; Feb. 1, 1947, \$500; June 1, 1947, \$600. Find the amount due Oct. 15, 1947.

Solve each of the following both by the United States Rule and by the Merchant's Rule. Can you explain the reason for the difference?

5. Note for \$4000 given Aug. 10, 1946, interest 6%. Payments: \$2500, Sept. 15, 1946; \$1000, July 15, 1947. Find amount due Sept. 1, 1947.

6. \$2600 Johnstown, Pa., July 1, 1947

One year after date for value received, we or either of us, promise to pay Lucy Ward — Twenty-six Hundred Dollars, with interest at 5%.

James Morrison
Roger Lewis

Indorsements: Apr. 10, 1948, \$1000; Sept. 25, 1948, \$500; Feb. 14, 1949, \$800. What was due on date of settlement Aug. 25, 1950?

7. Note for \$750. Date, Sept. 10, 1948. Interest 4%. Payments: Dec. 24, 1948, \$200; Jan. 30, 1949, \$200; July 1, 1949, \$200. Settlement Nov. 14, 1949. What was the amount due?

8. Note for \$10,000. Apr. 1, 1945. Interest 6%. Payments: Dec. 15, 1945, \$5000; Jan. 1, 1946, \$3000; Apr. 1, 1947, \$1000. Settlement Apr. 1, 1948. What was the amount due?

CHAPTER XXVIII

INSTALLMENT BUYING

414. Part Payment Buying. — Many articles are now sold on "part payment."

A rather small part of the purchase price is paid at the time of buying as a down payment and the remainder is paid in monthly, equalized installments. Furniture, clothes, automobiles, radios, washing machines, and even such automobile accessories as tires are sold in this way. "Pay from income," "Pay as you ride" (this for automobiles), "Budget buying," and other such expressions are constantly seen in advertising.

415. Examples in Installment Buying. — A common arrangement in installment buying is shown in the following examples.

Furniture, priced at \$100 if paid for in cash, is bought for \$10 in cash and ten \$10 monthly payments, making \$110 in all. For smaller amounts the number of equal monthly payments is usually smaller, and for larger amounts the payments may be spread over a longer period.

A remark often made by salesmen is, "Now \$10 interest on \$100 for 10 months is certainly not bad when you consider you can pay monthly." In this chapter we shall study the rates of interest actually paid on some typical purchases of this kind.

Example 1. Find the rate of interest when, instead of paying \$100 cash, \$10 is paid at once and then 10 monthly payments of \$10 each.

Solution: Since in borrowing money interest is always figured as payable at the *end* of the interest period, we will regard the last payment as interest. We will regard the other payments as payments of money borrowed, the time for which each \$10 is borrowed being 0 months, 1 month, 2 months, etc. As shown at the

right, we find that \$10 has been borrowed for 45 months, or \$100 for 4.5 months.

The problem then is: Find the rate of interest when the principal is \$100, the time $4\frac{1}{2}$ months, and the interest \$10. The time in terms of one year is $4\frac{1}{2} \div 12 = \frac{3}{8}$.

In the formula $r = \frac{i}{pt}$, we have $i = 10$, $p = 100$, $t = \frac{3}{8}$. We thus find that the actual rate is $26\frac{2}{3}\%$.

Payment	Time, Months
First	0
Second	1
Third	2
Fourth	3
Fifth	4
Sixth	5
Seventh	6
Eighth	7
Ninth	8
Tenth	9
Total	45

$$r = \frac{10}{100 \times \frac{3}{8}} = .26\frac{2}{3} = 26\frac{2}{3}\%$$

In this problem the interest is exactly the same as the last payment.

If we regard the first payment as the interest, then the total of the borrowing is somewhat larger, and hence the rate of interest is less. That the last payment, not the first, should be regarded as the interest is beyond question, since on all loans the interest is paid at the end of the interest period.

$$r = \frac{10}{100 \times \frac{3}{8} - \frac{80}{300}} = .26\frac{2}{3} = 26\frac{2}{3}\%$$

If the payments run more than one year, the problem is quite different, and is beyond us at this stage.

The computation at the right shows the rate when the first payment is regarded as interest. In this case we have \$10 borrowed for 55 months, or \$100 for 5.5 months, which is $\frac{55}{12}$ years. That is, $t = \frac{55}{12}$. Check the computation at the right.

Payment	Time, Months
First	1
Second	2
Third	3
Fourth	4
Fifth	5
Sixth	6
Seventh	7
Eighth	8
Ninth	9
Tenth	10
Total	55

$$r = \frac{10}{100 \times \frac{55}{12}} = .21\frac{9}{11}$$
$$= 21\frac{9}{11}\%$$

Example 2. Tom Wicks buys a bill of goods for \$50 on the installment plan, paying \$5 down, and five \$10 monthly payments. Find the rate of interest.

Solution: We will regard \$5 of the last payment as interest. The remaining \$5 of this payment has run 5 months; this is equivalent to \$10 for $2\frac{1}{2}$ months. The total "borrowing" amounts to \$10 for $12\frac{1}{2}$ months or $1\frac{1}{24}$ years.

Second payment	\$10,	1 mo.
Third payment	\$10,	2 mo.
Fourth payment	\$10,	3 mo.
Fifth payment	\$10,	4 mo.
Sixth payment	\$10,	$2\frac{1}{2}$ mo.
Total	\$10,	$12\frac{1}{2}$ mo.

If in this case \$5 of the first payment is regarded as interest, the rate turns out to be $41\frac{11}{29}\%$.

The difference between the rates, according to whether the interest is regarded as paid in advance or after the interest period, is greater for shorter periods.

$$r = \frac{5}{10 \times 1\frac{1}{24}} = \frac{12}{25} = 48\%$$

416. Cost of Installment Selling. — But, you will ask, why is it that with such high rates of actual interest, the dealer prefers to sell for cash rather than on partial payments? There are good reasons for this: In the first place, some interest is necessary to cover the use of the money until it is collected. But this is a small part of the real reason. Such accounts are expensive to collect and at times there are outright losses. If anything, the dealers lose by installment selling as compared with selling for cash, and the buyer pays a very high rate of interest.

The installment seller renders a service, which to him is very expensive, and the buyer necessarily has to pay for it. In some advertisements one sees "no extra charge on account of part payment." But if you step into a place using such advertisements, make a purchase on the deferred-payment basis, and then ask, "How much cash will you take," you will usually get a discount of 10%, which, considering the short time the payments cover, will be found equivalent to a very high rate of interest. Whoever offers something for nothing must plan to make up for it in some way, and the buyer is the one who pays him.

PROBLEMS

1. Proceeding as in Example 2 on page 327, find the rate of interest when 10% of the cost of the goods is paid down, and the rest in 3 equal monthly installments. Assume that the cost is \$30, the first payment \$3, and then 3 payments of \$10 each.

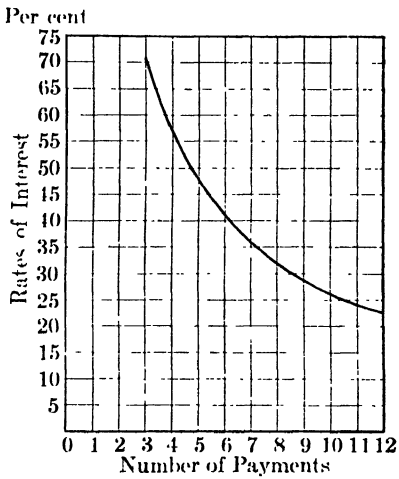
2. Find the rate for four payments, by using a \$40 purchase, \$4 down and four \$10 monthly payments.

3. Find the rate for six deferred payments, using a \$60 purchase, with \$6 down and six \$10 monthly payments.

4. Find the rates for seven, eight, and nine deferred payments, using purchases of \$70, \$80, and \$90. Note that the rates depend upon the number of deferred payments and, of course, also upon the apparent rate charged, but not upon the amount of the purchase.

417. Graphical Representation of Interest Rates. — The graph below shows the rate of interest actually paid for different numbers of monthly payments. It is assumed in each case that the interest comes out of the last payment. Also, in each case, it is assumed that 10% is added to the regular price and paid at once and that the remainder is spread over the monthly payments.

Number of Deferred Payments	Rates of Interest, Per cent
3	70.6
4	57.1
5	48
6	41.4
7	36.4
8	32.4
9	29.3
10	$26\frac{2}{3}$
11	24.4
12	22.6



A much more difficult and probably more just method can be used to compute these rates of interest, but the results differ but slightly from those found here.

When there are more then ten deferred payments, a complication arises because the interest amounts to more than the last payment.

418. Automobile Financing. — Of late years as many as three fourths of the automobiles have been sold on some plan of deferred payment. A substantial part of the price, one fourth or more, is paid in cash or by turning in a used car. A note is made out for the balance giving the car as security. Such notes are bought by certain finance corporations organized especially for this purpose. In recent years these corporations have operated substantially on a plan illustrated in the following example.

Example 1. The unpaid balance on a new car is \$600. The buyer wishes to pay this in six equal monthly installments. Six per cent interest on \$600 for six months is then added to the \$600 making \$618, which is divided by 6, making the monthly payments \$103. The note is made out on this basis, and the car is delivered. Our problem is to find the rate of interest actually paid.

Solution: The amount of interest paid each month is one sixth of the total interest (18 dollars), or \$3. Hence the amount paid on the car each month is \$100. The work is like that on ordinary installment payments, and is shown at the right. The rate, however, turns out to be decidedly lower than in the former case. This is due in part to the fact that the rate charged is 6% instead of 10%, and also to the fact that this is a yearly rate, and is charged only for the time up to the last payment. We note, however, that the interest is 6% on the sum of all the deferred payments, and not only on the unpaid balance for each month, which would be in the case of an ordinary loan. For this reason the rate of interest actually paid depends only slightly upon the number of deferred payments, being a little greater as the number of payments is increased.

1st p'mt on note	\$100	1 mo.
2nd p'mt on note	\$100	2 mo.
3rd p'mt on note	\$100	3 mo.
4th p'mt on note	\$100	4 mo.
5th p'mt on note	\$100	5 mo.
6th p'mt on note	\$100	6 mo.
Total	\$600	21 mo.

$$r = \frac{18}{100 \times 1\frac{3}{4}} = .103 - 10.3\%$$

Example 2. Under the conditions of Example 1 find the rate of interest if an unpaid balance of \$600 is paid in 4 monthly payments. Carry out the work in detail and show that the answer is 9.6%.

Example 3. Use an unpaid balance of \$600 and find the rate when there are 12 monthly payments. Show that the rate is 11.08%.

The rate does not depend upon the total unpaid balance and, as you see, only slightly upon the number of payments.

PROBLEMS

1. Find rates of interest for 3, 5, 8, 10 monthly payments on a car. Then make a table similar to that on page 329, using answers for 4, 6, 12 payments already obtained. (Note that 7, 9, 11 payments are omitted.)

2. Using the ordinary installment plan for selling furniture, etc. (see page 326), compute the actual rate of interest when only 6% is charged. Use a \$60 purchase with \$3.60 as the first payment and 6 monthly \$10 payments. Compare this rate with that found in Example 1, page 330.

TOPICS FOR DISCUSSION

Why should the interest on deferred automobile payments run about 10%, when banks are lending money at 6%?

419. Buying a House on Monthly Payments. — The following arrangement is not unusual in buying a house: A certain amount is paid down, and it is agreed that a definite amount is to be paid each month. From this monthly payment is deducted one twelfth of a year's interest on the unpaid amount, the rest being applied on the principal. An example will illustrate:

Example. Mr. Mason bought a house costing \$8800 and paid \$1600 in cash. He agreed to pay \$60 a month, the rate of interest being 5% on the unpaid balance. Mr. Mason then makes up a schedule, part of which is shown below.

SCHEDULE OF PAYMENTS ON HOUSE

PAYMENT	AMOUNT PAID	INTEREST	PAYMENT ON PRIN	UNPAID BALANCE
1	\$60	\$30	\$30	\$7170
2	\$60	29.88	30.12	7139.88
3	\$60	29.75	30.25	7109.63
4	\$60	29.62	30.38	7079.25
5	\$60	29.50	30.50	7048.75
6	\$60	29.37	30.63	7018.12

Clearly, the interest to be paid each month is $\frac{1}{12}$ of 5% or $\frac{5}{12}$% of the unpaid balance. Since the interest the first month is $\frac{5}{12}$% of \$7200, or \$30, this amount is written in the interest column, leaving \$30 of the \$60 payment to apply on the principal. This leaves \$7170 still unpaid, which is written in the last column. The interest for the second month is $\frac{5}{12}$% of \$7170 or $\frac{5}{12}$ of \$71.70 = \$29.88, which is written in the second line of the interest column. Then \$60.00 − \$29.88 = \$30.12 is applied on the principal, leaving \$7139.88.

By looking at the schedule on page 331, Mr. Mason can tell at a glance how much he still owes on his home, how much interest he is paying each month, and by how much he is reducing his indebtedness.

In the first column of such schedules the date of each payment, instead of the number of the payment, is usually given. As Mr. Mason makes a payment he fills out one line of this schedule, which he can do in a very few minutes.

420. Time Required to Pay for a House. — Of the first payment in the schedule on page 331, \$30 is applied on the principal, leaving \$30 for the interest. As the payments continue and the principal is reduced, the interest is also reduced and the amount applied on the principal is increased until the last payment, of which nearly all, or almost \$60, is applied on the principal. (Note that for the last payment the balance drawing interest is less than \$60 and hence the interest is less than 25 cents.) Therefore the average applied on the principal is about \$45. Then $7200 \div 45 = 160$ (months), which is about the time in which Mr. Mason will finish paying for his house.

PROBLEMS

1. Copy the schedule on page 331 and extend it to show the twentieth payment.

2. Mr. Tom Scott has an unpaid balance of \$2000 on his home. He agrees to pay \$60 each month to apply on interest and principal, at the rate of 5%. Make a schedule of Mr. Scott's payments. In this case you will have to find the interest of each month by computing on separate paper.

3. Using the method given above, find how many months it should take for Mr. Scott to pay for his home. How nearly does this agree with the result found in problem 2?

421. Buying a Farm on Deferred Payments. — Mr. Ed. Donlan buys a farm costing \$18,000 on which he pays \$3000 in cash, and agrees to pay \$1500 at the end of each year to apply on interest and principal, the rate of interest being $4\frac{1}{2}\%$. The beginning of Mr. Donlon's schedule of payments is as follows.

Payment	Amount Paid	Interest	Payment on Prin	Unpaid Balance
1	\$1500	\$675.00	\$825.00	\$14,175.00
2	\$1500	\$637.88	\$862.12	\$13,312.88
3	\$1500	\$599.08	\$900.92	\$12,411.96
4	\$1500	\$558.54	\$941.46	\$11,470.50

PROBLEMS

1. Check the figures given in the above schedule. Then copy and extend to twelve payments.

2. About how many years will Mr. Donlan require to pay for his farm?

3. The unpaid balance on a farm is \$9800. A loan is made at $4\frac{1}{2}\%$ with payments of \$500 every 6 months to apply on interest and principal. Start a schedule of payments and complete the first six lines. In about how many years will this debt be paid?

TOPICS FOR DISCUSSION

1. In making payments on the houses on pages 331, 332, interest is paid at the end of each month. Is this a little more burdensome on the debtor than paying the whole year's interest at the end of the year?

2. Which is more likely to be paid, \$30 in interest at the end of each month, or \$360 at the end of the year?

CHAPTER XXIX

BANKS AND BANK DISCOUNT

422. Functions of a Bank. — Banks are institutions whose business it is to receive money, to pay it out again when demanded, and to make loans.

Nowadays people seldom keep large sums of money on hand. It is deposited in banks and drawn out as needed.

The FIRST
NATIONAL BANK of
BOSTON

DEPOSITED for Account of

Allan B. Snyder
(Please Print Name)

Boston, February 20 1950

Items received for deposit or collection are accepted on the following terms and conditions. This Bank acts only as depositor's collecting agent and assumes no responsibility beyond its exercise of due care. All items are credited subject to final payment and to receipt of proceeds of final payment in cash or solvent credits by this Bank at its own office. This Bank may forward items to correspondents and shall not be liable for default or negligence of correspondents selected with due care nor for losses in transit and such correspondent shall not be liable except for its own negligence. Items and their proceeds may be handled by any Federal Reserve bank in accordance with applicable Federal Reserve rules and by this Bank or any correspondent in accordance with any common bank usage with any practice or procedure that a Federal Reserve bank may use or permit another bank to use or with any other lawful means. This Bank may charge back at any time prior to midnight on its business day next following the day of receipt any item drawn on this Bank which is ascertained to be drawn against insufficient funds or otherwise not good or payable. An item received after this Bank's regular afternoon cut off hour shall be deemed received the next business day

		DOLLARS	CENTS
CASH	Bills	85	00
	Coin	98	65
CHECKS: 1	Union	487	60
2			
3			
4			
5			
		671	25

423. Opening a Bank Account. — To open an account in a bank a man first makes a *deposit*.

To make a deposit a "deposit slip" is made out showing the amount deposited in checks, in bills, in currency, and in coin. Checks may be listed by the name of the bank if it is in the same city. The deposit slip also has written on it the name of the depositor.

When the account is first opened the depositor signs his name on a card which is deposited in the bank and kept to compare with the signatures on his checks which will come in later.

A bank usually requires references when opening an account with a stranger, since an irresponsible or dishonest depositor may be harmful to the bank.

EXERCISES (A)

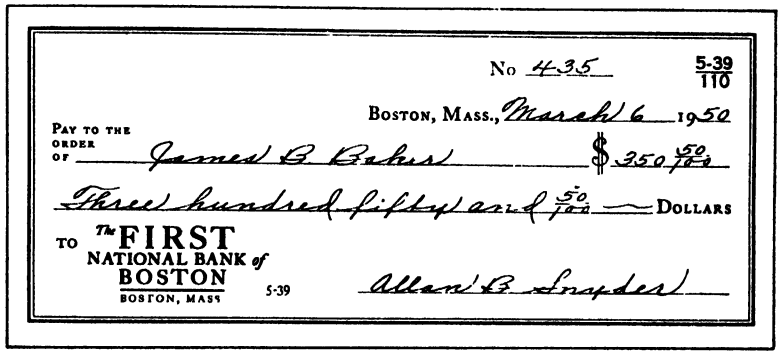
Make out deposit slips in your own name for the following deposits.

1. Check, $74.80; check, $107.60; check, $31.50; currency, $65; silver, $17.25.

2. Check, $75; check, $93.40; check, $140.26; gold, $20, $25, $10; silver, $20.25.

424. The Checking Account. — The usual account which a business house has in banks is what is called a "checking account."

Money in a checking account may be withdrawn at any time by presenting a *check* for the amount which it is desired to withdraw.



425. Making Out a Check. — There are certain features of a check which should be noticed: The amount of the check is written in figures and also in words. To make it difficult to raise the check the figures are written near the $ sign, and the words specifying the sum begin close to the left end of the check.

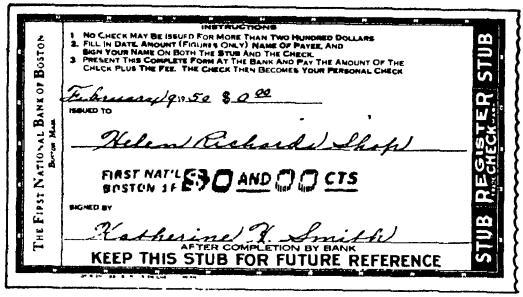
This check directs the bank to pay to James B. Baker the sum of $350.50. When the check is paid the bank deducts this amount from Mr. Snyder's account.

Checks are used very extensively because of their convenience and safety. In our larger cities over 90% of all bills are paid by check.

A rather new type of checking service, called "Register Check" service, has been instituted in many banks. It is adapted for the use of the many people who have not opened checking accounts and yet wish to pay "by check." The user fills out a check similar to the one on page 336, presents it to the teller with the amount of money the check calls for plus a small charge ranging from ten to twenty cents. The bank teller numbers the check, puts it through a protectograph

which cuts the amount payable on the check and stub and returns the check to the person who has made it out. He tears off the stub, which he retains, and sends the rest of the check to the payee.

Register Checks are personal checks and not checks of the bank's issue, and are, therefore, governed by laws and practices applying to



A REGISTER CHECK

(Above) The stub is torn off and kept by the payer; (below) the rest of the check is sent to the payee.

personal checks. Anyone may sign them. Anyone may "register" them. They may be certified. Payment may be stopped and refund may be made, if necessary, without requiring bond or surety.

426. Identification. — If Mr. Baker is not known at the bank, he must be *identified* before he can secure money on the check. That is, he must be introduced by some person known to the bank. If a check should be lost or stolen and presented for payment by one not entitled to receive it, the bank would be responsible to the rightful owner if the check were cashed. The bank requires the man who

cashes the check to write his name on the back of it. This serves the bank as a receipt. Any bank will cash a check drawn on it or any other bank provided the party presenting it is properly identified.

427. Clearing House. — In the large cities every bank during a day's business cashes checks on many other banks. At some time during the day, representatives of all the banks meet in what is called the *clearing house* to settle the mutual indebtedness thus resulting.

428. Balance in Bank. — The amount of money which one has in a bank is called his *balance* in the bank. The balance in the bank at any one time is obtained by adding all the deposits and subtracting the amount of all the checks drawn out. The balance is always recorded on the check stub. Every deposit is added and every check drawn is subtracted. This enables one to see at a glance how much is left.

WRITTEN EXERCISES (A)

Find the balance to be carried forward in each of the following.

1.	Bal. brought forward	$6829.80	**2.**	Bal. brought forward	$165.00
	Checks	120		Checks	17.50
		427			28.45
		600			19.46
		84.60			11.78
	Bal. carried forward			Bal. carried forward	
3.	Bal. brought forward	$17,500.45	**4.**	Bal. brought forward	$1984.10
	Checks	2,746.29		Checks	725.18
		1,749.41			29.46
		8,276.96			145.71
		548.10			87.16
	Bal. carried forward			Bal. carried forward	
5.	Bal. brought forward	$8765.10	**6.**	Bal. brought forward	$2996.40
	Checks	1849.11		Checks	784.29
		2548.10			1459.11
		76.96			29.29
		387.11			348.16
	Bal. carried forward			Bal. carried forward	

429. Banks Lending Money. — One of the chief functions of a bank is to lend money, and this is also the bank's chief source of income. A bank which has \$100,000 in deposits may lend out \$75,000, keeping only \$25,000 in cash. This is possible because experience has shown that only a small fraction of the total deposits will be drawn out in the course of a few weeks.

The loans which banks make are usually for a short time so that in case of need the bank can collect funds quickly.

430. Bank Discount. — The bank collects interest at the beginning of the period for which the loan is made instead of at the end as with ordinary loans. The deduction of interest at the beginning of the period is called *bank discount.*

The difference between ordinary interest and bank discount is shown by the following example.

(*a*) If a man borrows \$1000 at 6%, ordinary interest, for one year, he gets the use of \$1000 for one year and pays \$60 for that use.

(*b*) If a man borrows \$1000 at 6% bank discount for one year, the bank deducts the \$60 interest at the beginning of the year and the man gets the use of \$1000 − \$60 = \$940 for one year and pays \$60 for that use.

Hence we see that bank discount is more favorable to the lender than ordinary interest.

431. Proceeds of a Note. — The amount left after the bank discount is deducted is called the *proceeds* of a note. The time which the note has to run from the time it is discounted until it is due is called the term of discount.

WRITTEN EXERCISES (A)

Find the proceeds of each of the following notes.

	Face of Note	Rate of Discount	Term of Discount
1.	\$3250	6%	15 da.
2.	2600	5%	90 da.
3.	1950	4%	70 da.
4.	4570	$4\frac{1}{2}$%	63 da.
5.	3600	3%	30 da.

6.	5450	6%	75 da.
7.	7680	4%	40 da.
8.	19,500	7%	60 da.
9.	876	8%	90 da.
10.	4520	5%	100 da.
11.	890	6%	120 da.

432. Discounting Interest-bearing Notes. — When money is borrowed from a bank on ordinary bank discount the notes given do not bear interest. The bank collects its interest by discounting the face of the note. However, interest-bearing notes obtained in the course of business are frequently discounted at the bank. In this case the discount is computed on the amount of the note at maturity.

Problem. A note for \$7000 bearing interest at 5% due in 90 days is discounted at the bank at 5%, 45 days before it is due.

Find the proceeds of the note.

Solution.

Face of Note	\$7000
Int. at 5% for 90 days	87.50
Amt. of Note at Maturity	7087.50
Discount on Amt. for 45 days at 5%	44.30
Proceeds of Note	7043.20

EXERCISES (B)

Find the proceeds of the following.

	Face	Date	Due	Int.	Date of Dis.	Rate of Dis.
1.	\$12,500	May 1, 1937	Sept. 2, 1937	5%	June 16	6%
2.	350	Jan. 2	Apr. 15	5%	Feb. 10	6%
3.	760	Oct. 15	Feb. 12	6%	Jan. 6	5%
4.	1250	Nov. 1	Apr. 15	4%	Mar. 8	5%
5.	275.60	Aug. 8	Dec. 6	6%	Oct. 5	5%
6.	1450.25	May 4	Dec. 12	7%	Sept. 8	6%
7.	875.60	July 10	Nov. 15	5%	Aug. 15	7%
8.	976.75	Sept. 16	Jan. 7	6%	Dec. 1	5%

433. Call Loans. — Loans on which payment may be demanded at any time are called *call loans.* Notes given for call loans read "On demand I promise to pay." (See page 319.) The rates of interest on call loans are usually lower than on time loans. A bank can afford to make a call loan to a customer of unquestioned financial responsibility at a very low rate because the certainty that the money may be had any day makes it as good as cash in the vault for the purpose of the bank.

Interest on call loans is computed the same as interest on ordinary loans and not as bank discount, because the time of the loan is not known when it is made.

Problems of interest arising from call loans are more likely to involve difficulties arising from the number of days than those connected with ordinary time loans.

WRITTEN EXERCISES (B)

Find the amount due on each of the following loans.

	Principal	Rate	Time		Principal	Rate	Time
1.	$4500	$4\frac{1}{2}\%$	47 da.	16.	$7945	$2\frac{1}{2}\%$	37 da.
2.	2500	3%	25 da.	17.	8760	4%	29 da.
3.	1500	$3\frac{1}{2}\%$	45 da.	18.	4527	$3\frac{1}{2}\%$	41 da.
4.	1000	2%	40 da.	19.	1876	4%	54 da.
5.	7650	$4\frac{1}{4}\%$	27 da.	20.	9500	$4\frac{1}{2}\%$	63 da.
6.	2540	$4\frac{1}{2}\%$	30 da.	21.	8640	$3\frac{1}{2}\%$	18 da.
7.	1976	3%	28 da.	22.	9745	4%	75 da.
8.	8750	5%	42 da.	23.	7800	$3\frac{1}{2}\%$	47 da.
9.	2940	$2\frac{3}{4}\%$	61 da.	24.	18,700	$4\frac{1}{2}\%$	58 da.
10.	1250	$3\frac{1}{2}\%$	48 da.	25.	45,200	5%	41 da.
11.	9649.50	4%	37 da.	26.	12,500	$4\frac{1}{4}\%$	42 da.
12.	9600	4%	19 da.	27.	3540	$4\frac{3}{4}\%$	63 da.
13.	7525	3%	75 da.	28.	27,800	$3\frac{1}{2}\%$	71 da.
14.	7200	4%	21 da.	29.	75,600	$3\frac{3}{4}\%$	82 da.
15.	1450	$3\frac{1}{2}\%$	43 da.	30.	42,700	$3\frac{1}{2}\%$	91 da.

DRILL IN FUNDAMENTALS (A)

Find the net proceeds of the following bills and check.

	Gross Amount of Bill	Discount Series		Gross Amount of Bill	Discount Series
1.	$830	25%, 10%	16.	$1890	10%, 10%, 5%
2.	760	18%, 10%	17.	8600	15%, 15%, 10%
3.	480	15%, 10%	18.	3700	40%, 5%
4.	936	25%, 15%, 5%	19.	2950	45%, 10%
5.	478.50	30%, 20%, 10%	20.	940	40%, 15%, 5%
6.	880	40%, 15%	21.	780	15%, 5%
7.	460	35%, 20%, 5%	22.	290	18%, 10%, 5%
8.	1390	45%, 35%, 20%	23.	485	17%, 8%
9.	4270	50%, 10%	24.	475	15%, 5%
10.	8760	25%, 15%, 8%	25.	625	35%, 10%, 5%
11.	3498.50	35%, 18%, 5%	26.	938.75	40%, 25%, 5%
12.	4190	25%, $12\frac{1}{2}\%$	27.	1540	20%, 10%, 5%
13.	1490	20%, $16\frac{2}{3}\%$	28.	781.25	15%, 10%
14.	2780	20%, $8\frac{1}{3}\%$	29.	394.50	15%, 15%, 5%
15.	8417	15%, $6\frac{1}{3}\%$	30.	1876.80	20%, 20%

Reduce the fractions to percentages and check. Find each per cent to the nearest hundredths of one per cent.

31.	$\frac{3}{7}$	41.	$\frac{7}{15}$	51.	$\frac{7}{16}$	61.	$\frac{14}{19}$
32.	$\frac{5}{8}$	42.	$\frac{9}{19}$	52.	$\frac{9}{16}$	62.	$\frac{17}{24}$
33.	$\frac{3}{11}$	43.	$\frac{9}{21}$	53.	$\frac{3}{32}$	63.	$\frac{18}{31}$
34.	$\frac{5}{13}$	44.	$\frac{10}{29}$	54.	$\frac{5}{64}$	64.	$\frac{15}{23}$
35.	$\frac{7}{15}$	45.	$\frac{13}{31}$	55.	$\frac{11}{16}$	65.	$\frac{16}{27}$
36.	$\frac{4}{9}$	46.	$\frac{14}{17}$	56.	$\frac{7}{64}$	66.	$\frac{49}{97}$
37.	$\frac{1}{7}$	47.	$\frac{7}{17}$	57.	$\frac{11}{32}$	67.	$\frac{34}{53}$
38.	$\frac{11}{21}$	48.	$\frac{9}{28}$	58.	$\frac{9}{32}$	68.	$\frac{61}{97}$
39.	$\frac{4}{11}$	49.	$\frac{14}{19}$	59.	$\frac{9}{64}$	69.	$\frac{35}{47}$
40.	$\frac{12}{29}$	50.	$\frac{13}{27}$	60.	$\frac{11}{64}$	70.	$\frac{13}{67}$

CHAPTER XXX

CASH BALANCE IN BANK

434. Bankers' Cash Account. — A bank is constantly called upon to cash checks for its customers. It is therefore necessary for the banker to be able to tell by a mere glance at his book whether a certain customer has sufficient funds in the bank to pay a check which may be presented. For this reason the banker keeps a cash account with each customer which is in substance as shown on page 343.

At the close of the day's business all checks against the account of James B. Ward are added and total debited to his account. The account is credited with all deposits as shown by the cashier's deposit slips. The balance is then posted at once. Compare page 26.

WRITTEN EXERCISES (A)

During the month of January, W. R. Stokes & Co. make deposits and draw checks as follows: Deposits for the successive days beginning Jan. 3: \$4300, \$2450, \$3425, \$1375, \$1560, \$1250, \$600, \$1390, \$500, \$420, \$700, \$360, \$340, \$480, \$250, \$600, \$580, \$420, \$470, \$450, \$390, \$320, \$370, \$440. Withdrawals: 4th, \$2500; 7th, \$1980; 8th, \$260; 10th, \$2640; 11th, \$960; 14th, \$365; 15th, \$425.50; 17th, \$1945.80; 18th, \$2391.60; 20th, \$349.10; 22d, \$462.80; 24th, \$2360.40; 25th, \$1781.20; 28th, \$4960; 29th, \$320.15; 31st, \$2430.75. Balance Jan. 1 was \$5390.40. Arrange this account as on page 343.

435. Interest on Daily Balances. — Banks frequently allow a low rate of interest on daily balances in a checking account provided these exceed a certain amount.

Example. Find the interest on the daily balances on the opposite page, the rate being 2%.

Solution. The balances are given in the right-hand column of the account. Adding these we find they are equal to \$921,611.33 for one day. Multiplying this by $3\frac{1}{65} \times \frac{2}{100} = 18.\frac{1}{250}$ we have \$50.49, which is the required interest.

The balance in the bank draws interest every day of the month. Hence the balance is given for Sundays and other banking holidays though the balance cannot possibly be changed on such days.

In the case of an account drawing no interest the balance is posted only when deposits or withdrawals are made. See page 344.

In account with JAMES B. WARD

DATE	DEBITS		CREDITS		DAILY BALANCE
Jan. 1			By Balance	$54,865.40	$54,865.40
2					54,865.40
3	To Checks	$34,343.60	By Cash	1,360.50	21,882.30
4	To Checks	5,763.40	By Cash	1,539.65	17,658.55
5	To Checks	50.00	By Cash	1,267.85	18,876.40
6	To Checks	593.50	By Cash	1,438.65	19,721.55
7	To Checks	120.40	By Cash	1,794.36	21,395.51
8	To Checks	3,492.50	By Cash	1,647.60	19,550.61
9					19,550.61
10	To Checks	451.40	By Cash	2,192.75	21,291.96
11	To Checks	191.20	By Cash	1,819.15	22,919.91
12	To Checks	249.75	By Cash	1,917.80	24,587.96
13	To Checks	40.00	By Cash	2,008.65	26,556.61
14	To Checks	310.00	By Cash	1,391.75	27,641.36
15	To Checks	3,250.40	By Cash	2,038.25	26,129.21
16					26,129.21
17	To Checks	200.00	By Cash	1,840.90	28,070.11
18	To Checks	350.00	By Cash	1,472.45	29,192.56
19	To Checks	275.00	By Cash	1,064.87	29,982.43
20	To Checks	895.60	By Cash	1,937.65	31,024.48
21	To Checks	382.70	By Cash	2,268.27	32,910.05
22	To Checks	3,749.35	By Cash	1,497.65	30,658.35
23					30,658.35
24	To Checks	1,260.00	By Cash	1,924.60	31,322.95
25	To Checks	720.00	By Cash	1,764.80	32,367.75
26	To Checks	90.80	By Cash	1,891.60	34,168.55
27	To Checks	180.00	By Cash	2,678.20	36,666.75
28	To Checks	340.50	By Cash	1,946.80	38,273.05
29	To Checks	3,280.60	By Cash	2,194.60	37,187 05
30					37,187.05
31	To Checks	729.70	By Cash	1,261.95	37,719.30
		$61,310.40		$99,029.70	$921,611.33
	To Balance	37,769.79	By Interest	50.49	Total Daily Bal.

436. Simplified Account for No Interest on Daily Balances. — A very large number of bank accounts are small personal accounts in which only a few deposits and withdrawals are made each month and in which the balance is small. No interest is paid on these balances and hence the form of the account may be simplified.

Account with J. P. LAWRENCE

DATE	DR.		CR.		BALANCE
Jan. 1				$380.65	$380.65
3	To Checks	$85.70			
4	To Checks	42.25			
5	To Checks	7.80			
18			By Cash	25.00	
24	To Checks	16.50			
31			By Cash	250.00	
Feb. 1	To Checks	106.40			
2	To Checks	38.20			
3	To Checks	12.80			
15			By Cash	10.00	
27	To Checks	40.00			
March 1	To Checks	90 80	By Cash	250.00	
2	To Checks	72.40			
March 2	To Balance				

WRITTEN EXERCISES (B)

1. Copy the above account and fill in the daily balances.

Arrange cash accounts like the above and carry out the balances.

2. W. R. Smith: Balance Jan. 1, $1260.40. Deposits: Jan. 3, $94.60; Jan. 14, $106.20; Jan. 26, $49.80; Jan. 31, $200. Withdrawals were: Jan. 3, $260; Jan. 4, $139.20; Jan. 5, $27.80; Jan. 13, $49.60; Jan. 18, $23.50; Jan. 24, $57.40; Jan. 31, $194.20. Find balance Feb. 1.

3. R. S. Edwards: Balance Aug. 1, $1921.45. Deposits: Aug. 1, $27.16; Aug. 8, $121.45; Aug. 10, $246.53; Aug. 20, $548.16; Aug. 25, $1575.91. He withdrew as follows: Aug. 5, $111.46; Aug. 9, $124.95; Aug. 15, $93.71; Aug. 24, $250.17; Aug. 29, $981.42. Find balance Sept. 1.

CHAPTER XXXI

SAVINGS ACCOUNTS

437. Savings Banks. — Certain banks receive money and pay a low rate of interest, with the understanding that the money cannot be withdrawn by means of checks. Such banks are called *Savings Banks.*

Savings banks are required to invest their funds in high grade first mortgages or other "gilt edge" securities. This insures safety to the depositors and also compels a low rate of interest. The rate paid by savings banks in the United States varies from 2% to $3\frac{1}{2}$%.

438. Amount Required to Start Savings Account. — Savings banks will usually receive very small deposits on savings, but to start an account a deposit of at least one dollar is usually required.

439. Pass Book. — The depositor is given a *pass book* in which all deposits, withdrawals, and interest credits are recorded. The pass book therefore shows the condition of the account at any time.

440. Interest Term. Interest Days. — Savings banks compute interest quarterly or semiannually. The dates of computing interest are called *interest days,* and the time between successive interest days is the *interest term.* In case interest is computed semiannually, Jan. 1 and July 1 are usually interest days, and in case interest is computed quarterly, Jan. 1, April 1, July 1, and Oct. 1 are interest days.

441. Amount on Which Interest Is Computed. — Interest is usually computed on the lowest credit balance during the preceding interest term, but no interest is paid on a fractional part of a dollar. In some cases deposits made before the 10th or 15th of the first month of the interest term draw interest for the entire term.

442. Interest Compounded. — If the interest is not withdrawn when due, it is added to the principal and draws interest. That is, savings banks compounded interest semiannually or quarterly.

443. Banks May Demand Notice of Withdrawal. — The law usually permits savings banks to require a notice of from 30 to 60 days before money can be drawn from a savings account, but in normal times savings deposits may be withdrawn on demand. However, the amounts deposited from day to day tend to balance the amounts withdrawn, so the bank does not need to keep very much cash on hand. As fast as deposits accumulate they are invested in proper securities.

SAVINGS ACCOUNT

In Account with K. M. BRAINARD

DATE	DEPOSITS		INT. DAYS	SMALLEST QUARTERLY BALANCE	INTEREST 2%	WITHDRAWALS	BALANCE
6/15/37	By Cash	$50.00					$50.00
7/1	By Cash	75 00					125.00
8/15	By Cash	15.00					140.00
10/1	By Int.	0.62	Oct. 1	$125.00	$0.62		140.62
10/8						$25.00	115 62
10/15	By Cash	60.00					175.62
11/1	By Cash	10.00					185.62
12/1	By Cash	10.00					195.62
1/1/38	By Int.	0.88	Jan. 1/38	175.62	0.88		196.50

Interest is paid on all deposits up to the fifteenth of the first month of the interest term.

WRITTEN EXERCISES (A)

From each of the following sets of data make out a savings account.

1. Deposits: Dec. 12, 1936, $10; Jan. 7, 1937, $20; Jan. 27, $10; Feb. 1, $10; March 6, $15; May 1, $10; June 25, $50; July 10, $125; Aug. 6, $15; Nov. 5, $50; Dec. 23, $100. Withdrawals: Jan. 25, $15; April 3, $10; June 7, $15; Aug. 28, $25; Nov. 24, $35. Find balance Jan. 1, 1938, interest at 2% compounded quarterly.

2. Deposits: May 7, $200; July 5, $300; Sept. 4, $150; Oct. 2, $500. Withdrawals: Aug. 1, $50; Nov. 3, $75; Dec. 20, $100. Interest at 2% compounded quarterly. Find balance Jan. 1.

444. Postal Savings. - The postal saving system was established in the United States, Jan. 3, 1911. In the beginning forty-eight postal savings banks were established, one for each state and territory. In six months the number had grown to 500.

Postal savings certificates are issued in denominations of \$1, \$2, \$5, \$10, \$20, \$50, \$100. These bear the name of the depositor, the number of his account, the date issued, the name of the depository office and the date on which interest begins.

Interest at 2% is computed annually upon each certificate separately. Fractional parts of a year must be disregarded and deposits bear interest from the first day of the month next following the deposit.

EXERCISES

1. If a man deposits \$15 each month in a postal savings bank for four years, what amount will he have to his credit at the end of that period?

2. If a school boy deposits \$5 a month in a postal savings bank for five years, what amount will he have to his credit at the end of the five years?

Postal savings banks usually serve two other functions besides receiving deposits. These are: (1) the providing of annuities and the writing of life insurance; (2) serving their customers as agents in purchasing government securities.

DRILL IN FUNDAMENTALS (A)

Find the interest on the following and check results.

	Principal	Rate	Time
1.	\$1240	6%	3 mo. 7 da.
2.	2680	6%	4 mo. 14 da.
3.	1390	$6\frac{1}{2}\%$	5 mo. 15 da.
4.	2760	6%	2 mo. 18 da.
5.	31,900	5%	3 mo. 21 da.
6.	7840	$5\frac{1}{2}\%$	4 mo. 27 da.
7.	2980	6%	5 mo. 3 da.
8.	13,700	5%	7 mo. 6 da.
9.	14,920	5%	2 mo. 9 da.

CHAPTER XXXII

STOCKS AND SECURITIES

445. Corporations. — The laws of the various states permit people to organize for business as a *corporation*. To form such an organization, a *certificate of incorporation* must be filed with the proper authority, usually the secretary of state. When this certificate has been approved, the state gives the corporation a *charter*, granting it certain powers. Like an individual, the corporation may make contracts, sue and be sued in the courts, and carry on a business in its own name.

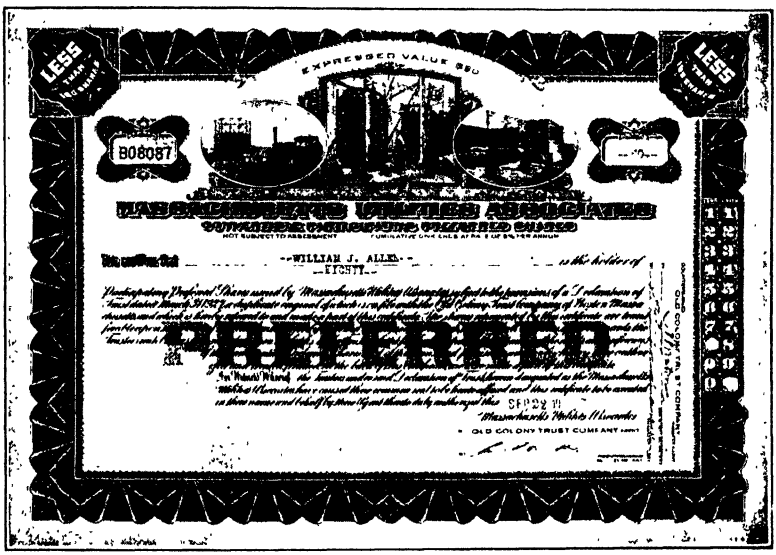
446. Capital Stock. — The ownership of a corporation is represented by *capital stock*. This stock is divided into *shares* which represent units of ownership. Sometimes each share has a nominal value, called *par value*, which may be any sum determined upon. This value is not an indication of real value. Shares frequently have no nominal value assigned to them and are said to be *no-par shares*.

447. Stockholders. — The *stockholders* of a corporation are those who own at least one share of stock in the organization. They have voting power in proportion to the number of shares held, usually one vote for each share owned. Thus, one stockholder, or a group of stockholders, may own enough stock to control the organization.

448. Officers and Directors. — At periodical meetings of the stockholders, which usually occur annually, a board of directors is elected to represent the stockholders in carrying on the affairs of the business. This board in turn elects the executive officers of the company who actually conduct the business. The directors usually serve for a year; that is, from one annual meeting of stockholders to the next.

449. Advantages of the Corporation. — Modern business has been showing a decided trend toward organization into larger units. Corporations and partnerships make it possible to secure capital for businesses of this sort. Partnerships are not so satisfactory as cor-

porations for two reasons. First, if a partner withdraws, sells out, or dies, the firm must be reorganized and a new agreement drawn up. The corporation continues to operate in spite of the death or bankruptcy of individual stockholders. Second, in a partnership, each partner is liable for the business acts of the others as well as for the



STOCK CERTIFICATE

debts of the firm. In a corporation, the owner of stock is not liable for the corporation's debts, except in the case of stock in banks or insurance companies where he has a limited legal liability.

450. Gross and Net Earnings. — The total income of a corporation is called its *gross earnings*. What remains from this after all expenses of operation, maintenance of the plant, salaries, taxes, interest on bonds, and other legitimate expenses have been paid is called *net earnings*.

451. Dividends. — *Dividends* represent distributions to stockholders from the net earnings of a corporation. They are declared payable by action of the board of directors. There are two common ways of paying dividends. This may be on a dollar basis per share

or on a percentage basis of the par value of each share. Thus, if the directors declare a dividend payable of three dollars a share, for each share owned the owner would receive three dollars. If the directors declare a dividend of four per cent *on stock having a $100 par value,* then for each share owned the owner would receive four dollars.

452. Par Value, Premium, Discount. — If a business is prosperous and the dividends constitute a higher rate of income on the par value of the shares than the usual returns on investment, then the shares will sell for more than their par value and are said to be *above par.* If the income from dividends is lower than the usual returns from investment, the stocks will sell for less than their par value and are then said to be *below par.*

The amount by which a stock sells above par is also called a *premium.* Thus, stock, par value 100, which sells at 124 is said to be at a premium of 24. If it sells at 92 it is said to be at a discount of 8.

The market values of shares are given in dollars and common fractional parts of a dollar. Thus $98\frac{3}{4}$ is written instead of 98.75 and $112\frac{3}{8}$ instead of 112.375. In giving values of shares the dollar sign is usually omitted.

453. Stock Dividends. — When a company is very prosperous, it may decide to invest part of its earnings in extending its plants, or it may wish to give the stockholders larger dividends without distributing more cash. In either of these cases, the amount of outstanding stock may be increased and distributed among the stockholders in proportion to the stock each already owns. Such a distribution is called a *stock dividend.*

454. Surplus. — The net earnings may not all be distributed among the stockholders as dividends. When a dividend is declared on the percentage basis, the rate of dividend is usually an integral number of per cent or a simple fractional number of per cent. When declared on the dollar basis, the dividend is an even part of the dollar or one or more dollars per share. Thus, if a corporation with a capital of $1,000,000 had net earnings of $87,500, the directors might declare a dividend of 5% or $5\frac{1}{2}$% on the par value of each share, or of $3 or $3.25 per share. The remainder would be carried as *surplus.*

455. Common and Preferred Stock. — A corporation usually issues two kinds of stock, *common* and *preferred*. Preferred stock has a fixed rate of dividend which must be paid before any earnings are declared on the common stock; it also receives preference in the distribution of assets if the company goes out of business. The rate of dividend on common stocks varies with the degree of success the company attains; therefore, the rate of dividend may be large or small, depending on the prosperity of the business.

456. Cumulative Preferred Stock. — In issuing preferred stock it is sometimes specified that the dividends on it are *cumulative*. In this case, if the company is unable to pay the regular dividends in any year, they accumulate and must be paid out of future profits before any dividends are paid on the common stock.

457. Transfer of Stock. — Stockholders hold *stock certificates* (page 349) indicating the amount of interest, that is, the value of the shares, each has in the corporation. These certificates may be sold, bequeathed, or pledged just as any other property. Corporations maintain facilities for recording the names of holders of stock and the transfers of ownership from one party to another.

458. To Find the Income from Shares When the Rate Is Known. — *Example.* What will be the yearly income from 150 shares of stock having a par value of \$20 and paying dividends of 6% per annum?

Solution. 6% of the par value of \$20 per share means \$1.20 per share. Hence, the required income is $150 \times \$1.20 = \$180$.

ORAL EXERCISES (A)

Find the income of each of the following, assuming that the par value of each share is \$100.

	No. of Shares	Rate of Dividend		No. of Shares	Rate of Dividend		No. of Shares	Rate of Dividend
1.	50	5%	**4.**	25	$6\frac{1}{2}$%	**7.**	225	5%
2.	75	6%	**5.**	45	$5\frac{1}{2}$%	**8.**	500	6%
3.	280	7%	**6.**	150	$4\frac{1}{2}$%	**9.**	350	7%

459. Buying and Selling Stocks for Own Account.

WRITTEN EXERCISES (A)

Find the cost of the shares in the following.

	No. of Shares	Value of a Share		No. of Shares	Value of a Share		No. of Shares	Value of a Share
1.	240	$112\frac{1}{4}$	4.	85	$124\frac{1}{4}$	7.	460	$49\frac{3}{4}$
2.	94	$38\frac{3}{8}$	5.	124	$85\frac{7}{8}$	8.	1045	$283\frac{1}{2}$
3.	45	$76\frac{7}{8}$	6.	500	$32\frac{3}{8}$	9.	930	$106\frac{1}{8}$

460. To Find the Rate of Return on an Investment in Stocks. — *Example.* What is the rate of income on an investment in 6% ($100 par) shares costing $106\frac{1}{4}$ a share?

Solution. 6% of $100 = $6. Therefore, the problem is: "$6 is how many % of 106.25?" (§ 300). 6 ÷ 1.0625 = 5.647 = the required rate.

WRITTEN EXERCISES (A)

Find the rate of income from each of the following, assuming that shares have a par value of $100.

	Rate of Dividend	Cost of a Share		Rate of Dividend	Cost of a Share		Rate of Dividend	Cost of a Share
1.	6%	$108\frac{3}{4}$	4.	7%	$159\frac{7}{8}$	7.	5%	120
2.	7%	$118\frac{1}{4}$	5.	6%	$89\frac{7}{8}$	8.	$4\frac{3}{4}$%	75
3.	6%	120	6.	6%	$105\frac{3}{8}$	9.	8%	160

461. To Find the Price per Share Which Will Yield a Given Rate on Investment. — *Example.* What must be paid for a 7% ($100 par) stock to realize $5\frac{1}{2}$% income on the investment?

Solution. 7% of $100 is $7. Therefore, the problem is: $7 is $5\frac{1}{2}$% of what number? (See § 302.) Hence, 7 ÷ .055 = 127.27 (dollars), which is the required amount.

WRITTEN EXERCISES (A)

Find the price of a share ($100 par) in each of the following.

	Rate on Investment	Rate of Dividend on Shares		Rate on Investment	Rate of Dividend on Shares
1.	$5\frac{1}{4}$%	6%	4.	$5\frac{1}{2}$%	8%
2.	5%	$6\frac{1}{2}$%	5.	$4\frac{1}{4}$%	5%
3.	$4\frac{3}{4}$%	7%	6.	7%	9%

Compare the problems of §§ 459–461 with the three problems in percentage on page 235.

462. Brokerage. — Stocks are usually bought and sold through an agent called a broker. His fee for buying and selling is called brokerage. The amount of the fee varies with the location of the market and the ease of completing a transaction. Brokers holding membership in the New York Stock Exchange charge the following fees:

Stocks selling at prices

from	1 to	$9\frac{7}{8}$	\$7.50 per 100 shares
from	10 to	$24\frac{7}{8}$	12.50 per 100 shares
from	25 to	$49\frac{7}{8}$	15.00 per 100 shares
from	50 to	$74\frac{7}{8}$	17.50 per 100 shares
from	75 to	$99\frac{7}{8}$	20.00 per 100 shares
from	100 to	$199\frac{7}{8}$	25.00 per 100 shares
from	200 to	$249\frac{7}{8}$	30.00 per 100 shares
from	250 to	$299\frac{7}{8}$	35.00 per 100 shares

NOTE. — The U. S. government places a tax on the sale of securities. Revenue stamps covering the amount of this tax must be affixed to the security or to the bill of sale. In New York state, there is also a state tax. For the purpose of this text, these taxes have been ignored.

PROBLEMS (A)

1–9. Solve the problems under § 459, adding brokerage at the New York Stock Exchange rates and showing total cost to buyer.

10. Bought 125 shares of U. S. Steel at $28\frac{1}{2}$ and sold same for $32\frac{1}{4}$. Find gain.

11. Sold 95 shares of American Telephone at $107\frac{5}{8}$. What did I receive for the stock?

12. If I purchase 325 shares of Packard Motor stock at $4\frac{7}{8}$, what will it cost me?

13. Your broker sells 89 shares of General Motors for you at $14\frac{3}{8}$. How much commission is he entitled to retain as his fee?

14. Bought 110 shares of United Aircraft Common at $19\frac{1}{4}$ and sold at $28\frac{1}{8}$. What was my profit?

15. How many shares selling at $68\frac{3}{4}$ can be bought for \$2757?

16. Sold my stock at $123\frac{5}{8}$, receiving \$5922. How many shares did I sell?

NEW YORK STOCK EXCHANGE TRANSACTIONS AUGUST 24, 1949
Partial List

STOCKS	SALES	HIGH	LOW	CLOSE
American Airlines	1,500	$9\frac{1}{2}$	$9\frac{3}{8}$	$9\frac{1}{2}$
American Can	200	93	93	93
American T. and T.	3,300	$144\frac{1}{8}$	$143\frac{3}{8}$	$144\frac{1}{8}$
Anaconda Copper	1,900	$28\frac{3}{4}$	$28\frac{1}{2}$	$28\frac{1}{2}$
Bethlehem Steel	3,600	$26\frac{5}{8}$	$26\frac{1}{8}$	$26\frac{1}{2}$
Case (J. I.) Co.	400	$38\frac{3}{8}$	$38\frac{1}{4}$	$38\frac{3}{8}$
Colgate Palmolive	300	$39\frac{1}{2}$	$39\frac{3}{8}$	$39\frac{3}{8}$
Consolidated Edison	3,400	$24\frac{7}{8}$	$24\frac{5}{8}$	$24\frac{3}{4}$
Consolidated Nat. Gas	800	42	$41\frac{3}{4}$	$41\frac{3}{4}$
Continental Can	500	$33\frac{1}{2}$	$33\frac{1}{4}$	$33\frac{1}{4}$
Continental Oil	1,000	$59\frac{1}{2}$	$58\frac{3}{8}$	$59\frac{1}{2}$
Du Pont	8,700	$50\frac{3}{4}$	50	$50\frac{1}{4}$
General Electric	3,100	$37\frac{1}{4}$	37	$37\frac{1}{8}$
General Motors	5,100	$60\frac{1}{8}$	$59\frac{5}{8}$	$59\frac{7}{8}$
Gillette	1,200	$28\frac{3}{4}$	$28\frac{1}{2}$	$28\frac{5}{8}$
Goodrich	600	$61\frac{5}{8}$	$61\frac{1}{2}$	$61\frac{5}{8}$
Goodyear	600	$40\frac{1}{8}$	$39\frac{5}{8}$	$40\frac{1}{8}$
Great Western Sugar	700	$17\frac{1}{8}$	$16\frac{3}{4}$	17
Greyhound	2,800	$11\frac{1}{2}$	$11\frac{1}{4}$	$11\frac{3}{8}$
Hudson Motors	700	$11\frac{7}{8}$	$11\frac{3}{4}$	$11\frac{3}{4}$
Illinois Central	3,300	27	$26\frac{1}{2}$	$26\frac{7}{8}$
International Harvester	2,300	$26\frac{5}{8}$	$26\frac{1}{2}$	$26\frac{1}{2}$
International T. and T.	2,700	$8\frac{1}{2}$	$8\frac{3}{8}$	$8\frac{3}{8}$
Kennecott Copper	1,800	$46\frac{1}{2}$	$46\frac{1}{4}$	$46\frac{1}{2}$
Marshall Field	400	$22\frac{5}{8}$	$22\frac{3}{8}$	$22\frac{1}{2}$
Montgomery Ward	1,200	$52\frac{3}{4}$	$52\frac{1}{2}$	$52\frac{3}{4}$
National Biscuit	600	$31\frac{5}{8}$	$31\frac{1}{2}$	$31\frac{5}{8}$
National Dairy	1,000	$33\frac{5}{8}$	$33\frac{1}{4}$	$33\frac{1}{2}$
New York Central R. R.	1,600	$10\frac{3}{8}$	$10\frac{1}{8}$	$10\frac{3}{8}$
Northern Pacific R. R.	3,100	$14\frac{1}{8}$	$13\frac{7}{8}$	$13\frac{7}{8}$
Packard	3,200	$3\frac{7}{8}$	$3\frac{3}{4}$	$3\frac{7}{8}$
Pan-American Airlines	2,300	$8\frac{7}{8}$	$8\frac{5}{8}$	$8\frac{7}{8}$
Penney, J. C.	600	50	$49\frac{1}{2}$	50
Pullman	1,100	$33\frac{1}{8}$	$32\frac{7}{8}$	$33\frac{1}{8}$
Safeway	900	$24\frac{1}{4}$	24	24
Standard Oil N. J.	4,600	$67\frac{3}{4}$	$67\frac{1}{2}$	$67\frac{3}{4}$
Swift Co.	500	$29\frac{3}{4}$	$29\frac{1}{2}$	$29\frac{3}{4}$
Texas Co.	2,000	$58\frac{3}{8}$	$57\frac{7}{8}$	$58\frac{3}{8}$
Union Pacific R. R.	500	$81\frac{1}{2}$	$81\frac{1}{4}$	$81\frac{1}{4}$
United Air Lines	1,400	$12\frac{3}{4}$	$12\frac{5}{8}$	$12\frac{5}{8}$
United Aircraft	500	$21\frac{3}{4}$	$21\frac{1}{4}$	$21\frac{5}{8}$
U. S. Rubber	1,800	33	$32\frac{1}{2}$	33
U. S. Steel	4,600	$22\frac{1}{2}$	$22\frac{1}{8}$	$22\frac{1}{2}$
Western Union Tel.	500	$14\frac{1}{4}$	$14\frac{1}{8}$	$14\frac{1}{4}$
Woolworth	600	$48\frac{3}{8}$	48	$48\frac{3}{8}$

17. Bought General Electric at $14\frac{3}{8}$ and sold at $19\frac{5}{8}$, making a gain of \$645. How many shares did I have?

18. Sold 175 shares of Anaconda Copper at $12\frac{7}{8}$ which had cost me $21\frac{1}{8}$. What did I lose?

19. On 265 shares of stock I lost \$397.50 net. If my selling price was $11\frac{3}{4}$, at what price did I buy?

20. If I purchase 35 shares of stock at $60\frac{1}{8}$, at what price will I sell to make a gain of \$346.50?

463. Prices of Stocks. — In the table on the opposite page there is reported a part of the transactions taking place on the New York Stock Exchange, August 24, 1949. The figures in the table show the number of shares of each stock changing hands, the highest and lowest prices during the day, and the price at which the final sale was made.

WRITTEN EXERCISES (A)

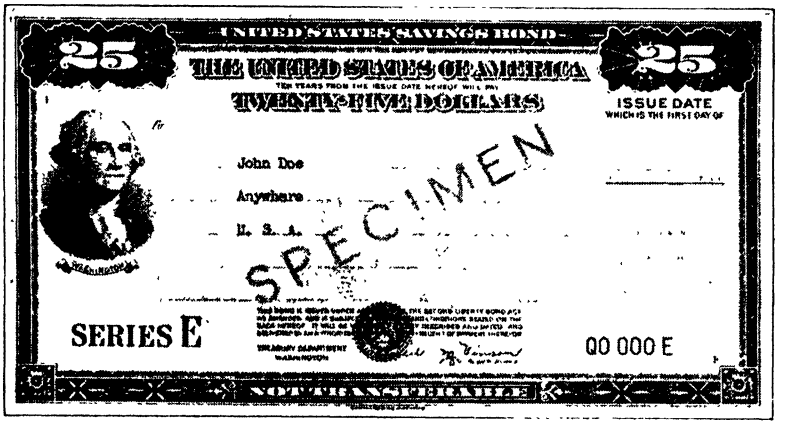
Find the gain of one who bought at the lowest price and sold at the highest on August 24, 1949 (see opposite page), deducting brokerage charges from the gross profit at the rates prescribed by the New York Stock Exchange.

	No. of Shares	Stock		No. of Shares	Stock
1.	700	Am. Air Lines	**13.**	400	Mont. Ward
2.	1400	Am. T. and T.	**14.**	350	Nat. Biscuit
3.	1000	Anaconda Cop.	**15.**	2800	Packard
4.	1450	Bethlehem Steel	**16.**	1500	Pan-Am. Air
5.	300	Continental Can	**17.**	600	Pullman
6.	2000	Du Pont	**18.**	250	Safeway
7.	1900	General Electric	**19.**	300	Swift Co.
8.	2100	General Motors	**20.**	900	Texas Co.
9.	400	Goodyear	**21.**	400	United Aircraft
10.	500	Hudson Motors	**22.**	1300	U. S. Rubber
11.	1600	Int. Harvester	**23.**	500	U. S. Steel
12.	900	Kennecott Cop.	**24.**	300	Woolworth

CHAPTER XXXIII

BONDS

464. Bonds, by Whom Issued. — Bonds are *promissory notes* issued by national and state governments, by municipal corporations such as counties and cities, and by private corporations. A bond bears a specified rate of interest and is payable at a definite time.



A United States Savings Bond*

465. Registered Bonds. — A corporation issuing registered bonds keeps a record of the owners of the bonds and the interest checks are mailed to the *registered bondholders*. Such bonds can be sold only by having the record of ownership transferred on the books of the company.

466. Coupon Bonds. — State and municipal bonds are usually *coupon bonds*. That is, the bond has attached to it little slips which are really promissory notes covering the interest from year to year.

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These slips, called coupons, may be cut off as they become due and deposited in the banks for collection. This saves the trouble of making transfers on the books of the issuing corporation in case the bonds are sold. The coupons and the bond, when due, will be paid to anyone having them in his possession.



BONDS AND THEIR COUPONS *H. Armstrong Roberts*

467. Mortgages. — A *mortgage* is an instrument made out in favor of one who lends money and giving to him a claim against the property of the borrower. If the loan is repaid according to the agreement, the mortgage becomes void and is canceled. If the loan is not repaid according to the terms of the mortgage, the mortgage holder may sue to *foreclose* on the borrower. If the property is foreclosed on, then it is usually sold at auction and the holder's claim is satisfied out of the proceeds. Bonds issued by private corporations are frequently secured by mortgage on the property of the issuing company.

468. Consols and Rentes. — The debt of the British government (up to the time of World War I) was represented by the so-called *Consols* (Consolidated Debts), bearing $2\frac{1}{2}\%$ interest and running indefinitely. That is, the consols differ from ordinary bonds in that they form a perpetual source of income, while bonds are payable at a fixed date. The bonds of the French government are called *rentes*, and bear interest at the rate of 3%.

469. Designation of Bonds. — Bonds are designated by the name of the issues, the date of maturity, and the rate of interest. Thus N. Y. City 4's 1956 means a bond issued by New York City bearing 4% interest and due in 1956.

470. Common Stocks and Bonds Compared.

Common Stocks	*Bonds*
1. Dividend depends upon prosperity of the company.	1. Bear a definite rate of interest payable at fixed times.
2. Stockholder has a voice in the management of the corporation by voting for directors.	2. Bondholder is merely a creditor and has no voice in the management of the corporation.
3. The stock is not payable at any time but remains as a part of the company's permanent capital.	3. The bond is payable at a definite time.

471. Problems on Long Term Bonds. — If the interest on a long term bond is very near the prevailing rates of income on similar investments, the problems on bonds are like those on stocks.

Example 1. New York City $4\frac{1}{4}$'s 1964 are sold at an average of \$101.45. Find the rate of income on the investment, allowing no brokerage.

Solution. This problem is exactly like that solved in § 460.

Example 2. Solve Example 1, allowing $\frac{1}{8}\%$ for brokerage.

Solution. $101.45 + .125 = 101.575$. $4.25 \div 101.575 = 4.18\%$.

Example 3. What must a long term $4\frac{1}{2}\%$ bond be bought for to give an income of 4.49% on the investment? (*a*) allowing no brokerage, (*b*) allowing $\frac{1}{8}\%$ brokerage.

Solution. (*a*) of this problem is just like that of § 461.

NOTE. — New York City, at one time, issued 45,000,000 $4\frac{1}{2}\%$ bonds, which were sold on an income basis of 4.49%.

472. Problems on Short Term Bonds (B). — When the remaining term of the bond is short or when great accuracy is required, the problems on bonds are very different from those on stocks.

Example 1. A bond bearing 3% interest matures one year from date. At what price must it be bought to make an income of $4\frac{1}{2}\%$ on the investment?

Solution. Let the par value of the bond be \$1000. Then at the end of the year it will be worth \$1030.

Hence \$1030 ÷ 1.045 = \$985.65 is the present value of the bond. That is, the bond should be bought at 98.565.

When the bond is due in several years this process must be repeated once for each year.

Example 2. Find the present value of the above bond if it is due in 4 years from date.

Solution. By example 1, the bond will be worth \$985.65 one year before maturity. Adding the \$30 interest for the second year we have \$1015.65.

Then \$1015.65 ÷ 1.045 = \$971.914 = value 2 years before maturity.

\$971.914 + \$30 = \$1001.914.

\$1001.914 ÷ 1.045 = \$958.77 = value 3 years before maturity.

\$958.77 + \$30 = \$988.77. \$988.77 ÷ 1.045 = \$946.191 = value 4 years before maturity.

Hence if this bond is bought four years before maturity, it should be bought at the rate of 94.619.

A perfectly secure permanent stock bearing 3% dividend would have to be sold at $66\frac{2}{3}$ to make an income of $4\frac{1}{2}\%$.

It is clear that if \$946.191 is invested at $4\frac{1}{2}\%$, the amount at the end of the year will be \$988.77. Of this \$30 is received in cash in the way of interest on the bond, the remaining \$958.77 is the present worth of the bond and continues to draw interest for the following year. The amount at the end of the year is \$1001.914, of which \$30 is again received in cash. The remainder, \$971.914, draws interest at the rate of $4\frac{1}{2}\%$, amounting to \$1015.65 at the end of the 3d year. \$30 received in cash and the remainder, which is \$985.65, draws interest at the rate of $4\frac{1}{2}\%$. At the end of the year the amount is \$1030, which is then received in full.

It will be noticed that part of the $4\frac{1}{2}\%$ on the investment is received in cash each year and part of it is received in increased value of the bond. If the cash received is invested promptly the total return will be $4\frac{1}{2}\%$ on the whole investment compounded annually.

The general theory of long and short term investments is beyond the scope of this book.

WRITTEN EXERCISES (B)

Find the cost of the bonds in each of the following. The interest in each case is payable annually. Use the method of § 472.

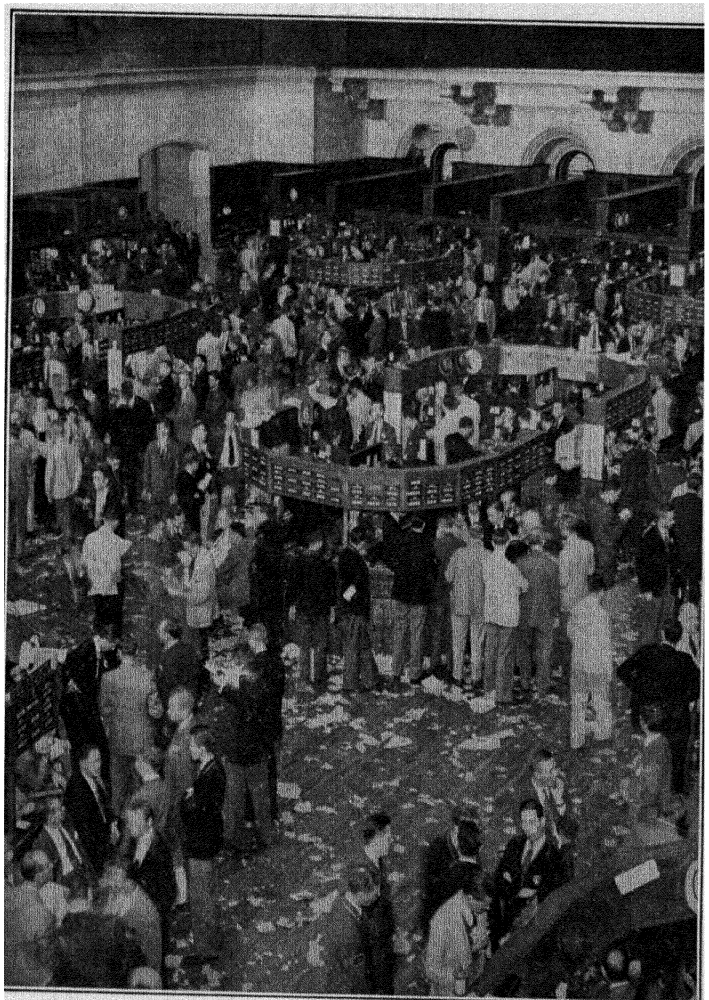
	Rate of Interest on Bond	Rate on Investment	Payable in
1.	4%	5%	3 years
2.	$4\frac{1}{2}$%	$5\frac{1}{2}$%	4 years
3.	$3\frac{1}{2}$%	$4\frac{3}{4}$%	3 years
4.	3%	4%	5 years

Bankers and investment brokers use tables which give the present value for a variety of rates of dividend and rates on the investment and for any number of years, say up to twenty.

DRILL IN FUNDAMENTALS (A)

Find the net proceeds of the following bills and check results.

	Gross Amount of Bill	Discount Series		Gross Amount of Bill	Discount Series
1.	$580.60	20%, 15%, 5%	17.	$780	40%, 10%, 5%
2.	1240.40	15%, 10%, 5%	18.	450	60%, 15%
3.	7800	35%, 25%	19.	275	40%, 15%
4.	4960.80	20%, 15%	20.	490	30%, 15%
5.	2937.86	15%, 10%, 5%	21.	1470	50%, 20%
6.	3786.40	25%, 20%, 10%	22.	1260	15%, 10%, 5%
7.	5137.60	40%, 20%	23.	3400	$12\frac{1}{2}$%, 5%
8.	2870	35%, 20%	24.	892	15%, 10%
9.	2930	25%, 15%, 5%	25.	290	20%, 15%
10.	1070	40%, 30%, 10%	26.	370	10%, 5%
11.	1180	35%, 25%	27.	1640	40%, 10%, 5%
12.	3860	20%, 10%	28.	3580	35%, 15%, 10%
13.	4270	15%, 5%	29.	3740	30%, 25%
14.	5630	25%, 18%, 5%	30.	2764	25%, 20%, 15%
15.	14,870	35%, 25%	31.	5897	30%, 10%, 15%
16.	350.60	15%, 10%	32.	950	15%, 12%, 5%



Ewing Galloway

THE BOARD ROOM OF THE NEW YORK STOCK EXCHANGE

CHAPTER XXXIV

THE STOCK EXCHANGE

473. The Stock Exchanges are associations of brokers who buy and sell stocks and bonds. Many cities have stock exchanges, but that in New York City is the best known and most important.

474. Hall of Exchange. — The business of the stock exchange is transacted in a large room called the *hall of exchange*. Brokers dealing in certain kinds of securities gather about certain posts in the room. These are designated as the "U. S. Steel Post," "R. R. Post," etc.

475. Strict Rules. — The rules of the stock exchange are very strict, and a high standard of integrity is maintained. Transactions involving thousands of dollars are made verbally. One broker shouts "1000 shares U. S. Steel at $85\frac{1}{4}$," another answers "Sold," and a transaction involving $85,250 is closed. Failure to live up to the terms of such deals is severely punished. The number of memberships on the exchange is limited, that of the New York Exchange being 1375. Memberships, called "seats," may be sold, but the new member is subject to election by the committee on memberships.

476. Rate of Brokerage. — The broker charges a commission, or brokerage, for buying and selling securities (§ 462). The standard lot in trading is usually 100 shares, smaller lots being known as "odd lots." Sometimes trading is not permitted in odd lots. In such cases, these small orders are filled through "odd lot brokers" who buy and sell them a fraction of a point away from the prices for regular 100 share lots. On securities which are not listed on any exchange, the rates of brokerage will vary with the ease or difficulty in completing the trade.

477. A Point. — When the price of a stock or bond changes, it must change by at least $\frac{1}{8}$ of a point. In the case of stocks, a point is one dollar per share, making the smallest change of price $12\frac{1}{2}$ cents per share. For bonds, a change of a point amounts to a change in price equal to 1% of the par value. A quotation of 86 for a $1000 bond indicates a price of 86% of $1000, or $860.00.

478. Kinds of Sales. — A sale "*buyers* 3, 10, 20, 30, or 60" means that the buyer may demand the stock at any time within 3, 10, 20, 30, or 60 days as the case may be, but is not required to take it before the last day of the period. "*Sellers*, 3, 10, 20, 30, 60" means that the seller may offer it before the last day of the period, but is not required to furnish it before that day.

For some days before and after interest and dividend day the books of a corporation are closed so that no transfers of registered bonds or stocks can be made. During this time securities are usually sold "ex interest" or "ex dividend," which means that the dividend just due is retained by the seller.

479. Wash Sales. — Sales arranged between different agents of the same party to influence the price of stocks are called *wash sales*.

480. Speculation. — A large part of the buying and selling of securities is for speculative purposes. A man who believes that a stock is going up in value buys it in the hope of selling at a gain. If he believes the stock is going down, he sells for a future delivery.

481. Buying and Selling Futures. — Stocks and bonds are sometimes bought and sold to be delivered at a future date. If on the day of delivery the market value of the stock is higher than the contract price, the buyer gains, and if the price is lower than the contract price, the seller gains. In such transactions the stock is usually not delivered, but the "difference" between the contract price and the market value is paid over in cash. This sort of transaction amounts to little more than betting on a future price.

482. Bulls, Bears, Longs, Shorts. — One who has bought stock for future delivery is said to be *long* on that stock and one who has sold stock for future delivery is said to be *short* on that stock. The shorts or *bears* try to force prices down and the longs or *bulls* try to force prices up.

483. Trading on Margins. — A sends $1000 to a broker who buys 100 shares of stock. The broker borrows money to pay the balance of the purchase price and uses the stock certificate as security. This is called *trading on margin*. A's margin is $10 a share.

484. Call Rates. — The broker borrows money *on call*. That is, the bank may demand payment at any time. The rate of interest on such loans is usually very low. (See § 433.) In time of panic, however, it may rise very high. "Money easy" means that call rates are low, say $\frac{1}{2}\%$ to $1\frac{1}{2}\%$. "Call rates stiff" means 4% to 8%.

485. Stockbroker's Account. — The form of the stockbroker's account may be studied by means of an example.

Example. On Jan. 12th, A. S. Walker sends $3000 to his broker, John Henry, instructing him to buy 300 shares of U. S. Steel as soon as it can be bought at 74. The broker buys the stock on Jan. 19. Under instruction from Mr. Walker, he sells it Jan. 31 at $75\frac{3}{8}$. How much is due Mr. Walker?

The following is the broker's statement: —

NEW YORK, January 31, 1919

MR. A. S. WALKER,

In account with JOHN HENRY & CO., (BROKERS)

1919 JAN.	BOUGHT OR REC'D	SOLD OR DEL'D	SECURITY	PRICE	CHARGES	CREDITS	BALANCE Debit	BALANCE Credit	DAYS	INTEREST @ 6% Charge	INTEREST @ 6% Cr't
12			Check on Acc't			3,000		3,000	7		3 50
19	300		U. S Steel	74	22,200		19,200				
			Brokerage		52 50		19,252 50		12	38 51	
31		300	U. S. Steel	75⅜		22,612 50		3,360			
			Brokerage		60 00			3,300			
			Interest @ 3%		17 51			3,282 49			
			To check		3,282 49						

Interest is charged on the cost of stocks from the time they are bought till the account is closed. Interest is allowed on the margin from the time it is received and on the proceeds from the sale of stocks from the day of sale.

If the market price of a stock bought on margin should fall, the broker will notify his customer to furnish additional margin. If this is not done, the broker may sell the stock to cover the loan he has made and to pay the brokerage.

Brokerage rates on the New York Stock Exchange are given in § 462.

WRITTEN EXERCISES (C)

1. August 1, R. A. Lord sent \$5000 to his broker, H. L. Moore. August 4, Moore bought 200 shares Pennsylvania Railroad at $14\frac{1}{2}$, 200 shares Baltimore & Ohio on August 7 at $17\frac{1}{4}$, and 100 shares of New York Central at $19\frac{3}{8}$ on August 8. Sold 100 shares New York Central August 12 at $24\frac{1}{2}$, 200 shares Pennsylvania on August 15 at $16\frac{3}{4}$, and 200 shares Baltimore & Ohio on August 18 at $18\frac{5}{8}$. Interest 4%. Prepare a statement dated August 18.

The receipts from the sales draw interest till the day of settlement.

2. A. R. Samuels furnished \$2500 margin for which his broker bought 250 shares of F. W. Woolworth Co. at $32\frac{1}{4}$, on June 7. On June 19 the broker sold the stock at $24\frac{3}{4}$ to protect himself. Rate of interest $4\frac{1}{2}$%. Setlement dated June 19. Prepare a statement.

3. A broker bought for M. A. Smith: February 6, 600 shares of General Electric at $18\frac{3}{4}$; February 7, 200 shares of General Motors at $14\frac{1}{4}$; February 11, 400 shares of U. S. Steel at $34\frac{5}{8}$. February 9, sold 200 shares General Motors at 15; February 13, sold 600 shares General Electric at $18\frac{3}{8}$; February 17, sold 400 shares U. S. Steel at $36\frac{1}{8}$. Margin of \$7500 forwarded February 3. Interest 3%. Prepare a statement dated February 20.

DRILL IN FUNDAMENTALS (A)

Find the rates of income to the nearest hundredth of one per cent.

	Investment	Yearly Income		Investment	Yearly Income
1.	\$108.50	\$6	**10.**	\$64.25	\$4
2.	189.25	9	**11.**	107.00	6
3.	114.00	7	**12.**	134.25	7
4.	204.75	10	**13.**	162.50	8
5.	94.50	5	**14.**	131.75	7
6.	88.25	5	**15.**	55.50	3
7.	75.75	4	**16.**	2560	465
8.	89.50	5	**17.**	3790	376
9.	80.75	4.50	**18.**	2340	186

CHAPTER XXXV

GRAPHIC REPRESENTATION

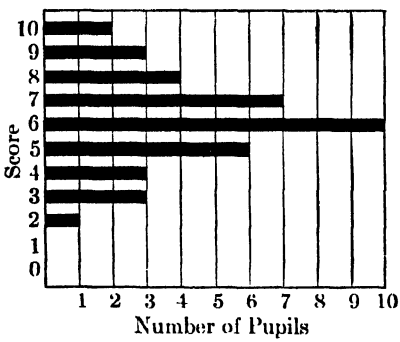
486. Graphs. — Graphs of different kinds are now in very general use. Probably you studied graphs to some extent in your earlier work in arithmetic. In a little greater detail we will now study such graphs as are needed in this course.

487. Bar Graphs. — Examples will illustrate the use of this kind of graph.

Example 1. In a test in arithmetic, the scores (numbers of problems worked correctly) were distributed as follows:

Number of pupils	1	3	3	6	10	7	1	3	2
Number of correct answers for each pupil	2	3	4	5	6	7	8	9	10

That is, one pupil had 2 correct answers, three pupils had 3 correct answers each, three had 4 correct answers, and so on. The horizontal bar graph shows the same facts that are given in the table above.



Notice the distribution shown in this example. There are only a few very low grades and only a few very high grades. By far the largest number had 5, 6, or 7 correct answers. This kind of distribution you will find repeatedly in your work.

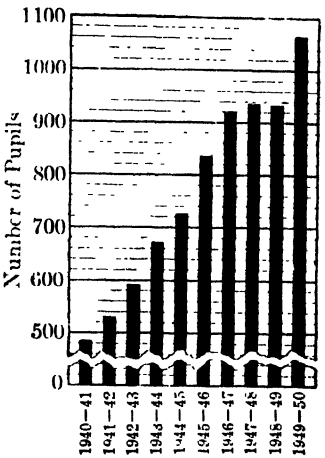
Example 2. In a rapidly growing high school in a small city, the attendance the last 10 years has been as shown in this table.

Year . . .	1940–1	41–42	42–3	43–4	44–5	45–6	46–7	47–8	48–9	19–50
Attendance	485	527	592	676	724	836	920	938	936	1065

In the graph at the right, the heights of the solid bars represent the number of pupils each year.

From this graph, the attendance to the nearest multiple of 10 can be read very easily (480, 530, 590, 680, etc.).

Such approximations are quite sufficient to give a clear idea of what the attendance is and how it has grown. Hence this graph gives at a glance the information that is really wanted.



EXERCISE

Construct bar graphs representing the following.

1. In a test in arithmetic the scores (number of problems worked correctly) were distributed as follows.

Number of pupils . .	2	3	4	8	6	5	3	1
Number of correct answers for each pupil	3	1	5	6	7	8	9	10

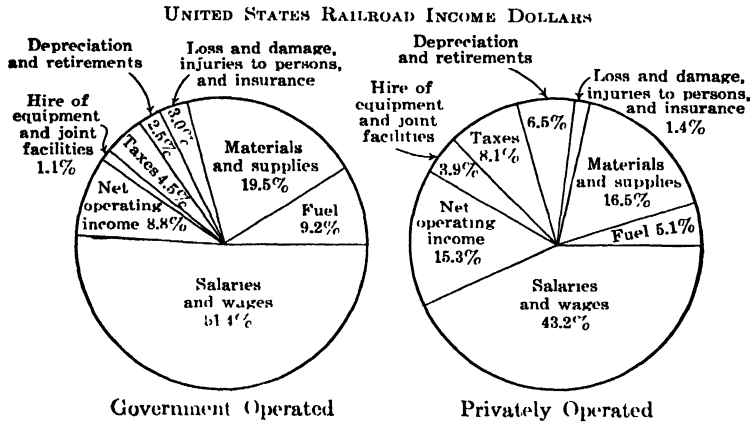
Construct a graph representing this distribution of scores. See page 366.

488. Circle Graphs. — The examples of circle graphs shown here will remind you of figures you have no doubt seen in the popular magazines.

These graphs were used in a published debate between a United States senator and a railway president on the question as to whether the government should take over the railroads. The point for us to notice is that they used these graphs to make themselves easily understood by readers who are not used to reading tables of figures.

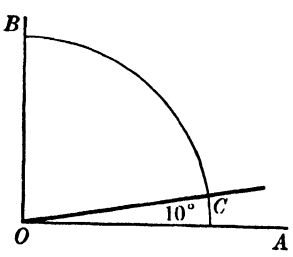
Notice that in each of these graphs the per cents are given. These contain all the information given in the graph, and, in fact, give the information more accurately. These per cents were obtained first

from tables of figures, and then the graphs were constructed. The purpose of such graphs is to make the facts more directly apparent to the



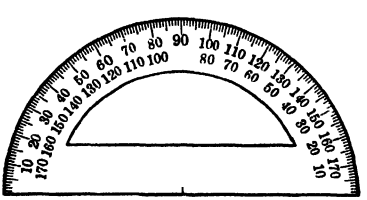
eye. We get the impression more easily from a graph than we do from a table of figures. It is for this reason that graphs are so much used.

489. Degree of Angle and of Arc. — In order to explain how circle graphs are made, we need to know what units of measure are used in measuring arcs and angles. A right angle, as angle AOB in the figure, is divided into 90 equal parts and each part is called a **degree** of angle. If a circle is constructed as indicated by the arc in the figure, then $\frac{1}{90}$ of the arc within the right angle AOB is a degree of arc. The part of the arc within the angle AOC is 10 degrees of arc.



The symbol ° is used to indicate degrees. That is, 10° means 10 degrees of angle, or 10 degrees of arc. Since the part of the circle lying within the right angle AOB is 90°, it follows that the whole circle equals 360°.

The graduated semicircle shown in the next figure is used to lay off angles and arcs of a given number



of degrees. It is called a **protractor.** You can probably buy a simple protractor at the school store for a few cents.

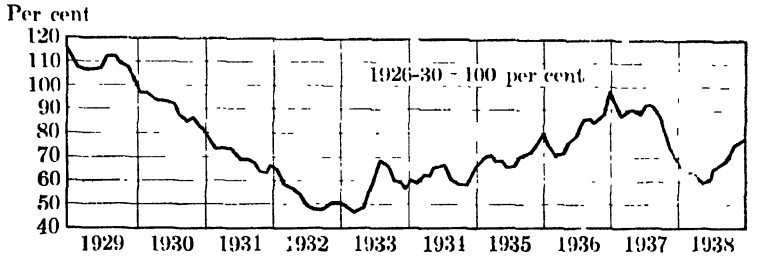
EXERCISES

1. Using a protractor, construct angles of 25°, 35°, 50°, 60°, 80°, and 120°.

2. Divide a circle into three parts of 75°, 110°, and 175° respectively. Why do these arcs cover the whole circle?

3. Divide a circle into five parts of 15°, 50°, 45°, 85°, and 165°.

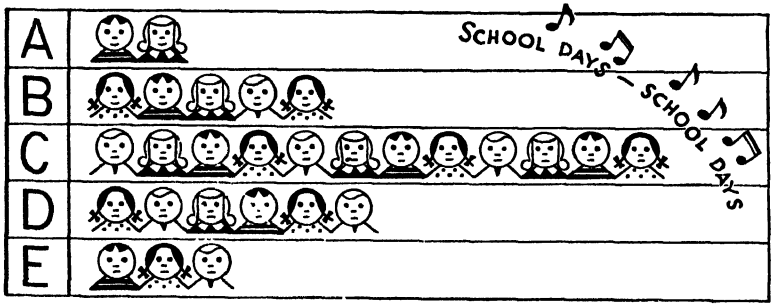
490. Line Graphs. — Here is a line graph representing general business activity in the United States from 1929–1938.



The line marked 100 represents the average of this activity for the years 1926–1930. In this graph the "depression" and various recoveries and "recessions" are shown very vividly: the high activity of 1929 reaching a peak in the summer of that year; the long downward slope reaching a bottom in the middle of 1932; a slight improvement in the latter part of that year, and then another slump reaching a new low early in 1933, then a rapid improvement, followed by a decline, which did not go as low as in 1932 and the early part of 1933. Then there is a fairly steady level on a low plane of between 60% and 70% of the high 1926–1930 level. In 1935 there is some improvement, reaching the 80% level, and still further improvement in 1936, in the autumn of that year reaching about 95%, then one year of a fairly sustained level. In the summer of 1937 we see a sharp and long decline reaching the 60% level in the summer of 1938. The last half of 1938 saw a definite improvement reaching close to the 80% level by the end of the year.

Graphs like the above are used extensively by those who are interested in the "trend," or the general way certain things are changing. In the offices of financial institutions, such graphs appear literally by the hundreds. You can find samples of them in many daily papers.

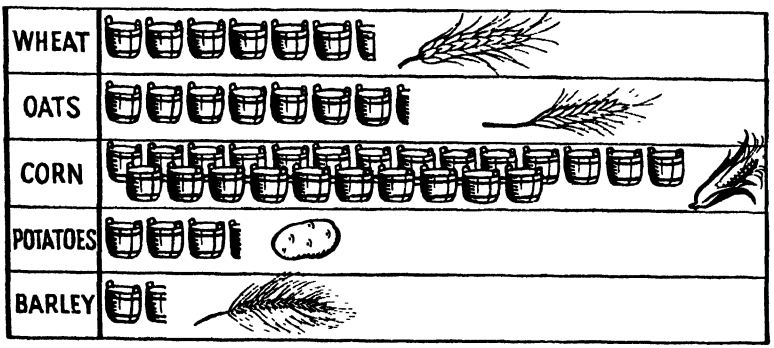
491. Pictorial Graphs. -- We will now study briefly still another kind of graph, called pictorial graph, that is sometimes used in books and reports. Examples will illustrate:



Example 1. In a class of 28 pupils, 2 received a grade of A, 5 a grade of B, 12 a grade of C, 6 a grade of D, and 3 a grade of E. These facts are shown in the figure above.

The meaning of this graph is apparent at once. Compare graphs on pages 366, 367, representing school grades.

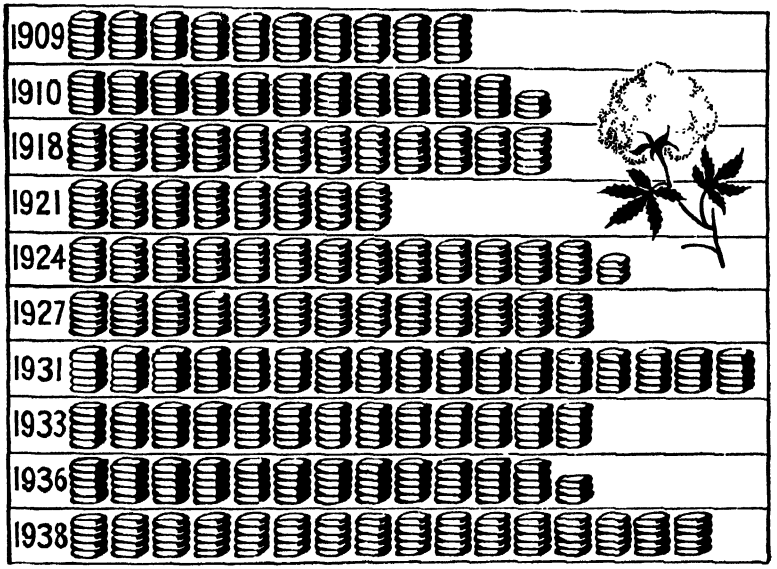
Example 2. In a recent year the principal crops (not including cotton) harvested in the United States are shown in the following figure. Each basket represents 100,000,000 bushels.



From the picture on page 370, give the approximate number of bushels of each crop.

Example 3. The cotton crop in the United States varies very considerably from year to year. The difference in the crops is due in part to natural conditions and in part to the amount of land used for raising cotton.

In the figure, each bale represents a million 500-pound bales.



From this picture, give the approximate number of bales of cotton produced for each of the years given.

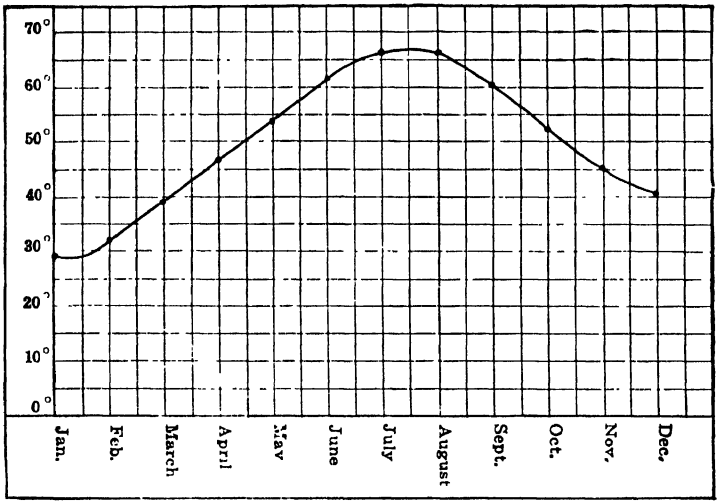
TOPICS FOR DISCUSSION

1. Of the four kinds of graphs you have studied, which do you think shows the facts most easily?

2. Which kind of graph would you think is used most often in practice? Watch papers and magazines for the different kinds of graphs.

492. Representation of Temperature. — The average monthly temperatures of New York City beginning with January are: 29°, 33°,

39°, 46°, 53°, 63°, 67°, 67°, 61°, 52°, 47°, 41°. These temperatures are represented in the graph below.

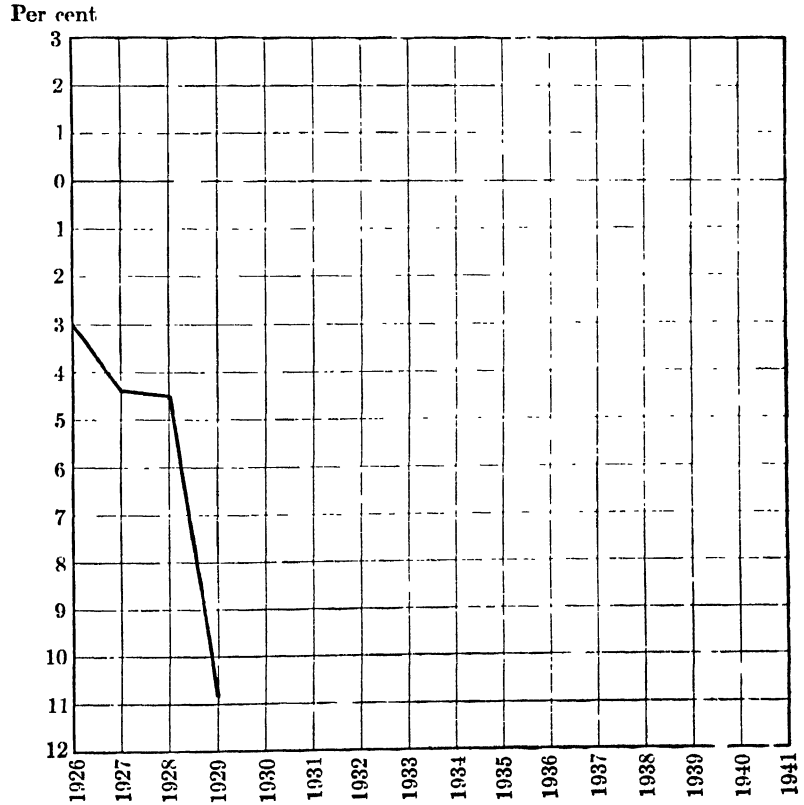


Year Ending June 30	Gross Revenue	Gross Expenditures	Surplus or Deficit	Gain or Loss %
1926	659,820	679,704	−19,884	−2 93
1927	683,122	714,577		
1928	693,634	725,700		
1929	696,948	782,344		
1930	705,484	803,667		
1931	656,463	802,485		
1932	588,172	793,684		
1933	587,631	699,887		
1934	586,733	630,733		
1935	630,795	696,503		
1936	665,343	753,616		
1937	726,201	772,713		
1938	728,634	772,308		
1939	745,955	784,550		
1940	766,949	807,629		
1941	812,828	836,859		
1942	859,817	873,950		
1943	966,227	952,535		
1944	1,112,877	1,068,986		
1945	1,314,240	1,145,101		
1946	1,224,572	1,353,654		

Vertical distances above the heavy horizontal line represent temperatures, and horizontal distances from the heavy vertical line represent length of time from Jan. 1.

493. Cost of Operating the United States Mail. — The table on the opposite page shows the total cost of operating our mail system and the total revenue. Both are given in thousands of dollars.

Example. Compute the surplus or deficit for each year and find the per cent loss or gain. Construct a graph to these per cents. A part of this graph is shown below. Note that deficit is represented by distance below the main horizontal line.



DRILL IN FUNDAMENTALS (A)

Copy from sight, add, and check.

1.	2.	3.	4.	5.	6.
13,927	64,390	76,408	24,231	38,941	49,840
8792	16,782	23,408	28,176	97,280	42,998
65,292	92,782	97,892	19,137	38,530	89,140
26,940	12,793	13,467	89,174	87,849	42,872
39,780	91,492	27,348	79,613	69,240	67,940
84,790	47,935	78,793	46,000	72,900	92,370
21,927	24,529	79,479	77,920	29,140	87,908
87,732	87,493	67,486	45,648	78,492	24,260
49,320	26,797	43,210	75,359	78,728	39,472
80,760	35,192	87,898	42,937	25,920	81,920
56,780	21,416	78,042	94,780	71,940	79,528
39,608	29,717	69,849	25,359	80,218	45,789
28,900	31,924	49,287	62,190	65,970	32,104
94,650	25,678	23,456	78,391	72,910	67,298
72,918	34,270	94,276	16,420	45,764	57,943
39,172	72,156	37,240	47,879	55,776	49,257
12,345	37,896	80,880	83,246	87,545	78,261
29,876	51,842	79,495	89,764	99,884	47,289

Copy from sight, multiply, and check.

1.	4.	7.	10.	13.
56,780 × 2975	78,042 × 3276	94,729 × 6472	71,420 × 1872	28,597 × 734
2.	**5.**	**8.**	**11.**	**14.**
64,390 × 2908	53,462 × 1249	53,162 × 6298	46,725 × 3192	64,289 × 1762
3.	**6.**	**9.**	**12.**	**15.**
49,267 × 1977	84,914 × 5627	39,276 × 8642	589,261 × 9753	145,926 × 9742

Copy from sight, divide, and check.

1. 84,628 ÷ 367	**4.** 17,104 ÷ 1941	**7.** 5317 ÷ 874
2. 59,143 ÷ 817	**5.** 21,607 ÷ 3140	**8.** 2987 ÷ 585
3. 91,417 ÷ 214	**6.** 92,140 ÷ 2516	**9.** 9376 ÷ 853

CHAPTER XXXVI

DOMESTIC EXCHANGE

494. Contracting Bills at a Distance. — People in one part of the country are constantly doing business with people in other parts. The North Central States produce yearly great quantities of cereals and meats which are sold to the other states. In New England and the Middle Atlantic States, clothing of all kinds, boots, shoes, cutlery, and other articles of manufacture are produced and sold in all sections of the country. This selling and buying between people living in different localities makes it necessary to pay bills at a distance.

495. The Problem of Domestic Exchange. — The problem which our bankers and others are trying to solve is to set off against one another the mutual indebtedness of different sections of the country and thus reduce the actual transportation of cash.

496. Means of Paying at a Distance. — In actual practice cash is seldom sent from one section of the country to another. The immediate means used for paying debts at a distance are:

1. Personal Checks.
2. Postal Money Orders.
3. Express Money Orders.
4. Telegraphic Money Orders.
5. Commercial Drafts.
6. Bankers' Checks or Drafts.

497. Personal Checks. — The simplest way to pay a bill at a distance is to send a personal check. However, the man who receives the check may not receive credit for it in his bank until enough time has elapsed to hear from the bank on which it is drawn.

498. The Certified Check. — If the bank on which the check is drawn *certifies* it, the check is known to be good so long as the bank is solvent. To certify a check the banker stamps "certified" across the face of it, and deducts the amount from the maker's account.

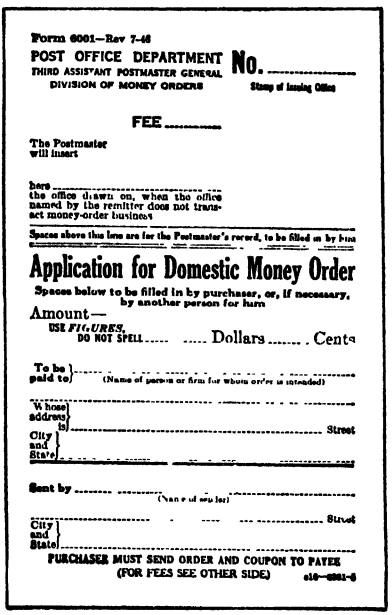
499. Postal Money Orders. — Payment of small bills such as subscriptions for magazines are often made by *postal money orders* or the

new *postal note*. The charges made by the Post Office for these orders are:

Postal Money Order:

Amounts from \$0.01 to \$5.00	10 cents
Amounts from \$5.01 to \$10.00	15 cents
Amounts from \$10.01 to \$50.00	25 cents
Amounts from \$50.01 to \$100.00	35 cents

A money order is made payable at a definite post office, and to a definite person. A money order can be signed over only *once* to another person or institution except by a banker. A money order is not issued for more than \$100. Large amounts are never sent by money order because the cost is high and also because it would be inconvenient to handle a large number of \$100 money orders to pay one large sum.



Form 6001—Rev 7-46
POST OFFICE DEPARTMENT No. _____
THIRD ASSISTANT POSTMASTER GENERAL
DIVISION OF MONEY ORDERS
Stamp of Issuing Office

FEE _____

The Postmaster will insert

here _____
the office drawn on, when the office named by the remitter does not transact money-order business

Spaces above this line are for the Postmaster's record, to be filled in by him

Application for Domestic Money Order

Spaces below to be filled in by purchaser, or, if necessary, by another person for him

Amount—
USE *FIGURES*, DO NOT SPELL..... .....Dollars..... Cents

To be paid to _____
(Name of person or firm for whom order is intended)

Whose address is _____ Street
City and State _____

Sent by _____
(Name and address)

_____ Street
City and State _____

PURCHASER MUST SEND ORDER AND COUPON TO PAYEE
(FOR FEES SEE OTHER SIDE) 610—6001-6

Postal Note: These notes in amounts from one cent to ten dollars, are for use in the United States proper only. The fee is eight cents for each, regardless of the amount of the note.

500. Express Money Orders.— Express companies sell *express money orders* at comparable rates to the postal charges. One company's rates are as follows:

From \$	.01 to \$	1.00 — Rate	5¢
" \$	1.01 to \$	5.00 — "	10¢
" \$	5.01 to \$	10.00 — "	15¢
" \$	10.01 to \$	50.00 — "	25¢
" \$	50.01 to \$	100.00 — "	30¢

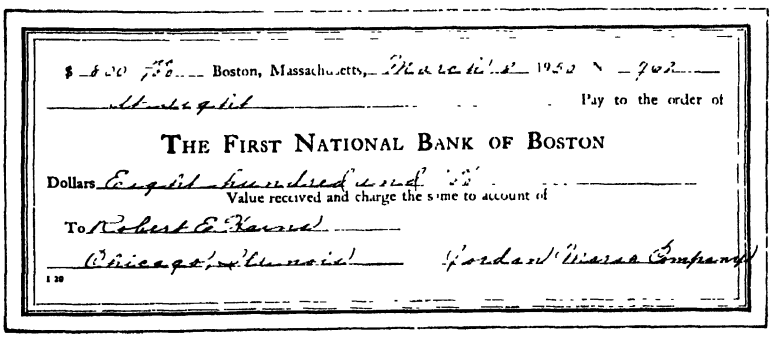
(Use the same schedule for sales in excess of \$100.00.)

These money orders have the advantage that they are payable at any office of the express company to a properly identified owner of the order. They may be indorsed and transferred any number of times the same as a check. Individual money orders are not issued for more than $50.

501. Travelers' Checks. — The express companies and some banks and travel agencies issue what are called *travelers' checks*. These checks are in denominations of $10, $20, $50, $100, and $200. The buyer signs his name on each check as he buys it, and when he wishes to use it, he identifies himself by duplicating this signature. Travelers' checks are drawn on certain banks but are accepted by the larger business concerns all over the civilized world. There is engraved on them their exact value in all the principal kinds of money and they are always accepted at their face value. A charge of $\frac{3}{4}$ of 1% of the value of the checks is usually made by the company issuing them, with a minimum charge of forty cents.

502. Sending Money by Telegraph. — The telegraph companies will forward money by telegram. The rates between points in the United States are: $10 and under — 15¢; $10.01 to $15.00 — 20¢; $15.01 to $20.00 — 25¢; increasing five cents for every ten dollars up to $100.00. From $100 the charge is 20 cents for every $25.

To this must be added the cost (plus tax) of a 15-word message to destination.


$ 600 [illegible] Boston, Massachusetts, March 1 1930 No 702

[illegible] Pay to the order of

THE FIRST NATIONAL BANK OF BOSTON

Dollars Eight hundred and [illegible]

Value received and charge the same to account of

To Robert E. Rand

Chicago, Illinois Gordan Wiese Company

130

A SIGHT DRAFT

Because of the very high rate, money is never sent by telegraph except when there is great need of instant payment.

503. Commercial Drafts. — One method of collecting money due at a distance is by means of a *commercial draft*. An example will show how this is done.

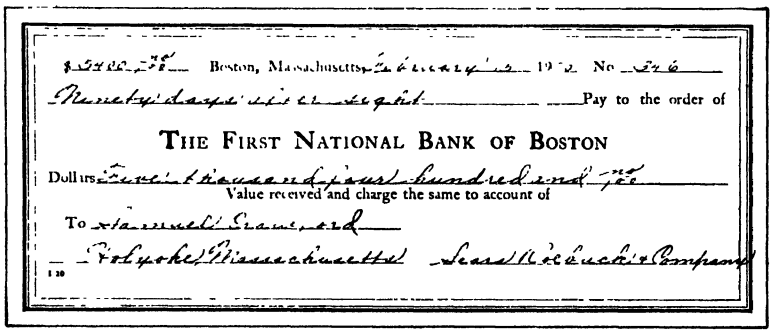
Robert E. Hains of Chicago owes $800 to Jordan Marsh Company of Boston. A draft like the one shown on page 377 is prepared by Jordan Marsh Company and deposited by them in their Boston bank. This bank gives Jordan Marsh Company credit for the face of the draft less whatever charge it may make for collecting, and forwards the draft to a Chicago bank for collection. Robert E. Hains is notified, and in case he acknowledges the debt, and wishes to pay it, he at once sends a check to the Chicago bank, which then notifies the Boston bank that the draft has been paid, and that the proceeds have been placed to its credit. The draft used in the example is payable at once, and is called a *sight draft*.

504. Bills of Lading Drafts. — The following example illustrates another use of the commercial draft.

Marshall Field & Co. of Chicago ship a bill of goods by freight to Gordon Smith & Co. of Omaha. A bill of lading is made out, which makes the goods deliverable in Omaha to the order of Marshall Field & Co.

Marshall Field & Co. indorse the bill to their bank, attaching to it a draft on Gordon Smith & Co. for the amount of the shipment. The Chicago bank forwards the draft and bill to their Omaha correspondent. Gordon Smith & Co. are notified, pay the draft to the Omaha bank, and receive the bill of lading signed over to them.

If Gordon Smith & Co. do not pay the draft, they cannot get the bill of lading from the bank, and cannot get the goods from the railway company.


$ 5200.75 Boston, Massachusetts, [illegible] 19 12 No 276

Ninety days after sight Pay to the order of

THE FIRST NATIONAL BANK OF BOSTON

Dollars Five thousand four hundred and 75/100

Value received and charge the same to account of

To Samuel Crawford

Holyoke Massachusetts Jesse McLeod & Company

130

A TIME DRAFT

505. Loaded Bills. — A draft with a bill of lading attached is called a *loaded bill.*

506. Clear Bills. — A draft not accompanied by a bill of lading or other such document is called a *clear bill.*

507. The Time Draft. — Another kind of commercial draft is what is called the time draft.

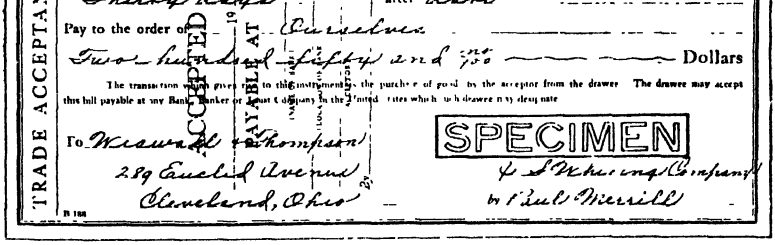
Samuel Crawford of Holyoke buys goods of Sears Roebuck & Company of Boston on ninety days' credit.

The draft on page 378 is made out by Sears Roebuck & Company, and deposited in their Boston bank, which forwards it to a bank in Holyoke. Samuel Crawford *accepts* the draft by writing across the face of it:

"Accepted, February 21, 1950, Samuel Crawford."

This draft now becomes the promissory note of Samuel Crawford, and he must pay it when due. The Boston bank discounts the draft from February 21 (the day it is accepted) to the date of maturity (the day when it is due), and credits Sears Roebuck & Company with the proceeds.

508. The Trade Acceptance. — A *trade acceptance* is a commercial draft used only in payment of a bill arising from the purchase of goods. When the purchaser accepts the trade acceptance, he writes *accepted,*


TRADE ACCEPTANCE

No. 120 Boston, Mass. February 8, 1950 $ 250.75

Thirty days after date

Pay to the order of Ourselves

Two hundred fifty and 75/100 Dollars

The transaction which gives rise to this instrument is the purchase of goods by the acceptor from the drawer. The drawer may accept this bill payable at any Bank, Banker or Trust Company in the United States which such drawee may designate

ACCEPTED PAYABLE AT

To Maxwell & Thompson
289 Euclid Avenue
Cleveland, Ohio

SPECIMEN

The Sherwin Company
by Paul Merrill

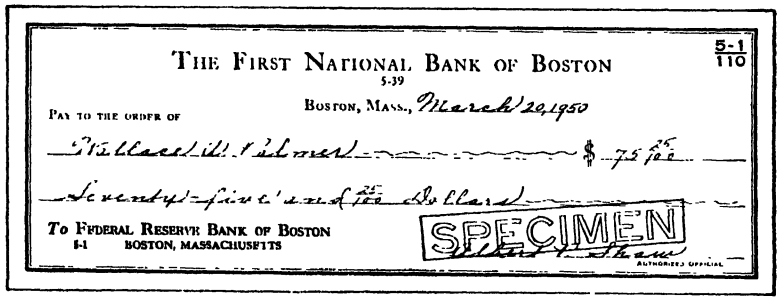
THE TRADE ACCEPTANCE

with the date, across its face and returns it to the seller of the goods. The purchaser is now legally responsible for its amount, for it is then, in effect, a note. The trade acceptance may be made for 30 or 60 days or for immediate payment.

509. Bank Draft. — Bills at a distance may also be paid by a *bank draft,* which is a check drawn by one bank on another.

New York, Chicago, and San Francisco are the most important financial centers and bills due in the general section of any of these cities may be paid by draft on it. Thus, a bill due in Milwaukee can be paid by draft on Chicago. Other large cities, such as Boston, Philadelphia, St. Louis, act in a similar capacity for more restricted regions.

New York drafts are usually accepted over the entire country. Hence New York banks sell cashier's checks on their own bank instead of drafts on distant banks, unless it happens that exchange on these places is at a premium, in which case regular drafts must be purchased.


THE FIRST NATIONAL BANK OF BOSTON
5-39
5-1
110
PAY TO THE ORDER OF
BOSTON, MASS., March 20, 1950
William H. Wilmer
$ 75 25/100
Seventy-five and 25/100 Dollars
To FEDERAL RESERVE BANK OF BOSTON
1-1 BOSTON, MASSACHUSETTS
SPECIMEN
AUTHORIZED OFFICIAL

A BANK DRAFT

510. Use of Commercial and of Bank Drafts. – When the creditor takes the initiative in settling the indebtedness, a commercial draft is used. When the debtor takes the initiative, he buys a bank draft.

Goods are often sold on a certain term of credit with the specific provision that the seller shall be permitted to draw on the buyer at the expiration of the time. In such cases the time draft is frequently used.

511. Transfer of Credits through Federal Reserve Banks. — At one time bankers charged a small premium for issuing checks payable in a city in some other section of the country. Establishment of the Federal Reserve Banks has made it possible for member banks to make immediate transfer of credit to any district of the United States. This fact has almost eliminated the item of exchange between cities.

512. Discounting Drafts. — The discount of a time draft is computed on the face of the draft the same as the bank discount of a note. The term of discount runs from the day the draft is discounted until it is due.

The discounting of time drafts is in all respects the same as the discounting of promissory notes bearing no interest. See page 339.

WRITTEN EXERCISES (A)

Find the proceeds of each of the following drafts. Use the exact number of days, and 360 days as one year.

	Face	Date Due	Date of Discount	Rate of Discount
1.	$2400	Aug. 1	June 7	6%
2.	$1300	Sept. 19	July 9	6%
3.	$540	July 13	May 7	$6\frac{1}{2}\%$
4.	$7800	Aug. 7	June 9	5%
5.	$200	Sept. 14	June 14	6%
6.	$39,400	Oct. 7	July 18	$4\frac{3}{4}\%$
7.	$490	Dec. 8	Aug. 13	$5\frac{1}{2}\%$
8.	$8300	July 14	April 24	$5\frac{1}{4}\%$

9. Find the proceeds of a draft for $7400 discounted 45 days before it is due, the rate of discount being $4\frac{1}{2}\%$.

10. How much would a merchant realize from his bank on a draft for $4900 due August 13 if he were to discount it June 30 at $5\frac{3}{4}\%$?

11. A merchant in Denver buys a bill of goods for $14,300 in Chicago, discounts being 10% and 5% (see page 250). For what face amount will the Chicago firm make draft and what will be the proceeds if the draft is discounted 17 days before due at The First National Bank at $4\frac{1}{2}\%$?

12. An automobile company in Detroit sells machines to a western dealer for $35,800. The bill is payable in 90 days. He accepts the draft 75 days before it is due. What are the proceeds of the draft if the rate of discount is $5\frac{1}{2}\%$?

13. The proceeds of a draft discounted at 6% thirty days before maturity amounted to $9896.29. What was the face amount?

14. The Union Trust Company discounted a draft 14 days before due at 5%, crediting the account of the drawer $3593. Find the amount of the draft.

15. A Chicago draft for $6500 is due in Salt Lake City in 120 days. It is accepted and discounted at Salt Lake City 112 days before it is due. What are the proceeds of the note, if the rate of discount is 8%?

16. A company purchased two airplanes for \$32,000 each, accepting the manufacturer's draft for this amount payable ninety days "after sight" on September 14. On September 29 the manufacturer discounted the draft with his bank at $4\frac{1}{4}\%$. What are the proceeds?

17. A lumber company in Montana sells a bill of lumber to a firm in St. Paul for \$28,400, payable in 90 days in St. Paul. Draft for this amount is accepted and discounted in St. Paul, at the rate of 5%, 83 days before it is due. What are the proceeds?

18. The draft of a tractor manufacturer in Cleveland is drawn on a Texas dealer for \$24,500 and discounted 72 days before due at $5\frac{1}{2}\%$. For what amount will the bank credit the tractor company?

513. Service of the Banks in Paying Bills at a Distance.—The full reason why payments are made by check, by bank draft, or by ordinary commercial drafts, is rather difficult to understand. We can acquire a partial comprehension of it, however, by studying a special case.

Throughout the year breadstuffs and meats are being shipped east from the North Central States. At the same time, clothing, shoes, and other manufactured goods are being shipped into the North Central States from New England and the Middle Atlantic States. Each shipment is made on a separate order, and payment must be made for it. If cash were to be sent in both directions to pay for each shipment, an immense amount of cash would be shipped in both directions each month or even each day. Indeed, it would be perfectly possible to have millions in actual money shipped from New York to Chicago the same day that an equal number of millions were shipped from Chicago to New York.

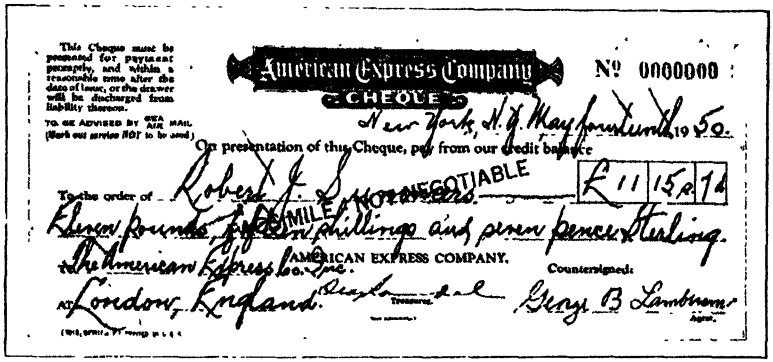
This waste of work and expense is saved by the banks. If merchants in Chicago owe one million dollars to merchants in New York, and at the same time other merchants in New York owe a million dollars to merchants in Chicago, the banks by means of their drafts enable both millions to be paid without transferring a single cent in cash.

The net result is that if the total of the payments to be made by Chicago people in New York is about the same as the total of the payments to be made by New York people in Chicago during any comparatively short period of time, say one or two months, then no cash is sent. If, for a long time, however, the total payments to be made by Chicago in New York exceed by a considerable amount the total payments to be made by New York in Chicago, then and only then does actual shipment of money take place. A full study of this subject is taken up in courses on Political Economy.

CHAPTER XXXVII

FOREIGN EXCHANGE

514. Reasons for Foreign Exchange. — Our immense foreign trade requires extensive means of settling international obligations. Foreign exchange exists, therefore, for the same reasons that domestic exchange exists (see § 513). Below is a customary form of a foreign draft or bill of exchange.


This Cheque must be presented for payment promptly, and within a reasonable time after the date of issue, or the drawer will be discharged from liability thereon.

TO BE ADVISED BY SEA / AIR MAIL (Mark out service NOT to be used)

American Express Company

CHEQUE

No 0000000

New York, N. Y. May fourteenth 1920.

On presentation of this Cheque, pay from our credit balance

To the order of Robert J. Simile NOT NEGOTIABLE £ 11 15s 1d

Eleven pounds fifteen shillings and pence Sterling.

The American Express Co. Inc.

AMERICAN EXPRESS COMPANY.

Countersigned:

At London, England. George B Lambrecht Agent.

515. Foreign Drafts. — Foreign drafts, or foreign bills of exchange, payable on demand for any amount desired, are issued by nearly all banks and financial institutions. Some banks in the larger cities keep funds deposited with banks at the principal foreign cities, against which these drafts are drawn direct, but most banks issue drafts through an arrangement with concerns having foreign accounts.

These drafts are usually issued in the currency of the country where payable, that is, for so many pounds, shillings, and pence on England, or so many marks and pfennigs if on a place in Germany, and so on.

A foreign bill of exchange formerly was made out in duplicate, sometimes in triplicate. In the latter case one was kept by the remitter as a memorandum. The first and second bills were sent in different

mails to insure that at least one should reach its destination. The payment of either bill invalidated the other. At present, only one bill is made in most cases.

516. Clear Bills and Documentary or Loaded Bills. — As in the case of domestic bills, a bill of exchange may be attached to some other document, such as a bill of lading, which latter will be turned over to the consignee only on condition that the bill is paid or accepted (see § 504). Such bills are called *documentary or loaded bills.* In case no document is attached to the bill it is a *clear bill.*

517. Standard of Exchange. — Until late in 1931, gold was the standard of exchange values over nearly the whole civilized world, and there were relatively stable and fixed rates for exchanging money between countries. The intrinsic par of exchange was determined by the ratio of the amount of pure gold in the principal coins. This arrangement still continues between a few nations, but economic disorders have forced many countries to abandon it.

518. Rate of Exchange. — The actual rate of exchange on any one country varies from time to time. The rate in force at any one time is called the rate of exchange at that time. At certain times of the year England buys large quantities of cotton and wheat from the United States. Before the seller is able to use the money thus realized, it must be changed from English pounds into American dollars. If the change takes place at a time when not sufficient amounts are being exchanged from dollars into pounds to offset it, there is a tendency toward lower quotations for the pound and higher corresponding quotations for the dollar.

519. Exchange Rate Quotations. — In recent years the fluctuations of currency from nation to nation have been so variable that it has been impossible to make new printings of this book rapidly enough to keep up with them. Your local bank can furnish you with information about the latest changes.

The rates of exchange are printed daily in various metropolitan newspapers, and these can always be found for reference at a city library.

520. Bankers' Fees. — To the rate of exchange, there is added a small charge to cover the banker's cost and his profit in making the

transfer. For small amounts, the fee may be only 50 cents or 75 cents, while on large transactions it may be as low as $\frac{1}{16}$ of 1% of the amount.

521. Cost of Shipping Gold. — When the rate of exchange on any country rises to a point where it is cheaper to transport the gold there than to buy exchange, New York bankers will arrange to ship the gold. Included in the cost of shipping the gold are such items as insurance, freight, packing, cartage, abrasion, and loss of interest during the period of transfer.

522. Export and Import Rate of Gold. — Adding together the actual costs of shipping gold to or from a country, the total represents the approximate amount that exchange should normally fluctuate above or below its par.

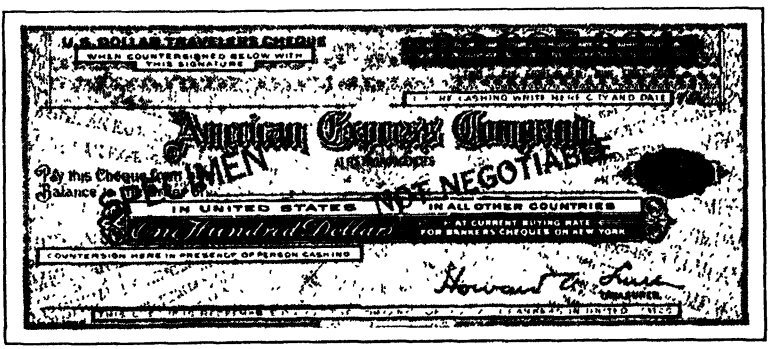
523. Bill Stamps. - Some foreign countries require revenue stamps to be affixed to bills of exchange.

Thus England requires $\frac{1}{20}$% on all bills of more than 3 days' sight.

524. Transfer of Money by Cable. — Money may now be ordered paid in all parts of the civilized world by telegram and cable. The rate to foreign countries is the course of exchange plus a small commission plus the usual rate for the cablegram.

The rate by cable is higher because the money is paid immediately, while a bill of exchange is not presented in Europe until from six to twenty-four days after it is drawn, depending on the place and the ship in which it is sent.

Similarly International Money Orders (see § 499) are issued by the Post Office Department and are available at any post office or branch.



A Travelers' Check

525. Travelers' Checks. — Travelers' checks are accepted in foreign countries as well as in the United States (see § 501).

526. Letter of Credit. — Banks issue traveling letters of credit in which they request their correspondents in the various parts of the world to pay to the owner of the letter any sums he may desire up to a total specified in the letter. The bank selling the letter of credit usually charges a fee of 1%, the minimum charge being $5.

When the owner of a letter of credit desires to obtain funds on it he goes to one of the banks named in the letter and makes out a draft on the bank which issued it. This draft is signed in the presence of an officer of the bank, who compares the signature with that on the letter of credit. The draft is then cashed and the amount of it indorsed on the back of the letter.

If the whole amount of the letter is drawn, the letter is surrendered by the owner and attached to the last draft.

527. Problem. To Find the Cost of a Bill of Exchange.

Example 1. A of New York has a bill of £1250 12*s*. 4*d*. to pay in London. What is the cost of a bill of exchange to cover this, the rate being $3.19\frac{3}{8}$?

Solution. £1250 12*s*. = £1250.60
$3.19375 \times 1250.60 = 3994.10$ (dollars)
Hence £1250 12*s*. 4*d*. = 3994.16 (4*d*. = 6¢)

Example 2. At $.0390\frac{7}{8} + \frac{1}{16}$, what is the cost of a bank check on Paris for 17,840 francs?

Solution. $17{,}840 \times .0390875 = \$697.32$
$\frac{1}{16}$ of 1% of 697.22 = $0.44
Hence the cost is $\$697.32 + \$.44 = \$697.76$

WRITTEN EXERCISES (B)

Using the rates of exchange as quoted in this week's reports, find the cost of the following bills of exchange.

1. £735 10*s*.
2. £14,300 16*s*. 4*d*.
3. 164,500 French francs.
4. 235,000 Swiss francs
5. 462,800 marks
6. 347,800 marks
7. 5625 guilders
8. 21,862 Spanish pesetas
9. 8563 Danish Kronen
10. 16,981 lira
11. 4917 Norwegian kroner
12. 7129 Swedish kroner

THE FIRST NATIONAL BANK OF BOSTON

CIRCULAR LETTER OF CREDIT

No. 00000

The FIRST
NATIONAL BANK of
BOSTON
BOSTON, MASSACHUSETTS

To our Correspondents November 15, 1949

Gentlemen:

We beg to introduce to you Mr. John Doe,***

528. Arbitrage of Exchange. — Sometimes rates of exchange are such that it is cheaper to send money to England, for example, by buying a bill on France and then one in France on England. The process of deciding which combination of rates is the most advantageous is called arbitrage of exchange.

529. Problem. To Find the Cost of a Foreign Time Draft.

Example. Find the cost of a bill of exchange for £2150 at 60 days' sight, rate 3.25, interest 3%.

NOTE. — Interest on London is counted 365 days in a year and 3 days' grace.

Solution. Interest on \$3.25 at 3% for 63 days = \$.016829
$\frac{1}{20}$% bill stamp on \$3.25 = .001625
Cost of £1 = rate − interest + bill stamp = \$3.234796
Cost of bill = 2150 × 3.234796 = \$7925.25

In practice bankers who do large volumes of foreign exchange business use tables giving cost of bills on England, France, and Germany. Such tables reduce the labor of computing the cost of bills to a minimum. The great majority of bankers, however, draw foreign bills only at rare intervals and find the cost as in the example just given.

530. Problem. To Find the Present Value or Proceeds of a Time Draft.

Example 1. Find the proceeds of a 60-day bill of exchange for £2450, rate \$3.25, interest 3%.

Solution.

Rate per pound		\$3.250000
Interest on \$3.25 at 3%, 63 days	.016829	
$\frac{1}{20}$% of bill stamp on 3.25	.001625	
		.018454
		\$3.231546

The banker will quote 3.2315
Hence, 2450 × 3.2315 = \$7917.18 = proceeds
The banker who buys the bill deducts interest and the cost of the bill stamp.

Example 2. Find the proceeds of a 90-day bill for 37,500 French francs, the rate being .0391, interest 3%.

NOTE. — Days of grace are not allowed in France. 360 days are counted as one year.

Solution.

Rate per franc	\$.0391
Interest at 3%, 90 days ($\frac{3}{4}$ of 1% of the rate)	.000293
The banker's quotation =	\$.038807

37400 × .038807 = \$1455.26. Hence the proceeds are \$1455.26.

PROBLEMS (B)

1. What will be the cost in Boston of a bill on Liverpool for £325 3*s*. 9½*d*., exchange being 3.18?

2. An agent in Philadelphia, having $7500 due his employer in England, is directed to remit a bill on Liverpool. What is the face of the bill which he can purchase for this money, exchange being 3.24½?

3. A sight draft on Manchester for £720 cost $2314.80. What was the course of exchange?

4. Find the cost of a draft of £540 8*s*. 6*d*. at 3.17¼.

5. How large a bill of exchange on Liverpool can be bought for $6000 at 3.22?

6. How many French francs can be bought for $4000 at .0392?

7. How many marks can be bought for $2000 at .2379?

8. I wish to send $750 to my son in Edinburgh. At $3.20 a pound, how large an express order can I buy?

9. An exporter in Boston drew a bill of exchange on a Paris firm for $28,560. If exchange in Paris on Boston is quoted at 25.51 francs to the dollar, how many francs will it take to pay the draft?

10. A commission merchant has drawn on him against an importation of gloves, a draft for £350 7*s*. 3*d*. If exchange is at 3.31, what will it cost him to pay the draft when it is due?

11. What will be the cost of a bill on Paris for 56,245 francs, exchange being .0391¼?

12. Change $40,535 to English money at 3.21; into French money at .0390⅝; into German money at .2379.

13. Change £5345 13*s*. 8*d*. to U. S. money at 3.21¼.

14. How much will a draft for 8500 francs cost at .0390¾?

15. A shipment of cotton to Liverpool sold for an amount of money which was exchanged into $3221.40. If the rate was $3.18¾, what was the amount realized in pounds, shillings, and pence?

16. Change 1,000,000 marks into U. S. money at .2378; into pounds sterling when the rate is 13.375 marks to the pound.

DRILL IN FUNDAMENTALS (A)

Find the interest on the following and check results.

	Principal	Rate	Time
1.	$780	$5\frac{1}{2}\%$	9 mo. 3 da.
2.	490	6%	3 mo. 9 da.
3.	16,080	$5\frac{1}{2}\%$	7 mo. 3 da.
4.	374	6%	6 mo. 18 da.
5.	2940	6%	5 mo. 6 da.
6.	360	7%	4 mo. 5 da.
7.	150	8%	7 mo. 27 da.
8.	145	8%	6 mo. 18 da.
9.	392	7%	4 mo. 13 da.
10.	765	$6\frac{1}{2}\%$	3 mo. 11 da.
11.	1892	6%	4 mo. 15 da.
12.	4930	6%	5 mo. 16 da.
13.	6780	$5\frac{1}{2}\%$	6 mo. 17 da.
14.	4560	$5\frac{1}{2}\%$	7 mo. 18 da.
15.	1720	6%	8 mo. 3 da.
16.	3920	$5\frac{1}{2}\%$	9 mo. 6 da.
17.	4937	$6\frac{1}{2}\%$	3 mo. 1 da.
18.	2978	6%	4 mo. 2 da.
19.	1390	6%	5 mo. 1 da.
20.	2820	$5\frac{1}{2}\%$	3 mo. 7 da.

Copy from sight, multiply, and check.

1. 89,764	**2.** 19,147	**3.** 98,765	**4.** 24,680	**5.** 45,782
398	940	439	908	478
6. 37,219	**7.** 24,356	**8.** 78,392	**9.** 70,249	**10.** 39,874
1418	2876	4648	4276	6587
11. 39,271	**12.** 24,567	**13.** 73,459	**14.** 128,492	**15.** 78,956
2418	9753	8642	4128	8642

CHAPTER XXXVIII

EQUATION OF PAYMENTS AND ACCOUNTS

531. Terms of Credit. — In the wholesale trade goods are usually sold on certain terms of credit. A bill for goods often specifies that it is due in 30 days, 60 days, 90 days, two months, etc. The time thus specified in which the bill must be paid is called the *term of credit.*

If the term of credit is given in days, the exact number of days must be counted; and if the number of months is given, the exact number of calendar months must be taken regardless of the number of days they contain.

532. Discount on Cash Payments and Interest on Late Payments. — If a bill is paid before it is due, this entitles the payer to a discount. If a bill is not paid when due, it will draw interest at the legal rate from the date it is due.

Thus, if a bill for $1000, due July 1, is paid June 1, the payee will have the use of the money for an extra month and can afford to pay interest for this use. Hence a discount will usually be allowed. If, on the other hand, it is paid August 1, the bill will draw interest for one month.

If $1000 is due June 1 and $1000 is due July 1 the whole debt may be discharged equitably by a payment of $2000 on June 16, that is, 15 days after the first date and 15 days before the second. If $2000 is due June 1 and $1000 July 1, the whole debt may be paid June 11, that is, 10 days after the first date and 20 days before the second. Note that the use of $2000 for 10 days is equivalent to the use of $1000 for 20 days.

533. Equation of Payments. — The finding of the date when several bills due at different times may all be paid, is called *equation of payments.*

ORAL EXERCISES

1. A bill for $2000 is due Aug. 1 and a bill for $4000 is due Sept. 1. What date may the two bills be paid equitably?

2. A bill for $10,000 is due Oct. 1 and another bill for $2000 is due October 24. What date may these bills both be paid?

534. Due Date. — The date on which the total payment may be made is called the *equated date* or the *due date.*

535. Focal Date. — The general plan in solving a problem like that given below is to find how many days before a certain date, as Aug. 5, the one payment is to be made. We could equally well find how many days after May 28 it is to be made. The date thus selected is called the focal date or pivotal date.

Example. Last year Ward & Co. sold goods to J. B. Ford as follows: April 15, 1933, $2800, terms 2 mo.; April 28, $3500, terms 1 mo.; May 5, $4500, terms 3 mo. Find the due date for the whole account.

Solution. The exact number of days is used as the time from the due date of each bill to the focal date. The exact number of days, or months and days, are used to determine the due date of each bill according as the term of credit of the bill is given in days or in months. In all cases the year is regarded as 360 days. We arrange the work as follows:

Date	Amount	Due	Focal Date	Interest to Focal Date at 6%	Average Time	Due Date
April 15	2,800	June 15		23.80		
April 28	3,500	May 28		40.25		
May 5	4,500	Aug. 5	Aug. 5	.00	36 da.	June 30
Total	10,800			64.05		

But the interest on $10,800 for one day is $1.80. Hence average time before Aug. 5 is 64.05 ÷ 1.8 = 35.6 or 36 days, and the due date is June 30. Notice that the due date does not depend on the rate of interest used. Hence 6% is used, being the most convenient rate.

WRITTEN EXERCISES

In this manner find the due date of each of the following:

1.

May 7	6890	Credit 45 da.
May 28	4800	Credit 30 da.
June 1	8200	Credit 60 da.

2.

Aug. 4	200	Credit 60 da.
Aug. 7	350	Credit 30 da.
Aug. 12	400	Credit 30 da.
Aug. 20	70	Credit 30 da.

3.

Dec. 1	250	Credit 60 da.
Dec. 8	45	Credit 40 da.
Dec. 16	75	Credit 30 da.

4.

June 7	700	Credit 2 mo.
June 10	900	Credit 3 mo.
June 13	450	Credit 2 mo.
June 23	340	Credit 30 da.

536. Equation of Accounts. — An account frequently consists of debit items designating goods sold on specified terms of credit and of credit items designating payments made on account. The finding of the date when such an account may be balanced equitably by one payment is called *equating the account.*

Example. Bought goods as follows: Aug. 15, 1949, $1260, 30 da.; Aug. 20, $2400, 60 da.; Sept. 1, $4300, 45 da.; Sept. 10, $6800, 90 da. Made payments as follows: Aug. 30, $2000, Sept. 5, $2500; Sept. 20, $2500. Find the due date.

Solution.

Bills				Payments				
Date	Amount	Due	Interest	Date	Amt	Int.	Focal Date	Due Date
Aug. 15	1260	Sept. 14	18.06	Aug. 30	2000	33.67		
Aug. 20	2400	Oct. 19	20.40	Sept. 5	2500	39.58		
Sept. 1	4300	Oct. 16	38.70	Sept. 20	2500	33.33		
Sept. 10	6800	Dec. 9					Dec. 9	Jan. 1, 1950
Totals	14760		77.16	Balance	7760	106.58		

Net interest Credit 29.42

But $29.42 is interest on $7760 at 6% for 22.8 or 23 days. Hence the due date is Jan. 1, 1950.

Since the interest on the payments is greater than the interest on the deferred payments, it follows that the due date is after the focal date.

Notice that in this example the exact number of days is used to find the due date of each bill. Compare the example on page 392.

WRITTEN EXERCISES

In this manner find the due date in each of the following.

1.

Bills			Payments	
Date	Amt	Terms	Date	Amt.
Dec. 1	3980	3 mo.	Jan. 5	1000
Dec. 15	5900	2 mo.	Feb. 2	2000
Dec. 20	3500	3 mo.	Mar. 1	4000

2.

Bills			Payments	
Date	Amt	Terms	Date	Amt.
Sept. 10	300	2 mo.	Sept. 25	200
Sept. 15	500	3 mo.	Oct. 15	200
Sept. 25	750	30 da.	Nov. 20	400

3.

Bills			Payments	
Date	Amt	Terms	Date	Amt.
Aug. 15	8900	90 da.	Nov. 1	10000
Sept. 1	7400	120 da.	Nov. 15	4000
Oct. 1	3200	60 da.		
Oct. 15	6000	90 da.		

4.

Bills			Payments	
Date	Amt	Terms	Date	Amt.
May 10	1000	10 da.	May 11	3000
May 20	2000	30 da.	May 30	1000
May 30	3000	2 mo.	July 1	2000
June 10	2500	1 mo.	July 8	2200

5.

Bills			Payments	
Date	Amt	Terms	Date	Amt
Apr. 7	7500	3 mo	June 1	5000
Apr. 25	6000	60 da.	June 15	5500
May 8	4000	2 mo.	July 1	3000
May 27	1800	30 da.	July 20	4500
June 7	3600	10 da.		

6.

Bills			Payments	
Date	Amt	Terms	Date	Amt.
Sept. 1	350	60 da.	Sept. 15	200
Sept. 20	400	30 da.	Oct. 10	300
Oct. 15	250	40 da.	Nov. 10	150
Nov. 1	100	25 da.	Nov. 20	350
			Nov. 25	200

7.

Bills			Payments	
Date	Amt	Terms	Date	Amt
Mar. 1	350	30 da.	Mar 15	200
May 15	400	60 da.	Apr. 10	350
May 25	675	40 da.	Apr. 25	500
May 30	850	30 da.		

8.

Bills			Payments	
Date	Amt	Terms	Date	Amt.
Jan. 10	1000	30 da.	Feb. 5	800
Feb. 1	800	60 da.	Feb. 15	700
Feb. 15	600	40 da.	Mar. 10	500
Feb. 25	700	50 da.		

9.

Bills			Payments	
Date	Amt	Terms	Date	Amt
July 1	1000	90 da.	Aug. 20	1000
July 20	1200	60 da.	Sept. 10	800
July 29	800	75 da.	Sept. 15	750
Aug 15	1500	2 mo.	Oct. 1	1200
Aug. 25	2000	3 mo.	Oct. 15	1900

10.

Bills			Payments	
Date	Amt.	Terms	Date	Amt.
Feb. 16	1400	30 da.	Mar. 10	1200
Apr. 10	1500	25 da.	May 1	1200
May 1	1600	40 da.	June 1	1500
May 20	1000	50 da.	July 1	800
June 15	800	60 da.	July 10	650

537. Cash Balance. — A slightly different problem arises when it is required to find the amount of cash needed to close an account on a

certain date. The amount needed is called the *cash balance* on that date.

538. Face Balance. — The difference between the debits and the credits of an account is the face balance of the account.

On the due date, the cash balance is the same as the face balance. If the payment is made after the due date, the cash balance is greater than the face balance, since the face balance draws interest from the due date. If the payment is made before the due date, the cash balance is less than the face balance, since early payment entitles the debtor to a discount.

Example. Find the cash balance of the following account, June 15, 1949, interest being 6%.

	Bills			Payments		
Date	Amount	Terms	Interest	Date	Amount	Interest
Jan. 15	24000	2 mo.	368	Feb. 1	5000	111 67
Feb 1	10000	3 mo.	75	Feb. 20	10000	191.67
Mar. 10	8500	2 mo.	51.00	April 15	6000	61
Mar. 10	1600	3 mo.	3 83	June 1	1606	9.33
Totals	47100		197.83		25000	373.67

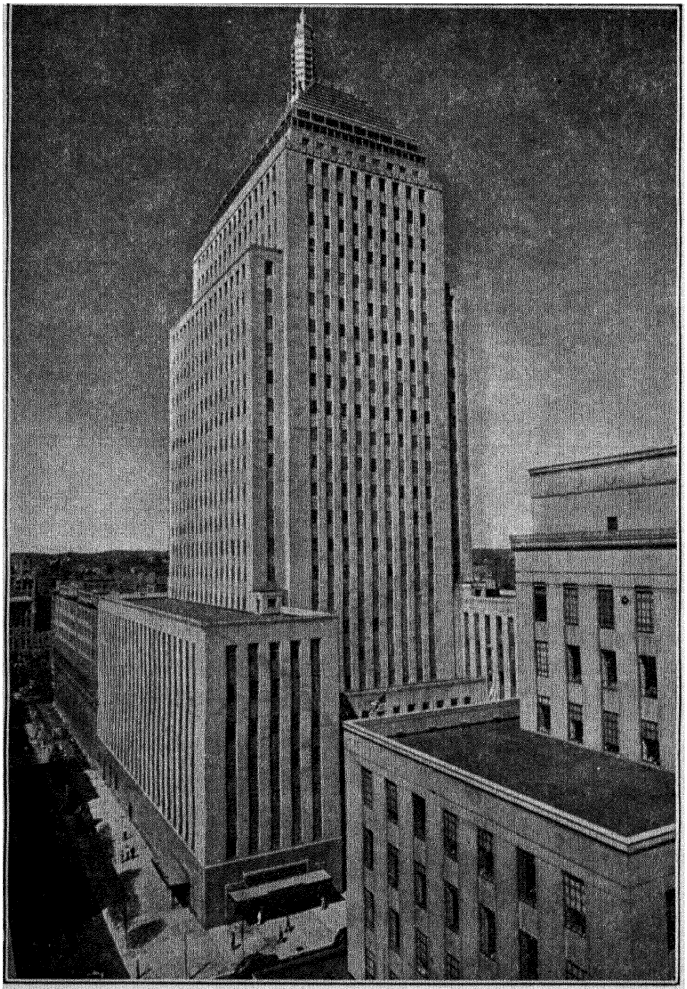
Net interest Balance 124.16 Face Balance 22,100

Face Balance + Int. Bal. = $22,224.16 = Cash Balance.

On bills due before the date of settlement, interest is charged from the date they are due to the date of settlement, while on bills due after the date of settlement, interest is credited from this date to the day they are due. Since all payments (except the final one) are made before the day of settlement interest is credited on all such payments.

WRITTEN EXERCISES

In the accounts on pages 393 and 394 find the cash balance of Example 1 on April 1, 1949; of Example 2 on Dec. 15, 1949; of Example 3 on Dec. 1, 1949; of Example 4 on Aug. 1, 1949; of Example 5 on Aug. 1, 1949; of Example 6 on Dec. 1, 1949; of Example 7 on June 1, 1949. Use 6% interest in all.



Courtesy John Hancock Mutual Life Insurance Co.

A Modern Life Insurance Building

CHAPTER XXXIX

PROPERTY INSURANCE

539. Insurance deals with problems arising from contracts in which one party agrees to idemnify another in case of damage from specified causes, to specified things or persons and in a specified time.

540. The Insurance Policy. — A written or printed contract containing the provisions of an agreement to insure is called an insurance policy.

541. Parties to an Insurance Contract. — The *insurer* is the party which agrees to idemnify another, and the party so idemnified is the *insured.*

542. Kinds of Policies. — A *valued* or *closed* policy agrees to pay a specified sum in case a certain damage is sustained.

An *open* policy provides insurance for an indeterminate amount, depending upon the quantity of goods insured at the time of damage.

Thus a fire insurance policy covering goods stored in a warehouse insures whatever goods may be in the warehouse at the time of fire. At the end of the year the average amount of goods in the warehouse is computed and the premium fixed accordingly.

543. Premium. Rate. — The amount paid by the insured to the insurer in consideration of the insurance is called *premium.*

In the case of property insurance the premium is usually figured as a certain rate per cent of the amount insured. This is called the *insurance rate.* The rate varies greatly with the kind of property insured, and the length of time for which it is insured.

544. Kinds of Property Insurance. — There is insurance against damage by fire, flood, and hail. There is marine insurance, river insurance, live stock insurance, and transit insurance which guarantees safe delivery. There is insurance against burglary and other theft, etc. This chapter treats particularly of fire insurance.

545. Long and Short Term Rates. — The standard period for which fire insurance rates are quoted is one year. Three-year policies are usually issued for a premium equal to two and a half times the yearly premium and five-year policies for a premium equal to four times the yearly premium.

For terms less than one year higher rates are charged. The following table shows the short term rates adopted by the Western Union of Underwriters.

1 Day	2% annual prem.	50 Days	28% annual prem.
2 Days	4% annual prem.	55 Days	29% annual prem.
3 Days	5% annual prem.	60 Days	30% annual prem.
4 Days	6% annual prem.	65 Days	33% annual prem.
5 Days	7% annual prem.	70 Days	36% annual prem.
6 Days	8% annual prem.	75 Days	37% annual prem.
7 Days	9% annual prem.	80 Days	38% annual prem.
8 Days	9% annual prem.	85 Days	39% annual prem.
9 Days	10% annual prem.	90 Days or 3 mo.	40% annual prem.
10 Days	10% annual prem.	105 Days	45% annual prem.
11 Days	11% annual prem.	120 Days or 4 mo.	50% annual prem.
12 Days	12% annual prem.	135 Days	55% annual prem.
13 Days	13% annual prem.	150 Days or 5 mo.	60% annual prem.
14 Days	13% annual prem.	165 Days	65% annual prem.
15 Days	14% annual prem.	180 Days or 6 mo.	70% annual prem.
16 Days	14% annual prem.	195 Days	73% annual prem.
17 Days	15% annual prem.	210 Days or 7 mo.	75% annual prem.
18 Days	16% annual prem.	225 Days	78% annual prem.
19 Days	16% annual prem.	240 Days or 8 mo.	80% annual prem.
20 Days	17% annual prem.	255 Days	83% annual prem.
25 Days	19% annual prem.	270 Days or 9 mo.	85% annual prem.
30 Days	20% annual prem.	285 Days	88% annual prem.
35 Days	23% annual prem.	300 Days or 10 mo.	90% annual prem.
40 Days	26% annual prem.	315 Days	93% annual prem.
45 Days	27% annual prem.	330 Days or 11 mo.	95% annual prem.
		360 Days or 12 mo.	100% annual prem.

546. Insurance Agent Broker. — Insurance companies are represented in various localities by agents who solicit insurance, write up policies, collect premiums, etc. These are called insurance agents and are regarded as the agents of the insurance company.

The insurance broker does pretty much the same kind of work as the insurance agent, but he is legally the agent of the insured.

547. Ordinary Policy. — An *ordinary* policy stipulates the amount to be paid in case of loss. Under such a policy the company pays the full amount of the loss up to the amount of the policy

548. The Average Clause. — One policy covering goods in different localities frequently contains a clause which stipulates that the company issuing the policy will pay only such proportion of the total loss as the amount insured bears to the total value of the property. Such a clause is called an *average clause*.

549. Three-fourths Value Clause or Coinsurance Clause. — In some insurance policies it is stipulated that the property shall be insured for at least three-fourths its actual value.

The purpose of the average clause is to guard against a small insurance policy on extensive property scattered in many different localities. Clearly such a property is very much more liable to a small fire than to one which will destroy the whole property.

The purpose of the three-fourths value clause is to guard against a small insurance on valuable property with good fire protection. As in the case of scattered property, such property is more liable to a small fire than to one which will destroy the whole property.

On buildings (brick or better) there is a reduction of 30% of premium if building is insured for 90% of value, a reduction of 25% if insured for 80% of value, and a reduction of 10% if insured for 70% of value. For contents of building these reductions are 20%, 15%, and 10% respectively.

550. Apportioning Loss. — In case there are several insurance policies on the same property the loss is apportioned among them in proportion to the amount of the policies.

Thus, a loss of \$21,000, covered by three policies of \$16,000, \$24,000, and \$30,000, will be apportioned among them in the amounts \$4800, \$7200, \$9000. The loss is $\frac{3}{10}$ of the total insurance and each company pays $\frac{3}{10}$ of the amount of its policy.

551. Canceling Policies. — Usually insurance policies contain a provision whereby they may be canceled at the option either of the insurer or the insured.

If the insurer requests the cancellation of the policy, he returns a part of the premium which bears the same ratio to the whole premium as the unexpired part of the term bears to the whole term.

If the insured requests the cancellation the premium charged for the expired part of the term is according to short term rates.

552. Reinsurance. — A small company accepting a large single risk may feel it is too large to carry. In that case the company will reinsure all or part of the risk in a larger company. Reinsurance is usually effected at a lower rate than original insurance because it is less expensive to the company than original insurance.

In this manner the smaller local companies become in a measure field agents for the larger companies.

553. Replacing Lost Property. — Insurance companies usually reserve to themselves the option of replacing the lost property, reserving a reasonable amount of time in which to do this.

554. The Arithmetic of Fire Insurance. — There are three elements involved in problems on insurance, viz. the amount insured, the rate, and the premium. The amount is the base in percentage, the rate of insurance is the rate, and the premium is the percentage.

The rate is fixed by the company according to the probability of loss as determined by the collective experience of all insurance companies. When the amount of insurance and the rate are known, the finding of the premium is the simplest case in percentage.

Example 1. Household goods valued at \$5000 are insured at the rate of 50¢ per \$100. What is the premium for a three-year period?

Solution. The premium for one year is \$25.
Premium for 3 years $= 2\frac{1}{2} \times \$25 = \$62.50$.

Example 2. An office building valued at \$175,000 is insured under the average clause for \$140,000. A fire damages the building to the extent of \$12,000. What is the amount of insurance paid?

Solution. The ratio of the total insurance to the value of the building is

$$\frac{140000}{175000} = \frac{4}{5}.$$

Hence the insurance paid is $\frac{4}{5} \times \$12{,}000 = \$9600$.

Example 3. Find the cost of insuring a stock of merchandise for \$45,000 for 90 days, the annual rate being $1\frac{1}{8}\%$.

Solution. Premium for one year $= 1\frac{1}{8}\%$ of \$45,000 $= \$506.25$.
Premium for 90 days $= 40\%$ of annual premium (§ 545).
Hence the required premium is 40% of \$506.25 $= \$202.50$.

Example 4. A building valued at \$240,000 is insured under the average clause as follows: Company A, \$60,000; Company B, \$45,000; Company C, \$100,000. The building is damaged by fire to the extent of \$145,000. How much must each company pay?

Solution. The total insurance is \$205,000.
Ratio of insurances $\frac{205000}{240000} = \frac{41}{48}$.
Total insurance to be paid is $\frac{41}{48} \times \$145{,}000 = \$123{,}854.17$.
A's share $= \frac{12}{41} \times \$123{,}854.17 = \$36.250$
B's share $= \frac{9}{41} \times \$123{,}854.17 = \$27{,}187.50$
C's share $= \frac{20}{41} \times \$123{,}854.17 = \$60{,}116.67$
$\$123{,}854.17$

Example 5. A factory building valued at \$48,000 is insured for three years, the annual rate being 65¢ per \$100. For how much must the building be insured to cover both building and premium in case of total loss?

Solution. The rate per \$100 for 3 years is \$1.625 (see § 545).
Net insurance for each \$1.00 = 1 − .01625 = .98375.
Hence 48,000 ÷ .98375 = \$48,792.88.
The amount of the policy could therefore conveniently be made \$48,800.

PROBLEMS (A)

1. A man owned $\frac{2}{5}$ of a shop and insured $\frac{3}{4}$ of his interest at $1\frac{1}{4}\%$, paying \$112.50 premium. What was the value of his interest in the shop? If the shop was damaged to the extent of \$4000, what did he receive from the insurance company?

2. A mill worth \$30,000 was insured for an annual premium of $1\frac{2}{3}\%$ on 90% of its value. In the second year it was injured by fire to the amount of \$1175. How much did the mill owner save by insuring?

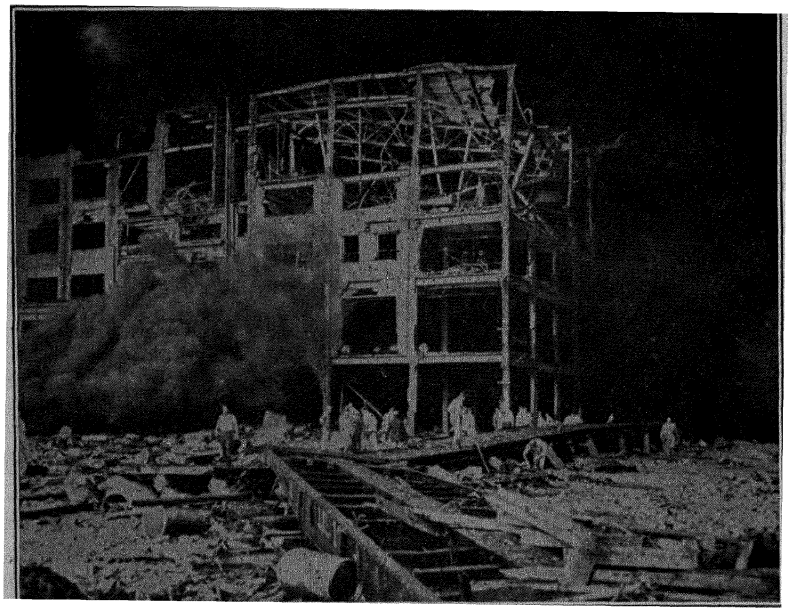
3. (B) The premium on a building, $\frac{5}{8}$ of which is insured at $1\frac{1}{3}\%$, is \$25.50. What is the value of the building?

4. Find the cost of insuring a grain elevator for \$12,000 for 3 months if the annual rate is $\frac{5}{8}\%$.

5. A house valued at \$4500 was insured for $\frac{2}{3}$ of its value for 1 year at 60¢ per \$100. At the end of 7 mo. the house was sold and the policy was canceled by the insured; how much should be returned?

6. Find the cost of insuring stored wheat valued at \$310,000 for 3 months at 80¢ per \$100 per annum.

7. Find the cost of insuring a factory for \$10,000 for 5 years if the annual premium is $\frac{3}{8}\%$.



WHY WE NEED FIRE INSURANCE

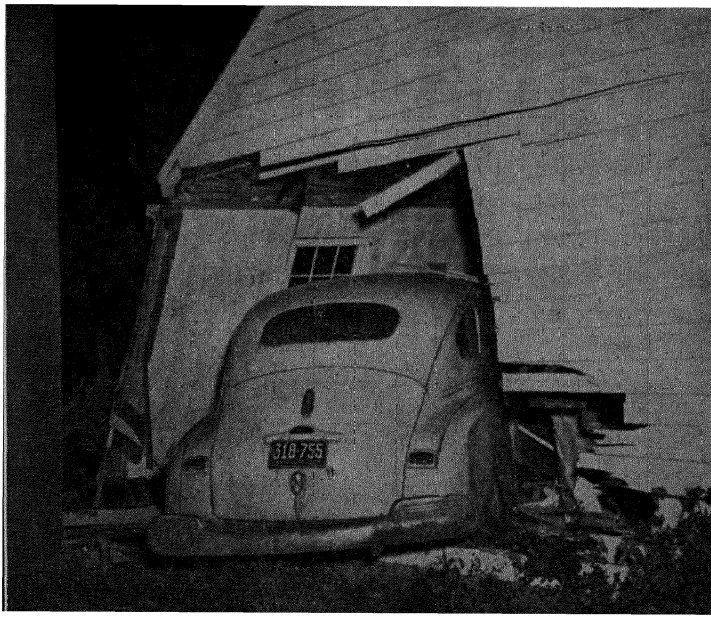
8. A manufacturing plant valued at \$20,000 is insured for \$12,000 at $\frac{1}{4}\%$ per annum. A fire causes a damage of \$8000. What amount would be paid by the insurance company (*a*) under an ordinary policy; (*b*) under an average clause policy?

9. For what sum must I get my shop insured at $\frac{7}{8}\%$ to cover a loss of \$7330 together with the premium?

10. A factory valued at \$9000 has been insured for \$5000 at $1\frac{1}{4}\%$, \$6 being charged for the policy and examination of the premises. If it should be destroyed by fire, what loss would the house suffer?

555. Automobile Insurance. — There are five things against which an automobile owner can insure himself: liability, fire, theft, property damage, and collision.

Liability insurance protects him against a limited amount of damages in case he injures someone.



AN ARGUMENT FOR AUTOMOBILE INSURANCE

Fire insurance protects him against damage to his automobile by fire.

Theft insurance protects him against loss by theft.

Property damage protects him in case his automobile damages any property such as another automobile, or a plate glass window, and so on.

Collision insurance protects him in case his automobile is damaged by another automobile. This is of especial benefit to one who has to leave his car standing in the street much of the time. For example, a doctor parks his car during his calls; he runs daily risk of damage to his car by careless drivers.

In order to be *fully* protected one would have to buy the five different kinds of automobile insurance. Probably the most important is liability insurance. If an owner injures or kills a person and is found to be at fault, he is liable for a very large amount of money. Perhaps fire and theft insurance would come next in importance.

In the case of property damage, the owner is liable for the amount of damage that he causes to other property. This is the kind of insurance most frequently collected. Collision insurance is probably the least necessary for the majority of people, particularly for those who drive carefully.

The rates for all types of automobile insurance vary in different localities depending upon the amount of traffic, the number of collectible cases per year, and other conditions. They depend also upon the horse power of the cars.

The following table shows the rates in three widely separated cities on a six-cylinder car selling for around $2200. The liability rate covers from $5000 to $10,000. $20,000 is the largest amount the company will pay in one accident on this particular policy. Under collision insurance the word *full* means that the insurance company pays all damage done to your car. $25 *deductible* (ded.) means that you assume all damages up to $25 yourself. In the same manner $50 *deductible* means that you assume all damage up to $50.

TABLE I

KINDS	RATES		
	City 1	City 2	City 3
Liability	$107	$38	$20
Fire	$.35 per $100	$.35 per $100	$.25 per $100
Theft	$2.30 per $100	$1.20 per $100	$.60 per $100
Property Damage	$30 per $1000	$16 per $1000	$11 per $1000
Collision			
Full	$297	$230	$60
$50 ded.	$107	$ 82	$25
$100 ded.	$ 47	$ 43	$12

Below is a table showing fire and theft insurance rates on cars of different prices when they are new and also after they have been run one or two years. These are limits that agents may work with, but we should understand that there are always exceptions, depending upon the condition of a used car.

TABLE II

COST OF CAR	$1900 TO $3000	$1300 TO $1900	LESS THAN $1300
New	Not over 90%	Not over 90%	Not over 90%
12 mo. old	Not over 70%	Not over 60%	Not over 60%
24 mo. old	Not over 50%	Not over 40%	Not over 40%

PRACTICE IN BUYING AUTOMOBILE INSURANCE (A)

1. In New York City the cost of fire insurance on a used automobile was $1.17 per $100 per year. What would it cost to insure an automobile against fire for $300?

2. In the same city the rate is 40¢ a $100 to insure a new car against fire. How much will it cost to insure a new automobile for $1500 against fire?

3. The rate of insurance against theft in that city for the car mentioned in example 2 is $1.30 per $100. How much will it cost to insure the machine for $1500 against theft?

4. The liability cost in that city to cover injuries or death to one person to the limit of $10,000 and injuries or death to more than one person to the limit of $20,000 is $128.40. How much is that per $1000 in case of death or injury to one person for the full limit?

5. How much would it cost to insure a car against fire for $1500, against theft for $1500, and against property damage for $2000, at the rates quoted in Table I for city 3?

6. Find the total cost of the insurance required in example 5 for city 2.

7. How much would it cost to insure a new $1200 car for all of the items mentioned in Table I, City 2? Use Table II to find the per cent of value for which it could be insured.

8. Suppose that you insured a new \$2000 car for each of the items in Table I, City 1, and also had to pay \$15 a month garage hire. How much a month would all of these items cost you?

DRILL IN FUNDAMENTALS (A)

Find the rates of income on the investments given below and check. Find each rate to the nearest hundredth of one per cent.

	Investment	Yearly Income		Investment	Yearly Income
1.	\$93.50	\$4.50	**16.**	3,900	260
2.	86.25	4.00	**17.**	4,900	340
3.	98.50	5.00	**18.**	8,670	520
4.	101.75	5.00	**19.**	10,800	480
5.	121.50	6.50	**20.**	27,300	1080
6.	43.50	3.00	**21.**	45,600	2160
7.	76.00	4.00	**22.**	50,000	3400
8.	$126.62\frac{1}{2}$	7.50	**23.**	74,990	4300
9.	$116.37\frac{1}{2}$	6.00	**24.**	34,980	1980
10.	$99.87\frac{1}{2}$	5.00	**25.**	18,146	937
11.	103.75	5.00	**26.**	85,000	4860
12.	141.25	7.00	**27.**	59,300	3190
13.	$201.12\frac{1}{2}$	10.00	**28.**	74,000	3980
14.	$93.37\frac{1}{2}$	4.50	**29.**	41,000	2108
15.	55.50	2.50	**30.**	14,500	980

CHAPTER XL

LIFE INSURANCE

556. Kinds of Personal Insurance. — There are several kinds of personal insurance, such as life insurance, health insurance, accident insurance, etc.

557. Life Insurance. — A life insurance policy is a contract whereby a company agrees, on consideration of a premium, to pay a certain sum at a certain time or on the death of the person insured.

558. Health Insurance. — A health insurance policy is a contract whereby a company agrees to pay a specified amount each week during any illness of the person insured.

559. Accident Insurance. — An accident insurance policy is a contract whereby a company agrees to pay a specified sum in case the insured meets with an accident which kills him or incapacitates him permanently or for a limited period.

An ordinary accident policy excepts accidents due to a variety of causes, such as extra hazardous occupations or carelessness.

560. Kinds of Life Insurance Policies. — There is a large variety of life insurance policies. The most important are straight life, limited payment life, term policies, and endowment policies.

561. Straight Life Policy. — A straight life policy is a contract to pay a specified sum at the time of death of the insured, the premium being payable throughout the life of the insured.

562. Limited Payment Life Policy. — A limited payment life policy is a contract to pay a specified sum on the death of the insured, the premium being payable for a specified number of years, say twenty, or during the life of the insured if his death occurs before the end of the specified period.

563. Endowment Policy. — An endowment policy is a contract to pay a certain amount to the insured at the end of a certain period or on his death if that occurs before the end of the specified period.

564. Term Policy. — A policy which is to run for a certain number of years and then cease to be in force is called a *term* policy. The payments continue during the time the policy is in force.

565. Beneficiary. — The policy may be payable to a person specified in the policy, called the *beneficiary*, or to the estate of the insured.

If the policy is payable to a beneficiary, it is not liable for the debt of the insured and he cannot divert it to his estate or to another person by will. If it is payable to the estate of the insured, it is liable for debt the same as any other property belonging to the estate.

566. Premiums. — The premiums are specified as so much per year per $1000 of insurance. They are based on mortality tables (expectation of life) and vary with the age of the insured, and the kind of policy. The younger a person is who takes out insurance the cheaper the rate he has to pay.

567. Reserve, Surplus. — An insurance company is compelled by law to set aside a certain per cent of the premiums as a reserve for the payment of death claims, endowment claims, etc. Any earnings of the company over and above the amount required are set aside as a surplus and may be used to pay dividend on stock in case the company is a stock company, or it may be apportioned among the policy holders as profits in the case of a mutual company.

568. Surrender and Loan Value. — For some reason a policy holder may decide not to continue paying his premium. In that case his policy has a certain specified *surrender value*. On turning the policy over to the company it will pay him the surrender value in cash. The insurance company will loan the insured an amount equal to the surrender value of the policy.

569. Continued Insurance, Paid-up Insurance. — If the holder of a policy fails to make a premium payment, it automatically converts into a paid-up policy for such an amount as the surrender value of the policy will buy. If he chooses, he may have instead an extension of the full amount of the policy for a certain specified time.

570. Days of Grace, Reinstatement. — Insurance companies usually allow 30 days of grace in the payment of premium. In some companies the insured may be reinstated on proof of insurable health and payment of the defaulted premiums.

571. Other Forms of Life Insurance. — There are many other forms of life insurance besides those mentioned above. Under life insurance there is educational insurance and annuities; under public liability there is insurance taken out by employers to cover accidents to employees; there is agricultural insurance (or crop) which insures the farmer, and machinery insurance which protects the owner of a plant against loss in case machinery breaks down, and marine insurance taken out by owners of ships and cargoes.

Group insurance has grown in importance in recent years. Employees in factories or offices take out health, accident, hospital or even life insurance as a group. Usually the premiums are lower in group insurance, and often the company employing the group pays part of the premium cost.

ANNUAL PREMIUMS IN AN OLD AMERICAN COMPANY

AGE	STRAIGHT LIFE	20-PAYM'T LIFE	20-YEAR ENDOWMENT	AGE	STRAIGHT LIFE	20 PAYM'T LIFE	20-YEAR ENDOWMENT
21	19.62	29.84	48.63	41	34.16	43.82	54.60
22	20.06	30.31	48.77	42	35.39	44.90	55.20
23	20.51	30.80	48.96	43	36.70	46.04	55.85
24	20.99	31.31	49.14	44	38.08	47 25	56.56
25	21.49	31.83	49.33	45	39.55	48.52	57.34
26	22.01	32.37	49.53	46	41.12	49.87	58.20
27	22.56	32 94	49.73	47	42.79	51 31	59.14
28	23.14	33.52	49.95	48	44.57	52.83	60.17
29	23.74	34.13	50.18	49	46.46	54.45	61.31
30	24.38	34.76	50.43	50	48.48	56.17	62.25
31	25.05	35 42	50.69	51	50.62	58.01	63.01
32	25.75	36.11	50.96	52	52.91	60.07	65.41
33	26.50	36.82	51.26	53	55.35	62.06	67.05
34	27.28	37.56	51.57	54	57.95	64.29	68.84
35	28.11	38.34	51 91	55	60.72	66.69	70 81
36	28 98	39 15	52.28	56	63.68	69.26	72.97
37	29.90	40.00	52.67	57	66.84	72.01	75.31
38	30.80	40.89	53.10	58	70.22	74.98	77.91
39	31.91	41.81	53.56	59	73.83	78.16	80.73
40	33.01	42.79	54.06	60	77.69	81.60	83.82

Social security provides two kinds of insurance: unemployment and old age. The social security insurance is obligatory on all workers in certain classes of employment, and the rates and benefits are fixed by the government.

ORAL EXERCISES (A)

1. What is the yearly premium on a \$5000 straight life policy taken out at the age of 25?

2. What is the yearly premium on a \$2000 20-payment life policy taken out at the age of 30?

3. What is the yearly premium on a \$10,000 20-year endowment policy taken out at 35?

PAID-UP VALUES AND LOAN VALUES

Ordinary Life, Age 30					20-Payment Life, Age 40				
Years in Force	Cash Surrender Values Loan Values	Paid-up Policy	Continued Insurance		Years in Force	Cash Surrender Values Loan Values	Paid-up Policy	Continued Insurance	
			Yr.	Mo.				Yr.	Mo.
	\$	\$				\$	\$		
3	26	65	3	2	3	61	126	6	1
4	36	86	4	4	4	90	182	8	8
5	45	108	5	6	5	120	239	10	11
6	57	135	6	11	6	149	290	12	8
7	72	167	8	8	7	181	346	14	5
8	88	199	10	4	8	214	401	15	10
9	104	230	11	11	9	248	456	17	0
10	120	261	13	3	10	283	510	18	1
11	134	286	14	3	11	316	559	18	11
12	148	311	15	0	12	350	607	19	7
13	163	335	15	9	13	385	656	20	4
14	178	359	16	3	14	422	701	21	0
15	193	383	16	9	15	459	753	21	8
16	209	407	17	1	16	498	801	22	6
17	225	430	17	4	17	538	850	23	5
18	242	453	17	6	18	579	899	34	8
19	258	475	17	8	19	622	940	26	7
20	276	497	17	8	20	666			
21	293	518	17	8	21	678			
22	311	539	17	8	22	689			
23	328	559	17	7	23	700			
24	346	579	17	5	24	712			
25	365	598	17	3	25	723			

ORAL EXERCISES (A)

1. An ordinary life policy for \$5000 is taken out at 30 years of age. What is the surrender value at the expiration of 15 years? For what paid-up policy may it be surrendered?

2. A 20-payment life policy for \$10,000 is taken out at the age of 40. What is its surrender value at the end of 12 years? What is its value in terms of paid-up policy?

3. A 20-year endowment policy for \$5000 is taken out at the age of 24. What is the yearly premium? How much will the premium amount to in 20 years?

PROBLEMS (B)

1. A \$5000 20-payment life policy is taken out at the age of 41. At 3% compound interest what will the premiums amount to in 10 years, in 15 years, in 20 years? (Use the table on page 314.)

2. In how many years will the premium on the policy in Example 1 amount to \$5000 (the face of the policy), interest being compounded yearly at 3%?

Also solve the same problem interest at 1% being compounded semiannually.

3. If in Example 1, the insured dies at the age of 57 years, how much will his premiums amount to, interest being compounded at $4\frac{1}{2}\%$?

Suggestion. First find the premiums at 41 as given by the table on page 403 and then find the amount of this annuity at $4\frac{1}{2}\%$ interest compounded annually for 16 years.

4. A man takes out a \$12,000 20-payment life at the age of 40. At the age of 48 he discontinues the policy and accepts the cash surrender value. How much does he get? If the premiums had been invested at 4% interest compounded yearly, how much would he have had?

5. A man takes out a \$10,000 20-payment life at the age of 42. If he dies at 49 how much more would his estate receive than if he had placed the premiums at 4% interest compounded annually?

6. A man takes out a 20-payment life policy for \$50,000 at 35 years of age. The dividends of the company reduce the premiums as follows: 4th year 4%, 5th year 5%, 6th year $5\frac{1}{2}$%, 7th year 6%, 8th year 7%, 9th year 7%, 10th year $7\frac{1}{2}$%, 11th year 8%, 12th year 9%, 13th year 9%, 14th year 9%, 15th year 9%, 16th year 9%, 17th year 10%, 18th year $10\frac{1}{2}$%, 19th year 11%, 20th year 12%. How much would the premiums amount to at the time of his death at 63 years of age if invested at $4\frac{1}{2}$% compounded yearly?

7. If the premiums actually paid on the policy in Example 6 had been invested at 4% compounded yearly, how much would they amount to at the time of the man's death?

DRILL IN FUNDAMENTALS (A)

Copy from sight, add, and check.

1.	2.	3.	4.	5.	6.
14,560	81,479	97,481	52,376	67,325	73,256
41,246	41,292	34,677	10,922	54,624	92,024
27,970	79,400	43,767	90,212	44,749	39,176
43,210	48,765	43,201	26,742	53,432	19,816
56,789	12,345	67,890	59,143	74,527	28,946
65,432	29,867	65,242	89,417	28,619	81,712
67,688	99,354	24,178	78,214	52,649	18,172
56,787	69,383	47,127	23,775	27,127	32,428
34,926	67,653	34,354	45,464	75,758	59,636
76,184	43,457	75,453	73,429	68,676	66,564
51,471	29,178	81,917	78,789	56,432	42,722
11,099	98,776	65,544	33,221	19,988	17,766
22,334	45,566	77,889	91,010	11,121	13,115
92,827	26,252	42,322	21,220	29,911	78,787
76,249	27,892	64,500	12,449	11,992	84,276
71,912	20,243	46,732	29,817	34,673	19,256
81,496	60,709	17,970	79,874	91,600	91,782
97,280	28,680	45,293	36,248	55,600	61,742
89,472	22,221	73,387	71,943	83,926	72,822

Copy from sight, divide, and check. Get quotients to the nearest integer.

1. $498\overline{)7,496,490}$
2. $819\overline{)17,948,200}$
3. $237\overline{)4,726,500}$
4. $456\overline{)819,247}$
5. $694\overline{)376,426}$
6. $745\overline{)6,924,980}$
7. $831\overline{)4,913,764}$
8. $1742\overline{)387,928}$
9. $1597\overline{)767,847}$
10. $987\overline{)374,592}$
11. $318\overline{)6,178,400}$
12. $392\overline{)17,748,792}$
13. $796\overline{)804,900}$
14. $2810\overline{)9,764,780}$
15. $3945\overline{)34,792,987}$
16. $844\overline{)8,792,704}$

CHAPTER XLI

DIRECT TAXATION

572. Public Expenditures. — Our various governments make great expenditures of money for many purposes. The city provides schools, police, and fire protection. It paves the streets, lights them, and builds sewer systems. The county and township governments build roads and bridges, etc. The state and the national government expend great sums for various public purposes. Nearly all the money needed for these expenditures is obtained through taxation.

573. Forms of Taxation. — Taxes may be in the form of direct taxes, which consist of property taxes, income taxes, license fees, and a personal tax called a poll tax, or they may be in the form of indirect taxes such as tariff on imports.

574. License Tax. — Many business concerns have to pay a yearly fee for permission to conduct their business. Saloons, where they are permitted, usually pay a large license fee. Hotels, drug stores, candy stores, circuses, auctioneers, street vendors, taxicabs, street cars, etc., frequently pay license fees.

575. Property Taxes. — By far the greater part of the funds of the local and state governments are derived from property taxes, and real estate, consisting of land and buildings, contributes most of these. Personal property, consisting of all kinds of property not real estate, is also taxed, but levying a tax on such property presents practical difficulties which greatly reduce the amount derived from it.

576. Assessors. — Men, called *assessors*, are appointed to make lists of the property of each person. The value put upon property by the assessors is called the *assessed valuation*.

577. Arithmetic of Taxation. — When it is decided that a certain sum is to be raised by means of a property tax, then the arithmetic

problem of taxation consists in apportioning this tax among the various persons in proportion to the assessed value of their property.

578. Rate of Taxation. — The rate of taxation is expressed as so many mills per dollar, cents per hundred dollars, or as a rate per cent of the assessed valuation.

Thus 12 mills, 120 cents, or $1\frac{1}{5}\%$, all express the same rate.

579. Problem. Given the Assessed Valuation and the Amount to Be Raised, to Find the Rate.

Solution. Find what per cent of the assessed valuation is equal to the amount to be raised.

580. Problem. Given the Assessed Valuation and the Rate, to Find the Tax.

Solution. Multiply the assessed valuation by the rate.

EXERCISES (A)

Find the rate of taxation in each of the following.

	Total Assessed Valuation	Total Tax		Total Assessed Valuation	Total Tax
1.	$12,860,000	$150,000	**4.**	$750,500	$5,628.75
2.	1,946,500	15,000	**5.**	1,924,000	24,050
3.	2,500,000	12,500	**6.**	2,750,600	8,526.86

Find the tax for each of the following.

	Assessed Valuation	Rate		Assessed Valuation	Rate
7.	$15,400	1.4%	**10.**	$79,000	1.35%
8.	28,600	.9%	**11.**	10,000	$2\frac{1}{4}\%$
9.	4980	1.6%	**12.**	5760	$1\frac{1}{2}\%$

581. Spreading the Tax. — After the assessor has made his list of assessed valuations the work of determining how much tax each person shall pay is called *spreading the tax*. This work consists in solving the two problems just given: (1) to find the rate, and (2) to find each person's tax, using this rate.

582. Tax Table. — The actual work of spreading a tax is greatly facilitated by what is called a *tax table*. This consists of a table giv-

ing the tax at the required rate on every dollar from \$10 to \$100. The tax on any valuation may then be obtained by properly placing the decimal point and adding.

TAX TABLE. RATE 13.5 MILLS

	0	1	2	3	4	5	6	7	8	9
1	1350	1485	1620	1755	1890	2025	2160	2295	2430	2565
2	2700	2835	2970	3105	3240	3375	3510	3645	3780	3915
3	4050	4185	4320	4455	4590	4725	4860	4995	5130	5265
4	5400	5535	5670	5805	5940	6075	6210	6345	6480	6615
5	6750	6885	7020	7155	7290	7425	7560	7695	7830	7965
6	8100	8235	8370	8505	8640	8775	8910	9045	9180	9215
7	9450	9585	9720	9855	9990	10125	10260	10395	10530	10665
8	10800	10935	11070	11205	11340	11475	11610	11745	11880	12015
9	12150	12285	12420	12555	12690	12825	12960	13095	13230	13365

The numbers in the first line of the table give the tax on amounts from \$10 to \$19, in the second line on amounts from \$20 to \$29, etc. Thus to find the tax on \$67, we go down the left column to 6, and then along this line until we get to the column under 7, finding \$.9045 as the required tax.

To find the tax on \$37,000 find the tax on \$37 and multiply by 1000 (move the decimal point three places to the right). To find the tax on \$860, find the tax on \$86 and multiply by 10.

The method of using the table is shown in the following.

Example. By means of the above tax table find the tax on an assessed valuation of \$37,860 at 13.5 mills.

Solution.

Tax on \$37,000 = \$499.50
Tax on 860 = 11.61
\$511.11

WRITTEN EXERCISES (A)

By means of the above tax table find the tax at 13.5 mills on each of the following assessed valuations.

1. \$14,640
2. 31,900
3. 64,850
4. \$8745
5. 5470
6. 6740
7. \$19,820
8. 91,280
9. 12,980
10. \$76,450
11. 16,500
12. 50,800

Example. Make a tax table if the rate is 11.7 mills.

Suggestions. The tax on \$10 is \$.117 and on \$1 is .0117. To find the tax on \$11 add .0117 to .117, obtaining .1287. To find the tax on \$12 add .0117 to .1287, obtaining .1404, etc. When the tax on amounts from \$10 to \$20 has been obtained in this way, check by multiplying .0117 by 20. This checks all amounts between \$10 and \$20. Similarly check the tax on \$30, \$40, etc. In this way we are certain we are right as we go along.

WRITTEN EXERCISES (A)

From the tax table made in the preceding example find the tax at 11.7 mills on each of the following assessed valuations.

1. \$64,410	**4.** \$5470	**7.** \$28,900
2. 19,300	**5.** 8540	**8.** 12,850
3. 54,860	**6.** 47,600	**9.** 21,430

PROBLEMS (B)

1. What is the value of my property, if I pay \$648.125 taxes at $15\frac{1}{4}$ mills on a dollar, the assessment being $\frac{5}{7}$ of the real value?

2. The tax rate of a city is $1\frac{3}{4}\%$ upon the assessed value of the property. If this value is 75% of the actual value, how much tax does a man pay upon property the actual value of which is \$24,000?

3. A city made the following appropriation for its public schools: supervision, \$70,000; instruction, \$2,756,344.15; debt service, \$335,855; contingent fund, \$1,475,108. The assessed valuation is \$898,204,300. What is the tax rate for school purposes?

4. In a town 2550 persons were subject to poll tax: the assessed valuation of the real estate was \$3,450,200 and of personal property \$350,000. The poll tax was \$1.50 each. The tax to be collected was \$55,000. What was the tax rate? What is a man's tax who has real estate to the value of \$5000, and personal property to the value of \$2500, and who pays for 2 polls?

5. A certain city spent \$64,519.08 for supervision, \$2,548,833.52 for salaries, \$1,878,471.51 for other school purposes. If the total valuation is \$1,750,421,250, what is the rate of school tax?

6. The tax rate on a certain property is 1.71% of its cash value; in how many years will the tax equal the cash value?

CHAPTER XLII

INDIVIDUAL INCOME TAX

583. Income Tax. On January 1, 1914, a law went into effect levying a tax on all persons in the United States having an income above a certain amount in any year. While the law has been amended many times, the changes have been mostly in the amount of tax imposed.

584. Income. According to the law, income consists of:

(1) Total wages, salaries, bonuses, commissions, and other compensation;

(2) Dividends;

(3) Interest;

(4) Profit from business (net income less business expenditures);

(5) Certain portions of rents, royalties, annuities, and profits from the sale of capital assets.

Since certain adjustments are permitted (particularly in (5) above), the income thus reported is called the Adjusted Gross Income.

585. Lawful Deductions, Net Income. Each taxpayer is permitted to deduct certain expenses from his income before computing the tax. Among these are taxes, interest, and losses not covered by insurance. Deductions for contributions up to a certain amount, and for unusually large medical expenses are also allowed. These may be itemized, and the total deducted from the adjusted gross income; or, with certain limitations, a standard deduction of ten per cent may be taken.

The result which is obtained by subtracting the total deductions from the adjusted gross income is called the *Net Income.*

586. Exemptions. An exemption of $600 is allowed for the taxpayer making a return, and of $1200 in the case of a joint return made by a husband and wife. A further exemption of $600 is also allowed for each dependent claimed by the taxpayer. In case the taxpayer is blind, the exemption for him is doubled. If he is 65 years old, or

older, the exemption is likewise doubled. If he is both blind and more than 65, the exemption is tripled. A similar arrangement is made in case of the wife of the taxpayer, but not for other dependents.

When the total of the exemptions is subtracted from the net income the remainder is the *Taxable Income*. This is the income referred to in the table below.

587. Normal Taxes and Surtaxes. A portion of the income tax paid is classed as normal tax, the remainder as surtax. In the computation of taxes it is not necessary to distinguish between these two classes except in the case of taxpayers who have partially tax-free interest on certain U. S. Bonds

TENTATIVE TAX RATES FOR 1949

If your taxed net income is: Not over $2,000	Your tentative tax is: 20% of the taxed net income (3% plus 17%)
Over $2,000 but not over $4,000	$400 plus 22% of excess over $2,000
" 4,000 " 6,000	840 " 26% " 4,000
" 6,000 " 8,000	1,360 " 30% " 6,000
" 8,000 " 10,000	1,960 " 34% " 8,000
" 10,000 " 12,000	2,640 " 38% " 10,000
" 12,000 " 14,000	3,400 " 43% " 12,000
" 14,000 " 16,000	4,260 " 47% " 14,000
" 16,000 " 18,000	5,200 " 50% " 16,000
" 18,000 " 20,000	6,200 " 53% " 18,000
" 20,000 " 22,000	7,260 " 56% " 20,000
" 22,000 " 26,000	8,380 " 59% " 22,000
" 26,000 " 32,000	10,710 " 62% " 26,000
" 32,000 " 38,000	14,460 " 65% " 32,000
" 38,000 " 44,000	18,360 " 69% " 38,000
" 44,000 " 50,000	22,500 " 72% " 44,000
" 50,000 " 60,000	26,820 " 75% " 50,000
" 60,000 " 70,000	34,320 " 78% " 60,000
" 70,000 " 80,000	42,120 " 81% " 70,000
" 80,000 " 90,000	50,220 " 81% " 80,000
" 90,000 " 100,000	58,620 " 87% " 90,000
" 100,000 " 150,000	67,320 " 89% " 100,000
" 150,000 " 200,000	111,820 " 90% " 150,000
" 200,000	156,820 " 91% " 200,000

NOTE. — Total tax for a single person or married couple may not be more than 77% of net income.

588. The Joint Return. A husband and wife are permitted to make a single return combining the income of both. In such a return, the total taxable income is divided equally. The tax for each is then computed as shown in the following paragraphs.

In practically all cases the joint return saves money for the taxpayer. It is permitted even though the wife actually has no income of her own. In the examples following, use the Joint Return method in every case in which the report is for a married couple.

589. Government Method of Computing the Tax. The method of computing the tax suggested in the instructions sent out by the United States government gives the combined normal tax and surtax — that is, the total income tax — by use of tables. From the table (§587), the so-called "tentative tax" is found as based on the taxable income. After the tentative tax has been found, the actual tax is found by *reducing* the tentative tax according to the following.

If the Tentative Tax is	The Reduction is
(*a*) Not over \$400	17% of the tentative tax
(*b*) Over \$400 but not over \$100,000	\$68 plus 12% of the excess of the tentative tax over \$400
(*c*) Over \$100,000	\$12,020 plus 9.75% of the excess of the tentative tax over \$100,000

590. Computing the Tax. The actual process will now be shown by means of examples.

Example 1. Find the income tax when the taxable income is \$114,000.

From the table we find that the tentative tax for an income between \$100,000 and \$150,000 is \$67,320 plus 89% of the excess over \$100,000.

We thus have:

Tentative tax on \$100,000	\$67,320.00
89% of \$14,000	12,460.00
Tentative tax	\$79,780.00

Since the tentative tax is more than \$400 and less than \$100,000, it is reduced by \$68 plus 12% of \$79,380, its excess over \$400.

12% of \$79,380.00 = \$9,525.60
\$68.00 + \$9,525.60 = \$9,593.60

Hence the tax is \$79,780.00 − \$9,593.60 = \$69,186.40

Example 2. Find the income tax for a single person with no dependents, if he has a net income of \$87,000.00.

From this net income, an exemption of \$600.00 is subtracted, leaving a taxable income of \$86,400.00. Following the procedure in Example 1:

Tentative tax on \$80,000	\$50,220.00
84% of \$6,400.00	5,376.00
Tentative tax	\$55,596.00

This exceeds \$400 by \$55,196.00.

12% of \$55,196.00 = \$6,623.52
\$68.00 + \$6,623.52 = \$6,691.52

Hence the income tax is:

\$55,596.00 − \$6,691.52 = \$48,904.48

Example 3. A married man with three dependent children has an adjusted gross income of \$37,650.00. Taxes, interest, and other deductible items amount to \$4,762.00. Find his income tax if he does not make a joint return.

There are five exemptions; hence \$3,000.00 will be subtracted from the net income.

Gross income	\$37,650.00
Deductions	4,762.00
Net income	\$32,888.00
Exemptions	3,000.00
Taxable income	\$29,888.00

Tentative tax on \$26,000.00	\$10,740.00
62% of \$3,888.00	2,410.56
Total tentative tax	\$13,150.56

This exceeds \$400 by \$12,750.56.

12% of \$12,750.56 = \$1,530.07
\$68.00 + \$1530.07 = \$1,598.07

Hence the income tax is:

\$13,150.50 − \$1,598.07 = \$11,552.43

Example 4. Compute the income tax for the family in Example 3 if a joint return is made.

Deductions and exemptions remain the same as above, giving as before a taxable income of $29,888.00

Divide by 2:	$14,944.00
Tentative tax on $14,000	$4,260.00
47% of $944.00	443.68
Total tentative tax	$4,703.68

This exceeds $400 by $4,303.68.

$$12\% \text{ of } \$4{,}303.68 = \$516.44$$
$$\$68.00 + \$516.44 = \$584.44$$
$$\text{Income tax, each, } = \$4{,}703.68 - \$584.44 = \$4{,}119.24$$
$$\text{Total income tax (multiply by 2)} = \$8{,}238.48$$

For the amount of saving from making the joint return, compare with the result above in Example 3.

591. Steps in Computation. In the computation, the following steps should be kept in mind.

1. From the Adjusted Gross Income subtract the total of the Deductions to obtain the Net Income.

2. From the Net Income subtract the Exemptions ($600 each) to obtain the Taxable Income.

3. Compute the Tentative Tax by means of the table in § 587.

4. From the Tentative Tax subtract the Reduction (based on the schedule in § 589) to obtain the Income Tax.

PROBLEMS (A)

1. Find the tax of a single person with no dependents if he has a net income of $3500.

2. Find the tax of a single person with no dependents if he has a net income of $6500.

3. Find the tax of a single person with no dependents if he has a net income of $12,500.

4. Find the tax of a single person with no dependents if his net income is $86,500.

5. Find the average rate of tax on the total net income in each of the four preceding problems. Compare these rates.

6. Find the tax of a married person with two dependent children if his net income is $7800.

7. Find the tax of a married person with five dependent children if his net income is $18,500.

8. Find the tax of a married person with four dependent children if his net income is $39,400.

9. Find the tax of a married person with one dependent child if his net income is $145,000.

10. Find the average rate of tax on the total net income in each of the four preceding problems. Compare these rates.

11. At about what yearly taxed income is tax one half of the taxed income?

CHAPTER XLIII

UNITED STATES CUSTOMS

592. Indirect Taxes. — Indirect taxes consist of internal revenue taxes and import duties.

593. Internal Revenue Tax. — For every gallon of alcohol distilled and sold in this country a tax (excise tax) is paid to the United States government. This is called an internal revenue tax. A similar tax is paid on fermented liquor, cigars, narcotics, amusements, and some so-called luxuries.

594. Import Duty. — Many different kinds of goods bought in foreign countries and brought into the United States are subject to a tax as they enter this country.

595. Kinds of Import Duty. — The tax so paid is called United States customs or import duty. Import duties are specific and ad valorem.

596. Specific Duty. — A specific duty is a fixed amount to be paid on a certain kind of article regardless of the price at which it was bought.

Thus, men's kid gloves of certain grade pay a duty of $6.00 per dozen pairs.

597. Ad Valorem Duties. — An ad valorem duty is fixed as so many per cent of the price actually paid for the goods in the country from which the goods are imported.

Ad valorem duties are not computed on fractions of a dollar. If the cents are less than fifty they are neglected, and if fifty or more they are considered an additional dollar.

Thus, imported ready-made cotton clothing is subject to a tax of 50% of the price at which they were bought in the foreign country.

Specific duties are usually levied on goods of fairly uniform value. Ad valorem duties are levied on goods whose value varies greatly. Thus, rugs or carpets vary greatly in price per square yard; and a specific duty might be exceedingly

high on the cheaper grades, while it would be insignificant on the very expensive grades.

598. Value of Goods, How Determined. — When goods are imported, an invoice is prepared by the seller in the foreign country specifying the kind of goods, the amount of them, and the exact price paid for them in the foreign money. In case of invoices amounting to more than $100 a United States consul or other agent must certify that the invoice is genuine. A copy of the certified invoice is forwarded by the consul to the collector of the United States port to which the goods are shipped. Such an invoice is taken as *prima facie* evidence of the value of the goods.

The value of foreign money in United States coin is determined by a statement issued every three months by the Secretary of the Treasury.

599. Tare, Leakage, Breakage. — Before specific duties are collected certain allowances are made. Tare is an allowance made for the weight of packing cases, etc. Leakage is an allowance made for the wastage of liquid goods. Breakage is allowed on certain goods which are liable to damage in shipment.

After allowances are made for tare, leakage, and breakage, specific duties are charged on the amount remaining.

600. Ports of Entry. — The United States is divided into collection districts in which one or more cities are designated as ports of entry. In each port of entry there is a United States custom-house at which goods are examined and duties paid.

601. The Long Ton. — In determining weights the long ton (2240 lb.) is used in the United States custom-house. The centiweight is 112 lb.

602. The Tariff. — The list giving the amount of duty on each article is called the tariff.

603. Port of Delivery. — A place where imported goods may be released into the full control of the owner is called a port of delivery. Every port of entry is also a port of delivery, while many places which are not ports of entry are ports of delivery.

A vessel pays a certain tax proportional to her tonnage for the mere privilege of entering a port of entry. This is called *tonnage.*

604. Bonded Warehouse. — In each port of entry there is a public store in which goods are received and examined. There is also a *bonded warehouse* in which dutiable goods may be stored, and from which they may be withdrawn on payment of the duty, storage, and cartage.

Goods imported may be brought in on what is called *warehouse entry*, in which case they must be stored in the government warehouse; or on *consumption entry*, in which case the duty must be paid at once and the goods removed.

Goods may be withdrawn from a bonded warehouse for the purpose of reëxportation, in which case no duty is charged. There are also arrangements for removing the goods to a bonded factory, where they are manufactured into other goods for export. In this case no duty is charged. In the case of reëxportation of goods on which duty has been paid this is refunded less one per cent.

605. Customs Officers. — In the larger ports of entry there are many general officers having charge of the various parts of the very large amount of work involved in the proper collection of duties on the goods imported. There are the *Collector of Customs*, the *Naval Officer*, the *Surveyor*, the *Appraiser*, etc.

606. Smuggling. — Bringing dutiable goods into the United States without paying full duty is called smuggling. This is a criminal act and is punishable by fines or imprisonment. Goods are smuggled by not entering them at the custom-house at all, in which case no duty is paid, or by making a false entry, in which case the government is defrauded of part of the tax.

607. Foreign and Domestic Systems of Measure. — Invoices are usually made out in the weights and measures of the country where the goods are bought. In case specific duties are imposed, the foreign weights or measures must be converted into United States weights and measures before the duties are computed. The metric system is in use in practically all European countries except England. See page 155.

A fraction of a unit of measure is neglected if it is less than $\frac{1}{2}$ and is regarded as a whole unit if equal to or greater than $\frac{1}{2}$.

608. The Tariff of 1930 (Revised). — The table given below gives the duties on some of the more important articles of import as fixed by the tariff of 1930 revised.

Automobiles	10%	Ready-made clothing (cotton)	50%
Carpets (velvet)	30%	Ready-made clothing (woolen), $37\frac{1}{2}$¢ lb. plus	25%
Cotton thread, $\frac{1}{4}$ cent per 100 yards	20%	Sewing silks	55%
Cotton stockings	50%	Shirts, collars, cuffs	25%
Jewelry	55%	Silk handkerchiefs 3¢ each plus	30%
Men's gloves (not under 25¢)	$6 per dozen	Silk ribbons	25%
Inlaid linoleum	25%	Tablecloths (cotton) not under $17\frac{1}{2}$ not over	30%
Olive oil	8 cents per lb.	Tin plate	1¢ per lb.
Oriental rugs (15¢ sq. ft.)	$22\frac{1}{2}$%	Trimmed hats, 20¢ each plus	15%
Painted china (50¢ a doz.)	70%	Views of landscapes	10%
Printing paper, $\frac{1}{5}$¢ lb. plus	5%	Wire (copper)	15%

Example 1. Find the import duty on 50 dozen pairs of men's gloves, the duty being a specific duty of $6.00 per dozen pairs.

Solution. $50 \times \$6.00 = \$300.00$, which is the required duty.

Example 2. Find the duty on an invoice of carpets (velvet) valued at 18,650 francs, the duty being 40% ad valorem.

40% of the invoice price is 7460.00 francs. To reduce to United States money multiply by .039, which is the value of a franc in dollars. This gives $290.94 as the duty.

Solution.
18650
.40
7460.00 francs

WRITTEN EXERCISES (C)

Find the import duty on each of the following, and tell which are specific, and which are ad valorem duties.

1. Imported 120 dozen pairs of men's gloves from Montreal, Canada.

2. Imported 1480 pounds of olive oil from Italy.

3. An invoice from Hamburg containing:

	Value
Shirts and collars	12,350 marks.
Cotton stockings	7,430 marks.

4. An invoice from Liverpool containing:

	Value
Velvet carpets	2,760 pounds.
Cotton clothing	10,890 pounds.

5. An invoice from Sheffield, England, containing:

	Value
Wire (copper)	4200 pounds.
Views of landscapes	1750 pounds.

6. An invoice from London containing:

	Value
Cotton thread	980 pounds.
Painted china	1060 pounds.

7. 8490 pounds of woolen clothing at 25 cents per pound.

8. Inlaid linoleum, value $45,820

9. Oriental rugs, value 27,400 francs.

10. 100 trimmed hats at 105 francs each.

11. Automobiles, value 75,350 francs.

12. Imported 2860 meters of carpets from Paris, invoiced at 13 francs per meter.

13. I bought an invoice of Oriental rugs, paying £386 16*s.* for them in London. What was the total cost of the rugs in United States money, including the duty?

14. An invoice of jewelry is bought in Holland, amounting to 17,150 florins. Find the cost of this invoice including the import duty.

CHAPTER XLIV

PARTNERSHIP

609. A partnership is an association of persons for the purpose of carrying on a business and sharing its profits and losses.

The persons forming the partnership are called *partners* and the business concern is called a *partnership, company, house,* etc. It is designated by giving the names of some or all of the partners, such as Harper & Baird, Lucy & Sons, etc.

610. Kinds of Partners. — There are four principal kinds of partners: (1) real or ostensible; (2) silent, dormant or secret; (3) limited; (4) nominal.

611. Real Partners. — A real partner is one who is known to be such and who is fully liable for all obligations contracted by the partnership.

612. Silent Partner. — A silent partner is in all respects a real partner except that his connection with the business is kept secret. If the fact that he is a partner is discovered, he is liable for all obligations of the partnership the same as a real partner.

613. Limited Partners. — In the articles of partnership it is sometimes specified that the liability of one or more partners shall be limited to a certain amount or in certain definite ways. Such partners are called *limited partners.*

614. Nominal Partner. — A person having no interest in the profits of a partnership may permit his name to be used by the partnership. Thus, a partner may sell his interest, one of the conditions of the sale being that his name shall continue in the firm name. This may be of considerable importance to the firm because of the good will connected with the name. Such partners are called nominal partners.

615. Contributions of the Partners. — Each partner contributes to the partnership something which is regarded as being commercially valuable. This may be cash, land and buildings, merchandise, personal services, good will, etc.

616. Sharing of Profits and Losses. — The articles of agreement usually specify the ratio in which profits and losses are to be shared by the partners. However, each real partner is liable to the full extent of his property for the debts of the firm. Limited and even nominal partners are liable to third persons who have no notice of the special kind of partnership.

617. Resources. — The resources of a company consist of all its property of whatever kind, such as cash, real estate, merchandise, etc.

618. Liabilities. The liabilities of a company consist of all debts of whatever kind, such as notes payable, accounts payable, etc.

619. Present Worth. — The difference between resources and liabilities is called present worth.

620. Apportioning Profits Equally or in a Fixed Ratio.

Example 1. A firm consisting of three partners makes \$8940 in profits. What is the share of each partner?

Solution. In the absence of any special agreement the profits are divided equally. Hence each partner's share is \$8940 ÷ 3 = \$2980.

Example 2. Two partners, A and B, agree to share all gains and losses in the ratio 3:2. During one year the business shows a loss of \$1860. How should the loss be shared?

Solution. A's share is $\frac{3}{5} \times \$1860 = \$1116$.
B's share is $\frac{2}{5} \times 1860 = 744$.

Example 3. In a partnership consisting of four persons, A contributes \$14,000; B, \$12,000; C, \$20,000; and D, \$25,000 in capital. At the end of the first year the net gain is \$16,500. Find the present worth of each partner.

Solution. The gain of each partner is \$16,500 ÷ 4 = \$4125.
Hence their present worth is

A's 14,000 + 4125 = 18,125 C's 20,000 + 4125 = 24,125
B's 12,000 + 4125 = 16,125 D's 25,000 + 4125 = 29,125

621. Profits Divided in Proportion to Investment.

Example 1. A contributes \$20,000; B, \$30,000; and C, \$35,000. Apportion among the partners in proportion to their investment a gain of \$11,800.

Solution. A's investment $= \frac{20000}{85000} = \frac{4}{17}$ of total investment.
B's investment $= \frac{30000}{85000} = \frac{6}{17}$ of total investment.
C's investment $= \frac{35000}{85000} = \frac{7}{17}$ of total investment.

A's share of profits $= \frac{4}{17} \times 11{,}800 = 2776.47$
B's share of profits $= \frac{6}{17} \times 11{,}800 = 4164.71$
C's share of profits $= \frac{7}{17} \times 11{,}800 = 4858.82$
Check: Total 11,800.00

Example 2. A contributes services equivalent to the income from an investment of \$40,000. B contributes \$35,000 for 5 months and then adds \$20,000 more. C contributes \$75,000 for 7 months and then withdraws \$40,000. Apportion a year's net profits of \$27,500 according to investment.

Solution.

A's investment $= 12 \times 40{,}000 = 480{,}000$ for one month
B's investment $= 5 \times 35{,}000 + 7 \times 55{,}000 = 560{,}000$ for one month
C's investment $= 7 \times 75{,}000 + 5 \times 35{,}000 = 700{,}000$ for one month
Total investment 1,740,000 for one month

A's investment $= \frac{84}{87}$ of total investment
B's investment $= \frac{28}{87}$ of total investment
C's investment $= \frac{35}{87}$ of total investment

A's profits $= \frac{24}{87} \times 27{,}500 = 7{,}586.21$
B's profits $= \frac{28}{87} \times 27{,}500 = 8{,}850.57$
C's profits $= \frac{35}{87} \times 27{,}500 = 11{,}063.22$
Check: Total 27,500.00

PROBLEMS (B AND C)

1. (B) A contributes \$8000; B, \$10,000; C, \$12,000. Later A withdraws \$1500; B, \$2000; C, \$1800. At the end of the year the net earnings are \$10,500. Find the present worth of each partner, the profits being divided equally.

2. (B) A contributes good will, services, and \$5000 cash. B contributes services and \$10,000. C contributes \$50,000 cash. Each partner is to receive 6% on the money invested. Of the remainder A receives 60%, B, 30%, and C 10%. If the net profits for the first year are \$12,000, find the present worth of each partner.

3. (C) A contributes \$25,000; B, \$30,000; C, \$35,000. Later A withdraws \$2500; B, \$2000; C, \$4000. At the end of the year the business shows the following conditions: Building and grounds, \$50,000; cash, \$6480; mdse., \$34,800; bills receivable, \$4200; accounts receivable, \$18,450; accounts payable, \$4360. What is the net gain, deducting 10% from the accounts receivable for bad debts? Dividing the gain equally, what is the present worth of the partners?

4. (B) A contributes his services; B contributes \$10,000 and good will; C contributes \$20,000. A is to receive 50% of the net profit and B and C share the remainder equally. The first year shows net profits of \$8000. Find the present worth of each partner.

5. (C) In a partnership four partners made installments and withdrawals of capital as follows: January 1, A put in \$15,000, May 1, added \$10,000, and withdrew \$8000 Oct. 1. B put in \$20,000 Jan. 1, withdrew \$5000 July 1, added \$2000 Nov. 1. C put in \$15,000 Jan. 1, withdrew \$5000 April 1, added \$20,000 Sept. 1. D put in \$25,000 Jan. 1, withdrew \$15,000 May 1, added \$5000 Aug. 1, added \$2000 Nov. 1. The net gain for the year was \$16,500. Find the present worth of each partner, the profits being apportioned according to monthly investment.

6. (B) A and B engaged in trade. A put in \$4500 at first, and 8 months afterward \$300. B put in at first \$2000 and at the end of 6 months took out \$300. At the end of 16 months their gain was \$2750. What was the share of each?

7. (C) X, Y, and Z form a company with investments of \$12,000, \$15,000, and \$18,000 respectively. X draws out \$2000 after 4 months, \$3000 in 6 months; Y puts in \$3000 after 6 months and draws out \$4000 at the end of 10 months; Z puts in \$2000 at the end of 6 months and draws out \$4000 at the end of 10 months. The gain at the end of 18 months was \$7800. Find each one's share of the gain.

8. (B) In a company Brown put in \$15,000 of the capital for six months; Jones put in \$25,000 for 9 months and Smith put in \$18,500 for one year. How much was each one's share of the gain if the total gains were \$45,000?

622. Partnership Statement (C). — A partnership statement is best understood from an example.

The firm of Davis, Iverson, and Co. consisted of three partners, Davis, Iverson, and Forbes, Forbes being a silent partner. The partners made investments and withdrawals as follows: January 1, Davis invested $20,000, April 1 invested $5000, July 1 withdrew $4000, November 1 withdrew $3000; January 1 Iverson invested $15,000, July 1 invested $5000, October 1 invested $10,000, February 1 withdrew $2500, August 1 withdrew $5000, and December 1 withdrew $1000; January 1, Forbes invested $10,000, June 1 invested $9000, August 1 invested $5000, March 1 withdrew $1000, May 1 withdrew $15,000, November 1 withdrew $1000.

From data below find net gain, and apportion it according to total monthly investment. Then copy and complete the partnership statement given below.

DAVIS, IVERSON AND CO.

Resources		
Cash	12,865 50	
Real Estate	35,000	
Merchandise	28,600	
Accounts receivable (less 10% deduction)	13,001 13	
Bills receivable (less 5% deduction)	8360	
Liabilities		
Accounts payable		8600
Bills payable		10,000
Taxes accrued but not due		480
Interest accrued but not due		245
Present worth of Davis, Iverson and Co.		

Davis' net investment
Davis' net gain
Davis' present worth
Iverson's net investment
Iverson's net gain
Iverson's present worth
Forbes' net investment
Forbes' net gain
Forbes' present worth
Davis, Iverson and Co.'s present worth

PROBLEMS (C)

For each of the following make a complete partnership statement.

1. Harper and Baird enter a partnership agreeing to bear all losses and share all profits in proportion to their investments. Harper invested, January 1, $8000; May 1, $1000; September 1, $3000; withdrew $1000, July 1; $1500 on November 1. Baird invested $12,000, January 1, and $4000, October 1, withdrew $1500, May 1, and $1000, August 1. At the end of the year their books showed:

Resources: cash, $2462.45; real estate, $8600; mdse., $10,000; equipment, $3500; bills receivable, $4200; accounts receivable, $15,840. Deduct 5% from bills receivable and 8% from accounts receivable.

Liabilities: bills payable, $1800, accounts payable, $8960.

2. A and B enter into a copartnership July 1, 1936. On this date A invests $40,000, and September 1, $5000 more. January 1 he withdraws $8000. B invests $45,000 on July 1, and withdraws $3000 October 1 and $2000 Feb. 1. Interest at 6% is allowed on investments and charged on withdrawals. A and B are to share equally in gains and losses. June 30, 1937, their ledger shows:

Resources: cash, $35,000; mdse., $20,000; bills receivable, $18,000.

Liabilities: bills payable, $12,000; accounts payable, $16,000; miscellaneous unpaid bills, $570. Determine the present worth of each partner June 30, 1937.

3. Martin & Baker enter into a partnership January 1, for 1 year on the basis of $\frac{5}{12}$ and $\frac{7}{12}$ of the losses and profits; Martin invests Jan. 1, $6000, February 1, $5000 more, June 1, $5000 more; Sept. 1, he withdraws $3000; Baker invests Jan. 1, $12,000, April 1, withdraws $3000; July 1, $2000; Sept. 1 he adds $4000. December 21 their books showed:

Resources: cash, $10,000; real estate, $12,000; fixtures, $1500; bills receivable, $8000; accounts receivable, $5000; mdse., $5000.

Liabilities: bills payable, $3500; accounts payable, $8000; bad debts, $1200. Determine the condition of the partners at the time of closing, showing the present worth of each. Allow 6% interest.

CHAPTER XLV

STORAGE

623. Storage is a term applied to the charges made for storing goods in warehouses.

624. Rates of Storage. — The rates charged in warehouses are in some cases fixed by law or by city ordinances. More frequently they are fixed by commercial bodies such as boards of trade and chambers of commerce. In the absence of any such regulations the charges are determined by the regular rates of the warehouse or by special agreement.

625. Term of Storage. — A certain length of time is fixed as the minimum time for which storage is charged. This is called the *term of storage*. Charges are made only for complete terms of storage. The charge for any fraction of a term is the same as the charge for a full term. In United States bonded warehouses the term is 30 days. (See § 604.)

Thus, if the term is 30 days, the charge for 47 days is the same as for 2 terms and the charge for 62 days is the same as for 3 terms.

626. Simple Storage. — In simple storage the charge on each amount of goods is paid as they are withdrawn from the warehouse. In fixing the charge to be paid on goods withdrawn it is assumed that these goods are those which have been in storage the longest. In the United States bonded warehouses goods may be withdrawn at any time in amount of whole packages and in whole tons if in bulk. Custom duties, warehouse charges, and labor must be paid on the goods as they are withdrawn.

An example will make clear the usage.

Example. Find the storage on the following, the rate being 5¢ per bbl. per term of 30 days.

May 1, received 400 bbl.; May 20, received 800 bbl.; June 15, delivered 400 bbl.; June 20, received 1400 bbl.; July 1, delivered 900 bbl.; July 30, delivered 1300 bbl.

Solution.

Date	Received	Delivered	Time	Charge
May 1	400 bbl.			
20	800 bbl.			
June 15		400 bbl.	2 terms	100 at 10¢ = $ 40.00
20	1400 bbl.			
July 1		900 bbl.	2 terms	800 at 10¢ = 80.00
			1 term	100 at 5¢ = 5.00
30		1300 bbl.	2 terms	1300 at 10¢ = 130.00
				Total charge $255.00

PROBLEMS (C)

Find the storage charges on each of the following.

1. May 20 received 80 cases; June 1 received 140 cases; June 10 received 200 cases; June 15 received 170 cases; June 25 received 380 cases; July 1 received 260 cases; Sept. 9 delivered 200 cases; Oct. 1 delivered 180 cases; Oct. 15 delivered 200 cases; Nov. 1 delivered 250 cases; Nov. 15 delivered 180 cases; Dec. 1 delivered 220 cases. Charges 3¢ per case per term of 30 days.

2. August 1 received 1850 bu.; Aug. 5 received 960 bu.; Aug. 7 received 1200 bu.; Aug. 10 received 2450 bu.; Aug. 14 received 4600 bu.; Sept. 8 delivered 800 bu.; Sept. 20 delivered 1500 bu.; Oct. 7 delivered 3700 bu.; Nov. 17 delivered 5060 bu. Charges $\frac{3}{4}$¢ per bushel per term of 30 days.

3. September 8 received 840 bbl.; Sept. 14 received 1340 bbl.; Sept. 18 received 2160 bbl.; Sept. 23 received 360 bbl.; Nov. 3 delivered 500 bbl.; Nov. 16 delivered 1200 bbl.; Nov. 30 delivered 900 bbl.; Dec. 7 delivered 1400 bbl.; Dec. 12 delivered 700 bbl. Charges 5¢ a barrel per term of 30 days.

627. Average Storage. — When goods are constantly being received and delivered *average storage* is more convenient. Storage is

paid on the amount of goods actually in storage for any one period. Settlement is made at stated intervals instead of on the withdrawal of goods.

Example. May 1 received 140 cases; May 4 received 90 cases; May 10 delivered 100 cases; May 20 delivered 60 cases; May 24 received 300 cases; May 28 delivered 270 cases. Storage $\frac{1}{5}¢$ per case per day, payable 1st of each month.

Solution.

Date	Received	Delivered	Balance	Time	Equivalent to 1 Case for - Days
May 1	140 cases		140 cases	3 days	420
" 4	90 cases		230 cases	6 days	1380
" 10		100 cases	130 cases	10 days	1300
" 20		60 cases	70 cases	4 days	280
" 24	300 cases		370 cases	4 days	1480
" 28		270 cases	100 cases	4 days	400
July 1			100 cases		5260
		Hence charges are 5260 × .002 = $10.520			

PROBLEMS (C)

Using Average Storage find the charges in each of the following.

1. September 7 received 430 bu.; Sept. 9 received 160 bu.; Sept. 10 received 445 bu.; Sept. 12 received 751 bu.; Sept. 13 delivered 1560 bu.; Sept. 15 received 1860 bu.; Sept. 17 received 2490 bu.; Sept. 18 delivered 1680 bu.; Sept. 24 delivered 1710 bu. Storage 1¢ per 25 bushel per day. Find storage due Oct. 1.

2. July 15 received 1000 bu. wheat; July 20 received 800 bu.; July 21 delivered 1200 bu.; July 25 received 7500 bu.; July 30 delivered 4000 bu.; Aug. 10 delivered 3000 bu. Storage 1¢ per 20 bushels per day. Find storage Sept. 1.

3. October 20 received 500 bu. of potatoes; Nov. 1 received 1200 bu.; Nov. 10 delivered 1000 bu.; Nov. 15 received 700 bu. Find average storage at 1¢ per 15 bushels per day, Dec. 1.

CHAPTER XLVI

BUILDING AND LOAN ASSOCIATIONS

628. A Building and Loan Association is a corporation whose purpose is "to encourage thrift and saving and to facilitate the owning of houses among its members." There are many thousands of such associations in existence in the United States and other countries. These associations are now regulated more or less closely by law in the various states.

629. Shares in Building and Loan Associations. — Shares ranging in par value from $25 to $500 (but generally $200) are issued by these associations and sold to subscribers or members on weekly or monthly installments of 25¢ (per week) or $1 per month for each share. Members continue to pay the installments until the sum paid in, together with the accumulated profits, amount to the face value of the shares, at which time they are entitled to withdraw the face value of the shares in cash.

630. Income. — The income of the association consists of dues paid each month, of fines, premiums, interest on loans, and profits from withdrawals.

631. Interest on Loans. — Members borrowing from the association are obliged to pay interest the same as if they borrow from any other source.

632. Fines. — Subject to general restrictions each association adopts its own by-laws. According to these, fines are usually imposed for failure to pay dues promptly. The fine may be a fixed amount for each share, say 10¢ a month per share, or it may be a certain per cent on the unpaid dues.

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* These institutions had their origin in England in 1795, were first established in the United States in 1831, and have become generally popular in Europe and the United States.

633. Premiums. — The method of making loans varies. In some associations the available money is loaned to the members offering the highest rate of interest. (Building and Loan Associations are usually exempt from usury laws when lending to their own members, since the members share in the profits.) In other cases the rate of interest is fixed, but the loan is made to the member offering the largest cash premium. The premium is deducted from the loan, thus forming a kind of bank discount. In still other cases the loan is made to the member offering to pay interest for the longest time in advance. In all cases there is some kind of competitive bidding for the loans.

634. Book Value of Shares. — The books of the association show at any time the amount paid in on each share together with the apportioned profits. This is called the book value of the share.

635. Profits from Withdrawals. — Members frequently find it necessary or desirable to withdraw the cash value of their shares before they mature. In such cases the amount actually paid in plus a certain low rate of interest is allowed. The by-laws usually specify that the withdrawal value shall be less than the true value or book value, the exact rate being fixed from time to time by the directors.

The rate of interest actually obtained by investors in building and loan associations is usually rather high. The reason is to be found in the many sources of revenue enumerated above.

636. Kinds of Associations. — There are four principal kinds of associations in operation in the United States: (1) Single Series, (2) Several Series, (3) Continuous, (4) The Dayton or Ohio Plan. Of these the Several Series is the most common and is the only one discussed here.

637. Several Series Plan. — Suppose an Association is started January 1, of this year. A series of 500 shares is issued. If these are fully subscribed for by April 1, another series, say of 500 shares, is issued and offered for subscription. In this manner new series are issued quarterly, semi-annually, or annually in such amounts as may be required to satisfy the demand.

638. Finding Book Values of Shares. — The method of finding the book value of shares is best shown by an example:

Example. Shares were issued and subscribed for as follows: January 1, 1937, 500 shares; April 1, 600 shares; July 1, 400 shares; October 1, 800 shares. Profits January 1, 1937, to July 1, 1937, \$180; July 1, 1937, to January 1, 1938, \$540. Find the book value of the shares January 1, 1938, the value of the shares being computed semi-annually.

We first find the book value July 1, 1937.

To find the investment for one month for one share of each series we notice that the investment for the first month is \$1.00, for the second month \$2.00, and so on. Hence in the case of the first series we need to find the sum $1 + 2 + 3 + 4 + 5 + 6 = 21$ and for the second series, $1 + 2 + 3 = 6$.

Solution. Investment for one mo. on a share of 1st series = 21
Investment for one mo. on a share of 2d series = 6
Total investment for one mo. 1st series = $21 \times 500 = 10{,}500$
Total investment for one mo. 2d series = $6 \times 600 = 3600$
Total investment for one mo. = 14,100

$\frac{180}{14100} = .012752$ = profit on one dollar for one month

Hence $21 \times .012752 = .26779$, profit on one share of 1st series
$6 \times .012752 = .076512$, profit on one share of 2d series

Hence the book values are:

$6 + .26779 = 6.27$ for 1st series
$3 + .076512 = 3.08$ for 2d series

Check:
$500 \times .26779 = 133.89$
$600 \times .076512 = 45.80$
Total profit 179.69

The difference of \$.31 is due to omission of decimals. The error is not sufficient to warrant the use of further decimal places.

To find the book value January 1, 1938, each share of the first series is credited with an investment of \$6.27 for the whole period and each share of the second series with \$3.08 for the whole period.

Investment for one mo. of 1st series = $6 \times 6.27 + 21 = 58.62$
Investment for one mo. of 2d series = $6 \times 3.08 + 21 = 39.48$
Investment for one mo. of 3d series = 21.
Investment for one mo. of 4th series = 6.

We now proceed exactly as above to find the book value on January 1, 1938.

The book value of each series of shares is computed in this manner twice a year, usually January 1 and July 1.

PROBLEMS (C)

1. In an association just starting, shares were issued and subscribed for as follows: January 1, 1937, 200 shares; February 1, 1937, 100 shares; April 1, 1937, 300 shares. Total profits July 1, 1937, $175. Find the book value on this date.

2. In an association just starting, shares were issued and subscribed for as follows: July 1, 1936, 150 shares; October 1, 300 shares; January 1, 1937, 250 shares; April 1, 1937, 350 shares. Profits July 1, 1936, to January 1, 1937, $200; from January 1, 1937, to July 1, 1937, $450. Find the book value of the shares on this last date, the profits being apportioned every six months.

Suggestion. First find the book value of the shares on January 1, 1937, and then on July 1. Proceed as in the example solved on page 449.

3. On January 1, 1937, in a certain building and loan association there were in force the following shares with book value as follows: 460 shares, $130.60 each; 340 shares, $112.50 each; 200 shares, $89.40 each; 150 shares, $58.50; 530 shares, $32.80; 112 shares, $15.50. Shares were subscribed for as follows: January 1, 1937, 200; March 1, 250; May 1, 300. Profits from January 1, 1937, to July 1, 1937, $2300. Find the book value of all these shares on the last-named date.

4. In an association starting business April 1, 1936, shares were subscribed for as follows: April 1, 1936, 400; July 1, 350; October 1, 500; January 1, 1937, 250; April 1, 1937, 700. Withdrawn October 1, 1936, 20 shares of the April 1 series and April 1, 1937, 50 shares of the July 1, 1936, series. Earnings April 1, 1936, to July 1, 1936, $50; July 1, 1936, to January 1, 1937, $350; January 1 to July 1, $850. Find book value of each series on this date.

Suggestion. The book value is computed for July 1, 1936, although the association has been in operation only three months, because January 1 and July 1 are customary dates for closing the books.

In computing the book value for January 1, 1937, consider only 380 shares of the April 1 series, since 20 shares were withdrawn during the period July 1, 1936, to January 1, 1937. Similarly, in computing the book value for July 1, 1937, consider only 300 shares of the July 1, 1936, series, since 50 shares of this series were withdrawn during this period.

CHAPTER XLVII

DEPRECIATION

639. Properties of various kinds, such as buildings and machinery, are constantly decreasing in value. In estimating the value of such properties from year to year it is therefore necessary to take account of such depreciation.

The following are the principal reasons for depreciation:

(1) Ordinary wear and tear due to use.

(2) Natural deterioration, such as rotting of wood and rusting of metals.

(3) New inventions and methods rendering machinery and buildings obsolete.

(4) Certain properties, such as mines and forests, become exhausted.

640. Reasons for Figuring Depreciation. — The chief purpose for figuring depreciation is to obtain the true value of buildings and plant; and this is important in ascertaining the condition of a business. The amount of depreciation must be charged as general or overhead expenses, and these forms are important elements in the cost of carrying on the business.

641. One Method of Charging Depreciation. - A method commonly used in charging depreciation is to deduct the same rate per cent each year of the value at the beginning of the year.

Example. An automobile is worth when new $1500. Charging a depreciation of 30% each year, what is its value at the end of three years? And what is each year's depreciation?

Solution. The value at the end of 1st year is 70% of $1500 = $1050
The value at the end of 2d year is 70% of $1050 = $ 735
The value at the end of 3d year is 70% of $ 735 = $ 514.5

The depreciations for the successive years are $150, $315, $220.50.

From this example it will be seen that when figured in this manner the depreciation decreases from year to year. This is as it should be, since the expense for

repairs increases from year to year. The depreciation plus repair expenses should be the same from year to year, it being assumed that the plant is kept in uniform efficiency. This method of computing depreciation more nearly fulfills this requirement than the other in general vogue. In many businesses special rules are required for computing depreciation from year to year.

It is obvious that this method of figuring depreciation is not applicable to the last few years before the building or machine becomes absolutely worthless, for by this method the property would never become worthless.

642. Uniform Depreciation. — In some cases the element of repairs does not enter at all.

Thus, suppose a team of draft horses is worth \$600 when 5 years of age and \$250 when 12 years of age, but that the immediate effectiveness of the team has not changed during this time. To distribute the depreciation uniformly over the 7 years, we have $350 \div 7 = 50$ as the yearly depreciation. In reality the problem is more complicated and the student is referred to any treatise on the mathematics of investment.

PROBLEMS (A)

In solving the following exercises use the method of § 641.

1. A building worth \$100,000 depreciates 4% yearly. How much is it worth at the end of 10 years?

2. A threshing machine and engine costing \$2500 depreciates 20% yearly for 5 years. Find its value at the end of this period.

3. A grading outfit consisting of steam shovels, teams, scrapers, plows, etc., costs \$150,000. If it depreciates at the rate of 25% per year, what is the daily charge for depreciation during the fourth year, the average number of days which it is used each being 150?

4. A dwelling costing \$12,000 depreciates at the rate of 6% per year. It is rented for \$100 per month. During the fifth year the cost of repairs is \$125 and taxes \$140. What rate per cent on the investment is realized this year?

Suggestion. Deduct the taxes, repairs, and depreciation from the rent.

5. An ocean liner costing \$1,250,000 depreciates at the rate of 8% per year for 10 years. How much is it worth at the end of this period?

SUPPLEMENTARY WORK

OPTIONAL WORK

The optional work given on the following pages is a little more general than the related problems of the main text and is intended for pupils who feel they are able to do better work than the average. The solution of these problems will test your mettle and at the same time show your teacher that you have a clear insight into the reason for the rules that are used. There is an old and very true saying that "He who knows how may have a job, but he who knows why will be his boss." So you should make your greatest effort to understand the "why" of things. In these problems, try to think things out for yourself — try this persistently. If this does not succeed, try to find explanations in the text or in some other book, and only as a last resort ask the teacher.

In working these special problems you should make an extra effort to check your work so you may be sure your answers are correct. A number of the problems require special forms or blanks for their completion. You should take pains to put them into the best possible form. This is sometimes quite difficult and you may have to ask help of the teacher. The many blanks in the main body of the textbook will help you with this part of your work.

All page references are to the main body of the book. They are used only when necessary to help you locate related matter which will assist you in your work, not to indicate when or in what order you should do the problems.

When you finally hand one of these problems to the teacher, it should be exactly right and in a form that makes it very easily understood. Also you should be ready to give good reasons for

every step in your work. If you get into the habit of doing all this well, you will know both the "how" and the "why," and in time you will stand a good chance of being the boss of some who have learned only the "how."

1. Pages 26 and 337. For each customer having a checking account, a bank keeps a record showing the balance on the first of the month, the deposits made, and the checks drawn. After receiving a deposit or cashing a check, the customer's new balance is entered in the right column of the record sheet. Construct such a statement of account, using the following data and ruling your blank appropriately:

Date	Checks	Deposits	Balance
		Balance brought forward	$486.81
		Jan. 3 $450.00	
Jan. 5	90.00, 20.00, 15.75		
Jan. 8	5.00, 4.00, 4.75		
		Jan. 12 46.13	
Jan. 14	7.34, 2.45, 16.10		
Jan. 15	10.50, 49.24		
		Jan. 16 124.65	
Jan. 20	30.00, 15.00, 2.75		
Jan. 25	25.00, 7.60, 3.00		
		Jan. 26 36.50	
Jan. 28	17.62 3.90		
Jan. 30	8.40, 2.71, 3.24		

2. Page 128. Solve the following pay-roll problem. Allow no time for fractions of $\frac{1}{4}$ hour. The wage per hour paid the different workers is: 1, 92¢; 2, 85¢; 3, 75¢; 4, 85¢; 5, $76\frac{2}{3}$¢; 6, 87¢; 7, 82¢; 8, $87\frac{1}{2}$¢; 9, 78¢; 10, 88¢. Time over 40 hours per week is overtime. Allow time and a half for overtime. Rule blank appropriately and make pay roll from the following time slips. Make pay-roll memorandum. Use separate columns to show overtime and wages for overtime.

MONDAY

	In	Out	In	Out
1.	7 : 56	11 : 59	1 : 05	4 : 30
2.	8 : 30	11 : 30	12 : 30	5 : 30
3.	8 : 15	11 : 45	12 : 50	5 : 20
4.	7 : 58	12 : 05	1 : 10	4 : 50
5.	7 : 30	12 : 00	1 : 00	5 : 30
6.	8 : 00	12 : 05	1 : 10	5 : 45
7.	9 : 00	11 : 30	12 : 30	5 : 30
8.	8 : 00	12 : 02	1 : 10	5 : 05
9.	8 : 05	12 : 20	1 : 20	5 : 40
10.	7 : 50	12 : 10	1 : 00	5 : 00

TUESDAY

	In	Out	In	Out
	8 : 10	12 : 15	1 : 30	5 : 15
	8 : 00	12 : 05	1 : 15	4 : 50
	7 : 45	11 : 50	1 : 00	5 : 05
	8 : 05	12 : 05	1 : 10	5 : 30
	8 : 10	12 : 15	1 : 15	5 : 00
	8 : 00	12 : 30	1 : 30	6 : 00
	8 : 30	12 : 30	1 : 15	5 : 30
	7 : 30	12 : 00	1 : 00	5 : 00
	7 : 40	11 : 50	1 : 06	4 : 50
	7 : 50	12 : 10	1 : 10	5 : 45

WEDNESDAY

	In	Out	In	Out
1.	9 : 10	12 : 10	1 : 20	5 : 10
2.	8 : 07	11 : 45	12 : 35	5 : 00
3.	7 : 50	12 : 05	1 : 10	4 : 45
4.	8 : 10	12 : 15	1 : 20	4 : 50
5.	7 : 55	11 : 05	12 : 00	5 : 10
6.	8 : 20	11 : 40	12 : 25	5 : 00
7.	9 : 00	12 : 10	1 : 05	5 : 50
8.	8 : 45	12 : 03	1 : 00	5 : 40
9.	8 : 50	11 : 30	12 : 15	4 : 50
10.	7 : 35	12 : 00	1 : 15	5 : 05

THURSDAY

	In	Out	In	Out
	7 : 30	11 : 20	12 : 10	5 : 15
	8 : 05	11 : 45	12 : 25	5 : 45
	9 : 00	12 : 15	1 : 10	5 : 05
	7 : 45	11 : 30	12 : 45	4 : 20
	8 : 10	11 : 45	12 : 30	4 : 30
	7 : 56	11 : 45	12 : 20	5 : 10
	8 : 30	12 : 10	1 : 20	5 : 25
	7 : 50	12 : 00	1 : 03	4 : 45
	7 : 56	12 : 30	1 : 25	5 . 15
	8 : 15	12 : 05	1 : 30	6 : 00

FRIDAY

	In	Out	In	Out
1.	7 : 53	12 : 00	1 : 10	4 : 50
2.	8 : 00	12 : 10	1 : 00	4 : 30
3.	7 : 45	11 : 15	12 : 30	5 : 05
4.	8 : 15	11 : 50	12 : 40	5 : 15
5.	8 : 45	12 : 05	1 : 15	5 : 00
6.	7 : 30	12 : 15	1 : 20	5 : 25
7.	7 : 55	11 : 55	12 : 35	5 : 30
8.	8 : 05	12 : 20	1 : 30	5 : 10
9.	9 : 00	12 : 30	1 : 16	5 : 45
10.	8 : 30	11 : 45	12 : 50	6 : 00

SATURDAY

	In	Out		In	Out
1.	8 : 06	11 : 40	*6.*	9 : 00	12 : 30
2.	8 : 30	1 : 00	*7.*	8 : 45	11 : 00
3.	7 : 54	1 : 30	*8.*	7 : 40	11 : 50
4.	8 : 15	12 : 45	*9.*	8 : 03	12 : 40
5.	7 : 50	12 : 15	*10.*	8 : 24	1 : 15

3. Page 129. In a certain factory, work is done by the piece. Since a worker occupies the same space, uses the same amount of light and heat, etc., whether he works fast or slow the company pays a higher price per piece to the fast worker. The average number of pieces for an eight-hour day is found to be 18 and the average worker is paid 35 cents a piece. He earns $18 \times 35¢ = \$6.30$ in 8 hours. The company adds $1\frac{1}{2}$ cents a piece for the worker who makes 19 pieces in 8 hours, 3 cents a piece for one who makes 20 pieces, and so on. Thus, one who makes 19 pieces, earns $19 \times 36\frac{1}{2}¢ = \$6.93\frac{1}{2}$ ($6.94), and one who makes 20 pieces earns $20 \times 38¢ = \$7.60$, and so on. There is a similar deduction for slower work. Thus one who makes 17 pieces earns $17 \times 33\frac{1}{2}¢ = \$5.70$, one who makes 16 pieces earns $16 \times 32¢ = \$5.12$, and so on.

Make out a pay roll with a pay-roll memorandum for the following workers. The average number of pieces on Saturday is 9.

	M.	T.	W.	Th.	F.	S.		M.	T	W.	Th.	F.	S.
1.	19	20	21	20	19	10	*11.*	21	18	20	17	18	10
2.	16	17	18	17	16	8	*12.*	20	19	18	19	20	9
3.	18	19	18	20	19	9	*13.*	17	18	15	16	18	7
4.	20	21	18	19	21	10	*14.*	18	17	19	18	19	9
5.	15	16	16	17	16	7	*15.*	19	19	20	20	21	10
6.	18	19	19	19	18	9	*16.*	16	18	17	18	16	8
7.	19	20	21	21	20	10	*17.*	20	20	21	19	20	10
8.	18	18	19	19	18	9	*18.*	17	17	16	16	17	8
9.	16	15	16	17	16	7	*19.*	21	21	20	20	21	10
10.	18	18	19	18	17	9	*20.*	20	20	19	21	20	10

4. A building site of a suburban home is 140 feet by 247½ feet. The house is 53 feet wide and 59 feet deep, and the garage is 22 feet by 32 feet. The walks and drives cover 1630 square feet, and the rest of the surface is to be covered to a depth of 8 inches with black dirt which costs $3.35 per cubic yard. What is the cost of this black dirt?

5. Page 191. Find the cost of the following bill of lumber:

84 pcs. 2″ × 4″ × 14′ at $56 per M.
160 pcs. 2″ × 4″ × 16′ at $54 per M.
48 pcs. 2″ × 4″ × 10′ at $54 per M.
2450 pcs. 1″ × 10″ × 12′ at $62 per M.
480 pcs. 1″ × 10″ × 14′ at $62 per M.
36 pcs. 2½″ × 8″ × 16′ at $68 per M.
42 pcs. 3″ × 10″ × 18′ at $70 per M.
24,350 linear feet 3″ wide and 1″ thick at $95 per M.
1840 board feet at $110 per M.
320 pcs. 2″ × 10″ × 16′ at $62 per M.

6. During the month of January, Stuart Miller delivered the following quantities of milk at the creamery: 564 lbs., 621 lbs., 556 lbs., 615 lbs., 550 lbs., 624 lbs., 637 lbs., 595 lbs., 700 lbs., 723 lbs., 584 lbs., 714 lbs., 572 lbs., 705 lbs., 649 lbs., 725 lbs., 650 lbs., 598 lbs., 702 lbs., 668 lbs., 694 lbs., 712 lbs., 588 lbs., 666 lbs., 689 lbs., 710 lbs.

At 53¢ a pound how much did the creamery pay for the butter fat in this milk, if it averaged 4.1% of butter fat?

If butter contains 84% of butter fat, how many pounds of butter could be made from this milk?

7. On January 5, the Stevensville Creamery received amounts of milk with butter fat contents as follows: 143 lbs., 3.7%; 424 lbs., 3.9%; 392 lbs., 4.5%; 564 lbs., 4.7%; 257 lbs., 3.8%; 820 lbs., 4.4%; 165 lbs., 3.9%; 236 lbs., 4.3%; 452 lbs., 4.4%; 865 lbs., 4.7%; 589 lbs., 3.7%; 384 lbs., 4.5%; 199 lbs., 4.1%; 354 lbs., 4.1%; 712 lbs., 3.8%; 764 lbs., 4.7%; 325 lbs., 3.8%; 187 lbs., 4.2%; 473 lbs., 4.6%; 287 lbs., 4.4%; 298 lbs., 4.4%; 495 lbs.,

3.9%; 652 lbs., 4.6%; 249 lbs., 4%; 565 lbs., 3.9%; 452 lbs., 3.7%; 656 lbs., 4.8%; 724 lbs., 4.1%; 840 lbs., 4.2%. At 54½¢ per pound how much did the creamery pay for this butter fat?

8. In the J. C. Haynes Store, clerks receive a salary, and commission on sales above $100 per week. On amounts between $100 and $200 the commission is 2% and on amounts above $200 the commission is 4%. Thus, a clerk who sells goods for $360 in one week receives as commission 2% of $100 + 4% of $160 = $8.40.

Find the weekly pay check of each of the following:

	M.	T.	W.	Th	F	S.	Salary	Com.	Total
Arthur Jones	$31 60	$47 50	$15.25	$27.60	$80 40	$76 10	$36.50		
Gertrude Coe	42 20	37 60	43.85	53.90	61 24	37 85	37.75		
Walter Smith	62.09	41 50	39 12	76 12	24.62	83 70	38 00		
Sam Thomas	37.10	8? 51	54 65	38 94	47 53	27 64	35 50		
Grace Hazlitt	9 80	63 .4	59 60	19.70	21.84	38 60	32 00		
Horace Worden	27 60	19 20	84.04	51 24	42 76	30.28	34.50		

9. Page 251. A wholesale house sells goods at discounts from advertised list prices, the discounts varying with the conditions of the sale. A table of discounts is made showing single discounts equal to the various discount series that are used by the house. Thus the series 30%, 5%, 2% is equal to a single discount of 34.83%.

Verify the statement just made and make a table showing the single discounts equal to each of the following series. Give each result to the nearest hundredth of one per cent. 25%, 10%, 2%; 15%, 5%, 3%; 10%, 6%, 2%; 40%, 10%, 5%; 50%, 10%, 10%, 2%; 30%, 15%, 15%, 3%; 25%, 20%, 10%, 2%; 60%, 5%, 3%, 2%; 33⅓%, 25%, 6%, 3%.

10. One day the following sales were made:

Gross Sales	Discounts	Gross Sales	Discounts
$840	30%, 5%, 2%	$425	15%, 5%, 3%
1,650	25%, 10%, 2%	975	10%, 6%, 2%
540	15%, 5%, 3%	2,500	40%, 10%, 5%
7,300	10%, 6%, 2%	5,280	40%, 10%, 5%
1,310	40%, 10%, 5%	640	33⅓%, 25%, 6%, 3%
3,625	50%, 10%, 10%, 2%	58	15%, 5%, 3%

Gross Sales	Discounts	Gross Sales	Discounts
4,200	30%, 15%, 15%, 3%	390	10%, 6%, 2%
915	25%, 20%, 10%, 2%	460	50%, 10%, 10%, 2%
12,400	60%, 5%, 3%, 2%	3,900	60%, 5%, 3%, 2%
2,950	$33\frac{1}{3}$%, 25%, 6%, 3%	8,250	25%, 20%, 10%, 2%

Make a list of these sales showing net sale and discount for each sale and the totals of these. To find the net amount of each sale use the table you have made.

11. Complete the following table of aliquot parts of 100 and multiples of such aliquot parts:

$\frac{1}{2}$ of 100 = 50	$\frac{1}{5}$ of 100 =	$\frac{5}{12}$ of 100 =	$\frac{1}{16}$ of 100 =
$\frac{1}{3}$ of 100 =	$\frac{1}{6}$ of 100 =	$\frac{7}{12}$ of 100 =	$\frac{3}{16}$ of 100 =
$\frac{2}{3}$ of 100 =	$\frac{5}{6}$ of 100 =	$\frac{11}{12}$ of 100 =	$\frac{5}{16}$ of 100 =
$\frac{1}{4}$ of 100 =	$\frac{1}{8}$ of 100 =	$\frac{1}{15}$ of 100 =	$\frac{7}{16}$ of 100 =
$\frac{3}{4}$ of 100 =	$\frac{3}{8}$ of 100 =	$\frac{4}{15}$ of 100 =	$\frac{9}{16}$ of 100 =
$\frac{1}{5}$ of 100 =	$\frac{5}{8}$ of 100 =	$\frac{7}{15}$ of 100 =	$\frac{11}{16}$ of 100 =
$\frac{2}{5}$ of 100 =	$\frac{7}{8}$ of 100 =	$\frac{11}{15}$ of 100 =	$\frac{13}{16}$ of 100 =
$\frac{3}{5}$ of 100 =	$\frac{1}{12}$ of 100 =	$\frac{13}{15}$ of 100 =	$\frac{15}{16}$ of 100 =

Use the table above to find the totals in each of the following:

1.	2.	3.
58 yds. at $33\frac{1}{3}$¢	980 lbs. at $87\frac{1}{2}$¢	260 pcs. at $91\frac{2}{3}$¢
294 yds. at $18\frac{3}{4}$¢	360 lbs. at $18\frac{1}{4}$¢	340 pcs. at $56\frac{1}{4}$¢
196 yds. at $43\frac{3}{4}$¢	2420 lbs at $66\frac{2}{3}$¢	860 pcs. at $68\frac{3}{4}$¢
93 yds. at $93\frac{3}{4}$¢	1760 lbs. at 40¢	294 pcs. at $13\frac{1}{8}$¢
136 yds. at $13\frac{1}{3}$¢	845 lbs. at $41\frac{2}{3}$¢	810 pcs. at $83\frac{1}{3}$¢
390 yds. at $31\frac{1}{4}$¢	2960 lbs. at $31\frac{1}{4}$¢	1240 pcs. at $37\frac{1}{2}$¢
275 yds. at $83\frac{1}{3}$¢	8870 lbs. at $43\frac{3}{4}$¢	350 pcs. at $26\frac{2}{3}$¢
1830 yds. at $37\frac{1}{2}$¢	4610 lbs. at $58\frac{1}{3}$¢	2104 pcs. at $81\frac{1}{4}$¢
450 yds. at $58\frac{1}{3}$¢	360 lbs. at 60¢	4100 pcs. at $93\frac{3}{4}$¢
830 yds. at $62\frac{1}{2}$¢	5480 lbs. at 80¢	920 pcs. at $26\frac{2}{3}$¢

12. Page 258. In a department store it is known from past experience what the approximate cost of doing business is in each

department. For various reasons the necessary rate per cent of profit is different for different kinds of goods.

Example. An article costs \$24.60, the cost of doing business is 18.3%, and the desired profit is 12%. Find the marking price to the nearest dollar.

Solution. 18.3% + 12% = 30.3%. Hence \$24.60 is 69.7% of the selling price which is \$24.60 ÷ .697 = \$35.3. Hence the marking price is \$35.00.

Find the marking price of each of the following.

Buying Cost	Cost of Doing Business	Rate Profits	Buying Cost	Cost of Doing Business	Rate Profit
\$12.50	21.8%	15%	\$ .75	24.5%	30%
27.40	20.1%	10%	1.80	23.4%	25%
57.80	19.6%	12%	9.15	22.8%	15%
31.60	18.2%	13%	12.80	18.9%	18%
51.30	22.3%	25%	4.60	22.6%	25%
120.60	17.6%	8%	21.25	14.5%	12%
3.90	20.9%	14%	37.50	23.7%	10%
5.34	24.6%	18%	.45	25.4%	35%
9.80	22.1%	9%	1.35	17.6%	25%
19.25	21.4%	17%	7.75	19.4%	16%

13. Farmer A sells his corn as soon as it is husked in October, receiving 96 cents a bushel; Farmer B sells his corn the following April, receiving \$1.14 cents a bushel. The items of expense for B in keeping the corn are: interest 4%, cost of storage and extra hauling 4½¢ per bushel. The corn shrinks 14.8%, from October to April. Suppose that each farmer has 1000 bushels in October, which makes the larger profit?

14. July 1 Walter Brooks bought a house for \$11,500, paying \$3500 in cash. Mr. Brooks agreed to pay \$500 every six months (\$500 January 1, \$500 July 1, etc.). Each payment was to be applied:

(*a*) to the payment of interest at 6% for the preceding six months on the unpaid balance,

(*b*) the remainder to part payment of the principal.

Write up a schedule showing the unpaid balance on each interest date, the amount of interest, and the amount applied to the reduction of the principal.

15. Mr. J. C. Phillips bought an apartment house for $48,600. Expenses for the year were: interest $4\frac{1}{2}\%$ of cost, taxes 34.5 mills on $33\frac{1}{3}\%$ of cost, depreciation and repairs 4% of cost, insurance $485, fuel $1725, janitor service $1200, water $144, light $108.

Apartments A, B, D were rented 12 months at $95 per month; apartments C, F, K, L were rented 12 months at $90 per month; apartment E was rented 10 months at $105 per month; apartments G and I, 11 months at $85; apartment H, 8 months at $75 per month; and apartment J, 6 months at $100 per month. Find the net rate of income earned by this investment.

16. The W. K. Dick Company sells goods to the Stowe Mercantile Company under an agreement that net sales are to carry 6% interest to date of final settlement as are also payments on account. Find the amount required to settle the following account:

Sales		Payments	
July 2	$560.00	August 1	$1000.00
July 31	1240.00	Sept. 1	500.00
August 14	395.50	Oct. 18	1000.00
Sept. 6	834.00	Nov. 20	600.00
Sept. 30	620.00	Dec. 1	500.00
Oct. 18	316.75	Jan. 3 payment to balance	
Nov. 14	2470.00		
Dec. 3	1525.00		

17. Example. Find the value of 8460 lbs. of wheat at 96 cents a bushel.

Solution.

84.60 (value at 60¢ a bu., or 1 cent per lb.)
42.30 (value at 30¢ a bu.)
8.46 (value at 6¢ a bu.)
135.36 = required value.

Solve the following problems, using the method illustrated:

a. Find the value of 10,280 lbs. of shelled corn at 94¢ a bushel.

Suggestion. First find the value at 56¢ a bushel. (One bushel of shelled corn weighs 56 lbs.)

b. Find the value of 4320 lbs. of oats at 64¢ a bushel. (One bushel of oats weighs 32 lbs.)

c. Find the totals of each of the following:

Wheat	Oats	Corn in Ear
4160 lbs. at $1.24 per bu.	5340 lbs. at 58¢ per bu.	6150 lbs. at 92¢ per bu.
6890 lbs. at $1.19 per bu.	8860 lbs. at 56¢ per bu.	5460 lbs. at 84¢ per bu.
3650 lbs. at $1.23 per bu.	3940 lbs. at 61¢ per bu.	5210 lbs. at 88¢ per bu.
4920 lbs. at $1.26 per bu.	2870 lbs. at 54¢ per bu.	6620 lbs. at 87¢ per bu.
5360 lbs. at $1.18 per bu.	1170 lbs. at 55¢ per bu.	4810 lbs. at 91¢ per bu.
7280 lbs. at $1.16 per bu.	5960 lbs. at 52¢ per bu.	1940 lbs. at 94¢ per bu.
9530 lbs. at $1.19 per bu.	1450 lbs. at 62¢ per bu.	5160 lbs. at 90¢ per bu.
6580 lbs. at $1.21 per bu.	8260 lbs. at 60¢ per bu.	3980 lbs. at 89¢ per bu.
4390 lbs. at $1.14 per bu.	7310 lbs. at 58¢ per bu.	4130 lbs. at 85¢ per bu.
7540 lbs. at $1.23 per bu.	9140 lbs. at 64¢ per bu.	6210 lbs. at 86¢ per bu.

18. Page 282. Explain the following solution fully and solve the problems by this method.

Example. Find the interest on $16,450 at 5% for 112 days.

Solution. At 5%, the interest on $100 for 72 days is $1.00.

$164.50	(interest for 72 days)
82.25	(interest for 36 days)
9.14	(interest for 4 days)
$255.89	112 days

a. Find the interest on $8260 at 4½% for 156 days. (Show that at 4½% the interest on $100 for 80 days is $1.00.)

b. Find the interest on $4280 at 7½% for 86 days. (Show that at 7½% the interest on $100 for 48 days is $1.00.)

c. Find the interest on $46,850 at 4% for 184 days. (Show that at 4% the interest on $100 for 90 days is $1.00.)

d. Using the same method, find the interest on the following:

	Principal	Rate	Time		Principal	Rate	Time
(1)	$1460	5%	94 da.	(4)	$2465	4%	121 da.
(2)	840	6%	112 da.	(5)	3120	5%	89 da.
(3)	1690	4½%	98 da.	(6)	790	7½%	114 da.

	Principal	Rate	Time		Principal	Rate	Time
(7)	$2548	6%	48 da.	(12)	$1,280	4½%	76 da.
(8)	8450	4½%	37 da.	(13)	1,540	5%	142 da.
(9)	265	7½%	54 da.	(14)	1,296	7½%	118 da.
(10)	391	6%	78 da.	(15)	7,350	4%	98 da.
(11)	820	4%	93 da.	(16)	15,640	4½%	37 da.

19. A city has the following debts:

$650,000 at 4.5%	$185,000 at 5%
260,000 at 4.8%	45,000 at 5.2%
1,345,500 at 4.2%	967,400 at 4.3%

What is the average rate of interest on this whole indebtedness? Find the rate to the nearest tenth of one per cent.

20. A man has investments yielding rates of income as follows:

$8,500 at 9%	$18,250 at 5½%
4,800 at 11.5%	5,840 at 5¾%
10,000 at 5%	45,300 at 6¼%
7,400 at 8.4%	27,600 at 4¾%

What is the average rate of income on these investments? Find rate to the nearest tenth of one per cent.

21. A department store building is valued at $650,000, on which 6% interest is charged; depreciation and repairs are 4% of the value of the building. Heat for the year costs $14,800; light, $5360; janitor work, $3830. Taxes are $12,865. These expenses are to be distributed among the various departments in proportion to the floor-space they occupy. Furniture occupies 12,800 sq. ft.; hardware, 3260 sq. ft.; crockery, 2840 sq. ft.; men's clothing, 3260 sq. ft.; women's clothing, 4840 sq. ft.; shoes, 1860 sq. ft.; notions, 2440 sq. ft. Make a table showing how much of each of the above items of expense is charged to each department and the total charged to each department. Check by showing that the sum of these amounts is equal to the sum of the expenses.

22. Pages 239, 240. In the store of the preceding example, additional costs were: taxes on stock, \$28,650; salaries to managers and sales people, \$623,960; delivery, \$51,430; miscellaneous, \$43,740. Find the total expense of running this store. The total net sales were \$4,962,800. How many per cent of the sales was the total cost?

23. Obtain the yearly budget for your school. Get an estimate of the present value of the building and compute interest at 5% and depreciation at 2%. Also get an estimate of the present value of the fixtures, furniture, and books and compute interest at 5% and depreciation at 10%. The total of these amounts will be an approximation of the yearly cost of the school to the community. Divide this total by the average daily attendance to find the cost per pupil. Discuss the possible errors in the data of this problem. To what extent do you judge these errors will affect the final result?

24. A car costs \$2450 and depreciates 30% the first year. Expenses for the year were 864 gallons of gasoline at $23\frac{1}{2}¢$; 172 quarts of oil at 32¢; repairs, \$43.60; one tire, \$31.40; insurance, \$83.50; license, \$15.00; interest on cost, 6% of \$2450. Find to the tenth of a cent the cost per mile, if the car was driven 13,286 miles.

If this car had been driven twice as far, the trip would have doubled the cost of the gasoline, oil, and repairs, and would have necessitated the purchase of 4 tires instead of one, the depreciation would have been 35% instead of 30%. What would have been the cost per mile? Compare with the cost per mile for the shorter distance. Discuss the reason for the difference.

25. Find the total attendance in your school for the last 10 years and also the total cost of running the school for each of these years. Then find the cost per pupil for each year and represent the results in a graph (use a form like that on page 372).

26. Pages 414-416. Find the amount of tax raised in your county for different purposes, the total assessed valuation, and compute the tax rate for each purpose. Then find the taxes on certain properties whose assessed value you have learned.

27. On July 7, 1930, when the partnership of John Harter and Fred Stokes was dissolved, it was found that Harter owed Stokes $17,840. Harter agreed to pay interest at the rate of $5\frac{3}{4}\%$ and to make payments on the principal from time to time, the accounting to be made according to the United States Rule, page 322.

Payments were made as follows: Nov. 1, 1930, $1000; May 5, 1931, $1500; July 7, 1931, $2000; Oct. 4, 1931, $1000; Jan. 10, 1932, $1650; June 15, 1932, $2500; Dec. 3, 1932, $800; April 24, 1933, $1000; Aug. 17, 1933, $3200; May 10, 1934, $1000; Jan. 4, 1935, $1000; Oct. 5, 1935, $500; March 15, 1936, $2500. Find balance Nov. 18, 1936.

28. Allan Swift and Company keep a large balance in the First National Bank and receive $1\frac{1}{4}\%$ interest on daily balances. For the month of November, the items involved were as shown below. **Find the total daily balance, using the form on page 343, and the interest on balance for this month.**

Credits: Nov. 1, balance, $64,840. Deposits: Nov. 3, $8,390; Nov. 4, $1,485.40; Nov. 5, $3,982.50; Nov. 6, $786.20; Nov. 7, $5,045.80; Nov. 8, $6,410.40; Nov. 10, $465.50; Nov. 11, $1,980.50; Nov. 12, $2,417.65; Nov. 13, $884.50; Nov. 14, $1,004.21; Nov. 15, $10,041.60; Nov. 17, $241.20; Nov. 18, $524.82; Nov. 19, $2,182.00; Nov. 20, $1,421.00; Nov. 21, $671.50; Nov. 22, $1,246.90; Nov. 24, $6,600.00; Nov. 25, $3,574.80; Nov. 26, $987.70; Nov. 28, $572.60; Nov. 29, $3,478.65.

Debits (checks): Nov. 3, $1,500.00; Nov. 4, $2,861.40; Nov. 6, $492.50; Nov. 8, $4,659.75; Nov. 10, $835.00; Nov. 12, $4,614.80; Nov. 13, $375.00; Nov. 14, $240.00; Nov. 15, $5,724.75; Nov. 18, $1,215.25; Nov. 19, $562.80; Nov. 19, $187.60; Nov. 21, $8,259.65; Nov. 22, $6,319.87; Nov. 24, $125.00; Nov. 25, $462.30; Nov. 26, $204.10; Nov. 28, $359.60; Nov. 29, $789.40.

29. Page 429. The law firm, Bryan and Smith, has three partners, Bryan, Smith, and Jameson. Under the agreement of partnership Bryan and Smith are to have 35% each of the net

earnings, and Jameson 30%. Interest at 6% is to be charged on all payments made to the partners from the time they are made to the end of the year. During the year payments to the partners are made as follows:

To Mr. Bryan: Feb. 12, $1500; April 1, $1000; July 6, $5000; Sept. 2, $500; Oct. 10, $2000; Nov. 20, $1500.

To Mr. Smith: Jan. 10, $500; Feb. 2, $500; March 8, $1000; May 6, $2000; July 1, $1000; Sept. 1, $2000; Nov. 2, $2500.

To Mr. Jameson: Jan. 3, $1000; March 5, $1500; April 1, $500; May 3, $2000; July 2, $1000; Aug. 3, $500; Oct. 1, $3000; Nov. 10, $500; Dec. 3, $500.

At the end of the year it is found that the net earnings of the firm during the year was $65,000. How much is due each of the partners?

30. A true story: Many years ago it was customary in the rural schools in some parts of the United States to change teachers very frequently. Only in rare cases did the same teacher teach in the same school two years in succession. In one such school district they had some "stickers" of problems in arithmetic which they tried on each new teacher. One young teacher was helped in retaining his "job" a second year by solving the following problem:

A farm was sold for $6000, $1000 to be paid down and the rest, principal and interest at 6%, to be paid in 5 equal annual installments. Find the amount of each installment.

This teacher had no tables of any kind. Solve this problem, using only the means he had.

CUMULATIVE REVIEWS

Your teacher will assign these exercises from time to time by the indicated groups of twenty. They may come any time *after* you have studied the section whose number heads one or more groups, or reviews. For example, your teacher may not assign the second and third reviews under § 136 until you are far past that section in your study. So keep in practice on these fundamental operations.

Review 1. § 20

1. $396 + 283 + 645$

2. $706 + 897 + 518$

3. $179 + 949 + 847 + 656$

4. $926 + 874 + 582 + 609$

5. $17.84 + 84.05 + 56.86 + 61.92$

6. $27.76 + 41.93 + 86.68 + 59.76$

7. $74.86 + 98.59 + 37.68 + 8.67 + 393.29$

8. $8.71 + 79.43 + 430.60 + 54.13 + 8.03$

9.

					Sums
	314	821	491	617	
	788	829	986	642	
	517	689	290	352	
	961	198	380	837	
	750	357	837	959	
	256	770	684	697	
Sums					= total

10.

					Sums
	9168	463	582	3717	
	632	870	604	1578	
	24	5840	56	861	
	289	277	641	839	
	686	97	597	586	
	499	789	59	37	
Sums					= total

11. 83
21
67
84

12. 46
24
51
68

13. 21
36
93
29
29

14. 62
15
37
46
22

15. $65 + 52 + 36 + 28$

16. $54 + 73 + 18 + 52$

17. $53 + 23 + 86 + 57 + 69$

18. $26 + 35 + 39 + 11 + 47$

19. $54 + 8 + 35 + 57 + 24 + 81$

20. $69 + 29 + 68 + 26 + 65$

Review 2. § 30

1. $386 + 897 + 752$

2. $79.75 + 68.99 + 94.86 + 89.53$

3. 91
43
27
46

4. $58 + 23 + 88 + 56 + 38$

5.

				Sums
	386	9479	475	4809
	25	899	1938	288
	781	506	5362	756
	5687	1490	179	6262
	556	99	865	9606
	59	5287	97	27
Sums				= total

6. 8471
− 2638

7. 3612
− 2947

8. 2312
− 1643

9. $846482 - 329136$

10. 58986
4219
878
4921
22471
12641

11. 80835
3859
475
6137
5875
44814

12. $234581 - 113827$

13. 394745 − 275123

14.
49781
2689
979
352
10014
1906

Find the difference in the sums in problems 15 to 20:

15.

4978	2653
7478	2864
9747	8537
4982	3988

16.

1502	1311
8372	6780
2980	2476
6262	5418

17.

7232	2620
8251	4162
2134	1963
4819	3123

18.

9150	2580
9180	2470
9660	2980
9790	2670
9680	2310
8960	2360

19.

8860	2880
9200	2470
9580	2310
9650	2780
9220	2520
9780	2450

20.

9820	2380
9600	2870
8730	2480
8660	2310
8970	2650
9620	2870

Review 3. § 51

1. 961+357+699+765

2. 405.69+28.78+4.01+70.43+832.87

3. 36 + 54 + 93 + 46

4. 26 + 89 + 53 + 74 + 8 + 77

5.
38
67
13
25
93

6.
67728
538
3891
823
5378
9637

7. 352765 − 191282

8.
4768
× 265

9.
5834
× 267

10.
4900
× 8760

11.
8500
× 6070

Find the difference in the sums:

12.		**13.**		**14.** 3×21	**15.** 2×51
3881	1242	9700	2610	**16.** 91×96	**17.** 81×34
4681	4121	9650	2950	**18.** 28×95	**19.** 9384×11
5654	2791	9870	2370	**20.** 7268×22	
8462	6939	9480	2630		
		9740	2160		
		9340	2250		

Review 4. § 54

1. $977 + 615 + 869$ **2.** $99.37 + 89.86 + 79.37 + 17.67$

3. $38 + 47 + 78 + 23 + 35$ **4.** $8216 - 3748$ **5.** $467267 - 248024$

6.

	62	385	7458	6793
	7396	760	769	5679
	968	75	929	926
	979	7065	2796	860
	7894	4587	52	4799
	9485	379	659	1589
Sums				= total

7. 6925×325 **8.** 6700×830 **9.** 9×94 **10.** 94×95

11. 47×68 **12.** 8294×11 **13.** 704×33 **14.** 9294×25

15. 4987×50 **16.** 372×75 **17.** 4795×201

18. 526×72 **19.** 894×299 **20.** 3864×972

Review 5. § 62

1. $299+887+579+836$ **2.** $797.58+2.96+82.77+6.46+107.59$

3.
36
21
84
43

4. $63 + 34 + 6 + 60 + 51 + 38$ **5.** 6925×325

6. $\begin{array}{r} 5600 \\ \times\ 480 \\ \hline \end{array}$

7. 87×48

8. $\begin{array}{r} 43962 \\ 853 \\ 673 \\ 1629 \\ 3057 \\ \hline 12964 \end{array}$

9. Find the difference in the sums:

9236	1936
3413	2876
3716	2943
6425	6281

10. 5477×11
11. 6837×44
12. 9294×25
13. 7352×50
14. 9289×75
15. 7157×302

16. 1687×198 **17.** 7065×568 **18.** $583\overline{)34687}$ **19.** $54286 \div 2$
20. $194\overline{)61847}$

Review 6. § 66

1. $378 + 799 + 247$

2. $90.34 + 27.96 + 26.81 + 69.87$

3. $24 + 38 + 19 + 61$

4. $\begin{array}{r} 1924 \\ -\ 1646 \\ \hline \end{array}$

5.

				Sums
821	2776	538	70	
637	49	3617	625	
9817	845	4753	9265	
69	704	859	4882	
6879	2897	607	869	
3565	7358	3158	359	
Sums				= total

6. Find the difference in the sums:

8860	2970
9660	2760
9600	2310
9590	2350
9640	2540
8370	2500

7. $\begin{array}{r} 7696 \\ \times\ 503 \\ \hline \end{array}$

8. $\begin{array}{r} 9050 \\ \times\ 7008 \\ \hline \end{array}$

9. 38×64
10. 486×11
11. 5189×55
12. 578×50
13. 8071×75
14. 7864×104
15. 29723×395
16. 1965×48
17. $2330 \div 10$
18. $37800 \div 100$
19. $738000 \div 1000$
20. $68780 \div 20$

Review 7. § 68

1. $570+405+478+896$ **2.** $29.64+485.15+79.06+2.54+70.89$

3. $52 + 46 + 48 + 37 + 96$ **4.** $517658 - 289289$

5. 8791×386 **6.** 8×59 **7.** $28 \times 56 \times 37 \times 42$ **8.** 97×94 **9.** 898×11

10. 879×66 **11.** 879×25 **12.** 17137×75

13. 4056×296 **14.** 1776×735 **15.** $67121 \div 8$

16. $9329000 \div 1000$ **17.** $87640 \div 30$ **18.** $29600 \div 400$

19. $5238 \div 56$ **20.** $49800 \div 25$

Review 8. § 68

1. $650 + 950 + 689$ **2.** $80.72 + 94.05 + 69.28 + 41.6$

3. $28 + 9 + 88 + 48 + 50 + 35$

4.

				Sums
879	98	693	62	
294	539	785	613	
7863	828	75697	358	
56	7629	53	2606	
702	8562	7297	607	
5629	956	5063	2479	
Sums				= total

5. Find the difference in the sums:

5003	3769
6731	2936
8274	5987
3720	2974

6. 4715×829 **7.** 68×57

8. 4998×11

9. 4752×50 **10.** 692×405

11. 16142×56 **12.** $489\overline{)25893}$

13. $86800 \div 10$ **14.** $935200 \div 100$

15. 2947×77 **16.** $2396 \div 48$

17. $287800 \div 25$ **18.** $9500 \div 50$

19. $6074000 \div 125$ **20.** $\dfrac{466 + 417 + 431 + 452 + 435}{5}$

Review 9. § 104

1. $817+629+704+357$

2. $346.60+128.41+6.07+53.83+160.87$

3.
53
81
25
49
57

4.
$$\begin{array}{r} 7123 \\ -\ 2862 \\ \hline \end{array}$$

5.
76489
2895
359
887
5057
31695

6.
$$\begin{array}{r} 4869 \\ \times\ 476 \\ \hline \end{array}$$

7.
$$\begin{array}{r} 4005 \\ \times\ 890 \\ \hline \end{array}$$

8. 3558×11

9. 3796×25

10. 4752×50

11. 16542×599

12. 3958×963

13. $398263 \div 6$

14. $948000 \div 300$

15. $76900 \div 50$

16. $683000 \div 125$

17. Find the L. C. M. of 8, 12, 18.

18. $36 \times 34 \times 72 \times 12 \div 48 \times 27 \times 14 \times 18$

19. $\frac{1}{3} + \frac{5}{9} + \frac{7}{18}$

20. $9\frac{4}{5} + 3\frac{3}{8} + 2\frac{2}{3}$

Review 10. § 120

1. $17.84 + 84.05 + 56.86 + 61.92$

2.

				Sums
81	53	299	890	
694	657	684	96	
597	817	978	439	
56	838	48	708	
871	437	266	638	
867	50	694	37	
Sums				= total

3.
19
37
43
82

4. $24 + 61 + 82 + 46 + 30$

5.
$$\begin{array}{r} 4680 \\ -\ 2182 \\ \hline \end{array}$$

6.
$$\begin{array}{r} 5066 \\ \times\ 529 \\ \hline \end{array}$$

7. 4×69

8. 85×38

9. 7605×88

10. 23739×59

11. 39622×35

12. $71259 \div 705$

13. $80200 \div 100$

14. $5277000 \div 125$

15. $14 \times 17 \times 21 \times 6 \div 9 \times 24 \times 42$

16. $8\frac{2}{3} + 7\frac{5}{8} + 9\frac{7}{12}$

17. $\frac{5}{8} + \frac{5}{16}$

18. $\frac{1}{4} + \frac{1}{11}$

19. $\frac{1}{4} + \frac{4}{5}$

20. $3\frac{1}{2} \div \frac{1}{4}$

Review 11. § 121

1. $396 + 482 + 654$

2. $39.68 + 58.21 + 454.51 + 529.42 + 2.98$

3. $46 + 23 + 41 + 13$

4. $9 + 47 + 84 + 76 + 45 + 36$

5. Find the difference in the sums:

9590	2870
9980	2680
9370	2640
9570	2450
9170	2650
9600	2850

6. 8346×407

7. 83020×1806

8. 659×11

9. 5461×75

10. 4795×201

11. $47284 \div 5$

12. $5740 \div 10$

13. $5554000 \div 1000$

14. $\dfrac{478 + 469 + 443 + 444 + 453}{5}$

15. $\frac{1}{2} + \frac{1}{16}$

16. $\frac{1}{2} + 1\frac{1}{2}$

17. $\frac{1}{4} \div \frac{3}{5}$

18. $97\frac{3}{4} \div 6$

19. $899\frac{1}{7} \times 49$

20. $60\frac{1}{4} \times 47\frac{2}{3}$

Review 12. § 128

1. $179 + 949 + 847 + 656$

2.

39
34
94
99

3. $45 + 69 + 91 + 76 + 12$

4.

5632
− 4398

5. $457312 - 288108$

6. 6023×750

7. 102×104

8. 65×27

9. 376×25

10. 23902×496

11. 9719×642

12. $43980 \div 40$

13. $5876 \div 72$

14. $\frac{1}{4} + \frac{1}{9}$

15. $\frac{5}{9} + \frac{5}{12}$

16. $\frac{1}{7} + \frac{4}{13}$

17. $675\frac{1}{5} \div 4$

18. $38\frac{1}{3} \div 3\frac{1}{4}$

19.

$2074.85
83.80
396.70
92.05
49.75
538.60

20. $38.15 + .756 + 296.7 + 4.98 + 598.1 + 3.749 + 50.89$

Review 13. § 129

1. $27.82+96.47+29.64+98.27$ **2.** $34+87+92+78+40+66$

3. 5176×658 **4.** 3×42 **5.** 81×26

6.

				Sums
258	98	597	489	
779	876	768	940	
185	607	51	1129	
709	901	169	486	
976	86	456	159	
543	307	60	4486	
Sums				= total

7. 8735×11 **8.** 14370×50 **9.** 43669×32

10. $100468 \div 189$ **11.** $35100 \div 100$ **12.** $498000 \div 125$

13. $\frac{1}{4} + \frac{1}{7}$ **14.** $\frac{1}{4} + \frac{2}{5}$ **15.**

75659
11523
8561
4007
38153
2658

16. $6\frac{1}{3} \div \frac{2}{5}$

17. 42.798×10

18. 549.03×100

19.

$1652.90
27.85
87.50
65.40
456.20
125.45

20. Find the totals for each day and for each item, and the grand total:

Day	Clothing	Groceries	Hardware	Furniture	Notions	Totals
Monday	$189.30	$230 55	$ 43.10	$520 75	$204 50	
Tuesday	241.25	218.22	82.43	129.91	195 95	
Wednesday	182.53	202.35	64.80	248.80	188 90	
Thursday	295.75	249.63	81.85	327.82	215 18	
Friday	300.65	284.70	91.45	289.50	268.78	
Saturday	375 80	300.10	115.30	321.10	284.49	
Totals						

Review 14. § 133

1. $98.59 + 8.96 + 863.48 + 416.28 + 49.26$

2.
$$\begin{array}{r} 34 \\ 42 \\ 46 \\ \underline{26} \end{array}$$

3. $34 + 97 + 68 + 8 + 86 + 45$

4. $647124 - 139489$

5.
$$\begin{array}{r} 7605 \\ \times\ 3007 \\ \hline \end{array}$$

6. 97×98

7. 932×11 **8.** 359×99 **9.** 4302×75 **10.** 7057×301

11. $24{,}000 \div 10$ **12.** $848400 \div 200$

13. $64 \times 36 \times 94 \times 18 \div 12 \times 34 \times 46$ **14.** $\frac{1}{4} + \frac{2}{5}$

15. $\frac{1}{2} + \frac{7}{12}$ **16.** $192\frac{5}{6} \div 17$

17. $.785 + 2.658 + 1819 + 890 + .856 + 64.05 + 1.789$

18. 246.25×10 **19.** 10018.5×1000 **20.** $96024 \div 10$

Review 15. § 133

1. $512 + 174 + 495$

2.
$$\begin{array}{r} 27 \\ 94 \\ 21 \\ 41 \\ \underline{98} \end{array}$$

3. $25 + 93 + 56 + 36$

4. Find the differences:

6748	3784
3927	2489
8192	7284
6984	3729

5.
$$\begin{array}{r} 3812 \\ \times\ 652 \\ \hline \end{array}$$

6. 65×27

7. 6804×25 **8.** 519×50

9. 3965×594 **10.** 484×749

11. $6488000 \div 1000$ **12.** $73100 \div 25$

13. Find the L. C. M. of 7, 28, 63 **14.** $\frac{1}{2} + \frac{5}{12}$ **15.** $\frac{1}{4} + \frac{1}{5}$

16. $179\frac{3}{5} \times 68$ **17.** $52\frac{1}{3} \times 19\frac{3}{4}$ **18.** 6225.3×100

19. $81790 \div 100$ **20.** $42937 \div 1000$

Review 16. § 136

1. $817 + 629 + 704 + 357$

2. $64 + 97 + 38 + 77 + 54$

3. Find the differences:

9280	2840
9130	2410
6500	2490
7720	2780
5660	2660
5020	2840

4. 57304×108

5. 2×42

6. 6978×22

7.
$$\begin{array}{r} 32 \\ 19 \\ 43 \\ 59 \\ 98 \\ \hline \end{array}$$

8. 11317×75 **9.** 7864×104 **10.** $32517 \div 3$

11. $20400 \div 100$ **12.** $4072 \div 48$ **13.** $\dfrac{672+654+658+649+654}{5}$

14. $\frac{3}{8} + \frac{3}{16}$ **15.** $\frac{4}{7} + \frac{4}{11}$ **16.** $4 \div 4\frac{1}{6}$ **17.** $19\frac{3}{4} \div 4\frac{1}{5}$

18. 1943.29×1000 **19.** $291)630.29$ **20.** $8133 \div 25$

Review 17. § 136

1. $27.76 + 41.93 + 86.68 + 59.76$

2.

				Sums
361	981	581	749	
2645	3926	676	582	
879	452	2754	587	
467	95	1889	690	
50	4847	567	47	
964	875	306	1859	
Sums				= total

3. $43 + 78 + 55 + 98 + 56 + 6$

4. $7483 - 1626$

5. 7396×767

6. 5389×11 **7.** 519×50 **8.** $57{,}736 \times 637$ **9.** 9122×36

10. $240793 \div 456$ **11.** $59980 \div 50$ **12.** $73100 \div 25$

13. $\frac{2}{3} + \frac{3}{4} + \frac{1}{8}$ **14.** $\frac{1}{2} + \frac{1}{12}$ **15.** $\frac{5}{3} + \frac{7}{8}$ **16.** $486\frac{1}{4} \div 15$

17. $126\frac{2}{3} \div 7$ **18.** 452.3×10 **19.** $294.93 \div 1000$

20. $308.08 \div 50$

Review 18. § 136

1. $774.88 + 2.29 + 82.86 + 26.37 + 385.59$

2.
$$\begin{array}{r} 46 \\ 34 \\ 55 \\ \underline{97} \end{array}$$

3.
$$\begin{array}{r} 4514 \\ \underline{-\ 3892} \end{array}$$

4.
$$\begin{array}{r} 9175 \\ \underline{\times\ 289} \end{array}$$

5.
$$\begin{array}{r} 80200 \\ \underline{\times\ 805} \end{array}$$

6. 97×99 **7.** 933×25 **8.** 692×105 **9.** 268×756

10. $2580 \div 10$ **11.** $34800 \div 500$ **12.** $25060000 \div 125$

13. $12\frac{3}{7} + 8\frac{4}{9} + 12\frac{4}{21}$ **14.** $\frac{1}{4} + \frac{7}{16}$ **15.** $\frac{4}{7} + \frac{4}{9}$

16. $587\frac{1}{5} \times 71\frac{2}{3}$ **17.** $1429.6 \div 100$ **18.** $74.73 \div 10$

19. $999.03 \div 125$

20. \$1793.00

$$\left\{\begin{array}{r} 91.50 \\ 79.35 \\ 366.50 \\ 297.40 \\ \underline{198.80} \end{array}\right.$$

Review 19. § 140

1. $88 + 65 + 91 + 39$

2.
$$\begin{array}{r} 32 \\ 13 \\ 54 \\ 76 \\ \underline{77} \end{array}$$

3.
$$\begin{array}{r} 26581 \\ \left\{\begin{array}{r} 1819 \\ 947 \\ 8105 \\ 3895 \\ \underline{8314} \end{array}\right. \end{array}$$

4. Find the difference in the sums:

8174	3004
6804	2768
5610	2984
7142	4908

5. 9×37

6. 73×97

7. 8147×11

8. 27839×50

9. 82248×35

10. $28943 \div 9$

11. $508900 \div 100$

12. $6836 \div 81$

13. $\frac{1}{1} + \frac{9}{10}$

14. $\frac{2}{3} + \frac{5}{8}$

15. $2\frac{1}{4} \div 6\frac{1}{3}$

16. $355\frac{5}{8} \times 39$

17. $8.039 + 367.6 + 780.9 + 29.78 + 6.201 + 46.84 + 7.115$

18. Find the totals for each day and for each item, and the grand total:

Day	Clothing	Groceries	Hardware	Furniture	Notions	Totals
Monday	$178.60	$241.61	$48.37	$522.52	$188.90	
Tuesday	249.70	216.20	84 28	137.40	175.32	
Wednesday	194.60	187.10	64.20	241.56	161.84	
Thursday	287.53	231.50	83.92	316.42	203.45	
Friday	305.65	276.93	94.80	276 55	238.60	
Saturday	361.89	298.34	104.80	312.55	277.45	
Totals						

19. $214\overline{)378.42}$ **20.** Reduce $\frac{2}{5}$ to a decimal.

Review 20. § 149

1. $448 + 289 + 968$ **2.** $\begin{array}{r} 54268 \\ -\ 32517 \\ \hline \end{array}$

3.

				Sums
361	981	581	749	
2645	3926	676	582	
7890	245	2475	785	
476	49	1998	690	
50	7478	567	74	
964	875	306	1589	
Sums				= total

4. $567426 - 286784$

5. $\begin{array}{r} 5539 \\ \times\ 705 \\ \hline \end{array}$

6. 4861×11

7. 1948×33 **8.** 37196×75 **9.** 69247×797

10. $169518 \div 3421$ **11.** $7763000 \div 1000$ **12.** $87200 \div 25$

13. Find the L. C. M. of 16, 48, 64. **14.** $\frac{3}{5} + \frac{1}{7}$ **15.** $\frac{3}{5} + \frac{3}{11}$

16. $5\frac{1}{4} \div 7\frac{1}{3}$ **17.** $45\frac{1}{7} \div 4\frac{3}{4}$ **18.** 61.751×100

19. $33\frac{1}{3} \times 72$ **20.** $66\frac{2}{3} \times 429$

Review 21. § 149

1. $375 + 601 + 127 + 934$ **2.** $87 + 67 + 99 + 66 + 50 + 19$

3. 95×97 **4.** 89276×25 **5.** $462724 \div 7$ **6.** $82860 \div 60$

7. $258800 \div 50$ **8.** $48 \times 36 \times 27 \div 32 \times 16 \times 42$ **9.** $\frac{1}{2} + \frac{7}{5}$

10. $\frac{2}{3} + \frac{3}{8}$ **11.** $7 \div 3\frac{3}{4}$ **12.** $847\frac{3}{4} \times 89\frac{5}{8}$ **13.** $976\frac{3}{4} \div 9$

14. 2851.8×10 **15.** $493.296 \div 94$ **16.** $28.112 \div 50$

17. $27139 \div 125$ **18.** $33\frac{1}{3} \times 864$ **19.** $3\frac{1}{3} \times 864$ **20.** $6\frac{1}{4} \times 268$

Review 22. § 149

1. $26.27 + 82.26 + 42.94 + 80.56$ **2.** 7897×357 **3.** 28×95

4. 89385×50 **5.** 2341×45 **6.** $864000 \div 600$

7. $1794000 \div 125$ **8.** $\frac{3}{4} + \frac{5}{16}$ **9.** $\frac{3}{5} + \frac{3}{8}$ **10.** $260\frac{2}{3} \div 12$

11. $68\frac{4}{5} \div 17\frac{1}{3}$ **12.** 2.1846×100 **13.** $5.8895 \div 10$

14. $1262.8 \div 25$ **15.** Reduce $\frac{5}{12}$ to a decimal.

16. $3\frac{8}{9} + 4\frac{2}{3} + 6\frac{4}{15}$ **17.** $66\frac{2}{3} \times 832$ **18.** $6\frac{1}{4} \times 874$

19. $6\frac{2}{3} \times 298$ **20.** $8\frac{1}{3} \times 933$

Review 23. § 150

1. $56.54 + 4.72 + 385.06 + 53.21 + 6.74$ **2.** 60050×1309

3. 754×11 **4.** 3792×401 **5.** $765\overline{)775234}$

6. $6998 \div 72$ **7.** $\frac{587 + 543 + 543 + 518 + 525}{5}$ **8.** $\frac{1}{2} + \frac{3}{10}$

9. $\frac{2}{3} + \frac{1}{6}$ **10.** $681\frac{2}{3} \times 29$ **11.** $509\frac{7}{2} \times 25\frac{2}{3}$

12. \$1905.35

379.20
387.55
97.60
446.70
82.10

13. 317.835×1000 **14.** $387.67 \div 1000$

15. $39.004 \div 125$ **16.** $33\frac{1}{3} \times 371$

17. $3\frac{1}{3} \times 576$ **18.** $12\frac{1}{2} \times 786$

19. $16\frac{2}{3} \times 592$ **20.** $490 \times 46\frac{2}{3}$

Review 24. § 174

1. 5×35 **2.** 53×98

3.

				Sums
987	604	48	4310	
576	617	269	1319	
986	89	753	523	
439	1537	3020	426	
298	498	1433	637	
87	630	63	93	
Sums				= total

4. 6254×75 **5.** 5062×637 **6.** $809500 \div 25$

7. $\frac{2}{9} + \frac{4}{15} + \frac{3}{5}$ **8.** $\frac{3}{4} + \frac{3}{16}$ **9.** $\frac{2}{5} + \frac{2}{11}$ **10.** $5\frac{1}{4} \div \frac{3}{8}$

11. $467\frac{3}{8} \div 6$ **12.** 4.3479×10 **13.** $27.395 \div 100$

14. $28.112 \div 50$ **15.** $66\frac{2}{3} \times 542$

16. Find the total bill:

15 bu. Potatoes	at	$1.75
5 bbl. Flour	at	1.80
144 doz. Oranges	at	.65
90 lb. Tea	at	.82
105 lb. Coffee	at	.74
120 Canned Goods	at	.28
80 bu. Apples	at	1.61

17. $8\frac{1}{3} \times 812$

18. Add:

8 yd.	2 ft.	3 in.
7	1	9
10	0	2

19. Subtract:

20 rd.	9 ft.	7 in.
8	10	11

20. Divide 18 rd. 2 ft. 3 in. by 5.

Review 25. § 293

1.
23
61
62
78

2.
8567
× 538

3. 4676×11 **4.** 1672×50

5. 16557×42 **6.** $38900 \div 10$ **7.** $31\frac{3}{8} + 42\frac{7}{11} + 1\frac{1}{24}$

8. $\frac{1}{2} + \frac{1}{16}$ **9.** $\frac{1}{3} + \frac{7}{8}$ **10.** $637\frac{1}{2} \div 11$ **11.** $495\frac{5}{6} \div 5\frac{1}{2}$

12. 7591.9×100 **13.** $89\overline{)2.7032}$ **14.** Reduce $\frac{7}{16}$ to a decimal.

15. $6\frac{1}{4} \times 506$ **16.** $16\frac{2}{3} \times 870$

17. Add:

30 mi.	1600 ft.	7 in.
120	1320	3
20	2100	6

18. Divide 130 mi. 35 rd. 9 ft. 4 in. by 4.

19. $\frac{1}{8} =$ %

20. $\frac{2}{3} =$ %

Review 26. § 293

1. 19
96
42
39
74

2. 37840 × 908

3. 6746 × 44

4. $765\overline{)775234}$

5. 86971 × 25

6. 829700 ÷ 100

7. Find the L. C. M. of 5, 25, 40.

8. $\frac{1}{4} + \frac{1}{16}$

9. $\frac{2}{5} \div \frac{2}{9}$

10. $786\frac{3}{8} \times 36$

11. 489.6 + .403 + 3.86 + 27.59 + 7.68 + 611.8 + 574.9

12. 21.8475 × 1000

13. 27.93 ÷ 25

14. 9607.1 ÷ 125

15. $6\frac{2}{3} \times 743$

16. $12\frac{1}{2} \times 693$

17. Subtract:

3 mi.	120 rd.	10 ft.
1	230	11

18. $\frac{1}{2}$ = %

19. $\frac{1}{4}$ = %

20. $\frac{1}{5}$ = %

Review 27. § 293

1. 706 + 897 + 518

2. 89 + 94 + 88 + 76 + 69 + 87

3. 89 × 73

4. 80807 × 101

5. 957407 ÷ 6

6. 604600 ÷ 20

7. 3991000 ÷ 125

8. $\frac{1}{2} + \frac{5}{6}$

9. $\frac{1}{3} + \frac{5}{8}$

10. $476\frac{3}{5} \times 64\frac{1}{3}$

11. 670.6 ÷ 10

12. 30.273 ÷ 50

13. \$3476.00
182.85
631.10
63.45
1247.35
949.85

14. $33\frac{1}{3} \times 219$

15. $3\frac{1}{3} \times 964$

16. $560 \times 41\frac{2}{3}$

17. $\frac{1}{3}$ = %

18. $\frac{2}{3}$ = %

19. $\frac{2}{5}$ = %

20. $\frac{3}{5}$ = %

Review 28. § 293

1. 926 + 874 + 582 + 609

2. 94280
− 67927

3. 7657
× 467

4. 853 × 11

5. 3577 × 49

6. 8055000 ÷ 1000

7. 631600 ÷ 700

8. $\frac{1}{4} + \frac{3}{8} + \frac{3}{16}$

9. $\frac{3}{4} + \frac{7}{16}$

10. $637\frac{1}{2} \div 14$ **11.** $182\frac{7}{8} \div 5$ **12.** 980.106×100

13. $864.91 \div 100$ **14.** Reduce $\frac{7}{16}$ to a decimal. **15.** $8\frac{1}{3} \times 731$

16. $\frac{1}{2} =$ % **17.** $\frac{1}{4} =$ % **18.** $\frac{1}{5} =$ %

19. $\frac{1}{6} =$ %

20. Find the total bill:

20 boxes Raisins	at	\$3 10
45 boxes Oranges	at	$3.87\frac{1}{2}$
50 boxes Lemons	at	4.56
15 bbl. Coffee Sugar	at	26 50
45 chests Green Tea	at	27.50
32 chests Black Tea	at	20 19

Review 29. § 293

1. $79.75 + 68.99 + 94.86 + 85.93$ **2.** 8297×418 **3.** 592×55

4.
67057
3465
597
2586
224165
11642

5. 23421×50 **6.** $94280 \div 4$

7. $651600 \div 700$ **8.** $\frac{2}{5} + \frac{2}{7}$

9. $\frac{1}{2} + \frac{1}{6}$ **10.** $4\frac{1}{2} \div 3$

11. $88\frac{2}{7} \div 15\frac{3}{5}$ **12.** 12.967×1000

13. $632\overline{)45.637}$ **14.** $66\frac{2}{3} \times 387$

15. $12\frac{1}{2} \times 785$

16. Add:

30 yd.	1 ft.	11 in.
15	2	5
8	2	6

17. $\frac{1}{5} =$ %

18. $\frac{3}{4} =$ %

19. $\frac{5}{6} =$ %

20. $\frac{1}{8} =$ %

Review 30. § 293

1. $634.79+6.28+26.42+341.79+26.37$ **2.** $312214 - 187493$

3. 4×33 **4.** 6075×11 **5.** 4525×75 **6.** $16920 \div 10$

7. $809500 \div 25$ **8.** $\frac{1}{3} + \frac{3}{8}$ **9.** $\frac{3}{4} + \frac{3}{16}$ **10.** $607\frac{4}{5} \times 46$

11. $37.285 \div 10$ **12.** $6252.4 \div 1000$

13. Find the totals for each day and for each item, and the grand total:

Day	Clothing	Groceries	Hardware	Furniture	Notions	Totals
Monday	$161.35	$250.64	$50.32	$557.30	$175.84	
Tuesday	259.70	222.35	80.44	600.50	183.90	
Wednesday	221.44	172.40	63.34	325.40	154.35	
Thursday	304.22	189.95	72.54	225.36	144.37	
Friday	270.08	200.55	91.35	325.84	137.62	
Saturday	384.71	341.35	66.75	333.25	255.66	
Totals						

14. $51.504 \div 125$ **15.** $6\frac{1}{4} \times 628$

16. Divide 123 mi. 30 rd. 7 ft. by 8. **17.** $\frac{2}{5} =$ %

18. $\frac{3}{8} =$ % **19.** $\frac{1}{12} =$ % **20.** $\frac{5}{8} =$ %

Review 31. § 293

					Sums
1.	969	3670	890	3943	
	385	526	756	670	
	1964	58	4005	62	
	567	974	357	978	
	489	818	869	1709	
	57	56	81	146	
Sums					= total

2. Find the difference in the sums:

9137	5321
7285	3646
6742	3679
5160	2467

3. 9230×652

4. 37×85

5. 43692×893

6. 918×648

7. $5082 \div 48$

8. $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8}$

9. $\frac{1}{8} + \frac{1}{11}$

10. $512\frac{1}{7} \times 66\frac{3}{4}$

11. 573.92×10

12. $6.1595 \div 25$

13. $3\frac{1}{3} \times 765$

14. $16\frac{2}{3} \times 601$ **15.** 1.75371×1000 **16.** $\frac{1}{3} =$ %

17. $\frac{1}{6} =$ % **18.** $\frac{7}{8} =$ % **19.** $\frac{5}{12} =$ % **20.** $\frac{7}{12} =$ %

Review 32. § 293

1. 78
65
58
73

2. Find the difference in the sums:

9850	2680
7980	2310
6830	5240
4690	2850
9560	5620

3. 17040 × 5020

4. 6478×66

5. 5396×56

6. $769\overline{)46882}$

7. $34 \times 97 \times 105 \times 16 \div (12 \times 28 \times 46)$

8. $\frac{2}{3} + \frac{3}{4}$

9. $\frac{1}{3} + \frac{1}{8}$

10. $863\frac{7}{8} \div 11$

11. $45900 \div 100$

12.
$6541.25
1812.45
546.65
621.75
1127.85
751.20

13. $3196.28 \div 50$

14. $6\frac{2}{3} \times 537$

15. $190 \times 83\frac{1}{3}¢$

16. $\frac{2}{3} =$ ___%

17. $\frac{11}{12} =$ ___%

18. $\frac{1}{15} =$ ___%

19. $\frac{2}{15} =$ ___%

20. $\frac{4}{15} =$ ___%

Review 33. § 293

1. 8994 × 658

2. 103×101

3. 9288×11

4. 3869×501

5. 34
65
81
38
74

6. $4267387 \div 2$

7. $\frac{5}{6} + \frac{3}{8} + \frac{5}{12}$

8. $\frac{1}{4} + \frac{9}{10}$

9. $\frac{1}{7} + \frac{1}{2}$

10. $5\frac{1}{4} \div 1\frac{3}{8}$

11. $328\frac{3}{5} \div 3$

12. 75637×1000

13. $42749 \div 125$

14. $6\frac{1}{4} \times 979$

15. Add:

50 rd.	8 ft.	3 in.
22	7	9
10	9	11

16. Subtract:

50 mi.	200 rd.	6 ft.
37	125	8

17. $\frac{1}{16} =$ ___%

18. $\frac{3}{16} =$ ___%

19. $\frac{5}{16} =$ ___%

20. $\frac{7}{16} =$ ___%

Review 34. § 293

1. $86 + 67 + 21 + 11$

2. $19 + 73 + 57 + 35 + 87 + 7$

3. 40768 × 3076

4. 6182×25

5. 56904×50

6. $680000 \div 100$

7. $5360000 \div 125$ **8.** $\frac{1}{3} + \frac{3}{4}$ **9.** $\frac{2}{3} + \frac{4}{5}$ **10.** $248\frac{1}{3} \times 76\frac{3}{8}$
11. $83\frac{1}{2} \times 8\frac{1}{5}$ **12.** 240.75×100 **13.** Reduce $\frac{1}{64}$ to a decimal.
14. $66\frac{2}{3} \times 294$ **15.** $8\frac{1}{3} \times 486$ **16.** $\frac{1}{2}$ = ____ % **17.** $\frac{7}{8}$ = ____ %
18. $\frac{9}{16}$ = ____ % **19.** $\frac{11}{16}$ = ____ % **20.** $\frac{13}{16}$ = ____ %

Review 35. § 296

1. $34.87 + 11 + 77 + 85$ **2.** $\begin{array}{r} 23172 \\ -\ 19684 \\ \hline \end{array}$ **3.** 92×99

4. 76×53 **5.** 23912×63 **6.** $7640 \div 10$
7. $\dfrac{318 + 334 + 313 + 308 + 321}{5}$ **8.** $\frac{1}{4} + \frac{7}{10}$ **9.** $\frac{1}{7} + \frac{1}{11}$
10. $527\frac{1}{3} \times 15$ **11.** $94.7 + .368 + 51.64 + 408.7 + .944 + 2.85 + 84.96$
12. $590.8 \div 10$ **13.** $33\frac{1}{3} \times 302$ **14.** $6\frac{2}{3} \times 659$
15. $12\frac{1}{2} \times 506$ **16.** $\frac{1}{4}$ = ____ % **17.** $\frac{4}{5}$ = ____ % **18.** $\frac{13}{16}$ = ____ %
19. $\frac{15}{16}$ = ____ % **20.** Express $6\frac{3}{4}$% as a decimal.

Review 36. § 296

1. $166 + 840 + 667$ **2.** $214378 - 152498$ **3.** $\begin{array}{r} 8994 \\ \times\ 652 \\ \hline \end{array}$

4. 537×11 **5.** 456×75 **6.** 7536×954
7. $5148400 \div 800$ **8.** $2\frac{3}{4} + 1\frac{1}{3} + 4\frac{7}{12} + 35\frac{5}{8}$ **9.** $\frac{2}{3} + \frac{1}{4}$
10. $512\frac{4}{9} \div 7$ **11.** $7.412 \div 100$ **12.** $607.137 \div 1000$
13. $\begin{array}{r} \$5496.30 \\ 617.50 \\ 452.00 \\ 927.00 \\ 1452.60 \\ 175.15 \\ \hline \end{array}$ **14.** $3\frac{3}{5} \times 984$ **15.** $16\frac{2}{3} \times 638$
16. $\frac{3}{4}$ = ____ % **17.** $\frac{1}{4}$ = ____ %
18. $\frac{11}{16}$ = ____ % **19.** $\frac{15}{16}$ = ____ %
20. Express $\frac{1}{8}$ as a per cent.

Review 37. § 297

1. $474 + 656 + 168 + 714$ **2.** $624357 - 593298$ **3.** $\begin{array}{r} 70605 \\ \times\ 3002 \\ \hline \end{array}$

4. 157×46 **5.** 4605×102 **6.** $385716 \div 4$ **7.** $7696 \div 56$
8. $\frac{1}{4} + \frac{3}{10}$ **9.** $\frac{1}{7} + \frac{1}{10}$ **10.** $10 \div 5\frac{1}{2}$ **11.** $54\frac{5}{6} \times 94\frac{3}{4}$

12. 15796.2×1000 **13.** $58.9\overline{)832.607}$ **14.** $66\frac{2}{3} \times 281$

15. $\frac{3}{5}$ = ___% **16.** $\frac{5}{6}$ = ___% **17.** $\frac{1}{15}$ = ___% **18.** $\frac{3}{16}$ = ___%

19. $7\frac{1}{4}\%$ of 625

20. Find the total bill:

15 bbl. Greenings	at	$4.25
24 bbl. Potatoes	at	5 63
220 lb. Dates	at	.28
30 boxes Floridas	at	5.12
42 bunches Bananas	at	3.02
150 doz. Pineapples	at	1.75

Review 38. § 297

1. $93.67 + 49.24 + 75.45 + 24.49$ **2.** $67121 - 28943$ **3.** 3×87

4. 506×77 **5.** 8933×50 **6.** $836\overline{)96278}$ **7.** $827900 \div 25$

8. $\frac{5}{8} + \frac{3}{10} + \frac{2}{5}$ **9.** $\frac{1}{3} + \frac{1}{4}$ **10.** $294\frac{2}{3} \times 48$ **11.** $6545\frac{5}{6} \div 8$

12. 14.7479×100 **13.** $400.14 \div 25$ **14.** $.90355 \div 125$

15. $6\frac{2}{3} \times 429$ **16.** Subtract:

11 rd.	7 ft.	10 in.
6	10	11

17. $\frac{1}{8}$ = ___%

18. $\frac{5}{12}$ = ___% **19.** $\frac{2}{15}$ = ___% **20.** Write $\frac{3}{8}$ as a per cent.

Review 39. § 297

1. $842.93 + 15.42 + 878.52 + 3.47 + 460.12$

2. Find the difference in the sums:

3190	2974
9874	2907
6479	3978
3942	3098

3. 817×685

4. 1264×11

5. 1577×995

6. 652×546

7. $38900 \div 50$

8. $\frac{2}{3} + \frac{2}{5}$

9. $\frac{1}{4} + \frac{1}{10}$

10. $776\frac{2}{7} \div 16$

11. $34\frac{7}{8} \div 3\frac{3}{5}$

12. $85.116 \div 10$

13. $708.17 \div 50$

14. Reduce $\frac{15}{16}$ to a decimal. **15.** $8\frac{1}{3} \times 556$ **16.** $\frac{1}{3}$ = ___%

17. $\frac{3}{8}$ = ___% **18.** $\frac{7}{12}$ = ___% **19.** $\frac{4}{15}$ = ___%

20. Express $\frac{8}{9}$ as a per cent.

Review 40. § 297

1. 517634 − 309950

2.

				Sums
920	652	454	589	
292	988	570	295	
4493	27	345	709	
1249	87	46	3773	
96	62020	645	709	
659	730	429	55	
Sums				= total

3. 9092 × 5004

4. 239 × 92

5. 61380 × 75

6. 548495 ÷ 7

7. 168800 ÷ 900

8. $\frac{1}{2} + \frac{4}{5}$

9. $\frac{2}{3} + \frac{1}{6}$

10. $67\frac{7}{8} \div 33\frac{3}{4}$

11. 75637 × 1000

12. 6134.6 ÷ 1000

13. $331\frac{1}{3} \times 379$

14. $6\frac{1}{4} \times 191$

15.

$9801.60
684.00
3242.90
1284.00
296.25
1122.50

16. $\frac{2}{3}$ = %

17. $\frac{1}{12}$ = %

18. $\frac{11}{12}$ = %

19. $\frac{1}{16}$ = %

20. Express $2\frac{1}{2}$% as a decimal.

Review 41. § 303

1.

59
21
81
38
47

2. 40035 × 706

3. 102 × 104

4. 94751 × 25

5. 8605)233605

6. 2520 ÷ 72

7. 15 × 24 × 32 ÷ 30 × 18 × 12

8. $\frac{3}{4} + \frac{7}{8}$

9. $\frac{1}{7} + \frac{1}{8}$

10. $385\frac{2}{7} \div 6$

11. 40.07+551+3.89+670+.576+6.796+279.8

12. 21.684 ÷ 100

13. 87575 ÷ 50

14. $81\frac{1}{3} \times 638$

15. Subtract:

11 mi.	260 rd.
7	285

16. $\frac{5}{8}$ = %

17. $\frac{9}{16}$ = %

18. Express $5\frac{3}{4}$% as a decimal.

19. $7\frac{5}{6}$% of 827

20. 26 is % of 328.

Review 42. § 303

1. $40 + 71 + 44 + 61$ **2.** 6×67 **3.** 2107×11

4. 2341×45 **5.** $65720 \div 10$ **6.** $58500 \div 25$

7. $\frac{2}{7} + \frac{1}{3} + \frac{1}{4}$ **8.** $\frac{1}{2} + \frac{3}{5}$ **9.** $206\frac{5}{8} \times 58$ **10.** $87\frac{1}{9} \div 21\frac{5}{6}$

11. 8654.3×10 **12.** $137.1\overline{)613.4616}$ **13.** $2.20172 \div 125$

14. $12\frac{1}{2} \times 750$ **15.** $\frac{1}{3} =$ ___% **16.** $\frac{1}{15} =$ ___% **17.** $\frac{11}{16} =$ ___%

18. Express $\frac{7}{9}$ as a per cent. **19.** 32 is ___% of 546.

20. 257 is 5% of ___.

Review 43. § 305

1. $60 + 23 + 60 + 75 + 37$ **2.** 1497×608 **3.** 2897×88

4. 24526×103 **5.** $972000 \div 1000$ **6.** $4075000 \div 125$

7. $45\frac{3}{5} + 19\frac{2}{3} + 1\frac{1}{4} + 2\frac{1}{10}$ **8.** $\frac{1}{3} + \frac{4}{5}$ **9.** $365\frac{3}{4} \times 54\frac{3}{5}$

10. Find the totals for each day and for each item, and the grand total:

Day	Clothing	Groceries	Hardware	Furniture	Notions	Totals
Monday	$169.43	$258.64	$52.35	$560.87	$149.50	
Tuesday	235.40	207.35	77.85	298.90	122.36	
Wednesday	241.30	167.22	71.24	302.60	145.75	
Thursday	236.72	215.35	91.27	346.55	209 54	
Friday	310.25	285.90	95.65	281.33	257.65	
Saturday	392.64	300.23	110 25	421.27	235.63	
Totals						

11. 3759×100 **12.** $607.23 \div 25$ **13.** $66\frac{2}{3} \times 690$

14. $16\frac{2}{3} \times 987$ **15.** $\frac{1}{2} =$ ___% **16.** $\frac{2}{15} =$ ___% **17.** $\frac{13}{16} =$ ___%

18. 9% of 846 is ___. **19.** 158 is 4% of ___. **20.** Find 25% of 1500.

Review 44. § 311

1. $78 + 66 + 84 + 17 + 67 + 8$ **2.** 5×94 **3.** 395×50

4. 7403×864 **5.** $69500 \div 80$ **6.** $\frac{318+334+322+329+331}{5}$

7. $\frac{3}{4} + \frac{5}{8}$ **8.** $\frac{1}{6} + \frac{1}{5}$ **9.** $26 \div 6\frac{1}{4}$ **10.** 657.352×1000

11. $9.3988 \div 10$ **12.** $8093.4 \div 50$ **13.** $3\frac{1}{3} \times 652$

14. $\frac{1}{4}$ = % **15.** $\frac{4}{15}$ = % **16.** $\frac{15}{16}$ = %

17. 6% of 321 **18.** 21 is % of 342

19. Cost \$24, 24% increase, find the selling price.

20. Find the total bill:

Jan.	4	8 Oakland Gas Ranges at	\$93.25
	12	10 Electric Ranges at	158.50
	26	12 Wood-coal Ranges at	60.50

Review 45. § 311

1.
$$\begin{array}{r} 84274 \\ -\ 39826 \\ \hline \end{array}$$

2. 89×73 **3.** 9372×11

4. $67927 \div 6$ **5.** $595\overline{)257678}$ **6.** $261600 \div 50$

7. $\frac{1}{6} + \frac{1}{13}$ **8.** $\frac{1}{6} + \frac{1}{11}$ **9.** $48\frac{2}{3} \div 7$ **10.** 1.9753×10

11. $727.67 \div 100$ **12.** $46.8307 \div 125$ **13.** $6\frac{1}{4} \times 768$

14. Add: **15.** $\frac{1}{5}$ = %

35 rd.	2 yd.	4 ft.	11 in.
21	3	1	7
13	1		5

16. $\frac{1}{16}$ = % **17.** Express $4\frac{1}{2}$ as a decimal. **18.** 7% of 546

19. 596 is 7% of . **20.** \$120 cost, decrease 10%, find the selling price.

Review 46. § 311

1. $589 + 901 + 413$ **2.**
$$\begin{array}{r} 913 \\ \times\ 573 \\ \hline \end{array}$$
3. 6182×99

4. 28504×193 **5.** $967600 \div 500$

6. $154 \times 124 \times 360 \div 105 \times 18 \times 6$ **7.** $\frac{1}{3} + \frac{3}{5}$ **8.** $\frac{3}{4} + \frac{3}{8}$

9. $358\frac{3}{4} \times 45$ **10.** $367\frac{4}{5} \div 8$ **11.** $56.987 \div 1000$

12. $33\frac{1}{3} \times 498$ **13.** $6\frac{2}{3} \times 583$ **14.** Divide 212 yd. 1 ft. by 14.

15. $\frac{1}{6}$ = % **16.** $\frac{3}{16}$ = % **17.** $\frac{11}{16}$ = % **18.** 8% of 689

19. 48 is % of 634. **20.** Selling price \$25, decrease 25%, find the cost.

Review 47. § 312

1. $345 + 249 + 427 + 394$ **2.** 958×209 **3.** 67299×25

4. 7864×54 **5.** $5695100 \div 100$ **6.** $\frac{4}{5} + \frac{9}{10} + \frac{11}{15}$

7. $\frac{1}{2} + \frac{1}{5}$ **8.** $\frac{1}{2} + \frac{2}{5}$ **9.** $371\frac{3}{4} \times 41\frac{2}{5}$ **10.** $94\frac{3}{4} \div 12\frac{1}{3}$

11. $3.86\overline{)12837.7}$ **12.** $66\frac{2}{3} \times 569$ **13.** $81\frac{1}{3} \times 264$

14. $\frac{2}{3} =$ ____% **15.** $\frac{1}{8} =$ ____% **16.** $\frac{5}{16} =$ ____% **17.** $\frac{13}{16} =$ ____%

18. 5% of 431 **19.** 419 is 8% of ____. **20.** Cost 50¢, selling price $1.00, find the margin per cent on the selling price.

Review 48. § 312

1. $99.37 + 89.86 + 79.37 + 17.67$ **2.** 105×109

3. 8162×11 **4.** 6084×25 **5.** $6950 \div 10$ **6.** $7857 \div 81$

7. $\frac{1}{3} + \frac{3}{5}$ **8.** $\frac{3}{4} + \frac{1}{8}$ **9.** $24 \div 6\frac{1}{4}$ **10.** $69.138 \div 10$

11. $3\frac{1}{3} \times 356$ **12.** $12\frac{1}{2} \times 691$ **13.** $\frac{3}{5} =$ ____% **14.** $\frac{3}{8} =$ ____%

15. $\frac{7}{16} =$ ____% **16.** Express $33\frac{1}{3}\%$ as a decimal.

17. 4% of 752 is ____. **19.** $11\frac{1}{2}\%$ of 360 is ____.

18. \$23049.20

$$-\begin{cases} 9214.10 \\ 197.80 \\ 286.40 \\ 2803. \\ \underline{1225.85} \end{cases}$$

20. Cost \$75, decrease 50%, find the selling price.

Review 49. § 319

1. $739.12 + 98.75 + 3.32 + 1.87 + 65.12$ **2.** 96×97

3. 538×22 **4.** 29619951767 **5.** 5874×981

6. $14600 \div 25$ **7.** $1\frac{1}{5} + 3\frac{3}{10} + 6\frac{1}{15}$ **8.** $\frac{1}{6} + \frac{1}{7}$ **9.** $11\frac{5}{8} \div 4$

10. $110.9 + .676 + 3.871 + 295.8 + 369 + 270 + 81.05$

11. $268.4 \div 100$ **12.** Reduce $\frac{11}{17}$ to a decimal. **13.** $16\frac{2}{3} \times 692$

14. $\frac{1}{3} =$ ____% **15.** $\frac{1}{12} =$ ____% **16.** $\frac{9}{16} =$ ____%

17. Express $12\frac{1}{2}\%$ as a decimal. **18.** 82 is ____% of 1238.

19. Selling price \$1050, decrease 10%, find the cost.

20. List price \$20, rate discount 25%, find the selling price.

Review 50. § 323

1. 46272 − 38481

2. 356 × 89

3. 16807 × 75

				Sums
4. 5586	634	937	2672	
797	552	1894	4191	
1753	3364	293	94	
6941	574	657	262	
896	476	1246	942	
65	98	492	78	
Sums				= total

5. 91589 ÷ 3

6. 2673000 ÷ 75

7. $\frac{1}{2} + \frac{2}{3}$

8. $\frac{1}{3} + \frac{1}{5}$

9. $602\frac{5}{7} \div 9$

10. 33.817 × 10

11. 1171.93 ÷ 25

12. $6\frac{1}{4} \times 699$

13. $1245 \times 62\frac{1}{2}¢$

14. $\frac{1}{2}$ = %

15. $\frac{5}{8}$ = %

16. $\frac{11}{16}$ = %

17. $4\frac{1}{4}$% of 7512

18. 299 is 9% of .

19. Cost $2.50, selling price $6, find the margin per cent, using the selling price as base.

20. Gross amount $2500, discounts 15%, 25%, 5%, find the net amount.

Review 51. § 323

1. 21
67
90
43

2. 431615 − 380600

3. 3552 × 33

4. 6957 × 50

5. 6835000 ÷ 1000

6. 58700 ÷ 50

7. $\frac{1}{4} + \frac{7}{8}$

8. $\frac{1}{5} + \frac{1}{13}$

9. $51\frac{3}{4} \times 24$

10. 2.8418 × 100

11. 3.02137 × 1000

12. $33\frac{1}{3} \times 405$

13. $\frac{1}{4}$ = %

14. $\frac{7}{8}$ = %

15. $\frac{15}{16}$ = %

16. 56 is % of 3724.

17. 20% of 2750

18. List price $175.50, rate discount 25%, find the selling price.

19. Gross amount $500, discounts 15%, 25%, 35%, find the net amount.

20. Find the total bill:

3 boxes Shredded Wheat	125 lb. (net)	at	$0.17½	
8 bbl. Sugar	640 lb.	at	.10¼	
6 boxes Tea	680 lb.	at	.75	
5 bbl. Flour	588 lb.	at	.09	

Review 52. § 323

1. 7286×11 **2.** 69296×64 **3.**

39			
10			
42			
87			
92			

4. Find the difference in the sums:

4720	4364
4932	2980
7984	3480
2400	1847

5. $43980 \div 90$ **6.** $\frac{2}{3} + \frac{5}{6} + \frac{7}{8}$

7. $\frac{1}{2} + \frac{1}{3}$ **8.** $\frac{1}{2} + \frac{15}{16}$

9. $8 \div 6\frac{1}{3}$ **10.** 64.815×100

11. $3714.9 \div 50$ **12.** $3\frac{1}{3} \times 998$

13. Add:

12 yd.	2 ft.	5 in.
13	1	8
5		7

14. $\frac{1}{5}$ = ____ % **15.** $\frac{5}{12}$ = ____ %

16. Express $5\frac{3}{4}\%$ as a decimal.

17. $5\frac{1}{2}\%$ of 6302 is ____.

18. 919 is $6\frac{1}{3}\%$ of ____.

19. Cost \$38, increase $33\frac{1}{3}\%$, find the selling price. **20.** Gross amount \$4500, discounts 40%, 25%, 5%, find the net amount.

Review 53. § 323

1. $45 + 69 + 90 + 44$ **2.** 65046×25 **3.** Find the differences:

9720	2670
7809	3210
5940	4250
6940	5280
5960	2850

4. 84765×295 **5.** $9540 \div 10$

6. $3928000 \div 500$ **7.** $\frac{1}{4} + \frac{5}{8}$

8. $\frac{1}{5} + \frac{1}{12}$ **9.** $119\frac{1}{3} \div 11$

10. $587.35 \div 10$

11. $190731.34 \div 125$

12. $6\frac{2}{3} \times 645$ **13.** Subtract:

9 mi.	1550 ft.
6	2000

14. $\frac{2}{5}$ = ____ % **15.** $\frac{7}{12}$ = ____ %

16. Express $\frac{5}{7}$ as a per cent. **17.** 19 is ____ % of 393. **18.** Selling price \$144, increase 44%, find the cost. **19.** Cost \$3.50, selling price \$10, find the margin per cent on the selling price. **20.** List price \$278.50, discount 40%, find the selling price.

Review 54. § 324

1. $92 + 55 + 68 + 95 + 68$ **2.** 678×396 **3.** 835×44

4. 5162×864 **5.** $8005)\overline{629389}$ **6.** $\frac{1}{5} + \frac{1}{11}$ **7.** $\frac{1}{4} + \frac{3}{8}$
8. $(82 \times 68 \times 72 \times 96) \div (48 \times 16 \times 27)$ **9.** $104\frac{2}{3} \div 9$
10. $213 \div 7\frac{1}{2}$ **11.** $57.286 \div 100$ **12.** $66\frac{2}{3} \times 811$
13. Multiply 15 yd. 2 ft. 6 in. by 65. **14.** $\frac{4}{5} =$ %
15. $\frac{11}{12} =$ % **16.** Cost \$75, selling price, \$80, find the rate of increase. **17.** $6\frac{3}{4}\%$ of 3408 is . **18.** 358 is $5\frac{1}{2}\%$ of .
19. Gross amount \$12000, discounts $8\frac{1}{2}\%$, 30%, 40%, find the net amount. **20.** Find the single discount equivalent to the discounts 10%, 5%.

Review 55. § 324

1. $10 + 97 + 71 + 90 + 54 + 90$ **2.** $\begin{array}{r} 756 \\ \times\ 508 \\ \hline \end{array}$ **3.** 674×45

4. 70538×108 **5.** $\dfrac{284 + 290 + 283 + 295 + 293}{5}$ **6.** $\frac{1}{2} + \frac{5}{8}$

7. $\frac{1}{2} + \frac{11}{16}$ **8.** $227\frac{3}{4} \div 6$ **9.** \$18635.70

10. $88)\overline{56.9128}$ **11.** $6\frac{1}{4} \times 976$

$$-\left\{\begin{array}{r} 7295.50 \\ 475.85 \\ 5724.00 \\ 526.00 \\ \underline{1369.55} \end{array}\right.$$

12. $8\frac{1}{3} \times 924$ **13.** $\frac{11}{4} =$ %
14. $\frac{1}{15} =$ % **15.** $\frac{13}{16} =$ %
16. 62 is % of 5284. **17.** $152941 \div 9$
18. 16 is 200% of what number?
19. Selling price \$81.25, increase 65%, find the cost.
20. Find the single discount equivalent to the discounts 25%, 20%.

Review 56. § 324

1. $56 + 94 + 40 + 87 + 51 + 8$ **2.** 4×56 **3.** 6200×11
4. 43617×81 **5.** $56900 \div 100$ **6.** $956000 \div 125$
7. $\frac{1}{2} + \frac{13}{16}$ **8.** $\frac{1}{4} + \frac{3}{8}$ **9.** $3\frac{2}{5} \div 3\frac{1}{3}$ **10.** 5674×100
11. $86.452 \div 25$ **12.** $3\frac{1}{3} \times 874$ **13.** $12\frac{1}{2} \times 759$
14. $\frac{5}{6} =$ % **15.** $\frac{2}{125} =$ % **16.** Express $2\frac{3}{4}\%$ as a decimal.
17. Express $\frac{1}{8}$ as a per cent. **18.** 620 is $6\frac{2}{3}\%$ of . **19.** Cost \$600, decrease $12\frac{1}{2}\%$, find the selling price. **20.** Find the single discount equivalent to the discounts 50%, 50%.

Review 57. § 324

1. 103×106 **2.** 9602×55 **3.** 87302×392

4.
$$\begin{array}{r} 81056 \\ - \quad 5281 \\ \quad\;\; 963 \\ \quad 30743 \\ \quad\;\; 7286 \\ \quad\;\; 3967 \\ \hline \end{array}$$

5. $1998000 \div 1000$

6. Find the L. C. M. of 15, 24, 60.

7. $\frac{1}{4} + \frac{1}{8}$ **8.** $\frac{1}{5} + \frac{1}{6}$

9. $219\frac{1}{5} \div 9$ **10.** 12485×10

11. $219390 \div 1000$

12. Reduce $\frac{5}{61}$ to a decimal.

13. $16\frac{2}{3} \times 196$ **14.** $\frac{1}{2}$ = ___ % **15.** $\frac{4}{15}$ = ___ %

16. $\frac{11}{16}$ = ___ % **17.** $7\frac{1}{4}\%$ of 8279 is ___. **18.** Find 75% of 2700.

19. Cost 20¢, selling price 50¢, find the margin per cent on the selling price. **20.** List price \$1250.40, rate discount 40%, find the selling price.

Review 58. § 324

1. $398260 - 300948$ **2.** $\begin{array}{r} 578 \\ \times\ 307 \\ \hline \end{array}$ **3.** 28975×25

4. 59788×75 **5.** $636\overline{)29684}$ **6.** $872700 \div 50$

7. $1\frac{7}{8} + 4\frac{3}{16} + 7\frac{7}{32}$ **8.** $\frac{1}{5} + \frac{1}{8}$ **9.** $62\frac{1}{2} \times 26$

10. $6657 + 5.06 + 467 + 6.403 + 198.1 + 2.987 + 200.9$

11. 775.39×1000 **12.** $33\frac{1}{3} \times 295$ **13.** $6\frac{2}{3} \times 146$

14. $\frac{1}{3}$ = ___ % **15.** $\frac{1}{16}$ = ___ % **16.** $\frac{15}{16}$ = ___ %

17. 45 is ___ % of 1379. **18.** 315 is $7\frac{1}{2}\%$ of ___. **19.** Cost \$14, selling price \$18, find the rate of increase. **20.** Gross amount \$650, discounts $8\frac{1}{2}\%$, 8%, 5%, find the net amount.

Review 59. § 324

1. $386 + 460 + 752$ **2.** $\begin{array}{r} 758 \\ \times\ 287 \\ \hline \end{array}$ **3.** 21×69 **4.** 39682×50

5. $231725 \div 6$ **6.** $250800 \div 25$ **7.** $\frac{3}{4} + \frac{4}{5}$ **8.** $4\frac{3}{4} \div 5\frac{1}{3}$

9. $21\frac{1}{3} \times 12\frac{1}{2}$ **10.** $348.88 \div 10$ **11.** $66\frac{2}{3} \times 123$

12. $1240 \times 83\frac{1}{3}$¢ **13.** $\frac{2}{3}$ = ___ % **14.** $\frac{5}{16}$ = ___ %

15. Express $1\frac{3}{4}\%$ as a decimal. **16.** Express $\frac{3}{7}$ as a per cent.

17. 60 is how many per cent greater than 50? **18.** Selling price

$735, decrease 5%, find the cost. **19.** Find the single discount equivalent to the series 12%, 10%, 2%.

20. Find the totals for each day and for each item, and the grand total:

Day	Clothing	Groceries	Hardware	Furniture	Notions	Totals
Monday	$173.36	$235.40	$42.33	$498.55	$158.60	
Tuesday	210.65	200.14	52.38	255.63	127.65	
Wednesday	199.57	250.35	77.50	384.25	99.36	
Thursday	283.59	288.70	62.75	505.39	133.74	
Friday	312.82	300.10	91.35	441.67	95.23	
Saturday	372.12	312.33	56.36	359.65	145.50	
Totals						

Review 60. § 324

1. $961.357 + 699 + 756$ **2.** 834×406 **3.** 3646×11

4. 62128×901 **5.** $6400 \div 10$ **6.** $7382 \div 56$ **7.** $\frac{1}{2} + \frac{9}{16}$

8. $214\frac{1}{8} \div 18$ **9.** $150\frac{4}{7} \div 7$ **10.** $91422 \div 100$ **11.** $3\frac{1}{3} \times 476$

12. $8\frac{1}{3} \times 294$ **13.** $\frac{1}{5}$ = ___% **14.** $\frac{5}{8}$ = ___% **15.** $\frac{13}{16}$ = ___%

16. $8\frac{3}{4}$% of 2505 is ___. **17.** 926 is $3\frac{1}{3}$% of ___. **18.** Cost $400, rate of decrease 10%, find the selling price. **19.** $219\frac{1}{2} \div 3\frac{4}{5}$

20. List price $272.50, rate discount 27%, find the selling price.

Review 61. § 344

1. 45.75 + 11.04 + 10.35 + 44.69 **2.** 2×97 **3.** 8174×22

4. 7047×636 **5.** $64091 \div 8$ **6.** $58000 \div 200$

7. 1408×872 **8.** $7\frac{7}{8} \div 2\frac{1}{3}$ **9.** $\frac{734 + 745 + 729 + 735 + 740}{5}$

10. $156\frac{5}{6} \times 43$ **11.** 398.16×100 **12.** $58611 \div 125$

13. $3\frac{1}{3} \times 843$ **14.** $1540 \times 16\frac{2}{3}$¢ **15.** $\frac{1}{8}$ = ___%

16. $\frac{11}{16}$ = ___% **17.** Express $4\frac{3}{4}$% as a decimal. **18.** 61 is ___% of 5375. **19.** Gross amount $76, discounts 50%, 20%, 10%, find the net amount. **20.** Selling price $150, rate commission 5%, find the commission

Review 62. § 344

1. $54.37 + 329.21 + 8.56 + 76.58 + 324.87$ **2.** 103×108

3. 506×25 **4.** 6972×54 **5.** $874000 \div 1000$

6. $480000 \div 125$ **7.** $\frac{1}{2} + \frac{3}{4} + \frac{5}{8}$ **8.** $32\frac{2}{3} \div 3$

9. $189.9 + 3.548 + 30.64 + 2798 + 640.3 + 20.09 + 711.5$

10. $64.17\overline{)2812.81}$ **11.** \$2846.40 − (63.40, 353.65, 24.30, 543.85, 37.50) **12.** Express $\frac{13}{10}$ as a decimal.

13. $12\frac{1}{2} \times 527$

14. $\frac{3}{8}$ = ____ %

15. $\frac{15}{16}$ = ____ %

16. $9\frac{1}{2}\%$ of 6982 is ____. **17.** 235 is $12\frac{1}{2}\%$ of ____. **18.** Find the single discount equivalent to the series 30%, 30%, 10%. **19.** Selling price \$900, rate commission 7%, find the commission.

20. Find the total bill:

		25 bbl. Greenings	at	\$4.25			
		24 bbl. Kings	at	5.25			
		20 crates Lemons	at	4.10			
		14 boxes Floridas	at	5.12			
		130 doz. Pineapples	at	1.75			
		32 bbl. Sweet Potatoes	at	5.63			

Review 63. § 362

1. 459 × 201

2. 2768×77

3.

					Sums
	834	819	877	1465	
	98	73	46	283	
	1425	624	9374	429	
	625	3505	264	93	
	267	361	984	1839	
	462	98	821	427	
Sums					= total

4. 19669×50 **5.** $41000 \div 100$ **6.** $147500 \div 50$

7. $1\frac{3}{4} + 2\frac{1}{5} + 3\frac{3}{7} + 1\frac{3}{8}$ **8.** $36\frac{5}{6} \times 21\frac{1}{3}$ **9.** $419\frac{1}{3} \div 18$

10. 357.34×10 **11.** $156.12 \div 25$ **12.** $6\frac{1}{4} \times 347$

13. $16\frac{2}{3} \times 345$ **14.** Subtract:

9 mi.	210 rd.	3 ft.
	320 rd.	8 in.

15. $\frac{1}{12} =$ %

16. Express $4\frac{1}{4}\%$ as a decimal. **17.** 54 is % of 2516. **18.** Cost \$2.50, selling price \$7.50, find the margin per cent on the selling price. **19.** Gross amount \$850, discounts 50%, 20%, 10%, find the net amount. **20.** Principal \$3860, rate 5%, time 30 days, find the interest.

Review 64. § 362

1. 4084×11 **2.**

$$\begin{array}{r} 70243 \\ -\left\{\begin{array}{r} 5281 \\ 963 \\ 30743 \\ 7286 \\ 3967 \end{array}\right. \end{array}$$

3.

76
19
75
98
37

4. 7296×75

5. $4981\overline{)950904}$ **6.** Find the L. C. M. of 24, 48, 192.

7. $\frac{1}{2} + \frac{3}{8}$ **8.** $201\frac{5}{8} \div 8$ **9.** $106\frac{2}{3} \div 8\frac{2}{5}$ **10.** 76.791×1000

11. $53.642 \div 50$ **12.** $6\frac{2}{3} \times 855$ **13.** Add:

5 mi.	50 rd.	3 ft.
7	80	9
12	240	10

14. $\frac{1}{2} =$ % **15.** $\frac{3}{16} =$ % **16.** Express $\frac{1}{9}$ as a decimal.

17. 427 is $6\frac{1}{4}\%$ of . **18.** Selling price \$1728, rate of increase $12\frac{1}{2}\%$, find the cost. **19.** Find the single discount equivalent to the series 25%, 10%, 5%. **20.** Principal \$5095, rate $4\frac{1}{2}\%$, time 90 days, find the interest.

Review 65. § 362

1. $304506 - 168492$ **2.** 94×28 **3.** 2826×499

4. 98
37
62
30
49

5. $647472 \div 4$

6. $12 \times 15 \times 25 \times 36 \div 8 \times 18 \times 46$

7. $\frac{1}{2} + \frac{1}{8}$ **8.** $524\frac{2}{3} \div 6$

9. 6.4815×10 **10.** $598.276 \div 10$

11. $33\frac{1}{3} \times 265$ **12.** $8\frac{1}{3} \times 476$

13. Divide 9 mi. 125 rd. 2 yd. by 6. **14.** $\frac{1}{4}$ = %

15. $\frac{5}{16}$ = % **16.** $5\frac{1}{8}$% of 3176 is . **17.** 8 is 400% of what number? **18.** Cost \$7.25, selling price \$12, find the margin per cent on the selling price. **19.** Selling or buying price \$76.50, rate commission $3\frac{1}{2}$%, find the commission. **20.** Principal \$7560, rate $6\frac{1}{4}$%, time 60 days, find the interest.

Review 66. § 362

1. $24 + 80 + 41 + 80$ **2.** 3×42 **3.** 43645×109

4. Find the differences:

7808	6008
9795	4796
8896	7949
5957	5886

5. $91000 \div 10$ **6.** $\frac{1}{2} + \frac{7}{16}$

7. $5\frac{1}{2} \div 2\frac{3}{4}$ **8.** $134\frac{1}{2} \div 7$

9. $297\frac{1}{2} \div 9\frac{1}{2}$ **10.** $3838.6 \div 100$

11. Reduce $\frac{11}{30}$ to a decimal.

12. $3\frac{1}{3} \times 967$ **13.** $\frac{1}{3}$ = %

14. $\frac{1}{5}$ = % **15.** $\frac{7}{16}$ = % **16.** 64 is % of 1573. **17.** 257 is $8\frac{1}{3}$% of . **18.** List price \$916.52, rate discount 25%, find the selling price. **19.** Selling or buying price \$960.40, rate commission $4\frac{1}{2}$%, find the commission. **20.** Principal \$9560, rate 6%, time 35 days, find the interest.

Review 67. § 352

1. $58 + 49 + 86 + 24 + 90$ **2.** 7142×44

3. Find the differences:

5089	2560
6940	5320
3590	2130
7270	6420
4990	3260
9540	2530

4. 4735×728 **5.** $49970 \div 20$

6. $691\overline{)676}23$ **7.** $2400 \div 25$

8. $\frac{3}{4} + \frac{3}{5}$ **9.** 7.5343×100

10. $66\frac{2}{3} \times 300$ **11.** $12\frac{1}{2} \times 398$

12. $\frac{2}{3}$ = % **13.** $\frac{4}{5}$ = %

14. $\frac{9}{16}$ = %

15. Express 84% as a fraction in its lowest terms. **16.**

$$\$1473.10 - \begin{Bmatrix} 31.20 \\ 124.55 \\ 35.90 \\ 63.60 \\ 325.85 \end{Bmatrix}$$

17. Find $62\frac{1}{2}\%$ of 800. **18.** Selling price \$425, rate of increase 11%, find the cost. **19.** Gross amount \$359.50, discounts 16%, 10%, 5%, find the net amount. **20.** Principal \$6175, rate $5\frac{1}{4}\%$, time 45 days, find the interest.

Review 68. § 362

1. $43 + 78 + 88 + 17 + 54 + 91$ **2.** 956×439 **3.** 91×37

4. 56991×49 **5.** $39000 \div 1000$ **6.** $\frac{7}{9} + \frac{2}{15} + \frac{4}{5}$

7. $421\frac{7}{9} \div 11$ **8.** $790+751+9.486+389.4+42.52+1655+.303$

9. 193.28×100 **10.** $839700 \div 1000$ **11.** $2867.9 \div 125$

12. $16\frac{2}{3} \times 696$ **13.** $\frac{1}{4}$ = ___ % **14.** $\frac{3}{4}$ = ___ % **15.** $\frac{11}{16}$ = ___ %

16. Express 42% as a fraction in its lowest terms. **17.** 13 is ___ % of 247. **18.** Cost \$13.50, selling price \$25, find the margin per cent on the selling price. **19.** Find the single discount equivalent to the series 40%, 15%, 5%. **20.** Principal \$4785, rate $6\frac{2}{3}\%$, time 80 days, find the interest.

Review 69. § 362

1. $15294 - 7695$ **2.** 546×306 **3.** 62074×25 **4.** 6218×598

5. $94600 \div 300$ **6.** $33700 \div 50$ **7.** $\frac{1}{5} + \frac{1}{7}$ **8.** $215\frac{1}{6} \times 72$

9. $6.3591 \div 10$ **10.** $5.67\overline{)28.823}$ **11.** $33\frac{1}{3} \times 789$

12. $2600 \times 41\frac{2}{3}¢$ **13.** Subtract:

	11 mi.	260 rd.
	8	300

14. $\frac{5}{6}$ = ___ %

15. $\frac{13}{16}$ = ___ % **16.** $6\frac{1}{4} \times 2691$ **17.** 498 is $33\frac{1}{3}\%$ of ___.

18. List price \$862.76, rate discount 40%, find the selling price.

19. Selling or buying price \$900, rate commission 5%, find the commission. **20.** Principal \$4750, rate $5\frac{1}{2}$%, time 63 days, find the interest.

Review 70. § 362

1. 7×61 **2.** 56993×75 **3.** $76954 \div 3$ **4.** $5900 \div 100$
5. $2846000 \div 125$ **6.** $8\frac{5}{6} \div 3$ **7.** $15\frac{1}{6} \times 41\frac{2}{7}$
8. $174 + 658 + 572 + 503$ **9.**

$$\begin{array}{r} 83{,}140 \\ -\left\{\begin{array}{r} 31496 \\ 2738 \\ 7871 \\ 7534 \\ 936 \end{array}\right. \end{array}$$

10. 5.6045×1000

11. Reduce $\frac{11}{48}$ to a decimal. **12.** $6\frac{2}{3} \times 497$ **13.** $\frac{3}{5} =$ %
14. $\frac{1}{8} =$ % **15.** $\frac{15}{16} =$ % **16.** 26 is % of 481.
17. 25 is 50% of what number? **18.** List price \$10.80, rate discount $37\frac{1}{2}$%, find the selling price. **19.** Gross amount \$775, discounts 40%, 10%, 5%, find the net amount. **20.** Principal \$6590, rate $4\frac{3}{4}$%, time 100 days, find the interest.

Review 71. § 362

1. $152084 - 119847$ **2.** 91×96 **3.** 6804×33
4. $738\overline{)64883}$ **5.** 674×109 **6.** $816000 \div 10$
7. $8\frac{4}{9} + 7\frac{3}{8} + 2\frac{1}{3} + 3\frac{3}{4}$ **8.** $\frac{1}{2} + \frac{3}{4}$ **9.** $48\frac{3}{4} \div 8$
10. $329\overline{)846.482}$ **11.** $6\frac{1}{4} \times 107$ **12.** $8\frac{1}{3} \times 388$ **13.** $\frac{1}{5} =$ %
14. $\frac{5}{8} =$ % **15.** Express 12% as a fraction in its lowest terms.
16. $5\frac{1}{3}$% of 7162 is . **17.** 286 is 11% of . **18.** Selling price \$9.60, rate of decrease 30%, find the cost. **19.** Find the single discount equivalent to the discount series 50%, 20%. **20.** Principal \$6590, rate $5\frac{1}{2}$%, time 100 days, find the interest.

Review 72. § 362

1. 286×71 **2.** $18.38 + 42.62 + 53.16 + 39.62$

3. Find the differences:

7141	1006
6247	4758
4759	2978
9150	4791

4. 6996×501

5. 842×427

6. $828640 \div 30$

7. $\dfrac{893 + 894 + 883 + 884 + 875}{5}$

8. $318\frac{2}{3} \div 14$ **9.** 62.856×10 **10.** $844.71 \div 100$

11. $3\frac{1}{3} \times 958$ **12.** $12\frac{1}{2} \times 769$ **13.** $\frac{1}{2}$ = % **14.** $\frac{7}{8}$ = %

15. Express $\frac{2}{9}$ as a per cent. **16.** 35 is % of 453. **17.** 481 is 6% of . **18.** Cost \$976, selling price \$800, find the rate of decrease. **19.** Cost \$14, selling price \$30, find the margin per cent on the selling price. **20.** Buying or selling price \$385, rate commission 8%, find the commission.

Review 73. § 362

1. 497 × 826 **2.** 47358×25 **3.** 5160×11 **4.** 4521×901

5. $894\overline{)78321}$ **6.** $3926 \div 27$ **7.** Find the L. C. M. of 3, 6, 9, 21.

8. $10\frac{2}{5} \div 4\frac{1}{5}$ **9.** $8.7113 \div 25$ **10.** $66\frac{2}{3} \times 943$

11. $16\frac{2}{3} \times 401$ **12.** $\frac{1}{4}$ = % **13.**

$$\begin{array}{r} \$956.40 \\ \left\{\begin{array}{r} 105.26 \\ 42.30 \\ 98.20 \end{array}\right. \\ - \\ \begin{array}{r} 317.65 \\ 25.85 \end{array} \end{array}$$

14. $7\frac{3}{8}$ of 1713 is . **15.** $\frac{5}{12}$ = %. **16.** 6218×598 **17.** 597 is $16\frac{2}{3}$% of .

18. Cost \$7560, rate of increase $16\frac{2}{3}$%, find the selling price. **19.** List price \$8.75, rate discount $6\frac{1}{4}$%, find the selling price. **20.** Gross amount \$5600, discounts 15%, 25%, 35%, find the net amount.

Review 74. § 362

1. $936 + 459 + 549$ **2.** 937×405 **3.** 4256×88

4. 40534×50 **5.** 2974×397 **6.** $844000 \div 1000$

7. $86000 \div 25$ **8.** $\frac{1}{2} + \frac{5}{16}$ **9.** $193\frac{5}{8} \div 16$

10. $412\frac{1}{2} \div 11\frac{3}{4}$ **11.** $875.94 \div 50$ **12.** $6\frac{2}{3} \times 826$

13. $\frac{2}{5} =$ % **14.** $\frac{7}{12} =$ % **15.** 562 is 25% of .

16. Find the total bill:

1200 yds. Cashmere	at $2.55	per yd.	
1000 yds. Gingham	at .66⅔	per yd.	
850 yds. Silk	at 2.22	per yd.	
650 yds. India Linen	at 1.09⅓	per yd.	
500 yds. Fancy Shirting	at .66⅔	per yd.	

17. 1600 is 45% of what number? **18.** List price \$25.40, rate discount $62\frac{1}{2}$%, find the selling price. **19.** Find the single discount equivalent to the discount series 30%, 20%, 10%. **20.** Principal \$19260, rate 7%, time 60 days, find the interest.

Review 75. § 362

1. $369 + 924 + 255 + 803$ **2.** 6×28 **3.** 753×75

4.
$$\begin{array}{r} 13259 \\ -\left\{\begin{array}{r} 3646 \\ 618 \\ 1236 \\ 3149 \\ \underline{2632} \end{array}\right. \end{array}$$

5. 4908×55 **6.** $196843 \div 7$

7. $(94 \times 36 \times 48 \times 56) \div (8 \times 9 \times 14)$

8. $\frac{3}{4} + \frac{5}{12} + \frac{7}{8}$ **9.** $511\frac{1}{2} \div 4$

10. 6739.5×1000 **11.** $846329 \div 1000$

12. $33\frac{1}{3}$% of 579 is

13. Multiply 15 miles 2200 feet 7 inches by 6. **14.** $\frac{2}{3} =$ %

15. $\frac{11}{12} =$ % **16.** $\frac{13}{16} =$ % **17.** 53 is % of 2631.

18. Cost \$10.50, rate of increase 85%, find the selling price. **19.** Selling price \$40, rate of increase 40%, find the cost. **20.** Selling or buying price \$3500, rate commission $4\frac{1}{2}$%, find the commission.

Review 76. § 362

1. $90.34 + 27.96 + 26.81 + 69.87$ **2.** $10 + 95 + 66 + 70$

3. 109×107 **4.** 7286×50 **5.** 89297×42

6. $5142\overline{)884951}$ **7.** $605875 \div 3$ **8.** $86000 \div 50$

9. $3\frac{3}{7} \div 8$ **10.** $164 + 81.01 + 2.24 + 5213 + 135.6 + 7.024 + 13.25$

11. $9182.3 \div 10$ **12.** $597.64 \div 125$ **13.** $530 \times 87\frac{1}{2}$

14. Reduce 180 square feet to square yards. **15.** $\frac{1}{15}$ = % **16.** $\frac{15}{16}$ = % **17.** Express $\frac{5}{9}$ as a per cent. **18.** 750 is $16\frac{2}{3}$% of . **19.** Cost \$11.25, selling price \$22, find the margin per cent on the selling price. **20.** Gross amount \$1200, discounts $8\frac{1}{2}$%, 8%, 5%, find the net amount.

Review 77. § 362

1. 816.94+97.67+8.71+6.64+43.97 **2.** 78+40+66+96+79

3. 51273 − 47284 **4.** 537 × 478 **5.** 41 × 35 **6.** 45200 ÷ 10

7. 6300000 ÷ 125 **8.** $\frac{3}{4} + \frac{2}{5}$ **9.** $109\frac{7}{8} \div 3$

10. 54610 × 100 **11.** Reduce $\frac{13}{16}$ to a decimal. **12.** $12\frac{1}{2} \times 266$

13. Multiply 8 rd. 2 ft. 7 in. by 12. **14.** $\frac{2}{15}$ = %

15. Express $\frac{7}{8}$ as a per cent. **16.** $8\frac{1}{3}$% of 5731 is . **17.** 500 is 250% of what number? **18.** List price \$1725.41, rate discount $17\frac{1}{2}$%, find the selling price. **19.** Find the single discount equivalent to the discount series 16%, 8%, 2%. **20.** Principal \$7468, rate $7\frac{1}{2}$%, time 42 days, find the interest.

Review 78. § 362

1. 50892 − 7852 **2.** 626 × 708

3.

				Sums
1396	269	718	67	
419	393	89	378	
48	4646	499	234	
3178	236	4643	4256	
276	158	627	977	
2604	73	821	865	
Sums				= total

4. 4230 × 44 **5.** 4968 × 64 **6.** 842744 ÷ 5

7. 41160 ÷ 20 **8.** $180\frac{1}{2} + 216\frac{3}{4} + 3\frac{5}{8} + 1\frac{3}{16}$ **9.** $14\frac{1}{8} \div 12\frac{1}{2}$

10. 5.967 × 10 **11.** $66\frac{2}{3} \times 601$ **12.** $6\frac{2}{3} \times 537$ **13.** $\frac{1}{3}$ = %

14. $\frac{4}{15}$ = % **15.** $\frac{11}{16}$ = % **16.** 75 is % of 3127.

17. 1132 is 75% of . **18.** Selling price \$810, rate of increase 22%, find the cost. **19.** Selling or buying price \$175, rate commission 5%, find the commission. **20.** Principal \$9915, rate $7\frac{3}{4}$%, time 27 days, find the interest.

Review 79. § 362

1. 5×67 **2.** 7883×25 **3.** $3126\overline{)703049}$ **4.**

$$\begin{array}{r} 71 \\ 88 \\ 37 \\ \underline{82} \end{array}$$

5.

$$\begin{array}{r} 75776 \\ -\left\{\begin{array}{r} 1560 \\ 3762 \\ 5798 \\ 32645 \\ \underline{8774} \end{array}\right. \end{array}$$

6. $840000 \div 600$ **7.** $\frac{1}{5} + \frac{1}{6}$

8. $10\frac{2}{3} \div 2\frac{1}{3}$ **9.** $450\frac{1}{2} \div 15$

10. 57980×1000 **11.** $.191\overline{)765.124}$

12. $33\frac{1}{3} \times 665$ **13.** $6\frac{1}{4} \times 972$

14. $\frac{1}{2} =$ % **15.** $\frac{1}{16} =$ %

16. $\frac{13}{16} =$ %

17. Express 65% as a fraction in its lowest terms. **18.** 3650 is $66\frac{2}{3}$% of . **19.** Cost \$4.50, selling price \$10, find the margin per cent on the selling price. **20.** Gross amount \$3330, discounts 15%, 5%, 2%, find the net amount.

Review 80. § 362

1. $753 + 479 + 692$ **2.** $300139 - 113452$ **3.** 105×109

4.

$$\begin{array}{r} 81 \\ 50 \\ 65 \\ 11 \\ \underline{64} \end{array}$$

5. 86×94 **6.** $70000 \div 100$

7. $50000 \div 25$ **8.** $5\frac{1}{4} \div 8\frac{2}{3}$

9. $318\frac{2}{7} \times 47$ **10.** $69246 \div 10$

11. $48.035 \div 125$ **12.** $8\frac{1}{3} \times 150$

13. $\frac{2}{3} =$ % **14.** $\frac{1}{12} =$ %

15. $\frac{15}{16} =$ % **16.** Express 46% as a fraction in its lowest terms **17.** 87 is % of 6236. **18.** Selling price \$7.25, rate of increase 35%, find the cost. **19.** Find the single discount equivalent to the discount series 2%, 5%, 10%. **20.** Principal \$5074, rate 8%, time 19 days, find the interest.

Review 81. § 372

1. $\begin{array}{r} 1234 \\ \times\ 325 \\ \hline \end{array}$ **2.** 8×29 **3.** 8029×99 **4.** 8742×497

5. $84900 \div 30$ **6.** $563000 \div 125$ **7.** $9\frac{1}{8} \div 3\frac{1}{4}$

8. $13.6\overline{)3942.75}$ **9.** $8\frac{1}{3} \times 283$ **10.**

$$\$863.50 - \left\{ \begin{array}{r} 283.00 \\ 29.55 \\ 296.70 \\ 17.85 \\ 129.75 \end{array} \right.$$

11. Find the total bill:

		2 Gloves	at	$2.50			
		3 Suits	at	42.50			
		4 Towels	at	.84			
		3 Blouses	at	2.75			
		2 Dresses	at	12.75			

12. Multiply 5 miles 23 rods by 15. **13.** $7\frac{1}{2}\%$ of 7324 is . **14.** 19 is % of 3121. **15.** $8\frac{1}{3}\%$ of 3600 is . **16.** Cost \$3.20, selling price \$8, find the margin per cent on the selling price. **17.** Find the single discount equivalent to the discount series 25%, 10%, 5%. **18.** Principal \$6233, rate $6\frac{2}{3}\%$, time 95 days, find the interest. **19.** Principal \$2740, rate 5%, time 64 days, find the interest. **20.** Find the exact number of days from November 4 to December 22.

Review 82. § 372

1. $\begin{array}{r} 803 \\ \times\ 809 \\ \hline \end{array}$ **2.** 86×42 **3.** 6084×25 **4.** $51273 \div 4$

5. $621700 \div 700$ **6.** $5250000 \div 125$ **7.** $4361\frac{1}{8} \div 16$

8. 2587.9×10 **9.** $2490 \div 25$ **10.** $12\frac{1}{2} \times 254$

11. Multiply 24 yd. 2 ft. 7 in. by 5. **12.** $\frac{1}{4} =$ % **13.** $4\frac{5}{8}\%$ of 4942 is . **14.** 37 is % of 113. **15.** Selling price \$96, rate of decrease 6%, find the cost. **16.** List price \$98.75, rate discount 50%, find the selling price. **17.** Selling or buying price \$76.50, rate

commission $3\frac{1}{2}\%$, find the commission. **18.** Principal \$3650, rate $5\frac{3}{4}\%$, time 104 days, find the interest. **19.** Principal \$7616, rate 5%, time 66 days, find the interest. **20.** Find the exact number of days from April 5 to September 3.

Review 83. § 372

1. $\begin{array}{r} 61184 \\ -\ 57493 \\ \hline \end{array}$ **2.** 9289×11 **3.** 9137×50 **4.** $295\overline{)19725}$

5. $4840 \div 48$ **6.** $3\frac{9}{10} + \frac{1}{5} + \frac{4}{15} + \frac{3}{20}$ **7.** $6\frac{7}{8} \div 9\frac{2}{3}$

8. $50.89 + 1.789 + 7.115 + 574.9 + 8.972 + 28.18 + 4973$

9. $38.131 \div 100$ **10.** Express $\frac{7}{16}$ as a decimal. **11.** $16\frac{2}{3} \times 620$

12. Divide 47 yd. 2 ft. 6 in. by 14. **13.** $\frac{7}{16} =$ ____ % **14.** $7\frac{1}{4}\%$ of 526 is ____. **15.** Cost \$8.50, selling price \$20, find the margin per cent on the selling price. **16.** Gross amount \$5500, discounts 40%, 25%, 15%, find the net amount. **17.** Find the single discount equivalent to the series 30%, 15%. **18.** Principal \$7052, rate $6\frac{2}{3}\%$, time 95 days, find the interest. **19.** Principal \$9875, rate 5%, time 62 days, find the interest. **20.** Find the exact number of days from January 6 to July 3.

Review 84. § 372

1. $17 + 50 + 61 + 51$ **2.** $79 + 47 + 98 + 89 + 55$ **3.** 7×68

4. 16807×75 **5.** 4850×648 **6.** $75000 \div 100$

7. $38700 \div 25$ **8.** $\frac{1}{2} + \frac{1}{4}$ **9.** $234\frac{1}{2} \div 7\frac{3}{4}$ **10.** $6984.7 \div 50$

11. $66\frac{2}{3} \times 317$ **12.** $980 \times 66\frac{2}{3}¢$ **13.** $\frac{2}{5} =$ ____ % **14.** Express .84 as a fraction in its lowest terms. **15.** 847 is 8% of ____. **16.** 25 is how many per cent greater than 10? **17.** List price \$17.50, discount 45%, find the selling price. **18.** Selling or buying price \$220, rate commission 8%, find the commission. **19.** Principal \$5376, rate $5\frac{1}{2}\%$, time 66 days, find the interest. **20.** Find the exact number of days from July 6 to October 17.

Review 85. § 372

1. $19 + 82 + 24 + 71 + 84 + 57$ **2.** $\begin{array}{r} 9008 \\ \times\ 807 \\ \hline \end{array}$ **3.** 426×32

4. 2097×108 **5.** $597911 \div 2$ **6.** $\frac{921+918+919+923+925}{5}$

7. $98\frac{5}{8} \div 12$ **8.** 2.7382×100 **9.** $6425.9 \div 25$ **10.** $3\frac{1}{3} \times 627$

11. Subtract:

16 mi.	235 rd.	9 ft.
7	315	6

12. $\frac{3}{8}$ = ____ %

13. Express $\frac{2}{7}$ as a per cent.

14. 43 is ____ % of 1136

15. Selling price \$25.12, rate of decrease 8%, find the cost. **16.** List price \$10.50, rate discount 30%, find the selling price. **17.** Gross amount \$12580, discounts 50%, 30%, 5%, find the net amount. **18.** Find the single discount equivalent to the series 5%, 10%, 20%. **19.** Principal \$2825, rate 5%, time 69 days, find the interest. **20.** Find the exact number of days from November 23 to March 5.

Review 86. § 372

1. 9402×639 **2.** 17542×798 **3.** $709\overline{)66036}$

4.
$$\begin{array}{r} 90059 \\ 5971 \\ 35796 \\ -\quad 480 \\ 8169 \\ \underline{24169} \end{array}$$

5. Find the L. C. M. of 17, 54, 108.

6. $3\frac{1}{2} \div 3\frac{3}{4}$ **7.** $48213.6 \div 1000$

8. $57900 \div 125$ **9.** $6\frac{1}{4} \times 268$

10. $\frac{1}{3}$ = ____ % **11.** $\frac{7}{8}$ = ____ %

12. $7\frac{5}{6}$ of 827 is ____ **13.** 626 is $16\frac{2}{3}$% of ____

14. Cost \$17.50, selling price \$50, find the margin per cent on the selling price. **15.** List price \$12, rate discount 22%, find the selling price. **16.** Find the single discount equivalent to the series 12%, 10%, 2%. **17.** Selling or buying price \$1845, rate commission 7%, find the commission. **18.** Principal \$5692, rate $6\frac{3}{4}$%, time 48 days, find the interest. **19.** Principal \$890, rate $5\frac{1}{4}$%, time 64 days, find the interest. **20.** Find the exact number of days from February 20 to August 5.

Review 87. § 372

1. $601337 - 335753$ **2.** 2122×66 **3.** 2160×25

4. 638×954 **5.** $600000 \div 1000$

6. $16 \times 36 \times 64 \times 96 \div 24 \times 8 \times 18$ **7.** $431\frac{3}{8} \div 5$

8. $2.75\overline{)74511.3}$ **9.** Reduce $\frac{7}{45}$ to a decimal. **10.** $6\frac{2}{3} \times 478$ **11.** $\frac{1}{4}$ = ___ % **12.** $\frac{5}{12}$ = ___ % **13.** 26 is ___ % of 328. **14.** Selling price \$73.50, rate increase 55%, find the cost. **15.** Gross amount \$5250, discounts 40%, 30%, 15%, find the net amount. **16.** Selling or buying price \$5000, rate commission $3\frac{1}{4}$%, find the commission. **17.** Principal \$17960, rate $7\frac{1}{2}$%, time 96 days, find the interest. **18.** $\begin{array}{r} 3507 \\ \times\ 491 \\ \hline \end{array}$ **19.** Principal \$1726, rate $5\frac{1}{2}$%, time 65 days, find the interest. **20.** Find the exact number of days from February 5 to June 16.

Review 88. § 372

1. 6×67 **2.** 82248×45 **3.** $70460 \div 40$

4. Find the differences:

8901	1036
5836	5286
8119	5651
2594	1675

5. $\frac{1}{2} + \frac{3}{16}$ **6.** $7\frac{1}{2} \div 9\frac{1}{3}$ **7.** $59\frac{1}{6} \times 217\frac{1}{8}$ **8.** $3572.98 \div 25$ **9.** $33\frac{1}{3} \times 581$ **10.** $8\frac{1}{3} \times 206$ **11.** $\frac{1}{5}$ = ___ %

12. Express 35% as a fraction in its lowest terms. **13.** $6\frac{5}{12} \times 372$ **14.** 15 is how many per cent less than 25? **15.** Cost \$2.50, selling price \$9, find the margin per cent on the selling price. **16.** List price \$1080, rate discount $26\frac{1}{2}$%, find the selling price. **17.** Buying or selling price \$1820, rate commission 4%, find the commission. **18.** Principal \$7062, rate $6\frac{2}{3}$%, time 95 days, find the interest. **19.** Principal \$7616, rate 5%, time 62 days, find the interest. **20.** Find the exact number of days from March 3 to October 9.

Review 89. § 372

1. $977 + 615 + 869$ **2.** 67×29 **3.** 1814×735 **4.** $490\overline{)47004}$ **5.** $215\frac{1}{8} \div 13$ **6.** $.52819 \times 10$ **7.** $69.507 \div 50$ **8.** $3\frac{1}{3} \times 894$ **9.** $12\frac{1}{2} \times 187$ **10.** $\frac{11}{16}$ = ___ % **11.** $\frac{1}{16}$ = ___ % **12.** Express $\frac{9}{7}$ as a per cent. **13.** 51 is ___ % of 625. **14.** Cost \$45, rate of increase 24%, find the selling price. **15.** Selling price \$52.50, rate of decrease 23%, find the cost. **16.** Gross amount \$2550, discounts 50%, 30%,

5%, find the net amount. **17.** Find the single discount equivalent to the series 40%, 30%. **18.** Principal \$31580, rate 6%, time 54 days, find the interest. **19.** Principal \$7540, rate 5%, time 67 days, find the interest. **20.** Find the exact number of days from July 15 to November 14.

Review 90. § 372

1. $356 + 788 + 232 + 908$ **2.** 4927×55 **3.** $816{,}608 \div 3$
4. $8764 \div 36$ **5.** $7\frac{1}{8} + 4\frac{5}{7}$ **6.** $9.2719 \div 100$ **7.** $66\frac{2}{3} \times 631$
8. $6\frac{2}{3} \times 678$ **9.** $16\frac{2}{3} \times 522$

10. Find the total bill (see page 162):

	Tubs Butter, 65–12, 68–12, 70–12, 72–12, 67 12, 68–12	$48\frac{1}{2}$¢	
	Kegs Pickled Pigs' Feet 85–21, 83–21, 88–21, 89–21	$19\frac{1}{2}$¢	
	Tubs Lard 80–15, 76–15, 69–15, 77–15, 80–15, 68–18	$28\frac{1}{2}$¢	

11. Express 44% as a fraction in its lowest terms. **12.** $8\frac{5}{8} \times 341$ **13.** 481 is $26\frac{2}{3}$% of . **14.** 76 is % of 846. **15.** Cost \$1.75, selling price \$9, find the margin per cent on the selling price. **16.** List price \$90.50, rate discount 33%, find the selling price. **17.** Selling or buying price \$900, rate commission $3\frac{1}{4}$%, find the commission. **18.** Principal \$2956, rate $5\frac{3}{4}$%, time 90 days, find the interest. **19.** Principal \$12650, rate $5\frac{1}{4}$%, time 60 days, find the interest. **20.** Find the exact number of days from June 6 to September 8.

Review 91. § 372

1. 1346×897 **2.** $82100 \div 100$ **3.** $663300 \div 50$
4. 12 **5.** $95\frac{1}{5} \div 9$ **6.** $394.275 \div 1000$ **7.** $6\frac{1}{4} \times 885$
58 **8.** $1460 \times 26\frac{2}{3}$¢
42 **9.** 54 is % of 512.
18 **10.** 291 is $12\frac{1}{2}$% of .

11. 120 is how many per cent greater than 100? **12.** Cost \$39, rate increase 25%, find the selling price. **13.** Cost \$4, selling price \$12, find the margin per cent on the selling price. **14.** Gross amount \$390, discounts 30%, 20%, 12%, find the net amount. **15.** Selling or buying price \$2786.90, rate commission $3\frac{1}{8}$%, find the commission. **16.** Principal \$7825, rate $5\frac{1}{2}$%, time 45 days, find the interest. **17.** $576\overline{)401068}$ **18.** Find the single discount equivalent to the series 60%, 10%. **19.** Principal \$9240, rate 5%, time 67 days, find the interest. **20.** Find the exact number of days from January 8 to April 10.

Review 92. § 372

1. 4×89 **2.** $90000 \div 1000$ **3.** Find the L. C. M. of 9, 27, 81. **4.** $5\frac{2}{3} \div 3\frac{1}{6}$ **5.** $119\frac{2}{7} \div 13$ **6.** $1.23\overline{)234.511}$ **7.** $6\frac{2}{3} \times 276$

8.
99
26
42
997
78

9. Divide 116 mi. 14 ft. 8 in. by 5.

10. $\frac{7}{12} =$ %

11. Express $\frac{5}{6}$ as a per cent.

12. 69 is % of 964.

13. 814 is $58\frac{1}{3}$% of .

14. Cost \$15.40, selling price \$27, find the rate of increase. **15.** Cost \$251.20, rate of increase 45%, find the selling price. **16.** List price \$3060, rate discount 40%, find the selling price. **17.** Find the single discount equivalent to the series 40%, 30%, 5%. **18.** Principal \$1725, rate $6\frac{3}{4}$%, time 87 days, find the interest. **19.** Principal \$2440, rate $5\frac{1}{2}$%, time 65 days, find the interest. **20.** Find the exact number of days from May 2 to August 3.

Review 93. § 372

1. $75 + 88 + 89 + 27$ **2.** 23×75 **3.** $78944 \div 4$ **4.** $\frac{3}{4} + \frac{1}{5}$ **5.** $9\frac{1}{2} \div 7\frac{1}{8}$ **6.** $312.944 \div 25$ **7.** $8\frac{1}{3} \times 780$ **8.** Divide 16 rd. 5 ft. 6 in. by 3. **9.** $\frac{11}{12} =$ % **10.** $8\frac{2}{3} \times 288$ **11.** 392 is $87\frac{1}{2}$% of . **12.** 16 is 45% of what number? **13.** Cost \$35, selling price \$75, find the rate of increase. **14.** Cost \$14.50, selling price \$30, find the margin per cent on the selling price. **15.** Gross amount \$10050, discounts 40%, 25%, 10%, find the net

amount. **16.** Selling or buying price \$1729.50, rate commission $3\frac{1}{8}\%$, find the commission. **17.** Principal \$17250, rate $7\frac{1}{2}\%$, time 80 days, find the interest. **18.** Principal \$3425, rate $4\frac{1}{2}\%$, time 68 days, find the interest. **19.** Find the exact number of days from April 1 to July 6. **20.** 302 is $16\frac{1}{4}\%$ of .

Review 94. § 372

1. $75 + 26 + 68 + 11 + 19$ **2.** 82125×108 **3.** $75000 \div 50$
4. $4\frac{2}{5} \div 5\frac{1}{4}$ **5.** $345\frac{1}{3} \div 17$ **6.** $94978 \div 50$ **7.** $12\frac{1}{2} \times 328$
8. $\frac{1}{2} =$ % **9.** $\frac{3}{16} =$ % **10.** 48 is % of 814.
11. 245 is $6\frac{2}{3}\%$ of . **12.** 6 is how many per cent less than 10?
13. Cost \$27.50, selling price \$80, find the rate of increase. **14.** Cost \$39.50, selling price \$72.50, find the margin per cent on the selling price. **15.** List price \$9.80, rate discount 12%, find the selling price.
16. Find the single discount equivalent to the discount series 20%, 15%. **17.** Principal \$2950, rate $8\frac{1}{2}\%$, time 48 days, find the interest.
18. Principal \$6774, rate $1\frac{3}{4}\%$, time 62 days, find the interest.
19. Find the exact number of days from May 30 to October 6.
20. $8\frac{2}{3}\%$ of 288 is .

Review 95. § 372

1.
$$\begin{array}{r} 833216 \\ -\ 579650 \\ \hline \end{array}$$

2. 5978×369 **3.** $402400 \div 900$ **4.** $11\frac{1}{6} \div 5\frac{3}{3}$
5. Reduce $\frac{7}{45}$ to a decimal. **6.** $16\frac{2}{3} \times 478$
7. $\frac{1}{5} =$ % **8.** $\frac{5}{16} =$ % **9.** \$2369.70

$$-\begin{cases} 984.20 \\ 57.60 \\ 389.55 \\ 61.80 \\ \hline 528.75 \end{cases}$$

10. Express 38% as a fraction in its lowest terms.
11. $7\frac{3}{16}\%$ of 678 is . **12.** 42 is % of 814.
13. 200 is what per cent of 800?
14. Selling price \$610, 12% decrease, find the cost.
15. Gross amount \$5600, discounts 25%, 10%, 8%, find the net amount. **16.** Buying or selling price \$1250, commission rate $3\frac{1}{2}\%$, find the commission. **17.** Principal \$978.60, rate $9\frac{3}{4}\%$, time 11 days, find the interest. **18.** Principal \$3760, rate $4\frac{1}{2}\%$, time 61 days, find the interest. **19.** Find the exact number of days from July 19 to December 5. **20.** 742 is $66\frac{2}{3}\%$ of .

Review 96. § 372

1. $264 + 787 + 458$ **2.** 4231×518 **3.** $362745 \div 5$ **4.** $7626 \div 54$

5. $\frac{7}{8} + \frac{7}{16}$ **6.** $190\frac{1}{2} \div 12$ **7.** $33\frac{1}{3} \times 934$ **8.** $\frac{3}{5} =$ %

9. $\frac{7}{16} =$ %

10. Find the total bill:

	1000 ft. 1″ Medium Quartered Sawed White Oak at	$105.00 per M			
	1800 ft. 2″ Com. White Pine at	50.00 per M			
	2400 ft. 2 × 4 Spruce at	35.00 per M			
	1200 ft. 1″ Plain White Oak A-1 at	68.00 per M			
	400 ft. $1\frac{1}{4}$″ Best Quartered Sawed White Oak at	100.00 per M			

11. Express 55% as a fraction in its lowest terms. **12.** $6\frac{3}{4}\%$ of 498 is . **13.** 63 is % of 768. **14.** Find $26\frac{2}{3}\%$ of 250. **15.** Cost \$22.50, selling price \$50, find the margin per cent using the selling price as the base. **16.** List price \$347.50, rate discount 25%, find the selling price. **17.** Find the single discount equivalent to the series 10%, 20%, 30%. **18.** Principal \$91.60, rate $5\frac{3}{4}\%$, time 96 days, find the interest. **19.** Find the exact number of days from August 9 to November 30. **20.** Principal \$7952, rate $3\frac{3}{4}\%$, time 66 days, find the interest.

Review 97. § 372

1. $246+61.27+877.2+33.09+3985+.231+682.2$ **2.** $659\overline{)59347}$

3. 8601×77 **4.** $894500 \div 25$ **5.** $9\frac{2}{3} \div 8\frac{1}{6}$ **6.** $745.112 \div 1000$

7. $87400 \div 25$ **8.** $\frac{2}{3} =$ % **9.** $\frac{4}{5} =$ % **10.** $\frac{9}{16} =$ %

11. Express $\frac{3}{5}$ as a per cent. **12.** 72 is % of 762. **13.** Cost \$1320, rate of decrease $33\frac{1}{3}\%$, find the selling price. **14.** Selling price \$395, rate of increase $16\frac{2}{3}\%$, find the cost. **15.** Gross amount \$5230, discounts 5%, 35%, 10%, find the net amount. **16.** Selling or buying price \$650, commission rate $4\frac{1}{2}\%$, find the commission.

17. Principal \$8756, rate $5\frac{3}{4}\%$, time 17 days, find the interest. **18.** Principal \$8640, rate $7\frac{1}{2}\%$, time 67 days, find the interest. **19.** Find the exact number of days from February 15 to July 31. **20.** $701\frac{3}{8} + 9\frac{4}{5}$

Review 98. § 372

1. $61.27 + 65.44 + 87.72 + 33.09$ **2.** 647×129 **3.** $655900 \div 50$ **4.** $16\frac{1}{2} \div 7\frac{1}{2}$ **5.** $27.3\overline{)815.352}$ **6.** $63870 \div 50$ **7.** $12\frac{1}{2} \times 512$ **8.** $\frac{3}{4} =$ ____ % **9.** $\frac{13}{16} =$ ____ % **10.** Express $\frac{8}{11}$ as a per cent. **11.** $9\frac{2}{3}\%$ of 3413 is ____. **12.** 47 is ____ % of 678. **13.** 35 is how many per cent less than 75? **14.** Selling price \$9.75, rate of decrease 22%, find the cost. **15.** List price \$32.50, rate discount 18%, find the selling price. **16.** Find the single discount equivalent to the series 20%, 30%, 40%. **17.** Selling or buying price \$3300, commission rate 8%, find the commission. **18.** Principal \$5412, rate $8\frac{3}{4}\%$, time 81 days, find the interest. **19.** Find the exact number of days from August 10 to December 24. **20.** Principal \$3474, rate $7\frac{1}{2}\%$, time 65 days, find the interest.

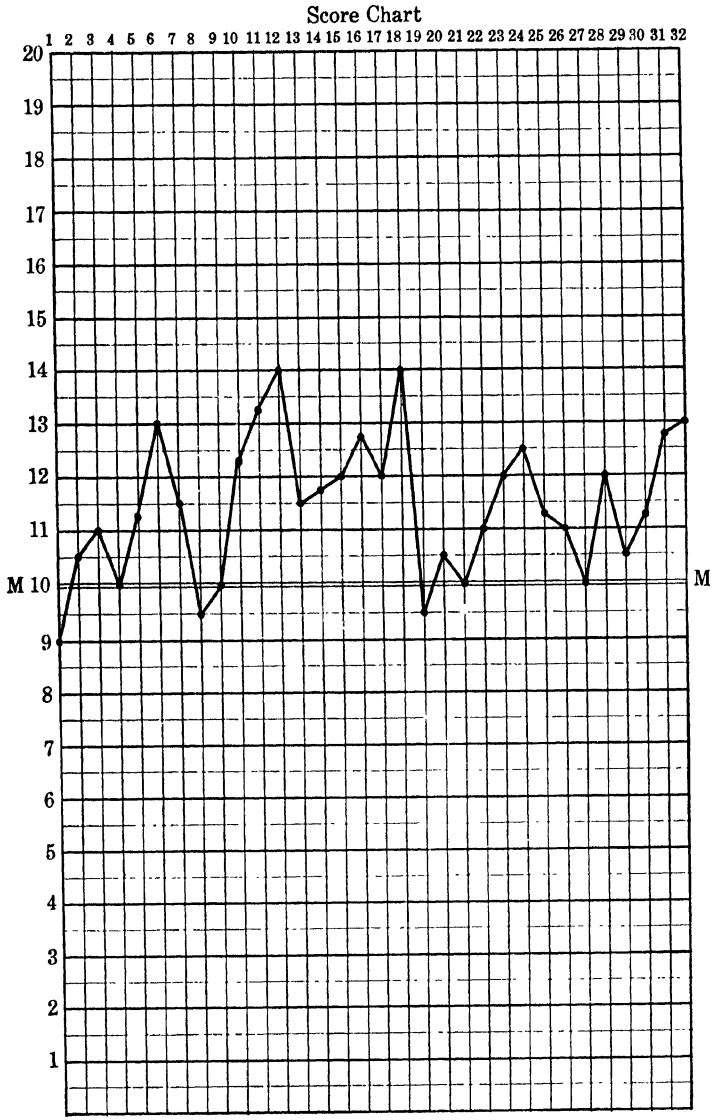
Review 99. § 372

1. $15 + 86 + 72 + 37$ **2.** 216×48 **3.** $939000 \div 1000$ **4.** $\frac{1}{2} + \frac{7}{8}$ **5.** $3\frac{2}{3} \div 5\frac{1}{6}$ **6.** $348\frac{2}{3} \div 12$ **7.** $852 + 60.74 + 261.4 + 8.913 + 3975 + 78.41$ **8.** $6\frac{1}{4} \times 494$ **9.** $16\frac{2}{3} \times 187$ **10.** $\frac{5}{6} =$ ____ % **11.** $\frac{15}{16} =$ ____ % **12.** Express $\frac{11}{12}$ as a per cent. **13.** $9\frac{5}{9}\%$ of 5028 is ____. **14.** 933 is $46\frac{2}{3}\%$ of ____. **15.** Cost \$15, selling price \$24.50, find the rate of increase. **16.** Selling price \$369, rate of decrease 11%, find the cost. **17.** Selling or buying price \$925.18, rate commission $3\frac{1}{4}\%$, find the commission. **18.** Principal \$1721, rate $6\frac{3}{4}\%$, time 84 days, find the interest. **19.** Principal \$8348, rate $7\frac{1}{4}\%$, time 66 days, find the interest. **20.** Find the exact number of days from June 8 to October 29.

Review 100. § 372

1. 817×75 **2.** $97400 \div 60$ **3.** $\frac{1}{2} + \frac{9}{11}$ **4.** $7\frac{3}{7} \div 2\frac{2}{3}$ **5.** $25\frac{1}{2} \times 102\frac{1}{3}$ **6.** $23.7234 \div 1000$ **7.** $16\frac{2}{3} \times 628$

8. $\frac{1}{6}$ = % **9.** $\frac{1}{8}$ = % **10.** Express 66$\frac{2}{3}$% as a fraction in its lowest terms. **11.** Express $\frac{5}{12}$ as a per cent. **12.** 39 is % of 453. **13.** 315 is 41$\frac{2}{3}$% of . **14.** Cost \$52.50, selling price \$45, find the rate of decrease. **15.** Cost \$29.75, selling price \$40, find the margin per cent on the selling price. **16.** 9$\frac{7}{8}$% of 2946 is . **17.** Selling or buying price \$1522, rate commission 3$\frac{1}{8}$%, find the commission. **18.** Principal \$1495, rate 6$\frac{2}{3}$%, time 21 days, find the interest. **19.** Principal \$2345, rate 4$\frac{3}{4}$%, time 67 days, find the interest. **20.** Find the exact number of days from January 2 to October 9.


Score Chart
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32
20
19
18
17
16
15
14
13
12
11
M 10
9
8
7
6
5
4
3
2
1
M

RECORD OF SCORES IN STANDARDIZED TESTS

Each student should rule a card like that shown on page 508, and on it keep a record of his scores in the Standardized Tests on Fundamentals. The total score for each test is twenty units. The scoring values for the examples are given in the table on page 510.

For example, a student who gets correct answers for Examples 1–14 in Test I will have a score of 10 which is a Median Standard Score. One who gets more than 10 units has a score higher than the median and one who gets less than 10 units has a score below the median. Only correct answers are counted, for in business a wrong answer is worse than no answer.

Some students work more rapidly and are more accurate than the average and they will regularly obtain scores higher than the median. The total work given in each test is just twice median.

In taking the tests, make your best effort to work rapidly and accurately; get the very highest scores that you can. Careful study of the various topics in the text and much practice on the exercises given there will be of great help.

If a pupil gets a score of 10 in the first test, he puts a dot or cross on the horizontal line marked 10 and in the first column. The score for the second test is marked in the second column and so on for the thirty-two tests. The dots or crosses may be connected by means of lines, making a graph which shows the scores.

Suppose that a pupil gets the scores 9, $10\frac{1}{2}$, 11, 10, $11\frac{1}{4}$, 13, $11\frac{1}{2}$, $9\frac{1}{2}$, 10, $12\frac{1}{4}$, $13\frac{1}{4}$, 14, $11\frac{1}{2}$, $11\frac{3}{4}$, 12, $12\frac{3}{4}$, 12, 14, $9\frac{1}{2}$, $10\frac{1}{2}$, 10, 11, 12, $12\frac{1}{2}$, $11\frac{1}{4}$, 11, 10, 12, $10\frac{1}{2}$, $11\frac{1}{4}$, $12\frac{3}{4}$, 13 on the thirty-two tests; then his record will be that shown on page 508.

Note that scores, such as $10\frac{1}{2}$, which contain a half are placed on the light lines, while scores containing $\frac{1}{4}$ and $\frac{3}{4}$ are placed midway between lines.

Scoring Values for Tests

Ex.	TESTS																													
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
1	½	1	1	1	1	½	1½	1	1	1	1	1	2	1	1	1½	1½	1½	1½	1½	1½	1½	1½	2	1	1½	½	2	1½	2	1½	2
2	½	½	1	1	1	½	1½	1	1	½	1	½	2	1	½	1½	1½	1½	1½	1½	1½	1½	1½	1½	1	1½	½	1½	1½	1½	1½	1½
3	½	1	½	1	1	½	1½	1½	1	1	1	½	1½	2	1	2	1½	1½	1½	1½	1½	1½	1½	1½	2	½	1½	1	1½	1	1½
4	½	½	½	1	1	½	1½	½	½	1	½	1	1	1½	1	1	1½	½	1½	1½	1½	1½	1½	1½	1	½	1½	1	1½	1	1½
5	1	½	½	1	1	½	½	½	½	½	½	1½	1	2	1½	½	½	½	½	1	1	1	1½	1	2	1	1	1½	1	1½	1	1½
6	1	1	½	1	½	½	½	½	½	1½	½	1	1	1	1	½	½	1	½	½	1	½	½	1	1½	1	1	1½	½	1½	1	1
7	1	½	½	½	½	½	½	1	½	1	1	1	½	½	1	1½	1	1	1	1	½	½	1½	½	1	½	¼	½	¼	½	1	1
8	½	½	½	½	½	½	½	1	½	½	1	1	½	½	2	½	1	1	1	1	1	½	½	1½	1½	1	1½	½	1	½	1	½
9	½	½	½	½	1	½	½	1	½	½	½	½	2	1	2	½	1	½	1	1	1½	½	½	1½	1	1½	1	½	1	½	½	1½
10	½	½	½	½	½	½	1	1½	½	½	½	½	2	1	1	½	1½	1½	½	1½	1½	½	½	2	1½	1½	1	1½	1	1½	½	1½
11	½	½	½	½	½	½	½	1½	½	½	½	½	1½	2	1	½	1½	1½	1½	1½	1½	1	1	1½	1	1	1	1½	½	2	1	2
12	1	½	½	½	½	½	1	½	1	1	½	½	1	1	1½	1	1½	1½	1½	1½	1½	1½	1½	1	1½	2	1	2	1½	1½	1	1½
13	1	½	½	1	1	1	1½	½	1	1	½	½	1	2	1	1½	1½	1½	1½	1½	1	1½	1½	1	1½	1	1	1½	1½	1½	1½	1
14	1	1	½	1	1	½	1½	½	½	½	1½	½	1	1½	½	1½	1½	½	½	1	1	1½	1½	1½	1½	1	1	1½	1	1½	1½	1½
15	1	½	1	1	1	½	1½	½	1	½	1	½	1	1	½	2	½	½	½	½	1	1½	1½		2	1	1	1½	1	1½	1	½
16	1	½	1	1	1	½	1½	½	½	½	½	½	½	1	1	1	½	½	1½	1	1	½	½		½	1	½	½	½	½	1	1
17	1	½	1	1	1	½	½	½	½	½	½			½	2	½	1	1½	½	1		½	½				1½	½	½	½	1	
18	½	½	1	1	1	½	½	1	½	1	½	1		1	2	½	1	1	1	1		½	½				1½	½	1	½	1	
19	½	1	½	½	½	1	½	1½	½	½	1	1½				1		1	1			1½	1½				1		1		½	
20	½	½	½	½	½	1	½	1	½	1	½					½		1	1			½	½				1		1		½	
21	½	½	½	1	1	½	½	1	1	1½	1	½				½						1	1				½		1			
22	1	1	½	1	½	½	1	½	1	1	1	1				½						½	½				½		½			
23	1	½	½	½	1	½	1	½	1	1	1	½				½											½					
24	1	1	½	½	½	½	½	1½	1	½	½	½				1											½					
25	½	½	½	½	½	½		½	1	1	½	1															1					
26	½	1	½	½	½	½		½	1	½	½	1															1					
27	½	½	1	1	1	½		1	½	½	½	½																				
28	½	½	1	1	1	½		½	½	½	½	½																				
29		½	½			½			½	½	½	½																				
30		½	½			½			½	1	½	½																				
31		½	½			½			½	½	1½	1½																				
32		½	½			½			½	½	1	½																				
33		½	1			1			½	½																						
34		½	1			½			1	½																						
35		½	1			½																										
36		½	1			½																										
37						½																										
38						½																										
39						1																										
40						1																										

STANDARDIZED TESTS ON FUNDAMENTALS

These tests may or may not come immediately following the study of the topics indicated by the accompanying page references. Your teacher will decide. Prepare for them, by faithful practice on the fundamentals represented, rather than by memorizing the tests themselves. The latter plan has disadvantages which will affect your scores.

Test 1

Time 8 min. **Page 18**

1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**
27	35	96	23	743	695	369
54	74	80	33	541	308	637
63	25	78	86	722	268	877
23	67	79	84	105	795	269
($\frac{1}{2}$) 64	($\frac{1}{2}$) 41	($\frac{1}{2}$) 67	($\frac{1}{2}$) 56	(1) 406	(1) 839	(1) 352
32	93	95	96	554	580	609
52	86	24	42	384	877	968
43	70	26	74	789	416	508
				778	369	279
				916	638	859

8. $27 + 86 + 40 + 58 + 63 + 55$ ($\frac{1}{2}$)

9. $38 + 87 + 70 + 89 + 98 + 64$ ($\frac{1}{2}$)

10. $96 + 49 + 76 + 48 + 95 + 56$ ($\frac{1}{2}$)

11. $73 + 99 + 58 + 95 + 57 + 46$ ($\frac{1}{2}$)

12. $929 + 947 + 768 + 976 + 887 + 577$ (1)

13. $459 + 809 + 660 + 787 + 850 + 489$ (1)

14. $684 + 509 + 578 + 468 + 370 + 409$ (1)

15.	**16.**	**17.**	**18.**	**19.**	**20.**	**21.**
987	359	168	59	37	67	92
840	690	259	66	92	69	41
678	869	386	97	69	44	87
(1) 584	(1) 778	(1) 887	($\frac{1}{2}$) 89	($\frac{1}{2}$) 53	($\frac{1}{2}$) 49	($\frac{1}{2}$) 91
679	747	824	42	76	86	78
578	987	964	98	76	29	96
869	587	793	67	95	87	16
400	580	750	78	96	53	73
405	789	777				
786	864	908				

22. 148 + 628 + 269 + 308 + 519 + 501
23. 462 + 284 + 149 + 532 + 651 + 980
24. 567 + 155 + 689 + 426 + 538 + 132
25. 31 + 61 + 50 + 81 + 37 + 98
26. 82 + 46 + 56 + 69 + 24 + 42
27. 48 + 43 + 12 + 26 + 53 + 44
28. 86 + 61 + 86 + 59 + 83 + 18

Test 2

Time 8 min. **Page 29**

1.
386
453
679
698
906
877
480
416
594
865

2.
50
96
79
51
35
19
47
35

3.
799
864
558
805
734
897
746
991
281
748

4.
65
24
50
46
96
82
78
16

5.
92
69
79
26
76
16
29
23

6.
607
849
556
105
868
209
371
328
678
439

7.
25
83
27
67
94
82
47
91

8. 45347 − 26089
9. 84652 − 79825

10.
57459
56193
17145
83238

11.
96754
79163
68495
17896

12. 97 + 49 + 62 + 43 + 38 + 58
13. 15 + 72 + 56 + 79 + 87 + 45
14. 969 + 647 + 456 + 888 + 517 + 763

15.
936128
− 577039

16. 34964 − 29147

17.
627856
− 286497

18. 40621 − 37859

19.	**20.**	**21.**	**22.**	**23.**	**24.**	**25.**
467	48	22	673	38	586	41
123	98	30	939	79	628	57
582	56	56	705	28	289	96
819	23	88	679	79	727	55
564	94	75	838	97	958	99
845	45	97	788	72	875	87
706	73	14	560	86	726	96
503	49	67	599	66	978	89
469			786		865	
394			446		689	

26. $386 + 257 + 465 + 198 + 594 + 428$

27. $71103 - 44056$

28. $27842 - 13867$

29. $815604 - 460599$

30. $357230 - 268973$

31. $38 + 62 + 57 + 81 + 56 + 40$

32. $46 + 50 + 94 + 73 + 89 + 53$

33. $79183 - 29979$

34. $59570 - 29648$

35.

45162
99739
19791
20595

36.

30271
62835
24785
30354

Test 3

Time 8 min. **Page 33**

1.	**2.**	**3.**	**4.**
582	248	43	89
397	321	86	21
426	898	38	76
540	789	17	99
898	355	17	42
570	903	50	60
507	580	33	55
866	499	51	61
773	119		
424	469		

5. $893476 - 539847$

6. $944578 - 511389$

7. $636595 - 328266$

8. $328644 - 229961$

9. $54641 - 31537$

10. $74853 - 36874$

11. $31585 - 25669$

12. 58322 − 18798

13.
97162
26578
31537
13842

14.
73368
32551
13626
10224

15.
4374
× 291

16.
8939
× 325

17.
824
× 306

18.
6214
× 923

19.
920372
− 759974

20.
557697
− 239789

21.
847069
− 574876

22.
623150
− 272955

23. 37853 − 23974

24. 68964 − 29978

25. 46826 − 37735

26. 79584 − 58759

27.
156
210
495
214
146
801
214
813
345
568

28.
486
359
868
933
598
178
913
554
955
508

29.
37
46
85
47
39
53
74
37

30.
94
86
49
38
87
89
27
74

31.
91858
32582
14637
21856

32.
83228
29659
46317
1268

33.
2264
× 821

34.
5175
× 127

35.
837
× 806

36.
7106
× 364

Test 4

Time 10 min. **Page 46**

1. 894468 ÷ 69

2. 594461 ÷ 37

3. 8393 ÷ 951

4. 7848 ÷ 397

5.
3264
× 2196

6.
6748
× 507

7. 311835 − 254329

8. 732832 − 217469

9.
65300
19762
8451
5932
8531
21400

10. 3916784
− 1908498

11. 6502184
− 2976937

12. 38 + 67 + 31 + 18 + 83 + 42 + 94 + 65

13. 8143
3122
5146
5714
2921
1744
2253
6218
3936
5094

14. 292+838+493+795+940+792+281+479

15. 7419 ÷ 283

16. 6673 ÷ 618

17. 475022 ÷ 78

18. 167670 ÷ 69

19. 211785 − 131891

21. 2635
× 3174

22. 4553
× 409

23. 79066
11567
8194
9407
23541
2478

20. 513774 − 292861

24. 72+85+37+86+90+46+59+15

25. 6421396
− 5832382

26. 7531250
− 3926845

27. 6665
6797
8395
8772
4858
4685
6458
7945
2707
7665

28. 745+775+469+428+953+104+977+456

Test 5

Time 10 min. **Page 57**

1. $517238 \div 48$

2. $823439 \div 53$

3. $6235 \div 157$

4. $27236 \div 529$

5. 3456×2535

6. 2346×503

7. $316534 - 197554$

8. $624173 - 283584$

9.
$$\begin{array}{r} 82421 \\ -\left\{\begin{array}{r} 4326 \\ 21622 \\ 8613 \\ 10862 \\ 31004 \end{array}\right. \end{array}$$

10. $4518236 - 2565945$

11. $6272463 - 3572319$

12. $28 + 19 + 63 + 34 + 81 + 95 + 37 + 84$

13.
$$\begin{array}{r} 8915 \\ 9123 \\ 4231 \\ 9729 \\ 2134 \\ 1975 \\ 3546 \\ 4867 \\ 4586 \\ 2145 \end{array}$$

14. $417 + 902 + 272 + 947 + 569 + 350 + 244 + 796$

15. $6327 \div 329$

16. $7384 \div 272$

17. $39432 \div 47$

18. $68206 \div 98$

19. $809482 - 738285$

20. $728433 - 667961$

21. 3522×1856

22. 8862×407

23.
$$\begin{array}{r} 94005 \\ -\left\{\begin{array}{r} 15575 \\ 12580 \\ 4259 \\ 1325 \\ 56178 \end{array}\right. \end{array}$$

24. $58 + 21 + 78 + 56 + 27 + 93 + 78 + 94$

25. $2860453 - 2337976$

26. $8227397 - 6594788$

27. $4978 + 7368 + 2865 + 7289 + 9246 + 6419 + 7558 + 2793 + 4375 + 5894$

28. $857 + 701 + 629 + 576 + 183 + 271 + 567 + 348$

Test 6

Time 10 min. **Page 73**

1. $6\frac{1}{4} + 5\frac{1}{3} + 3\frac{1}{6}$

2. $9\frac{5}{6} + 5\frac{3}{4} + 7\frac{1}{2}$

3. $1\frac{2}{3} + 4\frac{5}{6} + 6\frac{1}{2}$

4. $9\frac{5}{8} + 7\frac{3}{4} + 2\frac{1}{2}$

5. $12\frac{1}{6} + 7\frac{1}{3} + 18\frac{1}{8}$

6. $16\frac{1}{2} + 13\frac{1}{5} + 10\frac{1}{6}$

7. $11 - 3\frac{3}{4}$

8. $16\frac{1}{3} - 9\frac{1}{2}$

9. $17\frac{5}{6} - 13\frac{3}{4}$

10. $48\frac{1}{2} - 19\frac{2}{3}$

11. $52\frac{2}{3} - 37\frac{1}{4}$

12. $41\frac{3}{4} - 36\frac{5}{8}$

13. $4958 + 9389 + 2753 + 5074 + 5757 + 7963 + 7836 + 6965 + 2141 + 1844$

14. $84 + 39 + 83 + 78 + 76 + 24 + 37 + 75 + 27 + 73$

15. $17+39+41+36+85+44+62+83$

16. $47+92+74+17+58+40+68+33$

17. $25+56+67+19+94+73+59+87$

18. $73 + 98 + 65 + 29 + 57 + 64$

19. 8283×603

20. $9847 \div 733$

21. $8\frac{1}{5} + 3\frac{5}{6} + 7\frac{2}{3}$

22. $6\frac{4}{5} + 8\frac{1}{2} + 6\frac{7}{8}$

23. $21\frac{3}{5} + 17\frac{4}{7} + 4\frac{1}{2}$

24. $15\frac{1}{2} + 20\frac{2}{3} + 8\frac{3}{4}$

25. $17\frac{1}{2} + 22\frac{1}{5} + 33\frac{1}{6}$

26. $51\frac{3}{8} + 24\frac{1}{4} + 9\frac{1}{3}$

27. $15 - 8\frac{3}{8}$

28. $11 - 4\frac{3}{5}$

29. $27\frac{1}{4} - 18\frac{1}{3}$

30. $23 - 14\frac{5}{6}$

31. $72\frac{1}{2} - 37\frac{3}{4}$

32. $61\frac{1}{2} - 43\frac{3}{5}$

33.
6781
6287
3185
5849
3628
5439
8768
7043
5969
3280

34.
17
63
55
13
29
87
79
94
85
35

35. $77+80+45+89+47+93+48+54$

36. $42+90+83+17+76+24+38+47$

37. $83+46+73+47+56+78+67+49$

38. $26+73+28+56+41+52+54+84$

39. 3532×803

40. $2897 \div 423$

Test 7

Time 10 min. **Page 85**

1. $18\frac{1}{2} \times 21\frac{1}{3}$

2. $62\frac{1}{3} \times 25\frac{1}{2}$

3. $41\frac{1}{4} \div 2\frac{3}{5}$

4. $82\frac{1}{2} \div 7\frac{1}{3}$

5. $5794840 - 4797398$

6. $5269802 - 3974784$

7.
93192
16751
8268
6311
23646
10763

8. $392731 - 236611$

9. $876170 - 234139$

10. 3927×1049

11. $19+39+31+67+51+82+68+52$

12. $236+461+903+589+915+530+926+654$

13. $30\frac{5}{6} \times 21\frac{2}{5}$

14. $71\frac{1}{4} \times 36\frac{1}{7}$

15. $12\frac{4}{5} \div 8\frac{2}{3}$

16. $33\frac{3}{4} \div 8\frac{1}{3}$

17. $709576 - 690537$

18. $475867 - 358874$

19.
79843
5869
16078
3982
21673
4721

20.
4683956
$- 2795371$

21.
7122716
$- 4693618$

22. 2458×4529

23. $872+676+887+537+956+579+344+857$

24. $75+31+84+64+39+82+96+36$

Test 8

Time 10 min. **Page 91**

1. $685.97 + 47.28 + 627.69 + 585.96 + 804.28$

2.
$$\begin{array}{r} 84.65 \\ \times\ 122.6 \\ \hline \end{array}$$

3. $491.64 \div 2.16$

4. $16\frac{2}{3} \times 84$

5. $33\frac{1}{3} \times 284$

6. $16\frac{2}{3} \times 368$

7. $6\frac{1}{4} \times 684$

8.
$$\begin{array}{r} 820.45 \\ 197.46 \\ 80.42 \\ 28.67 \\ 312.19 \\ 94.32 \\ \hline \end{array}$$

9.
$$\begin{array}{r} 857.69 \\ 69.08 \\ 95.02 \\ 451.45 \\ 91.96 \\ 39.05 \\ \hline \end{array}$$

10. $44\frac{1}{3} \div 4\frac{1}{5}$

11.
$$\begin{array}{r} 76\frac{1}{2} \\ \times\ 62\frac{1}{3} \\ \hline \end{array}$$

12.
$$\begin{array}{r} 55 \\ -\ 38\frac{7}{8} \\ \hline \end{array}$$

13.
$$\begin{array}{r} 14\frac{3}{4} \\ 8\frac{3}{8} \\ 17\frac{1}{5} \\ \hline \end{array}$$

14. $13 + 89 + 96 + 66 + 74 + 45 + 20 + 78$

15. $33\frac{1}{3} \times 374$

16. $12\frac{1}{2} \times 497$

17. $83\frac{1}{3} \times 722$

18. $970.31 + 604.56 + 90.46 + 97.67 + 54.36$

19.
$$\begin{array}{r} 93.03 \\ \times\ 395.4 \\ \hline \end{array}$$

20.
$$\begin{array}{r} 290.44 \\ 97.51 \\ 36.96 \\ 17.67 \\ 10.88 \\ 9.27 \\ \hline \end{array}$$

21.
$$\begin{array}{r} 827.49 \\ 94.32 \\ 47.63 \\ 284.27 \\ 90.83 \\ 102.98 \\ \hline \end{array}$$

22. $37\frac{1}{2} \times 98$

23. $75\frac{1}{1} \div 9\frac{2}{3}$

24.
$$\begin{array}{r} 36\frac{1}{5} \\ \times\ 27\frac{3}{8} \\ \hline \end{array}$$

25.
$$\begin{array}{r} 63 \\ -\ 28\frac{5}{8} \\ \hline \end{array}$$

26.
$$\begin{array}{r} 44\frac{1}{2} \\ 75\frac{1}{5} \\ 39\frac{1}{6} \\ \hline \end{array}$$

27. $643.57 \div 3.65$

28. $21 + 65 + 18 + 45 + 36 + 18 + 76 + 93$

Test 9

Time 10 min. **Page 118**

1. $118916 \div 47$

2. $13006 \div 613$

3. 2863×3509

4. 4181×306

5. $672052 - 371876$

6. $461528 - 309675$

7.
$$\begin{array}{r} 70856 \\ -\ \left\{\begin{array}{r} 9431 \\ 27865 \\ 6988 \\ 2649 \\ 3178 \\ 13782 \end{array}\right. \end{array}$$

8. 48 yds. at $62\frac{1}{2}¢$

9. 350 yds. at $18\frac{3}{4}¢$

10. 380 lbs. at $66\frac{2}{3}¢$

11. 1600 lbs. at $68\frac{3}{4}¢$

12. 71.42×6.283

13. $194.14 \div 58.3$

14. $84 + 88 + 39 + 37 + 87 + 78 + 57 + 45$

15. $744 + 104 + 256 + 732 + 930 + 351 + 484 + 915$

16. $41\frac{3}{5} \times 350$

17. $56\frac{1}{4} \times 1694$

18. $542751 - 354893$

19. $272847 - 195778$

20. 8165×504

21. 6855×4127

22. $671758 \div 92$

23. $3078 \div 215$

24.
$$\begin{array}{r} 61820 \\ -\ \left\{\begin{array}{r} 18853 \\ 2158 \\ 2083 \\ 8918 \\ 5897 \\ 11528 \end{array}\right. \end{array}$$

25. 42.36×21.16

26. $89.796 \div 8.38$

27. 350 yds. at $50¢$

28. 870 lbs. at $41\frac{2}{3}¢$

29. 1540 yds. at $26\frac{2}{3}¢$

30. 9860 lbs. at $58\frac{1}{3}¢$

31. $18\frac{3}{4} \times 194$

32. $26\frac{2}{3} \times 390$

33. $42 + 36 + 18 + 85 + 78 + 58 + 97 + 96$

34. $273 + 489 + 762 + 615 + 976 + 680 + 473 + 265$

Test 10

Time 10 min. **Page 118**

1.
7654
6797
4788
2898
8641
2517
7692
9029
2536
1986

2.
68
75
36
98
36
60
57
38
64
70

3. $525.27+776.80+496.67+373.85+527.89$

4.
$$964.73 - \begin{cases} 85.97 \\ 298.18 \\ 26.40 \\ 49.37 \\ 336.07 \end{cases}$$

5. $15 - 8\frac{5}{6}$

6. $27\frac{2}{3} \times 14\frac{3}{4}$

7. $7\frac{1}{4} \div 3\frac{3}{5}$

8. $43\frac{3}{4} \times 794$

9. $58\frac{1}{3} \times 795$

10. 96 lbs. at $66\frac{2}{3}¢$

11. 64 yds. at $93\frac{3}{4}¢$

12. $476.89 \div 2.35$

13.
413.9
$\times$ 21.67

14.
$17\frac{3}{4}$
$15\frac{2}{5}$
$6\frac{1}{8}$

15.
6289507
$-$ 4761918

16.
2538618
$-$ 1826784

17.
4658
$\times$ 604

18. $853.79 + 881.96 + 652.77 + 358.89 + 476.18$

19. $9\frac{2}{3} \div 6\frac{7}{8}$

20. $31 - 17\frac{5}{9}$

21. $33\frac{1}{5} \times 25\frac{2}{3}$

22.
$$735.06 - \begin{cases} 237.69 \\ 35.89 \\ 26.53 \\ 158.73 \\ 39.68 \end{cases}$$

23.
4761
8253
8938
6470
5257
6915
8739
3968
2979
4976

24.
69
97
41
98
47
65
26
87
45
60

25.
825.38
$\times$ 7.39

26. 86 yds. at $13\frac{1}{3}¢$

27. 94 lbs. at $87\frac{1}{2}¢$

28. $91\frac{2}{3} \times 491$

29. $31\frac{1}{4} \times 126$

30. $831.18 \div 43.6$

31. 3878577 − 1863915

32. 5268704 − 3565457

33. $24\frac{1}{4}$, $27\frac{5}{6}$, $9\frac{1}{3}$

34. 8647 × 506

Test 11

Time 10 min. **Page 118**

1. 592746 ÷ 59

2. 7068 ÷ 616

3. 2589 × 7631

4. 552791 − 478179

5. 481572 − 365706

6. 90 + 45 + 89 + 71 + 30 + 28 + 96 + 16

7. 365 + 872 + 239 + 653 + 104 + 480 + 157 + 258

8.
$$\begin{array}{r} 54221 \\ -\ \left\{\begin{array}{r} 12907 \\ 6601 \\ 4552 \\ 1126 \\ 4591 \\ 20379 \end{array}\right. \end{array}$$

9. $13\frac{1}{3} \times 350$

10. $81\frac{1}{4} \times 483$

11. 75 lbs. at 80¢

12. 980 yds. at $66\frac{2}{3}$¢

13. $57 - 39\frac{2}{7}$

14. $8\frac{5}{7} \div 6\frac{4}{9}$

15. 542.21 + 29.07 + 594.46 + 85.23 + 21.78

16. 5458 × 907

17. 957836 − 584597

18. 891796 − 274735

19. 6527 ÷ 735

20. 8898 × 4172

21. 372829 ÷ 65

22.
$$\begin{array}{r} 69278 \\ -\ \left\{\begin{array}{r} 11273 \\ 8647 \\ 3567 \\ 20156 \\ 3693 \\ 2873 \end{array}\right. \end{array}$$

23. 368+445+762+869+472+339+471+968

24. 45 + 36 + 27 + 31 + 98 + 87 + 65 + 79

25. 350 yds. at $58\frac{1}{3}$¢

26. 86 lbs. at $37\frac{1}{2}$¢

27. $93\frac{3}{4} \times 460$

28. $46\frac{2}{3} \times 680$

29. 4536 × 806

30. $53 - 28\frac{5}{8}$

31. $7\frac{4}{5} \div 5\frac{1}{7}$

32. 598.49 + 47.19 + 657.35 + 98.17 + 706.52

Test 12

Time 10 min. **Page 118**

1.
4299
1563
6090
1378
8574
9055
2420
4852
1159
8063

2.
41
72
51
80
48
20
37
73
99
68

3.
95
15
91
80
61
78
95
75
87
61

4. $13\frac{3}{4}$
$28\frac{1}{3}$
$4\frac{5}{6}$

5. $35\frac{5}{8}$
$\times 57\frac{2}{3}$

6. $987.46 \div 6.09$

7.
89.17
$\times 1.642$

8.
987.92
$-$ (84.62, 74.85, 340.58, 217.07, 62.46)

9. $91\frac{2}{3} \times 560$

10. $68\frac{3}{4} \times 680$

11. 3800 lbs. at $46\frac{2}{3}¢$

12. 3280 yds. at $37\frac{1}{2}¢$

13. $5\frac{5}{8} \div 1\frac{2}{3}$

14.
9600162
$-$ 8543542

15. 960276 $-$ 747382

16.
$16\frac{1}{5}$
$-$ $9\frac{4}{7}$

17. $598.47 \div 4.96$

18.
$16\frac{2}{3}$
$32\frac{4}{9}$
$3\frac{5}{6}$

19.
$42\frac{3}{8}$
$\times 51\frac{4}{5}$

20.
52
35
76
20
40
19
37
86
23
36

21.
60
54
99
97
35
69
78
64
27
30

22.
3056
9432
5884
7940
2391
6804
5623
2134
2535
6784

23. $18\frac{3}{4} \times 128$

24. $66\frac{2}{3} \times 986$

25.
852.65
$-$ (186.34, 34.06, 197.07, 33.48, 273.26)

26.
970.6
$\times 85.15$

27. 3800 lbs. at $26\frac{2}{3}¢$

28. 1240 yds. at $83\frac{1}{3}¢$

29.
$26\frac{1}{2}$
$-$ $12\frac{1}{3}$

30.
4056257
$-$ 3546778

31. $8\frac{1}{4} \div 2\frac{1}{6}$

32. 358053 $-$ 197895

Test 13

Time 10 min. **Page 250**

	Principal	Rate	Time	Interest
1.	$6696	$6\frac{3}{4}\%$	29 da.	
2.	$19355	6%	61 da.	

	List Price	Discounts	Selling Price
3.	$155.00	15%, 10%, 5%	

	Base	Rate	Percentage
4.	6653	$4\frac{3}{4}\%$	

5. 1935
5643
4208
5769
7097
9477
8725
4136
9379
9166

6. 8468
$\times$ 6573

7. $82\frac{1}{5}$
$\times$ $75\frac{1}{3}$

8. $14\frac{2}{3}$
$19\frac{1}{4}$
$97\frac{5}{8}$

	Principal	Rate	Time	Interest
9.	$66120	6%	48 da.	
10.	$7640	$7\frac{1}{2}\%$	87 da.	

	List Price	Discounts	Selling Price
11.	$1250	13%, 15%, 5%	

	Base	Rate	Percentage
12.	8276	$5\frac{1}{4}\%$	

13. 2133
2023
5014
1579
2756
8151
3298
9332
9369
6962

14. 3293
$\times$ 5769

15. $32\frac{2}{3}$
$\times$ $96\frac{5}{8}$

16. $51\frac{1}{5}$
$32\frac{5}{8}$
$67\frac{1}{4}$

Test 14

Time 9½ min. **Page 250**

	Base	Rate	Percentage
1.		8%	$3800

2. $9332.30 is ____ % of $792.10

	Principal	Rate	Time	Interest
3.	\$689.30	6%	58 da.	

	List Price	Discounts	Selling Price
4.	\$800.00	25%, 10%, 2%	

	Principal	Rate	Time	Interest
5.	\$861.50	$7\frac{1}{2}$%	42 da.	

6. $436.24 - (10.14 + 21.18 + 91.80 + 132.10 + 51.56)$

7. $41\frac{2}{3} - 29\frac{7}{8}$

8. $81\frac{1}{3} - 12\frac{5}{6}$

9. $324\frac{4}{7} \times 56\frac{3}{5}$

	Base	Rate	Percentage
10.		$6\frac{2}{3}$%	\$7358

	Principal	Rate	Time	Interest
11.	\$7452	6%	85 da.	

12. \$63675 is ____ % of \$424.36

	Principal	Rate	Time	Interest
13.	\$29320	$8\frac{1}{2}$%	27 da.	

	List Price	Discounts	Selling Price
14.	\$142.50	15%, 10%, 10%	

15. $642.76 - (42.64 + 88.48 + 165.49 + 63.96 + 75.68)$

16. $47\frac{6}{7} - 39\frac{8}{9}$

17. $76\frac{3}{8} - 48\frac{5}{7}$

18. $36\frac{5}{9} \times 81\frac{5}{8}$

Test 15

Time 9 min. **Page 250**

1. $501 + 412 + 325 + 203 + 476 + 734 + 643 + 583$

2. $65 + 27 + 23 + 69 + 48 + 34 + 94 + 82$

3. $7535 \div 476$

4. 3032×5414

5. $7\frac{5}{6} \div 9\frac{1}{3}$

6. 35.18×4.853

7. $20\frac{1}{6} - 13\frac{1}{2}$

	Principal	Rate	Time	Interest
8.	$275100	$6\frac{3}{4}\%$	73 da.	

	Principal	Rate	Time	Interest
9.	$1948.25	$5\frac{1}{4}\%$	58 da.	

10. $5672 \div 618$

11. 4927×3278

12. $7\frac{3}{5} \div 8\frac{2}{3}$

13. 73.13×16.46

14. $75\frac{1}{4} - 52\frac{5}{8}$

15. $44 + 40 + 66 + 75 + 41 + 46 + 52 + 85$

16. $960 + 197 + 494 + 277 + 672 + 805 + 695 + 342$

	Principal	Rate	Time	Interest
17.	$7753.50	$9\frac{1}{2}\%$	89 da.	

	Principal	Rate	Time	Interest
18.	$2690.20	$5\frac{1}{2}\%$	60 da.	

Test 16

Time 9 min. **Page 250**

	List Price	Discounts	Selling Price
1.	$94.27	20%, 15%, 10%	

	Principal	Rate	Time	Interest
2.	$4440.50	6%	98 da.	

	Principal	Rate	Time	Interest
3.	$37580	$5\frac{3}{4}\%$	28 da.	

	Base	Rate	Percentage
4.	2658	$7\frac{1}{4}\%$	

5. $12\frac{1}{2} \times 5980$

6. $8\frac{1}{3} \times 85$

7. $49.01 + 37.29 + 26.58 + 8.45 + 10.46$

8. $6\frac{1}{9}$, $7\frac{5}{6}$, $9\frac{2}{3}$

9. $8139758 - 5819928$

10. $8062342 - 5499647$

11. 8139×908

12. $243 + 467 + 553 + 378 + 462 + 935 + 336 + 571$

	Principal	Rate	Time	Interest
13.	$84740	6%	76 da.	

	List Price	Discounts	Selling Price
14.	$3650	25%, 8%, 5%	

	Principal	Rate	Time	Interest
15.	$25000	$7\frac{3}{4}\%$	47 da.	

	Base	Rate	Percentage
16.	5627	$6\frac{1}{2}\%$	

17. $33\frac{1}{3} \times 870$

18. $16\frac{2}{3} \times 218$

19. $84.65 + 42.75 + 64.08 + 38.38 + 97.54$

20. $8\frac{1}{3}$, $16\frac{1}{4}$, $39\frac{8}{9}$

21. $9507419 - 8673286$

22. $5711488 - 1379743$

23. 3632×603

24. $356 + 718 + 849 + 594 + 736 + 264 + 172 + 507$

Test 17

Time 9 min. **Page 251**

1. Find the single discount equivalent to the discount series 25%, 10%, 5%.

2. Find the single discount equivalent to the discount series 30%, 10%, 10%.

	List Price	Discounts	Selling Price
3.	$1420	25%, 8%, 5%	
4.	$800	25%, 10%, 2%	

5. 630 yds. at 40¢

6. 780 lbs. at $31\frac{1}{4}$¢

7. $42\frac{3}{5} \times 24\frac{4}{5}$

8. 100.79 + 472.58 + 97.65 + 749.44 + 146.08

9. 876 + 683 + 207 + 785 + 489 + 694 + 262 + 726

10. Find the single discount equivalent to the discount series 20%, 10%, 10%.

11. Find the single discount equivalent to the discount series 40%, 10%, 5%.

	List Price	Discounts	Selling Price
12.	$940	20%, 15%, 10%	
13.	$125	13%, 15%, 5%	

14. $82\frac{2}{3} \times 34\frac{1}{4}$

15. 860 yds. at $46\frac{2}{3}$¢

16. 1860 lbs. at $31\frac{1}{4}$¢

17. 736 + 244 + 459 + 664 + 563 + 765 + 123 + 343

18. 62.57 + 76.14 + 375.49 + 786.41 + 678.71

Test 18

Time 10 min. **Page 251**

	Cost	Rate of Increase	Selling Price
1.	$270	28%	

2. Find the single discount equivalent to the discount series 40%, 20%, 5%.

	List Price	Discounts	Selling Price
3.	$185	25%, 5%, 2%	

4. $87\frac{1}{2} \times 870$

5. $83\frac{1}{3} \times 280$

	Base	Rate	Percentage
6.	155	$6\frac{1}{2}\%$	
7.	\$5740	%	\$52.30
8.		8%	\$51.25

	Cost	Rate of Increase	Selling Price
11.	\$540	14%	

9. $17\frac{3}{4}$, $12\frac{7}{8}$, $4\frac{5}{6}$

10. $81\frac{11}{12} \times 47\frac{2}{3}$

12. Find the single discount equivalent to the discount series 20%, 10%, 5%.

	List Price	Discounts	Selling Price
13.	\$775	40%, 10%, 5%	

14. $37\frac{1}{2} \times 346$ **15.** $43\frac{3}{4} \times 330$

16. $26\frac{1}{3}$, $47\frac{3}{4}$, $39\frac{5}{8}$

17. $31\frac{2}{5} \times 26\frac{5}{8}$

	Base	Rate	Percentage
18.	\$985.50	$5\frac{1}{2}\%$	
19.	\$5970	%	\$1990
20.		$6\frac{1}{4}\%$	\$136.20

Test 19

Time 10 min. **Page 251**

	Cost	Margin on Selling Price	Selling Price
1.	\$38	15%	

	Cost	Rate of Increase	Selling Price
2.	\$520	21%	

3. Find the single discount equivalent to the discount series 20%, 20%, 10%.

	List Price	Discounts	Selling Price
4.	\$4860	10%, 10%, 10%	

5. 650 yds. at $18\frac{3}{4}¢$

6. 2670 lbs. at $83\frac{1}{3}¢$

7. $570.36 \div 8.72$

8. 35.72×719.3

9.
$$\begin{array}{r} 912.37 \\ 197.35 \\ 89.15 \\ -\ 149.79 \\ 26.41 \\ \underline{263.93} \end{array}$$

10. $34 + 12 + 89 + 56 + 91 + 57 + 79 + 14$

11. Find the single discount equivalent to the discount series 20%, 30%, 40%.

	Cost	Rate of Increase	Selling Price
12.	\$600	$33\frac{1}{3}\%$	

	Cost	Margin on Selling Price	Selling Price
13.	\$144	22%	

14. 2600 lbs. at $18\frac{3}{4}¢$

15. 1245 yds. at $62\frac{1}{2}¢$

	List Price	Discounts	Selling Price
16.	\$425	8%, 8%, 5%	

17. $69 + 80 + 32 + 79 + 54 + 28 + 70 + 37$

18. 743.9×12.53

19. $894.14 \div 86.1$

20.
$$\begin{array}{r} 968.49 \\ -\left\{\begin{array}{r} 78.79 \\ 96.48 \\ 160.78 \\ 180.16 \\ 389.57 \end{array}\right. \end{array}$$

Test 20

Time 10 min. **Page 267**

	Total Sales	Expenses	Expense Per Cent
1.	\$208,150	\$53,400	

	Cost	Margin on Selling Price	Selling Price
2.	\$976	24%	

	Cost	Rate of Increase	Selling Price
3.	\$185	$37\frac{1}{2}\%$	

4. Find the single discount equivalent to the discount series 30%, 10%, 10%.

5. $395.82 + 863.58 + 461.52 + 84.97 + 45.73$

6. $43\frac{3}{4} \times 532$

7. $9225 \div 189$

8. 5629×4765

9.
$$\begin{array}{r} 92473 \\ -\left\{\begin{array}{r} 30025 \\ 2218 \\ 3947 \\ 4689 \\ 11302 \\ 25892 \end{array}\right. \end{array}$$

	Cost	Margin on Selling Price	Selling Price
10.	\$1760	32%	

	Total Sales	Expenses	Expense Per Cent
11.	\$400,960	\$95,300	

12. Find the single discount equivalent to the discount series 50%, 20%, 10%.

	Cost	Rate of Increase	Selling Price
13.	\$428	34%	

14. $897.36 + 394.35 + 546.92 + 668.98 + 798.13$

15. $81\frac{1}{4} \times 340$

16.
$$\begin{array}{r} 7469 \\ \times\ 1378 \\ \hline \end{array}$$

17. $66951 \div 742$

18.
$$\begin{array}{r} 82736 \\ -\left\{\begin{array}{r} 15849 \\ 9077 \\ 2865 \\ 4754 \\ 20076 \\ 3595 \end{array}\right. \\ \hline \end{array}$$

Test 21

Time 10 min. **Page 275**

	Sales	Rate Commission	Proceeds
1.	$760	6%	

	Cost	Margin on Selling Price	Selling Price
2.	$405	22%	

3. Find the single discount equivalent to the discount series 20%, 15%, 10%.

	List Price	Discounts	Selling Price
4.	$2450	10%, 8%, 6%	

	Base	Rate	Percentage
5.		$5\frac{1}{2}$%	$1760

	Base	Rate	Percentage
6.	$3650	%	$425.30

	Base	Rate	Percentage
7.	$456	$7\frac{1}{2}$%	

8. $506420 \div 43$

	List Price	Discounts	Selling Price
9.	$1500	20%, 12%, 6%	

10. Find the single discount equivalent to the discount series 40%, 25%, 10%.

	Cost	Margin on Selling Price	Selling Price
11.	$1050	$33\frac{1}{3}$%	

	Sales	Rate Commission	Proceeds
12.	$320	$5\frac{1}{2}$%	

	Base	Rate	Percentage
13.	$2512	$6\frac{1}{2}$%	

	Base	Rate	Percentage
14.		$7\frac{1}{4}$%	$172.80

15. $526144 \div 73$

	Base	Rate	Percentage
16.	$1820	%	$96.40

Test 22

Time 10 min. **Page 283**

	Principal	Rate	Time	Interest
1.	\$1845	6%	84 da.	

	Total Sales	Expenses	Expense Per Cent
2.	\$115,000	\$49,620	

	Cost	Rate of Increase	Selling Price
3.	\$4240	28%	

4. $95\frac{5}{6} \times 79\frac{1}{5}$

5. $8\frac{3}{4} \div 9\frac{3}{5}$

6. $25\frac{1}{3} - 18\frac{5}{7}$

7. $16\frac{4}{5} + 33\frac{1}{6} + 8\frac{2}{3}$

8. 8134131 − 4738974

9. 4972693 − 3959708

10. 46 + 85 + 17 + 64 + 37 + 29 + 38 + 19

11. 572 + 497 + 978 + 689 + 458 + 873 + 968 + 782

	Total Sales	Expenses	Expense Per Cent
12.	\$750000	\$122000	

	Principal	Rate	Time	Interest
13.	\$2580	6%	63 da.	

14. $91\frac{1}{5} \times 46\frac{2}{3}$

	Cost	Rate of Increase	Selling Price
15.	\$6145	32%	

16. 3882065 − 1987389

17. 5932754 − 4495178

18. $41\frac{4}{5} + 39\frac{2}{3} + 46\frac{1}{6}$

19. $9\frac{3}{4} \div 7\frac{4}{5}$

20. $31\frac{5}{8} - 14\frac{5}{6}$

21. 875 + 946 + 127 + 981 + 927 + 610 + 411 + 218

22. 16 + 21 + 56 + 78 + 41 + 24 + 57 + 74

Test 23

Time 10 min. **Page 283**

	Principal	Rate	Time	Interest
1.	\$2560	6%	82 da.	

	Total Sales	Expenses	Expense Per Cent
2.	\$451,500	\$95,555	

	Cost	Rate of Increase	Selling Price
3.	\$7624	16%	

4. $36\frac{2}{3} \times 48\frac{5}{6}$

5. $9\frac{1}{2} \div 2\frac{5}{8}$

6. $93\frac{1}{2} - 16\frac{4}{5}$

7. $25\frac{2}{3}$, $18\frac{5}{7}$, $9\frac{1}{6}$

8. 8912811 − 4351667

9. 4812455 − 1793117

10. 48 + 76 + 74 + 83 + 27 + 48 + 15 + 71

11. 196 + 743 + 469 + 384 + 463 + 174 + 151 + 918

	Total Sales	Expenses	Expense Per Cent
12.	$762,400	$145,750	

	Principal	Rate	Time	Interest
13.	$7500	6%	36 da.	

14. $92\frac{3}{8} \times 17\frac{4}{9}$

	Cost	Rate of Increase	Selling Price
15.	$4515	34%	

16. 7365376 − 5945972

17. 8069742 − 6296889

18. $45\frac{1}{2}$, $24\frac{2}{3}$, $9\frac{1}{12}$

19. $6\frac{4}{5} \div 2\frac{5}{8}$

20. $34\frac{5}{7} - 18\frac{5}{6}$

21. 230 + 722 + 657 + 804 + 969 + 867 + 415 + 684

22. 12 + 93 + 56 + 75 + 70 + 68 + 57 + 94

Test 24

Time 10 min. **Page 283**

	Principal	Rate	Time	Interest
1.	$6145	$5\frac{1}{2}$%	75 da.	

	Sales	Rate Commission	Proceeds
2.	$650	6%	

	Cost	Margin on Selling Price	Selling Price
3.	$825	43%	

4. Find single discount equivalent to the discount series 10%, 20%, 30%.

	Base	Rate	Percentage
5.	$7645	%	$124.60

	Base	Rate	Percentage
6.	$492	$6\frac{1}{2}$%	

	Cost	Margin on Selling Price	Selling Price
7.	$1944	$37\frac{1}{2}$%	

	Sales	Rate Commission	Proceeds
8.	$540	$5\frac{1}{2}$%	

9. Find single discount equivalent to the discount series 40%, 30%, 5%.

	Principal	Rate	Time	Interest
10.	$7580	$7\frac{1}{2}$%	10 da.	

	Cost	Margin on Selling Price	Selling Price
11.	$97.50	51%	

	Base	Rate	Percentage
12.	$5140	%	$87.60

	Base	Rate	Percentage
13.	$5750	$6\frac{2}{3}$%	

	Cost	Margin on Selling Price	Selling Price
14.	$984.20	24%	

Test 25

Time 10 min. **Page 283**

	Base	Rate	Percentage
1.	$894.20	%	$62.87

	Base	Rate	Percentage
2.		$8\frac{1}{2}$%	$164.80

	List Price	Discounts	Selling Price
3.	$836	15%, 10%, 10%	

4. Find single discount equivalent to the discount series 20%, 10%, 4%.

	Principal	Rate	Time	Interest
5.	$5406	$7\frac{1}{2}$%	17 da.	

	Total Sales	Expenses	Expense Per Cent
6.	$219,000	$50,450	

7. 1450 lbs. at $56\frac{1}{4}$¢

	Principal	Rate	Time	Interest
8.	$9540	6%	78 da.	

	Base	Rate	Percentage
9.		$6\frac{2}{3}$%	$325.75

10. Find the single discount equivalent to the discount series 40%, 20%, 10%.

	Base	Rate	Percentate
11.	$876.20		$47.22

	List Price	Discounts	Selling Price
12.	$378.50	20%, 10%, 4%	

	Principal	Rate	Time	Interest
13.	$451.50	6%	87 da.	

	Total Sales	Expenses	Expense Per Cent
14.	$916,500	$145,300	

	Principal	Rate	Time	Interest
15.	$6828	$7\frac{3}{4}$%	48 da.	

16. 3400 lbs. at $87\frac{1}{2}$¢

Test 26

Time 9½ min. **Page 303**

	Sales	Rate Commission	Proceeds
1.	$394	5%	

	Cost	Margin on Selling Price	Selling Price
2.	$790	24%	

	Principal	Rate	Time	Interest
3.	$6900	$7\frac{1}{2}$%	52 da.	

	Base	Rate	Percentage
4.	$925.40	$5\frac{3}{4}$%	

5. $941.55 \div 142.$

6.
$$\begin{array}{r} 30.99 \\ \times\ 75.42 \\ \hline \end{array}$$

7. $71.39 + 68.68 + 947.31 + 342.28 + 546.93$

8.
$$\begin{array}{r} 713.97 \\ -\ \left\{\begin{array}{r} 17.32 \\ 324.91 \\ 40.95 \\ 118.04 \\ 56.23 \end{array}\right. \end{array}$$

	Sales	Rate Commission	Proceeds
9.	$793	6%	

	Cost	Margin on Selling Price	Selling Price
10.	$842	$33\frac{1}{3}\%$	

	Base	Rate	Percentage
11.	$549	$6\frac{2}{3}\%$	

	Principal	Rate	Time	Interest
12.	$14760	$8\frac{3}{4}\%$	81 da.	

13. $651.74 \div 84.9$

14.
$$\begin{array}{r} 873.9 \\ \times\ 5.972 \\ \hline \end{array}$$

15. $38.67 + 693.71 + 49.37 + 743.83 + 597.42$

16.
$$\begin{array}{r} 733.42 \\ -\ \left\{\begin{array}{r} 86.37 \\ 139.62 \\ 129.75 \\ 86.37 \\ 201.54 \end{array}\right. \end{array}$$

Test 27

Time 9½ min. **Page 303**

1.
$$\begin{array}{r} 5719546 \\ -\ 3797569 \\ \hline \end{array}$$

2.
$$\begin{array}{r} 4569282 \\ -\ 2793696 \\ \hline \end{array}$$

3.
$$\begin{array}{r} 7213780 \\ -\ 6856092 \\ \hline \end{array}$$

4.
$$\begin{array}{r} 2784300 \\ -\ 1823618 \\ \hline \end{array}$$

5.
$$\begin{array}{r} 7365 \\ 3767 \\ 8069 \\ 2846 \\ 6546 \\ 9593 \\ 2307 \\ 1291 \\ 4373 \\ 4507 \\ \hline \end{array}$$

6.
$$\begin{array}{r} 68746 \\ -\ \left\{\begin{array}{r} 11442 \\ 6095 \\ 3428 \\ 2838 \\ 10967 \\ 4192 \end{array}\right. \end{array}$$

7. $38 - 23\frac{4}{9}$

8. $34\frac{5}{6} \times 30\frac{1}{5}$

9. $4\frac{2}{3} \div 7\frac{3}{5}$

10. $517.29 \div 6.08$

11. 482.3×234.5

12. $736 + 968 + 230 + 580 + 794 + 127 + 628 + 716$

13. $127 + 654 + 695 + 437 + 665 + 974 + 305 + 108$

14. $41.02 \div 2.93$

15. 2.658×386.5

16. $26\frac{1}{4} - 16\frac{5}{9}$

17. $26\frac{5}{6} \times 38\frac{3}{4}$

18. $8\frac{2}{3} \div 7\frac{3}{5}$

19. $56302 - (20225 + 1857 + 2173 + 6751 + 10045 + 5432)$

20. 1250, 6837, 9076, 9682, 2139, 4102, 3643, 7268, 3056, 5447

21. $8174731 - 7854936$

22. $6485251 - 2794856$

23. $7647882 - 5679844$

24. $8541732 - 4375768$

25. $935 + 617 + 811 + 211 + 747 + 261 + 247 + 478$

26. $184 + 371 + 401 + 471 + 346 + 298 + 764 + 841$

Test 28

Time 9 min. **Page 303**

	Principal	Rate	Time	Interest
1.	\$4962	$7\frac{1}{2}\%$	42 da.	

	Principal	Rate	Time	Interest
2.	\$8756	6%	17 da.	

	Total Sales	Expenses	Expense Per Cent
3.	\$171000	\$40000	%

4. Find the single discount equivalent to the discount series 50%, 20%, 5%.

	Cost	Rate of Increase	Selling Price
5.	$702	24%	

	List Price	Discounts	Selling Price
6.	$1550	15%, 10%, 5%	

7. 940 lbs. at $58\frac{1}{3}¢$

8. 640 yds. at $62\frac{1}{2}¢$

9. 330 lbs. at $68\frac{3}{4}¢$

10. Find the single discount equivalent to the discount series 13%, 15%, 5%.

	Principal	Rate	Time	Interest
11.	$9287	6%	90 da.	

	Principal	Rate	Time	Interest
12.	$1954	$7\frac{3}{4}\%$	85 da.	

	Total Sales	Expenses	Expense Per Cent
13.	$109220	$28500	

	List Price	Discounts	Selling Price
14.	$76.24	40%, 10%, 5%	

	Cost	Rate of Increase	Selling Price
15.	$408	46%	

16. 460 yds. at 40¢

17. 2400 lbs. at $46\frac{2}{3}¢$

18. 1694 yds. at $87\frac{1}{2}¢$

Test 29

Time 9 min. **Page 303**

	Sales	Rate Commission	Proceeds
1.	$3761	7%	

	Cost	Margin on Selling Price	Selling Price
2.	$61.45	36%	

	Base	Rate	Percentage
3.		$8\frac{3}{4}\%$	$49.62

	Base	Rate	Percentage
4.	$894	%	$51.40

	Base	Rate	Percentage
5.	$876.20	$6\frac{2}{3}\%$	

6. $31\frac{1}{4} \times 821$

7. $26\frac{2}{3} \times 430$

8. $714.24 + 924.41 + 926.31 + 872.33 + 737.67$

9. $2337.8 \div 17.8$

10.
$$\begin{array}{r} 2756 \\ \times\ 3835 \\ \hline \end{array}$$

11.
$$\begin{array}{r} 4048 \\ \times\ 305 \\ \hline \end{array}$$

	Cost	Margin on Selling Price	Selling Price
12.	$816.50	43%	

	Sales	Rate Commission	Proceeds
13.	$856	$5\frac{3}{4}\%$	

	Base	Rate	Percentage
14.	$8170	%	$628.30

	Base	Rate	Percentage
15.		$7\frac{1}{4}\%$	$19.21

16. $18\frac{3}{4} \times 922$

17. $83\frac{1}{3} \times 256$

	Base	Rate	Percentage
18.	$111.15	$6\frac{1}{4}\%$	

19. $246.97 + 193.62 + 811.69 + 757.52 + 369.24$

20. $491.37 \div 8.04$

21.
$$\begin{array}{r} 3901 \\ \times\ 4182 \\ \hline \end{array}$$

22.
$$\begin{array}{r} 4775 \\ \times\ 408 \\ \hline \end{array}$$

Test 30

Time 9 min. **Page 303**

	Principal	Rate	Time	Interest
1.	$1726	$7\frac{1}{2}\%$	102 da.	

	Principal	Rate	Time	Interest
2.	$2620	6%	45 da.	

	Total Sales	Expenses	Expense Per Cent
3.	$87200	$18400	

4. Find the single discount equivalent to the discount series 8%, 8%, 4%.

	Cost	Rate of Increase	Selling Price
5.	$927	53%	

	List Price	Discounts	Selling Price
6.	$3780	25%, 8%, 5%	

7. 980 yds. at $66\frac{2}{3}¢$

8. 941270 − 814956

9. 873541 − 589735

	Principal	Rate	Time	Interest
10.	$9456	6%	84 da.	

	Principal	Rate	Time	Interest
11.	$92.50	$7\frac{3}{4}\%$	23 da.	

	Cost	Rate of Increase	Selling Price
12.	$87.20	65%	

13. Find the single discount equivalent to the discount series 40%, 10%, 8%.

	List Price	Discounts	Selling Price
14.	$14.50	25%, 5%, 2%	

	Total Sales	Expenses	Expense Per Cent
15.	$111150	$35600	

16. 700127 − 496268

17. 437701 − 175685

18. 5300 sq. ft. at $58\frac{1}{3}¢$

Test 31

Time $8\frac{1}{2}$ min. **Page 303**

	Sales	Rate Commission	Proceeds
1.	$96.40	$5\frac{3}{4}\%$	

	Cost	Margin on Selling Price	Selling Price
2.	$7.75	44%	

	Base	Rate	Percentage
3.	$628	%	$57.32

	Base	Rate	Percentage
4.		$6\frac{1}{4}\%$	$27.18

5. 643.75 ÷ 37.6

6.
$$\begin{array}{r} 9.561 \\ \times\ 22.47 \\ \hline \end{array}$$

7.
$$\begin{array}{r} 817.36 \\ 107.85 \\ 78.52 \\ -\ 241.35 \\ 109.74 \\ 257.62 \end{array}$$

8. 239.45 + 231.29 + 35.74 + 835.23 + 589.24

9. 81 + 57 + 78 + 37 + 80 + 28 + 46 + 85

10. 25 + 47 + 22 + 54 + 91 + 63 + 41 + 83

	Base	Rate	Percentage
11.		$5\frac{3}{4}\%$	\$85.25

	Base	Rate	Percentage
12.	\$378.25	%	\$28.20

	Cost	Margin on Selling Price	Selling Price
13.	\$22.50	75%	

	Sales	Rate Commission	Proceeds
14.	\$194	$6\frac{2}{3}\%$	

15. 26.69×2.137

16. $4632.3 \div 20.9$

17. $724.80 - (303.76 + 29.74 + 49.96 + 121.45 + 58.28)$

18. $686.08 + 283.28 + 630.05 + 819.14 + 72.83$

19. $45 + 77 + 69 + 28 + 36 + 54 + 37 + 13$

20. $29 + 44 + 37 + 74 + 82 + 97 + 45 + 29.$

Test 32

Time 8½ min. **Page 303**

	Principal	Rate	Time	Interest
1.	\$629	$8\frac{3}{4}\%$	28 da.	

	Principal	Rate	Time	Interest
2.	\$826	6%	31 da.	

	Total Sales	Expenses	Expense Per Cent
3.	\$128200	\$36750	

4. Find the single discount equivalent to the discount series 25%, 5%, 2%.

	List Price	Discounts	Selling Price
5.	\$54	25%, $33\frac{1}{3}\%$, 10%	

	Base	Rate	Percentage
6.	\$390	$8\frac{1}{4}\%$	

7. $517 + 296 + 803 + 818 + 768 + 694 + 857 + 606$

8. $74\frac{1}{4} - 29\frac{5}{9}$

	Total Sales	Sales Expenses	Expense Per Cent
9.	\$364780	\$63756	

	Principal	Rate	Time	Interest
10.	\$3702	6%	53 da.	

	Principal	Rate	Time	Interest
11.	\$2932	$7\frac{1}{2}\%$	92 da.	

12. Find the single discount equivalent to the discount series $33\frac{1}{3}\%$, 25%, 5%.

	Base	Rate	Percentage
13.	\$95.20	$6\frac{2}{3}\%$	

	List Price	Discounts	Selling Price
14.	\$250.60	20%, $33\frac{1}{3}\%$, 10%	

15. $62\frac{3}{5} - 37\frac{5}{6}$

16. $562 + 643 + 730 + 826 + 294 + 493 + 618 + 465$

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